2 - Maxwell Equations & Boundary Conditions

- Microwaves correspond to: $f = [300\ MHz - 300\ GHz]$
  $\lambda = [1\ m - 1\ mm]$

- Maxwell Eqs in a medium

↳ Gauss: $\bar{\nabla} \cdot \bar{D} = \rho_f$; $\oint_S \bar{D} \cdot d\bar{s} = \int_V \rho_f \, dV = Q$

↳ Gauss: $\bar{\nabla} \cdot \bar{B} = 0$; $\oint_S \bar{B} \cdot d\bar{s} = 0$

↳ Faraday: $\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} - \bar{M}$; $\oint \bar{E} \cdot d\bar{l} = -\frac{d}{dt} \int_S \bar{B} \cdot d\bar{s} - \int_S \bar{M} \cdot d\bar{s}$

↳ Ampere: $\bar{\nabla} \times \bar{H} = \frac{\partial \bar{D}}{\partial t} + \bar{J}_f$; $\oint \bar{H} \cdot d\bar{l} = \frac{d}{dt} \int_S \bar{D} \cdot d\bar{s} + \int_S \bar{J}_f \cdot d\bar{s}$

⇒ Continuity equation: $\bar{\nabla} \cdot \bar{J} = -\frac{\partial \rho_f}{\partial t}$

- Harmonic time dependence: Phasors

$$\bar{E}(x,y,z,t) = A(x,y,z,t) \cos(\omega t + \phi)$$

$$\bar{E}(x,y,z,t) = \mathrm{Re}\left[\vec{E}(x,y,z,t)\, e^{j\omega t}\right]$$

If monochromatic:

$$\vec{E}(\bar{r},t) = \mathrm{Re}\left[\bar{E}(\bar{r},\omega)\, e^{j\omega t}\right]$$

$$= \frac{1}{2}\left[\bar{E}(\bar{r},\omega)\, e^{j\omega t} + \bar{E}^*(\bar{r},\omega)\, e^{-j\omega t}\right]$$

◦) Constitutive relations

Free space	Linear medium	
$\bar{D} = \varepsilon_0 \bar{E}$	$\bar{D} = \varepsilon_0 \bar{E} + \bar{P}_e$	$\bar{P}_e = \varepsilon_0 \chi_e \bar{E}$
$\bar{B} = \mu_0 \bar{H}$	$\bar{B} = \mu_0 \bar{H} + \bar{P}_m$	$\bar{P}_m = \mu_0 \chi_m \bar{H}$
$\bar{J} = 0$		

For χ_e:

$$\bar{D} = \varepsilon \bar{E} \rightarrow \varepsilon = \varepsilon_0 (1 + \chi_e) = \varepsilon' - j\varepsilon''$$

$$\bar{J} = \sigma \bar{E} \rightarrow \bar{\nabla} \times \bar{H} = j\omega \bar{D} + \bar{J} = j\omega \left(\varepsilon' - j\varepsilon'' - j\frac{\sigma}{\omega} \right) \bar{E}$$

loss tangent: $\tan\delta = \dfrac{\omega \varepsilon'' + \sigma}{\omega \varepsilon'}$

For χ_m:

$$\bar{B} = \mu \bar{H} \rightarrow \mu = \mu_0 (1 + \chi_m) = \mu' - j\mu''$$

ε in general is a tensor:

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

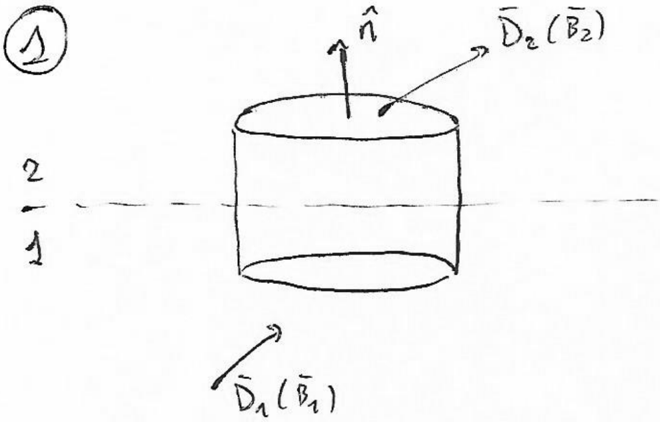
Isotropic if: $\varepsilon_{ij} = \varepsilon \quad \forall i = j$ and $\varepsilon_{ij} = 0 \quad \forall i \neq j$

Anisotropy: Polarization in a crystal varies depending on the direction of the applied field

Phase velocity can assume a different value depending on the direction of propagation and polarization

Boundary conditions

1



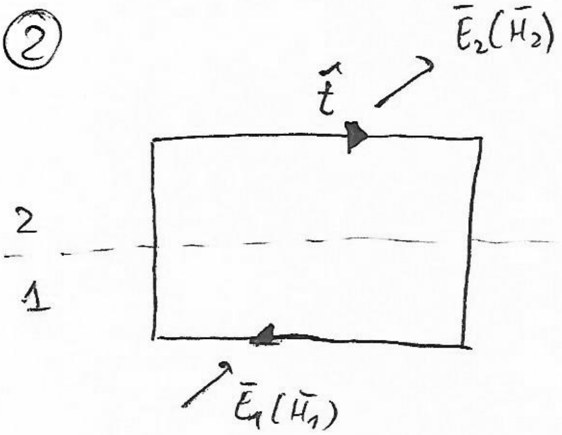
$$\boxed{\begin{aligned} \hat{n} \cdot (\bar{D}_2 - \bar{D}_1) &= 0 \\ \hat{n} \cdot (\bar{B}_2 - \bar{B}_1) &= 0 \end{aligned}}$$

Using gauss

$\oint_S \bar{D} \cdot d\bar{s} = \int_V \rho_f \, dV$

$\oint_S \bar{B} \cdot d\bar{s}$

2



$$\boxed{\begin{aligned} (\bar{E}_2 - \bar{E}_1) \times \hat{n} &= \bar{M}_s \\ \hat{n} \times (\bar{H}_2 - \bar{H}_1) &= \bar{J}_s \end{aligned}}$$

Using:
ampere
&
faraday

PEC

$\hat{n} \cdot \bar{D} = \rho_s$

$\hat{n} \cdot \bar{B} = 0$

$\hat{n} \times \bar{E} = 0$

$\hat{n} \times \bar{H} = \vec{J}_s$

PMC

$\hat{n} \cdot \bar{D} = 0$

$\hat{n} \cdot \bar{B} = 0$

$\hat{n} \times \bar{E} = -\bar{M}_s$

$\hat{n} \times \bar{H} = 0$

3 - Wave Equation & Plane Waves

∘) Maxwell eqs in freq. domain

$\bar{\nabla} \times \bar{E} = -j\omega \bar{B}$

$\bar{\nabla} \times \bar{H} = j\omega \bar{D} + \bar{J}$

$\bar{\nabla} \cdot \bar{D} = \rho$

$\bar{\nabla} \cdot \bar{B} = 0$

$$\xrightarrow[\text{Plane wave eqs.}]{} \begin{cases} \nabla^2 \bar{E} + \omega^2 \mu \varepsilon \bar{E} = 0 \\ \nabla^2 \bar{H} + \omega^2 \mu \varepsilon \bar{H} = 0 \end{cases}$$

$$k = \omega \sqrt{\mu \varepsilon}$$

∘) Plane wave in lossless medium:

$$\frac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0 \longrightarrow E_x(z,t) = E_x^+ \cos(\omega t - kz) + E_x^- \cos(\omega t + kz)$$

$k \in \mathbb{R}$

Foward propag Backwards propag.

↳ $\bar{\nabla} \times \bar{E} = -j\omega\mu \bar{H} \longrightarrow H_y(z) = \frac{1}{\eta}\left[E_x^+ e^{-jkz} - E_x^- e^{jkz}\right]$

$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$

$E \perp H$

↳ Phase velocity: Velocity at which the phase of any freq. component of the wave travels.
The ratio between the space travelled by a plane of the wave over the time it takes to travel that space

$$v_p = \frac{\Delta z}{\Delta t} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\varepsilon_r}}$$

↳ Wavelength : The distance between 2 maxima (or minima) on the wave at certain time

$$[\omega t - kz] - [\omega t - k(z+\lambda)] = 2\pi$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}} = \frac{2\pi v_p}{\omega} = \frac{v_p}{f}$$

•) Plane wave in lossy medium

$$\bar{\nabla}\times\bar{E} = -j\omega\mu\bar{H}$$
$$\bar{\nabla}\times\bar{H} = j\omega\varepsilon\bar{E} + \sigma\bar{E}$$

$$\Longrightarrow \quad \nabla^2\bar{E} + \omega^2\mu\varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)\bar{E} = 0$$

Wavenumber complex:

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}$$

Solution: $E_x(z) = E_x^+ e^{-\gamma z} + E_x^- e^{-\gamma z}$ $\rightarrow$ $e^{-\gamma z} = e^{-\alpha z} e^{-j\beta z}$

phase velocity : $v_p = \frac{\omega}{\beta}$

$$E_x(z,t) = E_x^+ e^{-\alpha z}\cos(\omega t - kz) + E_x^- e^{\alpha z}\cos(\omega t + kz)$$

$$H_y(z) = -\frac{j\gamma}{\omega\mu}\left[E_x^+ e^{-\gamma z} - E_x^- e^{\gamma z}\right]$$

↳ $\eta = j\frac{\omega\mu}{\gamma}$

If $\sigma = 0$ and $\varepsilon \in \mathbb{C}$ to take losses into account

$$\gamma = j\omega\sqrt{\mu\varepsilon} = jk = j\omega\sqrt{\mu\varepsilon'(1 - j\tan\delta)}$$

$$\tan\delta = \frac{\varepsilon''}{\varepsilon'}$$

•) Good conductor

$\sigma \gg \omega\varepsilon$ or $\varepsilon'' > \varepsilon'$ $\Rightarrow$ $\gamma = \alpha + j\beta \triangleq 1 + j\sqrt{\frac{\omega\mu\sigma}{2}}$

↳ Skind depth: Deph of penetration for which the amplitude decays to 1/e

$$\delta_s = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

↳ Wave impedance:

$$\eta = j\frac{\omega\mu}{\gamma} \simeq (1+j)\frac{1}{\sigma\delta_s}$$

•) General Plane Wave Solution

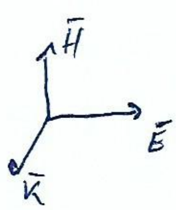
Using separation of variables; solution: $\bar{E}_x = \bar{E}_0\, e^{-j(k_x x + k_y y + k_z z)} + cc = \bar{E}_0\, e^{-j\bar{k}\cdot\bar{r}} + cc$

↳ Wavevector: $\bar{k}$

↳ Wavenumber: $|\bar{k}| = k = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r} = k_0 n$

↳ Dispersion relation: $k^2 = k_x^2 + k_y^2 + k_z^2$

→ E, H, k orthogonal



4- Energy and Power Polarization

•) Poynting Theorem

⊙ The power flowing out of a given volume V is equal to the time rate of the decrease in the energy stored within V minus the ohmic losses

$$\nabla \cdot (\bar{E} \times \bar{H}^*) = \nabla \cdot S$$

$$-\frac{1}{2} \iiint_V (\bar{E} \cdot \bar{J}_s^* + \bar{H}^* \cdot \bar{M}_s) \, dV = \frac{1}{2} \oiint_S \bar{E} \times \bar{H} \cdot \hat{n} \, ds + \frac{\sigma}{2} \iiint_V |\bar{E}|^2 dV + \frac{\omega}{2} \iiint_V (\varepsilon'' |\bar{E}|^2 + \mu'' |\bar{H}|^2) \, dV +$$

$$+ j\frac{\omega}{2} \iiint_V (\mu' |\bar{H}|^2 - \varepsilon' |\bar{E}|^2) \, dV$$

$$P_S = P_o + P_L + P_R$$

P_S	P_o	P_L	P_R
power provided by the source	Poynting vector Power flowing out of a closed surface	Power loss through Joule heating	Reactive power stored by the e.m. field in volume

$$P_r = 2 j\omega (W_m - W_e)$$

$$W_m = \frac{1}{4} \text{Re} \iiint_V \bar{H} \cdot \bar{B}^* dV$$

$$W_e = \frac{1}{4} \text{Re} \iiint_V \bar{E} \cdot \bar{D}^* dV$$

Surface resistivity:

$$R_s = \text{Re}\left[(1+j)\sqrt{\frac{\omega \mu}{2\sigma}}\right] = \frac{1}{\sigma \delta_s}$$

•) Polarization

The orientation of the E field vector at a position r and time t.

