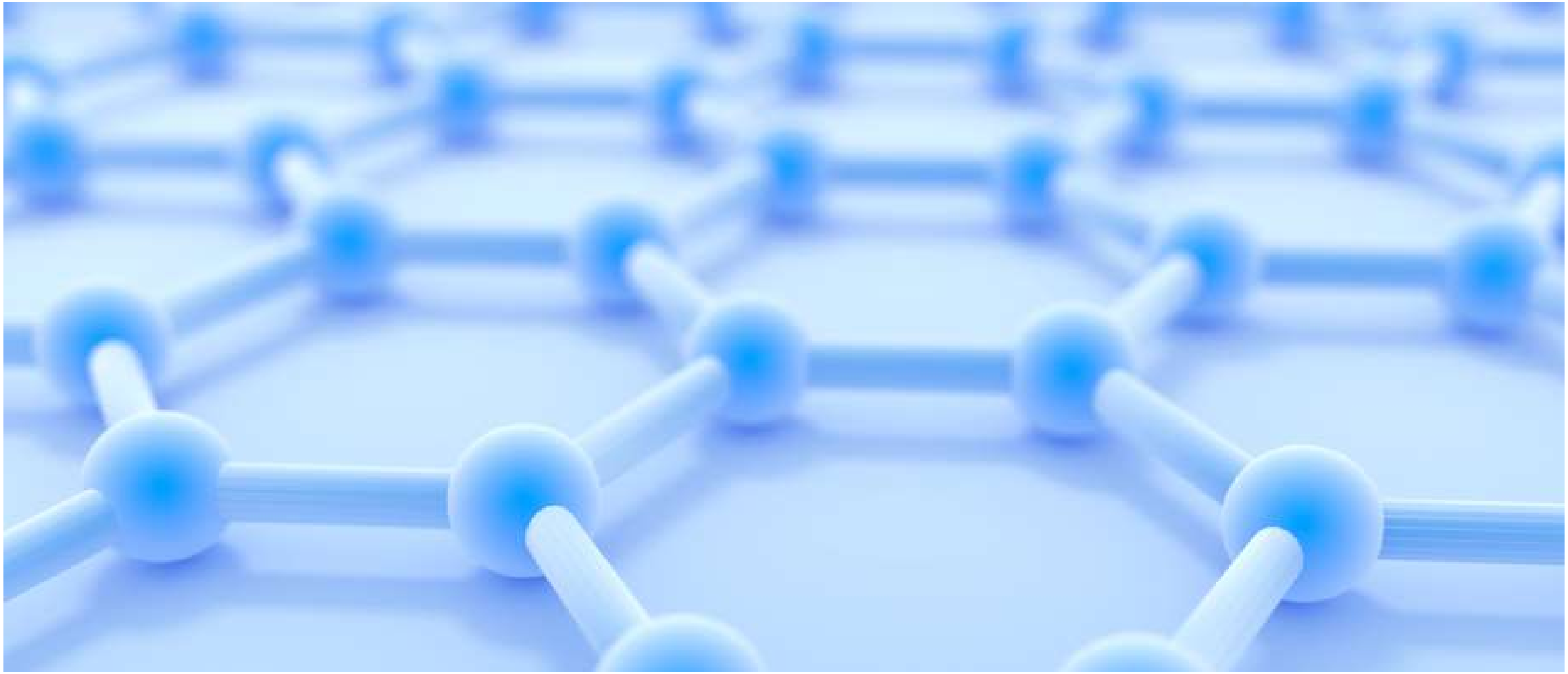
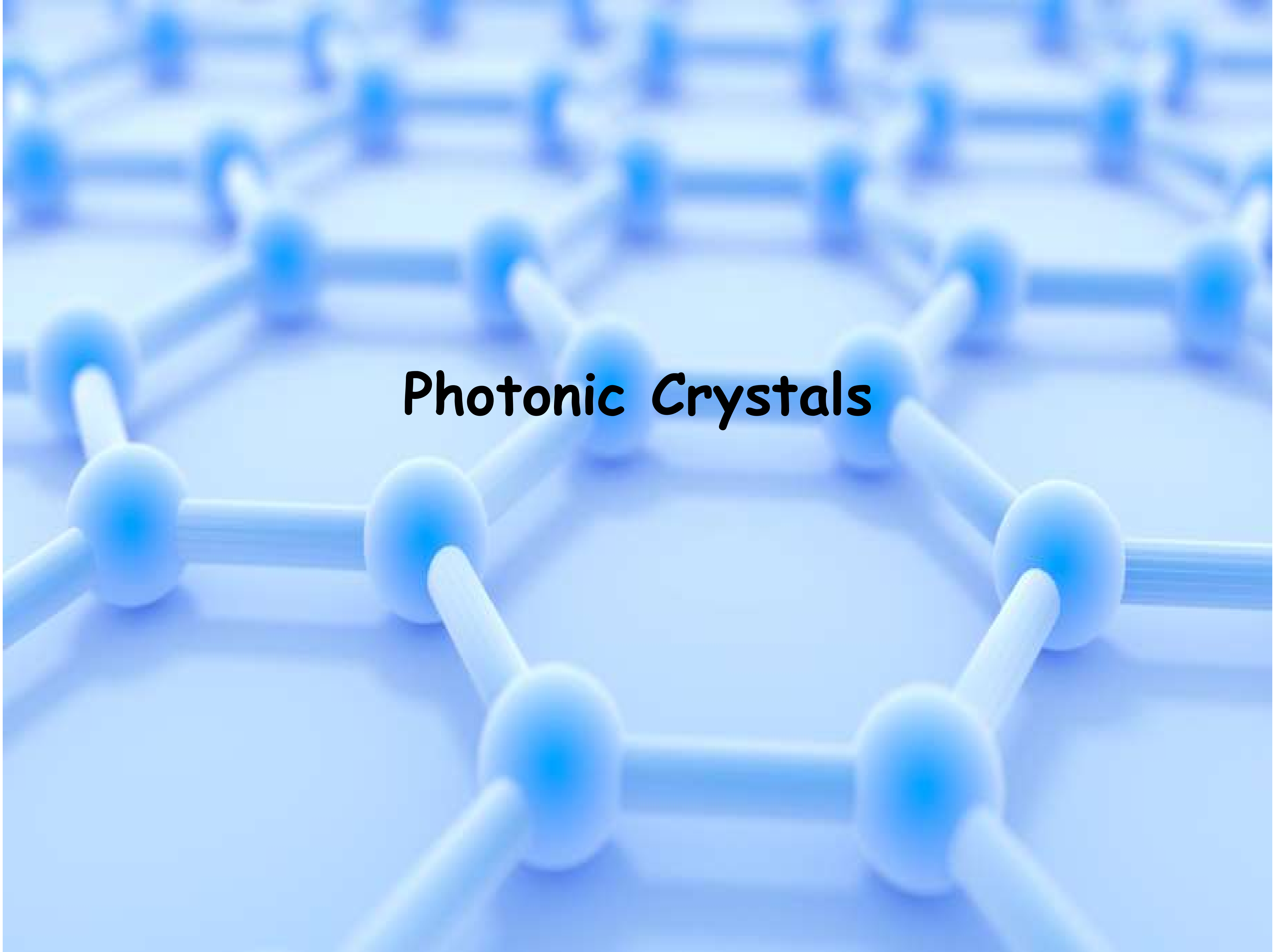


Photonics

aa 2021/2022

Prof. Maria Antonietta Vincenti
Università degli Studi di Brescia

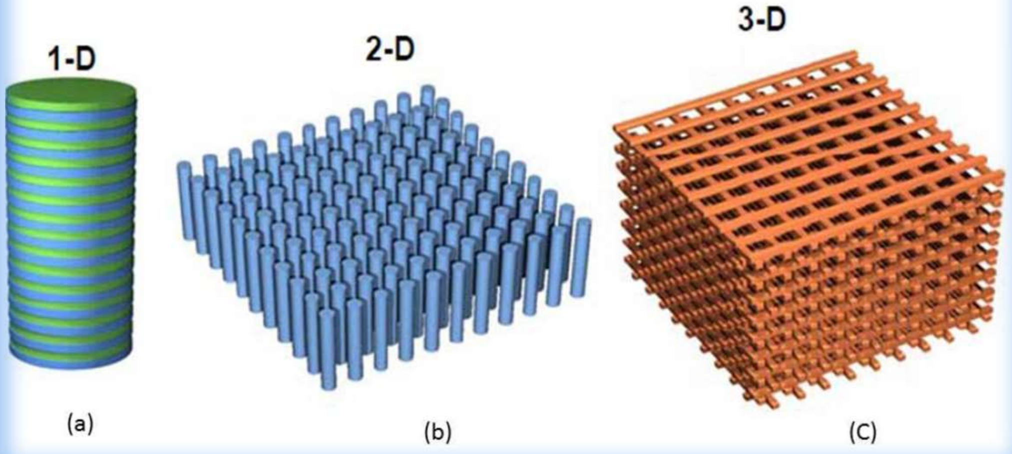
Photonic Crystals



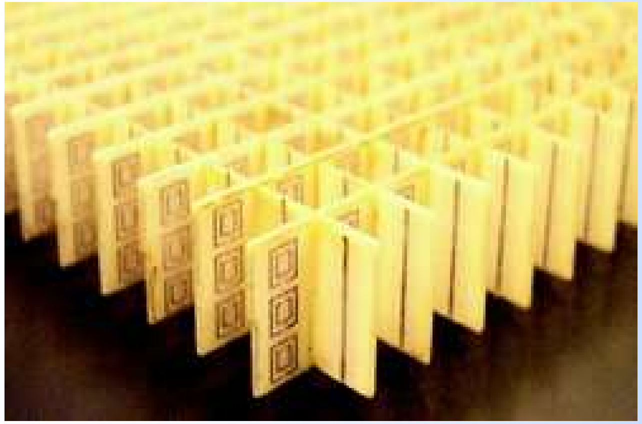
Introduction

Scientists and researchers refuse to be confined by the materials refined from naturally occurring compounds and they are creating new materials or nanostructured materials that exhibit unusual electronic and optical properties.

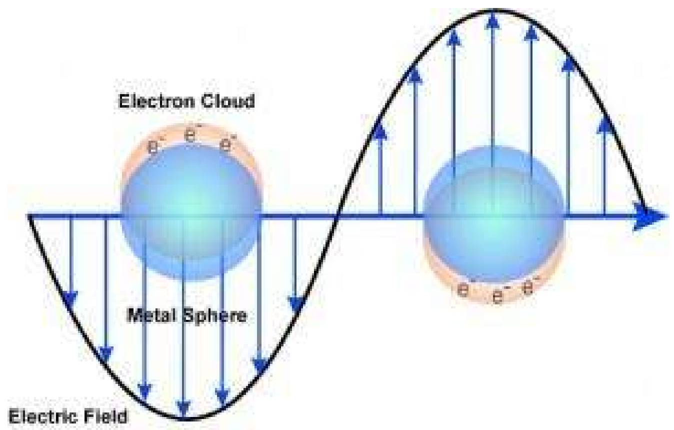
photonic crystals



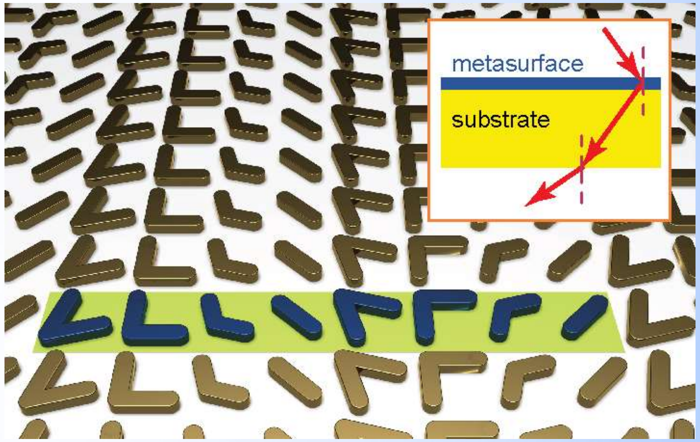
metamaterials



plasmonics



metasurfaces



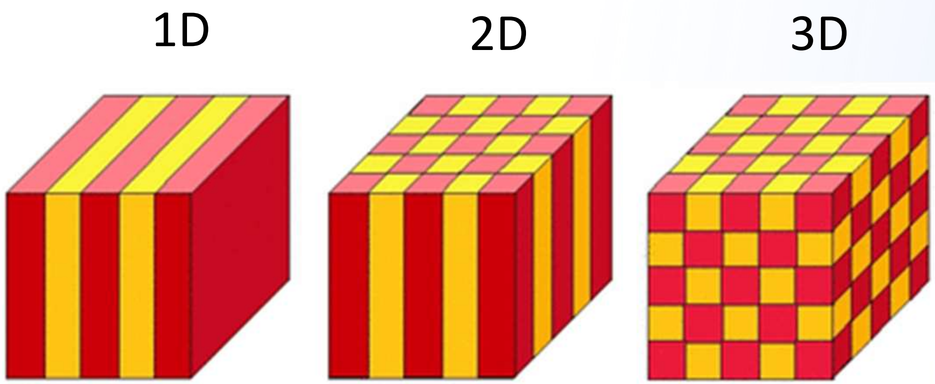
Introduction

WHAT ARE PHOTONIC CRYSTALS?

They are periodic dielectric structures that can interact resonantly with radiation with <u>wavelengths comparable to the periodicity length of the dielectric lattice.</u>

WHY ARE THEY CALLED PHOTONIC CRYSTALS?

<u>They can affect the properties of photons in much the same way that ordinary semiconductor and conductor crystals affect the properties of electrons.</u>
Properties of electrons in ordinary crystals are affected by parameters like lattice size and defects, for example. Crystals are made of periodic arrays of atoms at atomic length scales.



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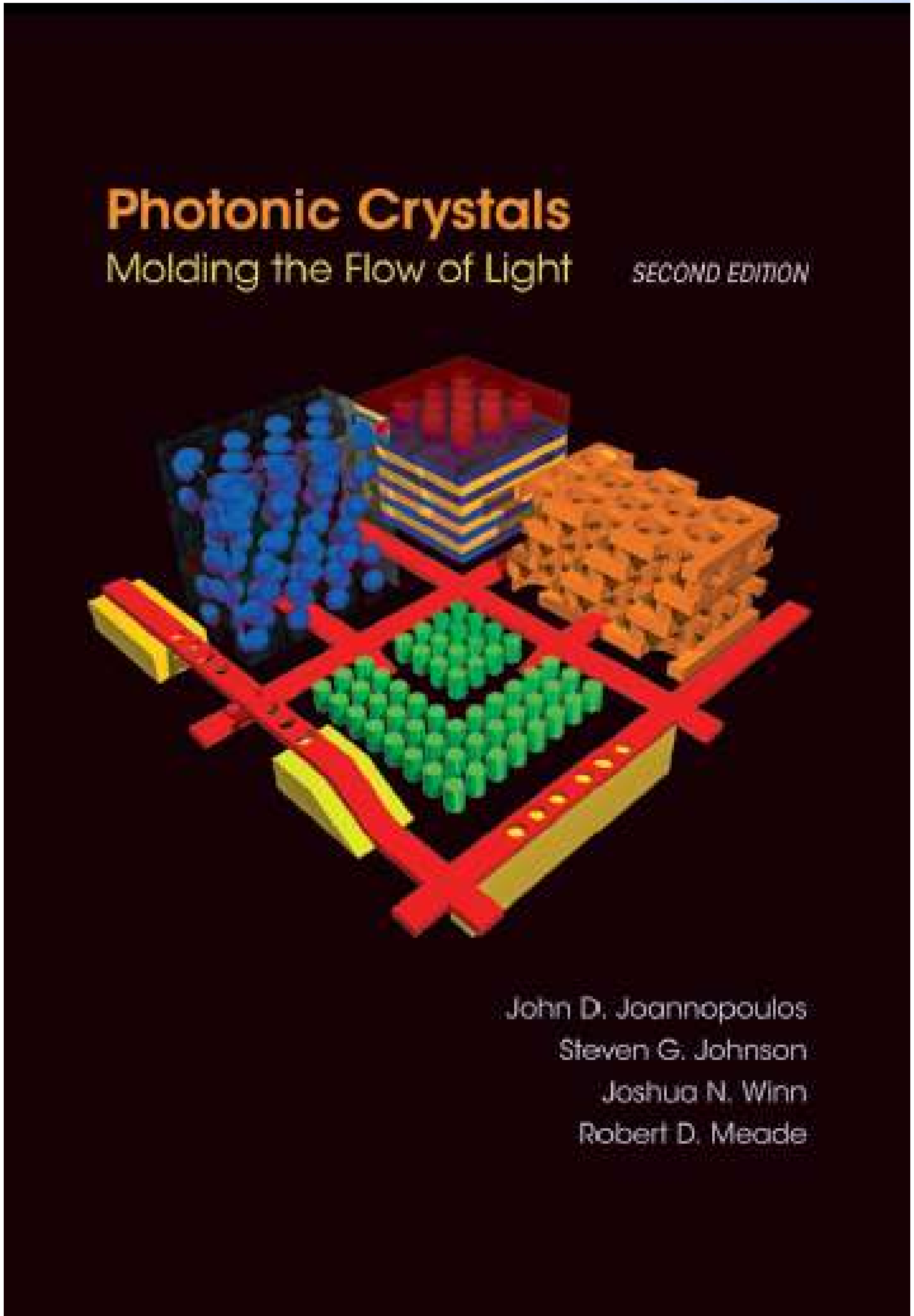
Useful read:
"Photonic Crystals: Molding the flow of light"
Joannopoulos, Johnson, Winn, Meade



Photonic Crystal History

1887: Lord Rayleigh stop bands 1D periodic multilayers
1987: Prediction of photonic crystals

S. John, Phys. Rev. Lett. **58**,2486 (1987), "*Strong **localization of photons** in certain dielectric superlattices*"
E. Yablonovitch, Phys. Rev. Lett. **58** 2059 (1987), "***Inhibited spontaneous emission** in solid state physics and electronics*"

