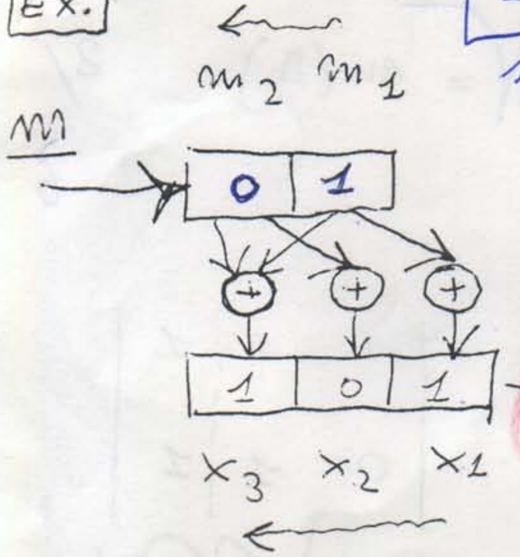
Ex.

$\boxed{\text{ES1}}$ 16/17 $\boxed{\text{E1}}$ 1/8



$$x_i = \sum_{j=1}^{K} g_{ji}\, m_j \qquad i = 1, \dots, N$$

$$\boxed{\begin{array}{l} N = 3 \\ K = 2 \end{array}}$$

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$$\begin{cases} x_1 = g_{11}\, m_1 + g_{21}\, m_2 = 1 \cdot m_1 + 0 \cdot m_2 \\ x_2 = g_{12}\, m_1 + g_{22}\, m_2 = 0 \cdot m_1 + 1 \cdot m_2 \\ x_3 = g_{13}\, m_1 + g_{23}\, m_2 = 1 \cdot m_1 + 1 \cdot m_2 \end{cases}$$

$$\begin{vmatrix} m_1 & m_2 \end{vmatrix} \cdot \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \end{vmatrix} = \begin{vmatrix} x_1 & x_2 & x_3 \end{vmatrix}$$

$$\begin{vmatrix} m_1 & m_2 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} m_1 & m_2 & m_1 + m_2 \end{vmatrix}$$

$$\begin{array}{cc} m_1 & m_2 \end{array} \qquad\qquad \begin{array}{ccc} x_1 & x_2 & x_3 \end{array}$$

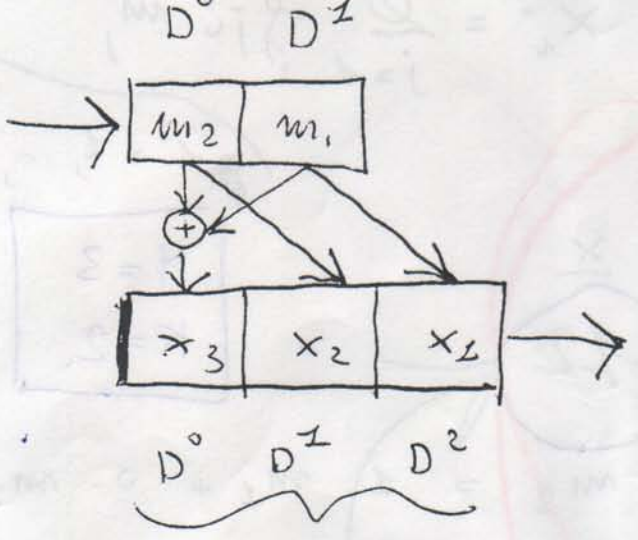
$$\begin{cases} \begin{vmatrix} 1 & 0 \end{vmatrix} \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 \end{vmatrix} \\ \begin{vmatrix} 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 \end{vmatrix} \end{cases}$$

