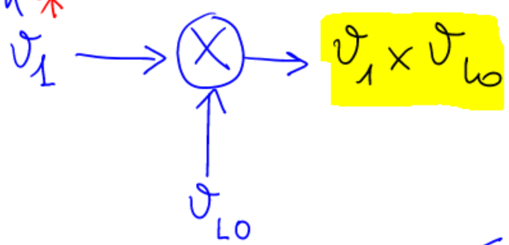
Spurs = Spurious = "parasitic" / "unwanted" frequencies

Principle of frequency conversion

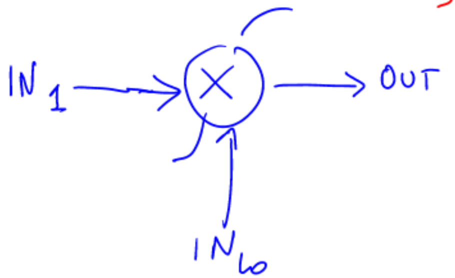
(1) Multiplication *



"Gilbert cell" only at low freq

→ only one spur

(2) Nonlinear device



$$OUT = f_{NL}(IN1 + IN_{LO})$$
$$= a(IN1 + IN_{LO}) + b(IN_1 + IN_{LO})^2 + \ldots$$

$$\ldots\ldots\ 2b\, IN_1 IN_{LO}\ \ldots\ldots$$

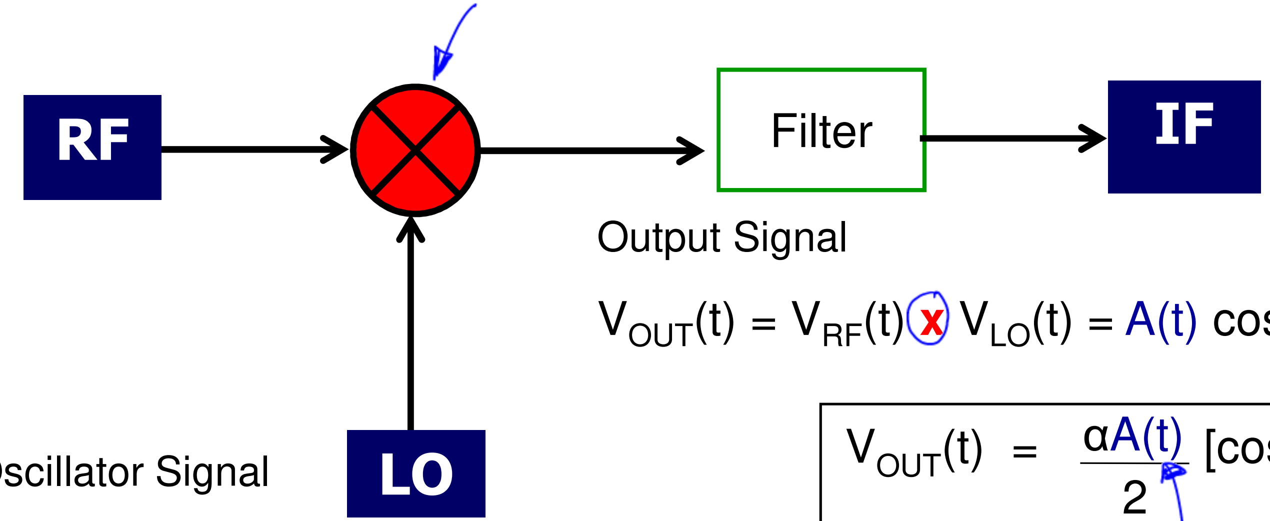
→ many spurs

(3) sign function (switching)

Mixing Function modelled by an ideal multiplier

Input Signal RF

$V_{RF}(t) = A(t)\cos(\omega_{RF}.t)$



Output Signal

$V_{OUT}(t) = V_{RF}(t) \times V_{LO}(t) = A(t)\cos(\omega_{RF}.t) \times \alpha\cos(\omega_{LO}.t)$

Oscillator Signal

$V_{LO}(t) = \alpha\cos(\omega_{LO}.t)$

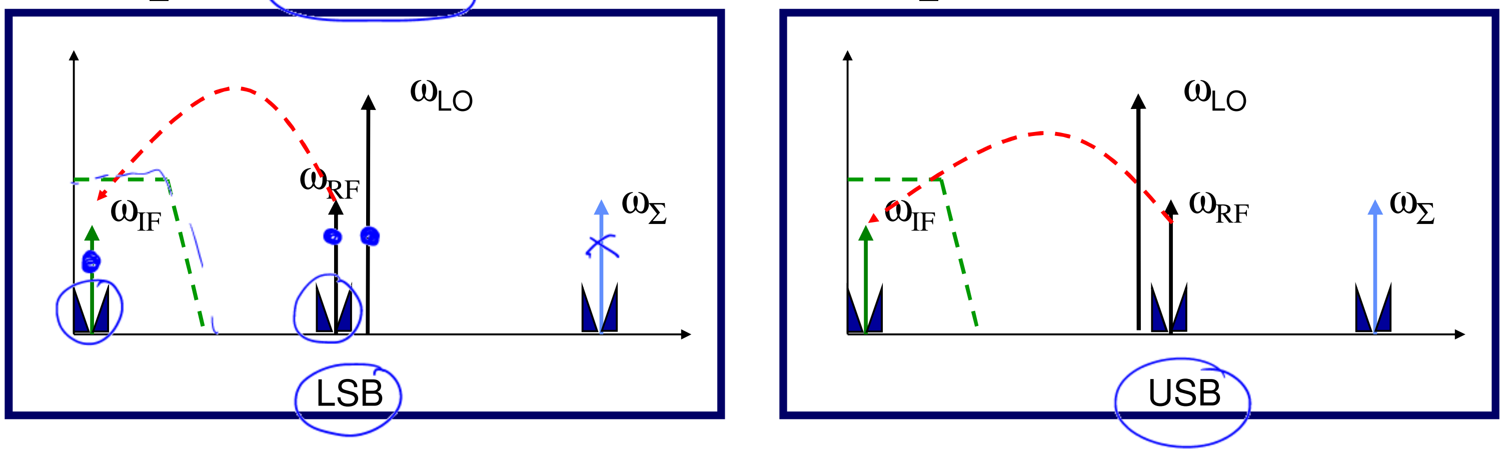
$$\boxed{V_{OUT}(t) = \frac{\alpha A(t)}{2}\left[\cos(\omega_{\Delta}t) + \cos(\omega_{\Sigma}t)\right]}$$

$\omega_{\Sigma} = \omega_{LO} + \omega_{RF}$

•<u>USB and LSB Modes</u>

$\omega_{\Delta\_LSB} = \omega_{LO} - \omega_{RF} = \omega_{IF}$

$\omega_{\Delta\_USB} = \omega_{RF} - \omega_{LO} = \omega_{IF}$



Ideal mixing because the only unwanted frequency is largely outside the useful bandwidth

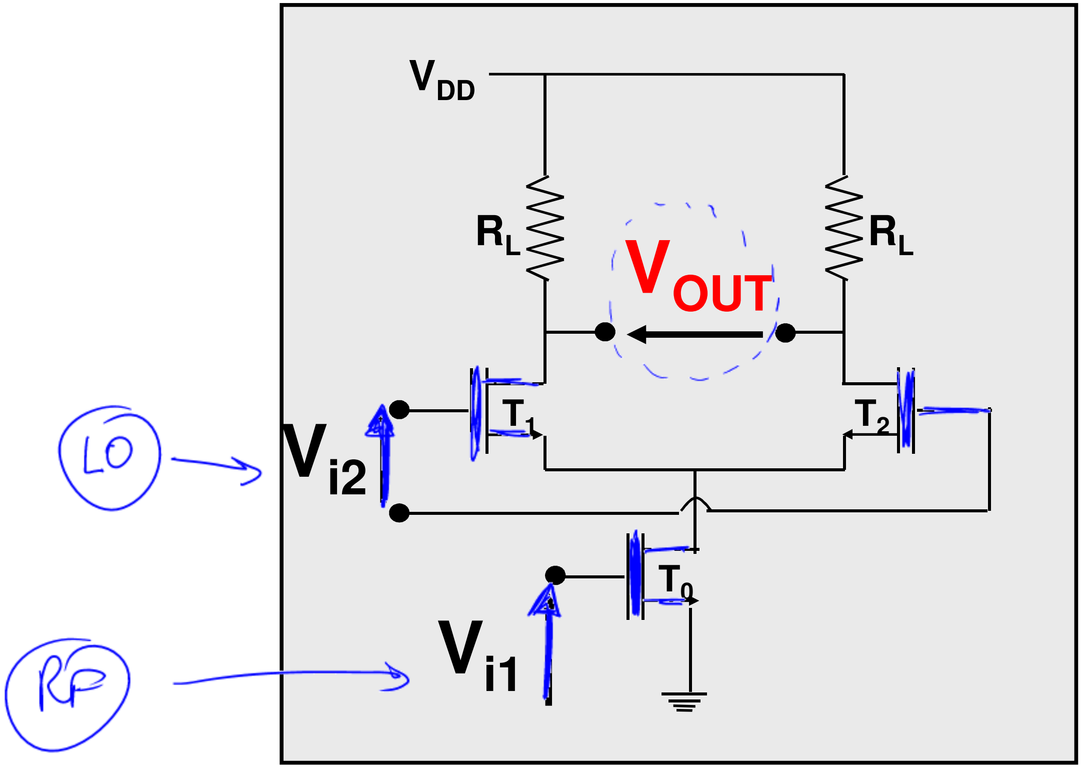
$\cos(a) \times \cos(b) = \frac{1}{2}\left(\cos(\underbrace{a+b}_{\Sigma}) + \cos(\underbrace{a-b}_{\Delta})\right)$

Example of Active Multiplier

- **Principle :** Voltage Multiplication of Input Voltages

$$V_{out}(t) = V_{i1}(t) \times V_{i2}(t)$$

- **Gilbert Cell**



Mixing Function performed by a nonlinear device

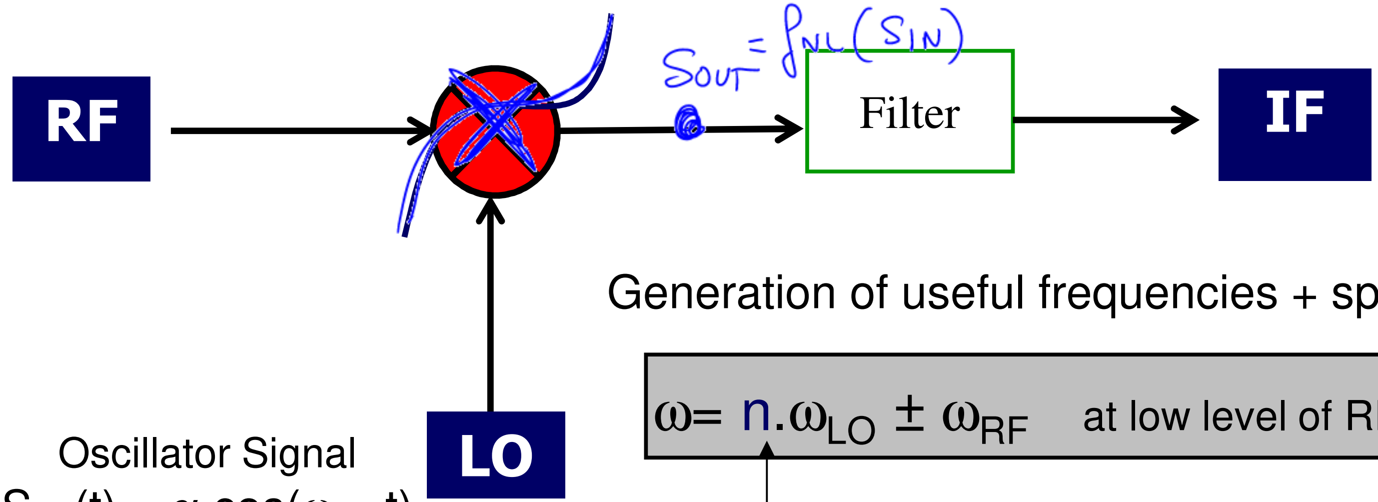
$$S_{OUT}(t) = \alpha_1 . S_{IN}(t)^{1} + \alpha_2 . S_{IN}(t)^2 + \alpha_3 . S_{IN}(t)^3 + \ldots$$

$$S_{IN}(t) = S_{LO}(t) \pm S_{RF}(t)$$

OUT = IN$_{lo}$ ± IN$_{RF}$

Input RF Signal

$$S_{RF}(t) = A(t) \cos(\omega_{RF}.t)$$



$$(x_1 + x_2)^2 \rightarrow x_1^2 + x_2^2 + 2x_1x_2$$

Oscillator Signal

$$S_{LO}(t) = \alpha \cos(\omega_{LO}.t)$$

Generation of useful frequencies + spurs:

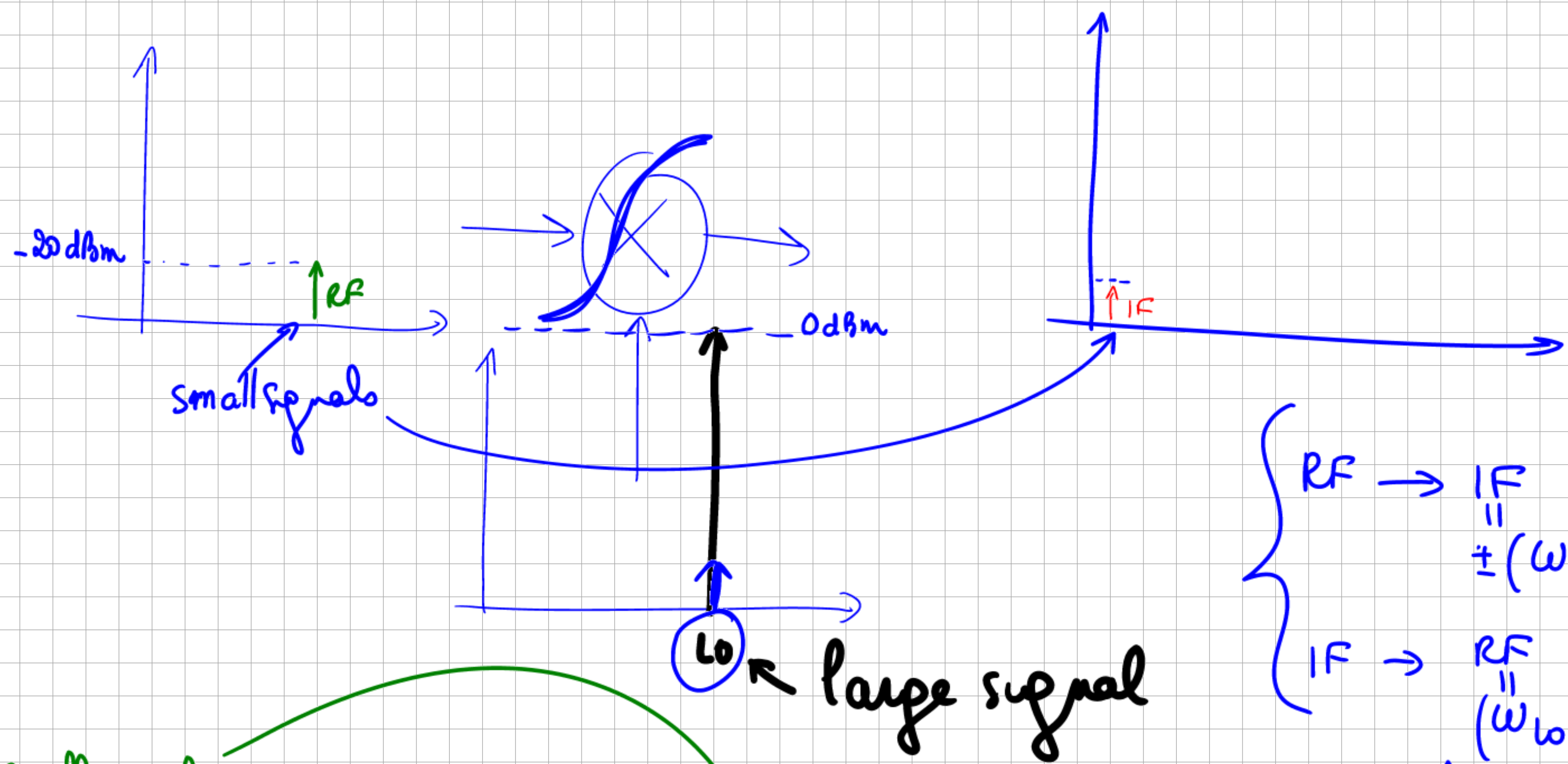
$\omega = n.\omega_{LO} \pm \omega_{RF}$ at low level of RF

Nonlinearity (Mixing with LO harmonics)

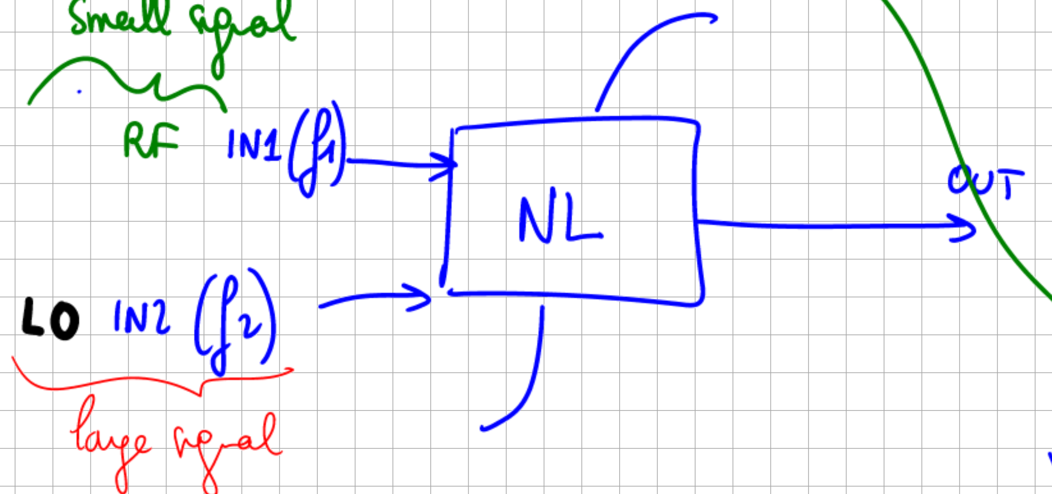
2nd Order Nonlinearity → useful signal @ $\pm(\omega_{LO} - \omega_{RF})$

but the nonlinearity gives a lot of parasitic frequencies (spurs)

Nonlinear devices used in High-Frequency Mixers : Diodes and Transistors



$$\begin{cases} RF \rightarrow IF = \pm(\omega_{LO} - \omega_{RF}) \\ IF \rightarrow RF = (\omega_{LO} \pm \omega_{IF}) \end{cases}$$



$$\pm p f_1 \pm \underbrace{q f_2}_{q f_2}$$

$(p = 1)$ f_1

$k = 1$ (usefull)

significant Spurs $= \boxed{k\,\omega_{LO} \pm \omega_{INPUT}}$

Mixing by LO switching : Frequency generation of a square signal « Sign Function»

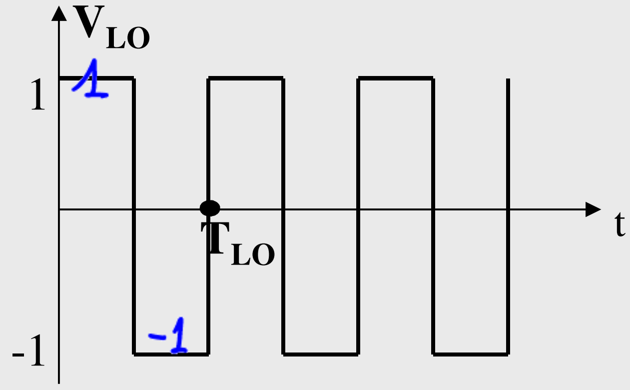
Switches controlled by the LO signal along the signal path RF→IF *(Gilbert Cell)*

$\omega_{lo} \pm \omega_{RF}$

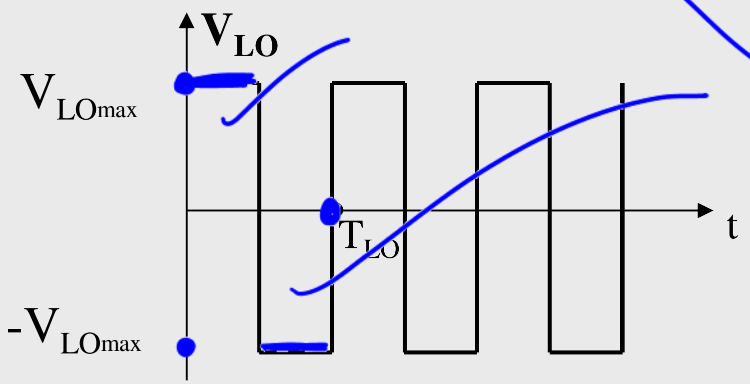
~~$2\omega_o \pm \omega_{RF}$~~

$3\omega_{lo} \pm \omega_{RF}$

X



$$V_{LO}(t) = \frac{4}{\pi}\left[\cos(\omega_{LO}.t) - \frac{1}{3}\cos(3\omega_{LO}.t) + \frac{1}{5}\cos(5\omega_{LO}.t) - \ \dots\right]$$



$$\text{sign}[V_{LO}(t)] = \frac{4}{\pi}\left[\cos(\omega_{LO}.t) - \frac{1}{3}\cos(3\omega_{LO}.t) + \frac{1}{5}\cos(5\omega_{LO}.t) - \ \dots\right]$$

$V_{OUT}(t) = V_{IN}(t)$. sign $(V_{LO}(t))$ → only odd mixing products $\underbrace{(2k+1)\omega_{LO}}_{k\omega_{lo}\,\pm\,\omega_{RF}} \pm \omega_{RF}$

→ less spurs

Characteristic Performances of Mixers

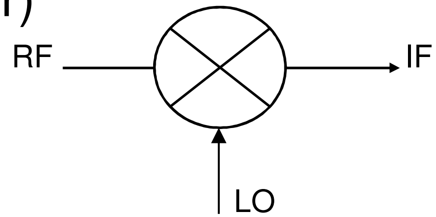
Conversion Gain (example of Receiver RF → IF)

• <u>Voltage Conversion Gain</u> and <u>Power Conversion Gain</u>

$$G_V = 20 \log \left(\frac{Vout_{IF}}{Vin_{RF}} \right) \qquad G_P = 10 \log \left(\frac{Pout_{IF}}{Pin_{RF}} \right)$$

Down Conversion (Receiver)

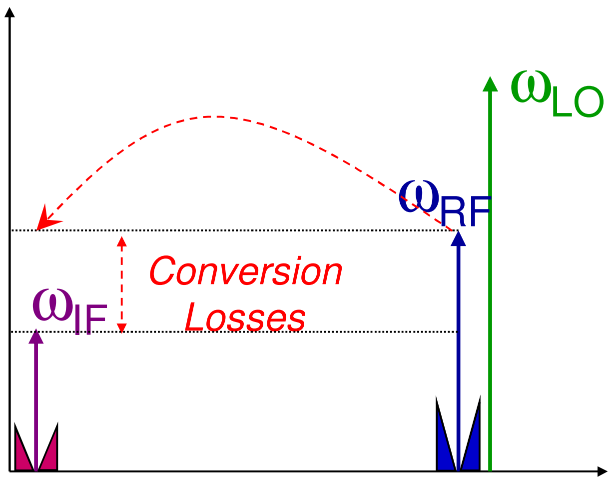
RF $\rightarrow$ IF



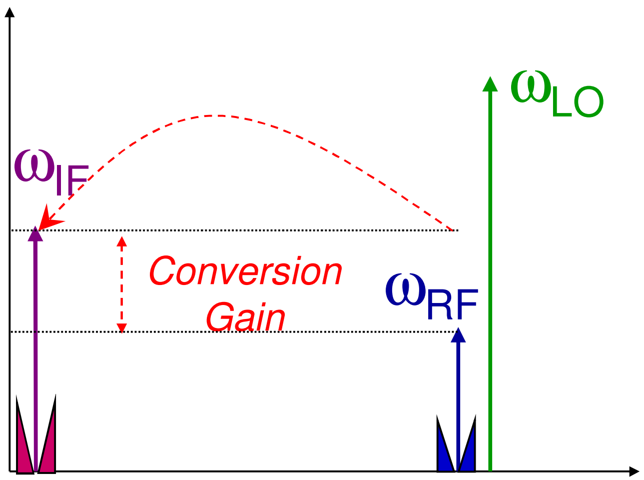
$$G_{dB\_down} = P_{OUTdBm}[\omega_{IF}] - P_{INdBm}[\omega_{RF}]$$

$$L_{dB\_down} = P_{INdBm}[\omega_{RF}] - P_{OUTdBm}[\omega_{IF}]$$

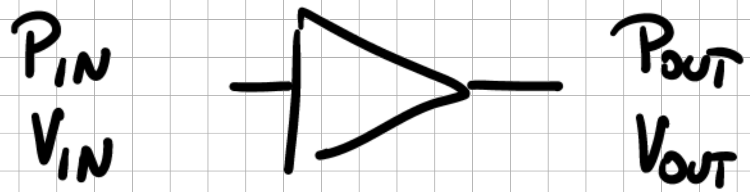
Example of passive down converter



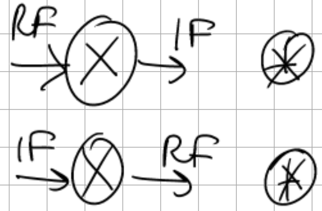
Example of active down converter

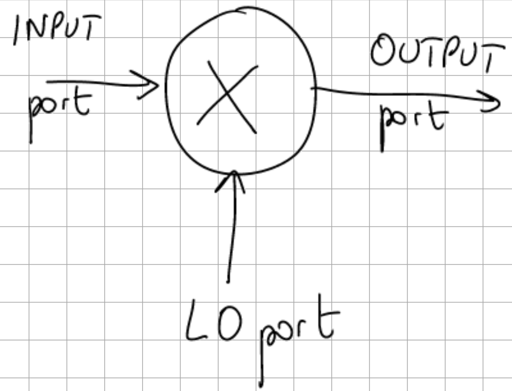


$$\text{Conversion Gain} = F(P_{LO})$$



$$G_p = \frac{P_{OUT}(f)}{P_{IN}(f)}$$





Voltage conversion gain

$$G_{CV} = \frac{V_{\substack{OUTPUT\\PORT}}(f_{OUT})}{V_{\substack{INPUT\\PORT}}(f_{IN})}$$

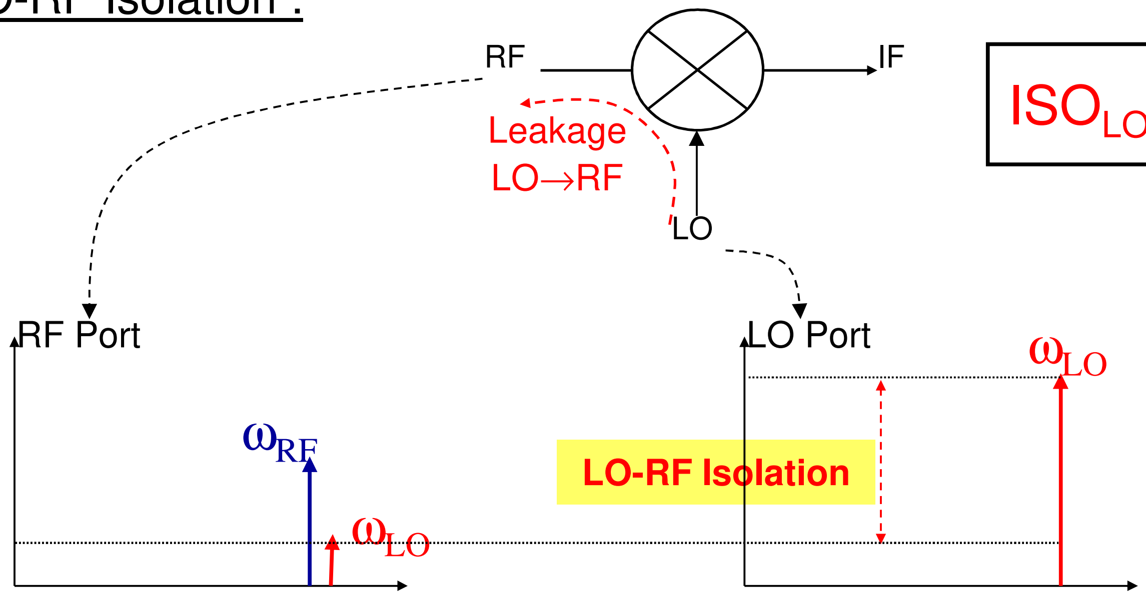
$$G_{CP} =$$

LO-RF Isolation

ISOLATION = power level coupled from one port to another one within the mixer

• <u>LO-RF Isolation :</u>

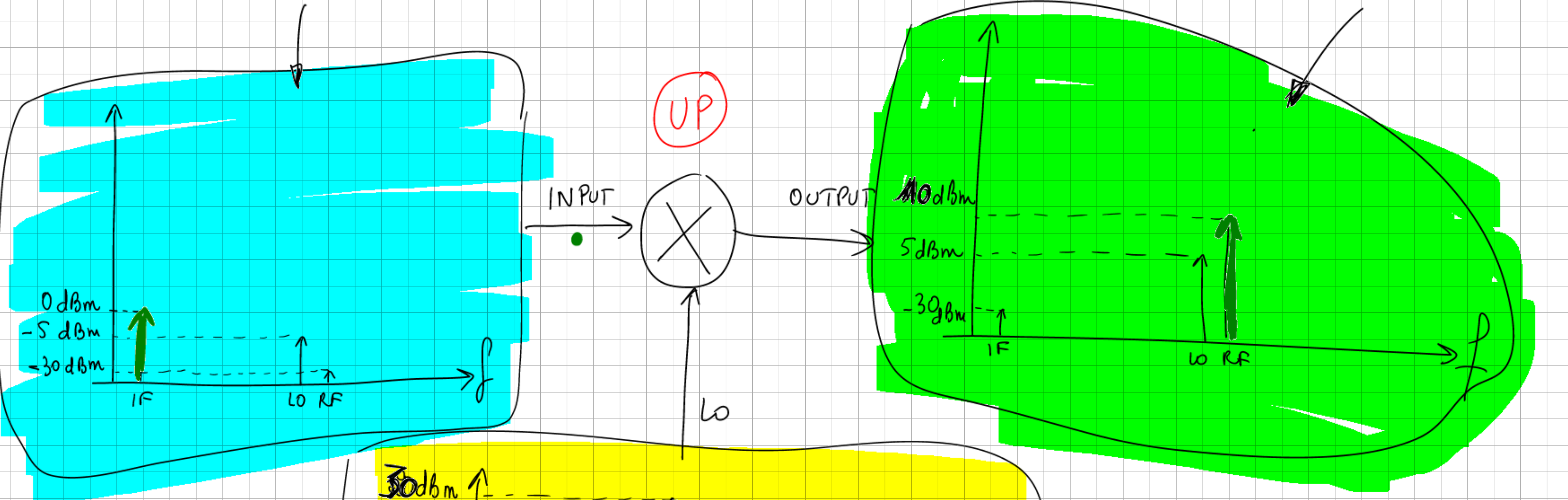
$$ISO_{LO\text{-}RF\ (dB)} = P_{dBm}(\omega_{LO})_{LO\ port} - P_{dBm}(\omega_{LO})_{RF\ port}$$

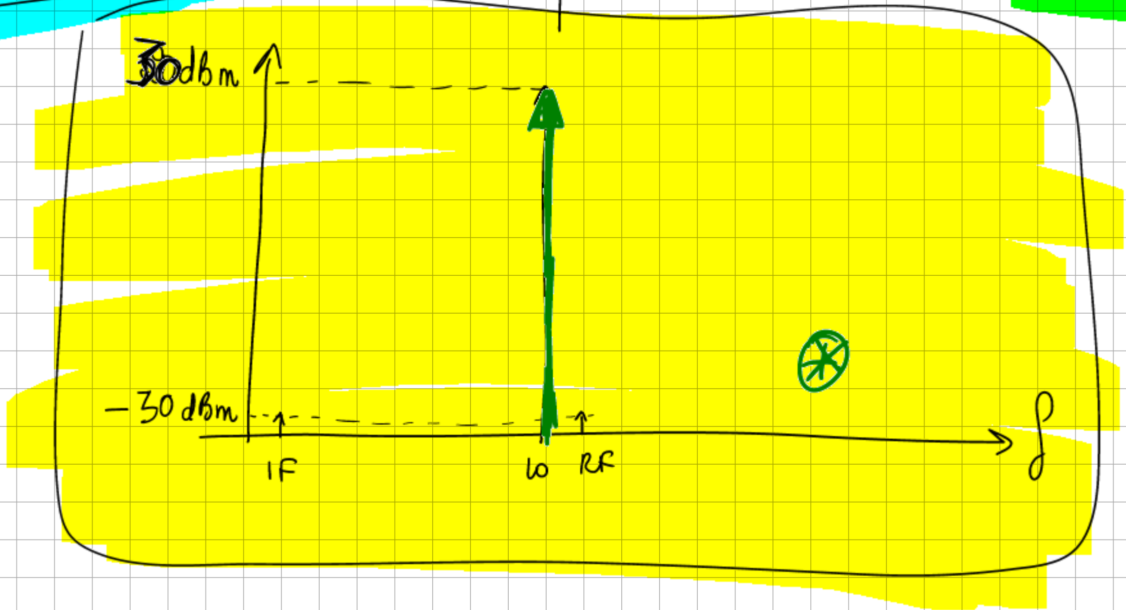


- Cannot be improved by filtering because RF and LO frequencies are too close

- Can be improved by Balanced Architectures

$ISO_{LO\text{-}RF\ (dB)}$ = LO leakage to the RF port (LNA, antenna)





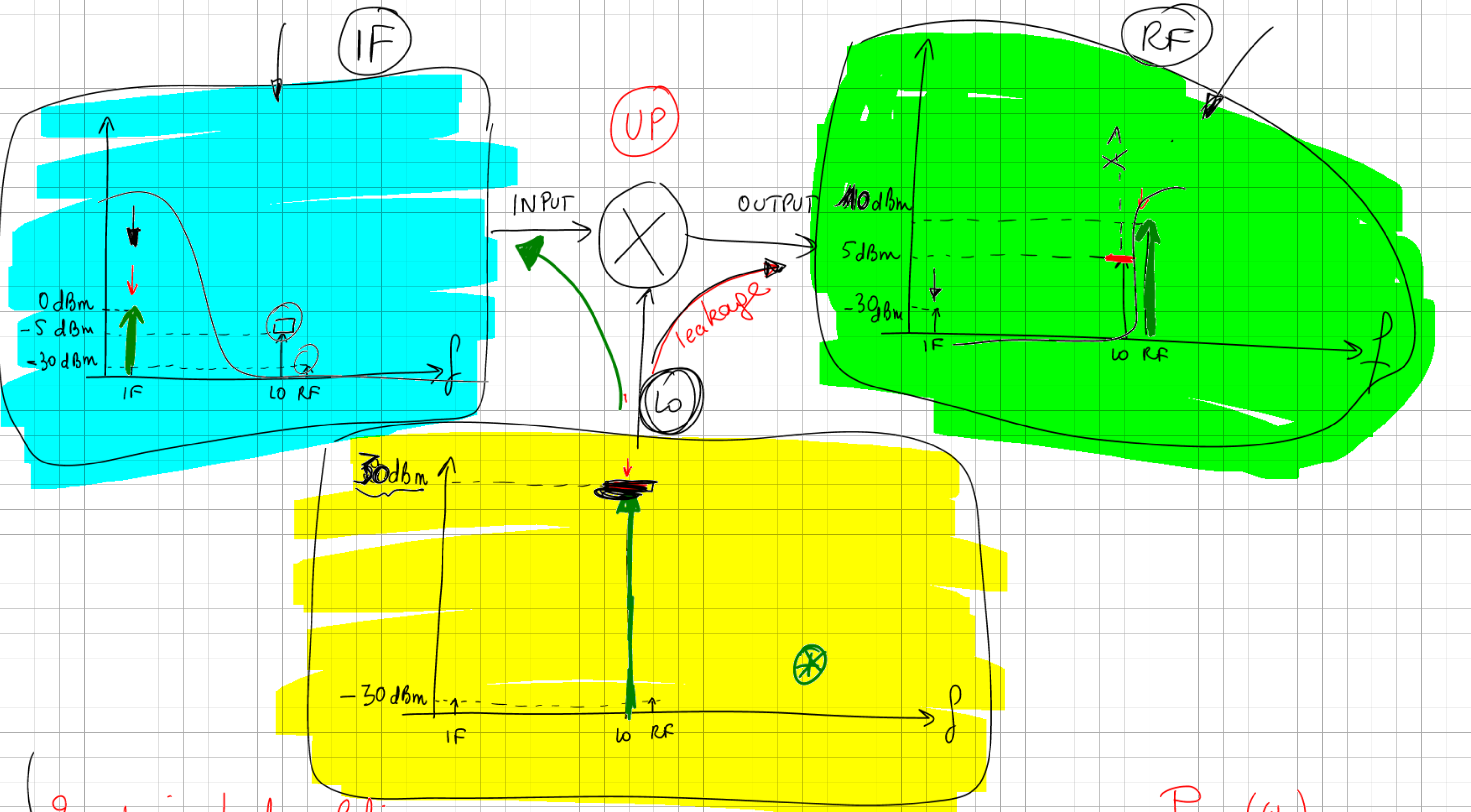
$$G_{CP_{UP}} = \frac{P_{OUTPUT}(\omega_{RF})}{P_{INPUT}(\omega_{IF})}$$

$$= 10\,dBm - 0\,dBm$$

down $\underbrace{RF}_{-30\,dBm} \rightarrow \otimes \rightarrow \underbrace{IF}_{30\,dBm}$

OR

up $\underset{0\,dBm}{IF} \rightarrow \underset{10\,dB}{\otimes} \rightarrow \underset{10\,dBm}{RF}$



2 most important isolations are:

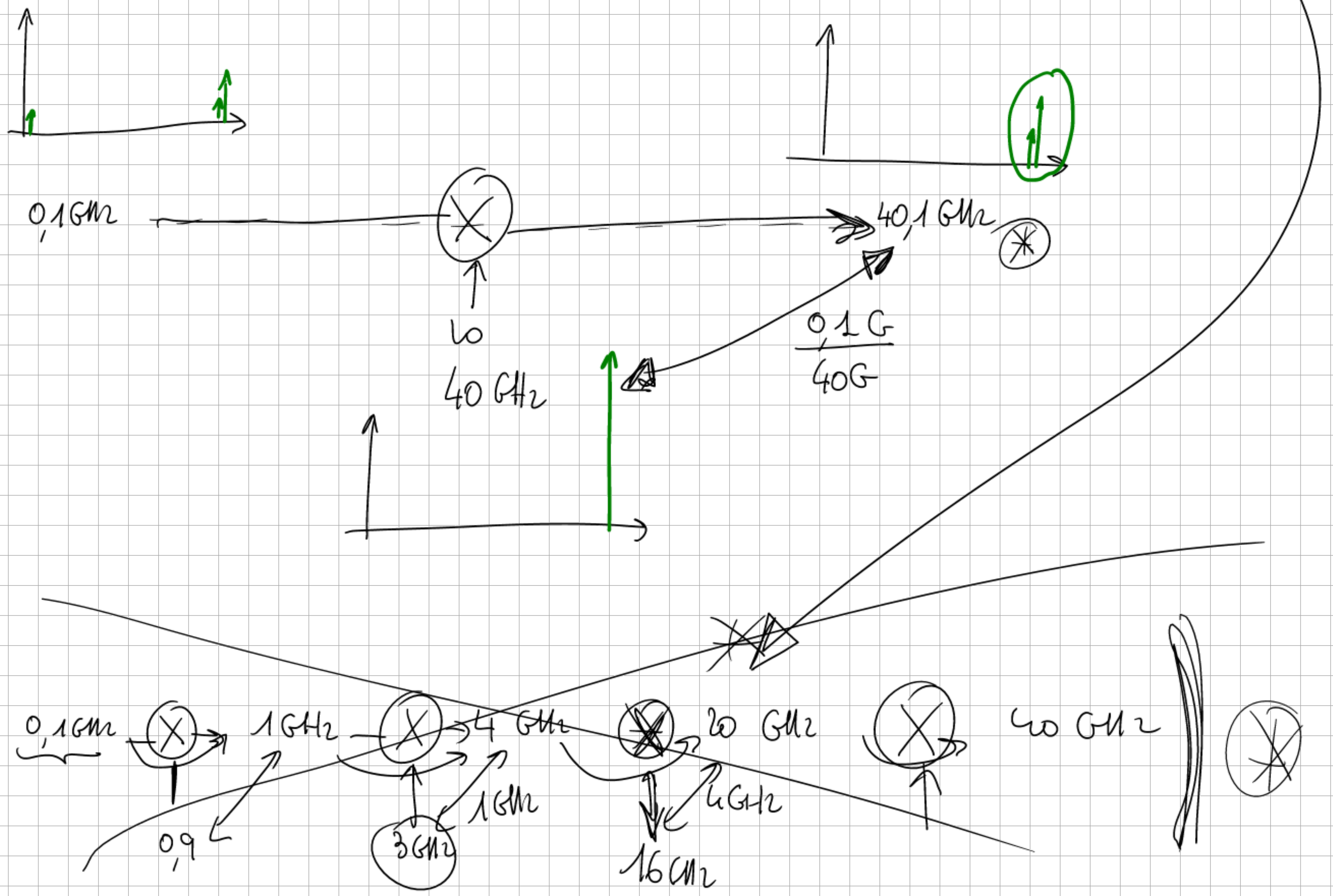
① LO-to-RF isolation = $ISO_{LO-RF} = \frac{P_{LO\,port}(\omega_{LO})}{P_{RF\,port}(\omega_{LO})} = 30\,dBm - 5\,dBm = 25\,dB$

② LO-to-IF isolation = $ISO_{LO-IF} = \frac{P_{LO\,port}(\omega_{LO})}{P_{IF\,port}(\omega_{LO})} = 30\,dBm - (-5\,dBm) = 35\,dB$

③ IF-to-RF isolation = $\frac{P_{IF\,port}(\omega_{IF})}{P_{RF_p}(\omega_{IF})} = 0\,dBm - (-30\,dBm) = 30\,dB$

superheterodyne

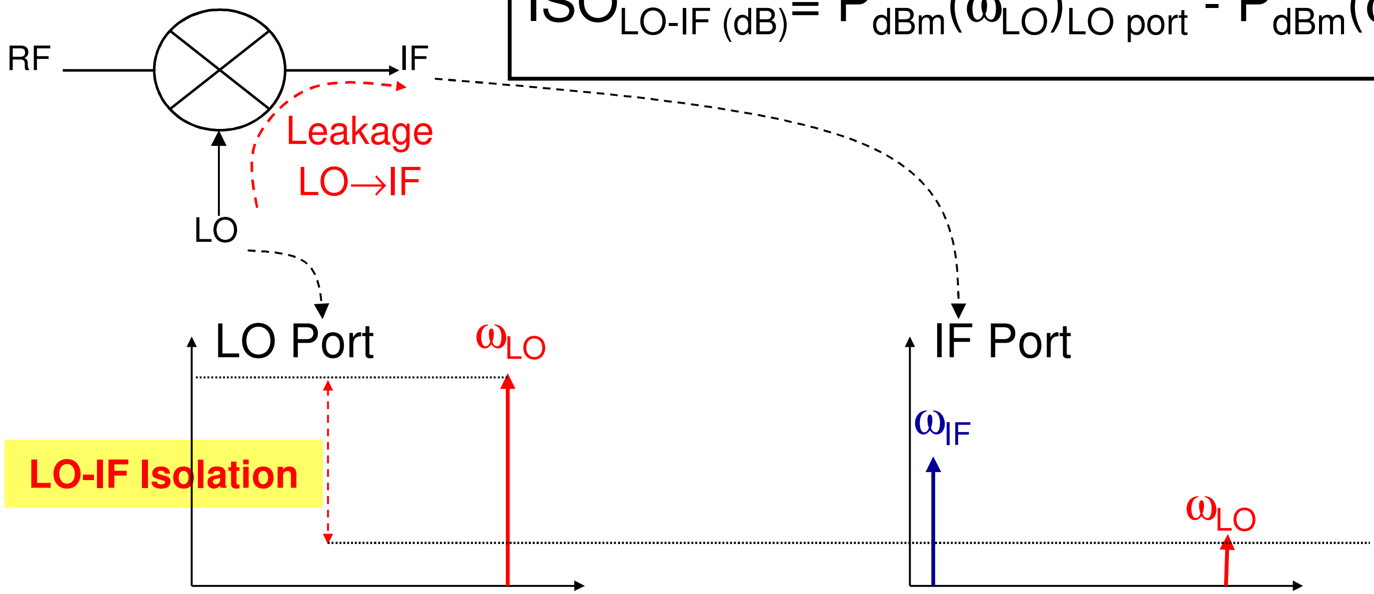
IOT



LO-IF and RF-IF Isolations

- <u>LO-IF Isolation :</u>

$$ISO_{LO\text{-}IF\ (dB)} = P_{dBm}(\omega_{LO})_{LO\ port} - P_{dBm}(\omega_{LO})_{IF\ port}$$



$ISO_{LO\text{-}IF\ (dB)}$ = LO leakage to the IF port (saturation of following stages due to large signal)

- <u>RF-IF and IF-RF Isolations :</u>

$$ISO_{RF\text{-}IF\ (dB)} = P_{dBm}(\omega_{RF})_{RF\ port} - P_{dBm}(\omega_{RF})_{IF\ port}$$

$$ISO_{IF\text{-}RF\ (dB)} = P_{dBm}(\omega_{IF})_{IF\ port} - P_{dBm}(\omega_{IF})_{RF\ port}$$

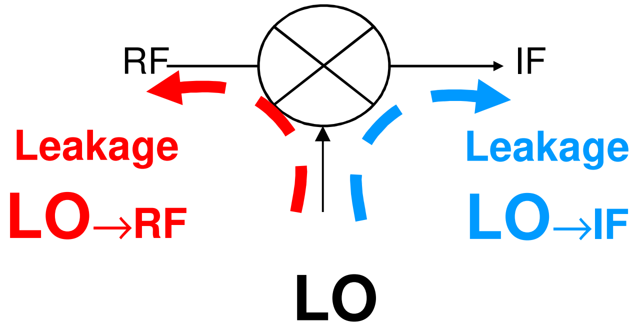
Less critical isolations because they can be improved by filtering

Isolations

•Two critical isolations due to the very large amplitude of the LO signal:

LO- IF Isolation (less critical because filtering is easy $\omega_{LO} >> \omega_{IF}$)

and LO-RF Isolation (the most critical isolation because of impossible filtering $\omega_{LO} \approx \omega_{RF}$)



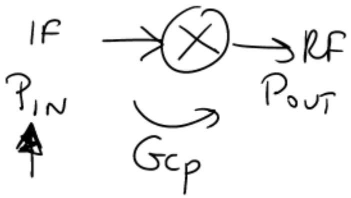
$$\text{Isolations} = F(P_{LO}\ ,\ P_{IN})$$

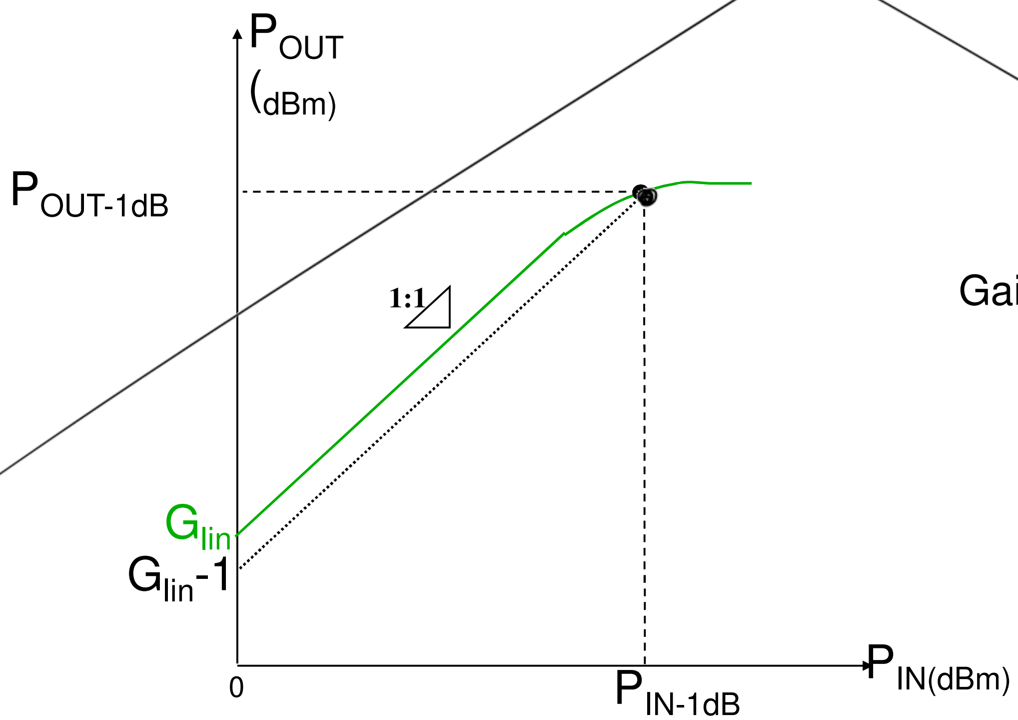
Linearity (ex : Single carrier amplifier)

<u>Nonlinearity</u> $y(t) = \alpha_1.x(t) + \alpha_2.\ x(t)^2 - \alpha_3.\ x(t)^3$

$$x(t) = A.\cos(\omega t) \ \rightarrow\ y(t) = \left(\frac{\alpha_2 A^2}{2}\right) + \left(\alpha_1 A - \frac{3\alpha_3 A^3}{4}\right)\cos(\omega t) + \left(\frac{\alpha_2 A^2}{2}\right)\cos(2\omega t) - \left(\frac{\alpha_3 A^3}{4}\right)\cos(3\omega t)$$

$$\rightarrow G_{lin} = \alpha_1 A$$





<u>Example (amplifier)</u>

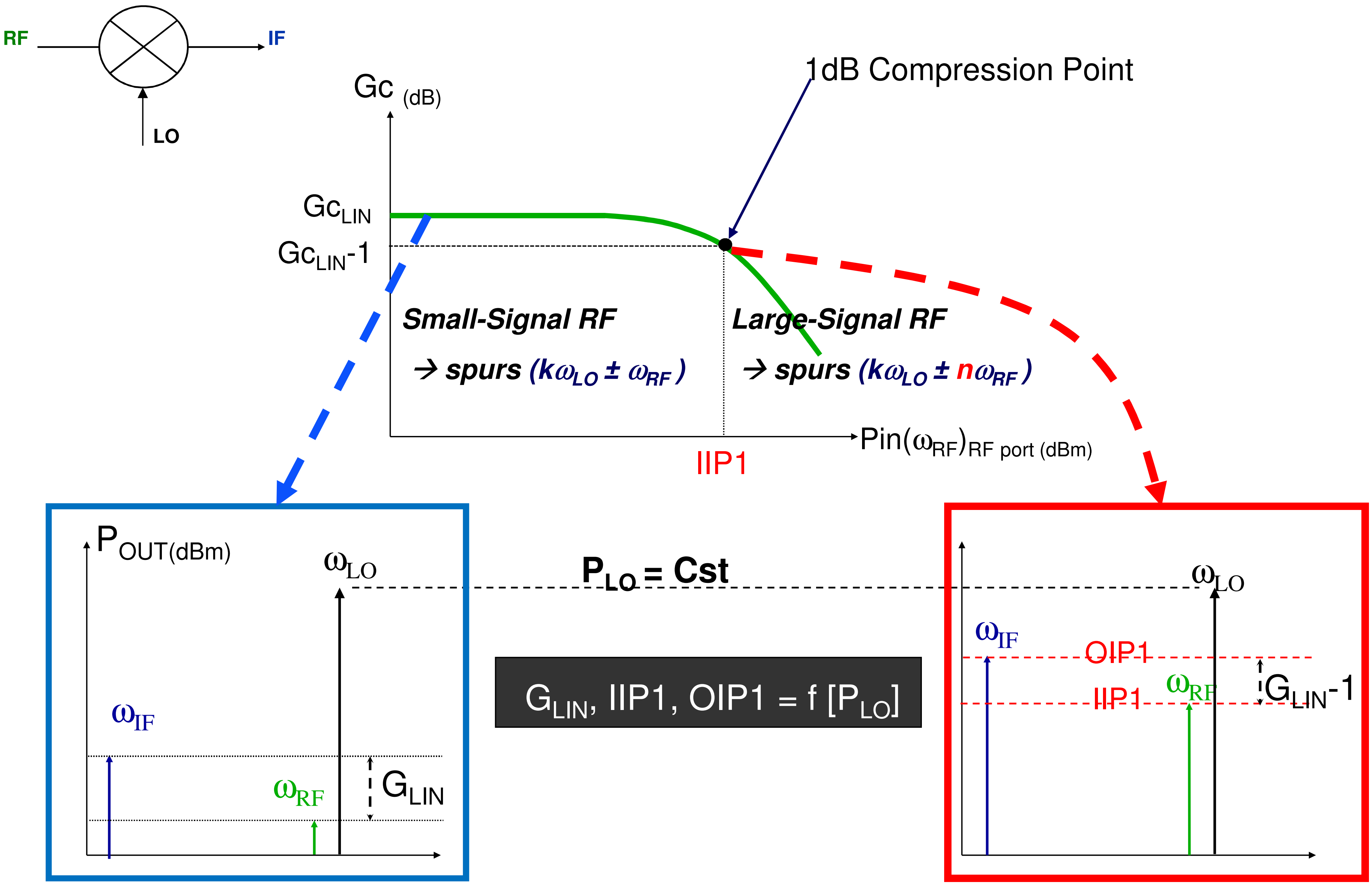
Gain Saturation at the input frequency ω

with

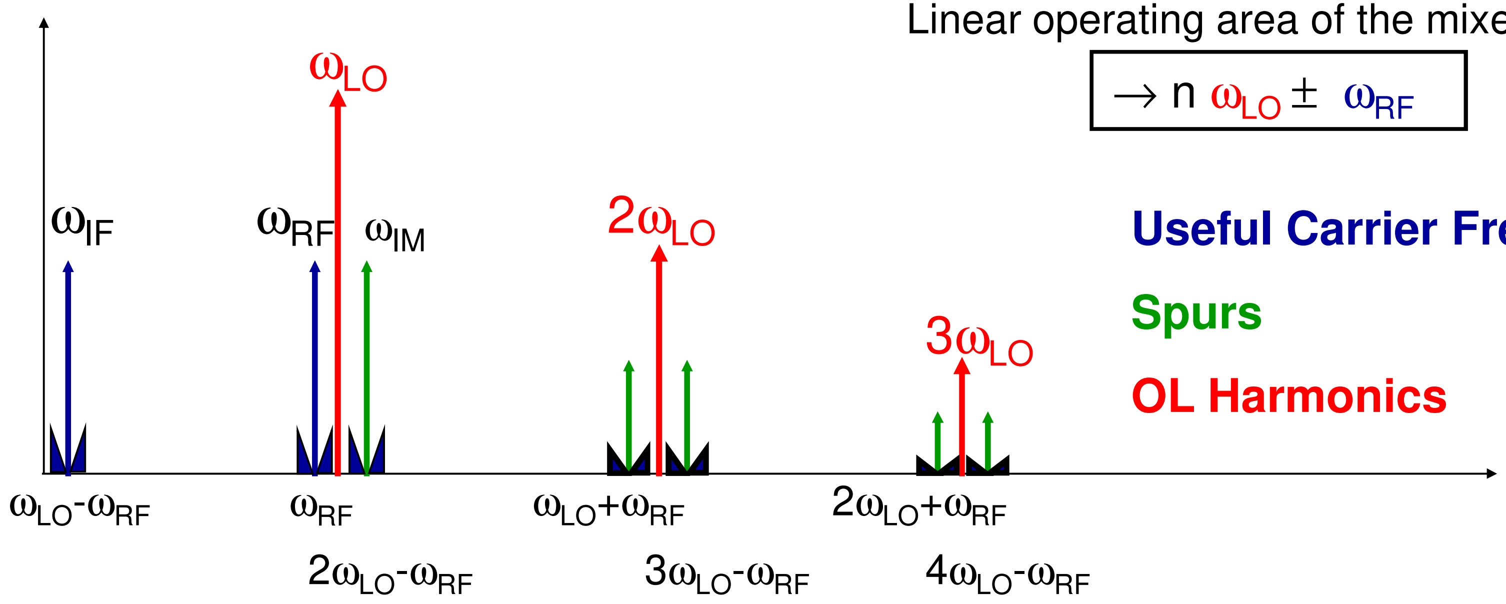
Harmonic Generation ($n.\omega$)

Gain compression of Active Mixers

• 1dB compression point at a given LO power



Simplified Output Spectrum



Linear operating area of the mixer

$\rightarrow n\ \omega_{LO} \pm\ \omega_{RF}$

Useful Carrier Frequencies

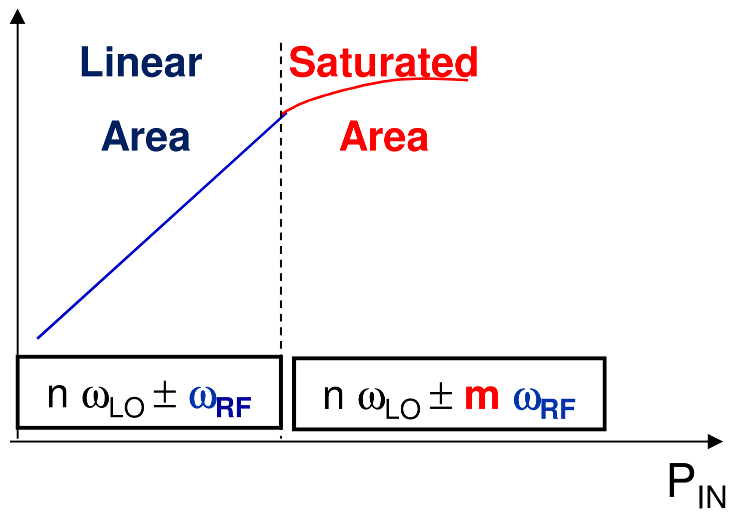
Spurs

OL Harmonics

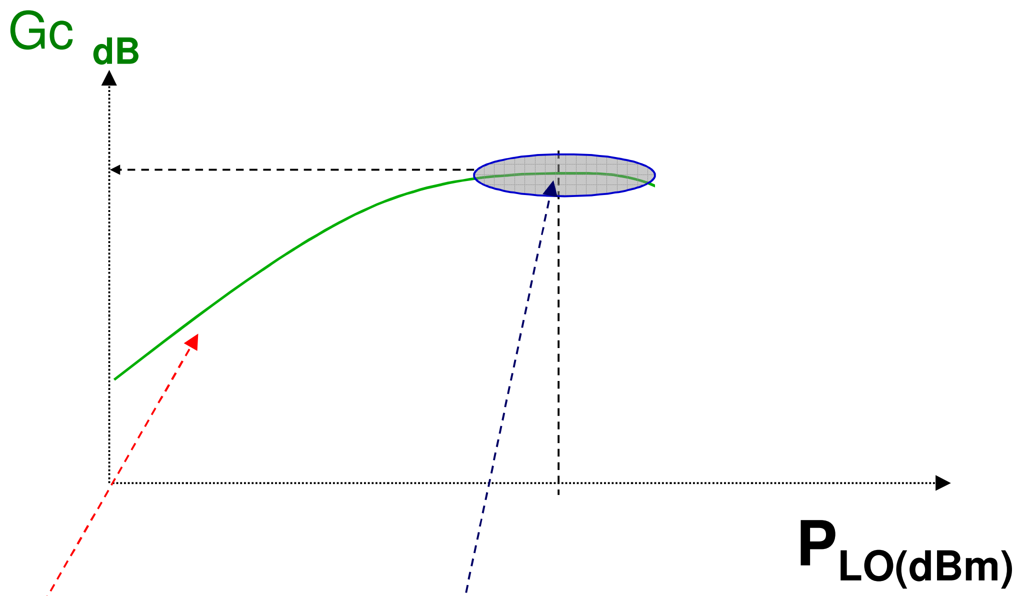
At a given level of P_{LO},

if P_{IN} increases $\rightarrow$ Mixer saturation

$\rightarrow n\ \omega_{LO} \pm\ m\ \omega_{RF}$



Gain Conversion as a function of P_{LO}



If P_{LO} is too low $\rightarrow$ Gc varies rapidly with P_{LO} ☹

If P_{LO} is high at its optimum value $\rightarrow$ Gc is optimal and remains constant with P_{LO} ☺

Outline

- **Introduction**
- **Theory of frequency conversion**
- **Characteristic performances of mixers**
- **Active mixers**
  - Single Ended (SEM) → Gate Mixer / Drain Mixer
  - Gilbert Cell→ DBDM Double Balanced Differential Mixer
- **Passive Mixers**
  - Diodes → SEM / SBM / DBM
  - Cold FETs → SEM / SBM
- **IRM: Image Reject Mixers**

Active Mixers

Active Mixers = Gain Modulation

The basic architecture of an active mixer is similar to that of an amplifier

except that the biasing conditions of the nonlinear device are chosen to get a 2nd order quadratic response

$$S_{OUT}(t) = \alpha_1 . S_{IN}(t) + \alpha_2 . S_{IN}(t)^2 + \alpha_3 . S_{IN}(t)^3 + \ldots$$

→ Active mixers can give high conversion gains

→ Active mixers require lower levels of LO power compared to passive mixers

However :

→ Active mixers saturate at very low powers

→ Active mixers require DC consumption

→ Active mixers are not well suited to IRM architectures

At high frequencies:

→ Performances of active mixers do not reach the required level → Passive mixers are preferred @ HF

♦ Active Mixers : Single Ended Mixers

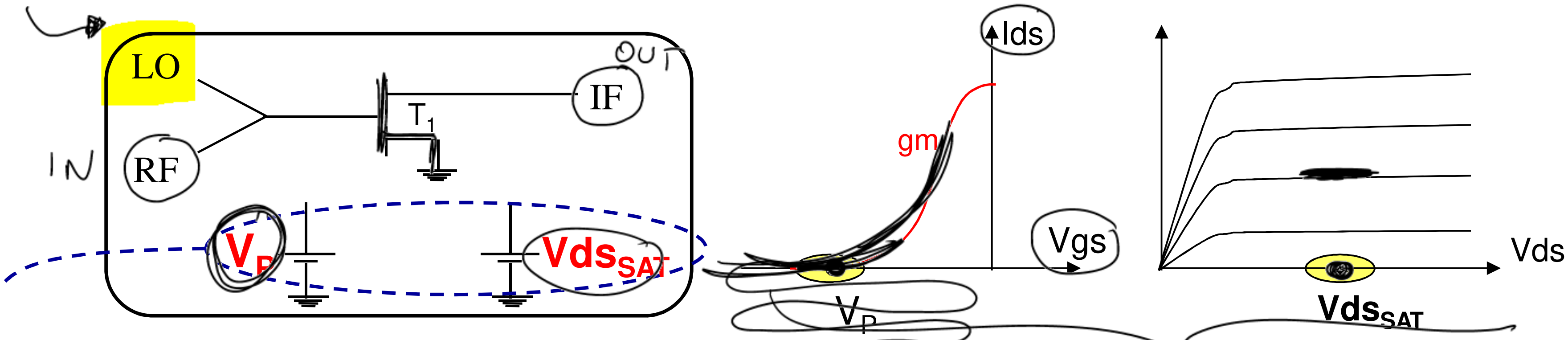
FET-based SEMs

SEM → Single Ended Mixer ← 1 NL device

SBM → " Balanced —— ← 2 mixers

DBM → Double balanced —— ← 4 mixers

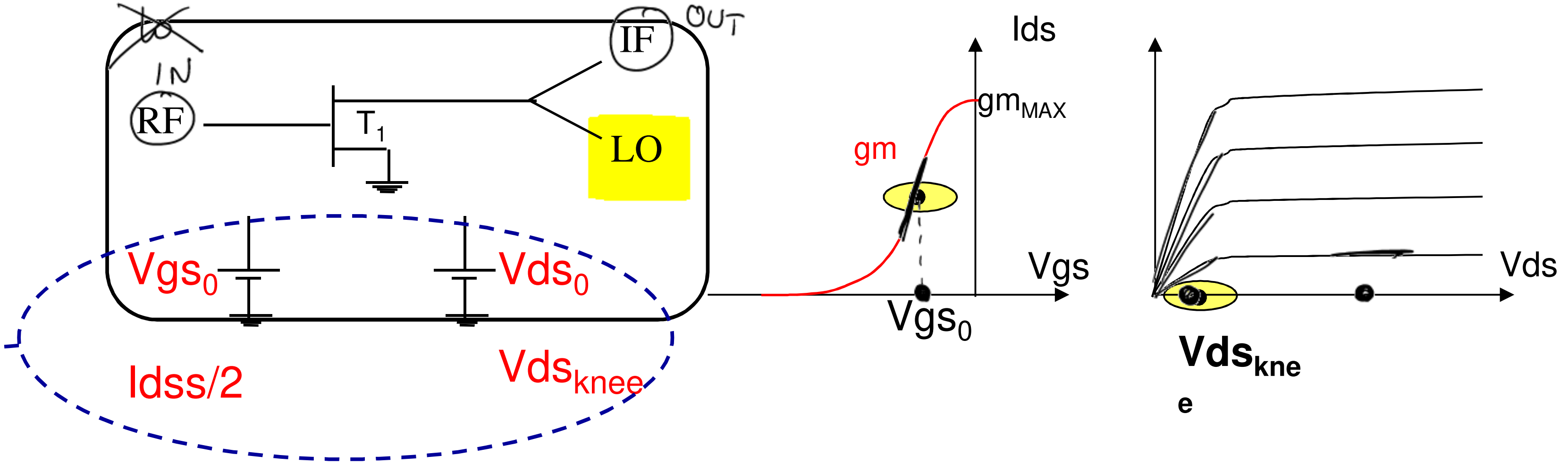
FET-based SEM (Gate Mixer)



Biasing conditions → High nonlinearity of the transconductance gm

$$gm(t) = gm_0 + gm_1.\cos(\omega_{LO}t) + gm_2.\cos(2\omega_{LO}t) + \ldots$$

FET-based SEM (Drain Mixer)



Biasing conditions → Nonlinearities of gm and gd)

Rds

$$gm(t) = gm_0 + gm_1.\cos(\omega_{LO}.t) + gm_2.\cos(2\omega_{LO}.t) + \ldots$$

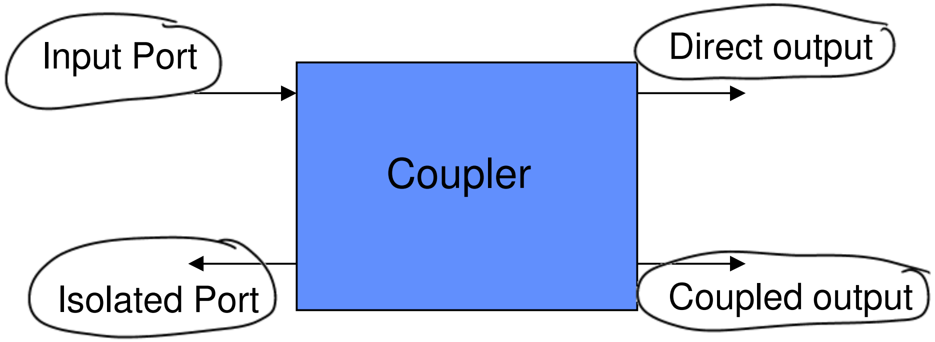
$$Gd(t) = Gd_0 + Gd_1.\cos(\omega_{LO}.t) + Gd_2.\cos(2\omega_{LO}.t) \ldots$$

Lower Conversion Gain compared to the Gate Mixer

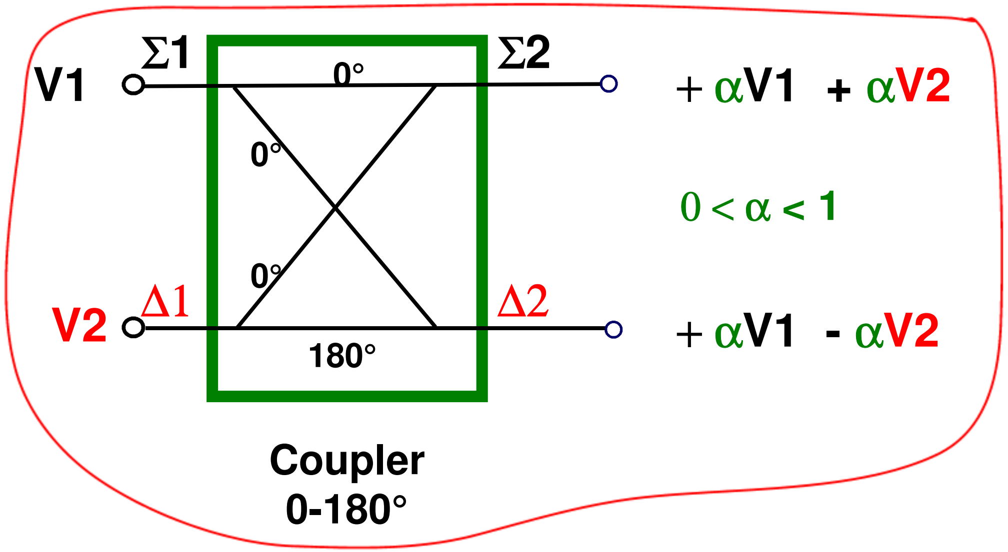
♦ Active Balanced Mixers

Reminder : Coupler

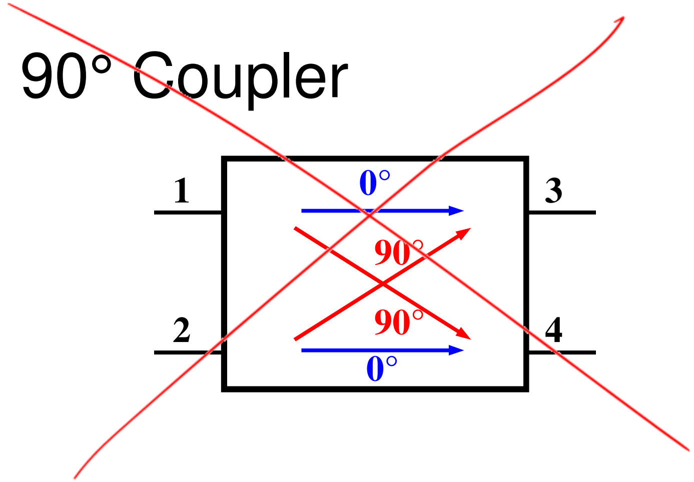
Coupler



ΣΔ Coupler

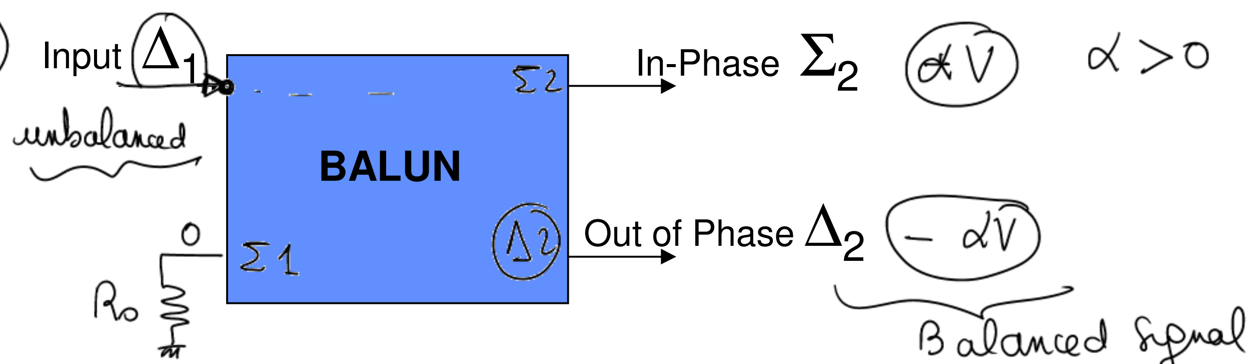


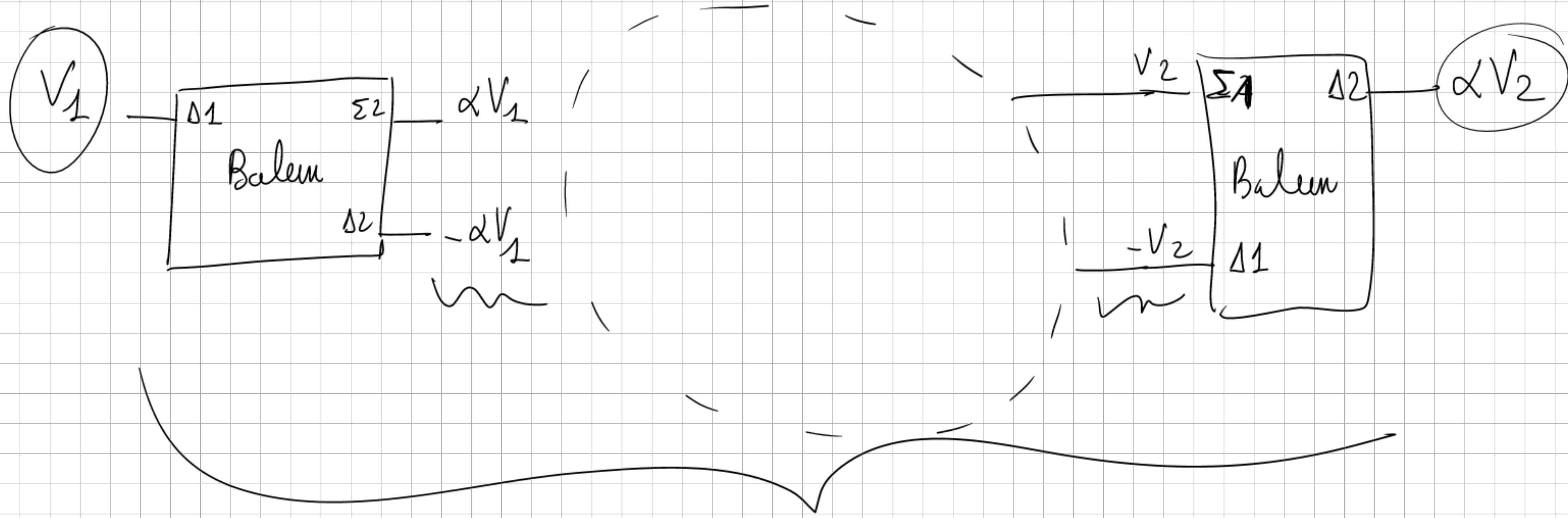
~~90° Coupler~~



BALUN

BALanced to UNbalanced





V1
Δ1
Σ2
Balun
Δ2
αV1
-αV1
V2
-V2
Σ1
Δ1
Balun
Δ2
αV2

Phase relationships of frequency products

$$2 \times \mathbf{cos}(\underbrace{\omega_{LO}t + \varnothing_{LO}}_{\text{LO}}) \times \mathbf{cos}(\underbrace{\omega_{RF}t + \varnothing_{RF}}_{\text{RF}}) = \mathbf{cos}[\underbrace{(\omega_{LO} + \omega_{RF})t + (\varnothing_{LO} + \varnothing_{RF})}_{\text{LO+RF}}] + \mathbf{cos}[\underbrace{(\omega_{LO} - \omega_{RF})t + (\varnothing_{LO} - \varnothing_{RF})}_{\text{LO-RF}}]$$

$$(\omega_{LO} + \omega_{RF}) \rightarrow (\varnothing_{LO} + \varnothing_{RF})$$

RF → ⊗ → IF

$$(\omega_{LO} - \omega_{RF}) \rightarrow (\varnothing_{LO} - \varnothing_{RF})$$

$$\text{If} \quad \omega_{IF} = \omega_{LO} - \omega_{RF} \rightarrow \varnothing_{IF} = (\varnothing_{LO} - \varnothing_{RF})$$

LSB

$$f_{IF} = f_{LO} - f_{RF} \longrightarrow \varnothing_{IF} =$$

RF → IF

SBM Active Mixers

$f_{IF} = f_{LO} - f_{RF}$ $\quad \phi_{IF} = \phi_{LO} - \phi_{RF}$

$\omega_{IF} = \omega_{LO} - \omega_{RF}$ → Phase(IF) = Phase(OL) - Phase(RF)

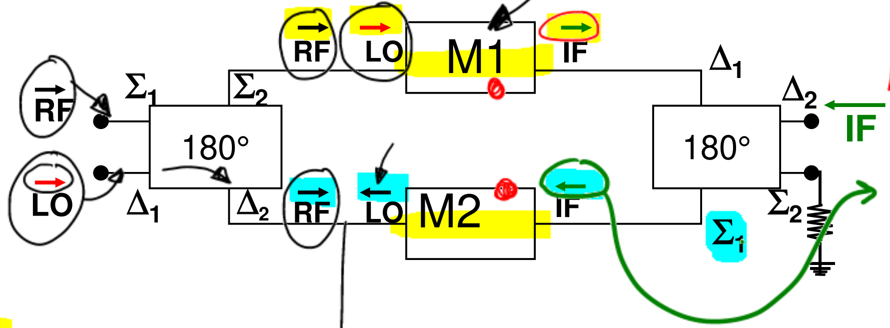
Notations

EH = Even Harmonic

OH = Odd Harmonic

Mix [EH(LO) , OH(RF)]= $\pm 2m\ \omega_{LO} \pm 2(n+1)\ \omega_{RF}$ with (m,n) integers

- SBM with 180° Couplers



- Good LO-RF isolation ⇔ Good Input Coupler isolation
- In all cases, **mix [EH(LO) , OH(RF)] are rejected at the output**
- LO at Δ port (the illustrated case in this slide)

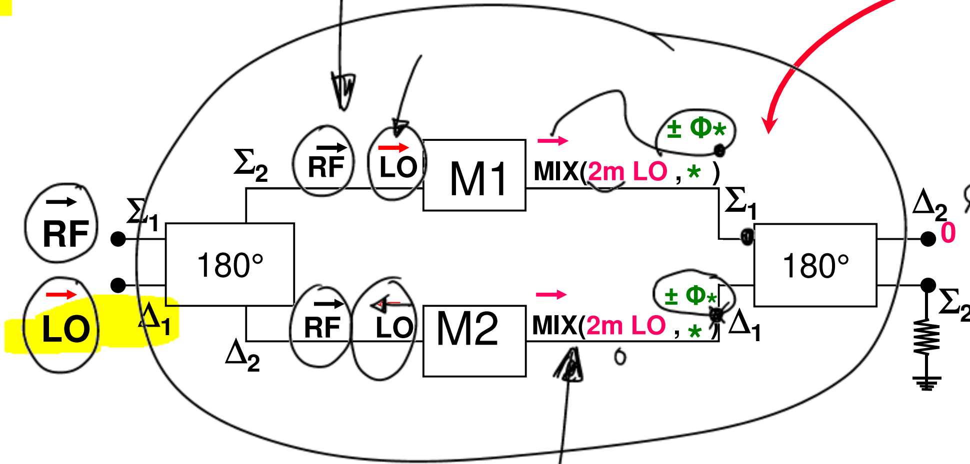
  → **mix [EH(LO) , *] are rejected** at the output

  $\pm 2m\ \omega_{lo} \pm p\omega_{RF}$

- LO at Σ port

  → **mix [* , OH (RF)] are rejected**

  $\pm p\omega_{OL} \pm (2k+1)\ \omega_{RF}$ rejected



$\phi_* \Leftrightarrow \phi_* + 180° \Rightarrow 0$

$\underbrace{2m\,\omega_{lo} \pm \omega_*}$

$2m\,\phi_{lo} \pm \phi$

RF → IF

if $2m = 0$

$p\omega_{RF}$ rejected

$Iso_{RF-IF} = \infty$

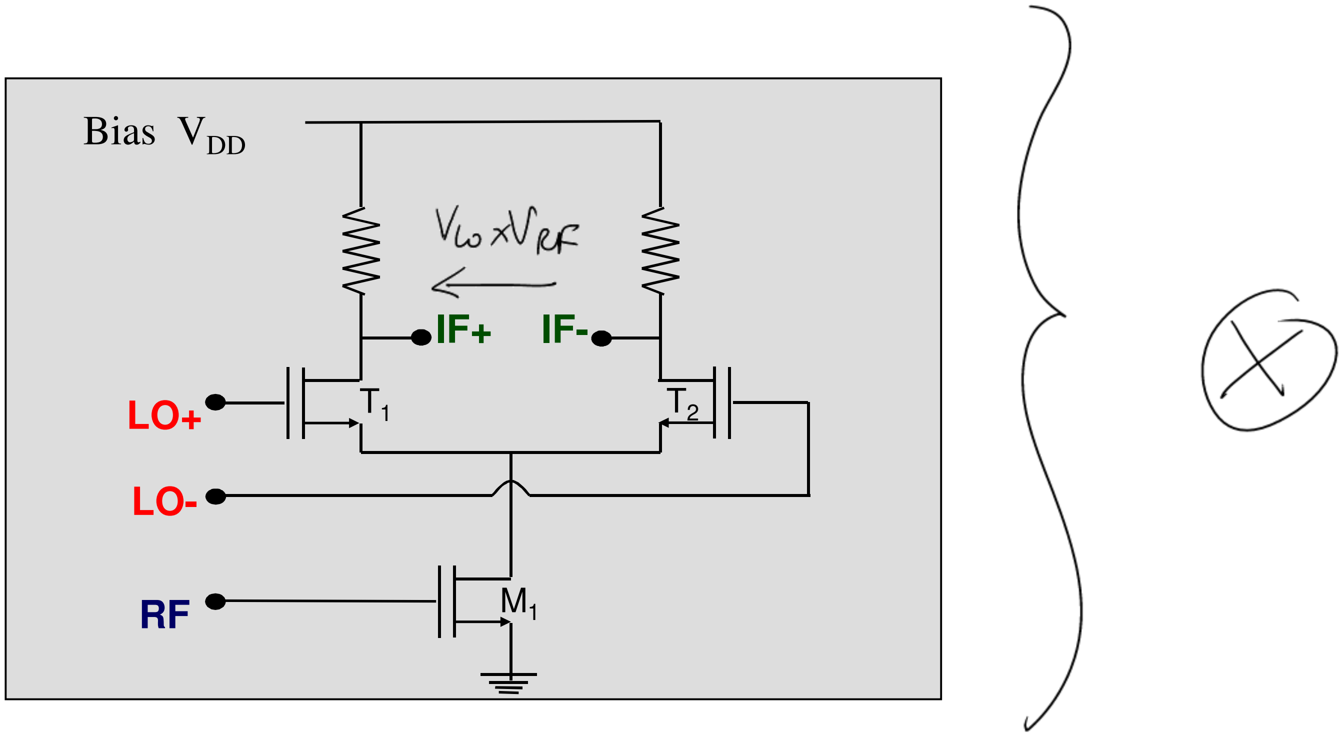
① LO - RF Iso = LO-RF Iso of coupler

② LO - IF Iso

③ RF - IF Iso = ∞

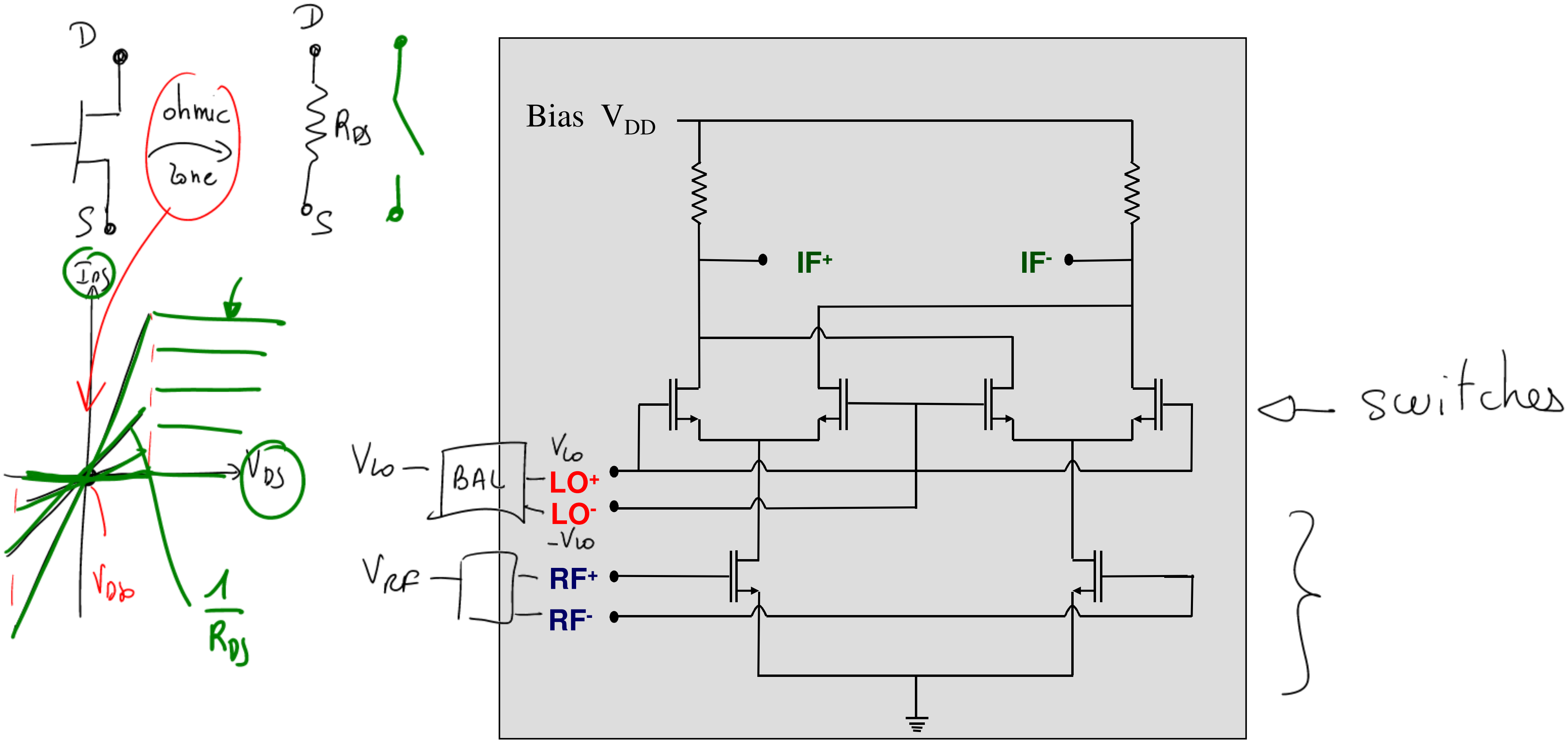
Example of Gilbert Cell Mixers

Single Balanced Gilbert Cell Mixer

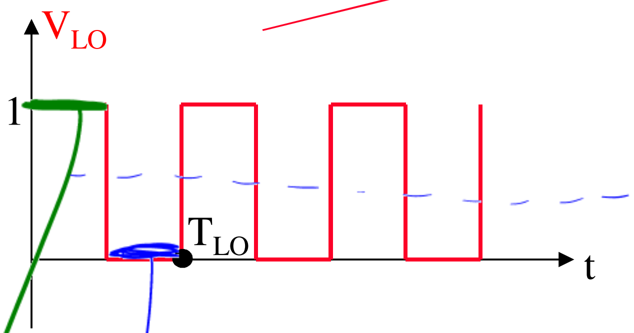


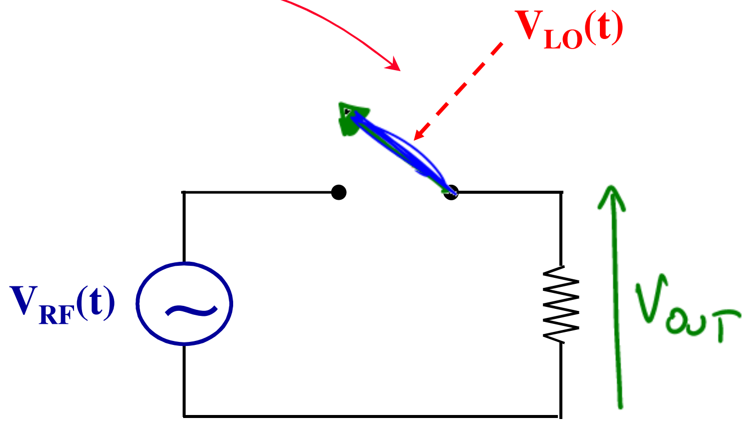
♦ Double Balanced Gilbert Cell Mixer

Double Balanced Gilbert Cell Mixer



Switch controlled by the LO signal
Square Signal (50% duty cycle)

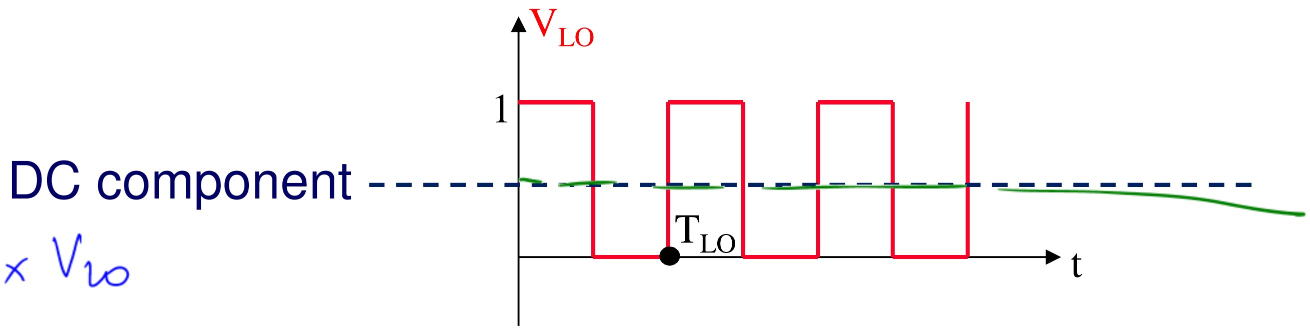




The linear signal path RF→IF is open and closed by a switch controlled at the LO rate

→ The output signal results from the multiplication of the RF signal by the square LO signal

→ Unfortunately, the output signal includes a parasitic RF component due to the DC component of the LO signal

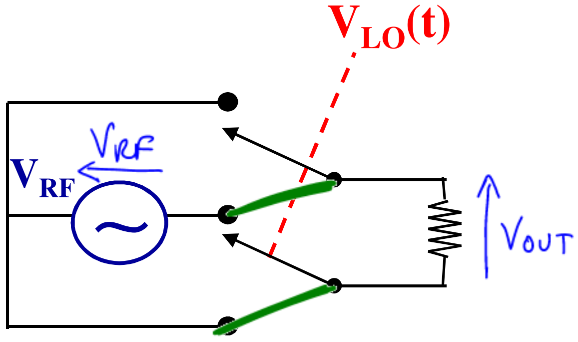


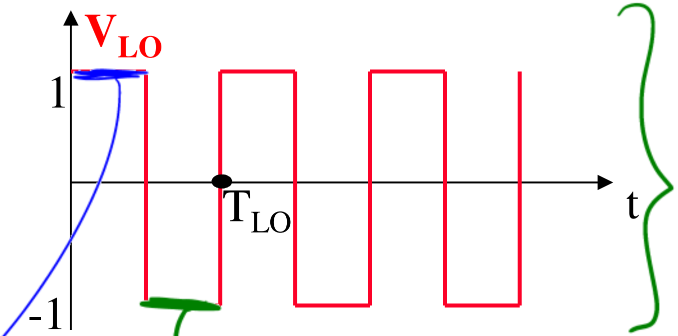
$V_{OUT} = V_{RF}$

$V_{OUT} = 0$

$V_{OUT} = V_{RF} \times V_{LO}$

Switch controlled by the LO signal with the suppression of its DC component





$$\text{sign}[V_{LO}(t)] = \frac{4}{\pi}\left[\cos(\omega_{LO}.t) - \frac{1}{3}\cos(3\omega_{LO}.t) + \frac{1}{5}\cos(5\omega_{LO}.t) - \ \dots\right]$$

$V_{OUT} = V_{RF}$

$V_{OUT} = -V_{RF}$

$V_{OUT} = V_{RF} \times \pm 1$

$\text{sign}(V_{LO})$

$\downarrow$

$(2k+1)\,\omega_{lo}$

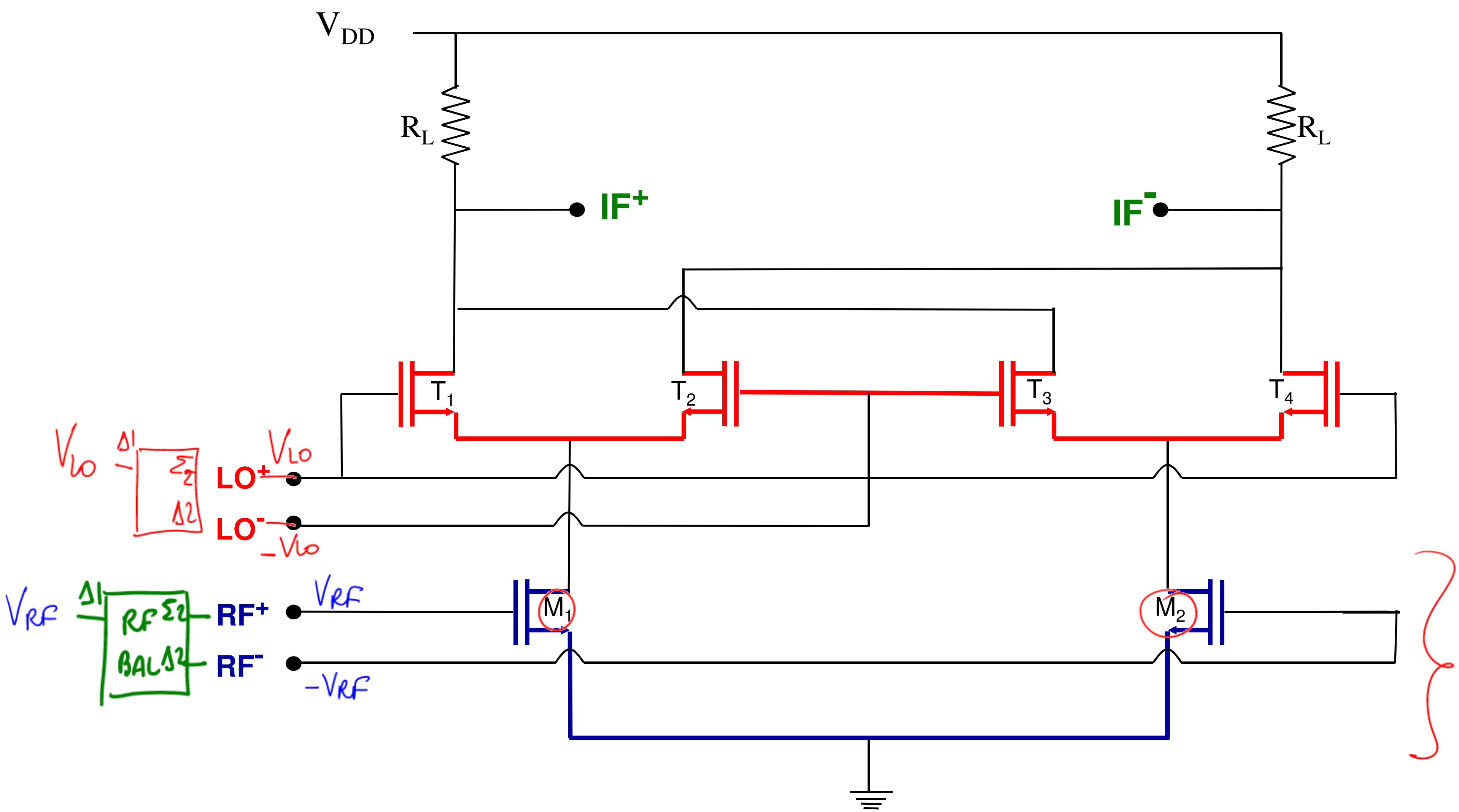
The sign of the RF signal is periodically changed at the LO rate instead of being cut

→ There is no DC component and the RF signal is fully rejected at the output

→ The RF signal is fully transposed at the IF frequency

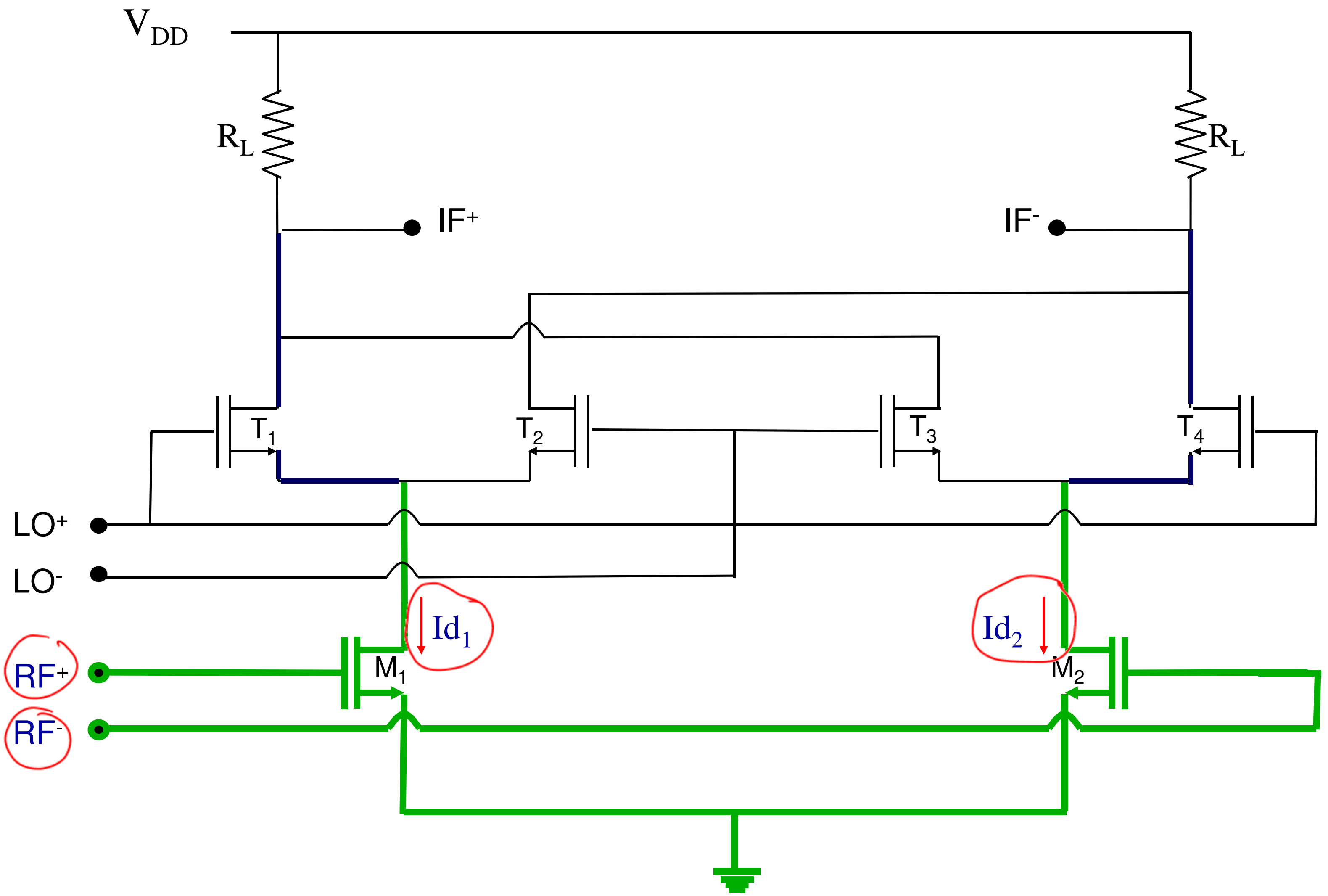
$(2k+1)\,\omega_{lo} \pm \omega_{RF}$

Double Balanced Active Gilbert Cell Mixer



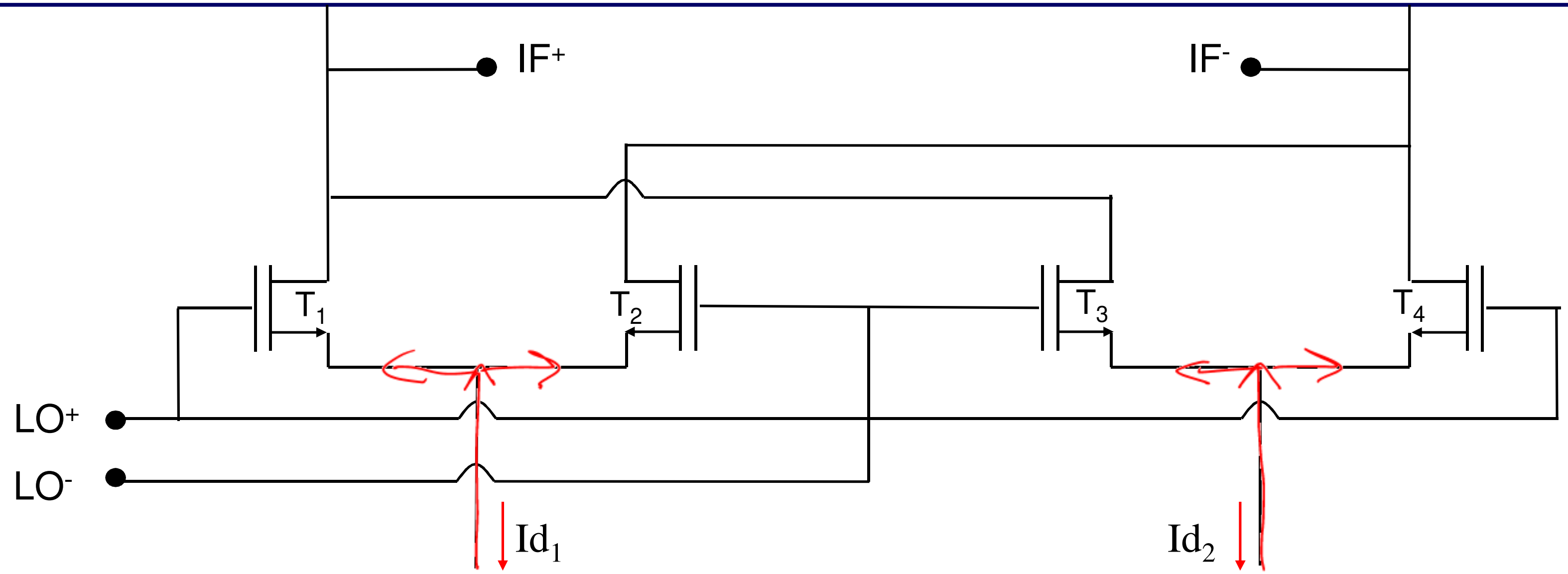
- **The differential RF pair** gives gain and the RF-IF path must be as linear as possible
- **The 2 LO pairs** work as switches to perform the frequency conversion

Differential RF pair : Voltage-Current conversion



The differential RF pair gives the conversion gain through the drain current Id (Vgs)

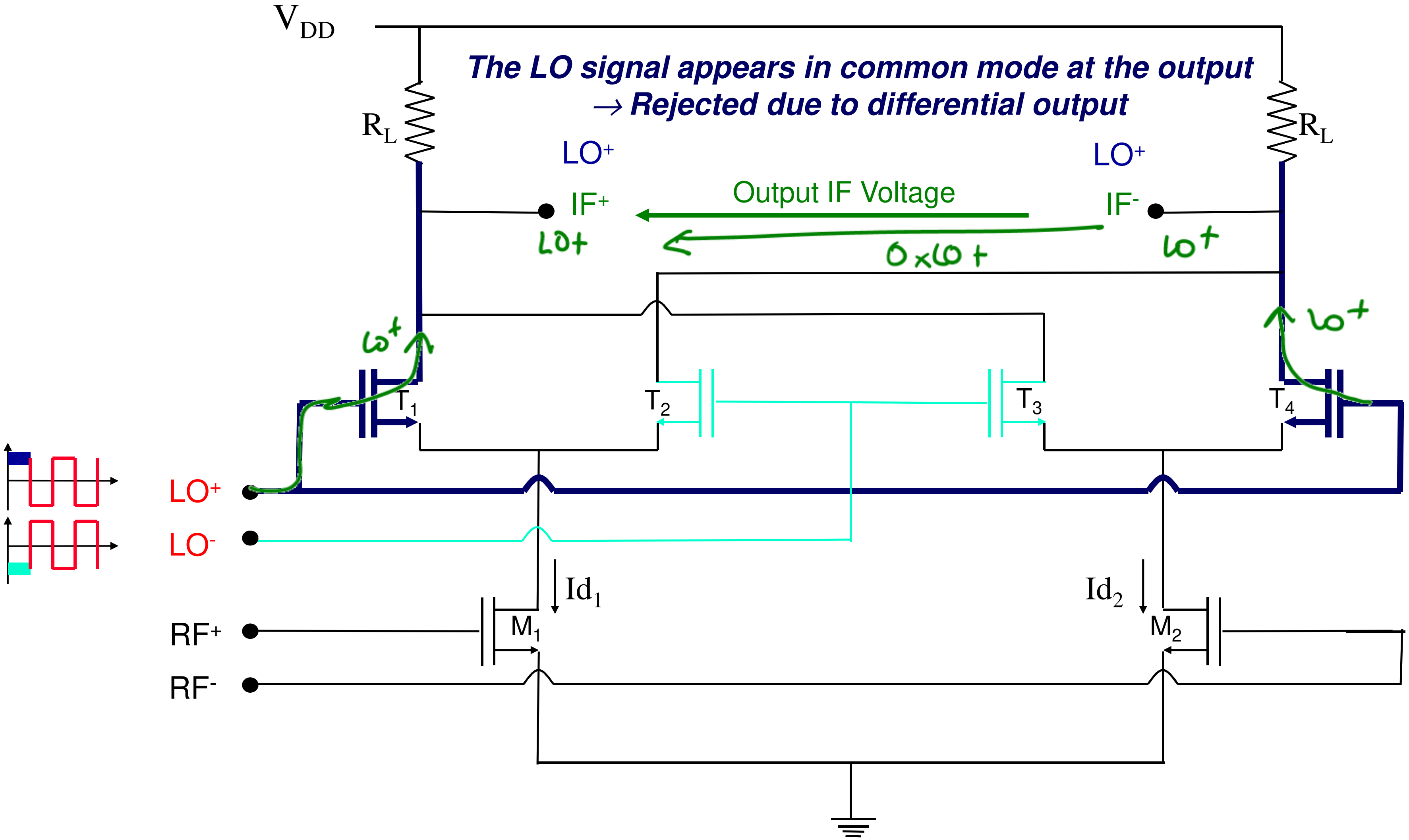
Differential LO pairs operate as switches : Mixing Core



• Id_1 and Id_2 currents of the RF pair are flowing towards the two LO pairs controlled by the outphased LO signals

• The 2 LO pairs operate as switches ⇔ Requirement of a large LO power to allow an efficient ON/OFF operation

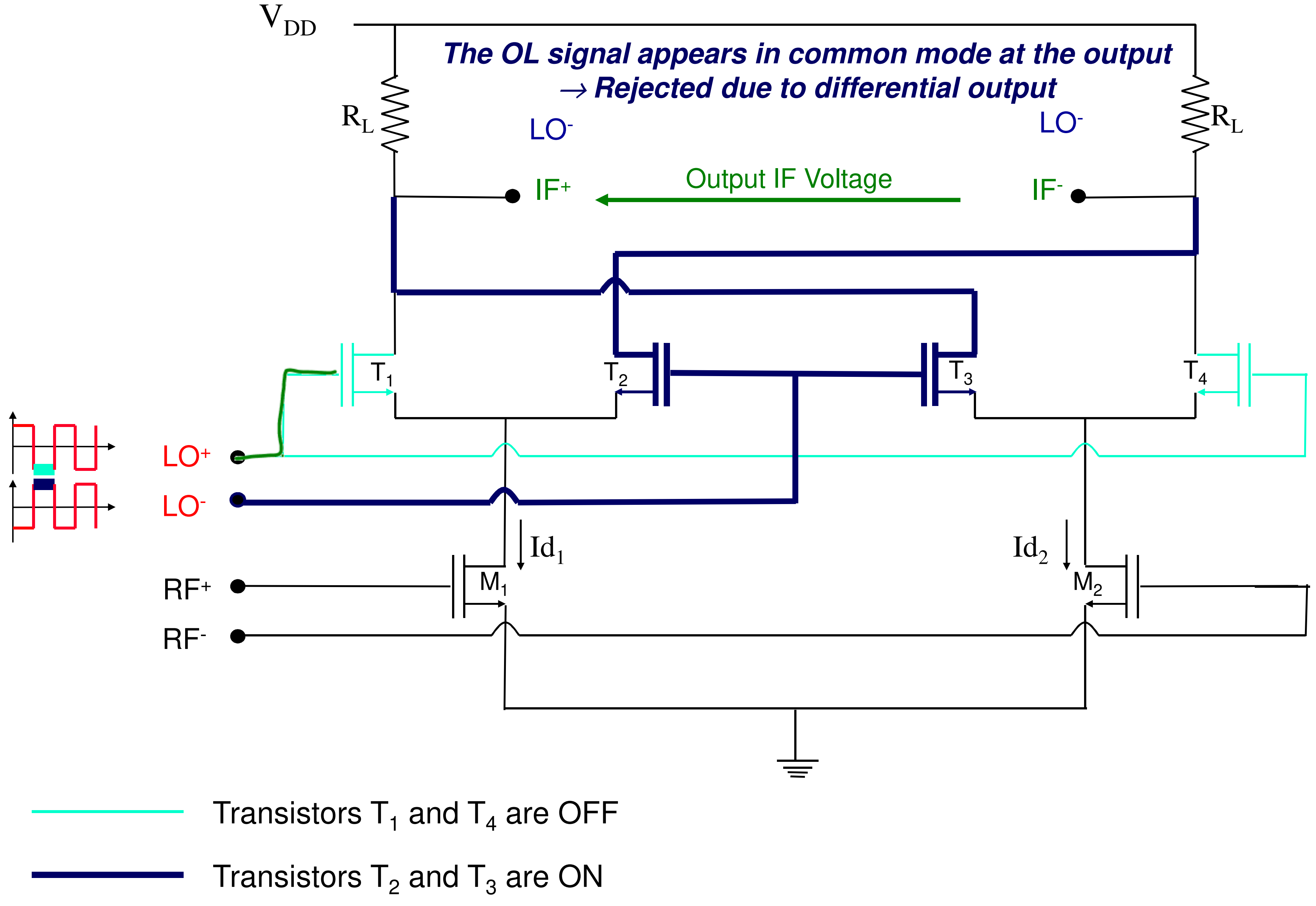
LO pair : Switching
1st Positive Alternation [0 - T_{LO}/2]



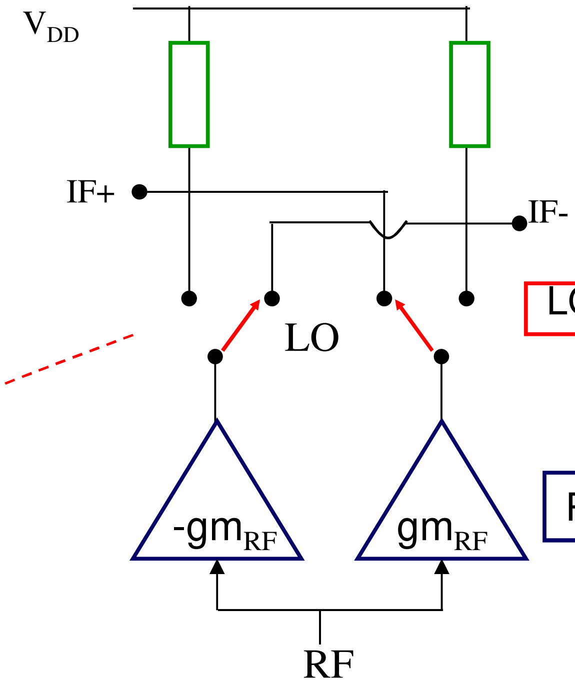
Transistors T_2 and T_3 are OFF

Transistors T_1 and T_4 are ON

LO pair : Switching
2^{nd} Negative Alternation $[T_{LO}/2 - T_{LO}]$



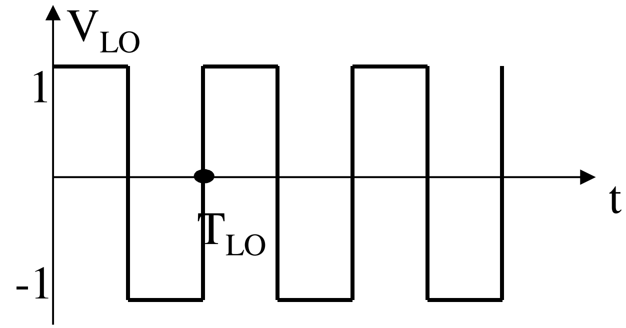
Operating mode of the Gilbert DBM



LO stage « switching - mixing »

RF stage « Linear Conversion V→I »

LO switching

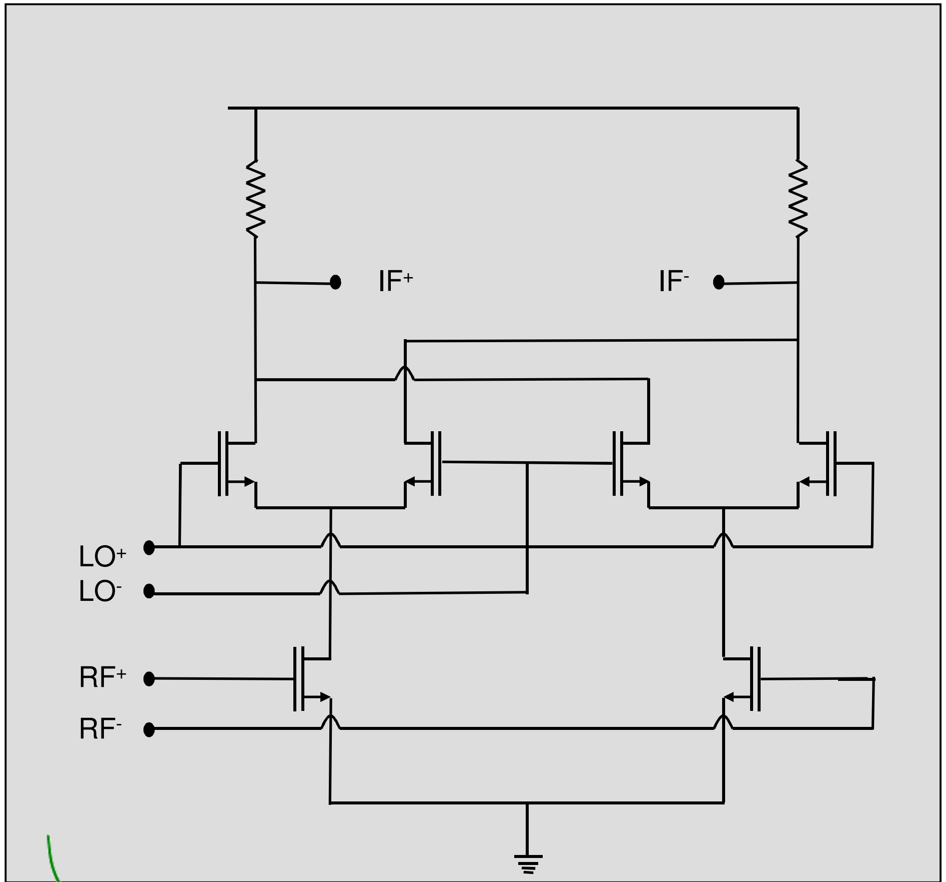


- *No generation of even LO Harmonics*

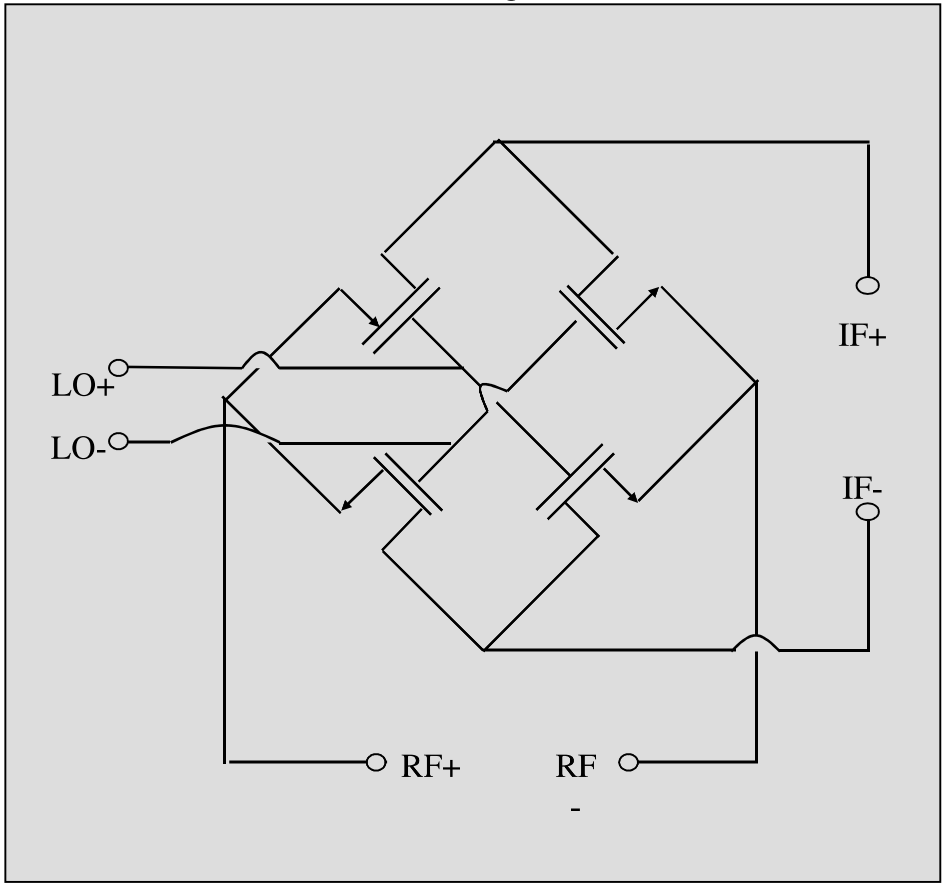
$$\text{sign}[V_{LO}(t)] = \frac{4}{\pi}\left[\cos(\omega_{LO}.t) - \frac{1}{3}\cos(3\omega_{LO}.t) + \frac{1}{5}\cos(5\omega_{LO}.t) - \ \ldots\right]$$

Example of two DBM (Active and Passive):
Active DBM mixer (Gilbert Cell) / Passive DBM mixer (Ring mixer of cold FETs)

Double Balanced Mixer « Gilbert Cell»



« Passive ring mixer »



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