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Lesson 2
Sept 10th 2020

Electromagnetism reminder

I Electrostatic

$$\overrightarrow{curl}\,\vec{E}(x,y,z) = \overrightarrow{rot}\,\vec{E}(x,y,z) = 0$$

$$\overrightarrow{curl}\,\overrightarrow{grad}\,V = 0$$

$$\vec{E}(x,y,z) = -\overrightarrow{grad}\,V(x,y,z)$$

$$\mathrm{div}\,\vec{D}(x,y,z) = \rho(x,y,z)$$

$$\vec{D}(x,y,z) = \varepsilon\,\vec{E}(x,y,z)$$

$$\vec{\nabla}\begin{vmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{vmatrix}$$

Nabla

$$\overrightarrow{curl}\,\vec{E}(x,y,z) = \vec{\nabla}\times\vec{E}(x,y,z) = \vec{\nabla}\wedge\vec{E}(x,y,z) = 0$$

$$\vec{\nabla}\cdot\vec{E}(x,y,z) = \frac{\rho(x,y,z)}{\varepsilon}$$

$$\vec{E}(x,y,z) = -\vec{\nabla}V(x,y,z)$$

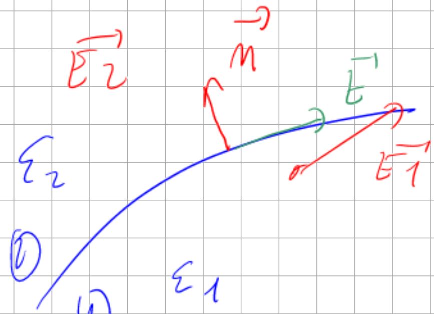
$$-\vec{\nabla}\cdot\vec{\nabla}V(x,y,z) = \frac{\rho(x,y,z)}{\varepsilon}$$

Δ Delta

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\Delta V(x,y,z) + \frac{\rho(x,y,z)}{\varepsilon} = 0$$

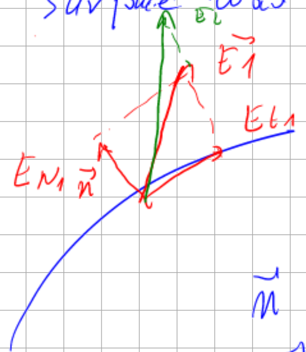
eq POISSON



$$\vec{n}\times(\vec{E_2}-\vec{E_1}) = 0 \quad (1)$$

$$\vec{n}\cdot(\vec{D_2}-\vec{D_1}) = \sigma_s$$

σ_s surface load density



$$\vec{E_1} = E_{N1}\,\vec{n} + E_{t1}\,\vec{t}$$

$$(1)\quad \vec{n}\times\left([E_{t2}-E_{t1}]\,\vec{t} + (E_{N2}-E_{N1})\,\vec{n}\right) = 0$$

$$\vec{n}\times\vec{t} = \vec{u}$$

$$\vec{n}\times\vec{n} = 0$$

$E_{t1} = E_{t2}$

$\vec{n} \cdot \left[(D_{t2} - D_{t1})\vec{t} + (D_{n2} - D_{n1})\vec{n} \right] = \sigma_S$

