(HARD) ERROR PROBABILITY

$$\underbrace{\boxed{\;|\;|\;\cdots\;|\;}}_{N} \longleftarrow t = \text{ERROR CORRECTING CAPABILITY}$$

* we have errors if we have $t+1$, $t+2$, ... errors

If we assume ERRORS ARE INDEPENDENT EVENTS $\longrightarrow$ let's model them as a BINOMIAL DISTRIBUTION

$$\varepsilon = Q\left(\sqrt{\frac{2E_b}{N_0}\cdot\frac{k}{N}(t+1)}\right)$$

$\frac{k}{N}$ $\longrightarrow$ RATE is used as a PENALTY ("wasted" energy)

$(t+1)$ $\longleftarrow$ Hamming distance

$$P_w(E) = \underbrace{\binom{N}{t+1}\varepsilon^{t+1}}_{\text{wrong bits}}\underbrace{(1-\varepsilon)^{N-(t+1)}}_{\text{correct bits}} \quad // \quad t+1 \text{ errors}$$

$$P(E) \underbrace{\leq}_{\text{(union bound)}} \sum_{h=t+1}^{N}\binom{N}{h}\varepsilon^{h}(1-\varepsilon)^{N-h}$$ PROB. OF ERROR (WORD)

!! $P_b(E) = \left(\frac{2t+1}{N}\right)P_w(E)$ PROB. OF ERROR (BIT)

$2t+1$ $\rightarrow$ min. distance

N $\rightarrow$ length

(SOFT) ERROR PROBABILITY

We need intuition on vector space ...

we can say $Q\left(\sqrt{\frac{2E_b}{N_0}\cdot\frac{k}{N}\cdot d^*}\right)$

d^* $\longrightarrow$ Euclidean distance

... very difficult to estimate ...

Let's compare soft-hard decision (using only the 1st term)

$$P_H(E) = Q\left(\sqrt{\frac{2E_b}{N_0}\cdot\frac{k}{N}(t+1)}\right) \qquad P_S(E) = Q\left(\sqrt{\frac{2E_b}{N_0}\cdot\frac{k}{N}\, d^*}\right)$$

$d^* \simeq 2t+1$

factor 2 is very effective since Q is exponential! $\longrightarrow$ $2t+1$

if $d_{MIN}\cdot R > 1$ $\longrightarrow$ we have a GAIN

otherwise (<1) $\rightarrow$ code isn't efficient

$d_{MIN}\cdot R$ = GAIN