CHAPTER 4

Dispersion in optical fibers

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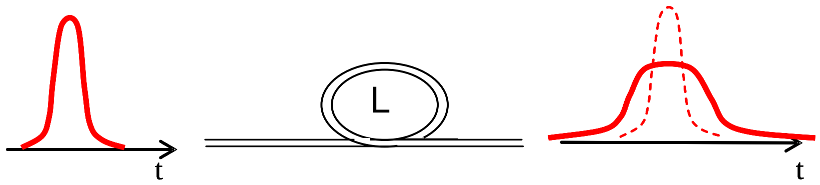
DEFINITION AND CAUSES OF DISPERSION IN OPTICAL FIBERS

DISPERSION : linear phenomenon resulting in a change (generally an increase) of the duration of a pulse when propagating in a fiber



3 causes of dispersion :

- **intermodal dispersion (in multimode regime**) → D_I
- **chromatic dispersion** → D_c
- **polarization mode dispersion** → PMD

* In the multimode regime : D_I >> D_c of each mode >> PMD of each mode

➔ D_c and PMD are neglected

* In the single mode régime : $D_I = 0$

- if D_c of the fundamental mode >> PMD (usual case) ➔ D_c only is taken into account and PMD is neglected
- si D_c of the fundamental mode ~0 ➔ PMD must be taken onto account (for very high bit rate transmissions)

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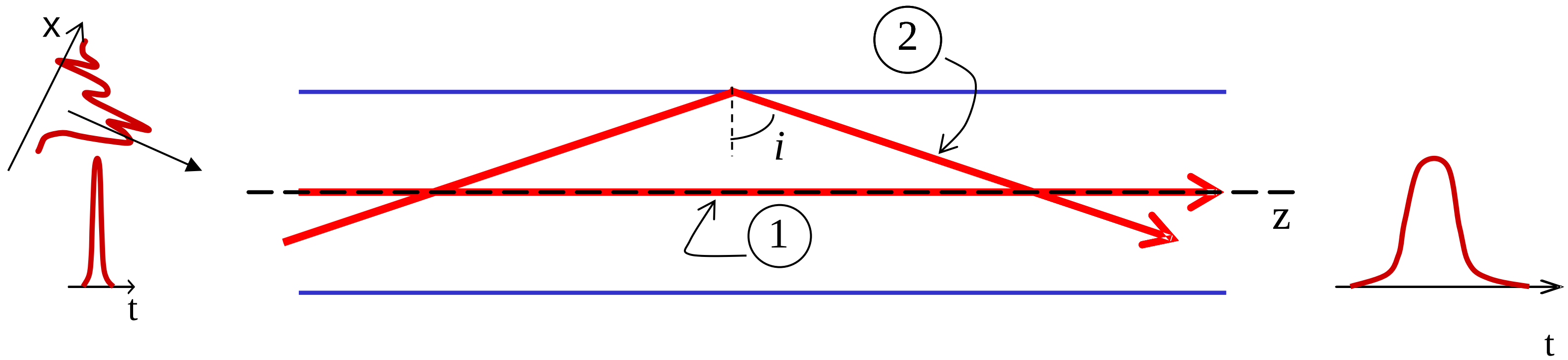
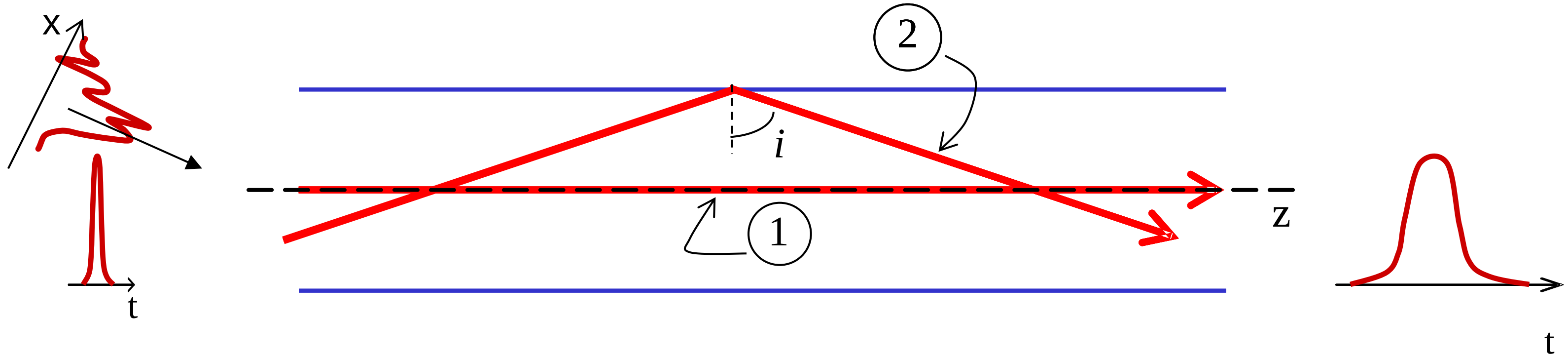
INTERMODALE DISPERSION



➔ due to the fact that the each excited mode has its own group velocity, different from that of the others

➔ D_I = pulse broadening per unit of length along which light propagates (ns/km)

INTERMODALE DISPERSION



➔ due to the fact that the each excited mode has its own group velocity, different from that of the others

➔ D_I = pulse broadening per unit of length along which light propagates (ns/km)

group delay: $$t_g = \frac{L}{v_g}$$

For a step index fiber → $$v_g \approx \frac{c}{n_1}\sin i$$

ray 1 → *sin i =1* → $v_g \approx \dfrac{c}{n_1}$ → $t_{g1} \approx \dfrac{L}{c} n_1$

ray 2 → $\sin i_{\min} = \dfrac{n_2}{n_1}$ → $v_g \approx \dfrac{c}{n_1}\dfrac{n_2}{n_1}$ → $t_{g2} \approx \dfrac{L}{c}\dfrac{n_1^2}{n_2}$

$$\Delta t_g = \tau = t_{g2} - t_{g1}$$

$$\approx \frac{L}{c}.n_1.\left(\frac{n_1}{n_2} - 1\right) \approx \frac{L}{c} n_1 \left(\frac{n_1 - n_2}{n_2}\right)$$

With $$\Delta = \left(\frac{n_1^2 - n_2^2}{2n_1^2}\right) = \frac{(n_1 + n_2)(n_1 - n_2)}{2n_1^2} \approx \frac{2n_1(n_1 - n_2)}{2n_1^2} \approx \frac{n_1 - n_2}{n_1}$$

$$\boxed{\tau = \frac{L}{c} n_1 \Delta}$$ (step index fiber)

INTERMODALE DISPERSION (step index fiber)

Definition : $$\boxed{D_I \triangleq \frac{\tau}{L} = \frac{n_1.\Delta}{c} \quad \text{(en ns/km)}}$$ $$\left(\text{reminder : } \tau = \frac{L}{c} n_1 \Delta \right)$$

Modulation bandwidth, for a fiber of length L : **B=1/τ** **(generally expressed in MHz)**

B.L = B_L =L/τ = 1/D_I = constant **(generally expressed in MHz.km)**

INTERMODALE DISPERSION (step index fiber)

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Modulation bandwidth, for a fiber of length L : **B=1/τ (generally expressed in MHz)**

B.L = B_L =L/τ = 1/D_I = constant **(generally expressed in MHz.km)**

Example :

Step index fiber, length L= 3km with n1 = 1,465 and n2 =1,45

$$\tau = 3 \cdot \frac{1}{3\ 10^5} \cdot 1{,}465 \cdot \left(\frac{1{,}465\text{-}1{,}45}{1{,}45}\right) = 1{,}52\ 10^{-7} s \quad = 152\ ns$$

$$B = \frac{1}{\tau} = \frac{1}{152\ 10^{-9}} = \ 6{,}58\ 10^6\text{Hz} \ = 6{,}58\ \text{MHz}$$

$$B_L = \frac{1}{D_I} = \frac{1}{51\ 10^{-9}} \simeq \ 20\ 10^6\text{Hz.km} \simeq \ 20\ \text{MHz.km}$$

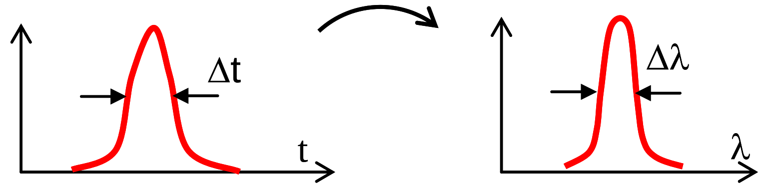
Optimized graded index fiber

τ' = τ/100

B' =100.B

B_L' =100.B_L

CHROMATIC DISPERSION



$\Delta\lambda = \lambda^2 . \Delta f/c$

where Δf = spectral bandwidth of the pulse with Δt. Δf = cte

if Δt=10ps with λ_0 = 1µm

$$\Delta\lambda = \frac{\left(10^{-6}\right)^2}{3.10^8} \text{ x } \frac{1}{10.10^{-12}} = 3.10^{-10} m = 0{,}3nm$$

(with $\Delta t.\Delta f$=1)

<u>causes of chromatic dispersion</u> :

- dispersive material ➔ $n=f(\lambda)$ ➔ $v_\varphi=c/n=f(\lambda)$ ➔ $v_g=f(\lambda)$ ➔ **material dispersion (D_{mat})**

- when the wave is guided, $\beta=f(V)=f(\omega)$ ➔ $v_g=d\omega/d\beta= f(\lambda)$ ➔ **guide dispersion (D_{gui})**

In first approximation : $D_c \approx D_{mat} + D_{gui}$

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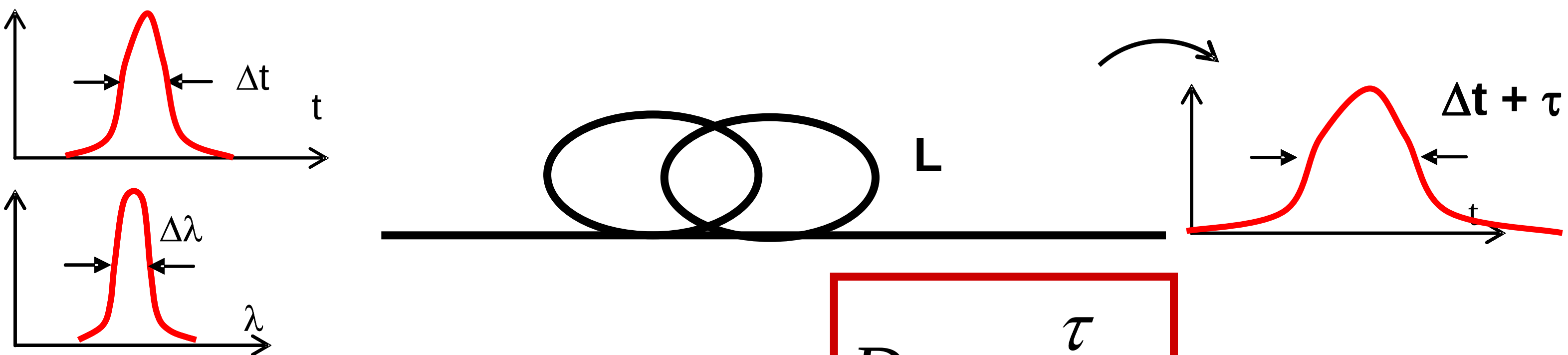
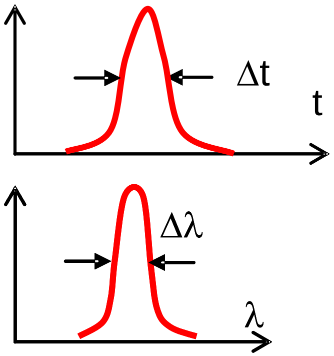
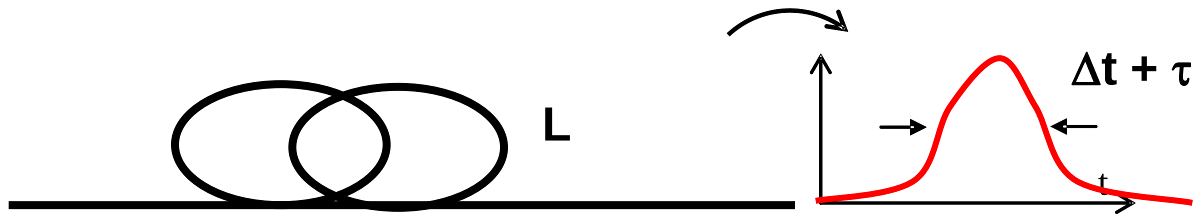
CHROMATIC DISPERSION



