ECOS3010

Monetary Economics – Notes

Brendan Chow

Student at the University of Sydney
Website
LinkedIn
GitHub

brendanchow@xagill.com

Based on the lecture notes by A/Prof Stella Huangfu

Contents

1
A Simple Model of Money

1.1 *Introduction*

- Money has always been important to people and to the economy.
- Money has a long history.
  - Commodity money: shells, beads, cigarettes, silver, gold,...
  - Fiat money: paper currency → intrinsically useless
  - Emoney: debit card, smart card, ecash,...
- Are we headed for a cashless society?
- Why do we need money?
  - Lack of double coincidence of wants.
  - Lack of record-keeping or credit: money is MEMORY!
- Functions of money:
  - a medium of exchange: primary function;
  - a unit of account;
  - a store of value.
- A suitable framework to study issues related to money: the overlapping generations (OLG) model.
  - highly tractable;
  - an elegant way to introduce money;
  - dynamic.

1.2 *Enviroment of the Model*

- The economy begins in period 1 and runs forever: $t = 1, 2, \ldots, \infty$.
- Individuals live for two periods: *young* and *old*.
- In period t, N_t individuals are born. In the first period, N_0 initial old.
- In each period t, N_t young individuals and N_{t-1} old individuals.
- Each individual receives y units of goods when young and nothing when old.
- Here is a summary of the model so far:

Generation	t=1	t=2	t=3	t=4	→
0	0				
1	y	0			
2		y	0		
3			y	0	
↓				…	…

- Preferences:
  - Individuals of all future generations value consumption both when young and when old.
  - Initial old value consumption only when old.
- Three assumptions about an individual's utility
  1. Utility is increasing with the consumption.
  2. Individuals value some consumption in both periods of life: the indifference curves never cross either axis.
  3. Diminished marginal rate of substitution

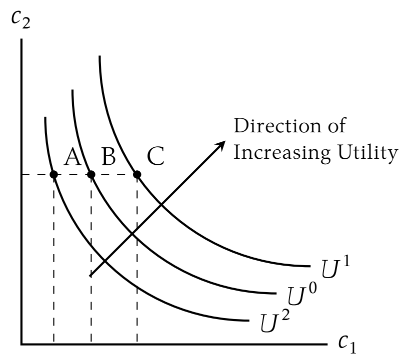


Figure 1.1. Increasing utility

1.3 *Centralised Solution: the Golden Rule Allocation*

- Suppose there is a central planner who can allocate the available goods among the young and the old in each period.
- Let $(c_{1,t}, c_{2,t+1})$ note the consumption bundle by individuals born in period t.
- Resource constraint in period t is

$$N_t c_{1,t} + N_{t-1} c_{2,t} \leqslant N_t y.$$

- Suppose for now that for all t,
  - the population is constant: $N_t = N$
  - we foucs on stationary allocations where $c_{1,t} = c_1$ and $c_{2,t} = c_2$.
- Resource constraint simplifies to

$$c_1 + c_2 \leqslant y.$$

- Graphically the feasible set is
- Within the feasible set, which allocation would the planner choose? The combination of (c_1, c_2) that maximizes an individual's utility.
- The golden rule allocation is the allocation within the feasible set that maximizes the utility of future generations. It occurs at the unique point of tangency between the feasible set line and an indifference curve
- Does the golden rule allocation maximize the utility of the initial old?
- Point A: the golden rule allocation; Point E: max utility of the initial old.

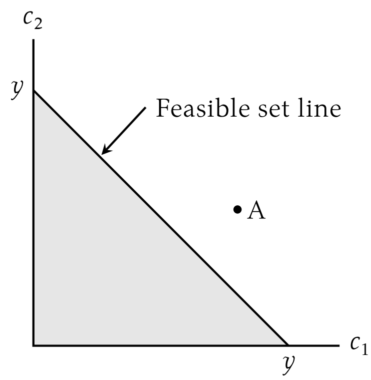


Figure 1.2. The feasible set

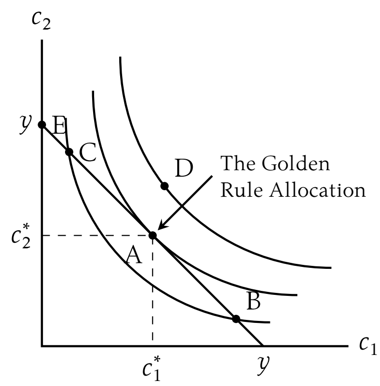


Figure 1.3. Golden Rule allocation

1.4 *Decentralised Solutions: a Competitive Equilibrium without Money*

- To achieve the golden rule allocation, the planner needs to redistribute c_2^* units of goods from each young to each old in every period.
- Strong assumptions about the power of central planners.
- When individuals trade among themselves → a competitive equilibrium.
  - Individuals maximise their own utilities.
  - Individuals are price takers.
  - Markets clear.
- No trade can occur in this economy → autarkic allocation: individuals have no economic interaction with others.
  - Lack of double coincidence of wants: the old would like to have some goods from the young, but they have nothing that the young want.
  - No record-keeping or credit.
- Each individual's consumption: $c_1 = y, c_2 = 0$. (Goods are non-storable!)
- Utility is low: both the future generations and the initial old are **worse off** than **almost** any other feasible consumption bundle.

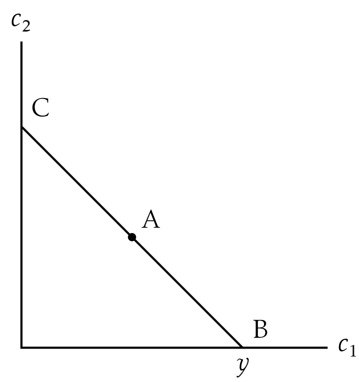


Figure 1.4. Decentralised solution

1.5 *Decentralised Solutions: a Monetary Equilibrium*

- How can the economy achieve a better allocation than the autarkic allocation?
- One way to allow some trading opportunities is to introduce **money**.
- Fiat money:
  - produced by the government (almost) costlessly;
  - cannot be counterfeited;
  - portable;
  - storable.
- **A monetary equilibrium** is a competitive equilibrium in which there is a **valued** supply of fiat money. that is, the fiat money can be traded for consumption good.

- For fiat money to have value, 2 conditions must be satisfied.
  - supply of money must be limited.
  - impossible (or very costly) to counterfeit.

Demand for Money

- There is a fixed stock of money: M units.
- Each of the initial old is endowed with M/N_0 units money.
- Are there potential trade opportunities?
  - At $t = 1$, the initial old have money and the young (newborn) have goods. Would they trade? Yes.
  - At $t = 2$, the old (who were young at $t = 1$) have money and the young (newborn) have goods. Would they trade? Yes.
  - At $t = 3, 4, \dots,$ the old in each period always have some money and the young always have goods.
  - Now each individual can consume in both periods of life.
- Consider an individual who is born at time t.
  - $c_{1,t}$: consumption when young;
  - $c_{2,t+1}$: consumption when old;
  - m_t: the number of dollars acquired when young (by giving up some of the endowed consumption good);
  - v_t: the value of money, which implies the price level $p_t = \frac{1}{v_t}$
- The individual's budget constraint in the first period of life

$$c_{1,t} + v_t m_t \leqslant y$$

- The individual's budget constraint in the second period of life

$$c_{2,t+1} \leqslant v_{t+1} m_t$$

- The individual's life-time budget constraint

$$c_{1,t} + \left(\frac{v_t}{v_{t+1}}\right) c_{2,t+1} \leqslant y.$$

  - $\left(\frac{v_t}{v_{t+1}}\right)$: the real return of money
- Graphically, we depict the budget set
- Within the budget set, point A maximises an individual's utility.

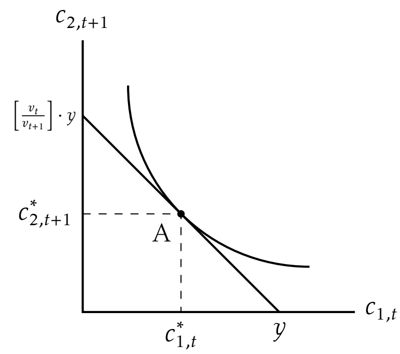


Figure 1.5. Demand for money

A Monetary Equilibrium

- It remains to find $\frac{v_{t+1}}{v_t}$. Recall that in any competitive market, the price (or value) of an object is determined as the price at which the supply of the object equals its demand.
  - demand for money at time t

$$N_t(y - c_{1,t});$$

  - supply of money at time t

$$v_t M_t;$$

  - v_t is determined through

$$N_t(y - c_{t,1}) = v_t M_t \to v_t = \frac{N_t(y - c_{1,t})}{M_t}$$

- From v_t and v_{t+1}, we can find

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y-c_1)}{M_{t+1}}}{\frac{N_t(y-c_1)}{M_t}} = \frac{\frac{N_{t+1}}{M_{t+1}}}{\frac{N_t}{M_t}}$$

- Let's further simplify our economy: suppose we focus on
  - stationary allocations where $c_{1,t} = c_1$ and $c_{2,t+1} = c_2$;
  - a constant population where $N_t = N$;
  - a constant money supply where $M_t = M$.

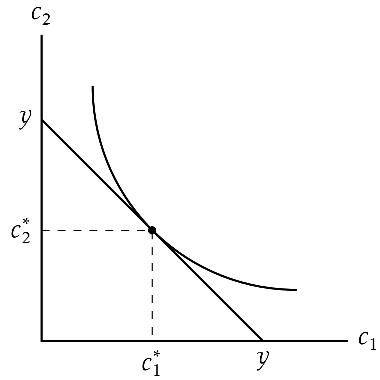
  for all t
- Now, we have

$$\frac{v_{t+1}}{v_t} = 1 \text{ or } v_{t+1} = v_t.$$

  The value of money is constant.
- Quantity theory of money: the price level is proportional to the quantity of money in the economy. In our economy, the price level is

$$p = \frac{1}{v} = \frac{M}{N(y-c_1)}.$$

- Neutrality of money: the nominal size (measured in dollars) of the stock of money M has no effect on the real (measured in goods) values of consumption (c_1, c_2) and real money demand $y - c_1$.



golden rule	monetary equilbrium
max utility subject to the *resource constraint*	max utility subject to the *budget constraint*
resource constraint: $c_1 + c_2 \leqslant y$	budget constraint: $c_1 + c_2 \leqslant y$
golden rule = monetary equilibrium	

- Compared to competitive equilibrium without money, the introduction of money allows all future generations to achieve the golden rule allocation. It also benefits the initial old, whose consumption increases from 0 to c_2^*.

1.6 *A Growing Economy*

- So far we have learned that the introduction of money opens up trade opportunities and monetary equilibrium coincides with the golden rule allocation. We have assumed **a constant money supply** and **a constant population**.
- Suppose that $N_t = nN_{t-1}$ where $n > 1$. How does a growing population affect the golden rule allocation and the monetary equilibrium?
- Golden rule allocation: the planner maximises an individual's utility subject to the resource constraint

$$N_t c_1 + N_{t-1} c_2 \leqslant N_t y \rightarrow c_1 + \frac{1}{n} c_2 \leqslant y$$

- A Monetary equilibrium: the individual maximises his own utility subject to the life-time budget constraint

$$c_1 + \left(\frac{v_t}{v_{t+1}}\right) c_2 \leqslant y.$$

  Recall that

$$v_t = \frac{N_t(y - c_{t,1})}{M} \rightarrow \frac{v_{t+1}}{v_t} = \frac{N_{t+1}}{N_t} = n$$

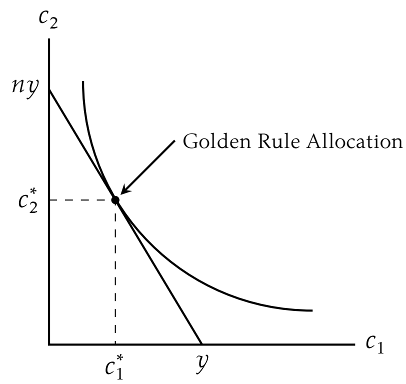


Figure 1.6. Golden rule allocation in a growing economy

When the population is growing, the value of money is also growing at the same speed. Therefore, the budget constraint simplifies to

$$c_1 + \frac{1}{n}c_2 \leqslant y.$$

- The budget constraint is identical to the resource constraint. So the monetary equilibrium coincides with the golden rule allocation. Again, the introduction of money helps the economy achieve the best possible allocation!

2
Inflation

2.1 *Measuring Money*

- The definition of money as anything that is generally accepted in payments for goods and services does not tell us how we should measure money.
- Which assets shall we include when we measure money? Each country's central bank provides precise definitions.
- In Australia, the Reserve Bank of Australia (RBA) is responsible for monetary policy.
- RBA's definition of monetary aggregates:
  - M0 or currency: notes and coins held by the private non-bank sector;
  - M1: currency + current deposits with banks;
  - M3: M1 + all other deposits at banks;
  - Broad money: M3 + other borrowings from private sector by AFIs.
- In general, currency < M1 < M3 < broad money.

2.2 *Introduction*

- In our simple model of money, money supply is constant. From data on monetary aggregates, money supply seems to grow over time.
- The supply of fiat money is usually controlled by the central bank. Printing new money is an important way to finance government spending needs.
- In this section, we will examine
  - the consequences of increasing money supply;
  - the link between government spending and inflation;
  - seigniorage: theory and evidence.

2.3 *Some Evidence*

- Money growth is the main determinant of inflation.
- A few examples of extraordinarily high inflation rates – hyperinflations:
  - Germany in 1923: inflation hits 3.25×10^6 percent per month $\rightarrow$ prices double every two days;

- Greece between 1941 and 1944: inflation hits 8.55×10^9 percent per month $\rightarrow$ prices double every 28 hours;
- Yugoslavia between Oct 1993 and Jan 1994: inflation hits 5×10^{15} percent per month $\rightarrow$ prices double every 16 hours;
- Hungary after the end of WWII: inflation peaks at 4.19×10^{16} percent per month $\rightarrow$ prices double every 15 hours;

- There could also be deflation. Examples:
  - United States from 1930 to 1933;
  - Hong Kong from late 1997 to 2004;
  - Japan in the early 1990s.
- To understand the causes and consequences of changing money growth rate, we will develop a theory.

