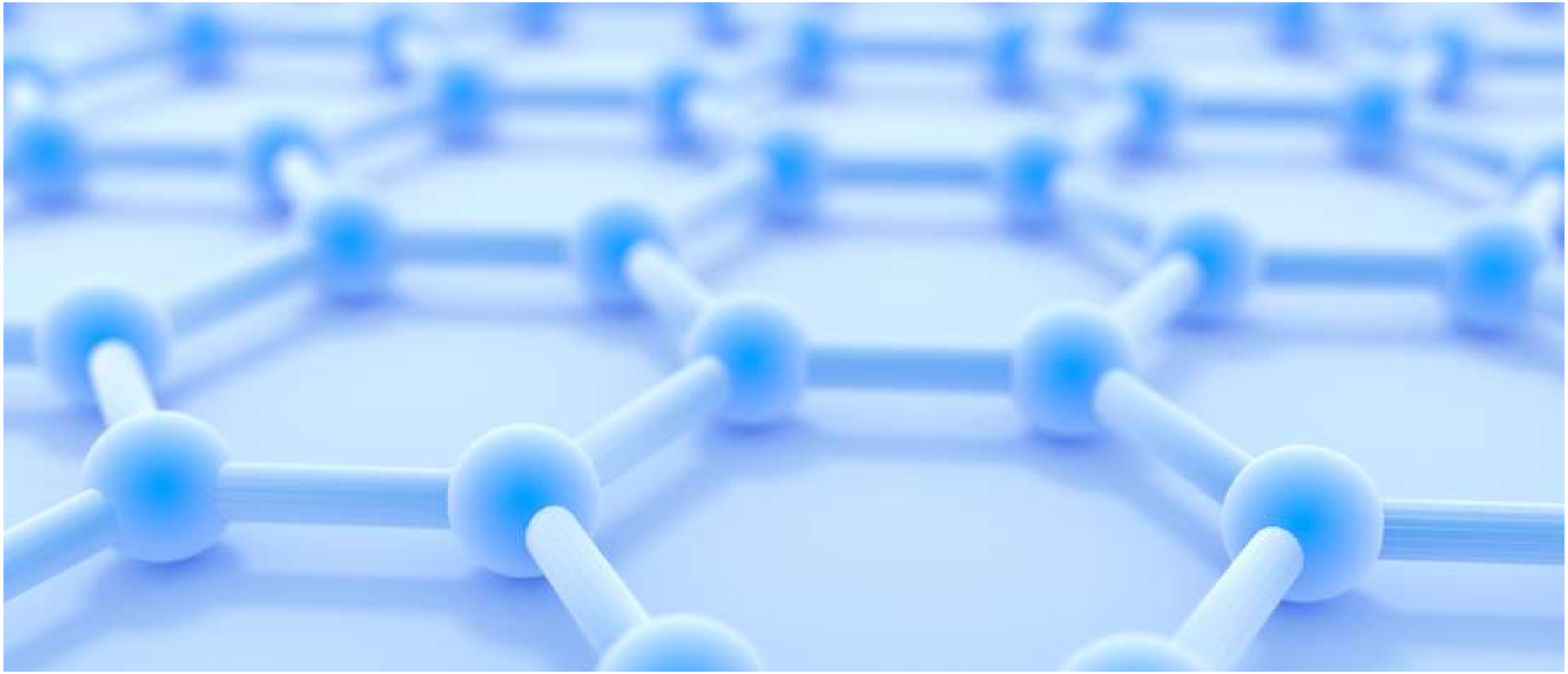


Photonics

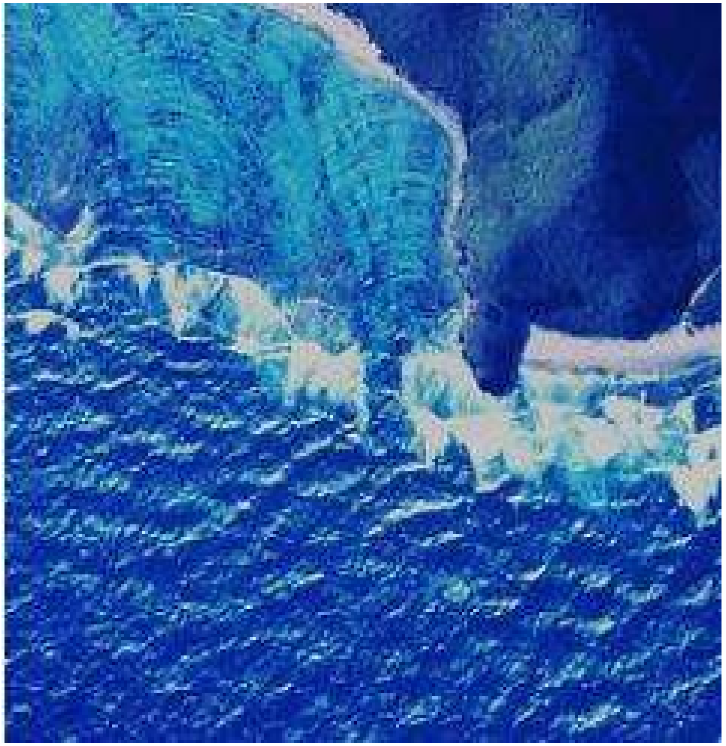
aa 2021/2022

Prof. Maria Antonietta Vincenti
Università degli Studi di Brescia

Electromagnetic Optics:
Reflection, Refraction and Scattering

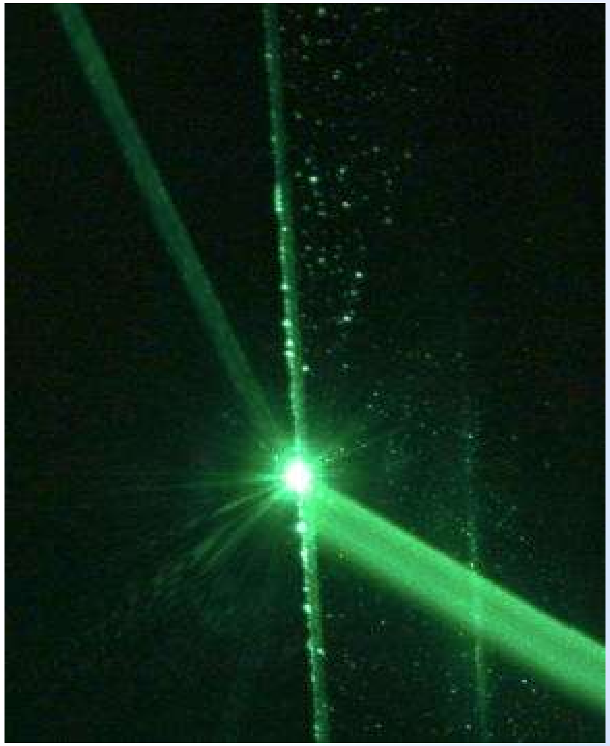
Refraction

Water Waves



Waves refract where the water is shallower

E&M Waves



Refraction involves a change in the direction of wave propagation due to a change in propagation speed. It involves the oblique incidence of waves on media boundaries, and hence wave propagation in at least two dimensions.

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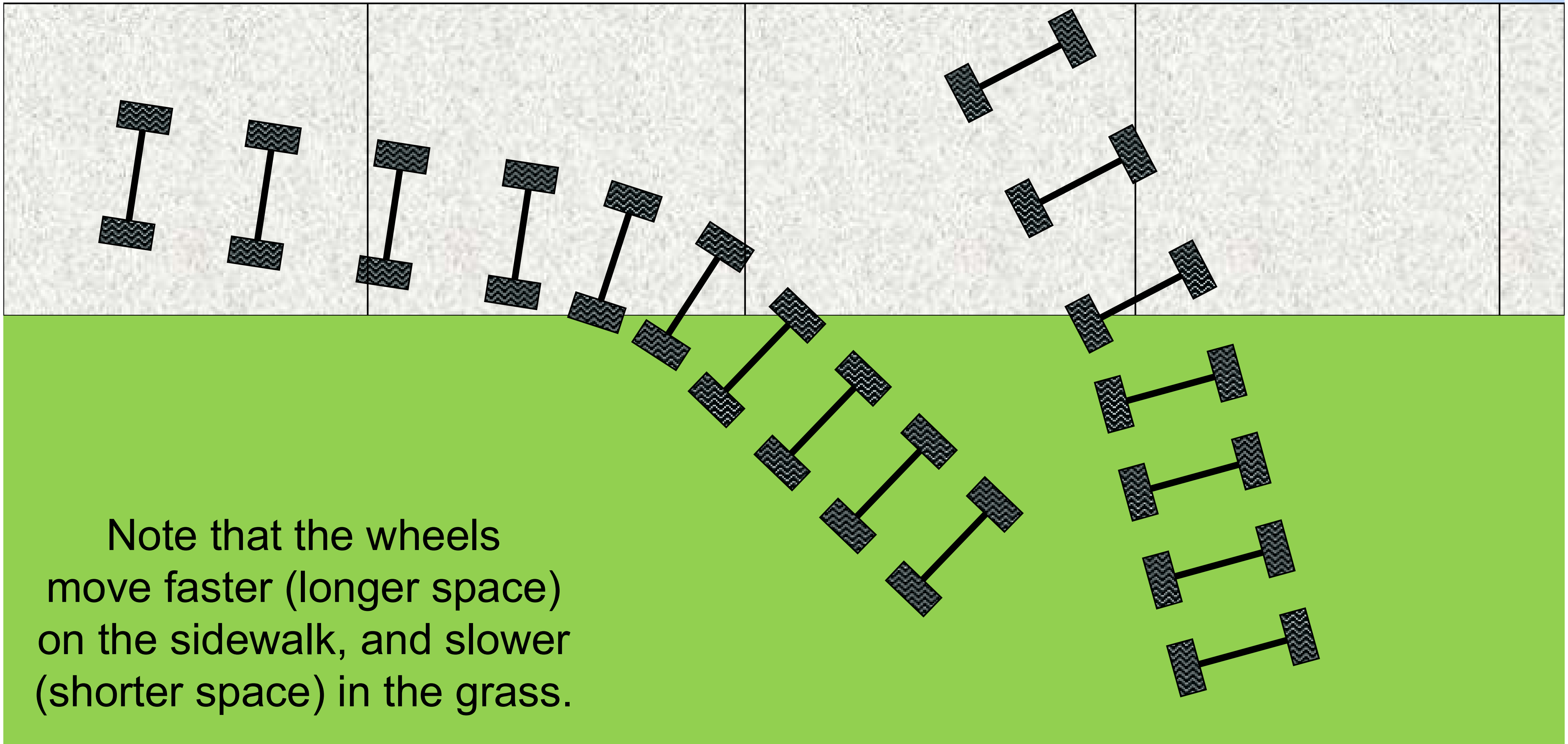
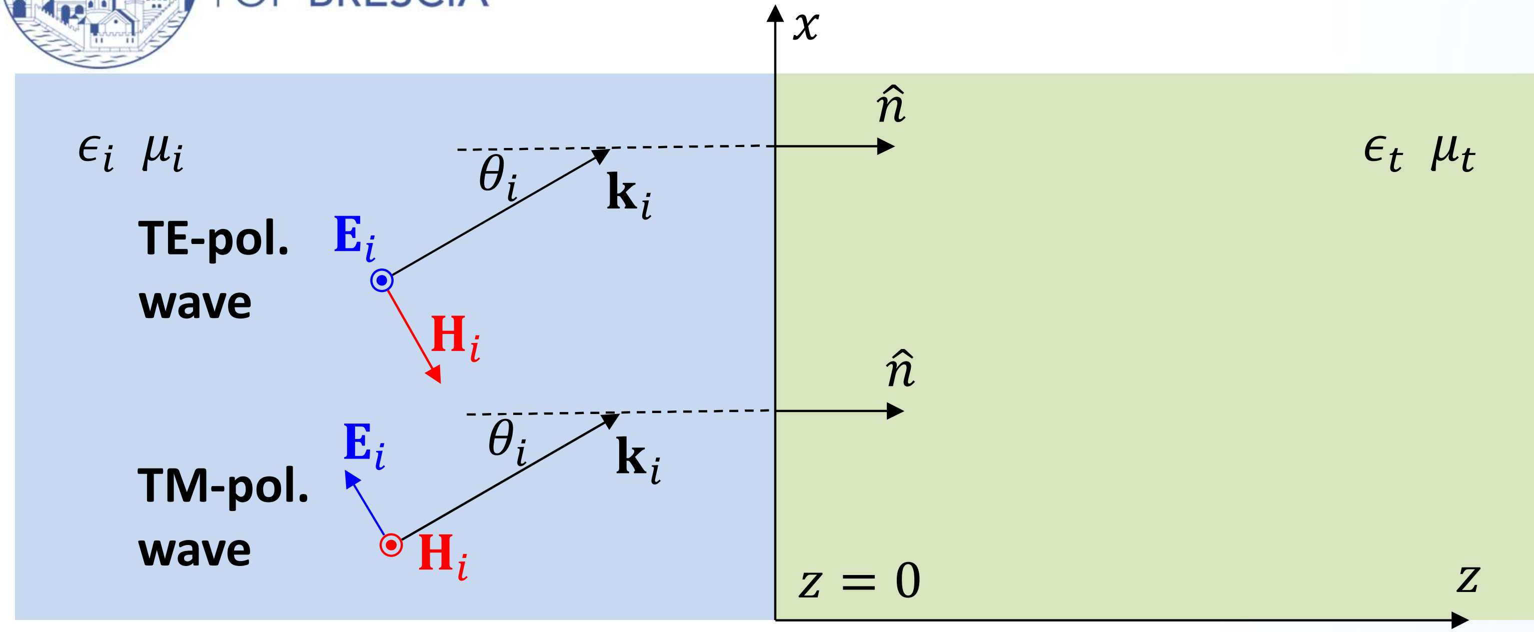
Refraction

Think of refraction as a pair of wheels on an axle going from a sidewalk onto grass. The wheel in the grass moves slower, so the direction of the wheel pair changes.



Reflection and Refraction at an interface



Incidence and transmission media are assumed isotropic.

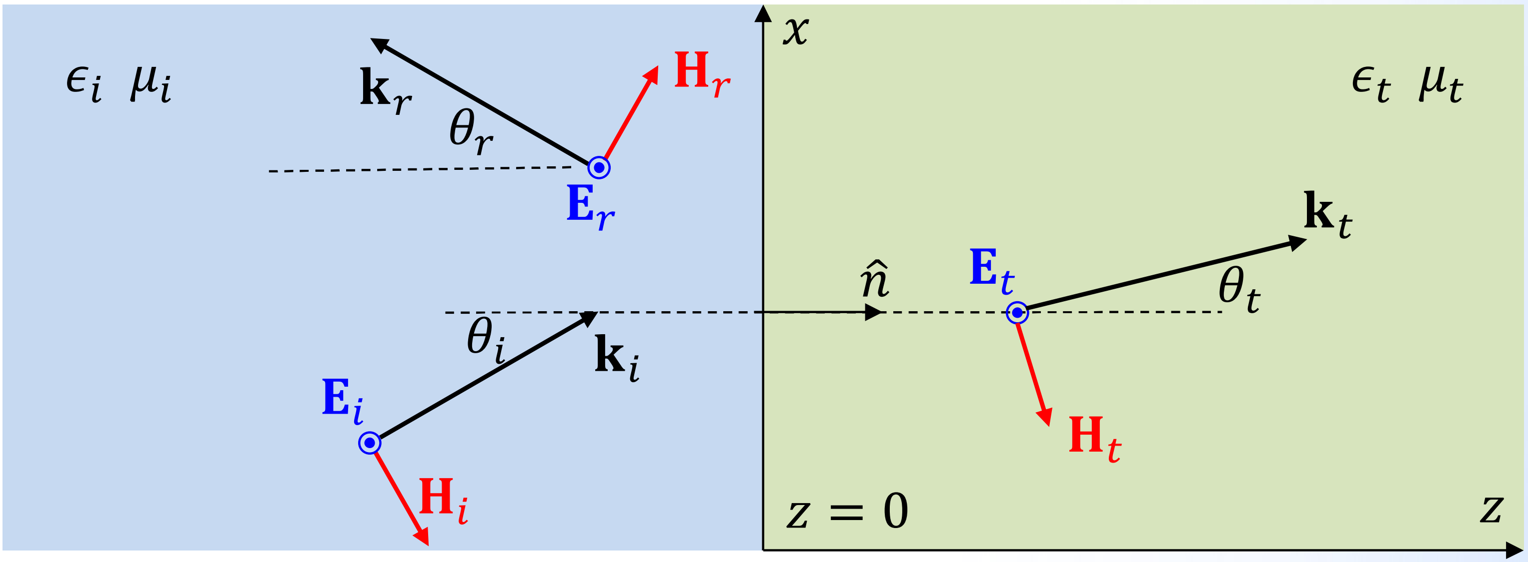
Plane of Incidence: The plane containing the incident wavevector $\boldsymbol{k}_i$ and a vector that is normal to the interface is called the plane of incidence (in the figure above, the normal to the interface is $\hat{z}$ and the x-z plane is the plane of incidence)

TE Wave: If the E-field of the wave is perpendicular to the plane of incidence then the wave is called a TE-wave (transverse electric polarization, also known as s-polarization)

TM Wave: If the H-field of the wave is perpendicular to the plane of incidence (i.e., if the E-field lies in the plane of incidence) then the wave is called a TM-wave (transverse magnetic polarization, also called p-polarization)

Reflection and Refraction at an interface

TE Wave



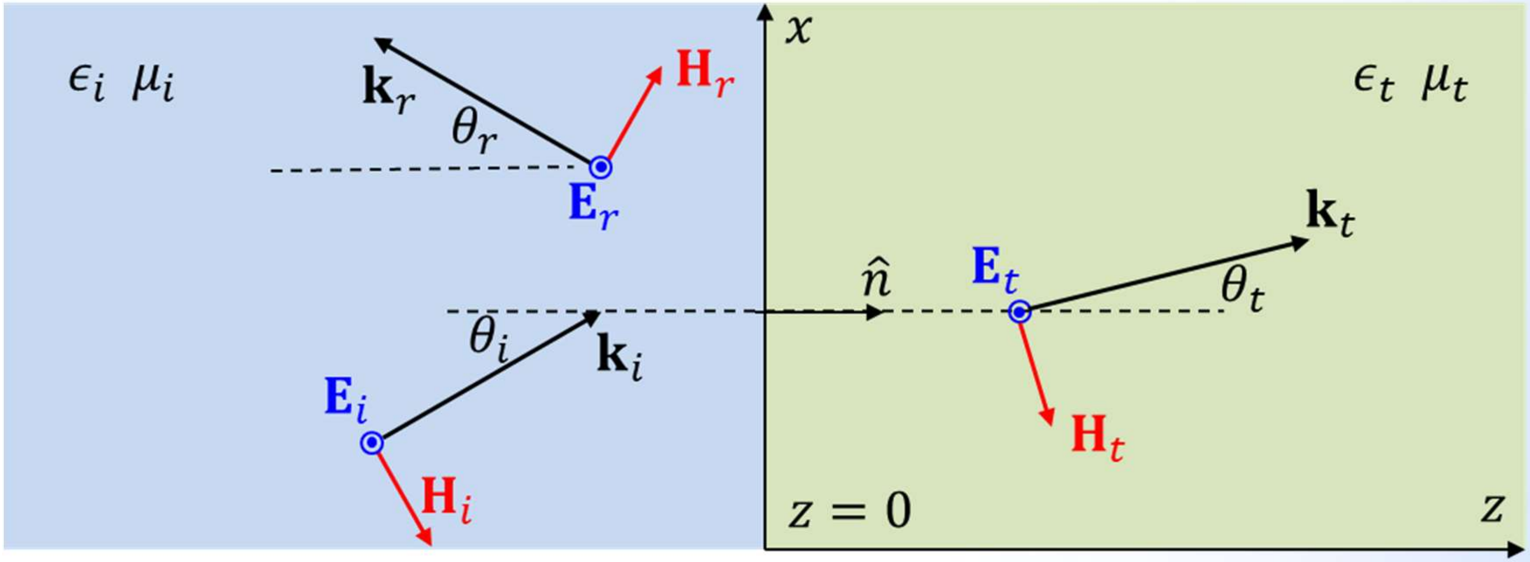
$$\mathbf{k}_i = k_{ix}\hat{x} + k_{iz}\hat{z} = k_i\left[\sin(\theta_i)\hat{x} + \cos(\theta_i)\hat{z}\right], \quad |\mathbf{k}_i| = \omega\sqrt{\mu_i\varepsilon_i} = (\text{if nonmagnetic}) = \omega\frac{n_i}{c}$$

$$\mathbf{k}_r = k_{rx}\hat{x} + k_{rz}\hat{z} = k_r\left[\sin(\theta_r)\hat{x} - \cos(\theta_r)\hat{z}\right], \quad |\mathbf{k}_r| = |\mathbf{k}_i| = \omega\sqrt{\mu_i\varepsilon_i} = (\text{if nonmagnetic}) = \omega\frac{n_i}{c}$$

$$\mathbf{k}_t = k_{tx}\hat{x} + k_{tz}\hat{z} = k_t\left[\sin(\theta_t)\hat{x} + \cos(\theta_t)\hat{z}\right], \quad |\mathbf{k}_t| = \omega\sqrt{\mu_t\varepsilon_t} = (\text{if nonmagnetic}) = \omega\frac{n_t}{c}$$

Reflection and Refraction at an interface

TE Wave – First Boundary Condition: continuity of E_y at $z = 0$



$$\mathbf{E}(\mathbf{r})\big|_{z<0} = \hat{y}E_i e^{-j\mathbf{k}_i\cdot\mathbf{r}} + \hat{y}E_r e^{-j\mathbf{k}_r\cdot\mathbf{r}}$$

$$\mathbf{E}(\mathbf{r})\big|_{z>0} = \hat{y}E_t e^{-j\mathbf{k}_t\cdot\mathbf{r}}$$

$$\begin{cases} \mathbf{k}_i = k_i\left[\sin(\theta_i)\hat{x} + \cos(\theta_i)\hat{z}\right] \\ \mathbf{k}_r = k_r\left[\sin(\theta_r)\hat{x} - \cos(\theta_r)\hat{z}\right] \\ \mathbf{k}_t = k_t\left[\sin(\theta_t)\hat{x} + \cos(\theta_t)\hat{z}\right] \end{cases}$$

(1) At *z* = 0 the E-field parallel to the interface must be continuous across the interface for all *x:*

$$E_i e^{-jk_i\sin(\theta_i)x} + E_r e^{-jk_r\sin(\theta_r)x} = E_t e^{-jk_t\sin(\theta_t)x}$$

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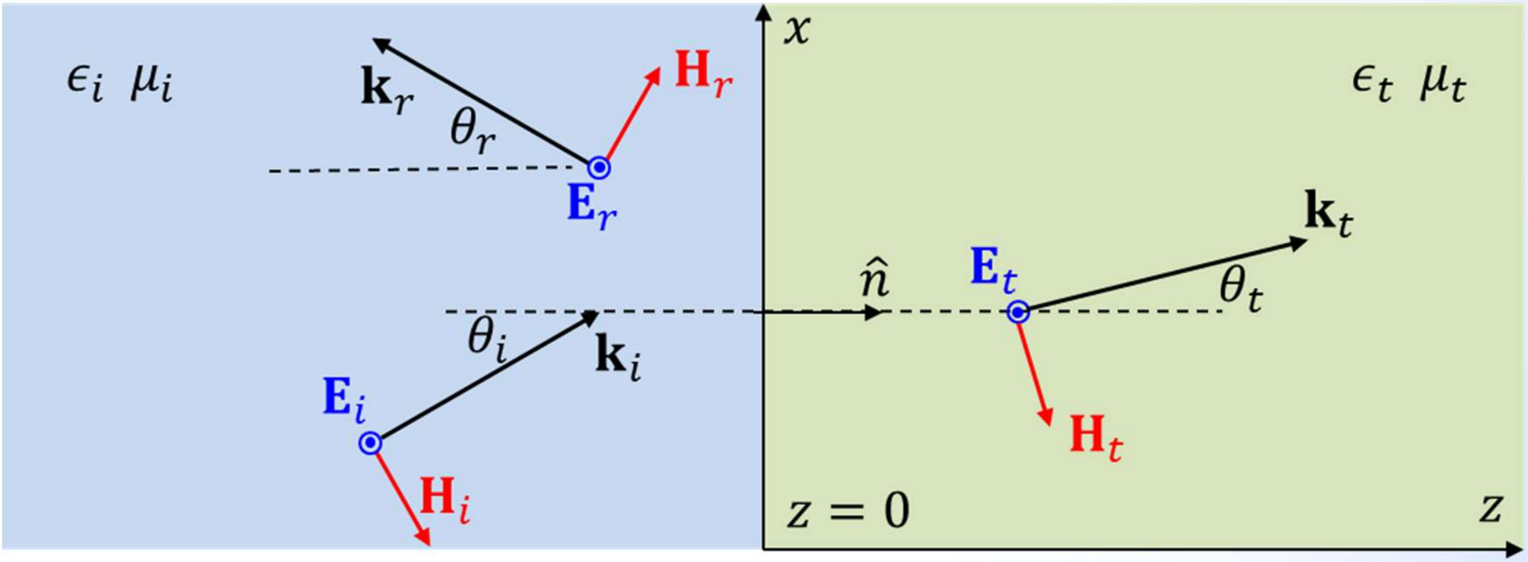
Reflection and Refraction at an interface

TE Wave – First Boundary Condition: continuity of E_y at $z = 0$



The only way the above boundary condition can be satisfied for all *x* , and at all times *t*, is if all the *x* dependent phase factors are the same. This condition is called "**phase matching condition**", or conservation of the transverse wavevector, or conservation of the transverse momentum.

$$k_i \sin(\theta_i) = k_r \sin(\theta_r) = k_t \sin(\theta_t) \quad \Longrightarrow \quad k_{ix} = k_{rx} = k_{tx}$$

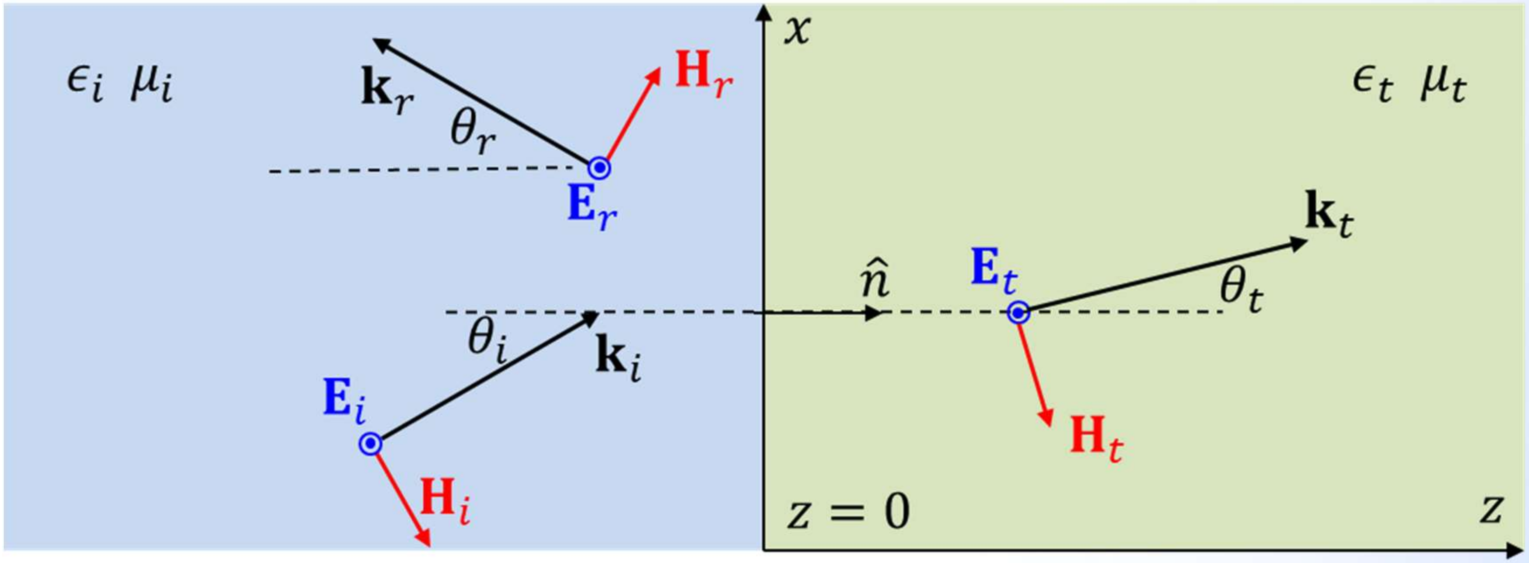
This also implies that:

$$\sin(\theta_i) = \sin(\theta_r) \quad \Longrightarrow \quad \theta_i = \theta_r$$

angle of incidence equals the angle of reflection

Reflection and Refraction at an interface

TE Wave – Snell's Law



$$k_i \sin(\theta_i) = k_t \sin(\theta_t) \quad \Longrightarrow \quad \omega \frac{n_i}{c} \sin(\theta_i) = \omega \frac{n_t}{c} \sin(\theta_t)$$

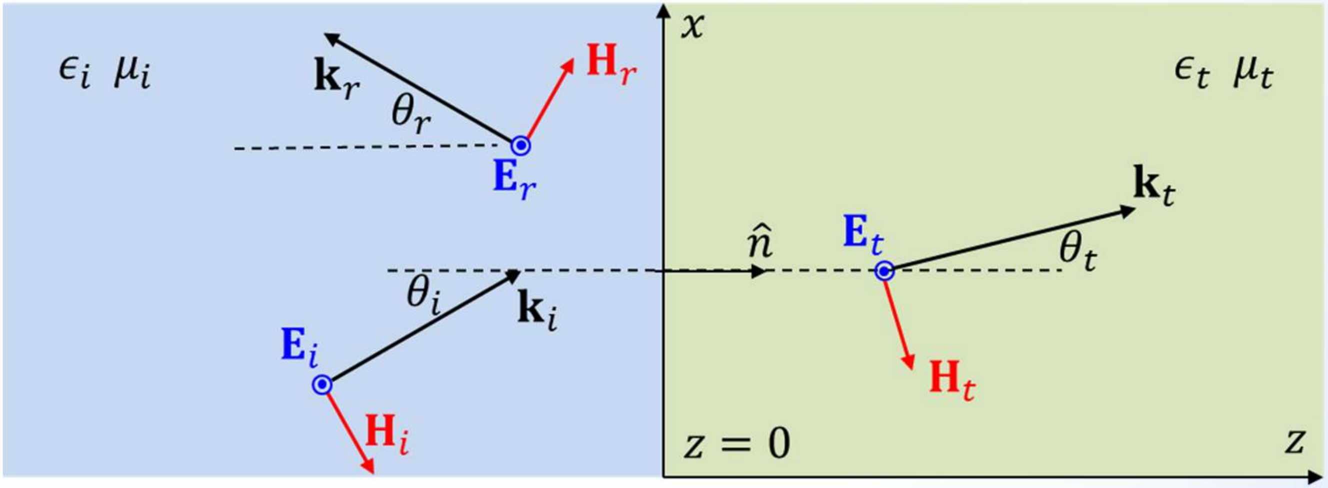
$$n_i \sin(\theta_i) = n_t \sin(\theta_t)$$ **Snell's law**

