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OPTOELECTRONICS

Exam 1

Grade:

Problem 1	*Problem 2*
a)	a)
b)	b)
c)	c)
	d)

Questions		
1) **b**	5) **d**	9) **b**
2) **c**	6) **c**	10) **c**
3) **c**	7) **d**	11) **c**
4) **a**	8) **a**	12) **d**

Problem 1 (4 pts).

We have a SLED with an emitting square surface of 0.09 mm^2 That we want to inject into a 175/62.5 μmfiber with numerical aperture 0.2. The device allows a distance between the fiber and the surface of the SLED of 3.5 cm.

a) **Determine the type of lens required, the focal, and the position of the lens (2.5 pts)**

We need to create a real image $q > 0$ of a real object $p < 0$, the only possibility using only one lens is a **convergent lens $f > 0$.**

Our object is the SLED and the image is needed to fit inside the fiber. Given that the SLED is square and the fiber is circular, we need to reduce the size of the image so the diagonal fits inside the area of the core.

$$2a = 62.5\ \mu m$$

$$A = d^2 = 0.09\ mm^2$$

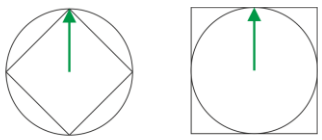
$$d = 0.3\ mm = 300\ \mu m$$

We center both object and image and we obtain:

$$h = d\sqrt{2}/2$$

$$h' = a$$

Note: Even the scribed square is optimal (all light is going inside the core), the circumscribed square ($h = d/2$) has been also considered a valid solution in the correction.



$h = d\sqrt{2}/2$ $\qquad$ $h = d/2$

Using the magnification coefficient we can create a relation between the position of the object p and the image q

$$M = \frac{q}{p} = \frac{h'}{h} = \pm\frac{\left(\frac{62.5}{2}\right)}{\left(\frac{300\sqrt{2}}{2}\right)} \approx \pm 0.147$$

As we know that $p < 0$ and $q > 0$, the image is flipped, therefore $M < 0$

$$q = Mp$$

$$M = -0.147$$

The other imposition given by the problem is that the distance between the SLED (the object) and the fiber (the image) is $L = 3.5\ cm$

$$L = q - p = p(1 - M)$$

So we can obtain both positions

$$p = \frac{L}{1 - M} = \mathbf{3.05\ cm}$$

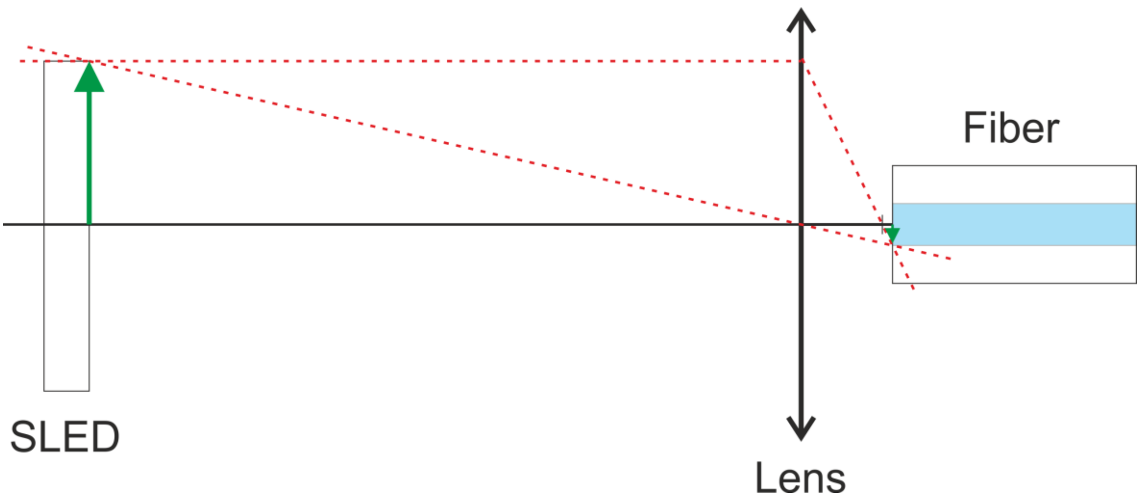
$$q = Mp = \frac{ML}{(1 - M)} = \mathbf{0.45\ cm}$$

To obtain the focal we can just apply the thin lens equation

$$\frac{1}{f} = \frac{1}{q} - \frac{1}{p}$$

$$f = \frac{qp}{p - q} = 0.39\ mm \approx \mathbf{4\ mm}$$

b) Draw the system and do the ray tracing (1pts)



Note: An approximated drawing showing the main rays will be enough

c) Determine the number of modes for a STEP and a GRIN fiber with the mentioned characteristics if the SLED emits at 1.05 μm (0.5pts)

The calculation is direct from the V number

$$V = \frac{2\pi}{\lambda} aNA = 37.39$$

And the expressions for STEP and GRIN fibers

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$$STEP \rightarrow N \approx \frac{V^2}{2} \approx \mathbf{700\ \textit{modes}}$$

$$GRIN \rightarrow N \approx \frac{V^2}{4} \approx \mathbf{350\ \textit{modes}}$$

(TIP: The SLED is symmetrical, there is no problem if the image is flipped)

Problem 2. (7pts)

We have an p-n homojunction based on GaAs, that we want to use as a detector.

a) **Determine the intrinsic carrier density and the potential bias V_D using the provided values. (2pts)**

The intrinsic carrier density is a constant value for a semiconductor.

$$n_i^2 = p * n$$

Also, we have a homo-junction, so the intrinsic carrier density is constant along the p-n junction. We can use the definitions of electrons and holes in the equilibrium

$$n_0 = N_c e^{-\frac{E_c - E_F}{kT}}$$

$$p_0 = N_v e^{\frac{E_v - E_F}{kT}}$$

These will lead to:

$$n_i^2 = N_v N_c e^{-\frac{E_c - E_v}{kT}}$$

Using the definition of the energy bandgap

$$\boldsymbol{n_i = \sqrt{N_v N_c} e^{-E_g/2kT} = 2.15 \cdot 10^{12}\ m^{-3}}$$

Given the high values of dopants, we can approximate the main carriers in the p and n regions as the dopant concentration

$$n_{n0} \approx N_d = 5 \cdot 10^{21}\ m^{-3}$$

$$p_{p0} \approx N_a = 5 \cdot 10^{23}\ m^{-3}$$

At this point we can either use the p or n region and the definition of the intrinsic carrier. For the n region this leads to:

$$n_i^2 = n_{n0} p_{n0} = N_d * N_a e^{\frac{-eV_D}{kT}}$$

We isolate the potential bias

$$\boldsymbol{V_D = -\frac{kT}{e} \ln\left(\frac{n_i^2}{N_a N_d}\right) = 1.234\ V}$$

b) **Determine the external potential required so 95% of the light is absorb in the depletion region. (Neglect absorption in the p or n regions) (2pts)**

As we have been told to ignore the p and n regions, we just apply the Beer's law to calculate the thickness of the depletion region

$$\frac{I}{I_0} = (1 - 0.95) = e^{-w\alpha}$$

$$w = -\frac{1}{\alpha} \ln(0.05) = 4.28\ mm = 4.28 \cdot 10^{-3}\ m$$

Now It is just applying the formula

$$w = \sqrt{\frac{2\epsilon}{e}(V_D + V)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$

$$\boldsymbol{V = \frac{e}{2\epsilon} w^2 \left(\frac{N_a N_d}{N_a + N_d}\right) - V_D = 8.28 \cdot 10^8\ V}$$

Note: This value is not realistic because there was a typo with the absorption coefficient, it should be at least in μm^{-1}

c) Calculate the efficiency of this detector knowing that the maximum theoretical responsibility is 0.33. (1pts)

Using the equation of the responsibility we see that the value increases with the wavelength.

$$\Re = \eta \frac{e}{hc} \lambda$$

The maximum wavelength that can be absorbed by the semiconductor is the so called cut wavelength, which is the one corresponding to the energy bandgap

$$\lambda_{cut} = \frac{hc}{E_g} = 867\ nm$$

Therefore, the maximum theoretical responsibility is at the band gap energy:

$$\Re_{max} = \eta \frac{e}{hc} \lambda_{cut} = \eta \frac{e}{E_g}$$

$$\boldsymbol{\eta = \Re_{max} \frac{E_g}{e} = 0.48}$$

d) We have two possible light sources, GaAs and InP, calculate the most intense wavelength of emission for each one at room temperature. If the proportionality constant α_{emi} is equal for both semiconductors, discuss which one is going to create more current in the detector. (2pt)

