Exam 2022

Exercise 1

a) Done. b) Done. d) Done e) Done.

c) Starting from definitions of emissivity and brightness temperature explain how the thermal radiation of any object can be described.

Emissivity: $L_{\varepsilon,\lambda} = \varepsilon(\lambda) \cdot L_\lambda$

Brightness temperature: $\varepsilon(\lambda) L_\lambda(\lambda, T) = L_\lambda(\lambda, T_b)$

$x = \dfrac{hc}{\lambda k_B T}$

$y = \dfrac{hc}{\lambda k_B T_b}$

$$\varepsilon \cdot \frac{2hc^2}{\cancel{\lambda^5}} \cdot \frac{1}{e^x - 1} = \frac{2hc^2}{\cancel{\lambda^5}} \cdot \frac{1}{e^y - 1} \Rightarrow e^y - 1 = \frac{e^x - 1}{\varepsilon} \Rightarrow e^y = \frac{e^x - 1}{\varepsilon} + 1$$

$$\Rightarrow y = \ln\left[\frac{e^x - 1}{\varepsilon} + 1\right] \Rightarrow \frac{hc}{\lambda k_B T_b} = \ln\left[\frac{e^x - 1}{\varepsilon} + 1\right] \Rightarrow \boxed{T_b = \frac{hc}{\lambda k_B} \frac{1}{\ln\left[\frac{\exp\left[\frac{hc}{\lambda k_B T}\right] - 1}{\varepsilon} + 1\right]}}$$

If $\lambda \to \infty$ (Rayleigh-Jeans approx)

$$\frac{hc}{\lambda k_B T} << 1 \Rightarrow \begin{cases} e^x = 1 + x \\ e^y = 1 + y \end{cases} \Rightarrow 1 + y - 1 = \frac{1 + x - 1}{\varepsilon} \Rightarrow \frac{\cancel{hc}}{\cancel{\lambda k_B} T_b} = \frac{\cancel{hc}}{\cancel{\lambda k_B} T} \cdot \frac{1}{\varepsilon} \Rightarrow \boxed{T_b = \varepsilon T}$$

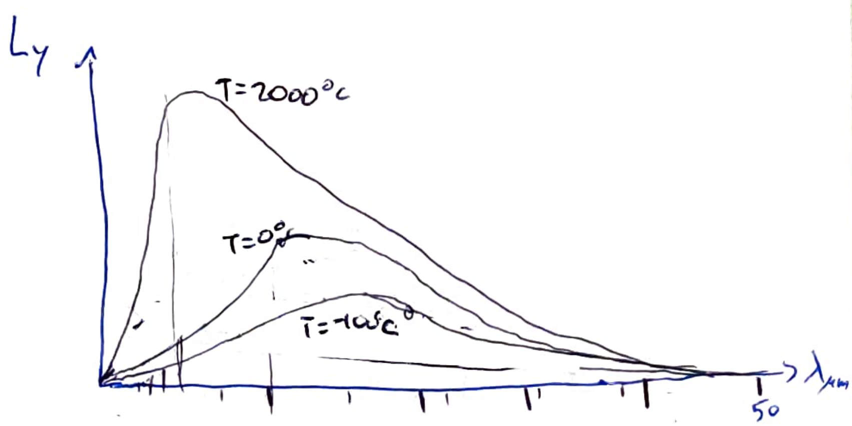
f) Plot qualitatively spectral radiance of black body for 3 temperatures $-100°c$, $0°c$ and $2000°c$.

$\lambda_{max} = \frac{A}{T}$, $A \approx 3 \cdot 10^{-3}$

$T_1 = -100°c = 173K \Rightarrow \lambda_{max_1} \simeq 18\,\mu m$

$T_2 = 0°c = 273K \Rightarrow \lambda_{max_2} \simeq 10\,\mu m$

$T_3 = 2000°c = 2273K \Rightarrow \lambda_{max_3} \simeq 1{,}2\,\mu m$



Exercise 2

a). Done b) Done. c) Done. d) Done.

e) Resolution limitation by diffraction or film?

Data:

$H = 200 \cdot 10^3\ m$ $\quad D = 8 \cdot 10^{-2}\ m$ $\quad$ Near infrared range: $0{,}75 \cdot 10^{-6}\ m$

$r = 150 \cdot 10^3\ lp/m$ $\quad f = 120 \cdot 10^{-3}\ m$ $\quad$ $8 \cdot 10^{-6}\ m$

$s = \frac{f}{H} = 6 \cdot 10^{-7}$

Film

$\delta x = \frac{1}{2r} = \frac{1}{2 \cdot 150 \cdot 10^3} \Rightarrow \delta x = 3{,}33 \cdot 10^{-6}\ m$

$\delta x_g = \frac{\delta x}{s} = \frac{3{,}33 \cdot 10^{-6}}{6 \cdot 10^{-7}} \Rightarrow \delta x_g = 5{,}55\ m$

Diffraction

① $\lambda = 0{,}75 \cdot 10^{-6}\ m$

$\Delta\theta = 1{,}22 \frac{\lambda}{D} = 1{,}22 \cdot \frac{0{,}75 \cdot 10^{-6}}{8 \cdot 10^{-2}} = 1{,}14375 \cdot 10^{-5}\ rad$

$R_F = f \Delta\theta = 120 \cdot 10^{-3} \cdot 1{,}14375 \cdot 10^{-5} \Rightarrow R_F = 1{,}3725 \cdot 10^{-6}\ m$

$R = \frac{R_F}{s} = \frac{1{,}3725 \cdot 10^{-6}}{6 \cdot 10^{-7}} \Rightarrow R = 2{,}2875\ m$

② $\lambda = 8 \cdot 10^{-6}\ m$

$\Delta\theta = 1{,}22 \cdot \frac{8 \cdot 10^{-6}}{8 \cdot 10^{-2}} = 1{,}22 \cdot 10^{-4}\ rad$

$R_F = 120 \cdot 10^{-3} \cdot 1{,}22 \cdot 10^{-4} \Rightarrow R_F = 14{,}64 \cdot 10^{-6}\ m$

$R = \frac{14{,}64 \cdot 10^{-6}}{6 \cdot 10^{-7}} \Rightarrow R = 24{,}4\ m$

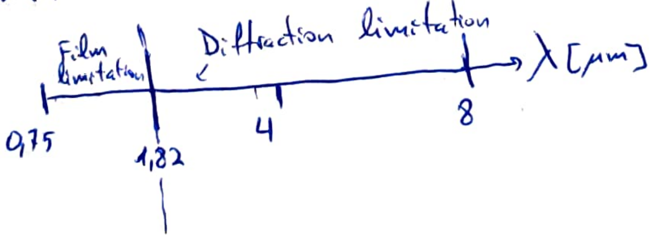
The limitation of the resolution on the ground is on the film for the shorter λ's of the range, while the diffraction limits the resolution for longer λ's

The λ that matches with film resolution is:

$\left.\begin{array}{l} R_F = \Delta\theta f \\ \Delta\theta = 1{,}22 \frac{\lambda}{D} \end{array}\right\rangle \lambda = \frac{R_F D}{1{,}22 f} = \frac{3{,}33 \cdot 10^{-6} \cdot 8 \cdot 10^{-2}}{1{,}22 \cdot 120 \cdot 10^{-3}} = 1{,}82 \cdot 10^{-6}\ m \Rightarrow \lambda = 1{,}82\ \mu m$

$R_F = \delta x$

So the limitation is like this

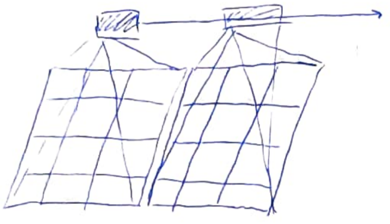


Exercise 3

a). Done

b) Two-dim. detector array

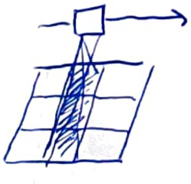
We can use the so-called Step-Stare imaging.
The detector array stares at a scene and the moves on to stare at next scene.



So we have to synchronize the exposition time we need for the system with the velocity of the satellite.
So the speed of the film is an important parameter

c) One-dim detector array

If the detector is a linear array we can use the so-called Push-Broom Imaging.
We synchonize the speed of the satellite and the detector to get the full picture



The line of detectors are arranged perpendicular to the direction of motion.

Another one is whisk-Broom Imaging