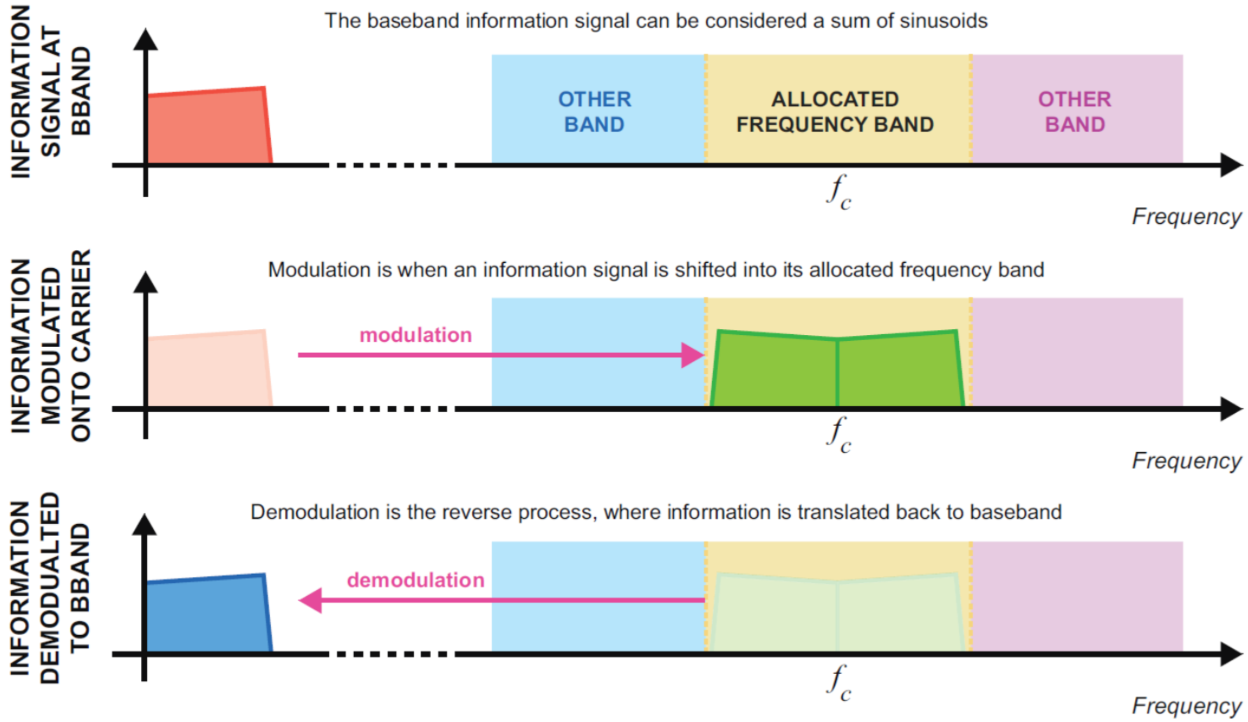
Digital Systems for Telecommunications

Software Defined Radio

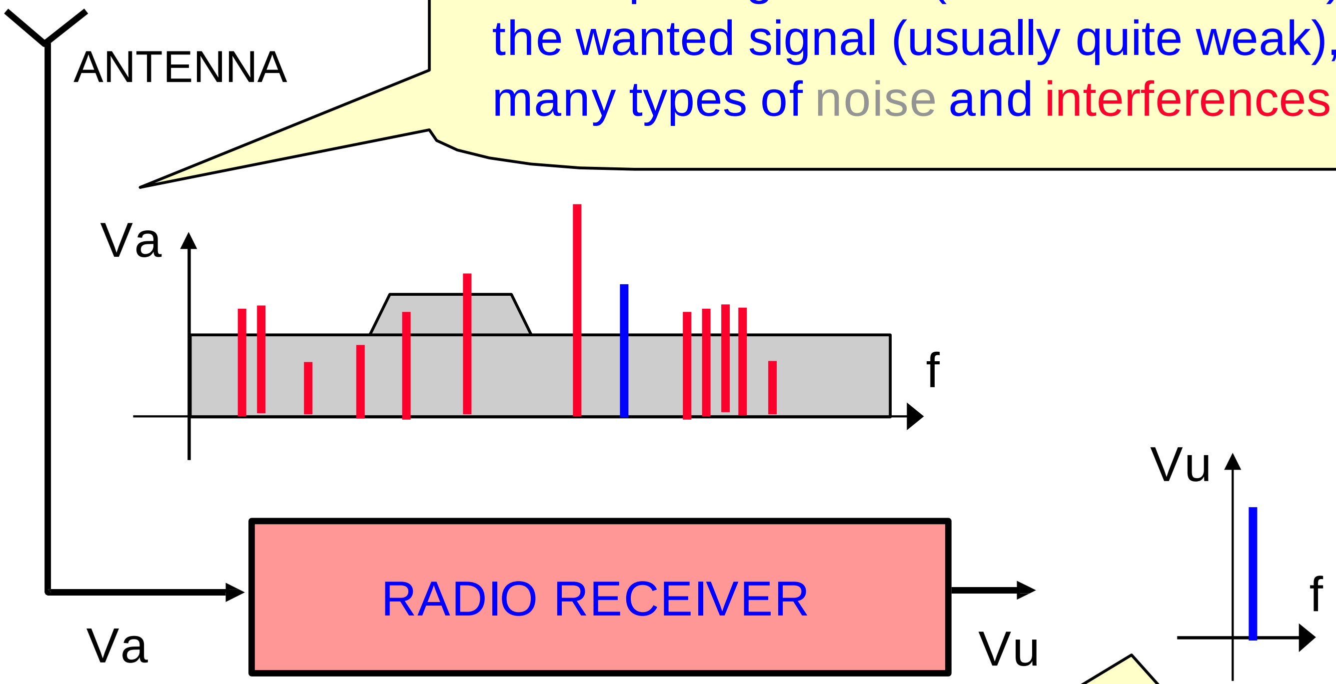
Our goal…

- Baseband information signals (which can be music, voice, or data) are *modulated* in RF transmitters to translate the baseband information into the allocated frequency band.
- The reverse process is carried out by RF receivers, and is referred to as demodulation.



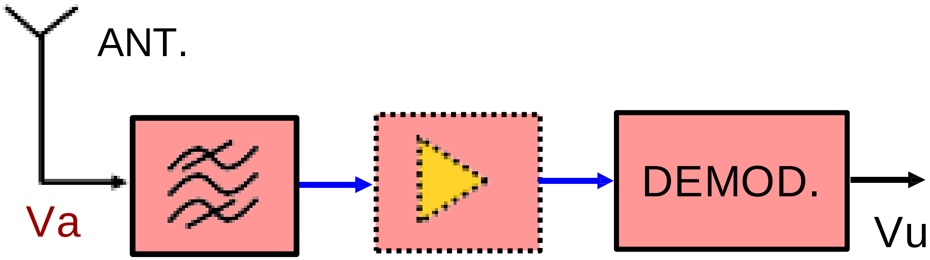
Our goal…

The input signal Va (from the antenna) contains the wanted signal (usually quite weak), plus many types of noise and interferences.

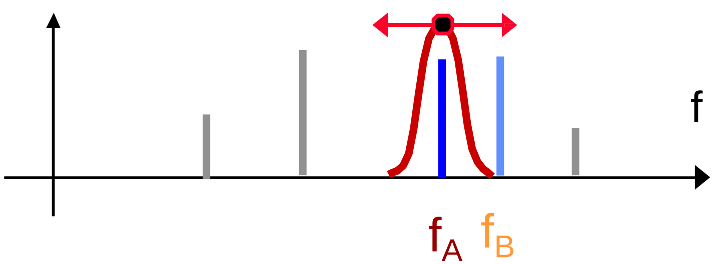


The receiver ouput signal Vu should be a copy of the original modulation, without noise and/or distortion.

Elementary receiver



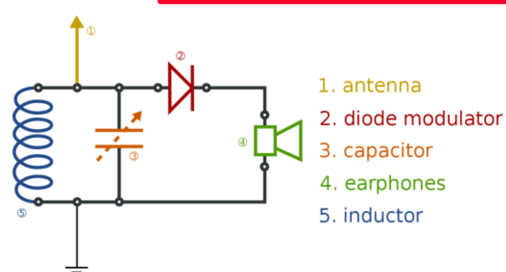
Input filter
(narrowband,
variable F)



Filter, amplifier, and demodulator must operate at variable frequency.

Tuning shifts the resonant frequency f_A of the filter
(e.g. from f_A to f_B)

- Example: crystal radio



Can we do it better?

- In the crystal radio the demodulator operates at RF…
- Generally it is cumbersome to effectively process signals at high frequency… how to move towards lower frequencies?

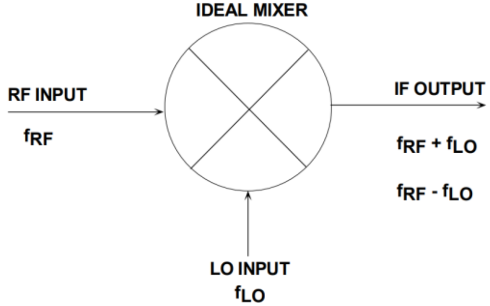
FT pair

$$\Im\{\cos(2\pi At)\} = \int_{-\infty}^{\infty} \frac{e^{i2\pi At}+e^{-i2\pi At}}{2} e^{-i2\pi ft}\,dt$$
$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{i2\pi At} e^{-i2\pi ft}\,dt + \int_{-\infty}^{\infty} e^{-i2\pi At} e^{-i2\pi ft}\,dt\right]$$
$$= \frac{1}{2}\left[\delta(f-A)+\delta(f+A)\right]$$

FT Mixing property

$$F^{-1}[V(f)*W(f)] = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} V(\lambda)\cdot W(f-\lambda)\cdot d\lambda\right]\cdot e^{j\cdot 2\pi\cdot f\cdot t}\cdot df$$
$$= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} W(f-\lambda)\cdot e^{j\cdot 2\pi\cdot f\cdot t}\cdot df\right]\cdot V(\lambda)\cdot d\lambda = \int_{-\infty}^{\infty}\left[w(t)\cdot e^{j\cdot 2\pi\cdot f\cdot \lambda}\right]\cdot v(\lambda)\cdot d\lambda$$
$$= w(t)\cdot \int_{-\infty}^{\infty} V(\lambda)\cdot e^{j\cdot 2\pi\cdot f\cdot \lambda}\cdot d\lambda = v(t)\cdot w(t)$$

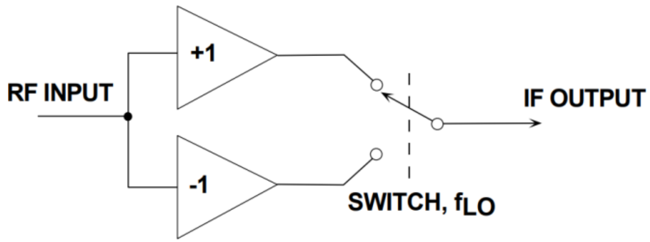
- A mixer takes an RF input signal at a frequency f_{RF}, mixes it with a LO signal at a frequency f_{LO}, and produces an IF output signal that consists of the sum and difference frequencies, $f_{RF} \pm f_{LO}$



It's a multiplier!

The easiest mixer

- For narrowband signal, the mixer can be reduced in a device splitting the RF signal into in-phase (0°) and anti-phase (180°) components;
- Now the product is with a square wave at f_{LO}
  - Higher order harmonics are present
  - Consider the Fourier series $S_{LO} = {}^4/_\pi \{ \sin\omega_{LO}t + {}^1/_3 \sin 3\omega_{LO}t + {}^1/_5 \sin 5\omega_{LO}t + \ldots \}$



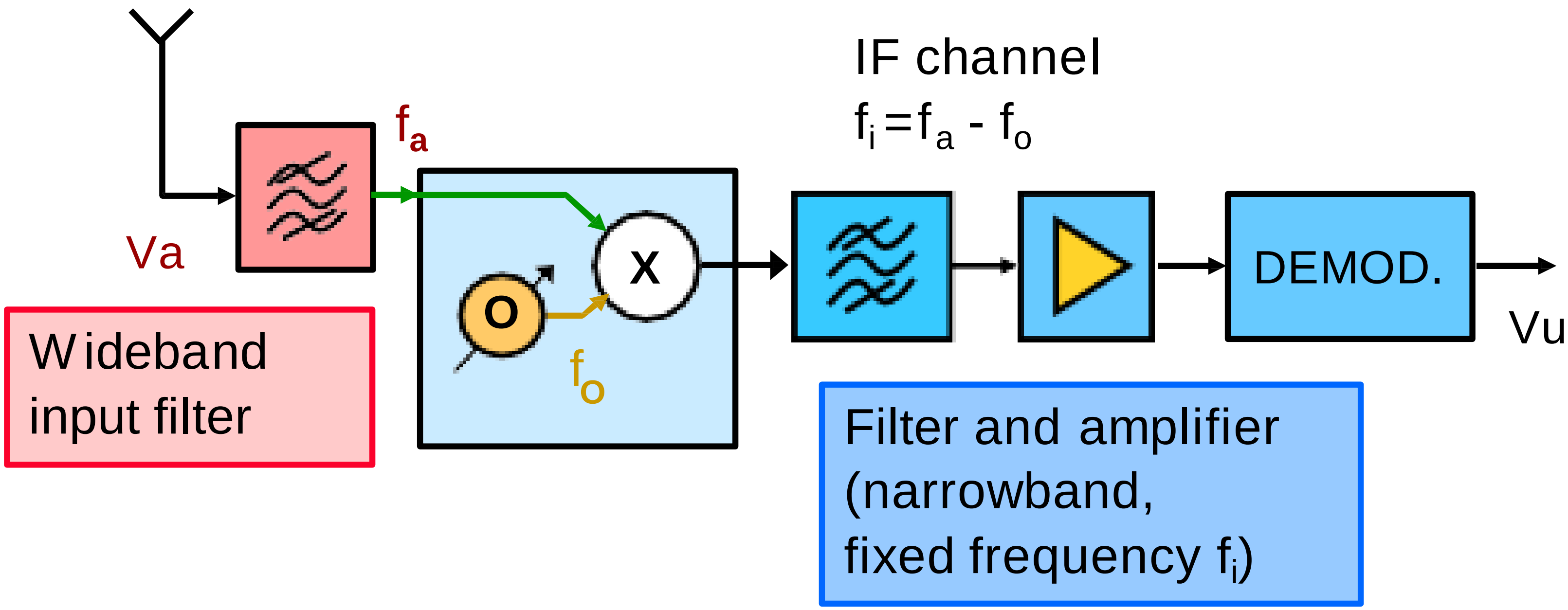
- Thus the output becomes:

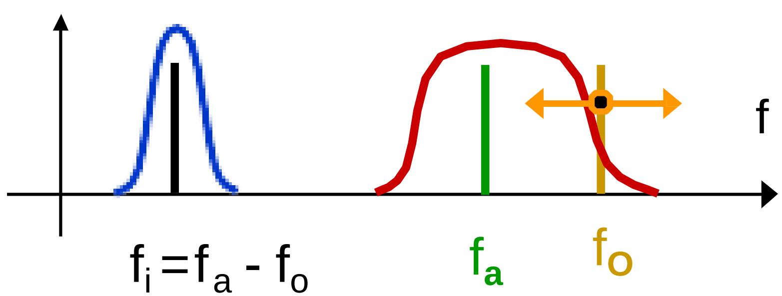
$$S_{IF} = {}^4/_\pi \{ \sin\omega_{RF}t \sin\omega_{LO}t + {}^1/_3 \sin\omega_{RF}t \sin 3\omega_{LO}t + {}^1/_5 \sin 5\omega_{RF}t \sin 5\omega_{LO}t + \ldots \}$$

$$S_{IF} = {}^2/_\pi \{ \sin(\omega_{RF} + \omega_{LO})t + \sin(\omega_{RF} - \omega_{LO})t + {}^1/_3 \sin(\omega_{RF} + 3\omega_{LO})t + {}^1/_3 \sin(\omega_{RF} - 3\omega_{LO})t + {}^1/_5 \sin(\omega_{RF} + 5\omega_{LO})t + {}^1/_5 \sin(\omega_{RF} - 5\,\omega_{LO})t + \ldots \}$$

harmonics

The heterodyne receiver





The input signal is shifted to a fixed frequency $f_i = f_a - f_o$. Tuning shifts the frequency f_o of the local oscillator

Benefits of heterodyne receivers

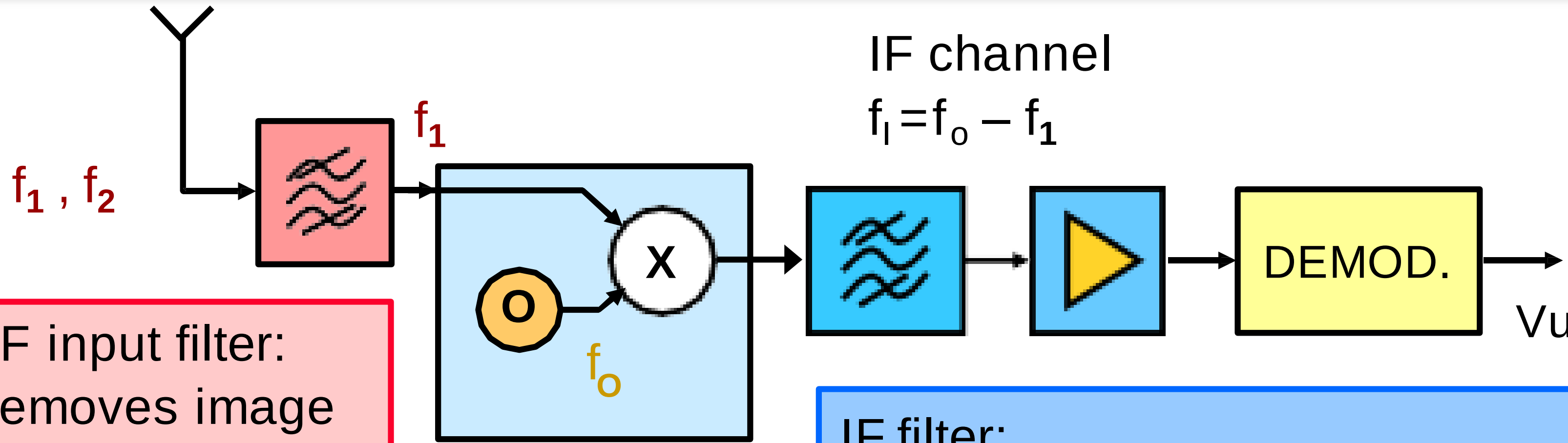
- Channel isolation achieved by fixed-frequency IF filter
  - No need for tunable narrowband filter

