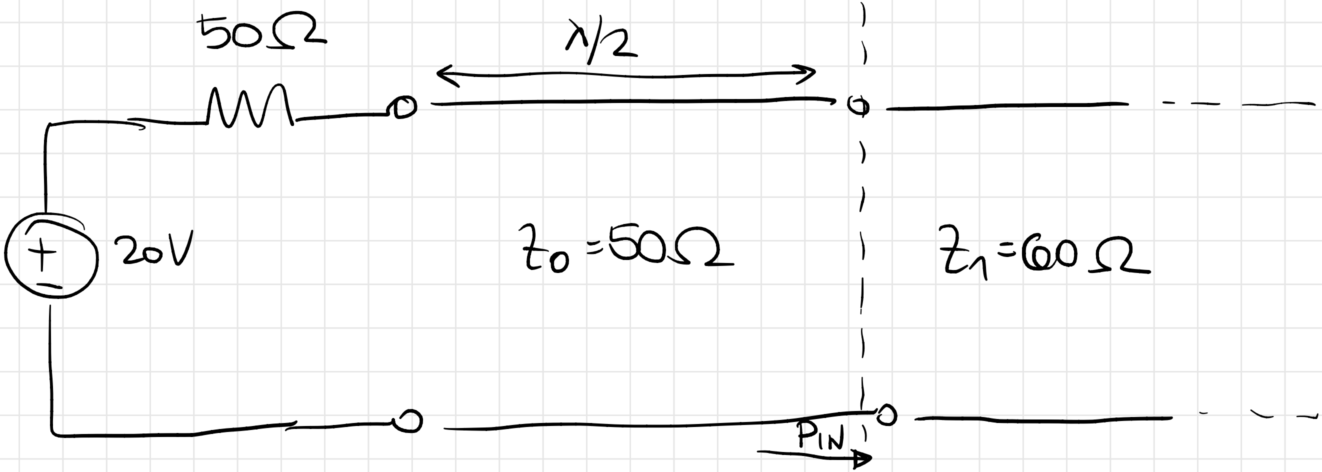
MICROWAVE ENGINEERING

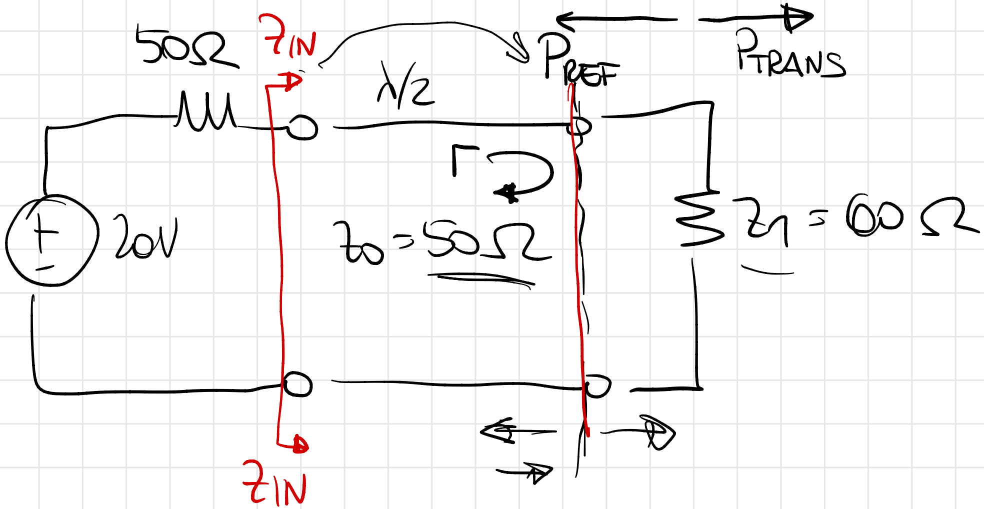
Homework 2

Solution

#1



50Ω
λ/2
20V
Z0 = 50Ω
Z1 = 60Ω
PIN



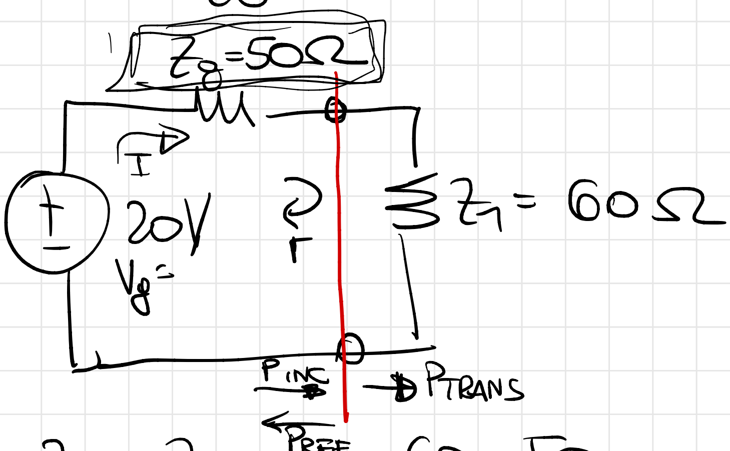
50Ω
ZIN
PREF
PTRANS
λ/2
Γ
20V
Z0 = 50Ω
Z1 = 60Ω
ZIN