$\vec{n} \cdot \vec{n} = 1 \qquad \vec{n} \cdot \vec{t} = 0$

$D_{n2} - D_{n1} = \sigma_S \neq 0$

Between 2 dielectric materials, $\sigma_S = 0$

$D_{n2} = D_{n1} \qquad \varepsilon_2 E_{n2} = \varepsilon_1 E_{n1}$

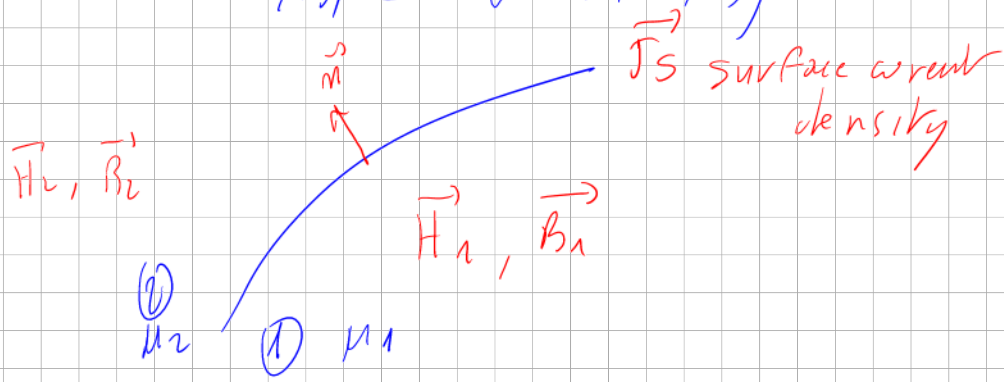
II Magnetostatic

$\vec{\mathrm{curl}}\ \vec{H}(x,y,z) = \vec{J}(x,y,z)$

$\mathrm{div}\ \vec{B}(x,y,z) = 0$

$\vec{B}(x,y,z) = \mu\ \vec{H}(x,y,z)$

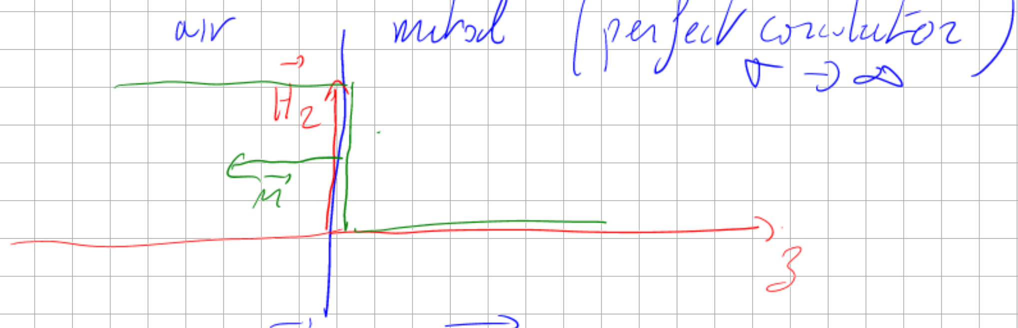
$\vec{B}(x,y,z) = \mathrm{curl}\ \vec{A}(x,y,z)$



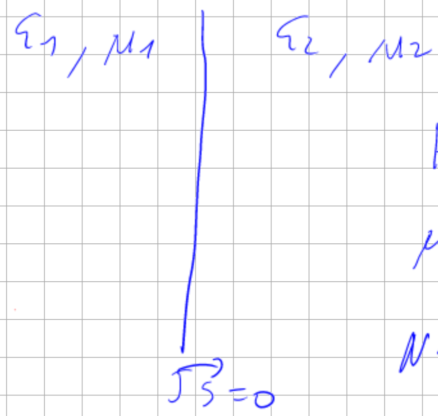
$\vec{n} \times (\vec{H_2} - \vec{H_1}) = \vec{J_S}$ (1)

$\vec{n} \cdot (\vec{B_2} - \vec{B_1}) = 0$ (2)

(2) $\Rightarrow B_{n2} = B_{n1}$



$\vec{n} \times \vec{H_2} = \vec{J_S}$

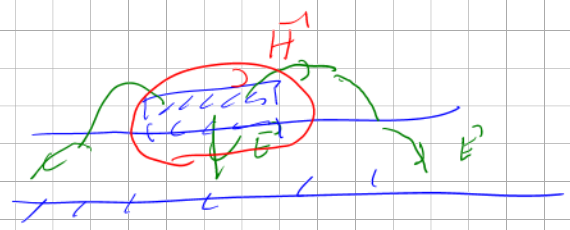


$H_{t2} = H_{t1}$

$\mu_2 H_{n2} = \mu H_{n1}$

$\mu_1 = \mu_2 \qquad H_{n2} = H_{n1}$

$\vec{H_2} = \vec{H_1}$



III Electromagnetism

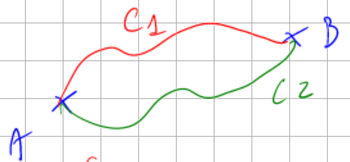
$$\vec{\nabla} \times \vec{e}(x,y,z,t) = -\frac{\partial \vec{b}(x,y,z,t)}{\partial t}$$

$$\vec{\nabla} \times \vec{h}(x,y,z,t) = \vec{j}(x,y,z,t) + \frac{\partial \vec{d}(x,y,z,t)}{\partial t}$$

$$\vec{\nabla} \cdot \vec{b}(x,y,z,t) = 0$$

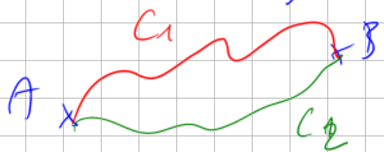
$$\vec{\nabla} \cdot \vec{d}(x,y,z,t) = \rho(x,y,z,t)$$

Electrostatic



$$-\int_{C_1} \vec{E} \cdot d\vec{C_1} = -\int \vec{E} \cdot d\vec{C_2}$$

Electromagnetism



$$-\int_{C_1} \vec{E} \cdot dC_1 \neq -\int_{C_2} \vec{E} \cdot dC_2$$

harmonic domain generator sin function

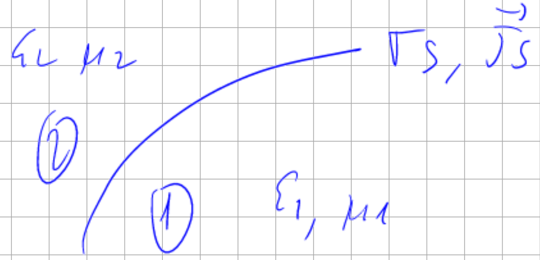
$$\vec{e}(x,y,z,t) = \mathrm{Re}\left[\vec{E}(x,y,z)\, e^{j\omega t}\right]$$

