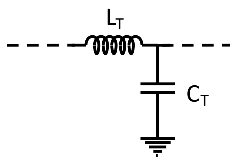
3. Lumped elements with transmission lines

3.1. Short transmission lines

A transmission line TEM consists of a series inductance per unit length, L_T and a parallel capacitance per unit length C_T.

The wave velocity is given by the following relation $v_p = \frac{c}{\sqrt{\varepsilon_{eff}}}$



Where c is the speed of light and ε_{eff} is the effective permittivity.

$$v_p = \frac{1}{\sqrt{L_T C_T}}$$

$$\frac{c}{\sqrt{(\varepsilon_{eff})}} = \frac{1}{\sqrt{L_T C_T}}$$

this leads to

$$C_T = \frac{\varepsilon_{eff}}{c^2 L_T}$$

Z_0 , the characteristic impedance of the line is:

$$Z_o = \sqrt{\frac{L_T}{C_T}}$$

We also derive:

$$L_T = Z_0^2 C_T$$

This expression can be placed in the C_T equation:

$$C_T = \frac{\varepsilon_{eff}}{c^2 Z_O^2 C_T}$$

$$C_T = \frac{\sqrt{\varepsilon_{eff}}}{cZ_O}\ F/m$$

$$C_T = \frac{1}{v_p Z_0}\ F/m$$

It's a capacitance per unit length

If Z_0 is small then C_T is large

One can show similarly that:

$$L_T = \frac{\sqrt{\varepsilon_{eff}}Z_0}{c} H/m$$

$$L_T = \frac{Z_0}{v_p}\ H/m$$

We can find the value of the inductance L for the physical length l of the transmission line (in Farads/m and Henry/m), but also from the electrical length E.

$$E = \frac{\omega l}{v_p}$$

with

$$L = L_T l = \frac{l}{v_p} Z_0 = \frac{E_L Z_0}{\omega}$$

$$E_L = \frac{L\omega}{Z_0}$$

Idem for C

$$C = C_T l = \frac{l}{Z_0 v_p} = \frac{E_C}{\omega Z_0}$$

$$E_C = Z_0 C\omega$$

From these simple formulas, it is possible to synthetize small inductive/capacitive series sections using integrated transmission lines.

3.2. Stubs as lumped elements

Transmission lines stubs can be used as lumped elements substitutes, with appropriate length. For instance, $\lambda/8$ stubs can serve as inductors or capacitors.

Short-circuited stubs as inductors:

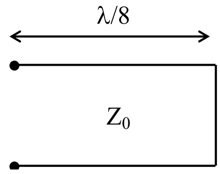


Figure 16 - Short circuited stub

The input impedance of this stub is:

$$Z_R = \frac{Z_0(0 + jZ_0\tan(\beta l))}{Z_0 + j0\tan(\beta l)} = jZ_0\tan(\beta l)$$

By using the Richards transformation, we substitute $tan\beta l = \Omega$, and we have:

$$Z_R = jZ_0\Omega$$

A $\lambda/8$ short-circuited stub is equivalent to an inductor so that $jL\omega = jZ_0$ around its center frequency.

Open-circuited stubs as capacitors:

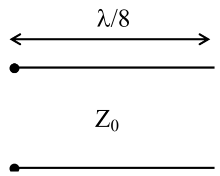


Figure 17 - Open circuit stub

The input impedance of this stub is:

$$Z_R = \frac{Z_0(\infty + jZ_0\tan(\beta l))}{Z_0 + j\infty\tan(\beta l)} = \frac{Z_0}{\mathrm{j}\tan(\beta \mathrm{l})}$$

By using the Richards transformation, we substitute $tan\beta l = \Omega$, and $\frac{1}{Z_0} = Y_0$ we have:

$$Y_R = jY_0\Omega$$

A $\lambda/8$ open-circuited stub is equivalent to an capacitor so that $jC\omega = jY_0$ around its center frequency.