1990: Computational demonstration of photonic crystal

K. M. Ho, C. T Chan, and C. M. Soukoulis, Phys. Rev. Lett. 65, 3152 (1990)

1991: Experimental demonstration of microwave photonic crystals

E. Yablonovitch, T. J. Mitter, K. M. Leung, Phys. Rev. Lett. 67, 2295 (1991)

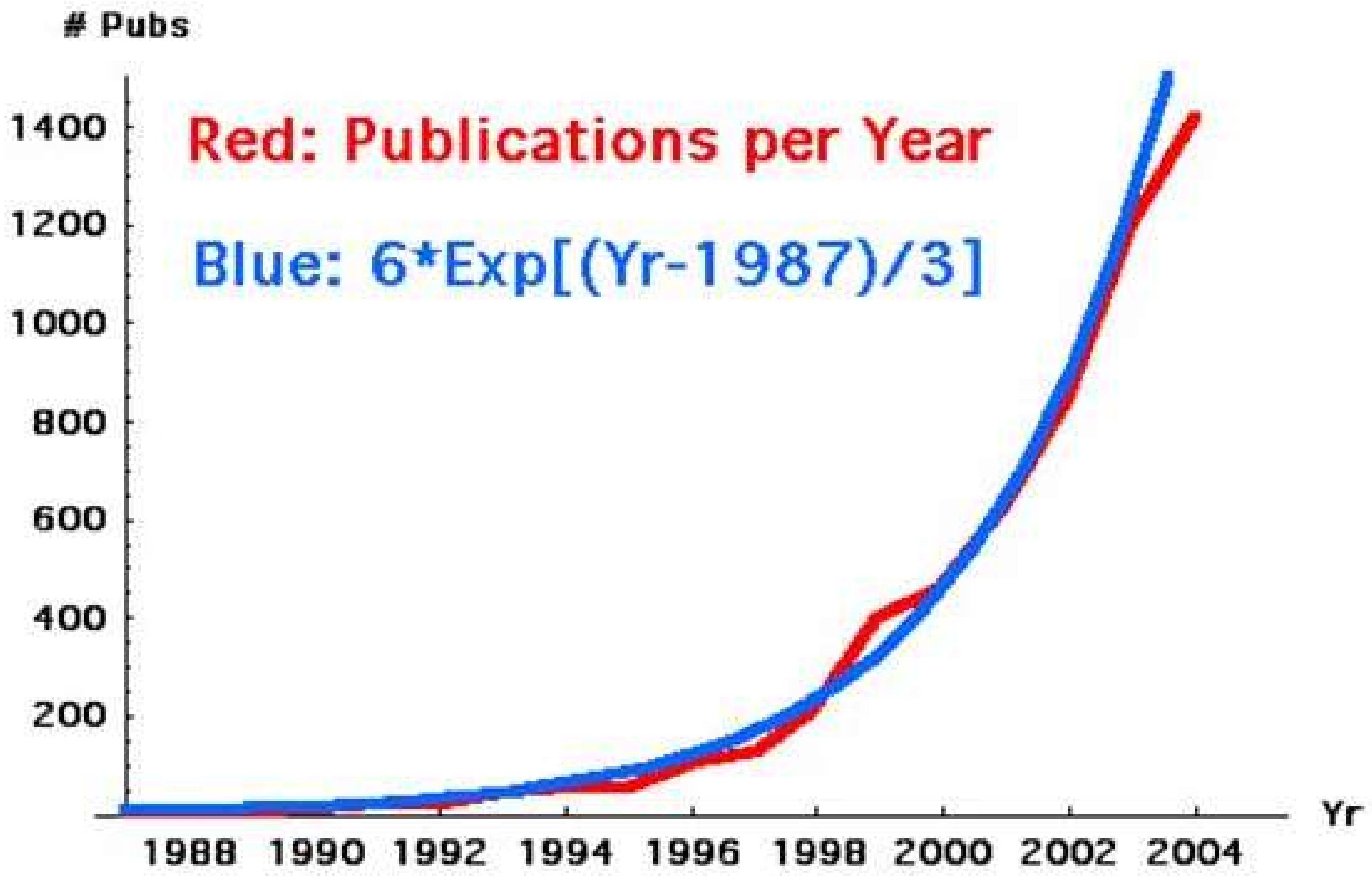
1995: "Large" scale 2D photonic crystals in Visible

U. Gruning, V. Lehman, C.M. Englehardt, Appl. Phys. Lett. 66 (1995)

1998: "Small" scale photonic crystals in near Visible; "Large" scale inverted opals

1999: First photonic crystal based optical devices (lasers, waveguides)

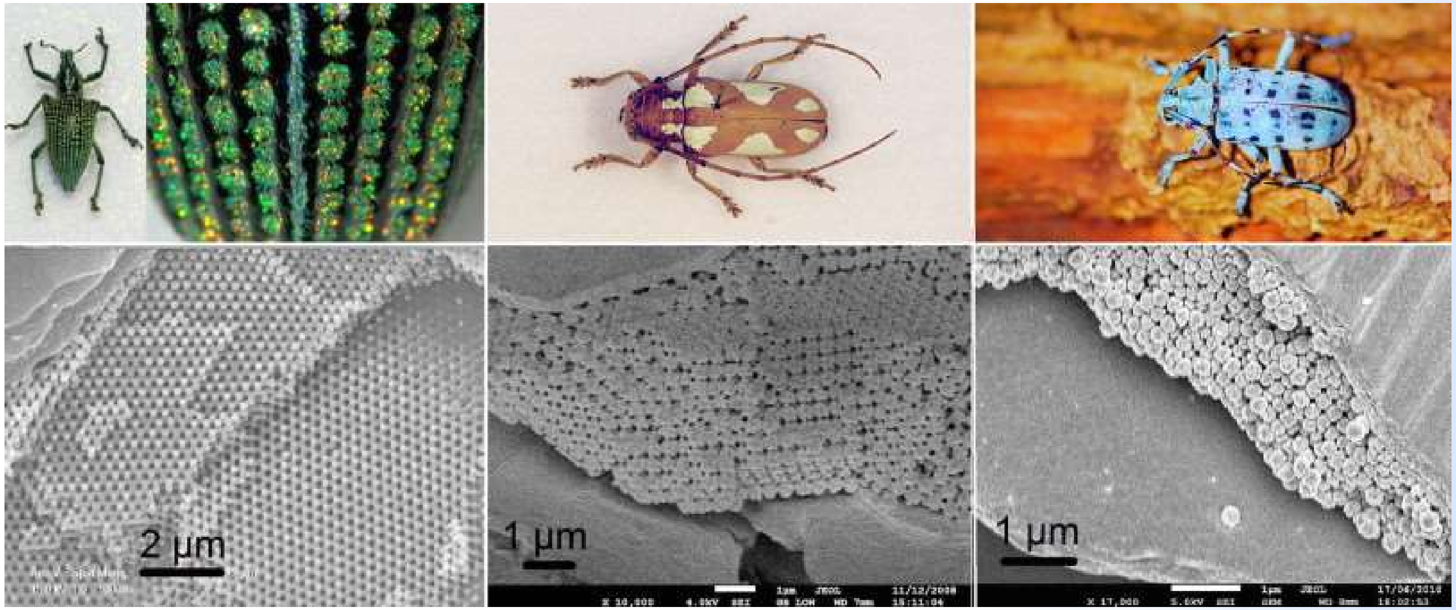
Photonic Crystal History



Natural Photonic Crystals

An example of nature's self-assembled periodic structures are *opaline materials*. The beautiful colors of opals are the result of light interference and diffraction from a periodic arrangement of silica spheres.

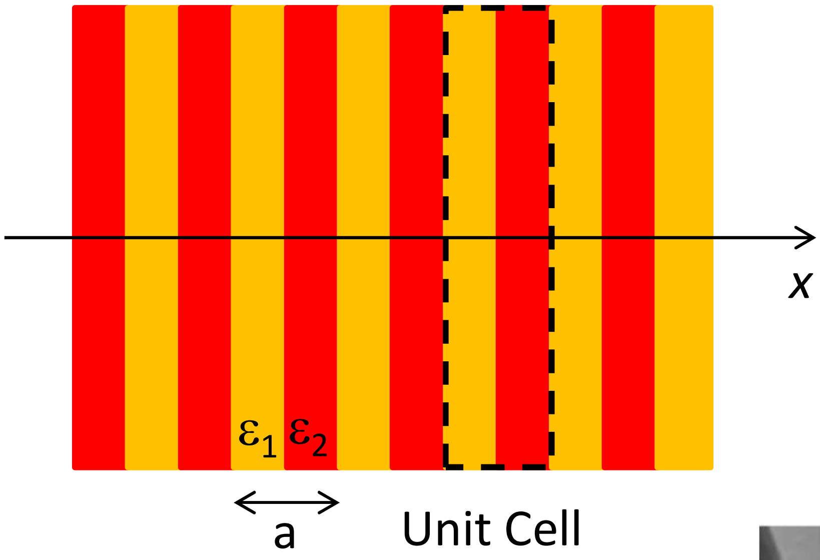
Other good examples are certain beetles' shells:



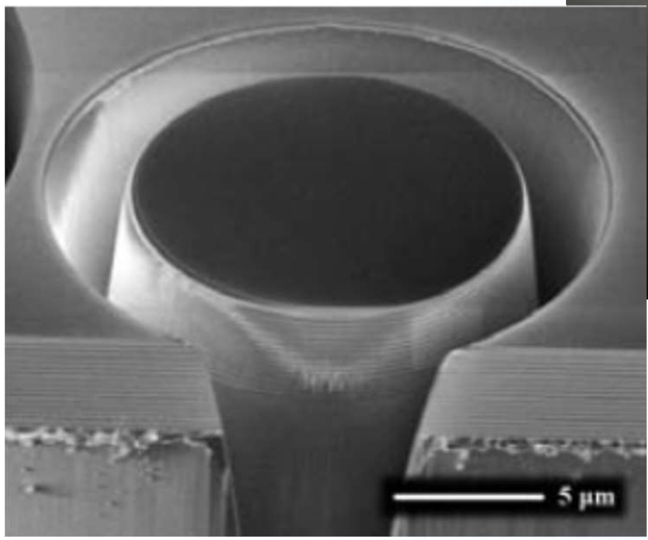
1D Photonic Crystals

The dielectric constant is periodically modulated in one dimension:

$$\varepsilon(\mathbf{r}) = \varepsilon(x) = \varepsilon(x + na), \quad n = 0, \pm1, \pm2, \ldots$$



They can be used to realize broadband dielectric mirrors. The characteristic color of the mirrors is caused by their wavelength dependent reflectivity.



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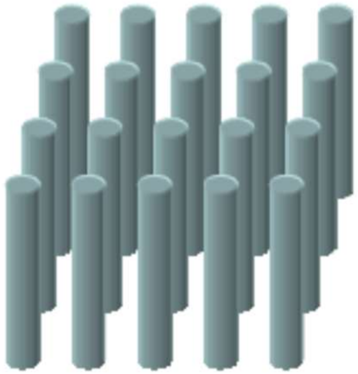
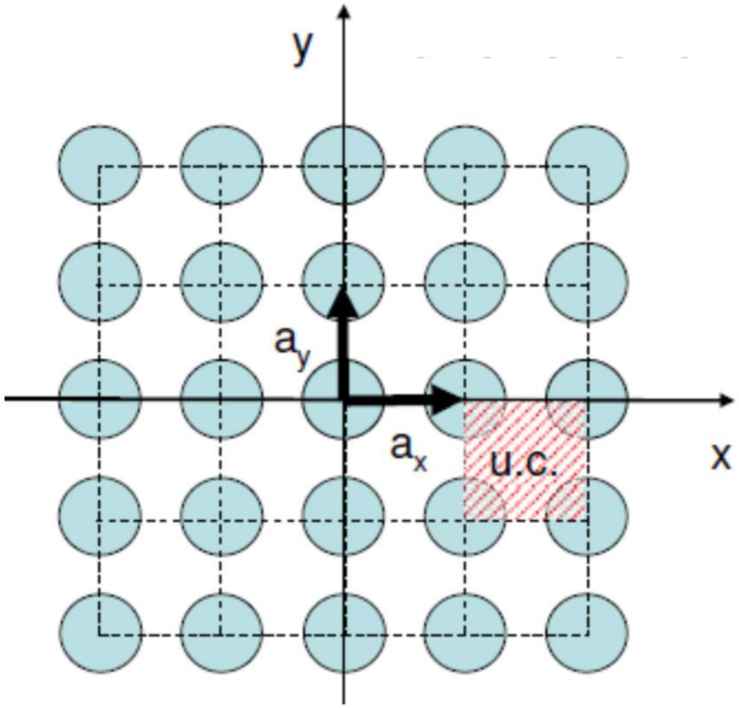
2D Photonic Crystals





The dielectric constant is periodically modulated in two dimensions:

$$\varepsilon(\mathbf{r}) = \varepsilon(x, y) = \varepsilon\left(x + na_x, y + ma_y\right), \quad n, m = 0, \pm 1, \pm 2, \ldots$$

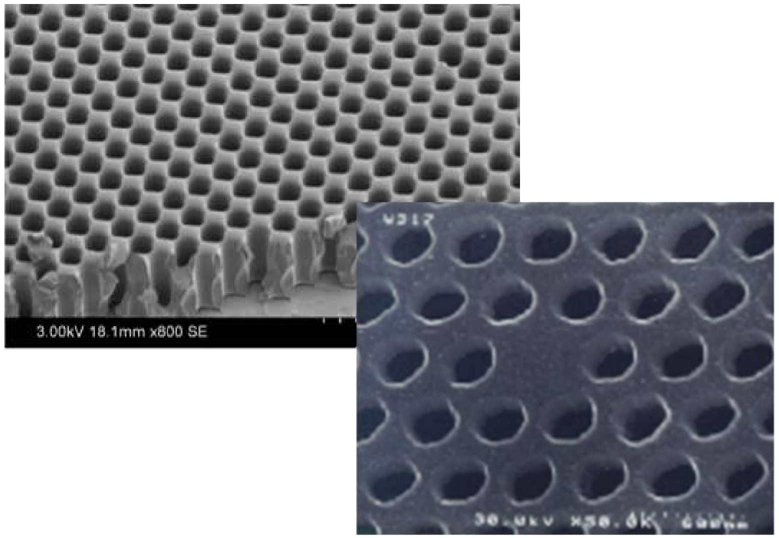
A cylinder is put at every lattice point. Any **Bravais** 2-D lattice is defined by 2 primitive vectors, for the square lattice they are:

$$\mathbf{a}_1 = a_x \hat{x}$$

$$\mathbf{a}_2 = a_y \hat{y}$$

Thus every lattice point can be written as:

$$\mathbf{R} = n\mathbf{a}_1 + m\mathbf{a}_2, \quad n, m = 0, \pm 1, \pm 2, \ldots$$

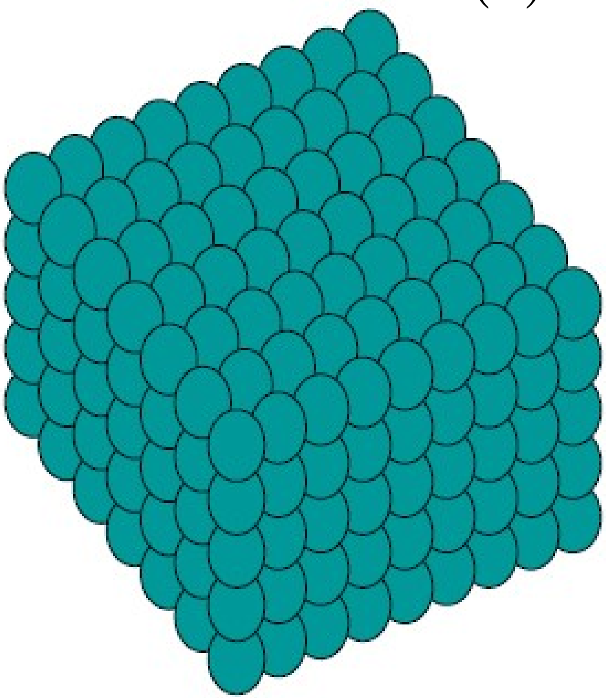


NOTE: There is no unique way of choosing a primitive cell for a given Bravais lattice. The choice would also depend on the type of periodicity of the crystal (cubic, hexagonal,…)

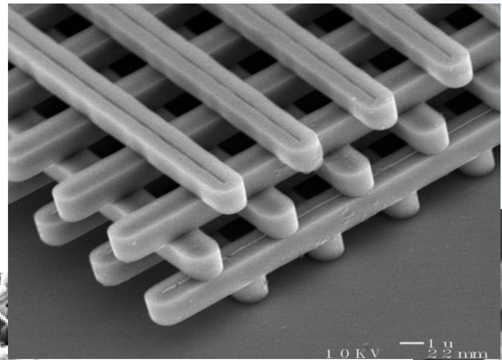
3D Photonic Crystals

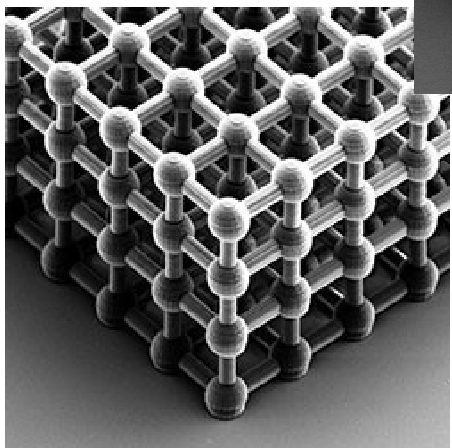
The dielectric constant is periodically modulated in three dimensions. Lattice is defined by 3 primitive vectors:

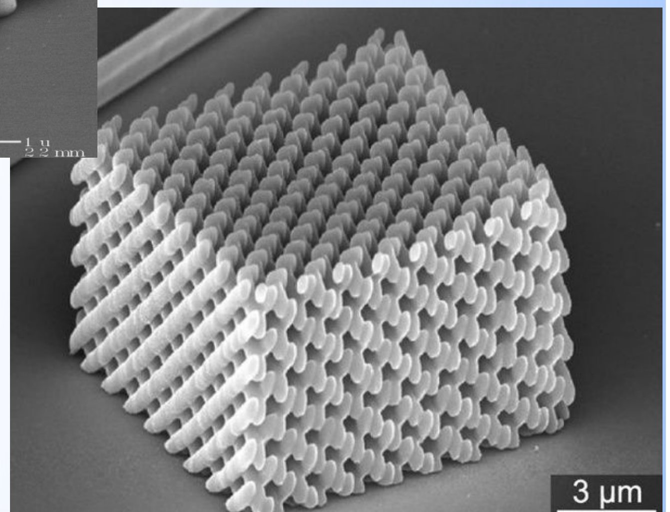
$$\varepsilon(\mathbf{r}) = \varepsilon(x, y, z) = \varepsilon\left(x + na_x, y + ma_y, z + la_z\right), \quad n, m, l = 0, \pm 1, \pm 2, \ldots$$



$$\mathbf{a}_1 = a_x \hat{x}$$
$$\mathbf{a}_2 = a_y \hat{y} \qquad \mathbf{R} = n\mathbf{a}_1 + m\mathbf{a}_2 + l\mathbf{a}_3, \quad n, m, l = 0, \pm 1, \pm 2, \ldots$$
$$\mathbf{a}_3 = a_z \hat{z}$$



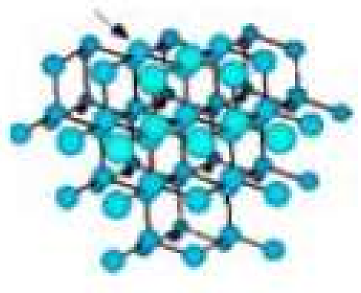




Propagation in periodic media

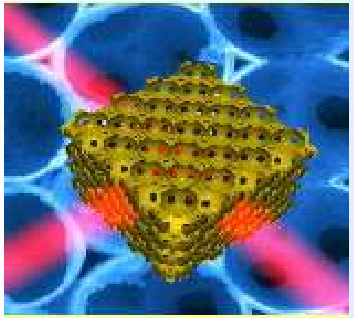
Waves in a periodic medium can propagate without scattering

ELECTRONS



If the atoms were randomly placed the free electrons would experience strong scattering with the lattice atoms and a short mean free path. This is in contrast with the high value of measured conductivity for some crystals. But crystal lattice are periodic with typical sizes of a few Å and electrons can be treated as waves (quantum mechanics) with energies corresponding to a typical wavelength comparable to lattice size.

