Optical communication components - Basic questions

Chapter 1

- What is the visible wavelength? Table 1.1 .

VISIBLE → $7{,}5 \times 10^{14}$ to 4×10^{14} 400 nm to 750nm VISIBLE

	Frequency (Hz)	Wavelength
GAMMA →	3×10^{20}	< 1 pm
X RAYS →	$< 3 \times 10^{17}$	1pm To 1nm
UV →	$< 7{,}5 \times 10^{14}$	1nm to 400nm
VISIBLE →	$7{,}5 \times 10^{14}$ to 4×10^{14}	400 nm to 750nm
NIR →	4×10^{14} To 1.2×10^{14}	2,5 μm to 25 μm
IR →	$1{,}2 \times 10^{14}$ TO $1{,}2 \times 10^{13}$	25 μm to 1 mm
MICROWAVE →	1.2×10^{13} To 3×10^{11}	
RADIO WAVE →	$< 3 \times 10^{11}$	$>$ 1mm

....

- Maxwell Equations... explain EF, MF .

$\nabla \times \vec{E}(r,t) = -\dfrac{\partial B(r,t)}{\partial t}$ → FARADAY'S LAW: Relation between time variant Magnetic flux and induced potential around a loop that generates $\vec{E}$

$\nabla \times \vec{H}(r,t) = -\dfrac{\partial D(r,t)}{\partial t}$ → AMPERE'S LAW: Relation between a displacement current and the magnetic field.

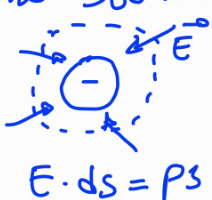
$\nabla \cdot \vec{D} = 0$ → GAUSS LAW: The Displacement lines begin/ends on free charges

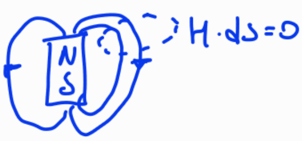
$\nabla \cdot \vec{B} = 0$ → GAUSS LAW of Magnetism: Lack of Magnetic monopole, There's no sources or sinks of magnetic field.

$P(r,t)$ → Electric Polarization

$M(r,t)$ → Magnetic "

↳ = 0





CONTINUITY EQUATION → The flux of current from a closed surface represents a decrease of the charge inside the surface

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- Wave equation .

$$\nabla^2 E + \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} - \cancel{\nabla(\nabla \cdot E)} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 P}{\partial t^2}$$

negligible due to the space dependence of the linear susceptibility.

$$\boxed{\nabla^2 E + \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 P}{\partial t^2}}$$

→ check deduction later

$$P = P^L(E) + P^{NL}(E)$$

....

- Definition of polarization (instaneous and non instantaneous... isotropic/no isotropic)

· $\vec{P} = \varepsilon_0 \chi \overset{\tau}{*} \vec{E}$

χ → susceptibility

Isotropic Media:
The response doesn't depend on the orientation of the source
↳ $\left.\begin{matrix} \chi_{xx} \neq 0 \\ _{yy} \neq 0 \\ _{zz} \neq 0 \end{matrix}\right\}$ χ matrix is diagonal
↳ $\bar{P} \parallel \bar{E}$

ISOTROPIC MEDIA: The way that the medium reacts to a modifiable electric field doesn't change.

$P = \varepsilon_0 \chi E$ → frequency domain

↳ This is true for time domain if the media responds instantaneously to the applied Electric field:

$\chi(r,\tau) = \chi\delta(\tau)$

....

-Refractive index

- $n(r,f) = \sqrt{\varepsilon_r(r,f)} = \sqrt{1+\chi(r,f)}$

Aspects:

$1 \leq n(\bar{r},f)$

$\lim_{f\to\infty} n(\bar{r},f) \to 1$ because $f \to \infty \therefore \lambda \to 0$ → The wave is so small that the media is like a vacuum media

....

- How refractive index is related to c

- $v = \dfrac{c}{n}$

↳ wave velocity in the medium

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- refractive index is a matrix, escalar, vector??

- Matrix
  if isotropic iT's a Scalar in each coordinate

....

-Whats non linear polarization?

- If the electric field applied is very intense, P is no longer proportional to $\vec{E}$.

$$\vec{P}_{(r,\tau)} = P^L_{(r,\tau)} + P^{NL}_{(r,\tau)}$$

$$P = \varepsilon_0 \chi^{(1)} E + \underbrace{\cancel{\varepsilon_0 \chi^{(2)} E^2}}_{SHG} + \underbrace{\varepsilon_0 \chi^{(3)} E^3}_{THG}$$

2nd harmonic Generation

Third harmonic Generation

....

-Second harmonic generation/third harmonic generation

- SECOND HARMONIC GENERATION
  - ↳ IT happens because of the quadratic term of the nonlinear induced polarization
  - ↳ Cannot be observed in centrosymmetric materials due symmetries in P and E $(r \to -r)$.
    - ↳ Ex: Silica
  - ↳ 27 $\chi^{(2)}_{jkl}$ are necessary to describe it
- THIRD HARMONIC GENERATION

$\varepsilon_0 \chi^{(3)} E^3$

81 $\chi^{(3)}_{jkl}$ are necessary to describe

$$P^{NL}_{(r,\tau)} = \varepsilon_0 \chi^{(3)} E(r,\tau) E(r,\tau) E(r,\tau)$$

....

- Optical parametric amplification (OPA)

·

....

- Centrosymmetric material -> cannot have x(2)

· Materials whose structure doesn't change upon the transformation $r \to -r$ (inversion with respect to the origin). EX: SILICA

· Centrosymmetric materials can't have second harmonic generation neither $\chi^{(2)}$ due the symmetries in polarization and E.

....

- What is Kerr effect?

· IT IS THE CHANGE OF REFRACTIVE INDEX CAUSED BY THE ELECTRIC FIELD, AND BEING PROPORTIONAL TO THE SQUARE OF THE ELECTRIC FIELD STRENGTH.

BRILLOUIN SCATTERING: The difference in energy generates acoustic phonons

RAMAN SCATTERING: The difference in energy generates a vibrational oscillation.

....

- We must to be able to use table 1.2

· $\chi^{(3)}_{xxyy}$ $\chi^{(3)}_{xyxy}$ $\chi^{(3)}_{xyyx}$ ARE INDEPEDENT

· 21 ELEMENTS ARE NON-ZERO.

} ISOTROPIC MEDIUM

....

- What is dirac

- It is a mathematical function that is infinite in $t=0$ and zero in all other time.

....

- Wave equation -> must know the equation

- $$\nabla^2 E = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left(E + \frac{1}{\varepsilon_0}P\right)$$

$\vec{B} \to \mu_0 \vec{H}$

$\vec{H} \to \frac{\partial D}{\partial t}$

$$\nabla \times E = -\frac{\partial B}{\partial t} \longrightarrow \nabla \times \nabla \times E = -\frac{\partial}{\partial t}\nabla \times \vec{B} = -\mu_0 \frac{\partial}{\partial t}\nabla \times \vec{H}$$

$$\nabla \times H = \frac{\partial D}{\partial t} \qquad \nabla \times \nabla \times E = -\mu_0 \frac{\partial^2}{\partial t^2} D$$

$$-\nabla^2 E + \cancel{\nabla(\nabla \cdot E)}^{\,0} = -\mu_0 \frac{\partial^2}{\partial t^2} D$$

$D = \varepsilon_0 E$

$= \varepsilon_0 E + \frac{1}{\varepsilon_0} P$

$\nabla \times \nabla \times E =$

$-\nabla^2 E + \nabla(\nabla \cdot E)$

$$-\nabla^2 E = -\mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} E - \mu_0 \frac{\partial^2 P}{\partial t^2}$$

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} E + \mu_0 \frac{\partial^2 P}{\partial t^2}$$

$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$$\nabla^2 E = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \left(E + \frac{1}{\varepsilon_0} P \right)$$

....

- Simple case in case of a plane wave... EF, MF, etc...

- IDEAL CASE

→ LINEAR MATERIAL ($P \sim E$)

→ HOMOGENEOUS (χ doesn't depend on $\bar{r}$)

→ LOSSLESS MATERIAL ($\mathrm{Im}(\chi(f)) = 0$)

→ $\vec{E}$ is a perfect sinusoidal electric field (f_0)

→ $\vec{E}(z,t)$

$$E(z,t) = E_M \cos(\omega_0 t + \beta_0 z + \phi_0) \left\{ \begin{array}{l} \omega_0 = 2\pi f_0 \quad \text{angular frequency} \\ \beta_0 = 2\pi q_0 = \frac{2\pi}{d} \quad \text{propagation constant} \\ \text{Spatial period} \to d. \end{array} \right.$$

DISPERSION RELATION: $\beta_0^2 = \frac{\omega_0^2}{c^2}(1 + \chi(f_0)) = \frac{\omega_0^2}{c^2} n^2(f_0)$

....

- Table 1.3 - Quantities should be known

	FREE SPACE	MEDIUM
REFRACTIVE INDEX	$n = 1$	$n(f_0)$
SPEED OF LIGHT	c	$v = \frac{c}{n(f_0)}$
WAVELENGTH	λ_0	$\lambda = \frac{\lambda_0}{n(f_0)}$
PROPAGATION CONSTANT	$\beta_0 = \pm \frac{2\pi}{\lambda_0}$	$\beta_0 = \pm \frac{2\pi}{\lambda} = \pm \frac{2\pi\, n(f_0)}{\lambda_0}$

$$\beta_0 = \frac{\omega_0}{c} n(f_0) = \frac{2\pi f_0}{c} \cdot n(f_0) = \frac{2\pi}{\lambda_0} \cdot n(f_0)$$

$\beta_0 > 0 \rightarrow$ wave propagates towards negative z

$\beta_0 < 0 \rightarrow$ wave propagates towards positive z.

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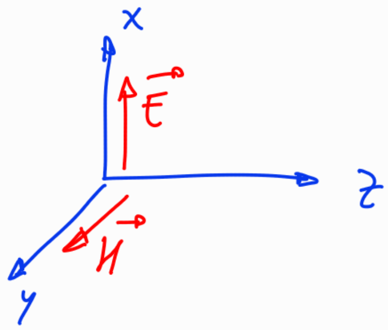
- How the magnetic field and electric field are related... the vectors, etc

$$\vec{W} = \vec{E} \times \vec{H}$$



$$\beta_0 = \frac{\omega_0}{c} = \omega_0 \sqrt{\mu_0 \varepsilon_0}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

....

- Poynting vector, etc

- $\vec{W} = \vec{E} \times \vec{H}$ quantify the quantity of EM intensity $\frac{W}{m^2}$

for a uniform plane wave:

$$W = -\frac{E_M^2 \beta_0}{\omega_0 \mu_0} \cos^2(\omega_0 t + \beta_0 z + \phi_0)\,\hat{k}$$

....

-Basics of Fourier Transformer... no mathematical...

- Used to provide a very useful to deal with derivatives and Integrals

$$\hat{E}(f_r, f) = \int_{R^3}\int_R E(r,t)\, e^{-j2\pi(f_r r + f t)}\, dt\, dr$$

....

- The envelope function

- $V_0(t) = V_0 \cos(2\pi f_0 t + \phi_0) \longrightarrow \bar{V}_0(f) = \frac{1}{2} V_0 e^{j\phi_0} \delta(f - f_0) + \frac{1}{2} V_0 e^{-j\phi_0} \delta(f + f_0)$

To obtain the complex envelope:

1. Remove from $V_0(t)$ the negative frequencies by using a frequency filter

$H(f) = 2 \cdot 1(f)$ $\quad 1(f) \rightarrow$ Heaviside function $\begin{cases} 0 & f < 0 \\ 1/2 & f = 0 \\ 1 & f > 0 \end{cases}$

2. Translate the positive frequency by $-f_0$ so the envelope will be at the origin.

* Instead of having 2 spectral modes, now we have only one with the same information. This envelope is a slowly variant in time too.

....

- Complex amplitude... meaning, why is it useful...

- $A(z,t)$

  ↳ slowly variant in z and t.

$E(z,t) = \mathrm{Re}\left[A(z,t)\, e^{j2\pi(q_0 z + f_0 t)}\right]$

$E(z,t) = |A(z,t)| \cos\left[2\pi(q_0 z + f_0 t) + \phi(z,t)\right]$

....

-Wave equation from the amplitude envelope

- Wave equation : $\nabla^2 E = \frac{1}{c^2}\frac{\partial^2}{\partial\tau^2}\left(E + \frac{1}{\varepsilon_0}P\right)$

$$\nabla^2 A + 2j\beta_0\frac{\partial A}{\partial z} - \beta_0^2 A$$
$$= -\frac{\omega_0^2}{c^2}\underbrace{\left(A + \frac{1}{\varepsilon_0}A_p\right)}_{A_1} + \frac{2j\omega_0}{c^2}\frac{\partial}{\partial\tau}\left(A + \frac{1}{\varepsilon_0}A_p\right) + \frac{1}{c^2}\frac{\partial^2}{\partial\tau^2}\left(A + \frac{1}{\varepsilon_0}A_p\right)$$

$$\nabla^2 A + 2j\beta_0\frac{\partial A}{\partial z} - \beta_0^2 A = -\frac{\omega_0^2}{c^2}A_1 + \frac{2j\omega_0}{c^2}\frac{\partial}{\partial\tau}A_1 + \frac{1}{c^2}\frac{\partial^2}{\partial\tau^2}A_1$$

....

-NLSE

- → linear contribution of the induced polarization

$$P(r,\tau) = \varepsilon_0\, \chi \overset{\tau}{*} E(r,\tau)$$

$$\bar{P}(r,f) = \varepsilon_0\, \chi(r,f)\, E(r,f) \rightarrow \text{unfriendly expression because it would be necessary to know } \chi(r,f)\ \forall f.$$

To overcome this the complex envelope of the induced polarization:

$$A_p(r,\tau) = \varepsilon_0 \sum_{n=0}^{\infty} \frac{\hat{\chi}^{(n)}(r,f_0)}{(j2\pi)^n n!}\frac{\partial^n A(r,\tau)}{\partial\tau^n}$$

$A(r,\tau)$ → complex amplitude of the electric field.

$$A_1(r,\tau) = A(r,\tau) + \frac{1}{\varepsilon_0}A_p(r,\tau) = \sum_{n=0}^{\infty}\frac{\hat{Q}^{(n)}(r,f_0)}{(j2\pi)^n n!}\frac{\partial^n A(r,\tau)}{\partial\tau^2}$$

$$\hat{Q}^{(n)}(r,f) = 1 + \chi(r,f)$$

$$A_1(r,\tau) = \sum_{n=0}^{\infty}\frac{\hat{Q}^{(n)}(r,\omega_0)}{j^n n!}\frac{\partial^n A(r,\tau)}{\partial\tau^n}$$

→ NONLINEAR CONTRIBUTION OF THE INDUCED POLARIZATION

$$P^{NL}(r,\tau) = \varepsilon_0 \underbrace{\chi^{(3)}_{(r,\tau)}}_{\text{CONSTANT}} \cdot E^3(r,\tau)$$

We need to find a function that: $P^{NL}(r,\tau) = \mathrm{Re}\left[A_p^{NL}(r,\tau)\, e^{j(\omega_0\tau+\beta_0 z)}\right]$

(...)

$$A_p^{NL}(r,\tau) = \frac{3}{4}\varepsilon_0 \hat{\chi}^{(3)} |A(r,\tau)|^2 A(r,\tau)$$

→ TOTAL CONTRIBUTION TO THE POLARIZATION

$$P = \varepsilon_0\left(\chi E + \chi^{(3)} E^3\right) = \left(A_p^{L} + A_p^{NL}\right) e^{j\alpha}$$

$$A_1 = A + \frac{1}{\varepsilon_0} A_p = A + \frac{1}{\varepsilon_0}\left(A_p^{L} + A_p^{NL}\right)$$

$$A_1 = A + \sum_{n=0}^{\infty} \frac{\chi^{(n)}(r,\omega_0)}{(j)^n\, n!} \frac{\partial^n}{\partial \tau^n} A(r,\tau) + \frac{3}{4}\varepsilon_0 \hat{\chi}^{(3)} |A(r,\tau)|^3 A(r,\tau)$$

NLSE

$$\nabla^2 A + 2j\beta_0 \frac{\partial A}{\partial z} - \beta_0^2 A = -\frac{\omega_0^2}{c^2}(1+\hat{\chi})A +$$

$$+ j\left(\frac{\omega_0^2}{c^2}\hat{\chi}' + \frac{2\omega_0}{c^2}(1+\hat{\chi})\right)\frac{\partial A}{\partial t} +$$

$$+\left(\frac{\omega_0^2}{2c^3}\hat{\chi}'' + \frac{2\omega_0}{c^2}\hat{\chi}' + \frac{1}{c^3}(1+\hat{\chi})\right)\frac{\partial^2 A}{\partial \tau^2}$$

$$-\frac{3}{4}\frac{\omega_0^2}{c^2}\hat{\chi}^{(3)}|A|^2 A$$

$$\chi' = \left.\frac{\partial \hat{\chi}}{\partial \omega}\right|_{\omega=\omega_0} \qquad \chi'' = \left.\frac{\partial^2 \hat{\chi}}{\partial \omega^2}\right|_{\omega=\omega_0}$$

WAVE NUMBER: $k^2 = \frac{\omega^2}{c^2}\left(1+\chi(r,\omega)\right) = \frac{\omega^2}{c^2} n^2(r,\omega)$

$$\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}+2j\beta_0\frac{\partial A}{\partial z}-\beta_0^2 A = -k_0^2 A + j(k_0^2)'\frac{\partial A}{\partial \tau}+\frac{1}{2}(k_0^2)''\frac{\partial^2 A}{\partial\tau}-\frac{3}{4}\frac{\omega_0^2}{c^2}\chi^{(3)}|A|^2A$$

3D NLSE (1.63)

$$2j\beta_0\frac{\partial A}{\partial z}+\underbrace{\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}}_{\text{DIFRACTION}}+\left(\underbrace{\frac{3}{4}\frac{\omega_0^2}{c^2}\hat{\chi}^{(3)}|A|^2}_{\text{KERR EFFECT}}+\underbrace{k_0^2-\beta_0^2}_{\text{WAVEGUIDING}}\right)A\underbrace{-2jk_0k_0'\frac{\partial A}{\partial\tau}}_{\text{GROUP VELOCITY}}\underbrace{-k_0k_0''\frac{\partial^2A}{\partial\tau^2}}_{\text{GVD}}=0$$

DIFRACTION $\rightarrow$ SPATIAL BROADENING

KERR EFFECT $\rightarrow$ nonlinear refractive index

GVD $\rightarrow$ DISPERSIVE TERM $\rightarrow$ TEMPORAL BROADENING ($k_0''\neq 0$)

$A(r,\tau) \rightarrow$ COMPLEX ENVELOPE FROM $\vec{E}$

$\beta_0 = 2\pi q_0 \rightarrow$ REFERENCE PROPAGATION CONSTANT

$\omega_0 = 2\pi f_0 \rightarrow$ REFERENCE PULSATION

$k_0 \rightarrow$ WAVENUMER

$$k_0^2 = \frac{\omega_0^2}{c^2}\left(1+\chi(r,\omega)\right) \qquad k_0' = \left.\frac{\partial k}{\partial\omega}\right|_{\omega=\omega_0} \qquad k_0'' = \left.\frac{\partial^2 k}{\partial\omega^2}\right|_{\omega=\omega_0}$$

....
- Terms appearing in the NLSE: 1st order, 2nd order, 3rd order, ... (3dNLSE)....

· 1 ST ORDER → negleting all the terms containing derivatives of slowly variant amplitude E and r dependence in X

We got: $\beta_0^2 = \frac{\omega_0^2}{c^2}\left(1+\hat{\chi}(f)\right)$ → DISPERSION RELATION.

2 nd order → negleting the terms involving the second order derivatives of A and the nonlinear contribution and 1 ST order

$$2j\beta_0\frac{\partial A}{\partial z} = + j\left(\frac{\omega_0^2}{c^2}\hat{\chi}' + \frac{2\omega_0}{c^2}(1+\hat{\chi})\right)\frac{\partial A}{\partial \tau}$$

$$K^2 = \frac{\omega_0^2}{c^2}\left(1+\hat{\chi}(\omega)\right)$$

$$K^{2'} = \frac{\omega_0^2}{c^2}\hat{\chi}' + \frac{2\omega_0}{c^2}(1+\hat{\chi})$$

$$2j\beta_0\frac{\partial A}{\partial z} = j\,(K^2)'\frac{\partial A}{\partial \tau}$$

$(K^2)' = 2K\,K'$

IF THE SPATIAL DEPENDENCE OF X is NEGLIBIBLE

⇓

$$K_0 \approx \pm\beta_0$$

$$\frac{\partial A}{\partial z} = \frac{\cancel{j}}{\cancel{2j}}\,\frac{\cancel{2}K}{K}\,K'\,\frac{\partial A}{\partial \tau} \rightarrow \boxed{\frac{\partial A}{\partial z} = \pm K'\frac{\partial A}{\partial \tau}}$$

GROUP VELOCITY

$$v_g = \frac{1}{K_0'} = \frac{1}{\frac{\partial K(\omega)}{\partial \omega}}$$

3 rd order → Last session

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- What is the correction due to the 3rd order non linearity

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$$\left(\frac{3}{4}\frac{\omega_0^2}{c^2}\chi^{(3)}|A(r,\tau)|^2 + k_0^2\right)A(r,\tau)$$

$$n_2 = \frac{3}{8n}\chi^{(3)}$$

$$\left(\frac{3}{4}\frac{\omega_0^2}{c^2}\underbrace{\frac{8n\,n_2}{3}}_{\chi^{(3)}}|A(r,\tau)|^2 + k_0^2\right)A(r,\tau)$$

$$k_0^2 = \frac{\omega_0^2}{c^2}n^2 \quad (\text{if } \hat{\chi}(\xi))$$

$$\left(2n\,n_2\frac{\omega_0^2}{c^2}|A(r,\tau)|^2 + \frac{\omega_0^2}{c^2}n^2\right)A(r,\tau)$$

ORDER of $1.2\times10^{-22}\,(m/V)^2$ for silica

$$\frac{\omega_0^2}{c^2}\left(2n\,n_2|A(r,\tau)|^2 + n^2\right)A(r,\tau)$$

INTENSITY OF E CHANGES n.

KERR EFFECT → The non-linear effect where the refractive index changes according to the field intensity

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What was neglected?

→ linear attenuation

$\hat{\chi} = \chi_R + \chi_I$ $\qquad \hat{\chi}(f)$ $\qquad A(z)$

↳ no time dependence

↳ no z dependence.

$$\frac{dA(z)}{dz} = \pm\frac{\omega_0}{2c}\frac{\chi_I(\omega_0)}{\sqrt{1+\hat{\chi}_R(\omega_0)}}A(z) = \pm\frac{\alpha}{2}A(z)$$

imaginary part

α → ATTENUATION COEFFICIENT

$$\alpha = \frac{\omega_0}{c}\frac{\chi_I(\omega_0)}{\sqrt{1+\chi_R(\omega_0)}}$$

$$A(z) = A(0)e^{\pm\frac{\alpha}{2}z}$$

$$\alpha_{dB/km} = 10\log_{10}\left(\left(\frac{e^{-\alpha z}I(0)}{I(0)}\right)\right)^{-1}\Bigg|_{z=1km}$$

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-Lamkabeer Lambert-Beer

*LINEAR ATTENUATION

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$$I(z) = I_0\, e^{-\alpha z}$$

↳ the intensity within a lossy medium decays exponentially

@ $L = \alpha^{-1} \rightarrow I(z=L) = 0{,}37\, I_0$

$L \rightarrow$ Penetration length

BIRIFRANGENCE → The medium is not isotropic. The $\chi_x \neq \chi_y$

DISPERSION OF POLARIZATION → For a birifrangence medium, the group velocity in x and y can be different.

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- Self-phase modulation

-Cross-phase modulations

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$$A_{px}^{NL} = \frac{3}{4}\varepsilon_0 \chi^{(3)} \Big[\underbrace{\left(|A_x|^2 + \frac{2}{3}|A_y|^2\right) A_x}_{\text{SELF PHASE MODULATION}} + \underbrace{\frac{1}{3} A_x^{*} A_y^{2} e^{2j(\beta_{0y} - \beta_{0x}) z}}_{\text{CROSS PHASE MODULATION}} \Big]$$

SELF PHASE MODULATION ↓ The electric field polarized in the same coordinates modifies it phase.

CROSS PHASE MODULATION ↓ The electric polarized in y changes the phase of Ex.

EFFECTS NEGLECTED:

→ linear attenuation → Lambert-Beer

→ Effects due the electric field polarization
- Polarization dispersion
- self phase modulation
- cross phase modulation

→ Higher order dispersion

→ Nonlinear attenuation → $\chi^{(3)}$ complex

→ Nonlinear dispersion → $\chi^{(3)}(\omega)$

} NEGLECTED

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for a non-dispersive medium → n doesn't depend on f.

$$K_0'(r) = \frac{n(r)}{c} = \frac{1}{v_g(r)} \qquad \boxed{K_0'' = 0} \text{ GVD}$$

↳ GROUP VELOCITY
↳ velocity for the energy transport.

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-Sellmeler equation

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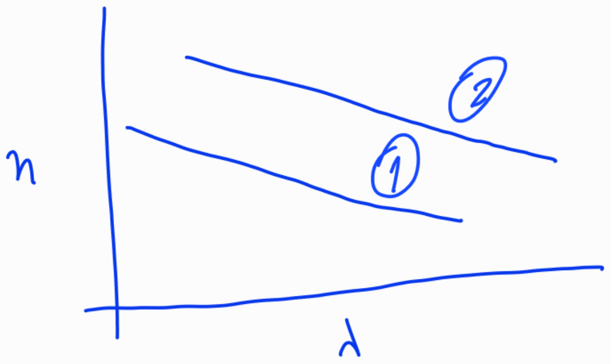
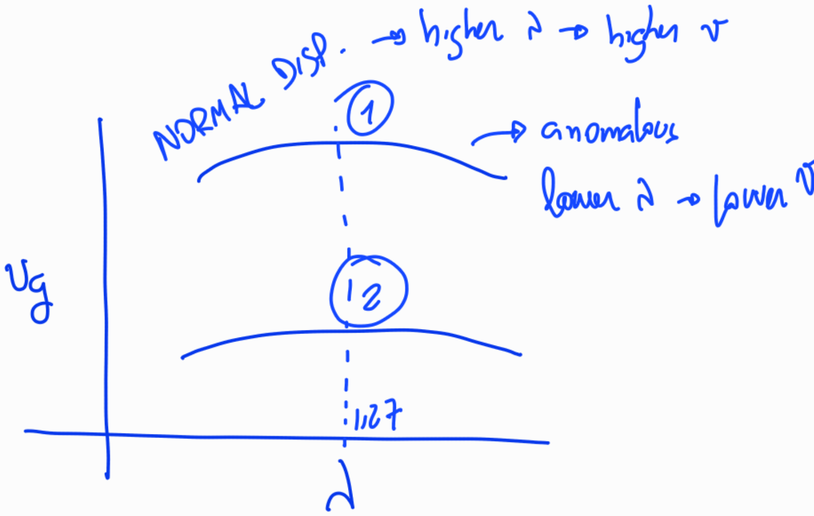
$$n^2(\lambda) - 1 = \sum_i A_i \frac{\lambda^2}{\lambda^2 - \lambda_i^2}$$

① 100% SiO_2	② 8% GeO_2, 92% SiO_2
A_1 A_2 A_3 λ_1 λ_2 λ_3	

$$v_g = \frac{1}{k_0'} = \left[\frac{\partial k}{\partial \omega}\bigg|_{\omega=\omega_0} \right]^{-1}$$





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-Show the graphics and explain what were done... (MATLAB)

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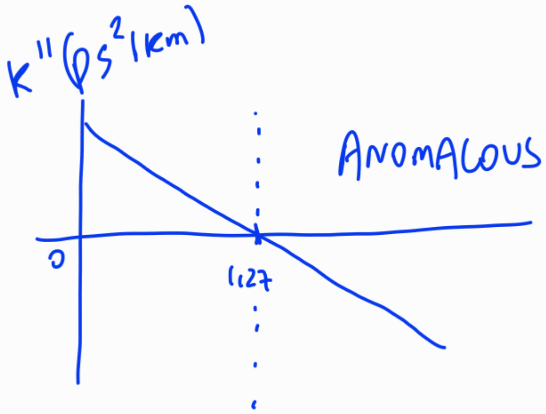
2 Regions of v_g:

1. v_g increases with λ ($\lambda < 1.27\ \mu m$) → for silica
   → NORMAL DISPERSION
   → higher λ are the fastest components

2. v_g decreases with λ ($\lambda > 1,27\ \mu m$ (for silica))
   → ANOMALOUS DISPERSION
   → lower λ are the fastest components

GVD



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- Non linear polarization

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- INTERACTION Length $\rightarrow$ The length where the nonlinear effect are active.

MONOMODAL FIBER — $a \sim 4 \mu m$

$\lambda = 1,55 \mu m \rightarrow \alpha \sim 0,2 dB/km$

Kerr effect in silica $\rightarrow$ $n = n_e + n_k I$ $\quad n_k = 3,2 \times 10^{-20} m^2/W$

$$I = \frac{2P}{\pi w_0^2}$$

can be neglected (very low)

Phase variation

$$\phi = \left(\frac{2\pi}{\lambda}\right)(n_e + n_k I)$$

$$\phi_{NL} = \left(\frac{2\pi}{\lambda}\right)(n_k I) z$$

$\hookrightarrow$ for $z = 60 km$ it can be introduced a phase shift of π.

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chapter 2

- definitions of optical fiber

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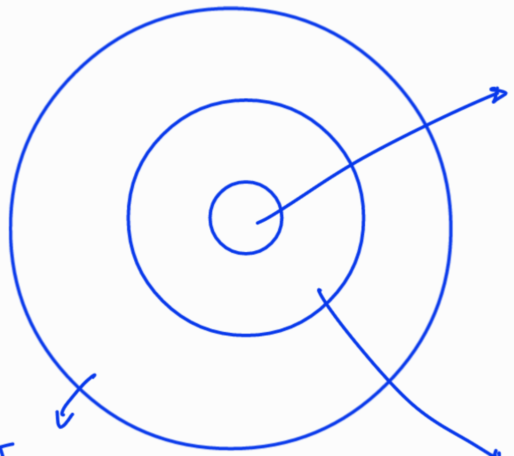

CORE

n_1 SiO_2 (can be doped)

To increase refractive index.

JACKET

Plastic Protection

CLADDING

n_2 SiO_2 with B_2O_3 To lower refractive index

MULTIMODE FIBER $\sim 2a \sim 50\mu m$ → INTERMODAL DISPERSION

MONOMODE FIBER $\sim 2a \sim 10\mu m$

$$NA = \sqrt{n_1^2 - n_2^2} = n_0 \sin \underbrace{\theta_{im}}_{\text{MAXIMUM ANGLE OF INCIDENCE.}} = n_1 \cos \phi_c$$

Snell's law: $n_0 \sin \theta_i = n_1 \sin \theta_r$

Total interna reflection : $\phi > \phi_c = a\sin\left(\frac{n_2}{n_1}\right)$

↳ critical angle

STEP REFRACTIVE INDEX PROFILE

SMOOTH " " "

↳ can be used for multimode fiber.

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- internal reflection

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- Attenuation of the fiber

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$$P(D) = P(0) \exp(-\alpha D)$$

LINEAR

$$\alpha\,[dB/km] = -\frac{10}{D} \log_{10} (\exp(-\alpha D))$$

ATTENUATION – 2 CONTRIBUTIONS
- INTRINSIC – properties of the medium
- EXTRINSIC – all other reasons.

Every material has the maximum absorption in proximity of its own resonance frequencies.

for silica : electronic resonance → $\lambda < 0.4\,\mu m$ (ultraviolet)
vibration resonance → $\lambda > 7\,\mu m$ (infrared)

Transmission window → $0{,}8\,\mu m < \lambda < 1{,}6\,\mu m$

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- Attenuation in dB/km... there are some examples

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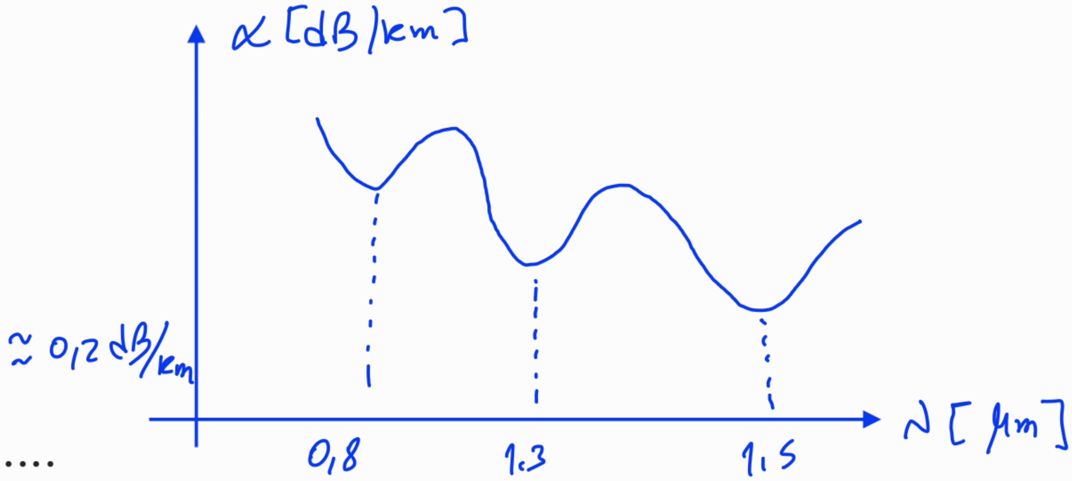

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-Types of dispersion

- Light is distorted during propagation. The different spectral components of the signal propagate with different group velocity.

DISPERSION PARAMETER : $D = \frac{d\left(\frac{dk}{d\omega}\right)}{d\lambda}$

$\lambda = \frac{c}{f} \rightarrow \lambda f = c \rightarrow 2\pi \lambda f = c \cdot 2\pi \rightarrow \lambda = \frac{2\pi \cdot c}{\omega}$

$d\lambda = -\frac{2\pi \cdot c}{\omega^2} d\omega \rightarrow d\lambda = -\frac{2\pi c}{\frac{(2\pi c)^2}{\lambda^2}} d\omega \rightarrow$

$d\lambda = -\frac{\lambda^2}{2\pi c} d\omega$

$D = \frac{\lambda}{\frac{-\lambda^2}{2\pi c}} \frac{d^2k}{d\omega^2} \rightarrow \boxed{D = \frac{-2\pi c}{\lambda} \frac{d^2k}{d\omega^2}}$ $\left[\frac{ps}{nm \cdot km}\right]$

→ MATERIAL DISPERSION

→ INTRAMODAL DISPERSION

→ POLARIZATION DISPERSION – Geometrical imperfections or small curvatures of the fiber may cause that 2 ortogonal polarizations travel with different group dispersions.

→ TRANSMITTED SIGNAL POWER

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- Guided modes

$$A(x,y,z,t) = \underbrace{F(z,t)}_{\text{Temporal evolution}} \underbrace{M(x,y)}_{\text{spatial}} e^{j\delta\beta z}$$

spatial → propagation along z as function of x, y.

$\delta\beta$ → takes into account the modification of the propagation constant of the wave due the presence of guiding structure.

$M(x,y)$ → Modal profile $M(r,\phi)$

$\beta = \beta_0 + \delta\beta$

Weakly guiding → $k_0 \simeq \beta_0$.

......

-what is M, mode profile

$$2j\beta_0 \frac{\partial A}{\partial z} + \underbrace{\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}}_{\text{Difraction}} + \Big(\underbrace{\frac{3}{4}\frac{\omega_0^2}{c^2}\chi^{(3)}|A|^2}_{\text{Kerr effect}} + \underbrace{k_0^2 - \beta_0^2}_{\text{guiding}}\Big)A - \underbrace{2jk_0k_0'\frac{\partial A}{\partial \tau}}_{\text{GV}} - \underbrace{k_0k_0''\frac{\partial^2 A}{\partial \tau^2}}_{\text{GVD}} = 0$$

$$n_2 = \frac{3}{8n}\chi^{(3)}$$

$$\begin{cases} 2j\beta_0 \dfrac{\partial F(z,\tau)}{\partial z} + 2\dfrac{k_0^2}{n} n_2 \left|F(z,\tau)M(x,y)\right|^2 F(z,\tau) - 2j\beta_0 k_0' \dfrac{\partial F(z,\tau)}{\partial \tau} - \beta_0 k_0'' \dfrac{\partial F}{\partial \tau} = 0 \\ \dfrac{\partial^2 M}{\partial x^2} + \dfrac{\partial^2 M}{\partial y^2} + (k_0^2 - \beta_0^2 - 2\beta_0\delta\beta)M = 0 \end{cases}$$

↳ LP MODES APPROXIMATION $\boxed{LP_{\ell m}}$

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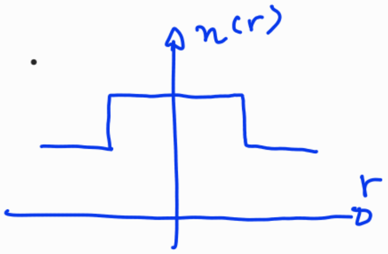
- Solution in dispersion relation for monomode



$$k_0^2(r) = \begin{cases} n_1^2 \omega^2 \mu_0 \varepsilon_0 & 0 \le r < a \\ n_2^2 \omega^2 \mu_0 \varepsilon_0 & a \le r < +\infty \end{cases}$$

$$M_\ell(r) = \begin{cases} A_1 J_\ell(k_t r) & 0 \le r < a \\ A_2 K_\ell(\gamma r) & a \le r < +\infty \end{cases}$$

$$k_t^2 = n_1^2 \omega^2 \mu_0 \varepsilon_0 - \beta_0^2 - 2\beta_0 \delta\beta_\ell \qquad \gamma^2 = \beta_0^2 + 2\beta_0 \delta\beta_\ell - n_2^2 \omega^2 \mu_0 \varepsilon_0$$

DISPERSION RELATION
$$\frac{X J_\ell'(X)}{J_\ell(X)} = \frac{Y K_\ell'(Y)}{K_\ell(Y)}$$

$$X = k_t a \qquad Y = \gamma a$$

$$X^2 + Y^2 = V^2 = \left(\frac{2\pi a}{\lambda}\right)^2 (n_1^2 - n_2^2) = \left(\frac{2\pi a}{\lambda} NA\right)^2$$

↳ NORMALIZED FREQUENCY

LHS RHS

$$-\frac{X J_{\ell-1}(X)}{J_\ell(X)} = \frac{Y K_{\ell-1}(Y)}{K_\ell(Y)}$$

J → FIRST ORDER BESSEL FUNCTION

K → SECOND " " "

......

- condition for single mode

· for $l = 0$

$LP_{01} \rightarrow$ FIRST MODE AND ALWAYS EXISTS

THE SECOND MODE ONLY OCCURS FOR $V > 3.832$

SO, $LP_{02} \rightarrow V = 3{,}832$

for $l = 1$

LP_{11} OCCURS AT $V = 2.405$

Apart from the fundamental mode LP_{01}, LP_{11} is the one with the lowest frequency

$$\boxed{V < 2.405 \Rightarrow \text{MONOMODAL FIBER}}$$

......

- propagation constant

· From The solution of the dispersion relation, (intersection between LHS and RHS)

we find $X \rightarrow$ find $k_T = \frac{X}{a}$

$$\beta_l = \beta_0 + \delta\beta_l = \frac{n_1^2 w^2 \mu_0 \varepsilon_0 - (k_T)^2}{2\beta_0} + \frac{\beta_0}{2}$$

$\hookrightarrow$ INITIAL GUESS: $\omega n_2 \sqrt{\mu_0 \varepsilon_0} \leq \beta_0 \leq \omega n_1 \sqrt{\mu_0 \varepsilon_0}$

From β_l (correct propagation constant)

↳ Find $n_e(\omega) = \dfrac{\beta(\omega)}{\omega\sqrt{\mu_0\varepsilon_0}}$

Now the fiber may be seen as an homogeneous material with effective index n_e.

......

- show the figure 2.7 in matlab... explain how it was done

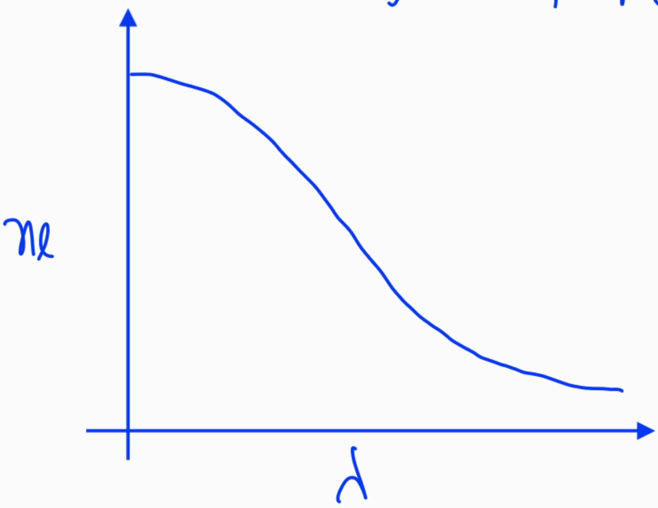
.

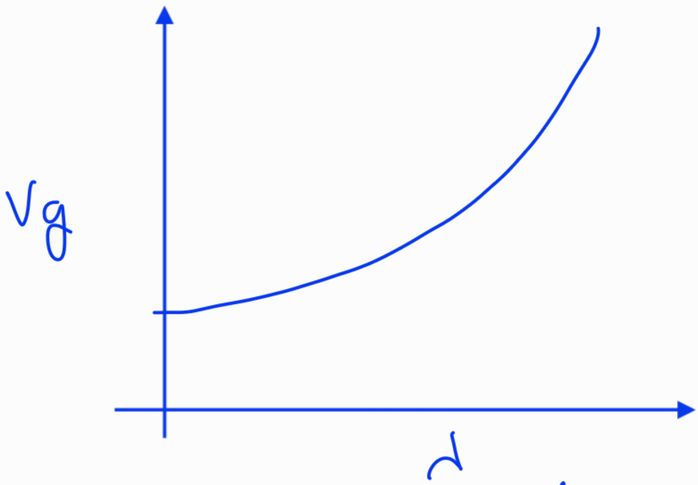
......

- effective refractive index, dependence with the wavelength

. If V increases the mode is more confined in the core (figure 2,8 b) → The mode is more guided

- V affects $\begin{cases} \text{the modal profile (eigenfunctions)} \\ \text{The propagation constant (eigenvalue)} \end{cases}$





$$V_g = \left[\frac{\partial\beta}{\partial\omega}\bigg|_{\omega=\omega_0}\right]^{-1}$$

......

- graphics and gaussian approximation... plot the graphics

. LP_{01} can be approximated with the gaussian function:

$$M(r) = \exp\left(-\left(\frac{r}{w_0}\right)^2\right)$$

$W_0 \to$ Modal Radius

$$\frac{W_0}{a} \simeq 0.65 + 1.619 V^{-\left(\frac{3}{2}\right)} + 2.879 V^{-6}$$

$$n_e \simeq n_2 + (n_1 - n_2)\left(1.1428 - \frac{0.996}{V}\right)^2$$

VALID FOR:

$2.0 < V < 2.4$

......

- what is the propagation equation, NLSE, explain each term

2DNLSE

$$j\frac{\partial F(z,\tau)}{\partial z} + \left(\frac{\omega_0 n_2}{c S_{eff}}\right)\left|F(z,t)\right|^2 F(z,\tau) - j\beta'\frac{\partial F(z,\tau)}{\partial \tau} - \beta''\frac{\partial^2 F}{\partial \tau^2} = 0$$

ANSWERING BY MYSELF

400nm − 700nm → visible light

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = \frac{\partial D}{\partial t}$$

$$\nabla \cdot D = 0$$

$$\nabla \cdot B = 0$$

$$\nabla^2 E = \frac{1}{c^2}\left(\frac{\partial E}{\partial t} + \frac{1}{\varepsilon_0} P\right)$$

$$P = P^L + P^{NL}$$

$$P(r,t) = \varepsilon_0 \chi \overset{t}{*} E(r,t)$$

$$P(r,f) = \varepsilon_0 \chi E(r,f)$$

if the medium responds instantaneously

ISOTROPIC:
The medium doesn't change the response if E changes.

$$v_p = \frac{c}{n(f)}$$

$n(f)$ → matrix (because χ)
→ Scalar if isotropic. χ is a identity matrix

$1 \leq n(f)$ $\quad \lim_{f \to \infty} n(f) \to 1$ } $f \to \infty \to d \to 0 \to$ vaccum.

P_{NL} occurs when $\vec{E}$ is very intense. Then P is no longer proportional to $\vec{E}$.

$$P = P^L + P^{NL} = \varepsilon_0 \chi^{(1)} E + \underbrace{\cancel{\varepsilon_0 \chi^{(2)}} E^2}_{SHG} + \underbrace{\varepsilon_0 \chi^{(3)} E^3}_{THG}$$

$$\hookrightarrow P^{NL} = \varepsilon_0 \chi^{(3)} E^3$$

Centrosymmetric material → The properties doesn't change if we make the transformation $r \to -r$. $P \parallel E$. There's no SHG due to the symmetricallity of P and E. EX: Silica

---

Kerr effect → the (high) intensity of $\vec{E}$ can change n. due $|A|^2$ n is modified

$$\left(\frac{3}{4}\frac{\omega_0^2}{c^2}\chi^{(3)}|A|^2 + k_0\right)A \qquad n_2 = \frac{3}{8n}\chi^{(3)}$$

$$\left(2\frac{\omega_0^2}{c^2} n_2 \cdot n |A|^2 + \frac{\omega_0^2}{c^2} n^2\right)A \to \frac{\omega_0^2}{c^2}\left(\underline{2 n_2 n |A|^2} + n^2\right)A$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \longrightarrow \nabla \times \nabla \times E = -\frac{\partial}{\partial t} \nabla \times B$$

$$\nabla \times H = \frac{\partial D}{\partial t} \qquad -\nabla^2 E - \cancelto{0}{\nabla(\nabla \cdot E)} = -\mu_0 \frac{\partial}{\partial t} \nabla \times H$$

$$\nabla^2 E = \mu_0 \frac{\partial^2 D}{\partial t^2}$$

$$\nabla^2 E = \mu_0 \frac{\partial}{\partial t^2}\left(\varepsilon_0 \vec{E} + P\right)$$

$$\boxed{\nabla^2 E = \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\left(\vec{E} + \frac{1}{\varepsilon_0} P\right)}$$

Simple case

- Linear $P \parallel E$
- Homogeneous $\chi(\ell) \to$ don't depend on r
- No losses $Im(\chi(\ell)) = 0$
- E sinusoidal

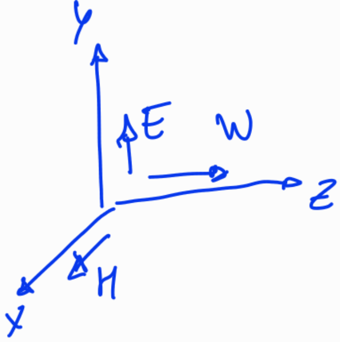
DISPERSION RELATION: $\beta_0^2 = \frac{\omega_0^2}{c^2} n^2(\ell) = \frac{\omega_0^2}{c^2}(1 + \chi(\ell))$

	Free Space	Medium
Refractive index	$n = 1$	$n(\ell)$
Speed of light	c	$v = \frac{c}{n(\ell)}$
wavelength	λ_0	$\lambda = \frac{\lambda_0}{n(\ell)}$
Propagation cte	$\beta = \pm\frac{2\pi}{\lambda_0}$	$\beta = \pm\frac{2\pi n(\ell)}{\lambda_0}$

$\beta < 0 \to$ prop. in z

$\beta > 0 \to$ prop. in $-z$

$$\beta = \frac{\omega^2}{c^2} = \frac{2\pi f^2}{\lambda f^2} = \frac{2\pi}{\lambda_0}$$



$$\vec{W} = \vec{E} \times \vec{H}$$

$$\beta_0 = \frac{\omega_0}{c} \qquad \eta_0 = \sqrt{\frac{\mu}{\varepsilon}}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Envelope function and complex amplitude

$A(r,\tau) \rightarrow$ slow variant in space and time

The sinusoidal in time $\vec{E}$ has two components in freq.

1. Filter the negative frequency component
2. Translate to the positive component

Same info but with only one component.

Wave equation for A.

$$\nabla^2 A + 2j\beta_0 \frac{\partial A}{\partial z} - \beta_0^2 A = \frac{-\omega_0^2}{c^2} A_1 + \frac{2j\omega_0}{c^2} \frac{\partial A_1}{\partial \tau} + \frac{1}{c^2} \frac{\partial^2 A_1}{\partial \tau^2}$$

NLSE

$\rightarrow$ linear polarization component + NL polariz. comp.

$$P = P^{L} + P^{NL}$$

$$= \varepsilon_0 \sum \frac{\hat{\chi}^{(n)}_{(r,f_0)}\, \partial^n A(r,\tau)}{(j2\pi)^n\, n!\, \partial t^n} + \frac{3}{4} \frac{\omega_0^2}{c^2} \chi^{(3)} |A|^2 A$$

Wave number:

$$k_0(\omega) = \frac{\omega_0^2}{c^2}(1+\chi(\omega)) \text{ if lossless, } k_0 \approx \beta_0$$

Homogeneous

$$k_0' = \frac{d k_0}{d\omega} = \frac{2\omega_0}{c^2}(1+\chi(\omega)) + \frac{\omega_0^2}{c^2} \hat{\chi}'$$

$$k_0'' = dk_0'/d\omega = \frac{\omega_0^2}{2c^3} \hat{\chi}'' + \frac{2\omega_0}{c^2} \chi' + \frac{1}{c^3}(1+\hat{\chi})$$

$$\underbrace{2j\beta_0\frac{\partial A}{\partial z}}\ +\underbrace{\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}}_{\text{DIFRACTION, Space broadening}}+\Big(\underbrace{\frac{3}{4}\frac{\omega_0^2}{c^2}\chi^{(3)}|A|^2}_{\text{kerr effect}}+\underbrace{k_0^2-\beta_0^2}_{\text{guiding}}\Big)A\underbrace{-2j\,k_0k_0'\frac{\partial A}{\partial \tau}}_{\text{Group Velocity}}\underbrace{-k_0k_0''\frac{\partial^2 A}{\partial \tau^2}}_{\text{GVD, Temporal Broadening}}=0$$

1^{st} order $\rightarrow$ neglect all derivatives and slow variant components.

$$\beta_0^2=\frac{\omega_0^2}{c^2}\left(1+\hat{\chi}(f)\right) \rightarrow \text{DISPERSION RELATION}$$

2^{n} order $\rightarrow$ $2j\beta_0\frac{\partial A}{\partial z} = -2j\,k_0k_0'\frac{\partial A}{\partial \tau}$

$\boxed{\beta_0 \sim k_0}$ lossless linear / spatial dependence of χ is negligible.

$$\frac{\partial A}{\partial z} = k_0'\frac{\partial A}{\partial \tau} \qquad v_g=\frac{1}{k'}$$

Lambert-Beer

$$I(z)=I(0)\,e^{\alpha z} \qquad \text{LINEAR ATTENUATION}$$

$L=\alpha \rightarrow I(z)=0{,}37\,I(0)$

$\hookrightarrow$ Penetration lenght

BIREFRINGENCE $\rightarrow$ $\chi_x \neq \chi_y$ $\rightarrow$ different fields in x and y can cause polarization dispersion,

CROSS-PHASE MODULATION

$$A^{NL}_{px}=\frac{3}{4}\varepsilon_0\chi^{(3)}\Big[\underbrace{|A_x|^2+\frac{2}{3}|A_y|^2}_{SPM}\Big)A_x+\underbrace{\frac{1}{3}A_x^*A_y^2e^{2j(\beta_{0y}-\beta_{0x})z}}_{XPM}$$

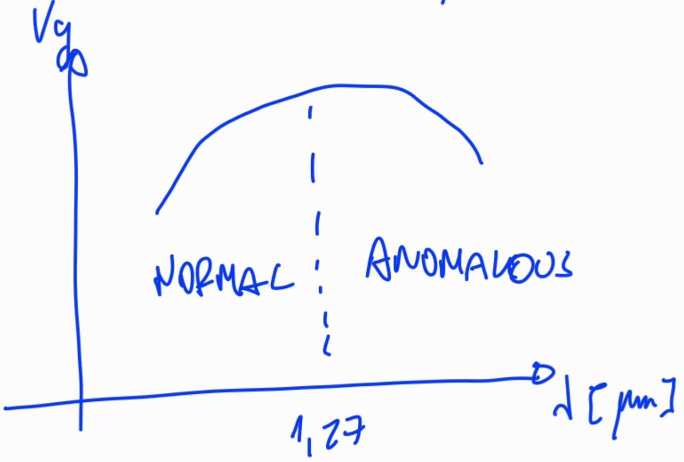
non-dispersive medium → n doesn't depend on f

$$k_0'(\nu) = \frac{n(\nu)}{c} = \frac{1}{v_g(\nu)}$$

v_g → velocity of energy transportation

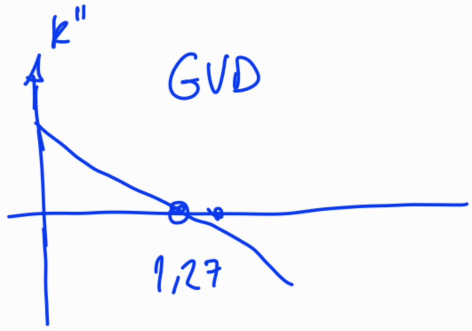
$$\boxed{k_0'' = 0}$$ GVD

$$n^2 - 1 = \sum_{i=0} A_i \frac{\lambda^2}{\lambda^2 - \lambda_i^2}$$ SELLMEIER EQUATION



NORMAL → bigger λ
↓
bigger v

ANOMALOUS → smaller λ
↓
bigger v

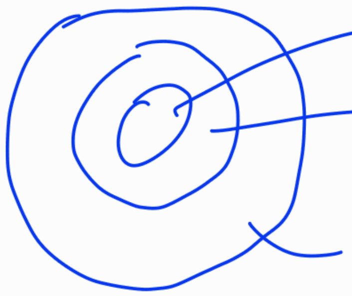


Interaction length → The length where the nonlinear effects are active

Phase variation → 2π @ 60km

Attenuation → 0,2dB/km

Chapter 2



core SiO_2 n_1

cladding n_2

JACKET

$$\theta_c = a\sin \frac{n_2}{n_1}$$

$$\alpha \left[\frac{dB}{km}\right] = -\frac{10}{D}\left(\log_{10} e\right.$$

$$P(D) = P(0)\, \exp(-\alpha D)$$

$$\alpha_{\frac{dB}{km}} = 10 \log_{10} \frac{P_0 \cdot e^{-\alpha D}}{P_0}$$

$$= -\alpha D \cdot 10 \log e$$