0.1 Basic Maths and Concepts

0.1.1 Quick Intro to the Chapter and Section

Digital modulation techniques are fundamental in modern communication systems, enabling the transmission of digital information over analog channels. Several key mathematical concepts and equations are essential to understanding and implementing digital modulation techniques. In this Section we shall present some of the most basic modulation techniques, concepts, and specifications, keep in mind though that specific modulation schemes may have variations and additional considerations depending on factors like channel characteristics and system requirements.

0.1.2

0.2 Baseband Representation of Digital Signals

Digital signals are typically represented in baseband using pulse waveforms. A simple rectangular pulse can be represented with the equation:

$$x(t) = \sum_{n=-\infty}^{\infty} A_n p(t - nT) \tag{1}$$

Where:

- A_n is the amplitude of the n^{th} pulse.
- p_t is the basic pulse shape (often a rectangular pulse, but it can be other shapes).
- T is the pulse duration or the time between the start of successive pulses.

Equation 0.2 essentially states that the digital signal is composed of a series of pulses, each shifted by multiples of T seconds. The amplitudes A_n determine the information content of each pulse. For a rectangular pulse $p(t)$ is defined as:

$$p(t) = \text{rect}\left(\frac{t}{T}\right) = \begin{cases} 1, & \text{if } |t| < \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}$$

To be clear, in equation 0.2, ”rect” is the rectangular function, and T is the duration of the pulse.
This representation is foundational for understanding how digital signals are formed and transmitted. It forms the basis for various modulation techniques, where the information is encoded in the characteristics of these pulses.

0.2.1 Amplitude Modulation

Amplitude Modulation (AM) is a modulation technique where the amplitude of a carrier signal is varied in proportion to the instantaneous amplitude of a modulating signal (message signal, $m(t)$). It is widely used in analog audio and video broadcasting and communication systems. The modulated signal $s(t)$ in the time domain is given by:

$$s(t) = A_c[1 + m \cdot x(t)] \cdot \cos(2\pi f_c t) \tag{2}$$

Where:

- A_c is the amplitude of the carrier signal,
- m is the modulation index, representing the extent of modulation,
- $x(t)$ is the baseband message signal (modulating signal),
- f_c is the frequency of the carrier signal.

In eq. 0.2.1, " $A_c[1 + m \cdot x(t)]$ ", represents the instantaneous amplitude of the modulated signal, and $cos(2\pi f_c t)$ is the carrier signal.
The modulation index, m, determines the depth of modulation. If $m = 0$, there is no modulation, and the output is just the carrier signal. As m increases, the amplitude of the carrier signal varies more with the message signal. It is also possible for the carrier to "cross" the time axis, resulting in a phase inversion, the circuit to transmit and receive this will be much more complex.

The modulation index is defined as the ratio of the peak amplitude of the modulating signal to the peak amplitude of the carrier signal. It is often expressed as a percentage. See equation 0.2.1:

$$m = \frac{\text{Amplitude of modulating signal}}{\text{Amplitude of carrier signal}} \times 100\% \tag{3}$$

Frequency components can be created from the expansion of the modulated signal, $s(t)$:

$$s(t) = A_c \cos(2\pi f_c t) + \frac{mA_c}{2}[x(t) \cos(2\pi(f_c - f_m)t) + x(t) \cos(2\pi(f_c + f_m)t)], \tag{4}$$

Note: f_m is the frequency of the modulating signal.

However, AM is somewhat inefficient; the **carrier doesn't contain any message information** but we are still using power to transmit i.
In AM, the frequency spectrum of the signal consists of the carrier frequency and two sidebands located above and below the carrier frequency, each containing the same information as the modulating signal. One can obtain the carrier from either of the bands (they are the conjugate of each other, thi is a Hermitian property). So if one wished to save more power on transmission, **on of the side bands can be removed.**