1 - Introduction

- Photonics → control of photons (in free space or matter)

- Nanophotonics → science that takes place on wavelength and sub-wavelength scales of light where physical, chemical or structural nature of natural or artificial nanostructured matter controls the interactions

- Visible light: $\lambda \in [400nm - 700nm]$

- Fields of photonics: Fabrication, Characterization & Materials
  - → Fabrication
    - Top-down: Wafer → Processing → Dicing → Packaging
      - ↳ For electr. circuits and MEMS device
    - Bottom-up: To fabricate novel materials with properties not available in nature
      - ↳ Molecular soup → self-assembly → Dicing → Functional device
    - Process of fabrication:
      - ↳ Lithography and patterning: UV light, x rays, electron beams...
      - ↳ Deposition techniques
      - ↳ Etching (grabado)

→ Characterization

After fabricating (writing) we have to be able to monitoring and verifing (read)

- Techniques:
  - ↳ Optical techniques
  - ↳ Electron microscopes
  - ↳ Scanning probe techniques

2 - Ray Optics

The simplification: If light is propagating through and around objects much bigger than the wavelength, the light can be described by rays obeying to a set of geometrical rules.

- **Geometrical Optics used for:**

  Location of light & Direction of light
  - → Image formation
  - → Guiding conditions
  - → Direction of optical energy

- **Postulates**

  → Light travels in form of rays emitted by light sources and can be observed by optical detector

  → Medium characterization: Refractive index: $n = \frac{c_0}{c}$

  Time taken by light to travel distance d: $\frac{d}{c} = \frac{nd}{c_0}$

  nd → optical path length

  → In an inhomogeneous medium n function of the position $n(r) = n(x,y,z)$

  Optical pathlength: $OP = \int_A^B n(r)\, ds$

  ↳ differential element of length along the path

→ Fermat's Principle: "The path taken by a ray between two given points is the path that can be traversed in the least time"

$$\delta \int_A^B n(\vec{r})\, ds = 0$$

If there are more than one path sharing the minimum time → they are all followed simultaneously by the rays.

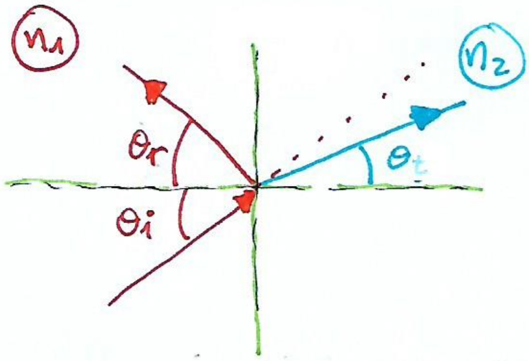
• $\boxed{\text{Propagation in a homogeneous medium}}$

n identical everywhere → c identical everywhere → Path of minimum time = Path of minimum distance → Light rays travels in straight lines

• $\boxed{\text{Reflection from a mirror, \& Reflection, Refraction at a boundary}}$

→ Geometrical reasoning to get: The reflected ray lies in the plane of incidence
The angle of reflection equal angle of incidence

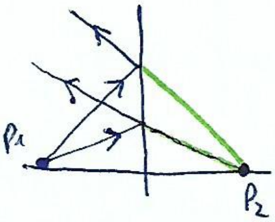
→ Law of refraction: Snell's law



$$\boxed{\begin{array}{c} \theta_i = \theta_r \\ n_1 \sin\theta_i = n_2 \sin\theta_t \end{array}}$$

• Mirrors

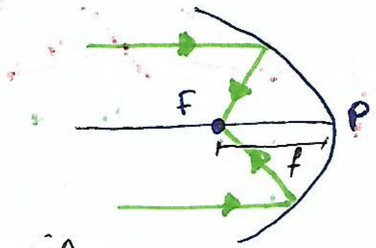
→ Planar mirror



Reflects rays originated at P_1 like they were originated at P_2

P_2 → image

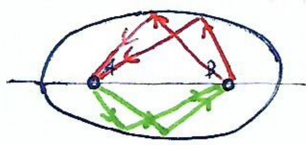
→ Paraboloidal mirror



It focuses all incident rays parallel to its axis to a single point called → FOCUS

Distance $\overline{PF} = f$ focal length

→ Elliptical mirror



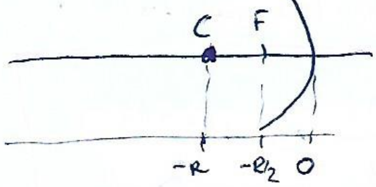
It reflects all the rays emitted from one two foci and images them onto the other focus

→ Spherical mirror

In paraxial approx. ($\sin\theta \simeq \theta$) rays are focused onto a single point F at $-R/2$ from the centre C

R → negative for concave mirrors

R → positive for convex mirrors

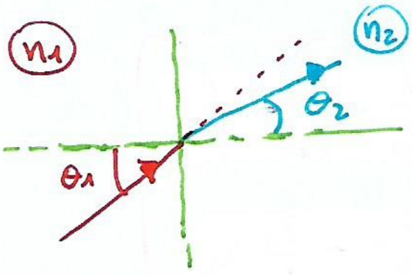


$$\boxed{\frac{1}{z_1} + \frac{1}{z_2} \simeq \frac{2}{-R} \simeq \frac{1}{f}}$$

→ Planar boundaries

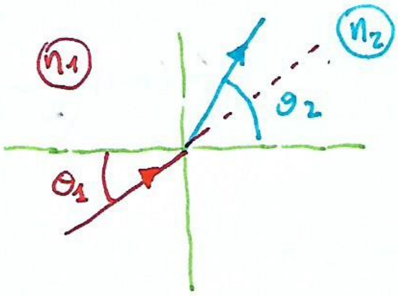
External refraction

$n_1 < n_2 \Rightarrow \theta_1 > \theta_2$



Internal refraction

$n_1 > n_2 \Rightarrow \theta_1 < \theta_2$



• Total internal Reflection can occur

$\theta_2 = 90°$

$\theta_1 = \theta_c$

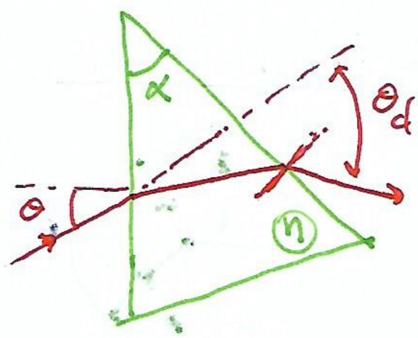
$$\boxed{\theta_c = \operatorname{asin}\left(\frac{n_2}{n_1}\right)}$$

Prisms

Apex: α

Refractive index: n

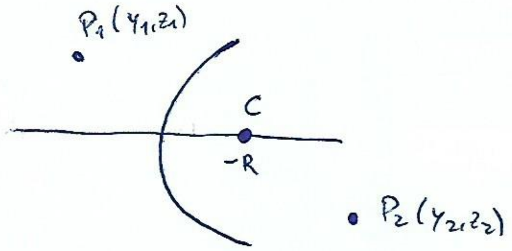
Incidence angle: θ



$$\theta_d = \theta - \alpha + \sin\left[\sqrt{n^2 - \sin^2\theta}\,\sin\alpha - \sin\theta\cos\alpha\right]$$

If α small $\Rightarrow$ $\theta_d \simeq (n-1)\,\alpha$

Spherical boundaries



$R \rightarrow \oplus$ convex boundary

$R \rightarrow \ominus$ concave boundary

$$\boxed{\frac{n_1}{z_1} + \frac{n_2}{z_2} \simeq \frac{n_2 - n_1}{R} \qquad y_2 = -y_1 \frac{n_1 z_2}{n_2 z_1}}$$

paraxial approx

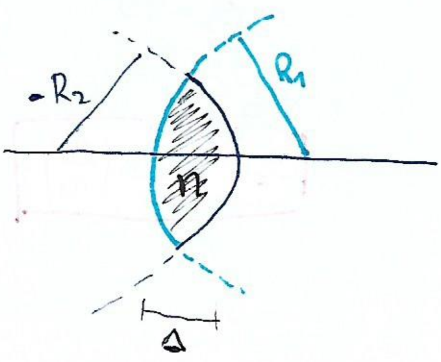
If not paraxial $\Rightarrow$ aberration

Lenses

- Defined by 2 spherical surfaces with 2 radii R_1 & R_2

Thickness $\rightarrow \Delta$

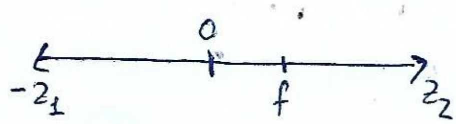
Refractive index $\rightarrow n$



If thin:

$$\theta_2 = \theta_1 - \frac{y}{f'}$$

$$\frac{1}{f} (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



$$\boxed{\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f'} \qquad y_2 = -\frac{z_2}{z_1} y_1}$$

•) Graded index Optics

GRIN (Graded index material) $\Rightarrow n(\vec{r})$

↳ In GRIN light follows curved trajectories

↳ Fermat's principle: $\delta \int_A^B n(\vec{r})\, ds = 0$

↳ Ray equation: $\frac{d}{ds}\left(n \frac{dr}{ds}\right) = \nabla n$

In paraxial: $\frac{d}{dz}\left(n \frac{dx}{dz}\right) \simeq \frac{dn}{dx}$

$$\frac{d}{dz}\left(n \frac{dy}{dz}\right) \simeq \frac{dn}{dy}$$

⊕ In homogeneous media n indep. of $\vec{r}:(x,y,z)$

$\frac{d^2x}{dz^2} = 0 \qquad \frac{d^2y}{dy^2} = 0$

trajectories: straight lines

→ Graded-index slab

$n = n(y)$

$$\frac{d^2y}{dz^2} = \frac{1}{n(y)} \cdot \frac{dn(y)}{dy}$$

→ Graded-index fiber

$n = n(\vec{r}) \qquad n^2 = n_0^2\left[1 - \underbrace{\alpha^2(x^2+y^2)}_{\ll 1 \text{ paraxial approx.}}\right]$

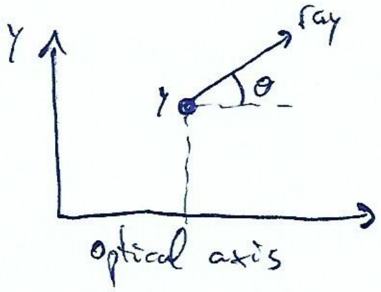
$$\begin{cases} \frac{d^2x}{dz^2} \simeq -\alpha^2 x \\ \frac{d^2y}{dz^2} \simeq -\alpha^2 y \end{cases} \xrightarrow[\text{solution}]{} \quad \begin{aligned} x(z) &= \frac{\theta_{x_0}}{\alpha}\sin(\alpha z) \\ y(z) &= \frac{\theta_{y_0}}{\alpha}\sin(\alpha z) + y_0\cos(\alpha z) \end{aligned}$$

• Matrix Optics

→ Ray is described by:
- Position
- Angle

with respect to the optical axis

→ The optical system is described by a 2x2 matrix: Ray Transfer Matrix



Two planes:

Input: z_1

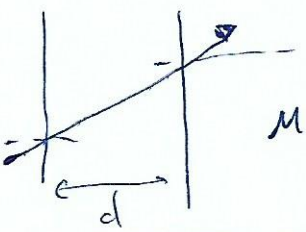
Output: z_2

From position (y_1, θ_1) to (y_2, θ_2)

System:

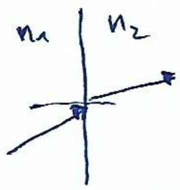
$$\boxed{\begin{aligned} y_2 &= A y_1 + B \theta_1 \\ \theta_2 &= C y_1 + D \theta_1 \end{aligned}} \qquad \boxed{\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = M \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}}$$

→ Free space propagation



$$M = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

→ Refraction at planar boundary



$$M = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

Reflection Planar Mirror

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

→ Refraction at spherical boundary

Convex: $R > 0$

Concave: $R < 0$

$y_2 \sim y_1$

$$M = \begin{bmatrix} 1 & 0 \\ -\frac{n_2 - n_1}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$$

→ Refraction thin lens

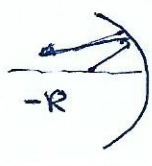
Convex $f > 0$

Concave $f < 0$

$y_2 = y_1$

$$M = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

Reflection spherical mirror



$$M = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$$

Convex: $R > 0$

concave: $R < 0$

→ Matrices of cascade optical comp

$$\to \boxed{M_1} \to \boxed{M_2} \to \boxed{M_3} \cdots \to \boxed{M_N} \to$$

$$\boxed{M = M_N \cdots M_2 M_1}$$

→ Periodic Optical Systems

Composed of a cascade of identical unit systems. If the matrix is the same just apply m times

$$\begin{matrix} y_0 \\ \theta_0 \end{matrix} \to \underset{1}{\boxed{M}} \xrightarrow{\begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}} \underset{2}{\boxed{M}} \xrightarrow{\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix}} \cdots \xrightarrow{\begin{matrix} y_{m-1} \\ \theta_{m-1} \end{matrix}} \underset{m}{\boxed{M}} \xrightarrow{\begin{matrix} y_m \\ \theta_m \end{matrix}}$$

$$\boxed{\begin{pmatrix} y_m \\ \theta_m \end{pmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^m \begin{pmatrix} y_0 \\ \theta_0 \end{pmatrix}}$$

Equation for the dynamic of position y_m

without knowing θ_m we can:

$$y_{m+2} = 2b\, y_{m+1} - F^2 y_m \qquad b = \frac{A+D}{2}$$

recurrence relation for ray position

$$F^2 = AD - BC = \det[M]$$

↳ One solution is the form: $y_m = y_{max} F^m \sin(m\varphi + \varphi_0)$ — y_{max} & φ_0 constant of initial conditions

↳ Regardless the system: $\det(M) = \dfrac{n_1 \rightarrow \text{input}}{n_2 \rightarrow \text{output}}$

if $\boxed{n_1 = n_2}$ → $y_m = y_{max} \sin(m\varphi + \varphi_0)$ — Ray position in a periodic system

↪

Y_m Harmonic

- It is harmonic if $\varphi = \cos^{-1} b$ is real

  stability Condition: $|b| \leq 1$ or $\frac{1}{2}|A + D| \leq 1$

- If $|b| > 1 \rightarrow$ solution is hyperbolic

Y_m periodic

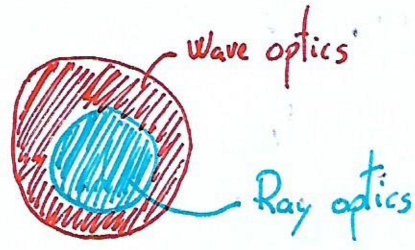
- If Y_m is harmonic and satisfy:

  $Y_{m+s} = Y_m$ $\quad S \in$ integer

  smallest integer $\Rightarrow$ Period

  $S\varphi = 2\pi q$, $q \in$ integer

3 - Wave Optics



Ray optics is the limit of wave optics when the wavelength is infinitesimally short.
Objects are much bigger than the incident wavelength

→ Wave optics:
Light is a wavefunction (scalar function) that obeys the wave equation (2nd order EDP)

→ Limits of wave optics
Not able to give the full picture of: reflection, refraction at interfaces
Not able to describe polarization effects

•) Postulates

→ Wave equation

$c = \frac{c_0}{n}$ wavefunction satisfies: $$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0$$

↳ Superposition principle: Sum of two solution is a solution

↳ Eq. valid for locally homogeneous media

→ Intensity

$$I(\vec{r},t) = 2\langle u^2(\vec{r},t)\rangle \quad \left[\frac{W}{cm^2}\right]$$

→ Power

$$P(t) = \int_A I(\vec{r},t)\, dA \quad [W]$$

→ Energy

The integral of optical power over a time interval [J]

Monochromatic Waves

$$u(\bar{r},t) = a(\bar{r}) \cos\left[2\pi f t + \varphi(\bar{r})\right]$$

Period: $T = \frac{2\pi}{\omega}$

Complex wavefunction

$$U(\bar{r},t) = U(\bar{r})\, e^{j\omega t}$$

$\hookrightarrow U(\bar{r}) = a(\bar{r})\, e^{j\varphi(\bar{r})}$

$$u(\bar{r},t) = \mathrm{Re}\left[U(\bar{r}) e^{j\omega t}\right] = \frac{1}{2}\left[U(\bar{r}) e^{j\omega t} + U^*(\bar{r}) e^{-j\omega t}\right]$$

Helmholtz Equation

$$\nabla^2 U + k^2 U = 0$$

wavenumber: $k = \frac{\omega}{c}$

intensity: $I(\bar{r}) = |U(\bar{r})|^2$

wavefronts: surfaces with $\varphi(\bar{r}) = $ constant.

$\boxed{\text{Solutions for wave equation}}$

→ Plane waves

$$U(\bar{r}) = A e^{-j\bar{k}\cdot\bar{r}} = A e^{-j(k_x x + k_y y + k_z z)}$$

wavevector $\bar{k}$

$\bar{k} = (k_x, k_y, k_z)$

Constant intensity $I(\bar{r}) = |A|^2$ everywhere in space ⇒ infinite power

wavelength

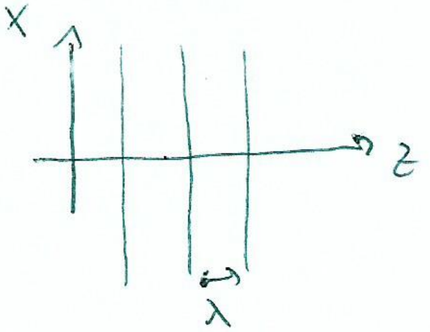
$\lambda = \frac{2\pi}{f} = \frac{c}{f}$

Propagating in direction z: $u(\bar{r},t) = |A| \cos\left[\omega t - kz + \arg(A)\right]$

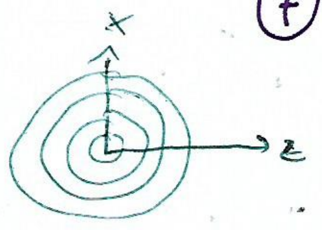
$$= |A| \cos\left[\omega\left(t - \frac{z}{c}\right) + \arg(A)\right]$$

phase velocity: $v_p = c$

$c = \frac{c_0}{n}$ $\quad k = k_0 n$ $\quad \lambda = \frac{\lambda_0}{n}$



→ Spherical waves



$$U(\bar{r}) = \frac{A_0}{r} e^{-jkr}$$

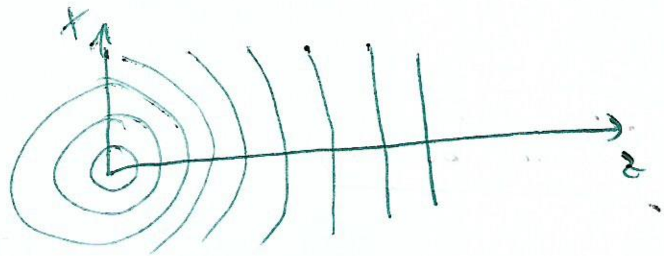
r: distance from the origin

$$I(\bar{r}) = \frac{|A_0|^2}{r^2}$$

$v_p = c$

wavefronts: $r = q\lambda$, $q \in \mathbb{Z}$

→ Paraboloidas waves



⊛ $\sqrt{x^2+y^2} \ll z$ paraxial approx.

$$U(\bar{r}) \simeq \frac{A_0}{r} e^{-jkz} e^{-jk\frac{x^2+y^2}{2z}}$$

Fresnel approx valid:

$$\frac{N_F \theta_m^2}{4} \ll 1$$

$N_F = \frac{a^2}{\lambda z}$ (Fresnel number)

$\theta_m = \frac{a}{z}$ (maximum angle)

→ Paraxial waves

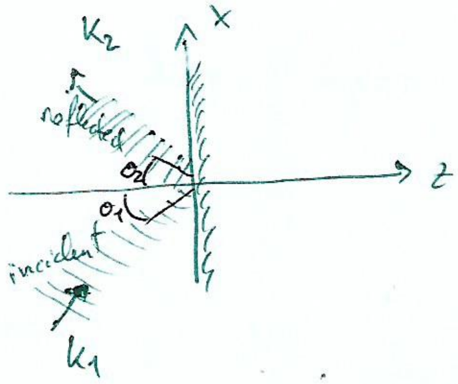
$$U(\bar{r}) = A(\bar{r}) e^{-jkz}$$

$A(\bar{r})$: complex envelope that varies slowly with the position

↳ satisfies Paraxial Helmholtz Eq. $\nabla_T^2 A - j2k\frac{\partial A}{\partial z} = 0$

Mirrors

In a plane mirror



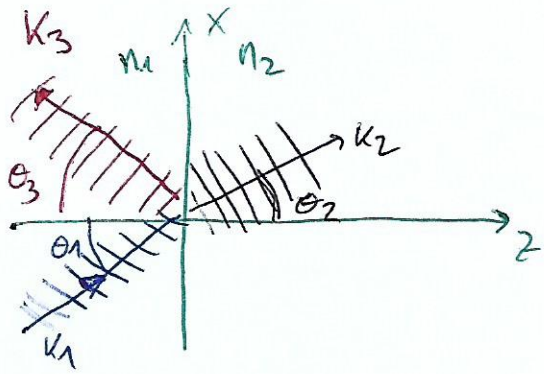
$$k_1 \cdot \bar{r} = k_2 \cdot \bar{r}$$

$$k_1 = (k_0 \sin\theta_1,\ 0,\ k_0 \cos\theta_1)$$

$$k_2 = (k_0 \sin\theta_2,\ 0,\ -k_0 \cos\theta_2)$$

$$k_0 \times \sin\theta_1 = k_0 \times \sin\theta_2 \longrightarrow \boxed{\theta_1 = \theta_2}$$

Planar boundary



$$\begin{cases} k_1 = k_3 = n_1 k_0 \\ k_2 = n_2 k_0 \end{cases}$$

$$k_1 \cdot \bar{r} = k_2 \cdot \bar{r} = k_3 \cdot \bar{r}$$

$$k_1 = (k_0 n_1 \sin\theta_1,\ 0,\ k_0 n_1 \cos\theta_1)$$

$$k_2 = (k_0 n_2 \sin\theta_2,\ 0,\ k_0 n_2 \cos\theta_2)$$

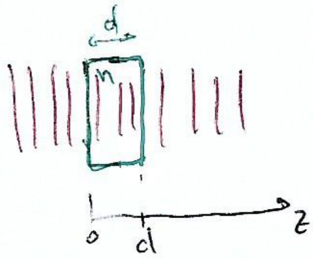
$$k_3 = (k_0 n_1 \sin\theta_3,\ 0,\ -k_0 n_1 \cos\theta_3)$$

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Transparent plates

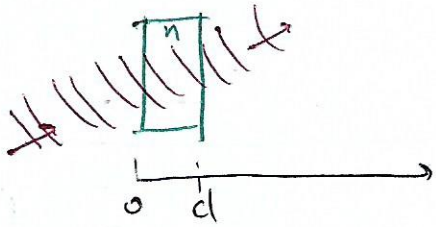
Transmission through a transparent plate of refractive index n and thickness d

$$\boxed{t(x,y) = \frac{U(x,y,d)}{U(x,y,0)} = e^{-jnk_0 d}}$$



↳ plate introduces a phase shift $nk_0 d = 2\pi \frac{d}{\lambda}$

↳ Incidence with angle θ



$$t(x,y) = e^{-jnk_0 d\cos\theta_1}$$

↳ if θ small ⇒ paraxial $\theta_1 \simeq \frac{\theta}{n}$

$$t(x,y) = e^{-jnk_0 d}\, e^{jk_0\theta^2 d/2n}$$

If thin and small angle
⇓
like normal incidence

•) $\boxed{\text{Thin Transparent Plate Varying thickness}}$

↳ Thickness that varies smoothly $d(x,y)$

↳ Paraxial wave

↳ Total thickness: $\underbrace{d_0 - d(x,y)}_{\text{air}}$

$$t(x,y) = e^{-jnk_0 d(x,y)}\, e^{-jk_0[d_0 - d(x,y)]}$$

$$\boxed{t(x,y) \triangleq h_0\, e^{-j(n-1)k_0 d(x,y)}} \qquad \boxed{h_0 = e^{-jk_0 d_0}}$$

↳ constant phase factor

•) $\boxed{\text{Thin lens}}$

with Fresnel approx: $d(x,y) \simeq d_0 - \frac{x^2+y^2}{2R}$

$$\boxed{t(x,y) \simeq h_0 \exp -jk_0\left[\frac{x^2+y^2}{2f}\right]} \qquad \boxed{f = \frac{R}{n-1}}$$

•) Diffraction Gratings

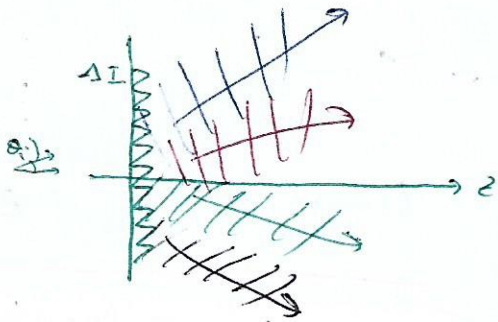
→ Periodically modulates the phase and amplitude of an incident wave

→ With period Λ the grating transforms an incident plane wave ($\lambda << \Lambda$ & small angle θ_i) into several plane waves

$$\boxed{\theta_q = \theta_i + q\frac{\lambda}{\Lambda}}$$ $q = 0, \pm 1, \pm 2$

if θ_i not small

$$\boxed{\sin\theta_q = \sin\theta_i + q\frac{\lambda}{\Lambda}}$$



•) Graded-index optical component

Basic effect of optical components with variable thickness is to introduce phase shift.
We can introduce phase shift by using a variable refractive index

$$t(x,y) = e^{-j\, n(x,y)\, k_0 d_0}$$

e) Interference

→ Total wavefunction = the sum of each individual wavefunction.

→ Superposition : Applies to complex amplitudes

DOES NOT apply to optical intensity

→ Interference depends on relative phase between waves

→ Monochromatic waves

$U(\vec{r}) = U_1(\vec{r}) + U_2(\vec{r})$

Interference eq. : $\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi}$ $\varphi = \varphi_2 - \varphi_1$

↳ if $I_1 = I_2 = I_0$

$I = 4 I_0 \cos^2(\varphi/2)$

$\varphi = 0 \Rightarrow I = 4 I_0$ Constructive

$\varphi = \pi \Rightarrow I = 0$ Destructive

$\varphi = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow I = 2 I_0$

→ Interferometer

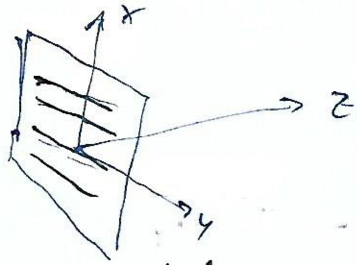
- split a wave into two waves and delays them by unequal distance
- Two waves, one delayed by distance d, equal intensity (I_0)

$$\left.\begin{aligned} U_1 &= \sqrt{I_0}\, e^{-jkz} \\ U_2 &= \sqrt{I_0}\, e^{-jk(z-d)} \end{aligned}\right\} \quad I = 2 I_0 \left[1 + \cos\left(\frac{2\pi d}{\lambda}\right)\right]$$

→ Oblique (plane) waves

- Two waves equal intensity two different angles at $z=0$ plane

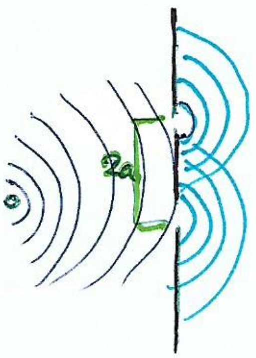
$$I = 2I_0\left[1+\cos(k\sin\theta\, x)\right]$$



interference pattern

→ Spherical wave

- Young's interference experiment



Intensity at $z=d$

$$I(x,y,d) \simeq 2I_0\left[1+\cos\left(\frac{2\pi x\theta}{\lambda}\right)\right]$$

$$\theta \sim 2a/d$$

intensity pattern periodic with periodo λ/θ

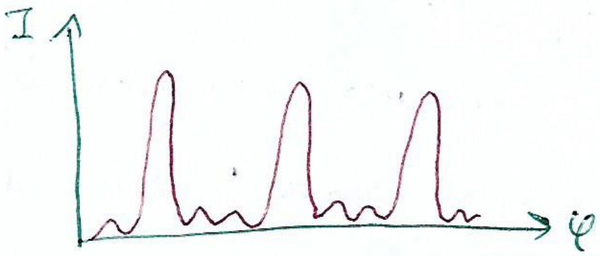
→ Multiple wave Interference

M monochromatic waves

$U = U_1 + U_2 + \dots + U_M$

If equal amplitude and equal phase difference

$$\boxed{I = I_0\,\frac{\sin^2(M\varphi/2)}{\sin^2(\varphi/2)}}$$



If equal phase difference and amplitudes that decrease geometrical

$$I = \frac{I_{max}}{1 + \left(\frac{2F}{\pi}\right)^2 \sin^2(\varphi/2)} \qquad I_{max} = \frac{I_0}{(1-|h|)^2}$$

$$F = \frac{\pi\sqrt{|h|}}{1-|h|} \qquad F: \text{finesse}$$

$h = |h| e^{j\varphi}$, $|h| < 1$ geometric

the larger F the sharper the peaks of intensity function

c) $\boxed{\text{Polychromatic light}}$

→ Polychromatic wave $u(\bar{r}, t)$ can be expanded as a superposition of monochromatic waves.

→ Superposition (integrals) $$u(t) = \int_{-\infty}^{\infty} v(f)\, e^{j2\pi ft}\, df$$

Fourier trans. → $$v(f) = \int_{-\infty}^{\infty} u(t)\, e^{-j2\pi ft}\, dt$$

only positive freq

$$U(t) = 2\int_0^{\infty} v(f)\, e^{j2\pi ft}\, df \quad \text{complex}$$

↳ $u(t) = \mathrm{Re}[U(t)]$ → satisfy wave eq

Intensity $$I(\bar{r}, t) = 2\langle u^2(\bar{r}, t)\rangle = 2 \cdot \tfrac{1}{2}\left\langle [U(t) + U^*(t)]^2 \right\rangle$$

if quasi-monochromatic

$$I(\bar{r}, t) = |U(\bar{r}, t)|^2$$

◦) Pulsed light

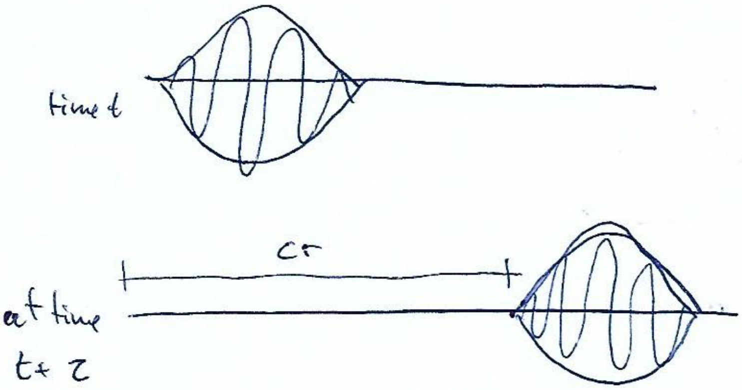
→ Pulsed plane wave $U(\bar{r}, t) = A\left(t - \frac{z}{c}\right) e^{j2\pi f_0 \left(t - \frac{z}{c}\right)}$

satisfies wave eq

↳ $A(t)$ complex envelope

↳ f_0 central freq

If $A(t)$ is finite → wavepacket



→ Fourier transform $V(\bar{r}, f) = A(f - f_0)\, e^{-j2\pi f \frac{z}{c}}$

↳ Spectral width inversely proportional to the temporal width

↳ Fourier transform of a Gaussian is still a Gaussian pulse

o) Interference : Two monochromatic waves different freqs.

$$U(t) = \sqrt{I_1}\, e^{j2\pi f_1 t} + \sqrt{I_2}\, e^{j2\pi f_2 t}$$

$$I(t) = I_1 + I_2 + 2\sqrt{I_1 I_2}\, \cos\left[2\pi (f_2 - f_1) t\right]$$

$|f_2 - f_1|$ beat freq.

o) $\boxed{\text{Gaussian Beam}}$

$$U(\bar{r}) = A_0 \frac{W_0}{W(z)}\, e^{-\frac{\rho^2}{W^2(z)}}\, e^{-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)}$$

$$A_0 = \frac{A_1}{jz_0}$$

$$W_0 = \sqrt{\frac{\lambda z_0}{\pi}}$$

$$W(z) = W_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$

$$R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right]$$

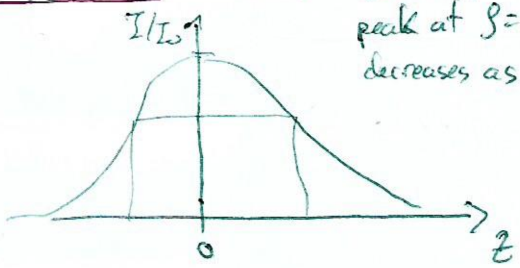
$$\zeta(z) = \tan^{-1}\frac{z}{z_0}$$

$$\rho^2 = x^2 + y^2$$

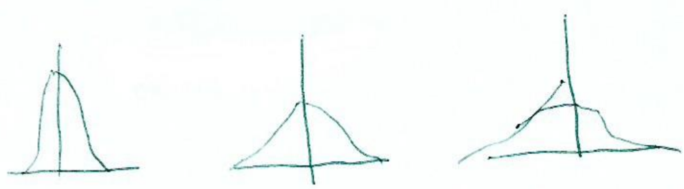
Intensity: $I(\bar{r}) = |U(\bar{r})|^2$

$$\boxed{I(\rho, z) = |A_0|^2 \left[\frac{W_0}{W(z)}\right]^2 e^{-2\rho^2/W^2(z)}}$$

Intensity peak at $\rho = 0$ decreases as ρ increases



The beam width $W(z)$ increases with increasing z



Power

$$P = \int_0^\infty I(\rho, z)\, 2\pi\rho\, d\rho = \frac{1}{2} I_0 \underbrace{\pi W_0^2}_{\text{beam area}}$$

Power does not depend on z.

$$I(\rho, z) = \frac{2P}{\pi W^2(z)} e^{-2\rho^2/W^2(z)}$$

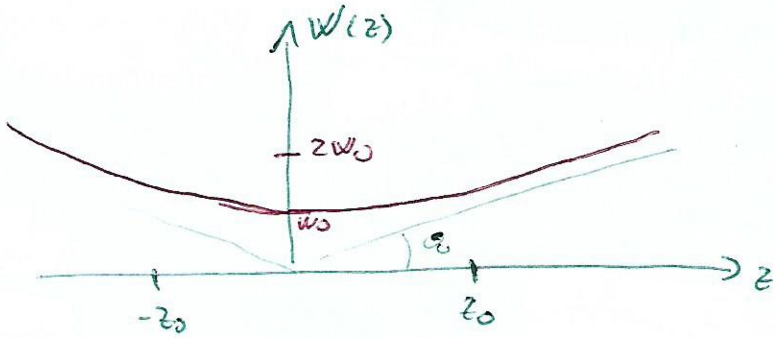
99% of power is carried in circle radius of 1.5 W(z)