Moreover:

$$E_i e^{-jk_i \sin(\theta_i)x} + E_r e^{-jk_r \sin(\theta_r)x} = E_t e^{-jk_t \sin(\theta_t)x} \quad \Longrightarrow \quad \boxed{E_i + E_r = E_t} \qquad (1)$$

Reflection and Refraction at an interface

TE Wave – Second Boundary Condition: continuity of H_x at $z = 0$



$$\mathbf{H}(\mathbf{r})\big|_{z<0} = \left(\hat{k}_i \times \hat{y}\right)\frac{E_i}{\eta_i}e^{-j\mathbf{k}_i\cdot\mathbf{r}} + \left(\hat{k}_r \times \hat{y}\right)\frac{E_r}{\eta_i}e^{-j\mathbf{k}_r\cdot\mathbf{r}}$$

$$\mathbf{H}(\mathbf{r})\big|_{z>0} = \left(\hat{k}_t \times \hat{y}\right)\frac{E_t}{\eta_t}e^{-j\mathbf{k}_t\cdot\mathbf{r}}$$

$$\mathbf{k}_i = k_i\left[\sin(\theta_i)\hat{x} + \cos(\theta_i)\hat{z}\right]$$

$$\mathbf{k}_r = k_r\left[\sin(\theta_r)\hat{x} - \cos(\theta_r)\hat{z}\right]$$

$$\mathbf{k}_t = k_t\left[\sin(\theta_t)\hat{x} + \cos(\theta_t)\hat{z}\right]$$

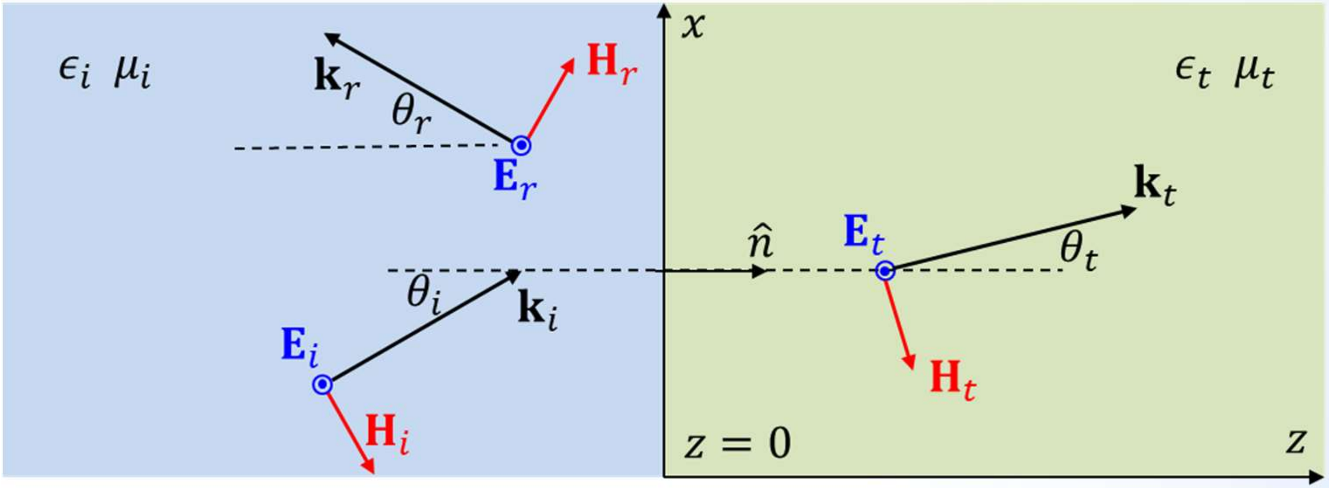
(2) At *z* = 0 the H-field component parallel to the interface must be continuous for all *x*

$$-\hat{x}\cos(\theta_i)\frac{E_i}{\eta_i}e^{-jk_i\sin(\theta_i)x} + \hat{x}\cos(\theta_r)\frac{E_r}{\eta_i}e^{-jk_r\sin(\theta_r)x} = -\hat{x}\cos(\theta_t)\frac{E_t}{\eta_t}e^{-jk_t\sin(\theta_t)x}$$

$$\boxed{\cos(\theta_i)\left[\frac{E_i}{\eta_i} - \frac{E_r}{\eta_i}\right] = \cos(\theta_t)\frac{E_t}{\eta_t}} \quad (2)$$

Reflection and Transmission Coefficients

TE Wave



$$\boxed{E_i + E_r = E_t} \quad \& \quad \boxed{\cos(\theta_i)\left[\frac{E_i}{\eta_i} - \frac{E_r}{\eta_i}\right] = \cos(\theta_t)\frac{E_t}{\eta_t}}$$

By solving the equation on the right using the equation on the left we get:

$$t_{\mathrm{TE}} = \frac{E_t}{E_i} = 2\frac{\eta_t \cos(\theta_i)}{\eta_i \cos(\theta_t) + \eta_t \cos(\theta_i)}$$

$$r_{\mathrm{TE}} = \frac{E_r}{E_i} = \frac{\eta_t \cos(\theta_i) - \eta_i \cos(\theta_t)}{\eta_t \cos(\theta_i) + \eta_i \cos(\theta_t)}$$

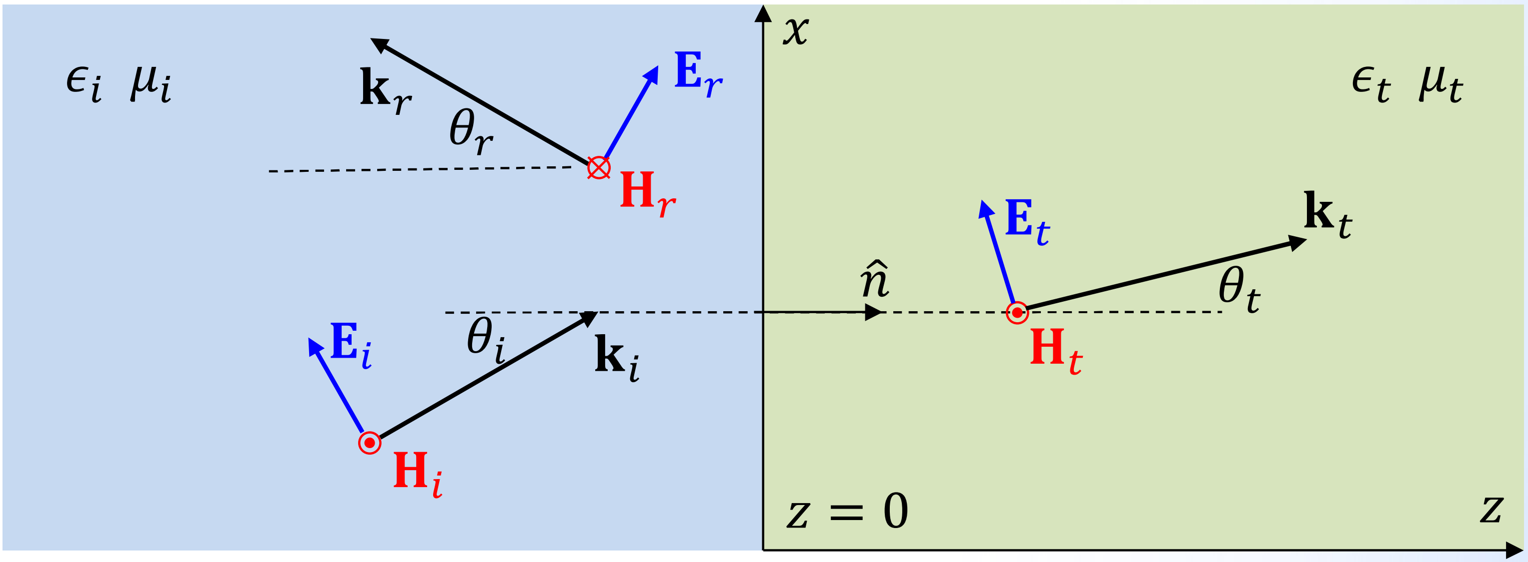
The two equations above can be re-written when considering $\mu = \mu_0 = 1$ as:

$$t_{\mathrm{TE}} = \frac{E_t}{E_i} = 2\frac{n_i \cos(\theta_i)}{n_t \cos(\theta_t) + n_i \cos(\theta_i)}$$

$$r_{\mathrm{TE}} = \frac{E_r}{E_i} = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

Reflection and Refraction at an interface

TM Wave



The definitions of the wavevectors are identical for both polarizations:

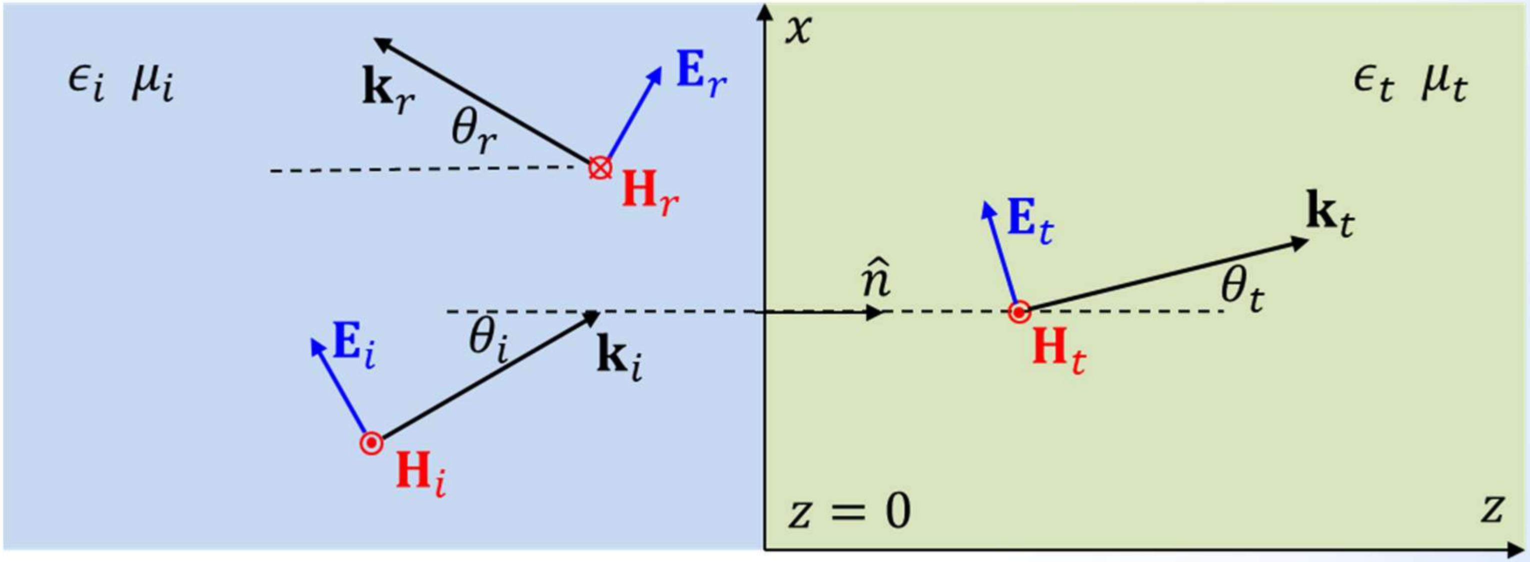
$$\mathbf{k}_i = k_{ix}\hat{x} + k_{iz}\hat{z} = k_i\left[\sin(\theta_i)\hat{x} + \cos(\theta_i)\hat{z}\right], \quad |\mathbf{k}_i| = \omega\sqrt{\mu_i\varepsilon_i} = (\text{if nonmagnetic}) = \omega\frac{n_i}{c}$$

$$\mathbf{k}_r = k_{rx}\hat{x} + k_{rz}\hat{z} = k_r\left[\sin(\theta_r)\hat{x} - \cos(\theta_r)\hat{z}\right], \quad |\mathbf{k}_r| = |\mathbf{k}_i| = \omega\sqrt{\mu_i\varepsilon_i} = (\text{if nonmagnetic}) = \omega\frac{n_i}{c}$$

$$\mathbf{k}_t = k_{tx}\hat{x} + k_{tz}\hat{z} = k_t\left[\sin(\theta_t)\hat{x} + \cos(\theta_t)\hat{z}\right], \quad |\mathbf{k}_t| = \omega\sqrt{\mu_t\varepsilon_t} = (\text{if nonmagnetic}) = \omega\frac{n_t}{c}$$

Reflection and Refraction at an interface

TM Wave – First Boundary Condition: continuity of H_y at $z = 0$



$$\mathbf{H}(\mathbf{r})\big|_{z<0} = \hat{y}\frac{E_i}{\eta_i}e^{-j\mathbf{k}_i\cdot\mathbf{r}} - \hat{y}\frac{E_r}{\eta_i}e^{-j\mathbf{k}_r\cdot\mathbf{r}}$$

$$\mathbf{H}(\mathbf{r})\big|_{z>0} = \hat{y}\frac{E_t}{\eta_t}e^{-j\mathbf{k}_t\cdot\mathbf{r}}$$

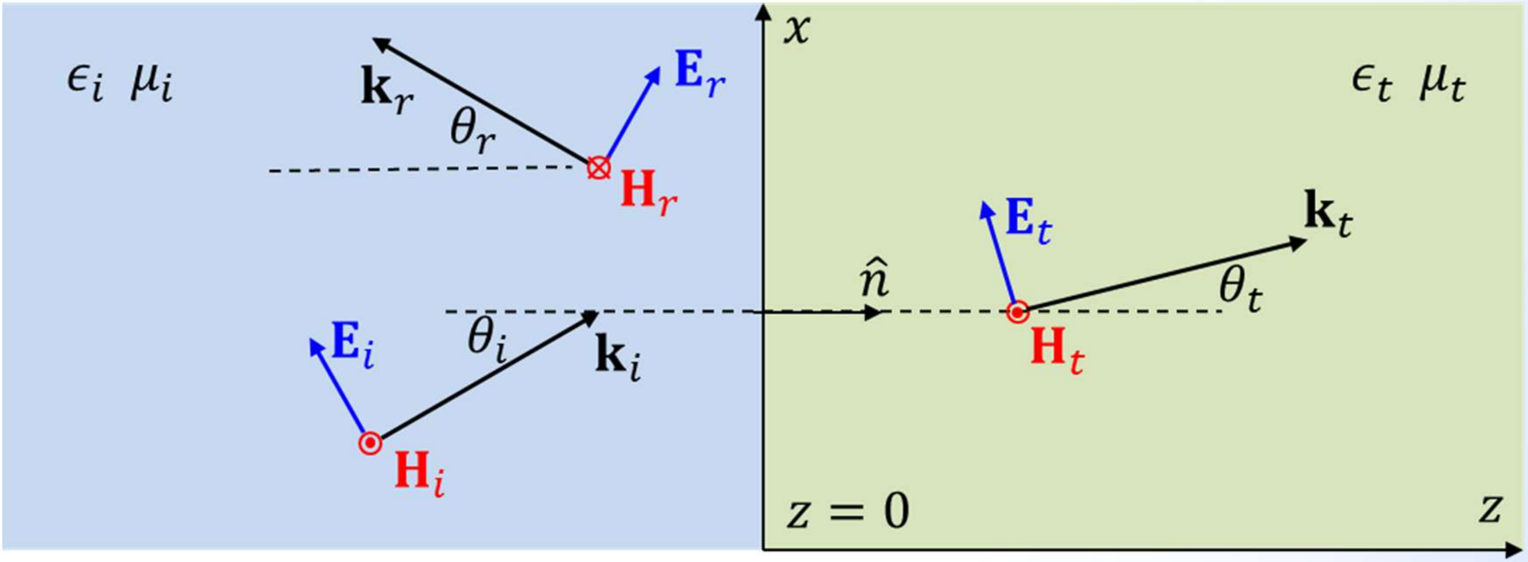
$$\begin{cases} \mathbf{k}_i = k_i\left[\sin(\theta_i)\hat{x} + \cos(\theta_i)\hat{z}\right] \\ \mathbf{k}_r = k_r\left[\sin(\theta_r)\hat{x} - \cos(\theta_r)\hat{z}\right] \\ \mathbf{k}_t = k_t\left[\sin(\theta_t)\hat{x} + \cos(\theta_t)\hat{z}\right] \end{cases}$$

