MICROWAVE ENGINEERING

Lecture 19:

Concept of Impedance

Z, Y and S matrices

Size of circuit << Wavelength $\Rightarrow$ model with lumped elements

$\Downarrow$

- negligible phase change between two points
- Fields can be considered as in TEM

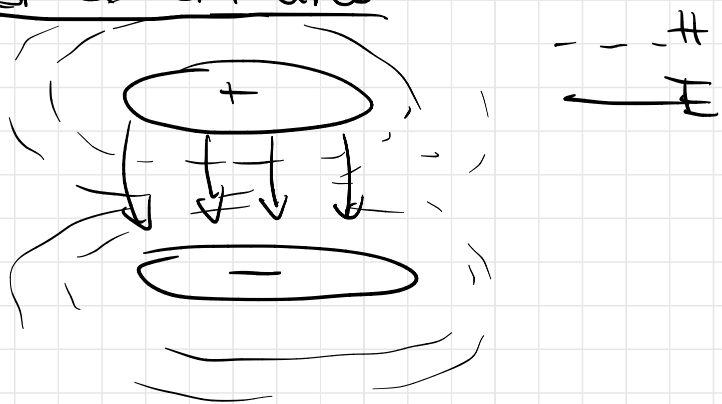
$\Leftarrow$ We can apply Kirchoff laws

Maxwell $\Rightarrow$ minute description (field distribution)

Circuit Theory $\Rightarrow$ Global behavior (Voltage, Current, I, R)

Impedance and equivalent voltage and current

For a TEM line:



$$V = \int_{+}^{-} \bar{E}\, d\bar{l}$$

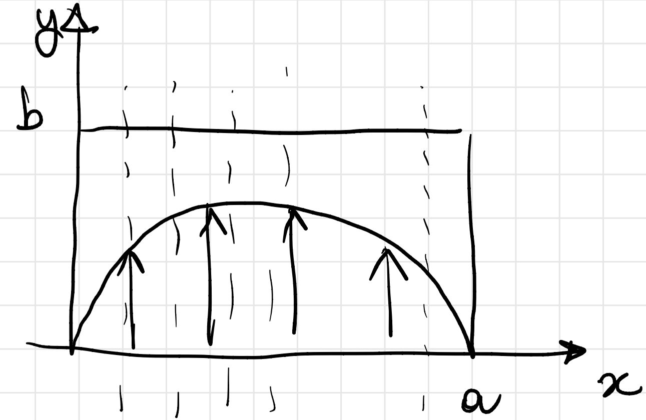
$$I = \oint_{C} \bar{H} \cdot d\bar{l}$$

Characteristic Impedance $Z_0 = \dfrac{V}{I}$

For a Waveguide :

CASE 1 RECTANGULAR WAVEGUIDE

The TE 10 mode :



$$E_y(x,y,z) = \frac{j\omega\mu a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z} =$$

$$= A\, e_y(x,y)\, e^{-j\beta z}$$

$$H_x(x,y,z) = \frac{j\beta a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z} = A\, h_x(x,y)\, e^{-j\beta z}$$

$$V = \int_{+} \bar{E}\, dl = \int_y \frac{j\omega\mu a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z} dy = \frac{-j\omega a\mu}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z} \int_y dy$$

VOLTAGE depends on both x and y $\Rightarrow$ There is NO UNIQUE value of Voltage

Rules to follow:

1 • Voltage and current refer to a specific mode

2 • Product of VI should always return the power flow of the mode

3 • $\frac{V}{I}$ has to be equal to the characteristic impedance of the line

For an arbitrary waveguide

$$\rightarrow \bar{E}_t(x,y,z) = \bar{e}(x,y)\left[A^+ e^{-j\beta z} + A^- e^{j\beta z}\right] =$$

$$= \frac{\bar{e}(x,y)}{C_1}\left[V^+ e^{-j\beta z} + V^- e^{j\beta z}\right] \qquad C_1 = \frac{V^+}{A^+} = \frac{V^-}{A^-}$$

$$\bar{H}_t(x,y,z) = \bar{h}(x,y)\left[A^+ e^{-j\beta z} + A^- e^{j\beta z}\right] =$$

$$= \frac{\bar{h}(x,y)}{C_2}\left[I^+ e^{-j\beta z} + I^- e^{j\beta z}\right]$$

$$C_2 = \frac{I^+}{A^+} = \frac{I^-}{A^-}$$

We know that generally

$$\bar{h}(x,y) = \frac{\hat{z} \times \bar{e}(x,y)}{Z_W}$$

Equivalent voltage and currents can be written as:

$$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$$

$$I(z) = I^+ e^{-j\beta z} + I^- e^{j\beta z}$$

$$\frac{V^+}{I^+} = \frac{V^-}{I^-} = Z_0$$

From rule No. 2 on Power

$$P^+ = \frac{1}{2}|A^+|^2 \iint_S \bar{e} \times \bar{h}^* \hat{z}\, dS = \frac{1}{2} \frac{V^+ I^{+*}}{C_1 C_2^*} \iint_S \bar{e} \times \bar{h}^* \hat{z}\, dS$$

We want $P = \frac{1}{2} V^+ I^{+*}$ $\Rightarrow$ $\boxed{C_1 C_2^* = \iint_S \bar{e} \times \bar{h}^* \hat{z}\, dS}$

S is the cross section of the WG

From rule No.3 on impedance

$$Z_0 = \frac{V^+}{I^+} = \frac{V^-}{I^-} = \frac{C_1}{C_2}$$

$$V^+ = C_1 A$$
$$I^+ = C_2 A$$

We want $Z_0 = Z_W \Rightarrow \frac{C_1}{C_2} = Z_W$ (Z_{TE} or Z_{TM})

CONCEPT OF IMPEDANCE

① $\eta = \sqrt{\frac{\mu}{\varepsilon}}$ INTRINSIC IMPEDANCE OF THE MEDIUM

EQUAL TO WAVE IMPEDANCE FOR PLANE WAVES

(2) $Z_W = \frac{E_t}{H_t}$ WAVE IMPEDANCE THAT DEPENDS ON THE TYPE OF WAVE, TYPE OF WG, MATERIAL AND FREQ.

(3) $Z_0 = \sqrt{\frac{L}{C}}$ CHARACTERISTIC IMPEDANCE, EQUAL TO V/I FOR TRAVELING WAVE ON Q TL.

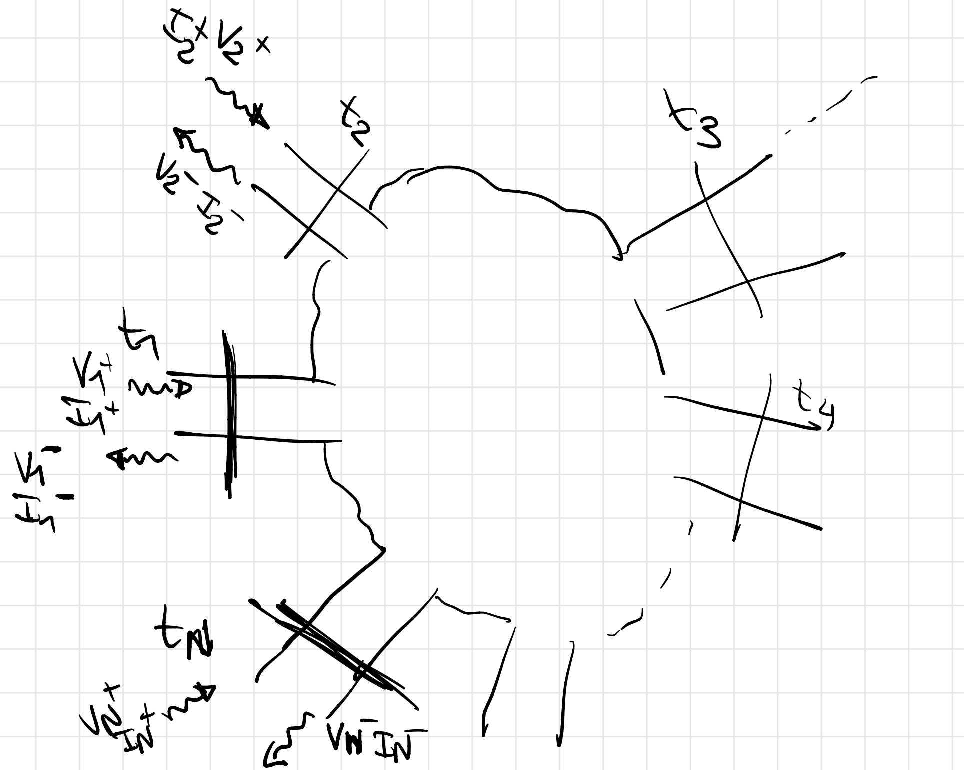
The IMPEDANCE IS UNIQUE ONLY FOR TEM WAVES!

IMPEDANCE AND ADMITTANCE MATRICES

At the n^{th} terminal plane:

$$V_N = V_N^+ + V_N^-$$

$$I_N = I_N^+ + I_N^-$$



The impedance matrix $[Z]$ is:

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & \ddots & & \\ \vdots & & \ddots & \\ Z_{N1} & \cdots & & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

$$\Downarrow$$

$$[V] = [Z]\,[I]$$

$$Z_{ij} = \frac{V_i}{I_j}\bigg|_{I_k = 0,\ k \neq j}$$

setting all other terminals to O.C.

Similarly we have the admittance matrix:

$$[I] = [Y][V]$$

$$Y_{ij} = \frac{I_i}{V_j}\bigg|_{V_k=0} \quad k \neq j \Rightarrow \text{SC all other terminals}$$

NOTE:

- $[Y] = [Z]^{-1}$
- z_{ij} and Y_{ij} are complex quantities

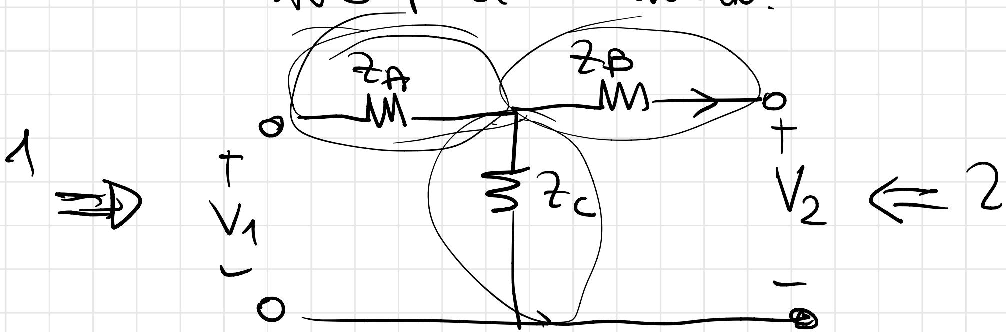
Properties of a network:

- RECIPROCAL: $z_{ij} = z_{ji}$ (matrix is symmetric)

• <u>LOSSLESS</u> : z_{ij} or Y_{ij} all purely imaginary

↳ the $Re\{P_{AV}\} = 0$ delivered to network

<u>Example</u> Find the Z parameters of the following Two port network:



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_A + Z_C$$

$$z_{21} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{V_2}{I_2}\,\frac{Z_C}{Z_B+Z_C} = \cancel{(Z_B+Z_C)}\,\frac{Z_C}{\cancel{(Z_B+Z_C)}} = Z_C$$

$$z_{12} = z_{21} \Rightarrow$$ network is reciprocal.

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_B + Z_C$$

THE SCATTERING MATRIX (Vector Network Analyzer)

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & \ddots & & \\ \vdots & & \ddots & \\ S_{N1} & \cdots & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

$$[V^-] = [S][V^+]$$

$$S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+ = 0 \text{ for } k \neq j}$$

- S_{ii} is the REFLECTION COEFFICIENT AT A SPECIFIC PORT

- S_{ij} is the transmission coefficient from j to i

Relation between $[S]$ and $[Z]$ or $[Y]$

let's assume that all characteristic impedances are identical Z_{0n} and we can set $Z_{0n} = 1$

$$V_N = V_N^+ + V_N^-$$

$$I_N = I_N^+ - I_N^- = V_N^+ - V_N^-$$

$$[Z][I] = [Z][V^+] - [Z][V^-] = [V] =$$

$$= [V^+] + [V^-]$$

$$([Z]+[U])([V^-]) = ([Z]-[U])[V^+]$$

Since $[V^-] = [S][V^+]$

$$([Z]+[U])([S]\cancel{[V^+]}) = ([Z]-[U])\cancel{[V^+]}$$

$$\boxed{[S] = ([Z]-[U])([Z]+[U])^{-1}}$$

For a 1 port network $\quad S_{11} = \dfrac{z_{11}-1}{z_{11}+1}$

Note! $\quad \boxed{[Z] = ([U]+[S])([U]-[S])^{-1}}$

- **Reciprocal**: $[S] = [S]^t$ | $[S]$ matrix is symmetrical

- **Lossless**: $[S]^t [S^*] = [U] \leftarrow$

or

$$[S]^* = \left\{ [S]^t \right\}^{-1}$$

SHIFT IN REFERENCE PLANES

We will consider a new ref. plane moved from $z_n = 0$ to $z_n = \ell_n$

The new scattering matrix will be $[V'^-] = [S'][V'^+]$

$$V_N'^+ = V_N^+ e^{j\theta_n} \qquad V_N'^- = V_N e^{-j\theta_n}$$

$\theta_n = \beta_n \ell_n$

In matrix form

$$\begin{bmatrix} e^{j\theta_1} & & & \\ & e^{j\theta_2} & & \\ & & \ddots & \\ & & & e^{j\theta_n} \end{bmatrix} [V'^-] = [S] \begin{bmatrix} e^{-j\theta_1} & & & \\ & e^{-j\theta_2} & & \\ & & \ddots & \\ & & & e^{-j\theta_n} \end{bmatrix} [V'^+]$$

$$[V'^-] = \begin{bmatrix} e^{-j\theta_1} & 0 & 0 & 0 \\ 0 & e^{-j\theta_2} & & \\ 0 & & \ddots & \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & & & \\ & e^{-j\theta_2} & & \\ & & \ddots & \\ & & & e^{-j\theta_n} \end{bmatrix} [V'^+]$$

$$\boxed{S'_{mn} = S_{mn} e^{-2j\theta_n}}$$