

Basics of Active and Nonlinear HF Electronics – Tutorial

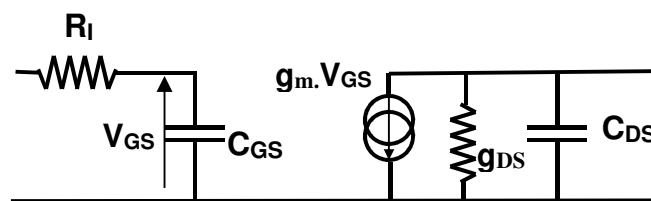
We consider a FET and its nonlinear drain current source I_{DS} modeled by the below equation as a function of its control voltages V_{GS} and V_{DS} :

$$I_{DS} = I_{DSS} \left[1 - \frac{V_{GS}(t)}{V_P} \right]^2 (1 + K \cdot V_{DS}(t))$$

The model is accurate in the variation range $-1.5V < V_{GS} < 0V$; $1V < V_{DS} < 7V$
with $I_{DSS} = 60 \text{ mA}$; $V_P = -1.5 \text{ V}$; $K = 0.05$; $C_{GS} = 0.5 \text{ pF}$; $C_{DS} = 0.1 \text{ pF}$; $R_I = 4 \Omega$.

The FET is biased at $V_{GS0} = -0.75 \text{ V}$ and $V_{DS0} = 4 \text{ V}$

The small signal linear model of the FET in these conditions is shown below:



Study of parallel-cells of FET

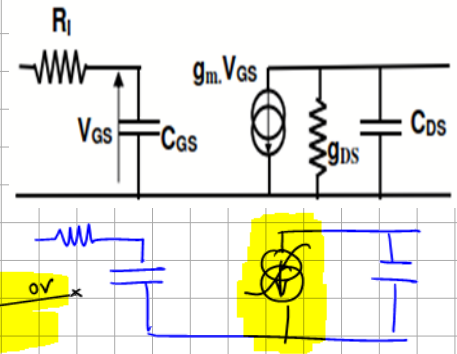
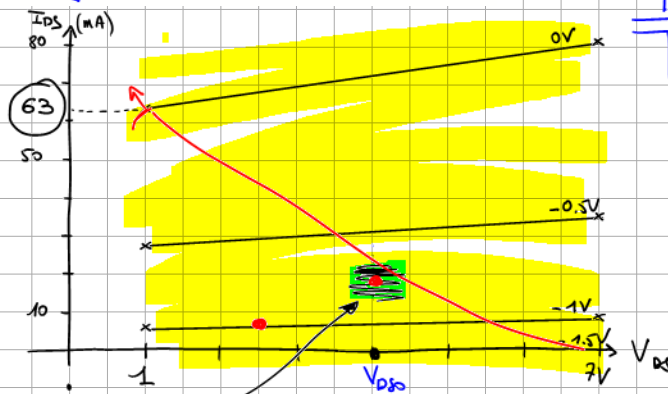
- 1) Draw the I_{DS} - V_{DS} curves in the variation range
- 2) Determine the values of transconductance g_m and drain conductance g_{DS} at the selected bias point.
- 3) Determine the ^{scaling rules} small-signal linear model corresponding to n parallel-cells of the FET. Determine the variation laws that give equivalent elements ($C_{GS}(n)$...) of the parallel-cells as a function of the number of cells n and the equivalent elements ($C_{GS}(1)$...) of a single-cell FET.
- 4) { Determine the maximum power gain G_{MAX} for an ideal power matching at small-signal (linear) level. Determine its cutoff frequency f_c and its maximum frequency f_{MAX} .
- 5) After determining the optimum power load in large signal class A operation, determine the maximum gain G_{MAX} in this loading condition. Estimate the maximum output power value in class A. Compare these values to G_{MAX} and its associated maximum output power when the FET is ideally matched to give optimum gain in small signal operation.
- 6) Same questions 4) and 5) in the case of n -parallel FETs

$$I_{DS} = I_{DSS} \left[1 - \frac{V_{GS}(t)}{V_P} \right]^2 (1 + K \cdot V_{DS}(t)) \quad *$$

The model is accurate in the variation range $-1.5V < V_{GS} < 0V$; $1V < V_{DS} < 7V$
 with $I_{DSS} = 60 \text{ mA}$; $V_P = -1.5 \text{ V}$; $K = 0.05$; $C_{GS} = 0.5 \text{ pF}$; $C_{DS} = 0.1 \text{ pF}$; $R_i = 4 \Omega$.

The FET is biased at $V_{GS0} = -0.75 \text{ V}$ and $V_{DS0} = 4 \text{ V}$ Bias point

V_{GS}	V_{DS}	$\left(1 - \frac{V_{GS}}{V_P}\right)^2$	$1 + K V_{DS}$	I_{DS}
0V	1V	1	1.05	63 mA
	7V		1.35	81 mA
-0.5V	1V	$\left(1 - \frac{-0.5}{-1.5}\right)^2 = \frac{4}{9}$	1.05	28 mA
	7V		1.35	36 mA
-1V	1V	$\left(1 - \frac{-1}{-1.5}\right)^2 = \frac{1}{9}$	1.05	7 mA
	7V		1.35	9 mA



$$f(x, y) = f(x_0, y_0) + df$$

$$\begin{cases} x = x_0 + dx \\ y = y_0 + dy \end{cases}$$

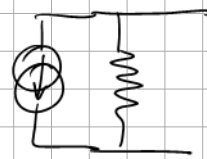
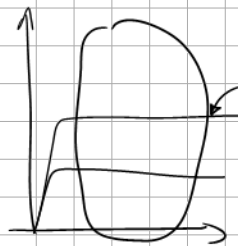
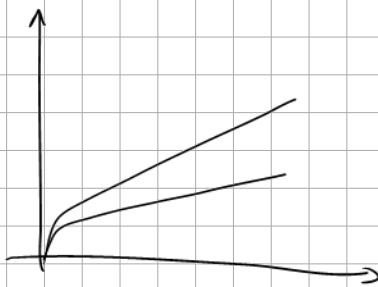
$$df = \left(\frac{\partial f}{\partial x} \right)_{(x_0, y_0)} dx + \left(\frac{\partial f}{\partial y} \right)_{(x_0, y_0)} dy$$

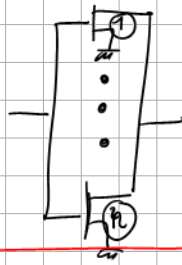
$$I_{DS} = I_{DSS} \left[1 - \frac{V_{GS}(t)}{V_P} \right]^2 (1 + K \cdot V_{DS}(t))$$

$$g_m(V_{GS}, V_{DS}) = \left(\frac{\partial I_{DS}}{\partial V_{GS}} \right)_{(V_{GS0}, V_{DS0})} = -2 \frac{I_{DSS}}{V_P} \left(1 - \frac{V_{GS0}}{V_P} \right) (1 + K V_{DS0}) = -2 \frac{60 \text{ mA}}{-1.5} \left(1 - \frac{-0.75}{-1.5} \right) (1 + 0.05 \times 4) = 48 \text{ m}\Omega^{-1}$$

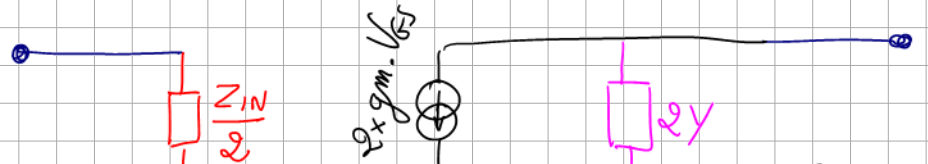
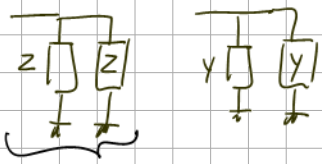
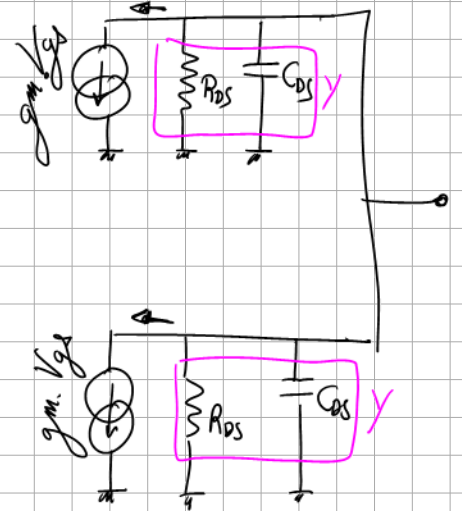
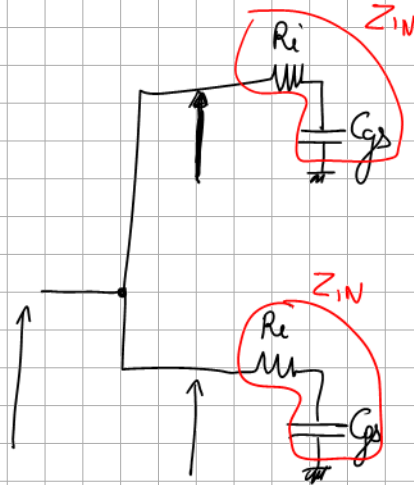
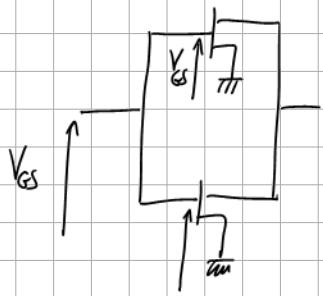
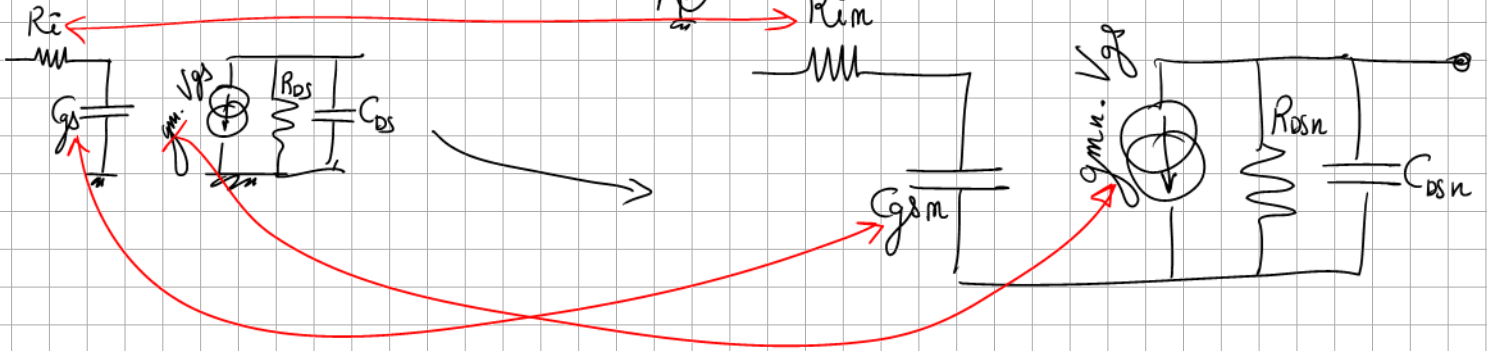
$$g_{DS} = \left(\frac{\partial I_{DS}}{\partial V_{DS}} \right)_{(V_{GS0}, V_{DS0})} = K I_{DSS} \left(1 - \frac{V_{GS0}}{V_P} \right)^2 = 0.005 \times 60 \text{ mA} \left(1 - \frac{-0.75}{-1.5} \right)^2 = 0.75 \text{ m}\Omega^{-1}$$

$$R_{DS} = \frac{1}{g_{DS}} = 1.3 \text{ k}\Omega$$



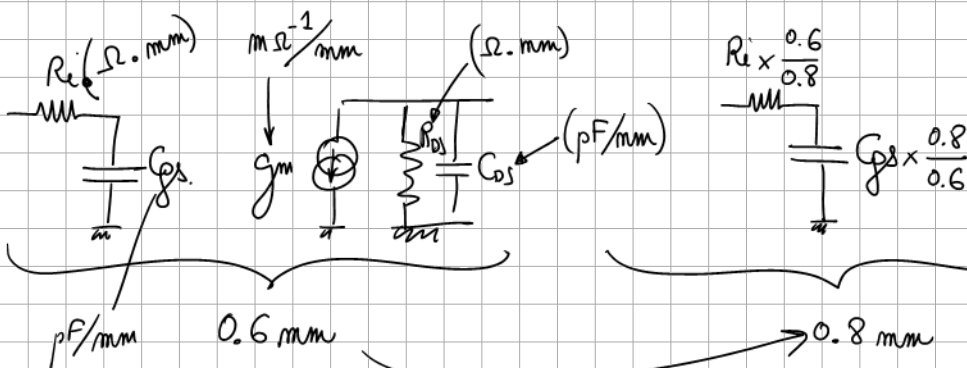


scaling rules



$$Z_{IN} = R_i + \frac{1}{j\omega C_{GS}} \rightarrow \frac{Z_{IN}}{2} = \frac{R_i}{2} + \frac{1}{j\omega 2C_{GS}} = R + \frac{1}{j\omega C}$$

$$Y = \frac{1}{R_{DS}} + j\omega C_{DS} \rightarrow 2Y = \frac{2}{R_{DS}} + j\omega 2C_{DS} = \frac{1}{R_p} + j\omega C_p \rightarrow R_p = \frac{R_{DS}}{2} \quad C_p = 2C_{DS}$$

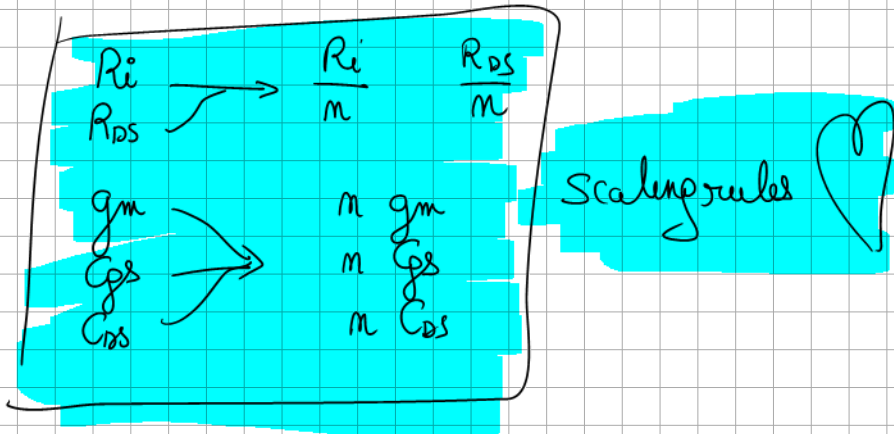


m

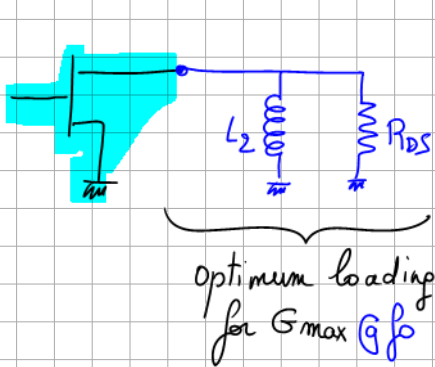
Technology :

$$\left. \begin{aligned} R_i &= 1 \Omega \cdot \text{mm} \times \\ R_{DS} &= 125 \Omega \cdot \text{mm} \times \\ C_{gs} &= 2 \text{ pF} / \text{mm} \times \end{aligned} \right\}$$

$$\begin{aligned} T_1 &\rightarrow 8 \times 75 \mu\text{m} = 0.6 \text{ mm} \\ R_{e1} &= \frac{1 \Omega \cdot \text{mm}}{0.6} \\ R_{DS1} &= \frac{125}{0.6} \\ C_{gs1} &= 2 \text{ pF} \times 0.6 \end{aligned}$$



