ECOS3018

Econoomics of Growth – Notes

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Based on the lecture notes by A/Prof Graham White

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Introduction to Growth Theory

1.1   *Growth, Distribution and Duality - One-Sector Case*

$$\underbrace{Y_c a_{cc}(1+g)}_{\text{total investment in corn}} + c_w L_c = Y_c \tag{1.1}$$

$$a_{cc}(1+g) + c_w l = 1 \tag{1.2}$$

where
- Y_c is the gross output of corn
- L_c is total labour required
- c_w is corn consumption per unit of labour
- a_{cc} is the unit input requirement of corn in the production of corn
- g is the rate of output of corn.

The price system is

$$p_c a_{cc}(1+r) + w_m l = p_c \tag{1.3}$$

so that with corn as numeraire

$$a_{cc}(1+r) + wl = 1 \tag{1.4}$$

The rate of profit (net profit of the value of commodity capital invested in production) associated with producing each unit of output is

$$r = \frac{p_c - w_m l - p_c a_{cc}}{p_c a_{cc}}$$

$$r = \frac{1 - wl - a_{cc}}{a_{cc}} \tag{1.5}$$

Respectively equations (1.2) and (1.4)

$$c_w = \frac{1 - a_{cc}(1+g)}{l} \tag{1.6}$$

$$w = \frac{1 - a_{cc}(1+r)}{l} \tag{1.7}$$

Note the symmetrical role of technical conditions in price and quantity systems. This is the duality of the price-quantity system. The maximum profit rate and the maximum growth rate are given by

$$G = R = \frac{1 - a_{cc}}{a_{cc}} \tag{1.8}$$

At maximum consumption per worker $g = 0$; at maximum real wage $r = 0$, so that

$$c_{\max} = w_{\max} = \frac{1 - a_{cc}}{l} \tag{1.9}$$

If the term $(1 - s_p)ra_{cc}Y_c$ is included on the left hand side of equation (1.1)

$$Y_c a_{cc}(1 + g) + c_w L + (1 - s_p)ra_{cc}Y_c = Y_c$$

then, with $c_w = w$

$$g = \frac{1 - a_{cc} - wl}{a_{cc}} - (1 - s_p)r$$

while, from equation (1.5)

$$r = \frac{1 - a_{cc} - wl}{a_{cc}}$$

so that

$$g = r - (1 - s_p)r \implies g = s_p r \tag{1.10}$$

Can arrive at equation (1.10) alternatively: assuming as before that all wages are spent

$$S = s_p \cdot P \tag{1.11}$$

where
- s_p is planned savings
- P is profit

With planned saving equal to planned investment

$$I = s_p \cdot P \quad \text{and} \quad \frac{I}{K} = \frac{s_p \cdot P}{K} \tag{1.12}$$

Assuming that output and the capital stock grow at the same rate, g, then can decompose equation (1.10)

$$g = s_p r = \frac{S}{P}\frac{P}{Y}\frac{Y}{K} = \frac{S}{Y}\frac{Y}{K} = s\frac{Y}{K} \tag{1.13}$$

so that, with

$$v = \frac{K}{Y} \tag{1.14}$$

then

$$g = \frac{s}{v} \tag{1.15}$$

1.2   Duality of Price and Quantity Systems in the Two-Sector Case

Consider the two-commodity "iron-corn model", where the economy is growing (uniformly across sectors) at the rate g per period Gross outputs for the two sectors are

$$\underbrace{(Y_c a_{cc} + Y_i a_{ci})(1 + g)}_{\text{total investment demand for corn}} + C_c = Y_c$$

$$\underbrace{(Y_c a_{ic} + Y_i a_{ii})(1 + g)}_{\text{total investment demand for iron}} + C_i = Y_i \tag{1.16}$$

Suppose the growth rate is such that consumption per capita remains constant $\implies \frac{C_{ct}}{N_t} = c_c$ remains constant where C_{ct} and N_t are corn consumption and population at time t. We can also assume:
- in each period output corresponds to demand;
- there is only consumption demand for corn; $C_i = 0$
- the labour force (L) seeking work is a constant percentage of population (N)

- growth is sufficient to provide for full-employment of labour

$$\frac{L_t}{N_t} \text{ is constant and } l_c Y_{ct} + l_i Y_{it} = L_t$$

- $\implies \frac{l_c Y_{ct} + l_i Y_{it}}{N_t}$ is constant

Consumption per worker can be expressed as

$$c_w = \frac{C_{ct}}{N_t} \frac{N_t}{l_c Y_{ct} + l_i Y_{it}} = c_c \frac{N_t}{l_c Y_{ct} + l_i Y_{it}} \tag{1.17}$$

Defining output per capita as

$$\frac{Y_{ct}}{N_t} = y_c \quad \frac{Y_{it}}{N_t} = y_i$$

one can write equation (1.17) and solve for c_c

$$c_c = c_w (l_c y_c + l_i y_i) \tag{1.18}$$

In turn, the quantity system equation (1.16) can be expressed as

$$(y_c a_{cc} + y_i a_{ci})(1 + g) + c_w (l_c y_c + l_i y_i) = y_c$$
$$(y_c a_{ic} + y_i a_{ii})(1 + g) = y_i \tag{1.19}$$

For a given structure or composition of output i.e. $\frac{y_c}{y_i} = \frac{Y_c}{Y_i}$, equation (1.19) yield the following relation betqeen g and c_w

$$c_w = \frac{(a_{cc} a_{ii} - a_{ci} a_{ic})(1 + g)^2 - (a_{cc} + a_{ii})(1 + g) + 1}{(l_i a_{ic} - l_c a_{ii})(1 + g) + l_c} \tag{1.20}$$

Consider the price system for this economy

$$(a_{cc} p_c + a_{ic} p_i)(1 + r) + w_m l_c = p_c$$
$$(a_{ci} p_c + a_{ii} p_i)(1 + r) + w_m l_i = p_i \tag{1.21}$$

Note the uniform rate of profit.
With corn as the numeraire

$$(a_{cc} p_c + a_{ic} p_i)(1 + r) + w_c l_c = 1$$
$$(a_{ci} p_c + a_{ii} p_i)(1 + r) + w_c l_i = p_{ic} \tag{1.22}$$

i.e $w_c = \frac{w_m}{p_c}$, $p_{ic} = \frac{p_i}{p_c}$. The real wage in terms of corn is given by

$$w_c = \frac{(a_{cc} a_{ii} - a_{ci} a_{ic})(1 + r)^2 - (a_{cc} + a_{ii})(1 + g) + 1}{(l_i a_{ic} - l_c a_{ii})(1 + g) + l_c} \tag{1.23}$$

Comparing with equation (1.20), this is the duality of price and quantity system, for every technique yielding a w-r relation there is a corresponding c_w-g relation.
The outmost envelope of the set of c_w-g represents the technological frontier of consumption-growth possibilities
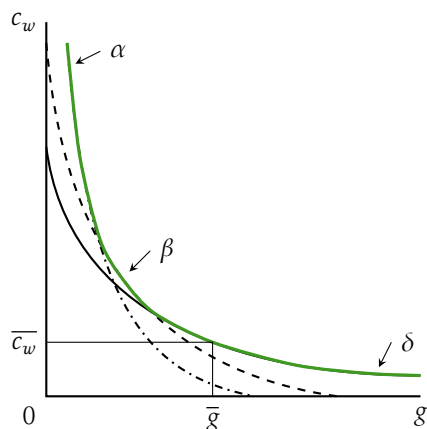
Figure 1.1. Technological Frontier of Consumption-Growth Possibilities

At a given growth rate, e.g. $\overline{g}$ the corresponding point on the frontier indicates the technique which maximises c_w for that growth rate.

- Preceding analysis assumes that g is determined by the requirement of full-employment.
- To the extent that one relaxes this assumption, this leaves up in the air the determination of g.
- Traditional orthodox growth theory accepts the notion that long-run growth will be consistent with the requirements of full-employment.
- Analogous arguments to those used in a static context (i.e. for a stationary economy).
- However, the history of formal growth theory (mainly through the 20th century) contains a rich undercurrent of dissent from this view.

2

Early Keynesian-Inspired Models of Growth: Harrod and Domar

2.1 *A Conceptual Issue in Moving to Growth Analysis: the Two Sides of Investment*

- Simple Keynesian multiplier (ignore autonomous components of planned aggregate expenditure (AE) other than investment):

$$Y = AE = \frac{1}{1-c}I \qquad (2.1)$$

- Given c, constant $I \implies$ constant level of expenditure. Thus the equilibrium Y is constant - a "stationary equilibrium" (i.e $Y = AE$).
- But suppose I includes positive net invest $\implies$ productive capacity is expanding. Producers must be anticipating growth in demand. How is this consistent with a constant AE?
- With demand constant producers would find their productive capacity underutilized. For equilibrium, increased output produced with larger capacity must be matched by increased demand. With positive net investment equilibrium requires that aggregate demand grow in line with productive capacity. From equation (2.1), if c is constant, I must be growing, so according to equation (2.1), Y must be growing - a "moving equilibrium".
- Hence, with a positive net investment, it is incorrect to speak of equilibrium **level** of income. The appropriate concept is instead an equilibrium **rate of growth** of income.

> Taking account of two sides of investment $\implies$ equilibrium must be one characterised by growing output. Along the equilibrium growth path, the demand effects from growing investment exactly balance capacity effects from that investment

- In other words, produces expand capacity by $x\%$ via increases investment on the basis of an expected expansion of demand at $x\%$.
- Increased investment orders $\implies$ a multiplied increase in aggregate expenditure

$$\Delta AE = \frac{1}{1-c}\Delta I \qquad (2.2)$$

- **Along the equilibrium growth path**, AE will actually expand by the rate which producers expected – i.e. $x\%$ – thereby justifying the $x\%$ expansion in capacity
- $\implies$ demand effects of expanding investment exactly match the capacity effects.

2.2   *The Harrod-Domar Approach*

- Can be seen as an attempt to extend Keynes's "principle of effective demand" (PED) (1936) into the "long-run", i.e. to a growing economy.
- PED is the level of output and employment are ultimately constrained by the level of aggregate demand in a capitalist economy – i.e. the effective limit on the level of output and employment is **not** the full-employment of labour, either in the short-run or the long-run.
- Alternatively put, investment governs the level of saving and there is no mechanism to ensure investment will be sufficient to warrant the full-employment level of output (i.e. to absorb the level of saving at the full-employment level of income).
- The simple multiplier can be used to illustrate

$$\Delta Y = \frac{1}{1-c}\Delta I \implies \Delta S = (1-c)\Delta Y \tag{2.3}$$
$$= \frac{1-c}{1-c}\Delta I$$
$$= \Delta I$$

- An autonomous increase in I generates sufficient change in Y, such that, given the propensity to save out of income $l-c$, S increases by a matching amount.
- Keynes's own formal statement dealt with a stationary economy – i.e. a given capital stock – thereby ignoring capacity effects of investment.
- But taking account of these effects $\implies$ need to recast the analysis for a growing economy.
- In essence both Harrod and Domar start with the following question:

    is there a rate of growth of output which would be self-sustaining in the sense that producers would be content to continue expanding capacity at that rate?

- Such a rate would require capacity and demand effects of growing investment to balance. From the multiplier relation

$$Y = \frac{1}{s}I \implies \Delta Y = \frac{1}{s}\Delta I \implies \Delta I = s\Delta Y \tag{2.4}$$

- Taking v as the desired "marginal" capital to output ratio, the accelerator is

$$I = v\Delta Y \implies \Delta K = v\Delta Y \tag{2.5}$$

- Combining equations (2.4) and (2.5):
  Firstly
$$\Delta I = s\Delta Y \quad \text{and} \quad I = v\Delta Y$$

  Secondly
$$Y = \frac{1}{s}I \quad \text{and} \quad I = v\Delta Y$$

  Therefore
$$\frac{\Delta I}{I} = \frac{s}{v} = \frac{\Delta Y}{Y} = g_w \tag{2.6}$$

- g_w is the equilibrium or "warranted" (Harrod) growth rate.
- Assume investment orders grow at this rate and producers utilize their expanding capacity so as to maintain the desired ratio of output to the capital stock equal to '$\frac{1}{v}$'.

- "On balance", according to Harrod, the growing demand generated through the multiplier process by growing investment orders will match the growing output produced with the expanding capital stock.
- The growth in investment orders and the consequent expansion of the capital stock will have been "warranted" (i.e. justified) by the growth in AE generated by those same investment orders.
- In effect, producers' expectations about future demand growth - on which investment is based – will have been realised.
- Along the warranted growth saving will adapt to growing investment via changes in output brought about by expanding productive capacity.
- Since $S = sY$, $\Delta S = s\Delta Y$ then

$$\frac{\Delta S}{S} = \frac{\Delta Y}{Y} = \frac{\Delta I}{I} \tag{2.7}$$

- Contrast with "static" Keynesian multiplier case where Y changes to bring S into line with I with given K.
- Along with the warranted path K is also growing.
- For Harrod, stable equilibrium growth consistent with continuous full-employment requires equality of actual, natural and warranted growth rates:

$$g_a = g_n = g_w \tag{2.8}$$

where

- g_a is the actual growth rate
- g_n natural growth rate
- g_w warranted growth rate
- $g_n = g_a$ required for the maintenance of full-employment
- g_n set maximum for g_a

- According to Harrod, the warranted growth path is an unstable equilibrium
- i.e. where $g_a \neq g_w$ the gap between g_a and g_w increases.
- Along the warranted growth path

$$g_a = \left(\frac{I}{K}\right)_{\text{actual}} = g_w = \frac{s}{v} = \frac{S}{Y}\frac{Y}{K} = \frac{S}{K} \tag{2.9}$$

- e.g. consider the case $g_a > g_w$

$$\left(\frac{I}{K}\right)_{\text{actual}} > \frac{s}{v} \quad \left(\frac{I}{K}\right)_{\text{actual}} > \frac{S}{K}$$

$$\implies \left(\frac{Y}{K}\right)_{\text{actual}} > \left(\frac{Y}{K}\right)_{\text{warranted}}$$

It implies that actual demand growth exceeds expected demand growth. Effectively two cases:

(a) $g_w > g_n$. Since $g_a \leqslant g_n$ then $g_a < g_w$.
   Hence, given the saving ratio and desired capital to output ratio, the growth of investment required for equilibrium is greater than the maximum possible $\implies$ chronic tendency to unemployment.

(b) $g_w < g_n$. Two possibilities arise:
   (i) $g_w < g_a \leqslant g_n \implies$ capacity continually overutilized $\implies$ growth rate of investment continually pressing against the full-employment ceiling $\implies$ inflationary pressures.

(i) $g_a \leqslant g_w < g_n$. Either mounting unemployment in the special case where
$g_a = g_w$ and thus $g_a < g_n$.
or, $g_a < g_w$ and hence case (a) above and therefore chronic unemployment.

3

Traditional Neo-Classical Growth Theory: Solow-Swan

3.1   Growth Without Technical Progress

- Assume constant returns to scale production technology using capital and labour:

$$Y = F(K, L) = AK^\alpha L^\beta \tag{3.1}$$

- Assume $\alpha + \beta = 1 \implies \alpha = 1 - \beta$. In per worker terms (dividing through by L)

$$\frac{Y}{L} = \frac{AK^\alpha L^\beta}{L} = AK^{1-\beta} L^{-(1-\beta)} = \frac{AK^{1-\beta}}{L^{1-\beta}} = \frac{AK^\alpha}{L^\alpha} \tag{3.2}$$

$$\therefore y = Ak^\alpha \quad \text{where } k = \frac{K}{L}, \text{ or } \quad y = f(k) \tag{3.3}$$

- Ignore technical progress $\implies$ in equilibrium output and the capital stock grow at the same rate, g_e
- Ignoring physical depreciation, $\%\Delta K \frac{I}{K}$ with depreciation at the rate $\delta \implies \%\Delta K = \frac{I}{K} - \delta$
- In this model, $g_e = g_n$ (i.e. $\%\Delta$ working age population)
- i.e. in equilibrium, output and the capital stock grow at the same rate as the labour force

$$g_e = \frac{\Delta Y}{Y} = \frac{I}{K} = \frac{s}{v} = \frac{S}{Y}\frac{Y}{K} = g_n \tag{3.4}$$

