

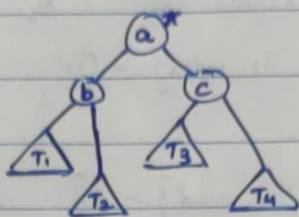
Q.1) Yes, it will be a red-black tree.

○* : Red

○ : Black

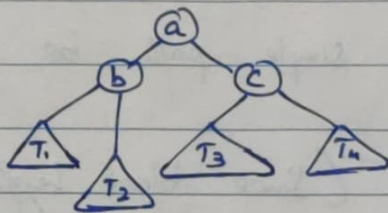
Let original tree be of the form:

We know that this is a relaxed binary tree.



For property (d) to hold, the heights of T_1 , T_2 , T_3 & T_4 must be equal, say h . {Heights of the leaves}

Now, colour a black:



Verifying all properties one by one

<@> Only @ is changing colour from red to black.

No other node is being touched.

Since property <@> held originally, it holds now as well.

<@> leaves are not being touched. Thus, if they were all black initially, they're all black now as well. <@> holds.

<@> Only problem may arise at @ since that is the only node going through a change. But a black node creates no problems with respect to <@> and since @ is turning black, <@> holds.

<@> The change in colour of @ increases black height of EVERY node in the tree by 1. \therefore EVERY leaf goes from the original h to $h+1$. Thus, even though their heights change, it still stays equal.

Hence <@> also holds.

Since the node is now black, all conditions required for a red-black tree are satisfied.