MICROWAVE ENGINEERING

Lecture 18:
Impedance
Matching and
Tuning Problems

Problem 1 : Design two lossless L-section (Lumped Elements) matching circuits to match each of the following loads to a 100 Ω generator at 3 GHz:

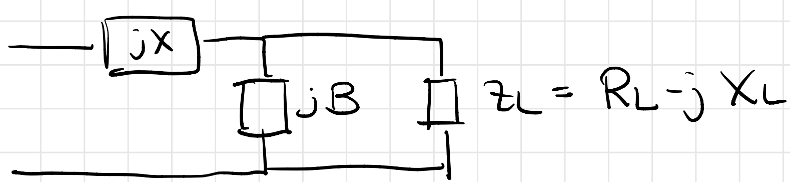
a) $Z_L = 150 - j200\ \Omega$

b) $Z_L = 20 - j90\ \Omega$

For load (a) we start by normalizing the load impedance to the line impedance.

$$z_L = \frac{Z_L}{Z_0} = \frac{150 - j200}{100} = 1.5 - j2$$

If $Z_L > Z_0$



The values of our components:

$$B = \frac{X_L \pm \sqrt{\frac{R_L}{Z_0}} \sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2} =$$

$$= \frac{-200 \pm \sqrt{1.5} \sqrt{150^2 + 200^2 - 100 \cdot 150}}{150^2 + 200^2} =$$

$$= \frac{-200 \pm 217.94 \cdot 122}{62500} = \begin{cases} 1.06 \cdot 10^{-3} \\ -7.5 \cdot 10^{-3} \end{cases}$$

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{B R_L} = \begin{cases} \frac{1}{1.06 \cdot 10^{-3}} + \frac{-200 \cdot 100}{150} - \frac{100}{1.06 \cdot 10^{-3} \cdot 150} = 181.13 \\ \ldots = -178.07 \end{cases}$$

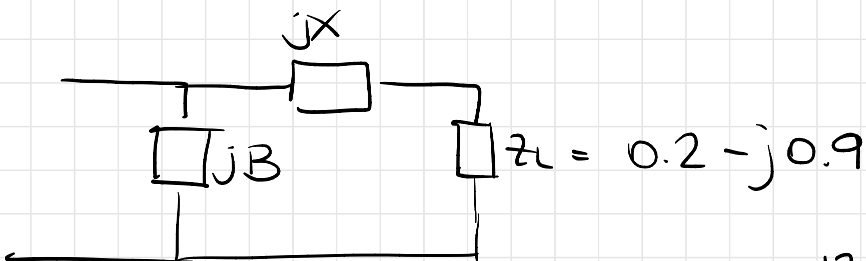
Sol 1 : $B = 1.06 \cdot 10^{-3}$ and $X = 181.13$

Sol 2 : $B = -7.5 \cdot 10^{-3}$ and $X = -178.07$

for load (a)

For load (b) : $Z_L = 20 - j90 \rightarrow z_L = \frac{Z_L}{Z_0} = 0.2 - j0.9$

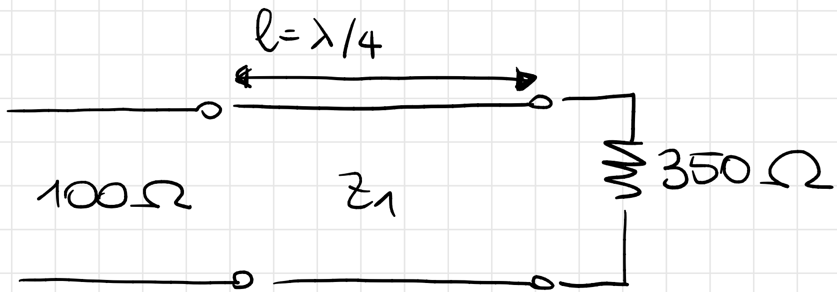
$Z_L < Z_0$



$$X = \pm\sqrt{R_L(Z_0 - R_L)} - X_L = \begin{cases} 130 \\ 50 \end{cases}$$

$$B = \frac{\pm\sqrt{(Z_0 - R_L)/R_L}}{Z_0} = \begin{cases} 0.02 \\ -0.02 \end{cases}$$

