Correction (Tutorial 4 SBM Single Balanced Mixer)

1) ***In the studied case, the input voltages are $V_{\Sigma 1}=(\sqrt{2}.V_{RF})$ and $V_{\Delta 1}=(\sqrt{2}.V_{LO})$. Therefore, express the input control voltages V_{IN1} and V_{IN2} of N as a function of V_{LO} and V_{RF}.***

Using the properties of the input ΣΔ coupler, one can write:

$$V_{IN1}=\frac{1}{\sqrt{2}}(V_{\Sigma 1}+V_{\Delta 1})=\frac{1}{\sqrt{2}}\left(\sqrt{2}V_{\mathrm{RF}}+\sqrt{2}V_{\mathrm{LO}}\right)=V_{RF}+V_{LO}$$

$$V_{IN2}=\frac{1}{\sqrt{2}}(V_{\Sigma 1}-V_{\Delta 1})=\frac{1}{\sqrt{2}}\left(\sqrt{2}V_{\mathrm{RF}}-\sqrt{2}V_{\mathrm{LO}}\right)=V_{RF}-V_{LO}$$

2) ***Deduce the expressions of output voltages (V_{OUT1}, V_{OUT2}) as a function of V_{LO} and V_{RF}.***

$$V_{OUT1}=aV_{IN1}^2-bV_{IN1}^3=a[V_{RF}+V_{LO}]^2-b[V_{RF}+V_{LO}]^3$$

$$V_{OUT2}=aV_{IN2}^2-bV_{IN2}^3=a[V_{RF}-V_{LO}]^2-b[V_{RF}-V_{LO}]^3$$

3) ***The SBM down-converter is designed at an IF angular frequency $\omega_{IF}=\omega_{LO}-\omega_{RF}$. What should be the type (Σ or Δ?) of port 4 in the output coupler?***

The properties of the square and the cube of sum and difference give:

$(X + Y)^2 = X^2 + 2XY + Y^2$ **(eq1)** and $(X + Y)^3 = X^3 + Y^3 + 3X^2Y + 3XY^2$ **(eq3)**

while

$(X - Y)^2 = X^2 - 2XY + Y^2$ **(eq2)** and $(X - Y)^3 = X^3 - Y^3 - 3X^2Y + 3XY^2$ **(eq4)**

As the output frequency of a mixer is obtained through the multiplication between LO and input signals, if X and Y stand for the OL and input signals of the mixer, respectively, one want to generate the product XY. Therefore, by looking at the preceding equations, one can deduce that only the difference between $(X + Y)^2$ and $(X - Y)^2$ gives the product XY whereas the sum would cancel the product XY.

➔ The port 4 should be a Δ **port** in the output coupler.

4) ***Given that $V_{LO}=V_0.\cos(\omega_{LO}.t)$ and $V_{RF}=V_1.\cos(\omega_{RF}.t)$, express the output voltage spectrum V_4 of the SBM as a function of V_0 and V_1. What are the remaining frequencies at the IF output port of the SBM?***

As the port 4 is a Δ port → $V_4=\frac{1}{\sqrt{2}}(V_{\mathrm{OUT1}}-V_{\mathrm{OUT2}})$

$$V_4=\frac{1}{\sqrt{2}}\left(a[V_{RF}+V_{LO}]^2-b[V_{RF}+V_{LO}]^3-a[V_{RF}-V_{LO}]^2+b[V_{RF}-V_{LO}]^3\right)$$

$$=\frac{1}{\sqrt{2}}a[(V_{RF}+V_{LO})^2-(V_{RF}-V_{LO})^2]-\frac{1}{\sqrt{2}}b[(V_{RF}+V_{LO})^3-(V_{RF}-V_{LO})^3]$$

