MGT 40750 – Quantitative Decision Modeling
Spring 2017

Spreadsheet Simulation Using @Risk

Professor Hong Guo

Spreadsheet Simulation Using @Risk

- Palisade and @Risk
  - Palisade Corporation is the maker of the market leading risk and decision analysis software @RISK and the DecisionTools® Suite.
  - Virtually all Palisade software adds in to Microsoft Excel, ensuring flexibility, ease-of-use, and broad appeal across a wide range of industry sectors.
  - Its flagship product, @RISK, debuted in 1987 and performs *risk analysis* using *Monte Carlo simulation*. With an estimated 150,000 users, Palisade software can be found in more than 100 countries and has been translated into seven languages.
  - Customers: http://www.palisade.com/industry/customers.asp

Competing Product

- Oracle and Crystal Ball
  - As a result of its acquisition of Hyperion, Oracle has also acquired Decisioneering, makers of Crystal Ball software.
  - Crystal Ball software is a leading spreadsheet-based software suite for predictive modeling, forecasting, Monte Carlo simulation and optimization.
  - With over 4,000 customers worldwide, including 85% of the Fortune 500, Crystal Ball is used by customers from a broad range of industries, such as aerospace, financial services, manufacturing, oil and gas, pharmaceutical and utilities.
  - Crystal Ball is used in over 800 universities and schools worldwide for teaching risk analysis concepts.

@Risk Features

- @Risk contains a good collection of *probability distribution functions* such as RISKNORMAL and RISKDISCRETE.
- Excel cells can be designated as *output cells*. @Risk automatically keeps summary measures such as averages, standard deviations, percentiles, etc. for the output cells.
- @Risk has a special function, *RISKSIMTABLE*, that allows you to run the same simulation several times, using a different value of some key input variable each time.

- Read more about @Risk on pages 561-572 in the text.

Example: Investing for retirement

Amanda has 30 years to save for her retirement. Initially, she invests $8,000 in her retirement account. Each subsequent year she invests 3% more. Estimate the balance of Amanda's retirement account at the end of 30 years.

Model 1: Suppose the *mean* annual return on stocks is 8% (a representative return for her investment style).

	A	B	C	D	E
1	**Investing for retirement**				
2					
3	Initial yearly investment	$8,000			
4	Annual increase in investmen	3.00%			
5					
6	Annual return on investment	8.00%			
7					
8		Amount	Beginning		Ending
9	Year	invested	balance	Return	balance
10	1	=B3	=B10	=B$6	=C10*(1+D10)
11	2	=B10*(1+B$4)	=E10+B11		
12	3				
13	4				
14	5				
15	6				
16	7				
17	8				
18	9				
19	10				
20	11				
21	12				
22	13				
23	14				
24	15				
25	16				
26	17				
27	18				
28	19				
29	20				
30	21				
31	22				
32	23				
33	24				
34	25				
35	26				
36	27				
37	28				
38	29				
39	30				

Model 1 Results:

	A	B	C	D	E
1	**Investing for retirement**				
2					
3	Initial yearly investment	$8,000			
4	Annual increase in investment	3.00%			
5					
6	Annual return on investment	8.00%			
7					
8		Amount	Beginning		Ending
9	Year	invested	balance	Return	balance
10	1	$8,000.00	$8,000	8.00%	$8,640
11	2	$8,240.00	$16,880	8.00%	$18,230
12	3	$8,487.20	$26,718	8.00%	$28,855
13	4	$8,741.82	$37,597	8.00%	$40,605
14	5	$9,004.07	$49,609	8.00%	$53,577
15	6	$9,274.19	$62,852	8.00%	$67,880
16	7	$9,552.42	$77,432	8.00%	$83,627
17	8	$9,838.99	$93,466	8.00%	$100,943
18	9	$10,134.16	$111,077	8.00%	$119,963
19	10	$10,438.19	$130,401	8.00%	$140,833
20	11	$10,751.33	$151,585	8.00%	$163,712
21	12	$11,073.87	$174,785	8.00%	$188,768
22	13	$11,406.09	$200,174	8.00%	$216,188
23	14	$11,748.27	$227,937	8.00%	$246,172
24	15	$12,100.72	$258,272	8.00%	$278,934
25	16	$12,463.74	$291,398	8.00%	$314,710
26	17	$12,837.65	$327,547	8.00%	$353,751
27	18	$13,222.78	$366,974	8.00%	$396,332
28	19	$13,619.46	$409,951	8.00%	$442,747
29	20	$14,028.05	$456,775	8.00%	$493,317
30	21	$14,448.89	$507,766	8.00%	$548,388
31	22	$14,882.36	$563,270	8.00%	$608,332
32	23	$15,328.83	$623,660	8.00%	$673,553
33	24	$15,788.69	$689,342	8.00%	$744,489
34	25	$16,262.35	$760,752	8.00%	$821,612
35	26	$16,750.22	$838,362	8.00%	$905,431
36	27	$17,252.73	$922,684	8.00%	$996,498
37	28	$17,770.31	$1,014,269	8.00%	$1,095,410
38	29	$18,303.42	$1,113,714	8.00%	$1,202,811
39	30	$18,852.52	$1,221,663	8.00%	$1,319,396

Problem?

<u>The flaw of averages</u>

Model 2: Suppose the annual return on stocks follows a *normal distribution* with a mean of 8% and a standard deviation of 25%.

	A	B	C	D	E
1	**Investing for retirement**				
2					
3	Initial yearly investment	$8,000			
4	Annual increase in investme	3.00%			
5					
6	Annual return on investment (normal dist.)				
7	Mean	8.00%			
8	St. dev.	25.00%			
9					
10		Amount	Beginning		Ending
11	Year	invested	balance	Return	balance
12	1	=B3	=B12	=RiskNormal(B$7,B$8)	=C12*(1+D12)
13	2	=B12*(1+B$4)	=E12+B13		
14	3				
15	4				
16	5				
17	6				
18	7				
19	8				
...					
39	28				
40	29				
41	30				output cell

Step 1: Loading @Risk.

Start → All Programs → Palisade Decision Tolls → @Risk

Step 2: Revise Excel worksheet.

RiskNormal

Step 3: Specify *output cells*.

Add output

Step 4: Choose simulation settings.

Iterations = <u>10,000</u>
Simulations = <u>1</u>

Step 5: Run simulation.

Start simulation

Step 6: Examine the results.