Problem 2 : Design a single section quarter-wave matching transformer to match a 350 Ω load to a 100 Ω line. What is the fractional bandwidth (in percent) of this transformer if SWR ≤ 2 ?



$$Z_1 = \sqrt{100 \cdot 350} = 187.1\ \Omega$$

From the value of the SWR:

$$\Gamma_m = \frac{SWR - 1}{SWR + 1} = \frac{1}{3}$$

The fractional bandwidth is:

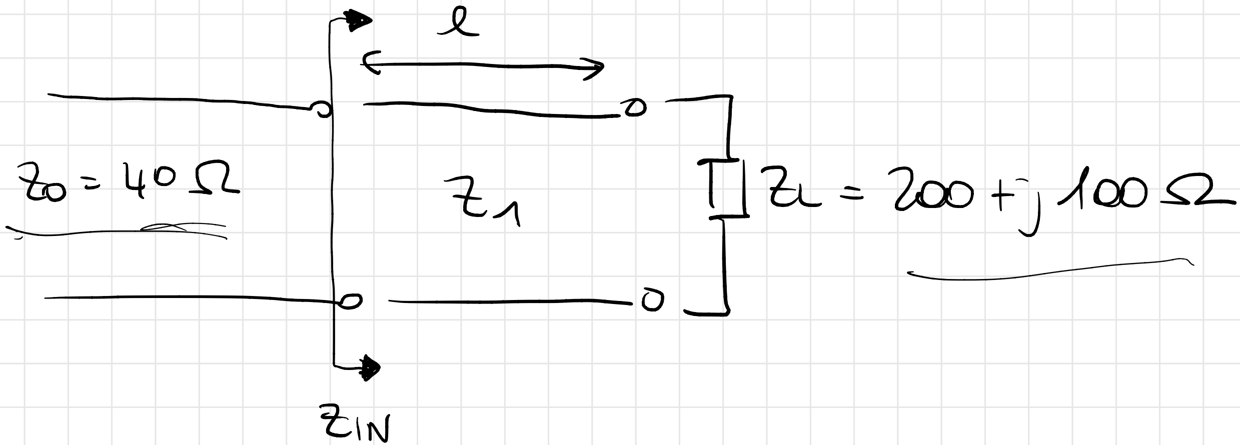
$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} \cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right] =$$

$$= 2 - \frac{4}{\pi} \cos^{-1}\left[\frac{1/3}{\sqrt{1-\frac{1}{9}}} \frac{2 \cdot 187.1}{|350-100|}\right] = 0.071$$

$$\rightarrow \frac{\Delta f}{f_0} \text{percentage} = 71\%$$

Problem 3 : In the circuit shown below a $Z_L = 200 + j100\,\Omega$ load is to be matched to a $40\,\Omega$ line using a length l of a lossless transmission line of impedance Z_1.

Find l and Z_1.



$$Z_{in} = Z_1 \frac{(200 + j100) + jZ_1 t}{Z_1 + j(200 + j100)t} = 40 = Z_0$$

$t = \tan\beta$

$$z_1(200+j100) + jz_1^2 t = 40z_1 + j40(200+j100)t$$

$$200z_1 + j(100+z_1 t)z_1 = (40z_1 - 4000t) + j8000t$$

By equating real and imaginary parts:

$$\begin{cases} 200z_1 = 40z_1 - 4000t \\ (100+z_1 t)z_1 = 8000t \end{cases}$$

$$\begin{cases} 160\, z_1 = -4000\, t & \Rightarrow \quad z_1 = -25t \\ (100 - 25t^2)(-25t) = 8000\, t \end{cases}$$

$$\begin{cases} z_1 = -25t \\ -25\,(100 - 25t^2) = 8000 \end{cases}$$

$$\begin{cases} z_1 = -25t \\ 8000 + 2500 = 625t^2 \end{cases} \Rightarrow t^2 = 16.8$$

$$\Downarrow$$

$$t = \pm\sqrt{16.8} = \pm 4.10$$

if $z_1 > 0$ then $t < 0$ $\Rightarrow$ $t = -4.10$

$t = \tan \beta l$

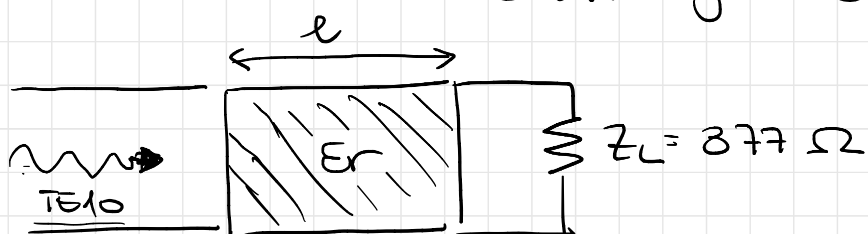
$$\beta l = \tan^{-1}(-4.10) = -76.3^\circ \simeq 104^\circ$$

$$\Rightarrow \quad l = \frac{104^\circ}{\beta} = \frac{104^\circ}{\underset{360^\circ}{2\pi}}\lambda = \underline{0.288\lambda}$$

$$z_1 = \underline{-25 \tan \beta l} = -25(-4.1) = 102.5\,\Omega$$

Problem 4 : A waveguide load with an equivalent TE_{10} wave impedance of 377 Ω must be matched to an air-filled X-band rectangular waveguide at 10 GHz.

A quarter wave matching transformer has to be used and consists of a section of a guide filled with dielectric. Find the required dielectric constant and physical length of the matching section. NOTE: All waveguides have the same geometrical dimensions.



X-band rectangular waveguide $\rightarrow$ $a = 2.286$ cm

NO NEED for b since we are propagating a TE_{10} mode!

$$k_0 = \frac{2\pi f}{c} = \frac{2\pi \cdot 10 \cdot 10^9}{3 \cdot 10^8} = 209.4 \ m^{-1}$$

In the air-filled waveguide

$$\beta_a = \sqrt{k_0^2 - \left(\frac{\pi}{a}\right)^2} = 158 \ m^{-1}$$

$$Z_a = \frac{k_0 \eta_0}{\beta_a} = \frac{(209.4)(377)}{158} = 499.6 \ \Omega$$

The matching section will have $Z_m = \sqrt{Z_a Z_L} = \sqrt{499.6 \cdot 377}$

$= \underline{434 \ \Omega}$

$$l = \frac{\lambda_g}{4} = \frac{2\pi}{4\beta_m}$$

$$Z_m = \frac{k_m \eta_m}{\beta_m} \Rightarrow \frac{k_0 \cancel{\sqrt{\varepsilon_r}} \, \frac{\eta_0}{\cancel{\sqrt{\varepsilon_r}}}}{\beta_m} \Rightarrow \beta_m = \frac{k_0 \eta_0}{Z_m} = \frac{(209.4)(377)}{436} = 181.9 \; m^{-1}$$

$$\underline{l} = \frac{2\pi}{4\beta_m} = \frac{\pi}{2\beta_m} = \underline{0.86 \text{ cm}}$$

$$\beta_m = \sqrt{\varepsilon_r k_0^2 - \left(\frac{\pi}{a}\right)^2} \Rightarrow \beta_m^2 = \varepsilon_r k_0^2 - \left(\frac{\pi}{a}\right)^2$$

$$\underline{\underline{\varepsilon_r}} = \frac{\beta_m^2 + \left(\frac{\pi}{a}\right)^2}{k_0^2} = \frac{(181.9)^2 + \left(\pi / 0.02286\right)^2}{(209.4)^2} = 1.18$$

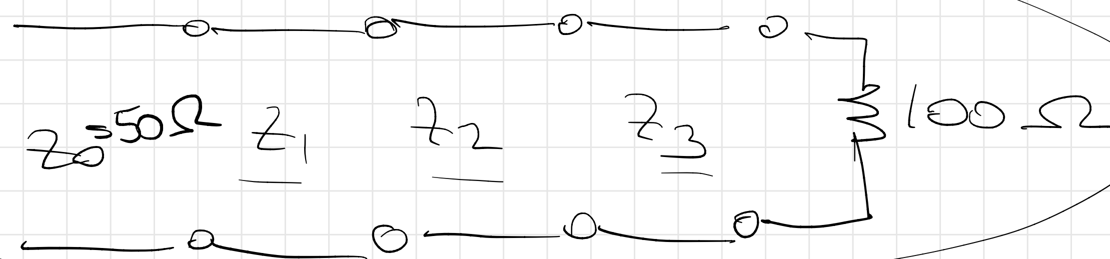
Problem 5: Design a three section binomial transformer To match a $Z_L = 100\ \Omega$ load to a $Z_0 = 50\ \Omega$ line and calculate the bandwidth for $\Gamma_m = 0.05$.

$N = 3$

$Z_L = 100\ \Omega$

$Z_0 = 50\ \Omega$

$\Gamma_m = 0.05$



$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} \cos^{-1}\left[\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{1/N}\right]$$

$$A = 2^{-N} \frac{Z_L - Z_0}{Z_L + Z_0} \simeq \frac{1}{2^{N+1}} \ln \frac{Z_L}{Z_0} \simeq 0.0433$$

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} \cos^{-1}\left[\frac{1}{2}\left(\frac{0.05}{0.0433}\right)^{1/3}\right] = 0.7$$

$$\boxed{70\%}$$

From Table:

$$Z_1 = 1.0907 \cdot Z_0 = 1.0907 \cdot 50 = 54.5\,\Omega$$

$$Z_2 = 1.4142 \cdot Z_0 = 70.7\,\Omega$$

$$Z_3 = 1.8337 \cdot Z_0 = 91.7\,\Omega$$

Problem 6   Design a three-section Chebyshev transformer to match a 100 Ω load to a 50 Ω line with $\Gamma_m = 0.05$.

$N = 3$

$Z_L = 100\,\Omega$

$Z_0 = 50\,\Omega$

$\Gamma_m = 0.05$

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi}$$

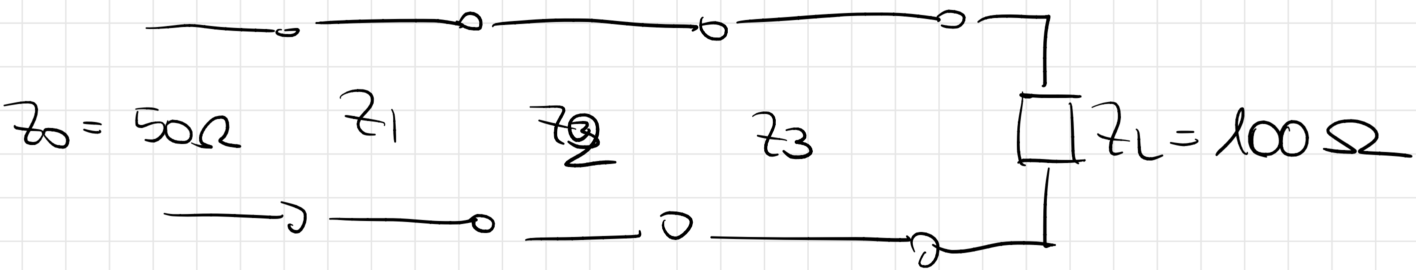
$$\sec\theta_m = \cosh\left[\frac{1}{N}\cosh^{-1}\left(\frac{\ln Z_L/Z_0}{2\Gamma_m}\right)\right] =$$

$$= \cosh\left[\frac{1}{3}\cosh^{-1}\left[\frac{\ln(100/50)}{2(0.05)}\right]\right] =$$

$$= 1.408$$

$\theta_m = 44.7°$

$$\frac{\Delta f}{f_0} = 2 - 4\left(\frac{44.7^\circ}{180^\circ}\right) = 1.01$$



From Table:

$Z_1 = 1.1475\, Z_0 = 57.5\,\Omega$

$Z_2 = 1.4142\, Z_0 = 70.7\,\Omega$

$Z_3 = 1.7429\, Z_0 = 87\,\Omega$