CHAPTER 6
POWER COUPLING
AT FIBERS CONNECTIONS

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Introduction

Introduction

Connections between fibers will induce losses and power coupling between transmitted modes

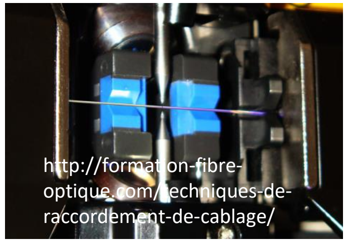
- → **Need to identify coupling conditions**
- → **Model of power coupling between fiber modes at connections**

Aim of the chapter 6

- → **Connections geometric defects definition**
- → **Power transfert between optical fibers**
- → **Connection losses in singlemode fibers (gaussian mode approximation)**
- → **Modal coupling at misaligned multimode fibers connections**

Fiber connections

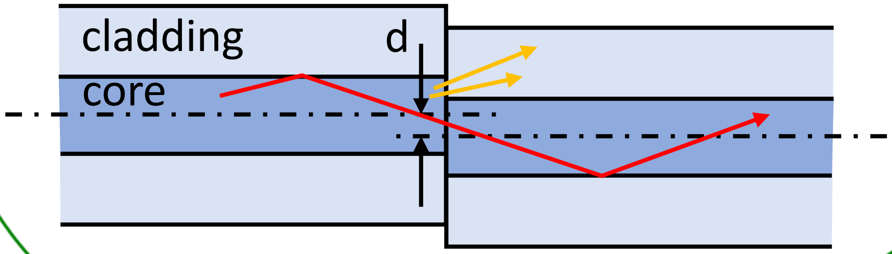
- Aims
  - Increasing the length of a fiber link
  - Allowing fiber network flexibility
  - Low loss
  - Reliability
- Technologies
  - Permanent link :
    Electrical arc fusion splices
  - Non-permanent link :
    Fiber connectors



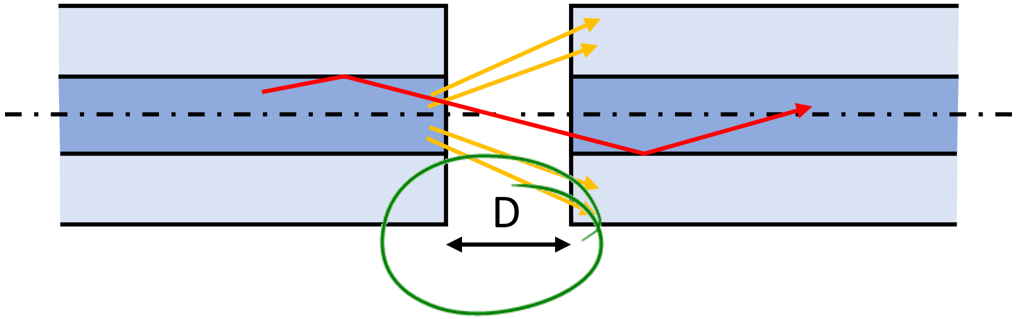
http://www.next-up.org/France/Fibre_optique.php

Fiber connections loss causes (1)

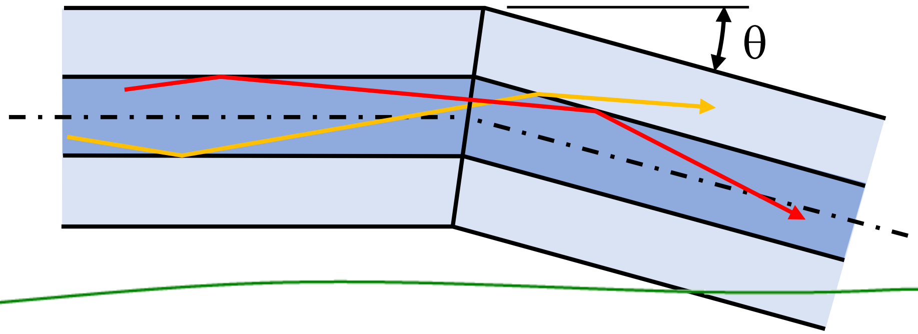
- Mechanical misalignments
  - Transversal offset d



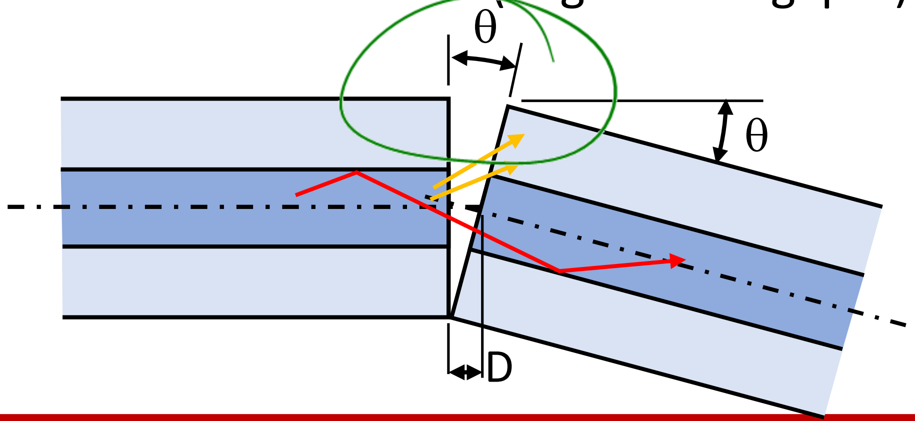
  - Longitudinal gap D



- Angular offset
  - ➢ Fusion splice (angle θ only)

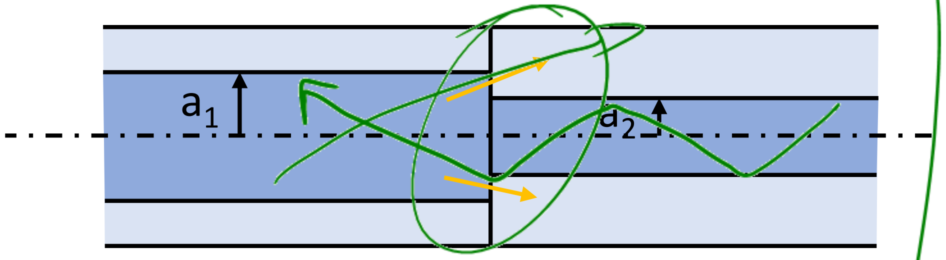


  - ➢ Connector (angle θ and gap D)

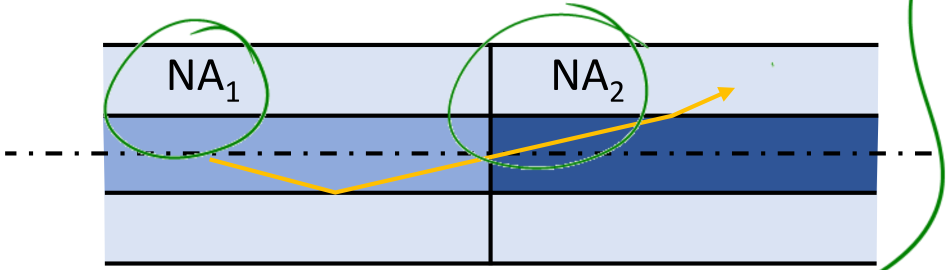


Fiber connections loss causes (2)

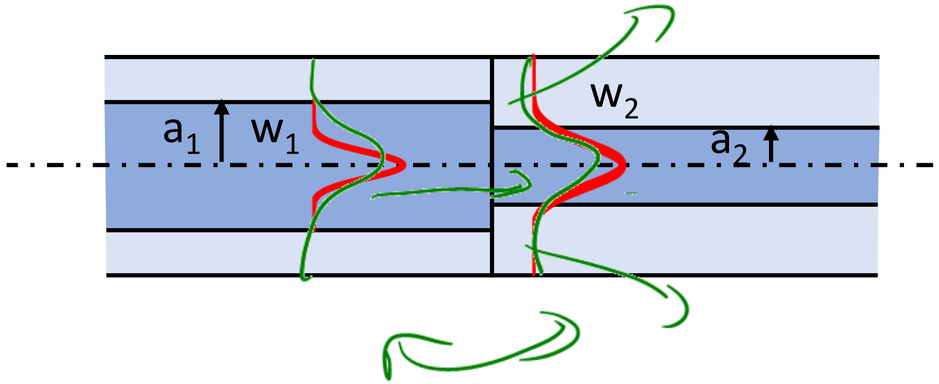
- Opto-geometrical mismatches
  - Core size mismatch (Multimode fiber)

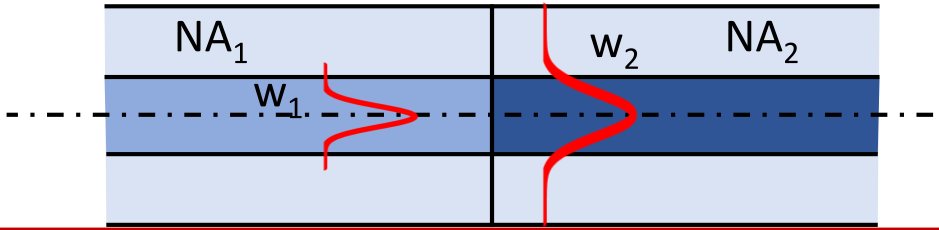


Numerical Aperture mismatch (Multimode fiber)



  - Mode field diameter mismatch (Singlemode fiber)





Power flow in optical fiber (1)

- Complex fields

Transverse field profile, Polarization — Propagation — spatio-temporal phase

$$\begin{cases}\vec{\mathcal{E}}(x,y,z,t) = \underbrace{\vec{E}(x,y)}_{\text{Transverse field Polarization}}\underbrace{e^{j(wt-\beta z)}}_{\text{Propagation}} = \vec{E}(x,y)\overbrace{e^{j\phi(z,t)}}^{\text{spatio-temporal phase}} \\ \vec{\mathcal{H}}(x,y,z,t) = \vec{H}(x,y)e^{j(wt-\beta z)} = \vec{H}(x,y)e^{j\phi(z,t)}\end{cases}$$

- Poynting vector

$$\vec{S} = \mathcal{R}e(\vec{\mathcal{E}}) \wedge \mathcal{R}e(\vec{\mathcal{H}}) = \overbrace{\frac{1}{2}(\vec{\mathcal{E}} + \vec{\mathcal{E}}^*)}^{2Re(\vec{\mathcal{E}})} \wedge \overbrace{\frac{1}{2}(\vec{\mathcal{H}} + \vec{\mathcal{H}}^*)}^{2Re(\vec{\mathcal{H}})}$$

$$\vec{S} = \frac{1}{4}(\vec{E} \wedge \vec{H}^* + \vec{E}^* \wedge \vec{H} + \vec{E} \wedge \vec{H} e^{j2\phi(z,t)} + \vec{E}^* \wedge \vec{H}^* e^{-j2\phi(z,t)})$$

- Time averaged Poynting vector

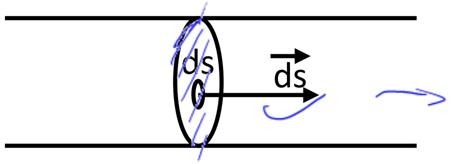
$$< \vec{S} >= \frac{1}{T}\int_0^T \vec{S}\,dt = \frac{1}{4}(\vec{E} \wedge \vec{H}^* + \vec{E}^* \wedge \vec{H}) = \boxed{\frac{1}{2}\mathcal{R}e\left(\vec{E}(x,y) \wedge \vec{\mathcal{H}}^*(x,y)\right)}$$

$V\,m^{-1}$ — $A\,m^{-1}$

$W\,m^{-2}$ *Surfacic power density*

Power flow in optical fiber (2)

- Power flow across the fiber section



$\rightarrow z$

$$< \vec{S} >= \begin{vmatrix} S_r \\ S_\theta \\ S_z \end{vmatrix} \qquad \vec{ds} \begin{vmatrix} 0 \\ 0 \\ ds \end{vmatrix}$$

The mean guided power

$$\bar{P} = \iint_{-\infty}^{+\infty} < \vec{S} > \vec{ds} = \iint_{-\infty}^{+\infty} S_z \, dx \, dy = \iint_{-\infty}^{+\infty} \frac{n}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}} |E(x,y)|^2 \, dx \, dy$$

We define $|\psi(x,y)|^2 = S_z = \frac{n}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}} |E(x,y)|^2$

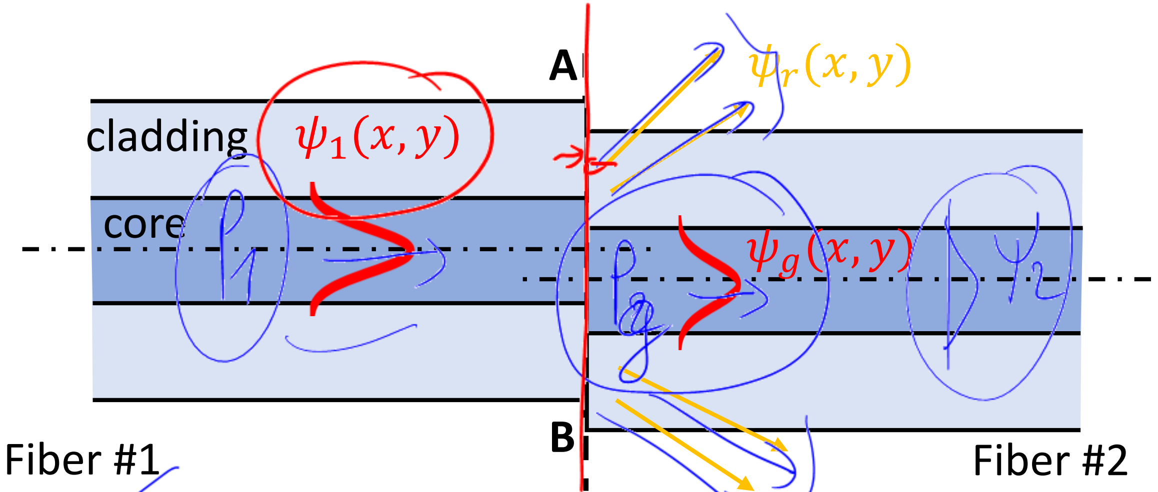
mode field transverse repartition

$$\boxed{\bar{P} = \iint_{-\infty}^{+\infty} |\psi(x,y)|^2 dx \, dy}$$

Fields overlap integral (1)

- Fields at the connection (plane AB)
  - Incident optical wave field : $\boldsymbol{\psi_1(x,y)}$ carrying a mean power $P_1 = \iint_{-\infty}^{+\infty} |\psi_1(x,y)|^2 dx\,dy$
  - Output mode field : $\boldsymbol{\psi_2(x,y)}$ with unknown amplitude and power
  - Guided output field : $\boldsymbol{\psi_g(x,y)} = \boldsymbol{\varepsilon\,\psi_2(x,y)}$ carrying a power $P_g = |\varepsilon|^2 \iint_{-\infty}^{+\infty} |\psi_2(x,y)|^2 dx\,dy$
  - Radiative (unguided) field : $\boldsymbol{\psi_r(x,y)}$ carrying a mean power $P_r = \iint_{-\infty}^{+\infty} |\psi_r(x,y)|^2 dx\,dy$

$P_1 = P_g + P_r$



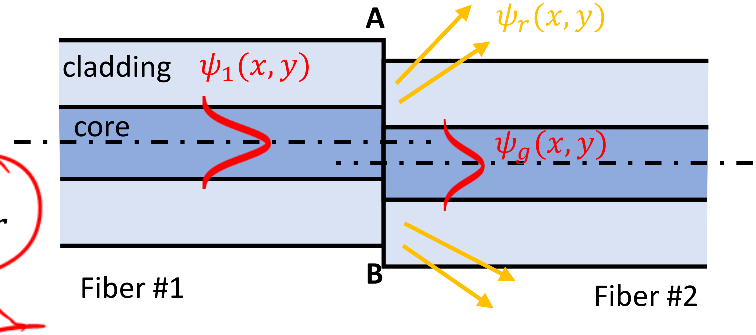
In AB plane

$$\boldsymbol{\psi_1(x,y)} = \boldsymbol{\psi_g(x,y)} + \boldsymbol{\psi_r(x,y)}$$

$$= \boldsymbol{\varepsilon\,\psi_2(x,y)} + \boldsymbol{\psi_r(x,y)} \quad \text{(Eqn 1)}$$

P_1

Fields overlap integral (2)



- Energy conservation
  - The total output power is equal to the input power : $P_{output} = P_1 = P_g + P_r$

$$\iint_{-\infty}^{+\infty}|\psi_1(x,y)|^2dx\,dy = |\varepsilon|^2\iint_{-\infty}^{+\infty}|\psi_2(x,y)|^2dx\,dy + \iint_{-\infty}^{+\infty}|\psi_r(x,y)|^2dx\,dy$$

P_1 P_g P_r

  - From equation 1 $\quad \psi_1(x,y) = \psi_g(x,y) + \psi_r(x,y) = \varepsilon\,\psi_2(x,y) + \psi_r(x,y)$

$$\iint_{-\infty}^{+\infty}|\psi_1(x,y)|^2dx\,dy = \iint_{-\infty}^{+\infty}|\varepsilon\,\psi_2(x,y) + \psi_r(x,y)|^2dx\,dy$$

