Tutorial 2 - Gaussian Beams

A.DB

Reminder:

$$E(x,y,z) = \frac{E_0}{\sqrt{1+z^2/\alpha^2}} \cdot \underbrace{e^{-\frac{(x^2+y^2)}{\omega^2(z)}}}_{\text{Gaussian amplitude}} \cdot \underbrace{e^{-j\frac{\pi(x^2+y^2)}{\lambda R(z)}}}_{\text{spherical phase term}} \cdot \underbrace{e^{-jkz}}_{\text{propagation term}} \cdot \underbrace{e^{-j\arctan(z/\alpha)}}_{\text{Gouy phase shift}}$$

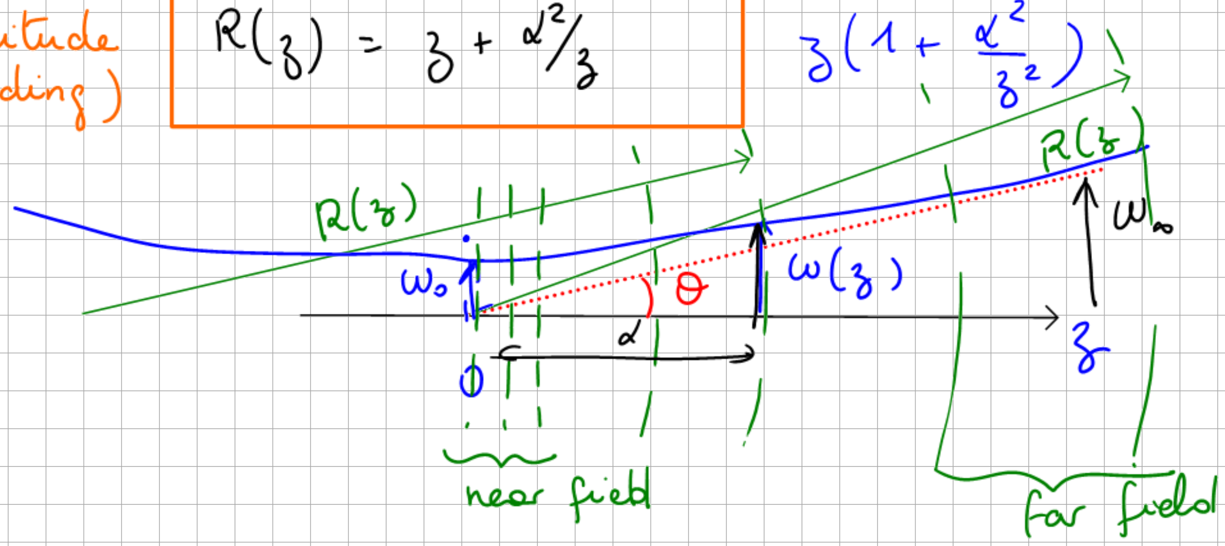
energy conservation (Amplitude spreading)

with

$$\alpha = \frac{\pi \omega_0^2}{\lambda}$$

$$\omega^2(z) = \omega_0^2\left(1 + z^2/\alpha^2\right)$$

$$R(z) = z + \alpha^2/z$$

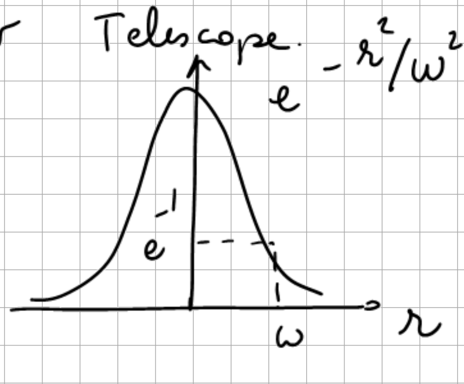


Ex 1.

1) $\omega^2(z=\alpha) = \omega_0^2(1+1) \Rightarrow \boxed{\omega(\alpha) = \sqrt{2}\,\omega_0}$

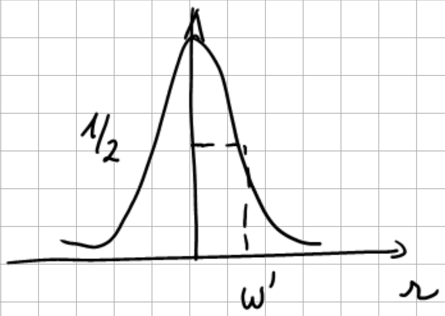
2) $\alpha = \frac{\pi \omega_0^2}{\lambda}$

ω_0	0.01 mm	1 mm	10 mm	1 m
α	0.3 mm	3 m	300 m	3 000. km.
	Single mode fiber	He-Ne Laser	Expander	Telescope.



$z \ll \alpha \rightarrow$ Near field

$z \gg \alpha \rightarrow$ Far field



$$e^{-2r^2/w^2} = 1/2 \quad \text{with } r = w'$$

$$r^2/w^2 = 2\ln 2 \Rightarrow \underset{w'}{\underset{\parallel}{r}} = w\sqrt{2\ln 2}$$

3) $\lambda = 633\,nm \qquad 2w_0 = 1\,mm. \qquad \alpha = \dfrac{\pi w_0^2}{\lambda}$

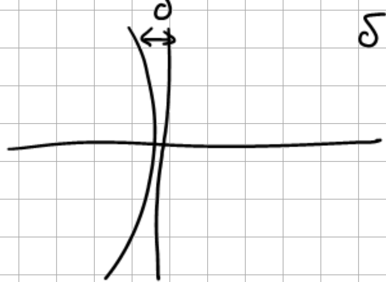
$\underline{z \gg \alpha}$ far field $\qquad w^2 = w_0^2\left(1 + z^2/\alpha^2\right) \qquad \underset{\theta}{\underset{\parallel}{\tan\theta}}$

$$w_\infty = w_0 z/\alpha = \frac{w_0 z \lambda}{\pi w_0^2} \Rightarrow w_\infty = \frac{\lambda z}{\pi w_0} \Rightarrow \theta = \frac{w_\infty}{z}$$

$$\Rightarrow \boxed{\theta = \frac{\lambda}{\pi w_0}}$$

NA: $\quad w_0 = 0.5\,mm \qquad \lambda = 633\,nm.$

$\Rightarrow \theta = 4 \cdot 10^{-4}\,rad. \longrightarrow 2\theta = \text{divergence} = 8 \cdot 10^{-4}\,rad$

$R(\alpha) \simeq$ 

$\delta \simeq \dfrac{\lambda}{6}$

b)

$$\alpha = \frac{\pi w_0^2}{\lambda} = 1.24\,m. \ll z \longrightarrow \text{Far Field}$$

$$w(z) = w_\infty = \theta \cdot z = 4 \cdot 10^{-3}\,m. = 4\,mm.$$

$$\phi = 8\,mm.$$

$$R(z) = z + \alpha^2/z \qquad \alpha \ll z$$

$$R(z) \underset{\text{far field}}{\underset{\uparrow}{\simeq}} z = 10\,m$$

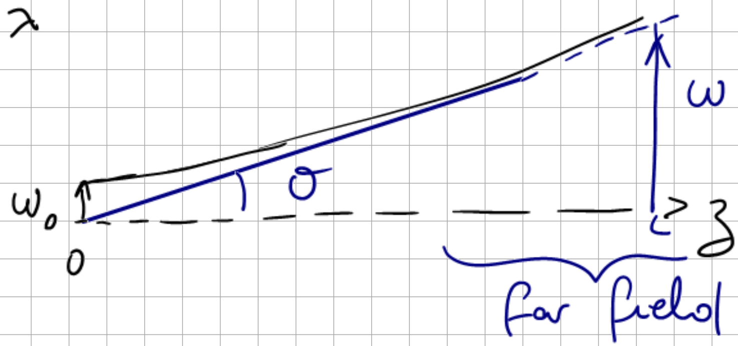
Ex2 :

1) Far field $\Rightarrow$ $w(L) = \theta . L$ with $\theta = \dfrac{\lambda}{\pi w_0}$

$\simeq 25\,km$

$\phi(L) \simeq 50\,km$

$\alpha = \dfrac{\pi w_0^2}{\lambda}$ $\qquad L >> \alpha$



$\dfrac{w}{L} = \theta$

$\theta = \dfrac{\lambda}{\pi w_0}$

2) $P_0 = k \displaystyle\iint_{-\infty}^{+\infty} |E|^2 \, dS$

$P_0 = \dfrac{\mathcal{E}}{\Delta t} = \dfrac{1}{2.10^{-9}} = 0.5\ GW$

$P_M = k \displaystyle\iint_M |E|^2 \, dS$

$E(z=0, x, y) = E_0 \, e^{-\left(\frac{x^2+y^2}{w_0^2}\right)}$

$\hookrightarrow = 1$

$P_0 = k \displaystyle\iint_{-\infty}^{+\infty} e^{-2\left(\frac{x^2+y^2}{w_0^2}\right)} \, dx\, dy$

$= k \displaystyle\int_0^{2\pi}\int_0^{+\infty} e^{-2\frac{\rho^2}{w_0^2}} \, \rho\, d\rho\, d\varphi$

$P_0 = k . 2\pi \left[-\dfrac{w_0^2}{4} \, e^{-2\rho^2/w_0^2} \right]_0^{+\infty}$

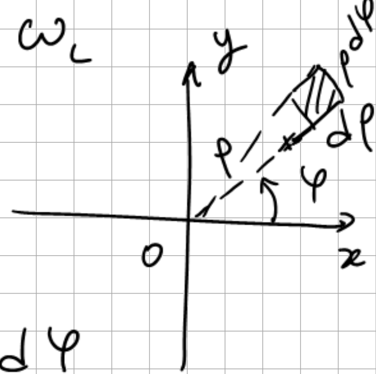
$$\boxed{P_0 = \frac{\pi}{2} k w_0^2}$$

Whole power on Moon : $T . P_0 = P_{Moon}$

Power on the mirror (without losses due to the atmosphere)

$$|E_L(z=L, x, y)| = \frac{E_0}{\sqrt{1 + L^2/\alpha^2}} e^{-(x^2+y^2)/w_L^2} = \frac{w_0}{w_L} e^{-\frac{(x^2+y^2)}{w_L^2}}$$

$$w_L^2 = w_0^2\left(1 + \frac{L^2}{\alpha^2}\right) \Rightarrow \frac{1}{\sqrt{1 + L^2/\alpha^2}} = \frac{w_0}{w_L}$$



$$P_{Mirror} = k\tau \cdot \iint_{Mirror} |E_{Moon}^2| \, dS$$

$$= k\tau \frac{w_0^2}{w_L^2} \int_0^{2\pi} \int_0^{r} e^{-2\rho^2/w_L^2} \rho \, d\rho \, d\varphi$$

r = radius of the mirror

$$P_{mirror} = 2\pi k \tau \frac{w_0^2}{w_L^2} \left[-\frac{w_L^2}{4} e^{-2\rho^2/w_L^2} \right]_0^r$$

$$P_{mirror} = \frac{\pi}{2} k \tau w_0^2 \left(1 - e^{-2r^2/w_L^2}\right)$$

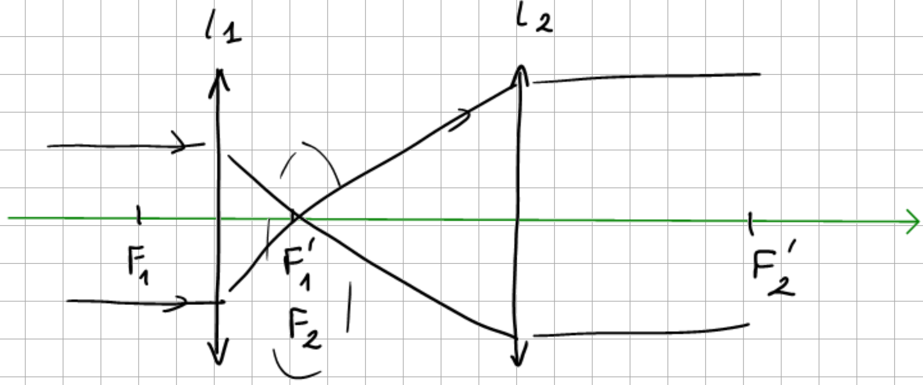
$$\boxed{P_{mirror} = \tau P_0 \left(1 - e^{-2r^2/w_L^2}\right)}$$

$r \ll w_L$ $\quad e^{-2r^2/w_L^2} \simeq 1 - \frac{2r^2}{w_L^2}$ Taylor expansio

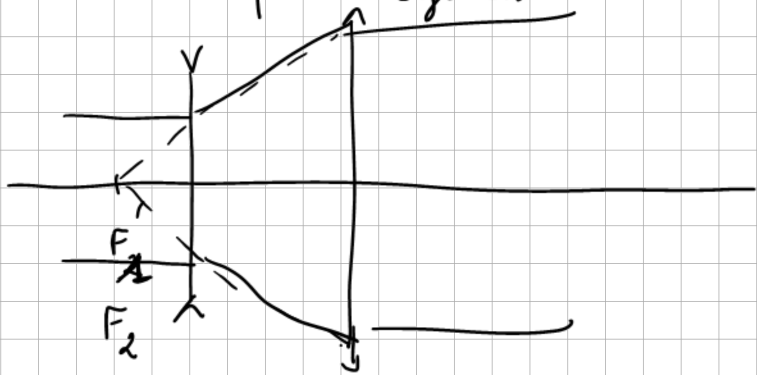
$$P_{mirror} = \tau P_0 \cdot \frac{2r^2}{w_L^2} \Rightarrow \boxed{2\tau P_0 \left(\frac{r}{w_L}\right)^2 = P_{mirror}}$$

$$\rightarrow \boxed{P_{mirror} = 0.18 \text{ W}}$$

3) Telescope to increase w_0 (i.e. increase α and decrease θ)



Afocal system

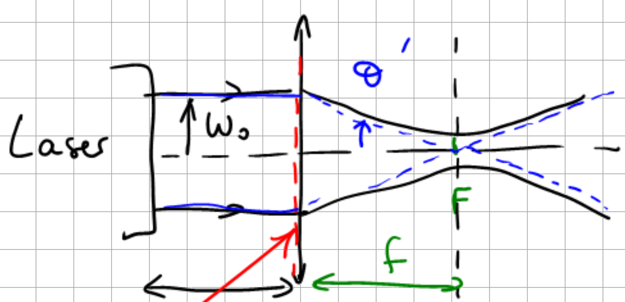


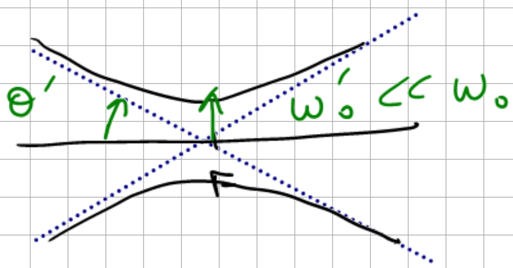
Ex : $\lambda_0 = 1.55\,\mu m$.

1) $\alpha = \dfrac{\pi w_0^2}{\lambda} = 2\,m$.

$w_0 = 1\,mm$.

2)





$10cm \ll \alpha$

plane wavefront $\longrightarrow$ focusing of the laser light in the focal plane

a) $w_0' = 4\,\mu m \Rightarrow \theta' = \dfrac{\lambda}{\pi w_0'} = 0.123$ rad

b) $\theta' <$ N.A $\rightarrow$ ok

c) $\theta' = \dfrac{w_0}{f} \Rightarrow f = \dfrac{w_0}{\theta'} = \dfrac{\pi w_0 w_0'}{\lambda} = \boxed{8\,mm}$

Ex 4

1. $\alpha = \frac{\pi w_0^2}{\lambda} = 38.7\,\mu m$

2. $z >> \alpha$

$$E(x,y,z) = j\frac{E_0\alpha}{z} e^{-(x^2+y^2)/\theta^2 z^2} e^{-j\pi(x^2+y^2)/\lambda z} e^{-jkz}$$

3. $$E_L = j\frac{E_0\alpha}{d} e^{-(x^2+y^2)/\theta^2 d^2} e^{-j\pi\frac{(x^2+y^2)}{\lambda}\left(\frac{1}{d}-\frac{1}{f}\right)} e^{-jkz}$$

4. $d = f$

$$E_L = j\frac{E_0\alpha}{f} e^{-(x^2+y^2)/\theta^2 f^2} e^{-jkf}$$

5. $w_L = \theta f = 30\text{ mm}$.

$\alpha_L = \frac{\pi w_L^2}{\lambda} = 2{,}2\,km \rightarrow$ Collimated beam.