Beam Width : W(z)

$$W(z) = W_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$

↳ W_0 waist radius

$2W_0$ spot size



Beam Divergence

If $z \gg z_0 \Rightarrow$ $W(z) \approx \frac{W_0}{z_0} z = \theta_0 z$

$$\theta_0 = \frac{W_0}{z_0} = \frac{\lambda}{\pi W_0}$$

angular divergence:

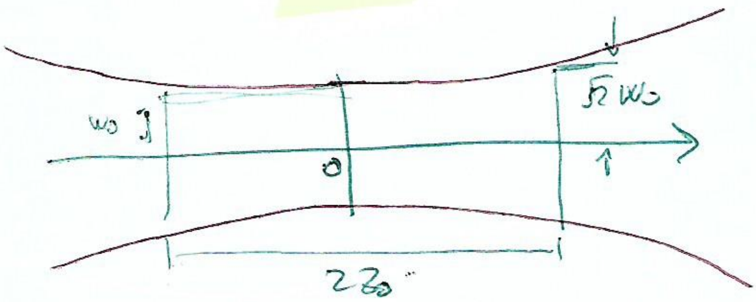
$$2\theta_0 = \frac{4\lambda}{2\pi W_0}$$

Depth focus

→ Minimum at $z=0$ → the best focus

The beam increases siz in both direction and at distance where beam radius is not greater than $\sqrt{2}$ of its minimum value is depth-of-focus or confocal parameter

$$2z_0 = \frac{2\pi W_0^2}{\lambda}$$



Phase

$$\varphi(\rho, z) = kz + k\frac{\rho^2}{2R(z)} - \zeta(z)$$

↳ on axis

$$\varphi(0, z) = kz - \zeta(z)$$

plane wave

↳ phase retardation (Gouy effect)

Wavefronts

The term: $k \dfrac{\rho^2}{2R(z)}$ represents a deviation of the phase

bending

at off axis points

$$z + \frac{\rho^2}{2R} \simeq q\lambda + \varphi \frac{\lambda}{2\pi}$$ → paraboloidal surface with radious of curvature R

Beam Quality

The deviation from ideal gaussian:

$$M^2 = \frac{2W_m \cdot 2\theta_m}{4\lambda/\pi}$$

smallest: $M = 1$

↳ $\frac{4\lambda}{\pi} = 2W_0 \cdot 2\theta_0$

4 – Fourier Optics

Harmonic analysis to describe propagation of light.

→ Expansion of a function $f(t)$ in a superposition of harmonic functions of time @ different freqs.

$$F(\nu) \xrightarrow[\text{of}]{\text{F.T.}} f(t)$$

→ Same with function of space $f(x,y)$

$$F(\nu_x, \nu_y)\, e^{-j 2\pi (\nu_x x + \nu_y y)}$$

Plane wave: $U(x,y,z) = A\, e^{-j(k_x x + k_y y + k_z z)}$

$k = \frac{2\pi}{\lambda}$, $A \in \mathbb{C}$

↓

Arbitrary traveling wave can be analyzed as a sum of plane waves

o) Free Space Propagation

plane wave: $U(x,y,z) = A e^{-j(k_x x + k_y y + k_z z)}$

Wavevector: $\bar{k} = (k_x, k_y, k_z)$

wavenumber: $k = \frac{2\pi}{\lambda} = \sqrt{k_x^2 + k_y^2 + k_z^2}$

$\Lambda_x = \frac{1}{\nu_x}$, $\Lambda_y = \frac{1}{\nu_y}$

$\nu_x = \frac{k_x}{2\pi}$ $\quad$ $\nu_y = \frac{k_y}{2\pi}$

for $z=0$

$U(x,y,0) = f(x,y) = A e^{-j2\pi(\nu_x x + \nu_y y)}$

$\bar{K}$ vector make angles

$$\boxed{\begin{aligned} \theta_x &= \sin^{-1}\left(\frac{k_x}{k}\right) = \sin^{-1}(\lambda \nu_x) \\ \theta_y &= \sin^{-1}\left(\frac{k_y}{k}\right) = \sin^{-1}(\lambda \nu_y) \end{aligned}}$$

Paraxial approx.

$$\boxed{\theta_x \simeq \frac{\lambda}{\Lambda_x} \qquad \theta_y \simeq \frac{\lambda}{\Lambda_y}}$$

→ Amplitude modulation

→ Fresnel zone plate

→ Frequency modulation

→ Transfer function

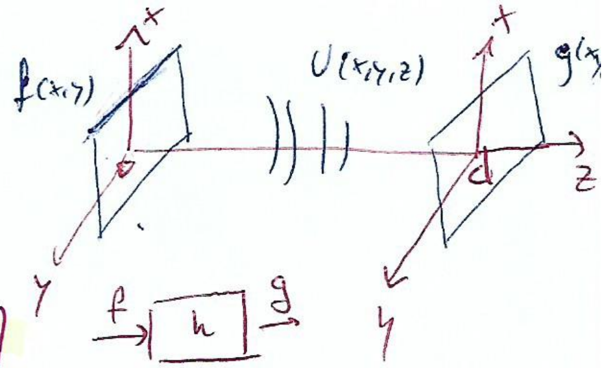
$f(x,y) = A e^{-j(k_x x + k_y y)}$

$g(x,y) = A e^{-j(k_x x + k_y y + k_z d)}$

$k_x = 2\pi\nu_x$

$k_y = 2\pi\nu_y$

$k_z = \sqrt{k^2 - k_x^2 - k_y^2}$



$$\boxed{H(\nu_x, \nu_y) = \frac{g(x,y)}{f(x,y)} = e^{-jk_z d} = e^{-j2\pi d\sqrt{\lambda^{-2} - \nu_x^2 - \nu_y^2}}}$$

* Sqrt can be real or imaginary
↓ propagating wave ↓ evanescence wave

⊛ λ^{-1} represents a cut-off spatial freq.

→ Fresnel Approx

$$\frac{N_F \, \theta_m^2}{4} \ll 1$$

Fresnel app. condition

$$N_F = \frac{a^2}{\lambda d}$$

$a \rightarrow$ the largest radial distance

$$\theta_m \simeq a/d$$

→ Impulse response

$h(x,y)$ is the response $g(x,y)$ when the input $f(x,y)$ is a point at the origin $(0,0)$ → So $h(x,y)$ is the inverse fourier transform

$$h(x,y) \simeq h_0 \, e^{-jk\frac{x^2+y^2}{2d}}$$

→ Huygens - Fresnel principle

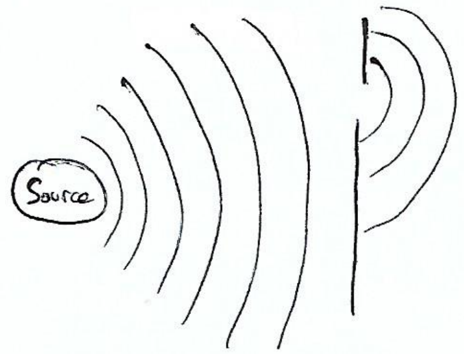
→ Optical Fourier Transform

$$\boxed{g(x,y) = h_0 F\left(\frac{x}{\lambda d}, \frac{y}{\lambda d}\right)}$$

$$h_0 = \frac{j}{\lambda d} e^{-jkd}$$

↳ lenses ☹

•) Diffraction



When a wave arrives to a hole to satisfy the wave equation & boundary conditions the solution is like we have a source in the hole.

Since wave eq. is linear we can add the amplitude of waves from a "source" at each hole → calculating the amplitude adding point sources: Huygens' Principle

The intensity distribution of a wave passing through an aperture

↳ Diffraction Pattern

Aperture function

$$p(x,y) = \begin{cases} 1 & \text{inside the aperture} \\ 0 & \text{outside the aperture} \end{cases}$$

$$f(x,y) = U(x,y)\, p(x,y)$$

$$I(x,y) = |g(x,y)|^2$$

↳ Fraunhofer Diffraction

~~Bragg~~

↳ Diffraction gratings

for atomic spectra

The slit spacing determines the location of peaks

d (inverse of slit spacing) → periodicity

angular despersing power

→ normal incidence : $d \sin\theta = m\lambda \quad m = 0, \pm 1, \pm 2, \ldots$

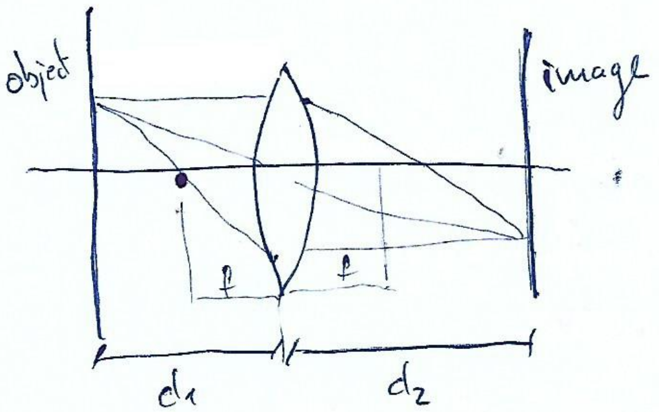
→ generic angle : $d(\sin\theta_m - \sin\theta_i) = m\lambda \quad m = 0, \pm 1, \pm 2, \ldots$

↳ Minimum wavelength separation a grating can resolve

$$\Delta(\sin\theta)_{min} = \frac{\lambda}{Nd}$$

$$\frac{\Delta(\lambda)}{\lambda} = \frac{1}{mN}$$

o) Single Lens Imaging System – Ray Optics



assuming focusing error:

$$\boxed{\varepsilon = \frac{1}{d_1} + \frac{1}{d_2} - \frac{1}{f}}$$

We say the system is focused

Pupil function (aperture function) $p(x,y) = \begin{cases} 1 & \text{inside aperture} \\ 0 & \text{outside aperture} \end{cases}$

impulse response function $h(x,y) \propto p\left(\frac{x}{\varepsilon d_2}, \frac{y}{\varepsilon d_2}\right)$

circular aperture of diameter D

the impulse response is

confined in a circle of radius: $\rho_s = \frac{1}{2} \varepsilon d_2 D$

o) Single Lens Imaging System – Wave Optics

→ Impulse response function

An impulse in object plane $\xrightarrow{\text{produces}}$ spherical wave $U(x,y)$ $\xrightarrow{\text{the wave crosses the aperture}}$ $U(x,y)$ multiplied by pupil function $p(x,y)$ and lens quadratic factor →

→ resultant $U_1(x,y)$ the propagates in free space to a distance d_2

$$\boxed{h(x,y) = h_1 h_2 \, e^{-j\pi \frac{x^2+y^2}{\lambda d_2}} \cdot P_1\left(\frac{x}{\lambda d_2}, \frac{y}{\lambda d_2}\right)}$$

$$h_{1,2} = \frac{j}{\lambda d_{1,2}} e^{-jkd_{1,2}}$$

P_1 is Fourier transform of $p_1(x,y) = p(x,y) \, e^{-j\pi\varepsilon \frac{x^2+y^2}{\lambda}}$

→ <u>Transfer function</u>

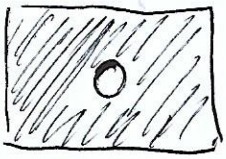
→ <u>4f imaging system</u>

Is a two-lens system with unity magnification

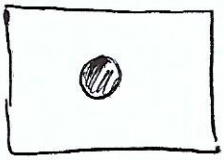
The analysis : We can consider the cascade of two Fourier transforming subsystems

Image : We get inverted replica of the object

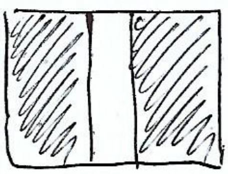
The 4f system can be used as a spatial filter



low pass



high pass



vertical pass

o) Near Field Imaging

5- EM Recap

$\mu_0 \simeq 4\pi \cdot 10^{-7}$ H/m

$\varepsilon_0 \triangleq 8.854 \cdot 10^{-12}$ F/m

$\eta_0 \simeq 120\pi$

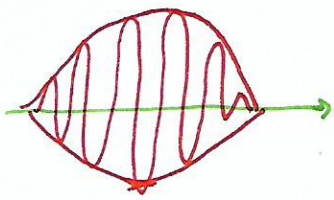
$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad k = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r} = k_0 n$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \qquad k_0 = \frac{\omega}{c}$$

Phase velocity : Velocity at which the phase of any frequency component of the wave travels.

$$v_p = \frac{\Delta z}{\Delta t} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{n}$$

Group velocity :



$$v_g = \frac{\Delta\omega}{\Delta k}$$

$$v_g = v_p\left(1 - \frac{k}{n}\frac{\partial n}{\partial k}\right)$$

* Group velocity & Phase velocity videos.

