EXAMS MICROWAVE - RAFAEL DOS SANTOS

6 JUN 2019

1) $Z_0 = 50\Omega$

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

a) Reciprocal $\Rightarrow$ SYMMETRICAL MATRIX $\Rightarrow$ $a_{ij} = a_{ji}$ $\Rightarrow$ this is TRUE FOR THIS NETWORK

$\Rightarrow$ THE NETWORK is RECIPROCAL

b) Lossless $\Rightarrow$ THE MATRIX MUST BE UNITARY $\Rightarrow$ FOR EACH ROW OR COLUMN THE MODULUS SQUARE SUM OF EACH ELEMENT HAS TO BE EQUAL TO 1.

$|a_{11}|^2 + |a_{12}|^2 + |a_{13}|^2 = 0^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2} \neq 1$ $\Rightarrow$ THE MATRIX ISN'T UNITARY.

$\Rightarrow$ THE NETWORK IS NOT LOSSLESS

c) MATCHED PORTS: $a_{ij} = 0 \ \forall \ i,j$ $\Rightarrow$ THIS IS TRUE FOR THIS MATRIX

$\Rightarrow$ THE PORTS ARE MATCHED.

d) $S_{23} \neq 0$ and $S_{32} \neq 0$ $\Rightarrow$ THE OUTPUT PORTS AREN'T ISOLATED.

e) $V_1^+ = 10V.$

$$S_{21} = \frac{V_2^+}{V_1^+} = \frac{1}{2} = \frac{V_2^+}{10} \longrightarrow \boxed{V_2^+ = 5V}$$

$$S_{31} = \frac{V_3^+}{V_1^+} = \frac{1}{2} = \frac{V_3^+}{10} \longrightarrow \boxed{V_3^+ = 5V}$$

f) $$P_2 = \frac{1}{2}\frac{{V_2^+}^2}{Z_0} = \frac{1}{2}\frac{5^2}{50} = \boxed{0{,}25\ W}$$

$$P_3 = \frac{1}{2}\frac{{V_3^+}^2}{Z_0} = \frac{1}{2}\frac{5^2}{50} = \boxed{0{,}25\ W}$$

g) $$P_1 = \frac{1}{2}\frac{{V_1^+}^2}{Z_0} = \frac{1}{2}\frac{10^2}{50} = 1W = P_{available}$$

$$P_{available} = P_{diss} + P_{OUT}$$

$$P_1 = P_{diss} + P_2 + P_3$$

$$\boxed{P_{diss} = 0{,}5W}$$

2) RESONATOR AIR-FILLED $Z_0 = 100\Omega$ $f_0 = 1GHz$ phase-shift $= \pi$

$\beta L = \pi$

a) $$\beta = \frac{2\pi}{\lambda} = \frac{2\pi \cdot 1 \times 10^9}{3 \times 10^8} = \boxed{20{,}94\ m^{-1}}$$

b) $$\beta L = \pi \longrightarrow L = \frac{\pi}{20{,}94} = 0.15\ m = \boxed{15cm}$$

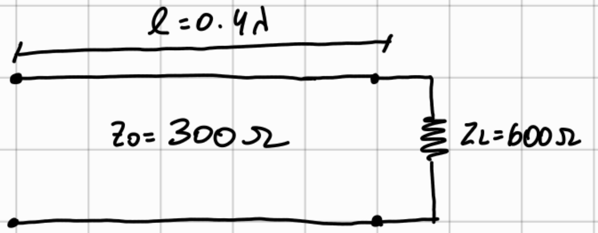
c) $f = f_0 + \Delta f$

$$\beta L = \frac{2\pi}{c}\cdot(f_0 + \Delta f)\cdot L = \left(\underbrace{\frac{2\pi f_0}{c} L}_{=\pi} + \frac{2\pi \Delta f}{c}\right) L = \pi + \frac{2\pi \cdot 0{,}15}{3 \times 10^8}\cdot \Delta f$$

$$\beta L = \pi + \pi \times 10^{-9} \Delta f \Rightarrow \boxed{\beta L = \pi + \frac{\pi}{f_0}\Delta f}$$

