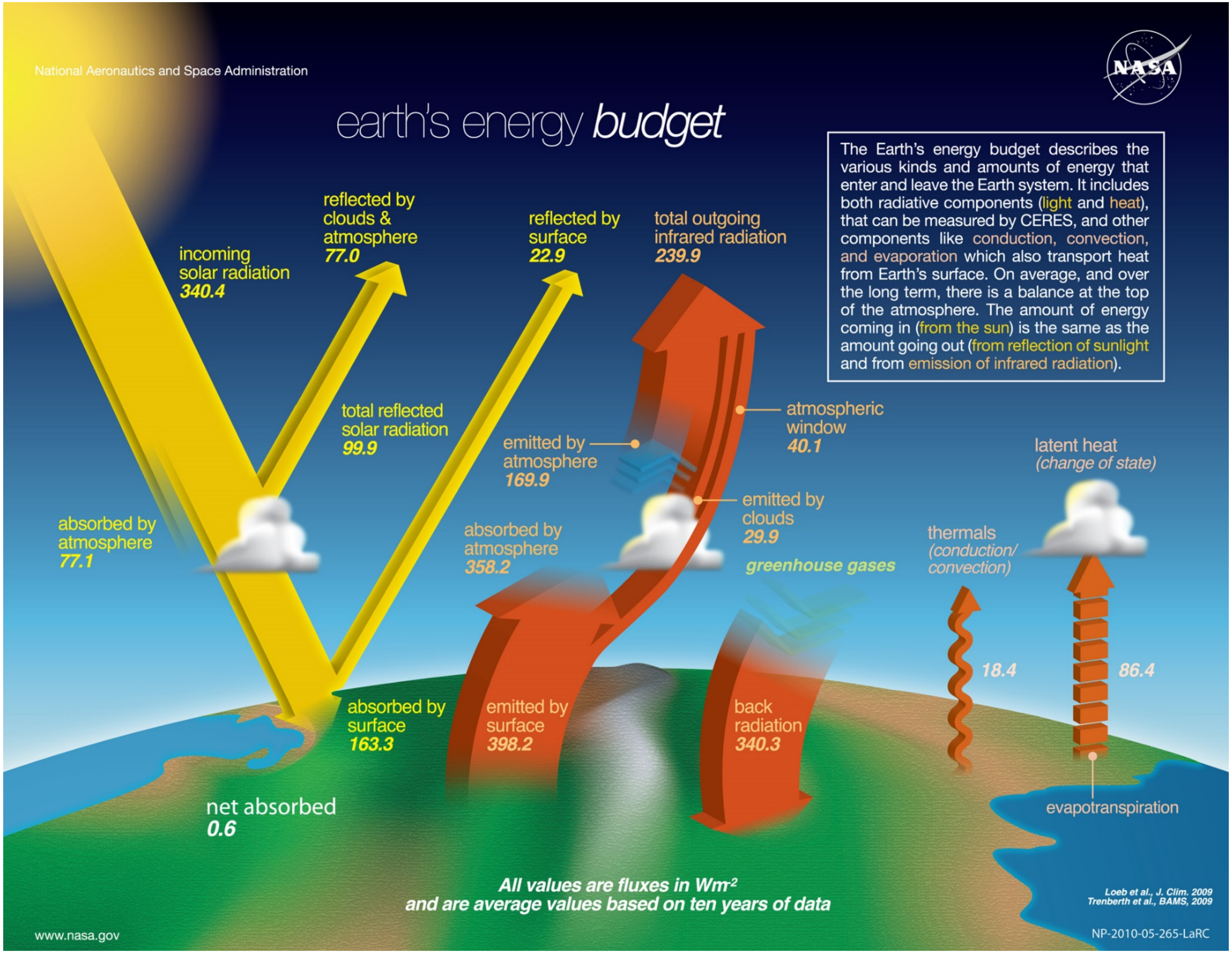
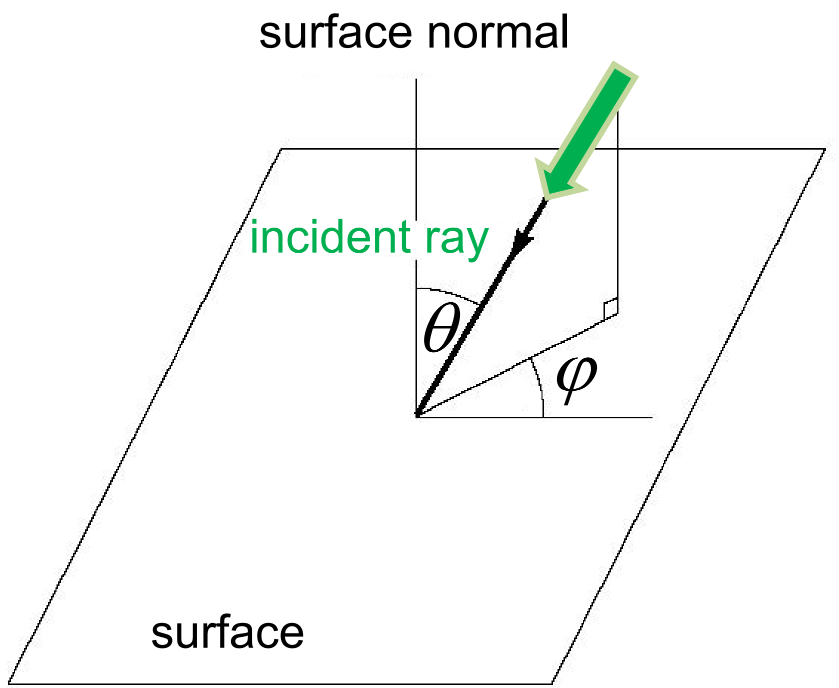
NATURAL ELECTROMAGNETIC RADIATION



