

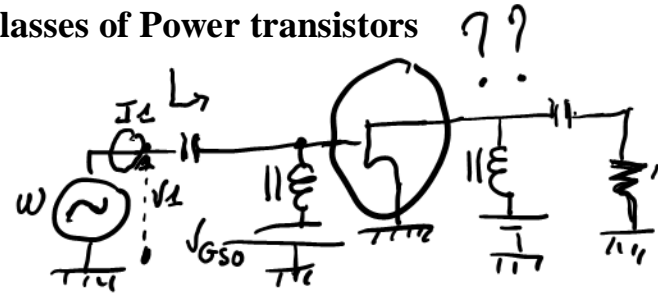
# **Semester S1 –Basics of active and non linear electronics**

## **RF Power amplifiers ( JM Nebus )**

### **COURSE N° 3**

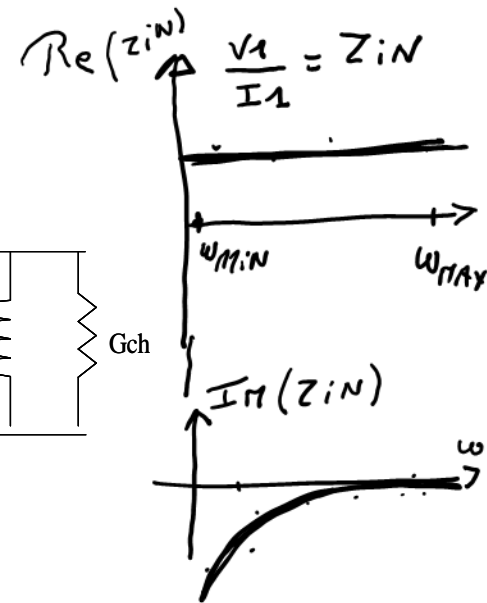
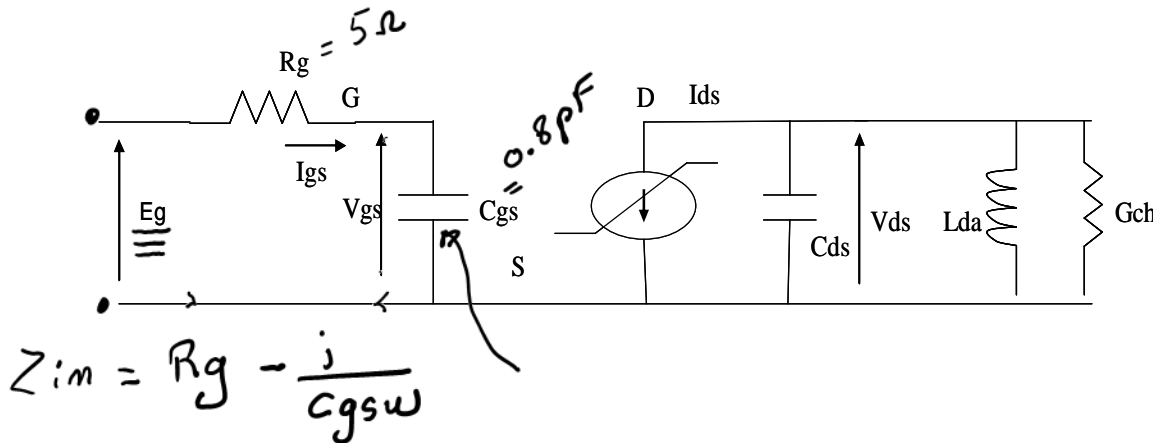
## Chapter III : Operating classes of Power transistors

### 1) Equivalent circuit of the transistor input

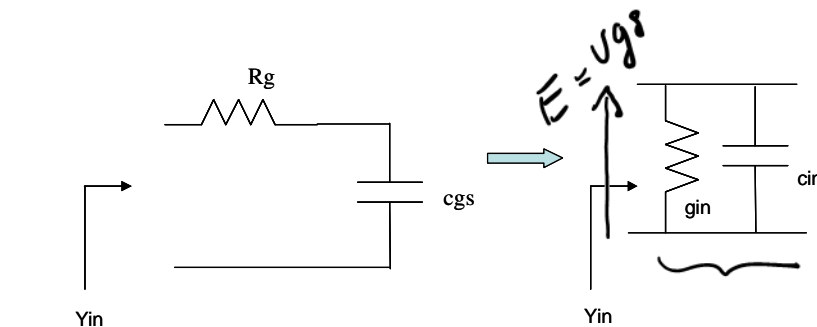


We have to consider the fact that at high frequency the input of a transistor (gate port) does not behave as an ideal open circuit.

It behaves as a first order low pass circuit



- Serie to parallel équivalence



$$Y_{in\text{serie}} = \frac{1}{R_g + \frac{1}{j\omega c_{gs}}} = \frac{j\omega c_{gs}}{1 + R_g^2 \cdot \omega^2 \cdot c_{gs}^2} =$$

$$Y_{in\text{parallele}} = g_{in} + j\omega c_{in}$$

$$\omega^2 \ll \frac{1}{R_g^2 c_{gs}^2}$$

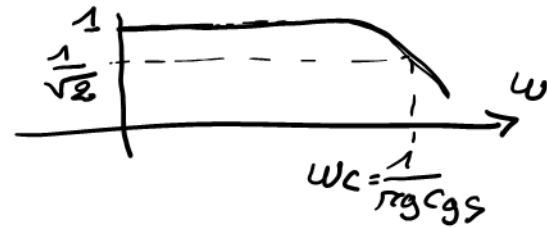
$$\omega \ll \frac{1}{R_g c_{gs}}$$

Considering that  $R_g^2 \cdot c_{gs}^2 \cdot \omega^2 \ll 1$  we can write  $g_{in} = R_g \cdot c_{gs}^2 \cdot \omega^2$  and  $c_{in} = c_{gs}$

If  $rg^2 \cdot cgs^2 \cdot \omega^2 \ll 1$   $\omega$  is slow compared to the cut off frequency

