<u>Exercice 5</u> : propagation and chromatic dispersion of LP modes in step index fibers

Data and curves likely to be used in this exercise are given at the end (pages 4-6). The two parts can be addressed separately.

<u>PART 1</u> :

When a powerful monochromatic optical wave is launched in an optical fiber, several different non-linear effects can occur in this fiber, resulting in the creation of new wevelengths. A research team takes advantage of one of these non-linear effects, which is called "Raman shift", for creating a light source in the visible range for applications to spectroscopy. The powerful laser source which is used is a YAG microchip laser emitting at the wavelength λ_p = 1064 nm (see figure 1). The emitted beam first crosses a non-linear crystal in which a part of the beam is converted at the wavelength $\lambda_m = \lambda_p /2$ = 532 nm. These two wavelengths are then injected in a step index optical fiber, called F1. We want to analyze the guiding behavior of this fiber.
Due to the powerful beam at λ_m propagating in the fiber F1, a set of 6 wavelengths λ_n (n= 1 to 6) are created by Raman shift. If we call ν_x the frequency of a radiation at the wavelength λ_x, the frequencies of the radiations at the wavelengths λ_n are given by $\nu_n = \nu_m$ - n.R where R is the Raman shift in silica which value is R = 13 THz.

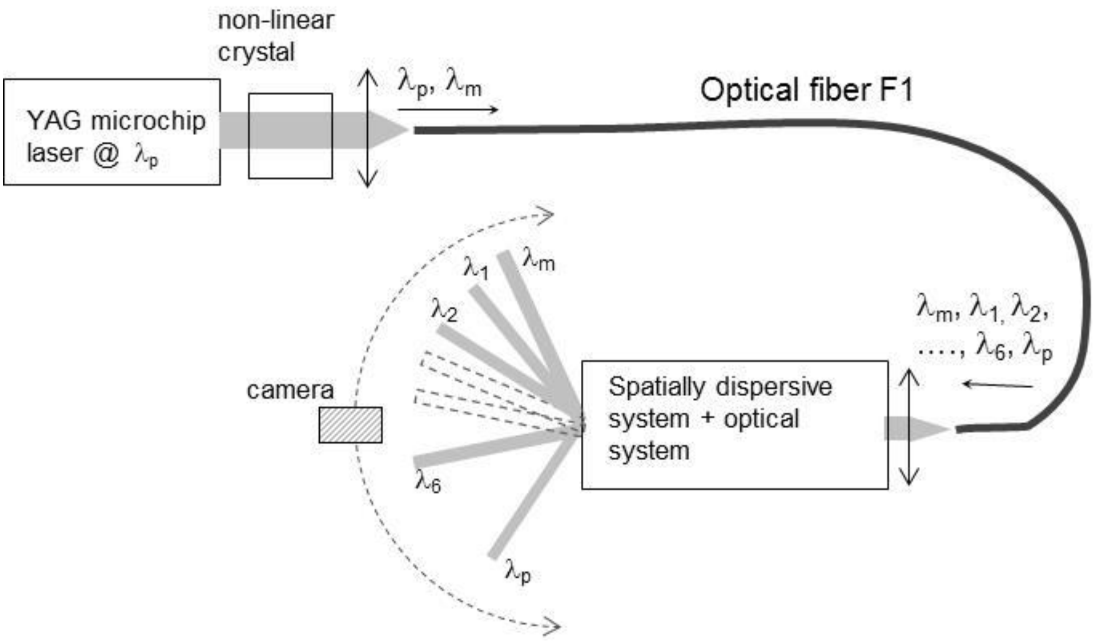
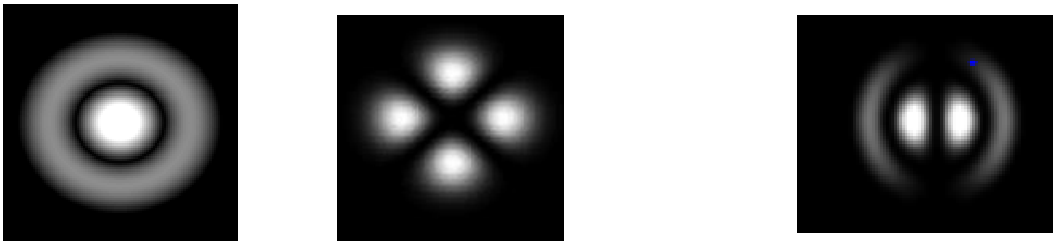


Figure 1

A dispersive system is set at the output of the fiber in order to separate the exiting wavelengths, as shown on the Figure 1. The spatial intensity distribution in a beam at a given wavelength can be recorded by means of a camera suitably positioned in front of this beam. Thanks to a well-designed optical system, set between the output end of the fiber and the camera, the intensity distribution detected by the camera at a given wavelength is homothetic of that existing in the fiber at this wavelength. In other words, it is the same, enlarged by a scaling factor.

