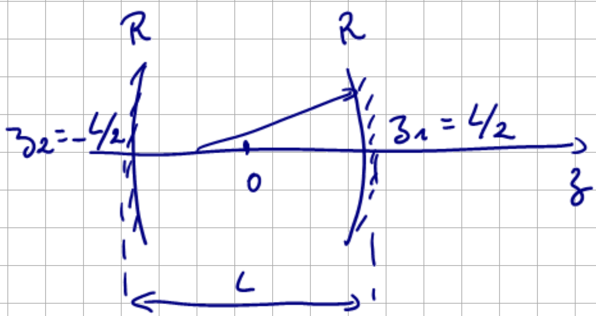
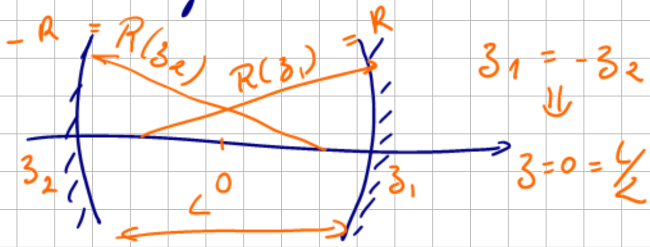
Correction of Tutorial 2 LASERS

1) $R(z) = z + \alpha^2/z$

$R(-z) = -z - \alpha^2/z = -R(z)$ Odd function $\rightarrow$ Waist located at the center of the cavity $\Rightarrow$ origin of z axis at the center of the cavity





Autocollimation condition : $\underbrace{R(L/2)}_{\text{beam}} = \underbrace{R}_{\text{Mirror}}$

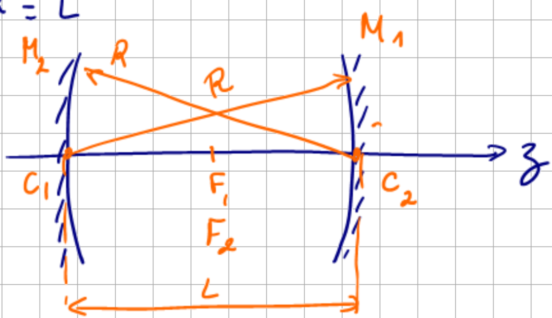
$$\frac{L}{2} + \frac{\alpha^2}{L/2} = R$$

$$\boxed{\alpha = \sqrt{\frac{L}{2}\left(R - \frac{L}{2}\right)}} \quad (1)$$

Stability condition : $R - L/2 \geqslant 0 \Rightarrow R \geqslant L/2$

$$\Rightarrow \boxed{0 < \frac{L}{R} \leqslant 2}$$

2) $R = L$



$F_1 \equiv F_2 \Rightarrow$ confocal cavity.

(1) $R = L \rightarrow \alpha = \sqrt{\frac{L}{2}\left(L - \frac{L}{2}\right)} = L/2$

Same case as question 1) $R_1 = R_2 \Rightarrow$ the waist is located at the center of the cavity

$$\boxed{\alpha = L/2} \Rightarrow \frac{\pi W_0^2}{\lambda} = L/2$$

$$\Rightarrow \boxed{W_0 = \sqrt{\frac{\lambda L}{2\pi}}}$$

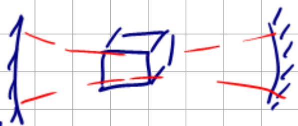
NA : $W_0 = 0.4$ mm $\Rightarrow \phi = 0.8$ mm.

$W^2(z) = W_0^2\,(1 + z^2/\alpha^2)$ $\quad z = L/2$

$W^2(-z) = W^2(z) \Rightarrow z = \pm L/2$ (Mirrors M_1 and M_2) $= \alpha$

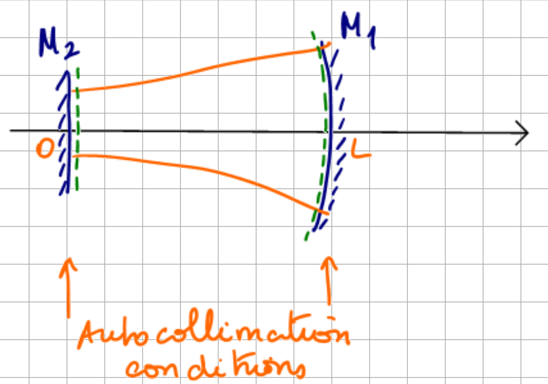
$$w^2(L/2) = w_0^2\left(1+\frac{\alpha^2}{\alpha^2}\right) \Rightarrow w(L/2) = \sqrt{2}\,w_0 = \sqrt{2}\cdot 0.4 = 0.56\text{ mm.}$$

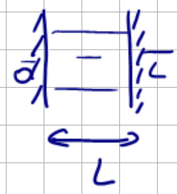
Comment on $w_0 = \sqrt{\frac{\lambda L}{2\pi}}$



If $w_0 \nearrow \Rightarrow \alpha \nearrow \Rightarrow L \nearrow$

3)





$L \ll \alpha$

on $M_2 \Rightarrow$ Plane wavefront $\Rightarrow w_0$ is on $M_2 \Rightarrow z=0$ on M_2

on $M_1 \Rightarrow \boxed{\underbrace{R(L)}_{\text{Beam}} = \underbrace{R}_{\text{Mirror}}}$

$$R(L) = L + \alpha^2/L = R \Rightarrow \alpha = \sqrt{L\cdot(R-L)} \qquad R \geq L$$

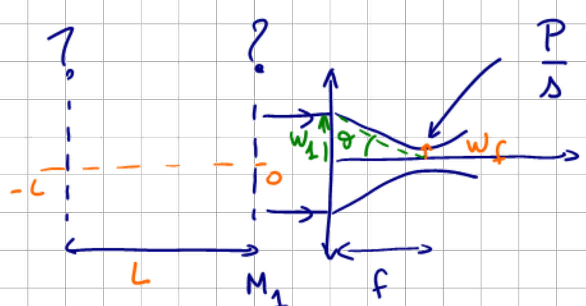
stability condition $\leftarrow$ $0 \leq \frac{L}{R} \leq 1$

$$\frac{\pi w_0^2}{\lambda} = \sqrt{L(R-L)} \Rightarrow \boxed{w_0 = \left(\frac{\lambda}{\pi}\right)^{1/2}\left(L(R-L)\right)^{1/4}}$$

If $R \nearrow$ $w_0 \nearrow$ $\Rightarrow$ way to increase w_0 without changing L

N.A : $w_0 = 0{,}56$ mm.

Ex 2 :



$\frac{P}{S} = 1.7\ \text{MW/cm}^2$ $\qquad P = 5\text{ W}$

$$S = \frac{5\text{ W}}{1.7\ 10^6} = \pi w_f^2$$

$\Rightarrow w_F = 10\ \mu\text{m}$

$$\left.\begin{array}{l}\tan\theta \simeq \theta = \frac{w_1}{f} \\ \theta = \frac{\lambda}{\pi w_F}\end{array}\right\} \Rightarrow w_1 = \frac{\lambda f}{\pi w_F} = 0.5\text{ mm} \Rightarrow \phi_1 = 1\text{ mm.}$$

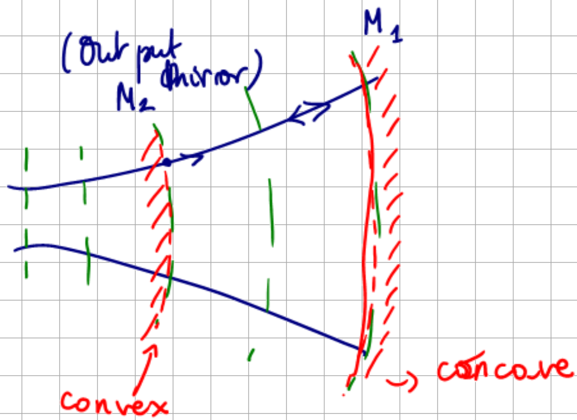
2) $\alpha = \frac{\pi w_1^2}{\lambda} = 0.7\text{ m} \gg 5\text{ cm}$ $\quad M_1$ in the near field $\Rightarrow$

M_1 plane mirror

$$R(-L) = -L + \frac{\alpha^2}{(-L)} = -1.5\ \text{m}$$

$$= -R_{M_2}$$

$R_{M_2} > 0 \Rightarrow$ Concave mirror



$$w^2(-L) = w_0^2\left(1 + L^2/\alpha^2\right) \Rightarrow w(-L) = 1.2\ \text{mm} \Rightarrow \phi_2 = 2.4\ \text{mm}.$$