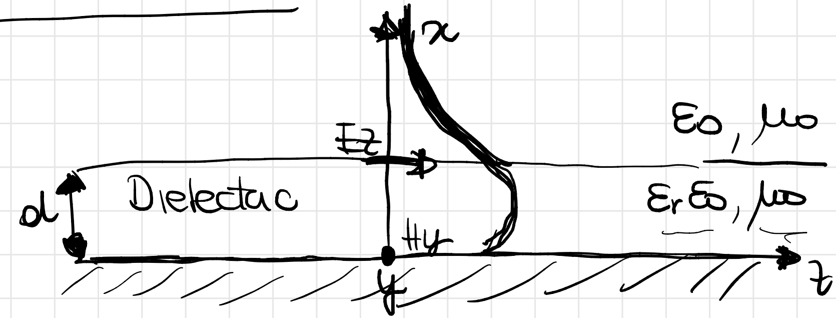
MICROWAVE ENGINEERING

Lecture 14:

Grounded dielectric slab and other waveguides

Surface waves of grounded dielectric slab



TM Modes

Propagating in z with
$e^{-j\beta z}$

No variation in y direction $\rightarrow \frac{\partial}{\partial y} = 0$

$$
\begin{cases}
0 \leq x < d & \left(\frac{\partial^2}{\partial x^2} + \varepsilon_r k_0^2 - \beta^2 \right) e_z(x,y) = 0 \\
d \leq x \leq \infty & \left(\frac{\partial^2}{\partial x^2} + k_0^2 - \beta^2 \right) e_z(x,y) = 0
\end{cases}
$$

$$\bar{E}(x,y,z) = e_z(x,y)\, e^{-j\beta z}$$

Two different cutoff wavenumbers :

a $k_c^2 = \varepsilon_r k_0^2 - \beta^2$

b $h^2 = \beta^2 - k_0^2$ ← we are anticipating that the field will decay exponentially for $x \geq d$

Solutions of the two-wave equations :

$0 \leq x < d \qquad e_z(x,y) = A \sin k_c x + B \cos k_c x$

$d \leq x \leq \infty \qquad e_z(x,y) = C e^{hx} + D e^{-hx}$

Boundary conditions:

(1) $E_z(x,y,z) = 0$ for $x=0$

(2) $E_z(x,y,z) < \infty$ for $x \to \infty$

(3) $E_z(x,y,z)$ continuous for $x=d$

(4) $H_y(x,y,z)$ continuous for $x=d$

$H_x = E_y = H_z = 0$

From (1) $\Rightarrow$ $B=0$

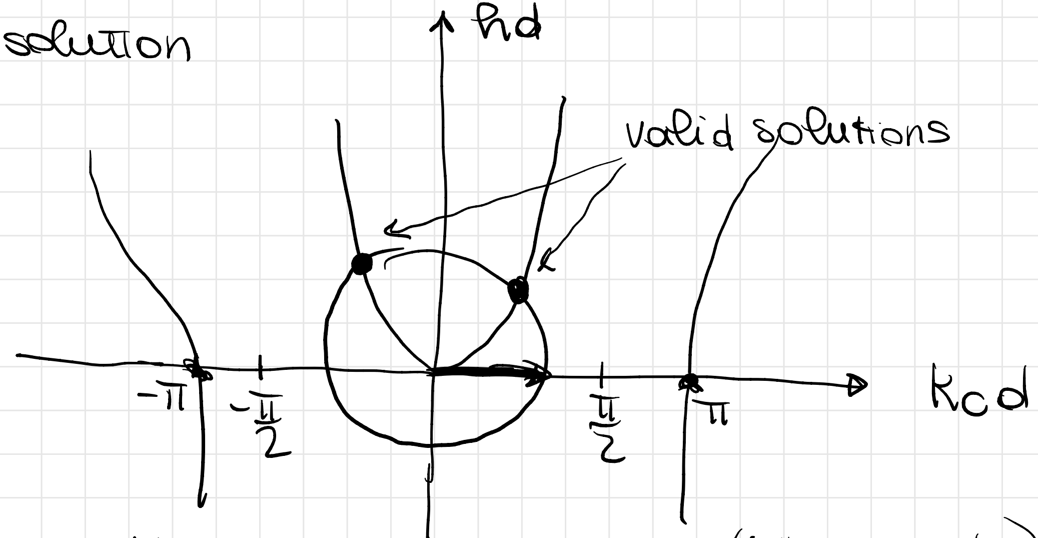
From (2) $\Rightarrow$ $C=0$

$$\begin{cases} \text{From (3)} \Rightarrow A \sin k_c d = D e^{-hd} \\ \text{From (4)} \Rightarrow \dfrac{\varepsilon_r A}{k_c} \cos k_c d = \dfrac{D}{h} e^{-hd} \end{cases}$$

Dividing ③ by ④ $\Rightarrow$ $\begin{cases} k_c \tan k_c d = \varepsilon_r h & \leftarrow \text{tan} \\ k_c^2 + h^2 = \boxed{(\varepsilon_r - 1)} k_0^2 & \leftarrow \text{circle} \end{cases}$

Sum a+b

Graphical solution



For $\varepsilon_r > 1$ $\Rightarrow$ At least 1 TM mode (TM_0 mode). Cutoff freq. is zero.

The next TM mode (TM_1) turn on only if the circle radius is $> \pi$

$$f_c = \frac{nc}{2d\sqrt{\varepsilon_r - 1}} \qquad n = 0, 1, 2 \ldots$$

The expressions for the other fields are:

$$E_z(x,y,z) = \begin{cases} A \sin k_c x \, e^{-j\beta z} & 0 \leq x < d \\ A \sin k_c d \, e^{-h(x-d)} e^{-j\beta z} & x \geq d \end{cases}$$

$$E_x(x,y,z) = \begin{cases} -\frac{j\beta}{k_c} A \cos k_c x \, e^{-j\beta z} & 0 \leq x < d \\ -\frac{j\beta}{h} A \sin k_c d \, e^{-h(x-d)} & x \geq d \end{cases}$$

$$H_y(x,y,z) = \begin{cases} \dfrac{-j\omega\varepsilon_0\varepsilon_r}{k_c^2} A \cos k_c x \, e^{-j\beta z} & 0 \leq x < d \\ \dfrac{-j\omega\varepsilon_0}{h} A \sin k_c d \, e^{-h(x-d)} e^{-j\beta z} & x \geq d \end{cases}$$

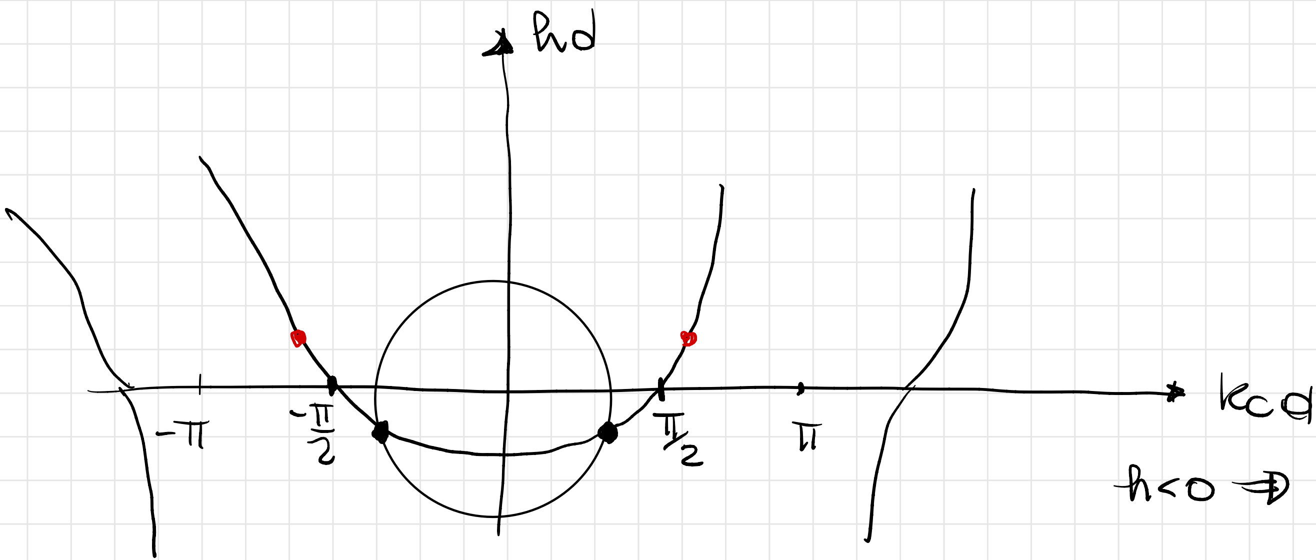
TE Modes

$$\begin{cases} \left(\dfrac{\partial^2}{\partial x^2} + k_c^2\right) h_z(x,y) = 0 & 0 \leq x < d \\ \left(\dfrac{\partial^2}{\partial x^2} - h^2\right) h_z(x,y) = 0 & x \geq d \end{cases}$$

$$H_z(x,y,z) = h(x,y)\, e^{-j\beta z}$$

After setting boundary conditions we get:

$$\begin{cases} -k_c \cot(k_c d) = h \\ k_c^2 + h^2 = (\varepsilon_r - 1) k_0^2 \end{cases}$$



$h<0 \Rightarrow$ invalid solution

The first TE mode starts to propagate when

$$\underbrace{\sqrt{\varepsilon_r - 1}\; k_0 d}_{\text{radius of circle}} > \frac{\pi}{2}$$

The cutoff frequencies for TE_n modes are

$$f_c = \frac{(2n-1)\, c}{4d\sqrt{\varepsilon_r - 1}} \qquad n = 1, 2, 3 \ldots$$

The order of appearance of the modes is TM_0, TE_1, TM_1, TE_2, TM_2 . . .

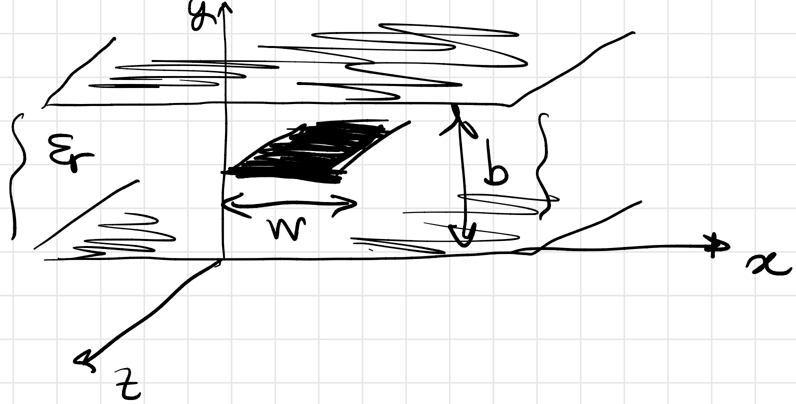
Fields for TE Modes:

$$H_z(x,y,z) = \begin{cases} B \cos k_c x \, e^{-j\beta z} & 0 \le x \le d \\ B \cos k_c d \, e^{-h(x-d)} e^{-j\beta z} & x \ge d \end{cases}$$

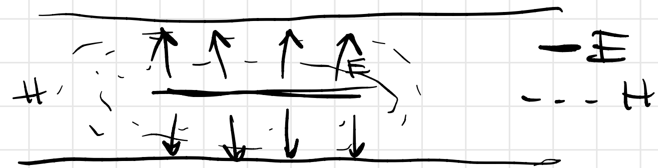
$$H_x(x,y,z) = \begin{cases} \dfrac{j\beta B}{k_c} \sin k_c x \, e^{-j\beta z} & 0 \le x \le d \\ -\dfrac{j\beta B}{h} \cos k_c d \, e^{-h(x-d)} e^{-j\beta z} & x \ge d \end{cases}$$

$$E_y(x,y,z) = \begin{cases} -\dfrac{j\omega\mu_0 B}{k_c} \sin k_c x \, e^{-j\beta z} & 0 \le x \le d \\ \dfrac{j\omega\mu_0 B}{h} \cos k_c d \, e^{-h(x-d)} e^{-j\beta z} & x \ge d \end{cases}$$

STRIPLINE



for TEM mode



Phase velocity

$$V_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}}$$

Propagation constant

$$\beta = \frac{\omega}{V_p} = \omega\sqrt{\mu_0 \varepsilon_0 \varepsilon_r} = k_0\sqrt{\varepsilon_r}$$

Characteristic Impedance

$$Z_0 = \frac{30\pi}{\sqrt{\varepsilon_r}} \frac{b}{W_e + 0.441b}$$

W_e is the effective width of strip:

$$\frac{W_e}{b} = \frac{W}{b} - \begin{cases} 0 & \frac{W}{b} \geq 0.35 \\ \left(0.35 - \frac{W}{b}\right)^2 & \frac{W}{b} < 0.35 \end{cases}$$

The strip width given Z_0 is:

$$\frac{W}{b} = \begin{cases} x & \sqrt{\varepsilon_r}\, Z_0 \leq 120 \\ 0.85 - \sqrt{0.6 - x} & \sqrt{\varepsilon_r}\, Z_0 > 120 \end{cases}$$

$$x = \frac{30\pi}{\sqrt{\varepsilon_r}\, Z_0} - 0.441$$

Attenuation

$$\alpha_d = \frac{k \tan\delta}{2} \quad \text{Np/m}$$

$$\alpha_c = \begin{cases} \dfrac{2.7 \cdot 10^{-3} R_s \varepsilon_r Z_0}{30\pi (b-t)} A & \sqrt{\varepsilon_r}\, Z_0 \leq 120 \\[2ex] \dfrac{0.16 R_s}{Z_0 b} B & \sqrt{\varepsilon_r}\, Z_0 > 120 \end{cases}$$

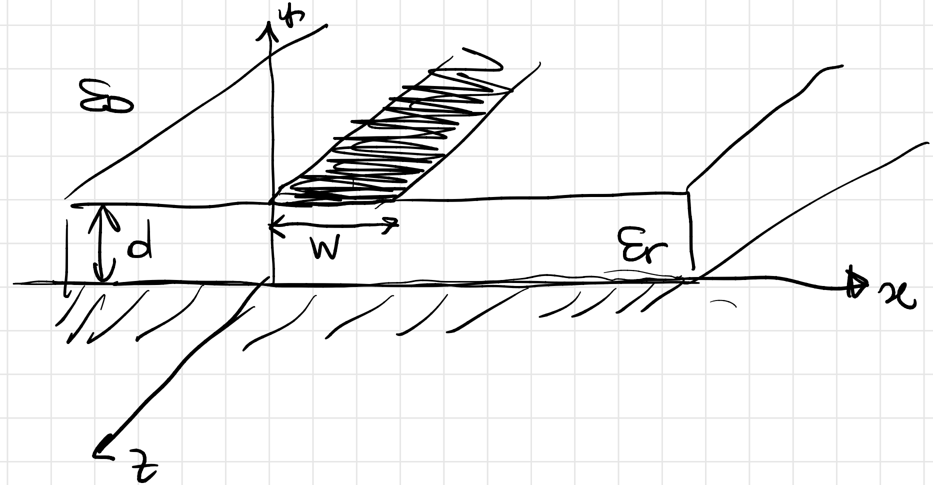
$$A = 1 + \frac{2W}{b-t} + \frac{1}{\pi}\frac{b+t}{b-t}\ln\left(\frac{2b-t}{t}\right)$$

$$B = 1 + \frac{b}{0.5W+0.7t}\left(0.5 + \frac{0.414t}{W} + \frac{1}{2\pi}\ln\frac{4\pi W}{t}\right)$$

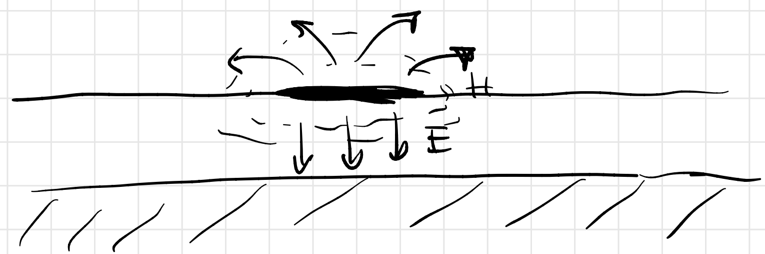
t is the thickness of the strip.

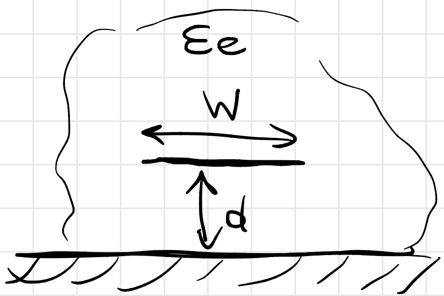
MICROSTRIP

The microstrip does not support pure TEM waves but supports hybrid TM-TE waves.



If $d \ll \lambda$ → quasi-TEM modes





→ Phase velocity $\qquad V_p = \dfrac{c}{\varepsilon_e}$

Prop. const $\qquad \beta = k_0 \sqrt{\varepsilon_e}$

effective permittivity $\quad 1 < \varepsilon_e < \varepsilon_r$

In the quasi-static approximation

$$\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12\, d/W}}$$

Characteristic impedance

$$Z_0 = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_e}} \ln\left(\dfrac{8d}{W} + \dfrac{W}{4d}\right) & \dfrac{W}{d} \le 1 \\[2ex] \dfrac{120\pi}{\sqrt{\varepsilon_e}\left[\dfrac{W}{d} + 1.393 + 0.667 \ln\left(\dfrac{W}{d} + 1.444\right)\right]} & \dfrac{W}{d} > 1 \end{cases}$$

For a given Z_0 and ε_r:

$$\frac{W}{d} = \begin{cases} \dfrac{8e^{A}}{e^{2A}-2} & \dfrac{W}{d} \leq 2 \\[2ex] \dfrac{2}{\pi}\left[B - 1 - \ln(2B-1) + \dfrac{\varepsilon_r - 1}{2\varepsilon_r}\left\{\ln(B-1) + 0.39 - \dfrac{0.61}{\varepsilon_r}\right\}\right] & \dfrac{W}{d} > 2 \end{cases}$$

$$A = \frac{Z_0}{60}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right)$$

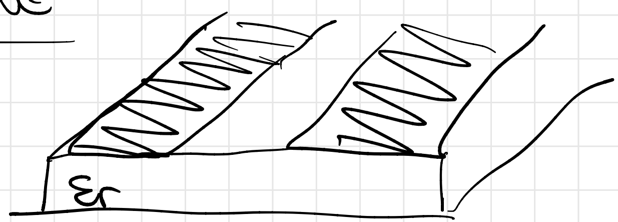
$$B = \frac{377\pi}{2 Z_0 \sqrt{\varepsilon_r}}$$

Attenuation

$$\alpha_d = \frac{k_0 \varepsilon_r (\varepsilon_e - 1) \tan\delta}{2\sqrt{\varepsilon_e}(\varepsilon_r - 1)} \quad Np/m$$

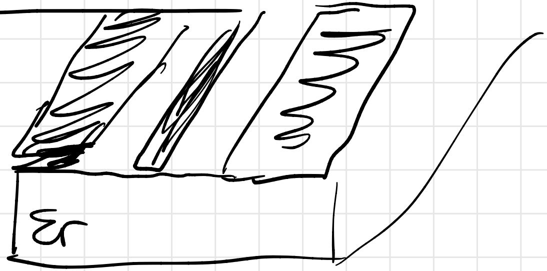
$$\alpha_c = \frac{R_s}{Z_0 W} \quad \frac{Np}{m} \qquad R_s = \sqrt{\frac{\omega \mu_0}{2\sigma}}$$

SLOT LINE



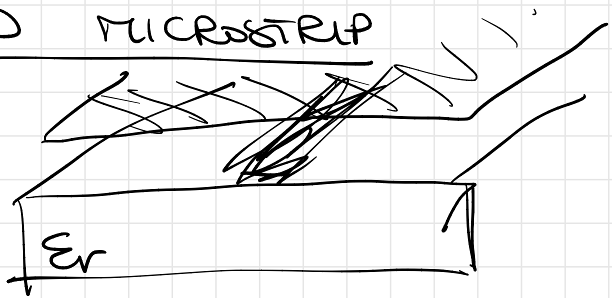
quasi-TEM modes

COPLANAR WAVEGUIDE

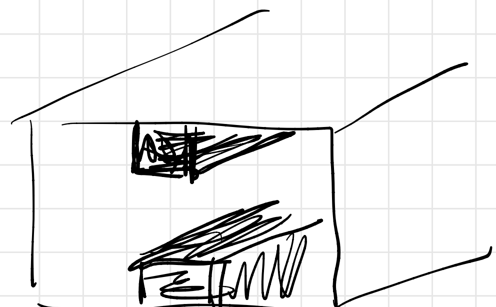


quasi-TEM modes

COVERED MICROSTRIP



RIDGE WAVEGUIDE



DIELECTRIC WAVEGUIDE

DISPERSION & WAVE VELOCITIES

Speed of light in a medium $\frac{1}{\sqrt{\mu\varepsilon}}$

Phase velocity $v_p = \frac{\omega}{\beta}$

If medium responds differently at different frequencies
$\rightarrow$ SIGNAL IS DISTORTED!

If Bandwidth of signal is SMALL
<u>and</u> Dispersion is not severe] $\Rightarrow$ we can define a <u>GROUP VELOCITY</u>

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

$$F(\omega) \longrightarrow \boxed{Z(\omega)} \longrightarrow F_o(\omega)$$

$$\underline{F_o(\omega)} = \underline{Z(\omega)} F(\omega)$$

if $Z(\omega)$ stands for a TL LOSSLESS AND MATCHED

$$\underline{Z(\omega)} = A e^{-j\beta z} = |Z(\omega)| e^{-j\psi}$$

the output signal in time domain is

$$f_o(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \, |Z(\omega)| \, e^{j(\omega t - \psi)} d\omega$$

if $|Z(\omega)| = A$ constant and $\psi = a\omega$ (LINEAR)

$$\underline{f_o(t)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} A \, F(\omega) \, e^{+j\omega(t-a)} d\omega = \underline{A \, f(t-a)}$$

If $Z(\omega) = A e^{-j\omega a}$ $\Rightarrow$ T.LINE DOES NOT DISTORT THE SIGNAL

(LOSSLESS TEM LINE ($\beta = \frac{\omega}{c}$))

If TEM line is LOSSY

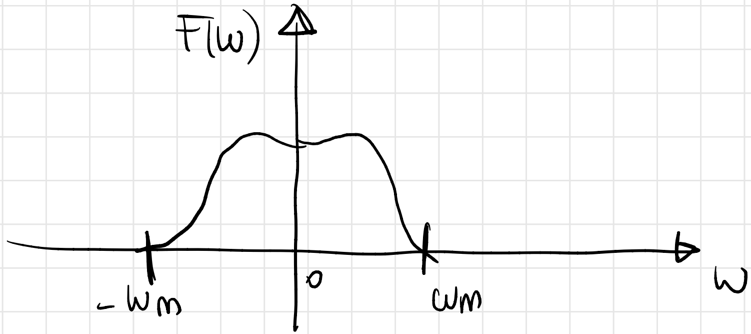
Assuming a narrowband signal

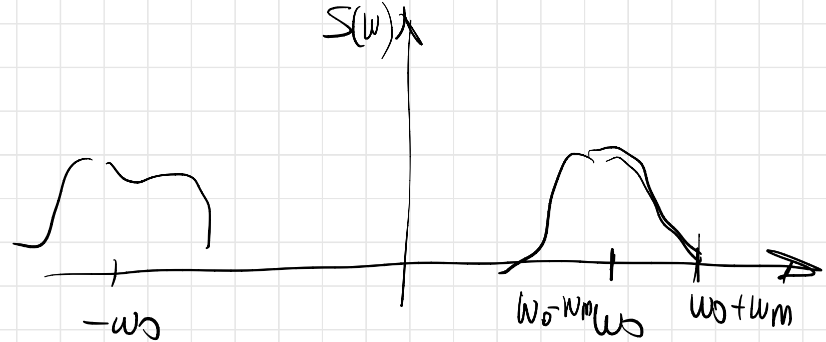
$$s(t) = \boxed{f(t) \cos(\omega_0 t)} = \operatorname{Re}\{ f(t)\, e^{j\omega_0 t} \}$$

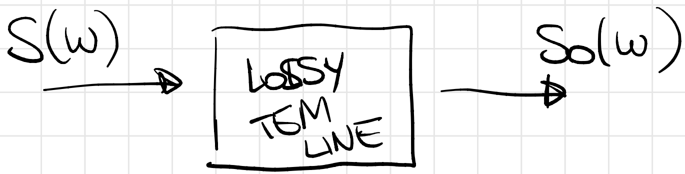
$\omega_m \ll \omega_0$

The Fourier Transform is:

$$S(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega_0 t}\, e^{j\omega t}\, dt = F(\omega - \omega_0)$$







$$S_o(\omega) = A\,F(\omega-\omega_0)\,e^{-j\beta z}$$

In the time domain

$$S_o(t) = \frac{1}{2\pi} \operatorname{Re} \int_{-\infty}^{\infty} S_o(\omega)\, e^{j\omega t}\, d\omega =$$

$$= \frac{1}{2\pi} \operatorname{Re} \int_{\omega_0 - \omega_m}^{\omega_0 + \omega_m} A\, F(\omega - \omega_0)\, e^{j(\omega t - \beta z)}\, d\omega$$

if bandwidth is narrow

$$\underbrace{\beta(\omega) = \beta(\omega_0) + \left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_0} (\omega - \omega_0)}_{\text{retain only 2 terms}} + \frac{1}{2} \left.\frac{d\beta}{d\omega^2}\right|_{\omega=\omega_0} (\omega - \omega_0)^2 + \dots$$

$$\beta(\omega) \simeq \underbrace{\beta_0}_{\beta(\omega_0)} + \underbrace{\beta_0'}_{\left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_0}} (\omega - \omega_0)$$

Imposing $y = \omega - \omega_0$

$$s_0(t) = \frac{A}{2\pi} \mathrm{Re}\left\{ e^{j(\omega_0 t - \beta_0 z)} \int_{-\omega_m}^{\omega_m} F(y)\, e^{j(t - \beta_0' z) y}\, dy \right\} =$$

$$= A \,\mathrm{Re}\left[f(t - \beta_0' z)\, e^{j(\omega_0 t - \beta_0 z)} \right] =$$

$$= A \underbrace{f(t - \beta_0' z)} \cos(\omega_0 t - \beta_0 z)$$

TIME SHIFTED REPLICA of $f(t)$ that travels at V_g (velocity of the envelope).

$$\boxed{V_g = \frac{1}{\beta_0'} = \left. \left(\frac{\partial \beta}{\partial \omega} \right)^{-1} \right|_{\omega = \omega_0}}$$