6 - Poynting Theorem & Material Properties

* "The power flowing out of a given volume V is equal to the time rate of the decrease in the energy stored within V minus the ohmic losses".

$\vec{B} = \mu \vec{H} = (\mu' - \mu'') \vec{H}$

$\bar{D} = \varepsilon \bar{E} = (\varepsilon' - \varepsilon'') \bar{E}$

Poynting vector: $\boxed{\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* \quad \left[\frac{W}{m^2}\right]}$ Power per unit surface flowing in the direction of $\vec{k}$

Interpretation of poynting theorem: (lecture 9 page 5)

* Light intensity at any point: $I = Re\left[\vec{S}\right] = \frac{1}{2} Re\left[\bar{E} \times \bar{H}^*\right]$

↳ plane wave: $\boxed{I = \frac{1}{2} \varepsilon_0 c n E^2}$

Polarization and Susceptibility

$\bar{D} = \varepsilon_0 \underbrace{(1 + \chi)}_{\varepsilon_r} \bar{E} \rightarrow \bar{P}_l = \varepsilon_0 \chi \vec{E}$

$\bar{B} = \mu_0 \underbrace{(1 + \chi_m)}_{\mu_r} \vec{H} \rightarrow \bar{P}_m = \mu_0 \chi_m \vec{H}$

Materials Properties

Linear / Nonlinear: Response is independent/dependent on field strength

General: $P = \varepsilon_0 \chi^{(1)} E + \varepsilon_0 \chi^{(2)} E^2 + \dots + \varepsilon_0 \chi^{(n)} E^n$

Linear: $\chi^{(2)} E \ll 1$ ...

$P \simeq \varepsilon_0 \chi^{(1)} E$

Homogeneous / Inhomogeneous: Response is independent/dependent on position within the medium

No homogeneous material exist bc they are finite.
We can consider homogen. as a collection of homogeneous materials

Isotropic / Anisotropic: Response is independent/dependent on direction of propagation

Isotropic: $\varepsilon \to$ scalar $\begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix}$

Anisotropic: $\varepsilon \to$ tensor

gas, liquid & amorphous solids $\to$ isotrop.

Stacionary / Non-stacionary: Response is independent / dependent on time

Stacionary medium ε, μ, σ don't change in time

stacionary: $D(t) = \varepsilon E(t)$

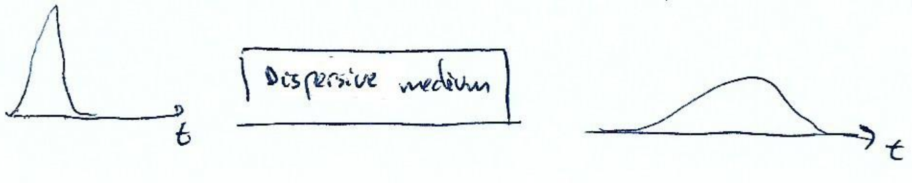
Non-stacionary: $D(t) = \varepsilon(t) E(t)$

Dispersive / Non-dispersive: Response is dependent / independent on time and space frequency

If ε, μ, σ don't depend on frequency

* Dispersion is very important bc light refracts at different angles depending on the wavelength

Also through a dispersive medium the freq. components of a short pulse will experience different time delay



Origin of dispersion

There are two kind of atoms in electrons: FREE & BOUND

⊛ Non-conducting media: Isotropic non-conducting medium such as dielectric, semiconductors, electrons are Bound

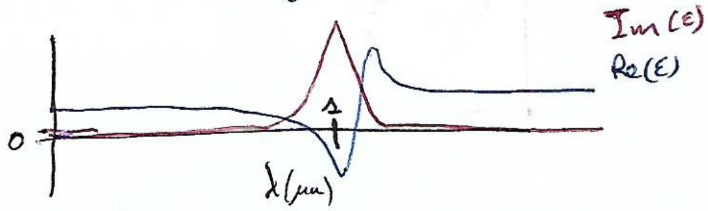
The eq. we use is a damped harmonic oscillator. If the force (E field) is harmonic ($e^{j\omega t}$) we get:

$$\boxed{\varepsilon = 1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 - j\gamma\omega}}$$

Lorentz oscillator

In the proximity of ω_0 is the resonance. There is a large change in the refractive index and strong light absorption

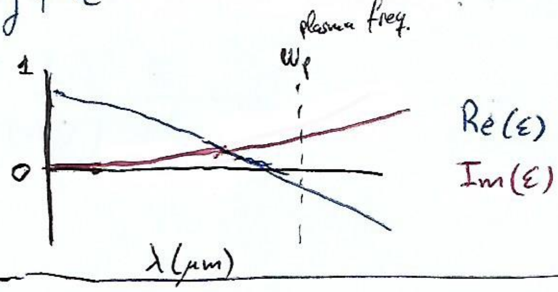
) Permitivity of a medium can be described as a superposition of Lorentz osci.



⊛ Conducting media: This is for e^- in the outer shells of atoms. Now we have a different equation of motion, however, there is not elastic restoring force

$$\boxed{\varepsilon = 1 - \frac{\omega_p^2}{\omega^2 - j\gamma\omega}}$$

Drude Oscillator



⊛ In General we have the contribution of both (superposition)

$$\varepsilon = 1 - \frac{\omega_{pD}^2}{\omega^2 - j\gamma_D\omega} - \sum_i \frac{\omega_{pi}^2}{\omega^2 - \omega_{0i}^2 - j\gamma_i\omega}$$

⊛ Sellmeier Equation

$$\varepsilon_r = n^2 = 1 + \chi$$

$$n^2 \simeq 1 + \sum_i \chi_{0i} \frac{\lambda^2}{\lambda^2 - \lambda_i^2}$$

For most optically transparent materials working far from absorption reson...

o) Wave velocities

(*) Non-dispersive media:

The wave velocity corresponds to the phase velocity:

$$v_p = \frac{\Delta z}{\Delta t} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{c}{n}$$

(*) Dispersive media:

In a dispersive medium each component of the pulse will travel at different speed.

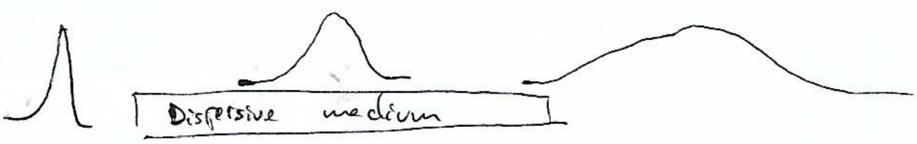
$$\boxed{v_g = \frac{c}{N} \qquad N = n - \lambda_0 \frac{\partial n}{\partial \lambda_0}}$$

$N \rightarrow$ group index

Each freq. component will undergo a different time delay, spreading the pulse
⇒ GVD (Group Velocity Dispersion)

→ Dispersion coeff. to stimate the spread of the pulse

$$D_v = \frac{\partial}{\partial \omega}\left(\frac{1}{v_g(\omega)}\right) = \frac{\partial^2 k}{\partial \omega^2}$$



Normal Dispersion: $D_v > 0$ → shorter λ arrive later than longer λ

Anomalous Dispersion: $D_v < 0$ → shorter λ arrive earlier than longer λ

7- Polarization

Polarization is the position of the tip of the E field vector in the plane xy (perpendicular to the propagation)

Monochromatic wave traveling in direction z: $\vec{E} = \left(a_x \hat{x} + a_y \hat{y} e^{j\varphi}\right) e^{-jkz}$

Real time expression of the field: $\bar{e}(t,z) = \mathrm{Re}\left[\bar{E} e^{j\omega t}\right] =$

$$\bar{e}(t,z) = \hat{x}\, a_x \cos(\omega t - kz) + \hat{y}\, a_y \cos(\omega t - kz + \varphi)$$

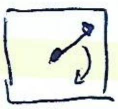
↳ Amplitude: $|\bar{e}(t,z)|$

↳ Polarization: (the "direction") $\psi = \arctan\left[\dfrac{e_y(z,t)}{e_x(z,t)}\right]$

⊛ Linear Polarization: $\varphi = 0$ or $\varphi = \pi$

⊛ RHCP : $\varphi = -\pi/2$ $a_x = a_y = a$. Counterclockwise

⊛ LHCP : $\varphi = \pi/2$ $a_x = a_y = a$. Clockwise



⊛ Elliptical

$a_x \neq a_y$

$\varphi \neq 0, \pm\pi, \pm\frac{\pi}{2}$

↳ A right (left)-circularly polarized wave is the superposition of two linearly-polarized waves. One along x and the other along Pi/2 and phase-shifted by $-(+)\frac{\pi}{2}$

↳ Circularly polarized: Sum of two linearly polarized with a phase-shift

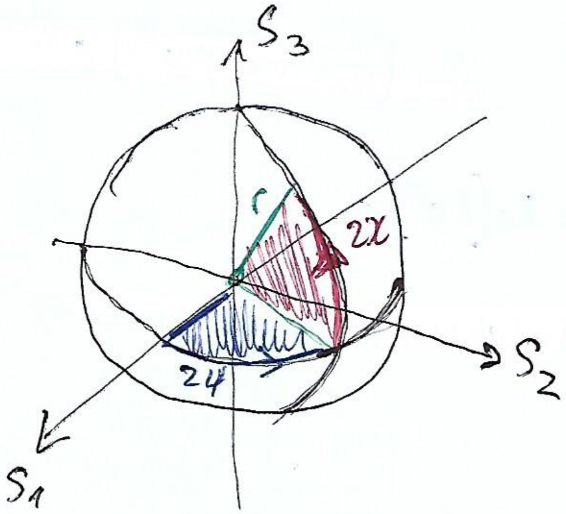
Linearly polarized: Sum of two circularly polarized waves
RHCP + LHCP

Poincarè Sphere

State of polarization described by two real parameters
$$\begin{cases} \text{Magnitude ratio: } R = \frac{a_x}{a_y} \\ \text{Phase difference: } \varphi = \varphi_y - \varphi_x \end{cases}$$

$\Downarrow$ Combined in: Complex polarization ratio

$R e^{j\varphi}$



$\chi = 0°$ → Linear Polarization

with $\phi = 0$ or $\phi = 180°$

linear along x and y

$\chi = \mp 90°$ → North → RH

South → LH

✻ Contains info about state of polarization but NO about intensity of the wave

Stokes Parameters

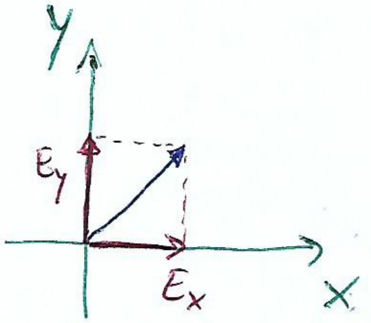
Info about state of polarization and intensity of the wave

