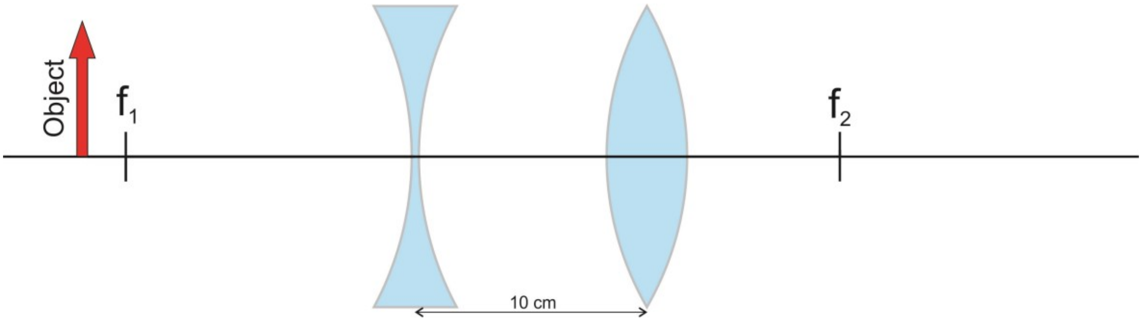
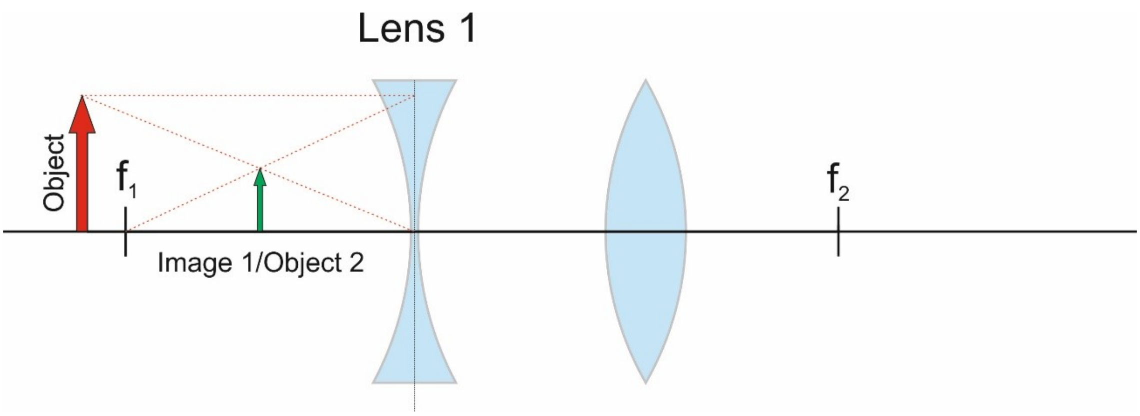
Exercise 1. An object of high 4 cm is placed at a distance of 17 cm from a divergent lens of focal 150 mm. A second convergent lens of focal 10 is placed at 12 cm of the first lens.

a) **Draw the system and realise the raytracing.**
b) **Calculate the position of the image and describe it.**

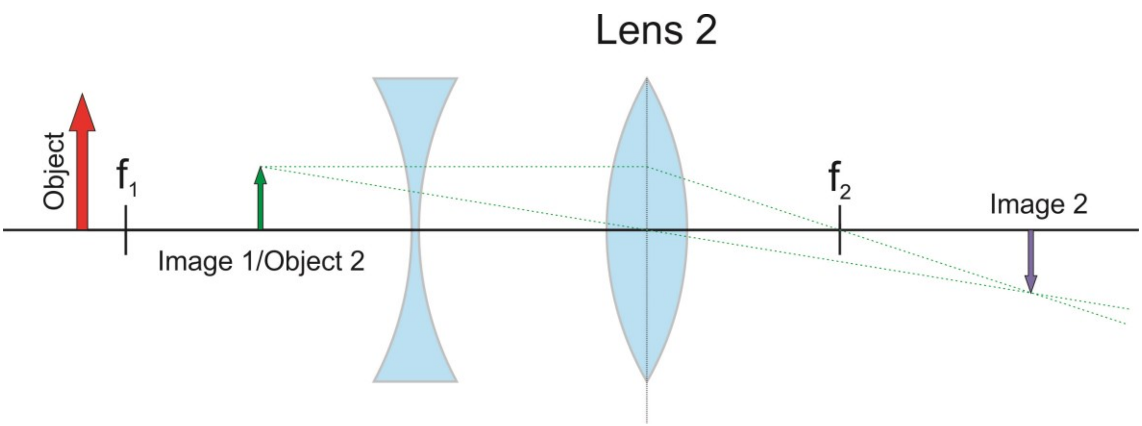
a) First of all, we must place all the items in the same frame reference, for that we can use the object as origin and start adding distances (the object is not scaled).



