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Problem 1

The penetration depth is calculated using the next expression:

$$\delta_m = \frac{\lambda}{2\pi}\sqrt{\left|\frac{\varepsilon'_m + \varepsilon_d}{\varepsilon_m^2}\right|} \quad \text{for metals}$$

$$\delta_d = \frac{\lambda}{2\pi}\sqrt{\left|\frac{\varepsilon'_m + \varepsilon_d}{\varepsilon_d^2}\right|} \quad \text{for dielectrics}$$

In the case of gold:

$$\delta_{gold} = \frac{\lambda}{2\pi}\sqrt{\left|\frac{\varepsilon'_{gold} + \varepsilon_{air}}{\varepsilon_{air}^2}\right|} = \frac{1064}{2\pi}\sqrt{\left|\frac{-43{,}8+1}{(-43{,}8)^2}\right|} = \frac{532}{\pi}\cdot 0{,}1494$$

$$\boxed{\delta_{gold} = 25{,}30\ \text{nm}}$$

In the case of air:

$$\delta_{air} = \frac{\lambda}{2\pi}\sqrt{\left|\frac{\varepsilon'_{gold} + \varepsilon_{air}}{\varepsilon_{air}^2}\right|} = \frac{532}{\pi}\sqrt{\left|\frac{-43{,}8+1}{1^2}\right|} = \frac{532}{\pi}\cdot 6{,}5422$$

$$\boxed{\delta_{air} = 11078{,}86\ \text{nm}}$$

Secondly, to excite the gold-air surface plasmon the SP wave has to match with: $k_0 \; n_d \; \sin(\theta_i) \pm \frac{2\pi m}{\Lambda}$

So:

$$k_0 \, n_{sp} = k_0 \, n_d \, \sin(\theta_i) \pm \frac{2\pi m}{\Lambda}$$

- Normal incidence: $\theta_i = 0 \Rightarrow \sin(\theta_i) = 0$
- $n_{sp} = \sqrt{\frac{\varepsilon' \varepsilon_d}{\varepsilon' + \varepsilon_d}} = \sqrt{\frac{\varepsilon'_{gold} \cdot \varepsilon_{air}}{\varepsilon'_{gold} + \varepsilon_{air}}} = \sqrt{\frac{-43,8 \cdot 1}{-43,8+1}} = \sqrt{\frac{43,8}{42,8}}$

$$\boxed{n_{sp} = 1,01161}$$

- First order gratting: $m = 1$

Using everything we get:

$$k_0 \, n_{sp} = \frac{2\pi}{\Lambda} \Longrightarrow \Lambda = \frac{2\pi}{k_0 \, n_{sp}} = \frac{2\pi}{\frac{2\pi}{\lambda} \cdot n_{sp}} = \frac{\lambda}{n_{sp}} = \frac{1064}{1,01161}$$

$$\boxed{\Lambda = 1051,8 \text{ nm}}$$

1 Problem 2

1.1 Maxwell Garnett

Used for media with ***small inclusions*** dispersed in a ***continuous host medium***. The grains of guest material's relative permittivity is ϵ_i and they are hosted in a continuous medium with relative permittivity ϵ_h. If grains are small enough we can assume quasi-static approximation. Also, if there is no information about the shape of the grains, a small sphere shape is assumed.

Limits of validity

- If $\epsilon_i > 0$ particle size should be $< \frac{1}{10}\lambda eff$
- If $\epsilon_i < 0$ the limits of validity are stricter

Modelling first step:
In quasi-static approximation external electric field E_{ext} is considered constant for each sphere. Also, each sphere behaves like a point source with electric dipole moment proportional to E_{ext}.

$$P_h = \epsilon_0 \epsilon_h \alpha E_{ext} \tag{1}$$

The field inside sphere E_i is uniform and parallel to E_{ext}. Polarizability is isotropic.
Modelling second step:
Create the model for a distribution of small spheres. First, several electric point dipoles radiating and influencing each other:

$$< D >= \epsilon_0 \epsilon_{MG} < E > \tag{2}$$

Then, the average medium response can be written as:

$$< D >= \epsilon_0 \epsilon_h < E > + < P > \tag{3}$$

The average dipole response can be calculated using a model of a sphere with a charge density in the surface:

$$< P >= \frac{3N\alpha\epsilon_0\epsilon_h}{3 - N\alpha} < E > \tag{4}$$

Finally, the expression for the effective permittivity is:

$$\epsilon_{MG} = \epsilon_h \left(1 + \frac{N\alpha}{1 - \frac{N\alpha}{3}}\right) \tag{5}$$

For diluted media $1 - \frac{N\alpha}{3} \approx 1$ so we get $\epsilon_{MG} = \epsilon_h(1 + N\alpha)$. Also, introducing the parameter f which relates the volume fraction of the inclusions gives the next expression:

$$\epsilon_{MG} = \epsilon_h \left[1 + 3f \frac{\epsilon_i - \epsilon_h}{\epsilon_i + 2\epsilon_h - f(\epsilon_i - \epsilon_h)} \right] \tag{6}$$

The limitations of the model

- Values of f higher than 0.5 make predictions questionable, because the interparticle distances decrease and trigger high-order multipole effects.
- This approach only can be used in a quasi-static state with dipole approximation. Higher-order terms are excluded.
- The role of the host and guests particles cannot be exchanged. If there is not a clear distinction, the validity of the formula cannot be ensured.

1.2 Bruggeman Theory

This theory is used when the medium is aggregate mixture with random distributions. Statistical formulation theories are used to model it.

In the Bruggeman theory the mixture is modeled as a continuous medium hosting a distribution of small spherical inclusions. Both, host medium and spherical inclusions, have different dielectric permittivities. Because of the fact that it is a statistical formulation based theory, the probabilities of finding spheres with permittivity ϵ_i and ϵ_h are assigned as f and $1-f$ respectively.

Assuming quasi-static approximation we get the formula:

$$f \frac{\epsilon_i - \epsilon_{Br}}{\epsilon_i + 2\epsilon_{Br}} + (1 - f) \frac{\epsilon_h - \epsilon_{Br}}{\epsilon_h + 2\epsilon_{Br}} = 0 \tag{7}$$

Where ϵ_{Br} is the unknown permittivity.

This formula is symmetric so the roles of inclusions and host are exchangeable. It can be also be extended to multi-phase aggregates just adding more terms:

$$\sum_{M}^{m=1} f_m \frac{\epsilon_m - \epsilon_{Br}}{\epsilon_m + 2\epsilon_{Br}} = 0 \tag{8}$$

In this theory, shape effects can be included, such as ellipsoidal inclusions.

1.3 Differences between Maxwell Garnett and Bruggeman theories

- For situations with large inclusions' fill factor, Bruggeman formula gives a better modelling approach.
- Bruggeman theory provides a more realistic description when the mixture has large difference in the permittivities of the constituents.
- In the Bruggeman theory the roles of the host and inclusions are exchangeable, whereas, they are not in Maxwell Garnett.
- Maxwell Garnett theory suits better when there is a clear distinction between the host and inclusions.

Problem 3

The expression of optimal thickness for antireflection is:

$$d_{AR} = \frac{\lambda_{eff}}{4}$$

Developing it:

$$d_{AR} = \frac{\lambda_{eff}}{4} = \frac{\lambda_0}{4 \cdot n_{cR}} = \frac{500}{4 \cdot 1{,}3} \Rightarrow \boxed{d_{AR} = 96{,}15\,nm}$$