Photonics

Lecture 4

Problems

on Ray Optics

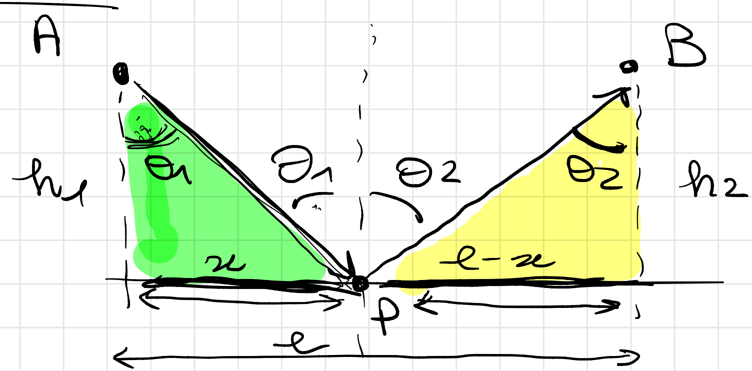
Theory

PROBLEM 1 : Demonstrate the laws of reflection and refraction using Fermat's principle.

Per Fermat's principle → OPTICAL RAYS TRAVEL BETWEEN TWO POINTS USING THE SHORTEST TIME.

IF THE MEDIUM IS HOMOGENEOUS → MINIMUM TIME = MINIMUM OPTICAL PATH

LAW OF REFLECTION



$$\overline{AP} = \sqrt{x^2 + h_1^2}$$

$$\overline{PB} = \sqrt{(\ell - x)^2 + h_2^2}$$

Total time = $\dfrac{\text{total travel space}}{\text{speed of light}}$

$$t = \frac{\sqrt{x^2 + h_1^2}}{c} + \frac{\sqrt{(\ell - x)^2 + h_2^2}}{c}$$

Minimize time by imposing $\dfrac{dt}{dx} = 0 \Rightarrow$

$$\cancel{\frac{1}{c}} \, \frac{x}{\sqrt{x^2 + h_1^2}} + \cancel{\frac{1}{c}} \, \frac{-(\ell - x)}{\sqrt{(\ell - x)^2 + h_2^2}} = 0$$

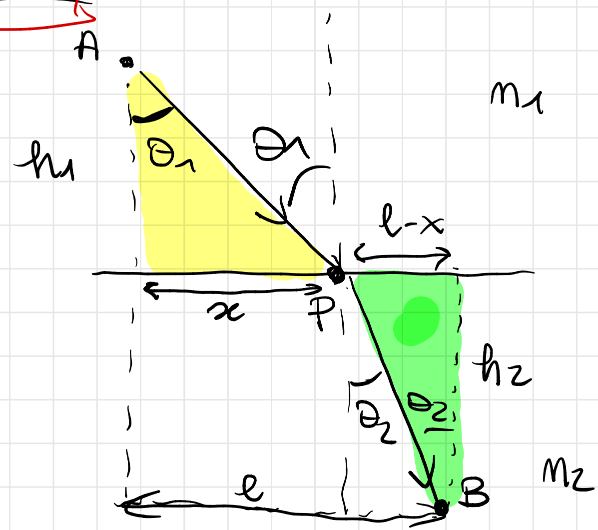
$$\Downarrow$$

$$\frac{x}{\sqrt{x^2+h_1^2}} = \frac{l-x}{\sqrt{(l-x)^2+h_2^2}}$$

$$\sin\theta_1 = \sin\theta_2$$

$$\Downarrow$$

$$\boxed{\theta_1 = \theta_2}$$



LAW OF REFRACTION

$$\overline{AP} = \sqrt{x^2+h_1^2}$$

$$\overline{PB} = \sqrt{(l-x)^2+h_2^2}$$

Total time

$$t = \frac{\sqrt{x^2 + h_1^2}}{c/n_1} + \frac{\sqrt{(l-x)^2 + h_2^2}}{c/n_2}$$

$$\frac{dt}{dx} = 0 \Rightarrow \frac{n_1 x}{\cancel{c}\sqrt{x^2 + h_1^2}} + \frac{-n_2 (l-x)}{\cancel{c}\sqrt{(l-x)^2 + h_2^2}} = 0$$

$$\Downarrow$$

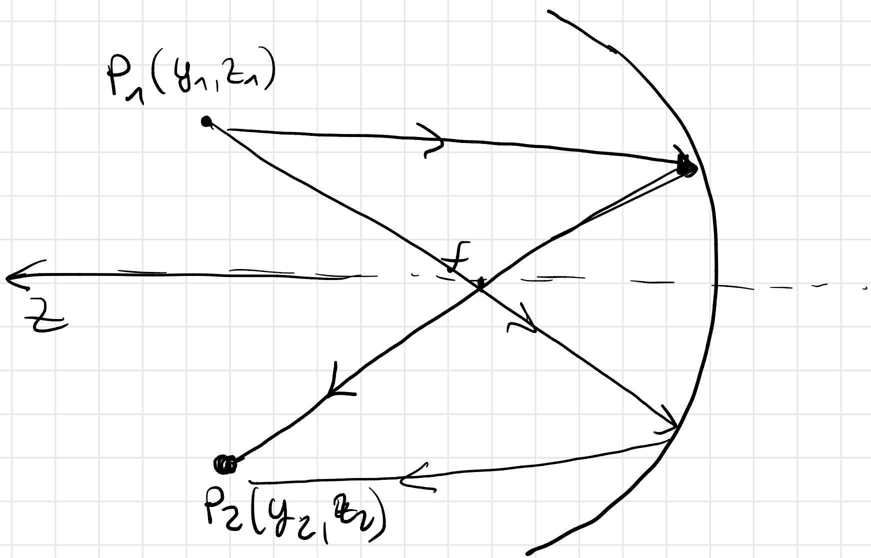
$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

PROBLEM 2

An object is placed 10 cm from a concave mirror. The focal length is 5 cm. Determine:

(a) the image distance

(b) the image magnification



$f = 5$ cm

$z_1 = 10$ cm

The imaging equation is:

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

$$\Downarrow$$

$$\frac{1}{z_2} = \frac{1}{f} - \frac{1}{z_1} = \frac{1}{5} - \frac{1}{10} = \frac{1}{10} \Rightarrow \boxed{z_2 = 10\ \text{cm}}$$

IMAGE DISTANCE

The magnification is:

$$M = -\frac{z_2}{z_1} = -\frac{10}{10} = -1$$

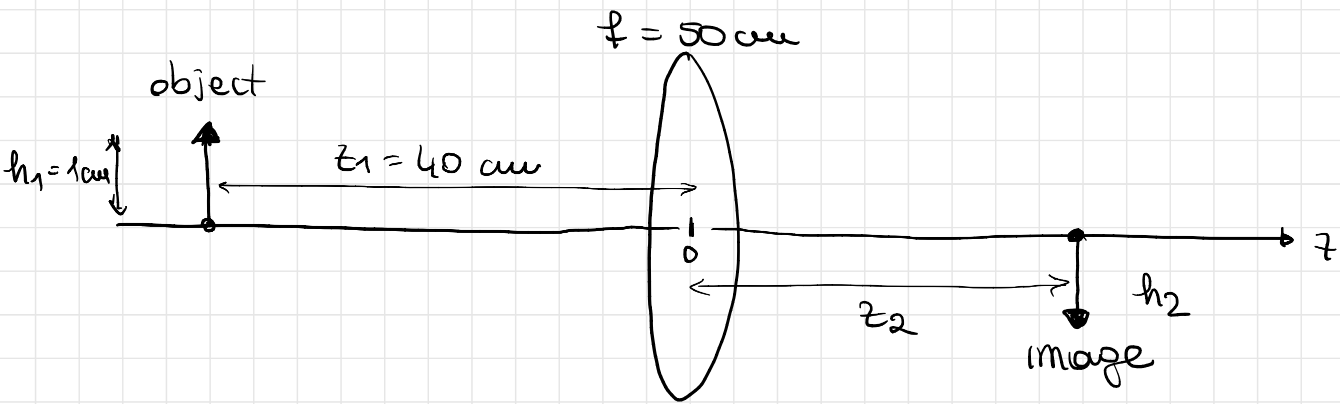
The image is INVERTED

PROBLEM 3

An object is placed 40 cm in front of a lens with focal length $f = 50$ cm.

(a) Where is the image located?

(b) If the height of the object is 1 cm, what is the height of the image?



To calculate z_2 we use the imaging equation:

$$\frac{1}{z_1}+\frac{1}{z_2}=\frac{1}{f} \Rightarrow \frac{1}{z_2}=\frac{1}{50}-\frac{1}{40}=-\frac{1}{200} \Rightarrow \boxed{z_2=-200\ \text{cm}}$$

NEGATIVE SIGN MEANS IMAGE IS ON THE <u>LEFT</u> OF THE LENS

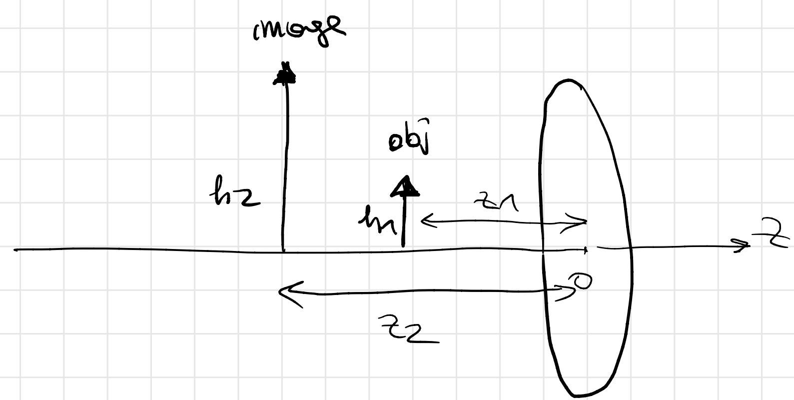
To calculate height let's calculate the magnificationfactor.

$$M = -\frac{z_2}{z_1} = -\left(\frac{-200}{40}\right) = 5$$

$M>0$ the image IS NOT inverted

$$M = \frac{h_2}{h_1} \Rightarrow$$

$$h_2 = h_1 M = 5\,\text{cm}$$



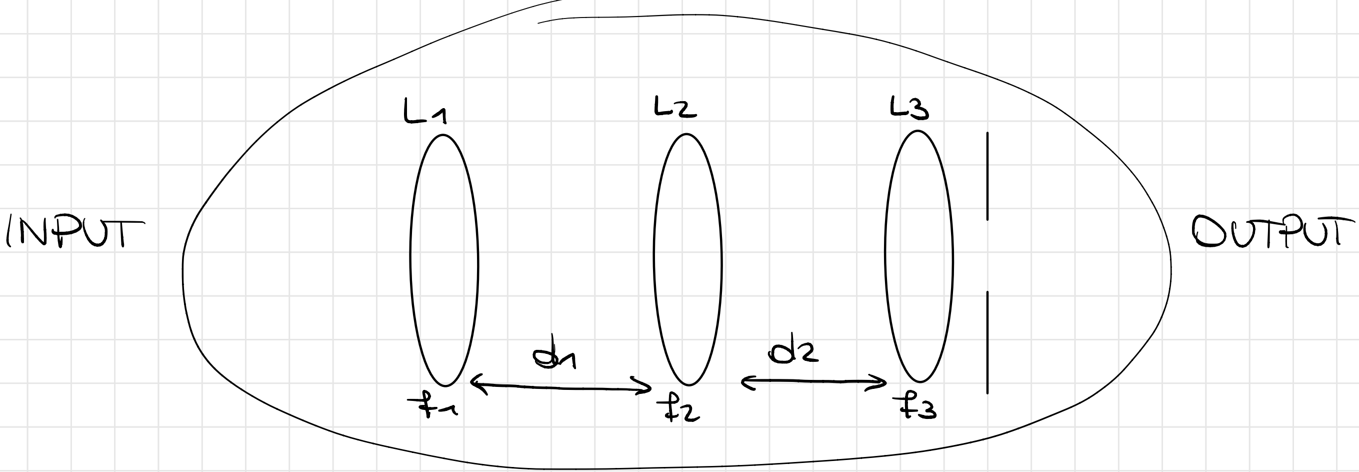
PROBLEM 4

A given optical system is made of 3 lenses and an aperture. Let's assume we want this system to be a telescope, that means that the rays that enter with a certain angle will exit the system with the same angle. Calculate:

(a) The focal length of the second lens f_2

(b) The angular magnification power of the system

Assume that the aperture does not introduce an additional matrix and that $f_1 = 4\,cm$, $f_3 = 3\,cm$, $d_1 = 1\,cm$, $d_2 = 1\,cm$.



The generic lens transfer matrix is: $$L_i = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_i} & 1 \end{pmatrix}$$

The free space propagation matrix is: $$M_i = \begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix}$$

The overall matrix for the system is:

$$M = L_3 M_2 L_2 M_1 L_1 =$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_3} & 1 \end{pmatrix} \begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 - \frac{d_1+d_2}{f_1} - \frac{d_2}{f_2} + \frac{d_1 d_2}{f_1 f_2} & d_1 + d_2 - \frac{d_1 d_2}{f_2} \\ -\frac{1}{f_1} - \frac{1}{f_2} - \frac{1}{f_3} + \frac{d_1}{f_1 f_2} + \frac{d_2}{f_2 f_3} + \frac{d_1+d_2}{f_1 f_3} - \frac{d_1 d_2}{f_1 f_2 f_3} & 1 - \frac{d_1}{f_2} - \frac{d_1+d_2}{f_3} + \frac{d_1 d_2}{f_2 f_3} \end{pmatrix}$$

$$\begin{pmatrix} y_{OUT} \\ \theta_{OUT} \end{pmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{pmatrix} y_{IN} \\ \theta_{IN} \end{pmatrix}$$

$$\underline{\theta_{OUT}} = \underbrace{\cancel{C\,y_{IN}}}_{=0} + \underline{D\theta_{IN}}$$

$C = 0 \Rightarrow$ Telescope behavior

Solving for $C=0$ we get:

$$-d_1 d_2 + d_1 f_3 + d_2 f_1 + (d_1 + d_2)\underline{f_2} - f_1 \underline{f_2} - \underline{f_2} f_3 - f_3 f_1 = 0$$

$$f_2 = \frac{d_1 d_2 + f_1 f_3 - d_1 f_3 - d_2 f_1}{d_1 + d_2 - f_1 - f_3} = \frac{1 + 12 - 3 - 4}{1 + 1 - 4 - 3} = -\frac{6}{5} \text{ cm}$$

$f_1 = 4 \text{ cm}$

$f_3 = 3 \text{ cm}$

$d_1 = d_2 = 1 \text{ cm}$

The angular magnification power is

$$M_\theta = \frac{\theta_{OUT}}{\theta_{IN}} = D = 1 - \frac{d_1}{f_2} - \frac{d_1 + d_2}{f_3} + \frac{d_1 d_2}{f_2 f_3} =$$

$$= 1 + \frac{5}{6} - \frac{2}{3} - \frac{5}{18} = \frac{18 + 15 - 12 - 5}{18} = \frac{16}{18} = \frac{8}{9}$$