Definition : $$\boxed{D_c = \frac{\tau}{L.\Delta\lambda}}$$ expressed in ps/(nm.km)

CHROMATIC DISPERSION





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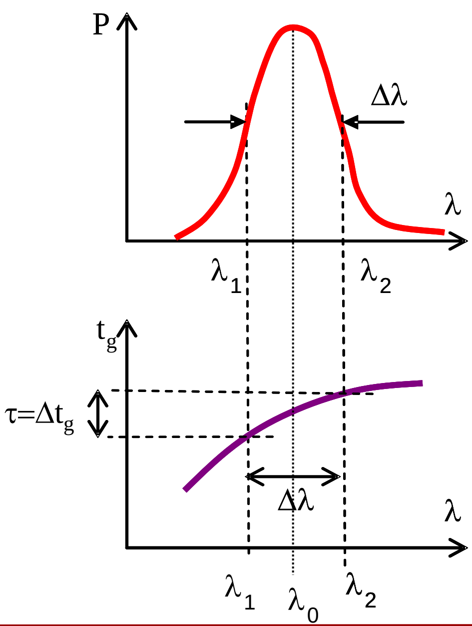

$$\tau = \left| t_g(\lambda_2) - t_g(\lambda_1) \right| \simeq \frac{dt_g}{d\lambda}\Delta\lambda \quad \Rightarrow \quad \boxed{D_c = \frac{1}{L}\frac{dt_g}{d\lambda}}$$

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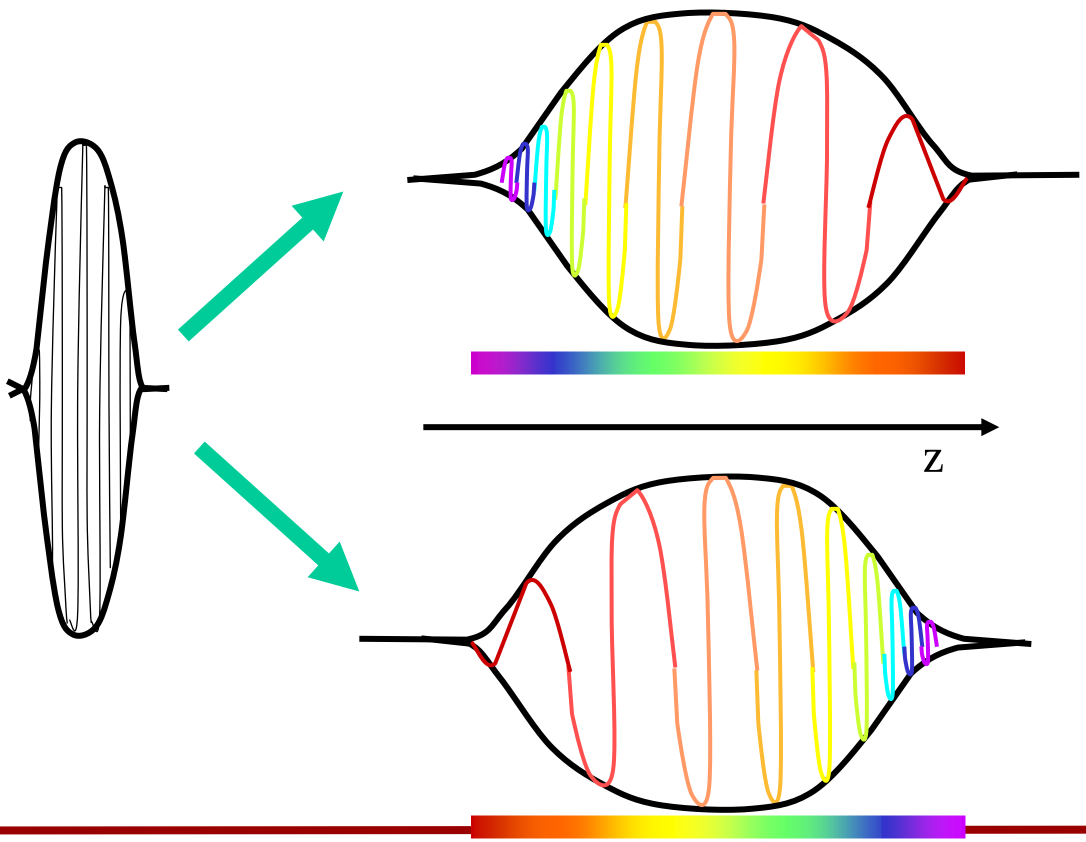
CHROMATIC DISPERSION

$$\boxed{D_c = \frac{1}{L}\frac{dt_g}{d\lambda}} \Rightarrow D_c = \frac{t_{gR} - t_{gB}}{L\,(\lambda_R - \lambda_B)} \quad \text{en ps/(nm.km)}$$



$v_{gR} > v_{gB} \Rightarrow t_{gR} < t_{gB}$

$D_c < 0$
Normal dispersion

$v_{gB} > v_{gR} \Rightarrow t_{gB} < t_{gR}$

$D_c > 0$
Anomalous dispersion

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CHROMATIC DISPERSION : remarks

$$D_c = \frac{1}{L}\frac{dt_g}{d\lambda} = \frac{1}{L}\frac{d}{d\lambda}\left(\frac{L}{v_g}\right) = \frac{d}{d\lambda}\left(\frac{d\beta}{d\omega}\right) = \frac{d}{d\omega}\left(\frac{d\beta}{d\omega}\right).\frac{d\omega}{d\lambda} = -\frac{2\pi c}{\lambda^2}\frac{d^2\beta}{d\omega^2}$$

➔ the dispersion curve β=f(ω) allows to calculate the chromatic dispersion (taking into account the actual values $n_1(\lambda)$ and $n_2(\lambda)$ at each wavelength)

CHROMATIC DISPERSION : remarks

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➔ the dispersion curve $\beta=f(\omega)$ allows calculating the chromatic dispersion (taking into account the actual values $n_1(\lambda)$ and $n_2(\lambda)$ at each wavelength)

Taylor expansion of the spectral phase of the guided wave :

$$\varphi(\omega) = \beta L = \beta(\omega_0).L + L.\frac{d\beta}{d\omega}(\omega-\omega_0) + \frac{L}{2}\frac{d^2\beta}{d\omega^2}(\omega-\omega_0)^2 + \dots$$

out of the "guided optics" community, $\frac{d^2\beta}{d\omega^2}$ is often called "group velocity dispersion"

→ inappropriate denomination and risk of confusion with D_c

in fact $\frac{d^2\beta}{d\omega^2}$ is proportional to the dispersion of group delay

CALCULATION OF THE MATERIAL DISPERSION

→ plane wave (= propagating wave NOT GUIDED)
→ dispersive propagation medium : $n_1 = f(\lambda)$

$$t_g = t_{mat} = \frac{L}{v_g} = \frac{L}{c} N_g = \frac{L}{c}\left(n_1 - \lambda \frac{dn_1}{d\lambda} \right)$$

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$$\tau_{mat} = \Delta t_g = \frac{dt_{mat}}{d\lambda}\Delta\lambda = \frac{L\Delta\lambda}{c}\frac{d}{d\lambda}\left(n_1 - \lambda \frac{dn_1}{d\lambda} \right)$$

$$= \frac{L}{c}\Delta\lambda.\left(\frac{dn_1}{d\lambda} - \left(1x\frac{dn_1}{d\lambda} + \lambda \frac{d^2 n_1}{d\lambda^2} \right) \right)$$

$$= -\frac{\lambda L}{c}\Delta\lambda.\frac{d^2 n_1}{d\lambda^2}$$

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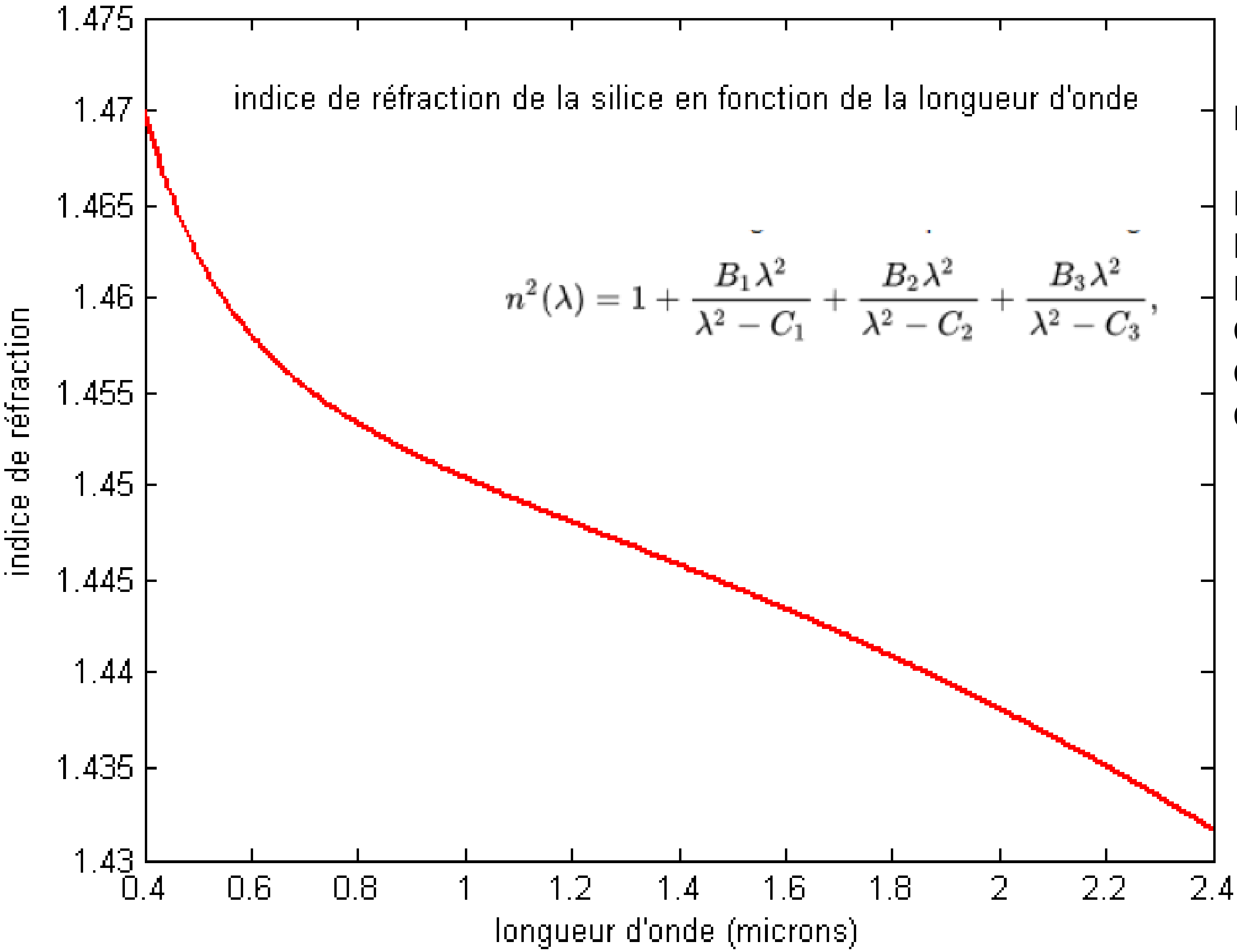
$$\boxed{D_{mat} = \frac{\tau_{mat}}{L.\Delta\lambda} = -\frac{\lambda}{c}\frac{d^2 n_1}{d\lambda^2}}$$

