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Lesson 6
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II Definition of the characteristic impedance $Z_c(p)$ and of the reflexion coefficient $\Gamma(p)$

2-1 $Z_c(p)$

$$\begin{cases} V(z,p) = V_i(p)\, e^{-\gamma(p) z} + V_r(p)\, e^{\gamma(p) z} \\ I(z,p) = I_i(p)\, e^{-\gamma(p) z} + I_r(p)\, e^{\gamma(p) z} \end{cases}$$

$$Z_c(p) = \frac{V_i(p)}{I_i(p)}$$

Impedance at point z of the line $Z(p) = \frac{V(z,p)}{I(z,p)}$

$$-\frac{\partial v(z,t)}{\partial z} = R\, i(z,t) + L \frac{\partial i(z,t)}{\partial t}$$

$$-\frac{\partial V(z,p)}{\partial z} = (R + Lp)\, I(z,p)$$

$$\gamma(p) V_i(p)\, e^{-\gamma(p) z} - \gamma(p) V_r(p)\, e^{\gamma(p) z} = (R+Lp)\left(I_i(p)\, e^{-\gamma(p) z} + I_r(p)\, e^{\gamma(p) z}\right)$$

$$\gamma(p)\, V_i(p) = (R+Lp)\, I_i(p)$$

$$-\gamma(p)\, V_r(p) = (R+Lp)\, I_r(p)$$

$$\frac{V_i(p)}{I_i(p)} = \frac{R+Lp}{\gamma(p)} = \frac{R+Lp}{\sqrt{(R+Lp)(G+Cp)}} = \sqrt{\frac{R+Lp}{G+Cp}}$$

$$Z_c(p) = \sqrt{\frac{R+Lp}{G+Cp}}$$

$$\frac{V_r(p)}{I_r(p)} = -Z_c(p)$$

$$I(z,p) = \frac{1}{Z_c(p)}\left(V_i(p)\, e^{-\gamma(p) z} - V_r(p)\, e^{\gamma(p) z}\right)$$

$$V(z,p) = V_i(p)\, e^{-\gamma(p) z} + V_r(p)\, e^{\gamma(p) z}$$

lossless line : $Z_c(p) = \sqrt{\frac{L}{C}}$ $\quad R = G = 0$

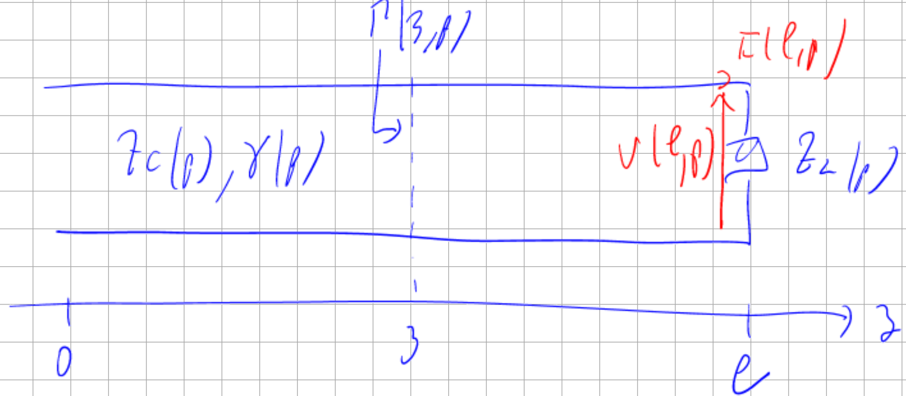
harmonic domain : $p = j\omega$

$$Z_c = \sqrt{\frac{R + jL\omega}{G + jC\omega}}$$

2-2- $\Gamma(p)$

$$\Gamma(z,p) = \frac{V_r(p)\,e^{\gamma(p)z}}{V_i(p)\,e^{-\gamma(p)z}} = \frac{V_r(p)}{V_i(p)}\,e^{2\gamma(p)z}$$

$$V(z,p) = V_i(p)\,e^{-\gamma(p)z} + V_r(p)\,e^{\gamma(p)z}$$



$$V(\ell,p) = Z_L(p)\,I(\ell,p)$$

$$V_i(p)\,e^{-\gamma(p)\ell} + V_r(p)\,e^{\gamma(p)\ell} = \frac{Z_L(p)}{Z_c(p)}\left(V_i(p)\,e^{-\gamma(p)\ell} - V_r(p)\,e^{\gamma(p)\ell}\right)$$

$$V_i(p)\,e^{-\gamma(p)\ell}\left[1 - \frac{Z_L(p)}{Z_c(p)}\right] = -V_r(p)\,e^{\gamma(p)\ell}\left[1 + \frac{Z_L(p)}{Z_c(p)}\right]$$

$$\frac{V_r(p)}{V_i(p)} = \frac{Z_L(p) - Z_c(p)}{Z_L(p) + Z_c(p)}\,e^{-2\gamma(p)\ell}$$

$$\Gamma(z,p) = \frac{Z_L(p) - Z_c(p)}{Z_L(p) + Z_c(p)}\,e^{2\gamma(p)(z-\ell)}$$

on the load $z = \ell$

$$\Gamma(\ell,p) = \Gamma_L(p) = \frac{Z_L(p) - Z_c(p)}{Z_L(p) + Z_c(p)}$$

- $Z_L(p) = Z_c(p)$
  $\Gamma_L(p) = 0$ incident wave
  the line is matched
- $Z_L(p) = 0$ short circuit
  $\Gamma_L(p) = -1$
- $Z_L(p) \to \infty$ open circuit
  $\Gamma_L(p) = +1$

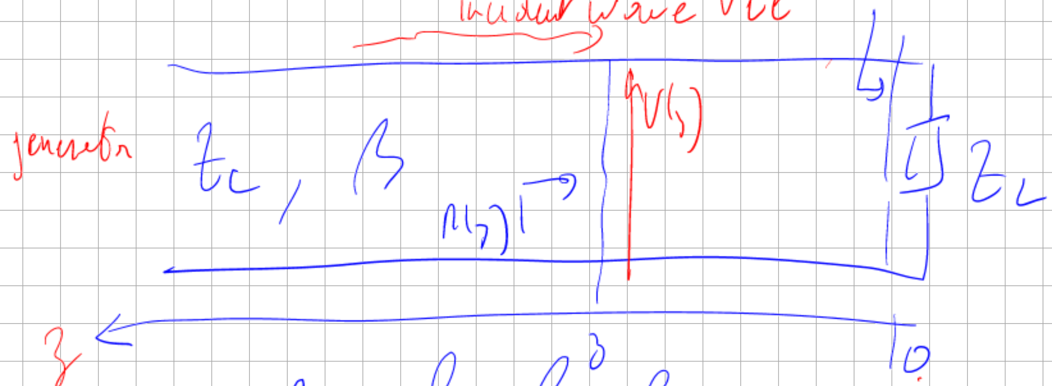
2-3. harmonic state

$$\gamma = j\omega$$

$$V(z) = V_i e^{+j\beta z} + V_r e^{-j\beta z}$$

$$I(z) = \frac{1}{Z_c}\left(V_i e^{+j\beta z} - V_r e^{-j\beta z}\right)$$

incident wave $V_i e^{j\beta z}$ $\quad \Gamma_L$



Z_c real lossless line

$$V(z) = V_i e^{j\beta z}\left(1 + \Gamma(z)\right)$$

$$\Gamma(z) = \frac{V_r}{V_i} e^{-2j\beta z} \qquad \Gamma(0) = \frac{V_r}{V_i} = \frac{Z_L - Z_c}{Z_L + Z_c}$$

$$\Gamma(z) = \Gamma(0) e^{-2j\beta z} = \Gamma_L e^{-2j\beta z}$$

$$\Gamma_L = |\Gamma_L| e^{j\varphi_L}$$

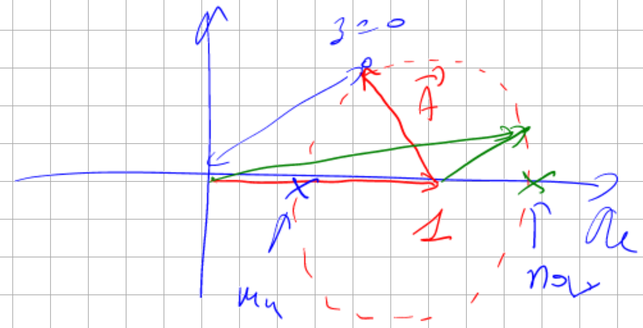
$$\Gamma(z) = |\Gamma_L| e^{j(\varphi_L - 2\beta z)}$$

$$V(z) = V_i e^{j\beta z}\left(1 + |\Gamma_L| e^{j(\varphi_L - 2\beta z)}\right)$$

$$|V(z)| = |V_i| \left| \underbrace{1}_{\vec{1}} + \underbrace{|\Gamma_L| e^{-j(\varphi_L - 2\beta z)}}_{\vec{A}} \right|$$

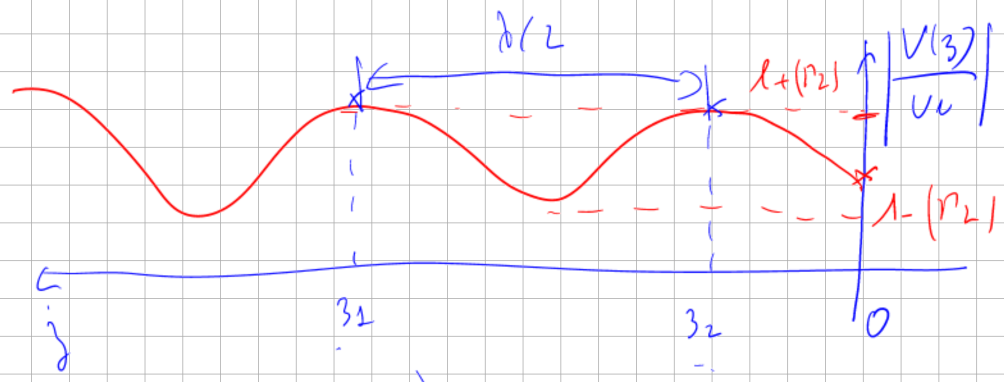
$$|V(z)|_{max} = |V_i|\left(1 + |\Gamma_L|\right)$$

$$\varphi_L - 2\beta z = 2k\pi$$



$$|V(z)|_{min} = |V_i|\left(1 - |\Gamma_L|\right)$$

$$\varphi_L - 2\beta z = 2(k+1)\pi$$



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between two max of $|V(z)|$

$$(\phi_L - 2\beta z_1) - (\phi_L - 2\beta z_2) = 2\pi$$

$$|2\beta(z_2 - z_1)| = 2\pi \qquad \beta = \frac{2\pi}{\lambda}$$

$$|z_2 - z_1| = \frac{\lambda}{2}$$

SWR standing wave ratio

$$S = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$Z_L = 0 \qquad \Gamma_L = -1 \qquad V_r = -V_i$$

$$V(z) = V_i e^{j\beta z} + V_r e^{-j\beta z}$$

$$= V_i \left(e^{j\beta z} - e^{-j\beta z}\right)$$

$$= 2j V_i \sin \beta z$$

$$v(z,t) = \mathrm{Re}\left(2j V_i \sin \beta z \, e^{j\omega t}\right)$$

$$= -2 V_i \sin \beta z \sin \omega t$$

$$\neq \sin(\omega t - \beta z)$$

$S \to \infty$ no propagation