Department of Electrical and
Electronic Engineering

Embedded Systems Project 2022-23

DESIGN REPORT #1

Title: Motor Characterisation Lab

Group Number: 12

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1. Introduction

This report describes the procedures and calculations done to decide the optimal gearbox ratio to be used in the design of the ESP buggy. This is done by analysing the various relationships between the electrical properties and mechanical parameters of the motor so that the buggy's movement can be controlled by varying certain electrical parameters. This is necessary so that constraints of the project can be acknowledged and incorporated into the gearbox choice.

The largest constraint in this project is the inclined path that the buggy will likely take. The buggy's motors would have to provide enough torque to overcome its own weight and the frictional force of the tracks - however, as mentioned by the project technical handbook [1], the buggy's motors itself cannot provide enough torque to climb the slope, and so a gearbox is needed to provide the additional torque required. Additionally, a gearbox that provides too much excess torque will lower the average speed of the motor, resulting in an unnecessarily slower buggy. Thus, a balance between the speed and torque provided by the gearbox would have to be found and an appropriate gearbox to be selected to design a buggy that is optimal in both speed and performance.

In addition, a gearbox that is optimal in both speed and torque would possibly reduce the power consumption of the motor, which reduces the chances of failure by battery flattening. The motor speed and torque are controlled by the pulse width modulation (PWM) signal created on the microcontroller. From this, the signal generated in the microcontroller will then be fed to the DC motor through the motor drive board, and the speed and torque can be varied by changing the outputs of the microcontroller.

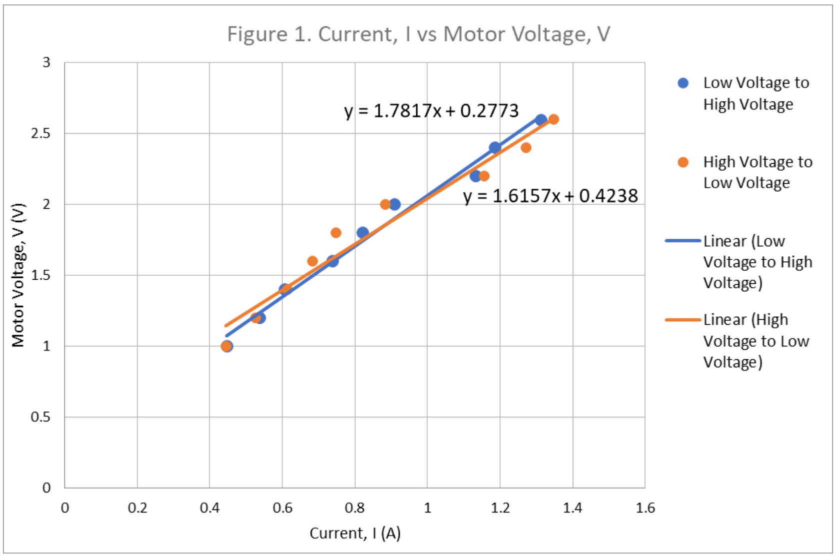
2. Motor characterisation

The characterisation of the motor is a significant prerequisite to the gearbox selection process, allowing motor torque measurements and therefore torque when paired with each of the three gearboxes. Important characteristics such as the relationships between motor torque and speed, between motor current and voltage and between motor EMF and motor speed are also measured or calculated. These results are used in conjunction with the load measurement results to calculate which of the three gearboxes, in combination with the motor, provides torque equal or larger than that needed to drive up the sloped track.

Two large constraints of the motor are the maximum current that can be supplied to the motor and the maximum voltage. The maximum current the motor can use is 1.4 A - this is due to the motor driver board only being able to output a maximum of 1.4 A to each motor, as stated in the technical handbook [1]. The maximum voltage is 9.6 V as that is approximately the maximum supplied by the batteries when fully charged.

2.1 Armature resistance measurement and brush voltage

With the motor held in stall position, the voltage was increased from approximately 1 V to 2.6 V, providing measurements for the current at each voltage value. These values, when entered into a graph, provided a linear graph with positive correlation between the current and motor voltage - the y-intercept representing the brush voltage V_b. Two graphs were created by measuring current, starting at a lower voltage and increasing to a higher voltage and vice versa.