MATERIAL DISPERSION

$$D_{mat} = -\frac{\lambda}{c}\frac{d^2 n_1}{d\lambda^2}$$



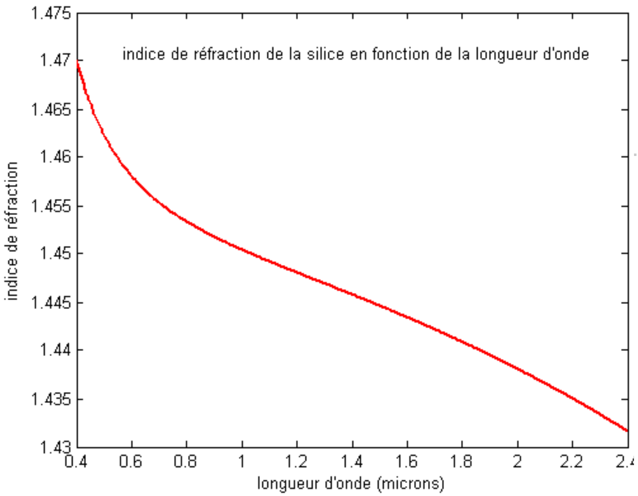
For pure silica :

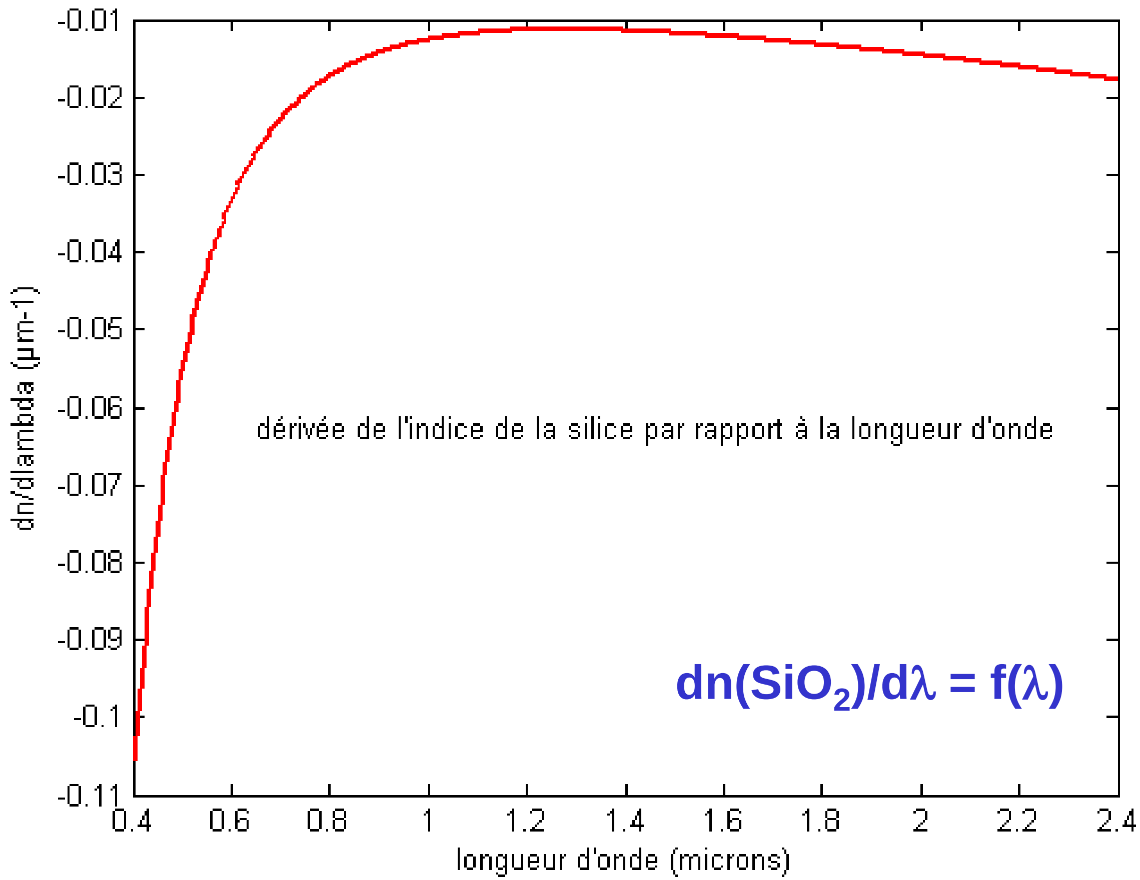
B_1 = 0.696166300
B_2 = 0.407942600
B_3 = 0.897479400
C_1 = $4.67914826 \times 10^{-3}$ µm^2
C_2 = $1.35120631 \times 10^{-2}$ µm^2
C_3 = 97.9340025 µm^2