$\omega_c$  of the circuit

$$\omega_c = \frac{1}{rg \cdot cgs}$$



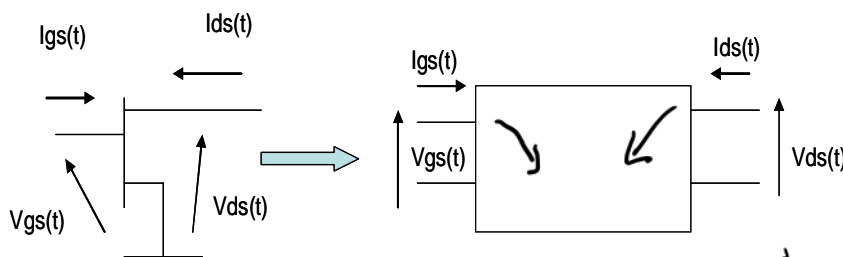
The RF input power is

$$P_{in} = \frac{1}{2} \cdot \text{Re}(V_{gs1} \cdot I_{gs1}^*) = \frac{1}{2} \cdot \text{Re}(V_{gs1} \cdot Y_{in}^* V_{gs1}^*) = \frac{1}{2} \cdot |V_{gs1}|^2 \text{Re}(Y_{in}^*) = \frac{1}{2} \cdot g_{in} \cdot |V_{gs1}|^2 = \frac{1}{2} \cdot rg \cdot cgs^2 \cdot \omega^2 \cdot |V_{gs1}|^2$$

$$P_{in} = \frac{1}{2} R_g C_{gs}^2 \omega^2 |V_{gs1}|^2$$

## 2) Power budget

If we consider that a parallel resonant circuit (having a high quality factor) is connected at the output of the transistor, the drain source voltage  $V_{ds}(t)$  is only the sum of a DC component and a fundamental frequency component.



$$V_{gs}(t) = V_{gs0} + V_{gs1} \cdot \cos(\omega t)$$

$$I_{gs}(t) = I_{gs0} + I_{gs1} \cdot \cos(\omega t + \rho)$$

$$V_{ds}(t) = V_{ds0} - V_{ds1} \cdot \cos(\omega t)$$

$$I_{ds}(t) = I_{ds0} + I_{ds1} \cdot \cos(\omega t) + I_{ds2} \cdot \cos(2\omega t) + \dots$$

$$p(t) = V_{gs}(t) \cdot I_{gs}(t) + V_{ds}(t) \cdot I_{ds}(t)$$

$$P = P_{diss} = \frac{1}{T} \int_0^T p(t) \cdot dt$$

$$P_{diss} = \underbrace{|V_{gs0}| \cdot I_{gs0}}_{=0} + \underbrace{V_{ds0} \cdot I_{ds0}}_{P_{DC}} + \frac{1}{2} \cdot V_{gs1} \cdot I_{gs1} \cdot \cos(\rho) \xrightarrow{\text{Power}} \frac{1}{2} \cdot \text{Re}(V_{gs1} \cdot I_{gs1}^*) = P_{in} - \frac{1}{2} \cdot V_{ds1} \cdot I_{ds1} = P_{DC} + P_e - P_{out}$$

$$P_{diss} = P_{in} + P_{DC} - P_{out}$$

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$$\eta_{\text{Added}} = \eta_{\text{Aj}} \left\{ \begin{array}{l} P_{\text{diss}} = P_{\text{dc}} \left( 1 - \frac{P_{\text{out}} - P_{\text{in}}}{P_{\text{dc}}} \right) = P_{\text{dc}} (1 - \eta_{\text{aj}}) \end{array} \right.$$

PAE  
Power added efficiency

### 3) Operating classes and efficiency performances

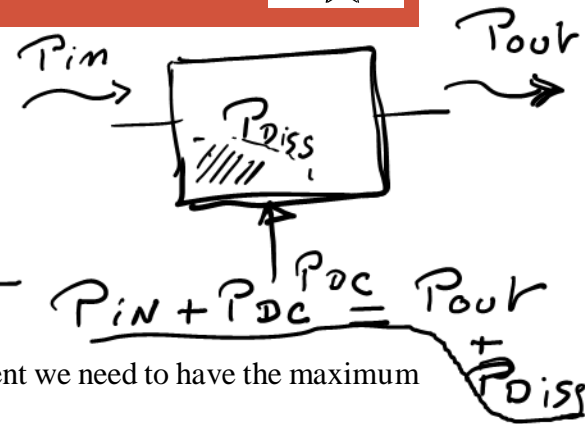
To reach the maximum value of the output drain current we need to have the maximum voltage swing at the input

$$\eta_D = \frac{P_{\text{out}}}{P_{\text{dc}}}$$

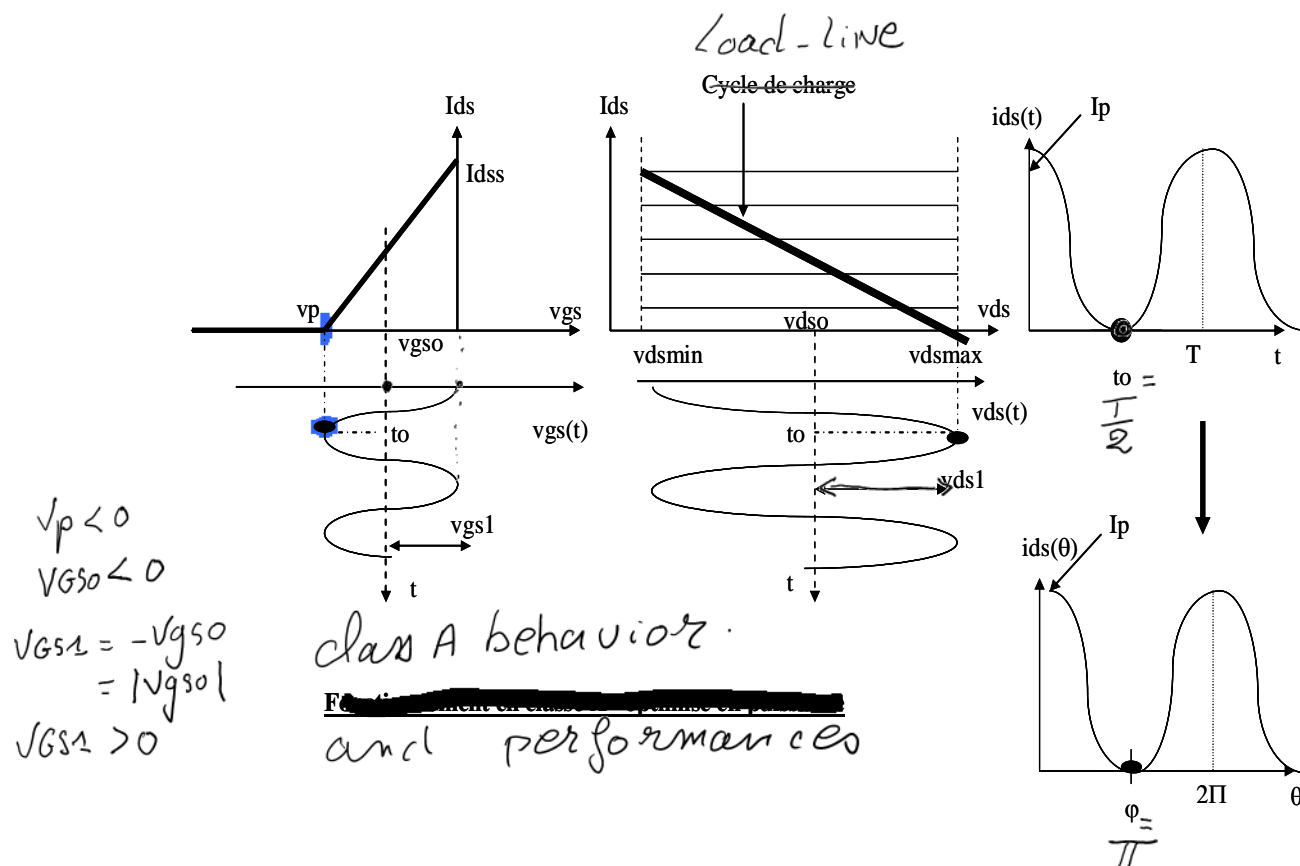
Consequently:  $V_{\text{gs1}} = |V_{\text{gs0}}| = -V_{\text{gs0}}$  and  $I_{\text{dss}} = I_p$

