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Lesson 3
Sept 13rd 2020

II Maxwell equations

$$\vec{E}(x,y,z) = \vec{E}_t(x,y)\, e^{-\gamma z}$$

$$\vec{H}(x,y,z) = \vec{H}_t(x,y)\, e^{-\gamma z}$$

$$* \quad \vec{\nabla} \times \vec{E}(x,y,z) = -j\omega\mu\, \vec{H}(x,y,z) \times$$

$$\vec{\nabla} = \vec{\nabla}_t + \frac{\partial}{\partial z}\vec{u} \qquad \vec{\nabla}_t \left| \begin{matrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{matrix} \right.$$

$$\left(\vec{\nabla}_t + \frac{\partial}{\partial z}\vec{u}\right) \times \left[\vec{E}_t(x,y)\, e^{-\gamma z}\right] = -j\omega\mu\, \vec{H}_t(x,y)\, e^{-\gamma z}$$

$$\vec{\nabla}_t \times \left(\vec{E}_t(x,y)\, e^{-\gamma z}\right) + \frac{\partial}{\partial z}\vec{u} \times \vec{E}_t(x,y)\, e^{-\gamma z}$$

$$= -j\omega\mu\, \vec{H}_t(x,y)\, e^{-\gamma z}$$

$$(O, \vec{e_x}, \vec{e_y}, \vec{u})$$

$$\vec{u} \times \vec{e_x} = \vec{e_y}$$

$$\vec{u} \times \vec{e_y} = -\vec{e_x}$$

$$\vec{e_x} \times \vec{e_x} = 0$$

$$\vec{e_x} \times \vec{e_y} = \vec{u}$$

$$\vec{e_y} \times \vec{e_x} = -\vec{u}$$

$$\vec{e_y} \times \vec{e_y} = 0$$

$$\cancel{e^{-\gamma z}}\, \vec{\nabla}_t \times \vec{E}_t(x,y) = 0$$

longitudinal

$$\gamma\, \cancel{e^{-\gamma z}}\, \vec{u} \times \vec{E}_t(x,y) = -j\omega\mu\, \vec{H}_t(x,y)\, \cancel{e^{-\gamma z}}$$

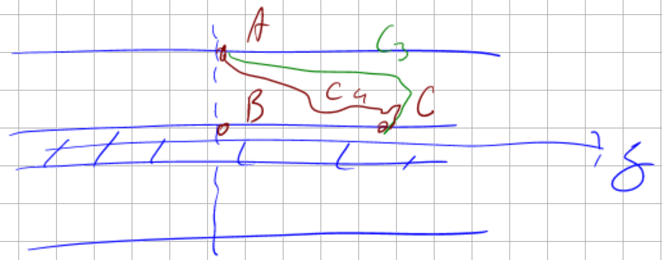
transversal

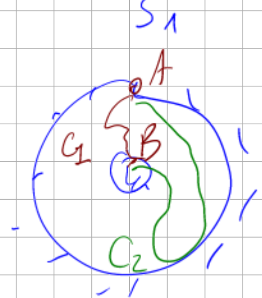
$$\vec{\nabla}_t \times \vec{E}_t(x,y) = 0 \qquad (1)$$

$$\gamma\, \vec{u} \times \vec{E}_t(x,y) = -j\omega\mu\, \vec{H}_t(x,y) \qquad (2)$$

$$(1) \Rightarrow \quad \vec{E}_t(x,y) = -\vec{\nabla}_t V(x,y)$$

Potential $\quad V(x,y,z) = V(x,y)\, e^{-\gamma z}$





$$-\int_{C_1} \vec{E}\cdot \vec{dC_1} = -\int_{C_2} \vec{E}\cdot \vec{dC_2}$$

$$-\int_{C_3} \vec{E}\cdot \vec{dC_3} \neq -\int_{C_4} \vec{E}\cdot \vec{dC_4}$$

$$\vec{\nabla} \times \vec{E}_t(x,y,z) \neq 0$$

* $\vec{\nabla} \times \vec{H}(x,y,z) = \cancel{\vec{J}(x,y,z)} + j\omega\varepsilon \vec{E}(x,y,z)$

$\vec{J}(x,y,z) = 0 \qquad \rho(x,y,z) = 0$

$\vec{E} \to \vec{H} \qquad \vec{H} \to \vec{E} \qquad \mu \to -\varepsilon$

$\vec{\nabla}_t \times \vec{H}_t(x,y) = 0 \quad (3)$

$\gamma\, \vec{u} \times \vec{H}_t(x,y) = j\omega\varepsilon \vec{E}_t(x,y) \quad (4)$

* $\vec{\nabla} \cdot \vec{E}(x,y,z) = 0$

$\left(\vec{\nabla}_t + \frac{\partial}{\partial z}\vec{u}\right) \cdot \left(\vec{E}_t(x,y) e^{-\gamma z}\right) = 0$

$\vec{e_x} \cdot \vec{e_x} = 1 \qquad \vec{e_x} \cdot \vec{e_y} = 0 \qquad \vec{e_y} \cdot \vec{e_x} = 0$

$\vec{e_y} \cdot \vec{e_y} = 1$

$\vec{u} \cdot \vec{i_x} = 0 \qquad \vec{u} \cdot \vec{i_y} = 0$

$\vec{\nabla}_t \cdot \vec{E}_t(x,y) e^{-\gamma z} = 0$

$\vec{\nabla}_t \cdot \vec{E}_t(x,y) = 0 \quad (5)$

(1) $\vec{\nabla}_t \times \vec{E}_t(x,y) = 0 \Rightarrow \vec{E}_t(x,y) = -\vec{\nabla} V(x,y)$

$-\vec{\nabla}_t \cdot \vec{\nabla}_t V(x,y) = 0$

$\Delta_t V(x,y) = 0 \qquad$ POISSON equation

* $\vec{\nabla} \cdot \vec{H}(x,y,z) = 0$

$\vec{\nabla}_t \cdot \vec{H}_t(x,y) = 0 \qquad (6)$

electrostatic, magnetostatic laws

$\Delta_t V(x,y) \to \Rightarrow \frac{\partial^2 V(x,y)}{\partial x^2} + \frac{\partial^2 V(x,y)}{\partial y^2} = 0$

* (1) and (4)

(1) $\gamma\, \vec{u} \times \vec{E}_t(x,y) = -j\omega\mu \vec{H}_t(x,y)$

$\vec{u} \times (4) \quad \gamma\, \vec{u} \times [\vec{u} \times \vec{E}_t(x,y)] = -j\omega\mu\, \underbrace{\vec{u} \times \vec{H}_t(x,y)}_{(4)}$

$\vec{A} \times \vec{B} \times \vec{C} = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

$\gamma\left[\vec{u}\,\overbrace{(\vec{u} \cdot \vec{E}_t(x,y))}^{=0} - \vec{E}_t(x,y)\,\overbrace{(\vec{u} \cdot \vec{u})}^{1}\right] =$

$-j\omega\mu \left[\frac{j\omega\varepsilon}{\gamma} \vec{E}_t(x,y)\right]$

$\gamma^2 \vec{E}_t(x,y) = -\omega^2 \varepsilon\mu\, \vec{E}_t(x,y)$

$\gamma^2 = -\omega^2 \varepsilon\mu \qquad \gamma = j\omega\sqrt{\varepsilon\mu}$

$$\vec{E}_t(x,y) = E_x(x,y)\,\vec{e}_x + E_y(x,y)\,\vec{e}_y$$

$$E_x(x,y,z) = E_x(x,y)\, e^{-j\beta z}$$

$$e_x(x,y,z,t) = \Re\left(E_x(x,y)e^{-j\beta z}\, e^{j\omega t}\right)$$

$$E_x(x,y) = |E_x(x,y)|\, e^{j\varphi_x(x,y)}$$

$$e_x(x,y,z,t) = |E_x(x,y)| \underline{\cos\left(\omega t - \beta z + \varphi_x(x,y)\right)}$$

$$\cos\Big[\omega\big(\underset{s}{t} - \underset{s/m}{\tfrac{\beta}{\omega}}\underset{m}{z}\big) + \varphi_x(x,y)\Big]$$

$$\beta = \omega\sqrt{\varepsilon\mu} = \frac{\omega}{v} \quad \text{velocity} \quad v = \frac{1}{\sqrt{\varepsilon\mu}}$$

lossless lines
$$v = \frac{1}{\sqrt{\varepsilon\mu}} \quad \text{non dispersive line}$$
$$\beta = \frac{\omega}{v}$$

$$\times \quad \gamma\, \vec{u}\times\vec{E}_t(x,y) = j\omega\varepsilon\, \vec{H}_t(x,y)$$

$$\vec{H}_t = \frac{\gamma}{j\omega\varepsilon}\,\vec{u}\times\vec{E}_t(x,y) = \frac{j\omega\sqrt{\varepsilon\mu}}{j\omega\varepsilon}\,\vec{u}\times\vec{E}_t(x,y)$$

$$\vec{H}_t(x,y) = \sqrt{\frac{\varepsilon}{\mu}}\;\vec{u}\times\vec{E}_t(x,y)$$

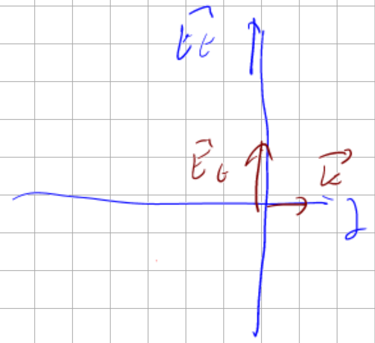
$$Z = \sqrt{\frac{\mu}{\varepsilon}} \quad \text{wave impedance}$$

$$\vec{H}_t(x,y) = \frac{1}{Z}\,\vec{u}\times\vec{E}_t(x,y)$$

$$\vec{E}_t(x,y) = -Z\left(\vec{u}\times\vec{H}_t(x,y)\right)$$

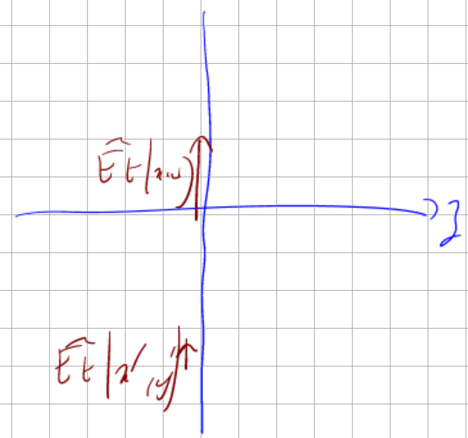
$$\Rightarrow \vec{E}_t(x,y) \perp \vec{H}_t(x,y)$$

planar wave



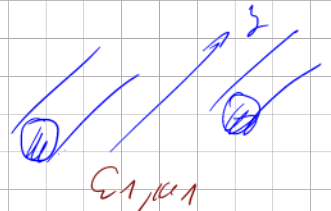
$$k = \frac{\omega}{v}$$

TEM wave

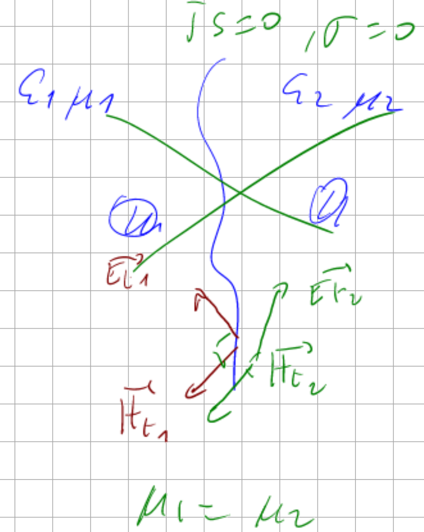


III When a TEM wave exists?

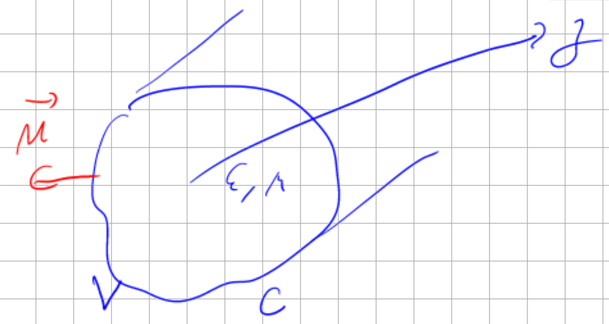
* homogeneous material between the metallic parts.



hp 2 materials



* We need at least 2 conductors. It's impossible with only 1 conductor.



$$\int_C V(x,y)\,\vec{\nabla}_t V^*(x,y)\cdot\vec{m}\,dC$$

$$= V\int_C \underline{\vec{\nabla}_t V^*(x,y)\cdot\vec{m}}\,dC$$

$$= V\iint_S \underbrace{\vec{\nabla}_t\cdot\vec{\nabla}_t V^*(x,y)}\,dS$$

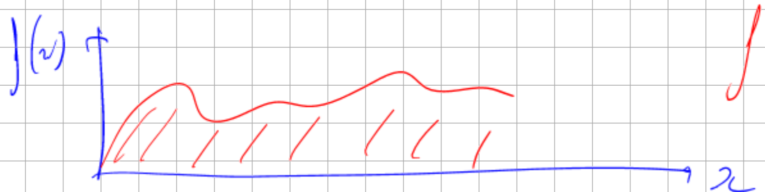
$$\Delta_t V^*(x,y) = 0$$

$$= 0$$

$$= \iint_S \vec{\nabla}_t\cdot\left(V(x,y)\,\vec{\nabla}_t V^*(x,y)\right)dS$$

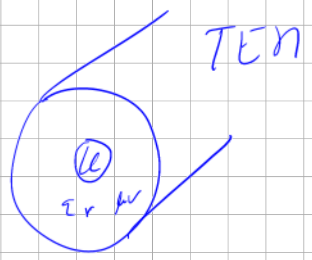
$$= \iint_S \left[V(x,y)\,\underbrace{\vec{\nabla}_t\cdot\vec{\nabla}_t V^*(x,y)}_{=0} + \vec{\nabla}_t V(x,y)\cdot\vec{\nabla}_t V^*(x,y)\right]dS$$

$$= \iint_S \|E_t(x,y)\|^2\,dS = 0$$

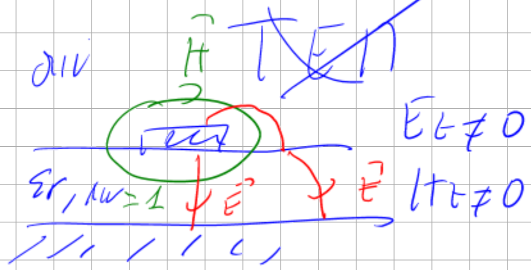


$f(x) \geq 0$

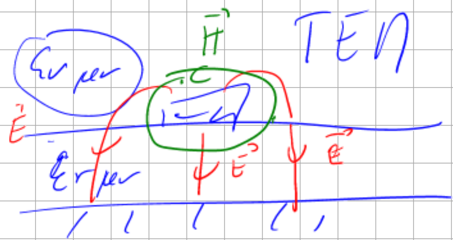
$\Rightarrow \vec{E}_t(x,y) = 0$



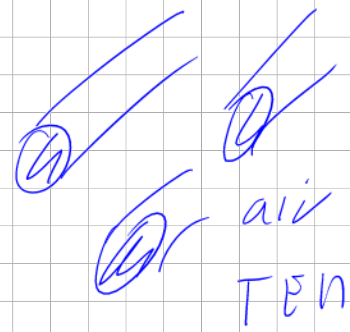
Coaxial line



quasi TEM



ε_{eff}



rectangular waveguide