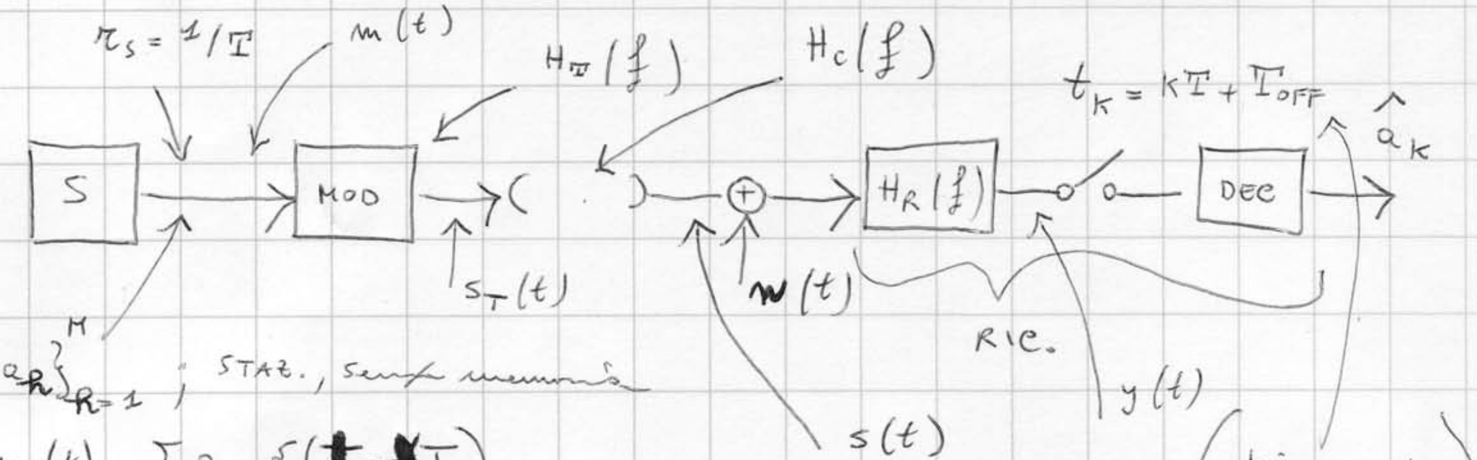
Base-band digital communication over band-limited channels

TRASM. NUMERICA in BANDA BASE
su canali REALI a BANDA limitata



$\{a_k\}_{k=1}^{M}$; STAZ., senza memoria

tiene conto dei vari ritardi $T_T, T_c, T_R, \ldots$ T_0 del FA, ...

$$\begin{cases} m(t) = \sum_k a_k \delta(t-kT) \\ s_T(t) = \sum_k a_k g_T(t-kT) \end{cases} ; \quad g_T(t) \longleftrightarrow H_T(f)$$

T_T

$$s(t) = s_T(t) * h_c(t) = \sum_k a_k g(t-kT)$$

T_c

$$g(t) = g_T(t) * h_c(t) = h_T(t) * h_c(t)$$

NB: the time delays due to the channel and to the various filters have been understood

Ip: we consider linear phase filters

IP: Filtri a FASE LINEARE

$$\Downarrow$$

$$H_1(f) = |H_1(f)|\, e^{-j 2\pi f T_1}$$

T1: Delay due to the filter

$$y(t) = \big(s(t) + w(t)\big) * h_R(t) =$$

$$y(t) = \sum_k a_k\, p(t-kT) + n_0(t) \quad ; \quad p(t) = g(t) * h_R(t)$$

$$y(\underset{\uparrow}{t_K}) = \sum_i a_i \, p(\underset{\substack{\uparrow \\ t_K}}{kT} - iT) + n_0(\underset{\substack{\uparrow \\ t_K}}{kT})$$

Istante di campionamento : $t_K = kT + ---$

Sampling instant

SOTTINTENDO un ritardo ----

Delays are understood

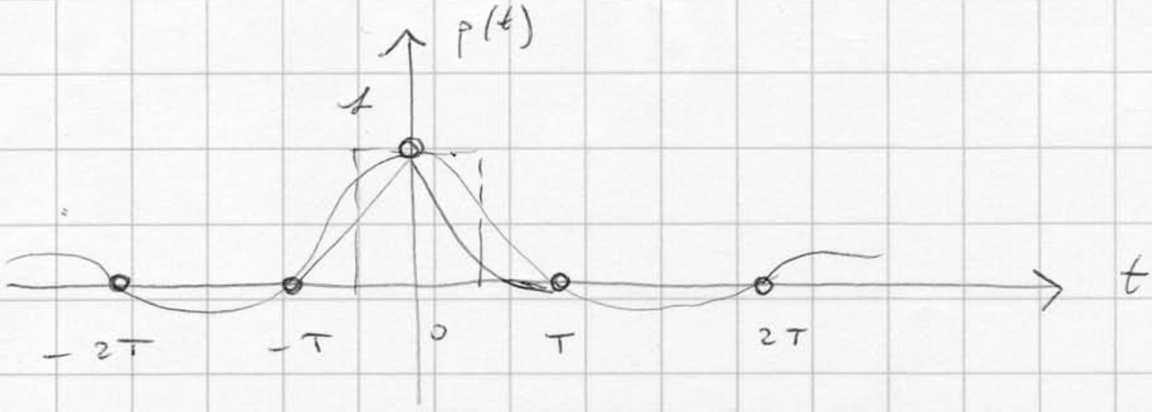
$$y(\underset{\substack{\uparrow \\ t_K}}{kT}) = a_k \, p(0) + \sum_{h \neq k} a_h \, p((k-h)T) + n_0(kT)$$

