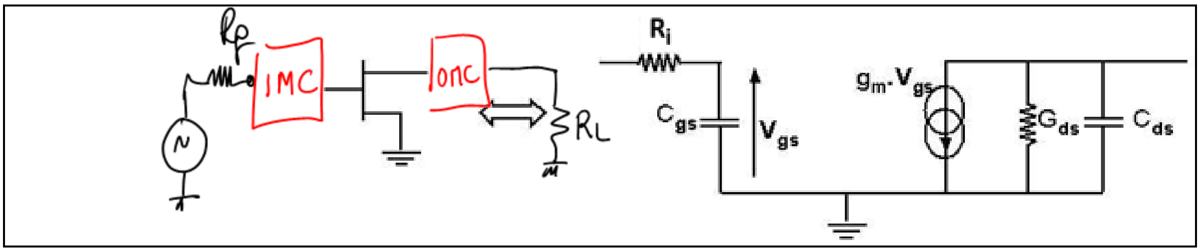
Basics of Active and Nonlinear HF Electronics – Tutorial 2

A) Power gain matching of a single-stage narrow-band amplifier

This figure represents the simplified electrical model of an FET.

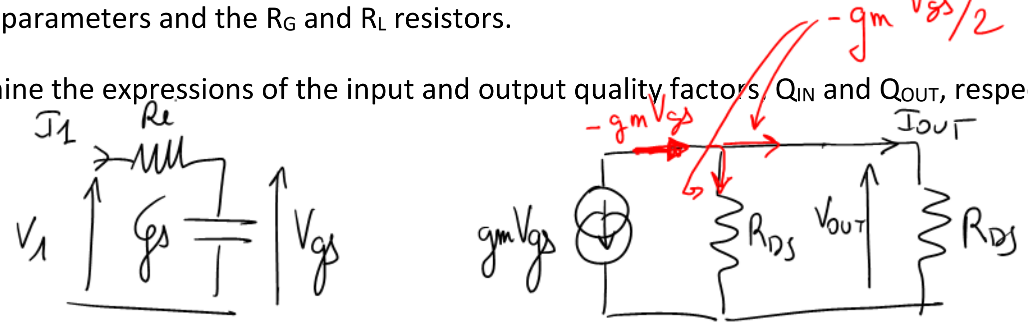


The model parameters are given at a bias point of (V_{GS0} = -2 V and V_{DS0} = 20 V)

Ri = 5 Ω ; C_{GS} = 0.5 pF ; g_m = 100 ms ; R_{DS} = 200 Ω ; C_{DS} = 0.15 pF

The amplifier environment is : R_G = 50 Ω ; R_L = 50 Ω ; f_0 = 10 GHz

1) Determine the expression of G_{MAX} : maximum power gain of the transistor as a function of frequency
2) We consider the design of a linear amplifier, which is loaded by a resistor R_L, while the input generator has an internal resistor R_G.
   a) Draw the ideal input and output matching circuits of a narrow-band amplifier which is matched to its maximum power gain G_{MAX} at f_0.
   b) Determine the expressions of each passive matching elements as a function of f_0, the model parameters and the R_G and R_L resistors.
3) Determine the expressions of the input and output quality factors, Q_{IN} and Q_{OUT}, respectively.



$$P_{IN} = \frac{1}{2}\mathrm{Re}\left(V_1 I_1^*\right)$$

$$= \frac{1}{2}\mathrm{Re}\left(\underbrace{\left(R_i - j\frac{1}{C_{gs}\omega}\right) I_1}_{V_1} \overbrace{I_1 I_1^*}^{|I_1|^2}\right) = \frac{1}{2} R_i |I_1|^2$$

$$I_1 = j C_{gs}\omega V_{gs} \rightarrow |I_1| = C_{gs}\,\omega |V_{gs}|$$

$$\hookrightarrow \boxed{P_{IN} = \frac{1}{2} R_i C_{gs}^2 \omega^2 |V_{gs}|^2}$$

$$\boxed{G_{MAX} = \frac{1}{4}\frac{R_{DS}\, g_m^2}{R_i C_{gs}^2 \omega^2}}$$

$$P_{OUT} = \frac{1}{2}\mathrm{Re}\left(V_{OUT} I_{OUT}^*\right)$$

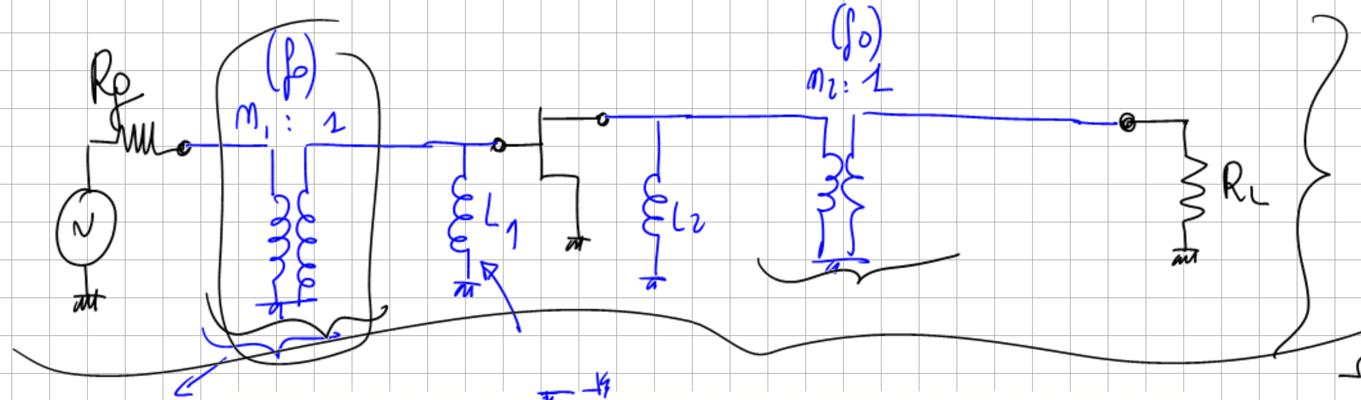
$$= \frac{1}{2}\mathrm{Re}\left(R_{DS} I_{OUT} I_{OUT}^*\right)$$