(S_0, S_1, S_2, S_3) Real numbers

$S_0 = a_x^2 + a_y^2$ → Intensity

$$\left.\begin{aligned} S_1 &= S_0 \cos(2\chi)\cos(2\psi) \\ S_2 &= S_0 \cos(2\chi)\sin(2\psi) \\ S_3 &= S_0 \sin(2\chi) \end{aligned}\right\}$$ Cartesian coordinates of the point on the Poincarè sphere

o) Jones Matrix Description



Monochromatic wave: $\bar{E} = (a_x \hat{x} + a_y \hat{y} e^{j\varphi})$

$\varphi = \varphi_y - \varphi_x$

Jones vector:

$$\vec{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} a_x \\ a_y e^{j\varphi} \end{bmatrix}$$

⊛ Linearly polarized: $A \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$ horizontally (x) polarized $\theta = 0$ $A \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

vertically (y) polarized $\theta = \pi$ $A \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

⊛ Circularly polarized

→ RH: $\varphi = -\frac{\pi}{2} \Rightarrow e^{-j\frac{\pi}{2}} = \cos(-\frac{\pi}{2}) + j\sin(-\frac{\pi}{2}) = -j \rightarrow A \begin{bmatrix} 1 \\ -j \end{bmatrix}$

→ LH: $\varphi = \frac{\pi}{2} \Rightarrow e^{j\frac{\pi}{2}} = \cos(\frac{\pi}{2}) + j\sin(\frac{\pi}{2}) = j \rightarrow A \begin{bmatrix} 1 \\ j \end{bmatrix}$

⊛ Elliptically-polarized

$$\begin{bmatrix} A \\ B(\cos\varphi + j\sin\varphi) \end{bmatrix}$$

→ Linear polarization in x, y direction are orthogonal

→ RHCP and LHCP are orthogonal

Jones Matrix Description

Incident light: $J_1 = \begin{bmatrix} A \\ B \end{bmatrix}$

Emerging light: $J_2 = \begin{bmatrix} A' \\ B' \end{bmatrix}$

$$J_2 = T J_1$$

$$\begin{bmatrix} A' \\ B' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

Elements

↳ Linear polarizers

↳ Wave retarder

↳ Rotator

Cascaded elements

System composed by element with T_1 and then element with T_2:

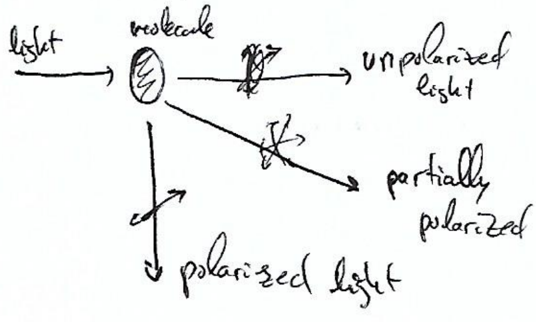
$$T = T_2 T_1$$

•) Light Polarization in Nature

Some tissues or molecules more sensitive to specific polarization orientations. Act like anisotropic media. (ε is a tensor: $\vec{D} = \varepsilon \cdot \vec{E}$)
respond. diff. depending on polatrizatoon.
Birefringent medium: Shows two different refractive indeces

•) Polarizatoon by Scattering

Sunlight is unpolarized. But sunlight scattered from the sky or sunlight reflected by surfaces.



⊙ Polarization by reflection

Sunlight reflected from surfaces is partially polarized.

The polaried sunglasses block horizontal polarized light.

8 - Reflection, Refraction & Scattering

• Refraction / Reflection

Refraction: Involves a change in the direction of wave propagation due to a change in propagation speed.

TE wave

$$E_i + E_r = E_t$$

$$\cos(\theta_i)\left[\frac{E_i}{\eta_i} - \frac{E_r}{\eta_i}\right] = \cos(\theta_t)\cdot\frac{E_t}{\eta_t} \xrightarrow{\mu=\mu_0=1}$$

$$\boxed{t_{TE} = \frac{E_t}{E_i} = 2\,\frac{n_i\cos(\theta_i)}{n_t\cos(\theta_t) + n_i\cos(\theta_i)} \qquad r_{TE} = \frac{E_r}{E_i} = \frac{n_i\cos(\theta_i) - n_t\cos(\theta_t)}{n_i\cos(\theta_i) + n_t\cos(\theta_t)}}$$

$\vec{E} \perp$ plane of incidence

s-polarization

TM wave

$$E_i - E_r = \frac{\eta_i}{\eta_t}E_t$$

$$\cos(\theta_i)(E_i + E_r) = \cos(\theta_t E_t) \xrightarrow{\mu=\mu_0=1}$$

$$\boxed{t_{TM} = \frac{E_t}{E_i} = 2\,\frac{n_i(\cos\theta_i)}{n_i\cos(\theta_t) + n_t\cos(\theta_i)} \qquad r_{TM} = \frac{E_r}{E_i} = \frac{n_i\cos(\theta_t) - n_t\cos(\theta_i)}{n_i\cos(\theta_t) + n_t\cos(\theta_i)}}$$

→ Reflectance & Transmittance

Reflectance: $\boxed{R = \frac{P_r}{P_i}}$

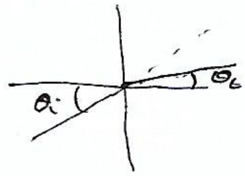
Transmittance: $\boxed{T = \frac{P_t}{P_i}}$

$$\boxed{\begin{array}{l} R = \dfrac{|E_r|^2}{|E_i|^2} = |r|^2 \\[2ex] T = 1 - R = |t|^2 \dfrac{\eta_i \cos\theta_t}{\eta_t \cos\theta_i} \end{array}}$$

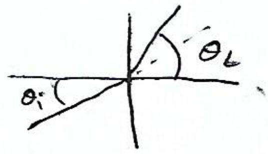
→ Snell's Law

$$\boxed{\begin{array}{c} \theta_i = \theta_r \\[1ex] n_i \sin(\theta_i) = n_t \sin(\theta_t) \end{array}}$$

If $n_i < n_t \rightarrow \theta_i > \theta_t$



If $n_i > n_t \rightarrow \theta_i < \theta_t$



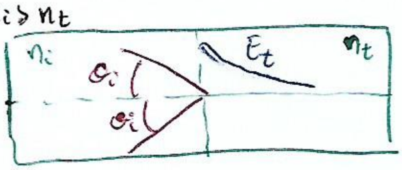
→ Total internal reflection

$$\boxed{\begin{array}{l} n_i > n_t \\[1ex] \theta_c = \operatorname{asin}\left(\dfrac{n_t}{n_i}\right) \end{array}}$$

→ Phase matching.

when $\theta_i > \theta_c$ → Evanescence field

$n_i > n_t$



$\longrightarrow \Gamma = e^{i\varphi}$

→ Brewster angle

⊛ Reflection is never ZERO in TE waves

⊛ Reflection is ZERO in TM waves when:

$$\boxed{\theta_B = \operatorname{atan}\left(\frac{n_t}{n_i}\right)}$$

•) Scattering

Def: Redirection of radiation out of the original direction of propagation

Different light phenomena like reflection, refraction or diffraction can be assumed as forms of scattering.

→ Scattering is negligible when the gain due to scattering is small compared to:
- Losses
- Gains due to thermal emission
- Direct radiation from a point source (the sun)

→ Types:

Elastic scattering λ of scattered light $=$ λ of the incident light

Inelastic scattering emitted radiation $\lambda \neq$ incident radiation λ

Quasi-elastic scattering λ of scattered light shifts (Doppler in matter)

Single-scattering photons scatters only once (thin mat.)

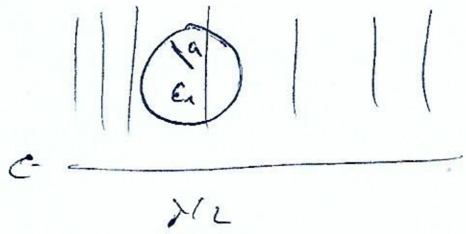
Multiple-scattering Photons can scatter hundreds times before emerging

→ Parameters

1. λ of incident radiation
2. Size of the scattering particle
3. Particle optical properties relative to surrounding media

Scattering regimes

Depending on the size of the scattering particle and the wavelength of the incident wave we have diff. regimes:



$$x = \frac{2\pi}{\lambda} \cdot a = ka$$

→ $ka \ll 1 \Rightarrow$ Rayleigh scattering

→ $ka \sim 1 \Rightarrow$ Mie scattering

→ $ka \gg 1 \Rightarrow$ Geometrical scattering

9 - Anisotropic Media

Anisotropic medium is the one which macroscopic properties depend on direction

Isotropic: $D = \underset{\substack{\downarrow \\ \text{scalar}}}{\varepsilon} E$ $\qquad \varepsilon = \begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}$

Anisotropic: $D = \underset{\substack{\downarrow \\ \text{tensor}}}{\varepsilon} \cdot E$ $\qquad \varepsilon = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$

→ Polarization in a crystal varies depending on the direction of applied field.

⇒ Phase velocity can assume diff. values depending on the direction of propag. and polarization.

→ Propagation in anisotropic mat.

Ordinary wave: $\bar{E}$ and $\bar{D}$ are parallel and both are $\perp$ to $\vec{k}$

Extraordinary wave: $\vec{D}$ is normal to $\vec{k}$ but $\bar{D}$ and $\vec{E}$ are not parallel

→ Dispersion relation

10- Liquid Crystals and Polarization Devices

•) Liquid Crystals

Is a collection of elongated organic molecules (typical cigar-shape)

→ Molecules ⟶ Not positional order (like liquids)

⟶ Yes orientation order (like crystals)

→ Types (phases)

increasing opacity ↓

- ⊙ Nematic liquid crystal: Orientation the same, position random
- ⊙ Smectic liquid crystal:
- ⊙ Cholestic liquid crystal:

•) Polarization devices

$I = I_0 \cos^2(\theta_0 - \theta_1)$

(1) → Polarizers: Is a device that allows transmission only for a specific polarization. Allow to pass the component in the transmission axis while blocking the orthogonal component.

⊛ Blocking Action: Selective absorption: Absorb light in certain directions
i.e. Polaroid H-sheet

⊛ Blocking Action: Selective reflection: Reflectance of light at boundary is dependent on its polarization. i.e. Brewster TM vanish and TE is reflected

⊛ Blocking Action: Selective refraction: When light enters an anisotropic crystal the ordinary and extraordinary waves refract at different angles.
This is a way to obtain polarized light from unpolarized one.

(2) → Wave retarders: Convert a wave with one form of polarization into another form.

characterized by: (1) Retardation Γ

(2) Fast and slow axes

Made by: birefringent material, a material that shows two refractive indices

→ ordinary (n_o)

→ extraordinary (n_e)

(3) → Rotators: Rotate the plane of polarization of linearly polarized light by a fixed angle.

(4) → Non Reciprocal devices:

Reciprocal: The effect they do on polarization state is invariant respect the propagation direction

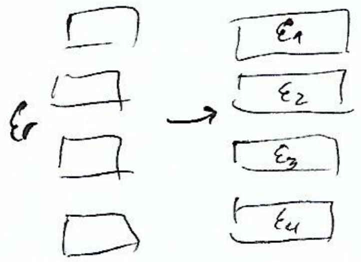
Non reciprocal: Don't have directional invariance

ie. optical isolator: transmit light only in one direction

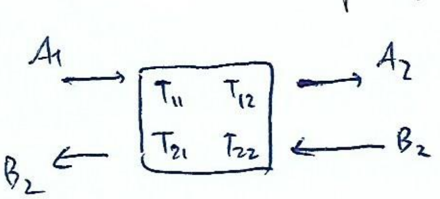
11 - Introduction to Periodic Nanostructures

o) Transfer Matrix Method (TMM)

Used to calculate transmission and reflection from a planar layered structure in which refractive index vary in one dimension.



Reduces numerical complexity



$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} A_2 \\ B_2 \end{pmatrix}$$

We can obtain the transmission matrix of a system just multiplying matrices.

→ Single slab - TE polarization

$$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

transmission coeff: $t = \left.\frac{a_2}{a_0}\right|_{b_2=0} = \frac{1}{M_{11}}$

refraction coeff: $r = \left.\frac{b_0}{a_0}\right|_{b_2=0} = M_{21} / M_{11}$

→ Fabry - Perot Resonator

Distance of 2 mirror is a multiple of $\lambda/2$

Resonance if: $2kd = 2m\pi$

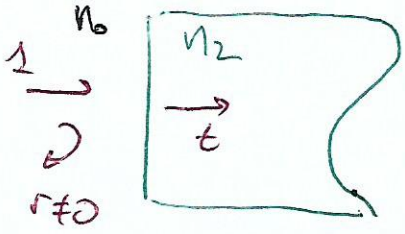
↳ In a slab of material with n_1, thickness d a sorounded by another material n_0

$$d = m \frac{\lambda_0}{2n_1} \quad , m = 1, 2 \ldots$$

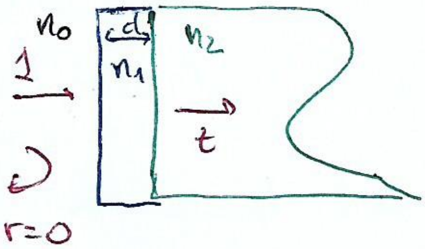
$n_1 d$ → optical length of the film

On FP resonance the slab becomes highly transmissive (transparent)

→ Aplication: Antireflection coating



Adding anti-ref. coating



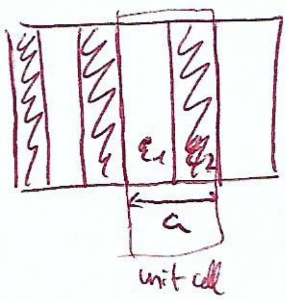
$$\boxed{\begin{aligned} n_1 d &= \lambda_{AR}/4 \\ n_1 &= \sqrt{n_0 n_2} \end{aligned}}$$

we can inhibit reflection in at least a wavelength bandwidth near λ_{AR}

12 - Photonic Crystal

Def: Periodic dielectric structures that interact resonantly with radiation with λ comparable to the periodicity length of the dielectric lattice.

1D Photonic crystals



$\varepsilon(\vec{r}) = \varepsilon(x) = \varepsilon(x + na)$

$n = 0, \pm 1, \pm 2$

2D Pho Crys

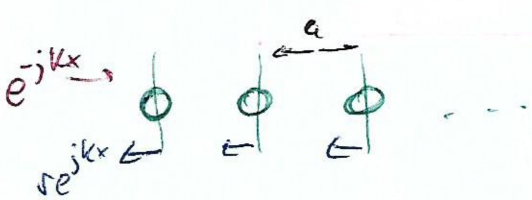
Dielectric constant periodically modulated in two dim

$\varepsilon(\vec{r}) = \varepsilon(x,y) = \varepsilon(x + na_x, y + ma_y)$

$n, m = 0, \pm 1, \pm 2$

3D Pho Crys

- Total reflection from a periodic lattice.



$R = re^{jkx} \dfrac{1}{1 - e^{2jka}}$

This diverges if:

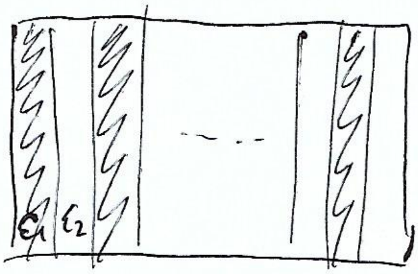
$e^{2jka} = 1 \qquad k = \frac{\pi}{a}$

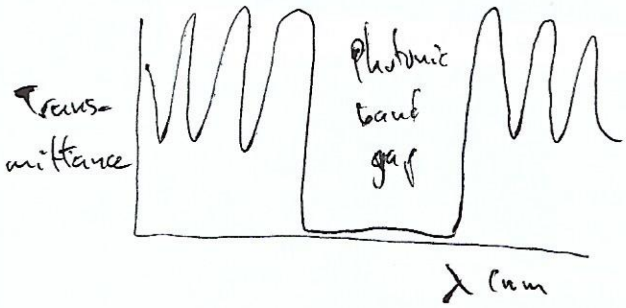
$(\lambda \sim 2a)$

Light cannot propagate in crystal if the Bragg condition is satisfied (Photonic bandgap)

→ Propagation in periodic media $\Longrightarrow$ Bloch's Theorem

a) Analysis of 1D Photonics Crystals





Def: $\boxed{\text{gap-midgap ratio}}$: to meassure the size of the gap

$$\frac{\Delta\omega}{\omega_c} \simeq \frac{\Delta n^2}{n^2}\,\frac{\sin(\pi a/\Lambda)}{\pi} \qquad \text{with weak periodicity}$$

Center of bandgap

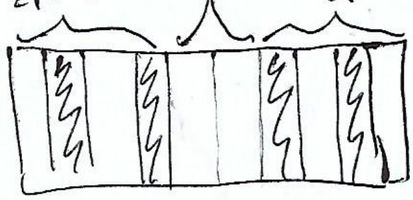
Normal incidence gap is maximized when

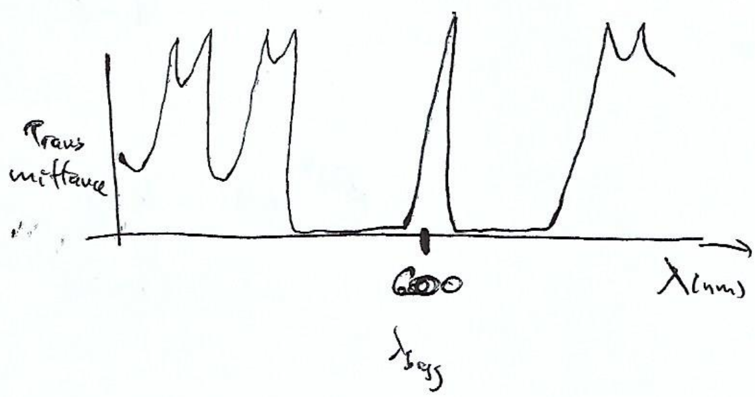
$$\boxed{n_1L_1 = n_2L_2 = \frac{\lambda_{bragg}}{4}}$$

$\xrightarrow{\text{midgap freq}}$

$$\boxed{\omega_{bragg} = \frac{n_1+n_2}{4n_1n_2}\,\frac{2\pi c}{\Lambda} \qquad \lambda_{bragg} = \frac{2\pi c}{\omega_{bragg}}}$$

Defects





13 - Plasmonics

→ Plasmonics refers to optical phenomena resulting from interaction of EM fields with conduction electrons in metals

→ Surface Plasmons: The collective vibrations of an electron gas (or plasma) sorrounding the atomic lattice sites of a metal

when plasmons couple with a photon, the resulting particle is called a polariton.

SSP (surface plasmon polariton) They are EM waves that travel along a metal-dielectric or metal-air, practically in the IR or visible freq. The term SSP explains that the wave involves both charge motion in the metal (surface plasmon) and EM waves in the air or dielectric (polariton)

i) Optical Properties of metals

Two types of e^- in metal → Bound, → Free

→ Bound: Lorentz oscillator

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 - j\gamma\omega}$$

→ Free: Drude oscillator

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2 - j\gamma\omega}$$

In general, both contributions:

$$\varepsilon = 1 - \frac{\omega_{pf}^2}{\omega^2 - j\gamma_f\omega} - \sum_i \frac{\omega_{pb,i}^2}{\omega^2 - \omega_{0b,i}^2 - j\gamma_{b,i}\omega}$$

In Visible → bound e^- (Lorentz)

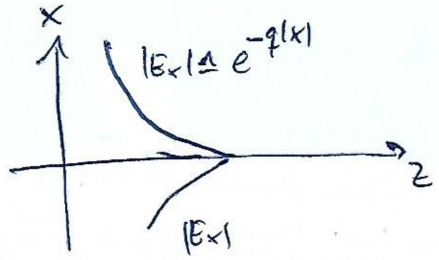
In Infrared → free e^- Drude

•) SSP at metal-dielectric interfaces

e^- charges on a metal boundary can perform coherent fluctuations (surface plasma oscillations)

$\Downarrow$

These are accompanied by mixed Tranversal and Longitudinal EM that disappears at $x \to \infty$. Max at $x=0$ (surface wave)



ε_d for $x>0$

ε_m for $x<0$

SSP only possible for TM polarized field. not for TE

→ Dispersion relation: $\boxed{k_z = \beta = k_{sp} = \frac{\omega}{c}\sqrt{\frac{\varepsilon_d \varepsilon_m}{\varepsilon_d + \varepsilon_m}}}$

$$q_d = \sqrt{\beta^2 - k_d^2}$$

$$q_m = \sqrt{\beta^2 - k_m^2}$$

$$k_{m,d} = \frac{\omega}{c}\sqrt{\varepsilon_{m,d}} = k_0\sqrt{\varepsilon_{m,d}}$$

Low absorption: $\varepsilon_m = \varepsilon' - j\varepsilon'' \to \varepsilon' \gg \varepsilon''$

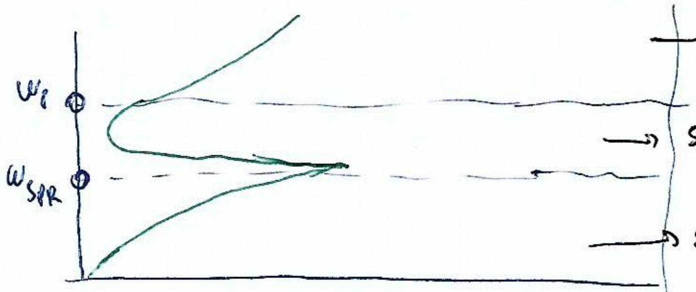
$$k'_{sp} \simeq \frac{\omega}{c}\sqrt{\frac{\varepsilon' \varepsilon_d}{\varepsilon' + \varepsilon_d}} = k_0 n_{sp}$$

k'_{sp} to be real we need $\underline{\varepsilon' < 0}$

this occurs in metals and

For $\varepsilon' = -\varepsilon_d$, $n_{sp} \to \infty$
this cond. occurs at specific freq. ω_{sp}
(surface plasmon resonance SPR)
At ω_{sp} plasmon becomes extremely confined
Effective wavelength $\frac{\lambda_0}{n_{sp}}$ becomes extremely small
If losses → Absorption very large

~ at drude-metal \ dielectric interface



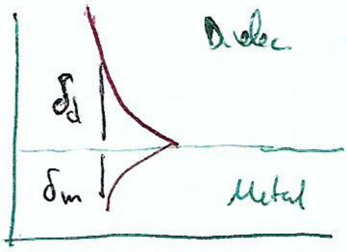
→ SPP becomes a Volume Plasmon Polariton that can propagate in metal.

