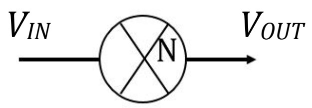
$(x+y)^2 = x^2 + y^2 + 2xy$

$(x-y)^2 = x^2 + y^2 - 2xy$

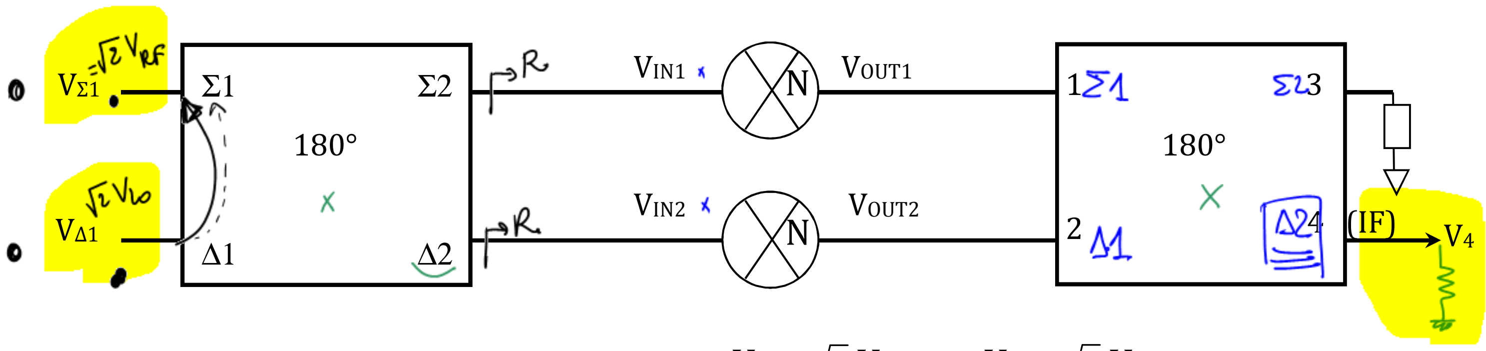
Tutorial (Balanced Mixer SBM)

■The mixer is based on a **nonlinear device N** characterized with the following 3rd order transfer function :



$$V_{OUT} = a.V_{IN}^2 - b.V_{IN}^3$$ where (a and b) are constants

■The following block diagram represents a **single balanced mixer SBM** consisting of two 180°-couplers and two identical mixers N.



1) In the studied case, the input voltages are $V_{\Sigma 1} = (\sqrt{2}.V_{RF})$ and $V_{\Delta 1} = (\sqrt{2}.V_{LO})$. Therefore, express the input control voltages V_{IN1} and V_{IN2} of N as a function of V_{LO} and V_{RF}.

$V_{IN1} = \frac{1}{\sqrt{2}}(V_{\Sigma 1} + V_{\Delta 1}) = V_{RF} + V_{Lo}$ $\quad V_{IN2} = V_{RF} - V_{Lo}$

2) Deduce the expressions of output voltages (V_{OUT1}, V_{OUT2}) as a function of V_{LO} and V_{RF}.

$V_{OUT1} = a(V_{RF} + V_{Lo})^2 - b(V_{RF} + V_{Lo})^3$ $\quad V_{OUT2} = a(V_{RF} - V_{Lo})^2 - b(V_{RF} - V_{Lo})^3$

3) The SBM down-converter is designed at an IF angular frequency $\omega_{IF} = \omega_{LO} - \omega_{RF}$. What should be the type (Σ or Δ?) of port 4 in the output coupler?

Δ port $\quad$ ↳ I want $V_{Lo} \times V_{RF}$ at the output → only $(V_{RF}+V_{Lo})^2 - (V_{RF}-V_{Lo})^2 \Longrightarrow$

4) Given that $V_{LO} = V_0.\cos(\omega_{LO}.t)$ and $V_{RF} = V_1.\cos(\omega_{RF}.t)$, express the output voltage spectrum V_4 of the SBM as a function of V_0 and V_1. What are the remaining frequencies at the IF output port of the SBM?

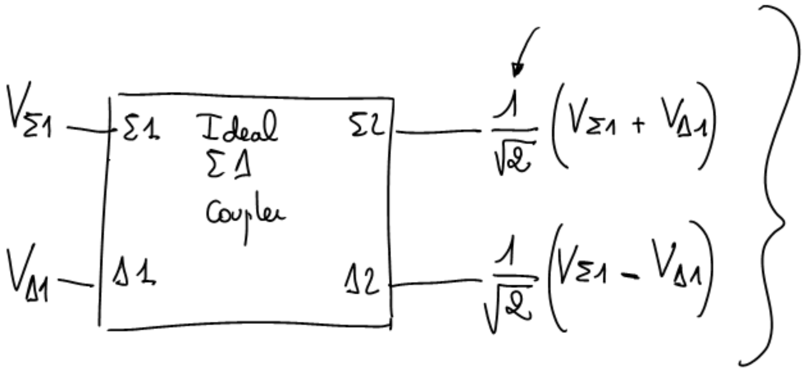
5) What are the advantages of this SBM configuration (LO input at Δ port)?

6) Express the voltage conversion gain G_{CV} of the SBM. What is the power conversion gain G_{CP} if all 180° couplers are matched to 50Ω ?

LO-RF isolation of SBM = LO-RF isolation of input coupler

LO-IF isolation

7) What are the values of LO-to-RF and LO-to-IF isolations?



ideal operation

$$\left.\begin{aligned}(x+y)^2 &= \cancel{x^2}+\cancel{y^2}+2xy\\ (x-y)^2 &= \cancel{x^2}+\cancel{y^2}-2xy\end{aligned}\right\}$$

$\Delta \rightarrow$ $\ominus$ $\downarrow$ $4xy$

$$(x+y)^3 = \cancel{x^3}+y^3+3x^2y+\cancel{3xy^2}$$
$$(x-y)^3 = \cancel{x^3}-y^3-3x^2y+\cancel{3xy^2}$$

$\ominus$ $\downarrow$ $\boxed{2y^3+6x^2y}$

$$\boxed{\begin{aligned}\cos^2(x) &= \frac{1}{2}+\frac{1}{2}\cos(2x)\\ \cos^3(x) &= \frac{3}{4}\cos(x)+\frac{1}{4}\cos(3x)\end{aligned}}$$

$$\cos^3(x) = \cos^2(x)\cos(x)$$
$$= \frac{1}{2}\cos(x)+\frac{1}{2}\cos(2x)\cos(x)$$
$$= \underbrace{\frac{1}{2}\cos(x)}+\frac{1}{4}\cos(3x)+\underbrace{\frac{1}{4}\cos(x)}$$

$$V_4 = \frac{1}{\sqrt{2}}\left(V_{OUT1}-V_{OUT2}\right) = \frac{1}{\sqrt{2}}\Big[a\underbrace{(V_{RF}+V_{LO})^2}_{x+y} - b\underbrace{(V_{RF}+V_{LO})^3}_{(x+y)^3} - a\underbrace{(V_{RF}-V_{LO})^2}_{x-y} + b\underbrace{(V_{RF}-V_{LO})^3}_{(x-y)^3}\Big]$$

$$= \frac{1}{\sqrt{2}}\Big[\underbrace{4aV_{RF}V_{LO}}_{\omega_{IF}\ \&\ \omega_\Sigma}\ \ \underbrace{-2bV_{LO}^3}_{\omega_{LO}\quad 3\omega_{LO}}\ -\ \underbrace{6bV_{RF}^2V_{LO}}_{\omega_{LO},\ 2\omega_{RF}\pm\omega_{LO}}\Big]$$

(A) (B) (C)

$$V_{RF} = V_1\cos(\omega_{RF}t)$$
$$V_{LO} = V_0\cos(\omega_{LO}t)$$
$$\omega_{IF} = \omega_{LO}-\omega_{RF}$$
$$\omega_\Sigma = \omega_{LO}+\omega_{RF}$$
$$\omega_1 = 2\omega_{RF}+\omega_{LO}$$
$$\omega_2 = 2\omega_{RF}-\omega_{LO}$$

