

From the assigned words, $d_{\min} = 7$. $R = k/N = 16/31$.
and $N = 31$.

To find $P(E)$.

$$K = 31 - 15 = 16$$

For soft decision.

After the code extension, $R = 16/32 = 1/2$ and $d_{\min} = 7 + 1 = 8$.

$$P(E) \approx Q\left(\sqrt{\frac{2E_b}{N_0} \cdot R \cdot d_{\min}}\right) \quad (\text{in general})$$

$$= 155 Q\left(\sqrt{\frac{2E_b}{N_0} \cdot \frac{1}{2} \times 8}\right) + 465 Q\left(\sqrt{\frac{2E_b}{N_0} \cdot \frac{1}{2} \times 9}\right) + \dots \quad t = \left\lfloor \frac{d_{\min}-1}{2} \right\rfloor$$

$$= \frac{7-1}{2} = 3$$

For Hard decision.

$$P(E) \approx Q\left(\sqrt{\frac{2E_b}{N_0} \cdot R \cdot d_{\min}}\right)$$

$$P(E) = \sum_{h=t+1}^N \binom{N}{h} \epsilon^h (1-\epsilon)^{N-h}$$

$$= \sum_{h=4}^{32} \binom{N}{h} \epsilon^h (1-\epsilon)^{N-h}$$

Where $\epsilon = Q\left(\sqrt{\frac{2E_b}{N_0} R d}\right)$ and $P(E) \approx Q\left(\sqrt{\frac{2E_b}{N_0} R (t+1)}\right)$

$$= Q\left(\sqrt{\frac{2E_b}{N_0} \cdot \frac{1}{2} \times (3+1)}\right)$$

(b) To design (6,2) cyclic code.

$$N = 6, K = 2.$$

$$\text{Thus, } D^6 + 1 = (D+1)(D+1)(D^2+D+1)(D^2+D+1).$$

There are 2 possible ways.

$$(i) (D+1)(D+1)(D^2+D+1) = (D^2+D+1)(D^2+D+1)$$

$$= D^4 + D^3 + D^2 + D + 1 = D^4 + D^3 + D + 1.$$

$$(ii) (D^2+D+1)(D^2+D+1) = D^4 + D^3 + D^2 + D + 1$$

$$= D^4 + D^2 + 1 \quad \text{is the shortest general polynomial.}$$

= # possible = $2^2 = 4$ possible codewords.

0 0 0 0 0 0	3	$d_{\min} = 3$
0 1 0 1 0 1		
1 0 1 0 1 0		
0 1 0 1 0 1		