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Problem 1

The expressions of both circulary polarized waves with the same amplitude are:

RHC: $\bar{e} = a\cos(\omega t - kz)\,\hat{e}_x + a\cos(\omega t - kz - \pi/2)\,\hat{e}_y$

LHC: $\bar{e} = a\cos(\omega t - kz)\,\hat{e}_x + a\cos(\omega t - kz + \pi/2)\,\hat{e}_y$

when we add them:

$$\bar{e}_{total} = \bar{e}_{RHC} + \bar{e}_{LHC} = 2a\cos(\omega t - kz)\,\hat{e}_x + a\left[\cos(\omega t - kz - \pi/2) + \cos(\omega t - kz + \pi/2)\right]\hat{e}_y =$$

$$= 2a\cos(\omega t - kz)\,\hat{e}_x + a\left[\cos(\omega t - kz)\underbrace{\cancel{\cos\left(\tfrac{\pi}{2}\right)}}_{0} + \sin(\omega t - kz)\cdot\underbrace{\cancel{\sin\left(\tfrac{\pi}{2}\right)}}_{1} +\right.$$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$$\left. + \cos(\omega t - kz)\underbrace{\cancel{\cos\left(\tfrac{\pi}{2}\right)}}_{0} - \sin(\omega t - kz)\underbrace{\cancel{\sin\left(\tfrac{\pi}{2}\right)}}_{1}\right]\hat{e}_y =$$

$$= 2a\cos(\omega t - kz)\,\hat{e}_x + 2\left[\sin(\omega t - kz) - \sin(\omega t - kz)\right]\hat{e}_y$$

$$\boxed{\bar{e}_{total} = 2a\cos(\omega t - kz)\,\hat{e}_x}$$

Using Jones vector representation:

•) In general: $\bar{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} a_x \\ a_y e^{j\varphi} \end{bmatrix}$

→ For RHC:

$$\left.\begin{matrix} a_x = a_y = a \\ \varphi = -\frac{\pi}{2} \end{matrix}\right\} \Rightarrow e^{j\varphi} = e^{-j\frac{\pi}{2}} = \underbrace{\cos\left(-\frac{\pi}{2}\right)}_{0} + j\underbrace{\sin\left(-\frac{\pi}{2}\right)}_{-1} = -j$$

$$\bar{E}_{RHC} = a \begin{bmatrix} 1 \\ -j \end{bmatrix}$$

→ For LHC

$$\left.\begin{matrix} a_x = a_y = a \\ \varphi = \frac{\pi}{2} \end{matrix}\right\} \Rightarrow e^{j\varphi} = e^{j\frac{\pi}{2}} = \underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} + j\underbrace{\sin\left(\frac{\pi}{2}\right)}_{1} = j$$

$$\bar{E}_{LHC} = a \begin{bmatrix} 1 \\ j \end{bmatrix}$$

Adding both of them

$$\bar{E}_{total} = \bar{E}_{RHC} + \bar{E}_{LHC} = a \begin{bmatrix} 1 \\ -j \end{bmatrix} + a \begin{bmatrix} 1 \\ j \end{bmatrix} =$$

$$\boxed{\bar{E}_{total} = a \begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

Problem 2

a)

For angle of reflection: $\theta_i = \theta_r \Rightarrow \boxed{\theta_r = 60°}$

For angle of transmission we use Snell's law:

$$n_i \sin\theta_i = n_t \sin\theta_t$$

$$n_a \sin\theta_i = n_w \sin\theta_t \longrightarrow \theta_t = \operatorname{asin}\left(\frac{n_a}{n_w}\sin\theta_i\right) = \operatorname{asin}\left[\frac{1}{1{,}33}\sin 60°\right]$$

$$\boxed{\theta_t = 40{,}63°}$$

For reflectance we have:

$$R = |r|^2$$

$$r_{TE} = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} = \frac{n_a\cos\theta_i - n_w\cos\theta_t}{n_a\cos\theta_i + n_t\cos\theta_t} = \frac{1\cdot\cos 60° - 1{,}33\cdot\cos 40{,}63°}{1\cdot\cos 60° + 1{,}33\cdot\cos 40{,}63°}$$

$$r_{TE} = -0{,}3375 \Rightarrow \boxed{R = 0{,}1139}$$

For transmittance:

$$T = 1 - R \Rightarrow \boxed{T = 0{,}8861}$$

b)

By definition we have: $T = \frac{P_t}{P_i}$ and $I = P \cdot A$

$$T = \frac{\frac{I_t}{A}}{\frac{I_i}{A}} = \frac{I_t}{I_i} \Rightarrow I_t = T \cdot I_i = 0{,}8861 \cdot 2$$

$$\boxed{I_t = 1{,}772 \ MW/cm^2}$$

c)

In this case we have the light coming from the "slower" medium (the speed of light is slower in water than in the air) to a "faster" medium. Because of that the angle of transmission is larger than the angle of incidence. Thus, the total internal reflection can happen. Calculating the critical angle we get:

$$\theta_c = \operatorname{asin}\left(\frac{n_t}{n_i}\right) = \operatorname{asin}\left(\frac{n_a}{n_w}\right) = \operatorname{asin}\left(\frac{1}{1{,}33}\right) \Rightarrow \theta_c = 48{,}75°$$

So as $\theta_i > \theta_c$ there will be no transmission. Therefore, the values of angles of reflection & transmission and R & T are:

$$\theta_r = \theta_i \Rightarrow \boxed{\theta_r = 60°}$$

$$\boxed{\theta_t = 90°}$$

$$\boxed{R = 1}$$

$$\boxed{T = 0}$$

Problem 3

Generally speaking scattering is defined as: redirection of radiation out of the original direction of propagation.

Different light phenomena such as reflection, refraction or diffraction can be assumed as forms of scattering.

Therea are some different types of scattering such as: Elastic scattering, inelastic scattering, quasi-elastic scattering, single scattering or multiple scattering.

There are three important parameters that governs scattering:

- The wavelength of the incident radiation
- The size of the object (particle)
- The optical properties relative to sorrounding medium

Depending of the relation between the size of the scattering particle and the wavelength of the incident wave we have different light scattering regimes.

Problem 4

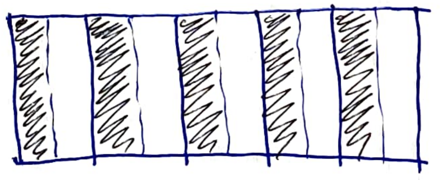
Calculating the central wavelength:

$$\lambda_{Bragg} = \frac{2\pi c}{\omega_{Bragg}} = \frac{2\pi c}{\frac{(n_1+n_2)\,2\pi c}{4n_1n_2\Lambda}} = \frac{4n_1n_2\Lambda}{(n_1+n_2)} = \frac{4n_1n_2a}{(n_1+n_2)\,2} =$$

$$= \frac{4\cdot 1{,}5\cdot 3\cdot 300}{(1{,}5+3)\cdot 2}$$

$$\boxed{\lambda_{Bragg} = 600\,\text{nm}}$$

The original structure is:



The modified structure: