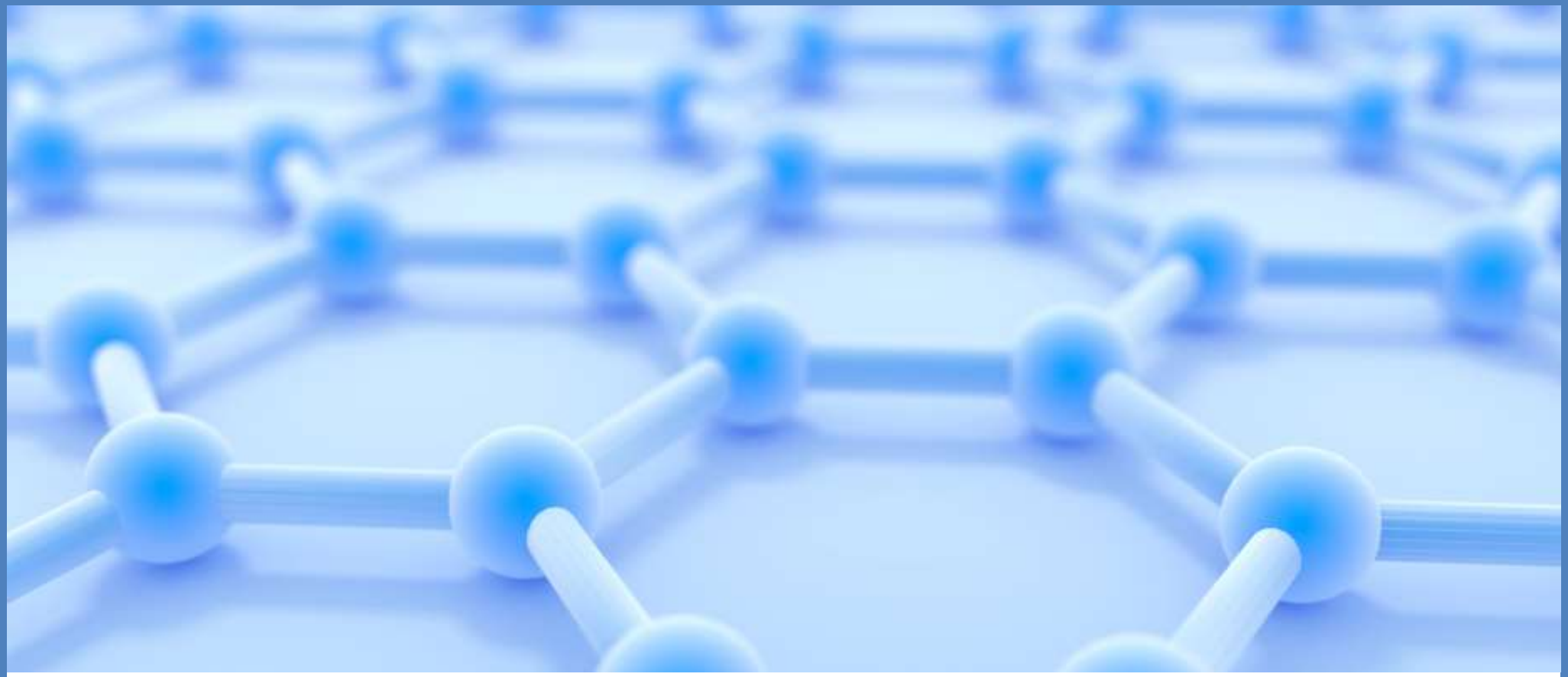
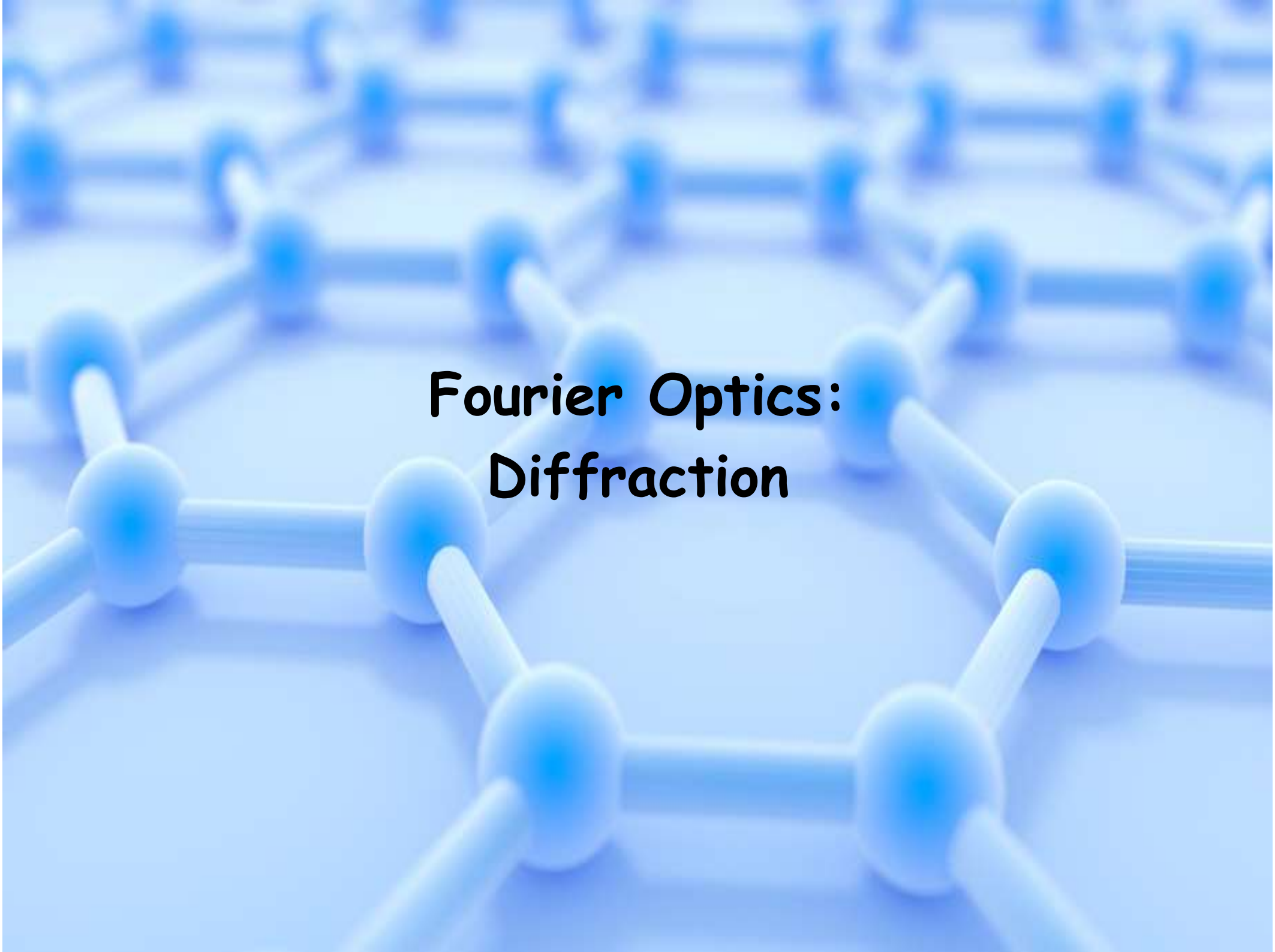


Photonics
aa 2021/2022

Prof. Maria Antonietta Vincenti
Università degli Studi di Brescia



Fourier Optics: Diffraction

DIFFRACTION

Generally, it refers to what happens to a wave when it hits an obstacle.

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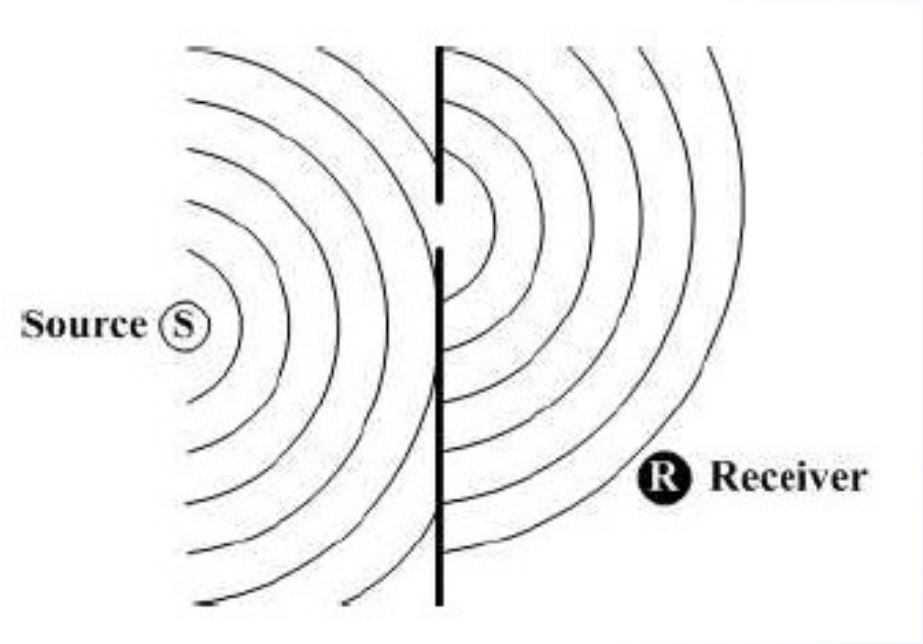
Huygens' Principle

The key to understanding diffraction is a very simple observation first due to Huygens in 1678. Say a wave arrives at an opaque screen with a little hole in it. On the other side of the screen, the wave equation must still be satisfied with boundary conditions given by the motion of the wave in the hole. In other words, the solution is identical to a situation where there was a source in the hole. The wavefront diagram looks like this:



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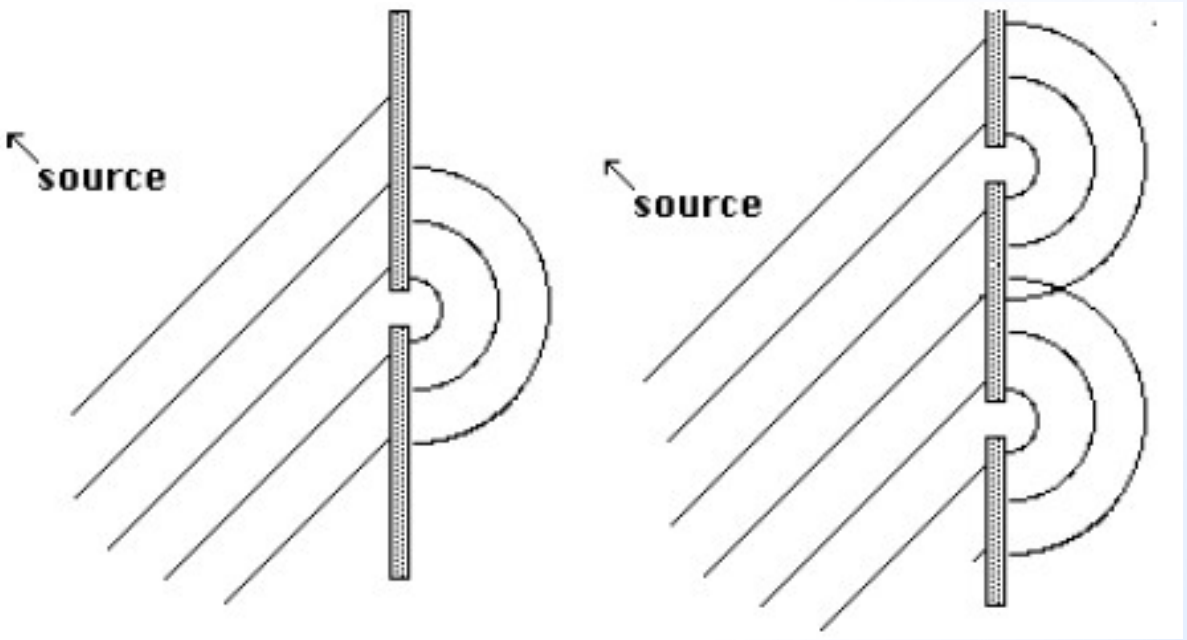
Huygens' Principle

The great thing about this way of thinking about diffraction is that, since the wave equation is linear, you can use this trick for any number of holes. You simply add the amplitude for the waves produced from a "source" at each hole:



Calculating the amplitude by adding point sources in this way is known as **Huygens' principle**. Huygen's principle works even if the holes are very close together. In fact, it works if they are connected, so instead of a hole, it's a slit.

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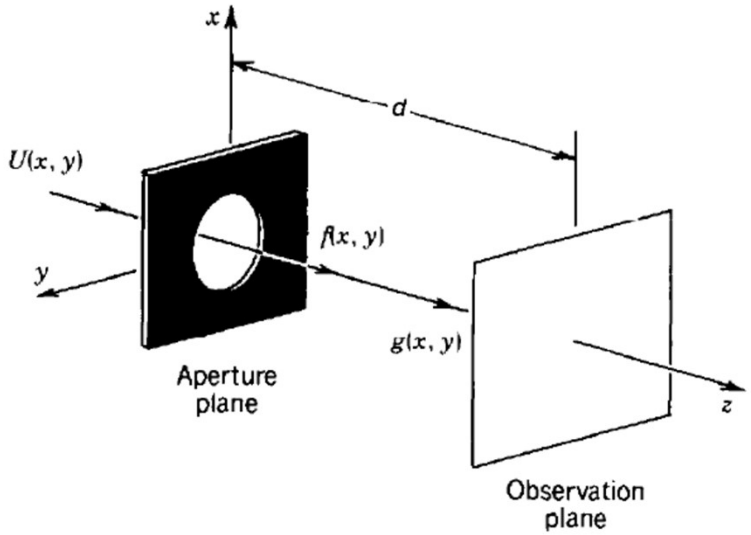
Diffraction of light

When an optical wave passes through an aperture and light propagates in free space, its intensity distribution is called diffraction pattern. Note that ray optics fails to predict the diffraction pattern correctly.

The simplest approach to diffraction considers the so-called aperture function:

$$p(x,y) = \begin{cases} 1, & inside\ the\ aperture \\ 0, & outside\ the\ aperture \end{cases}$$

Using this definition, the complex amplitude on the right side of the screen is:



$$f(x,y) = U(x,y)p(x,y)$$

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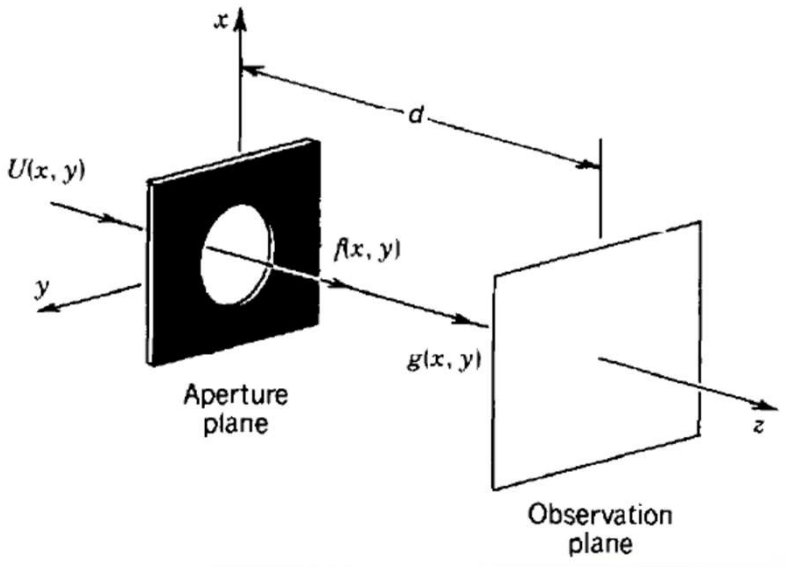
Diffraction of light

After passing through the aperture, we can calculate the complex amplitude at an observation plane at a distance $g(x, y)$ as described previously.
The diffraction pattern known as **Fraunhofer diffraction or Fresnel diffraction** (depending on the chosen approximation) has an intensity:

$$I(x, y) = |g(x, y)|^2$$

NOTE: The assumption that the complex amplitude vanishes outside the aperture is a simplification that allows to find a solution for the diffraction pattern. A different approach that considers the wave extending outside the aperture region is extremely complicated and does not always return an exact Solution if not for some specific cases.



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Fraunhofer diffraction

The Fraunhofer diffraction theory assumes that, after passing through the aperture, the wave propagates following the Fraunhofer approximation. Such approximation is valid if the observation plane is sufficiently far from the aperture so that the Fresnel number calculated at the aperture section is <<1.

If the incident wave has intensity I_i and travels along z so that $U(x,y) = \sqrt{I}_i$, then $f(x,y) = \sqrt{I_i}p(x,y)$. After propagation in the Fraunhofer approximation we have:

$$g(x,y) \approx \sqrt{I_i} h_0 P\left(\frac{x}{\lambda d}, \frac{y}{\lambda d}\right)$$

Where:

$$P(\nu_x, \nu_y) = \int_{-\infty}^{\infty} p(x,y) \exp\left[j2\pi(\nu_x x + \nu_y y)\right] dxdy$$

$$h_0 = \left(\frac{j}{\lambda \mathrm{d}}\right) \exp(-jkd)$$

The diffraction pattern is therefore:

$$I(x,y) = \frac{I_i}{(\lambda d)^2} \left| P\left(\frac{x}{\lambda d}, \frac{y}{\lambda d}\right) \right|^2$$

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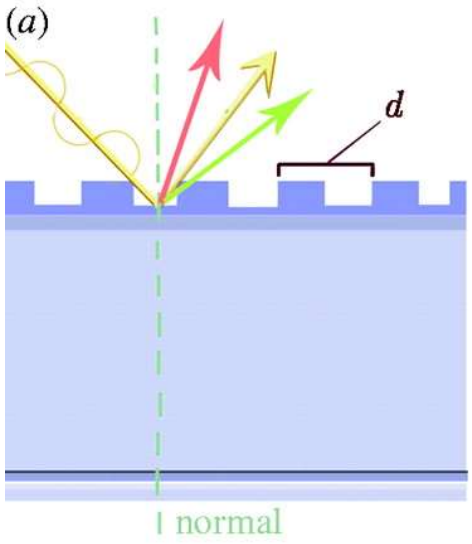
Diffraction Gratings

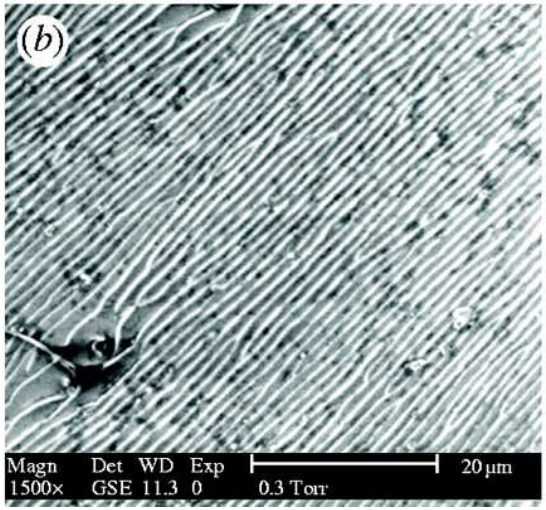
Examples around us.

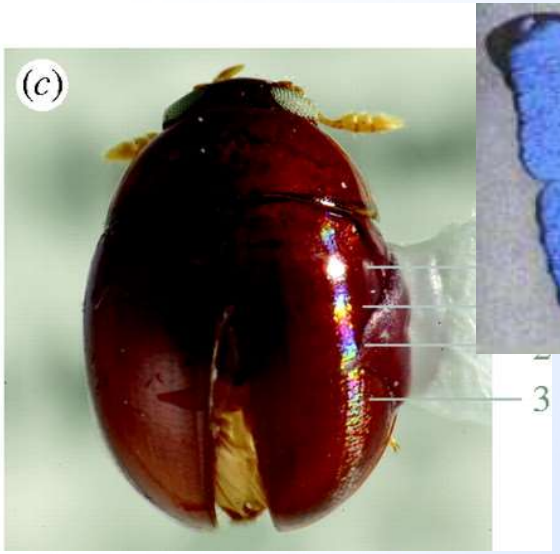
CD – grooves spaced by wavelength of visible light.

The color of some butterfly wings and reflection from other animals.

They are not pigmented! The colors come from interference of the reflected light from the pattern of scales on the wings or shells – a grating with spacing of order the wavelength of visible light!







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Diffraction Gratings

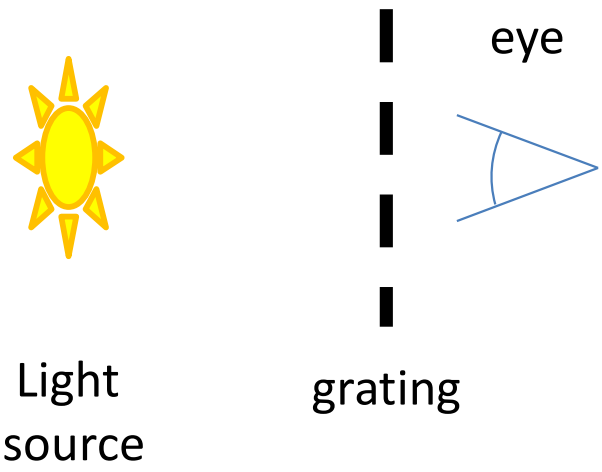
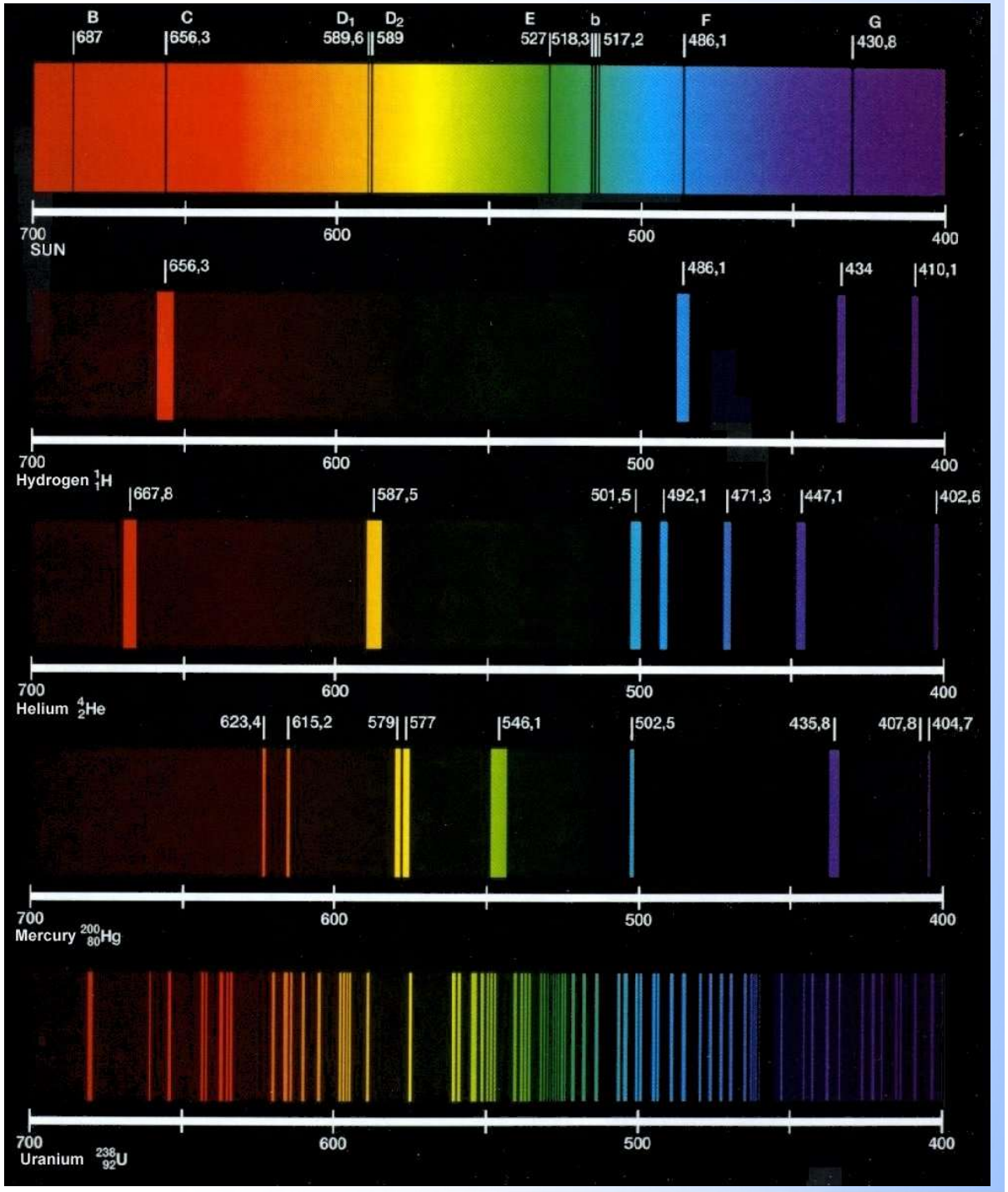


Figure shows examples of **atomic spectra** for the Sun, Hydrogen, Helium, Mercury and Uranium.



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Diffraction Gratings

The slit spacing determines the location of the peaks (NOTE: *d* is the inverse of the slit spacing, i.e. ***d* = periodicity**!)

The angular dispersing power $\theta(\lambda)$ of the grating can be then inferred from the grating formula. At normal incidence we have:

$$d \sin\theta = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

Note that the positions of the principal interference maxima are the *same* for any number of slits!

At a generic angle of incidence, we have:

$$d\left(\sin\theta_m - \sin\theta_i\right) = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

The number of slits/beam size determines the *width* of the peaks (narrower peaks easier to resolve).

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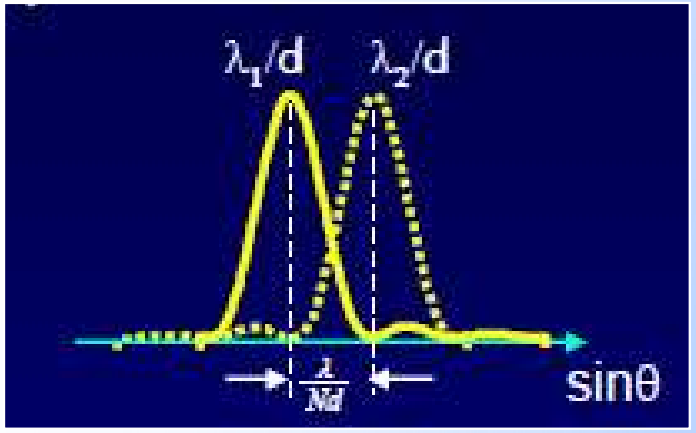
Diffraction Gratings

Diffraction gratings rely on N-slit interference. They consist of a large number of evenly spaced parallel slits.

How effective are diffraction gratings at resolving closely-spaced 'spectral lines'? Are these two lines distinguishable using a particular grating?

Rayleigh's criterion

The minimum wavelength separation a grating can resolve occurs when the λ_2 peak coincides with the first zero of the λ_1 peak



So, the Rayleigh criterion can be written as:

$$\Delta(\sin\theta)_{\min} = \frac{\lambda}{Nd}$$

The location of the peak is $\sin\theta = \frac{m\lambda}{d}$

Therefore $\Delta(\lambda)_{\min} = \frac{d}{m}\Delta(\sin\theta)_{\min} = \frac{\lambda}{mN}$ → $\frac{\Delta(\lambda)}{\lambda} = \frac{1}{mN}$

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Image formation

An ideal imaging system is an optical system that can replicate the distribution of light at the object plane into the image plane.

Obviously, it is not possible to obtain a perfect replica of the object and other than magnification, blurring that results from imperfect focusing should be considered.

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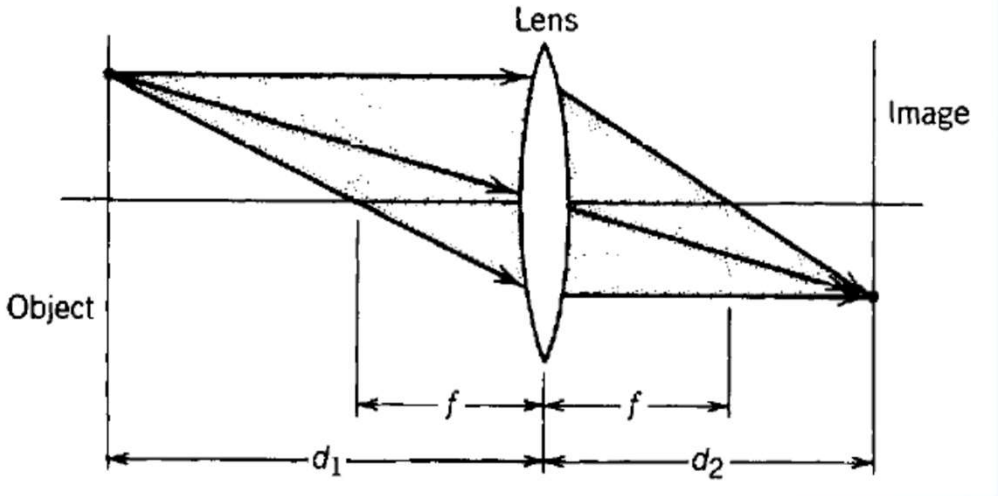
Single lens imaging system (ray optics)

Let's consider an imaging system using a lens with focal length f at distances d1 and d2 from object and image planes. The system is focused only if $1/d_1$ and $1/d_2$ are equal to f.



If we now assume the system is out of focus and define a **focusing error** as:

$$\epsilon = \frac{1}{d_1} + \frac{1}{d_2} - \frac{1}{f}$$

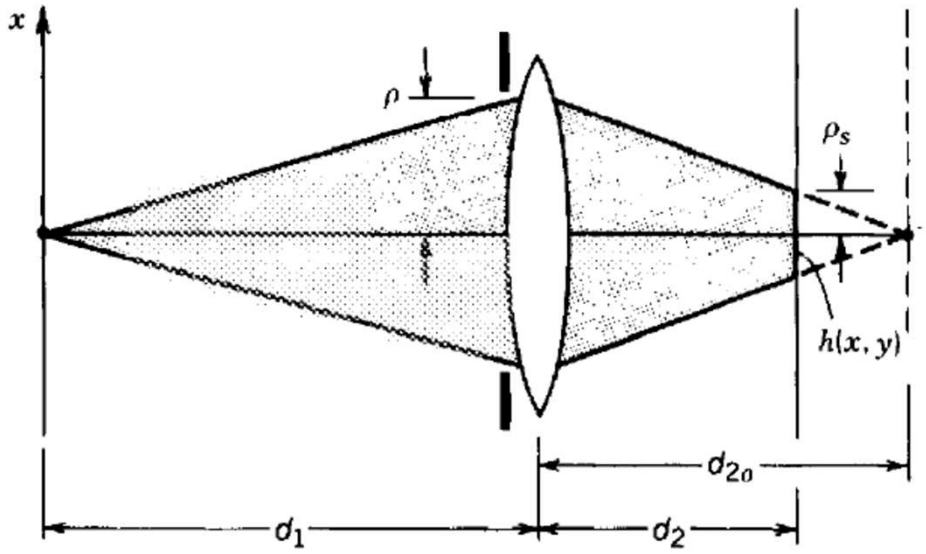
A single point on the object plane will not correspond to another point on the image plane.

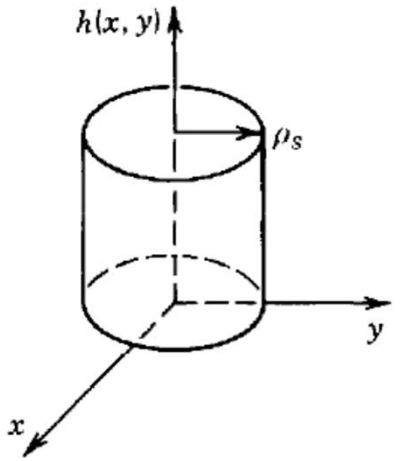
Single lens imaging system (ray optics)

If the system is out of focus, we can consider the projection of the point of the object plane as the image of an aperture on the image plane. The result is a patch of light on the image plane. If we assume that the plane of the focused image is at a distance d_{20} from the lens it means that $\frac{1}{d_1} + \frac{1}{d_{20}} = \frac{1}{f}$.

The shadow of a point at the edge of the aperture at a radial distance ρ is a point in the image plane at distance ρ_s where the ratio

$$\frac{\rho}{\rho_s} = \frac{d_{20} - d_2}{d_{20}} = 1 - \frac{d_2}{d_{20}} = 1 - d_2\left(\frac{1}{f} - \frac{1}{d_1}\right) = 1 - d_2\left(\frac{1}{d_2} - \epsilon\right) = \epsilon d_2$$





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Single lens imaging system (ray optics)

If the **pupil function** or aperture function is

$$p(x,y) = \begin{cases} 1, & inside\ the\ aperture \\ 0, & outside\ the\ aperture \end{cases}$$

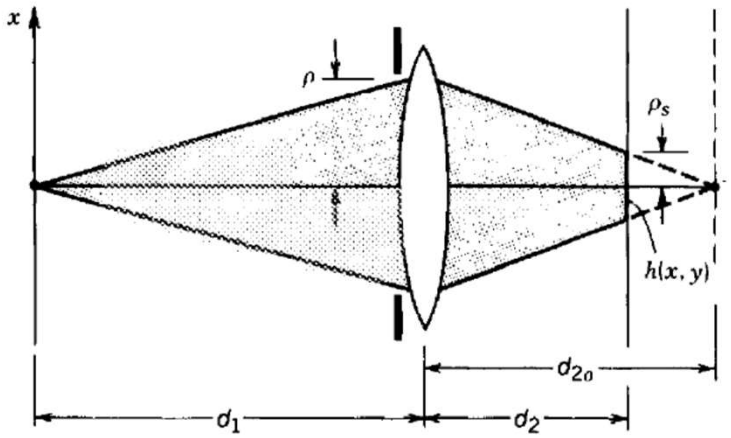
Then

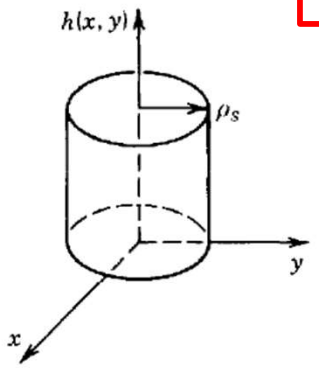
$$h(x,y) \propto p\left(\frac{x}{\epsilon d_2}, \frac{y}{\epsilon d_2}\right)$$

Impulse response function

For example, a circular aperture of diameter D corresponds to an impulse response function confined in a circle of radius:

$$\rho_s = \frac{1}{2}\epsilon \mathrm{d}_2 D$$





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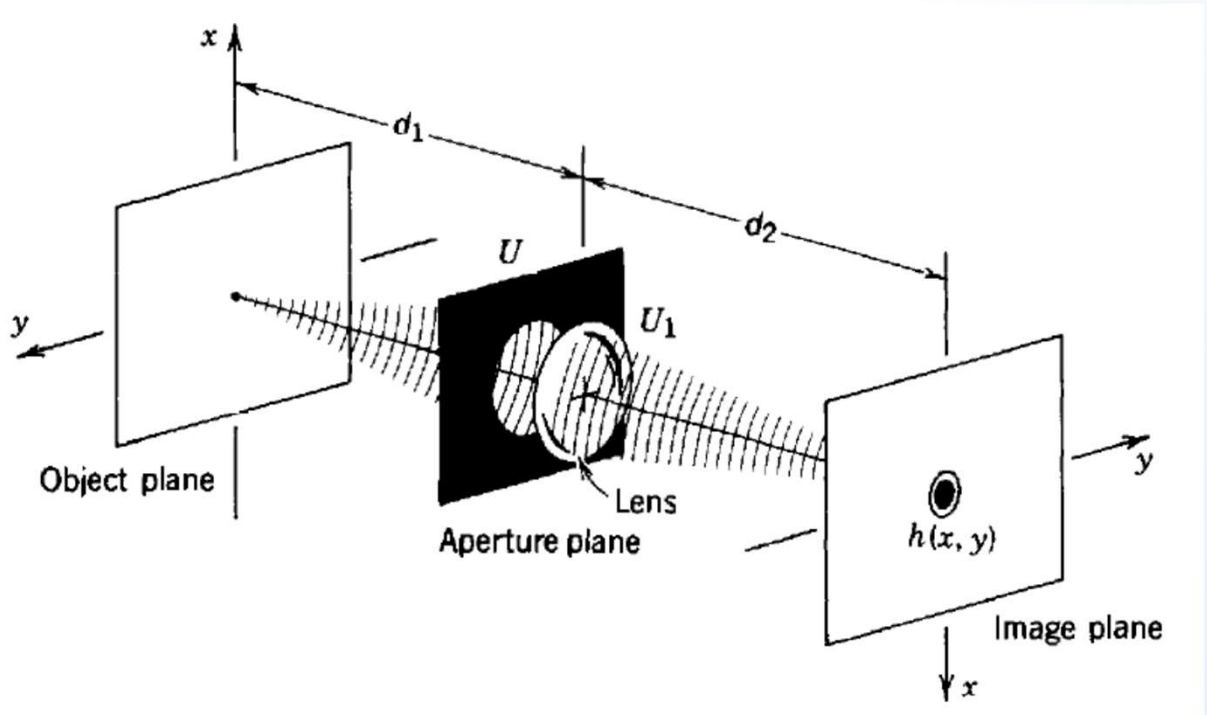
Single lens imaging system (wave optics)

We can re-analyze the single lens imaging system with a wave optics approach determining the impulse response function and from that derive the system's transfer function. Both these functions are related to the focusing error ε and the pupil function $p(x, y)$.



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Single lens imaging system (wave optics)

Impulse response function

To determine the impulse response function, we consider a single point (an impulse) on the optical axis at point (0,0) and then follow the wave to the image plane. The resultant complex amplitude is the impulse response function $h(x, y)$.

An impulse in the object plane produces a spherical wave $U(x, y)$. The wave then crosses the aperture and the function $U(x, y)$ is multiplied by the pupil function $p(x, y)$ and the lens quadratic phase factor.

The resultant field $U_1(x, y)$ then propagates in free space a distance d_2 so that the overall impulse response function is:

$$h(x,y) = h_1 h_2 \exp\left[-j\pi \frac{(x^2+y^2)}{\lambda d_2}\right] P_1\left(\frac{x}{\lambda d_2}, \frac{y}{\lambda d_2}\right), \quad \text{with } h_{1,2} = \left(\frac{j}{\lambda d_{1,2}}\right)\exp(-jkd_{1,2})$$

P_1 is the Fourier transfer function of:

$$p_1(x,y) = p(x,y)\exp\left[-j\pi\epsilon\frac{(x^2+y^2)}{\lambda}\right]$$

Generalized pupil function

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Single lens imaging system (wave optics)

In a high-quality imaging system, the impulse response is a narrow function extending over a very small range of x and y.

If the phase factor can be neglected (i.e., it's much smaller than 1 for all values of x and y) then the impulse function reduces to:

$$h(x,y) = h_0 P_1\left(\frac{x}{\lambda d_2}, \frac{y}{\lambda d_2}\right),$$

with $h_0 = h_1 h_2 = \frac{1}{\lambda d_1}\frac{1}{\lambda d_2}$.

If the system is also focused (ε=0) then

$$p_1(x,y) = p(x,y)$$

$$h(x,y) \approx h_0 P\left(\frac{x}{\lambda d_2}, \frac{y}{\lambda d_2}\right)$$

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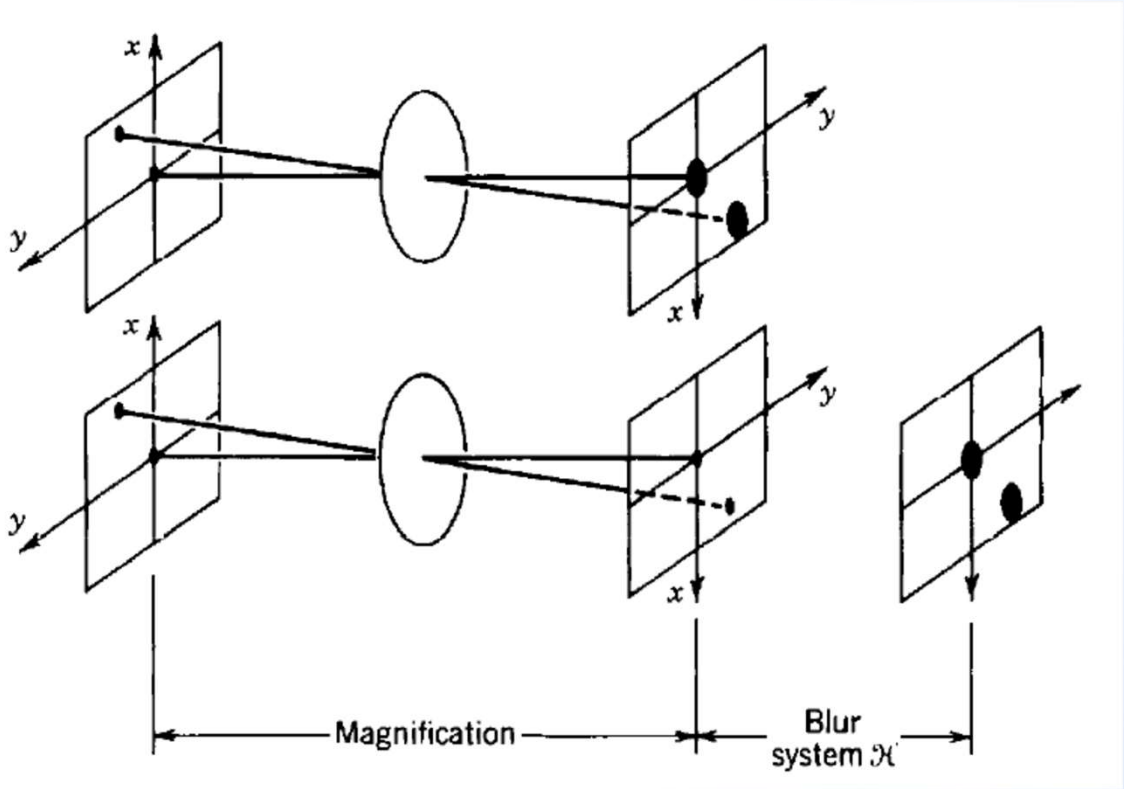
Single lens imaging system (wave optics)

Transfer function

The transfer function of a linear system can be only defined if the system is shift invariant. The single lens system is not shift invariant since a shift of a point in the object plane is accompanied by a different shift in the image plane.
The image will be magnified and blurred.



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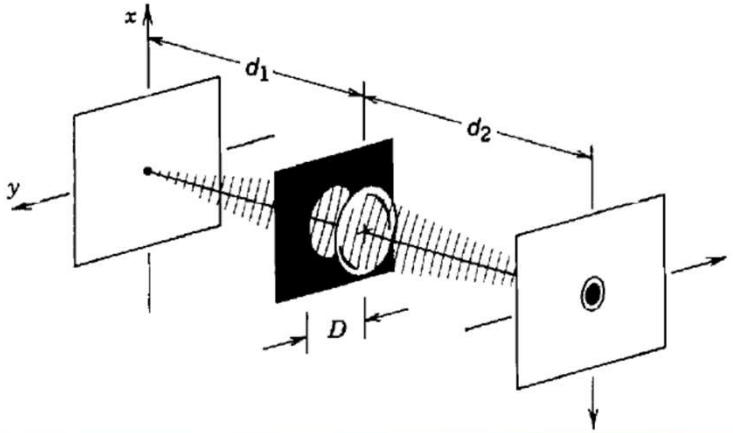
Single lens imaging system (wave optics)

While the transfer function of the single lens system is not shift invariant a magnification system and a blurring system are shift invariant for points near the optical axis, therefore one can split the system into two steps and calculate the overall transfer function.



The transfer function of the blurring system is:

$$H(\nu_x, \nu_y) \approx p_1(\lambda d_2 \nu_x, \lambda d_2 \nu_y),$$

Where p_1 is the generalized pupil function and we ignored the phase factors. If the system is focused, then

$$H(\nu_x, \nu_y) \approx p(\lambda d_2 \nu_x, \lambda d_2 \nu_y)$$

If the aperture is a circle of diameter D then the transfer function is constant on a circle of radius $\nu_s = \frac{D}{2\lambda d_2}$ and zero elsewhere.

The transfer function of a single lens system is identical to the transfer function of the 4-f imaging system.

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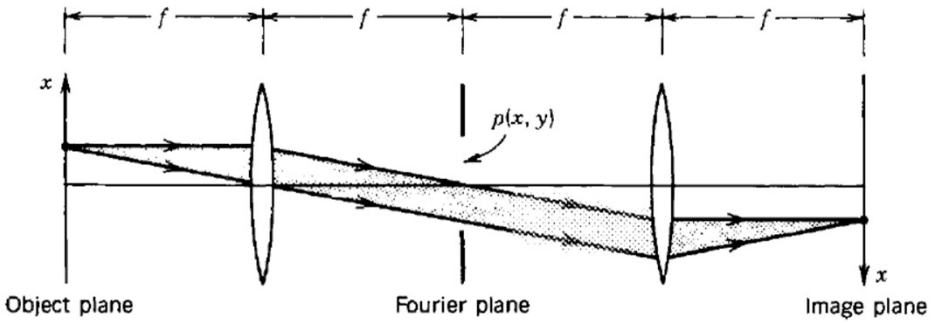
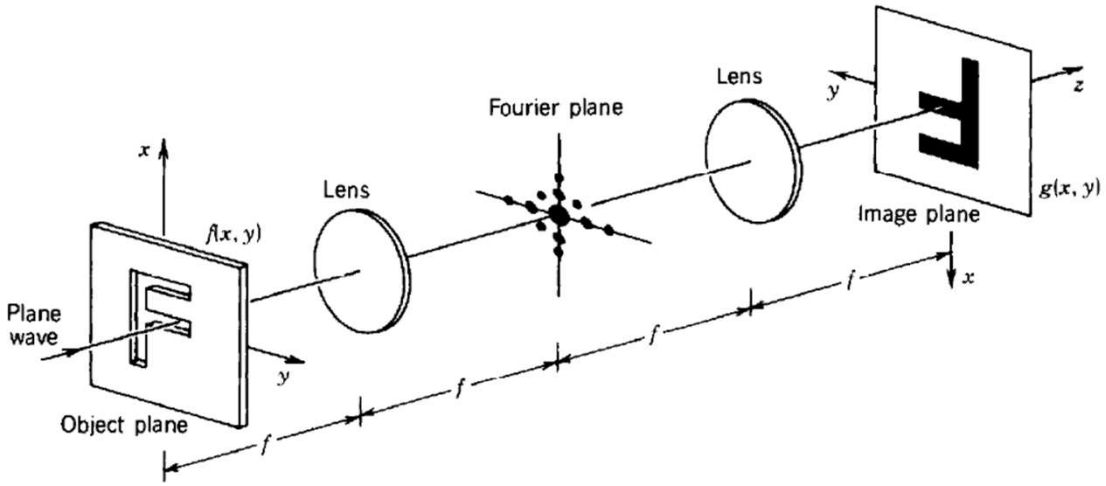
4-f imaging system (wave optics)

A 4-f imaging system is a two-lens system with unity magnification. The analysis of this system can be solved by considering the cascade of two Fourier transforming sub-systems. In the absence of an aperture the image is a perfect, inverted replica of the object.

If $f(x, y)$ is the complex amplitude transmittance of a transparency placed at the object plane and we illuminate the transparency with a plane wave traveling in the z direction $\exp(-jkz)$ we will find a complex amplitude $g(x, y)$ at the image plane.





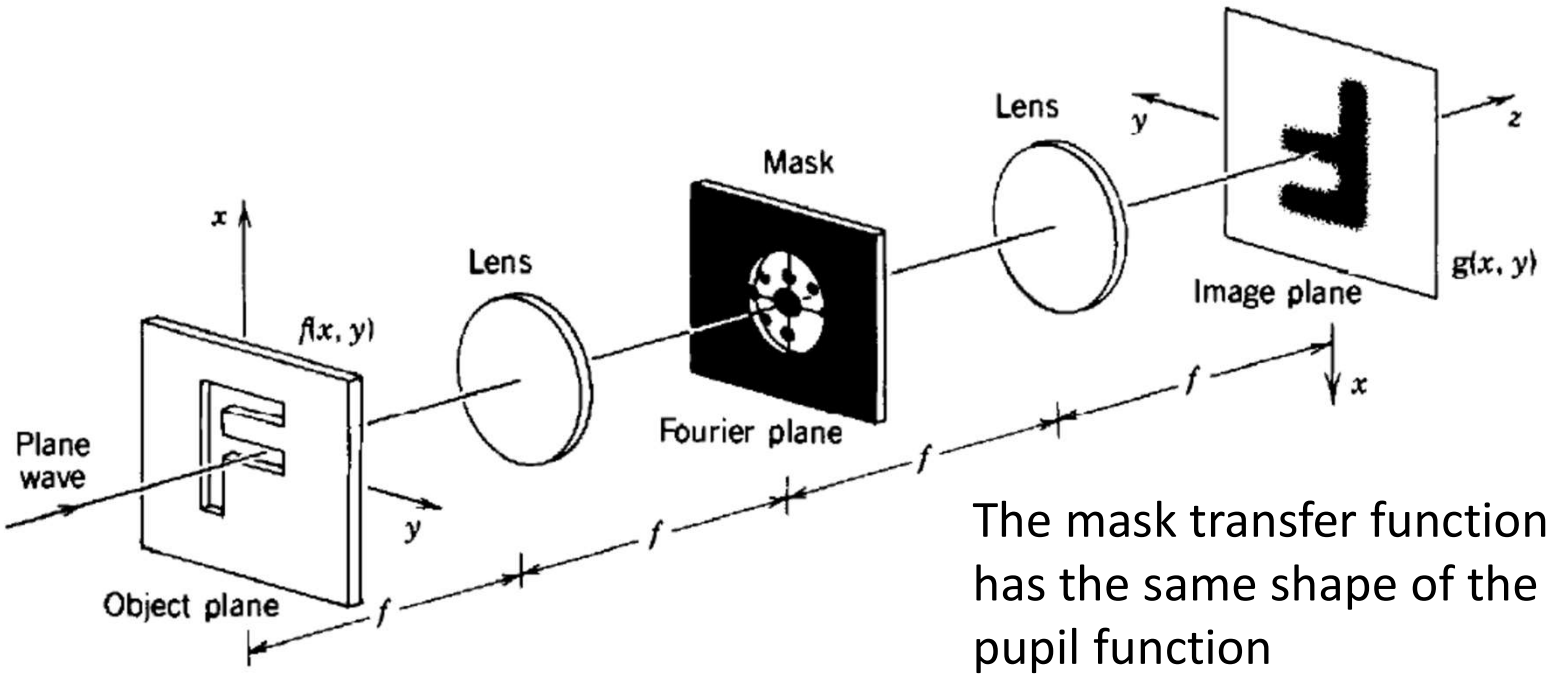
4-f imaging system (wave optics)

A 4-f imaging system can be used as a spatial filter in which the image $g(x, y)$ is a filtered version of the object $f(x, y)$. A mask can be used to adjust the wanted components selectively. The transfer function of the filter realized by a mask of transmittance $p(x, y)$ is:

$$H(\nu_x, \nu_y) = p(\lambda f \nu_x, \lambda f \nu_y)$$

So that:

$$G(\nu_x, \nu_y) = H(\nu_x, \nu_y) F(\nu_x, \nu_y)$$



The mask transfer function has the same shape of the pupil function

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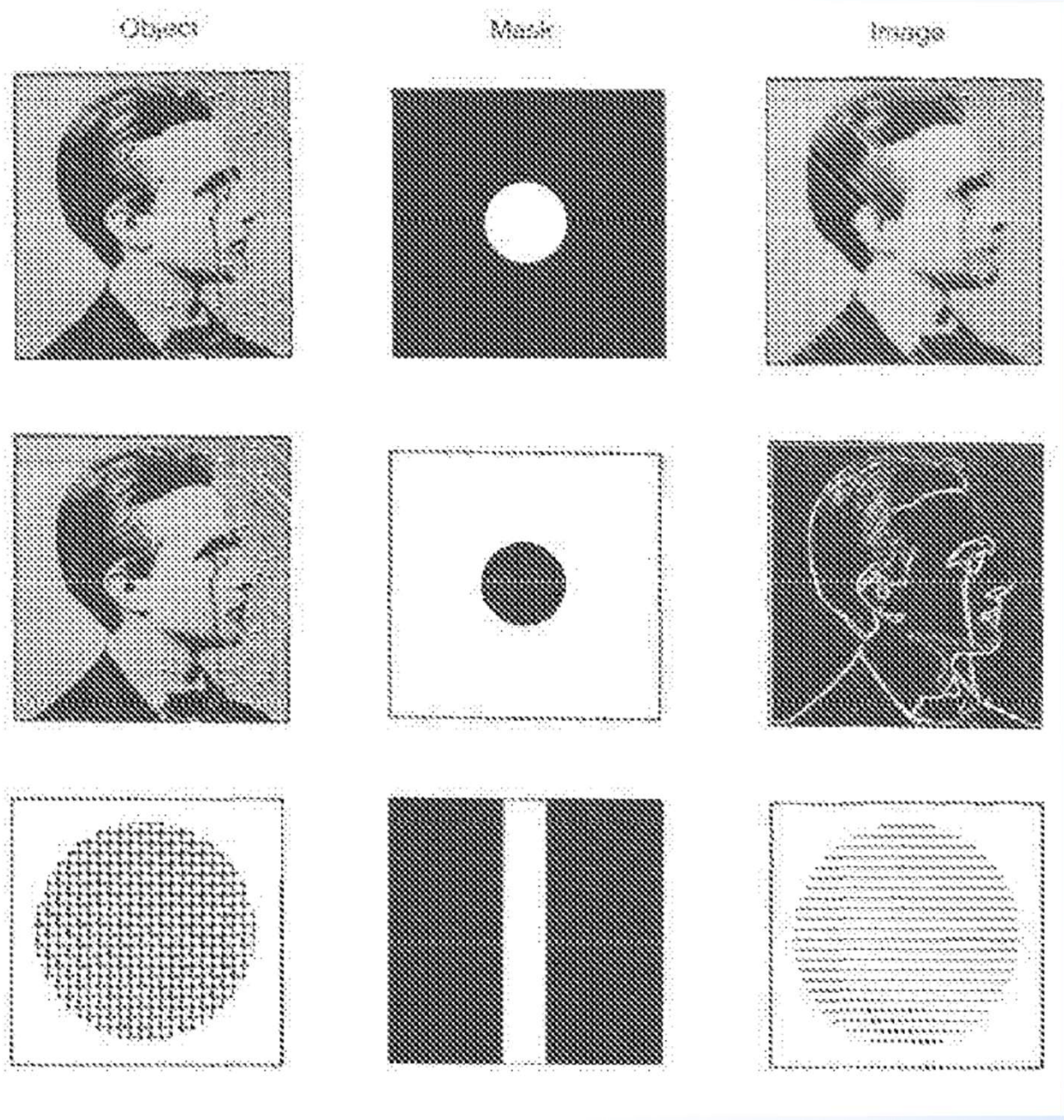
Spatial filter examples



Low-pass

High-pass

Vertical-pass

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Near field imaging

We showed that Fourier components of an object distribution with spatial frequencies λ^{-1} lead to evanescent waves, that decay rapidly. This means that object features smaller than the wavelength cannot be transmitted.

If a lens is focused at infinity, i.e. $d_2=f$ then the spatial bandwidth is:

$$\nu_s = \frac{1}{2\lambda F_{\#}}$$

Where $F_{\#}=f/D$ is the lens F number. ν_s is the highest spatial frequency that the imaging system can transmit.

If the system has a circular aperture and a certain $F_{\#}$ then the radius of its impulse response function is $1.22\lambda F_{\#}$. Points separated by distances smaller that this value cannot be discriminated. This is also referred to as the system **DIFFRACTION LIMIT.**

The diffraction limit can be circumvented if the imaging system is brought in the very vicinity of the object. At sub-wavelength distances evanescent waves have not decayed yet and can help to form the image.

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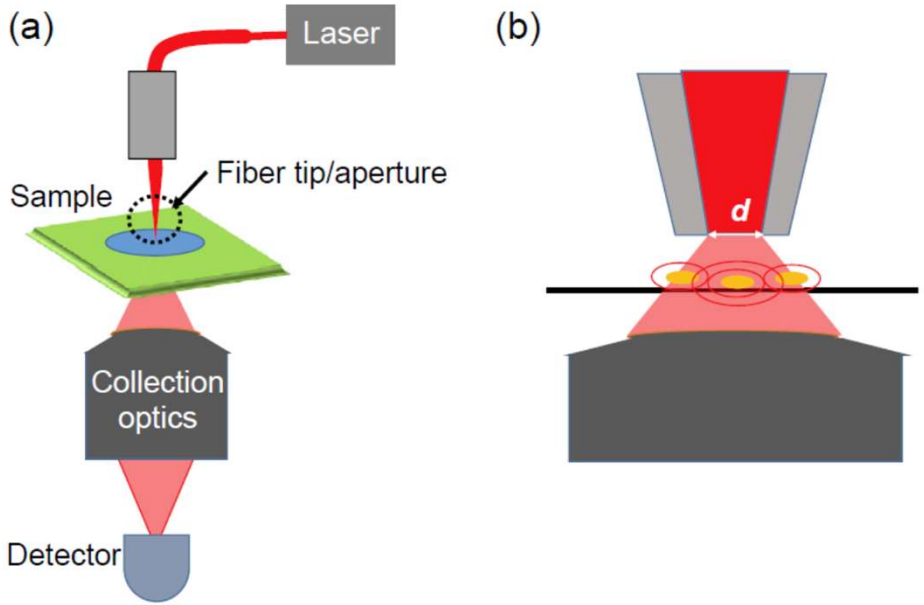
Near-Field Scanning optical microscopy

NSOM (also called SNOM) is an optical technique that achieves high resolution with **mechanical elements** that are similar to the AFM and STM. For instance, it has a tip that also scans the surface and maintains a constant height by using piezoelectric steering elements. A **laser source** illuminates the sample through an **optical fiber**. The end of the **fiber is tapered and coated with metal**; the light squeezes through a small, subwavelength aperture at the tip of the fiber. The NSOM fiber construction and electronics are similar to other SPM techniques, but with the metal tip replaced by a metal film-coated, tapered optical fiber. The fiber position is manipulated using **piezoelectric elements** attached to the fiber and the detector measures **variations in the scattered light** due to tiny features on the surface. The **small aperture** at the fiber tip typically has a diameter of approximately **50 nm**. The size of the aperture, which is much smaller than the wavelength, determines the ultimate resolution of the NSOM. **Subwavelength resolution** is maintained by keeping the fiber **tip sufficiently close to the surface to minimize diffractive beam spreading**. The scattered light transmitted through the sample is collected by a sensitive detector. The smallest aperture size is determined by a compromise between the desired resolution and the signal-to-noise ratio.



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Near-Field Scanning optical microscopy

NSOMs are used in several different operational modes. **Three modalities** are illustrated in the Figure.

(a) **Transmission mode** with light passing through the small aperture of a tapered fiber, and **scattered light** from the surface or near-surface volume irregularities is **collected in the far field** by optical means. For opaque samples, the reflected light can also be collected by designing collection optics above the sample.

(b) The **sample is illuminated from below at an angle that exceeds the total internal reflection**; in this modality, the scattered light is collected through the small aperture at the end of the tapered fiber. The fiber must be in the near field for location-sensitive detection of the scattered light.

(c) **The sample is illuminated from above** by light and a near-field tip collected the scattered light from local irregularities on the surface.