$$Z_{IN} = Z_0 \frac{Z_1 + j Z_0 \tan \beta l}{Z_0 + j Z_1 \tan \beta l} =$$

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

$$\tan \pi = \emptyset$$

$$= Z_0 \frac{Z_1}{Z_0} = Z_1$$



$$\Gamma = \frac{Z_1 - Z_g}{Z_1 + Z_g} = \frac{60-50}{60+50} = \frac{10}{110} = 0.09$$

Power available :

$$P_{SOURCE} = \frac{1}{2} V_{SOURCE} I^*_{SOURCE} \overset{*}{=} \frac{1}{2} V_{SOURCE} \left(\frac{V_{SOURCE}}{Z_{TOT}} \right)^* =$$

see note

$$= \frac{1}{2} |V_g|^2 / Z_{TOT} = \frac{1}{2} \frac{20^2}{(50+60)} = \underline{1.82 \text{ W}}$$

For energy conservation

$$P_{TRANS} = P_{INC} - P_{REF}$$

$$P_{REF} = P_{INC} |\Gamma|^2$$

$$\Downarrow$$

$$\underline{P_{TRANS}} = P_{INC} \left(1 - |\Gamma|^2 \right)$$

$$\underline{P_{TRANS}} = \frac{1}{2} Z_1 I^2 = \frac{1}{2} Z_1 \left(\frac{V_g}{Z_{TOT}}\right)^2 = \frac{1}{2}\, 60\, \frac{20^2}{110^2} = 0.9917\,W$$

total current in the loop

$$\underline{P_{INC}} = \frac{P_{TRANS}}{(1-|\Gamma|^2)} = \frac{0.9917}{1-0.0083} = 1\,W$$

$$\underline{P_{REF}} = P_{INC}\,|\Gamma|^2 = 0.0081\,W$$

To check power conservation:

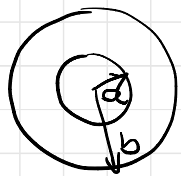
$$\underline{P_{SOURCE}} = P_{DISSIPATED} + \underline{P_{TRANS}}$$

$$P_{DISSIPATED} = \frac{1}{2} Z_g I^2 = \frac{1}{2} Z_g \left(\frac{V_g}{Z_{TOT}}\right)^2 = \frac{1}{2} 50 \left(\frac{20}{110}\right)^2 =$$

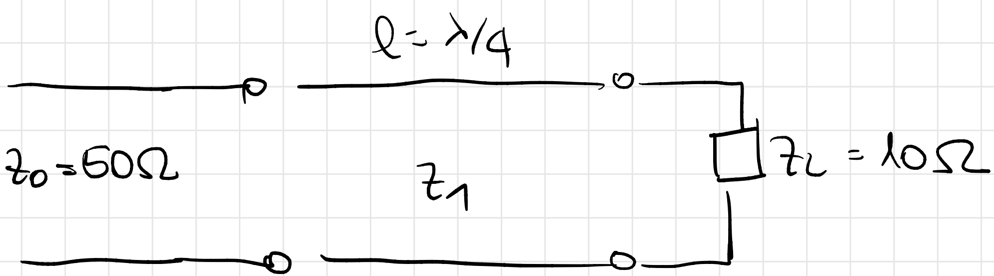
$$= 0.8264 \text{ W}$$

$$\underline{P_{DISS} + P_{TRANS}} = 0.8264 + 0.9917 = 1.82 \text{ W} = \underline{P_{SOURCE}}$$

#2



$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ \ln\left(\frac{b}{a}\right)$$



The length of the quarter wave transformer is:

$$\underline{Z_1} = \sqrt{Z_0 Z_L} = \sqrt{50 \cdot 10} = \underline{22.36\ \Omega}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\frac{2\pi f}{v_p}} = \frac{v_p}{f} = \frac{c/\sqrt{\varepsilon_r}}{f} = \frac{3\cdot 10^8/\sqrt{2.25}}{2\cdot 10^9} = 0.1\,m = 10\,cm$$

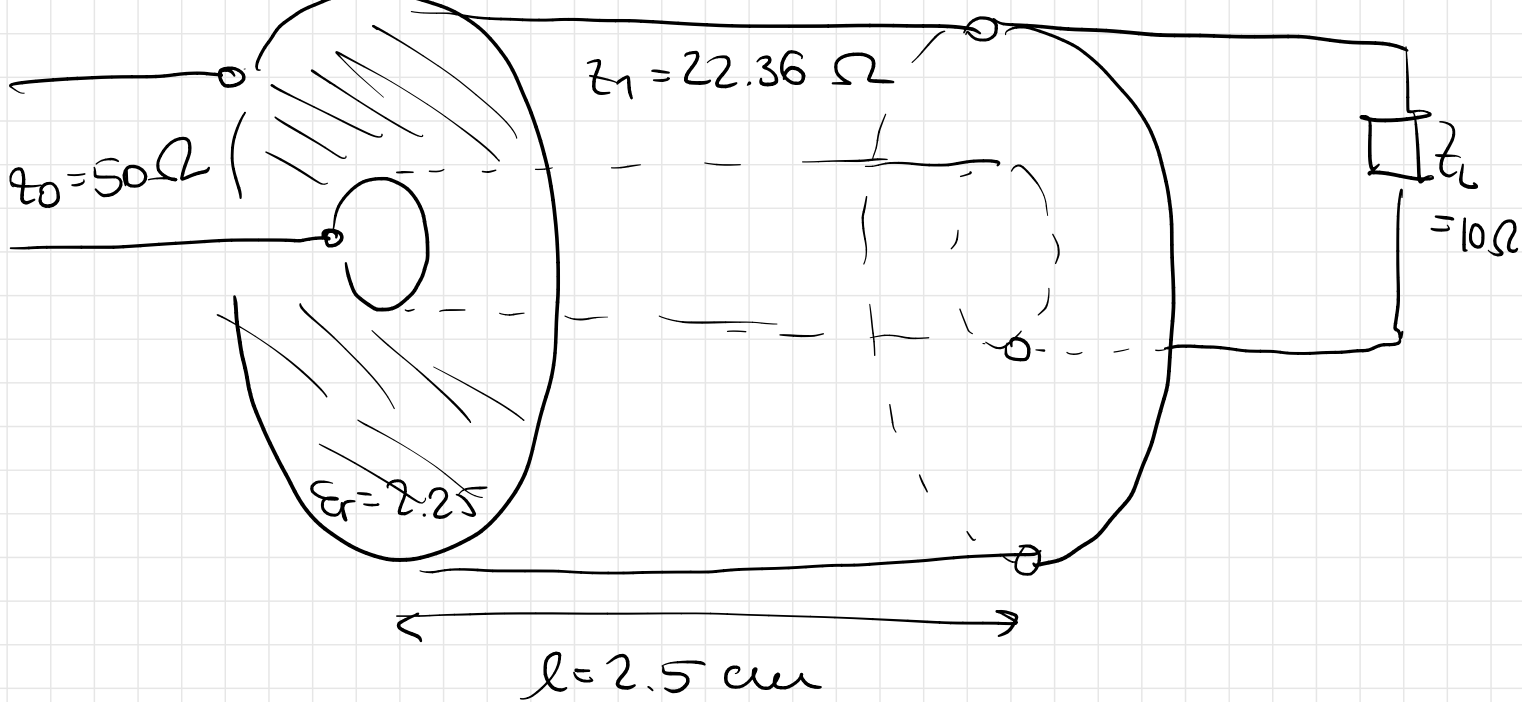
$$l = \frac{\lambda}{4} = 2.5\ \text{cm}$$

$$Z_1 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ \ln\left(\frac{b}{a}\right)$$

$$\ln\left(\frac{b}{a}\right) = 2\pi Z_1 \sqrt{\frac{\varepsilon}{\mu}} = 2\pi Z_1 \left(\sqrt{\frac{\varepsilon_0}{\eta_0}}\right)\sqrt{\varepsilon_r} \Rightarrow$$

$$1/\eta$$

$$\frac{b}{a} = e^{\frac{2\pi Z_1 \sqrt{\varepsilon_r}}{\eta}} = 1.7489$$



Z0 = 50 Ω
Z1 = 22.36 Ω
εr = 2.25
ZL = 10 Ω
l = 2.5 cm

3

W-band $\rightarrow$ W(H): 7.05 - 10 GHz $\rightarrow$ $a = 2.85$ cm, $b = 1.262$ cm

$$\boxed{f_{c_{mn}} = \frac{c}{2\pi\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

$$f_{TE_{10}} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2} = \frac{3\cdot 10^8}{2\pi}\sqrt{\left(\frac{\pi}{2.85\cdot 10^{-2}}\right)^2} = 5.26 \text{ GHz}$$

$$f_{TE_{20}} = \frac{c}{2\pi}\sqrt{\left(\frac{2\pi}{a}\right)^2} = 10.53 \text{ GHz}$$

$$f_{TE_{11}} = f_{TM_{11}} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2} = 12.99 \text{ GHz}$$

$$f_{TE01} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{b}\right)^2} = 11.89 \text{ GHz}$$

$$TE_{10} \rightarrow TE_{20} \rightarrow TE_{11}/TM_{11} \rightarrow TE_{01}$$

When we fill the WG with Teflon ($\varepsilon_r = 2.08$)

$$f_{TE10\,Teflon} = \frac{f_{TE10\,air}}{\sqrt{\varepsilon_r}} = 3.65 \text{ GHz}$$

$$f_{TE20\,tef} = 7.3 \text{ GHz} \qquad f_{TE01\,Tef} = 8.24 \text{ GHz} \qquad \boxed{f_{TE_{11}/TM_{11}\,Tef} = 9.01 \text{ GHz}}$$

The speed of light in Teflon:

$$V_{Teflon} = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \cdot 10^8}{\sqrt{2.08}} = 2.08 \cdot 10^8 \text{ m/s}$$

The phase velocity associated with each mode:

$$V_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} \qquad \beta_{mn} = \sqrt{k_{Tef}^2 - k_{c_{mn}}^2}$$

$$k_{Tef}^2 = \frac{\omega^2}{c^2}\varepsilon_r = \left(\frac{2\pi f}{c}\right)^2 \varepsilon_r = 7.39 \cdot 10^4 \ m^{-2} \Leftarrow$$

$$k_{c_{10}}^2 = \left(\frac{\pi}{a}\right)^2 = \left(110.2 \ m^{-1}\right)^2$$

$$V_{p10} = \frac{2\pi f}{\sqrt{k_T^2 - k_{c10}^2}} = 2.27 \cdot 10^8 \ m/s$$

$$k_{c20}^2 = \left(\frac{2\pi}{a}\right)^2 = \left(220.4 \ m^{-1}\right)^2$$

$$k_{c01}^2 = \left(\frac{\pi}{b}\right)^2 = \left(248.8 \ m^{-1}\right)^2$$

$$V_{p20} = \frac{2\pi f}{\sqrt{k_T^2 - k_{c20}^2}} = 3.55 \cdot 10^8 \ m/s$$

$$V_{p01} = \frac{2\pi f}{\sqrt{k_T^2 - k_{c01}^2}} = 5.16 \cdot 10^8 \ m/s$$

$$k_{c_{11}}^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 = 7.41 \cdot 10^4 > k_T^2$$

$$\sqrt{\underbrace{k_T^2 - k_{c_{11}}^2}_{<0}} \Rightarrow \beta \text{ imaginary}$$

The group velocity :

$$V_g = \frac{d\omega}{d\beta} = \left(\frac{d\beta}{d\omega}\right)^{-1}$$

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{\left(\frac{\omega}{c}\right)^2 \varepsilon_r - k_c^2}$$

$$\frac{d\beta}{d\omega} = \frac{1}{\cancel{2}} \frac{\cancel{2}\frac{\omega}{c^2}\varepsilon_r}{\underbrace{\sqrt{\left(\frac{\omega}{c}\right)^2 \varepsilon_r - k_c^2}}_{\Rightarrow \beta}} =$$

$$\frac{d\beta}{d\omega} = \frac{\omega \varepsilon_r}{c^2 \beta} \Rightarrow V_g = \frac{c^2 \beta}{\omega \varepsilon_r}$$

$$V_{g_{TE_{10}}} = \frac{c^2 \sqrt{k_T^2 - k_{c10}^2}}{\omega \varepsilon_r} = 1.9 \cdot 10^8 \ m/s$$

$$V_{g_{TE_{20}}} = \frac{c^2 \sqrt{k_T^2 - k_{c20}^2}}{\omega \varepsilon_r} = 1.22 \cdot 10^8 \ m/s$$

$$V_{g_{TE_{01}}} = \frac{c^2 \sqrt{k_T^2 - k_{c01}^2}}{\omega \varepsilon_r} = 8.38 \cdot 10^7 \ m/s$$

* NOTE

Power definition in transmission lines refers to the phasor form of the fields while in circuit theory the time domain expression of V and I is considered! P_{SOURCE} in this case is the real part of the complex Poynting vector which refers to the power that can actually propagate in the line.