UNIVERSITÀ DEGLI STUDI DI BRESCIA

Microwave Engineering Final Test
Novembre 15, 2021

Student ID:
Name:
Instructions: You have 2 hours to complete the test. Please write everything with blue or black ink pen so that all your work can be read easily. You can use your calculator. If you don't have a calculator, you can leave the formulas in expression forms and still get full score for the questions/exercises. Use of course notes or internet resources will invalidate the results of the test.
<u>VERY IMPORTANT: Please WRITE YOUR FULL NAME AND STUDENT ID on this sheet and all your sheets where the problems are solved!</u>

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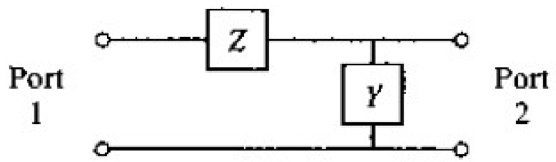
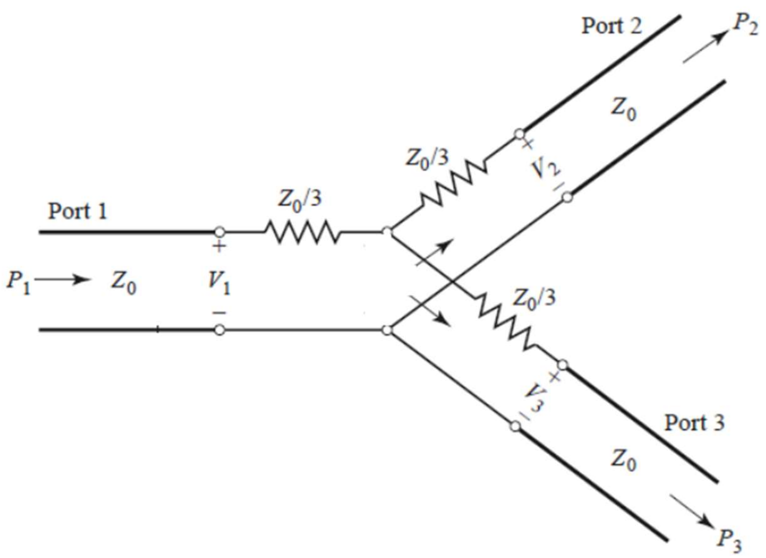
1. A terminated lossless transmission line with Z_0 = 60 Ω has a reflection coefficient at the load of Γ = 0.4. Calculate:
   a. The load impedance.
   b. The reflection coefficient 0.3λ away from the load.
   c. The input impedance at 0.3λ.

2. Find the ABCD matrix for the circuit shown below by direct calculation using the definition of the ABCD matrix:



3. Consider the equal split three ports resistive power divider in figure below. If port 3 is matched, calculate the output power at port 3 when:
   a. port 2 is connected to a matched load;
   b. port 2 is connected to a load having a mismatch of Γ = 0.3.

   Assume the input power at port 1 is P1=1W.



a). Port 2 connected to a matched load

$$P_1 = \frac{1}{2}\frac{|V_1|^2}{Z_0} = 1\,W$$

$$P_2 = \frac{1}{2}\frac{|V_2|^2}{Z_0}, \quad P_3 = \frac{1}{2}\frac{|V_3|^2}{Z_0}$$

$$V_2^+ = V_3^+ = 0$$

$$V_2^- = V_3^- = \frac{1}{2}V_1$$

$$V_3 = V_3^+ + V_3^- = \frac{1}{2}V_1$$

$$P_3 = \frac{1}{2}\frac{|V_3|^2}{Z_0} = \frac{1}{4}P_1$$

$$P_3 = \frac{1}{4}\times 1 = 0.25\,W$$

b). Port 2 connected to a load having a mismatch of $\Gamma = 0.3$

$$V_2^+ = \Gamma V_2^-$$

$$V_3^- = \frac{1}{2}\left[V_1 + V_2^+\right]$$

$$V_3^- = \frac{1}{2}\left[V_1 + \Gamma V_2^-\right]$$

$$\boxed{\begin{aligned} V_2^- &= \frac{1}{2}\left[V_1 + V_3^+\right] \\ V_2^- &= \tfrac{1}{2} V_1 \end{aligned}}$$

