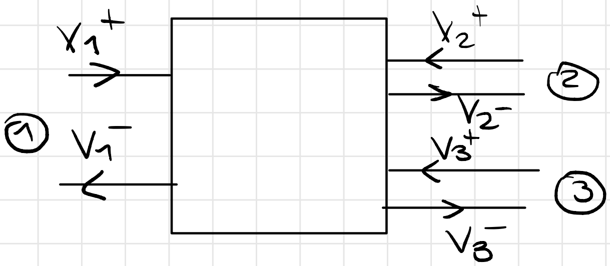
Microwave Engineering

Homework 3

Solution

#1

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$



a) The device is RECIPROCAL since $S_{ij} = S_{ji}$

b) The device is LOSSY since

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 \neq 1$$

(c) The device IS MATCHED at all ports since $S_{ii} = 0$

$(S_{11} = S_{22} = S_{33} = 0)$

(d) Output ports 2 and 3 are NOT ISOLATED since

$$S_{23} = S_{32} \neq 0$$

Let's consider $V_1^+ = 10V$ and output ports terminated with matched loads.

(e) Calculate voltage at ports (2) and (3):

$$S_{21} = S_{12} = \frac{V_2^+}{V_1^+}$$

$$V_2^- = S_{21} V_1^+ = \frac{1}{2} V_1^+ = 5V$$

$$V_3^- = S_{31}V_1^+ = \frac{1}{2}V_1^+ = 5V$$

$V_2 = V_2^-$ and $V_3 = V_3^-$ since ports 2 and 3 are matched

(f) Power available at ports (2) and (3) is:

$$P_2 = \frac{1}{2}\frac{|V_2^2|}{Z_0} = \frac{1}{2}\frac{25}{50} = 0.25W$$

$P_3 = P_2 = 0.25W$ since $V_2 = V_3$

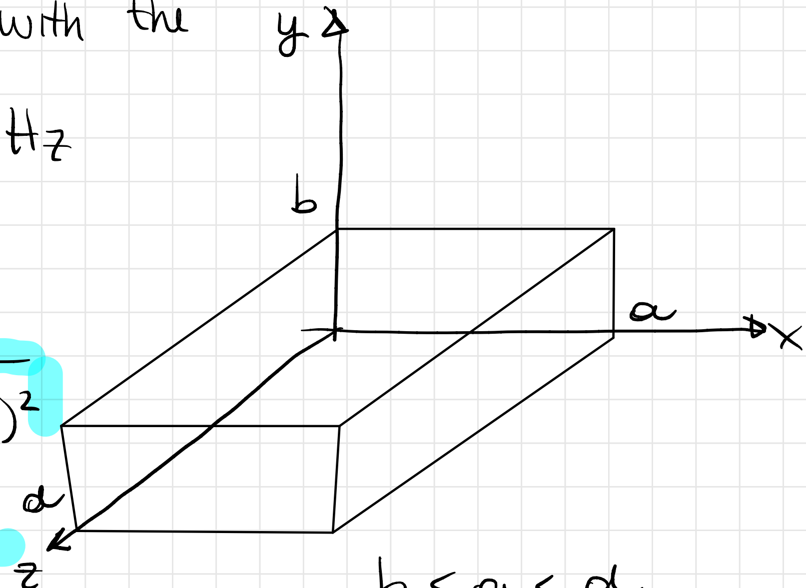
(g) Dissipated power

$$P_1 = \frac{1}{2}\frac{|V_1|^2}{Z_0} = \frac{1}{2}\frac{100}{50} = 1W$$

$$P_{loss} = P_1 - P_2 - P_3 = 0.5\ W$$

#2 First mode is associated with the fundamental mode TE_{10}.

$$f_{c101} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{d}\right)^2} = 5\,GHz$$



$$\Downarrow$$

$$\boxed{\frac{1}{a^2}+\frac{1}{d^2} = \left(\frac{2{,}5\cdot10^9}{3\cdot10^8}\right)^2 = (33{,}3)^2}$$

$b < a < d$

The modes that follow are either TM_{110}, TE_{102} or TE_{011} with associated frequencies:

$$f_{110} = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$$

$$f_{102} = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{4}{d^2}}$$

Since $d > a$

$f_{011} < f_{110}$

$$f_{011} = \frac{c}{2}\sqrt{\frac{1}{b^2} + \frac{1}{d^2}}$$

From geometrical considerations it follows that the second and third mode are f_{102} and f_{011}.