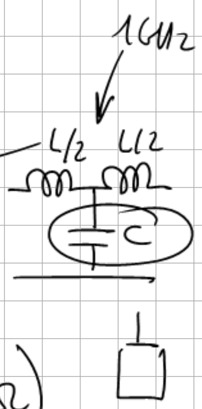
$$= \frac{1}{2} R_{DS} |I_{OUT}|^2$$

$$= \frac{1}{2} R_{DS} \left|-g_m \frac{V_{gs}}{2}\right|^2$$

max gain

$$\boxed{P_{OUT} = \frac{1}{8} R_{DS}\, g_m^2 |V_{gs}|^2}$$





$R_i = 5\,\Omega$; $C_{GS} = 0.5$ pF ; $g_m = 100$ ms ; $R_{DS} = 200\,\Omega$; $C_{DS} = 0.15$ pF

The amplifier environment is : $R_G = 50\,\Omega$; $R_L = 50\,\Omega$; $f_0 = 10$ GHz

1) • INPUT MATCHING

(R_i series or R_{IN} parallel ? to transform to $\underbrace{R_G = 50\,\Omega}$)

$$R_{IN}(10\,GHz) = \frac{1}{R_i C_{gs}^2 \omega^2} = \frac{1}{5\,(0{,}5\,10^{-12}\,2\pi\,10^{10})^2} = \frac{1}{5 \times \pi^2\,10^{-4}} = \frac{10000}{5\pi^2}$$
$$= 202\,\Omega$$

$$\frac{R_{IN}}{R_G} = \frac{202}{50} \simeq 4 \quad \text{and} \quad \frac{R_G}{R_i} = \frac{50}{5} = 10 \rightarrow \text{Best choice // matching} \quad \boxed{m_1 = \sqrt{\frac{202}{50}} = 2}$$

$$\boxed{L1 = \frac{1}{C_{gs}\,\omega^2} = 0{,}5\,nH}$$

• OUTPUT MATCHING

$$\boxed{L_2 = \frac{1}{C_{DS}\,\omega^2} = 1{,}7\,nH} \qquad \boxed{m_2 = \sqrt{\frac{R_{DS}}{R_L}} = \sqrt{\frac{200}{50}} = 2}$$

2) Quality factors

$$Q_S = \frac{1}{RC\omega_0}$$

$$Q_{//} = RC\omega_0$$

• INPUT

$$\boxed{Q_{IN}} = \frac{1}{R_i C_{gs} \omega_0} = \frac{1}{5\,(0{,}5\,10^{-12}\,2\pi\,10^{10})} = \frac{100}{5\pi} = \boxed{6{,}4}\,*$$

• OUTPUT

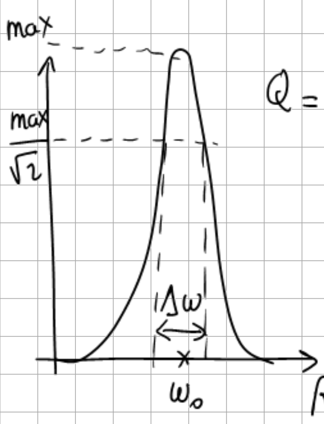
in our case (matching to R_{DS})

$$\boxed{Q_{OUT}} = R_{DS}\,C_{DS}\,\omega_0 = 200 \times 0{,}15\,10^{-12}\,2\pi\,10^{10} = \boxed{1{,}9}$$

or

$$= R_{opt}\,C_{DS}\,\omega_0$$

if you match to R_{opt}



$$Q = \frac{\omega_0}{\Delta\omega}$$

$$\Delta\omega = \frac{\omega_0}{Q}*$$

$$\Delta f_{max} \simeq \frac{f_0}{Q_{max}} = \frac{10\,GHz}{6{,}4} = 1{,}6\,GHz \qquad 2\,GHz$$

$R_{IN}(9\,GHz)$ $\qquad$ $R_{IN}(11\,GHz)$

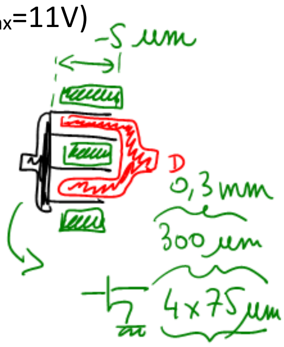
B) Power gain matching of a two stage narrow-band amplifier

Main pecifications of the narrow-band amplifier:

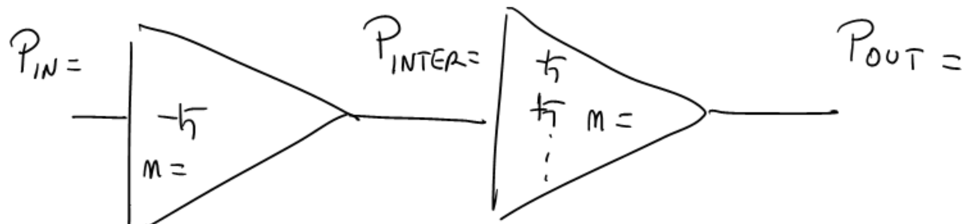
Output Power P_{OUT} > 1 W ; Center frequency f_0 = 15 GHz ; Power Gain G_P > 15 dB

Selected MMIC foundry (0.15 µm GaAs HEMT) :

Power Density PD = 1 W/mm ; Measured Gain $G_{MAX(@10GHz)}$ = 16 dB ;
Saturated Drain Current I_{DSS} = 800 mA/mm ; Drain Voltage range (V_{DSmin}=1V and V_{DSmax}=11V)
Pinch-off Voltage : V_P = -4 V
Optimum load impedance R_{OPT} = 12.5 Ω.mm
Small-signal model: C_{GS} = 2.7 pF/mm ; Ri =1 Ω.mm ; g_m = 200 mS/mm ;
R_{DS} = 125 Ω.mm ; C_{DS} = 0.6 pF/mm
Selected device size @ 15 GHz = 4x75 µm GaAs HEMT



1) Given the required specifications and the selected transistor technology, determine the initial sizing of the power amplifier (number of stages, number of amplifying cells per stage, power gain and input/output powers of each stage) using the selected device size (4x75 µm) as the unitary amplifying cell.
2) Given that only the last stage should be matched to its optimum output power while all the preceding stages should be matched to their maximum power gain, draw the input and output matching circuits of this two-stage narrow-band amplifier and calculate the values of each matching elements.
3) Calculate the input and output quality factors of each stage to assess what will be the bandwidth behavior.
4) Can you comment on the power limitations and the power added efficiency of this two-stage amplifier?



You have only to know how to find the Gmax value

$$G_p(R_{opt}) = \frac{\frac{G_{opt}}{(G_{opt}+G_{DS})^2}\, g_m^2}{R_i\, C_{GS}^2\, \omega^2} = \frac{G_{opt}}{(G_{opt}+G_{DS})^2} \times \frac{4\, G_{max}}{R_{DS}}$$

$$g_m^2 \rightarrow \frac{4\, G_{max}}{R_{DS}}$$

$$\boxed{G_{max} = \frac{1}{4}\, \frac{R_{DS}\, g_m^2}{R_i\, C_{GS}^2\, \omega^2}} \; * \; = \frac{1/R_{opt}}{\underbrace{(1/R_{opt} + 1/R_{DS})^2}_{\left(\frac{R_{opt}+R_{DS}}{R_{opt} R_{DS}}\right)^2}} \times \frac{4\, G_{max}}{R_{DS}}$$

$$G_P = \frac{\frac{G_2}{[G_2+g_{DS}]^2}\, g_m^2}{R_i\, C_{GS}^2\, \omega^2} \; *$$

$$\boxed{G_p(R_{opt}) = 4\, \frac{R_{opt}\, R_{DS}}{(R_{opt}+R_{DS})^2}\, G_{max}}$$

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Saturated Drain Current $I_{DSS} = 800$ mA/mm ; Drain Voltage range (V_{DSmin}=1V and V_{DSmax}=11V)

Pinch-off Voltage : $V_P = -4$ V

Optimum load impedance $R_{OPT} = 12.5$ Ω.mm

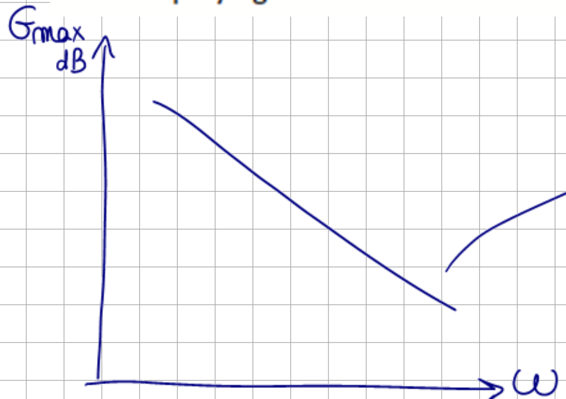
Small-signal model: $C_{GS} = 2.7$ pF/mm ; Ri =1 Ω.mm ; $g_m = 200$ mS/mm ;

$R_{DS} = 125$ Ω.mm ; $C_{DS} = 0.6$ pF/mm

Selected device size @ 15 GHz = 4x75 µm GaAs HEMT

1) Given the required specifications and the selected transistor technology, determine the initial sizing of the power amplifier (number of stages, number of amplifying cells per stage, power gain and input/output powers of each stage) using the selected device size (4x75 µm) as the unitary amplifying cell.

$$G_{max} = \underbrace{\frac{g_m^2 R_{DS}}{4 R_i C_{gs}^2}}\,\frac{1}{\omega^2}$$



$$G_{max} = \frac{K}{\omega^2}$$

$$\begin{cases} G_{max}(f_1) = \dfrac{K}{\omega_1^2} \\ G_{max}(f_2) = \dfrac{K}{\omega_2^2} \end{cases}$$

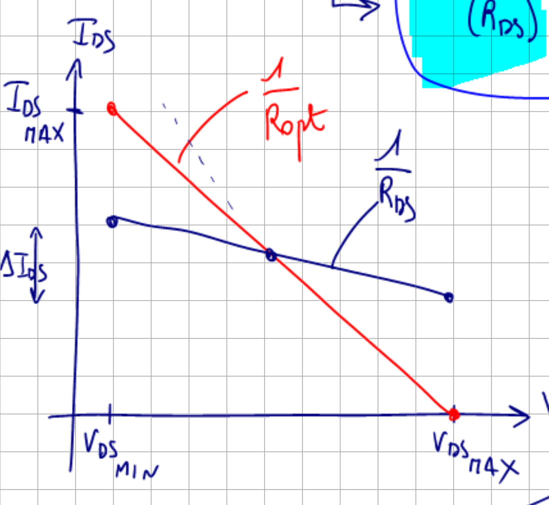
$$\boxed{G_{max}(f_1) = G_{max}(f_2) \times \left(\frac{f_2}{f_1}\right)^2}$$

In our case : $G_{max}(15G) = G_{max}(10G) \times \left(\dfrac{10}{15}\right)^2 = 39{,}8 \times \left(\dfrac{10}{15}\right)^2 = 17{,}7 = 12{,}5\,dB$