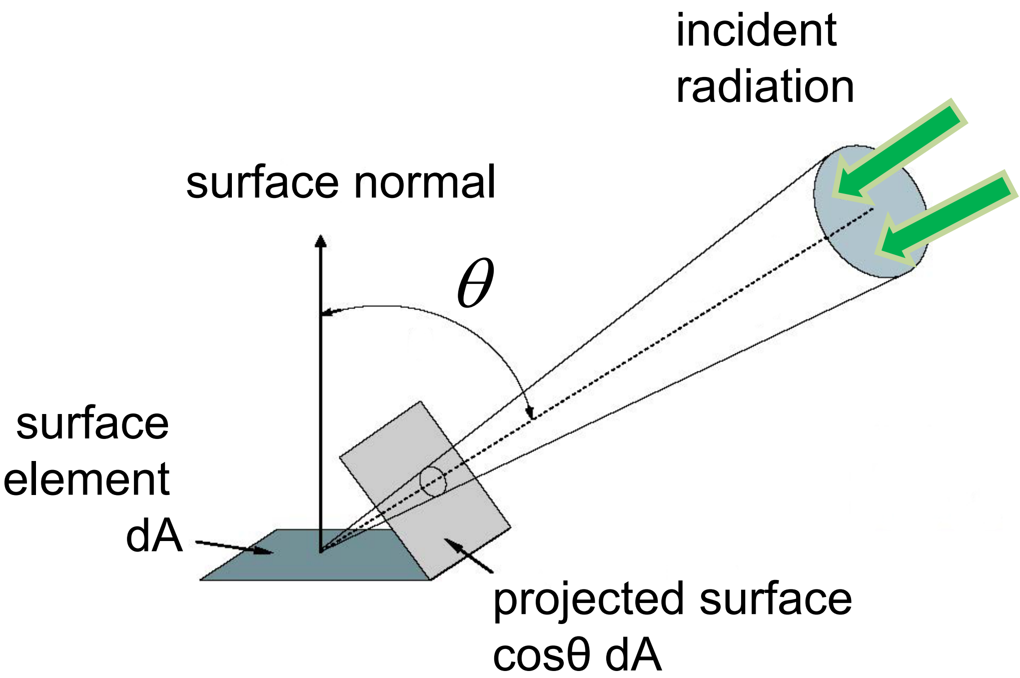
DEFINITION OF RADIATION QUANTITIES

We are used to consider plane waves or collimated beams (i.e. radiation travelling in a single direction), but we need to describe radiation distributed over a range of directions in the 3D space.

Let us consider a plane surface that is illuminated by radiation from a variety of directions; to specify a particular direction of incident radiation we need two angles: θ (the angle between the propagation direction and the normal to the surface) and φ, the azimuth angle, measured around the normal in the plane of the surface:



If we consider an element of the surface dA and radiation incident from a solid angle dΩ the power incident must be proportional to the area dA, to the solid angle dΩ and to a function giving the radiation strength



$$dP = L\cos\theta\, dA\, d\Omega$$

L is the RADIANCE of the incident radiation in the direction given by θ,φ

$$L = \frac{dP}{\cos\theta\, dA\, d\Omega}$$

the radiance unit of measurement is $W\ m^{-2}\ sr^{-1}$

When the directions can vary in the intervals θ,θ+dθ and φ,φ+dφ the solid angle reads as:

$$d\Omega = \sin\theta\, d\theta\, d\varphi$$

E is the IRRADIANCE at a surface: total <u>incident power per unit area</u>

$$E = \int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{2\pi} L_{in} \cos\theta \, d\Omega$$

the irradiance unit of measurement is W m^{-2}

If we consider the emitted radiation, we can define in a similar way the RADIANT EXITANCE M

$$M = \int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{2\pi} L_{out} \cos\theta \, d\Omega$$

For <u>isotropic radiation</u>, the radiance is independent of direction and the relationship between the radiance and the exitance is given by

$$M = \int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{2\pi} L \cos\theta \, d\Omega = \pi L$$

THERMAL RADIATION

Thermal radiation is emitted by all the objects above absolute zero (0 K=-273.15 °C) and is the radiation that is detected by the great majority of passive remote sensing systems. To describe this radiation we use the radiometric quantities we have already defined but we must take into account the variation with wavelength (or with frequency).

The spectral radiance L_λ is such that the radiance contained in a small range of wavelengths $\Delta\lambda$ is

$$\Delta L = L_\lambda \, \Delta\lambda$$

the unit of measurement of the spectral radiance is W m^{-3} sr^{-1} and the radiance in the interval (λ_1, λ_2) can be easily calculated

$$L = \int_{\lambda_1}^{\lambda_2} L_\lambda \, d\lambda$$

The spectral radiance can also be defined in terms of the frequency L_f (with unit W m^{-2} sr^{-1} Hz^{-1})

$$\Delta L = L_f \, \Delta f \qquad L = \int_{f_2}^{f_1} L_f \, df \qquad \frac{L_\lambda}{L_f} = \frac{c}{\lambda^2}$$

A black-body is an object absorbing all the radiation incident on it; it can be approximated by a closed cavity with a small hole in one of its walls (the radiation enters through that hole). The spectral radiation inside the closed cavity (and also emitted by the hole) is known as black body radiation.

