PROBLEM P2

AN ANTENNA EXHIBITS THE FOLLOWING NORMALIZED POWER PATTERN

$$i(\theta,\varphi) = \sin^2\theta \, |\cos\varphi| \qquad 0 \leq \theta \leq \pi \qquad 0 \leq \varphi \leq 2\pi$$

A) CALCULATE THE DIRECTIVITY

B) THE RADIATION EFFICIENCY IS 0,75 ; CALCULATE THE GAIN AND EXPRESS IT IN A LOGARITHMIC SCALE (IN dB)

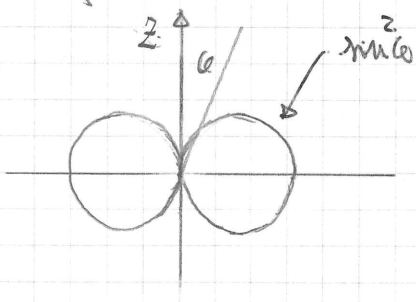
C) CALCULATE THE POWER DENSITY IN A POINT Q AT A DISTANCE $R = 1\,km$ FROM THE ORIGIN, IN A DIRECTION (θ,φ) GIVING $i(\theta,\varphi)=1$ (I.E. IN A MAXIMUM DIRECTION) BY ASSUMING THAT THE POWER RADIATED BY THE ANTENNA IS $P = 100\,W$

SOLUTION

A) FROM THE TEXT OF THE PROBLEM WE LEARN THAT $|F(\theta,\varphi)|^2 = i(\theta,\varphi)$ AND WE OBSERVE THAT THE POWER PATTERN IS OBTAINED BY MULTIPLYING THE POWER PATTERN OF THE SHORT DIPOLE $\sin^2\theta$ BY $|\cos\varphi|$.

$|F(\theta,\varphi)|^2 = 1$ FOR $\theta = \pi/2$ AND $\varphi = 0$ OR $\varphi = \pi$

FOR $\varphi = 0$ (IN THE PLANE $\varphi = 0$)



THE DIRECTIVITY IS EASILY CALCULATED FROM THE POWER PATTERN

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\iint_{4\pi} |F|^2 \, d\Omega} = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} |F(\theta,\varphi)|^2 \sin\theta \, d\theta \, d\varphi}$$

$$D = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} \sin^3\theta \, |\cos\varphi| \, d\theta \, d\varphi}$$

SINCE WE ARE DEALING WITH A DOUBLE INTEGRAL OF A FUNCTION WHICH IS THE PRODUCTION OF A FUNCTION OF θ AND A FUNCTION OF φ, THE DOUBLE INTEGRAL CAN BE REARRANGED AS THE PRODUCT OF TWO ONE VARIABLE INTEGRALS

$$\int_0^{2\pi}\int_0^{\pi} \sin^3\theta \, |\cos\varphi| \, d\theta \, d\varphi = \int_0^{2\pi} |\cos\varphi| \, d\varphi \int_0^{\pi} \sin^3\theta \, d\theta$$

$$\int_0^{2\pi} |\cos\varphi| \, d\varphi = 4\int_0^{\frac{\pi}{2}} \cos\varphi \, d\varphi = 4\left(\sin\varphi\right|_0^{\frac{\pi}{2}} = 4$$

$$\int_0^{\pi} \sin^3\theta \, d\theta = \int_0^{\pi} \sin^2\theta \, \sin\theta \, d\theta = \int_0^{\pi} \left(1-\cos^2\theta\right) \sin\theta \, d\theta$$

WE CAN CHANGE THE INTEGRATION VARIABLE $\cos\theta = x$ $\quad d\cos\theta = -\sin\theta \, d\theta = dx$

AND WE OBTAIN $\displaystyle\int_0^{\pi} \left(1-\cos^2\theta\right) \sin\theta \, d\theta = \int_1^{-1} \left(1-x^2\right)\left(-dx\right) = \int_{-1}^{1} \left(1-x^2\right) dx =$

$$= \left(x - \frac{x^3}{3}\right|_{-1}^{1} = 2 - \frac{2}{3} = \frac{4}{3}$$

AND FINALLY WE CAN WRITE $\displaystyle\int_0^{2\pi}\int_0^{\pi} \sin^3\theta \, |\cos\varphi| \, d\theta \, d\varphi = 4 \cdot \frac{4}{3} = \frac{16}{3}$

THE DIRECTIVITY IS $\displaystyle D = \frac{4\pi}{\frac{16}{3}} = \frac{3}{4}\pi \simeq 2{,}356$

B) THE GAIN IS OBTAINED BY MULTIPLYING THE DIRECTIVITY BY THE RADIATION EFFICIENCY $G = D e_r = \frac{3}{4}\pi\ 0{,}75 \simeq 1{,}767$

GAIN IN dB IS $G_{dB} = 10 \log_{10} G \simeq 2{,}473\ dB$ ($\log_{10}$ IS THE BASE 10 LOGARITHM)

WE CAN ALSO OBSERVE THAT

$D_{dB} = 10 \log_{10} D = 3{,}7221\ dB$ $\quad e_{r\,dB} = 10 \log_{10} e_r = -1{,}2494\ dB$

$G_{dB} = D_{dB} + e_{r\,dB} \simeq 2{,}473\ dB$

C) SINCE THE POINT Q IS IN THE MAXIMUM DIRECTION, THE GAIN IN THAT DIRECTION IS G, AS CALCULATED IN THE PREVIOUS POINT B), AND IN THE SAME DIRECTION THE DIRECTIVITY IS D, AS CALCULATED IN A)

SINCE THE TEXT IS REFERRING TO THE RADIATED POWER, WE HAVE TO USE D TO CALCULATE THE POWER DENSITY S

$$D = \frac{U_m}{\frac{P}{4\pi}} = \frac{\frac{U_m}{R^2}}{\frac{P}{4\pi R^2}} = \frac{S}{\frac{P}{4\pi R^2}}$$

$$S = \frac{P}{4\pi R^2} D = \frac{100}{4\pi\ 10^6}\ 2{,}356 = 18{,}75\ \mu W/m^2$$
$$\stackrel{!}{=} 18{,}75\ 10^{-6}\ W/m^2$$

WE OBSERVE THAT THE POWER DENSITY CAN BE CALCULATED FROM THE INPUT POWER P_{IN} (AT THE ANTENNA PORT) BY RESORTING TO THE FOLLOWING FORMULA

$$S = \frac{P_{IN}}{4\pi R^2} G = \frac{P_{IN}}{4\pi R^2} e_r D = \frac{P}{4\pi R^2} D$$

AND OBVIOUSLY $P = P_{IN}\, e_r$