

Non inverting amplifier

$$V^- = V_{out} \frac{R_1}{R_1 + R_2}$$

$$V_{out} = \frac{R_1 + R_2}{R_1} V_i$$

$$\text{if } V^+ = V^- \quad (V^+ = V_i)$$

$$\begin{cases} I_1 = \frac{V_i}{R_1} \\ V_{R2} = R_2 \cdot I_1 \end{cases}$$

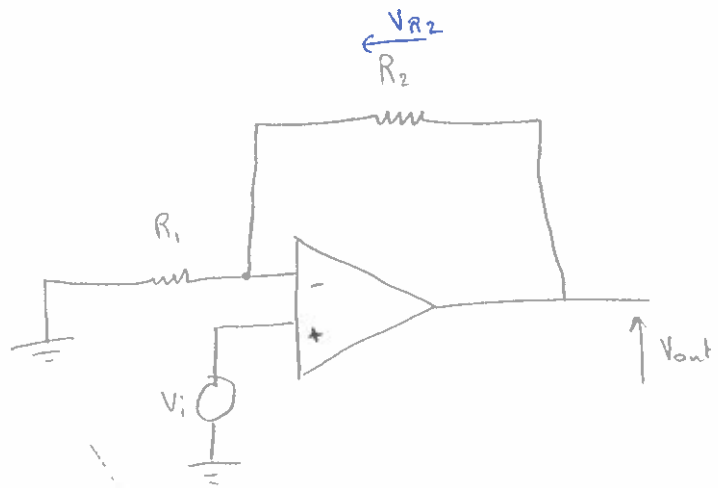
$$\rightarrow V_{out} = V_i + V_{R2}$$

$$= V_i + R_2 I_1$$

$$= V_i + R_2 \frac{V_i}{R_1}$$

$$V_{out} = V_i \left(1 + \frac{R_2}{R_1} \right)$$

$$\rightarrow G = 1 + \frac{R_2}{R_1}$$

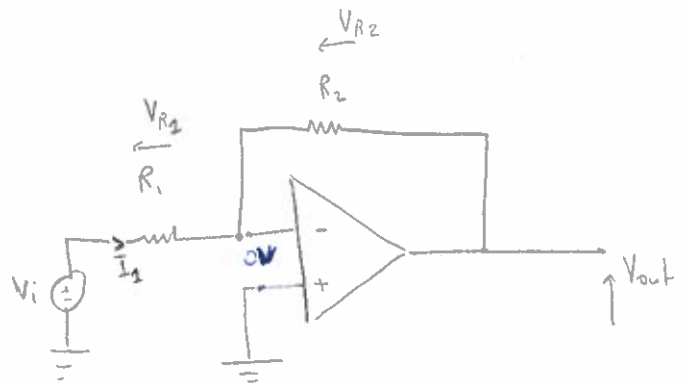


Inverting amplifier

assume $V^+ = V^-$ [when $G \rightarrow \infty$]

$$I_1 = \frac{V_i - 0}{R_1} = \frac{0 - V_{out}}{R_2}$$

$$V_{out} = - \frac{R_2}{R_1} V_{in}$$

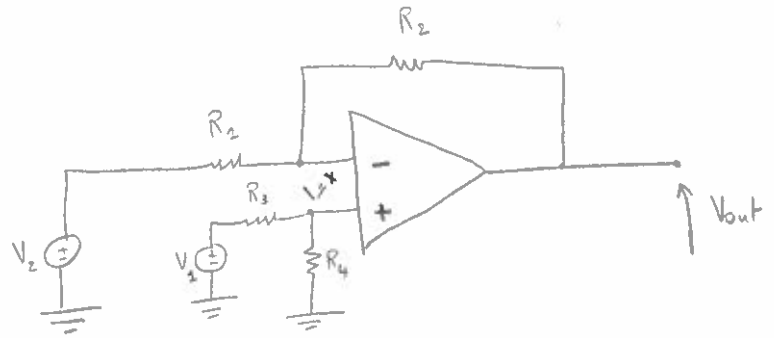


Difference amplifier

use of superposition principle

① $V_2 = 0$

$$V^+ = V_1 \frac{R_4}{R_3 + R_4}$$



$$V_{out}' = V^+ \cdot \left(1 + \frac{R_2}{R_2}\right) \quad [\text{non-inverting amplifier}]$$

$$V_{out}' = V_1 \frac{R_4}{R_3 + R_4} \left(1 + \frac{R_2}{R_2}\right)$$

② $V_1 = 0$ (similar to inverter amplifier)

$$V_{out}'' = - \frac{R_2}{R_1} V_2$$

$$\begin{aligned} \Rightarrow V_{out} &= V_{out}' + V_{out}'' \\ &= V_1 \underbrace{\frac{R_4}{R_3 + R_4} \left(1 + \frac{R_2}{R_2}\right)}_{G^+} - \underbrace{\frac{R_2}{R_1} V_2}_{G^-} \end{aligned}$$

$$G = G^+ = G^- \rightarrow V_{out} = G (V_1 - V_2)$$

↓

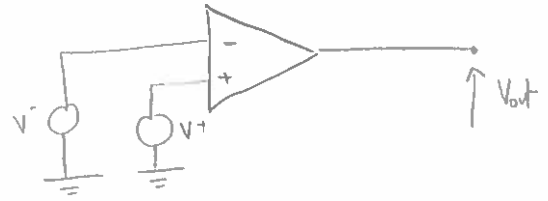
$$\begin{cases} R_2 = R_4 \\ R_1 = R_3 \end{cases}$$

Common Mode Rejection Ratio

$$V_{out} = G^+ V^+ - G^- V^-$$

$$\begin{cases} G_d = \frac{G^+ + G^-}{2} \\ G_c = G^+ + G^- \end{cases}$$

$$CMRR = \frac{G_d}{G_c}$$

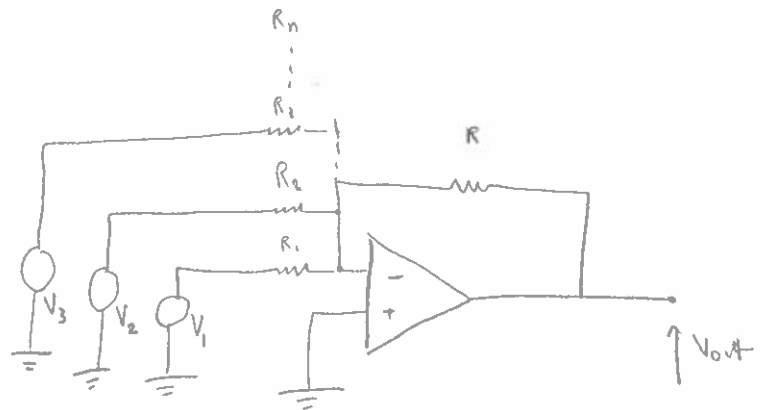


Summing amplifier

assume

$$R_1 = R_2 = R_3 = \dots = R_n$$

$$V_{out} = -R \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots + \frac{V_n}{R_n} \right)$$



$$V_{out} = -\frac{R}{R_1} (V_1 + V_2 + \dots + V_n)$$

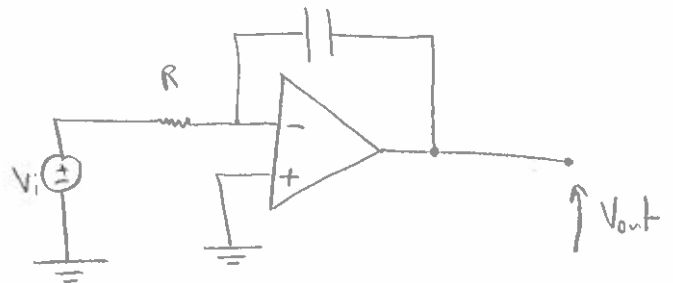
Integrator

↳ similar to inverter amplifier

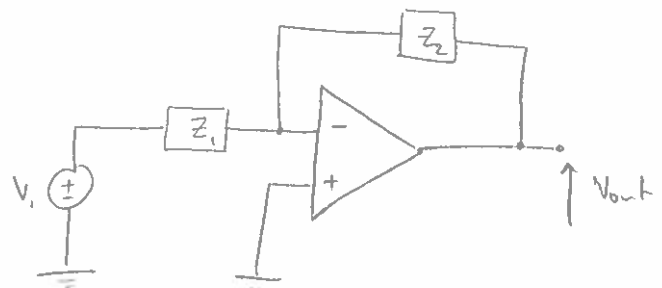
$$V_{out} = -\frac{Z_2}{Z_1} V_i$$

$$V_{out}(s) = -\frac{1}{sC \cdot R} V_i(s)$$

$$V_{out}(s) = -\frac{1}{s} \cdot \frac{1}{RC} V_i(s)$$



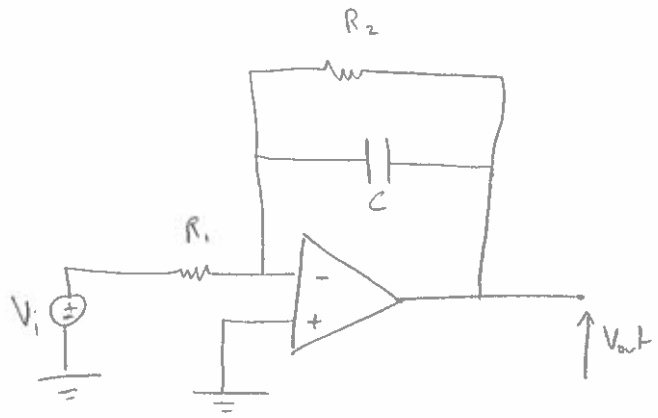
→ if there is a continuous component of the input → the system will saturate
 ↳ the solution is approximate integrator.



Approximate integrator

$$V_{out} = - \frac{Z_{eq}}{Z_1} \cdot V_i$$

$$Z_{eq} = \frac{Z_2 Z_3}{Z_2 + Z_3} \quad \left/ \begin{array}{l} Z_1 = R_1 \\ Z_2 = R_2 \\ Z_3 = \frac{1}{sC} \end{array} \right.$$



$$V_{out}(s) = - \frac{1}{R_1} \cdot \frac{\frac{1}{sC} \cdot R_2}{\frac{1}{sC} + R_2} \cdot V_i(s)$$

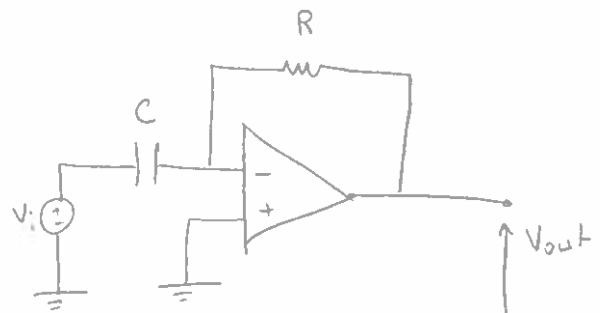
$$V_{out}(s) = - \frac{1}{R_1} \cdot \frac{R_2}{1 + s R_2 C} \cdot V_i(s)$$

Approximate differentiator

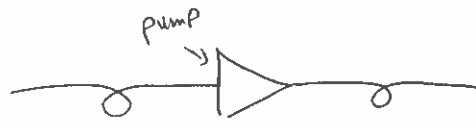
$$V_{out}^{(H)} = -RC \frac{dV_i(t)}{dt}$$

$$V_{out}(s) = - \frac{R}{1/sC} V_i(s)$$

$$V_{out}(s) = -sRC V_i(s)$$

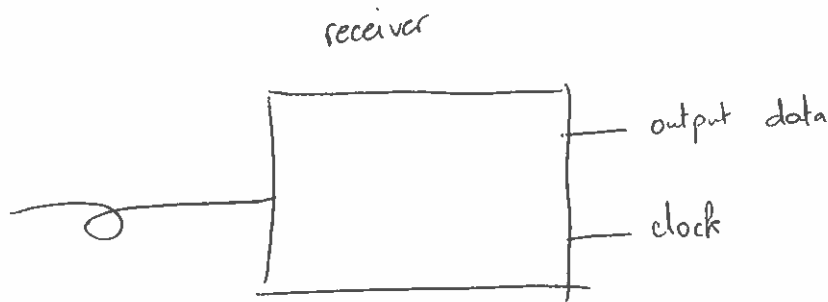


OA : to increase the power of the optical signal.

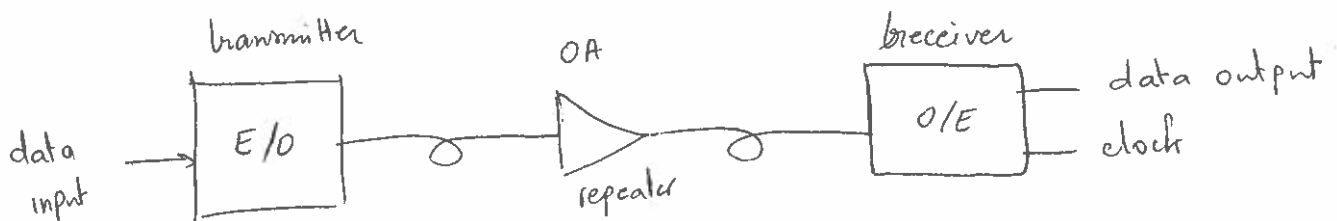


→ noisy component. → high output power.
→ worse SNR.

Repeater: optical signal → converted to electrical signal, it is reshaped
→ then converted back to optical signal.
→ SNR same as original signal from transmitter.

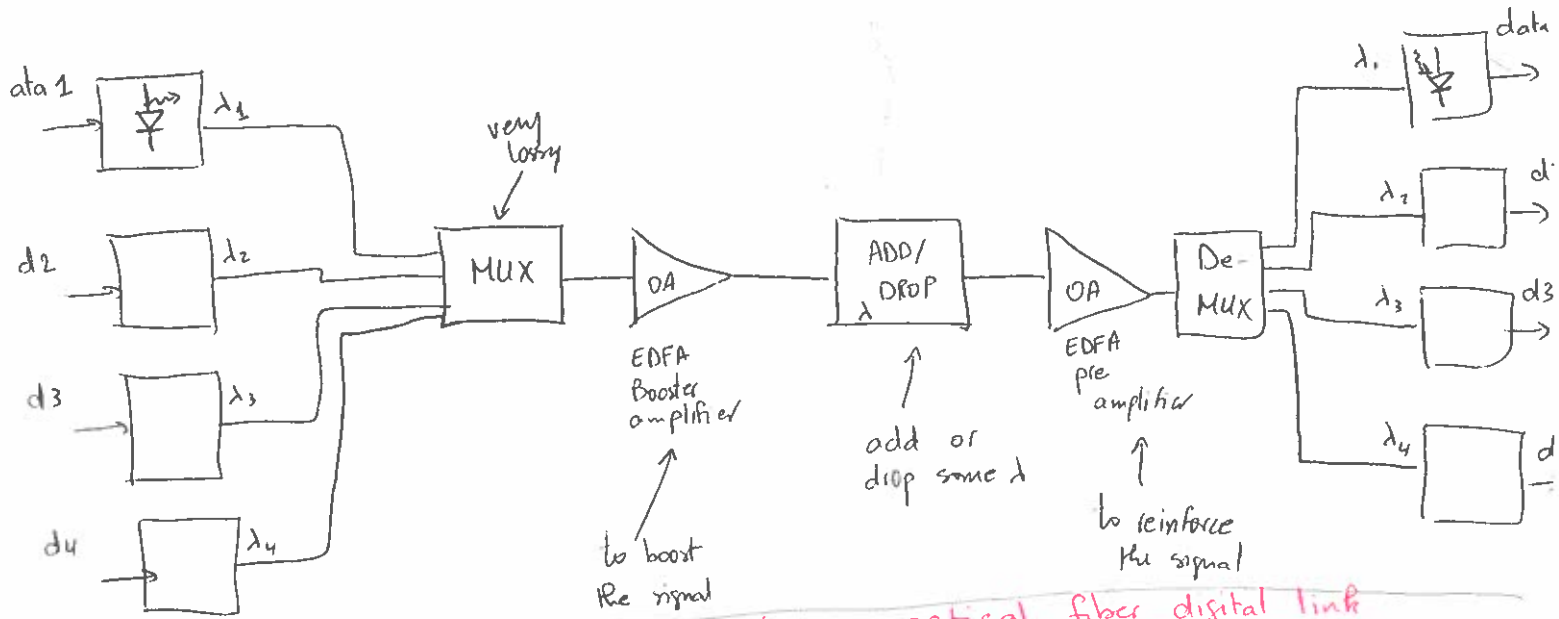


→ convert optical power → to electrical power: (based on photodetectors)



WDM

EDFA \rightarrow 1530 ~ 1565 nm

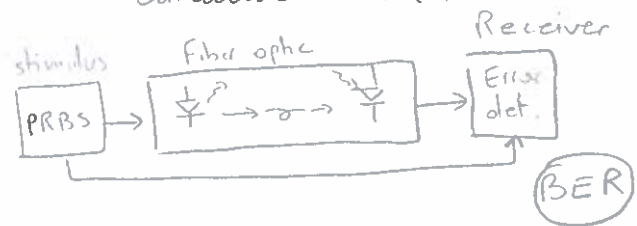


characterization of an optical fiber digital link

\rightarrow Send: pseudo Random Binary sequence: to compare with the output [receiver] and calculate BER.
length: $2^{23} - 1$

$$\text{BER} \approx 10^{-9} - 10^{-13}$$

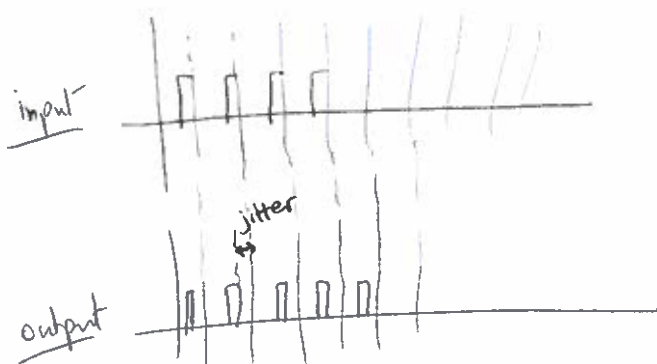
BER \propto losses of the fiber



\rightarrow Waveform analysis: compare the in and out signals to find where is the loss coming from. [check the form of the signal].

Clock jitter: how much the period of the clk is changing
tolerance: limit of max jitter \rightarrow guarantees that the receiver is working as expected

and: BER $< 10^{-13}$



Optical power \rightarrow ~~optical~~ power meter

optical power meters

	thermoelectric detectors	electronic photo-detectors
λ	low dependence	high dependence.
auto-calibration	YES	NO
sensitivity	low / $> 10 \text{ mW}$	high / $< 1 \text{ pW}$
uncertainty	1%	2%

idea: absorb the light.

thermopile
 \downarrow
measures ΔT

\rightarrow when light = off $\rightarrow \Delta T \rightarrow 0$.

heater: to increase T_1 and $\Delta T \nearrow$

\rightarrow thermal equilibrium $\rightarrow \Delta T$ stable.

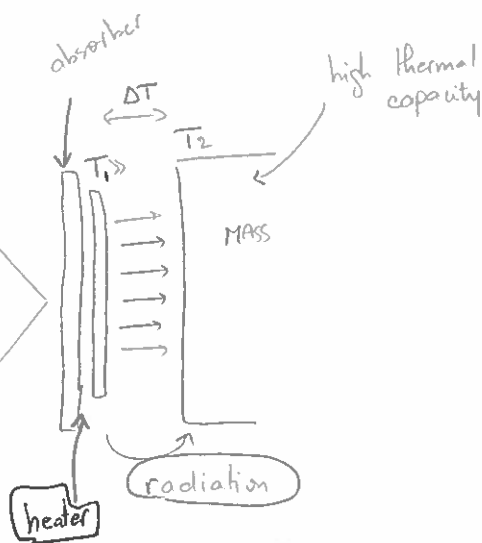
\rightarrow measure the P_{opt} AND $P_{\text{electrical}}$ and compare.

$$P_{\text{opt}} = P_e$$

\rightarrow so, we have

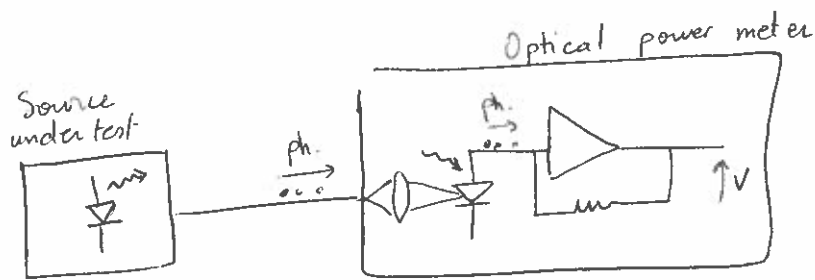
$$P_{\text{opt}}$$

$$P_e = R \cdot I^2$$



PIN Photo-detectors

- ↳ converts photons to electrons → generates current
- ↳ photons absorbed by "intrinsic" layer Ge, Si, InGaAs
- ↳ efficiency 90% ↳ fast response time.
- ↳ Avalanche Photo-diode (APD)
 - ↳ lower speed ↳ higher sensitivity.

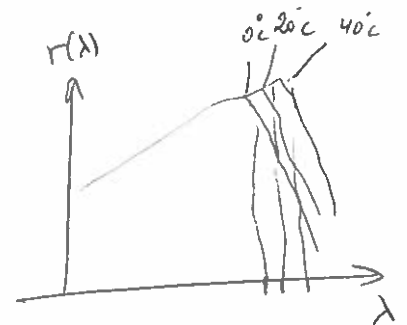


PIN photodiode

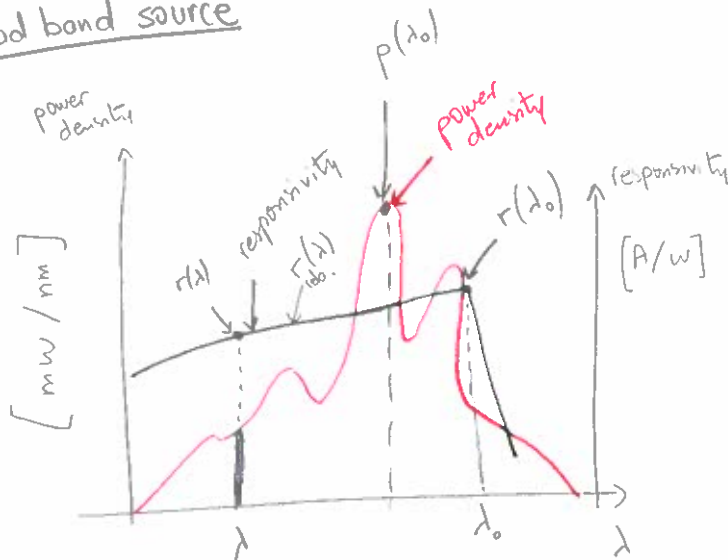
responsivity: how much current produced ~~per~~ ~~unit area~~ per unit of optical power.

$$r = \frac{I}{P_{opt}} = r(\lambda) \quad \left[\frac{A}{W} \right]$$

→ responsivity depends on the wave length.



Broad band source



→ The total photo-current

$$I_{Tot} = \int r(\lambda) \cdot p(\lambda) d\lambda$$

→ Relative responsivity:

$$r(\lambda) = r(\lambda_0) \cdot \underbrace{r_{rel} f(\lambda)}_{\text{relative resp.}}$$

→ relative power spectral density

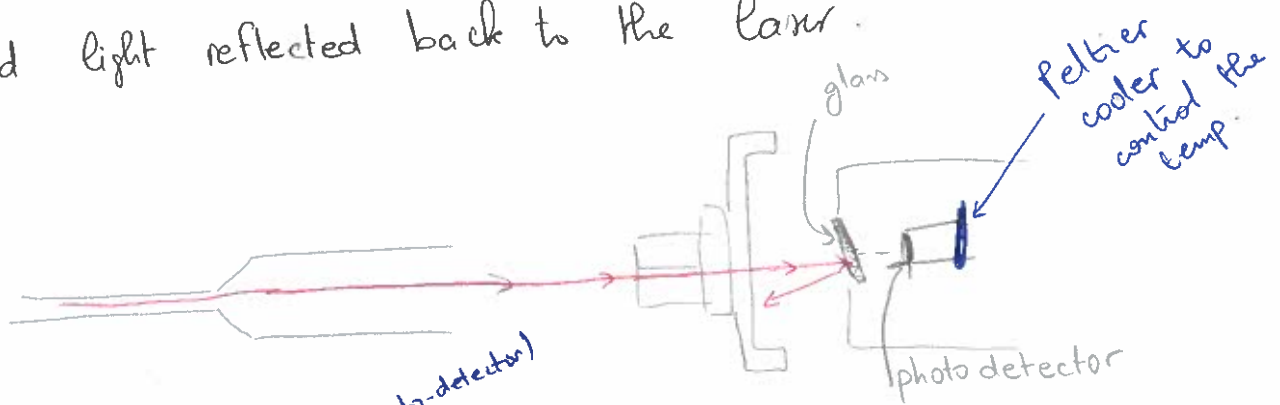
$$p(\lambda) = p(\lambda_0) \cdot \underbrace{f(\lambda)}_{\text{relative power density}}$$

$r(\lambda_0)$: sensitivity

$p(\lambda_0)$: absolute power of the light source

Power meters with photo-detectors

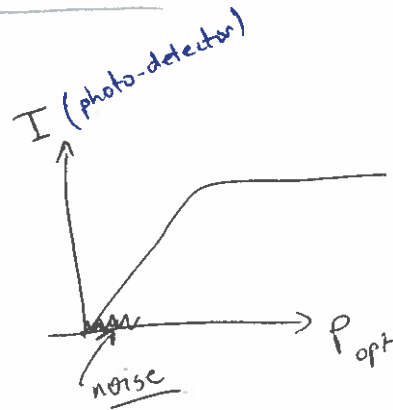
→ avoid light reflected back to the laser.



High dynamic range :
ratio of $\max I$ / I_{noise}

$$\left\{ \frac{I_{\max}}{I_{\text{noise}}} = \text{dynamic range.} \right.$$

$$\approx 10^{-5}$$



power noise → [Photo-detector] noise equivalent power (NEP)

$$NEP = \frac{1}{r} \sqrt{[i_n^2]} = \frac{1}{r} \sqrt{2eB_n (2I_{\text{dark}} + r \cdot P_{\text{opt}})}$$

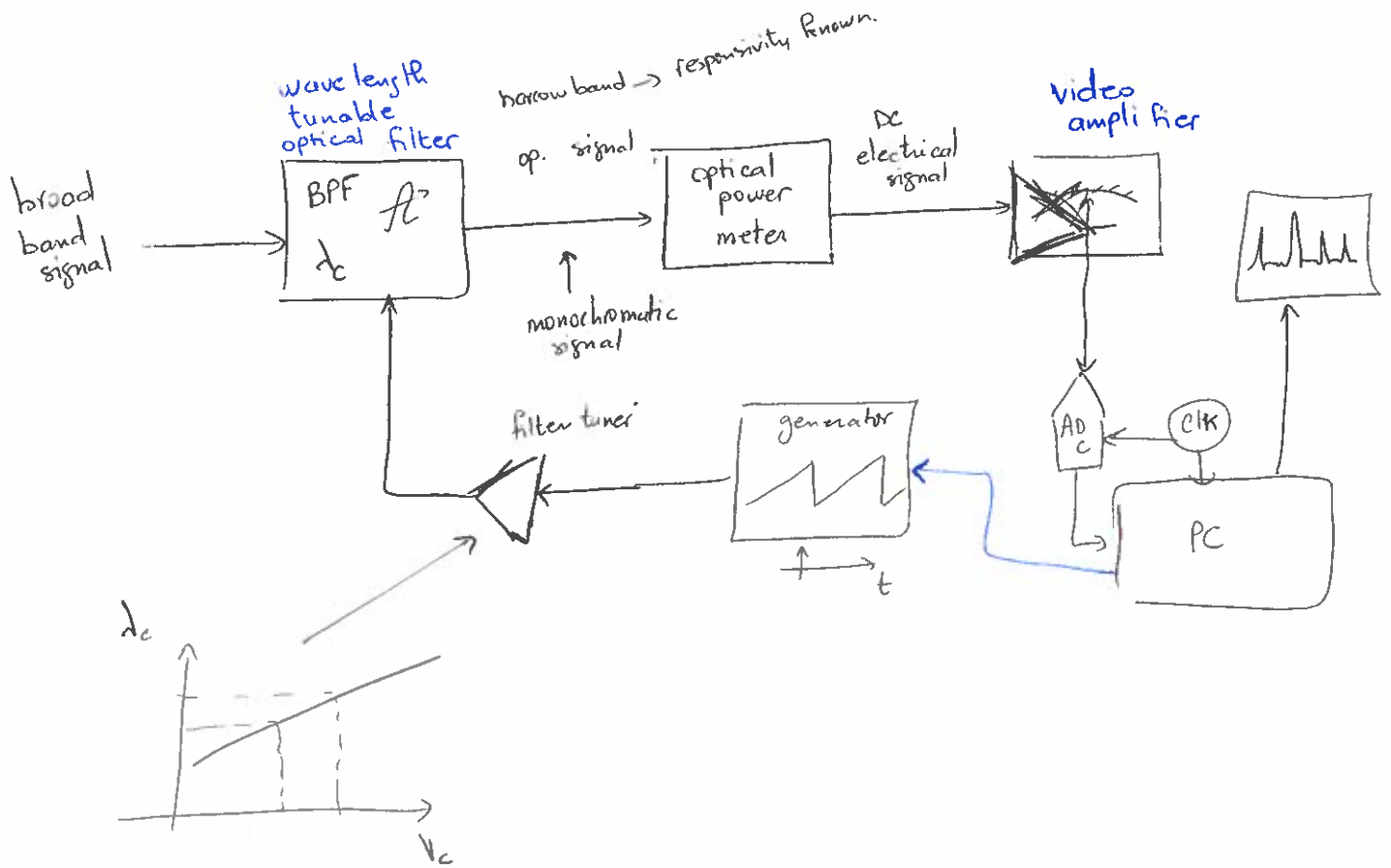
I_{dark} : dark current even in the output even when $i_n = 0$.

$$SNR = \frac{P_{\text{opt}}}{NEP}$$

I_{dark} : dark current is the current at the output ~~even~~ when we have no input optical power.

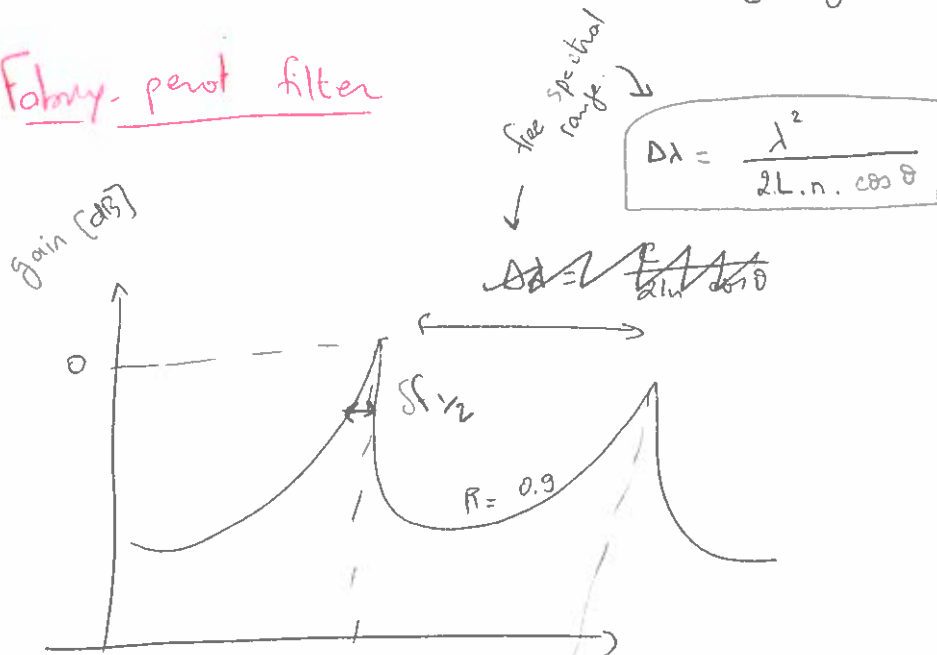
OSA

VBW: BW of the final stage of the processing chain.



related to the velocity of the scan [the generator].

Fabry-perot filter



Finesse $F \approx 10^3$
→ good F.P. filter

$$F = \frac{\Delta f}{\delta f_{1/2}}$$

resolution

- very high λ resolution
- narrow λ range due to the periodicity of Transmission function

Diffraction grating

$$\overset{\substack{\uparrow \\ \text{periodic}}}{n} \cdot \lambda = d (\sin \beta - \sin \alpha)$$

divergence angle: $D\beta_{\min} = \frac{\lambda}{N \cdot d \cdot \cos \beta}$

N : nbr of illuminated lines

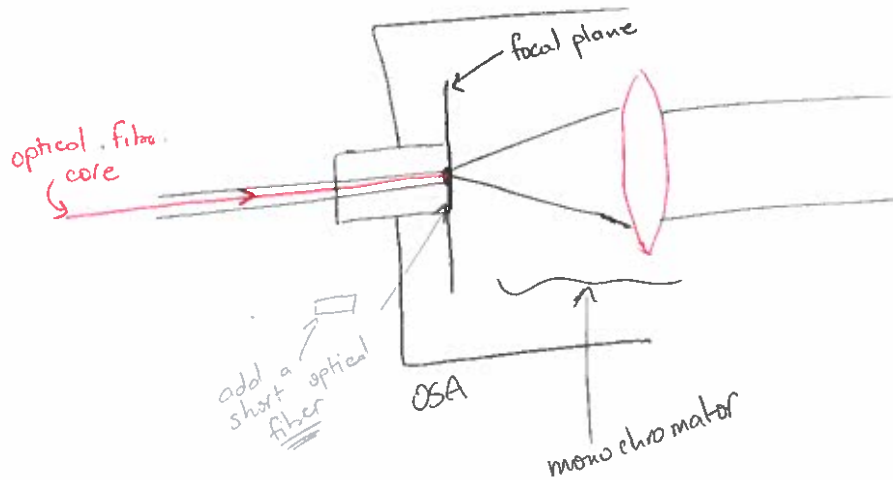
dispersion $D = \frac{D\beta}{d\lambda} = \frac{n}{d \cos \beta}$

best resolution: $\Delta\lambda_{\min} = \frac{\lambda}{N \cdot n}$

monochromator: input: many wavelengths
output: one single λ

monochromator + photo-detector \Rightarrow spectrometer

Input stage of OSA



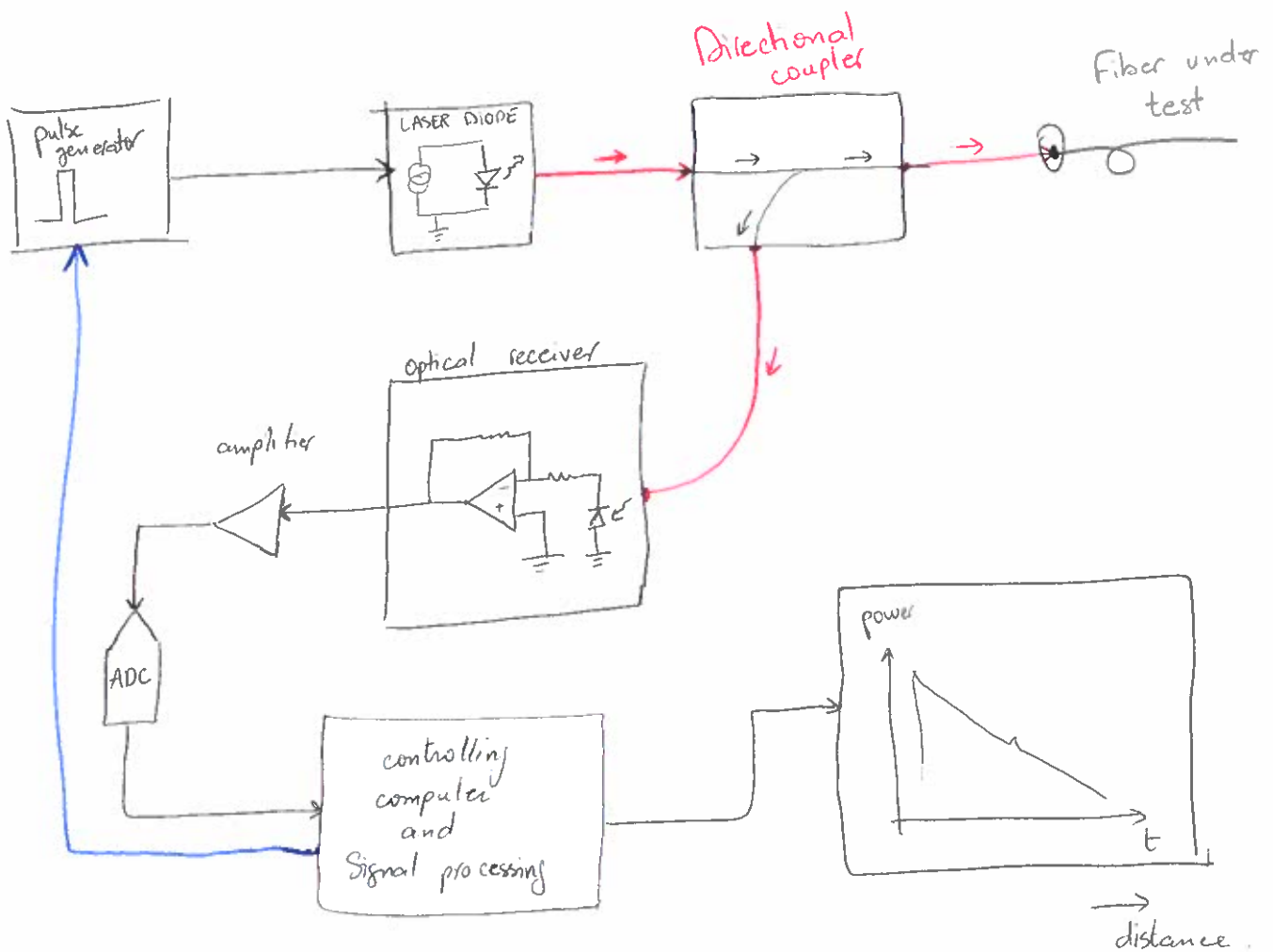
solution: add an optical fiber

- \rightarrow no insertion loss
- \rightarrow no damage of connector
- \rightarrow dirt can enter to the monochromator.
- \rightarrow low accuracy
- \rightarrow fiber/air interface causes a return loss = 14 dB.

- ↳ λ resolution of OSA: determined by input slit + diffraction grating + output slit
- ↳ BW of amplifier \rightarrow determines the sweep velocity AND sensitivity

↳ the Total loss determines the quality of the transmission system.

↳ OTDR: solution for monitoring installed optical fibers.



\rightarrow back propagation light: loss in fiber

↳ because of macro discontinuities: connectors / splices / bending...

↳ back scattering

$$P(z) = P_0 \cdot e^{-\alpha z}$$

α : attenuation coeff [km⁻¹]



$$\alpha_{[dB/km]} = 10 \log \left(\frac{P_0}{P(z)} \right) \cdot \frac{1}{z}$$

$$\alpha = \alpha_a + \alpha_s \leftarrow \text{scattering}$$

↑
absorbance

the scattered intensity

↓

$$\alpha_s \approx \frac{1}{\lambda^4}$$

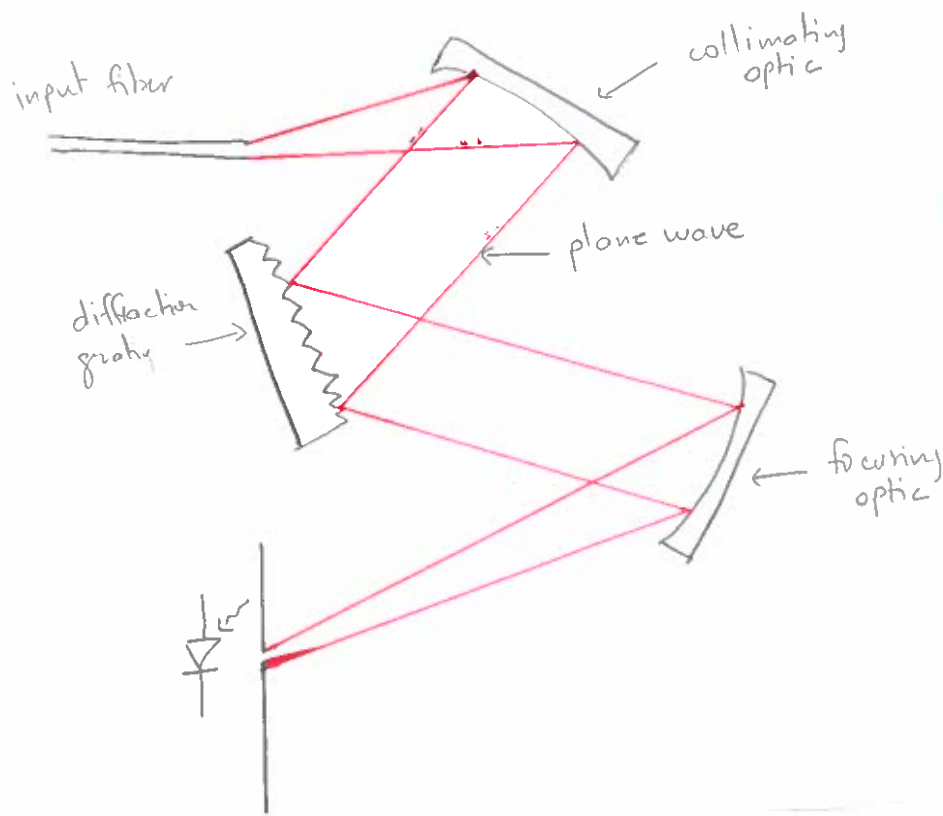
backscatter factor $S = 10 \log \frac{P_0}{P_s(0)} = -10 \log (k.w)$

↪ a parameter characterizing the fiber.

Sensitivity of the OSA: limiting factors:

- the power loss of the monochromator (3 → 8 dB)
- sensitivity of the detector
- BW of the photodetector - detector signal (Video BW)

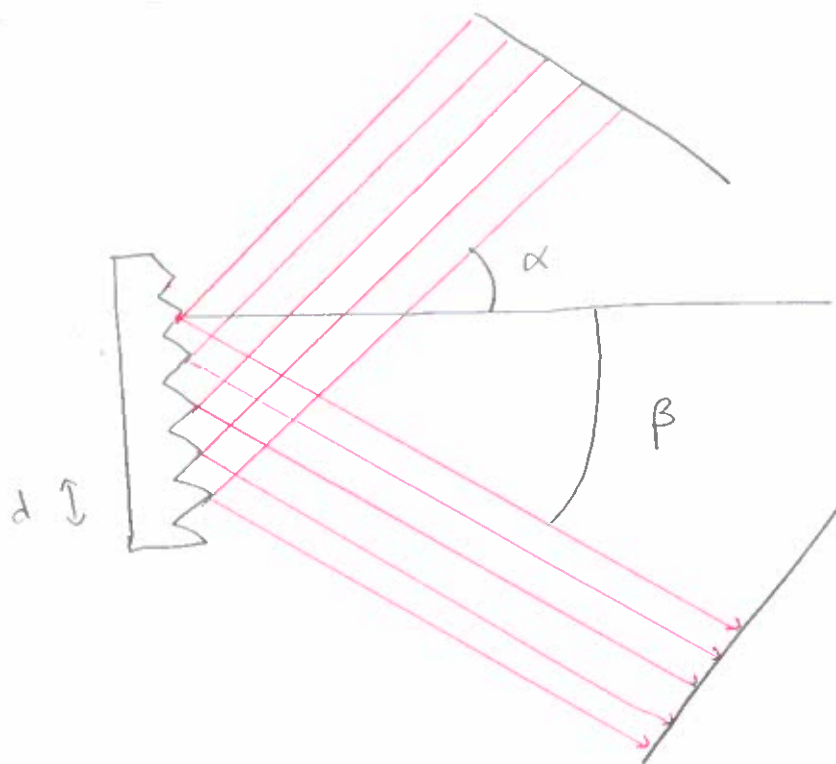
S: frac



Diffraction grating

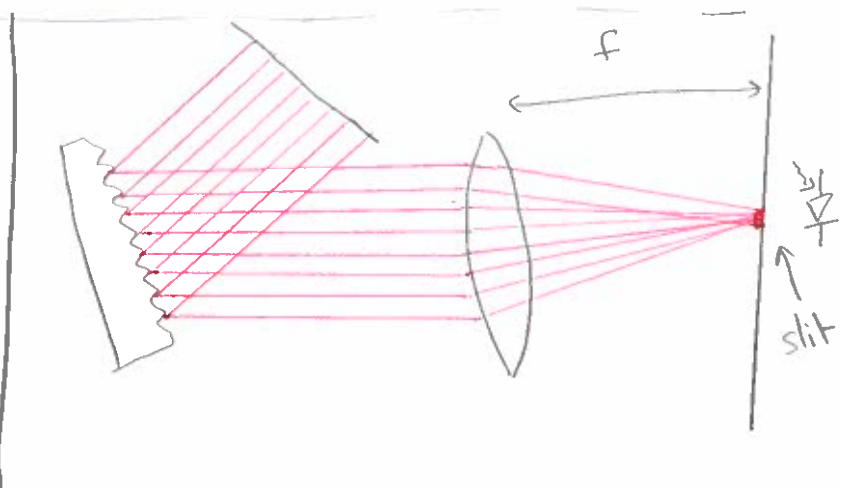
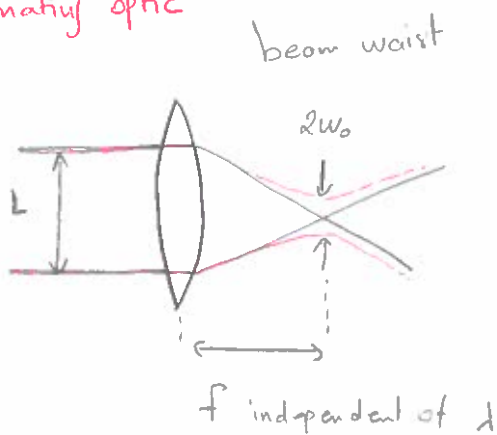
→ output & input slit + the diffraction grating → determine the resolution of the OSA.

→ BW of the amplifier determines the sweep velocity & the instrument sensitivity.



$$n\lambda = d (\sin \beta - \sin \alpha)$$

collimating optic



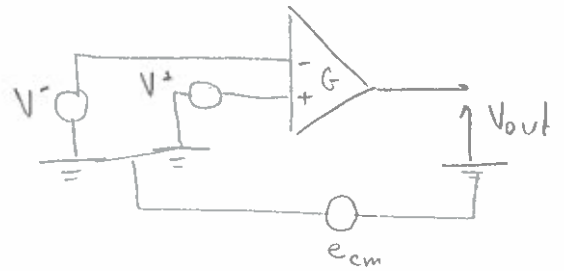
Difference amplifier

$$V_{out} = V_1 G^+ - V_2 G^-$$

→ Study of the unwanted source (e_{cm})

→ the difference amplifier will amplify $(V_1 - V_2)$ but will reject (e_{cm})

$$\begin{cases} G_d = \frac{G^+ + G^-}{2} \\ G_c = G^+ - G^- \end{cases}$$



$$\begin{cases} G^+ = G_d + \frac{G_c}{2} \\ G^- = G_d - \frac{G_c}{2} \end{cases}$$

→ we replace in V_{out}

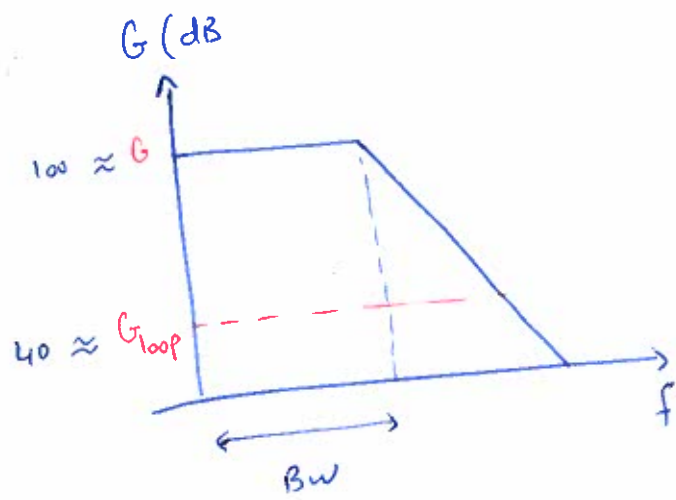
$$V_{out} = V_1 \left(G_d + \frac{G_c}{2} \right) - V_2 \left(G_d - \frac{G_c}{2} \right)$$

$$V_{out} = G_d (V_1 - V_2) + G_c \left(\frac{V_1 + V_2}{2} \right)$$

$$V_{out} = G_d (V_1 - V_2) + G_c (e_{cm})$$

$$CMRR = \frac{G_d}{G_c}$$

(Common Mode
Rejection Ratio)



es:

$$V_{out} = - \overbrace{\frac{R_2}{R_1}}^{G_{loop}} V_{in}$$

$$= -100 V_{in}$$

$$G_{loop} = 100 = 20 \log(100)$$

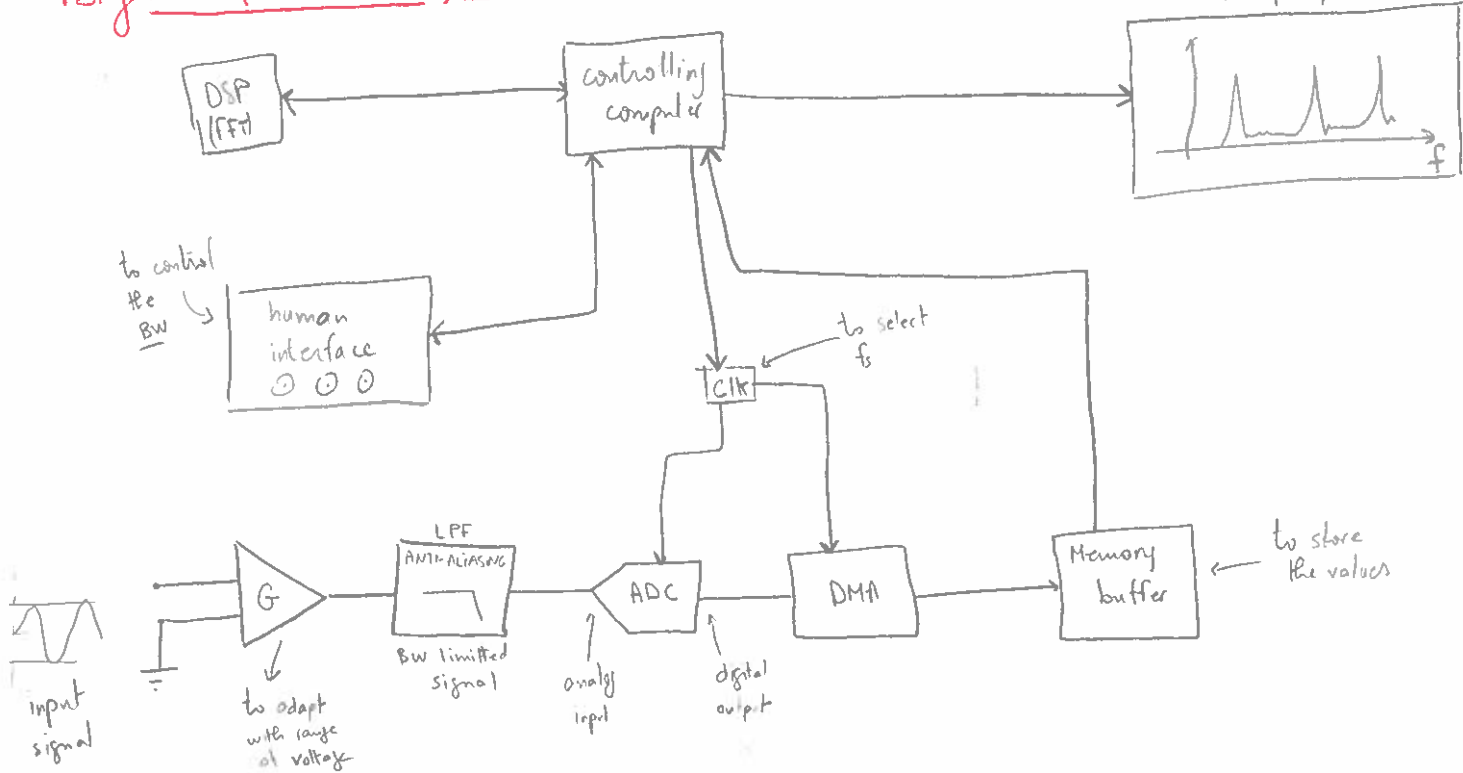
$$= 40 \text{ dB}$$

→ The signals used by the Network analyzer to stimulate the DUT during the measurement of its TF:

- ① Sinusoid signal: we have to iteratively set the ~~center~~ frequency of the sinusoid at the center freq. of every filter. we have to make as many measurements as there are bins.
- ② Broadband signal: consimultaneously produce energy in each of the FFT bins, which can be captured in one FFT measurement
- ③ Chirp sine signal: is a swept sine burst designed to fill the time record of the FFT analyzer. It has a relatively high average power → produces a better SNR compared to random noise.
- ④ Broadband random noise: has equal energy in all of the ~~frequency~~ ^{FFT} bins.
- ⑤ PRN signal: periodic ~~data~~ within the time record of the analyzer so that it does not produce leakage.
- ⑥ Random noise source: usefull with non-linear networks. The non-linear measurements can be averaged out since they produce a different response for each measurement

Digital (FFT based) spectrum analyzer

Display.



FFT : works in the Baseband $f = [0 \text{ Hz} \rightarrow \frac{f_s}{2}]$.

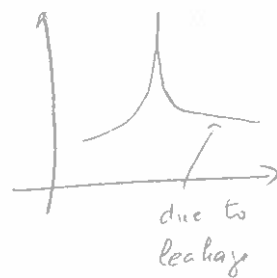
resolution : $\Delta f = \frac{f_s}{N}$ / $f_n = n \frac{f_s}{N}$ / $f_{\max} = \frac{f_s}{2}$

computation time : [number of computations = $N \log_2(N)$].

Leakage: discontinuity \rightarrow source of new frequency components.

\rightarrow use windows to force the ends of waveform = 0.

\rightarrow reduce leakage \rightarrow better resolution.



Mean : $\bar{x} = E(x) = \int_{-\infty}^{+\infty} x \cdot \text{PDF}(x) \cdot dx$.

Variance : $S^2 = E(x - \bar{x})^2 = \overline{x^2} - \bar{x}^2$

Noise power : when $R = 1 \Omega$ $P_{\text{noise}} = V^2 = I^2$

signal power $\rightarrow P_{12} = \int_{f_1}^{f_2} S_x(f) df$ (power spectral density)

Wiener - Khinchin Theorem

→ auto correlation function : $R_{xx}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t) x(t+\tau) dt$

→ power spectral density $S_x(f) = 2 \int_{-\infty}^{+\infty} R_{xx}(\tau) e^{-j2\pi f\tau} d\tau$

→ Noise power for ideal filter

$$P_n = N_0 \cdot G_0 \cdot BW_N$$

↑
input PSD

Noise Equivalent BW

$$BW_N = \frac{1}{G_0} \int_0^{\infty} G(f) df$$

power gain of ideal filter power gain of the filter

Heterodyne (swept) spectrum analyzer [analog heterodyne]

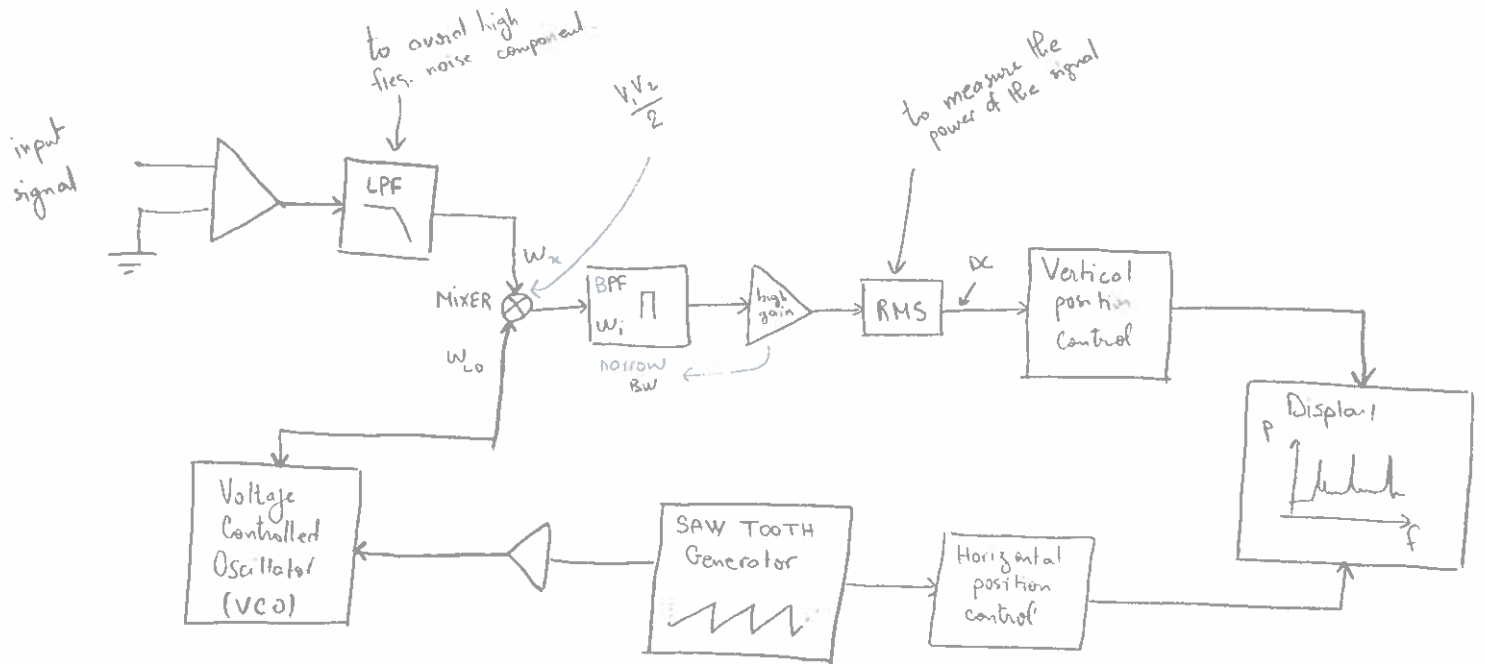
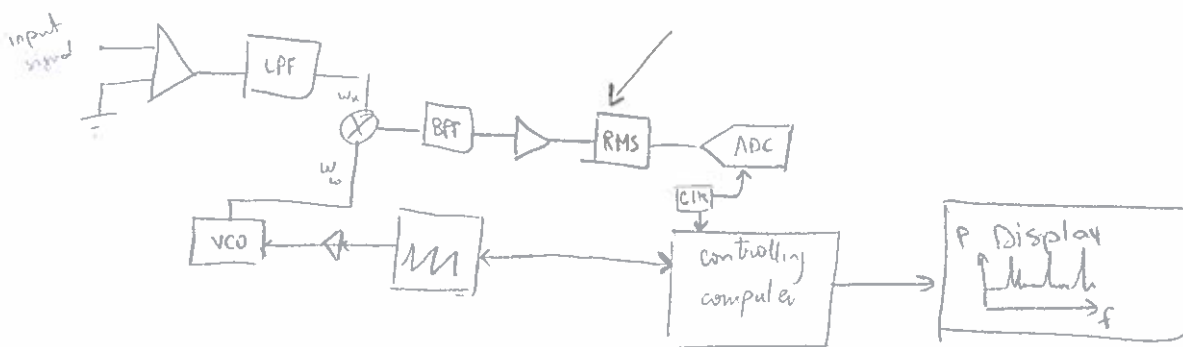
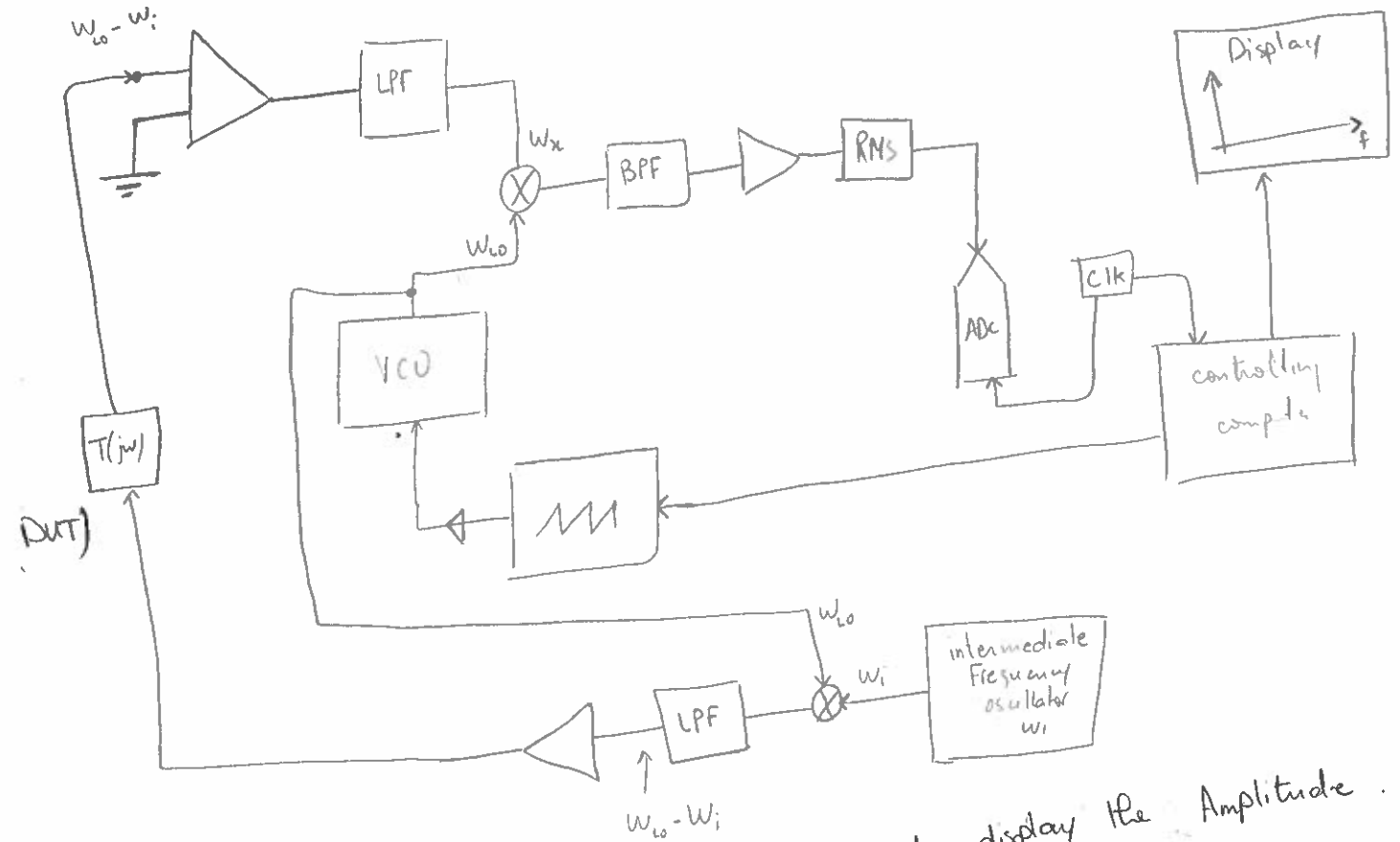


image frequency : high freq. component that should be taken away using a LPF.

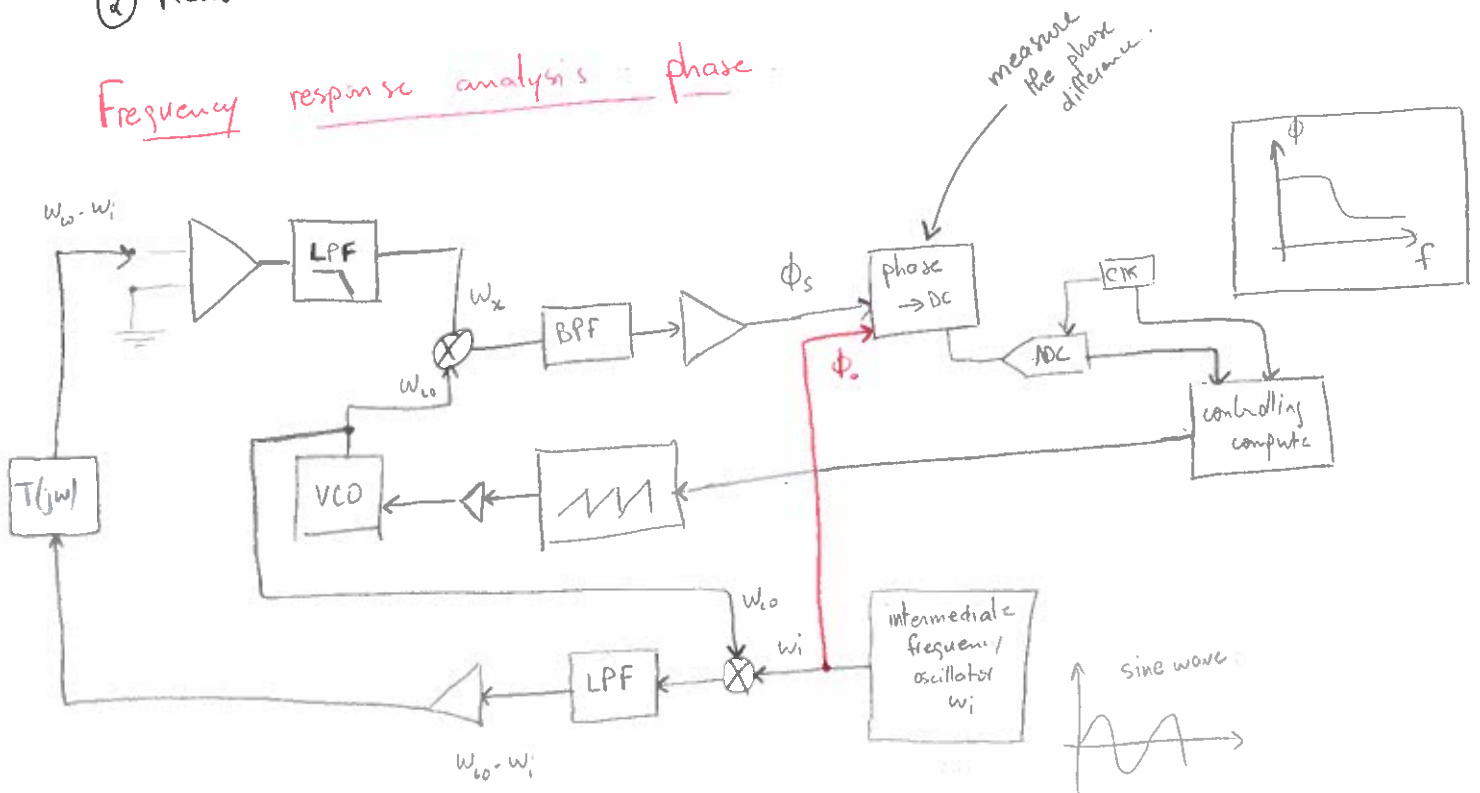


Frequency response analysis : Amplitude

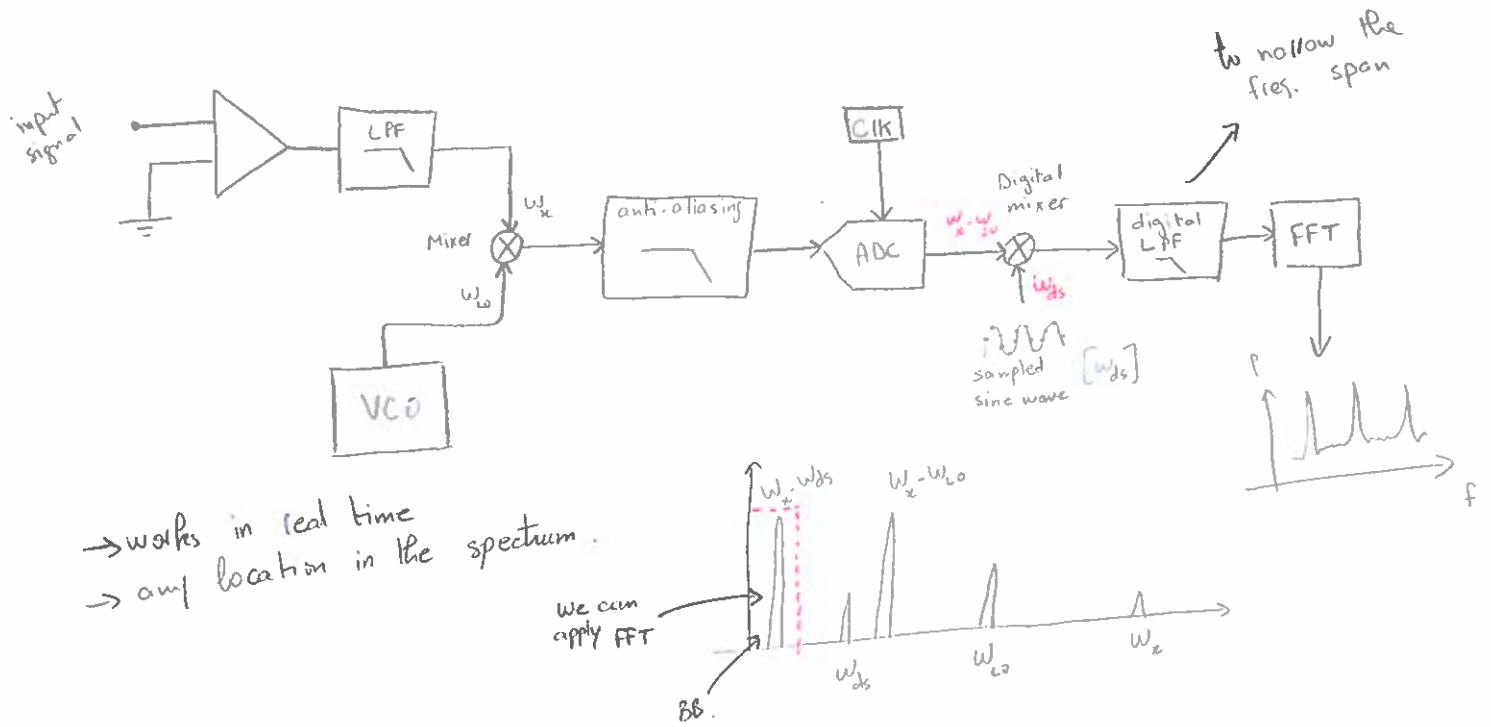


- ① Calibration: we take off the DUT and display the Amplitude.
- ② Measurement: add the DUT and measure the displayed Amplitude.

Frequency response analysis : phase



Real time spectrum analyzer : combination of heterodyne and FFT.



- works in real time
- any location in the spectrum

Distortion : [single tone input]. $V_{in} = A \cos(\omega t)$.

- reduce the input amplitude A by 1 dB
- reduce of 1 dB the fundamental harmonic.
- " " 2 dB " 2nd " "
- " " 3 dB " 3rd " "

(intermodulation) distortion [two tone input] $V_{in} = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)$

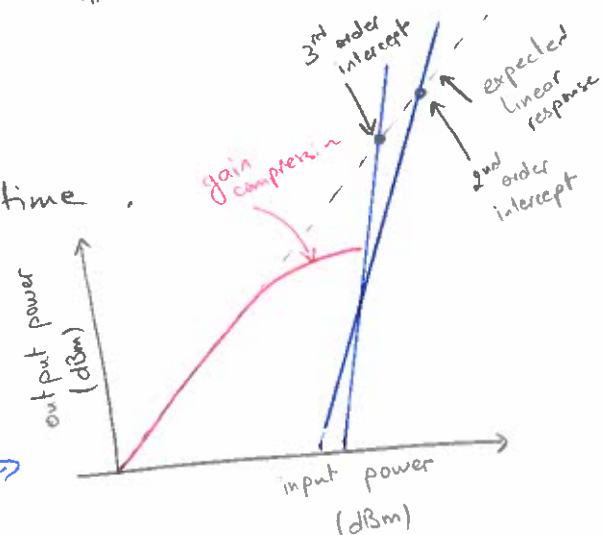
→ the frequencies in the output : $\omega_{nm} = |n\omega_1 \pm m\omega_2|$

order of component = $n+m$ → f_{21} → 3rd order comp.
 f_{11} → 2nd " " "

→ To increase the sensitivity :

- ↳ improve the SNR
- ↳ reduce the VBW.
- ↳ the measurement will take more time.

The intercept concept

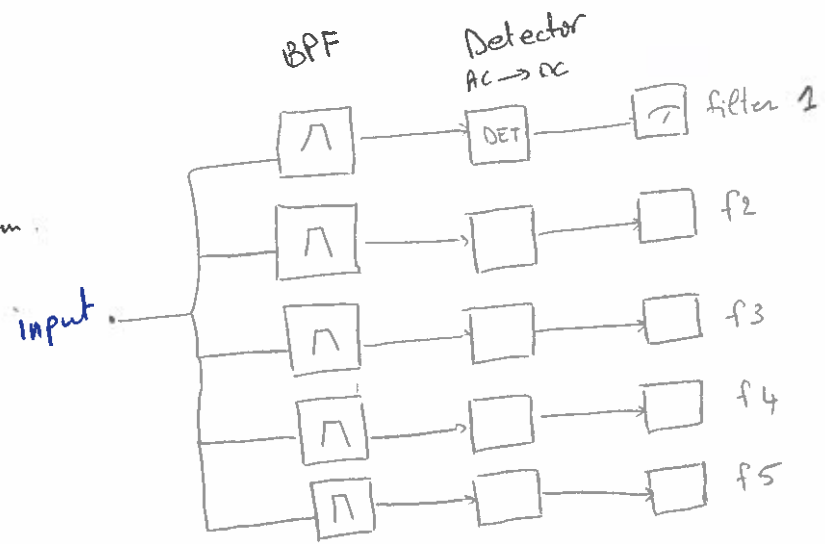


Bank of filters

→ simple.

→ fast

→ ideal time measurement system



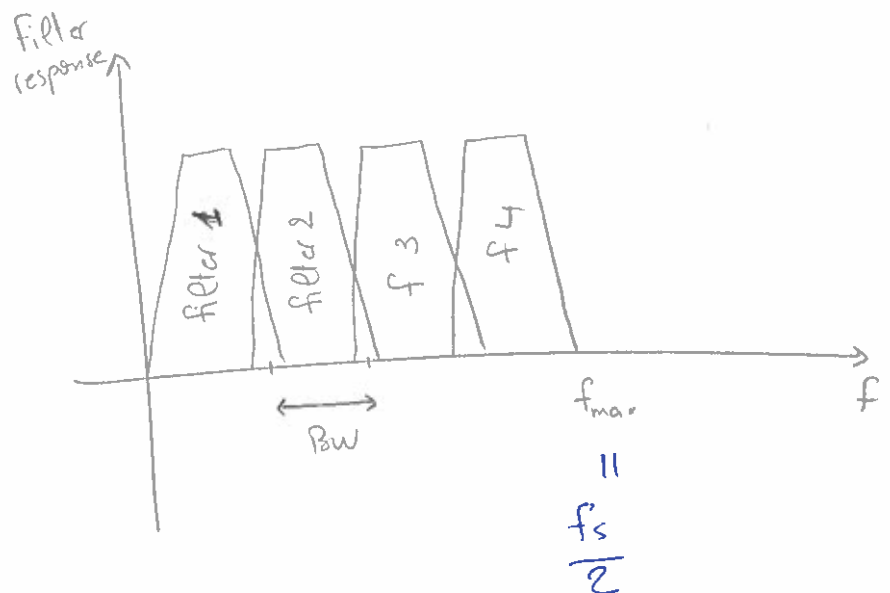
To implement a spectrum analyzer, we connect a bank of electronic filters together, each with its own output.

Each filter is a Band pass filter tuned at a different center frequency. The Bandwidth and center frequencies are aligned to cover the entire range of frequency of interest with minimal overlap of filter shape.

The output of the filters are connected to detectors that convert the AC signal to DC level, then displayed by a meter.

$$\Delta f = \frac{f_s}{N}$$

↑
resolution



→ Noise level of the analyzer depends on the frequency resolution of the measurement.

OTDR

- provide info about attenuation & loss of the fiber
- by exploiting backscattered light from fiber.
- we use "directional coupler" to prevent the laser signal from saturating the receiver.

→ $\approx 10 \mu\text{s} / \text{km}$.

→ Dynamic range: the difference between the initial backscattered level & the noise level after 3 min of measurement time.

→ High spatial resolution → short pulse width → wide receiver BW. \Rightarrow leads to a reduced SNR

Long pulse wide → low noise \Rightarrow improve sensitivity but bad spatial resolution

→ S: the fraction of light scattered

$$S = \left(\frac{NA}{n_0}\right)^2 \cdot \frac{1}{m}$$