→ SPP cannot propagate in z (β imaginary)

→ SPP propagates in z and is confined in x direction

$$\omega_{SPR} = \frac{\omega_p}{\sqrt{1+\varepsilon_d}}$$

•) Penetration depth

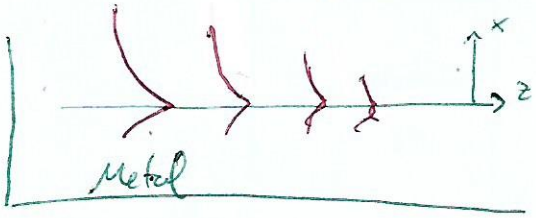


$$\delta_d = \frac{\lambda}{2\pi}\left[\left|\frac{\varepsilon_m' + \varepsilon_d}{\varepsilon_d^2}\right|\right]^{1/2}$$

$$\delta_m = \frac{\lambda}{2\pi}\left(\left|\frac{\varepsilon_m' + \varepsilon_d}{\varepsilon_m'^2}\right|\right)^{1/2}$$

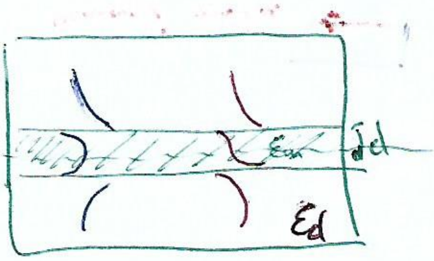
•) Propagation length

$$L_{sp} = \frac{1}{2K_{sp}''} = \frac{c}{\omega}\,\frac{\sqrt{|\varepsilon_m'|}\,(|\varepsilon_m'| - \varepsilon_d)^{3/2}}{\varepsilon_d^{3/2}\,\varepsilon_m''}$$



•) Surface plasmonics on metallic thin films

Can be seen as a system of coupled waveguides. Each isolated interface is a waveguide that supports propag. of SPP.



Symmetric antisymmetric

Symmetric mode: Long-range SPP (extends more into air than in the metal)

Asymmetric mode: Short-range SSP

→

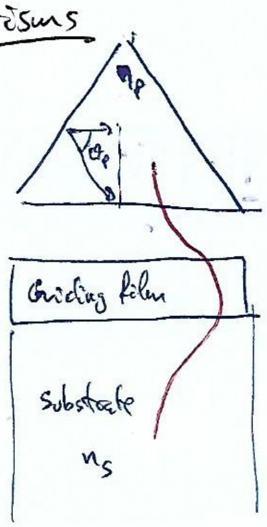
a) $\boxed{\text{Excitation of SPP}}$

We can use:

$e^- \rightarrow$ shooting into a metal, e^- scatter and energy is tranferredinto the plasma.

photons $\longrightarrow$ Using prisms and gratings

$\rightarrow$ Prisms



Incident transverse wave number

+

Lattice transverse momentum

$\downarrow$

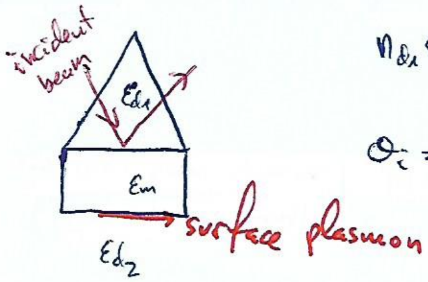
Plane wave excitation of the guided mode

$|k_{z,inc} + G| = \beta$

$k_{z,inc} = \frac{\omega}{c} n_d \sin\theta$

$G = \frac{2\pi m}{\Lambda} \qquad m = \pm 1, \pm 2, \pm 3, \dots$

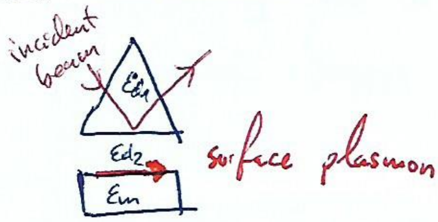
Kretschmann configuration



$n_{d1} \sin(\theta_i) = n_{sp} = \mathrm{Real}\left[\sqrt{\frac{\varepsilon_m \varepsilon_{d2}}{(\varepsilon_m + \varepsilon_{d2})}}\right]$

$\theta_i = \mathrm{asin}(n_{sp}/n_{d1})$

Otto configuration



→ Gratings

Evanescence fields can be obtained in grating when a diffraction order assumes a wave vector larger than the incident radiation.

$$\boxed{k_0 n_{sp} = k_0 n_d \sin(\theta_i) \pm 2\pi m / \Lambda}$$

c) Localized Surface Plasmons (LSP)

→ Planar, metal-dielectric interfaces act as waveguides and support the propag of surface waves coupled to the e^- plasma

→ Small particles may host non-propag. EM modes (LSP). These modes induce resonances observable as peaks in the scattering and absorption

14 - Metamaterials

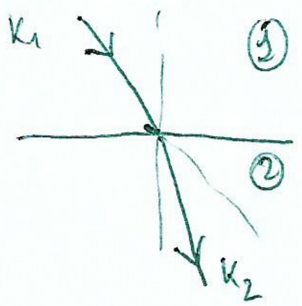
→ Structures that exhibit properties not observed in nature

Types:

Resonant: ∘ Period comparable to λ
∘ Oscillating currents emulate atomic resonances
⇒ { ∘ Freq. selective
∘ Sensitive to fabrication defects
∘ Can be design to achieve certain properties }

Non-Resonant: ∘ Period much smaller than λ
∘ Nothing resonates or scatters from unit cells
⇒ { ∘ Greatest potential to be broadband
∘ " " for wide field-of-view
∘ " tolerance to structural deform.
∘ Fewer magical properties tha resonant }

→ Remember what we studied



$k_1 = \frac{2\pi}{\lambda_1}$ $\quad v_{p1} = \frac{c}{n_1}$ $\qquad V_p = \frac{\omega}{K}$ $\quad V_g = \frac{\partial\omega}{\partial K}$

$k_2 = \frac{2\pi}{\lambda_2}$ $\quad v_{p2} = \frac{c}{n_2}$

from EM theory: $k^2 = \frac{\omega^2}{c^2}\varepsilon \rightarrow k = \pm\frac{\omega}{c}\sqrt{\varepsilon} \rightarrow n = \pm\sqrt{\varepsilon}$

$$\Downarrow$$

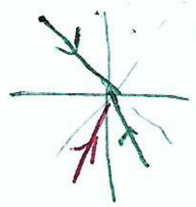
$$\boxed{n = \pm\sqrt{\varepsilon\mu}}$$

negative refractive index

we have $\vec{k} = \pm\frac{\omega}{c}\sqrt{\mu\varepsilon}\,\hat{k}$

if ε and μ are simultaneously negative then we have to chose $\vec{k} = -\frac{\omega}{c} n \hat{k}$ to ensure positive energy flow $\vec{S} = \frac{|E|^2}{\omega\mu}\hat{k}$

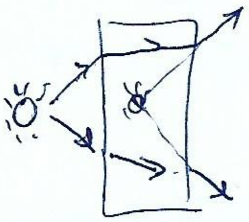
if neg. backward wave

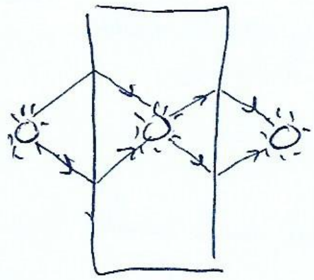


Flat Lenses

Normal positive ⇒ Virtual image

Metamaterial: Negative ⇒ Real image





Reversal snell law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow n_2 < 0 \Rightarrow \theta_2 < 0$$

negative phase vel.

$$v_p = \frac{\omega}{k} < 0$$

15 - Effective Medium Theories

→ What are they?: Models of inhomogeneous media based on analytical, numerical or experimental techniques. We try to describe a composite material in terms of effective medium approximation.

→ Why are helpful? Allow to write the constitutive relations in a very simple way

→ What are their limits? EMT usually depends on:

- → Electric & Magnetic properties
- → Volume fraction of each constituent
- → Geometry of the structure @ constituent level.

Fundamentally, λ of field <u>much larger</u> than characteristic scale of inhomogeneity.

- Maxwell Garnett Theory
- Bruggeman Theory

} Done in HW3

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1 Problem 2

1.1 Maxwell Garnett

Used for media with ***small inclusions*** dispersed in a ***continuous host medium***. The grains of guest material's relative permittivity is ϵ_i and they are hosted in a continuous medium with relative permittivity ϵ_h. If grains are small enough we can assume quasi-static approximation. Also, if there is no information about the shape of the grains, a small sphere shape is assumed.

Limits of validity

- If $\epsilon_i > 0$ particle size should be $< \frac{1}{10}\lambda eff$
- If $\epsilon_i < 0$ the limits of validity are stricter

Modelling first step:
In quasi-static approximation external electric field E_{ext} is considered constant for each sphere. Also, each sphere behaves like a point source with electric dipole moment proportional to E_{ext}.

$$P_h = \epsilon_0 \epsilon_h \alpha E_{ext} \tag{1}$$

The field inside sphere E_i is uniform and parallel to E_{ext}. Polarizability is isotropic.
Modelling second step:
Create the model for a distribution of small spheres. First, several electric point dipoles radiating and influencing each other:

$$<D>= \epsilon_0 \epsilon_{MG} <E> \tag{2}$$

Then, the average medium response can be written as:

$$<D>= \epsilon_0 \epsilon_h <E> + <P> \tag{3}$$

The average dipole response can be calculated using a model of a sphere with a charge density in the surface:

$$<P>= \frac{3N\alpha\epsilon_0\epsilon_h}{3 - N\alpha} <E> \tag{4}$$

Finally, the expression for the effective permittivity is:

$$\epsilon_{MG} = \epsilon_h \left(1 + \frac{N\alpha}{1 - \frac{N\alpha}{3}}\right) \tag{5}$$

For diluted media $1-\frac{N\alpha}{3}\approx 1$ so we get $\epsilon_{MG}=\epsilon_h(1+N\alpha)$. Also, introducing the parameter f which relates the volume fraction of the inclusions gives the next expression:

$$\epsilon_{MG}=\epsilon_h\left[1+3f\frac{\epsilon_i-\epsilon_h}{\epsilon_i+2\epsilon_h-f(\epsilon_i-\epsilon_h)}\right] \tag{6}$$

The limitations of the model

- Values of f higher than 0.5 make predictions questionable, because the interparticle distances decrease and trigger high-order multipole effects.
- This approach only can be used in a quasi-static state with dipole approximation. Higher-order terms are excluded.
- The role of the host and guests particles cannot be exchanged. If there is not a clear distinction, the validity of the formula cannot be ensured.

1.2 Bruggeman Theory

This theory is used when the medium is aggregate mixture with random distributions. Statistical formulation theories are used to model it.

In the Bruggeman theory the mixture is modeled as a continuous medium hosting a distribution of small spherical inclusions. Both, host medium and spherical inclusions, have different dielectric permittivities. Because of the fact that it is a statistical formulation based theory, the probabilities of finding spheres with permittivity ϵ_i and ϵ_h are assigned as f and $1-f$ respectively.

Assuming quasi-static approximation we get the formula:

$$f\frac{\epsilon_i-\epsilon_{Br}}{\epsilon_i+2\epsilon_{Br}}+(1-f)\,\frac{\epsilon_h-\epsilon_{Br}}{\epsilon_h+2\epsilon_{Br}}=0 \tag{7}$$

Where ϵ_{Br} is the unknown permittivity.

This formula is symmetric so the roles of inclusions and host are exchangeable. It can be also be extended to multi-phase aggregates just adding more terms:

$$\sum_{M}^{m=1} f_m\frac{\epsilon_m-\epsilon_{Br}}{\epsilon_m+2\epsilon_{Br}}=0 \tag{8}$$

In this theory, shape effects can be included, such as ellipsoidal inclusions.

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1.3 Differences between Maxwell Garnett and Bruggeman theories

- For situations with large inclusions' fill factor, Bruggeman formula gives a better modelling approach.
- Bruggeman theory provides a more realistic description when the mixture has large difference in the permittivities of the constituents.
- In the Bruggeman theory the roles of the host and inclusions are exchangeable, whereas, they are not in Maxwell Garnett.
- Maxwell Garnett theory suits better when there is a clear distinction between the host and inclusions.