III Population inversion

III.1 Two-level laser system

At thermal equilibrium, $\frac{N_2}{N_1} = exp\left(-\frac{E_2 - E_1}{kT}\right)$

At T = 300 K, kT = 4.14×10^{-21} J = 0.026 eV

At λ = 1 µm, wavelength corresponding to the lasing transition, $\Delta E = hc/\lambda$ = 1.24 eV >> kT

$$\frac{N_2}{N_1} \rightarrow 0^+$$

There are no atoms in the upper level.

$T \rightarrow \infty$, $1/kT \rightarrow 0$,

$$\frac{N_2}{N_1} = 1$$

N_2 never exceeds N_1. So, absorption dominates.

In a two-level system, the probability of a photon causing stimulated emission is the same as that of a photon being absorbed. **There is no net gain.**

If $N_2 = N_1$ the material is transparent. In general $N_1 > N_2$ then absorption dominates.

III.2 Three-level laser system

An extra level is required to ensure that $N_2 > N_1$.

Consider the same group of N atoms but able to exist in either state (1), or state (2) or state (3). Here $E_3 > E_2 > E_1$. Populations are N_3, N_2, N_1 such that, at all times, $N = N_3 + N_2 + N_1$.

The atoms in level (1) are now subject to light at frequency ν_{13}. The process of absorption will populate level (3). This process is called **pumping**. However, level (3) is not the excited state for the laser transition. So, the excited atoms must de-excite quickly to level (2). The energy released in this process may be emitted in the form of a photon via spontaneous emission (radiative transition) or in the form of a lattice vibration (a phonon) (non-radiative transition in terms of light) when the energy gap is small. At this point, atoms in level (2) can either decay spontaneously to level (1) or via stimulated emission if photons with frequency ν_{12} pass through the medium in due time. It the lifetime of level (2) $\tau_{21} = 1/A_{21}$, where A_{21} is Einstein's coefficient A, is much longer than the lifetime of the non-radiative transition $3 \rightarrow 2$, population of level (3) can be considered as essentially zero, while population of level

(2) will be high. If $N_2 > N/2$, then $N_2 > N_1$, net gain can be obtained. This condition is called **population inversion**.

In such a 3-level system, the lower level of the laser transition is the ground level, which is naturally highly populated. Since more than 50% of the atoms must be in the upper state, very strong pumping rate is needed in 3-level laser systems. Er^{3+}, emitting at 1.55 µm when pumped at 0.98 µm, is an example of such a 3-level laser system.

III.3 Four-level laser system

The energy level diagram of a 4-level laser system, Nd^{3+} pumped at 808 m and emitting at 1064 nm, is shown in Fig. 6. The pumping transition excites atoms in the ground level to the pump level. From level 4, atoms decay rapidly to level 3. Since the lifetime of the laser transition is much longer than that of level 4, a population accumulates in level 3. These atoms may relax to level 2, the lower level of the laser transition, via spontaneous or stimulated emission of photons. If the lower level of the laser transition has a short lifetime, much shorter than that of the upper level, the population N_2 is negligible. $N_2 = N_4 = 0$. Then, as long as $N_3 > 0$, population inversion is achieved. This is possible because the lower level of the laser transition is an excited state from which atoms quickly relax to the ground state in a non-radiative (fast) fashion.

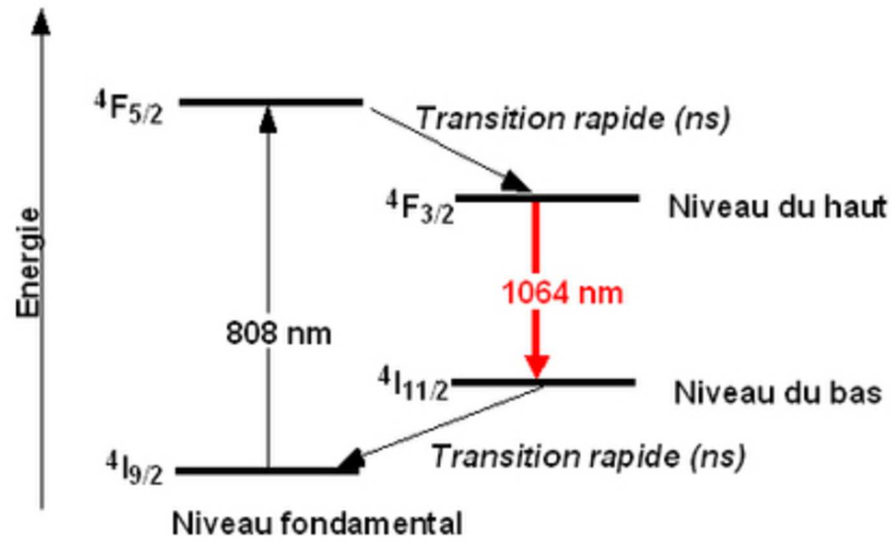


Fig. 6 Energy level diagram of Nd^{3+} acting as a 4-level laser system.

