PROBLEM P4

AN ANTENNA WORKING AT THE FREQUENCY $f = 3$ GHz HAS A RADIATION RESISTANCE EQUAL TO $R = 50\,\Omega$, A RADIATION EFFICIENCY $e_r = 1$ AND A GAIN $G_{dB} = 9$ dB

CALCULATE

A) EFFECTIVE AREA

B) EFFECTIVE HEIGHT

SOLUTION

A) WE START BY CALCULATING THE WAVELENGTH $\lambda = \frac{c}{f} = \frac{3\,10^8}{3 \cdot 10^9} = 0{,}1$ m

WE OBSERVE THAT SINCE THE RADIATION EFFICIENCY IS UNITARY GAIN EQUALS DIRECTIVITY $G = D$

THE TEXT GIVES THE GAIN IN A LOGARITHMIC SCALE $G_{dB} = 9$ dB

AND WE CAN EASILY OBTAIN G IN THE LINEAR SCALE

$$G_{dB} = 10 \log_{10} G \quad \Rightarrow \quad G = 10^{\frac{G_{dB}}{10}}$$

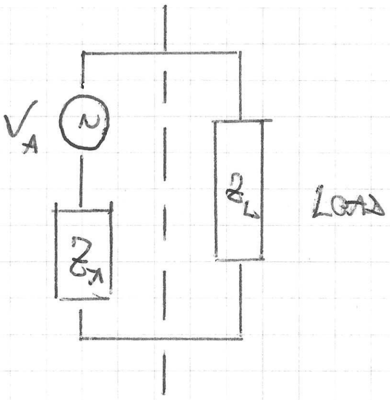
IN OUR PROBLEM $G = 10^{\frac{9}{10}} = 7{,}943$

THE EFFECTIVE AREA CAN BE DERIVED FROM THE GAIN:

$$G = \frac{4\pi}{\lambda^2} A_e \qquad A_e = G \frac{\lambda^2}{4\pi}$$

IN OUR PROBLEM $A_e = 7{,}943 \frac{0{,}01}{4\pi} = 0{,}0063$ m^2

B) IN ORDER TO DERIVE THE EFFECTIVE HEIGHT, LET US CONSIDER THE EQUIVALENT CIRCUIT OF A RECEIVING ANTENNA



RECEIVING ANTENNA

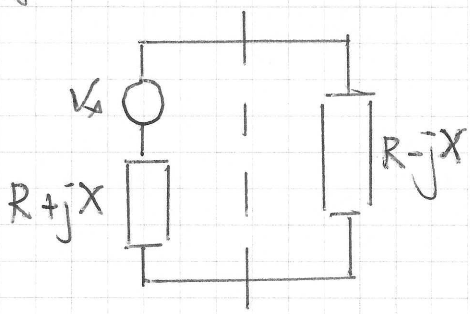
V_A = OPEN-CIRCUIT VOLTAGE

THE EFFECTIVE HEIGHT IS $h = \dfrac{V_A}{\mathcal{E}}$ WHERE $\mathcal{E}$ IS THE ELECTRIC FIELD ILLUMINATING THE ANTENNA

IF THE CONJUGATE MATCHING CONDITION IS FULFILLED $Z_A = Z_L^* = R + jX$

AND THE MAXIMUM POWER DELIVERED TO THE LOAD IS

$$P = \frac{1}{2} R |I|^2 = \frac{1}{2} R \left| \frac{V_A}{2R} \right|^2 = \frac{1}{8} \frac{|V_A|^2}{R}$$



AND SINCE $V_A = h\mathcal{E}$

$$P = \frac{1}{8} \frac{h^2 |\mathcal{E}|^2}{R}$$

THE MAXIMUM POWER DELIVERED TO THE LOAD CAN ALSO BE EXPRESSED BY USING THE EFFECTIVE AREA $P = A_e S$, WHERE S IS THE POWER DENSITY

$$P = A_e \frac{1}{2\eta} |\mathcal{E}|^2$$

WE OBTAIN $\dfrac{1}{8} \dfrac{h^2 |\mathcal{E}|^2}{R} = A_e \dfrac{1}{2\eta} |\mathcal{E}|^2 \Rightarrow h^2 = 4 \dfrac{R}{\eta} A_e$

AND WE CAN WRITE AN EXPLICIT FORMULA FOR THE EFFECTIVE HEIGHT h

$$h = 2 \sqrt{\frac{R}{\eta} A_e} = 2 \sqrt{\frac{R}{\eta} \frac{G\lambda^2}{4\pi}} = 2 \sqrt{\frac{R}{120\pi} \frac{G\lambda^2}{4\pi}} = \frac{\lambda}{\pi} \sqrt{\frac{RG}{120}}$$

AND THE NUMERICAL RESULT IS

$$h = \frac{0{,}1}{\pi} \sqrt{\frac{50 \cdot 7{,}943}{120}} = 0{,}0579 \text{ m}$$