By using the quantization of energy, it can be proved that the **spectral radiance** of any **black-body** is given by the **Planck formula**

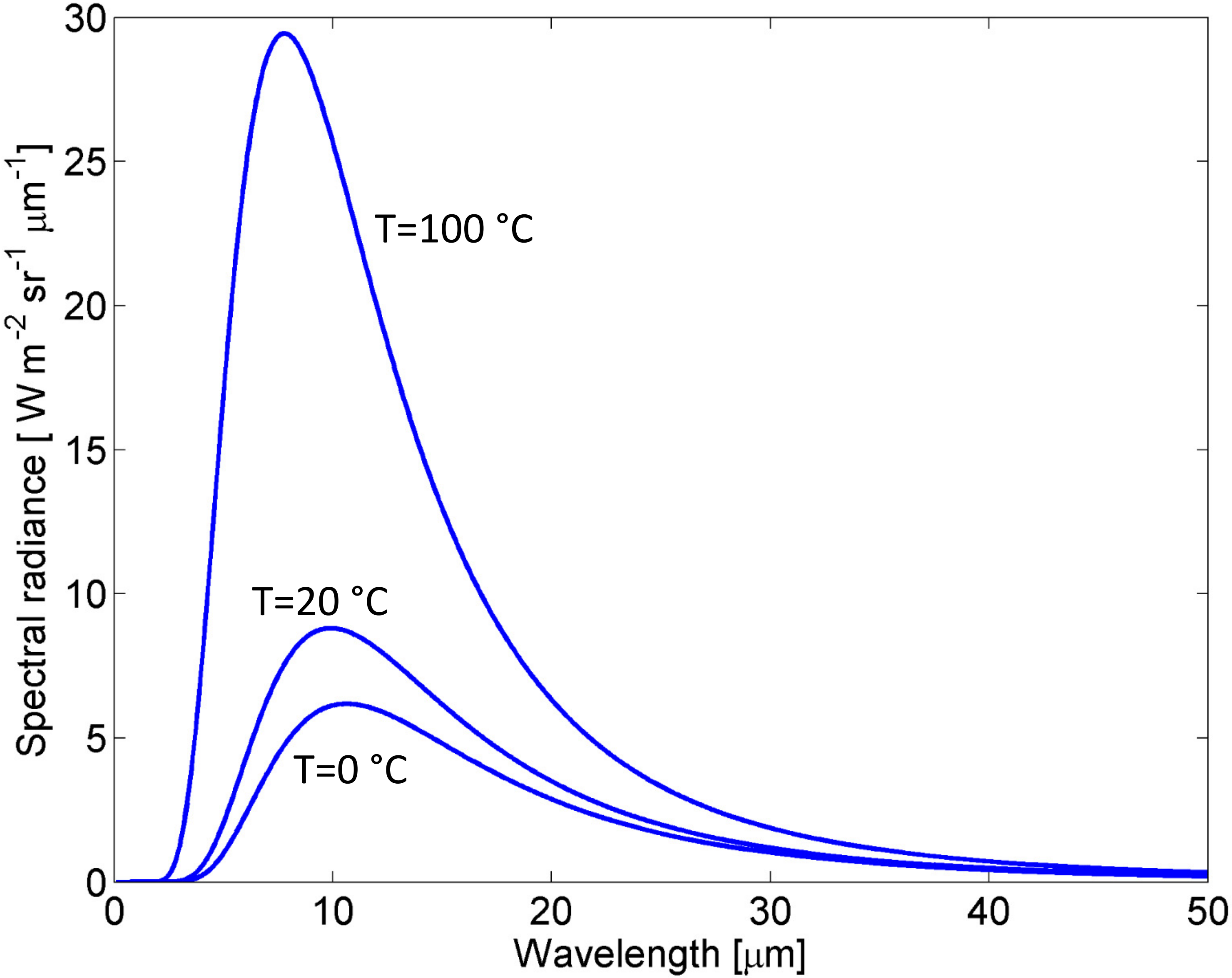
$$\boxed{L_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\dfrac{hc}{\lambda k_B T}\right) - 1}} \qquad L_f = \frac{2hf^3}{c^2} \frac{1}{\exp\left(\dfrac{hf}{k_B T}\right) - 1}$$

$h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$ Planck constant

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$ Boltzmann constant

$c = 299792.458\ \mathrm{km/s}$ speed of light in vacuum

SPECTRAL RADIANCE OF THE BLACK-BODY



At sufficiently long wavelengths we can approximate the exponential with its first order Taylor expansion and we obtain the **Rayleigh-Jeans approximation**

$$L_f \approx \frac{2k_B T f^2}{c^2} = \frac{2k_B T}{\lambda^2}$$

$$\boxed{L_\lambda \approx \frac{2k_B Tc}{\lambda^4}}$$

By integrating the Planck formula the total radiance of the black-body is obtained

$$L = \int_0^\infty L_\lambda \, d\lambda = \frac{2\pi^4 k_B^4}{15c^2h^3} T^4$$

Stefan's law

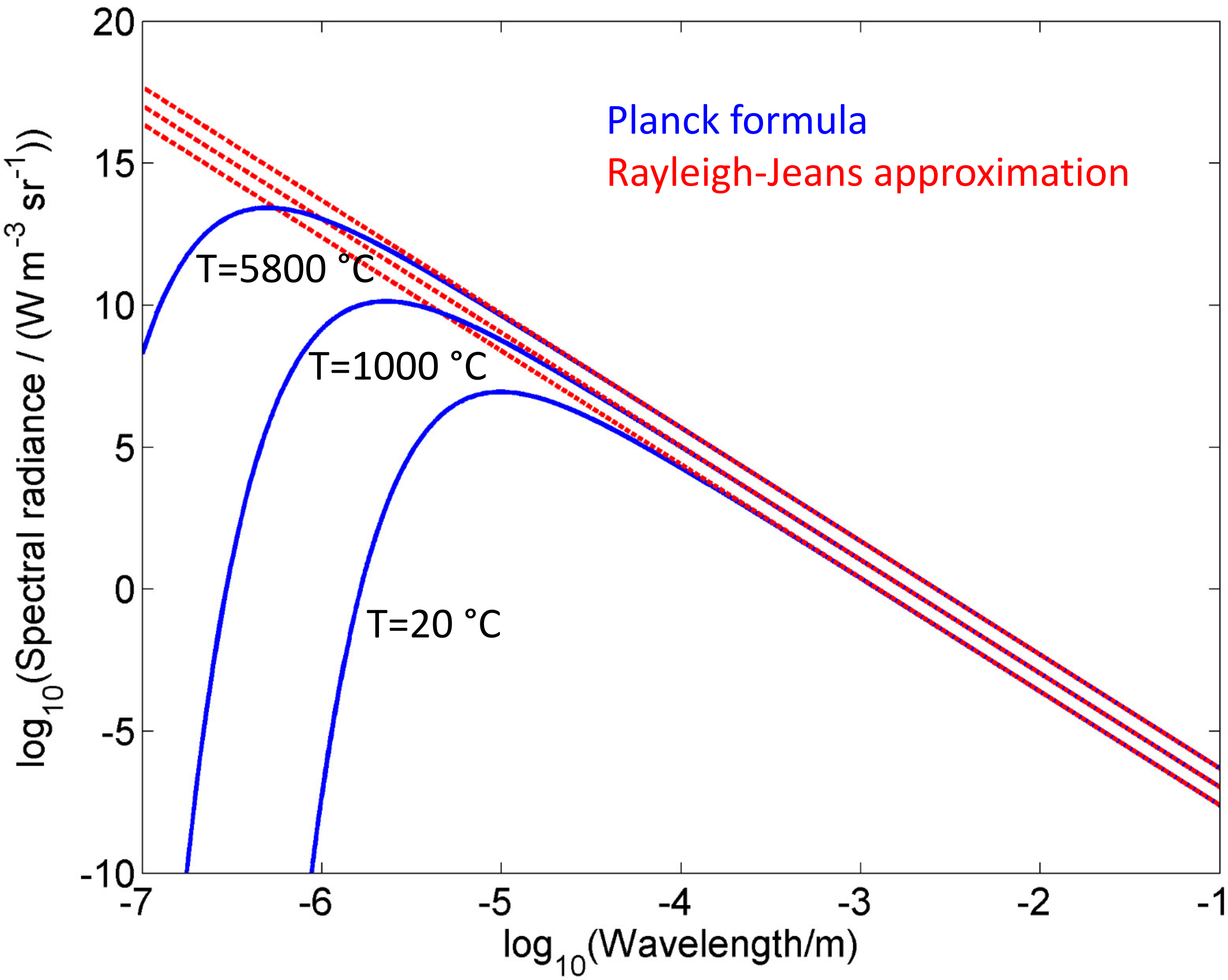
The radiation from the black-body is isotropic and the total radiant exitance reads as

$$M = \pi L = \frac{2\pi^5 k_B^4}{15c^2h^3} T^4$$

$$\boxed{M = \sigma T^4}$$

$$\sigma = 5.67 \times 10^{-8} \ \mathrm{W \cdot m^{-2} \cdot K^{-4}}$$ Stefan-Boltzmann constant

SPECTRAL RADIANCE OF THE BLACK-BODY



It is easy to find the wavelength at which the spectral radiance reaches its maximum

$$\boxed{\lambda_{\max} = \frac{A}{T}}$$

Wien's displacement law

$$A = 2.898 \times 10^{-3}\ \mathrm{K \cdot m}$$

temperature on the Sun's surface $T = 5800\ \mathrm{K} \Rightarrow \lambda_{\max} = 0.5\,\mu\mathrm{m}$

temperature on the Earth's surface $T = 293\ \mathrm{K} \quad \Rightarrow \lambda_{\max} = 9.9\,\mu\mathrm{m}$

Most materials do not behave as a black-body, but their radiance can be obtained from the Planck formula by introducing the **emissivity** ε (which is wavelength dependent)

$$\boxed{L_{\lambda,\varepsilon} = \varepsilon(\lambda) L_\lambda}$$

Brightness temperature of a body T_b: temperature of the equivalent black-body that would give the same radiance at the wavelength under consideration

$$\varepsilon \, L_\lambda(\lambda, T) = L_\lambda(\lambda, T_b)$$

$$\varepsilon \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\dfrac{hc}{\lambda k_B T}\right) - 1} = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\dfrac{hc}{\lambda k_B T_b}\right) - 1}$$