We assume here that the maximum acceptable value of  $V_{\text{gs}}(t)$  is equal to 0V

$V_{\text{gs}}(t) > 0V$  can lead to a non-reliable behavior of the transistor.



### Class A operation



We have

$$t_o = \frac{T}{2}$$

$$V_{gs0} = \left(\frac{V_p}{2}\right), V_{gs1} = -\left(\frac{V_p}{2}\right), V_p \text{ is a negative value while } V_{gs1} \text{ is positive}$$

$$V_{gs0} + V_{gs1} \cdot \cos(\varphi) = V_p$$

$$I_p = I_{dss}$$

$$\frac{V_p}{2} - \frac{V_p}{2} \cos \varphi = V_p$$

$$\cos(\varphi) = -1 \quad \text{soit} \quad \varphi = \pi \rightarrow \begin{cases} I_{D50} = \frac{I_{DSS}}{2} \\ I_{D51} = \frac{I_{DSS}}{2} \end{cases}$$

Using the equations of  $I_{d50}$  and  $I_{d51}$  obtained in chapter II we obtain :

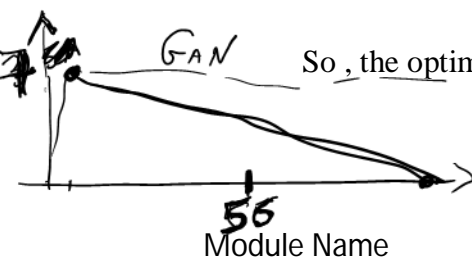
$$I_{d50} = \left(\frac{I_{dss}}{2}\right) \quad \text{and} \quad I_{d51} = \left(\frac{I_{dss}}{2}\right) \quad \leftarrow$$

In order to have the maximum drain source voltage swing  $V_{ds1}$ , we choose

$$\begin{aligned} R_{opt} &= \frac{V_{ds1}}{I_{d51}} \\ &= \frac{28}{7} \\ &= 4 \Omega \end{aligned}$$

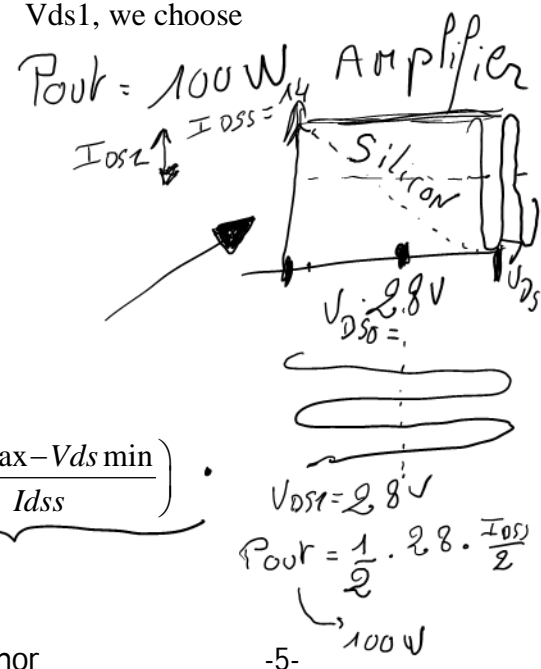
$$V_{ds0} = \left(\frac{V_{ds \max} + V_{ds \min}}{2}\right) \quad \leftarrow$$

$$\text{Therefore} \quad V_{ds1} = \left(\frac{V_{ds \max} - V_{ds \min}}{2}\right) \quad \leftarrow$$



$$\text{So, the optimal load resistance is } R = \left(\frac{V_{ds1}}{I_{d51}}\right) = \left(\frac{V_{ds \max} - V_{ds \min}}{I_{dss}}\right)$$

$$R_{opt} = \frac{56}{3.5} = 16 \Omega$$



The output RF power absorbed by the load resistance R is

$$P_{out} = \frac{1}{2} \cdot V_{ds1} \cdot I_{ds1} = \frac{1}{8} \cdot (V_{ds \max} - V_{ds \min}) \cdot I_{dss}$$

The required DC power is :

$$P_{dc} = \underline{V_{dso}} \cdot \underline{I_{dso}} = \frac{1}{4} \cdot (V_{ds \max} + V_{ds \min}) \cdot I_{dss}$$

$\eta_D =$   
output efficiency

We define the output efficiency (also named the drain efficiency):

$$\eta_D = \frac{P_s}{P_{dc}} = \left( \frac{V_{ds \max} - \cancel{V_{ds \min}}}{2 \cdot (V_{ds \max} + \cancel{V_{ds \min}})} \right) \quad \eta_D = 50\%$$

If we assume that  $V_{ds \min}$  is very low ( close to 0 ) or  $V_{ds \min}$  is very small compared to  $V_{ds \max}$

We have :