So we can write:

$$f_{102} = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{4}{d^2}} = 6.5 \text{ GHz} \Rightarrow \boxed{\frac{1}{a^2} + \frac{4}{d^2} = (43.3)^2}$$

$$f_{011} = \frac{c}{2}\sqrt{\frac{1}{b^2} + \frac{1}{d^2}} = 7.2 \text{ GHz} \Rightarrow \boxed{\frac{1}{b^2} + \frac{1}{d^2} = (48)^2}$$

Solving for a, b and d we get

$$\boxed{\begin{aligned} d &= 6.25 \text{ cm} \\ a &= 3.42 \text{ cm} \\ b &= 2.2 \text{ cm} \end{aligned}}$$

$d > a > b$

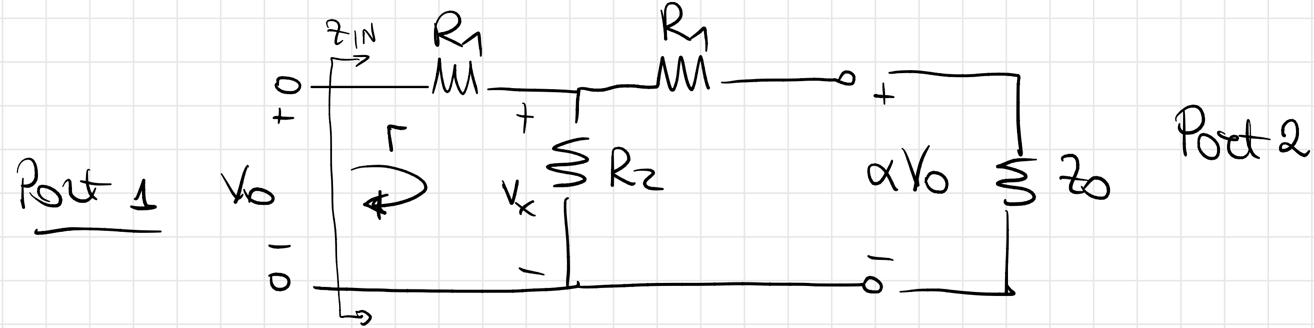
We can verify that

$$f_{110} = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = 8.1 \text{ GHz}$$

> 7.2 GHz

(third mode)

#3 T network



$$S_{11} = \frac{V_1^-}{V_1^+}\Bigg|_{V_2^+=0} = \Gamma = 0$$

$$\Gamma = \frac{Z_{IN} - Z_0}{Z_{IN} + Z_0}\Bigg|_{V_2^+=0} = \frac{\left[R_1 + R_2 /\!/ (R_1 + Z_0)\right] - Z_0}{\left[R_1 + R_2 /\!/ (R_1 + Z_0)\right] + Z_0} = 0$$

$$R_1 + \frac{R_2(R_1+Z_0)}{R_2+R_1+Z_0} - Z_0 = 0$$

$$R_1R_2 + R_1^2 + \cancel{Z_0R_1} + R_1R_2 + \cancel{R_2Z_0} - \cancel{R_2Z_0} - \cancel{R_1Z_0} - Z_0^2 = 0$$

$$R_1^2 + 2R_1R_2 - Z_0^2 = 0$$

$$\boxed{R_2 = \frac{Z_0^2 - R_1^2}{2R_1}}$$

$$S_{12} = \frac{V_2^-}{V_1^+}\bigg|_{V_2^+=0} = \alpha$$

Ports are matched so $V_2^- = V_2 = \alpha V_0$

$$V_2 = \frac{Z_0}{R_1+Z_0} V_x$$

$$V_x = \frac{(R_1+Z_0)//R_2}{[(R_1+Z_0)//R_2]+R_1} V_1$$

$$S_{12} = \frac{V_2^-}{V_1^+} = \frac{V_2}{V_1} = \frac{Z_0}{\cancel{R_1+Z_0}} \; \frac{\dfrac{\cancel{(R_1+Z_0)}R_2}{R_1+Z_0+R_2}}{\dfrac{(R_1+Z_0)R_2}{R_1+Z_0+R_2}+R_1} = \alpha$$

$$S_{12} = \frac{Z_0 R_2}{(R_1+Z_0)R_2 + R_1^2 + Z_0 R_1 + R_1 R_2} = \alpha$$

$$\frac{Z_0 R_2}{R_1R_2 + Z_0R_2 + R_1^2 + Z_0R_1 + R_1R_2} = \alpha$$

$$Z_0^2$$

$$\frac{\cancel{Z_0} R_2}{Z_0^{\cancel{2}} + \cancel{Z_0}R_2 + \cancel{Z_0}R_1} = \alpha$$

