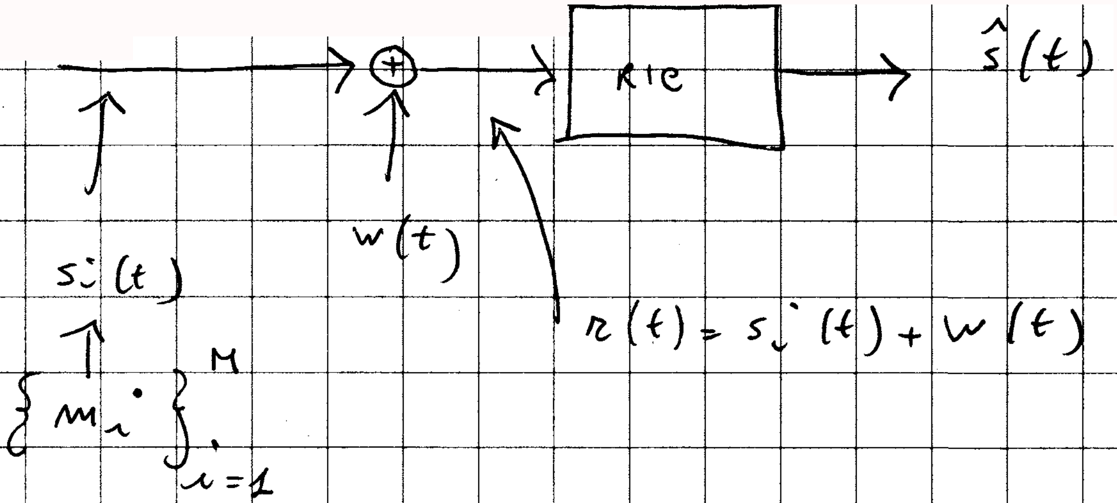
Principles of digital Communications in presence of AWGN



R/e: min $\boxed{}$ $P(\varepsilon)$ → The receiver choses the message which maximize the "a posteriori" probabilty

BAYES: $P(A_i/B) = \dfrac{P(B/A_i)\,P(A_i)}{P(B)}$

$$P(A_i/X) = \frac{f_X(x/A_i)\,P(A_i)}{f_X(x)}$$

$f_X(x)$ → It is independent from "i"

$A_i \to s_i(t)$; $B \to r(t) = s_i(t) + n(t)$

Infinit dimensions (n(t))

($n(t)$ è) Se AWGN, si ha:

$$E[r_k / s_i] = \begin{cases} s_{ik}; & \boxed{k} \le N \\ 0; & \boxed{k} > N \end{cases}$$

N is the signal space dimension.
rk is the coordinate of r in the base function k

Considering n components …

$$f\left(r_1, r_2, \dots, r_n \,/\, s_i\right) =$$

$$= \prod_{K=1}^{N} \left(\frac{1}{\sqrt{2\pi \frac{N_0}{2}}} \exp\left[- \frac{(r_K - s_{iK})^2}{2\frac{N_0}{2}} \right] \right) \cdot$$

$$\cdot \left(\prod_{K=N+1}^{n} \frac{1}{\sqrt{2\pi\, \sigma_K^2}} \exp\left[- \frac{r_K^2}{2\sigma_K^2} \right] \right) =$$

We are looking for the maximum with respect to "i" … therefore the terms that are independent of "i" are not relevant … (irrelevant)

We can say that the component of the received signal in the signal space represent a "sufficient statistic" for the optimal detection … therefore the component of the received signal out of the signal space are "irrelevant" (beeing orthogonal and therefore (beeing gaussian) statistically independent) …

$$f(r/s_i)$$

is proportional to ...

$$f(r/s_i) \equiv \exp\left[-\frac{1}{N_0}\sum_{k=1}^{N}(r_k - s_{ik})^2\right] =$$

$$= \exp\left[-\frac{1}{N_0}\|r - s_i\|^2\right]$$

square distances between r(t) and si(t), in the signal vector space (N domensional)

$$\|r - s_i\|^2 = \int (r(t) - s_i(t))^2\, dt =$$

$$= \sum_{k=1}^{\infty}(r_k - s_{ik})^2 = \sum_{k=1}^{N}(r_k - s_{ik})^2 + \sum_{k=N+1}^{\infty} r_k^2$$

$$f(r/s_i)\, P(s_i) \equiv \exp\left[-\frac{1}{N_0}\|r - s_i\|^2\right] P(s_i)$$

PROB. a PRIORI ...

Likelihood function ...

Using log() ...

$$\underbrace{\|r - s_i\|^2}_{} - N_0 \log P(s_i)$$

$$\underbrace{\text{ML}}_{} \quad \text{MAP}$$

$\underbrace{}_{\text{MAP :}}$ ~~richiede~~: $\frac{N_0}{2}$, $P(s_i)$!!!

ML : richiede solo la DISTANZA minima ....

I can predefine some "decision regions", associated to each possible transmitted signal ...

In generale :

$$\|r - s_i\|^2 = \|r\|^2 + \|s_i\|^2 - 2\langle r, s_i\rangle$$

$$\langle r, s_i\rangle - \frac{1}{2}\|s_i\|^2$$

we have ro evaluate the maximum

$$\langle r, s_i\rangle = \sum_{k=1}^{N} r_k s_{ik}$$

$$r_k = \int r(t)\, \phi_k(t)\, dt$$

$$\langle r, s_i\rangle = \int r(t)\, s_i(t)\, dt$$

in case of band-pass signals ... $\longrightarrow$ $r_k = \frac{1}{2}\int z(t)\, z_k^*(t)\, dt$ ...

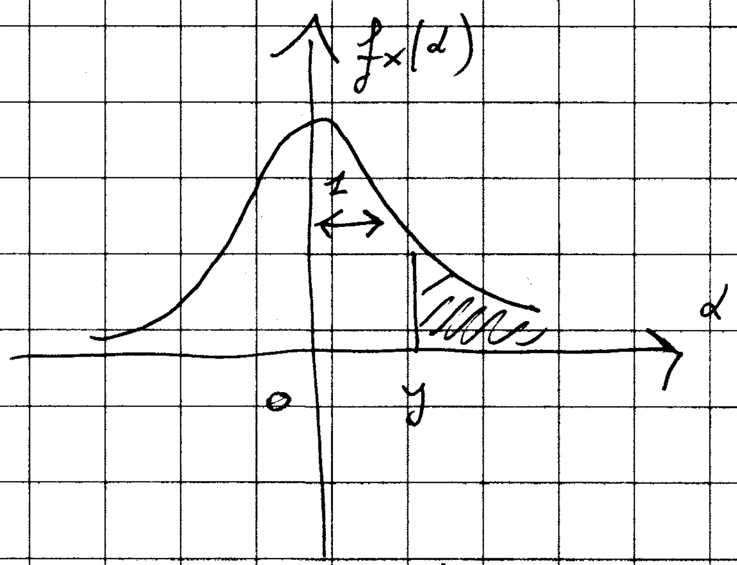
Error probabilities

$$P(E) = \sum_{i=1}^{M} P(s_i)\, P(E/s_i) = \sum_{i=1}^{M} P(s_i) \sum_{j \neq i} P(s_j/s_i)$$

Tr. Binaria

$$P(s_2/s_1) = P(s_1/s_2) = P(E) = Q\left(\frac{d/2}{\sqrt{N_0/2}}\right) = Q\left(\frac{d}{\sqrt{2N_0}}\right)$$

$$Q(y) = \int_y^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$$



Se $s_2 = -s_1$

$$P(E) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$P(E)$	10^{-3}	10^{-5}	10^{-7}	10^{-10}	10^{-13}
E_b/N_0 [dB]	6,79	9,59	11,31	13,06	14,31

$$\left(Q(y) \approx \frac{1}{\sqrt{2\pi}\, y} e^{-y^2/2} \qquad y > 3 \right)$$

$$\log_{10} Q(\gamma) \approx -0{,}22\,\gamma^2 - 1{,}04$$

$$P_b(E) = \frac{1}{\log_2 M} \sum_{1}^{M} {}_i\, P(s_i) \sum_{j \neq i} n_{ij}\, P(s_j / s_i)$$

number of bits differing between the bit mapping associated to si and sj

$$\frac{P(E)}{\log_2 M} \leq P_b(E) \leq P(E)$$

P errore sul bit

P(E) sul SIMBOLO

UNION BOUND

$$P(E) = \sum_{1}^{M} {}_i\, P(s_i) \sum_{j \neq i} P(s_j / s_i) \leq \sum_{i=1}^{M} P(s_i) \sum_{j \neq i} Q\left(\frac{d_{ij}}{\sqrt{2N_0}}\right)$$

$$P_b(E) = \frac{1}{\log_2 M} \sum_{1}^{M} {}_i\, P(s_i) \sum_{j \neq i} n_{ij}\, P(s_j / s_i) \leq (\cdot)$$

$$\leq \frac{1}{\log_2 M} \sum_{1}^{M} {}_i\, P(s_i) \sum_{j \neq i} n_{ij}\, Q\left(\frac{d_{ij}}{\sqrt{2N_0}}\right)$$