Photonics / Nanophotonics

Lecture 25:
Problems on
Nanostructures

Problem 1 (Photonics AND Nanophotonics students)

Calculate the Bragg frequency and the gap-midgap ratio for the lowest bandgap of a 1D periodic structure comprising a stack of dielectric layers of equal optical thickness with $n_1 = 1.5$ and $n_2 = 3.5$ and $\Lambda = 2\ \mu m$. Assume the structure is a quarter-wave stack.

$$\omega_{Bragg} = \frac{n_1 + n_2}{4 n_1 n_2} \cdot \frac{2\pi c}{\Lambda} = \frac{1.5 + 3.5}{4 \cdot 1.5 \cdot 3.5} \cdot \frac{2\pi \cdot 10^8}{2 \cdot 10^{-6}} = 2.24 \cdot 10^{14} \frac{rad}{s}$$

$$\lambda_{Bragg} = \frac{2\pi c}{\omega} = 8.41\ \mu m$$

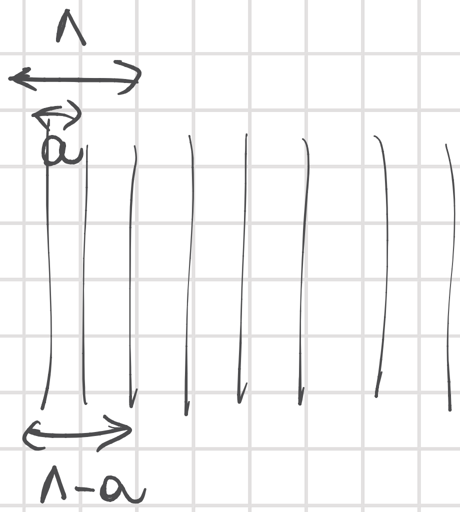
If $\frac{\Delta \varepsilon}{\varepsilon} < 1$ then we can assume to have weak periodicity

$$\frac{\Delta\varepsilon}{\varepsilon} = \frac{4}{12.25} = \underline{\underline{0.32 < 1}}$$

n_2^2

$$\Delta\varepsilon = \Delta n^2$$

$$\frac{\Delta\omega}{\omega_{Bragg}} \approx \frac{\Delta n^2}{n^2} \, \frac{\sin\frac{\pi a}{\Lambda}}{\pi}$$



The two layers componing the stack are

$$L_1 = a$$

$$L_2 = \Lambda - a$$

$$n_1 L_1 = n_2 L_2 \Rightarrow n_1 a = n_2 \Lambda - n_2 a \Rightarrow$$

$$a = \frac{n_2 \Lambda}{n_1+n_2} = \frac{3.5}{5} 2 \cdot 10^{-6} = 1.4 \cdot 10^{-6} = L_1$$

$$L_2 = \Lambda - a = 0.6 \cdot 10^{-6} = 0.6\ \mu m$$

$$\frac{\Delta\omega}{\omega_{Bragg}} \approx \frac{4}{12.25} \frac{\sin\left(\frac{\pi \cdot 1.4 \cdot 10^{-6}}{2 \cdot 10^{-6}}\right)}{\pi} = 0.0841$$

What changes if we consider $n_1 = 3.4$ and $n_2 = 3.6$

$$\omega_{Bragg} = \frac{n_1+n_2}{4 n_1 n_2} \cdot \frac{2\pi c}{\Lambda} = \frac{7}{4 \cdot 3.4 \cdot 3.6} \cdot \frac{2\pi\, 3 \cdot 10^{8}}{2 \cdot 10^{-6}} = 1.34 \cdot 10^{14} \frac{rad}{s}$$

$$\lambda_{Bragg} = 14.03\ \mu m$$

$\varepsilon = 3.6^2 = 12.96$

$\Delta\varepsilon = 0.04$

$\frac{\Delta\varepsilon}{\varepsilon} = 0.0031 \ll 1$

$\frac{\Delta\omega}{\omega_{Bragg}} \doteq \frac{\Delta n^2}{n^2} \quad \frac{\sin \pi a/\Lambda}{\pi} =$

$a = \frac{m_2 \Lambda}{m_1 + m_2} = 1.028 \cdot 10^{-6} \simeq \frac{\Lambda}{2}$

$L_2 \simeq 1 \cdot 10^{-6}$

$= \frac{0.04}{12.96} \cdot \frac{\sin \frac{\pi\, 1.028 \cdot 10^{-6}}{2 \cdot 10^{-6}}}{\pi} =$

$= 9.81 \cdot 10^{-4}$

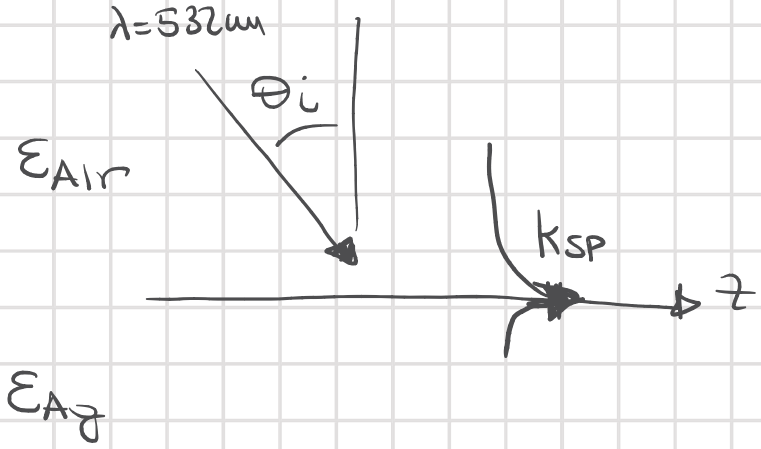
PROBLEM 2 (Photonics and Nanophotonics students)

Calculate the phase velocity of a surface plasmon propagating on a flat silver-air interface at $\lambda = 532$ nm and compare it to the phase velocity of light in air.

Assume the dielectric permittivity of silver at $\lambda = 532$nm is

$\varepsilon_{Ag} = -9.3 - j\,0.87$ whereas the permittivity of air is $\varepsilon_{Air} = 1$.

Determine the decay length of the surface plasmon along the propagation direction.



$$k_{sp} = k_0\, n_{sp}$$

Since we can satisfy $|\varepsilon'| \gg |\varepsilon''|$

$$n_{SP} = \sqrt{\frac{\varepsilon'_{Ag}\,\varepsilon_{Air}}{\varepsilon'_{Ag} + \varepsilon_{Air}}} = \sqrt{\frac{-9.3}{-8.3}} = 1.0585$$

$$n_{SP} > n_{Air}$$

The phase velocity of the SP is: $v_{p_{SP}} = \frac{c}{n_{SP}} = \frac{3 \cdot 10^8}{1.0585} = 2.83 \cdot 10^8 \frac{m}{s}$

$$< v_{p_{air}} = c$$

The propagation constant of the SP is a complex quantity

$$k_{SP} = \frac{\omega}{c} n_{SP} = \frac{\omega}{c} \sqrt{\frac{\varepsilon_{Ag}\,\varepsilon_{Air}}{\varepsilon_{Ag} + \varepsilon_{Air}}} = k'_{SP} - j\,k''_{SP}$$

The propagation length (decay length)

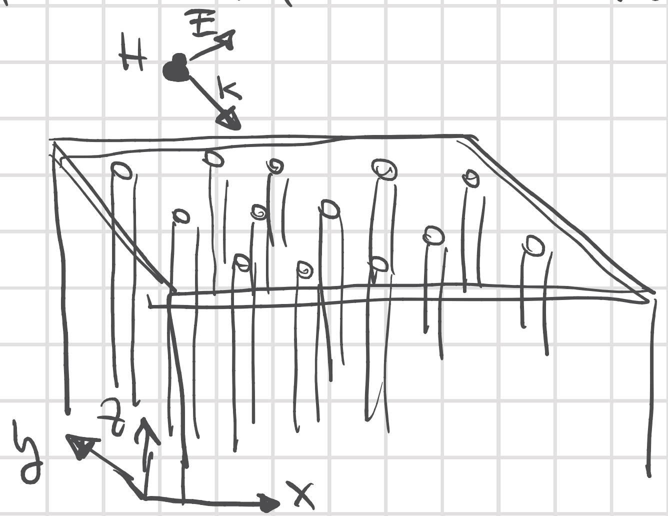
$$L_{sp} = \frac{1}{2 k_{sp}''} = \frac{c}{\omega} \underbrace{\frac{\sqrt{|\varepsilon_m'|} \left(|\varepsilon_m'| - \varepsilon_d\right)^{3/2}}{\varepsilon_m'' \, \varepsilon_d^{3/2}}}_{1/n_{sp}''} =$$

$$= \frac{1}{2} \frac{\lambda}{2\pi} \frac{1}{n_{sp}''} = 7.1 \cdot 10^{-6}\,m$$

Problem 3 (Photonics and Nanophotonics students)

Consider a wire medium with metal fill factor $f = 0.1$. Assume for metal a complex, frequency dependent dielectric constant that can be modeled with a single Drude oscillator with $\omega_p = 2\pi \cdot 2.18 \cdot 10^{15}\ s^{-1}$ and damping $\gamma = 2\pi \cdot 4.35 \cdot 10^{12}\ s^{-1}$ and for the dielectric a dispersion-free permittivity $\varepsilon_d = 2.25$.

Can this structure be considered a hyperbolic metamaterial for a TM-polarized wave at 500 nm?



$$\omega_p = 2\pi \cdot 2.18 \cdot 10^{15}\ s^{-1}$$

$$\gamma = 2\pi \cdot 4.35 \cdot 10^{12}\ s^{-1}$$

$$\varepsilon_d = 2.25$$

$$\varepsilon_m = \varepsilon_i = 1 - \frac{\omega_p^2}{\omega^2 - j\omega\gamma} = 1 - \frac{\left(2\pi \cdot 2.18 \cdot 10^{15}\right)^2}{\left(\frac{2\pi c}{500 \cdot 10^{-9}}\right)^2 - j\left(\frac{2\pi c}{500 \cdot 10^{-9}}\right)\left(2\pi \cdot 4.35 \cdot 10^{12}\right)}$$

$$= \underline{\underline{-12.2 - j\,0.09}}$$

We can assume the wire medium to be an effective uniaxial medium

$$\bar{\varepsilon} = \varepsilon_\perp \left(\hat{x}\hat{x} + \hat{y}\hat{y}\right) + \varepsilon_\parallel \hat{z}\hat{z}$$

parallel to the wire axis

$$\langle D_\parallel \rangle = \varepsilon_0 \underline{\varepsilon_\parallel} \langle E_\parallel \rangle = \varepsilon_0 f \varepsilon_i E_{\parallel,i} + \varepsilon_0 (1-f)\,\varepsilon_h E_{\parallel,h}$$

$$\boxed{\varepsilon_{\parallel} = f\,\varepsilon_i + (1-f)\,\varepsilon_h}$$

$\varepsilon_{\perp}$ can be calculated looking at a section in the xy plane.

$L_x = L_y = \frac{1}{2}$

$$\boxed{\varepsilon_{\perp} = \varepsilon_h \left(1 + 2f \, \frac{\varepsilon_i - \varepsilon_h}{\varepsilon_i + \varepsilon_h - f(\varepsilon_i - \varepsilon_h)} \right)}$$

When $f = 0.1$ $\longrightarrow$

$$\varepsilon_{\parallel} = 0.805 - j\,0.0096$$
$$\varepsilon_{\perp} = 3.01 - j\,0.0027$$

Our wire medium

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{\perp} & 0 & 0 \\ 0 & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon_{\parallel} \end{bmatrix}$$

The dispersion relation assuming k in the x,z plane

$$\boxed{\frac{k_x^2}{\varepsilon_{\parallel}} + \frac{k_z^2}{\varepsilon_{\perp}} = \left(\frac{\omega}{c}\right)^2}$$

for $f=0.1$ $\qquad$ $\underline{\varepsilon_{\parallel}\varepsilon_{\perp} > 0}$ $\Rightarrow$ THE MEDIUM IS NOT HYPERBOLIC

Problem 5

Photonics and Nanophotonics Students

A camera lens ($n_{ge} = 1.55$) is coated with a cryolite film ($n_{CR} = 1.3$) as an antireflection coating. a) What is the optimal film thickness for incident green light ($\lambda_0 = 532\,nm$)?

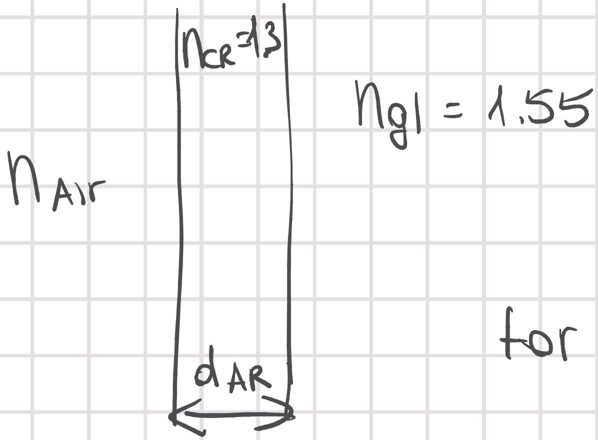
b) Under this scenario and with the calculated thickness, are we able to completely suppress reflection? Assume the medium outside the camera is air ($n_{Air} = 1$) and $\lambda = 532\,nm$.

a) Optimal thickness for antireflection coating

$$d_{AR} = \frac{\lambda_{eff}}{4} \qquad \lambda_{eff} = \frac{\lambda_0}{n_{CR}} = \frac{532 \cdot 10^{-9}}{1.3} = 409.23 \cdot 10^{-9}$$

$$d_{AR} = \frac{409.23 \cdot 10^{-9}}{4} = 102.3\ nm$$

(b)



for $R = 0$

$$n_{CR} = \sqrt{n_{AIR} n_{GL}}$$

this is not satisfied

So the right medium for an AR coating for the camera in air

$$n_x = \sqrt{1.55} = 1.245$$

$$d_x = \frac{\lambda_x}{4} = \frac{532 \cdot 10^{-9} / n_x}{4} = 106.8 \text{ nm}$$