$G_{max}(10G) = 16\,dB = 10^{1,6} = 39{,}8$

$\boxed{1^{st}\text{ case}}$ = Optimize a stage to get $G_{max} = 12{,}5\,dB = 17{,}7$ and R_{DS} load

$$\boxed{PD_{(R_{DS})} = PD_{(R_{opt})} \times \frac{R_{opt}}{R_{DS}} = (1\,W/mm) \times \frac{12{,}5}{125} = 0{,}1\,W/mm}$$



$$P_{OUT} = \frac{1}{2}\,\frac{|V_{DS\,MAX} - V_{DS\,MIN}|}{2} \times \frac{\Delta I_{DS}}{2} = \frac{1}{2}\,\frac{\Delta V_{DS}}{2}\,\frac{\Delta V_{DS}/R}{2} = \frac{1}{8}\,\frac{\Delta V_{DS}^2}{R}$$

$$\begin{cases} P_{OUT}(R_{opt}) = \dfrac{1}{8}\,\dfrac{|V_{DS\,MAX} - V_{DS\,MIN}|^2}{R_{opt}} \\ P_{OUT}(R_{DS}) = \dfrac{1}{8}\,\dfrac{|V_{DS\,MAX} - V_{DS\,MIN}|^2}{R_{DS}} \end{cases}$$

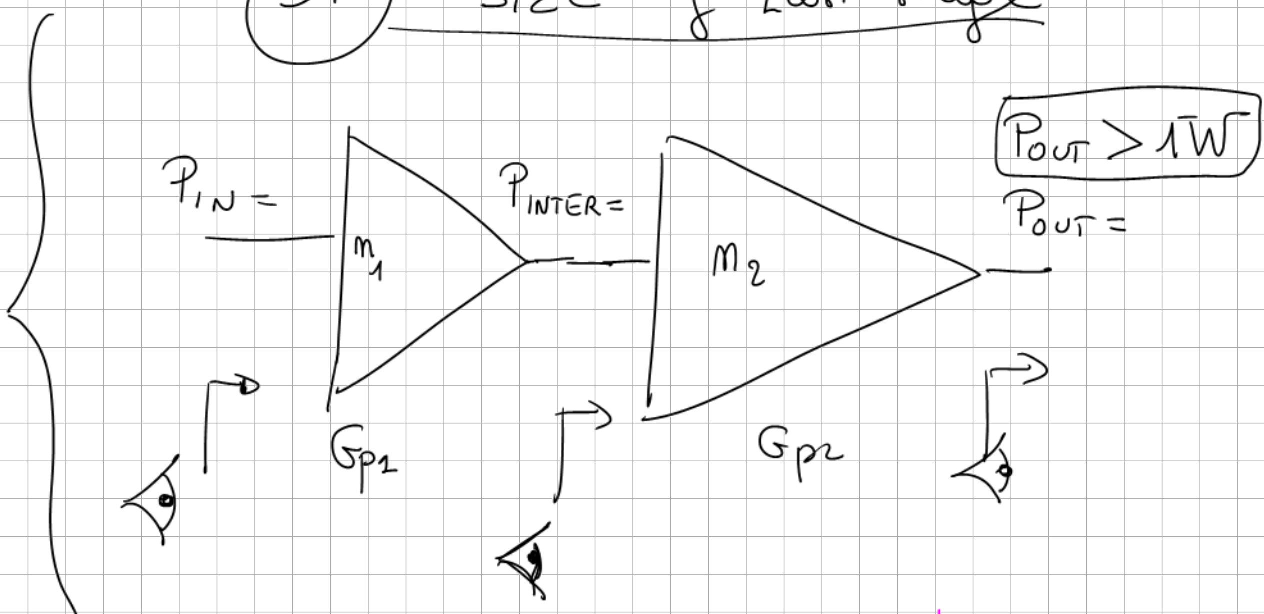
$$\boxed{PD_{(R_{DS})} = PD_{(R_{opt})} \times \frac{R_{opt}}{R_{DS}}}$$

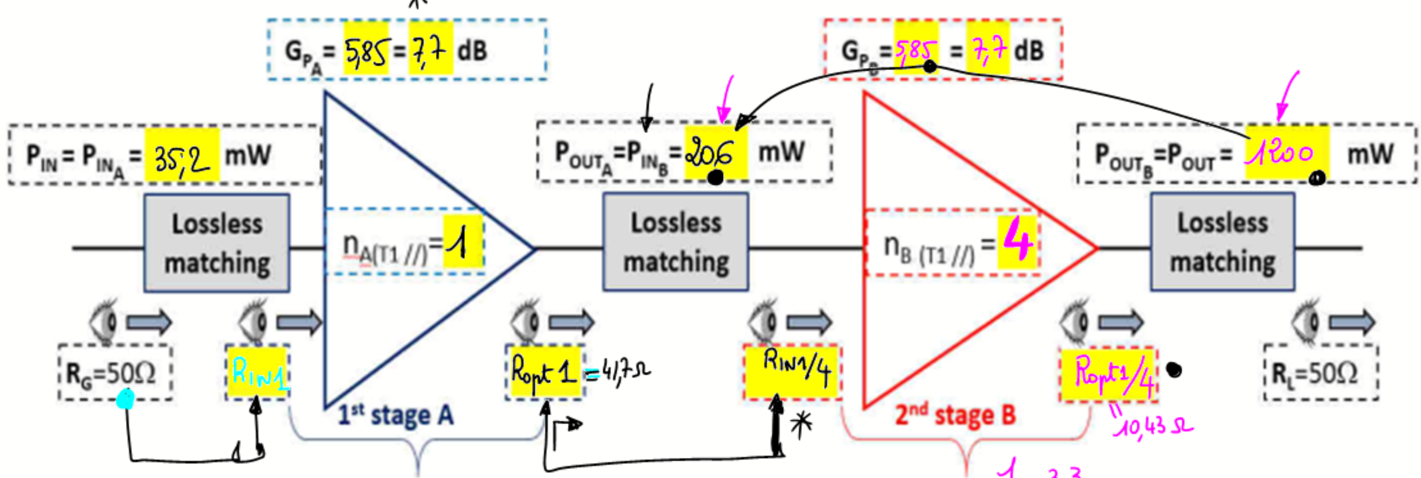
$\boxed{2^{nd}\text{ case}}$ Optimize a stage to get $P_{OUT_{MAX}} \Rightarrow PD(R_{opt}) = 1\,W/mm$ and R_{opt} load

$$\boxed{G_p(R_{opt}) = 4\,\frac{R_{opt}R_{DS}}{(R_{opt}+R_{DS})^2}\,G_{max}} = 4 \times \frac{12.5 \times 125}{(12{,}5+125)^2} \times 17{,}7$$

$$\boxed{G_p(R_{opt}) = 5{,}85 = 7{,}7\,dB}$$

1 Size of Last stage





$P_{OUT} > \overset{2W}{1W}$

$PD(R_{opt}) = 1\,W/mm$

$PD(R_{DS}) = 0{,}1\,W/mm$

$W_T(R_{opt}) = \overset{2\,mm}{1\,mm} \rightarrow$ I need $4 \times 0{,}3\,mm = 1.2\,mm$

$\frac{1}{0{,}3} = 3{,}3\ldots$

$W_T(R_{DS}) = \underbrace{\overset{20\,mm}{10\,mm}}_{\text{each device is }0{,}3\,mm} \rightarrow$ I need $\underbrace{34}_{INT\left(\frac{10}{0{,}3}\right)} \times 0{,}3\,mm = 10{,}2\,mm$

$R_{opt} = 12,5\,\Omega.mm \longrightarrow R_{opt_1}(0,3\,mm) = \frac{12,5}{0,3} = 41,7\,\Omega$

$$\boxed{\underbrace{\frac{R_{opt1}}{4}}_{R_{opt}(1,2mm)} = \frac{41,7}{4} = \frac{12,5}{1,2} = 10,43\,\Omega}$$

$$\boxed{P_{OUT_B} = 1,2\,mm \times PD(R_{opt}) = 1,2\,mm \times 1\,W/mm = 1,2\,W}$$

$$P_{IN_B} = \frac{P_{OUT_B}}{G_p(R_{opt})} = \frac{1,2\,W}{5,85} = 0,206\,W = \boxed{206\,mW}$$

1st stage   to reach 206 mW $\Longrightarrow$ loading by $R_{opt} \rightarrow PD = 1W/mm \rightarrow$ I need $W_T = \left(\frac{0,206\,W}{1\,W/mm}\right) = 0,206\,mm$

↳ I need only one device of 0,3 mm

transistors in // are 2^p $\begin{cases} p=0 & 1\text{ device} \\ p=1 & 2 \\ p=2 & 4 \\ & \vdots \end{cases}$

~~if the load is $R_{DS} \rightarrow PD = 0,1W/mm \rightarrow$ I need $W_T = \left(\frac{0,206W}{0,1/mm}\right) = 2,06\,mm$~~

~~I need $\frac{2,06\,mm}{0,3\,mm} \Rightarrow 7$ transistors~~

~~$\Rightarrow 8$ transistors $= 2^3$~~

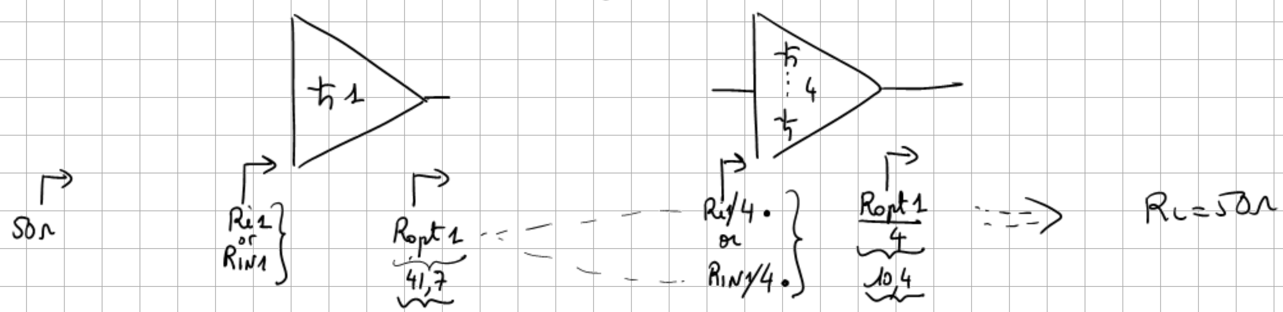
not efficient

$$P_{INA} = \frac{P_{OUTA}}{G_p(R_{opt})} = \frac{206\,mW}{5,85} = 35,2\,mW$$

• To determine the input matching and interstage matching $\rightarrow$ Calculate R_{IN}

$$R_{IN} = \frac{1}{R_i\, C_{gs}^2\, \omega^2} \longrightarrow R_{IN}(15G) = \frac{1}{1 \times (2,7\,10^{-12} \times 2\pi \times 15\,10^9)^2} = 15,4\,\Omega.mm$$

$$0,3\,mm \Rightarrow \begin{cases} R_{IN_1} = \frac{15,4\,\Omega}{0,3} = 51,5\,\Omega \\ \end{cases} \qquad R_{i_1} = \frac{1}{0,3} = 3,33\,\Omega$$



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Optimum load impedance $R_{OPT} = 12.5$ Ω.mm

Small-signal model: $C_{GS} = 2.7$ pF/mm ; Ri =1 Ω.mm ; $g_m = 200$ mS/mm ;

$R_{DS} = 125$ Ω.mm ; $C_{DS} = 0.6$ pF/mm

Selected device size @ 15 GHz = 4x75 µm GaAs HEMT

First calculations:

a) Scaled parameters for the unit size of $\boxed{4x75\,\mu m = 0{,}3\,mm}$

- $C_{GS1} = 2{,}7\,pF \times 0{,}3 = 0{,}81\,pF$ $\quad Ri_1 = \frac{1}{0{,}3} = 3{,}33\,\Omega$ $\quad Ropt_1 = \frac{12{,}5}{0{,}3} = 41{,}7\,\Omega$
- $C_{DS1} = 0{,}6\,pF \times 0{,}3 = 0{,}18\,pF$ $\quad R_{DS1} = \frac{125}{0{,}3} = 417\,\Omega$

$$R_{IN1} = \frac{1}{R_{i_1} C_{gs_1}^2 \omega^2} = \frac{1}{3{,}33 \times (0{,}81\,10^{-12} \times 2\pi\,15\,10^9)^2}$$

$\rightarrow R_{IN1} = 51{,}5\,\Omega$ *

b) Power gain for optimum Power matching: $G_p(Ropt)$ @ 15 GHz

* $G_{max}(10\,GHz) = 16\,dB = 10^{1{,}6} = 39{,}8 \longrightarrow G_{max}(15G) = G_{max}(10G) \times \left(\frac{10}{15}\right)^2 = 39{,}8 \times \left(\frac{10}{15}\right)^2 = 17{,}7 = 12{,}5\,dB$

given → $\boxed{G_p(Ropt) = 4\,\frac{R_{DS}\,Ropt}{(R_{DS} + Ropt)^2}\,G_{max}}$ $\rightarrow G_p(Ropt_{15G}) = 4 \times \frac{125 \times 12{,}5}{(125 + 12{,}5)^2} \times 17{,}7 = 5{,}85 = 7{,}7\,dB$

c) Power density for maximum gain matching $PD(R_{DS})$

$$PD(R_{DS}) = PD(Ropt) \times \frac{Ropt}{R_{DS}} = (1\,W/mm) \times \frac{12{,}5\,\Omega.mm}{125\,\Omega.mm} \Rightarrow PD(R_{DS}) = 0{,}1\,W/mm$$

$PD(Ropt) = 1\,W/mm$

$(P_{OUT} > 1W)$

Power specifications → 2nd stage dimensions

STAGE B • 2nd stage loaded by Ropt ⇒ $PD(Ropt) = 1\,W/mm$, $P_{OUT} > 1W$ → $Size_B > \frac{1W}{1W/mm} = 1\,mm$

Possible sizes:
- N = 1 → 0,3 mm
- N = 2 → 0,6 mm
- N = 4 → 1,2 mm
- N = 8 → 2,4 mm
- N = 16 → 4,8 mm

→ $m_B = 4$

→ $P_{OUT_B} = (4 \times 0{,}3\,mm) \times 1\,W/mm = 1{,}2\,W$

→ $G_{P_B} = G_p(Ropt) = 5{,}85 = 7{,}7\,dB$

→ $P_{IN_B} = P_{OUT_A} = \frac{P_{OUT_B}}{G_{P_B}} = \frac{1{,}2\,W}{5{,}85} = 0{,}205\,W$

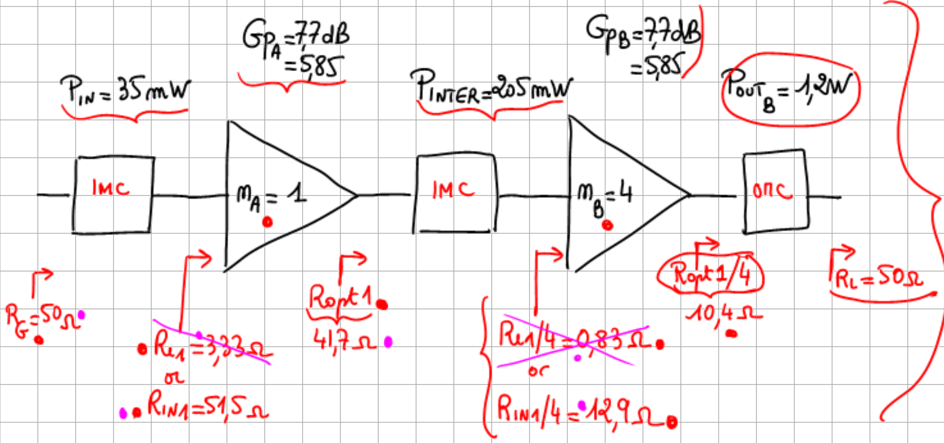
STAGE A • 1st stage loaded by Ropt ⇒ $PD(Ropt) = 1\,W/mm$, $P_{OUT} > 0{,}205\,W$ → $Size_A > \frac{0{,}205\,W}{1\,W/mm} = 0{,}205\,mm$

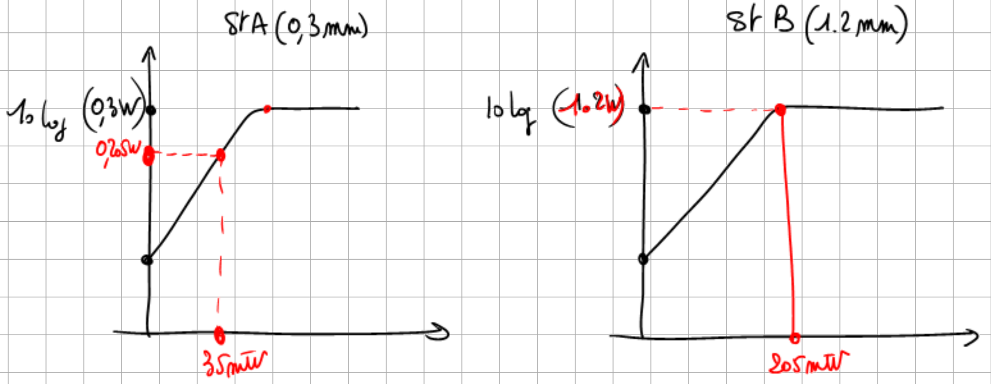
$m_A = 1$

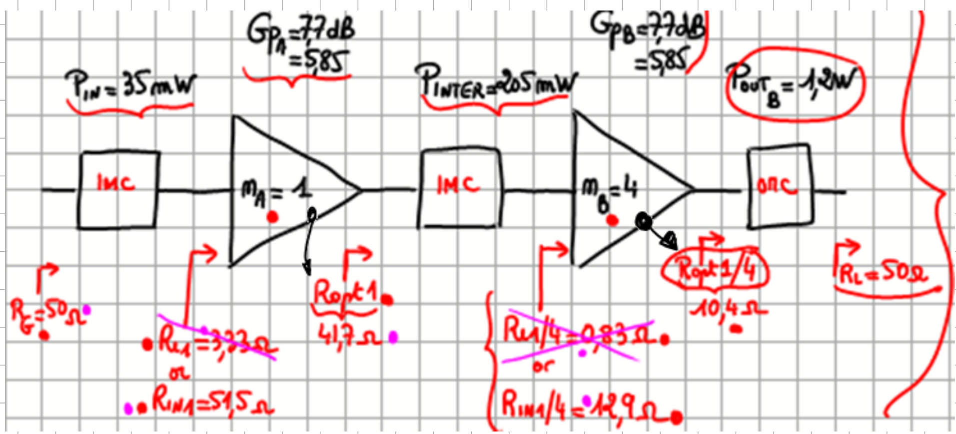
→ $P_{OUT_A} = 0{,}205\,W$ ~~≠ 0,3W~~

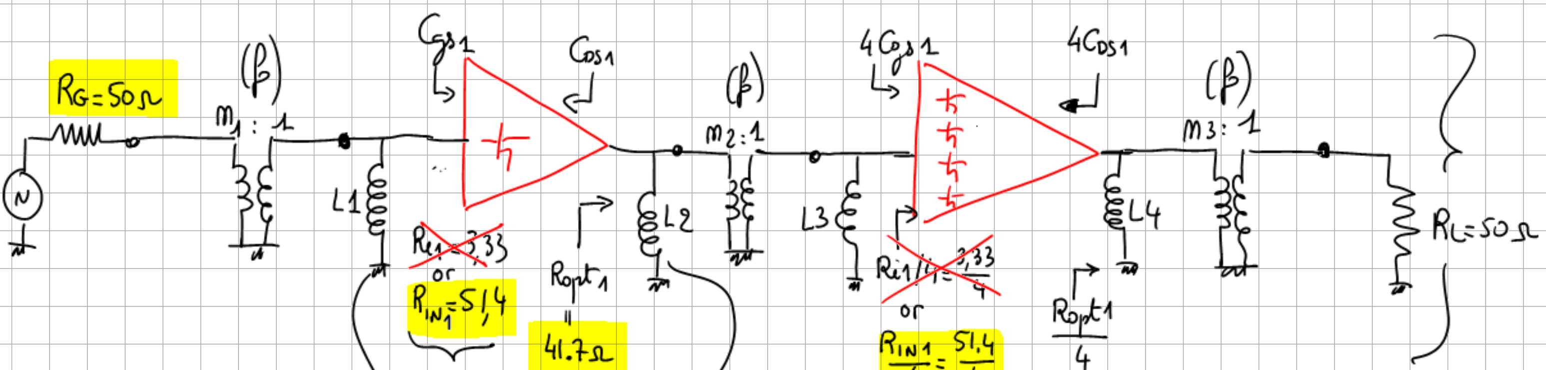
→ $G_{P_A} = G_p(Ropt) = 5{,}85 = 7{,}7\,dB$

→ $P_{IN_A} = \frac{P_{OUT_A}}{G_{P_A}} = \frac{0{,}205}{5{,}85} = 35\,mW$









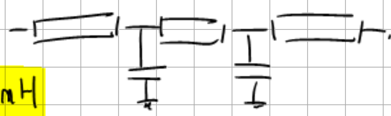
$$m_1 = \sqrt{\frac{R_G}{R_{IN1}}} = \sqrt{\frac{50}{51,4}} \simeq 1$$

$$L_2 = \frac{1}{C_{DS1}\,\omega_0^2} = \frac{1}{0,18\ 10^{-12}(2\pi 15\, 10^9)^2} = 0,62\ mH$$

$$L1 = \frac{1}{C_{gs1}\,\omega_0^2} = \frac{1}{0.81\ 10^{-12}\,(2\pi\ 15\ 10^9)^2} = 0,14\ nH$$

$$m_2 = \sqrt{\frac{R_{opt1}}{\left(\frac{R_{IN1}}{4}\right)}} = \sqrt{\frac{41.7}{\frac{51.4}{4}}} = 1.8$$

$$\rightarrow\quad L_3 = \frac{1}{(4C_{gs1})\,\omega_0^2} = \frac{L1}{4} = 0,035\ nH$$



$$L_4 = \frac{1}{(4C_{DS1})\,\omega_0^2} = \frac{L2}{4} = 0,156\ mH$$

$$m_3 = \sqrt{\frac{\frac{R_{opt1}}{4}}{R_L}} = \sqrt{\frac{\left(\frac{41.7}{4}\right)}{50}} = 0.46$$

3) Q doesnot depend on size

maximum value → Bandwidth

$$Q_{IN} = \frac{1}{R_{i_1} C_{gs_1} \omega_0} = R_{IN1} C_{gs_1} \omega_0 = 51{,}5 \times 0{,}81\,10^{-12} \times 2\pi\, 15\, 10^{9}$$