To calculate the most intense wavelength we need to find the maximum of the power emitted. The derivative is easier to do in energies.

$$\frac{dP}{dE_{ph}} = 0$$

$$e^{-\frac{(E_{ph}-E_g)}{kT}} + (E_{ph} - E_g)\left(-\frac{1}{kT}\right) e^{-\frac{(E_{ph}-E_g)}{kT}} = 0$$

$$1 + (E_{ph} - E_g)\left(-\frac{1}{kT}\right) = 0$$

$$1 + (E_{ph} - E_g)\left(-\frac{1}{kT}\right) = 0$$

$$E_{ph}^{max} = E_g + kT$$

At the given temperatures the values for both semiconductors are:

$$\begin{cases} \boldsymbol{GaAs} \ \rightarrow \boldsymbol{\lambda_{max}} = \mathbf{851}\ \boldsymbol{nm} \\ \ \boldsymbol{InP} \ \rightarrow \boldsymbol{\lambda_{max}} = \mathbf{956}\ \boldsymbol{nm} \end{cases}$$

Now, to properly calculate the current generated by each light source we should use the responsibility and integrate over the whole spectrum emitted

$$I_p = \Re P_{in} = \int_{E_g}^{\infty} \frac{e}{E_{ph}} \alpha_{emi}(E_{ph} - E_g) e^{-\frac{(E_{ph}-E_g)}{kT}} dE$$

A more direct solution is given by the cut wavelength. From the previous calculus we see that the most intense wavelength emitted by InP is not going to be absorbed because

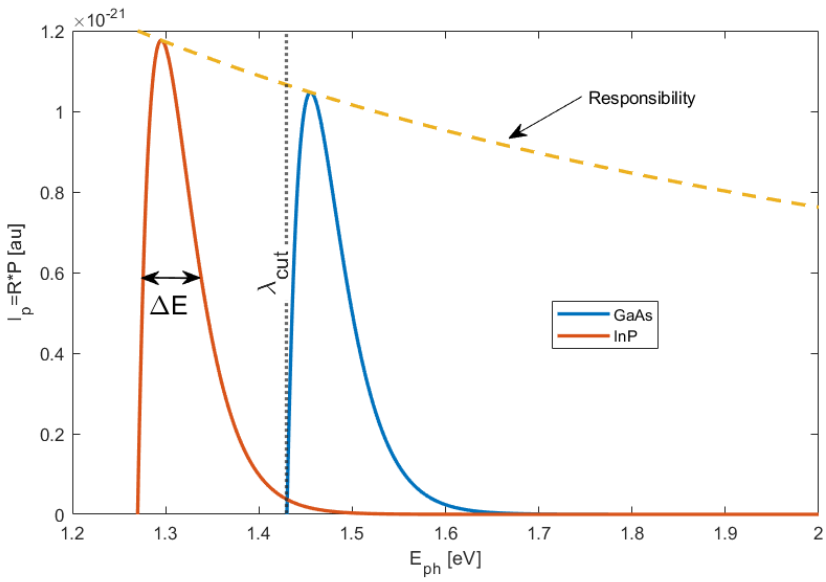
$$\lambda_{max}(InP) > \lambda_{cut}(GaAs) = 867\ nm$$

Also, if we check the width of the emission bell

$$\Delta E = 2.4kT$$

$$E_{ph}^{max}(InP) + \frac{\Delta E}{2} = E_g(InP) + 2.2kT \approx 1.33\ eV < E_g(GaAs)$$

What is still lower than the energy band gap from GaAs, that means that most of the light emitted by the InP is not going to be absorbed. **In conclusion, a GaAs light source is going to produce more current because most of the light emitted by InP is under the cut wavelength**.



Note: You were not supposed to do the integral or to draw the figure, just to point out the cut wavelength limitation and the width of the bell.

