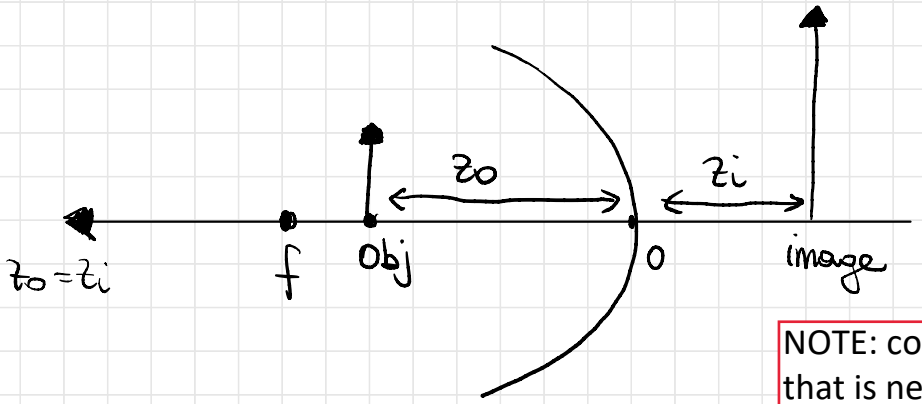
Photonics

Homework 1

Solution

PROBLEM 1 : $M = 4$ $R = 20$ cm



$$f = -\frac{R}{2} = -\left(\frac{-20}{2}\right) =$$

10 cm ↑

NOTE: concave mirror have radius of curvature that is negative by convention, therefore f>0. However, if the z axis is properly flipped then the opposite convention can be assumed.

$$M = -\frac{z_i}{z_o} = 4$$

$$-z_i = 4 z_o$$

NOTE: Magnification sign is tipically given in the problem. For those of you that made the calculations with both positive and negative signs of M, they were still evaluated as correct.

$$\frac{1}{z_i} + \frac{1}{z_o} = \frac{1}{f}$$

$$z_i = -4 z_o$$

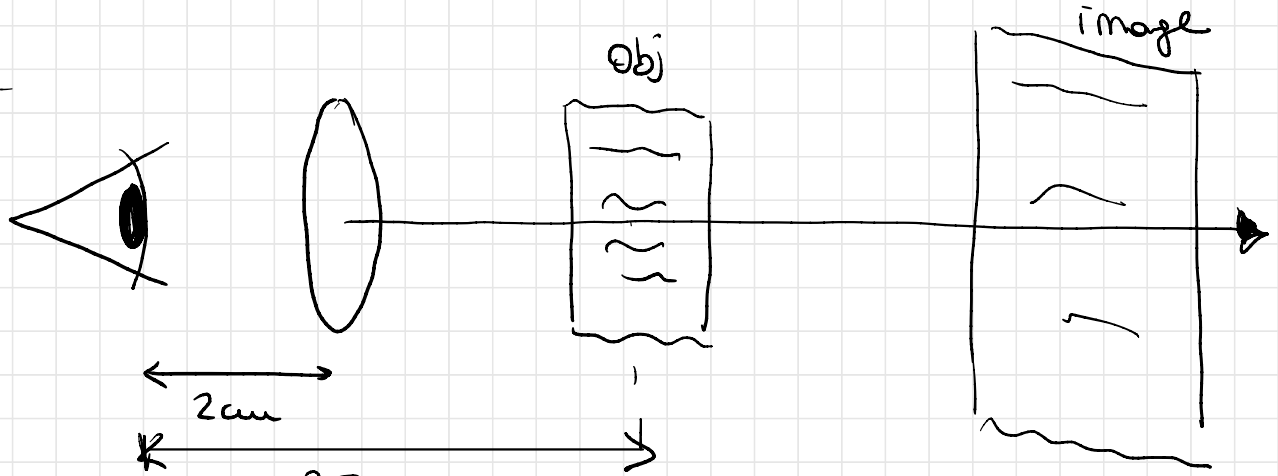
$$-\frac{1}{4z_0} + \frac{1}{z_0} = \frac{1}{f}$$

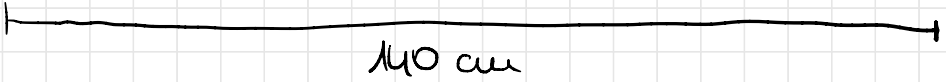
$$\frac{-1+4}{z_0} = \frac{4}{10} \Rightarrow z_0 = 7.5\ cm$$

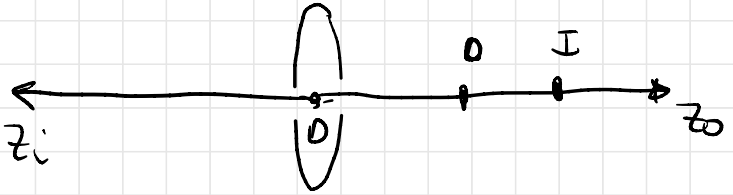
$$z_i = -4z_0 = -30\ cm$$

IMPORTANT: I noticed that most of the times the drawing is missing. If the drawing is not done or it is not clear then it is not possibile to establish if the signs in the problem are correct or not.

