# CHAPTER 4

# Dispersion in optical fibers

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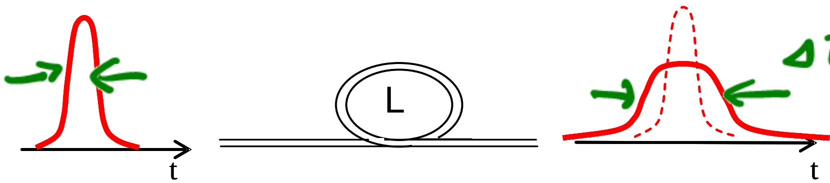
With the support of the Erasmus+ Programme of the European Union

# DEFINITION AND CAUSES OF DISPERSION IN OPTICAL FIBERS

**DISPERSION : linear phenomenon resulting in a change (generally an increase) of the duration of a pulse when propagating in a fiber**

$\Delta t$




$\Delta t'$

most of the time

$\Delta t' > \Delta t$

3 causes of dispersion :

- **intermodal dispersion (in multimode regime)** → $D_i$
- **chromatic dispersion** → $D_c$
- **polarization mode dispersion** → PMD

$N_{gx} \neq N_{gy}$

* In the multimode regime : $D_i$ >> $D_c$ of each mode >> PMD of each mode

➔ $D_c$ and PMD are neglected

* In the single mode régime : $D_i = 0$

- if $D_c$ of the fundamental mode >> PMD (usual case) ➔ $D_c$ only is taken into account and PMD is neglected
- si $D_c$ of the fundamental mode ~0 ➔ PMD must be taken onto account (for very high bit rate transmissions)

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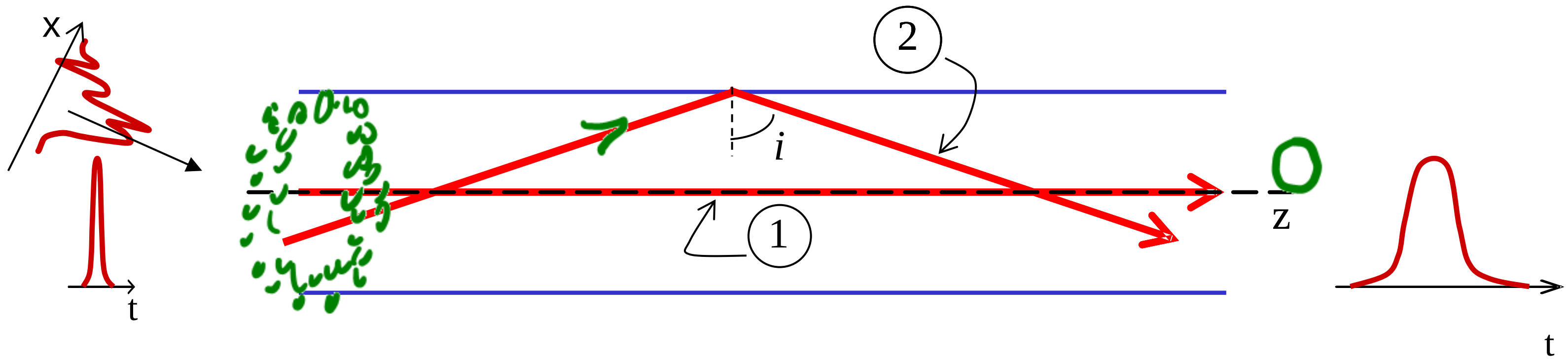
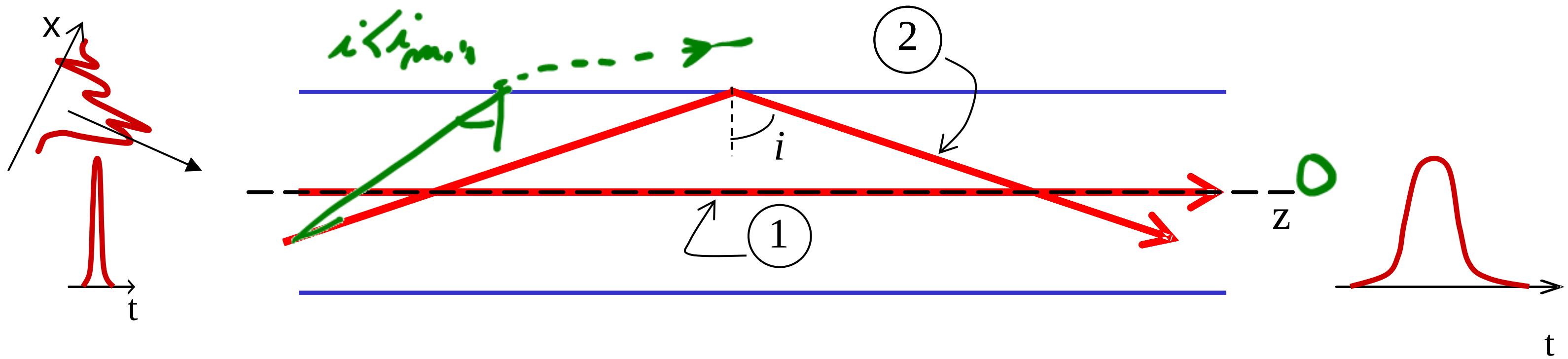
## INTERMODALE DISPERSION




➔ due to the fact that the each excited mode has its own group velocity, different from that of the others

➔ $D_I$ = pulse broadening per unit of length along which light propagates (ns/km)

# INTERMODALE DISPERSION




➔ due to the fact that the each excited mode has its own group velocity, different from that of the others

➔ $D_I$ = pulse broadening per unit of length along which light propagates (ns/km)

group delay: $t_g = \dfrac{L}{V_g}$    For a step index fiber → $v_g \approx \dfrac{c}{n_1}\sin i$

ray 1 → sin i = 1 → $v_g \approx \dfrac{c}{n_1}$ → $t_{g1} \approx \dfrac{L}{c} n_1$

ray 2 → $\sin i_{\min} = \dfrac{n_2}{n_1}$ → $v_g \approx \dfrac{c}{n_1}\dfrac{n_2}{n_1}$ → $t_{g2} \approx \dfrac{L}{c}\dfrac{n_1^2}{n_2}$

⇒ $\Delta t_g = \tau = t_{g2} - t_{g1}$

$$\approx \frac{L}{c}.n_1.\left(\frac{n_1}{n_2} - 1\right) \approx \frac{L}{c} n_1 \left(\frac{n_1 - n_2}{n_2}\right)$$

With $\Delta = \left(\dfrac{n_1^2 - n_2^2}{2n_1^2}\right) = \dfrac{(n_1+n_2)(n_1-n_2)}{2n_1^2} \approx \dfrac{2n_1(n_1-n_2)}{2n_1^2} \approx \dfrac{n_1 - n_2}{n_1}$ ⇒ $\boxed{\tau = \dfrac{L}{c} n_1 \Delta}$ (step index fiber)

## INTERMODALE DISPERSION (step index fiber)

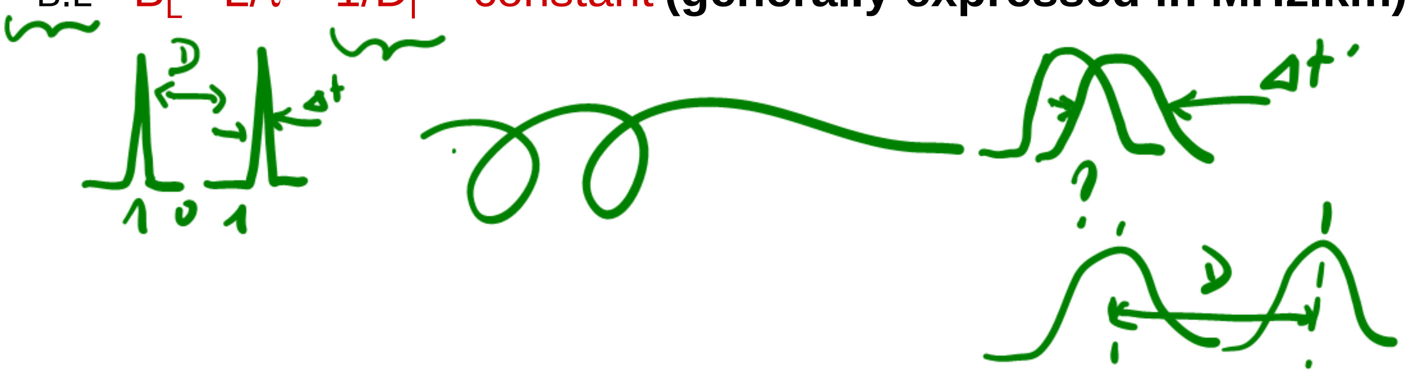
Definition :

$$\boxed{D_I \triangleq \frac{\tau}{L} = \frac{n_1.\Delta}{c} \quad \text{(en ns/km)}}$$

$$\left(\text{reminder : } \tau = \frac{L}{c} n_1 \Delta\right)$$

Modulation bandwidth, for a fiber of length L : **B=1/τ** **(generally expressed in MHz)**

B.L = $B_L$ =L/τ = 1/$D_I$ = constant **(generally expressed in MHz.km)**



# INTERMODALE DISPERSION (step index fiber)

Définition : $\boxed{D_I \triangleq \frac{\tau}{L} = \frac{n_1.\Delta}{c} \quad \text{(en ns/km)}}$

$\left(\text{reminder : } \tau = \frac{L}{c} n_1 \Delta\right)$

Modulation bandwidth, for a fiber of length L : **B=1/τ** **(generally expressed in MHz)**

B.L = $B_L$ =L/τ = 1/$D_I$ = constant **(generally expressed in MHz.km)**

$\lambda \sim 0.8\,\mu m$

Example :

**Step index fiber, length L= 3km with n1 = 1,465 and n2 =1,45**

$$\tau = 3 \,.\, \frac{1}{3\ 10^5} \,.\, 1{,}465 \,.\, \left(\frac{1{,}465\text{-}1{,}45}{1{,}45}\right) = 1{,}52\ 10^{-7} s \quad = 152\ ns$$

$$B = \frac{1}{\tau} = \frac{1}{152\ 10^{-9}} = \ 6{,}58\ 10^6\text{Hz} \ = 6{,}58\ \text{MHz}$$

$$B_L = \frac{1}{D_I} = \frac{1}{51\ 10^{-9}} \simeq \ 20\ 10^6\text{Hz.km} \simeq \ 20\ \text{MHz.km}$$

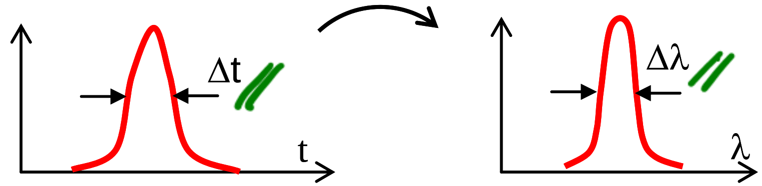
**Optimized graded index fiber**

τ' = τ/100

B' =100.B

$B_L$' =100.$B_L$

## CHROMATIC DISPERSION




$\Delta\lambda = \lambda^2 . \Delta\nu / c$

where $\Delta\nu$ = spectral bandwidth of the pulse with $\Delta t . \Delta\nu$ = cte

**if Δt=10ps with $\lambda_0$ = 1µm**

$$\Delta\lambda = \frac{\left(10^{-6}\right)^2}{3.10^8} \text{ x } \frac{1}{10.10^{-12}} = 3.10^{-10}\, m = 0{,}3 nm$$

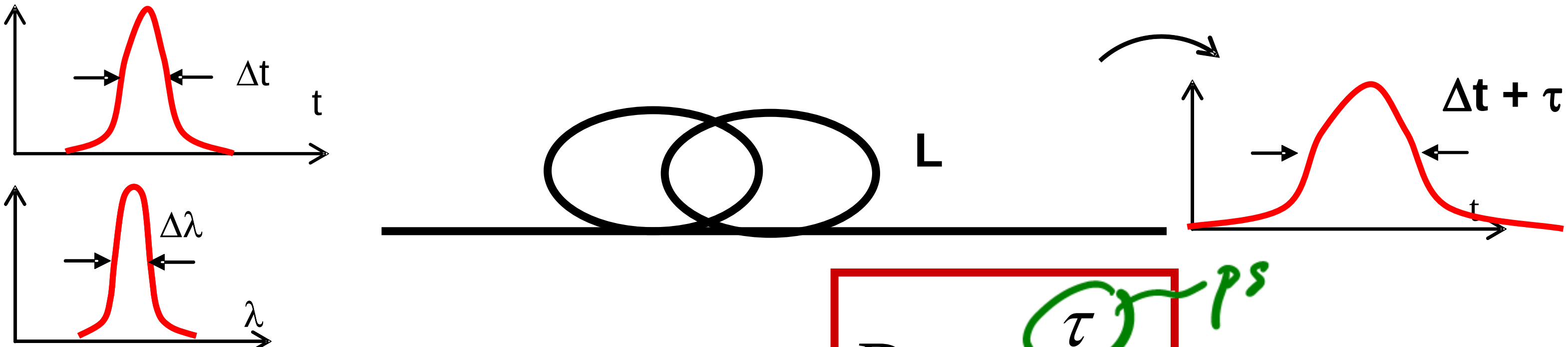
(with $\Delta t . \Delta\nu$=1)

