Spatial Optics
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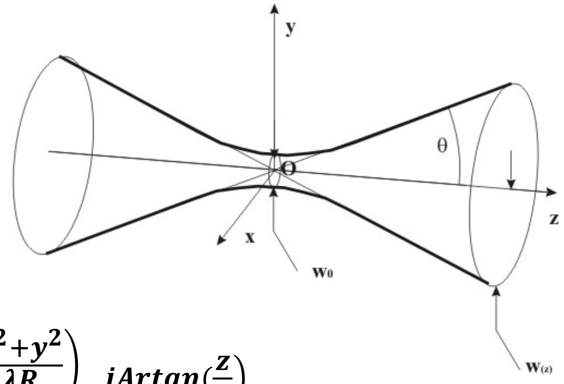
Gaussian beams

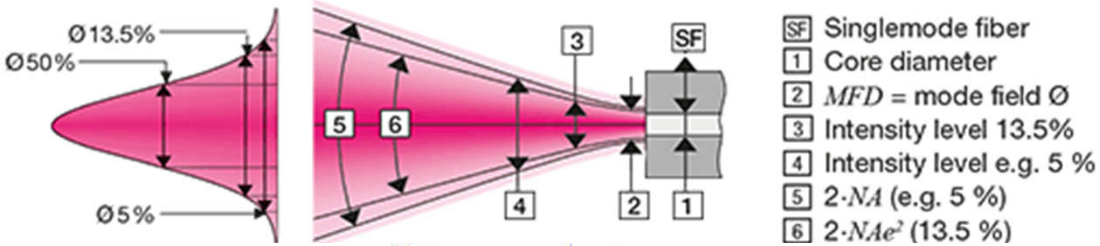


Why?

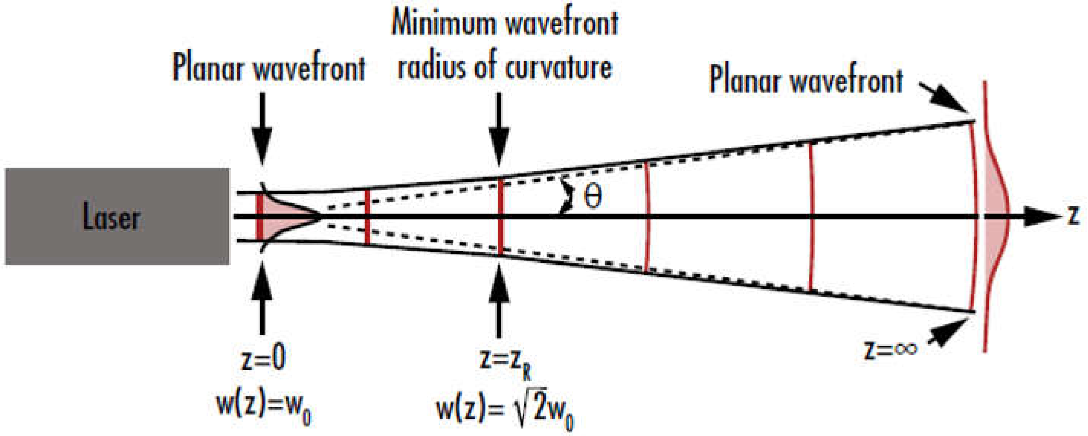
The only realistic solution for a free beam propagation

$$E_z(x,y) = E_0 \frac{e^{-(\frac{x^2+y^2}{w^2})} e^{-jkz} e^{-j\pi(\frac{x^2+y^2}{\lambda R})} e^{jArtan(\frac{z}{\alpha})}}{\sqrt{1+\frac{z^2}{\alpha^2}}}$$

Modélisation of the single mode Optical fibre beams



Modélisation of the single mode Laser beams



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Propagation of a gaussian beam

Gaussian beam

$$E_0(x,y) = \boldsymbol{E}_0\, e^{-\left(\frac{x^2+y^2}{w_0^2}\right)}$$

$$\Downarrow$$

$$E_z(x,y) = \boldsymbol{E}_0 \frac{e^{-\left(\frac{x^2+y^2}{w^2}\right)} e^{-jkz} e^{-j\pi\left(\frac{x^2+y^2}{\lambda R}\right)} e^{jArtan\left(\frac{z}{\alpha}\right)}}{\sqrt{1+\frac{z^2}{\alpha^2}}}$$

with

$$\alpha = \frac{\pi . w_0^2}{\lambda}$$

$$\boldsymbol{R} = z + \frac{\alpha^2}{z}$$

$$\boldsymbol{w}^2 = \boldsymbol{w}_0^2\left(1 + \frac{z^2}{\alpha^2}\right)$$

Two aspects :

