S. Verdeyme
Tutorial1
Sept 17th 2020

Tutorial 1

Microstrip

$\beta = \omega / v$ $\qquad v = ?$

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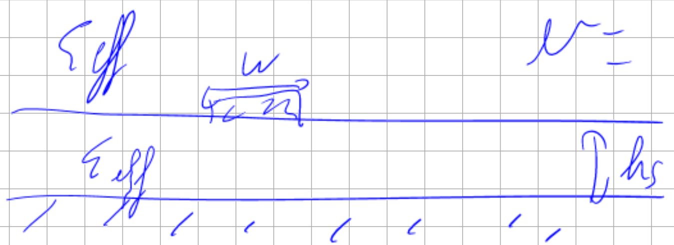
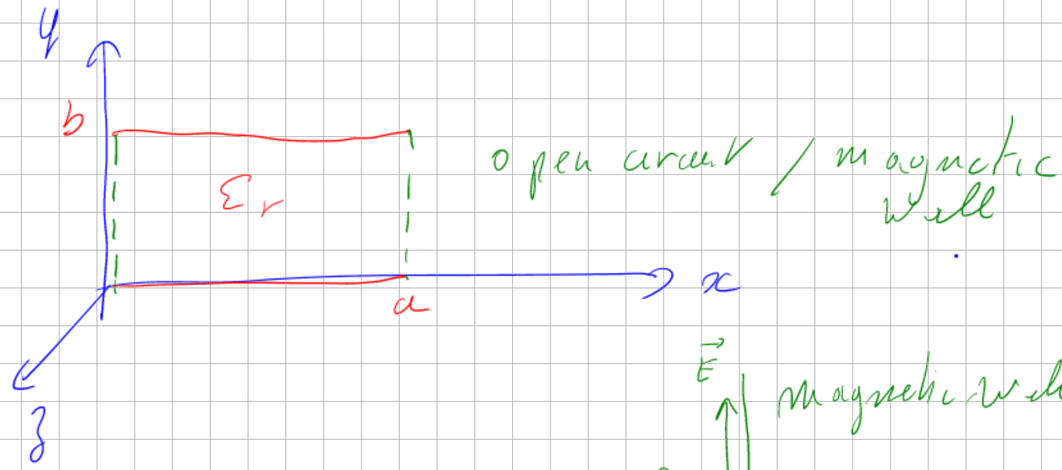
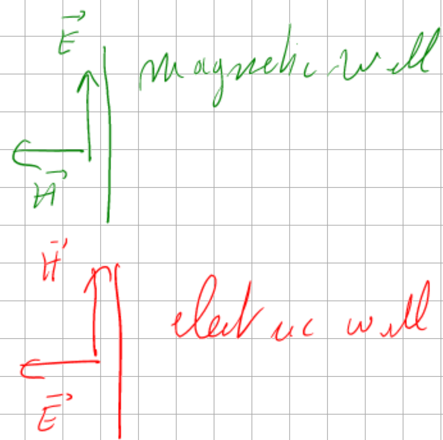


$v = \frac{c}{\sqrt{\varepsilon_{eff}}}$

alumina $\varepsilon_s = 9.8$ $\qquad \varepsilon_{eff} \approx 7$

$v = \frac{1}{\sqrt{LC}}$ $\qquad Z_c = \sqrt{\frac{L}{C}}$

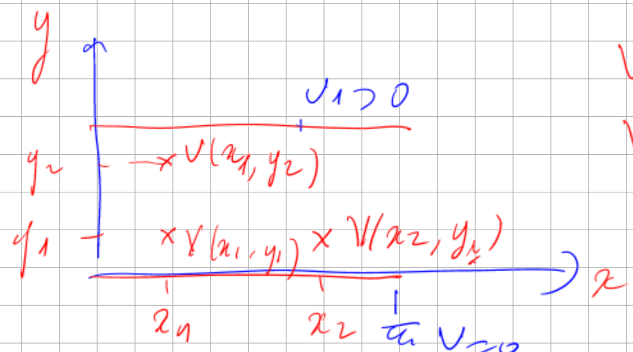




1) TEM exists?

2 conductors
1 material) TEM ok

2) Considering that the 'edge effects' are neglectible on the MSC as for the electrostatic computation of the capacitance between two metallic plates, do the voltage and EM field vary along both the x and y directions ?



$$V(x_1, y_2) \neq V(x_1, y_2)$$

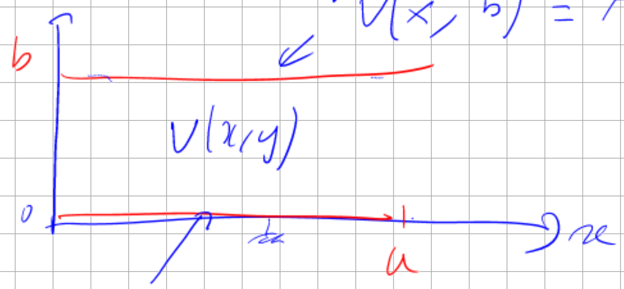
$$V(x_1, y_1) = V(x_2, y_1)$$

$$V(y)$$

$$V(x,y) = Ay + B$$

$$V(x, b) = Ab + B = V_1$$

$$A = \frac{V_1}{b}$$



$$V(x, 0) = A \times 0 + B = 0 \Rightarrow B = 0$$

$$V(x,y) = \frac{V_1}{b}\, y$$

3) From the POISSON equation, compute the voltage variation along the propagation axis

$$\Delta_t V(x,y) = 0$$

$$\frac{\partial^2 V(x,y)}{\partial x^2} + \frac{\partial^2 V(x,y)}{\partial y^2} = 0$$

$$\frac{\partial^2 V(x,y)}{\partial y^2} = 0$$

$$\frac{\partial f(x)}{\partial x} = 0$$

$$f(x) = A$$

$$\frac{\partial^2 f(x)}{\partial x^2} = 0 \Rightarrow f(x) = Ax + B$$

4) Give then the electrical and magnetic fields. Do they respect the 'right hand rule'?

$$\vec{E}_t(x,y) = -\vec{\nabla}_t V(x,y)$$

$$(O, \vec{e_x}, \vec{e_y}, \vec{u})$$

$$(O, \vec{i}, \vec{j}, \vec{k})$$

$$(O, \vec{x}, \vec{y}, \vec{z})$$

$$\vec{E}_t(x,y) = -\left(\frac{\partial V(x,y)}{\partial x}\vec{e_x} + \frac{\partial V(x,y)}{\partial y}\vec{e_y} \right)$$

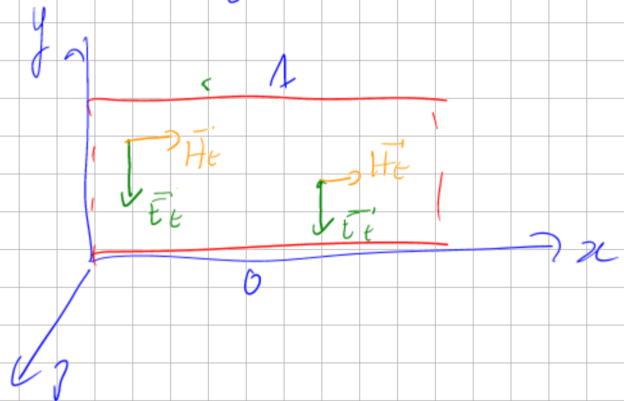
$$= -\frac{V_1}{b}\vec{e_y}$$

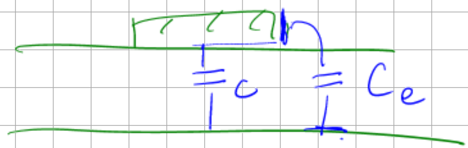
$$\vec{H}_t(x,y) = \frac{1}{Z}\left(\vec{n} \times \vec{E}_t(x,y)\right)$$

$Z = \sqrt{\frac{\mu}{\varepsilon}}$ wave impedance

$$= \frac{1}{Z}\left(\vec{n} \times \left(-\frac{V_1}{b}\vec{e_y}\right)\right)$$

$$= -\frac{1}{Z}\frac{V_1}{b}\left(\vec{n} \times \vec{e_y}\right) = \frac{1}{Z}\frac{V_1}{b}\vec{e_x}$$





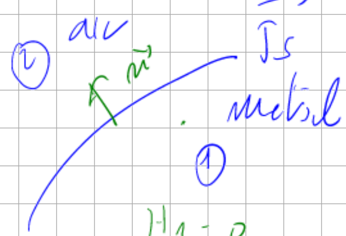
5) Compute the surface current density vector and then the current flowing along the upper conductor (y=b)

6) Compute the characteristic impedance of the line, the inductance and capacitance values per length unit.

7) What is the velocity of the wave in this line ?

5) $\vec{J_S}\cancel{(x,y)}$

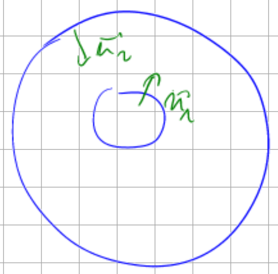
$$\vec{J_{S1}} = \vec{n_1} \times \vec{H_t}(x,y)$$



$$\vec{n} \times (\vec{H_2} - \vec{H_1}) = \vec{J_S}$$

$H_1 = 0$

$$I_1 = \int_{C_1} \vec{J_{S1}} \cdot \vec{u} \, dC_1$$



$$\vec{J}s_1 = -\vec{e}_y \times \frac{1}{Z}\frac{V_1}{b}\vec{e}_x$$

$$= \frac{1}{Z}\frac{V_1}{b}\vec{u}$$

$$\vec{J}s_0 = \vec{e}_y \times \frac{1}{Z}\frac{V_1}{b}\vec{e}_x$$

$$= -\vec{J}s_1$$

$$I_1 = \int_0^a \vec{J}s_1 \cdot \vec{u}\, dx$$

$$I_1 = \int_0^a \frac{1}{Z}\frac{V_1}{b}\vec{u}\cdot\hat{u}\, dx$$

$$= \frac{1}{Z}\frac{a}{b}V_1$$

* Z_c characteristic impedance

$$Z_c = \frac{V_1}{I_1} = \frac{\cancel{V_1}}{\frac{1}{Z}\frac{a}{b}\cancel{V_1}} = \frac{b}{a}Z$$

* $L = \frac{Z_c}{v}$ $\quad v = \frac{1}{\sqrt{\varepsilon\mu}}$

$$L = \frac{b}{a}\sqrt{\frac{\mu}{\varepsilon}}\sqrt{\varepsilon\mu} = \mu\frac{b}{a} \text{ H/m}$$

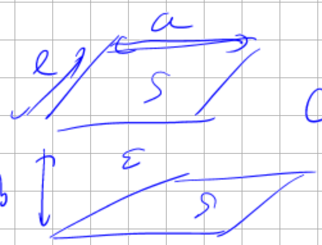
$$C = \frac{1}{Z_c v} = \frac{a}{bZ v}$$

$$= \frac{a}{b}\sqrt{\frac{\varepsilon}{\mu}}\sqrt{\varepsilon\mu} = \varepsilon\frac{a}{b} \text{ F/m}$$

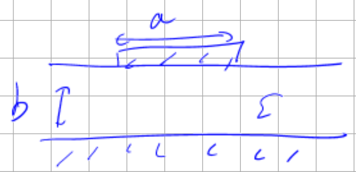
$$v = \frac{1}{\sqrt{LC}}$$

8) Modifying the line physical and geometrical dimensions, how could you increase or decrease the characteristic impedance value ?

$$C = \varepsilon \frac{a}{b}$$



$$C = \frac{\varepsilon S}{b} = \frac{\varepsilon a l}{b}$$



$Z_c = 50\,\Omega$

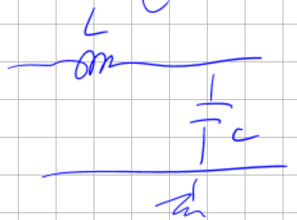
$Z_c = 10\,\Omega$

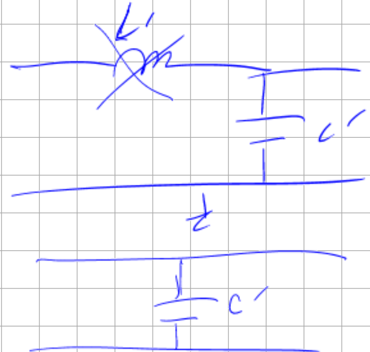
$$Z_c = \sqrt{\frac{L}{C}}$$

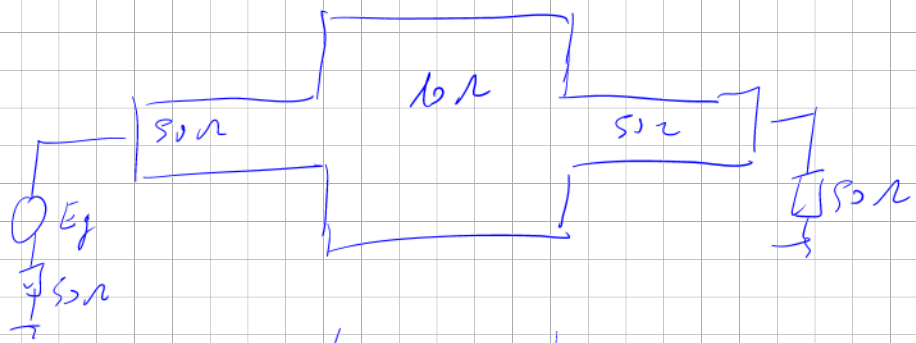
$Z_c \downarrow \Rightarrow C \nearrow$

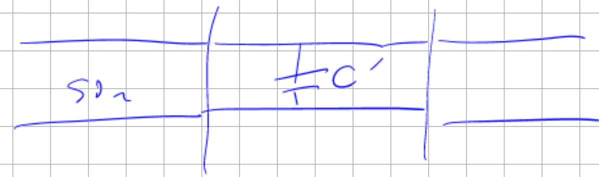
$|Z_{L'}| = L'\omega$

$|Z_{C'}| \leq 1\,\Omega$

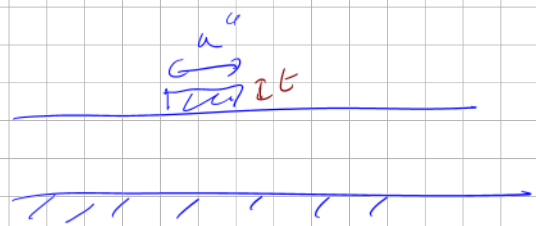


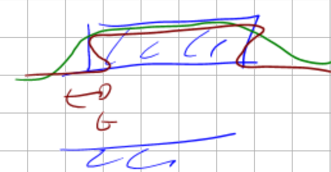






$a'' \gg \delta$

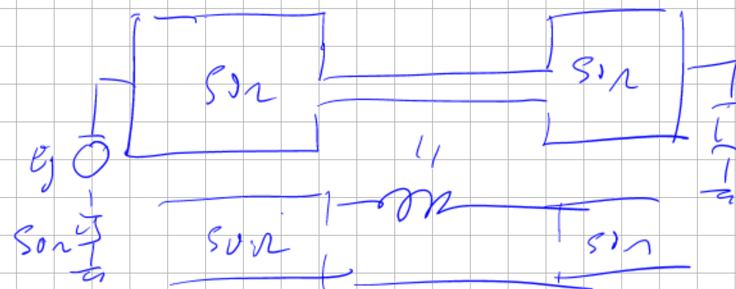




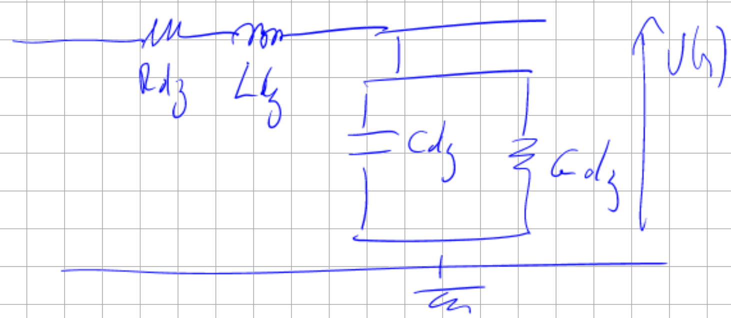
$Z_c'' = 100\,\Omega$

$a'' \sim 20\,\mu m$

$C'' \downarrow \quad |Z_{C''}| = \frac{1}{C''\omega} \nearrow \gg 1\,k\Omega$



9) losses in the line

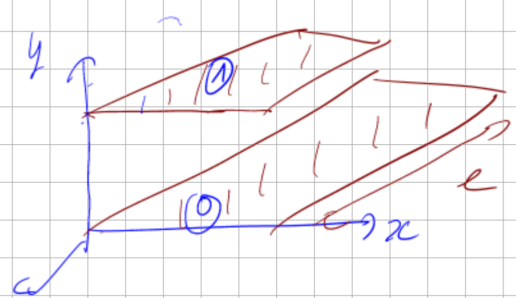


$$\overline{P_{met}} = \frac{1}{2} \ell R |I|^2 = \frac{1}{2} R_s \iint_{S_{met}} |H|^2 \, dS_{met}$$

$$I_1 = \frac{1}{Z} \frac{a}{b} V_1 \qquad \vec{H}_t = \frac{1}{Z} \frac{V_1}{b} \vec{e_x}$$

$$R_s = \frac{1}{\sigma \delta}$$

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$



$$\overline{P_{met_1}} = \frac{1}{2} R_s \int_0^a \int_0^\ell \left(\frac{1}{Z} \frac{V_1}{b}\right)^2 dx\, dz$$

$$\overline{P_{met\,1}} = \frac{1}{2} \frac{R_s}{Z^2} \frac{V_1^2}{b^2} a \ell$$

$$\overline{P_{met\,0}} = \overline{P_{met_1}}$$

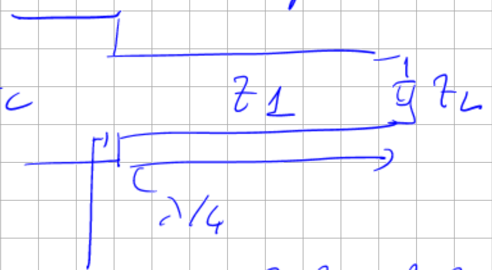
$$\overline{P_{met}} = \frac{R_s}{Z^2} \frac{V_1^2}{b^2} a \ell$$

$$\overline{P_{met}} = \frac{R_s}{Z^2} \frac{V_1^2}{b^2} a \ell = \frac{1}{2} R \ell |I|^2$$

$$\frac{R_s}{\cancel{Z^2}} \frac{\cancel{V_1^2}}{\cancel{b^2}} a = \frac{1}{2} R \frac{1}{Z^2} \left(\frac{a}{b}\right)^2 \cancel{V_1^2}.$$

$$R = \frac{2 R_s}{a} \quad \Omega/m$$

losses in capacitor << losses in inductor



$$Z_e = Z_1 \frac{Z_L + j \tan\beta\ell \, Z_1}{Z_1 + j Z_L \tan\beta\ell}$$

$$\beta \ell = \frac{2\pi}{\lambda} \frac{\lambda}{4} = \frac{\pi}{2}$$

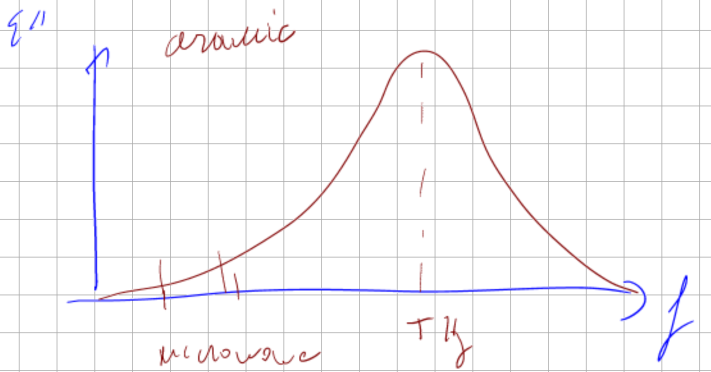
$$Z_e = \frac{Z_1^2}{Z_L} = \frac{Z_1^2}{jL\omega} = \frac{-j Z_1^2}{L\omega} = \frac{-j}{C'\omega}$$

$$* \quad \overline{P}_{diel} = \frac{1}{2} G \ell |V_1|^2 = \frac{1}{2} \omega \varepsilon'' \iiint |E_t|^2 dV$$

$$G \ell |V_1|^2 = \varepsilon'' \omega \int_0^a \int_0^b \int_0^\ell \left(\frac{V_1}{b}\right)^2 dx\,dy\,dz$$

$$= \varepsilon'' \omega \left(\frac{V_1}{b}\right)^2 a b \ell$$

$$= \omega \varepsilon'' \frac{a}{b}$$



$$f_0 \frac{\varepsilon''}{\varepsilon'} = cte$$