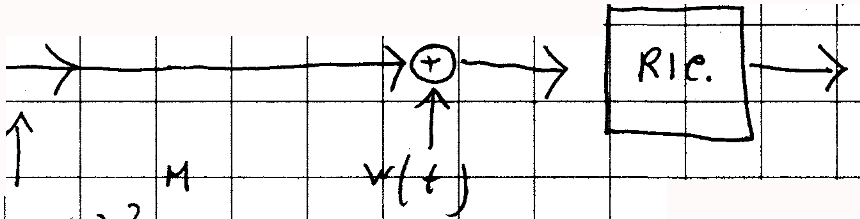
Signal spaces



$\{s_i(t)\}_{i=1}^{M}$ ⟶ N is the dimensionality of the vector space

$$s_i(t) = \sum_{k=1}^{N} s_{ik}\,\phi_k(t) \quad ; \quad s_{ik} = \int_0^T s_i(t)\,\phi_k(t)\,dt$$

$$\langle \phi_i(t), \phi_k(t) \rangle = \delta_{ik} = \begin{cases} 0 ; & k \neq i \\ 1 ; & k = i \end{cases}$$ ← (PRODOTTO SCALARE) Inner product

$$\langle s_i(t), s_j(t) \rangle = \int_0^T s_i(t)\,s_j(t)\,dt = \sum_{k=1}^{N} s_{ik}\,s_{jk}$$

(Disug. di Schwartz)

$$|\langle s_i, s_j \rangle| \leq \|s_i\|\,\|s_j\|$$

$$\big|\, \|s_i\| - \|s_j\| \,\big| \leq \|s_i \pm s_j\| \leq \|s_i\| + \|s_j\|$$

(Dis. triangolare)

AWGN noise

Rumore AWGN in $[0, T]$

$$n(t) = \sum_k n_k\,\phi_k(t) \quad ; \quad n_k = \int_0^T n(t)\,\phi_k(t)\,dt$$

$$E[n_k] = 0 \quad ; \quad E[n_k n_j] = \begin{cases} 0 ; & k \neq j \\ N_0/2 ; & k = j \end{cases}$$

$$E\left[\left(n(t) - \sum_{k=1}^{\infty} n_k\,\phi_k(t)\right)^2\right] = 0$$

Segnali PASSA-BANDA

Pass-Band signals

$$s(t) = \mathrm{Re}\left\{ z(t)\, e^{j 2\pi f_0 t} \right\} = \frac{z(t)}{2} e^{j(\cdot)} + \frac{z^*(t)}{2} e^{-j(\cdot)} =$$

$$= |z(t)| \cos\left(2\pi f_0 t + \arg z(t)\right)$$

$$z(f) = 2\, S(f+f_0)\, \underbrace{U(f+f_0)}_{\text{gradino}}$$

U(f) is the unit step function (U(f)=0 if f<0, U(f)=1 if f>0)

Se $\underbrace{s(t) = A(t) \cos(2\pi f_0 t + \varphi(t))}$ con A e φ lentamente VARIABILI rispetto ad f_0

$$z(t) = A(t)\, e^{j\varphi(t)}$$

A and phi change slowly with respect to f0 (low frequncy funtions)

$$z(t) = x(t) + j\, y(t)$$

Phase and quadrature components

$$s(t) = \underbrace{x(t)\cos(2\pi f_0 t) - y(t)\sin(2\pi f_0 t)}_{\text{fase e quadratura}}$$

2.2 !!!

$$s_1(t)\, s_2(t) = \frac{1}{2} \mathrm{Re}\left\{ z_1(t)\, z_2^*(t) \right\} + \frac{1}{2} \mathrm{Re}\left\{ z_1(t) z_2(t)\, e^{j 4\pi f_0 t} \right\}$$

$$\int s_1(t)\, s_2(t)\, dt = \frac{1}{2} \mathrm{Re}\left\{ \int z_1(t)\, z_2^*(t)\, dt \right\}$$

$$\int s^2(t)\, dt = \frac{1}{2} \int |z(t)|^2\, dt \longrightarrow \text{Energia}$$

!!!

$$\int A(t) \cos\left(2\pi f_0 t + \varphi_1(t)\right) \cos\left(2\pi f_0 t + \varphi_2(t)\right) dt =$$

$$= \frac{1}{2} \int A(t) \cos\left(\varphi_2(t) - \varphi_1(t)\right) dt$$

$$\int A^2(t) \cos^2\left(2\pi f_0 t + \varphi(t)\right) dt = \frac{1}{2} \int A^2(t) dt$$

Funzioni base passa banda

Pass-band base functions

$$\phi_k(t) = \mathrm{Re}\left\{ z_k(t) e^{j 2\pi f_0 t} \right\} = A_k(t) \cos\left(2\pi f_0 t + \varphi_k(t)\right)$$

Se consideriamo $j z_k(t) \longrightarrow$

$$\phi_{k'}(t) = \mathrm{Re}\left\{ j z_k(t) e^{j(\cdot)} \right\} = -\mathrm{Im}\left\{ z_k(t) e^{j(\cdot)} \right\}$$

$$= -A_k(t) \sin\left(2\pi f_0 t + \varphi_k(t)\right)$$

Risulta $\phi_k(t) \perp \phi_{k'}(t)$!!!

Se abbiamo un segnale con componenti lungo k e k', si ha:

If we have a signal with components along the axis k and k' we obtain:

$$s_{ik} \phi_k(t) + s_{ik'} \phi_{k'}(t) = \mathrm{Re}\left\{ \left(s_{ik} + j s_{ik'}\right) z_k(t) e^{j(\cdot)} \right\}$$

$\boxed{\text{es: QAM}}$

$$s_i(t) = a\, g(t) \cos(2\pi f_0 t) - b\, g(t) \sin(2\pi f_0 t)$$

$$\underbrace{a = \pm 1, \pm 3, \ldots}$$

$$d = a + j\, b$$

In generale

$$s_i(t) = \mathrm{Re}\left\{ \sum_k d_k\, g(t - kT)\, e^{j 2\pi f_0 t} \right\} =$$

$$= \underbrace{\sum_k a_k\, g(t - kT) \cos(\cdot)} - \sum_k b_k\, g(t - kT) \sin(\cdot)$$

$$\underbrace{d_k = a_k + j\, b_k}$$