## Set #8

**24.**

Electrons, protons and neutrons have spin 1/2. There are two values of $m_s$ for s=1/2. These are $m_s$=1/2 and $m_s$=-1/2. The corresponding eigenstates $\chi_\uparrow$ (+$\hbar$/2) and $\chi_\downarrow$ (-$\hbar$\2) obey the following eigenvalue eq.s

$$S^2\chi_\uparrow = \frac{3\hbar^2}{4}\chi_\uparrow \quad S_z\chi_\uparrow = \hbar/2\chi_\uparrow$$

One can conveniently represent the two eigenstates as column matrices

$$\chi_\uparrow = \begin{pmatrix}1\\0\end{pmatrix} \quad \text{and} \quad \chi_\downarrow = \begin{pmatrix}0\\1\end{pmatrix}$$

a. Compute the matrix form of $S^2$.
b. Compute the matrix form of $S_z$.
c. Are these two eigenstates normalized?

## Set #3

**25.** One can conveniently represent the two spin-eigenstates as column matrices

$$\chi_\uparrow = \begin{pmatrix}1\\0\end{pmatrix} \quad \text{and} \quad \chi_\downarrow = \begin{pmatrix}0\\1\end{pmatrix}$$

Using $\chi_\uparrow$ and $\chi_\downarrow$ as spin basis states, the most general state of a spin 1/2 particle can be expressed as two-element column matrix

$$\chi = \begin{pmatrix}a\\b\end{pmatrix} = a\ \chi_\uparrow + b\ \chi_\downarrow$$

a. If you measured $S_z$ on a particle in the state $\chi$ what is the probability of measuring +$\hbar$/2 or -$\hbar$/2?

b. What is the sum of the probabilities?

c. What does it mean?

**26.** Compute the angles that the intrinsic angular momentum (*spin*) forms about a given direction z. These angles define the two eigenstates $\chi_\uparrow$ e $\chi_\downarrow$ of the operator $S_z$.

Write the relevant eigenvalue eq. for $\chi_\uparrow$ e $\chi_\downarrow$ and the orthogonality condition.

Write the total electron spin wavefunction: this is often used an archetype of a spin q-bit (spin quantum-bit)

Write the relation the coefficients of the spin wavefunction have to satisfy. Briefly explain how such a q-bit differ from a classical bit.

***Extra*.**

Suppose the electron is a classical solid "sphere" with radius $r \sim 5 \times 10^{-17} m$, what would be the speed a point placed on the equator of the sphere? Is such a spherical model for the electron sound?