# MICROWAVE ENGINEERING

HOMEWORK 1

SOLUTION

Problem 1 : A plane wave traveling along z in a medium with $\varepsilon_r = 2.55$ has an electric field $E_y = E_0 \cos(\omega t - kz)$. Frequency is 2.4 GHz and $E_0 = 30 \frac{V}{m}$. Find:

a) Amplitude and direction of H;

b) Phase velocity;

c) Phase shift between $z_1 = 0.5$ m and $z_2 = 1.7$ m.

a) The impedance of the medium $\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{2.55}} = 236\ \Omega$

$$\bar{E} = E_0 e^{-jkz} \hat{y}$$

$$\nabla \times \bar{E} = -j\omega\mu\bar{H} \Rightarrow \cancel{-}\hat{z}\,\cancel{j}k \times E_0 e^{-jkz}\,\hat{y} = \cancel{-j}\omega\mu\bar{H}$$

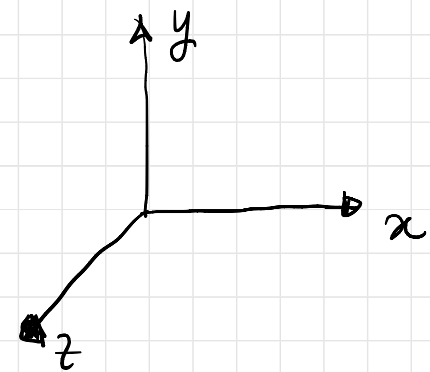
$$\nabla = jk$$

$$H = -\hat{x}\frac{k}{\omega\mu} E_0 e^{-jkz} =$$

$$= -\hat{x}\frac{\cancel{\omega}\sqrt{\mu\varepsilon}}{\cancel{\omega}\mu} E_0 e^{-jkz} =$$

$$= -\hat{x}\underbrace{\sqrt{\frac{\varepsilon}{\mu}}}_{1/\eta} E_0 e^{-jkz} \Rightarrow$$

$$\boxed{H_x = -\frac{E_0}{\eta}\cos(\omega t - kz)}$$




$x \times y \Rightarrow z$

$z \times x \Rightarrow y$

$y \times z \Rightarrow x$

Direction: $-x$

$$\text{Amplitude} = \frac{\bar{E}_0}{\eta} = \frac{30}{236} = 0.127 \ \frac{A}{m}$$

$$\boxed{H_x = -0.127\cos(\omega t - kz)}$$

b) Phase velocity:

$$V_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \cdot 10^8}{\sqrt{2.55}} = 1.88 \cdot 10^8 \ m/s$$

c) $$e^{-j\underline{kz}} = e^{-j\phi}$$

$$\phi_2 - \phi_1 = kz_2 - kz_1 = k(\Delta z) = \frac{\omega}{V_p}(\Delta z) =$$

$$= \frac{2\pi f}{V_p} \Delta z = \frac{2\pi \cdot 2.4 \cdot 10^9}{1.88 \cdot 10^8} (1.7 - 0.5) = 96.25 \text{ rad}$$

$$\simeq 114°$$

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② Compare the polarization of the following plane waves:

a)

Wave 1: $E_1 = \hat{x}\, 2\cos(\boxed{\omega t} - kz) + \hat{y}\, 2\sin(\omega t - kz)$

Wave 2: $E_2 = \hat{x}\, 2\cos(\omega t + kz) + \hat{y}\, 2\sin(\omega t + kz)$

<u>WAVE 1</u> $\hat{y}$ component is shifted of $-\frac{\pi}{2}$ with respect to $\hat{x}$ component.

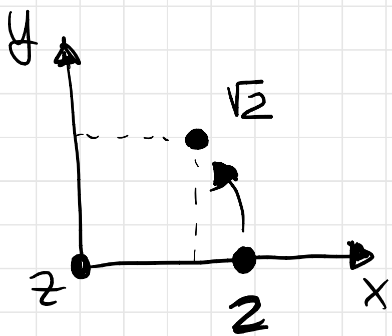
Propagates in the positive z direction

$\longrightarrow$ RHCP +z direction

At z=0

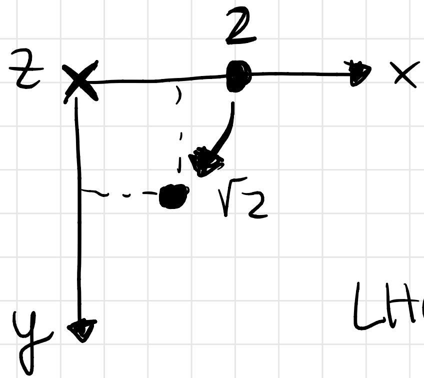
$z=0 \quad \omega t = 0 \quad \rightarrow \quad \bar{E}_1 = 2\hat{x}$

$z=0 \quad \omega t = \frac{\pi}{4} \quad \rightarrow \quad \bar{E}_1 = \sqrt{2}\,\hat{x} + \sqrt{2}\,\hat{y}$




WAVE 2 $\quad \hat{y}$ comp has $-\frac{\pi}{2}$ phase shift with resp. to $\hat{x}$

Propagates in the negative z direction.




$z=0 \quad \omega t=0 \quad \rightarrow E_2 = 2\hat{x}$

$z=0 \quad \omega t = \frac{\pi}{4} \quad \rightarrow E_2 = \sqrt{2}\,\hat{x} + \sqrt{2}\,\hat{y}$

LHCP travelling in $-z$ direction

b) Wave 1 : $E_1 = \hat{x}\, 2\cos(\omega t - kz) - \hat{y}\, 2\sin(\omega t - kz)$

$E_2 = \hat{x}\, 2\cos(\omega t + kz) - \hat{y}\, 2\sin(\omega t + kz)$

WAVE 1 : LHCP $+z$

WAVE 2 : RHCP $-z$

Problem 3

The critical angle for an interface is 45°. What is the Brewster angle for the same interface?

$$\theta_c = \operatorname{asin}\left(\frac{n_2}{n_1}\right) = 45^\circ$$

$$\frac{n_2}{n_1} = \sin 45^\circ = \frac{\sqrt{2}}{2}$$

$$\theta_B = \operatorname{atan}\left(\frac{n_2}{n_1}\right) = \operatorname{atan}\left(\frac{\sqrt{2}}{2}\right) = 35.26^\circ$$