IV Rate equations

<u>*Applying a signal*</u> to a collection of atoms with the frequency ω tuned near one of the atomic transition frequencies ω_a, will cause the population $N_1(t)$ and $N_2(t)$ in the collection of atoms to begin changing slowly because of *stimulated transitions between the two levels E_1 and E_2*. The rate of changes of the populations are given by the atomic rate equations, which contain both stimulated terms (absorption and emission) or relaxation (or energy-decay) terms. These atomic rate equations are of great value in analyzing pumping and population inversion in laser systems. Solution of the rate equations for strong applied signals also lead to population saturation effects, which are of very great importance in understanding the large-signal saturation behavior of laser amplifiers and the power output of laser oscillators.

IV.1. Spontaneous energy decay or relaxation

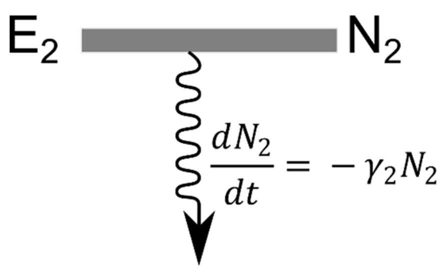
Suppose that a certain number N_2 of atoms have been pumped into some upper level E_2. These atoms will then spontaneously drop down or relax to lower energy levels, giving up their excess internal energy in the process. The rate at which atoms spontaneously decay or relax downward from any upper level E_2 is given by a *spontaneous energy-decay rate*, often called γ_2, times the instantaneous number of atoms in the level, or:

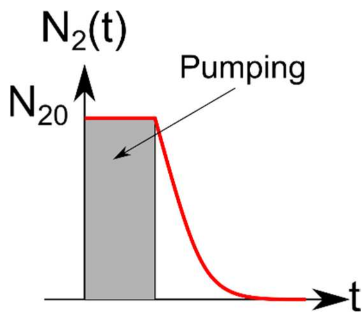
$$\frac{dN_2}{dt} = -\gamma_2 N_2(t) = -\frac{N_2(t)}{\tau_2} \qquad \text{(IV.1)}$$

If an initial number of atoms N_{20} are pumped into the level at t = 0 by a short intense pumping pulse, and the pumping process is turned off, the number of atoms in the upper level will decay exponentially in the form:

$$N_2(t) = N_{20} e^{-\gamma_2 t} = N_{20} e^{-\frac{t}{\tau_2}} \qquad \text{(IV.2)}$$

where $\tau_2 = 1/\gamma_2$ is the lifetime of the upper level E_2 for energy decay to *all lower levels*.





Spontaneous energy decay when pumping is turned off.

IV.2. Stimulated transition rates

Suppose now an optical signal applied to these atoms to cause stimulated transitions. The strength of the signal is characterized by its intensity I (in W per m^2), or by the magnitude of its E field (or H field). It can also be characterized by the number of photons n(t) per unit volume in the applied signal. n(t) is the electromagnetic energy density of the applied signal divided by the quantum energy unit hν.

Such a signal will cause upward transitions at a rate proportional to the applied signal intensity times the number of atoms in the starting level. The population of the upper level will then change according to:

$$\frac{dN_2}{dt} = Kn(t)N_1(t) \qquad \text{(IV.3)}$$

The same applied signal will also cause any atoms initially in the upper level to begin making downward transition at a rate which is proportionnal to the population of the upper level times the number of photons:

$$\frac{dN_2}{dt} = -Kn(t)N_2(t) \qquad \text{(IV.4)}$$

The constant K measures the strength of the stimulated response on that particular atomic transition.

So, in total, the rate equation for the atomic population is given by:

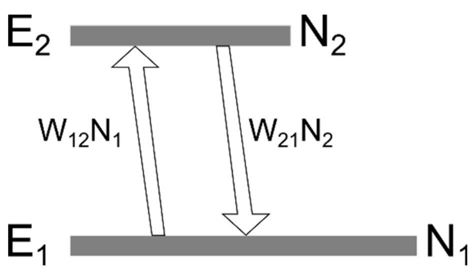
$$\frac{dN_2}{dt} = Kn(t)N_1(t) - Kn(t)N_2(t) - \gamma_2 N_2(t) \qquad \text{(IV.5)}$$

where n(t) is directly proportional to the applied signal intensity or power density.

Neglecting spontaneous emission (IV.5) becomes:

$$\frac{dN_2}{dt} = -\frac{dN_1}{dt} = W_{12}N_1 - W_{21}N_2 \qquad \text{(IV.6)}$$

The quantities W_{12} and W_{21} in units of [s^{-1}] are referred to as the *upward* and *downward stimulated-transition probabilities* per atom per unit time, produced by the applied signal acting on the lower-level and upper-level atoms, respectively.



Upward and downward stimulated transitions between two energy levels.

with

$$W_{12} = W_{21} = \frac{3}{4\pi}\frac{\lambda^3}{\tau h\Delta\omega}\frac{\varepsilon|\tilde{E}|^2}{1+\frac{4(\omega-\omega_a)^2}{\Delta\omega^2}} \qquad \text{(IV.7)}$$

Here, we can say that ***the applied signal gives*** *each atom in the lower level E_1 a probability W_{12} per unit time of making a stimulated transition to the upper level, absorbing a quantum of energy in the process. Similarly, the applied signal gives each atom in the upper level an equal probability W_{21} per unit time of making a transition downward to the lower level, giving up one quantum of energy in the process.*