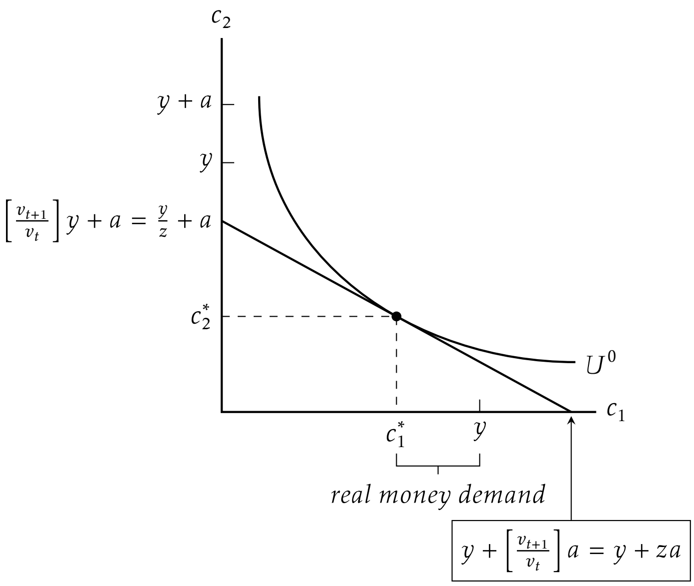
2.4 *A Growing Money Supply: New Money to the Public*

- Consider the OLG economy that we developed so far. Suppose that money supply grows at a rate z:
$$M_t = zM_{t-1}$$
- The amount of new money introduced into the economy in period t is:
$$M_t - M_{t-1} = M_t - \frac{M_t}{z} = \left(1 - \frac{1}{z}\right)M_t$$
- New money is introduced into the economy by means of *lump-sum* transfers to each **old** individual in every period t worth a_t units of consumption goods.
- To find the value of a_t in aggregate the government budget constraint is:
$$N_{t-1}a_t = \left(1 - \frac{1}{z}\right)v_tM_t$$
$$\Rightarrow a_t = \frac{\left(1 - \frac{1}{z}\right)v_tM_t}{N_{t-1}}$$

A Monetary Equilibrium

- Budget constraints:
  - first period of life:
$$c_{1,t} + v_tm_t \leqslant y;$$
  - second period of life:
$$c_{2,t+1} \leqslant v_{t+1}m_t + a_{t+1};$$
  - lifetime budget constraint
$$c_{1,t} + \frac{v_t}{v_{t+1}}c_{2,t+1} \leqslant y + \frac{v_t}{v_{t+1}}a_{t+1}.$$
- What is the value of $\frac{v_{t+1}}{v_t}$? As before, we first solve for v_t from money market clearing condition.
$$v_tM_t = N_t(y - c_{1,t}) \rightarrow v_t = \frac{N_t(y - c_{1,t})}{M_t}$$
- As usual, we focus on stationary allocations. **Assume for now that the population is constant.**

- The value of money is then
$$v_t = \frac{N(y-c_1)}{M_t}.$$
- It follows that
$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y-c_1)}{M_{t+1}}}{\frac{N_t(y-c_1)}{M_t}} = \frac{M_t}{M_{t+1}} = \frac{M_t}{zM_t} = \frac{1}{z}$$
- Furthermore, the price level
$$\frac{p_{t+1}}{p_t} = \frac{\frac{1}{v_{t+1}}}{\frac{1}{v_t}} = \frac{v_t}{v_{t+1}} = z.$$
- When money supply is growing at a rate z, the price level increases at a rate z. **Quantity Theory of Money!**
- An individual's budget constraint simplifies to
$$c_1 + zc_2 \leqslant y + za$$



- The solution $(c_1^* c_2^*)$ are functions of (z, y, a). To close the model, we need to find the value of a. Recall that from the government budget constraint
$$N_{t-1}a_t = \left(1 - \frac{1}{z}\right) v_t M_t.$$
a is solved from
$$a = \frac{\left(1-\frac{1}{z}\right) v_t M_t}{N} = \frac{\left(1-\frac{1}{z}\right)\frac{N(y-c_1)}{M_t} M_t}{N} = \left(1 - \frac{1}{z}\right)(y - c_1^*).$$
- In a monetary equilibrium, (c_1^*, c_2^*) maximises an individual utility subject to the lifetime budget constraint is **satisfied** in every period.
- An example: if $u(c_1, c_2) = c_1 c_2$, an individual
$$\max_{c_1, c_2} c_1 c_2 \quad \text{subject to } c_1 + zc_2 \leqslant y + za$$

We have

$$c_1 = \frac{y + za}{2} \text{ and } c_2 = \frac{y + za}{2z}$$

We can also find a by solving

$$a = \left(1 - \frac{1}{z}\right)\left(y - \frac{y + za}{2}\right) \rightarrow a = \frac{y\left(1 - \frac{1}{z}\right)}{1 + z}$$

Substituting a into (c_1, c_2), we have

$$c_1 = \frac{yz}{1 + z} \text{ and } c_2 = \frac{y}{1 + z}.$$

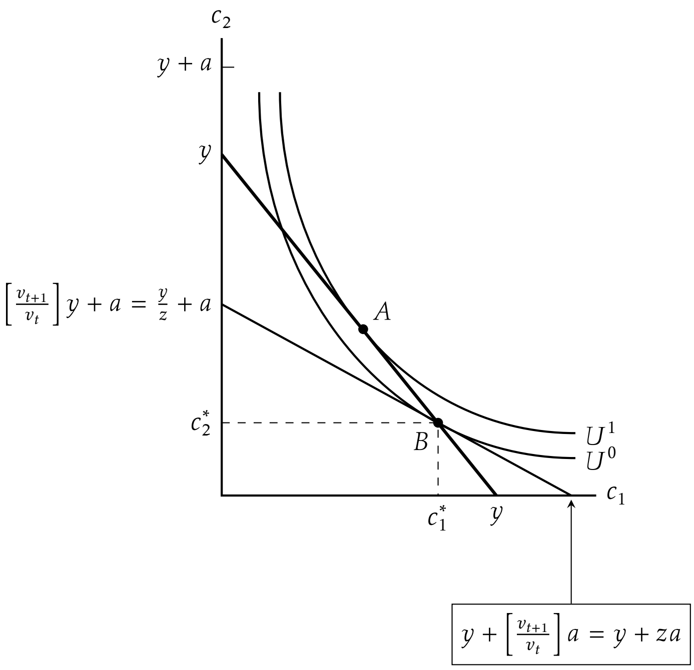
Is the Monetary Equilibrium Efficient?

- Consider the model with a constant population and a growing money supply, $M_t = zM_{t-1}$
- An individual's budget constraint

$$c_1 + zc_2 \leqslant y + za$$

- The golden allocation: a planner maximises and individual's utility subject to the resource constraint.

$$Nc_1 + Nc_2 \leqslant Ny \;\rightarrow\; c_1 + c_2 \leqslant y$$



- Compare monetary equilibrium allocation at point B with the golden rule allocation at point A.
- Monetary equilibrium at point B: intersection of the budget constraint and the resource constraint.
- With a growing money supply, the allocation in a monetary equilibrium is not the golden rule allocation.
  - Young consume more → noncash goods.
  - Old consume less → cash goods.

- In a monetary equilibrium, all future generations are worse off: utility at point B is lower than utility at point A. The initial old are also worse off.

Cost of Inflation

- In general, effects of inflation:
people are less willing to hold money and economise the use of money,

$$\downarrow$$

transactions that are conducted using money are adversely affected,

$$\downarrow$$

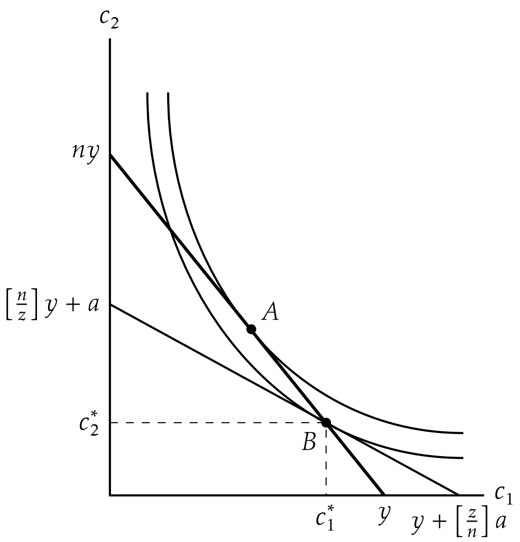
violates "smooth consumption" assumption

$$\downarrow$$

welfare fall.
- Inflation is effectively a tax

A Growing Population

- Suppose that population grows such that $N_t = nN_{t-1}$
- Budget constraints:
  - first period budget constraint:
$$c_1 + v_t m_t \leqslant y;$$
  - second period budget constraint:
$$c_2 \leqslant v_{t+1} m_t + a;$$
  - lifetime budget constraint
$$c_1 + \frac{v_t}{v_{t+1}} c_2 \leqslant y + \frac{v_t}{v_{t+1}} a.$$
- Value of money v_t:
$$N_t(y - c_1) = v_t M_t \to v_t = \frac{N_t(y - c_1)}{M_t}$$
- Money's rate of return $\frac{v_t+1}{v_t}$:
$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y-c_1)}{M_{t+1}}}{\frac{N_t(y-c_1)}{M_t}} = \frac{N_t}{N_{t+1}} \frac{M_t}{M_{t+1}} = \frac{n}{z}$$
The value of money may increase or decrease over time depending on the values of n and z.
- An individual's lifetime budget constraint simplifies to
$$c_1 + \frac{z}{n} c_2 \leqslant y + \frac{z}{n} \cdot a.$$
- Graphically, we depict the budget constraint and the allocation B that is chosen in a monetary equilibrium. Allocation A is the golden rule allocation.



- Golden rule allocation: a planner's resource constraint

$$N_t c_1 + N_{t-1} c_2 \leqslant N_t y \quad \rightarrow \quad c_1 \frac{1}{n} c_2 \leqslant y$$

- The resource constraint is different from the individual's budget constraint. The allocation in a monetary equilibrium is not the golden rule allocation. Again, the expansion of money supply makes individuals consume more when young and less when old. The overall utility is lower than the utility at the golden rule allocation.
- What is the optimal growth rate of money supply in an economy with a growing population? To make the individual's budget constraint identical to the resource constraint, it requires

$$\frac{v_{t+1}}{v_t} = \frac{n}{z} = n$$

  It means that $z = 1$. A constant money supply allows the economy to achieve the golden rule allocation.
  - Planner's resource constraint: if each young gives up 1 unit of consumption, the old can receives n units.
  - Individual's budget constraint: if the young gives up 1 unit of consumption, he will receive n/z units when old.
  - To convey the message that the economy can offer n units of goods to the old for each good not consumed by the young, the budget constraint has to be adjusted so that it coincides with the resource constraint.
  - The value of money needs to increase at a rate n. That is $\frac{v_{t+1}}{v_t} = n$

Summary

- So far, we have shown that when money supply grows at a rate z with $z > 1$, the allocation in a monetary equilibrium generally differs from the golden rule allocation.
- Inflation reduces individuals incentives to hold money and adversely affects transactions using money. As a result, inflation may reduce output and welfare.
- In our model, the optimal growth rate of money supply is **always** $z = 1$, no matter the population is constant or growing. That is, **a constant money supply is the best policy**.

2.5 *A Growing Money Supply: New Money to Finance Government Purchases*

A Monetary Equilibrium

- Government needs to create revenue to finance various types of expenditures. The use of money creation as a revenue device is called **"seigniorage"**
- We focus on stationary allocations and a constant population.
- Suppose that money supply grows at a constant rate z : $M_t = zM_{t-1}$
  - The amount of new money created in period t is

$$M_t - M_{t-1} = M_t - \frac{1}{z}M_t = \left(1 - \frac{1}{z}\right)M_t.$$

  - The amount of goods that the government can purchase in period t is

$$G_t = v_t(M_t - M_{t-1}) = \left(1 - \frac{1}{z}\right)v_t M_t.$$

    This is also the government budget constraint.
  - Suppose that G_t does not affect an individual's consumption choice.
- Budget constraints:
  - first period budget constraint:

$$c_1 + v_t m_t \leqslant y;$$

  - second period budget constraint:

$$c_2 \leqslant v_{t+1} m_t;$$

  - lifetime budget constraint

$$c_1 + \frac{v_t}{v_{t+1}} c_2 \leqslant y.$$

  - Notice that in this model, **individuals do not receive government transfers**.
- Money's rate of return $\frac{v_{t+1}}{v_t}$:
  - value of money v_t is determined when money market clears

$$N(y - c_1) = v_t M_t \quad \rightarrow \quad v_t = \frac{N(y - c_1)}{M_t}$$

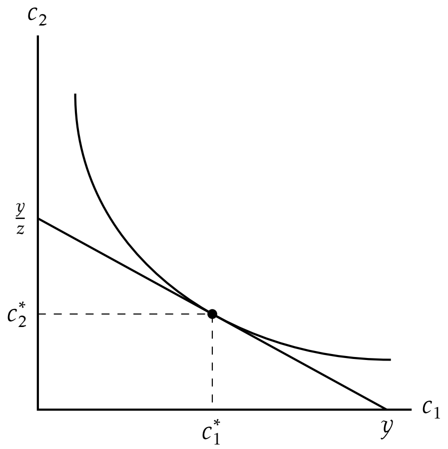
  - money's rate of return

$$\frac{v_t + 1}{v_t} = \frac{\frac{N_{t+1}(y-c_1)}{M_{t+1}}}{\frac{N_t(y-c_1)}{M_t}} = \frac{M_t}{M_{t+1}} = \frac{1}{z}$$

- We simplify the individual's budget constraint to

$$c_1 + zc_2 \leqslant y$$

- Graphically, we depict the budget constraint and add a typical indifference curve.



- In a monetary equilibrium, the amount of goods that the government can purchase in period t can be found from

$$G_t = \left(1 - \frac{1}{z}\right) v_t M_t = \left(1 - \frac{1}{z}\right) N(y - c_1).$$

  Notice that G_t is also stationary because $G_t = G_{t+1}$ for any t.

Golden Rule Allocation

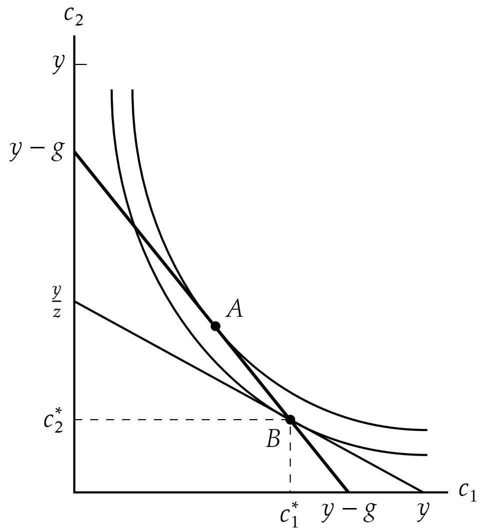
- To discuss the optimality of monetary equilibrium, we need to find the golden rule allocation.
- The planner's resource constraint

$$Nc_1 = Nc_2 + G \leqslant Ny$$

  where $G_t = G$ for stationary allocations. Divide both sides by N and let $g \equiv \frac{G}{N}$. The resource constraint can be rewritten as

$$c_1 + c_2 + g \leqslant y$$

  Notice that when the government uses new money to finance its own purchases, G or g is in the resource constraint. The government competes with individuals for resources.
- Graphically, we depict the resource constraint and add a typical indifference curve.



- We compare monetary equilibrium at point B with the golden rule allocation at point A.
- When the government prints new money to finance its own purchases, the allocation in a monetary equilibrium achieves a lower level of utility than the golden rule allocation.
- Inflation makes individuals trade less goods for money when young, which leads to
  - higher consumption when young;
  - lower consumption when old.
- Note that in comparison with the golden rule allocation, inflation hurts all future generations, as well as the initial old because c_2^* is lower in a monetary equilibrium.

Inflation Tax v.s. Nondistorting Tax

- Creating new money is one way to finance government purchases → effectively an inflation tax. As we have shown, inflation leads to the monetary equilibrium allocation at point B, which is inferior to the golden rule allocation at point A.
- Given the need for the government to raise revenue, are there other ways to raise revenue and make the golden rule allocation attainable?
- Consider a **lump-sum tax**. Suppose that the government collects a tax of τ goods from each old individual in every period.
- Monetary equilibrium:
  - first-period budget constraint

$$c_1 + v_t m_t \leqslant y;$$

  - second-period budget constraint

$$c_2 \leqslant v_{t+1} m_t - \tau;$$

  - lifetime budget constraint

$$c_1 + \frac{v_t}{v_{t+1}} c_2 \leqslant y - \frac{v_t}{v_{t+1}} \tau.$$

- How can the government choose the values of $\frac{v_t}{v_{t+1}}$ and τ so that
  - monetary equilibrium can be the same as the golden rule allocation;

- the government can still finance its own purchases G?
- The government can keep a constant money supply by imposing $\tau = g$ In this case, we can verify $\frac{v_t}{v_{t+1}} = 1$ and the individual's budget constraint becomes

$$c_1 + c_2 \leqslant y - g$$

Now the budget constraint is identical to the planner's resource constraint. The allocation in a monetary equilibrium is the same as the golden rule allocation.
- Inflation tax (creating new money) and lump-sum taxes:
  - inflation tax: inferior equilibrium allocation but easy to implement – low cost
  - lump-sum taxes: golden rule allocation but hard to implement in reality.
- Money creation has been a popular means to raise government revenue.