The cladding of the fiber is made of pure silica having a refractive index of n_2. In the wavelength range of interest, this index decreases from $n_2(\lambda_m)$ =1,461 down to $n_2(\lambda_p)$ = 1,45 whereas the numerical aperture remains constant and is equal to NA = 0.11.

1/ calculate the refractive index of the core of the fiber at λ_p and at λ_m with a precision of 10^{-3}.
2/Are we allowed to make use of the weak guidance approximation in this fiber, in the range of wavelengths $[\lambda_m, \lambda_p]$? What does it means ?
3/ When we create perturbations in this fiber (bendings for example), the spatial intensity distribution recorder by the camera at λ_p does not changes. What can you conclude of this observation? Deduce a condition concerning the core radius of the fiber.
4/ Show that the wavelength of the 5th Raman line generated in the fiber is λ_5 = 601 nm.
5/ We place the camera in front of the beam at λ_5. For particular arrangements of the fiber (bends, twists…) we sometimes observe intensity distributions on this camera corresponding to that of pure LP modes. For example, we can obtain the intensity distributions D1 or D2 shown on Figure 2. What are the names of the LP modes corresponding to these two intensity distributions ? What other mode(s) should we also observe? Why ? Schematically represent the spatial intensity distribution in this (these) other mode(s).



D1 **D2** **D3**

Figure 2

6/ What new condition on the core radius can we deduce from the above observations ? For the previous results, find an estimation of the core radius to the nearest ±0,2µm.
7/ The camera is now set in front of the beam at λ_m. Could we observe the distribution D3, corresponding the the LP_{12} mode, on the camera? Justify your answer.
8/ The core radius is in fact equal to 3.5µm. Evaluate the effective index of the fundamental mode at λ_p and at λ_m. Deduce the phase velocity of this mode at the two wavelengths.
9/ At the output of the non-linear crystal, we place a dichroic filter which lets pass only the wave at λ_m and strongly attenuates it. The exiting beam is now too weak for creating Raman effect. It is launched into a pie-ce of fiber called F2, single mode at λ_m, so that the beam exiting this fiber is monochromatic (wavelength at λ_m), and perfectly Gaussian. The output end of the fiber F2 is spliced to the input of the fiber F1 by means of a fusion splicer. The two fibers are supposed to be perfectly aligned (no shift or angle between their axes). We also suppose that there is no mode coupling along the propagation in the fiber F1. What mode(s) can we observe at the output? Justify your answer.

PART 2

We make use again of the fiber F1 studied in the part 1. We directly inject in this fiber the beam emitted by the microchip laser at $\lambda_p = 1064$ nm, in order to produce new wavelengths by means of non-linear effects. The spectrum created around $\lambda_p = 1064$ nm depends on the nature of the non-linear effects involved, themselves being dependent on the chromatic dispersion of the fiber at 1064 nm. It is the reason why it is necessary to determine the value of this dispersion. It is the aim of this part of the problem.

Let us remind the opto-geometrical characteristics of the fiber F1at λ_p : core radius a = 3.5µm; numerical aperture NA = 0.11; refractive index of the cladding $n_2(\lambda_p) = 1.45$

1/ at λ_p verify that the normalized spatial frequency of the fiber is V = 2.27 and that the refractive index of the core is $n_1(\lambda_p) = 1,454$

2/ Why can we assert that there is no modal dispersion in the fiber F1 at λ_p?

3/ The polarization mode dispersion is considered as negligible in this fiber. What are the two contributions that are added for constituting the chromatic dispersion of the fiber?

4/ Remind the relationships existing in a material of thickness L, between the group velocity v_g, the group index N_g, and the group delay t_g. Show that the expression of chromatic dispersion in a this bulk material having a refractive index $n_1(\lambda)$ is $D_m = -\dfrac{\lambda}{c}\dfrac{d^2 n_1}{d\lambda^2}$

5/ with the data of this problem, together with the formulas and documents provided at the end, evaluate as precisely as possible:

- the waveguide dispersion (or dispersion of the guide);
- the material dispersion;
- the chromatic dispersion of the fiber.