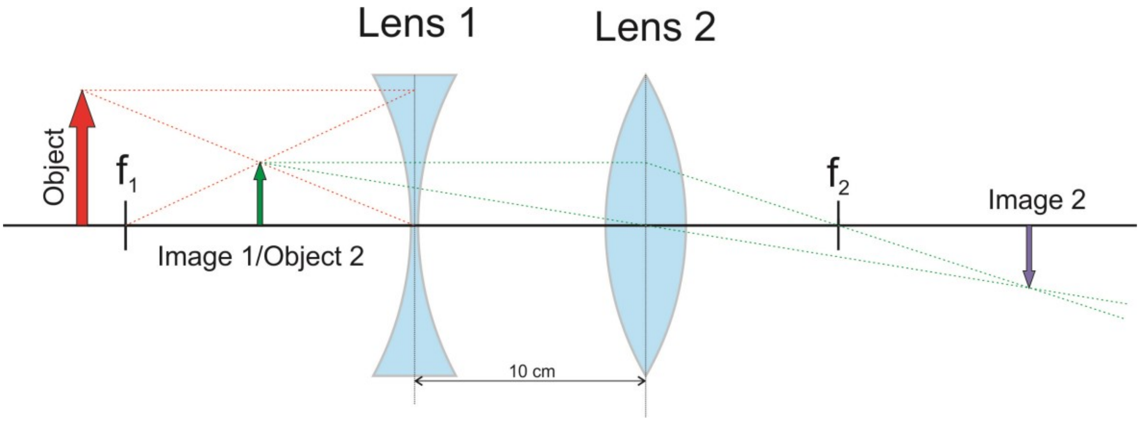
As we have been told , the first lens is divergent (so negative) $f_1 = -15\ cm$ and the second one is convergent (positive) $f_2 = 10\ cm$. These distances are relative to the centre of the lens. Also the displacement between lens is 10 cm. After this we can perform the ray tracing by using the main rays that cross the centre of the lens and the one that goes parallel to the axis and then crosses the focal. For the first part we have:



And the image of the first lens acts as object of the second one, therefore:



So putting all together we end with the following drawing.



b) From the ray tracing we have an idea of the values that our position must have. The first image is not flipped, it is behind the lens (virtual) and it is smaller than the original. Meanwhile, the final image is flipped, after the second lens (real) and almost the same size as the second image. Following the same path as before we calculate in two steps.

For lens 1:

$$p_1 = -17\ cm$$
$$f_1 = -15\ cm$$

$$\frac{1}{q_1} - \frac{1}{p_1} = \frac{1}{f_1} \quad ,, \quad \frac{1}{q} = \frac{1}{f} + \frac{1}{p} \;=\; \frac{1}{-15} + \frac{1}{-17} \approx \frac{1}{-8}$$

$$q_1 = -8\ cm$$

As there is a displacement between both lens, the position of the object for the second one is displaced. For lens 2:

$$p_2 = q_1 - 10 = -20\ cm$$
$$f_2 = 10\ cm$$

$$\frac{1}{q_2} - \frac{1}{p_2} = \frac{1}{f_2} \quad ,, \quad \frac{1}{q} = \frac{1}{f} + \frac{1}{p} \;=\; \frac{1}{10} + \frac{1}{-20} = \frac{1}{20}$$

$$q_1 = 20\ cm$$

Finally, the magnifying factor for each case is:

$$M_1 = \frac{q_1}{p_1} = \frac{-8}{-15} \approx 0.53$$

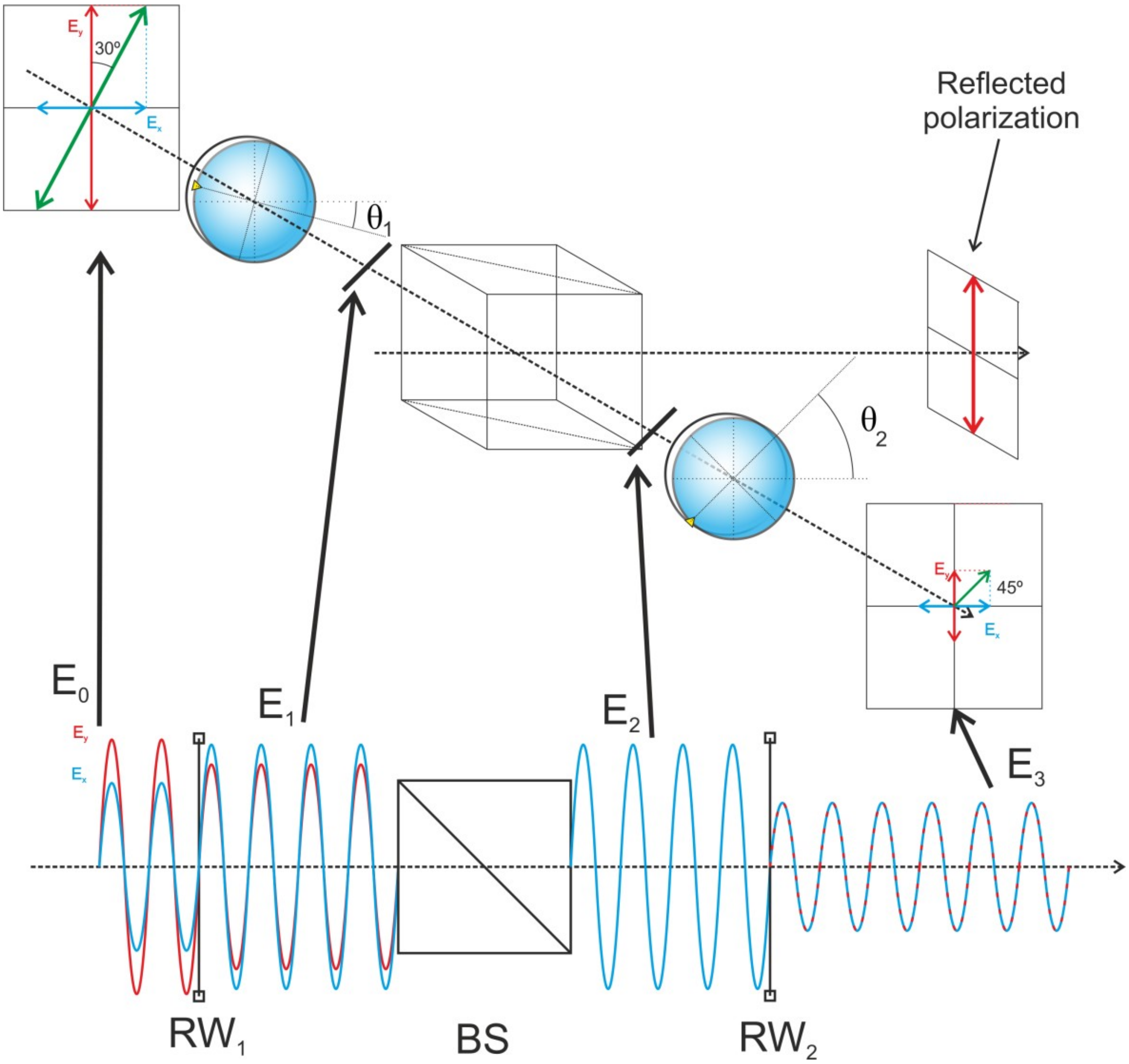
$$M_2 = \frac{q_2}{p_2} = \frac{20}{-20} = -1$$

$$M = M_1 M_2 = -0.53$$

$$h_2 = M h_0 = -0.53 * 4 \approx -2.1\ cm$$

Exercise 2. We have a linearly polarized beam at 30º respect to the vertical, we want to reduce its intensity by a 75% and change the angle of polarization to 45º. For this purpose, we use two half-lambda wave plates and a polarised beam splitter with 100% transmission in the horizontal polarization and 100% reflectance in the vertical polarization What are the angles of the wave-plates needed to obtain this result.

First off all we should draw a schematic system of what we have like in the following image.



This will help us to identify each angle and position. Note that we have designed with the letter θ_i the angel of rotation of the waveplates and φ_i the angle of polarization in each case.

$$rotation \longrightarrow \theta_i$$

$$delay\ in\ the\ field \longrightarrow \phi_i$$

$$angle\ of\ the\ field \longrightarrow \varphi_i$$

Now we must about the conditions that we have impose.

1) The final intensity must be 25% (100%-75%) of the initial. In other words

$$I_3 = 0.25 * I_0$$

2) The final polarization angle should be 45º to the vertical. This is going to translate into that both polarizations, $E_{x,3}$ and $E_{y,3}$, are equal. Wrote properly as

$$\overrightarrow{E_3} = \begin{bmatrix} E_{x,3} \\ E_{y,3} \end{bmatrix} = \begin{bmatrix} E_3 \cos\varphi_3 \\ E_3 \sin\varphi_3 \end{bmatrix} = \frac{\sqrt{2}}{2} E_3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

3) The initial beam is linearly polarized and has a 30º with the vertical, so $\varphi_0 = 90 - 30$. As it is linearly polarized, the delay between both polarization angles $\phi = 0$ and this is going to be preserved in all the system because there is no element introducing a delay other than the half-plates.

$$\overrightarrow{E_0} = \begin{bmatrix} E_{x,0} e^{i\phi} \\ E_{y,0} \end{bmatrix} = \begin{bmatrix} E_{x,0} \\ E_{y,0} \end{bmatrix} = \begin{bmatrix} E_0 \cos\varphi_0 \\ E_0 \sin\varphi_0 \end{bmatrix} = E_0 \frac{1}{2} \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$$

Schematically now we can propose the values of the field at the different positions as:

$$0) \Rightarrow \begin{cases} \overrightarrow{E_0} = E_0 \frac{1}{2} \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} \\ I_0 = E_0^2 \end{cases}$$

$$1) \Rightarrow \begin{cases} \overrightarrow{E_1} = \begin{bmatrix} E_{x,1} \\ E_{y,1} \end{bmatrix} \\ I_1 = E_{x,1}^2 + E_{y,1}^2 \end{cases}$$

$$2) \Rightarrow \begin{cases} \overrightarrow{E_2} = \begin{bmatrix} E_{x,2} \\ E_{y,2} \end{bmatrix} \\ I_2 = E_{x,2}^2 + E_{y,2}^2 \end{cases}$$

$$3) \Rightarrow \begin{cases} \overrightarrow{E_3} = \frac{\sqrt{2}}{2} E_3 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ I_3 = E_3^2 \end{cases}$$

Now we need to describe mathematically the different components. Our set-up is simple and it includes 2 half-lambda waveplates that can be rotated an angle. We can describe a rotated waveplate RW multiplying a rotation matrix RT with an angle θ_i and the waveplate matrix WP with a delay $\phi_i = \pi$

$$RW_i = RT_i * WP = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix} * \begin{bmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\cos\theta_i & -\sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{bmatrix} \quad i = 1,2$$

The other component is a polarised beam splitter that let cross all the horizontal component ($A = 1$) and reflects all the vertical ($B = 0$).

$$BS = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

The relation between fields is given by these matrixes

$$\overrightarrow{E_3} = RW_2 * \overrightarrow{E_2}$$

$$\overrightarrow{E_2} = BS * \overrightarrow{E_1}$$

$$\overrightarrow{E_1} = RW_1 * \overrightarrow{E_0}$$

It is important to know that, since its determinant is 1, both RW_i are conservative matrixes while the beam splitter is not.

$$\det(RW_i) = 1$$

$$\det(BS) \neq 1$$

That means that the intensity is conserved but between $\overrightarrow{E_2}, \overrightarrow{E_1}$

$$I_0 = I_1$$

$$I_2 = I_3$$

At this point we can just apply all the equations and calculated the angles θ_i

$$\begin{cases} \overrightarrow{E_3} = RW_2 * BS * RW_1 * \overrightarrow{E_0} \\ I_3 = 0.25 * I_0 \end{cases}$$

Substituting

$$\begin{cases} \frac{\sqrt{2}}{2} E_3 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\cos\theta_2 & -\sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -\cos\theta_1 & -\sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix} E_0 \frac{1}{2} \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} \\ E_3^2 = \frac{1}{4} * E_0^2 \end{cases}$$

$$\begin{cases} \sqrt{2} E_3 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\cos\theta_2 & -\sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} -\cos\theta_1 & -\sin\theta_1 \\ 0 & 0 \end{bmatrix} E_0 \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} \\ E_3 = \frac{1}{2} * E_0 \end{cases}$$

$$\frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 \cos\theta_2 & \sin\theta_1 \cos\theta_2 \\ \cos\theta_1 \sin\theta_2 & \sin\theta_1 \sin\theta_2 \end{bmatrix} \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$$

Which leads to 2 equations:

$$\begin{cases} \frac{\sqrt{2}}{2} = \cos\theta_1 \cos\theta_2 + \sqrt{3} \sin\theta_1 \cos\theta_2 \\ \frac{\sqrt{2}}{2} = \cos\theta_1 \sin\theta_2 + \sqrt{3} \sin\theta_1 \sin\theta_2 \end{cases}$$

$$\begin{cases} \frac{\sqrt{2}}{2} = (\cos\theta_1 + \sqrt{3} \sin\theta_1) * \cos\theta_2 \\ \frac{\sqrt{2}}{2} = \left(\cos\theta_1 + \sqrt{3} \sin\theta_1\right) * \sin\theta_2 \end{cases}$$

Diving both equations, we obtain the second angle:

$$1 = \frac{\cos\theta_2}{\sin\theta_2} \Longrightarrow \theta_2 = \frac{\pi}{4} = 45º$$

And with this result we obtain the second condition:

$$\frac{\sqrt{2}}{2} = (\cos\theta_1 + \sqrt{3} \sin\theta_1) * \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} (\cos\theta_1 + \sqrt{3} \sin\theta_1)$$

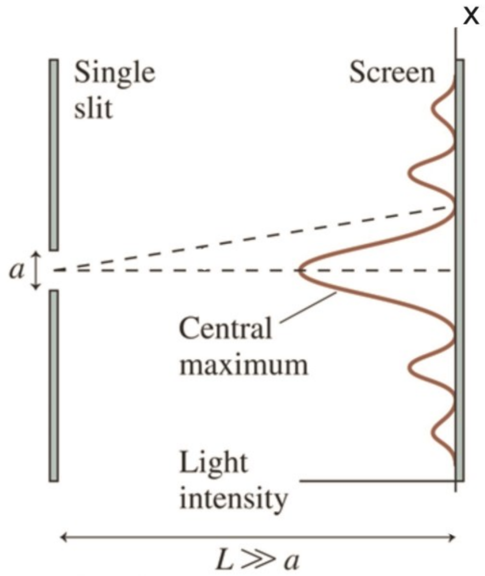
$$1 = (\cos\theta_1 + \sqrt{3} \sin\theta_1)$$

$$\frac{1}{\sqrt{3}} = -\tan \theta_1$$

$$\theta_1 = -\frac{\pi}{6} = -30º$$

Exercise 3. Deduce the intensity profile at a distance $z = L \gg x, y$ produced by the diffraction of a planar wave on a single slit of width $a \sim \lambda$

The experiment is as simple as the figure shows. A planar wave faces a wall with a slit in one direction with a width similar to the wavelength $a \sim \lambda$. We analyse the intensity pattern in a plane at a distance L in the z axis (which is the propagation axis).



