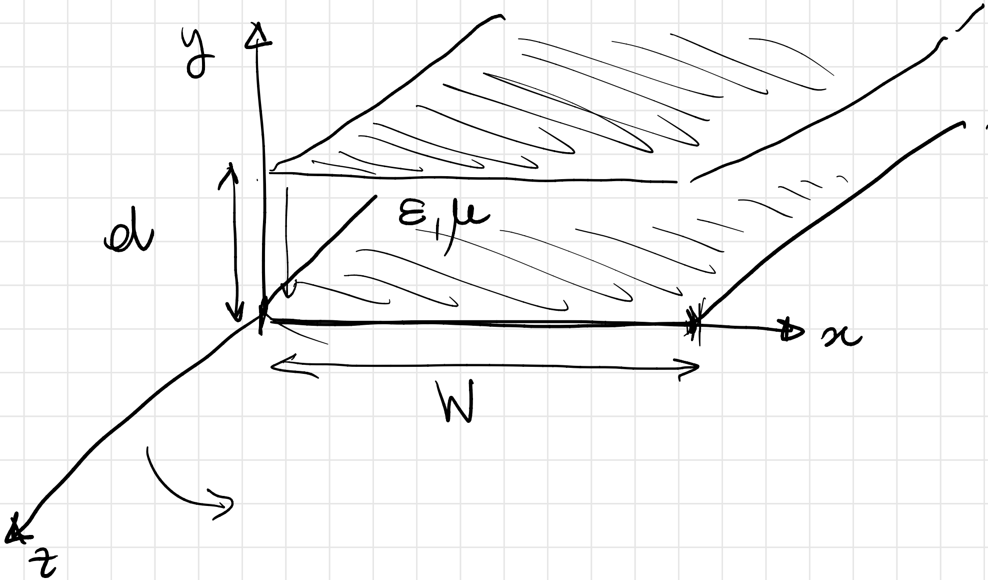
MICROWAVE ENGINEERING

Waveguides

PARALLEL PLATE WAVEGUIDE



$\phi(x,d)=V_0$

$\boxed{W \gg d}$

$\phi(x,0)=0$

TM, TE, TEM

TEM Modes $E_z=H_z=0$

$\phi(x,y)$

$$\boxed{\nabla_t^2 \phi(x,y)=0}$$

$0 \le x \le W$

$0 \le y \le d$

$$\phi(x,0) = 0$$
$$\phi(x,d) = V_0$$

$$\phi(x,y) = A + By \xrightarrow{y=0} \boxed{A = 0}$$

$$\phi(x,y) = A + By \xrightarrow{y=d} A + Bd = V_0 \Downarrow \boxed{B = \frac{V_0}{d}}$$

$$\boxed{\phi(x,y) = \frac{V_0}{d} y}$$

The transverse field

$$\bar{e}(x,y) = -\nabla_t \phi(x,y) = -\hat{y}\frac{V_0}{d}$$

The Total electric field

$$\bar{E}(x,y,z) = \bar{e}(x,y)\cdot e^{-jkz} = -\hat{y}\,\frac{V_0}{d}\,e^{-jkz}$$

$$k = \omega\sqrt{\mu\varepsilon}$$

The magnetic field is:

$$\bar{H}(x,y,z) = \frac{1}{\eta}\,\hat{z}\times\bar{E}(x,y,z) = \hat{x}\,\frac{V_0}{\eta d}\,e^{-jkz} =$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$= \hat{x}\,\frac{V_0}{Z_{TEM}\,d}\,e^{-jkz}$$

