

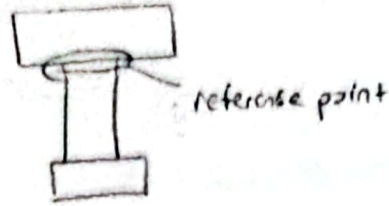
Question-1:1- Areas and Centroid (\bar{y})

$$A_t = b_t \cdot t_t = 300 \cdot 20 = 6000 \text{ mm}^2$$

$$A_{web} = t_w \cdot h_w = 10 \cdot 760 = 7600 \text{ mm}^2$$

$$A_{bottom} = 100 \cdot 20 = 2000 \text{ mm}^2$$

$$A = 6000 + 7600 + 2000 = 15600 \text{ mm}^2$$



$$\bar{y} = \frac{6000 \cdot 10 + 7600 \cdot 400 + 790 \cdot 2000}{15600} = 300 \text{ mm}$$

1.2 - Moment of Inertia:

Elastic Section Modulus: $W_e = \frac{I}{C}$

$$I = I_0 + A d^2$$

$$\frac{bh^3}{12} = I$$

$$I_{x,pt} = \frac{300 \cdot 20^3}{12} = 2 \cdot 10^5 \quad \left\{ \begin{array}{l} I_{x,ow} = \frac{10 \cdot 760^3}{12} = 3,658 \cdot 10^8 \\ I_{x,ob} = \frac{100 \cdot 20^3}{12} = 6,67 \cdot 10^4 \text{ mm}^4 \end{array} \right.$$

$$A d^2 = 6000 \cdot 290^2 = 50416 \cdot 10^6$$

$$A d^2 = 7600 \cdot 100^2 = 76 \cdot 10^6$$

$$A d^2 = 2000 \cdot 490^2 = 480,2 \cdot 10^6$$

$$I_x = 1,4268 \cdot 10^9$$

1.3 Elastic Section Module and elastic moment capacities:

$$W_{e,top} = \frac{I_x}{300} = 4,756 \cdot 10^6 \text{ mm}^3$$

$$W_{e,bottom} = \frac{I_x}{500} = 2,854 \cdot 10^6 \text{ mm}^3$$

$$M_{e,x} = C_y \cdot W_{e,bottom} = 355 \cdot 2,854 \cdot 10^6 = 1,013 \cdot 10^9 \text{ N/mm} = 1013 \text{ kNm}$$

1.4 Plastic Neutral Axis (PNA) and Plastic Section modulus $W_{p,x}$

PNA is located so that compression area = tension area = $A/2 = 7800 \text{ mm}^2$

Accumulating area from the top:

Top flange gives 6000

$$\text{Need} = 7800 - 6000 = 1800 \text{ more} = 10 \cdot t \Rightarrow t = 180 \text{ mm of web}$$

Hence PNA is 200 mm below the top (20+180)

Compression above PNA

- Top flange $A = 6000$, centroid $200 - 10 = 190 \rightarrow 6000 \cdot 190$.

- Top web part $A = 10 \cdot 180 = 1800$ centroid distance = 90. $\rightarrow 1800 \cdot 90$

$$Q_c = 1.302.000 \text{ mm}^3$$

Tension (Below PNA)

- Bottom web part: $A = 10 \cdot 580 = 5800$, centroid $d_1 = 290 \rightarrow 5800 \cdot 290$

- Bottom flange $A = 2000$ centroid dis. = 590 $\rightarrow 590 \cdot 2000$

$$Q_t = 2.862.000 \text{ mm}^3$$

Plastic modulus (x-axis):

$$W_{px} = Q_c + Q_t = 4.164.10^6 \text{ mm}^3$$

Plastic moment:

$$M_{px} = f_y \cdot W_{px} = 355 \cdot 4.164.10^6 = 1.478.10^9 = 1478 \text{ kNm}$$

Shape factor (k):

$$\frac{W_{px}}{W_{e,bot}} = \frac{4.164}{2854} = 1.459 > 1 \quad \checkmark$$

Weak-axis (y) properties

$$2.1) I_{y,top} = \frac{20 \cdot 300^3}{12} = 45 \cdot 10^6$$

$$I_{y,web} = \frac{760 \cdot 10^3}{12} = 63.333$$

$$I_{y,bot} = \frac{20 \cdot 100^3}{12} = 1.67 \cdot 10^7$$

$$\left. \begin{array}{l} I_{y,top} \\ I_{y,web} \\ I_{y,bot} \end{array} \right\} I_y = 4.673 \cdot 10^7 \text{ mm}^4$$

2.2) Elastic and plastic modulus about y

$$W_{ey} = I_y / c_y = \frac{4.673 \cdot 10^7}{150} = 3.115 \cdot 10^5 \text{ mm}^3$$

Plastic modulus about y (rectangle shortcuts) $\left[Z_y = \frac{tb^2}{4} \right]$

$$Z_y = \frac{20 \cdot 300^2 + 20 \cdot 100^2 + 760 \cdot 10^2}{4} = 5.19 \cdot 10^5 \text{ mm}^3$$

$$M_{ey} = f_y \cdot W_{ey} = 355 \cdot 3.115 \cdot 10^5 = 1.106 \cdot 10^8 = 110.6 \text{ kNm}$$

$$M_{py} = f_y \cdot W_{py} = 355 \cdot 5.19 \cdot 10^5 = 1.842 \cdot 10^8 = 184.2 \text{ kNm}$$

$$k = \frac{W_{py}}{W_{ey}} = \frac{5.19}{3.115} = 1.67$$

②

Question - 2

TUBO 323 12:

$$D_o = 323 \text{ mm} \rightarrow R_o = 161.5 \text{ mm}$$

$$t = 12 \text{ mm} \rightarrow R_i = 149.5 \text{ mm}$$

$$I_{tube} = \frac{\pi}{3} (R_o^4 - R_i^4) = 1.4186 \cdot 10^8 \text{ mm}^4$$

$$W_{p,tube} = \frac{4}{3} (R_o^3 - R_i^3) = 1.161 \cdot 10^6 \text{ mm}^3$$

UPE 300

$$A = 5660 \text{ mm}^2$$

$$I_x = 3.74 \cdot 10^7 \text{ mm}^4$$

$$I_y = 1.52 \cdot 10^6 \text{ mm}^4$$

$$W_{ex} = 2.16 \cdot 10^6 \text{ mm}^3$$

Horizontal CG offset

$$x_c = R_o + 28.9 + 27.5 = 217.9 \text{ mm}$$

Moments of inertia:

$$I_x = I_{tube} + 2I_{upe} = 1.4186 \cdot 10^8 + 2 \cdot (3.74 \cdot 10^7) = 2.0676 \cdot 10^8$$

$$I_y = I_{y,tube} + 2(I_{y,upe} + Ax_c^2) =$$

$$1.4186 \cdot 10^8 + 2(1.52 \cdot 10^6 + 5660 (217.9^2)) = 6.874 \cdot 10^8 \text{ mm}^4$$

Elastic Section Module $W_e = I/c$

Extreme fibers

$$c_x = 161.5 \text{ mm} \quad c_y = (217.9 + 50) = 267.9 \text{ mm}$$

$$W_{e,x} = \frac{2.0676 \cdot 10^8}{161.5} = 1.2803 \cdot 10^6 \text{ mm}^3$$

$$W_{e,y} = \frac{6.8748 \cdot 10^8}{267.9} = 2.5675 \cdot 10^6 \text{ mm}^3$$

Plastic section Modul W_p

x-axis: use tube W_p + 2 (UPE strong axis W_{px})

$$W_{px} = 1.161 \cdot 10^6 + 2 \cdot (2.419 \cdot 10^5) = 1.6448 \cdot 10^6 \text{ mm}^3$$

~ approximate value

y-axis

$$W_{py} = 1.161 \cdot 10^6 + 2 \cdot (5660 \cdot 217.9) = 3.6276 \cdot 10^6 \text{ mm}^3$$

Capacities

$$M_e = f_y W_e$$

$$M_p = f_y W_p$$

$$k = \frac{W_p}{W_e}$$

	$W_e (mm^3)$	$W_p (mm^3)$	$M_e (kNm)$	$M_p (kNm)$	k
X	$1.2893 \cdot 10^6$	$1.6448 \cdot 10^6$	454.51	583.19	1.285
Y	$2.5425 \cdot 10^6$	$3.6276 \cdot 10^6$	824.37	1287.81	1.424

Question 3

Geometry and Positions

TUBE : $D_o = 323 \rightarrow R_o = 161.5$ $t = 12mm$ $R_i = 149.5$ $I_{tube} = 1.4186 \cdot 10^8$

UPE : $A = 5660 mm^2$

Vertical centroid spacing :

$$I_x = 3.174 \cdot 10^8 mm^2$$

$$y_c = 161.5 + 281.9 = 443.4 mm$$

$$I_y = 1.52 \cdot 10^8 mm^2$$

Centroid of the build up section is at the tube center (0,0)

Moment of Inertias :

$$I_x = I_{tube} + 2(I_{xupe} + A y_c^2)$$

$$1.4186 \cdot 10^8 + 2 \cdot (3.174 \cdot 10^7 + 2.052 \cdot 10^8) = 6.17 \cdot 10^8 mm^4$$

$$I_y = I_{tube} + 2I_{upe} = 1.4186 \cdot 10^8 + 2(1.52 \cdot 10^8) = 3.45 \cdot 10^8 mm^4$$

Elastic Section Modules; $W_e = I/c$

X-axis extreme fiber is the top of the UPE

$$c_x = y_c + \frac{h}{2} = 443.4 + 150 = 593.4 mm$$

$$W_{ex} = \frac{6.17 \cdot 10^8}{593.4} = 1.04 \cdot 10^6 mm^3$$

Y-axis extreme half-width governed by the tube radius

$$c_y = 161.5$$

$$W_{ey} = \frac{3.45 \cdot 10^8}{161.5} = 2.14 \cdot 10^6 mm^3$$

Plastic Section Modulus W_p

X-axis : $A_{yc} = 5660 \cdot 190.4 \approx 1.078 \cdot 10^6$ $W_{px} = W_{ptube} + 2A_{yc}$

$$W_{px} = 1.161 \cdot 10^6 + 2 \cdot (1.078 \cdot 10^6) = 3.32 \cdot 10^6 \text{ mm}^3$$

Y-axis: $W_{py_{upc}} = 2 \cdot \left(\frac{t_f \cdot b^2}{4} \right) + \frac{h_i \cdot t_w^3}{4} = 75.000 + 6.092 = 81.092 \text{ mm}^3$

$$W_{py} = W_{ptube} + 2W_{py_{upc}} = 1.161 \cdot 10^6 + 2 \cdot (81.092) = 1.32 \cdot 10^6 \text{ mm}^3$$

Capacities and Shape factors

$$M_e = f_y \cdot W_e, \quad M_p = f_y \cdot W_p, \quad \phi = \frac{W_p}{W_e}$$

	$W_e \text{ (mm}^3\text{)}$	$W_p \text{ (mm}^3\text{)}$	$M_e \text{ (kNm)}$	$M_p \text{ (kNm)}$	k
X	$1.81 \cdot 10^6$	$3.32 \cdot 10^6$	643	1178	1.83
Y	$8.97 \cdot 10^5$	$1.32 \cdot 10^6$	318	469	1.47