# MICROWAVE ENGINEERING

Lecture 11:
Transmission lines
and Waveguides

WAVEGUIDES

Pros: - high power handling capabilities
- low loss

Cons: - Bulky
- Expensive

COAXIAL LINES

Pros: - high bandwidth
- convenient

Cons: - Difficult for integration

PLANAR TRANSMISSION LINES:

(Stripline, Microstrips, Slotline, Coplanar Waveguides)

→ Planar

→ Low Cost

→ Easy to integrate

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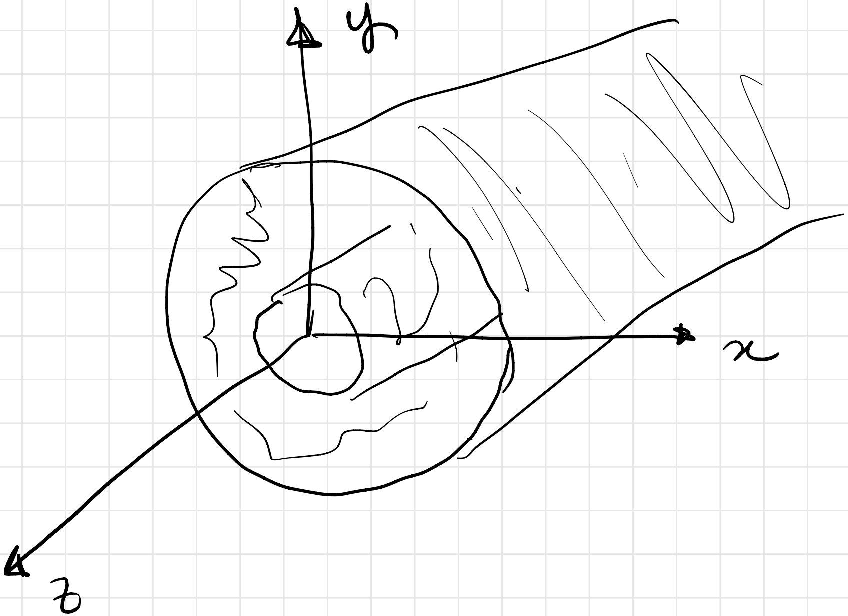
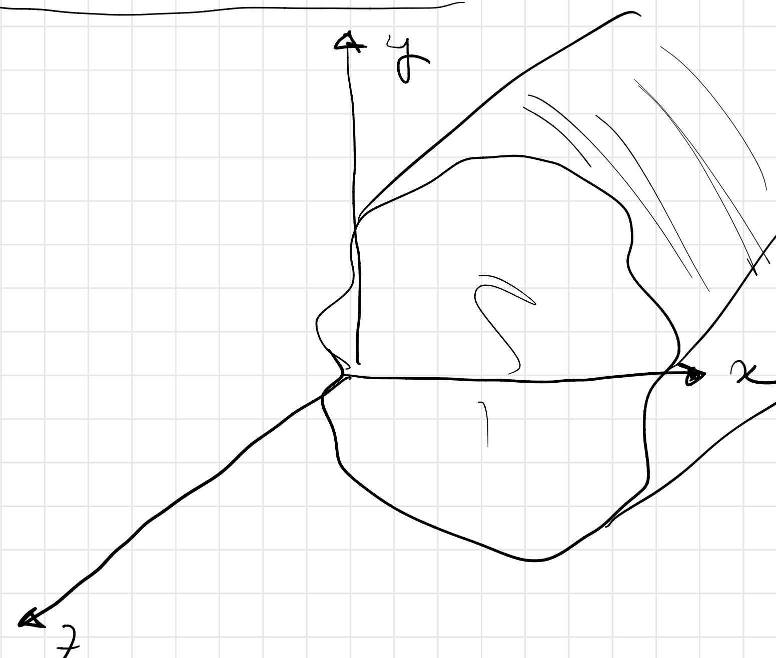
If TL has 2 or more conductors → we can have TRANSVERSE ELECTROMAGNETIC WAVES (TEM Modes)

If TL has only 1 conductor → the WG/TL supports TE and/or TM waves.

