Digital Modulation and Channel Coding

Review – Analogue Signal Modulation

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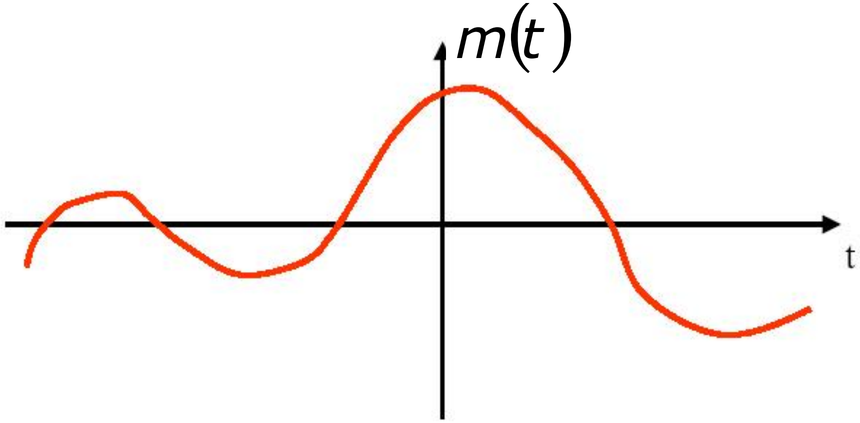
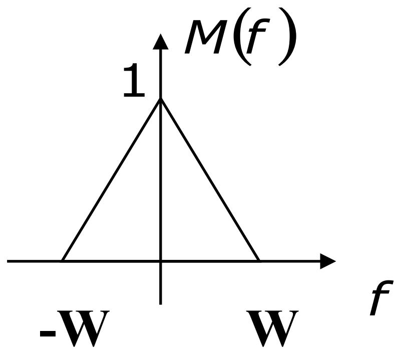
Analogue signal modulation

- Problem: we need to send the signal m(t) from TX to RX



- Spectrum in bandbase (band W) generally low-pass type



Modulation with one carrier

- **Amplitude modulation (AM)**

  → translation of the signal spectrum

  → linear

- **Angle modulation (PHASE/FREQUENCY)**

  → changes the argument of the carrier

  → not linear

- **Application**

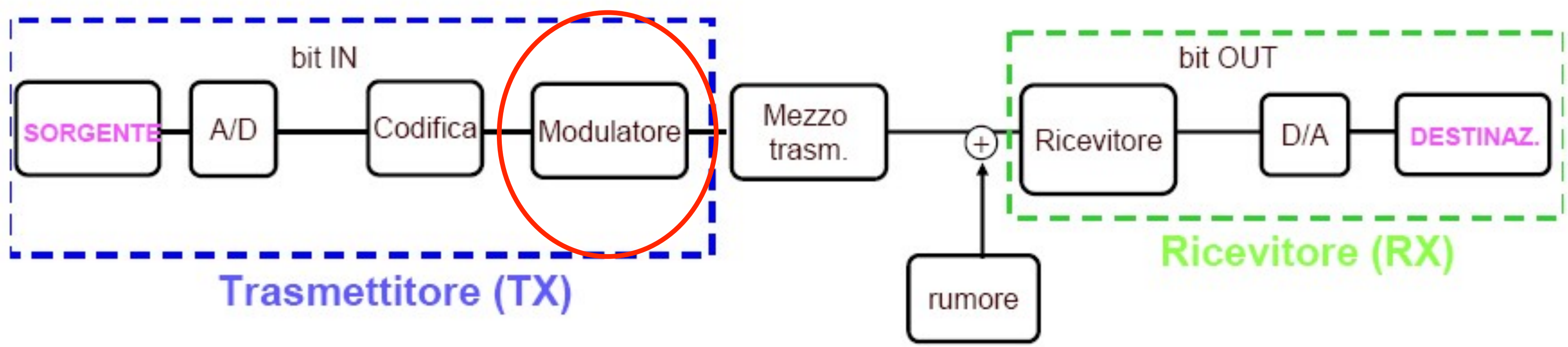
  → radio AM, FM

  → TV

  → traditional telephony

Modulation

- Signal modulation: turns the baseband bandwidth signal into a passband bandwidth signal, so the signal will be transmitted in a more easy way (it uses in an efficient way the frequency characteristics of the channel)



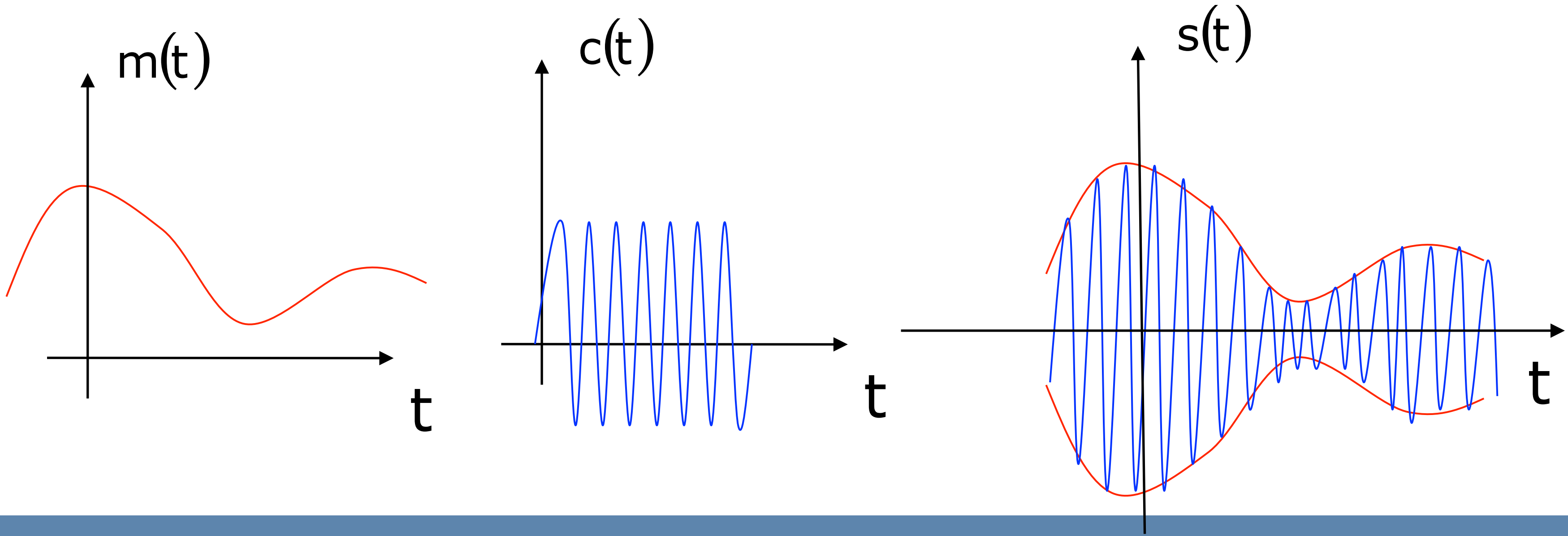
Amplitude Modulation AM

- Amplitude Modulation - AM

INFORMATION: m(t) is the "message" that must be sent

CARRIER: $c(t) = A_c \cos(2 \pi f_c t)$

MODULATED signal: s(t) is the signal that "travels" on the channel



AM with transmitted carrier (1)

