2. Common integrated transmission lines

Integrating transmission lines on chip integrated circuits is relying on planar structures, derived from coaxial cables.
Two examples are shown below, with transformation from a conventional coaxial cable section and a microstrip and coplanar waveguide line.

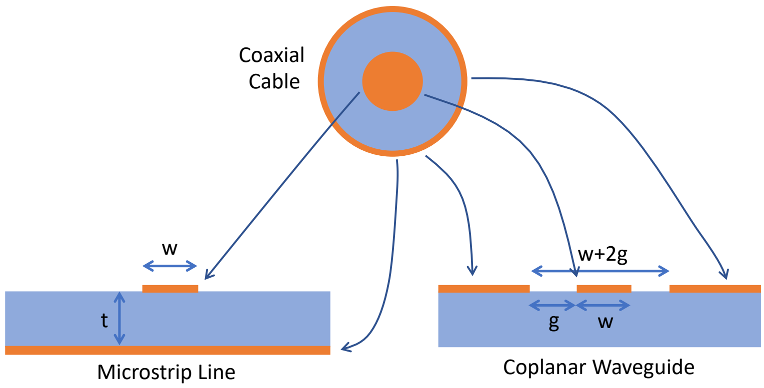


Figure 12 – Microstrip line and coplanar waveguide transmission line compared to a coaxial cable

The microstrip transmission line is shown above, along with coplanar waveguide. These guides are "quasi" TEM transmission lines, i.e. the electric field is divided between air above the substrate and the dielectric slab where the circuit is deposited.

2.1. Microstrip transmission lines

On a microstrip transmission line, the electromagnetic fields are divided between air and the substrate, and the quasi-TEM wave velocity can be derived from the concept of effective permittivity, taking into account the division between air and the substrate.
On a first approximation, the effective permittivity can be approximated by the following formula:

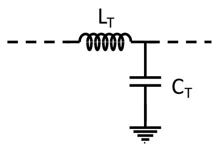
$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2}$$

And the wave velocity is defined as:

$$v_p = \frac{c}{\sqrt{\varepsilon_{eff}}}$$

Where *c* is the speed of light in vacuum.

The line characteristics can be approximated using the following formulas, and the L-C ideal line:



$$v_p = \frac{1}{\sqrt{L_T C_T}}$$

Where C_T and L_T are the inductance and capacitance per unit length.

It is possible to derive the line characteristics using quasi static equations. C_T can be readily computed from a parallel-plate approximation:

$$C_T = \frac{\varepsilon_0 \varepsilon_r w}{t}$$

Where w is the width of the microstrip line, and t and ε_r the thickness and relative permittivity of the substrate.
The inductance per unit length can be computed using the fact that the wave velocity is

$$v_p = \frac{c}{\sqrt{\varepsilon_{eff}}}$$

We can deduce:

$$L_T = \frac{\varepsilon_{eff}}{c^2 C_T}$$

$$L_T = \frac{\varepsilon_{eff} t}{c^2 \varepsilon_0 \varepsilon_r w}$$

2.2. Limitations of microstrip transmission lines

The microstrip transmission lines are very useful in many practical applications. However, they have limitations due to the fact that they can support a dielectric slab mode, propagating inside the substrate. This parasitic propagation causes leakage of the energy from the main microstrip mode into the substrate mode.

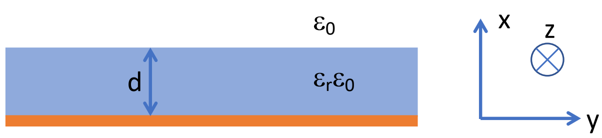


Figure 13- Cross section of a microstrip substrate

We will derive the TM_0 mode EM fields, assuming that the propagation is along the *z* axis.

$$E_z \rightarrow e_z(x,y)e^{-j\beta z}$$

Inside the dielectric $0 < x < d$:

$$\left(\frac{\partial^2}{\partial x^2}+\varepsilon_r k_0^2-\beta^2\right)e_z(x,y)=0$$

In air $d < x < \infty$:

$$\left(\frac{\partial^2}{\partial x^2}+k_0^2-\beta^2\right)e_z(x,y)=0$$

k_0 is the wavenumber and β is the propagation constant.

$$k_c^2=\varepsilon_r k_0^2-\beta^2$$
$$h^2=\beta^2-k_0^2$$

h is the propagation constant in air, and note that $k^2-\beta^2$is negative in this media.
Eliminating β in these two equations gives:

$$k_c^2=\varepsilon_r k_0^2-h^2-k_0^2$$

$$k_c^2+h^2=(\varepsilon_r-1)k_0^2 \qquad (1)$$

This is the first equation that we will us to determine the propagation constant of the TM mode.
The general solutions of the propagation equation in air and in the dielectric are:

$$e_z(x,y)=Asin(k_c x)+Bcos(k_c x) \text{ for } 0 < x < d$$

$$e_z(x,y)=Ce^{hx}+De^{-hx} \text{ for } d < x < \infty$$

k_c , h are real but the solutions are also valid if k_c and h are complex.

In order to solve the equations for E_z, we apply the following boundary conditions:

(a) $E_Z(x,y,z)=0$ at $x=0$
(b) $E_Z(x,y,z)=0$ at $x=\infty$
(c) $E_Z(x,y,z)$ continuous at $x=d$
(d) $H_Y(x,y,z)$ continuous at $x=d$

From (a) and (b), we can deduce that $B=0$ and $C=0$. E_z can be written as:

$$e_z(x,y)=Asin(k_c x) \text{ in the dielectric and } e_z(x,y)=De^{-hx} \text{ in air}$$

$$h_Y(x,y,z)=-\frac{j\omega\varepsilon_0\varepsilon_r}{k_c^2}.\frac{\partial e_Z(x,y,z)}{\partial x}=-\frac{j\omega\varepsilon_0\varepsilon_r}{k_c}Acos(k_c x) \text{ in the dielectric}$$
$$\text{and } h_Y(x,y,z)=\frac{j\omega\varepsilon_0}{k_c^2}.\frac{D}{h}e^{-hx} \text{ in Air.}$$

For the tangential continuities, we match H_Y and E_Z at the dielectric/air interface. The continuity of e_z leads to:

$$Asink_c d=De^{-hd}$$

For H_y we obtain:

$$\frac{\varepsilon_r A}{k_c} cos k_c d = \frac{D}{h} e^{-hd}$$

Taking the ratio of the two equations leads to:

$$k_c tan k_c d = \varepsilon_r h \qquad (2)$$

By combining this equation with equation (1), we have to solve this set of two transcendental equations:

$$k_c^2 + h^2 = (\varepsilon_r - 1) k_0^2$$

And

$$k_c tan(k_c d) = \varepsilon_r h$$

These two equations can be expressed in terms of hd:

$$(k_c d)^2 + (hd)^2 = (\varepsilon_r - 1)(k_0 d)^2$$

$$k_c d tan(k_c d) = \varepsilon_r hd$$

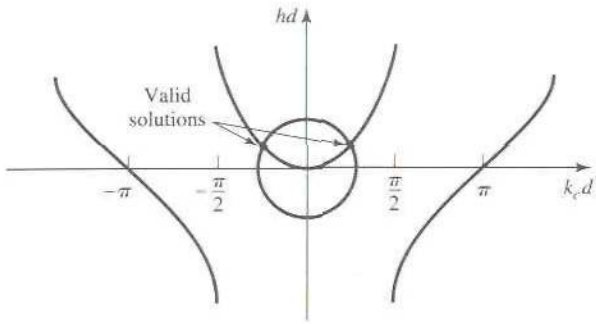


Figure 14- Plot of the two transcendental equations (D.M. Pozar, Microwave Engineering 2nd Ed. pp. 166-167)

On the plot above, the first equation is a circle, and the second equation is the periodic tangent plot. The intersections correspond to the solutions of the transcendental equations, for hd and $k_c d$.

One should note that the TM_0 mode has no frequency cutoff, and that it is present jointly with the main microstrip mode. The solutions for the transcendental equations are shown in the figure below:

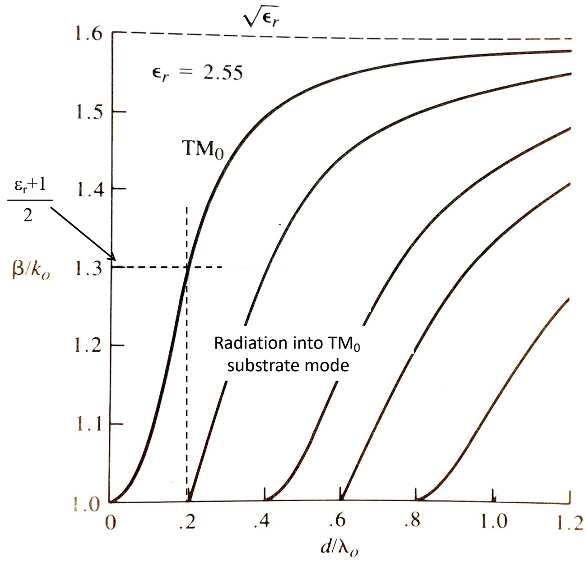


Figure 15- Plot of dispersion diagram for a grounded dielectric slab (D.M. Pozar, Microwave Engineering 2nd Ed. pp. 166-167)

β/k_0 is equivalent to the effective index of the mode, and when it becomes higher than the microstrip effective index, the energy leaks from the microstrip mode into the TM_0 substrate. This phenomenon is similar Snell/Descartes law, with refraction of the microstrip mode into the higher index TM_0 mode.

On the graph above, β/k_0 for the TM_0 mode becomes higher than the effective permittivity of the microstrip mode when the substrate thickness is more than $0.2\lambda_0$.