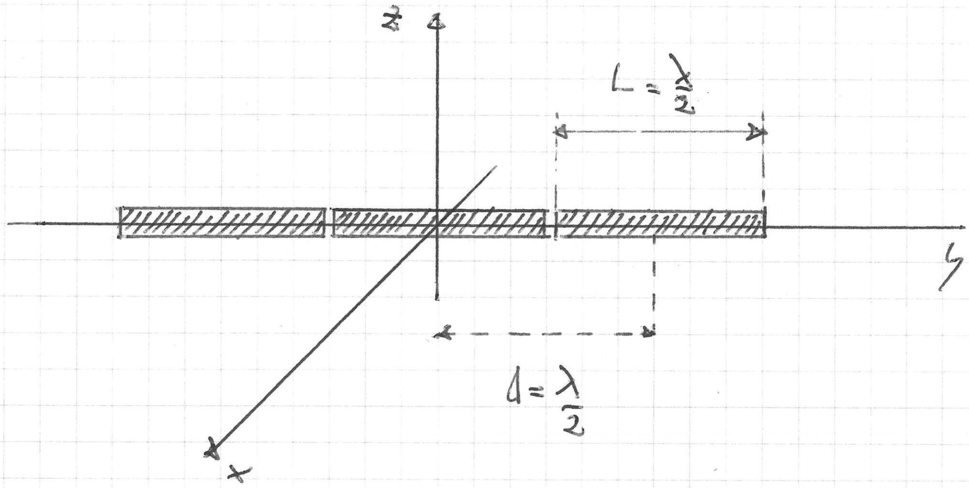
PROBLEM A2

LET US CONSIDER A LINEAR UNIFORM ARRAY CONSISTING OF THREE HALF-WAVE DIPOLES, ARRANGED IN A COLLINEAR CONFIGURATION AND PLACED AT A DISTANCE $d = \frac{\lambda}{2}$ (THE DISTANCE IS MEASURED FROM THE ANTENNAS' CENTRES)

THE PHASE DIFFERENCE BETWEEN NEIGHBOURING HALF-WAVE DIPOLES IS $\alpha = \pi$ AND THE WORKING FREQUENCY IS $f = 2{,}44$ GHz



FIND MAXIMA AND NULL DIRECTIONS IN THE (x, y) PLANE AND PLOT THE CORRESPONDING RADIATION PATTERN (IN THE x, y PLANE)

SOLUTION

$$f = 2{,}44 \text{ GHz} \qquad \lambda = \frac{c}{f} = \frac{3 \cdot 10^8}{2{,}44 \cdot 10^9} = 12{,}3 \text{ cm}$$

THE LENGTH OF EACH HALF WAVE ANTENNA IS $L = \frac{\lambda}{2} = 6{,}15$ cm AND THE SEPARATION BETWEEN THE ANTENNAS' CENTRES IS $d = \frac{\lambda}{2} = 6{,}15$ cm

IN ORDER TO PLOT THE RADIATION PATTERN WE NEED FIRST TO STUDY THE ARRAY ANTENNA FACTOR (AF)

$$|AF| = \left| \frac{\sin\left(N\frac{\varphi}{2}\right)}{\sin\left(\frac{\varphi}{2}\right)} \right| \qquad \varphi = \frac{2\pi}{\lambda} d \cos\psi + \alpha$$

$N=3$ $$\psi = \frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\theta + \pi = \pi\cos\theta + \pi = \pi\left(\cos\theta+1\right)$$

WE UNDERSTAND THAT THE RADIATION PATTERN OF THE ARRAY IS OBTAINED BY MULTIPLYING THE ARRAY FACTOR BY THE RADIATION PATTERN OF THE HALF-WAVE DIPOLE, AND WE OBSERVE THAT IN THE (x,y) PLANE THE HALF-WAVE DIPOLE IS NOT OMNIDIRECTIONAL

- MAXIMA OF THE ARRAY FACTOR FOR $\psi = 0, \pm 2\pi, \pm 4\pi, \pm 6\pi, \dots$

$\psi = 0 \Rightarrow$ $\psi = \pi\left(\cos\theta+1\right) = 0$ $\cos\theta = -1$

$\theta = \pi$ $\left(\theta = 180^\circ\right)$

$\psi = \pm 2\pi$ $\psi = \pi\left(\cos\theta+1\right) = \pm 2\pi$ $\cos\theta + 1 = \pm 2$

$\cos\theta = 1$ $\theta = 0$ $\left(\theta = 0^\circ\right)$

$\cos\theta = -3$ NO SOLUTION

$\psi = \pm 4\pi$ $\psi = \pi\left(\cos\theta+1\right) = \pm 4\pi$ $\cos\theta + 1 = \pm 4$

$\cos\theta = -1 \pm 4$ NO SOLUTION

WE FOUND ALL THE MAXIMA OF THE AF $\theta = 0, \pi$ $\left(\theta = 0, 180^\circ\right)$

- NULL DIRECTIONS OF THE ARRAY FACTOR FOR $\psi = \pm \frac{2k\pi}{N}$ $k \neq N, 2N, \dots$

$k=1$ $\psi = \pm\frac{2\pi}{N} = \pm\frac{2\pi}{3}$ $\psi = \pi\left(\cos\theta+1\right) = \pm\frac{2\pi}{3}$

$\cos\theta + 1 = \pm\frac{2}{3}$

$\cos\theta = -1 + \frac{2}{3} = -\frac{1}{3}$ $\theta = a\cos\left(-\frac{1}{3}\right)$ WHERE $a\cos$ IS THE ARCCOSINE FUNCTION

$\theta = a\cos\left(-\frac{1}{3}\right) = \pm 109{,}5^\circ$

$\cos\theta = -1 - \frac{2}{3}$ NO SOLUTION

$k=2$ $\quad \psi = \pm \frac{4\pi}{N} = \pm \frac{4}{3}\pi \qquad \psi = \pi(\cos\theta + 1) = \pm \frac{4}{3}\pi$

$\cos\theta + 1 = \pm \frac{4}{3}$

$\cos\theta = \frac{1}{3} \qquad \theta = a\cos\left(\frac{1}{3}\right) \qquad \theta = \pm 70{,}5^\circ$

$\cos\theta = -1 - \frac{4}{3}$ NO SOLUTION

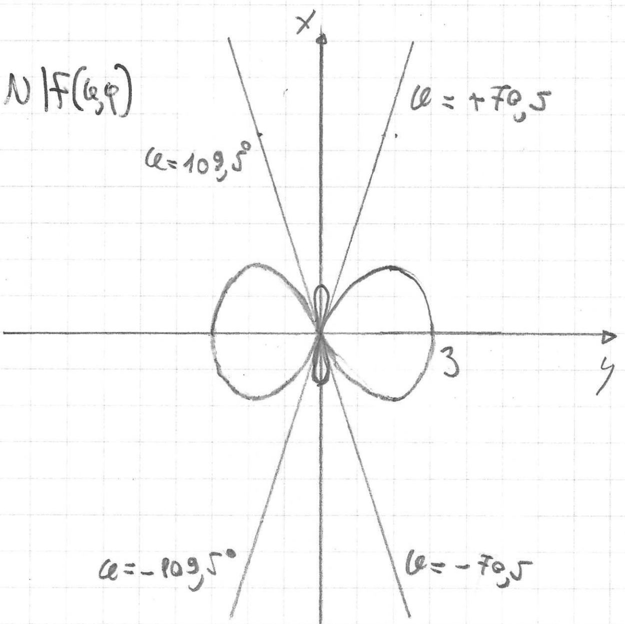
$k=3$
$N=3$ $\quad \psi = \pm \frac{6\pi}{N} = \pm 2\pi \qquad \psi = \pi(\cos\theta + 1) = \pm 2\pi$

$\cos\theta + 1 = \pm 2$ WRONG

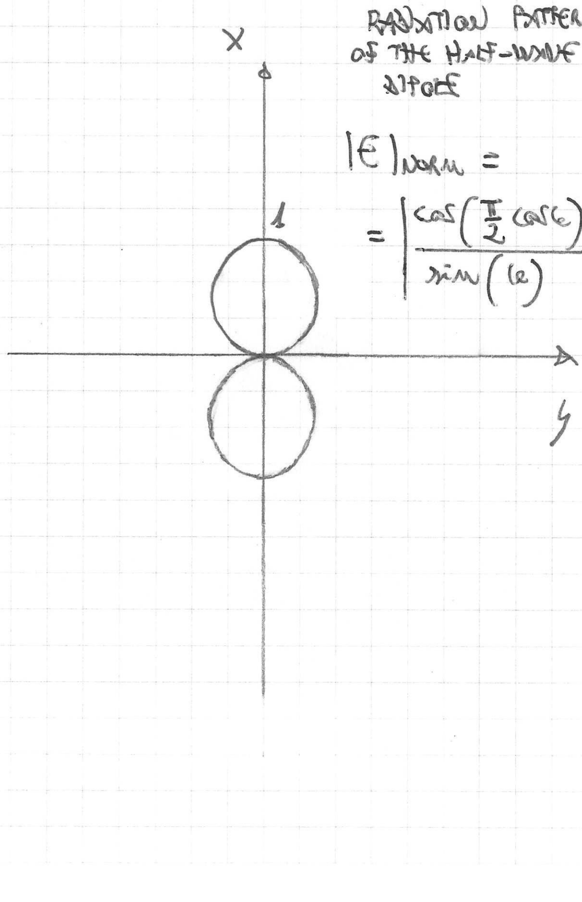
$k=4$ $\quad \psi = \pm \frac{8\pi}{N} = \pm \frac{8}{3}\pi \qquad \psi = \pi(\cos\theta + 1) = \pm \frac{8}{3}\pi$

