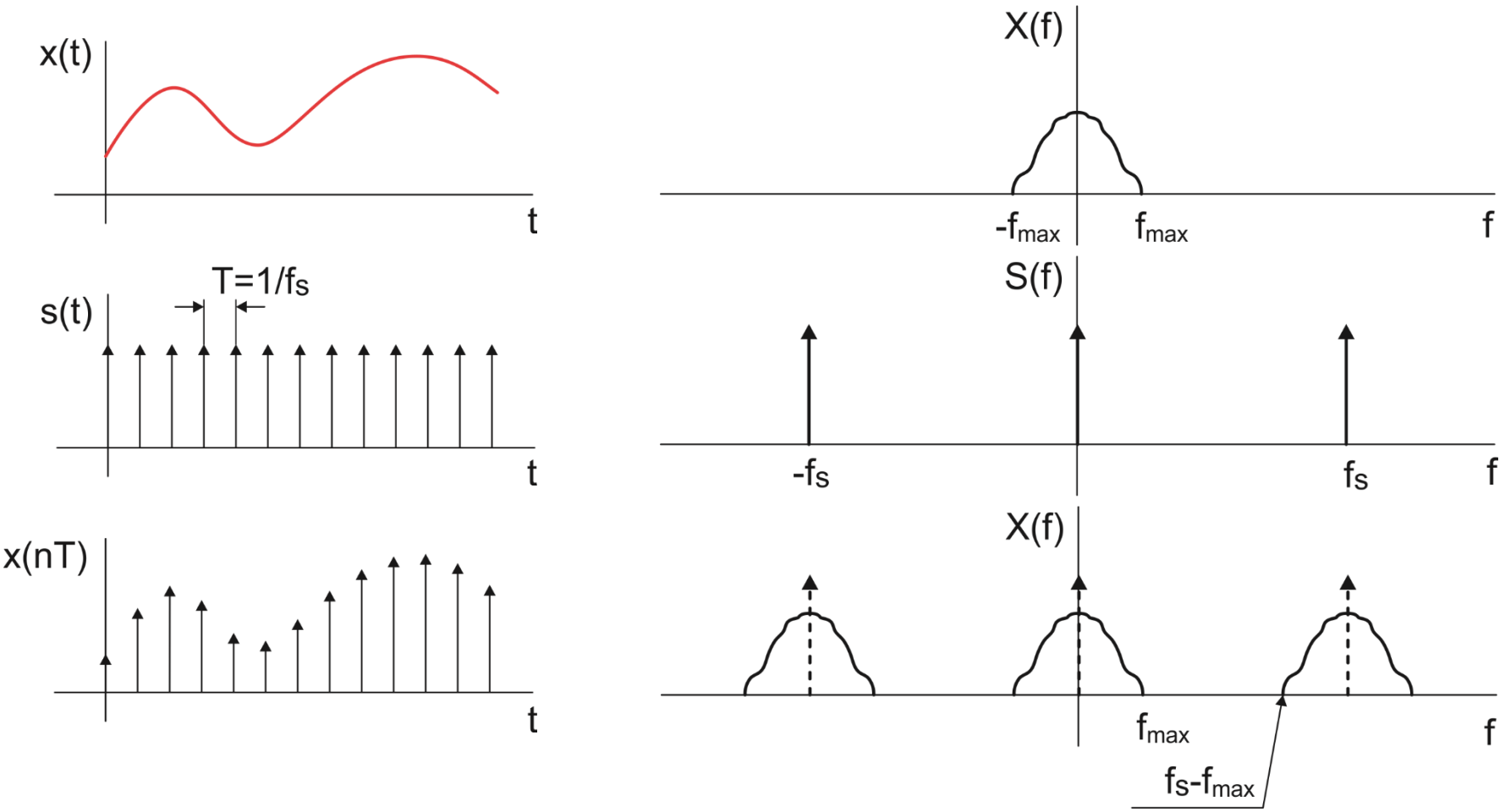
Sampled waveform



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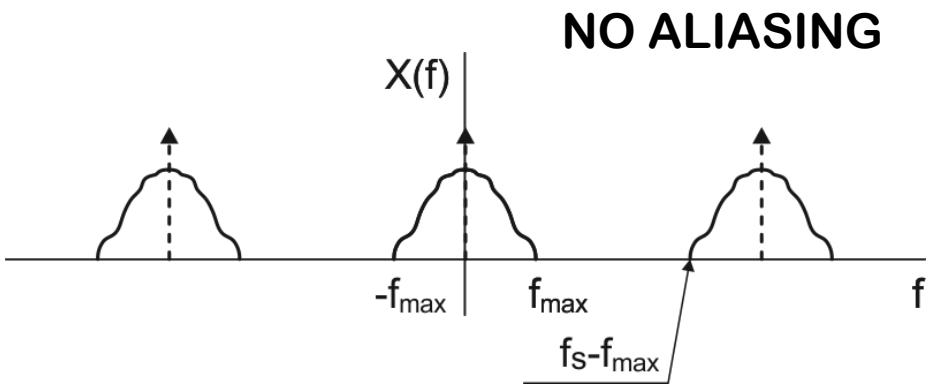
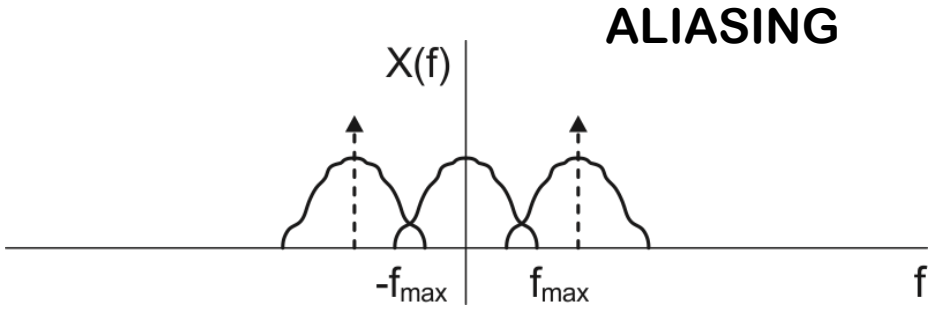
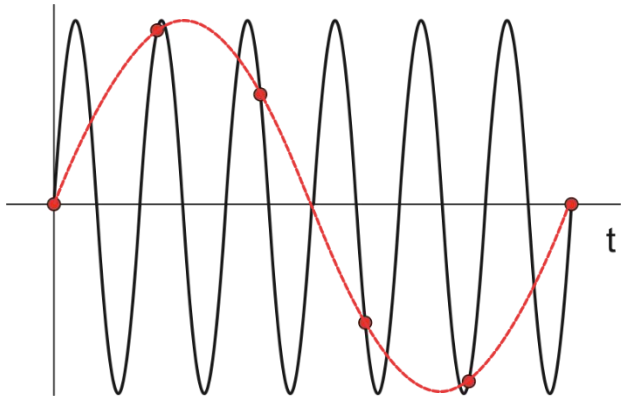
Aliasing







- to avoid aliasing:
  - 1) digitize only bandwidth-limited signals
  - 2) use a sample rate greater than 2 f_{max}

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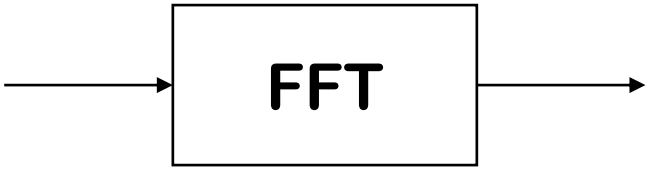
FFT properties

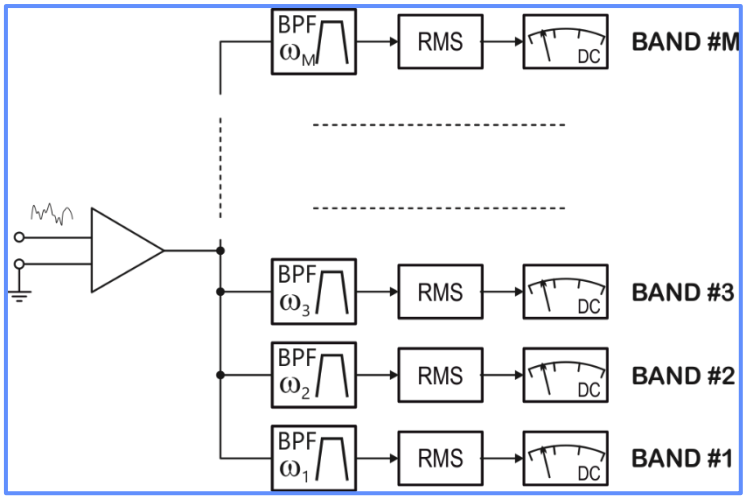
time record
$N=2^n$ real values



frequency record
N/2+1 complex values

- each frequency point (bin) is the equivalent of the filter/detector output in the bank-of-filters analyser

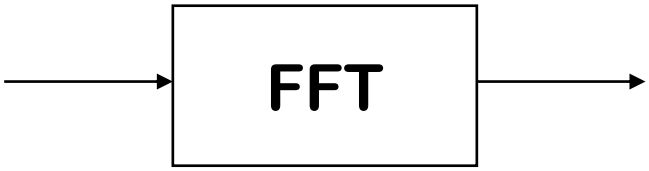
≈



- the module value of each complex number is equivalent to the bank-of-filter channel readout

FFT properties

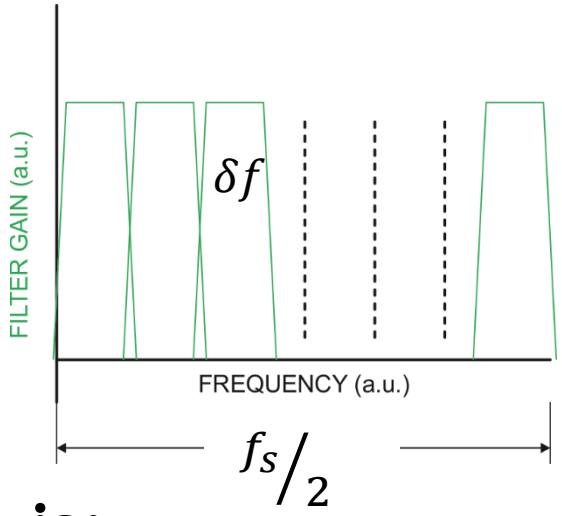
time record
$N=2^n$ real values



frequency record
N/2+1 complex values

- the bins are equally spaced in the frequency:

$$\delta f = {}^{f_S}\!/_{\frac{N}{2}} = \frac{f_S}{N} = \frac{1}{\Delta T}$$



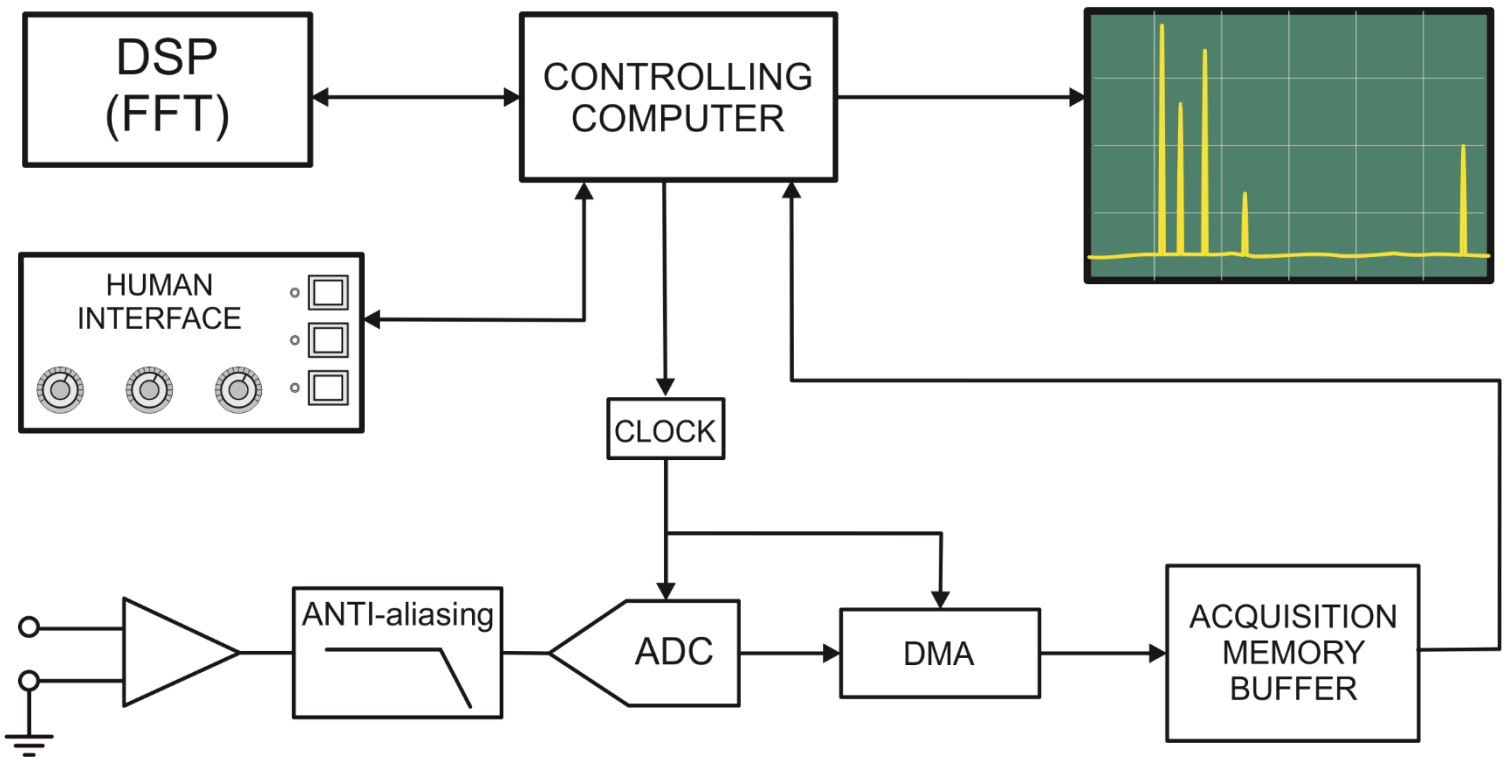
- the frequency associated with bin n is:

$$f_n = n \cdot \frac{f_S}{N}$$

- the highest frequency, that of the last bin, is:

$$f_{max} = f_S/2$$

Digital (FFT based) spectrum analyzer



Frequency span and frequency resolution

- the FFT is an inherently base-band transformation: the frequency interval always starts at $0\ Hz$ and ends at $f_s/2$
- total frequency span: $0 \div BW = 0 \div f_s/2$
- frequency resolution: $\delta f = f_s/N$
- given the dimension N of the time record, to improve the resolution we must decrease the sampling frequency f_s and, therefore, the working bandwidth
- given the bandwidth, to improve the resolution we must increase the time record length N, and therefore augment the computing resource
- FFT number of computations: $N \cdot \log_2(N)$

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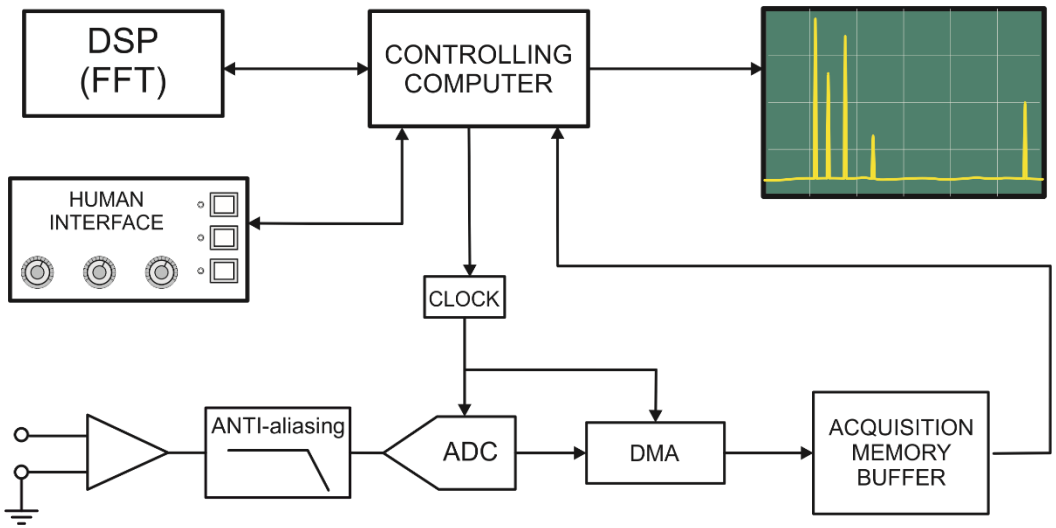
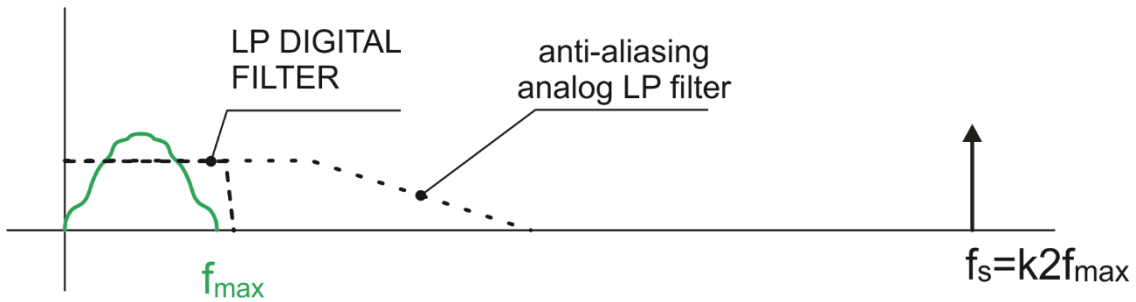
Oversampling and digital filtering



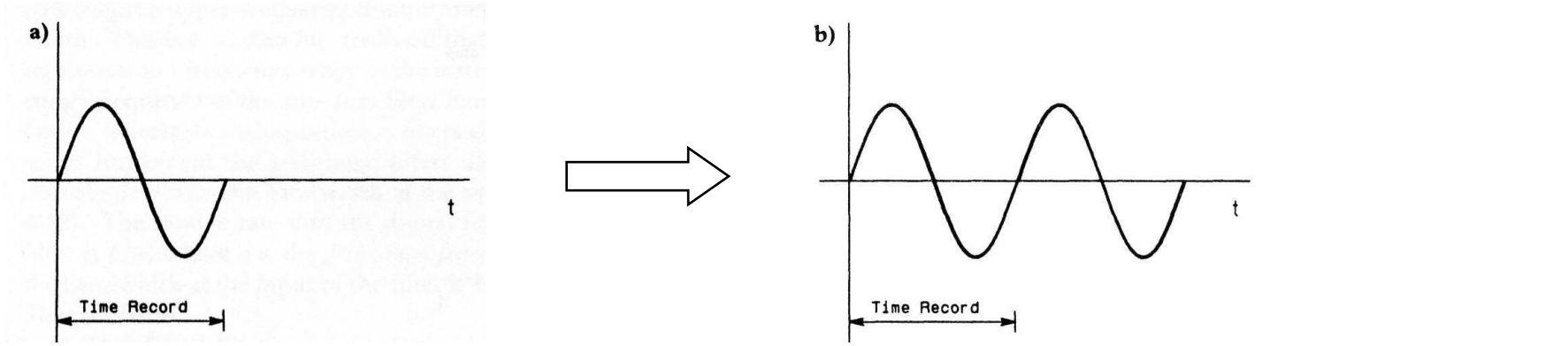


- the input signal is oversampled
- a digital low-pass filtering (LPF) is applied to limit the signal bandwidth to the desired analysis frequency range
- the signal is decimated before applying the FFT algorithm

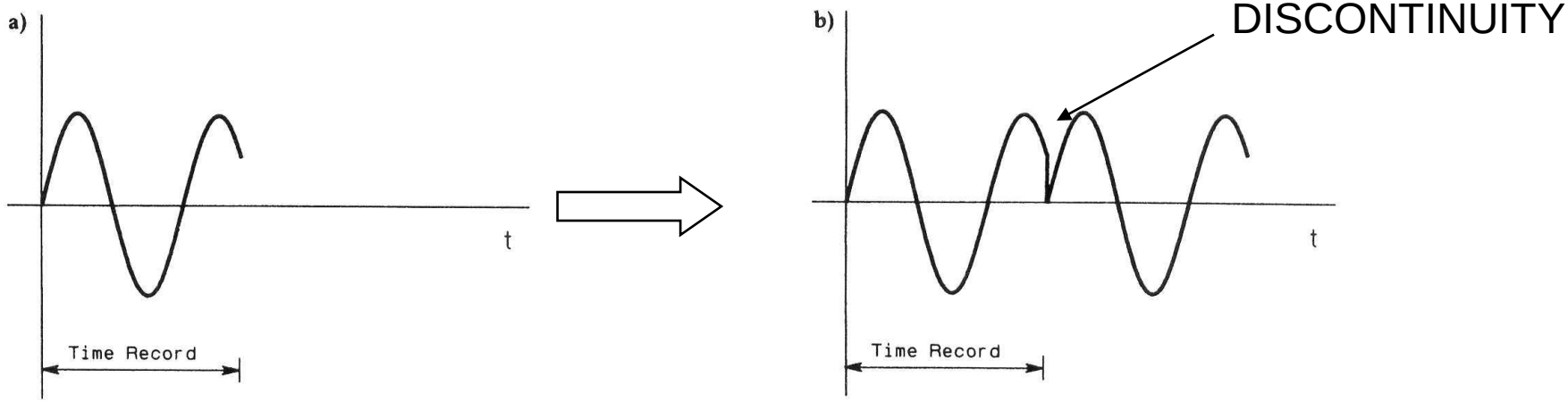
Leakage (frequency dispersion)

- FFT actually replicates the finite time record indefinitely:

A)

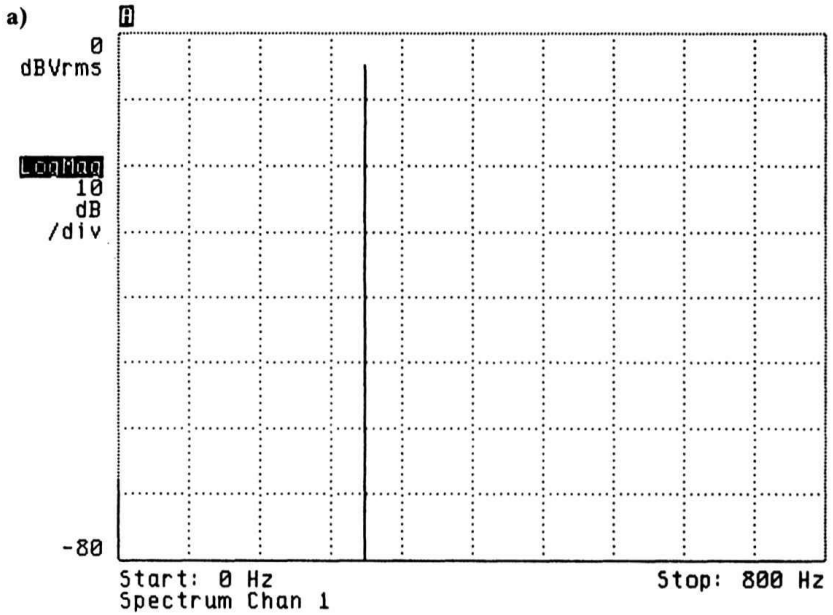


B)

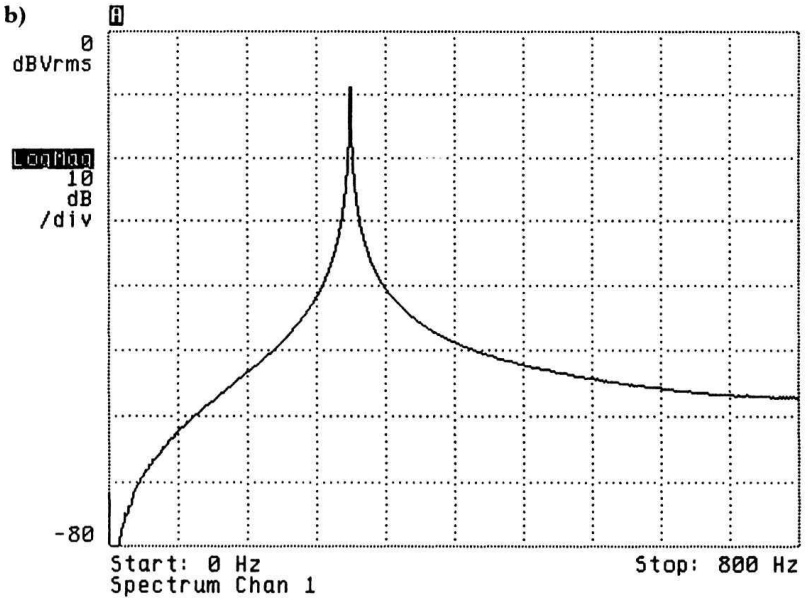


cont.

A)

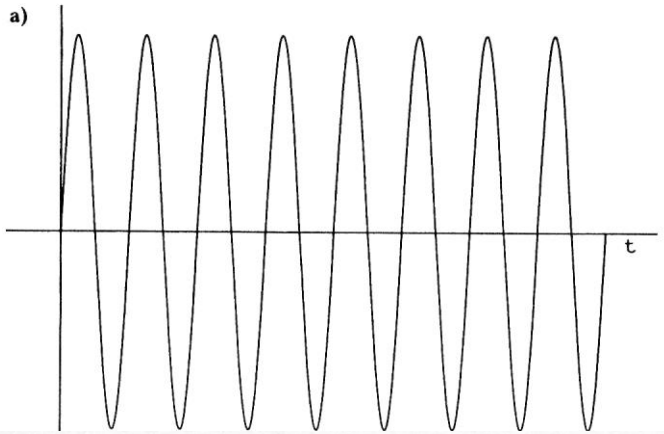


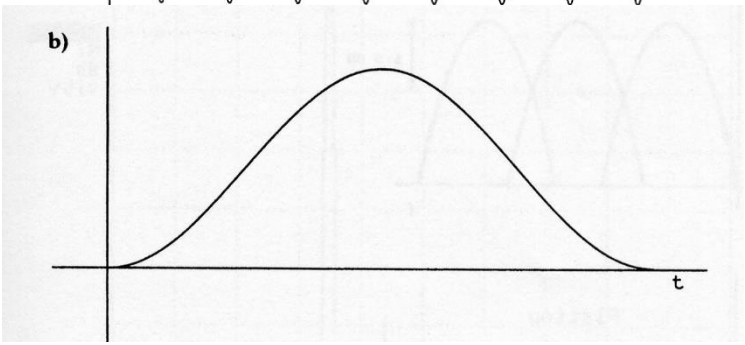
B)

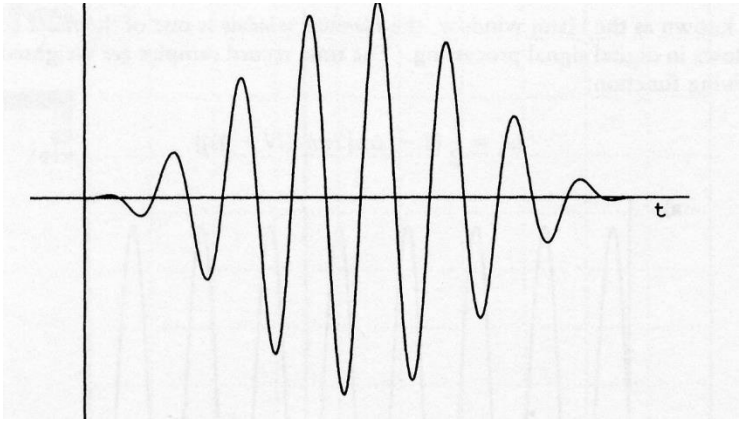


- the leakage must be reduced
- to avoid discontinuities the sampled waveform is forced to zero at the ends of the sample

