In-Switch Traffic Distribution Approximation

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Overview

Motivation

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- Keep operators of algorithm simple to allow for implementation in P4

Background Information

- Streaming Algorithms
- Sketches
- Stochastics

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- Stream usually only examined once

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- Count-Min Sketch[3]

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- Every timestep X chooses some packet σ across all possible ones it could choose Ω from some distribution
- X forms an input stream of observed packets that S reads from

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Packet Type	Buffer	Group
σ_0	0	19
σ_1	0	20
σ_2	1	13
σ_3	2	19
σ_4	1	19
σ_5	0	16

Streaming Algorithm

Algorithm 1 $\mathcal{A}$

```
n ← ℕ                                                   ▷ n is window size
m ← ℕ, m < n                                            ▷ m is max group value
Sketch s is empty initially
while Stream not empty do
    for i = 0; i < n; i++ do
        Read next packet p, σ_p ← "type"(p)             ▷ Type could be srcIP, dstIP, ...
        if σ_p ∉ s then
            Add ⟨σ_p, 1, m⟩ to s                        ▷ σ_p acts as a key to s
        else
            s[σ_p] buffer + 1, s[σ_p] group = max(1, group - 1)
        end if
        for All other σ ≠ σ_p ∈ s do
            s[σ] buffer - 1
            if s[σ] buffer < 0 then
                s[σ] buffer = 1, s[σ] group = min(m, group + 1)
            end if
        end for
    end for
    Send/Save/Process s, then clear for next window
end while
```

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- Saved sketch for each of the 200 windows

Interesting Theoretical Results

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Theorem

The maximum number of different σ that can get to group 1 for a given n, m is $\lfloor \log\left(\frac{n}{m}\right) \rfloor + 1, n, m \in \mathbb{N}, n > m$

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Remark

For $m = 2$, there is a tight bound where if the number of different σ is $\geq \lfloor \log\left(\frac{n}{m}\right) \rfloor + 2$ then there will always be at least σ in group m

More Theory Results

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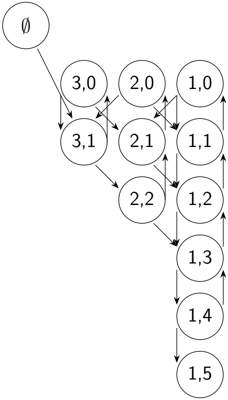

Using Markov Chain Properties

Consider $p(x) = 0.4$ for a packet in the previous example:

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\begin{array}{cccccccccccc}
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\end{array}
$$

$$
\begin{bmatrix}
0.6 & 0 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.6 & 0 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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\begin{bmatrix}
0.07776 & 0.10368 & 0.22464 & 0.06912 & 0.13824 & 0.06912 & 0 & 0.1152 & 0.06912 & 0.10752 & 0.01536 & 0.01024 \\
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0 & 0.07776 & 0 & 0.20736 & 0 & 0.05184 & 0.1728 & 0 & 0.38016 & 0 & 0.08448 & 0.0256 \\
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We can compute the expected value of the class $\mathbb{E}(x) = 1.85536$

Experimental Results

Future Work and Open Questions

- Tighter bound on interval before group m packets are guaranteed
- P4 implementation of sketch that chooses σ as srcIP (in progress)
- Adding time dynamics with minimal increase to computational complexity
- Determining sweet spot for m, n in relation to each other

References

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The space complexity of approximating the frequency moments.
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This can be generalized to the following equation:

$$\sum_{i=0} 2^i \cdot (m-1) \leq n-(m-1)$$

$$= \lfloor \log \left(\frac{n}{m-1} \right) \rfloor + 1$$

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Can we do better? Yes, this is an extremely loose bound, my intuition is that the lower bound before m is guaranteed is also logarithmic because of the buffers of each σ that we add in a window