Sound propagation in a wedge-shaped ocean with a penetrable bottom

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Modal cutoff during up-slope propagation in a wedge-shaped ocean is studied using the parabolic equation model; theoretical results are compared with some model tank experiments.

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INTRODUCTION

Sound propagation in a range-dependent ocean environment has received considerable attention within the acoustic modeling community during recent years. Various approaches have been attempted for a theoretical description of range-dependent acoustic propagation, but the most promising wave-theoretical approach still seems to be the parabolic equation (PE) method,[1] which includes both diffraction and mode-coupling effects. The main theoretical limitation on the PE method is that it is restricted to horizontal ray angles of less than ~20°; it also has a small inherent phase error resulting in a slightly different spatial acoustic interference pattern than that calculated from the exact normal-mode theory. However, for most practical purposes these limitations are of minor importance.

A considerable practical limitation on the PE method is the excessive computer running times required when dealing with shallow-water environments.[2] However, this problem has partly been overcome by recent advances in computer technology: by running the PE model on a dedicated computer system in connection with an array processor one can increase the speed by a factor of 100 compared with that obtained by using a general-purpose, time-shared computer.[3] Hence, the PE method is also becoming practical to use for shallow-water environments.

In this paper we use the PE method to study sound propagation in a wedge-shaped ocean. This problem has earlier been treated for the simple case of a rigid ocean bottom.[4] It was found that an approximate solution based on adiabatic mode theory agreed well with the exact solution for the acoustic field for gradual bottom slopes. Here we are going to consider a physically realistic, penetrable ocean bottom, which is a case that adiabatic mode theory will not be able to handle, since, as we will find, propagation here is mainly associated with energy conversion from the discrete to the continuous mode spectrum.

TABLE I. Environmental parameters.

	Water	Bottom
Sound speed (m/s)	1500.0	1704.5
Density (g/cm^3)	1.00	1.15
Attenuation (dB/λ)	0	0.5

I. THEORETICAL RESULTS FROM PE MODEL

We are considering up-slope propagation in an isovelocity, wedge-shaped ocean with a penetrable, isovelocity bottom. The PE model used in this study is a deep-water version[5] modified to include the ocean bottom characterized by an arbitrary sound-speed profile, a density, and a wave attenuation.[6] All environmental parameters given in Table I were chosen to correspond to the parameters used in a model tank experiment reported by Coppens and Sanders.[7] The actual environment used here consists of an initial 5-km flat stretch of 200-m depth, followed by a bottom slope of 1.55°. Figure 1 shows the three discrete normal modes that exist in the flat region for a frequency of 25 Hz. Figures 2 and 3 show contoured propagation loss versus depth and range for two different source depths, as obtained from the PE model. The water/bottom interface is here indicated by the heavy line starting at 200-m depth and moving towards the surface beyond a range of 5 km.

Before interpreting the contour plots, let us have a look at the simplified sketch in the upper part of Fig. 2. Using the ray/mode analogy, a given mode can be associated with up- and down-going rays with a specific grazing angle. Here is indicated a ray corresponding to a given mode. As sound propagates up the slope, the grazing angle for that particular ray (mode) increases, and at a certain point in range the angle exceeds the critical angle at the bottom, meaning that the reflection loss becomes very large and that the ray essentially leaves the water column and starts propagating in the bottom. In the ray picture the point in range where this

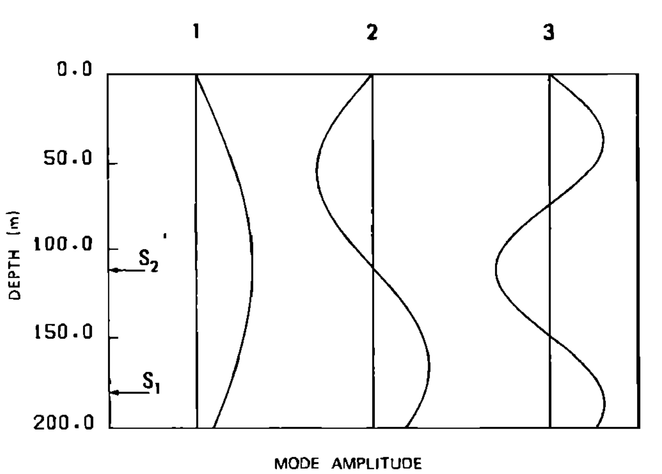

FIG. 1. Modal depth functions in flat part of the ocean.

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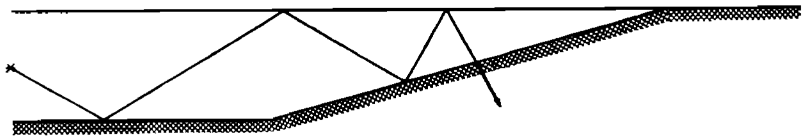

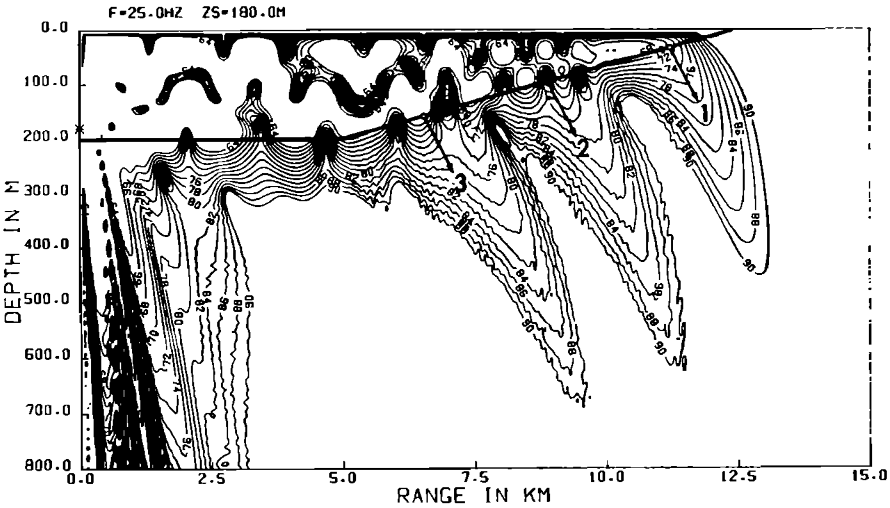

FIG. 2. Contoured loss versus depth and range with all three modes excited (source at 180 m).

happens corresponds to the cutoff depth for the equivalent mode.

In order to emphasize the main features in Figs. 2 and 3 we have chosen to display contour levels between 64 and 90 dB in 2 dB intervals. Thus, the uncontoured regions in the water indicate losses less than 64 dB whereas the uncontoured regions in the bottom indicate losses greater than 90 dB. In Fig. 2 the source depth is 180 m, which means that all three modes are excited in the flat region, as seen from Fig. 1 (source S_1). Figure 2 shows four regions of significant intensity in the bottom. The nearfield region out to about 3 km corresponds (to within the validity of the PE approximation) to the radiation of the "continuous modes" into the bottom. As sound propagates up the slope we see three well-defined beams in the bottom. The arrows originate at the theoretical cutoff depths for the three modes, computed as if the bottom was locally flat. We see that the PE model predicts cutoff at approximately the depths predicted by normal-mode theory. Of course, as one expects, the cutoff is not abrupt but takes place over a finite distance, which essentially provides an aperture for radiation of a beam into the bottom.

In order to further investigate the relation between the mode and PE pictures, we present, in Fig. 3, results for a source depth of 112 m. As seen from Fig. 1, that particular source depth (S_2) corresponds to a null for the second mode, which, therefore, should not be excited. We see in Fig. 3 that beam number 2 is conspicuously absent. Furthermore, Fig. 4 shows the field as a function of depth in the water column after the third mode cuts off by the PE model at a range of 8 km. The line is the plot of the first mode as calculated by a mode program,[6] and we see exact agreement in shape between the two methods, indicating that there is no second mode present. (For the mode plot we used normalized modes expressed in arbitrary decibels.) Thus we have here a very strong indication that energy contained in a given mode does not couple into the next lower mode but couples almost entirely into the con-