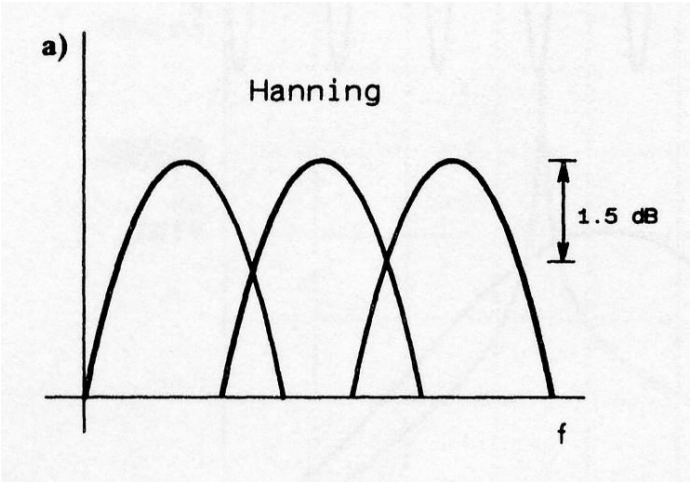
Example: Hanning window



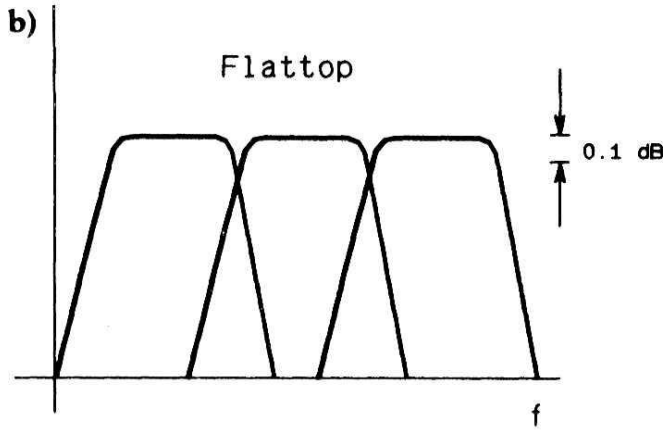




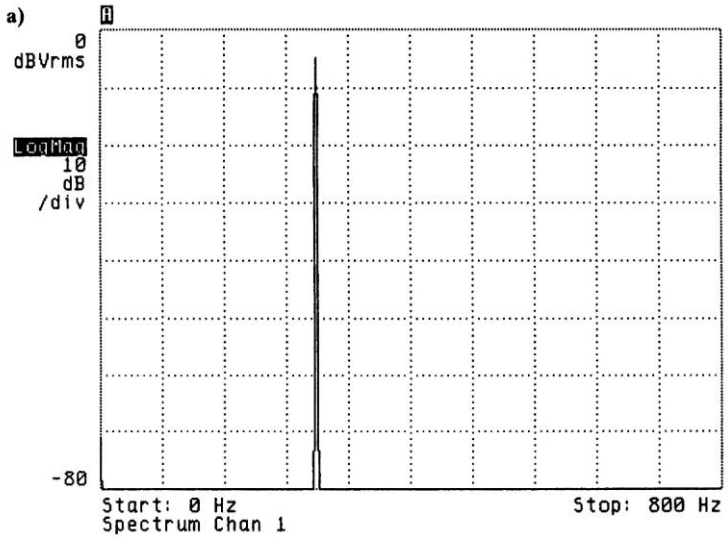
$$w_n = \frac{1}{2}\{1 - \cos[2\pi n/(N-1)]\}$$



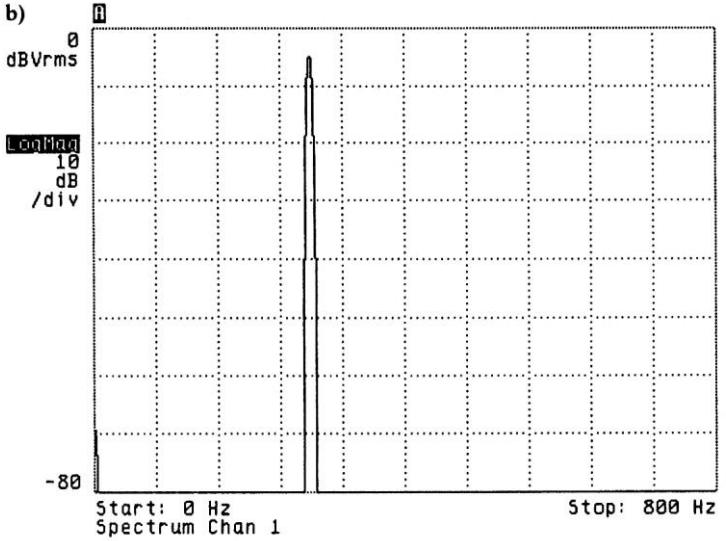
Flat top window



Sine wave spectrum



Hanning



Flat top