Quantum Technologies

Class Notes

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EMIMEO Programme

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Contents

Explanation and Introduction of this Document

I wrote this document for the students studying Quantum Technologies to have a nice set of notes, and correct reference code and graphs for the module. I hope that it is sufficient for this task and it helps all of your studies.

I spent have spent a lot of time developing the template used to make this LaTeX document, I want others to benefit from this work so the source code for this template is available on GitHub [**?**].

1 Introduction

1.1 Introduction

Richard Feynman was a nobel prize winner (for the formulation of QED) and one of the many physicists involve in the Manhattan project (among many, many other things). He gave a set of rather famous, public lectures at Cornell in 1964 called the *Messenger Lectures*. At the beginning of his 6th lecture, when comparing the number of people who understand GR to the number that understand quantum physics, he famously said:

> ***"... I think I can safely say that nobody understands quantum mechanics!"***

This quote is exceedingly well known and it is very likely I am not the first person to present it to you. A more interesting quote of his that less people seem to know, despite it being something rather interesting for *any* physicist to say, is what he said immediately following:

> *"Now, if you appreciate this, and don't take the lecture too seriously that you really have to understand, in terms of some model, what I'm going to describe, and just relax and enjoy it, I'm going to tell you what nature behaves like, and if you will simply admit that maybe she does behave like this, you will find her a delightful, entrancing thing."*

This course is, in many ways, designed as such. Your background likely isn't suitable to make a successful attempt at truly understanding Quantum Mechanics (even the experts don't understand it). At many times (if you are like me) you will find yourself completely lost or wondering where some piece of mathematics or physics came from during the adventure of this course. And if you take an overly serious approach to it, you will find yourself pulling your hair out (I made this document in an attempt to better understand and I failed three times).
But if you suspend, for just a moment (or quite a few), your disbelief, you will find the lectures of Professor Artoni fascinating and thoroughly enjoyable.

TODO - Not quite sure what should be put here.

1.2 Max Planck, The Concept of "Quanta", and Planck's Constant

Quantum theory and mechanics was initially developed and discovered by one man:
Max Karl Ernst Ludwig Planck (23/04/1858 Kiel, Duchy of Horstein ~ 04/10/1957, Göttingen, West Germany). Or, as he is more commonly known, simply, ***Max Planck***.

He happened upon the principle phenomena involved in quantum physics during research on black body radiation. He published these papers from 1900 - 1901 and they earned him the Nobel prize for physics in 1918.
Planck's initial discovery is the foundation of all of quantum physics and is known as "Planck's Postulate", it states that all electromagnetic radiation is made of very small "particles" known as quanta. These quanta have an energy given by the quanta's frequency and a constant, known as "Planck's Constant":

$$E_h = h\nu = \frac{hC}{\lambda} \rightarrow \text{Energy of One Single Quanta} \tag{1}$$

So if one was to "send" 10 quanta the energy delivered will be 10 quanta, discrete and finite, and given by $10 \cdot h\nu = 10 \cdot E_h$. To be absolutely clear at a given frequency, ν, the amount of energy that can be sent will be integer multiples of Planck's equation, it will be *discrete*.
Quanta are also very commonly referred to by another name; *Photons*. Photons are what make up light, and the discoveries of Planck and others were incredibly important to forwarding science to what it is today.
Another incredibly important discovery of quantum physics which is very commonly now is the dual wave-particle nature or behaviour of very small particles. But we shall discuss all of this in more detail photons, quanta and some of the very first implications of Planck's equation very soon.

1.3 Black Body Radiation

A black body is something which is in complete temperature equilibrium, i.e. it's temperature is not changing, it is emitting as much temperature as it is receiving. The earliest form, which likely would have provided the measurements that allowed Planck's to develop his theorem, is shown in Figure 1[?].

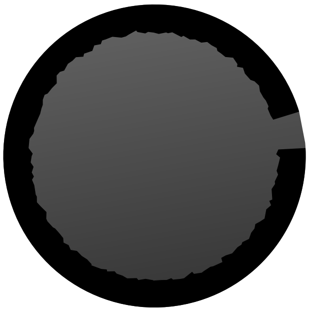


Figure 1: The Construction of an Ideal Black Body, a Platinum Cavity with a Small Hole.

The sun is (nearly) a black body and will absorb EM radiation (in the form of photons) and then emit radiation according to a curve, as shown in Figure 2. But, the interesting, and rather baffling, thing about black bodies is that they will emit photons even when in equilibrium and when no photon has impinged upon them!

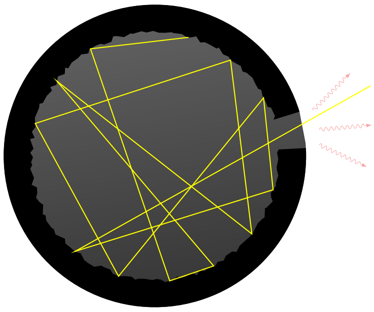


Figure 2: The Ideal Black Body Radiating after Receiving In-Falling Light (One Photon). Note; the yellow ray is the incident photon.

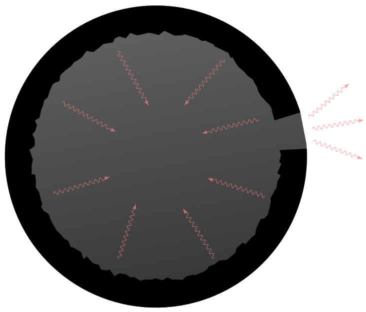


Figure 3: The Ideal Black Body still Radiating without In-Falling Light.

From at least around the mid-1800s scientists had been trying to describe the spectrum of black body radiation, the curve of radiated power versus wavelength (or the spectrum) is shown in Figure 2. They could quite well describe and model the higher wavelengths of this curve (and the very low wavelengths) but they couldn't yet find a way to describe the "middle" portion, roughly around the wavelengths of visible light and IR radiation.

The classical curve of black body radiation is given by the Rayleigh-Jeans equation, two very important figures in the field of optics and physics. They could only describe the upper wavelengths of black body radiation, that would soon change.

It is said that Planck's mentor (in a story that must have occurred a hundred times before in science) told him it was not possible to describe the black body phenomena and that the limits of science had been reached. He ultimately revolutionised this field of study, based on the experimental results observed by others he formulated Planck's Law for black body radiation, given by Equation 2.

$$q_\lambda = \frac{2\pi c^2 h \lambda^{-5}}{e^{\frac{ch}{k_B \lambda T}} - 1} \qquad (2)$$

Where:

λ = Wavelength

k_B = Boltzmann's Constant

c = Celerity, Speed of Light in a Vacuum

q_λ = Energy Flux

h = Planck's Constant

T = Temperature

This equation perfectly described the emission curve of a black body based on temperature, shown below in Figure 2. Planck had unwittingly stumbled upon the basis of one of the most incredible fields of study in physics, and arguably one of the most important. He would go on to use this to develop quantum theory and the rest is history!

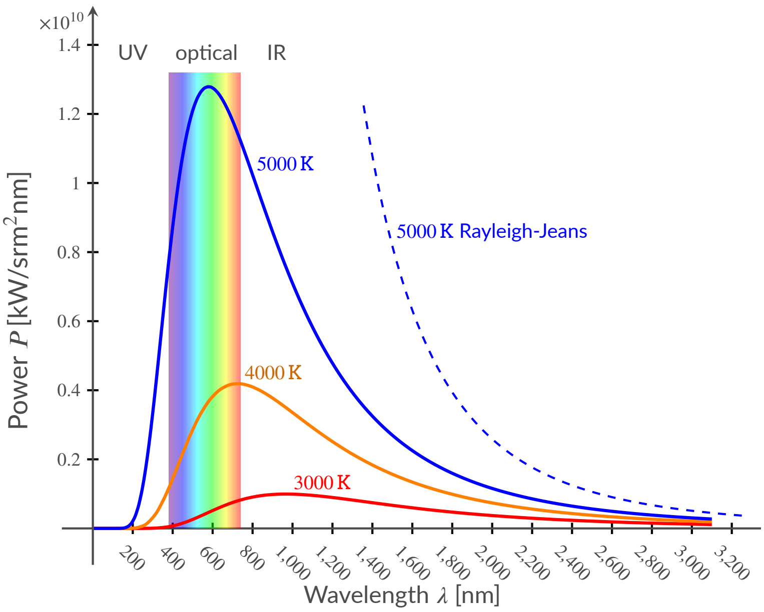


Figure 2: Black Body Radiation Curve at Different Temperatures, the Classical Model (Rayleigh-Jones Curve) is Marked with a Dashed Line.

2 The Photoelectric Effect

In 1905 Albert Einstein would use the findings of Planck's papers to finally describe the results of an experiment which had, until then, not been fully understood. The resulting effect was named the "Photoelectric Effect" [?], it states "that electrons are emitted when electromagnetic radiation, such as light, hits a material". Electrons emitted in this way are known as "Photoelectrons" and this discovery was another key step in the development of quantum theory. It still finds uses in practical applications for chemistry and solid-state electronics today.

2.1 The Photoelectric Experiment

The diagram of the photoelectric experiment is shown in Figure 3, it consists of an ammeter, controlled DC voltage source, a collector, a photocathode, and a light source emitting monochromatic light.
If the DC voltage is kept constant and lower frequency light, of frequency ν_1, is emitted towards the photocathode (as in Figure 3) there is no current measured at the ammeter, no mater how long the light is shone or how intensely.
However if light of a higher frequency, ν_2, is shone at the photocathode (as in Figure 4), the ammeter begins to immediately show current!

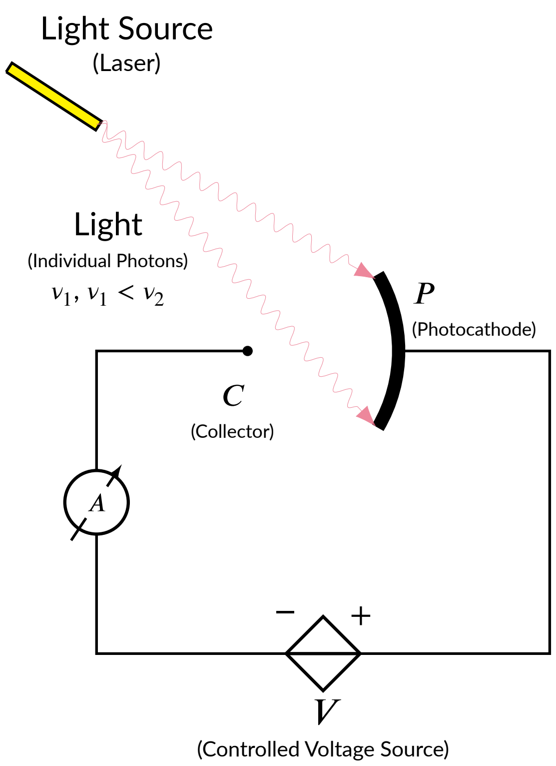


Figure 3: A Diagram of the Photoelectric Experiment. Lower frequency light is being emitted and there is no activity in the experiment.

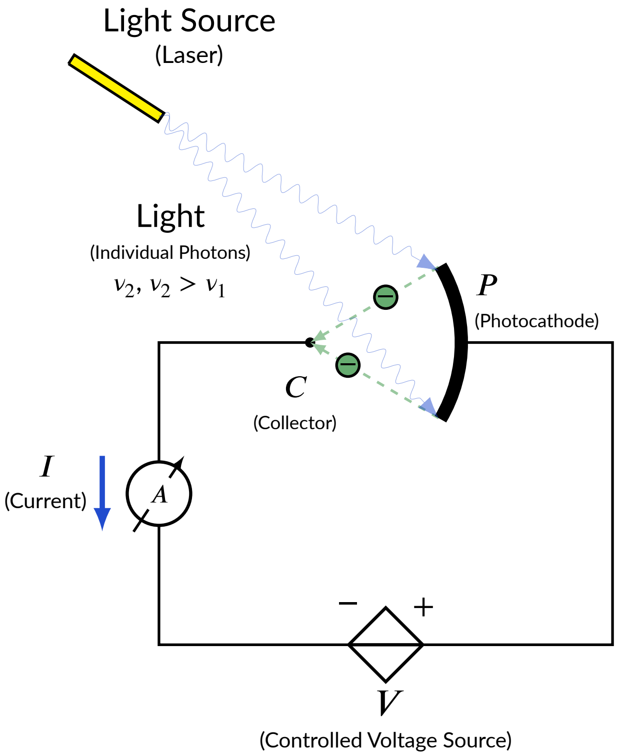


Figure 4: A Diagram of the Photoelectric Experiment. Higher frequency light is being emitted and electrons are passing across to the collector, and current is measured at the ammeter.

The experimental results described by the photoelectric effect inherently disagree with, and were unexplained by, classical electromagnetics which predicts that *continuous* light waves transfer energy to electrons, which would then be emitted when they accumulate enough energy. An alteration in the intensity of light would theoretically (according to classical EM) change the kinetic energy of the emitted electrons, with sufficiently dim light resulting in a delayed emission. The experimental results instead showed that electrons are dislodged **only** when the light exceeds a certain frequency - regardless of the light's intensity or duration of exposure.

Another thing to note here is that, if we wished, we could increase the potential of the DC voltage source the flow of electrons would eventually stop again, and a higher frequency would be required to cause the transport of electrons across the gap.

2.2 The Photoelectric Equation

So, as previously stated, in 1905 Albert Einstein used the findings of Planck to finally describe the results of the photoelectric experiment. He made an approximation, and stated the one quanta is absorbed by one electron (this is an approximation that he made to simplify his calculations, it is not true in every case). Then he stated that before the quanta is absorbed by the electron it's total energy is given by Equation 3. This is a simple, linear equation, rearranged in eq. 4 it states what the *kinetic* energy of the quanta is before interacting with the electron.

$$h\nu = E_k + \Phi \tag{3}$$

$$E_k = h\nu - \Phi \tag{4}$$

Where:

$$E_k = \text{Kinetic Energy}$$
$$\Phi = \text{Potential Energy (work function)}$$
$$\nu = \text{Frequency}$$
$$h = \text{Planck's Constant}$$

Equation 4 provides a clearer explanation for why we were not propagating any photons across the gap when the frequency of the light was too low, we had to be over a threshold frequency, ν_{th}, in order to overcome the potential!
Figure 5 shows this rather simple relationship. There are also some other insights to be gleamed here, namely, that E_K cannot be negative, that the work function is clearly material dependent, and that we can state a relationship to calculate the threshold frequency, given in Equation 5:

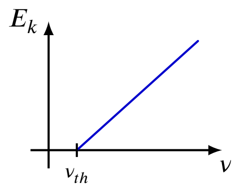


Figure 5: Something... TODO

$$\begin{aligned} h\nu &\geq 0 \\ \therefore\ h\nu &\geq \Phi \\ \implies \nu &\geq \frac{\Phi}{h} \\ \therefore\ \nu_{th} &= \frac{\Phi}{h} \end{aligned} \tag{5}$$

Since we can also state the potential energy, E_k in terms of charge and voltage, this means that we can obtain another equation (6) to express E_k based on the biasing voltage required to stop the electrons from flowing when particular frequencies of light are incident on the plates.

$$\begin{aligned} E_k &= C \times V_{stop} \\ \therefore\ E_k &= e \times V_{stop} \end{aligned} \tag{6}$$

2.3 A Quick Example

Let's (as a demonstrative example) calculate how many quanta we might have traveling in a given beam of light...
You probably know the meaning of flux quite well in classical electromagnetic theory (namely a vector quantity describing the magnitude and direction of the flow of electromagnetic energy). But we can (an probably must) redefine the meaning of flux with our new-found knowledge of quanta, considering the discrete definition of quanta instead of non-discrete EM energy, see Equation 7:

$$\text{Flux, } F = \frac{\text{no. of quanta, } n}{\text{area, } a \times \text{time, } t} \tag{7}$$

Given the definition of intensity, shown in 9, let's say the intensity and wavelength of the light beam are $I = 2w/m^2$ and $\lambda = 250nm$ respectively. We'll begin by calculating the energy of one single quanta (Equation 8), then we'll calculate the number of quanta emitted by our light beam (Equation 10), and finally we could combine those two equations to get the total energy once given beam diameter (or area) and time.

$$\begin{aligned}
&\text{Energy of a single quanta} \\
&E = h\nu = \frac{hc}{\lambda} = \frac{12.4 \times 10^3 eV \cdot \text{Å}}{250 \times 10^{-9} m} \quad \text{Remember an Angstrom is } 1 \times 10^{-10} \text{ metres} \\
&\implies \text{1 Quanta} = 4.96\, eV \qquad (8) \\
&\text{Number of quanta in the beam} \\
&\text{Intensity} = 2\,\frac{w}{m^2} = 2\,\frac{J}{m^2 \times s} \qquad (9) \\
&\therefore \frac{N}{m^2 s} = 2\,\frac{J}{m^2 \times s} \times \frac{1}{h\nu} = 2\,\frac{J}{m^2 \times s} \times \frac{1}{4.96 \cdot 1.602 \times 10^{-19} J} \\
&\text{No. of Quanta } /m^2 s \approx 2.52 \times 10^{18} \qquad (10)
\end{aligned}$$

3 Bohr's Model of the Hydrogen Atom

3.1 Bohr's Hypothesis - Angular Momentum is Quantised

In 1913 Niels Bohr took a visit to Ernest Rutherford, Hans Geiger, and Ernest Marsden's lab in Manchester as a result of this visit he offered an answer to some strange and, at the time, unexplained measurements they had made involving electrons.
Bohr hypothesized that the energies of the atom are quantised (along with a number of other properties) and that as a consequence this would result in a particular model or interpretation of the hydrogen atom. Any transition up to a higher energy level requires absorption of the appropriate frequency photon, and any down transition the emission of a photon of similar frequency. Explicitly, Bohr's hypothesis is derived below, then written in Equation 11.

$$\text{Classically, Angular Momentum would be} \int L_\theta \, d\theta = hn$$

$$\text{Bohr's Hypothesis} \implies \int L_\theta \, d\theta = L_\theta \oint_0^{2\pi} d\theta = L_\theta \cdot 2\pi$$

$$\therefore L_\theta = \frac{h}{2\pi} n = \hbar n \quad \text{Where } n \in \mathbb{N} \tag{11}$$

Thus, in quantum physics, the angular momentum is quantised and an integer multiple of Planck's reduced constant, $\hbar$. We will soon look at the implications of this hypothesis and how this helped Bohr construct his model of the Hydrogen atom.

3.2 Bohr's Model - An Intro for the Unfamiliar and Recap for the Familiar

Hydrogen Atoms are a very simple system consisting of a single proton (with a positive charge) and a single electron (with a negative charge). We say that the electron orbits with *tangential* velocity, V_R, it sweeps and angle, θ, and it orbits with a radius, r. As a quick reminder to classical physics, the *angular momentum* of the electron with thus be:

$$\text{Angular Momentum, } \vec{L} = \text{radius, } \vec{r} \wedge \text{Momentum, } \vec{P}$$

In the case of a single particle, in circular motion; Momentum has the scalar value of $\vec{P} = p = m \cdot V_R$ and the Equation becomes:

$$\vec{L} = r \cdot m \cdot V_R \cdot \hat{\theta} = L_\theta \hat{\theta}$$

And we eventually obtain the result as per Equation 11 given Bohr's hypothesis. But let's elaborate the consequences fully, and provide some simple proof...

TODO: Nice Bohr Model and Angular Momentum Diagram

3.3 Force Balance - The Bohr Radius

In order to have electrons orbiting a central nucleus they must be under a “centrifugal” force.

$$\text{Force, } F = \text{mass, } m \cdot \frac{\text{Instant. Velocity, } V_R^2}{\text{radius, } r}$$

As we assume the only attractive/repellent forces in play here must be electromotive, the centrifugal force must therefore be equal to the electromotive forces. Given our previous definitions regarding E.M.F we can state:

$$\text{Force, } F = \text{mass, } m \cdot \frac{\text{Instant. Velocity, } V_R^2}{\text{radius, } r} = \frac{e^2}{r^2}$$

$$\therefore r = \frac{e^2}{mV_R^2}$$

$$\because V_R = \frac{L_\theta}{m \cdot r}, \quad \frac{L_\theta^2}{m \cdot r^3} = \frac{e^2}{r^2} \rightarrow \therefore r = \frac{L_\theta}{m \cdot e^2}$$

$$\because L_\theta = \hbar n \quad :$$

$$r_n = \frac{\hbar^2}{m \cdot e^2} \cdot n^2 \quad \text{OR,} \quad r_n = r_0 \cdot n^2 \quad \text{Where } n \in \mathbb{N} \tag{12}$$

r_n for the value $n = 1$ in Equation 12 above is usually termed r_0 and is known as the ”Bohr Radius” ($r_0 \approx 0.5$Å), it is the inner most orbit radius an electron can have in a hydrogen atom. All other orbits are square integer multiples of this value, giving us a way of calculating the other possible orbits. It should be apparent this means that all electron orbit radii are **quantised**, we will explore the implications of this discovery in the following subsections. Table 1 below shows some of the radii:

Orbit One, r_1	”r_0” ≈ 0.5Å
Orbit Two, r_2	$r_0 \times 4 \approx 2$Å
Orbit Three, r_3	$r_0 \times 9 \approx 4.5$Å

Table 1: *The first three Bohr model radii*

3.4 Velocity

As the velocity of the "orbiting" particle is dependent on the angular momentum it will be discretized also. The result of the first orbit is derived below in Equation 13, with the general case found in Equation 14.

Recall:

$$V_R = \frac{L_\theta}{m \cdot r} = \frac{\hbar \cdot n}{\frac{\hbar^2}{m \cdot e^2} \cdot n^2 \cdot m} = \frac{e^2}{\hbar \cdot n}$$

$$\therefore \text{ For the first orbit we have: } \quad V_{R,1} = \frac{e^2}{\hbar}$$

But there is a more "convenient"/beautiful definition:

$$V_{R,1} = \frac{e^2}{\hbar \cdot n} = \frac{e^2}{\hbar} \quad \text{(Multiply by } \tfrac{c}{c} \text{ for a nice result)} \quad V_{R,1} = \frac{e^2}{\hbar \cdot c} \cdot c \ldots$$

$$V_{R,1} \equiv \alpha \cdot c \approx 3.8 \cdot 10^6 \;\; m/s \tag{13}$$

$$V_{R,n} = \frac{\alpha \cdot c}{n} \quad \text{Where } n \in \mathbb{N} \tag{14}$$

Where α is the Fine Structure constant ($\alpha = \frac{1}{137}$), and there are other names for the orbits; $V_{R,1}$ is also called the "ground state orbit", $V_{R,2}$ is also known as the "first excited state orbit".

3.5 Energy and Energy Levels

We will now show one of the most important implications of the quantization of angular momentum, which is that all electrons of an atom have a specific "energy level" corresponding to each orbit. As usual, the derivation is below, with the first orbit value found in Equation 16 and the general case in Equation 15:

Recall:

We have energy from an attractive force, $E_q = -\frac{e^2}{r}$

Also kinetic energy, $E_k = \frac{1}{2} m \cdot V_R^2$

$$E = \frac{1}{2} \cdot m \cdot V_R^2 - \frac{e^2}{r}$$

Sub V_R & r from Equations 14 & 12:

$$\therefore E = \frac{1}{2} \cdot m \cdot \left(\frac{e^2}{\hbar \cdot n}\right)^2 - \frac{e^2}{\frac{\hbar^2}{m \cdot e^2} \cdot n^2} E = \frac{1}{2} \cdot \frac{m \cdot e^4}{\hbar^2 \cdot n^2} - \frac{me^4}{\hbar^2 \cdot n^2}$$

$$E_n = -\frac{1}{2} \cdot \frac{m \cdot e^4}{\hbar^2 \cdot n^2} = -\frac{m \cdot e^4}{2 \cdot \hbar^2} \cdot \frac{1}{n^2} = -\frac{R}{n^2} \tag{15}$$

Where $R = \frac{m \cdot e^4}{2 \cdot \hbar^2}$ is the Rydberg Constant and $n \in \mathbb{N}$

$$E_1 = -\frac{m \cdot e^4}{2 \cdot \hbar^2} \cdot \frac{1}{1^2} = -\frac{R}{1^2} = -\frac{m \cdot |e|^4}{2 \cdot \hbar^2} = -R$$

$$E_1 \approx -13.6 \text{ eV} = 13.6 \cdot 1.602 \cdot 10^{-19} \text{ Joules (J)} \tag{16}$$

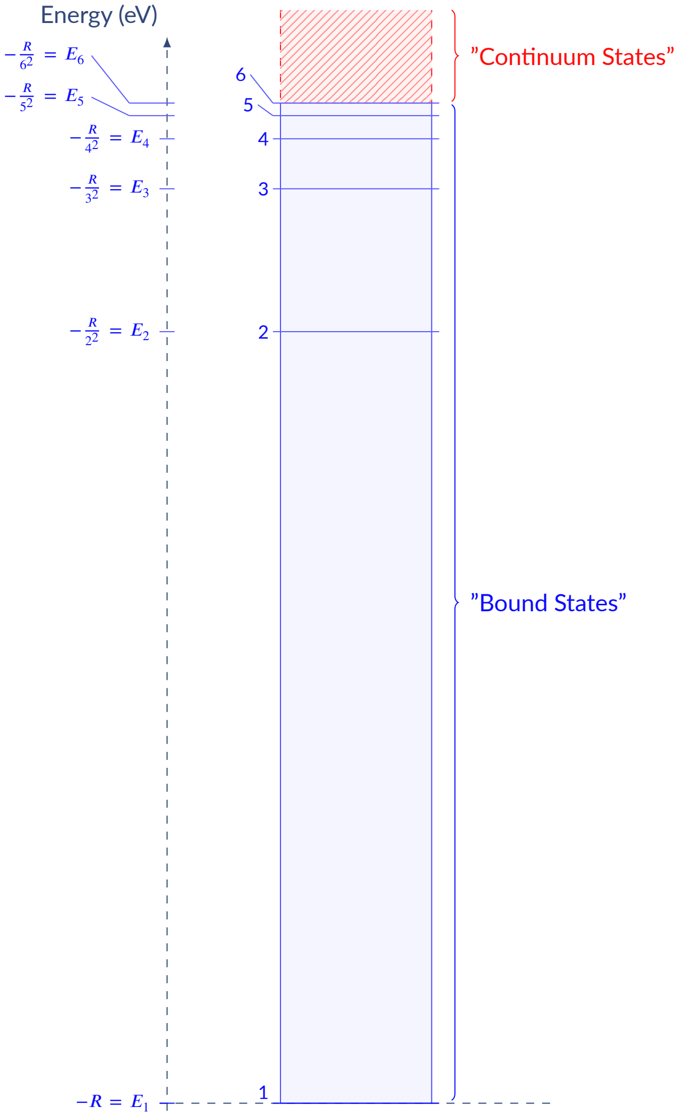


Figure 5: Something... TODO

4 The Wave Nature of Matter

4.1 De Broglie and The De Broglie Wavelength

4.2 Uses - Electron Microscope vs Conventional Light Microscope

4.3 The Double Slit Experiment

4.4 Augen's Principle

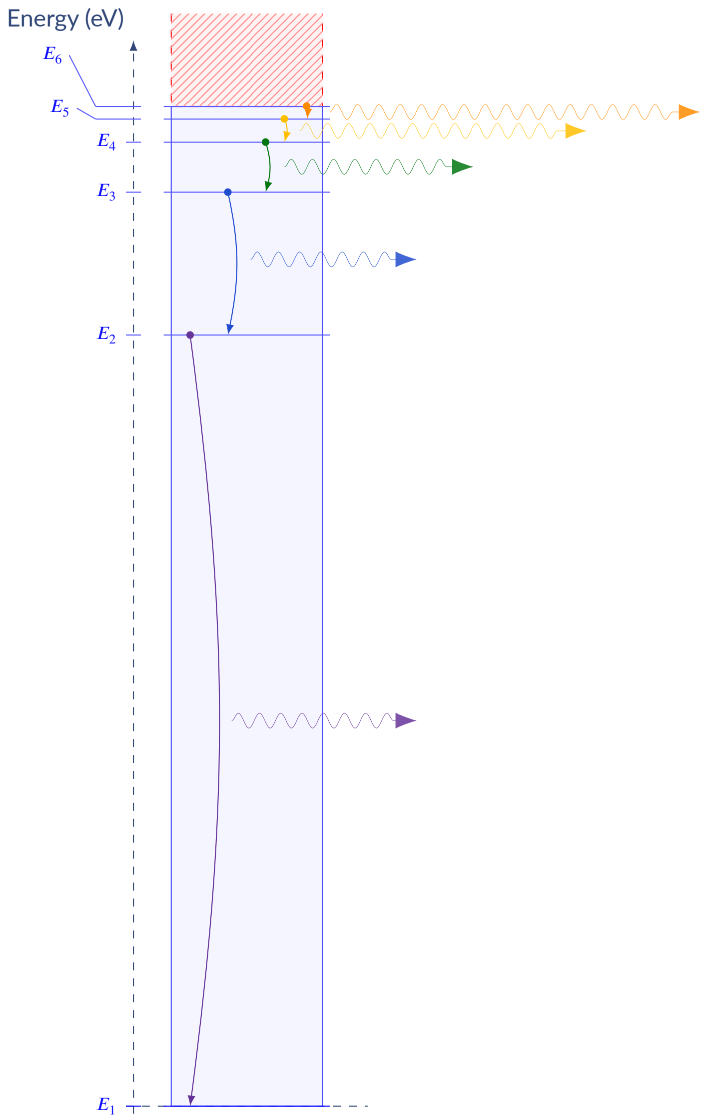
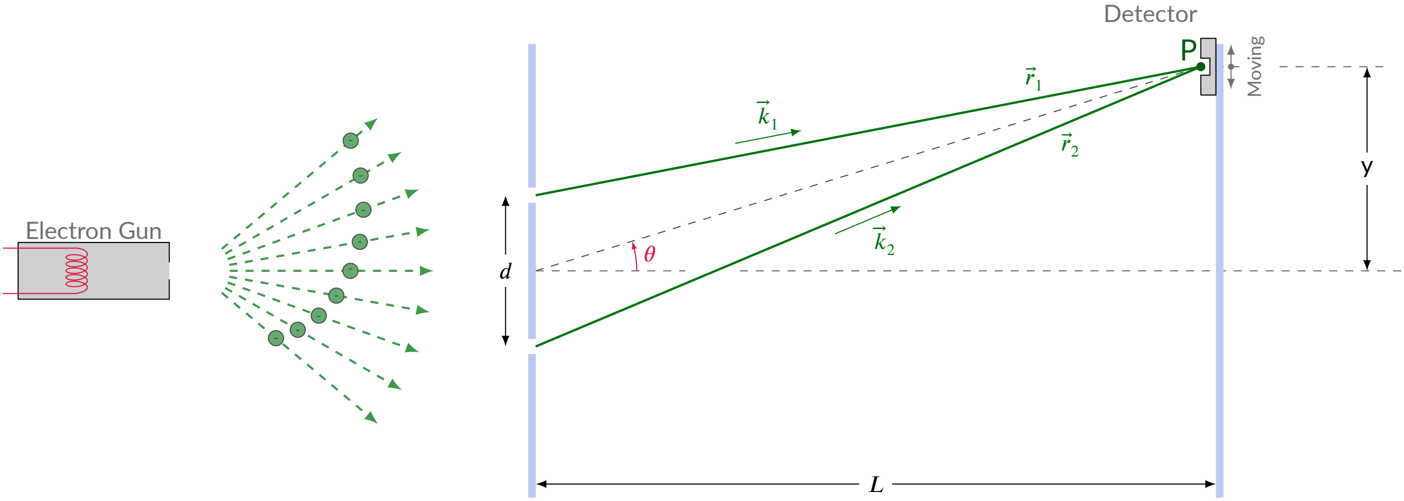
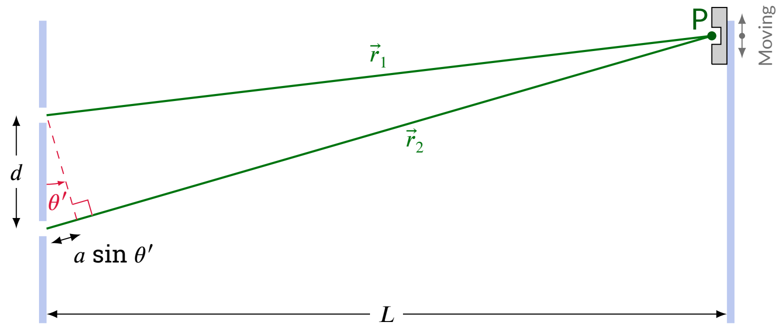


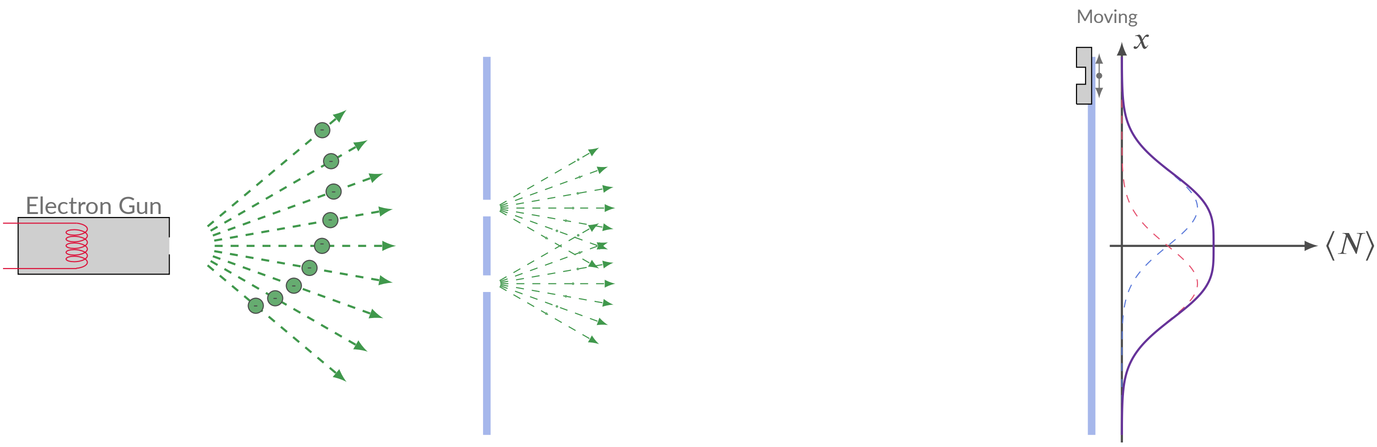
Figure 6: Something... TODO

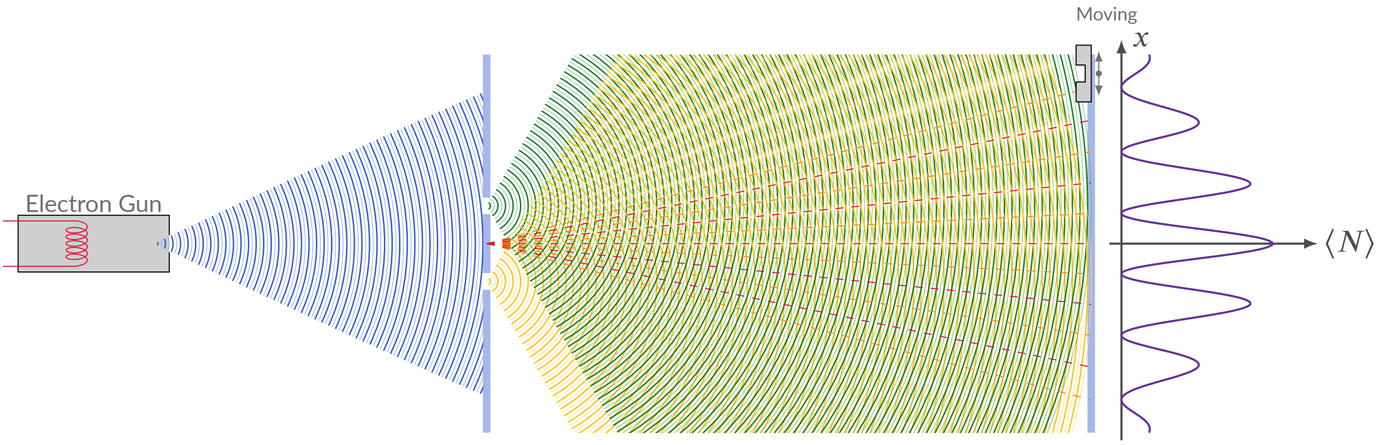
5 Particle Interference

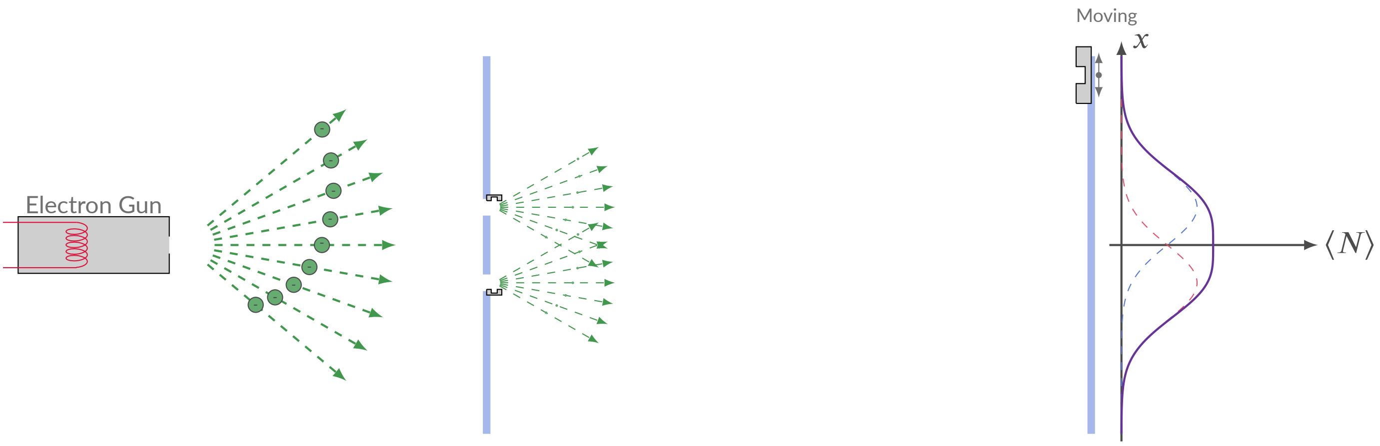
5.1 The Double Slit Experiment Explored









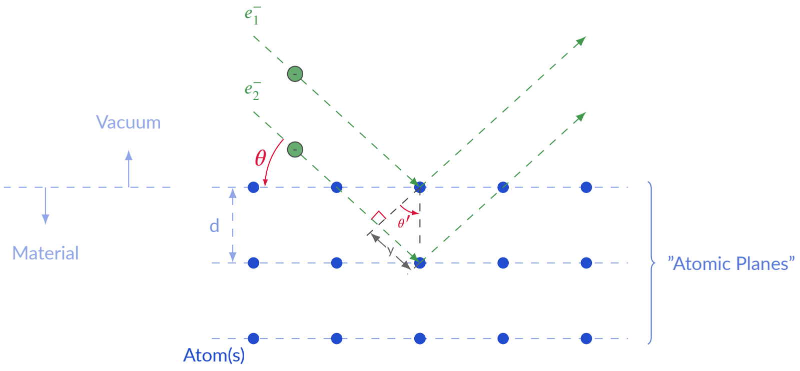


5.2 Possible Solutions

5.3 Superposition of Solutions

5.4 Final Solution

5.5 Diffraction of Particles

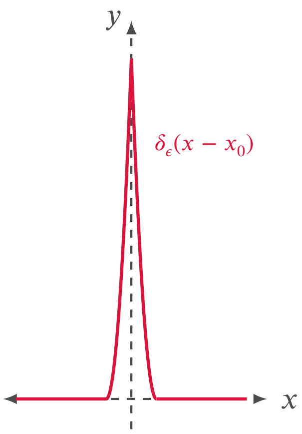


6 The Schrödinger Equation

6.1 The general Schrödinger Equation in time and space

6.2 The Superposition Principle

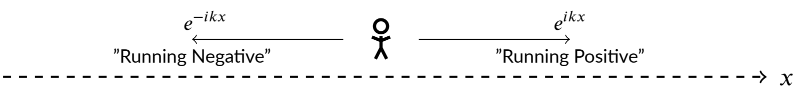
6.3 The S.E.'s Eigenfunction and its Properties



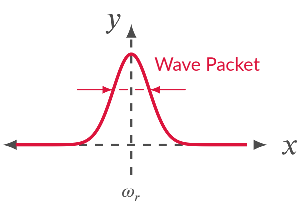
6.4 The Wave function; its Properties and Conditions

6.5 Possible Solutions to the Eigen and Wave functions

6.6 The Kroncker Equation



6.7 A particle with mass (m) moving in one dimension according to the S.E.

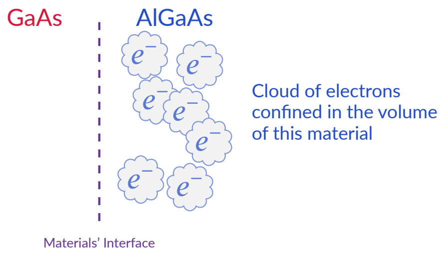


7 Observables

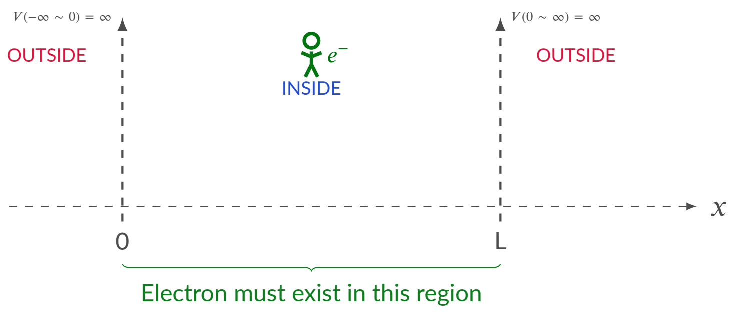
7.1 What are Observables?

7.2 Calculating Observables, Step-by-Step

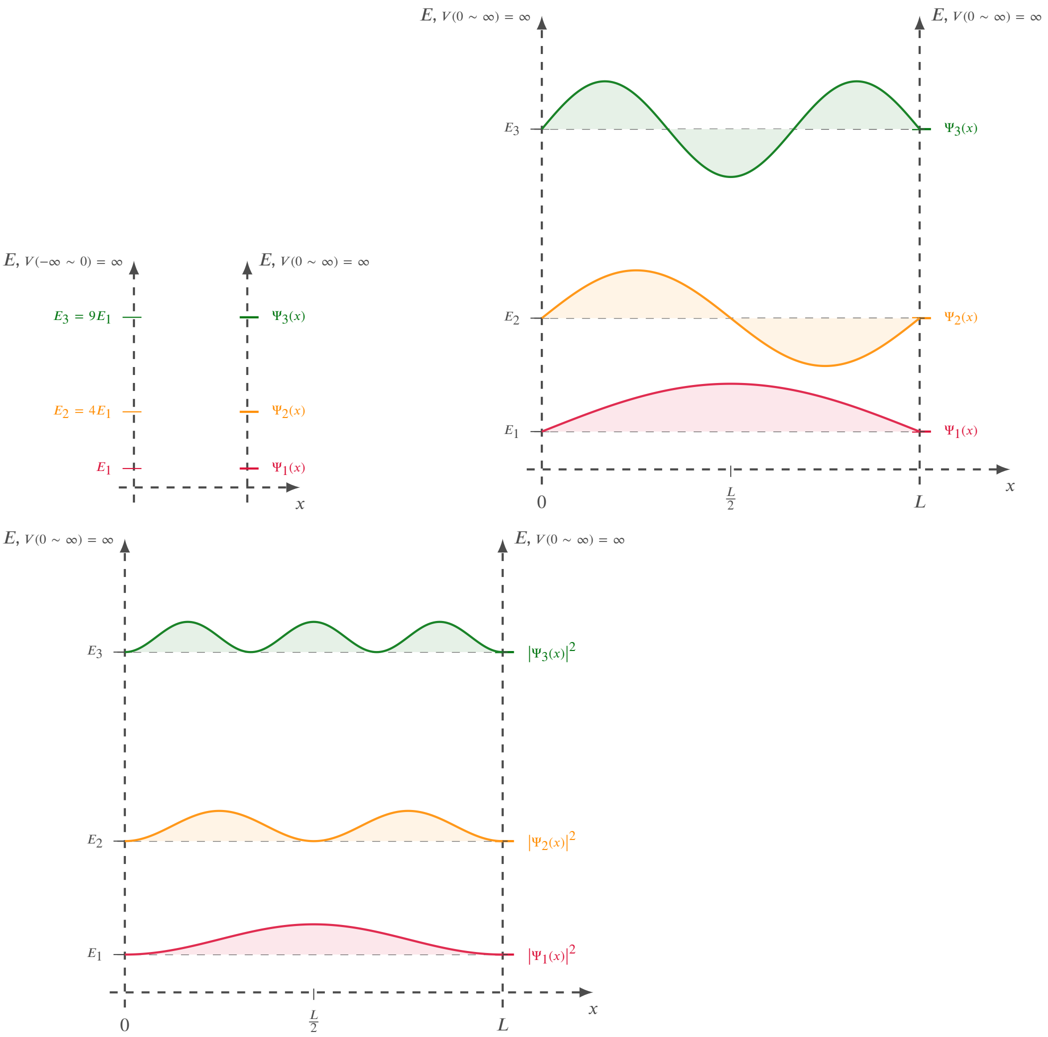
8 Confinement



8.1 Confined Particles in 1D



8.1.1 The Quantum Well



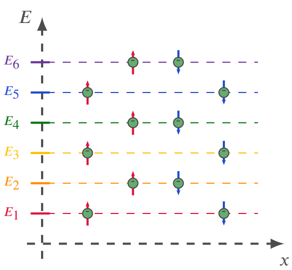
8.1.2 Using the S.E., Eigen, and Wave Functions to Find Solutions to observables

8.1.3 Conditions

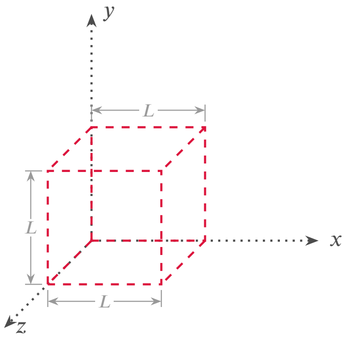
8.1.4 Superposition of Solutions

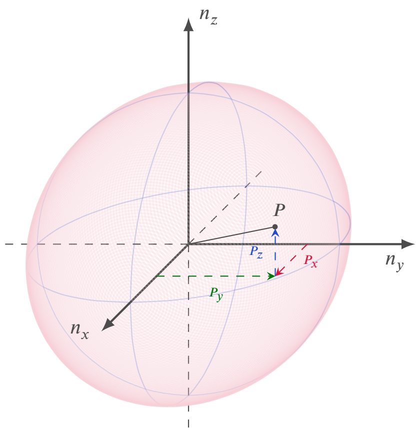
8.2 Hisenburg Principle

8.3 Paul Exclusion Principle



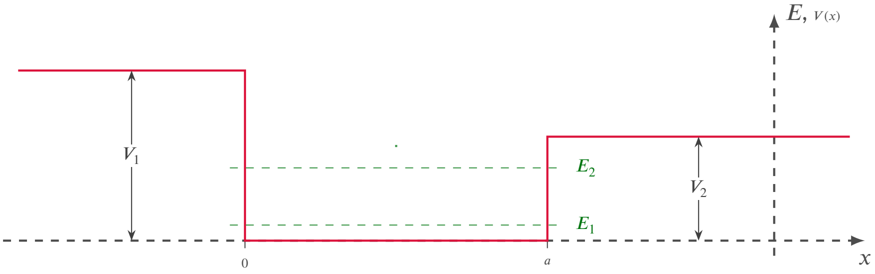
8.4 Confined Particles in 3D

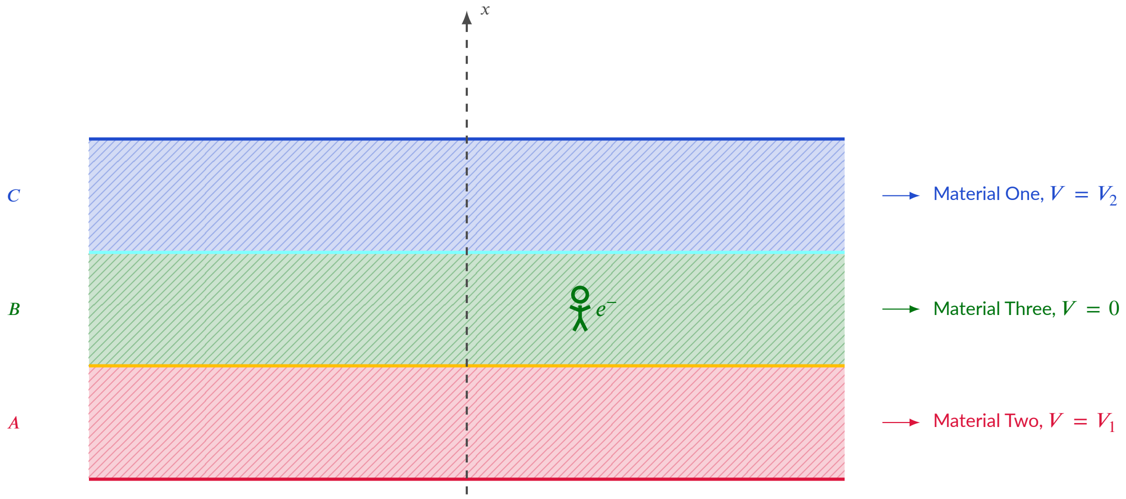


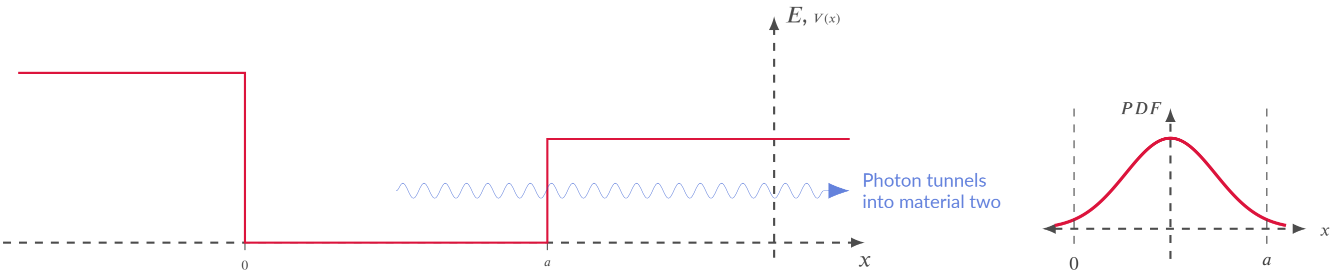


8.5 The Fermi Level

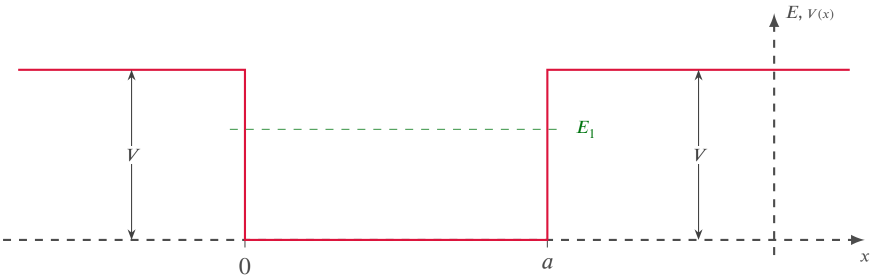
8.6 Confined Particles in 1D - Realistic (Finite Potential) Boundaries



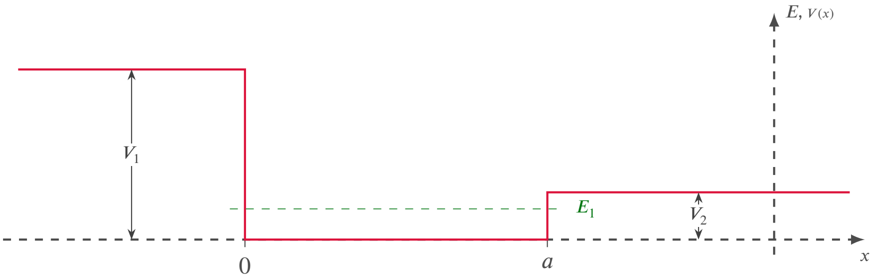




8.6.1 Symmetric QW

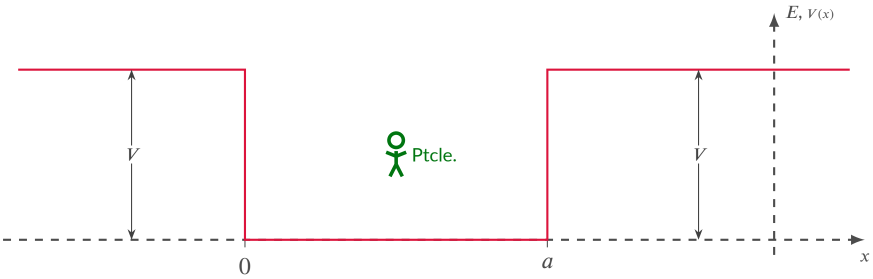


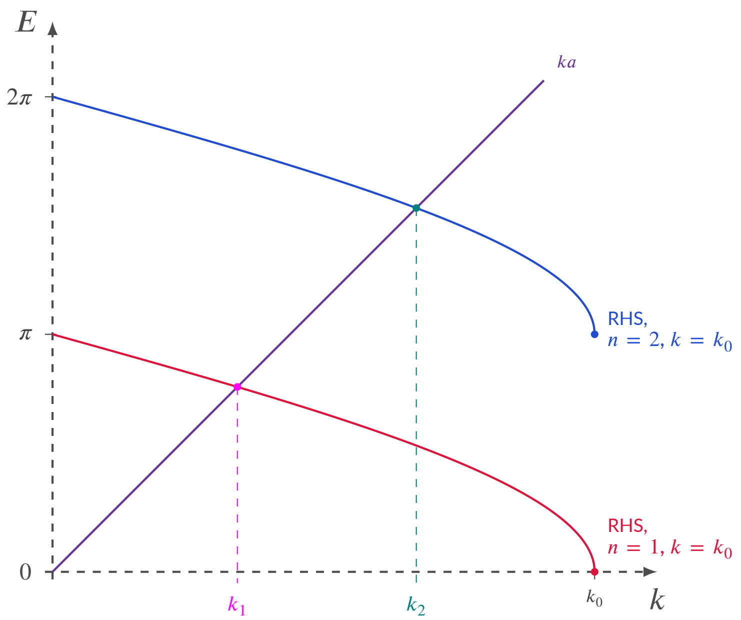
8.6.2 Asymmetric QW

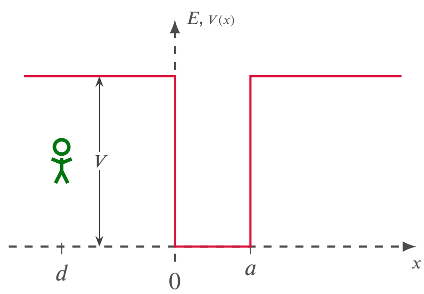


8.6.3 The Wave Vector

8.6.4 Examples







8.7 Quantum Tunnelling

8.7.1 General Example and Solution for Tunneling Across a 1D Boundary

8.7.2 Electron Microscope

8.8 Quantum Oscillators - Parabolic QW/Confinement

9 Periodic Photonic Structures

Block Modes in Periodic, Quantum Structures

9.1 The Transfer Matrix

9.2 Applying the Transfer Matrix

9.3 Block Theorem

9.4 Solution Cases/Types for the Quantum Structure

9.5 The Quantum Bandgap

9.6 2D Periodic Structure for Electron Containment

9.7 Time Reversal of the Transfer Matrix

10 Angular Momentum and Observable Commutation

10.1 Angular Momentum - Classical Perspectives

10.2 Angular Momentum - Quantum Interpretation

10.3 What is Commutation?

10.4 Commutation Examples and Useful Results

10.5 The Meaning of Commutation - Common Sets of Eigen Functions

10.6 Energy Levels in the Presence of a Magnetic Field - The Zeeman Effect

10.7 The Zeeman Effect and Free Angular Momentum

10.8 Orbital Angular Momentum

10.9 Orbital Angular Momentum - Quantisation

11 Some Interesting Applications

11.1 NMR - Nuclear Magnetic Resonance

11.2 Quantum Bit

11.3 Spontaneous Parametric Down Conversion

11.4 No-Cloning Theorem

11.5 Shor’s Algorithm

12 Appendix

12.1 Constants & Relevant Definitions

12.1.1 Constants

12.1.2 Relevant Classical Definitions

12.2 Units Involved and Some Important Starting Equations

12.3 Conversions

12.4 Properties of Elemental Particles