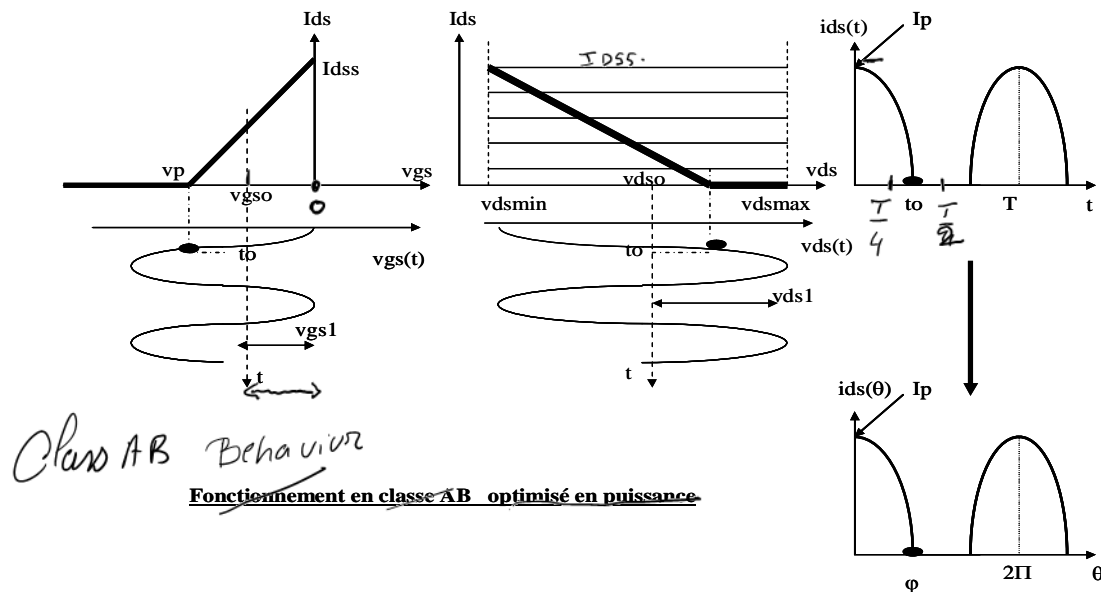
$$R = \frac{V_{ds \max}}{I_{dss}} \quad , \quad P_s = \frac{1}{8} \cdot V_{ds \max} \cdot I_{dss} \quad , \quad \eta_s = 50\%$$

Nevertheless, note that we need to calculate also the power added efficiency which depends on the input RF power

We will see several numerical applications during the tutorials .

$$\left\{ \begin{array}{l} P_{\eta} = \frac{1}{2} \cdot r_g \cdot c g s^2 \cdot w^2 \cdot \left| \frac{V_p}{2} \right|^2 \\ \eta_{aj} = \frac{P_{out} - P_{in}}{P_{dc}} \quad \rightarrow \underline{V_{GS1}} \\ G_p = \frac{P_{out}}{P_{in}} \rightarrow 10 \log \left( \frac{P_{out}}{P_{in}} \right) \end{array} \right.$$

### Class AB operation



For Class AB operation we have :

$$\frac{T}{4} \leq t_o \leq \frac{T}{2}$$

$$V_p \leq V_{gs0} \leq \frac{V_p}{2}, \text{ and } \underline{V_{gs1} = -V_{gs0}} \text{ to have the maximum swing of the input}$$

Voltage  $V_{gs}$

Let us take for example  $\boxed{V_{gs0} = \frac{2}{3} \cdot V_p}$

The aperture angle  $\varphi$  can be calculated by using the following equation .

$$\frac{2V_p}{3} - \left( \frac{2V_p}{3} \right) \cdot \cos(\varphi) = V_p \quad \text{soit} \quad \cos[\varphi] = -\frac{1}{2}$$

$V_{gs0} + V_{gs1} \cos \varphi = V_p$

$I_{D50} = \frac{I_p}{\pi} \frac{\sin \varphi - \varphi \cos \varphi}{(1 - \cos \varphi)}$

$I_{D51} = \dots \dots \dots$

Therefore

$$\boxed{\varphi = \frac{2\pi}{3}}$$

As  $I_p = I_{DSS}$  , we can calculate the DC drain current  $I_{D50}$  and the drain current component at the fundamental frequency  $I_{D51}$

$$\underline{I_{D50} = 0.4 \cdot I_{DSS}} \quad \text{and} \quad \underline{I_{D51} = 0.54 \cdot I_{DSS}}$$

We still choose  $\underline{V_{D50} = \left( \frac{V_{D5 \max} + V_{D5 \min}}{2} \right)}$

so the maximum value of  $V_{D51}$  is  $\underline{V_{D51} = \left( \frac{V_{D5 \max} - V_{D5 \min}}{2} \right)}$

The optimal load resistance is now :  $\underline{R = \left( \frac{V_{D51}}{I_{D51}} \right) = \left( \frac{V_{D5 \max} - V_{D5 \min}}{1.08 \cdot I_{DSS}} \right)}$





The output RF power absorbed by R is

$$P_{out} = 0.135 \cdot (V_{ds\ max} - V_{ds\ min}) \cdot I_{DSS}$$

The DC power supplied by the DC power supply is :

$$P_{dc} = 0.2 \cdot (V_{ds\ max} + V_{ds\ min}) \cdot I_{DSS}$$

The drain efficiency is :

$$\eta_s = 0.675 \cdot \left( \frac{V_{ds\ max} - V_{ds\ min}}{V_{ds\ max} + V_{ds\ min}} \right)$$

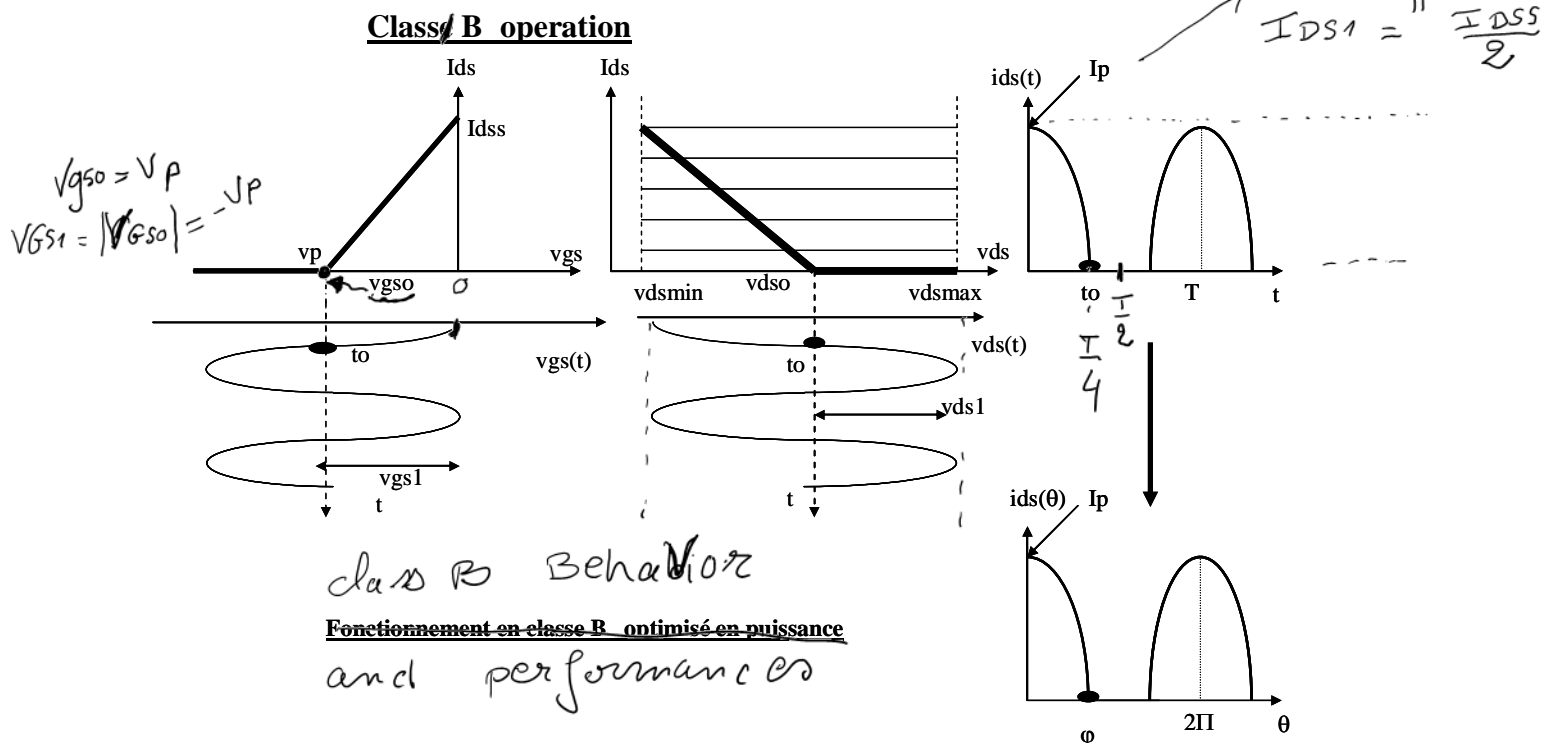
In an ideal case where  $V_{ds\ min} \ll V_{ds\ max}$

