Journal of Computational Acoustics, Vol. 9, No. 1 (2001) 133–148

GAUSSIAN RAY BUNDLE (GRAB) MODEL SHALLOW WATER ACOUSTIC WORKSHOP IMPLEMENTATION

RUTH ETA KEENAN

Science Applications International Corporation,
P.O. Box 658, 13 Steeple Street, Mashpee, MA 02649, USA
E-mail: rkeenan@capecod.net

HENRY WEINBERG

Integrated Performance Decisions, 23 Colonial Drive,
Waterford, CT 06385, USA
E-mail: weinbergh@home.com

Received 1 January 2000
Revised 1 June 2000

The Shallow Water Acoustic Workshop (SWAM) test cases stress low frequency propagation in shallow water environments dominated by energy refracted through the bottom. Ray based models are not normally considered applicable for such problems. The implementation and accuracy of the range dependent, Gaussian Ray Bundle eigenray model is examined for a subset of these test cases.

1. Introduction

The Gaussian Ray Bundle (GRAB) eigenray propagation model is the Navy Standard model for propagation in the 10–100 kHz frequency band.[1] GRAB has compared well with data and academic models at frequencies between 600 Hz and 30 kHz[2] and has been given interim approval for fleet use down to 600 Hz.[3] The GRAB methodology[4] specifies a set of test rays traced from a source position through the range dependent environment the target range. The amplitude of the test rays are Gaussian weighted by their distance from the target depth. GRAB then organizes the test rays into families of ray paths with similar path histories and generates an eigenray for each ray family. Each is described by eigenray source angle, target angle, arrival time, level, phase and path history. The eigenrays can be added in phase or power added to produce the coherent or random propagation loss or they can be convolved with a source function to produce arrival structure.

Inherent in the GRAB method is the specular reflection of each test ray at the boundaries. Because GRAB does not propagate the test rays through the bottom, it requires a bottom loss and phase associated with the grazing angle at the bottom to describe the bottom interaction. GRAB has several bottom loss options available to the user. Of interest are the table inputs and Rayleigh bottom reflection model.[5] The Rayleigh bottom reflection model is applicable to a homogeneous half-space. The Shallow Water Acoustic

Modeling (SWAM) workshop test cases focus on sediment conditions which not only reflect energy at the sediment interface, but refract energy through the sediment and back into the water column. To represent the SWAM bottom conditions for GRAB the OASES[6] equivalent plane wave reflection coefficient algorithms were used to collapse this complicated bottom interaction into a suitable tabular representation of bottom loss versus grazing angle.

This report discusses how well GRAB matches the benchmark ASA wedge test and a simpler homogeneous environment test. Weaknesses of GRAB exposed by the SWAM test cases are examined. Since there were no benchmark solutions for the workshop test cases and the inter-model comparisons are presented elsewhere as part of the workshop documentation, none of the GRAB results for the SWAM test cases are presented in this paper. However, the Appendix does list background information used to generate the SWAM GRAB test case results.

2. Baseline Cases

It is difficult to benchmark an acoustic model because there are so few relevant analytic solutions available and model to model comparisons must account for the different underlying assumptions and numerical implementations. However, the Acoustical Society of America (ASA) wedge problem[7] is an accepted benchmark. Another classic example is an isovelocity environment with homogeneous bottom conditions. In this case there is excellent agreement between the FEPE[8] and OASES solutions whose different numerical techniques to solve the wave equation lend credibility to the solution. GRAB compares favorably with both baseline cases.

2.1. *Benchmark wedge*

The ASA benchmark wedge problem is a low frequency, shallow water problem that is not particularly well-suited to ray theory. A homogeneous ocean (sound speed 1500 m/s) overlies a homogeneous sediment (sound speed 1700 m/sec, attenuation 0.5 dB/λ and density of 1.5 g/cm^3). The bottom depth is 200 m at the source decreasing linearly to 0 m at a 4 km range. The 25 Hz source at 100 m generates an acoustic field that is a classic example of bathymetric mode stripping. Figure 1 illustrates the wedge problem using the Navy Standard PEv5 RAM[9] model.

More out of curiosity than any expectation that GRAB should be able to model propagation under these conditions, Fig. 2 compares the GRAB predictions with the 30 m and 150 m receiver depth benchmark solutions.[a] GRAB matches the benchmark solution quite well as long as the receiver depth remains in the water column (remember that GRAB does not propagate energy through the bottom).

[a]Provided by Richard Evans, September 15, 1999.

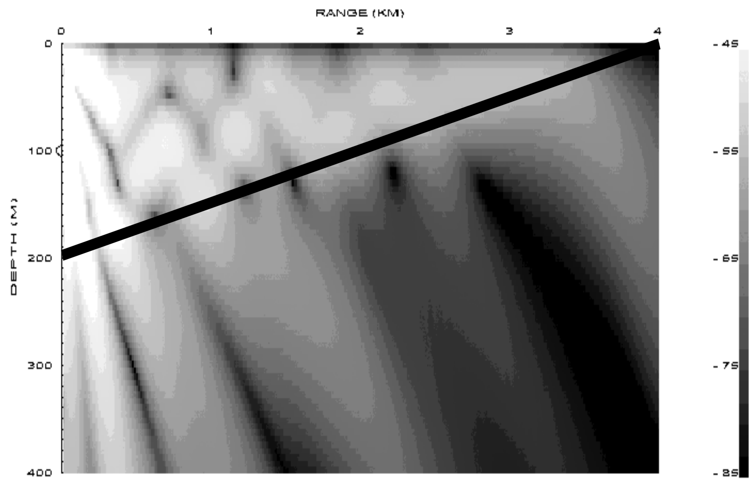
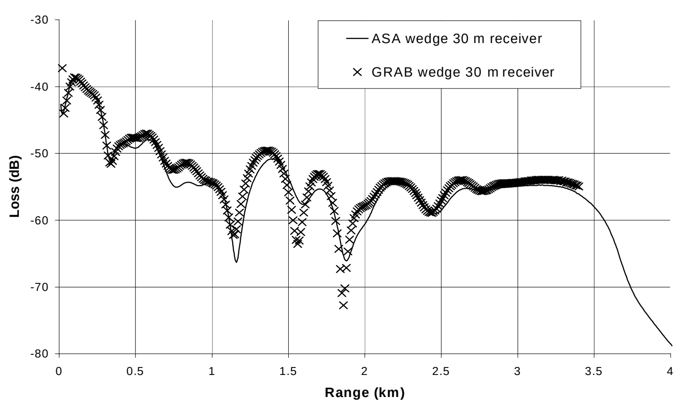
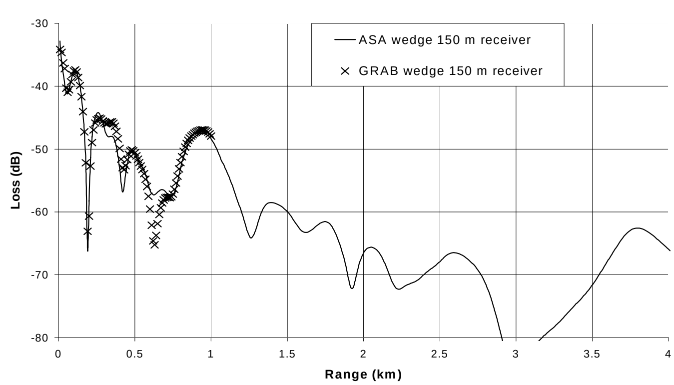


Fig. 1. ASA benchmark wedge 25 Hz propagation using RAM.



(a)



(b)

Fig. 2. ASA benchmark wedge 25 Hz solutions versus GRAB predictions.

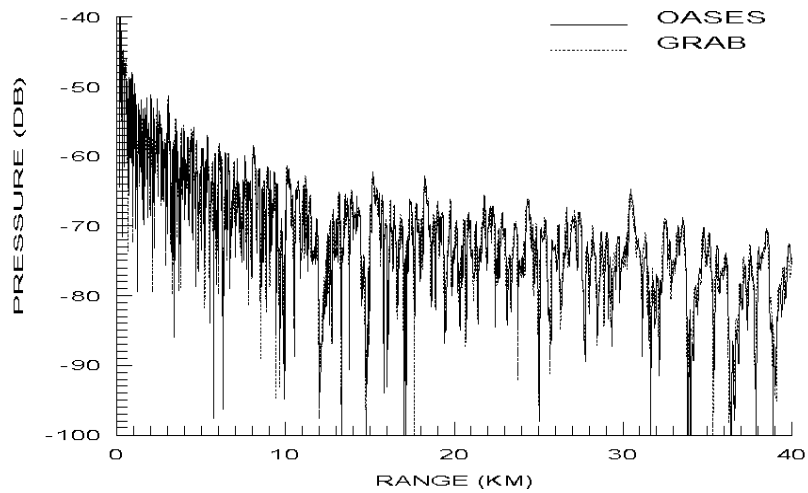


Fig. 3. SRS test case GRAB versus OASES 1 kHz propagation loss.

2.2. *GRAB Software Requirements Specification example*

The Software Requirements Specification[4] (SRS) illustrated how well GRAB compared with FEPE predictions at 1 kHz in a 150 m water depth, isovelocity (1491 m/s water) environment without volume attenuation for a 30.48 m source and receiver depth combination. The sediment was described to GRAB with the Rayleigh bottom reflection coefficient and phase model option. The sediment sound speed was 1565.55 m/s with a 1.268 g/cm^3 density and 0.6 dB/λ attenuation. This test case is reconstructed here using OASES. As with FEPE, the match between the two models in terms of level and interference pattern is excellent (Fig. 3).

2.3. *Simplified SWAM environment example*

If the SWAM Case 1, range 0 km bottom sediment sound speed were homogeneous (constant with depth), it would be similar to the previous SRS example. The difference is the SWAM water depth is 100 m versus the SRS 150 m depth and the source/receiver depths are 30/35 m, respectively versus 30.48 m for the SRS source/receiver depths. Figure 4 compares the plane wave equivalent bottom loss for the SRS case versus the homogeneous SWAM case 1, 0 km bottom sediment. The initial bottom condition for the SWAM test case is much lossier than that used for the SRS example. Figures 5 and 6 illustrate the OASES and GRAB propagation loss predictions at 1 kHz, 500 Hz, 250 Hz and 25 Hz, respectively.

The test ray spacing limits the accuracy of the GRAB eigenrays; the eigenray source angles can be no less accurate than the test ray spacing. In Fig. 6, the GRAB 250 Hz coherent predictions using a 0.1° test ray spacing are becoming "ragged", but decreasing the test ray spacing to 0.01° improves the predictions. The OASES predictions at 25 Hz show a monotonic decay with levels greater than 140 dB by 10 km. GRAB however is having difficulty modeling this modal decay at 25 Hz. Note that in the absence of destructive

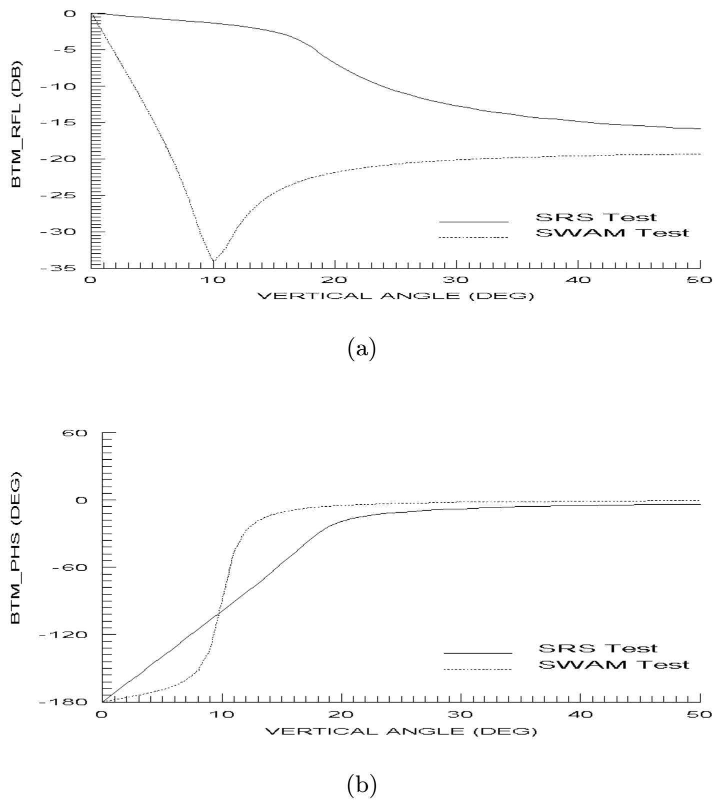
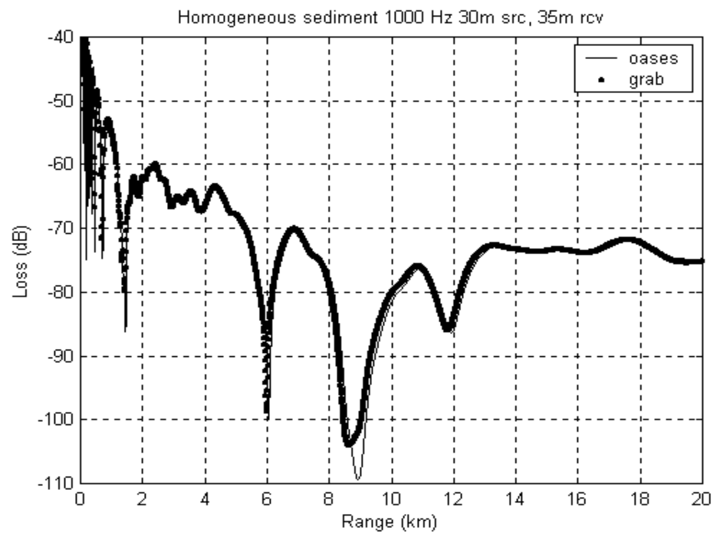


Fig. 4. SRS versus SWAM homogeneous bottom loss and phase.

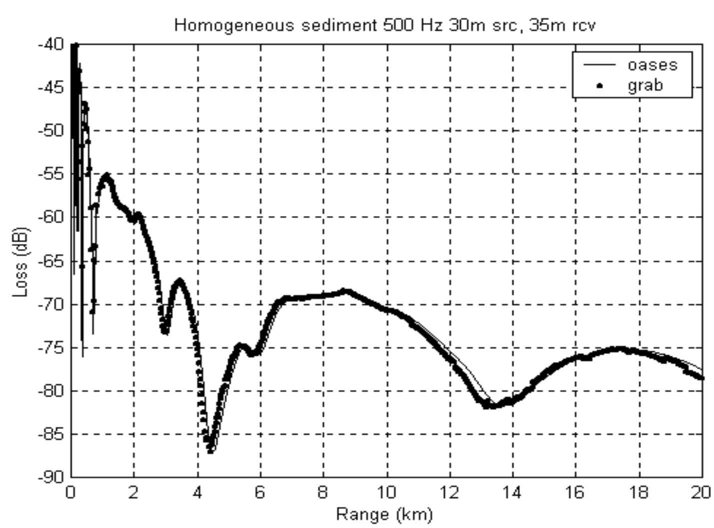
interference (the GRAB random addition solution) the levels are less than 80 dB. Adding the eigenrays with the default test ray spacing of 0.1° coherently increases the loss by 30 dB. Increasing the numerical precision of the eigenrays by calculating them with a test ray spacing of 0.01° increases the coherent loss by another 10 dB. The fact that GRAB does not predict loss levels greater than 110 dB at 10 km indicates it is unable to add all the eigenrays together with enough precision to generate the destructive interference effects of this acoustic field. At 25 Hz the water depth is only 1.67λ and destructive interference effects are an important physical mechanism that the numerical precision in GRAB is struggling to construct.

3. Swam Bottom Interaction

The OASES generated bottom loss and phase sampled every 0.25° at 25 Hz and 1000 Hz are plotted in Fig. 7 for the case 1 bottom region at 0 km. The bottom sediment for the SWAM environments was defined for the OASES reflection coefficient calculation in 100 m layers, down to a 6500 m basement depth, to generate the bottom loss curves. Although the plane wave reflection coefficient algorithm was sensitive to the sediment thickness at the higher grazing angles, it was found that the 25 Hz RAM and OASES propagation



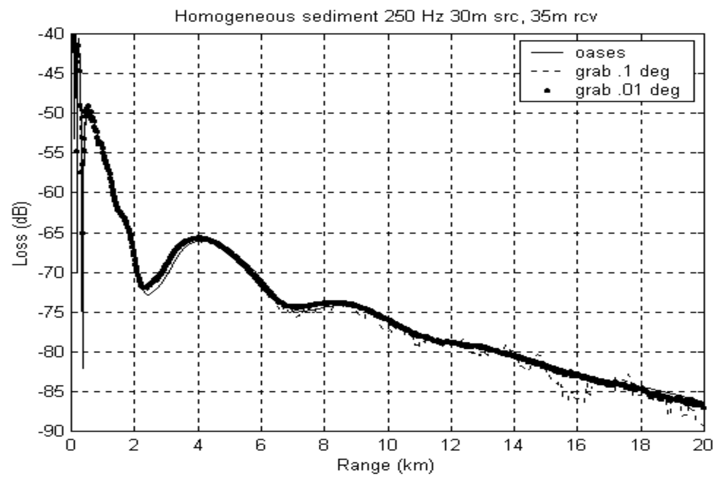
(a)



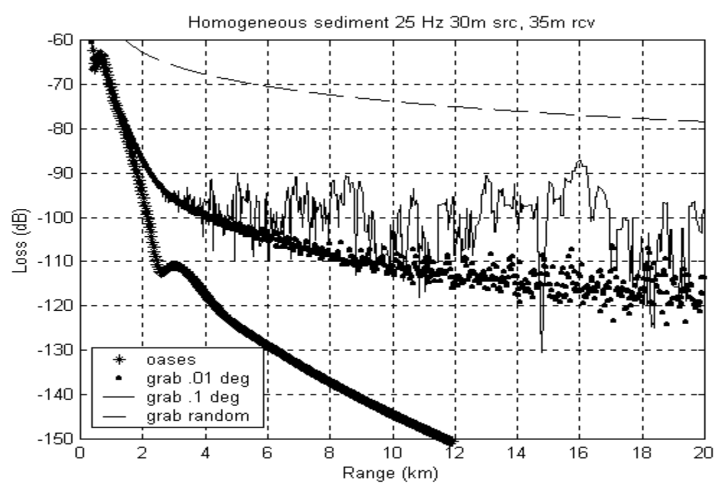
(b)

Fig. 5. SWAM homogeneous sediment 1000 Hz and 500 Hz propagation.

predictions were not sensitive to sediment depths thicker than 300 m, indicating the higher angles were insignificant in the overall propagation. Based upon this observation, the GRAB predictions limited the test ray aperture to ±45°. The interference pattern of the refracted sediment paths generate rapid amplitude and phase fluctuations. This presents a challenge for GRAB to model because destructive interference effects are sensitive to the precision of the phase.



(a)

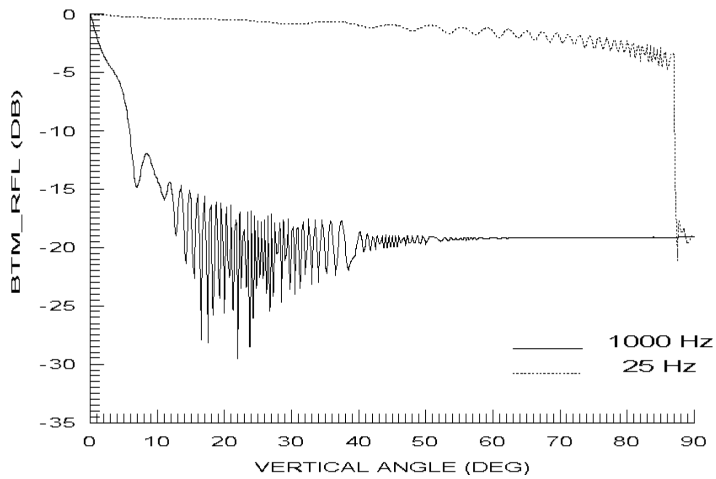


(b)

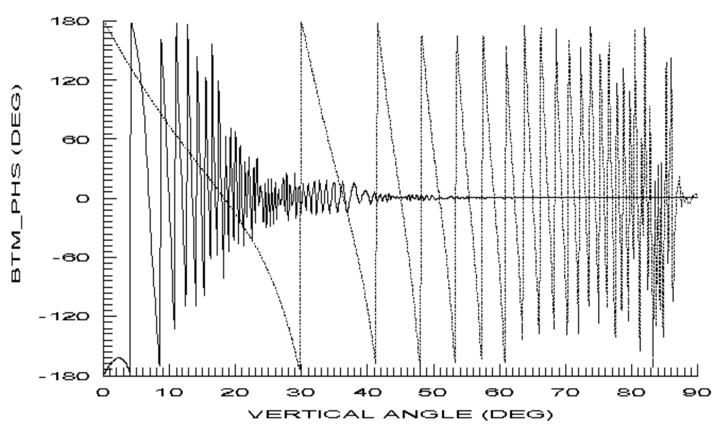
Fig. 6. SWAM homogeneous sediment 250 Hz and 25 Hz propagation.

GRAB interpolates table inputs to obtain loss and phase for the test ray interface grazing angle. The interpolation method will introduce errors. Note the phase at 4° for 1 kHz is about $-179°$ and the phase at 4.25° is about $+179°$. GRAB will interpolate a phase of 0° at a 4.125° grazing angle instead of a more appropriate 180° phase angle. To avoid this kind of phase error the bottom phase curves were unfolded by 360° when the phase changed direction. A more exact method would be to incorporate the OASES reflection coefficient model into GRAB so that actual loss and phase values can be calculated for the desired angles. However desirable this feature may be, it is not currently available.

The previous discussion focused on isovelocity sediment conditions that do not refract energy back into the water column. Most of the sediment descriptions for the SWAM workshop



(a)



(b)

Fig. 7. Bottom loss and phase case 1, range 0 km SWAM bottom.

involve bottom conditions with strong positive gradients. Figure 8 illustrates the differences in the ray paths if the bottom is treated as a boundary (solid paths) and as part of the water column (dotted paths) with the sediment sound speed gradient. In both cases the $-2°$, $-3°$, $-4°$, $-5°$, $-10°$, $-20°$, $-30°$, and $-40°$ rays are traced.

This section examines the GRAB and OASES propagation for the case 1, region 0 km bottom for 25, 250, 500 and 1000 Hz (Figs. 9 and 10). In Fig. 9, "GRAB table" was run with the bottom loss/phase tables illustrated in Fig. 7. It is not a surprise that the ray based GRAB model is unable to construct the interference pattern in the 1.6λ water depths that is observed in the OASES predictions. However, GRAB does much better predicting the appropriate level and interference pattern when the bottom is incorporated as part of

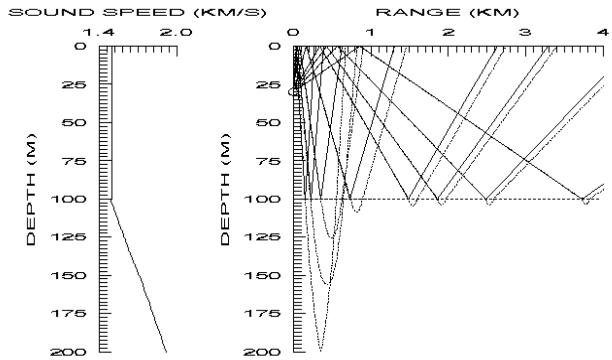
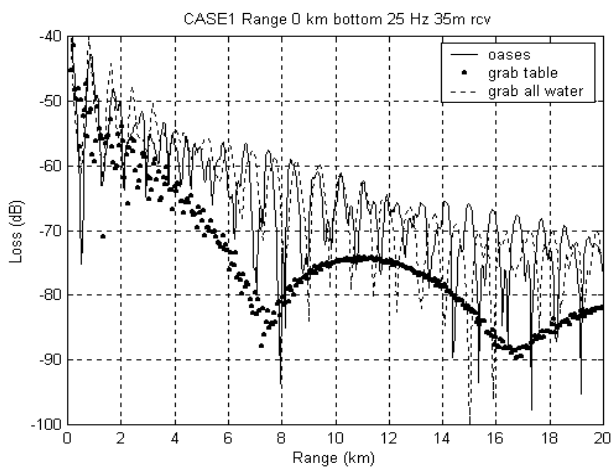
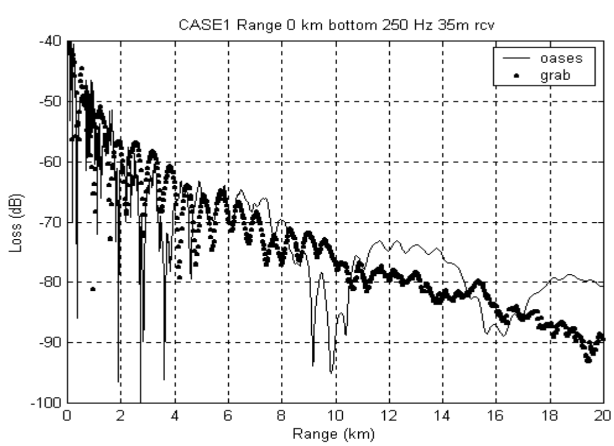


Fig. 8. Case 1, range 0 km ray trace.

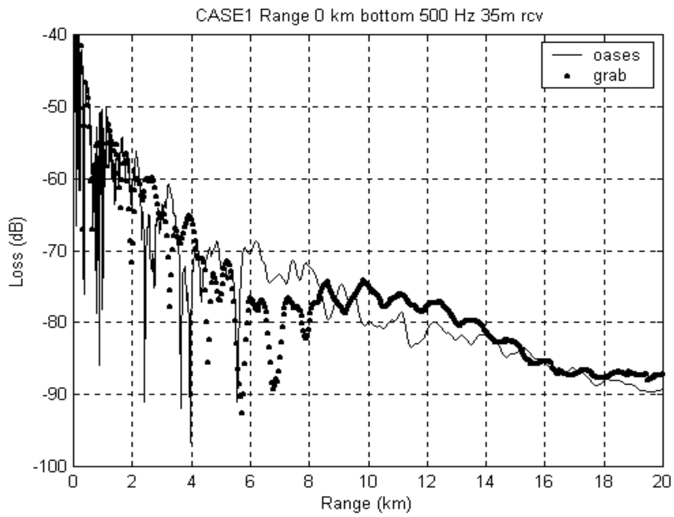


(a)

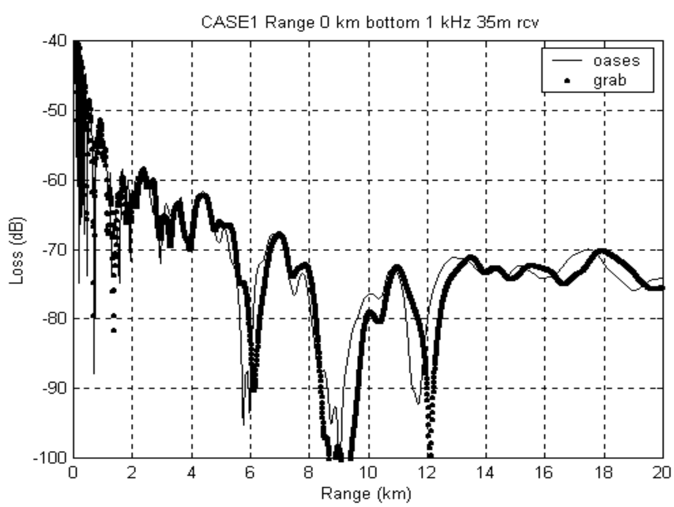


(b)

Fig. 9. Case 1, range 0 km sediment 25 Hz and 250 Hz.



(a)



(b)

Fig. 10. Case 1, range 0 km sediment 500 Hz and 1 kHz.

the water column with the appropriate attenuation and sound speed (GRAB all water). The comparison between GRAB and OASES improves with increasing frequency as the importance of the bottom refracted paths diminishes.

4. Reciprocity

The SWAM downslope case 2 (30 m source depth and 35 m target depth) and upslope case 3 (35 m source depth and 30 m target depth) tested reciprocity. Theoretically, reciprocity must hold, but the SWAM example demonstrates that there are some numerical implementation considerations in GRAB that may degrade the results of that theorem. Given the phase errors introduced by the tabular representation of the bottom loss, there is no reason to expect reciprocity to hold for the coherent propagation loss for these test cases. Given the sensitivity of the GRAB coherent prediction to small phase errors it would be a better measure of the model's ability to handle reciprocity if we looked at the random phase addition of the eigenrays. The following table lists the difference in the 1 kHz transmission loss at 20 km for the reciprocal points for various conditions using the test case 2 and 3 bathymetry. The reference in the table to linear or cubic refers to how the bathymetry points are connected to define the bottom depth. Under the homogeneous SRS test case bottom conditions (see Fig. 4) the coherent propagation loss using the default linear and $0.1°$ test ray spacing is an unacceptable 13.6 dB different. By decreasing the test ray spacing the accuracy of the eigenrays improves but the difference is still an unacceptable 8.5 dB. By using the cubic interpolation of the bathymetry the first derivative of the slope is continuous and the error could be reduced to 1.47 dB. The SWAM bottom loss descriptions introduce phase errors and degrade the GRAB reciprocity results. (The case A bottom conditions are illustrated in Fig. A.2 and the range dependent case B bottom conditions are illustrated in Fig. A.3.) However, GRAB satisfies reciprocity for all cases if the phase information had been ignored and the random summation of eigenrays examined.

Test Case	Case 2 Up (dB)	Case 3 Down (dB)	Difference (dB)
Homogeneous, $0.1°$, linear, coherent	75.91	89.53	13.6
Homogeneous, $0.01°$, linear, coherent	92.89	84.37	8.5
Homogeneous, $0.01°$, cubic, coherent	75.76	74.29	1.47
Homogeneous, $0.1°$, linear, random	73.18	73.41	0.23
Homogeneous, $0.01°$, linear, random	73.148	73.140	0.008
Homogeneous, $0.01°$, cubic, random	73.35	73.19	0.16
SWAM case A, $0.01°$, cubic, coherent	67.81	73.32	5.5
SWAM case B, $0.01°$, cubic, coherent	84.15	84.48	0.33
SWAM case A, $0.01°$, cubic, random	67.64	67.91	0.27
SWAM case B, $0.01°$, cubic, random	77.18	76.83	0.35

5. Conclusions

The SWAM test cases present a challenge to the GRAB model. The complexity of the equivalent plane wave reflection coefficient to describe the bottom interaction introduces phase errors into the GRAB eigenrays. As a result, it is expected the coherent propagation loss will have phase errors. In fact, at the SWAM September 8–9, 1999 workshop none of the models phase predictions agreed with each other and they were not starting with the GRAB handicap. However, the power summation of the eigenrays should provide a good measure of the mean energy levels and an "eyeball" average of the various SWAM results indicates GRAB is predicting reasonable mean levels.

When the bottom interaction is simplified to a homogeneous bottom condition, GRAB matches the more precise OASES and FEPE models in level and phase as long as the water depth was in excess of 10λ. When the water depth is less than 10λ, the propagation is usually dominated by 1–2 modes. This is a situation better suited to full wave models rather than a ray based model.

Reciprocity tests demonstrated that how the bathymetry was modeled (linear segments or cubic spline) in the segmented upslope and downslope test cases affects the accuracy of the GRAB phase information. The smoother cubic spline fit with it's continuous first derivative gave better results than the simple linear segments. Overall the coherent GRAB solutions for these test cases did not demonstrate reciprocity due to the phase errors. However, the GRAB random addition solutions did satisfy the reciprocity tests.

Keenan approached these workshop test cases with trepidation because they emphasize the weaknesses of the GRAB model. However, both authors believe documenting the models strengths and weaknesses will educate users how to use the model advantageously.

Appendix A. Model Settings for SWAM Test Cases

A.1. *Test case 1: Flat bottom*

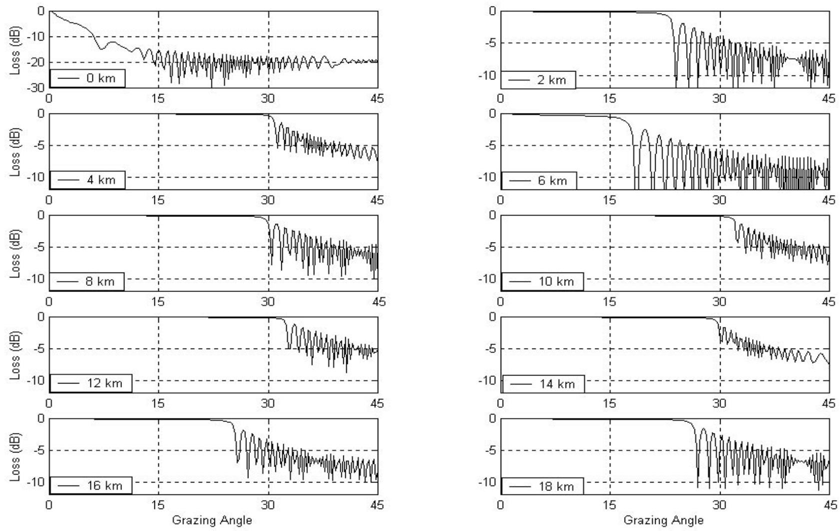
The OASES bottom loss/phase table descriptions at 1 kHz for test case 1 are illustrated in Fig. A.1 for each bottom type region. GRAB was set up to transition suddenly between each bottom region in a 1 m range. For instance, the first bottom region was applied to the GRAB model between 0 and 1.999 km, the transition to the second bottom region occurred between 1.999 and 2.0 km. GRAB was set up with a 0.01° test ray spacing for a $\pm 45^\circ$ test ray aperture at a 30 m source depth. The propagation loss at the 35 m and 70 m receiver depths from 0–20 km was sampled every 10 m and the propagation loss from 0–100 m every 1 m at the 5 km and 15 km ranges.

A.2. *Test case 2: Downslope*

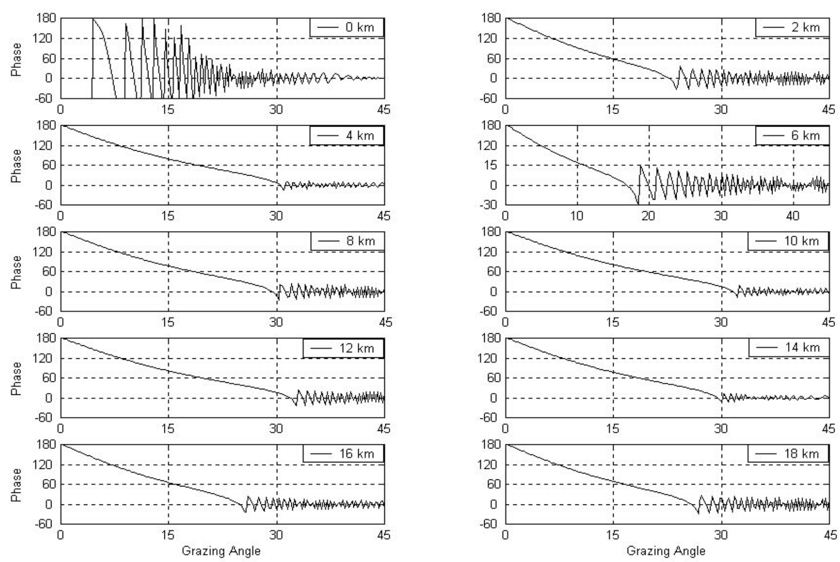
Test case 2 is a downslope environment. Case 2A has constant sediment properties whose equivalent plane wave reflection coefficient level and phase is illustrated in Fig. A.3. Case 2B uses the same bottom loss environment used in case 1. GRAB was run at 1 kHz with a $\pm 45^\circ$ aperture and 0.01° test ray spacing for a 30 m source depth with a cubic interpolation between the bathymetry points. The transmission loss for the 35 m and 70 m receivers for ranges 0–20 km was sampled every 10 m and the transmission loss for the 5 km and 15 km ranges between 0–220 m was sampled every 1 m.

A.3. *Test case 3: Upslope*

Case 3 is the reciprocal of case 2 for the 35 m receiver at 20 km. The source depth is now 35 m and the receiver depths are 30 m and 70 m. The order of the bottom loss tables



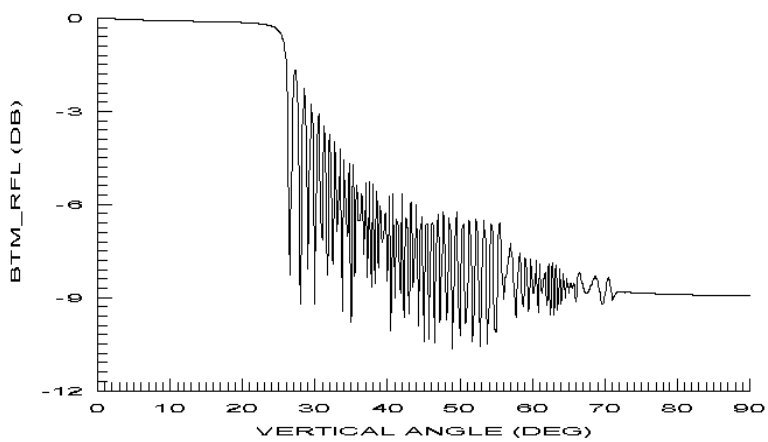
(a)



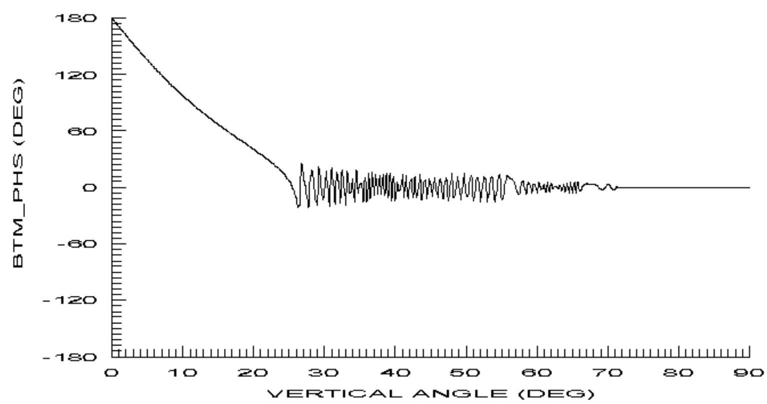
(b)

Fig. A.1. Case 1, 1 kHz bottom loss and phase.

has been reversed for the case 3B environment. Thus the lossy bottom conditions between 0–2 km for case 2B is found at 18–20 km for case 3B. GRAB was run at 1 kHz with a cubic interpolation of the bathymetry points using a ±45° aperture sampled every 0.01°. The propagation loss at the 30 m and 70 m receivers out to 20 km was sampled every 10 m and the 5 km and 15 km ranges sampled from 0 to 220 m every 2 m.



(a)



(b)

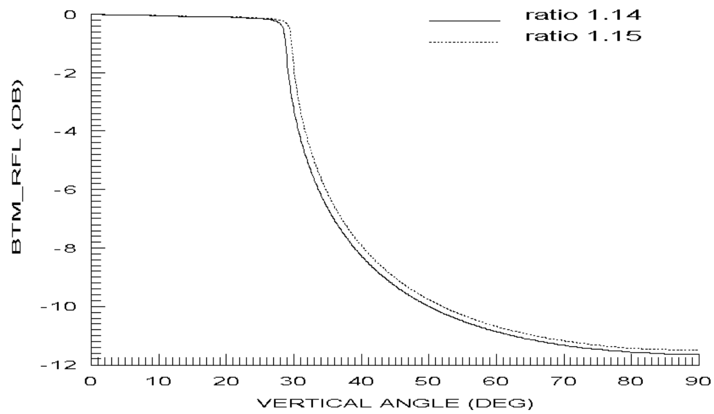
Fig. A.2. Case 2A, 1 kHz bottom loss and phase.

A.4. *Test case 4: 3-D Bathymetry*

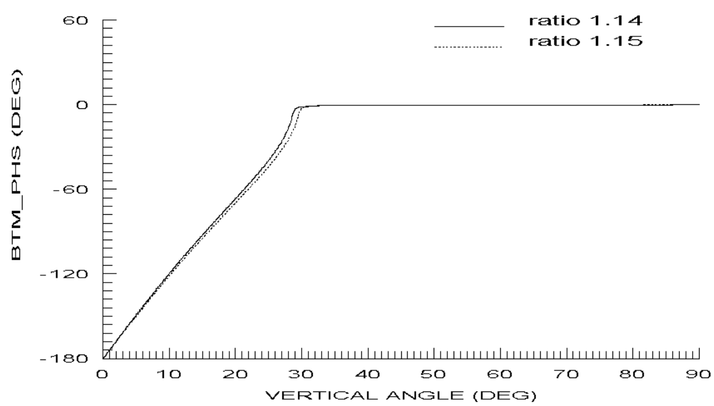
GRAB is a N $\times$ 2D model and was not exercised for test case 4.

A.5. *Test case 5: Internal waves*

Test case 5 is the internal wave test case. This test case is particularly difficult for GRAB to model precisely because the sound speed varies continuously with range but was sampled every 200 m. GRAB was given 100 sound speed tables and used a default linear interpolation between tables. GRAB was run at 1 kHz using a ± 45 aperture sampled every 0.1° for a 30 m source depth. The sound speed at the bottom of the water column varies, the sediment to water sound speed was modeled with an average value of 1.14 using the Rayleigh bottom model. The equivalent bottom loss/phase for this test case is illustrated in Fig. A.3.



(a)



(b)

Fig. A.3. Effect of ratio difference bottom loss and phase.

A.6. *Test case 6: Shelf break*

Test case 6 varies the bathymetry and sound speed. The bottom properties are the same as in test case 5, but there is a different sound speed variation at the bottom of the water column. The GRAB Rayleigh bottom model option was used with an average sediment to water sound speed ratio of 1.15 (Fig. A.3). As with the internal wave case, the sound speed profile varies continuously with range and must be sampled for GRAB. Fortunately this sound speed environment does not vary as rapidly as the internal wave case. For environment 1 the sound speed profile at 0 km was used until 8.99 km. Between 8.99 and 9.0 km the SVP at 9 km is introduced. GRAB then smoothly transitions to profiles at 9.5 km, 10 km, 10.5 km, 11 km, 11.5 km and 12 km. Beyond 12 km the sound speed profile at 12 km is used. For environment 2 the sound speed profile at 0 km was used until 13.99 km. Between 13.99 and 14 km the SVP at 14 km is introduced. GRAB then smoothly transitions to profiles at

14.5 km, 15 km, 15.5 km, 16 km and 16.5 km. Beyond 16.5 km the sound speed profile at 16.5 km is used.

GRAB was run at 1 kHz using a ±45° test ray spacing aperture sampled every 0.01° for a 30 m source depth. The downward refracting environment allows the deeper source depth to propagate energy further up the shelf than the shallow source depth. The propagation loss as a function of range was sampled every 10 m at the 35 m and 70 m receiver depths. The propagation loss was sampled every 2 m as a function of depth at 5 km and 15 km.

References

1. Oceanographic and Atmospheric Master Library Summary, Naval Oceanographic Office, Systems Integration Division, Stennis Space Center, MS, OAML-SUM-21G, April 1999, p.7.
2. M. Boyd, "Gaussian Ray Bundle (GRAB)-Independent Verification and Validation (IVV)," Applied Physics Laboratory, University of Washington, Seattle, WA, March 1998. http://pluto.apl.washington.edu/GRAB_IVV
3. Oceanographic and Atmospheric Master Library April 1999 Software Review Board proceedings.
4. R. E. Keenan, H. Weinberg, and F. Aidala, "Software requirements specification GRAB: Gaussian Ray Bundle eigenray propagation model," July 1997, prepared for Naval Oceanographic Office System Integration Department, Stennis Space Center, MS.
5. ______, "APL-UW high-frequency ocean environmental acoustic models handbook," Technical Report APL-UW TR 9407, Section IV, October 1994.
6. H. Schmidt, "OASES v2.2 users guide and reference manual," Department of Ocean Engineering, Massachusetts Institute of Technology, July 9, 1999.
7. F. B. Jensen and W. A. Kuperman, "Sound penetration in a wedge-shaped ocean with a penetrable bottom," *J. Acoust. Soc. Am.* **67**, 1564 (1980).
8. M. D. Collins, "FEPE user's guide," Naval Research Laboratory, Stennis Space Center, MS, NORDA Technical Note 365, 1988.
9. ______, "Software requirement specification for the parabolic equation/finite element parabolic equation model, version 5.0," prepared for the Naval Oceanographic Office, Systems Integration Department, October 1999.