$n(SiO_2) = f(\lambda)$

MATERIAL DISPERSION

$$D_{mat} = -\frac{\lambda}{c}\frac{d^2 n_1}{d\lambda^2}$$

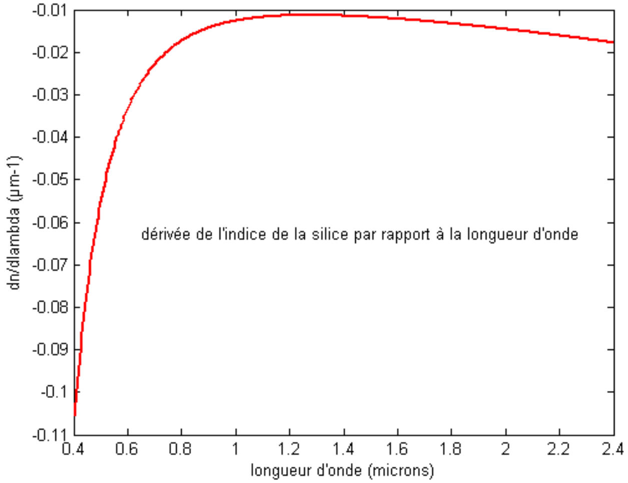


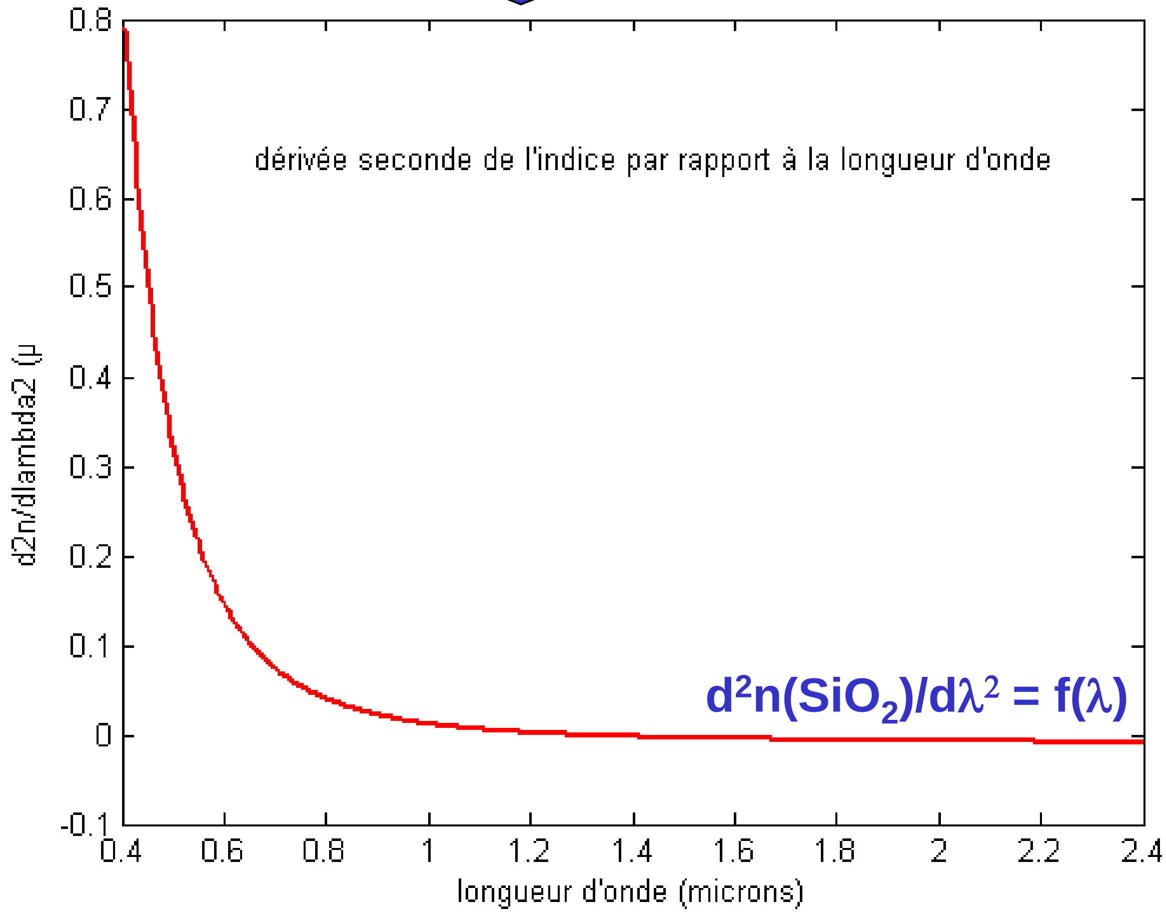


$dn(SiO_2)/d\lambda = f(\lambda)$

MATERIAL DISPERSION

$$D_{mat} = -\frac{\lambda}{c}\frac{d^2 n_1}{d\lambda^2}$$





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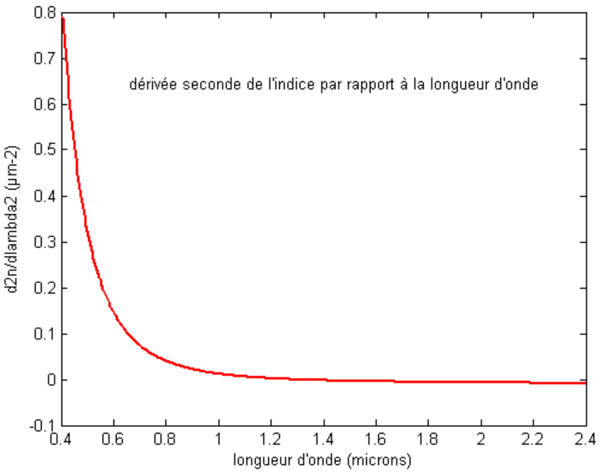
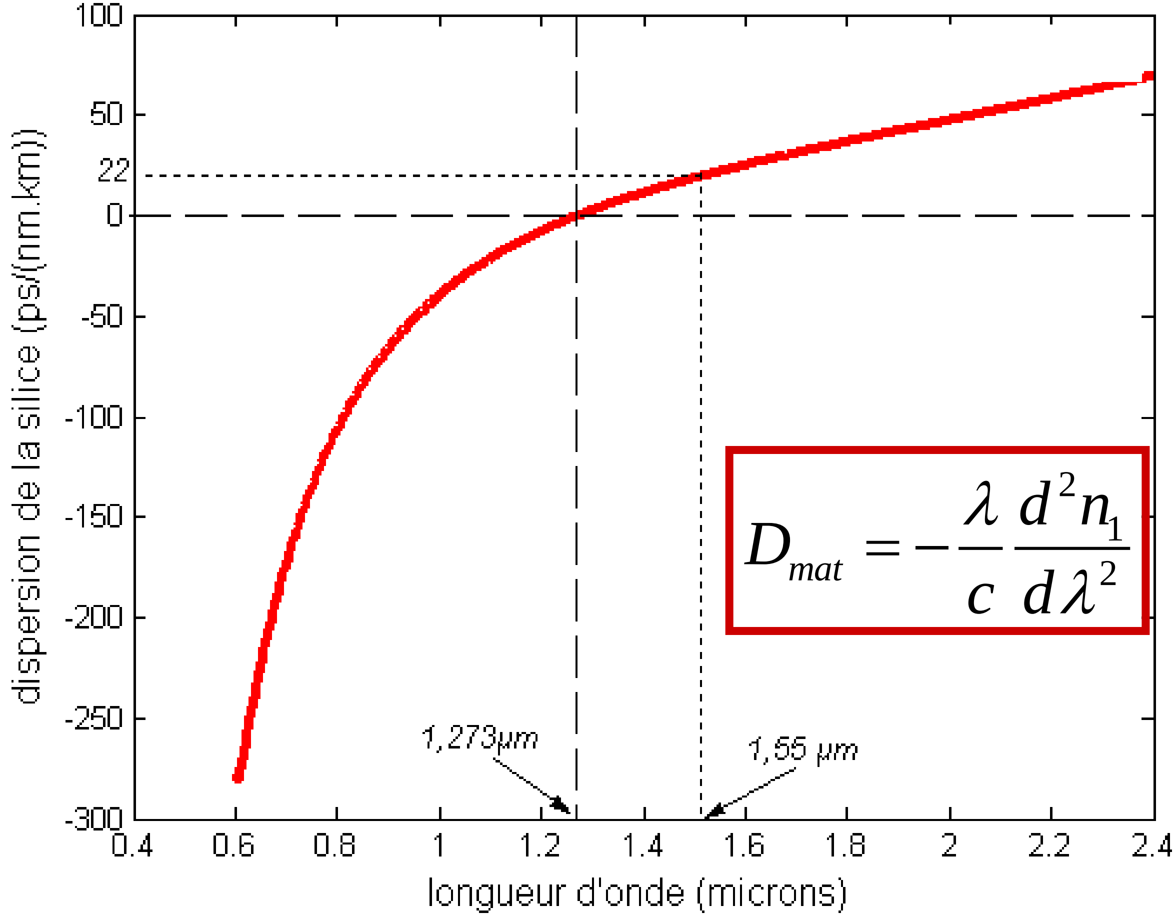
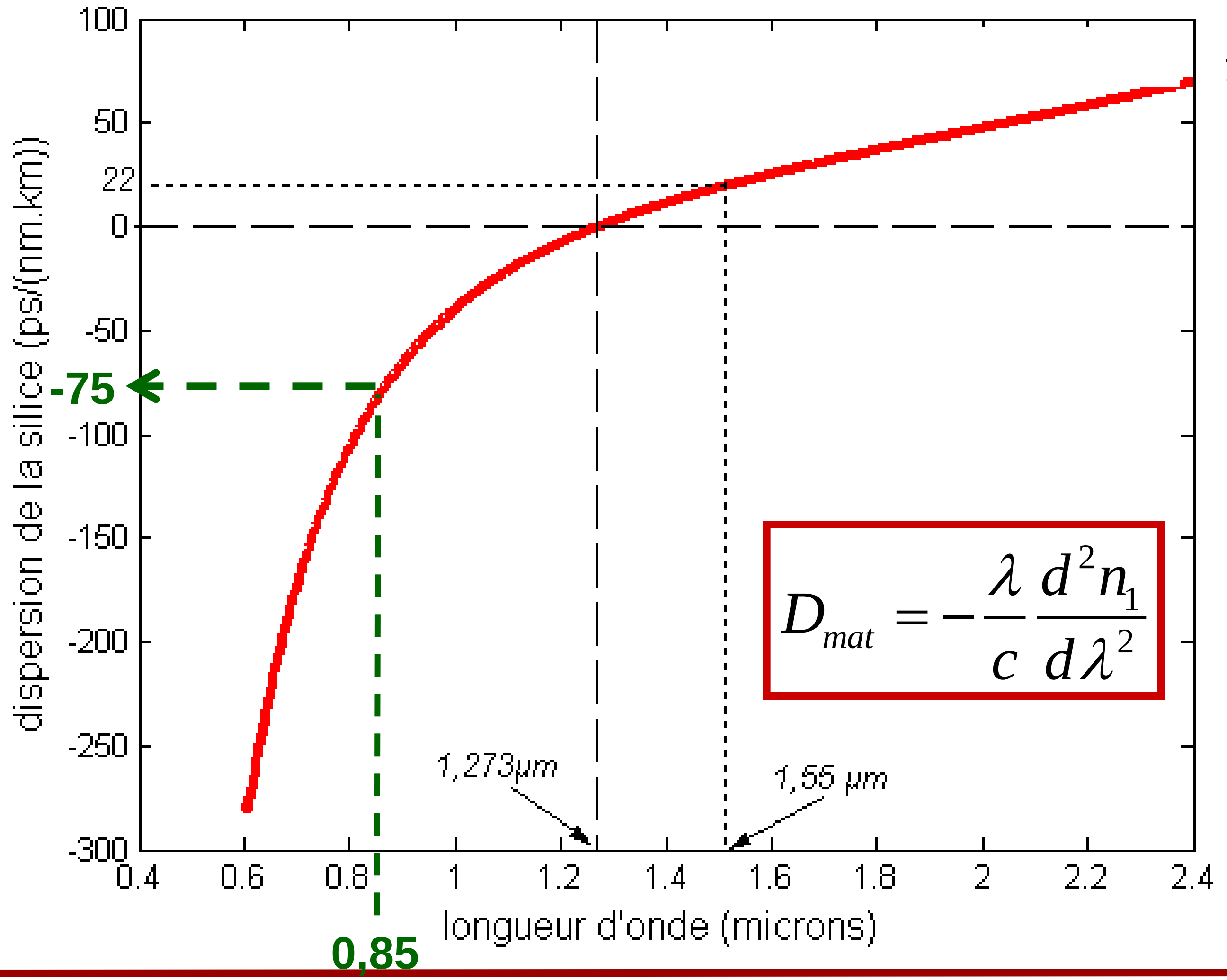
MATERIAL DISPERSION





$$\boxed{D_{mat} = -\frac{\lambda}{c}\frac{d^2 n_1}{d\lambda^2}}$$

MATERIAL DISPERSION



$$D_{mat} = -\frac{\lambda}{c}\frac{d^2 n_1}{d\lambda^2}$$

<u>example</u> let us consider a beam from a laser diode emitting at 850nm, with $\Delta\lambda$ = 40nm, launched in a fiber with a length L= 2km

D_{mat} = -75ps/(nm.km)

$$\tau_{mat} = L.\Delta\lambda.|D_{mat}|$$
$$= 2\text{x}40\text{x}75 = 6000\,ps = 6ns$$

CALCULATION OF THE GUIDE DISPERSION

→ guided wave
→ non dispersive propagation medium $\Rightarrow \dfrac{dn_1}{d\lambda}=0\ ;\ \dfrac{dn_2}{d\lambda}=0\ ;\ \dfrac{d\Delta}{d\lambda}=0$

$$v_g=\frac{d\omega}{d\beta} \quad \text{et} \quad t_g=\frac{L}{v_g}=L\frac{d\beta}{d\omega}=\frac{L}{c}\frac{d\beta}{dk_0}$$ (car $\omega=k_0.c$)

<u>Goal</u> : express t_g as a function of B and V (➔ allowing to exploit the dispersion curves B=f(V))

One easily shows that :

