Exam 2021 - June - 18

Problem 1

$R = 8\,cm$, $\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$

$z_1 \to$ object dist.

$z_2 \to$ image dist.

$f = \frac{-R}{2}$

A) Concave mirror & $z_1 < 4\,cm$

$R = -8\,cm \to \frac{1}{f} = \frac{2}{-R} = \frac{2}{-(-8)} = \frac{1}{4}$ | $z_1 < 4\,cm$ for instance $z_1 = 3\,cm$

$\frac{1}{z_2} = \frac{1}{4} - \frac{1}{3} \Rightarrow z_2 = -12\,cm \to \boxed{M = +4 \to \text{Valid!}}$

B) Convex mirror & $z_1 < 4\,cm$

$R = 8\,cm \to \frac{1}{f} = -\frac{1}{4}$ | $z_1 < 4\,cm \Rightarrow z_1 = 3\,cm$

$\frac{1}{z_2} = -\frac{1}{4} - \frac{1}{3} = -\frac{7}{12} \to z_2' = -\frac{12}{7}\,cm \to$ image diminish

C) Concave mirror & $z_1 > 4\,cm$

$R = -8\,cm \to \frac{1}{f} = \frac{1}{4}$ | $z_1 = 5\,cm$

$\frac{1}{z_2} = \frac{1}{4} - \frac{1}{5} = \frac{1}{20} \to z_2 = 20\,cm \to$ image amplified but inverted

D) Convex & $z_1 > 4cm$

$R = 8cm \rightarrow \frac{1}{f} = -\frac{1}{4}$ | $z_1 = 5cm$

$\frac{1}{z_2} = -\frac{1}{4} - \frac{1}{5} = \frac{-9}{20} \rightarrow z_2 = -\frac{20}{9}$ cm $-$ image diminish

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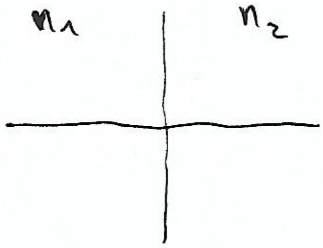
$\boxed{\text{Problem 2}}$ $n_2 = \frac{n_1}{3}$



$\Rightarrow n_1 > n_2$

a)

Calculate θ_t with snell:

$n_i \sin(\theta_i) = n_t \sin(\theta_t) \rightarrow \sin\theta_t = \frac{n_i}{n_t}\sin(\theta_i) \overset{\theta_i = 0}{=} 0$

$\theta_t = 0$

$$r = \frac{n_i\cos(\theta_i) - n_t\cos(\theta_t)}{n_i\cos(\theta_i) + n_t\cos(\theta_t)} = \frac{n_1 \cdot 1 - \frac{n_1}{3}\cdot 1}{n_1 \cdot 1 + \frac{n_1}{3}\cdot 1} = \frac{\frac{2}{\cancel{3}}}{\frac{4}{\cancel{3}}} = \frac{1}{2}$$

$R = |r|^2 = \frac{1}{4} \longrightarrow T = 1 - R = \frac{3}{4}$

b)

To have zero reflectance:

$$r_{TE} = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

We cannot achieve zero reflectance with TE waves

$$r_{TM} = \frac{n_i \cos(\theta_t) - n_t \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

The condition is

$$n_i \cos\theta_t = n_t \cos\theta_i$$

for TM waves

Brewster: $\theta_B = \operatorname{atan}\left(\frac{n_2}{n_1}\right) = \operatorname{atan}\left(\frac{1/3}{1}\right) = \operatorname{atan}\left(\frac{1}{3}\right)$

$$\boxed{\theta_B \simeq 18{,}44^\circ}$$

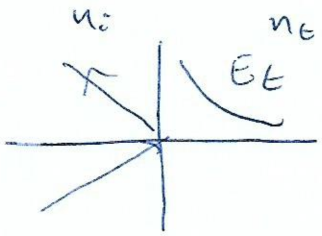
c) the critical angle is when we are travelling from medium 1 to medium 2 and: $n_1 > n_2$ and $\theta_t = \frac{\pi}{2}$

So;

$$n_1 \sin(\theta_i) = n_2 \sin(\theta_t) \underset{\theta_t = \frac{\pi}{2}}{=} n_2 \rightarrow \sin\theta_c = \frac{n_2}{n_1} = \frac{1/3}{1}$$

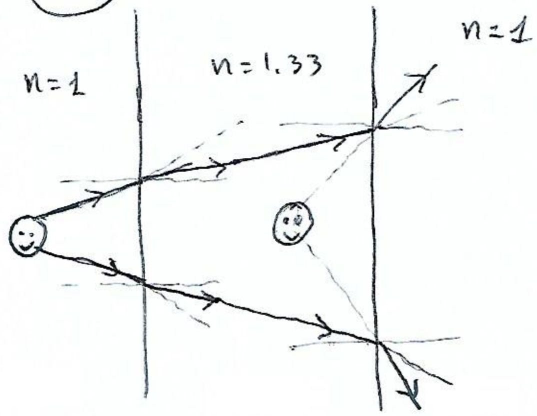
$$\boxed{\theta_c = 19{,}47^\circ}$$

d) Under these circumstances we have the evanescence wave that is different from zero.



Problem 3

(1a)

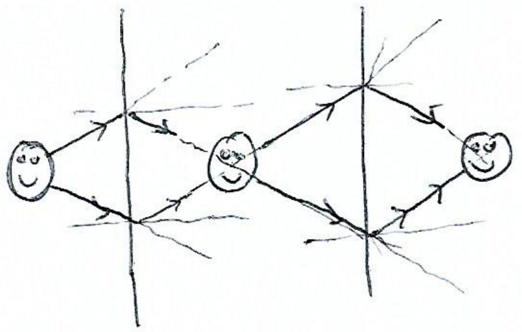


First: Going from $n_i < n_t \Rightarrow \theta_i > \theta_t$

Second: Going from $n_i > n_t \Rightarrow \theta_i < \theta_t$

Virtual image

(1b)



First: Same angle as before but k opposite sign

Second: Same angle, k opposite sign

Real image.

To achieve this we need to have a medium with $\epsilon < 0$ and $\mu < 0$.