Seigniorage: Theory and Evidence

- The use of seigniorage as a source of government revenue varies from country to country and from time to time.
  - For most developed countries during normal times: seigniorage contributes little to government revenue. For example, seigniorage in U.S. accounted for about 2% of total government revenue and for about 0.3% of gross national product from 1948 to 1989.
  - For countries that experience high inflation episodes like Argentina, Chile and etc., seigniorage contributes significantly to government revenue. For example, seigniorage accounted for about 46% of Argentinian government revenue and 6.2% of gross national product from 1960 to 1975.
  - An extreme case: Germany during its hyperinflation of the early 1920s. Seigniorage was about 10% to 15% of gross national product.
- Can the government simply print enough money to finance any purchase without the bother of direct taxation?
  - The government can print any amount of dollars.
  - The value of those dollars may shrink as the supply of money increases.
  - Seigniorage revenue in terms of real goods is limited by the real value of money.
- To formally examine how seigniorage revenue depends on the speed of money creation, we revisit the government budget constraint

$$G = (M_t - M_{t-1})v_t = \underbrace{\left(1 - \frac{1}{z}\right)}_{\text{tax rate}} \underbrace{v_t M_t}_{\text{tax base}}$$

- There are two terms in G.
  - $1 - \frac{1}{z}$: *tax rate* the fraction of the real value of the money stock that becomes government revenue;
    - example: if $z = 1.05$, then $1 - 1/z = 1 - 1/1.05 = 0.0476$.
  - $v_t M_t$: *tax base* – the real value of the money stock (the value of the money stock in terms of goods).
- When the government increases the speed of printing money by raising z,
  - the tax rate $1 - 1/z$ will increase;
  - but what is the effect of z on the tax base $v_t M_t$?
- Recall: from the money market clearing condition

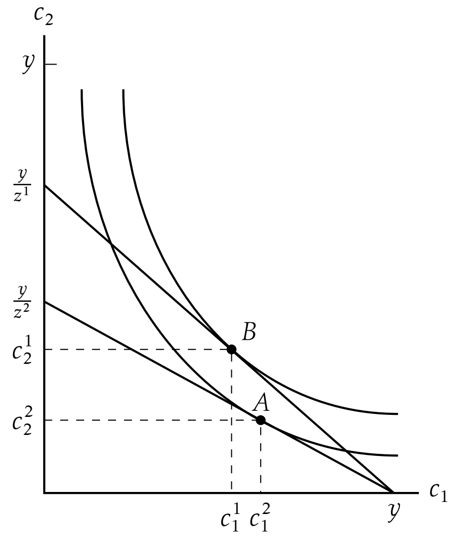
$$v_t M_t = N(y - c_1).$$

- We need to know how c_1 depends on z.

- Consider z^1 and z^2 where $z^2 > z^1$. How does c_1 respond to an increase in z from z^1 to z^2?
  - (c_1, c_2) in a monetary equilibrium is determined by the tangency point between the indifference curve and the budget constraint.
  - The budget constraint in this economy is

$$c_1 + zc_2 \leqslant y$$

An increase in z would affect the individual's budget constraint.
- Graphically, when z increases from z^1 to z^2, c_1 increases from c_1^1 to c_1^2



- When z increases, the inflation rate increases. Higher inflation induces young to trade less goods for money so that c_1 increases and c_2 decreases.
- Now, back to our money market clearing condition

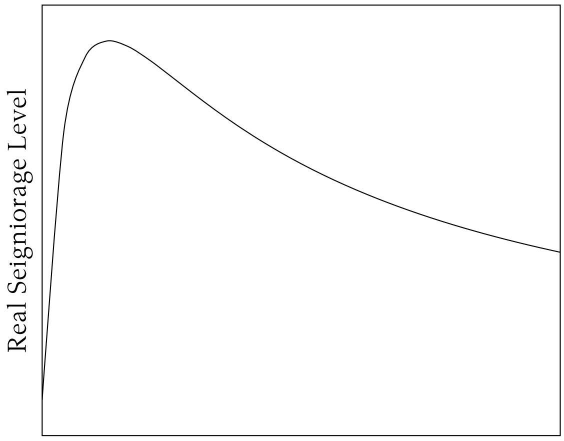
$$c_1 + zc_2 \leqslant y$$

When c_1 increases, the aggregate demand for money in real terms $N(y - c_1)$ decrease. Therefore, the aggregate supply of money in real terms $v_t M_t$ also decrease. The tax base $v_t M_t$ decreases.
- Economists have found evidence that higher inflation leads to lower real demand for money
- So far, we have found that when growth rate of money supply z increases,
  - the tax rate $1 - \frac{1}{z}$ increases;
  - the tax base $v_t M_t$ decreases;
- Overall, seigniorage revenue

$$G = \left(1 - \frac{1}{z}\right) v_t M_t = \left(1 - \frac{1}{z}\right) N(y - c_1),$$

may or may not increase as z increases. The exact relationship between the seigniorage revenue G and the growth rate of money supply z depends on the utility function of individuals and anything else that affects the demand for money.
- The general shape of G as a function of z **resembles** the Laffer curve:
  - at low growth rates of money supply, a higher growth rate leads to a higher level of seigniorage;

  - at high growth rates of money supply, a higher growth rate leads to a lower level of seigniorage;
  - there exists a growth rate of money supply that maximizes seigniorage.
- The original Laffer curve describes the relationship between income tax rate and income tax revenue.



3
Price Surprises

3.1 *Introduction*

- The relationship between inflation and unemployment – the Phillips curve.
- Cross country evidence on the relationship between inflation and output.
- We develop a theory to rationalise the empirical observations?
  - **Unanticipated** changes in money supply. In previous sections, we consider **anticipated** increases in money supply.
  - How do unanticipated fluctuations in money supply affect output?
  - Can the government exploit such a relationship?
- The Lucas (1972) model and the Lucas critique.

3.2 *The Data*

The Phillips Curve

- The original Phillips curve suggests that there is a negative relationship between inflation and unemployment, or there is a positive relationship between inflation and output.
- Does it imply that there maybe an exploitable trade-off between inflation and unemployment? Can the government reduce unemployment and increase output by increasing inflation?
- In the following decades, many governments tried to use monetary policy to stimulate the economy. Suddenly, the Phillips curve, a stable relationship for

more than a century, disappeared. Inflation occurs with no gains in output or employment.

Cross-country Comparisons

- The Phillips curve is a **time series** correlation between inflation and unemployment in different periods of the same country.
- If we compare across countries, inflation rates are on average higher in countries with lower average real GDP growth rates. Note: its a **cross-section** comparison here.
- How can there exist seemingly contradictory correlations
  - Time series of the same country: **-ve** correlation between inflation and unemployment, that is, **+ve** correlation between inflation and output. Cross-country comparisons: **-ve** correlation between inflation and output.

3.3 *The Lucas Model*

Basic Environment

- Consider the standard OLG model with money. Now assume that individuals live on two spatially separated islands.
- N_t individuals are born in each period. N_t is constant. In each period,
  - half of the old live on each of the islands;
  - 1/3 of the young live on one island and 2/3 live on the other island;
  - the allocation of the young and the old is random.
    * the old are randomly distributed across the two islands, regardless of where they lived when young
    * in any single period, each island has an equal chance of having the large population of young.
  - Money supply grows at a rate z_t in period $t, M_t = z_t M_{t-1}$. The new money is distributed to each old person as a lump-sum transfer in every period t worth at units of the consumption good.
- Money supply grows at a rate z_t in period t, $M_t = z_t M_{t-1}$. The new money is distributed to each **old** person as a lump-sum transfer in every period t worth at units of the consumption good.

$$a_t = \left(1 - \frac{1}{z_t}\right) \frac{v_t M_t}{N}$$

- Informational assumptions: in any period,
  - the young cannot observe the number of young individuals on their island;
  - the young cannot observe the size of the transfers to the old;
  - the nominal stock of money supply is known with a delay of one period; e.g., in period t, individuals know M_{t-1}, but not M_t;
  - the price of goods on an island is observed but only by the individuals on that island;
  - no communication between islands within a period.
- We assume that individuals are rational.
  - They may not have complete information, but they can infer whatever they can from the information they have and they make the most correct inference possible given the explicitly specified limits on what they can observe.
  - The assumption of "rational expectation", first introduced by Muth (1961): people understand the probabilities of outcomes important to their welfare.

  - In our model, individuals do not observe z_t and the population of the young on each island, but they know the prices. They know 1/3 of the young are on one island and the rest 2/3 are on the other island. They will try to infer z_t and the distribution of the young population.
- A reinterpretation of an individual's problem.
  - y: individuals are endowed when young with y units of time (instead of goods): think of y as 24 hours;
  - c_1: consumption of **leisure** (instead of consumption of goods); $y - c_1$ is spent working;
  - c_2: consumption of goods;
  - $I = y - c_1$: labour supply by the individual when young; $l_t^i = l(p_t^i)$: the choice of labour by an individual born in period t for a given price of goods, p_t^i on island i;
  - production function: 1 unit of l can be used to produce 1 unit of the consumption goods
- Consumption

Generation	t=1	t=2	t=3	t=4	$\longrightarrow$
0	c_2				
1	c_1	c_2			
2		c_1	c_2		
3			c_1	c_2	
$\downarrow$				$\ldots$	$\ldots$

- In our model, the young are endowed with time and the old are endowed with nothing.
- In period t, a young individual on island i
  - chooses between **working** l_t^i and **leisure** $c_{1,t}^i$

$$c_{1,t}^i + l_t^i \leqslant y,$$

  where l_t^i units of goods are produced and sold to the old to acquire m_t^i units of money,

$$l_t^i = v_t^i m_t^i.$$

  Here $v_t^i m_t^i$ still represents the individual's real demand for money.
- In period $t+1$, the young individual born in period t becomes old and could be on island j, where j may or may not be the same island as i. His consumption $c_{2,t+1}^{i,j}$ comes from his own saving and government transfers,

$$\begin{aligned} c_{2,t+1}^{i,j} &= v_{t+1}^j m_t^i + a_{t+1} \\ &= \frac{v_{t+1}^j}{v_t^i} l_t^i + a_{t+1} \\ &= \frac{p_t^i}{p_{t+1}^j} l_t^i + a_{t+1} \end{aligned}$$

- Note that when the young individual decides to supply 1 more unit of labour by increasing l_t^i by 1, he will be able to produce 1 more unit of good and acquire $1/v_t^i$ more units of money. Then he can use the $1/v_t^i$ units of money to buy v_{t+1}^j / v_t^i units of goods when old. This implies that the rate of return to labour is

$$\frac{v_{t+1}^j}{v_t^i} = \frac{p_t^i}{p_{t+1}^j}$$

- We assume that an increase in the current price of goods p_t^i, other things being equal, will induce the young to work more, that is, l_t^i increases.
  - When p_t^i increases, the rate of return to labour increases.
    - Substitution effect: work more because working is more profitable.
    - Income effect: work less because the higher return from labour means higher income and less need to work.
  - We are assuming that the substitution effect of an increase in price **dominates** the income effect.

Nonrandom Inflation

- Before we examine a random z_t, we begin with a constant growth rate of money supply $z_t = z$ in all periods.
- Rational individuals infer the current stock of money. Individuals know M_{t-1} and z. So they can infer $M_t - zM_{t-1}$
- In period t, money market clearing condition on on island i with N^i young individuals is:

$$N^i\left(y - c_{1,t}^i\right) = v_t^i \frac{M_t}{2},$$

  or equivalently

$$N^i l_t^i = v_t^i \frac{M_t}{2} = \frac{1}{p_t^i}\frac{M_t}{2}.$$

- We label the island with 1/3 young individuals as island A and the other island as island B. We have

$$p_t^A = \frac{\frac{M_t}{2}}{N^A l_t^A} = \frac{\frac{M_t}{2}}{\frac{1}{3}N l_t^A} \tag{3.1}$$

$$p_t^B = \frac{\frac{M_t}{2}}{N^B - l_t^B} = \frac{\frac{M_t}{2}}{\frac{2}{3}N l_t^B} \tag{3.2}$$

  Claim: $p_t^A > p_t^B$ – the price level is higher on the island with less young individual.
  Why? By contradiction. If $p_t^A \leqslant p_t^B$ then the rate of return to labour is lower on island A which implies that $l_t^A \leqslant l_t^B$. However, from equation (3.1) and equation (3.2), $l_t^A \leqslant l_t^B$ implies that $p_t^A > p_t^B$, which is a contradiction to the assumption of $p_t^A \leqslant p_t^B$. So it is only possible that $p_t^A > p_t^B$.
- We find that the price of goods is high on the island with relatively **less** young individuals and is low on the island with relatively **more** young individuals.
  - Intuition: when there are less young people, there are less people supplying labour to produce the good. With the same number of old individuals, the demand for goods is relatively high on the island with less young individuals. Therefore, the price is high on the island with less young individuals.
  - Further implication: since we know that $p_t^A > p_t^B$ (and *all else equal*), the rate of return to labour is high on island A, young individuals work more on island A with less young individuals. That is, $l_t^A > l_t^B$.
  - These implications depend critically on the assumption that the substitution effect dominates the income effect.
- Prices here signal the true state of the economy: the young can infer that
  - they are on the island with a smaller population if they observe the high price;
  - they are on the island with a larger population if they observe the low price.
- Money supply can also affect the price level.
  - When money supply increases, the price level increases.

  - When money supply decreases, the price level decreases.
- Suppose $z = 1$. What if there is a permanent (once-and-for-all) increase in the money stock? That is, M increases permanently. Recall that

$$p_t^i = \frac{M_t/2}{N^i l_t^i}$$

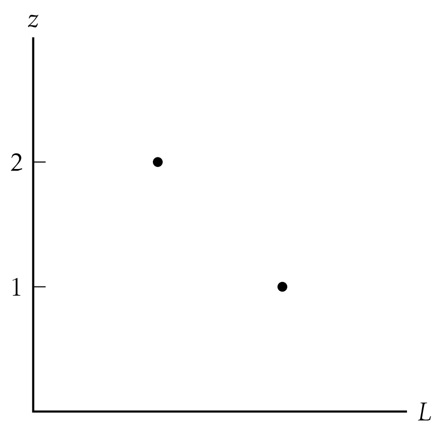
Once M increases permanently, p_t^i will increase but p_{t+1}^j will also increase. Overall, the rate of return to labour.

$$\frac{v_{t+1}^j}{v_t^i} = \frac{p_t^i}{p_{t+1}^j} = \frac{\frac{M_t/2}{N^i l_t^i}}{\frac{M_{t+1}/2}{N^j l_{t+1}^j}} = \frac{N^j l_{t+1}^j}{N^i l_t^i}\frac{M_t}{M_{t+1}}$$

is not affected by the level of money supply. Therefore, a permanent increase in money supply does not affect employment and output in this economy.
- Money is **neutral** in this economy: a permanent change in M does not affect the real economic variables.
- What if there is a permanent increase in z? Now the rate of return to labour is

$$\frac{v_{t+1}^j}{v_t^i} = \frac{p_t^i}{p_{t+1}^j} = \frac{\frac{M_t/2}{N^i l_t^i}}{\frac{M_{t+1}/2}{N^j l_{t+1}^j}} = \frac{N^j l_{t+1}^j}{N^i l_t^i}\frac{M_t}{M_{t+1}} = \frac{N^j l_{t+1}^j}{N^i l_t^i}\frac{1}{z}$$

An increase in z lowers the rate of return to labour, which discourages working because the money earned from labour is now taxed by the government through inflation. Lower l_t^i leads to lower output.
- Money is **not superneutral** in this economy: a permanent change in z affects the real economic variables.
- We can construct a graph plotting output as a function of z.