PHOTONS



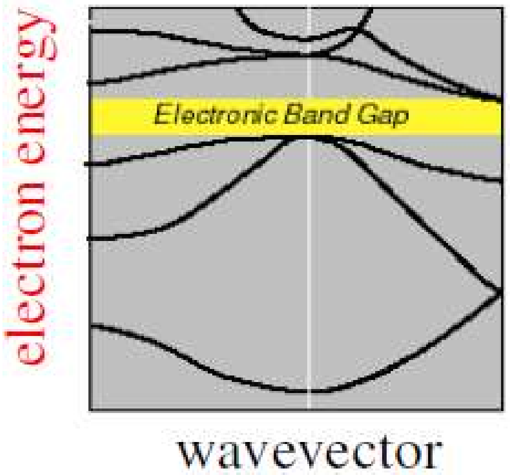
Light scattered from a random media: The effect can be weak or strong depending on density of scatterers. Size and shape of the scatterers produce wavelength and angular dependence on the efficiency of scattering. Photonic crystals are characterized by primitive cells of a size comparable to the wavelength of the photon. Resonant scattering can occur as a function of frequency and wavevector.

Propagation in periodic media

ELECTRONS

Electrons can propagate in a periodic lattice without experiencing scattering with a proper dispersive relation (energy as a function of the wave vector).
Band diagrams can be calculated using Bloch theorem

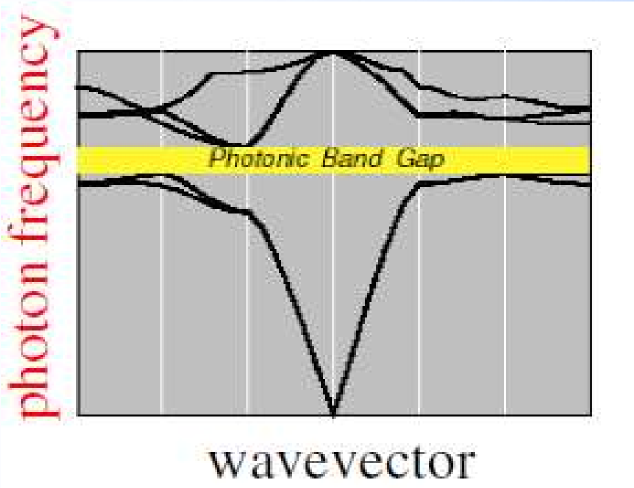
Electrons with energies in the ***Electronic Band Gap*** cannot propagate inside the crystals.



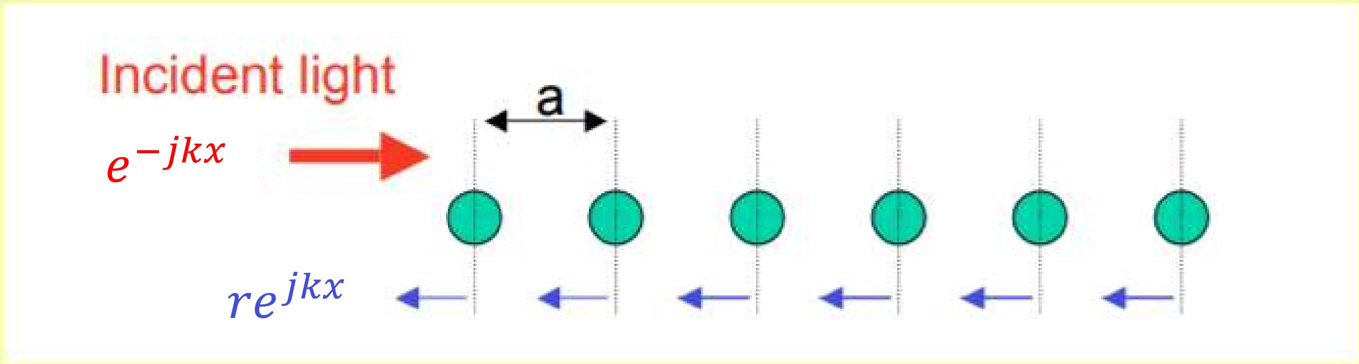
PHOTONS

Periodic modulation of the dielectric constant can affect the properties of photons in much the same way that ordinary semiconductor and conductor crystals affect the properties of electrons.

For certain geometries it exists a range of frequencies called ***Photonic Band Gap*** for which light is forbidden inside the crystal



BRAGG condition – Total reflection from a periodic lattice



Regardless of how small the reflectivity r is from an individual scatter, the total reflection R from a semi infinite structure:

$$R = re^{jkx} + re^{2jka}e^{jkx} + re^{4jka}e^{jkx} + \cdots = re^{jkx}\frac{1}{1-e^{2jka}}$$

Diverges if

$e^{2jka}=1 \qquad k = \frac{\pi}{a}$ ← Bragg condition $(\lambda \sim 2a)$

Light can not propagate in a crystal, when the frequency of the incident light is such that the Bragg condition is satisfied → Origin of the photonic band gap

An important issue that we need to address is: What is $k = 2\pi/\lambda$? What is the λ here?

But there are few Issues...

1. If the photonic crystal is made of materials with different indices, how is k defined in our simple BRAGG formula? In other words, what is the wavevector in a composite medium?

2. What form does the solution of Maxwell equations get in a periodic medium? E.g., is the field periodic? If so, what is the periodicity?

3. What is the speed of light in these structures (phase and group velocity)?

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Propagation in periodic media
BLOCH's THEOREM

Our starting point are Maxwell's equations and constitutive relations with no sources.

$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

$$\nabla \times \mathbf{h} = \frac{\partial \mathbf{d}}{\partial t} + \cancel{\mathbf{j}}$$

$$\nabla \cdot \mathbf{d} = \cancel{\rho}$$

$$\nabla \cdot \mathbf{b} = 0$$

and

σ~0

$$\mathbf{j} = \cancel{\sigma}\mathbf{e}$$

$$\mathbf{d} = \varepsilon_0 \varepsilon_r \mathbf{e}$$

$$\mathbf{b} = \mu_0 \mu_r \mathbf{h}$$

μ_r=1

If we seek solutions of the form:

$$\mathbf{e}(\mathbf{r},t) = \mathbf{E}(\mathbf{r}) e^{j\omega t} \qquad \mathbf{h}(\mathbf{r},t) = \mathbf{H}(\mathbf{r}) e^{j\omega t}$$

We have the following eigenvalue equations (in practice, these are the Helmoholtz equations without sources):

$$\hat{\Gamma}_E \mathbf{E}(\mathbf{r}) = \frac{1}{\varepsilon_r(\mathbf{r})} \nabla \times \left[\nabla \times \mathbf{E}(\mathbf{r})\right] = \frac{\omega^2}{c^2} \mathbf{E}(\mathbf{r})$$

$$\hat{\Gamma}_H \mathbf{H}(\mathbf{r}) = \nabla \times \left[\frac{1}{\varepsilon_r(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r})\right] = \frac{\omega^2}{c^2} \mathbf{H}(\mathbf{r})$$

If ε_r is a periodic function of the spatial coordinate, Bloch's Theorem states that fields solutions (eigensolutions, without sources) are characterized by a **Bloch wave vector K**, a band index n and have the form:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_{\mathbf{K}n}(\mathbf{r}) e^{-j\mathbf{K}\bullet\mathbf{r}} \qquad \mathbf{H}(\mathbf{r}) = \mathbf{H}_{\mathbf{K}n}(\mathbf{r}) e^{-j\mathbf{K}\bullet\mathbf{r}}$$

With:

$$\varepsilon_r(\mathbf{r}) = \varepsilon_r(\mathbf{r}+\mathbf{a}_i),\ \mathbf{E}_{\mathbf{K}n}(\mathbf{r}) = \mathbf{E}_{\mathbf{K}n}(\mathbf{r}+\mathbf{a}_i),\ \mathbf{H}_{\mathbf{K}n}(\mathbf{r}) = \mathbf{H}_{\mathbf{K}n}(\mathbf{r}+\mathbf{a}_i);$$

i=1,2,3. $\quad$ **$\mathbf{a}_i$** are the primitive vectors of the periodic lattice

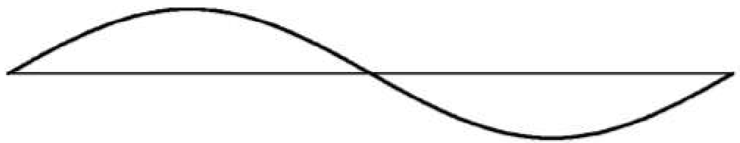
Bloch waves

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_{\mathbf{K}n}(\mathbf{r})e^{-j\mathbf{K}\bullet\mathbf{r}}$$

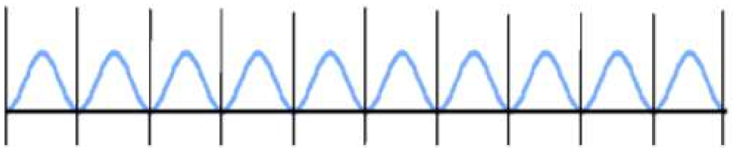
PERIODIC FUNCTIONS
(SAME PERIODICITY OF ϵ,μ,n)

PLANE WAVE

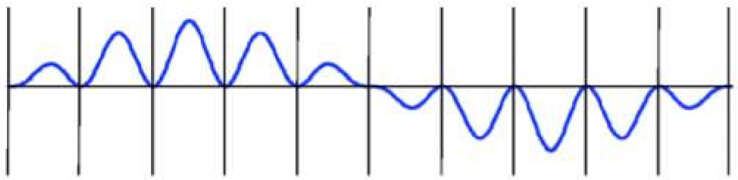
$$e^{-j\mathbf{K}\cdot\mathbf{r}}$$



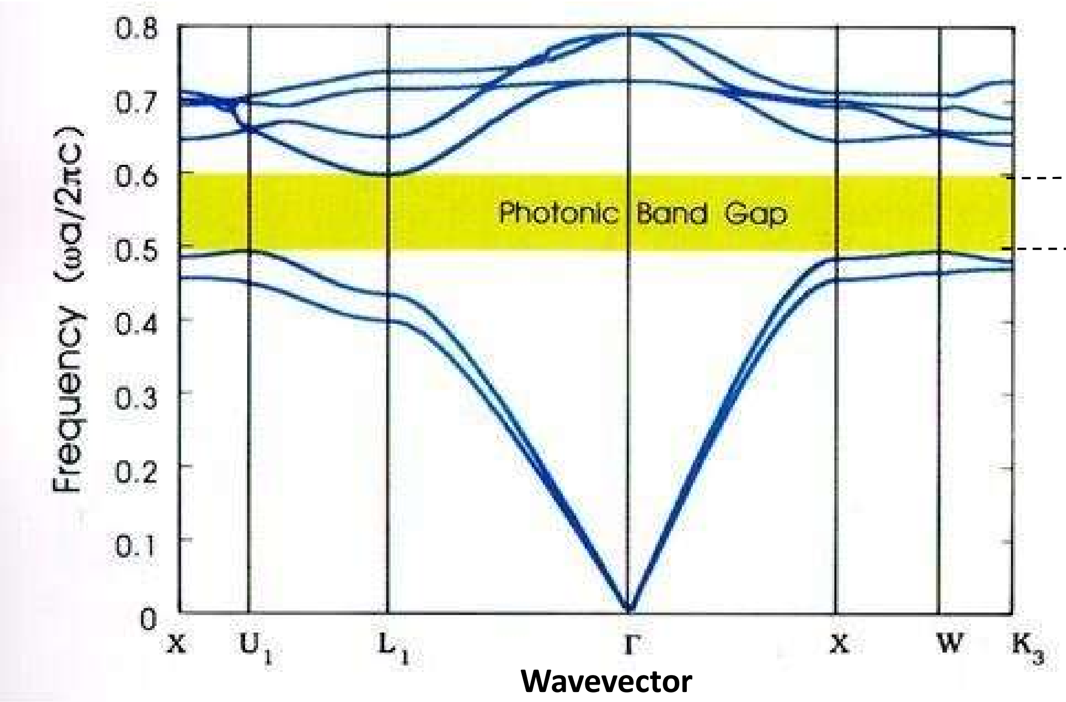
$$\mathbf{E}_{\mathbf{K}n}(\mathbf{r})$$

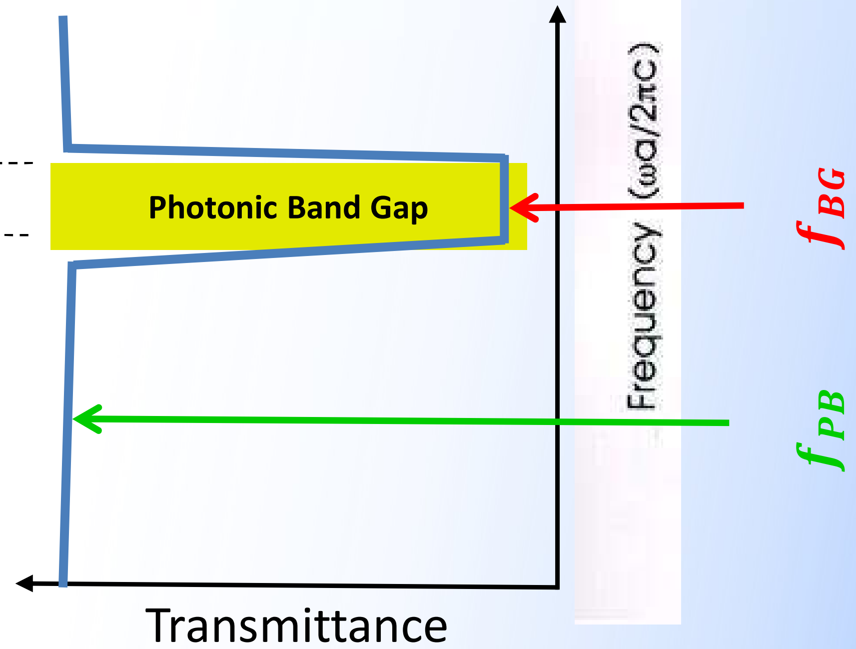


$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_{\mathbf{K}n}(\mathbf{r})e^{-j\mathbf{K}\cdot\mathbf{r}}$$



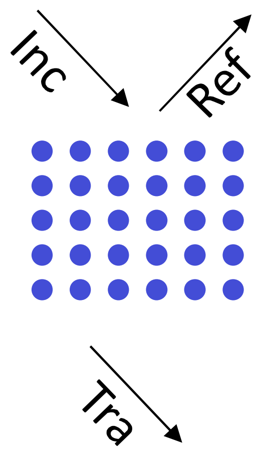
Bloch waves in action: band gap (BG) and pass band (PB)

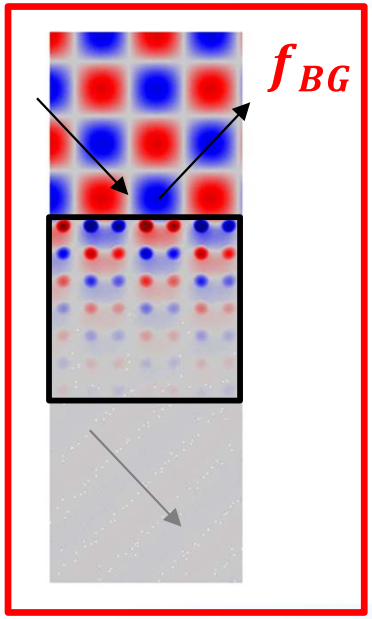


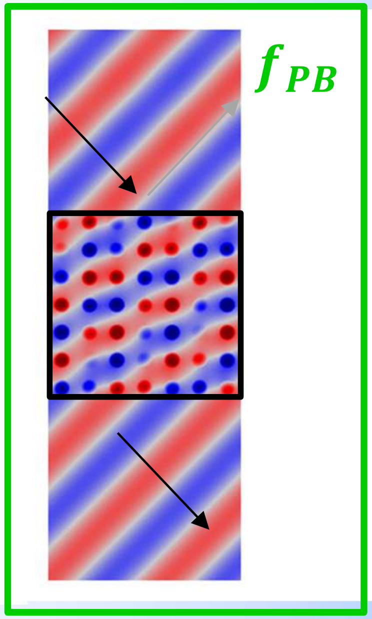


Photonic crystal:

Silicon Spheres (n~3.5) surrounded by air (n=1)







Propagation in periodic media

Substituting Bloch's solution into eigenvalue equations it is possible to calculate:

Eigen-angular frequencies $\omega_{\mathbf{K}n}$

Bloch modes (eigenvectors): $\mathbf{E}_{\mathbf{K}n},\ \mathbf{H}_{\mathbf{K}n}$

If we consider an infinite periodic structure and real values of ε (i.e lossless case), Γ_E and Γ_H are Hermitian eigen-operators and : $\omega_{\mathbf{K}n} \in R$

$\mathbf{E}_{\mathbf{K}n},\ \mathbf{H}_{\mathbf{K}n}$ are a complete set of orthogonal eigen-functions

Solving equations for several values of K and ω one can calculate and plot band diagrams (**dispersion relations**).

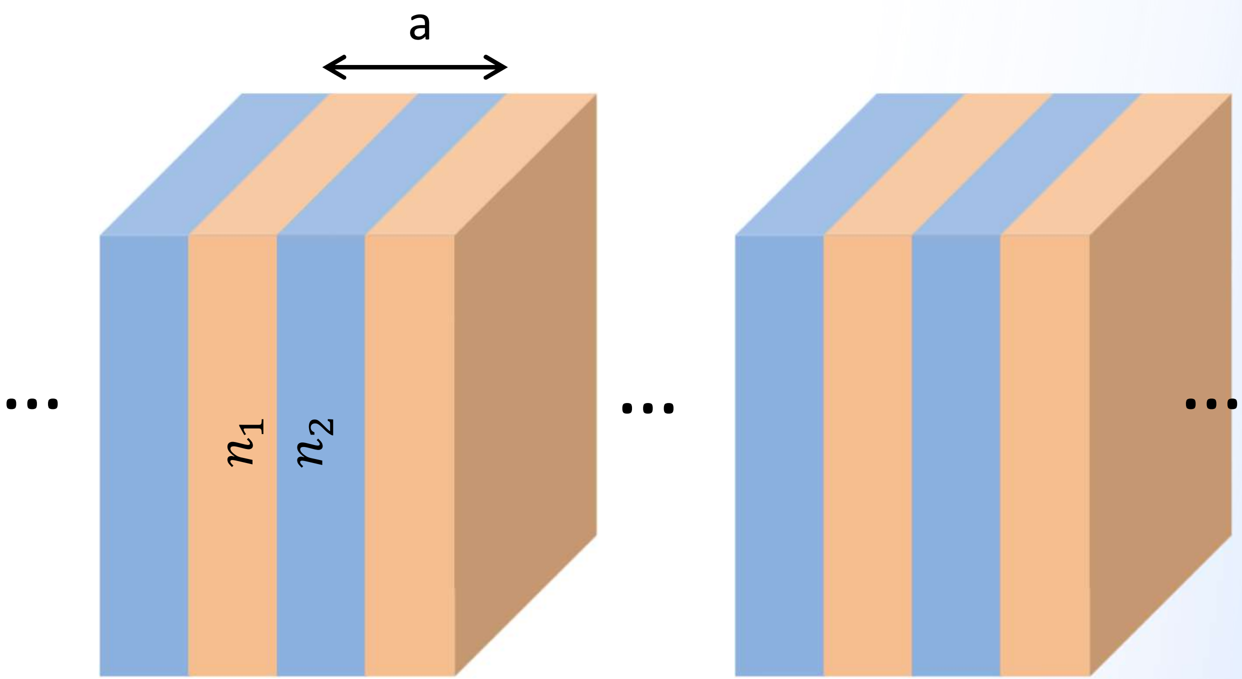
Some Computational tools:
Plane Wave Expansion (PWE) Method **, Transfer matrix method**, Rigorous Coupled-wave analysis

NOTE: **Phase velocity CANNOT be defined appropriately in photonic crystals** because eigenfunctions are superposition of plane waves and phase surfaces cannot be defined properly.

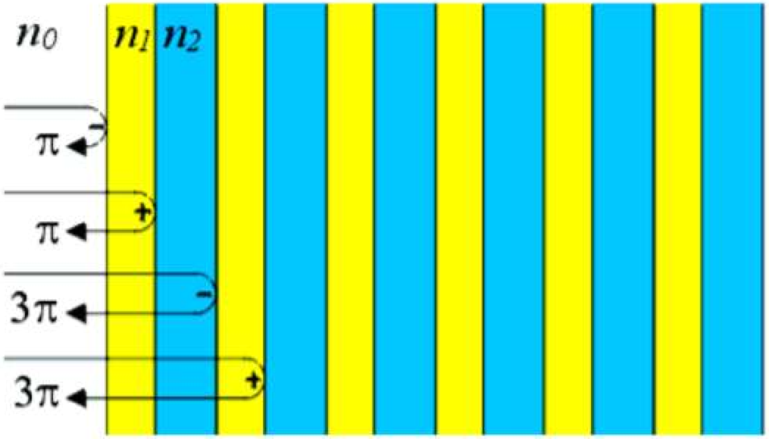
Analysis of 1D Photonic Crystals

The simplest possible photonic crystal is a **multilayer film**. This arrangement was discussed for the first time by Lord Rayleigh (1887).
This type of photonic crystal can act as a *mirror* (a **Bragg mirror**) for light with a frequency within a specified range, and it can localize light modes if there are any defects in its structure.

The traditional way to analyze this system, pioneered by Lord Rayleigh (1917), is to imagine that a plane wave propagates through the material and to consider the sum of the multiple reflections and refractions that occur at each interface.



Intuitive explanation of stop band formation in periodic structures



Let's consider a 1D periodic structure at the Bragg condition, i.e., $ka = m\pi$, where $L_1 + L_2 = a$ is the period, n_1 and n_2 the indices of the two materials in the structure and m is an integer (e.g., m=1).

Here $\lambda_{BRAGG} = \frac{2\pi}{k_{BRAGG}}$ will be the wavelength where reflectivity is 100%.

The bandgap bandwidth (or gap size) is maximized with quarter-wave layers (**QUARTER-WAVE STACK**):

$$k_1 L_1 = k_{Bragg} n_1 L_1 = \frac{\pi}{2} = k_2 L_2 = k_{Bragg} n_2 L_2$$

If $n_0 < n_2 < n_1$, then, AT THE BRAGG WAVELENGTH, the waves reflected at the interfaces will add up in phase, and the reflectivity will be maximum (=100%), as indicated in the figure.

Remember that, assuming normal incidence, $r_{01} = (n_0 - n_1)/(n_0 + n_1) < 0$, $r_{12} = (n_1 - n_2)/(n_1 + n_2) > 0$, and $r_{21} = -r_{12} < 0$.

Analysis of 1D Photonic Crystals: Brillouin Zone

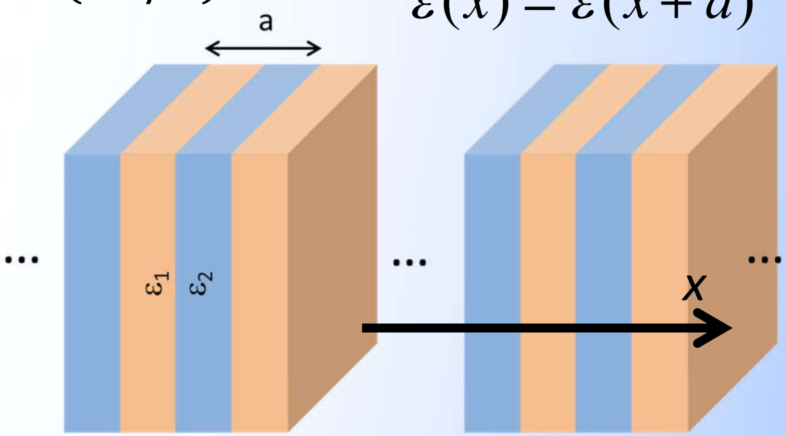
For 1D systems we can write the field as:

$$\mathbf{E}(x) = \mathbf{E_K}(x)e^{-jKx}, \quad \text{where } \mathbf{E_K}(x) = \mathbf{E_K}(x+ma), \quad m = 0, \pm1, \pm2, \ldots$$

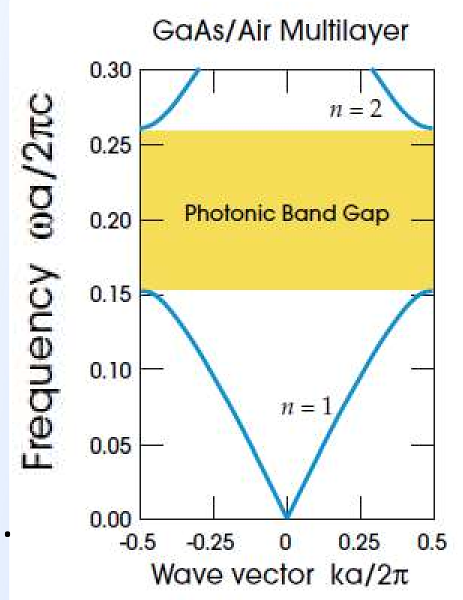
If we consider the Bloch mode with a wavevector $K' = K + m(2\pi/a)$

$$\varepsilon(x) = \varepsilon(x+a)$$

$$\mathbf{E}(x) = \mathbf{E_{K'}}(x)e^{-j\frac{2\pi x}{a}}e^{-jKx} = \mathbf{E_{K'}}(x)e^{-j(K+G)x}$$



This is a periodic function with period a, like $\mathbf{E_K}(x)$
The Bloch modes with wavevector $\mathbf{K}$ and $\mathbf{K}'$
are the same, and, therefore the eigenfrequencies for
$\mathbf{K}'$ are the same as those for $\mathbf{K}$. In other words, the
dispersion diagram repeat itself with periodicity $G = \frac{2\pi}{a}$

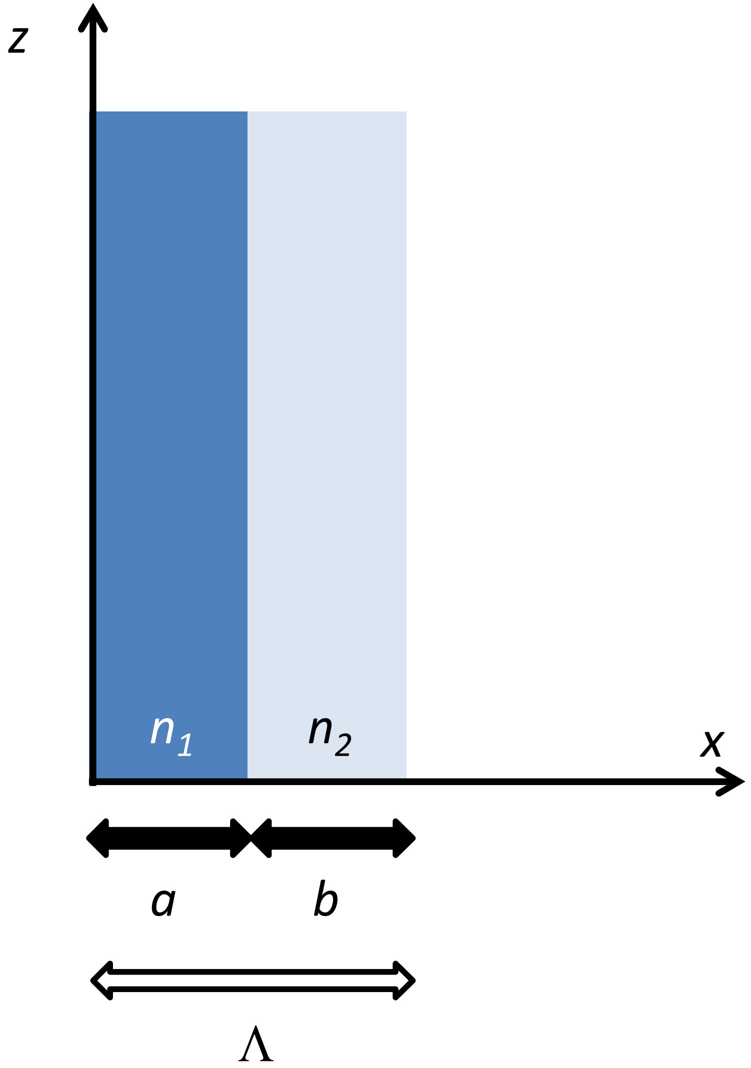


$G = 2\pi/a$ is the reciprocal lattice vector

- The primitive cell of the lattice is $a\hat{x}$
- The primitive cell of the reciprocal lattice is G=2π/a.
- Dispersion curves are periodic with a unit cell G = 2π/a called **Brillouin zone.**
- **The Brillouin zone is a primitive cell of the reciprocal lattice ($|k| < \pi/a$)**

Periodic Structure

Let's go back to the transfer matrix approach and generalize the expressions for the single slab in the case of a periodic structure with period Λ, composed of two media with refractive indices n_1 and n_2 and thicknesses *a* and *b*, respectively.



The refractive index of this structure can be easily expressed in an analytical form as:

$$n(x) = \begin{cases} n_2, & 0 < x < b \\ n_1, & \mathrm{b} < x < \Lambda \end{cases}$$

Where:

$$n(x) = n(x + \Lambda)$$

Periodic Structure

As for the case of the single layer, one can calculate the exact solution of the wave equations propagating in such periodic structure by imposing the usual boundary conditions on the electric field and its derivative at each interface.

Without making any assumption on the propagation direction we can write the electric field inside the structure in the following form:

$$E(x,z) = E(x)e^{-j\beta z}$$

The propagation inside the structure can be evaluated once again by considering the superposition of incident and reflected waves so that **in the layer having refractive index** α one can write:

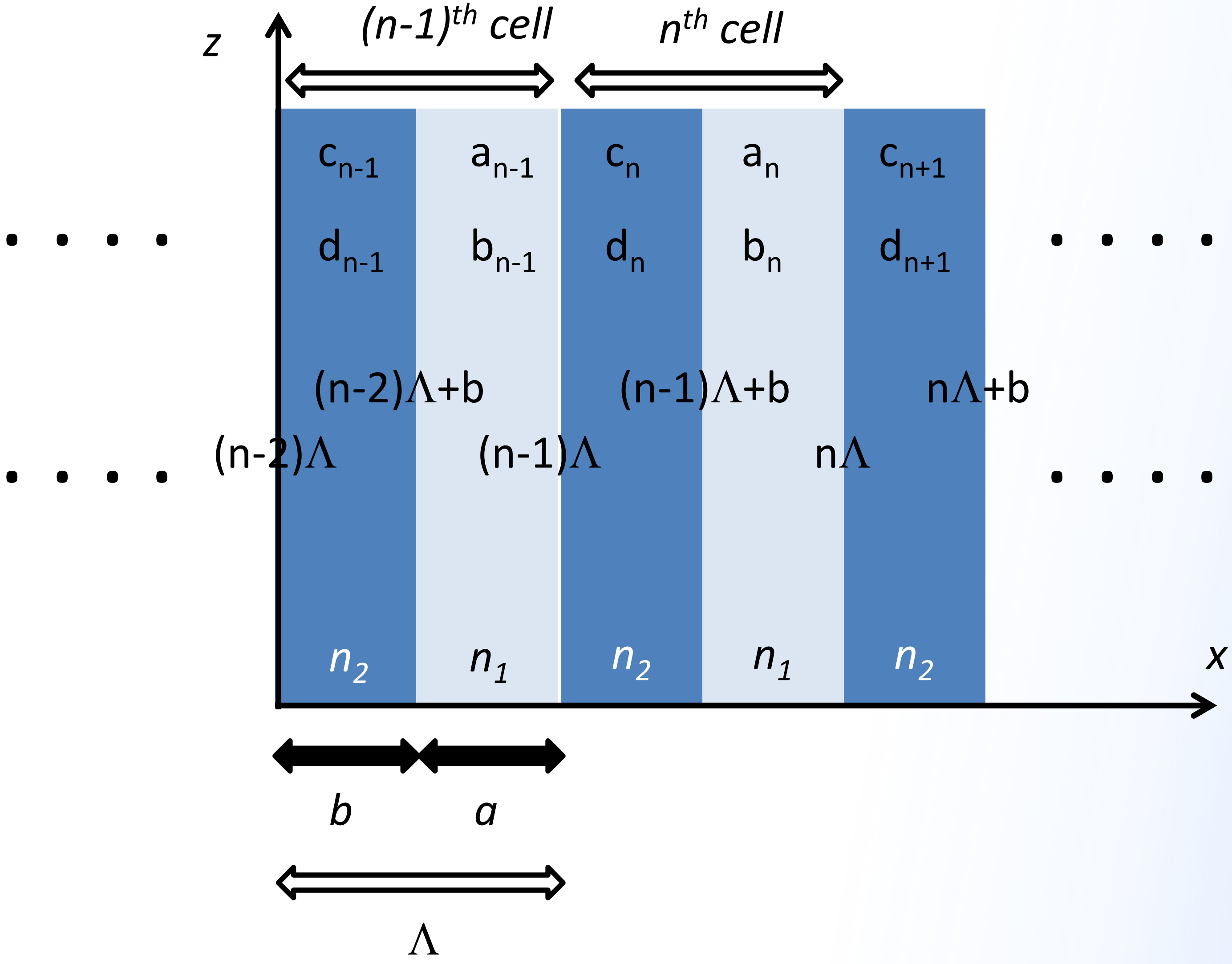
$$E(x,z) = \left(a_n^{(\alpha)} e^{-jk_{\alpha x}x} + b_n^{(\alpha)} e^{jk_{\alpha x}x}\right) e^{-j\beta z}$$

Where:

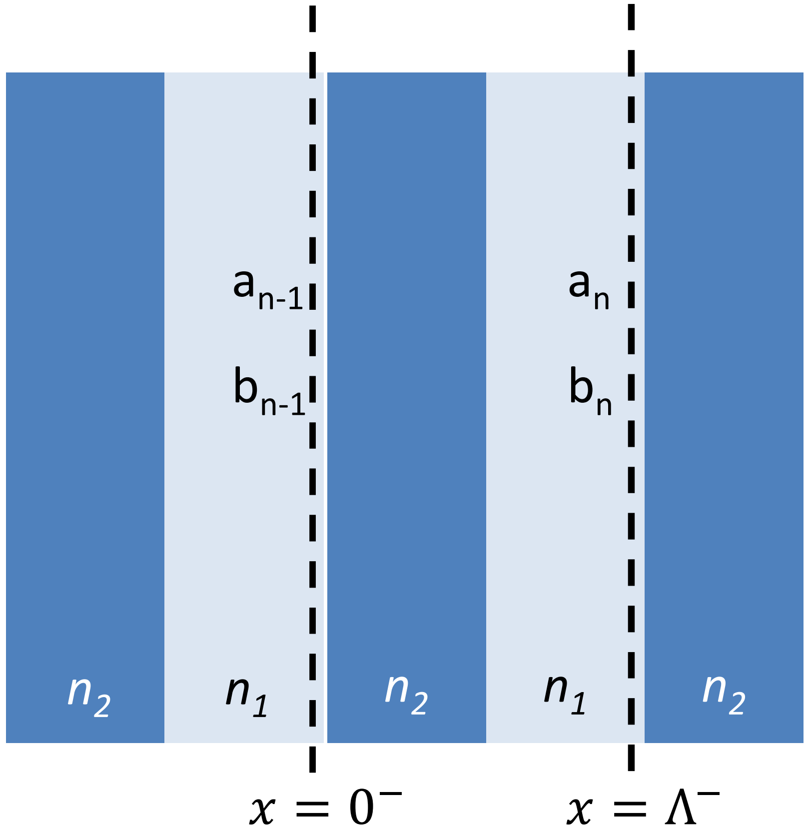
$$k_{\alpha x} = \sqrt{\left(\frac{\omega}{c} n_\alpha\right)^2 - \beta^2}, \qquad \alpha = 1,2$$

Periodic Structure

If we extend our structure to an infinite number of cells we can define the ratio between the incident fields and the reflected and transmitted fields for each cell.



Periodic Structure



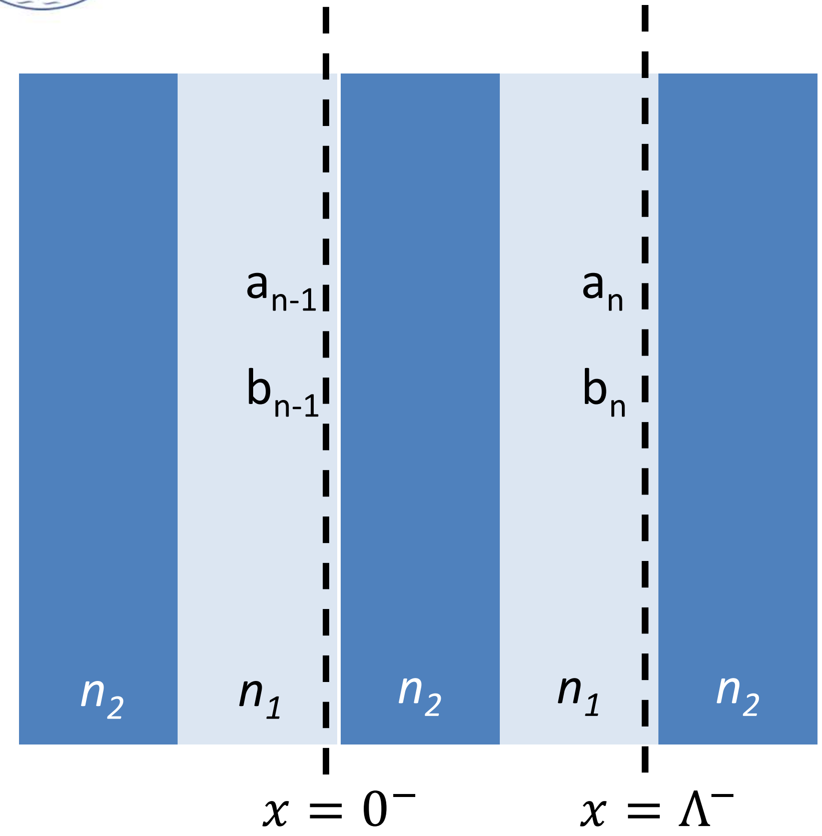
$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

We are only interested in finding a matrix relation (transfer matrix) between the amplitudes of the forward and backward waves in the cell n-1, i.e., a_{n-1} and b_{n-1}, and the amplitudes of the forward and backward waves in the cell n, i.e., a_n and b_n.

We know how to do this for a single interface, by using the discontinuity matrix D.

We also know that the propagation matrix P can be applied to let waves propagate in a portion of a homogeneous material.

Periodic Structure



$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = D_{21} P_2^{-1} D_{12} P_1^{-1} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

Hence $\boldsymbol{M} = \boldsymbol{D_{21} P_2^{-1} D_{12} P_1^{-1}}$

$$D_{m,n} = D_n^{-1} D_m = \frac{1}{2} \begin{bmatrix} 1 + k_{mx}/k_{nx} & 1 - k_{mx}/k_{nx} \\ 1 - k_{mx}/k_{nx} & 1 + k_{mx}/k_{nx} \end{bmatrix} \qquad P_n^{-1} = \begin{bmatrix} e^{jk_{nx}d} & 0 \\ 0 & e^{-jk_{nx}d} \end{bmatrix}$$

Periodic Structure

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = M \begin{pmatrix} a_n \\ b_n \end{pmatrix} \quad \text{with } \boldsymbol{M} = \boldsymbol{D_{21}} \boldsymbol{P_2^{-1}} \boldsymbol{D_{12}} \boldsymbol{P_1^{-1}} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

$$M = \frac{1}{2} \begin{bmatrix} \left(1 + \frac{k_{2x}}{k_{1x}}\right) e^{jk_{2x}b} & (1 - \frac{k_{2x}}{k_{1x}}) e^{-jk_{2x}b} \\ (1 - \frac{k_{2x}}{k_{1x}}) e^{jk_{2x}b} & \left(1 + \frac{k_{2x}}{k_{1x}}\right) e^{-jk_{2x}b} \end{bmatrix} \frac{1}{2} \begin{bmatrix} \left(1 + \frac{k_{1x}}{k_{2x}}\right) e^{jk_{1x}a} & (1 - \frac{k_{1x}}{k_{2x}}) e^{-jk_{1x}a} \\ (1 - \frac{k_{1x}}{k_{2x}}) e^{jk_{1x}a} & \left(1 + \frac{k_{1x}}{k_{2x}}\right) e^{-jk_{1x}a} \end{bmatrix}$$

After simple algebra one finds the 4 entries of the matrix (TE polarization):

$$M_{11} = e^{jk_{1x}a} \left[cos(k_{2x}b) + \frac{1}{2} j \left(\frac{k_{2x}}{k_{1x}} + \frac{k_{1x}}{k_{2x}} \right) sin(k_{2x}b) \right]$$

$$M_{12} = e^{-jk_{1x}a} \left[\frac{1}{2} j \left(\frac{k_{2x}}{k_{1x}} - \frac{k_{1x}}{k_{2x}} \right) sin(k_{2x}b) \right]$$

$$M_{21} = e^{jk_{1x}a} \left[-\frac{1}{2} j \left(\frac{k_{2x}}{k_{1x}} - \frac{k_{1x}}{k_{2x}} \right) sin(k_{2x}b) \right]$$

$$M_{22} = e^{-jk_{1x}a} \left[cos(k_{2x}b) - \frac{1}{2} j \left(\frac{k_{2x}}{k_{1x}} + \frac{k_{1x}}{k_{2x}} \right) sin(k_{2x}b) \right]$$

Periodic Structure

We can make the same analysis for the TM polarization and we will find the following coefficients for the ABCD matrix:

$$M_{11\,TM} = e^{jk_{1x}a}\left[\cos(k_{2x}b) + \frac{1}{2}j\left(\frac{n_2^2 k_{2x}}{n_1^2 k_{1x}} + \frac{n_1^2 k_{1x}}{n_2^2 k_{2x}}\right)\sin(k_{2x}b)\right]$$

$$M_{12\,TM} = e^{-jk_{1x}a}\left[\frac{1}{2}j\left(\frac{n_2^2 k_{2x}}{n_1^2 k_{1x}} - \frac{n_1^2 k_{1x}}{n_2^2 k_{2x}}\right)\sin(k_{2x}b)\right]$$

$$M_{21\,TM} = e^{jk_{1x}a}\left[-\frac{1}{2}j\left(\frac{n_2^2 k_{2x}}{n_1^2 k_{1x}} - \frac{n_1^2 k_{1x}}{n_2^2 k_{2x}}\right)\sin(k_{2x}b)\right]$$

$$M_{22\,TM} = e^{-jk_{1x}a}\left[\cos(k_{2x}b) - \frac{1}{2}j\left(\frac{n_2^2 k_{2x}}{n_1^2 k_{1x}} + \frac{n_1^2 k_{1x}}{n_2^2 k_{2x}}\right)\sin(k_{2x}b)\right]$$

P and M are unimodular matrices

PROPERTY 1:

When a transfer matrix translates forward/backward waves from one plane to another plane of a reciprocal, lossless material, i.e., performing a translation operation, then this matrix must be unimodular, i.e., the determinant is equal to 1. You can trivially demonstrate this for the propagation matrix P, which translates light from two planes of a homogeneous material that are separated by any arbitrary distance d. The same happens in a photonic crystal if you translate light (forward/backward wave) from one unit cell to the next one. In fact, you can easily verify that $M_{11}M_{22} - M_{12}M_{21} = 1,$ for both polarizations.

PROPERTY 2:

An important property of unimodular matrices is that they have reciprocal eigenvalues, i.e., if $\lambda_{1,2}$ are the eigenvalues of the unimodular matrix M, then $\lambda_1 \lambda_2 = 1 = \det M$

Bloch Theorem: finding the eigenvalues of the matrix M

We know the matrix M and that $\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = M \begin{pmatrix} a_n \\ b_n \end{pmatrix}$.

As a consequence of the Bloch's theorem, we can also write $\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = e^{jK\Lambda} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$. Therefore

$$M \begin{pmatrix} a_n \\ b_n \end{pmatrix} = e^{jK\Lambda} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

Now we have an explicit eigenvalue problem,

$$\det \begin{bmatrix} M_{11} - e^{jK\Lambda} & M_{12} \\ M_{21} & M_{22} - e^{jK\Lambda} \end{bmatrix} = 0$$

whose eigenvalues, $\lambda_{1,2}$ are reciprocal ($\lambda_1 \lambda_2 = 1$, because M is unimodular, i.e., $det(M) = M_{11} M_{22} - M_{12} M_{21} = 1$), and they can be written as follows:

$$\lambda_{1,2} = e^{\pm jK} = \frac{1}{2}(M_{11} + M_{22}) \pm \sqrt{\left[\frac{M_{11} + M_{22}}{2}\right]^2 - 1}$$

The corresponding eigenvectors are:

$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = const * \begin{pmatrix} M_{12} \\ e^{\pm jK\Lambda} - M_{11} \end{pmatrix}$, where $const$ is an arbitrary constant.

Dispersion relation for the Bloch wave

If we take the sum of the two eigenvalues: $e^{\pm jK\Lambda} = \frac{1}{2}(M_{11} + M_{22}) \pm \sqrt{\left[\frac{M_{11}+M_{22}}{2}\right]^2 - 1}$

we obtain an explicit expression of the dispersion relation:

$$cosK\Lambda = \frac{1}{2}(M_{11} + M_{22})$$

Example: at normal incidence, $\beta = 0$ and $cosK\Lambda = \cos k_1 a \cos k_2 b - \frac{1}{2}\left(\frac{n_2}{n_1} + \frac{n_1}{n_2}\right)\sin k_1 a \sin k_2 b$

Where of course, $k_1 = \frac{\omega}{c} n_1$ and $k_2 = \frac{\omega}{c} n_2$.

Hence the explicit expression of the Bloch wavevector: $K(\beta, \omega) = \frac{1}{\Lambda}\cos^{-1}\left[\frac{1}{2}(M_{11} + M_{22})\right]$

It is important to remember that the wavevector is in general a function of ω and β (angle of incidence), that is the wavevector component in the z direction (a conserved quantity in our problem). It is obvious now that one can observe different regimes of propagation in a periodic medium:

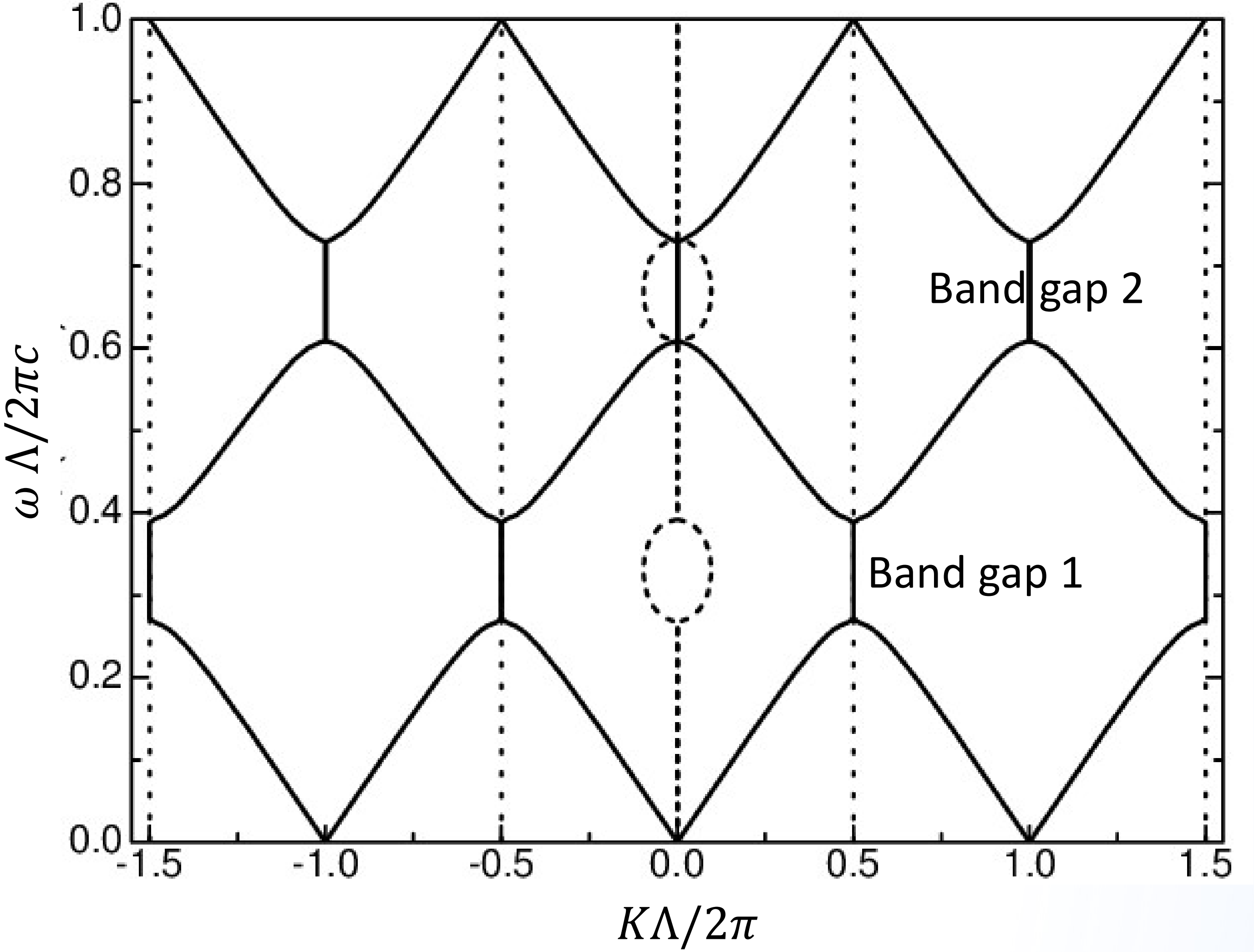
Scenario 1. When $\left|\frac{1}{2}(M_{11} + M_{22})\right| < 1$, then K is real and the Bloch wave can propagate (PASS BAND)

Scenario 2. When $\left|\frac{1}{2}(M_{11} + M_{22})\right| > 1$, then $K = \frac{m\pi}{\Lambda} - jK_i$ and the Bloch wave becomes evanescent (see now that there is an imaginary part). This regime is the PHOTONIC BAND GAP or STOP BAND

Scenario 3. When $\left|\frac{1}{2}(M_{11} + M_{22})\right| = 1$, then $K = \frac{m\pi}{\Lambda}$. These are the BAND EDGES

Dispersion relation for the Bloch wave

$$K(\beta = 0, \omega) = \frac{1}{\Lambda}\cos^{-1}\left[\cos k_1 a \cos k_2 b - \frac{1}{2}\left(\frac{n_2}{n_1} + \frac{n_1}{n_2}\right)\sin k_1 a \sin k_2 b\right]$$



——— $Real[K(\omega)]$

------ $|Imag[K(\omega)]|$

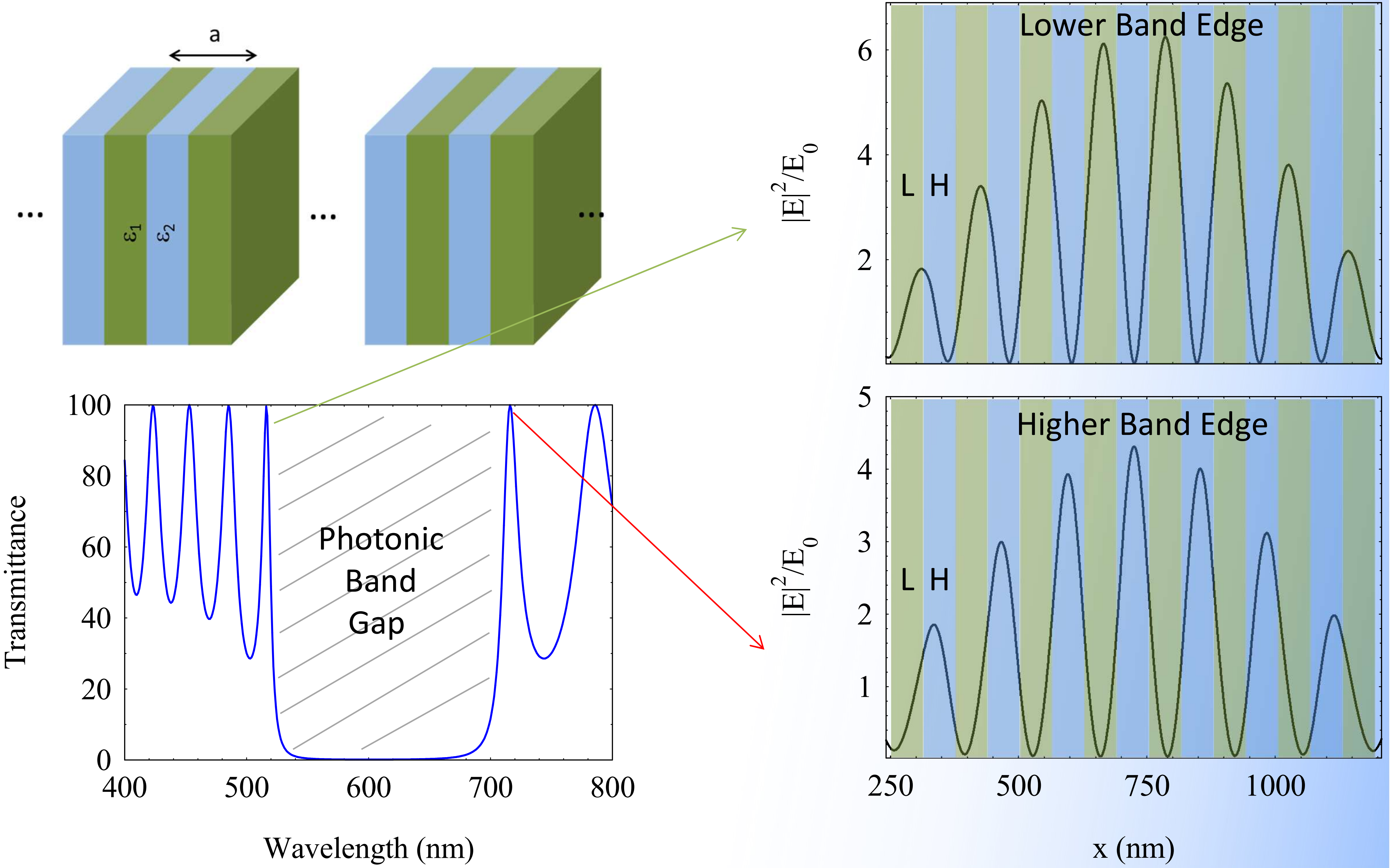
K is purely imaginary in the band gap and real in the pass bands.

Here K and ω are written in normalized units

- **K in units of $\frac{2\pi}{\Lambda}$**
- **ω in units of $\frac{2\pi c}{\Lambda}$**

Analysis of 1D Photonic Crystals

Field localization at the band edges of the gap assume a very specific form:



Analysis of 1D Photonic Crystals

How we define the size (bandwidth) of the band gap?

One way to define the size of the band gap independently of the scale of the crystal, is the **gap–midgap ratio**. If ω_c is the frequency at the middle of the gap, we define the gap–midgap ratio as:

$$\Delta\omega/\omega_c$$

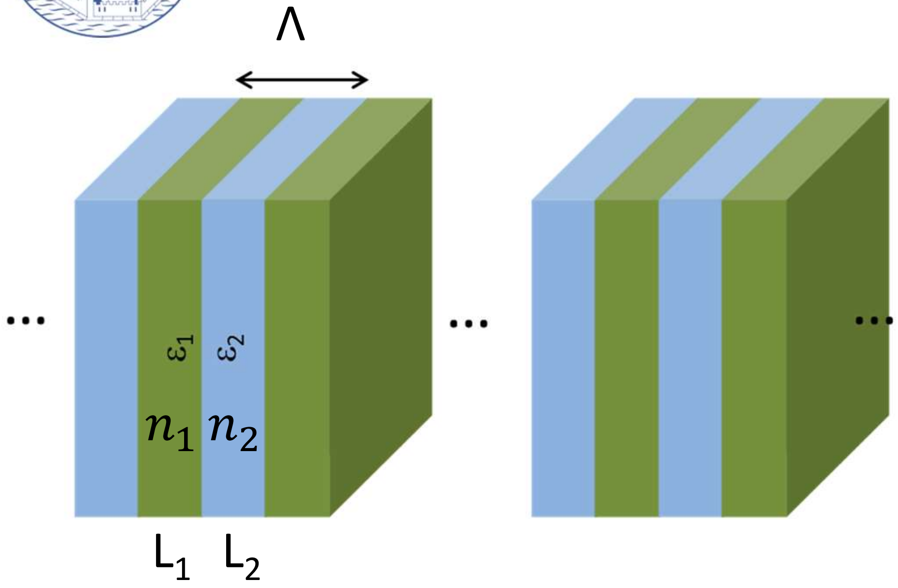
Generally expressed as a percentage. If the system is scaled up or down, all of the frequencies scale accordingly, but the gap–midgap ratio remains the same. Thus, when we refer to the "size" of a gap, we are generally referring to the gap–midgap ratio.