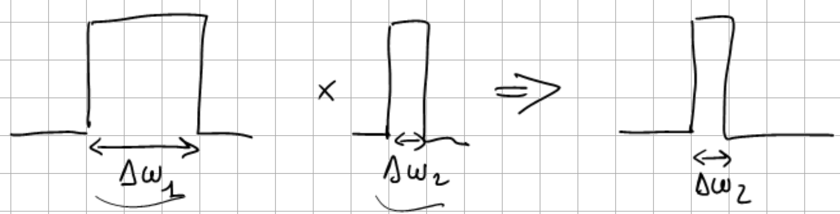
$$\boxed{Q_{IN}(15\,GHz) = 3{,}9}$$

$$* \quad Q_{OUT}(R_{opt_1}, 15\,GHz) = \underline{R_{opt_1}} \times C_{DS1} \times \omega_0 = 41.7 \times 0{,}18\,10^{-12} \times 2\pi\, 15\,10^{9} = 0{,}7$$

$$\rightarrow \quad Q_{OUT}(R_{DS1}, 15\,GHz) = \underline{R_{DS1}} \times C_{DS1} \times \omega_0 = 417 \times 0{,}18\,10^{-12} \times 2\pi\, 15\,10^{9} = 7$$

$$Q = \frac{\omega_0}{\Delta\omega} \rightarrow \Delta f = \frac{f_0}{Q}$$

The limiting factor of the maximum achievable bandwidth is Q_{max}



in our case stage 1 → Ropt, stage 2 → Ropt

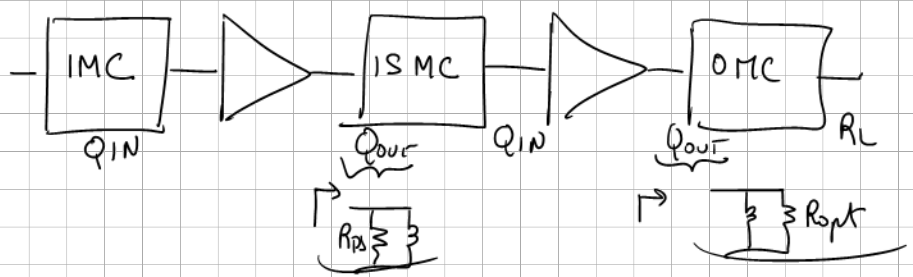
$$\Longrightarrow \quad Q_{IN} = 3{,}9 \leftarrow \text{limiting factor } Q_{max}, \qquad Q_{OUT}(R_{opt}) = 0{,}7$$

$$\Delta f_{max} < \frac{f_0}{Q_{MAX}} = \frac{15G}{3{,}9} = 3{,}8\,GHz$$

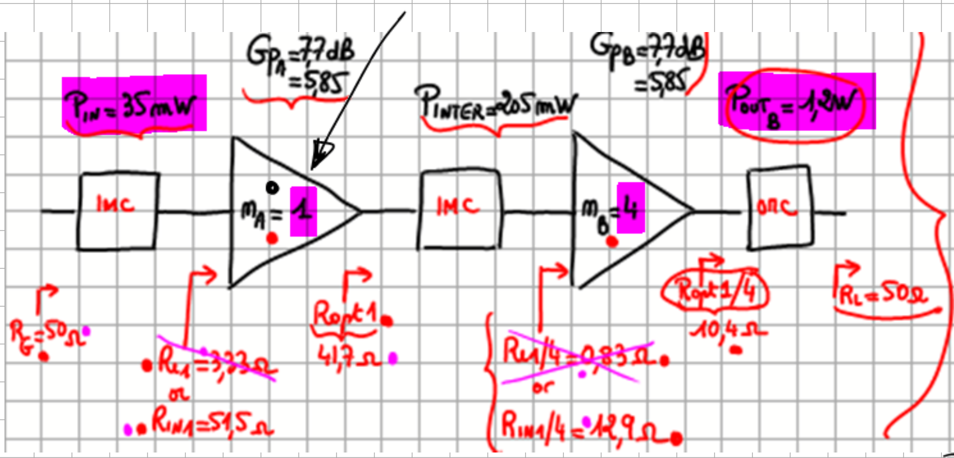
was not asked

in another case stage 1 → R_{DS}, stage 2 → R_{opt}

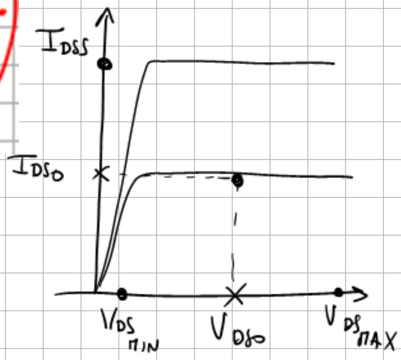
$$\Longrightarrow \begin{cases} Q_{IN} = 3{,}9 \\ Q_{OUT}(R_{opt}) = 0{,}7 \\ Q_{OUT}(R_{DS}) = 7 \end{cases} \Longrightarrow \Delta f_{max} < \frac{f_0}{Q_{MAX}} = \frac{15G}{7} = 2{,}1\,GHz$$



4)



Class A



$$V_{DS0} = \frac{V_{DS_{MIN}} + V_{DM_{MAX}}}{2} = \frac{1 + 11}{2} = 6\,V$$

$$I_{DS0_1} = \frac{I_{DS\,MAX\,1}}{2} = \frac{(800\,mA/mm) \times (0{,}3\,mm)}{2} = 120\,mA$$ ← for one transistor of 0,3mm size

$$P_{DC_{HPA}} = (4 + 1) \times V_{DS0} \times I_{DS01} = 5 \times 6 \times 120\,mA = 3{,}6\,W$$

$P_{IN} = 35\,mW$ $\qquad$ $P_{OUT} = 1{,}2\,W$

$Ropt_A = Ropt_B$

$$PAE = \frac{P_{OUT} - P_{IN}}{P_{DC}} = \frac{1{,}2 - 0{,}035}{3{,}6} = 32\%$$