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Tutorial 3
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$$\vec{E}_t = -2j\, E_0 \sin(kx\sin\theta)\, e^{-jk\cos\theta\, z}\, \vec{e}_y$$

$\vec{E}_t = 0$ $\quad x = 2d$

$$\sin(k\,2d\,\sin\theta) = 0$$

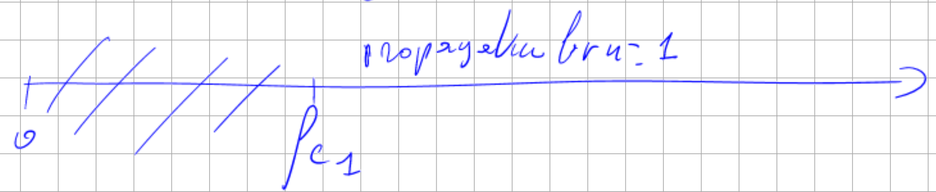
* $n = 1$ $\quad k\,2d\,\sin\theta = \pi \Rightarrow \sin\theta = \frac{\pi}{k\,2d}$

$$k = \frac{\omega}{v}$$

$\omega \to \infty$ $\quad \sin\theta = 0 \Rightarrow \theta = 0$

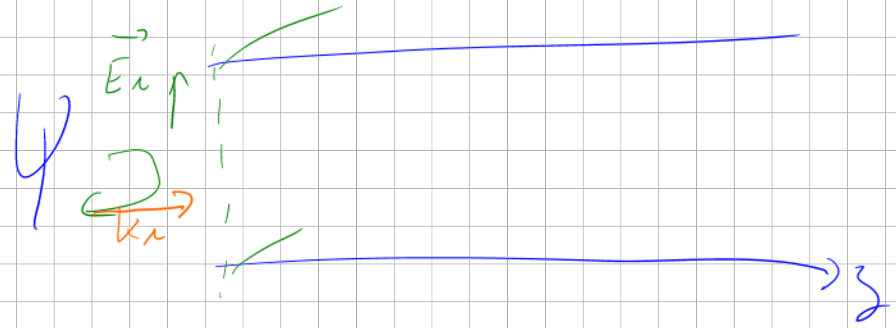
$\omega_{c1} \to \sin\theta = 1 = \frac{\pi}{\frac{\omega_{c1}}{v}\,2d}$

$\to \theta = \frac{\pi}{2}$



$$k^2 = \left(\frac{\omega}{v}\right)^2 = (k\cos\theta)^2 + (k\sin\theta)^2$$

$$\left(\frac{\omega}{v}\right)^2 = \beta^2 + (k_{c1})^2$$



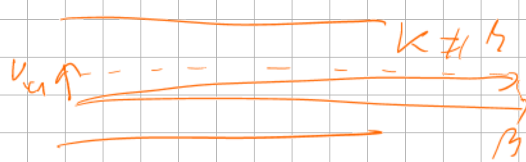
$\omega < \omega_{c1}$ $\quad \gamma = \alpha$

$\omega = \omega_{c1}$ $\quad \theta = \frac{\pi}{2}$

$\beta = k\cos\theta = 0$

$\omega > \omega_{c1}$ $\quad \theta \downarrow$

$\omega \gg \omega_{c1}$ $\quad \theta \to 0$



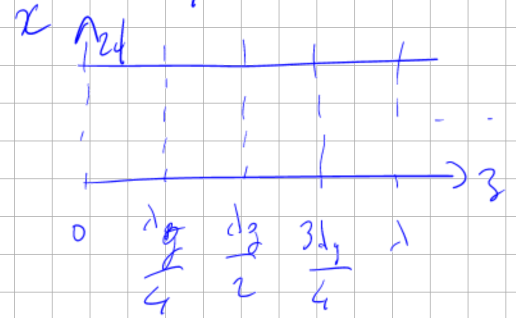
$$\vec{E}_t = -2j\, E_0 \sin\frac{\pi}{2d}x\; e^{-j\beta z}\;\vec{e_y}$$

E_0 real

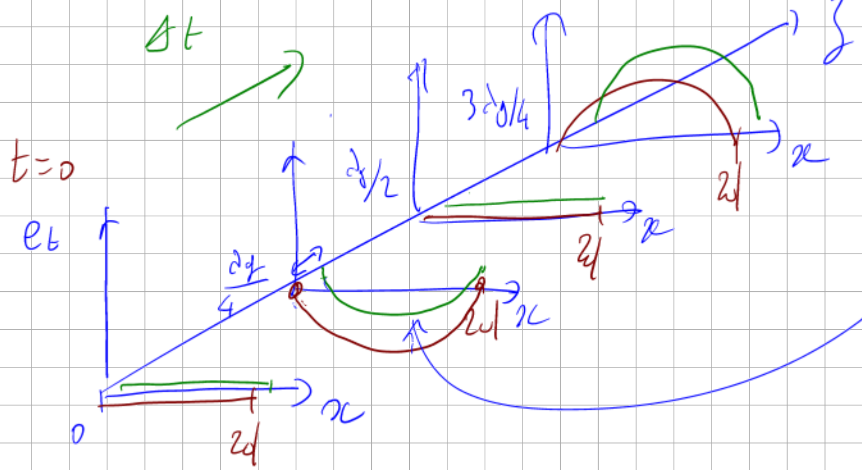
$$e_t = \Re\left(E_t\, e^{j\omega t}\right)$$