$$\vec{\nabla} \times \vec{E}(x,y,z) = -j\omega\mu\, \vec{H}(x,y,z)$$

$$\vec{\nabla} \times \vec{H}(x,y,z) = \vec{J}(x,y,z) + j\omega\varepsilon\, \vec{E}(x,y,z)$$

$$\vec{\nabla} \cdot \vec{H}(x,y,z) = 0$$

$$\vec{\nabla} \cdot \vec{E}(x,y,z) = \frac{\rho(x,y,z)}{\varepsilon}$$



$$\vec{n} \times (\vec{E_2} - \vec{E_1}) = 0$$

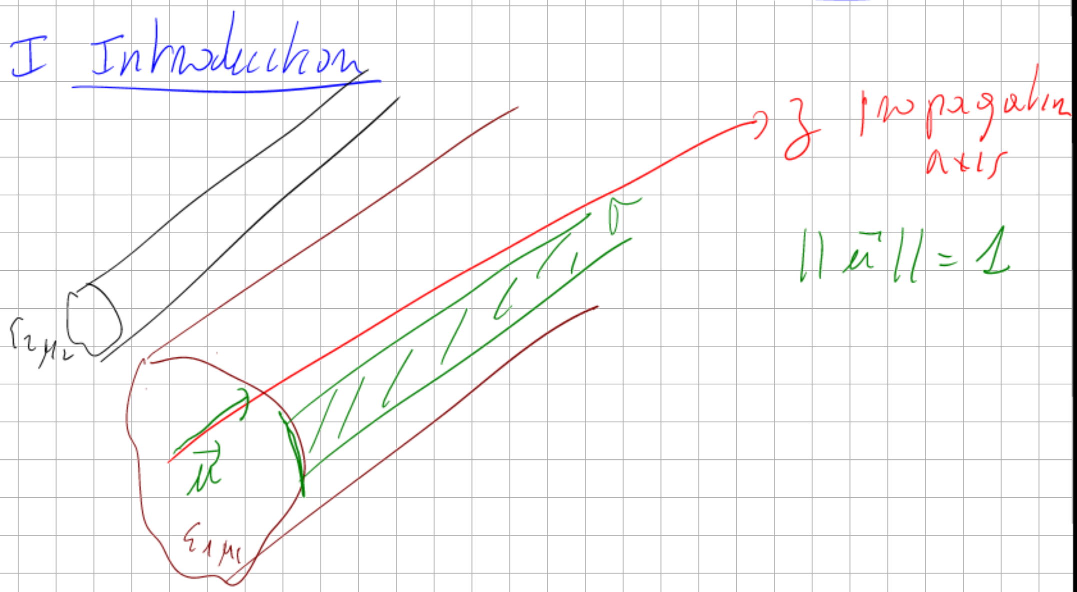
$$\vec{n} \cdot (\vec{D_2} - \vec{D_1}) = \rho_S$$

$$\vec{n} \times (\vec{H_2} - \vec{H_1}) = \vec{J_S}$$

$$\vec{n} \cdot (\vec{B_2} - \vec{B_1}) = 0$$

Transmission line theory

I Introduction

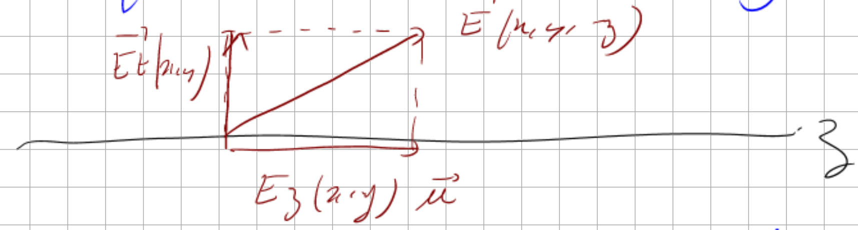


H_p : Material :
- linear : ε, μ don't depend of level of E and H
- non dispersive : ε, μ don't depend of the frequency
- isotropic : ε, μ are with the direction.
- homogeneous by parts
- lossless : ε, μ real

$$\vec{E}(x,y,z) = \vec{E}(x,y)\, e^{-\gamma z}$$

$$\gamma = \alpha + j\beta$$

$$\vec{E}(x,y,z) = \left[\vec{E}_t(x,y) + E_z(x,y)\,\vec{u} \right] e^{-\gamma z}$$



E_t transversal field

E_z longitudinal "

$$\vec{H}(x,y,z) = \left[\vec{H}_t(x,y) + H_z(x,y)\,\vec{u} \right] e^{-\gamma z}$$

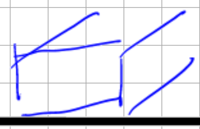
3 types of Maxwell equations solutions:

- $E_z(x,y) = H_z(x,y) = 0$

TEM line : transverse electromagnetic

- $E_z(x,y) = 0$ but $H_z(x,y) \neq 0$

TE mode transverse electric



$H_z(x,y) = 0$ but $E_z(x,y) \neq 0$

TM mode transverse magnetic

- $E_z(x,y) \neq 0$ and $H_z(x,y) \neq 0$

hybrid modes