$$R = \frac{V_{ds\ max}}{1.08 \cdot I_{DSS}}, \quad P_s = 0.135 \cdot V_{ds\ max} \cdot I_{DSS}, \quad \boxed{\eta_s = 67.5\%}$$

$$V_{DS1} = \frac{V_{DS\ max} - V_{DS\ min}}{2}$$

$$I_{DS1} = 0.54 I_{DSS}$$

$$R = \frac{V_{DS1}}{I_{DS1}} = \frac{(V_{DS\ max} - V_{DS\ min})}{1.08 I_{DSS}}$$



For Class B operation :

$$t_o = \frac{T}{4}$$

$V_{gs0} = V_p$  , and  $V_{gs1} = -V_{gs0}$  for a maximum input voltage RF swing

Concerning the aperture angle , we have now the equation :

$$V_{gs0} + V_{gs1} \cos \varphi = V_p$$

$$V_p - V_p \cdot \cos(\varphi) = V_p \quad \text{therefore} \quad \cos[\varphi] = 0$$

And  $\boxed{\varphi = \frac{\pi}{2}}$

We still have

$$I_p = I_{dss}$$

$$I_{Dso} = \frac{I_p}{\pi} \frac{\sin \varphi - \varphi \cos \varphi}{1 - \cos \varphi}$$

$$I_{Ds1} = \frac{I_p}{\pi} \frac{\varphi - \sin \varphi \cos \varphi}{1 - \cos \varphi}$$

!!!  $\varphi$  in rad

The Drain current components at Dc and at the fundamental frequency are :

$$I_{dso} = \frac{I_{dss}}{\pi} \quad \text{et} \quad I_{ds1} = \frac{I_{dss}}{2}$$

$$V_{dso} = \left( \frac{V_{ds \max} + V_{ds \min}}{2} \right) \quad \text{and} \quad V_{ds1} = \left( \frac{V_{ds \max} - V_{ds \min}}{2} \right)$$

$$\text{The optimal load resistance is } R = \left( \frac{V_{ds1}}{I_{ds1}} \right) = \left( \frac{V_{ds \max} - V_{ds \min}}{I_{dss}} \right)$$

The output RF power is

$$P_s = \frac{1}{8} \cdot (V_{ds \max} - V_{ds \min}) \cdot I_{dss}$$

The DC power is

$$P_{dc} = \frac{1}{2\pi} \cdot (V_{ds\max} + V_{ds\min}) \cdot I_{dss} = V_{DSO} \cdot I_{DSO}$$

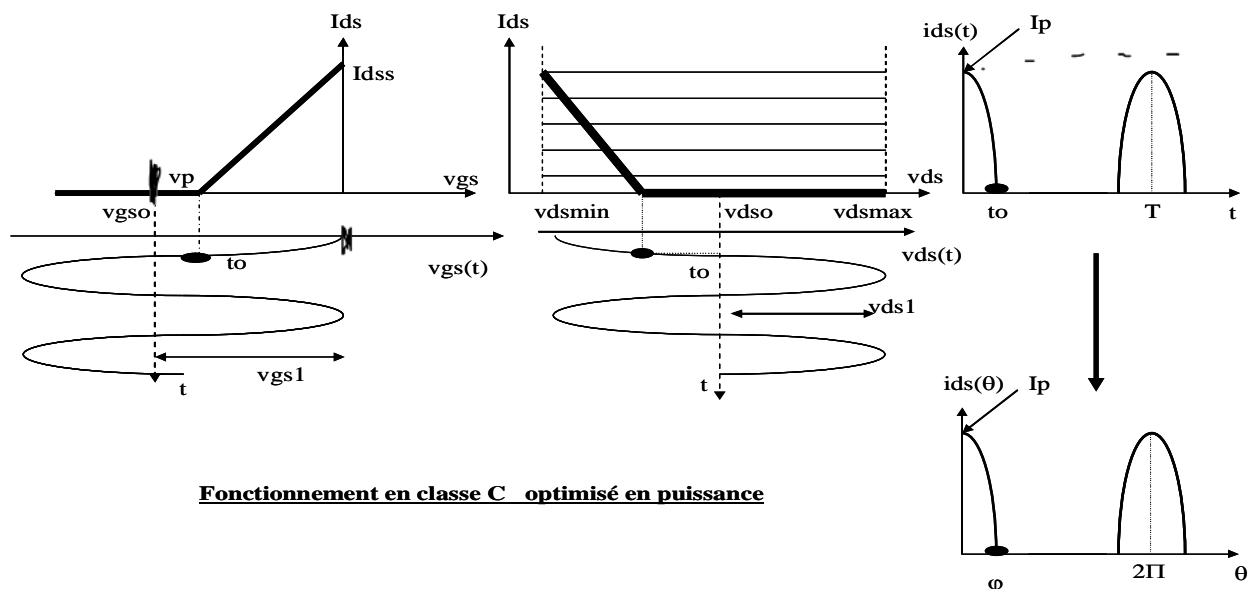
The drain efficiency is :  $= \frac{P_{out}}{P_{DC}}$

$$\rightarrow \eta_D = \left(\frac{\pi}{4}\right) \cdot \left(\frac{V_{ds\max} - V_{ds\min}}{(V_{ds\max} + V_{ds\min})}\right)$$

$\rightarrow$  If  $V_{ds\min} \ll V_{ds\max}$

$$R = \frac{V_{ds\max}}{I_{dss}}, \quad P_{s\text{ ou }r} = \frac{1}{8} \cdot V_{ds\max} \cdot I_{dss}, \quad \eta_s = 78.5\% = \frac{\pi}{4}$$

### Classe C operation



For Class C

$$t_o \leq \frac{T}{4}$$

$V_{gso} \leq V_p$  and  $V_{gs1} = -V_{gso}$  to have a maximum input voltage swing

Let us take for example :  $V_{gso} = \frac{3}{2} \cdot V_p$

The equation used to calculate the aperture angle is :

$$\frac{3}{2} \cdot V_p - \frac{3}{2} \cdot V_p \cdot \cos(\varphi) = V_p \rightarrow \cos[\varphi] = \frac{1}{3}$$

$$\varphi = 1.23Rd$$

$$I_p = I_{dss} \quad ; \quad I_{dso} = 0.25 \cdot I_{dss} \quad \text{and} \quad I_{ds1} = 0.44 \cdot I_{dss}$$

$$V_{dso} = \left( \frac{V_{ds \max} + V_{ds \min}}{2} \right) \quad \text{and} \quad V_{ds1} = \left( \frac{V_{ds \max} - V_{ds \min}}{2} \right)$$

$$\text{The optimal load resistance is } R = \left( \frac{V_{ds1}}{I_{ds1}} \right) = \left( \frac{V_{ds \max} - V_{ds \min}}{0.88 \cdot I_{dss}} \right)$$

The output RF power is :

$$P_s = 0.11 \cdot (V_{ds \max} - V_{ds \min}) \cdot I_{dss}$$

The DC power is :

$$P_{dc} = 0.125 \cdot (V_{ds \max} + V_{ds \min}) \cdot I_{dss}$$

The drain efficiency is :

$$\eta_s = 0.88 \cdot \left( \frac{V_{ds \max} - V_{ds \min}}{(V_{ds \max} + V_{ds \min})} \right)$$



If  $V_{dsmin} \ll V_{dsmax}$

$$R = 1.136 \cdot \frac{V_{dsmax}}{I_{dss}} \quad , \quad P_s = 0.11 \cdot V_{dsmax} \cdot I_{dss} \quad , \quad \eta_s = 88\%$$

#### 4) The load line equation:

During the conduction time of the transistor

$$I_{ds}(\theta) = \frac{I_p}{1 - \cos(\varphi)} (\cos(\theta) - \cos(\varphi))$$

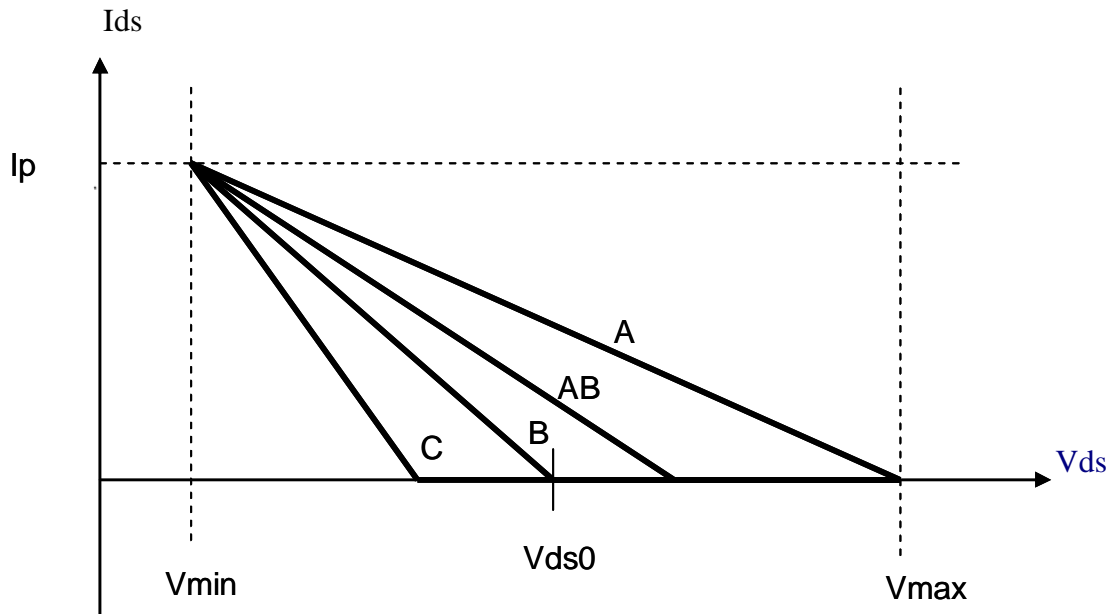
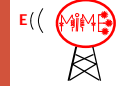
$$V_{ds}(\theta) = V_{ds0} - V_{ds1} \cdot \cos(\theta)$$