$$= 2E_0 \sin\frac{\pi}{2d}x\,\sin(\omega t - \beta z)$$

× $t = 0$



Δt

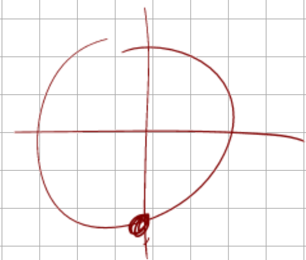


$\lambda_g \neq \lambda = \frac{c}{f}$ $\qquad \lambda_g = \frac{2\pi}{\beta}$, $\quad k^2 = \beta^2 + k_c^2$

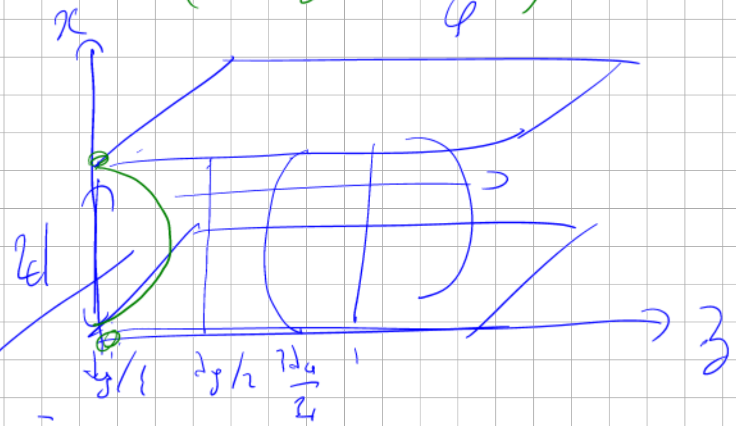
$z = \frac{\lambda_g}{4}$ $\qquad \beta z = \frac{2\pi}{\lambda_g}\frac{\lambda_g}{4} = \frac{\pi}{2}$

$z = \frac{\lambda_g}{2}$ $\qquad \beta z = \pi$

$z = \frac{3\lambda_g}{4}$ $\qquad \beta z = \frac{3\pi}{2}$

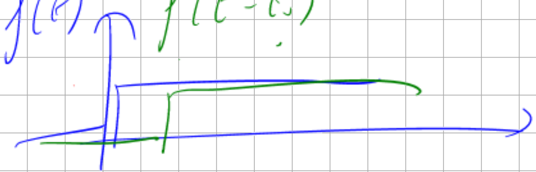


× $\underline{\Delta t}$ $\qquad \sin(-\beta z + \underbrace{\omega\Delta t}_{\varphi})$



$$\sin(-\beta z + \varphi) = -\sin(\beta z - \varphi)$$

$f(t)$ $\qquad f(t - t_0)$



× $n = 2$ $\qquad k\,2d\sin\theta = n\pi$, $\; n \in \mathbb{N}$

$$\sin(k\,2d\sin\theta) = 0$$

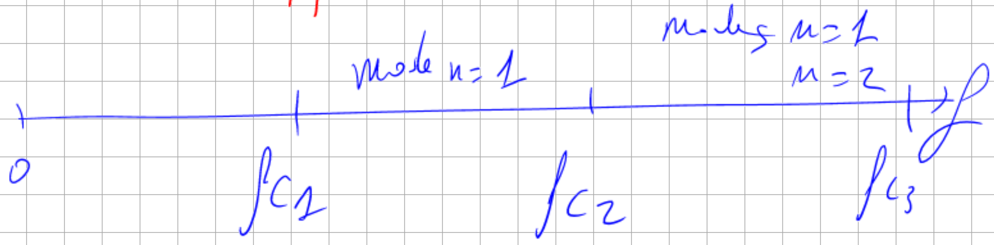
$$k\,2d\sin\theta = 2\pi \qquad (n = 2)$$