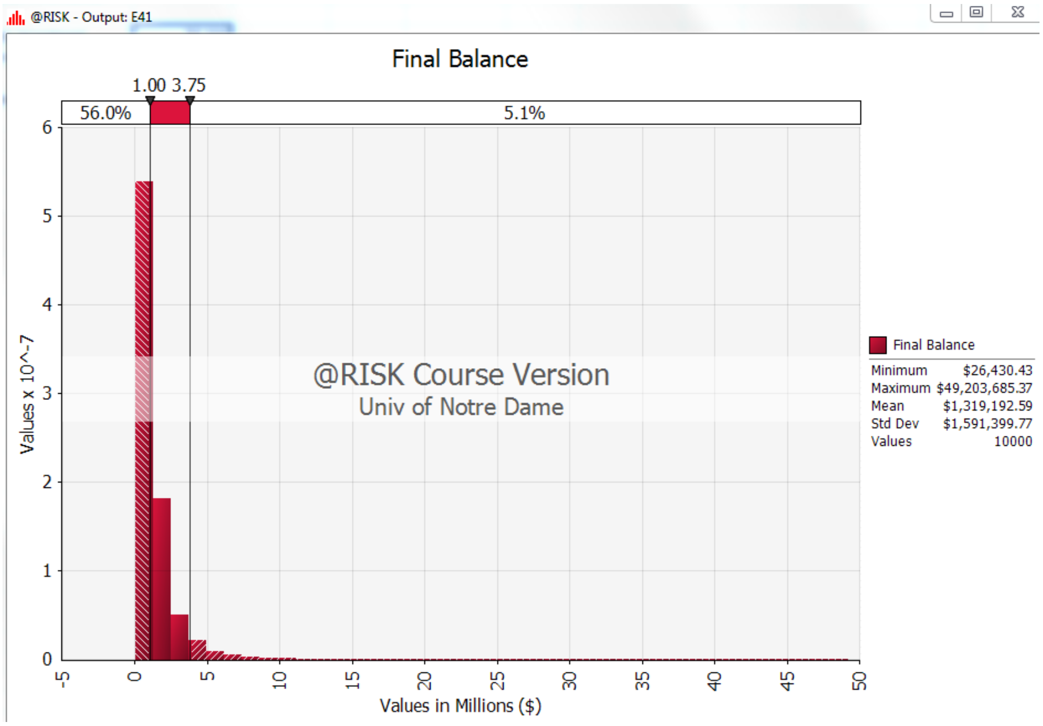
Model 2 Results:

One particular simulation result for Model 2:

Investing for retirement				
Initial yearly investment	$8,000			
Annual increase in investment	3.00%			
Annual return on investment (normal dist.)				
Mean	8.00%			
St. dev.	25.00%			
	Amount	Beginning		Ending
Year	invested	balance	Return	balance
1	$8,000.00	$8,000	16.93%	$9,354
2	$8,240.00	$17,594	45.27%	$25,558
3	$8,487.20	$34,045	17.07%	$39,856
4	$8,741.82	$48,597	16.28%	$56,508
5	$9,004.07	$65,512	9.55%	$71,766
6	$9,274.19	$81,041	-46.97%	$42,979
7	$9,552.42	$52,531	1.02%	$53,066
8	$9,838.99	$62,905	-12.15%	$55,265
9	$10,134.16	$65,399	43.95%	$94,145
10	$10,438.19	$104,583	-10.11%	$94,005
11	$10,751.33	$104,757	7.12%	$112,217
12	$11,073.87	$123,291	25.08%	$154,212
13	$11,406.09	$165,619	-7.83%	$152,650
14	$11,748.27	$164,398	15.43%	$189,758
15	$12,100.72	$201,859	14.61%	$231,356
16	$12,463.74	$243,820	-29.55%	$171,766
17	$12,837.65	$184,603	-29.66%	$129,842
18	$13,222.78	$143,065	48.88%	$213,002
19	$13,619.46	$226,621	11.11%	$251,806
20	$14,028.05	$265,834	-8.06%	$244,409
21	$14,448.89	$258,858	23.71%	$320,221
22	$14,882.36	$335,103	49.16%	$499,854
23	$15,328.83	$515,183	44.20%	$742,884
24	$15,788.69	$758,673	29.47%	$982,240
25	$16,262.35	$998,503	26.65%	$1,264,613
26	$16,750.22	$1,281,364	11.56%	$1,429,518
27	$17,252.73	$1,446,771	42.92%	$2,067,653
28	$17,770.31	$2,085,423	-13.63%	$1,801,123
29	$18,303.42	$1,819,426	41.19%	$2,568,768
30	$18,852.52	$2,587,621	42.40%	$3,684,843

<u>@Risk output for Model 2:</u>

Browse results:



Simulation Detailed Statistics:

@RISK - Detailed Statistics

Name	Final Balance
Description	Output
Cell	Model 2 Results!E..
Minimum	26430.43
Maximum	4.920368E+07
Mean	1319193
Std Deviation	1591400
Variance	2.532553E+12
Skewness	8.066064
Kurtosis	159.3809
Errors	0
Mode	473230.4
5% Perc	240736.9
10% Perc	313589.4
15% Perc	377160.6
…	
Target #1 (Value)	1000000
Target #1 (Perc%)	56%

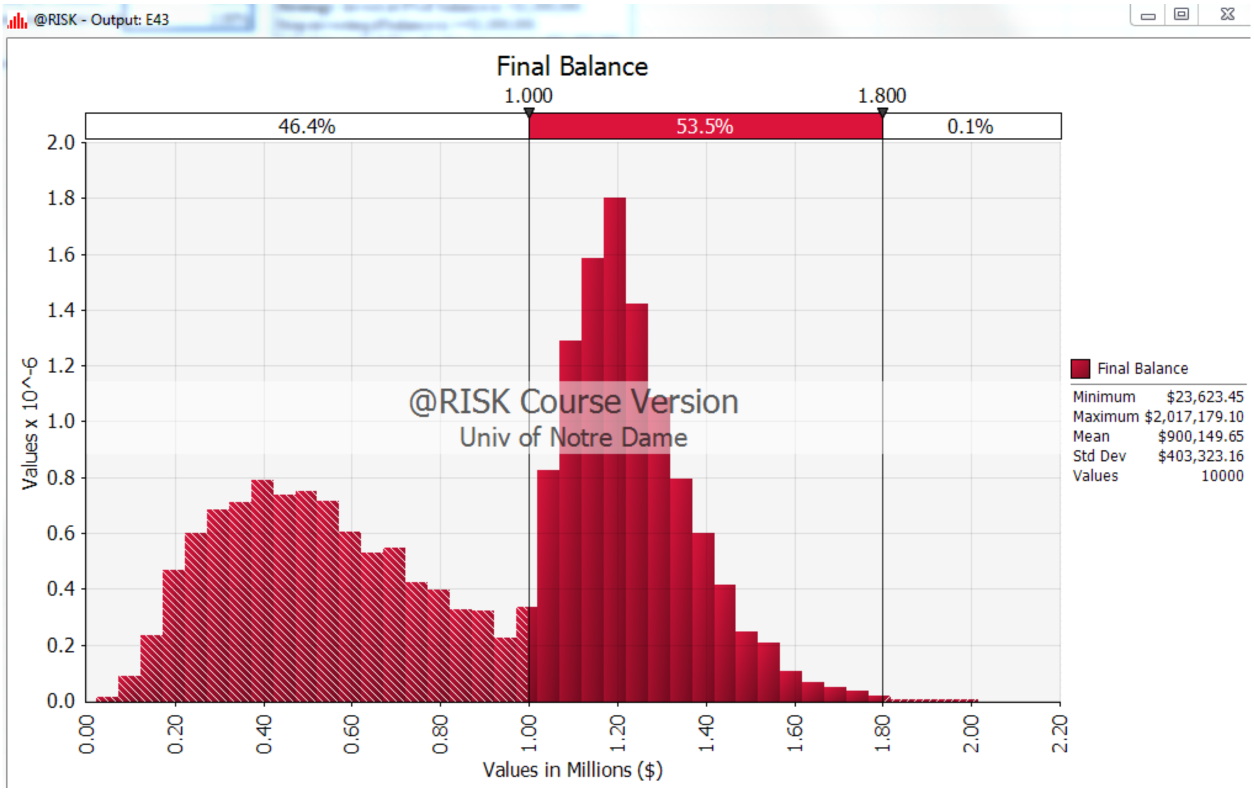
Question: What's the probability of getting a final balance of more than $1,000,000?