*** Demonstration of the analytic solution**

*** Analysis of the formula**

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Demonstration of the gaussian beam formula

General principle

LTI system Σ

$$\begin{tikzcd}
in(x,y) \arrow[r, "* PSF"] \arrow[d, "FT"', "\textcircled{1}"] & out(x,y) = in * PSF = FT^{-1}(\widetilde{in}.TF) \\
\widetilde{in}(N_x,N_y) \arrow[r, ".\mathrm{TF}\ \textcircled{2}"] & \widetilde{out}(N_x,N_y)=\widetilde{in}(N_x,N_y).\mathrm{TF} \arrow[u, "\textcircled{3}", "\mathrm{FT}^{-1}"']
\end{tikzcd}$$

❶ **Input field spectrum**

② **Transfert function and output spectrum determination**

③ **Output field derivation**

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❶ Input field spectrum calculation

$$E_0(t) = E_0\, e^{-\pi t^2} \xrightarrow{\text{FT}} \widetilde{E_0}(v) = E_0\, e^{-\pi v^2}$$

$$f(t) \xrightarrow{\text{FT}} \tilde{f}(v)$$

$$f(kt) \xrightarrow{\text{FT}} \frac{1}{k}\tilde{f}\left(\frac{v}{k}\right)$$

$$f(kx) \xrightarrow{\text{FT}} \frac{1}{k}\tilde{f}\left(\frac{N_x}{k}\right)$$

$$f(kx, ky) \xrightarrow{\text{FT}} \frac{1}{k^2}\tilde{f}\left(\frac{N_x}{k}, \frac{N_y}{k}\right)$$

$$E_0(x,y) = E_0\, e^{-\left(\frac{x^2+y^2}{w_0^2}\right)} \xrightarrow{\text{FT}} \widetilde{E}_0(N_x, N_y) = E_0\, \pi w_0^2 e^{-\pi^2 w_0^2 (N_x{}^2 + N_y{}^2)}$$

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❷ Transfert function an output spectrum determination

$$PW_{z=0}(x,y) = e^{-j(k_x.x+k_y.y)}$$

$$PW_{z\neq 0}(x,y) = e^{-j(k_x.x+k_y.y)}\boxed{e^{-j(k_z.z)}}$$

Transfert function

$$e^{-j\vec{k}.\overrightarrow{\mathbf{OM}}}$$

$$\vec{k} = \frac{2\pi}{\lambda}\vec{n}$$

$$\vec{n} = \begin{matrix}\sin(\alpha)\\ \sin(\beta)\\ \cos(\gamma)\end{matrix}$$

$$N_x = \frac{k_x}{2\pi} = \frac{sin(\alpha)}{\lambda}$$

$$N_y = \frac{sin(\beta)}{\lambda}$$

$$cos(\gamma) = \sqrt{1-(sin^2(\alpha)+sin^2(\beta))} = 1 - 1/2(sin^2(\alpha)+sin^2(\beta))$$

$$k_z = \frac{2\pi}{\lambda}(1 - \frac{1}{2}\lambda^2({N_x}^2 + {N_y}^2))$$

$$TF(N_x, N_y) =. e^{-j(k_z.z)} = e^{-j\frac{2\pi}{\lambda}z}.e^{+j\frac{\pi}{\lambda}\lambda^2({N_x}^2+{N_y}^2)z}$$

Transfert function

$$\widetilde{E}_0(N_x, N_y) = E_0\,\pi w_0^2 e^{-\pi^2 w_0^2({N_x}^2+{N_y}^2)}$$

Propagation = x Transfert function

$$\widetilde{E}_z(N_x, N_y) = E_0\,\pi w_0^2 e^{-\pi^2 w_0^2({N_x}^2+{N_y}^2)}\,.\,e^{-j\frac{2\pi}{\lambda}z}\,.\,e^{+j\pi\lambda z({N_x}^2+{N_y}^2)}$$

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❸ Output field derivation

$$\tilde{E}_z\left(N_x, N_y\right) = E_0\,\pi w_0^2 e^{-\pi^2 w_0^2 (N_x{}^2+N_y{}^2)} \,.\, e^{-j\frac{2\pi}{\lambda}z} \,.\, e^{+j\pi\lambda z (N_x{}^2+N_y{}^2)}$$

$$= E_0\,\pi w_0^2 e^{-\pi\underbrace{((\pi w_0^2 - j\lambda z)}_{k'^2})(N_x{}^2+N_y{}^2)} \,.\, e^{-j\frac{2\pi}{\lambda}z}$$

$$\Big\downarrow \text{FT}^{-1}$$

$$E_z(x,y) = E_0\,\frac{\pi w_0^2}{\pi w_0^2 - j\lambda z}\, e^{-\pi\frac{(x^2+y^2)}{\pi w_0^2 - j\lambda z}} \,.\, e^{-j\frac{2\pi}{\lambda}z}$$