<u>causes of chromatic dispersion</u> :

- dispersive material ➔n=f(λ)➔ $v_\varphi$=c/n=f(λ) ➔ $v_g$=f(λ)➔ **material dispersion ($D_{mat}$)**
- when the wave is guided, β=f(V)=f(ω)➔ $v_g$=dω/dβ= f(λ)➔ **guide dispersion ($D_{gui}$)**

**In first approximation : $D_c \approx D_{mat} + D_{gui}$**

## CHROMATIC DISPERSION




<u>Definition</u> :

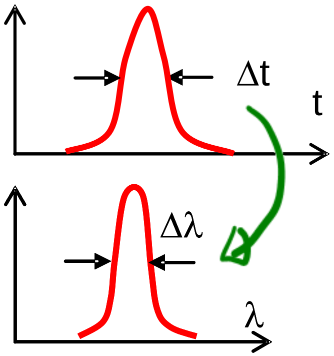
$$\boxed{D_c = \frac{\tau}{L.\Delta\lambda}}$$

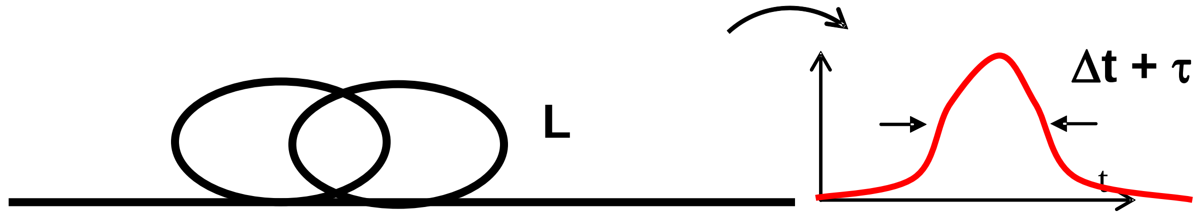
ps

km nm

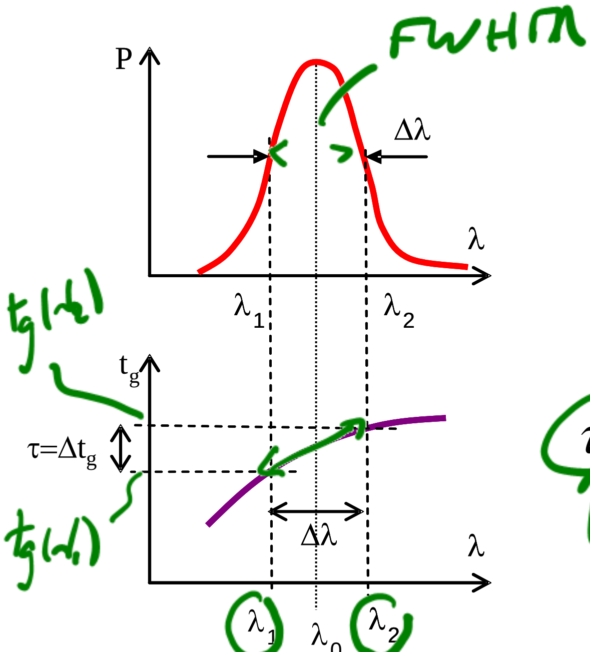
expressed in ps/(nm.km)

## CHROMATIC DISPERSION




Definition : $$\boxed{D_c = \frac{\tau}{L.\Delta\lambda}}$$ expressed in ps/(nm.km)




$$\tau = \left| t_g(\lambda_2) - t_g(\lambda_1) \right| \simeq \frac{dt_g}{d\lambda} \Delta\lambda$$

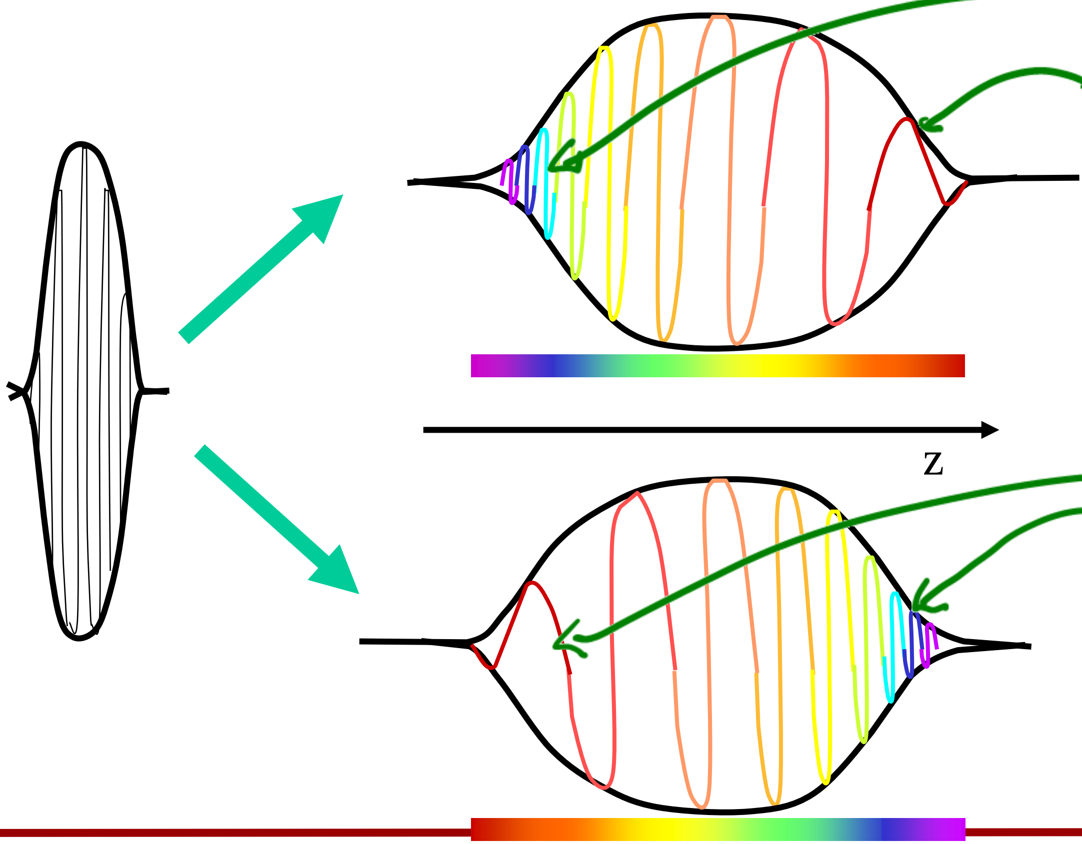
$$\Rightarrow \boxed{D_c = \frac{1}{L} \frac{dt_g}{d\lambda}}$$

R → red → long wavelengths.
B → blue → short "

## CHROMATIC DISPERSION

$$\boxed{D_c = \frac{1}{L}\frac{dt_g}{d\lambda}}$$

$$D_c = \frac{t_{gR} - t_{gB}}{L\,(\lambda_R - \lambda_B)} \quad \text{en ps/(nm.km)}$$




$v_{gR} > v_{gB} \Rightarrow t_{gR} < t_{gB}$

$D_c < 0$ //
Normal dispersion //

$v_{gB} > v_{gR} \Rightarrow t_{gB} < t_{gR}$

$D_c > 0$
Anomalous dispersion //

## CHROMATIC DISPERSION : remarks

$v_g = \frac{d\omega}{d\beta}$

$$D_c = \frac{1}{L}\frac{dt_g}{d\lambda} = \frac{1}{\cancel{L}}\frac{d}{d\lambda}\left(\frac{\cancel{L}}{v_g}\right) = \frac{d}{d\lambda}\left(\frac{d\beta}{d\omega}\right) = \frac{d}{d\omega}\left(\frac{d\beta}{d\omega}\right).\frac{d\omega}{d\lambda} = -\frac{2\pi c}{\lambda^2}\frac{d^2\beta}{d\omega^2}$$

➔ the dispersion curve $\beta=f(\omega)$ allows to calculate the chromatic dispersion (taking into account the actual values $n_1(\lambda)$ and $n_2(\lambda)$ at each wavelength)

## CHROMATIC DISPERSION : remarks

$$D_c = \frac{1}{L}\frac{dt_g}{d\lambda} = \frac{1}{L}\frac{d}{d\lambda}\left(\frac{L}{v_g}\right) = \frac{d}{d\lambda}\left(\frac{d\beta}{d\omega}\right) = \frac{d}{d\omega}\left(\frac{d\beta}{d\omega}\right).\frac{d\omega}{d\lambda} = -\frac{2\pi c}{\lambda^2}\frac{d^2\beta}{d\omega^2}$$

➔ the dispersion curve β=f(ω) allows calculating the chromatic dispersion (taking into account the actual values $n_1(\lambda)$ and $n_2(\lambda)$ at each wavelength)

$e^{j(\omega t-\beta z)}$

Taylor expansion of the spectral phase of the guided wave :

$$\varphi(\omega) = \beta L = \beta(\omega_0).L + L.\frac{d\beta}{d\omega}(\omega-\omega_0) + \frac{L}{2}\frac{d^2\beta}{d\omega^2}(\omega-\omega_0)^2 + \ldots$$

out of the "guided optics" community, $\frac{d^2\beta}{d\omega^2}$ is often called "group velocity dispersion"

→ inappropriate denomination and risk of confusion with $D_c$

in fact $\frac{d^2\beta}{d\omega^2}$ is proportional to the dispersion of group delay

## CALCULATION OF THE MATERIAL DISPERSION

$D_{mat}$

$v_g = \frac{c}{N_g}$ $N_g = n_1 - \lambda \frac{dn_1}{d\lambda}$

→ plane wave (= propagating wave NOT GUIDED)
→ dispersive propagation medium : $n_1 = f(\lambda)$

$$t_g = t_{mat} = \frac{L}{v_g} = \frac{L}{c} N_g = \frac{L}{c}\left(n_1 - \lambda \frac{dn_1}{d\lambda}\right)$$

## CALCULATION OF THE MATERIAL DISPERSION

→ plane wave (= propagating wave NOT GUIDED)
→ dispersive propagation medium : $n_1 = f(\lambda)$

$$t_g = t_{mat} = \frac{L}{v_g} = \frac{L}{c} N_g = \frac{L}{c}\left( n_1 - \lambda \frac{dn_1}{d\lambda} \right)$$

$$\tau_{mat} = \Delta t_g = \frac{dt_{mat}}{d\lambda}\Delta\lambda = \frac{L\Delta\lambda}{c}\frac{d}{d\lambda}\left( n_1 - \lambda \frac{dn_1}{d\lambda} \right)$$

$$= \frac{L}{c}\Delta\lambda.\left( \frac{dn_1}{d\lambda} - \left( 1x\frac{dn_1}{d\lambda} + \lambda \frac{d^2 n_1}{d\lambda^2} \right) \right)$$

$$= -\frac{\lambda L}{c}\Delta\lambda.\frac{d^2 n_1}{d\lambda^2}$$

# CALCULATION OF THE MATERIAL DISPERSION

→ plane wave (= propagating wave NOT GUIDED)
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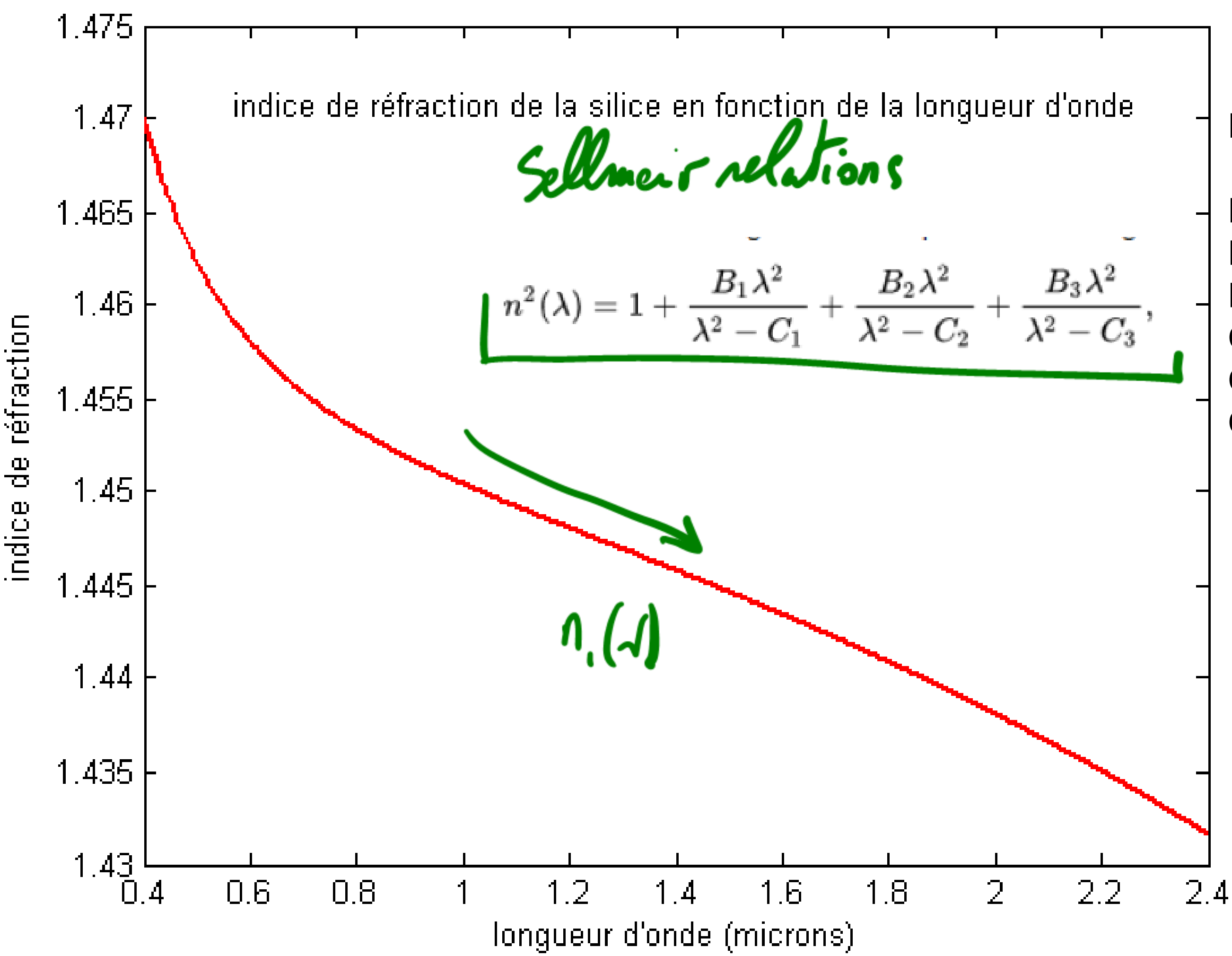
$$= -\frac{\lambda \cancel{L}}{c}\cancel{\Delta\lambda}.\frac{d^2 n_1}{d\lambda^2}$$

$$\boxed{D_{mat} = \frac{\tau_{mat}}{L.\Delta\lambda} = -\frac{\lambda}{c}\frac{d^2 n_1}{d\lambda^2}}$$

# MATERIAL DISPERSION

$$D_{mat} = -\frac{\lambda}{c}\frac{d^2 n_1}{d\lambda^2}$$




Sellmeir relations

$$\boxed{n^2(\lambda) = 1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3},}$$

$n_1(\lambda)$

For pure silica :

$B_1$ = 0.696166300
$B_2$ = 0.407942600
$B_3$ = 0.897479400
$C_1$ = 4.67914826x10$^{-3}$ µm$^2$
$C_2$ = 1.35120631x10$^{-2}$ µm$^2$
$C_3$ = 97.9340025 µm$^2$

**$n(SiO_2) = f(\lambda)$**

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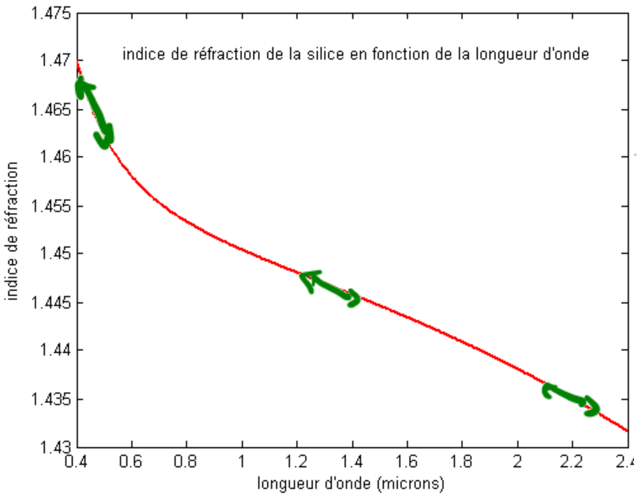
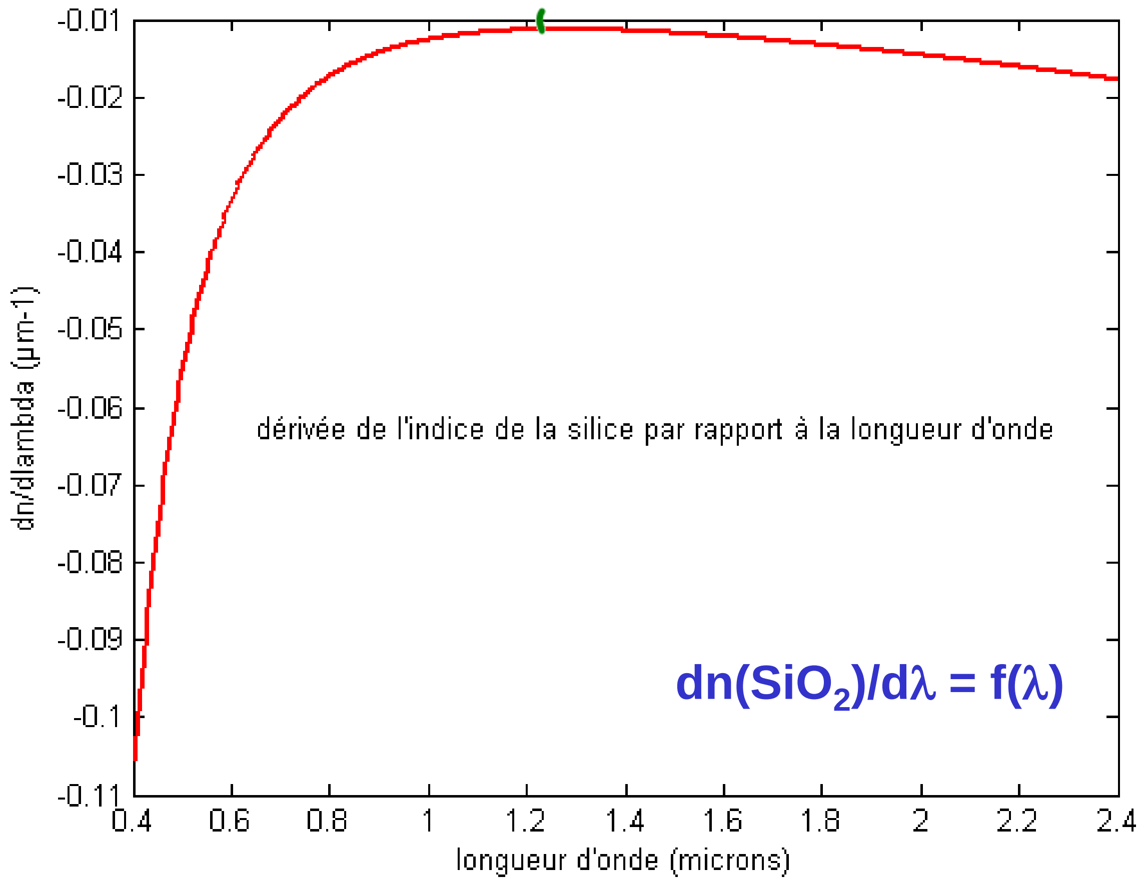
# MATERIAL DISPERSION

$$D_{mat} = -\frac{\lambda}{c}\frac{d^2 n_1}{d\lambda^2}$$

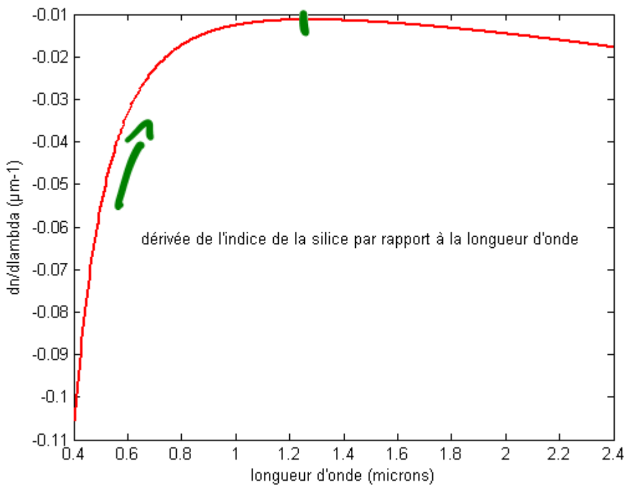


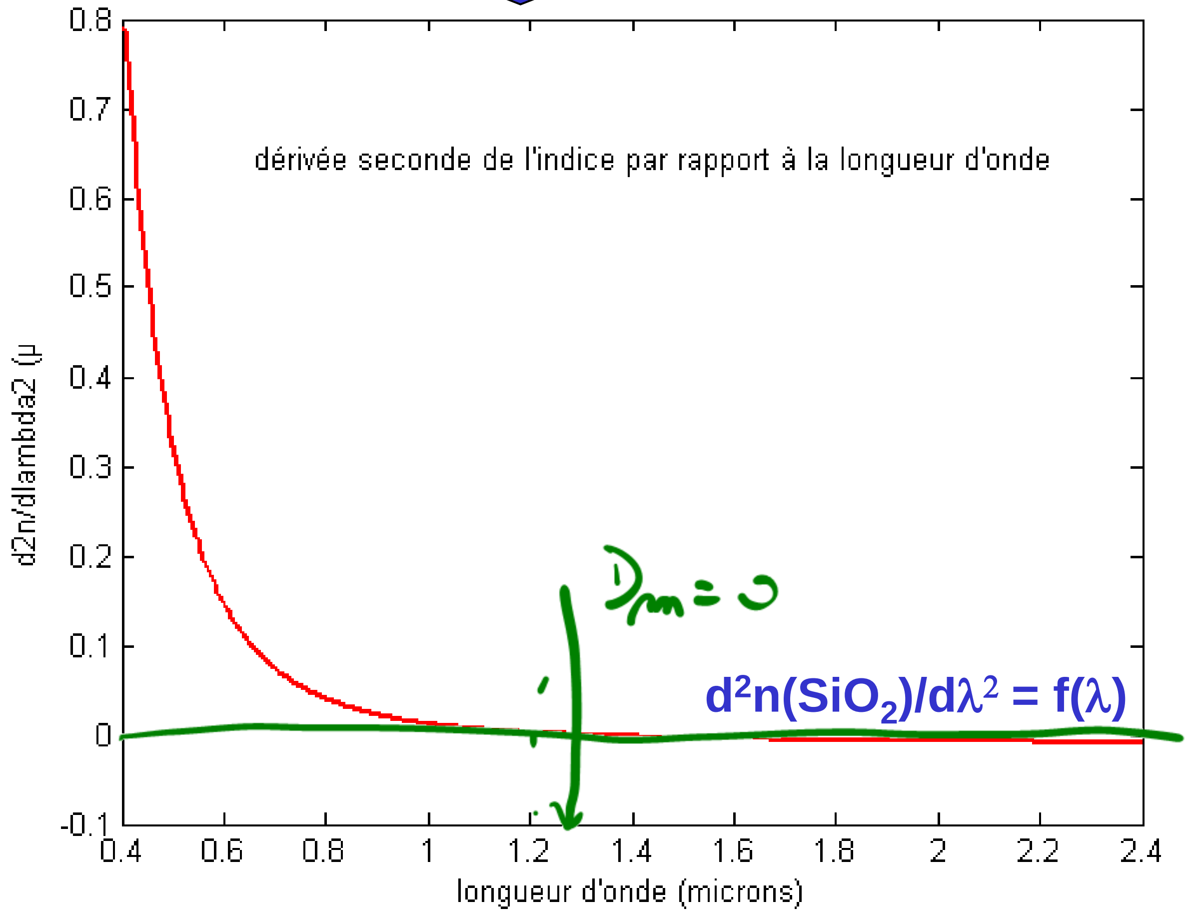




**$dn(SiO_2)/d\lambda = f(\lambda)$**

# MATERIAL DISPERSION

$$D_{mat} = \boxed{-\frac{\lambda}{c}} \frac{d^2 n_1}{d\lambda^2}$$

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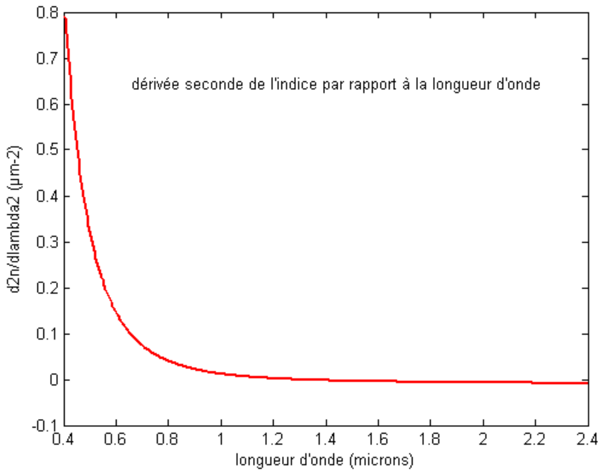
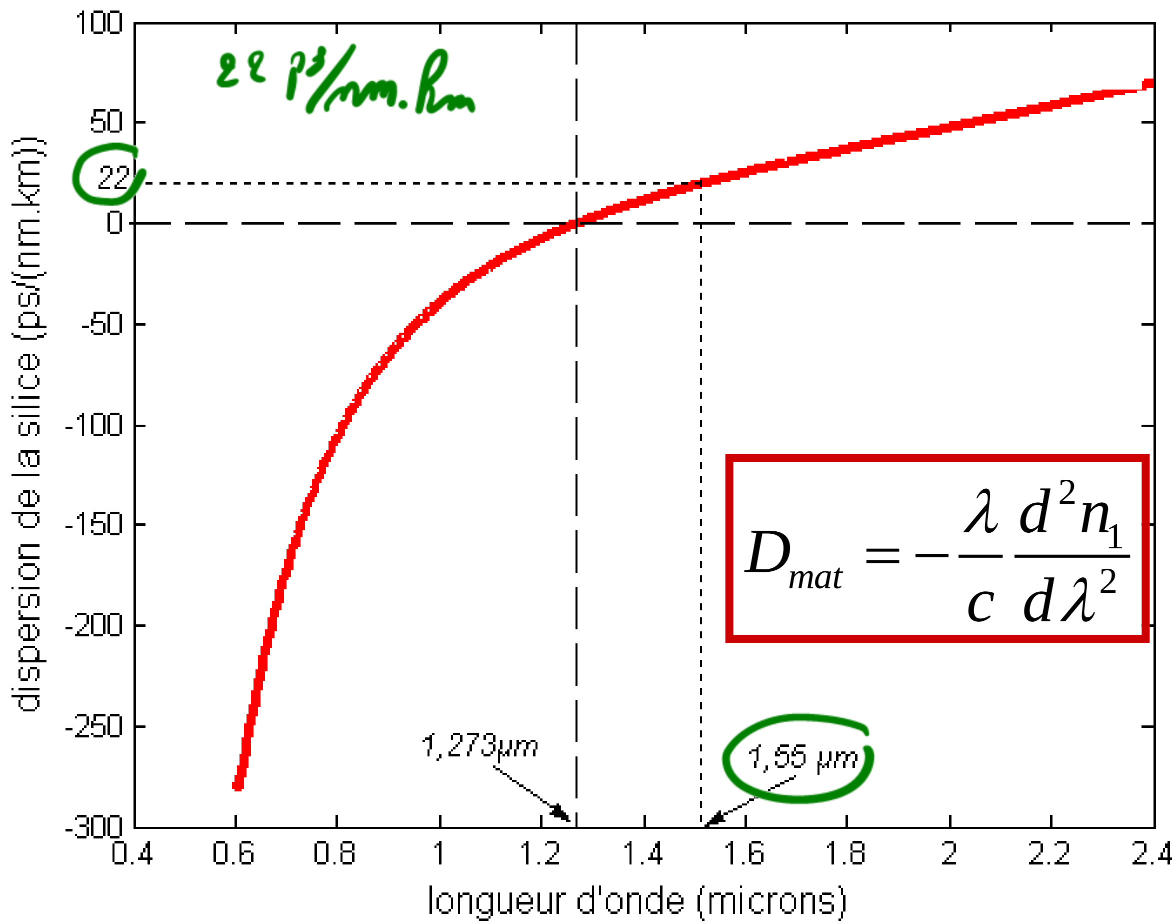
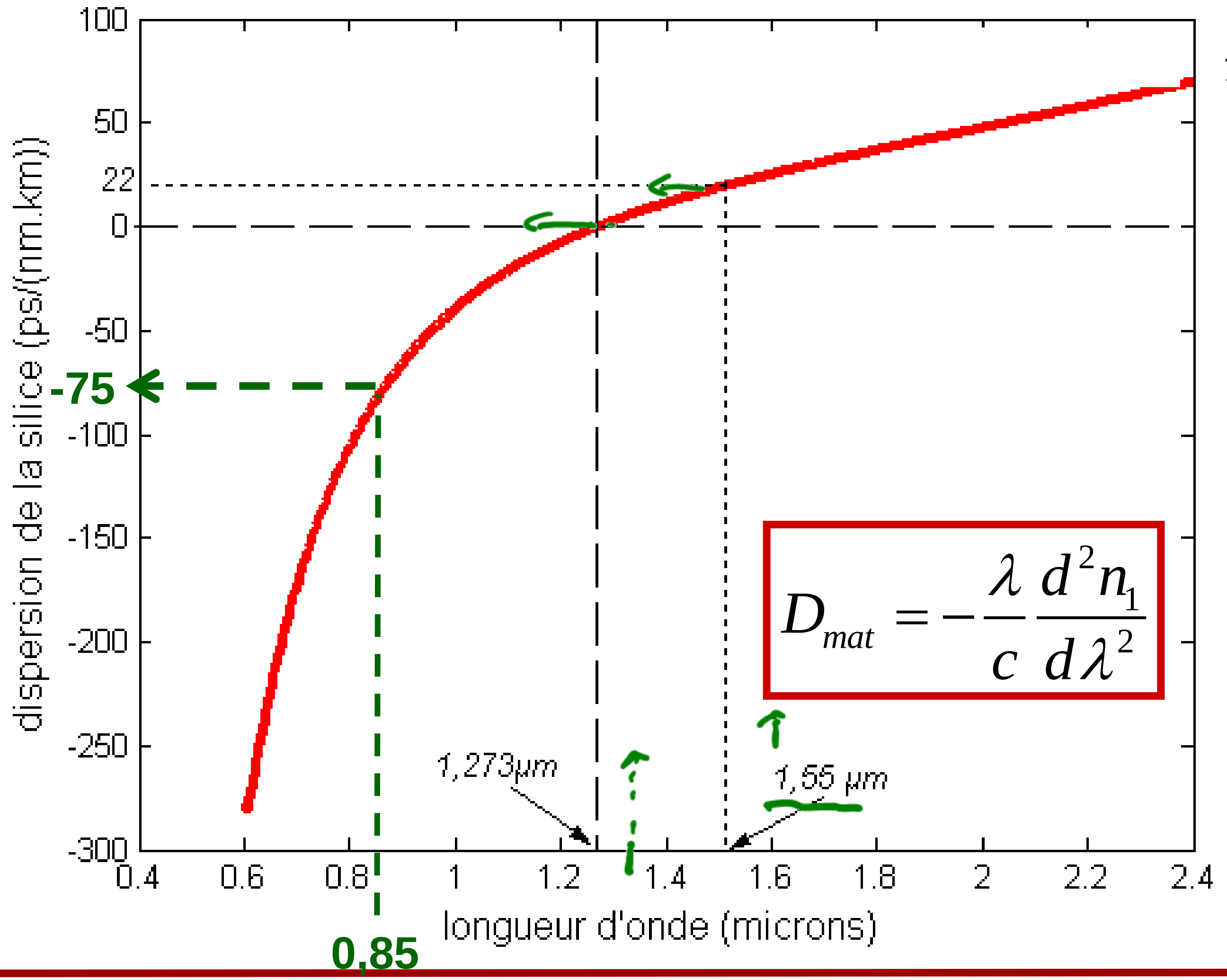
## MATERIAL DISPERSION




$$D_{mat} = -\frac{\lambda}{c}\frac{d^2 n_1}{d\lambda^2}$$

# MATERIAL DISPERSION

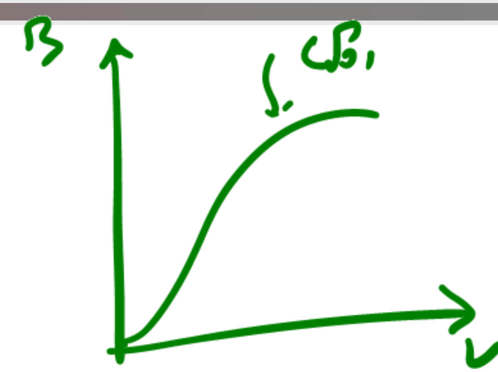



$$D_{mat} = -\frac{\lambda}{c}\frac{d^2 n_1}{d\lambda^2}$$

example let us consider a beam from a laser diode emitting at 850nm, with $\Delta\lambda$ = 40nm, launched in a fiber with a length L= 2km

$D_{mat}$ = -75ps/(nm.km))

$$\tau_{mat} = L.\Delta\lambda.|D_{mat}|$$
$$= 2\text{x}40\text{x}75 = 6000\,ps = 6ns$$




# CALCULATION OF THE GUIDE DISPERSION

→ guided wave
→ non dispersive propagation medium $\Rightarrow \dfrac{dn_1}{d\lambda}=0\ ;\ \dfrac{dn_2}{d\lambda}=0\ ;\ \dfrac{d\Delta}{d\lambda}=0$

$D_{gui} \leftarrow \tau$

$$v_g=\frac{d\omega}{d\beta} \qquad \text{et} \qquad t_g=\frac{L}{v_g}=L\frac{d\beta}{d\omega}=\frac{L}{c}\frac{d\beta}{dk_0} \qquad \text{(car } \omega=k_0.c\text{)}$$

Goal : express $t_g$ as a function of B and V (➔ allowing to exploit the dispersion curves B=f(V) )

One easily shows that : $\beta=k_0\left[n_2+n_1\Delta B\right]$

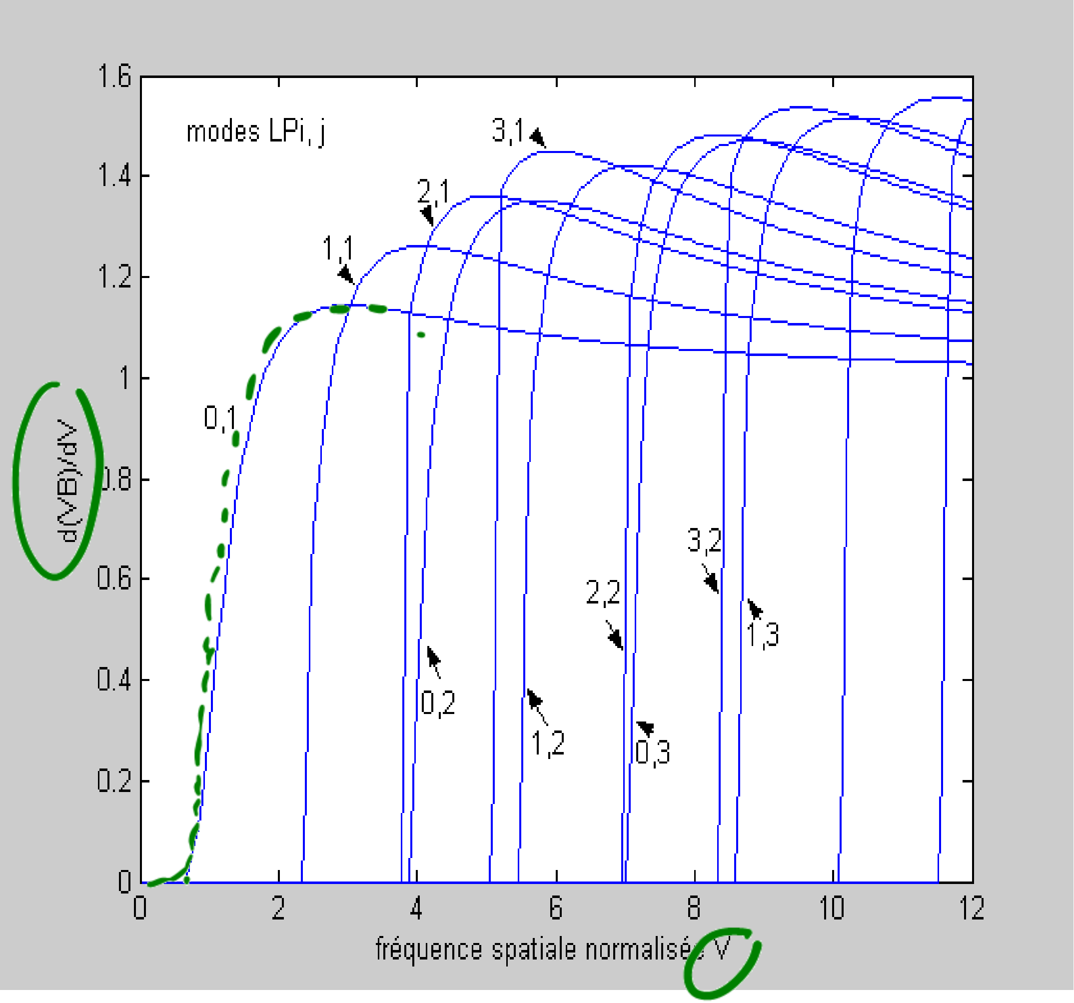
$$k_0=\frac{V}{a.n_1\sqrt{2\Delta}}$$

$$\boxed{t_g=t_{gui}=\frac{L}{c}\left[n_2+n_1\Delta\frac{d(VB)}{dV}\right]}$$

see the development of the calculations in ***pdf pages 10 et 11***

# CALCULATION OF THE GUIDE DISPERSION

$$t_{gui} = \frac{L}{c}\left[n_2 + n_1\Delta\frac{d(VB)}{dV}\right]$$



## CALCULATION OF THE GUIDE DISPERSION

$$t_{gui} = \frac{L}{c}\left[n_2 + n_1\Delta\frac{d(VB)}{dV}\right]$$

temporal broadening : $\tau_{gui} = \Delta t_{gui} = \dfrac{dt_{gui}}{d\lambda}\Delta\lambda$ (see slide number 9)

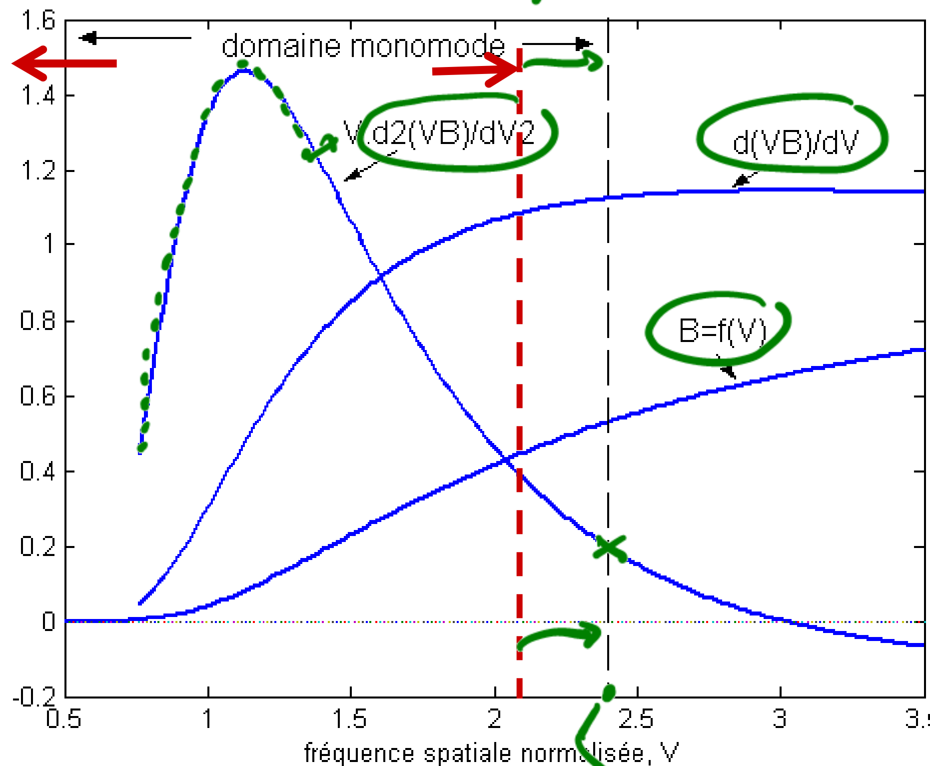
Dispersion due to guiding effect : $D_{gui} = \dfrac{\tau_{gui}}{L.\Delta\lambda}$

for LP01

resulting in :

$$\boxed{D_{gui} = -\frac{n_1\Delta}{c\lambda}V\frac{d^2(VB)}{dV^2}}$$

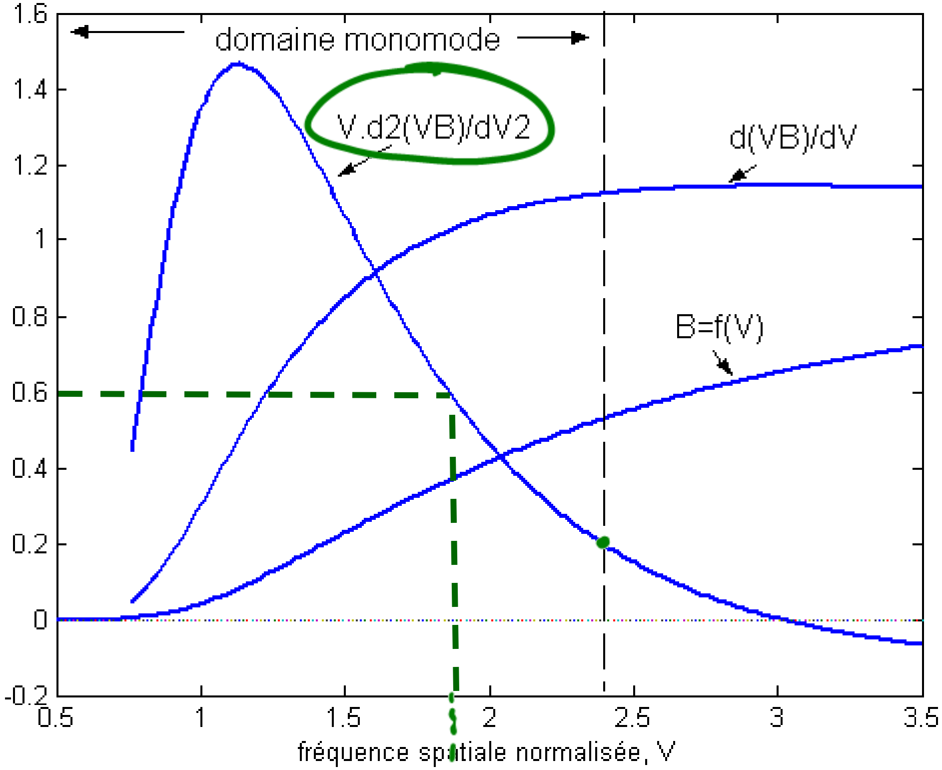
(see the development of the calculations in ***pdf page 12)***




2.405

# CALCULATION OF THE GUIDE DISPERSION

$$D_{gui} = -\frac{n\Delta}{c\lambda} V \frac{d^2(VB)}{dV^2}$$




1.91

Example: case of a fiber with

**a = 4,5µm    ~~ON~~ NA = 0,105    n1 = 1,46**

**à λ= 1,55µm**

$$\rightarrow \quad \Delta = \frac{\cancel{ON}^2 \; NA^2}{2n_1^2} = 2{,}83.10^{-3}$$

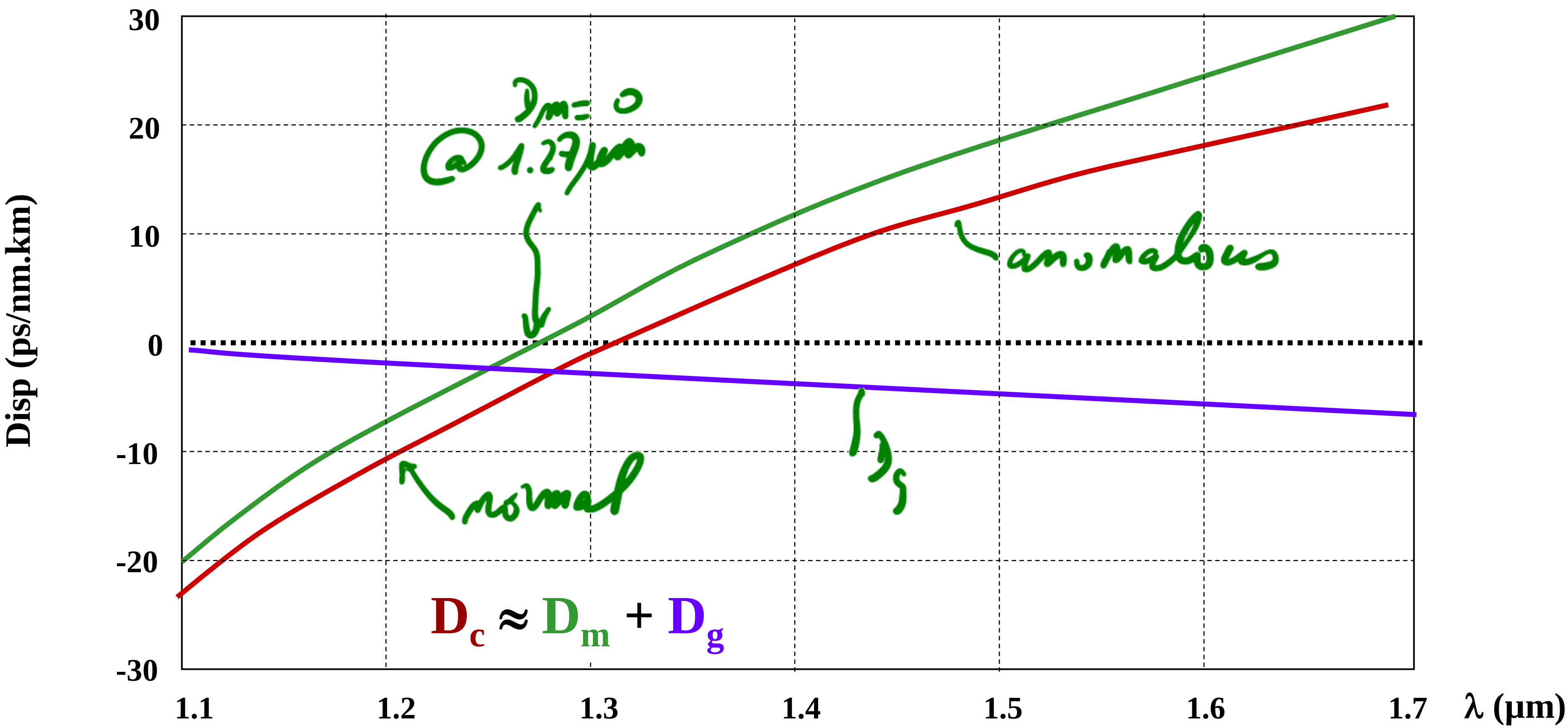
$$\rightarrow \quad V = \frac{2\pi}{\lambda} a \cancel{ON} \; NA = 1{,}91$$

$$\rightarrow \quad V\frac{d^2(VB)}{dV^2} \simeq 0{,}6$$

$$\boxed{D_{gui} = -\frac{1{,}46 \text{ x } 2{,}83.10^{-3}}{3.10^{8} \text{ x } 1{,}55.10^{-6}} \text{ x } 0{,}6 = -\,5 10^{-6} s/(m.m) = -\,5 ps/(nm.km)}$$

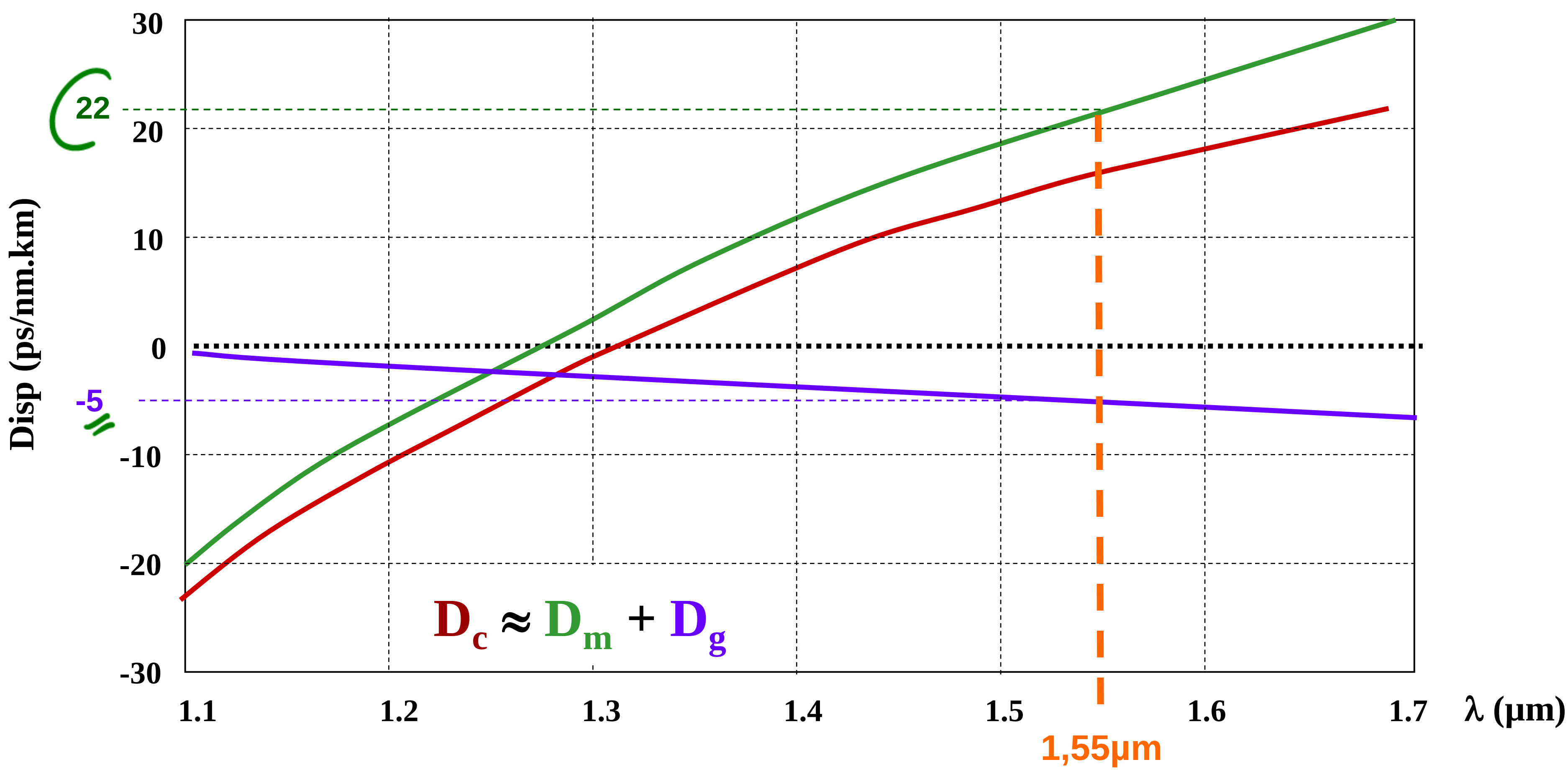
## CURVES OF THE CHROMATIC DISPERSION versus WAVELENGTH

**Example with a step index fiber : $n_1 = 1.46$   $n_2 = 1.455$   $a = 4\ \mu m$**



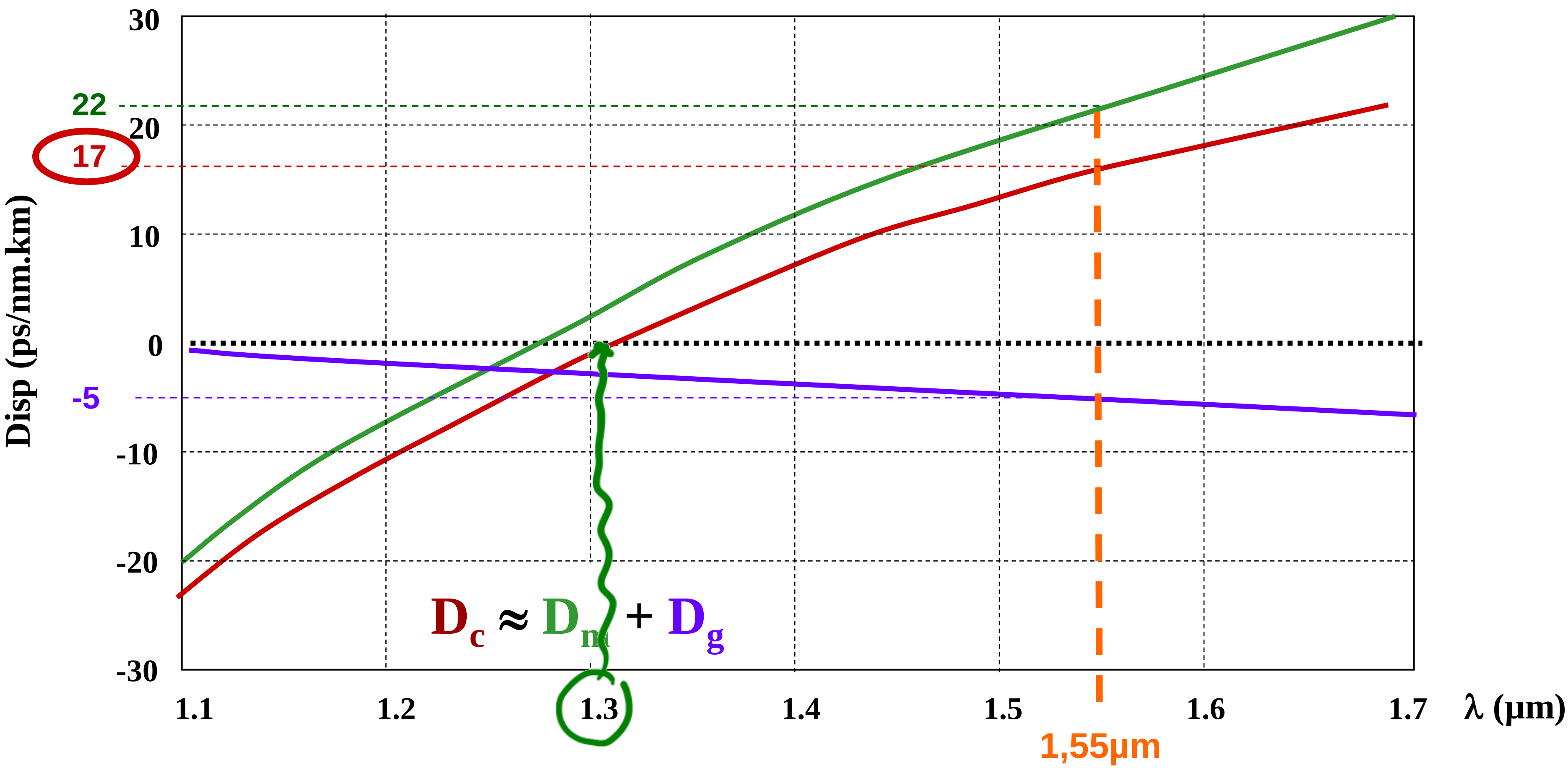
# CURVES OF THE CHROMATIC DISPERSION versus WAVELENGTH

**<u>Example with a step index fiber</u> : $n_1$ = 1.46 $n_2$ = 1.455 a = 4 µm**

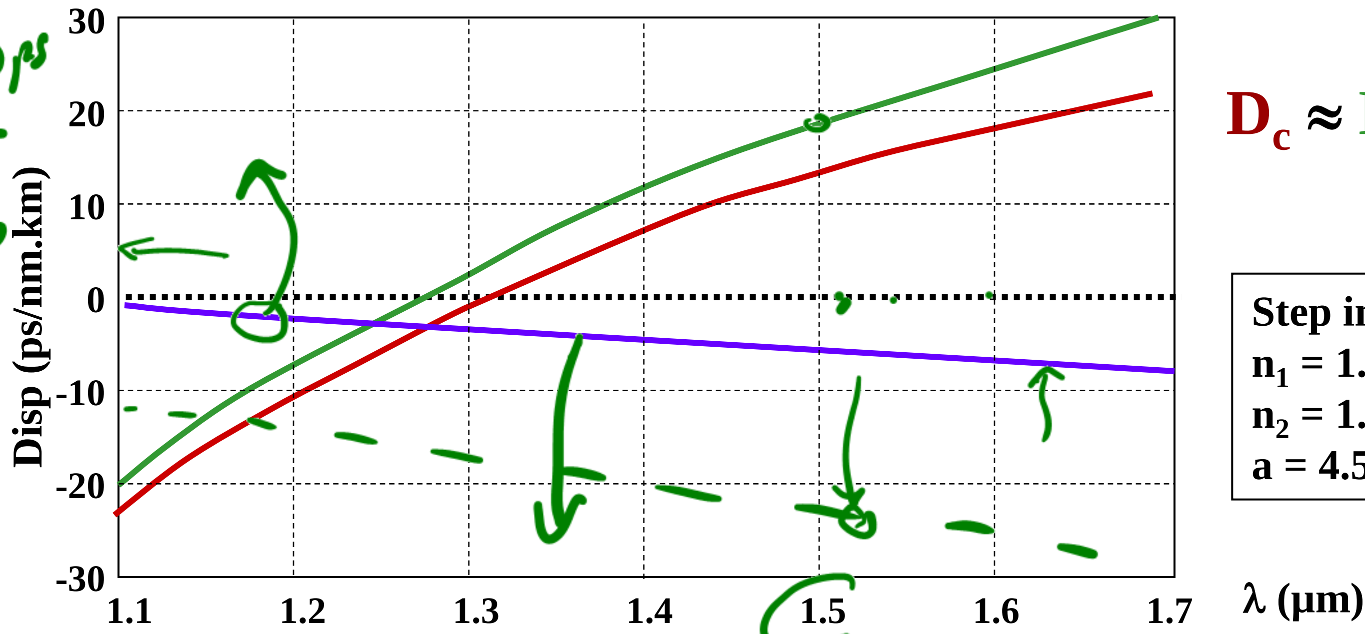


# CURVES OF THE CHROMATIC DISPERSION versus WAVELENGTH

**Example with a step index fiber : $n_1$ = 1.46 $n_2$ = 1.455 a = 4 µm**



## CURVES OF THE CHROMATIC DISPERSION VERSUS WAVELENGTH




$$D_c \approx D_m + D_g$$

**Step index fiber**
$n_1 = 1.46$
$n_2 = 1.456$
$a = 4.5\mu m$

**How can we change the chromatic dispersion of an optical fiber ?**

→ **By changing the material dispersion ???? → no**

→ **By changing the dispersion of the guide !!!**

⇨ **Working with higher order modes**

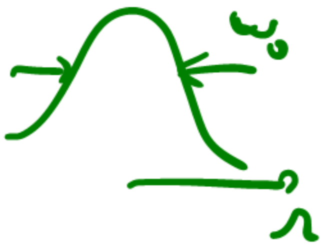
⇨ **Working in the single mod regime, but with a fiber having a modified index profile**

- **multiclad fibers ("DS fibers", "DF fibers…)**
- **Air silica microstructured optical fibers (MOFs so called "PCFs")**
- **Bragg fibers or photonic bandgap fibers**

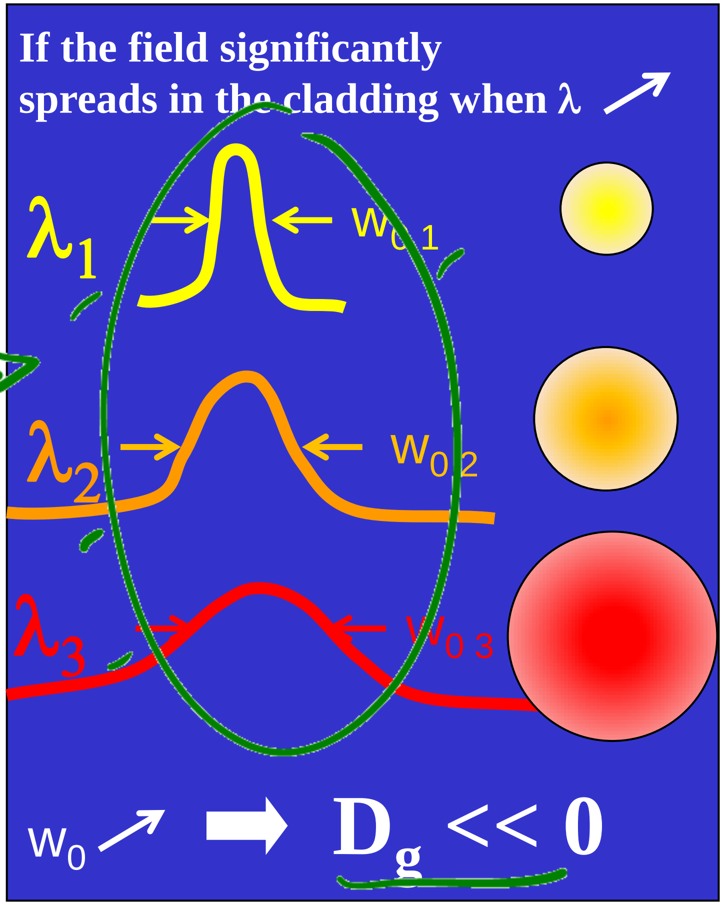
# HOW CAN WE CHANGE THE DISPERSION OF THE GUIDE ?

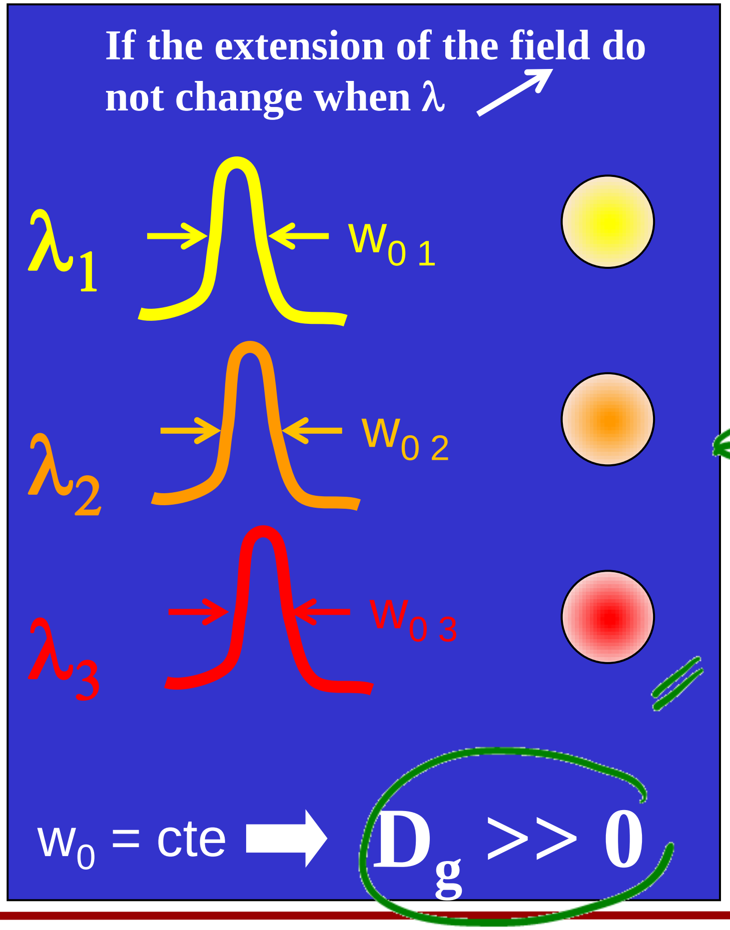
$$D_g = -\frac{1}{\pi^2 n_2 c}\frac{\lambda}{w_0^2}\left(\frac{\lambda}{w_0}\frac{dw_0}{d\lambda}-\frac{1}{2}\right)$$

Pierre Sansonetti, Elect. Letters, vol 18, n°15, pp 647-648 (1982)

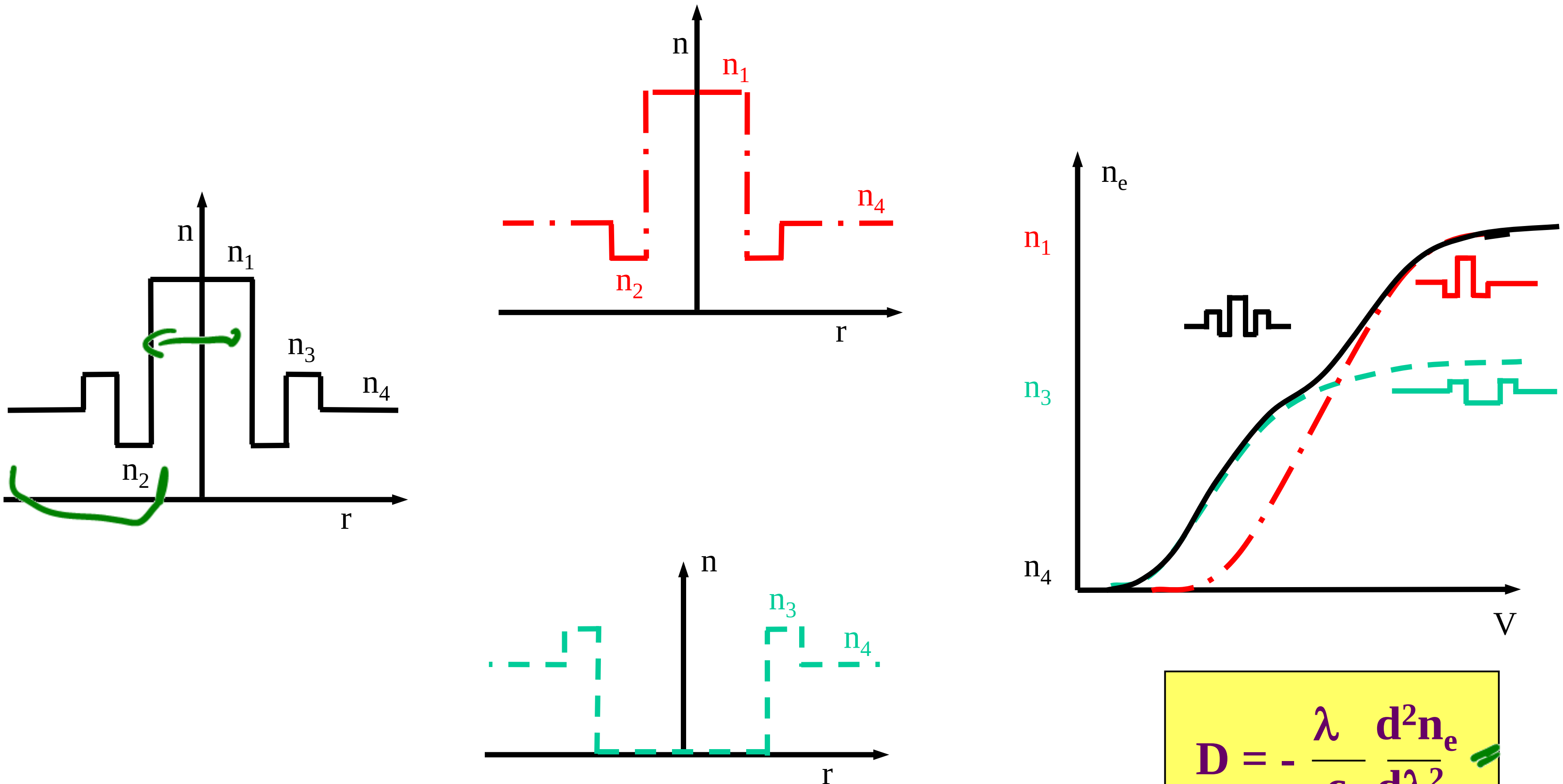



$\lambda_1 < \lambda_2 < \lambda_3$



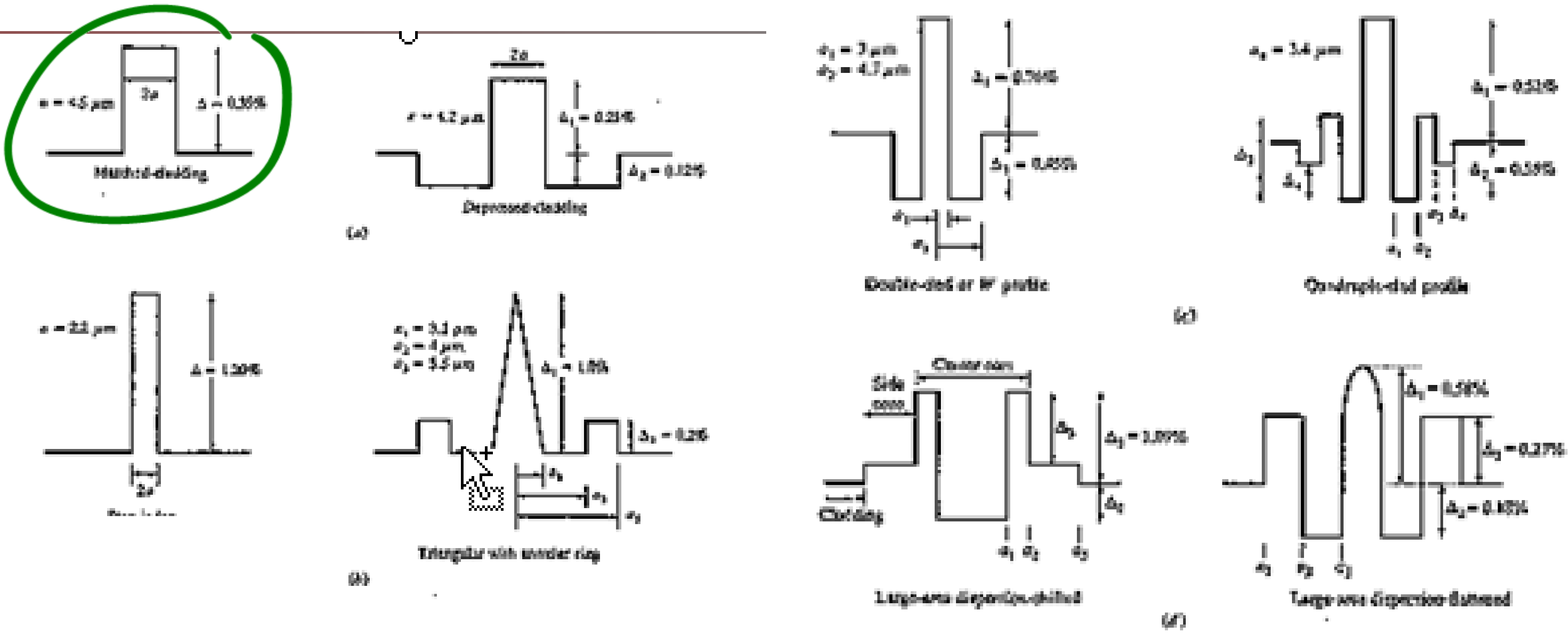



# DISPERSION SHIFTED FIBERS : MULTICLAD FIBERS

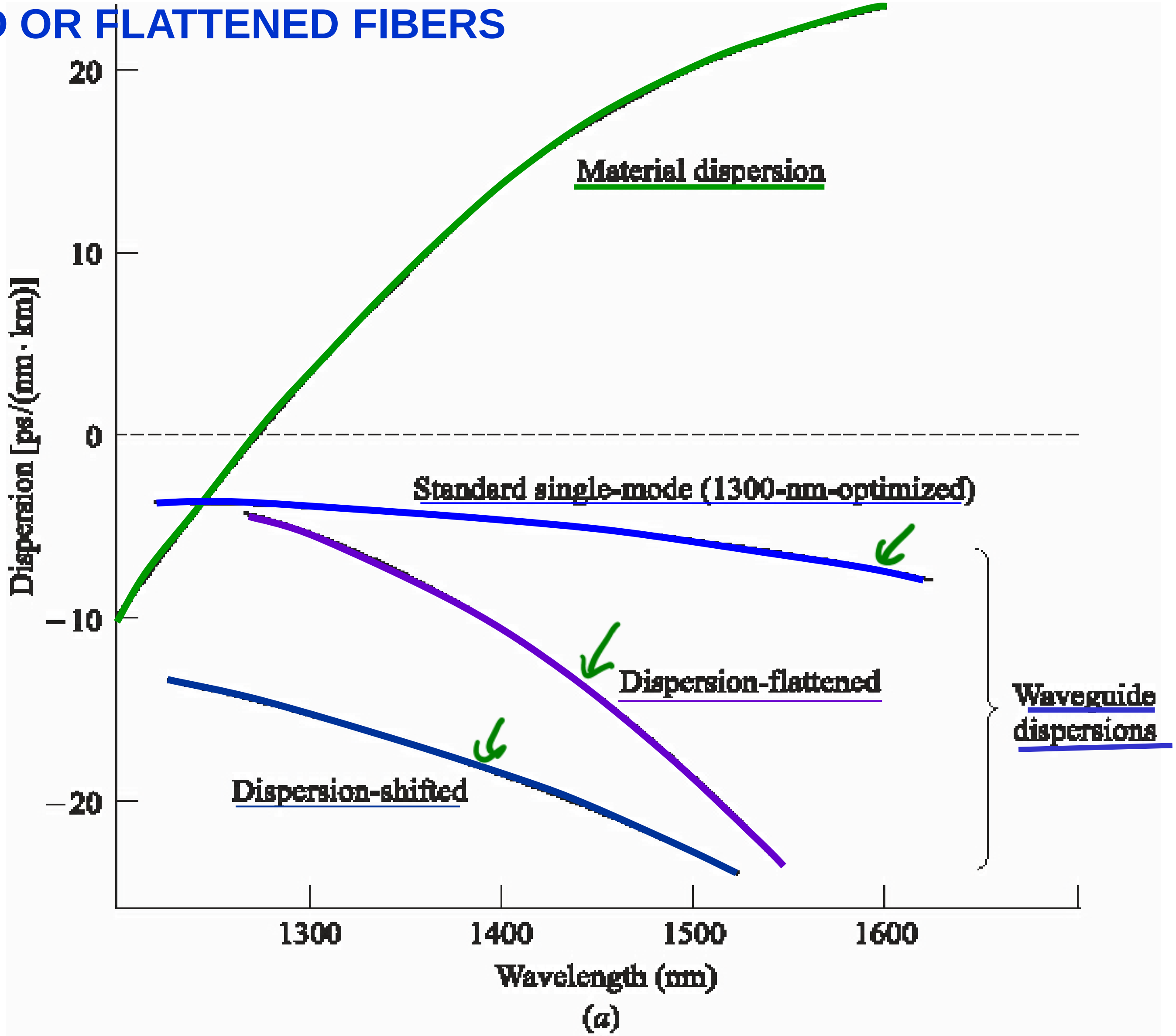



$$D = -\frac{\lambda}{c}\frac{d^2 n_e}{d\lambda^2}$$

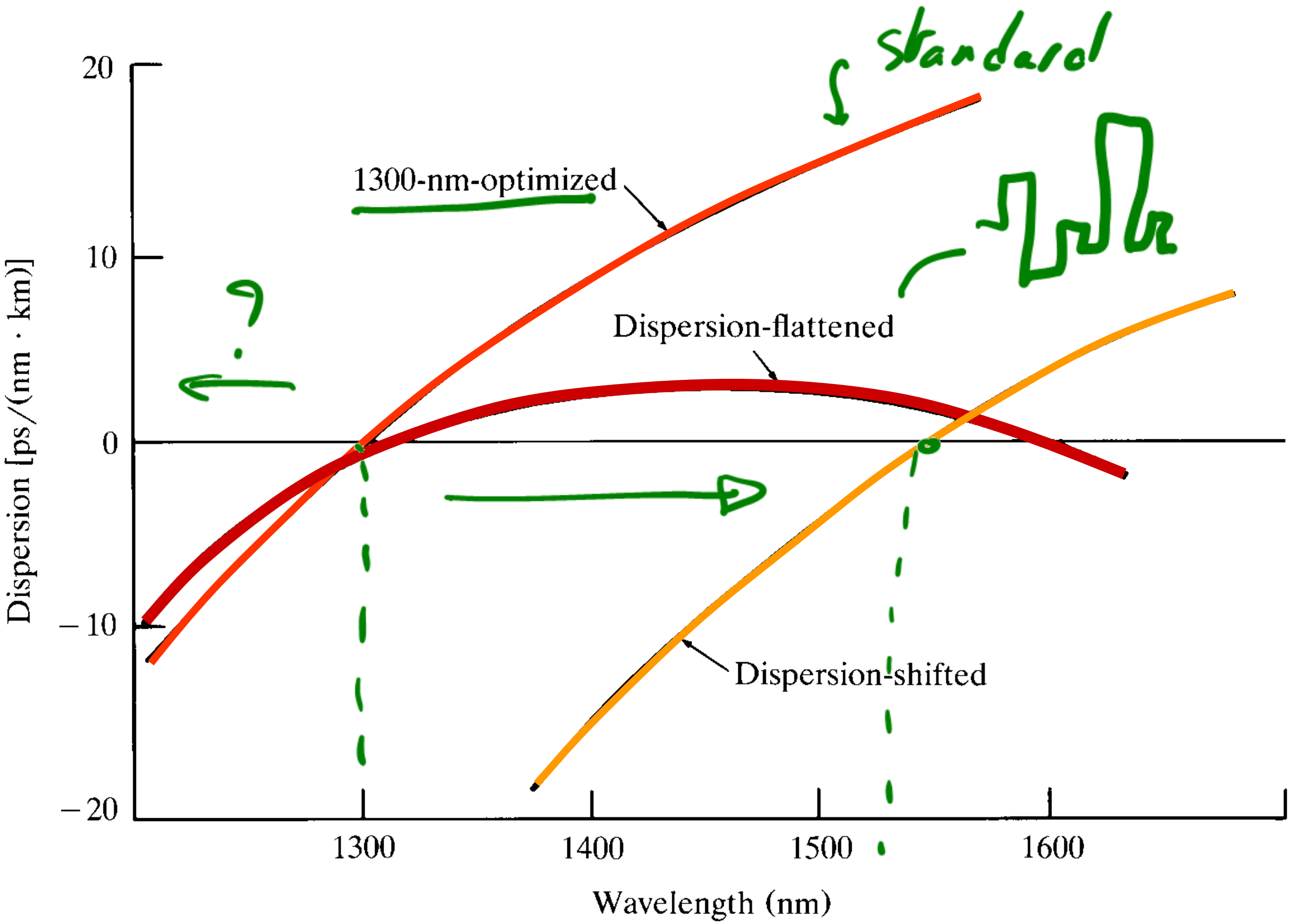
# DISPERSION SHIFTED FIBERS : A LARGE VARIETY OF INDEX PROFILES



# DISPERSION SHIFTED OR FLATTENED FIBERS



# DISPERSION SHIFTED OR FLATTENED FIBERS

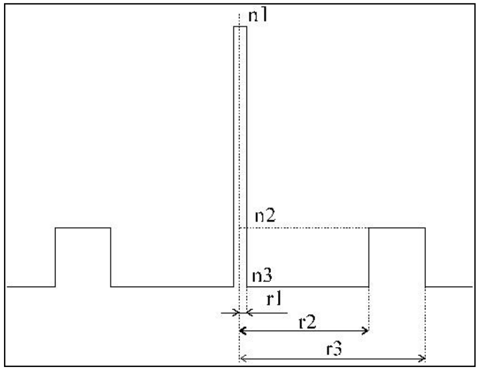


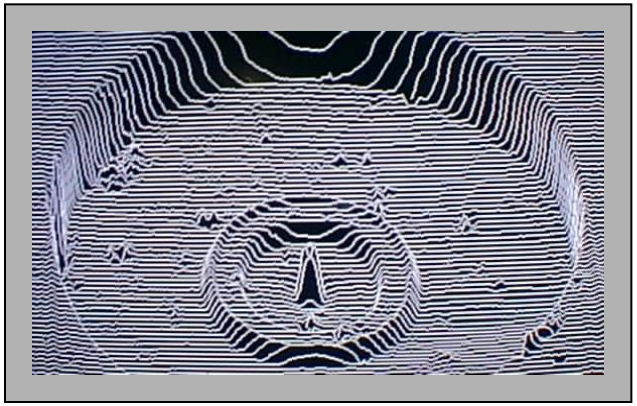

(*b*)

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

# OTHER KINDS OF FIBERS FOR THE MANAGEMENT OF THE DISPERSION (1)

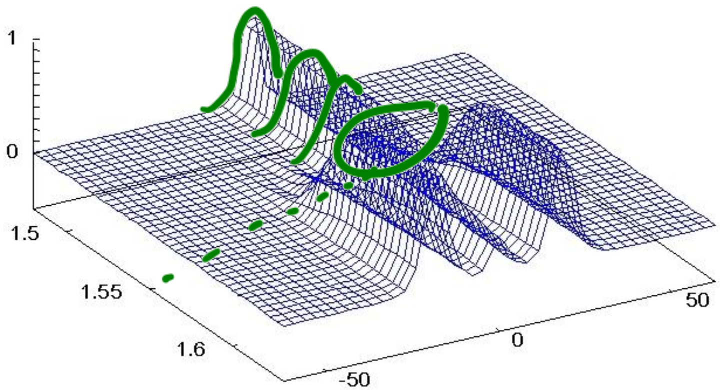
## * fibers with Dg << 0 (compensating fibers)



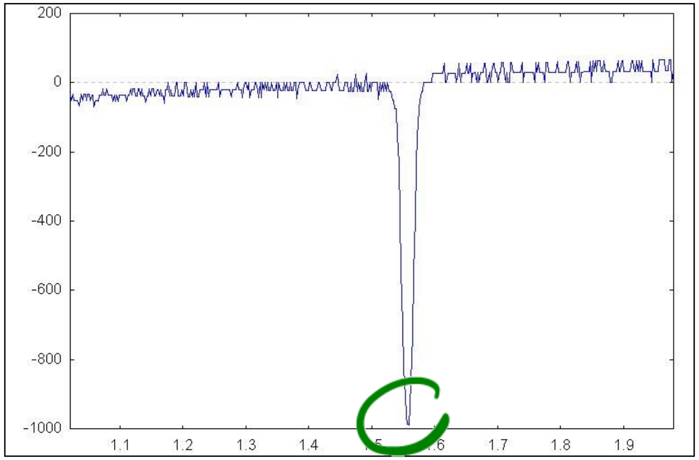



JL Auguste et al.
Optical fiber technology
Vol. 24, issue 1, pp. 442- (2006)

**index profile**




**Distribution of the field versus wavelength**

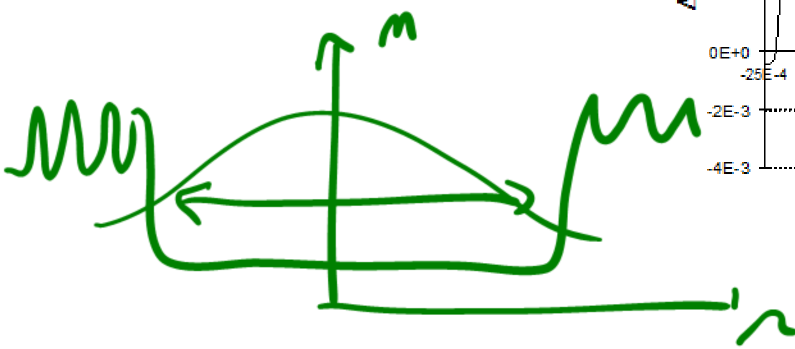


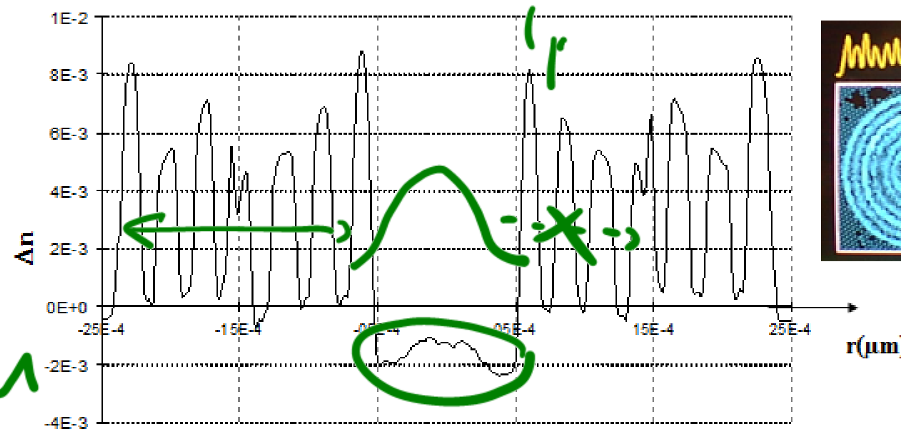

**measured chromatic dispersion**

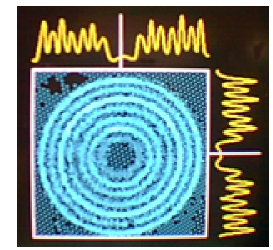
# OTHER KINDS OF FIBERS FOR THE MANAGEMENT OF THE DISPERSION (2)

## * fibers with Dg > 0 at short wavelengths

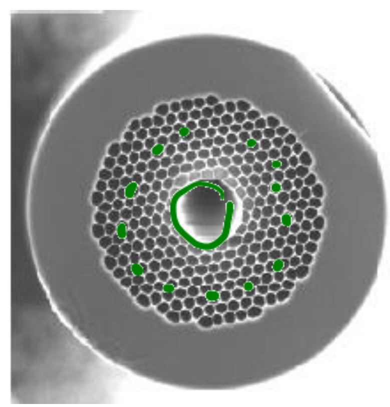
→ **Bragg Fibers**





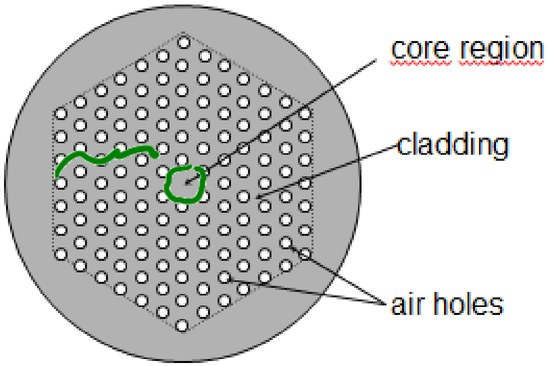
→ **Hollow core photonic crystal fibers**



# OTHER KINDS OF FIBERS FOR THE MANAGEMENT OF THE DISPERSION (3)

## * fibers with Dg specially managed for particular applications

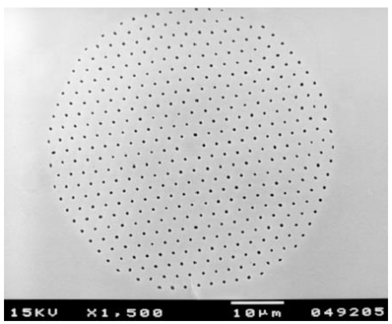
→ **Air silica microstructured optical fibers :**

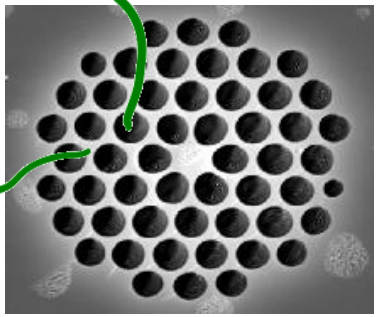


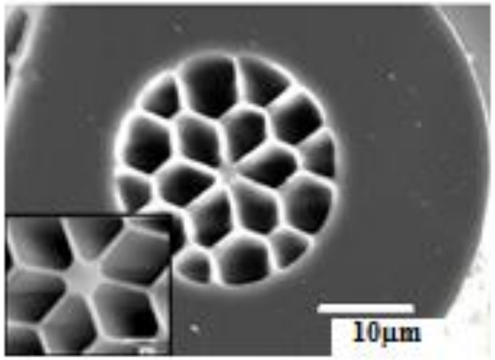

diameter of holes :d
pitch : $\Lambda$

air

silica






**d= 0,6µm $\Lambda$=2,6µm**

flattened dispersion
1100 nm – 1600 nm

**d= 1,9µm $\Lambda$=2,3µm**

Dc = 0 @ 1,06µm

**core diameter = 1,5µm $\Lambda$=2µm**

Dc = 0 @ 0,56µm

**End of chapter 4**

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Erasmus+ Programme
of the European Union