For the low voltage to high voltage graph, when the trendline is set to a y-intercept of 0 V, the resistance (gradient of the trendline) was measured as 2.073 Ω. The brush voltage was also measured as 0.277 V. Although the shape of the graph matches that of the expected shape in the Week 2 ESP lecture notes [2], the resistance is lower than the 3 Ω expected - there is a percentage error of 30.9%. This discrepancy is most likely due to a high motor temperature as the measured voltage is altered by the motor's temperature - although the motor was unpowered between tests it may not have fully cooled down between the tests, leading to some of the voltage readings being less accurate. Another cause could be due to the motor base being at a slight angle during the experiment, which may have altered the

For the high voltage to low voltage graph, when the trendline is set to a y-intercept of 0 V, the resistance (gradient of the trendline) was measured as 2.056 Ω. The brush voltage was also measured as 0.424 V. The percentage error of the resistance is 31.5%, with the brush voltage differing from the previous measurement by 0.147 V, indicating that there is little difference in accuracy between the two methods, however both graphs give a lower value for resistance than expected - indicating that this is a systematic error and must be accounted for in the gearbox selection process.

2.2 Estimating K_E and K_T at a constant motor voltage

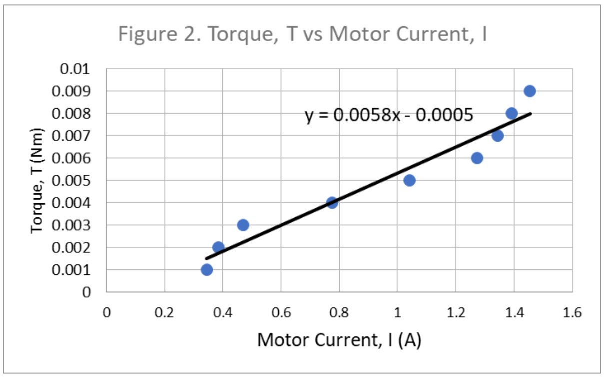
For the second part of the experiment, at constant motor voltage, two force gauges were used simultaneously to provide a torque on the motor, increasing the torque in equal steps. The force from each gauge was measured and torque calculated using the equation

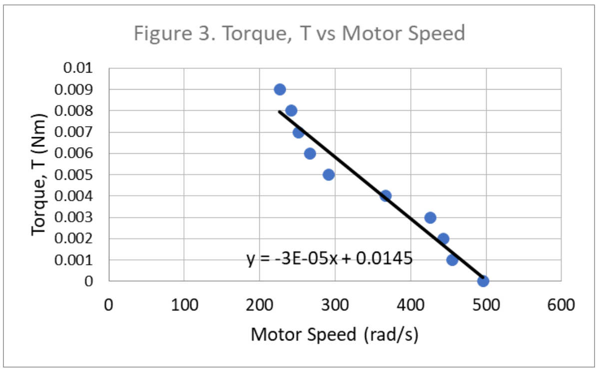
$$T = K_T \cdot I = \frac{(F1 - F2)\cdot d}{2} \qquad (2.1)$$

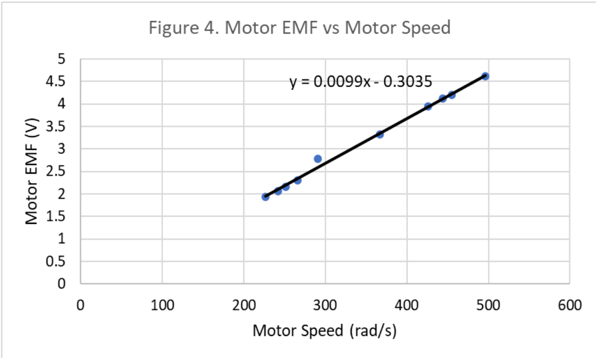
where d = motor shaft diameter (m), measured to be 0.01 m and F_1 and F_2 = the forces of the two force gauges (N). At each step the motor speed is measured in rpm using a tachometer pointing at a reflective dot on the wheel section of the shaft. The motor speed is then converted to radians/second and the emf is calculated using the previously measured value of current and the brush voltage - the equation being

$$E = V - V_b - I \cdot R \qquad (2.2)$$

where E = emf (V), V = motor voltage (V), V_b = brush voltage (V), I = motor current (A) and R = armature resistance (Ω). From these results, figures 2, 3 and 4 were made.







