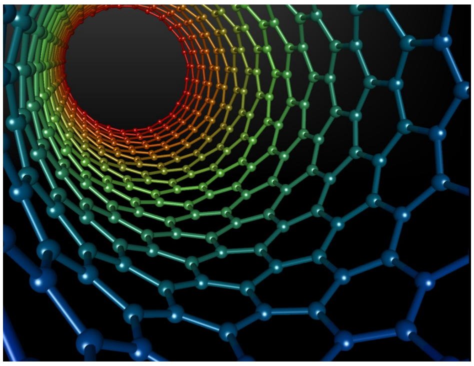
Set #3

8.

Suppose a particle is moving freely inside a carbon nanotube. If the particle if moving with a given momentum p (eigenvalue), find its quantum state (eigenfunction) by directly solving the corresponding eigenvalue equation for the momentum operator:

$$\hat{p} = -i\hbar\hat{\nabla}$$



b. Show that these particle states are orthogonal (in the Dirac sense)

c. Show that such states are also complete with respect to any square integrable function.

d. Apply part c. to a Gaussian function of width σ and centred at the origin and one centred at some position $x = a$

9.

a. For an electron moving inside a tiny nanotube (1D) give the general solution of the Schroedinger equation.

b. Discuss (briefly) whether such a solution is acceptable. In the negative what would you foresee as a way out?

10.

Work out eigenfunctions and eigenvalues of a free electron inside a box of sides {a,b,c} with impenetrable walls using the method of separation of variables for (partial) differential equations.

11.

Find under what condition the solutions of the Schroedinger eq.

$$i\hbar\,\partial_t\Psi(x,t) = -\frac{\hbar^2}{2m}\partial_x^2\Psi(x,t) + V(x,t)\Psi(x,t)$$

can be factorized as $\psi(x,t) = \psi(x)\,T(t)$. Determine the form of $T(t)$.