(A) term

$$4aV_{RF}V_{LO} = \boxed{2aV_0V_1}\left(\cos(\omega_{IF}t)+\cos(\omega_\Sigma t)\right)$$

(B) term

$$-2bV_{LO}^3 = \boxed{-\frac{1}{2}bV_0^3}\left[3\cos(\omega_{LO}t)+\cos(3\omega_{LO}t)\right]$$

(C) term

$$-6bV_{RF}^2V_{LO} = -6bV_1^2V_0\left[\frac{1}{2}+\frac{1}{2}\cos(2\omega_{RF}t)\right]\left[\cos(\omega_{LO}t)\right]$$
$$= \boxed{-3bV_1^2V_0}\cos(\omega_{LO}t)\ \boxed{-\frac{3}{2}bV_1^2V_0}\left[\cos(\omega_1t)+\cos(\omega_2t)\right]$$

$$V_4 = \boxed{\frac{1}{\sqrt{2}}}\left(\text{(A)}+\text{(B)}+\text{(C)}\right)$$

$f_{IF}\ \&\ f_\Sigma$ $\quad 3f_{LO}$ $\quad f_{LO}$ $\quad f_1\ \&\ f_2$

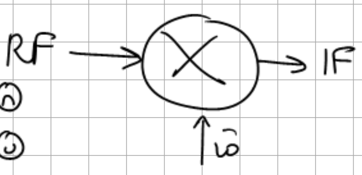
(1) LO → RF
(2) LO → IF (not good)
(3) RF → IF (down) or IF → RF (up)

good

if inputs are reversed (good)

RF @ Σ port, LO @ Δ port } tutorial
→ LO-IF isolation poor ☹
→ RF-IF isolation good ☺

reverse
LO @ Σ port, RF @ Δ port } → LO-IF isolation good ☺, RF-IF isolation poor ☹



$$\text{LO-IF}_{\text{isolation}_{dB}} = 10\log\left(\frac{P_{LO\,port}(\omega_{LO})}{P_{IF\,port}(\omega_{LO})}\right) = 10\log\left(\frac{\frac{1}{2}\frac{|V_{\Delta 1}(\omega_{LO})|^2}{50\Omega}}{\frac{1}{2}\frac{|V_4(\omega_{LO})|^2}{R_L}}\right)$$

$$= 10\log\left(\frac{\cancel{\frac{1}{2}}\frac{1}{50}(\sqrt{2}V_0)^2}{\cancel{\frac{1}{2}}\frac{1}{R_L}\left[\frac{1}{\sqrt{2}}\left(-\frac{3}{2}bV_0^3-3bV_1^2V_0\right)\right]^2}\right) = 10\log\left(\frac{\frac{1}{50}\times 2}{\frac{1}{R_L}\frac{9}{2}b^2\left(\frac{1}{2}V_0^2+V_1^2\right)^2}\right)$$

$$G_{CV} = \frac{V_{OUT}(\omega_{OUT})}{V_{IN}(\omega_{IN})} = \frac{V_4(\omega_{IF})}{V_{\Sigma 1}(\omega_{RF})} = \frac{\frac{1}{\cancel{\sqrt{2}}}\cancel{2} a V_0 \cancel{V_1}}{\cancel{\sqrt{2}} \times \cancel{V_1}} = aV_0$$

$$G_{CP} = \frac{P_{OUT}(\omega_{OUT})}{P_{IN}(\omega_{IN})} = \frac{\frac{1}{2}\frac{|V_4(\omega_{IF})|^2}{R_L}}{\frac{1}{2}\frac{|V_{\Sigma 1}(\omega_{RF})|^2}{50}} = \frac{\frac{1}{2R_L} \times \left(\frac{1}{\sqrt{2}} 2 a V_0 V_1\right)^2}{\frac{1}{2}\frac{(\sqrt{2}V_1)^2}{50}}$$

$$= \frac{50}{R_L} a^2 V_0^2$$