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Problem 1

Firstly, we have the generator matched with the first transmission line because $Z_g = 50\,\Omega = Z_0$, so there is not reflection and all the power is transmitted

$$P_{generator} = \frac{1}{2} |V_g|^2 \frac{1}{4 R_g} = \frac{1}{2} \cdot 20^2 \frac{1}{4 \cdot 50} = 1\,W \Rightarrow \boxed{P_g = 1\,W}$$

And because all the power is transmitted

$$\boxed{P_{inc} = P_g = 1\,W}$$

Secondly, in the junction between two transmission lines we know that:

$$T = 1 - \Gamma = \frac{2 Z_1}{Z_1 + Z_0} \Longrightarrow \Gamma = \frac{2 Z_1}{Z_1 + Z_0} - 1 = \frac{2 \cdot 60}{60 + 50} - 1 \Rightarrow \boxed{\Gamma = 0{,}0909}$$

And the power transmitted is calculated.

$$P_{trans} = \frac{1}{2} |V_g|^2 \cdot \frac{R_{in}}{(R_{in} + R_g)^2 + (X_{in} + X_g)^2}$$

In this case $R_{in} = 60\,\Omega$ because $\lambda/2$ TL $\Rightarrow$

$\Rightarrow Z_{in} = Z_L = Z_1 = 60\,\Omega$

And $X_{in} = X_g = 0$

$$P_{trans} = \frac{1}{2} \cdot |20|^2 \cdot \frac{60}{(50+60)^2} = 0,9917 \, W$$

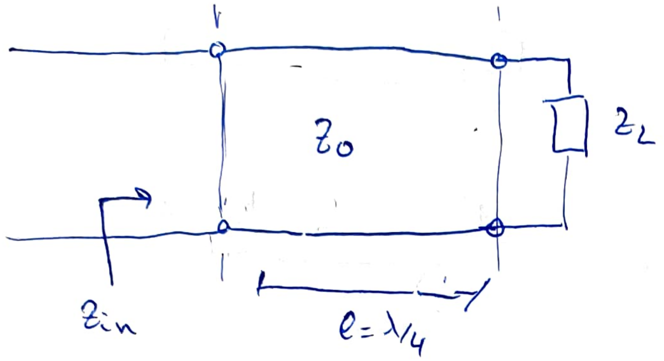
$$\boxed{P_{trans} = 0,9917 \, W}$$

The reflected power:

$$P_{ref} = |\Gamma|^2 P_{inc} = 0,0909^2 \cdot 1 \Rightarrow \boxed{P_{ref} = 0,0083 \, W}$$

Problem 2

The schematic will be:



$Z_{in} = 50\,\Omega$

$Z_L = 10\,\Omega$

So we have:

$$Z_{in} = \frac{Z_0^2}{Z_L} \Rightarrow Z_0 = \sqrt{Z_{in} Z_L} = \sqrt{50 \cdot 10} \Rightarrow \boxed{Z_0 = 22{,}36\,\Omega}$$

$$\ell = \frac{\lambda_g}{4} = \frac{\lambda_0}{\sqrt{\varepsilon_r}\cdot 4} = \frac{c}{4\cdot f\cdot\sqrt{\varepsilon_r}} = \frac{3\cdot 10^8}{4\cdot 2\cdot 10^9\cdot\sqrt{2{,}25}} \Rightarrow \boxed{\ell = 2{,}25\,cm}$$

Using the given equation:

$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\left(\frac{b}{a}\right) \Rightarrow \ln\left(\frac{b}{a}\right) = 2\pi Z_0\sqrt{\frac{\varepsilon_0\varepsilon_r}{\mu_0\mu_r}} \underset{\mu_r = 1}{=} 2\pi Z_0\sqrt{\frac{\varepsilon_0}{\mu_0}}\sqrt{\varepsilon_r} \underset{\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}}{=}$$

$$= \frac{2\pi Z_0}{\eta_0}\sqrt{\varepsilon_r} \underset{\eta_0 = 120\pi}{=} \frac{2\pi Z_0}{120\pi}\sqrt{\varepsilon_r} = \frac{Z_0}{60}\sqrt{\varepsilon_r}$$

$$\Rightarrow \frac{b}{a} = e^{\frac{Z_0}{60}\sqrt{\varepsilon_r}} = e^{\frac{22{,}36}{60}\sqrt{2{,}25}} = 1{,}74895$$

$$\boxed{\frac{b}{a} = 1{,}74895} \rightarrow \boxed{\begin{array}{l} a = 1\,cm \\ b = 1{,}74895\,cm \end{array}}$$

Problem 3

The general formula for cut-off frequency:

$$f_{c_{nm}} = \frac{1}{2\pi} \cdot \frac{c}{\sqrt{\varepsilon_r}} \sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}$$

Working in W-band the dimensions are:

$a = 2{,}850$ cm

$b = 1{,}262$ cm

① 4 first modes

medium: air $\Rightarrow \varepsilon_r = 1$

$f_{c_{10}} = 5{,}26$ GHz $\qquad f_{c_{01}} = 11{,}89$ GHz

$f_{c_{20}} = 10{,}53$ GHz $\qquad f_{c_{11}} = 13$ GHz

② 4 first modes

medium: teflon $\Rightarrow \varepsilon_r = 2{,}08$

$f_{c_{10}} = \frac{5{,}26}{\sqrt{\varepsilon_r}} = 3{,}65$ GHz $\qquad f_{c_{01}} = \frac{11{,}89}{\sqrt{\varepsilon_r}} = 8{,}24$ GHz

$f_{c_{20}} = \frac{10{,}53}{\sqrt{\varepsilon_r}} = 7{,}3$ GHz $\qquad f_{c_{11}} = \frac{13}{\sqrt{\varepsilon_r}} = 9{,}01$ GHz

If we operate at $f = 9\,GHz$:

→ $V_{teflon} = \frac{c}{\sqrt{\varepsilon_r}} = 2{,}08 \cdot 10^8\ m/s$

→ Phase velocity:

$$V_{\phi_{TE_{nm}}} = \frac{\omega}{\beta_{nm}} \quad \text{with} \quad \beta_{nm} = \sqrt{\left(\frac{\omega}{c/\sqrt{\varepsilon_r}}\right)^2 - \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}$$

$V_{\phi_{TE_{10}}} = 2{,}276 \cdot 10^8\ m/s$

$V_{\phi_{TE_{20}}} = 2{,}557 \cdot 10^8\ m/s$

$V_{\phi_{TE_{01}}} = 5{,}178 \cdot 10^8\ m/s$

→ Group velocity

$$V_{g_{TE_{nm}}} = \frac{\beta_{nm}}{2\pi f} \frac{c^2}{\varepsilon_r}$$

$V_{g_{TE_{10}}} = 1{,}901 \cdot 10^8\ m/s$

$V_{g_{TE_{20}}} = 1{,}216 \cdot 10^8\ m/s$

$V_{g_{TE_{01}}} = 0{,}836 \cdot 10^8\ m/s$