↳ $\bar{E} = \left(a_x \hat{x} + a_y \hat{y} e^{j\varphi}\right) e^{-jkz}$

Time domain expression ↳ $\bar{e}(t,z) = \mathrm{Re}\left\{\bar{E} e^{j\omega}\right\} = \hat{x}\, a_x \cos(\omega t - kz) + \hat{y}\, a_y \cos(\omega t - kz + \varphi)$

amplitude: $|\bar{e}(t,z)| = \sqrt{e_x^2(z,t) + e_y^2(z,t)}$

direction of polarization given by the angle: $\psi = \arctg\left(\frac{e_y(z,t)}{e_x(z,t)}\right)$

→ Linear Polarization: $\varphi = 0$ or $\varphi = \pi$

→ RHCP : $\varphi = -\frac{\pi}{2}$, $a_x = a_y = a$

- Counterclockwise
- Superposition of two linearly pol. waves

→ LHCP : $\varphi = \frac{\pi}{2}$, $a_x = a_y = a$

- Clockwise
- Super

↳ RHCP + LHCP ⇒ linearly polarized wave

→ Elliptical Polarization: $\varphi \neq 0, \pm\pi, \pm\frac{\pi}{2}$ $\quad a_x \neq a_y$

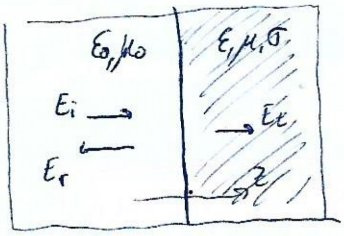
·Polarizer: A filter that allows transmission only for a specific polarizatio

5 - Transmission, Reflection @ normal oblique incidence

Reprocity theorem

Images theory

•) Interface $\boxed{\text{Lossy}}$ Medium



$$\eta = j\frac{\omega\mu}{\gamma}$$

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}$$

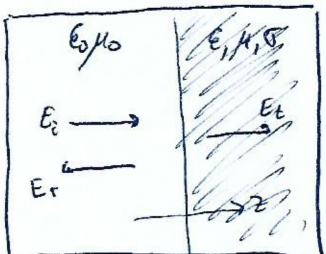
↳ Imposing continuity of tangencial compo:

$$\begin{cases} E_i + E_r = E_t \\ H_i + H_r = H_t \end{cases} \Rightarrow \quad \Gamma = \frac{\eta - \eta_0}{\eta + \eta_0}$$

$$T = 1 + \Gamma = \frac{2\eta}{\eta + \eta_0}$$

Γ and $T \in \mathbb{C}$

•) Interface $\boxed{\text{Good}}$ Conductor



$$\eta = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}} = (1+j)\frac{1}{\sigma\delta_s}$$

$$\gamma = \alpha + j\beta = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}} = (1+j)\frac{1}{\delta_s}$$

Γ and $T \in \mathbb{C}$

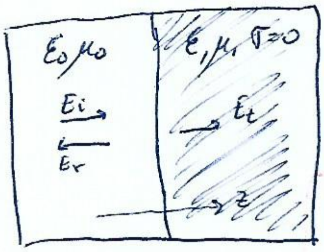
↳ Power balance not obtained if separet incid. and reflec. Poynting vectors. Only we recover balance with time average quantities

↳ Power in lossy conductor decays exponentially $\sim e^{-2\alpha z}$

$J_t = \sigma E_t = \hat{x}\,\sigma E_0 T e^{-\gamma z}$ $[A/m^2]$ → volume current in conducting region

$$P_t = \frac{1}{2}\int_V E_t \cdot J_t \, dv = \frac{\sigma |E_0|^2 |T|^2}{4\alpha}$$

•) Interface $\boxed{\text{Loss less}}$ Medium

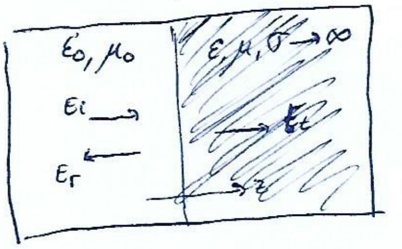


$$\eta = \frac{\omega\mu}{\gamma}$$

$$\gamma = j\beta = j\omega\sqrt{\mu\varepsilon} = jk_0\sqrt{\mu_r\varepsilon_r}$$

Γ and $T \in \mathbb{R}$

•) Interface $\boxed{\text{Perfect}}$ Conductor



$$\sigma \to \infty$$

$$\alpha \to \infty$$

$$\eta = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}} \to 0$$

$$\delta_s = \sqrt{\frac{2}{\omega\mu\sigma}} \to 0$$

$T \to 0$, $\Gamma \to -1$

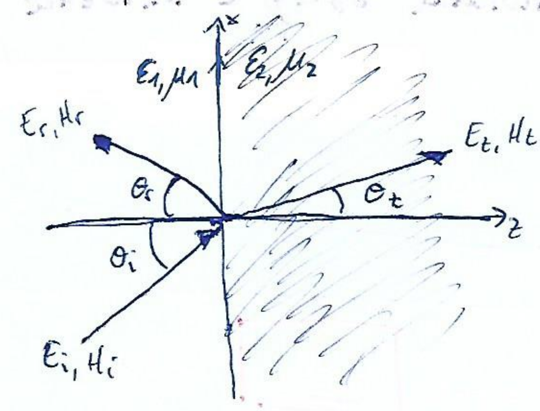
$J_s = \hat{n} \times \vec{H} = \hat{x}\,\frac{2}{\eta_0}E_0$ $[A/m]$ → volume current density reduces to a surface current
No fields propagate into perfect conductor.

↳ Most of the power transmitted is rapidly dissipated into heat.

$$P_t = \frac{2|E_0|^2 R_s}{\eta_0}$$

$$R_s = \frac{1}{\sigma\delta_s} = \sqrt{\frac{\omega\mu}{2\sigma}}$$

o) Oblique incidence @ dielectric interface



Parallel Polarization (TM)

If the E-field of the wave is in the plane of incidence then the wave is called a TM-wave. $\boxed{\text{E-field in the xz plane}}$

Perpendicular Polarization (TE)

If the E-field of the wave is perpendicular to the plane of incidence then the wave is called TE-wave

$\boxed{\text{E-field vector orthogonal to the xz plane } \perp}$

↳ Snell's law

$$\boxed{\begin{gathered}\theta_i = \theta_r \\ k_1 \sin\theta_i = k_2 \sin\theta_t\end{gathered}}$$

↳ Parallel Polarization (TM)

$$\Gamma = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$T = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

Brewster: Angle for $\Gamma = 0$

No reflection of parallel component

$$\boxed{\theta_b = a\tan\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)}$$

↳ Perpendicular Polarization (TE)

$$\Gamma = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$T = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

Not Brewster angle here

o) Total Internal Reflection

When: $\varepsilon_1 > \varepsilon_2$ Everything is reflected back to the medium of incidence

$\theta_t = 90°$

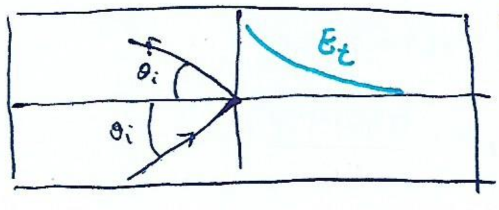
$$\boxed{\theta_c = a\sin\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)}$$

Happens for both parallel and perpendicular polarization

Surface waves: Lecture 5 pages: 21-22

$\varepsilon_1 > \varepsilon_2$

$\theta_i > \theta_c$



•) Lorentz Reciprocity Theorem (Lecture 5 - pages: 23 - 24)

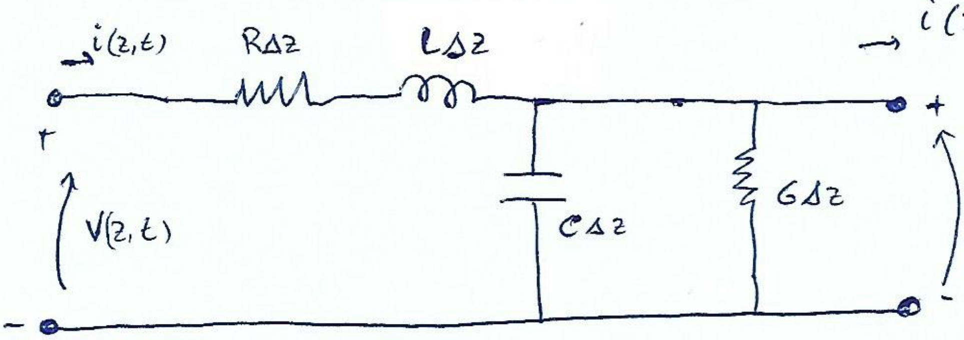
You can exchange the receiver and the transmitter

•) Image Theory (Lecture 5 - pages: 25 - 26 - 27)

↳ We use it when we have a current source (electric or magnetic) in the vecinity of a conducting ground plane. Then, we can remove it by placing a virtual image source.

6 & 7 - Transmission Line Theory

∘) Model



$R = \Omega/m$

$L = H/m$

$C = F/m$

$G = S/m$

↳ Applying kirchoff and $z \to 0$ we get telegrapher's equations

$$\begin{cases} \dfrac{\partial V(z,t)}{\partial z} = -R\, i(z,t) - L\, \dfrac{\partial i(z,t)}{\partial t} \\ \dfrac{\partial i(z,t)}{\partial z} = -G\, V(z,t) - C\, \dfrac{\partial V(z,t)}{\partial z} \end{cases}$$

∘) In Harmonic domain eqs:

$$\begin{cases} \dfrac{d^2 V(z)}{dz^2} - \gamma^2 V(z) = 0 \\ \dfrac{d^2 I(z)}{dz^2} - \gamma^2 I(z) = 0 \end{cases} \xrightarrow[\text{solutions}]{} \boxed{\begin{aligned} V(z) &= V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \\ I(z) &= \frac{1}{Z_0}\left(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right) \end{aligned}}$$

↳ propagation constant: $\boxed{\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}}$

↳ characteristic impedance $\boxed{Z_0 = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}}$

- Lossless Line $G = R = 0$

$$\gamma = j\omega\sqrt{LC}$$

$\hookrightarrow \beta = \omega\sqrt{LC}$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\lambda = \frac{2\pi}{\omega\sqrt{LC}}$$

$$V_p = \frac{1}{\sqrt{LC}}$$

- Field analysis

$\hookrightarrow$ Stored Magnetic Field

$$W_m = L\frac{|I_0|^2}{4}$$

$$L = \frac{\mu}{|I_0|^2}\int_S H \cdot H^* \, ds \quad [H/m]$$

$\hookrightarrow$ stored Electric Field

$$W_e = C\frac{|V_0|^2}{4}$$

$$C = \frac{\varepsilon}{|V_0|^2}\int_S E \cdot E^* \, ds \quad [F/m]$$

- Power Loss

$\hookrightarrow$ Due to a conductor

$$P_c = R\frac{|I_0|^2}{2}$$

$$R = \frac{R_s}{|I_0|^2}\int_{C_1+C_2} H \cdot H^* \, dl$$

$$R_s = \frac{1}{\sigma\delta_s}$$

$\hookrightarrow$ Due to the material

$$P_d = G\frac{|V_0|^2}{2}$$

$$G = \frac{\omega\varepsilon''}{|V_0|^2}\iint_S E \cdot E^* \, ds$$

◦) Coaxial Line

↳ Telegrapher's eq. for coaxial line

$$\begin{cases} \dfrac{\partial V(z)}{\partial z} = -j \dfrac{\omega\mu \ln(b/a)}{2\pi} I(z) \\[2ex] \dfrac{\partial I(z)}{\partial z} = -j\omega \underbrace{(\varepsilon' - j\varepsilon'')}_{\varepsilon} 2\pi \dfrac{1}{\ln(b/a)} \cdot V(z) \end{cases} \quad \Rightarrow \gamma = \omega\sqrt{\mu\varepsilon}$$

↳ Anyway you get the same wave equation ⇒ Wave eq. doesn't depend on the material.

↳ Lossless media

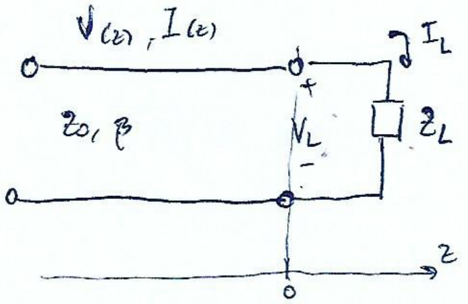
$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{LC}$ $\qquad$ $Z_0 = \eta \dfrac{\ln(b/a)}{2\pi}$

$\eta = \sqrt{\dfrac{\mu}{\varepsilon}}$ $\qquad$ ↓ geometry dependend

↳ Power flow: $P = \frac{1}{2}\int_S E \times H^* ds = \frac{1}{2} V_0 I_0^*$

◦

c) Terminated Lossless T.L.



$$\begin{cases} V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \\ I(z) = \frac{1}{Z_0}\left(V_0^+ e^{-j\beta z} - V_0^- e^{j\beta z}\right) \end{cases}$$

$$Z_L = \frac{V(0)}{I(0)} \Rightarrow \Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

↳ Power AV

$$P_{av} = \frac{1}{2} \operatorname{Re}\left[V(z) I^*(z)\right] = \frac{1}{2} \frac{|V_0|^2}{Z_0}\left(1 - |\Gamma|^2\right)$$

↳ Return Loss

$$RL = -20 \log |\Gamma| \quad (dB)$$

↳ Some results of Γ

* If $\Gamma = 0 \rightarrow |V(z)| = |V_0^+|$ → constant along the line
* If $\Gamma \neq 0 \rightarrow |V(z)| = |V_0^+| \, |1 + \Gamma e^{2j\beta z}|$ → not constant anymore

↳ SWR: A measure of the mismatch of the line

$$V_{max} = |V_0^+| (1 + |\Gamma|)$$

$$SWR = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \qquad 1 < SWR < \infty$$

$$V_{min} = |V_0^-| (1 - |\Gamma|)$$

↳ To remember

$$\boxed{V(z) = V_0^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)}$$

$$\boxed{I(z) = \frac{V_0^+}{Z_0} \left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)}$$

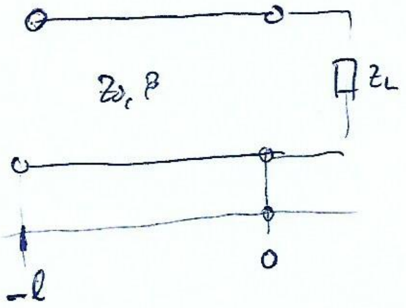
↳ Expression to find max and min

$$|V(z)| = |V_0^+| \left|1 + |\Gamma| e^{j(\theta - 2\beta l)}\right|$$

* Distance between 2 maxima (or minima): $\lambda/2$

* Distance between max and min: $\lambda/4$

•) Transmission Line Impedance



$$Z_{in} = Z(-l) = \frac{V(-l)}{I(-l)}$$

$$\boxed{Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}}$$

$$Z_{in} = \frac{1+\Gamma e^{-2j\beta l}}{1-\Gamma e^{-2j\beta l}}$$

↳ Case: Short Circuit $Z_L = 0$

$\Gamma = -1$ $\quad V(z) = -2j V_0^+ \sin(\beta z)$ $\quad Z_{in} = j Z_0 \tan(\beta l)$ purely imaginary

$SWR = \infty$ $\quad I(z) = 2 \frac{V_0^+}{Z_0} \cos(\beta z)$

↳ Case: Open circuit $Z_L = \infty$ $\quad Z_{in} = -j Z_0 \cot(\beta l)$ purely imaginary

$\Gamma = 1$ $\quad V(z) = 2 V_0^+ \cos(\beta z)$

$SWR = \infty$ $\quad I(z) = -2j \frac{V_0^+}{Z_0} \sin(\beta z)$

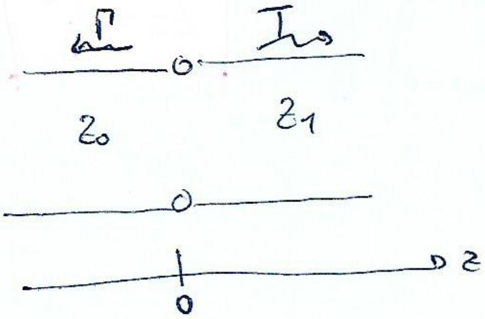
↳ Length $l = \frac{\lambda}{2}$

$$l = \frac{\lambda}{2} \Rightarrow \beta l = \pi \Rightarrow Z_{in} = Z_L$$

↳ Length : Quarter wavelength transformer

$$l = \frac{\lambda}{4} + n\frac{\lambda}{2} \Rightarrow \beta l = \frac{\pi}{2} \Rightarrow Z_{in} = \frac{Z_0^2}{Z_L}$$

↳ Junction of two lines



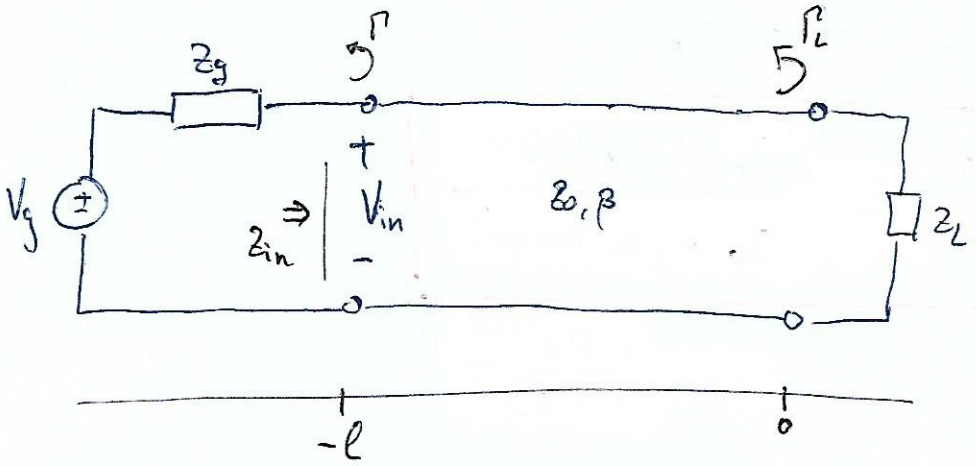
$$\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$T = \frac{2Z_1}{Z_1 + Z_0}$$ the portion of the wave that is transmitted

Insertion Losses: $IL = -20 \log |T|$

o) Generator and load mismatches

$Z_g, Z_L \in \mathbb{C}$



$$V_{in} = V(-\ell) = V_g \frac{Z_{in}}{Z_{in}+Z_g}$$

$$V(z) = V_0^+ \left(e^{-j\beta z} + \Gamma_L e^{j\beta z}\right) \quad \text{in } z=-\ell \;\; \& \;\; Z_{in} = Z_0 \frac{1+\Gamma}{\dots}$$

combine with this eq and V_{in}

$$V_0^+ = V_g \frac{Z_0}{Z_0+Z_g} \frac{e^{-j\beta\ell}}{1-\Gamma_L \Gamma_g e^{-j\beta\ell}}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0}$$

↳ Power delivered to the load

$$\boxed{P = \frac{1}{2} \mathrm{Re}\left\{V_{in} I_{in}^*\right\} = \frac{1}{2} |V_g|^2 \frac{R_{in}}{(R_{in}+R_g)^2 + (X_{in}+X_g)^2}}$$

* Load matched $Z_{in} = Z_0$

$$P = \frac{1}{2} |V_g|^2 \frac{Z_0}{(Z_0+R_g)^2 + X_g^2}$$

* Generator matched $Z_{in} = Z_g$

$$P = \frac{1}{8} |V_g|^2 \frac{R_g}{R_g^2 + X_g^2}$$

↳ Conjugate matching

Z_g is fixed, we vary Z_{in}:

$$\frac{\partial P}{\partial R_{in}} = 0 \implies R_g^2 - R_{in}^2 + (X_{in}+X_g)^2 = 0$$

$$\frac{\partial P}{\partial X_{in}} = 0 \implies R_{in}(X_{in}+X_g) = 0$$

$$\left.\begin{aligned} R_{in} &= R_g \\ X_{in} &= -X_g \end{aligned}\right\} \implies \boxed{Z_{in} = Z_g^*}$$

Conjugate matching condition

↳ Max available power

$$\boxed{P = \frac{1}{2} |V_g|^2 \frac{1}{4 R_g}}$$

o) Lossy Transmission Lines

↳ Low loss line

$R \ll \omega L$
$G \ll \omega C$ $\Rightarrow$ $RG \ll \omega^2 LC$ Using Taylor:

$\beta \simeq \omega\sqrt{LC}$ $Z_0 \simeq \sqrt{\frac{L}{C}}$

$\alpha \simeq \frac{1}{2}\left(\frac{R}{Z_0} + G Z_0\right)$

↳ Distorsionless Line

→ Unless line is lossless β is not linear function of ω

→ The phase velocity $v_p = \frac{\omega}{\beta}$ will be different for diff freqs ⇒ diff. signal wavelength traveling on the line will arrive at diff. times

⇓

$\boxed{\text{Dispersion}}$

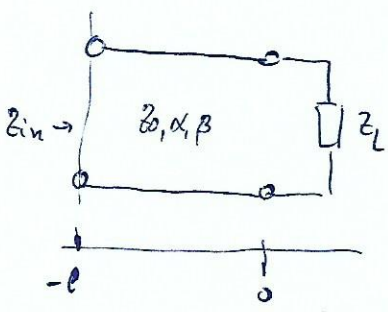
* Distorsionless line $\frac{R}{L} = \frac{G}{C}$

linear
↳ $\beta = \omega\sqrt{LC}$

⇒ $\alpha = R\sqrt{\frac{C}{L}}$ } $\boxed{\text{No Distorsion}}$

not freq. dependency

o) Terminated lossy Line



$$\boxed{Z_{in} = \frac{V(-\ell)}{I(-\ell)} = Z_0 \frac{Z_L + Z_0 \tanh \gamma\ell}{Z_0 + Z_L \tanh \gamma\ell}}$$

* Power @ $z=-\ell$ ⇒ $P_{in} = \frac{1}{2}\,\mathrm{Re}\{V(-\ell)\cdot I(-\ell)^*\} = \frac{|V_0^+|^2}{2Z_0}\left[e^{2\alpha\ell} - |\Gamma(\ell)|^2 e^{-2\alpha\ell}\right]$

* Power @ load $z=0$ ⇒ $P_L = \frac{1}{2}\,\mathrm{Re}\{V(0) + I^*_{(0)}\} = \frac{|V_0^+|^2}{2Z_0}\left(1 - |\Gamma|^2\right)$

Power loss: $P_{Loss} = P_{in} - P_L$

9 - Transmission Lines and Waveguides

◦) Intro

Waveguides , coaxial lines, planar TL

→ If a TL has 2 or more conductors: Can have TEM modes

→ If a TL has only 1 conductor: The WG/TL supports TE and/or TM waves

◦) General sols. for TEM, TE and TM waves

→ $\bar{E}(x,y,z) = [\bar{e}(x,y) + \hat{z}\, e_z(x,y)]\, e^{-j\beta z}$

$\bar{H}(x,y,z) = [\bar{h}(x,y) + \hat{z}\, h_z(x,y)]\, e^{-j\beta z}$

$\bar{e}(x,y)$, $\bar{h}(x,y)$ transverse field compo.

$e_z(x,y)$, $h_z(x,y)$ field compo in the prop. direction.

↳ cutoff wavenumber : $\boxed{k_c^2 = k^2 - \beta^2}$

$k = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda}$

* if losses in the dielectric

$\varepsilon = \varepsilon_0 \varepsilon_r (1 - j\tan\delta)$

◦) TEM waves

$E_z = H_z = 0 \longrightarrow \beta = \omega\sqrt{\mu\varepsilon} = k \Rightarrow k_c = 0$

$Z_{TEM} = \eta = \sqrt{\frac{\mu}{\varepsilon}} \rightarrow \bar{h}(x,y) = \frac{1}{Z_{TEM}}\, \hat{z} \times \bar{e}(x,y)$

◦) TE waves

$E_z = 0 \quad H_z \neq 0 \qquad k_c^2 \neq 0 \rightarrow \beta^2 = \sqrt{k^2 - k_c^2}$

$Z_{TE} = \frac{k}{\beta}\eta \qquad \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) H_z = 0$

◦) TM waves

$E_z \neq 0 \quad H_z = 0 \qquad k_c^2 \neq 0 \rightarrow \beta = \sqrt{k^2 - k_c^2}$

$Z_{TM} = \frac{\beta}{k}\eta \qquad \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) H_y = 0$

- Attenuation

$$\alpha = \alpha_d + \alpha_c$$

dielectric conductor

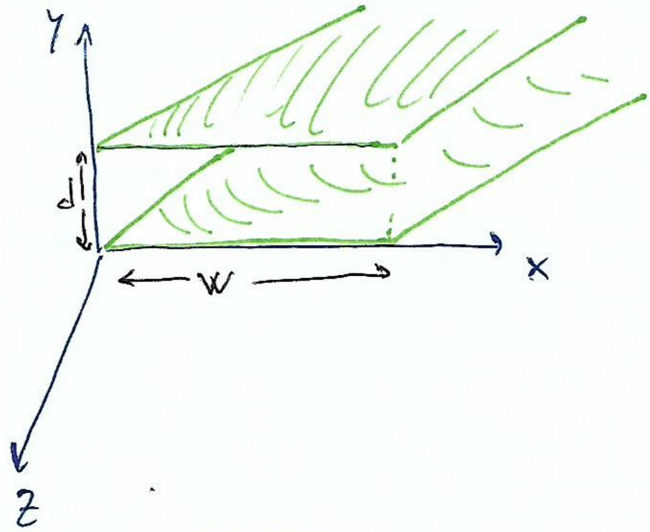
↳ $\alpha_d = \dfrac{k^2 \tan\delta}{2\beta}$ $\left[\frac{Np}{m}\right]$ → TE or TM

$\alpha_d = \dfrac{k \tan\delta}{2}$ → TEM ($k_c = 0$ & $\beta = k$)

↳ $\alpha_c = \dfrac{P_e(z=0)}{2P_0}$ (lecture 9, page 19)

10 - Waveguides

•) Parallel waveguide



$W \gg d$

* TEM mode

$E_z = H_z = 0$

$$Z_0 = \eta \frac{d}{W}$$

$$v_{phase} = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

* TM mode

$H_z = 0 \quad E_z \neq 0 \qquad k_c^2 = k^2 - \beta^2$

↳ prop. const.

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{n\pi}{d}\right)^2}$$

$$k_c = \frac{n\pi}{d}$$

↳ note 1: $TM_0 = TEM$

↳ Note 2: β real if $k < k_c$

cutoff freq. $k = k_c$ $\quad f_c = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{n}{2d\sqrt{\mu\varepsilon}}$

$$\lambda_c = \frac{2d}{n}$$

↳ wave impedance: $Z_{TM} = \eta \frac{\beta}{k}$

↳ phase velocity: $v_p = \frac{\omega}{\beta}$

↳ guide wavelength $\lambda_g = \frac{2\pi}{\beta}$

↳ attenuation (dielectric) $\alpha_d = \frac{k^2 \tan\delta}{2\beta}$

↳ attenuation (conductor) $\alpha_c = \frac{P_\ell}{2P_0} = \frac{2kR_s}{\beta\eta d}$ $n \neq 0$

$$\alpha_c = \frac{R_s}{\eta d} \quad n = 0$$

* TE mode

$E_z = 0 \quad H_z = 0 \qquad k_c^2 = \sqrt{k^2 - \beta^2}$

↳ prop. conts

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{n\pi}{d}\right)^2}$$

$$k_c = \frac{n\pi}{d}$$

↳ cuttoff freq $f_c = \dfrac{n}{2d\sqrt{\mu\varepsilon}}$

↳ wave impedance $Z_{TE} = \dfrac{k\eta}{\beta}$

↳ Attenuation (conductor)

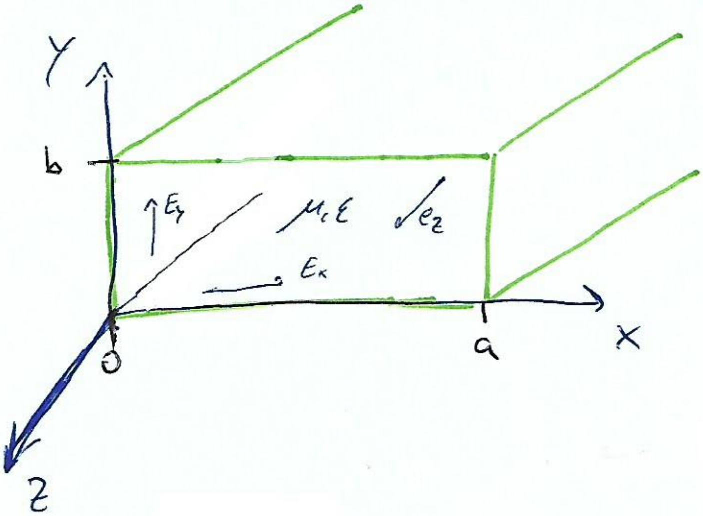
$$\alpha_c = \frac{P_e}{2P_0}$$

$$P_0 = \frac{\omega\mu d w}{4k_c^2} |B_n|^2 \, Re(\beta) \quad n>0$$

$P_0 = 0 \qquad n=0$

a/o TE_0 mode

11 - Rectangular Waveguide



$\boxed{a > b}$

- TE modes ($E_z = 0$)

prop. constant: $\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$

$k_c^2 = k_x^2 + k_y^2$ $\qquad \beta \in \mathbb{R}$ propagating if $k > k_c$

cut-off freq.

$$\boxed{f_{c_{mn}} = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

↳ TE_{10} lowest cut-off freq

↳ if $f < f_c$ all modes evanescence
β imaginary

wave impedance

$$Z_{TE} = \eta \frac{k}{\beta}$$

guide wavelength

$$\lambda_g = \frac{2\pi}{\beta} \longrightarrow \lambda_g > \lambda$$

↳ $\lambda = \frac{2\pi}{k}$

phase velocity

$$v_p = \frac{\omega}{\beta} \longrightarrow v_p > \frac{\omega}{k}$$

↳ $\frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}}$

o) TM modes ($H_z = 0$)

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

f_c, λ_g, v_p $\longrightarrow$ same as TE

There is no TM_{00}, TM_{01}, TM_{10}

The lowest order is TM_{11} $\longrightarrow$ $f_{c_{TM_{11}}} > f_{c_{TE_{10}}}$

wave impedance: $Z_{TM} = \eta \frac{\beta}{k}$

11 bis - Circular Waveguides

o) TE Modes $E_z = 0$

P'_{nm} roots of $J'_m(x)$

→ cut-off freq. $f_{c_{nm}} = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{P'_{nm}}{2\pi a\sqrt{\mu\varepsilon}}$

→ prop. constant $\beta_{nm} = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{P'_{nm}}{a}\right)^2}$

→ The dominant mode : $\boxed{TE_{11}}$

No TE_{10} but is a TE_{01}

→ Wave imp. $Z_{TE} = \eta \frac{k}{\beta}$

o) TM Modes $H_z = 0$

→ cut-off freq. $f_{c_{nm}} = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{P_{nm}}{2\pi a\sqrt{\mu\varepsilon}}$

P_{nm} roots of $J_m(x)$

→ prop. const. $\beta_{nm} = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{P_{nm}}{a}\right)^2}$

→ First TM mode → $\boxed{TM_{01}}$

TM_{01} comes after TE_{11}

No TM_{10} mode $m \geq 1$

→ wave imp. $Z_{TM} = \eta \frac{\beta}{k}$

- Attenuation for both TE & TM

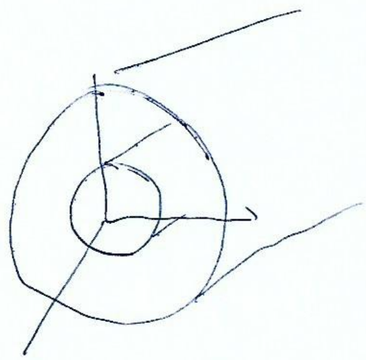
→ dielectric: $\alpha_d = \dfrac{k^2 \tan\delta}{2\beta}$

→ conductor $\alpha_c = \dfrac{R_s}{a k \eta \beta}\left(k_c^2 + \dfrac{k^2}{p_{11}'^2 - 1}\right)$

$$R_s = \sqrt{\frac{\omega \mu_0}{2\sigma}}$$

total att: $\alpha = \alpha_d + \alpha_c$

- Coaxial lines

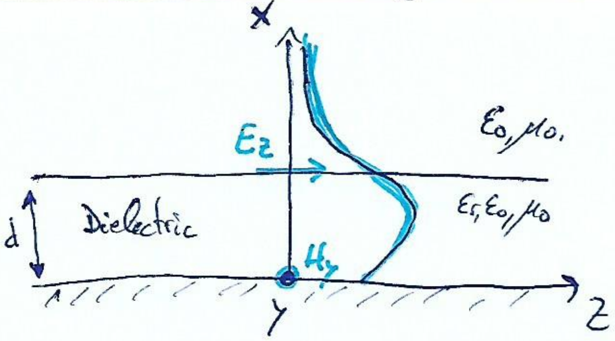


TEM can propagate

(lecture 11 page 19)

13- Grounded Slab and others waveguides

1) Surface waves of grounded dielec. slab



→ TM modes

$$\begin{cases} 0 \leq x < d & \left(\dfrac{\partial^2}{\partial x^2} + \varepsilon_r k_0^2 - \beta^2\right) e_z(x,y) = 0 \\ d \leq x \leq \infty & \left(\dfrac{\partial^2}{\partial x^2} + k_0^2 - \beta^2\right) e_z(x,y) = 0 \end{cases}$$

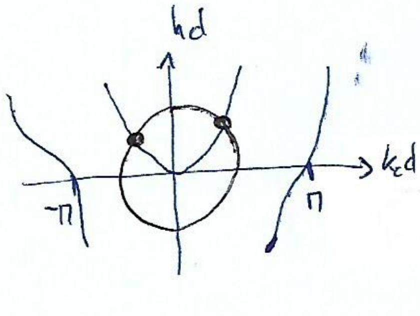
$$E(x,y,z) = e_z(x,y)\, e^{-j\beta z}$$

Two different cut-off freqs.

$$\begin{cases} k_c^2 = \varepsilon_r k_0^2 - \beta^2 & 0 \leq x < d \\ h^2 = \beta^2 - k_0^2 & d \leq x < \infty \end{cases}$$

The two solutions are:

$$\begin{cases} k_c \tan(k_c d) = \varepsilon_r h \\ k_c^2 + h^2 = (\varepsilon_r - 1) k_0^2 \end{cases}$$



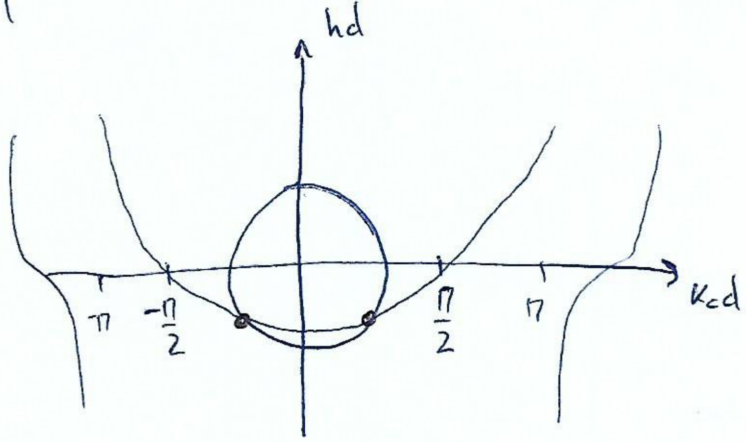
- If $\varepsilon_r > 1 \rightarrow$ At least 1 TM modes (TM_0 mode) $\Rightarrow$ cut-off freq $= 0$

- Next TM mode only if circle radius $> \pi$ (TM_1)

$$f_c = \frac{nc}{2d\sqrt{\varepsilon_r - 1}}$$

→ TE modes

$$\begin{cases} \left(\frac{\partial^2}{\partial x^2} + k_c^2\right) h_z(x,y) = 0 & 0 \leq x < d \\ \left(\frac{\partial^2}{\partial x^2} - h^2\right) h_z(x,y) = 0 & x \geq d \end{cases} \xrightarrow[\text{Solutions}]{} \begin{cases} -k_c \cot(k_c d) = h \\ k_c^2 + h^2 = (\varepsilon_r - 1) k_0^2 \end{cases}$$



$h < 0$ invalid sol.

No TE_0 mode

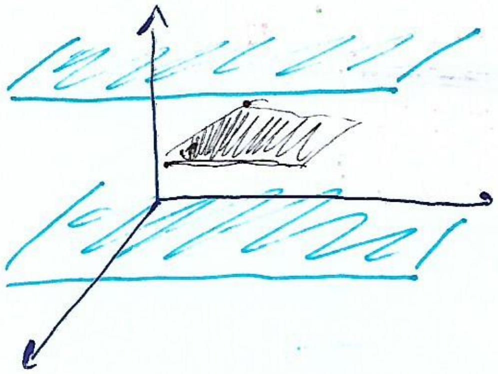
① First TE mode propagates when $\sqrt{\varepsilon_r - 1}\, k_0 d > \frac{\pi}{2}$

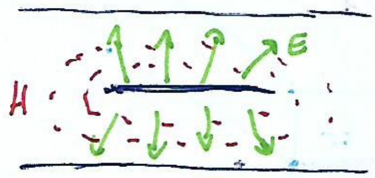
② Cut-off freqs.

$$f_c = \frac{(2n-1)c}{4d\sqrt{\varepsilon_r - 1}} \quad n = 1, 2 \dots$$

③ Order: TM_0, TE_1, TM_1, TE_2, TM_2 ....

2.) Stripline





•) TEM mode

↳ phase velocity: $V_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}}$

↳ Propagation constant: $\beta = \frac{\omega}{v_p} = \omega \sqrt{\mu \varepsilon_0 \varepsilon_r} = k_0 \sqrt{\varepsilon_r}$

↳ characteristic impedance: $Z_0 = \frac{30\pi}{\sqrt{\varepsilon_r}} \cdot \frac{b}{W_e + 0{,}441 \cdot b}$

W_e effective width → $\frac{W_e}{b} = \frac{W}{b} - \begin{cases} 0 & W/b \geq 0{,}35 \\ \left(0{,}35 - \frac{W}{b}\right)^2 & W/b < 0{,}35 \end{cases}$

↳ step width given Z_0:

$\frac{W}{b} = \begin{cases} x & \sqrt{\varepsilon_r}\, Z_0 \leq 120 \\ 0{,}85 - \sqrt{0{,}6 - x} & \sqrt{\varepsilon_r}\, Z_0 > 120 \end{cases}$ $\qquad x = \frac{30\pi}{\sqrt{\varepsilon_r}\, Z_0} - 0{,}441$

↳ Attenuation

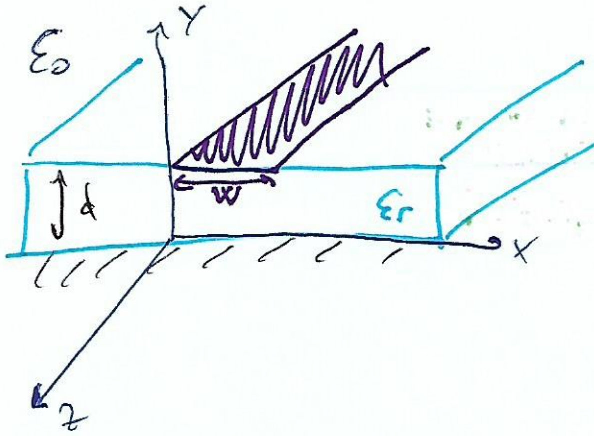
$\alpha_d = \frac{k \tan\delta}{2}$

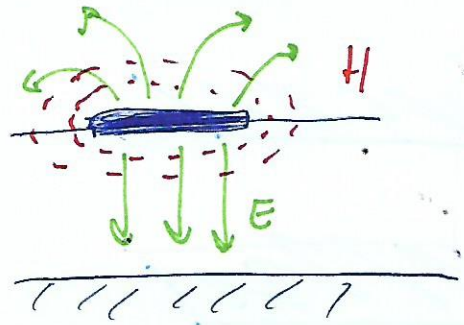
$\alpha_c = \begin{cases} \frac{2{,}7 \cdot 10^{-3} R_s \varepsilon_r Z_0}{30\pi (b-t)} A & \sqrt{\varepsilon_r}\, Z_0 \leq 120 \\ \frac{0{,}16 R_s}{Z_0 b} B & \sqrt{\varepsilon_r}\, Z_0 > 120 \end{cases}$

A & B expressions at page 14, lecture 12

t → thickness

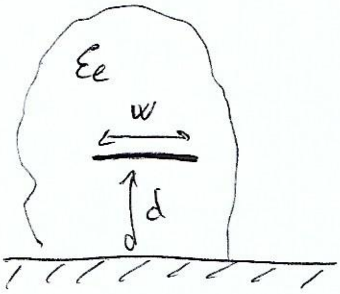
3) Microstrip





② Doesn't support pure TEM waves but yes hybrid TM-TE modes

If $d \ll \lambda$ ⇒ quasi-TEM modes



phase velocity: $V_p = \frac{c}{\varepsilon_e}$

Prop. const. $\beta = k_0 \sqrt{\varepsilon_e}$

ε_e: effective permittivity : $1 < \varepsilon_e < \varepsilon_r$

→ Expressions of

ε_e, Z_0, $\frac{W}{d}$ (for given Z_0 & ε_r), attenuation (α_d, α_c) are in: lecture 12 pages: 16 to 18

→ Slot line, Coplanar waveguide ⇒ quasi-TEM modes

→ covered microstrip, ridge wg, dielectric wg

4o) Dispersion and Wave Velocities

- speed of ligth in a medium $c = \frac{1}{\mu\varepsilon}$
- Phase velocity $v_p = \frac{\omega}{\beta}$

→ If a medium responds diff @ diff freqs. ⇒ Signal Distorsion

→ Small bandwidth ⊕ Dispersion not severe ⇒ Group velocity can be defined

Fourier: $$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

$$F(\omega) \longrightarrow \boxed{Z(\omega)} \xrightarrow{F_o(\omega)} \qquad F_o(\omega) = Z(\omega)\, F(\omega)$$

↳ $Z(\omega)$ lossless and match ⇒ $Z(\omega) = A\, e^{-j\beta z} = |Z(\omega)|\, e^{-j\psi}$

output signal ⇒ $$f_o(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, |Z(\omega)|\, e^{j(\omega t - \psi)}\, d\omega$$

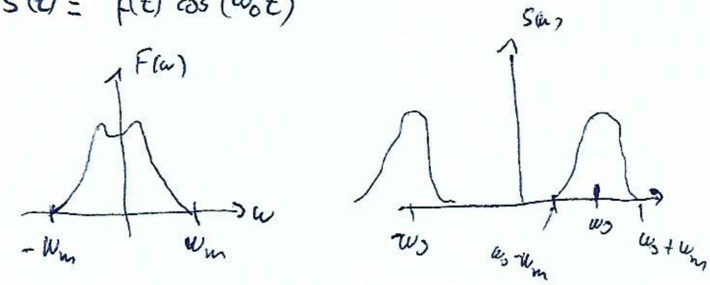
① $|Z(\omega)| = A$ constant
$\psi = a\omega$ linear $\Big\rangle$ $f_o(t) = A\, f(t-a)$

No distorsion $\beta = \omega/c$

② If TEM lossy : Assuming narrowband $s(t) = f(t)\cos(\omega_0 t)$

$$S(\omega) = F(\omega - \omega_0)$$

$$S_o(\omega) = A\, F(\omega - \omega_0)\, e^{-j\beta z}$$



In time dom.

$$s_o(t) = \frac{1}{2\pi} \mathrm{Re} \int_{\omega_0 - \omega_m}^{\omega_0 + \omega_m} A\, F(\omega - \omega_0)\, e^{j(\omega t - \beta z)}\, d\omega$$

if narrow bandwidth

$$\beta(\omega) \simeq \beta(\omega_0) + \left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_0} (\omega-\omega_0) = \beta_0 + \beta_0' (\omega-\omega_0)$$

result

$$S_0(t) = A\, f(t-\beta_0' z) \cos(\omega_0 t - \beta_0 z)$$

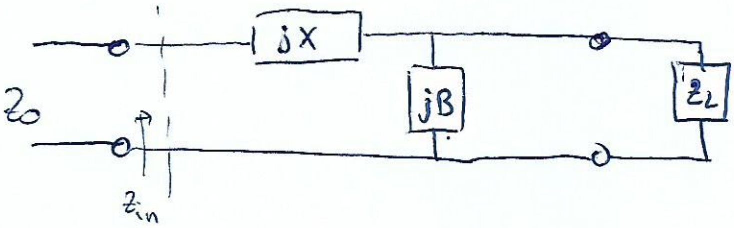
Time shifted replica of $f(t)$ that travels at V_g (velocity of the envelope)

$$\boxed{V_g = \frac{1}{\beta_0'} = \left.\left(\frac{\partial \beta}{\partial \omega}\right)^{-1}\right|_{\omega=\omega_0}}$$

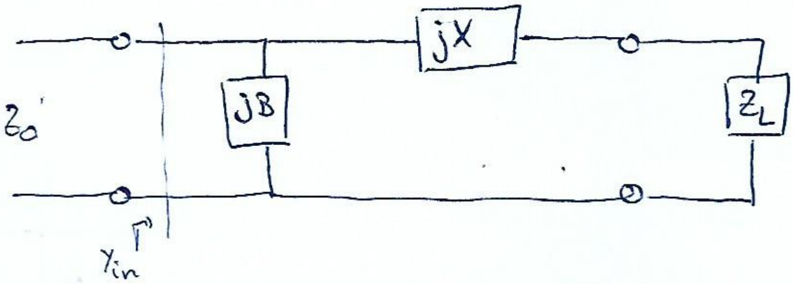
14- Impedance Matching

•) Matching with lumped elements

→ ① $\boxed{z_\ell = \frac{Z_L}{Z_0} > 1}$



→ ② $\boxed{z_\ell = \frac{Z_L}{Z_0} < 1}$



↳ Values

→ ① $\boxed{R_L > Z_0}$

$$Z_{in} = Z_0 = jX + \cfrac{1}{jB + \cfrac{1}{R_L + jX_L}}$$

$$B = \frac{X_L \pm \sqrt{R_L/Z_0}\sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2}$$

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{B R_L}$$

$X > 0 \rightarrow$ inductor $\quad B > 0 \rightarrow$ capacitor

$X < 0 \rightarrow$ capacitor $\quad B < 0 \rightarrow$ inductor

→ ②

$$Y_{in} = \frac{1}{Z_0} = jB + \frac{1}{R_L + j(X + X_L)}$$

$$X = \pm\sqrt{R_L(Z_0 - R_L)} - X_L$$

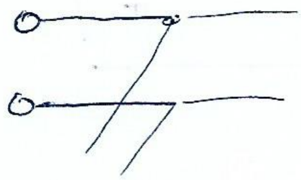
$$B = \pm\frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0}$$

if ⊕ for top ⊕ for bot

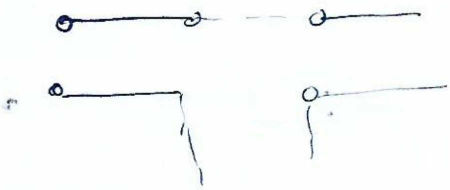
if ⊖ for top ⊖ for bot

•) <u>Single stub</u>

↳ Shunt / Parallel



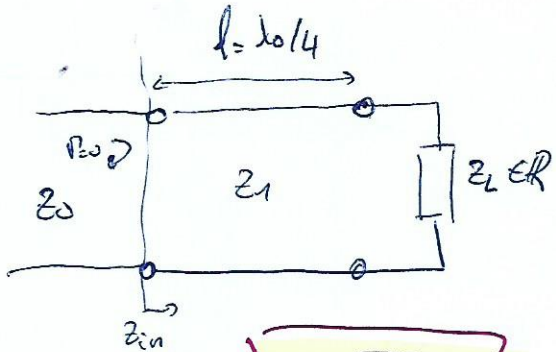
↳ Series



We get 2 values for d and 2 values for l_s depending on
if l_s neg. add $\lambda/2$ length to make it positive
open-circuit or short circuit

(eqs. lecture 14)

•) Quarter-wave transformer

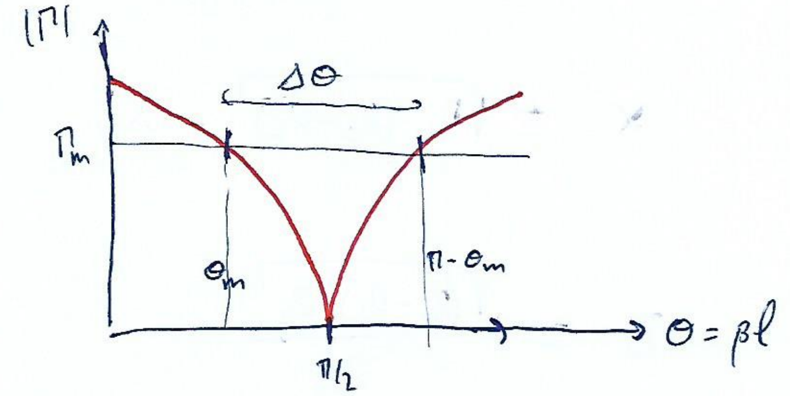


$$\boxed{Z_1 = \sqrt{Z_0 Z_L}}$$

(+)	(−)
- Simple design	- Matching for real loads
- Can be modified to multi-section	- Single freq. design

Calculating:

$$|\Gamma| \simeq \frac{|Z_L - Z_0|}{2\sqrt{Z_0 Z_L}} |\cos\theta| \quad , \theta \sim \frac{\pi}{2}$$



Expressions: (Lecture 14 (bis) Pag: 7-10)
$\theta_m, \Gamma_m,$

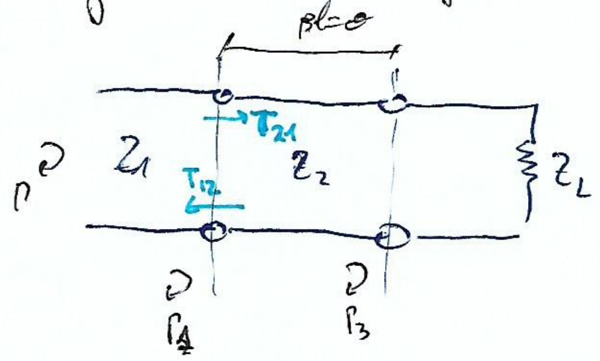
$$f_m = \frac{2\theta_m f_0}{\pi}$$

$$BW = \frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0}$$

Valid for TEM lines with no dispersion

•) Theory of small reflections

→ Single section transformer

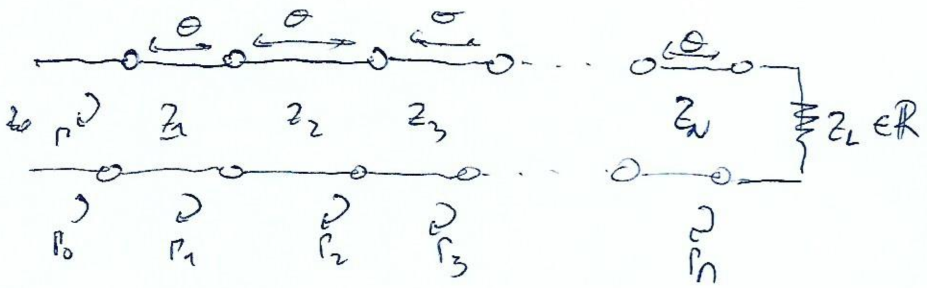


$$\Gamma = \frac{\Gamma_1 + \Gamma_3 e^{-2j\theta}}{1 + \Gamma_1 \Gamma_3 e^{-2j\theta}}$$

if small discontinuities $|Z_1 - Z_2| \ll$, $|Z_2 - Z_L| \ll$ $\Rightarrow |\Gamma_1 \Gamma_3| \ll 1$

$$\Gamma \simeq \Gamma_1 + \Gamma_3 e^{-2j\theta}$$

→ Multisection transformer



$$\theta = \beta l$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 e^{-2j\theta} + \dots + \Gamma_N e^{-2jN\theta}$$

↳ if transformer symmetrical: $\Gamma_0 = \Gamma_N$, $\Gamma_1 = \Gamma_{N-1}$ ... (does not imply that Z_n symm.)

(*) Binomial Multisection Transformer Γ_N, Γ_m, θ_m, $BW = \frac{\Delta f}{f_0}$
↳ flatest response

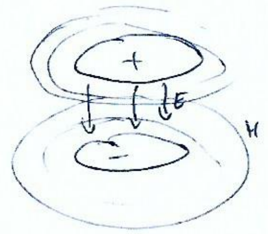
(*) Chebischev Multisection Transf.
↳ optimized bandwidth
at expences of passband ripples

15- MW Networks

◦) Defining: Impedance, Voltage and Current

→ TEM line:

- 2 conductors



$V = \int_{+}^{-} \bar{E}\cdot d\bar{l}$ $I = \oint_{C} \bar{H}\cdot d\bar{l}$ $Z_0 = \frac{V}{I}$

→ Waveguide

- Voltage depends on x, y ⇒ $\nexists$ unique voltage value

→ generally: $\bar{h}(x,y) = \frac{\hat{z} \times \bar{e}(x,y)}{Z_w}$

→ $C_1 = \frac{V^+}{A^+} = \frac{V^-}{A^-}$

$C_2 = \frac{I^+}{A^+} = \frac{I^-}{A^-}$

◦) Concept of Impedance

→ Intrinsic impedance of the medium: $\eta = \sqrt{\frac{\mu}{\varepsilon}}$
Equal to wave impedance for plane waves

→ Wave impedance that depends on the type of wave, type of WG material, freq $Z_w = \frac{E_t}{H_t}$

Unique impedance only for TEM waves

→ Characteristic impedance equal to V/I traveling wave on a TL $Z_0 = \sqrt{\frac{L}{C}}$

Impedance and Admittance matrices

- $[V] = [Z][I]$ $\qquad Z_{ij} = \left.\dfrac{V_i}{I_j}\right|_{I_k=0} \quad k \neq j$

  $I_k=0$ ↓ setting all other terminals to o.c.

- $[I] = [Y][V]$ $\qquad Y_{ij} = \left.\dfrac{I_i}{V_j}\right|_{V_k=0} \quad k \neq j$

  $V_k=0$ ↓ setting all other terminals to s.c.

Properties

- Reciprocal

  $Z_{ij} = Z_{ji}$

- Lossless

  Z_{ij} or Y_{ij} are all imaginary

Scattering matrix

→ $[V^-] = [S][V^+]$ $\qquad S_{ij} = \left.\dfrac{V_i^-}{V_j^+}\right|_{V_k^+=0}$ for $k \neq j$

S_{ii} reflection coef. at port i

S_{ij} transmission coef. from j to i

Properties

- Reciprocal

  $[S] = [S]^t$

- Lossless

  $[S]^t [S^*] = [U]$

  2x2

  $|S_{11}|^2 + |S_{12}|^2 = 1$

  $|S_{11}|^2 + |S_{21}|^2 = 1$

→ shifted in reference plane

$$S'_{nm} = S_{nm}\, e^{-2j\theta_n}$$

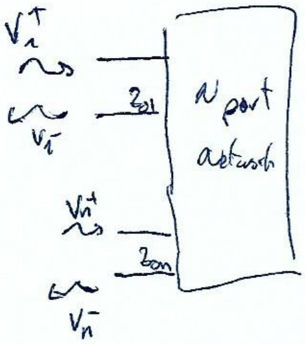
$\theta_n = \beta_n l_n$

→ Relation $[Z]$ & $[S]$

$$[S] = ([Z] - [U])([Z] + [U])^{-1}$$

$$[Z] = ([U] + [S])([U] - [S])^{-1}$$

•) Generalized S-param



$$a_n = \frac{V_n^+}{\sqrt{Z_{0n}}} \qquad V_n = \sqrt{Z_{0n}}\,(a_n + b_n)$$

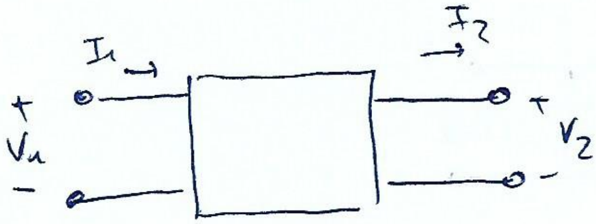
$$b_n = \frac{V_n^-}{\sqrt{Z_{0n}}} \qquad I_n = \frac{1}{\sqrt{Z_{0n}}}\,(a_n - b_n)$$

→ power: $P_n = \frac{1}{2}\left(|a_n|^2 - |b_n|^2\right)$

→ matrix:

$[b] = [S][a]$ $\qquad S_{ij} = \left.\frac{b_i}{a_j}\right|_{\substack{a_k=0 \\ k\neq j}} = \left.\frac{V_i^-\sqrt{Z_{0j}}}{V_j^+\sqrt{Z_{0i}}}\right|_{\substack{V_k^+=0 \\ k\neq j}}$

•) ABCD matrix

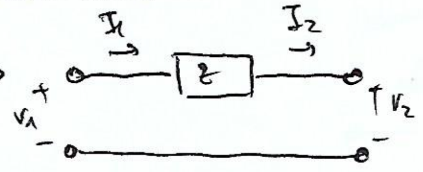


$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

If network reciprocal

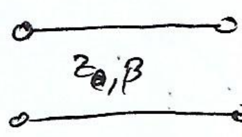
$AD - BC = 1$

① ↳



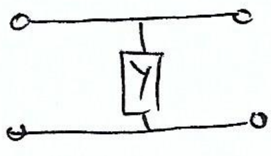
$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

③



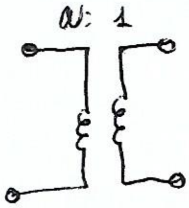
$$\begin{bmatrix} \cos\beta l & jZ_0\sin\beta l \\ jY_0\sin\beta l & \cos\beta l \end{bmatrix}$$

②



$$\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

④

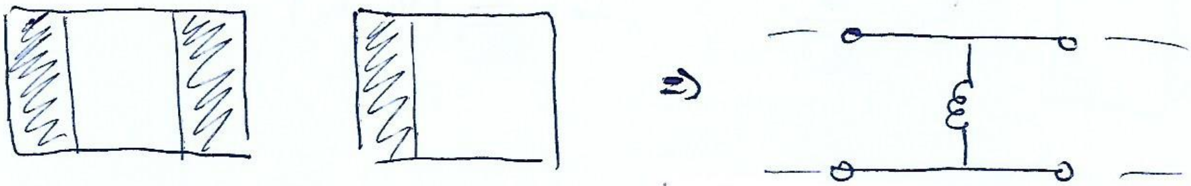


$$\begin{bmatrix} N & 0 \\ 0 & \frac{1}{N} \end{bmatrix}$$

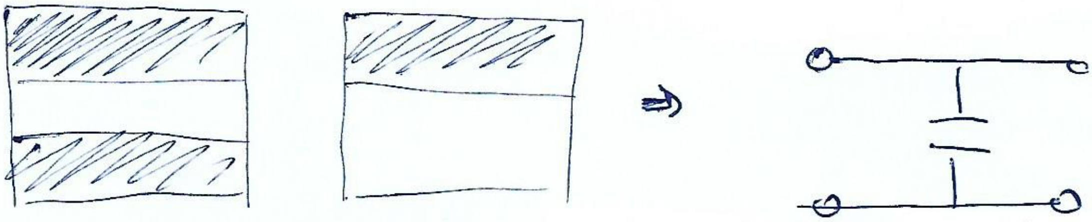
•) Discontinuities and Modal Analysis

→ Rectangular waveguides

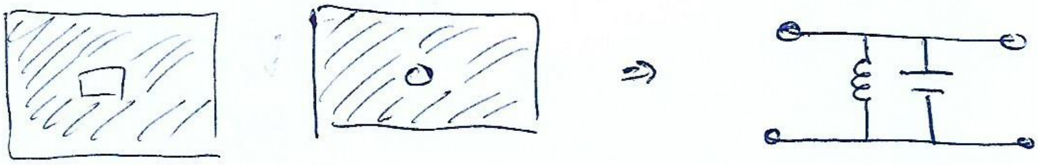
Inductive



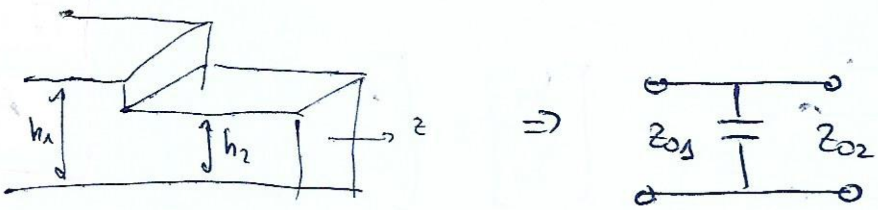
Capacitative



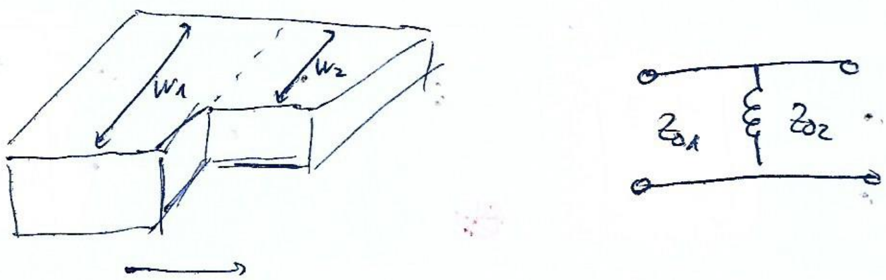
Resonant



Change height

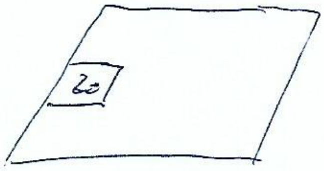


change width

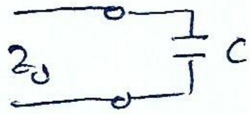


a) Microstrip Discontinuities

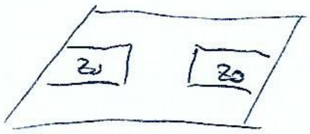
① Open-ended MS



⇒



② Gap in MS



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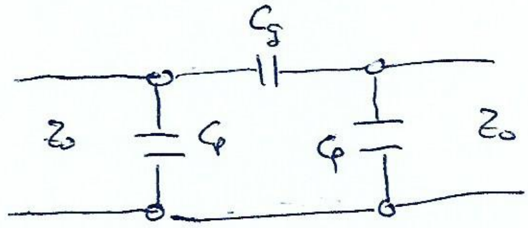
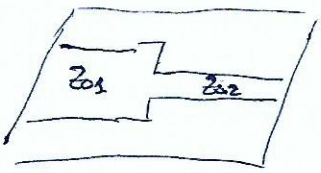
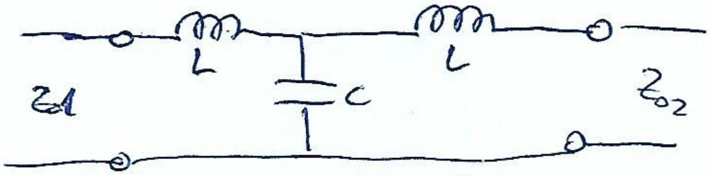


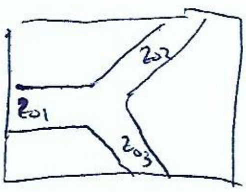
③ Change in width



⇒



④ T-Junction



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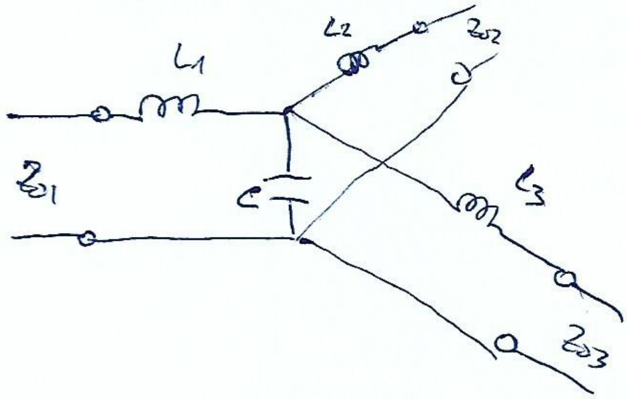
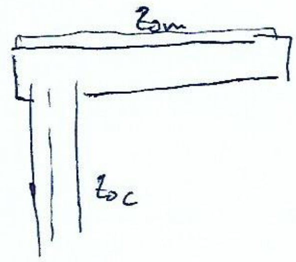
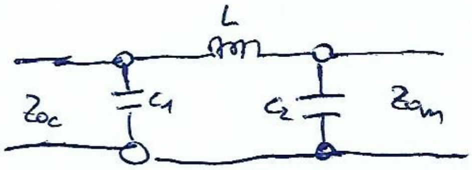


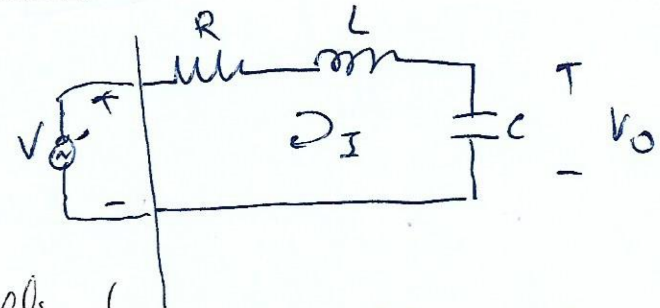
⑤ Coax to MS junction





19- Resonators

•) Series resonator circuit



$$Z_{in} = R + j\omega L - \frac{j}{\omega C}$$

Power delivered to the reson. $P_{in} = \frac{1}{2}|I|^2\left(R + j\omega L - \frac{j}{\omega C}\right) = P_{loss} + 2j\omega(W_m - W_e)$

$$W_m = \frac{1}{4}|I|^2 L$$

Power dissipated $P_{Loss} = \frac{1}{2}|I|^2 R$

$$W_e = \frac{1}{4}|V_C|^2 C = \frac{1}{4}|I|^2 \frac{1}{\omega^2 C}$$

Resonance when $W_m = W_e \Rightarrow P_{in} = P_{loss}$

$\Rightarrow Z_{in} = R$, purely real

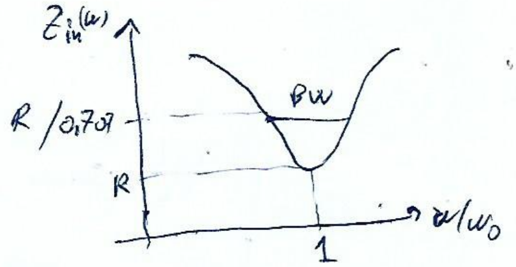
$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

Q factor: $Q = \omega \dfrac{W_m + W_e}{P_{loss}}$

↳ at resonance

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

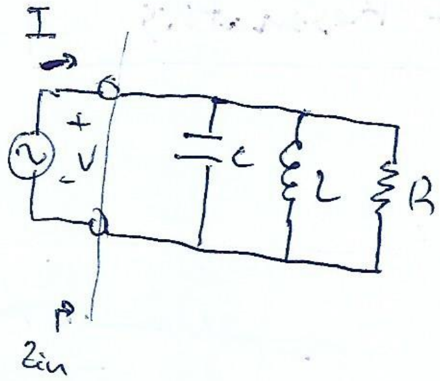
↳ BW



$$BW = \frac{1}{Q}$$

Parallel resonant circuit

$$Z_{in} = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)^{-1}$$



power delivered at the resonator

$$P_{in} = P_{loss} + \frac{1}{2}\frac{|V|^2}{R}$$

power loss

$$P_{loss} = \frac{1}{2}\frac{|V|^2}{R}$$

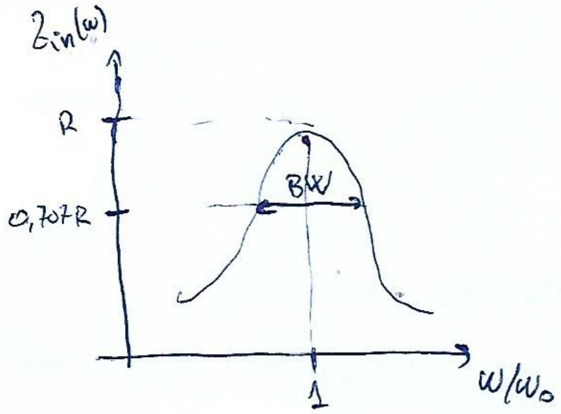
stored elec. energy: $W_e = \frac{1}{4}|V|^2 C$

stored mag. energy $W_m = \frac{1}{4}|I|^2 L = $

$$= \frac{1}{4}|V|^2 \frac{1}{\omega^2 L}$$

Q-factor

$$\boxed{Q = R\sqrt{\frac{C}{L}}}$$



$$BW = \frac{1}{Q}$$

Loaded, unloaded Q

→ Unloaded: Intrinsic property of resonator

→ Depending on the resonator (series or parallel) the load adds in the same way so:

$R_{serie} = R + R_L$

$R_{parallel} = RR_L/(R+R_L)$

→ External Q: $Q_E = \begin{cases} \frac{\omega_0 L}{R_L} & \text{series} \\ \frac{R_L}{\omega_0 L} & \text{parallel} \end{cases}$

→ Loaded Q $\quad \frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_0}$

Transmission line resonators

→ Short-circuit λ/2 line

$$Z_{in} = R + 2jL\Delta\omega \qquad R = Z_0\alpha l \qquad L = \frac{Z_0\pi}{2\omega_0} \rightarrow C = \frac{1}{\omega_0^2 L}$$

↳ circuit resonates at $\Delta\omega = 0$ ($l = \lambda/2$) : $l = n\frac{\lambda}{2}$, $n = 1, 2, 3, \ldots$

$$Z_{in} = R + Z_0\alpha l$$

Q factor

$$Q = \frac{\omega_0 L}{R} = \frac{\pi}{2\alpha l} = \frac{\beta}{2\alpha} \qquad \beta l = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2} = \pi \text{ at first resonance}$$

→ Short circuited λ/4 line

$$Z_{in} = \frac{1}{\frac{1}{R} + 2j\Delta\omega C} \qquad R = Z_0/\alpha l \qquad C = \frac{\pi}{4\omega_0 Z_0} \rightarrow L = \frac{1}{\omega_0^2 C}$$

Q factor

$$Q = \omega_0 RC = \frac{\beta}{2\alpha}$$

→ Open circuited λ/2 line

$$Z_{in} = \frac{Z_0}{\alpha l + j\left(\frac{\Delta\omega\pi}{\omega_0}\right)} \qquad R = \frac{Z_0}{\alpha l} \qquad C = \frac{\pi}{2\omega_0 Z_0} \longleftrightarrow$$

capacitance of equ. circuit

Q factor

$$Q = \omega_0 RC = \frac{\beta}{2\alpha}$$

c) Rectangular waveguide cavities

→ Wave number $\boxed{k_{mnl} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}}$

→ freq. (TE_{mnl}, TM_{mnl})

$$\boxed{f_{mnl} = \frac{c\,k_{mnl}}{2\pi\sqrt{\mu_r\varepsilon_r}} = \frac{c}{2\pi\sqrt{\mu_r\varepsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}}$$

if $b < a < d$ → Dominant: TE_{101} → for TE

TE_{110} → for TM

→ Q factor of T_{10l} mode

W_e

W_m

power loss cond. → $P_c = \frac{R_s E_0^2 \lambda^2}{8\eta^2}\left(\frac{l^2 ab}{d^2} + \frac{bd}{a^2} + \frac{l^2 a}{2d} + \frac{d}{2a}\right)$ $\quad Q_c = \frac{2\omega_0 W_e}{P_c}$

power loss dielec → $P_d = \frac{abd\,\omega\varepsilon''|E_0|^2}{8}$ $\quad Q_d = \frac{2\omega_0 W_e}{P_d} = \frac{\varepsilon'}{\varepsilon''} = \frac{1}{\tan\delta}$

Total power loss

$P_{loss} = P_c + P_d$

total Q factor

$$Q = \left(\frac{1}{Q_c} + \frac{1}{Q_D}\right)^{-1}$$

21- Dividers & Couplers

•) Basic properties

- 3 ports → T-junction
- 4 ports → directional couplers or hybrids
- We cannot have a reciprocal, lossless and matched T-junction

Matched: $S_{ii} = 0$

Reciprocal: $[S] = [S]^t$, $[S]$ symmetric

Lossless: $[S]$ unitary : sum columns rows real part = 1
multiplication imaginary

$$[S]^t [S]^* = [U]$$

$$\sum_{k=1}^{N} S_{ki} S_{kj}^* = \delta_{ij}$$

if $i = j \rightarrow \delta_{ij} = 1$

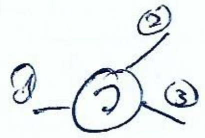
if $i \neq j \rightarrow \delta_{ij} = 0$

•) Circulator

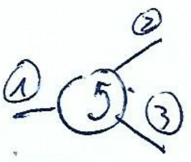
$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

Lossless, non-reciprocal, matched:

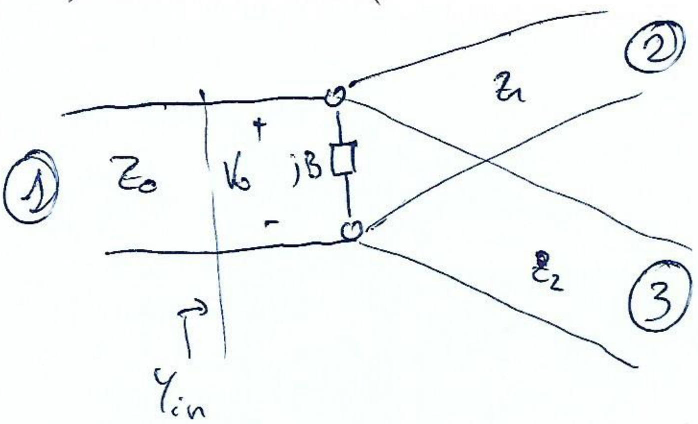
$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



∘) Lossless power divider



$$Y_{in} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

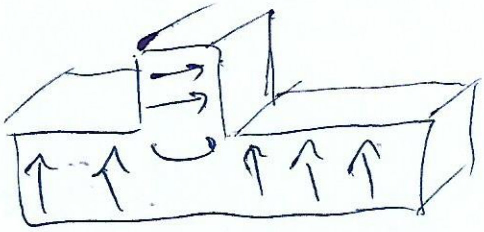
∘) Resistive power divider

- Lossy
- Reciprocal
- matched

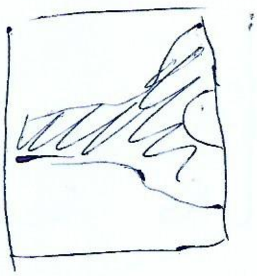
* Example (to it)

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

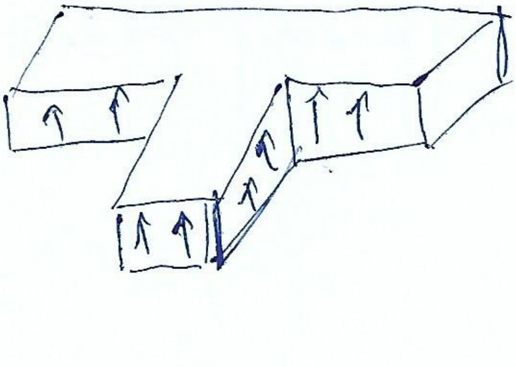
∘) T-junction implementation



E plane WG



Microstrip junction



H plane WG

◦) Four Ports Networks – Directional couplers

Reciprocal
Matched
Lossless

$$[S] = \begin{bmatrix} 0 & \alpha & \beta e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta e^{j\phi} \\ \beta e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta e^{j\phi} & \alpha & 0 \end{bmatrix}$$

$\alpha, \beta \in \mathbb{R}$

θ, ϕ constants

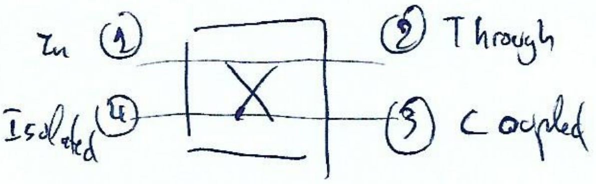
$\theta + \phi = \pi \pm 2n\pi$

$|\alpha|^2 + |\beta|^2 = 1$

→ Symmetrical: $\theta = \phi = \frac{\pi}{2}$

→ Antisymmetrical $\theta = 0$, $\phi = \pi$

→ How a directional coupler works



Coupling factor: $|S_{13}|^2 = \beta^2$

Port 2: $|S_{12}|^2 = \alpha^2 = 1 - \beta^2$

No power at port 4

$$C = 10 \log \frac{P_1}{P_3} = -20 \log \overset{S_{31}}{\beta} \quad \text{(coupling)}$$

$$D = 10 \log \frac{P_3}{P_4} = 20 \log \frac{\beta}{|S_{14}|} \quad \text{(Directivity)}$$

$I = D + C$

$$I = 10 \log \frac{P_1}{P_4} = -20 \log \underset{s}{|S_{14}|} \quad \text{(Isolation)}$$

Ideal coupler

$D = I = \infty$

Special couplers

Hybrid coupler

① Quadrature hybrid (90° phase shift between P2 and P3 when input at P1

② Magic T-hybrid

○) Wilkinson power divider

* Odd & Even mode analysis

* S parameters

lossy only if P2 and P3 are matched

○) Bethe hole coupler (see picture)

○) Multi hole coupler (see picture)

○) Coupled line directional coupler (see picture)

23-Filters

- Introduction

→ Responses: $\begin{cases} \text{Low-Pass} \\ \text{High-Pass} \\ \text{Band-Pass} \\ \text{Band-Reject} \end{cases}$

→ Composed of waveguide or transmission line periodically loaded with reactive elements

→ Model: Infinitely long periodic structure with small cells of d length and b susceptance

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \left[\frac{d}{2}\right][b]\left[\frac{d}{2}\right] = \underbrace{\begin{pmatrix} \cos\theta/2 & j\sin\theta/2 \\ j\sin\theta/2 & \cos\theta/2 \end{pmatrix}}_{d/2} \underbrace{\begin{pmatrix} 1 & 0 \\ jb & 1 \end{pmatrix}}_{\text{shunt impedance}} \begin{pmatrix} \cos\theta/2 & j\sin\theta/2 \\ j\sin\theta/2 & \cos\theta/2 \end{pmatrix}$$

$\theta = kd$

- $$\begin{bmatrix} A - e^{\gamma d} & B \\ C & D - e^{\gamma d} \end{bmatrix}$$

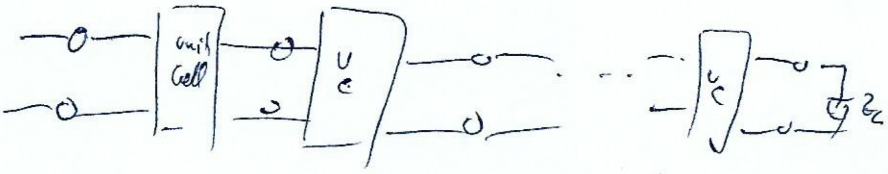
- if $\alpha = 0, \beta \neq 0$: not attenuated propagating waves : Pass-Band
- if $\alpha \neq 0, \beta = 0$: Attenuated non-propagating waves : Stop-Band

Bloch impedance:

$$Z_B^{\pm} = \frac{\pm B Z_0}{\sqrt{A^2 - 1}}$$

$\pm$ → indicate positive/negative travelling waves

$$\Gamma = \frac{Z_L - Z_B}{Z_L + Z_B}$$



c) Image parameter method

Summary

<u>Low-Pass</u>