For these calculations, clearly explain the approach followed and your calculations. The results will be given in ps/(nm.km).

6/ An optical pulse centered around λ_p is launched in a long sample of fiber. Its power is small enough, so that no non-linear effect occurs (no generation of new frequencies or wavelengths). The spectrum of this pulse broadens from λ_1 to λ_2 around λ_p with $\lambda_1 < \lambda_2$. Let us call P1 and P2 two wave packets respectively centered around λ_1 and λ_2. Which among these two packets will exit the fiber first ? Justify your answer.

Data and curves : see pages 4 to 6

With the support of the Erasmus+ Programme of the European Union

Data for the whole problem (parts 1 and 2)

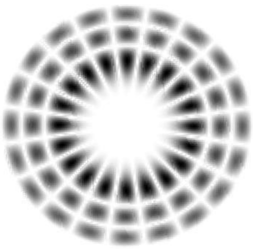
fonction	$J_0(x)$	$J_1(x)$	$J_2(x)$	$J_3(x)$	$J_4(x)$
premier zéro en x =	2,405	0	0	0	0
deuxième zéro en x =	5,52	3,83	5,14	6,38	7,59

First zeros of some Bessel functions of the first kind

mode	LP_{21} et LP_{02}	LP_{31}	LP_{12}	LP_{41}	LP_{22} et LP_{03}	LP_{32}	LP_{13}
V_c	3,83	5,14	5,52	6,38	7,02	8,41	8,65

Normalized cutoff frequencies V_c for some LP modes of step index fibers

$LP_{10,3}$ mode of a step index fiber :



Formulas :

π=3,14159 relative index difference : $\Delta = \dfrac{n_1^2 - n_2^2}{2n_1^2}$

Normalized propagation constant for a given mode, β being the propagation constant of this mode :

$$B = \frac{\beta^2 - k_0^2.n_2^2}{k_0^2.(n_1^2 - n_2^2)}$$

Group index of a material which refractive index is $n_1(\lambda)$: $N_g = n_1 - \lambda \dfrac{dn_1}{d\lambda}$

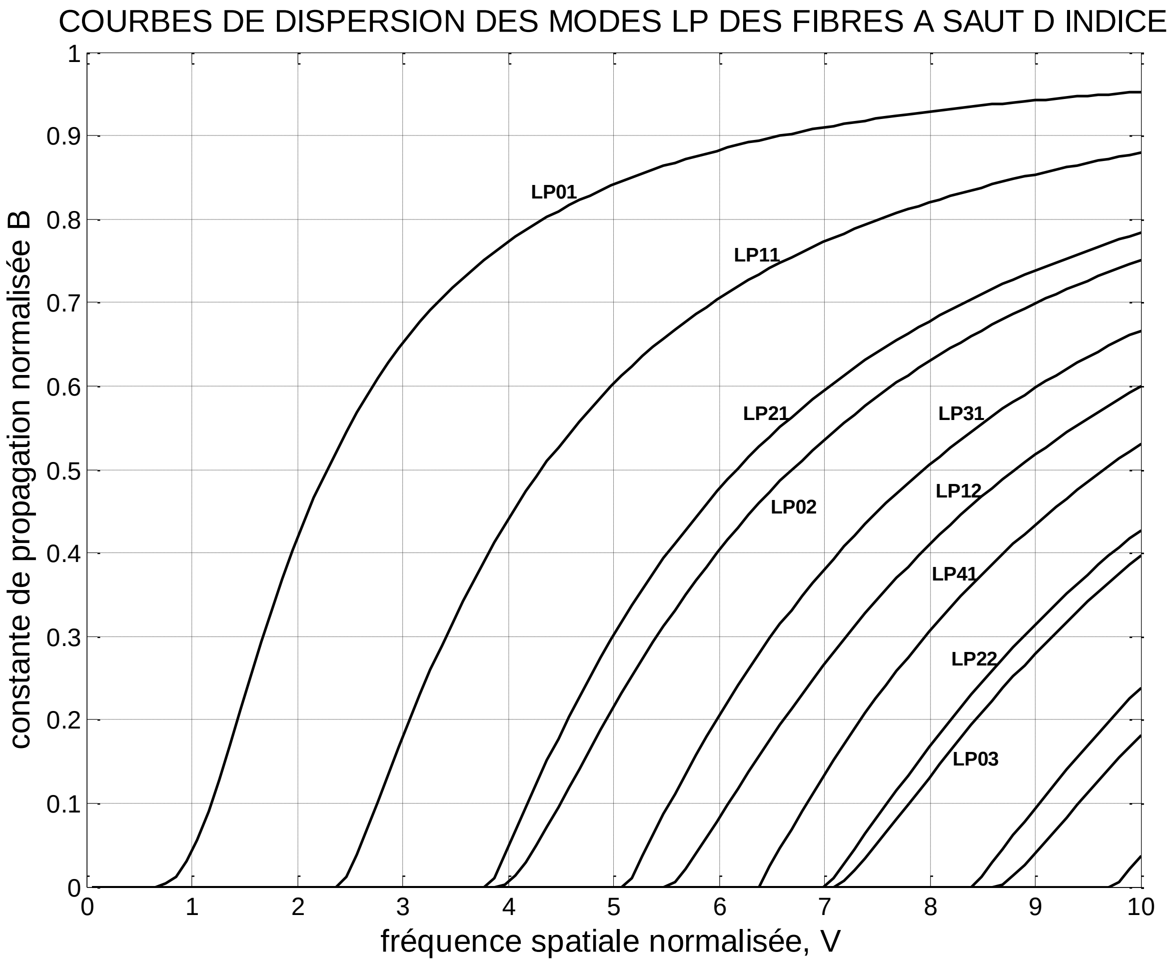
Temporal broadening of a pulse having a spectral width $\Delta\lambda$: $\Delta\tau = \dfrac{dt_g}{d\lambda}.\Delta\lambda$ where t_g is the group delay of the pulse

