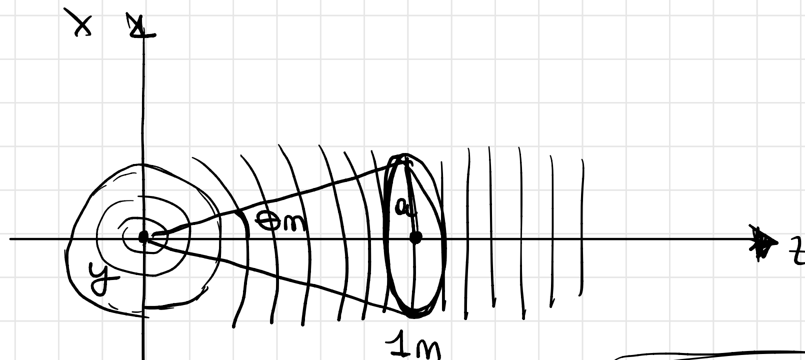
Photonics

Lecture 7:

Problems on Wave and Beam Optics

PROBLEM 1 : Determine the radius of a circle within which a spherical wave of wavelength $\lambda = 633$ nm, origniating at a distance 1 m away, may be approximated by a paraboloidal wave. Determine the maximum angle θ_m and the Fresnel number N_F.



Validity of Fresnel approximation is that

$$\boxed{\frac{N_F \theta_m^2}{4} << 1}$$

$$\theta_m = \frac{a}{z} = \frac{a}{1m} = a$$

$$N_F = \frac{a^2}{\lambda z} = \frac{a^2}{633 \cdot 10^{-9} \cdot 1}$$

$$\frac{\frac{a^2}{633 \cdot 10^{-9}} \cdot \frac{a^2}{}}{4} \ll 1$$

$$a^4 \ll 4 \cdot 633 \cdot 10^{-9}$$

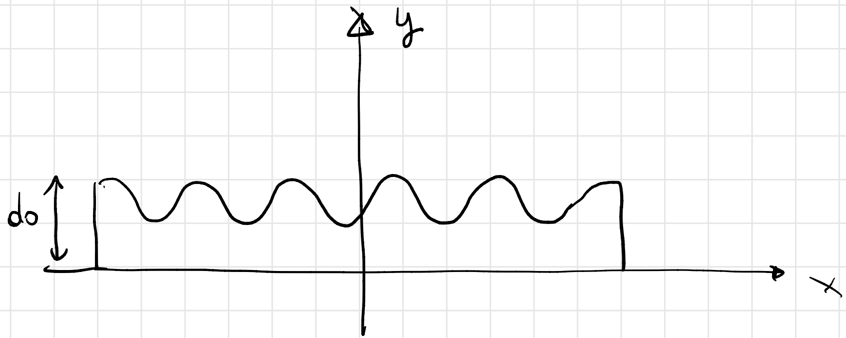
$$a \ll \sqrt[4]{4 \cdot 633 \cdot 10^{-9}} = 0.039\,m = 3.9\,cm$$

$$\underline{\theta_m} = \frac{0.039}{1} = 0.039 \text{ rad}$$

$$\underline{N_F} = \frac{0.039^2}{633 \cdot 10^{-9}} = 2513.8$$

PROBLEM 2 : Calculate the complex amplitude transmittance of a diffraction grating in which the thickness varies sinusoidally in the x direction with the function :

$$d(x,y) = \frac{1}{2} d_0 \left[1 + \cos\left(\frac{2\pi x}{\Lambda} \right) \right]$$