(1) At *z* = 0 the H-field parallel to the interface must be continuous across the interface for all *x:*

$$\frac{E_i}{\eta_i}e^{-jk_i\sin(\theta_i)x} - \frac{E_r}{\eta_i}e^{-jk_r\sin(\theta_r)x} = \frac{E_t}{\eta_t}e^{-jk_t\sin(\theta_t)x}$$

Reflection and Refraction at an interface

TM Wave – First Boundary Condition



The only way the above boundary condition can be satisfied for all *x* is if all the *x* dependent phase factors are the same. This condition is called “phase matching”.

$$k_i \sin(\theta_i) = k_r \sin(\theta_r) = k_t \sin(\theta_t) \quad \Rightarrow \quad k_{ix} = k_{rx} = k_{tx}$$

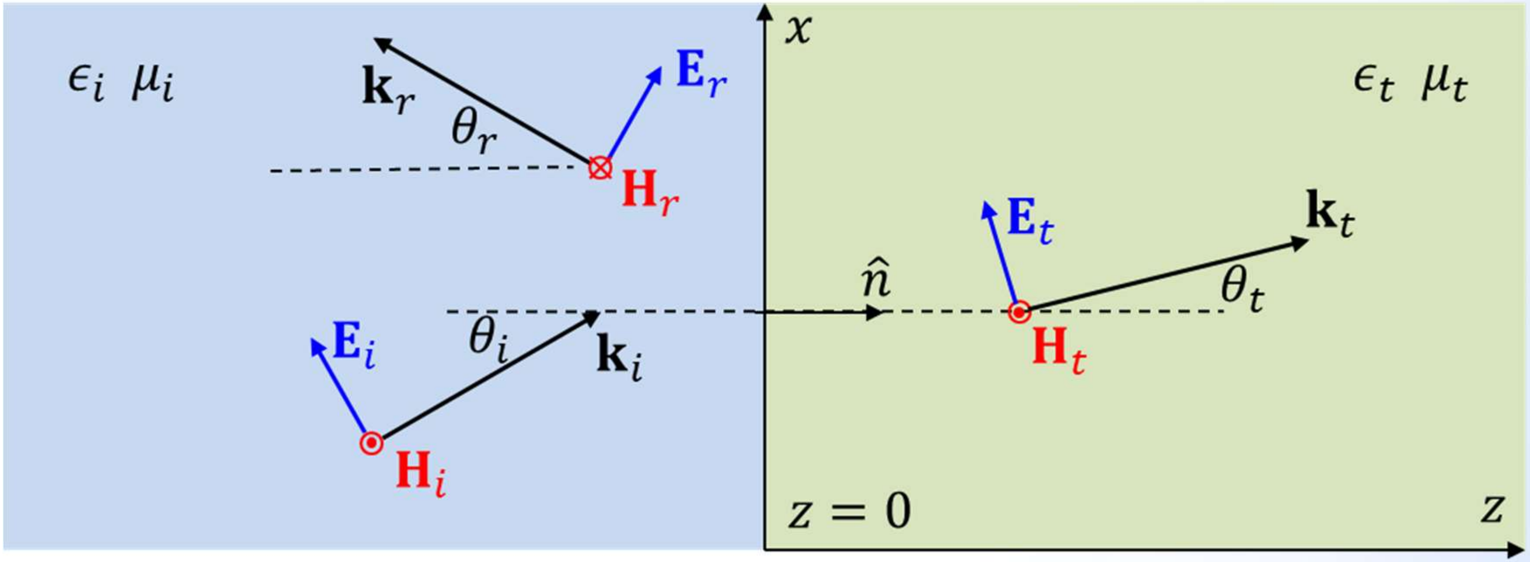
This also implies that:

$$\sin(\theta_i) = \sin(\theta_r) \quad \Rightarrow \quad \theta_i = \theta_r$$

angle of incidence equals the angle of reflection

Reflection and Refraction at an interface

TM Wave – Snell's Law



$$k_i \sin(\theta_i) = k_t \sin(\theta_t) \quad \Rightarrow \quad \omega \frac{n_i}{c} \sin(\theta_i) = \omega \frac{n_t}{c} \sin(\theta_t)$$

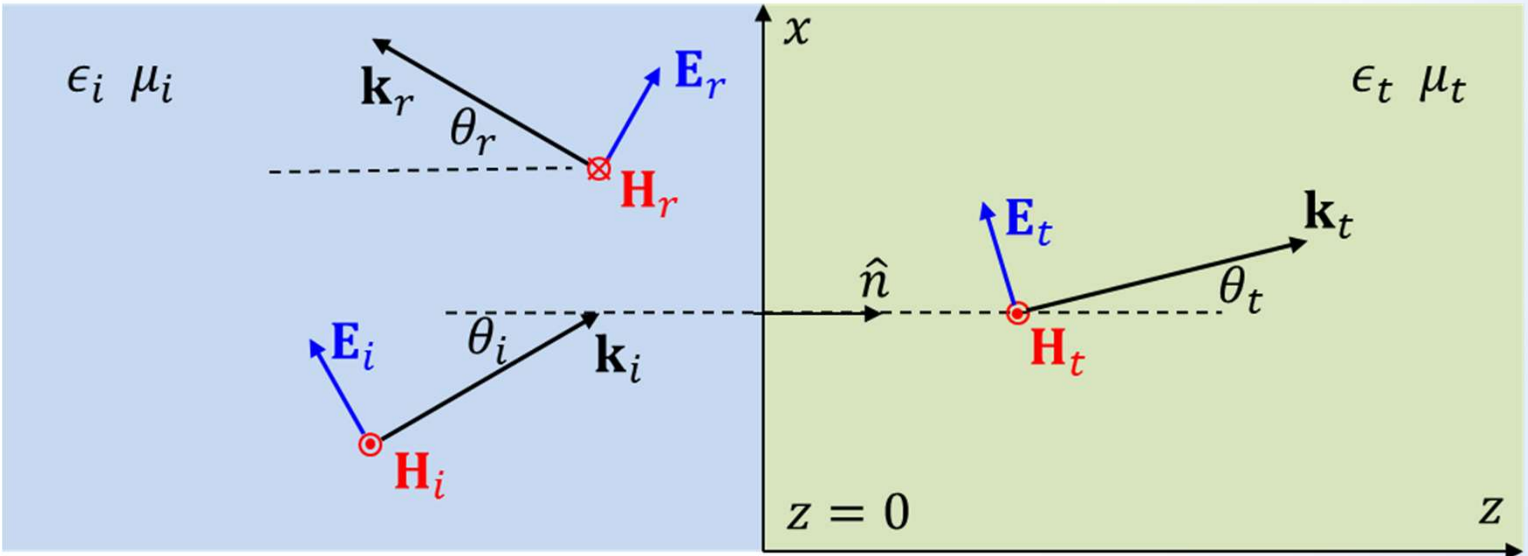
$$n_i \sin(\theta_i) = n_t \sin(\theta_t)$$ **Snell's law**

Moreover:

$$\frac{E_i}{\eta_i} e^{-jk_i \sin(\theta_i)x} - \frac{E_r}{\eta_i} e^{-jk_r \sin(\theta_r)x} = \frac{E_t}{\eta_t} e^{-jk_t \sin(\theta_t)x} \quad \Rightarrow \quad \boxed{E_i - E_r = \frac{\eta_i}{\eta_t} E_t} \qquad (1)$$

Reflection and Refraction at an interface

TM Wave – Second Boundary Condition



$$\mathbf{k}_i = k_i\left[\sin(\theta_i)\hat{x} + \cos(\theta_i)\hat{z}\right]$$

$$\mathbf{k}_r = k_r\left[\sin(\theta_r)\hat{x} - \cos(\theta_r)\hat{z}\right]$$

$$\mathbf{k}_t = k_t\left[\sin(\theta_t)\hat{x} + \cos(\theta_t)\hat{z}\right]$$

$$\mathbf{E}(\mathbf{r})\big|_{z<0} = -\left(\hat{k}_i \times \hat{y}\right)E_i e^{-j\mathbf{k}_\mathrm{i}\cdot\mathbf{r}} + \left(\hat{k}_r \times \hat{y}\right)E_r e^{-j\mathbf{k}_\mathrm{r}\cdot\mathbf{r}}$$

$$\mathbf{E}(\mathbf{r})\big|_{z>0} = -\left(\hat{k}_t \times \hat{y}\right)E_t e^{-j\mathbf{k}_\mathrm{t}\cdot\mathbf{r}}$$

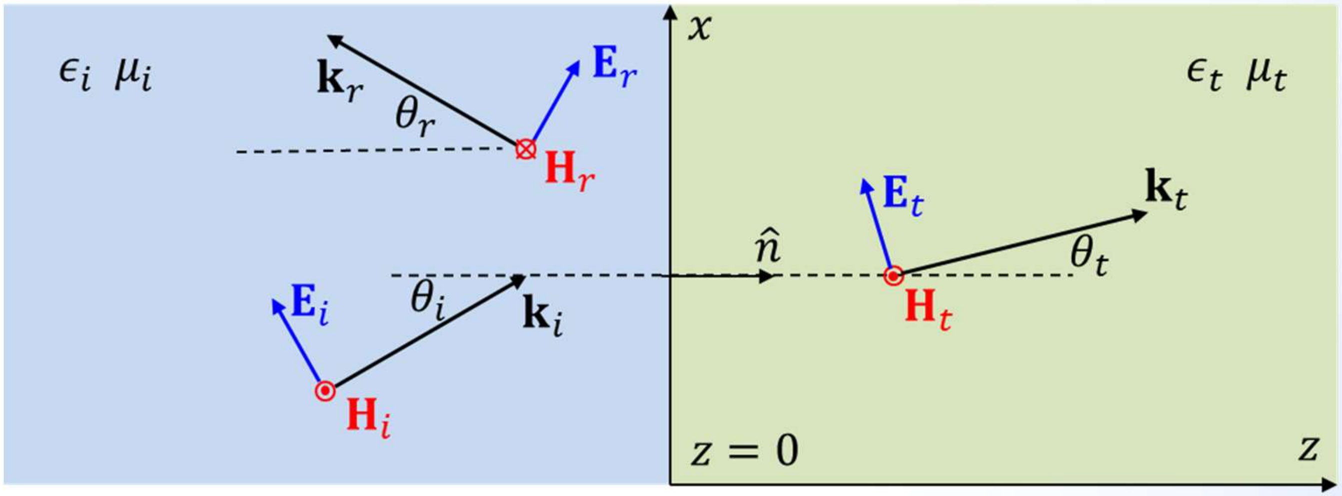
(2) At z = 0 the E-field component parallel to the interface must be continuous for all x

$$-\hat{x}\cos(\theta_i)E_i e^{-jk_i\sin(\theta_i)x} - \hat{x}\cos(\theta_r)E_r e^{-jk_r\sin(\theta_r)x} = -\hat{x}\cos(\theta_t)E_t e^{-jk_t\sin(\theta_t)x}$$

$$\boxed{\cos(\theta_i)(E_i + E_r) = \cos(\theta_t)E_t} \qquad (2)$$

Reflection and Transmission Coefficients

TM Wave



$$\boxed{E_i - E_r = \frac{\eta_i}{\eta_t} E_t} \quad \& \quad \boxed{\cos(\theta_i)(E_i + E_r) = \cos(\theta_t) E_t}$$

By solving the equation on the right using the equation on the left we get:

$$t_{\mathrm{TM}} = \frac{E_t}{E_i} = 2\frac{\eta_t \cos(\theta_i)}{\eta_t \cos(\theta_t) + \eta_i \cos(\theta_i)}$$

$$r_{\mathrm{TM}} = \frac{E_r}{E_i} = \frac{\eta_t \cos(\theta_t) - \eta_i \cos(\theta_i)}{\eta_t \cos(\theta_t) + \eta_i \cos(\theta_i)}$$

The two equations above can be re-written when considering $\mu = \mu_0 = 1$, as:

$$t_{\mathrm{TM}} = \frac{E_t}{E_i} = 2\frac{n_i \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

$$r_{\mathrm{TM}} = \frac{E_r}{E_i} = \frac{n_i \cos(\theta_t) - n_t \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

Reflection and Transmission Coefficients

For non magnetic materials

$$\boldsymbol{\mu} = \boldsymbol{\mu_0}$$

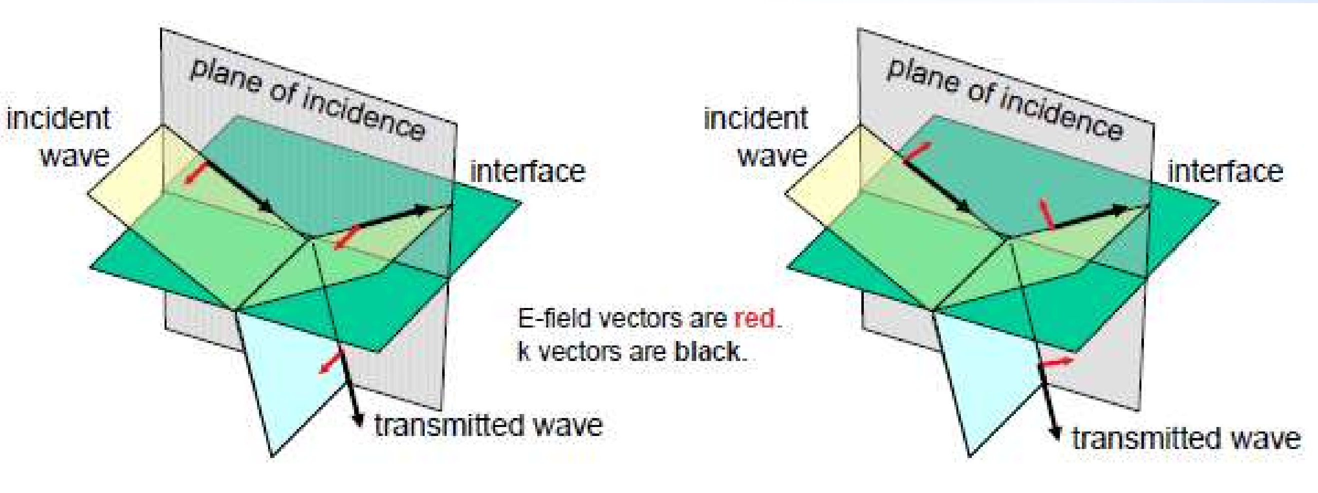
$$\boxed{r_{\mathrm{TE}} = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}}$$

$$\boxed{r_{\mathrm{TM}} = \frac{n_i \cos(\theta_t) - n_t \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}}$$

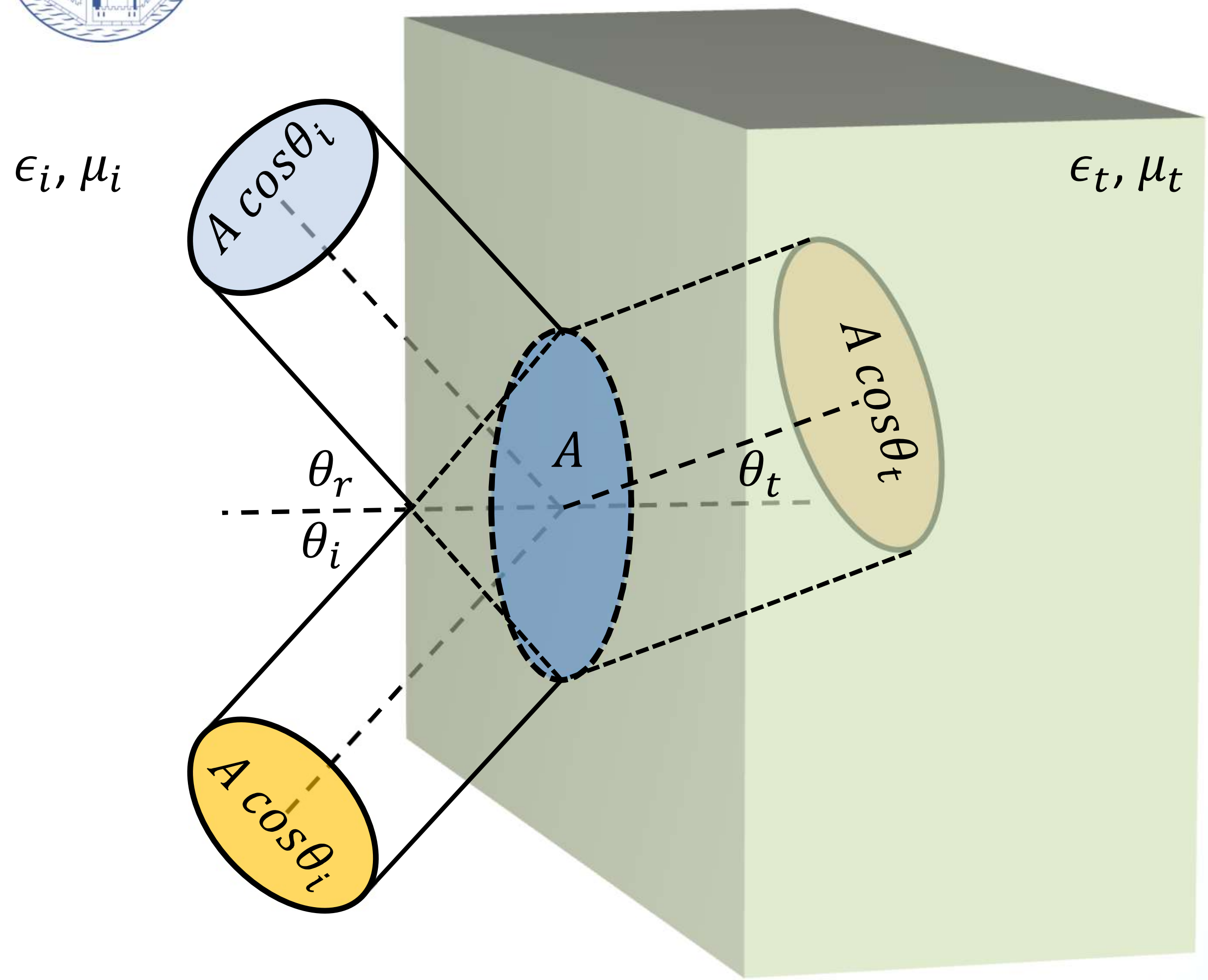
$$\boxed{t_{\mathrm{TE}} = 2\frac{n_i \cos(\theta_i)}{n_t \cos(\theta_t) + n_i \cos(\theta_i)}}$$

$$\boxed{t_{\mathrm{TM}} = 2\frac{n_i \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}}$$

TE Wave **TM Wave**



Reflectance and Transmittance



Consider a finite beam of uniform intensity, equal to the intensity of the input plane wave, $I_i = |Re(S_i)| = \frac{|E_i|^2}{2\eta_i}$

Here we assume the input medium be lossless.

Reflectance or Reflectivity

$$R = P_r/P_i$$

Transmittance or transmittivity

$$T = P_t/P_i$$

Power of the input beam

$$P_i = I_i A_i = \frac{|E_i|^2}{2\eta_i} A\, cos\theta_i$$

Power of the reflected beam

$$P_r = I_r A_r = \frac{|E_r|^2}{2\eta_i} A\, cos\theta_i$$

Power of the transmitted beam

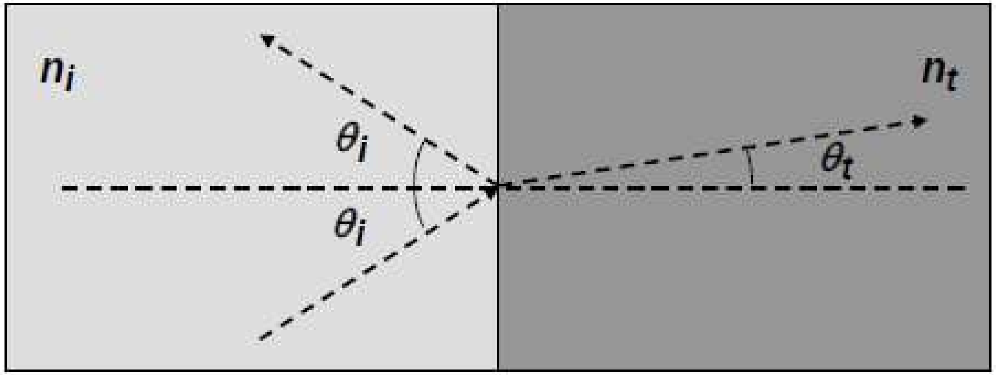
$$P_t = I_i A_i = \frac{|E_t|^2}{2\eta_t} A\, cos\theta_t$$

$$\boxed{R = \frac{|E_r|^2}{|E_i|^2} = |r|^2}$$

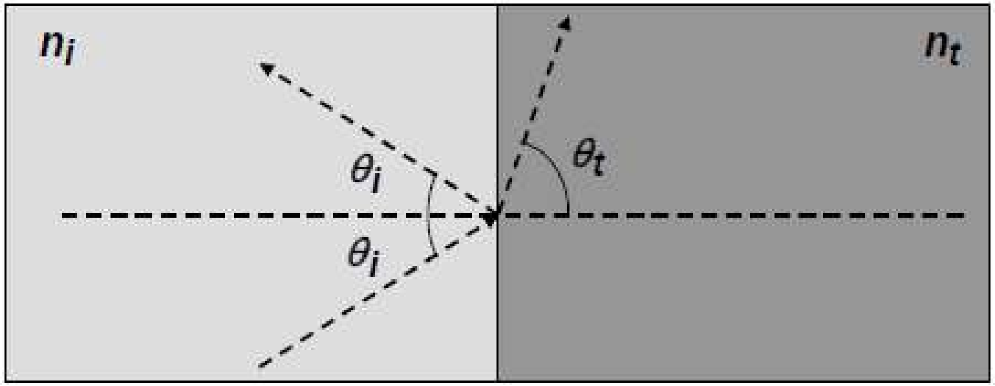
$$\boxed{T = 1 - R = |t|^2 \frac{\eta_i cos\theta_t}{\eta_t cos\theta_i}}$$

Snell's Law

$$n_i \sin(\theta_i) = n_t \sin(\theta_t)$$



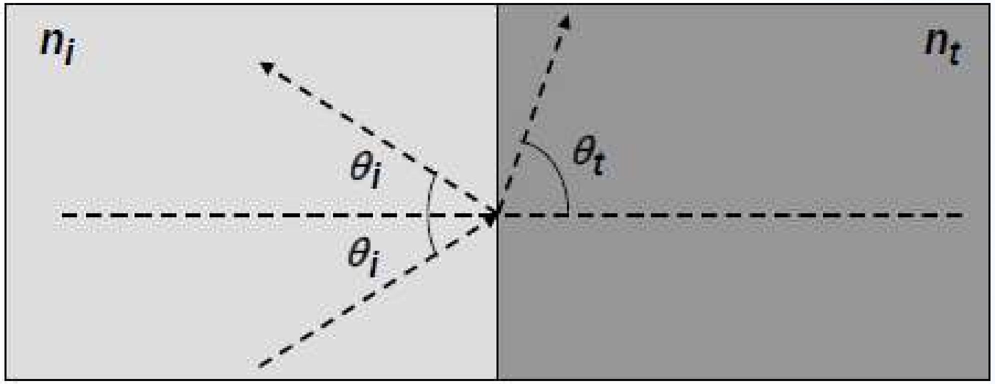
If $n_i < n_t$ then $\theta_t < \theta_i$ and the transmitted wave bends towards the normal



If $n_i > n_t$ then $\theta_t > \theta_i$ and the transmitted wave bends away the normal

Total Internal Reflection & Critical Angle

$n_i > n_t$



If θ_i is increased, then θ_t will eventually become 90°. The value of θ_i for which θ_t is 90° is called the critical angle θ_c

$$n_i \sin(\theta_c) = n_t \sin\left(\frac{\pi}{2}\right)$$

$$\boxed{\theta_c = a\sin\left(\frac{n_t}{n_i}\right)}$$

If θ_i is increased beyond θ_c the wave is not transmitted but is completely (100%) reflected at the interface back into the medium of incidence.

This phenomenon is called TOTAL INTERNAL REFLECTION and it happens for both TE and TM waves

Total Internal Reflection
Phase Matching

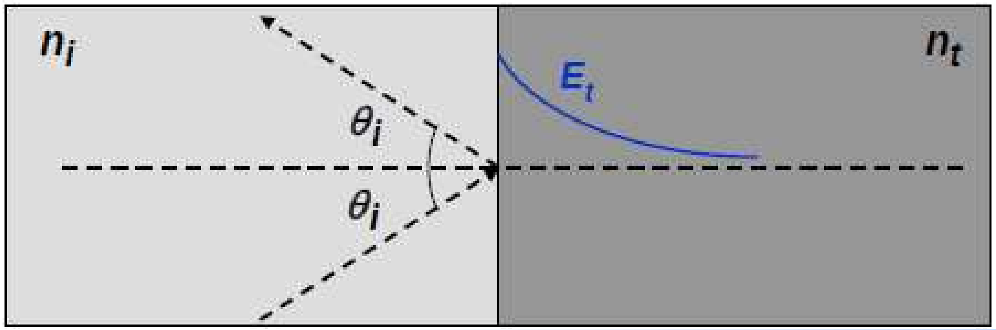
$n_i > n_t$ and $\theta_i > \theta_c$

The phase matching condition gives:

$$k_{ix} = k_{rx} = k_{tx}$$

$$k_{tx} = k_{ix} = k_i \sin(\theta_i)$$

$$k_{tx}^2 = k_i^2 \sin^2(\theta_i) = \frac{\omega^2}{c^2} n_i^2 \sin^2(\theta_i)$$



The dispersion relation in the medium "t" is: $k_t^2 = \frac{\omega^2}{c^2} n_t^2 \Rightarrow k_{tx}^2 + k_{tz}^2 = \frac{\omega^2}{c^2} n_t^2$

$$k_{tz}^2 = \frac{\omega^2}{c^2} n_t^2 - k_{tx}^2 = \frac{\omega^2}{c^2}\left[n_t^2 - n_i^2 \sin^2(\theta_i)\right] \Rightarrow k_{tz} = \frac{\omega}{c}\sqrt{n_t^2 - n_i^2 \sin^2(\theta_i)}$$

This value is NEGATIVE when $\theta_i > \theta_c$

In other words the z-component of the wavevector has become completely imaginary, therefore we can write the E-field (assuming TE polarization) as:

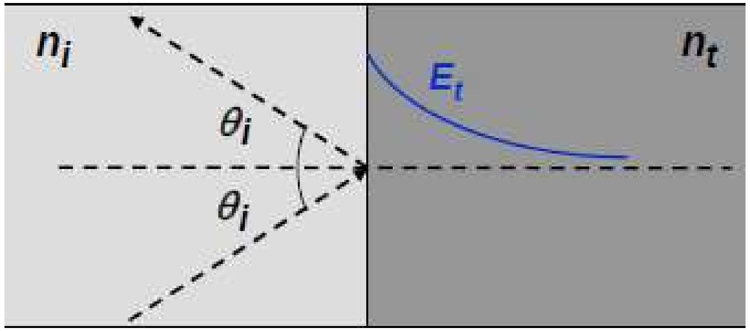
$$k_{tz} = -j\frac{\omega}{c}\sqrt{n_i^2 \sin^2(\theta_i) - n_t^2} = -jk_{tz}^{"}$$

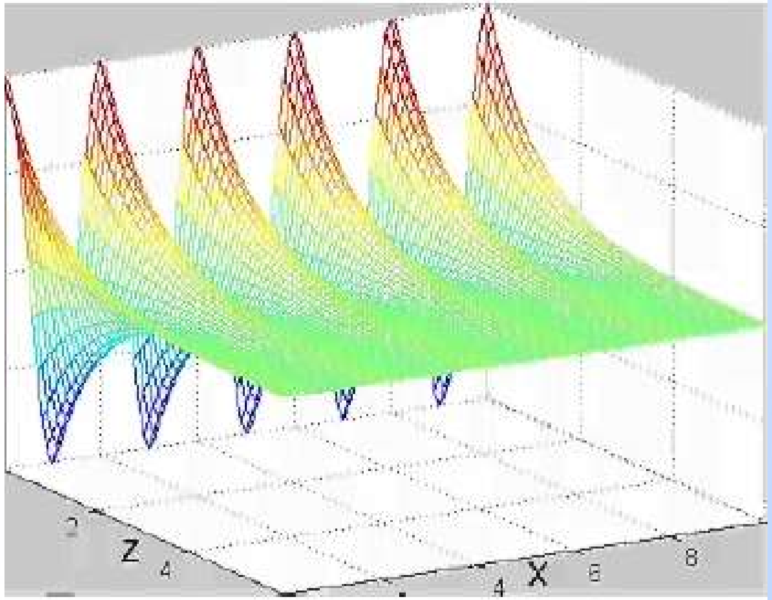
$$\mathbf{E}(\mathbf{r})\big|_{z>0} = \hat{y} E_t e^{-jk_{tx}x} e^{-k_{tz}^{"}z}$$

EVANESCENT FIELD in the z-direction

Total Internal Reflection
Electric Field Profile

$n_i > n_t$ and $\theta_i > \theta_c$





the E-field in the medium "t"(assuming TE polarization)is:

$$\mathbf{E}(\mathbf{r})\big|_{z>0} = \hat{y}E_t e^{-jk_{tx}x}e^{-k_{tz}^{"}z} = \hat{y}\left|E_t\right|e^{-j\varphi}e^{-jk_{tx}x}e^{-k_{tz}^{"}z}$$

$$\boxed{\mathbf{E}(\mathbf{r})\big|_{z>0} = \hat{y}\left|E_t\right|e^{-k_{tz}^{"}z}\cos\left(\omega t - k_{tx}x - \varphi\right)}$$

The wave is propagating along the interface (in the x-direction) but decaying (without spatial oscillations) in the z-direction

If $\theta_i > \theta_c$ the wave is completely reflected back into the medium of incidence and we have:

$$k_{tz} = -j\frac{\omega}{c}\sqrt{n_i^2\sin^2(\theta_i) - n_t^2} = -jk_{tz}^{"}$$

The reflection coefficient for the E-field (assuming TE wave) is:

$$\boxed{\Gamma = \frac{E_r}{E_i} = \frac{k_{iz} - k_{tz}}{k_{iz} + k_{tz}} = \frac{k_{iz} + jk_{tz}^{"}}{k_{iz} - jk_{tz}^{"}} = e^{i\varphi}}$$

$$|\Gamma| = 1, \quad \varphi \text{ Goos-Hanschen phase-shift}$$

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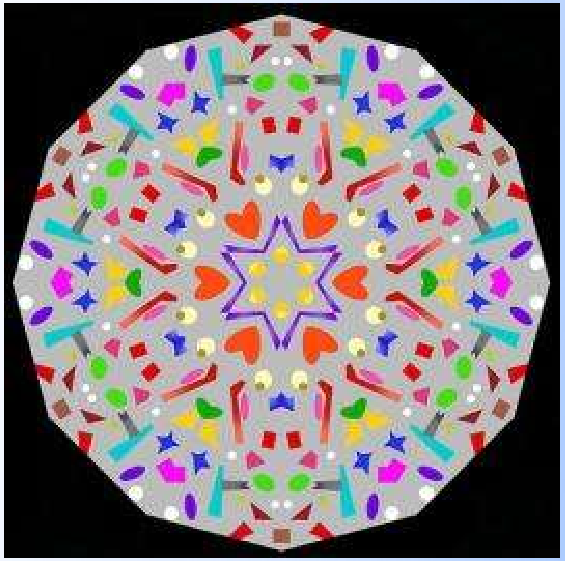
Brewster’s Angle

Today’s Culture Moment

Sir David Brewster

1781 –1868

- Scottish scientist
- Studied at University of Edinburgh at age 12
- Independently discovered Fresnel lens
- Editor of *Edinburgh Encyclopedia and contributor to Encyclopedia Britannica (7th and 8th editions)*
- Inventor of the Kaleidoscope
- Nominated (1849) to the National Institute of France.



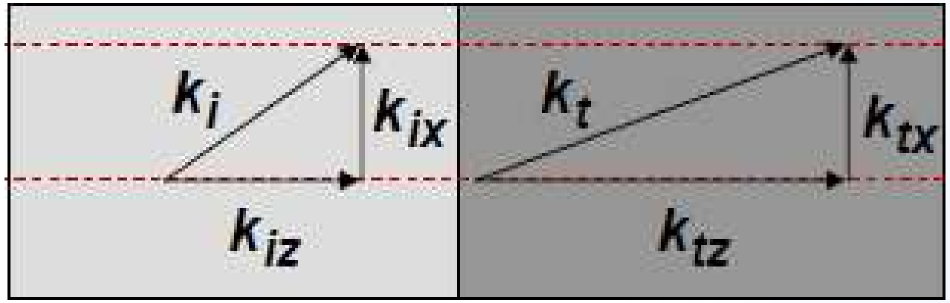
Kaleidoscope

Brewster's Angle

<u>Question</u>: Can one ever get the reflection coefficient to go to zero (very desirable to get rid of unwanted reflections in optics)?

$$r_{\mathrm{TE}} = \frac{E_r}{E_i} = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

$$r_{\mathrm{TM}} = \frac{H_r}{H_i} = \frac{n_i \cos(\theta_t) - n_t \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$



For a TE wave reflection is zero if : $n_i \cos(\theta_i) = n_t \cos(\theta_t) \Rightarrow k_i \cos(\theta_i) = k_t \cos(\theta_t)$

If: $n_i \neq n_t$ then $k_{iz} \neq k_{tz}$ since $k_{ix} = k_{tx}$ for the phase-matching condition.

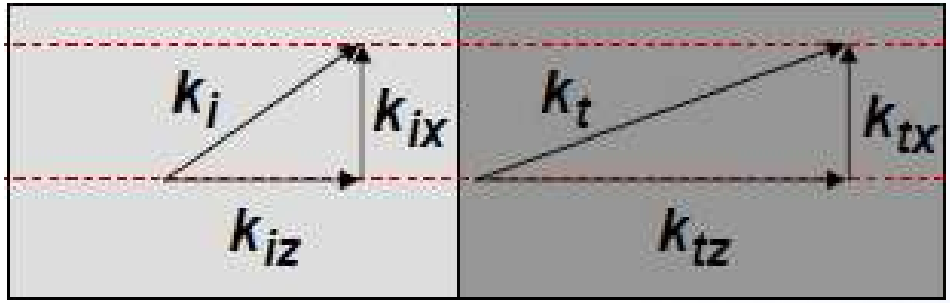
Therefore REFLECTION IS NEVER ZERO FOR TE WAVES

Brewster's Angle

<u>Question</u>: Can one ever get the reflection coefficient to go to zero (very desirable to get rid of unwanted reflections in optics)?

$$\boxed{r_{\mathrm{TE}} = \frac{E_r}{E_i} = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}}$$

$$\boxed{r_{\mathrm{TM}} = \frac{H_r}{H_i} = \frac{n_i \cos(\theta_t) - n_t \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}}$$



For a TM wave reflection is zero if : $n_i \cos(\theta_t) = n_t \cos(\theta_i)$

For Snell's law: $n_i \sin(\theta_i) = n_t \sin(\theta_t)$

By squaring both equations, solving for the terms in θ_i and then adding each corresponding term one can obtain:

$$\boxed{\theta_B = a\tan\left(\frac{n_t}{n_i}\right)}$$

Practical Example: The Lens Flare!

If θ_i=0 reflectance and transmittance are:

$$\boxed{R = |r|^2 = \left(\frac{n_i - n_t}{n_i + n_t}\right)^2}$$

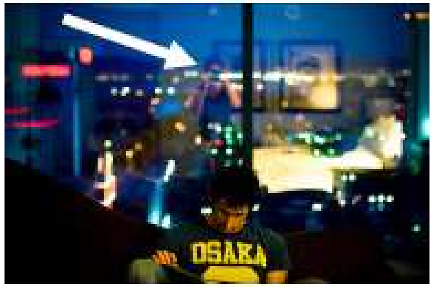
$$\boxed{T = \frac{n_t}{n_i}|t|^2 = 4\left(\frac{n_i}{n_t + n_i}\right)^2 \frac{n_t}{n_i}}$$

For a camera lens reflection/transmission occurs at the interface between glass and air.
If n_i = 1 and n_t = 1.5:

R = 4% and T = 96%

Reflectance is "only" 4% but has big implication for photography

Practical Example: Fresnel Equations in action!

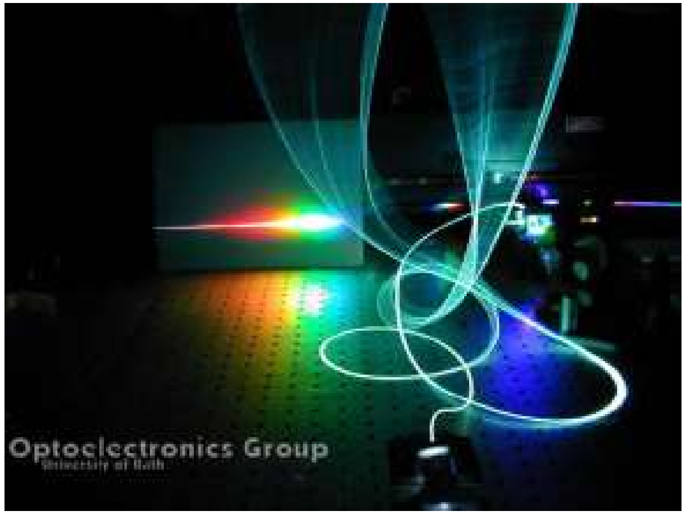


Windows look like mirrors at night (when you're in a brightly lit room).

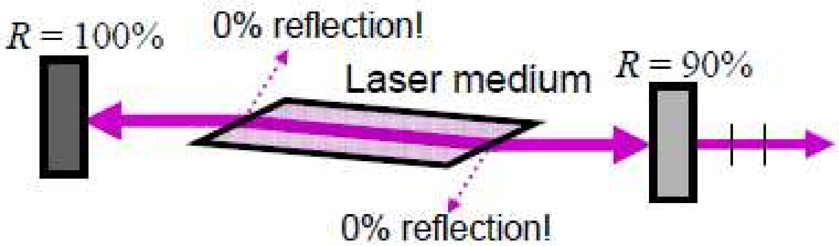
One-way mirrors (used by police to interrogate bad guys) are just partial reflectors (actually, with a very thin aluminum coating).

Smooth surfaces can produce pretty good mirror-like reflections, even though they are not made of metal.

Practical Example: Fresnel Equations in action!



Optical fibers only work because of total internal reflection.



Many lasers use Brewster's angle components to avoid reflective losses

Scattering

Scattering can be broadly defined as the

redirection of radiation out of the original direction of propagation

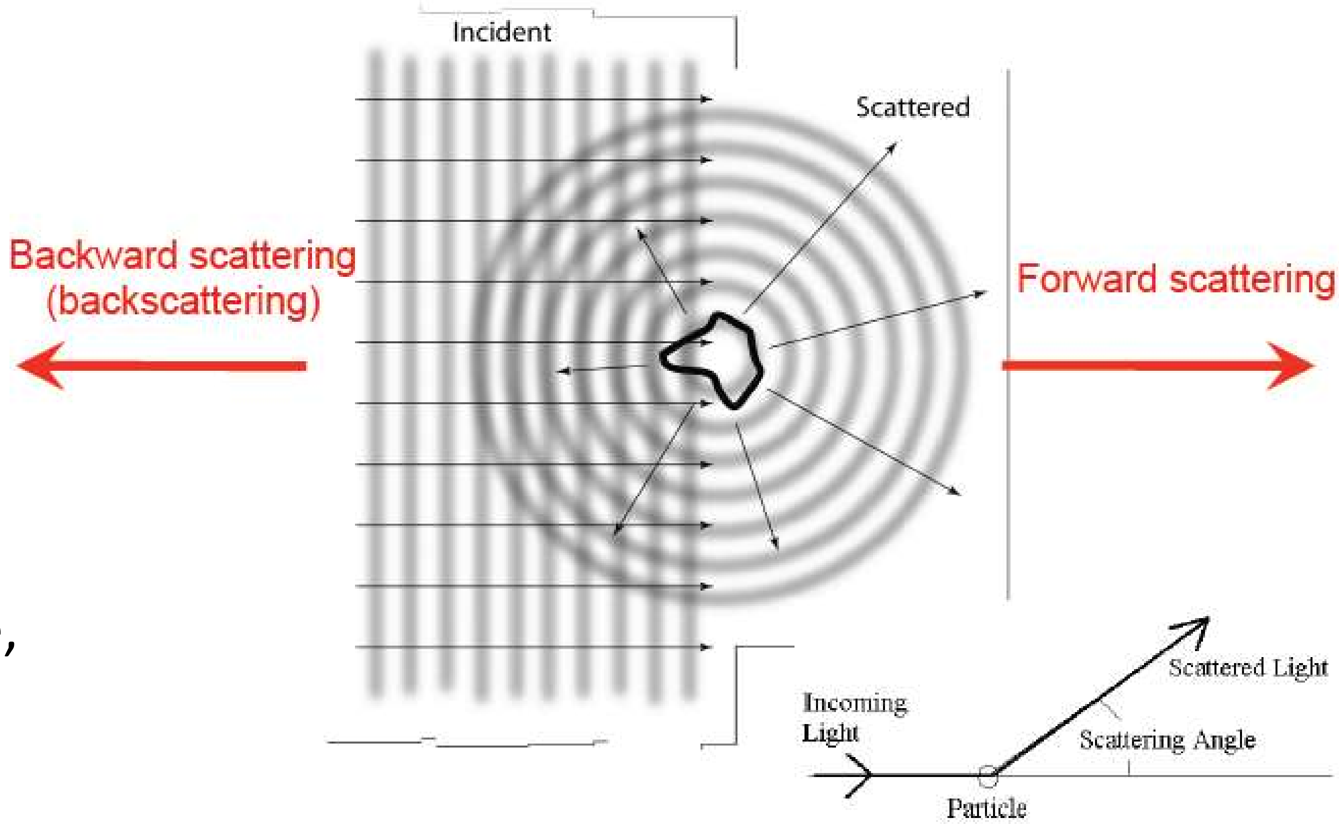
usually due to interactions with molecules and particles

Reflection, refraction, diffraction etc. are actually all just forms of scattering. The *superposition of incident and scattered waves* is what is actually observed.

When is scattering important?

Scattering is negligible whenever gains in intensity due to scattering are small compared to:

- Losses due to extinction
- Gains due to thermal emission
- When considering direct radiation from a point source, such as the sun



In the UV, visible and near-IR bands, scattering is the dominant source of radiation along any line of sight, other than looking directly at the sun.

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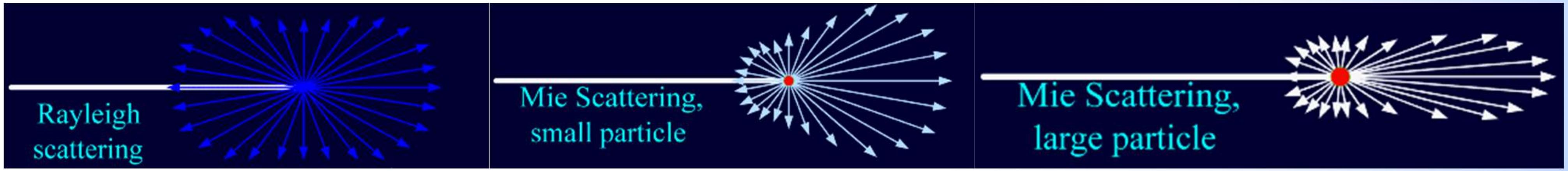
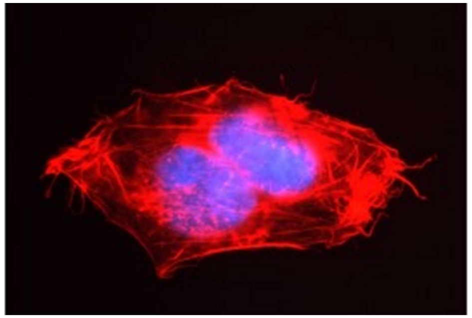
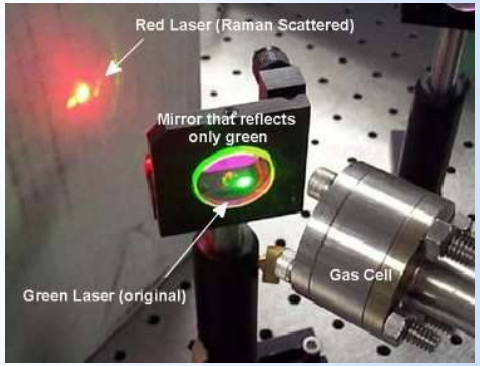
Types of Scattering

Elastic scattering – the wavelength (frequency) of the scattered light is the same as the incident light (*Rayleigh and Mie scattering*)

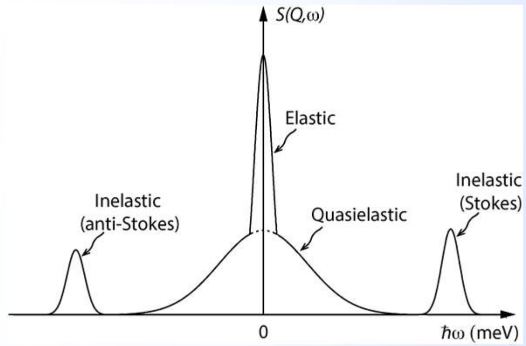


Inelastic scattering – the emitted radiation has a wavelength different from that of the incident radiation (*Raman scattering, fluorescence*)





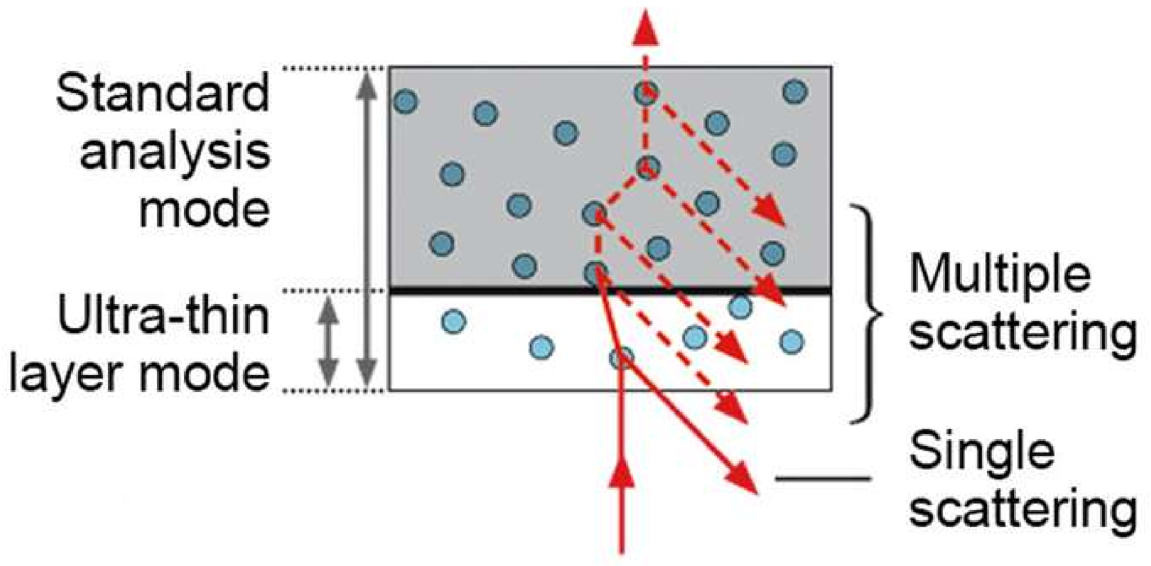
Quasi-elastic scattering – the wavelength (frequency) of the scattered light shifts (e.g., in moving matter due to Doppler effects)



Types of Scattering

Single scattering: photons scattered only once prevail in optically thin media, since photons have a high probability of exiting the medium (e.g., a thin cloud) before being scattered again . Also favored in strongly absorbing media

Multiple scattering: prevails in optically thick, strongly scattering and non-absorbing media. Photons may be scattered hundreds of times before emerging



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Practical example: Variation in sky brightness

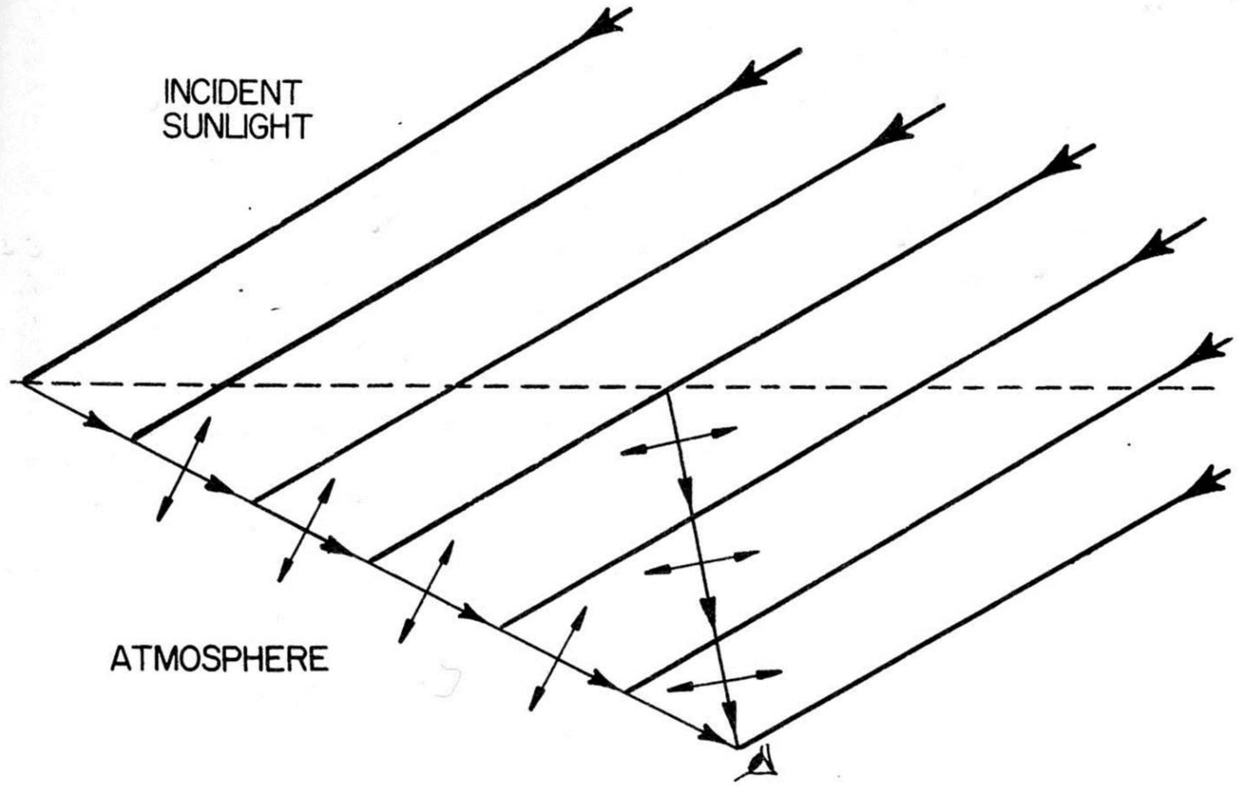


Figure 20.3 Path lengths in the atmosphere. An observer receives light scattered by all the molecules and particles along the line of sight. Paths near the horizon are longer than those near the zenith, hence the horizon sky is brighter. From *The Physics Teacher*, C. F. Bohren and A. B. Fraser, May 1985.

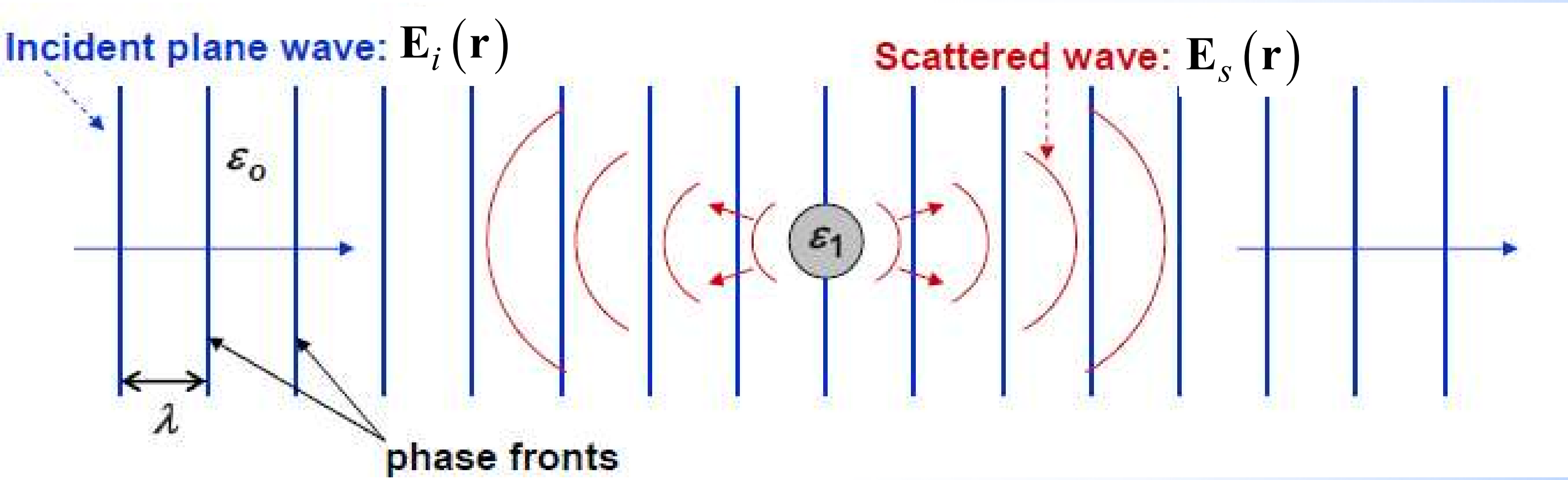
NOTE: The horizon sky is usually brighter than the zenith sky.
This is a result of single scattering (zenith) vs. multiple scattering (horizon)

Parameters governing scattering

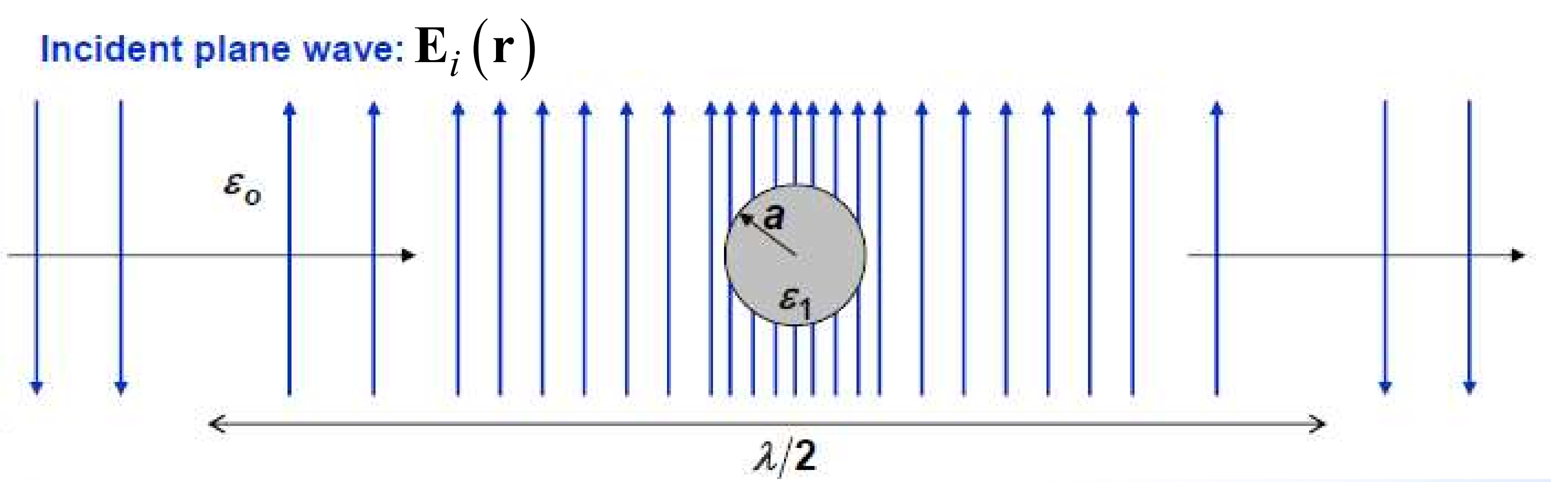
(1) The wavelength (λ) of the incident radiation

(2) The size of the scattering particle

(3) The particle optical properties relative to the surrounding medium



Scattering from spherical particles



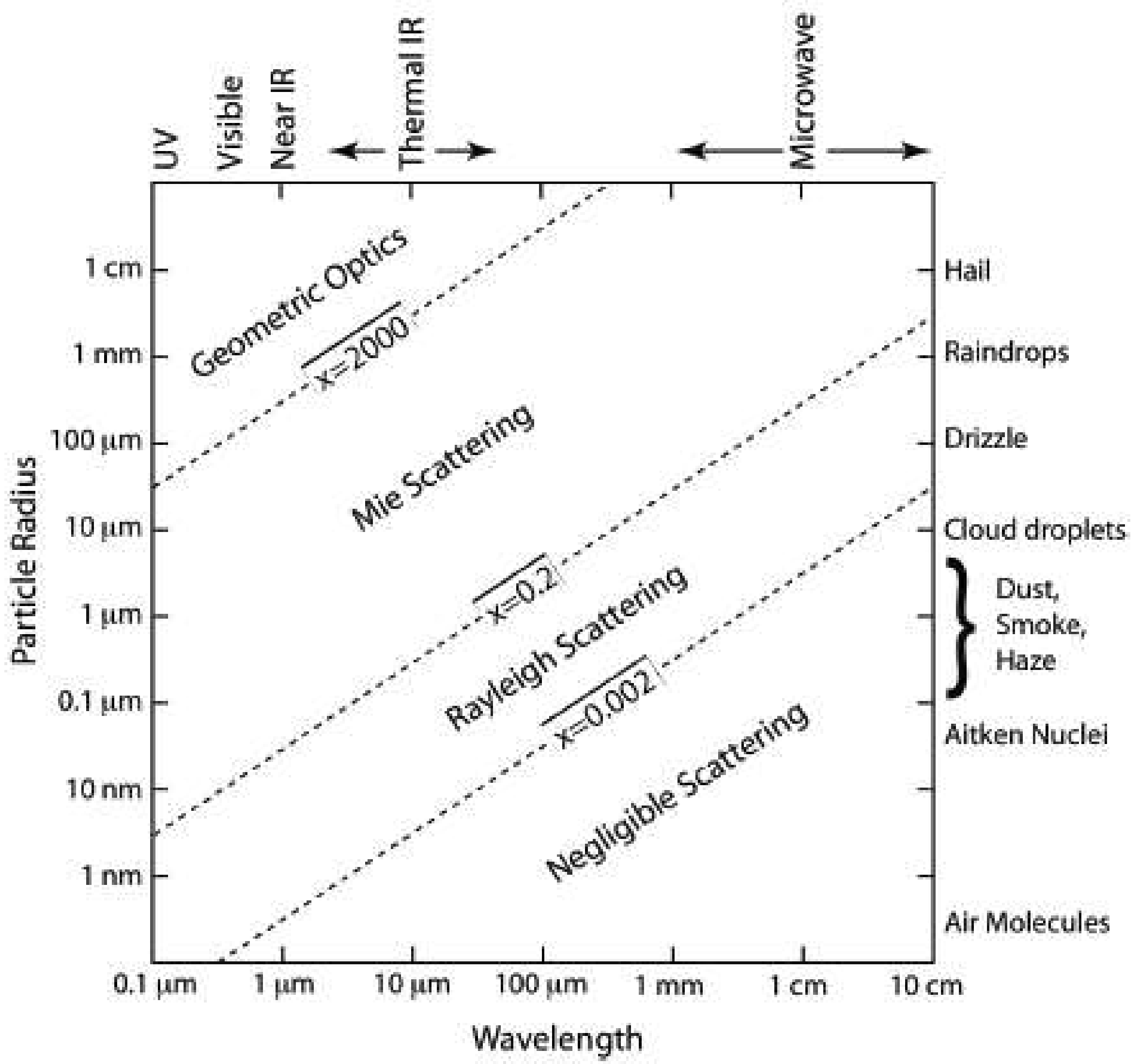
Different scattering conditions can be identified depending on their geometrical size in relation with the incident wavelength. Let's define the adimensional parameter:

$$\boxed{x = \frac{2\pi a}{\lambda} = ka}$$

If ka << 1 the scattering is called Rayleigh Scattering
If ka ~ 1 the scattering is called Mie Scattering
If ka >> 1 the scattering is called Geometrical Scattering

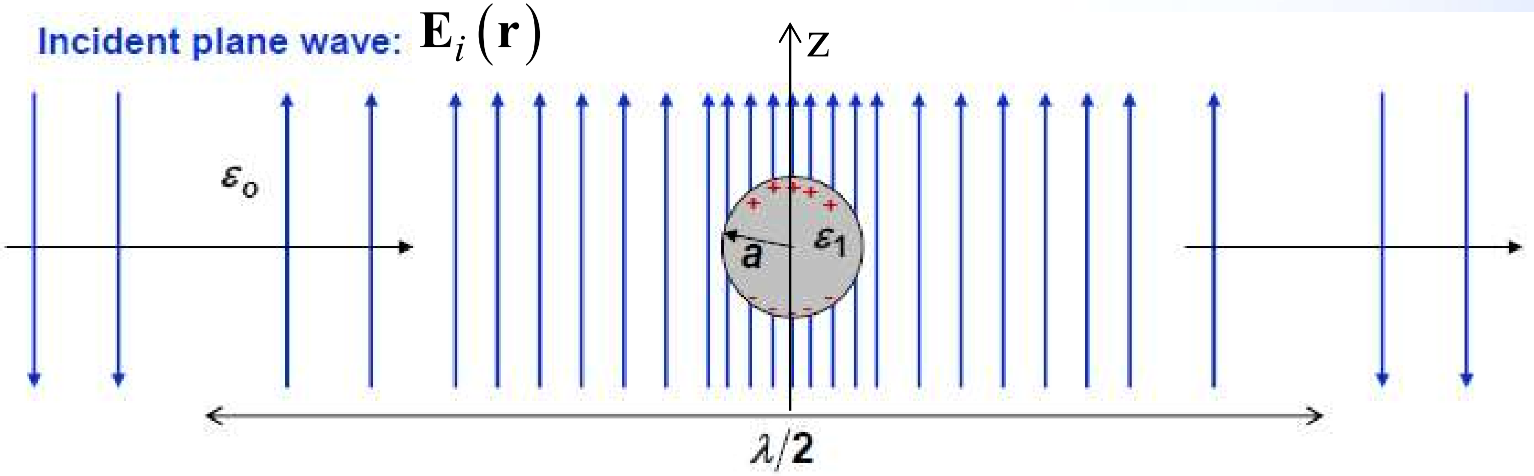
When *ka* << 1, the particle sees a uniform E-field that is slowly oscillating in time

Light scattering regimes



NOTE: This plot considers only single scattering by spheres. Multiple scattering and scattering by non-spherical objects can get really complex!