$$E_z(x,y) = E_0\,\frac{e^{-(\frac{x^2+y^2}{w^2})} e^{-jkz} e^{-j\pi(\frac{x^2+y^2}{\lambda R})} e^{jArtan(\frac{z}{\alpha})}}{\sqrt{1+\frac{z^2}{\alpha^2}}}$$

$$\alpha = \frac{\pi . w_0^2}{\lambda}$$

$$R = z + \frac{\alpha^2}{z}$$

$$w^2 = w_0^2\left(1+\frac{z^2}{\alpha^2}\right)$$

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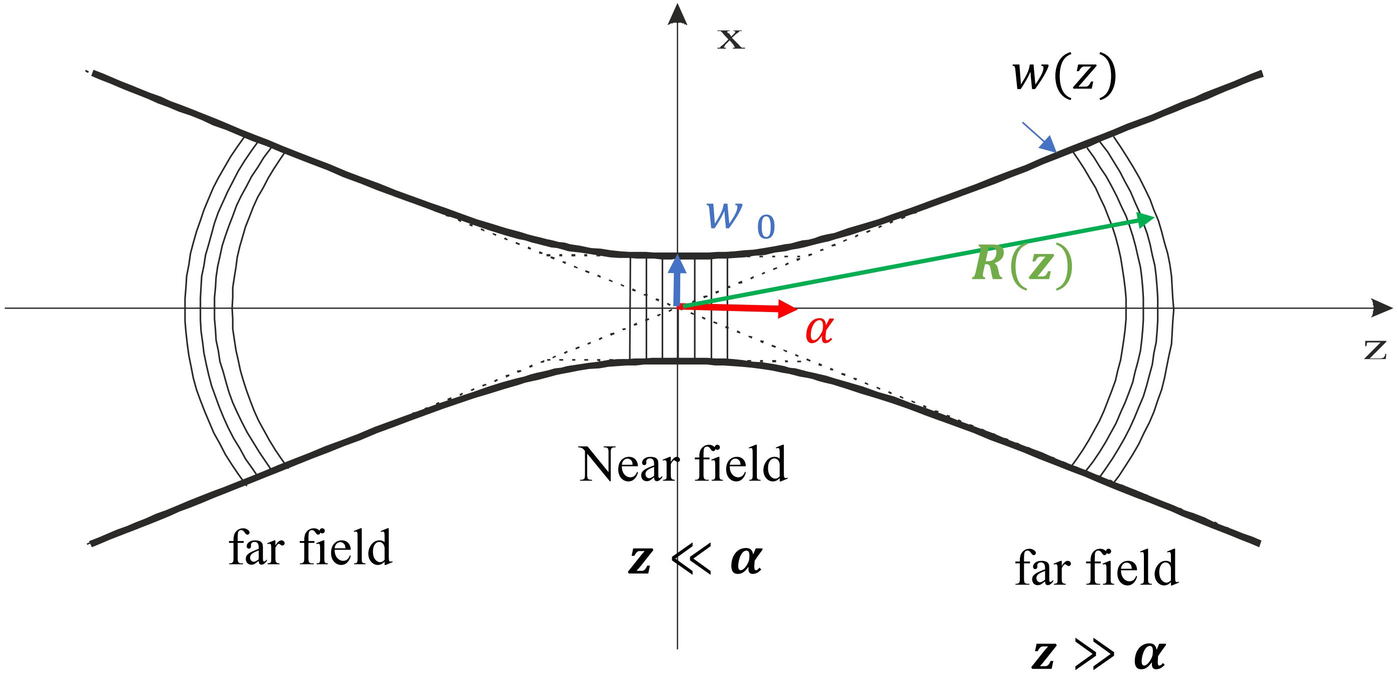
Analysis of the gaussian beam formula

Quite in all part of the formula comparison between z and $\boldsymbol{\alpha} = \frac{\pi w_0^2}{\lambda}$

$$w^2\,(z) = w_0^2\sqrt{1+\frac{z^2}{\alpha^2}}$$

$$R = z + \frac{\alpha^2}{z}$$



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Near field

$z \ll \alpha$

$$E_z(x,y) = E_0 \frac{e^{-(\frac{x^2+y^2}{w_0^2})} e^{-jkz}}{1}$$

far field

$z \gg \alpha$

$$E_z(x,y) = E_0 \frac{e^{-(\frac{x^2+y^2}{(\theta z)^2})} e^{-jkz} e^{-j\pi(\frac{x^2+y^2}{\lambda z})} j}{\frac{z}{\alpha}}$$

$$\theta = \frac{\lambda}{\pi w_0}$$

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