Using the variable thickness plate complex amplitude Transmittance and including $d(x,y)$ in the expression:

$$t(x,y) \simeq h_0 \, e^{-j(n-1)k_0 d(x,y)} \qquad h_0 = e^{-jk_0 d_0}$$

we get:

$$t(x,y) = e^{-jk_0 d_0}\, e^{-j(n-1)k_0 \frac{1}{2} d_0 \left[1+\cos\frac{2\pi x}{\Lambda}\right]} =$$

$$= \underbrace{e^{-jk_0 d_0}\, e^{-j(n-1)\frac{k_0}{2} d_0}}_{h_0}\, e^{-j(n-1)\frac{k_0 d_0}{2}\cos\frac{2\pi x}{\Lambda}} =$$

$$= \boxed{h_0\, e^{-\frac{j}{2}(n-1)k_0 d_0 \cos\frac{2\pi x}{\Lambda}}}$$

$$h_0 = e^{\left(-jk_0 d_0 - jn\frac{k_0}{2}d_0 + j\frac{k_0}{2}d_0\right)} = e^{-\frac{j}{2}k_0 d_0 - jn\frac{k_0}{2}d_0} =$$

$$= e^{-\frac{j}{2}(n+1)k_0 d_0}$$

PROBLEM 3 A 1 mW He-Ne laser produces a Gaussian Beam at $\lambda = 633$ nm with a spot size $2W_0 = 0.1$ mm.

Determine:

- the angular divergence of the beam
- depth of focus
- diameter at $z = 3.5 \cdot 10^5$ km (approximately the distance to the moon)

• the Angular divergence of a gaussian beam is:

$$2\theta_0 = \frac{4}{\pi}\frac{\lambda}{2W_0} = \frac{4}{\pi}\frac{633 \cdot 10^{-9}}{0.1 \cdot 10^{-3}} = 0{,}0081 \text{ rad} = 8.1 \text{ mrad}$$

• the depth of focus is:

$$2z_0 = \frac{2\pi W_0^2}{\lambda} = \frac{2\pi\,(0.05 \cdot 10^{-3})^2}{633 \cdot 10^{-9}} = 0.025 \text{ m}$$

- beam diameter at $z = 3.5 \cdot 10^5$ km

$$2W(z) = 2W_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2} =$$

$$= 0.1 \cdot 10^{-3} \sqrt{1 + \left(\frac{3.5 \cdot 10^8}{0.0125}\right)} = 2.8 \cdot 10^6 \text{ m} =$$

$$2.8 \cdot 10^3 \text{ km} =$$

$$\underline{2800 \text{ km}}$$

PROBLEM 4

For the same laser of Problem 3, what is the radius of curvature of the wavefront at $z=0$, $z=z_0$ and $z=2z_0$?

The expression of the radius of curvature for a G. Beam is

$$R(z) = z\left[1+\left(\frac{z_0}{z}\right)^2\right] = z + z\frac{z_0^2}{z^2} = z + \frac{z_0^2}{z}$$

If $z=0$ $\rightarrow$ $R(z)=\infty$ looks like a plane wave

If $z=z_0$ $\rightarrow$ $R(z) = z_0 + \frac{z_0^2}{z_0} = 2z_0 = 0.025\,m$ $\rightarrow$ point of higher curvature for the beam

If $z=2z_0$ $\rightarrow$ $R(z) = 2z_0 + \frac{z_0^{\cancel{2}}}{2\cancel{z_0}} = \frac{5}{2}z_0 = \frac{5}{2}\,0.0125 = 0.0313$

PROBLEM 5

For the same laser of problems 3 and 4: What is the value of optical intensity at beam center ($z=0$, $\rho=0$) and at the axial point ($z=z_0$, $\rho=0$)?

The laser has power $P = 1\,mW$

$$I(\rho, z) = \frac{2P}{\pi W^2(z)} e^{-\frac{2\rho^2}{W^2(z)}}$$

$$I(0,0) = \frac{2P}{\pi W_0^2} = \frac{2 \cdot 10^{-3}}{\pi (0.05 \cdot 10^{-3})^2} = 2.54 \cdot 10^5 \frac{W}{m^2}$$

$$= 2.54 \frac{GW}{cm^2}$$

$$W(z) = W_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$

$$W_0 = \sqrt{\frac{\lambda z_0}{\pi}}$$

$$I(0, z_0) = \frac{\cancel{2} P}{\pi \cancel{2} W_0^2} = \frac{10^{-3}}{\pi (0.05 \cdot 10^{-3})^2} =$$

$$= 1.27 \cdot 10^5 \frac{W}{m^2} =$$

$$= 1.27 \frac{GW}{cm^2}$$