CHANGES IN PULSE SPECTRA DUE TO SPM

An estimate of the magnitude of SPM-induced spectral broadening can be obtained from the peak value of $\delta\omega$. Quantitatively, we can calculate the peak value by maximizing $\delta\omega(t)$ from eq. (9):

$$\delta\omega(t) = \frac{2m}{t_0} L\gamma \left(\frac{t}{t_0}\right)^{2m-1} \exp\left(-\left(\frac{t}{t_0}\right)^{2m}\right) \qquad (9)$$

By setting its time derivative to zero, the maximum value of $\delta\omega$ is given by

$$\delta\omega_{MAX} = \frac{m\, f(m)}{T_0}\, \phi_{MAX} \qquad (10)$$

where $\phi_{MAX} = \gamma L$, and $f(m)$ is defined as

$$f(m) = 2\left(1 - \frac{1}{2m}\right)^{1-1/2m} \exp\left[-\left(1 - \frac{1}{2m}\right)\right] \qquad (11)$$

The numerical value of f depends on m only slightly; For $m=1$, $f = 0.86$ and Tends Toward 0.74 for large values of m. To obtain the broadening factor, The width parameter t_0 should be related to The initial spectral width $\Delta\omega_0$ of The pulse.

For an unchirped Gaussian pulse, $\Delta\omega_0 = t_0^{-1}$.

Equation (10) becomes (with $m=1$):

$$\delta\omega_{MAX} = 0.86\,\Delta\omega_0\,\phi_{MAX} \qquad (12)$$

showing that the spectral broadening factor is approximately given by the numerical value of the maximum phase shift ϕ_{MAX}.

In the case of a super-gaussian pulse, it's difficult to estimate $\Delta\omega_0$ because it's spectrum is not gaussian. However, if we refer to the rise time $T_r = t_0/m$, and assume the $\Delta\omega_0$ approximately equals T_r^{-1}, Eq. (10)

shows that the broadening factor of a super Gaussian pulse is also given by ϕ_{MAX}.

With $\phi_{MAX} \simeq 100$ possible for high-intensity pulses or long fibers, SPM can broaden the spectrum considerably.

The actual shape of the pulse spectrum $\hat{F}(\omega)$ is obtained by taking the fourier transform of Eq. (4) $\left[F(z,t) = F(0,t) \exp\left[i\, \phi_{NL}(z,t) \right] \right]$.

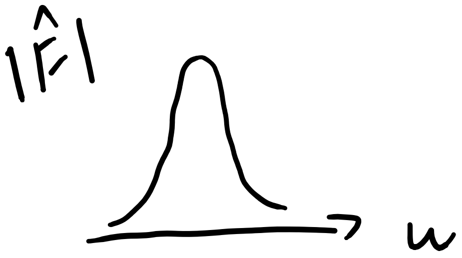
setting $S(\omega) = |\hat{F}(z=L,\omega)|$ we obtain

$$S(\omega) = \left| \int_{-\infty}^{+\infty} F(0,t) \exp\left[i\phi_{NL}(L,t) + i(\omega-\omega_0)t \right] dt \right|^2 \tag{13}$$

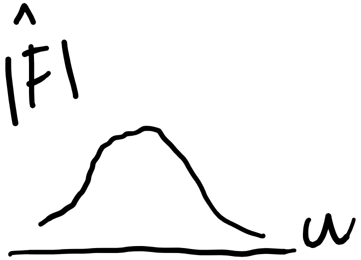
In general the spectrum depends not only on the pulse shape but also on the initial chirp of the pulse.

At different length L we have different spectra of an unchirped Gaussian pulse, due to the fact that the pulse has different phase shift ϕ_{MAX}.

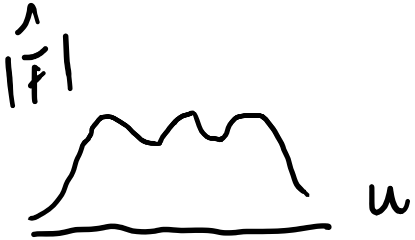
From numerics one can derive these plots, as to the output spectrum:



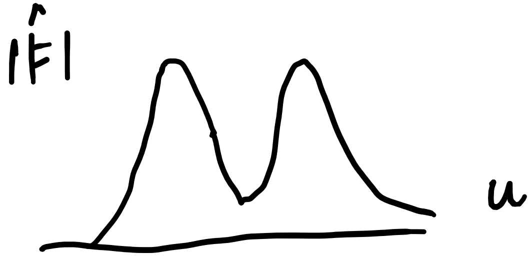
$\phi_{MAX} = 0$



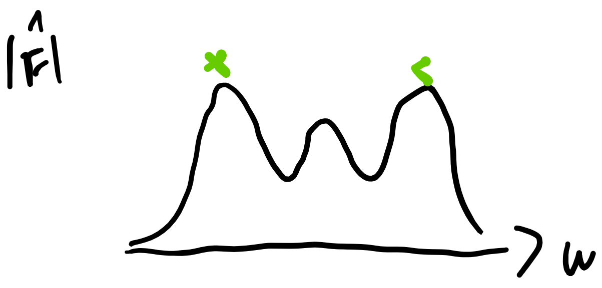
$\phi = 0.5\pi$



$\phi_{MAX} \simeq \pi$



$\phi_{MAX} \simeq 1.5\pi$



$\phi_{MAX} = 2.5\pi$

$\phi_{MAX} = 3.5\pi$

The most notable feature of these figures is that SPM-induced spectral broadening is accompanied by an oscillatory structure covering the entire frequency range. In general, the spectrum consists of many peaks, and the outermost peaks are the most intense.

The number of peaks depends on ϕ_{MAX} and increases linearly with it.

One can derive that the number of peaks M in the SPM-broadened spectrum is given approximately by the relation

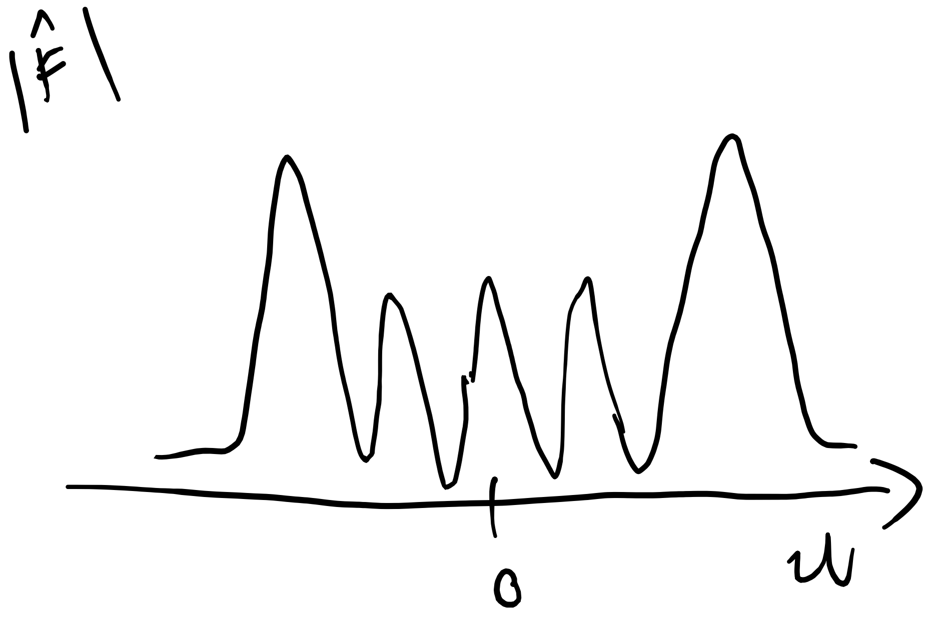
$$\phi_{MAX} \simeq \left(M - \frac{1}{2}\right)\pi \qquad (14)$$

(derived from Fourier integral (13).).

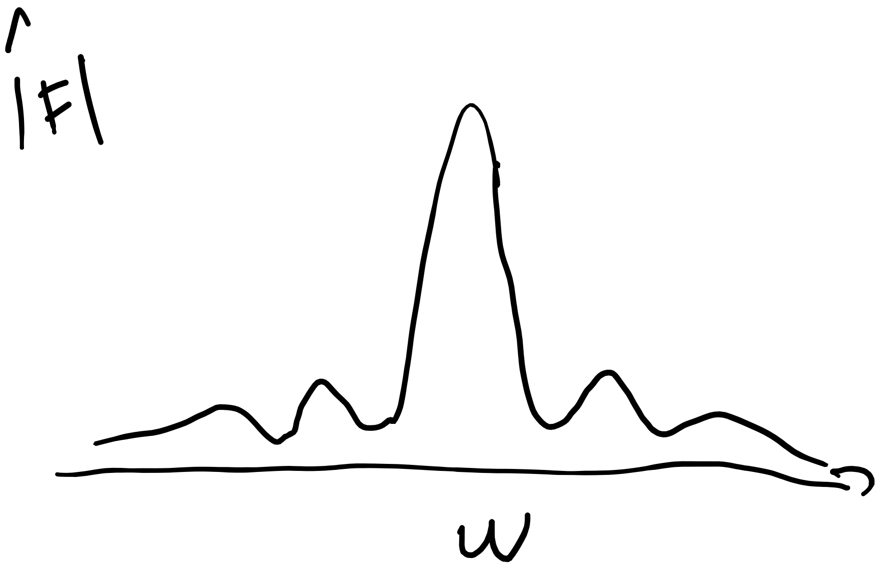
As mentioned before in the numerics, The shape of the SPM-broadened spectrum depends on:

1) The pulse shape
2) on the initial chirp (if the input pulse is chirped).

We spend some time to compare the pulse spectra for gaussian ($m = 1$) and super-Gaussian ($m = 3$) pulses.



unchirped gaussan pulse ($m = 1$) and distance z; which corresponds to $\phi_{max} \simeq 4.5\pi$



unchirped super gaussian pulse ($m = 3$), $\phi_{max} \simeq 4.5$

An initial frequency chirp can also lead to drastic changes in the SPM-broadened pulse spectrum.

We analyse numerically the case of a Gaussian pulse with positive ($C>0$) and negative ($C<0$) chirps.

The chirp influences the broadened pulse spectrum.

You can play with numerics, and see what happens with supergaussian chirped pulses.

SPM induces spectral broadening, and the spectrum depends on the nonlinear max phase shift, on the initial pulse profile, and the chirp.