The Huygens principle says that each point of the front-wave in the slit will act as a new source like:

$$E(r,t) = \frac{A}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + z^2}} e^{i(k\sqrt{(x-x_0)^2+(y-y_0)^2+z^2} - \omega t)}$$

$$x_0 \ni \left[-\frac{a}{2}, \frac{a}{2}\right] \qquad\qquad y_0 \ni [-\infty, \infty]$$

In order to calculate the total field as at distance $z = L$ of the slit plane we need to integrate over the whole slit extension

$$E(x,y,L,t) = \int_{-a/2}^{a/2} \int_{-\infty}^{\infty} \frac{A}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + L^2}} e^{i(k\sqrt{(x-x_0)^2+(y-y_0)^2+L^2} - \omega t)} dx_0 dy_0$$

To perform this integral, we need to consider that we are working close to the axis of propagation, that means that the distance between the slit and our plane is a lot bigger than the range of x and y that we are observing, this is reference as the *paraxial approximation* or the *Fraunhofer regime*. Under this regime we can approximate the value of r to:

$$L \gg x, y, x_0, y_0$$

$$r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + L^2} = L\sqrt{1 + \frac{(x-x_0)^2}{L^2} + \frac{(y-y_0)^2}{L^2}}$$

$$r \approx L - \frac{1}{2}L\left[\frac{(x-x_0)^2}{L^2} + \frac{(y-y_0)^2}{L^2}\right] = L - \frac{(x-x_0)^2 + (y-y_0)^2}{2L}$$

So the integral ends as:

$$E(x,y,L,t) = \frac{Ae^{i(kL-\omega t)}}{L} \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\infty}^{\infty} e^{-ik\frac{(x-x_0)^2+(y-y_0)^2}{2L}} dx_0 dy_0$$

Expanding the terms in x and solving the integral in y, $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$, we obtain:

$$E(x,y,L,t) = \frac{Ae^{i(kL-\omega t)}e^{-\frac{ikx^2}{2L}}}{2L}\sqrt{\frac{\pi}{ik}} \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{-ik\frac{(x_0^2-2xx_0)}{2L}} dx_0$$

Once again, as $kx_0^2 \ll L$, we can approximate $e^{-\frac{ikx_0^2}{2L}} \approx 1$

$$E(x,y,L,t) = \frac{Ae^{i(kL-\omega t)}e^{-\frac{ikx^2}{2L}}}{2L}\sqrt{\frac{\pi}{ik}}\left[\frac{e^{\frac{ikax}{2L}} - e^{\frac{-ikax}{2L}}}{\frac{ikx}{L}}\right]$$

Finally we just use the definitions of $\sin x = \frac{e^{ix}-e^{-ix}}{2i}$, $sinc(x) = \frac{\sin x}{x}$ and $k = \frac{2\pi}{\lambda}$.

$$E(x,y,L,t) = \frac{Ae^{i(kL-\omega t)}e^{-\frac{ikx^2}{2L}}}{2L} a\sqrt{\frac{\pi}{ik}} sinc\left(\frac{\pi a}{\lambda L}x\right)$$

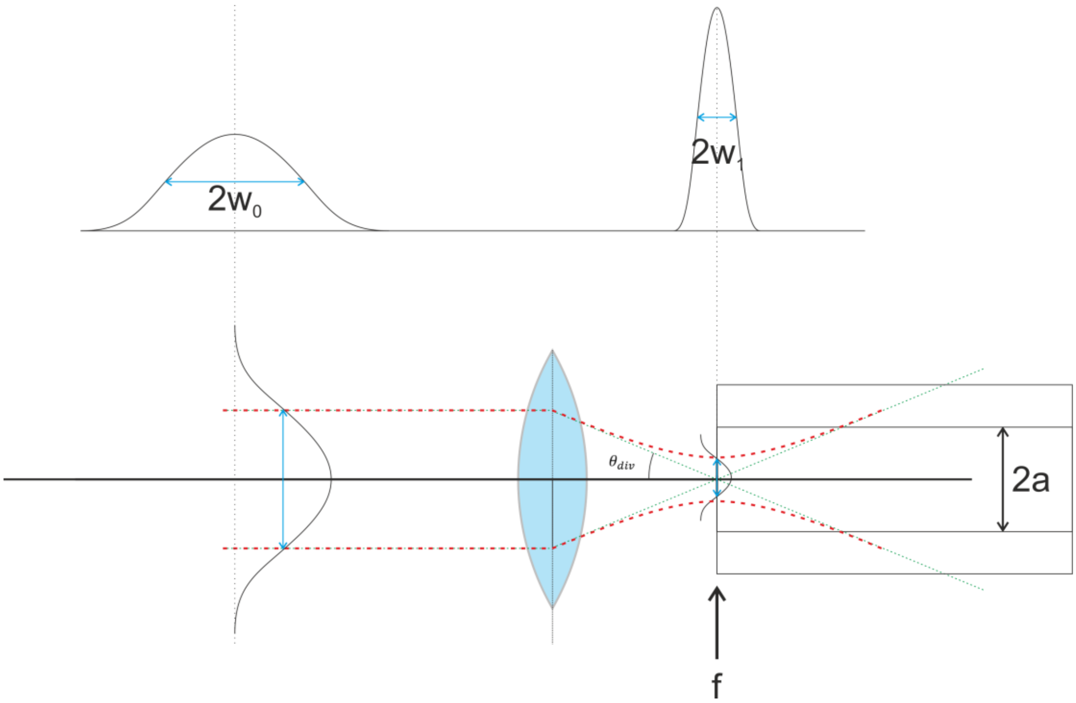
So finally the intensity:

$$I = E \cdot E^* \propto\ sinc^2\left(\frac{\pi a}{\lambda L}x\right)$$

4. We want to couple a collimated Gaussian beam ($\lambda = 1.55\ \mu m$) with a waist of 4 mm into a given a GRIN parabolic fiber with a core diameter of 50 microns and a cladding diameter of 125 microns. The refractive index of the cladding is $n_{clad} = 1.457$ and the maximum value in the core of $n_{core} = 1.46$. Determine

a) The lens required to fit the beam so it covers half of the core.

b) Is the beam divergence after the lens small enough to couple with the fiber.



a) First we are asked to inject the given Gaussian beam in the fiber. We have been told that is collimated so its divergence is 0, other way to think about it is that its object position is in the infinite $p = -\infty$.

$$\frac{1}{q} - \frac{1}{p} = \frac{1}{f}$$

$$q = f$$

Whith this we know that the lens must be positioned at a distance f from the fiber. As it is a Gaussian beam, its power distribution as function of the radial distance r is:

$$P = P_0 e^{-\frac{r^2}{2\omega^2}}$$

Where 2ω is the waist. The divergence relation of a Gaussian beam is:

$$\theta_{div} = \frac{4\lambda}{2\pi\omega}$$

Using trigonometry, we can approximate that the beam divergence after the lens is close to (small angles):

$$\theta_{div} \approx \tan\theta_{div} = \frac{\omega_0}{f}$$

Now we put all together and apply the condition that we have been told, that the beam cover half of the core (diameter $2a = 50\mu m$)

$$2\omega_1 = a = 25\ \mu m$$

$$2\omega_0 = 4\ mm = 4000\ \mu m$$

$$\frac{\omega_0}{f} = \frac{4\lambda}{2\pi\omega_1}$$

$$f = \frac{2\pi\omega_0\omega_1}{4\lambda} = \frac{\pi\omega_0 a}{8\lambda} \approx 13\ mm$$

b) In this case we must remember that the angles accepted θ_{acept} by the fiber so total internal reflexion is achieved is defined by the numerical aperture

$$NA = sin\theta_{acept} = \sqrt{n_{core}^2 - n_{clad}^2} = 0.0935$$

So as $\theta_{div} < \theta_{acept}$

$$\sin\theta_{div} < \sin\theta_{acept} = NA$$

$$\sin\frac{\omega_0}{f} \approx 0.3$$

So the divergence of the beam is too big to couple properly. Instead of a single lens a telescope system or any other alternative must be used to couple them properly.

5. We have a ILD with the structure is the provided in the image. One of the output faces in metalized $R_1 = 1$ and the other completely uncover (semiconductor-air). Considering that the refractive index and the energy gap of the alloys $Ga_{(1-x)}Al_xAs$ follow a linear relation for small values of x. Determine:

a) The gain threshold and the numerical aperture of the output beam

b) Is it 300K an optimal temperature? If not calculate the closest optimal temperature and determine if the lasing is monochromatic.

c) Which is the voltage that must be applied in order to have an internal current 3dB under the saturation current.

d) Determine the injection efficiency and the quantum efficiency.

Provided values:

$$L = 0.5\,mm\ ,, \qquad x = 0.1\ ,, \qquad \alpha_{eff} = 0.15\,mm^{-1}$$

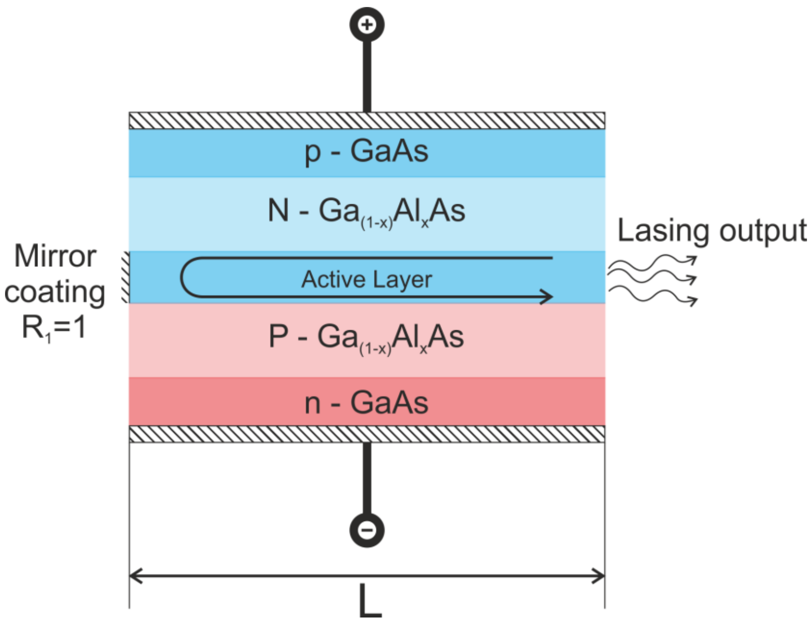
$$E_g(AlAs) = 2.16\,eV\ ,, \qquad E_g(GaAs) = 1.43\,eV$$

