Lecture 2

1

- Signal/System
- Time, freq domain
- Decibels
→ Power Voltage, blocks
- Bank of filters spectrum analyzer
- Sampling aliasing
- FFT
- Digital FFT spect an.
- oversampling

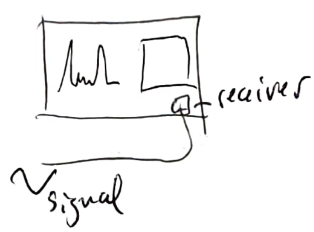
1

S1 → signals carry the info
system manage several signals
Time domain → Oscilloscope, (instrument)
represent signal as function of time

~~Freq domain~~
we can represent signal and syst in freq domain.
we represent System in freq dom → using transfer funct.
Freq domain → main instrument: spectrum analyzer

we usually represent power and no amplitude $P \sim A_{mp}^2$

S4 Spectrum measurements

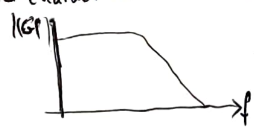


We use dB scale because
→ Very very sensitive instruments that are able to measure very low freq components
→ Very wide measuring range

S2 S3

Why measurements in freq. domain
- BW can be narrowed at will
  we can decide the region to analyze
  ↳ Reducing the effect of the noise bc we narrow BW
  ↳ Remove the freq. components outside the BW we are interested in
- Narrow-band measurements more sensitive
  ⇒ improvement signal to noise ratio
- Some systems are inherently freq. dom systems. (FDM freq div. multiplex)
- For systems not oriented to freq-dom can benefit
  ↳ measurement of BW
  ↳ complex signals analysis: split in freq component

S → network analysis
To analyse the system → transfer function
to characterize " → stimulate the system with a signal and read the response of the system
so we have the source and receiver
to provide stimulus — to measure the response



- Network analyser: Spectrum analyzer + source + software to get transfer func. having the signals in freq dom

S5 S6 S7 → signore

Def decibel $A_{(dB)} = 10 \log_{10}\left(\frac{P_2}{P_1}\right)$ ratio between to powers

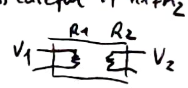
In electrical signals the power is the dissipated in a load (optical i.e. the power is absorbed)

P_1 → dissipated in load R_1 $V \updownarrow R$
P_2 → " " " R_2

$$A_{dB} = 10 \log_{10}\left(\frac{V_2^2/R_2}{V_1^2/R_1}\right) = 10 \log_{10}\left(\frac{V_2}{V_1}\right)^2 + 10 \log_{10}\left(\frac{R_1}{R_2}\right)$$

$$= 20 \log_{10}\left(\frac{V_2}{V_1}\right) + 10 \log\left(\frac{R_1}{R_2}\right)$$ → careful if $R_1 \neq R_2$

0 if $R_1 = R_2$



Every 6dB we double acoustic pressure

V_2/V_1	P_2/P_1	dB
1	1	0dB
2	4	6dB

signore

S8 S9 S10

Absolute bc we deal with i.e. 1mW $\rightarrow$ Fix the reference value

$\dfrac{\text{Power I want}}{\text{given reference}} \Rightarrow P_{dB_m} = 10 \log_{10}\left(\dfrac{P}{P_{ref}}\right)$

$P_{ref} = 1mW \rightarrow dB_m$

dBm $\longrightarrow$ $P_{ref} = \dfrac{V_{ref}^2}{R} \Rightarrow V_{ref} = \sqrt{P_{ref} \cdot R}$ $\hookrightarrow$ depend on R

$R = 50\Omega \rightarrow V_{ref}$ 0,2236

$R = 75\Omega \rightarrow V_{ref} = 0,2739$

to pass from power ref to voltage ref we change the ref: Vref

$P_{dBm} = 20 \log_{10} (V_{rms}/V_{ref})$

dBV $\quad V_{dBV} = 20 \log (V/V_{ref})$

$V_{ref} = 1V \rightarrow P_{ref} = \dfrac{V_{ref}^2}{R} =$ $50\Omega \rightarrow 1/50 = P_{ref}$; $75 \rightarrow 1/75 = P_{ref}$

$V_{(dBV)} = 10 \log_{10} P/P_{ref}$

From dBV to dBm

$P_{dBm} = 10 \log_{10}\left(\dfrac{V_{rms}^2}{R} \cdot \dfrac{1}{0{,}001}\right) =$

$= 10 \log_{10}(V_{rms}^2) + 10 \log_{10}\left(\dfrac{1}{R \cdot 0{,}001}\right)$

$= V_{dBV} + 10 \log_{10}\left(\dfrac{1}{R \cdot 0{,}001}\right)$

S11 S12

Def power gain: $G_p = \dfrac{P_{out}}{P_{in}}$ $\quad G_p > 0 \rightarrow$ power gain ; $G_p < 0 \rightarrow$ power loss

$G_{PdB} = 10 \log\left(\dfrac{P_{out}}{P_{in}}\right)$

Def power loss: $L_p = \dfrac{1}{G_p} = \dfrac{P_{in}}{P_{out}}$ $\quad L_p > 0 \rightarrow$ power loss ; $L_p < 0 \rightarrow$ power gain

$L_{PdB} = 10 \log\left(\dfrac{P_{in}}{P_{out}}\right)$

def Voltage gain $G_V = \dfrac{V_{out}}{V_{in}}$ $\quad G_V > 0 \rightarrow$ gain

$G_{VdB} = 20 \log\left(\dfrac{V_{out}}{V_{in}}\right)$ $\quad G_V < 0 \rightarrow$ attenuation

def voltage attenuation

$A_V = \dfrac{1}{G_V} = \dfrac{V_{in}}{V_{out}}$ $\quad A_V > 0 \rightarrow$ attenuat

$A_{V_{dB}} = 20 \log\left(\dfrac{V_{in}}{V_{out}}\right)$ $\quad A_V < 0 \rightarrow$ gain

S13 S14 S15

Multiple blocks

$P_{in} \rightarrow [G_{p1}] - [G_{p2}] - [G_{p3}] \rightarrow P_{out}$ $\quad$ Gain: 1

$G_{pT} = G_{p1} \cdot G_{p2} \cdot G_{p3}$

$G_{pT dB} = 10 \log_{10}(G_{p_1} \cdot G_{p_2} \cdot G_{p_3}) = G_{p_1 dB} + G_{p_2 dB} + G_{p_3 dB}$

$P_{out} = G_{pT} P_{in} \rightarrow \dfrac{P_{out}}{P_{ref}} = G_{pT} \dfrac{P_{in}}{P_{ref}}$

$P_{out\,dBm} = P_{in\,dBm} + G_{pT\,dB}$

(S16) (S17)

Bank-of-filters spectrum analyser

Idea: I take a number of BPF and I will analyse the overall BW by subdividing it into small sub-bands and I will measure how much power is inside each band.

The BPF are characterise by a transfer function ideally rectangular

and place the filters after other, *meaning centered at different frequencies

→ we point out here that the real components aren't rectangular

and the filters are in parallel, working together at the same time

I take the BW I want to investigate and I divide it in M sub-bands

Then, at the output of the filter I want to measure how much power is passing through the filter: I use a block $\boxed{RMS}$

Then a meter that measures it

* Careful: each BPF — RMS — meter block contents the contribution of all freq. components included in filter band

→ Resolution: The filter band-width determinates the freq resolution of the analyser

For M filters $\quad BW_{res} = \dfrac{f_{max}}{M}$

- Increase the freq. resolution
  - → shrink the BW (=> f_{max})
  - (or)→ Increase number of filters (: increase M)

→ This instrument is working in real time (advantage)

→ Poor frequency resolution (disadvantage)

→ cannot be applied to signal with large BW only for signals with narrow BW

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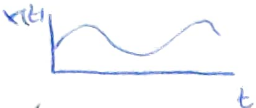
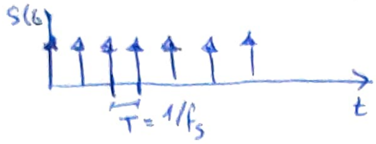
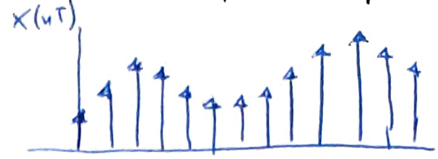
Sampling (S1)

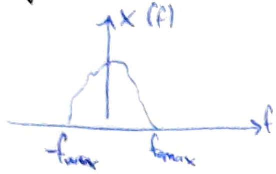
We have a signal → We want to represent in digital way so we sample it with delta functions (sampling function) → We multiply: amplitude of delta by amplitude of signal a_i



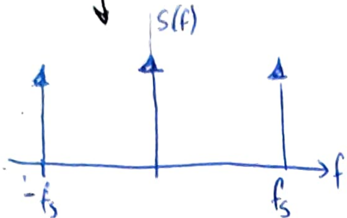




Signal limited in the BW
spectrum is

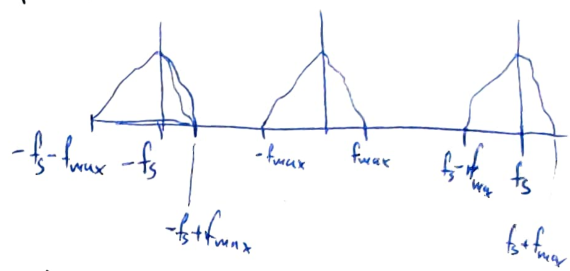


Transfer function of signal



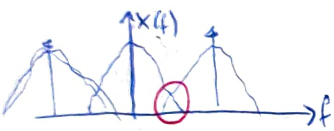
Transfer function of sampling signa

Convolution (in time dom. is multiplication) → The convolution means I have to replicate the signal centered is $-f_s$ and f_s



(S2) Aliasing

If the sampling freq. is not enough when we do the convolution we will have an overlaping of the replicas and the info will be lost



To avoid aliasing

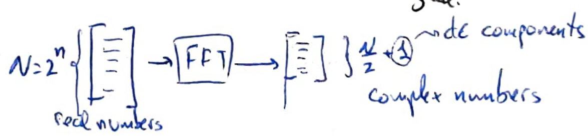
1. Digitize only BW limited signals: If the spectrum of my signal is not BW limited there is no sampling freq f_s that guarantee that the replica will not overlap. Typically LPF and remove high freqs.
2. $f_s > 2 f_{max}$

(S3) (S4) FFT

In digital world: We have to apply DFT (discrete) to the sampling signal to obtain the spectrum of the signal.

Compute FFT takes a lot of resources → we use FFT (fast)

Start: power of 2 numbers of samples



→ How understand the meaning of those complex numbers?

We compare this technique (FFT) with the technique of bank of filters.: Each complex number of the output of the algorithm can be seen as the output of a channel of bank of system.

The module of a complex number is equivalent of the readout of one channel in bank of fil

FFT → Digital

Bank of filter → Analog

→ We have a fixed number $\left(\frac{N}{2}+1\right)$ of points ⇒ fix number of freq. channels $= (n^\circ \text{channels}) \frac{N}{2}$

→ freq. resolution $\delta f = \frac{BW_{max}}{\text{Number of channels}} = \frac{f_s/2}{N/2} = \frac{f_s}{N}$ $f_s = \frac{1}{T_s}$ | i.e. 1 msec of time record length $\Delta T = T_s \cdot N$ (sampling period, number of samples)

↳ freq res δf increases by increasing the time record ΔT $\delta f = \frac{1}{\Delta T}$

for very high freq. resol. I have to take long time record.

→ freq for bin n:

$f_n = n \cdot \frac{f_s}{N} = n \, \delta f$

→ highest freq (last bin)

$f_{max} = \frac{f_s}{2}$

Digital spectrum analyser makes the analysis applyin FFT algorithms

Digital (FFT based) Spectrum Analyser

- The system takes our signal in time domain, sample it then on the samples we will apply FFT algorithms to get the spectrum of the signal.

- How to build?

① → We need to digitalise our signal → ADConverter

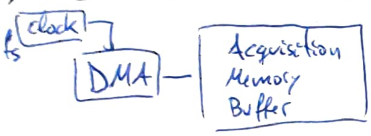
ADC block: Represented with 2 functions
- Sampling function → Take samples (sampler)
- Quantising function → for each sample assign a value (quantizer)

↳ Before entering the ADC the signal has to be adapted to the ADC dynamic range (i.e. it goes from ø to 25V) That's why we need an amplifier (G can be 20 or 40)

↳ After ampli and before ADC I put a LPF to assure that the input signal is BW limited (even if signal is inherently BW limited there can be high freq. components (noise) that overlap the signal)

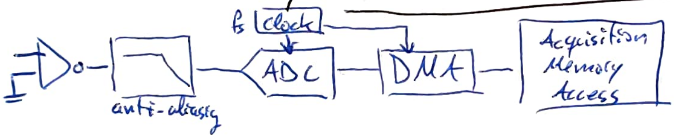
↳ ADC works with a sampling freq. clock f_s

↳ We need to record the the values of sampled signal, so we have a digital memory buffer.



So the block is: We have a Direct-Memory-Access that allows to then record the value on the memory buffer.
The values at the exit of the ADC are recorded directly into the buffer memory without the intervetion of controlling computer

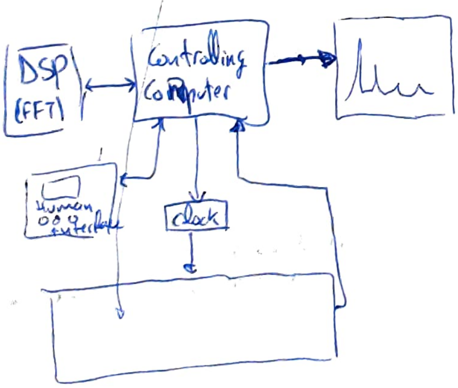
↳ Diagram:



This is to do the acquisition of the time record.

② → With the time record we can apply FFT algorithm and construct the spectrum of the signal

↳ Using a controlling computer: It takes the content of memory buffer (the time record) and apply FFT algorithms.



- We apply FFT algorithms with the help of a Digital Signal Processor DSP because the computing is heavy
- Then represents the vector of complex numbers → modules of the complex number
- The computer must know the sampling freq. f_s and the number and the samples N ⟹ then we know ΔT time record length. We know all the parameters to reconstruct of the spectrum.
- Finally, human interface of the instrument
  ↳ i.e. control the BW analyse
  N is fixed (the power of the hardware DSP will have a N_{max})
  I can change the freq. span ⟹ change the clock freq f_s

S6 About frequency

→ FFT is inherently base-band transformation ⇒ We must start from 0 Hz to $f_s/2$
it is required for the algorithm.

→ Total freq. span ⇒ $0 \xrightarrow{to} BW \equiv 0 \longrightarrow f_s/2$

→ Freq. resolution ⇒ $\delta f = \frac{f_s}{N}$

If N is fix
Improve resolution means decrease f_s ⇒ decrease the BW

If BW is fix
Improve the resolution means increase time record length N ⇒ More powerful hardware (more computational power)
↳ Number of computations to solve FFT is $N\log_2(N)$

S7 Oversampling and digital filtering

→ This technique is to improve performances and make the use of the instrument more flexible and better adapt the configuration of the instrument to the different signals.

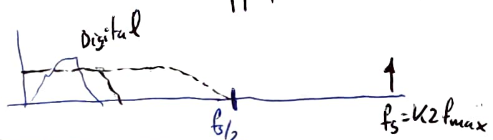
⇒ Why we need to apply? ⇒ ① Avoid aliasing
② If I want to change freq. resolution (δf) or freq. BW ($BW = f_s/2$) I have to change sampling freq f_s

But if I change sampling freq (f_s) ⇒ I must change the BW of antialiasing filter

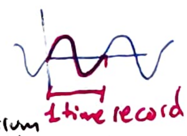
③ Changing the BW of an analog LPF → complex to build LPF tunnable.

④ Much easier oversample analog signal and the apply digital filtering to reduce the BW before apply FFT algorithms

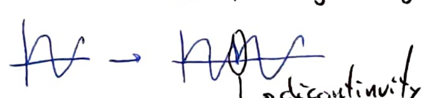
Procedure
① Input signal oversampled
the antialiasing analog filter has to be compatible with the oversampling freq f_s
② Apply Digital LPF to limit the BW. D-LPF is easy to tune to match with the actual signal.
③ Apply to the FFT algorithm



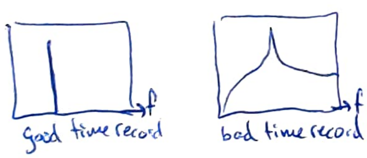
S8 Leakage (freq. dispersion)

→ what happens? Using digital approach we take a sample of our signal → finite time record


Then replicate it indefinitely
so the FFT provides the spectrum of this signal replicated not of the original signal but the spectrum

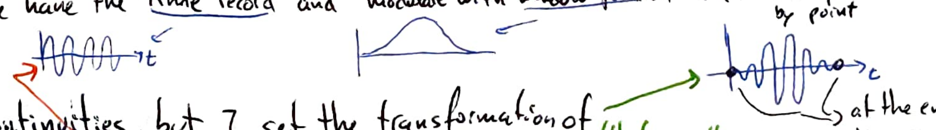
⇒ So if I take a time record differently 

⇒ The spectrum will be different
Discontinuity means high freq components

⇒ 

a) To avoid leakage
- synchronize the acquisition with the input signal (if I can know it → not in general)
→ Avoid discontinuities: Reduce the amplitude of the time record to zero at the end

⇒ The technique is: Windowing: We have the time record and modulate with window function and do multiplication point by point

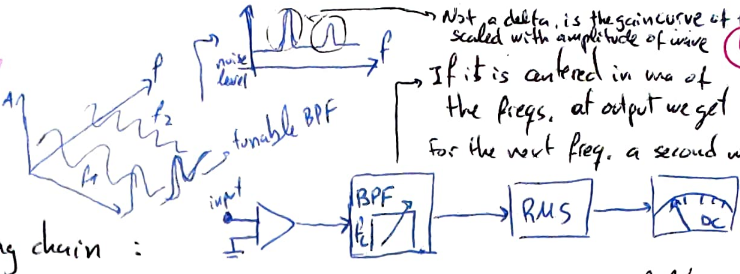


↳ It solves the problem of discontinuities, but I get the transformation of (that is the convolution of the two signals) instead of the transformation of

Find balance the effect of leakage and the effect of windowing

↳ Functions: Hanning window
Flat top window

Another technique for spectrum analysis



→ Not a delta, is the gain curve of filter scaled with amplitude of wave (4)

→ If it is centered in one of the freqs. at output we get a maximum for the next freq. a second maximum

(S1)

- Starting point: (Approach 1)

→ A signal can be represented as a sum of waves

↳ To measure the two components: Measuring chain:

→ This is similar to the "Bank of filters" but we only have one filter and we tune the filter

Difference: Bank of filters ⇒ all filters work in parallel

This case ⟹ I select just one freq. at a time a I do measurement freq. by freq.

→ Freq. resolution ⇒ depends on BW of the filter I am using

→ Difficult to build ~~[illegible]~~, narrow BW (high freq. resolution) and tunable in a wide freq. range

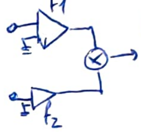
(S2)

$$A\cos x \cdot B\cos y = \frac{A \cdot B}{2}\left[\cos(x+y) + \cos(x-y)\right]$$

- (Approach 2)

⇒ Instead of tuning the filter → shift the freq. of the signal

↳ Filter's center freq f_c fixed and we move the signal to f_c ⟹ Multiplication of signals



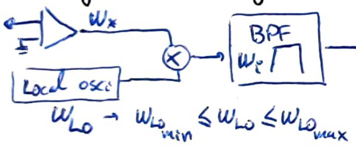
↳ The new signal (sum) brings the same information: amplitudes, phases, freqs.

⇒ We make one of the two freq. changable: i.e. f_1 varies in a freq. range

↳ In the output of the BPF we will get a max. value when $(f_1-f_2) = f_c$

(S3)

⇒ Building the processing chain



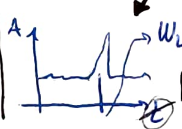
max when $W_{LO} - W_x = W_i$

$W_{LO} \rightarrow W_{LO_{min}} \leq W_{LO} \leq W_{LO_{max}}$

- Heterodyne spectrum analyser is like a receiver but without demodulat.
- After mixer we have $\frac{V_x V_{LO}}{2}$

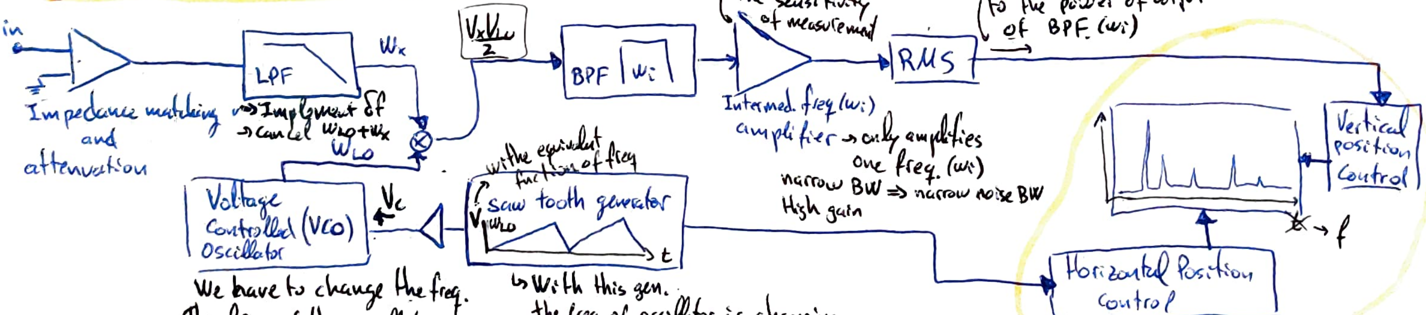
$\frac{V_{LO}}{2}$ is heterodyne gain

The graph has the freq f in the horizontal but the measure is done in time



→ I know W_i, W_{LO} so I can calibrate to have $t \rightarrow f$

⇒ Heterodyne (swept) spectrum analyzer (It's is an analog instrument)



We have to change the freq. The freq. of the oscillator is controlled by a voltage

↳ With this gen. the freq of oscillator is changing in a linear way

Advantage: High sensitivity because $\frac{V_{LO}}{2}$ can be very high and we have the amplifier for W_i in a narrow BW. Therefore improves SNR

With HSA: move freq range.

Sensitivity of vertical scale change

↳ Reduce velocity of scan ⇒ then we can reduce VBW ⇒ obtain better SNR but we have to pay in longer measuring time

Image frequency:

Why LPF in front of mixer?

If we have a noise component W_n at high freq. (far from our signal)

If W_n enters the mixer can happen: $W_n - W_{LO} = W_i$

And the output of measuring chain we add the power of W_n component

This happens when: $W_n - W_{LO} = W_i$

LPF have to attenuate W_n

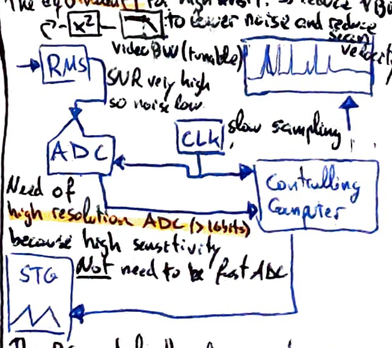
Decrease velocity scan
Increase freq. res
Increase sensitivity

$W_n - (W_i + W_x) = W_i$

$W_n = W_x + 2W_i$

The equivalent

For high sensit. ⇒ reduce VBW to lower noise and reduce scan velocity



Need of high resolution ADC (> 16 bits) because high sensitivity

Not need to be fast ADC

The PC controls the slope, period ⇒ so it controls the freq range and scan

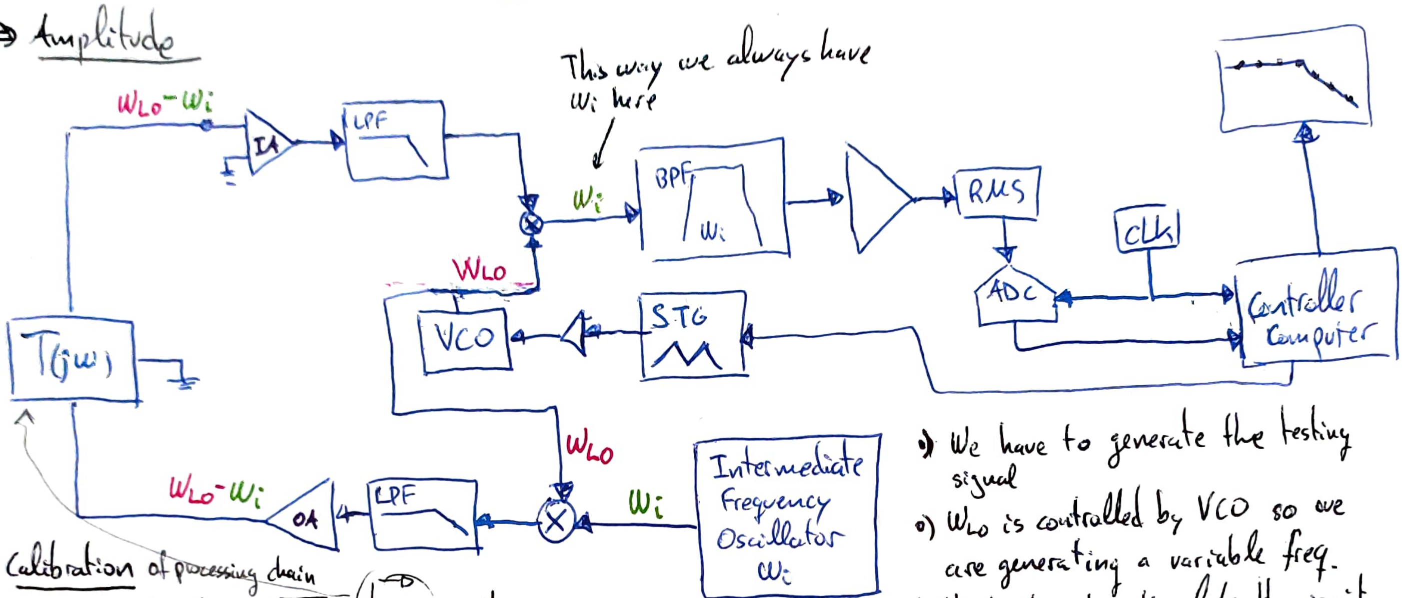
⇒ We can measure networks also, the freq. response of a device using HSA