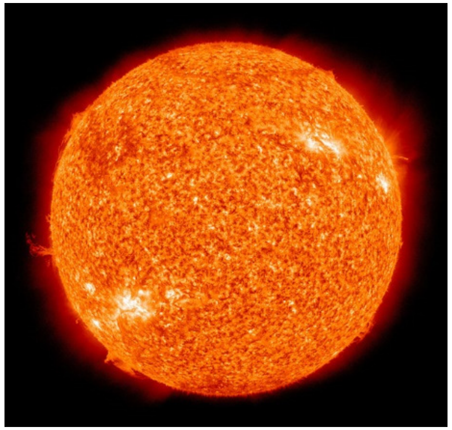
$$T_b = \frac{hc}{k_B \lambda} \frac{1}{\ln\left(1 + \frac{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1}{\varepsilon}\right)}$$

At sufficiently long wavelengths we can write the **Rayleigh-Jeans approximation**

$$\boxed{T_b = \varepsilon T}$$

SOLAR RADIATION

The Sun can be taken to be a black-body with a temperature of T=5800 K and it can be modeled as a sphere of radius r=6.96x10^{8} m at a distance D=150x10^{9} m from the Earth



Sun's radiant exitance

$$M = \sigma T^4 = 6.35 \times 10^7 \, \mathrm{Wm}^{-2}$$

Total power radiated by the Sun (luminosity)

$$P = 4\pi r^2 M = 3.87 \times 10^{26} \, \mathrm{W}$$

Sun's irradiance above the Earth atmosphere (mean exoatmospheric solar irradiance)