$$\frac{R_2}{Z_0 + R_2 + R_1} = \alpha$$

$$R_2 = \alpha Z_0 + \alpha R_2 + \alpha R_1$$

$$R_2 (1-\alpha) = \alpha (Z_0 + R_1)$$

since $R_2 = \dfrac{Z_0^2 - R_1^2}{2R_1}$

$$\frac{(Z_0+R_1)(Z_0-R_1)}{2R_1}(1-\alpha) = \alpha (Z_0+R_1)$$

$$\boxed{R_1 = \frac{1-\alpha}{1+\alpha} Z_0}$$

if $Z_0 = 50\ \Omega$

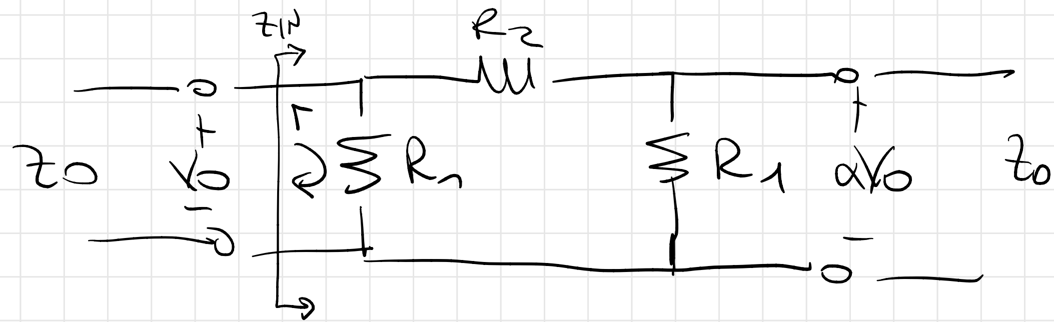
$$\alpha_{dB} = 20 \log(\alpha_{lin}) \qquad \alpha_{lin} = 10^{-\frac{\alpha_{dB}}{20}}$$

$$\alpha_{dB} = -3\,dB \Rightarrow \alpha_{lin} = 0.708 \Rightarrow \begin{cases} R_1 = 8.65\,\Omega \\ R_2 = 142\,\Omega \end{cases}$$

$$\alpha_{dB} = -10\,dB \Rightarrow \alpha_{lin} = 0.316 \Rightarrow \begin{cases} R_1 = 26\,\Omega \\ R_2 = 35.1\,\Omega \end{cases}$$

$$\alpha_{dB} = -20\,dB \Rightarrow \alpha_{lin} = 0.1 \Rightarrow \begin{cases} R_1 = 40.9\,\Omega \\ R_2 = 10.1\,\Omega \end{cases}$$

For the π-network we repeat the same procedure



$$S_{11} = \frac{V_1^-}{V_1^+}\Bigg|_{V_2^+=0} = \Gamma = 0 \qquad \Gamma = \frac{Z_{IN} - Z_0}{Z_{IN} + Z_0}$$

$$Z_{IN} = (R_1 // Z_0 + R_2) // R_1$$

$$Z_{IN} = \frac{(R_1 Z_0 + R_1 R_2 + Z_0 R_2) R_1}{R_1 Z_0 + R_1 R_2 + Z_0 R_2 + R_1^2 + Z_0 R_1}$$

$$S_{11} = \frac{Z_{IN} - Z_0}{Z_{IN} + Z_0} = 0 \Rightarrow \boxed{R_2 = \frac{2 R_1 Z_0}{R_1^2 - Z_0^2}}$$

$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+ = 0} = \frac{\alpha V_0}{V_0} = \alpha$$

$$V_2 = V_1 \frac{R_1 // Z_0}{R_1 // Z_0 + R_2} = \frac{\frac{R_1 Z_0}{R_1 + Z_0}}{\frac{R_1 Z_0}{R_1 + Z_0} + R_2} V_1$$

$$\frac{V_2}{V_1} = \alpha \implies \frac{R_1 Z_0}{R_1 Z_0 + R_2 R_1 + Z_0 R_2} = \alpha$$

Replacing $R_2 = \dfrac{2R_1 Z_0^2}{R_1^2 - Z_0^2}$ we get

$$\cancel{R_1}\cancel{Z_0}(1-\alpha) = \alpha \frac{2\cancel{R_1} Z_0^{\cancel{2}}}{\cancel{(R_1+Z_0)}(R_1 - Z_0)} \cancel{(R_1+Z_0)}$$

$$(1-\alpha)(R_1 - Z_0) = 2\alpha Z_0$$

$$\boxed{R_1 = \frac{1+\alpha}{1-\alpha} Z_0}$$

So if $Z_0 = 50\Omega$

$\alpha_{dB} = -3\,dB \Rightarrow \alpha_{lin} = 0.708 \Rightarrow \begin{cases} R_1 = 292.5\,\Omega \\ R_2 = 17.6\,\Omega \end{cases}$

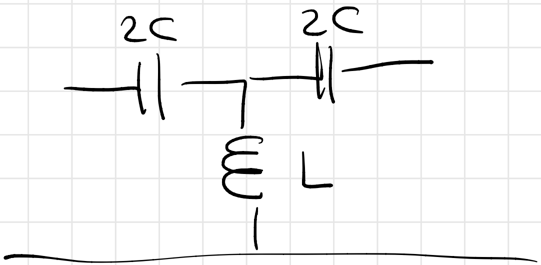
$\alpha_{dB} = -10\,dB \Rightarrow \alpha_{lin} = 0.316 \Rightarrow \begin{cases} R_1 = 96.2\,\Omega \\ R_2 = 71.2\,\Omega \end{cases}$

$\alpha_{dB} = -20\,dB \Rightarrow \alpha_{lin} = 0.1 \Rightarrow \begin{cases} R_1 = 61.1\,\Omega \\ R_2 = 247.5\,\Omega \end{cases}$

#4

$R_0 = 75\ \Omega$ $\quad f_c = 50\ \text{MHz}$ $\quad f_\infty = 48\ \text{MHz}$

First we calculate the parameters for constant-k section for high-pass filter

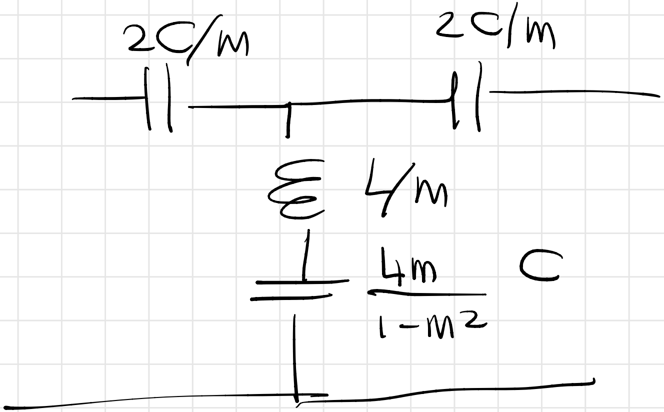


$$L = \frac{R_0}{2\omega_c} = \frac{R_0}{2 \cdot 2\pi f_c} = 119\ \text{nH}$$

$$C = \frac{1}{2R_0\,\omega_c} = \frac{1}{2 \cdot 75 \cdot 2\pi f_c} = 21.2\ \text{pF}$$

$$2C = 42.4\ \text{pF}$$

We add the m-derived section for sharp cut-off.



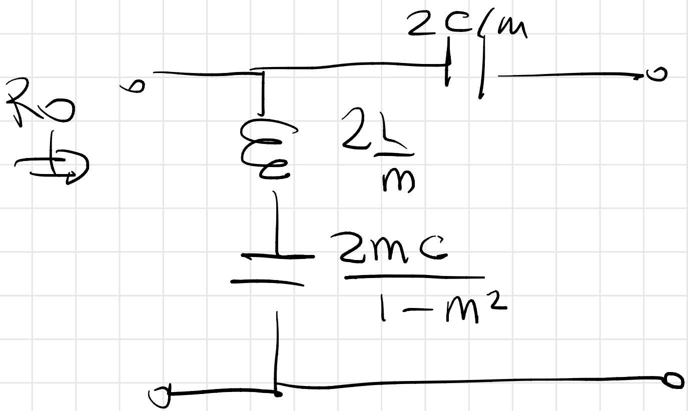
$$m = \sqrt{1 - \left(f_\infty / f_c\right)^2} = 0.28$$

$$\frac{2C}{m} = 151 \text{ pF}$$

$$\frac{L}{m} = 425 \text{ nH}$$

$$\frac{4mC}{1-m^2} = 25.8 \text{ pF}$$

Finally we add the matching section $(m=0.6)$



$$\frac{2C}{m} = 70.7\ pF$$

$$\frac{2L}{m} = 397\ nH$$

$$\frac{2mC}{1-m^2} = 39.8\ pF$$

The filter is then: