Optical Communication Networks

Notes 1 Introduction

Main goal: Investigation of the evolution of optical pulses propagating in an optical fiber.

Topics: → Group Velocity Dispersion (Linear effect)

→ Self-Phase Modulation (Nonlinear effect)

→ Optical self-trapped waves, Solitons

→ Abnormal, Extreme waves

→ Optical shocks

Nonlinear Schrödinger Equation (NLSE)

(3+1)D

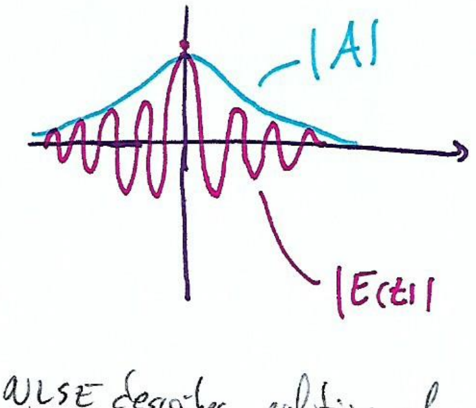
$$\underbrace{j\frac{\partial A(r,t)}{\partial z}}_{\text{propagation along } z} + \underbrace{\frac{1}{2\beta}\frac{\partial^2 A(r,t)}{\partial x^2}}_{\text{diffraction along } x} + \underbrace{\frac{1}{2\beta}\frac{\partial^2 A(r,t)}{\partial y^2}}_{\text{diffraction along } y} - \overbrace{\frac{\beta''}{2}\frac{\partial^2 A(r,t)}{\partial t^2}}^{\text{dispersion term along } t} + \overbrace{\chi^{(3)}|A(r,t)|^2 A(r,t)}^{\text{nonlinear term}} = 0$$

→ $E(r,t) = \mathrm{Re}\left[A(r,t)\, e^{i(\omega_0 t + \beta_0 z)}\right]$

$E(r,t)$ → electric field of pulses

$A(r,t)$ → slowly varying envelope

$e^{i(\omega_0 t + \beta_0 z)}$ → optical carrier at freq ω_0 and wavenumber β_0



β → propagation constant

β'' → group velocity dispersion

$\chi^{(3)}$ → nonlinear cubic response of the medium

NLSE describes evolution of an optical light bullet in space and time

The envelope $A(r,t)$: $A(r,t) = A(x,y,z,t) = F(z,t)\, M(x,y)\, e^{i\delta\beta z}$

→ $M(x,y)$: Modal profile

→ $\delta\beta$: Correction of the propag. constant. $(\beta = \beta_0 + \delta\beta)$

⟹ $M(x,y)\, e^{i\delta\beta z}$: describes the modal distribution in the plane (x,y)

→ $F(z,t)$: Describe the slowly varying pulse envelope in the plane (z,t)

Focus on spatio-temporal evolution of $F(z,t) \xRightarrow{\text{we get}}$ NLSE (1+1)D

$$j \frac{\partial F(z,t)}{\partial z} - \frac{\beta^{4}}{2} \frac{\partial^2 F(z,t)}{\partial t^2} + \gamma |F(z,t)|^2 F(z,t) = 0$$

NLSE (1+1)D

group velocity dispersion inside the fiber

effective nonlin. term related to $\chi^{(3)}$

→ For some cases NLSE (1+1)D and NLSE (3+1)D lead to analytical solutions

Notes 2 | Numerical techniques

- Split-step Fourier method.

NLSE → $\frac{\partial F}{\partial z} = (\hat{D} + \hat{N}) F$

In general dispersion and nonlinearity act together along the fiber

$\hat{D} = -i \frac{\beta^4}{2} \frac{\partial^2}{\partial t^2}$ → Differential operator that account for dispersion (absorptio, higher order dispersions too...) in a <u>linear</u> medium

$\hat{N} = i \gamma |F|^2$ → Nonlinear operator that governs the effect of fiber nonlinearity on pulse propagation (Kerr nonlin, Raman effect)

→ The method: Obtain approx sol. assuming that for small dist. $\hat{D}$ and $\hat{N}$ act independently. Small distance h

$F(z) \longrightarrow F(z+h)$ in two steps

1st $\hat{N} = 0$ & $\hat{D} \neq 0$

2nd $\hat{N} \neq 0$ & $\hat{D} = 0$

Approx sol. → $F(z+h, t) \triangleq \exp(h\hat{D}) \exp(h\hat{N}) F(z,t)$

Exact sol. → $F(z+h, t) = \exp\left[h \underbrace{(\hat{D} + \hat{N})}_{z\text{-independent}}\right] F(z,t)$

The dominant error between exact and approx is: $\frac{1}{2} h^2 [\hat{D}, \hat{N}]$ second order of h

h small ⟶ small errors

$$\boxed{\frac{\partial}{\partial t} \longrightarrow i\omega \qquad \frac{\partial^2}{\partial t^2} \longrightarrow -\omega^2}$$

↪

→ Dispersive step.

$$\frac{\partial F(z,t)}{\partial z} = -i \frac{\beta''}{2} \frac{\partial^2 F(z,t)}{\partial t^2}$$

Fourier transform of F:

$$\hat{F}(z,\omega) = \int_{-\infty}^{+\infty} F(z,t)\, e^{-i\omega}\, dt$$

$$\Big\downarrow \text{Fourier}$$

$$\frac{\partial \hat{F}(z,\omega)}{\partial z} = \left(i\, \omega^2 \frac{\beta''}{2} \right) \hat{F}(z,\omega)$$

Analytically:

$$\hat{F}(z+h,\omega) = \hat{F}(z,\omega) \exp\left[+i \frac{\omega^2}{2} \beta'' \cdot h \right]$$

$$F(z+h,t) = \mathcal{F}^{-1}\left[\hat{F}(z+h,\omega) \right]$$

→ Nonlinear term

$$\frac{\partial F(z,t)}{\partial z} = i \gamma |F|^2 F$$

$|F|^2$ z-independent

$$F(z+h,t) = F(z,t) \exp\left[i \gamma |F(z,t)|^2 \cdot h \right]$$

↳ step in z

Notes 3 | Group Velocity Dispersion (GVD)

⊛ Dispersion induces pulse broadening

To study dispersion → $\gamma = 0$ in NLSE ⟹ $i\dfrac{\partial F}{\partial z} = \dfrac{\beta_2}{2}\dfrac{\partial^2 F}{\partial t^2}$

Using fourier $F(z,t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{+\infty} \hat{F}(z,\omega)\, e^{-i\omega t}\, d\omega$

$$i\frac{\partial \hat{F}}{\partial z} = -\frac{1}{2}\beta_2 \omega^2 \hat{F}$$

solution

$$\rightarrow \boxed{\underset{\text{in}}{\hat{F}(z,\omega)} = \underset{\text{out}}{\hat{F}(z=0,\omega)}\ \underbrace{\exp\left(\frac{i}{2}\beta_2\omega^2 z\right)}_{\text{changes}}} \quad (1)$$

→ GVD changes the phase of each spectral component of the pulse by amount that depends on : freq. ω, propagated dist. z

Solution →

$$\boxed{F(z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{F}(0,\omega)\, \exp\left(\frac{i}{2}\beta_2\omega^2 z - i\omega t\right) d\omega} \quad (2)$$

→ For input impulses of arbitrary shapes

fourier transform at input $\hat{F}(0,\omega) = \mathcal{F}^{-1}[F(0,t)]$

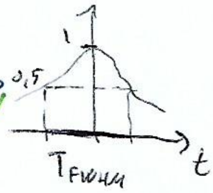
→ Gaussian pulses

$t_0 \to$ half-width (@ 1/e intensity)

Input envelope: $F(0,t) = \exp\left(-\dfrac{t^2}{2t_0^2}\right)$

More use the FWHM

$$T_{FWHM} = 2(\ln 2)^{1/2}\, t_0 \simeq 1{,}665\, t_0$$



full width at half maximum

Doing the calculation in ②

$$F(z,t) = \frac{t_0}{(t_0^2 - i\beta_2 z)^{1/2}} \cdot \exp\left[-\frac{t^2}{2(t_0^2 - i\beta_2 z)}\right]$$

↳ Gaussian pulse maintains the shape but its width is increased

GVD broadens a gaussian pulse.

$$T_1(z) = t_0\left[1 + (z/L_D)^2\right]^{1/2} \;;\; L_D = t_0^2/|\beta_2|$$

Temporal broadening
Amplitude reduction } energy conservation

↳ The extent of broadening governed by L_D.

→ short pulses broaden more bc smaller L_D (dispersion lenght)

↳ Although input pulse is unchirped (no phase modulation) transmitted pulse becomes chirped.

$$F(z,t) = |F(z,t)| \exp\left[i\,\phi(z,t)\right]$$

Time dependence of phase $\phi(z,t)$ implies that across the pulse the instant. freq. differs from central freq. ω_0

$$\partial\omega(t) = -\frac{\partial\phi}{\partial t}$$

is the chirp and depends on the sign of β_2

↳ Dispersion-induced pulse broadening can be understood if we remeber that different freq. components of a pulse travel a slighly diff speed bc of GVD

→ Red components travel faster than blue in normal GVD ($\beta_2 > 0$)

→ The opposite in anomalous regime ($\beta_2 < 0$)

Notes 4

→ Chirped gaussian pulses

↳ In Unchirped gaussian pulse dispersion-induced broadening does not depend on sign of β_2

↳ For chirped: $$F(0,t) = \exp\left[-\frac{(1+iC)}{2}\cdot\frac{t^2}{t_0^2}\right]$$

↳ instantaneous freq. increases linearly from the leading to the trailing edge when $C>0$

↳ instant. freq. decreases lin. when $C<0$

↳ Fourier: $$\hat{F}(0,\omega) = \left(\frac{2\pi t_0^2}{1+iC}\right)^{1/2} \exp\left[-\frac{\omega^2 t_0^2}{2(1+iC)}\right]$$; spectral half-width $\Delta\omega = (1+C^2)^{1/2}/T_0$

- if $C=0 \rightarrow \Delta\omega T_0 = 1$
- spectral width enhanced by a factor $(1+C^2)^{1/2}$ in linear chirp

↳ Transmited field: $$F(z,t) = \frac{t_0}{[t_0^2 - i\beta_2 z(1+iC)]^{1/2}} \exp\left[-\frac{(1+iC)\,t^2}{2\,[t_0^2 - i\beta_2 z\,(1+iC)]}\right]$$

↳ A chirped gaussian pulse maintains its gaussian shape.

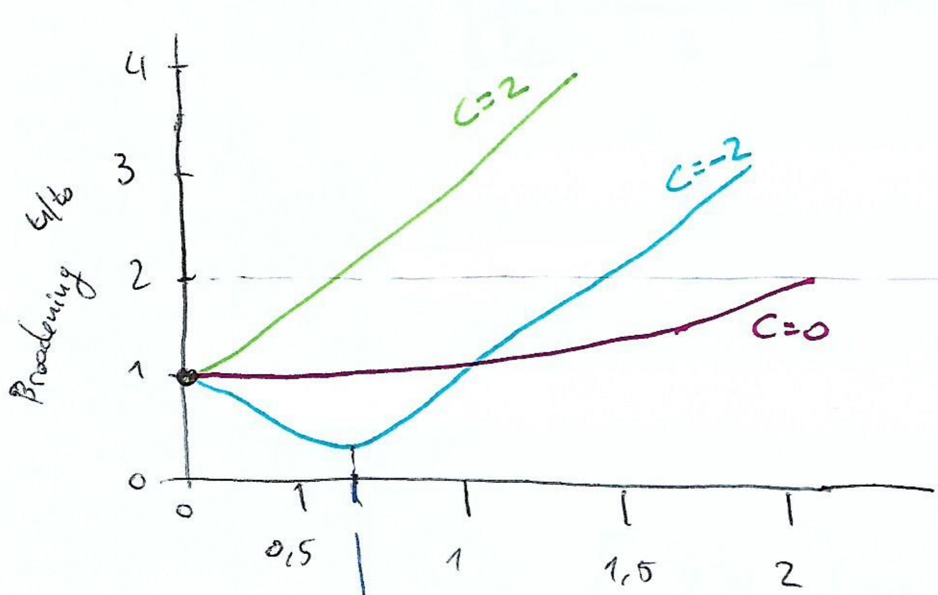
↳ The width t_1 after propagating a distance z

$$\frac{t_1}{t_0} = \left[\left(1 + \frac{C\beta_2 z}{t_0^2}\right)^2 + \left(\frac{\beta_2 z}{t_0^2}\right)^2 \right]^{1/2}$$

→ Broadening depends on signs of β_2 and C

⊛ Gaussian pulse broadens monotonically with z if $\boxed{\beta_2 C > 0}$

⊛ Gaussian pulse goes through an initial narrowing stage wit z if $\boxed{\beta_2 C < 0}$



↳ $z_{min} = \frac{|C|}{1+C^2} L_D$

$$t_1^{min} = \frac{t_0}{(1+C^2)^{1/2}}$$

Notes 5

→ Hyperbolic - Secant pulses

⊛ Pulses emitted from many lasers are appoximated by chirped gaussian shape

⊛ Hyperbolic-secant: For optical solitons and pulses emitted by high-performance mode-locked lasers.

⊛ Optical envelope → $F(0,t) = \mathrm{sech}\,(t/t_0)\, \exp\left[-\dfrac{iCt^2}{2t_0^2}\right]$

C → chirp param. controls initial phase.

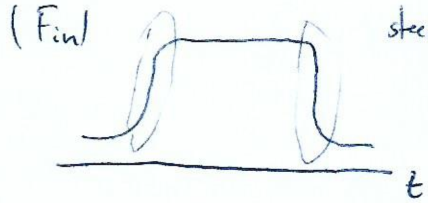
Transmitted envelope: → $F(z,t)$ → difficult to solve → Numerics

↳ Qualitative features of dispersion induced broadening nearly identical between Gaussian and "sech"

↳ t_0 is not FWHM, but: $t_{FWHM} = 2\ln(1+\sqrt{2})\,t_0 \simeq 1.763\,t_0$

→ Super gaussian Pulses

* Used to model the effects of steep leading and trailing edges on dispersion-induced pulse broadening.
Generally with steeper leading and trailing edges it broadens more rapidly with propag.



* Eq. $F(0,t) = \exp\left[-\frac{1+iC}{2} \cdot \left(\frac{t}{t_0}\right)^{2m} \right]$

m controls the degree of edge sharpness
($m=1 \rightarrow$ gaussian)
The larger m → pulse becomes squared shape

* Differences between gaussian and super gaussian
  → Gaussian pulse maintains its shape during propagation
  → Super gaussian: broadens at a faster rate
  distorts in shape

The enhanced broadening of a super-gaussian pulse can be understood by noting that its spectrum is wider than that of a Gaussian pulse bc of steeper leading and trailing edges

Notes 6

→ GVD Induced Limitations - Dispersion Management

In fiber optics communications we transmit info coded using a sequence of optical pulses. The width is determinated by bit rate B of the system.

Dispersion-induced broadening is an inconvenence bc it can lead to errors.

* Dispersion Compensation
  - → Combine fibers with different characteristics such that the average GVD of the entire link is low.
  - → Is used a periodic dispersion map. If average GVD is zero → dispersion is compensated

→ For a dispersion map of 2 fibers segments:

$$F(L_m,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{F}(0,\omega)\, \exp\left[\frac{i}{2}\,\omega^2\left(\beta_{21}L_1+\beta_{22}L_2\right) - i\omega t\right] d\omega$$

$L_m = L_1 + L_2$

β_{2j} → GVD parameter of fiber segment L_j $(j=1,2)$

→ Condition for dispersion compensation: with: $D_j = -\frac{2\pi c}{\lambda^2}\beta_{2j}$

$$D_1L_1 + D_2L_2 = 0$$

when the condition is satisfied → $A(L_m,t) = A(0,t)$

The pulse recovers its initial width after each map period

Notes 7 — Self-Phase Modulation SPM

Interesting manifestation of the intensity dependence of the refractive index in nonlin. optical media occurs through SPM

Simplification: Effect of GVD on SPM is negligible so $\beta_2 \to 0$ in NLSE

$$j \frac{\partial F(z,t)}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 F(z,t)}{\partial t^2} + \gamma |F(z,t)|^2 F(z,t) = 0$$

$$\downarrow \beta_2 \to 0$$

$$\frac{\partial F(z,t)}{\partial z} = i \gamma |F(z,t)|^2 F(z,t)$$

↳ Nonlinear length: $L_{NL} = 1/\gamma$ — γ effective nonlin. coeff related to $\chi^{(3)}$ nonlinearity

↳ The eq can be solved if:

$$F = V \exp(i\phi_{NL}) \rightarrow \frac{\partial V}{\partial z} = 0$$

→ The amplitude does not change along the fiber of length L

$$\frac{\partial \phi_{NL}}{\partial z} = \gamma V^2$$

$$\downarrow$$

$$F(z,t) = F(0,t) \exp\left[i \phi_{NL}(z,t)\right]$$

Nonlin. phase shift ϕ_{NL} increases with fiber length z ($z=L$) $\longrightarrow$ $\phi_{NL}(z,t) = |F(0,t)|^2 \gamma z$

Maximum: ϕ_{max} at $t=0$

$$\phi_{max} = \gamma L$$

SPM gives rise to an intensity-dependent phase shift but the pulse shape remains unaffected in temporal domain

⊛ Nonlin. length L_{NL} is the effective propagation dist at which $\phi_{max} = 1$

The SPM-induced spectral broadening is a consequence of the time dependence of ϕ_{NL}

↳ Temporally varying phase implies instant. opt. freq. differs across the pulse from central value ω_0 → $\delta\omega$

$$\delta\omega = \pm \frac{\partial \phi_{NL}}{\partial t} = \pm \gamma L \frac{\partial}{\partial t}\left(|F(0,t)|^2\right)$$

↳ Time dependence of $\delta\omega$ is: Frequency chirping
The chirped induced by SPM increases in magnitude with propagated dist.

↳ In other words: New freqs. are generated continously as the pulse propagates in the fiber.
SPM-generated freq. comps. broaden the spectrum over initial width
The broadening depends on the pulse shape

→ Gaussian & Supergaussian diff.

→ Interesting features of the temporal variation of the induced chirp ($\delta\omega$)

1. $\delta\omega$ negative near leading edge (red shift)
   positive near trailing edge (blue shift)
2. Chirp linear and positive over a large central regime of gaussian pulse
3. Chirp larger for pulses with steeper and trailing edges.

Notes 8

→ Changes in pulse spectra due to SPM

* stimation of magnitude of SPM-induced spectral broadening from peak value of $\delta\omega$ (expression of $\delta\omega$ super gaussian time derivative to zero)

$$\delta\omega_{max} = \frac{m\, f(m)}{t_0}\, \phi_{max}$$

$$\phi_{max} = \gamma L$$

$$f(m) = 2\left(1-\frac{1}{2m}\right)^{1-1/2m} \exp\left[-\left(1-\frac{1}{2m}\right)\right]$$

* To obtain the broadening factor we need to relate width parameter t_0 to initial spectral width of the pulse $\Delta\omega_0$

↳ For unchirped gaussian pulse : $\Delta\omega_0 = t_0^{-1}$

⇒ $\delta\omega_{max} = 0{,}86\ \Delta\omega_0\ \phi_{max}$

↳ For supergaussian difficult to stimate $\Delta\omega_0$, but

⇒ if we refer to rise time: $T_r = t_0/m$

⇒ and assume $\Delta\omega_0 \simeq T_r^{-1}$

} approx. given by ϕ_{max}

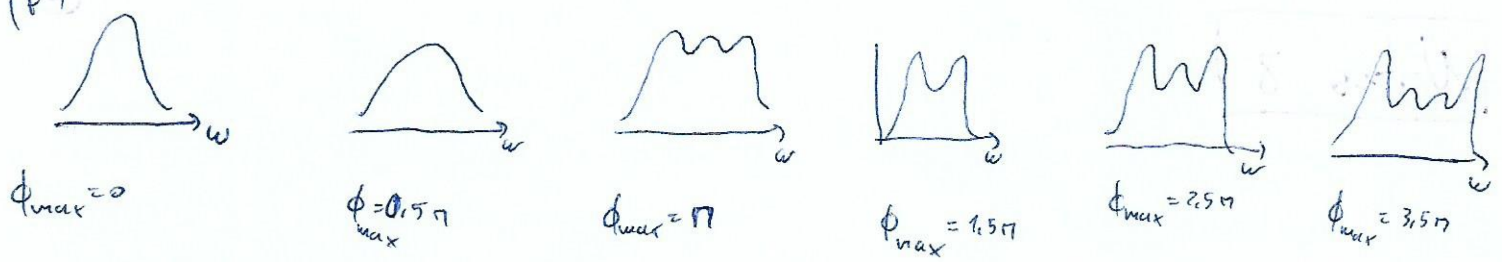
* We obtain pulse spectrum by using Fourier

$$S(\omega) = \left| \int_{-\infty}^{+\infty} F(0,t) \exp\left[i\,\phi_{NL}(L,t) + i\,(\omega-\omega_0)\,t \right] dt \right|$$

→ Spectrum depends on
↳ pulse shape
↳ initial chirp

@ diff. length → diff. spectra of unchirped gaussian pulse
due to pulse has diff phase shift ϕ_{max}

(F1)



→ The SPM - induced spectral broadening is accompanied by oscilatory structure.

→ In general spectrum consist in many peaks with outermost peaks the most intense.

→ The number of peaks depends on ϕ_{max} and increases linearly

Number of peaks (approx): $\phi_{max} \simeq (M - \frac{1}{2})\pi$

* Comparision unchirped gaussian and unchirped supergaussian

Notes 9 — Competition of SPM and GVD

* SPM effects discussed before work for long pulses ($t_0 > 100ps$). When L_D (dispersion length) is much larger than L (fiber length) and L_{NL} (nonlin. length)

$$L_D \gg L \quad \text{and} \quad L_D \gg L_{NL}$$

L_D, L_{NL} provide scales over which dispersive or nonlin. effects become important for pulse evolution.

↳ When $L_D \sim L$ we have to consider the combination of GVD and SPM

→ In anomalous dispersion regime: The combination may lead to a pulse propagation without distorsion

→ In normal disp. regime: The combination can be used for pulse compression

* Starting point: NLSE

$$j\frac{\partial F}{\partial z} - \frac{\beta''}{2}\frac{\partial^2 F}{\partial t^2} + \gamma |F|^2 F = 0$$

NLSE normalised:

$$j\frac{\partial F}{\partial \xi} = \frac{\operatorname{sgn}(\beta'')}{2}\frac{\partial^2 F}{\partial \tau^2} - N^2 |F|^2 F = 0$$

$\xi = z/L_D$

$\tau = t/t_0$

$$N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma t_0^2}{|\beta''|}$$

N parameter rules the importance of GVD or SPM effects on pulse evolution on fiber

- $N \ll 1$ → Dispersion dominates
- $N \gg 1$ → SPM dominates
- $N \sim 1$ → GVD and SPM important

⊛ Example: Evolution of shape and spectrum of unchirped gaussian pulse in normal dispersion

($N=1$; $\beta_2=\gamma=1$, $t_0=1$)

↳ The pulse broadens much more rapidly (compared with $N=0$, no SPM)
This is bc SPM generates new freq. components : red shifted near leading edge & blue shifted near trailing edge

↳ Red components travel faster than blue ones ($\beta_2>0$)
⇒ SPM leads to an enhanced rate of pulse broadening comparing to GVD alone
⇒ This↑ affects spectral broadening : SPM phase shift (ϕ_{NL}) becomes less

⊛ Anomalous dispersion ($N=1$, $\boxed{\beta_2=-1}$, $\gamma=1$, $t_0=1$)

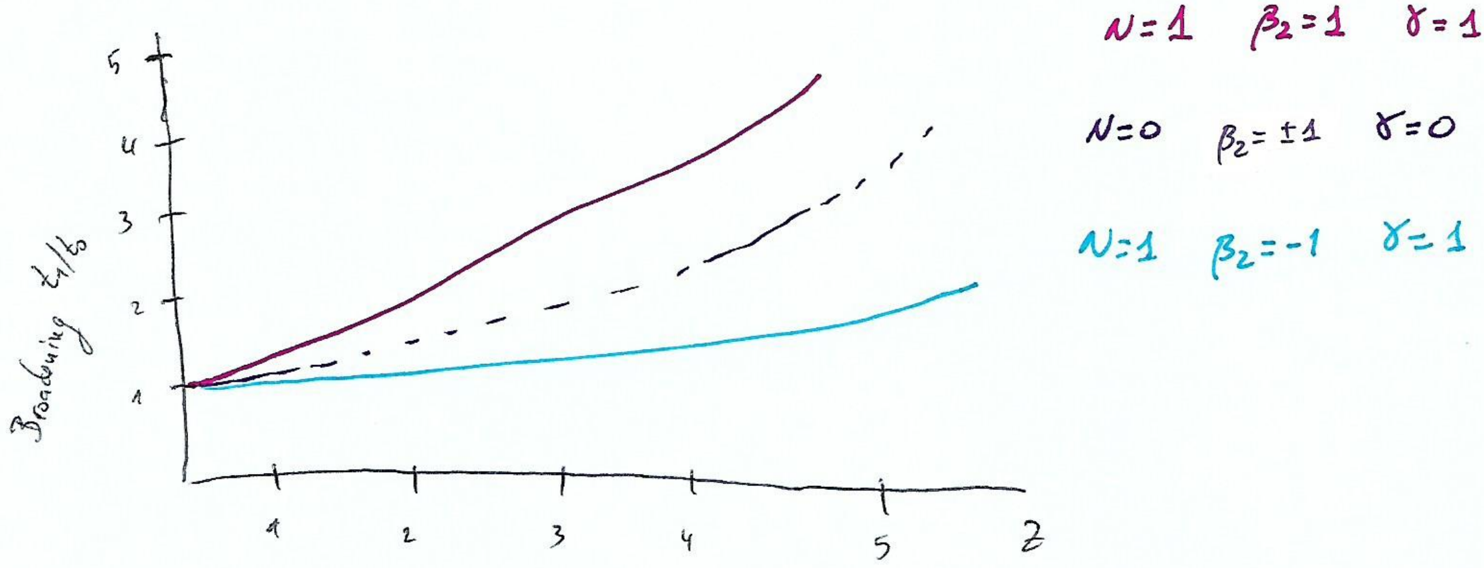
↳ Pulse broadens initially at a much lower rate as expected with no SPM and seems to reach stationary state for $z>4$

↳ Spectrum narrows rather than broadening expected by SPM (in absence of GVD)
It is understood bc $\left.\begin{array}{l}\text{SPM-induced chirp} \to \text{positive} \\ \text{Dispersion-induced chirp} \to \text{negative } (\beta_2<0)\end{array}\right\}$ nearly canceled when $L_D=L_{NL}$ ($N=1$)

→ Conclusion: GVD and SPM compete or colaborate to maintain a chirp-free pulse (solitons)

The main effect of SPM is to alter the temporal broadening rate imposed by GVD alone

* Broadening factor



The SPM enhances the broadening in the normal dispersion regime and decrease it in the anomalous disp. regime

Notes 11 | Optical Solitons

→ Formed as a result of the interplay between the dispersive and nonlinear effects.

Solitons are special kind of wave packet that can propagate undistorted over long distances.

The GVD and the SPM are equally important and must be considered simultaneously.

→ History : 1834 - First time J. Scott Russell Wave of Translation

1965 - Soliton term coined

1973 - Use of solitons for optical communications

1999 - Exploited in real systems

⊛ Starting point: NLSE $i\dfrac{\partial F}{\partial z} - \dfrac{\beta_2}{2}\dfrac{\partial^2 F}{\partial t^2} + \gamma |F|^2 F = 0$

↳ anomalous dispersion regime: $\beta_2 = -1$, $\gamma = 1$

$$i\frac{\partial F}{\partial z} + \frac{1}{2}\frac{\partial^2 F}{\partial t^2} + |F|^2 F = 0$$

⊛ Fundamental soliton:

$$F(z,t) = \eta \operatorname{sech}(\eta t) \exp\left(i\eta^2 z/2\right)$$

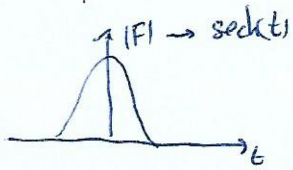
$\eta \rightarrow$ determinates the amplitude
the temporal width
(also the phase ??)

The canonical form: $\eta = 1 \rightarrow \quad F(z,t) = \text{sech}(t)\exp(i\,z/2)$

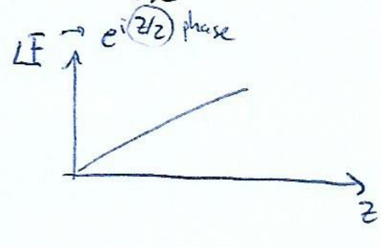
It is a solution of NLSE

→ Study numerically the dynamics of soliton in:

Amplitude



Phase



→ Higher-Order Solitons

* Special role is played by solitons whose: $F(0,t) = N\,\text{sech}(t)$
(a subset of possible solitons)

$N \rightarrow$ integer, soliton order

* Second order soliton ($N=2$)

$$F(z,t) = \frac{4\left[\cosh(3t) + 3\exp(4iz)\cosh(t)\right]\exp(iz/2)}{\left[\cos(4t) + 4\cosh(2t) + 3\cos(4z)\right]}$$

→ $|F(z,t)|^{(2)?}$ is periodic in z with the period $z_0 = \pi/2$ $\left(z_0 = \frac{\pi}{2} L_D\right)$

High-order solitons behave periodically (when $N \geq 2$)

→ It first contracts a fraction of initial width, splits and broadens recovering the original shape at the end of period $z = \frac{\pi}{2}$

→ Spectral evolution: Broadening, the two peaks and then a spectral narrowing untill original spectral shape

↘

* For fundamental soliton ($N=1$)

GVD and SPM balance each other so that neither the pulse shape nor the pulse spectrum changes along the fiber length.

* Summary:

→ High-order solitons: SPM dominates initially
GVD catches up soon and leads too pulse contractio

→ Pulses with hyperbolic-secant shape GVD & SPM cooperate to ~~get~~ have an evolution shape that repeats the original shape in a period of $z = \pi/2$

→ Dark Solitons

* solution of NLSE with $\beta_2 = 1$, in normal GVD regime
  The intensity profile exhibits a dip in an uniform background.
  Previous solitons are called → bright

* NLSE:

$$i \frac{\partial F}{\partial z} - \frac{1}{2} \frac{\partial^2 F}{\partial t^2} + |F|^2 F = 0 \qquad \beta_2 = +1 \qquad \gamma = 1$$

* The solution:

$$|F(z,t)| = \underbrace{\eta}_{\text{soliton amplitude}} \left[1 - B^2 \operatorname{sech}^2 [\eta B (t - \underbrace{t_s}_{\text{can be set at } t_s = 0})] \right]^{1/2}$$

Main diff. with bright is that the solution becomes constant when $|t| \to \infty$ (instead of zero like brights)

phase

$$\phi(z,t) = \frac{1}{2} \eta^2 (3 - B^2) z + \eta \sqrt{1 - B^2}\, t + \tan^{-1} \left[\frac{B \tanh(\eta B t)}{\sqrt{1 - B^2}} \right]$$

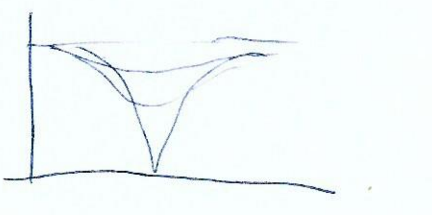
→ η: amplitude of soliton

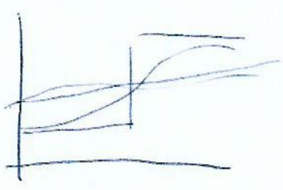
→ t_s: dip location

→ B: $|B| \leq 1$ governs the depth of the dip $\qquad |B| < 1$ gray soliton

$|B| = 1$ dark soliton

For a given value of η $|F(z,t)|$ represents a family of dark solitons





→ Phase of dark solitons change
i.e. dark solitons are chirped

* Dark Soliton in canonical form

$\eta = 1 \quad B = 1$

$$F(z,t) = \tanh(t)\exp(iz)$$

→ The phase jump of π at $t=0$ is included in the amplitude $\tanh(t)$

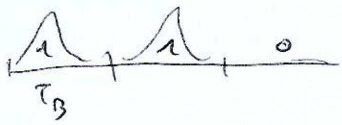
→ This dark soliton would propagate unchanged in normal dispersion region

Notes 12

→ Soliton interaction

* The bit rate is determined by the interval T_B between two pulses (bits)

$B = 1/T_B$



We would like to know how close two solitons can come without affecting each other.

* Total envelope: $F = F_1 + F_2$

$$F_j(z,t) = \eta_j \operatorname{sech}\left[\eta_j (t - q_j)\right] \exp\left(i\phi_j - i\delta_j t\right) \qquad j = 1,2$$

Substituting $F = F_1 + F_2$ in NLSE we get 2 eqs. for F_1 & F_2

$$i\frac{\partial F_1}{\partial z} + \frac{1}{2}\frac{\partial^2 F_1}{\partial t^2} + |F_1|^2 F_1 = -2|F_1|^2 F_2 - F_1^2 F_2^*$$

$$i\frac{\partial F_2}{\partial z} + \frac{1}{2}\frac{\partial^2 F_2}{\partial t^2} + |F_2|^2 F_2 = -2|F_2|^2 F_1 - F_2^2 F_1^*$$

Two NLSE and terms in right-hand act as perturbations responsible of nonlin interactions between two solitons.

* Numerical exploration:

$$F(0,t) = \operatorname{sech}(t - q_0) + r\operatorname{sech}\left[r(t + q_0)\right]\exp(i\theta)$$

r → relative amplitude

$\theta = 2\psi_0$ → initial phase diff.

$2q_0$ → initial separation between two solitons

Case: $r = 1$ & $\theta = 0$ → Solitons attract each other and collide periodically along fiber

$r = 1$ & $\theta = \pi/2$ → Solitons repeal each other, spacing increases monotonically

Case: $s = 1,1$ & $\theta = 0$ $\Rightarrow$ Two in-phase solitons oscillate periodically
never collide or move far away
from each other

Notes 13

Extreme Wave Events