00	00 0
01	01 1
10	10 1
11	11 0

$$\begin{vmatrix} 0\,0 \end{vmatrix} \rightarrow \begin{vmatrix} 0 & 0 & 0 \end{vmatrix} \qquad \begin{vmatrix} 1\,1 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & 1 & 0 \end{vmatrix}$$

$$m_1 D^1 + m_2 \cdot D^0 = m(D)$$

g



$$G = \left|\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right|$$

$\cancel{D+1} \rightarrow g(D)$

$$\underbrace{x_1 D^2 + x_2 D^1 + x_3 \cdot D^0} = x(D)$$

$$x(D) = m_1 D^2 + m_2 D^1 + (m_1 + m_2) \cdot D^0$$

$$\cancel{(m_1 D + m_2)}\cancel{(D+1)} = \cancel{m_1 D^2 + m_2 D +}$$
$$\cancel{+\, m_1 D + m_2}$$

$$\begin{cases} |1 \;\; 1| \cdot \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = |1 \;\; 1 \,|\, 0| \\ |0 \;\; 0| \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = |0 \;\; 0 \,|\, 0| \end{cases}$$

$$X = \left\{\begin{array}{cc|cl} 0 & 0 & 0 & \ldots \\ 0 & 1 & 1 & \cdot \\ 1 & 0 & 1 & \cdot \\ 1 & 1 & 0 & (+) \end{array}\right.$$

(*)

$$g(D) = D + 1$$

$$\cancel{D \cdot (D+1) = D^2 + D}$$
$$\cancel{D^2 (D+1) = D^3 + D^2}$$

$$m(D) = (m_1 D + m_2) \cdot (\cdot) = \begin{array}{l} m_1 D^2 + m_2 D + \\ + m_1 D + m_2 \end{array}$$

$$m_1 D^2 + (m_1 + m_2) D + m_2 = x'(D)$$

$x'(D) \neq x(D)$???
???

$(N-K \to 1$

$$D \cdot m(D) = m_1 D^2 + m_2 \cdot D =$$

$$\begin{array}{l|l} m_1 D^2 + m_2 D & D + 1 \\ \hline m_1 D^2 + m_1 D & m_1 D + (m_1 + m_2) \\ \hline 0 + (m_2 + m_1) D & \\ (m_1 + m_2) D + m_1 + m_2 & \end{array}$$

$D^{N-M} \cdot (m(D))$

$$m_1 D^2 + m_2 D = m_1 D \cdot (D+1) + (m_2 + m_1)(D+1) = m_1 D^2 + \cancel{m_1 D} + \cancel{m_1 D} + m_2 D$$

$$NUM = q \cdot DEN + r$$

$$\frac{N}{D} = q + \frac{r}{D}$$

$$m_1 D^2 + m_2 D = (D+1) \cdot (m_1 D + [m_1 + m_2]) + (m_1 + m_2) =$$
$$= m_1 D^2 + \cancel{m_1 D} + \cancel{m_1 D} + m_2 D + \cancel{m_1} + \cancel{m_2} + \cancel{m_1} + \cancel{m_2}$$

$$m_1 D \cdot (D+1) + (m_2 + m_1) D =$$

$$= m_1 D^2 + \cancel{m_1 D} + m_2 D + \cancel{m_1 D}$$

$$D \cdot m^{(N-K)}(D) + r_{g(D)}\left(D \cdot m^{(N-K)}(D)\right) =$$

$$= m_1 D^2 + m_2 D + (m_1 + m_2)$$

(*) $g(D) = D + 1 \longrightarrow D^N + 1 = D^3 + 1$

	$D^2 + D + 1$
$D+1$	$D^3 + 1$
	$D^3 + D^2$
	$0 + D^2 + 1$
	$D^2 + D$
	$0 + D + 1$

è un divisore di $D^N + 1$

$$D^3 + 1 = \underbrace{(D^2 + D + 1) \cdot (D + 1)}_{}$$ è ciclico

• RIPETERE l'esempio con $(D^2 + D + 1)$

$m_1(D) = D+1 \longrightarrow 1\,1$

$$m(D) \cdot g(D) = (D+1)(D+1) = D^2 + D + D + 1 = \underbrace{D^2+1}_{1\,0\,1}$$

$m_2(D) = D \rightarrow 1\,0$

$D \cdot (D+1) = D^2 + D \longrightarrow 1\,1\,0$

$m_3(D) = 1 \rightarrow 0\,1$

$1 \cdot (D+1) = D+1 \longrightarrow 0\,1\,1$

$m_3(D) = 0 \longrightarrow 0\,0\,0$

$\searrow 0\,0$

VERSIONE NON SIST. del CODICE

$m_1(D) \longrightarrow$ SIST.

$$D \cdot (D+1) + R_{g(D)}\left[(D+1)\,D\right] = \underbrace{D^2 + D + 0}_{1\,1\,0}$$

$$\begin{array}{r|l} D+1 & D \\ \hline & D^2 + D \\ & D^2 + D \\ \hline & 0 + 0 \end{array}$$

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$$m_2(D) = D + 0 \xrightarrow{10} D\cdot(D+1) = D^2 + D \longrightarrow \cancel{110}$$

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NO SIST.

SIST

$$D\cdot D + R_{g(D)}\left[D\cdot D\right] = \underbrace{D^2 + 1}$$

$$\underbrace{101}$$

$$\begin{array}{r|l} & D+1 \\ \hline D+1 & D^2 \\ & D^2 + D \\ \hline & 0 + D \\ & D+1 \\ \hline & 1 \end{array}$$

OK

$$m_3(D) = 1 \xrightarrow{01} D+1 \longrightarrow 011$$

$$D + R_{g(D)}\left[D\right] = D+1$$

$$\begin{array}{r|l} & 1 \\ \hline D+1 & D \\ & D+1 \\ \hline & 1 \end{array}$$

$m_1(D) = D+1 \longrightarrow 11$

$m(D)\cdot g(D) = (D+1)(D+1) = D^2+D+D+1 = \underbrace{D^2+1}_{101}$

$m_2(D) = D \longrightarrow 10$

$D\cdot(D+1) = D^2+D \longrightarrow 110$

$m_3(D) = 1 \longrightarrow 01$

$1\cdot(D+1) = D+1 \longrightarrow 011$

$m_3(D) = 0 \longrightarrow 000$

$\searrow 00$

VERSIONE NON SIST. del CODICE

$m_2(D) \longrightarrow$ SIST.

$$D\cdot(D+1) + R_{g(D)}\left[(D+1)D\right] = \underbrace{D^2+D+0}_{110}$$

$$\begin{array}{r|l} & D \\ \hline D+1 & D^2+D \\ & D^2+D \\ \hline & 0+0 \end{array}$$

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$$\overset{M-K}{(m_1 D + m_2)\cdot(D)} = m_1 D^2 + m_2 D$$

$$\begin{array}{r|l} & m_1 D + m_1 + m_2 \\ \hline D+1 & m_1 D^2 + m_2 D \\ & m_1 D^2 + m_1 D \\ \hline & 0 + \cancel{m_1 D} + \cancel{m_2 D} \\ & m_2 D + \cancel{m_1 D} + m_1 + m_2 \\ \hline & m_2 \cancel{D} + m_1 \end{array}$$

$$\underbrace{m_1 D^2 + m_2 D + m_1 + m_2}_{\text{SIST.}} = X(D)$$

$$\begin{vmatrix} 1 & 1 & 0 \\ & 1 & 1 \end{vmatrix} \oplus \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = G$$

$g(D)$

$1+1$ $0+1$

$$G = \left|\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right| \qquad \underbrace{I_2}_{} \quad P$$

codice $(3,2)$ → matrice $2 \cdot 3$

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$$H^T = \left|\begin{array}{c} P \\ I_{N-K} \end{array}\right| = \left|\begin{array}{c} 1 \\ 1 \\ 1 \end{array}\right| \longrightarrow H^T: \text{matrice } (N, N-K) \Rightarrow$$

matrice $3 \cdot 1$

$$x H^T = 0 \longrightarrow |x_1 \ x_2 \ x_3| \left|\begin{array}{c} 1 \\ 1 \\ 1 \end{array}\right| =$$

$$\Rightarrow \boxed{x_1 + x_2 + x_3 = 0}$$

NUMERO PARI di "UNI"

(controllo di parità semplice...)

$$d_{MIN} = 2 \longrightarrow t = 0$$

↓

RIVELATORE di "UNO" errore