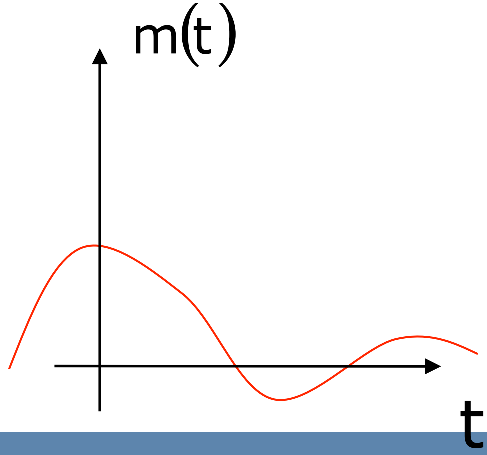
- Amplitude modulation with transmitted carrier (Classic Amplitude Modulation)

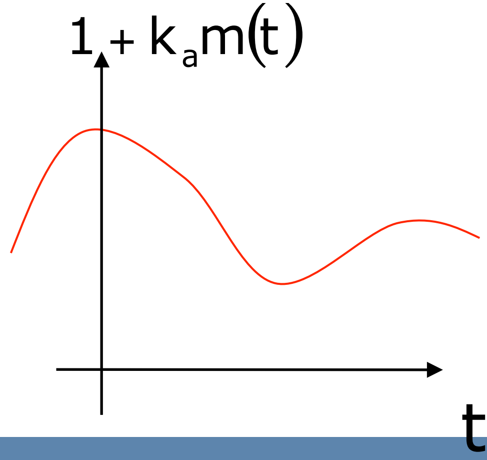
  INFORMATION: $m(t)$ is the "message" that must be sent

  CARRIER: $c(t) = A_c \cos(2 \pi f_c t)$

  MODULATED signal: $s(t)$ is the signal that "travels" on the channel

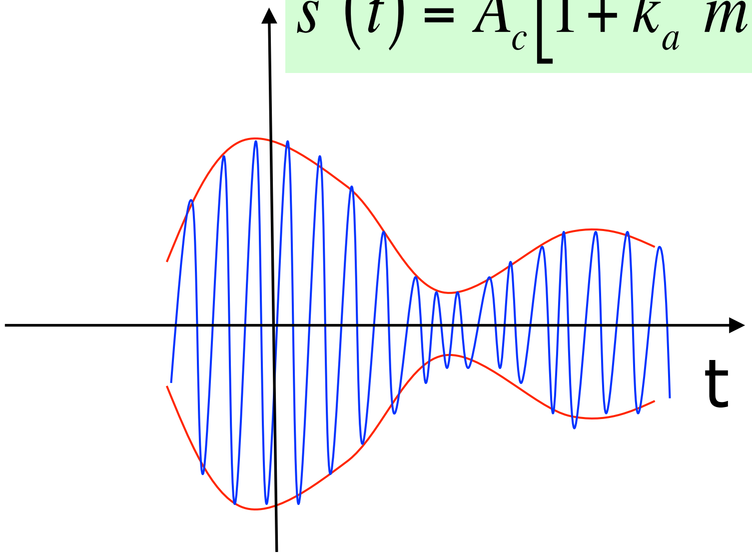
$$s(t) = A_c \left[1 + k_a\, m(t)\right] \cos\left(2\, \pi\, f_c\, t\right)$$





AM with transmitted carrier (2)

$$s(t) = A_c\left[1 + k_a\, m(t)\right]\cos\left(2\pi f_c t\right)$$



$$\mu \le 1$$

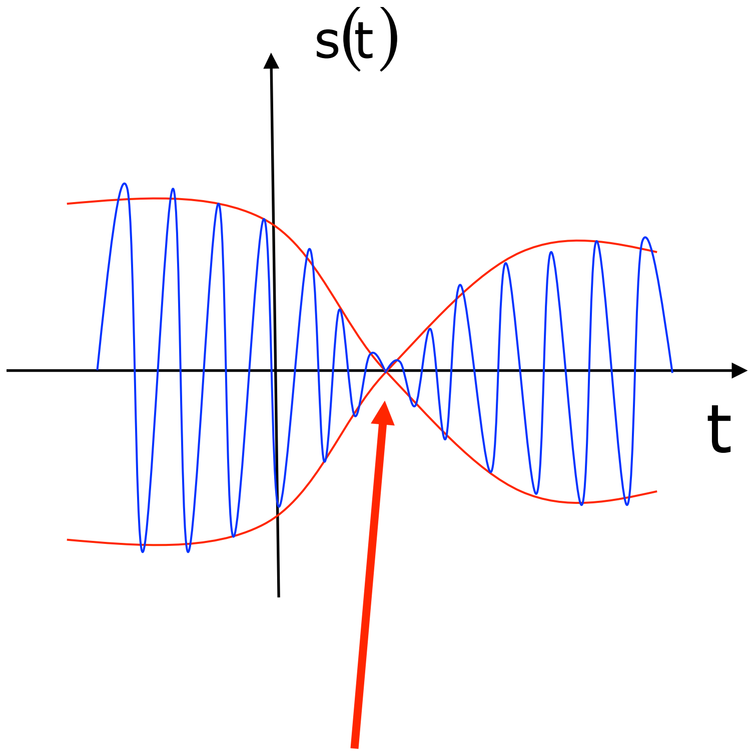
- The functioning limit is given from the index of the modulation.

$$\mu = \left|k_a\, m(t)\right|_{\max}, \quad 0 \le \mu \le 1$$

- N.B. The envelope doesn't cross zero!

Over-modulation

- In the case that the modulation index is bigger then 1 we are in presence of over-modulation



$\mu > 1$

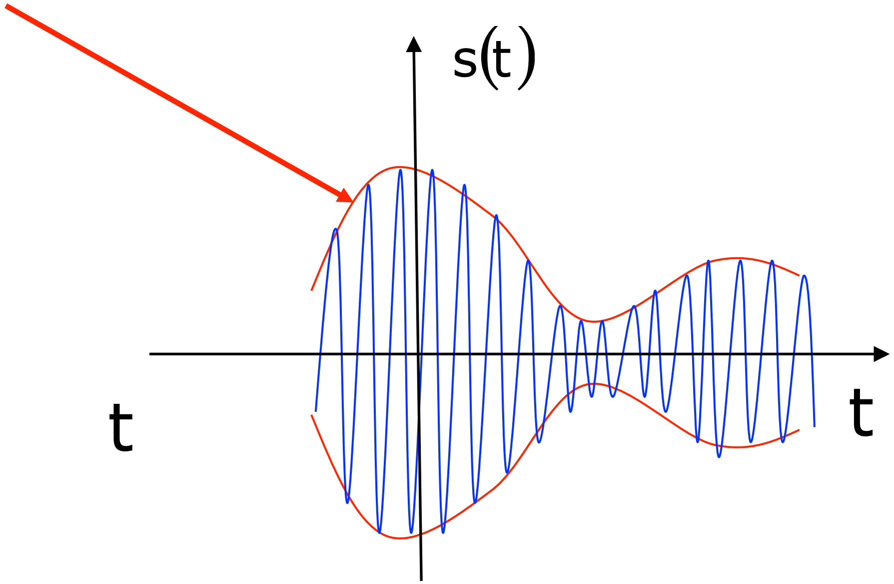
- N.B. There is a phase inversion in the point where the function crosses zero!

Amplitude Modulation

- Another condition: the carrier frequency must be much bigger than the maximum frequency of the "information":

$$f_c >> W$$

- Under these conditions, the amplitude of m(t) modules the instant amplitude of the carrier c(t): The information is hidden in the envelope of the modulated signal s(t)



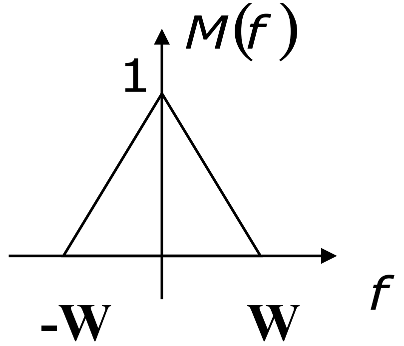
AM Signal's spectrum (1)

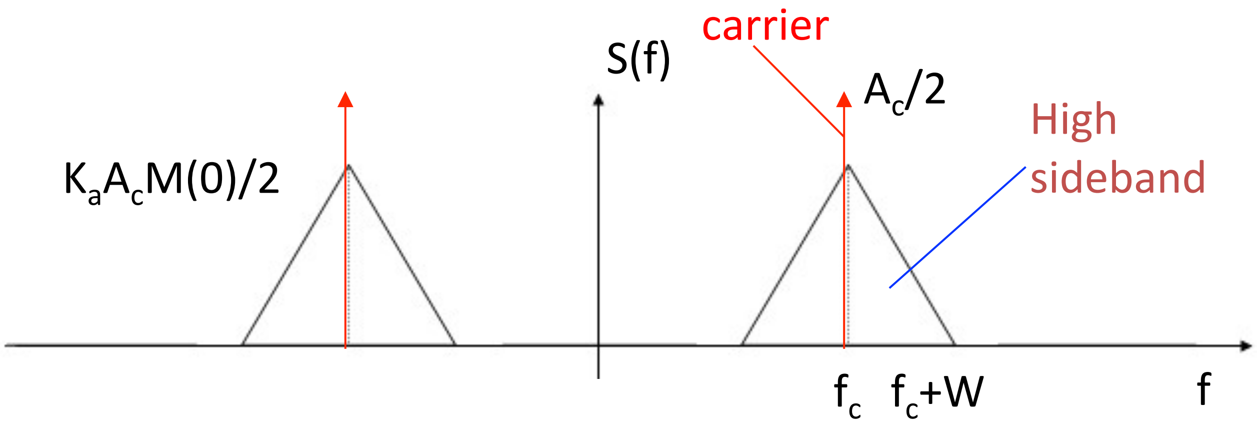
- AM Spectral characteristic

$$s(t) = A_c\,[1 + k_a\, m(t)]\,\cos(2\pi f_c t)$$

$\Updownarrow$

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{A_c\, k_a}{2}\left[M(f - f_c) + M(f + f_c)\right]$$





AM signal spectrum (2)

- AM Spectral characteristic

  It's a simple translation of the signal's spectrum around the frequency of the carrier!!

- Bandwidth

$$\boxed{B_T = 2W}$$

- It's a LINEAR modulation

- It is DISTORTED from NON-LINEAR systems!!

It's easy to recovery (demodulate) the original signal

→ demodulation = envelope detector

Bandwidth and power efficiency

- The efficiency of the bandwidth in the AM is equal to

$$\eta_B = \frac{W}{2W} = \frac{1}{2}$$

- The power efficiency is

$$\eta_P = \frac{P_s}{P_s + P_c}$$

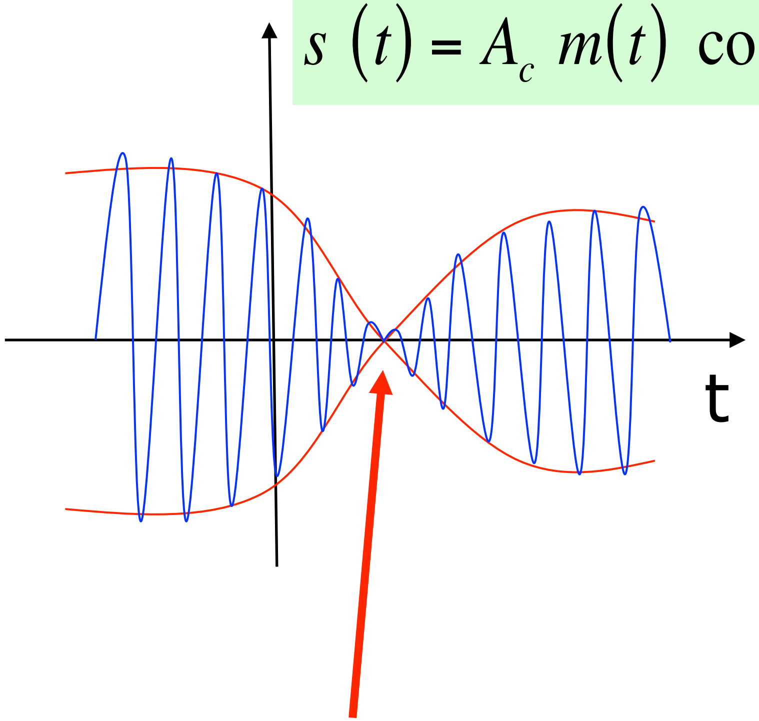
N.B. The transmitted carrier WASTES POWER! → double-sideband suppressed-carrier transmission

AM with Suppressed Carrier (1)

- Modulation DSB-SC (Double Side Band Suppressed Carrier)

  Called "AM with Suppressed Carrier"

$$s(t) = A_c\, m(t)\, \cos(2\pi f_c t)$$

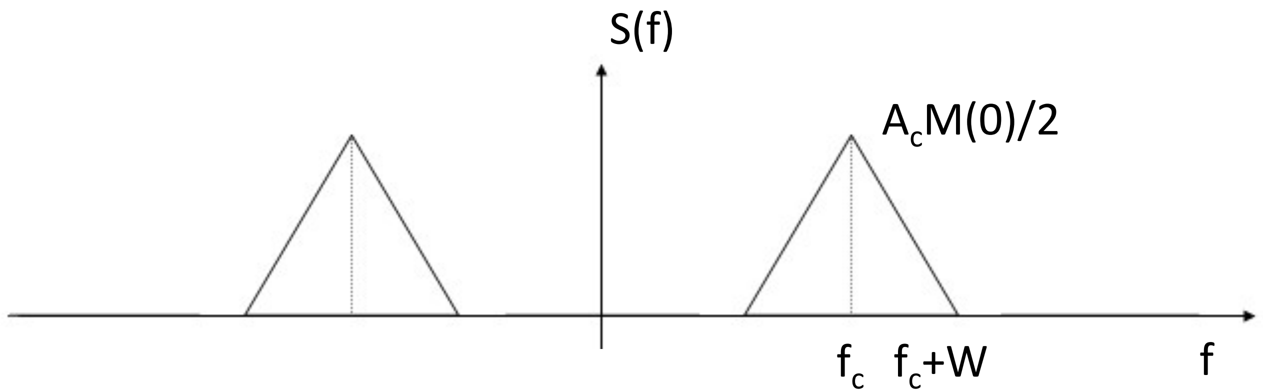


- There is a phase inversion in the point where the function crosses zero!

AM with Suppressed Carrier (2)

- It's more difficult to recover (demodulate) the original signal
- It uses less power then the system with TRANSMITTED CARRIER
- The bandwidth is the same as before B_T=2W, in fact the spectrum is

$$s(t) = A_c\, m(t)\, \cos(2\pi f_c t) \Longleftrightarrow S(f) = \frac{A_c}{2}\left[M(f - f_c) + M(f + f_c)\right]$$



AM with Suppressed Carrier (3)

- The efficiency of the bandwidth in the AM with suppressed carrier is equal to

$$\eta_B = \frac{W}{2W} = \frac{1}{2}$$

- The power efficiency is

$$\eta_P = 1$$

- For recovering the signal without transmitting the carrier we will need, at the demodulator, a sinusoid with the same frequency and phase of the carrier

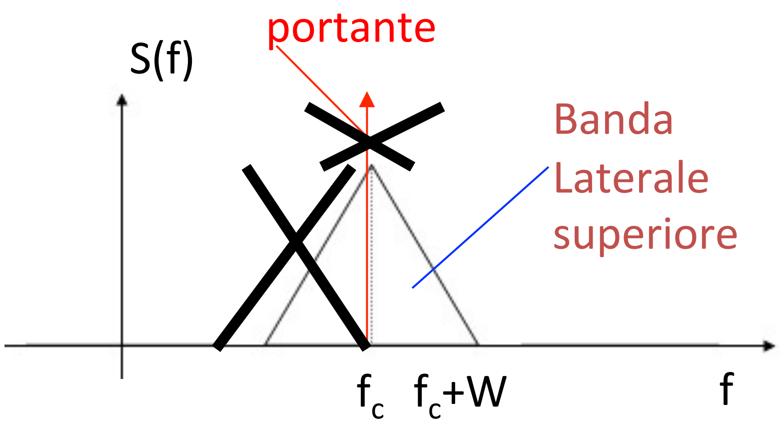
- This system is called SYNCHRONOUS (expensive synchronization circuits based on PLL at the demodulator)

AM with Unique SideBand

- There are some techniques of modulation that remove also ONE sideband of the two
  → used of the spectrum band is reduced!

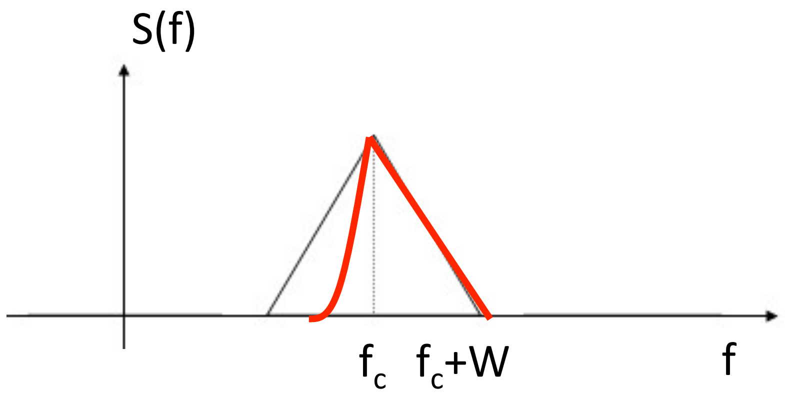
- AM with Suppressed Side Band (SSB)
- Bandwidth

$$B_T = W$$



Sideband partially suppressed

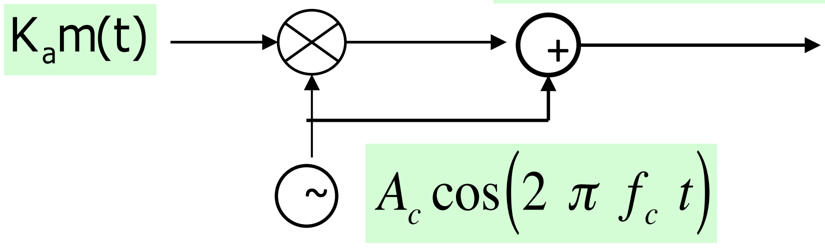
- It could be really difficult delete completely the sideband
- There are some modulation techniques that partially suppress the sideband
- Amplitude modulation with Vestigial Side Band (VSB)



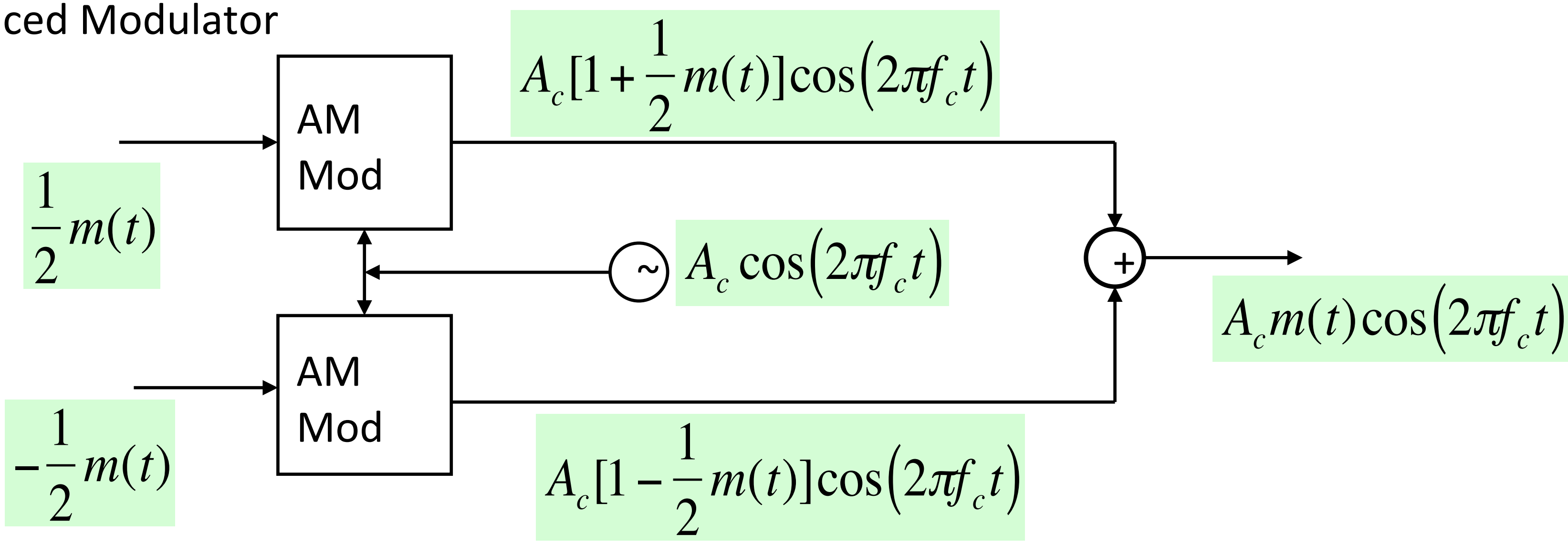
Modulator (1)

- Modulator with "product"

$$s(t) = A_c \cos(2\pi f_c t) + K_a m(t) \cdot A_c \cos(2\pi f_c t)$$



- Balanced Modulator



Modulator (2)

- **Kind of modulator:**

  → Variable Transconductance

  → Quadratic Modulator (Square law)

  → Switching Modulator

“Envelope” Demodulator (1)