# GENERAL SOLUTIONS FOR TEM, TE and TM WAVES




$$\bar{E}(x,y,z) = \left[\bar{e}(x,y) + \hat{z}\, e_z(x,y)\right] e^{-j\beta z}$$

$$\bar{H}(x,y,z) = \left[\bar{h}(x,y) + \hat{z}\, h_z(x,y)\right] e^{-j\beta z}$$

$\bar{e}(x,y)$, $\bar{h}(x,y)$ TRANSVERSE FIELD COMPONENTS

$e_z(x,y)$, $h_z(x,y)$ FIELD COMPONENTS in the propagation direction

in case either the conductor is lossy or the dielectric filling the WG is lossy $\longrightarrow$ $\boxed{\beta \neq \gamma}$

Assuming we have NO SOURCES:

$$\begin{cases} \nabla \times \bar{E} = -j\omega\mu\bar{H} & (1) \\ \nabla \times \bar{H} = j\omega\varepsilon\bar{E} & (2) \end{cases}$$

for our fields $\dfrac{\partial}{\partial z} = -j\beta$

(1)
$$\begin{cases} \dfrac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu H_x \\ -j\beta E_x - \dfrac{\partial E_z}{\partial x} = -j\omega\mu H_y \\ \dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = -j\omega\mu H_z \end{cases}$$

(2)
$$\begin{cases} \dfrac{\partial H_z}{\partial y} + j\beta H_y = j\omega\varepsilon E_x \\ -j\beta H_x - \dfrac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \\ \dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \end{cases}$$

Extracting $E_x$, $E_y$, $H_x$, $H_y$ as a function of $E_z$ and $H_z$:

$$\begin{cases} H_x = \dfrac{j}{K_c^2}\left(\omega\varepsilon \dfrac{\partial E_z}{\partial y} - \beta \dfrac{\partial H_z}{\partial x}\right) \\[2ex] H_y = \dfrac{-j}{K_c^2}\left(\omega\varepsilon \dfrac{\partial E_z}{\partial x} + \beta \dfrac{\partial H_z}{\partial y}\right) \\[2ex] E_x = \dfrac{-j}{K_c^2}\left(\beta \dfrac{\partial E_z}{\partial x} + \omega\mu \dfrac{\partial H_z}{\partial y}\right) \\[2ex] E_y = \dfrac{j}{K_c^2}\left(-\beta \dfrac{\partial E_z}{\partial y} + \omega\mu \dfrac{\partial H_z}{\partial x}\right) \end{cases}$$

$$\boxed{k_c^2 = k^2 - \beta^2}$$ CUTOFF WAVENUMBER

$k = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda}$

If losses are present in the dielectric then it is sufficient to define

$$\boxed{\varepsilon = \varepsilon_0 \varepsilon_r (1 - j \tan\delta)}$$

**TEM Waves :** $E_z = H_z = 0$

$$\begin{cases} j\beta E_y = -j\omega\mu H_x \\ -j\beta H_x = j\omega\varepsilon E_y \end{cases}$$

$$\begin{cases} H_x = \dfrac{\gamma \beta E_y}{-j\omega\mu} = -\dfrac{\beta E_y}{\omega\mu} \\ \cancel{-j}\beta\left(-\dfrac{\beta E_y}{\omega\mu}\right) = \cancel{j}\omega\varepsilon E_y \end{cases}$$

$$\Downarrow$$

$$\beta^2 E_y = \omega^2 \mu \varepsilon E_y$$

$$\Downarrow$$

$$\boxed{\beta = \omega\sqrt{\mu\varepsilon} = k} \Rightarrow \boxed{k_c = 0}$$

The Helmholtz equation for $E_x$ :

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+k^2\right)E_x=0$$

$$\frac{\partial^2}{\partial z^2}E_x=-\beta^2 E_x=-k^2 E_x$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)E_x=0$$

solve for $E_y$

$$\boxed{\nabla_t^2\, \bar{e}\,(x,y)=0}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)$$

The transverse electric fields of a TEM wave satisfy the laplace equation

Similarly we find $\boxed{\nabla_t^2 h(x,y) = 0}$

TRANSVERSE FIELDS of a TEM wave are the same as the STATIC FIELDS BETWEEN 2 CONDUCTORS

In the electrostatic case ELECTRIC FIELDS CAN BE EXPRESSED AS The gradient of the scalar potential $\phi(x,y)$

$$\boxed{\bar{e}(x,y) = -\nabla_t \phi(x,y)}$$

$$\nabla_t \times \bar{e} = -j\omega\mu h_z \hat{z} = 0$$

$$\nabla \bar{D} = \varepsilon \nabla_t \bar{e} = 0 \quad \Rightarrow \quad \boxed{\nabla_t^2 \phi(x,y) = 0}$$

