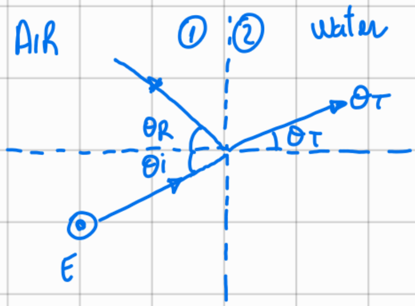
Nanophotonics Test

June 7, 2018

1)



$n_1 = 1$

$n_2 = 1.33$

$\theta_i = 60^\circ$

$$n_1 \sin\theta_i = n_2 \sin\theta_T$$

$$\sin\theta_T = \frac{1 \sin 60^\circ}{1.33} \rightarrow \boxed{\theta_T = 40.6^\circ}$$

$$\boxed{\theta_R = \theta_i = 60^\circ}$$

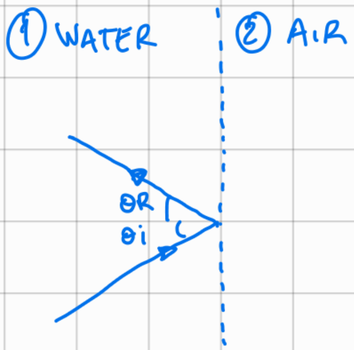
$$r_{TE} = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)} \rightarrow r_{TE} = -0.3375$$

$$\text{Reflectance} \rightarrow R = r_{TE}^2 \rightarrow \boxed{R = 11{,}39\%}$$

$$t_{TE} = 2\frac{n_1 \cos(\theta_i)}{n_2 \cos(\theta_i) + n_1 \cos(\theta_t)} = 0{,}6625$$

$$T = t_{TE}^2 \cdot \frac{n_1 \cos\theta_t}{n_2 \cos\theta_i} \Rightarrow \boxed{T = 88{,}61\%}$$

If the light impinges from the water side:



First we need to check the critical angle

$$\theta_c = a\sin\left(\frac{n_2}{n_1}\right) = a\sin\left(\frac{1}{1.33}\right) = 48{,}75^\circ < 60^\circ \Rightarrow \text{NO TRANSMISSION}$$

$$\theta_i = \theta_R = 60^\circ$$

Since $\theta_i > \theta_c \Rightarrow$ total reflection, $R=1, t=0$.

2) SCATTERING → Redirection of radiation out of the original direction due to interactions with molecules and particules.
→ Refraction, Reflection, diffraction, etc are all different forms of scattering
→ Types: elastic, inelastic, Quasi-elastic, Single and Multiple.

3) DIFFRACTION GRADING

$$\sin \theta_q = \sin \theta_i + q\frac{\lambda}{\Lambda}$$

$\theta_i = 0°$ (normal incidence)
$\theta_q = 33° \approx 0.578$ rad
$q = 2$ (diffraction order)
$\lambda = 600$ nm

Periodicity: $\Lambda = \dfrac{q\lambda}{\sin\theta_q} = \dfrac{2 \cdot 600\times10^{-9}}{\sin 33°} \rightarrow \Lambda = 2{,}2\,\mu m$

Number of Slits: $N = \dfrac{3\times10^{-2}}{2{,}2\times10^{-6}} = 14000$

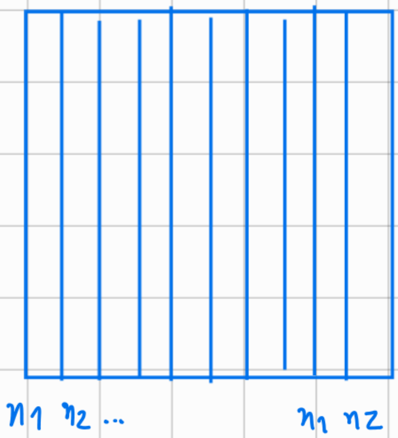
4) 1D PHOTONIC CRYSTAL

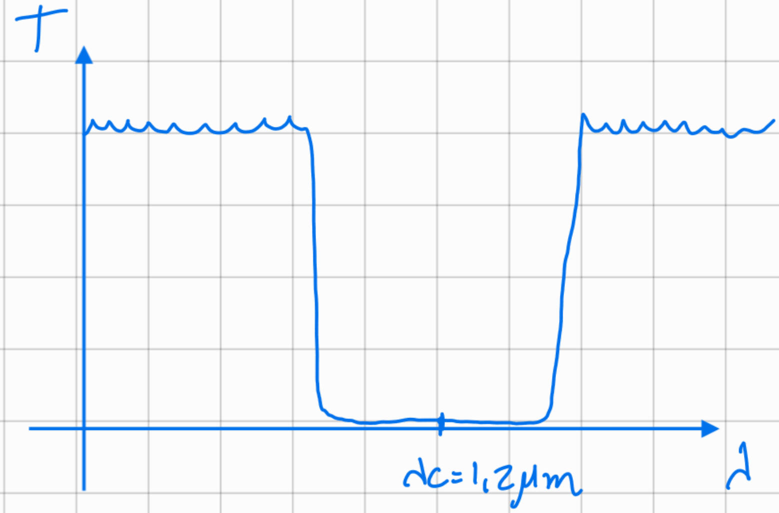
CENTRAL FREQUENCY

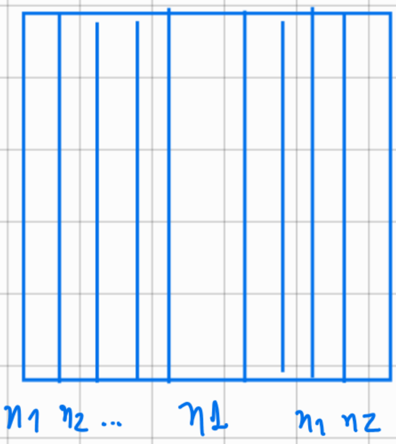
$$\omega_c = \frac{n_1+n_2}{4n_1n_2}\,\frac{2\pi c}{\Lambda}$$

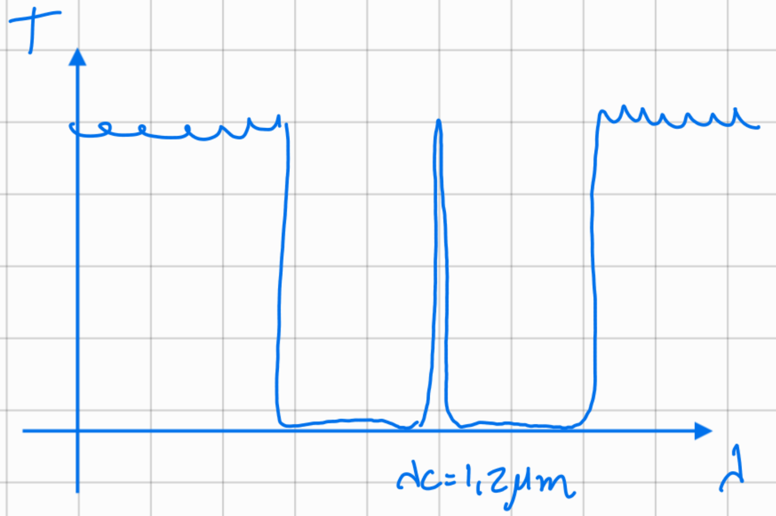
$n_1 = 1.5$
$n_2 = 3$
$\Lambda = 300$nm

$f_c = 2.5\times10^{14}$ Hz $\rightarrow \lambda_c = \dfrac{c}{f_c} = 1{,}2\,\mu m$









4.5 periods total

5) SURFACE PLASMON

$\lambda = 532\,nm$

$\varepsilon_{Ag} = -9.3 + i\,0{,}87$ $\qquad$ $\varepsilon_d = 1$ (A.R)

SURFACE-PLASMON EFFECTIVE INDEX

$$n_{sp} = \sqrt{\frac{\varepsilon' \varepsilon_d}{\varepsilon' + \varepsilon_d}}$$

$$n_{sp} = 1{,}059$$

phase velocity in air.

$$v_p = \frac{c}{n_{sp}} = 2{,}83 \times 10^8 \text{ m/s} < c$$

6)

$$\varepsilon_m = \varepsilon_i = 1 - \frac{\omega_p^2}{\omega^2 - j\omega\gamma}$$

$\varepsilon_h = 2.25$

$\varepsilon_m = -12.2 - j0.09$

$\varepsilon_m = -12,2 - j0.09$

Assuming the wire medium is effective uniaxial medium:

$$\bar{\varepsilon} = \varepsilon_\perp(\hat{x}\hat{x} + \hat{y}\hat{y}) + \varepsilon_\parallel \hat{z}\hat{z}$$

$\varepsilon_\parallel$ → parallel to the wire axis

$$\langle D_\parallel \rangle = \varepsilon_0 \varepsilon_\parallel \langle E_\parallel \rangle = \varepsilon_0 f \varepsilon_i E_{\parallel,i} + \varepsilon_0 (1-f) \varepsilon_h E_{\parallel,h}$$

$$\boxed{\varepsilon_\parallel = f\varepsilon_i + (1-f)\varepsilon_h}$$

$\varepsilon_\perp$ can be calculated looking at a section in the xy plane

$$\varepsilon_\perp = \varepsilon_h \left(1 + 2f \frac{\varepsilon_i - \varepsilon_h}{\varepsilon_i + \varepsilon_h - f(\varepsilon_i - \varepsilon_h)} \right)$$

When $f = 0.1$ → $\varepsilon_\parallel = 0.805 - j0.0096$

$\varepsilon_\perp = 3.01 - j0.0027$

One wire medium

$$[\varepsilon] = \begin{bmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{bmatrix}$$

The dispersion relation assuming k in the x,z plane

$$\frac{k_x^2}{\varepsilon_\parallel} + \frac{k_z^2}{\varepsilon_\perp} = \left(\frac{\omega}{c}\right)^2$$

For $f = 0.1$ $\quad \varepsilon_\perp \varepsilon_\parallel > 0$ ⇒ THE MEDIUM IS NOT HYPERBOLIC.

Nanophotonics Final Test

June 9, 2020.

1)

$$n_2 = \frac{n_1}{3}$$

a) $\theta_i = 0°$

$$n_1 \sin(\theta_i) = n_2 \sin(\theta_t)$$

$$n_1 \sin 0° = \frac{n_1}{3} \sin(\theta_t) \rightarrow \theta_t = \operatorname{asin}(3 \sin 0°)$$

$$\boxed{\theta_t = 0°}$$

$$R = r_T^2 = \left(\frac{n_2 \cos(\theta_t) - n_1 \cos(\theta_1)}{n_2 \cos(\theta_t) + n_1 \cos(\theta_i)} \right) = \left(\frac{n_1 \left(\frac{1}{3} \cos 0° - \cos 0° \right)}{n_1 \left(\frac{1}{3} \cos 0° + \cos 0° \right)} \right)^2$$

TM:

$$R = \left(\frac{\frac{1}{3} - 1}{\frac{1}{3} + 1} \right)^2 = \left(\frac{-\frac{2}{3}}{\frac{4}{3}} \right)^2 = \left(-\frac{1}{2} \right)^2 \rightarrow \boxed{R = 0{,}25}$$

$$t = 2 \frac{n_2 \cos(\theta_i)}{n_2 \cos \theta_T + n_1 \cos \theta_i} \rightarrow T = t^2 \frac{n_1 \cos \theta_T}{n_2 \cos \theta_i}$$

$$t = 2 \frac{n_2 \cos(0°)}{n_2 \cos 0° + \frac{1}{3} n_1 \cos 0°} = 2 \cdot \frac{1/3}{1 + \frac{1}{3}} = \frac{1/3}{\frac{4}{3}} = \frac{1}{2}$$

$$T = \frac{1}{4} \frac{\cos 0°}{\frac{1}{3} \cos 0°} = \frac{3}{4} \rightarrow \boxed{T = 0{,}75}$$

b) For $R=0 \rightarrow$ Brester angle of incidence :

$$\theta_B = \operatorname{atan}\left(\frac{n_2}{n_1}\right)$$

Since $n_2 \neq n_1$, $R=0$ is possible!

$$\theta_B = \operatorname{atan}\left(\frac{1}{3}\right) = 18{,}43^\circ$$

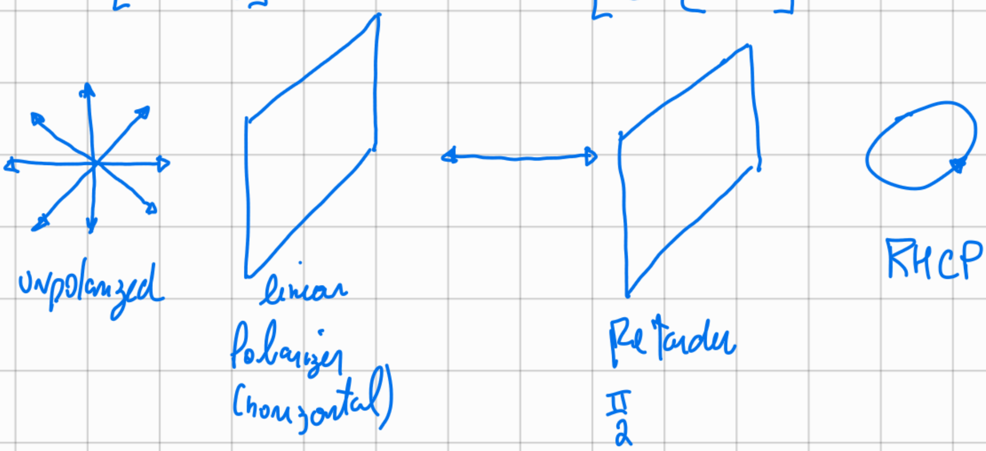
c) $$\theta_c = \operatorname{asin}\left(\frac{n_2}{n_1}\right) = 19{,}47^\circ$$

d) if $\theta_i > \theta_c$ the transverse wave is completely reflected but there's still an evanescence wave in the z direction

2) We can use a linear polarizer and after that a wave retarder to generate a circular polarized wave from an unpolarized wave.

The Jones matrix of each element used is:

$$M_{horizontal} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad M_{retarder} = \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Gamma} \end{bmatrix} \quad \text{with } \Gamma = -\frac{\pi}{2}$$



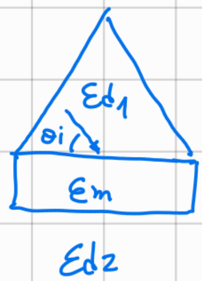
3) $\lambda = 532nm$

$\varepsilon_m = -9.3 + i0.5$

$\varepsilon_{glass} = 2.25 = \varepsilon_{d1}$ $\qquad \varepsilon_{d2} = 1$ (AIR)

$\theta_i = ?$ to excite a surface plasmon air/metal

$L_{sp} = ?$ PROPAGATION LENGHT



Kretschmann configuration

$$n_{sp} = Re\left[\sqrt{\frac{\varepsilon_m \varepsilon_{d2}}{\varepsilon_m + \varepsilon_{d2}}}\right]$$

$$n_{sp} = 1.058$$

$$\theta_i = a\sin\left(\frac{n_{sp}}{n_{d1}}\right) = a\sin\left(\frac{1.058}{\sqrt{2.25}}\right) \Rightarrow \boxed{\theta_i = 44.85^\circ}$$

PROPAGATION LENGHT

$$L_{sp} = \frac{1}{2k''_{sp}} = \frac{c}{\omega} \frac{\sqrt{|\varepsilon_m'|}\left(|\varepsilon_m'| - \varepsilon_d\right)^{3/2}}{\varepsilon_d^{3/2}\,\varepsilon_m''}$$

$$\omega = 2\pi f = \frac{c}{\frac{2\pi c}{\lambda}} = \frac{\lambda}{2\pi}$$

$$L_{sp} = 2.86\,\mu m$$

4)

Nanophotonics Final Test

23 June 2020

1) a)

b) LINEAR : $E = (a_x \hat{x} + a_y e^{j\phi} \hat{y}) e^{-jk}$

$a_x = a_y$ and $\phi = 0$ or $\pm\pi$

CIRCULAR

$a_x = a_y$ $\phi = -\pi/2 \rightarrow$ RHCP

$\phi = \pi/2 \rightarrow$ LHCP

ELLIPTICAL

$a_x \neq a_y$ $\phi \neq 0, \pm\pi, \pm\pi/2$

c) $M = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow$ HORIZONTAL LINEAR POLARIZER

2)

DIFFRACTION GRADING

$$\sin\theta_q = \sin\theta_i + q\frac{\lambda}{\Lambda}$$

Number of Slits = 12600

$\Lambda = 25.4\,mm$

$\lambda_1 = 589\,nm$

$\lambda_2 = 589.59\,nm$

$q = 1$

$\theta_i = 0^\circ$ (NORMAL INCIDENCE)

$$\theta_{q1} = a\sin\left(0 + \frac{1 \cdot 589\times10^{-9}}{25{,}4\times10^{-3}}\right) = 1{,}3286\times10^{-3\,\circ}$$

$$\theta_{q2} = a\sin\left(0 + \frac{1 \cdot 589{,}59\times10^{-9}}{25{,}4\times10^{-3}}\right) = 1{,}3299\times10^{-3\,\circ}$$

3)