Chromatic dispersion of a pulse having a spectral width $\Delta\lambda$, and experiencing a time broadening $\Delta\tau$ in a fiber of length L: $D = \dfrac{\Delta\tau}{L.\Delta\lambda}$

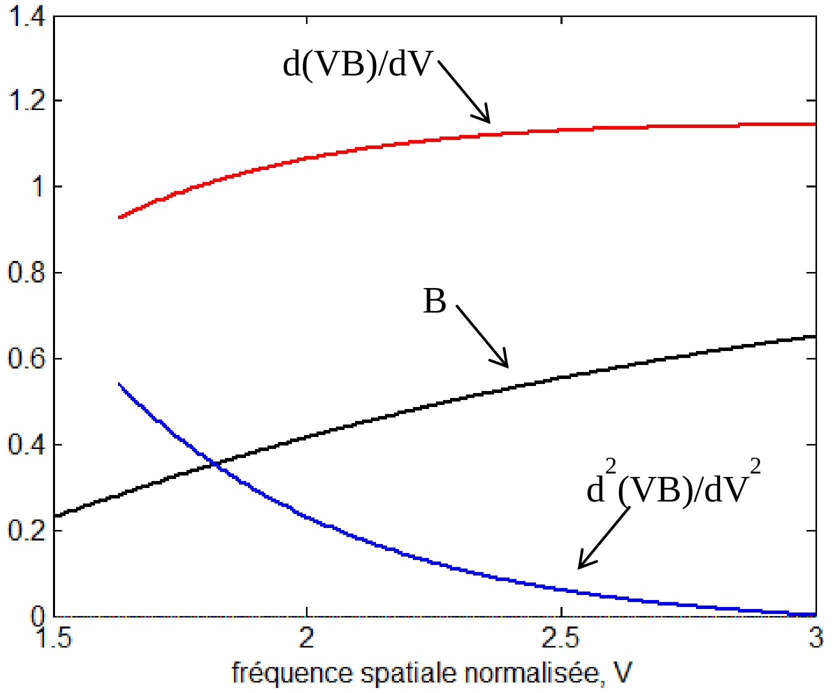
Dispersion of the guide: $D_g = -\dfrac{n_1\Delta}{c\lambda}\left[V\dfrac{d^2(VB)}{dV^2}\right]$

Curves:

Network of dispersion curves B=f(V) for the first LP modes, where V is the normalized spatial frequency and B is the normalized propagation constant: $B = \frac{\beta^2 - k_0^2.n_2^2}{k_0^2.(n_1^2 - n_2^2)}$, β being the propagation constant of the considered mode.



Normalized curves for the fundamental mode :



Curve of $\frac{d^2 n_1}{d\lambda^2} = f(\lambda)$ for the pure silica of refractive index $n_1(\lambda)$: