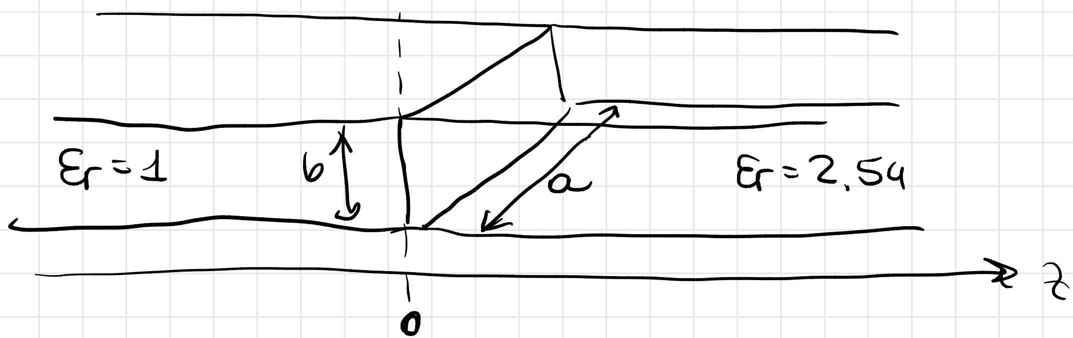
# MICROWAVE ENGINEERING

Lecture 21:

Microwave Networks

Problems

1. Consider a rectangular waveguide in the X-band ($a = 2.286$ cm and $b = 1.016$ cm) filled with air for $z<0$ and Rexolite ($\varepsilon_r = 2.54$) for $z>0$. If the operating frequency is $f = 10$ GHz calculate, using an equivalent TL model, the reflection coefficient of a $TE_{10}$ wave at the interface.

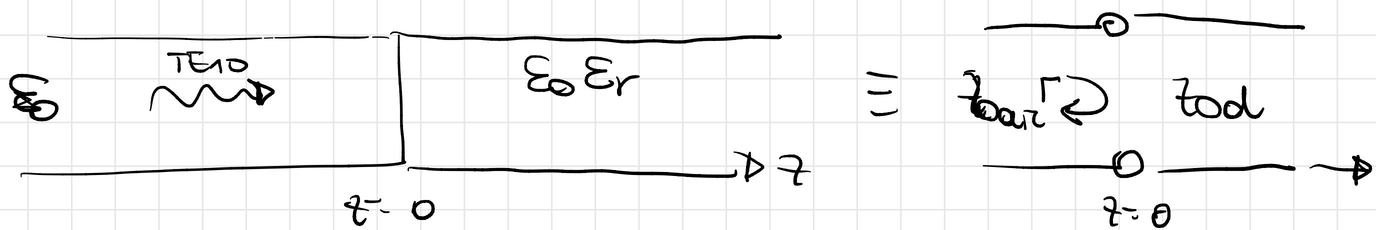



Calculate the propagation constant in the two regions for TE 10 wave:

$$\beta_{air} = \sqrt{\underbrace{k_0}_{\uparrow}{}^2 - \left(\frac{\pi}{a}\right)^2} = \sqrt{\left(\frac{2\pi f}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2} = 158\ m^{-1}$$

$$k_0 = \frac{2\pi f}{c} = \frac{2\pi \cdot 10 \cdot 10^9}{3 \cdot 10^8} = 209.4\ m^{-1}$$

$$\beta_d = \sqrt{\underbrace{\varepsilon_r}_{2.54}\, k_0^2 - \left(\frac{\pi}{a}\right)^2} = 304.1\ m^{-1}$$



For a rectangular waveguide $\quad Z_{TE} = \dfrac{k\eta}{\beta}$

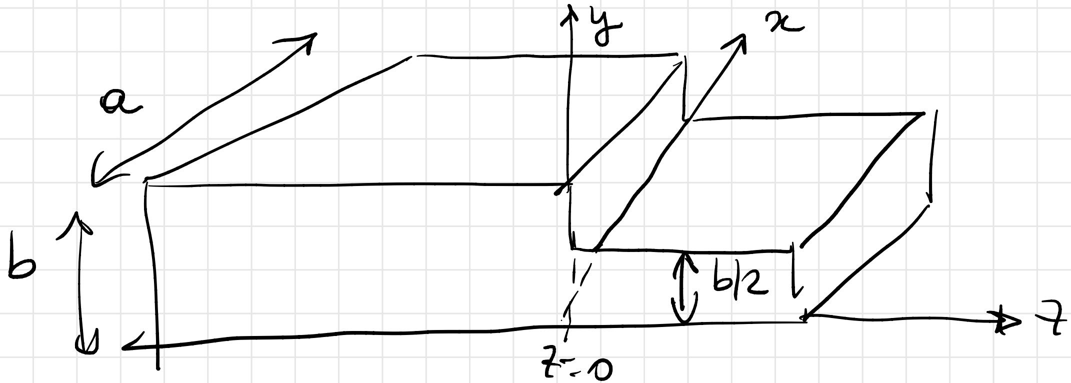
$$Z_{0\,air} = \frac{k_0\eta_0}{\beta_{air}} = \frac{(209.4)(377)}{158} = 500\ \Omega$$

$$Z_{0d} = \frac{\overset{k_0\sqrt{\varepsilon_r}}{k}\ \overset{\eta_0/\sqrt{\varepsilon_r}}{\eta}}{\beta_d} \quad \frac{k_0\eta_0}{\beta_d} = \frac{(209.4)(377)}{304.1} = 259.6\ \Omega$$

The reflection coefficient:

$$\Gamma = \frac{Z_{0d} - Z_{0\,air}}{Z_{0d} + Z_{0\,air}} = -0.316$$

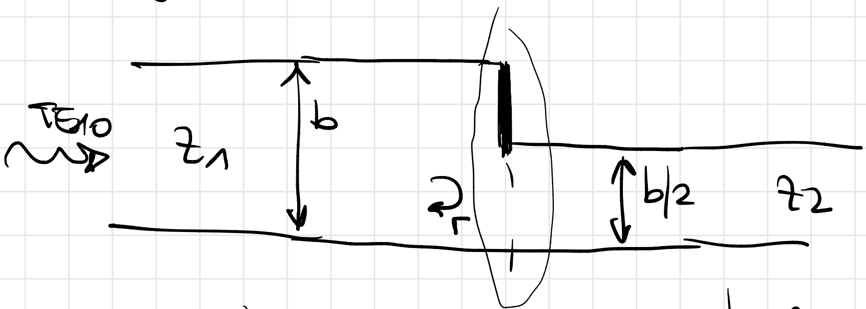
③ Where the TL analogy fails!




Consider the reflection of a $TE_{10}$ mode incident from $z<0$ at a step change in height of a rectangular waveguide.

What $\Gamma$ do we get by using the TL approach? Is that correct?

Using the TL analogy




$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$Z_1 = \frac{k_0 \eta_0}{\beta_1} \qquad Z_2 = \frac{k_0 \eta_0}{\beta_2}$$

$$\beta_1 = \beta_2 = \sqrt{k_0^2 - \left(\frac{\pi}{a}\right)^2}$$

for TE10

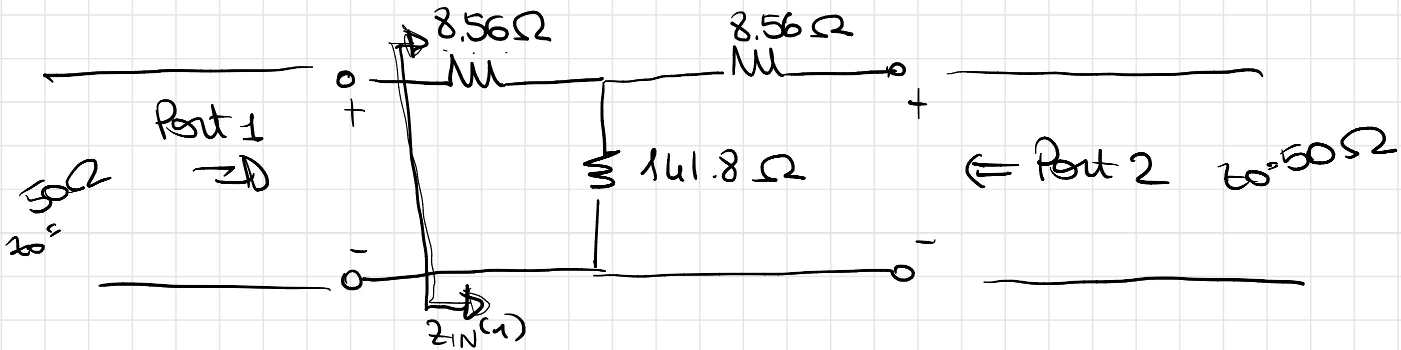
It's easy to conclude that $\Gamma = 0$.

That's not correct!

$\Downarrow$

We cannot consider that at the interface only TE10 is reflected!

(3) Find the S parameters of the 3dB attenuator:




The S matrix for a 2 port system is:

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{ij} = \frac{V_i^-}{V_j^+} \Bigg|_{\substack{V_k^+ = 0 \\ k \neq j}}$$

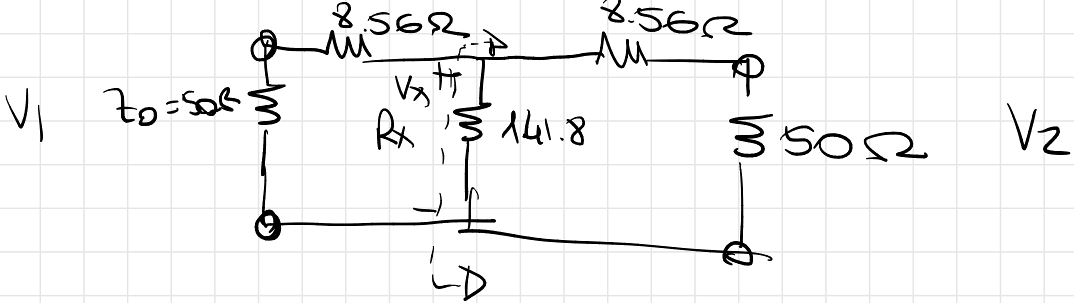
$$S_{11} = \frac{V_1^-}{V_1^+}\Bigg|_{V_2^+=0} = \Gamma^{(1)}\Big|_{V_2^+=0} = \frac{Z_{IN}^{(1)} - Z_0}{Z_{IN}^{(1)} + Z_0} \qquad Z_0 = 50\,\Omega$$

$$Z_{iN}^{(1)} = 8.56 + \frac{141.8\,(8.56+50)}{141.8 + 8.56 + 50} = 50\,\Omega$$

$$S_{11} = \Gamma^{(1)} = 0$$

$$S_{22} = \Gamma^{(2)} = 0 \qquad \text{for symmetry}$$

$$S_{21} = S_{12} = \frac{V_2^-}{V_1^+}\Bigg|_{V_2^+=0}$$




$$V_2 = \frac{50}{50+8.56} V_x$$

$$V_x = \frac{R_x}{R_x+8.56} V_1$$

$$R_x = \frac{141.8\,(8.56+50)}{141.8+8.56+50} = 41.44\,\Omega$$

$$V_2 = \left(\frac{50}{50+8.56} \cdot \frac{41.44}{41.44+8.56}\right) V_1 \Rightarrow S_{21} = \frac{V_2^-}{V_1^+} = 0.707 = S_{12}$$

We can verify this is a 3dB attenuator:

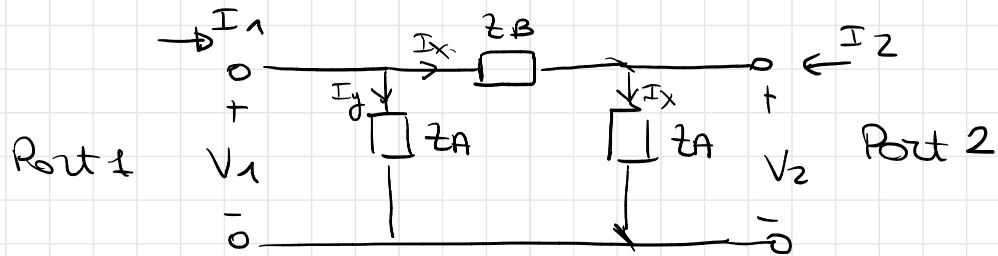
If my input power is $\frac{|V_1^+|^2}{2Z_0}$ then the output power is:

$$\frac{|V_2^-|^2}{2Z_0} = \frac{|S_{21}V_1^+|^2}{2Z_0} = \frac{\overset{=0.707}{|S_{21}|}^2}{2Z_0}|V_1^+|^2 = \frac{|V_1^+|}{4Z_0}$$

$\downarrow$

is $\frac{1}{2}$ of the input power!
(−3dB)

4. Derive the impedance matrix for the following 2-port network




The impedance matrix $[V] = [Z][I]$ $\qquad Z_{ij} = \left.\dfrac{V_i}{I_j}\right|_{\substack{I_k=0 \\ k\neq j}}$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{\cancel{V_1}}{\dfrac{\cancel{V_1}}{\left[\dfrac{Z_A(Z_A+Z_B)}{2Z_A+Z_B}\right]}} = \boxed{\frac{Z_A(Z_A+Z_B)}{2Z_A+Z_B}} = Z_{22} \text{ for symmetry}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$V_2 = z_A I_x$$

$$V_1 = I_x\left(z_B + z_A\right) = I_y z_A$$

$$I_x = \frac{z_A}{z_A + z_B} I_y$$

$$\boxed{I_x + I_y = I_1}$$

$$I_1 = I_y\left(1 + \frac{z_A}{z_B + z_A}\right)$$

$$z_{21} = \frac{V_2}{I_1} = \frac{z_A\left[\frac{z_A}{z_A + z_B}\right]\cancel{I_y}}{\cancel{I_y}\left(1 + \frac{z_A}{z_A + z_B}\right)} = \boxed{\frac{z_A^2}{2z_A + z_B}} = z_{12}$$

(5) A four-port network has the following S matrix :

$$[S] = \begin{bmatrix} 0.1\angle 90^\circ & 0.8\angle -45^\circ & 0.3\angle -45^\circ & 0 \\ 0.8\angle -45^\circ & 0 & 0 & 0.4\angle 45^\circ \\ 0.3\angle -45^\circ & 0 & 0 & 0.6\angle -45^\circ \\ 0 & 0.4\angle 45^\circ & 0.6\angle -45^\circ & 0 \end{bmatrix}$$

a) Is this network lossless?

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \; ?$$

$$|0.1|^2 + |0.8|^2 + |0.3|^2 = 0.74 \neq 1$$

NO Network is LOSSY!

b) Is the network reciprocal? Yes, S matrix is symmetric

c) What is the Return loss at port 1 if all other ports are matched?

$\Gamma = S_{11} \Rightarrow \underline{RL} = -20 \log|\Gamma| = -20 \log(0.1) = 20\ dB$

d) What is the insertion loss between ports 2 and 4 when all other ports are terminated with matched loads?

$\underline{IL} = -20 \log|S_{42}| = -20 \log(0.4) = 8\ dB$

e) What is the reflection coef. at port 1 if a short circuit is placed at the terminal plane of port 3 and all other ports are matched.

If we place a sc at port 3 and watch other ports

$$V_2^+ = V_4^+ = 0 \qquad V_3^+ = -V_3^-$$

$$\downarrow$$

$$[S] = \begin{bmatrix} 0.1 \angle 90^\circ & 0 & 0.3 \angle -45^\circ & 0 \\ 0 & 0 & 0 & 0 \\ 0.3 \angle -45^\circ & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Gamma^{(1)} = \frac{V_1^-}{V_1^+}$$

$$V_1^- = S_{11} V_1^+ + S_{13} V_3^+ =$$

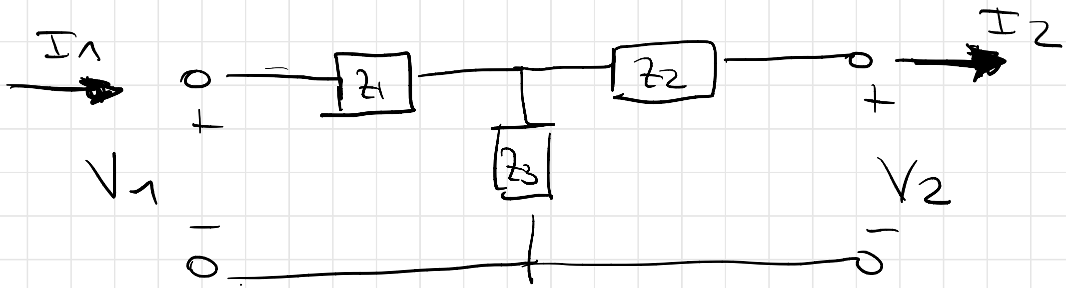
$$= S_{11} V_1^+ - \underline{S_{13} V_3^-}$$

$$V_3^- = S_{31} V_1^+$$

$$V_1^- = S_{11} V_1^+ - S_{13} S_{31} V_1^+$$

$$\Gamma^{(1)} = \frac{(S_{11} - S_{13} S_{31}) \cancel{V_1^+}}{\cancel{V_1^+}}$$

(7) Calculate the ABCD entries for the following circuit:




$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{\cancel{I_1}(Z_1+Z_3)}{\cancel{I_1} Z_3} = 1 + \frac{Z_1}{Z_3}$$

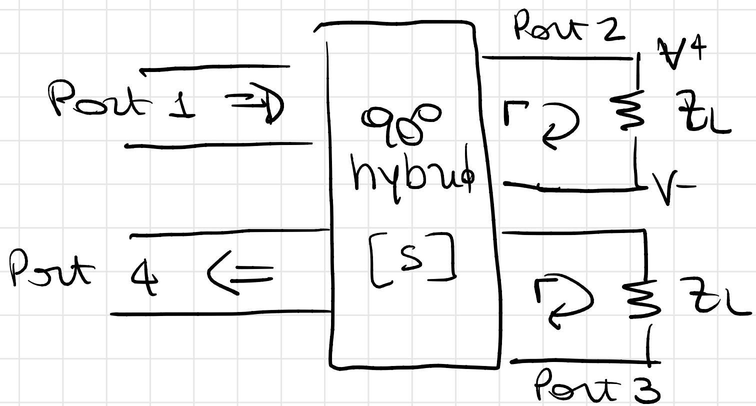
$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{I_1\left(Z_1 + \dfrac{Z_2 Z_3}{Z_2+Z_3}\right)}{I_1\,\dfrac{Z_3}{Z_2+Z_3}} =$$

$$= \frac{Z_2 Z_1 + Z_3 Z_1 + Z_2 Z_3}{Z_3} = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{\dfrac{\cancel{V_1}}{\cancel{Z_1+Z_3}}}{\cancel{V_1}\,\dfrac{Z_3}{\cancel{Z_1+Z_3}}} = \frac{1}{Z_3}$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{\cancel{I_1}}{\cancel{I_1}\,\dfrac{Z_3}{Z_2+Z_3}} = \frac{Z_2+Z_3}{Z_3} = 1+\frac{Z_2}{Z_3}$$

(8) A variable attenuator can be implemented with equal but adjustable loads. Using the S matrix for the coupler, show that the transmission coefficient between port 1 and port 4 is $j\Gamma$, with $\Gamma$ reflection coefficient of the mismatch at ports 2 and 3.




$$[S] = -\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

Transmission from port 1 to port 4 is $T = \dfrac{V_4^-}{V_1^+}$

$$T = \frac{V_4^-}{V_1^+} = \frac{1}{V_1^+}\left(-\frac{1}{\sqrt{2}}\right)\left(V_2^+ + jV_3^+\right)$$

$$V_2^+ = \Gamma V_2^-$$

$$V_3^+ = \Gamma V_3^- \Rightarrow T = \frac{1}{V_1^+}\left(-\frac{1}{\sqrt{2}}\right)\left[\Gamma V_2^- + j\Gamma V_3^-\right]$$

$$V_2^- = -\frac{1}{\sqrt{2}}\, jV_1^+$$

$$V_3^- = -\frac{1}{\sqrt{2}}\, V_1^+$$

$$T = \frac{1}{\cancel{V_1^+}}\left(-\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right)\left[\Gamma j \cancel{V_1^+} + j\Gamma \cancel{V_1^+}\right] = \frac{1}{\cancel{2}}\cancel{2}j\Gamma = j\Gamma$$