$T = 300\ K$	$m_e = 0.07m_e^0$	$m_h = 0.56m_e^0$
$\alpha = 0.7\ mm^{-1}$	$N_d = 5 * 10^{21}\ m^{-3}$	$N_a = 5 * 10^{23}\ m^{-3}$
	$\mu_e = 0.85\ m^2 \cdot V^{-1} \cdot s^{-1}$	$\mu_h = 0.04\ m^2 \cdot V^{-1} \cdot s^{-1}$
	$E_g(GaAs) = 1.43\ eV$	$E_g(InP) = 1.27\ eV$

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Questions. (0.75 pts each, 12 in total)

1. **After a given lens the beam rays converge. The image will be**
   a) Always virtual
   b) **Always real**
   c) Real only if the lens is convergent
   d) Real only if the object is virtual

C is not correct because a divergent lens with a virtual object can still produce a real image. For example $f = -2$, $q = 2$ and $p = 1$

2. **A planar monochromatic beam crosses the interface between two media with the same refractive index for that given wavelength $n_1 = n_2$, but opposite dispersion regime (one is anomalous and the other is normal dispersion). The beam direction will remain constant, $\theta_1 = \theta_2$,…**
   a) Only if the first medium has anomalous dispersion and the second normal dispersion
   b) Only if the first medium has normal dispersion and the second anomalous dispersion
   c) **Always**
   d) Never

There is no influence of the dispersion in the Snell's law

3. **The reflected beam obtained at the Brewster angle will be:**
   a) Always linear polarized
   b) Always perpendicular to the plane of propagation
   c) **Both a) and b)**
   d) Neither a) nor b)

In the Brewster angle, the component perpendicular **to the interface** is not reflected. That means that is linear polarized. B is always true for light $\vec{k} \cdot \vec{E} = 0$

4. **Two given fields such as $\overrightarrow{E_1}$ and $\overrightarrow{E_2} = \overrightarrow{E_1} e^{i\phi}$ interfere at a given plane. This interference is:**
   a) **Always constructive if $\phi = 0$ and destructive if $\phi = \pi$**
   b) Always constructive if both waves are spherical
   c) Always destructive if both waves are planar
   d) It depends on the plane of observation

Both fields are exactly equal but delayed so $I = |(E_1 + E_2)|^2 = |E_1|^2 \left(1 + e^{i\phi}\right)^2$. The only dependency is ϕ

5. **Given injection only in the center of a generic circular GRIN fiber, which is the relation between the parameter g and the numerical aperture of that fiber.**
   a) The numerical aperture is linear with g, $NA \propto g$
   b) The numerical aperture is quadratic with g, $NA \propto g^2$
   c) The numerical aperture is inversely proportional to g, $NA \propto g^{-1}$

d) **The numerical aperture is independent of g**

The paramenter g changes the shape of the profile but it was demonstrated in class that for the numerical aperture it only matters n_{core} and n_{clad}.

6. Mark the true affirmation

a) Single mode optical fiber have always one mode independently of the signal wavelength
b) Optical fibers are very sensitive to external electric fields
c) **Given a STEP and a GRIN fibers with the same parameters, the GRIN fiber will have less modes at any given wavelength**
d) A drawback in optical fibers is their huge signal attenuation

The number of modes depends on the V number which is a function of wavelength but for any given wavelength a GRIN fiber will have half of the modes of a STEP

7. The band-gap in an homojunction

a) Is determined by the dopant concentrations
b) Is greater in the depletion region
c) Is smaller in the depletion region
d) **Is constant**

You cannot compare bandgap and depletion region (one is an energy and the other a distance or volume). The dopants vary the carrier density but not the bandgap.

8. The internal quantum efficiency

a) **Is only determined by the lifetime of holes and electrons**
b) Is only determined by the physical structure of the device
c) Is a combination of the internal and external efficiencies
d) This parameter does not exist

The physical structure determines the external quantum efficiency, both together define the quantum efficiency.

9. Mark the correct one.

a) Surface and edge emitting LEDs have always the same efficiency
b) **Edge emitting LEDs have a linear response over a wide range.**
c) Injection laser diodes produce mainly incoherent light
d) The difference between SLED, ELED and ILD is the wavelength emitted

It cannot be a) as the external quantum efficiency is define by the structure of the LED, nor c) ILDs are coherent, nor d).

10. In order to reduce the gain coefficient threshold, you can:

a) Increase the longitudinal length
b) Reduce the transmission in the edge faces
c) **Either a) and/or b)**
d) Neither a) nor b)

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$$g_{th} = \alpha_{eff} + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)$$

It is possible to arrive to this expression from the gain. Increasing either L or R_i will reduce the second term value, which is always positive.

11. What is the main difference between photodiodes and photoconductors?

a) The photodiodes require an external potential but the photoconductors not

b) The photodiodes can detect high energy photons but the photoconductors not

c) The photodiodes involve at least two differently doped semiconductors but the photoconductors only one

d) The photodiodes are forward biased and photoconductors are reverse biased

12. Photodiodes are not sensitive to what kind of noise

a) Shot noise

b) Thermal noise

c) Dark noise

d) They are sensitive to all of them

Data.

$$e = -1.6 * 10^{-19}\ C$$

$$m_e^0 = 9.11 * 10^{-31}\ kg$$

$$\epsilon_0 = 8.854 * 10^{-12}\ \ F \cdot m^{-1}$$

$$\mu_0 = 1.257 * 10^{-6}\ N \cdot A^{-2}$$

$$h = 6.626 * 10^{-34}\ \ J \cdot s$$

$$c_0 = 3 * 10^{8}\ \ m \cdot s^{-1}$$

$$k = 1.381 * 10^{-23}\ J \cdot K^{-1}$$

$$N_A = 6.022 * 10^{23}$$

Equations.

Snell's Law

$$n_1 sin\theta_1 = n_2 sin\theta_2$$

Thin lens

$$\frac{1}{q} - \frac{1}{p} = \frac{1}{f}$$

Magnification

$$M = \frac{h'}{h} = \frac{n_1}{n_2}\frac{q}{p}$$

Brewster eq.

$$\theta_B = \arctan\frac{n_2}{n_1}$$

Fresnel coeffs.

$$r_\perp = \frac{n_1 cos\theta_1 - n_2 cos\theta_2}{n_1 cos\theta_1 + n_2 cos\theta_2} = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}$$

$$t_\perp = \frac{2 n_1 cos\theta_1}{n_1 cos\theta_1 + n_2 cos\theta_2}$$

$$r_{||} = \frac{n_2 cos\theta_1 - n_1 cos\theta_2}{n_1 cos\theta_2 + n_2 cos\theta_1} = -\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)}$$

$$r_{||} = \frac{2 n_1 cos\theta_1}{n_1 cos\theta_2 + n_2 cos\theta_1}$$

Current carrier densities

$$n_0 = N_c e^{-\frac{E_c - E_F}{kT}}$$

$$N_c = 2\left(2\pi m_e \frac{kT}{h^2}\right)^{3/2}$$

$$p_0 = N_v e^{\frac{E_v - E_F}{kT}}$$

$$N_v = 2\left(2\pi m_h \frac{kT}{h^2}\right)^{3/2}$$

$$n_{p0} = N_d e^{\frac{-eV_D}{kT}}$$

$$p_{n0} = N_a e^{\frac{-eV_D}{kT}}$$

Diffusion parameters

$$D_{h/e} = \frac{\mu_{h/e} kT}{e}$$

$$L_{h/e} = \sqrt{D_{h/e}\tau_{h/e}}$$

Power emission

$$P = \alpha_{emi}\left(E_{ph} - E_g\right)e^{-\frac{\left(E_{ph} - E_g\right)}{kT}}$$

Gain

$$G = R_1 R_2 e^{2\left(g - \alpha_{eff}\right)L}$$

ILD Efficiency

$$\eta = \eta_{int}\eta_{ext} = \eta_{int}\frac{\left(g_{th} - \alpha_{eff}\right)}{g_{th}}$$

Responsibility

$$\Re = \frac{I_p}{P_{in}} = \eta\frac{e}{h\nu} = \eta\frac{e}{hc}\lambda$$

Depletion region width

$$w = \sqrt{\frac{2\epsilon}{e}\left(V_d + V\right)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$