- Tuning achieved by shifting the LO frequency
  - Possible to cover wide frequency range

- Amplifiers and demodulator operate at fixed IF
  - Narrowband circuits more easy to design and test

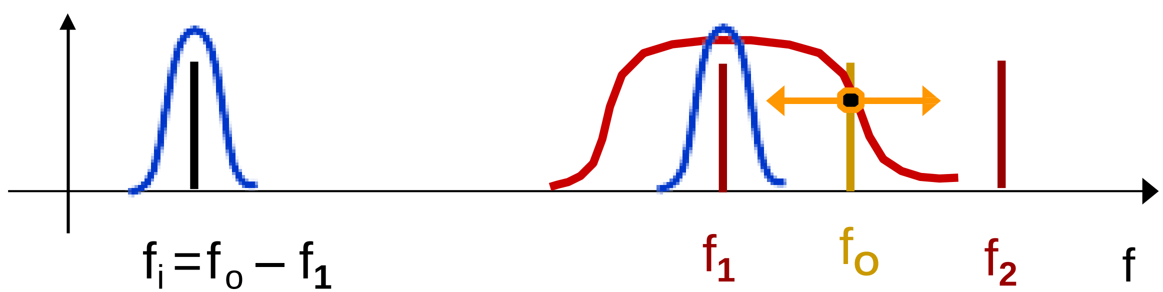
- But …
  - Fi = Fo – Fa1 or Fa2 – Fo
  - both Fa1 and Fa2 enter the IF chain: image frequency problem

Filters in the heterodyne receiver



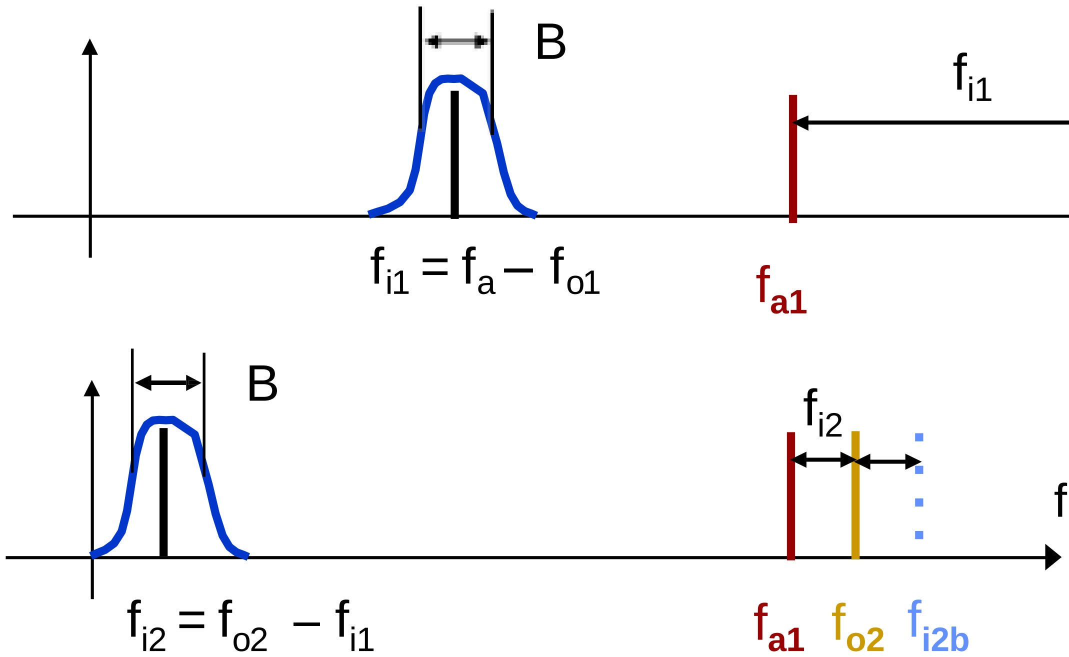
RF input filter:
Removes image frequency f_2

IF filter:
Removes adjacent channels (narrowband, fixed frequency f_i)



Same effect as narrowband RF filter

Problems with high IF frequency

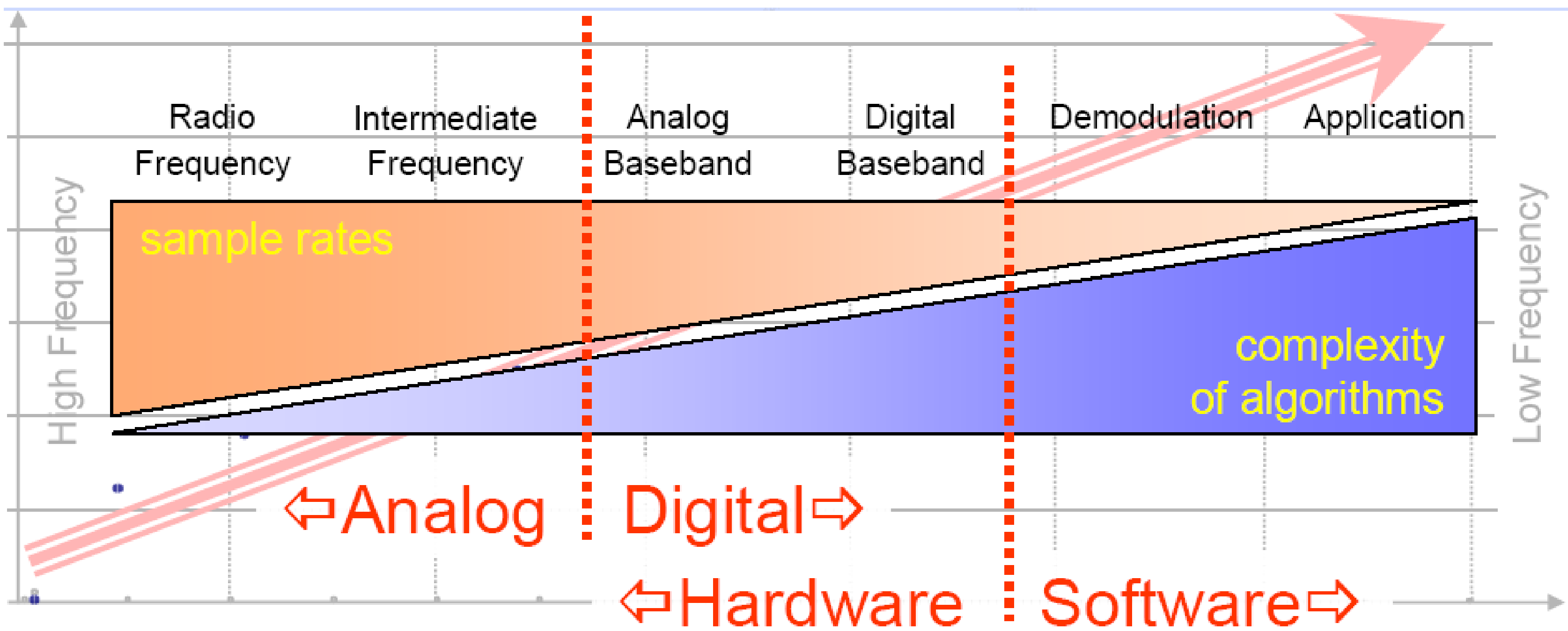


Q = f/B
With high IF, a given bandwidth B needs high Q (for IF filter)

The IF channel is moved to a lower frequency with a second beat: from fi1 to fi2. Since fi2 is lower, the same bandwidth B can be achieved with lower Q

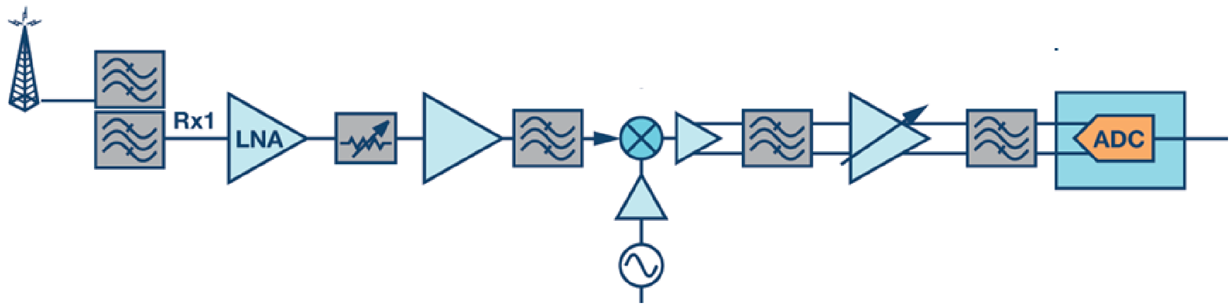
Where does «digital» start?

- Digital radios: from Radio Frequency to Baseband:
  - Where to put A/D Converters?
  - How to split Hardware and Software?



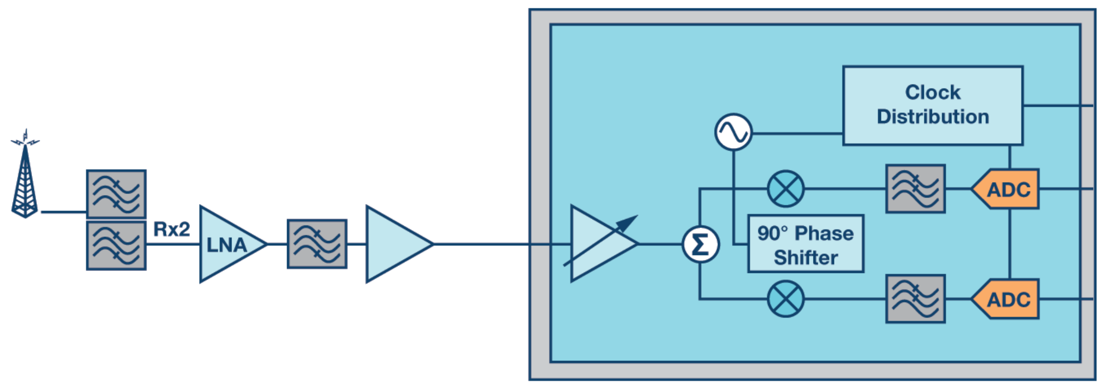
Architectures

- In a traditional IF sampling receiver there are 4 basic stages: low noise gain and RF selectivity, frequency translation, IF gain and selectivity, and detection: it is hard to integrate all of them in a single IC!



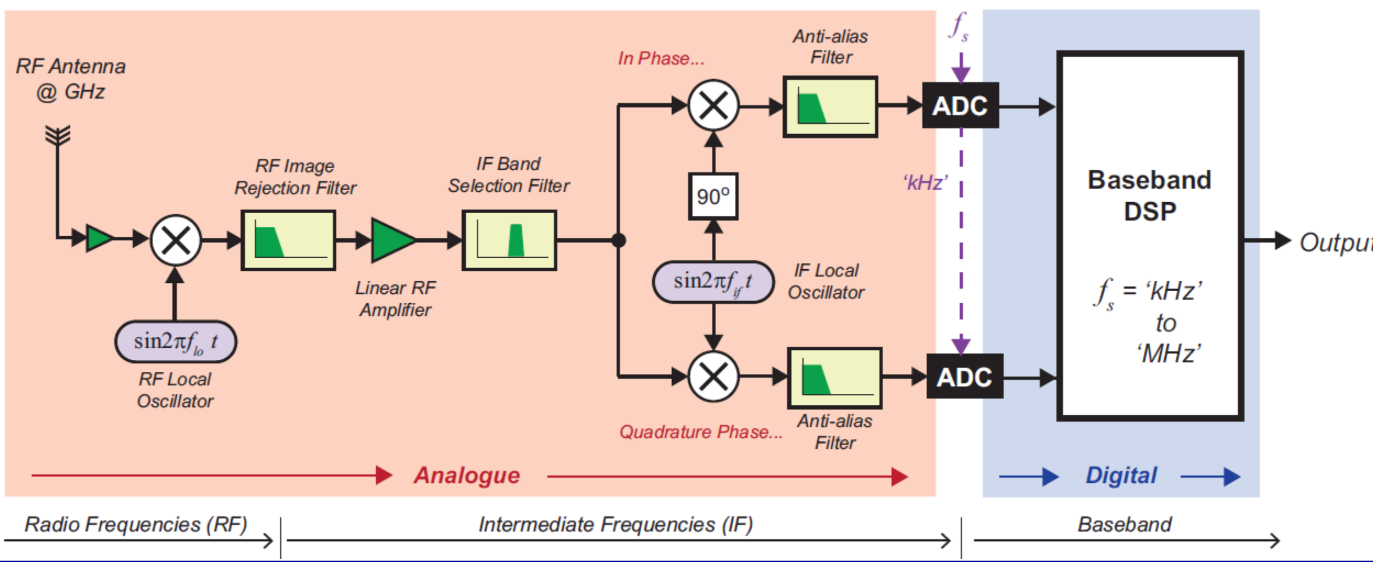
- In a typical zero-IF signal chain the RF signal is translated directly to (a complex) baseband, completely eliminating the need for an IF filter and IF amplifiers.

Zero-IF places extreme demands on both the LO performance and the mixer performance. Any instabilities or imperfections in the LO, and any DC offsets or imperfections in the mixer, quickly corrupt the resulting zero-IF output and so result in poor system performance.



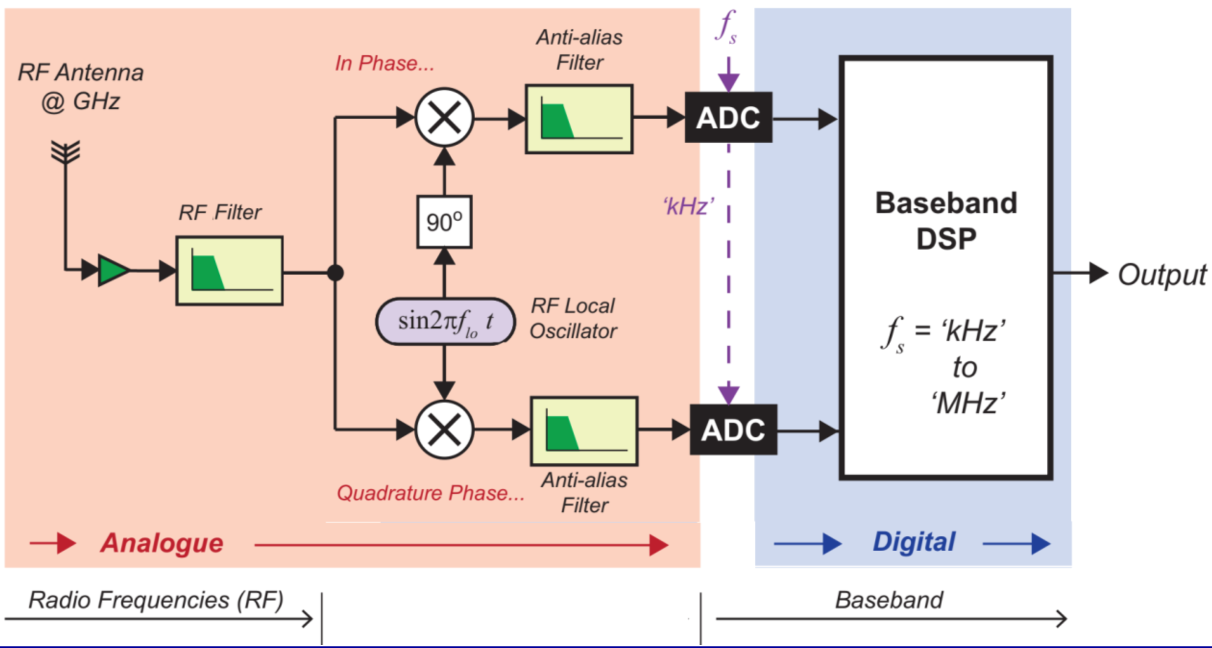
Real world digital radio architecture

- First generation "digital radios" appeared back in the mid–1990s
  - Analogue part of this radio architecture downconverted signals from their RFs to an IF using a Local Oscillator (LO), and then, using a second LO, further downconverted the IF signal to baseband.
  - Baseband signal was then sampled and digitised using an ADC (≈ 10-100kHz), and then DSP was used to perform the final processing stages to recover the transmitted information.
  - 2G mobile phones of the 1990s have this architecture



Real world digital radio architecture

- “Zero IF” of digital radios emerged in the 2000s
  - sampling and digitization is performed IFs (e.g., IFs of around 40MHz could be supported by an ADC that sampled at, say, 125MHz).
  - Zero-IF places extreme demands on both the LO performance and the mixer performance. Any instabilities or imperfections in the LO, and any DC offsets or imperfections in the mixer, quickly corrupt the resulting zero-IF output and so result in poor system performance.



Some terms…

IEEE Project 1900.1 - Standard Definitions and Concepts for Dynamic Spectrum Access: Terminology Relating to Emerging Wireless Networks, System Functionality, and Spectrum Management

https://standards.ieee.org/develop/project/1900.1.html.

