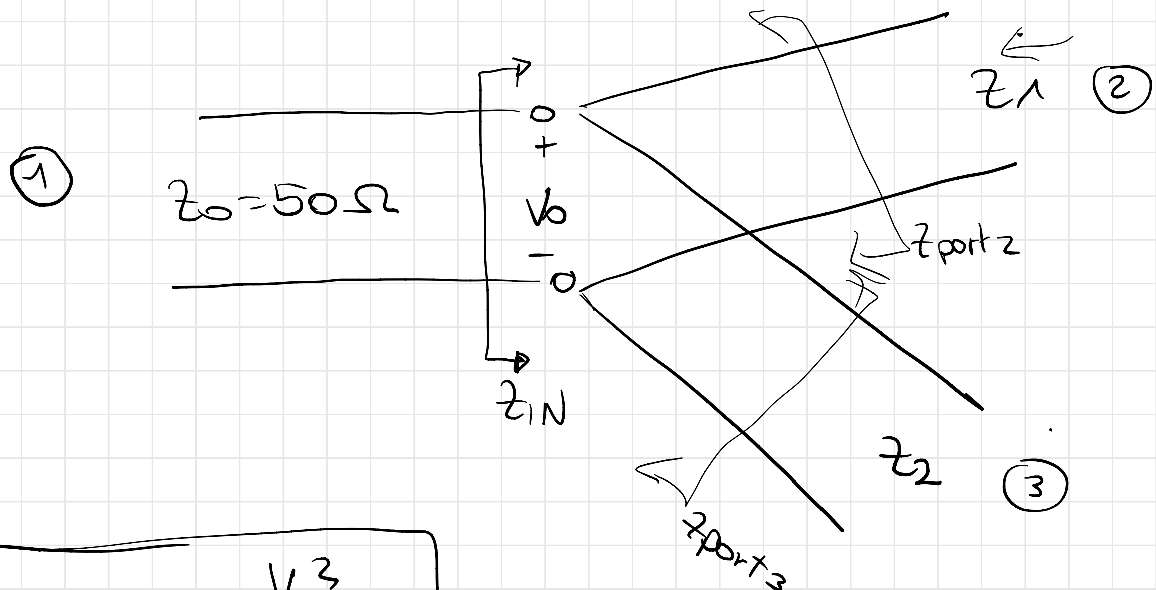
MICROWAVE ENGINEERING

Lecture 26:
Problems on power dividers and combiners

<u>Problem 1</u> A lossless T-Junction power divider has a source impedance of $50\,\Omega$. Find the output impedances so that P_{IN} is divided in a 2:1 ratio. Calculate the reflection coefficients at the output ports.



$$\boxed{P_{IN} = \frac{1}{2}\,\frac{V_0^3}{Z_0}}$$

If we want a 2 : 1 ratio

$$P_1 = \frac{1}{2}\frac{V_0^2}{Z_1} \qquad \Rightarrow \qquad P_2 = 2P_1 = \frac{V_0^2}{Z_1}$$

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_1}$$

$$P_{IN} = P_1 + P_2 = \frac{1}{2}\frac{V_0^2}{Z_1} + \frac{V_0^2}{Z_1} = \frac{3}{2}\frac{V_0^2}{Z_1}$$

but

$$P_{IN} = \frac{1}{2}\frac{V_0^2}{Z_0}$$

$$\Downarrow$$

$$\frac{1}{\cancel{2}}\frac{\cancel{V_0^2}}{Z_0} = \frac{3}{\cancel{2}}\frac{\cancel{V_0^2}}{Z_1} \Rightarrow \boxed{Z_1 = 3Z_0 = 150\,\Omega}$$

$$P_2 = \frac{1}{2}\frac{\cancel{V_0^2}}{Z_2} = 2P_1 = \frac{\cancel{V_0^2}}{Z_1} \Rightarrow \quad Z_1 = 2Z_2 \Rightarrow \boxed{Z_2 = 75\Omega}$$

$$Z_{IN} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{75 \cdot 150}{75 + 150} = 50\ \Omega$$

matched with the input line

looking into port 2 our $Z_{port\,2} = \dfrac{50 \cdot 75}{50 + 75} = 30\,\Omega$

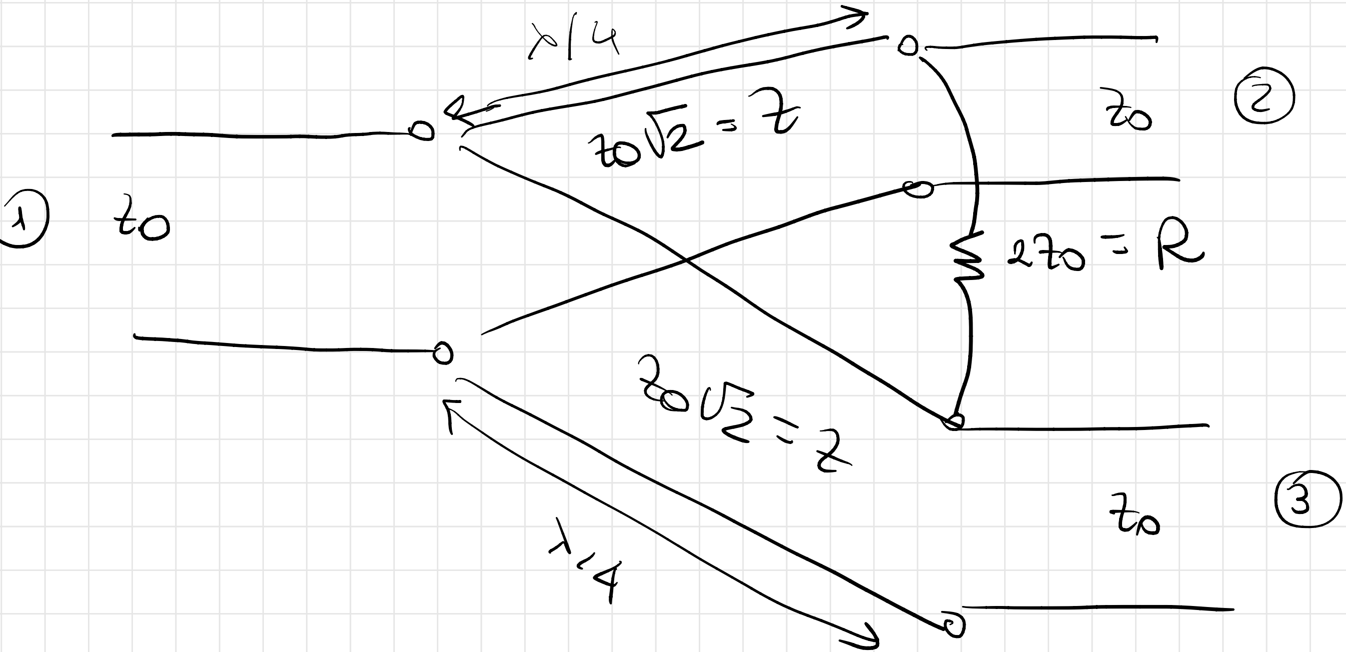
looking into port 3 $Z_{port\,3} = \dfrac{50 \cdot 150}{50 + 150} = 37.5\,\Omega$

The reflection coefficients at ports 2 and 3 are:

$$\Gamma_{port2} = \frac{30-150}{30+150} = -0.67$$

$$\Gamma_{port3} = \frac{37.5-75}{37.5+75} = -0.33$$

Problem 2 Design an equal split Wilkinson power divider for $50\,\Omega$ system impedance at $f_0 = 5$ GHz.



$$Z = Z_0\sqrt{2} = 50\sqrt{2} = 70.7\,\Omega \qquad R = 2Z_0 = 100\,\Omega$$

$$f = \frac{c}{\lambda} \Rightarrow \lambda = \frac{c}{f}$$

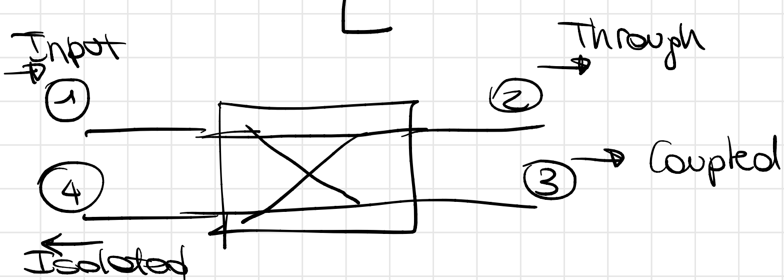
$$l = \frac{\lambda}{4} = \frac{c}{4f} = \frac{3 \cdot 10^8}{4 \cdot 5 \cdot 10^9} =$$

$$= 0.015\ m = \underline{1{,}5\ cm}$$

Problem 3

A directional coupler has the following scattering matrix. Find Directivity, Coupling, Isolation and Return loss at the input port when other ports are terminated with matched loads.

$$[S] = \begin{bmatrix} 0.05\angle 30^\circ & 0.96\angle 0^\circ & 0.1\angle 90^\circ & 0.05\angle 90^\circ \\ 0.96\angle 0^\circ & 0.05\angle 30^\circ & 0.05\angle 90^\circ & 0.1\angle 90^\circ \\ 0.1\angle 90^\circ & 0.05\angle 90^\circ & 0.04\angle 30^\circ & 0.96\angle 0^\circ \\ 0.05\angle 90^\circ & 0.1\angle 90^\circ & 0.96\angle 0^\circ & 0.05\angle 30^\circ \end{bmatrix}$$



The Directivity is

$$D = 10 \log \frac{P_3}{P_4} = 20 \log \frac{\beta}{|S_{14}|} = \qquad \beta^2 = |S_{13}|^2$$

coupling factor

$$= 20 \log \left| \frac{S_{13}}{S_{14}} \right| = 20 \log \frac{|0.1|}{|0.05|} = 6\,dB$$

Coupling is:

$$C = 10 \log \frac{P_1}{P_3} = -20 \log \beta = -20 \log |S_{13}| =$$

$$= -20 \log |0.1| = 20\,dB$$

Isolation is:

$$I = 10 \log \frac{P_1}{P_4} = -20 \log |S_{14}| = -20 \log |0.05|$$

$$= 26 \text{ dB}$$

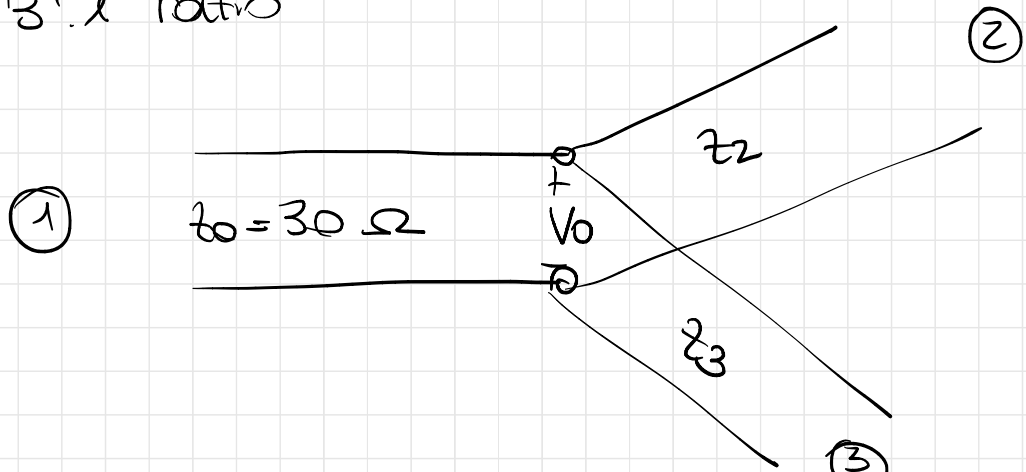
The Return loss:

$$RL = -20 \log |\Gamma| = -20 \log |S_{11}| = -20 \log |0.05|$$

$$= 26 \text{ dB}$$

Problem 4 Design a lossless T-junction divider with a 30 Ω source impedance to give a 3:1 power split. Design quarter-wave matching transformers to convert the impedances of the output lines to 30 Ω.

First step we find port 2 and 3 impedances to realize 3:1 ratio



$$\frac{P_2}{P_3} = 3$$

$$P_1 = P_2 + P_3$$

$$\boxed{P_2 = \frac{3}{4} P_1 \qquad P_3 = \frac{1}{4} P_1}$$

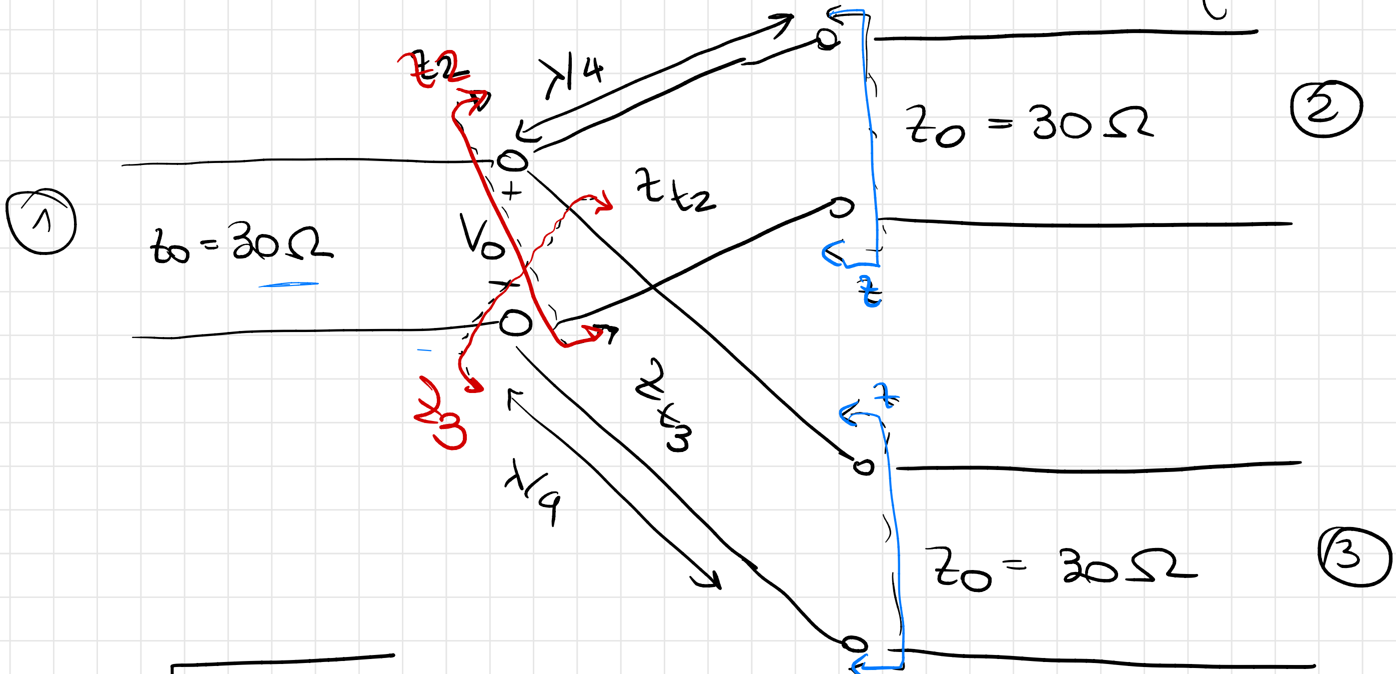
$$P_1 = \frac{1}{2}\frac{V_0^2}{Z_0}$$

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_2} = \frac{3}{4}P_1 = \frac{3}{4}\left(\frac{1}{2}\frac{V_0^2}{Z_0}\right) \Rightarrow \frac{1}{Z_2} = \frac{3}{4}\frac{1}{Z_0} \Rightarrow$$

$$\Rightarrow \boxed{Z_2 = \frac{4}{3}Z_0 = 40\Omega}$$

$$P_3 = \frac{1}{2}\frac{V_0^2}{Z_3} = \frac{1}{4}P_1 = \frac{1}{4}\left(\frac{1}{2}\frac{V_0^2}{Z_0}\right) \Rightarrow \frac{1}{Z_3} = \frac{1}{4}Z_0 \Rightarrow \boxed{Z_3 = 4Z_0 = 120\Omega}$$

If we now want our output port 2 and 3 to be on output lines 30 Ω our circuit need to be changed:

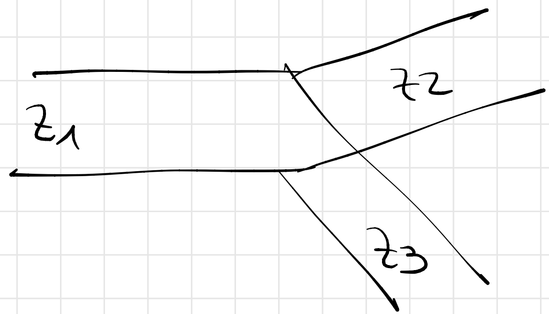


$$Z_{t_2} = \sqrt{30 \cdot \underset{Z_2}{\underset{\parallel}{40}}} = 34.64\ \Omega$$

$$Z_{t_3} = \sqrt{30 \cdot \underset{Z_3}{\underset{\parallel}{120}}} = 60\ \Omega$$

Determine the S parameters magnitude for the circuit.

If we analyze the original network



$Z_1 = 30\,\Omega$

$Z_2 = 40\,\Omega$

$Z_3 = 120\,\Omega$

$$|S_{11}| = \left| \frac{Z_2 \| Z_3 - Z_1}{Z_2 \| Z_3 + Z_1} \right| = \left| \frac{30-30}{30+30} \right| = \emptyset$$

$$|S_{22}| = \left| \frac{Z_1 \| Z_3 - Z_2}{Z_1 \| Z_3 + Z_2} \right| = \left| \frac{30 \| 120 - 40}{30 \| 120 + 40} \right| = 0.25$$

$$|S_{33}| = \left| \frac{Z_1 \| Z_2 - Z_3}{Z_1 \| Z_2 + Z_3} \right| = \left| \frac{40 \| 30 - 120}{40 \| 30 + 120} \right| = 0.75$$

If we analyze the modified network:

$$|S_{11}| = \left| \frac{Z_2 \| Z_3 - Z_1}{Z_2 \| Z_3 + Z_1} \right| = \emptyset$$

$$|S_{22}| = \left| \frac{Z - 30}{Z + 30} \right| = \left| \frac{50 - 30}{50 + 30} \right| = 0.25$$

$$Z = \frac{Z_{t_2}^2}{Z_1 \| Z_3} = \frac{\cancel{30} \cdot \cancel{40}}{\frac{\cancel{30} \cdot \cancel{120}\, 3}{30 + 120}} = \frac{150}{3} = 50$$

$$|S_{33}| = \left|\frac{Z-30}{Z+30}\right| = \left|\frac{210-30}{210+30}\right| = \left|\frac{180}{240}\right| = 0.75$$

$$Z = \frac{Z_3}{Z_1 \parallel Z_2} = \frac{30 \cdot 120}{\frac{30 \cdot 40}{30+40}} = 210$$

$$|S_{21}| = |S_{12}| = \left|\sqrt{\frac{P_2}{P_1}}\right| = \sqrt{\frac{3}{4}} = 0.866$$

$$|S_{31}| = |S_{13}| = \left|\sqrt{\frac{P_3}{P_1}}\right| = \sqrt{\frac{1}{4}} = 0.5$$

S matrix is unitary since the network is lossless

$$|S_{21}|^2 + |S_{22}|^2 + |S_{23}|^2 = 1$$

$$|S_{23}| = |S_{32}| = \sqrt{1 - |S_{21}|^2 - |S_{22}|^2} =$$

$$= \sqrt{1 - (0.25)^2 - (0.866)^2} = 0.433$$

$$\boxed{|S| = \begin{bmatrix} 0 & 0.866 & 0.5 \\ 0.866 & 0.25 & 0.433 \\ 0.5 & 0.433 & 0.75 \end{bmatrix}}$$