PROBLEM 2







$d_{oe} = d_{oe} - d_{ee} = 35 - 2 = 33\ cm$

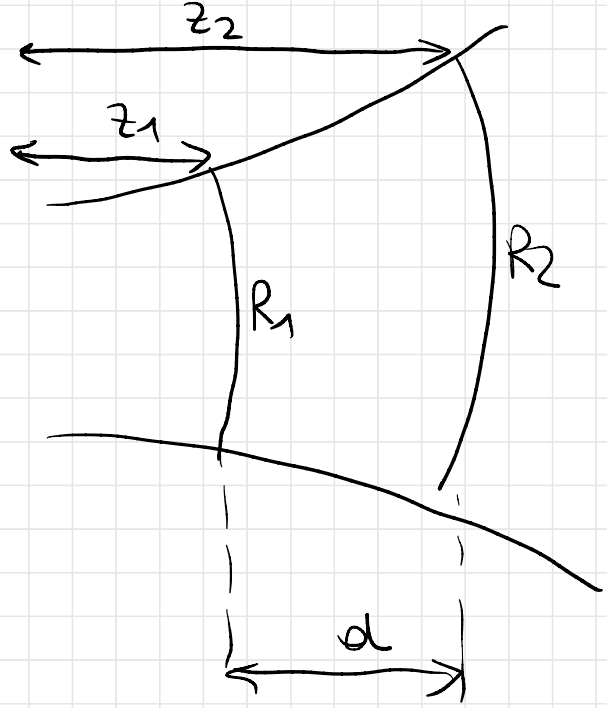
$d_{il} = d_{ie} - d_{le} = 140 - 2 = 138\ cm$ ← $d_{ie} = -138\ cm$ because is on the side of the obj!

Thin lens equation:

$$\frac{1}{d_{ie}} + \frac{1}{d_{oe}} = \frac{1}{f}$$

$$-\frac{1}{138} + \frac{1}{33} = \frac{1}{f} \Rightarrow f = 43.37\ cm$$

PROBLEM 3



$$R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right] = z + z\frac{z_0^2}{z^2} = \frac{z^2 + z_0^2}{z}$$

$$\begin{cases} z_0^2 = z_1 R_1 - z_1^2 \\ z_0^2 = z_2 R_2 - z_2^2 \end{cases} \qquad z_2 = z_1 + d$$

$$\begin{cases} z_0^2 = z_1 R_1 - z_1^2 \\ z_0^2 = (z_1 + d) R_2 - (z_1 + d)^2 \end{cases}$$

Subtracting we get:

$$(z_1 + d) R_2 - (z_1 + d)^2 - z_1 R_1 + z_1^2 = 0$$

$$z_1 R_2 + d R_2 - \cancel{z_1^2} - d^2 - 2 z_1 d - z_1 R_1 + \cancel{z_1^2} = 0$$

$$z_1 (R_2 - R_1 - 2d) = d^2 - d R_2$$

$$z_1 = \frac{d^2 - d R_2}{R_2 - R_1 - 2d}$$

$$z_2 = d + z_1$$

$$z_0^2 = z_1 R_1 - z_1^2 \Rightarrow \boxed{z_0 = \sqrt{z_1 R_1 - z_1^2}}$$

$$W_0 = \sqrt{\frac{\lambda z_0}{\pi}}$$

PROBLEM 4 $\lambda = 600\ nm$ $\theta = 33°$ $m = 2$ Width = 4.4 cm

(a)

$$d \sin\theta = m\lambda$$

if $m = 2$

$$d = \frac{m\lambda}{\sin\theta} = \frac{2 \cdot 600 \cdot 10^{-9}}{\sin(33°)} = 2.2 \cdot 10^{-6}\ m$$

Total number of slits:

$$N = \frac{\text{total width}}{\text{periodicity}} = \frac{4.4 \cdot 10^{-2}}{2.2 \cdot 10^{-6}} = 20000$$

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{mN}$$

$$\Delta\lambda = \frac{\lambda}{mN} = \frac{600 \cdot 10^{-9}}{2 \cdot 20000} = \frac{6 \cdot 10^{-7}}{4 \cdot 10^{4}} = 1.5 \cdot 10^{-11}\ m$$

$$= 15\ pm$$