UNIVERSITÀ DEGLI STUDI DI BRESCIA

Photonics Homework 1
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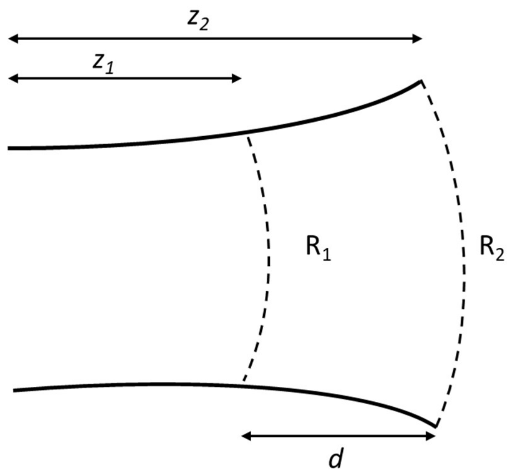
Instructions: Please write your answer on a file or piece of paper and return it to me by uploading it in the Assignment Moodle section (or in person if you don't have access to Moodle yet). I will look at all homework but will consider for evaluation only those retuned **not later than Thursday March 31st** . I will then upload the solutions of the homework so you will have the chance to check the solution if something was not clear.
IMPORTANT: This homework is not mandatory so you will still be able to get your full score at the final test even if you can't do it or can't return it on time!

Questions:

1. The image produced by a concave mirror is 4 times greater than the object. If the radius of curvature is 20 cm, determine the object distance from the mirror.

2. A farsighted person cannot focus on objects closer to his eyes than 140cm. What focal length eyeglass lenses are needed to focus on a newspaper held at 35cm from the person's eyes, if the glasses are worn 2cm from his eyes? (HINT: The newspaper is the object and 140 cm is the distance where the image of the newspaper needs to be virtually located from the eye).

3. A Gaussian beam has radii of curvature R_1 and R_2 at two points on the beam axis, separated by a distance *d* as in Figure 1. Calculate the location of the two points (z_1 and z_2), the location of the beam center z_0 and the beam waist expression as a function of R_1 and R_2.



4. A diffraction grating, illuminated at normal incidence with light at λ=600 nm, produces a second order diffracted wave at θ=33°.
   a. What is the total number of slits in the structure if the grating is 4.4 cm wide?
   b. Applying the Rayleigh criterion, calculate the resolution ability ($\Delta\lambda$) of this grating for the same order of diffraction and the same incident wavelength.

1. The image produced by a concave mirror is 4 times greater than the object. If the radius of curvature is 20 cm, determine the object distance from the mirror.

$R = 20\,cm \; ; \; |M| = 4$

- Considering the paraxial approximation we have the imaging equation:

$$\frac{1}{z_1} + \frac{1}{z_2} \approx \frac{1}{F} \qquad \text{with} \qquad f = \frac{R}{2}$$

- The equation for magnification is:

$$M = -\frac{z_2}{z_1}$$

We have 3 equations and 3 onknowns so we can solve the problem

$$\frac{1}{z_1} = \frac{1}{F} - \frac{1}{z_2} = \frac{2}{R} + \frac{1}{M z_1} \Rightarrow \frac{1}{z_1}\left(1 - \frac{1}{M}\right) = \frac{2}{R}$$

$$\left.\begin{aligned} z_1 &= \frac{R}{2}\left(1 - \frac{1}{M}\right) \\ z_2 &= -z_1 M \end{aligned}\right\}$$

We have 2 cases for the magnification. One with a virtual image and one with a real image

- If $M < 0$: Real image
- If $M > 0$: Virtual image

$$\boxed{\begin{aligned} &\bullet \text{ Real image} \Rightarrow z_1^{object} = -12.5\,cm \text{ and } z_2 = -50\,cm \\ &\bullet \text{ Virtual image} \Rightarrow z_1^{object} = -7.5\,cm \text{ and } z_2 = 30\,cm \end{aligned}}$$

2. A farsighted person cannot focus on objects closer to his eyes than 140cm. What focal length eyeglass lenses are needed to focus on a newspaper held at 35cm from the person's eyes, if the glasses are worn 2cm from his eyes? (HINT: The newspaper is the object and 140 cm is the distance where the image of the newspaper needs to be virtually located from the eye).

$|d| > 140\,cm$ $\quad f = ?$

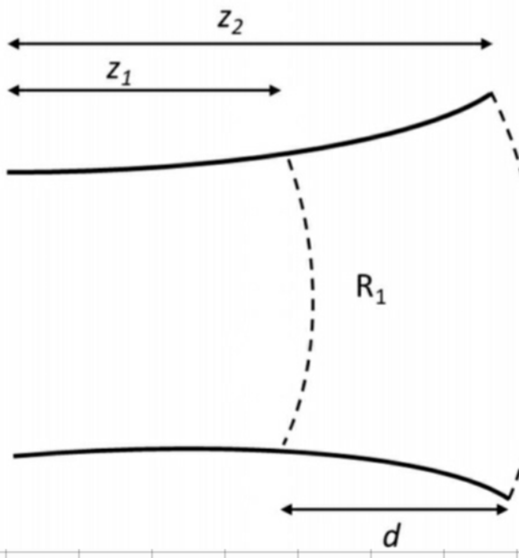
$z_1^{object} = 33\,cm \quad z_2^{image} = -138\,cm$

- Considering the same approximations as before, we have:

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} \Rightarrow \boxed{f = -43.37\,cm}$$

- We will need a concave lens with $|f| = 43.37\,cm$. This agrees with the lens properties studied.

3. A Gaussian beam has radii of curvature R_1 and R_2 at two points on the beam axis, separated by a distance *d* as in Figure 1. Calculate the location of the two points (z_1 and z_2), the location of the beam center z_0 and the beam waist expression as a function of R_1 and R_2.



$R_2 < R_1$; $z_2 - z_1 = d$; $z_1 = ?$, $z_2 = ?$, $z_0 = ?$, $W(z) = ?$

For Gaussian beams we have:

$$R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right]$$

$$\left.\begin{aligned} R(z_2) &= z_2\left[1 + \left(\frac{z_0}{z_2}\right)^2\right] = R_2 \\ R(z_1) &= z_1\left[1 + \left(\frac{z_0}{z_1}\right)^2\right] = R_1 \\ z_2 - z_1 &= d \end{aligned}\right\} \quad \text{3 equations with 3 unknowns.}$$

$$\Rightarrow \frac{R_2}{z_2} - 1 = \left(\frac{z_0}{z_2}\right)^2 \Rightarrow z_0^2 = R_2 z_2 - z_2^2$$

$$\Rightarrow R_1 = z_1\left[1 + \frac{(R_2 z_2 - z_2^2)}{z_1^2}\right] \Rightarrow \frac{R_1}{z_1} = \frac{z_1^2 + R_2(z_1+d) - (z_1+d)^2}{z_1^2}$$

$$\Rightarrow z_1 R_1 = z_1^2 + R_2 z_1 + R_2 d - z_1^2 - 2z_1 d - d^2$$

$$\Rightarrow z_1 (R_1 - R_2 + 2d) = d(R_2 - d)$$

$$\Rightarrow z_1 = \frac{d(R_2 - d)}{R_1 - R_2 + 2d} \; ; \; z_2 = z_1 + d \Rightarrow z_2 = \frac{d(R_1 + d)}{R_1 - R_2 + 2d}$$

$$z_0^2 = R_2 z_2 - z_2^2 \Rightarrow z_0^2 = \frac{R_2 d (R_1 + d)}{R_1 - R_2 + 2d} - \frac{d^2 (R_1 + d)^2}{(R_1 - R_2 + 2d)^2}$$

$$\Rightarrow z_0^2 = \frac{R_2 d (R_1 + d)(R_1 - R_2 + 2d) - d^2 (R_1 + d)^2}{(R_1 - R_2 + 2d)^2}$$

$$W(z) = W_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2} \quad \text{with} \quad W_0 = \sqrt{\frac{\lambda z_0}{\pi}}$$

$$W(z) = \sqrt{\frac{\lambda}{\pi}} \left[\frac{R_2 d (R_1 + d)(R_1 - R_2 + 2d) - d^2 (R_1 + d)^2}{(R_1 - R_2 + 2d)^2} \right]^{\frac{1}{4}} \cdot \sqrt{1 + z^2 \frac{(R_1 - R_2 + 2d)^2}{R_2 d (R_1 + d)(R_1 - R_2 + 2d) - d^2 (R_1 + d)^2}}$$

Results:

$$z_1 = \frac{d(R_2 - d)}{R_1 - R_2 + 2d} \qquad z_2 = \frac{d(R_1 + d)}{R_1 - R_2 + 2d}$$

$$z_0 = \left[\frac{R_2 d (R_1 + d)(R_1 - R_2 + 2d) - d^2 (R_1 + d)^2}{(R_1 - R_2 + 2d)^2} \right]^{\frac{1}{2}}$$

$$W(z) = W_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2} = \sqrt{\frac{\lambda}{\pi}} \left[\frac{R_2 d (R_1 + d)(R_1 - R_2 + 2d) - d^2 (R_1 + d)^2}{(R_1 - R_2 + 2d)^2} \right]^{\frac{1}{4}} \cdot \sqrt{1 + \frac{z^2 (R_1 - R_2 + 2d)^2}{R_2 d (R_1 + d)(R_1 - R_2 + 2d) - d^2 (R_1 + d)^2}}$$

4. A diffraction grating, illuminated at normal incidence with light at λ=600 nm, produces a second order diffracted wave at θ=33°.
   a. What is the total number of slits in the structure if the grating is 4.4 cm wide?
   b. Applying the Rayleigh criterion, calculate the resolution ability (Δλ) of this grating for the same order of diffraction and the same incident wavelength.

$\lambda = 600\,nm$, Second order diffracted wave @ $\theta_q = 33°$

We have: $\sin(\theta_q) = \sin(\theta_i) + q\frac{\lambda}{\Lambda}$; $\theta_i = 0$

$\Rightarrow \sin(\theta_q) = q\frac{\lambda}{\Lambda}$ $q = \pm 2$

$$\Rightarrow \Lambda = \frac{q\lambda}{\sin(\theta_q)}$$

a. Number of slits:

$\Lambda = 2203.29\,nm$ $l = 4.4\,cm$

$\Rightarrow$ # of slits $= \frac{l}{\Lambda}$ $\Rightarrow$ $\boxed{\text{\# of slits} = N = 19970}$

b. Resolution ability $(\Delta\lambda)$

$\Delta(\sin\theta)_{min} = \frac{\lambda}{N\Lambda}$; with $\sin\theta = \frac{q\lambda}{\Lambda}$ $\Rightarrow \frac{\Delta\lambda}{\lambda} = \frac{1}{qN}$

$\Rightarrow \Delta\lambda = 0.015\,nm$ $\Rightarrow$ $\boxed{\Delta\lambda = 0.015\,nm \text{ @ around } \lambda = 600\,nm}$