Introduction

- Electromagnetic spectrum for remote sensing:
  - Microwave: P, L, S, C, X, K
  - Infrared (IR):
    - Visible light: 380 - 750 nm
    - Near Infrared: 0,75 - 8 μm or (0,75 - 3) μm
    - Thermal Infrared: 8 - 15 μm
    - Far Infrared: 15 μm - 1 mm

- Transparency of Earth's atmosphere as a function of λ
  - Three transparent windows, the rest opaque
    - Visible and near infrared: (VNIR): $\lambda = 0{,}38 - 3\ \mu m$
    - Thermal infrared radiation (TIR): $\lambda = 8 - 15\ \mu m$
    - Microwave region (μW): $\lambda = 1\ mm - 1\ m$

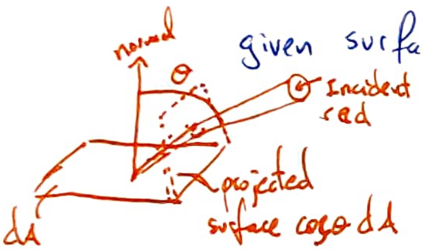
Natural EM Radiation

radiant energy: Energy of electromagnetic or gravitational radiation [J]

radiant flux (or power): radiant energy emitted, reflected, transmitted or received per unit time [J/s] = [W]

⊛ Radiation Quantities

↳ **Radiance:** Is the radiant ~~flux~~ power emitted, reflected, transmitted or received (incident) by a given surface per unit solid angle per unit projected area

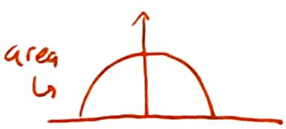


→ Power incident: $dP = L \cos\theta \, dA \, d\Omega$

→ Solid angle: $d\Omega = \sin\theta \, d\theta \, d\varphi$

→ Radiance: $\boxed{L = \dfrac{dP}{\cos\theta \, dA \, d\Omega} \quad [W \, m^{-2} \, sr^{-1}]}$ $\rightsquigarrow dP(\theta,\varphi) \rightsquigarrow L(\theta,\varphi)$

↳ **Irradiance:** Is the total incident power per unit area



$$\boxed{E = \int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{2\pi} L_{in} \cos\theta \, d\Omega \quad [W.m^{-2}]}$$

↳ **Radiant Excitance:** Is the total emitted power per unit area

$$\boxed{M = \int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{2\pi} L_{out} \cos\theta \, d\Omega \quad [W.m^{-2}]}$$

•) Isotropic radiation: Radiance (L) is independent of the direction, i.e it's constant $L = L_0$

Therefore, radiant existance:

$$M = L_0 \int_0^{\pi/2} \int_0^{2\pi} \cos\theta \, d\Omega = L_0 \int_0^{\pi/2} \int_0^{2\pi} \cos\theta \sin\theta \, d\theta \, d\varphi = \pi L_0 \Rightarrow \boxed{M = \pi L_0}$$

⊛ Thermal Radiation

→ Thermal radiation is emitted by all the objects above 0K

→ Is the radiation detected by majority of passive remote sensing systems

↳ Spectral radiance: L_λ or L_f

Is the radiance contained in a small range of wavelengths (freqs) $\Delta\lambda$ (Δf)

$$\boxed{\Delta L = L_\lambda \, \Delta\lambda} \qquad \boxed{\Delta L = L_f \, \Delta f}$$

$L_\lambda = W m^{-3} sr^{-1}$ $\qquad$ $L_f = W m^{-2} Hz^{-1} sr^{-1}$

⇓ interval λ_1, λ_2 $\qquad$ ⇓

$$\boxed{L = \int_{\lambda_1}^{\lambda_2} L_\lambda \, d\lambda} \qquad \boxed{L = \int_{f_2}^{f_1} L_f \, df}$$

$\longrightarrow$ $\lambda = \frac{c}{f}$

$$L_\lambda \, d\lambda = -L_\lambda \left(-\frac{c}{f^2}\right) df = L_f \, df$$

↳ $\boxed{\frac{L_\lambda}{L_f} = \frac{f^2}{c} = \frac{c}{\lambda^2}}$

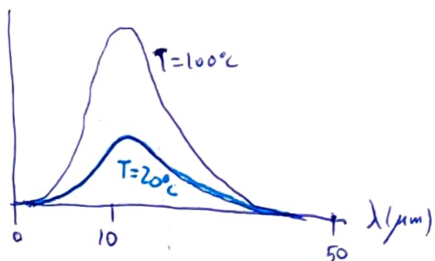
⊛ Black Body

→ Object that absorb all the radiation incident on it.

→ Spectral radiance does not depend on θ, φ but it depends on wavelength λ

→ Planck formula

$$\boxed{L_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left[\frac{hc}{\lambda k_B T}\right] - 1}} \Rightarrow L_f = L_\lambda \, (c/f)$$



⊛ Increasing T shifts to a shorter wavelengths

→ At long $\lambda: \to \infty$ $\Rightarrow$ $\lim_{\lambda \to \infty} \boxed{L_\lambda \simeq \frac{2 k_B T c}{\lambda^4}}$ Rayleigh-Jeans approx.

→ Integrating total radiance of Black-body $\longrightarrow$ $L = \int_0^\infty L_\lambda \, d\lambda = \frac{2\pi^4 k_B^4}{15 c^2 h^3} T^4$

⇒ Total radiant exitance B-B $\xrightarrow[\text{radiation from B-B is isotropic}]{}$ $M = \pi L \Rightarrow \boxed{M = \sigma T^4}$ Stefan's Law

→ Wavelength at which spectral radiance is maximum

$\max(L_\lambda) \Rightarrow \frac{dL_\lambda}{d\lambda} = 0 \Rightarrow \boxed{\lambda_{max} = \frac{A}{T}}$ Wien's displacement Law

→ Black Body model is useful bc any radiance from a body can be modeled as B-B with correction factor: emissivity ε

$\varepsilon(\lambda) \longrightarrow \boxed{L_{\lambda,\varepsilon} = \varepsilon(\lambda) L_\lambda}$

→ Brightnes temperature : T_b

Is the temperature of the equivalent B-B would give the same radiance @ a λ

$\varepsilon L_\lambda(\lambda,T) = L_\lambda(\lambda, T_b) \underset{\lambda\to\infty}{\Longrightarrow} \boxed{T_b = \varepsilon T}$

⊛ Solar Radiation

$\underset{Wm^{-2}}{M} = \sigma T^4$

$\underset{W}{P} = 4\pi r^2 M$ power radiated

$\underset{Wm^{-2}}{E} = \frac{P}{4\pi D^2}$ sun's irradiance
$D \rightarrow$ Earth–Sun dist

$\underset{sr}{\Delta\Omega} = \frac{\pi r^2}{D^2}$

$\underset{Wm^{-2}sr^{-1}}{L} = \frac{E}{\Delta\Omega} = \frac{\sigma T^4}{\pi}$

⊛ Absorption in the atmosphere

→ $\varepsilon_r = \varepsilon' - j\varepsilon'' = \varepsilon'(1 - j\tan\delta)$

→ Intensity decays exponentially: $I(z) = I_0 e^{-z/L_a}$

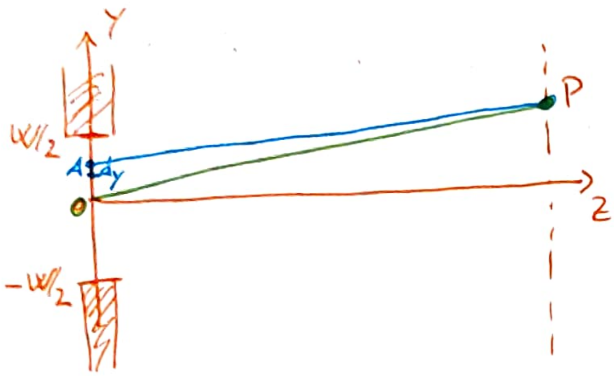
→ Absorption length $L_a = \frac{\lambda}{4\pi K}$

If $\gamma_a = \frac{1}{L_a} \Rightarrow I(z) = I_0 e^{-\tau}$

$\tau = \int_0^z \gamma_a(z')\,dz'$

optical thickness

⊛ Diffraction



↳ Contribution on P of an element dy is proportional to $\propto \exp(jky\sin\theta)\,dy$

↳ All the contributions:

$$a(\theta) = \int_{-W/2}^{W/2} \exp(jky\sin\theta)\,dy$$

↳ In general there is a transmittance function f(y) ande we get a fourier transf.

$$\boxed{a(\theta) = \int_{-\infty}^{\infty} f(y)\, e^{jky\sin\theta}\, dy}$$

Conjugated magnitudes y , $\frac{k\sin\theta}{2\pi}$

⊛ Slit case:

$$f(y) = \text{rect}\left(\frac{y}{W}\right) \Rightarrow a(\theta) = W \,\text{sinc}\left(\frac{Wk\sin\theta}{2\pi}\right)$$

↳ Direction of two zeros of main peak

$$\boxed{\sin\theta = \pm\frac{\lambda}{W} \underset{\text{if } W \gg \lambda}{\Longrightarrow} \theta \simeq \pm\frac{\lambda}{W}}$$

⊛ Diffraction for circular aperture

↳ amplitude is proportional to first-order Bessel function

↳ The first zero: $\boxed{\theta_r = 1{,}22\,\frac{\lambda}{D}}$ D → diameter of aperture

↳ Approximation valid if: $\frac{W^2}{8z} < \frac{\lambda}{4}$

↳ Accurate if: $z > \frac{W^2}{2\lambda} = z_F$ → Fresnel distance

⇒ Diffraction puts a limit in the resolution and it depends on the ratio between λ and D

Aerial Photography

⊛ Context

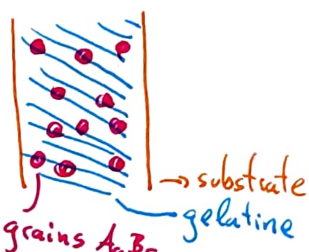
→ Photographic films sensitive to visible and near infrared.

→ Aerial photgraphy is a passive technique ⇒ It detects existing radiation and give 2D representation

Film-based preferred over digital due to better resolution

→ Traditional process: chemical reaction

① Film made of many crystals of a salt embedded in gelatin with plastic base
② Absorption of sufficient energetic photon converts the grain into metallic silver
③ Unexposed grains are removed by chemical process
④ The result: Negative ⇒ Bc areas that received light during exposure stage will appear dark
↳ Visible photography: 0,3 - 0,7 μm
↳ True infrared photography: 0,7 - 0,9 μm

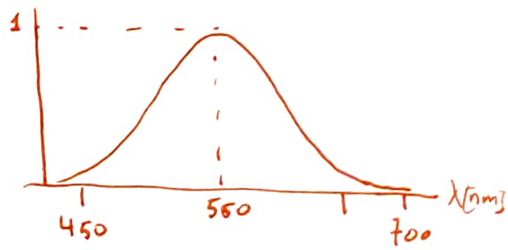


→ Two Limits
↳ Diffraction: I can resolve two points if the angular separation is greater than $1{,}22 \frac{\lambda}{D}$
↳ Quality of material: Size of grain, distribution of the grains in the gelatine.

→ Response of photographic film.
↳ In terms of photometric units: weighted with nominal spectral sensitivity of human eye
↳ Photometric unit corresponding to irradiance is → illuminance

$$E_v = K \int_0^\infty E_\lambda V(\lambda)\, d\lambda$$

illuminance (E_v)
spectral irradiance (E_λ)
$V(\lambda)$ → nominal sensitivity of the light adapted to human eye
Max at green



⊛ Speed of a film

→ It is the time duration a film has to be exposed to light of a given illuminance to get a significant change of opacity after processing

→ Aerial Film Speed (AFS) index is used
Larger numbers ⇒ Faster films → Shorter exposure times

→ The grain size controls the speed:
Hige speed films ⇒ large grains

⊛ Contrast of a film

→ It is the effect of changing the exposure time (or illuminance)

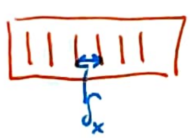
→ if Small change of illuminance ⇒ large change of opacity ⇒ film with high contrast

→ The grain size controls the contrast:
High contrast ⇒ specific range of grain size

