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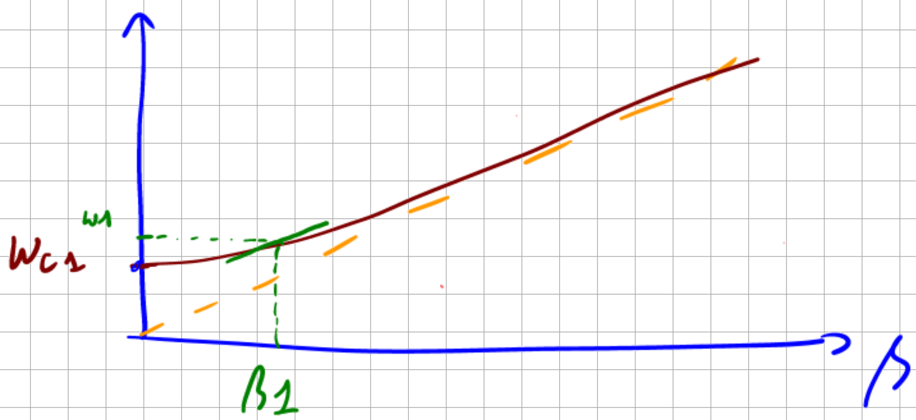
Lesson 10
Oct 6th 2021

$2 \to 3$
ω



$\cos(\omega t - \beta z)$

phase velocity $v_{\varphi 1} = \frac{\omega_1}{\beta_1}$ dispersive

group velocity $v_{g1} = \left.\frac{\partial \omega}{\partial \beta}\right|_1$

at $f_{c\,TE_{10}}$ $v_g = 0$

$f \gg f_{c\,TE_{10}}$ $v_g = v_0 = \frac{1}{\sqrt{\varepsilon\mu}}$

$$k_c^2 = \left(\frac{\omega}{v_0}\right)^2 - \beta^2$$

$$\omega = v_0 \sqrt{k_c^2 + \beta^2}$$

$$v_g = \frac{\partial \omega}{\partial \beta}$$

2.4. Computation of the transversal components of the field

$$H_z(x,y) = H_0 \cos\frac{m\pi}{a}x \cos\frac{n\pi}{b}y$$

$\gamma = j\beta$ $\quad k_c^2 = k_0^2 + \gamma^2$

$$E_z(x,y) = 0$$

$$\overline{H_t}(\xi,\eta) = -\frac{\gamma}{K_c^2}\overline{\nabla_t}H_z(\xi,\eta) - \frac{j\omega\varepsilon}{K_c^2}\vec{u}\wedge\overline{\nabla_t}E_z(\xi,\eta)$$

And for the electrical component :

$$\overline{E_t}(\xi,\eta) = -\frac{\gamma}{K_c^2}\overline{\nabla_t}E_z(\xi,\eta) + \frac{j\omega\mu}{K_c^2}\vec{u}\wedge\overline{\nabla_t}H_z(\xi,\eta)$$

$$\vec{H_t}(x,y) = -\frac{\gamma}{k_c^2}\vec{\nabla_t} H_z(x,y)$$

$$= -\frac{\gamma}{k_c^2}\left(\frac{\partial}{\partial x}H_z(x,y)\vec{e_x} + \frac{\partial}{\partial y}H_z(x,y)\vec{e_y}\right)$$

$$H_x(x,y) = \frac{\gamma}{k_c^2} H_0 \frac{m\pi}{a} \sin\frac{m\pi}{a}x \cos\frac{n\pi}{b}y$$

$$H_y(x,y) = \frac{\gamma}{k_c^2} H_0 \frac{n\pi}{b} \cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y$$

$$\vec{E}_t(x,y) = \frac{j\omega\mu}{k_c^2}\left(\vec{u}_z \times \vec{\nabla}_t H_z(x,y)\right)$$

$$\vec{\nabla}_t H_z(x,y) = -\frac{k_c^2}{\gamma}\vec{H}_t(x,y)$$

$$\vec{E}_t(x,y) = \frac{-j\omega\mu}{\gamma}\left(\vec{u}_z \times \vec{H}_t(x,y)\right)$$

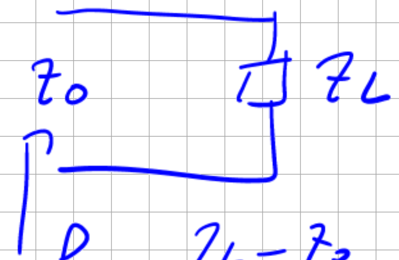
$Z_{TE_{mn}} = \frac{j\omega\mu}{\gamma}$ impedance of TE_{mn} mode

if $\gamma = j\beta$, for $f > f_{cTE_{mn}}$

$$Z_{TE_{mn}} = \frac{\omega\mu}{\beta} \quad \text{real}$$

$\omega = \omega_{cTE_{mn}}$ $\quad Z_{TE_{mn}} \to \infty$

$\beta = 0$



$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$E_x(x,y) = -\frac{j\omega\mu}{j\beta}\left(\vec{u}_z \times H_y(x,y)\,\vec{e}_y\right)$$

$$= \frac{\omega\mu}{\beta}\frac{j\beta}{k_c^2} H_0 \frac{n\pi}{b}\cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y$$

$$E_y(x,y) = \frac{-j\omega\mu}{k_c^2} H_0 \frac{m\pi}{a}\sin\frac{m\pi}{a}x \cos\frac{n\pi}{b}y$$

$$H_z(x,y) = H_0 \cos\frac{m\pi}{a}x \cos\frac{n\pi}{b}y$$

$$h_z(x,y,z,t) = \mathrm{Re}\left(H_z(x,y)\,e^{j\omega t}e^{-j\beta z}\right)$$

H_0 real

$$h_z(x,y,z,t) = H_0 \cos\frac{m\pi}{a}x \cos\frac{n\pi}{b}y \cos(\omega t - \beta z)$$

$$h_x(x,y,z,t) = \frac{-\beta}{k_c^2} H_0 \frac{m\pi}{a}\sin\frac{m\pi}{a}x \cos\frac{n\pi}{b}y \sin(\omega t - \beta z)$$

$$h_y(x,y,z,t) = \frac{-\beta}{k_c^2} H_0 \frac{n\pi}{b}\cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y \sin(\omega t - \beta z)$$

$$e_x(x,y,z,t) = -\frac{\omega\mu}{k_c^2} H_0 \frac{m\pi}{a} \sin\frac{m\pi}{a}x \cos\frac{n\pi}{b}y \sin(\omega t-\beta z)$$

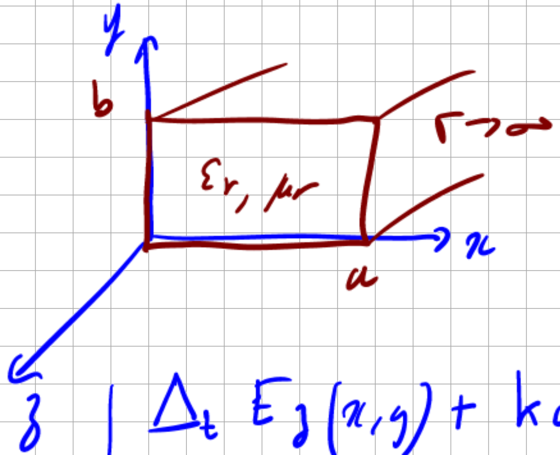
$$e_y(x,y,z,t) = +\frac{\omega\mu}{k_c^2} H_0 \frac{n\pi}{b} \cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y \sin(\omega t-\beta z)$$

$\vec{E}_t$, $\vec{H}_t$ are in phase

quadrature of phase for H_z and $(\vec{E}_t, \vec{H}_t)$

$$P = \frac{1}{2} \mathrm{Re} \int_0^a\!\!\int_0^b (\vec{E}_t \times \vec{H}_t^*)\cdot\vec{n}\, dS$$

III TM modes − $\gamma = j\beta$



$H_z(x,y) = 0$

$$\Delta_t E_z(x,y) + k_c^2 E_z(x,y) = 0$$

$$k_c^2 = k_0^2 - \beta^2$$

for $x=0$, $x=a$, $y=0$, $y=b$ $\quad E_z = 0$

Same propagation equation to solve ≠ boundary conditions.

$$E_z(x,y) = f(x)\,g(y)$$
$$= (A\cos k_x x + B\sin k_x x) \times (C\cos k_y y + D\sin k_y y)$$

$x=0 \quad A=0$

$x=a \quad B\sin k_x a = 0$

$k_x = \frac{m\pi}{a}, \; m \in \mathbb{N}$

$y=0 \quad C=0$

$y=b \quad D\sin k_y b = 0$

$k_y = \frac{n\pi}{b}, \; n \in \mathbb{N}$

$$k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$k_{c\,TE_{mn}} = k_{c\,TM_{mn}}$$

$$E_z(x,y) = E_0 \sin\frac{m\pi}{a}x \sin\frac{n\pi}{b}y$$

First TM mode $n \neq 0$ and $m \neq 0$

~~$n = 1$, $m = 0$~~

TM_{11} $a > b$

Second mode TM_{21}

Cut-off frequencies for the TM modes

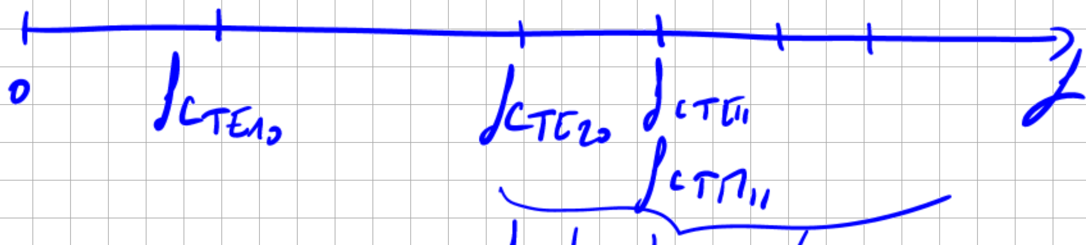
$k_c^2 = k_0^2 - \beta^2$, $\beta = 0$

$$f_{c\,TM_{mn}} = \frac{c}{2\pi\sqrt{\varepsilon_r \mu_r}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$= f_{c\,TE_{mn}}$$

$a > 2b$
$a < 3b$

only TE_{10} propagate: monomode



TE_{11} and TM_{11} are degenerate.

Fundamental mode : TE_{10}

$$\overline{H_t}(\xi,\eta) = -\frac{\gamma}{K_c^2}\overline{\nabla_t}H_z(\xi,\eta) - \frac{j\omega\varepsilon}{K_c^2}\vec{u} \wedge \overline{\nabla_t}E_z(\xi,\eta)$$

And for the electrical component :

$$\overline{E_t}(\xi,\eta) = -\frac{\gamma}{K_c^2}\overline{\nabla_t}E_z(\xi,\eta) + \frac{j\omega\mu}{K_c^2}\vec{u} \wedge \overline{\nabla_t}H_z(\xi,\eta)$$

$$\vec{H_t}(x,y) = -\frac{j\omega\varepsilon}{k_c^2}\left(\vec{u} \times \vec{\nabla_t} E_z(x,y)\right)$$

$$\vec{E_t}(x,y) = -\frac{\gamma}{k_c^2}\vec{\nabla_t} E_z(x,y)$$

if $\gamma = j\beta$, same phase for $\vec{E_t}$ and $\vec{H_t}$

$$\vec{\nabla_t} E_z(x,y) = -\frac{k_c^2}{\gamma}\vec{E_t}(x,y)$$

$$\vec{H_t}(x,y) = \frac{j\omega\varepsilon}{\gamma}\left(\vec{u} \times \vec{E_t}(x,y)\right)$$

$$Z_{TM_{mn}} = \frac{\gamma}{j\omega\varepsilon}$$

$\gamma = j\beta$ $\quad Z_{TM_{mn}} = \frac{\beta}{\omega\varepsilon}$

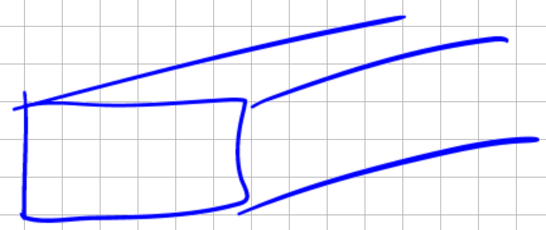
$\rho = -1$
In $f = f_{cTM_{mn}}$
short circuit

IV Evanescent modes – $\gamma = \alpha$

First TE_{10} mode

$$f_{cTE_{10}} = \frac{c}{2\sqrt{\varepsilon_r \mu_r}} \frac{1}{a}$$

Ψ



$$k_c^2 = \left(\frac{\pi}{a}\right)^2 = k_0^2 + \gamma^2$$

for $f > f_{cTE_{10}}$, $\gamma = j\beta$

so $f < f_{cTE_{10}}$, $\left(\frac{\pi}{a}\right)^2 > k_0^2$

$$\gamma = \alpha$$

$$k_c^2 = k_0^2 + \alpha^2$$

lossless waveguide

TE_{10} $H_z(x,y) = H_0 \cos\frac{\pi}{a}x$

$f < f_{cTE_{10}}$ $H_z(x,y,z) = H_0 \cos\frac{\pi}{a}x \, e^{-\alpha z}$

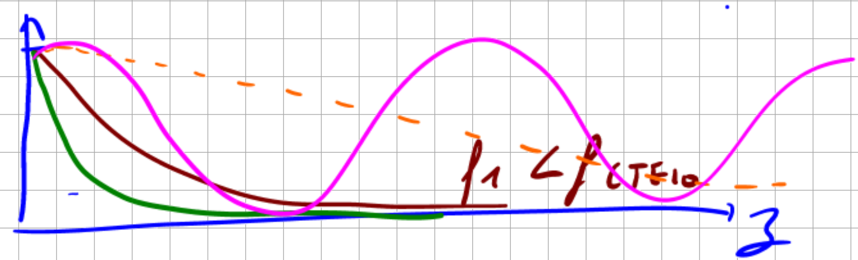
$$h_z(x,y,z,t) = H_0 \cos\frac{\pi}{a}x \, e^{-\alpha z} \cos\omega t.$$

$f > f_{cTE_{10}}$ $H_z(x,y,z) = H_0 \cos\frac{\pi}{a}x \, e^{-j\beta z}$

$$h_z(x,y,z,t) = H_0 \cos\frac{\pi}{a}x \cos(\omega t - \beta z)$$

h_z

Ψ



$\gamma = \alpha$

$f_2 < f_1$

$f_3 \neq f_{cTE_{10}}$

$f_4 > f_{cTE_{10}}$

$$Z_{TE_{nm}} = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\alpha}$$

$$\vec{E}_t = -Z_{TE_{nm}} (\vec{n} \times \vec{H}_t)$$

$\sin\omega t$ $\cos\omega t$

$\gamma = \alpha$ $\quad f_{c\,TE_{10}}$ $\quad \gamma = j\beta$

$f = f_c \, , \quad \gamma = \alpha + j\beta$
$= 0$