$$E = \frac{P}{4\pi D^2} = 1.37 \times 10^3 \, \mathrm{Wm}^{-2}$$

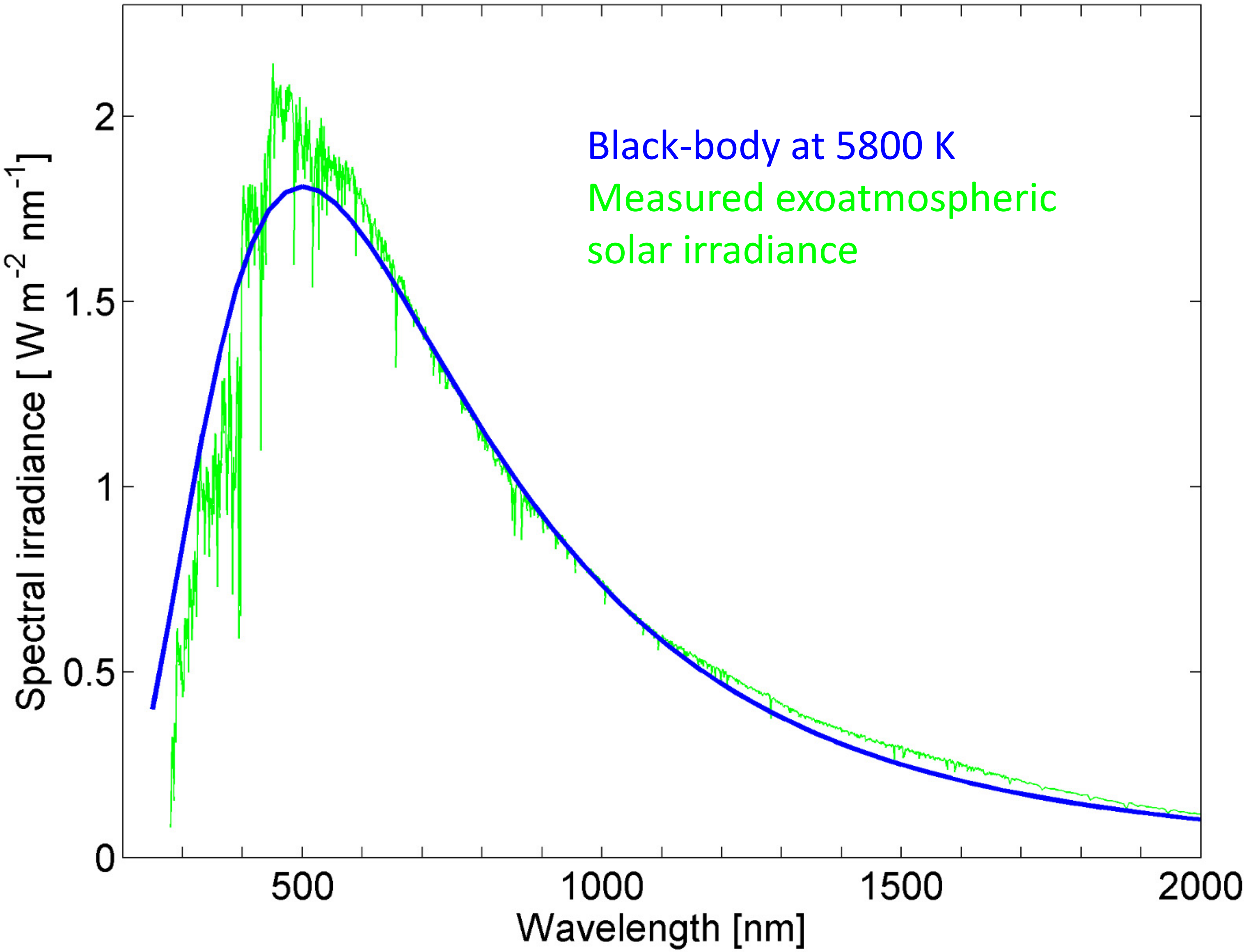
Solid angle subtended by the Sun

$$\Delta\Omega = \frac{\pi r^2}{D^2} = 6.76 \times 10^{-5} \, \mathrm{sr}$$

Sun's radiance

$$L = \frac{E}{\Delta\Omega} = \frac{\sigma T^4}{\pi} = 2.02 \times 10^7 \, \mathrm{Wm}^{-2} \mathrm{sr}^{-1}$$

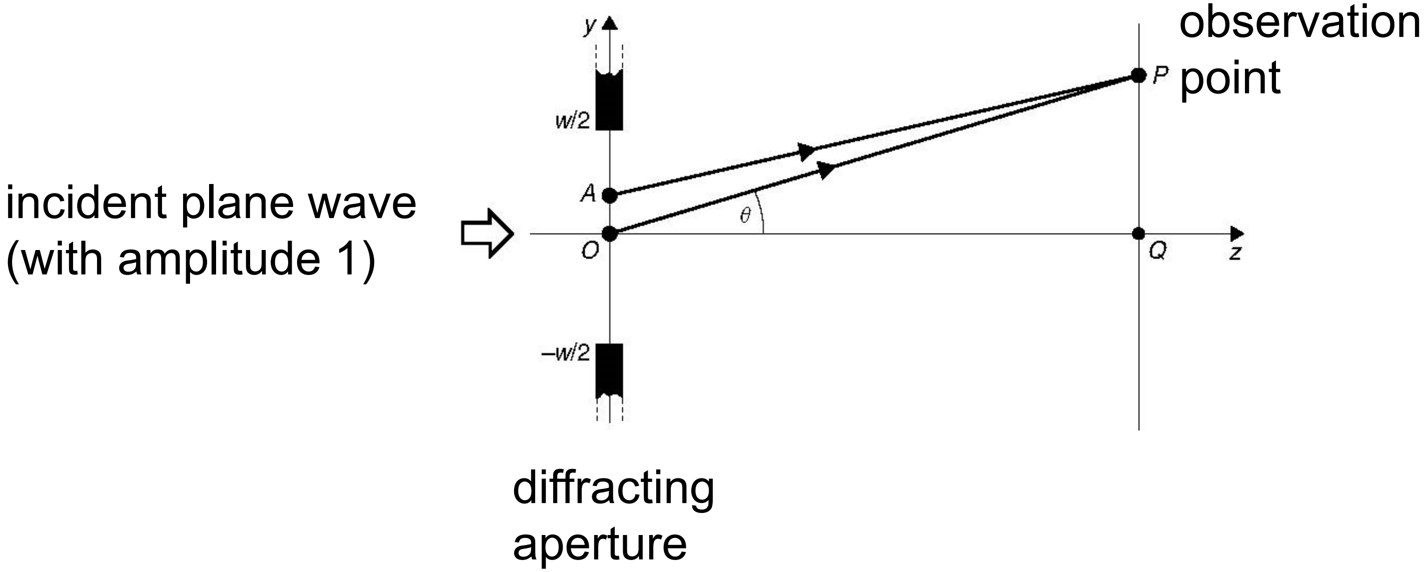
SPECTRAL IRRADIANCE OF THE SUN



http://rredc.nrel.gov/solar/spectra/am1.5/

DIFFRACTION

Let us consider a plane wave travelling in the direction z and incident on a very long slit, of width W, in an infinite opaque screen. The slit has its long axis parallel to the x axis of a Cartesian coordinate system, and the center of the slit is located at the origin of this coordinate system. We wish to determine the amplitude of the electric field at the point P.



The complex amplitude at P contributed by an element of the slit of width dy, located at A, is thus proportional to: $\exp\left(jk\,y\sin\theta\right)dy$

By integrating all the contributions over the entire slit, the total amplitude at P is found

$$a(\theta) = \int_{-W/2}^{W/2} \exp(jk\,y\sin\theta)\,dy$$

By introducing an amplitude transmittance function f(y) which defines the fraction of the incident amplitude that is transmitted, the expression for the complex amplitude in the direction θ becomes a Fourier transform (the conjugate variables are y and ksinθ)

$$\boxed{a(\theta) = \int_{-\infty}^{\infty} f(y)\exp(jk\,y\sin\theta)\,dy}$$

Fraunhofer diffraction integral

In the case of the slit of width W, the diffraction integral is

$$f(y) = \mathrm{rect}\left(\frac{y}{W}\right) \qquad \boxed{a(\theta) = W\,\mathrm{sinc}\left(\frac{Wk\sin\theta}{2\pi}\right)} \qquad \mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

The directions of the two zeros delimiting the sinc main peak are

$$\sin\theta = \pm\frac{\lambda}{W}$$

If W is much larger than λ, the previous formula can be approximated as

$$\theta \approx \pm\frac{\lambda}{W}$$

A plane wave impinging on an opaque screen with an aperture of width W spreads in a diverging beam whose angular width will be in the order of λ/W radians.

We can consider a transmittance function that depends on both x and y and a two-dimensional diffraction pattern is obtained.

$$a\left(\theta_x, \theta_y\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\exp\left(jkx\sin\theta_x + jky\sin\theta_y\right)dx\,dy$$

If we consider the diffraction pattern of a circular aperture of diameter D, the amplitude of the diffracted wave is proportional to the first-order Bessel function

$$a(\theta_r) \propto J_1\left(\frac{kD\sin\theta_r}{2}\right) \bigg/ \frac{kD\sin\theta_r}{2}$$

The first zero occurs when: $J_1(3.832) = 0 \Rightarrow \boxed{\theta_r = 1.22\frac{\lambda}{D}}$

In the previous analysis we assumed that the two rays OP and AP were parallel: this approach is valid if the error in the phase difference is less than π/2

$$AQ - OQ < \frac{\lambda}{4} \Rightarrow \sqrt{\left(\frac{W}{2}\right)^2 + z^2} - z < \frac{\lambda}{4} \Rightarrow \frac{W^2}{8z} < \frac{\lambda}{4}$$

Our description gives accurate results if the distance z is larger than the Fresnel distance

$$z > \frac{W^2}{2\lambda} = z_F$$ **Fresnel distance**

ABSORPTION

In order to model the absorption of the electromagnetic field by a medium, an imaginary part is added in the relative dielectric constant of the medium itself

$$\varepsilon_r = \varepsilon' - j\varepsilon'' = \varepsilon'(1 - j\tan\delta)$$

Losses can be equivalently included by making the refractive index complex

$$n = m - j\kappa \qquad \varepsilon_r = n^2 \Rightarrow \begin{array}{l} \varepsilon' = m^2 - \kappa^2 \\ \varepsilon'' = 2m\kappa \end{array}$$

Due to the absorption the intensity of the electromagnetic field decreases exponentially with the distance z

$$I(z) = I_0 \exp\left(-\frac{4\pi\kappa}{\lambda} z\right) = I_0 \exp\left(-\frac{z}{L_a}\right)$$

Beer-Lambert law

absorption length $\quad L_a = \dfrac{\lambda}{4\pi\kappa}$

$$I(z) = I_0 \exp\left(-\frac{z}{L_a}\right) = I_0 \exp(-\gamma_a z)$$

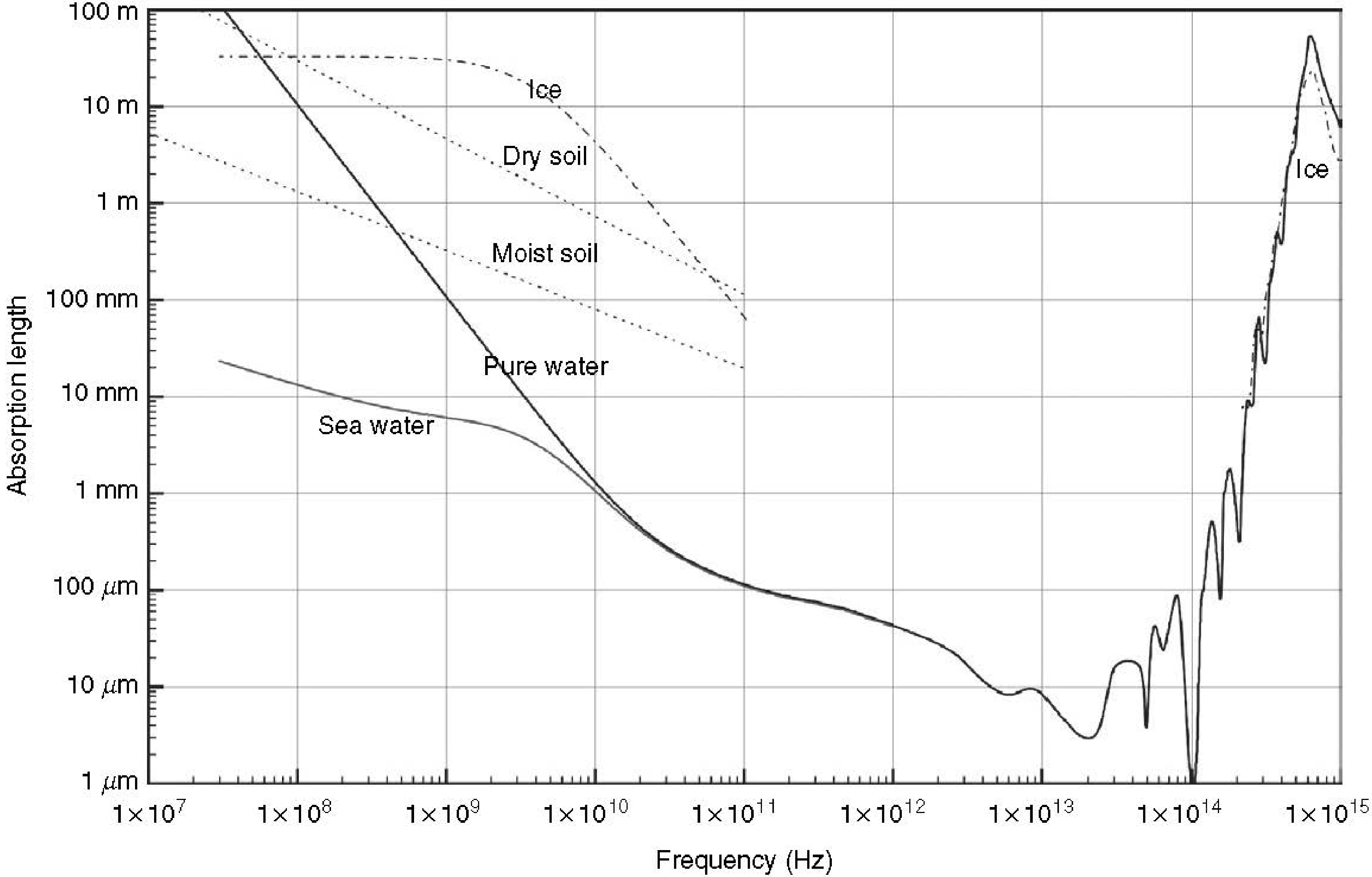
<u>absorption coefficient</u> $\gamma_a = \frac{1}{L_a}$

If the absorption coefficient depends on the propagation distance, the intensity reads as

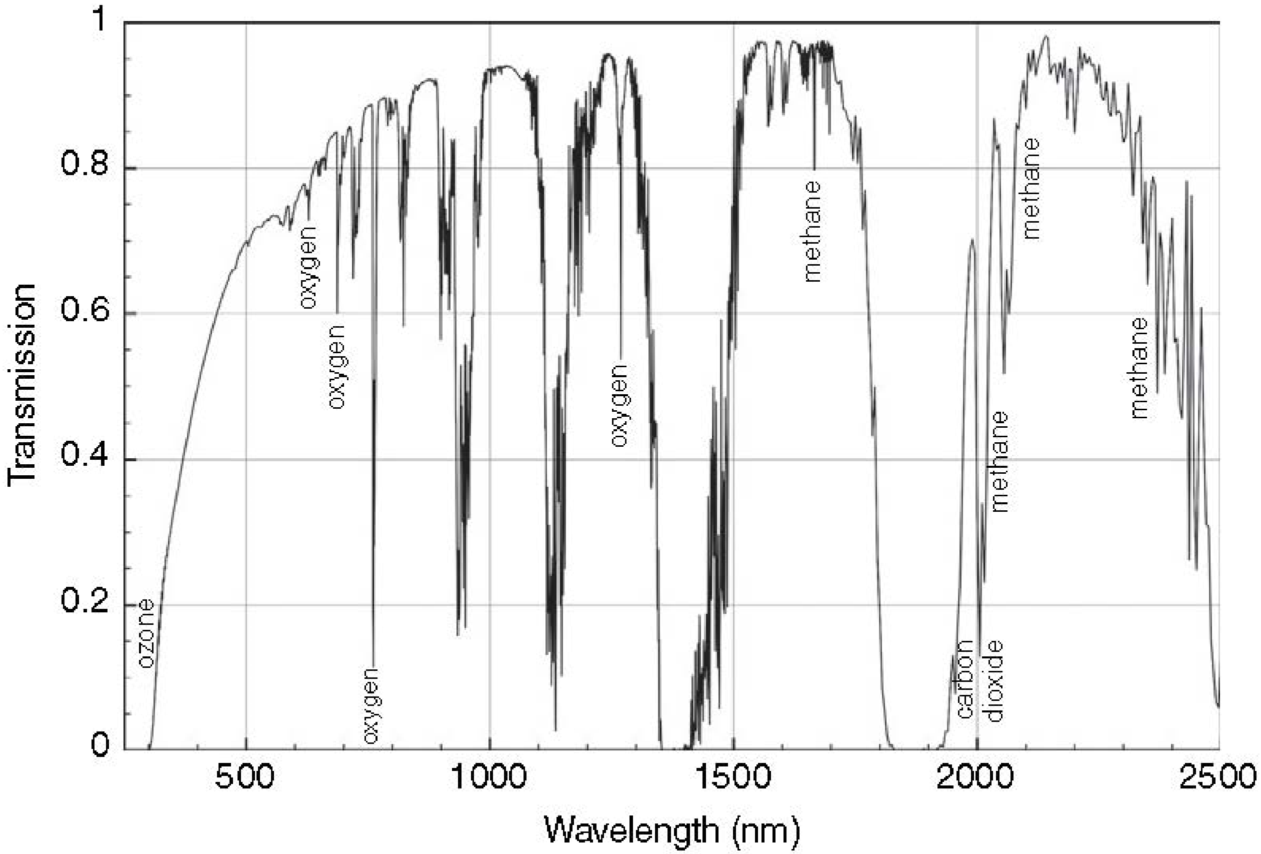
$$\boxed{I(z) = I_0 \exp(-\tau)}$$

optical thickness $\boxed{\tau = \int_0^z \gamma_a(z')dz'}$

Absorption lengths of various materials. Note that the absorption lengths are strongly influenced by such factors as temperature and the content of trace impurities, especially at low frequencies.



Typical zenith atmospheric transmittance between 250 nm and 2500 nm.



The main absorption features are labelled, with the exception of water vapour, which is responsible for most of the unlabelled features.

Typical optical thickness of the clear atmosphere at the zenith.
The main absorption peaks are identified for wavelengths longer than 2 μm.