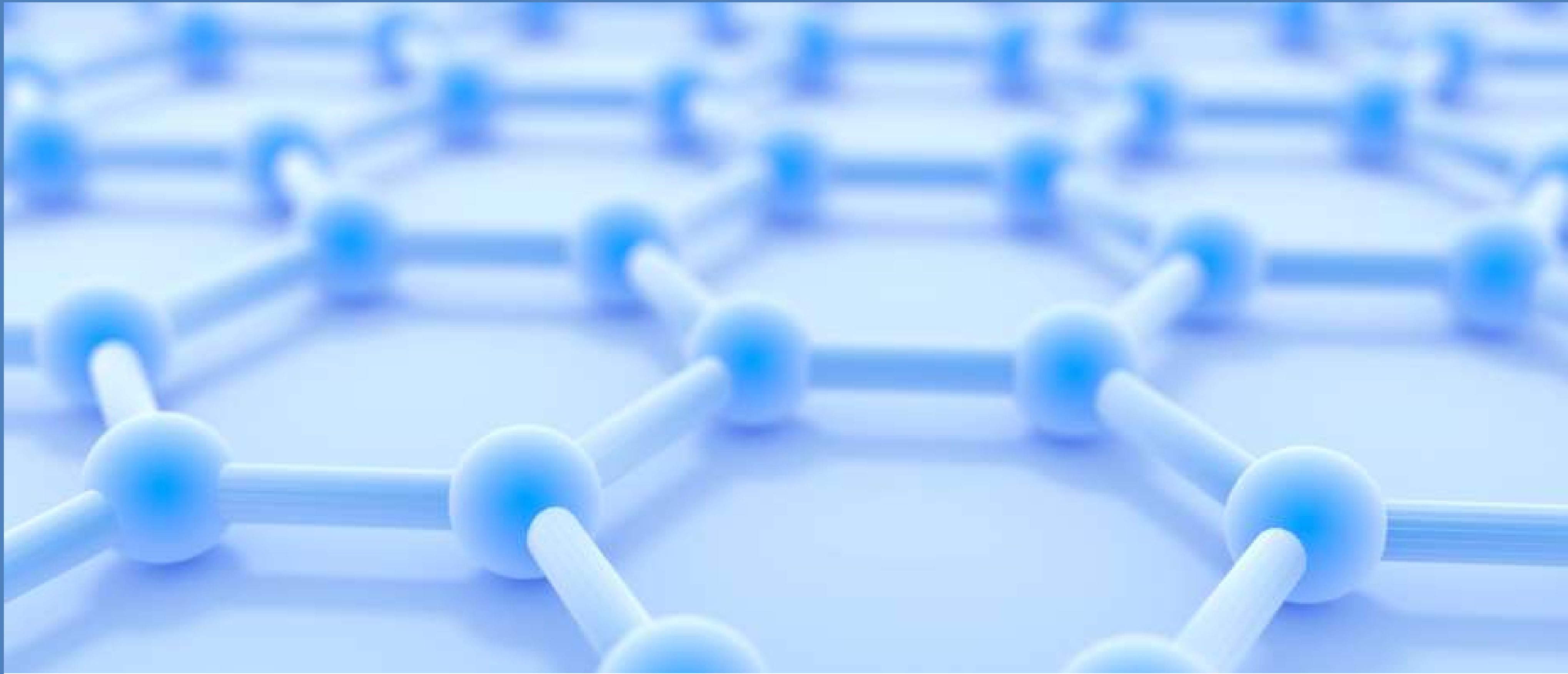

Photonics

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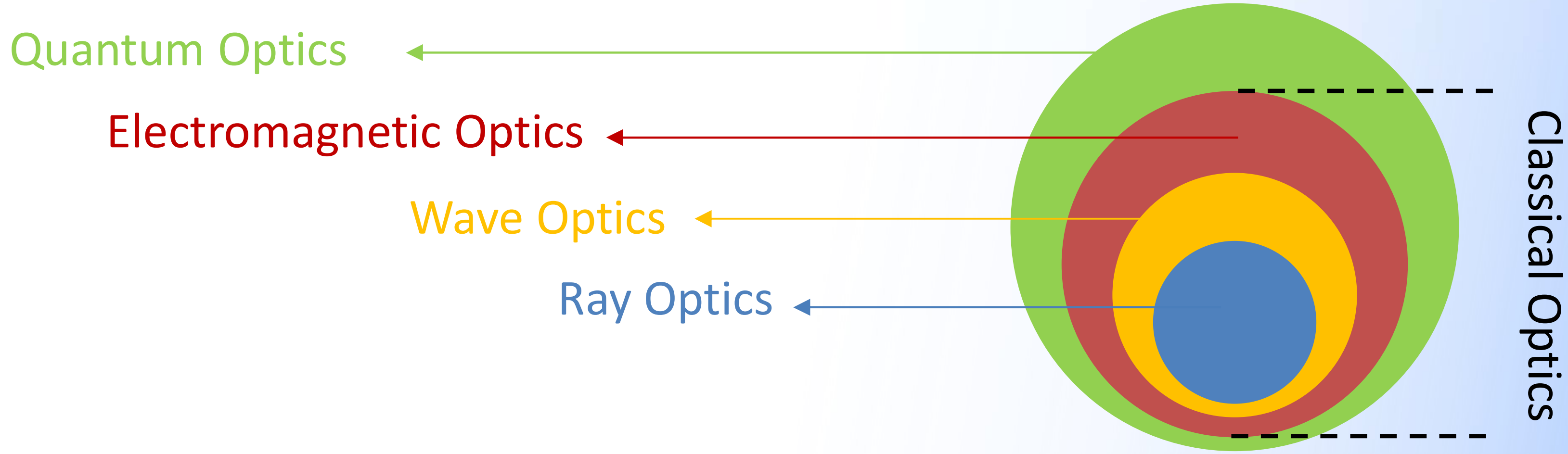
Università degli Studi di Brescia

Ray Optics:
Postulates and
Simple Optical Components

Introduction



Light can be described as a wave phenomenon. Electromagnetic radiation, which is well described by **Electromagnetic Optics**, propagates in the form of two mutually coupled vector waves (electric and magnetic field waves).
Some optical phenomena allow to treat light as a single scalar wavefunction. This approximation is what we call **Wave Optics**.
Further simplification can be done if light is propagating through and around objects much bigger than the wavelength so that light can be described by rays obeying to a set of geometrical rules. This model is called **Ray Optics**.
Those three theories fall in the realm of **Classical Optics.**
However, certain optical phenomena cannot be explained classically, but they need to be described by a quantum version of the electromagnetic theory known as **Quantum Electrodynamics**.

Introduction

Theories of Optics history

Ray Optics → **Wave Optics** → **Electromagnetic Optics** → **Quantum Optics**

complexity →

Theories evolved in time to provide explanations for the outcomes of increasingly precise optical experiments.

Which model for which problem?

It is difficult to say *a priori* which model is the most appropriate, but the optimal choice of a model is the simplest one that can describe properly a particular phenomenon.

Ray Optics

- **Simplest and approximate** theory of light;
- Also known as **Geometrical Optics** (because the rays obey to certain geometrical rules);
- Sufficient to describe most of our **daily experience with light**;

What is ray optics used for?

Location of light

Direction of light

Image Formation

Guiding conditions

Direction of optical energy

Postulates of Ray Optics

- Light travels in the form of **rays**, which are emitted by **light sources** and can be observed by an **optical detector**;

- An optical medium is characterized by a quantity $n \geq 1$, called the **refractive index**. The refractive index is $\mathbf{n = c_0/c}$, where c_0 is the speed of light in vacuum and c is the speed of light in the medium. The **time** taken by light to travel a distance **d** is $d/c = nd/c_0$. The product **nd** is also called **optical pathlength**.

- In an inhomogeneous medium the refractive index n(**r**) is a function of the spatial position **r** = (x, y, z). The optical pathlength between two points A and B is therefore:

$$\text{Optical Pathlength} = \int_A^B n(\boldsymbol{r})\,dS$$

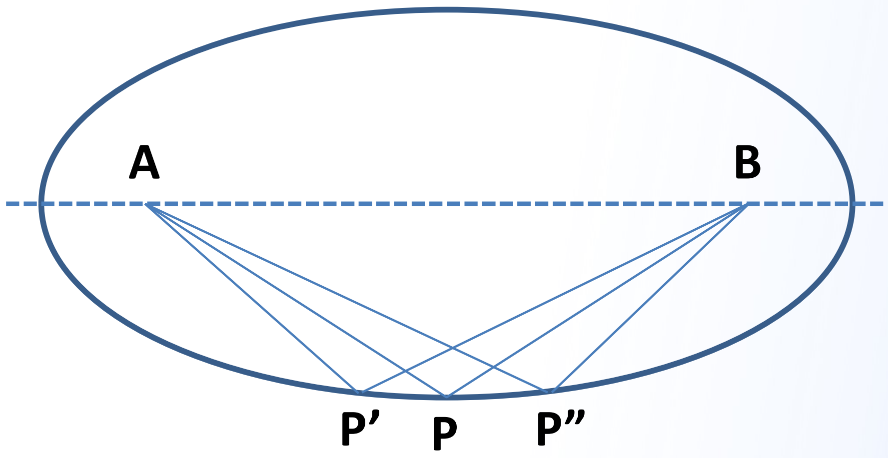
where *dS* is the differential element of length along the path.

Postulates of Ray Optics

- **Fermat's Principle**: "the path taken by a ray between two given points is the path that can be traversed in the least time".
  This concept is expressed mathematically as:

$$\delta \int_A^B n(\boldsymbol{r})dS = 0$$

NOTE: Sometimes the minimum time is shared by more than one path, which are then all followed simultaneously by the rays.



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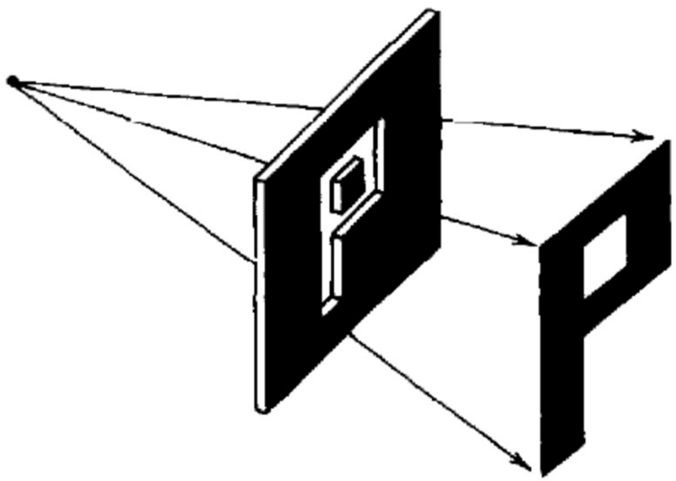
Propagation in a Homogeneous Medium

In a homogeneous medium the refractive index is identical everywhere

Speed of light is identical everywhere

Path of minimum time = Path of minimum distance
(Hero’s principle)



Light rays travel
in straight lines

Reflection from a mirror

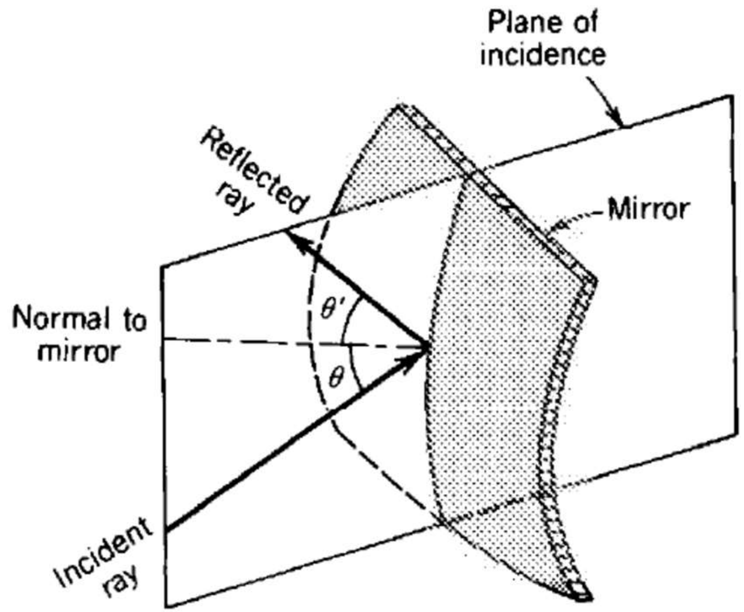
To prove the law of reflection we use **Hero's principle.**

Mirrors are made of highly polished metals, or metallic or dielectric films on glass. The **law of reflection** states:

The reflected ray lies in the plane of incidence.
The angle of reflection equals the angle of incidence.

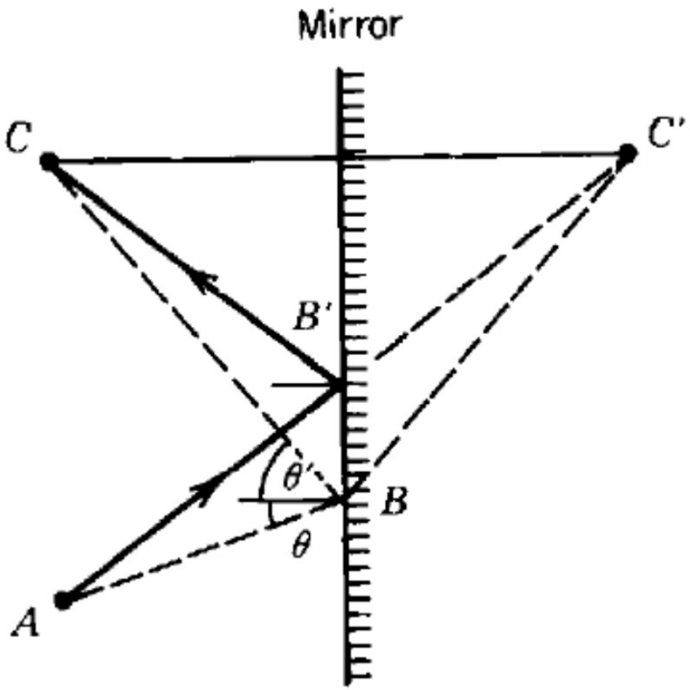
Plane of incidence: plane formed by the incident ray and the normal to the mirror at the point of incidence.

Angles of incidence and reflection: angles formed by the incident/reflected ray with the normal to the mirror at the point of incidence.



Reflection from a mirror

To prove the law of reflection we use Hero’s principle.



What happens when we go from point A to point C after reflection from an infinitesimally thin mirror?

Since we are traveling in a homogeneous medium, minimum path is equal to minimum distance, therefore we need to verify under what circumstances $\overline{AB} + \overline{BC}$ is a minimum.

If C’ is a mirror image of C, then $\overline{BC} = \overline{BC'}$ so that also $\overline{AB} + \overline{BC}'$ has to be a minimum.
This occurs only when $\overline{ABC'}$ is a straight line.

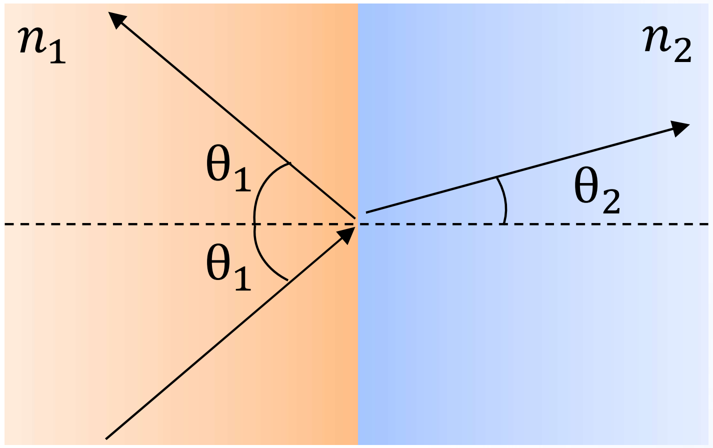
This is true only if B = B’ and, therefore, θ = θ’.

Reflection and Refraction at a boundary

At the boundary between two media with refractive indices n_1 and n_2 an incident ray is split into two rays: a **reflected ray** and a **refracted ray.**
The reflected ray always obeys to the law of reflection, while the refracted ray obeys to the **law of refraction**:

The refracted ray lies in the plane of incidence.
The angle of refraction θ_2 is related to the angle of incidence θ_1 by means of the Snell's law:

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$



NOTE: Ray Optics does not describe the proportion in which light is reflected or refracted

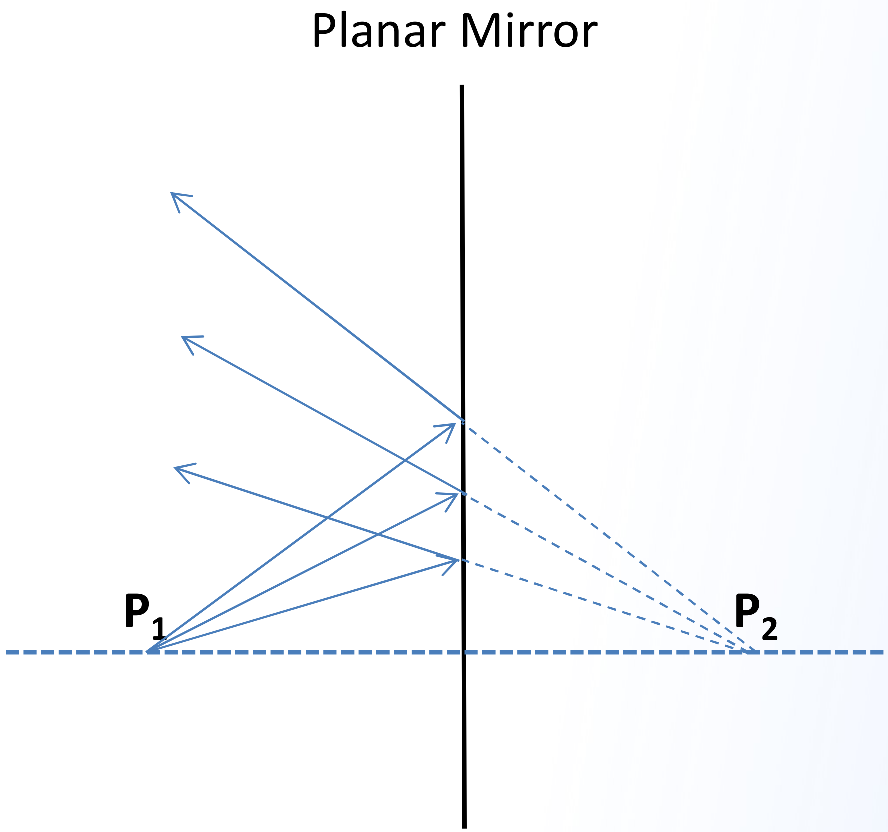
Simple optical Components

- MIRRORS
- PLANAR BOUNDARIES
- SPHERICAL BOUNDARIES AND LENSES
- LIGHT GUIDES

Mirrors

PLANAR MIRRORS

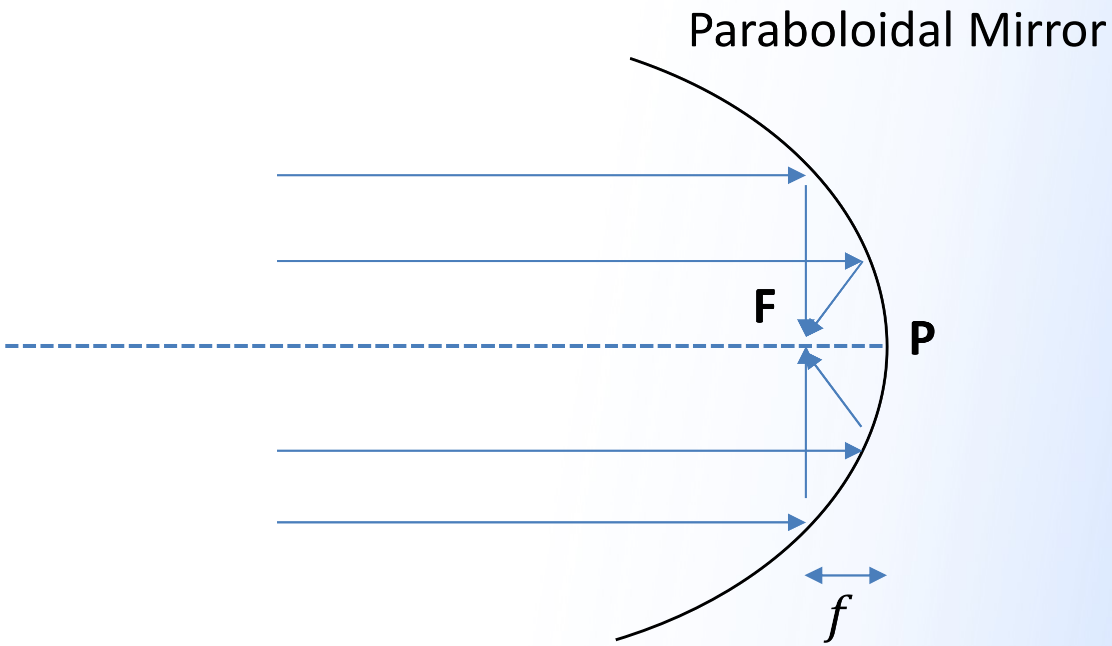
Reflects the rays originating at a point P_1 . The reflected rays look like they are originated from a point P_2 behind the mirror. Point P_2 is called **image**.



Mirrors

PARABOLOIDAL MIRRORS

The surface of a paraboloidal mirror is a paraboloid of revolution. It focuses all incident rays parallel to its axis to a single point called **focus**. They are used to collect light in telescopes or to make parallel beams from point sources as in flashlights.

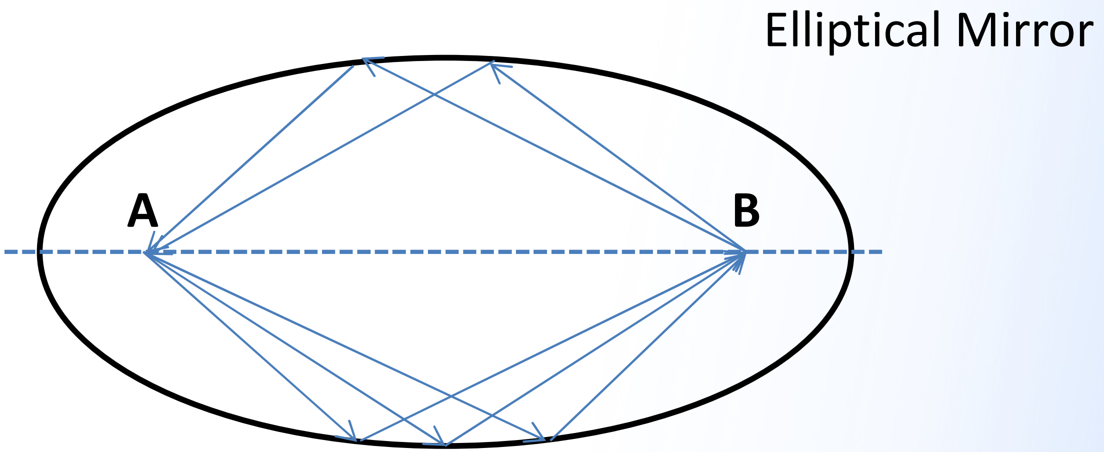


The distance $\overline{PF} = f$ is called **focal length**.

Mirrors

ELLIPTICAL MIRRORS

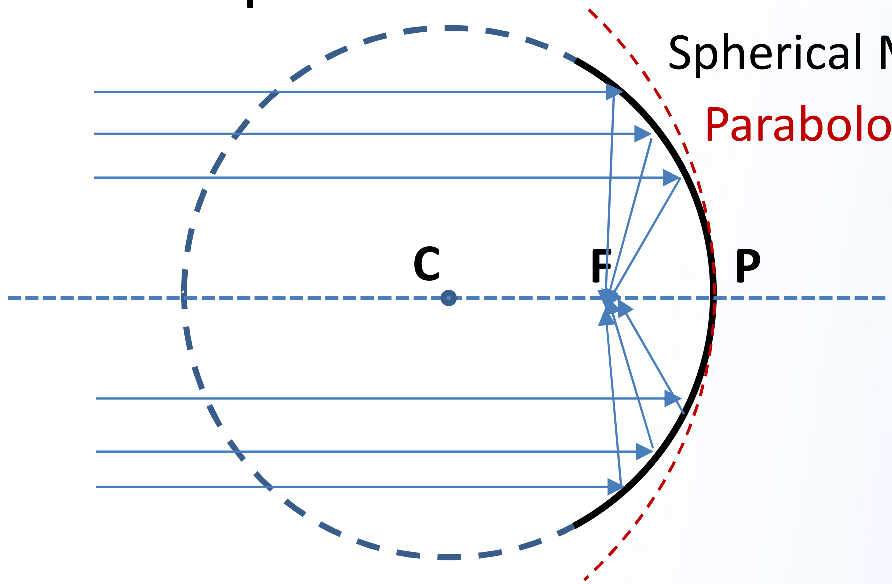
It reflects all the rays emitted from one of its two foci and images them onto the other focus. The distances traveled from one focus to the other are identical (according to Hero's principle).



Mirrors

SPHERICAL MIRRORS

It does not have the same focusing properties of paraboloidal or elliptical mirrors. However, **paraxial rays**, which are rays that form a small angle with the mirror axis ($\sin\theta \sim \theta$), are *approximately* focused onto a single point F at distance $-R/2$ from the mirror center C. By convention R is negative for concave mirrors and positive for convex mirrors. In the **paraxial approximation** a spherical mirror has also the imaging properties of an elliptical mirror.



A spherical Mirror behaves like a paraboloidal mirror with f = R/2 for paraxial rays

The envelope of reflected rays (dashed curve) is called **caustic curve**.

Mirrors

SPHERICAL MIRRORS

All paraxial rays originating on each point on the axis of a spherical mirror are reflected and focused onto a single corresponding point on the axis. From geometrical consideration and considering that all angles are small so that $\tan\theta \sim \theta$ we can extract the imaging equation for spherical mirrors.

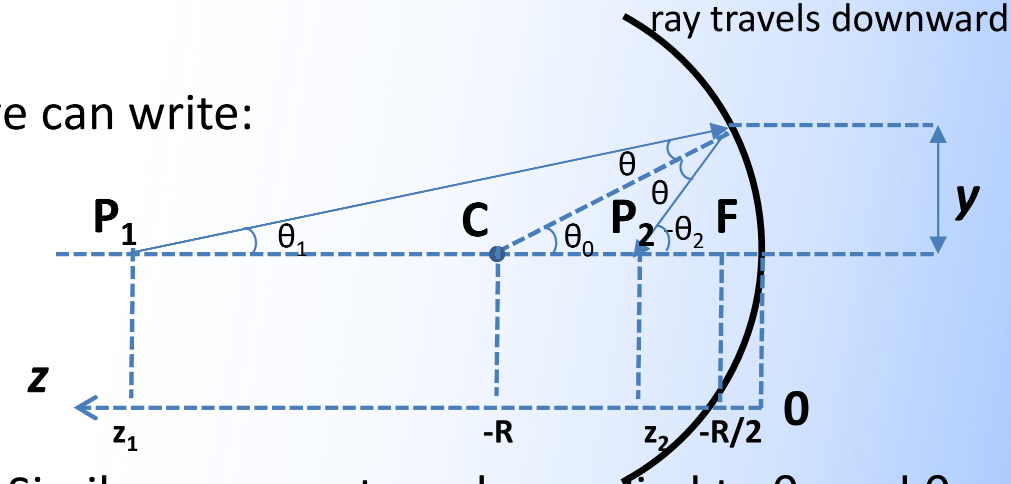
NOTE: θ_2 is negative because the ray travels downward

Since the sum of the angle in a triangle are 180° we can write:

$$\theta_0 = \theta_1 + \theta$$

$$(-\theta_2) = \theta_0 + \theta$$

$$(-\theta_2) + \theta_1 = 2\theta_0$$



If θ_0 is small then $\tan\theta_0 = \theta_0$, therefore $\theta_0 \sim y/(-R)$. Similar argument can be applied to θ_1 and θ_2, from which follows:

$$\boxed{\frac{1}{z_1} + \frac{1}{z_2} \approx \frac{2}{-R} \approx \frac{1}{f}}$$

If z_1 goes to infinity the focus falls at $-R/2$. So, within the paraxial approximation $f = -R/2$ (focal length of a spherical mirror).

Planar Boundaries

The relation between the incident and refracted field is governed by Snell's law $(n_1 \sin\theta_1 = n_2 \sin\theta_2)$. We can then distinguish two different scenarios:

- External refraction $(n_1 < n_2)$: $\theta_1 > \theta_2$, rays bend away from the boundary;

- Internal refraction $(n_1 > n_2)$: $\theta_1 < \theta_2$, rays bend toward the boundary;

NOTE: rays always bend to minimize optical pathlength!

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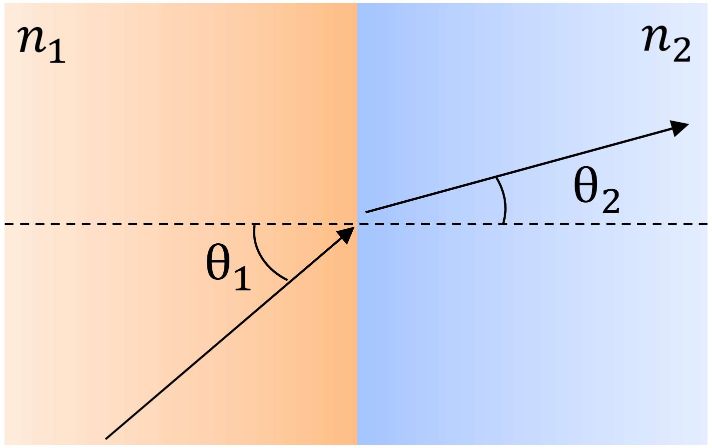
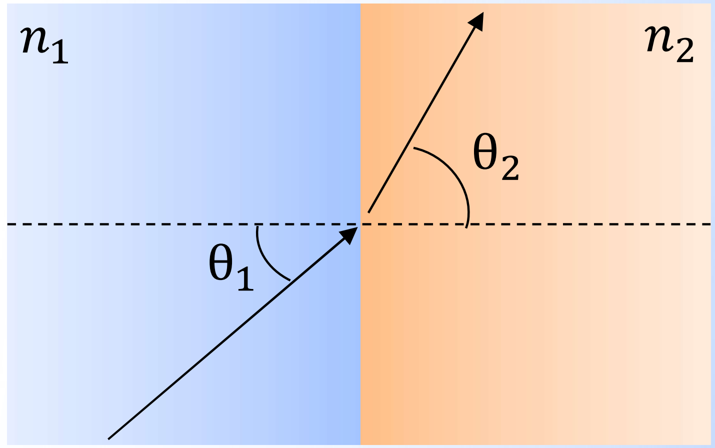
Planar Boundaries

External refraction ($n_1 < n_2$)



$\theta_1 > \theta_2$

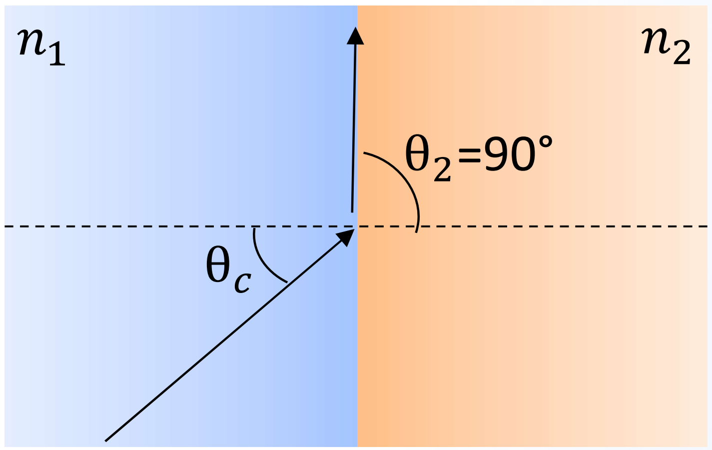
Internal refraction ($n_1 > n_2$)



$\theta_1 < \theta_2$

Total internal Reflection

If $n_1 > n_2$ and θ_1 is increased, then θ_2 will eventually become 90°. The value of θ_1 for which θ_2 is 90° is called the **critical angle** θ_c. Prism and optical fibers exploit the total internal reflection concept.



$$\boxed{\theta_c = a\sin\left(\frac{n_2}{n_1}\right)}$$

If $\theta_1 > \theta_c$ the boundary behaves like a perfect mirror.

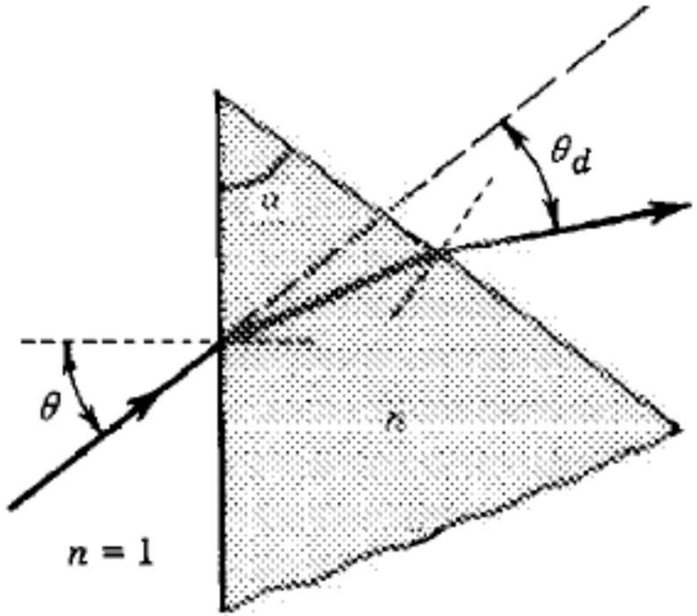
Planar Boundaries

PRISMS

A prism with apex angle α and refractive index n deflects a ray incident by angle θ by an angle:

$$\theta_d = \theta - \alpha + \sin[\sqrt{n^2 - sin^2\theta}\, sin\alpha - sin\theta cos\alpha]$$

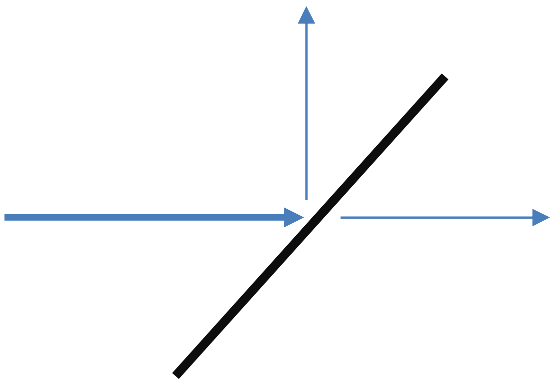
This equation is obtained by applying Snell's law twice. If α is very small (thin prism) then $\theta_d \approx (n-1)\alpha$.



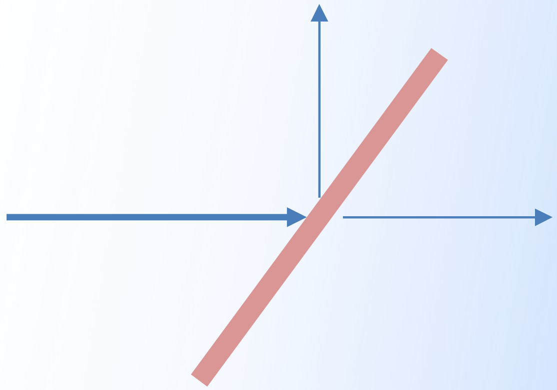
Planar Boundaries

BEAM SPLITTERS

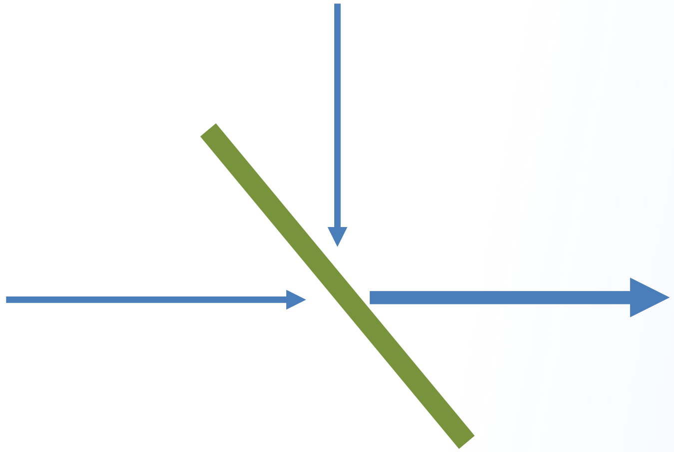
A beam splitter is an optical component that separates the incident beam into a reflected beam and a transmitted beam. They can also be used to combine two beams into one.



Partially reflective mirror



Thin glass plate

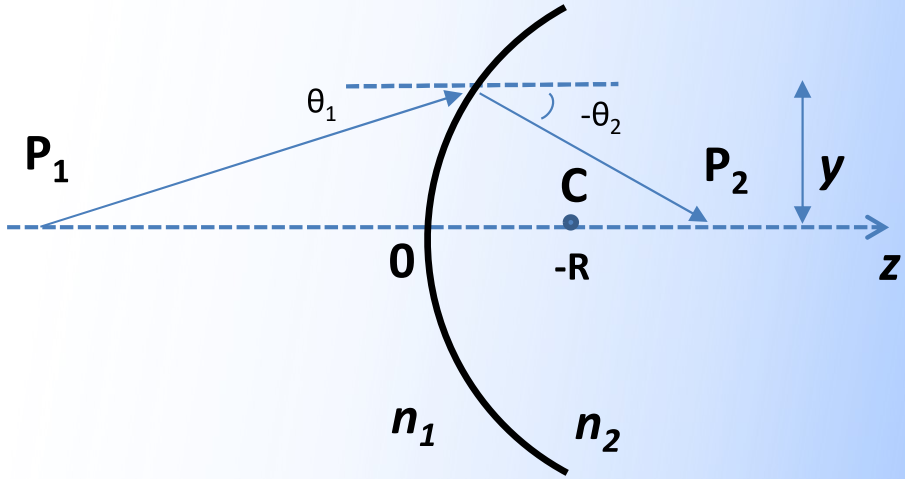


Beam combiner

Spherical Boundaries

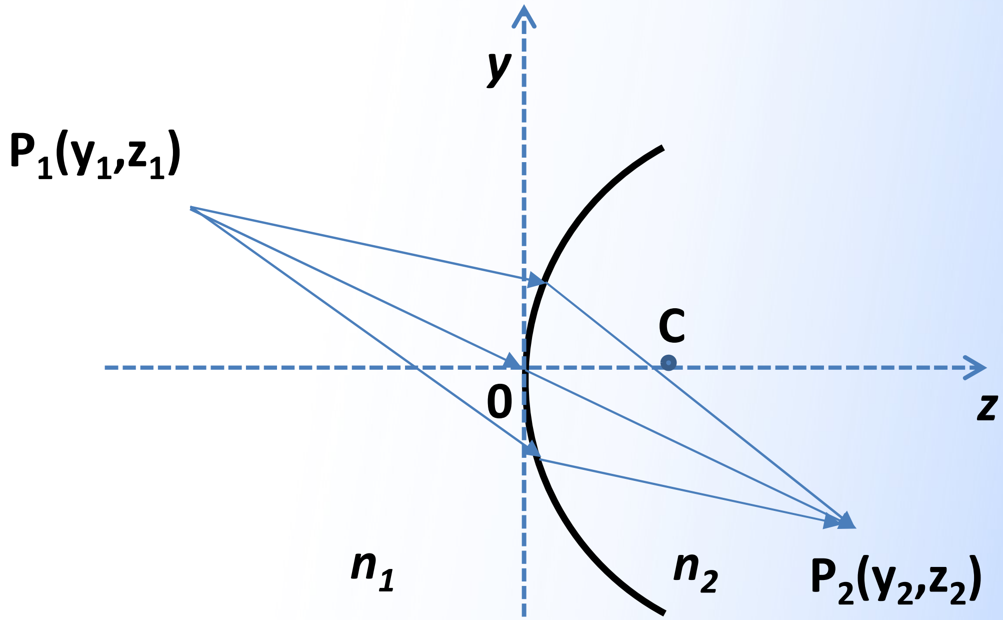
A refraction of rays from a spherical boundary between to media with refractive indexes n_1 and n_2 follows the application of Snell's law. By convention, R is positive for a convex boundary and negative for a concave boundary. Note that the angles formed with respect to the normal to the surface are different from the angles made with respect to the z axis. In the paraxial rays' approximation the following properties are valid:

- $$\theta_2 \approx \frac{n_2}{n_1}\theta_1 - \frac{n_2 - n_1}{n_2}\frac{y}{R}$$



Spherical Boundaries

All paraxial rays originating from P_1 in a plane z_1 meet at a point P_2 in a plane z_2 where:



- $\frac{n_1}{z_1} + \frac{n_2}{z_2} \approx \frac{n_2 - n_1}{R}$
- $y_2 = -y_1 \frac{n_1 z_2}{n_2 z_1}$

z_1 and z_2 are said to be conjugate planes. The **magnification** of the image in the second plane is $-\frac{n_1 z_2}{n_2 z_1}$. The minus sign means that the image is inverted.

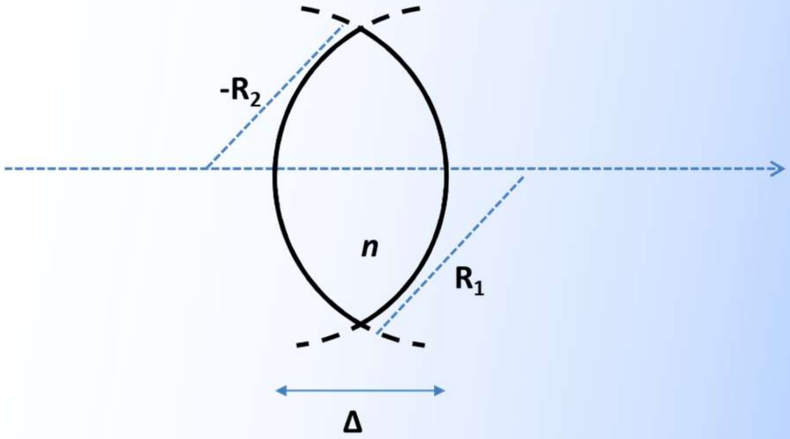
NOTE: if the rays are not paraxial the image will be distorted. This phenomenon is known as **aberration**.

Spherical Boundaries

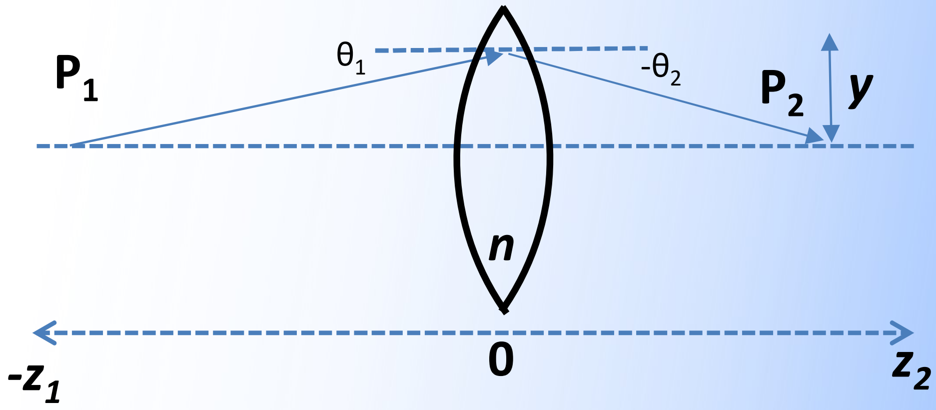
LENSES

A spherical lens is defined by two spherical surfaces. It is defined by its two radii R_1 and R_2, its thickness Δ and its refractive index n.

When the lens is thin the refraction throughout the lens can be simplified assuming the exit point is identical in height to the incident point so that the following relations can be written:



- $\theta_2 = \theta_1 - \frac{y}{f},$



Where the **focal length *f*** is defined as:

- $\frac{1}{f} = (n-1)(\frac{1}{R_1} - \frac{1}{R_2})$

Spherical Boundaries

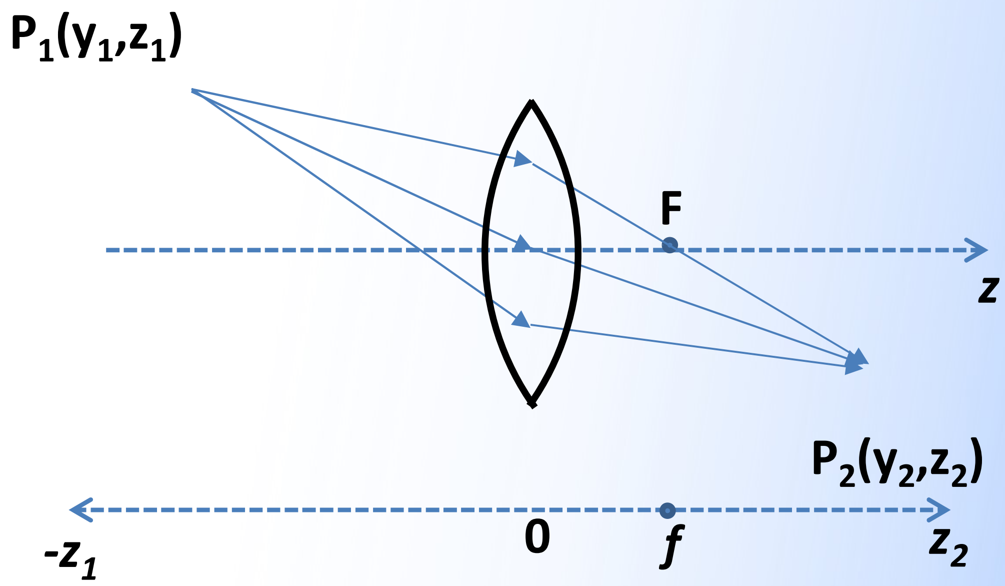
LENSES

All rays originating at P_1 meet at point P_2 where

- $\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$,

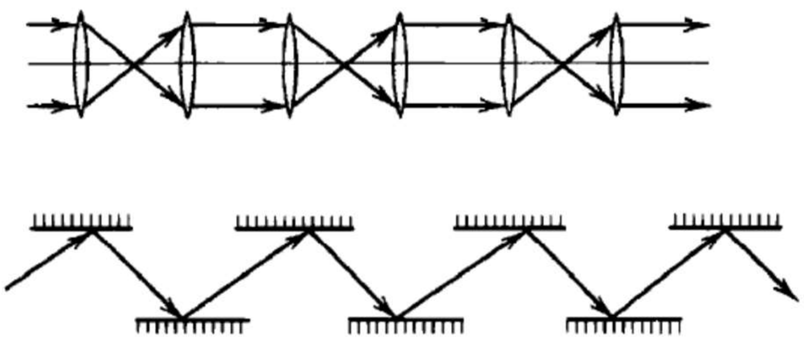
With a magnification:

- $y_2 = -\frac{z_2}{z_1} y_1$

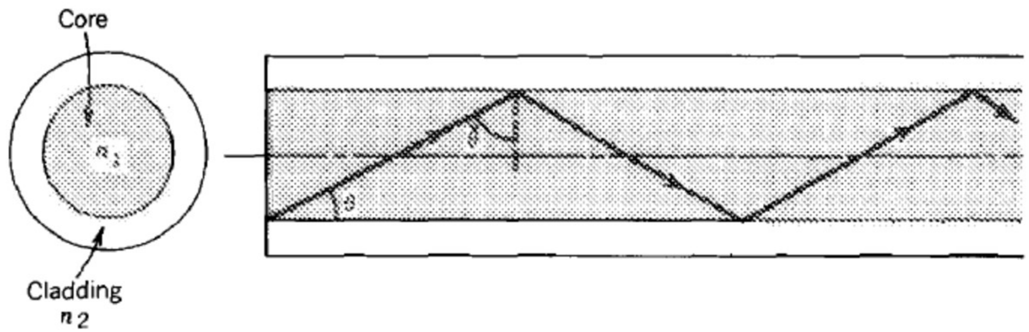


Light guides

Light may be guided from one point to another by using a set of lenses or mirrors with some loss in power due to the partial reflectivity of each component.



Total internal reflection is also an effective way to guide light as it is done in optical fibers.



Light is guided in the core of the fiber since the cladding has a refractive index smaller than the core.