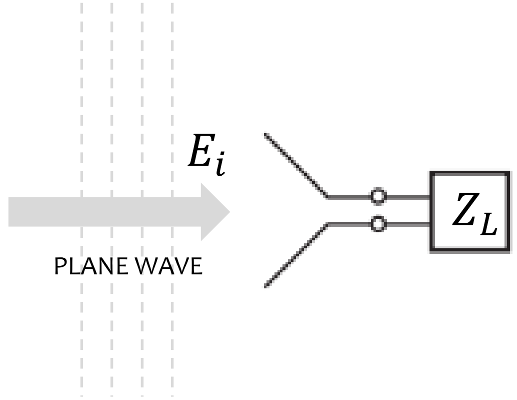
ANTENNAS IN COMMUNICATION SYSTEMS

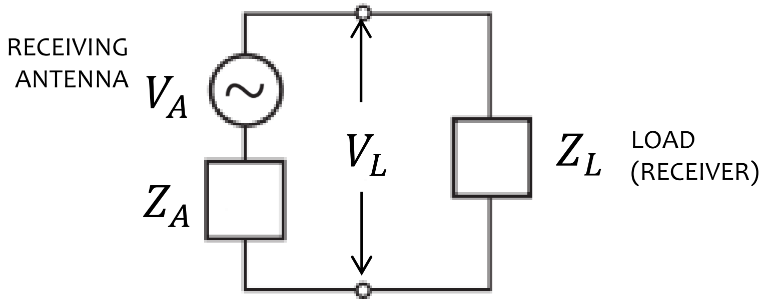
An antenna is the interface between the radio-frequency circuits of the transmitter or receiver and free space. Antenna specifications must be developed in concert with the full system specifications and any constraints due to limits on power consumption or radiated power.

EFFECTIVE LENGTH AND EFFECTIVE AREA

A receiving antenna converts the power density arriving from a distant source to a current on the connected circuit or transmission line. We can assume that the receiving antenna is in the far-field region of the transmitting antenna and thus the receiving antenna is illuminated by a plane wave. The receiving antenna transforms the incoming plane wave in an open-circuit voltage at its port/terminals: the receiving antenna is equivalent to a generator having an impedance Z_A and an open-circuit voltage V_A that is obtained by multiplying the amplitude E_i of the incoming plane wave by the **effective length** (or **effective height**) h of the antenna:

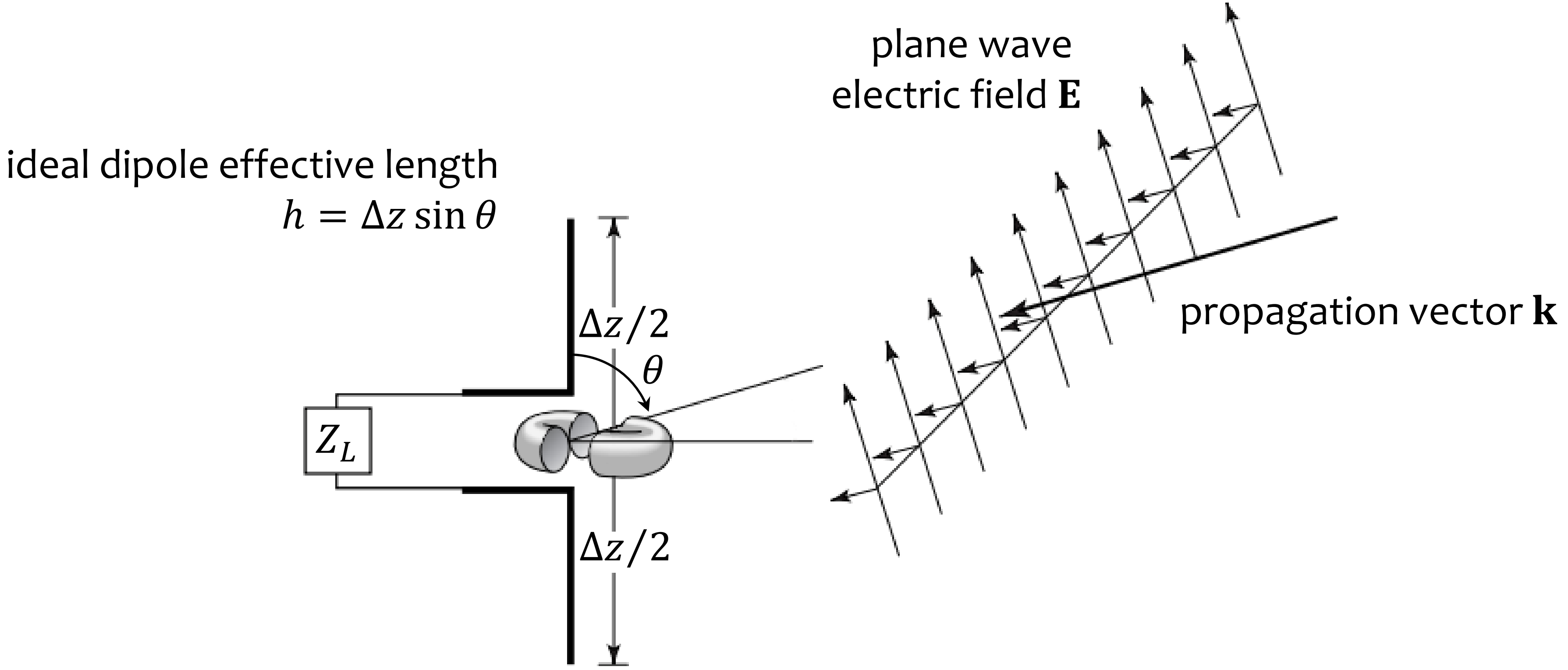
$$\boxed{h = \frac{V_A}{E_i}}$$





In other words, h (measured in meter) is the ratio of the induced open-circuit voltage V_A to incident electric field E_i. Here we assume that the incoming wave is linearly polarized and the antenna is oriented in such a way that the maximum of the main lobe of the radiation pattern is directed towards the approaching wave.

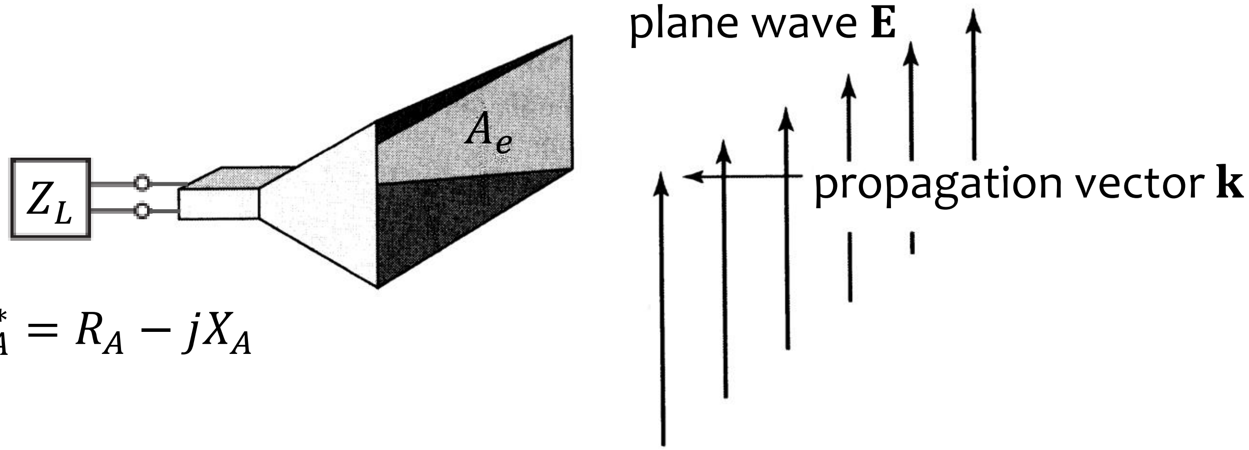
In the case of the ideal dipole it can be proved that the effective length is $h = \Delta z \sin\theta$, where θ is the tilt of the incoming wave propagation vector with respect to the dipole axis: h is the same as the projection of the physical length viewed from angle θ. When $\theta = 90°$ the effective length of the dipole is equal to its physical length $h = \Delta z$; in general, for a given antenna, the effective length does not coincide with one of its physical "lengths" or "widths".



The receiving antenna can also be viewed as converting the incident power density (magnitude of the Poynting vector, measured in W/m²) $S = |\mathbf{S}|$ into received power by the collecting area of the antenna, called **effective area** (or **effective aperture**) A_e (measured in m²), as expressed by the following relation where P_{rm} is the maximum available received power:

$$\boxed{P_{rm} = A_e S}$$

P_{rm} is realized if the main lobe is in the direction of the incoming wave, the antenna is polarization matched to the wave, and the antenna load impedance is conjugate matched to the antenna impedance (as this is the condition for the maximum power transfer to the load): $Z_L = Z_A^*$



$$Z_L = Z_A^* = R_A - jX_A$$

power density

$$S = \frac{1}{2\eta}|E|^2$$

The received real power (i.e. the power delivered to the load) reads as

$$P_L = \frac{1}{2} R_L |I_L|^2 = \frac{1}{2} R_L \left| \frac{V_A}{Z_A + Z_L} \right|^2 = \frac{1}{2} R_L \frac{|V_A|^2}{|(R_A + R_L) + j(X_A + X_L)|^2} = \frac{1}{2} R_L \frac{|V_A|^2}{(R_A + R_L)^2 + (X_A + X_L)^2}$$

If the conjugate matching condition is fulfilled $R_L = R_A$, $X_L = -X_A$ the received power is the maximum available power $P_L = P_{rm}$

$$P_{rm} = \frac{1}{2} R_L |I_L|^2 = \frac{1}{2} R_A \frac{|V_A|^2}{(R_A + R_A)^2 + (X_A - X_A)^2} = \frac{1}{8} \frac{|V_A|^2}{R_A}$$

Moreover, by definition of effective area the maximum received power is

$$P_{rm} = A_e \frac{1}{2\eta} |E|^2$$

By equating the two expression for P_{rm} the effective length h can be written as a function of the effective area A_e

$$P_{rm} = \frac{1}{8}\frac{|V_A|^2}{R_A} = \frac{1}{8}\frac{|hE|^2}{R_A} = A_e S = A_e \frac{1}{2\eta}|E|^2 \quad \Longrightarrow \quad \frac{1}{4}\frac{h^2}{R_A} = A_e \frac{1}{\eta}$$

$$h = 2\sqrt{\frac{R_A A_e}{\eta}}$$

Example: effective area of an ideal dipole (in spite of the fact that a short wire hasn't got a physical area where it collects the power of the electromagnetic field!)

$$A_e = \frac{P_{rm}}{S} = \frac{\frac{1}{8}\frac{|V_A|^2}{R_A}}{\frac{1}{2\eta}|E|^2} = \frac{1}{4}\frac{\eta}{R_A}\frac{h^2|E|^2}{|E|^2} = \frac{1}{4}\frac{\eta}{R_A}h^2 = \frac{1}{4}\frac{\eta}{2\frac{\pi}{3}\eta\frac{\Delta z^2}{\lambda^2}}\Delta z^2 = \frac{3}{8\pi}\lambda^2 \cong 0.119\lambda^2$$

By using the reciprocity of Maxwell's equations it can be proved that the following relation between gain and effective area is always true:

$$\boxed{G = \frac{4\pi}{\lambda^2} A_e}$$

In the lossless case, it follows that $G = D$ and $\lambda^2 = \Omega_A A_e$. For a fixed wavelength, effective area A_e and beam solid angle Ω_A are inversely proportional: as the maximum antenna effective area increases (for instance, as a result of increasing antenna physical size) the beam solid angle decreases, which means power is more concentrated in angular space.

Effective area and gain depend on the angular direction (i.e. the direction of the incoming wave or of the radiated far-field) and contain power pattern information:

$$G(\theta,\varphi) = G|F(\theta,\varphi)|^2 = \frac{4\pi}{\lambda^2} A_e(\theta,\varphi) = \frac{4\pi}{\lambda^2} A_e |F(\theta,\varphi)|^2 \qquad G = \max[G(\theta,\varphi)] \qquad A_e = \max[A_e(\theta,\varphi)]$$

We introduced the relationship between G and A_e having in mind a receiving antenna, but it also applies to transmitting antennas as ensured by reciprocity.

Example: let's verify the relationship between G and A_e for a lossless ideal dipole

$$A_e = \frac{3}{8\pi}\lambda^2 = \frac{\lambda^2}{4\pi}\frac{3}{2} = \frac{\lambda^2}{4\pi}D = \frac{\lambda^2}{4\pi}G$$

Effective length h and effective area A_e can be used to describe the behavior of any receiving or transmitting antenna, but we can observe that for wire antennas h is proportional to the wire length (and it is equal to the wire length if the current is uniform) and that for antennas having a large aperture, as horns and parabolic antennas, the effective area is proportional to the area of the antenna aperture.

FRIIS TRANSMISSION FORMULA

We want to describe the power transfer in a radio link: we consider a transmitting and a receving antenna, separated by a distance R, and we need to calculate the received power P_R as a function of the power P_T fed to the transmitting antenna. Initially, we assume that the main lobes of both antennas are aligned.

If the transmitting antenna were an isotropic radiator the power density S at the position of the receiver would be readily calculated

$$S = \frac{P_T}{4\pi R^2}$$

Since the transmitting antennas has a gain G_T the power density S on the receiving antenna is increased by the gain

$$S = G_T \frac{P_T}{4\pi R^2}$$

By using the effective area A_{eR} of the receiving antenna, the available received power P_R reads as

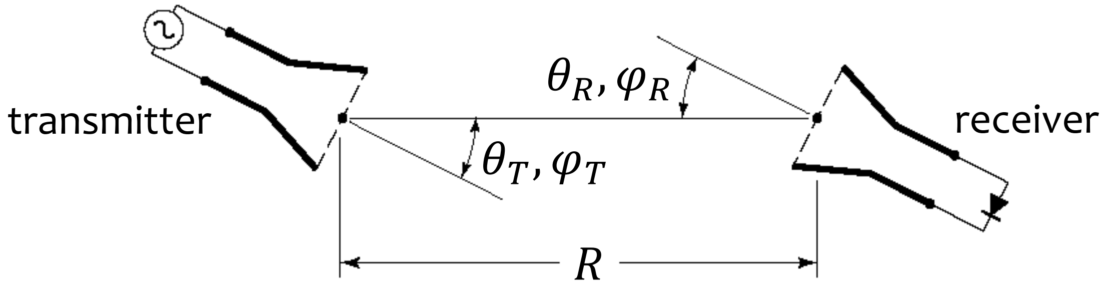
$$P_R = A_{eR} S = A_{eR} G_T \frac{P_T}{4\pi R^2}$$

and since the effective area is proportional to gain $A_{eR} = G_T\,\lambda^2/4\pi$, the formula for the power budget can be rearranged to show the gain of both antennas and it is named **Friis transmission formula**

$$\boxed{P_R = G_T G_R \left(\frac{\lambda}{4\pi R}\right)^2 P_T}$$

Sometimes the factor $(\lambda/4\pi R)^2$ is called free space loss, but this term is misleading because no power is dissipated (free space has no ohmic losses).

If the antennas are not aligned in the direction of their main lobes, the maximum gains have to be replaced by the gains in the corresponding directions: in the following drawing the transmission is in the direction θ_T, φ_T and the receiving antenna is irradiated from the direction θ_R, φ_R (in the reference frame of the receiver)

