Fundamentals on coherent optics: linear propagation in optical waveguides exercice 2 correction

1: region Zext = optical cladding ; region Zin = core

2: $NA = (n_1^2 - n_2^2)^{1/2} = (1.463^2 - 1.458^2)^{1/2} = 0.12$

3: the volume of silice is not changed.

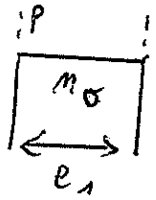
$\Rightarrow$ volume of the preform V_p = Volume of the fiber V_f

$$\Leftrightarrow \frac{\pi D_p^2}{4} L_p = \frac{\pi D_f^2}{4} L_f$$ (D → Diameter ; index p → preform ; L → length ; index f → fiber)

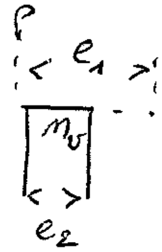
$$\Leftrightarrow L_f = L_p \left(\frac{D_p}{D_f}\right)^2 = 0{,}5 \left(\frac{3\,10^{-2}}{125\,10^{-6}}\right)^2 = 28\,800\ m = 28.8\ km$$

4/ a) $\Delta = \frac{NA^2}{2n_1^2} = \frac{0.12^2}{2\times 1.463^2} \sim 3.4\ 10^{-3} < 10^{-2}$

The weak guidance approximation applies → LP. (= linearly polarized) modes can be considered. In a LP mode at given time and place (t and z fixed), the polarization remains linear (= in a given direction) in all the cross section.

b) 

$\delta\varphi_1 = \frac{2\pi}{\lambda_T} n_v e_1$

c) 

$\delta\varphi_2 = \frac{2\pi}{\lambda_T}[n_v . e_2 + 1.(e_1 - e_2)]$

d) $\Delta\varphi = \delta\varphi_1 - \delta\varphi_2 = \frac{2\pi}{\lambda_T}[n_v e_1 - n_v e_2 - (e_1 - e_2)] = \frac{2\pi}{\lambda_T}(n_v - 1)(e_1 - e_2) = \frac{2\pi}{\lambda_T}(n_v - 1)\Delta e$

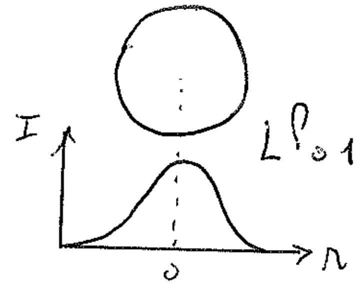
5/ a) $\frac{d\Delta\varphi}{d\lambda} = \frac{-2\pi}{\lambda^2}(n_v - 1)\Delta e \Big|_{\lambda=\lambda_T} \Rightarrow d\lambda = \frac{-\lambda_T^2}{2\pi(n_v - 1)\Delta e} d\Delta\varphi$

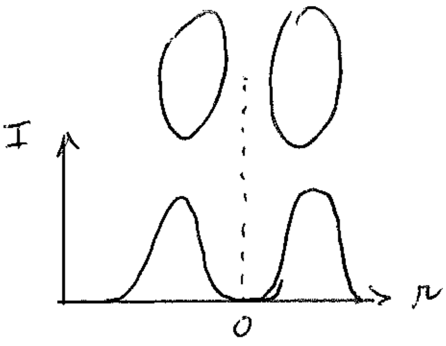
b) $\Delta\lambda = \frac{\lambda_T^2}{2\pi(n_v - 1)\Delta e} \times 2\pi = \frac{\lambda_T^2}{(n_v - 1)\Delta e}$

c) $\Delta e = e_1 - e_2 = 0{,}5\ mm$ $n_v = 1{,}5$ $\lambda_{T1} = 1.1\ \mu m$

$$\Delta\lambda = \frac{(1{,}1\ 10^{-6})^2}{(1.5 - 1)\times 0{,}5\ 10^{-3}} = 4.8\ 10^{-9} m = 4.8\ nm$$

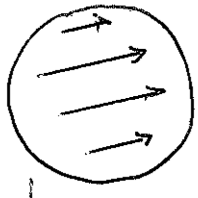
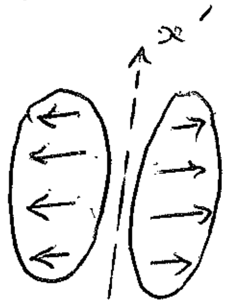
6/ a) The fiber is able to guide only two modes. They are the modes with the lowest cutoff spatial frequency, i.e. LP_{01} mode ($V_c = 0$) and LP_{11} mode ($V_c = 2.405$), (see the dispersion curves, with $B = 0$ and the table of zeros of J_ν Bessel functions).





b) Generally speaking, the direction of the polarization of the field is the same all over the cross section of the mode, at a given z, at a given time. This direction depends on the polarization of the input beam and on the birefringence of the fiber.

Let us assume that it is $\longleftrightarrow$:

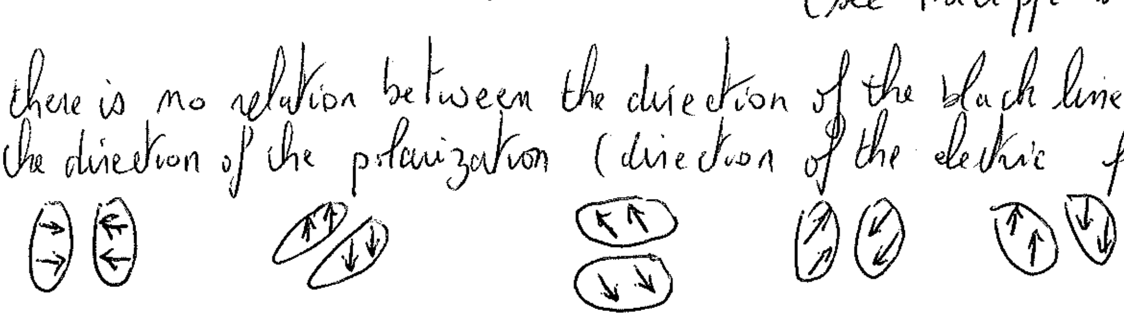
LP_{01} mode :  ; LP_{11} mode 

Let us note that the direction x' of the black line of the LP_{11} mode depends on the field distribution Ψ_e of the exciting beam. It is the one for which the normalized integral overlap

$$\alpha^2 = \frac{\left|\iint \Psi_e \, LP_{11} \, ds\right|^2}{\iint |\Psi_e|^2 ds \iint |LP_{11}|^2 ds}$$

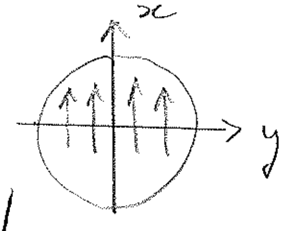
is maximum (see Philippe DiBin lessons).

$\Rightarrow$ there is no relation between the direction of the black line and the direction of the polarization (direction of the electric field):

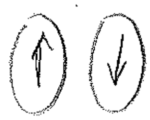
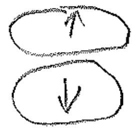
 is possible!

7/ $\Delta\varphi_1 = 2m\pi \rightarrow$ no additional phase shift between the upper and the lower parts of the beam after crossing the plates

$\Rightarrow$ the exciting beam (after the plates) is an even beam



The LP_{01} mode (which is even ) can be excited

The LP_{11} mode (which is odd  or )

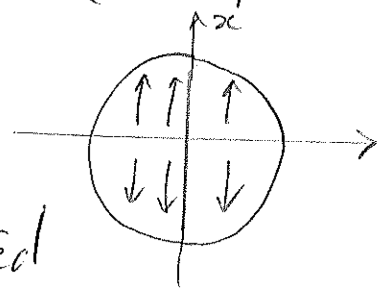
is not excited at all $\Rightarrow \alpha^2 \big|_{LP_{11}\text{ mode}} = 0$

8/ wavelength shift $1102.4 - 1100 = 2.4\text{ nm} = \frac{4.8}{2}\text{ nm}$

$= \frac{1}{2}(\Delta\lambda$ of question 5c$)$

$\Rightarrow$ the phase difference between the upper part and the lower part of the beam, after the plates is $\Delta\varphi_2 = \Delta\varphi_1 \pm \pi = (2m \pm 1)\pi$

$\rightarrow$ The exciting beam (after the plates) is odd



The LP_{11} mode () will be excited

The LP_{01} mode (which is even ) is not excited at all

$\Rightarrow \alpha^2 \big|_{LP_{01}\text{ mode}} = 0$

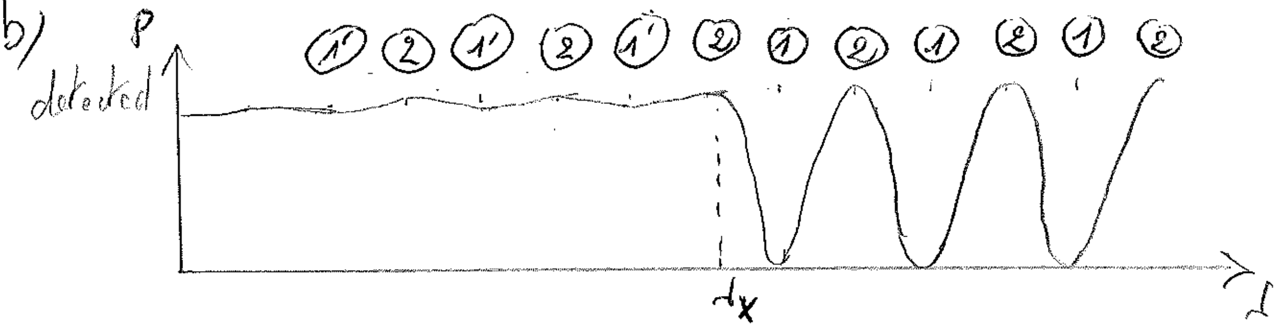
9/ a) beyond a certain wavelength, the LP_{11} mode is no more guided

thus, when $\Delta\varphi = (2m'+1)\pi$ — LP_{01} not excited; LP_{11} mode excited but not guided) no power detected at the fiber output ①

when $\Delta\varphi = 2m'\pi$ — LP_{01} excited (and always guided); LP_{11} not excited) power detected ②

① and ② : fluctuations of the detected power when λ is swept

b)



② = LP_{01} excited and guided
①' = LP_{11} excited and guided
① = LP_{11} excited but NOT guided (P detected ~ 0)

λ_X is the wavelength corresponding to the cutoff wavelength λ_c of the LP_{11} mode

→ $\lambda_X = \lambda_c$: the device allows measuring the cutoff wavelength of the LP_{11} mode → limit of the single mode regim

in other words: below λ_c, the LP_{01} and LP_{11} modes are excited alternately when λ is swept and, as they are both guided, $P(\lambda)$ is relatively stable

Above λ_c, the LP_{01} and LP_{11} modes are excited alternately when λ is swept and, as LP_{01} mode is guided but not the LP_{11} mode, $P(\lambda)$ deeply fluctuates.

1d) a $\lambda_c / LP_{11} = 1240\ nm$ @ λ_c : $V = 2.405$

$$\frac{2\pi}{\lambda_c} a\, NA = 2.405 \Rightarrow a = \frac{2.405\, \lambda_c}{2\pi\, NA} = \frac{2.405 \times 1.24}{2\pi \times 0.12} = 3.96\ \mu m \simeq 4\ \mu m$$

b) $V = \frac{2\pi}{\lambda_{T_3}} a\, NA = \frac{2\pi}{0.633} \times 4 \times 0.12 \simeq 4.75$

On the set of dispersion curves we can see that $V > V_c$ for the modes LP_{01} LP_{11} LP_{21} and LP_{02}. Among these modes:

c) the lowest order mode M_L is the LP_{01} mode (highest B)
the highest order mode M_H is the LP_{02} mode (lowest B)

For $V = 4.75$ we can read on the normalized dispersion curves:

$B(LP_{01}) \simeq 0.82$ and $B(LP_{02}) \simeq 0.13$

$$B = \frac{\beta^2 - k_0^2 n_2^2}{k_0^2 \underbrace{(n_1^2 - n_2^2)}_{NA^2}} \quad \text{with } \beta = k_0 n_e \Rightarrow B = \frac{n_e^2 - n_2^2}{NA^2}$$

$$\Rightarrow n_e = (B\, NA^2 + n_2^2)^{1/2}$$

calculations: $n_e(LP_{01}) = (0.82 \times 0.12^2 + 1.458^2)^{1/2} = 1.462$

$n_e(LP_{02}) = (0.13 \times 0.12^2 + 1.458^2)^{1/2} = 1.486$

d) $v_\varphi = \frac{c}{n_e}$ $v_\varphi(M_L) = v_\varphi(LP_{01}) = \frac{c}{n_e(LP_{01})} = \frac{3\,10^8}{1.462} \simeq 2.052\,10^8\ ms^{-1}$

$v_\varphi(M_H) = v_\varphi(LP_{02}) = \frac{c}{n_e(LP_{02})} = \frac{3\,10^8}{1.486} \simeq 2.057\,10^8\ ms^{-1}$