Para estimar $p(y_6 | y_1, y_2, y_3, y_4, y_5)$, utilizamos o Teorema de Bayes:

$$p(y_6 | y_1, y_2, y_3, y_4, y_5) = \frac{p(y_1, y_2, y_3, y_4, y_5 | y_6)\, p(y_6)}{p(y_1, y_2, y_3, y_4, y_5)}$$

Uma vez que $\{y_1, y_2\}$, $\{y_3, y_4\}$, $\{y_5\}$ são conjuntos de variáveis independentes, podemos reescrever a expressão da seguinte forma:

$$p(y_6 | y_1, y_2, y_3, y_4, y_5) = \frac{p(y_1, y_2 | y_6)\, p(y_3, y_4 | y_6)\, p(y_5 | y_6)\, p(y_6)}{p(y_1, y_2)\, p(y_3, y_4)\, p(y_5)}$$

Dada uma nova observação D, esta pode ser classificada, calculando $p(y_6 | D)$ para os valores do domínio de y_6 e escolhendo como previsão o valor associado à maior probabilidade

$$\hat{z} = \underset{y_6 \in \{A,B\}}{\arg\max}\ p(y_6 | D) = \underset{y_6 \in \{A,B\}}{\arg\max} \frac{p(y_1, y_2 | y_6)\, p(y_3, y_4 | y_6)\, p(y_5 | y_6)\, p(y_6)}{p(y_1, y_2)\, p(y_3, y_4)\, p(y_5)} =$$

$$= \underset{y_6 \in \{A,B\}}{\arg\max}\ p(y_1, y_2 | y_6)\, p(y_3, y_4 | y_6)\, p(y_5 | y_6)\, p(y_6)$$

(uma vez que o argmax de uma função é o mesmo a menos de constantes multiplicativas positivas)

Calculam-se, portanto, estes parâmetros, com base nas observações de treino:

①: $p(y_6)$: $p(y_6 = A) = \frac{3}{7}$ $p(y_6 = B) = \frac{4}{7}$

②: $p(y_1, y_2 | y_6)$: $y_1 \times y_2 \in \mathbb{R}^2 \sim \mathcal{N}(\underline{\mu}, \underline{\Sigma})$

$$\mu(y_1) = \frac{1}{7} \sum_{i=1}^{7} x_{i1} = \frac{1}{7}(0.24 + \dots + 0.42) = \frac{1}{7} \times 3{,}09 = 0.4414 = \mu_1$$

$$\mu(y_2) = \frac{1}{7} \sum_{i=1}^{7} x_{i2} = \frac{1}{7}(0.36 + \dots + 0.59) = \frac{1}{7} \times 2{,}87 = 0.41 = \mu_2$$

$$\underline{\mu} = \begin{bmatrix} 0.4414 \\ 0.41 \end{bmatrix}$$

$$Var(y_1) = \frac{1}{7-1} \sum_{i=1}^{7} (x_{i1} - \mu_1)^2 = \frac{1}{6} \times 49{,}399$$

$$Var(y_2) = \frac{1}{7-1} \sum_{i=1}^{7} (x_{i2} - \mu_2)^2 = \frac{1}{6} \times 42.5864$$

$$Cov(y_1, y_2) = \frac{1}{7-1} \sum_{i=1}^{7} (x_{i1} - \mu_1)(x_{i2} - \mu_2) = \frac{1}{6}\, 45{,}482$$

$$\underline{\Sigma} = \frac{1}{6} \times \begin{bmatrix} 49{,}399 & 45{,}482 \\ 45{,}482 & 42{,}5864 \end{bmatrix} = \begin{bmatrix} 8{,}2332 & 7{,}5803 \\ 7{,}5803 & 7{,}0977 \end{bmatrix}$$

$$|\underline{\Sigma}| = 0{,}9754$$

$$p(y_1, y_2) = \mathcal{N}(y_1, y_2 | \underline{\mu}, \underline{\Sigma}) = \frac{1}{2\pi \sqrt{|\underline{\Sigma}|}} \exp\left(-\frac{1}{2}\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \underline{\mu}\right)^T \cdot \underline{\Sigma}^{-1} \cdot \left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \underline{\mu}\right)\right)$$

(3) $P(y_3, y_4)$:

$y_3 \backslash y_4$	0	1	
0	2/7	2/7	4/7
1	2/7	1/7	3/7
	4/7	3/7	1

$P(y_3, y_4 \mid y_6 = A)$:

$y_3 \backslash y_4$	0	1	
0	0	1/3	1/3
1	1/3	1/3	2/3
	1/3	2/3	1

$P(y_3, y_4 \mid y_6 = B)$:

$y_3 \backslash y_4$	0	1	
0	2/4	1/4	3/4
1	1/4	0	1/4
	3/4	1/4	1

(4) $p(y_5)$:

y_5	0	1	2
p	2/7	3/7	2/7

$p(y_5 \mid y_6 = A)$:

y_5	0	1	2
p	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$p(y_5 \mid y_6 = B)$:

y_5	0	1	2
p	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

I.2b.

$\hat{z}_8 = \underset{y_6 \in \{A,B\}}{\arg\max}\ p(y_6 \mid x_8)$

para $y_6 = A$:

$$\left.\begin{array}{l} p(y_1, y_2 \mid A) = 0.9847 \\ p(y_3, y_4 \mid A) = \frac{1}{3} \\ p(y_5 \mid A) = \frac{1}{3} \\ y(y_6 = A) = \frac{3}{7} \end{array}\right\rangle \Pi = 0.0469$$

para $y_6 = B$:

$$\left.\begin{array}{l} p(y_1, y_2 \mid B) = 1.9624 \\ p(y_3, y_4 \mid B) = \frac{1}{4} \\ p(y_5 \mid B) = \frac{1}{4} \\ p(y_6 = B) = \frac{4}{7} \end{array}\right\rangle \Pi = 0.07$$

$\hat{z}_8 = \underline{B}$

$\hat{z}_9 = \underset{y_6 \in \{A,B\}}{\arg\max}\ p(y_6 \mid x_9)$

para $y_6 = A$:

$$\left.\begin{array}{l} p(y_1, y_2 \mid A) = 0.4031 \\ p(y_3, y_4 \mid A) = 1/3 \\ p(y_5 \mid A) = 1/3 \\ p(y_6 = A) = 3/7 \end{array}\right\rangle \Pi = 0.0192$$

para $y_6 = B$:

$$\left.\begin{array}{l} p(y_1, y_2 \mid B) = 1.7286 \\ p(y_3, y_4 \mid B) = 1/4 \\ p(y_5 \mid B) = 2/4 \\ p(y_6 = B) = 4/7 \end{array}\right\rangle \Pi = 0.1235$$

$\hat{z}_9 = \underline{B}$

I 1.c.

Maximum likelihood assumption: $p(x|y_c) = p(y_1, y_2|y_c)\, p(y_3, y_4|y_c)\, p(y_5|y_c)$

para x_8: com $y_c = A$:

$$0.9847 \times \frac{1}{3} \times \frac{1}{3} = 0.1094$$

com $y_6 = B$:

$$1.9624 \times \frac{1}{4} \times \frac{1}{4} = \underline{0.1227} \qquad z_8 = B$$

para x_9: com $y_6 = A$:

$$0.4031 \times \frac{1}{3} \times \frac{1}{3} = 0.0448$$

com $y_c = B$:

$$1.7286 \times \frac{1}{4} \times \frac{1}{4} = \underline{0.2161} \qquad z_9 = B$$

Normalização:

$$p(A|x_8) = \frac{p(x_8|A)}{p(x_8|A) + p(x_8|B)} = \frac{0.1094}{0.1094 + 0.1227} = 0.4713$$

$$p(A|x_9) = \frac{p(x_9|A)}{p(x_9|A) + p(x_9|B)} = \frac{0.0448}{0.0448 + 0.2161} = 0.1717$$

Para $\theta < 0.1717$: $\hat{z}_8 = A$, $\hat{z}_9 = A$ accuracy $= \frac{0}{2} = 0$

Para $\theta \in]0.1717; 0.4713[$: $\hat{z}_8 = A$, $\hat{z}_9 = B$ accuracy $= \frac{1}{2}$

Para $\theta > 0.4713$: $\hat{z}_8 = B$, $\hat{z}_9 = B$ accuracy $= 1$

Threshold: $\theta \in]0.4713; 1.00]$

	y1	y2	y3	y4	y5	y6
0	0.24	0	1	1	0	A
1	0.16	0	1	0	1	A
2	0.32	1	0	1	2	A

	y1	y2	y3	y4	y5	y6
3	0.54	0	0	0	1	B
4	0.66	0	0	0	0	B
5	0.76	0	1	0	2	B

	y1	y2	y3	y4	y5	y6
6	0.41	1	0	1	1	B
7	0.38	1	0	1	0	A
8	0.42	1	0	1	1	B

Train observations: folds $x_1 - x_3$, $x_4 - x_6$

I.2.b.

Test observations: fold $x_7 - x_9$

Hamming distances:

	x_1	x_2	x_3	x_4	x_5	x_6
x_7	4	4	2	2	3	4
x_8	2	4	1	4	3	5
x_9	4	4	2	2	3	4

$w_{3,7} = \frac{1}{2}$ $\quad w_{4,7} = \frac{1}{2}$ $\quad w_{5,7} = \frac{1}{3}$

$w_{1,8} = \frac{1}{2}$ $\quad w_{3,8} = \frac{1}{1}$ $\quad w_{5,8} = \frac{1}{3}$

$w_{3,9} = \frac{1}{2}$ $\quad w_{4,9} = \frac{1}{2}$ $\quad w_{5,9} = \frac{1}{3}$

Normalização:

$w_{3,7} = \frac{3}{8}$ $\quad w_{4,7} = \frac{3}{8}$ $\quad w_{5,7} = \frac{1}{4}$

$w_{1,8} = \frac{3}{11}$ $\quad w_{3,8} = \frac{6}{11}$ $\quad w_{5,8} = \frac{2}{11}$

$w_{3,9} = \frac{3}{8}$ $\quad w_{4,9} = \frac{3}{8}$ $\quad w_{5,9} = \frac{1}{4}$

$$\hat{z}_7 = w_{3,7}\, x_{3,2} + w_{4,7}\, x_{3,4} + w_{5,7}\, x_{3,5} = \frac{3}{8} \times 0.32 + \frac{3}{6} \times 0.54 + \frac{1}{4} 0.66 = 0.4875$$

$$\hat{z}_8 = \frac{3}{11} \times 0.24 + \frac{6}{11} \times 0.32 + \frac{2}{11} \times 0.66 = 0.36$$

$$\hat{z}_9 = \frac{3}{8} \times 0.32 + \frac{3}{8} \times 0.54 + \frac{1}{4} \times 0.66 = 0.4875$$

$$MAE = \frac{1}{k} \sum_{i=7}^{9} |x_{i,1} - \hat{z}_i| = \frac{1}{3} \left(|0.41 - 0.4875| + |0.38 - 0.36| + |0.42 - 0.4875| \right) =$$

$$= 0.055$$