Prob(Final Balance >= 1,000,000) = 1 – 56% = 44%

Model 3: Examine a strategy to invest at 8% if balance is <$1,000,000. Stop investing if balance is >=$1,000,000. Calculate the probability of achieving target of $1,000,000.

	A	B	C	D	E
1	**Investing for retirement**				
2					
3	Initial yearly investment	$8,000	**Strategy**: Invest at 8% if balance is <$1,000,000.		
4	Annual increase in invest	3.00%	Stop investing if balance is >=$1,000,000.		
5			Calculate probability of achieving target of $1,000,000.		
6	Annual return on investment (normal dist.)				
7	Mean	8.00%			
8	St. dev.	25.00%			
9					
10	Target	$1,000,000			
11					
12		Amount	Beginning		Ending
13	Year	invested	balance	Return	balance
14	1	=B3	=B14	=If(C14>=1,000,000,0,	=C14*(1+D14)
15	2	=B14*(1+B$4)	=E14+B15	RiskNormal(B$7,B$8))	
16	3				
17	4				
18	5				
19	6				
20	7				
21	8				
22	9				
23	10				
24	11				
25	12				
26	13				
27	14				
28	15				
29	16				
30	17				
31	18				
32	19				
33	20				
34	21				
35	22				
36	23				
37	24				
38	25				
39	26				
40	27				
41	28				
42	29				
43	30				

<u>@Risk output for Model 3:</u>



Question: What is the probability of achieving target of $1,000,000?

Prob(Final Balance >= 1,000,000) = 53.5% + 0.1% = 53.6%
or = 1 – 46.4% = 53.6%

The New York Times

In Search of Better Odds in Retirement Planning

By ELIZABETH HARRIS Published: October 06, 2002

BEFORE Walt Huckabee, a retired engineer, buys mutual fund shares, he likes to take a test drive using a computer-based statistical technique known as Monte Carlo simulation.

He contends that crunching numbers online, using a service available through his account at Vanguard, gives him a better sense of the range of possible financial returns for his investments, in good and bad markets. "It's the difference between theory and practice," said Mr. Huckabee, 60, of Placerville, Calif.

The bear market has made people like him look for more precise methods of assessing whether their portfolios will provide enough retirement income. Companies seeking independent advice to help employees invest for retirement have made Monte Carlo simulation widely available.

The developers of Monte Carlo simulation warn that the technique is imperfect at best, but they say it helps display the range of possible returns more accurately than do other methods.

Traditional financial projections generally assume a fixed annual compounded return, and fixed interest and inflation rates. Monte Carlo simulations, which use random numbers to imitate behavior, run investments through thousands of situations to assess the probability of reaching a financial goal. Calculations assume variations in inflation, interest rates and market returns based in part on history's wide range of returns, going back to the 1920's.

The largest companies that generate Monte Carlo simulations for investors include Financial Engines of Palo Alto, Calif., Morningstar of Chicago and Fidelity Investments, the biggest mutual fund company.

Financial Engines supplies online simulations to 20 financial institutions, including Vanguard, which generally make the techniques available free to their investors. Financial Engines also provides number-crunching services to 800 companies, including McDonald's and Occidental Petroleum, for their employees. Together, these investors number more than three million.

Fidelity has made its Monte Carlo simulations available to 15 million customers. Investors may also get the services directly from Morningstar and Financial Engines.

Mathematicians developed the statistical technique in the 1940's, and it was used in thousands of calculations to develop the first atomic bomb in the Manhattan Project during World War II, said Christopher Jones, executive vice president for financial research and strategy at Financial Engines.

As computer power has become cheaper, many professions have adopted Monte Carlo simulation to describe the behavior of complex systems. It is used to manage traffic, search for oil and determine proper doses for radiation cancer therapy.

"It's useful where the problem you are studying is so complex, it's analytically very hard to determine the overall behavior of the system," Mr. Jones said.

William F. Sharpe, the chairman of Financial Engines and a Nobel winner in economics in 1990, began applying Monte Carlo simulations to individual investors' portfolios later in the decade. With Mr. Jones, he created a software recipe that mixes thousands of pieces of data by tracking 15 types of investments, or asset classes, like long-term government bonds or large-cap value stocks, with financial and economic factors like interest rate shifts and their effects on capital markets. Information about 20,000 stocks and mutual funds contributes in forecasting performance.

Because Monte Carlo simulation takes into account the inherent uncertainty of financial markets, Dr. Sharpe regards it as superior to other ways of gauging future performance -- like calculating a linear, or fixed rate of return. One such linear calculator, found on Quicken.com, tallies investment performance by assuming an annual average rate of return, like 8 percent, and projecting it forward. Monte Carlo simulations use more varied assumptions. All presume differing rates of return over time, but Morningstar's ClearFuture projects returns for entire asset classes, while Financial Engines projects returns for individual mutual funds and equities.

WHEN using an online Monte Carlo program, an investor saving for retirement enters information about when he wishes to retire, how much he has saved and what investments he owns. The results show potential outcomes based on the best, worst and most likely results.

For example, using Financial Engines, a 48-year-old investor with a $300,000 portfolio of stock and bond mutual funds who saves an additional $6,500 annually has an 83 percent likelihood of retiring with at least a $50,000 income at age 65 (including about $20,000 in Social Security income). The results of the calculation would be a range of outcomes, the most likely being $64,700. The calculation also indicates a 5 percent likelihood of hitting $103,000 a year, and an equal probability of receiving $42,100 a year.

With the Quicken calculator, the same investor would get one outcome. Using fixed projections, the investor would retire at 65 with an average income of $71,240, based on an 8 percent expected return and a 3 percent inflation rate. (That includes the same Social Security income.)

"In the past, people were happy to project a linear model, but the problem with that is, it's linear," said Ray Martin, president of CitiStreet Advisors. "If you're projecting a 10 percent return, the problem is it has to be 10 percent. I call it a perfect 10 and there's no investment that even grows at a perfect 3 -- there's always variability."

Understanding variable market returns helped Mr. Huckabee, the retired engineer, reshape his portfolio. When he retired in 1998 as a system engineer at Lockheed Martin, he began to understand the risks of his growth-heavy 401(k) portfolio. Financial Engines calculations suggested that adding more value stocks would lower his risk; he says his portfolio has been beating the market. "I've been treading water in this downturn," he said, "but it's better than the significant declines you would have if you were just taking the total market approach."

But the Monte Carlo method has its critics. David Nawrocki, a finance professor at Villanova University, challenges the simulations' reliance on normal distributions, or results that resemble bell curves, when those are not typical in financial markets.

"The probability results from Monte Carlo simulation may look impressive to a client," said Dr. Nawrocki, a consultant to PIE Technologies, a company that developed MoneyGuidePro, financial planning software. "However, if that number is derived from assumptions that are not realistic, there is no value to the number." He said he is working on alternatives.

Dr. Sharpe acknowledges that market returns do not always mirror the pattern of a bell curve. So, he said, Financial Engines makes more complex assumptions and mixes in asymmetric patterns to mimic real life. "I don't see how anyone can be a critic of trying to project that there is uncertainty," he said. "If you're going to project anything like the complexity of the market, you're almost doomed to Monte Carlo rather than using analytic projections."

Andrew Clark, a senior research analyst at Lipper, points out that the calculations are based on limited financial market history. "From my standpoint, it is more of a learning tool -- it may give you a bounce, the top and bottom, but please do not take it as gospel," Mr. Clark said.

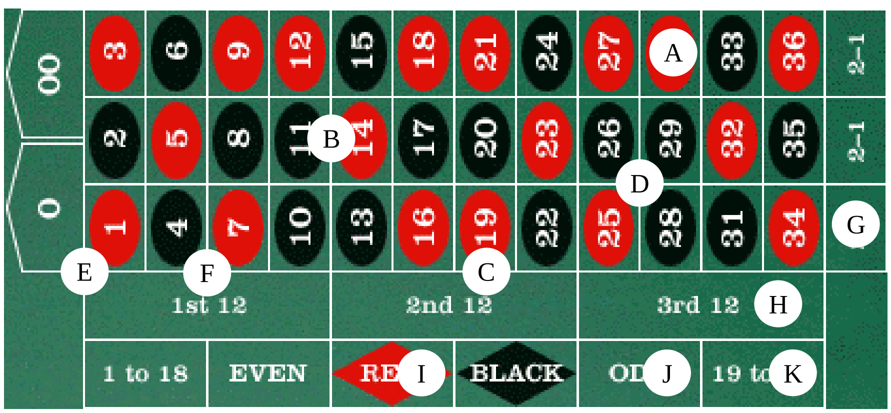
Chris Cordaro, chief investment officer at RegentAtlantic Capital, a wealth management firm in Chatham, N.J., says that while the approach does not yield precise predictions, it gives investors a more sophisticated picture. "There's no perfect forecasting tool, but Monte Carlo simulations are the best ones we have," Mr. Cordaro said. "At least you're demonstrating there's a probability for different returns, and we're at least acknowledging it's not going to come out perfectly."

Example: American Roulette

<u>American Roulette Wheel</u>

- In American Roulette the wheel consist of 38 identical slots, numbered from 0, 00, 1 through 36.
- Zero and double zero are both green pockets, while the remaining 36 are split evenly between red and black.
- On the standard roulette wheel the numbers are not distributed in increasing series or randomly.
- On the contrary, the numbers are ordered to achieve a certain mathematical balance between high and low, red and black and even and odd.
- Numbers face the outside of the wheel.

<u>American Roulette Table Layout and Payoffs</u>



Roulette Inside Bets

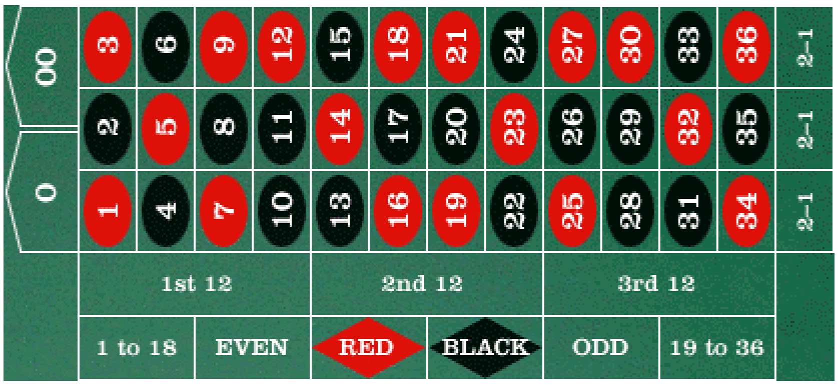
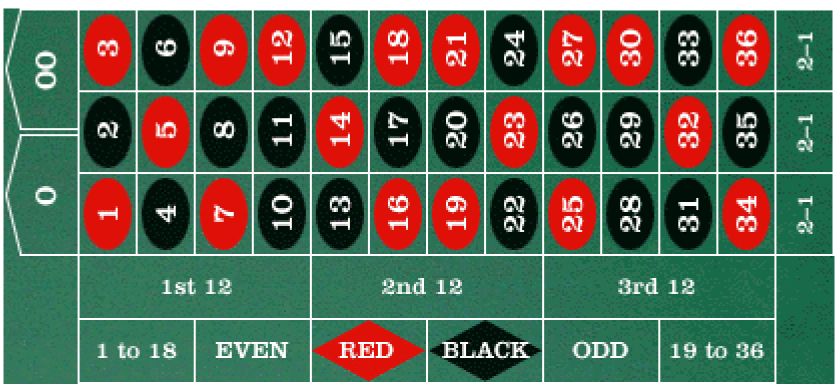
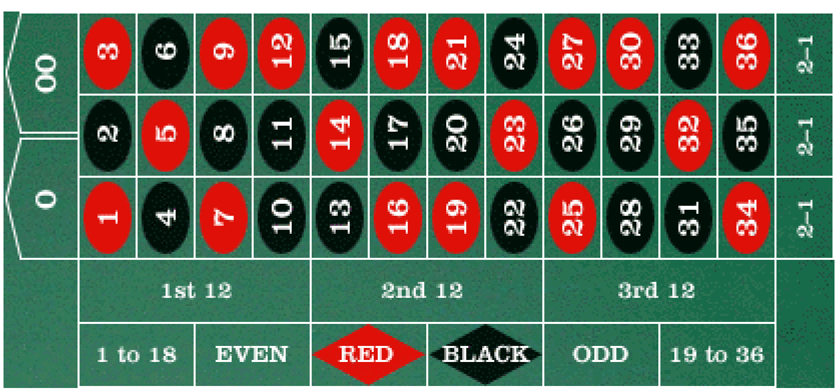
Wager	*Example*	*Bet on*	*Payoff*	*Probability*
Straight up	A	30	35:1	1/38
Split Bet	B	11 or 14	17:1	2/38
Street Bet	C	19, 20, 21	11:1	3/38
Corner	D	25, 26, 28, 29	8:1	4/38
Five Numbers	E	0, 00, 1, 2, 3	6:1	5/38
Line Bet	F	4, 5, 6, 7, 8, 9	5:1	6/38

Roulette Outside Bets

Wager	*Example*	*Bet on*	*Payoff*	*Probability*
Column	G	Set of column numbers	2:1	12/38
Dozen	H	25 through 36	2:1	12/38
Red or Black	I	Red numbers	1:1	18/38
Even or Odd	J	Odd numbers	1:1	18/38
Low or High	K	19 through 36	1:1	18/38

<u>Let’s play three rounds of Roulette:</u>

Suppose you have 20 one dollar chips.

<table>
<tr><td>Round</td><td>Place bets on the Roulette Table</td><td>Record your winnings</td></tr>
<tr><td>1</td><td>
</td><td></td></tr>
<tr><td>2</td><td>
</td><td></td></tr>
<tr><td>3</td><td>
</td><td></td></tr>
</table>

Optimal Target Hitting Strategy:

Suppose you have $80 to bet and you would like to hit the target of $100. Suppose we focus on the Red or Black bet.

	A	B	C	D	E	F	G	H	I	J
1	**Optimal Target Hitting Strategy**									
2										
3	Outcome		Prob.		Target		Bet #	Bet size	Spin result	Winnings
4	1	(your chosen color)	0.4737		100		0			80
5	2		0.5263				1	20	2	60
6							2	40	2	20
7	Final outcome	100					3	20	1	40
8							4	40	1	80
9	Number of bets	6					5	20	2	60
10							6	40	1	100
11							7	0	2	100
12							8	0	2	100
13							9	0	1	100
14							10	0	1	100
15							11	0	2	100
16							12	0	2	100
17							13	0	2	100
18							14	0	2	100
19							15	0	1	100
20							16	0	2	100
21							17	0	1	100
22							18	0	1	100
23							19	0	1	100
24							20	0	1	100
25							21	0	2	100
26							22	0	1	100
27							23	0	2	100
28							24	0	1	100
29							25	0	2	100
30							26	0	2	100
31							27	0	2	100
32							28	0	1	100
33							29	0	2	100
34							30	0	2	100

Strategy:

- Bet the difference between 100 and your balance (what you have) if you have enough money.
- If your balance is not enough to get you to 100, then bet all you have.

The idea is to hit the target as soon as possible.

Specify the @Risk Model in Excel:

	A	B	C	D	E	F	G	H	I	J
1	**Optimal Target Hitting Strategy**									
2										
3	Outcome		Prob.		Target		Bet #	Bet size	Spin result	Winnings
4	1	(your chosen color)	0.4737		100		0			80
5	2		0.5263				1	↓	↓	↓
6							2			
7	Final outcome	=J34					3			
8							4			
9	Number of bets	=Countif(H5:H34,">0")					5			
10							6			
11							7			
12							8			
13							9			
14							10			
15							11			
16							12			
17							13			
18							14			
19							15			
20							16			
21							17			
22							18			
23							19			
24							20			
25							21			
26							22			
27							23			
28							24			
29							25			
30							26			
31							27			
32							28			
33							29			
34							30			output cell

H5: =If(J4<100-J4,J4,100-J4) = min(J4,100-J4)
I5: =RiskDiscrete(A$4:A$5,C$4:C$5)
J5: =If(I5=1,J4+H5,J4-H5)

What's the probability of hitting the target?

Detailed Statistics: Target (Value) = 0 → Target (%) = 22.55%
Prob(Final Winning = 0) = Prob(Final Winning <= 0) = 22.55%
Prob(Final Winning = 100) = 1 – Prob(Final Winning = 0) = 1 – 22.55% = 77.45%

Example: Reservation Management

Marty Ford is an operations analyst for Piedmont Commuter Airlines (PCA). Recently, Marty was asked to make a recommendation on how many reservations PCA should book on Flight 343 – a flight from a small regional airport in New England to a major hub at Boston's Logan airport. The plane used on Flight 343 is a small twin-engine turbo-prop with 19 passenger seats available. PCA sells nonrefundable tickets for Flight 343 for $150 per seat.

Industry statistics show that for every ticket sold for a commuter flight, a 0.10 probability exists that the ticket holder will not be on the flight. Thus, if PCA sells 19 tickets for this flight, there is a fairly good chance that one or more seats on the plane will be empty. Of course, empty seats represent lost potential revenue to the company. On the other hand, if PCA overbooks this flight and more than 19 passengers show up, some of them will have to be bumped to a later flight.

To compensate for the inconvenience of being bumped, PCA gives these passengers vouchers for a free meal, a free flight at a later date, and sometimes also pays for them to stay overnight in a hotel near the airport. PCA pays an average of $325 (including the cost of lost goodwill) for each passenger that gets bumped. Marty wants to determine if PCA can increase profits by overbooking this flight and, if so, how many reservations should be accepted to produce the maximum average profit. To assist in the analysis, Marty analyzed market research data for this flight that reveals the following probability distribution of demand for this flight:

Seats Demanded	14	15	16	17	18	19	20	21	22	23	24	25
Probability	0.03	0.05	0.07	0.09	0.11	0.15	0.18	0.14	0.08	0.05	0.03	0.02

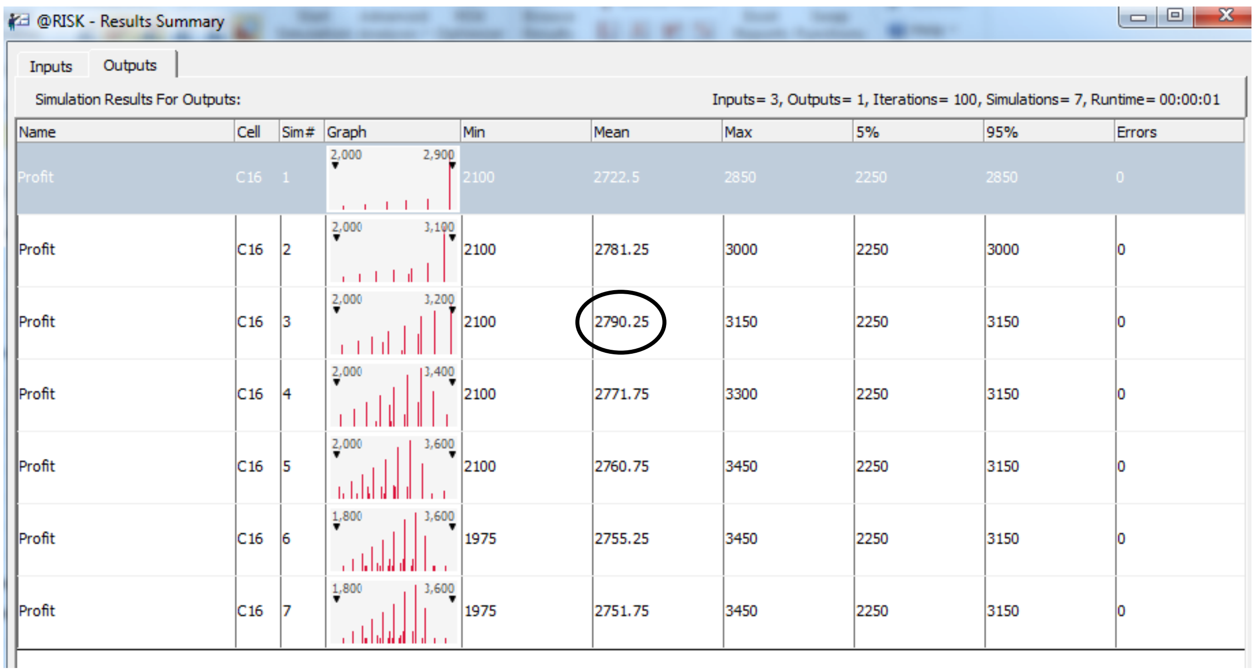
<u>Specify the following @Risk Model in Excel:</u>

	A	B	C	D	E	F	G	H
1	**Piedmont Commuter Airlines**							
2	(from Spreadsheet Modeling and Decision Analysis by Ragsdale)							
3								
4		Seats Available	19		Demand	Probability		Max # of Reservations to Accept
5		Ticket Price per Seat	$150		14	0.03		19
6		Prob. of No-Show	0.1		15	0.05		20
7		Cost of Bumping	$325		16	0.07		21
8		Max # of Reservations to Accept	=RiskSimTable(H5:H11)		17	0.09		22
9					18	0.11		23
10		Seats Demanded	=RiskDiscrete(E5:E16,F5:F16)		19	0.15		24
11		Tickets Sold	=min(C8,C10)		20	0.18		25
12		Passengers Wanting to Board	=RiskBinomial(C11,1-C6)		21	0.14		
13					22	0.08		
14		Ticket Revenue	=C5*C11		23	0.05		
15		Cost of Bumping	=C7*max(C12-C4,0)		24	0.03		
16		Profit	=C14-C15		25	0.02		

Choose simulation settings:

Iterations = 10,000
Simulations = 7

@Risk Results Summary:



@RISK - Results Summary

Inputs | Outputs

Simulation Results For Outputs: Inputs= 3, Outputs= 1, Iterations= 100, Simulations= 7, Runtime= 00:00:01

Name	Cell	Sim#	Graph	Min	Mean	Max	5%	95%	Errors
Profit	C16	1	2,000 2,900	2100	2722.5	2850	2250	2850	0
Profit	C16	2	2,000 3,100	2100	2781.25	3000	2250	3000	0
Profit	C16	3	2,000 3,200	2100	2790.25	3150	2250	3150	0
Profit	C16	4	2,000 3,400	2100	2771.75	3300	2250	3150	0
Profit	C16	5	2,000 3,600	2100	2760.75	3450	2250	3150	0
Profit	C16	6	1,800 3,600	1975	2755.25	3450	2250	3150	0
Profit	C16	7	1,800 3,600	1975	2751.75	3450	2250	3150	0

What's the optimal maximum # of reservations to accept?

The optimal maximum # of reservations to accept is 21 because it yields the highest mean profit.

The New York Times

April 5, 2010

Airlines Look to Limit Bumping

By JAD MOUAWAD and MICHELLE HIGGINS

Ryan Kingsbury is the rare flier who's actually looking to be bumped from a flight.

"If I see a big weather system, I see big dollar signs," said Mr. Kingsbury, an aeronautical engineer from Boston who claims he has earned about $6,700 in flight vouchers over the last three years. His latest coup was picking up $600 in flight vouchers and a hotel room after giving up his seat on an overbooked red-eye flight from Los Angeles to Washington in bad weather in January.

But savvy travelers like Mr. Kingsbury are having to work harder than ever to exploit loopholes in the travel system. Even as airlines have substantially cut capacity — meaning fewer but more crowded flights — they have so refined their computer tools over the last decade that they can pretty much predict which passengers will show up for a flight and which ones won't. In the process, they have gotten better at the science of overbooking a flight.

Last year, 13 out of every 10,000 passengers were bumped on domestic flights — or 762,422 out of a total of 582 million. That was down from over 20 per 10,000 passengers in 1999, according to the Department of Transportation. In over 90 percent of cases, airlines found volunteers to give up their seats in exchange for some compensation.

The airlines argue that they must overbook to make up for passengers who fail to show up. For an industry desperate to return to profitability after losing $60 billion in the last decade, an empty seat at takeoff equals one thing: lost revenue.

"A seat is a perishable item," said Leon Kinloch, the senior vice president for pricing and revenue management at Continental Airlines. "It's like a fruit that spoils. The moment the door is closed, that item has perished."

So the airlines are imposing more restrictive booking policies. Most airlines, for instance, require travelers to buy their tickets within 24 hours of booking them, forcing travelers to stick with their plans or risk steep penalties to change tickets.

And the airlines have invested in new software to get a better sense of how many passengers will actually show up for a flight. They look at historical data on specific routes, the time of the day, whether there is a holiday, what fares passengers paid and how many business-class travelers are booked with refundable tickets.

Mr. Kingsbury, the Boston flier who tries to get bumped, says he books flights he thinks are likely to be oversold. He will pick the last flight of the day, for instance, and then tell the check-in clerks or gate agents that he is willing to give up his seat should it be needed. He also prays for bad weather, which increases his chances of getting bumped.

Not all airlines practice the art of overbooking. JetBlue Airways offers only one class of service and most of its tickets are not refundable, meaning passengers are more likely to show up. As a result, last year, it had only one oversold seat for 5.1 million passengers.

"It's like a theater overselling tickets for a show," said Dave Barger, the president and chief executive of JetBlue. "It's wrong."

Don Casey, the vice president of revenue management at American Airlines, said, "As an industry under such financial duress, we have had to come up with ways of making ourselves more efficient."

About 60,000 passengers boarded 450 American Airlines flights on a sunny day at the end of March in Dallas-Fort Worth. Seventeen flights had a total of 50 oversold seats; 48 people volunteered for a later flight. Two people were bumped involuntarily.

Thanks to a better understanding of its booking patterns, Mr. Casey said American now overbooked about 5 percent of its seats, down from about 12 percent a decade ago.

In 1999, the company said it had on average about 72 percent of its seats filled and 35.2 out of every 10,000 seats were oversold. Last year, the company filled 82 percent of its seats while the number of oversold seats had dropped to 8.3 for each 10,000 passengers.

Bill S. Swelbar, a research engineer with M.I.T.'s International Center for Air Transportation, called this "the dark art of revenue management."

The trouble, of course, is that the airlines' mathematical wizards don't always get it right.

Tiffany Sumlin, a stay-at-home mother from Fresno, Calif., nearly missed her grandmother's funeral viewing after she was involuntarily bumped from her Delta Air Lines flight to Houston last month when connecting in Salt Lake City.

"They had overbooked the flight and I was not going to get on," said Mrs. Sumlin, who was left behind with another passenger when no one volunteered to give up a seat in exchange for flight vouchers. She was initially offered $400 and a flight out the next day — but that meant being late.

"I would have literally been landing during my grandmother's viewing," Mrs. Sumlin said. She eventually made it in time — but not before flying first to Atlanta, spending the night there, and getting an early-morning flight to Houston.

Not every passenger is equal when it comes to being bumped: business-class travelers and frequent fliers holding elite status are much less likely to get bumped. The last in line are leisure travelers holding discounted fares. (Online check-in reduces somewhat your chances of being bumped. Most airlines offer that option up to 24 hours before departure and strongly suggest that passengers use it.)

Passengers who are involuntarily bumped, like Mrs. Sumlin, and rebooked on another flight within two hours after their original domestic flight time (or within four hours for international flights) are entitled to $400 in cash — double the compensation offered two years ago, according to Department of Transportation regulations. They are eligible for up to $800 if they are not rerouted by then.

But getting people to volunteer is tricky these days. Full planes mean that the next flight for bumped passengers may be the next day.

While fewer people are getting bumped over all, the share of passengers being denied boarding involuntarily is going up. Last year, 1.19 of every 10,000 passengers had their seats taken away outright, the highest rate since 1996.

One reason is that airlines are flying fewer planes in a bid to cope with high fuel costs and lower demand in the recession. Domestic capacity has fallen for five of the last nine years, the most sustained cutback in the history of commercial aviation.

The winter storms that led to chaos in the nation's air transportation system demonstrated just how little wiggle room airlines had.

"I think this summer is going to be pretty good for airlines but the flip side is it is going to be awful for travelers," said Tim Winship, the editor at large for SmarterTravel.com, a Web site offering travel advice. That is good news for Mr. Kingsbury, who says he is looking forward to the coming months when tight capacity, the spring and summer travel crunch and weather cancellations could increase bumping.

"I think this coming summer it will be very lucrative for people doing bumps," he said. "I'll just sit back and watch the chaos."

Example: When to pull the goalie in hockey

When his team is behind late in the game, a hockey coach usually waits until there is one minute left before pulling the goalie out of the game. Using simulation, it is possible to show that coaches should pull their goalies sooner. Suppose that if both teams are at full strength, each team scores an average of 0.05 goal per minute. Also, suppose that if you pull your goalie you score an average of 0.08 goal per minute and your opponent scores an average of 0.12 goal per minute. Suppose you are one goal behind. Compare the following two strategies:

- Pull your goalie if you are behind at any point in the last minute of the game; put him back in if you tie the score.
- Pull your goalie if you are behind at any point in the last two minutes of the game; put him back in if you tie the score.

Simulate the game using 10-second increments of time. Use the RISKBINOMIAL function to determine whether a team scores a goal in a given 10-second segment. This is reasonable because the probability of scoring two or more goals in a 10-second period is near zero.

Which strategy provides a higher probability of winning or tying the game?

<u>Specify the following @Risk Model in Excel:</u>

	A	B	C	D	E	F
1	**When to pull the goalie in hockey**					
2				Possible strategies (seconds left when goalie is pulled)		
3	Seconds left when goalie is pulled	=RiskSimTable(D3:E3)		60	120	
4						
5	**Inputs**					
6	Prob of goal full-strength	=.05/6				
7	Prob of our goal (our goalie pulled)	=.08/6				
8	Prob of their goal (our goalie pulled)	=.12/6				
9						
10	Current score (ours minus theirs)	-1				
11						
12	**Simulation**					
13	Time left (seconds)	Our score minus theirs	Are we behind?	Goalie Pulled?	We score?	They score?
14	120	=B10	=If(B14<0,1,0)			
15	110	=B14+E14-F14				
16	100					
17	90			D14:	=If(And(A14<=B$3,C14=1),1,0)	
18	80			E14:	=If(D14=0,RiskBinomial(1,B$6), RiskBinomial(1,B$7))	
19	70			F14:	=If(D14=0,RiskBinomial(1,B$6), RiskBinomial(1,B$8))	
20	60					
21	50					
22	40					
23	30					
24	20					
25	10					
26	0					
27						
28	We win or tie?	=If(B26>=0,1,0)				
29						
30	**Selected summary measures from @RISK**					
31		Prob(we win or tie)				
32	Strategy	1 minute	2 minutes			
33	Mean	=RiskMean(B28,1)	=RiskMean(B28,2)			

Results:

	A	B	C	D	E	F	G
1	**When to pull the goalie in hockey**						
2				Possible strategies (seconds left when goalie is pulled)			
3	Seconds left when goalie is pulled	60		60	120		
4							
5	**Inputs**						
6	Prob of goal full-strength	0.0083					
7	Prob of our goal (our goalie pulled)	0.0133					
8	Prob of their goal (our goalie pulled)	0.02					
9							
10	Current score (ours minus theirs)	-1					
11							
12	**Simulation**						
13	Time left (seconds)	Our score minus theirs	Are we behind?	Goalie Pulled?	We score?	They score?	
14	120	-1	1	0	0	0	
15	110	-1	1	0	0	0	
16	100	-1	1	0	0	0	
17	90	-1	1	0	0	0	
18	80	-1	1	0	0	0	
19	70	-1	1	0	0	0	
20	60	-1	1	1	0	0	
21	50	-1	1	1	0	0	
22	40	-1	1	1	0	0	
23	30	-1	1	1	0	0	
24	20	-1	1	1	0	1	
25	10	-2	1	1	0	0	
26	0	-2	1	1	0	0	
27							
28	We win or tie?	0					
29							
30	**Selected summary measures from @RISK**						
31		Prob(we win or tie)					
32	Strategy	1 minute	2 minutes				
33	Mean	0.112	0.132				

Which strategy provides a higher probability of winning or tying the game?

<u>The strategy of pulling with 2 minutes to go seems to increase the probability slightly of us winning or tying.</u>

The New York Times

April 7, 2009

Answering Baseball's What-Ifs

By ALAN SCHWARZ

You can learn a lot during a major league baseball game. Like Ukrainian, if it is a particularly slow nine innings.