The voltage of the top plate is:

$$V = -\int_{y=0}^{d} E_y\,dy = V_0\,e^{-jkz}$$

The current on the top plane:

$$I = \int_{x=0}^{W} \bar{J}_S \, \hat{z} \, dx = \int_{x=0}^{W} \left(-\hat{y} \times \bar{H}\right) \cdot \hat{z} \, dx = \int_{x=0}^{W} H_x \, dx$$

$$= \frac{W V_0}{\eta d} e^{-jkz}$$

The characteristic impedance

$$Z_0 = \frac{V}{I} = \frac{\cancel{V_0} \cancel{e^{-jkz}}}{\frac{W \cancel{V_0}}{\eta d} \cancel{e^{-jkz}}} = \frac{\eta d}{W}$$

constant that depends on geometry

Phase velocity

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

← CONSTANT does not depend on geometry

TM Modes

$H_z = 0 \qquad E_z \neq 0$

$$\left(\frac{\partial^2}{\partial y^2} + k_c^2\right) e_z(x,y) = 0 \qquad k_c^2 = k^2 - \beta^2$$

$$E_z(x,y,z) = e_z(x,y)\, e^{-j\beta z}$$

The general solution for the wave equation

$$e_z(x,y) = A \sin k_c y + B \cos k_c y$$

Boundary conditions $e_z(x,y) = 0$ at $y = 0, d$

$$e_z(x,y) = A \sin k_c y + \underline{B \cos k_c y}$$

$y = 0 \rightarrow B = 0$

$y = d \rightarrow A \sin k_c d + \underset{0}{\underset{\shortparallel}{B}} \cos k_c d = 0$

$$\Downarrow$$

$$A \sin k_c d = 0$$

$$\Downarrow$$

$$\boxed{k_c = \frac{n\pi}{d}} \quad n = 0, 1, 2$$

The propagation constant

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{n\pi}{d}\right)^2}$$

The solution of the field is:

$$e_z(x,y) = A_n \sin \frac{n\pi y}{d}$$

$$\underbrace{E_z(x,y,z) = A_n \sin \frac{n\pi y}{d} e^{-j\beta z}}$$

the transverse field components:

$$H_x = \frac{j\omega\varepsilon}{k_c^2} \frac{\partial E_z}{\partial y} \rightarrow \underline{H_x} = \frac{j\omega\varepsilon}{k_c} A_n \cos \frac{n\pi y}{d} e^{-j\beta z}$$

$$E_y = \frac{-j\beta}{k_c^2} \frac{\partial E_z}{\partial y} \rightarrow \underline{E_y} = \frac{-j\beta}{k_c} A_n \cos \frac{n\pi y}{d} e^{-j\beta z}$$

$H_y = E_x \propto \frac{\partial E_z}{\partial x} \rightarrow E_x = H_y = 0$

<u>NOTE 1</u>: for $n=0 \Rightarrow \beta = k = \omega\sqrt{\mu\varepsilon} \Rightarrow \boxed{TM_0 = TEM}$

For $n = 1, 2, 3 \dots$ $TM_1, TM_2, TM_3 \dots$ modes.

<u>NOTE 2</u> β is real only if $k > k_c \Rightarrow$ We can find a frequency for which the propagation occurs only above such frequency

<u>CUTOFF FREQUENCY</u>

$$\boxed{f_c = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{n}{2d\sqrt{\mu\varepsilon}}}$$

$k = k_c$

$\omega\sqrt{\mu\varepsilon} = k_c \Rightarrow 2\pi f\sqrt{\mu\varepsilon} = k_c$

<u>CUTOFF WAVELENGTH</u> $\boxed{\lambda_c = \frac{2d}{n}}$

For example

$$f_{c\,TM_1} = \frac{1}{2d\sqrt{\mu\varepsilon}}$$

$$f_{c\,TM_2} = \frac{1}{d\sqrt{\mu\varepsilon}} \quad \ldots$$

if $f \geq f_c$
I have
a propagating
mode

if $f < f_c \Rightarrow \beta$ IS IMAGINARY $\rightarrow$ cutoff-mode
evanescent mode

WAVE IMPEDANCE

$$Z_{TM} = \frac{-E_y}{H_x} = \frac{\beta}{\omega\varepsilon} = \frac{\beta\eta}{k}$$