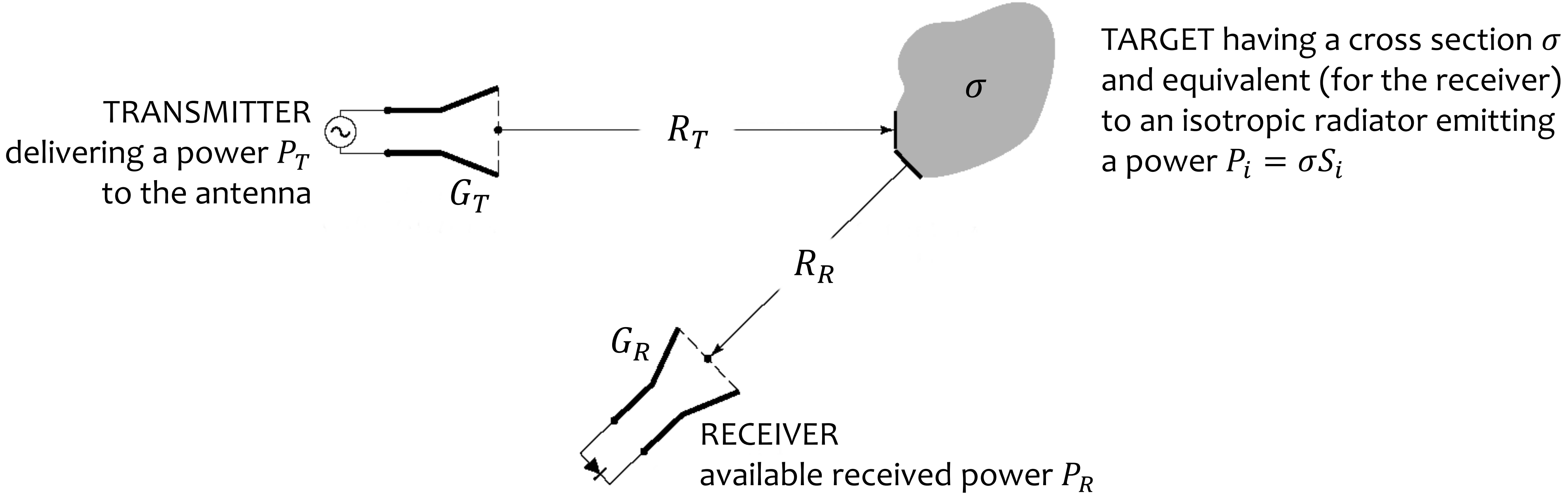


$$\boxed{P_R = G_T(\theta_T, \varphi_T) G_R(\theta_R, \varphi_R) \left(\frac{\lambda}{4\pi R}\right)^2 P_T}$$

RADAR EQUATION

The power budget can be easily calculated for a RADAR (RAdio Detection And Ranging), as well.

We consider the general case of a bistatic radar in which the transmitting and receiving antennas are physically separated.



The power density incident on the radar target is given by $S_i = G_T P_T/(4\pi R_T^2)$ and the power intercepted by the target is proportional to S_i

$$\boxed{P_i = \sigma S_i} \qquad\qquad S_R = \frac{P_i}{4\pi R_R^2} = \sigma \frac{1}{4\pi R_R^2} S_i$$

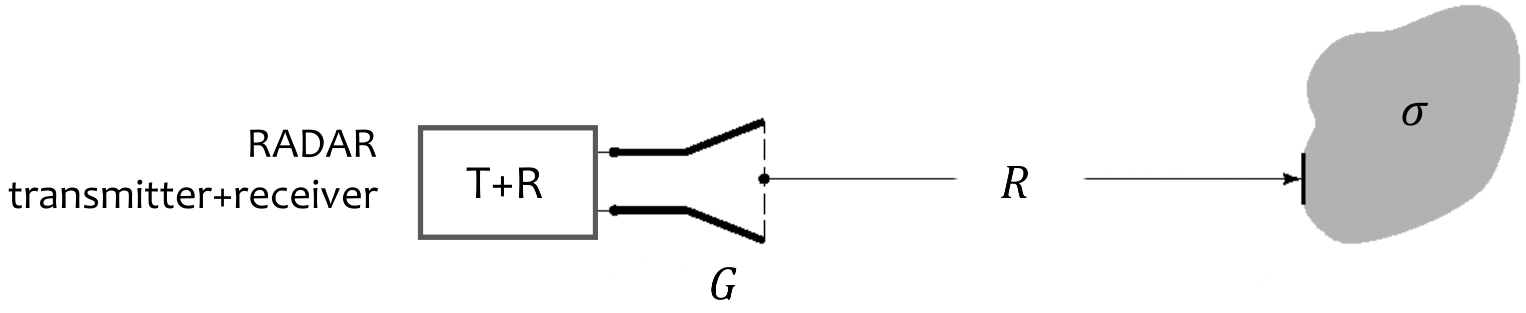
where σ is the **radar cross section** (measured in m²), defined as the area intercepting that amount of power P_i which, when scattered isotropically, produces at the receiving antenna a power density S_R which is equal to that scattered by the actual target. Although the intercepted power P_i is not really scattered isotropically, we are only concerned about the power scattered in the direction of the receiver and can assume the target scatters isotropically. The power density illuminating the receiver reads as

$$P_R = A_{eR} S_R = G_R \frac{\lambda^2}{4\pi} \sigma \frac{1}{4\pi R_R^2} S_i = G_R \frac{\lambda^2}{4\pi} \sigma \frac{1}{4\pi R_R^2} G_T \frac{1}{4\pi R_T^2} P_T$$

Finally the **radar equation** is

$$\boxed{P_R = \sigma G_T G_R \frac{\lambda^2}{(4\pi)^3 R_T^2 R_R^2} P_T}$$

If the transmitter and the receiver of the radar share the same antenna we obtain a <u>monostatic radar</u>



TARGET having a cross-section σ and equivalent (for the receiver) to an isotropic radiator emitting a power $P_i = \sigma S_i$

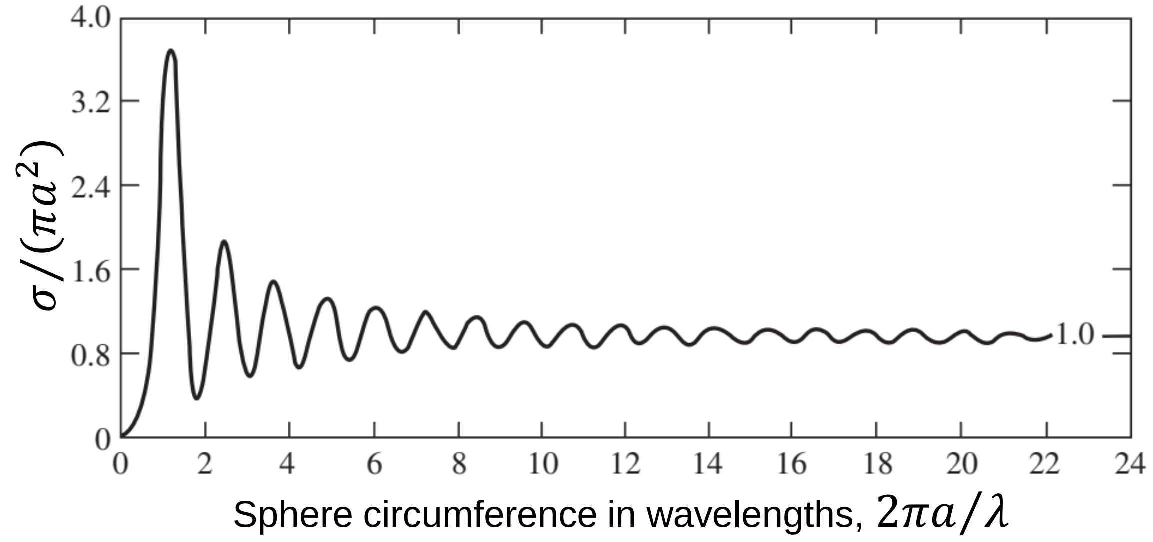
and the radar equation reads as follows

$$\boxed{P_R = \sigma G^2 \frac{\lambda^2}{(4\pi)^3 R^4} P_T}$$

We underline that the received power is inversely proportional to the fourth power of the distance R between radar and target.

The radar cross section (RCS) σ of a target is a function of the angle of incidence, the angle of observation, the shape of the target, the electromagnetic parameters of the target (conductivity, dielectric permittivity, magnetic permeability) and the frequency of operation. Only for a few basic geometries it is possible to calculate the RCS: for instance, for a metallic sphere whose radius a is much larger than the wavelength the RCS is equal to the sphere physical cross section $\sigma = \pi a^2$. The RCS can be obtained by means of numerical simulations and by performing measurements on the target or on a small-scale model of the target itself.

RCS of a metallic sphere



Typical monostatic RCS in the X band (8-12 GHz)

Target	Radar Cross Section
Truck	200 m^2
Car	100 m^2
Jumbo jet airliner	100 m^2
Cabin cruiser boat	10 m^2
Fighter aircraft	6 m^2
Adult man	1 m^2
Bird	10^{-2} m^2
Insect	10^{-5} m^2

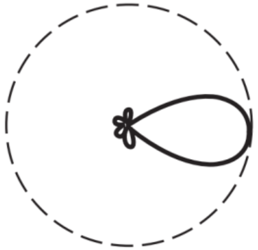
EFFECTIVE ISOTROPICALLY RADIATED POWER

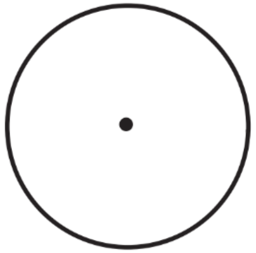
A frequently used concept in radiofrequency systems is that of **effective isotropically radiated power** (**EIRP**): which is the amount of power emitted from an isotropic antenna to obtain the same power density in the direction of the actual antenna pattern peak with gain G_T. EIRP is simply the gain of the transmitting antenna multiplied by the net power accepted by the antenna from the connected transmitter:

$$\boxed{\mathrm{EIRP} = G_T P_T}$$

In other words, to obtain the same radiation intensity produced by the directional antenna in its pattern maximum direction, an isotropic antenna would require an input power G_T times greater.

directional antenna with gain G_T and input power P_T





isotropic antenna with gain 1 and input power $G_T P_T$

If the reference antenna is the half-wavelength resonant wire and the losses in the transmission line from the transmitter to the antenna are included, we obtain a new parameter called **effective radiated power** (**ERP**).