$\cos\theta + 1 = \pm \frac{8}{3}$ NO SOLUTION

WE FOUND ALL THE NULL DIRECTION OF THE AF $\theta = \pm 70{,}5^\circ, \pm 109{,}5^\circ$



RADIATION PATTERN OF THE HALF-WAVE DIPOLE

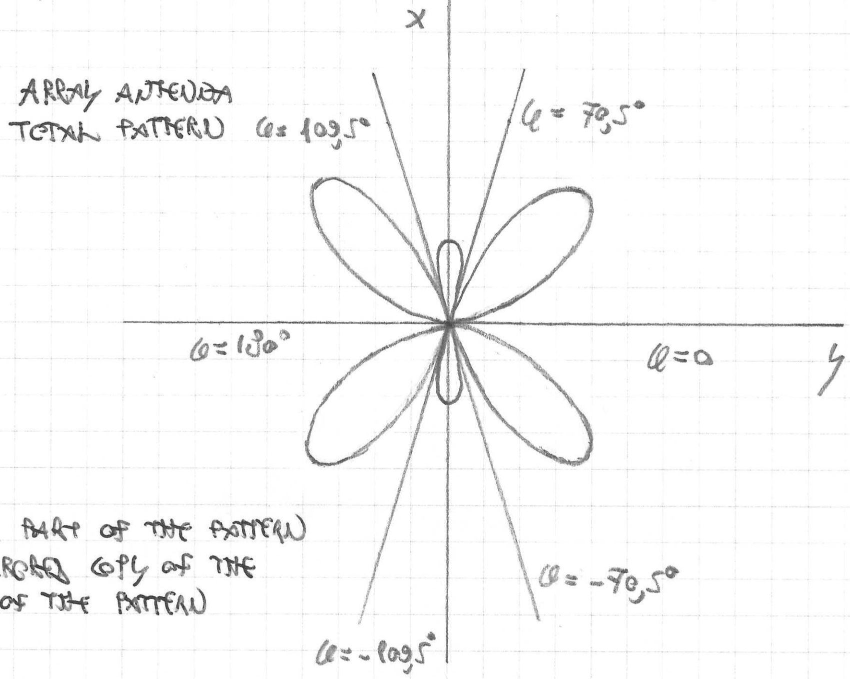


$|E|_{NORM} = \left| \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin(\theta)} \right|$

TWO MAIN LOBES AND
TWO SECONDARY LOBES

THE TOTAL PATTERN IS OBTAINED BY MULTIPLYING THE AF BY THE PATTERN OF THE HALF-WAVE DIPOLE, AND WE COULD NORMALIZE THE PATTERN AS WELL

THE TOTAL PATTERN CAN BE CONSTRUCTED FROM THE NULL DIRECTIONS $\theta = 0, \pm 70{,}5^\circ, \pm 109{,}5^\circ, \pm 180^\circ$



THE LOWER PART OF THE PATTERN IS THE MIRRORED COPY OF THE UPPER PART OF THE PATTERN

WE ALSO OBSERVE THAT THE CONSIDERED ARRAY IS EQUIVALENT TO A RESONANT DIPOLE HAVING A TOTAL LENGTH $L = \frac{3}{2}\lambda$

AND IN FACT THE TOTAL NORMALIZED RADIATION PATTERN IS

$$|F(\theta,\varphi)| = 0{,}7148 \left| \frac{\cos\left(\frac{3}{2}\pi\cos\theta\right)}{\sin(\theta)} \right|$$