Lecture 2: OFDM (Orthogonal Frequency Division Multiplexing)

* We will see that in reality is an amplitude modulation, is the channel that is divided in different subchannels based on the frequency.
* Also called ⇒ Multi-Carrier or Multi-Tone modulation
* Used for:
  - Digital Audio Broadcasting
  - Digital Video Broadcasting
  - ADSL
  - Wireless LAN

Wireless Channel

This channel is characterize by a multipath. If we send one impulse at the receiver reaches a lot of impulses that are different from each other. (due to different propagation)

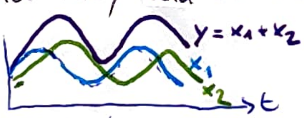
→ Impulse response of the channel is composed by many impulses

→ The channel is also time variant (we can move or the thing that provokes the interference can move)

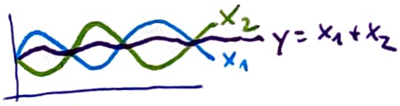
* **Problem: Multipath**

When we transmit the signal there are many different path, with different lengths that reach the receiver

The effect of multipath is that if we send a sinusoidal signal the same sinusoidal signal will reach the receiver with different delays due to the different paths. It means that we will have interferences and they could be destructive interferences so the signal will be quasi-cancelled



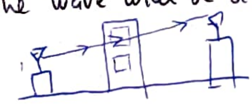
constructive



destructive

* **Problem: Shadowing**

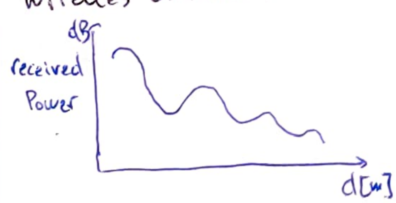
When a wave passes through something (i.e. a building) the wave will be attenuated



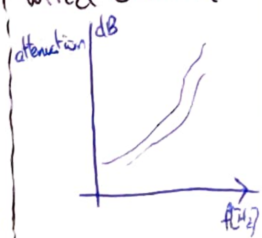
* **Pathloss**

The loss of power due to the propagation. If we move further the wave will be more attenuated

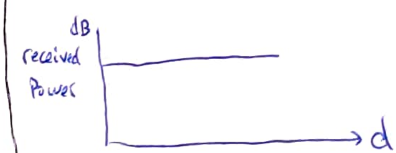
Wirelles channel:



Wired Channel

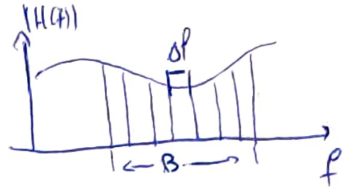


Ideal channel



Basic Idea

* We have wired and wireless channels that are non-lineal channels (freq. response no constant) therefore is difficult to do a modulation on non-lineal channel



① : We sub-divide the non-ideal channel into many sub-channels if Δf is small the subchannels will be similar to ideal channels

② : If N big (number of sub-channels) ⇒ Every sub-channel is considered flat-fading sub-channel ⇒ Flat-fading: Freq. response quite constante

↳ Ideal channel: { Constant Amplitude
Linear Phase ⇒ Group delay constant

③ : Our sub-channels are quasi-ideal ⇒ Easier equalization (in OFDM it can be done in freq. domain)

↳ Equalization: It's the produce that tries to transform a non-ideal channel into an ideal channel

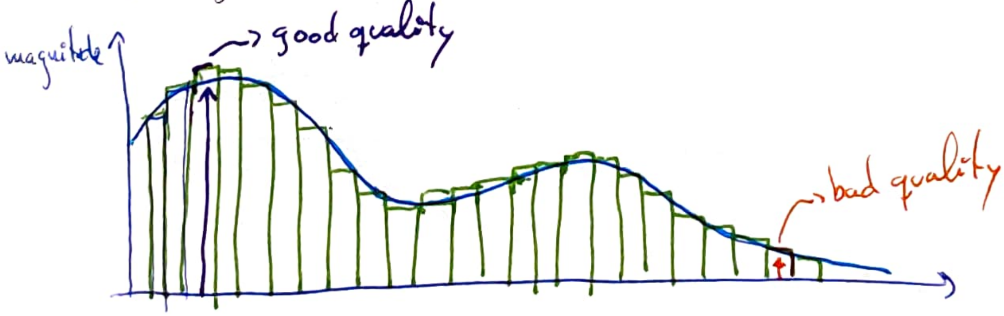
④ : Being that all of the sub-channels are quasi-ideal ⇒ Easier to get close to Shannon Limits

⑤ : For each sub-channel I will use a different carrier (Multi carrier mod.)
Instead of having one serial transmission, we have many parallel transmissions

⑥ : ↳After applying the division I can use different modulation for <u>each</u> sub-channel
i.e. QAM, PSK, Binary Antipolar Modulation (BAM)

↳ This flexibility is very strong because we can use a multilevel modulation for good quality channels (great capacity → send many bits i.e. 1024-QAM) and for the poor quality channels (not able to send many bits) we can use a simpler modulation (i.e. BAM).

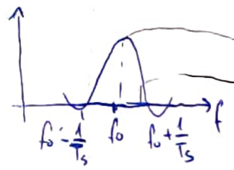
↳ This can be done dinamically, we can change the type of modulation for each sub-channel depending on the conditions of the channel (i.e. if it is rainning or a sunny day)



Idea of Modulation

* OFDM is an evolution of the linear modulation ⇒ Amplitude modulation
  Each carrier will be modulated using QAM → Depending on quality of channel (BAM or 16-QAM ...)

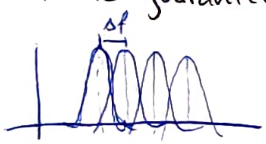
  ↳ Small remember QAM
  - Start: basic PAM → spectrum centered around zero
  - Then: Multiply by (cosine) carrier → shift the spectrum around the freq. of the carrier



The spectrum is not very compact
→ Here is good
→ Here has very small signal
The freqs. are not used very well

* In the case of OFDM we have thousands of different spectrums
  ⇒ Spectrum is very compact, very good use of the frequencies

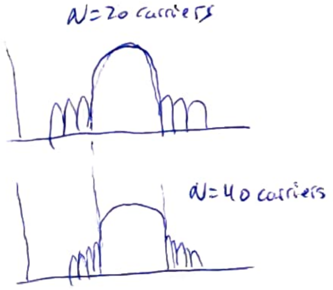
* Each channel's spectrum interferes with the others ⇒ It could create problems
  We need orthogonality to guarantee no interferences between channels
  ⇒ To guarantee orthogonality between carriers, the distance (in freq): $\Delta f = \frac{1}{T_s}$ → symbol time



→ The max of one sinc overlap with the zero of the neighbour sinc

↳ Take into account that in the time domain the signals are mixed. At the receiver due to orthogonality principle I split the signal into the basic components ⇒ I get the original information

* About spectrum:
  → If I increase the number of carriers ⇒ I increase the freq. of oscillations ⇒ Amplitude <u>is not</u> reduced
  → We cannot change the shape of the spectrum increasing the number of carriers



* Problem: OFDM is a linear modulation
  If the system <u>is not</u> linear ⇒ Orthogonality will be destroyed ⇒ Spectrum will increase (BW is increased)

Transmitter Block (Basic)

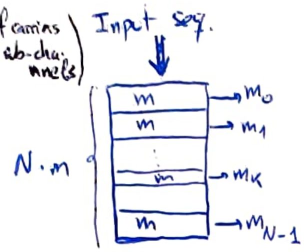
① We start from an input sequence, we want to transmit a set of bits.

We fill a memory that is organized in blocks

$N \rightarrow$ number of sequences (Number of carriers / Number of sub-channels)

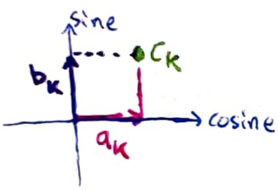
$m \rightarrow$ bits per sequence (symbol)

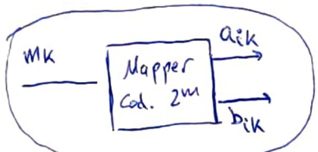
$N \cdot m \rightarrow$ number of bits



② Any m_K go to a mapper where a complex number c_K is produced.

$c_K = a_K + j\,b_K$



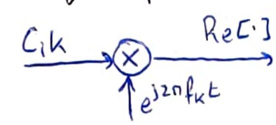


no complex envelope notation

For the block diagram we use the complex envelope notation because after OFDM uses DFT and the complex notation is required.

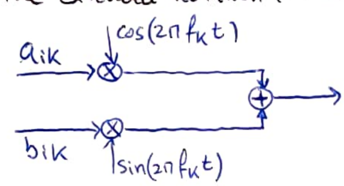
③ Now, using the obtained complex number and taking into account the freq. of the carrier f_K I generate the modulated signal.

∘) Using complex envelope notation



I have to take the real part

∘) The extended notation (to understand what is happening)



It is the exact same thing as before.

⊙ The real part of c_{iK} will modulate the cosine

⊙ The imaginary part of c_{iK} will modulate the cosine.

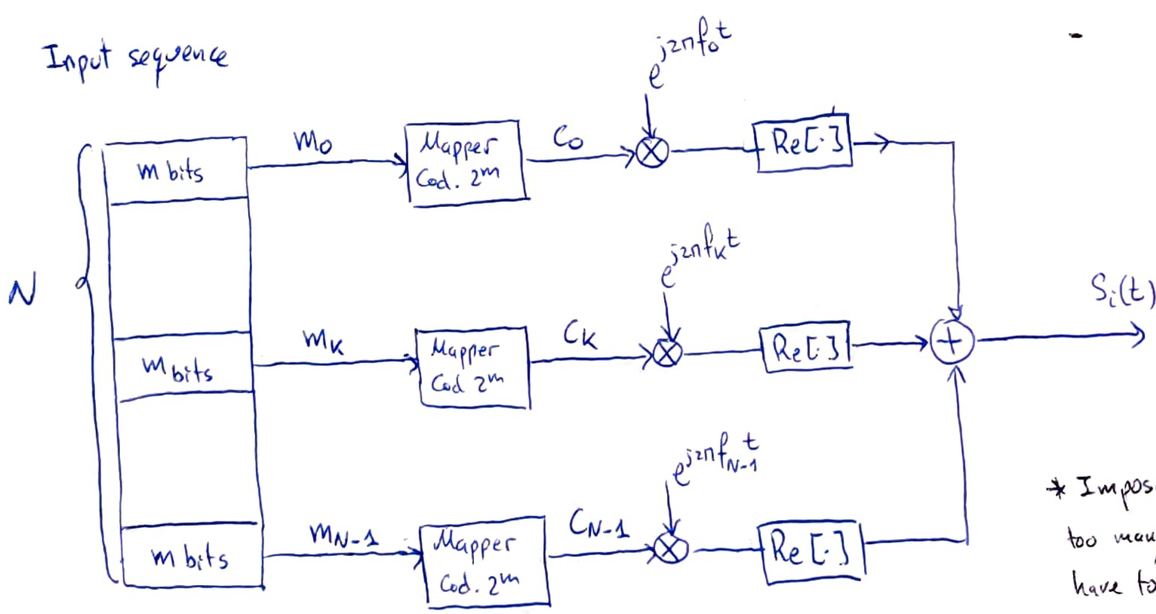
④ Bandwidth $B_T \triangleq (N+1)\cdot\frac{1}{T_S}$

∘) $T_S = N\,T \rightarrow T = m\,T_b$: $\begin{cases} T_S \rightarrow \text{symbol time} \\ T_b \rightarrow \text{basic time related to 1-bit} \end{cases}$

$\hookrightarrow T_S = N \cdot m\, T_b$

∘) $f_i = f_{i-1} + \frac{1}{T_S}$ $\begin{cases} f_i \rightarrow \text{freq. of the carrier} \\ \frac{1}{T_S} \rightarrow \text{To assure the orthogonality the shift has to be } 1/T_S \end{cases}$

∘) Increase the number of carriers ⇒ increase BW



* Impossible to be implemented too many oscillators that have to be synchronized one with respect to the others

OFDM Signal Expression

$$S_i(t) = \mathrm{Re}\left[\sum_{k=0}^{N-1} c_{ik}\, e^{j2\pi f_k t} \cdot \mathrm{rect}\left(\frac{t-\frac{T_s}{2}-iT_s}{T_s}\right)\right]$$

•) S_i is one OFDM symbol in i-th time position

↳ The symbol is the real part of the sum along all the carriers.

↳ The symbol is multiplied by the rect. The duration of the rect is related to the symbol time
I am not transmitting from $-\infty$ to $+\infty$, I am transmitting in a time frame

•) f_k is the frequency of the carrier

$f_k = f_0 + \frac{k}{T_s}$ → with f_0 the first carrier ⟹ Start from first carrier

alternatively: $f_k = f_c + \frac{1}{T_s}\left(k - \frac{N-1}{2}\right)$ → $k = 0, 1, \dots, N-1$; → f_c is the central freq. } ⟹ Start from the middle

•) c_{ik} is a complex number that decides the in-phase and in-quadrature component of tha carrier

Analysis of a Symbol

I take for example $i=1$ ⟹ $S_1(t)$ and f_k centered in central freq. ⟹ $f_k = f_c + \frac{1}{T_s}\left(k - \frac{N-1}{2}\right)$

from $S_1(t)$ → $\underbrace{\sum_{k=0}^{N-1} c_{1k}\, e^{j2\pi \frac{kt}{T_s}}}_{\tilde{u}_1(t)} \cdot \underbrace{e^{-j2\pi t \cdot \frac{N-1}{T_s}}\, e^{j2\pi f_c t}}_{\text{independent from } k}$; $u_1(t)$

⟹ We focus on $\tilde{u}_1(t)$, is the complex envelope and it keeps the information (depends on k)

We sample $t = nT = n\frac{T_s}{N}$ ⟶ $\tilde{u}_1(nT) = \sum_{k=0}^{N-1} c_{1k}\, e^{j2\pi \frac{kn}{N}}$

Sampled complex envelope $\tilde{u}(nT)$ ⟶ Related to DFT {

DFT: $X(k) = \sum_{i=0}^{N-1} x(i)\, e^{-j2\pi ik/N}$ $\{k = 0, 1, \dots, N-1\}$

IDFT: $x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j2\pi \frac{nk}{N}}$

⟹ **Important result!** We can build up the OFDM symbol using the DFT which implementation is simpler

Real Transmitter Block

① We do the same as in the previous one to obtain the set of complex numbers c_{ik}

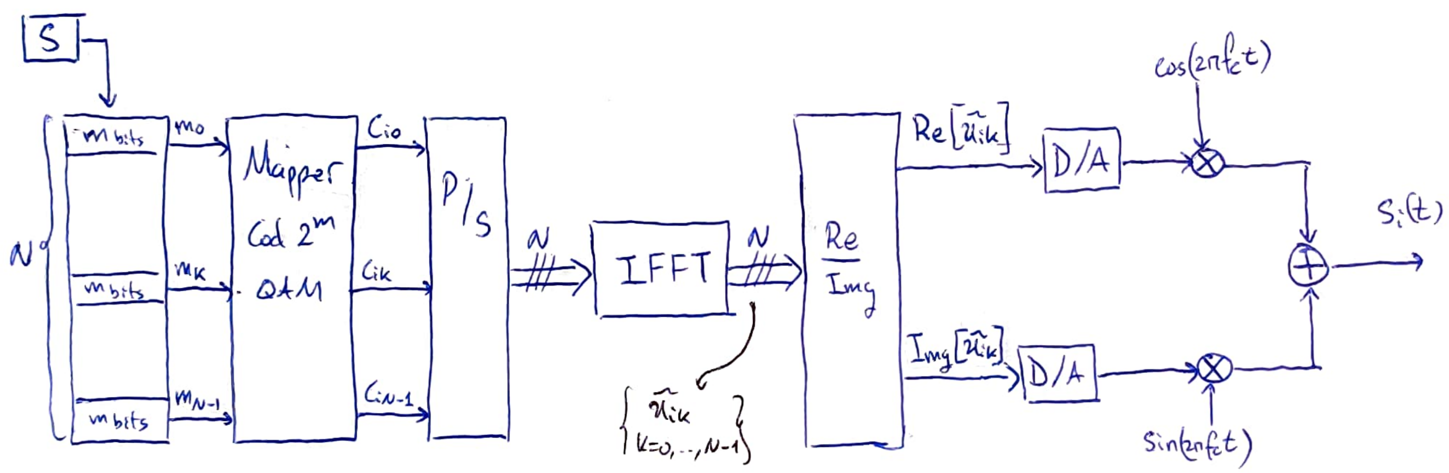
② Then, we do a $\boxed{\text{parallel-to-serial Conversion}}$ and we get a vector of N complex numbers

③ We give the N-dim vector to the IDFT using $\boxed{\text{IFFT}}$ ⇒ we get the samples of the complex envelope of the signal $\tilde{u}_{ik}$

④ We take the $Re[\tilde{u}_{ik}]$ and $Im[\tilde{u}_{ik}]$ parts of the complex envelope

⑤ We need to convert to analog using $\boxed{\text{Digital-to-Analog Converter}}$ ⇒ Do the modulation with the cosine carrier and with sine carrier.

⑥ Add both contributions and obtain $S_i(t)$

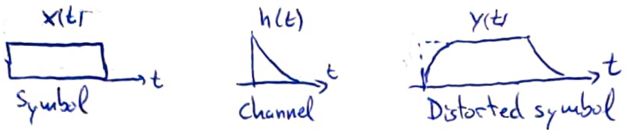


* This implementation is feasible, we only have one oscillator
Also, the FFT chip is cheap and easy to make

Interferences of a Non-Ideal Channel

* We consider that a non-ideal channel $\xrightarrow{\text{can be modelled}}$ Linear system $x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$
  - ↳ $x(t)$: input signal
  - ↳ $h(t)$: impulse response of the channel
  - ↳ $y(t)$: output signal

↳ When we transmit a symbol, the effect of the channel is the convolution between the input signal and the impulse response. Finally, because of non-ideal channel the output signal is distorted

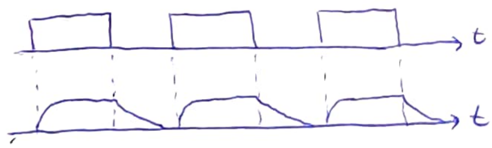


* When we send many symbols because of the channel ⇒ Intersymbol Interference (ISI)

① Solution 1: Guard Interval

↳ I know that the symbol will be enlarged by the channel ⇒ I introduce intervals where I am not transmitting anything

↳ The time I have to wait depends on the length of the channel response. ⇒ I reduce the bit rate



↳ I only remove (ISI) but the distorsion of the symbol remains ⇒ Produces Inter Carrier Interference (ICI) ⇒ When the channel is non-ideal I need an equalization strategy

Equalize the response of the channel to avoid ICI

② Regarding Equalization: Equalization in Freq. Domain

* I transmit C_{ik}, which is the useful information and the receiver have to understand C_{ik} to do the demapping
* Then, the transmission on the channel → Produces a product (convolution in time-dom) between: FT of signal × FT impulse response

So, depending on FT of channel I have a modification of the spectrum of the original signal

$$C'_{ik} = C_{ik} H_{ik}$$

→ C'_{ik}: The received

→ C_{ik}: Is the DFT of the signal. C_{ik} produces the signal making an IDFT (easier: If I give you the signal $x(n)$ and you take DFT you obtain C_{ik})

→ H_{ik}: Is the Freq. Response of the channel corresponding to k freq. Actually it is the sample of freq. response

Remember

$$C''_{ik} = \frac{C'_{ik}}{\hat{H}_{ik}}$$

→ It is an equalization carried out using DFT. Therefore, it compensates for a circular convolution

* The problem →

* **The problem:**

↳ The signal is working with Discrete FT. In the domain of DFT the product in freq. domain is not related to Linear Convolution but to Circular Convolution. Because DFT is working on periodic signals so the convolution is the circular (or periodic) convolution

↳ The channel is analog and works with the classic FT. Meaning that the channel produces a Linear Convolution between the input signal and impulse response

⇒ We have a mixing between the Lin. Conv. given by the channel and Circ. Conv. given by $C'_{ik} = C_{ik} \cdot H_{ik}$

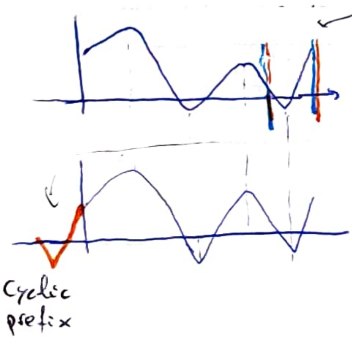
* **The solution:**

↳ Idea: To force the channel to make a circular convolution instead a linear convolution, so I can work completely with DFT

↳ The way to force the channel to produce a circular conv. ⇒ Introduce a sort of periodititization on the transmitted signal. (Cyclic Prefix)

③ Cyclic Prefix

Idea: To make a sort of periodic signal in the input of the channel



Instead of transmitting this signal, I take the last part of the signal and I repeat at the beginning. That part will be transmitted twice

This way I have produced a symbol that is periodic.

The length of cyclic prefix is the length of the channel impulse response

⇒ Drawbak: We are using the gap interval ⇒ meaning we have to give power
↳ Ineffective use of power (no useful info is transmitted)

④ Equalization Procedure

First: With the strategy to make a periodic signal ⇒ I can us DFT for { Signal and Channel } ⇒ The channel does a product with the rule of DFT

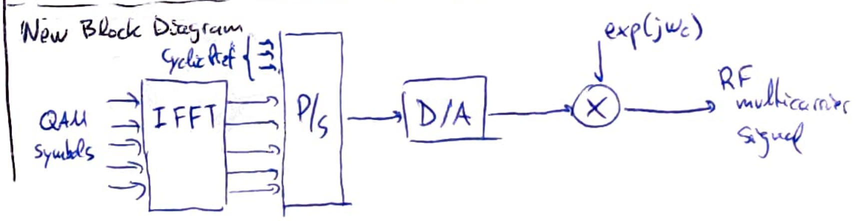
Second: If this is true ⇒ I can do equalization in freq. Domain

Equalization:

- Transmitted signal: C_{ik}
- Due to channel, received signal ⇒ $C'_{ik} = C_{ik} H_{ik}$
- At receiver, equalization ⇒ $C''_{ik} = \frac{C'_{ik}}{\hat{H}_{ik}}$ (division to try to compensate distorsion)

The problem is that I don't know the freq. response of the channel so I have to use a stimation of freq. response ⇒ $\hat{H}_{ik}$

New Block Diagram



Stimation of Frequency Response : $\hat{H}_{ik}$

* The freq. response is related to the behaviour of the channel when I transmit a sinusoidal signal. The Fourier Transform is the decomposition of the signal in many sinusoids and the freq. response say what is happening when I concentrate those sinusoids in one frequency
→ So, freq response is the response to a signal compose by 1-freq. that is the sinusoidal signal

Idea: Transmit pilot carriers

↳ Pilot carrier is a carrier that does not carry useful info, but is set to an amplitude that is known by the receiver.

* So, the receiver know that in at certain position it have to receive the specified amplitude. If it receives something different ⇒ It is due to freq. response ⇒ I can stimate the value

* I will try to reconstruct the freq. response by some sampling given by pilot carriers
The positions that the pilot carrier are change with time to be able to sample all freqs. of channel

Efficiency of OFDM

* Shanon Limit: Ideal Channel (Flat)

$$C = W \log_2\left(\frac{P+N}{N}\right)$$

↳ channel capacity (bit/sec)
W → Bandwidth
P → Signal Power
N → Noise Power

* Shanon Limit: Non-ideal Channels
↳ We make and integral ⇒ the channel was divided in small channels (OFDM)

$$C = \int_0^\infty \log_2\left[1 + \frac{S_s(f)\,C(f)}{N_0(f)}\right] df$$

OFDM Main Problems

(1) Synchronization is a critical issue.

↳ We are working with QAM (Linear Mod.) ⇒ I need a coherent receiver (freq and phase)

↳ If I am not able to make the stimation in a proper way → I will lose the orthogonallity between the carrier → I will lose information

(2) The OFDM signal is composed by addition of many independent signals ⇒

⇒ Probability Density Function (PDF) has a gaussian shape

⇒ The gaussian signal has a very large peak factor } ⇒ the gaussian variable can reach very high amplitudes

↳ peak factor: takes into account the dynamic of the signal

⇒ So because the modulation is Linear Mod. I need a linear system for this high dynamic range (amplitude). If the system is not linear → the non-linearity will mix the carrier and the info will be lost P(E)

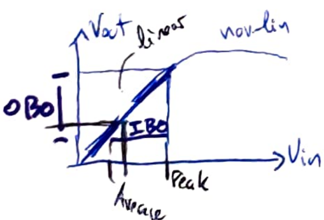
(3) The power spectrum is strongly affected by non-linearities

(4) Small problem of inefficiency due to guard interval and cyclic prefix

Waste of time and energy

* Regarding non-linearities

↳ The transmitter uses an amplifier



Input Back-Off (IBO)

Ratio between $\frac{Peak}{Average}$ in linear region

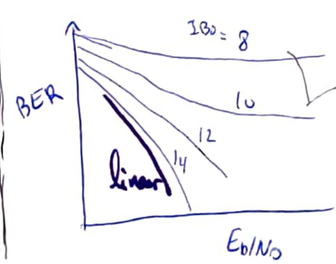
$IBO = \frac{Vin\ Peak}{Vin\ Average}$

IBO big ⇒ Very linear amplifier (good)

↳ Prob of Error P(E)

BER → bit error ratio

Eb/No → signal to noise ratio



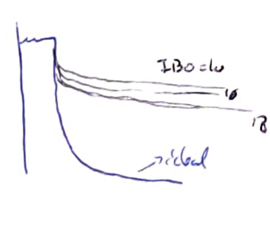
These errors aren't due to noise. It's because non-linearity so if I increase SNR the error (BER) reamains

↳ If the system is not linear we have intermodulation between carriers ⇒

⇒ Produces enlargement in the spectrum

You can go out of spectral mask

This problem is the biggest the linearity



OFDM Advantages

→ Efficient with multipath fading

→ Efficient with channel delay spread

→ Enhanced channel capacity

→ Adaptively modifies modulation density

→ Robust to narrowband interferences