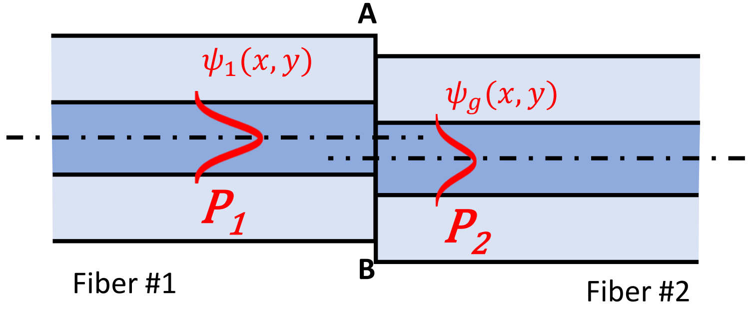
guided mode   radiative mode

$$= |\varepsilon|^2\iint_{-\infty}^{+\infty}|\psi_2(x,y)|^2dx\,dy + \iint_{-\infty}^{+\infty}|\psi_r(x,y)|^2dx\,dy + \underbrace{\varepsilon\iint_{-\infty}^{+\infty}\psi_2(x,y)\,\psi_r^*(x,y)\,dx\,dy + \varepsilon^*\iint_{-\infty}^{+\infty}\psi_2^*(x,y)\,\psi_r(x,y)\,dx\,dy}_{=0}$$

$$\Leftrightarrow \iint_{-\infty}^{+\infty}\boldsymbol{\psi_2(x,y)\,\psi_r^*(x,y)\,dx\,dy = 0}$$

$\Leftrightarrow \psi_2\,(x,y)$ and $\psi_r(x,y)$ are orthogonal fields
Guided and radiative modes are orthogonal

Fields overlap integral (3)



- Power transfer
  - Coupling power coefficient definition

$$\alpha^2 = \frac{P_g}{P_1} = \frac{P_2}{P_1} = \frac{|\varepsilon|^2 \iint_{-\infty}^{+\infty} |\psi_2(x,y)|^2 dx\, dy}{\iint_{-\infty}^{+\infty} |\psi_1(x,y)|^2 dx\, dy}$$

Eqn(2)

  - Value of ε

Multiplying by $\psi_2^*(x,y)$ and integrating eqn. 1 $\psi_1(x,y) = \varepsilon\, \psi_2(x,y) + \psi_r(x,y)$

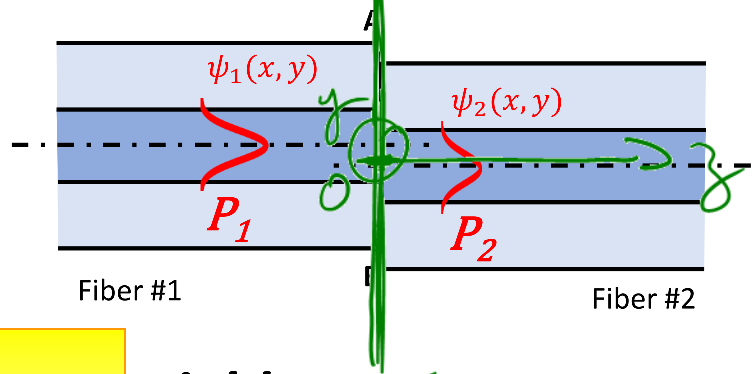
$$\Leftrightarrow \iint_{-\infty}^{+\infty} \psi_1(x,y)\, \psi_2^*(x,y)\, dx\, dy = \varepsilon \iint_{-\infty}^{+\infty} |\psi_2(x,y)|^2 dx\, dy + \underbrace{\iint_{-\infty}^{+\infty} \psi_r(x,y)\, \psi_2^*(x,y) dx\, dy}_{=0}$$

$$\Leftrightarrow \varepsilon = \frac{\iint_{-\infty}^{+\infty} \psi_1(x,y)\, \psi_2^*(x,y)\, dx\, dy}{\iint_{-\infty}^{+\infty} |\psi_2(x,y)|^2 dx\, dy}$$

ε^2

By introducing this expression in Eqn(2) we get the final expression of α^2

Fields overlap integral (4)



- Power transfer
  - Coupling power coefficient – Fields overlap integral

$$\alpha^2 = \frac{P_2}{P_1} = \frac{\left|\iint_{-\infty}^{+\infty} \psi_1(x,y)\,\psi_2^*(x,y)\,dx\,dy\right|^2}{\iint_{-\infty}^{+\infty}|\psi_1(x,y)|^2 dx\,dy \;\; \iint_{-\infty}^{+\infty}|\psi_2(x,y)|^2 dx\,dy}$$

Fields overlap integral

Power normalization terms

Fields are expressed in the plane AB and in the same coordinates axes

For normalized fields

$$\iint_{-\infty}^{+\infty}|\psi_1(x,y)|^2 dx\,dy = 1 \text{ and } \iint_{-\infty}^{+\infty}|\psi_2(x,y)|^2 dx\,dy = 1$$

$0 \leq \alpha^2 \leq 1$

$P_2 = \alpha^2 \cdot P_1$

$|\psi_1|^2$

$$\alpha^2 = \frac{P_2}{P_1} = \left|\iint_{-\infty}^{+\infty} \psi_1(x,y)\,\psi_2^*(x,y)\,dx\,dy\right|^2$$

Normalized fields overlap integral

Fields overlap integral (5)

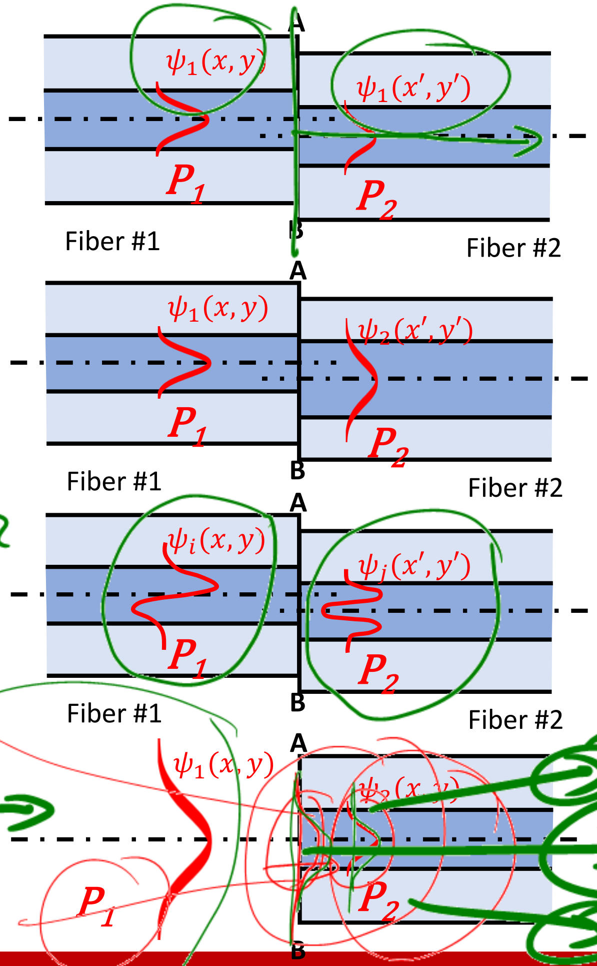
$$P_2 = \alpha^2 P_1$$

- Power transfer
  - Between fibers
    - Identical singlemode fibers :
      mode of fiber #1 : $\boldsymbol{\psi_1(x,y)}$ with (x,y) coordinates of the fiber #1
      mode of fiber #2 : $\boldsymbol{\psi_2(x',y')}$=$\boldsymbol{\psi_1(x',y')}$ with (x',y') coordinates of the fiber #2
    - Non – identical singlemode fibers
      mode of fiber #1 : $\boldsymbol{\psi_1(x,y)}$
      mode of fiber #2 : $\boldsymbol{\psi_2(x',y')}$
    - Multimodes fibers
      mode #i of fiber #1 : $\boldsymbol{\psi_1(x,y) = \psi_i(x,y)}$ M
      mode #j of fiber #2 : $\boldsymbol{\psi_2(x',y') = \psi_j(x',y')}$ N

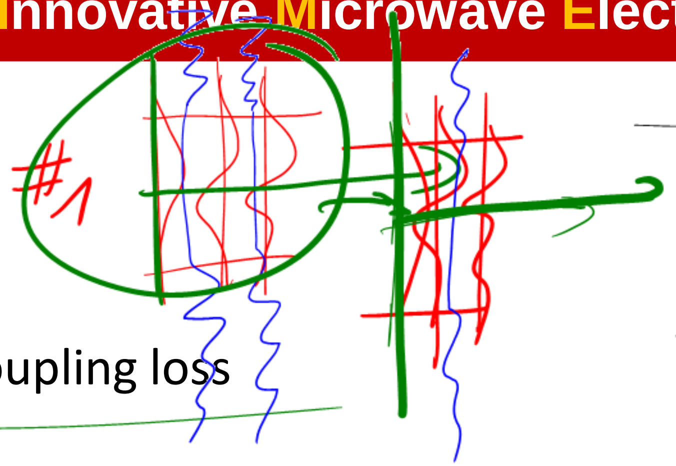
M×N values of α^2

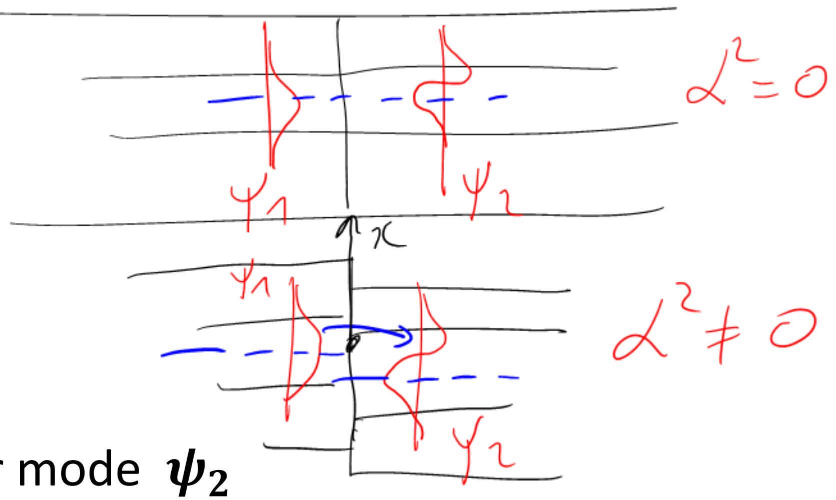
$\alpha^2_{i,j}$

- At fiber input face
  - Power injection in fiber
    input field : $\boldsymbol{\psi_1(x,y)}$ with (x,y) coordinates of the fiber
    mode of fiber : $\boldsymbol{\psi_2(x,y) = \psi_j(x,y)}$

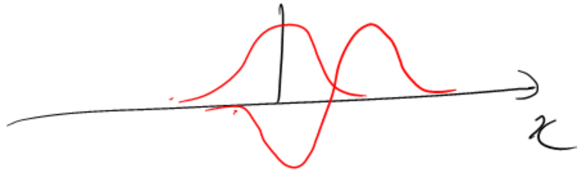


Fields overlap integral (6)





- Power transfer
  - If $\boldsymbol{\psi_1(x,y)} = \boldsymbol{\psi_2(x,y)}$ then
    $\boldsymbol{\alpha^2 = 1 \Leftrightarrow P_2 = P_1}$: no coupling loss
  - If $\boldsymbol{\psi_2(x,y)}$ is orthogonal to $\boldsymbol{\psi_1(x,y)}$ (guided or radiative modes)
    $\boldsymbol{\alpha^2 = 0 \Leftrightarrow P_2 = 0}$: no coupled power to the second fiber mode $\boldsymbol{\psi_2}$



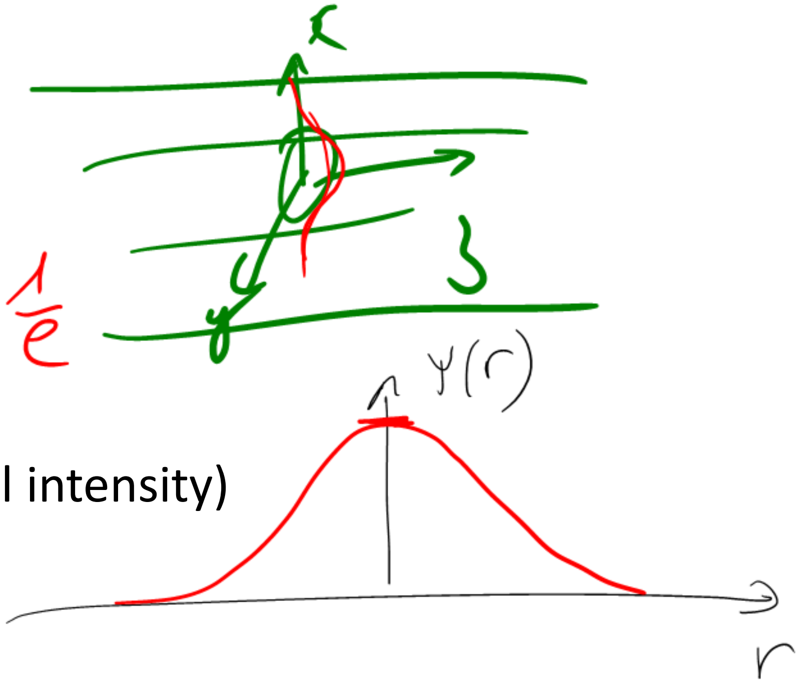
  - Schwarz inequality

$$\mathbf{0} \leq \left| \iint_{-\infty}^{+\infty} \boldsymbol{\psi_1(x,y)\, \psi_2^*(x,y)\, dx\, dy} \right|^{\mathbf{2}} \leq \iint_{-\infty}^{+\infty} |\boldsymbol{\psi_1(x,y)}|^{\mathbf{2}} \boldsymbol{dx\, dy} \iint_{-\infty}^{+\infty} |\boldsymbol{\psi_2(x,y)}|^{\mathbf{2}} \boldsymbol{dx\, dy}$$

$$\Leftrightarrow \boxed{\mathbf{0} \leq \boldsymbol{\alpha^2} \leq \mathbf{1}} \quad \Leftrightarrow \boxed{\boldsymbol{P_2 \leq P_1}}$$

  - Independence of propagation direction (from fiber #1 to #2 or #2 to #1)

Connexion losses between singlemode fibers (1)



- Gaussian guided mode fields :

$$-\frac{r^2}{w_0^2}$$

$$\psi(x,y) = A_0\, e^{-\frac{x^2+y^2}{w_0^2}} = A_0 e^{-\frac{r^2}{w_0^2}} \to e^{-1} = \frac{1}{e}$$

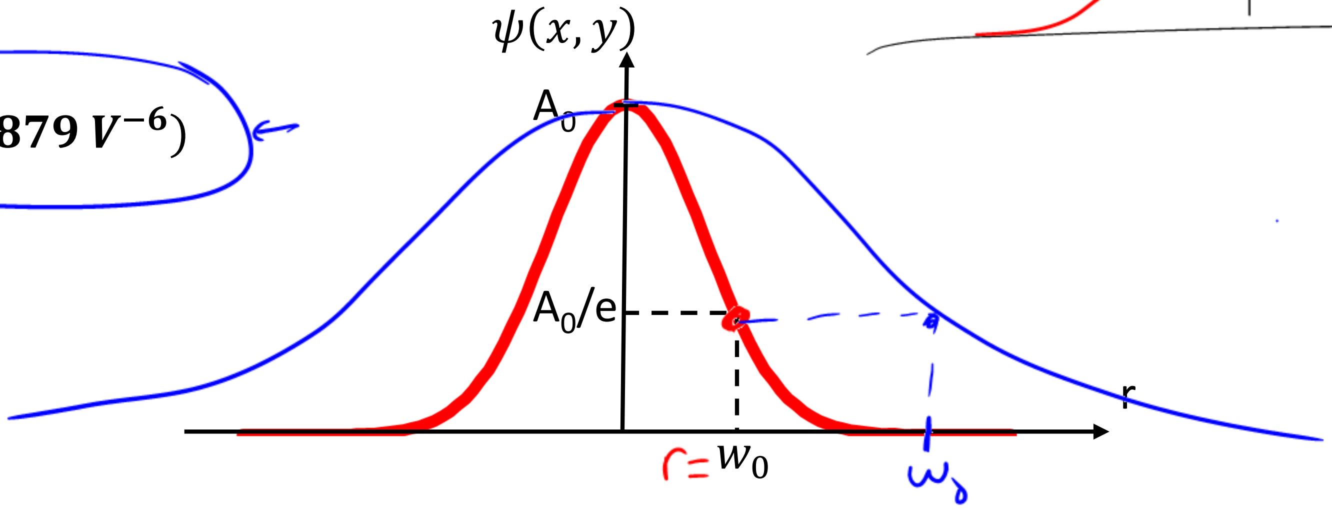
$r = w_0 \to$

w_0 is the mode field radius @ 1/e of the maximal amplitude (1/e² of the maximal intensity)

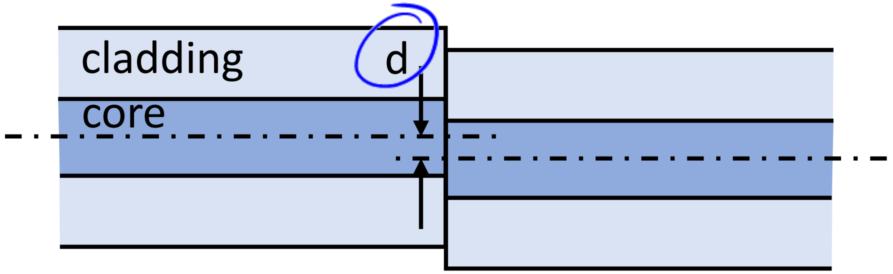
$$w_0 \approx \mathrm{a}\,(0,65 + 1.619\, V^{-\frac{3}{2}} + 2.879\, V^{-6})$$

with $V = k\,a\sqrt{n_1^2 - n_2^2}$

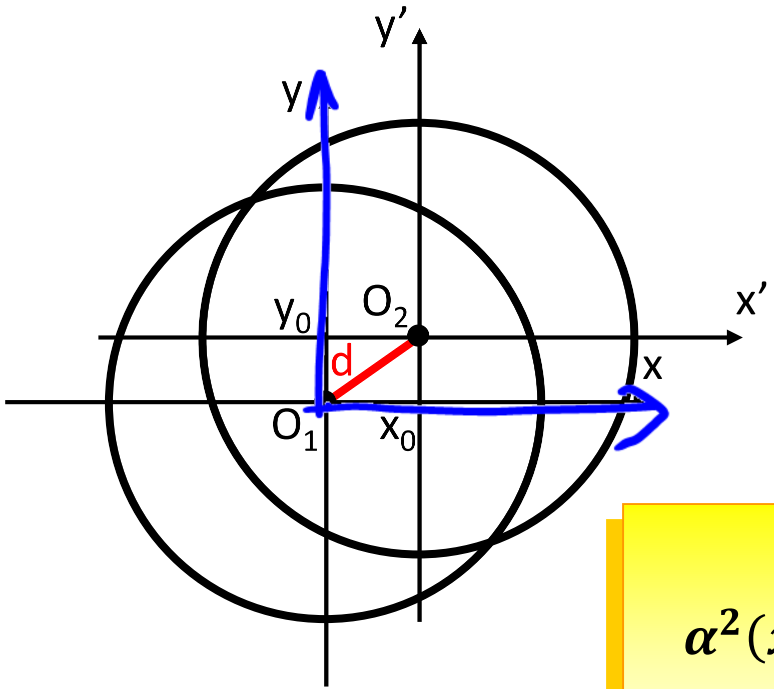
For $1.2 \le V \le 4$



Connexion losses between singlemode fibers (2)



- Tranversal offset (1)



$$\psi_1(x,y) = \psi(x,y)$$

$$\psi_2(x,y) = \psi(x-x_0, y-y_0)$$

are real fields

$$d = \sqrt{x_0^2 + y_0^2}$$

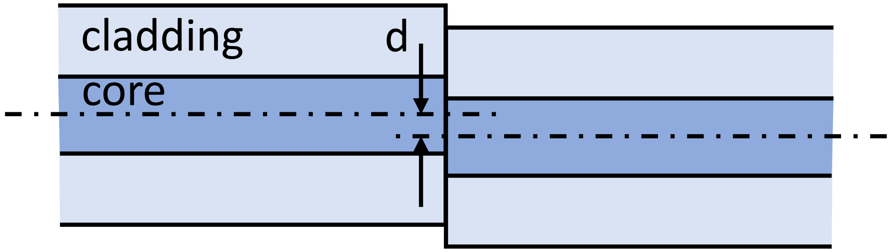
$$\alpha^2 = \frac{P_2}{P_1} = \frac{\left| \iint_{-\infty}^{+\infty} \psi(x,y)\psi(x-x_0,y-y_0)\,dx\,dy \right|^2}{\iint_{-\infty}^{+\infty} |\psi(x,y)|^2 dx\,dy \ \iint_{-\infty}^{+\infty} |\psi(x-x_0,y-y_0)|^2 dx\,dy}$$

$$= \iint_{-\infty}^{+\infty} \psi(x,y)$$

$$\boldsymbol{\alpha^2(x_0,y_0) = \frac{\left(\psi(x,y) * \psi(x,y)\right)^2_{x_0,y_0}}{\left(\psi(x,y) * \psi(x,y)\right)^2_{0,0}}}$$

0,0

Connexion losses between singlemode fibers (3)

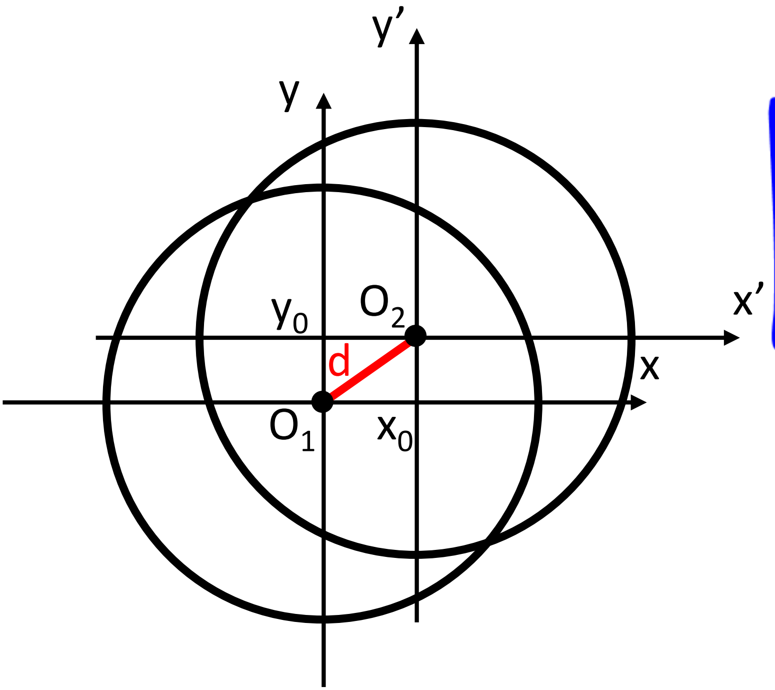


- Tranversal offset (2)

with

$$e^{-\pi x^2} \rightleftharpoons e^{-\pi u^2}$$

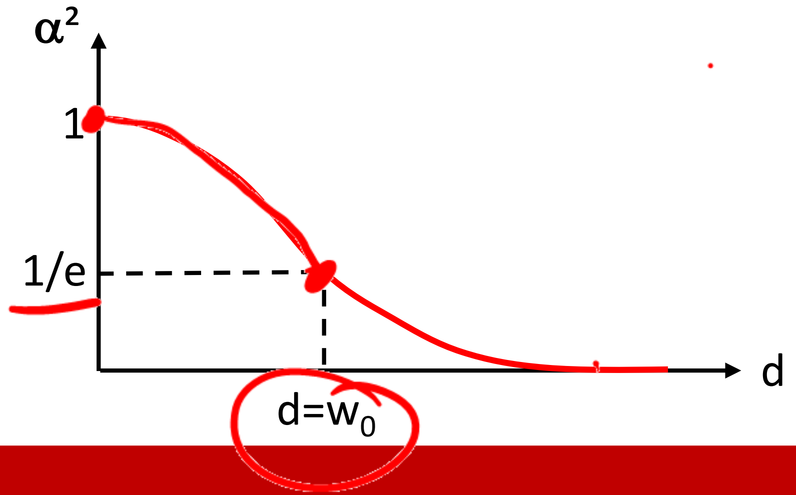
$$f(ax) \rightleftharpoons \frac{1}{|a|} F\left(\frac{u}{a}\right)$$



$$\left(\psi(x,y) * \psi(x,y)\right)^2_{x_0,y_0} = \frac{\pi w_0^2}{2} e^{-\frac{x_0^2+y_0^2}{w_0^2}}$$

$$\left(\psi(x,y) * \psi(x,y)\right)^2_{0,0} = \frac{\pi w_0^2}{2}$$

$$\boldsymbol{\alpha^2(x_0,y_0) = \frac{P_2}{P_1} = e^{-\frac{x_0^2+y_0^2}{w_0^2}} = e^{-\frac{d^2}{w_0^2}}}$$



Offset $\frac{d}{w_o}$	Case w_0=4.75µm	Losses (dB)
0.1	0.95 µm	0.04
0.15	1.4 µm	0.1
0.2	1.9 µm	0.17
0.3	2.85 µm	0.4
0.5	4.75 µm	1

Connexion losses between singlemode fibers (4)

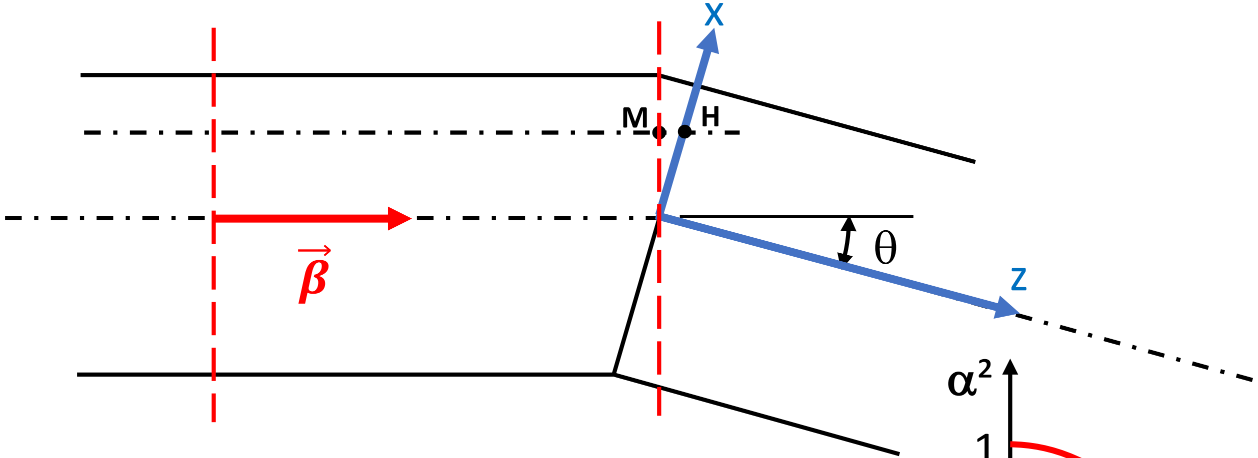
- Angular offset

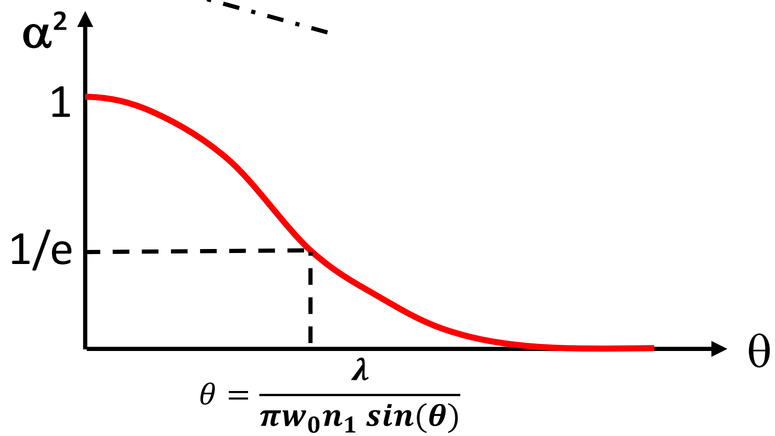
In the coordinates axes of fiber #2 :

$\psi_2(x,y) = \psi(x,y) = \psi_2^*(x,y)$ because is real

$\psi_1(x,y) = \psi(x,y)\, e^{j\beta\, z(x,y))}$ with $z(x,y) = HM$

$$\alpha^2(\theta) = \frac{P_2}{P_1} = e^{-\left(\frac{\pi w_0 n_1 \sin(\theta)}{\lambda}\right)^2}$$

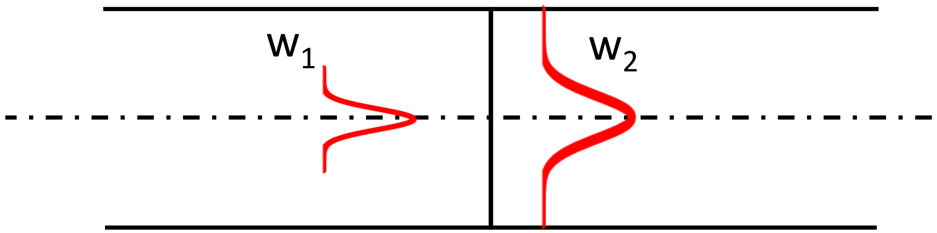




θ (deg) w_0=4.75µm L=1.55µm, n_1=1,45	Losses (dB)
0.1	0.0025
0.2	0.01
0.5	0.06
1	0,27
2	0,92

Connexion losses between singlemode fibers (5)

- Mode field diameter mismatch



In fiber #1

$$\psi_1(x,y) = A_0\, e^{-\frac{x^2+y^2}{w_1^2}}$$

In fiber #2

$$\psi_2(x,y) = A_0\, e^{-\frac{x^2+y^2}{w_2^2}}$$

$$\alpha^2 = \frac{P_2}{P_1} = \left(\frac{2w_1w_2}{w_1^2+w_2^2}\right)^2$$

if $w_1 = w_2 \rightarrow \alpha^2 = 1$

$\frac{w_2}{w_1}$	Losses (dB)
1.1	0.04
1.3	0.3
1.63	1
2	2

Connexion losses between singlemode fibers (6)

- Triple defects connection (transversal+angular+mode mismatch)

$$\boldsymbol{\alpha^2(d,\theta,w_1,w_2) = \frac{P_2}{P_1} = \left(\frac{2w_1w_2}{w_1^2+w_2^2}\right)^2 e^{-\left(\frac{\pi w_0 n_1 \sin(\theta)}{\lambda}\right)^2} e^{-\frac{d^2}{w_m^2}}}$$

With

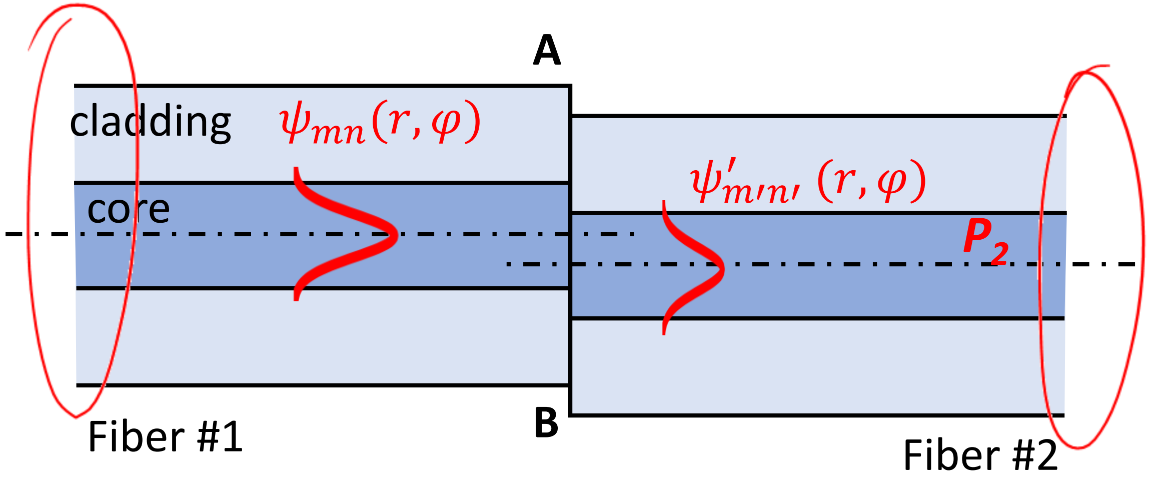
$$w_m^2 = \frac{w_1^2+w_2^2}{2}$$

$$w_0^2 = \frac{2w_1^2w_2^2}{w_1^2+w_2^2}$$

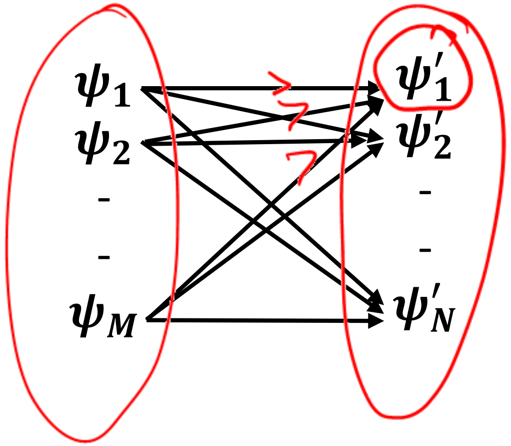
Connexion losses between multimode fibers (1)

- Power transfer at connection
  - Multimode fibers connection
    - M $LP_{m,n}$ modes guided in fiber #1
      $\psi_{mn}(r,\varphi)$
    - N $LP_{m',n'}$ modes guided in fiber #2
      $\psi'_{m'n'}(r,\varphi)$

MxN



    - MxN power coupling coefficients



$$\left(\alpha_{mn}^{m'n'}\right)^2 = \frac{P_{m'n'}}{P_{mn}} = \left|\iint_{-\infty}^{+\infty} \psi_{mn}(r,\varphi)\, \psi'^{*}_{m'n'}(r,\varphi)\, r\, dr\, d\varphi\right|^2$$

$$P_2 = \sum_{j=1}^{N} P_{m'n'} = \sum_{i=1}^{M} \sum_{j=1}^{N} \left(\alpha_{mn}^{m'n'}\right)^2 P_{mn}$$

Connexion losses between multimode fibers (3)

- Power coupling at transversally misaligned connection

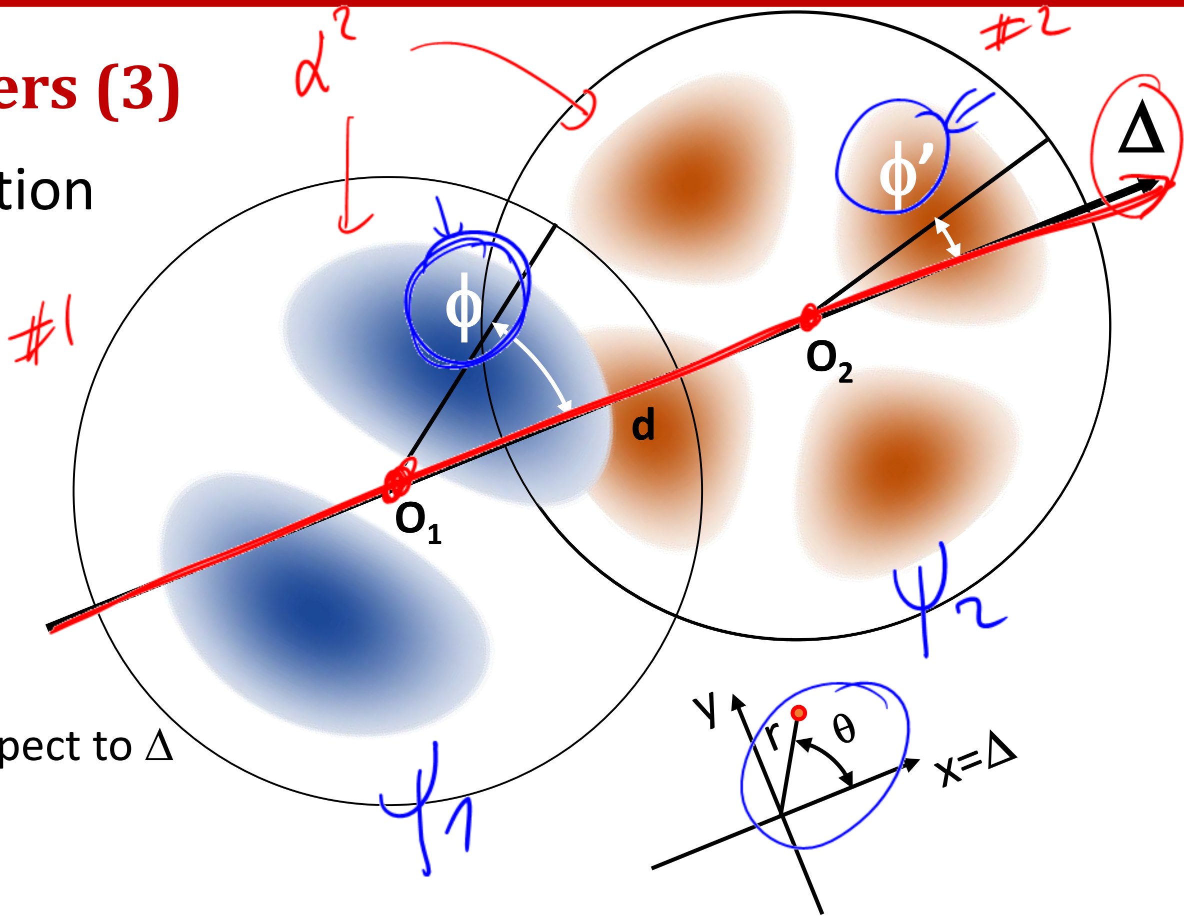
  - Connection description

    - d=O_1O_2
    - Δ : axis of the transversal misalignment
    - ϕ : azimuthal orientation of mode LP_{mn} (fiber #1) with respect to Δ

$$\Psi_{mn} = R(r).\cos(m(\theta - \phi))$$

    - ϕ' : azimuthal orientation of mode $LP_{m'n'}$ (fiber #2) with respect to Δ

$$\Psi'_{m'n'} = R'(r).\cos(m'(\theta - \phi'))$$



r
θ
Cylindrical coordinates

Connexion losses between multimode fibers (3)

- Fields decomposition

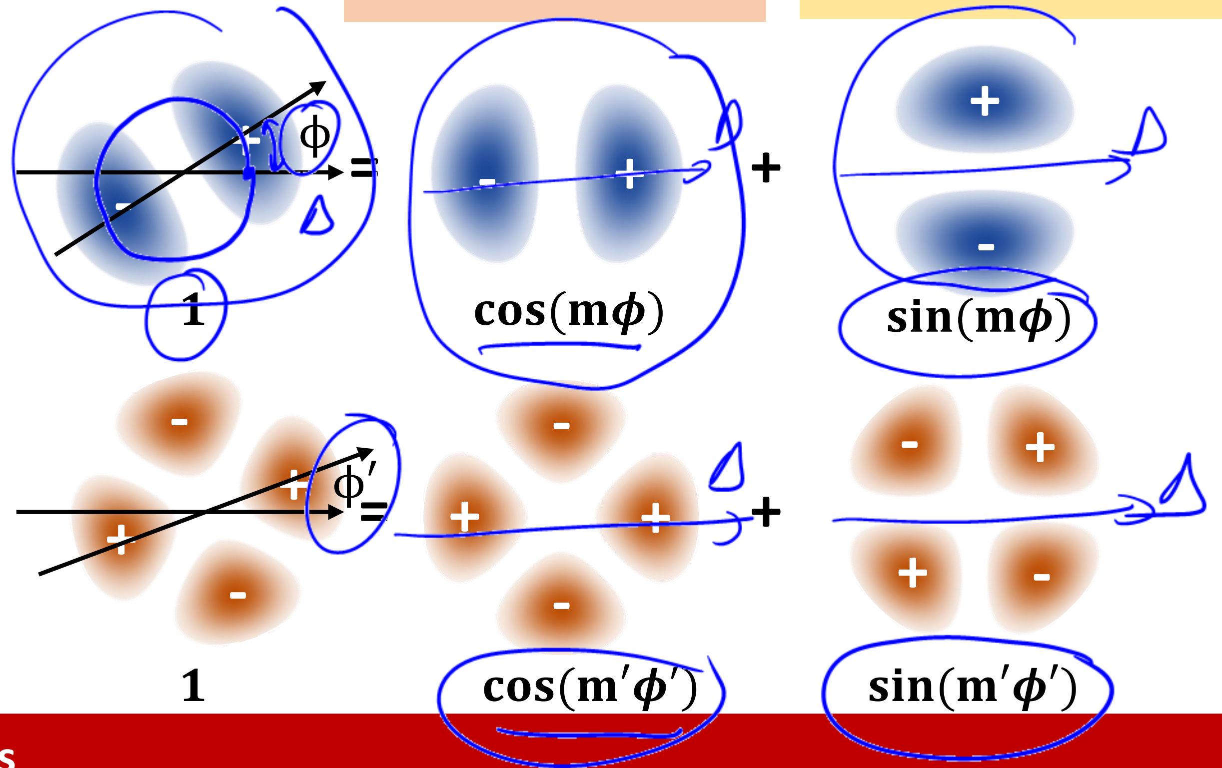
$$\Psi_{mn} = R(r)\,.\,\left(\cos(m\theta)\,\boldsymbol{\cos(m\phi)} + \sin(m\theta)\,\boldsymbol{\sin(m\phi)}\right)$$

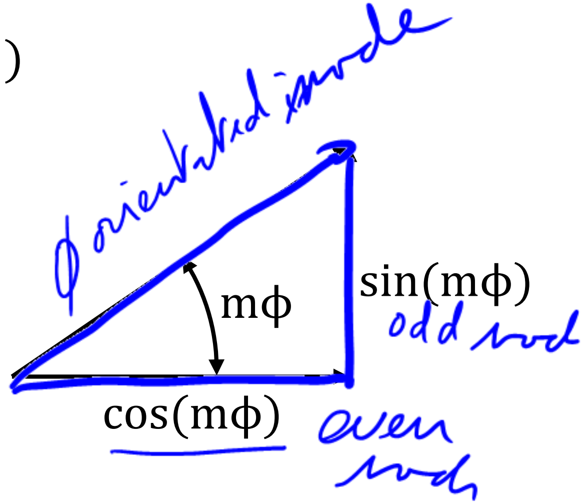
$$\Psi_{m'n'} = R'(r).\left(\cos(m'\theta)\,\boldsymbol{\cos(m'\phi')} + \sin(m'\theta)\,\boldsymbol{\sin(m'\phi')}\right)$$

Even modes | **Odd modes**

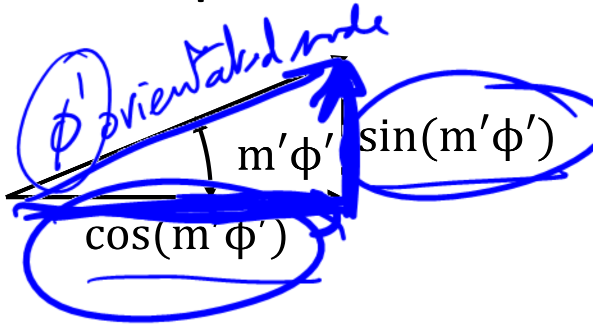
Input fiber

Output fiber





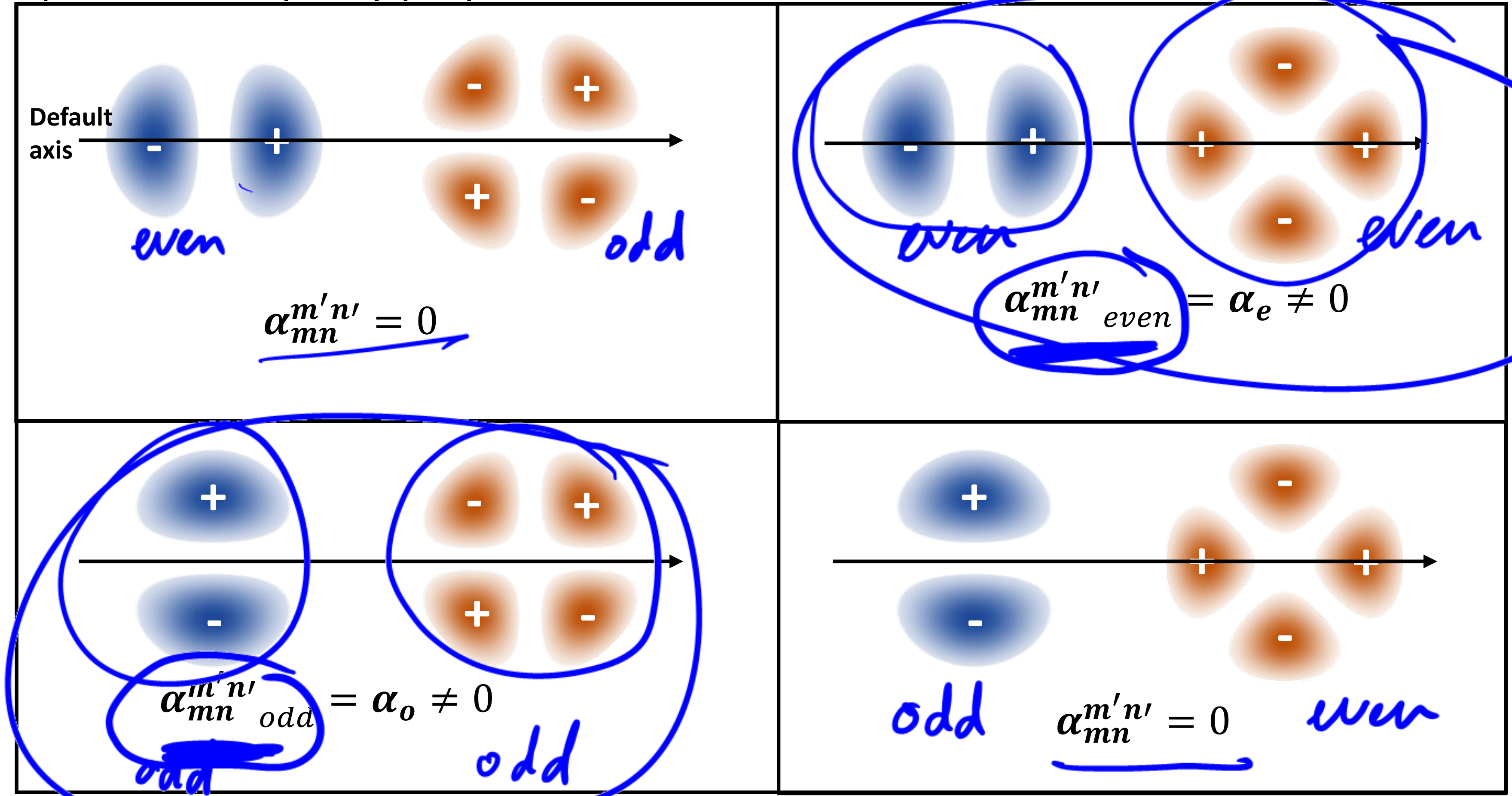
Fresnel representation



Connexion losses between multimode fibers (4)

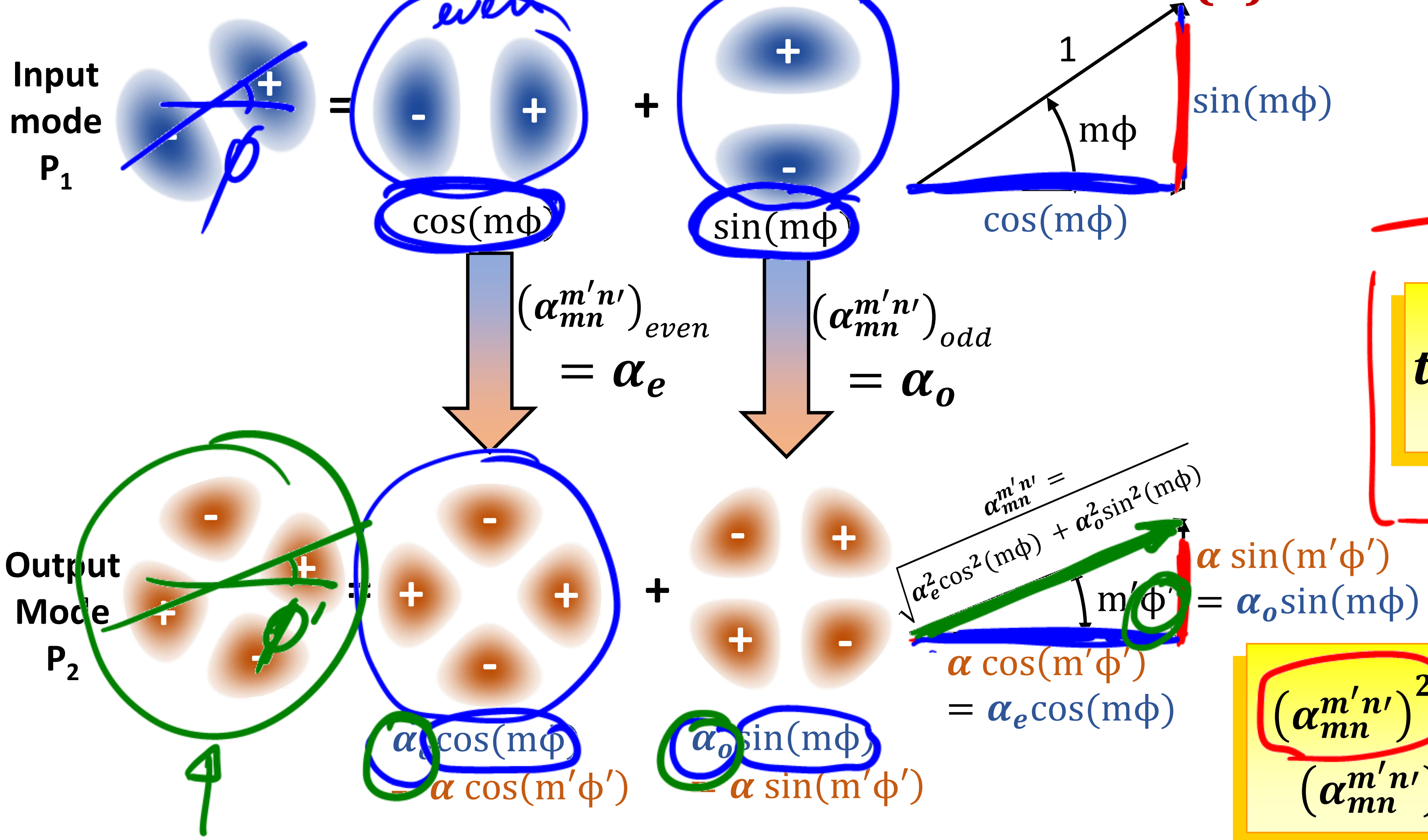
$$\alpha^2 = \frac{P_2}{P_1} = \left| \iint_{-\infty}^{+\infty} \psi_1(x,y)\, \psi_2^*(x,y)\, dx\, dy \right|^2$$

- Coupling properties from parity properties – 4 cases



$\alpha = \sqrt{\alpha^2}$

Connexion losses between multimode fibers (5)



$$P_2 = \left(\alpha_{mn}^{m'n'}\right)^2 P_1$$

$$tan(m'\phi') = \frac{\alpha_o}{\alpha_e} tan(m\phi)$$

The « tangents law »

$$\left(\alpha_{mn}^{m'n'}\right)^2 = \alpha_e^2\, cos^2(m\phi) + \alpha_o^2\, sin^2(m\phi)$$

$$\left(\alpha_{mn}^{m'n'}\right)^2 = \alpha_o^2 + \left(\alpha_e^2 - \alpha_o^2\right) cos^2(m\phi)$$