

8

Optoelectronic Detectors

After reading this chapter you will be able to understand the following:

- Principles of optoelectronic detection
- Defining parameters of optoelectronic detectors: quantum efficiency, responsivity, cut-off wavelength
- Types of photodiodes
- Noise considerations

8.1 INTRODUCTION

In any fiber-optic system, it is required to convert the optical signals at the receiver end back into electrical signals for further processing and display of the transmitted information. This task is normally performed by an optoelectronic detector. Therefore, the overall system performance is governed by the performance of the detector. The detector requirements are very similar to those of optoelectronic sources; that is, they should have high sensitivity at operating wavelengths, high fidelity, fast response, high reliability, low noise, and low cost. Further, its size should be comparable with that of the core of fiber employed in the optical link. These requirements are easily met by detectors made of semiconducting materials, and hence we will discuss the detection principles and design of only such detectors.

8.2 THE BASIC PRINCIPLE OF OPTOELECTRONIC DETECTION

Figure 8.1 shows a reverse-biased p - n junction. When a photon of energy greater than the band gap of the semiconductor material (i.e., $h\nu \geq E_g$) is incident on or near the depletion region of the device, it excites an electron from the valence band into the conduction band. The vacancy of an electron creates a hole in the valence band. Electrons and holes so generated experience a strong electric field and drift rapidly towards the n and p sides, respectively. The resulting flow of current is proportional

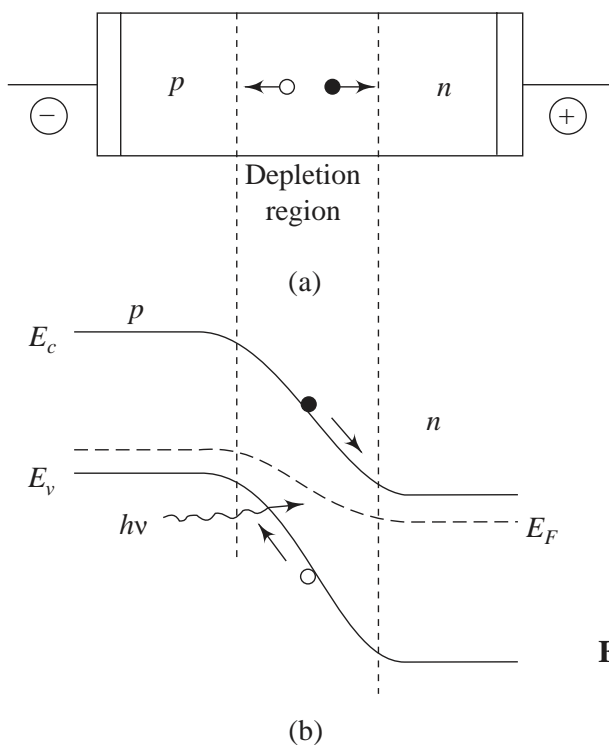


Fig. 8.1 (a) A reverse-biased p - n junction. (b) Energy band diagram showing carrier generation and their drift.

to the number of incident photons. Such a reverse-biased p - n junction, therefore, acts as a photodetector and is normally referred to as a p - n photodiode.

8.2.1 Optical Absorption Coefficient and Photocurrent

The absorption of photons of a specific wavelength in a photodiode to produce electron-hole pairs and thus a photocurrent depends on the absorption coefficient α of the semiconductor for that particular wavelength. If we assume that the total optical power incident on the photodiode is P_{in} and that the Fresnel reflection coefficient at the air-semiconductor interface is R , the optical power entering the semiconductor will be $P_{\text{in}}(1 - R)$. The power absorbed by the semiconductor is governed by Beer's law. Thus, if the width of the absorption region is d and α is the absorption coefficient of the semiconductor at the incident wavelength, the power absorbed by this width will be given by

$$P_{\text{abs}} = P_{\text{in}}(1 - R) [1 - \exp(-\alpha d)] \quad (8.1)$$

Let us assume that the incident light is monochromatic and the energy of each photon is $h\nu$, where h is Planck's constant and ν is the frequency of incident light. Then the rate of photon absorption will be given by

$$\frac{P_{\text{abs}}}{h\nu} = \frac{P_{\text{in}}(1 - R)}{h\nu} [1 - \exp(-\alpha d)]$$

The dependence of α on wavelength for some commonly used semiconductors is shown in Fig. 8.2. It is clear from these curves that α depends strongly on the wavelength of incident light. Thus, a specific semiconductor material can be used

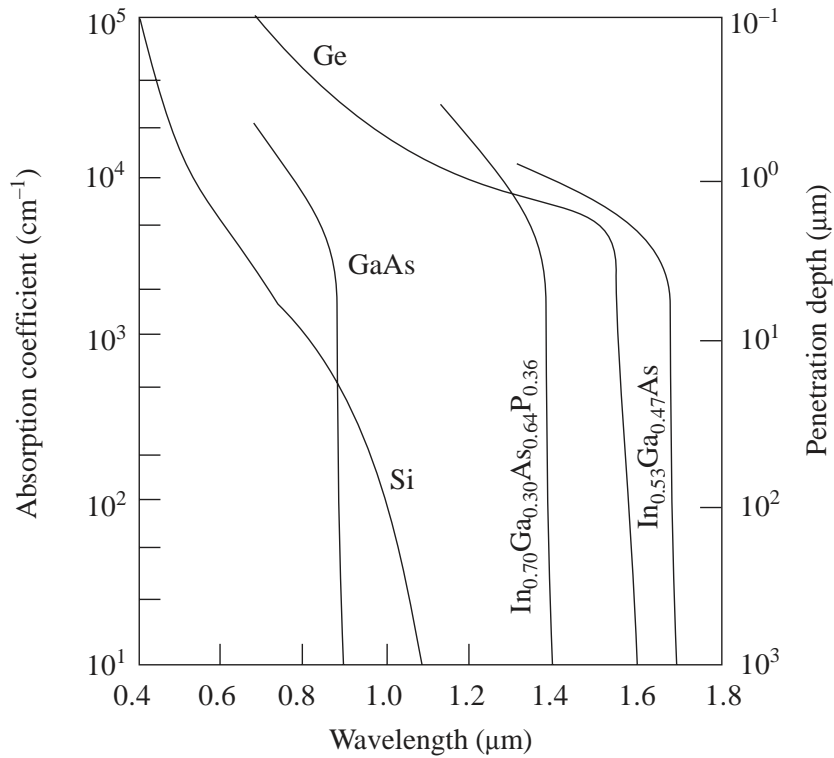


Fig. 8.2 Wavelength dependence of the absorption coefficient α for some semiconductors (Lee & Li 1979)

only in a specific range. There is also an upper wavelength limit (explained in Sec. 8.2.4) for each semiconductor material.

Further, if we assume that (i) the semiconductor is an intrinsic absorber (i.e., the absorption of photons excites the electrons from the valence band directly to the conduction band), (ii) each photon produces an electron–hole pair, and (iii) all the charge carriers are collected at the electrodes, then the photocurrent (rate of flow of charge carriers) I_p so produced will be given by

$$I_p = \frac{P_{in}(1-R)e}{h\nu} [1 - \exp(-\alpha d)] \quad (8.2)$$

where e is the electronic charge.

8.2.2 Quantum Efficiency

Quantum efficiency η is defined as the ratio of the rate (r_e) of electrons collected at the detector terminals to the rate r_p of photons incident on the device. That is,

$$\eta = \frac{r_e}{r_p} \quad (8.3)$$

η may be increased if the Fresnel reflection coefficient R is decreased and the product αd in Eq. (8.2) is much greater than unity. It must be noted, however, that η is also a function of the photon wavelength.

8.2.3 Responsivity

The responsivity \mathfrak{R} of a photodetector is defined as the output photocurrent per unit incident optical power. Thus, if I_p is the output photocurrent in amperes and P_{in} is the incident optical power in watts,

$$\mathfrak{R} = \frac{I_p}{P_{\text{in}}} \text{ (in A W}^{-1}\text{)} \quad (8.4)$$

The output photocurrent I_p may be written in terms of the rate, r_e , of electrons collected as follows:

$$I_p = e r_e \quad (8.5)$$

where e is the electronic charge. Combining Eqs (8.3) and (8.5), we get

$$I_p = e \eta r_p \quad (8.6)$$

Now the rate of incident photons is given by

$$r_p = \frac{\text{Incident optical power}}{\text{Energy of the photon}} = \frac{P_{\text{in}}}{h\nu} \quad (8.7)$$

Thus

$$I_p = \frac{\eta e P_{\text{in}}}{h\nu} \quad (8.8)$$

Substituting for I_p from Eq. (8.8) in Eq. (8.4), we get an expression for \mathfrak{R} in terms of η as follows:

$$\mathfrak{R} = \frac{\eta e}{h\nu} = \frac{\eta e \lambda}{hc} \quad (8.9)$$

where λ and c are the wavelength and speed of the incident light in vacuum, respectively. Equation (8.9) shows that the responsivity is directly proportional to the quantum efficiency at a particular wavelength and in the ideal case, when $\eta = 1$, \mathfrak{R} is directly proportional to λ . For a practical diode, as the wavelength of the incident photon becomes longer, its energy becomes smaller than that required for exciting the electron from the valence band to the conduction band. The responsivity thus falls off near the cut-off wavelength λ_c . This can be seen in Fig. 8.3(b).

8.2.4 Long-wavelength Cut-off

In an intrinsic semiconductor, the absorption of a photon is possible only when its energy is greater than or equal to the band gap energy E_g of the semiconductor used to fabricate the photodiode. That is, the photon energy $hc/\lambda \geq E_g$. Thus, there is a long-wavelength cut-off λ_c , above which photons are simply not absorbed by the semiconductor, given by

$$\lambda_c = \frac{hc}{E_g} \quad (8.10)$$

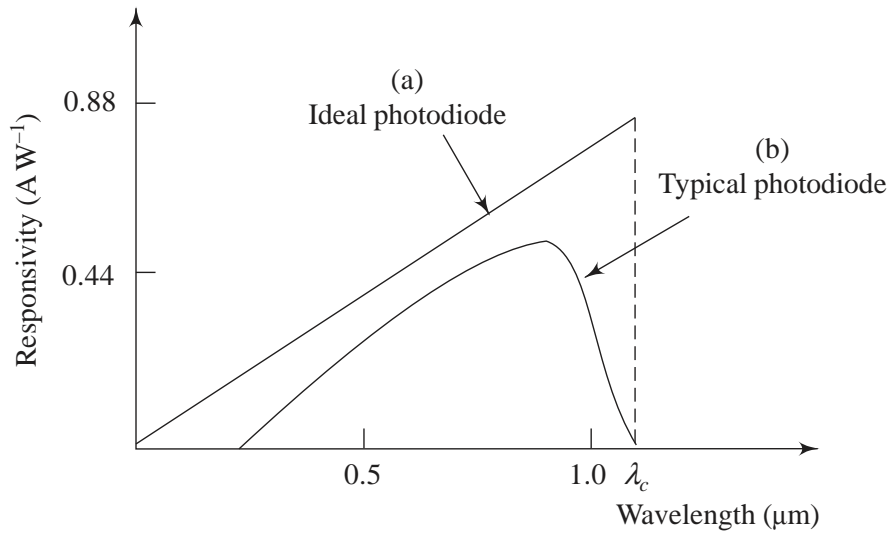


Fig. 8.3 Responsivity as a function of wavelength for (a) an ideal Si photodiode and (b) a practical Si diode

If E_g is expressed in eV, substituting the values of $h = 6.626 \times 10^{-34}$ Js, $c = 3 \times 10^8$ ms⁻¹, and 1 eV = 1.6×10^{-19} J, we get

$$\lambda_c (\mu\text{m}) \approx \frac{1.24}{E_g (\text{eV})} \quad (8.11)$$

This expression allows us to calculate λ_c for different semiconductors in order to select them for specific detection purposes.

Example 8.1 A p - n photodiode has a quantum efficiency of 70% for photons of energy 1.52×10^{-19} J. Calculate (a) the wavelength at which the diode is operating and (b) the optical power required to achieve a photocurrent of 3 μ A when the wavelength of incident photons is that calculated in part (a).

Solution

(a) The photon energy

$$E = h\nu = \frac{hc}{\lambda}$$

$$\text{Therefore } \lambda = \frac{hc}{E} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.52 \times 10^{-19}} = 1.30 \times 10^{-6} \text{ m} \\ = 1.30 \mu\text{m}$$

$$(b) \mathfrak{R} = \frac{\eta e}{h\nu} = \frac{0.70 \times 1.6 \times 10^{-19}}{1.52 \times 10^{-19}} = 0.736 \text{ A W}^{-1}$$

$$\text{Since } \mathfrak{R} = \frac{I_p}{P_{\text{in}}} \\ = 4.07 \times 10^{-6} \text{ W}$$

$$\text{or } P_{\text{in}} = 4.07 \mu\text{W}$$

Example 8.2 A *p-i-n* photodiode, on an average, generates one electron–hole pair per two incident photons at a wavelength of $0.85\ \mu\text{m}$. Assuming all the photo-generated electrons are collected, calculate (a) the quantum efficiency of the diode; (b) the maximum possible band gap energy (in eV) of the semiconductor, assuming the incident wavelength to be a long-wavelength cut-off; and (c) the mean output photocurrent when the incident optical power is $10\ \mu\text{W}$.

Solution

$$(a) \quad \eta = \frac{1}{2} = 0.5 = 50\%$$

$$(b) \quad E_g = \frac{hc}{\lambda_c} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{0.85 \times 10^{-6}} = 2.33 \times 10^{-19} \text{ J} \\ = 1.46 \text{ eV}$$

$$(c) \quad I_P = \Re P_{\text{in}} = \frac{\eta e}{h\nu} P_{\text{in}} = \frac{0.5 \times 1.6 \times 10^{-19}}{2.33 \times 10^{-19}} \times 10 \times 10^{-6} = 3.43 \times 10^{-6} \text{ A} \\ = 3.43 \mu\text{A}$$

Example 8.3 Photons of wavelength $0.90\ \mu\text{m}$ are incident on a *p-n* photodiode at a rate of $5 \times 10^{10} \text{ s}^{-1}$ and, on an average, the electrons are collected at the terminals of the diode at the rate of $2 \times 10^{10} \text{ s}^{-1}$. Calculate (a) the quantum efficiency and (b) the responsivity of the diode at this wavelength.

Solution

$$(a) \quad \eta = \frac{2 \times 10^{10}}{5 \times 10^{10}} = 0.40$$

$$(b) \quad \Re = \frac{\eta e \lambda}{hc} = \frac{0.40 \times 1.6 \times 10^{-19} \times 0.90 \times 10^{-6}}{6.626 \times 10^{-34} \times 3 \times 10^8} = 0.29 \text{ A W}^{-1}$$

8.3 TYPES OF PHOTODIODES

8.3.1 *p-n* Photodiode

The simplest structure is that of a *p-n* photodiode, shown in Fig. 8.4(a). Incident photons of energy, say $h\nu$, are absorbed not only inside the depletion region but also outside it, as shown in Fig. 8.4(b). As discussed in Sec. 8.2, the photons absorbed within the depletion region generate electron–hole pairs. Because of the built-in strong electric field [shown in Fig. 8.4(c)], electrons and holes generated inside this region

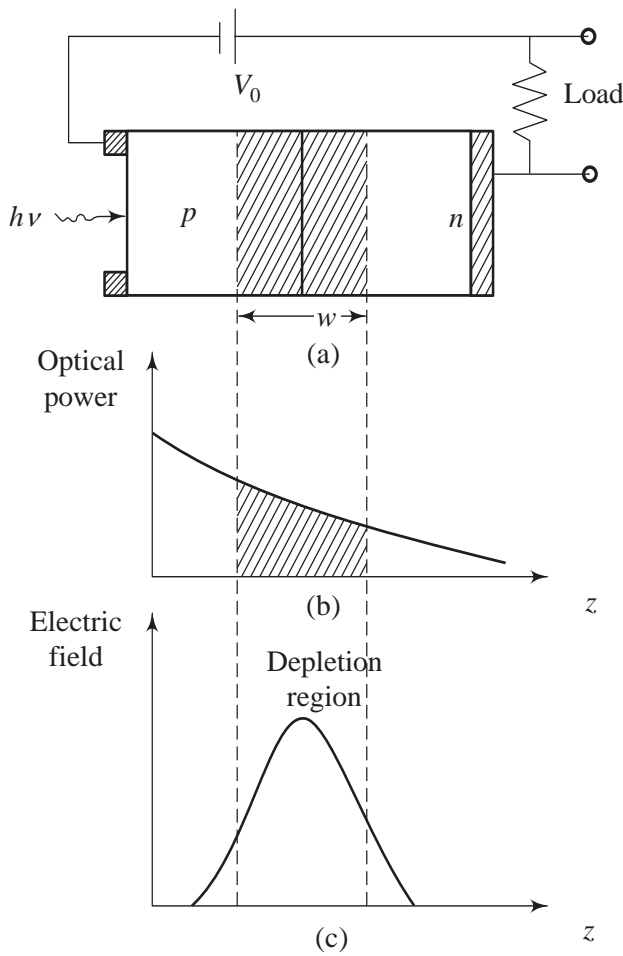


Fig. 8.4 (a) Structure of a p - n photodiode and the associated depletion region under reverse bias. (b) Variation of optical power within the diode. (c) Variation of electric field inside the diode.

get accelerated in opposite directions and thereby drift to the n -side and the p -side, respectively. The resulting flow of photocurrent constitutes the response of the photodiode to the incident optical power. The response time is governed by the transit time τ_{drift} , which is given by

$$\tau_{\text{drift}} = \frac{w}{v_{\text{drift}}} \quad (8.12)$$

where w is the width of the depletion region and v_{drift} is the average drift velocity. τ_{drift} is of the order of 100 ps, which is small enough for the photodiode to operate up to a bit rate of about 1 Gbit/s. In order to minimize τ_{drift} , both w and v_{drift} can be tailored. The depletion layer width w is given (Sze 1981) by

$$w = \left[\frac{2\epsilon}{e} (V_{\text{bi}} + V_0) \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2} \quad (8.13)$$

where ϵ is the dielectric constant, e is the electronic charge, V_{bi} is the built-in voltage and depends on the semiconductor, V_0 is the applied bias voltage, and N_a and N_d are the acceptor and donor concentrations used to fabricate the p - n junction. The drift velocity v_{drift} depends on the bias voltage but attains a saturation value depending on the material of the diode.

As shown in Fig. 8.4(b), incident photons are absorbed outside the depletion region also. The electrons generated in the p -side have to diffuse to the depletion-region boundary before they can drift (under the built in electric field) to the n -side. In a similar fashion, the holes generated in the n -side have to diffuse to the depletion-region boundary for their drift towards the p -side. The diffusion process is inherently slow and hence the presence of a diffusive component may distort the temporal response of a photodiode, as shown in Fig. 8.5.

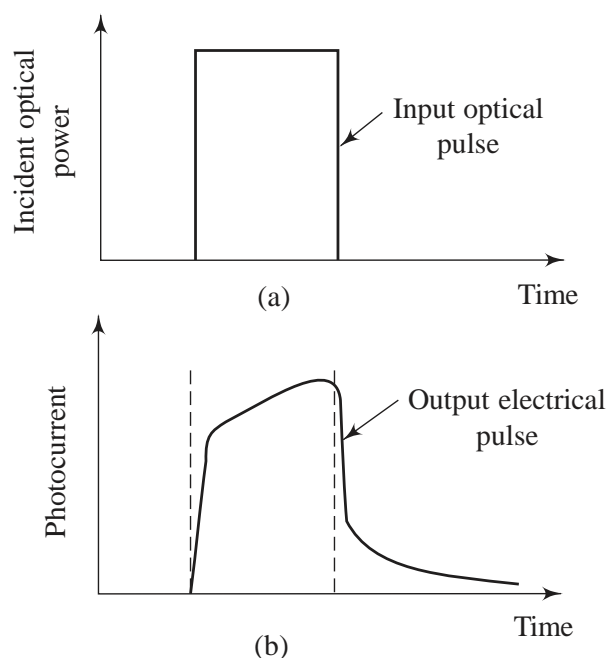


Fig. 8.5 Response of a typical p - n photodiode to a rectangular optical pulse when both drift and diffusion contribute to the photocurrent

8.3.2 p - i - n -Photodiode

The diffusion component of a p - n photodiode may be reduced by decreasing the widths of the p -side and n -side and increasing the width of the depletion region so that most of incident photons are absorbed inside it. To achieve this, a layer of semiconductor, so lightly doped that it may be considered intrinsic, is inserted at the p - n junction. Such a structure is called a p - i - n photodiode and is shown in Fig. 8.6 along with the electric-field distribution inside it under the reverse bias. As the middle layer is intrinsic in nature, it offers high resistance, and hence most of the voltage drop occurs across it. Thus, a strong electric field exists across the middle i -region. Such a configuration results in the drift component of the photocurrent dominating the diffusion component, as most of the incident photons are absorbed inside the i -region.

A double heterostructure, similar to that discussed in Chapter 7 for sources, improves the performance of the p - i - n photodiodes. Herein, the middle i -region of a material with lower band gap is sandwiched between p - and n -type materials of higher band gap, so that incident light is absorbed only within the i -region. Such a configuration is shown in Fig. 8.7. The band gap of InP is 1.35 eV and hence it is

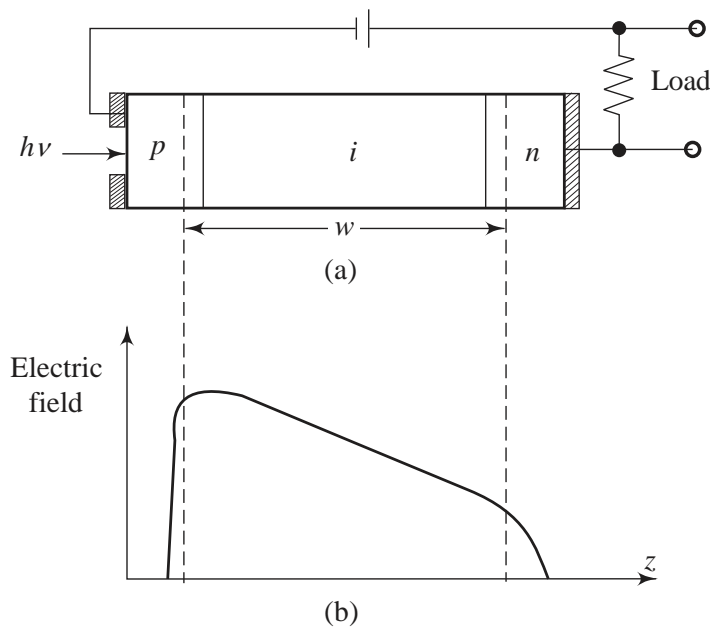


Fig. 8.6 (a) Structure of a p - i - n photodiode. (b) Electric-field distribution inside the device under reverse bias.

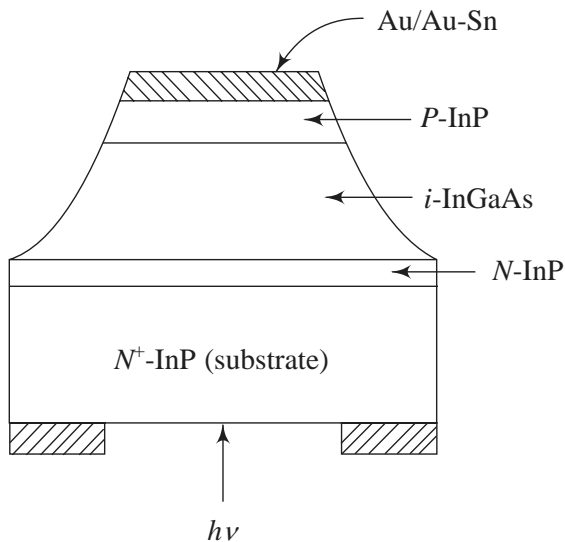


Fig. 8.7 A double heterostructure design of a p - i - n photodiode using InGaAs/InP

transparent for light of wavelength greater than $0.92\text{ }\mu\text{m}$, whereas the band gap of lattice-matched InGaAs is about 0.75 eV , which corresponds to a λ_c of $1.65\text{ }\mu\text{m}$. Thus the intrinsic layer of InGaAs absorbs strongly in the wavelength range $1.3\text{--}1.6\text{ }\mu\text{m}$. The diffusive component of the photocurrent is completely eliminated in such a heterostructure simply because the incident photons are absorbed only within the depletion region. Such photodiodes are very useful for fiber-optic systems operating in the range $1.3\text{--}1.6\text{ }\mu\text{m}$.

8.3.3 Avalanche Photodiode

Avalanche photodiodes (APDs) internally multiply the primary photocurrent through multiplication of carrier pairs. This increases receiver sensitivity because the photocurrent is multiplied before encountering the thermal noise associated with the receiver circuit. Through an appropriate structure of a photodiode, it is possible to

create a high-field region within the device upon biasing. When the primary electron–hole pairs generated by incident photons pass through this region, they get accelerated and acquire so much kinetic energy that they ionize the bound electrons in the valence band upon collision and in the process create secondary electron–hole pairs. This phenomenon is known as impact ionization. If the field is high enough, the secondary carrier pairs may also gain sufficient energy to create new pairs. This is known as the avalanche effect. Thus the carriers get multiplied, all of which contribute to the photocurrent.

A commonly used configuration for achieving carrier multiplication with little excess noise is shown in Fig. 8.8 and is called a reach through avalanche photodiode (RAPD). It is composed of a lightly doped p -type intrinsic layer (called a π -layer) deposited on a p^+ (heavily doped p -type) substrate. A normal p -type diffusion is made in the intrinsic layer, which is followed by the construction of an n^+ (heavily doped n -type) layer. Such a configuration is called a $p^+-\pi-p-n^+$ reach through structure. When the applied reverse-bias voltage is low, most of the potential drop is across the $p-n^+$ junction. As the bias voltage is increased, the depletion layer widens and the latter just reaches through to the π -region when the electric field at the $p-n^+$ junction becomes sufficient for impact ionization. Normally, an RAPD is operated in a fully depleted mode. The photons enter the device through the p^+ -layer and are absorbed in the intrinsic π -region. The absorbed photons create primary electron–hole pairs, which are separated by the electric field in this region. These carriers drift to the $p-n^+$ junction,

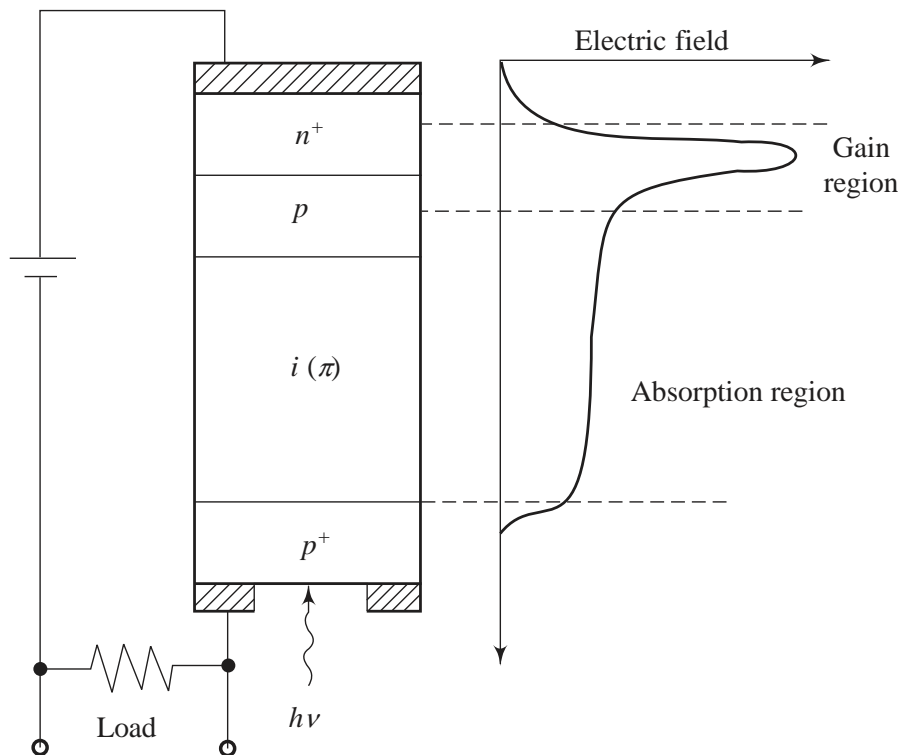


Fig. 8.8 Schematic configuration of RAPD and the variation of electric field in the depletion and multiplication regions

where a strong electric field exists. It is in this region that the carriers are multiplied first by impact ionization and then by the avalanche breakdown.

The average number of carrier pairs generated by an electron per unit length of its transversal is called the electron ionization rate α_e . Similarly, the hole ionization rate α_h may also be defined. The ratio $K = \alpha_h/\alpha_e$ is a measure of the performance of APDs. An APD made of a material in which one type of carrier dominates impact ionization exhibits low noise and high gain. It has been found that there is a significant difference between α_e and α_h only in silicon and hence it is normally used for making RAPDs for detection around 0.85 μm . For longer wavelengths, advanced structures of APDs are used.

The multiplication factor M of an APD is a measure of the internal gain provided by the device. It is defined as

$$M = \frac{I_M}{I_p} \quad (8.14)$$

where I_M is the average output current (after multiplication) and I_p is the primary photocurrent (before multiplication) defined by Eq. (8.2).

Analogous to p - n and p - i - n photodiodes, the performance of an APD is described by its responsivity $\mathfrak{R}_{\text{APD}}$, given by

$$\mathfrak{R}_{\text{APD}} = \frac{\eta e}{h\nu} M = M \mathfrak{R} \quad (8.15)$$

where Eq. (8.9) has been used.

Example 8.4 An APD has a quantum efficiency of 40% at 1.3 μm . When illuminated with optical power of 0.3 μW at this wavelength, it produces an output photocurrent of 6 μA , after avalanche gain. Calculate the multiplication factor of the diode.

Solution

$$\begin{aligned} M &= \frac{I}{I_p} = \frac{I}{P_{\text{in}} \mathfrak{R}} = \frac{I}{P_{\text{in}} \left(\frac{\eta e \lambda}{hc} \right)} = \frac{I (hc)}{P_{\text{in}} (\eta e \lambda)} \\ &= \frac{6 \times 10^{-6} \times (6.626 \times 10^{-34} \times 3 \times 10^8)}{0.3 \times 10^{-6} \times (0.4 \times 1.6 \times 10^{-19} \times 1.3 \times 10^{-6})} \\ &= 47.6 \end{aligned}$$

Example 8.5 A silicon RAPD, operating at a wavelength of 0.80 μm , exhibits a quantum efficiency of 90%, a multiplication factor of 800, and a dark current of 2 nA. Calculate the rate at which photons should be incident on the device so that the output current (after avalanche gain) is greater than the dark current.

Solution

$$\begin{aligned}
 I &= I_p M = P_{\text{in}} \Re M = P_{\text{in}} \left(\frac{\eta e \lambda}{hc} \right) M \\
 &= \left(\frac{P_{\text{in}}}{(hc/\lambda)} \right) \eta e M \\
 &= [(\text{photon rate})e] \eta M
 \end{aligned}$$

For $I = 2 \text{ nA}$,

$$\begin{aligned}
 \text{Photon rate} &= \frac{I}{e \eta M} = \frac{2 \times 10^{-9}}{1.6 \times 10^{-19} \times 0.90 \times 800} \\
 &= 1.736 \times 10^7 \text{ s}^{-1}
 \end{aligned}$$

For $I > 2 \text{ nA}$,

$$\text{Photon rate} \approx 1.74 \times 10^7 \text{ s}^{-1}$$

8.4 PHOTOCONDUCTING DETECTORS

The basic detection process involved in a photoconducting detector is raising an electron from the valence band to the conduction band upon absorption of a photon by a semiconductor, provided the photon energy is greater than the band gap energy (i.e., $h\nu \geq E_g$). As long as an electron remains in the conduction band, it will contribute toward increasing the conductivity of the semiconductor. This phenomenon is known as photoconductivity.

The structure of a typical photoconducting detector designed for operation in the long-wavelength range (typically $1.1\text{--}1.6 \text{ }\mu\text{m}$) is shown in Fig. 8.9. The device comprises a thin conducting layer, $1\text{--}2 \text{ }\mu\text{m}$ thick, of n -type InGaAs that can absorb photons in the wavelength range $1.1\text{--}1.6 \text{ }\mu\text{m}$. This layer is formed on the lattice-matched semi-insulating InP substrate. A low-resistance interdigital anode and cathode are made on the conducting layer.

In operation, the incident photons are absorbed by the conducting layer, thereby generating additional electron–hole pairs. These carriers are swept by the applied field towards respective electrodes, which results in an increased current in the external circuit. In the case of III-V alloys such as InGaAs, electron mobility is much higher than hole mobility. Thus whilst the faster electrons are collected at the anode, the corresponding holes are still proceeding towards the cathode. The process creates an absence of electrons and hence a net positive charge in the region. However, the excess charge is compensated for, almost immediately, by the injection of more electrons from the cathode into this region. In essence, this process leads to the generation of more electrons upon the absorption of a single photon. The overall

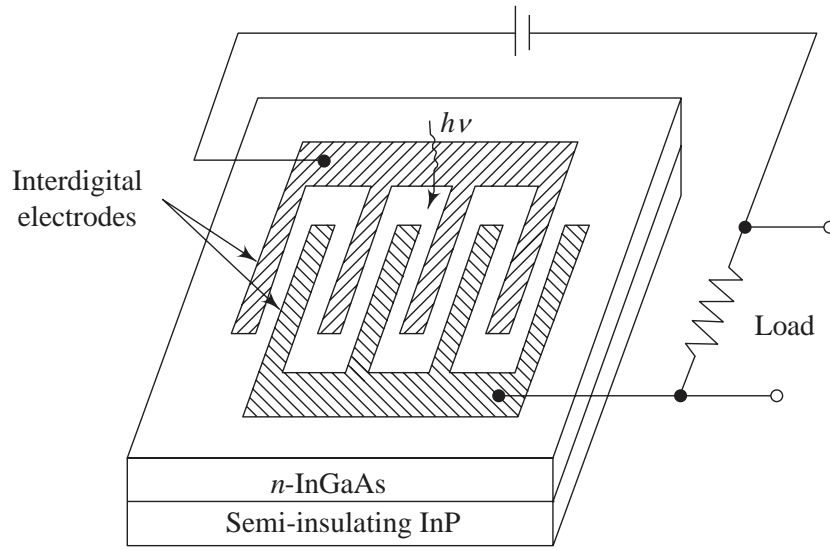


Fig. 8.9 A photoconducting detector for long-wavelength operation

effect is that it results in a photoconductive gain G , which may be defined as the ratio of transit time t_s for slow carriers to the transit time t_f for fast carriers. Thus,

$$G = \frac{t_s}{t_f} \quad (8.16)$$

The photocurrent I_g produced by the photoconductor can be written as

$$I_g = \frac{\eta P_{\text{in}} e}{h\nu} G = I_p G \quad (8.17)$$

where η is the quantum efficiency of the device, P_{in} is the incident optical power, and I_p is the photocurrent in the absence of any gain. Gain in the range 50–100 and a 3-dB bandwidth of about 500 MHz are currently achievable with the InGaAs photoconductors discussed above.

Example 8.6 The maximum 3-dB bandwidth permitted by an InGaAs photoconducting detector is 450 MHz when the electron transit time in the device is 6 ps. Calculate (a) the gain G and (b) the output photocurrent when an optical power of 5 μW at a wavelength of 1.30 μm is incident on it, assuming quantum efficiency of 75%.

Solution

The current response in the photoconductor decays exponentially with time once the incident optical pulse is removed. The time constant of this decay is equal to the slow carrier transit time t_s . Therefore, the maximum 3-dB bandwidth $(\Delta f)_m$ of the device will be given by

$$(\Delta f)_m = \frac{1}{2\pi t_s} = \frac{1}{2\pi t_f G}$$

where we have used Eq. (8.16).

$$(a) \quad G = \frac{1}{2\pi t_f (\Delta f)_m} = \frac{1}{2\pi \times 6 \times 10^{-12} \times 450 \times 10^6} = 58.94$$

$$(b) \quad I = GI_P = \frac{G\eta P_{in} e\lambda}{hc} = \frac{58.94 \times 0.75 \times 5 \times 10^{-6} \times 1.6 \times 10^{-19} \times 1.3 \times 10^{-6}}{6.626 \times 10^{-34} \times 3 \times 10^8} \\ = 232.1 \mu A = 2.321 \times 10^{-4} A$$

8.5 NOISE CONSIDERATIONS

Optical signals at the receiver end in a fiber-optic communication system are quite weak. Therefore, even the simplest kind of receiver would require a good photodetector to be followed by an amplifier. Thus the signal power to noise ratio (S/N) at the output of the receiver would be given by

$$\frac{S}{N} = \frac{\text{Signal power from photocurrent}}{\text{Photodetector noise power} + \text{amplifier noise power}} \quad (8.18)$$

It is obvious from Eq. (8.18) that to achieve high S/N , (i) the photodetector should have high quantum efficiency and low noise so that it generates large signal power and (ii) the amplifier noise should be kept low. In fact the sensitivity of a detector is described in terms of the minimum detectable optical power, which is defined as the optical power necessary to generate a photocurrent equal in magnitude to the rms value of the total noise current. Therefore, in order to evaluate the performance of receivers, a thorough understanding of the various types of noises in a photodetector and their interrelationship is necessary. In order to see these interrelationships, let us analyse the simplest type of receiver model (Keiser 2000), shown in Fig. 8.10.

Herein, the photodiode has a negligible series resistance R_s , a load resistance R_L , and a total capacitance C_d . The amplifier has an input resistance R_a and capacitance C_a . There are three principal noises arising from the spontaneous fluctuation in a photodetector. These are (i) quantum or shot noise, (ii) dark current noise, and (iii) thermal noise. Thus, a current generated by a p - n or p - i - n photodiode in response to an instantaneous optical signal may be written as

$$I(t) = \langle i_p(t) \rangle + i_s(t) + i_d(t) + i_T(t) \quad (8.19)$$

where $\langle i_p(t) \rangle = I_p = \Re P_{in}$ is the average photocurrent, and $i_s(t)$, $i_d(t)$, and $i_T(t)$ are the current fluctuations related to shot noise, dark current noise, and thermal noise, respectively.

Quantum or shot noise arises because of the random arrival of photons at a photodetector and hence the random generation and collection of electrons. The mean square value of shot noise is given by

$$\langle i_s^2(t) \rangle = 2eI_p \Delta f M^2 F(M) \quad (8.20)$$