

Formulas

$$n = \frac{c}{v} \quad \lambda = \frac{\lambda_0}{n} \quad k = nk_0 \quad \lambda = \frac{c}{f}$$

Snell
 $\theta_i = \theta_r$

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$n_1 < n_2 \Rightarrow \theta_i > \theta_t$$

$$n_1 > n_2 \Rightarrow \theta_i < \theta_t$$

total reflect

$$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right)$$

Spherical mirror

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{2}{-R} = \frac{1}{f}$$

$R \oplus$ if convex

$R \ominus$ if concave

Spherical boundaries

$$\frac{n_1}{z_1} + \frac{n_2}{z_2} = \frac{n_2 - n_1}{R}$$

$$y_2 = -y_1 \frac{n_1 z_2}{n_2 z_1}$$

Thin lens

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

$$y_2 = -\frac{z_2}{z_1} y_1$$

Matrix optics

$$y_2 = Ay_1 + B\theta_1$$

$$\theta_2 = Cy_1 + D\theta_1$$

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

Cascade:

$$\rightarrow [M_1] \rightarrow [M_2] \dots \rightarrow [M_N] \rightarrow$$

$$M = M_N M_{N-1} \dots M_2 M_1$$

Wave optics

$$\nabla^2 u - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 0$$

wave function

$$U(\vec{r}, t) = U(\vec{r}) e^{i\omega t}$$

$$u(\vec{r}, t) = \text{Re}[U(\vec{r}, t)]$$

Transparent plate

$$b(x, y) = e^{-jk_0 d}$$

$$\text{phase shift } \text{mod} = 2\pi \frac{d}{\lambda}$$

Diffraction grating

$$\sin \theta_g = \sin \theta_i \pm g \frac{\lambda}{\Lambda}$$

if θ_i small

$$\sin \theta_i = \theta_i$$

$$\sin \theta_g = \theta_g$$

$$\text{Intensity: } I(\vec{r}, t) = 2 \langle u^2(\vec{r}, t) \rangle$$

$$\text{Power: } P(t) = \int_A I(\vec{r}, t) dA$$

Interference

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \varphi$$

$$I_1 = I_2 = I_0$$

$$I = 4I_0 \cos^2(\varphi/2)$$

$$\varphi = 0 \rightarrow I = 4I_0 \text{ constructive}$$

$$\varphi = \pi \rightarrow I = 0 \text{ destructive}$$

$$\varphi = 0, \frac{2}{3}\pi \rightarrow I = 2I_0$$

Gaussian Beam

Plane wave

$$u(\vec{r}, t) = |A| \cos[\omega t - kz + \arg(A)]$$

$$I(\vec{r}) = |A|^2$$

Spherical wave

$$U(\vec{r}) = \frac{A_0}{r} e^{-jkr}$$

wavefront
 $r = q\lambda$

$$I(\vec{r}) = \frac{|A_0|^2}{r^2}$$

Paraboloidal

paraxial approx

$$U(\vec{r}) \approx \frac{A_0}{r} e^{-jkz} e^{-jk \frac{x^2 + y^2}{2z}}$$

condition

$$\frac{r_F}{u} \ll 1$$

Lens equation

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

u : objects dist. from the lens

v : Image dist. from the lens

f : focal length

Sign conventions

Right of lens \rightarrow Positive

Left of lens \rightarrow Negative

Above principal axis \rightarrow Positive

Below principal axis \rightarrow Negative

Focal length

Convex lens \rightarrow Positive

Concave lens \rightarrow Negative

Mirror equation

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

