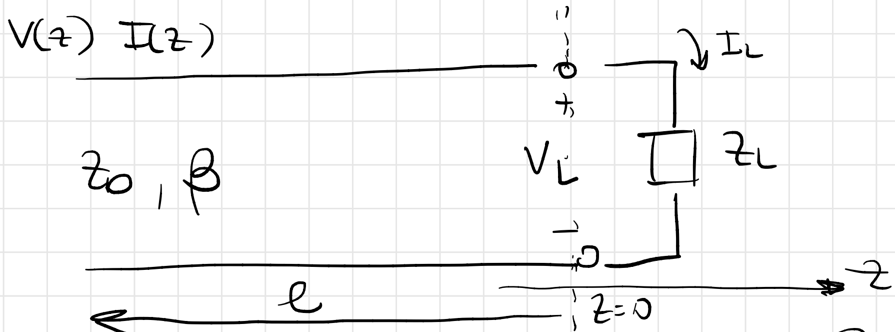
MICROWAVE ENGINEERING

Lecture 8:

Terminated Transmission Lines

TERMINATED LOSSLESS TRANSMISSION LINE



If the line is terminated on a load $Z_L \neq Z_0 \Rightarrow$ The ratio $\frac{V}{I}$ @ the load is Z_L

a Reflected wave is generated

$$\begin{cases} V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \\ I(z) = \dfrac{V_0^+}{Z_0} e^{-j\beta z} - \dfrac{V_0^-}{Z_0} e^{j\beta z} \end{cases}$$

At the load $(z=0)$ $\dfrac{V(0)}{I(0)} = Z_L$

It follows:

$$Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0 \Rightarrow V_0^- = \frac{Z_L - Z_0}{Z_L + Z_0} V_0^+$$

VOLTAGE REFLECTION COEFFICIENT Γ $(z=0)$

$$\boxed{\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

So we can re-write:

$$V(z) = V_0^+ \left[e^{-j\beta z} + \Gamma e^{j\beta z} \right]$$

$$I(z) = \frac{V_0^+}{Z_0} \left[e^{-j\beta z} - \Gamma e^{j\beta z} \right]$$

Standing waves = superposition of FW and BW waves

If the LOAD IS MATCHED to the line, i.e. $\boxed{Z_L = Z_0}$

$$\Downarrow$$

$$\boxed{\Gamma = 0}$$

Time averaged power flow at a generic point (z):

$$P_{av} = \frac{1}{2} \mathrm{Re} \left[V(z) I(z)^* \right] =$$

$$= \frac{1}{2}\frac{|V_0^+|^2}{Z_0}\,\mathrm{Re}\Big\{1 \underbrace{- \Gamma^* e^{-2j\beta z} + \Gamma e^{2j\beta z}}_{\text{purely imaginary terms}} - |\Gamma|^2\Big\}$$

$$P_{av} = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}\left[1 - |\Gamma|^2\right]$$

NOTE 1 — P_{AV} IS CONSTANT ALONG THE LINE

NOTE 2 — Max power transfer occurs for $\Gamma = 0$

No power is delivered if $\Gamma = 1$

The power that is NOT DELIVERED TO THE LOAD is called

RETURN LOSS $(RL) = -20 \log |\Gamma|$ dB if $\Gamma = 0 \rightarrow RL = \infty$ dB

$\Gamma = 1 \rightarrow RL = 0$ dB

if $\Gamma = 0$ $\quad |V(z)| = V_0^+ \rightarrow$ constant along the line

if $\Gamma \neq 0$ we have a standing wave

$$|V(z)| = |V_0^+| \, |1 + \Gamma e^{2j\beta z}| = |V_0^+| \, |1 + \Gamma e^{-2j\beta \ell}| =$$

$z = -\ell$

$$= |V_0^+| \, |1 + |\Gamma| e^{j(\theta - 2\beta\ell)}|$$

distance from the load

$$\Gamma = |\Gamma| e^{j\theta}$$

The voltage oscillates between

$V_{max} = |V_0^+| (1 + |\Gamma|)$ $\qquad$ $V_{min} = |V_0^+| (1 - |\Gamma|)$

A measure of the mismatch on the line is the

STANDING WAVE RATIO $(SWR) = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$

$$1 < SWR < \infty$$

matched mismatched

The distance between two voltage maxima or minima is:

imposing $2\beta l = 2\pi$

$$\underline{l} = \frac{2\pi}{2\beta} = \frac{\pi\lambda}{2\pi} = \underline{\frac{\lambda}{2}}$$

The distance between a maximum and a minimum by imposing $2\beta l = \pi$

$$\underline{l} = \frac{\pi}{2\beta} = \underline{\frac{\lambda}{4}}$$

At any point on the line we can calculate the reflection coefficient at $z = -\ell$

$$\Gamma(-\ell) = \frac{V_0^- e^{-j\beta\ell}}{V_0^+ e^{j\beta\ell}} = \Gamma(0)\, e^{-2j\beta\ell} = \frac{Z_L - Z_0}{Z_L + Z_0} e^{-2j\beta\ell}$$

We can also calculate the impedance at any point on the line:

$$Z_{IN} = Z(-\ell) = \frac{V(-\ell)}{I(-\ell)} = \frac{\cancel{V_0^+}\left[e^{j\beta\ell} + \Gamma e^{-j\beta\ell}\right]}{\cancel{V_0^+}\left[e^{j\beta\ell} - \Gamma e^{j\beta\ell}\right]} Z_0 =$$

$$= Z_0 \frac{1 + \Gamma e^{-2j\beta\ell}}{1 - \Gamma e^{-2j\beta\ell}}$$

knowing

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

we replace Γ $\rightarrow$

$$\underline{Z_{IN}} = Z_0 \frac{(Z_L + Z_0)e^{j\beta l} + (Z_L - Z_0)e^{-j\beta l}}{(Z_L + Z_0)e^{j\beta l} - (Z_L - Z_0)e^{-j\beta l}} =$$

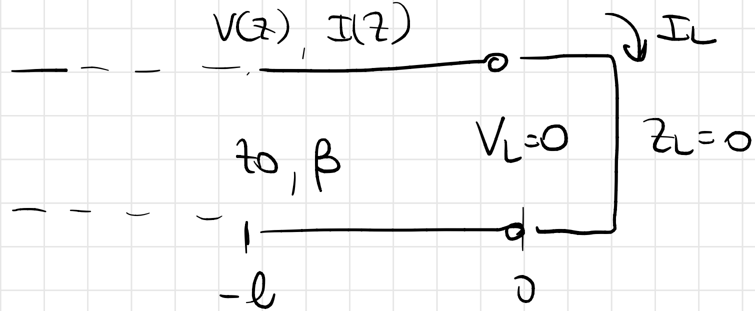
$$= Z_0 \frac{Z_L \cos\beta l + j Z_0 \sin\beta l}{Z_0 \cos\beta l + j Z_L \sin\beta l} = \boxed{Z_0 \frac{Z_L + j Z_0 \tan\beta l}{Z_0 + j Z_L \tan\beta l}}$$

TRANSMISSION LINE IMPEDANCE

SPECIAL CASES

① $\boxed{Z_L = 0}$

SHORT CIRCUIT TERMINATION

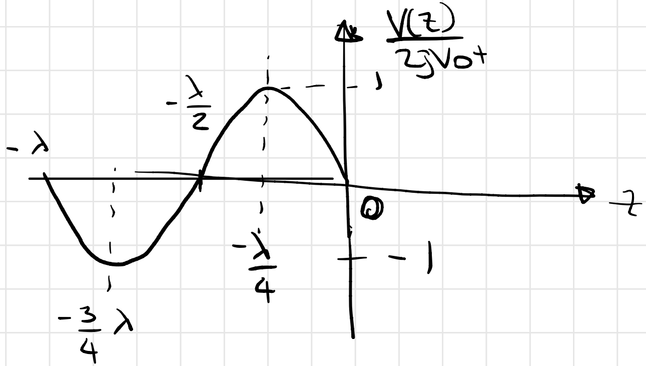


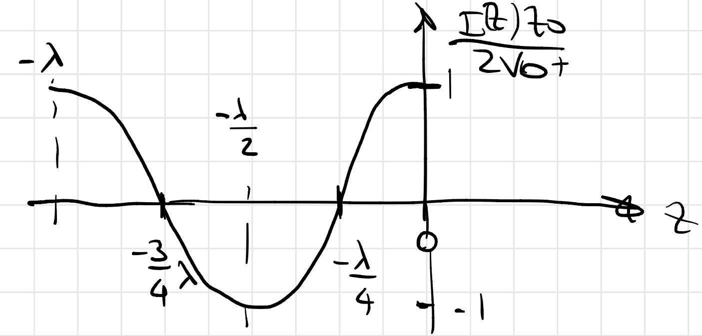
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$$

$$SWR = \frac{1+\Gamma}{1-\Gamma} = \infty$$

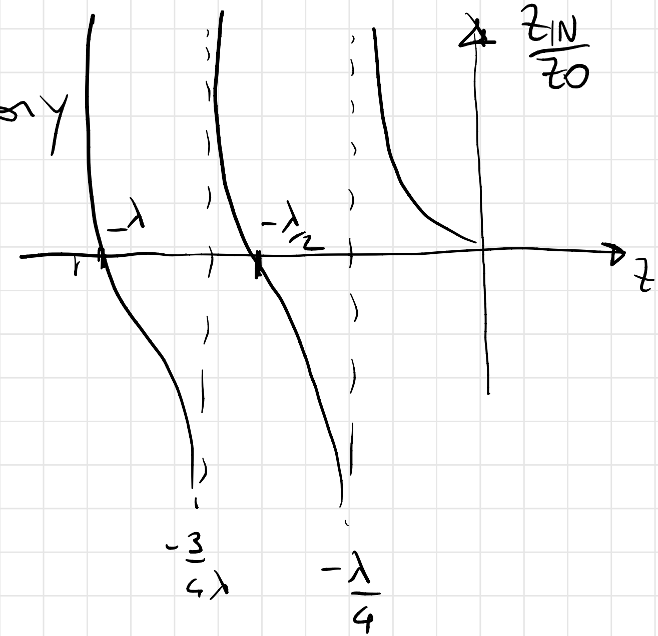
$$V(z) = V_0^+ \left[e^{-j\beta z} - e^{j\beta z} \right] = -2jV_0^+ \sin\beta z$$

$$I(z) = \frac{V_0^+}{Z_0} \left[e^{-j\beta z} + e^{j\beta z} \right] = 2\frac{V_0^+}{Z_0} \cos\beta z$$



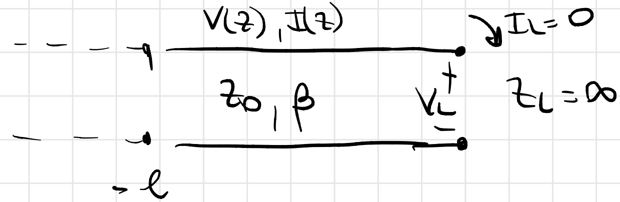


$Z_{IN} = jZ_0 \tan \beta l$ ← purely imaginary



② $\boxed{Z_L = \infty}$

OPEN CIRCUIT TERMINATION



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1 - \cancel{\frac{Z_0}{Z_L}}}{1 + \cancel{\frac{Z_0}{Z_L}}} = 1$$

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \infty$$

$$V(z) = V_0^+ \left[e^{-j\beta z} + e^{j\beta z} \right] = 2V_0 \cos\beta z$$

$$I(z) = \frac{V_0^+}{Z_0} \left[e^{-j\beta z} - e^{j\beta z} \right] = -\frac{2j V_0^+}{Z_0} \sin\beta z$$

$Z_{IN} = -jZ_0 \cot \beta l$ ← purely imaginary

③ $\boxed{l = \frac{\lambda}{2}}$

$$\beta l = \frac{2\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{\cancel{2}} = \pi \Rightarrow \tan \beta l = \tan \pi = 0 \Rightarrow \boxed{Z_{IN} = Z_L}$$

④ $\boxed{l = \frac{\lambda}{4}}$

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2} \Rightarrow \tan \beta l = \tan \frac{\pi}{2} = \infty$$

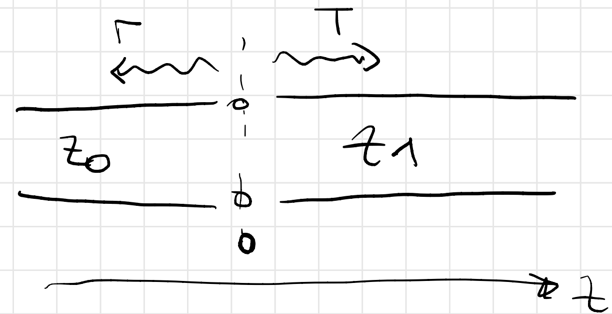
$$\underline{Z_{IN}} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = Z_0 \frac{\cancel{\frac{Z_L}{\tan \beta l}} + \cancel{j} Z_0}{\cancel{\frac{Z_0}{\tan \beta l}} + \cancel{j} Z_L} = \frac{Z_0^2}{Z_L}$$

$$\boxed{Z_{IN} = \frac{Z_0^2}{Z_L}}$$

QUARTER WAVE TRANSFORMER

(5) JUNCTION BETWEEN TWO LINES

$$\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$



How do we calculate the portion of wave that is transmitted

for $z < 0$ $\qquad V(z) = V_0^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$

for $z > 0$ $\qquad V(z) = V_0^+ T e^{-j\beta z}$

At $z = 0$ $\qquad T = 1 + \Gamma = 1 + \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{2Z_1}{Z_1 + Z_0}$

INSERTION LOSS

$$IL = -20 \log_{10} |\bar{T}| \quad dB$$