Se $p(t)$: $\begin{cases} p(0) = 1 \\ \sum_{h \neq k} p((k-h)T) = 0 \end{cases} \Longrightarrow p(iT) = 0 \; ; \; \forall i \neq 0$

$$\underbrace{y(kT)}_{y_k} = \overbrace{a_k}^{(\cdot p(0))} + \underbrace{n_0(kT)}_{n_K}$$

ISI = 0

$$\hookrightarrow \begin{cases} p(t) = 1 \; ; \; t = 0 \\ p(t) = 0 \; ; \; \forall t = kT \; ; \; k \neq 0 \end{cases}$$



IMPULSI di NYQUIST

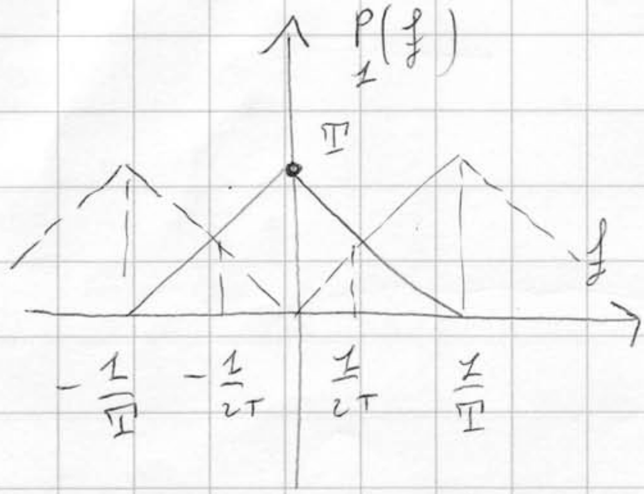
Nyquist pulses

$$ISI = 0 \Longrightarrow \begin{cases} p(t) = 1 \,; & t = 0 \,. \\ p(t) = 0 \,; & t = kT \,,\ k \neq 0 \,. \end{cases}$$

$$ISI = 0 \longrightarrow \underbrace{p(t) \cdot \delta_T(t) = \delta(t)}$$

$$\underbrace{P(f) * \frac{1}{T}\,\delta_{1/T}(f)} = 1$$

$$\frac{1}{T} \sum_k P\left(f - \frac{k}{T}\right) = 1$$



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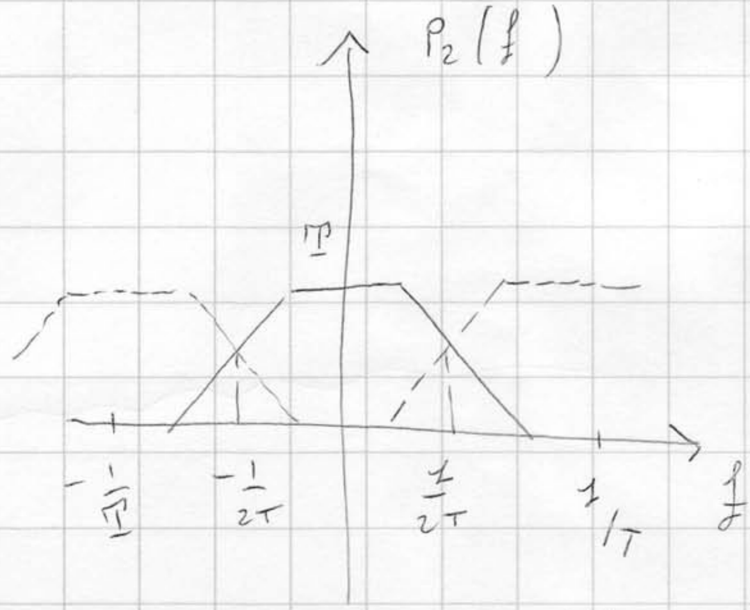
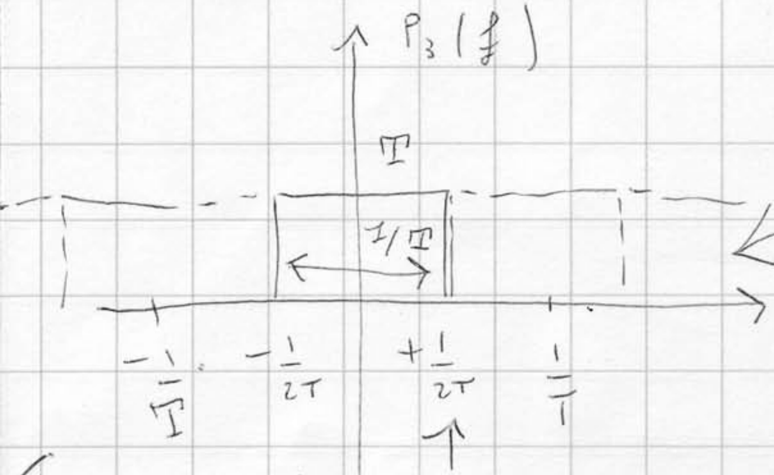
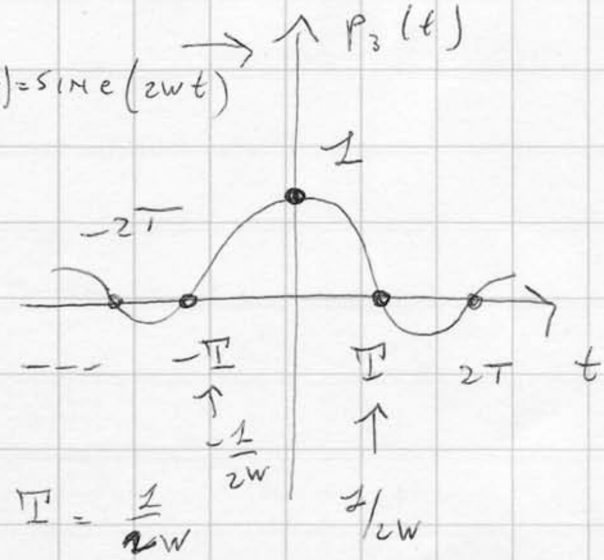




$\mathcal{F}$

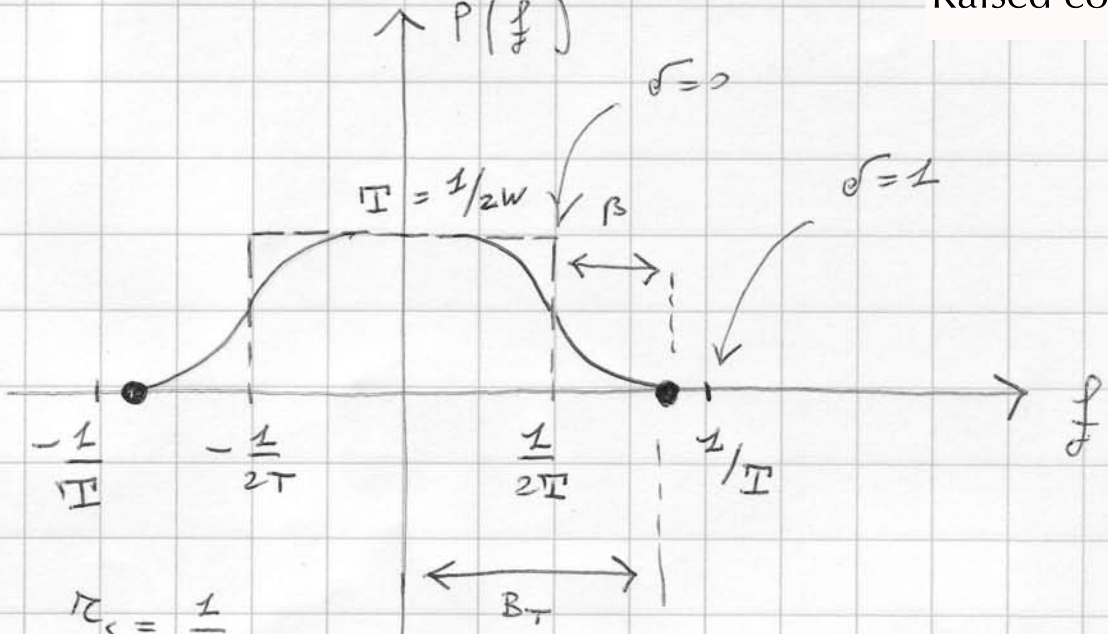
$p_3(t) = \mathrm{sinc}(2Wt)$



$\left(r_s = \frac{1}{T} = 2W\right)$ $\quad \frac{1}{2T} = W \longrightarrow T = \frac{1}{2W}$

FUNZIONI A "COSENO RIALZATO"

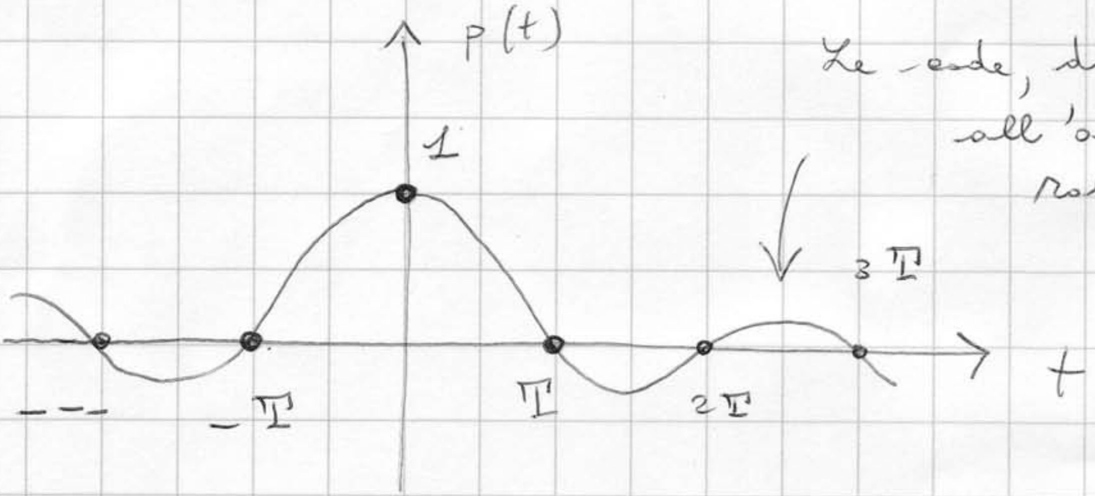
Raised cosine functions



$r_s = \frac{1}{T}$

Fattore di ROLL-OFF (δ)

$$B_T = \frac{1}{2T}(1+\delta) = \frac{1}{2T} + \beta = \frac{r_s}{2}(1+\delta)$$



Le code diminuiscono all'aumentare del roll-off ...

The tails reduces as the roll-off factor (delta) increases

$$P(f) = \begin{cases} \frac{1}{2W}\,; & 0 \le |f| < f_1 \\ \frac{1}{4W}\left[1 - \sin\left[\frac{\pi(|f| - W)}{2W - 2f_1}\right]\right]; & f_1 \le |f| < 2W - f_1 \\ 0\,; & |f| \ge 2W - f_1 \end{cases}$$

$$p(t) = \mathrm{sinc}(2Wt)\left[\frac{\cos(2\pi\delta W t)}{(1 - 16\delta^2 W^2 t^2)}\right]$$

FILTRO di RIC. OTTIMA per il PAM

Optimal receiver filter for the PAM modulation

$$\begin{cases} H_T(f)\, H_C(f)\, H_R(f) = P_N(f)\, e^{-j2\pi f T_{TOT}} \longrightarrow ISI = 0 \\ H_R(f) = K\, G^*(f)\, e^{-j2\pi f T_0} \longrightarrow F.A. \end{cases}$$

$$\left(h_R(t) = K g(T_0 - t) \text{ !!!} \right)$$

Matched Filter: Filtro Adattato (Fa)

$$G(f) = H_T(f)\, H_C(f)$$

Assume linear phase filters and consider the Amplitude response of the filter, i.e., |H(f)|

Se consideriamo filtri a fase lineare e analizziamo i moduli ..... (IP: $K = 1$)

$$\begin{cases} |H_T(f)| \cdot |H_C(f)| \cdot |H_R(f)| = |P_N(f)| \\ |H_R(f)| = |H_T(f)| \cdot |H_C(f)| \end{cases}$$

SOLUZIONE : (Con $|H_C(f)| = 1$ (e fase lin.): canale ideale)

$$\begin{cases} |H_T(f)| = |H_R(f)| = \sqrt{|P_N(f)|} \\ |H_R(f)| = |H_T(f)| \end{cases}$$

Per la fase: devono essere a FASE LINEARE!!!

Solution in the case of ideal channel and linear phase filters

In generale:

$$\begin{cases} |H_T(f)| = \dfrac{|P_N(f)|}{|H_C(f)\, H_R(f)|} \\[2ex] |H_R(f)|^2 = \dfrac{k\,|P_N(f)|}{\sqrt{G_n(f)}} \end{cases}$$

DSP del rumore gaussiano

Power Spectral Density of the gaussian noise

Se canale ideale e rumore AWGN

$$\boxed{|H_T(f)| = |H_R(f)| = k_1 \sqrt{|P_N(f)|}}$$

Solution in case of ideal channel and AWGN

Pulses with amplitude spectrum that is a square root of a Nyquist pulse

Impulsi con SPETTRO a radice di Nyquist