In $f = f_{c2}$: $k\,2d = 2\pi$

$k = \frac{\pi}{d}$

$\frac{2\pi f_{c2}}{v} = \frac{\pi}{d}$

$$f_{c2} = \frac{v}{2d} = 2 f_{c1}$$



n=1

f_{c1}

$\beta_1 = \sqrt{k^2 - k_{c1}^2}$

$k_{c1} = \frac{\pi}{2d}$

$\lambda_{g1} = \frac{2\pi}{\beta_1}$

$k^2 = \left(\frac{\omega}{v}\right)^2 = k_{c1}^2 + \beta_1^2$

n=2

$f_{c2} = 2 f_{c1}$

$\beta_2 = \sqrt{k^2 - k_{c2}^2} \neq \beta_1$

$k_{c2} = \frac{\pi}{d} = 2 k_{c1}$

$\lambda_{g2} = \frac{2\pi}{\beta_2} \neq \lambda_{g1}$

$k^2 = \left(\frac{\omega}{v}\right)^2 = k_{c2}^2 + \beta_2^2$

n=1

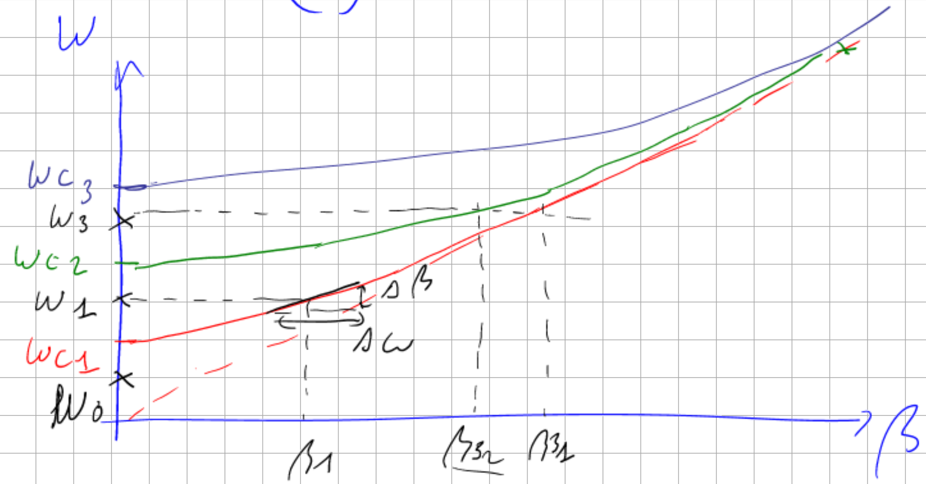
$k^2 = \underbrace{\left(\frac{\pi}{2d}\right)^2}_{(k_{c1})^2} + \beta_1^2$

$k_{c1} = k$ at f_{c1} ($\frac{\omega}{v}$)

$k^2 = k_{c1}^2 + \cancel{(k \cos\theta)}^2$

$k^2 = k_{c2}^2$ at f_{c2}

$= \left(\frac{\pi}{d}\right)^2$



$\omega \to \infty$ $\beta_{1,2,3,4} = k = \frac{\omega}{v}$

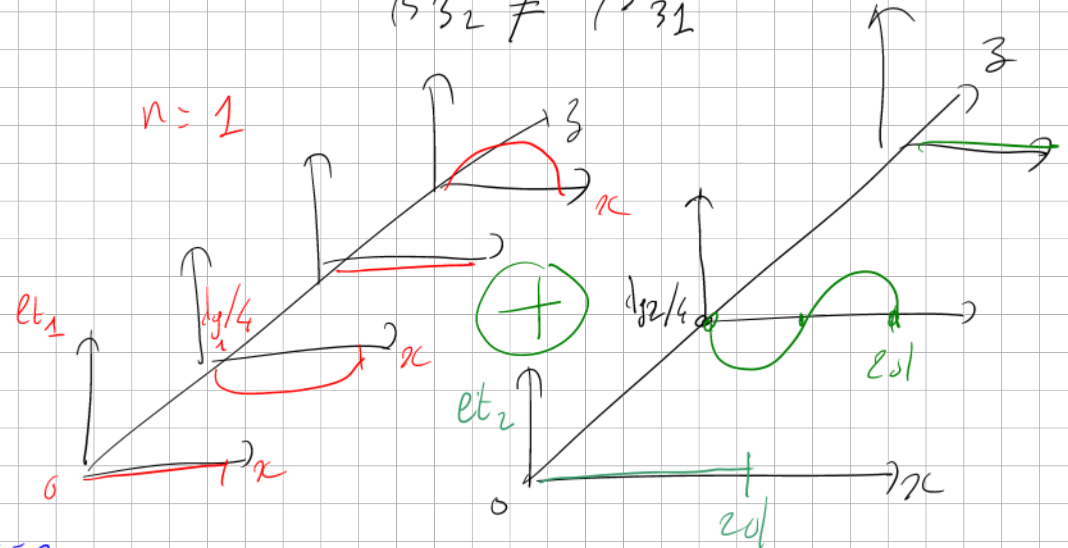
at f_0 no propagation

at f_1 mode 1 propagates

$\beta_1 = \frac{\omega_1}{v_{\varphi 1}}$ group velocity $v_{g1} = \frac{\Delta\omega}{\Delta\beta}$

at f_3 2 modes propagate

$\beta_{32} \neq \beta_{31}$



$t = 0$

$e_{t1} = 2E_{01} \sin \frac{2\pi}{d}x \, \sin(\omega t - \beta_1 z)$

$e_{t2} = 2E_{02} \sin \frac{\pi}{d}x \, \sin(\omega t - \beta_2 z)$

at f_3

$e_t = e_{t1} + e_{t2}$

$\beta_2 < \beta_1$

$\lambda_{g2} > \lambda_{g1}$