Spatial Optics

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Spatio temporal analogy

Applied to Free space propagati

MONOCHROMATIC PLANE WAVE STRUCTURE

$$MPW(t,M) = e^{j(2\pi\nu t - \vec{k}.\overrightarrow{\mathbf{OM}})}$$

$$MPW(t,M) = e^{j2\pi\nu t}.e^{-j\vec{k}.\overrightarrow{\mathbf{OM}}}$$

Temporal

Spatial

Propagation along the Oz axis

$$MPW_{z=0}(t,x,y) = e^{j2\pi\nu t}.e^{-j(k_x.x+k_y.y)}$$

variable	t	x ; y
Frequency	ν	$N_x = \frac{k_x}{2\pi}$; $N_y = \frac{k_y}{2\pi}$

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First analogy about the propagation

Temporal

Monochromatic wave

$$MW_{z=0}(t) = e^{j2\pi\upsilon t}.$$

$$\Downarrow$$

$$MW_{z\neq0}(t\ ) = e^{j2\pi\upsilon t}.\boxed{e^{-j\beta.z)}}$$

Spatial

Plane wave

$$PW_{z=0}(x,y) = e^{-j(k_x.x+k_y.y)}$$

$$\Downarrow$$

$$PW_{z\neq0}(x,y) = e^{-j(k_x.x+k_y.y)}.\boxed{e^{-j(k_z.z)}}$$

Transfert function

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Analogy about the function decomposition : Fourier analysis

Temporal

Spatial

$$f(t) = \int_{-\infty}^{+\infty} \tilde{f}(\nu)\, e^{j2\pi\nu t} dt$$

FT-1

$$f(x,y) = \int_{-\infty}^{+\infty} \tilde{f}(N_x, N_y) \overbrace{e^{-j(k_x.x+k_y.y)}}^{\text{Plane wave}} dN_x dN_y$$

$2\pi(x N_x + y N_y)$

$$\tilde{f}(\nu) = \int_{-\infty}^{+\infty} f(t)\, e^{-j2\pi\nu t} dt$$

FT

$$\tilde{f}(N_x, N_y) = \int_{-\infty}^{+\infty} f(x,y)\, e^{j(k_x.x+k_y.y)} dxdy$$

1D

2D

!!signs

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Analogy about general skills of the data processing

Applied to linear and translation invariant systems

Linearity

time invariance

$$\begin{tikzcd}
in(t) \arrow[r, "* IR\ (impulse\ response)"] \arrow[d, "FT"] & out(t) = in * IR = FT^{-1}(\tilde{in}.TF) \\
\tilde{in}(\upsilon) \arrow[r, ".TF"] & \tilde{out}(\upsilon)=\tilde{in}(\upsilon).TF \arrow[u, "FT^{-1}"']
\end{tikzcd}$$

LTI system Σ

Linearity

space invariance

$$\begin{tikzcd}
in(x,y) \arrow[r, "* PSF"] \arrow[d, "FT"] & out(x,y) = in * PSF = FT^{-1}(\tilde{in}.TF) \\
\tilde{in}(N_x,N_y) \arrow[r, ".TF"] & \tilde{out}(N_x,N_y)=\tilde{in}(N_x,N_y).TF \arrow[u, "FT^{-1}"']
\end{tikzcd}$$

LTI system Σ

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Application to the diffraction grating formula

$$\sin(\theta) = \sin(\theta_0) + \frac{k\lambda}{a}$$

$$\frac{\sin\theta}{\lambda} = \frac{\sin\theta_0}{\lambda} + k\,\boxed{\frac{1}{a}}$$

Λ

= Frequency of the grating

$$\Lambda = \frac{1}{a}$$

$N_{x\,\text{out}} = N_{x\,\text{in}} + k$ spatial frequency grating

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