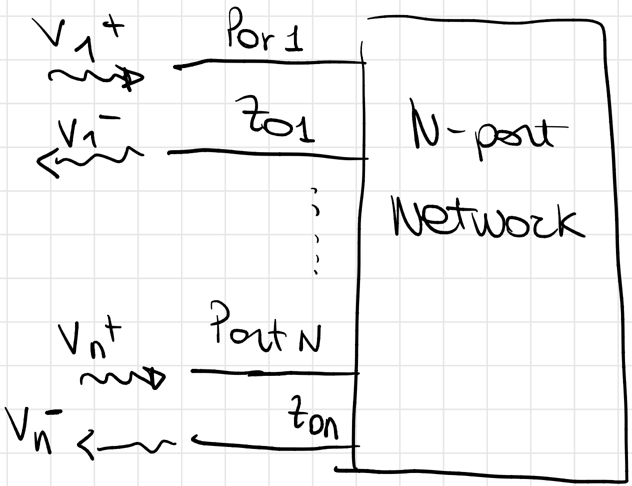
MICROWAVE ENGINEERING

Lecture 20: Generalized S parameters, ABCD Matrix, Discontinuities and Modal Analysis

Generalized S parameters

Z_{0n} are all real



At each port we will define:

$$a_n = \frac{V_n^+}{\sqrt{Z_{0n}}}$$

$$b_n = \frac{V_n^-}{\sqrt{Z_{0n}}}$$

$$V_n = V_n^+ + V_n^- = \sqrt{Z_{0n}}\,(a_n + b_n)$$

$$I_n = \frac{1}{Z_{0n}}\,(V_n^+ - V_n^-) = \frac{1}{Z_{0n}}\,(a_n - b_n)$$

The average power delivered at the n^{th} port :

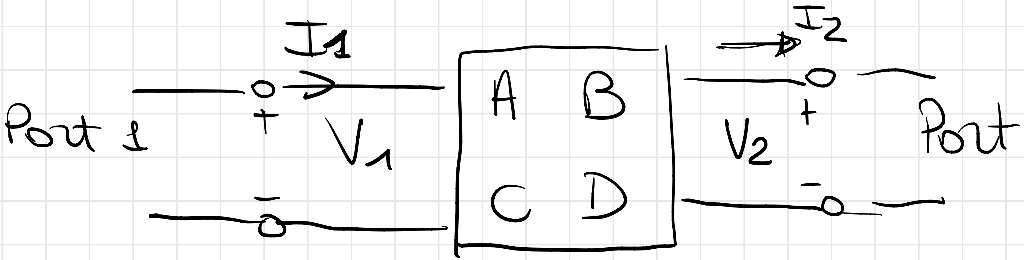
$$P_n = \frac{1}{2}\mathrm{Re}\{V_n I_n^*\} = \frac{1}{2}\mathrm{Re}\left[|a_n|^2 - |b_n|^2 + \overbrace{(b_n a_n^* - b_n^* a_n)}^{\text{purely imaginary}}\right]$$

$$= \frac{1}{2}|a_n|^2 - \frac{1}{2}|b_n|^2 \Leftarrow$$ Power delivered = incident power − reflected power

The generalized [S] matrix is :

$$[b] = [S][a] \quad \text{where} \quad S_{ij} = \left.\frac{b_i}{a_j}\right|_{\substack{a_k=0 \\ \text{for } k\neq j}} = \left.\frac{V_i^- \sqrt{Z_{0j}}}{V_j^+ \sqrt{Z_{0i}}}\right|_{\substack{V_k^+=0 \\ j\neq k}}$$

The ABCD (Transmission) matrix:



if I have n networks

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} - \dots \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} \begin{bmatrix} V_n \\ I_n \end{bmatrix}$$

$$V_1 = A V_2 + B I_2$$

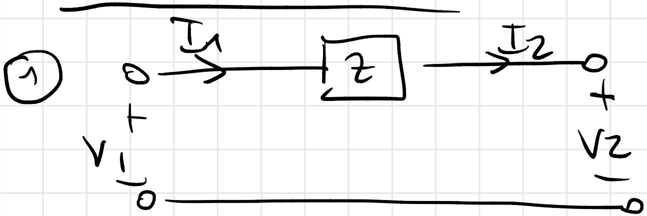
$$I_1 = C V_2 + D I_2$$

$$\Downarrow$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Useful Examples

(1)



$A = 1 \qquad B = Z$

$C = 0 \qquad D = 1$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1$$

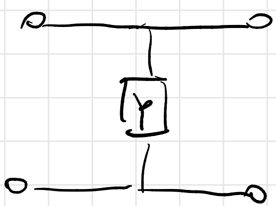
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{V_1/Z} = Z$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{I_1}{I_1} = 1$$

$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

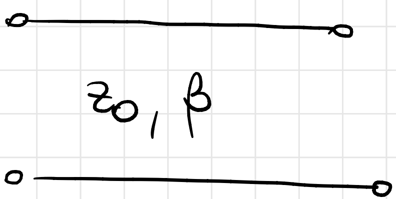
2



$A = 1 \qquad B = 0$

$C = Y \qquad D = 1$

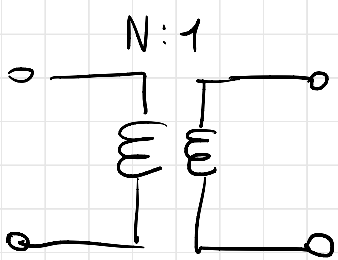
3



$A = \cos \beta l \qquad B = j Z_0 \sin \beta l$

$C = j Y_0 \sin \beta l \qquad D = \cos \beta l$

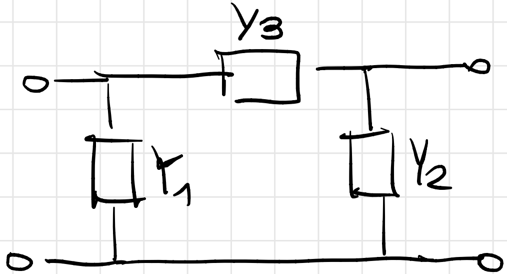
4



$A = N \qquad B = 0$

$C = 0 \qquad D = \frac{1}{N}$

⑤ Pi-Network



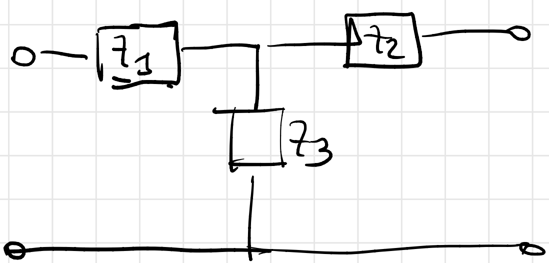
$$A = 1 + \frac{Y_2}{Y_3}$$

$$B = \frac{1}{Y_3}$$

$$C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3}$$

$$D = 1 + \frac{Y_1}{Y_3}$$

⑥ T-network



$$A = 1 + \frac{Z_1}{Z_3}$$

$$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$

$$C = \frac{1}{Z_3}$$

$$D = 1 + \frac{Z_2}{Z_3}$$

Relation between ABCD and Z matrix

For Z matrix

$$\begin{cases} V_1 = I_1 Z_{11} - I_2 Z_{12} \\ V_2 = I_1 Z_{21} - I_2 Z_{22} \end{cases}$$

$$\underline{A} = \frac{V_1}{V_2}\bigg|_{I_2=0} = \underline{\frac{Z_{11}}{Z_{21}}}$$

$$\underline{B} = \frac{V_1}{I_2}\bigg|_{V_2=0} = \frac{I_1 Z_{11} - I_2 Z_{12}}{I_2} = Z_{11}\frac{I_1}{I_2}\bigg|_{V_2=0} - Z_{12} =$$

$$= z_{11} \frac{\cancel{I_1} z_{22}}{\cancel{I_1} z_{21}} - z_{12} = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{21}}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{z_{21}}$$

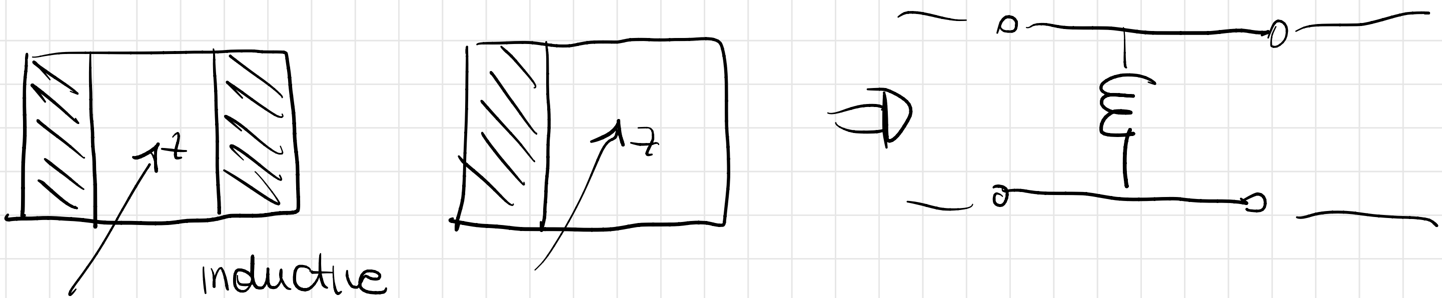
$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{z_{22}}{z_{21}}$$

If the network is reciprocal we know $z_{12} = z_{21}$ and it gives

$$\boxed{AD - BC = 1}$$

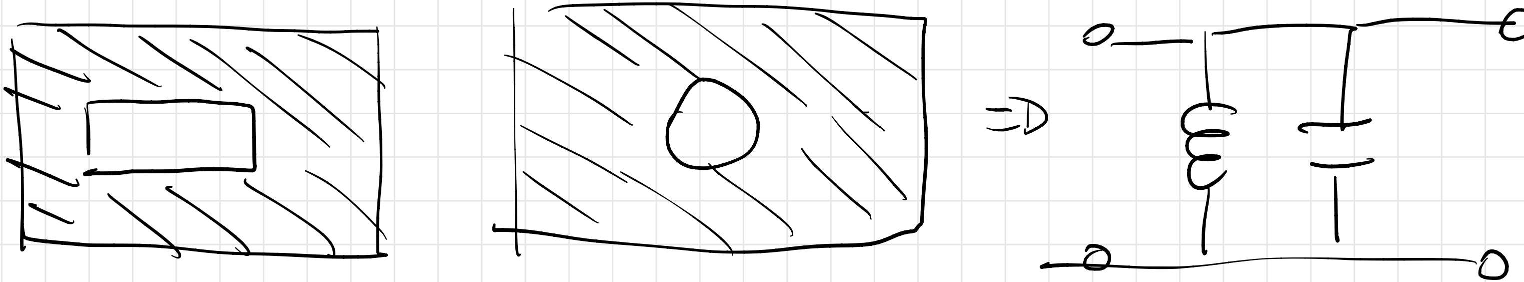
DISCONTINUITIES AND MODAL ANALYSIS

Rectangular waveguides

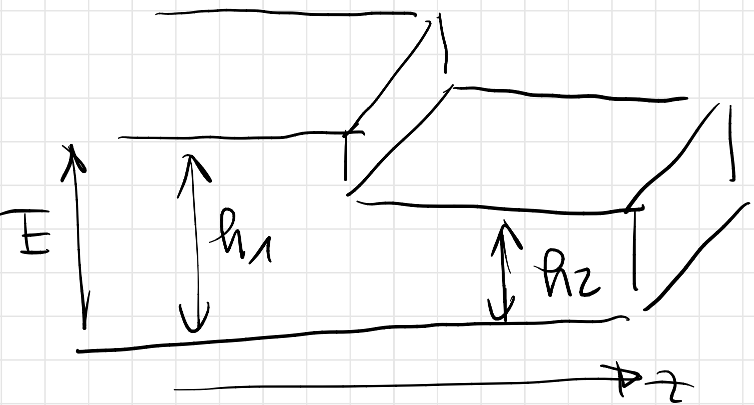


inductive diaphragm

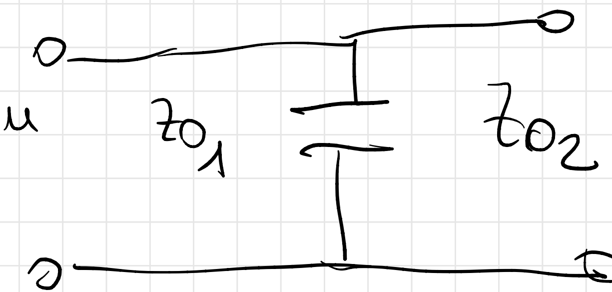
capacitive diaphragm

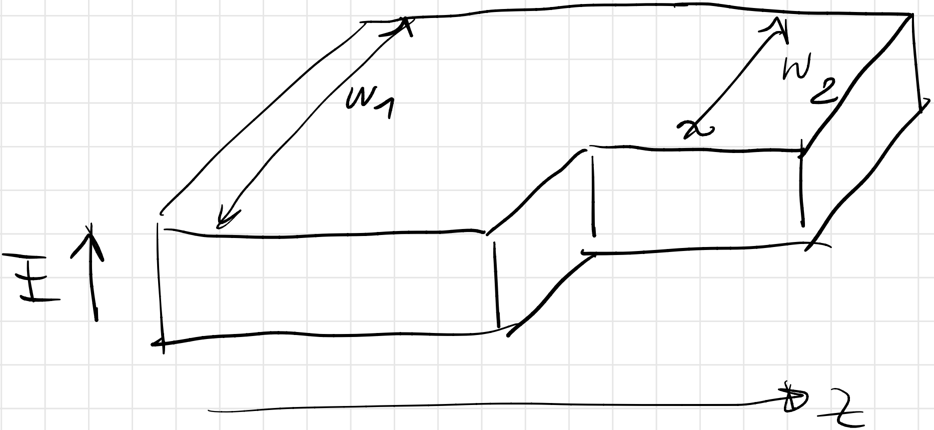


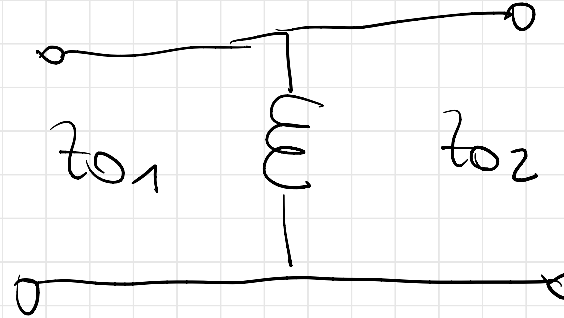
Resonant inserts



change in height

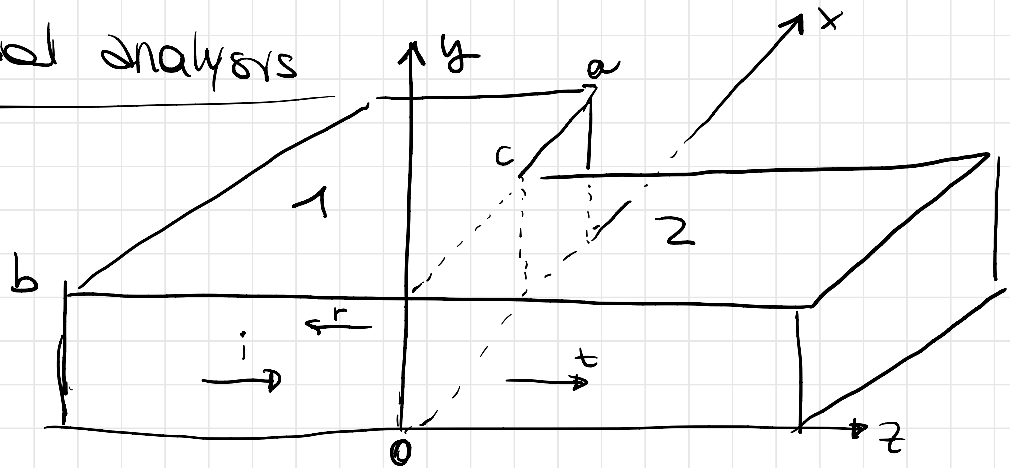




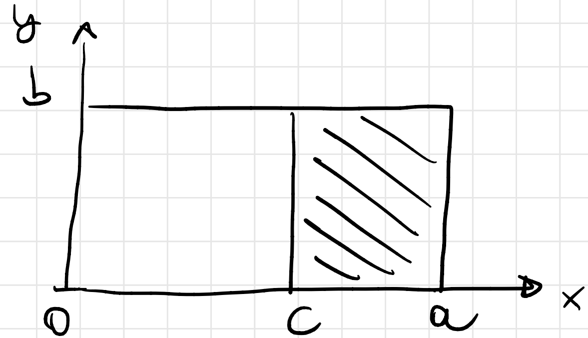


Change in width

Modal analysis



CHANGE IN WIDTH



For $z<0$ we assume only TE_{10} mode propagating.

For $z>0$ we assume NO modes are propagating

For $z<0$ the incident modes are:

$$\begin{cases} E_y^i = \sin\frac{\pi x}{a}\, e^{-j\beta_1^a z} \\ H_x^i = -\frac{1}{Z_1^a}\sin\frac{\pi x}{a}\, e^{-j\beta_1^a z} \end{cases}$$

$$\beta_1^a = \sqrt{k_0^2 - \left(\frac{n\pi}{a}\right)}$$

TE_{n0} mode in guide 1

$$Z_n^a = \frac{k_0 \eta_0}{\beta_n^a}$$

wave impedance of TE_{n0} mode in WG 1

The reflected modes ($z<0$)

$$E_y^r = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{a}\, e^{j\beta_n^a z}$$

$$H_x^r = \sum_{n=1}^{\infty} \frac{A_n}{Z_n^a} \sin\frac{n\pi x}{a}\, e^{j\beta_n^a z}$$

The transmitted modes are ($z>0$)

$$E_y^t = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{c} e^{-j\beta_n^c z}$$

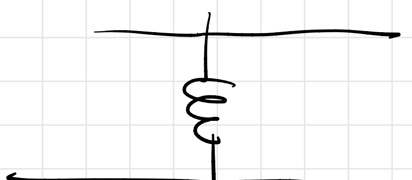
$$H_x^t = \sum_{n=1}^{\infty} \frac{B_n}{Z_n^c} \sin \frac{n\pi x}{c} e^{-j\beta_n^c z}$$

$$\beta_n^c = \sqrt{k_0^2 - \left(\frac{n\pi}{c}\right)^2} \qquad Z_n^c = \frac{k_0 \eta_0}{\beta_n^c}$$

At $z=0$ the fields must be continuous and $E_y = 0$ for $c < x < a$

$$
\begin{cases}
E_y = \sin\dfrac{\pi x}{a} + \displaystyle\sum_{n=1}^{\infty} A_n \sin\dfrac{n\pi x}{a} = \begin{cases} \displaystyle\sum_{n=1}^{\infty} B_n \sin\dfrac{n\pi x}{c} & 0 < x < c \\ 0 & c < x < a \end{cases} \\[2ex]
H_x = -\dfrac{1}{Z_1^a} \sin\dfrac{\pi x}{a} + \displaystyle\sum_{n=1}^{\infty} \dfrac{A_n}{Z_n^a} \sin\dfrac{n\pi x}{a} = -\displaystyle\sum_{n=1}^{\infty} \dfrac{B_n}{Z_n^c} \sin\dfrac{n\pi x}{c} \qquad 0 < x < c
\end{cases}
$$

System can be solved numerically but if we assume waveguide 2 does not support ANY MODE $\rightarrow$

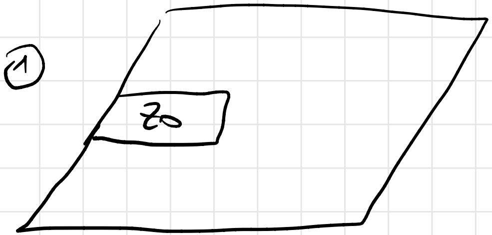


$$\boxed{X = -j Z_1^a \frac{1+A_1}{1-A_1}}$$

A_1 is the reflection coeff. of the TE_{10} mode.

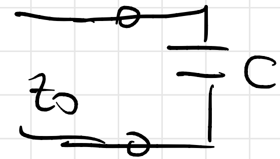
Microstrip discontinuities

①

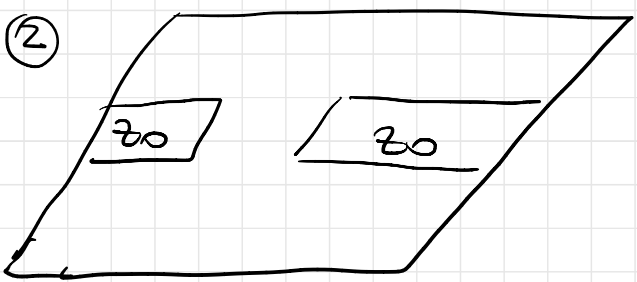


Open-ended MS

$\Rightarrow$

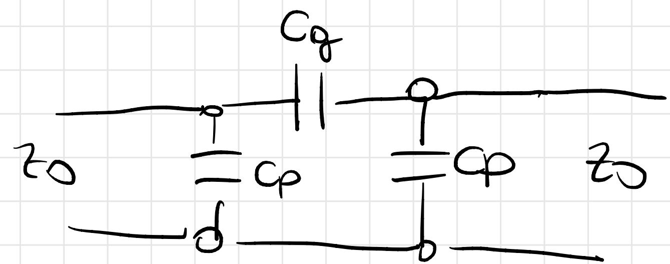


②



Gap in MS

$\Rightarrow$



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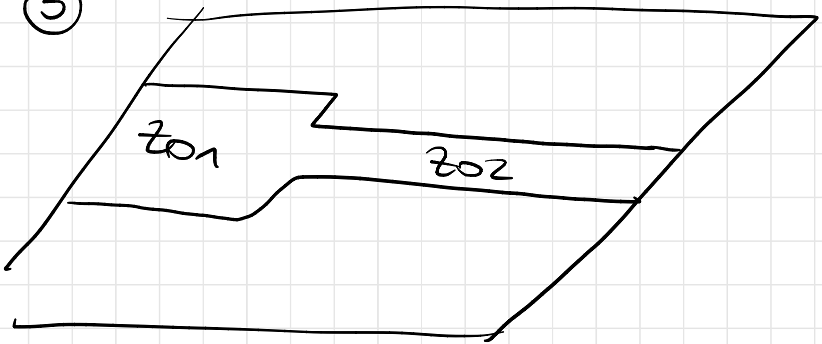
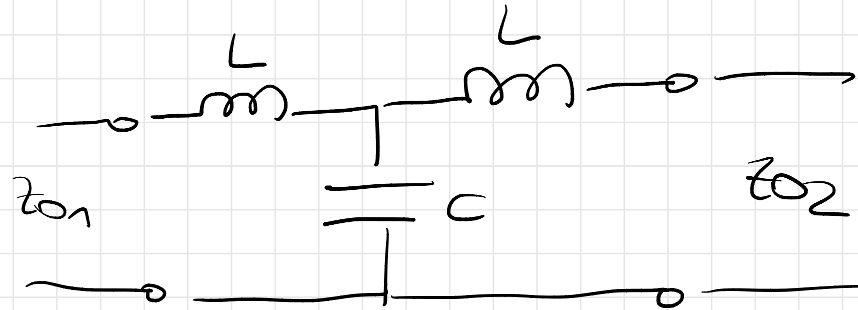
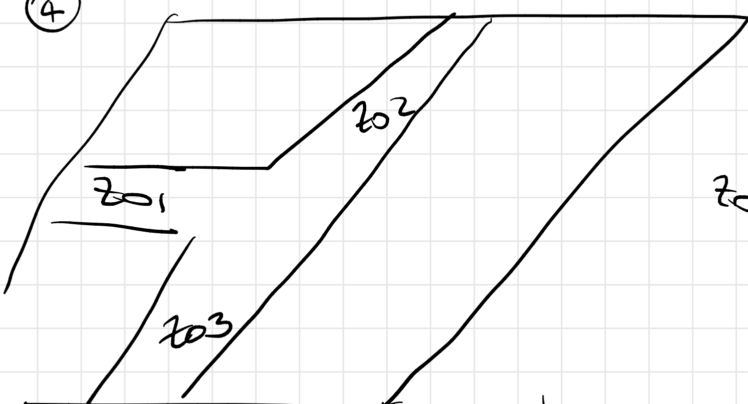


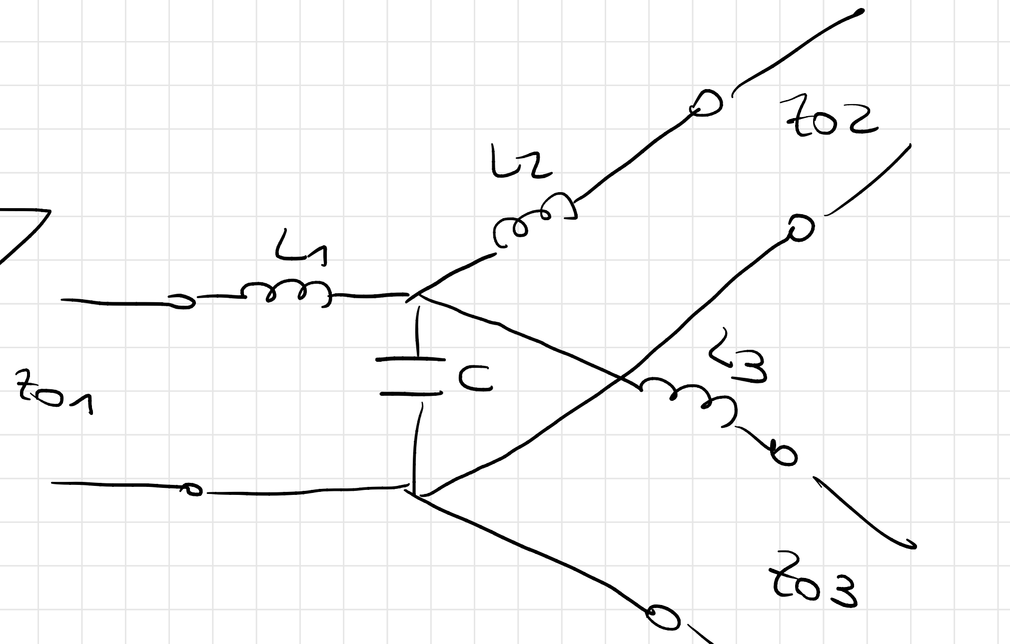


change in width

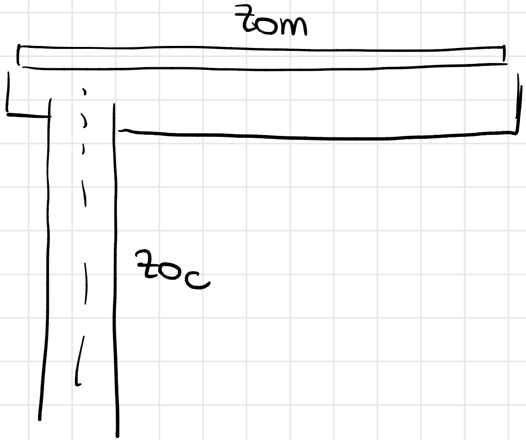
4



T-junction



(5)



Coax-to-MS junction