So

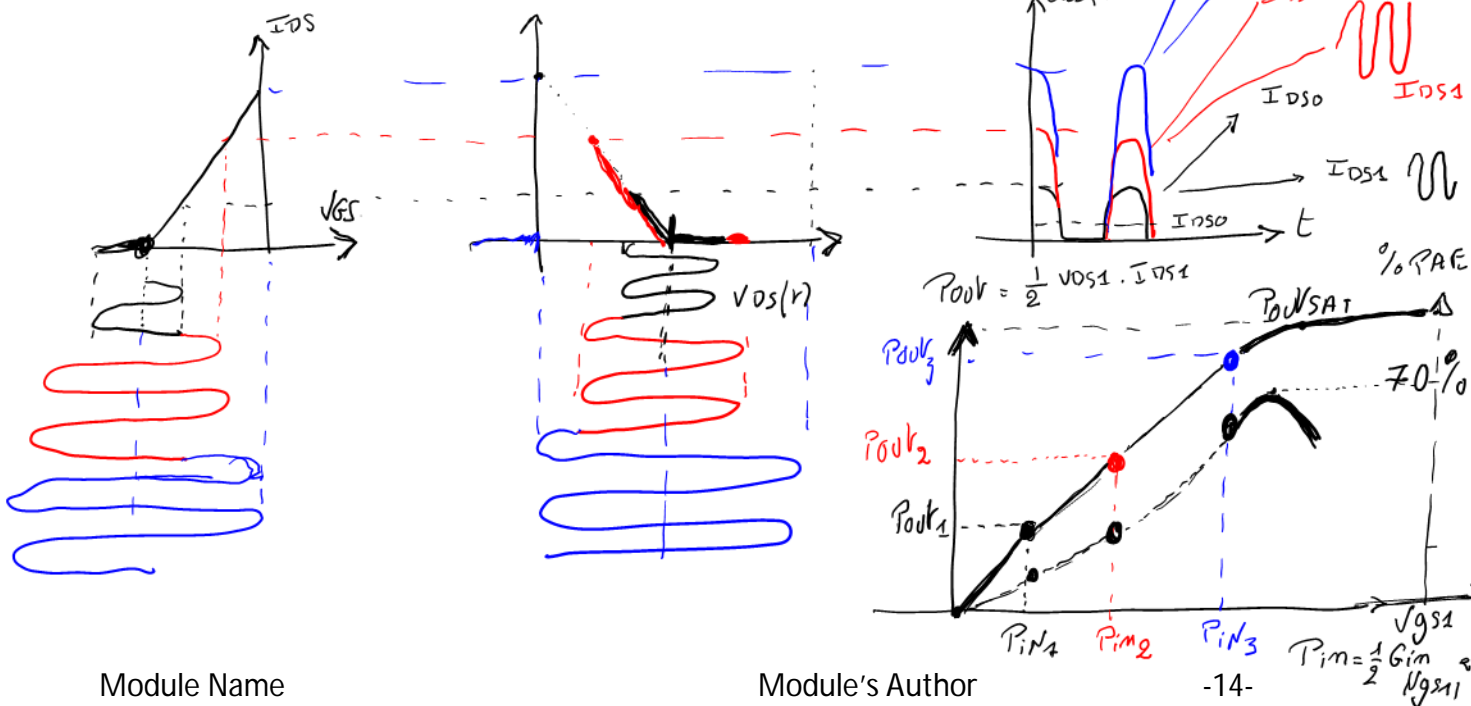
$$\cos(\theta) = \frac{V_{ds0} - V_{ds}}{V_{ds1}}$$

$$I_{ds} = \frac{I_p}{1 - \cos(\varphi)} \cdot \left( \frac{V_{ds0} - V_{ds}}{V_{ds1}} - \cos(\varphi) \right) = \frac{I_p}{1 - \cos(\varphi)} \cdot \left( \frac{V_{ds0}}{V_{ds1}} - \cos(\varphi) \right) - \frac{I_p}{(1 - \cos(\varphi)) \cdot V_{ds1}} \cdot V_{ds}$$