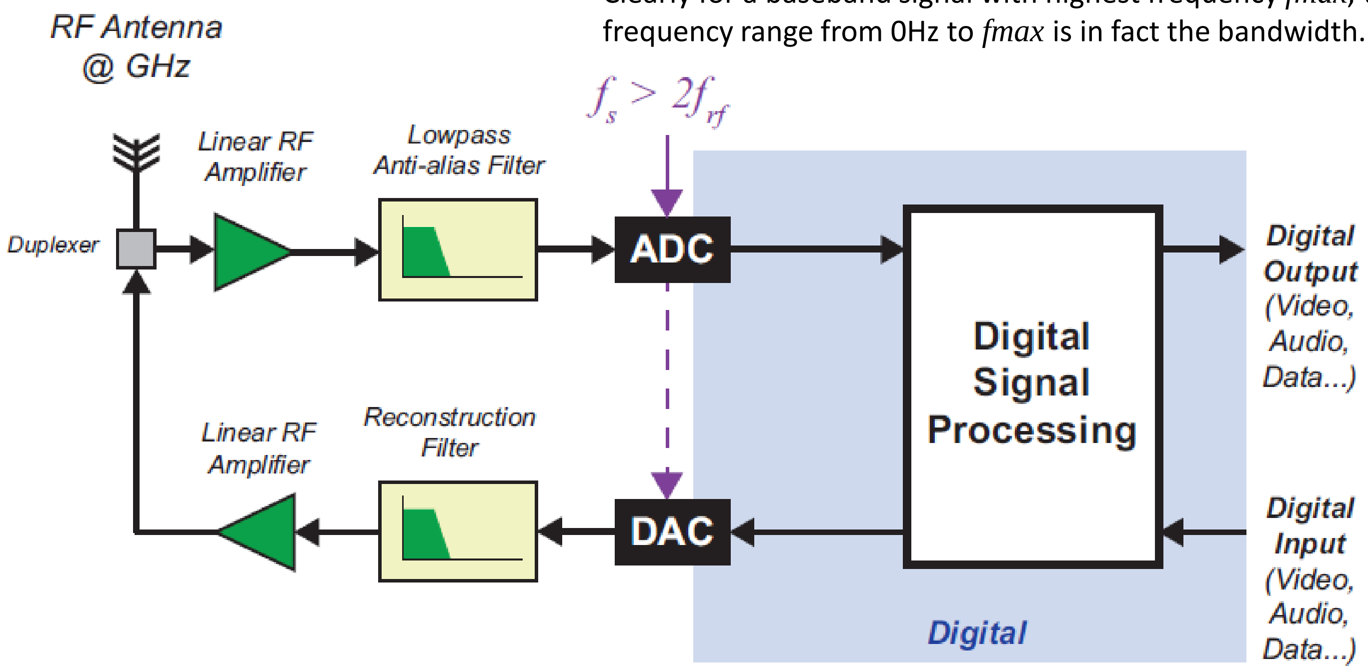
Control vs Define

- Software Controlled: refers to the use of software processing within the radio system or device to select the parameters of operation.
- Software Controlled Radio: Radio in which some or all of the physical layer functions are software controlled.
- **Software Defined**: refers to the use of software processing within the radio system or device to implement operating (but not control) functions.
- **Software-Defined Radio (SDR)**: Radio in which some or all of the physical layer functions are software defined.

The ultimate SDR architecture

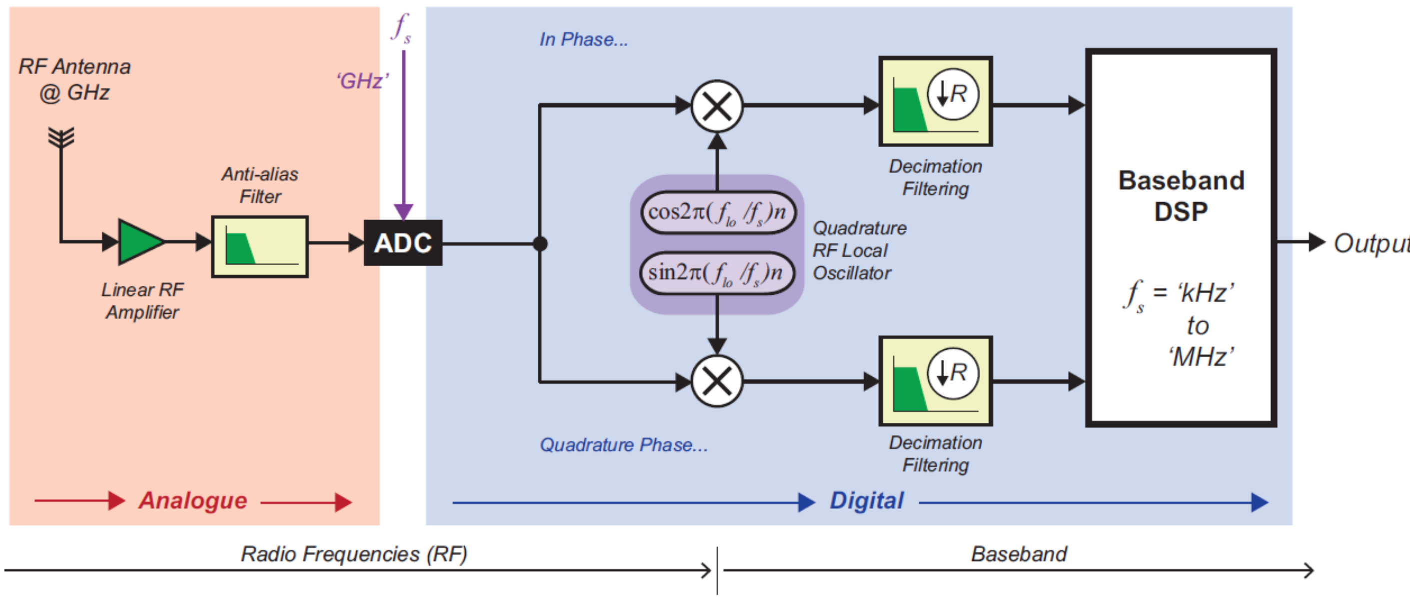
- At its very simplest conceptual level, SDR comprises of an RF section (antenna, amplifiers and filters) and a very high-speed Analogue-to-Digital Converter (ADC) and Digital-to-Analogue Converter (DAC) pair, interfaced with a powerful DSP processor and/ or computing system.

The Nyquist sampling theorem requires *twice the signal bandwidth*. Clearly for a baseband signal with highest frequency *fmax*, the frequency range from 0Hz to *fmax* is in fact the bandwidth.



Ideal digital radio architecture

- Ultimately the move has been made to sample RF signals directly, and downconvert them from RF frequencies to baseband in a single stage, using DSP
  - a digital down-converter (DDC) converts a digitized, band-limited signal to a lower frequency signal at a lower sampling rate in order to simplify the subsequent radio stages



- This is a shift towards an "universal" receiver architecture!

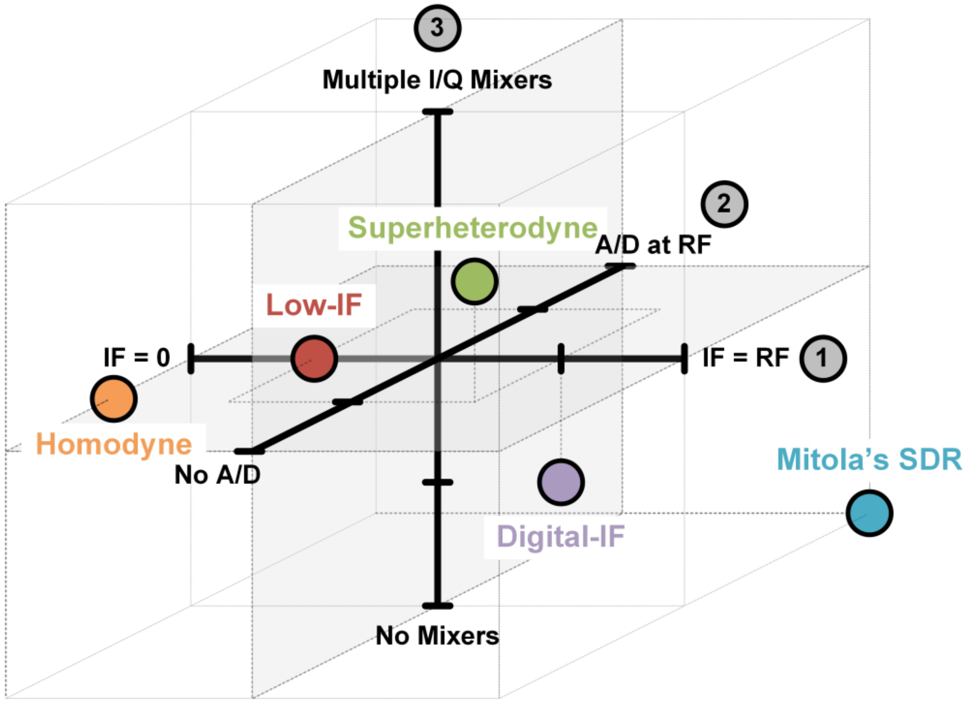
SDR Levels

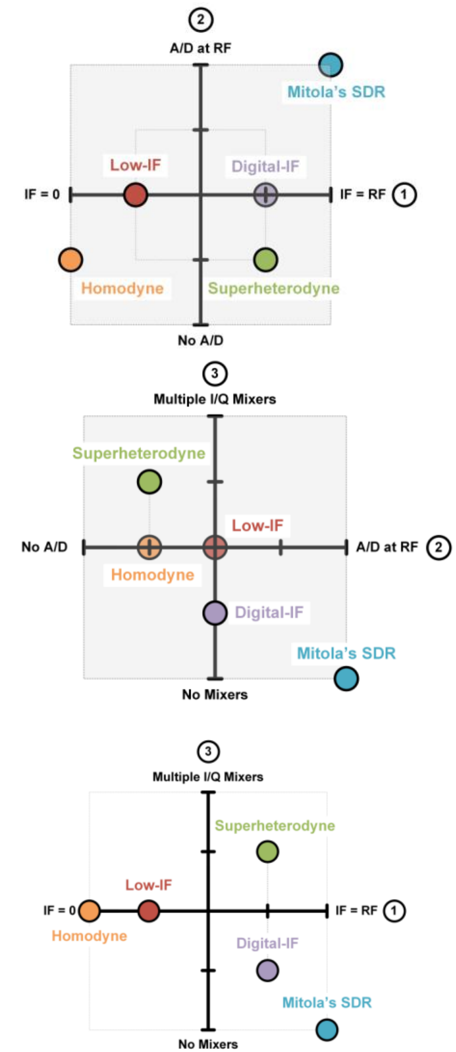
<source:http://www.sdrforum.org>

Tier	Name	Description
Tier 0	Hardware Radio (HR)	Implemented using hardware components. Cannot be modified
Tier 1	Software Controlled Radio (SCR)	Only control functions are implemented in software: inter-connects, power levels, etc.
Tier 2	**Software Defined Radio (SDR)**	**Software control of a variety of modulation techniques, wide-band or narrow-band operation, security functions, etc.**
Tier 3	Ideal Software Radio (ISR)	Programmability extends to the entire system with analog conversion only at the antenna.
Tier 4	Ultimate Software (Cognitive) Radio (USR)	Not only does this form of software defined radio have full programmability, but it is also able to support a broad range of functions and frequencies at the same time.

Architectures

- Resume

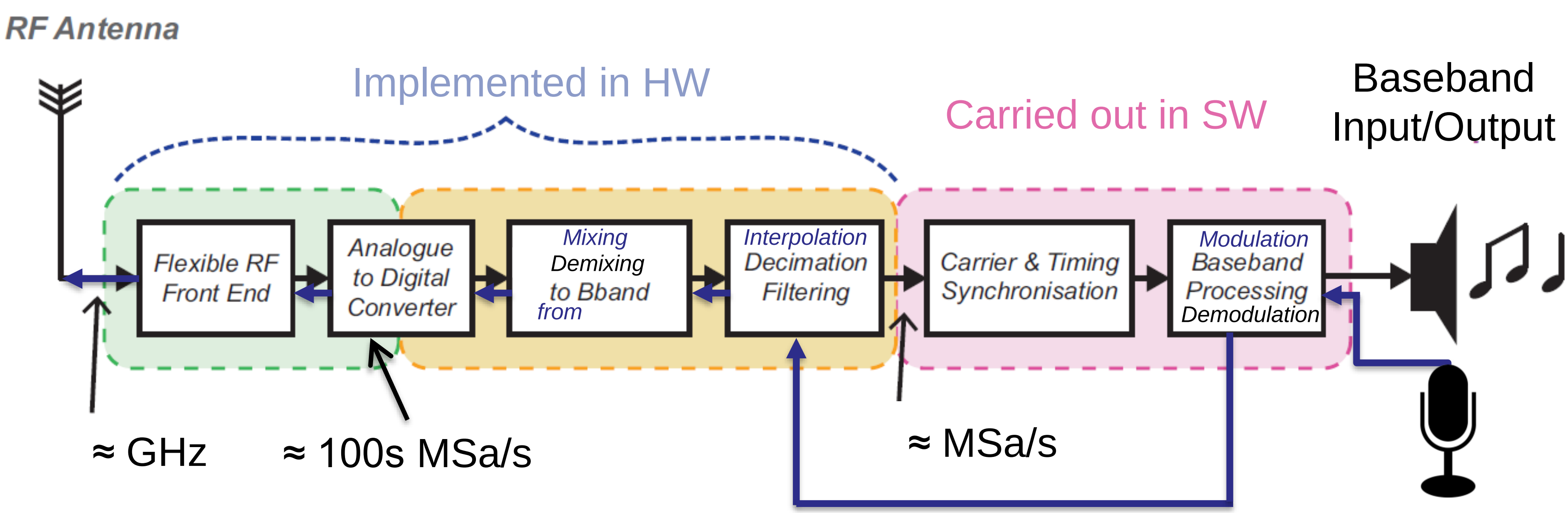




Bronckers, L. A., Roc'h, A., & Smolders, A. B. (2017). Wireless receiver architectures towards 5G: where are we? IEEE Circuits and Systems Magazine, 17(3), 6-16. https://doi.org/10.1109/MCAS.2017.2713306

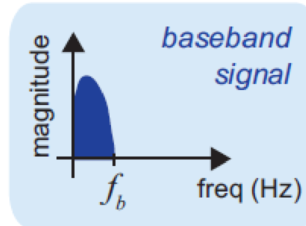
Where the HW actually meets the SW today

- In a real-world SDR, RF signals are:
  - received at the antenna,
  - quadrature up/downconverted by the HW,
  - and In Phase/ Quadrature Phase samples are processed by SW



Real world SDR and I/Q paths

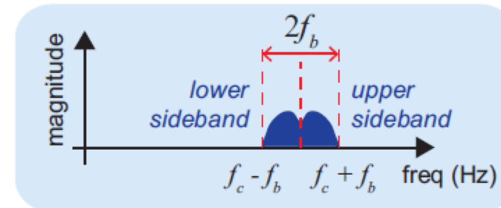
- To transmit a baseband signal of bandwidth fb Hz, using simple AM modulation, we require 2fb Hz.



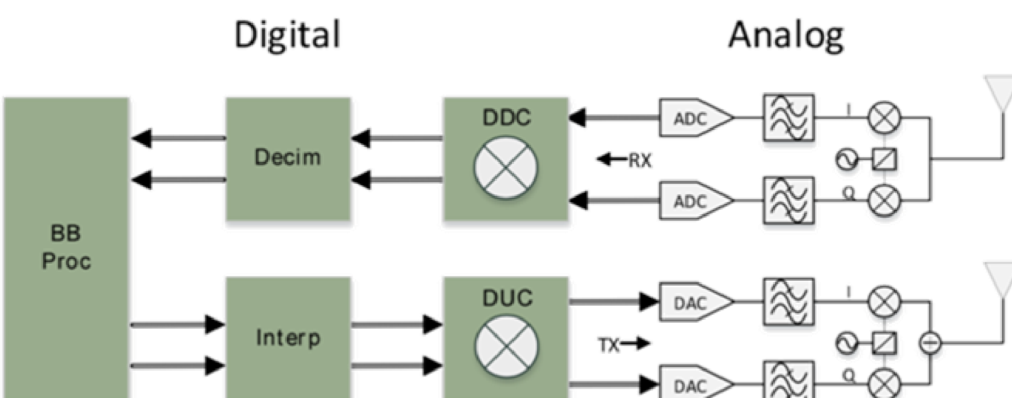
(bandwidth = f_b Hz)

Modulated Signal

(bandwidth = $2f_b$ Hz)



- More bandwidth-efficient signaling exploits quadrature (de)modulation, where we transmit two signals of bandwidth fb Hz
  - We use the same carrier frequency, but we separate the carrier phases by 90° (i.e. quadrature carrier), without causing interference to each other.



✓ "Zero IF" architecture: sampling and digitization is performed at baseband.

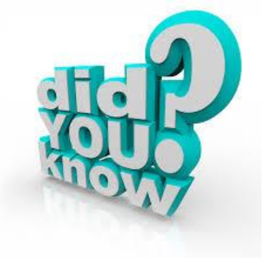
✓ Direct Digital Down/Up-converter (DDC/DUC) is used to shift "signal portion" of interest to/from baseband using (de)modulation and decimation (interpolation) filtering.

Digital Systems
for
Signal Processing

Quadrature Amplitude Modulation

Knowledge test…

Time Domain				Frequency Domain	
Continuous	x (t)	FT → / ← IFT	x^(ω)	Continuous	
Continuous periodic	x (t)	FS → / ← IFS	X [n]	Discrete	
Discrete	x [n]	DTFT → / ← IDTFT	x^(ω)	Continuous periodic	
Discrete Finite Support	x [n]	DFT → / ← IDFT	X [k]	Discrete Finite Support	



Narrowband signal model

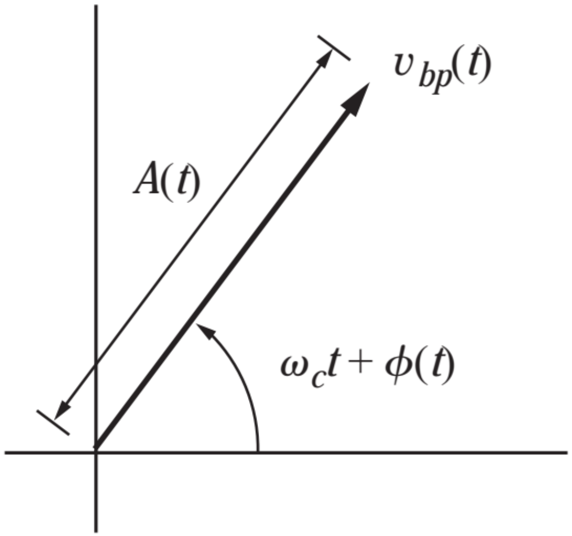
- If we consider the generic signal: $x(t) = A\cos(2\pi fc \cdot t + \varphi)$, exploiting the identity $\cos(a + b) = \cos(a)\cos(b) - \sin(b)\sin(a)$ it is possible to express it as:

$$x(t) = [A\cos(\varphi)] \cdot \cos(2\pi fc \cdot t) - [A\sin(\varphi)] \cdot \sin(2\pi fc \cdot t)$$

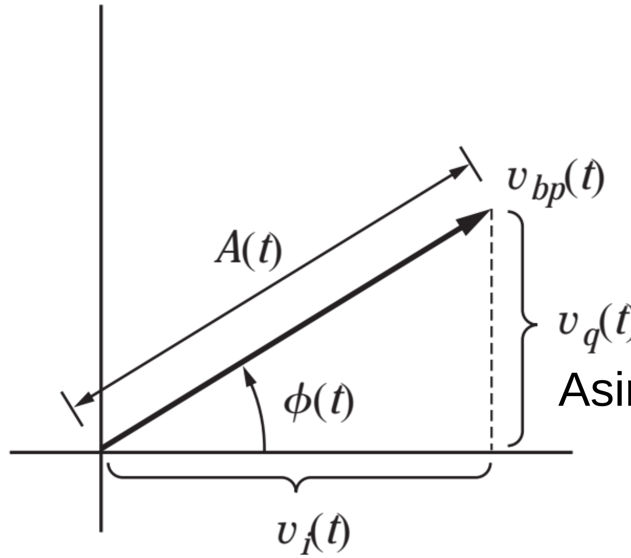
- This is known as the quadrature or IQ form, where the first term is named in-phase (I) and the second term is the quadrature (Q). In this case, I and Q are orthogonal.
- If A(t) and φ(t) are slowly varying functions compared to 2πfc · t, the assumption of I and Q orthogonality is also considered. Authors often call it a narrowband assumption, or a narrowband signal model.
- I and Q comprise a high-frequency sinusoid (or carrier) that is amplitude-modulated by a relatively low-frequency baseband function, usually conveying some sort of information: QAM or quadrature amplitude modulation.

Phasor interpretation

- The quadrature-carrier designation comes about from the fact that the two terms may be represented by phasors with the second at an angle of +90° compared to the first.



Envelope and phase



Asin(φ)

Acos(φ)

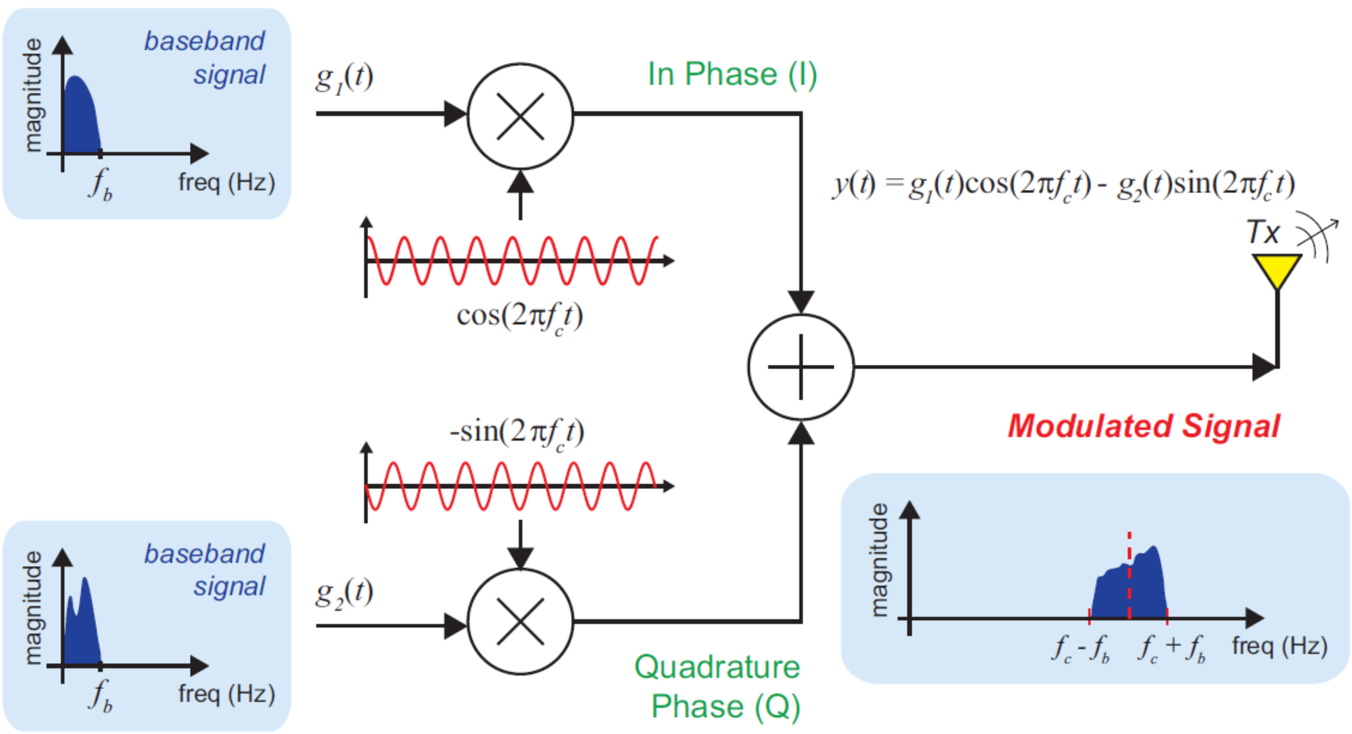
Quadrature carriers

$$A = \sqrt{I^2 + Q^2}$$

$$\varphi = \tan^{-1}\left(\frac{Q}{I}\right)$$

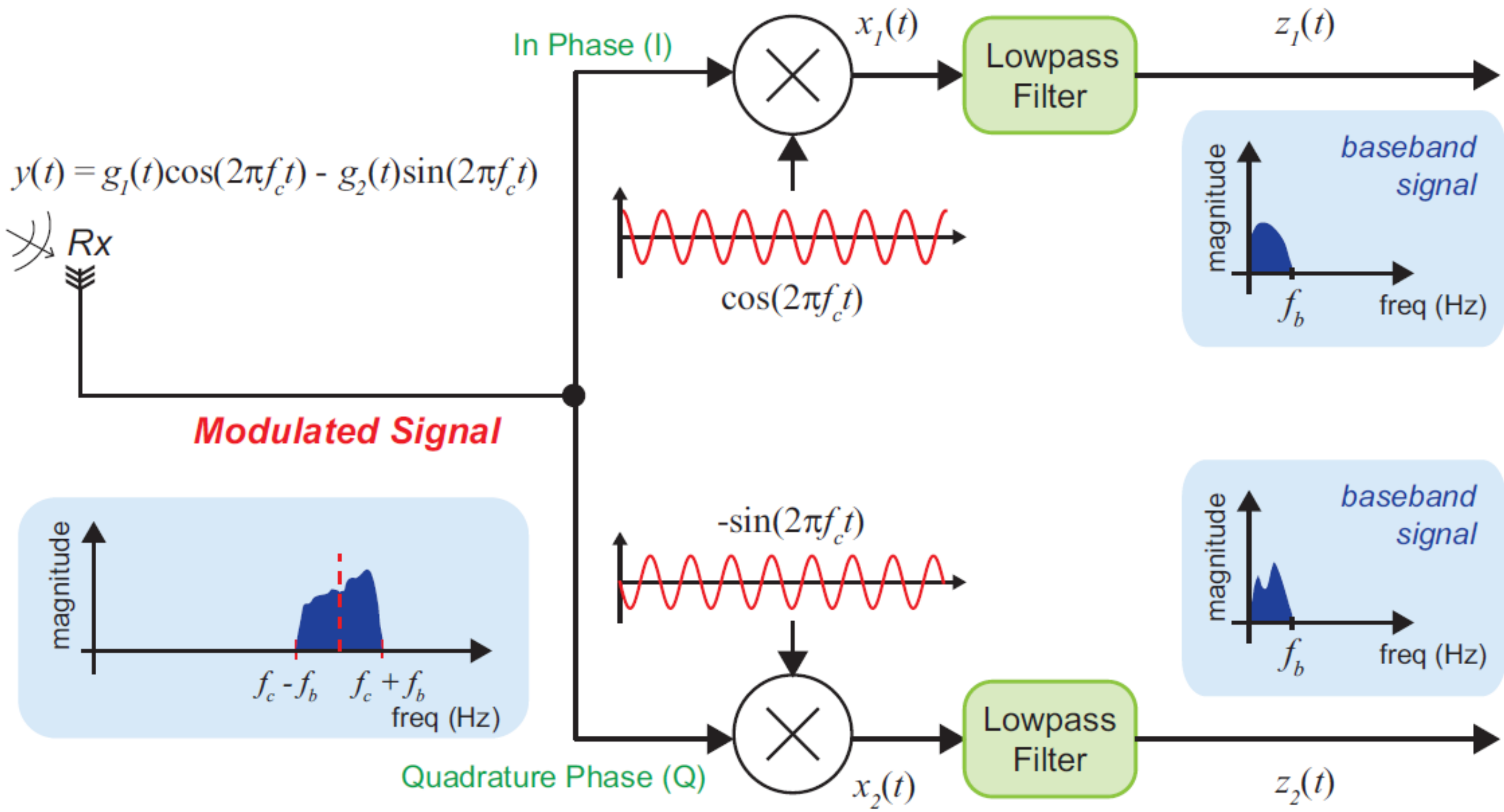
Quadrature Modulation and Demodulation (QAM)

- Suppose we have two independent baseband signals: $g_1(t)$ and $g_2(t)$. If we have now quadrature modulated our baseband signals the transmitter will produce: $y(t) = g_1(t)\cos(2\pi fct) – g_2(t)\sin(2\pi fct)$



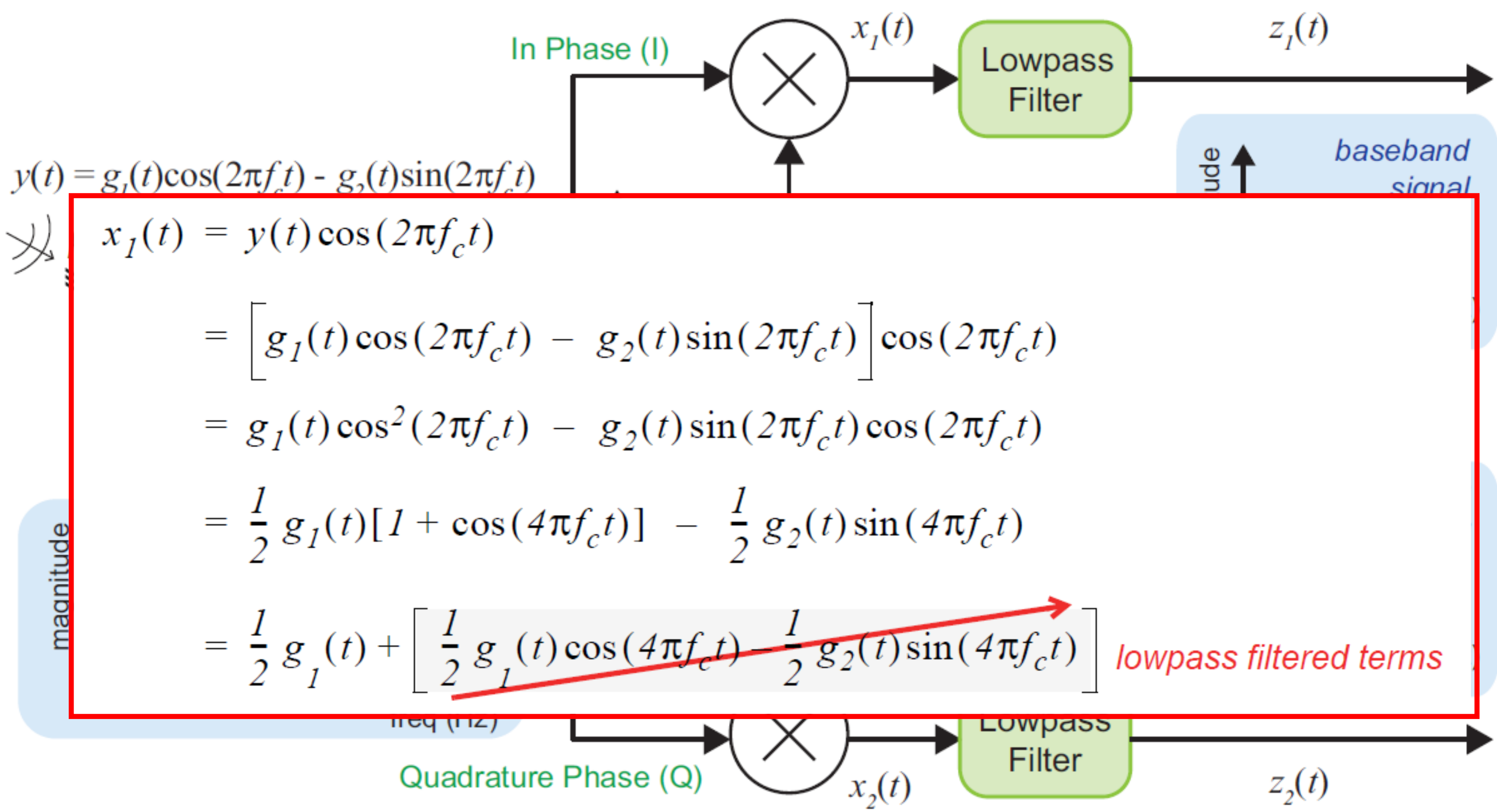
Quadrature Modulation and Demodulation (QAM)

- Can we quadrature demodulate the received signals and get the baseband signals back?



Quadrature Modulation and Demodulation (QAM)

- For the I (In Phase, or cosine) channel, the output after the cosine demodulator and the lowpass filter is:



$$
\begin{aligned}
x_1(t) &= y(t)\cos(2\pi f_c t) \\
&= \Big[g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\Big]\cos(2\pi f_c t) \\
&= g_1(t)\cos^2(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\cos(2\pi f_c t) \\
&= \frac{1}{2} g_1(t)[1 + \cos(4\pi f_c t)] - \frac{1}{2} g_2(t)\sin(4\pi f_c t) \\
&= \frac{1}{2} g_1(t) + \cancel{\left[\frac{1}{2} g_1(t)\cos(4\pi f_c t) - \frac{1}{2} g_2(t)\sin(4\pi f_c t)\right]}
\end{aligned}
$$

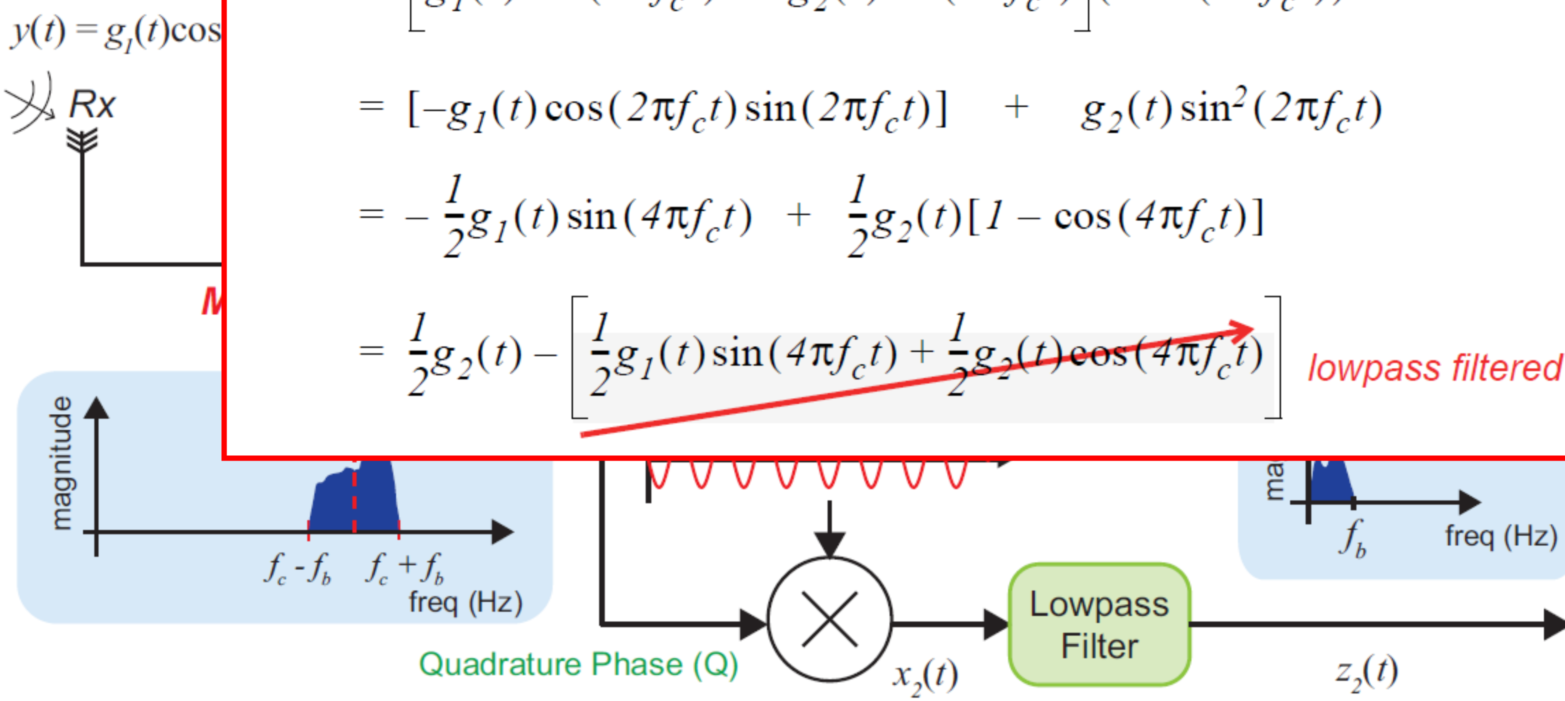
lowpass filtered terms

Quadrature Modulation and Demodulation (QAM)

- While the output of the quadrature (sine) demodulator and lowpass filter is:

$$
\begin{aligned}
x_2(t) &= y(t)(-\sin(2\pi f_c t)) \\
&= \left[g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\right](-\sin(2\pi f_c t)) \\
&= [-g_1(t)\cos(2\pi f_c t)\sin(2\pi f_c t)] + g_2(t)\sin^2(2\pi f_c t) \\
&= -\frac{1}{2}g_1(t)\sin(4\pi f_c t) + \frac{1}{2}g_2(t)[1-\cos(4\pi f_c t)] \\
&= \frac{1}{2}g_2(t) - \cancel{\left[\frac{1}{2}g_1(t)\sin(4\pi f_c t) + \frac{1}{2}g_2(t)\cos(4\pi f_c t)\right]}
\end{aligned}
$$

lowpass filtered terms



Quadrature Modulation and Demodulation (QAM)

- If there is a phase shift of on the local oscillator, then the quadrature output signal will be mixed with the In Phase component

$$x_1(t) = y(t)\cos(2\pi f_c t + \theta)$$

$$= \left[g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\right]\cos(2\pi f_c t + \theta)$$

$$= g_1(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \theta) \quad - \quad g_2(t)\sin(2\pi f_c t)\cos(2\pi f_c t + \theta)$$

$$= \frac{1}{2}g_1(t)\left[\cos(-\theta) + \cos(4\pi f_c t + \theta)\right] \quad - \quad \frac{1}{2}g_2(t)\left[\sin(-\theta) + \sin(4\pi f_c t + \theta)\right]$$

Noting that $\cos(-x) = \cos(x)$ and $\sin(-x) = -\sin(x)$,

$$= \frac{1}{2}g_1(t)\left[\cos(\theta) + \cos(4\pi f_c t + \theta)\right] \quad - \quad \frac{1}{2}g_2(t)[-\sin(\theta) + \sin(4\pi f_c t + \theta)]$$

lowpass filtered terms

$$= \underbrace{\frac{1}{2}\left[g_1(t)\cos(\theta) + g_2(t)\sin(\theta)\right]}_{z_1(t)} + \left[\cancel{\frac{1}{2}g_1(t)\cos(4\pi f_c t + \theta) - \frac{1}{2}g_2(t)\sin(4\pi f_c t + \theta)}\right]$$

Quadrature Modulation and Demodulation (QAM)

- If there is a phase shift of on the local oscillator, then the quadrature output signal will be mixed with the In Phase component

$$x_2(t) = y(t)(-\sin(2\pi f_c t + \theta))$$

$$= \left[g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\right](-\sin(2\pi f_c t + \theta))$$

$$= -g_1(t)\cos(2\pi f_c t)\sin(2\pi f_c t + \theta) + g_2(t)\sin(2\pi f_c t)\sin(2\pi f_c t + \theta)$$

$$= -\frac{1}{2}g_1(t)[-\sin(-\theta) + \sin(4\pi f_c t + \theta)] + \frac{1}{2}g_2(t)[\cos(-\theta) - \cos(4\pi f_c t + \theta)]$$

Noting that $\cos(-x) = \cos(x)$ and $\sin(-x) = -\sin(x)$,

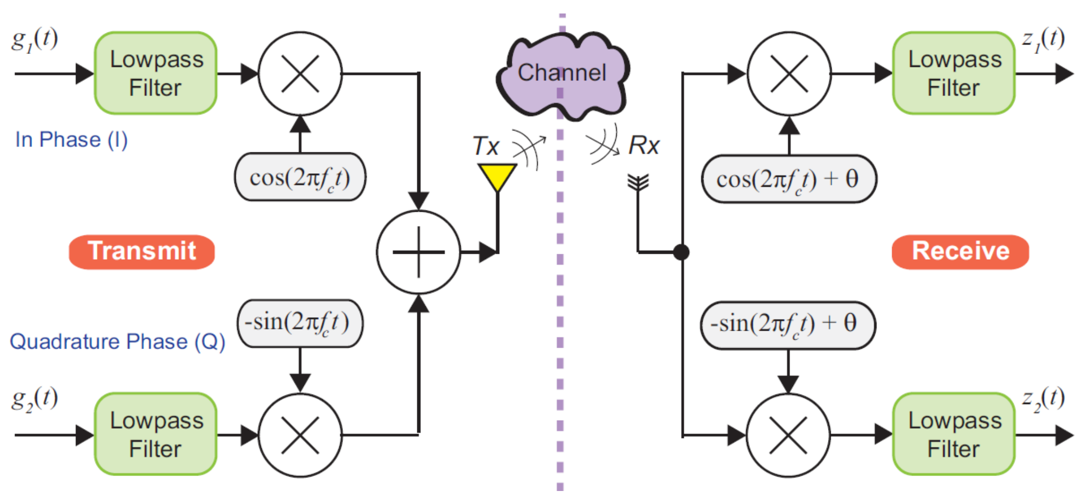
$$= -\frac{1}{2}g_1(t)[\sin(\theta) + \sin(4\pi f_c t + \theta)] + \frac{1}{2}g_2(t)[\cos(\theta) - \cos(4\pi f_c t + \theta)]$$

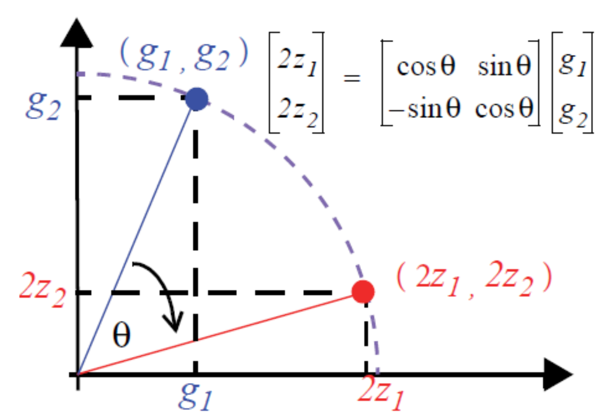
lowpass filtered terms

$$= \underbrace{\frac{1}{2}\left[-g_1(t)\sin(\theta) + g_2(t)\cos(\theta)\right]}_{z_2(t)} - \cancel{\left[\frac{1}{2}g_1(t)\sin(4\pi f_c t + \theta) + \frac{1}{2}g_2(t)\cos(4\pi f_c t + \theta)\right]}$$

Quadrature Modulation and Demodulation (QAM)

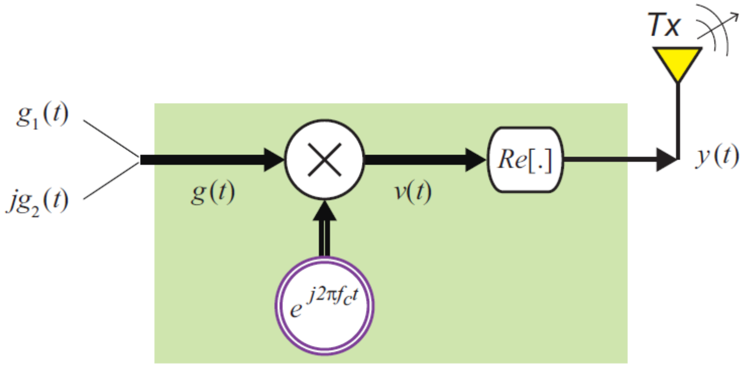
- If g_1 and g_2 were interpreted as points (at a given sample time) in the Cartesian (x-y) plane, the resulting points $2z_1$ and $2z_2$ are just the points rotated about the origin by θ degrees.
  - We multiply by 2 to account for the 0.5 scaling in previous eq.s
  - a phase error means that the received points are 'rotated'
  - if the carrier has a small frequency error, then the phase error $\theta(t)$ is constantly changing and we see a spinning constellation.





QAM using complex notation

- Consider a complex baseband signal: $g(t) = g_1(t) + jg_2(t)$, and a complex exponential carrier frequency at fc Hz: $e^{j2\pi fct} = \cos(2\pi fct) + j\sin(2\pi fct)$



$$v(t) = g(t)e^{j2\pi f_c t} = \left[g_1(t) + jg_2(t)\right]e^{j2\pi f_c t}$$

$$= \left[g_1(t) + jg_2(t)\right]\left[\cos(2\pi f_c t) + j\sin(2\pi f_c t)\right]$$

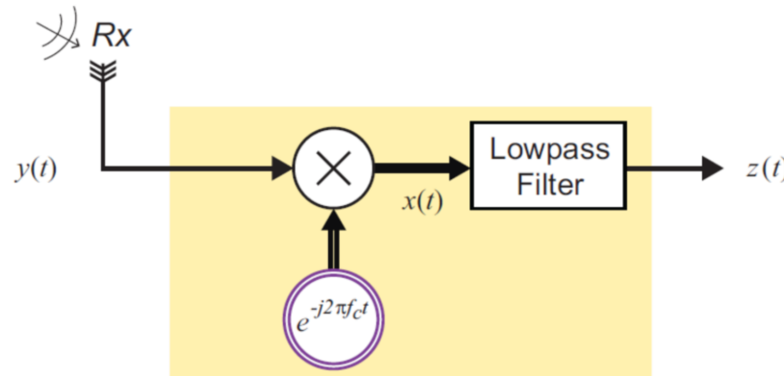
$$= g_1(t)\cos(2\pi f_c t) + jg_2(t)\cos(2\pi f_c t) + jg_1(t)\sin(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)$$

$$= \underbrace{\left[g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\right]}_{\textbf{Real}} + \underbrace{j\left[g_1(t)\sin(2\pi f_c t) + g_2(t)\cos(2\pi f_c t)\right]}_{\textbf{Imaginary}}$$

This y(t) is the same signal output that we achieved from the simple QAM modulation

QAM using complex notation

- The input to the complex demodulator is the real signal from the real-world y(t)



$$
\begin{aligned}
x(t) &= y(t)e^{-j2\pi f_c t} \\
&= \left[g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\right] \; e^{-j2\pi f_c t} \\
&= \left[g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\right] \left[\cos(2\pi f_c t) - j\sin(2\pi f_c t)\right] \\
&= \left[A\cos(\phi) - B\sin(\phi)\right] \left[\cos(\phi) - j\sin(\phi)\right] \\
&= A\cos(\phi)\left[\cos(\phi) - j\sin(\phi)\right] - B\sin(\phi)\left[\cos(\phi) - j\sin(\phi)\right] \\
&= A\cos^2(\phi) - jA\cos(\phi)\sin(\phi) - B\sin(\phi)\cos(\phi) + jB\sin^2(\phi) \\
&= A\cos^2(\phi) \; + jB\sin^2(\phi) - jA\cos(\phi)\sin(\phi) - B\sin(\phi)\cos(\phi)
\end{aligned}
$$

QAM using complex notation

- Using trigonometric identities gives:

$$= \frac{A}{2}\left[1+\cos(2\phi)\right]+j\frac{B}{2}\left[1-\cos(2\phi)\right]-j\frac{A}{2}\sin(2\phi)-\frac{B}{2}\sin(2\phi)$$

$$= \frac{A}{2}+\frac{A}{2}\cos(2\phi)+j\frac{B}{2}-j\frac{B}{2}\cos(2\phi)-j\frac{A}{2}\sin(2\phi)-\frac{B}{2}\sin(2\phi)$$

$$= \frac{A}{2}+j\frac{B}{2}+\frac{A}{2}\cos(2\phi)-j\frac{B}{2}\cos(2\phi)-j\frac{A}{2}\sin(2\phi)-\frac{B}{2}\sin(2\phi)$$

- And substituiting back for $g_1(t)$, $g_2(t)$ and $2\pi f_c t$

$$x(t) = \frac{1}{2}\left[g_1(t)+jg_2(t)\right]+\cancel{\frac{1}{2}g_1(t)\cos(4\pi f_c t)}-j\cancel{\frac{1}{2}g_2(t)\cos(4\pi f_c t)}$$

$$-j\cancel{\frac{1}{2}g_1(t)\sin(4\pi f_c t)}-\cancel{\frac{1}{2}g_2(t)\sin(4\pi f_c t)}$$

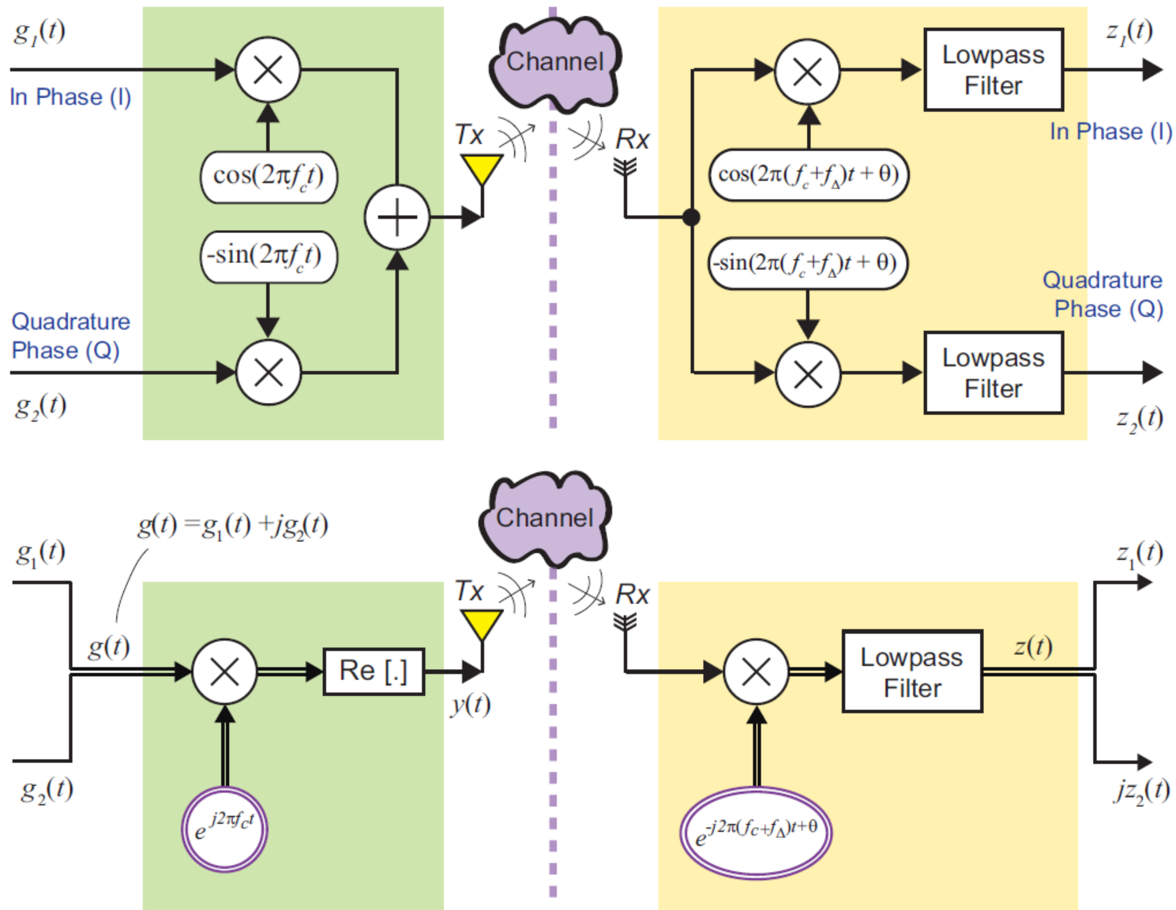
lowpass filtered terms

- Hence the output z(t) is given by:

$$z(t) = \frac{1}{2}\left[g_1(t)+jg_2(t)\right]$$

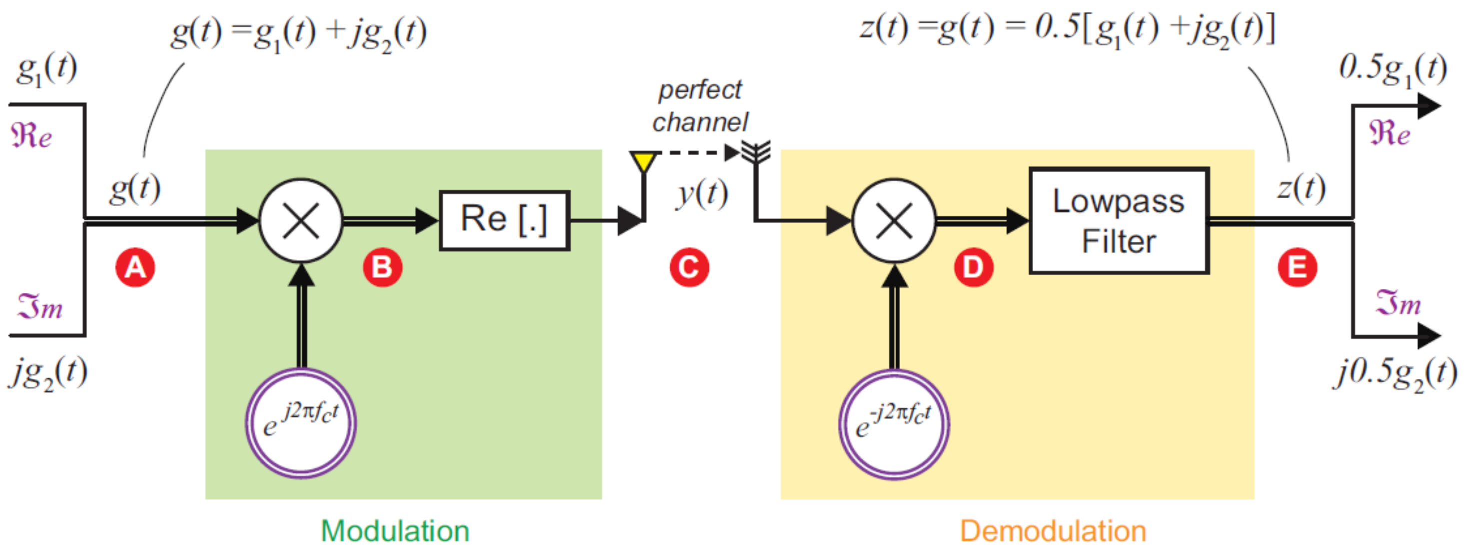
QAM using complex notation

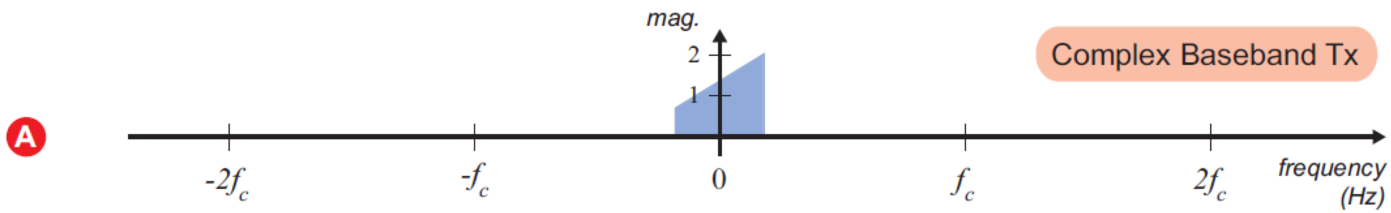
- The mathematical equivalence of the "standard" QAM and the complex one is therefore confirmed:



Spectral Representation for Complex QAM

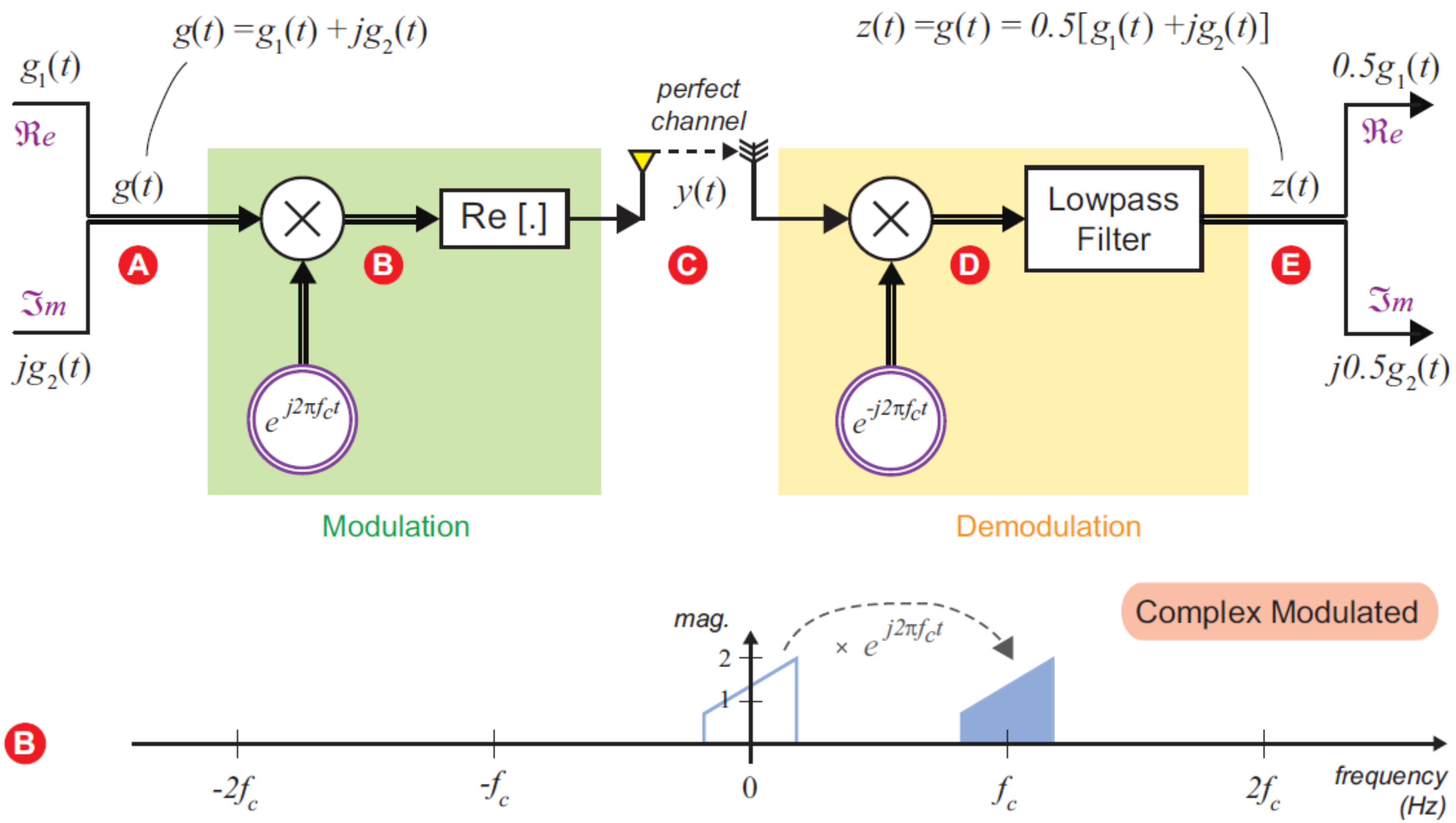
- If we represent $g_1(t)$, $g_2(t)$ as $g(t)=g_1(t)+jg_2(t)$, such a complex signal generally has a non-symmetric spectra!





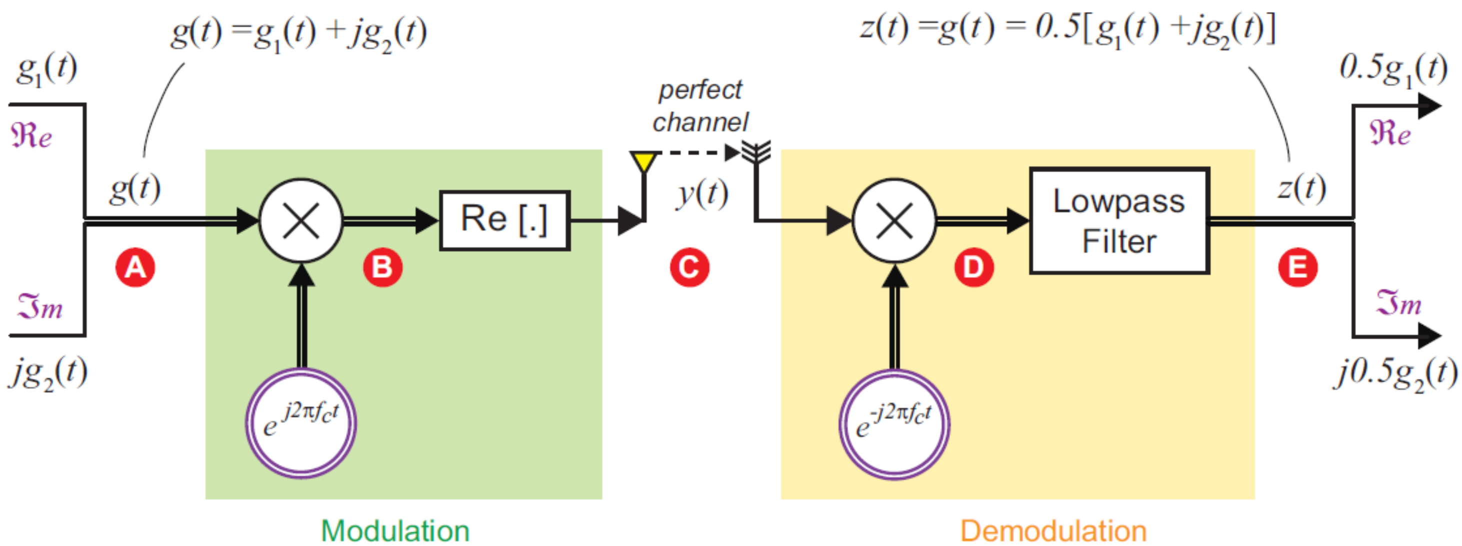
Spectral Representation for Complex QAM

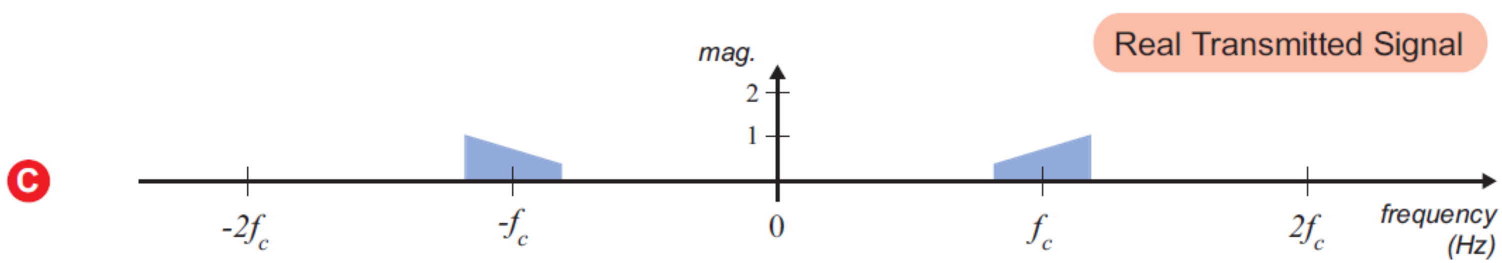
- The spectra at B then shows the complex baseband signal modulated by the complex carrier $v(t) = g(t)\, e^{j2\pi fct}$
- Spectrum magnitude only exists around fc!



Spectral Representation for Complex QAM

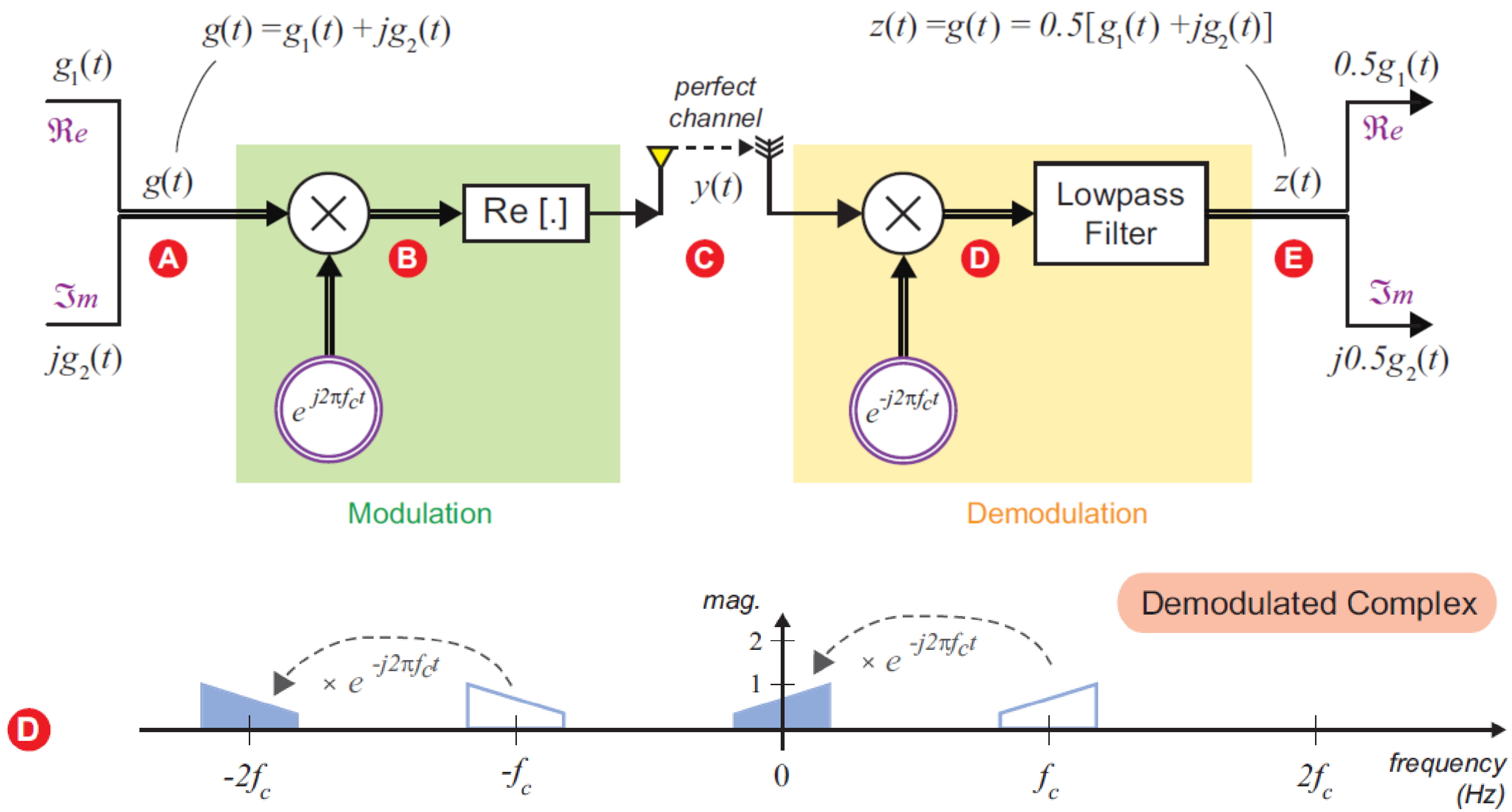
- When we take the real part only, which is the real signal for transmission, $y(t) = g_1(t)\cos(2\pi fct) - g_2(t)\sin(2\pi fct)$ and of course this signal is now (Hermitian) symmetric in the frequency domain.





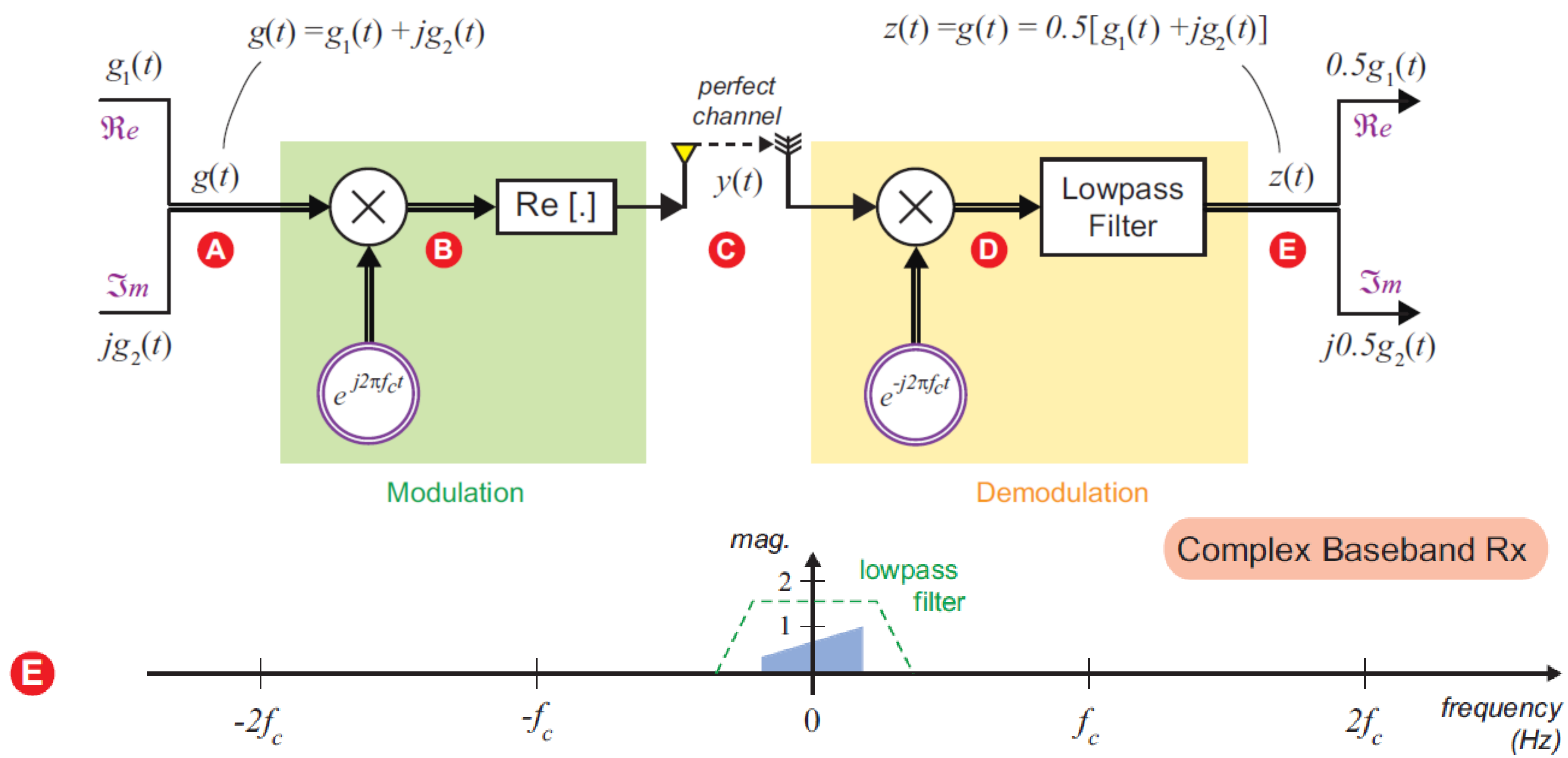
Spectral Representation for Complex QAM

- After applying the complex exponential multiplier $e^{j2\pi fct}$, at the receive side, the positive and negative frequencies of *y(t)* are both shifted by fc Hz to realise the analytic complex spectra.



Spectral Representation for Complex QAM

- Finally both the real and imaginary signals are filtered by a suitable low pass filter and the complex baseband received signal is obtained: $z(t) = 0.5g(t) = 0.5g_1(t) + j0.5g_2(t)$.



An example

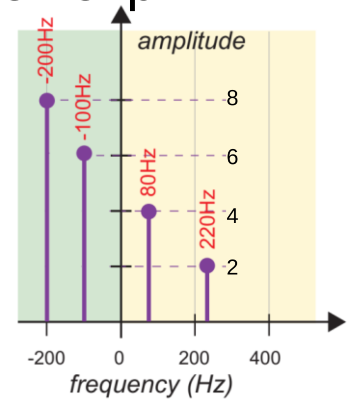
- Consider a base-band signal x(t), consisting of 4 complex exp signals at different frequencies:

$8e^{-j2\pi\cdot 200t}$

$6e^{-j2\pi\cdot 100t}$

$4e^{j2\pi\cdot 80t}$

$2e^{j2\pi\cdot 220t}$



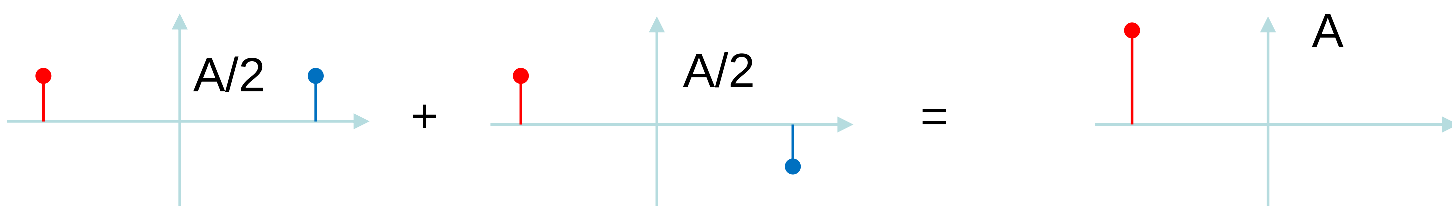
$8\cos(2\pi\cdot 200t) - \mathrm{j}8\sin(2\pi\cdot 200t)$

$4\cos(2\pi\cdot 80t) + \mathrm{j}4\sin(2\pi\cdot 80t)$

$6\cos(2\pi\cdot 100t) - \mathrm{j}6\sin(2\pi\cdot 100t)$

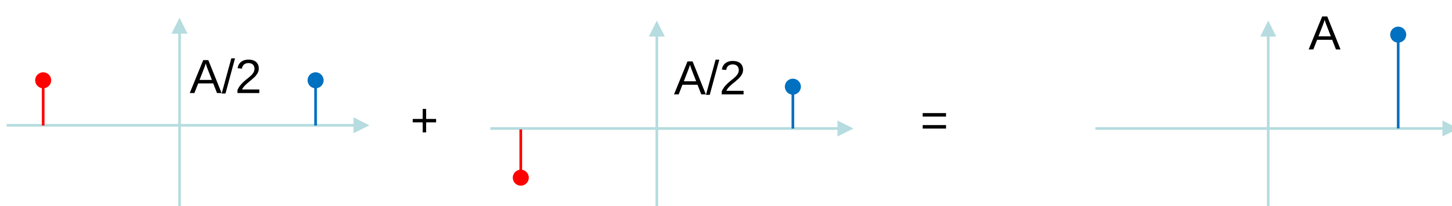
$2\cos(2\pi\cdot 220t) + \mathrm{j}2\sin(2\pi\cdot 220t)$

$\mathrm{A}\cos(2\pi\cdot ft) - \mathrm{jA}\sin(2\pi\cdot ft)$



$\mathrm{A}\exp(-j2\pi ft)$

$\mathrm{A}\cos(2\pi\cdot ft) + \mathrm{jA}\sin(2\pi\cdot ft)$



$\mathrm{A}\exp(j2\pi ft)$

An example

- The band-pass real signal, consists of 4 cosine signals at the different frequencies (e.g. generated and modulated onto a carrier at fc = 1kHz): y(t) = Real{x(t)*exp(j2π1000t)}

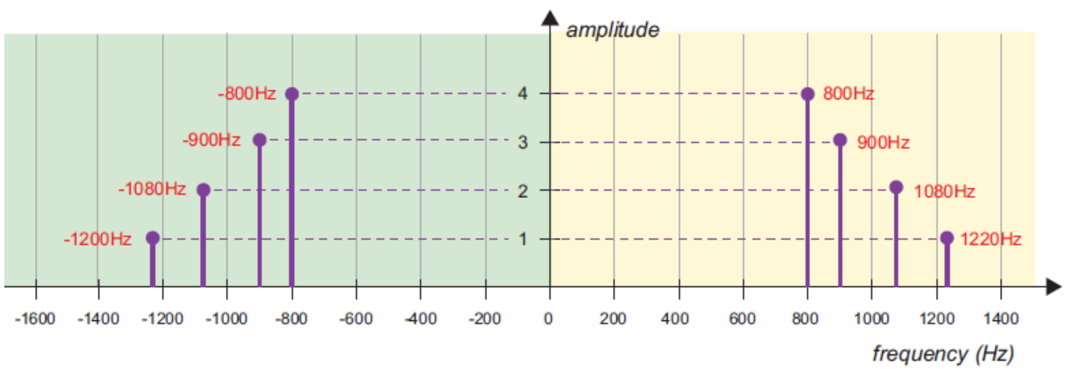
$$y(t) = 8\cos(2\pi 800t) + 6\cos(2\pi 900t) + 4\cos(2\pi 1080t) + 2\cos(2\pi 1220t)$$

$$= \frac{8}{2}(e^{j2\pi 800t} + e^{-j2\pi 800t}) + \frac{6}{2}(e^{j2\pi 900t} + e^{-j2\pi 900t})$$

$$+ \frac{4}{2}(e^{j2\pi 1080t} + e^{-j2\pi 1080t}) + \frac{2}{2}(e^{j2\pi 1220t} + e^{-j2\pi 1220t})$$

$$= 4e^{j2\pi 800t} + 3e^{j2\pi 900t} + 2e^{j2\pi 1080t} + e^{j2\pi 1220t}$$

$$+ 4e^{-j2\pi 800t} + 3e^{-j2\pi 900t} + 2e^{-j2\pi 1080t} + e^{-j2\pi 1220t}$$



The signal is a real cosine, then the complex frequency spectra is symmetric, and the imaginary amplitude spectra has no non-zero components (and is not drawn)

An example

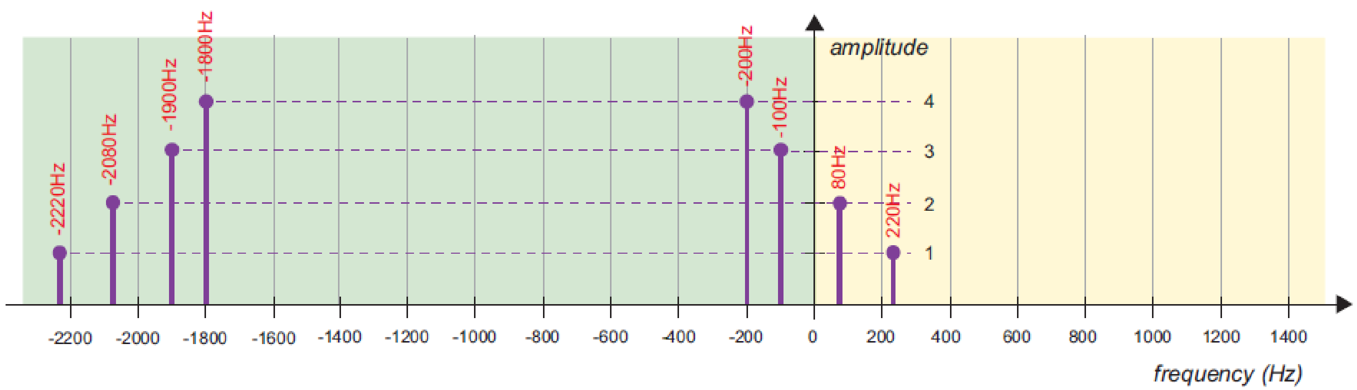
- If we demodulate (multiply by a complex exponential), we get:

$$y(t)e^{-j2\pi 1000t} = 4e^{j2\pi(800-1000)t} + 3e^{j2\pi(900-1000)t} + 2e^{j2\pi(1080-1000)t} + e^{j2\pi(1220-1000)t}$$

$$+ 4e^{-j2\pi(800+1000)t} + 3e^{-j2\pi(900+1000)t} + 2e^{-j2\pi(1080+1000)t} + e^{-j2\pi(1220+1000)t}$$

$$= 4e^{-j2\pi 200t} + 3e^{-j2\pi 100t} + 2e^{j2\pi 80t} + e^{j2\pi 220t}$$

$$+ 4e^{-j2\pi 1800t} + 3e^{-j2\pi 1900t} + 2e^{-j2\pi 2080t} + e^{-j2\pi 2220t}$$



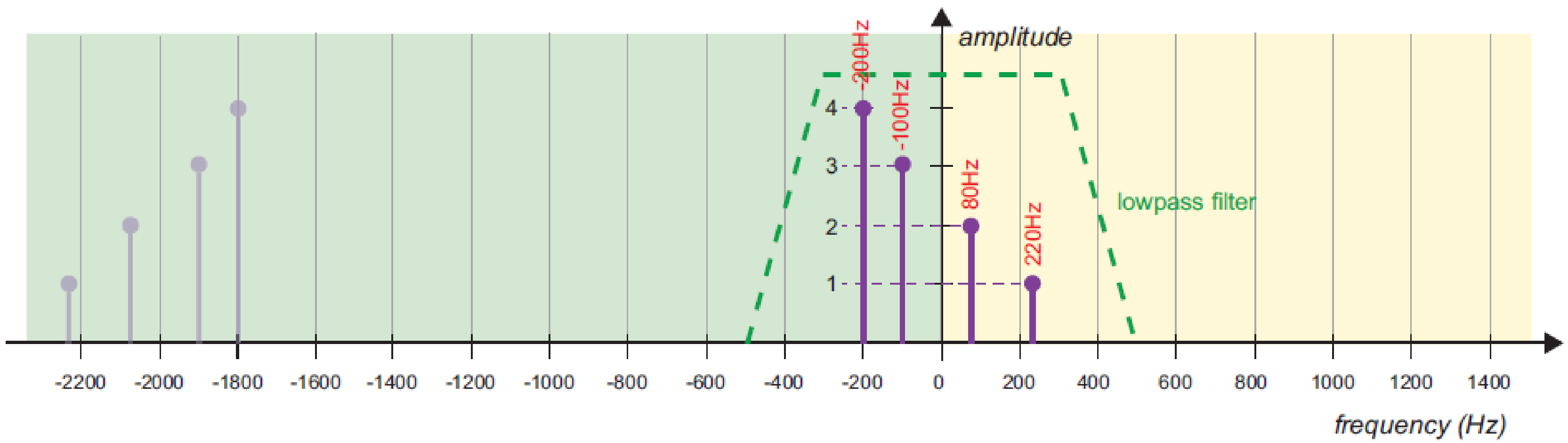
The bandpass signal (centred around 1000Hz) is complex demodulated by a negative 1000Hz complex exponential

An example

- Finally, after passing both the real and imaginary parts through a lowpass filter, we get just the complex baseband signal (with a 0,5 scaling factor):

$$z(t) = \mathrm{LPF}\{y(t)(e^{-j2\pi 1000t})\} = 4e^{-j2\pi 200t} + 3e^{-j2\pi 100t} + 2e^{j2\pi 80t} + e^{j2\pi 220t}$$

$$\cancel{+ 4e^{-j2\pi 1800t} + 3e^{-j2\pi 1900t} + 2e^{-j2\pi 2080t} + e^{-j2\pi 2220t}}$$



The bandpass signal (centred around 1000Hz) is complex demodulated by a negative 1000Hz complex exponential

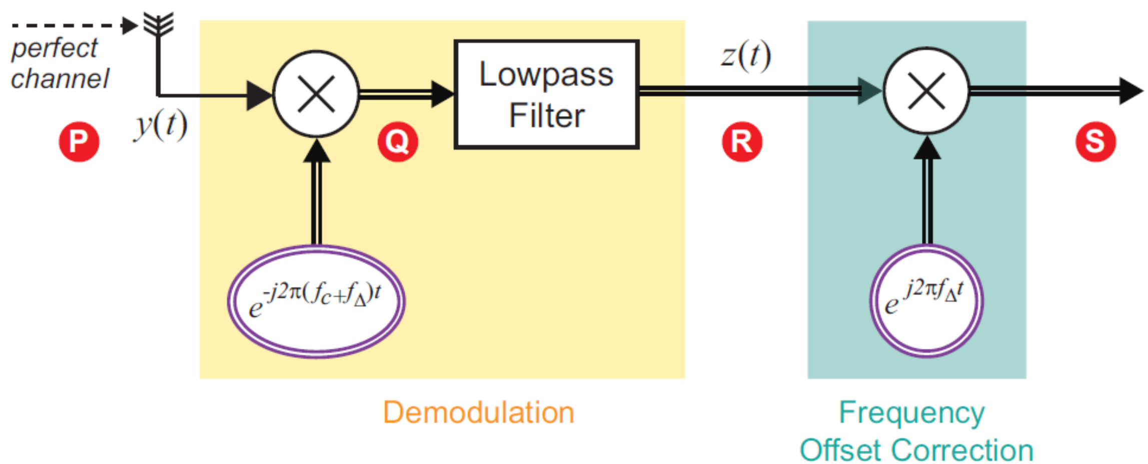
Frequency offset

- A potential errors may be that the receiver carrier frequency has a slight offset *fc* + Δ*f*.

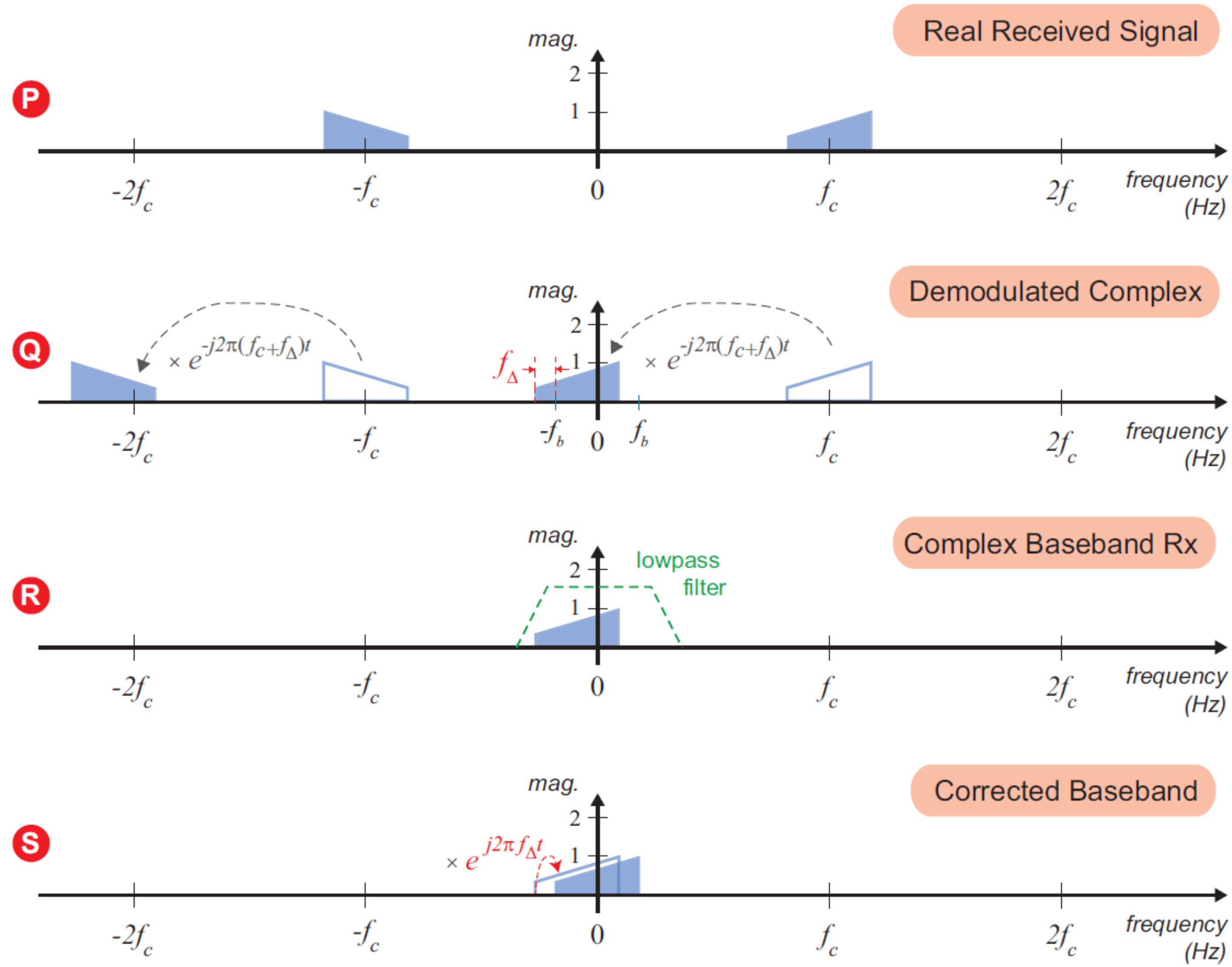
$$x(t) = y(t)e^{-j2\pi(f_c+\Delta f)t}$$

$$= \left[g_1(t)\cos(2\pi f_c t) - g_2(t)\sin(2\pi f_c t)\right] e^{-j2\pi f_c t} \cdot e^{-j2\pi f_\Delta t}$$

- The effect is to shift the spectrum by Δ*f* Hz from 0 Hz
  - the first processing stage in the receiver may be a frequency correction which will multiply the incoming complex spectrum by $e^{j2\pi\Delta f t}$



Frequency offset



Analytic signal

- The negative frequency components of the Fourier transform (or spectrum) of a real-valued function are superfluous, due to the Hermitian symmetry of such a spectrum: S(f) = S(f)*
- An analytic signal is a **complex-valued** function that has no negative frequency components. The real and imaginary parts of an analytic signal are real-valued functions related to each other by the Hilbert transform
  - Hilbert transform is given by convolution with the function $1/(\pi t)$
  - It imparts a phase shift of $\pm 90^\circ$ ($\pi/2$ radians) to every frequency component of a function, the sign of the shift depending on the sign of the frequency

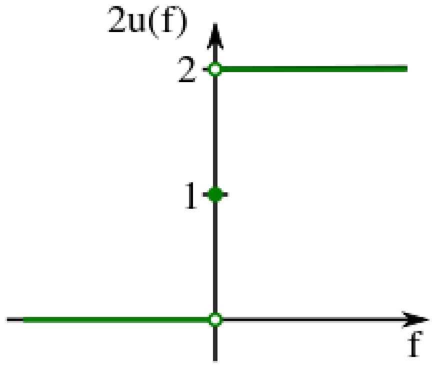
Analytic signal

If $s(t)$ is a *real-valued* function with Fourier transform $S(f)$, then the transform has Hermitian symmetry about the $f = 0$ axis:

$$S(-f) = S(f)^*,$$

where $S(f)^*$ is the complex conjugate of $S(f)$. The function:

$$\begin{aligned} S_a(f) &\triangleq \begin{cases} 2S(f), & \text{for } f > 0, \\ S(f), & \text{for } f = 0, \\ 0, & \text{for } f < 0 \end{cases} \\ &= \underbrace{2\,\mathrm{u}(f)}_{1+\operatorname{sgn}(f)} S(f) = S(f) + \operatorname{sgn}(f) S(f), \end{aligned}$$



where

- $\mathrm{u}(f)$ is the Heaviside step function,
- $\operatorname{sgn}(f)$ is the sign function,

contains only the *non-negative frequency* components of $S(f)$. And the operation is reversible, due to the Hermitian symmetry of $S(f)$:

$$\begin{aligned} S(f) &= \begin{cases} \frac{1}{2} S_a(f), & \text{for } f > 0, \\ S_a(f), & \text{for } f = 0, \\ \frac{1}{2} S_a(-f)^*, & \text{for } f < 0 \text{ (Hermitian symmetry)} \end{cases} \\ &= \frac{1}{2}[S_a(f) + S_a(-f)^*]. \end{aligned}$$

Analytic signal

The **analytic signal** of $s(t)$ is the inverse Fourier transform of $S_a(f)$:

$$\begin{aligned}
s_a(t) &\triangleq \mathcal{F}^{-1}[S_a(f)] \\
&= \mathcal{F}^{-1}[S(f) + \operatorname{sgn}(f) \cdot S(f)] \\
&= \underbrace{\mathcal{F}^{-1}\{S(f)\}}_{s(t)} + \overbrace{\underbrace{\mathcal{F}^{-1}\{\operatorname{sgn}(f)\}}_{j\frac{1}{\pi t}} * \underbrace{\mathcal{F}^{-1}\{S(f)\}}_{s(t)}}^{\text{convolution}} \\
&= s(t) + j\underbrace{\left[\frac{1}{\pi t} * s(t)\right]}_{\mathcal{H}[s(t)]} \\
&= s(t) + j\hat{s}(t),
\end{aligned}$$

where

- $\hat{s}(t) \triangleq \mathcal{H}[s(t)]$ is the Hilbert transform of $s(t)$;
- $*$ is the binary convolution operator;
- j is the imaginary unit.

Noting that $s(t) = s(t) * \delta(t)$, this can also be expressed as a filtering operation that directly removes negative frequency components:

$$s_a(t) = s(t) * \underbrace{\left[\delta(t) + j\frac{1}{\pi t}\right]}_{\mathcal{F}^{-1}\{2u(f)\}}.$$

Analytic signal

An analytic signal can also be expressed in polar coordinates:

$$s_{\mathrm{a}}(t) = s_{\mathrm{m}}(t)e^{j\phi(t)},$$

where the following time-variant quantities are introduced:

- $s_{\mathrm{m}}(t) \triangleq |s_{\mathrm{a}}(t)|$ is called the *instantaneous amplitude* or the *envelope*;
- $\phi(t) \triangleq \arg[s_{\mathrm{a}}(t)]$ is called the *instantaneous phase* or *phase angle*.

In the accompanying diagram, the blue curve depicts $s(t)$ and the red curve depicts the corresponding $s_{\mathrm{m}}(t)$.

The time derivative of the unwrapped instantaneous phase has units of *radians/second*, and is called the *instantaneous angular frequency*:

$$\omega(t) \triangleq \frac{d\phi}{dt}(t).$$

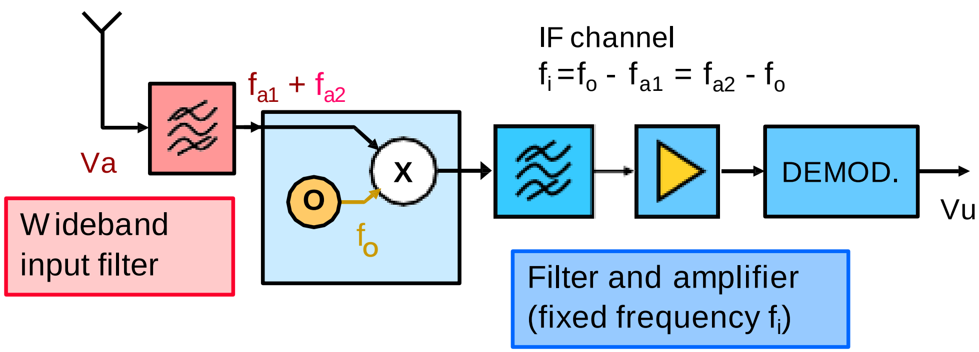
The *instantaneous frequency* (in hertz) is therefore:

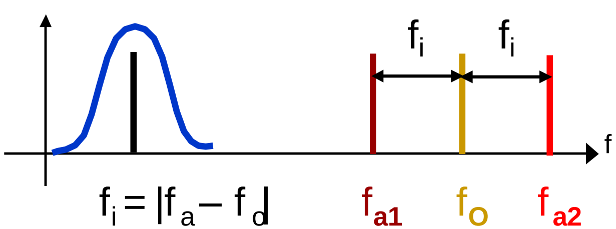
$$f(t) \triangleq \frac{1}{2\pi}\omega(t).$$ [3]

The polar coordinates conveniently separate the effects of amplitude modulation and phase (or frequency) modulation

The image frequency

- When a mixer translates a signal in frequency, it does so for every spectral component that lies within its bandwidth: all input signals at $f_a = f_o \pm f_i$ are shifted to the IF frequency
- An image frequency is one which lies equidistant from the LO frequency, but on the opposite side

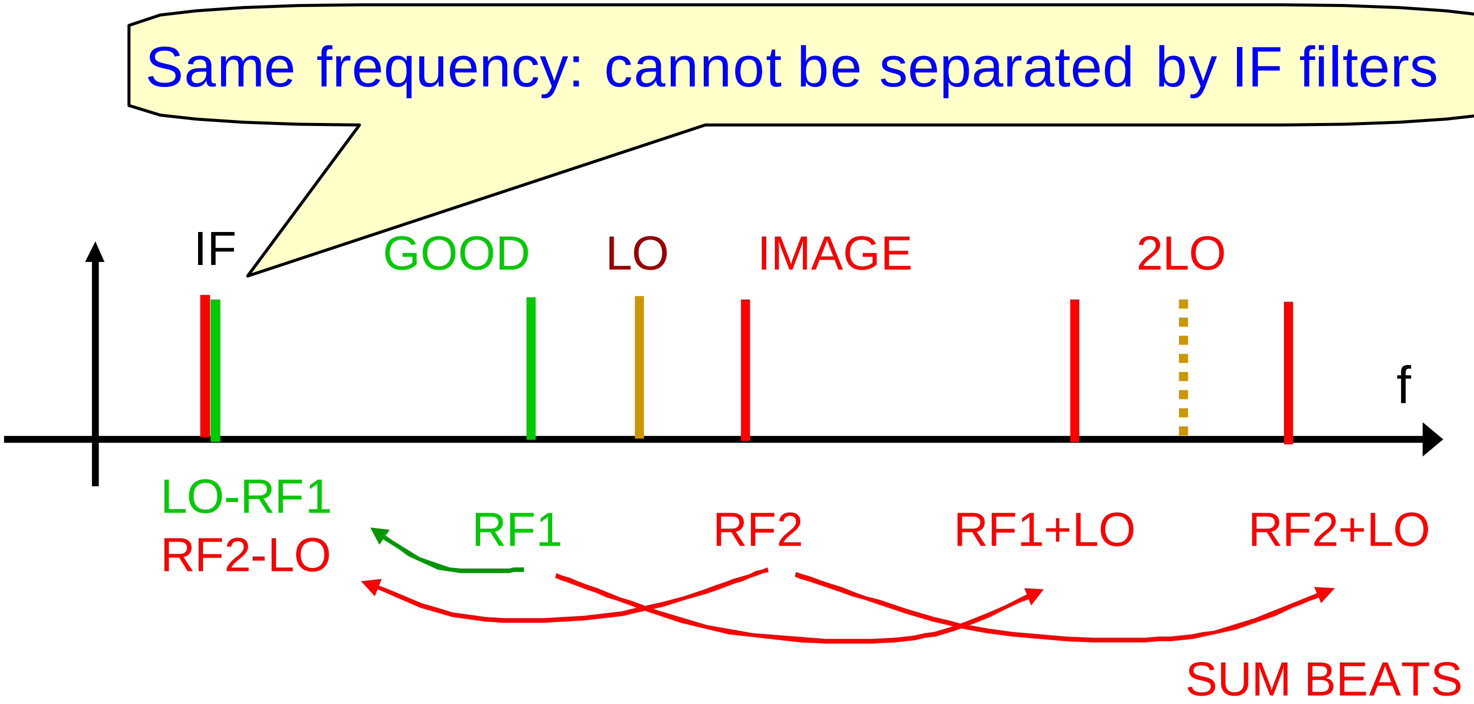




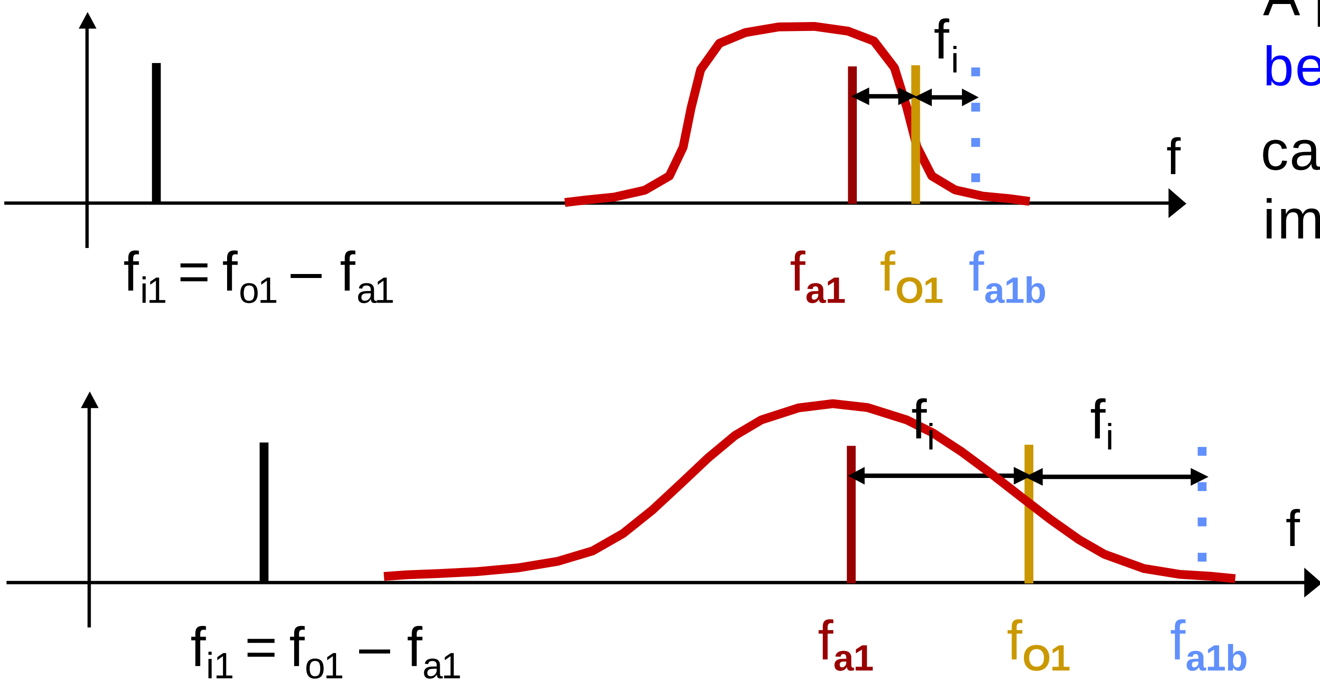
- The image could come from
  - Nearby transmitter
  - Other channels
- Can be stronger than the useful signal, and cause
  - Interference (high noise)
  - Blocking (drive LNA/mixer into saturation)

Complete spectrum

- The mixer generates sum and difference beats
  - Sum beats can be easily filtered
- good (RF1) and image (RF2) both folded to IF



Remove the image by filtering

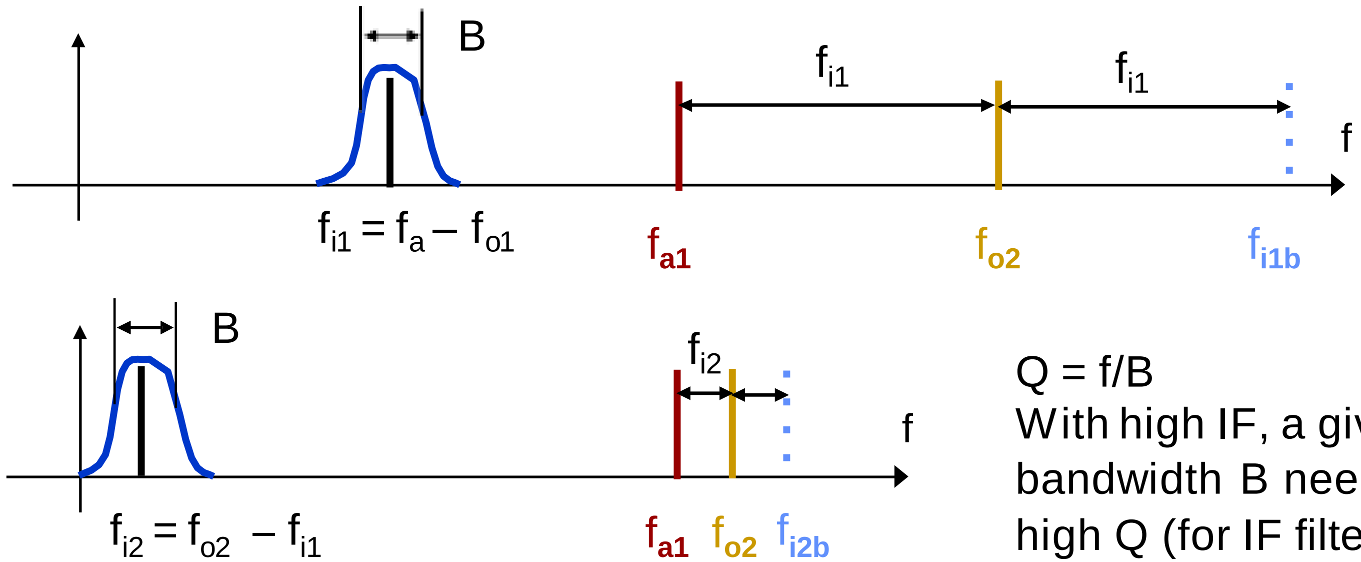


A passband filter before the mixer can remove the image f_{a1b}.

With high IF, the image f_{a1b} is more far away, and the filter can be less steep.

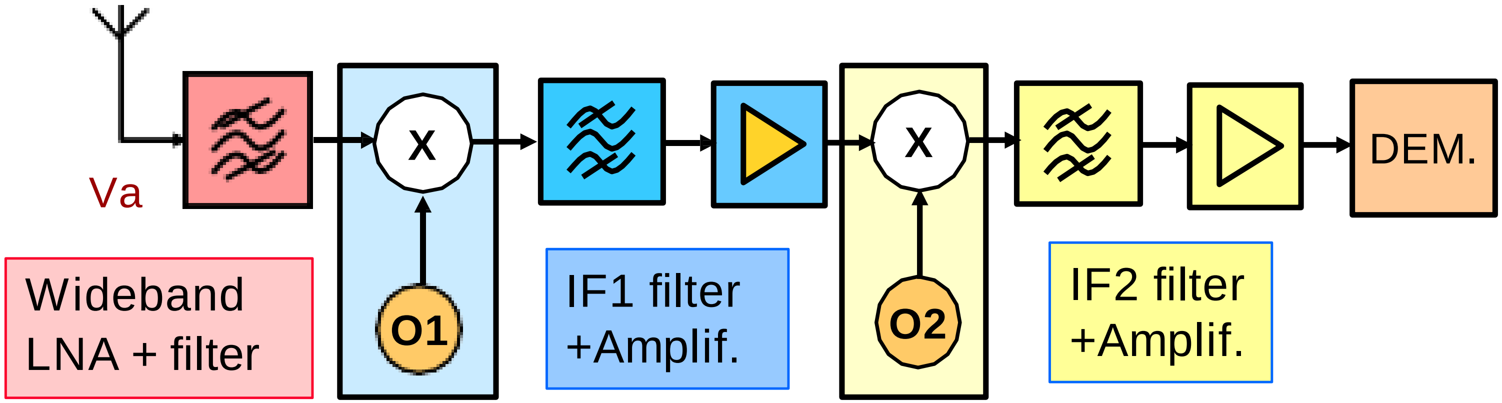
Problems with high IF frequency

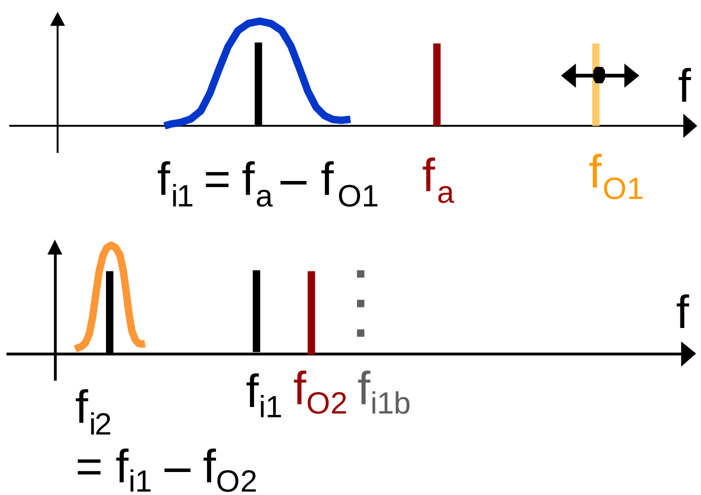
- RF Input channel: 2.5 GHz
- IF channel: 1 MHz
  - LO frequency: 2.5 GHz + 1 MHz = 2,501 MHz; Image at: 2.502 GHz
  - Q of RF image removal filter: 2,500/2 = 1250 !!!
- IF channel: 10 MHz
  - LO frequency: 2.5 GHz + 10 MHz = 2,510 MHz; Image at: 2.52 GHz
  - Q of RF image removal filter: 2,500/20 = 125



Q = f/B
With high IF, a given bandwidth B needs high Q (for IF filter)

Solution: dual-conversion heterodyne receiver





Second frequency translation to a low IF (IF2):
- Simple channel filter (IF2)

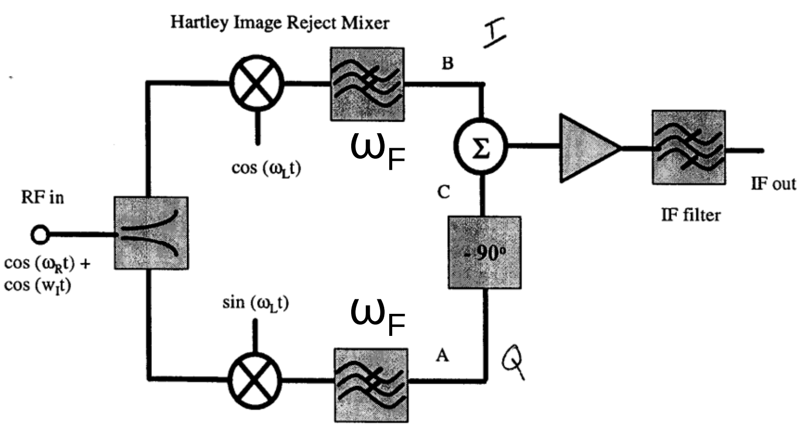
Tuning by shifting O1 or O2

IF1 image risk on IF2 (f_{i1b})!

- Need good bandpass filters at IF: expensive & bulky and difficult to shape the passband

Better: Image rejection mixer

- Nowadays image rejection is achieved canceling out unwanted components by means of phase shifting in a quadrature mixer
- If “sum” components are filtered out (remember that 2cos(a)cos(b)=cos(a+b)+cos(a-b), 2sin(a)cos(b)=sin(a+b)+sin(a-b) and filter cutoff $\omega_F < \omega_L + \omega_R$) :



$$\cos(\omega_L t)\cos(\omega_R t) + \cos(\omega_L t)\cos(\omega_I t)$$

$$X_B = \frac{1}{2}\left[\cos(\omega_L - \omega_R)t + \cos(\omega_L - \omega_I)t\right]$$

$$X_C = +\frac{1}{2}\cos(\omega_R - \omega_L)t - \frac{1}{2}\cos(\omega_L - \omega_I)t$$

$$\sin(\omega_L t)\cos(\omega_R t) + \sin(\omega_L t)\cos(\omega_I t)$$

$$X_A = \frac{1}{2}\left[\sin(\omega_L - \omega_R)t + \sin(\omega_L - \omega_I)t\right]$$

- If $\omega_I < \omega_L$ and $\omega_R > \omega_L$

$$\frac{1}{2}\sin(\omega_L - \omega_R)t \;=\; -\frac{1}{2}\sin(\omega_R - \omega_L)t$$

$$\sin(\omega t - 90) = -\cos(\omega t)$$

$$\boxed{X_{IF} = X_B + X_C = \cos(\omega_R - \omega_L)t}$$