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Photonics group (xlim)

Fiber optics

Fiber laser and amplifiers

Nonlinear optics in fiber

The idea is to turn an optical fiber into an active device (a laser) amplifier

Thank to the fluorescence phenomena we can fabricate a fiber oscillator

The physical principles at stake in the fiber amplifier

Examples: Erbium - doped fiber amplifier

I Fabrication of optical fiber

Glass : • Inorganic product of fusion cooled to a rigid compound

• amorphous ≠ crystalline

• SiO_2 ↳ silica

↳ synthetized

↳ purest material ever created by humankind

↳ pollution < 1 ppb (part per billion) ⇒ used for long - haul telecommunic.

attenuation $\alpha \simeq 0.2$ dB/Km

$Loss = \alpha \underset{100\,km}{L} = 20\,dB \simeq 1\%$ transmission

• GeO_2 also used in optical fibers

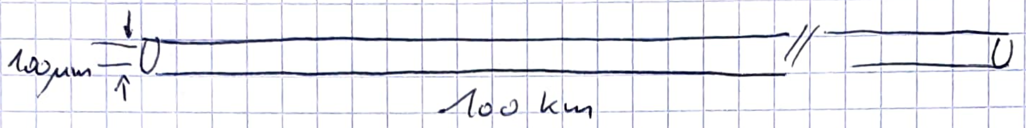
• GeO_2 or SiO_2 : passive glasses, no fluorescence

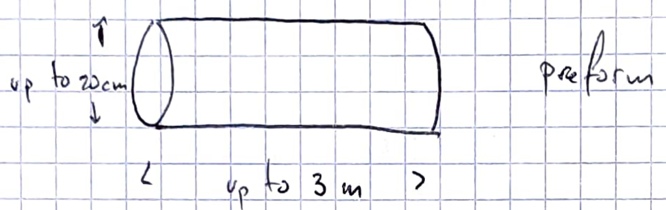
A few elements emit light when they are incorporated into such glasses

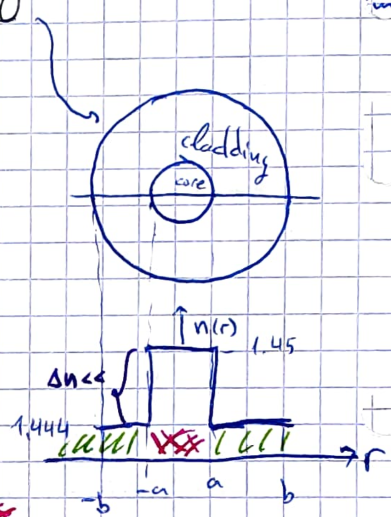
Er^{3+} Yb^{3+} Nd^{3+} Ho^{3+} Tb^{3+} ...

We have to put (to dope) with this element to produce light

The long is not higher than 100km, after that you should put something to increase light







pure fused silica
SiO_2

$(1-x)\,SiO_2 + x\,GeO_2$

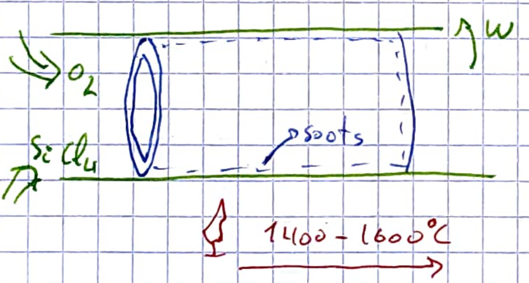
$x \simeq$ a few %

$SiCl_4 + O_2 \xrightarrow{1400°C} SiO_2 + 2Cl_2$

$GeCl_4 + O_2 \xrightarrow{1400°C} GeO_2 + 2Cl_2$

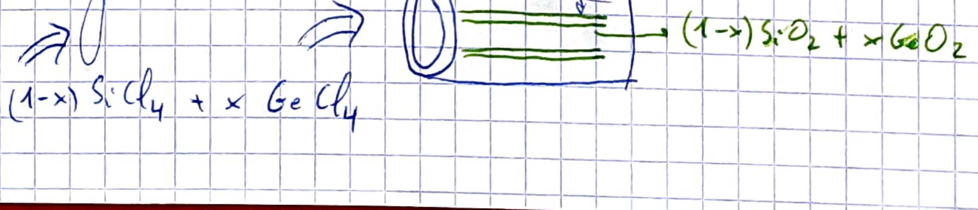
Soots

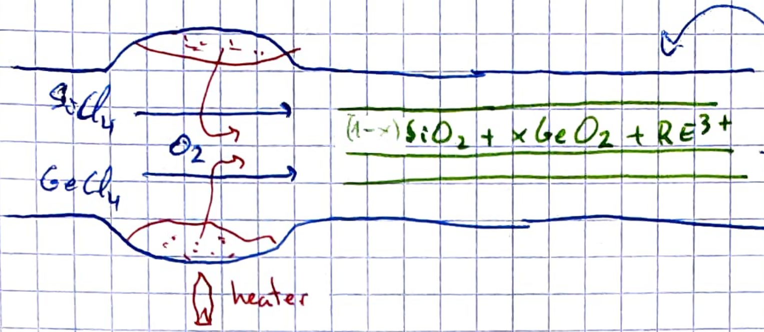
The temperature is raised again to utility the soots



Charles KAO Nobel Physics 2009

After you use:



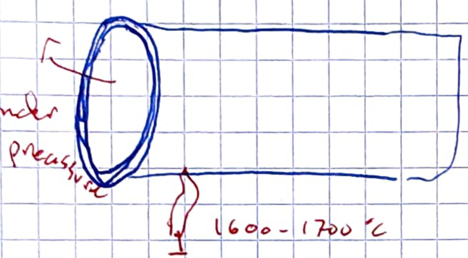


Impurity sought for

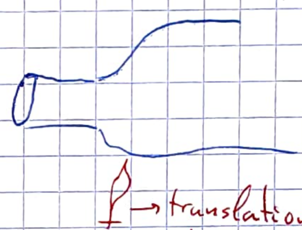
RE^{3+} a few ppm

RE chloride
(rare earth)

$RE - Cl_3\ 6H_2O$

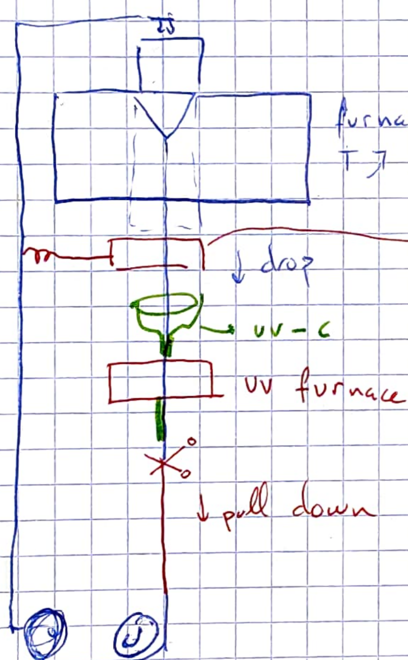


it collapse



rod

strand of glass



$\boxed{\text{Fiber drawing tower}}$

diameter measurement

125 µm

polymer

II Spectroscopy of rare-earths: Light matter interactions

An ensemble of N ions : our lasing medium

at least 2 energy levels such that their difference in energy is $\boxed{\Delta E = h\nu_{12} = h\frac{c}{\lambda_{12}}}$ (Planck's formula)

The population of the upper level is N_2

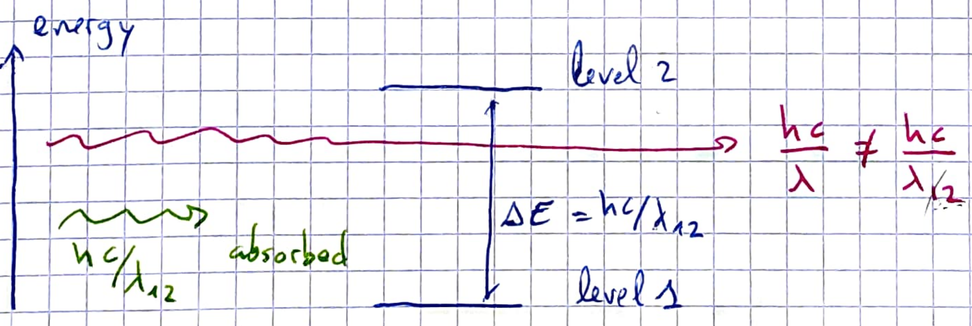
——— lower ——— N_1

$\forall t \quad N_1 + N_2 = N$

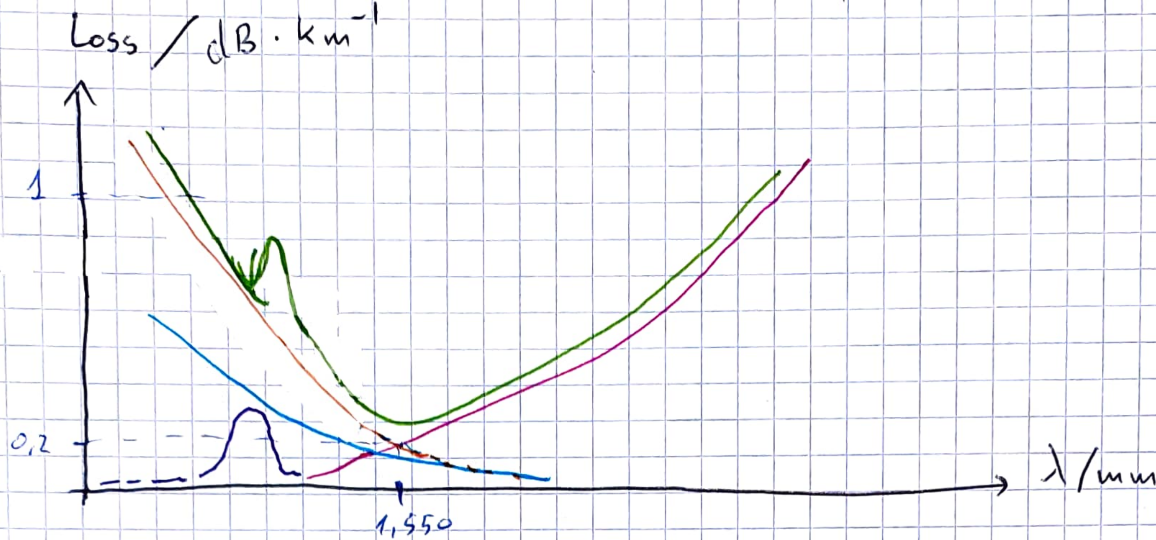
material

- We consider photons travelling in this medium
- The 3 light-matter interactions
  - → absorption
  - → stimulated emission
  - → spontaneous emission

II.1 Absorption



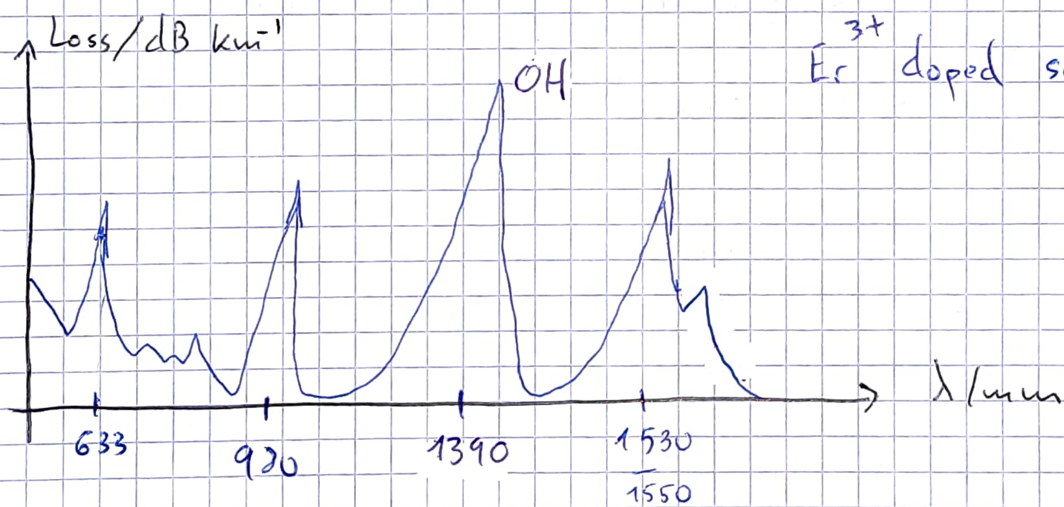
Rare-earth doped silica glass (made by MCVD)

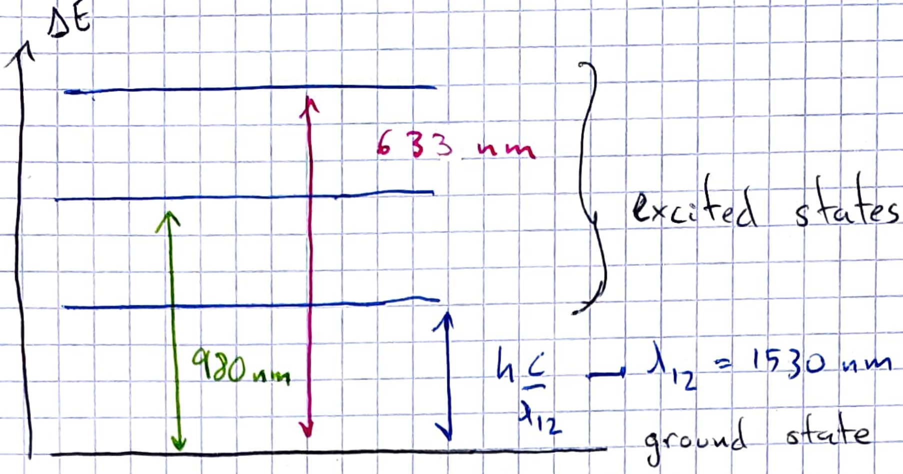


The higher the probability of absorption the higher the loss

- Rayleigth scattering (λ^{-4})
- UV absorption
- impurities OH^-
- IR absorption
- pure Silica

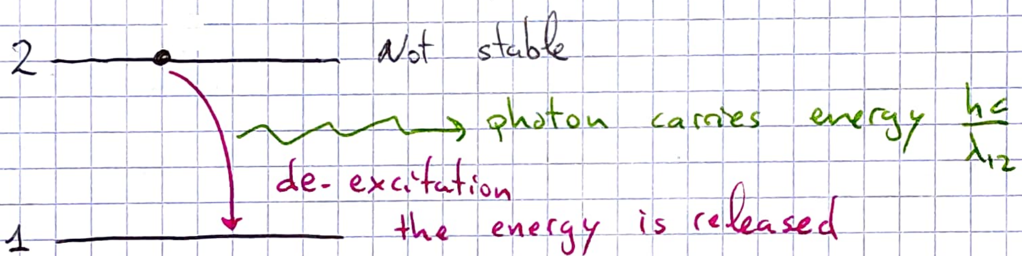
Er^{3+} doped silica fiber

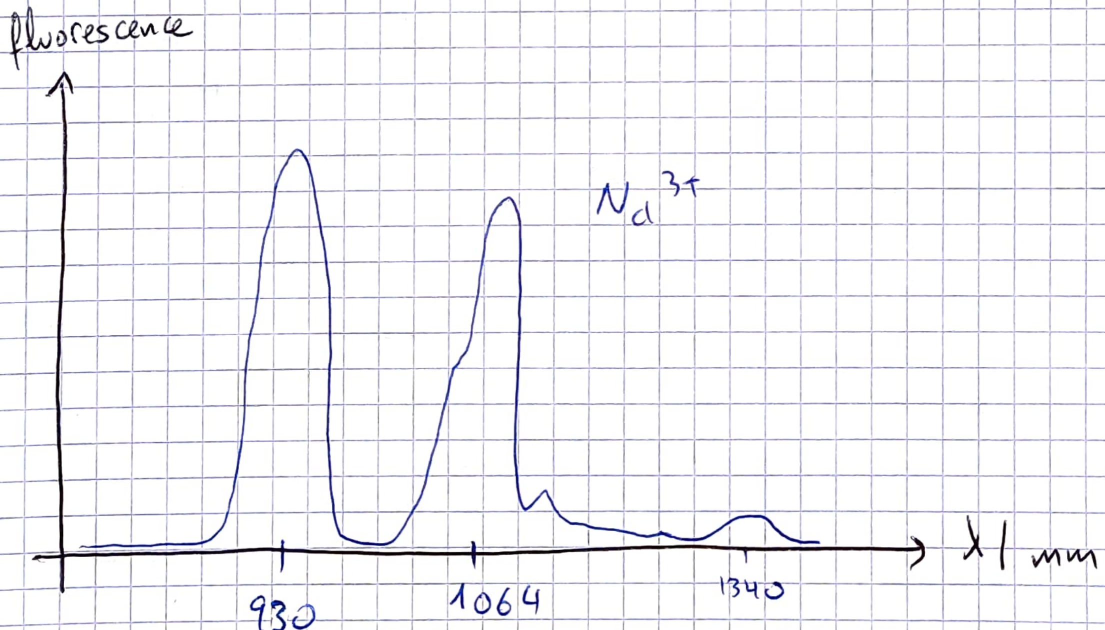




We will use these absorption transition to transfer energy from the outside to the fiber.

II.2 Spontaneous Emission



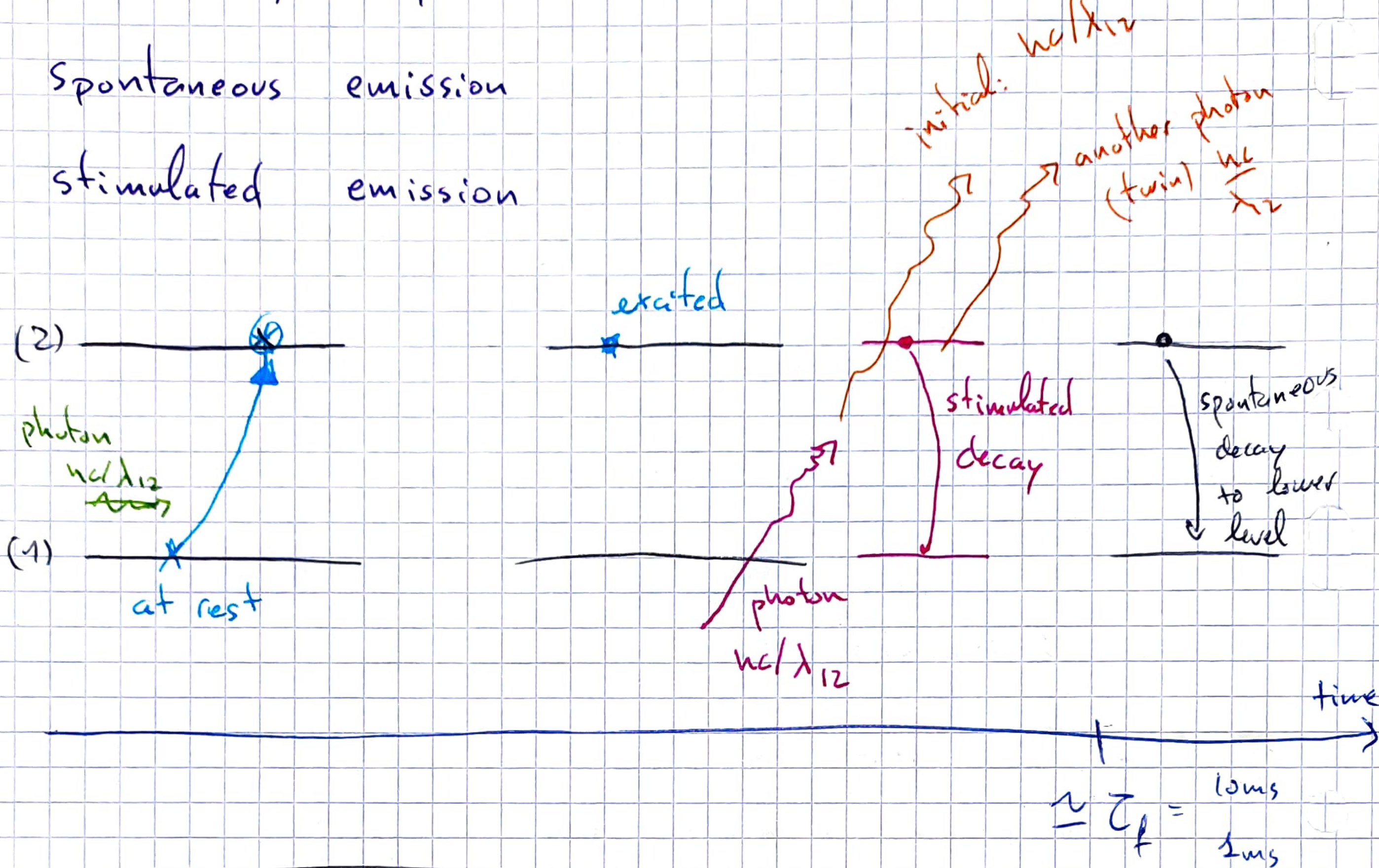


Nd : YAG

(stimulated) absorption

spontaneous emission

stimulated emission



The initial and twin

same: energy / frequency / λ
direction
polarization

Light amplification by stimulated emission of radiation.

III Population Inversion

III.1 Two-level laser system

N_1, N_2 population densities

At thermal equilibrium (Boltzmann's laws)

$$\frac{N_2}{N_1} = \exp - \left(\frac{E_2 - E_1}{kT}\right)$$

at $T = 300\,K$ $kT = 0.026\,eV$

Consider $\lambda_{lasing} = 1\,\mu m$

Planck's law $\Delta E = \frac{hc}{\lambda} = 1.29\,eV \gg kT$

$$\frac{N_2}{N_1} \to 0$$

There are no atoms in the higher level

$T \to \infty$ $\frac{1}{kT} \to 0$ $\frac{N_2}{N_1} \to 1$

In a 2-level laser system, the probability of a photon causing stimulated emission is (at best) the same as that of a photon causing absorption.

There is no net gain

if $N_2 = N_1$ the material is transparent

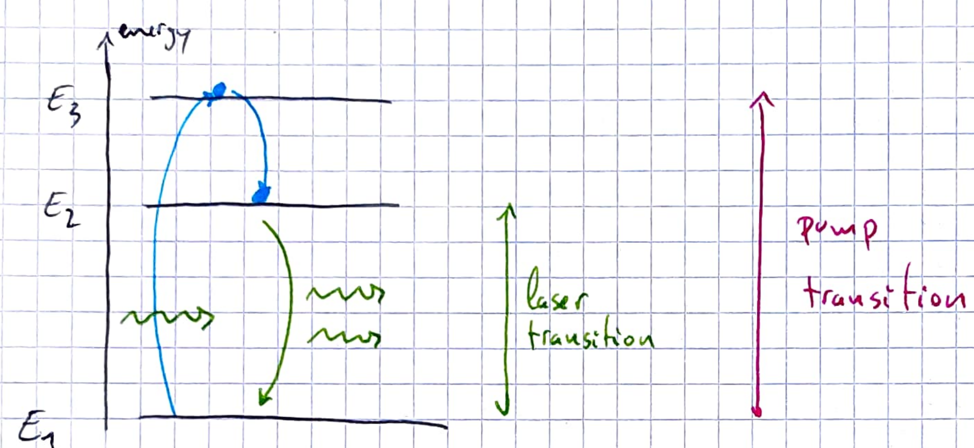
In general $N_2 < N_1 \Rightarrow$ Absorption dominates

III. 2. Three-level laser system

We need an extra level to ensure $N_2 > N_1$ a group of N atoms but able to exist in one of 3 possible levels.

N_1, N_2, N_3 such that $N_1 + N_2 + N_3 = N$ at al times.

$E_3 > E_2 > E_1$



a photon with freq. ν_{13} comes first.
It is absorbed. The e^- is promoted to E_3

The energy is released so that the e^- goes down to E_2

$\Rightarrow N_2 > N_1$ This is population inversion

For this to happen: Transition from (3) $\rightarrow$ (2) must be very fast

(1) $\rightarrow$ (2) is not an optical transition

is accompanied with the emission of an acustic particle

a phonon

GHz $\leftrightarrow$ ns

IV. Rate Equations

Applying a signal to a collection of atoms which freqency is tuned to the resonance frequency of the atomic transition will cause variations of the populations $N_1(t)$ and $N_2(t)$

The rates of changes are given the (atomic) rate equations (some kind of prey-predator model or Lotka-Volterra equations)

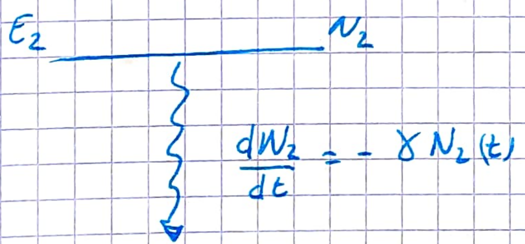
These equations are very useful to predict the laser behaviour
- what is the power threshold?
- Saturation effects (gain contraction)

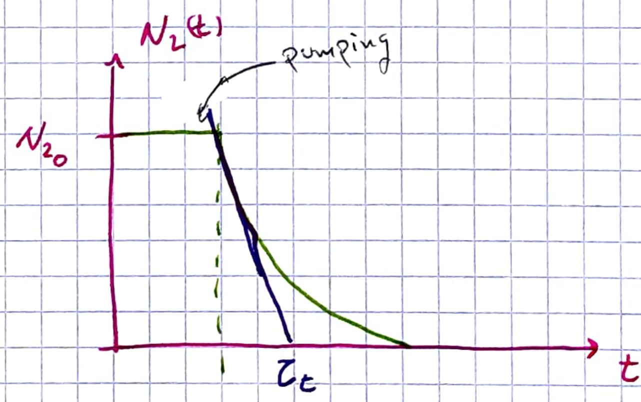
IV 1 Spontaneous energy decay or relaxation

$$\frac{dN_2}{dt} = -\gamma_2 N_2(t) = -\frac{N_2(t)}{\tau_2}$$

If an initial number of atoms N_{20} are pumped into the level 2 at $t=0$, and the pumping process is turned off:

$$N_2(t) = N_{20}\, e^{-t/\tau_2}$$



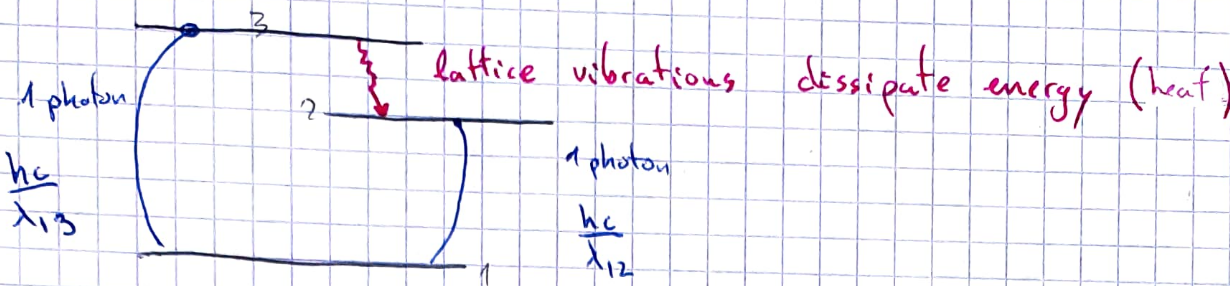


IV. 2 Stimulated transition rates

$$\frac{dN_2}{dt} = \underbrace{\overset{\text{increase } N_2}{+} W_{12} N_1}_{\text{absorption from level 1}} \underbrace{\overset{\text{decrease } N_2}{-} W_{21} N_2}_{\text{stimulated emission depopulates level 2}}$$

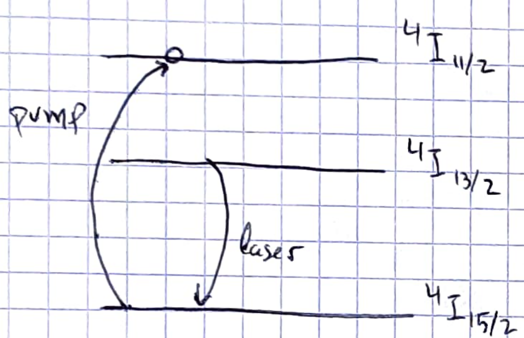
W_{ij} units is: s^{-1}

There is some energy losses:



$$\eta = \frac{\lambda_{12}}{\lambda_{13}} = 60\%$$

Example: Er^{3+} : SiO_2 (Er diluted in silica)

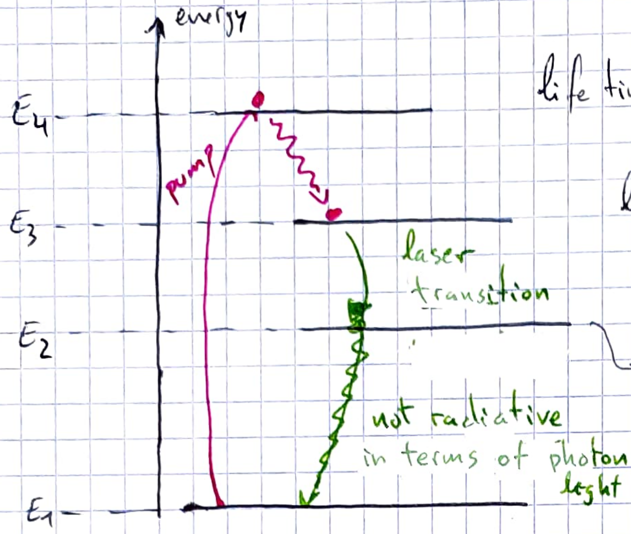


pumping transition: $\lambda_{13} = \lambda_p = 0.98\,\mu m$ (see absorption spectrum)

laser : $\lambda_{12} = \lambda_s = 1.55\,\mu m$ (see fluorescence spectrum)

III. 3 : 4-level laser system

Ex : Nd^{3+} : YAG



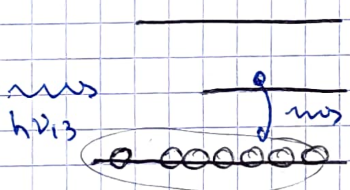
life time of this level is very short (ms)

lifetime of the higher level of the laser transition is long (ms)

↳ life time is extremely small << 1 ⟺ population is 0.

The advantage over 3-level system

↳ 3-level laser system

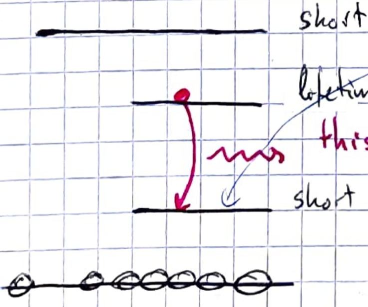


lower state of the laser transition

fundamental ground rest : 10^{24} atom/m^3

In this system absorption fights against emission, is very likely the absorption to happen. These atoms can absorb

↳ 4-level laser system



So because this level is not populated there won't be absorption

lifetime of this one is long

this one will propagate (and won't be absorb)

The ground state is not involved in the laser transition

V Stimulated transition cross-section

Suppose a particle with a capture area (cross-section, σ) illuminated with optical wave having an intensity

$$I = \frac{P}{A}$$

P → power

A → surface area

The net absorbed power ΔP_{abs} by the object is defined:

$$\boxed{\Delta P_{abs} = \sigma \frac{P}{A}}$$

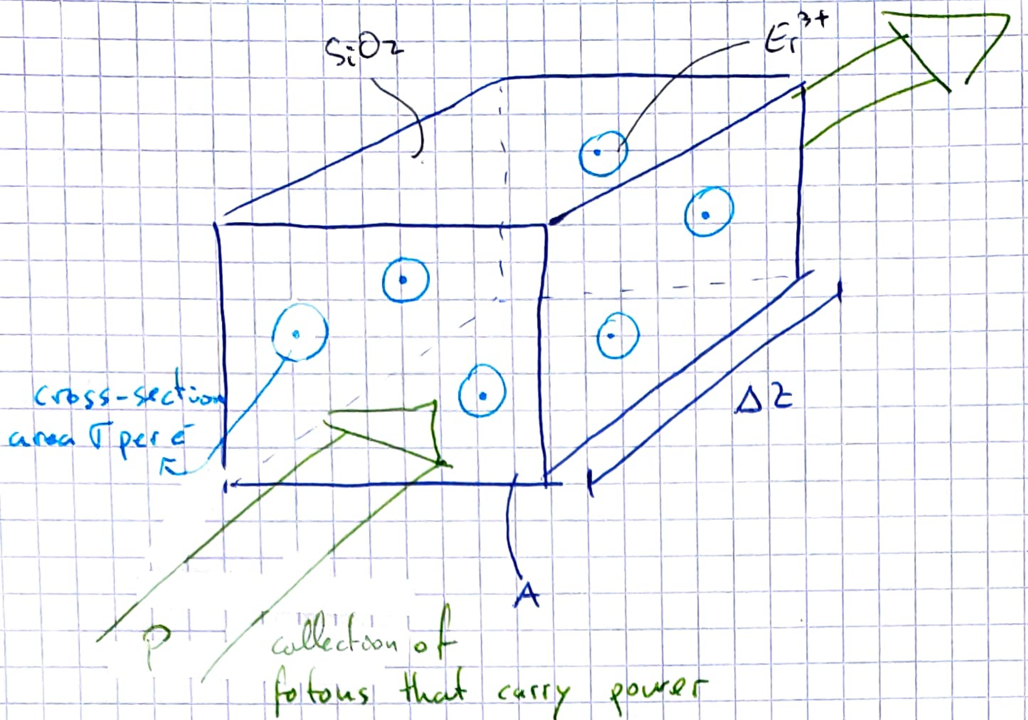
V.1 Power equations

Consider a thin slab of thickness Δz and transverse area A, containing densities of N_1 and N_2. Each atom in the lower state has an absorption cross-section σ_{12} and an emission cross-section σ_{21}

Each electron has its own cross-section but for simplicity we assume it is the same.

We have a piece of material and its electrons

↘



The total number of e^- in the slab: $N_1 A\, dz$

Then the total absorbing area $S_{abs} = N_1 A\, dz * \sigma_{12}$

The absorbed power $\Delta P_{abs} = P \cdot \frac{S_{abs}}{A} = \frac{P N_1 \cancel{A} dz\, \sigma_{12}}{\cancel{A}}$

Similarly, the total emitting area is $N_2 A\, dz\, \sigma_{21}$

The emitted power: $\Delta P_{em} = P N_2 \sigma_{21} dz$

$\Rightarrow \quad \Delta P = (N_2 \sigma_{21} - N_1 \sigma_{12})\, dz\, P$

The net growth or decay with distance caused by an atomic transition

$$\frac{dP}{dz} = (N_2 \sigma_{21} - N_1 \sigma_{12})\, P$$

$$\Updownarrow$$

$$\frac{dI}{dz} = (N_2 \sigma_{21} - N_1 \sigma_{12})\, I$$

V.2 Amplification coefficient

$$I(z) = I(z_0)\, e^{-2\alpha_m (z-z_0)}$$

This corresponds to the ODE $\frac{dI}{dz} = -2\alpha_m I$

↳ the attenuation coefficient : $2\alpha_m$

$$2\alpha_m = N_1 \sigma_{12} - N_2 \sigma_{21} = -(N_2 - N_1)\sigma = -\Delta N \sigma$$

if $2\alpha_m > 0$ → no inversion → exponential decay ↘

if $2\alpha_m < 0$ → inversion → exponential increament ↗

V.3 Relation between W_{ij} and σ_{ij}

Consider absorption in a 2 level system

$$\frac{dN_1}{dt} = -W_{12} N_1$$

For a fixed z distance, this variation is proportional to:

- → the density of e^- in the lower state
- → the absorption cross-section σ_{12}
- → the number of photons N_{ph} per unit of time also called the photon flux Φ per unit of surface

$$\frac{dN_1}{dt} = -N_1 \sigma_{12} \Phi$$

where $\Phi = \dfrac{\text{Total energy carried by the wave}}{\text{the energy of a single photon}} \times \dfrac{1}{\text{unit of time}} \times \dfrac{1}{\text{unit of surface}}$

$$\Rightarrow \Phi = \frac{P \times dt}{h\nu} \times \frac{1}{dt} \times \frac{1}{A}$$

$$\boxed{\Phi = \frac{\overbrace{P}^{\to I}}{A h\nu}}$$

$$\Rightarrow \frac{dN_1}{dt} = - N_1 \sigma_{12} \frac{P}{A h\nu}$$

$$\boxed{W_{12} = \sigma_{12} \frac{P}{A h\nu}} \qquad \boxed{W_{21} = \sigma_{21} \cdot \frac{P}{A h\nu}}$$

In general:

$$\boxed{W_{ij} = \sigma_{ij} \frac{P}{A h\nu}}$$

$W \to$ the ratio of a photon to jump to another level (transition probability)

V.4 Saturation intensity in laser materials

$$\frac{dI}{I} = \pm 2\alpha_m I = \pm \Delta N \sigma I$$

$\to dz$?

For strong enough signal, the stimulated transition rate may become large enough to saturate the population difference and thus reduce the gain.

$$\Delta N = \frac{\Delta N_0}{1 + \tau_{eff} W_{sig}}$$ $\longrightarrow$ W of the signal.

We can write

$$\Delta N = \frac{N_0}{1 + \frac{I}{I_{sat}}}$$

where I_{sat}, the saturation intensity or the value of the signal intensity that will saturate the gain (or loss) to half its small-signal or unsaturated value.

with I_{sat}: $\boxed{I_{sat} = \frac{h\nu}{\sigma_{sig}\, \tau_{eff}}}$

For a 3-level system: (and the same for a 4 level system)

$$I_{sat} = \frac{h\nu}{\sigma_{21}\, \tau_{21}}$$

For example:

Er^{3+} $\quad \tau_{21} = 10 \text{ ms}$

$\sigma_{21} = 5 \cdot 10^{-25} \text{ m}^2$

$\lambda = 1.55 \ \mu m$

$$\Rightarrow I_{sat} = \frac{hc}{\lambda\, \sigma_{21}\, \tau_{21}} = \frac{6.62 \cdot 10^{-34} \cdot 3 \cdot 10^{8}}{1{,}55 \cdot 10^{-6} \cdot 0{,}01 \cdot 5 \cdot 10^{-25}} = 2{,}56 \cdot 10^{7} \ W/m^2$$

$A = 1 \text{ m}^2 \longrightarrow I_{sat} = 25{,}6 \ MW$ average intensity

$\downarrow$

For fiber amplifier

$$A = \pi a^2 = 100\ \mu m^2 \Rightarrow P = 2{,}56\ mW$$

Another example:

I_{sat} for Yb^{3+} $\quad \tau_{21} = 1\,ms$

$\lambda = 1{,}05\ \mu m$

$$\Rightarrow I_{sat} = 38\ MW/m^2$$

In a fiber $\Rightarrow P_{sat} = 38\ mW$

V-5 Homogeneous saturation in laser amplifiers

$$\frac{dI}{I} = 2\alpha_m \, dz = \frac{\Delta N_0 \, \sigma \, dz}{1 + \frac{I}{I_{sat}}} = \frac{2\alpha_{m_0} \, dz}{1 + \frac{I}{I_{sat}}}$$

where $2\alpha_{m_0} = \Delta N_0 \sigma$ is the unsatured gain.

To get $G(z)$ we need to integrate this equation.

We assume I_{out} at the output

I_{in} " " input

$$\frac{dI}{I}\left(1 + \frac{I}{I_{sat}}\right) = 2\alpha_{m_0} \, dz$$

Integrate:

$$\int_{I_n}^{I_{out}} \left(\frac{1}{I} + \frac{1}{I_{sat}}\right) dI = 2\alpha_{m_0} \int_0^L dz$$

$$\Rightarrow \left[\ln(I)\right]_{I_{in}}^{I_{out}} + \frac{1}{I_{sat}}\left[I\right]_{I_{in}}^{I_{out}} = 2\alpha_{m_0} L$$

$$e^{\left[\ln\left(\frac{I_{out}}{I_{in}}\right) + \frac{I_{out} - I_{in}}{I_{sat}}\right]} = e^{2\alpha_{m_0} L}$$

$$\frac{I_{out}}{I_{in}} \, e^{\frac{I_{out}-I_{in}}{I_{sat}}} = \underbrace{e^{2\alpha_{m0}L} = G_0}_{\text{small signal gain}}$$

$$\Rightarrow \quad G = \frac{I_{out}}{I_{in}} = G_0 \, e^{-\left(\frac{I_{out}-I_{in}}{I_{sat}}\right)}$$

$$\frac{I_{extracted}}{I_{sat}}$$

$$\Rightarrow \quad \text{(curve)} \; ? \qquad \frac{I_{out}}{I_{in}} = f_n\left(\frac{I_n}{I_{sat}}\right) \; ?$$

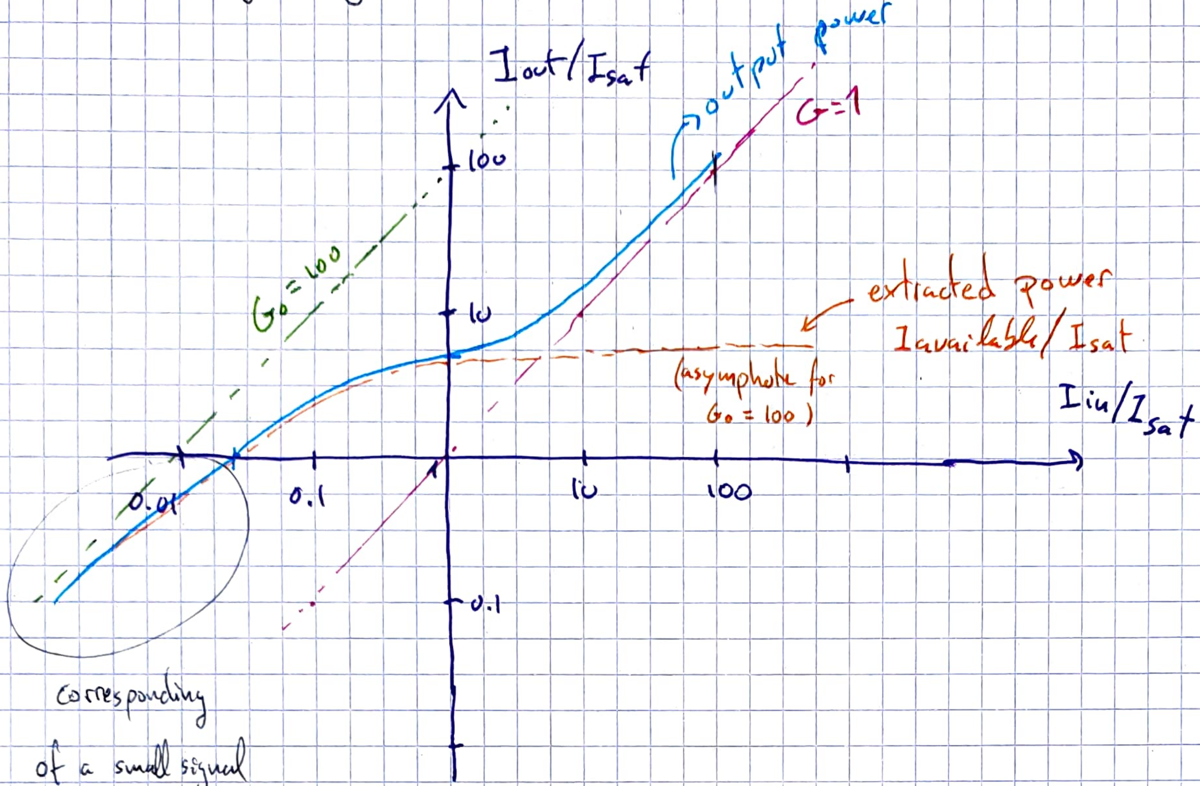
$$\frac{I_{out}}{I_{in}} = G \, \frac{I_{in}}{I_{sat}}$$

$$\Rightarrow \quad \frac{G}{G_0} = \exp -\left(\frac{I_{out}-I_{in}}{I_{sat}}\right)$$

$$\frac{I_{in}}{I_{sat}} = \frac{1}{G-1} \ln\left(\frac{G_0}{G}\right)$$

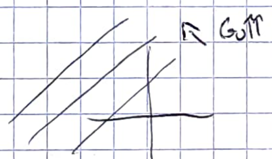
$$\frac{I_{out}}{I_{sat}} = \frac{G}{G-1} \ln\left(\frac{G_0}{G}\right)$$

The amplifier output-versus-input curve for a given small-signal gain $G_0 = 100$



We have 2 asymphotes, corresponding to the limits of the gain
If you increase too much you don't have photons to use ? so you saturate the medium.

If we increase G_0 :
we increase the limit.



→ How much power per unit area can be extracted from such an amplifier at different input power?

$$I_{extract} = I_{out} - I_{in} = \frac{I_{out}}{I_{sat}} I_{sat} - \frac{I_{in}}{I_{sat}} I_{sat} =$$

$$= \frac{G-1}{G-1} \ln\left(\frac{G_0}{G}\right) \times I_{sat} = \ln\left(\frac{G_0}{G}\right) \times I_{sat}$$

At low intensities and high Gain (G_0)

$$I_{extract} \doteq I_{out}$$

However, as the input intensity $\nearrow$, the extracted power reaches a limit, which is the max power available from the laser medium

$$\lim_{G \to 1} \ln\left(\frac{G_0}{G}\right) \times I_{sat} = \ln(G_0)\, I_{sat}$$

↳ for example: $\ln(G_0) = \ln(100) = 4.34$

$$G_0 = \exp(2\alpha_m L)$$

$$\ln G_0 = 2\alpha_m L = \Delta N_0 \sigma L$$

and $$I_{sat} = \frac{h\nu}{\sigma_{sig}\,\tau_{eff}}$$

$$I_{available} = \Delta N_0 \sigma L \frac{h\nu}{\sigma\,\tau_{eff}} = \Delta N_0 L \frac{h\nu}{\tau_{eff}} = \Delta N_0 L h\nu\, \gamma_{eff}$$

we know $I = \frac{P}{A}$

$$\frac{I}{L} = \frac{P}{AL} = \frac{P}{V} \Rightarrow \frac{I_{available}}{L} = \frac{\Delta N_0 L h\nu}{L\,\tau_{eff}} = \frac{P_{available}}{V}$$

The max power per unit volume is given by the small-signal inversion energy stored in the medium $\Delta N_0 h\nu$ times an effective recovery time $\gamma_{eff} = \frac{1}{\tau_{eff}}$ ~~x~~ $\longrightarrow$

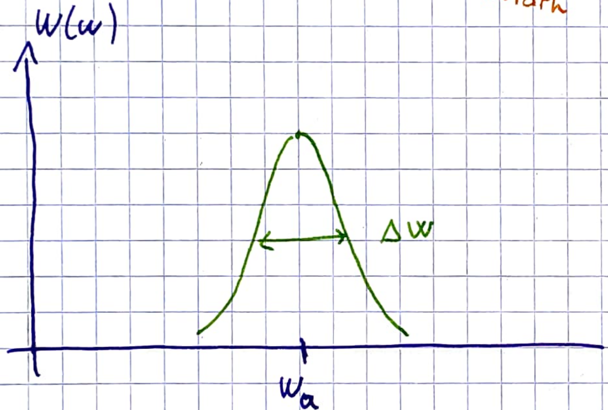
In other words, one can extract the initial inversion energy $\Delta N_0 h\nu$ every τ only

VI. 6 Linewidth broadening mechanisms

The probability of stimulated transitions depends on the frequency and flows a Lorentzian function or lineshape

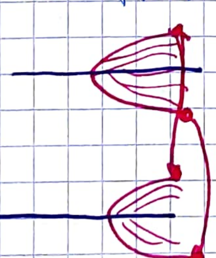
$$W_{12} = W_{21} = \frac{3}{4\pi} \frac{\lambda^3}{\underbrace{\tau}_{\text{lifetime}}\, h\, \underbrace{\Delta\omega}_{\text{linewidth}}} \frac{\overbrace{\varepsilon}^{\text{permittivity}}|E|^2}{1 + \frac{4(\omega - \omega_a)^2}{\Delta\omega}}$$

central freq



Actually, in disordered solids such as glasses and the lineshape will be modified and the linewidth is ↗

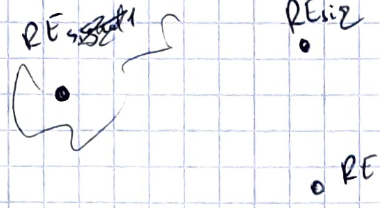
Effect 1 → Stark effect



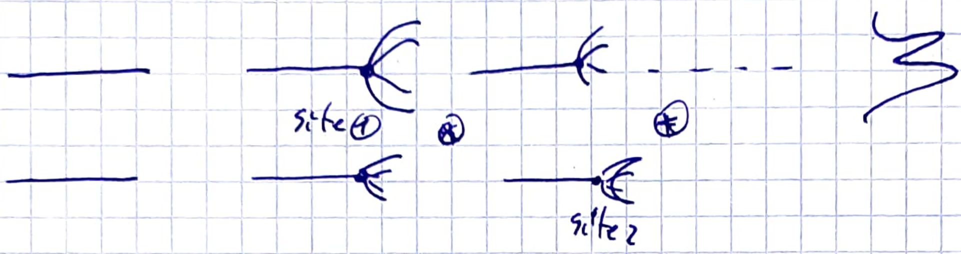
More laser transitions likely to occur

$\boxed{\text{Effect 2}}$

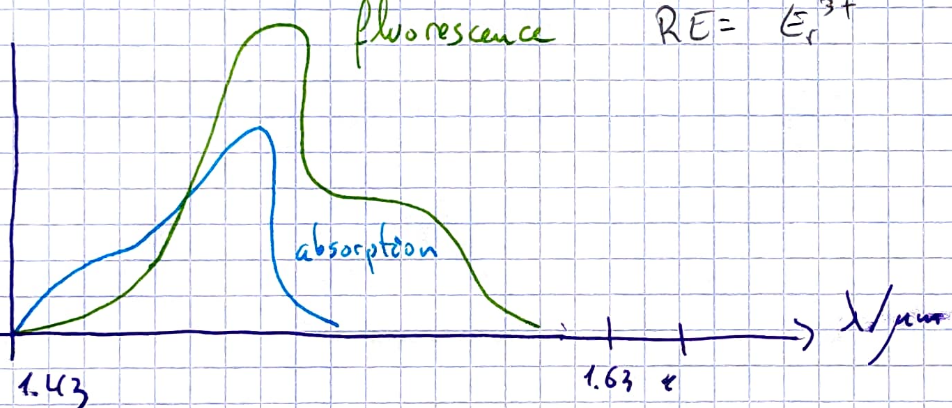
→ Various sites in the amorphous medium



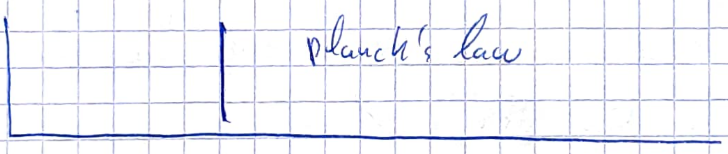
Environment is "random"



Example in slices (last)



RE = Er^{3+}



All these phenomena (stochastic phenomena) are encapsulated in a single quantity : the cross-sections

- absorption
- emission

From the fluorescence and absorption spectra, which are measurable quantities, one can extract:

$$\sigma_{12}(\omega) \neq$$
and $$\sigma_{21}(\omega)$$

→ used in numerical software to evaluate the performance of the lasers.

VI Rate equation Modeling

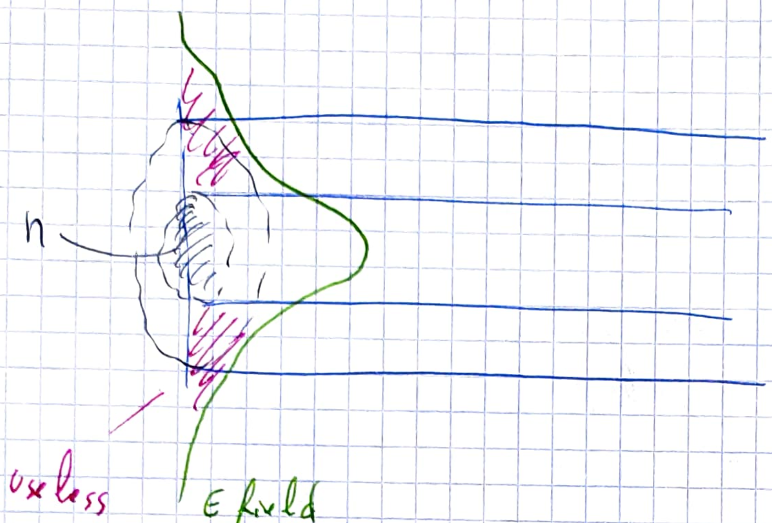
Using the transition cross-sections $\sigma_{em}(\lambda)$ and $\sigma_{abs}(\lambda)$ one can develop a model where the dynamic variables are the population densities and powers.

The ensemble of laser-active ions can be described with the following set of data:

- their density N, which in fibers is usually depending on the radius (see tutorial #1)
- transitions rates A_{ij} caused by spontaneous emission or $1/A_{ij} = \tau_{ij}$ (measurable quantities)
- the λ-dependent cross-sections $\sigma_{ij}(\lambda)$ [for stimulated transitions]

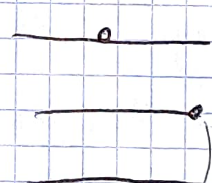
Quick note: Tutorial 1

We calculated Γ: the overlap between the gaussian-like mode of the fiber and the density of RE ions



3 level laser system

$$W_{13}(z) = \sigma_{abs}(\lambda_{pump}) * \frac{I_{pump}(z)}{h\nu_{pump}}$$



applied signal see tutorial #3 exercise IV

$$W_{21}(z) = \sigma_{em}(\lambda_{signal}) * \frac{I_{signal}(z)}{h\nu_{signal}}$$

$$W_{12}(z) = \sigma_{abs}(\lambda_{signal}) * \frac{I_{signal}(z)}{h\nu_{signal}}$$

$$A_{21} = 1/\tau_{21}$$

$W_{21}(z) \neq W_{12}$ because $\sigma_{em} \neq \sigma_{abs}$

$$N_1(z) = \frac{A_{21} + W_{21}(z)}{A_{21} + W_{21}(z) + W_{12}(z) + W_{13}(z)}$$

$$N_2(z) = N - N_1(z)$$

from V - 1

$$\frac{d I_{signal}(z)}{dz} = -\left(N_1(z)\,\sigma_{abs}(\lambda_{pump}) - N_2(z)\,\sigma_{em}(\lambda)\right]\, I_{signal}(z)$$

$$\frac{d I_{pump}}{dz} = - N_1(z)\,\sigma_{abs}(\lambda_{pump})\, I_{pump}(z)$$

↳ They are ODE (solve with the Runge-Kutta 4 routine
ode 45 solver