$$\beta=k_0\left[n_2+n_1\Delta B\right]$$

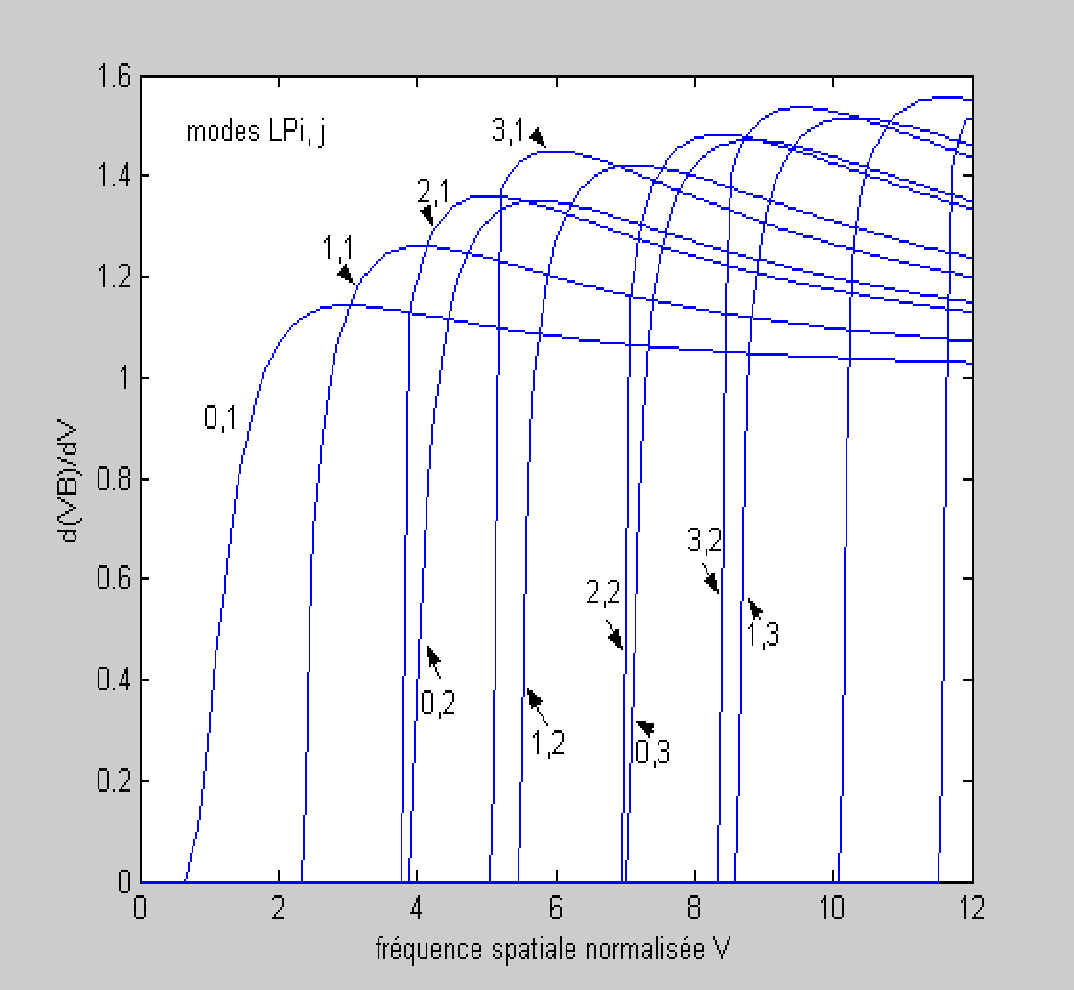
$$k_0=\frac{V}{a.n_1\sqrt{2\Delta}}$$

$$\boxed{t_g=t_{gui}=\frac{L}{c}\left[n_2+n_1\Delta\frac{d(VB)}{dV}\right]}$$

see the development of the calculations in ***pdf pages 10 et 11***

CALCULATION OF THE GUIDE DISPERSION

$$t_{gui} = \frac{L}{c}\left[n_2 + n_1 \Delta \frac{d(VB)}{dV} \right]$$



CALCULATION OF THE GUIDE DISPERSION

$$t_{gui} = \frac{L}{c}\left[n_2 + n_1\Delta\frac{d(VB)}{dV}\right]$$

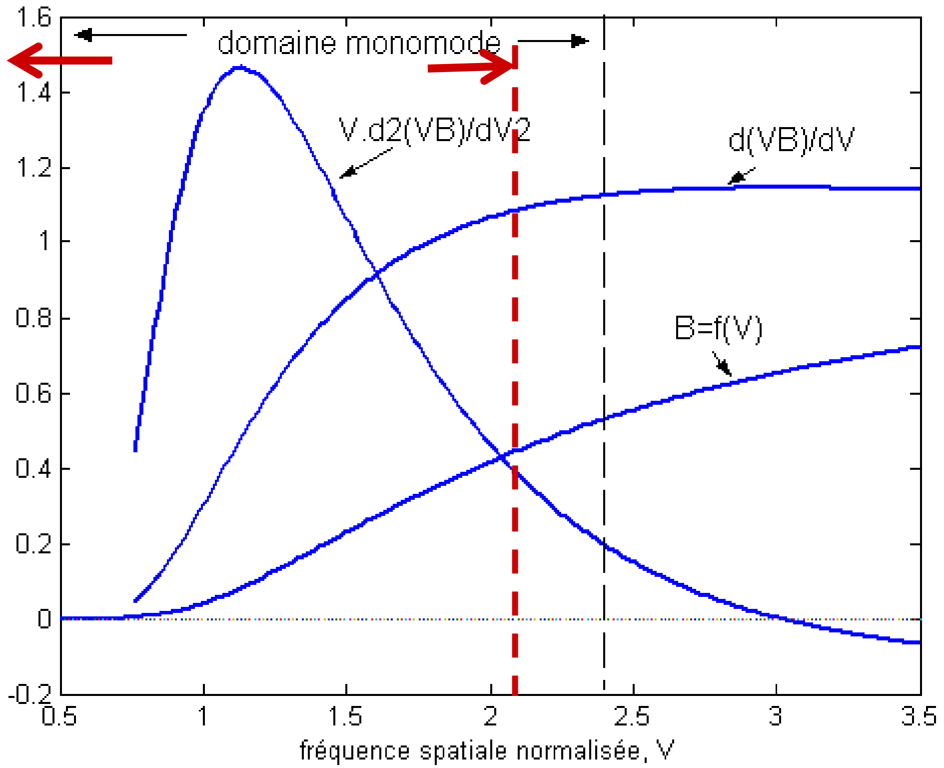
temporal broadening : $\tau_{gui} = \Delta t_{gui} = \dfrac{dt_{gui}}{d\lambda}\Delta\lambda$ (see slide number 9)

Dispersion due to guiding effect : $D_{gui} = \dfrac{\tau_{gui}}{L.\Delta\lambda}$

resulting in :

$$\boxed{D_{gui} = -\frac{n_1\Delta}{c\lambda}V\frac{d^2(VB)}{dV^2}}$$

(see the development of the calculations in ***pdf page 12)***



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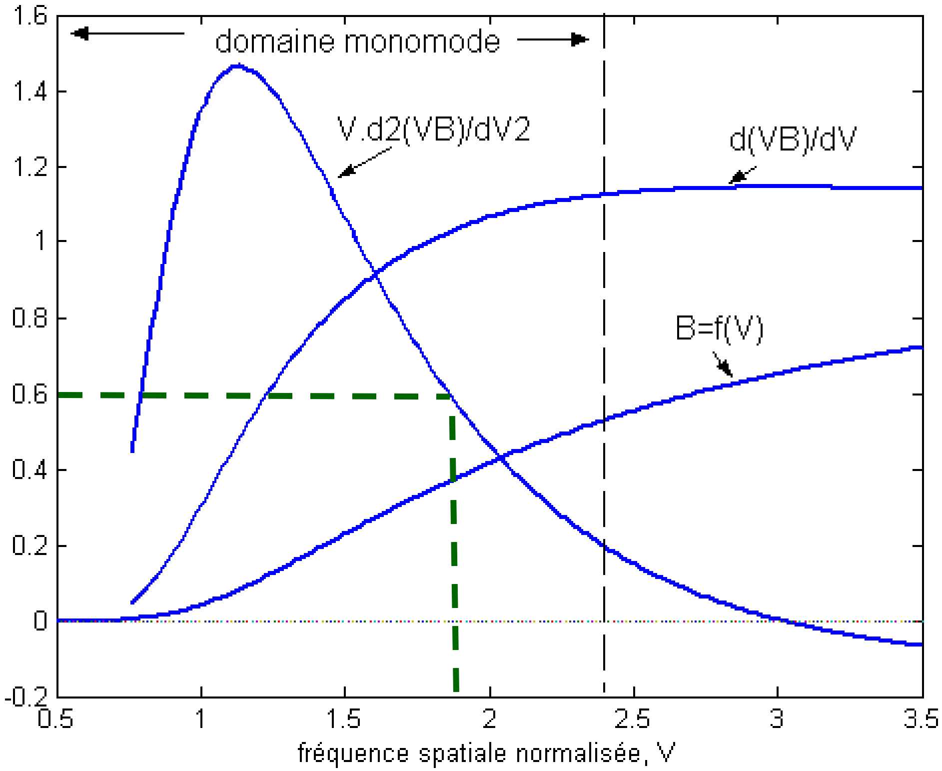


Example: case of a fiber with

a = 4,5µm   ON = 0,105   n1 = 1,46

à λ= 1,55µm

$\rightarrow \quad \Delta = \frac{ON^2}{2n_1^2} = 2,83.10^{-3}$

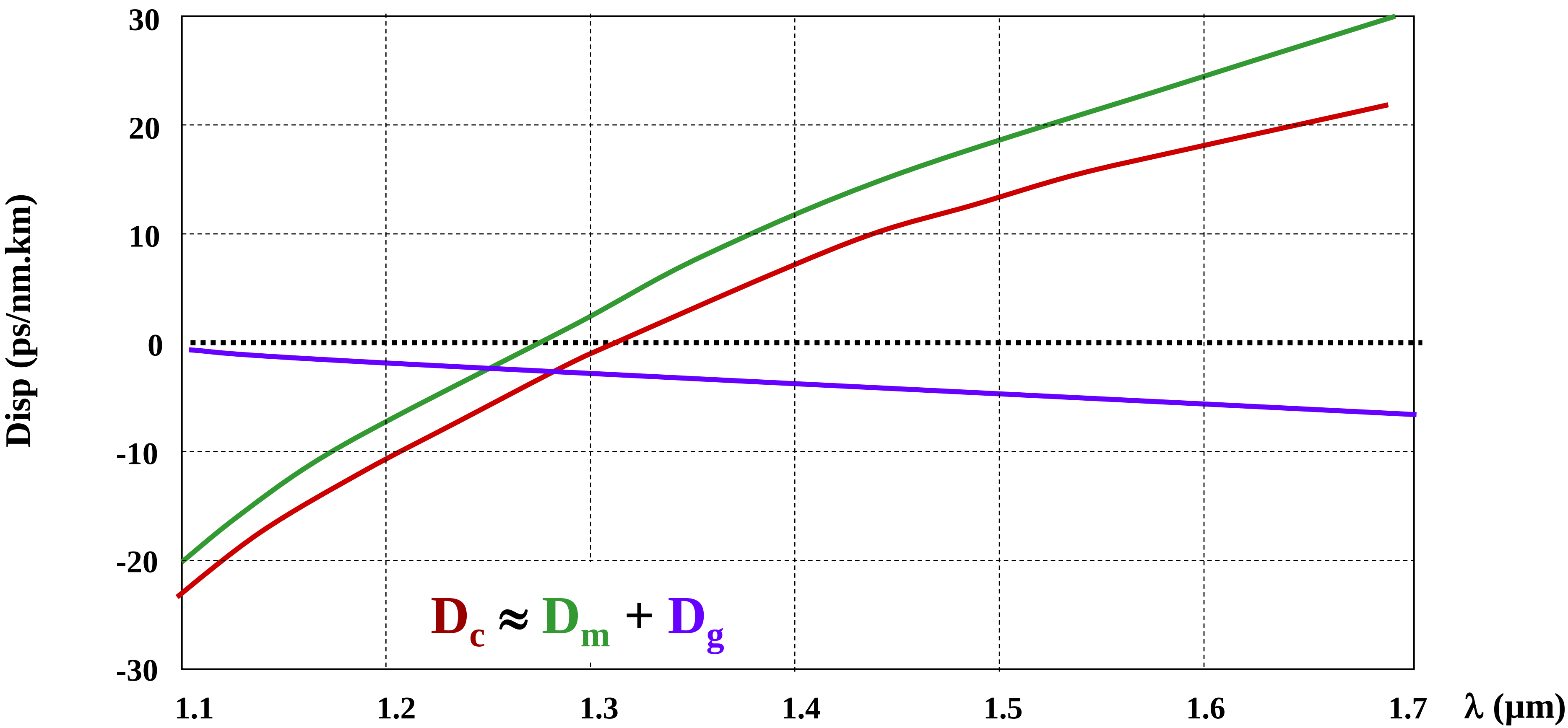
$\rightarrow \quad V = \frac{2\pi}{\lambda} aON = 1,91$

$\rightarrow \quad V \frac{d^2(VB)}{dV^2} \simeq 0,6$

$$\boxed{D_{gui} = -\frac{1,46 \text{ x } 2,83.10^{-3}}{3.10^8 \text{ x } 1,55.10^{-6}} \text{ x } 0,6 = -5 10^{-6} s/(m.m) = -5 ps/(nm.km)}$$

CURVES OF THE CHROMATIC DISPERSION versus WAVELENGTH

Example with a step index fiber : $n_1 = 1.46$ $n_2 = 1.455$ $a = 4\ \mu m$



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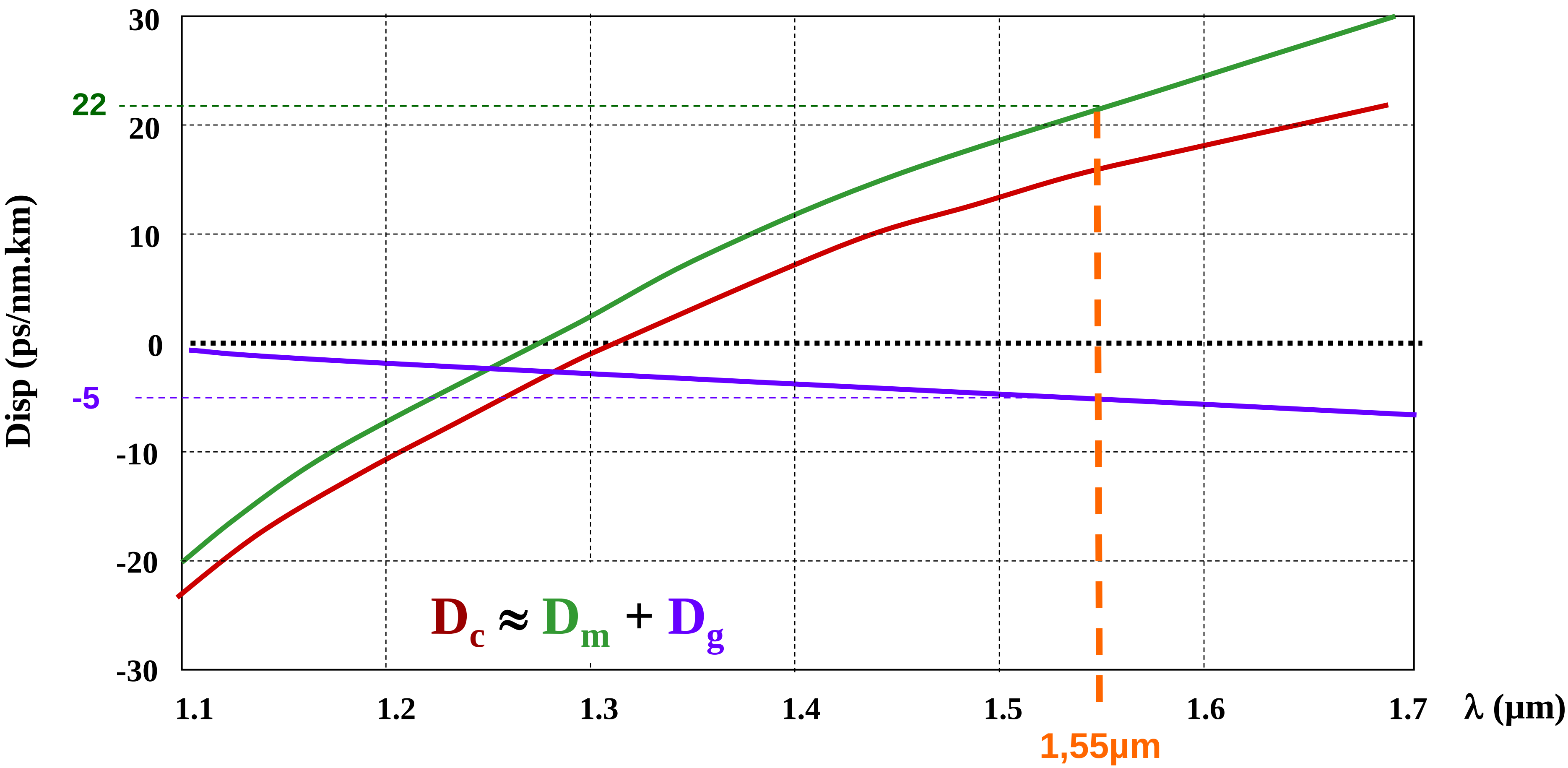
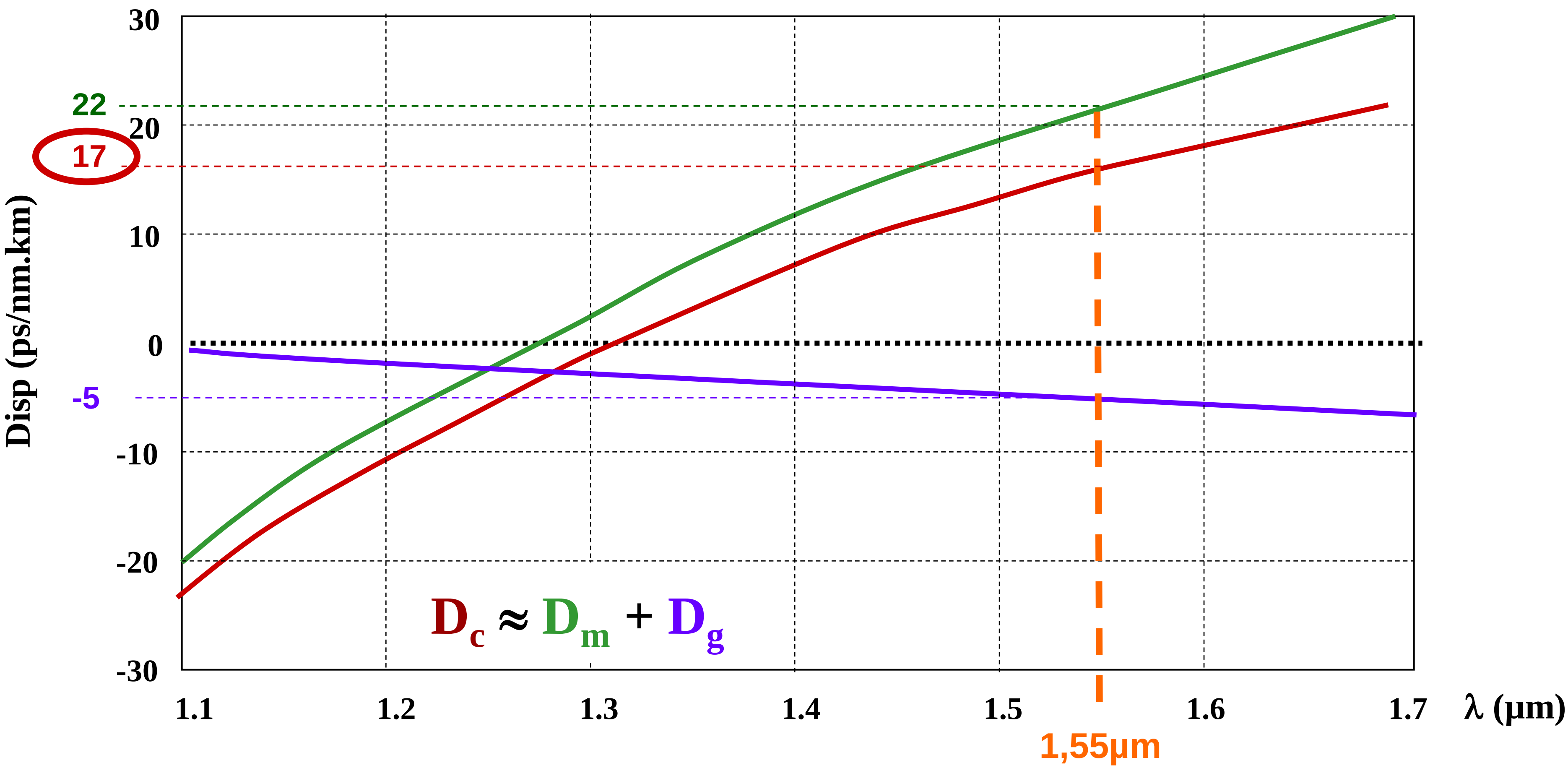
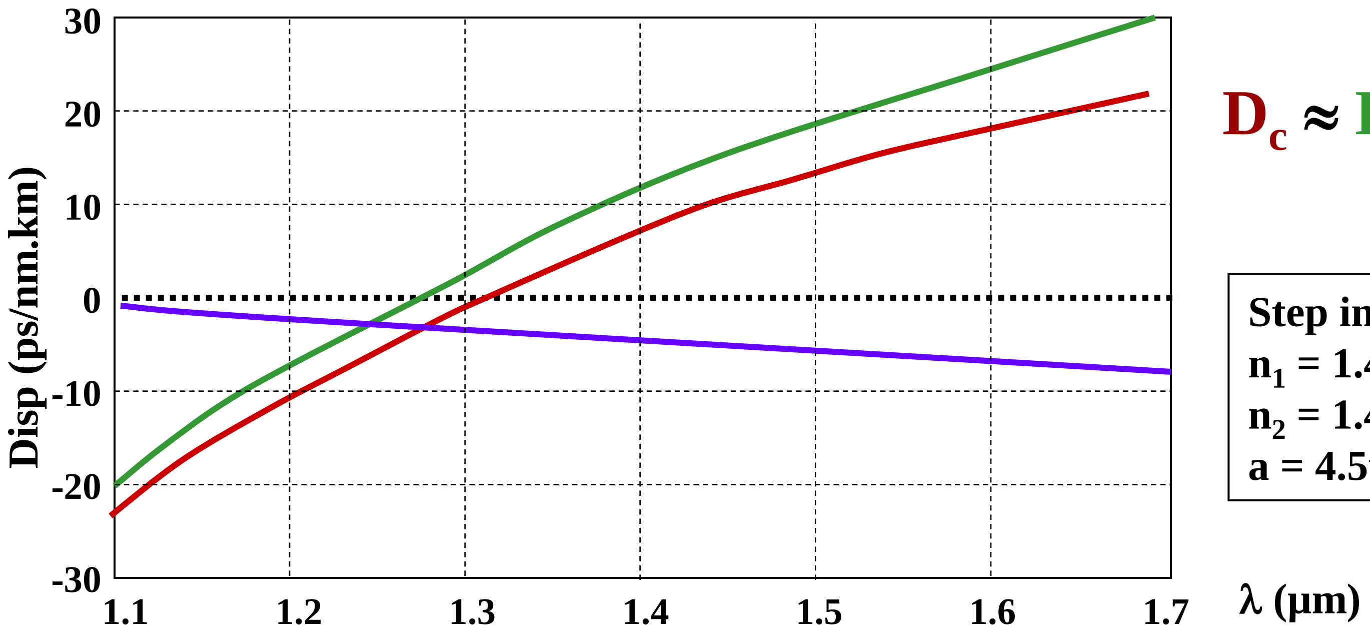


CURVES OF THE CHROMATIC DISPERSION versus WAVELENGTH

Example with a step index fiber : n_1 = 1.46 n_2 = 1.455 a = 4 µm



CURVES OF THE CHROMATIC DISPERSION VERSUS WAVELENGTH



$D_c \approx D_m + D_g$

Step index fiber
$n_1 = 1.46$
$n_2 = 1.456$
$a = 4.5 \mu m$

How can we change the chromatic dispersion of an optical fiber ?

→ **By changing the material dispersion ???? → no**

→ **By changing the dispersion of the guide !!!**

⇨ **Working with higher order modes**

⇨ **Working in the single mod regime, but with a fiber having a modified index profile**

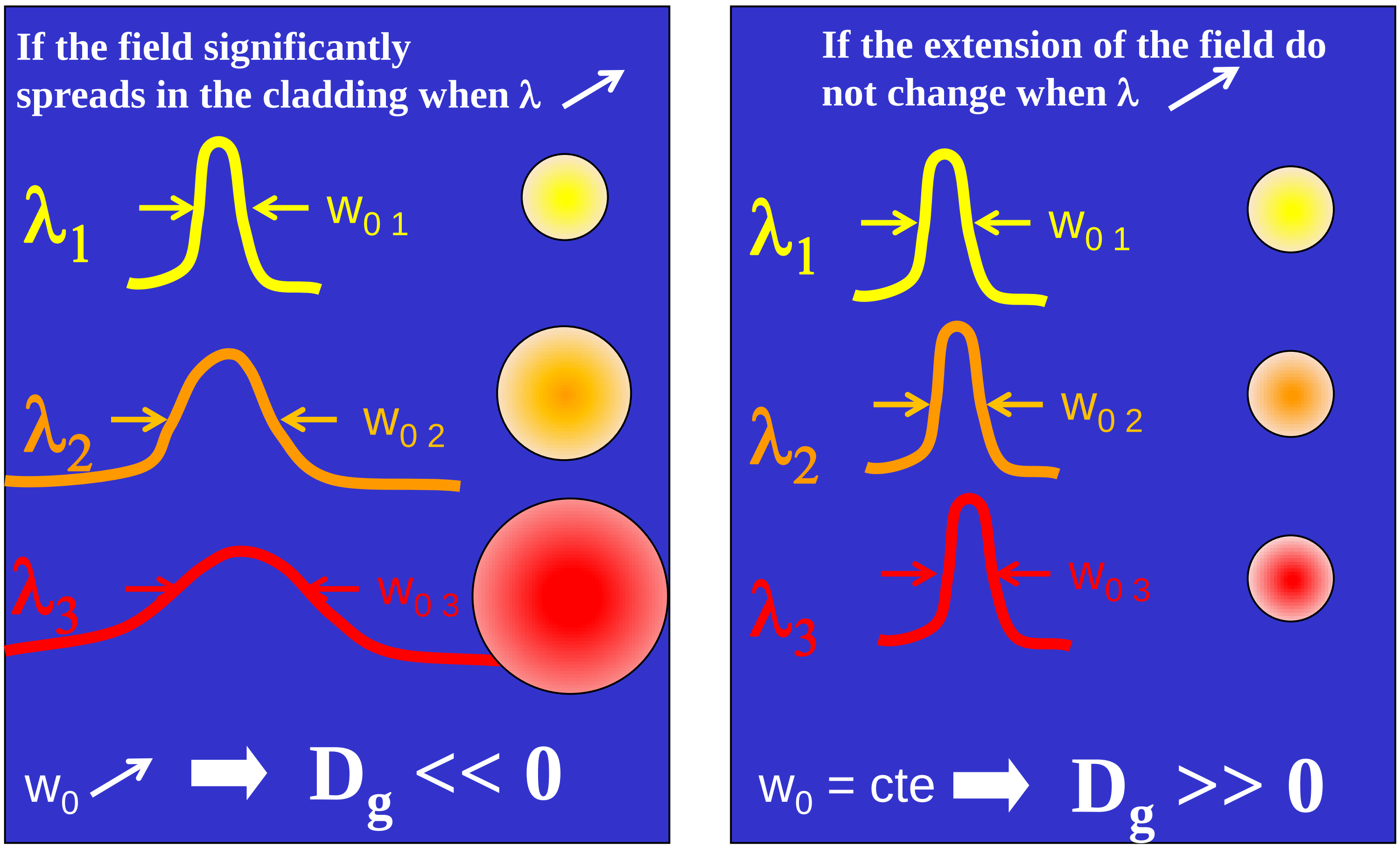
- **multiclad fibers ("DS fibers", "DF fibers…)**
- **Air silica microstructured optical fibers (MOFs so called "PCFs")**
- **Bragg fibers or photonic bandgap fibers**

HOW CAN WE CHANGE THE DISPERSION OF THE GUIDE ?

$$D_g = -\frac{1}{\pi^2 n_2 c}\frac{\lambda}{w_0^2}\left(\frac{\lambda}{w_0}\frac{dw_0}{d\lambda} - \frac{1}{2}\right)$$

Pierre Sansonetti, Elect. Letters, vol 18, n°15, pp 647-648 (1982)

$$\lambda_1 < \lambda_2 < \lambda_3$$



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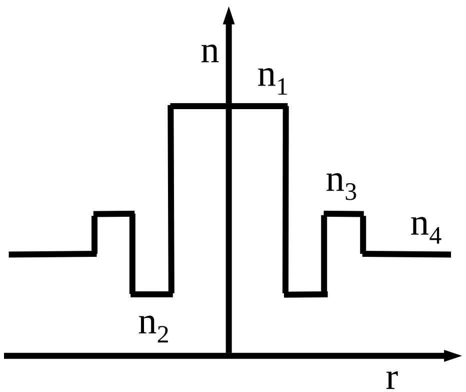
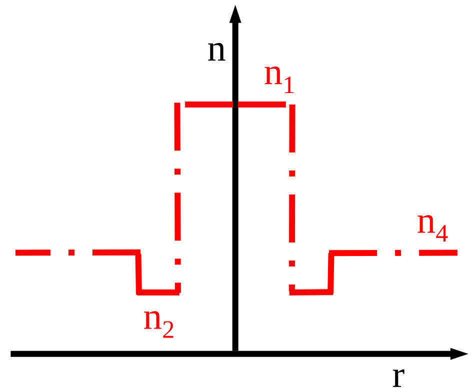
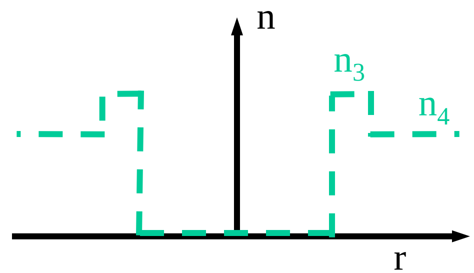
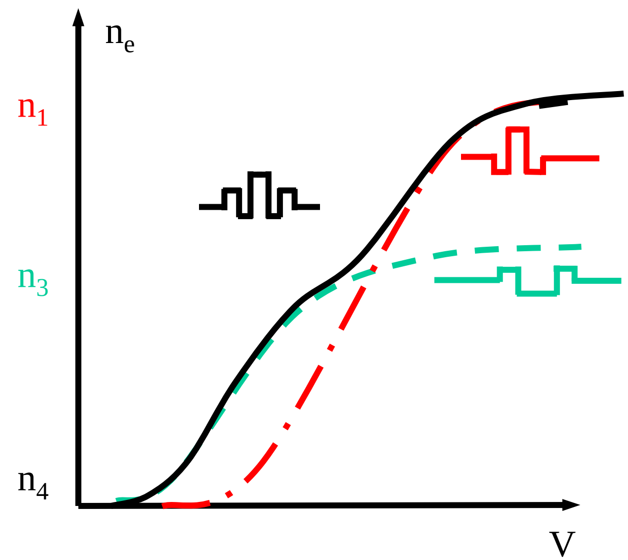
DISPERSION SHIFTED FIBERS : MULTICLAD FIBERS



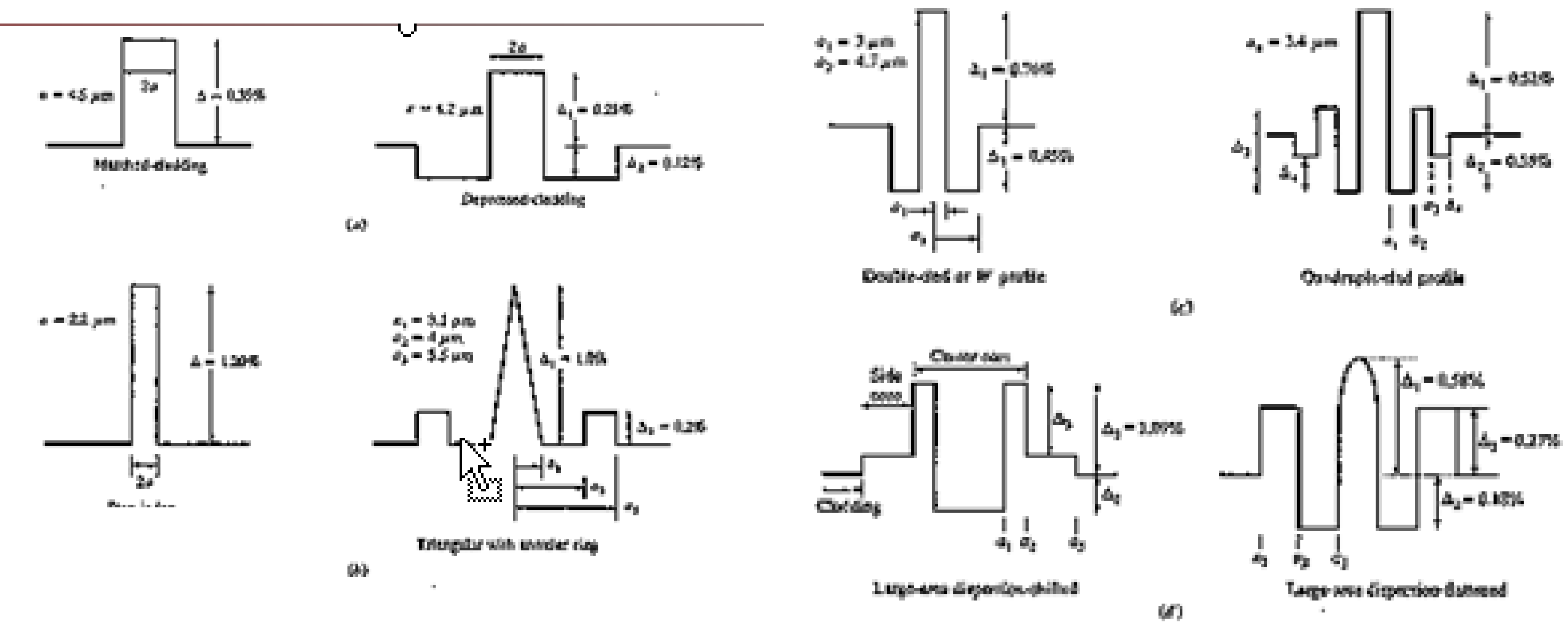




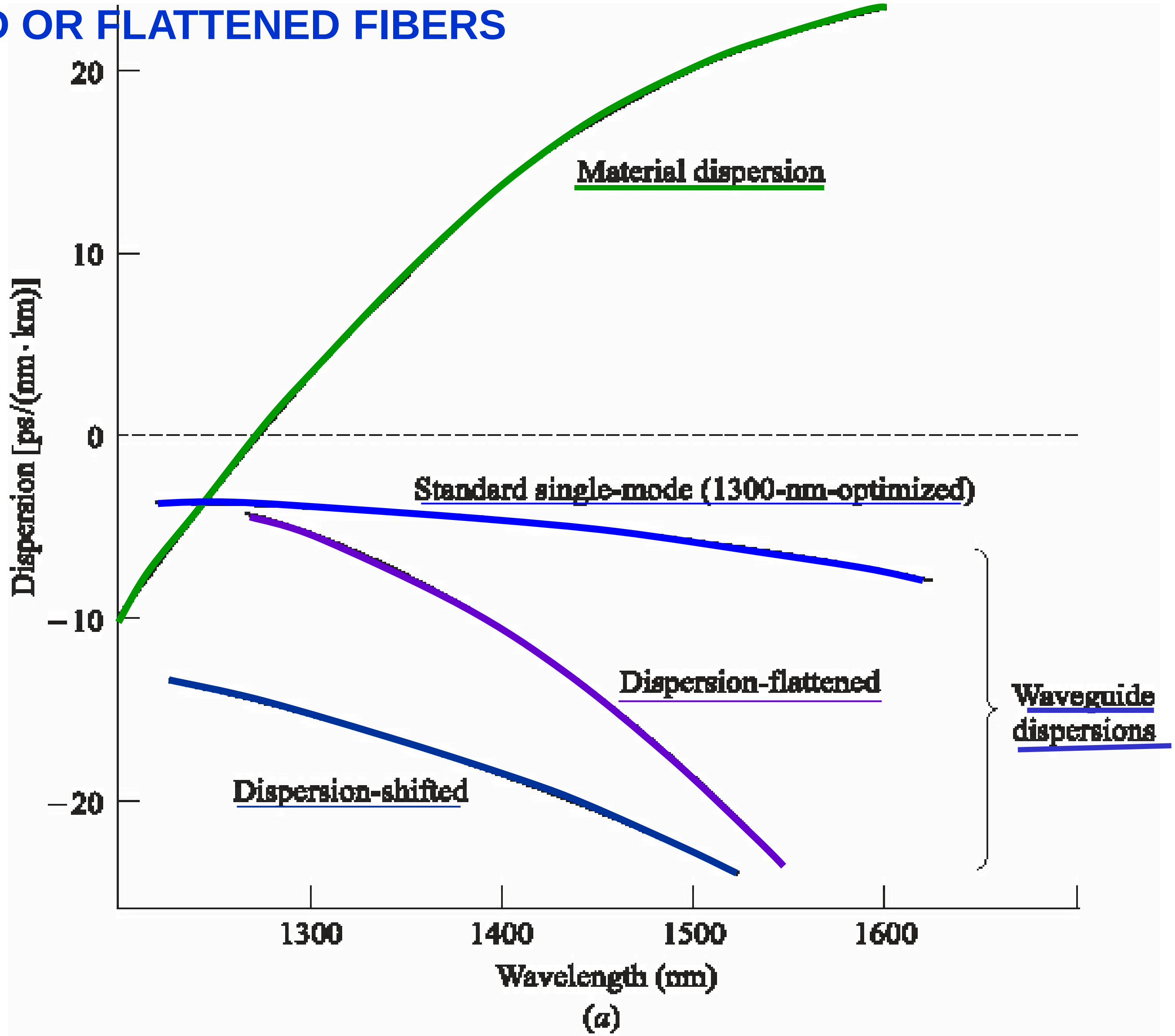


$$D = -\frac{\lambda}{c}\frac{d^2 n_e}{d\lambda^2}$$

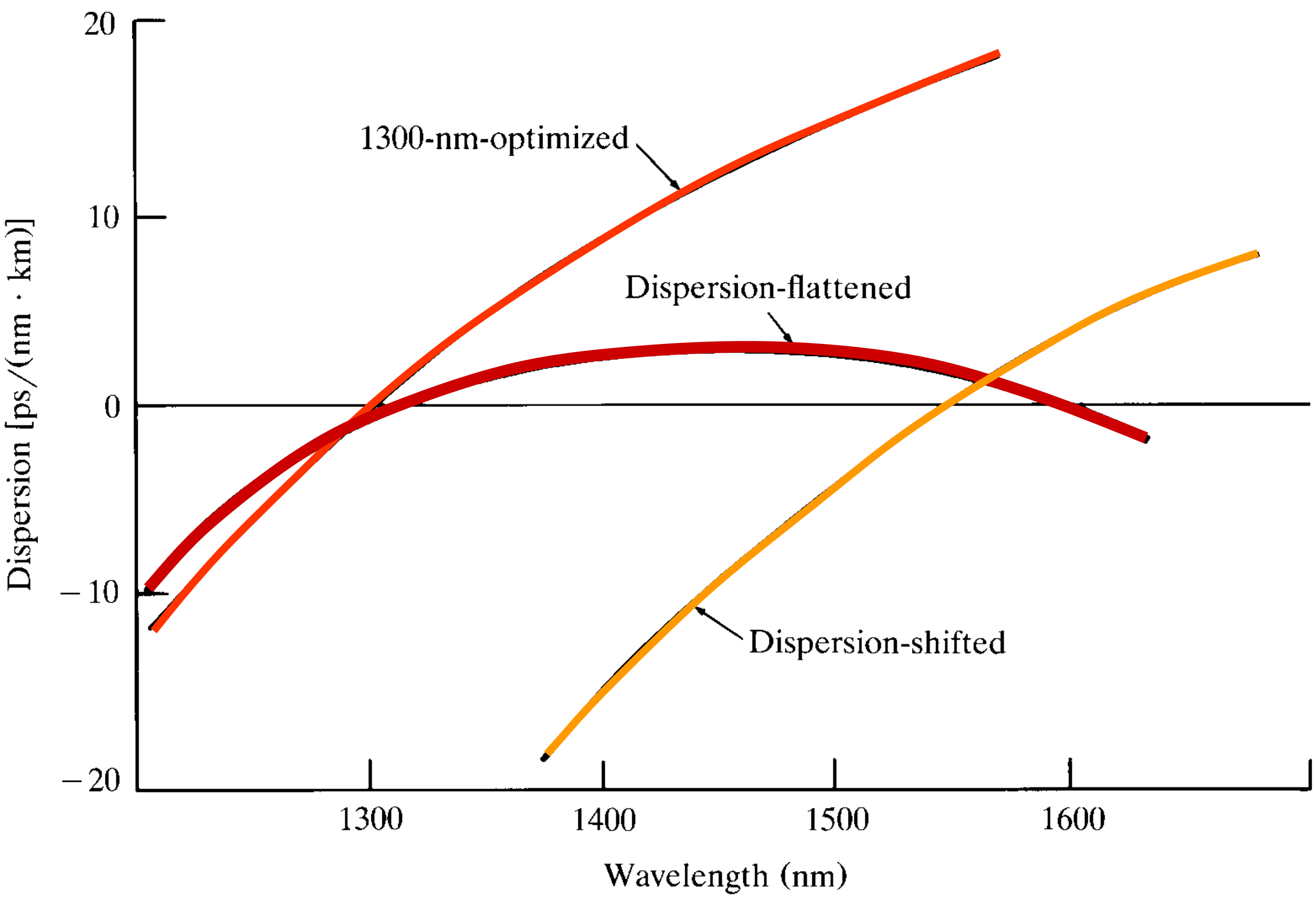
DISPERSION SHIFTED FIBERS : A LARGE VARIETY OF INDEX PROFILES



DISPERSION SHIFTED OR FLATTENED FIBERS



DISPERSION SHIFTED OR FLATTENED FIBERS

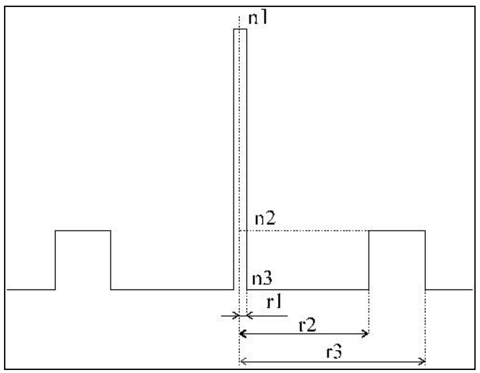


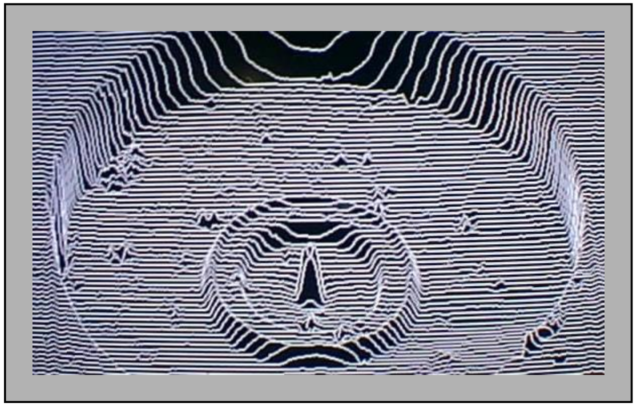
(b)

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

OTHER KINDS OF FIBERS FOR THE MANAGEMENT OF THE DISPERSION (1)

* fibers with Dg << 0 (compensating fibers)





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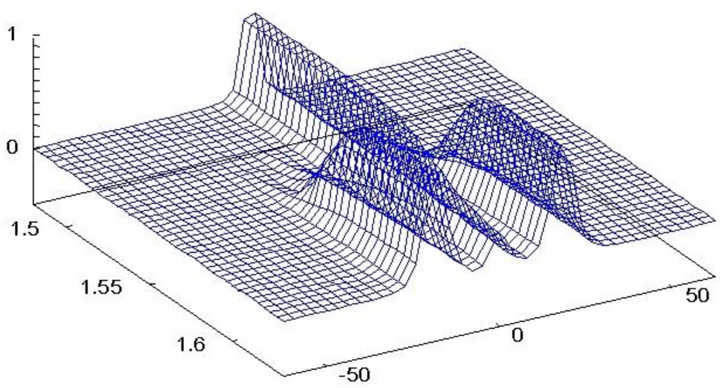
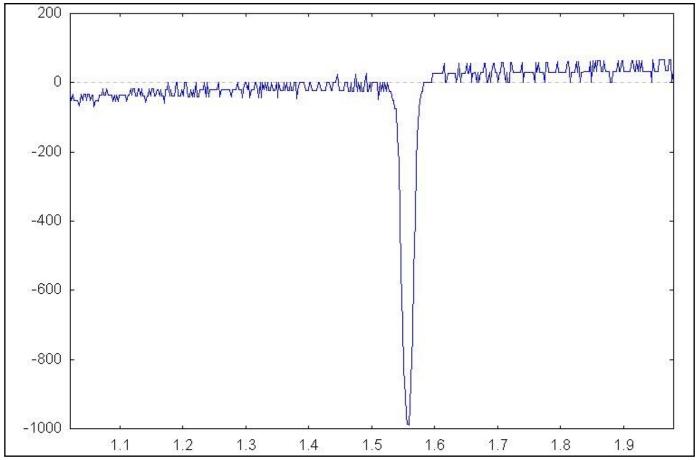
Vol. 24, issue 1, pp. 442- (2006)

index profile



Distribution of the field versus wavelength

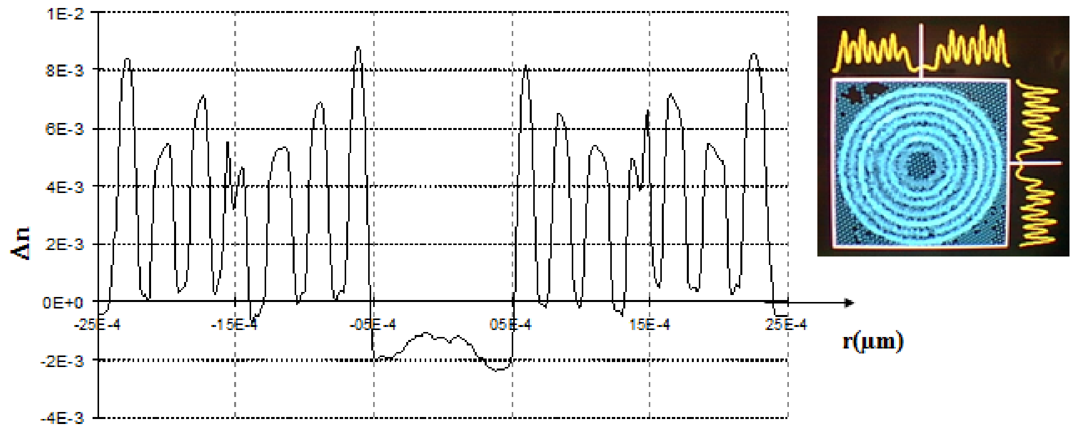


measured chromatic dispersion

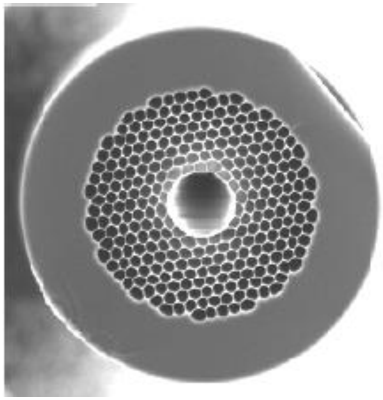
OTHER KINDS OF FIBERS FOR THE MANAGEMENT OF THE DISPERSION (2)

* fibers with Dg > 0 at short wavelengths

→ **Bragg Fibers**



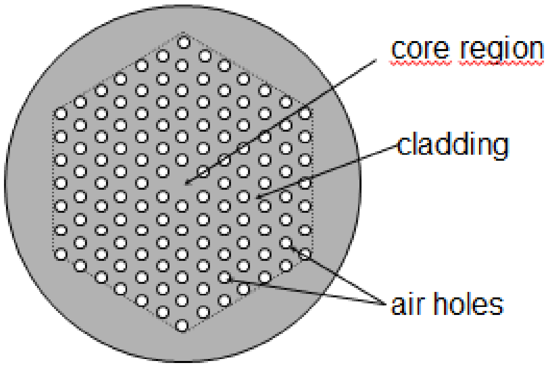
→ **Hollow core photonic crystal fibers**



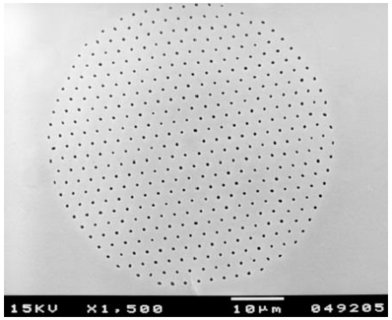
OTHER KINDS OF FIBERS FOR THE MANAGEMENT OF THE DISPERSION (3)

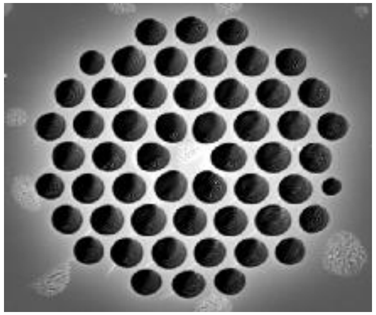
* fibers with Dg specially managed for particular applications

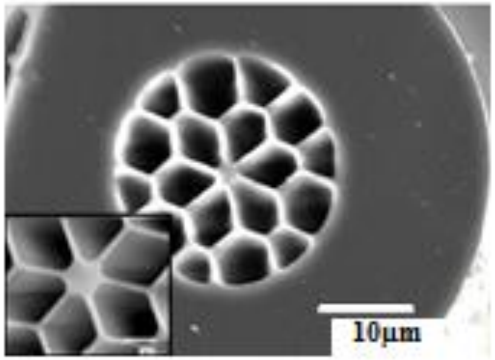
→ **Air silica microstructured optical fibers :**



diameter of holes :d
pitch : Λ







d= 0,6µm Λ=2,6µm

flattened dispersion
1100 nm – 1600 nm

d= 1,9µm Λ=2,3µm

Dc = 0 @ 1,06µm

core diameter = 1,5µm Λ=2µm

Dc = 0 @ 0,56µm

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End of chapter 4

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of the European Union