

Surname, Name, Matr. (ID): ..... Signature: .....

Answer to the questions carefully, and according to the order assigned in the text. An answer consisting of ONLY FEW LINES of text will be considered NOT sufficient. Therefore, try to describe the considered topic with a bit of details (it is expected an average value of (circa) one half /one page for every sub-question (circa 18 sub-questions), including diagrams and figures). If the hand written text and the general organization of the answers on the paper will not be CLEAN and CLEARLY written, and therefore difficult to be properly read and interpreted, the answer would NOT BE TAKEN into account in the final evaluation. Any NOT GIVEN or COMPLETELY WRONG answer will be taken into account negatively (i.e., producing a penalty (negative marks)) in the overall evaluation.

## Part1 (Exercises on Error Control Codes)

1.
  - A block code is characterized by the Generator matrix given in Figure. Determine the possible codewords. Is this a cyclic code ? What is the generator polinomial ? What is the minimum distance ?

$$G = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

- A (7,4) cyclic linear block code is described by the generator polynomial  $g(D) = D^3 + D^2 + 1$ . Indicate the values assumed by the "syndrome" associated to a possible single bit error, a possible two bits error, a possible three bits error. In case of 2 errors, how many different syndromes are possible ?
  - Consider the Hamming code with  $N = 127$ . Determine the number of possible codewords, the error probability in case of both hard (use the more precise estimation) and soft decoding, and the minimum bandwidth required to transmit 10 Mbit/sec.
2. Consider the linear block code of length  $N = 31$  and generator polynomial (in octal description) 107657. In this code there is 1 word composed by all zeros, 155 words with 7 ones, 465 with 8 ones, 5208 with 11 ones, ...
  - What is the generator polynomial of this code ( $g(D) = \dots$ ) ? Determine the number of possible codewords, and the probability of error (in case of hard and soft decision, using the more precise estimation).
  - The code is now extended adding a final parity check bit (imposing an EVEN number of "1"). Determine the new probability of error (in case of hard and soft decision, using the more precise estimation).
  - Consider the following possible codewords (of the extended code).
    - 000000000000001000111110101111001 is a valid codeword ?
    - 000000000000001111010111110001001 is a valid codeword ?
3.
  - Consider a convolutional code with  $R = 1/3$ , and octal generators (1, 2, 3). Determine and draw the trellis and tree diagram of the code.

- Determine the bit-error probability, considering at least 3 non zero terms in the union bound.
- Determine the code word associated to the information sequence: 010101100, and the minimal bandwidth required in case of an information bit-rate equal to 1 Mbit/sec.

## **Part2 (Theoretical Description)**

### 1. Optimal receiver

- Define and describe the likelihood function related to the optimal receiver in case of AWGN noise.
- Define and describe the equations related to the concept of "complex envelope".

### 2. OFDM

- Describe the analytical expression of an OFDM symbol and the block diagram of an OFDM encoder.
- Describe the channel equalization procedure performed in the OFDM modulation systems.

### 3. DSSS

- Describe why and when a DSSS modulation system is robust against multi-path fading.
- Describe the basic idea of the Rake Receiver, indicating also why this is working properly in the case of DSSS modulation.

### 4. CPM

- Describe the analytical expression of the likelihood function that should be maximized by the optimal receiver in the case of MSK modulation.

### 5. Turbo-LDPC Codes

- Describe the basic idea of the bit-flipping algorithm for the decoding of an LDPC code.
- Describe the basic idea and the motivation of the EXIT charts.