$$I_{ds} = A - B \cdot V_{ds}$$



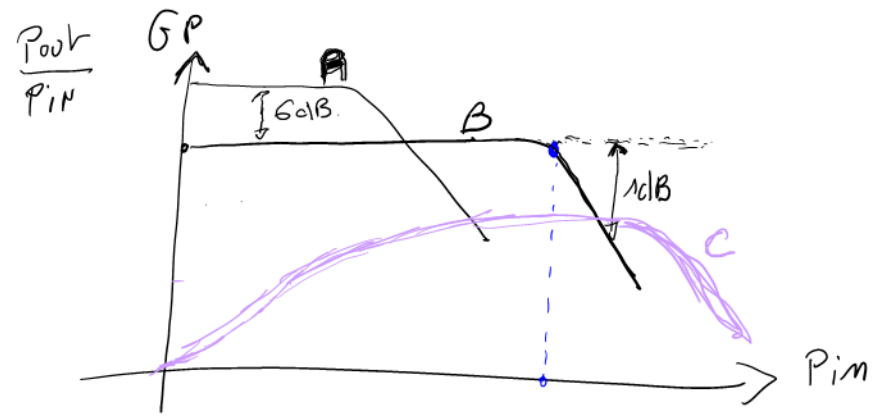
### 5) Power gain versus input power



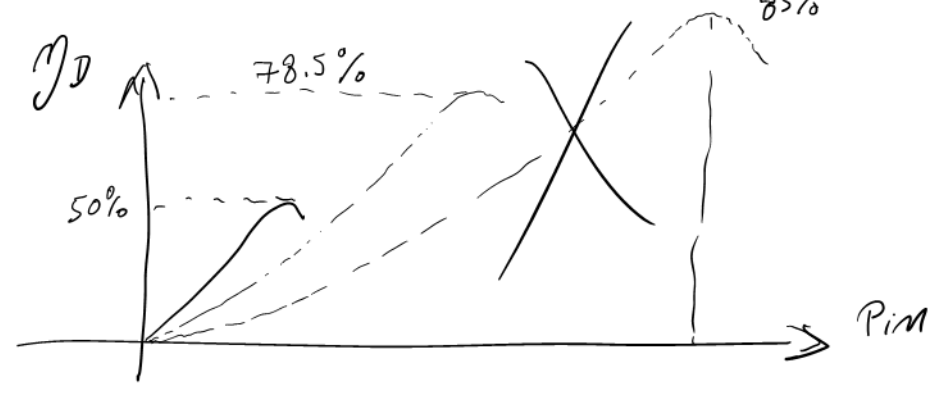
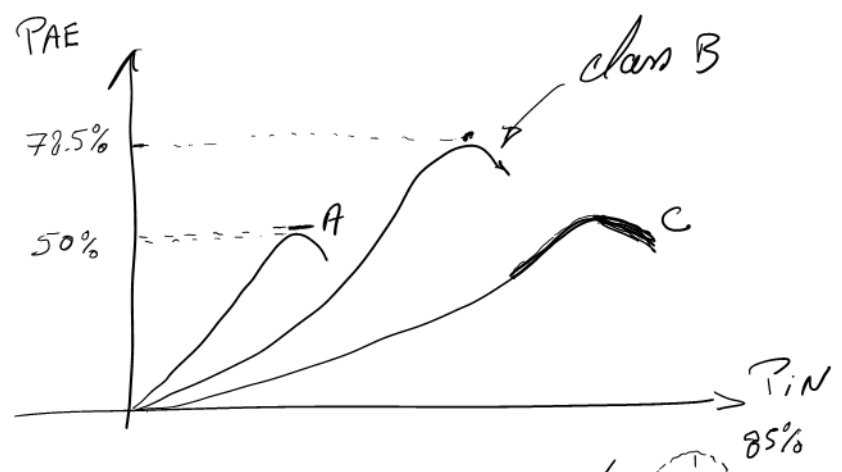
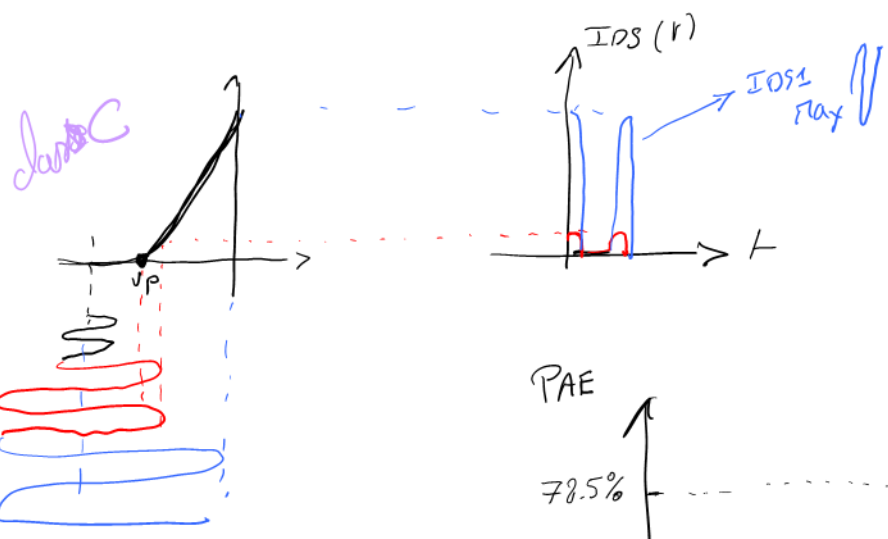
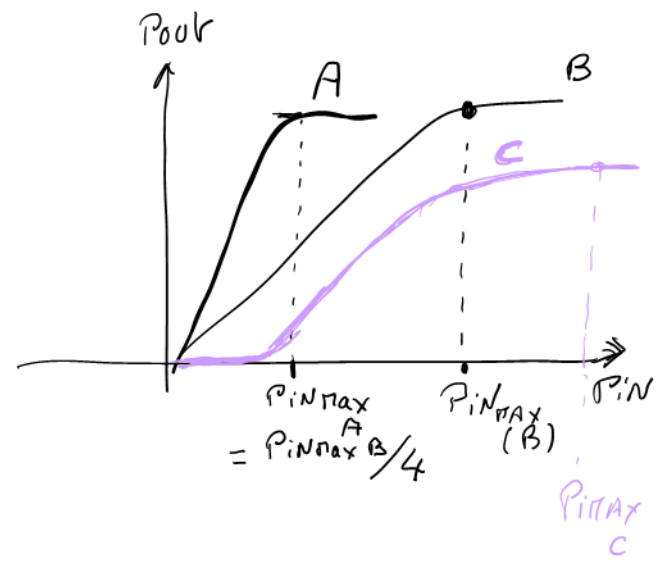
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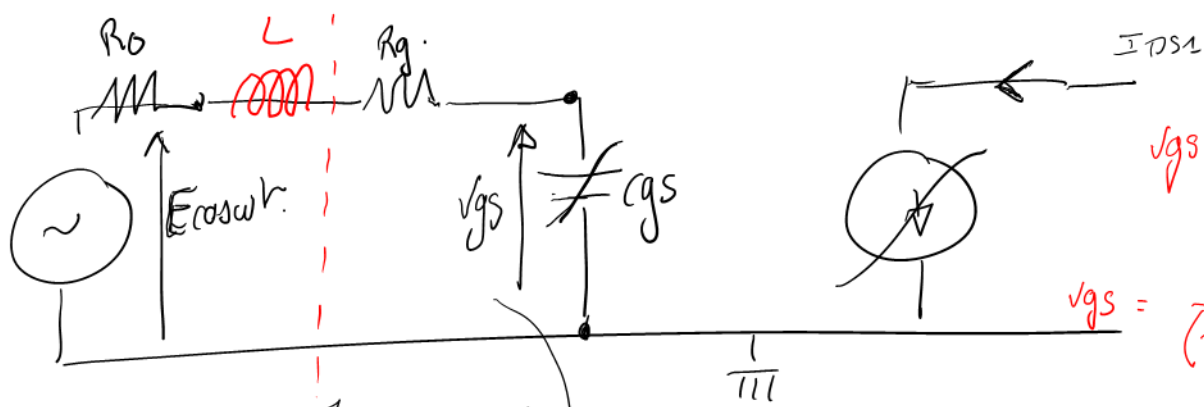
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$$G_P = 10 \log \left( \frac{P_{out}}{P_{in}} \right)$$





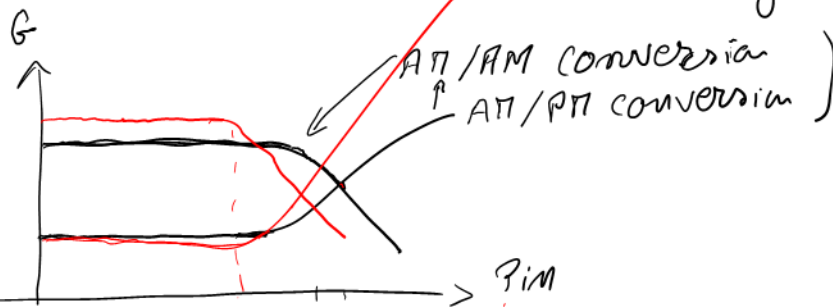
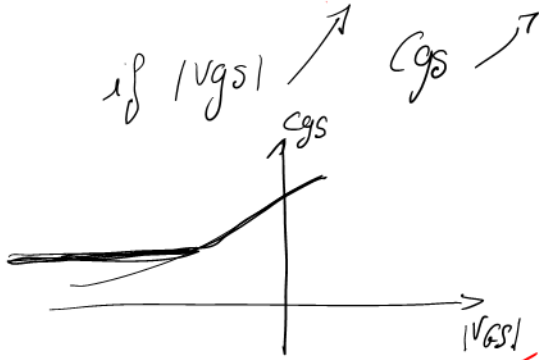
$$v_{gs} = \frac{\frac{1}{j\omega C_{gs}} E}{(R_0 + R_g) + j\omega L + \frac{1}{j\omega C_{gs}}}$$

$$v_{gs} = \frac{E}{(1 - L C_{gs} \omega^2) + j C_{gs} (R_0 + R_g) \omega}$$

$$v_{gs} = \frac{E \times \frac{1}{j\omega C_{gs}}}{R_g + \frac{1}{j\omega C_{gs}}} = \frac{-E}{1 + j R_g C_{gs} \omega}$$

$$|v_{gs}| = \frac{E}{\sqrt{1 + R_g^2 C_{gs}^2 \omega^2}}$$

$$\angle v_{gs} = -\text{Arctg}(R_g C_{gs} \omega)$$



Non linear distortion.

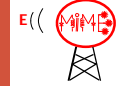
$$\angle v_{gs} = -\text{Arctg}\left(\frac{C_{gs} (R_0 + R_g)}{1 - L C_{gs} \omega^2}\right)$$





Erasmus+

E(rasmus) M(undus) on I(nnovative) M(icrowave)  
Electronics and O(ptics) M(aster)



## 6) Drain efficiency and power added efficiency versus input power

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