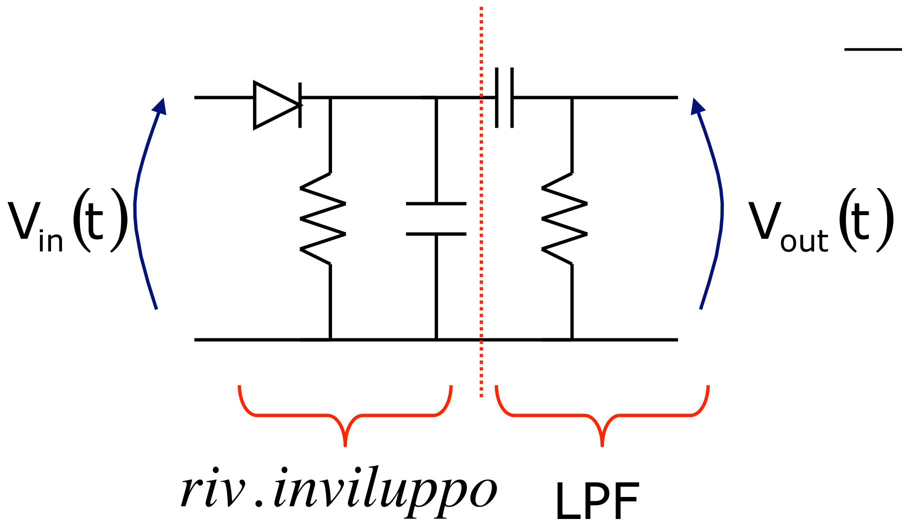
- Demodulation: it is the operation that allows for recovering m(t) from s(t)

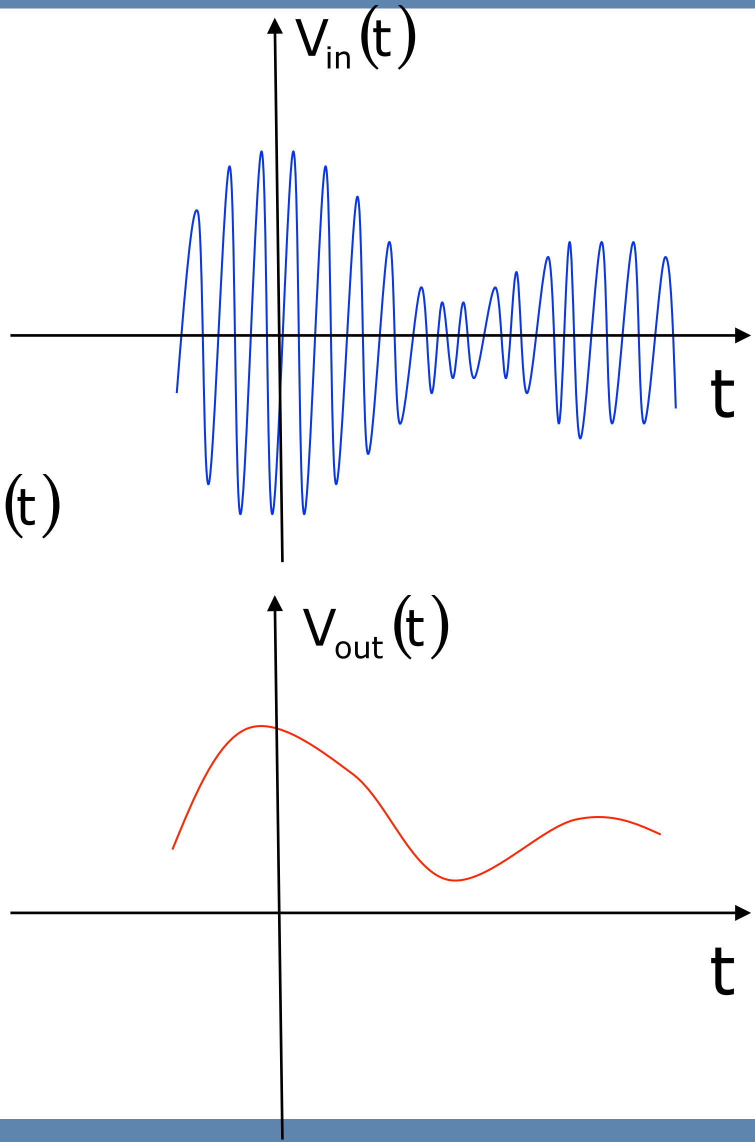
- In the case of amplitude modulation with transmitted carrier, we can obtain the “information” with a very simply demodulator, called “envelope” demodulator

- This demodulator is composed from a peak detector that discovers the envelope of the modulated signal.

- In the case of small modulation index (μ<<1) the performance are really good.

"Envelope" Demodulator (2)





Synchronous Demodulator (1)

- In the case of amplitude modulation with suppressed carrier, we can obtain the "information" with a synchronous demodulator.

- The modulated signal is multiplies with a co-sinusoidal at the same frequency and phase of the carrier

$$s(t) = A_c m(t) \cdot \cos(2\pi f_c t) \longrightarrow \bigotimes \longrightarrow d(t) = A_c m(t) \cdot \cos^2(2\pi f_c t)$$

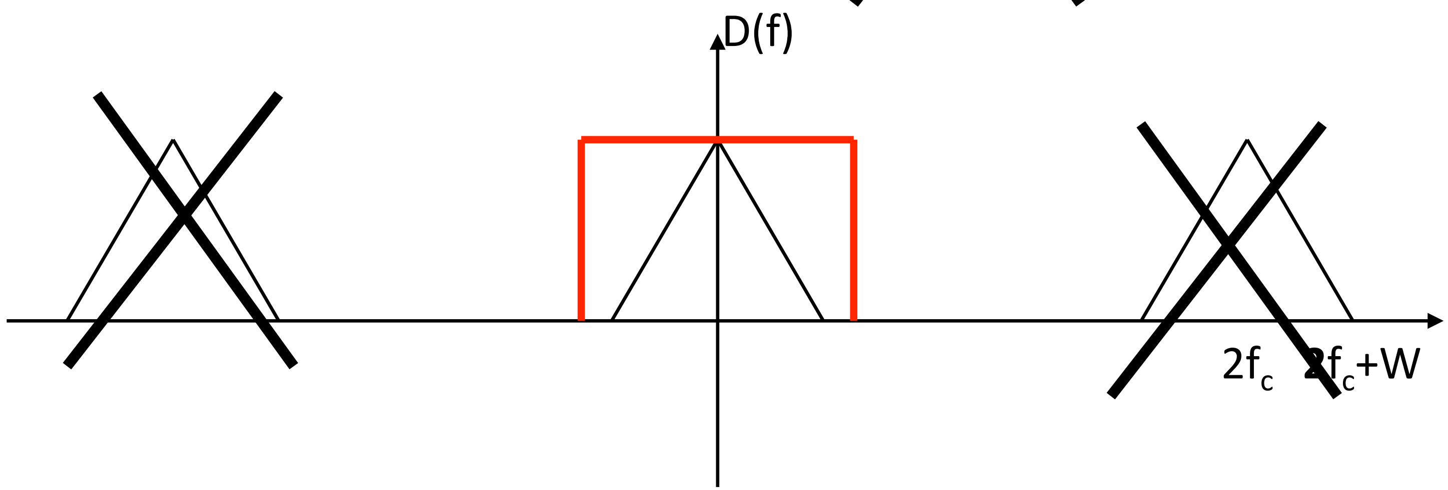
$$\cos(2\pi f_c t) \uparrow$$

- Since $\cos^2(\alpha) = \dfrac{(1+\cos(2\alpha))}{2}$ we have $d(t) = \dfrac{A_c}{2} m(t) \cdot \left(1 + \cos(2 \cdot 2\,\pi\, f_c\, t)\right)$

Synchronous Demodulator (2)

- Signal spectrum d(t)

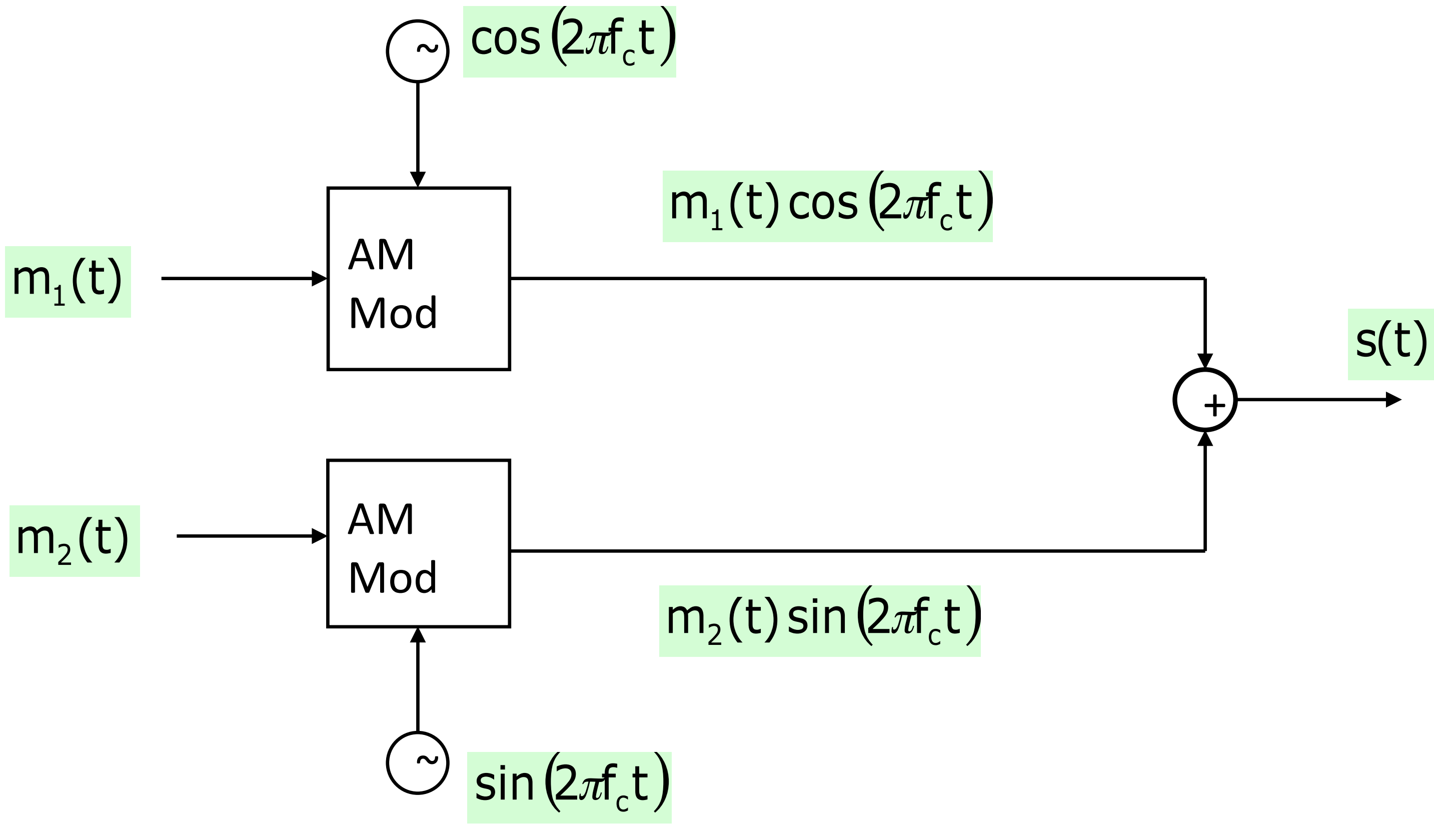
$$d\,(t) = \frac{A_c}{2} m(t) \cdot \left(1 + \cancel{\cos\left(2 \cdot 2\pi f_c\, t\right)}\right)$$



- The signal can be obtained by filtering the signal with a simple low pass filter
- N.B. I MUST know the characteristics of the carrier!!!

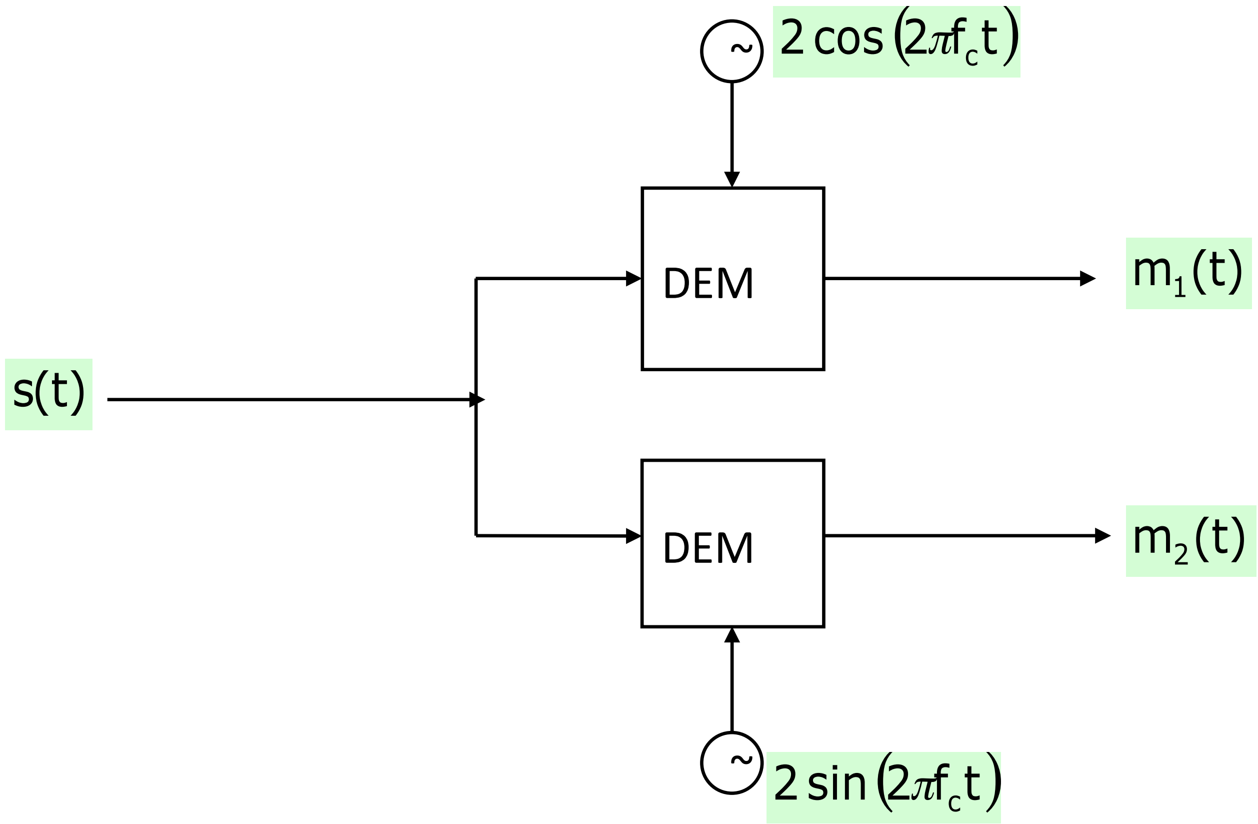
Quadrature Amplitude Modulation (QAM)

- The modulator simultaneously modules two signals $m_1(t)$ and $m_2(t)$ with two carries with a phase difference of $\pi/2$



QAM Demodulator (1)

- The demodulator simultaneously demodules s (t) with two carries with a phase difference of $\pi/2$



Demodulator QAM (2)

- It's extremely important the SYNCRONISM, otherwise the two signal $m_1(t)$ and $m_2(t)$ will mix together.
- The sinusoidal signals must have not only the same FREQUENCY but also the same PHASE!!
- With these technique we have $B_T = W$ like in the case of SSB
- In the case of SSB we CAN'T send two signals with the quadrature technique because the signal SSB occupies already both the orthogonal carriers

Angular Modulation

- Angular Modulation

$$s\,(t) = A_c \cos\left(2\ \pi\ f_0\ t + \varphi(t)\right)$$

Where φ(t) is the "information" that we must transmit

- PHASE modulation

$$\varphi\ (t) = k_P \cdot m(t)$$

- FREQUENCY modulation

$$\varphi\ (t) = \int_{-\infty}^{t} k_F \cdot m(t')\ dt'$$

Frequency Modulation FM

- Frequency Modulation (FM)

$$s_{FM}(t) = A_c \cos(\vartheta_i(t))$$

where $\vartheta_i(t)$ is the instantaneous phase

- The instantaneous frequency of the FM signal changes in function of m(t)

$$f_i(t) = f_c + k_F m(t)$$

where the instantaneous frequency is defined as the derivative of the instantaneous phase

$$f_i(t) = \frac{1}{2\pi}\frac{d\vartheta_i(t)}{dt}$$

FM signal

- The instantaneous phase becomes

$$\vartheta_i(t) = 2\pi\ f_c t + 2\pi\ k_F \int_0^t m(\tau)\ d\tau$$

- So the FM signal is given by

$$s_{FM}(t) = A_c \cos\left(2\pi\ f_c t + 2\pi\ k_F \int_0^t m(\tau)\ d\tau\right)$$

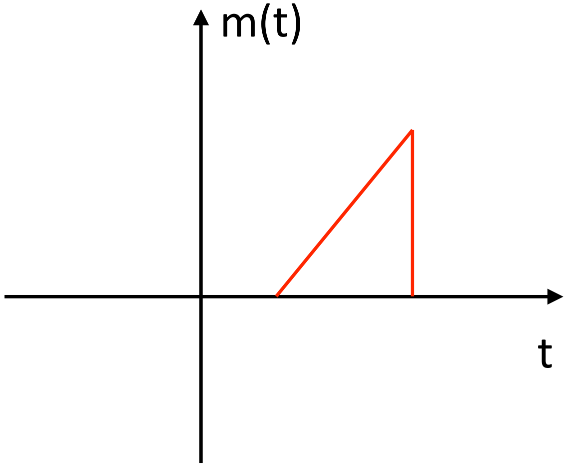
- The frequency deviation Δf is

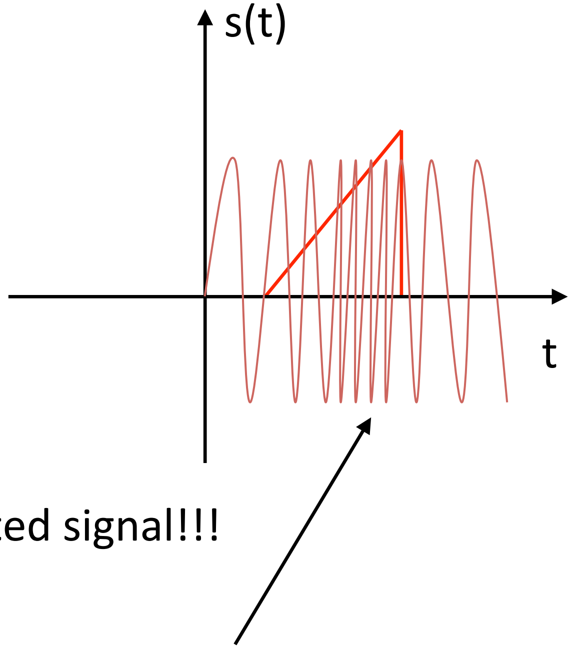
$$\Delta f = \left|k_F m(t)\right|_{\max}$$

Example

- The instantaneous frequency of the FM signal changes in function of m(t)

$$f_i(t) = f_c + k_F\ m(t)$$





- There is a frequency increment of the modulated signal!!!
- s(t) has CONSTANT AMPLITUDE

  → FM is ROBUST against NOT-LINEARITY!!!

FM: single tone

- Particular case: single tone

$$m(t) = A_m \cos(2\pi \; f_m t)$$

- The FM signal becomes

$$s_{FM}(t) = A_c \cos\left(2\pi \; f_c t + \frac{k_f A_m}{f_m} \sin(2\pi \; f_m t)\right)$$

namely

$$s_{FM}(t) = A_c \cos\left(2\pi \; f_c t + \frac{\Delta f}{f_m} \sin(2\pi \; f_m t)\right), \quad dove \; \frac{\Delta f}{f_m} = \beta$$

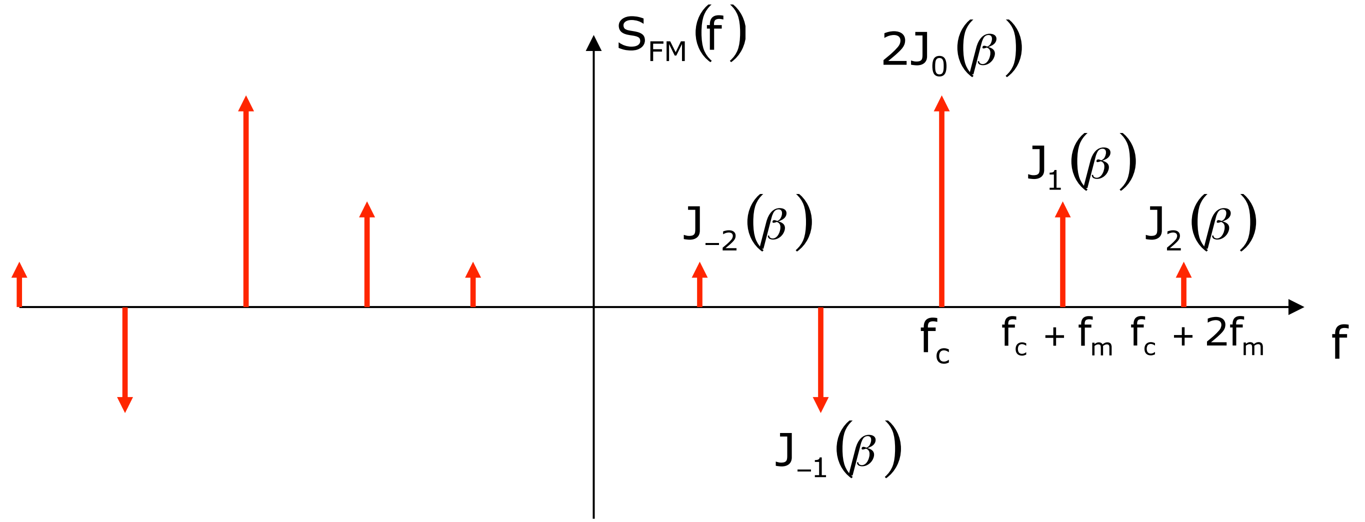
β is the maximum frequency deviation

FM signal's spectrum

- Ideally the spectrum has infinite band

$$S_{FM}(f) = \frac{A_c}{2} \sum_{n=-\infty}^{+\infty} J_n(\beta) \left[\delta\left(f - f_c - nf_m\right) + \delta\left(f + f_c + nf_m\right)\right]$$

where $J_n(\beta)$ are the Bessel's functions, that decrease very fast at the increasing of 'n' (even functions for even n, odd for odd n)



Occupied band (1)

- In general the signal's band modulated with angular modulation is bigger than in the case of amplitude modulation.

- An approximated formula that returns the bandwidth is the Carson's formula:

$$B_T = 2f_{max} + 2\Delta f$$

where Δf is the maximum frequency deviation and f_{max} is the maximum frequency of the "information"

- The angular modulation needs more bandwidth than amplitude one, but has a better response to **noise** and at the **non-linearity** in the transmission channel.

Occupied band (2)

- In the case β<<1 (narrow band) $B_T = 2f_m$

  If the signal isn't at single frequency $B_T = 2f_{\max}$

- In the case β>>1 (strong amplitude deviation) $B_T = 2\Delta f$

- Bandwidth (worst case): CARSON's approximation

$$B_T = 2f_{\max} + 2\Delta f$$

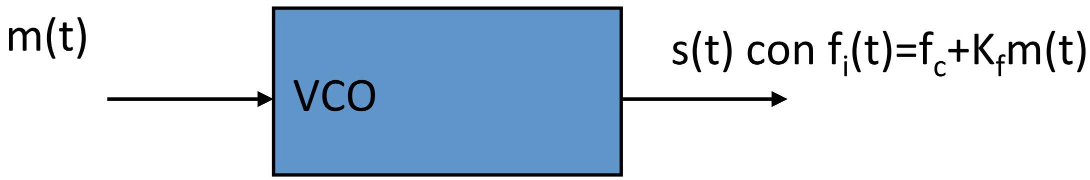
- In the case of single tone the Carson's bandwidth is equal to:

$$B_T = 2f_m + 2\Delta f$$

FM Modulator

- VCO

  Oscillator controlled in voltage



- VCO issues:

  Stability in frequency

  → Usually the VCO is controlled/implemented with a feedback loop

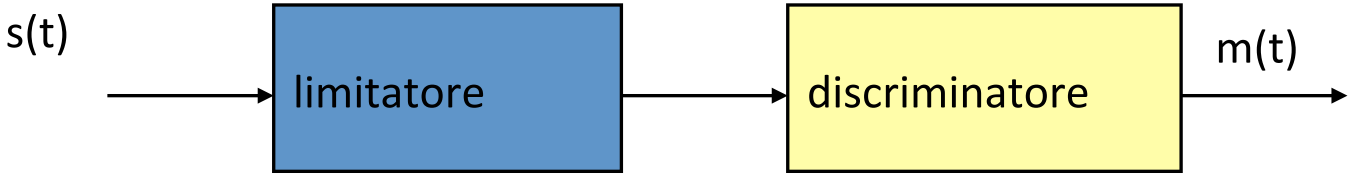
FM De-modulator

- It's a kind of inverse VCO made of

  → clipping stage

  → frequency discriminator

  always in series



Limit control:

→ it's necessary to made the amplitude of the FM signal constant (clipping)

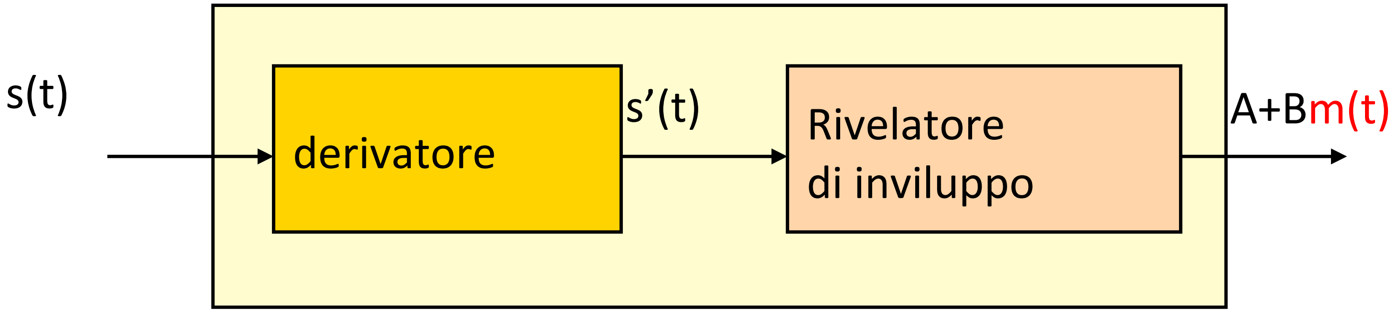
Frequency discriminator (1)

- The discriminator is made of

  → derivator

  → envelope detector

  always in series



Frequency discriminator (2)

- Now the FM signal is equal

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_F \int_0^t m(\tau)d\tau\right)$$

- At the output of the derivator

$$s'(t) = -A_c \sin\left(2\pi f_c t + 2\pi k_F \int_0^t m(\tau)d\tau\right) \cdot [2\pi f_c t + 2\pi K_F m(t)]$$

- The calculation of the complex envelope of s'(t) return the signal

$$A_c \cdot [2\pi f_c t + 2\pi K_F m(t)] = A + B\, m(t)$$

And we recovered the information m(t)!

Comparison between modulations (1)

- **AMPLITUDE MODULATION (AM)**

- classic AM (with transmitted carrier)

  → Envelope Demodulator (simple)

  → B_T=2W

  → Waste of carrier's power

  → Sensible to the NON-linearity

  → Use: AM radio

- DSB-SC (Double SideBand Suppressed Carrier transmission)

  → Synchronous Demodulation

  → B_T=2W, but I can implement the QAM

  → Sensitive to the NON-linearity

Comparison between modulations (2)

- SSB (AM with Suppressed Side Band)

→ Filter is difficult to assemble

→ B_T=W, it ISN'T possible use the QAM

- **FREQUENCY MODULATION**

- FM advantage

→ Robust against the NON-linearity and to the NOISE

→ Not too difficult to implement

→ Particularly indicated when we have LITTLE power (saturation amplifier)

$$B_T = 2f_{max} + 2\Delta f$$

- FM disadvantage

→ Huge use of the bandwidth!!!

Use of the analogue modulations (1)

- AM
  - → Radio Broadcasting
- SSB
  - → TV signal distribution
  - → Telephone channels on coaxial cable (FDM)
- QAM
  - → Numeric Transmission

Use of the analogue modulations (2)

- FM

  → Radio Stereo

  → Transmission by SATELLITE and RADIO BRIDGE

  → Contribution in the TV signal

  → Optic and magnetic recording

  → Audio for TV

  …

  → SPACE transmissions

  → Radio mobile Transmissions

- PM

  → Numeric Transmission