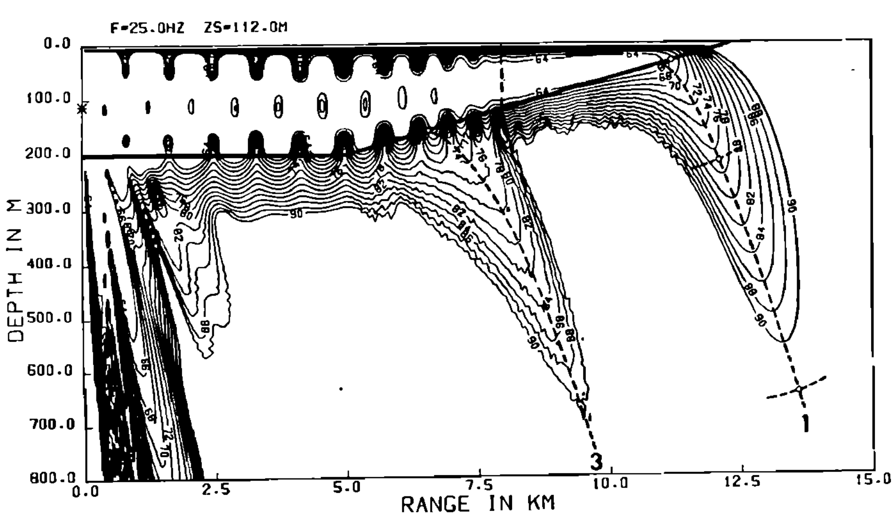

FIG. 3. Contoured loss versus depth and range with only two modes excited (source at 112 m).

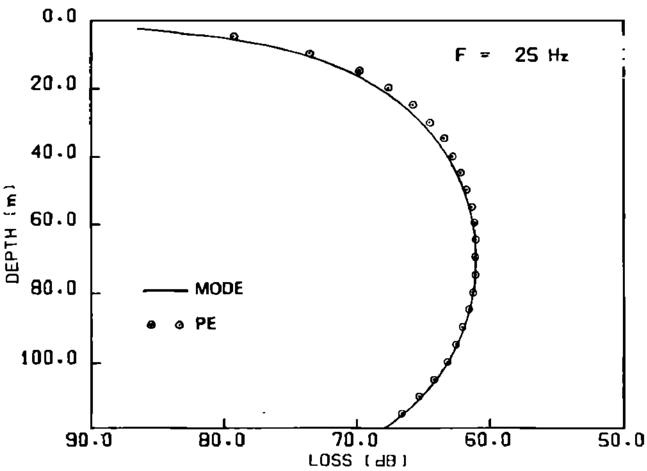

FIG. 4. Loss versus depth at a range of 8 km; comparison of modal and PE results.

tinuous mode spectrum.

Figures 2 and 3 are particularly interesting since this PE program models the source as a gaussian beam and does not use a normal-mode calculation for the initial field. Hence the results presented here seem to indicate that the gaussian representation of a point source in the presence of a bottom is an adequate approximation.

II. COMPARISON WITH EXPERIMENTAL RESULTS

A model tank experiment for studying the physical mechanism of modal cutoff during up-slope propagation in a wedge has been carried out by Coppens and Sanders.[7] They used the same environmental parameters as given in Table I, but used a source frequency of 150 kHz. Therefore, to compare the experimental data with the PE predictions, the geometry must be scaled with the acoustic wavelength.

The experimental data show that modal cutoff does indeed take place in the form of a discrete beam being radiated into the bottom. However, the measured beam angle is approximately 20% higher than predicted by the PE model. This discrepancy is probably due to the fact that the bottom speed is such that the critical angle is around 28°, which leads to some inaccuracies in the PE computations, where angles should be limited to ~20°.

Returning to Fig. 3, the two dashed lines perpendicular to beam 1 indicate where experimental measurements were made. We see that the data actually were obtained in the nearfield of the radiating aperture, where the beam path is curved and where the asymptotic beam angle has not yet been reached. If we define the aperture size to be the distance between the 3-dB down points from the center of the beam along the bottom, the aperture size is of the order of 40 wavelengths. Hence the measurements were made at distances of 0.4 and 1.25 aperture sizes, which is clearly in the nearfield. This fact seems to explain the discrepancy found by Coppens and Sanders[7] between measured beam angles and angles predicted from an asymptotic theory (the stationary phase approximation used is essentially a farfield, high-frequency approximation). Thus measured angles were generally 20%–30% lower than predicted by the asymptotic theory.

In measuring the amplitude across the beam, Coppens and Sanders[7] found a second peak on the high-angle side (to the left of beam 1 in Fig. 3). This could be interference with the second mode, and Fig. 3 dramatically demonstrates that the beam can be isolated when the source is placed at the null of the second mode.

III. CONCLUSIONS

This study clearly demonstrates the power of the parabolic equation method in handling propagation in a depth-varying, range-dependent ocean environment. From this study we may also draw the following specific conclusions:

(1) The output from the PE model agrees with an experimental result that mode cutoff in a wedge-shaped ocean occurs by the radiation of a discrete beam into the bottom. Thus there is very little conversion of energy to the next lower mode.

(2) A corollary of (1) is that mode-coupling theories that attempt to describe propagation in a wedge must include coupling into the continuum.

(3) The Gaussian representation of a point source in the PE model also gives very good results in the presence of a bottom.

(4) An experimental study of this mode cutoff phenomenon would be facilitated by placing the source at the null of the second mode.

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