In a multilayer film with *weak periodicity* we can derive a simple formula for the size of the band gap from the perturbation theory. Suppose that the two materials have dielectric constants $\varepsilon = n^2$ and $\varepsilon + \Delta\varepsilon = n^2 + \Delta n^2$, and thicknesses $\Lambda - a$ and a. If *either* the dielectric contrast is weak ($\Delta\varepsilon/\varepsilon$<<1) *or* the thickness a/Λ is small, then the

$$\boxed{\frac{\Delta\omega}{\omega_c} \approx \frac{\Delta n^2}{n^2} \frac{\sin(\pi a/\Lambda)}{\pi}}$$

The equation also tells us that the gap–midgap ratio is maximized for $a = \Lambda/2$, but this is valid only for small $\Delta\varepsilon/\varepsilon$.

Analysis of 1D Photonic Crystals



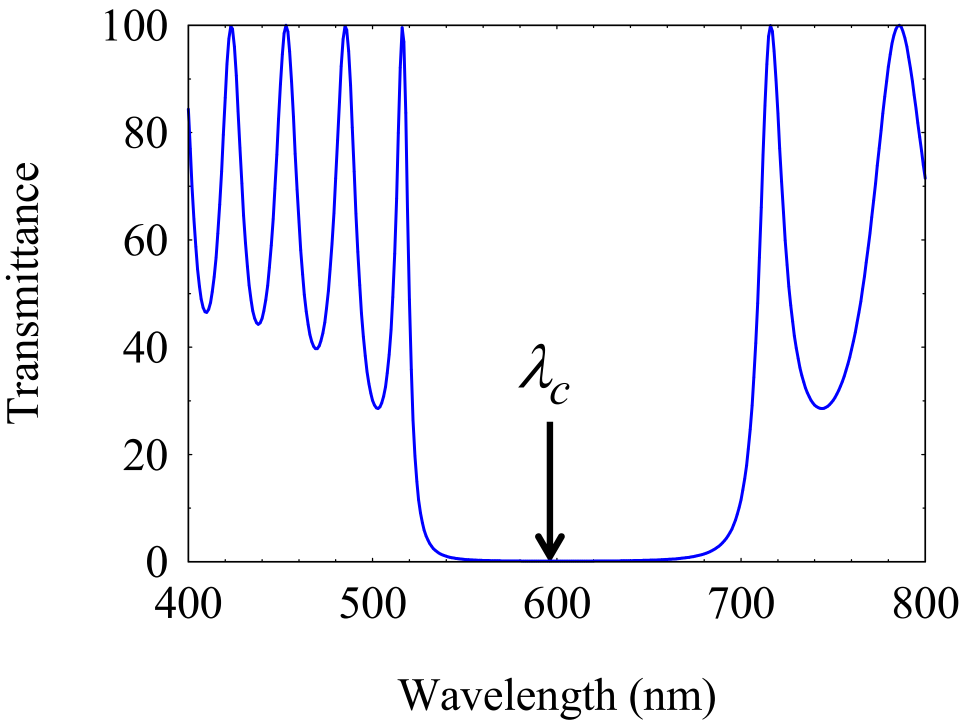
How can we calculate the center of the band gap?

For two materials with indices n_1 and n_2 and thicknesses L_1 and $L_2 = \Lambda - L_1$, respectively, the normal-incidence gap is maximized when:

$$\boxed{n_1 L_1 = n_2 L_2 = \lambda_{Bragg} / 4}$$

The midgap frequency ω_{Bragg} is:

$$\boxed{\omega_{Bragg} = \frac{n_1 + n_2}{4 n_1 n_2} \frac{2\pi c}{\Lambda}, \quad \lambda_{Bragg} = \frac{2\pi c}{\omega_{Bragg}}}$$



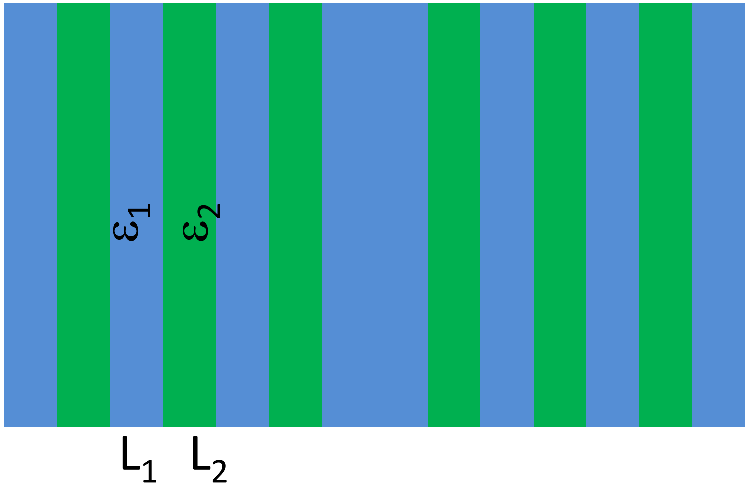
Example of transmission for a stack with λ_{Bragg}=600nm:

$$n_1 = 2$$
$$n_2 = 3$$
$$L_1 = 75nm$$
$$L_2 = 50nm$$
$$m = 1$$

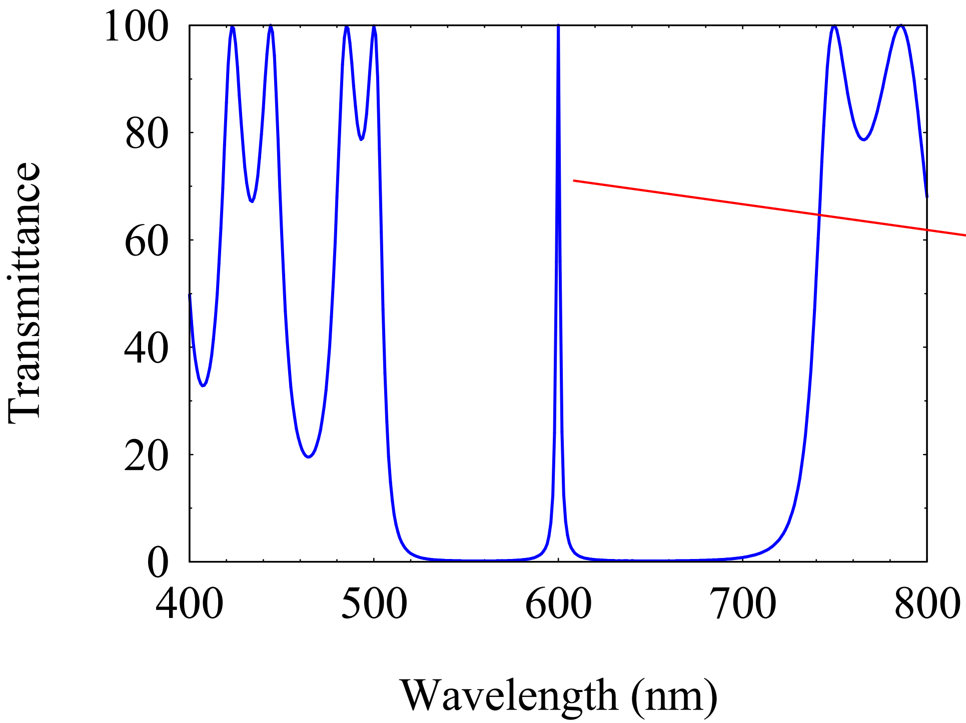
The reason why the gap is maximized for a quarter-wave stack is related to the property that the reflected waves from each layer are all exactly in phase at the mid-gap frequency.

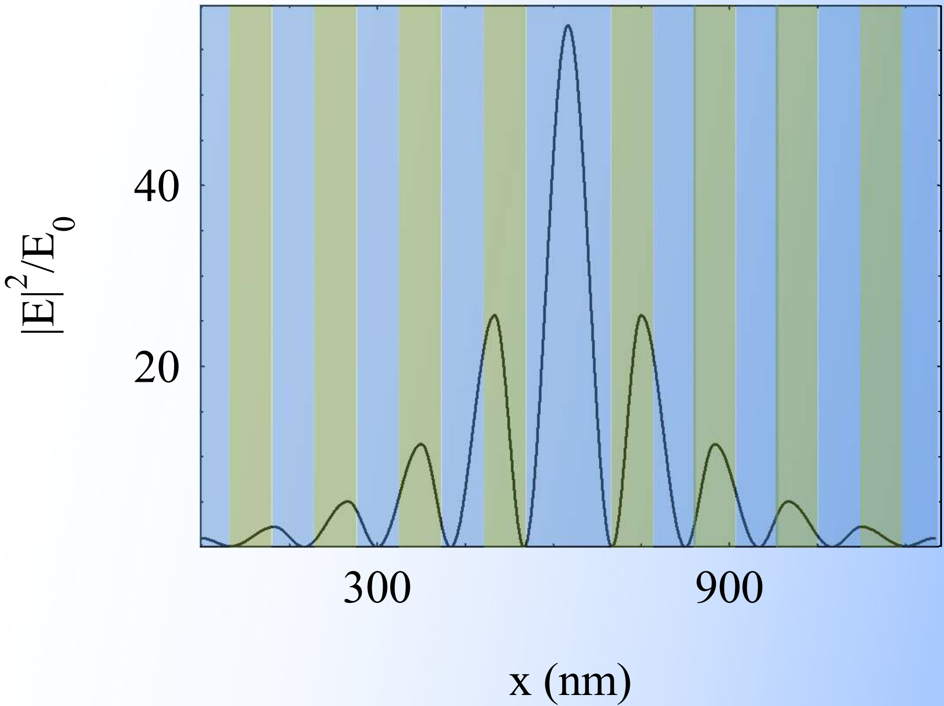
Defect States

What happens if we introduce a defect in the periodicity?

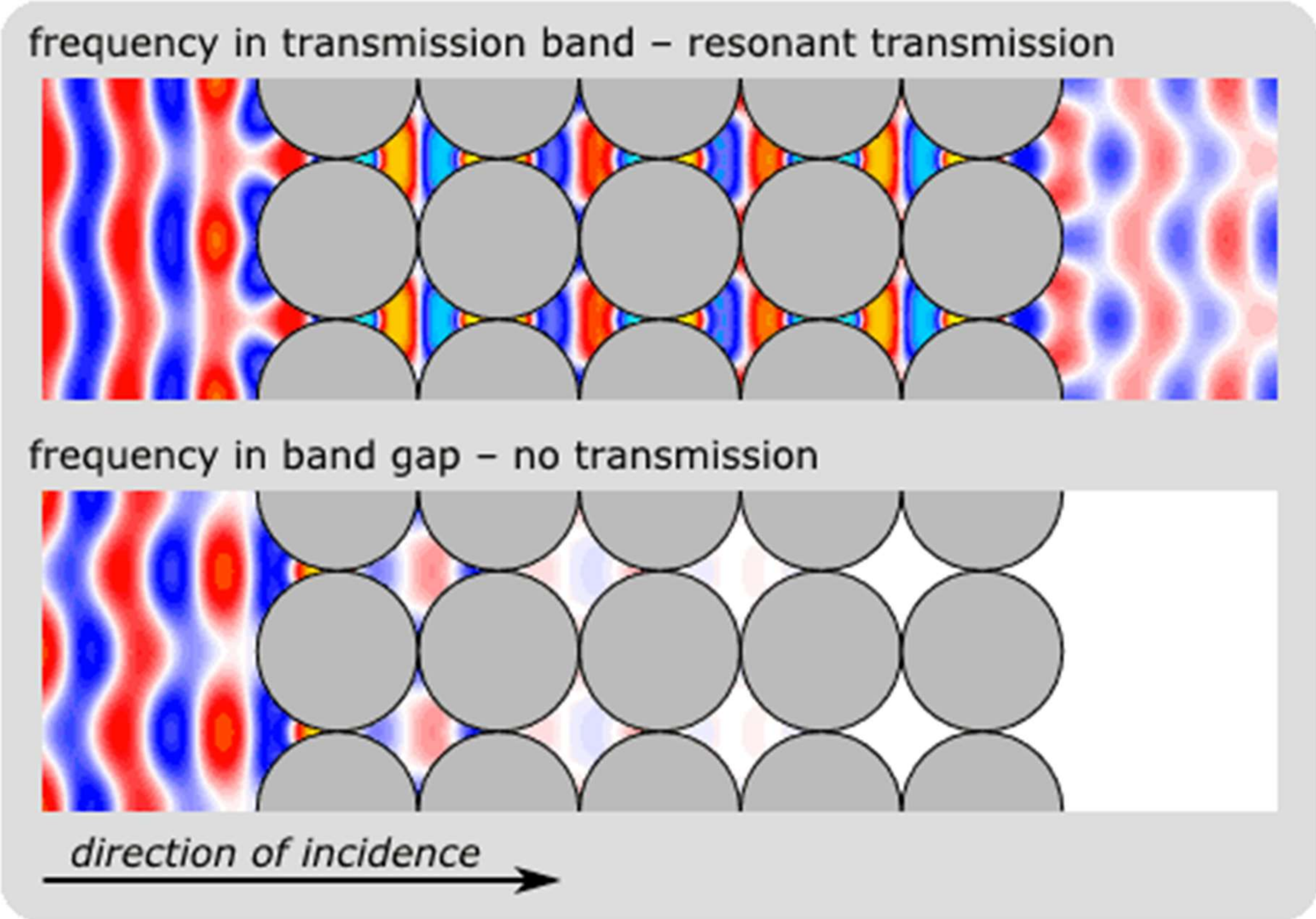


Defects may permit *localized* modes to exist, with frequencies inside photonic band gaps. If a mode has a frequency in the gap, then it must exponentially decay once it enters the crystal. The multilayer films on both sides of the defect behave like frequency-specific mirrors, and the system is indeed a Fabry-Perot resonator. If two such films are oriented parallel to one another, any *x*-propagating light trapped between them will just bounce back and forth between these two mirrors.





Photonic crystal as a **filter**



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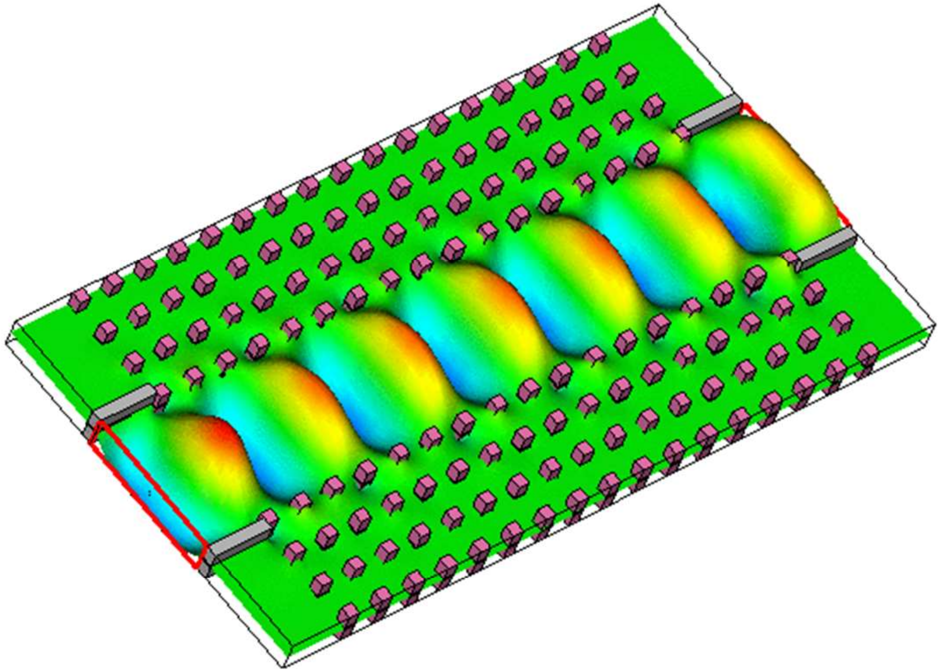
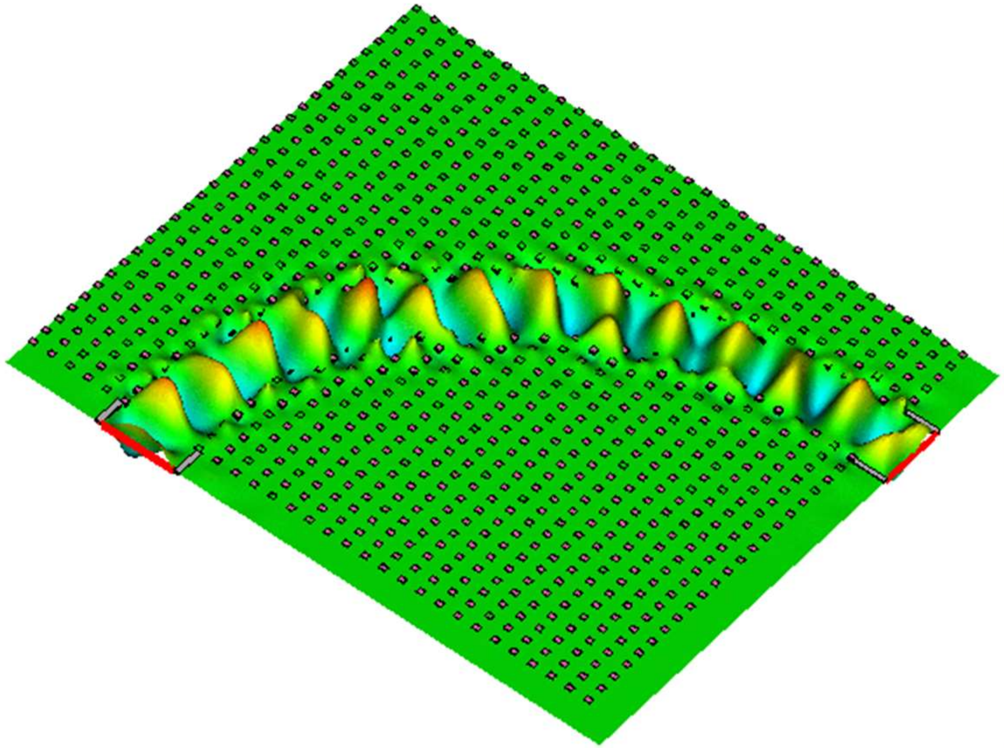
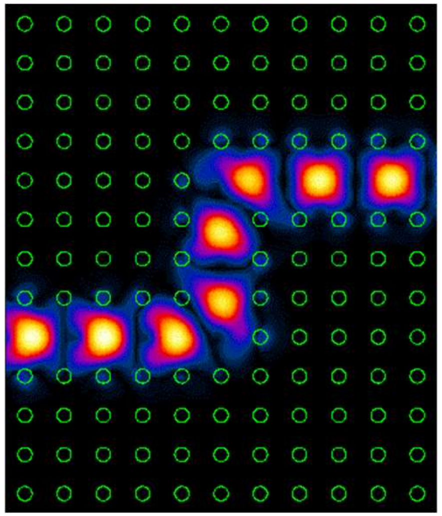
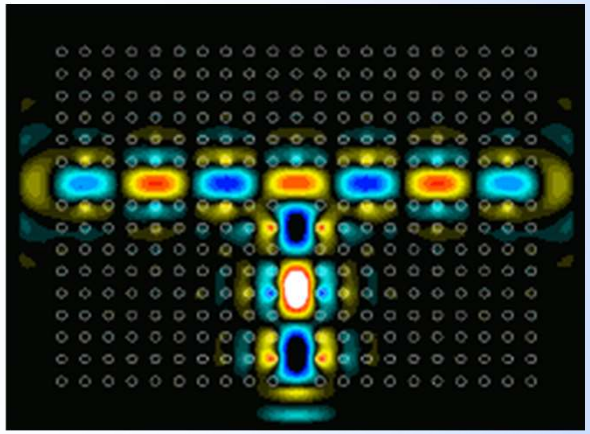
Photonic crystal to **guide and route light** in Photonic Integrated Circuits (PICs)





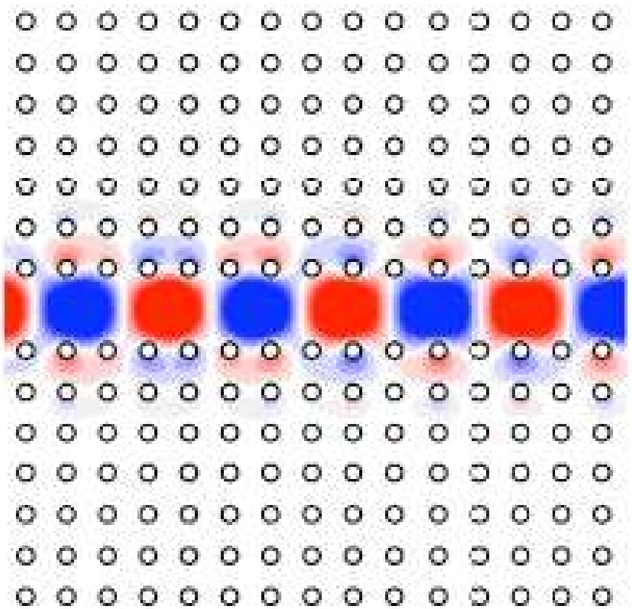


Beam splitters

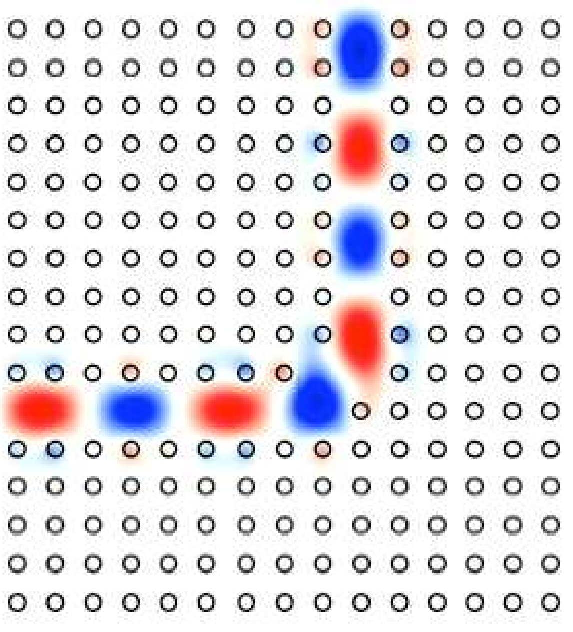


Photonic Crystal Based Devices – 2D

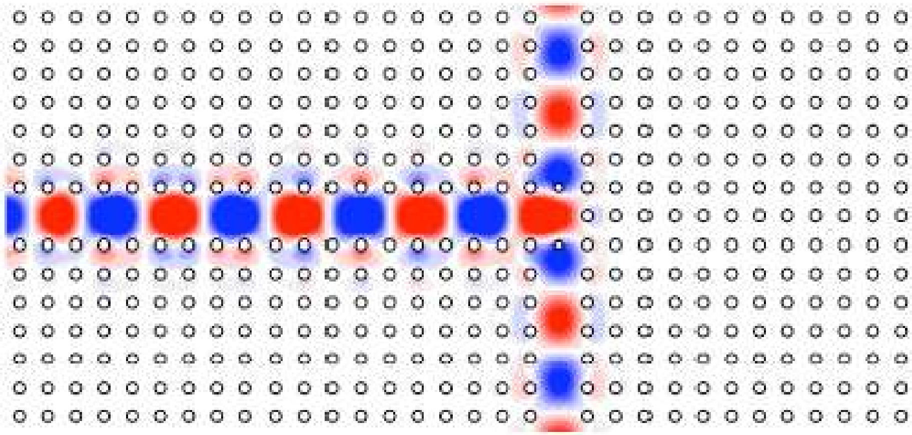
Waveguides



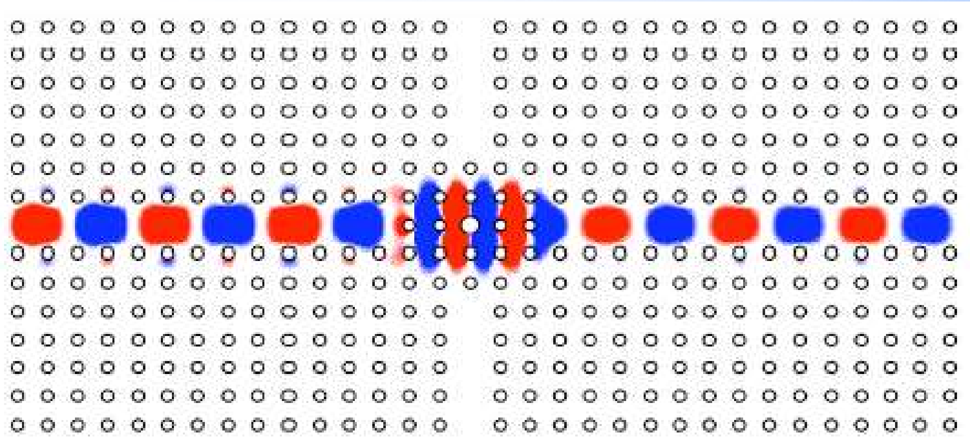
Sharp bends



Beam splitters



Waveguide Crossing



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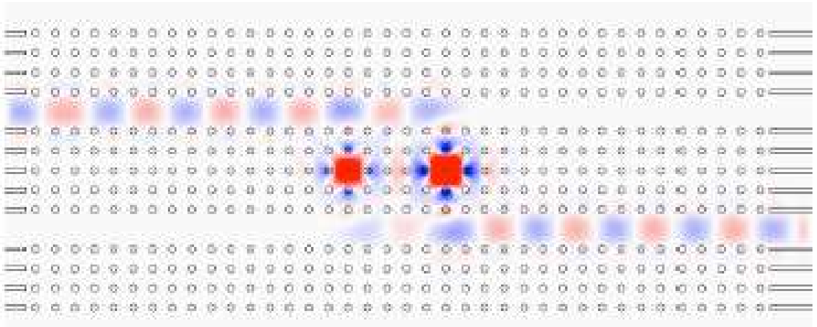
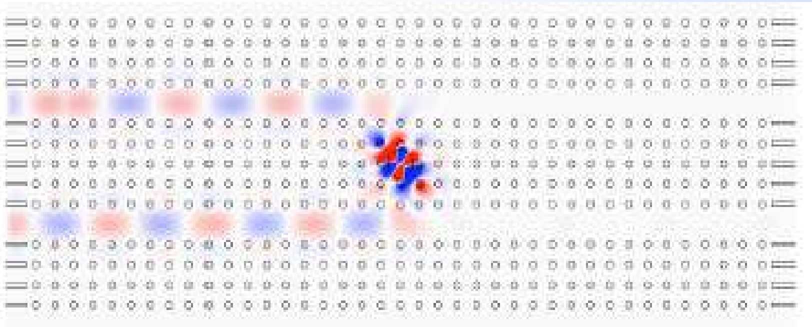
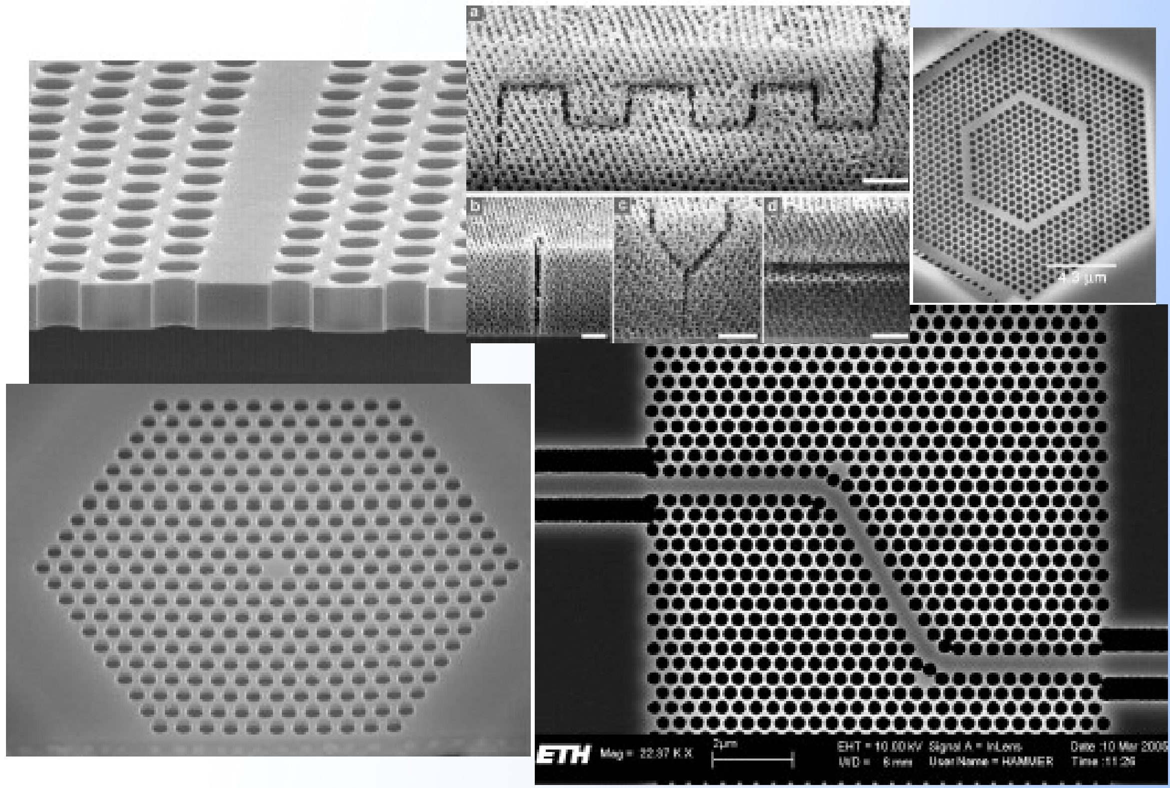
Photonic Crystal Based Devices – 2D

Channel Drop Filters

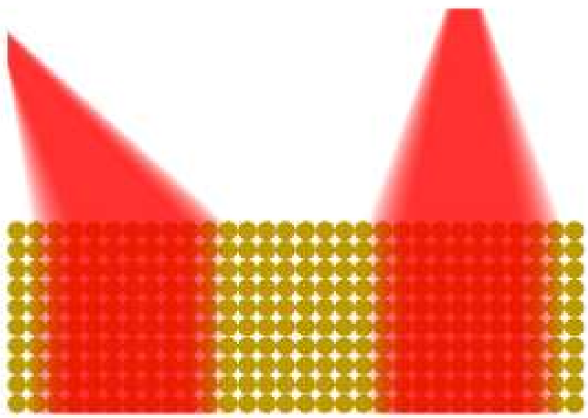




Some examples

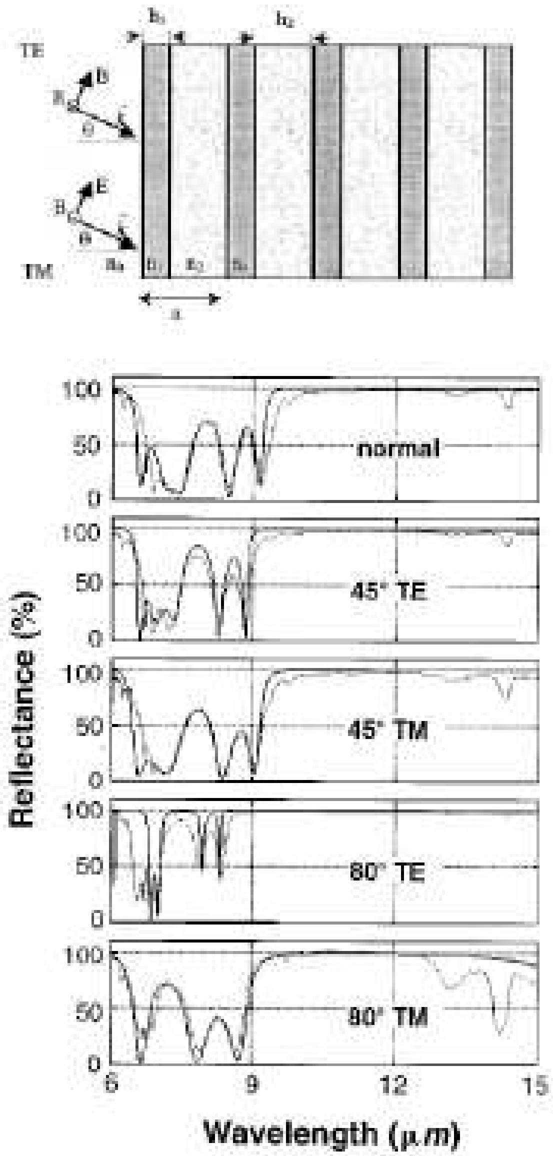


Self Collimator

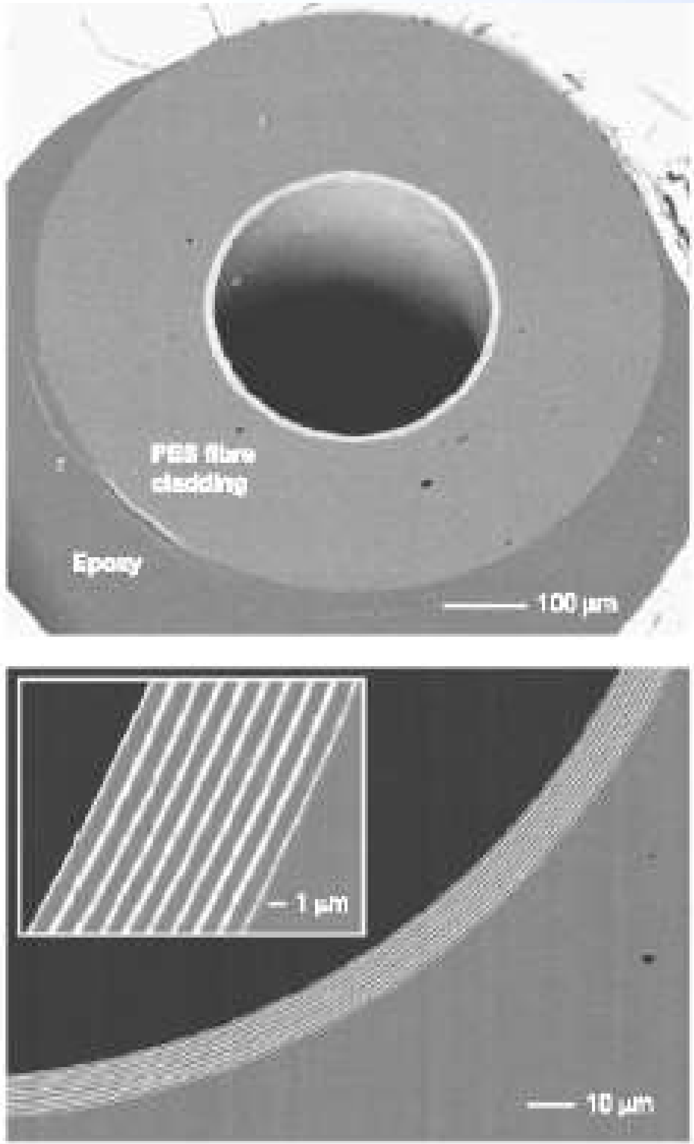


Photonic Crystal Based Devices – 1D

Omnidirectional Reflector



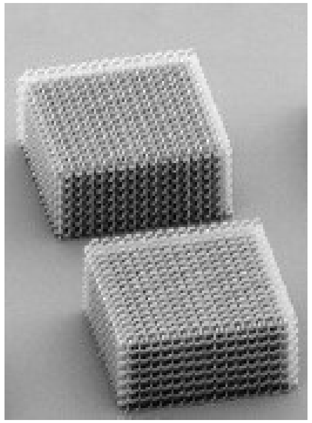
Y. Fink, et al, Science, vol.282, p.1679 (1998)

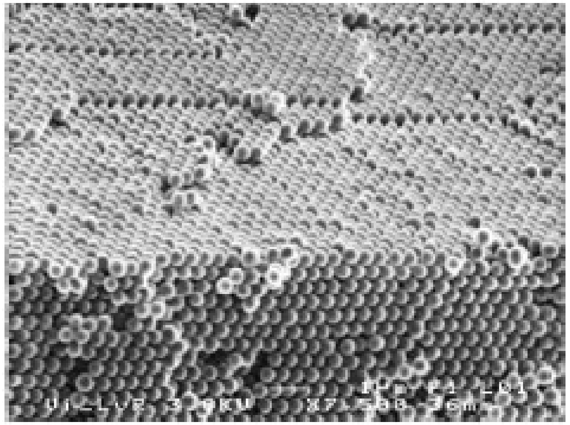


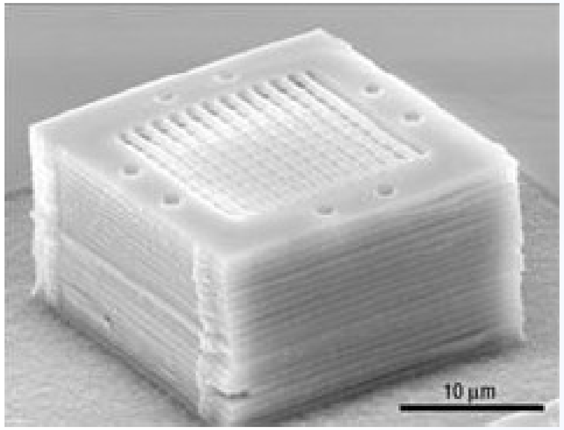
B. Temelkuran et al, Nature, vol.420, p.650 -3 (2002)

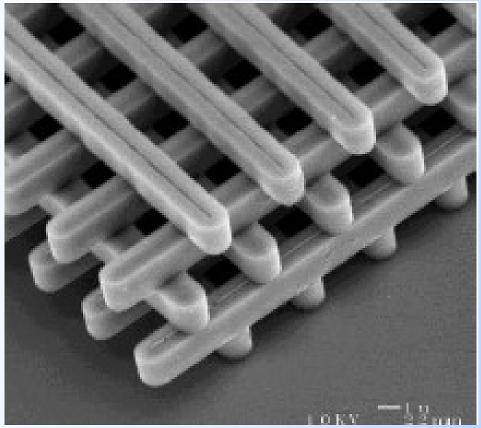
Photonic Crystal Based Devices – 3D

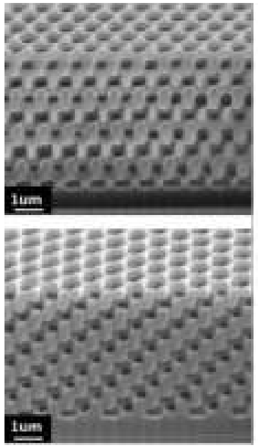
Some examples

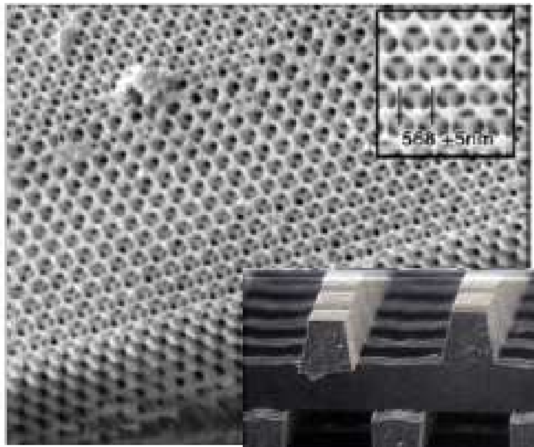


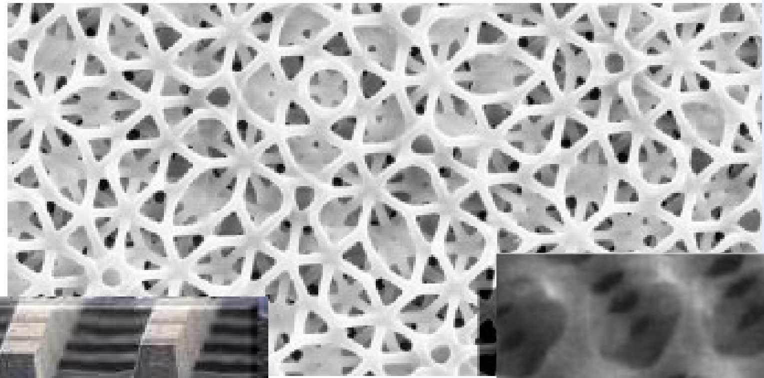


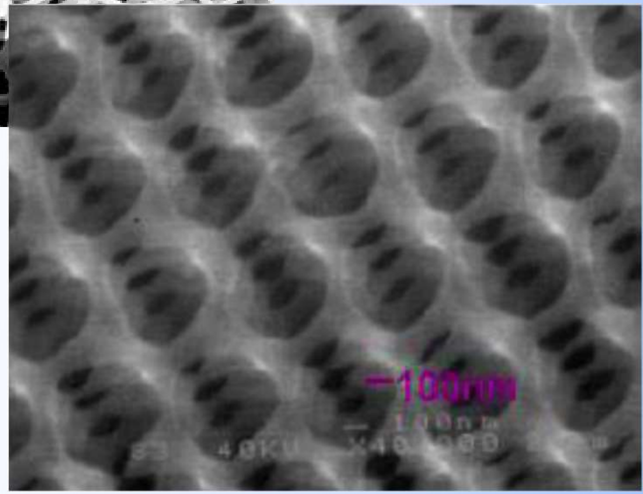






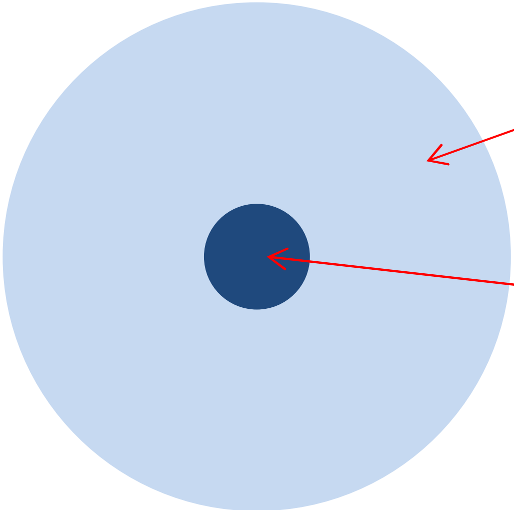






Photonic Crystal Fibers

In conventional optical fibers mode is confined by *total internal reflection*

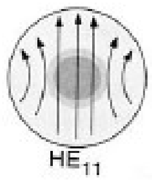


Silica cladding: n_1 = 1.44 to 1.46

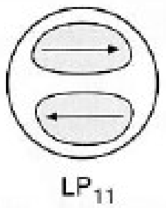
Silica core doped with Germanium, Boron or Titanium

$$\frac{n_2 - n_1}{n_2} = 0.001 \text{ to } 0.02$$

Core diameter for single mode fiber about 8 μm

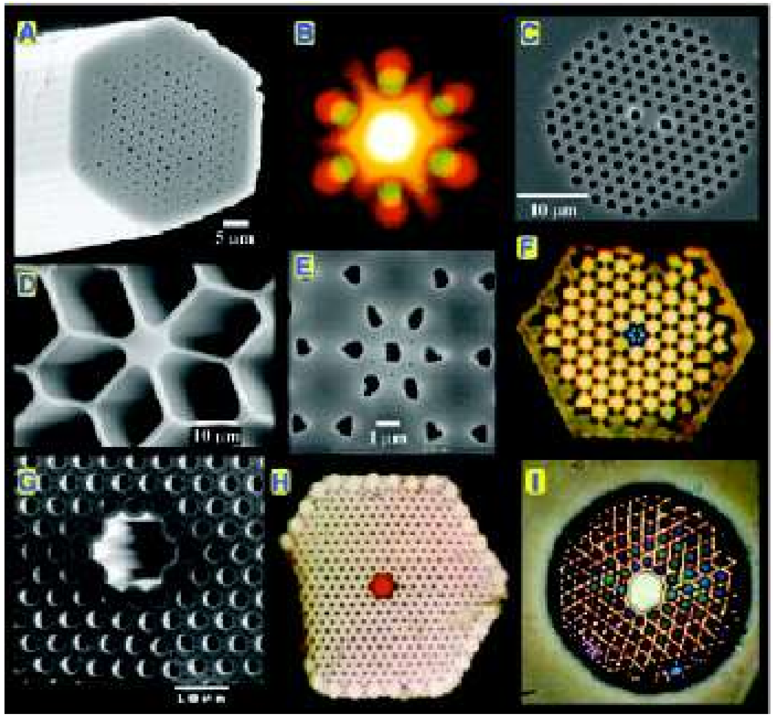


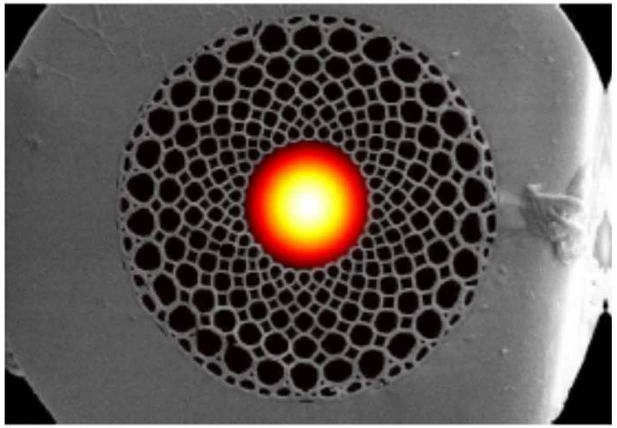
Fundamental mode



Higher order mode

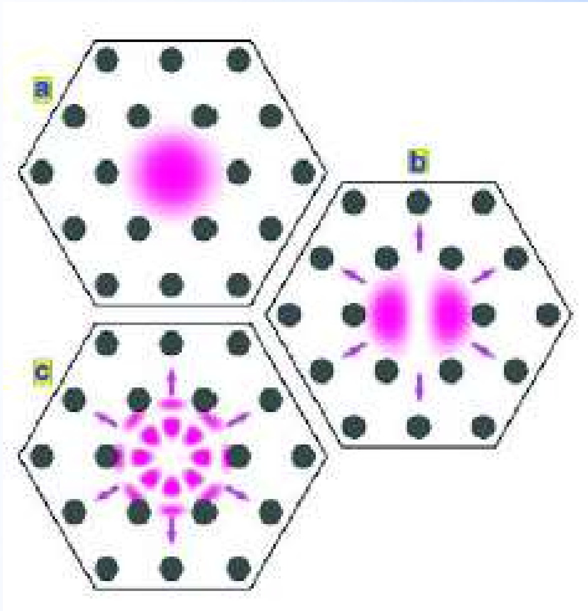
Photonic Crystal Fibers





PCF can have a dielectric core or an air core. Modes are confined in a region with lower refractive index thanks to the photonic band gap surrounding the core.

They can be single or multi-mode depending on the ratio of the hole diameters over the periodicity and the core size.

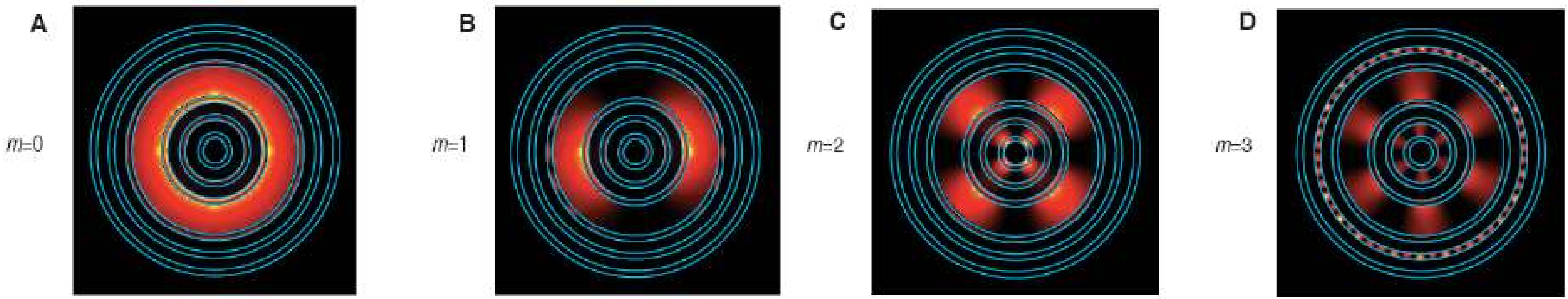


Photonic Crystal Fibers

Bragg fiber

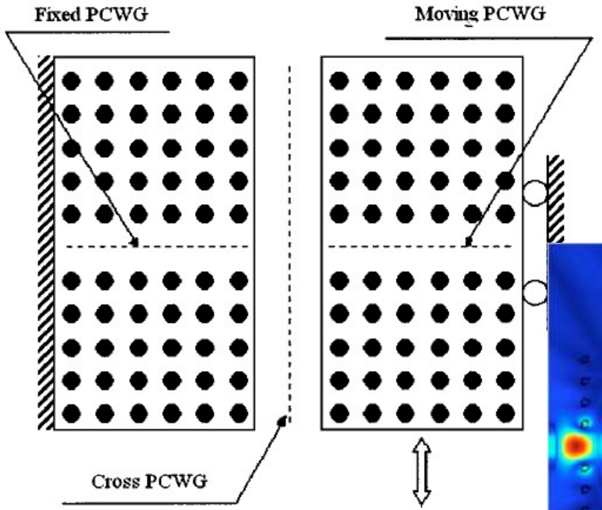
Multilayer reflection can be used to replace metal and create a light pipe. The boundary condition for the field at the core-cladding boundary can be designed to be similar to metal.

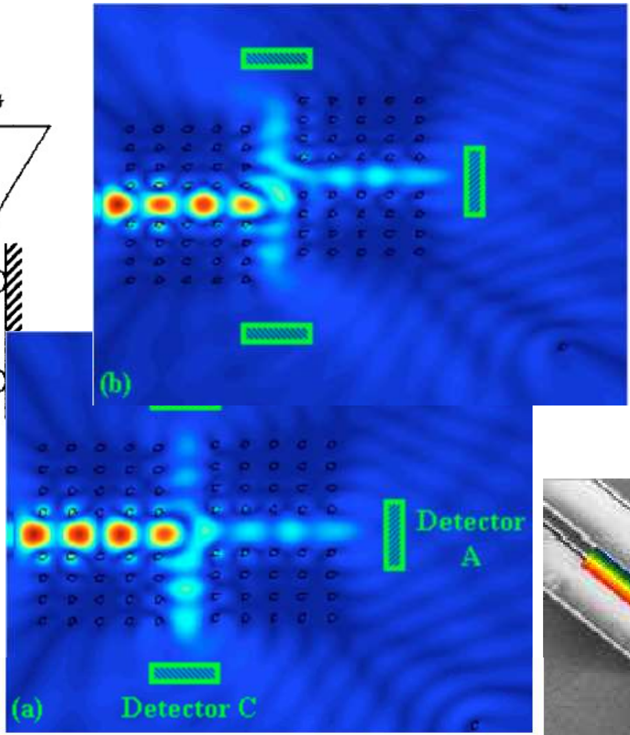
All dielectric co-axial waveguide



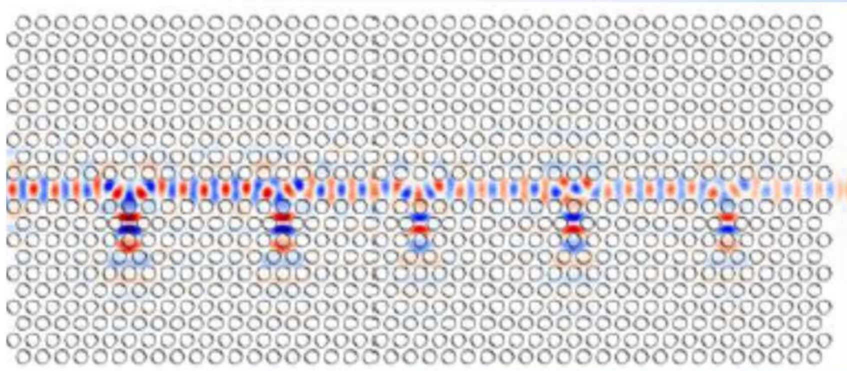
Photonic Crystal Sensors

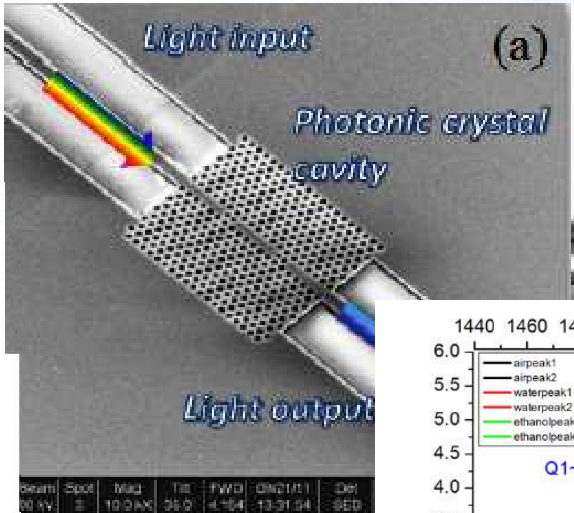
Displacement sensor



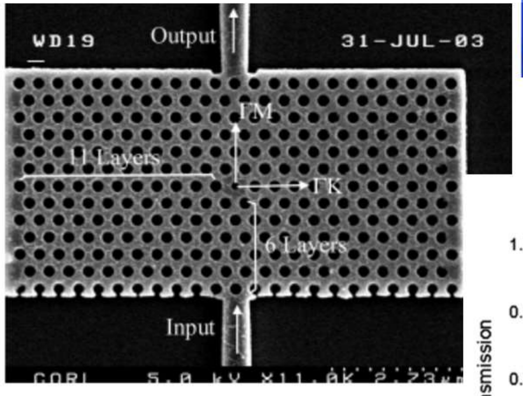


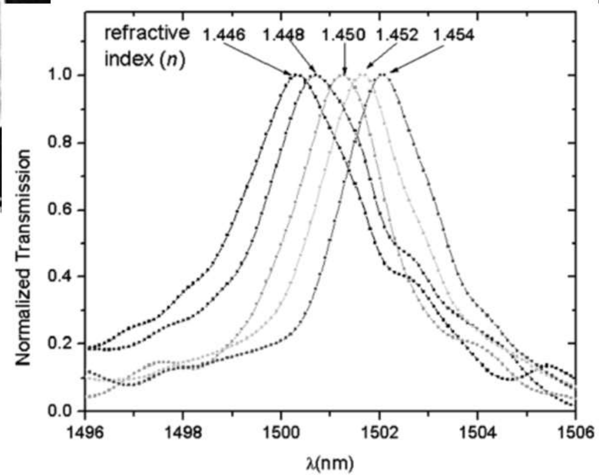
Sensor arrays

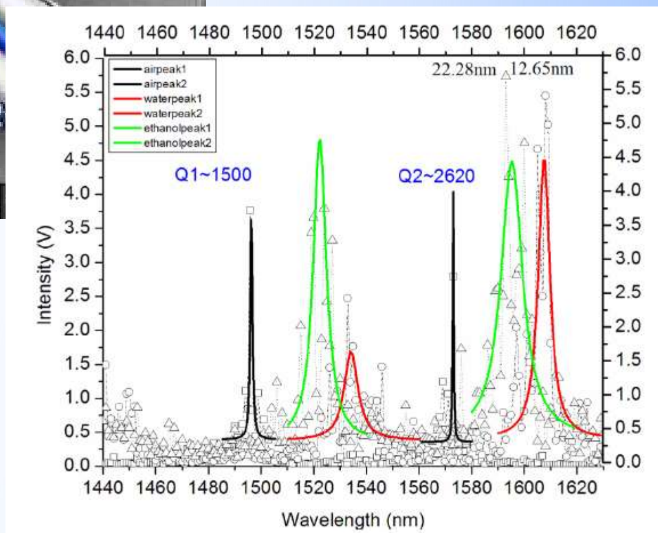




Microfluidic sensor







Biochemical sensor