<u>Exercice 3</u> : polarization mode dispersion and chromatique dispersion in a single mode fiber

<u>Part 1</u> : birefringence and polarization mode dispersion

One can show that the normalized phase birefringence B_φ of an optical fiber having an optical core and working in the single mode regime depends on the wavelength following the relation : $B_\varphi = \sigma.\lambda^2$ where σ is a constant.

1) What is the relation existing between B_φ and the effective indices n_{ex} and n_{ey} of the two polarization modes (HE_{11x} and HE_{11y}, x and y being the directions of the two neutral axes (or eigen axes) of the fiber)?

2) show that the beat length L_B between these two modes can be written : $L_B = \dfrac{\lambda}{B_\varphi}$.

3) We work at λ= 1.55µm. The effective index of the mode polarized along one of the eigen axis is 1.4455 and L_B= 3mm. Calculate the constant σ (USI) and deduce the possible values of the effective index of the mode polarized in the orthogonal direction, rounded to the nearest 10^{-5}.

4) show that, for this fiber, the group birefringence B_G can be expressed very simply versus B_φ. Give its value at λ= 1,55µm.

5) The group index of the mode polarized in the direction ① is $N_{g_①}$=1.4722 (① = x or y). Calculate $N_{g_②}$ (② = y or x, respectively) to the nearest 10^{-5}, $N_{g_②}$ being the group index of the mode polarized in the orthogonal direction, knowing that $N_{g_②} > N_{g_①}$.

6) A short light pulse with a triangular P(t) shape, P being the power, is emitted by a laser. The full width at half maximum (FWHM) of this pulse is 1 ps. The pulse is launched into the studied fiber after crossing a polarizer oriented at 45° to the neutral axes of the fiber. What proportion of the energy propagating in the fiber is carried by the HE_{11x} mode and by the HE_{11y} mode?
We neglect the effects of the chromatic dispersion and we consider that the fiber behaves as a polarization maintaining fiber: represent the temporal shape of the signal detected at the output, after a 10 m long propagation in the fiber.

<u>Part 2</u> : chromatique dispersion

A silica step index fiber is used for a high bit rate transmission @ λ_T =1.55µm. This fiber is single mode at the working wavelength λ_T. The effective index of the fundamental mode can take the following form around λ_T : $n_e(\lambda) = A_2\lambda^2 + A_1\lambda + A_0$ where A_2, A_1 and A_0 are constant values.

1/ A short pulse centered at λ_T takes exactly 0,491317 ms for travelling over a distance of 100 km in the fiber.

a- What is the value of the group velocity in the fiber @ λ_T ?
b- Calculate the group index at λ_T, with a precision of 10^{-5}.

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2/ a- Show that the expression of the group index as a function of the effective index n_e and the wavelength is $n_g = n_e - \lambda \frac{dn_e}{d\lambda}$

b- express the group index versus A_2, A_1, A_0 and λ.

c- the parameter A_0 is equal to 1,47. Show that A2= -16,45 10^{-4} μm^{-2}

3/ The chromatic dispersion D of the fundamental mode can be expressed under the form $D_c = \frac{1}{c}\frac{dn_g}{d\lambda}$

a- Show that $D_c = -\frac{\lambda}{c}\frac{d^2 n_e}{d\lambda^2}$

b- Calculate the chromatic dispersion of the fiber at λ_T, expressed in the usual unit system: ps/(km.nm).

4/ At λ_T, the dispersion of silica is D_m= 22 ps/(km.nm).

a- Why is the chromatic dispersion of the fiber different from the dispersion of silica ($D_c \neq D_m$) ?

b- With what means can the manufacturers adjust the chromatic dispersion of the guided mode, at a given wavelength?

5/ Calculate the propagation length L at the end of which a pulse having an initial duration Δt = 150 ps and a spectral width equal to σ_λ = 0,3nm will have its duration increased by 50%.

Some formulas which can be useful in part 1 or in part 2

Phase velocity: $v_\varphi = \frac{\omega}{\beta}$ Group velocity : $v_g = \frac{d\omega}{d\beta}$

Group index : $N_g = n_e - \lambda \frac{dn_e}{d\lambda}$ (n_e = effective index for the considered mode)

Phase birefringence : $b_\varphi = |\beta_x - \beta_y|$ where β_x and β_y are the propagation constants of the HE_{11x} and HE_{11y} modes, respectively.

Normalized phase birefringence : $B_\varphi = \frac{b_\varphi}{k_0}$, k_0 being the modulus of the wave vector in the vaccum

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