⊛ Spatial resolution

→ The ability of a RS system to distinguish two points: line-pairs (lp)

→ lp per unit length ⇒ greatest number of lin-pairs per unith length that can be resolved

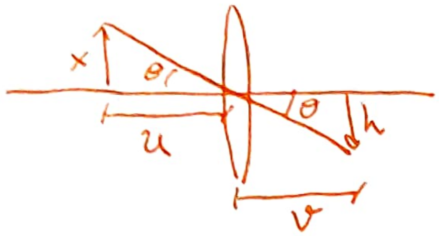


$$\boxed{\delta_x = \frac{1}{2r}}$$

δ_x → smallest distance between 2 points that can be resolved

↳ resolution [1/length] 10 lp/mm $= 10^4\ m^{-1}$

⊛ Optics of photography systems



$$\boxed{\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$$

$$\boxed{h = v \tan\theta} \xrightarrow[\text{remote sensing}]{u \to \infty} \left.\begin{array}{l} v = f \\ \tan\theta = \frac{x}{u} \end{array}\right\} \boxed{h = f\frac{x}{u} \simeq f\theta}$$

↳ What happens with radiance:

- Object with uniform exitance so radiance (luminance) incident at lens → L
- object subtends small solid angle Ω

> Irradiance at lens: $L\,\Omega$

⇒ Total power by lens diameter D: $P_{tot} = L\Omega \cdot \pi \frac{D^2}{4}$

⇒ Solid angle for object and image is the same: $\Omega = \frac{A_{image}}{f^2}$

⇒ Irradiance on the film: $E_{film} = \frac{P_{Tot}}{A_{image}} \Rightarrow E_{film} = \frac{\pi}{4} L \left(\frac{D}{f}\right)^2$

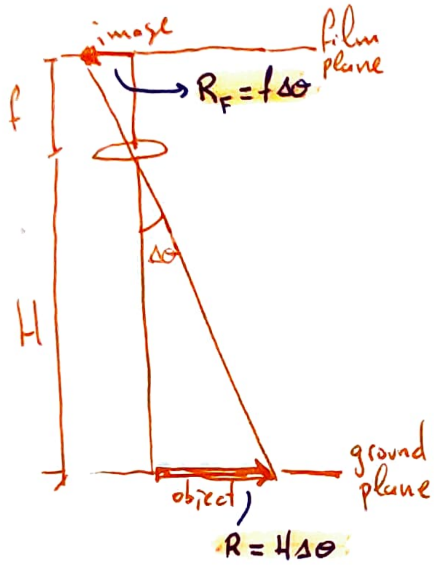
⇒ Ratio irradiance (illuminance) at film to radiance (luminance) at lens $\boxed{\frac{E_{film}}{L} = \frac{\pi}{4}\left(\frac{D}{f}\right)^2}$

↳ f/number of lens $\left(\frac{f}{D}\right)$

- the smaller f/number the larger the lens the brighter the image

⊛ Scale of vertical aerial photography

↳ Scale of the image: The ratio of the size of the representation of object on the map to the size of real object



$$\boxed{S = \frac{f}{H}}$$ $S < 1$

$$S = \frac{R_F}{R}$$

↳ Negative with width w

$$S = \frac{w}{W_g} \Rightarrow \boxed{W_g = w\,\frac{H}{f}}$$

↳ region of the ground

↳ Spatial resolution

∘) Distance on film: $\delta_x = \frac{1}{2r}$

∘) Distance on the ground: $\boxed{\delta x_g = \frac{1}{2r} \cdot \frac{H}{f}}$

$$S = \frac{\delta x}{\delta x_g}$$

Electro-Optical Systems

⊛ **Detectors for visible and near infrared**

- Minimum photon energy ⇒ max photon λ depend on semiconductor

↳ Photodiode

Silicon (Si) → up to 1,1 μm

Germanium (Ge) → up to 1,7 μm

Lead sulfide (PbS) → up to 3 μm

Indium Antimonide (InSb) → up to 5 μm

Mercury Cadmium Telluride (MCT) → up to 15 μm

Mercury doped Germanium (Ge:Hg) → up to 15 μm

↳ Thermal detectors

- Wider spectral range ⇒ when $\lambda > 15\,\mu m$
- Lower sensitivity

↳ Charge Coupled Devices (CCDs)

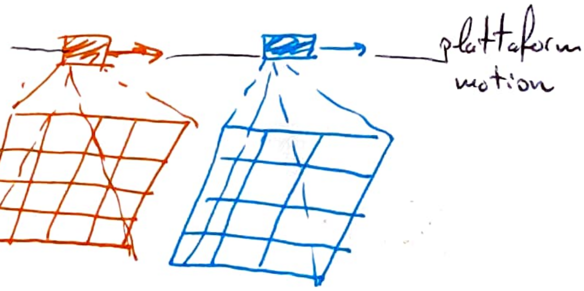
Si → 1,1 μm

⊛ The electro-optical system can be in motion respect to the target so it may be necessary to compensate for the movements to avoid blurring the image.

⊛ **Step-Stare Imaging**

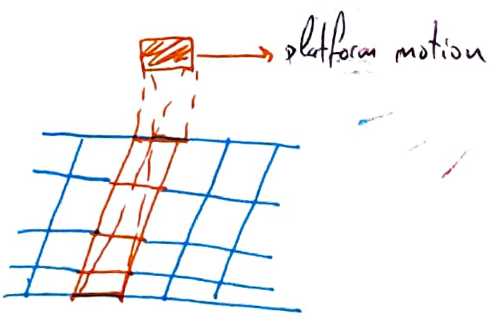
The detector array stares at a scene and then moves on to stare at next scene

We get a strip picture (franja)

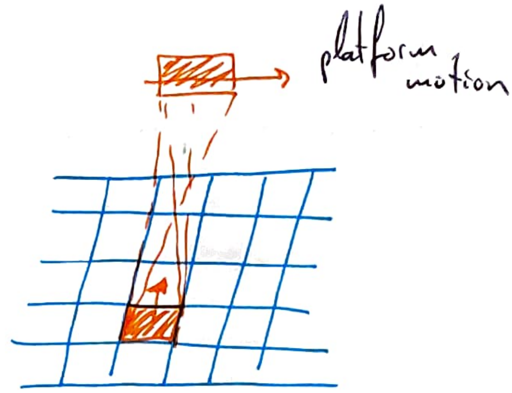


⊛ **Push-Broom Imaging**

If the detector is a linear array and we synchronise the speed of the plane and the detector we get the full picture



⊛ Whisk-Broom Imaging

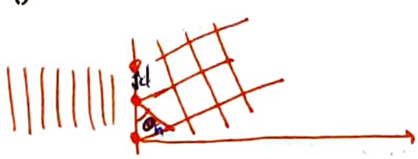


There is a single detector scanning in the direction perpendicular to the motion of platform.
We have to synchronize two speeds, the one of the flight and the scanner.

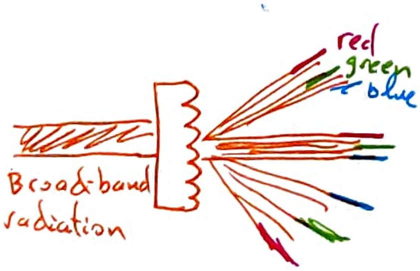
⊛ Spin-Scan Imaging

⊛ Diffraction Gratting

It is difficult to scan a wide spectrum so the detectors are often optimized for a specific bandwidth. Using a diffaction gratting we can focus on a BW



$$\boxed{\sin \theta_n = n \frac{\lambda}{d}}$$



Satellites

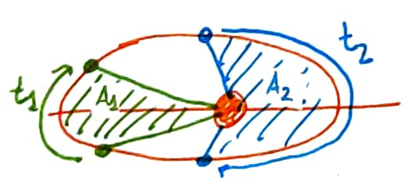
Atmosphere Layers

- Troposphere (0-12 km) ⟶ commercial aircrafts (~10km)
  (more gass density ⇒ we can fly)
- Stratosphere (12-50 km)
- Mesosphere (50-80 km)
- Thermosphere (80-700 km) ⟶ satellites (~400 km)
  (low gass density ⇒ low friction)
  (too much friction ⇒ satellite fall)
- Exosphere (700-10 000 km)

Kepler's Laws

1. Planets move in a plane. Elliptical orbits being the Sun at one focus

2. Law of areas



$$\boxed{A_1 t_1 = A_2 t_2}$$

A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.

3.

$$\boxed{\frac{T^2}{a^3} = \text{Constant}}$$

$T \rightarrow$ period of revolution of a planet around the Sun

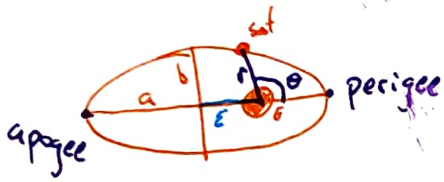
$a \rightarrow$ semi-major axis

Law of Gravitation

$$\boxed{\vec{F} = -G \frac{Mm}{r^2}\hat{r}}$$

- Earth radious: $R = 6371$ km
- Earth mass: $M = 5974 \cdot 10^{24}$ kg
- Gravitational constant: $G = 6{,}672 \cdot 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$
- $\mu = GM$

Elliptical orbit



- Distance of the satellite
  - respect major axis
  - from perigee

$$r = \frac{a(1-\varepsilon^2)}{1+\varepsilon\cos\theta}$$

$$\varepsilon = \sqrt{1-\frac{b^2}{a^2}}$$

- Satellite velocity

$$v = \sqrt{\mu\left(\frac{2}{r}-\frac{1}{a}\right)}$$

- Period of satellite motion

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

→ $v_{perigee} > v_{apogee}$ ⇒ 2nd Kepler Law

Circular orbit

$$F_c = \frac{m v^2}{r}$$

$$F_G = G\frac{Mm}{r^2}$$

- Velocity: Balance of forces

$$\boxed{v = \sqrt{\frac{gR^2}{R+h}}}$$

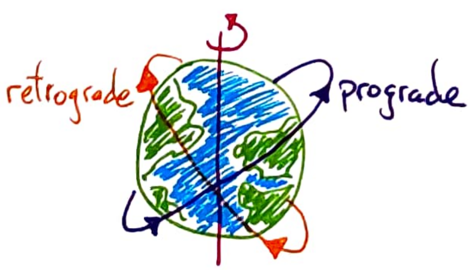
$r = R + h$

$$g = \frac{GM}{R^2}$$

- Period of satellite
  - Time of one round: T

$$v = \frac{\text{distance}}{\text{time}} \Rightarrow T = \frac{2\pi r}{v} \rightarrow \boxed{T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}}$$

Prograde and Retrograde orbits

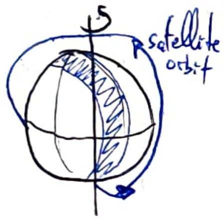


Useful Orbits

- Sub-satellite position: The point on the Earth's surface which is directly below the satellite.

Polar orbit

The satellite orbits the Earth in such a way to cover the north and the south polar regions.



- Usages: Earth-mapping
  Weather satellites

Ground-track

It is the path on the surface of a planet directly below a satellite trajectory

Geostationary satellite

It is a satellite in a circular orbit above the equator. The period is equal to a Earth's rotational period, one sideral day: $\boxed{86164 \text{ seconds} = T}$

It is also referred as geosynchronous equatorial orbit (GEO)

It has a fixed position in the sky so its ground-track is just a fixed point on the Earth surface

Geosynchronous satellite

It is a satellite with a period of a sideral day but it rotates in any axis, Not necessarily the same rotation axis of the Earth

Sub-satellite path traces a lemniscate 8 8 ....

⊛ Addional things

- Velocity of the lowest altitude $h=0$

- Escape velocity: It is the minimum speed needed to stop escape from the gravitational influence of a planet.

↳ Energy balance

$$\left.\begin{array}{l} E_c = \frac{1}{2} m v^2 \\ E_p = \frac{-GMm}{r} \end{array}\right\} \quad U_{in} = U_{fin}$$

$$E_c + E_p = 0 \Rightarrow \frac{1}{2}\cancel{m} v^2 = \frac{GM\cancel{m}}{r} \qquad v = \sqrt{\frac{2GM}{r}}$$