Random Inflation

- So far we find that inflation reduces employment and output in our economy.
- Now consider the following random monetary policy.

$$\begin{aligned} M_t &= M_{t-1} \quad \text{with probability } \theta \quad (z_t = 1) \\ &= 2M_{t-1} \quad \text{with probability } 1-\theta \quad (z_t = 2) \end{aligned}$$

The realisation of z_t is kept secrete from the young until the end of period t. Can the young still infer the current money supply? **Maybe not**.

- As before, we will focus on how the young's labour supply decisions depend on monetary policy.
- Again, using the equation that determines the price level on island i,

$$p_t^i = \frac{\frac{M_T}{2}}{N^i l_t^i}$$

Notice that individuals do not know M_t and N^i, but they know M_{t-1}. We can rearrange the price equation as

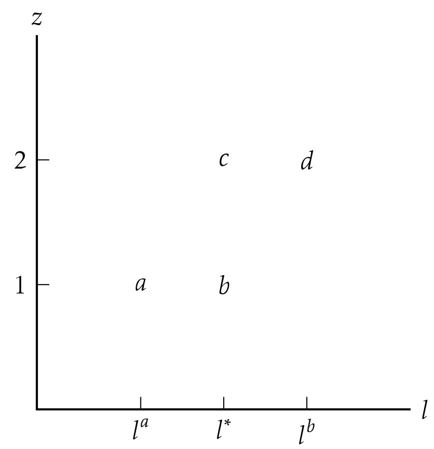
$$p_t^i = \frac{z_t M_{t-1}/2}{N^i l_t^i}$$

Individuals know that with probability $\theta, z_t = 1$ and with probability $1-\theta, z_t = 2$.

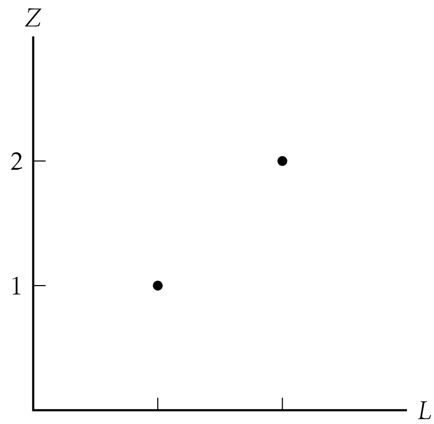
- Let's think about the potential prices. Depending on the values of z_t and N^i,

	$\frac{2}{3}N$	$\frac{1}{3}N$
$z_t = 1$	$p_t^a = \frac{M_{t-1}/2}{\frac{2}{3}Nl(p_t^a)}$	$p_t^b = \frac{M_{t-1}/2}{\frac{1}{3}Nl(p_t^b)}$
$z_t = 2$	$p_t^c = \frac{2M_{t-1}/2}{\frac{2}{3}Nl(p_t^c)}$	$p_t^d = \frac{2M_{t-1}/2}{\frac{1}{3}Nl(p_t^d)}$

- For any young individual, he does not know the population of the young on his island. He also does not know the current money supply in the economy. Can he still infer z_t and N^i from the prices?
  - If the young individual observes p_t^a, he will know that he is on the island with $N^i = 2N/3$ and $z_t = 1$.
  - If the young individual observes p_t^d, he will know that he is on the island with $N^i = N/3$ and $z_t = 2$.
  - If the young individual observes p_t^b, what can he infer?
  - If the young individual observes p_t^c, what can he infer?
- There are two factors that affect the price level: N^i and z_t.
  - If $N^i = N/3$ (island with less young individuals), it contributes to a higher p_t^i. If $N^i = 2N/3$ (island with more young individuals), it contributes to a lower p_t^i.
  - If $z_t = 1$, it contributes to a lower p_t^i. If $z_t = 2$, it contributes to a higher p_t^i.
- Two of the four possible prices are unique: (p_t^a, p_t^d). Each can have occurred in only one particular combination of events.
  - If observing the low price p_t^a, the young would supply labour l_t^a (a low level).
  - If observing the high price p_t^d, the young would supply labour l_t^d (a high level).
- If the young observe p_t^b or p_t^c, the young cannot infer whether they are on the island with $N/3$ young and $z_t = 1$ or they are on the island with $2N/3$ young and $z_t = 2$. Therefore, the young decide to supply labour l^*.
- If we graph output and inflation on two islands separately,



- If we graph aggregate output and inflation, we have



The Lucas Critique

- Imagine that an economy's time series plot of inflation and output resembles our previous figure.
  - The historical correlation suggest that the government can not control aggregate output through its control of the money supply.
  - If the government wants to achieve a higher level of output, The government should not print money to stimulate output in every period
  - The policy would not work.
  - The growth rate of money supply becomes constant, people can perfectly infer current money supply. A higher growth rate of money supply leads to lower labour supply and lower output. The positive correlation between inflation and output disappears!
- The correlation of money and output or any set of variables results from the reaction of decision makers to the environment they face. An important feature of this environment is government policies.
  - In our example, the relation between inflation and output depends on the monetary policy being followed.
    * Random inflation: positive correlation between inflation and output.
    * Steady inflation (nonrandom inflation): negative correlation between inflation and output.
    * When monetary policy changes from random to nonrandom, the labour supply decisions by the young change as well.

- A correlation between variables that is the result of equilibrium interactions of an economy can be called a **reduced-form** correlation.
  - In our example, it is the correlation between inflation and output.
- The Lucas Critique: these reduced form correlations are subject to change when the government changes its policies.
  - In our example, the positive correlation between inflation and output disappears when the government changes from random inflation to nonrandom inflation because young individuals change their labour supply decisions.
- How can we evaluate policies?
  - Econometric policy evaluation is useful.
  - But we also need a theory to help us understand people's motives (preferences) and constraints (physical limitations, informational restrictions, and government policies).
  - It is not sufficient just to look at the data.

4
International Monetary Systems

4.1 *Introduction*

- In the first three sections, we have examined closed economies – economies that operate entirely in isolation with a single fiat money.
- In modern world, trade and financial links between countries are increasingly important. We turn our focus to the role of money in economies that encompass more than one country and currency. In this section, we will examine
  - how exchange rates are determined;
  - different types of international monetary system: fixed exchange rate, flexible exchange rate and etc.;
  - the rationales for the European countries to adopt a single currency – Euro;
  - when a country's currency is more likely to be subject to speculative attack: the Asian Financial Crisis.

4.2 *A Model of International Exchange*

- Based on our standard OLG model with money: suppose there are two countries, country a and country b, each with its own money/currency.
- Assume that endowments in each country consist of the same goods (a good in country a is indistinguishable from a good in country b). Individuals are indifferent to the origin of the goods they purchase. There is free international trade.
- We use superscripts a and b to identify the parameters and variables of each country.
  - growth rates of population: n^a and n^b;
  - growth rates of money: z^a and z^b.
- For simplicity, assume that any new money created by the government is used to finance the government's own purchases.

- Let e_t denote the exchange rate: the units of country b money that can be purchased with one unit of country a money,

$$e_t = \frac{\text{country } b \text{ money}}{1 \text{ unit of country } a \text{ money}}.$$

  - For example, country a is Australia and country b is the U.S.:

$$e_t = \frac{\text{U.S dollar}}{1 \text{ Australian dollar}}.$$

  - The inverse of e_t indicates the number of Australian dollar per U.S. dollar.
  - For each pair of currencies, there are always two exchange rates, depending on which currency serves as the base currency.
- Consider an old individual in period t who was born in period $t-1$.
  - If the old individual owns 1 unit of country a money, he can
    * use country a money to buy v_t^a units of goods;
    * or exchange 1 unit of country a money for e^t units of country b money and buy $e_t v_t^b$ units of country b goods.
  - If the old individual owns 1 unit of country b money, he can
    * use country b money to buy v_t^b units of goods;
    * or exchange 1 unit of country b money for $1/e^t$ units of country a money and buy v_t^a/e_t units of country a goods.
- No matter which money the old individual holds, he always compares v_t^a with $e_t v_t^b$ when deciding which money to use to purchase the goods.
  - If $v_t^a > e_t v_t^b$, everyone prefers to use country a money. Country b money is not valued by anyone.
  - If $v_t^a < e_t v_t^b$, everyone prefers to use country b money. Country a money is not valued by anyone.
  - Only if $v_t^a = e_t v_t^b$ all individuals are indifferent between the two monies. For both monies to be valued in equilibrium, the exchange rate must be

$$v_t^a = e_t v_t^b \text{ or } e_t = \frac{v_t^a}{v_t^b}.$$

  - We will examine the behaviour of this exchange rate under alternative international monetary arrangements.

4.3 *Foreign Currency Controls*

- The first international monetary system that we consider is called "foreign currency controls" a policy that completely separates the monetary sectors of the two countries:
  - the citizens of each country are permitted to hold over time only the money of their own country;
  - free international trade.
- In our model, the policy of foreign currency controls implies that
  - the young of each country can hold only their country's money from one period to the next;
  - the old can buy goods from any country, but if he wishes to buy goods from foreign country he needs to exchange his money for the foreign currency and then make the purchase.
- With foreign currency controls, demand for country a money comes from country a young individuals and demand for country b money comes from country

b young individuals. The money market clearing conditions for country a and country b are

$$v_t^a M_t^a = N_t^a \left(y^a - c_{1,t}^a\right);$$
$$v_t^b M_t^b = N_t^b \left(y^b - c_{1,t}^b\right).$$

It follows that

$$e_t = \frac{v_t^a}{v_t^b} = \frac{\frac{N_t^a\left(y^a - c_{1,t}^a\right)}{M_t^a}}{\frac{N_t^b\left(y^b - c_{1,t}^b\right)}{M_t^b}} = \frac{N_t^a\left(y^a - c_{1,t}^a\right)}{N_t^b\left(y^b - c_{1,t}^b\right)} \frac{M_t^b}{M_t^a}$$

The exchange rate e_t depends on the relative values of the demand for money and the supply of money in the two countries.

- Growth rates of population: (n^a, n^b) and growth rates of money supply: (z^a, z^b) – we consider **constant growth rates of money supply** in this section. Suppose that we focus on stationary allocations. We have

$$\frac{v_{t+1}^a}{v_t^a} = \frac{\frac{N_{t+1}^a\left(y^a - c_1^a\right)}{M_{t+1}^a}}{\frac{N_t^a\left(y^a - c_1^a\right)}{M_t^a}} = \frac{N_{t+1}^a}{N_t^a} \frac{M_t^a}{M_{t+1}^a} = \frac{n^a}{z^a},$$
$$\frac{v_{t+1}^b}{v_t^b} = \frac{\frac{N_{t+1}^b\left(y^b - c_1^b\right)}{M_{t+1}^b}}{\frac{N_t^b\left(y^b - c_1^b\right)}{M_t^b}} = \frac{N_{t+1}^b}{N_t^b} \frac{M_t^b}{M_{t+1}^b} = \frac{n^b}{z^b}.$$

The path of the exchange rate can be expressed as

$$\frac{e_{t+1}}{e_t} = \frac{\frac{v_{t+1}^a}{v_{t+1}^b}}{\frac{v_t^a}{v_t^b}} = \frac{v_{t+1}^a}{v_t^a} \frac{v_t^b}{v_{t+1}^b} = \frac{n^a}{z^a} \frac{z^b}{n^b} = \frac{n^a}{n^b} \frac{z^b}{z^a}$$

- What are the factors that determine how the exchange rate changes over time? From

$$\frac{e_{t+1}}{e_t} = \frac{n^a}{n^b} \frac{z^b}{z^a}$$

growth rates of population and growth rates of money supply affect the path of the exchange rate.
- Population growth: the greater the growth rate of country a's population relative to country b's, the greater the growth rate of the exchange rate.
  - Greater growth of population in one country → higher demand for the country's money → the value of the country's money increases → the country's money appreciates over time.
  - In general, any factor that contributes to increase in the **demand** for a country's money will drive up the value of the country's money.
- Money growth: the greater the growth rate of country a's money supply relative to country b's, the lower the growth rate of the exchange rate.
  - Greater growth of money in one country → higher demand for the country's money → the value of the country's money decreases → the country's money depreciates over time.
  - In general, any factor that contributes to increase in the **supply** for a country's money will drive up the value of the country's money.
- A special case is $e_{t+1} = e_t$ – fixed exchange rate.

Fixed Exchange Rates

- A special case: $e_t = e_{t+1}$, it requires that

$$z_a = \frac{n^a}{n^b} z^b \tag{4.1}$$

- If country a choose to keep a fixed exchange rate with country b, it needs to set its growth rate of money supply according to equation (4.1). Country a loses its independence in monetary policy.
  - Country a money and country b money have the same rate of return.
  - If country b increases its growth rate of money supply z^b, country a will be forced to increase z^a to keep the fixed exchange rate.
  - Country a government cannot acquire its preferred level of seigniorage revenue.
  - With foreign currency controls, a country can choose the growth rate of money supply either to fix the exchange rate or to acquire its preferred level of seigniorage, it cannot meet both objectives.

4.4 *The Indeterminacy of the Exchange Rate*

- Suppose now that people are free to hold and use the money of any country. We can no longer have two separate money market clearing conditions. Instead,
  - the world's supply of money

$$v_t^a M_t^a + v_t^b M_t^b;$$

  - the world's demand for money

$$N_t^a \left(y^a - c_{1,t}^a\right) + N_t^b \left(y^b - c_{1,t}^b\right);$$

  - the world's money market clearing condition

$$v_t^a M_t^a + v_t^b M_t^b = N_t^a \left(y^a - c_{1,t}^a\right) + N_t^b \left(y^b - c_{1,t}^b\right) \tag{4.2}$$

  - How can we determine the exchange rate?

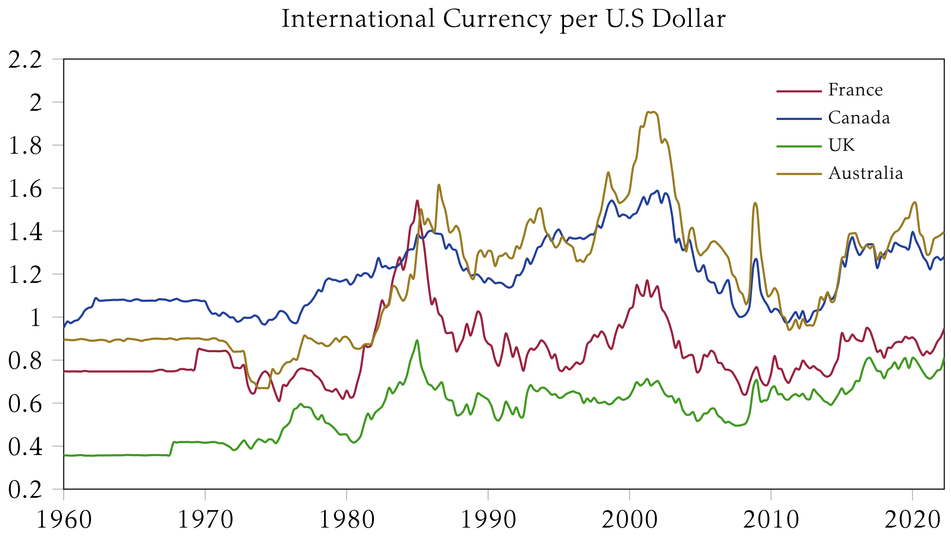
$$e_t = \frac{v_t^a}{v_t^b}$$

- To find e_t, we need to know v_t^a and v_t^b. However, with one money market clearing condition, how can we solve for two unknowns $\left(v_t^a, v_t^b\right)$
  - One cannot solve for two unknowns with one equation.
  - There exists an infinite combinations of $\left(v_t^a, v_t^b\right)$ that satisfy equation (4.2)
  - In other words, for any positive exchange rate e_t, we can find an equilibrium in which equation (4.2) is satisfied.
  - The exchange rate is indeterminate!