Rayleigh Scattering



One way to understand scattering is as follows:
i) The incident E-field induces a time-varying dipole moment in the sphere
ii) The time-varying dipole radiates like a Hertzian dipole and this is the scattered radiation

Let's now suppose the z-directed E-field phasor for the incident plane wave at the location of the particle is:

$$\mathbf{E}(\mathbf{r}=0)=\hat{z}E_i$$

The z-directed dipole moment *p* induced in a sphere in the presence of E-field *E* is:

$$p=4\pi\varepsilon_0 a^3\left(\frac{\varepsilon_1-\varepsilon_0}{\varepsilon_1+2\varepsilon_0}\right)E_i$$

Rayleigh Scattering

Total scattered power P_s from a dielectric sphere is:

$$P_s = \int_0^{2\pi}\int_0^{\pi} \frac{|\mathbf{E}_s(\mathbf{r})|^2}{2\eta_0} r^2 \sin(\theta) d\theta d\phi = \frac{4\pi}{3\eta_0} k^4 a^6 \left(\frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_1 + 2\varepsilon_0}\right)^2 |E_i|^2$$

The incident power per unit area is the Poynting vector of the incident wave: $\dfrac{|E_i|^2}{2\eta_0}$

The scattering cross-section σ_s of a scatterer is defined as the area of a plane oriented perpendicular to the direction of incident wave that would intercept the same total incident power as the power P_s that the scatterer radiates:

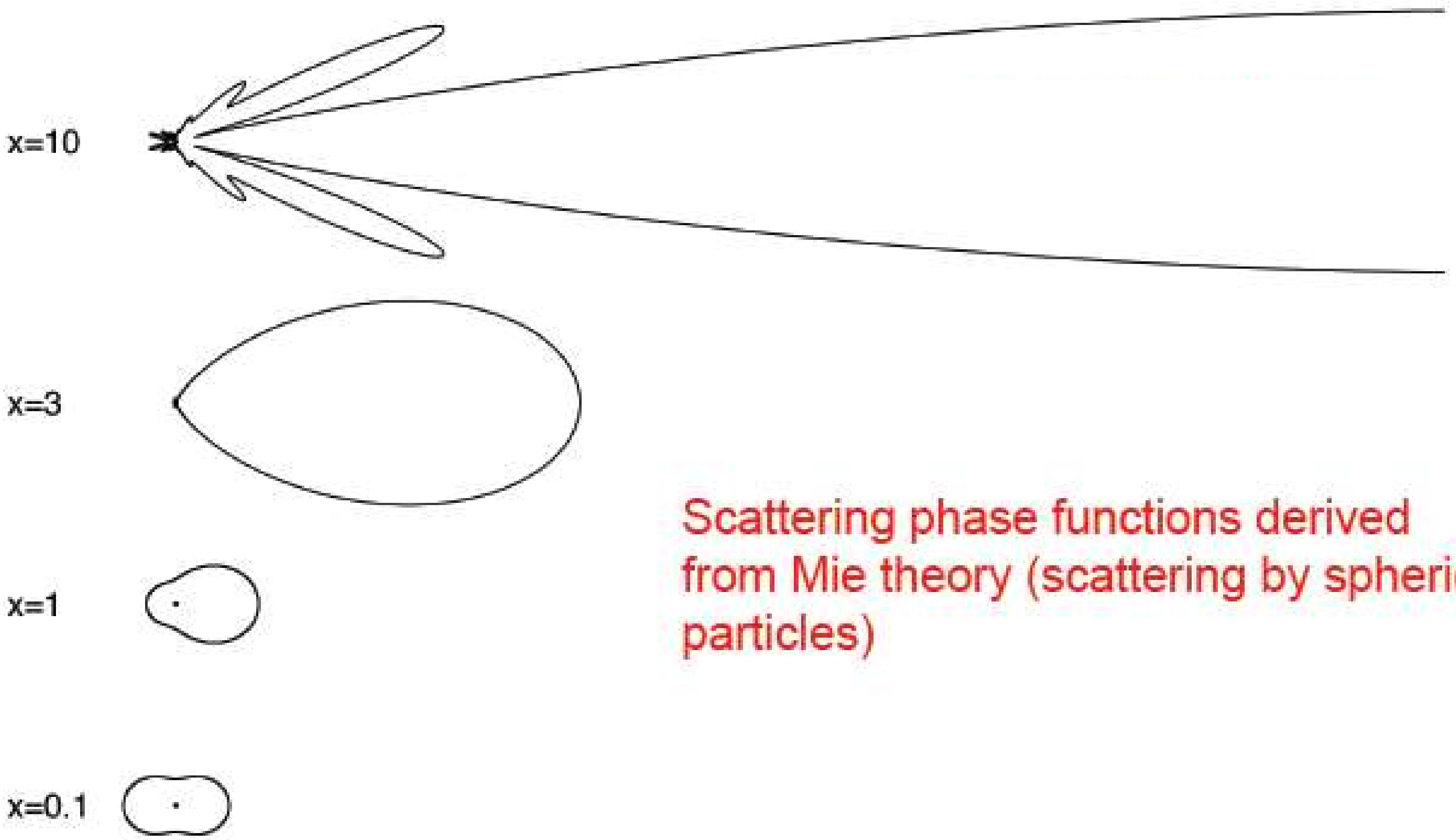
$$\sigma_s = \frac{P_s}{|\mathbf{E}_i(\mathbf{r})|^2 / 2\eta_0}$$

σ_s is also the ratio of the total scattered power to the power per unit area of the incident wave at the location of the scatterer

For the dielectric sphere: $$\sigma_s = \frac{8\pi}{3} k^4 a^6 \left(\frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_1 + 2\varepsilon_0}\right)^2$$

Scattering phase functions

The scattering phase function, or phase function, gives the angular distribution of light intensity scattered by a particle at a given wavelength.

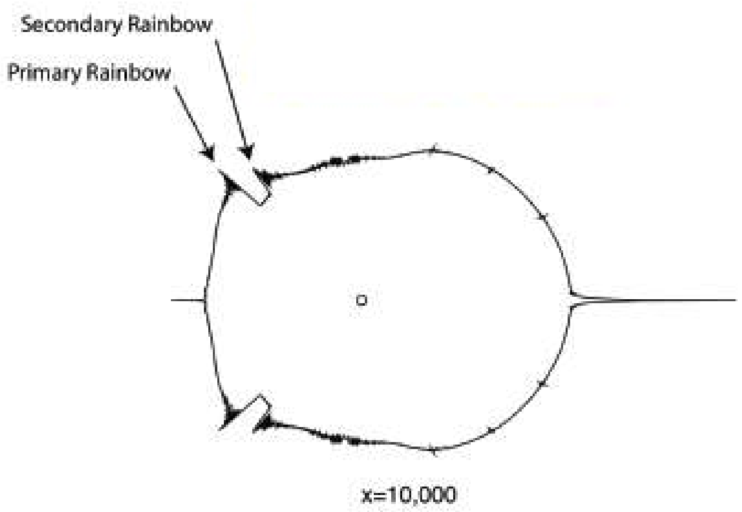


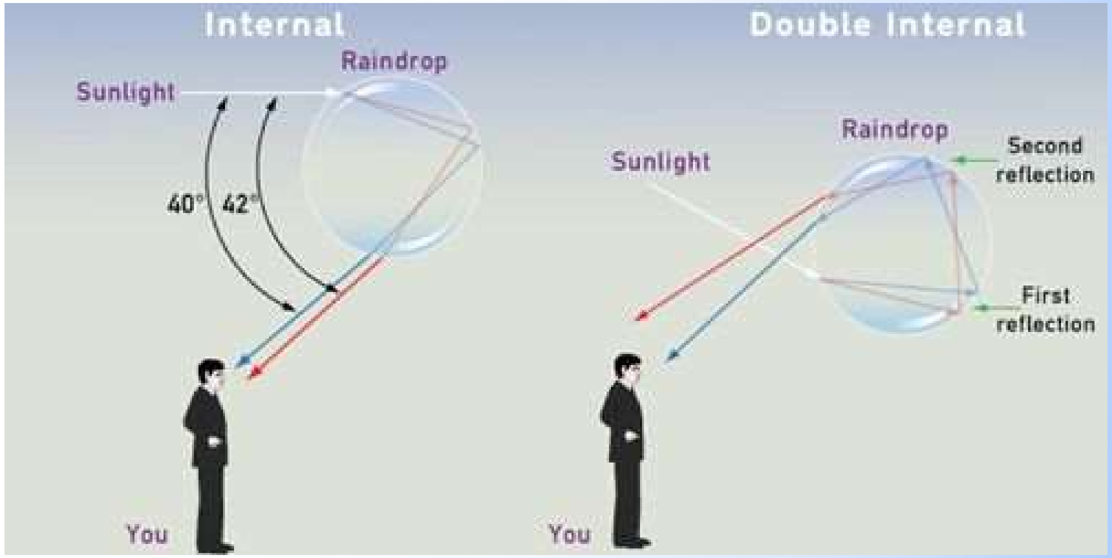
Scattering phase functions derived from Mie theory (scattering by spherical particles)

Practical example: Rainbows

Rainbow: for **large particles** (x = 10,000), the forward and backward peaks in the scattering phase function become very narrow (almost non-existent). Light paths are best predicted using geometric optics and ray tracing.

Primary rainbow: single internal reflection
Secondary rainbow: double internal reflection





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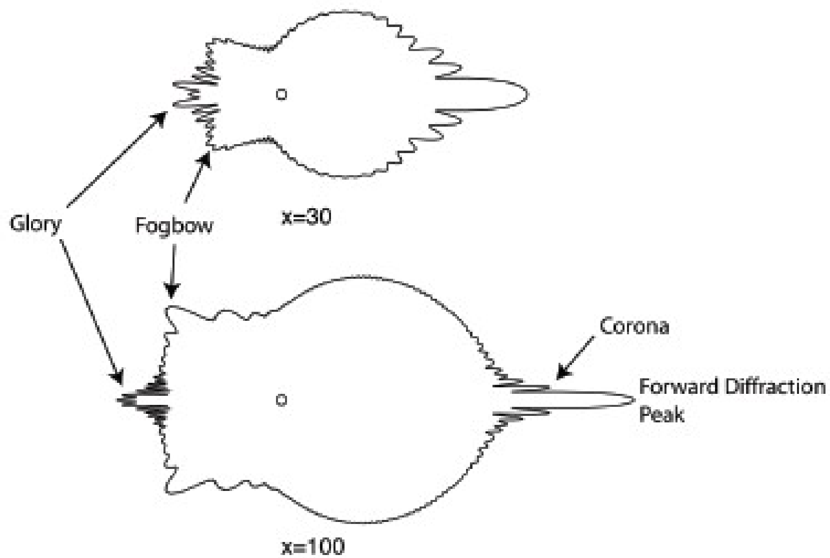
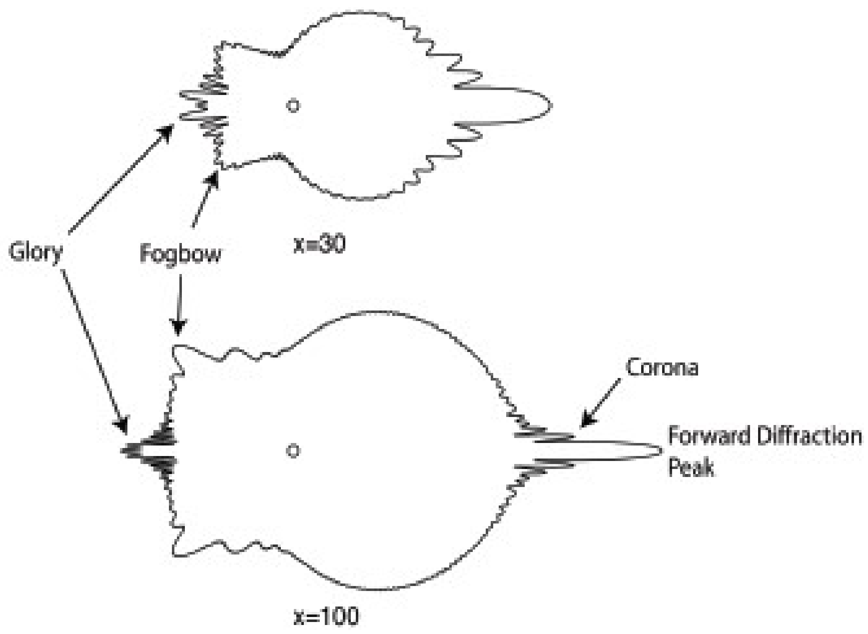
Practical example: Glory and Fogbow

Fogbow: spikes in scattering phase function present but not sharp as for rainbows. Hence the separation of colors (due to varying refractive index) is not as vivid as a normal rainbow. A whitish ring centered on one's shadow (i.e. opposite the sun) is seen. Arises when **water droplets have a size characteristic of fog and clouds** rather than rain.



Glory: opposite end of the phase function from the corona. Seen as a 'halo' around one's shadow when looking at a fog bank with the sun at your back. Also seen from aircraft. **Glories have vivid colors if the range of drop sizes in the fog is relatively narrow, otherwise they are whitish**.

Practical example: Corona



Corona: for **intermediate values of the size parameter (x)**, the forward scattering peak is accompanied by weaker *sidelobes*. If you were to view the sun through a thin cloud composed of identical spherical droplets (with x = 100 or less), you would see closely spaced rings around the light source. The angular position of the rings depends on wavelength, so the rings would be colored. This is a *corona*. Because few real clouds have a sufficiently narrow distribution of drop sizes, coronas are usually more diffuse and less brightly colored.