PHASE VELOCITY

$$V_p = \frac{\omega}{\beta}$$

GUIDE WAVELENGTH

$$\lambda_g = \frac{2\pi}{\beta}$$

Attenuation due to dielectric

$$\alpha_d = \frac{k^2 \tan\delta}{2\beta} \quad \frac{Np}{m}$$

Attenuation due to the conductor

$$\alpha_c = \frac{P_e}{2P_o}$$

$P_e \rightarrow$ Power dissipated per unit length

$$P_e = 2\left(\frac{R_s}{2}\right)\int_{x=0}^{W} |J_s|^2\,dx = \frac{\omega^2 \varepsilon^2 R_s W}{k_c^2}|A_n|^2 \quad \frac{W}{m}$$

$$\boxed{\alpha_c = \frac{P_\ell}{2P_o} = \frac{2kR_s}{\beta\eta d}} \quad Np/m \qquad \underline{(n>0)}$$

if $\underline{n=0}$ $$\boxed{\alpha_c = \frac{R_s}{\eta d}} \quad Np/m$$

TE Modes

$E_z = 0 \qquad H_z \neq 0$

$$\left(\frac{\partial^2}{\partial y^2} + k_c^2\right) h_z(x,y) = 0 \qquad k_c^2 = k^2 - \beta^2$$

$$H_z(x,y,z) = h_z(x,y)\, e^{-j\beta z}$$

The generic solution:

$$h_z(x,y) = A \sin k_c y + B \cos k_c y$$

Boundary conditions

$E_z = 0$ everywhere

$E_x = 0 \quad y = 0, d$

$$E_x = -\frac{j\omega\mu}{k_c}\left[A \cos k_c y - B \sin k_c y\right] e^{-j\beta z}$$

$y = 0$

$A = 0$

$y = d$

$$\frac{j\omega\mu}{k_c} B \sin k_c d \, e^{-j\beta z} = 0$$

$$\boxed{k_c = \frac{n\pi}{a}} \qquad n = 1, 2, 3 \ldots$$

The solution is:

$$H_z(x, y, z) = B_n \cos\left(\frac{n\pi y}{a}\right) e^{-j\beta z}$$

Transverse fields can be calculated:

$$E_x = \frac{-j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial y} \Rightarrow E_x = \frac{j\omega\mu}{k_c} B_n \sin \frac{n\pi y}{a} e^{-j\beta z}$$

$$H_y = \frac{-j\beta}{k_c^2} \frac{\partial H_z}{\partial y} \Rightarrow H_y = \frac{j\beta}{k_c} B_n \sin \frac{n\pi y}{a} e^{-j\beta z}$$

$$E_y = H_x \propto \frac{\partial H_z}{\partial x} = 0 \Rightarrow E_y = H_x = 0$$

$$\beta = \sqrt{k^2 - \left(\frac{n\pi}{a}\right)^2}$$

Cutoff frequency is

$$\boxed{f_c = \frac{n}{2d\sqrt{\mu\varepsilon}}}$$

WAVE IMPEDANCE

$$Z_{TE} = \frac{E_x}{H_y} = \frac{\omega\mu}{\beta} = \frac{k\eta}{\beta}$$

Attenuation due to conductor

$$\alpha_c = \frac{P_\ell}{2P_0}$$

$$P_0 = \frac{1}{2}\,\mathrm{Re}\int_{x=0}^{W}\int_{y=0}^{d} E \times H^{*}\,\hat{z}\,dy\,dx = \frac{1}{2}\,\mathrm{Re}\int_{x=0}^{W}\int_{y=0}^{d} E_x H_y^{*}\,dy\,dx =$$

$$= \frac{\omega \mu d W}{4 k_c^2} |B_n|^2 \, Re(\beta) \qquad \underline{n > 0}$$

if $n = 0$ $\quad E_x = H_y = 0 \Rightarrow P_0 = 0 \Rightarrow$ There is NO TE_0 mode

$$\alpha_c = \frac{2 k_c^2 R_s}{\omega \mu \beta d} = \frac{2 k_c^2 R_s}{k \beta \eta d} \qquad Np/m$$

$\underline{\beta \to 0}$ as $k \to k_c$

$\hookrightarrow \alpha_c \to \infty$

$$\beta = \sqrt{k^2 - k_c^2}$$