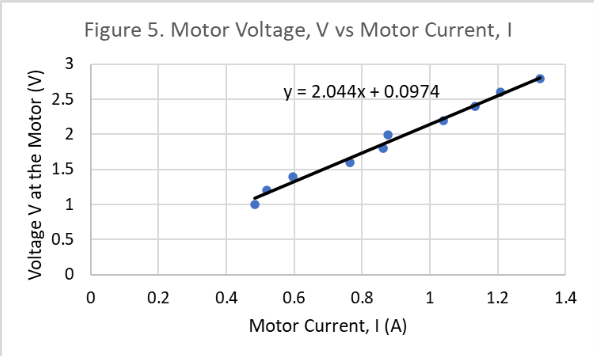
In figure 2 the gradient represents the motor constant K_T, whilst the y-intercept represents the friction torque, T_f. From the graph the constant K_T was measured as 0.0058 and T_f as 0.0005 Nm. These measurements can be compared with the corresponding set of measurements taken in section 2.3.

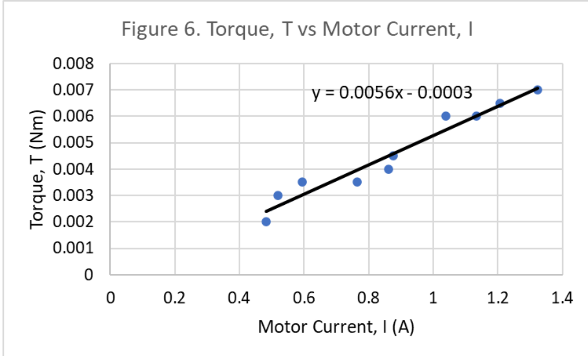
Figure 3 provides a representation of the relationship between torque and motor speed. As the motor speed increases, the torque decreases and vice versa. This is an essential feature of the motor as it demonstrates the need for a balance between speed and torque.

From figure 4 it is made clear that the emf increases linearly with motor speed. It also provides the constant K_E, which is equal to the gradient of the line. Measured as 0.0099, K_E should ideally be equal to the constant K_T, however there is a percentage error of 41.4% between K_T and K_E, likely due to the torque vs current graphs being less accurate than the motor emf vs motor speed graph - therefore K_E is the preferred value to use for the current and voltage gearbox calculations.

2.3 Estimating KT again but at stall (high torque)

In the third part of the experiment the motor was stalled using the friction of the string connected to the force gauges against the motor shaft, decreasing the motor voltage in equal steps and recording the current. The highest voltage used was restricted to ensure the motor did not exceed its maximum input current by calculating the current based on the previous calculation for resistance. The force on each gauge was also measured, allowing the torque at each voltage to be calculated using the torque equation (2.1). This allows a graph of motor voltage vs motor current to be created, which can be compared with figure 1. A second torque vs motor current graph can also be made, allowing comparison between this graph and figure 2.





As with figure 1, figure 6 presents a linear relationship between the current and motor voltage. This provides a resistance value of 2.145 Ω, which is closer to the expected value in comparison to figure 1, with a percentage error of 28.5% between this resistance and the expected resistance. The measured brush voltage is 0.0974 V, which is consistent with the expectation set out in the week 2 ESP lecture slides [2].

As with figure 2 in the previous section the gradient represents the motor constant K_T, whilst the y-intercept represents the friction torque, T_f. From this graph the constant K_T was measured as 0.0056 and T_f as 0.0003 Nm. These measurements are close to those found in the previous section - there is a percentage error of 3.4%, whilst the y-intercept is 0.0002 Nm larger than in figure 2, indicating that they are accurate.

3. Load measurements

The load measurement section compares the difference in the measurements and calculation of different forces needed at different sections of the track. This aids in the selection of the most optimal gearbox as the precise amount of torque needed for different sections can be calculated based on the measurement of forces, resulting in better designs. This helps tackle the challenges brought forth by the inclined slope.