$$n(GaAs) = 3.66\ ,, \qquad n(AlAs) = 2.95$$

$$\mu_e = 0.85\frac{m^2}{Vs}\ ,, \qquad \mu_h = 0.04\frac{m^2}{Vs}$$

$$N_a = 10^{23}\,m^{-3}\ ,, \qquad N_d = 10^{21}\,m^{-3}$$

$$\tau_e = \tau_h$$



a) We have been provided with the gain equation

$$G = R_1 R_2 e^{2(g-\alpha_{eff})L}$$

We know that the threshold for obtaining amplification is if this gain is at least 1 so we isolate the value of gain coefficient threshold g_{th}

$$1 = R_1 R_2 e^{2(g_{th}-\alpha_{eff})L}$$

$$g_{th} = \alpha_{eff} + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)$$

We know all these parameters except R_2, which we have been told that is the interphase between air($n_a = 1$) and the semiconductor($n_s = n(GaAs)$). The semiconductor in the active layer is pure $GaAs$, so the reflectivity coefficient is

$$R_2 = \left(\frac{n_s - n_a}{n_s + n_a}\right)^2 = \left(\frac{2.66}{4.66}\right)^2 \approx 0.326$$

What finally leads to a gain coefficient threshold of:

$$g_{th} = 0.15\, mm^{-1} + \frac{1}{2 * 0.5\, mm}\ln\left(\frac{1}{1 * 0.326}\right) \approx 1.27\, mm^{-1}$$

b) When asked if a temperature is optimal for a laser diode, we are being asked indirectly if the output power is going to be maximum for a given wavelength. The amplification in the cavity is a function of the power emitted by the semiconductor so we must assure that the maximum of this spectrum is a wavelength supported by the cavity.

The spectrum of emission is defined by:

$$P(E) \propto \left(E_{ph} - E_g\right)e^{-\frac{(E_{ph}-E_g)}{kT}}$$

So the maximum if emission is for

$$\frac{dP}{dE} \propto e^{-\frac{(E_{ph}-E_g)}{kT}} + \left(E_{ph} - E_g\right) * \left(-\frac{1}{kT}\right)e^{-\frac{(E_{ph}-E_g)}{kT}}$$

$$P|_{max} \Rightarrow \frac{dP}{dE} = 0 = 1 - \left(E_{ph} - E_g\right) * \left(\frac{1}{kT}\right)$$

$$E_{ph}\big|_{max} = E_g + kT$$

On the other hand, this wavelength must be supported by the cavity. The frequencies supported by a laser cavity are defined by:

$$\nu_m = \frac{mc}{2n_sL}$$

Multiplying by h and replacing:

$$E_{ph}\big|_{max} = E_g + kT = h\nu_m = \frac{mch}{2n_sL}$$

If this condition is fulfilled, m should be a integer

$$m = \left(E_g + kT\right)\frac{2n_sL}{ch} \approx 4297.66$$

Which is not, so we round this number ($m = 4298$) in order to obtain the closest optimal temperature.

$$T_{opt} = \left(\frac{mch}{2n_sL} - E_g\right)\frac{1}{k} \approx 301.34\, K$$

To check if the amplified light is, monochromatic we should compare the separation between supported spectral lines

$$\Delta\nu_m = \frac{c}{2n_s L}$$

And the FWHM of the emitted light $\Delta E \approx 2.4kT$, a good approximation is that all the light outside this central values is negligible for lasing. In other words, the laser is going to be monochromatic if:

$$\Delta E_{ph} < h\Delta\nu_m$$

$$\Delta E_{ph} = 2.4kT = 0.062\ eV$$

$$\mathrm{h\Delta\nu_m} = 0.34\ meV$$

the light is not monochromatic.

c) we can calculate the current intensity as a function saturation current intensity by

$$I = I_s\left[e^{\frac{eV}{kT}} - 1\right]$$

Using that

$$3dB = 10\log_{10}\frac{I}{I_s}$$

$$10^{\frac{3}{10}} = \frac{I}{I_s} = \left[e^{\frac{eV}{kT}} - 1\right]$$

$$V = \frac{kT}{e} ln\left(10^{\frac{3}{10}} + 1\right) = 28.4\ mV$$

d) first we need to identify if the current is carried by holes or electrons. This is determined by the concentrations of the dopants, as the concentration of acceptor is greater than the donor, the current is dominated by holes so the intrinsic efficiency is defined as:

$$\eta_{inj} = \frac{1}{1 + \frac{J_e}{J_h}} = \left(1 + \left(\frac{D_e}{D_h}\right)\left(\frac{L_h}{L_e}\right)\left(\frac{N_d}{N_a}\right)\right)^{-1}$$

