

Spatial Optics

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Spatio temporal analogy

MONOCHROMATIC PLANE WAVE STRUCTURE

$$MPW(t, M) = e^{j(2\pi vt - \vec{k} \cdot \vec{OM})}$$

$$MPW(t, M) = e^{j2\pi vt} \cdot e^{-j\vec{k} \cdot \vec{OM}}$$

Temporal

Spatial

*Applied to
Free space
propagation*

Propagation along the Oz axis

$$MPW_{z=0}(t, x, y) = e^{j2\pi vt} \cdot e^{-j(k_x x + k_y y)}$$

variable	t	x ; y
Frequency	v	$N_x = \frac{k_x}{2\pi} ; N_y = \frac{k_y}{2\pi}$

First analogy about the propagation

Temporal

Monochromatic wave

$$MW_{z=0}(t) = e^{j2\pi vt}$$



$$MW_{z \neq 0}(t) = e^{j2\pi vt} \cdot e^{-j\beta \cdot z}$$

Spatial

Plane wave

$$PW_{z=0}(x, y) = e^{-j(k_x \cdot x + k_y \cdot y)}$$



$$PW_{z \neq 0}(x, y) = e^{-j(k_x \cdot x + k_y \cdot y)} \cdot e^{-j(k_z \cdot z)}$$

Transfert function

Analogy about the function decomposition : Fourier analysis

Temporal

Spatial

$$f(t) = \int_{-\infty}^{+\infty} \tilde{f}(\nu) e^{j2\pi\nu t} d\nu$$

$$\tilde{f}(\nu) = \int_{-\infty}^{+\infty} f(t) e^{-j2\pi\nu t} dt$$

FT⁻¹

FT

1D

$$f(x, y) = \int_{-\infty}^{+\infty} \tilde{f}(N_x, N_y) e^{-j(k_x x + k_y y)} dN_x dN_y$$

Plane wave
2D (2 N_x + y N_y)

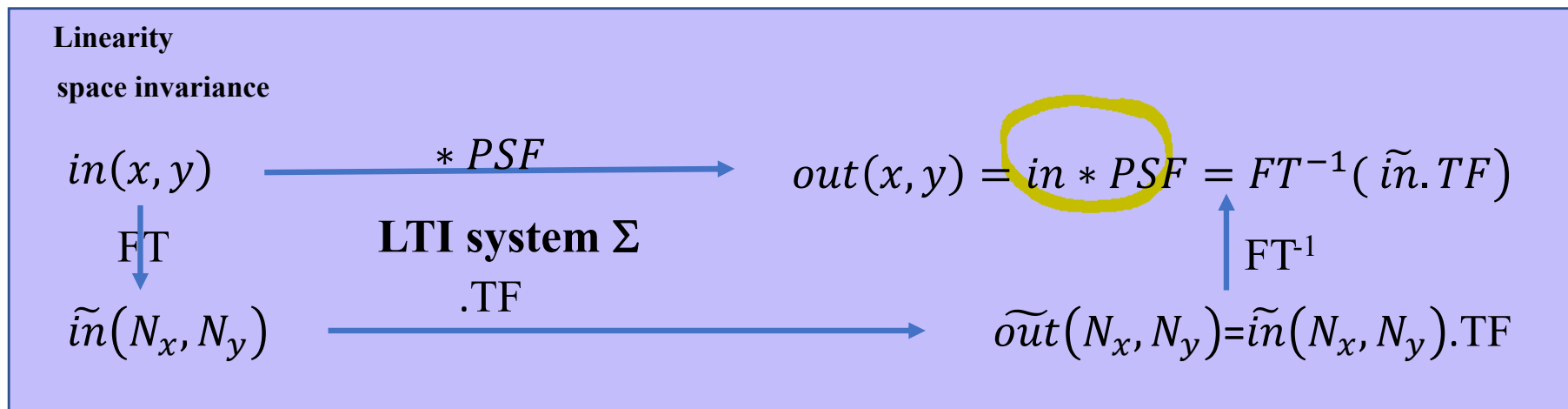
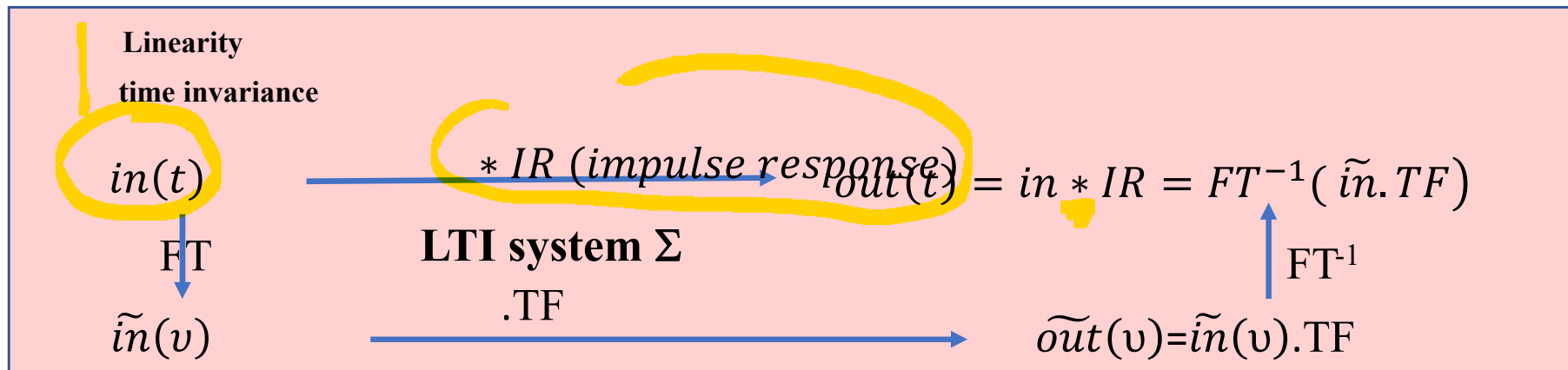
$$\tilde{f}(N_x, N_y) = \int_{-\infty}^{+\infty} f(x, y) e^{j(k_x x + k_y y)} dx dy$$

2D

!!signs

Analogy about general skills of the data processing

Applied to linear and translation invariant systems



Application to the diffraction grating formula

$$\sin(\theta) = \sin(\theta_0) + \frac{k\lambda}{a}$$

$$\frac{\sin \theta}{\lambda} = \frac{\sin \theta_0}{\lambda} + k \frac{1}{a}$$

Λ
= Frequency
of the
grating

$$\Lambda = \frac{1}{a}$$

$$N_{\text{out}} = N_{\text{in}} + k \text{ spatial frequency grating}$$