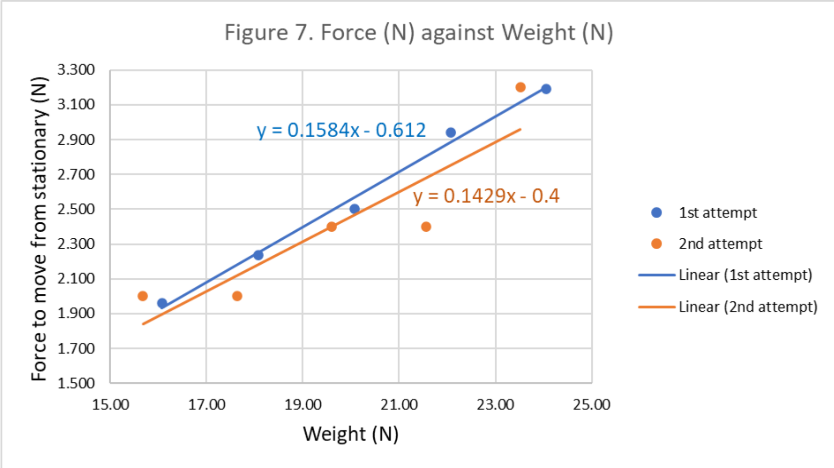
3.1 Estimated forces to drive buggy up inclined slope

The estimated force required to drive the buggy up the slope was calculated to be 7.479 N for a buggy of mass 1594 g. This is calculated by first finding the coefficient of static friction by plotting a graph of force needed to move buggy and the buggy's weight, the gradient is then the coefficient of static friction based on equation (3.1)

$$\mu = \frac{F}{W} \qquad (3.1)$$

where μ = coefficient of static friction, F = Force (N), W = Weight (N)

As shown on the graph below, the coefficient of static friction is 0.1584 and 0.1429 respectively. Additionally, the 1st graph shows greater systematic error, as it has a larger offset from the origin, while the 2nd graph shows greater random error, as there are more points where the line is far away from it. Subsequently a free body diagram as shown in figure 1 can be used to model the scenario. By using equation (3.2) the force required for the buggy to go up the slope can be calculated.



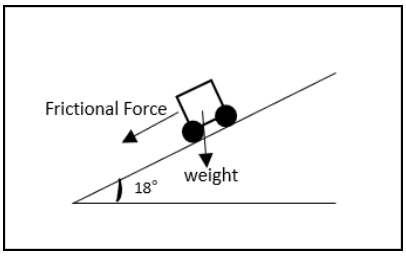


Figure 8. Free body diagram of the buggy going up the inclined path.

$$F = W \cdot sin(18^{\circ}) + \mu \cdot W \qquad (3.2)$$

$$F_1 = 15.94 \cdot sin(18^{\circ}) + 0.1584(15.94) = 7.451\,N$$

$$F_2 = 15.94 \cdot sin(18^{\circ}) + 0.1429(15.94) = 7.204\,N$$

The estimated force was chosen to be 7.451 N instead of 7.204 N, as the first μ had far less random error and was more precise. In addition, a greater force estimation would allow greater room of error for the design later, as the torque produced would more likely be greater than required. The mass of 1594 g was also greater than the previous speculated buggy mass of 1500 g which was based on the addition of different individual buggy components. Allowing even greater room for error.

3.2 Estimated forces to drive buggy at flat land

	Experimental Force (N)	Theoretical Force (N)
First attempt	1.96	2.52
Second attempt	2.00	2.28
Average Force	1.98	2.40

Table 1. experimental force and theoretical force

The force required to steadily move the buggy on flat land was measured to be 1.98 N. This is estimated by measuring the force required to move the buggy by pulling it with a force gauge. The measurements are very prone to human error, so the measurements are repeated twice with similar buggy weights and averaged to minimise the amount of error. The average theoretical force was found to be 2.40 N for a buggy of mass 1594 g using equation 3.1. The percentage error is then given by:

$$percentage\ error\ =\ \frac{|Theoretical\,force - Experimental\,force|}{Theoretical\,force}\ \times\ 100\% \qquad (3.3)$$

The percentage error is found to be roughly 17.5%, which was believed to be reasonable. The theoretical force is higher than the experimental force. This was expected as the method used to measure μ produces random and systematic errors. Its random error was reduced significantly by plotting a graph, however, there is no way to eliminate its systematic error. Thus, the theoretical force is higher.

3.3 Required Torque at wheel shaft

The torque can be found by using equation (3.4)

$$T\ =\ \frac{F\cdot d}{2} \qquad (3.4)$$

where T = torque (Nm), F = force (N), d = diameter (m).

The diameter was measured using a vernier calliper to be 0.08071 m. From the equation above, $T_{ramp}\ =\ \frac{7.450\times0.08071}{2}$ = 0.301 Nm. Only torque required to climb the slope was to be found as the torque to move buggy on flat land will always be lesser.

4. Gear ratio selection

The gear ratio selection is based on the motor characterisation and the load measurements. The former shows how much torque the motor provides, while the latter identifies the minimum torque required to overcome the steepest slope of the track. A gearbox is needed to provide a torque gain with minimised speed loss as the motor cannot directly offer the necessary torque.

4.1 Current/Voltage required without gearbox

From section 3.3 it is apparent the torque required on the steepest hill is 0.301 Nm, which means each motor should provide at least 0.1505 Nm torque. By applying equations:

$$I\ =\frac{T}{K_T}\quad (4.1) \text{ and } V\ =\ IR\ +V_b+K_E\omega\ =\frac{T}{K_T}R\ +V_b+K_E\omega \quad (4.2)$$

where I = current (A), T = torque required on the ramp (Nm), R = Armature resistance (Ω), V_b = brush voltage (V) and ω = angular speed of the motor (rad/s).

The current required: $I\ =\frac{0.301}{0.0058}\ =\ 51.90\ A$, and the corresponding minimum voltage $V\ =\frac{0.301}{0.0058}\times\ 2.145\ +\ 0.0973\ =\ 111.42\ V$ when angular speed $\omega\ =\ 0$. It is apparent that the voltage source on the buggy cannot supply such a high voltage.

4.2 Available motor torque

The maximum voltage available from the battery held by the buggy is 9.6 V (since the battery holder can contain 8✕1.2 V batteries). The available motor torque can be obtained by applying the function:

$$T\ =\frac{K_T}{R}(V\ -V_b-K_E\omega) \quad (4.3)$$

where T = torque (Nm), R = Armature resistance (Ω), V = voltage (V), V_b = brush voltage (V) and ω = the angular speed of the motor (rad/s).

$T = \frac{0.0058}{2.145} \times (9.6 - 0.0973) = 0.0257\,Nm$ for a single motor and 0.0514 Nm for both. This figure is far less than the torque needed to drive the buggy up the slope, which means extra torque needs to be obtained from the gearbox.

4.3 Gear ratio selection

Three different intermediate gears are provided to choose. In order to select the ideal one for the buggy, the gear ratio needs to be known, which is also the torque gain of the gearbox. The parameters can be found using:

$$G_{ratio} = \frac{T_3}{T_1} = \frac{\omega_1}{\omega_3} = \frac{N_3 N_{2A}}{N_{2B} N_1} \quad (4.4)$$

where N_1, N_{2A}, N_{2B}, N_3, are the number of teeth of each gear respectively.

Besides, the provided gearboxes have three gear stages, and assuming the efficiency of each stage is 85%, then 72.25% ($0.85^2 = 0.7225$) efficiency should be applied to the output torque T_3.

	Gearbox 1 (orange*)	Gearbox 2 (orange)	Gearbox 3 (red)
Gear Ratio	12	15	18.75
T_3 with efficiency/Nm	0.223	0.279	0.348

Table 2. gear ratio and output torque for a single motor for each gearbox

It seems each gearbox is capable of driving the buggy up the slope. The current should be considered as it positively correlated with torque, and its value should be under the maximum of 1.4 A. As the output torque needed is 0.1505 Nm, the current needed to drive a single motor can be found by applying:

$$I = \frac{T_m}{K_E} = \frac{0.1505}{0.7225 \times Gear\ Ratio \times K_E} \quad (4.4)$$

$$I_1 = \frac{0.1505}{0.7225 \times 12 \times 0.0099} = 1.75\,A \qquad I_2 = \frac{0.1505}{0.7225 \times 15 \times 0.0099} = 1.40\,A$$

$$I_3 = \frac{0.1505}{0.7225 \times 18.75 \times 0.0099} = 1.12\,A$$

where T_m is the required torque of each motor.

This shows that only the last two gearboxes are fit for this specific condition. Since the buggy needs to run as fast as possible in the final race, some torque has to be exchanged for speed. Overall, gearbox 2 is the optimal selection for the buggy.

4.4 Position of the intermediate shaft

The intermediate shaft position should be calculated as the motor and drive shaft is fixed but the intermediate one is varied. For the chosen gearbox, Pitch Circle Diameter (PCD) must be calculated using:

$$PCD = No.\ of\ teeth \times MOD \quad (4.5)$$

Where the MOD is claimed to be 0.5mm in the technical handbook [1]. Because two gears are used on the intermediate shaft, 4 gear wheels need to be considered.

$$PCD(1) = 16 \times 0.5 = 8\,mm \qquad PCD(2A) = 50 \times 0.5 = 25\,mm$$

$$PCD(2B) = 10 \times 0.5 = 5\,mm \qquad PCD(3) = 48 \times 0.5 = 24\,mm$$

And the central distance between pinion and intermediate gear depends on gear 1 and gear 2A, while the distance between intermediate and press-fit gear depends on gear 2B and gear 3. The central distance can be acquired by applying:

$$centre\ distance = \frac{PCD(A)+PCD(B)}{2} + 0.1\,mm \quad (4.6)$$

$$d_{12} = \frac{8+25}{2} + 0.1 = 16.6\,mm \qquad d_{23} = \frac{5+24}{2} + 0.1 = 14.6\,mm$$

The centre of the motor shaft could be set as the origin and the centre of the drive shaft is located at (31.1, 0). Assuming the desired point is (x,y):

$$\begin{cases} x^2 + y^2 = 16.6^2 \\ y^2 + 17.6^2 = (31.1 - x)^2 \end{cases} \Rightarrow \begin{cases} x = 16.553 \\ y = 1.245 \end{cases}$$

The centre of the desired shaft should be (16.55,1.25), measured in millimetres.

4.5 Maximum speed on the slope and on the flat

The maximum speed of the buggy can be calculated according to the data processed in the previous section. The torque required on the slope for each motor with gearbox 2 is 0.0139 Nm. And by using a variant of the EMF equation:

$$\omega = \frac{V - V_b}{K_E} - \frac{TR}{K_T K_E} \quad (4.7)$$

where ω = angular speed (rad/s), V = voltage (V), V_b = brush voltage (V), K_T = torque constant, K_E = emf constant, T = torque (Nm) and R = Armature resistance (Ω).

$$motor\ angular\ velocity:\ \omega_m = \frac{9.6-0.0973}{0.0099} - \frac{0.0139\times 2.145}{0.0099\times 0.0058} = 440.61\,rad/s$$

$$wheel\ angular\ velocity\ with\ efficiency:\ \omega_w = \frac{0.7225\omega_m}{15} = 21.22\,rad/s$$

$$buggy\ velocity\ on\ ramp:\ v_r = \omega_w r = 0.86\,m/s$$

The torque required on the flat is 0.0074 Nm. The process is the same as the one before, and for this case $\omega_m = 683.43\,rad/s$, $\omega_w = 32.92\,rad/s$ and $v_f = 1.33\,m/s$.

5. Summary

To summarise, the report's aim was to select an optimal gearbox to create a buggy that is fast, energy efficient and can overcome the obstacles of the track. Additionally, it is assumed that the air drag is insignificant due to the size and speed of the buggy, and thus is omitted from the calculations. An interesting thing to note is the fact that theoretical and experimental values differ by a sizable margin, due to the constraints of the measurement methods used. The gearbox that was chosen was gearbox 2 due to various factors such as, its ability to provide sufficient torque and decent speed, and is within the current limit of the motor drive board of 1.4 A. It fits the design criteria best and thus was chosen. Below is a table of key values:

K_T	K_E	V_b(V)	R(Ω)	T_f(Nm)	G_{ratio}	T_{ramp}(Nm)	v_r(m/s)	v_f(m/s)
0.0058	0.0099	0.0974	2.145	0.0005	15	0.301	0.86	1.33

6. References

[1] ESP Technical Handbook – EEEN21000 Embedded Systems ... (manchester.ac.uk) [Accessed 3rd November 2022]

[2] Lecture Slides - Lecture 2 Motor Characterisation and Gearbox ... (manchester.ac.uk) [Accessed 1st November 2022]