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Exam correction
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Consider the waveguide presented figure 1. The material embedded in this support is characterized by its relative permeability $\mu_r = 1$ and its relative permittivity $\varepsilon_r = 1$. It is bounded by perfect magnetic walls (PMW), placed in x=0, x=a, and by perfect electric walls (PEW), placed at y=0 and y=b. We will work in the frequency domain.

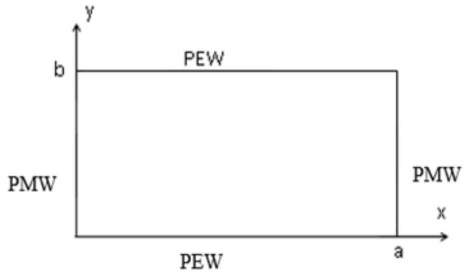


Figure 1

1) TEM Mode

We give for this TEM mode :

$$\overrightarrow{H(x,y)} = \frac{V}{bZ_0}\overrightarrow{e_x} \qquad \overrightarrow{E(x,y)} = -\frac{V}{b}\overrightarrow{e_y}$$

where : Z_0 is the plane wave impedance

V is the voltage on the conductor placed in the plane y=b (the other conductor is grounded)

Compute the expression of the characteristic impedance Z_c of this line.

1) $\vec{J_s} = \vec{u} \times \vec{H}$

on $y = b$ conductor, $\vec{J_s} = -\vec{e_y} \times \frac{V}{bZ_0}\vec{e_x} = \frac{V}{bZ_0}\vec{u}$

$$I = \int_0^a \vec{J_s}\cdot\vec{u}\,dl = \int_0^a \frac{V}{bZ_0}\,dx = \frac{a}{b}\frac{V}{Z_0}$$

$$Z_c = \frac{V}{I} = \frac{bZ_0}{a}$$

2) TM Mode

This waveguide is now excited on TM modes. From the propagation equation, considering the waveguide lossless :

a) compute the expressions of the first TM mode cutoff frequency and the E_z component of this mode.

$$\begin{cases} \Delta_t E_z(x,y) + k_c^2 E_z(x,y) = 0 \\ E_z(x,0) = E_z(x,b) = 0 \\ \dfrac{\partial E_z(x,y)}{\partial x} = 0 \text{ on } x = 0 \text{ and } x = a \end{cases}$$

$\rightarrow$ $E_z(x,y) = E_0 \cos\frac{m\pi}{a}x \, \sin\frac{n\pi}{b}y$

$$f_{c\,TM_{mn}} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

First TM mode : TM_{01}

$E_z(x,y) = E_0 \sin\frac{\pi}{b}y$ for this mode

b) From the following expressions, compute all the E and H field components of the first TM mode.

$$\left(\gamma^2+k_o^2\right)\overline{E_t}(\xi,\eta)=-\gamma\overline{\nabla_t}E_z(\xi,\eta)+j\omega\mu\overline{u}\wedge\overline{\nabla_t}H_z(\xi,\eta)$$

$$\left(\gamma^2+k_o^2\right)\overline{H_t}(\xi,\eta)=-\gamma\overline{\nabla_t}H_z(\xi,\eta)-j\omega\varepsilon\overline{u}\wedge\overline{\nabla_t}E_z(\xi,\eta)$$

With $\vec{u}$ the unitary vector in the z direction.

$E_z(x,y) = E_0 \sin\frac{\pi}{b}y$ $\quad H_z(x,y)=0$ $\quad k_c=\frac{\pi}{b}$

$$k_c^2\left(E_x(x,y)\vec{e_x} + E_y(x,y)\vec{e_y}\right) = -j\beta\left(\cancel{\frac{\partial E_z(x,y)}{\partial x}\vec{e_x}} + \frac{\partial E_z(x,y)}{\partial y}\vec{e_y}\right)$$

$$E_y(x,y) = -j\frac{\beta E_0}{k_c}\cos\frac{\pi}{b}y \qquad E_x(x,y)=0$$

$$k_c^2\left(H_x(x,y)\vec{e_x}+H_y(x,y)\vec{e_y}\right) = -j\omega\varepsilon\,\vec{u}\times\left(\cancel{\frac{\partial E_z(x,y)}{\partial x}\vec{e_x}}+\frac{\partial E_z(x,y)}{\partial y}\vec{e_y}\right)$$

$$H_x(x,y) = j\omega\varepsilon\frac{E_0}{k_c}\cos\frac{\pi}{b}y \qquad H_y(x,y)=0$$

c) A surface impedance condition is now imposed in the plane z=0, at the end of the waveguide which length is L(figure 2). In this plane, the following relation between the tangential total H and E fields is defined as :

$$\overrightarrow{H_t}=\frac{1}{Z_s}(\vec{n}\times\overrightarrow{E_t})$$

With Z_s the surface impedance, $\vec{n}$ the unitary vector normal to the plane z=0, directed toward the waveguide.

$E_z^+(x, y, z) = E_0^+f_1^+(x, y, z)$ is defined as the expression of the incident part of the total field $E_z(x, y , z)$

$E_z^-(x, y, z) = E_0^-f_1^-(x, y, z)$ is defined as the expression of the reflected part of the total field $E_z(x, y , z)$

Give $f_1^+(x, y, z)$ and $f_1^-(x, y, z)$ as a function of a, b, x, y, z, β(propagation constant).

We define the reflection coefficient by $\rho(z) = - E_z^-(x, y, z) / E_z^+(x, y , z)$

Give $\rho(0)$ as a function of Z_s and $Z_{TM}= \beta/\omega\varepsilon$, and then $\rho(L)$

v

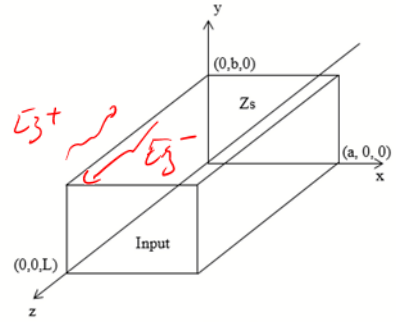


Figure 2

$* z=0$ $\quad f_1^+(x,y,z)=\sin\frac{\pi}{b}y\,e^{+j\beta z}$ $\quad f_1^-(x,y,z)=\sin\frac{\pi}{b}y\,e^{-j\beta z}$

$E_z^+(x,y,z)=E_0^+\sin\frac{\pi}{b}y\,e^{+j\beta z}$ $\quad E_z^-(x,y,z)=E_0^-\sin\frac{\pi}{b}y\,e^{-j\beta z}$

$\rho(0)=\frac{-E_0^-}{E_0^+}$ $\qquad \vec{H_t}=\frac{1}{Z_s}(\vec{n}\times\vec{E_t})$

$$\Rightarrow H_x(x,y)\vec{e_x}=\frac{1}{Z_s}\left(\vec{u}\times E_y(x,y)\vec{e_y}\right)$$

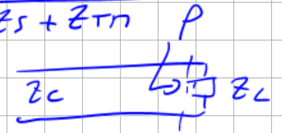
$$\Rightarrow \frac{j\omega\varepsilon}{\cancel{k_c}}\cancel{\cos\frac{\pi}{b}y}\left(E_0^++E_0^-\right) = \frac{1}{Z_s}\frac{\cancel{j}\beta}{\cancel{k_c}}\cancel{\cos\frac{\pi}{b}y}\left(E_0^+-E_0^-\right)$$

$$\Rightarrow E_0^++E_0^- = \frac{\beta}{\omega\varepsilon Z_s}\left(E_0^+-E_0^-\right) = \frac{Z_{TM}}{Z_s}\left(E_0^+-E_0^-\right)$$

$$E_0^+\left(\frac{Z_s-Z_{TM}}{Z_s}\right) = -E_0^-\left(\frac{Z_s+Z_{TM}}{Z_s}\right)$$

$$E_0^-/E_0^+ = \frac{Z_{TM}-Z_s}{Z_{TM}+Z_s} \qquad \rho(0)=\frac{Z_s-Z_{TM}}{Z_s+Z_{TM}}$$

equivalent to $\rho=\frac{Z_{load}-Z_c}{Z_{load}+Z_c}$



$$\rho(L) = -\frac{E_0^-}{E_0^+}e^{-2j\beta L} = \frac{Z_s-Z_{TM}}{Z_s+Z_{TM}}e^{-2j\beta L}$$