The voltage between the two conductors is:

$$V_{12} = \Phi_1 - \Phi_2 = \int_1^2 E\,dl$$

The current is

$$I = \oint_C H\,dl$$

C is the contour of the conductor

## TEM WAVE IMPEDANCE

$$Z_{TEM} = \frac{E_x}{H_y} = \frac{j\beta}{j\omega\varepsilon} = \frac{\omega\sqrt{\mu\varepsilon}}{\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}} = \eta$$

$$j\beta H_y = j\omega\varepsilon E_x$$

$$Z_{TEM} = -\frac{E_y}{H_x} = \sqrt{\frac{\mu}{\varepsilon}} = \eta$$

$$\boxed{\bar{h}(x,y) = \frac{1}{Z_{TEM}} \hat{z} \times \bar{e}(x,y)}$$

## TE WAVES

$E_z = 0 \quad H_z \neq 0$

$$\begin{cases} H_x = \dfrac{-j\beta}{k_c^2} \dfrac{\partial H_z}{\partial x} \\[2ex] H_y = \dfrac{-j\beta}{k_c^2} \dfrac{\partial H_z}{\partial y} \\[2ex] E_x = \dfrac{-j\omega\mu}{k_c^2} \dfrac{\partial H_z}{\partial y} \\[2ex] E_y = \dfrac{j\omega\mu}{k_c^2} \dfrac{\partial H_z}{\partial x} \end{cases}$$

$k_c^2 \neq 0 \qquad \beta^2 = \sqrt{k^2 - k_c^2}$ function of $\omega$ and geometry

To find $h_z$ we solve:

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \underbrace{\frac{\partial^2}{\partial z^2}}_{-\beta^2 H_z} + k^2 \right) H_z = 0$$

$$H_z = h_z e^{-j\beta z}$$

$$\boxed{\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2 \right) H_z = 0}$$

Solved by imposing proper boundary conditions for the specific WG

$\eta = \sqrt{\frac{\mu}{\varepsilon}}$

TE WAVE IMPEDANCE

$$Z_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\omega\mu}{\beta} = \frac{k\eta}{\beta}$$

# TM WAVES

$E_z \neq 0 \qquad H_z = 0$

$$\begin{cases} H_x = \dfrac{j\omega\varepsilon}{k_c^2} \dfrac{\partial E_z}{\partial y} \\[2ex] H_y = \dfrac{-j\omega\varepsilon}{k_c^2} \dfrac{\partial E_z}{\partial x} \\[2ex] E_x = \dfrac{-j\beta}{k_c^2} \dfrac{\partial E_z}{\partial x} \\[2ex] E_y = \dfrac{-j\beta}{k_c^2} \dfrac{\partial E_z}{\partial y} \end{cases}$$

$k_c \neq 0 \qquad \beta = \sqrt{k^2 - k_c^2}$

Solving Helmholtz eq. for $E_z$ we get:

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \underbrace{\frac{\partial^2}{\partial z^2}}_{-\beta^2 E_z} + k^2 \right) E_z = 0$$

$$\downarrow$$

$$\boxed{\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2 \right) E_z = 0}$$

TM Wave Impedance

$$Z_{TM} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\beta}{\omega \varepsilon} = \frac{\beta \eta}{k}$$

ATTENUATION $\rightarrow$ DIELECTRIC $(\alpha_d)$
$\searrow$ CONDUCTOR $(\alpha_c)$

$$\alpha = \alpha_d + \alpha_c$$

- the attenuation of the dielectric can be introduced through the dielectric permittivity

$$\gamma = \alpha_d + j\beta = \sqrt{k_c^2 - k^2} =$$

$$= \sqrt{k_c^2 - \omega^2 \mu_0 \varepsilon_0 \varepsilon_r (1 - \tan\delta)}$$

Since $\tan\delta \ll 1$ $\quad \sqrt{a^2 + x^2} \simeq a^2 + \frac{1}{2}\left(\frac{x^2}{a}\right)$ if $x \ll a$

It follows that:

$$\gamma = \sqrt{k_c^2 - k^2 + j k^2 \tan\delta} \simeq \sqrt{k_c^2 - k^2} + \frac{j k^2 \tan\delta}{2\underbrace{\sqrt{k_c^2 - k^2}}_{j\beta}} =$$

$$= j\beta + \boxed{\frac{k^2 \tan\delta}{2\beta}}$$

$$\boxed{\alpha_d = \frac{k^2 \tan\delta}{2\beta}} \quad Np/m \qquad \text{(TE or TM)}$$

for TEM waves $\quad k_c = 0 \Rightarrow \beta = k$

$$\boxed{\alpha_d = \frac{k \tan\delta}{2}} \quad \frac{Np}{m} \quad \text{(TEM)}$$

- Attenuation due to conductors can be included by using the PERTURBATION METHOD

$$P(z) = P_0 \, e^{-2\alpha z}$$

Power at $z=0$

We can define the power loss per unit length

$$P_\ell(z) = -\frac{\partial P}{\partial z} = 2\alpha P_0 e^{-2\alpha z} = 2\alpha P(z)$$

$$\boxed{\alpha_c = \frac{P_\ell(z)}{2P(z)} = \frac{P_\ell(z=0)}{2P_0}}$$