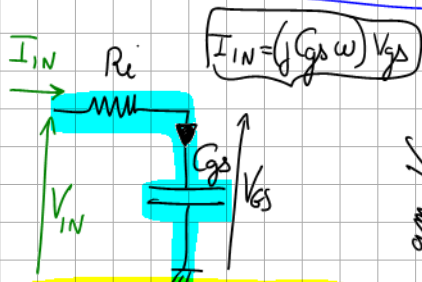
- 4) Determine the maximum power gain G_{MAX} for an ideal power matching at small-signal (linear) level. Determine its cutoff frequency f_c and its maximum frequency f_{MAX} .
- 5) After determining the optimum power load in large signal class A operation, determine the maximum gain G_{MAX} in this loading condition. Estimate the maximum output power value in class A. Compare these values to G_{MAX} and its associated maximum output power when the FET is ideally matched to give optimum gain in small signal operation.
- 6) Same questions 4) and 5) in the case of n-parallel FETs



$$Y_{CDS} = j C_{DS} \omega_0 \quad \oplus \quad Y_{L2} = \frac{-j}{L_2 \omega_0} = 0$$

$$C_{DS} \omega_0 = \frac{1}{L_2 \omega_0}$$

$$L_2 = \frac{1}{C_{DS} \omega_0^2}$$



$$P_{IN} = \frac{1}{2} \text{Re}(V_{IN} I_{IN}^*)$$

$$P_{OUT} = \frac{1}{2} \text{Re}(V_{OUT} I_{OUT}^*)$$

$$I_{IN} = (j C_{gs} \omega) V_{GS}$$

$$P_{IN} = \frac{1}{2} \text{Re} \left(\left[R_i + \frac{1}{j C_{gs} \omega} \right] I_{IN} \times I_{IN}^* \right) = \frac{1}{2} R_i |I_{IN}|^2 = \frac{1}{2} R_i |j C_{gs} \omega V_{GS}|^2$$

$$= \frac{1}{2} R_i C_{gs}^2 \omega^2 |V_{GS}|^2$$

$$P_{OUT} = \frac{1}{2} \text{Re} \left(V_{OUT} \frac{V_{OUT}^*}{R_{DS}} \right) = \frac{1}{2} \frac{|V_{OUT}|^2}{R_{DS}} = \frac{1}{2} \frac{g_m^2 R_{DS}^2}{4 R_{DS}} |V_{GS}|^2$$

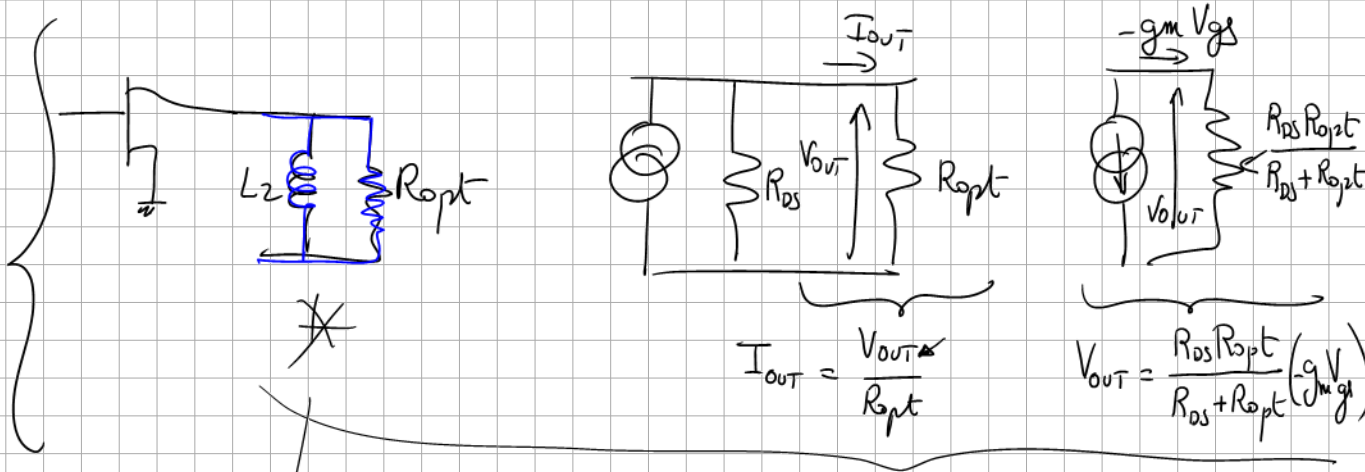
$$= \frac{1}{8} g_m^2 R_{DS} |V_{GS}|^2$$

$$G_{max} = \frac{P_{OUT}}{P_{IN}} = \frac{g_m^2 R_{DS}}{4 R_i C_{gs}^2 \omega^2}$$

$$V_{OUT} = - \frac{g_m R_{DS}}{2} V_{GS}$$

POWER

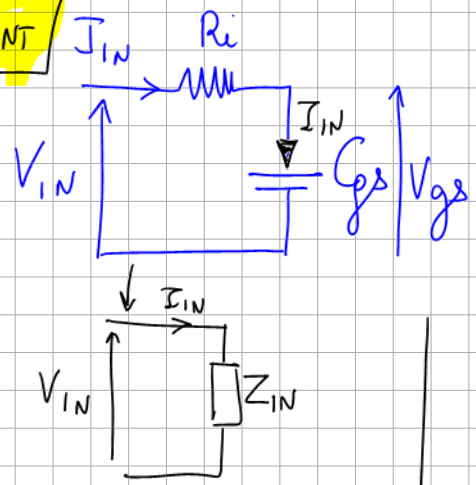
MATCHING



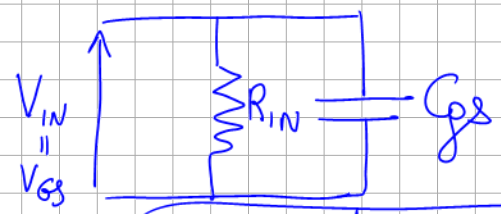
$$G_p(R_{opt})$$

$$G_{max} = G_p(R_{ds})$$

INPUT EQUIVALENT

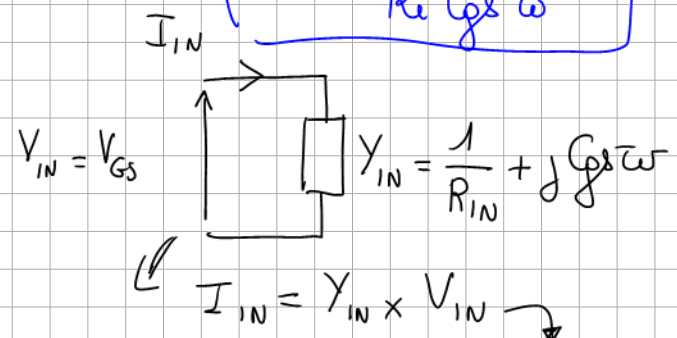


$\equiv G\omega$



$$R_{IN} = \frac{1}{R_i G_{ps}^2 \omega^2}$$

$$V_{IN} = Z_{IN} \times I_{IN} \Leftarrow V_{IN}$$



$$P_{IN} = \frac{1}{2} \text{Re}(V_{IN} I_{IN}^*) = \frac{1}{2} \text{Re}(Z_{IN} I_{IN} I_{IN}^*)$$

$|I_{IN}|^2$
real

$$P_{IN} = \frac{1}{2} \text{Re}(V_{IN} I_{IN}^*) = \frac{1}{2} \text{Re}(V_{IN} Y_{IN}^* V_{IN}^*)$$

$$= \frac{1}{2} \text{Re}(Y_{IN}^*) \times |V_{gs}|^2$$

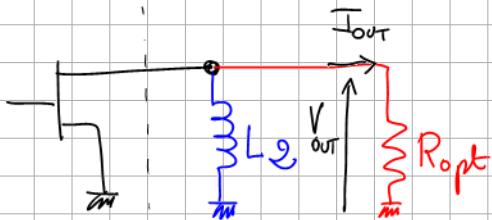
$$= \left(\frac{1}{2}\right) \frac{1}{R_{IN}} \times |V_{gs}|^2$$

$$P_{IN} = \frac{1}{2} R_i G_{ps}^2 \omega^2 |V_{gs}|^2$$

$$P_{IN} = \frac{1}{2} R_i G_{ps}^2 \omega^2 |V_{gs}|^2$$



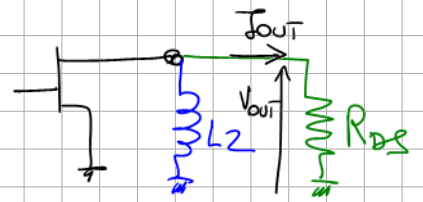
INPUT POWER



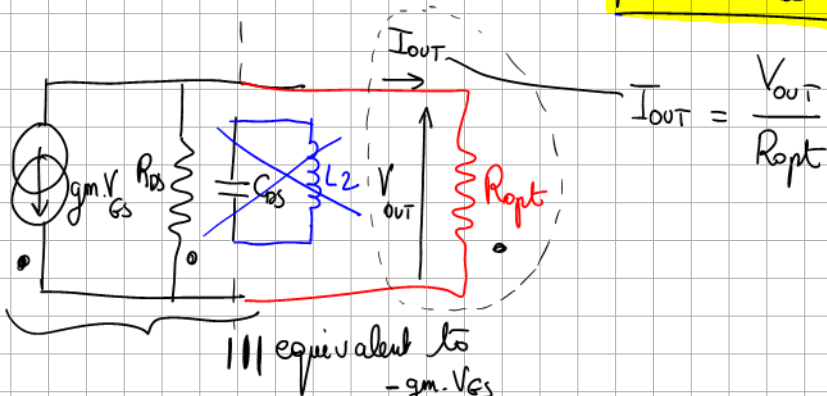
\downarrow
 $GP(R_{opt})$

In both cases,

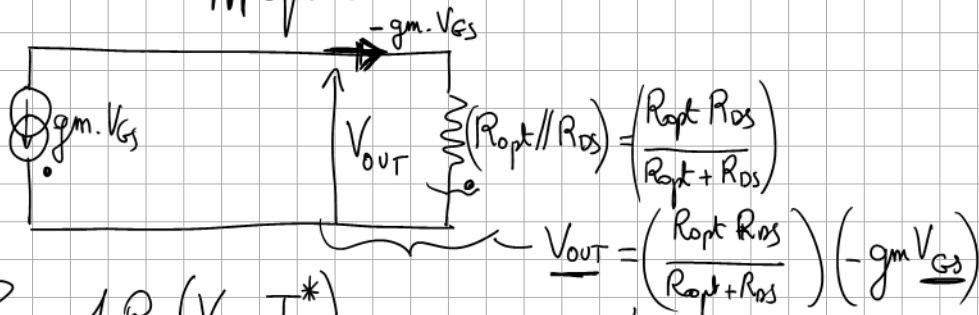
$$L_2 = \frac{1}{G_{ss} \omega_o^2}$$



\downarrow
 $G_{max} = GP(R_{DS})$



equivalent to



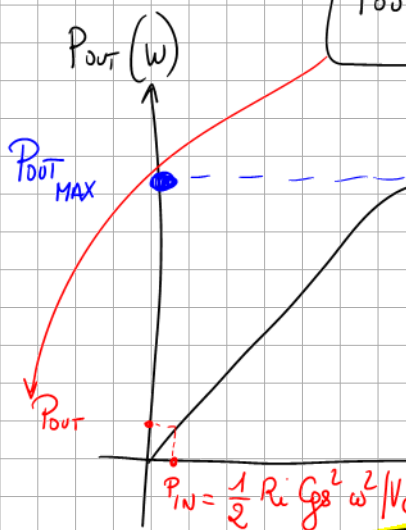
$$P_{out} = \frac{1}{2} \operatorname{Re}(V_{out} I_{out}^*) =$$

$$= \frac{1}{2} \operatorname{Re}\left(V_{out} \frac{V_{out}^*}{R_{opt}^*}\right)$$

$$= \frac{1}{2} \frac{|V_{out}|^2}{R_{opt}} = \frac{1}{2} \frac{\left(\frac{R_{opt} R_{DS}}{R_{opt} + R_{DS}}\right)^2 g_m^2 |V_{GS}|^2}{R_{opt}}$$

$$P_{out} = \frac{1}{2} R_{opt} \left(\frac{R_{DS}}{R_{opt} + R_{DS}}\right)^2 g_m^2 |V_{GS}|^2$$

$$P_{in} = \frac{1}{2} \operatorname{Re} G_{ss}^2 \omega^2 |V_{GS}|^2$$

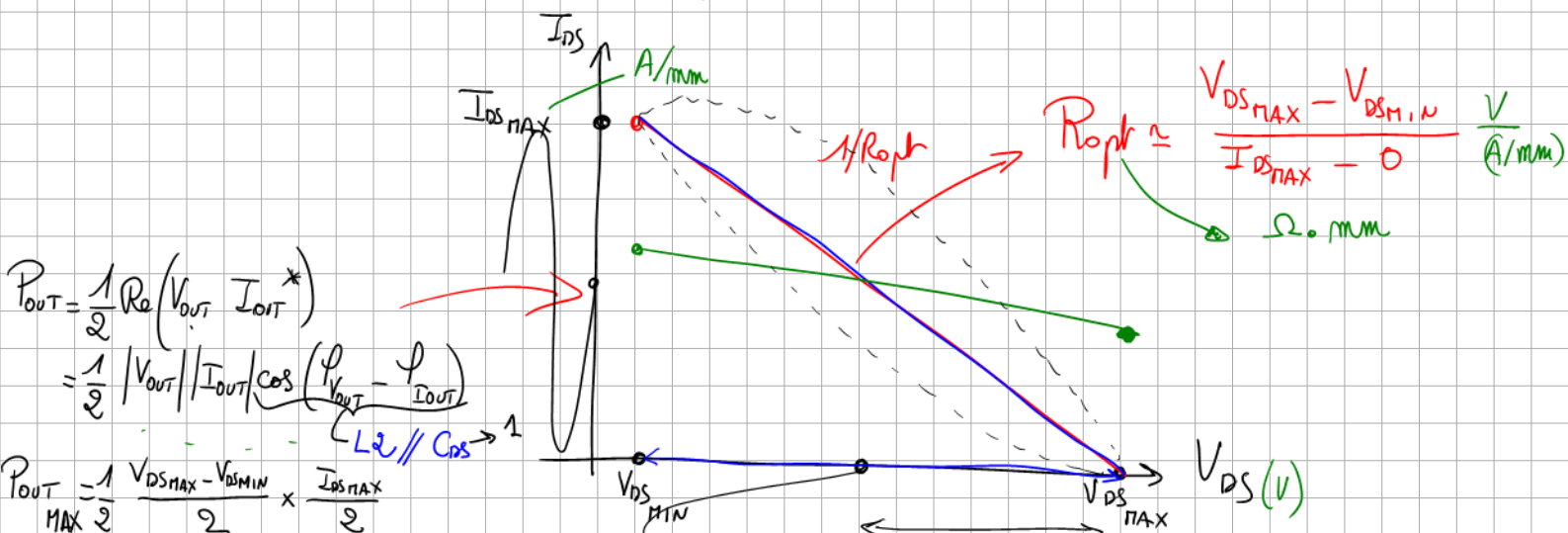
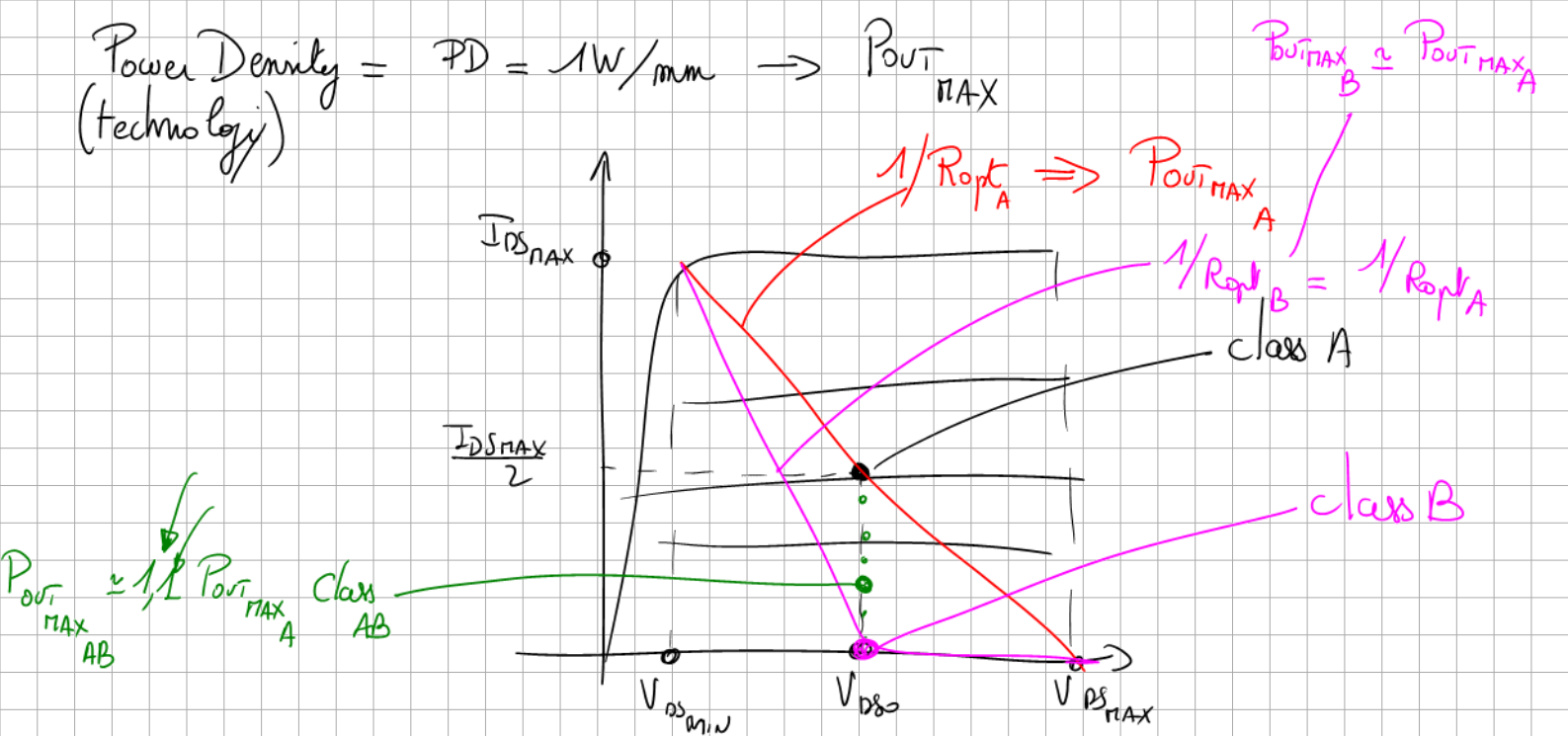


$$G_p(R_{opt}) = 4 \frac{R_{opt} R_{DS}}{(R_{opt} + R_{DS})^2} G_{max}$$

$$G_p(R_{opt}) = \frac{R_{opt} \left(\frac{R_{DS}}{R_{opt} + R_{DS}}\right)^2 g_m^2}{R_{in} G_{ss}^2 \omega^2}$$

$$G_{max} = GP(R_{DS}) = \left(\frac{1}{4}\right) \frac{R_{DS} g_m^2}{R_{in} G_{ss}^2 \omega^2}$$

Power Density = PD = 1W/mm \rightarrow $P_{OUT\ MAX}$
(technology)



$$P_{OUT\ MAX\ A} = \frac{1}{8} (V_{DS\ MAX} - V_{DS\ MIN}) I_{DS\ MAX}$$

W/mm

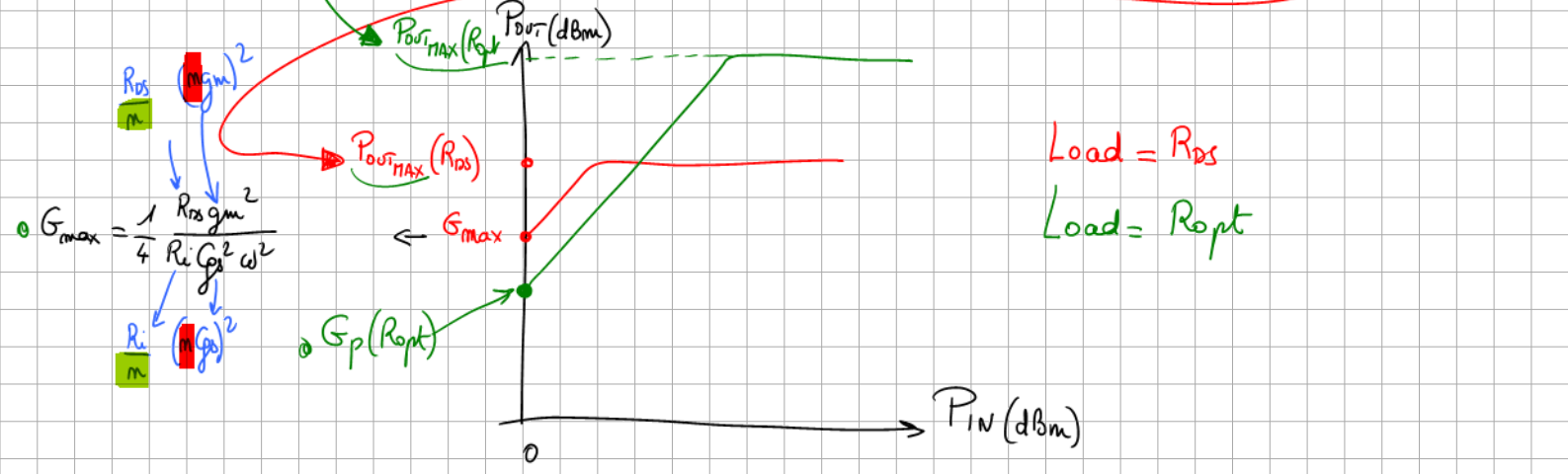
V

A/mm

$\frac{V_{DS\ MAX} - V_{DS\ MIN}}{R_{opt}}$

$P_{OUT\ MAX} = \frac{1}{8} \frac{(V_{DS\ MAX} - V_{DS\ MIN})^2}{R_{opt}}$

$P_{OUT\ MAX}(R_{DS}) = \frac{1}{8} \frac{(V_{DS\ MAX} - V_{DS\ MIN})^2}{R_{DS}}$



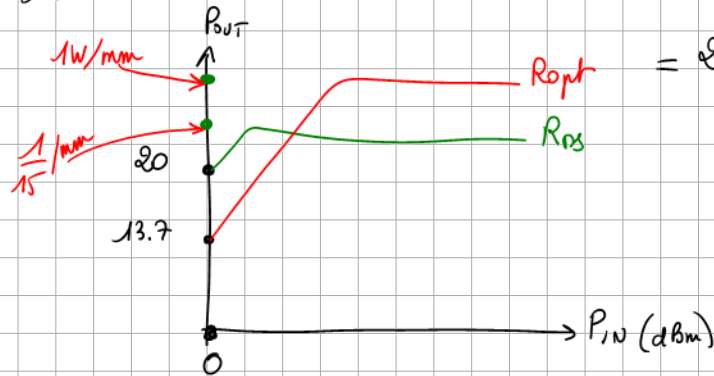
Technology \rightarrow $PD_0 = 1 \text{ W/mm}$ \leftrightarrow $R_{opt} = 10 \Omega \cdot \text{mm}$
 $R_{DS} = 150 \Omega \cdot \text{mm}$

$G_{max} = 20 \text{ dB}$
 $= 100$

$PD(R_{DS}) = \frac{R_{opt}}{R_{DS}} PD(R_{opt})$ \leftarrow $\underbrace{\frac{R_{opt}}{R_{DS}}}_{\frac{1}{15}}$

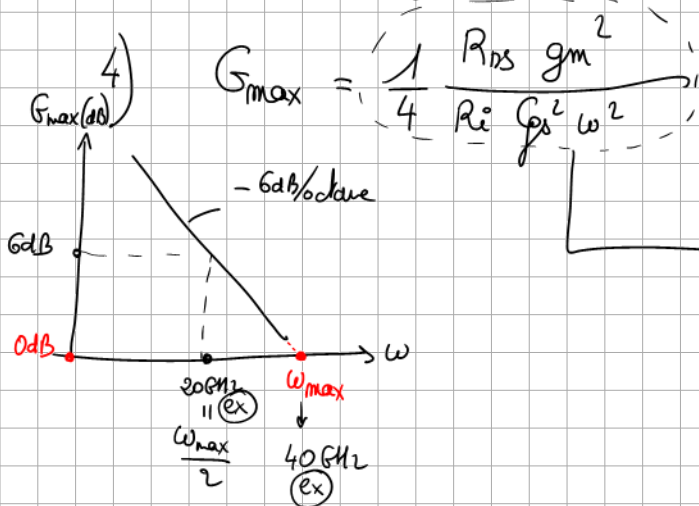
$PD(R_{DS}) = \frac{10}{150} 1 \text{ W/mm} =$

$G_p(R_{opt}) = 4 \frac{R_{opt} R_{DS}}{(R_{opt} + R_{DS})^2} G_{max}$
 $= 4 \times \frac{10 \times 150}{(10 + 150)^2} \times 100$
 $= 23.4 = 13.7 \text{ dB}$



- 4) Determine the maximum power gain G_{MAX} for an ideal power matching at small-signal (linear) level. Determine its cutoff frequency f_c and its maximum frequency f_{MAX} .
- 5) After determining the optimum power load in large signal class A operation, determine the maximum gain G_{MAX} in this loading condition. Estimate the maximum output power value in class A. Compare these values to G_{MAX} and its associated maximum output power when the FET is ideally matched to give optimum gain in small signal operation.
- 6) Same questions 4) and 5) in the case of n-parallel FETs

Numerical values of tutorial



$$f_c = \frac{g_m}{2\pi C_{gs}} = \frac{48 \cdot 10^{-3}}{2\pi \cdot 0.5 \cdot 10^{-12}} = 15 \text{ GHz}$$

$$G_{max}(\omega_{max}) \triangleq 1$$

$$\omega_{max} = \frac{1}{2} \sqrt{\frac{R_{DS}}{R_i}} \frac{g_m}{C_{gs}}$$

$$f_{max} = \frac{g_m}{4\pi C_{gs}} \sqrt{\frac{R_{DS}}{R_i}} = \frac{48 \cdot 10^{-3}}{4\pi \cdot 0.5 \cdot 10^{-12}} \sqrt{\frac{1300}{4}} = 137 \text{ GHz}$$

e.g. calculation of G_{max} @ (10 GHz)

$$G_{max}(10 \text{ GHz}) = \frac{1}{4} \frac{R_{DS} g_m^2}{R_i C_{gs}^2 \omega^2} = \frac{1}{4} \frac{1300 (48 \cdot 10^{-3})^2}{4 \times (0.5 \cdot 10^{-12} \times 2\pi \cdot 10^{10})^2}$$

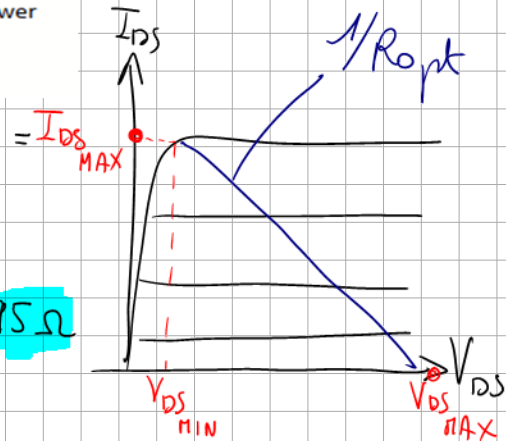
$$G_{\max}(10 \text{ GHz}) = \frac{1300 \times 48^2 \cdot 10^{-6}}{16 \times \pi^2 \cdot 10^{-4}} = \frac{13 \times 48^2}{16 \times \pi^2} = 189,67$$

$$= 10 \log(189,67) = 22,78 \text{ dB}$$

- 4) Determine the maximum power gain G_{\max} for an ideal power matching at small-signal (linear) level. Determine its cutoff frequency f_c and its maximum frequency f_{\max} .
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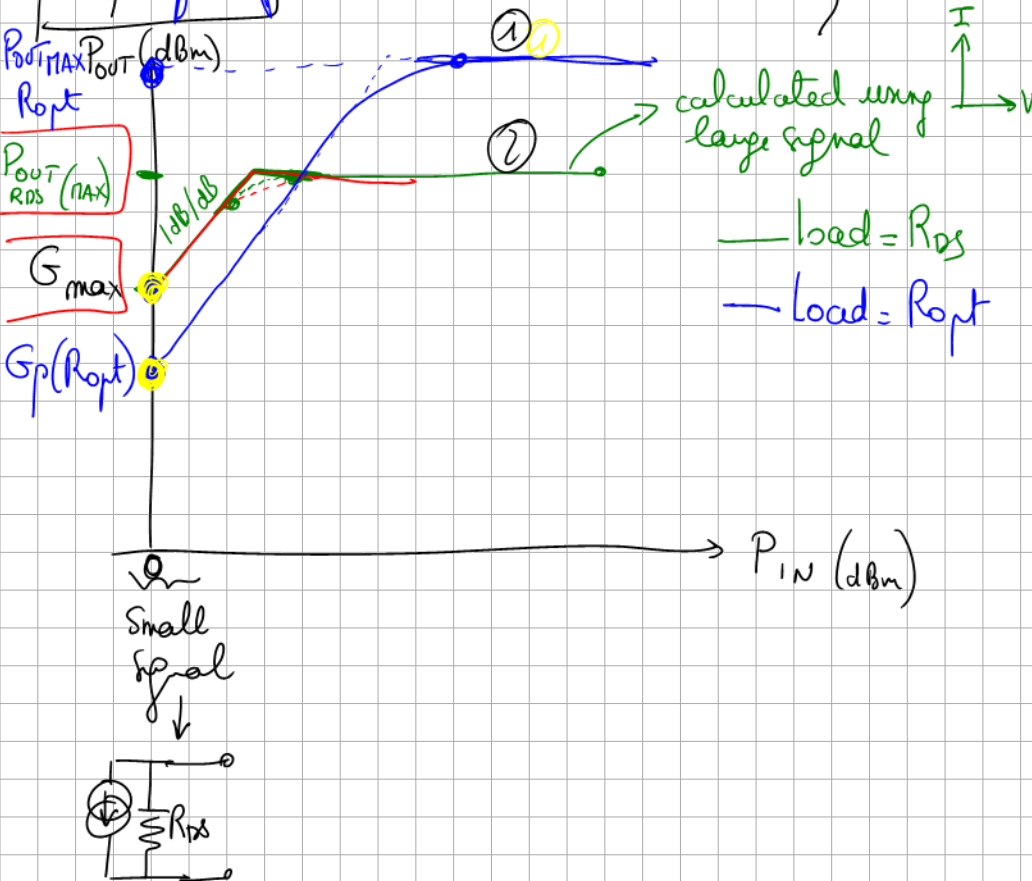
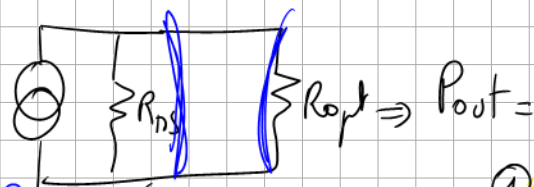
• Optimum power load = R_{opt} in class A

$$R_{opt} = \frac{V_{DS \max} - V_{DS \min}}{I_{DS \max}} = \frac{7 - 1}{63 \cdot 10^{-3}} = \frac{6000}{63} = 95 \Omega$$



What is the power gain $G_P(R_{opt})$

$$P_{IN} = \frac{1}{2} R_i G_s^2 \omega^2 |V_{gs}|^2$$



$$G_P(R_{opt})_{(10 \text{ GHz})} =$$

$$= \frac{R_{opt} \left(\frac{R_{DS}}{R_{opt} + R_{DS}} \right)^2 g_m^2}{R_i G_s^2 \omega^2}$$

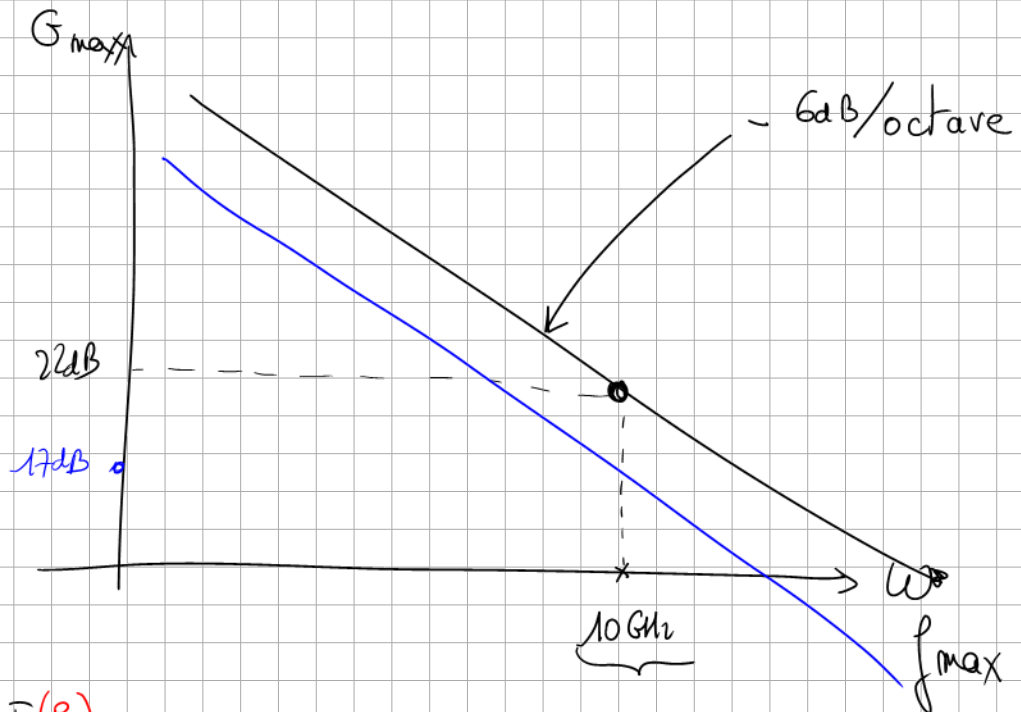
$$= \frac{95 \left(\frac{1300}{95 + 1300} \right)^2 (48 \cdot 10^{-3})^2}{4 \times (0,5 \cdot 10^{-12} \times 2\pi \cdot 10^{10})^2}$$

$$= \frac{95 \times \left(\frac{1300}{1395} \right)^2 (48)^2 \cdot 10^{-6}}{4 \times \pi^2 \cdot 10^{-2}}$$

$$= 48$$

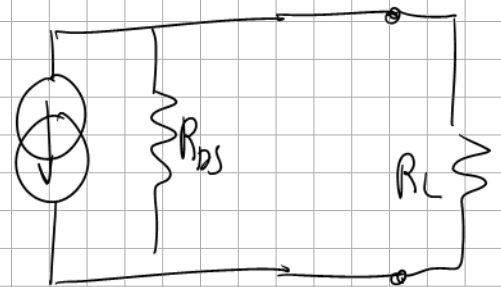
$$= 16,8 \text{ dB}$$

$GP(R_{opt})$

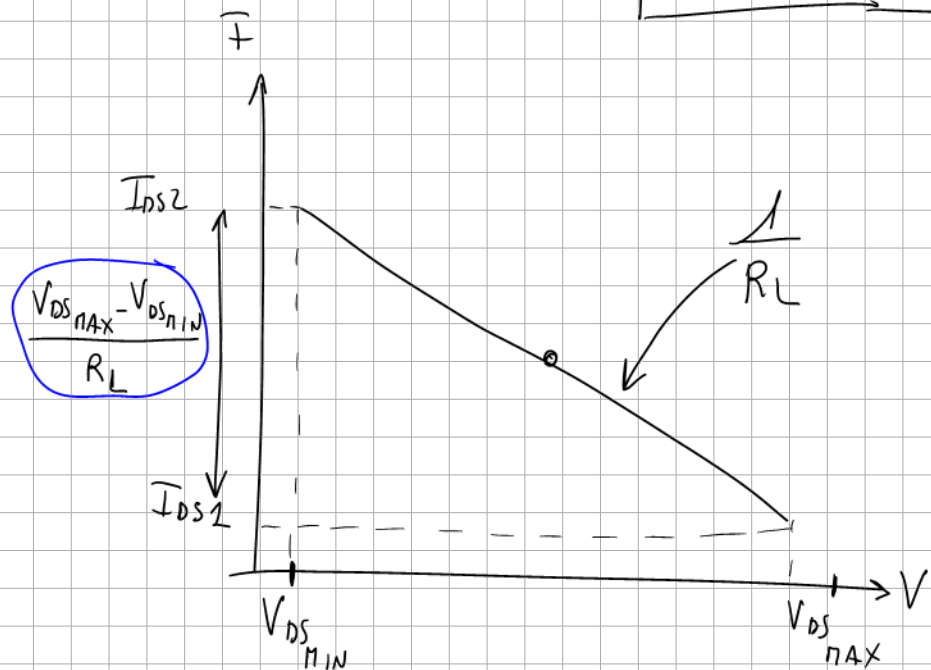


$$GP(\omega)_{(R_L)} = \frac{P_{OUT}(R_L)}{P_{IN}(\omega^2)}$$

$$P_{IN} = \frac{1}{2} R_i G_p^2 \omega^2 |V_{gs}|^2$$



$$P_{OUT} =$$

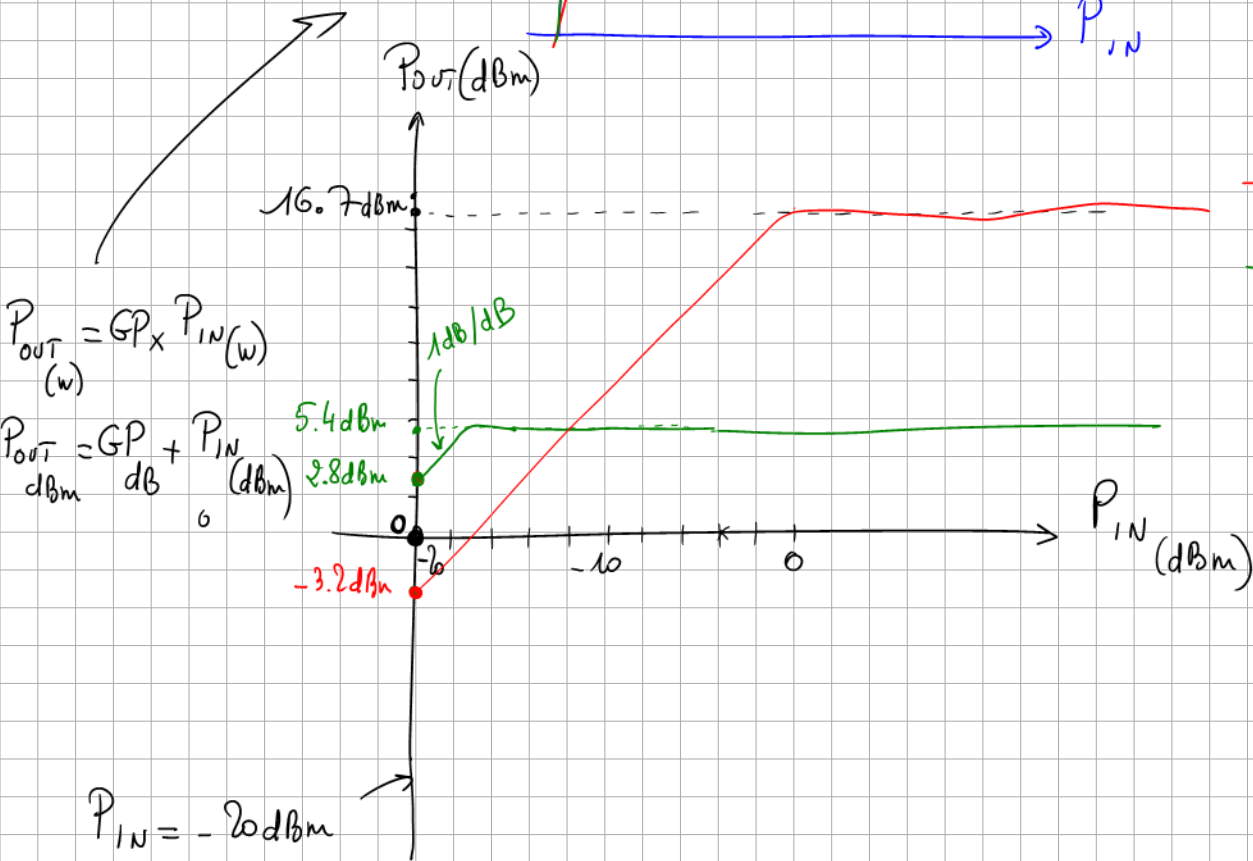
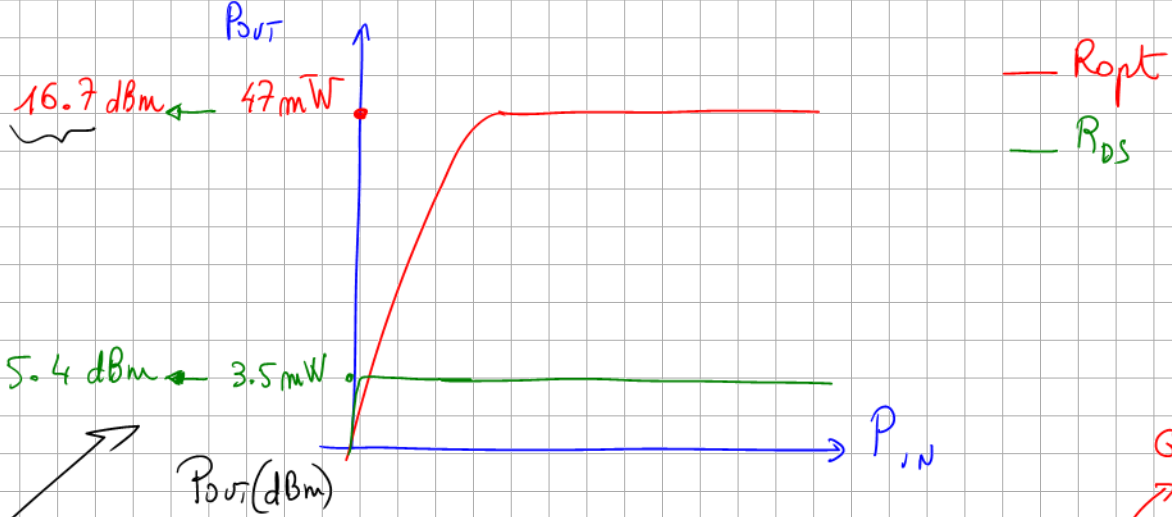


$$P_{OUT} = \frac{1}{2} \times \frac{V_{DSMAX} - V_{DSMIN}}{2} \times \frac{V_{DSMAX} - V_{DSMIN}}{2 R_L}$$

$$P_{OUT_{MAX}}(R_{opt}) = \frac{1}{2} \frac{V_{DSMAX} - V_{DSMIN}}{2} \quad \left(\frac{I_{DSMAX}}{2} \right) = \frac{1}{8} (7-1) 63 \cdot 10^{-3}$$

$$= 47 \text{ mW} =$$

$$P_{OUT_{MAX}}(R_{DS}) = \frac{1}{2} \frac{V_{DSMAX} - V_{DSMIN}}{2} \quad \frac{V_{DSMAX} - V_{DSMIN}}{2 R_{DS}} = \frac{1}{8} \frac{(7-1)^2}{1300} = 3.5 \text{ mW}$$

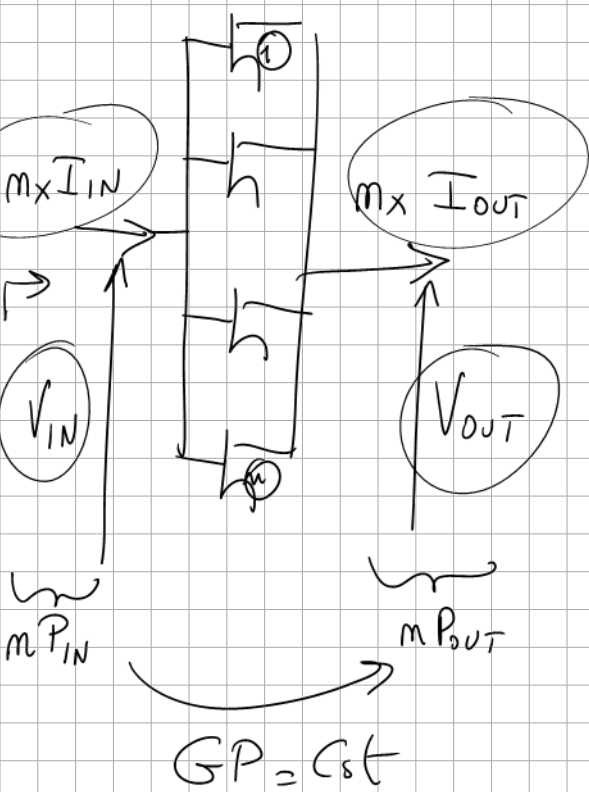


Annotations for R_{opt} and R_{ds} :

- $GP (10 \text{ GHz}) = 16.8 \text{ dB}$
- $GP_{max} (10 \text{ GHz}) = 22.8 \text{ dB}$
- -20

$$P_{out} = GP + P_{in}$$

$$= GP - 20$$



$m = 1$

m devices

R_i	\longrightarrow	R_i/m
C_{gs}	\longrightarrow	$m C_{gs}$
C_{ds}	\longrightarrow	$m C_{ds}$
R_{DS}	\longrightarrow	R_{DS}/m
g_m	\longrightarrow	$m \cdot g_m$