

E. Useful Mathematical Formulas

E.1. Summation Formulas

$$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} \frac{1 - \alpha^N}{1 - \alpha} & \alpha \neq 1 \\ N & \alpha = 1 \end{cases}$$

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1 - \alpha} \quad |\alpha| < 1$$

$$\sum_{n=k}^{\infty} \alpha^n = \frac{\alpha^k}{1 - \alpha} \quad |\alpha| < 1$$

$$\sum_{n=0}^{\infty} n\alpha^n = \frac{\alpha}{(1 - \alpha)^2} \quad |\alpha| < 1$$

$$\sum_{n=0}^{\infty} n^2 \alpha^n = \frac{\alpha^2 + \alpha}{(1 - \alpha)^3} \quad |\alpha| < 1$$

E.2. Euler's Formulas

$$e^{\pm j\theta} = \cos \theta \pm j \sin \theta$$

$$\cos \theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$

$$\sin \theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$$

E.3. Trigonometric Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$a \cos \alpha + b \sin \alpha = \sqrt{a^2 + b^2} \cos \left(\alpha - \tan^{-1} \frac{b}{a} \right)$$

E.4. Power Series Expansions

$$e^{\alpha} = \sum_{k=0}^{\infty} \frac{\alpha^k}{k!} = 1 + \alpha + \frac{1}{2!} \alpha^2 + \frac{1}{3!} \alpha^3 + \dots$$

$$(1 + \alpha)^n = 1 + n\alpha + \frac{n(n-1)}{2!} \alpha^2 + \dots + \binom{n}{k} \alpha^k + \dots + \alpha^n$$

$$\ln(1 + \alpha) = \alpha - \frac{1}{2} \alpha^2 + \frac{1}{3} \alpha^3 - \dots + \frac{(-1)^{k+1}}{k} \alpha^k + \dots \quad |\alpha| < 1$$

E.5. Exponential and Logarithmic Functions

$$e^{\alpha} e^{\beta} = e^{\alpha + \beta}$$

$$\frac{e^{\alpha}}{e^{\beta}} = e^{\alpha - \beta}$$

$$\ln(\alpha\beta) = \ln \alpha + \ln \beta$$

$$\ln \frac{\alpha}{\beta} = \ln \alpha - \ln \beta$$

$$\ln \alpha^{\beta} = \beta \ln \alpha$$

$$\log_b N = \log_a N \log_b a = \frac{\log_a N}{\log_a b}$$

E.6. Some Definite Integrals

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}} \quad a > 0$$

$$\int_0^{\infty} e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} \quad a > 0$$

$$\int_0^{\infty} x e^{-ax^2} dx = \frac{1}{2a} \quad a > 0$$