Using the preceding equations (eq1) to (eq4), one can deduce:

$$V_4=\frac{1}{\sqrt{2}}a[4V_{RF}V_{LO}]-\frac{1}{\sqrt{2}}b[6V_{RF}^2V_{LO}+2V_{LO}^3]$$

$$V_4=\sqrt{2}a[2V_{RF}V_{LO}]-\sqrt{2}b[3V_{RF}^2V_{LO}+V_{LO}^3]$$

The following framed text corresponds only to comments for helping students to understand but should not be integrally written in the exam because it is over the actual question that only requires the calculation of V_4

Before expanding the output voltage V_4 as a function of its spectral components, one can deduce its frequencies using its polynomial terms:

The term ${V_{RF}}^{1}\,{V_{LO}}^{1}$ gives two mixing frequencies between (1st order of RF) and (1st order of LO)

→ $\omega_{IF} = 1\omega_{LO} - 1\omega_{RF}$ and $\omega_{\Sigma} = 1\omega_{LO} + 1\omega_{RF}$

The term $V_{RF}^{2}V_{LO}^{1}$ gives three mixing frequencies between (0 and 2nd orders of RF) and (1st order of LO)

→ $1\omega_{LO}$; $\omega_1 = 2\omega_{RF} - 1\omega_{LO}$ and $\omega_2 = 2\omega_{RF} + 1\omega_{LO}$

The term $V_{LO}^{3} = V_{RF}^{0}V_{LO}^{3}$ gives two mixing frequencies between (0 order RF) and (1st and 3rd orders LO)

→ $1\omega_{LO}$; $3\omega_{LO}$

All possible output frequencies of mixers are: $\omega_{OUT} = \pm\, m\, \omega_{LO} \pm n\, \omega_{IN}$ with (m,n)= positive integers

As we are designing a down-converter mixer (RF → IF), one can denote $\omega_{IN} = \omega_{RF}$

→ $\omega_{OUT} = \pm\, m\, \omega_{LO} \pm n\, \omega_{RF}$

Looking at the preceding equation of V_4, one can note that there are only have **odd orders** of ω_{LO} in V_4

→ Therefore, one can deduce that all output mixing frequencies $\omega_{OUT} = \pm m\, \omega_{LO} \pm n\, \omega_{RF}$ using even orders of ω_{LO} are rejected. If we consider the simplest even order **0** of ω_{LO}, this means that all output frequencies $\omega_{OUT} = \pm \mathbf{0}\, \omega_{LO} \pm n\, \omega_{RF} = n\, \omega_{RF}$ are rejected

One can conclude that there is no harmonics of the RF frequency at the output of the mixer

→ This means that the isolation RF—IF will be infinite

Before expanding V_4, one can define simplified frequency notations:

$\omega_{IF} = \omega_{LO} - \omega_{RF}$; $\omega_{\Sigma} = \omega_{LO} + \omega_{RF}$; $\omega_1 = 2\omega_{RF} - \omega_{LO}$ and $\omega_2 = 2\omega_{RF} + \omega_{LO}$

$$V_4 = \sqrt{2}a[2V_{RF}V_{LO}] - \sqrt{2}b[3V_{RF}^2V_{LO} + V_{LO}^3]$$

→ One can make separate calculations of each terms in V_4:

- $\sqrt{2}a[2V_{RF}V_{LO}] = \sqrt{2}\,a\,2\,V_1 cos(\omega_{RF}t)\,V_0 cos(\omega_{LO}t) = \sqrt{2}\,a\,V_1\,V_0[cos(\omega_{IF}t) + cos(\omega_{\Sigma}t)]$
- $\sqrt{2}b[3V_{RF}^2V_{LO}] = \sqrt{2}\,b\,3V_1^2 cos^2(\omega_{RF}t)\,V_0 cos(\omega_{LO}t) = \sqrt{2}\,b\,\frac{3}{2}V_1^2V_0[1 + cos(2\omega_{RF}t)]cos(\omega_{LO}t)$
  $= \sqrt{2}\,b\,\frac{3}{2}V_1^2V_0\left[cos(\omega_{LO}t) + \frac{1}{2}cos(\omega_1 t) + \frac{1}{2}cos(\omega_2 t)\right]$
  $= \frac{3\sqrt{2}}{2}bV_1^2V_0 cos(\omega_{LO}t) + \frac{3\sqrt{2}}{4}bV_1^2V_0[cos(\omega_1 t) + cos(\omega_2 t)]$
- $\sqrt{2}b[V_{LO}^3] = \sqrt{2}\,b\,V_0^3 cos^3(\omega_{LO}t) = \sqrt{2}\,b\,V_0^3\left[\frac{3}{4}cos(\omega_{LO}t) + \frac{1}{4}cos(3\omega_{LO}t)\right]$ ■

→ $V_4 = \sqrt{2}\,a\,V_1\,V_0[cos(\omega_{IF}t) + cos(\omega_{\Sigma}t)] + \frac{3\sqrt{2}}{4}bV_1^2V_0[cos(\omega_1 t) + cos(\omega_2 t)]$
$+ \frac{3\sqrt{2}}{2}bV_1^2V_0 cos(\omega_{LO}t) + \sqrt{2}\,b\,V_0^3\left[\frac{3}{4}cos(\omega_{LO}t) + \frac{1}{4}cos(3\omega_{LO}t)\right]$

➔ $V_4 = \sqrt{2}\, a\, V_1\, V_0[cos(\omega_{IF}t) + cos(\omega_\Sigma t)] + \frac{3\sqrt{2}}{4} bV_1^2V_0[cos(\omega_1 t) + cos(\omega_2 t)]$
$+\frac{3\sqrt{2}}{2} bV_1^2V_0 cos(\omega_{LO}t) + \frac{3\sqrt{2}}{4} b\, V_0^3 cos(\omega_{LO}t) + \frac{\sqrt{2}}{4} b\, V_0^3 cos(3\omega_{LO}t)$

➔ $V_4 = \sqrt{2}\, a\, V_1\, V_0[cos(\boldsymbol{\omega_{IF}}t) + cos(\boldsymbol{\omega_\Sigma}t)] + \frac{3\sqrt{2}}{4} bV_1^2V_0[cos(\boldsymbol{\omega_1}t) + cos(\boldsymbol{\omega_2}t)]$
$+ \frac{3\sqrt{2}}{4} b\, V_0\,[2\,V_1^2 + V_0^2] cos(\boldsymbol{\omega_{LO}}t) + \frac{\sqrt{2}}{4} b\, V_0^3 cos(\boldsymbol{3\omega_{LO}}t)$

The remaining frequencies at the IF output port are : $\boldsymbol{\omega_{IF}}$; $\boldsymbol{\omega_\Sigma}$; $\boldsymbol{\omega_1}$; $\boldsymbol{\omega_2}$; $\boldsymbol{\omega_{LO}}$ and $\boldsymbol{3\omega_{LO}}$

5) *What are the advantages of this SBM configuration (LO input at Δ port)?*

One have to look at the main isolations:

- The most important isolation is LO-to-RF because LO is a large signal which cannot be improved by filtering since LO and RF are very close frequencies. As the LO generator is applied to an input port (Δ1) of the passive ΣΔ coupler while the RF generator is applied to the other input port (Σ1) of this coupler, one can conclude that the LO-to-RF isolation is imposed by the isolation between both input ports of the ΣΔ coupler and not by the own isolation of individual mixers N → The LO-to-RF isolation of the SBM mixer is equal to that of the input ΣΔ coupler and thus very good.
- The second criterion is the LO-to-IF isolation because LO is a large signal. However, it is less critical than LO-to-RF since it can be improved by filtering because LO and IF are very distant frequencies. As one can note in V_4, a part of the LO signal is generated at the IF output so that this SBM mixer does not improve the LO-to-IF isolation.
- In our case of down-conversion mixer, the last criterion is RF-to-IF isolation (it would be IF-to-RF isolation in the case of an up-conversion mixer). In our actual case, one can note in the equation of V_4 that there is no RF signal leakage at the output IF port so that this SBM configuration gives an infinite RF-to-IF isolation.

6) *Express the voltage conversion gain G_{CV} of the SBM. What is the power conversion gain G_{CP} if all 180° couplers are matched to 50Ω ?*

$$G_{CV} = \frac{V_{OUT}(\omega_{OUT})}{V_{IN}(\omega_{IN})}$$

As this mixer is a down-converter (RF → IF), one can denote $\omega_{IN} = \omega_{RF}$ and $\omega_{OUT} = \omega_{IF}$

$$➔\; G_{CV} = \frac{V_{OUT}(\omega_{IF})}{V_{IN}(\omega_{RF})} = \frac{V_4(\omega_{IF})}{V_{\Sigma 1}(\omega_{RF})} = \frac{\sqrt{2}\, a\, V_1\, V_0}{\sqrt{2}V_1} = a\, V_0$$

To calculate powers, one can also consider that the IF output port 4 is loaded by a R_L resistor while the couplers are matched to R_{50}=50Ω. Therefore, the power conversion gain is:

$$G_{CP} = \frac{P_{OUT}(\omega_{OUT})}{P_{IN}(\omega_{IN})} = \frac{P_4(\omega_{IF})}{P_{\Sigma 1}(\omega_{RF})} = \frac{\frac{1}{2}\frac{|V_4(\omega_{IF})|^2}{R_L}}{\frac{1}{2}\frac{|V_{\Sigma 1}(\omega_{RF})|^2}{R_{50}}} = \frac{R_{50}}{R_L}\frac{\left[\sqrt{2}\, a\, V_1\, V_0\right]^2}{\left[\sqrt{2}\; V_1\right]^2} = a^2V_0^2\frac{R_{50}}{R_L}$$

7) What are the values of LO-to-RF and LO-to-IF isolations?

➔ The LO-to-RF isolation is defined by:

$$\boldsymbol{ISO_{LO-RF}\ of\ SBM} = \frac{P_{LO-port}(\omega_{LO})}{P_{RF-port}(\omega_{LO})} = \frac{P_{\Delta 1}(\omega_{LO})}{P_{\Sigma 1}(\omega_{LO})} = ISO_{\Delta 1-\Sigma 1}\ of\ the\ coupler$$

$$= \infty\ in\ the\ case\ of\ an\ ideal\ coupler$$

➔ The LO-to-IF isolation is defined by:

$$\boldsymbol{ISO_{LO-IF}\ \ of\ SBM} = \frac{P_{LO-port}(\omega_{LO})}{P_{IF-port}(\omega_{LO})} = \frac{P_{\Delta 1}(\omega_{LO})}{P_4(\omega_{LO})} = \frac{\frac{1}{2}\frac{|V_{\Delta 1}(\omega_{LO})|^2}{R_{50}}}{\frac{1}{2}\frac{|V_4(\omega_{LO})|^2}{R_L}} = \frac{R_L}{R_{50}}\ \frac{|V_{\Delta 1}(\omega_{LO})|^2}{|V_4(\omega_{LO})|^2}$$

$$= \frac{R_L}{R_{50}}\ \frac{\left|\sqrt{2}\,V_1\right|^2}{\left|\frac{3\sqrt{2}}{4}b\,V_0\,[2\,V_1^2+V_0^2]\right|^2} = \frac{16}{9}\ \frac{R_L}{R_{50}}\ \frac{V_1^2}{b^2\,V_0^2\left[2V_1^2+V_0^2\right]^2}$$

➔ The RF-to-IF isolation is not asked because it is also infinite since there is no RF signal at the output IF port. It is defined by:

$$\boldsymbol{ISO_{RF-IF}\ \ of\ SBM} = \frac{P_{RF-port}(\omega_{RF})}{P_{IF-port}(\omega_{RF})} = \frac{P_{\Sigma 1}(\omega_{RF})}{P_4(\omega_{RF})} = \frac{P_{\Sigma 1}(\omega_{RF})}{0} = \infty$$