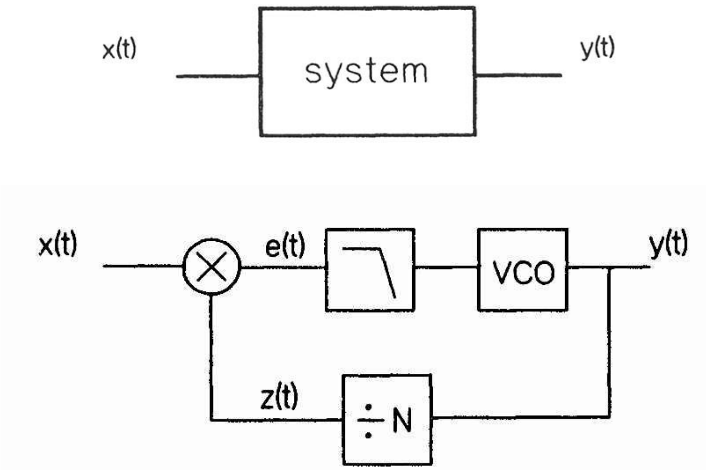
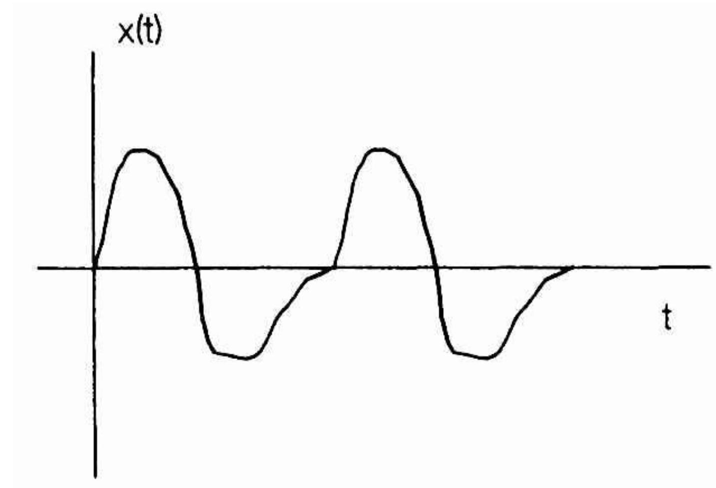
# Signals and Systems

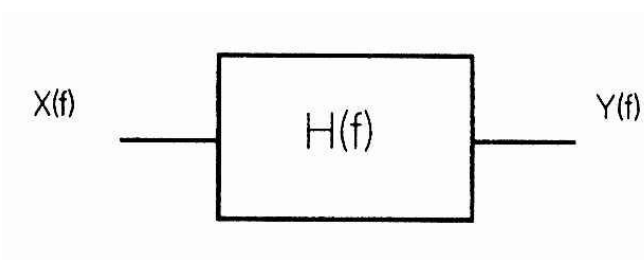
## ❑ Time domain representation



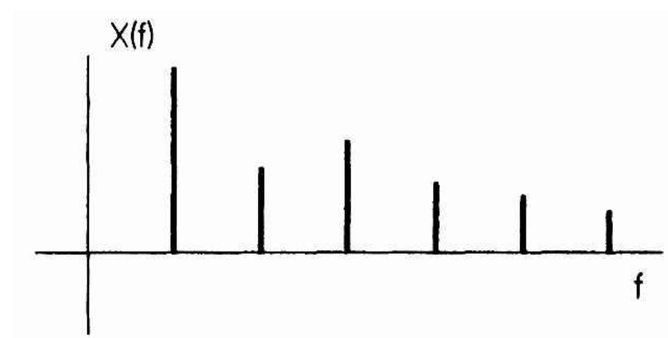

## oscilloscope




## ❑ Frequency domain representation




## spectrum analyzer



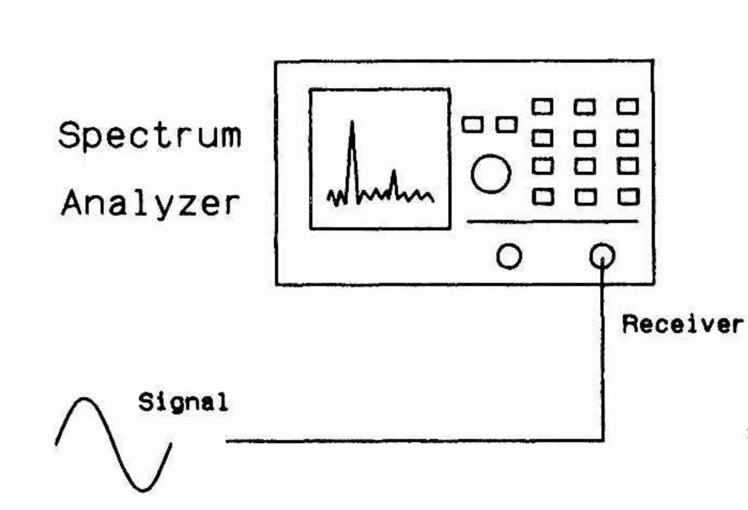
# Frequency-domain measurements

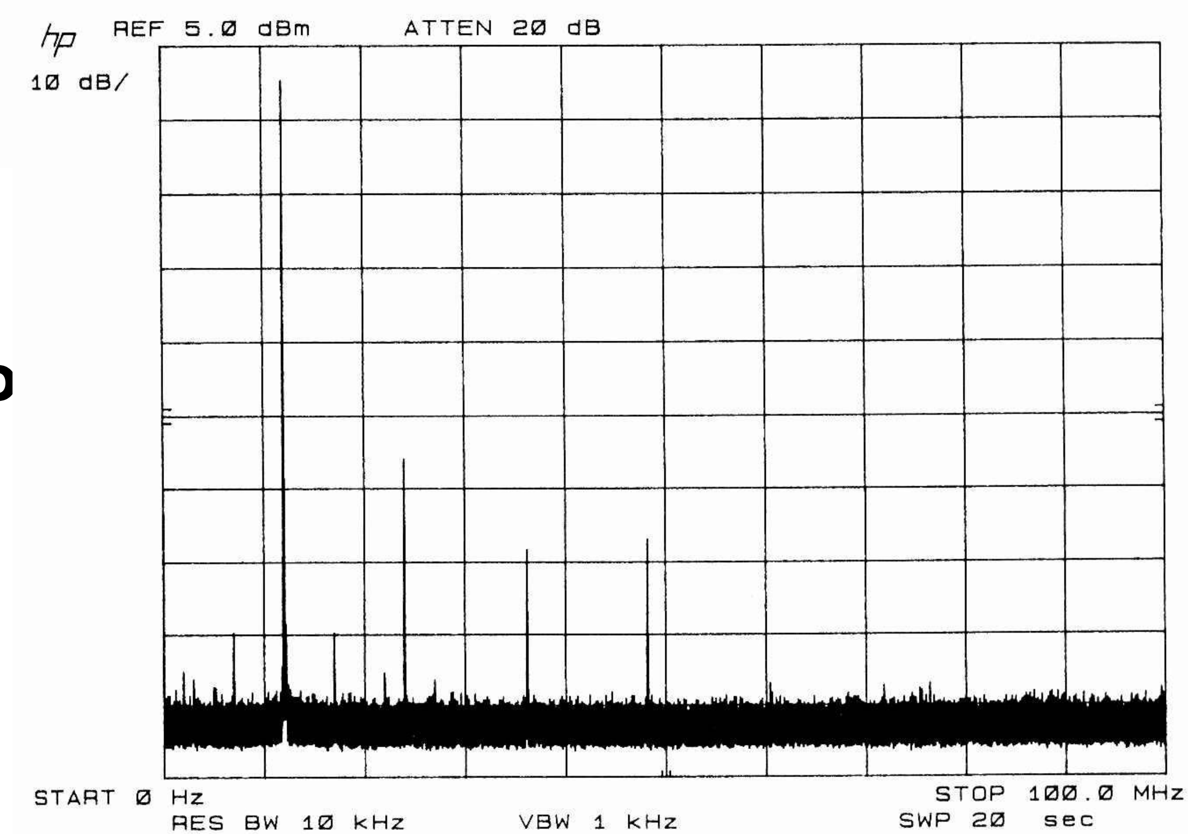
- in frequency domain measurements, the bandwidth can be narrowed at will, therefore:
  - great reduction of the amount of noise present in the measurement
  - removal from the measurements of the interfering frequencies located on the spectrum far away from the bandwidth of interest
- the narrow-band measurements are more sensitive (it is possible to have a high gain with a small noise component addition)
- some systems, such as frequency division multiplexing (FDM) systems, are inherently frequency-domain systems

# cont.

- even systems that are not oriented to the frequency-domain can benefit from frequency measurements:
  - digital transmission lines: measurement of the bandwidth
  - complex signal analysis: often it is easy to separate the different components in the frequency domain
  - linear system characterization
  - ……

# Spectrum measurements




**e.g. harmonic content o signal as provided by a function generator**

# Decibels

- **definition:** $A_{(dB)} = 10\log_{10}(P_2/P_1)$

  **the power $P_2$ is expressed relative to $P_1$**

- **if $P_1$ is the dissipated power over a load $R_1$ and $P_2$ is the power dissipated over a load $R_2$ then:**

$$A_{(dB)} = 10\log_{10}\frac{\left({V_2}^2/R_2\right)}{\left({V_1}^2/R_1\right)} = 10\log_{10}(V_2/V_1)^2 + 10\log_{10}(R_1/R_2)$$

$$\Rightarrow \quad \boxed{A_{(dB)} = 20\log_{10}(V_2/V_1) + 10\log_{10}(R_1/R_2)}$$

# decibel ...

- **if** $R_1 = R_2$**:** $A_{(dB)} = 20\log_{10}(V_2/V_1) + 10\,\cancel{\log_{10}(R_1/R_2)}$

- **IMPORTANT: if the voltage/current gain is expressed in decibels and, as often happens, the input impedance and the output impedance are significantly different then the power gain can not be (directly) derived from the voltage/current gain**

# Cardinal values

| V2/V1 | P2/P1 | Decibel | |
|---|---|---|---|
| | | | |
| 1.000 | 1 | 0 | dB |
| 1.414 | 2 | 3 | dB |
| 2.000 | 4 | 6 | dB |
| 3.162 | 10 | 10 | dB |
| 10.000 | 100 | 20 | dB |
| | | | |

- **Clearly:**
  - **the changing of the sign to a decibel value is equivalent of taking the reciprocal of the power ratio**
  - **summing up two decibel values is equivalent to multiplying the power ratios**

# "Absolute" decibel values (1)

$$P_{(dB)} = 10\log_{10}(P/P_{REF})$$

$$\boldsymbol{P_{REF}} = 1\,mW \Longrightarrow \boldsymbol{dBm}$$

$$P_{(dBm)} = 10\log_{10}(P/0.001)$$

$$P_{REF} = \frac{(V_{REF})^2}{R} \Rightarrow V_{REF} = \sqrt{P_{REF}\cdot R} = \begin{cases} 50\Omega \Rightarrow \sqrt{0.001\cdot 50} = 0.2236V_{rms} \\ 75\Omega \Rightarrow \sqrt{0.001\cdot 75} = 0.2739V_{rms} \end{cases}$$

$$50\Omega \Rightarrow P_{(dBm)} = 20\log_{10}(V_{rms}/0.2236)$$

$$75\Omega \Rightarrow P_{(dBm)} = 20\log_{10}(V_{rms}/0.2739)$$

# "Absolute" decibel values (2)

$$V_{(dB)} = 20\log_{10}(V/V_{REF})$$

$$\boldsymbol{V_{REF}} = 1\,V \quad \Longrightarrow \quad \boldsymbol{dBV}$$

$$V_{(dBV)} = 20\log_{10}(V_{rms}/1) = 20\log_{10}(V_{rms})$$

$$P_{REF} = \frac{(V_{REF})^2}{R} = \begin{cases} 50\Omega \Rightarrow 1/50 = 0.0200W \\ 75\Omega \Rightarrow 1/75 = 0.0134W \end{cases}$$

$$50\Omega \Rightarrow V_{(dBV)} = 10\log_{10}(P/0.0200)$$

$$75\Omega \Rightarrow V_{(dBV)} = 10\log_{10}(P/0.0134)$$

# dBV <--> dBm Conversions

$$P_{(dBm)} = 10\log_{10}\left[\frac{(V_{rms})^2}{R}\cdot\frac{1}{0.001}\right] = 10\log_{10}\left((V_{rms})^2\right) + 10\log_{10}\left(\frac{1}{R\cdot 0.001}\right) =$$

$$= V_{(dbV)} + 10\log_{10}\left(\frac{1}{R\cdot 0.001}\right)$$

$$50\Omega \Rightarrow P_{(dBm)} = V_{(dbV)} + 13.01$$

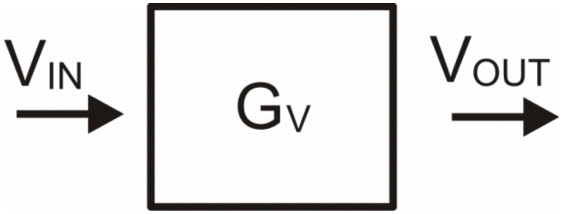
$$75\Omega \Rightarrow P_{(dBm)} = V_{(dbV)} + 11.25$$

# Gain and loss calculation (power)


- **definition of power gain:** $G_P = \frac{P_{out}}{P_{in}}$
- **in decibels:** $G_{P(dB)} = 10\ log_{10}\left(\frac{P_{out}}{P_{in}}\right)$

- $G_{P(dB)} > 0$**: true power gain**
- $G_{P(dB)} < 0$**: power loss**
- **definition of power loss:** $L_P = \frac{P_{in}}{P_{out}}$
- **in decibels:** $L_{P(dB)} = 10\ log_{10}\left(\frac{P_{in}}{P_{out}}\right)$
- $L_{P(dB)} > 0$**: true power loss**
- $L_{P(dB)} < 0$**: power gain**

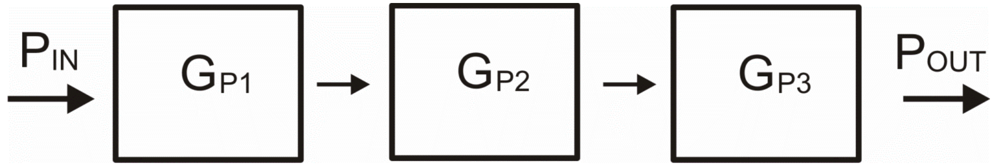
# Gain and attenuation calculation (voltage)




- definition of voltage gain: $G_V = \frac{V_{out}}{V_{in}}$
- in decibels: $G_{V(dB)} = 20\ log_{10}\left(\frac{V_{out}}{V_{in}}\right)$

- $G_{V(dB)} > 0$: true voltage gain
- $G_{V(dB)} < 0$: voltage attenuation
- definition of voltage attenuation: $A_V = \frac{V_{in}}{V_{out}}$
- in decibels: $A_{V(dB)} = 10\ log_{10}\left(\frac{V_{in}}{V_{out}}\right)$
- $A_{V(dB)} > 0$: true voltage attenuation
- $A_{V(dB)} < 0$: voltage gain

# Multiple blocks



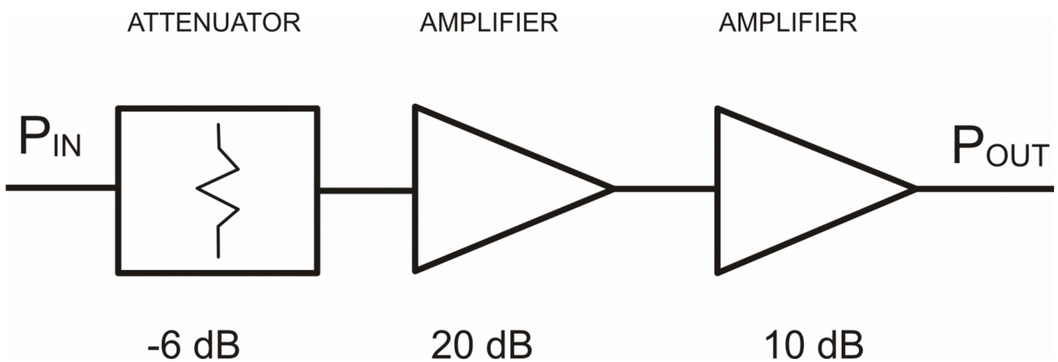

- **the total power gain is** $G_{PT} = \frac{P_{out}}{P_{in}} = G_{P1} \cdot G_{P2} \cdot G_{P3}$
- **in decibels:**

$$G_{PT(dB)} = 10\, log_{10}(G_{P1} \cdot G_{P2} \cdot G_{P3}) = G_{P1(dB)} + G_{P2(dB)} + G_{P3(dB)}$$

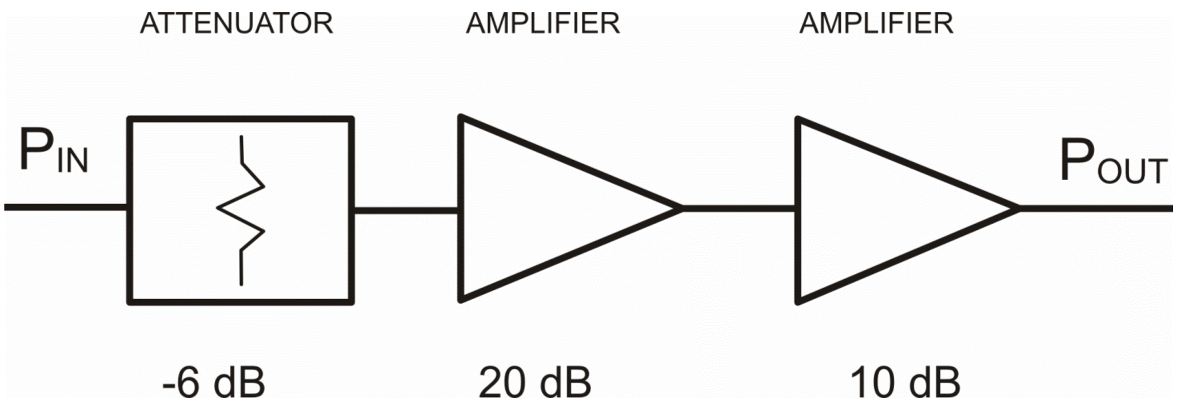
- **by definition of gain:** $P_{out} = G_{PT} \cdot P_{in} \;\Rightarrow\; \frac{P_{out}}{P_{ref}} = G_{PT} \cdot \frac{P_{in}}{P_{ref}}$
- **in decibels:** $10\, log_{10}\left(\frac{P_{out}}{P_{ref}}\right) = 10\, log_{10}\left(G_{PT} \cdot \frac{P_{in}}{P_{ref}}\right)$

$$\Rightarrow\; P_{out(dBm)} = P_{in(dBm)} + G_{PT(dB)}$$

# Example



# Example

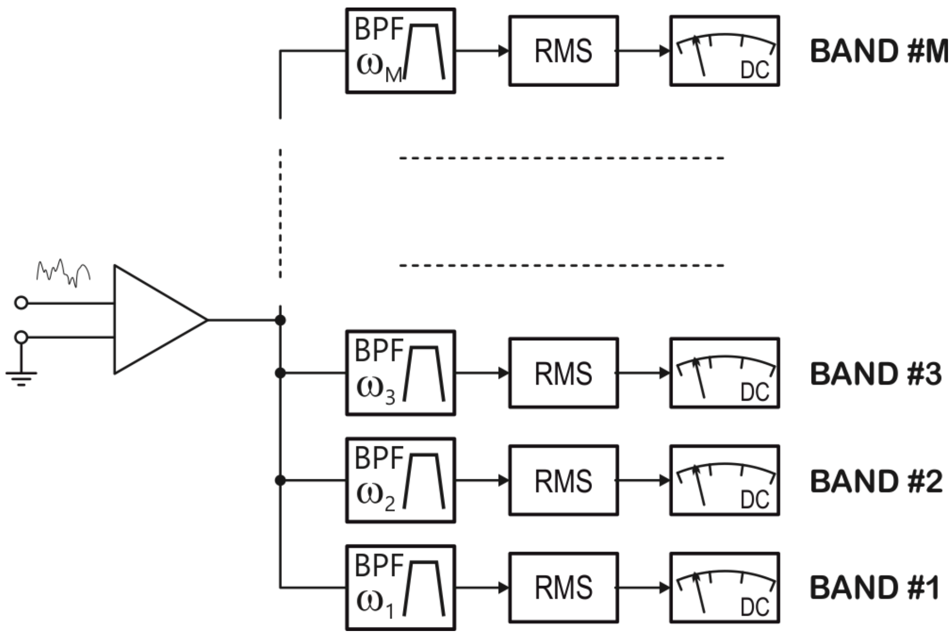


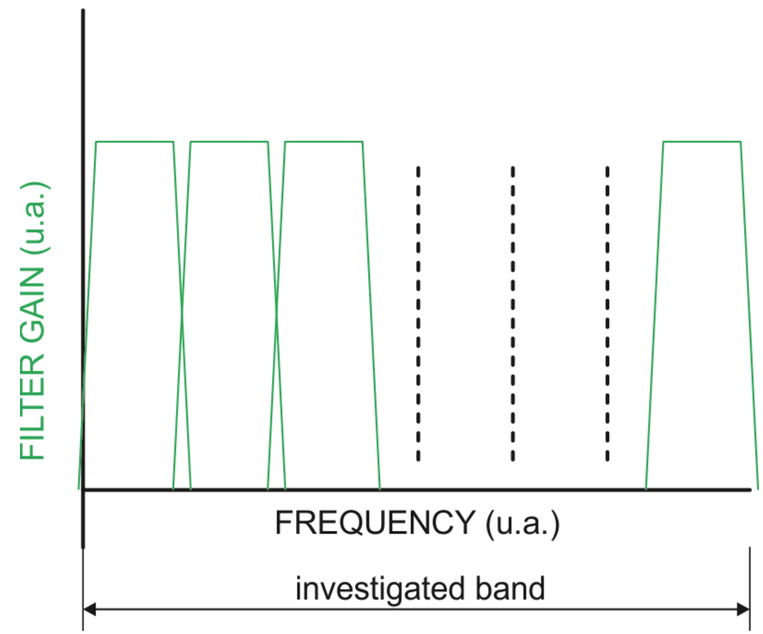

- $G_{T(dB)} = (-6 + 20 + 10\ )\ (dB) = 24\ dB$

- if $P_{IN} = 0\ dBm$

$$\Rightarrow \quad P_{OUT} = (0 + 24)\ dBm = 24\ dBm$$

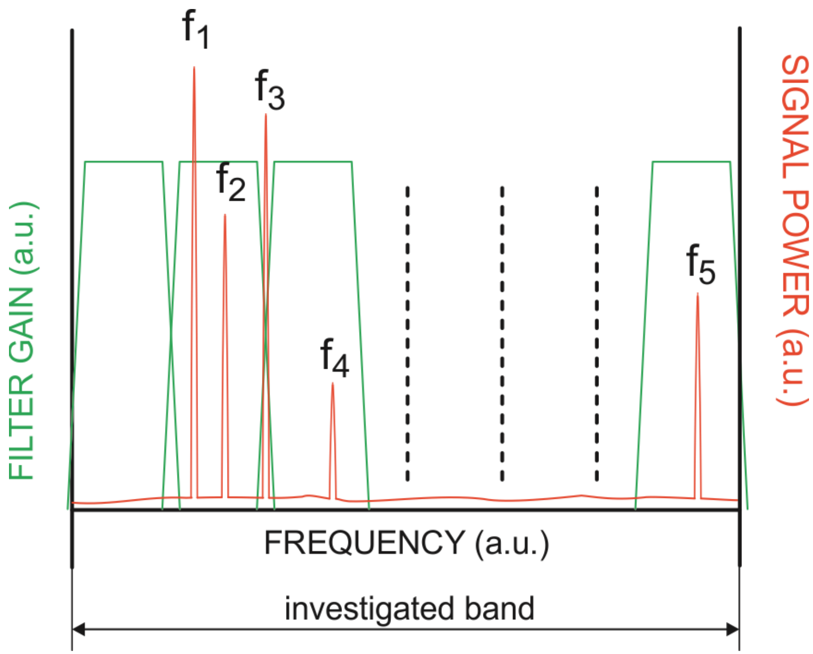
# Bank-of-filters spectrum analyser




- several band-pass filters are centered at different frequencies and working in parallel
- ideally, the filters should have a rectangular shaped transfer function
- each filter can have its presentation device or there could be a unique presentation device preceded by a multiplexer

# Bank-of-filters spectrum analyser




- each set of filter/rms-indicator shows the energy content of the frequency components included in the filter band
- the filter band-width determines the frequency resolution of the analyzer
- in case of M filters with adjacent transfer functions:

$$BW_{RES} = f_{MAX}/M$$