$$V_3^- = \frac{1}{2}\left[V_1 + \Gamma \tfrac{1}{2} V_1\right]$$

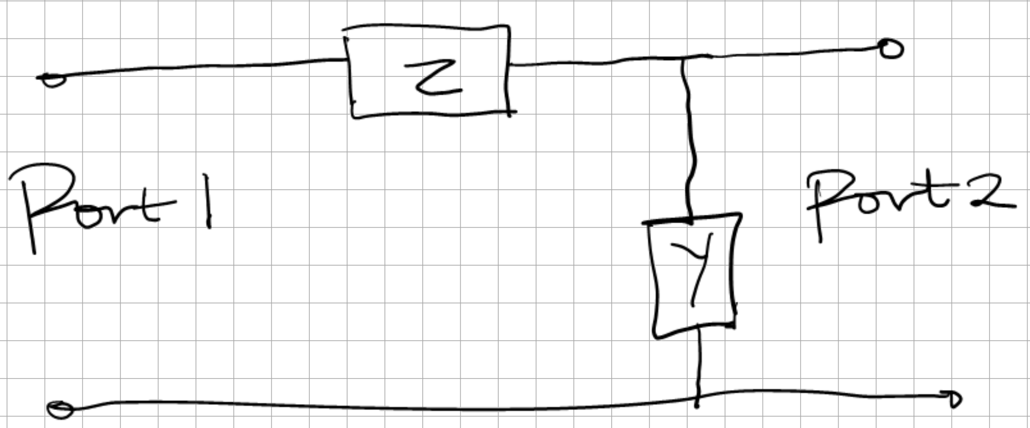
$$V_3^- = \frac{1}{2} V_1 \left[1 + \Gamma/2\right] = \tfrac{1}{2} V_1 \left[1 \cdot 15\right]$$

$$V_3^- = \tfrac{1}{2} V_1 \left[1 \cdot 15\right], \quad V_3 = \tfrac{1}{2} V_1 \left[1 \cdot 15\right]$$

$$P_3 = \frac{1}{2} \frac{|V_3|^2}{Z_0} = \frac{1}{4}\left[1 \cdot 15\right]^2 P_1$$

$$P_3 = 0 \cdot 3306 P_1 = \underline{\underline{0 \cdot 3306\, W}}$$

2) ABCD Matrix



$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{V_1}{\frac{1/Y}{1/Y + Z}\cdot V_1} = \frac{1/Y}{\frac{1+ZY}{Y}}$$

$$= 1 + ZY$$

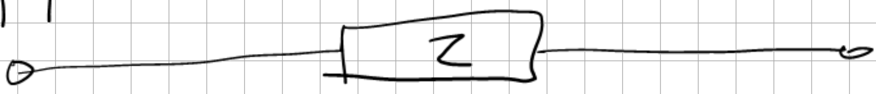
$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{\frac{V_1}{Z}} = Z$$

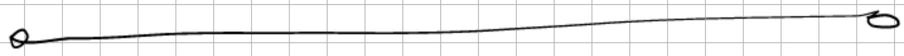
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{I_1 \cdot 1/Y} = Y$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{V_1/Z}{V_1 Y} = \frac{1/Z}{Y} = 1$$

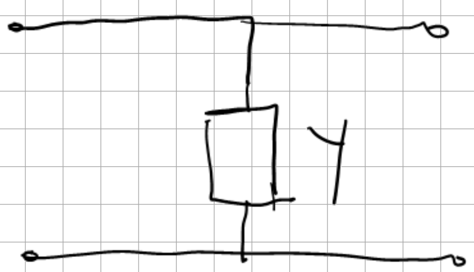
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+ZY & Z \\ Y & 1 \end{bmatrix}$$

You can verify your answer using this approach.





$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} 1+ZY & Z \\ Y & 1 \end{bmatrix}$$

i) $\Gamma = 0.4$, $Z_0 = 60\Omega$

a) Load Impedance

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma(Z_L + Z_0) = Z_L - Z_0$$

$$\Gamma Z_L + \Gamma Z_0 = Z_L - Z_0$$

$$\Gamma Z_0 + Z_0 = Z_L - \Gamma Z_L$$

$$Z_L(1-\Gamma) = Z_0(1+\Gamma)$$

$$Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma}$$

$$Z_L = 60 \times \frac{1+0.4}{1-0.4}$$

$$Z_L = 140\Omega$$

b). Reflection coefficient 0.3λ away from the Load.

$$\Gamma(L) = \Gamma(0) e^{-2j\beta L}$$

$-L = -0.3\lambda$ [Distance away from the load]

$L = 0.3\lambda$

$$\Gamma(0.3\lambda) = 0.4 e^{-2j\frac{2\pi}{\lambda} \cdot 0.3\lambda}$$

$\Gamma(0.3\lambda) = 0.4e^{-j1.2\pi}$

c). Input Impedance at 0.3λ

$$Z_{IN} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$$\tan\beta l \rightarrow \tan\left(\frac{2\pi}{\lambda} \cdot 0.3\lambda\right) \rightarrow \tan(0.6\pi)$$

$$Z_{in} = 60 \frac{140 + j60\tan(0.6\pi)}{60 + j140\tan(0.6\pi)}$$

$$Z_{IN} = 27.89 + j15.61\ \Omega$$