Exchange Rate Fluctuations

- **In the absence of the government determination of the exchange rate**, the exchange rate in a unified world economy can be whatever people believe it to be. The exchange rate could fluctuate because these beliefs fluctuate. Exchange rate fluctuations need not be tied to changes in real economic conditions.

- Before 1971, the U.S. dollar is pegged to gold at 35 dollars per ounce of gold (the Bretton Woods System). In 1971, the U.S. abandoned the effort to control exchange rates. Afterwards, the world has seen tremendous volatility in exchange rates.



International Currency Traders

- Even with foreign currency controls sometimes the exchange rate can be indeterminate.
- A model by King, Wallace and Weber (1992) about international currency traders. Three types of individuals:
  - citizens of country a, forced by law to hold only country a's money;
  - citizens of country b, forced by law to hold only country b's money;
  - multinational people, free to hold either currency.
- The numbers of each type individuals born in period t are denoted as N_t^a, N_t^b and N_t^c.
- Each country's money is held by its own citizens and perhaps by multinational people as well. Let λ_t be the fraction of country a money in the multinational people's real money balances.
- Money market clearing conditions for country a money and country b money:

$$
\begin{aligned}
v_t^a M_t^a &= N_t^a \left(y^a - c_{1,t}^a\right) + \lambda_t N_t^c \left(y^c - c_{1,t}^c\right), \\
v_t^b M_t^b &= N_t^b \left(y^b - c_{1,t}^b\right) + (1-\lambda_t) N_t^c \left(y^c - c_{1,t}^c\right).
\end{aligned}
$$

The value of country a is

$$
v_t^a = \frac{N_t^a \left(y^a - c_{1,t}^a\right) + \lambda_t N_t^c \left(y^c - c_{1,t}^c\right)}{M_t^a}
$$

The value of country b money is:

$$
v_t^b = \frac{N_t^b \left(y^b - c_{1,t}^b\right) + (1-\lambda_t) N_t^c \left(y^c - c_{1,t}^c\right)}{M_t^b}
$$

- The exchange rate in the world economy is

$$e_t = \frac{v_t^a}{v_t^b} = \frac{\frac{N_t^a(y^a - c_{1,t}^a) + \lambda_t N_t^c(y^c - c_{1,t}^c)}{M_t^a}}{\frac{N_t^b(y^b - c_{1,t}^b) + (1-\lambda_t) N_t^c(y^c - c_{1,t}^c)}{M_t^b}}.$$

- A simple case: suppose that preferences are such that the total real demand for money is identical across the different types of people. That is, $N_t^a(y^a - c_{1,t}^a) = N_t^b(y^b - c_{1,t}^b) = N_t^c(y^c - c_{1,t}^c)$. The exchange rate can be simplified to

$$e_t = \frac{\frac{1+\lambda_t}{M_t^a}}{\frac{1+(1-\lambda_t)}{M_t^b}} = \frac{\frac{1+\lambda_t}{M_t^a}}{\frac{2-\lambda_t}{M_t^b}}$$

  - When λ_t increases, e_t increases. When λ_t decreases, e_t decreases.
  - The change in λ_t will cause change in e_t.
  - If $M_t^a = M_t^b$, $e^t = (1 + \lambda_t)/(2 - \lambda_t)$, $1 < e_t < 2$
- With international currency traders, the exchange rate is still indeterminate. There exist multiple exchange rates that satisfy the money market clearing conditions. Exchange rates may fluctuate dramatically as multinationals change the composition of their money balances.
- The fluctuations in exchange rates make each country's money a risky asset.
  - Multinationals may be able to free themselves from this risk if they hold a balanced portfolio of both monies.
  - Citizens of each country suffer the risk associated with exchange rate fluctuations.
- Maybe monetary authorities would like to stabilise the exchange rate to reduce the risk associated with exchange rate fluctuations.

4.5 *Fixing the Exchange Rate*

- Monetary authorities may want to stabilise the exchange rate to reduce exchange rate fluctuations.
- How shall we organise the world to maintain a stable exchange rate?
  - Cooperative stabilisation: countries coordinate to fix the exchange rate.
  - Unilateral defence: unilateral commitment to a fixed exchange rate.

Cooperative Stabilisation

- If we take a cue from the monetary organisation of national economies, what determines the exchange rate between two different bills in a **single** national economy?
  - The government tells us the exchange rate by printing the denomination on each bill.
  - The government also stands ready to exchange the bills at that rate.
- For the **world** economy, if the two governments stand ready to exchange their currencies at some given rate, can they determine the exchange rate? We rarely see countries cooperatively fix the exchange rate.
  - European Economic Community (now the European Union) had tremendous difficulties in maintaining fixed exchange rates.
- One major impediment:
  - the strong incentive to inflate when exchange rate is fixed.

Unilateral Defence of the Exchange Rate

- For the world economy, fixing the exchange rates requires cooperation of foreign central banks. In the absence of such cooperation, the government can keep a fixed exchange rate through unilateral defence of the exchange rate.
- When a government commits to a fixed exchange rate unilaterally, the government needs to tax its citizens to acquire enough resources to defend the exchange rate.
  - If such a commitment is believed, there will be little incentive for anyone to turn in one form of money for the other.
  - Is it believable that the government can defend the fixed exchange rate by taxing its citizens?
- Consider an OLG model with two countries. No foreign currency controls is in effect. The government of a pledges to tax the old in order to defend a fixed exchange rate. The world money market clearing condition is

$$v_t^a M_t^a + v_t^b M_t^b = N_t^a \left(y^a - c_{1,t}^a\right) + N_t^b \left(y^b - c_{1,t}^b\right)$$

  Or with a fixed exchange rate $\bar{e}$

$$\bar{e} v_t^a M_t^a + v_t^b M_t^b = N_t^a \left(y^a - c_{1,t}^a\right) + N_t^b \left(y^b - c_{1,t}^b\right)$$

- Consider a specific example to have a better understanding of the differences between cooperative stabilisation versus unilateral defence of the exchange rate.
  - Suppose country a (Australia) and country b (the U.K.) are identical. In each country, the population of every generation is 100, $N^a = N^b = 100$.
  - Each young wants to hold real money balances worth 10 goods. It follows that the aggregate demand for money in real terms in each country is

$$N^a \left(y^a - c_1^a\right) = N^b \left(y^b - c_1^b\right) = 100 \times 10 = 1000$$

  - Assume that the total money supply in country a is \$800 and in country b is £600.
  - In the first period, each initial old holds \$4 and £3, regardless of citizenship.
  - The exchange rate is fixed at $\bar{e} = 1/2$ – \$1 trades for £0.5.
- We can derive the value of country b money v_t^b

$$\begin{aligned}
\bar{e} v_t^b M_t^a + v_t^b M_t^b &= N_t^a \left(y^a - c_{1,t}^a\right) + N_t^b \left(y^b - c_{1,t}^b\right) \\
\frac{1}{2} v^b (800) + v^b (600) &= 1000 + 1000 \\
1000 v^b &= 2000 \\
v_t^b &= 2
\end{aligned}$$

- and the value of country a money v_t^a

$$v_t^a = \bar{e} v_t^b = \frac{1}{2}(2) = 1$$

- Consumption of each old is

$$c_2^a = c_2^b = v^a(4) + v^b(3) = (1)(4) + (2)(3) = 10 \text{ goods.}$$

Cooperative Stabilisation

- Now suppose that every member of the initial old of both countries decides to cut their balances of country a money in half. Each member of the initial old turns in \$2 to the monetary authority of country a to acquire £1.
- **Cooperative stabilisation**: if monetary authority of country b agrees to cooperate by printing the amount of its currency demanded. At the end of currency exchange, the stock of dollars is \$400 and the stock of pounds is £800.
  - The value of country b money is

$$\begin{aligned}\bar{e}v_t^b M_t^a + v_t^b M_t^b &= N_t^a\left(y^a - c_{1,t}^a\right) + N_t^b\left(y^b - c_{1,t}^b\right)\\ \frac{1}{2}v^b(400) + v^b(800) &= 1000 + 1000\\ 1000v_t^b &= 2000\\ v_t^b &= 2\end{aligned}$$

  - Since the exchange rate is fixed at 1/2, the value of country a money is the same:

$$v_t^a = 1$$

  - Consumption of each old is

$$c_2^a = c_2^b = v_t^a(4) + v_t^b(3) = (1)(2) + (2)(4) = 10 \text{ goods.}$$

- With a fixed exchange rate, the world money market clearing condition suggests that

$$v_t^b = \frac{N_t^a\left(y^a - c_{1,t}^a\right) + N_t^b\left(y^b - c_{1,t}^b\right)}{\bar{e}M_t^a + M_t^b}$$

  - An increase in one country's money supply reduces the value of both currencies.
  - This is mainly because without foreign currency controls, two currencies are perfect substitutes.
- If one country's government wants to inflate to collect seigniorage, **both** countries's citizens are taxed. If both governments wish to inflate to collect seigniorage, a large inflation of the world's money stock will result.
- This inflation can be prevented if governments are willing to agree to limit their own growth rates of money supply. This can be difficult if some countries rely on seigniorage far more than others.

Unilateral Defence of the Exchange Rate

- **Unilateral defense**: if country b refuses to print money to accommodate the demand for its currency, country a has to attempt a unilateral defense of the exchange rate. To do this, country a government must raise tax revenue to provide all of the country b currency demanded.
  - Total amount of pounds needed by country a government:

$$200 \times 2 \times \frac{1}{2} = 200,$$

  which implies each initial old of country a has to pay a tax of

$$\frac{200}{100} = 2v_t^b.$$

– Total stock of money:

$$
\begin{aligned}
M_t^a &= (N^a + N^b) \\
(2 &= (100 + 100) \\
)2 &= \$400 \\
M_t^b &= 600,
\end{aligned}
$$

Since country b refuses to print additional money.

– The value of country b money is

$$
\begin{aligned}
\bar{e}v_t^b M_t^a + v_t^b M_t^b &= N_t^a \left(y^a - c_{1,t}^a\right) + N_t^b \left(y^b - c_{1,t}^b\right) \\
\frac{1}{2}v^b(400) + v^b(600) &= 1000 + 1000 \\
800v_t^b &= 2000 \\
v_t^b &= 2.5
\end{aligned}
$$

– The value of country a money is

$$v_t^a = \bar{e}v_t^b = \frac{1}{2}(2.5) = 1.25$$

– Consumption of each old is

$$c_t^b = v_t^a(2) + v_t^b(4) = (1.25)(2) + (2.5)(4) = 12.5 \text{ goods.}$$

and

$$c_t^a = v_t^a(2) + v_t^b(4) - (tax) = (1.25)(2) + (2.5)(4) - 2(2.5) = 7.5 \text{ goods.}$$

- In general, whenever some people decide to exchange their holdings of country a money for country b money and country a government honors these requests,
  – M^a falls and M^b stays the same;
  – values of both monies increase;
  – country b old's consumption increases;
  – country a initial old's consumption decreases because of the tax payment.
  – Overall, the unilateral defense policy has resulted in a transfer of goods from each old in country a to each old in country b.

Speculative Attacks on Currencies

- The unilateral defence of the fixed exchange rate relies on the government's willingness to take actions (taxation) that make its citizens worse off.
  – Would the government follow such a commitment?
  – If the government lacks the will to take any of the actions to defend the exchange rate, what would people anticipate?
- More realistically, the government is prepared to take limited action to defend the exchange rate.
  – For example, the government is willing to tax its citizen a limited amount – F goods. The government is committed to defending the exchange rate until the tax bill of this policy reaches F goods.
    ∗ If fewer than F goods worth of domestic currency are turned in for exchange, the fixed exchange rate is maintained.

        * If more than F goods worth of domestic currency are turned in for exchange, the government abandons its efforts to fix the exchange rate. Domestic currency will depreciate.
- A limited government commitment may encourage speculative attacks in foreign currency markets in a way that does not occur when the government's commitment is total.
- Consider the two country OLG model. Suppose that country a keeps a fixed exchange rate with country b with a limited commitment to defend its exchange rate. Speculators may want to exchange country a currency for country b currency.
    - For people who hold country a currency, exchanging country a currency for country b currency is a can't-lose action.
        * If country a government's commitment is sufficient to defend the exchange rate, the exchange rate does not change.
        * If country a government's commitment is too small to defend the exchange rate, country a currency will depreciate. For those who have exchanged country a currency for country b currency, they benefit because country b currency appreciates.
    - For country a citizens, they are in a can't-win situation.
        * If country a government's commitment is sufficient to defend the exchange rate, country a citizens need to pay the tax.
        * If country a government's commitment is too small to defend the exchange rate, country a currency will depreciate. If country a citizens hold most of the country a currency, their money's value decreases.
- Examples of speculative attacks:
    - Mexico in 1994;
    - the Asian Financial Crisis in 1997;
    - Brazil in 1999;
    - Argentina in 2002.
- The Asian Financial Crisis in 1997:
    - In the 1990s, Thailand, Malaysia, Indonesia and the Philippines were added to the list of Asian Tigers (Hong Kong, South Korea, Singapore and Taiwan, all of which displayed high rates of economic growth from the early 1960s to the 1990s).
    - In July 1997, the Thai baht which was fixed at 25 baht for 1 USD came under speculative attack. The Baht depreciated from 25 Baht per USD to around 65 Baht per USD in 1997.
    - The crisis spread to other southeast Asian countries, including Malaysia, Indonesia, the Philippines and South Korea.
    - The crisis brought to an end a period of extraordinary economic growth in southeast Asia.
- In general, a country's currency is more likely to be subject to speculative attacks when
    - the government adopts a fixed exchange rate;
    - the government lacks sufficient reserves and cannot resort to unlimited taxation of its citizens;
    - there is some change of the economic condition which causes concerns about the value of the country's currency;
- These concerns could trigger speculative attacks on the country's currency.
    - If speculative attacks are successful, the country's currency depreciates.
    - If speculative attacks are not successful, the country defends its exchange rate.

- Speculative attacks on currency is called currency crisis. Sometimes currency crisis can lead to financial crisis.

4.6 *The Optimal International Monetary System*

Currency Union

- What is the optimal international monetary system?
  - Cooperative stabilisation: hard to implement because of country's incentive to inflate.
  - Unilateral defences of the exchange rate: speculative attacks on currencies.
- A currency union is where a group of countries share a single currency. Unlike a multilateral fixed exchange rate regime (cooperative stabilisation), the control of the money supply is taken out of the hands of individual member countries and relegated to a central authority. This arrangement avoids the issue of member countries' incentives to inflate.
- The European Currency Union (ECU) adopted the Euro in 1999. The central bank is called the European Central Bank.
  - The ECB is governed by a board of directors and headed by a president.
  - Each country's central bank does not have independent influence on its domestic monetary policy.
- Having a centralised monetary authority help to
  - reduce the costs of conducting international trade;
  - mitigate the lack of coordination in domestic monetary policies.
- Some problems associated with the ECB:
  - feeling of the ECB members that the central authority neglects the "special" concerns of their respective countries;
  - how much seigniorage to collect and distribute among member countries;
  - how should monetary policy help to solve countries' fiscal problems.
- The success of a currency union depends largely on the ability of the centralised monetary authority to deal with issues of competing political interests.

5
Capital and Private Debt

5.1 *Introduction*

- So far, the only asset in our model is fiat money. In the real world, there are many other assets:
  - capital: goods that are saved for production;
  - private debt: loans that facilitate borrowing and lending;
  - and more.
- In this section, we will examine the interaction between money and other assets (capital and private debt).
  - A model with capital.
  - A model with private debt.

- How can money and other assets coexist?
- Inflation rate and interest rate.

5.2 *A Model with Capital*

- Consider the following production technology:
  - at time t, k_t units of consumption goods are saved as capital goods at time t;
  - at time $t+1$, xk_t units of consumption goods can be produced with k_t units of capital; x is a constant;
  - capital fully depreciates after production.
- Back to our basic OLG model, **suppose that there is no money**. Instead, technology allows the young to save in the form of capital. The old can use capital to produce consumption goods. In addition,
  - each inital old is endowed with k_0 units of capital;
  - population is growing at a constant rate, $n, N_t = nN_{t-1}$.
- Suppose we focus on stationary allocations. An individual faces
  - the first-period budget constraint

$$c_1 + k \leqslant y;$$

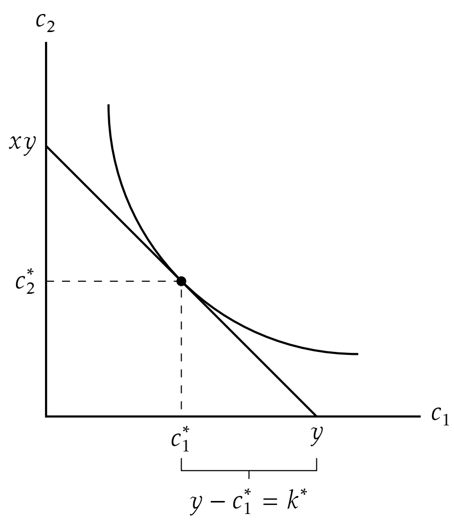
  - the second-period budget constraint

$$c_2 \leqslant xk$$

  - the lifetime budget constraint

$$c_1 + \frac{c_2}{x} \leqslant y$$

  combining the two period budget constraints.
- For any given x, we can depict the lifetime budget constraint.

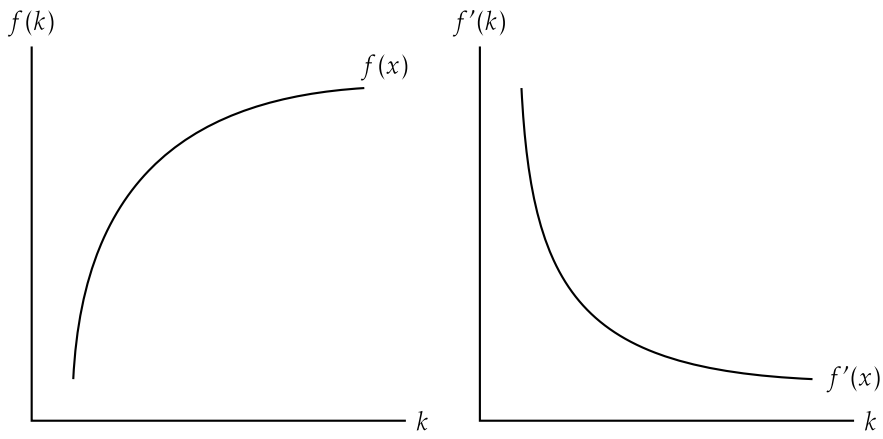


- The individual chooses (c_1, c_2) such that the indifference curve is tangent to the budget constraint. The optimal choice of k^* is

$$k^* = y - c_1^*$$

- This simple model assumes that the marginal product of capital is a constant x. A more realistic assumption is that capital exhibits a "diminishing marginal product". Let $f(k)$ denote a general production function. In general, $f'(k) > 0$. A diminishing marginal product of capital means that

$$f''(k) < 0$$



5.3 *A Model with Private Debt*

- Consider private debt as IOUs issued by individuals – private loans. Let there be two types of individuals:
  - borrowers: endowed with nothing when young and y units of goods when old;
  - lenders: endowed with y units of goods when young and nothing when old.
- In each generation, half of the people are borrowers and the rest half are lenders.
- To begin with, suppose that private debt is the only asset in the economy. **There is neither money nor capital.**
- For a lender,
  - the first-period budget constraint is

$$c_{1,L} + l \leqslant y,$$

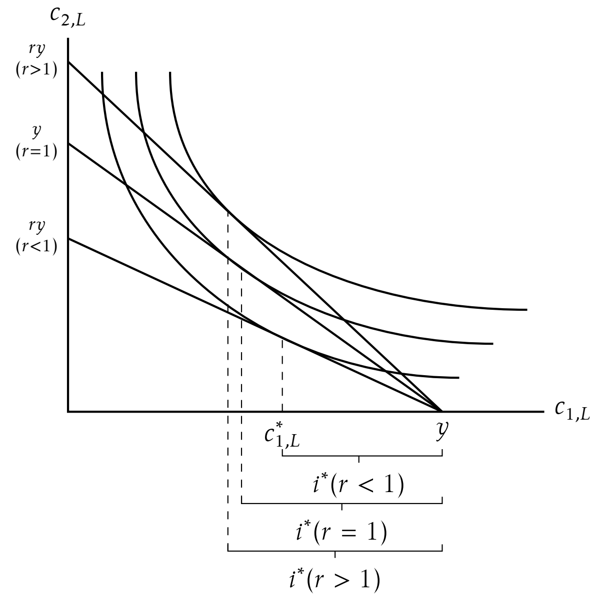
    where l demontes the amount of loans;
  - the second-period budget constraint Is

$$c_{2,L} \leqslant rl,$$

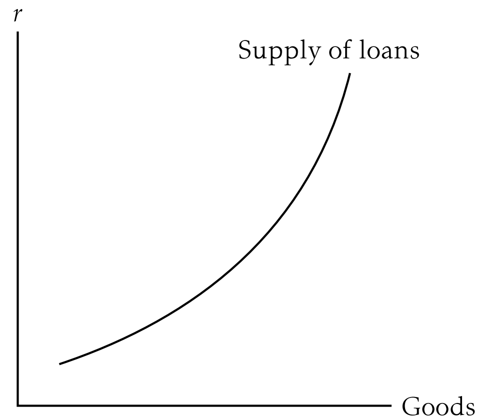
    where r is the gross, real interest on loans;
  - the lifetime budget constraint is

$$c_{1,L} + \frac{c_2, L}{r} \leqslant y.$$

    by combining the two period budget constraints.
- We can depict the lender's lifetime budget constraint.



- The lender chooses (c_1, c_2) such that the indifference curve is tangent to the budget constraint.
- We assume that preferences are such that as r increases, $c_{1,L}$ decreases so that l increases. Let L denote the aggregate supply of loans.



- For a borrower,
  - the first-period budget constraint is

$$c_{1,B} \leqslant b,$$

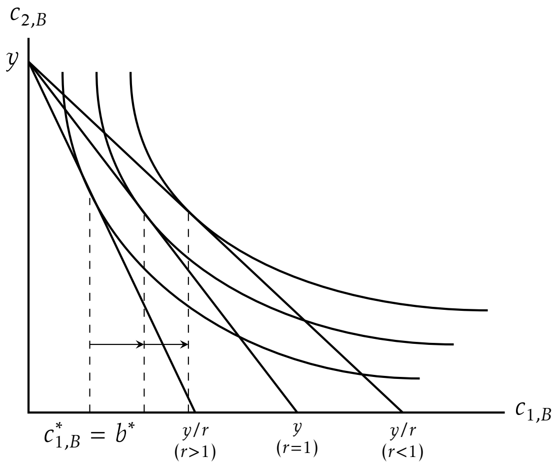
    where b denotes the amount of debt (loans);
  - the second-period budget constraint is

$$c_{2,B} \leqslant y - rb$$

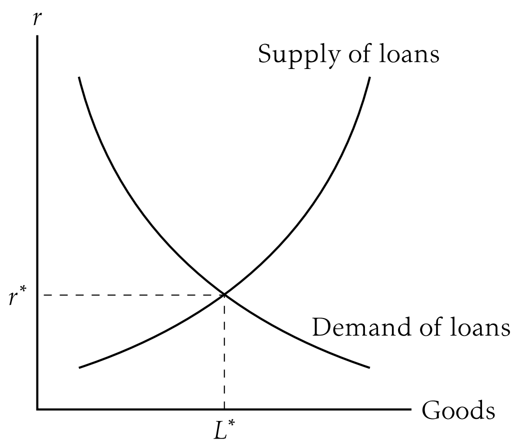
  - the lifetime budget constraint is

$$c_{1,B} + \frac{c_{2,B}}{r} \leqslant \frac{y}{r}.$$

    by combining the two period budget constraints.
- We can depict the borrower's lifetime budget constraint.



- The borrower chooses (c_1, c_2) such that the indifference curve is tangent to the budget constraint.
- We assume that preferences are such that as r increases, $c_{1,B}$ decreases so that b decreases. Let B denote the aggregate demand for loans.



5.4 *Rate of Return Equality*

- Suppose that we introduce capital to our model with private debt.
  - The marginal product of capital is x.
  - The rate of return on loans is r^*.
- How should **lenders** choose between capital and private loans?
  - What would happen if $x > r^*$?
  - What would happen if $x < r^*$?
  - What would happen if $x = r^*$?
- For people to be willing to hold both capital and loans as assets, we must have

$$x = r$$

  or more generally

$$f'(k) = r$$

- Suppose that there are many assets available to individuals. Without any uncertainty about returns and any government restrictions (**perfect substitutes**), the

rate of return on these assets must be identical for people to hold all available assets simultaneously.
- We refer to this as the principle of "**rate-of-return equality**".

5.5 *Coexistence of Money and Other Assets*

- If we introduce money into our model with capital and private debt, the rate-of-return equality requires that for all assets to be held by individuals,

$$\frac{n}{z} = r = z$$

Here n is the growth rate of population and z is the growth rate of money supply; so n/z is the rate of return on money.
- **If all assets are perfect substitutes**, then rate-of-return equality holds for all assets to coexist. In this case, how does money interact with other assets?

The Tobin Effect

- Consider a standard OLG model with money and capital. Each young is endowed with y units of consumption goods when young and nothing when old.
- Suppose that capital displays a diminishing marginal product. That is, $f''(k) < 0$.
- For both money and capital to be valued, we must have

$$f'(k) = \frac{n}{z}$$

For any given (n, z) we can find a desired level of capital stock.
- What if there is a permanent increase in z?
- Graphically, we show the determination of k.
- When z increases, n/z decreases and k^* increases.
- The substitution of capital for money in reaction to an increase in inflation, described by Tobin (1965), is called the "**Tobin effect**".
- Does the Tobin effect suggest that an increase in z could help to increase output? In our model, the output in period t is

$$GDP_t = N_t y + N_{t-1} f(k_{t-1}).$$

If we live in a world **where money and capital are perfect substitutes, an increase in z would raise k and hence output**. Should the policy maker use z as a tool to raise output?
  - Output ≠ Welfare.
  - The Tobin effect is not large in the real world.

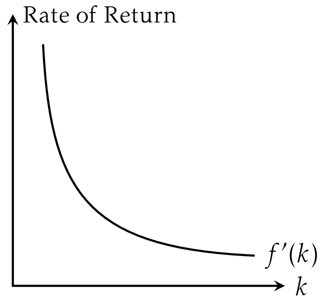


Figure 5.1. Tobin effect

5.6 *The Golden Rule Capital Stock*

- Let the feasible set be

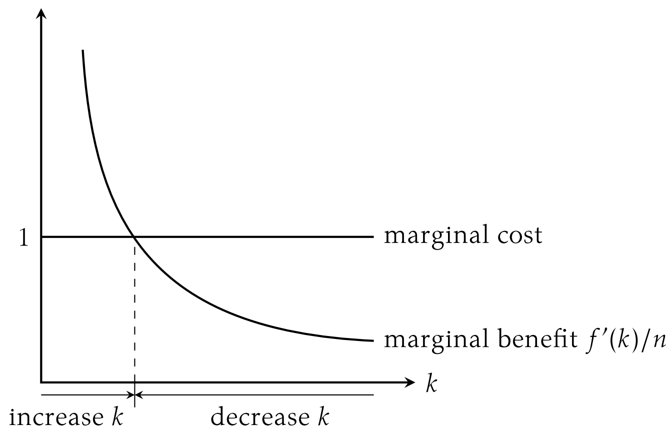
$$N_t c_{1,t} + N_{t+1} c_{2,t} + N_t k_t \leqslant N_t y + N_{t-1} f(k_{t-1})$$

Dividing by N_t to find the feasible set for per person. If we also restrict ourselves to stationary solutions, we can elimiate the time subscriptss. These simplifications results in

$$c_1 + \frac{c_2}{n} + k \leqslant y + \left[\frac{f(k)}{n}\right]$$

or

$$c_1 + \frac{c_2}{n} \leqslant y + \left[\frac{f(k)}{n} - k\right]$$



5.7 *When Money and Other Assets are not Perfect Substitutes*

- In the real world, fiat money and other assets **are not perfect substitutes**. In particular, the rate of return on money is generally lower than the rate of return on other assets. What are the effects of anticipated inflation on interest rate, capital and output?
- This raises an obvious question: why would money still be valued? We postpone this question for later sections. For now, consider a simple argument: legal restriction. Each young is required by law to acquire money worth a fixed number of goods q^*
- Nominal interest rate and real interest rate.
  - **Nominal** interest rates: the number of **dollars** paid in interest for each dollar lent.
  - **Real** interest rates: the number of **goods** paid in interest for each good lent.
  - Nominal interest rates are the ones cited by financial intermediaries and the press.
  - In times of inflation, nominal intrest rates do not reflect the real rate of return.
  - Let R_t and r_t denoate the nominal interest rate and the real interest rate. Let p_t denote the price of a good.
- We can express the gross interest rate r_t as

$$r_t = \frac{R_t p_t}{p_{t+1}} = R_t \frac{p_t}{p_{t+1}} \tag{5.1}$$

If we arrange equation (5.1), we can obtain

$$\underbrace{R_{t-1}}_{\substack{\text{net nominal}\\\text{interest rate}}} = \underbrace{(r_t - 1)}_{\substack{\text{net real}\\\text{interest rate}}} + \underbrace{\left(\frac{p_{t+1}}{p_t} - 1\right)}_{\text{net inflation rate}} + (r_t - 1)\left(\frac{p_{t+1}}{p_t} - 1\right)$$

- For low values of the real interest rate and the inflation rate, we have approximately:

  net nominal interest rate = net real interest rate + net inflation rate.

- **Anticipated inflation and the nominal interest rate:** the predicted full adjustment of the nominal interest rate to anticipated inflation is called the *"Fisher effect"* - named after Irving Fisher
- Consider a model with moneym capital and private debt. Suppose that the marginal product of capital is x. The rate-of-return equality implies that

$$x = r_t = R_t \frac{p_t}{p_{t+1}}$$

Note that here money is held by people **because of the legal restriction. The rate-of-return equality applies to capital and private debt**. As the rate of return on money is $p_t/p_{t+1} = n/z$, we have

$$x = R_t \frac{n}{z} \quad \text{or} \quad R_t = x\frac{z}{n}$$

- The real interest rate is a constant x. The nominal interest rate rises with anticipated inflation to keep the real interest rate constant x.
- There is a tendency for nominal interest rates and inflation rates to move together in accordance with the Fisher effect. However, the gap between the nominal interest rate and the inflation rate is not cosntant due to changes in the real interest rate.
- Will inflation affect the real interest rate?
  - In our previous example,

$$x = r_t = R_t \frac{n}{z}.$$

    The real interest rate is always a constant x by the rate-of-return equality. An increases in inflation will only affect the nominal interest ratem but not the real interest rate. The key assumption is a constant marginal product of capital.
  - What if capital exhibits a diminishing marginal product?
- **Anticipated inflation and the real interest rate:** an exception to the Fisher effect could occur **if money, private debt, and capital are subsitutes and capital has a diminshing marginal product.** In this case,
  - an increase in the inflation rate leads to an increase in capital - the Tobin effect;
  - an increase in capital lends to a decline in the real interest rate because of diminishing marginal product of capital;
  - overall, an increase in inflation will still lead to a rise in the nominal interest rate, however, beacuse of the simultaneous decrease in the real interest rate, the nominal interest rate will not rise by the full amount of the rise in inflation.

5.8 *Risk*

- So far, we have assumed that all assets pay a rate of return that is known with complete certainty. What would happen to rate-of-return equality if instead one asset has a random rate of return?
  - For example: there is some positive probability that a loan will not be repaid by the borrower. If the marginal product of capital is a constant x, then capital and private debt are **not** perfect substitutes.
- If people do not care about risk (are "risk neutral"), then the rate-of-return equality still holds on average. Suppose that an asset pays return $r_1, r_2, \ldots, r_n$ with probabilities $\pi_1, \pi_2, \ldots, \pi_n$,respectively. The expected rate of return on this asset

can be calculated as

$$E(r) = \pi_1 r_1 + \pi_2 r_2 + \cdots + \pi_n r_n$$

The rate-of-return equality is modified to

$$E(r) = x$$

- Are people risk neutral? Probability not.
- If people are risk averse, people may still accept a risky asset. However, the rate-of-return equality will not hold in this case. In fact, the expected rate of return on this risky asset must exceed that of the risk-free asset, compensating for the risk.
- The extra average rate of return that is necessary to entice people to hold a risky asset is called a risk premium,

$$\text{risk premium} = E(r_{\text{risky}}) - r_{\text{safe}}$$

  The greater the potential loss and the greater the probability of the loss, the larger the risk premium must be.
- In many real economies, why do people choose to hold fiat money when many alternative assets appear to offer greater rates of return?

6
Liquidity and Financial Intermediation

6.1 *Introduction*

- We have discussed why money is valued in many real economies in which other assets have a greater return than money:
  - risk premium
- In this section, we consider an additional explanation that money is valued despite its low return: liquidity.
  - Money is a liquid asset.
  - Illiquid assets offer a higher rate of return than liquid assets.
  - Financial intermediation could emerge by borrowing at a low rate of return while investing at a high rate of return.

6.2 *Money as a Liquid Asset*

- People might hold money and other assets for different motives.
  - Why do people hold money?
  - Why do people hold assets such as houses?
- Compared with other assets, money
  - change hands much more frequently;
  - is held for shorter periods of time;
  - is less costly to exchange.

- We say that an asset is "liquid" if it is exchanged easily, quickly, and at little cost. Money is the most liquid asset.

6.3 *A Model of Illiquidity*

- We develop a model based on Freeman (1985) to capture the essential distinctions between liquid and illiquid assets. The model is consistent with the following observations:
  1. money and capital are both valued;
  2. the rate of return of capital exceeds that of money;
  3. money is exchanged more often than capital.
- Consider an economy of overlapping generations in which people live for three periods: young, middle age and old.
- People are endowed with y units of the consumption good when young and nothing in the other two periods of life. Preferences are such that people value consumption in all three periods.
- Let N_t represent the number of people in the generation born at time t, with $N_t = nN_{t-1}$.
- There is a constant supply of money M in the economy. Money is distributed equally among the initial middle-aged individuals.
- There is a single physical asset, capital.
  - A unit of capital may be created from a unit of the consumption good in any period t.
  - Capital may be created in any amount.
  - Two periods after capital is created, a unit of capital produces X units of the consumption goods and then fully depreciates. Let $X > n^2$
  - Each initial old begins with a stock of capital that produces Xk_0 goods in the first period.
- Two assumptions about information: we assume that
  - it is expensive/impossible to observe the capital created by others, which implies capital cannot be traded in the 2nd period of life for consumption – capital is illiquid.
  - it is impossible to enforce repayment of IOUs because people can costlessly hide from anyone looking for them and in this way avoid repaying their IOUs – no private debt.
- The second assumption ensures that private borrowing and lending is not feasible in this economy. We will relax this assumption later.
- Let $c_{1,t}, c_{2,t+1}, c_{3,t+2}$ denote the consumption in the first, second and third periods of life for an individual born in period t, respectively.
- Given the pattern of endowments, individuals need to find ways to provide consumption in the second and third periods of their lives. Individuals can choose between capital and money.
  - Second period: can use only money.
  - Third period: can use both money and capital. Should an individual use capital or money to finance the third-period consumption?
    * If the individual holds money from the first period to the third period, the two-period rate of return of money is

$$\frac{v_{t+2}}{v_t} = \frac{v_{t+2}}{v_{t+1}}\frac{v_{t+1}}{v_t} = n^2$$

    * If the individual holds capital from the first period to the third period, the two-period rate of return of capital is X.

    * Which asset would the individual choose to hold to finance the third-period consumption?

- We can summarise the budget constraints faced by the individual as
  - the first-period budget constraint

$$c_{1,t} + v_t m_t + k_t \leqslant y,$$

  - the second-period budget constraint

$$c_{2,t+1} \leqslant v_{t+1} m_t$$

  - the third-period budget constraint

$$c_{3,t+2} \leqslant X k_t$$

  - Combining the three period budget constraints, we have

$$c_{1,t} + \frac{v_t}{v_{t+1}} c_{2,t+1} + \frac{1}{X} c_{3,t+2} \leqslant y,$$
$$\text{or } c_{1,t} + \frac{1}{n} c_{2,t+1} + \frac{1}{X} c_{3,t+2} \leqslant y$$

- The frontier of the budget set would be a plane in a three-dimensional space. The optimal $c^*_{1,t}, c^*_{2,t+1}, c^*_{3,t+2}$ combination would be located where an indifference curve is tangent to this plane.
- Some observations from the model:
  1. money and capital are both valued – money is used to finance the second-period consumption while capital is used to finance the third-period consumption;
  2. the rate of return of capital exceeds that of money – the rate of return on capital is X, which is greater that the rate of return of money n^2;
     - the rate of return of equality is violated: because money and capital are not perfect subsitutes;
     - Money: liquid. Capital: illiquid
  3. money is exchanged more often than capital – how should we compare the trading frequency of these two assets?
- Define the velocity of an asset as the amount of the asset that is exchanged in a given period of time divided by the total stock of that asset.
  - In any period t, the total stock of money change hands. It implies that eh velocity of money is 1.
  - If we view a young individual's creation of capital as an exchange, the amount of new capital created in period t is $N_t K_t$. The total stock of capital in period t is $N_t k_t + N_{t-1} k_{t-1}$. The velocity of capital is thus

$$\frac{N_t k_t}{N_t k_t + N_{t-1} k_{t-1}} < 1$$

    In this simple case that the total capital stock does not change in size over time, the velocity of capital is 1/2.
  - It verifies that the velocity of money is higher than the velocity of capital.

6.4 *The Business of Banking*

- Suppose that now at least some people cannot hide from their creditors so that enforcement of their IOUs is possible. → This allows the emergence of banking and a new form of liquid asset – private debt (IOUs).

- Suppose that you are the only one in the economy who can issue IOUs. How might you use your ability to make profits?

A Simple Arbitrage Plan

- Here is a plan:
  - In period t, borrow one good from the young in period t (and issue a one-period IOU) and invest the good in the creation of capital. To induce the young to lend, you need to promise to pay the rate of return at least n (rate of return on money)
  - In period $t+1$, repay n to your lender (and take back the one-period IOU issued at period t) by borrowing n from the young born in period $t+1$ (and issuing another one period IOU). To induce the young to lendm you need to promise to pay thre rate of return at least n.
  - In period $t+2$, repay $n \times n = n^2$ to your lender (and take back the one-period IOU issued at period $t+1$) from production using capital. total output produced is X. Net profit is $X - n^2 > 0$
- In this case, you are essentially a third party that channels economy's saving to assets with the highest rate of return

The Effect of Arbitrage on Equilibrium

- Suppose that you are not the only one that is able to ussye IOUs. A large number of competitive people (labelled as banks or intermediaries) can borrow goods from the younf (by costlessly issuing IOUs) and invest the goods in capital creation. How does this affect our equilibrium outcome> In particular, what will the one-period rate of return (call it r) paid on these IOUs in a competitive equilibrium be?
  - If you are the only one that issues IOUs, the one-period interest rate on IOUs is $r = n$. (The minimum rate to entice lenders to lend.)
  - With a large number of people issuing IOUs, they compete for lenders till $r = X^{1/2}$. Then there eill be no incentive for intermediaries to offer higher interest rates on IOUs. Perfectly competitive intermediaries drive their profits to 0.
- In the absence of financial intermediation, capital was held only to acquire consumption in the third period of life; money was held to acquire consumption in the second period of life.
- With financial intermediation, IOUs (inside money) replaces money (outside money) in the acquisition of consumption in the second period of life. Inside money: "money" issued by private financial intermediaries. Outside money: fiat money issued by central bank.
  - People invest in capital directly to acquire consumption in the third period of life.
  - People invest in capital indirectly through intermediaries to acquire consumption in the second period of life.
- Financial intermediation serves to mobilize all the savings of the economy for investment in the asset that generates a greater rate of return. It implies that output will be higher in an economy with financial intermediation.
- In terms of welfare, all future generations benefit. Only the initial middle-aged are worse off (money loses value as people abandon it).

6.5 *Summary*

This section focuses on models where money and other assets are **not** perfect substitutes.

- In the model of illiquidity, money serves as the liquid asset and capital serves as the illiquid asset. Money earns a lower rate of return, but is exchanged more frequently.
- If some people can issue private IOUs, financial intermediaries naturally emerge to take advantage of the rate-of-return differences in money and capital. These intermediaries provide a service by correcting the mismatch of maturities between liquid money and illiquid capital.
- There are other roles for banks or financial intermediaries
  - Banks serve as monitors of risky investment: diversification and lower monitoring cost.
  - Banks allow people to insure each other.
  - Banks can help reduce the cost of evaluating loans.
  - Banks can enjoy other economies of scale.

7
Bank Risk

7.1 *Introduction*

- In our model of banks, banks take deposits and invest in interest-bearing assets. Are there any risk that banks face?
- In the real world, there are many instances of bank failures.
- In this chapter, we focus on two possible reasons for bank failures:
  - a sudden rush of withdrawals;
  - unexpectedly low return on interest bearing assets.

7.2 *A Model of Demand Deposit Banking*

- Banks have liabilities that are payable on demand but assets that are not. This mismatch of bank assets and liabilities raises the possibility of a "bank panic" or "bank run".
  - If depositors all withdraw at once, a bank must borrow or sell its assets to pay them off.
  - If all depositors fear that the rush of others to withdraw will leave them with nothing, they will rationally join the rush. This is called a "bank run".
- Assume that N three-period-lived individuals are born each period (in overlapping generations).
- Each individual is endowed with y units of consumption goods only when young.
- No one consumes when young. Everyone wants to consume in one of the next two periods of life, depending on their type.
  - Early consumer: with 0.5 probability, the young becomes a type 1, who consumes in the second period of life.

  - Late consumer: with 0.5 probability, the young becomes a type 2, who consumes in the third period of life.
- No one knows his type when young. Everyone learns his type in the second period of life. **An individual's type is not observed by anyone else, including banks.**
- Individuals have access to two assets: storage and capital
  - Storage: the gross rate of return is 1 over one period.
  - Capital: produces X goods for each good invested two periods after its creation, where $X > 1$.
- Early liquidation of capital: capital can be sold one period after its creation.
  - The price is v^k, where $v^k \leqslant X$. (why this must be true?)
  - There is a verification cost θ per unit of capital. We assume $1 + \theta > X$
- Assume that private IOUs are possible among members of the same generation, but not between generations.
- Effective rates of return on storage and capital:

Effective rates of return on:	One Period	Two Periods
Storage	**1**	1
Capital	$v^k - \theta$ (early liquidation)	X

- This structure of returns ensures that $v^k - \theta < 1$
- **Individuals do not know their type at the time he chooses his asset.** How should individuals select assets to save for future consumption?
  - Storage: offers a **better** one-period return, but **worse** two-periods return.
  - Capital: offers a **worse** one-period return, but **better** two-periods return.
- Suppose that there exist banks. Banks know that in each generation, half of its people will be of each type. If all individuals deposit their endowments at the bank, the bank can invest half of its deposits $Ny/2$ in storage and half in capital. A bank can offer the rate of return on deposits
  - 1 after one period;
  - X after two periods.
- Individuals enjoy the higher rate of return in both periods.
- Banks provide liquidity by taking advantage of the fact that there is more randomness for an individual than for the aggregate economy.
- In our model, the bank provides demand deposits: the bank relies on the word of the depositor and make returns available to any depositor who asks for them. Would all depositors tell the truth?
  - Type 1 people will not pretend to be type 2.
  - Type 2 people do not want to claim to be type 1: if they do so, they will withdraw their deposits earlier and store the goods by themselves. Overall, the rate of return is 1, which is less than X.

7.3 *Bank Runs*

- Suppose you are a type-2 consumer and now you hear a **rumor** that every other type 2 is going to pretend to be type 1 in order to withdraw his deposits from the bank. Would you rush to the bank and try to withdraw early?
  - The bank has $Ny/2$ in storage which allows the bank to payoff $N/2$ people. Each withdraws y units of goods.
  - If you do not withdraw and a large number of type 2 withdraw early, the bank has to sell some of its capital to meet the demand. For every 1 unit of capital sold, the bank receives $v^k - \theta$ goods. Therefore, the total capital stock $Nt/2$ can be sold to pay off $Ny\left(v^k - \theta\right)/2$. Given that $v^k - \theta < 1$, **the**

bank cannot pay off all type 2 people. An honest type 2 may get nothing if everyone else withdraws early and he does not!
  - If every type 2 believes that all others will rush to the bank to withdraw early, he will also rush to the bank to withdraw early.
  - In this way, the bank would not be able to meet all its obligations and a bank run occurs.

7.4 *Preventing Panics*

- How can the bank in our model avoid the bank run? There are several ways that economists have identified.
- Interbank lending: if a bank faces a run can borrow enough to meet all withdrawals, it can avoid the losses from the sale of its capital.
  - If a bank is threatened by a run, it could borrow from other banks or people who are not experiencing runs.
- Identifying unnecessary withdrawals: if the bank can identify an individual's type, the bank can stop the bank run by refusing to allow type 2 people to withdraw early.
- Suspension of withdrawals: the bank can temporarily close its doors when its reserves of the liquid short-term asset (storage) have been used up, and then reopen in the next period when its long-term capital pays its return.
  - If the bank follows such a policy, it will never be required to sell its capital at a loss.
  - It follows that if a bank has the right to suspend withdrawals, it may never actually need to do so because depositors will no longer panic.
  - An example: during the bank panics of 1893 and 1907, banks in the U.S. restricted convertibility of deposits into currency.
  - This policy may not work well if the number of type 1 people is **random**.

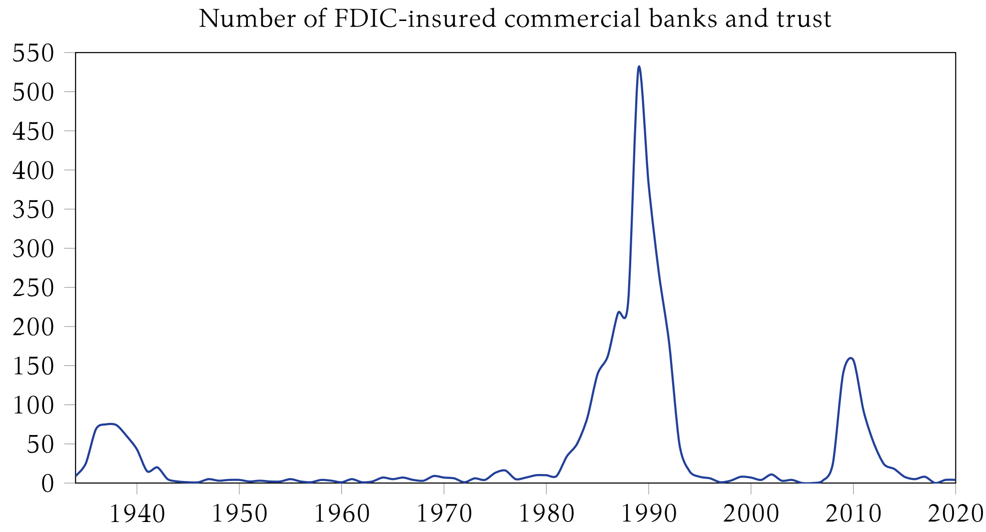
Government Deposit Insurance

- Why should the government care more about bank failures?
- Government deposit insurance: the government can help prevent bank runs by guaranteeing type 2 people that they will receive their promised return even if the bank becomes insolvent.
  - How can the government back up its guarantee? By taxing the endowment of the currently young generation.
  - **If the government guarantee is believed by all people**, no type 2 would want to withdraw early. There will be no bank runs. In this case, the government will never have to use its power of taxation.
  - The government guarantee prevents bank runs costlessly.
  - The government may need to tax people to provide deposit insurance if
    * the number of type 1 people is random and is unusually large;
    * or if bank assets are risky.
  - In the U.S., the Federal Deposit Insurance Corporation (FDIC) gives the resolution costs of $197.68 billion from 1980 to 1994.

7.5 *Bank Failures*

- The U.S. history of bank failures.
  - From 1930 to 1933, bank failures averaged more than 2000 per year.
  - From 1941 to 1981, bank failures averaged five per year.
  - From 1982, bank failures rose and peaked at more than 200 failures in 1988

- From 2007, bank failures rose again.



- Government deposit insurance may actually induce banks to take greater risks than they would if they are not insured by the government – Moral hazard of deposit insurance!

A Bank's Balance Sheet

Assets		Liabilities	
Reserves	γH	Deposits	H
Interest-bearing assets	$(1-\gamma)H + W$	Net worth	W
Total assets	$H + W$	Total liabilities	$H + W$

- Deposits are protected from changes in the value of bank assets by the positive net worth of a bank. We use an example for an illustration.
- An example:

Assets		Liabilities	
Reserves	\$2M	Deposits	\$20M
Interest-bearing assets	\$22M	Net worth	\$4M
Total assets	\$24M	Total liabilities	\$24M

- Suppose that the bank loses 5 percent of its interest-bearing assets because of an unexpected surge in loan defaults.

Assets		Liabilities	
Reserves	\$2M	Deposits	\$20M
Interest-bearing assets	\$20.9M	Net worth	\$2.9M
Total assets	\$22.9M	Total liabilities	\$22.9M

- Although the bank lost only 5% of its interest-bearing assets, the shareholders lost 27.5% (2.9/4=0.725) of their investment in the bank because the entire loss is subtracted from net worth.
- What if the bank loses 20 percent of its interest-bearing assets?

Assets		Liabilities	
Reserves	$2M	Deposits	$20M
Interest-bearing assets	$17.6M	Net worth	$0
Total assets	$19.6M	Total liabilities	$20M

- Insolveny occurs since there are not enough assets to pay off the liabilities.

7.6 *Moral Hazard of Deposit Insurance*

- A bank that takes on too much risk will be unable to attract shareholders or depositors.
- How will a bank attract depositors if there is government depositor insurance?
  - If the government insures depositors against all losses, depositors will no longer be exposed to risk.
  - Depositors will care about only return.
  - Therefore, banks have an incentive to offer high return to attract depositors.
  - High return is generally associated with high risk.
  - A **moral hazard** problem of insurance.

7.7 *Importance of Capital Requirement*

- What can the government do to limit the risk taking that deposit insurance encourages?
- A "capital requirement" forces banks to maintain a net worth no less than some fraction of their assets.
  - Capital requirement provides a cushion to absorb asset losses before depositors or the insurer of the deposits suffers any losses.
- For example, the US government increased the core capital requirements from 3 percent of total assets to **8** percent in 1989;

7.8 *Summary*

In this section, we
- examined two sources of bank failures: runs on banks and bank holding risky assets;
- analyzed four ways that banks or the government can prevent bank runs;
- identified the moral hazard problem associated with government deposit insurance;
- Notice that so far our model of banks does not have money. The available assets are storage and capital. We proceed to the next section to include money and discuss the bank risk in a banking model with money.

8

Liquidity Risk and Bank Panics

8.1 *Introduction*

- Our previous model of bank run: there is no money in the model economies – the model economy focused on real factors.
- In the real economy: liquidity shortages, in the form of too little money (cash), are frequently associated with widespread bank failures, which can turn into bank panics.
- In this section, we develop a model where money and bank panics are clearly linked. We will use the model to examine
  - how money withdrawals are associated with bank panics;
  - the optimal monetary policy.

8.2 *A Model of Random Relocation*

- There are two islands: island 1 and island 2.
- On each island, N (a large number) of two-period lived individuals are born in each period. In the first period, each island has N initial old who live only in the first period.
- Each individual is endowed with y units of a perishable consumption good when young and nothing when old.
- Each individual wants to consume **only when old**.
- When born, each individual faces a risk that he will spend the second period of life on the other island. Each young is notified whether he will be relocated or not at the beginning the second period of life. For a young individual,
  - With probability π, he will be relocated to the foreign island.
  - With probability $1 - \pi$, he will stay on the home island.
- We assume that the relocation probability is the same on both islands. In aggregate, πN individuals will move from island 1 to island 2 and vice versa.
- Money: there is a central monetary authority that controls the money supply.
  - In period t, let M_t denote the aggregate supply of money.
  - Money supply grows at a constant rate, $M_t = zM_{t-1}$. Newly created money is used to finance a lump-sum transfer to young individuals.
- Capital: capital matures in one period with the marginal product x. We assume that $x > \frac{v_{t+1}}{v_t}$, but
  - it cannot move across locations;
  - there is limited communication across islands, which renders claims against capital worthless.

Without Banks

- Decisions of an individual who is born in period t:
  - when young, the individual has y units of good that will be divided between money and capital,

$$c_t m_t + k_t \leqslant y + \tau,$$

    where v_t denotes the value of money in period t and τ denotes the lump-sum transfer from the government;

- when old, the individual's consumption depends on his relocation status:
  * with probability π, the individual is relocated and his second-period budget constraint is

$$c^m \leqslant v_{t+1} m_t;$$

  * with probability $1 - \pi$, the individual stays on the same island and his second period budget constraint is

$$c^n \leqslant v_{t+1} m_t;$$

• Money market clearing condition

$$v_t M_t = 2N(y + \tau - k_t);$$

• We focus on stationary allocations

$$\frac{v_{t+1}}{v_t} = \frac{\frac{2N(y+\tau-k)}{M_{t+1}}}{\frac{2N(y+\tau-k)}{M_t}} = \frac{M_t}{M_{t+1}} = \frac{1}{z}$$

• Since we assume that individuals consume only when old, the young save everything to finance the second-period consumption. The young need to decide how to divide his saving between money and capital.
  – If a young individual chooses to hold only money to finance his second-period consumption, we have

$$c^m = c^n = v_{t+1} m_t = \frac{v_{t+1}}{v_t}(y + \tau) = \frac{y + \tau}{z}.$$

  – If a young individual chooses to hold only capital to finance his second-period consumption, we have

$$c^m = 0 \quad \text{and} \quad c^n = x k_t = x(y + \tau).$$

  – Generally, a risk-averse young individual balances the risk against the return of each asset and acquires a combination of money and capital.
• An individual's problem is

$$\begin{aligned} \max_{m_t, k_t} \quad & \pi u(c^m) + (1 - \pi) u(c^n) \\ \text{s.t} \quad & v_t m_t + k_t \leqslant y + \tau \\ & c^m \leqslant v_{t+1} m_t \\ & c^n \leqslant v_{t+1} m_t + x k_t \end{aligned}$$

• What is the individual's expected utility?

$$\pi u(v_{t+1} m_t) + (1 - \tau) u(v_{t+1} m_t + x k_t)$$

With Banks

• Suppose that banks exist on each island. They can accept deposits and use the deposits to acquire assets.
• We assume that banks can identify who is a mover and who is a non-mover.
• How can banks help individuals achieve a better allocation?
  – Imagine that young individuals deposit their endowments at the banks.
  – On each island, the bank knows that a fraction of π people are movers and a fraction of $1 - \pi$ people are non-movers.

  - After knowing whether to move, movers can go to the banks and withdraw their deposits under the rules established between the bank and the depositor.
  - When old, movers finance their second-period consumption with money and non-movers finance their second-period consumption with capital.
- After accepting the deposits, the bank's problem is to choose the combination of money and capital such that
  - the asset allocation maximizes the expected utility of the individuals;
  - there is enough money to meet the liquidity needs of the movers.
- Formally, we consider the bank's decisions.
  - On each island, the bank needs to decide how to allocate deposits between money and capital. The bank's balance-sheet constraint in period t is

$$v_t m_t + k_t \leqslant y + \tau = d_t$$

    where d_t stands for the quantity of goods deposited by each individual.
- Note that d_t, m_t, k_t in this balance-sheet constraint are expressed as per capita variables. We can define the reserve-to-deposit ratio $\gamma = \frac{v_t m_t}{d_t}$
- The bank's decisions
  - The bank can pay movers up to the amount of real money balances that the bank possesses. Let r^m be the rate of return on deposits for movers. The deposit contract for movers is

$$r^m d_t = v_{t+1} \frac{m_t}{\pi} \quad \text{or} \quad r^m \pi d_t = v_{t+1} m_t.$$

  - Let r^n be the rate of return on deposits for non-movers. The deposit contract for non-movers is

$$r^n d_t = x \frac{k_t}{1-\pi} \quad \text{or} \quad r^n (1-\pi) d_t = x k_t.$$

  - The bank chooses (m_t, k_t) and (r^m, r^n) to maximize the expected utility of an individual.
- We rewrite an individual's budget constraints when there exist banks:
  - when young, each individual deposits his endowment at the bank

$$d_t \leqslant y + \tau,$$

    when old, a mover's budget constraint is

$$c^m \leqslant r^m d_t,$$

    and a non-mover's budget constraint is

$$c^n \leqslant r^n d_t.$$

- The money market clearing conditions is unchanged.
- A bank's problem is

$$\max_{m_t, k_t, r^m, r^n} \quad \pi u(c^m) + (1-\pi) u(c^n)$$

$$\begin{aligned} \text{s.t} \qquad c^m &\leqslant v_{t+1} m_t \\ c^n &\leqslant v_{t+1} m_t + x k_t \\ v_t m_t + k_t &\leqslant y + \tau \\ r^m \pi(y+\tau) &\leqslant v_{t+1} m_t \\ r^n (1-\pi)(y+\tau) &\leqslant x k_t. \end{aligned}$$

- Key features of the model with banks:
  - important assumption – banks **can costlessly** distinguish a mover from a non-mover;
  - bank contract – deposit contract distinguish between movers and non-movers;
  - role of banks – banks provide liquidity insurance for depositors.

Comparison of the cases with and without Banks

- Expected utility in the case with banks

$$\pi u(r^m d_t) + (1-\pi) u(r^n d_t)$$

- Expected utility in the case without banks

$$\pi u(v_{t+1} m_t) + (1-\pi) u(v_{t+1} m_t + x k_t)$$

- Which one dominates?
  - For non-movers
$$\frac{v_{t+1}}{v_t} v_t m_t \leqslant \frac{v_{t+1}}{v_t}(y+\tau)$$
  - For movers
$$\frac{v_{t+1}}{v_t} v_t m_t + x k_t \leqslant x(y+\tau)$$

  Hence expected utility of banks dominates for both non-movers and movers.

8.3 *Optimal Allocation*

- What is the optimal allocation in this economy? The planner maximizes aggregate welfare
$$\pi u(c^m) + (1-\pi) u(c^n)$$
  subject to the resource constraint
$$\pi c^m + (1-\pi) c^n + s = y + xs.$$

- Two observations: the planner
  - should invest all endowment in capital;
  - should not have a preference of one type over the other: perfect risk-sharing;

  It follows that
$$s = y \quad \text{and} \quad c^m = c^n = c^* = xy.$$

8.4 *Optimal Allocation and Optimal Monetary Policy*

- What would monetary policy have to be to achieve perfect risk sharing?
  - Recall that the deposit contract implies that

$$c^m = r^m d_t \quad \text{and} \quad c^n = r^n d_t.$$

To achieve perfect risk sharing, we need to have

$$r^m = r^n.$$

One way to achieve this is to set the rate of return on money equal to the rate of return on capital

$$\frac{1}{z} = x$$

which is also labelled as the **Friedman rule**.

- In general, the choice of the monetary policy that achieves full risk sharing **is not** the policy that achieves the most effcient allocation: it benefits the initial old.
- The monetary policy that maximizes welfare for all future generations is the one that keeps the money supply constant. The intuition follows from the OLG model of money.
  - But the allocation associated with constant money supply may not achieve full risk sharing.
- To summarize, to achieve full risk sharing, the optimal monetary policy is to set $z = \frac{1}{x}$; to achieve the highest expected utility of all future generations, the optimal monetary policy is to set $z = 1$.

8.5 *Bank Risk*

- So far, there is no aggregate uncertainty. Banks do not have any risk. To consider bank risk, we modify our model and assume that π is a random variable.
  - The fraction of people who move is drawn from a distribution of possible outcomes.
- Suppose that the realisation of π could be
  - with probability ε, a high fraction (π^H) of people are going to become movers;
  - with probability $1 - \varepsilon$, a low fraction (π^L) of people are going to become movers;
- Let

$$E(\pi) = \varepsilon\pi^H + (1 - \varepsilon)\pi^L.$$

- How should a bank choose its portfolio then? The bank chooses the amount of money (cash reserve) in the presence of uncertain demand.
- Comparing an economy with a certain π and an economy with a random π, we know that
  - The bank will hold a larger amount of money in the uncertain environment. The reserve-to-deposit ratio is higher when π is random.
  - Since capital offers a higher rate of return, the bank **will not choose** the amount of money to insure against the worst possible shock.
  - The marginal condition balances the value of the insurance against uncertain demand with the opportunity cost of holding low-return money instead of high-return capital.
  - With some probability, the bank's money holding could be too small to meet the realized liquidity demand. In case that the worst possible shock (highest demand for liquidity) is realised, the bank may not have enough cash reserve to meet the demand.