Both diffusion coefficients can be calculated from the values provided:

$$D_{e/h} = \mu_{e/h}\frac{kT}{e}$$

Also, the expression for the diffusion length is

$$L_{e/h} = \sqrt{D_{e/h}\tau_{e/h}}$$

We do not know the value of the lifetime for electrons nor holes but the coefficient $\tau_e/\tau_h = 1$

$$\eta_{inj} = \left(1 + \left(\frac{D_e}{D_h}\right)\left(\frac{\sqrt{D_h \tau_h}}{\sqrt{D_e \tau_e}}\right)\left(\frac{N_d}{N_a}\right)\right)^{-1} = \left(1 + \left(\frac{\sqrt{D_e}}{\sqrt{D_h}}\right)\left(\frac{N_d}{N_a}\right)\right)^{-1} = \left(1 + \left(\frac{\sqrt{\mu_e}}{\sqrt{\mu_h}}\right)\left(\frac{N_d}{N_a}\right)\right)^{-1}$$

$$\eta_{inj} = \left(1 + \sqrt{\frac{0.85}{0.04}} * 10^{-2}\right)^{-1} = 0.956$$

For the quantum efficiency the calculus is direct:

$$\eta_{int} = \frac{1}{1 + \tau_e/\tau_h} = \frac{1}{1+1} = 0.5$$

$$\eta_{ext} = \frac{g_{th} - \alpha_{eff}}{g_{th}} = 0.88$$

$$\eta = \eta_{int}\eta_{ext} = 0.44$$

6. A detector is designed using the structure p-i-n with a hetero junction of InGaAs/InP. Determine:

a) The structure of the detector and the wavelength window that it can measure.

b) The size of the intrinsic window so the 90% of the light is absorbed in the intrinsic region.

c) Draw the (approximated) responsibility function between 0.8 and 2 microns. Consider the absorption coefficients constant for the absorbed wavelengths in that window and the thickness of the p and n regions is a 10% of the intrinsic.

Provided values:

$$n(InGaAs) = 3.52 \text{ ,, } \quad n(InP) = 3.45$$

$$E_g(InGaAs) = 0.75\ eV \text{ ,, } \quad E_g(InP) = 1.35\ eV$$

$$\alpha(InGaAs) = 0.85\ mm^{-1} \text{ ,, } \quad \alpha(InP) = 0.7\ mm^{-1}$$

a) The structure of the device is the same as the one shown in the keynotes. Here the question is which one of the semiconductors is used for the p and n regions and which is for the intrinsic region. To determine this, we must check the value of the energy gap, the material with the bigger energy gap is going to be used for the p and n regions as the cut-off wavelength is shorter than the other one.

$$i \Rightarrow InGaAs\ //\ E_g = 0.75\ eV\ \ //\ \lambda_c = 1.65\ \mu m$$

$$p, n \Rightarrow InP\ //\ E_g = 1.35\ eV\ \ //\ \lambda_c = 0.92\ \mu m$$

So the wavelengths between both cut off are not going to be absorbed by the regions p nor n and are going to be strongly absorbed in the intrinsic region.

$$\Delta\lambda = [0.92 - 1.65]\mu m$$

b) We simply consider that the absorbed light in the intrinsic region is

$$(1 - 0.9) = e^{-\alpha_i w}$$

So the width of this region is:

$$w = -\frac{1}{\alpha_i}\ln(0.1) = 2.71\ mm$$

c) To represent the responsibility in the intrinsic region

$$\Re = \frac{I_p}{P_{in}} = \frac{(1 - R)e}{hc}\lambda(1 - e^{-\alpha_i w})$$

We need to take into account that for values higher than the cut off wavelength it is 0.

$$\Re(\lambda > \lambda_c = 1.65\ \mu m) = 0$$

Also, the parameter α_1 is not the absorption in the intrinsic region but in the p or n region. The parameter $\left(e^{-\alpha_1 d}\right)$ determines how many light arrives to the intrinsic region. As we have chosen InP as the semiconductor for the p and n regions, $\alpha = \alpha(InP) = 0.7$.

Finally, the transmission is between air-InP-InGaAs, so it must be written as:

$$(1-R) = T = \left(1 - R_{\frac{air}{InP}}\right)\left(1 - R_{\frac{InP}{InGaAs}}\right) =$$

$$\left[1 - \left(\frac{n_{InP} - n_a}{n_{InP} + n_a}\right)^2\right]\left[1 - \left(\frac{n_{InGaAs} - n_{InP}}{n_{InGaAs} + n_{InP}}\right)^2\right] \approx 0.7$$

So putting all together

$$\Re \begin{cases} (1-R)\left(1-e^{-\alpha_1 d}\right)\dfrac{e}{hc}\lambda & \lambda < 0.92\ \mu m \\ (1-R)(1-e^{-\alpha_i w})\dfrac{e}{hc}\lambda & 0.92 < \lambda < 1.65\ \mu m \\ 0 & \lambda > 1.65\ \mu m \end{cases}$$

Where d is the thickness of the p and n regions that we were told is 10% of the intrinsic region, $d = 0.1w = 0.271\ mm$. The second region substitutes the term $(1 - e^{-\alpha_1 d})$ because the p and n regions do not absorb that wavelengths (longer than the cut-off).

An approximated draw is expected as: