

Surname, Name, Matr.: Signature:

Answer to the questions according to the order assigned in the text.

Any “not given” answer will be taken into account (producing a penalty) in the overall evaluation.

Any “schematic and clearly described” answer will be appreciated.

The given answers should be reported in the original signed exam document. The minutes will not be taken into account in the final evaluation.

Questions

1.
 - A block code is described by the parity check matrix indicated in Fig. 1.
 - Indicate the possible code-words. Is this a cyclic code ?
 - What is the probability of error in case of hard and soft decision ?
 - What is the minimum required bandwidth (in case of binary modulation) if the information bit-rate is equal to 10 Mbit/s.

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

- Consider the Hamming code with $N = 127$. Determine the error probability in case of both hard (use the more precise estimation) and soft decoding.
 - Design a $(6, 2)$ cyclic code by choosing the shortest possible generator polynomial¹. Determine the generator matrix G (in the systematic form) for this code and find all the possible codewords. How many errors can be corrected by this code ?
2. Consider a convolutional code with $R = 1/2$, and octal generators $(5, 2)$.
 - Determine and draw the trellis diagram of the code.
 - Determine the code word associated to the information sequence: 010101100.
 - Determine the bit-error probability (considering at least 3 non zero terms in the union bound), and the minimal bandwidth required in case of an information bit-rate equal to 10 Mbit/sec.
 3. Turbo-LDPC Codes
 - Describe the curves that represents the performance $(P(E))$ as a function of E_b/N_0 of a turbo code, indicating the role of the iterations and of the interleaver.
 - Describe the basic idea of the bit-flipping algorithm for the hard decoding of an LDPC code.
 - Describe the basic idea of the EXIT charts.

¹Hint: $D^6 + 1 = (D + 1)^3 \cdot (D + 1)^3 = (D + 1) \cdot (D + 1) \cdot (D^2 + D + 1) \cdot (D^2 + D + 1)$.