↳ To measure a network: Put a signal at the input of the device and measure the response. By comparing the input and the output we can determine the freq. response.

Get the response over a range of frequencies

↳ The device is characterize by the transfer function $\boxed{T(j\omega)}$ → Linear circuit $freq_{input} = freq_{output}$

⇒ Amplitude



•) We have to generate the testing signal

•) W_{LO} is controlled by VCO so we are generating a variable freq.

•) The trick is to stimulate the circuit with a variable freq. and measure the response to a fix freq.

•) The freq. is fixed but the amplitude is modulated by the gain of the transfer function at testing signal $(W_{LO}-W_i)$ (remember W_{LO} is variable) and we measure the amplitude

•) Calibration of processing chain

① Put a short instead of $\boxed{T(j\omega)}$  ⇒ Measure

② Remove the short and put transfer function ⇒ Do the ratio freq. by freq. of the two measurements is the Transfer Func

⇒ Phase

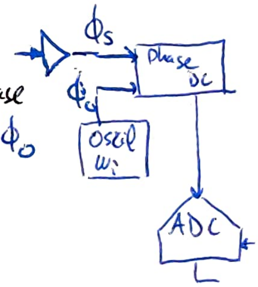
•) To measure the phase: compare two signals of the same freq and evaluate the relative phase of one signal with respect the other one. Phase not absolute measurement. We measure the phase difference

•) We substitute the block $\boxed{\text{RMS}}$ for the block of phase meter $\boxed{\text{PHASE DC}}$

↳ We have the phase ϕ_s due to all the measuring chain

↳ We have the phase ϕ_0 coming directly from intermediate oscillator

⇒ We measure the phase difference $\phi_s - \phi_0$



•) We need to calibrate:

$$\phi_s = \phi_0 - \phi_{LO} + \underline{\phi_{OA}} + \arg[T(j\omega)] + \phi_{IA} + \phi_{LO} + \phi_{\text{Intermediate Freq Filter}}$$

ϕ_0: Source of intermediate

and at phase meter: $\phi_s - \phi_o = \arg[T(j\omega)] + \underbrace{\Phi_{OA} + \Phi_{IA} + \Phi_{IFF}}_{\text{We don't want}}$

↳ calibrate puting a short, memorise, do the difference with the device as in amplitude

Probability density function (PDF)

- We use PDF to describe a non predictable (random) signal, in a statistical way.
- Definition: the probability that the actual value x is included in the interval x_1, x_2 $P(x_1, x_2)$

- Expected value, (the average) $\bar{x} = \int_{-\infty}^{\infty} x \, PDF(x)\, dx$
- Variance: $\sigma^2 = \int_{-\infty}^{\infty} (x-\bar{x})^2 \, PDF(x)\, dx = \overline{x^2} - (\bar{x})^2$
- Noise is assumed: $\bar{x} = 0 \Rightarrow \sigma^2 = \overline{x^2}$

Noise Power

- Noise represents unwanted fluctuations of voltages or currents. These voltages or currents disipates into the resistance. So we deal with the power of the noise
- By definition the noise power is the power dissipated in a load of $R = 1\Omega$
- $\sigma^2 = \overline{x^2} - (\bar{x})^2 = \overline{x^2}$ → Represents the average power of the noise

$x \to$ is voltage or current; $\bar{x}$: ϕ bc is noise

Power spectra density (PSD)

- Because we can't define X (time domain) we cannot do Fourier Transf
- We define power spectral density (S_x) $P_{12} = \int_{f_1}^{f_2} S_x(f)\, df$
- P_{12} is the power dissipated by the freq. components of the noise included in freq range f_1, f_2
- Total power $P_{tot} = \int_0^{\infty} S_x(f)\, df = \overline{x^2(t)}$

- Wiener-Kinchin theorem: Gives us the connection between the time representation and the freq. repre. of the noise
- Noise power in a given BW ⇒ $\overline{x^2_{BW}(t)} = \int_{f_1}^{f_2} S_x(f)\, df$ — Voltage units: $\left[\frac{V^2}{Hz}\right]$
- Power density: How much power is dissipated over a range of 1 Hz over a load of 1 ohm — Current units: $\left[\frac{A^2}{Hz}\right]$

Noise Equivalent Bandwidth (NEBW)

Input noise PSD

- We try to find how much power noise we will have at the output of a filter and the noise: $P_n = \int_0^{\infty} G(f) \cdot N_0\, df = N_0 \int_0^{\infty} G(f)\, df$

To get the total output power: Multiply (the power gain of the device)

① Ideal BPF

$P_n = G_0 \cdot N_0 \cdot BW$

② One pole LPF

$T(s) = \frac{1}{1+s\tau}$ $\quad T(f) = \frac{1}{f_c + jf}$

$G(f) = |T(f)|^2 = \frac{f_c^2}{f_c^2 + f^2}$ ⇒ $BW_n = \frac{\pi}{2} f_c$

$BW_n = \frac{1}{G_0} \int_0^{\infty} G(f)\, df$

Def: NEBW
The power of the noise of the ideal filter equal to that of the real filter $N_0 G_0 BW_n = \int_0^{\infty} G(f) \cdot N_0\, df$

- The noise BW of the ideal filter producing the noise power of the real filter ≡ NEBW

Noise and decibel

- Def: Noise bandwidth of 1 Hz

$$P_n(dB_m, 1Hz) = 10 \log\left(\frac{N_0 \cdot 1^{(W)}}{0{,}001^{(W)}}\right)$$

- Generic BW_N (with N_0 = cte)

$$P_n(dB_m, BW_N) = 10 \log(BW_N) + P_n(dB_m, 1Hz)$$

- From BW_1 to BW_2

$$P_n(dB_m, BW_2) = 10 \log\left(\frac{BW_2}{BW_1}\right) + P_n(dB_m, BW_1)$$

Noise measurement with Spectrum Analyser

⇒ By construction the spectrum analyser measures the power of the input signal content in the resolution bandwidth → and we create the plot

$P_{readout}(f) = \frac{P_{out}(f)}{BW_S}$

⇒ The output of the Spec. Anal. is power → then we normalise with signal BW → BW_S

def. of power spectrum density

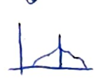
⇒ For the noise: the noise BW is different from the signal BW $\quad P_{readout}(f) = P_{out}(f) / BW_N$

Usually $BW_N > BW_S$

⇒ Noise level of measuring chain is depending on the resolution BW

Phase Noise: Random phase variation of the signal

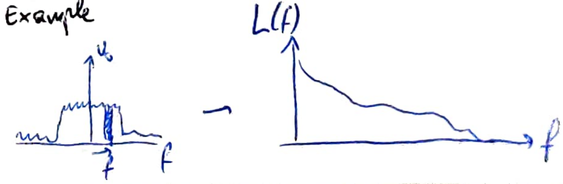
$V(t) = V_0 \sin[2\pi f_0 t + \phi_N(t)]$ — The result is two side bands symmetrical to the carrier



To give a value: Ratio between: noise level in BW of 1 Hz at f Hz away from carrier / amplitude of the carrier

$$L(f) = \frac{V_N(1Hz\,BW)}{V_0} \rightarrow L(f)_{dB} = 20 \log\left[\frac{V_N(1Hz\,BW)}{V_0}\right]$$

Example



Real-Time Spectrum Analysers

Reminders:

→ FFT-based: First digitalise the signal (ADC) and the apply FFT algorithm we compute the spectrum of the signal
- ∘) It is able to do real-time measures
- ∘) Only can work in base band always start from 0 Hz

→ Heterodyne: Analog measurement, we shift the freq. using LO and mixer then it pass through a very selective BPF. Then we digitalise the output
- ∘) It do NOT do real-time measures
- ∘) Can work in a BW located anywhere in the spectrum

Combine both advantages
Digital Heterodyning

① We need band limited signal

f_{min} f_{max}

② Sample the signal, at least $f_s = 2f_{max}$

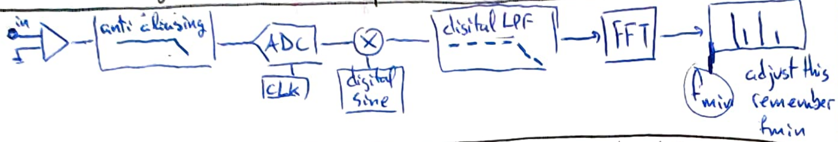
③ Shift the signal: Multiply the sampled signal with a sampled sine

④ Use a LPF. Eliminate High freq. components and gives us a base band base limited signal

⑤ Apply FFT to base band signal and obtain spectrum. careful, it is the spectrum of the shifted signal

⑥ The result must be shifted again to the right position

Process chain



Real-time: ∘) Parallelise ↳ During acquisition of record N we can digitaly process the record N-1

∘) Real time when: Time it takes to process FFT is ≤ time it takes to acquire the record

$t_r = 2N \cdot \frac{1}{f_s}$

$t_p \propto N \log_2(N)$

Real time cond. $t_p < t_r$

↳ We have a lower limit to the time interval

$\Delta t_r = 2N \frac{1}{f_s} > t_p \Rightarrow \frac{N}{t_p} > \frac{f_s}{2}$ → BW of FFT

Then we have a maximum to work in real time $RTBW = \frac{f_s}{2} = \frac{N}{t_p}$

RTBW is the max freq. we can analyse in real time

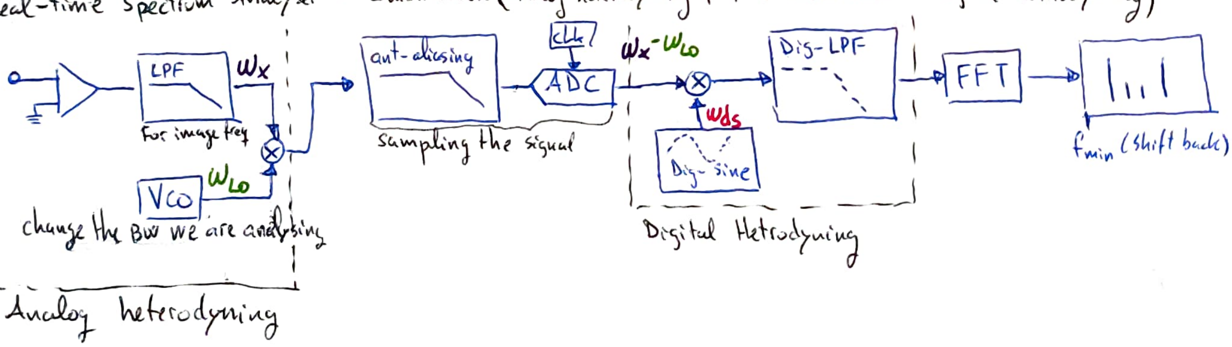
increase RTBW ⇒ increase processing power

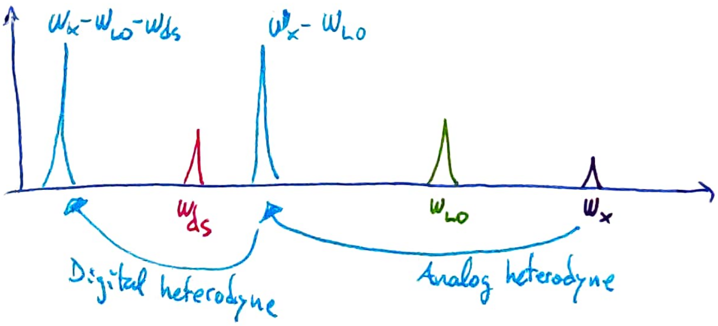
∘) Why work in real time? → Transient must be acquire enterely with no interruptions

→ Long transient with high freq. components require wide RTBW

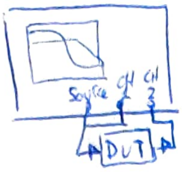
∘) Overlapping

Real-time Spectrum Analyser = Combination (Analog heterodyning + FFT-based S.A. + Digital heterodyning)





Network Analysis



We supply a sinusoidal to the DUT and see the output

⇒ Swept Spectrum Analyser: We supply a sinusoidal to the DUT and see the output (Not real time)

⇒ FFT analyser: Faster and in real-time. (Behaves as bank of filters)

◦) Internal source:

(1) → PRN (pseudo-random noise): It has constant power density over a freq range of interest. Out of the range we do not know the distribution of power.

Better for non perfect linear systems

Using PRN we can sychronize signal and time record ⇒ avoid freq leakage and windowing

(2) → Chirp sine: Sum of sine waves centered in FFT bins. We use to produce the testing signal

↳ By a given peak to peak voltage ⇒ better SNR, better characterise

(3) → Real white noise: From thermal noise for example

Transfer Function measurements

$$X(f) \rightarrow \boxed{H(f)} \rightarrow \Sigma \rightarrow Y(f) \quad \uparrow N(f) \quad \Rightarrow \frac{Y(f)}{X(f)} = H(f) + \frac{N(f)}{X(f)}$$

$Y = XH + N$

Compute energy spectral density of output: $G_{yy}(f) = Y(f)\,Y^*(f) = G_{xx}(f)|H(f)|^2 + |N(f)|^2 + \underbrace{\text{terms}}_{\text{average make it zero}}$

$$\therefore \quad \hookrightarrow \frac{\overline{G_{yy}}}{\overline{G_{xx}}} = |H(f)|^2 + \frac{|N(f)|^2}{G_{xx}}$$ Not good approach

Compute the cross spectrum X and Y: $\overline{G_{yx}}(f) = \overline{Y(f)X^*(f)} = \overline{G_{xx} \cdot H(f)} + \overline{N(f)X^*(f)} \Rightarrow \frac{\overline{G_{yx}}}{\overline{G_{xx}}} = H(f) + \frac{N(f)X^*(f)}{G_{xx}(f)}$ negligible.

⇒ Complex calculations but with digital signals can be done

Correlation measurements

with the network analyser we can obtain the correlation between two signals.

↳ correlation function between $x(t), y(t)$

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(\tau)\, y(t+\tau)\, dt$$

$$R_{xy} = F^{-1}\left[X(f) \cdot Y^*(f)\right]$$