- Writing for v,

$$v = \frac{K}{Y} = \frac{K}{L}\frac{L}{Y} = \frac{k}{y} \therefore s\frac{y}{k} = g_n \tag{3.5}$$

$$\implies s\frac{f(k)}{k} = g_n \tag{3.6}$$

$$\boxed{sf(k) = sy = g_n k} \quad \text{or} \quad \boxed{f(k) = \frac{g_n k}{s}} \tag{3.7}$$

- An example: assume $g_L = g_n$ (i.e. labour employment grows at the same rate as the labour force – why this is, we will take up shortly).
- Suppose at a point time $K = 1000, L = 100$, therefore $k = 10$, and $g_L = 0.2 = g_n$.
- To keep k constant requires, $g_n k$ investment per worker.
- $0.2(\frac{1000}{100} = 2)$ units of capital of investment per worker.
- Equations (3.6) and (3.7) imply that $s\frac{Y}{L} = sy$ (i.e saving per worker) must be equal to 2 in order to keep k constant i.e. if $sy = g_n k$, k will be constant.
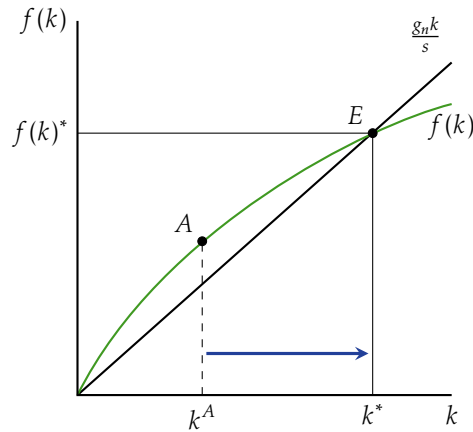- With k constant, y is constant, given $f(k)$.

- From equation (3.6)

$$s\frac{f(k)}{k} = g_n \implies s\frac{Y/L}{K/L} = g_n \implies \frac{sY}{K} = \frac{I}{K} = g_n \tag{3.8}$$

- Along the equilibrium path, S, I, Y and K all grow at the same rate - g_n
- Where $\delta > 0 \; \frac{I}{K} - \delta = g_n$ in the steady state; and hence

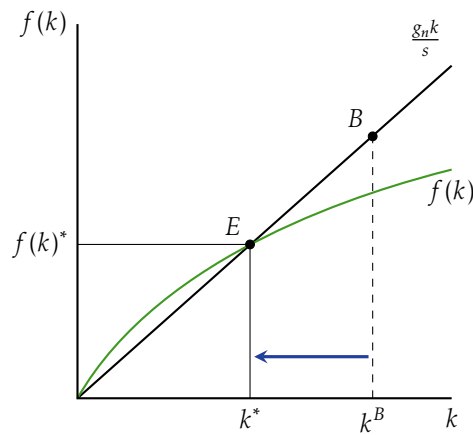$$s\frac{Y}{K} - \delta = g_n \quad \text{and} \quad s\frac{y}{k} = g_n + \delta$$

so that equation (3.7) becomes $sy = (g_n + \delta)k$

- At k^A,

$$f(k) > \frac{g_n k}{s} \implies s\frac{f(k)}{k} > g_n \implies \frac{I}{K} > g_n \tag{3.9a}$$

- Note $g_L < g_n$
- If $\frac{\Delta K}{K} > g_n$, then $\frac{\Delta K}{K} > g_L \implies \uparrow k$
- At E, $f(k) = \frac{g_n k}{s}$
- $\implies$ if g_n is exogenous, the transition from A to E (and thus the rise) in k from $k^A \to k^*$ must result in a fall in $\frac{I}{K}$.

- At k^B,

$$f(k) < \frac{g_n k}{s} \implies s\frac{f(k)}{k} < g_n \implies \frac{I}{K} < g_n \tag{3.9b}$$

- If $g_L < g_n \implies$ rising unemployment, $\downarrow w$ and incentive to reduce k.
- If $g_L = g_n$, while $\frac{\Delta K}{K} < g_n, k$ would be falling.
- Transition from B to E (and thus fall in k from $k^B \to k^*$) results in a rise in $\frac{I}{K}$ so that at E

$$f(k) = \frac{g_n k}{s}$$

- At the steady state $k = k^*$, there must be no incentive to alter k, i.e. to alter the employment of capital relative to labour.
- This implies no pressure on real wages via excess demand or supply in the labour market. In turn, this requires full-employment and growth of labour employment equal to the rate of growth of the labour force.
- i.e. $g_L = g_n$

3.2   Diminishing Returns and the Steady State

- Falling marginal and average productivities $\implies \downarrow \frac{\Delta Y}{\Delta K}$ and $\downarrow \frac{Y}{K}$ as $\uparrow k$
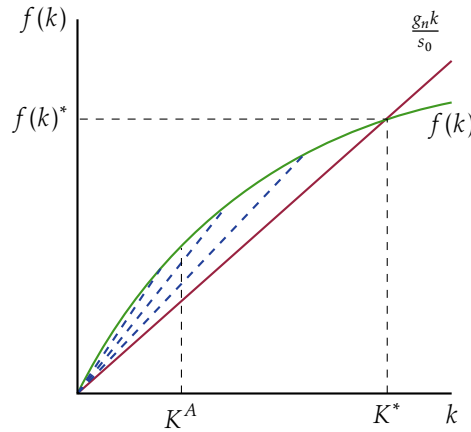
- e.g. at k^A, $\uparrow k \implies \downarrow \frac{Y}{K} \implies \frac{sY}{K} \downarrow \implies \downarrow \frac{I}{K} \implies \downarrow \frac{\Delta K}{K}$ until k stops rising at k^* where $\frac{\Delta K}{K} = g_n$

3.3   Growth and Income Distribution
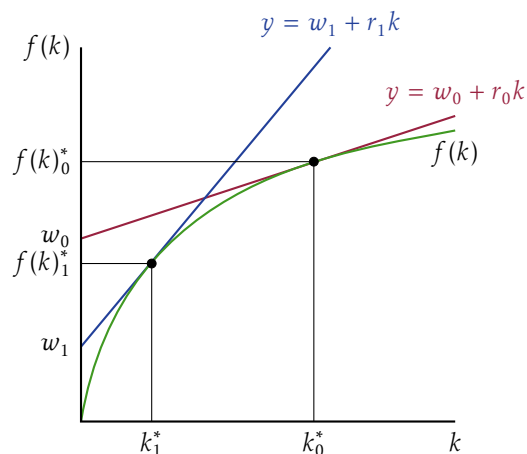
- The constant returns to scale production function (3.1) can be rewritten as

$$Y = \frac{\partial Y}{\partial L}L + \frac{\partial Y}{\partial K}K \tag{3.10}$$

- Assuming factors are paid their marginal products
- $\implies Y = wL + rK$

$$y = w + rk \tag{3.11}$$

- $\implies$ income consists of wages and profit

- Note the implications of marginal productivity theory regarding the connection between production, distribution and growth.
- A higher k (lower $\frac{L}{K}$) is associated with a lower profit rate and a higher real wage.
- Consider an ↑ in $g_n \implies \frac{g_n k}{s}$ line pivots to the left.
- New steady state at B: higher growth rate; lower y, k and v lower w, higher r
- Note: equilibrium growth rate in the Solow-Swan model is given by $g_e = \frac{s}{v} = g_n$.
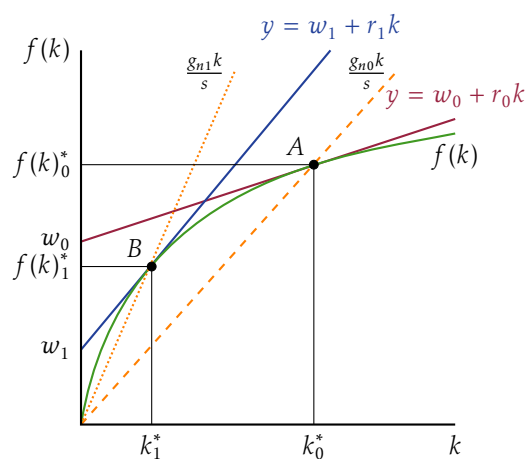
- In this case, an ↑ $g_n \implies$ ↑ g_e.
- But for this to be consistent with the fact that the equilibrium growth rate is equal to $\frac{s}{v}$, either 's' or 'v' must change.
- For a given 's', 'v' must fall.
- Contrast this with Harrod: both 's' and 'v' are fixed $\implies$ if g_n rises above $\frac{s}{v}$ there is no automatic mechanism to push $\frac{s}{v}$ in line with the higher g_n
- Consider a rise in the savings ratio, $s_0 \rightarrow s_1$
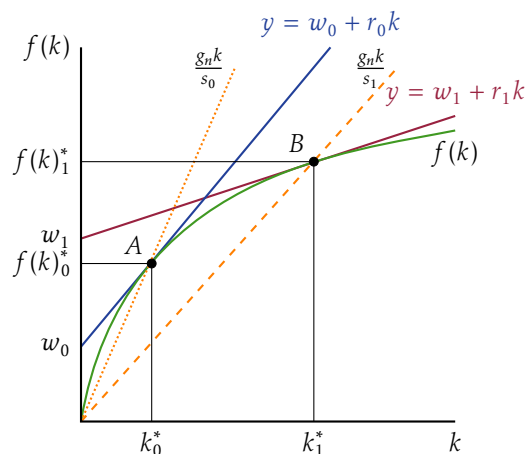
- New steady state associated with same growth rate, but higher y, k and v (lower $\frac{Y}{K}$).
- Adjustment to new steady state involves involves a transition to a higher real wage (move to a lower $\frac{L}{K}$) and a lower rate of profit.
- Note again how this contrasts with Harrod's analysis.
- In the Solow-Swan case, $\frac{s}{v} = g_n$. $\frac{s}{v}$ adapts to $g_n \implies$ with a given g_n, if $'s'$ rises, $'v'$ rises so that $\frac{s}{v}$ remains equal to g_n.
- For Harrod an increase is $'s'$ with a given $'v'$ increases the warranted rate g_w.
- If g_w rises above g_n the economy would suffer from a chronic tendency to unemployment

4

Optimality and Growth Theory

4.1   *Growth, Saving and the "Golden Rule"*

- What is the impact of different saving ratios on the level of consumption in a growing economy?
- Consider this in the context of the Solow-Swan model: how is the steady state consumption per head affected by changing the saving ratio?
- The $'k^*'$ and $'s'$ associated with the highest steady state consumption per head known as the *golden-rule* case.
- Steady state consumption per worker can be expressed as

$$f(k^*) - sf(k^*) = c^* = f(k^*) - g_n k^* \tag{4.1}$$

- To find the maximum c^* set

$$\frac{dc^*}{dk^*} = 0 \implies \frac{dc^*}{dk^*} = f'(k^*) - g_n = 0 \tag{4.2}$$

and thus

$$MP_k = g_n \tag{4.3}$$

which is **"golden rule"**

- Thus consumption per worker and hence (with a fixed ratio of L to N) consumption per capita is maximised where the marginal product of capital is equal to the growth rate (g_n in the Solow-Swan model)
- In the Solow-Swan model

$$k^* = \frac{sf(k^*)}{g_n} \tag{4.4}$$

- $\implies$ if g_n is fixed and the production function is unchanging, k^* is determined by $'s'$
- $\implies$ $'s'$ has to "adjust" to generate a k^* consistent with $MP_K = g_n$
- This golden rule also implies a connection between distribution and growth
- Assuming $MP_K = r$, this rule implies $r = g_n$
- Different values of $'s'$ imply different k^*'s and MP_K's and hence different r's, for a given technology and given g_n
- One of these $'s'$ will imply $r = g_n$
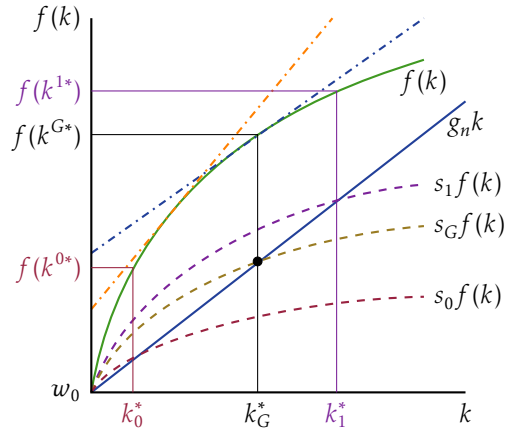
- With $s_0 \implies k^{0^*}$ and $MP_K > g_n$
- $\implies$ for a small $\uparrow$ in k, the $\uparrow$ in y exceeds the $\uparrow$ in saving and investment per worker required for the new higher $k \implies$ allows for an $\uparrow c^*$
- But higher k^* requires, ceterius paribus, a higher s
- With $s_1 = k^{l^*}$ and $MP_K < g_n$
- $\implies$ small $\downarrow$ in $k \implies \downarrow$ in $y <\downarrow$ in required saving and investment per worker $\implies \uparrow c^*$
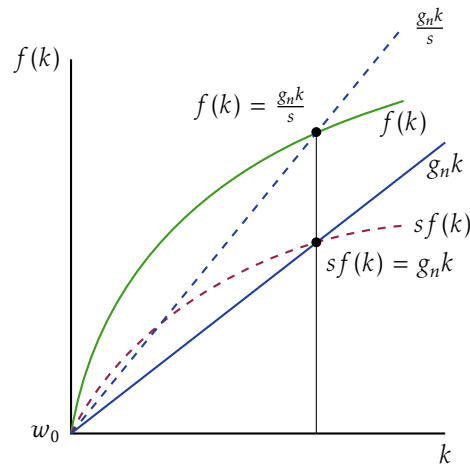
- At $k^{0^*} MP_K > g_n$
- $\implies$ for a small Δk^*

$$\frac{\Delta y}{\Delta k^*} = MP_K$$

- But higher k^* requires higher saving and investment per worker - call it $s\Delta y$ to keep it constant
- For a small Δk^* the required change in saving and investment per worker is giver by

$$\frac{\Delta(sy)}{\Delta k^*}$$

Since $\frac{Y}{L} = \frac{C}{L} + \frac{S}{L}$
- At k_0^* a small increase in k^* increases $\frac{Y}{L}$ more than it does required $\frac{S}{L} \implies \frac{C}{L}$ can increase

- The range of $k^* > k^{G^*}$ sometimes referred to as region of "dynamic inefficiency"
- Moving to a lower steady state k would unambiguously allow for higher steady state consumption per head
- Seen to reflect "over-saving" and "over-accumulation"
- $k^* < k^{G^*}$ by contrast represent the area of "dynamic efficiency": an increase in s results in higher steady state c, though not at all points in the transition
- Wider significance: if r and i are related dynamic efficiency implies a relation between i and g.
- $\implies$ implications about fiscal sustainability

4.2 Competitive Outcomes, Optimal Growth and Dynamic Efficiency

- Consider the two-commodity "iron-corn" model, with 3 available techniques differing in the method used to produce commodity 2
- e.g. for the α technique, the price system is

$$(a_{cc}p_c + a_{ic}p_{21})(1+r) + w_c l_c = 1$$
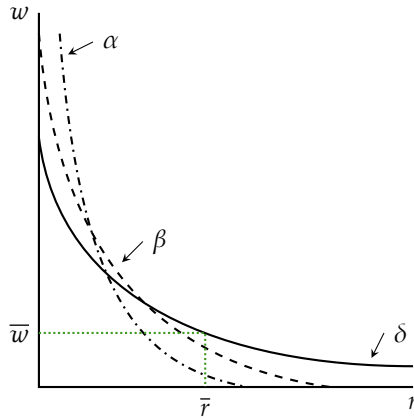$$(a_{ci}^\alpha p_c + a_{ii}^\alpha p_{21})(1+r) + w_c l_i = p_{ic}$$

- The real wage in terms of corn is given by

$$w_c^\alpha = \frac{(a_{cc}a_{ii} - a_{ci}a_{ic})(1+r)^2 - (a_{cc} + a_{ii})(1+g) + 1}{(l_i a_{ic} - l_c a_{ii})(1+g) + l_c}$$

- For β and δ techniques, price systems are

$$(a_{cc}p_c + a_{ic}p_{21})(1+r) + w_c l_c = 1 \qquad (a_{cc}p_c + a_{ic}p_{21})(1+r) + w_c l_c = 1$$
$$(a_{ci}^\beta p_c + a_{ii}^\beta p_{21})(1+r) + w_c l_i = p_{ic} \qquad (a_{ci}^\delta p_c + a_{ii}^\delta p_{21})(1+r) + w_c l_i = p_{ic}$$

- For any given w_c, presumably technique yielding highest rate of profit. Alternatively, for any given r...?

- Note: adjacent techniques at a switch point will in general differ in the method of producing **only one** of the commodities
- $\implies$ in the 2-commodity case, at the switch point there are three unknowns between the two price systems – p_{21}, w_1 and r

- At $r = \overline{r_1}$

$$(a_{11}^{\beta} + a_{21}^{\beta} p_{21})(1 + r) + w_1 l_1 = 1$$
$$(a_{12}^{\beta} + a_{22}^{\beta} p_{21})(1 + r) + w_1 l_2 = p_{21}$$
$$(a_{11}^{\delta} + a_{21}^{\delta} p_{21})(1 + r) + w_1 l_1 = 1$$
$$(a_{12}^{\delta} + a_{22}^{\delta} p_{21})(1 + r) + w_1 l_2 = p_{21}$$

- $\implies$ require three equations from the two systems $\implies$ only one equation can be different between the two systems
- Applying this choice of technique analysis to basic commodities, a lower price for one commodity under one method $\implies$ a lower price for all commodities using that method
- $\implies$ ranking of techniques according to profitability as w falls and r rises is the same regardless of the numeraire
- Moreover, the ranking of techniques is independent of the technique in which prices are expressed
- e.g. at r^* the relative cheapness of techniques δ and β should be the same whether costs of production are compared using p^{δ} or p^{β}
- $\implies$ at r^* using either the real wage w^{β} or w^{δ} to calculate prices for each technique would show that technique δ yields lower prices than technique β
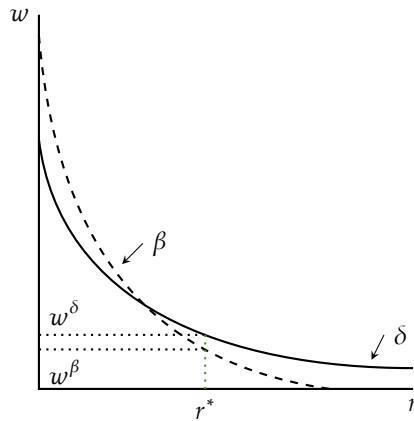
- $\implies$ at r^* using either the real wage w^{β} or w^{δ} to calculate prices for each technique would show that technique δ yields lower prices than technique β (see for example, Garegnani, 1970, p. 411n)
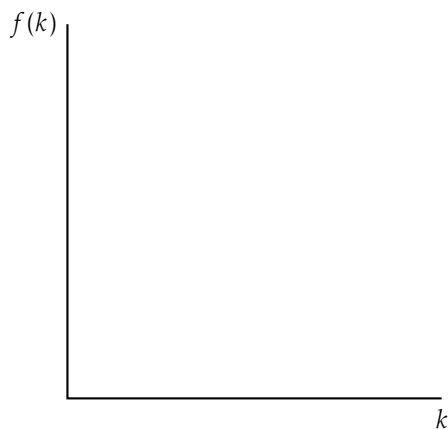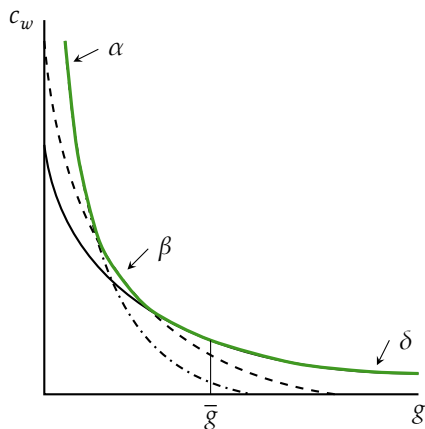
- Suggests the technique which would dominate in a long-period equilibrium, for a given rate of profit, generates the highest real wage
- $\implies$ *dominant technique will be the technique generating the highest rate of profit at the given real wage or the highest real wage at the given rate of profit*
- Suggests that the relevant portion of the set of w-r curves (representing available techniques) is the outermost envelope of this set
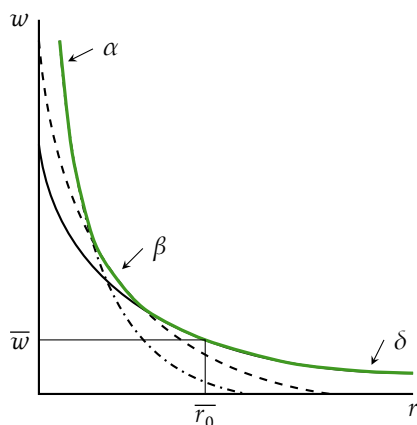
- Recall the discussion of duality from section 1: for every technique yielding a $w - r$ relation there is a corresponding c_{w-g} relation
- $\implies$ technological frontier of income distribution $(w - r)$ possibilities should correspond to the technological frontier of consumption-growth possibilities
- Recall also that for a given $g = \bar{g}$ the corresponding point on the frontier indicate the technique which maximises c_w for that growth rate

In the golden-rule case ($r = g$) technique which maximises c_w is also cost-minimizing

- At the rate of profit, r, under competitive conditions, technique δ would dominate
- However with the growth rate given at $\bar{g}$ the technique which maximises consumption per worker (and consumption per head) is β
- $\implies$ "optimal" technique need not correspond with the cost-minimizing technique under competitive conditions
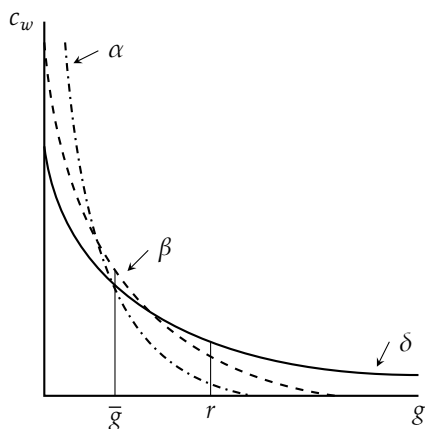
5

Growth and Issues in Capital Theory

5.1 *Distribution, Input Proportions and the Choice of Technique*

- Challenge to orthodox growth theory (Solow-Swan) in the 1950's and 1960's
- Based to a significant extent on criticism of orthodox capital theory $\implies$
  - (i) attack on the notion of an aggregate production function
  - (ii) attack on orthodox view of the relation between relative factor prices and factor proportions
  - (iii) attack on marginal productivity theory as a basis for the theory of distribution
- In general, in a competitive capitalist economy the technique in use is lowest cost
- Cost of production of a particular technique depends on relative prices
- Relative prices depend on income distribution
- Do techniques as well as factor proportions change in an orderly way as the distribution of income - i.e w and r-changes?
- Criticism between the mid-1950's and early 1970's suggests this cannot be assumed for a multi-commodity economy
- Significant implications for traditional (and arguably more recent) orthodox growth theory
- Consider a simplified corn-machine model, with only labour and a machine used as inputs

$$a_c p_m (1 + \delta) + w_m l_1 = p_c$$
$$a_m p_m (1 + \delta) + w_m l_2 = p_m \tag{5.1}$$

- With corn as numeraire and assuming everlasting machines

$$a_c p_{mc} + w l_c = 1$$
$$a_m p_{mc} + w l_m = p_{mc} \tag{5.2}$$

- where
$$w = \frac{1 - a_m r}{l_c + r(1_m a_c - l_c a_m)} \tag{5.3}$$

and
$$\frac{dw}{dr} = -\frac{a_c l_m}{[l_c + r(l_m a_c - l_c a_m)]^2} < 0 \tag{5.4}$$

and
$$p_{mc} = \frac{l_m}{l_c + r(l_m a_c - l_c a_m)} \tag{5.5}$$

so that
$$\frac{dp_{mc}}{dr} = \frac{1_m(l_c a_m - l_m a_c)}{[l_c + r(l_m a_c - l_c a_m)]^2} \tag{5.6}$$

- It follows from equation (5.6) that

$$\frac{dp_{mc}}{dr} \gtreqless 0 \quad \text{where} \quad l_c a_m - l_m a_c \gtreqless 0 \tag{5.7}$$

- and therefore

$$\frac{dp_{mc}}{dr} \gtreqless 0 \quad \text{where} \quad \left(\frac{a_m}{l_m}\right) \gtreqless \left(\frac{a_c}{l_c}\right) \tag{5.8}$$

- For the quantity system, with a constant growth rate, g, and writing all quantities as ratios to total labour employed in the economy

$$l_m(g + \delta)M_L + l_c c = 1$$
$$a_m(g + \delta)M_L + a_c c = M_L \tag{5.9}$$

- where M_L is the stock of machines (per unit of labour employed in the economy) in place at the end of a period and $(g + \delta)M_L$ is the gross investment demand for machines per unit of labour
- $\implies (g+\delta)M_L$ is output of new machines per unit of aggregate labour employment, call it Y_m/L
- 'c' $\implies$ output of corn per unit of aggregate labour employment, call it Y_c/L
- Decomposing equation (5.9):

$$l_m \frac{Y_m}{L} + l_c \frac{Y_c}{L} = 1$$
$$a_m \frac{Y_m}{L} + a_c \frac{Y_c}{L} = M_L \tag{5.10}$$

$$\frac{L_m}{Y_m}\frac{Y_m}{L} + \frac{L_c}{Y_c}\frac{Y_c}{L} = 1 = \frac{L_m + L_c}{L}$$
$$\frac{M_m}{Y_m}\frac{Y_m}{L} + \frac{M_c}{Y_c}\frac{Y_c}{L} = M_L = \frac{M_m + M_c}{L} \tag{5.11}$$

6

Modern Marginalist Growth Theory - "Endogenous" Growth Models

6.1   Technical progress in the Solow-Swan model

- Simplest case: labour-augmenting ("Harrod-neutral") technical progress.
- $\implies$ same machines, but more productive workers from one period to the next
- "Effective" labour force E grows at the rate $g_n + \lambda$
- Production function is $Y = (F, E)$
- Relevant capital to labour ratio is now $k_g = \frac{K}{E}$ with $y_e = f(k_e)$ representing output per "efficiency unit" of labour
- Steady state growth consistent with full-employment requires a constant k_e and y_e
- In this situation the capital stock per worker is expanding at the same rate as labour productivity, so capital per efficiency unit of labour (i.e. k_e) is constant
- $\implies$ marginal productivity of capital is constant
- With constant k_e, y_e is constant
- Maintenance of full-employment requries $g_L = g_n$
- But for employment to grow at this rate output must grow at a rate equal to $g_n + \lambda$, since the quantity of labour required to produce a unit of output is falling by $\lambda\%$ each year
- Hence in the steady state:

$$\frac{SY}{K} = \frac{I}{K} = g_n + \lambda \implies s\frac{\frac{Y}{E}}{\frac{K}{E}} = g_n + \lambda \implies \boxed{f(k) = \frac{(g_n + \lambda)k_e}{s}} \qquad (6.1)$$

- $\implies$ growth in steady state output per worker and capital per worker equal to the exogenous rate of growth of labour productivity
- Exogenous technical progress can "overcome" diminishing returns and generate long-run growth in output per head
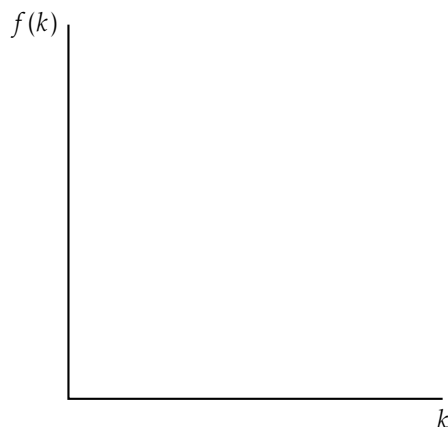
6.2   *Marginalist growth theory and "convergence"*

- Solow-Swan model suggests that for the same saving propensity, population growth and technology, growth rate of poor countries exceeds that of rich countries in the transition to the steady state

$$g = \frac{I}{K} = \frac{SY}{K} = s\frac{\frac{Y}{L}}{\frac{K}{L}}$$

$$g = \frac{sf(k)}{k} = \frac{sy}{k} \qquad (6.2)$$

- S-S model also suggests however that poorer countries need not have a higher transition g if their 's' is lower (e.g. sP)
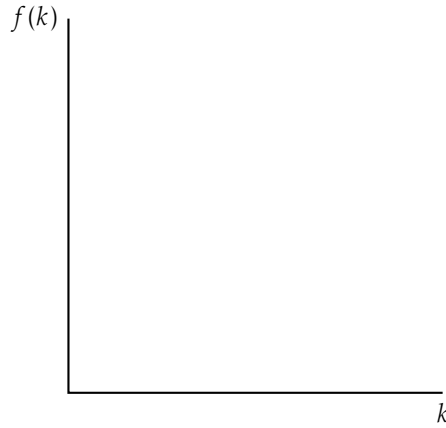
- More generally, Solow-Swan model would suggest that if technology is shared, differences in steady state growth rates should reflect only differences in g_n
- $\implies$ "conditional convergence hypothesis"
- Suggests also that 's' does not influence steady state growth rate of output per head
- Some have argued that evidence does not support the latter suggestion - viz., s appears to influence growth in output per head
- $\implies$ development of "endogenous" growth model, where growth is not governed by an "endogenous" changes in technology

6.3   *Endogenous growth theory*

- Representative agent is assumed to maximise the following intertemporal utility function

$$\int_0^\infty e^{-\rho t} u(c_t)\, dt = \int_0^\infty e^{-\rho t} \frac{1}{1-\sigma}(c^{1-\sigma} - 1)\, dt \qquad (6.3)$$

such that

$$y_t = f(k) = c_t + \Delta k_t + \delta k_t \qquad (6.4)$$

- The optimum consumption plan will entail that capital per worker and consump-

tion per worker both growth at the rate given by

$$\frac{\Delta k_t}{k_t} = \frac{\Delta c_t}{c_t} = (\frac{1}{\sigma}f'(k) - \rho - \delta)$$ (6.5)

$$\implies \rho + \left(\rho\frac{\Delta c_t}{c_t}\right) = f'(k) - \delta$$ (6.6)

- What is the meaning of equation (6.6)?
- $\sigma \implies$ elasticity of marginal utility with respect to a given proportionate increase in per capita consumption
- $\implies \sigma\frac{\Delta c_t}{c_t}$ shows the decrease in marginal utility as consumption per head grows from one period to the next
- $\rho + \left(\rho\frac{\Delta c_t}{c_t}\right)$ represents the 'cost' of deferring consumption from the present into the future
- Hence $\rho + \left(\rho\frac{\Delta c_t}{c_t}\right) = f'(k) - \delta \implies$ cost of deferring consumption is equal to the gain from additional saving
- With zero population growth, the question is why there would be steady state growth in output and the capital stock?
- Consider the "AK" model: linear production function

$$Y = f(k) = AK$$ (6.7)

$$MP_K = f'(k) = A \quad r = A - \delta$$ (6.8)

- Substituting for $f'(k)$ in equations (6.5) and (6.6) yields for the steady state growth rate

$$g = \frac{1}{\sigma}(A - \rho - \delta) \quad \therefore g = \frac{r - \rho}{\sigma}$$ (6.9)

- Given the elasticity of substitution between present and future consumption (and hence σ), the rate of accumulation is governed by the rate of profit compared with the discount rate
- The endogenous case of saving with the technology of the AK model (assuming $\delta = 0$)

$$s = \frac{I}{Y} = \frac{\Delta K}{K}\frac{K}{Y} = \frac{\Delta k}{k}\frac{K}{Y} = \frac{1}{\sigma}\frac{1}{A}(A - \rho)$$ (6.10)

so that

$$g = \frac{1}{\sigma}(A - \rho) = sA = sr$$ (6.11)

- With s and r determined independently of g, then both g and hence the **growth rate of output per head are "endogenous"**
- With the saving ratio set exogenously

$$\frac{I}{K} = s\frac{Y}{K} = s(A) = sr$$ (6.12)

6.4   *Endogenous growth with human capital: Lucas*

- Emphasis on positive externalities associated with the accumulation of human capital
- In the absence of such externalities the production function is

$$Y = AK^\beta[uhL]^{1-\beta}$$ (6.13)

- With the positive "spin-off" associated with accumulating human capital the production function becomes

$$Y = AK^{\beta}[uhL]^{1-\beta}h^{*\gamma} \tag{6.14}$$

- with the average "quality" of labour 'h' growing according to the expression:

$$\frac{\Delta h}{h} = v(1-u) \tag{6.15}$$

- $1 - u \implies$ % of non-lesiure time in education
- With the intertemporally optimising representative agent, steady state growth in output per worker is given by (Sala-i-Martin, 1990)

$$g = \frac{(v-\rho)(1-\beta)}{\sigma(1+\gamma-\beta)} = \frac{\Delta c}{c} = \frac{\Delta k}{k} \tag{6.16}$$

- Note, in the absence of the externatlity (i.e. $\gamma = 0$) equation (6.16) simplifies to

$$g = \frac{v-\rho}{\sigma} \tag{6.17}$$

- Note the similarity with equation (6.9) for the AK model
- The steady state growth rate of human capital depends on the nature of the externality, specifically, on the coefficient γ so that

$$\frac{\Delta h}{h} = \frac{\Delta k}{k}\frac{(1-\beta)}{(1+\gamma-\beta)} \tag{6.18}$$

- If $\gamma = 0$, $\frac{\Delta h}{h} = \frac{\Delta k}{k}$
- Endogenous growth models however retain a couple of troublesome features of earlier marginalist growth theory:
    - failure to consider effective demand, reflected in the assumed relation between S and I;
    - the use of aggregate production functions to represent multi-commodity systems

7

"Cambridge" growth theory and modern post-Keynesian/Kaleckian Models

7.1 *Cambridge growth theory*

- Early Cambridge/post-Keynesian analysis (1950's – 60's):
  - heavily influenced by Keynes and Kalecki: accept a demand-driven view of output
  - rejection of marginal productivity theory of distribution
  - dissatisfaction with the uniqueness of Harrod's warranted growth path (though not the notion that g_w need not to coincide with g^n)
- Along the warranted path producers accumulate capital at the rate $\frac{s}{v}$
- If their actual rate differs from $\frac{s}{v}$ $\implies$ instability in Harrod's analysis
- For early Cambridge theorists
  - 's' depended on distribution between wages and profits;
  - distribution was influenced by the rate of growth, in particular 'r' was influenced by 'g'
- Hence if $\frac{I}{K} > \frac{s}{v}$ the higher growth rate could become the warranted path if 'r' increased and in turn 's'
- $\implies$ g_w is no longer unique
- Cambridge theorists arrived at the so-called Cambridge equation

$$g = s_c r \tag{7.1}$$

- *Causation is seen to run from left to right – 'g' is exogenous and determines 'r'*

7.2 *Modern Post-Keynesian/Kaleckian Models*

- Modern Kaleckian theorists take 'g' as endogenous, by combining the Cambridge growth equation (7.1) with an explanation of the rate of accumulation
- Consider a stylised one-sector version of modern post-Keynesian/Kaleckian "growth" models

$$p = wl\varphi \quad \text{with} \quad \varphi > 1 \tag{7.2}$$

- Profit share π is given by

$$\pi = \frac{p - wl}{p} = 1 - \frac{w}{p}l = \frac{\varphi - 1}{\varphi} \tag{7.3}$$

- Profit rate r is given by

$$r = \left(\frac{p - wl}{p}\right)\frac{Y}{K} = \left(\frac{\varphi - 1}{\varphi}\right)u = \pi u \tag{7.4}$$

- The rate of accumulation which would absorb exactly the flow of savings is

$$g^s - s_c r \left(= \frac{S}{\pi} \frac{\Pi}{K} \right) \tag{7.5}$$

- On the basis of an investment function like

$$I_t = A + \alpha \Pi_t + \beta Y_t - \gamma K_t \tag{7.6}$$

- one can write (dividing throgh by K) a "desired accumulation" function in the form

$$g^i = f_0 + f_1 r + f_2 u_t \tag{7.7}$$

- For equilbrium growth $g^s = g^i$

$$\implies \quad \frac{S}{K} = \frac{I}{K} \quad \text{and therefore} \quad s_c r = f_0 + f_1 r + f_2 u_t \tag{7.8}$$
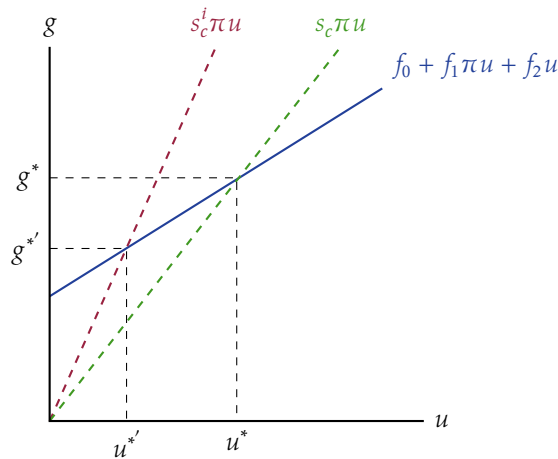
- Substituting πu for r in equation (7.7) gives

$$s_c \pi u = f_0 + f_1 \pi u + f_2 u \tag{7.9}$$

- and therefore steady-state u is given by

$$u^* = \frac{f_0}{(s_c - f_1)\pi - f_2} \tag{7.10}$$

- Stability of steady state growth requires saving to react to changes in utilisation more than investment
- $\uparrow s_c$ leads to a fall in g^*, u^* and $r^* \implies$ "paradox of thrift"

- In this model a (assuming a stable g^*) shift in distribution towards profit reduces utilisation, the profit rate and g^* - referred to as the "stagnationist" case
- This case has non-orthodox implications about the relation between the real wage and employment. From equation (7.10)

$$\frac{\mathrm{d}u^*}{\mathrm{d}\pi} = \frac{-(s_c - f_1)f_0}{[(s_c - f_1)\pi - f_2]^2} \tag{7.11}$$

- Since, for stability, $f_1, \pi + f_2 \leqslant s_c\pi$, and thus $\frac{f_2}{\pi} < (s_c - f_1)$, then, with f_2 and $\pi > 0$, $(s_c - f_1) > 0$ and thus $\frac{du^*}{d\pi} < 0$
- In turn, since

$$I = \frac{L}{Y}\frac{Y}{K}K = l \cdot u \cdot K \qquad (7.12)$$

- $\implies$ L per unit of K rises with utilisation, and thus with a fall in π and thus with a **rise** in the real wage $\frac{w}{P}$
- This case also suggests a positive relation between the steady state rate of profit and the real wage
- i.e. a fall in the profit share – rise in the real wage – increases aggregate demand, utilisation and accumulation
- The increase in utilisation outweighs effect of a fall in π on the rate of profit, r, so that r rises
- Hence rate of return to capital and growth are stimulated by a shift in distribution towards wages
- Argument by later theorists in the post-Keynesian Kaleckian tradition that the model above exaggerated the effects of changes in utilization rates (u) on investment demand
- A more general alternative suggested for the desired accumulation function was

$$g^i = h(\pi, u) \quad h_\pi, h_u > 0 \qquad (7.13)$$

- $\implies$ an increase in profit share, for a given utilisation and an increase in utilisation for a given profit share would both increase the desired rate of accumulation
- Steady state utilisation rate is the solution to

$$s_c\pi u = h(\pi, u) \quad \text{with} \quad \frac{du}{d\pi} = \frac{-(s_c u - h_\pi)}{s_c\pi - h_u} \qquad (7.14)$$

- Stability of the steady state requires

$$s_c\pi - h_u > 0$$

- Whether 'u^*' rises or falls with a rise in the profit share turns on whether $s_c u - h_\pi$ is < 0 or > 0
- If $s_c u - h_\pi > 0$, $\frac{du}{d\pi} < 0 \implies$ "stagnationist" case
- Although a rise in the profit share, π, has a positive effect, *ceteris paribus*, on the desired rate of accumulation (and potentially on u), there is a greater negative effect on utilisation (via the fall in the real wage depressing consumption demand)
- $\implies$ negative impact on the steady state rate of accumulation and on the steady state rate of profit
- Alternative possibility, referred to in the literature as the "exhilarationist" case
- Suppose that $s_c\pi - h_u < 0$, then $\frac{du}{d\pi} > 0$
- $\implies$ a redistribution of income towards profits has a net stimulatory effect on aggregate demand
- $\implies$ stimulus to utilisation via the positive effect of a rise in π on capitalists' consumption and accumulation outweighs negative impact on utilisation from a falling real wage reducing workers' consumption
- The steady state rate of accumulation and the steady state rate of profit both rise
- These different cases entail two different relations between steady state distribution and the steady state

- The stagnationist case: a rise in g^*, a fall in the π and **a rise in the real wage**, steady state utilisation **and rate of profit**
- The exhilarationist case: a rise in g^*, a rise in π, **a fall in the real wage, a rise in the rate of profit** and a rise in steady state utilisation
- The "stagnationist" case may also be referred to as "wage-led" growth while the "exhilarationist" case may be referred to as "profit-led" growth
- As Blecker (2003) notes, there are intermediate cases between the stagnationist and exhilarationist possibilities

8
Growth and Policy

8.1 *Concluding Remarks*

- What, if any, policy implications can be drawn from the discussion in previous sections.
- Have noted two "grand traditions" in growth theory
- Generate different perspectives on policy
- In relation to marginalist growth theory (Solow-Swan, endogenous growth models (EGT)) - ??
- In Solow-Swan: assuming a higher growth rate is desirable $\implies$ increasing the growth rate of the labour force and/or rate of technical progress
- Both are exogenous; though policy makers could make use of labour market policies which increase gn and policies to stimulate productivity?
- Note: higher long-run growth rate is not to prevent rising unemployment – this is not a long-run problem with flexible factor prices and competitive factor markets
- $\implies$ possible role for micro-based policy?
- Higher steady state growth may be desired for long-run increases in income per capita
- If the object is to maximise consumption per head $\implies$ a role for saving in the Solow-Swan model
- $\implies$ a role for policy designed to influence saving (e.g. pensions, superannuation) depending on initial situation (i.e. economy is initially dynamically efficient or inefficient)
- However in EGT, in contrast to Solow-Swan, saving has a role in governing the long-run growth rate
- EGT suggests also a role for policy in enhancing growth in human capital (education policy)
- Also potentially a role for industry/innovation policy?
- So what of non-orthodox growth theory – Harrod-Domar, post-Keynesian/Kaleckian models?
- More immediate implication of modern post-Keynesian/Kaleckian models of a role for distribution
- But how? Effect on steady state growth of shifting distribution between wages and profits turns on parameters
- i.e. depends whether growth is wage or profit-led
- More generally with both old and new non-orthodox models – growth is demand-driven!
- And there is no presumption of a mechanism bringing g^* into line with g^n
- $\implies$ a role for policy in adjusting growth to deal with unemployment
- **AND** (in light of the capital-theoretic criticisms of orthodox theory) regardless of the competitiveness of factor markets, esp. the labour market
- $\implies$ a role potentially for policy in manipulating the rate of growth of aggregate demand

- ⟹ completely contrasting to marginalist growth theory which would not see demand as a constraint on long-run growth

8.2   *Demand-Led Theory of Growth – the "supermultiplier approach"*

- A relatively recent view of growth as a demand-led process is the so-called "super-multiplier" approach
- Consists of extending the Keynesian short-run theory of output to a long-run theory of output growth. In the long run the growth of productive capacity adjusts to the requirements of demand growth
- In this theory the growth of output, employment and capital depend on the growth in aggregate demand where the growth path is not characterised by the full-employment of labour
- Capacity adjustment is incorporated by way of the accelerator theory of investment
- From the basic Keynesian income-expenditure model

$$AE = A + (c - ct - m)Y + I \tag{8.1}$$

- Induced investment is given by:

$$I_t = v(Y_{t+1}^e - Y_t) \tag{8.2}$$

- In turn

$$AE = A + (c - ct - m)Y + v(Y_{t+1}^e - Y_t) \tag{8.3}$$

In equilibrium, $Y = AE$ so that

$$Y_t = A_t + c^d Y_t + v(Y_{t+1}^e - Y_t) \tag{8.4}$$

where $c^d = (c - ct - m)$ is the propensity to spend on domestically produced goods.
- The expected growth rate is given by

$$g_{t+1}^e = \frac{(Y_{t+1}^e - Y_t)}{Y_t} \tag{8.5}$$

so that equation (8.4) can be rewritten as

$$Y_t = A_t + c^d Y_t + v g_{t+1}^e Y_t \tag{8.6}$$

- In turn

$$Y_t = A_t \frac{1}{1 - c^d - v g_{t+1}^e} \tag{8.7}$$

- $\frac{1}{1-c^d-vg_{t+1}^e}$ represents the "supermultiplier"
- i.e. the multiplier effect of a change in autonomous demand 'A'is not just induced consumption deman (via 'c^d') but also induced investment demand (via 'vg^e')
- Equation (8.7) implies that with a given supermultiplier, the rate of growth of output over time reflects the rate of growth of autonomous demand
- This approach is in the spirit of Keynes and later post-Keynesian and Kaleckian approaches in suggesting that there is no mechanism to guarantee that the growth of autonomous demand will be sufficient for the maintenance of full-employment
- Apart from consumption proprensities and technology (i.e. 'v') the supermultiplier itself depends on expectations about the future rate of growth

- $\implies$ implications of the supermultiplier approach depend in part on the theoretical approach regarding the formation of expectations
- BUT, the demand-led growth thesis raises an additional complex theoretical question (not entirely without practical policy interest): what is the autonomous element(s) of demand which drives growth?
- If output and output capacity are ultimately dictated by demand, there must be an independent or autonomous component?
- i.e. independent of current and anticipated growth rate of income
- Not easy to conceive of expenditures autonomous in this sense for **all time**
- Problems with the concept of autonomous investment and to a lesser extent with consumption
- Government expenditure also less of a problem, but arguably subject to at least a long-run intertemporal government budget constraint
- And seen as having a one-way relation with public debt and running a sustainable fiscal policy
- But a demand-led approach suggests this is an over-simplified view
- Consider a simplified version of the standard fiscal sustainability condition:

$$\frac{S}{PY} \geqslant -g\frac{B}{PY} \qquad \frac{S^p}{PY} \geqslant (i-g)\frac{B}{PY}$$

- Latter condition shows how big the primary surplus as a % of GDP must be to prevent debt rising faster than nominal GDP
- $\implies$ if $i > g$, $\implies$ fiscal sustainability requires a primary budget surplus
- From an orthodox standpoint – specifically, Solow-Swan – 'g' is exogenous
- If 'i' is governed by the rate of profit then in a Solow-Swan world, 'i' is governed by 's', '$f(k)$' and 'g'
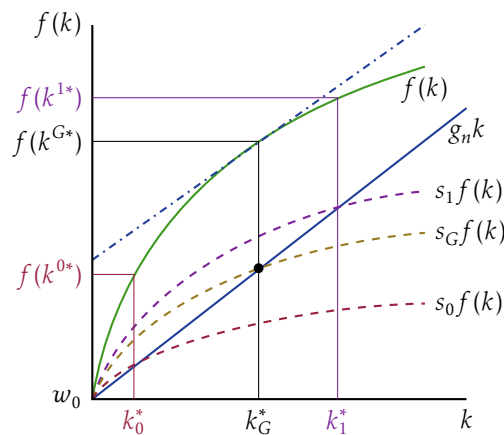
- $\implies$ if 's' is manipulated to keep the economy in the region of "dynamic efficiency" so that $(i \geqslant g)$ primary surplus will be required for fiscal sustainability (defined as a stable debt to income ratio)
- From a non-orthodox, demand-led perspective things are more complex
- 'g' is not independent of the growth rate of govt. expenditure
- $\implies$ although the primary deficit is ceteris paribus positively influenced by the govt. expenditures, so is the growth rate of nominal GDP

- $\implies$ a rise in the rate of growth of govt. expenditure will ceteris paribus act negatively on 's^P' but also positively on 'g' relative to 'i' (assuming the economy is not too close to full-employment)
- Moreover, non-orthodox theorists would reject the implication of the Solow-Swan model that a change in 's' would influence 'i' while either boosting 'g' or leaving it unchanged
- $\implies$ from non-orthodox, demand-led perspective, 'i' is not governed by demand and supply of saving; but a rise in saving could depress the rate of growth of effective demand and in turn the rate of growth of output