16JUN 2020

1) $Z_0 = 300\,\Omega$ $R_L = 600\,\Omega$ $\ell = 0{,}4\lambda$ $f = 500\,MHz$



a) Γ ? $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{600-300}{600+300} = \frac{1}{3}$

b) SWR? $SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+\frac{1}{3}}{1-\frac{1}{3}} = 2$

c) Z_{IN} ? $Z_{IN} = Z_0 \cdot \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\ell}$

$\beta\ell = \frac{2\pi}{\lambda} \cdot 0{,}4\lambda = 2{,}51$

$\rightarrow \tan\beta\ell = -0.726.$

$Z_{IN} = 300 \cdot \frac{600 + j\cdot 300\cdot(-0{,}726)}{300 + j\,600\cdot(-0{,}726)}$

$Z_{IN} = 294{,}77 + j\,210{,}2\ \Omega$

d) $\ell = \frac{\lambda}{4} \rightarrow Z_1 = \sqrt{Z_0 \cdot Z_L} = \sqrt{300\cdot 600} = 424{,}26\ \Omega$

e) Fractional BW:

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\ \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right]$$

$SWR = 1{,}5 \rightarrow \Gamma_m = \frac{SWR-1}{SWR+1} = \frac{1{,}5-1}{1{,}5+1} = \frac{1}{5} = 0{,}2$

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{0{,}2}{\sqrt{1-0{,}2^2}}\ \frac{2\ 424{,}6}{|600-300|}\right]$$

$$\frac{\Delta f}{f_0} = 0{,}7836 \simeq 78{,}36\%$$

2) $\varepsilon_r = 2,25$ $\tan\delta = 0.0004$ $f_0 = 10\,GHz$.

a) a, b ?

$$f_{c_{10}} = 0,75 f_0 = 7,5\,GHz$$

$$f_{c_{01}} = 1,25 f_0 = 12,5\,GHz$$

$$f_{c_{10}} = \frac{c}{2\pi\sqrt{\varepsilon_r}} \cdot \left(\frac{\pi}{a}\right) \Rightarrow a = 1,33\,cm$$

$$f_{c_{01}} = \frac{c}{2\pi\sqrt{\varepsilon_r}} \left(\frac{\pi}{b}\right) \Rightarrow b = 0,8\,cm$$

b) RECTANGULAR WAVEGUIDE CAVITY.

Perfectly conducting walls $\rightarrow \sigma \rightarrow \infty$

$\downarrow$

$R_s = 0$

$\downarrow$

$Q_c \rightarrow \infty$

$$Q = \left(\cancel{\frac{1}{Q_c}}^{\,0} + \frac{1}{Q_d}\right)^{-1}$$

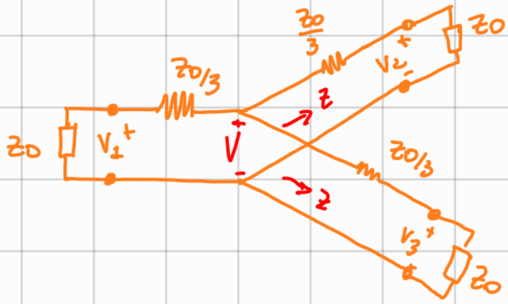
$$Q_d = \frac{1}{\tan\delta} = 2500$$

$$\boxed{Q = 2500}$$

3)

a) $S_{33}=0 \quad S_{22}=0$ (Ports 2 and 3 matched)

$$S_{31} = \frac{1}{2} = \frac{V_3^+}{V_1^+}$$



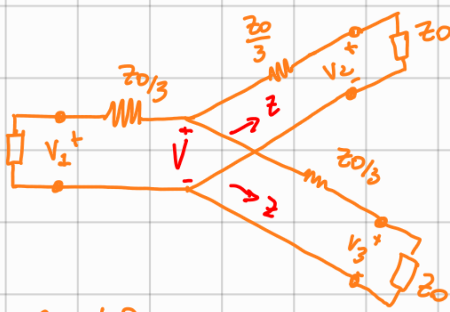
$$Z = \frac{Zo}{3} + Zo = \frac{4}{3} Zo$$

$$V = V_1^+ \cdot \frac{Z \| Z}{Z\|Z + \frac{Zo}{3}} = V_1^+ \cdot \frac{\frac{2}{3} Zo}{\frac{2}{3} Zo + \frac{Zo}{3}} = V_1^+ \frac{\frac{2}{3} Zo}{Zo} \Rightarrow V = \frac{2}{3} V_1^+$$

$$V_3^+ = V \cdot \frac{Zo}{Zo + \frac{Zo}{3}} = \frac{2}{3} V_1^+ \cdot \frac{Zo}{\frac{4}{3} Zo} = \frac{2}{3} V_1^+ \cdot \frac{3}{4} \rightarrow \boxed{V_3^+ = \frac{V_1^+}{2}}$$

$$P_3 = \frac{1}{2} \frac{{V_3^+}^2}{Zo} = \underbrace{\frac{1}{2} \frac{V_1^2}{Zo}}_{P_1} \cdot \frac{1}{4}$$

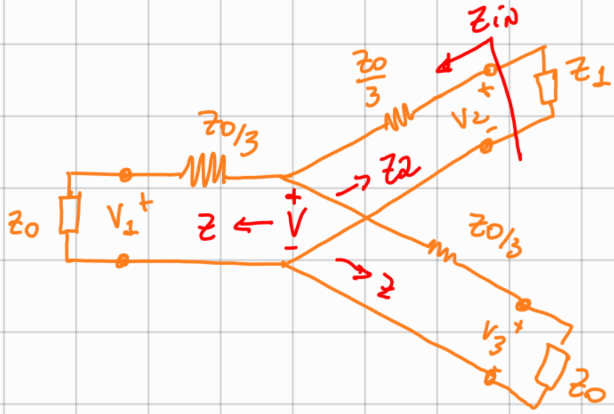
$$P_3 = \frac{1}{4} P_1$$



$$\frac{P_3}{P_1} = \frac{1}{4}$$

$$\left[\frac{P_3}{P_1}\right]_{dB} = 10 \log \frac{1}{4} = -6\,dB$$

b) $\Gamma_2 = 0.3 = S_{22}$



$$\Gamma_2 = \frac{Z_1 - Z_{IN}}{Z_1 + Z_{IN}}$$

$$Z_{IN} = \frac{Z_0}{3} + Z \| Z$$

$$Z_{IN} = \frac{Z_0}{3} + \frac{2}{3} Z_0 = Z_0$$

$$0,3 = \frac{Z_1 - Z_0}{Z_1 + Z_0} \rightarrow 0,3 Z_1 + 0,3 Z_0 = Z_1 - Z_0$$

$$1,3 Z_0 = 0,7 Z_1$$

$$\boxed{Z_1 = 1,86 Z_0}$$

$$Z_2 = \frac{Z_0}{3} + Z_1 = 2,20 Z_0 \qquad Z = \frac{Z_0}{3} + Z_0 = \frac{4}{3} Z_0$$

$$Z \| Z_2 = \frac{\frac{4}{3} Z_0 \cdot 2,20 Z_0}{\frac{4}{3} Z_0 + 2,20 Z_0} = 0,83 Z_0$$

$$V = V_1 \cdot \frac{Z \| Z_2}{\frac{Z_0}{3} + Z \| Z_2} = V_1 \cdot \frac{0,83 Z_0}{\frac{Z_0}{3} + 0,83 Z_0} \Rightarrow V = 0,71 V_1$$

$$V_3 = V \cdot \frac{Z_0}{Z_0 + \frac{Z_0}{3}} = 0,71 V_1 \cdot \frac{Z_0}{\frac{4}{3} Z_0} = 0,71 V_1 \cdot \frac{3}{4}$$

$$\boxed{V_3 = 0,535 V_1}$$

$$\alpha_{dB} = 20 \log (0,535) = -5,43 \text{ dB}$$

29 JUN 2020

1) for $l = \frac{\lambda}{2}$, $Z_{IN} = Z_L$ and is reasonable to ignore the presence of the transmission line.

a) $30 \times 10^{-2} \neq \frac{1}{2} \cdot \frac{3 \times 10^8}{20 \times 10^3}$ → can't ignore

b) $50 \times 10^3 \neq \frac{1}{2} \frac{3 \times 10^8}{60}$ → can't ignore

c) $30 \times 10^{-2} \neq \frac{1}{2} \frac{3 \times 10^8}{600 \times 10^6}$ → can't ignore

2) a) Lossless → [S] must be unitary.

$$(0,178 < 90^\circ)^2 + (0.6 < 45^\circ)^2 + (0.4 < 45^\circ)^2 \neq 1$$

↳ the network has losses.

b) Reciprocal → $a_{ij} = a_{ji}$ → TRUE → NETWORK is RECIPROCAL.

c) $RL = -20 \log(|\Gamma|) = -20 \log 0,178 = 15\,dB.$

d) Ports 2 and 4 $S_{24} = S_{42} = 0.3 < -45^\circ$

$IL = -20 \log(|S_{24}|) = -20 \log 0.3 = 10.45\,dB.$

Phase delay ⇒ -45°.

e) SC @ PORT 3 $\Rightarrow V_3^+ = V_3^-$

MATCHED @ PORTS 2 and 4 $\quad V_2^+ = V_4^+ = 0$

$S_{11} = ?$

$$S = \begin{bmatrix} S_{11} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

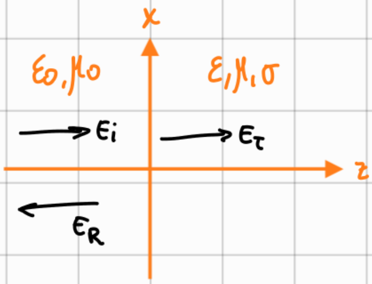
?

$$[V^-] = [S]\,[V^+]$$

3) RHCP plane wave

$$E_i = E_0(\hat{x} - j\hat{y})e^{-jk_0 z}$$

Interface with a good conductor



$$E_T = T\, E_0(\hat{x} - j\hat{y})e^{-jk_0 z}$$

$$H_T = T\, \frac{E_0}{\eta}(\hat{x} - j\hat{y})e^{-jk_0 z}$$

$$\gamma = \alpha + j\beta = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}} = (1+j)\frac{1}{\delta_s}$$

$$\eta = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}} = (1+j)\frac{1}{\sigma\delta}$$

17 JUL 2020

1)

a) $f_0 = \frac{1}{2\pi}\frac{1}{\sqrt{LC}} = \frac{1}{2\pi}\frac{1}{\sqrt{50\times10^{-9}\cdot 0{,}79\times10^{-12}}} = 800\ \text{MHz}$

b) $Z_{IN} = R + j\omega L - \frac{j}{\omega C} = 2{,}5\ \Omega$

c) $Q = \omega \frac{\text{AVERAGE ENERGY STORED}}{\text{ENERGY LOSS / SEC}} = \omega \frac{W_M + W_e}{P_{loss}}$

at Resonance:

$$Q = \omega_0 \frac{2W_M}{P_{loss}} = \omega_0 \frac{2\frac{1}{4}|I_0|^2 L}{\frac{1}{2}|I_0|^2 R} = \omega_0 \frac{L}{R}$$

$$Q = \frac{1}{\sqrt{LC}}\cdot\frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$Q = \frac{1}{2{,}5}\sqrt{\frac{50\times10^{-9}}{0{,}79\times10^{-12}}} = 100{,}64.$ (UNLOADED)

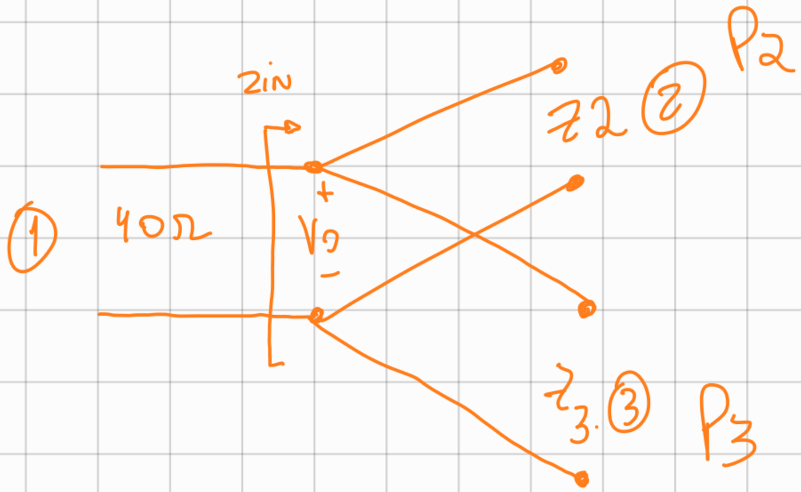
$Q_E = \frac{\omega_0 L}{R_L}$ (SERIES)

$Q_E = \frac{2\pi\cdot 800\times10^{6}\cdot 50\times10^{-9}}{2{,}5} = 100$

$\frac{1}{Q_L} = \frac{1}{Q} + \frac{1}{Q_E} \rightarrow \boxed{Q_L = 50}$

2) T-JUNCTION 4:1 power split

LOSSLESS 40Ω source impedance.



$$P_{IN} = \frac{1}{2}\frac{V_0^2}{Z_0}$$

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_2}$$

$$P_3 = \frac{1}{2}\frac{V_0^2}{Z_3}$$

$$\boxed{P_2 = 4P_3} \rightarrow P_2 = 2\frac{V_0^2}{Z_3}$$

$$P_{IN} = P_2 + P_3 = 2\frac{V_0^2}{Z_3} + \frac{1}{2}\frac{V_0^3}{Z_3} = \frac{5}{2}\frac{V_0^2}{Z_3}$$

$$\frac{1}{2}\frac{V_0^2}{Z_1} = \frac{5}{2}\frac{V_0^2}{Z_3}$$

$$Z_3 = 5Z_1 = \boxed{200\Omega = Z_3}$$

$$\frac{1}{2}\frac{V_0^2}{Z_2} = 2\frac{V_0^2}{Z_3} \rightarrow 4Z_2 = Z_3$$

$$\boxed{Z_2 = 50\Omega}$$

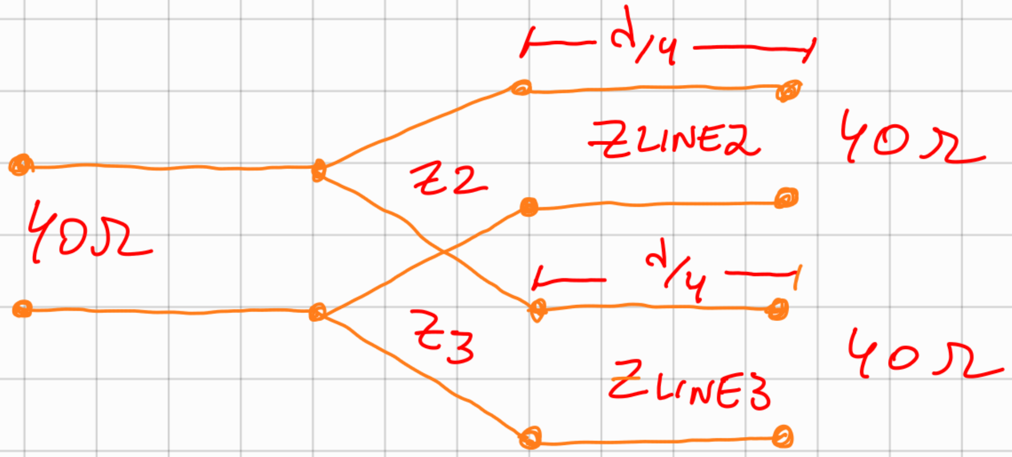
$\lambda/4$ LINES MATCHING:

FOR Z_2 BRANCH:

$$Z_{LINE2} = \sqrt{50 \cdot 40} = 44.72\,\Omega$$

FOR Z_3 BRANCH:

$$Z_{LINE3} = \sqrt{200 \cdot 40} = 89{,}44\,\Omega$$



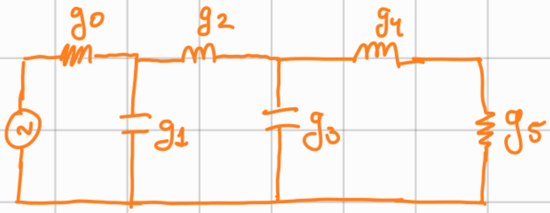
$$|S| = \begin{bmatrix} 0 & 0.8 & 0.2 \\ 0.8 & 0.11 & \\ 0.2 & & 0.67 \end{bmatrix}$$

$$|\Gamma_2| = \frac{50-40}{50+40} = 0.11$$

$$|\Gamma_3| = \frac{200-40}{200+40} = 0.67$$

3) FOUR-SECTION BANDPASS   Fractional BW = 5% $f_0 = 2GHz$ $Z_0 = 50\Omega$

Prototype:



$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0} = 0,05$$

$g_0 = 1$

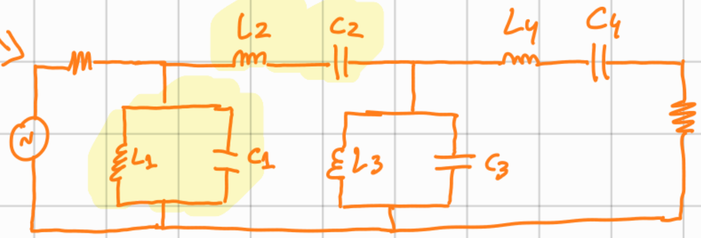
$g_1 = 1.0598$

$g_2 = 0.5116$

$g_3 = 0.3181$

$g_4 = 0.1104$

$g_5 = 1$



$$L_1 = \frac{\Delta}{\omega_0 C} = \frac{0,05}{2\pi \cdot 2 \cdot 10^9 \cdot 1,0598} \longrightarrow \boxed{L_1 = 3,75\,pH}$$

$$C_1 = \frac{C}{\omega_0 \Delta} = \frac{1,0598}{2\pi \cdot 2 \times 10^9 \cdot 0,05} \longrightarrow \boxed{C_1 = 1,69\,nF}$$

$$L_2 = \frac{L}{\omega_0 \Delta} = \frac{0,5116}{2\pi 2 \times 10^9 \cdot 0,05} \longrightarrow \boxed{L_2 = 0,81\,nH}$$

$$C_2 = \frac{\Delta}{\omega_0 L} = \frac{0,05}{2\pi \cdot 2 \times 10^9 \cdot 0,5116} \longrightarrow \boxed{C_2 = 7,7\,pF}$$

$L_3, C_3, L_4, C_4 \rightarrow$ Same procedure!

15 NOV 2021

1) $Z_0 = 60\Omega$ $\Gamma = 0{,}4$

a) $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} \rightarrow 0{,}4\, Z_L + 60 \cdot 0{,}4 = Z_L - 60$

$84 = 0.6\, Z_L$

$\boxed{Z_L = 140\,\Omega}$

b) $\Gamma(\ell) = \Gamma e^{-2\gamma \ell}$

$\underline{\ell = 0.3\lambda}$ $\beta\ell = \dfrac{2\pi}{\lambda} \cdot 0{,}3\,\lambda = 0{,}6\pi$

$\Gamma(0.3\lambda) = 0.4\, e^{-2j\beta \cdot \ell}$

$\Gamma(0.3\lambda) = 0.4\, e^{-2 \cdot j \cdot 0{,}6\pi}$

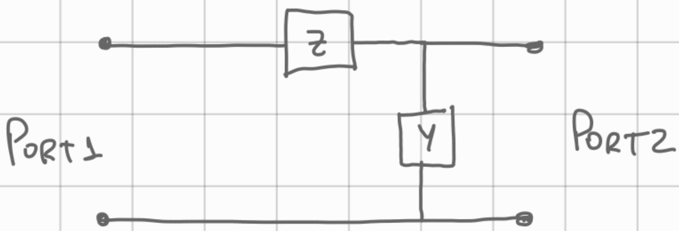
$\Gamma(0.3\lambda) = -0{,}324 + j\, 0.235$

c) $Z_{IN} = Z_0 \dfrac{Z_L + j Z_0 \tan \beta\ell}{Z_0 + j Z_L \tan \beta\ell}$

$Z_{IN} = 60 \cdot \dfrac{140 + j\, 60 \tan(0{,}6\pi)}{60 + j\, 140 \tan(0{,}6\pi)}$

$Z_{IN} = 27{,}88 + j\, 15{,}61\ \Omega$

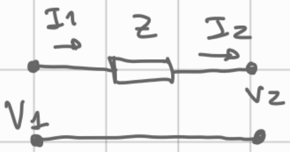
2)



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}}_{\boxed{Z}} \cdot \underbrace{\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}}_{\boxed{Y}} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$

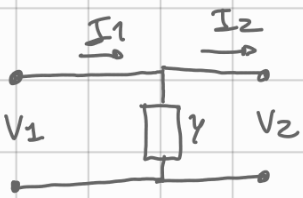
Z :



$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = Z \qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = 1$$

Y:



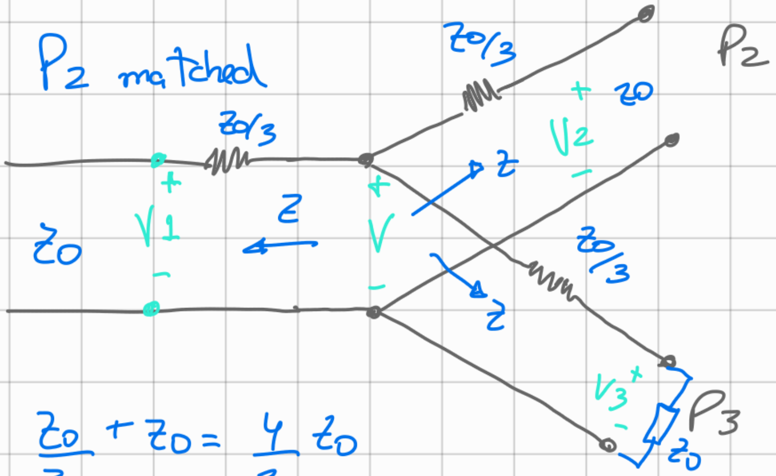
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = Y$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = 0 \qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = 1$$

$$ABCD = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} 1+ZY & Z \\ Y & 1 \end{bmatrix}$$

3)

a) P_2 matched



$$Z = \frac{Z_0}{3} + Z_0 = \frac{4}{3} Z_0$$

$$V = V_1 \cdot \frac{Z \| Z}{\frac{Z_0}{3} + Z \| Z} = V_1 \cdot \frac{\frac{2}{3} Z_0}{\frac{Z_0}{3} + \frac{4}{3} Z_0} \rightarrow V = \frac{2}{3} V_1$$

$$V_3 = V \cdot \frac{Z_0}{\frac{Z_0}{3} + Z_0} = \frac{2}{3} V_1 \cdot \frac{Z_0}{\frac{4}{3} Z_0} = \frac{2}{3} V_1 \cdot \frac{3}{4}$$

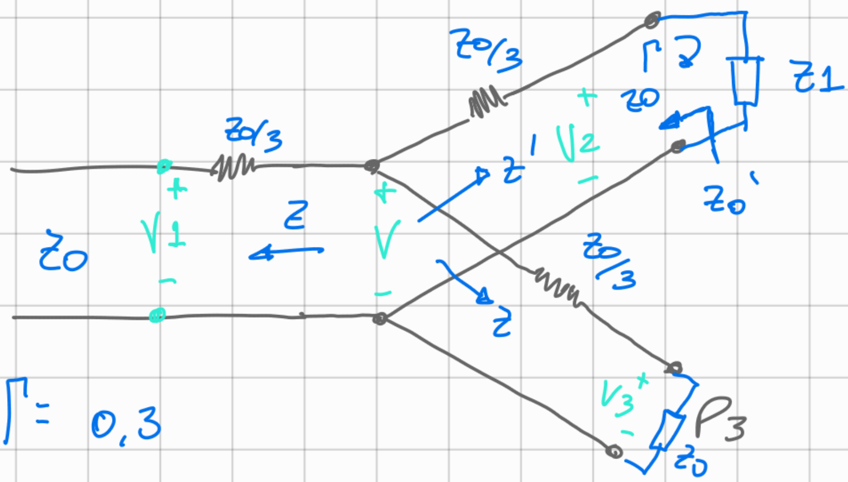
$$\boxed{V_3 = \frac{V_1}{2}}$$

$$P_1 = 1W \rightarrow P_1 = \frac{1}{2} \frac{V_1^2}{Z_0} \rightarrow V_1^2 = 2 Z_0 P_1$$

$$P_3 = \frac{1}{2} \frac{V_3^2}{Z_0} = \frac{1}{2} \frac{V_1^2}{4} \cdot \frac{1}{Z_0} = \frac{1}{2} \frac{2 Z_0 P_1}{4} \cdot \frac{1}{Z_0}$$

$$\boxed{P_3 = \frac{P_1}{4} = 0{,}25 W}$$

b)



$\Gamma = 0{,}3$

$$Z_0' = \frac{Z_0}{3} + Z \| Z = \frac{Z_0}{3} + \frac{2}{3} Z_0 = Z_0$$

$$\Gamma = \frac{Z_1 - Z_0'}{Z_1 + Z_0'} \rightarrow 0{,}3\, Z_1 + 0{,}3 \cdot Z_0 = Z_1 - Z_0$$

$$1{,}3\, Z_0 = 0{,}7\, Z_1$$

$$\boxed{Z_1 = 1{,}86\, Z_0}$$

$$Z \| Z' = \frac{\left(\frac{Z_0}{3} + Z_0\right) \cdot \left(\frac{Z_0}{3} + 1{,}86\, Z_0\right)}{\frac{Z_0}{3} + Z_0 + \frac{Z_0}{3} + 1{,}86\, Z_0} = \frac{2{,}92\, Z_0^2}{3{,}53\, Z_0}$$

$$Z \| Z' = 0{,}828\, Z_0$$

$$V = \frac{V_1 \cdot (Z \| Z')}{(Z \| Z') + \frac{Z_0}{3}} = \frac{V_1 \cdot 0{,}828\, Z_0}{0{,}828\, Z_0 + Z_0/3} \qquad V = 0{,}713\, V_1$$

$$V_3 = V \cdot \frac{Z_0}{Z_0 + \frac{Z_0}{3}} = 0{,}713\, V_1 \cdot \frac{3}{4} = 0{,}535\, V_1$$

$$P_1 = \frac{1}{2} \frac{V_1^2}{Z_0} \rightarrow V_1^2 = 2 P_1 Z_0$$

$$P_3 = \frac{1}{2} \frac{V_3^2}{Z_0} = \frac{1}{2} \frac{0{,}535^2 V_1^2}{Z_0} = \frac{1}{2} \frac{0{,}535^2}{Z_0} \cdot 2 P_1 Z_0 = \underline{0{,}286\ \text{W}}$$