Q.2) Let the length of the longest simple path be L.
Let no. of black nodes be b_L

$b_L \geq \frac{L}{2}$ {if L is even} – ①

$b_L \geq \frac{L+1}{2}$ {if L is odd} – ②

(In order to not violate the double red property.)

Combining ① and ②:

$$b_L \geq \left[\frac{L+1}{2}\right] \qquad \{[\ ] \text{ is the floor function}\}$$

Let length of shortest simple path be S.

$\therefore\ S \geq b_L$ (Since black heights must be equal)

$\therefore\ S \geq \left[\frac{L+1}{2}\right]$

Suppose $L > 2S \Rightarrow L \geq 2S+1$

$$\Rightarrow S \geq \left[\frac{L+1}{2}\right] \geq \left[\frac{(2S+1)+1}{2}\right] = [S+1] = \underline{\underline{S+1}}$$

But $S \geq S+1$ is clearly false.

Hence, by contradiction, we can say that $L \leq 2S$

OR

The longest simple path from a node x in a red-black tree to a descendant leaf has a length at most twice that of the shortest simple path from node x to a descendant leaf.