As for the science of baseball strategy, one game teaches precious little. A well-timed sacrifice bunt can backfire and lose the game; a foolish steal can appear brilliant. The vagaries of randomness — the way Sandy Koufax got battered occasionally and a pipsqueak named Bucky Dent hit one of the most famous home runs ever — camouflage the game's inner forces, which for 150 years have operated somewhere between fact and fable.

One game has little meaning. A thousand seasons can take a while. Thank goodness for quad-core processors.

"Computer simulations work pretty well in baseball for two reasons," said Carl Morris, a professor of statistics at Harvard University who has written several papers that commingled baseball and formal statistical theory. "In general, they allow you to study fairly complicated processes that you can't really get at with pure mathematics. But also, sports are great for simulations — you can play 10,000 seasons overnight."

No one can afford to wait less than major league teams, which crave every extra run or victory they can wring from their $100 million rosters. John Abbamondi, the assistant general manager for the St. Louis Cardinals, says his team and about 10 others use simulations to evaluate potential trades and how they might affect the pennant race.

"It's all part of the statistical analysis that complements the more traditional scouting we do," he said.

Using computer simulations to explore in-game and other baseball strategies is by no means new. As early as 1958, a professor at the Massachusetts Institute of Technology programmed a behemoth I.B.M. 704 mainframe to investigate whether the sacrifice bunt was a smart play. (More on that later.) Simulators have since grown so complex that the most sophisticated one available to the public, called Diamond Mind, not only runs lickety-split on laptops but even considers minutiae like the effects of wind in individual ballparks.

Under what conditions is bunting advantageous? When does trying to steal make sense, and when does it decrease the chances of scoring? Questions like these turn out to be ideally suited to computer programs through which millions of iterations can smooth out the peaks and valleys of randomness, and converge toward a reliable approximation.

Known among formal statisticians as the Monte Carlo method, this approach takes spectacularly complex phenomena like weather patterns and stock performance and allows their behavior to be approximated, if not determined.

What are the chances of winning a game of solitaire? Rather than writing an equation that tries to take into account the trillions of trillions of possible hands and moves, a statistician can run a computer program that simply plays the game a few million times in minutes to see how often it wins. Dr. Morris says he has seen the Monte Carlo method used to improve computer graphics and explore gene sequences.

Like such competitors as Strat-O-Matic — which made its debut in 1961 with at-bats determined by cards and dice, and remains popular on the personal computer — Diamond Mind is designed to allow fans to play fictional games and seasons, exploring what-if scenarios that real life would be too slow and controversial to allow.

Take the age-old question of how much difference a team's lineup order makes. This issue so vexed the former manager Billy Martin that he once literally picked his Detroit Tigers batting order out of a hat.

Luke Kraemer of Imagine Sports, which owns Diamond Mind, programmed the simulator to force the 2008 Yankees to bat their best hitter and cleanup man, Alex Rodriguez, ninth — to see how scoring was affected. Mr. Kraemer got the run total not for just one season, which can fluctuate as much as 80 runs in each direction from simple randomness, but for 100 seasons — more than 16,000 Yankees games in all.

The result? The Yankees scored 747 runs per season, 40 fewer than their real-life 787. (Diamond Mind was so accurate that 100 seasons with A-Rod batting fourth averaged 789, almost dead-on.) Most research suggests that those 40 runs would mean only about four fewer victories, for a strategy no manager would ever consider; so the difference with Rodriguez batting third or fifth would be insignificant, and nowhere near worth the forests of trees that would give their lives to the ensuing sports-page debate.

Diamond Mind took its cuts at several other baseball knucklers, running 100 full seasons of games for each:

- The intentional walk. This frequently used defensive strategy avoids dangerous hitters and can set up a double play, but it also awards a free base, and even the best hitters usually make an out. So is it smart in the long run? Diamond Mind found that it was not, though the difference was only about five runs per team per season.

- The stolen base. Advancing from first to second puts the runner in scoring position, but he — and the rest of your hitters — will have a hard time scoring if he gets thrown out. Mr. Kraemer looked at a recent team that ran wild (the 2008 Tampa Bay Rays) and one that barely stole at all (the 2005 Oakland A's) and switched their mind-sets to see what happened. The A's scored 20 runs fewer, which probably says more about their players' inability to run in the first place. But when the speedy Rays stole sparingly, they increased their scoring by 47 runs per season — suggesting that perhaps the Rays were running too often in real life.

- The sacrifice bunt. Is it worth making an out intentionally to move a runner from first to second? Forcing a team that hated that maneuver (the 2005 Boston Red Sox) to do it a lot cost them 19 runs per season. But making a bunting team (the 2008 New York Mets) avoid it also cost them — by 15 runs on average — suggesting that the Mets' managers, Willie Randolph and Jerry Manuel, used it quite intelligently. (The 1958 M.I.T. statisticians found that the sacrifice was rarely a good move; major league managers paid little attention.)

One problem with computer simulations is that no matter how realistically they might be programmed, they can say more about the programmer than baseball itself. A computer, after all, cannot feel human emotions like pressure or the will to hit in the clutch.

"We can run the experiment in the simulation environment and think we're measuring the effect of a great defense on a pitching staff, but it might tell us more about how we modeled defense," said Tom Tippett, who wrote the original Diamond Mind code in the early 1980s. "The simulation is real close to real-life baseball, but in the end it isn't real-life baseball."

After developing Diamond Mind into the industry standard, Mr. Tippett was hired a few years ago by the Boston Red Sox — a sign of how much some teams have come to value simulation research. While none will discuss exactly what they model and how, Mr. Abbamondi, of the Cardinals, said they could provide objective insight into how an offense might be affected by trading for a hitter in midseason; how many games that might improve the team; and how that hitter might improve or deteriorate as he ages. Many of these measurements come in the form of scenarios of increasing uncertainty, not unlike the projection of hurricane paths.

As Mr. Tippett suggested, however, simulations have inherent limits, and probably will not ever model baseball's vicissitudes of fate — how scrubs morph into all-stars and some teams just collapse. (Indeed, fans of the recent New York Mets would be relieved that some things defy re-creation.) Tony La Russa, the Cardinals' manager, who is a sure bet for the Hall of Fame, said the value of computer simulations in baseball tended to stop at the dugout entrance.

"There's way too much importance given to what you can produce from a machine," he said. "These are human beings, and I don't think any computer is going to model that close to what we deal with at this level."

That can be as true now as it was 25 years ago, when a Tank McNamara cartoon captured it best. A downtrodden manager peered over his computer. He asked plaintively, "But will it take the blame?"

List of @Risk Functions

- RiskNormal(mean, standard deviation)
- RiskDiscrete(outcomes, probabilities)
- RiskBinomial(# of trials, probability of success)
- RiskOutput ("output cell name")
- RiskSimTable(list of parameter values)
- RiskMean(cell reference, Sim#)