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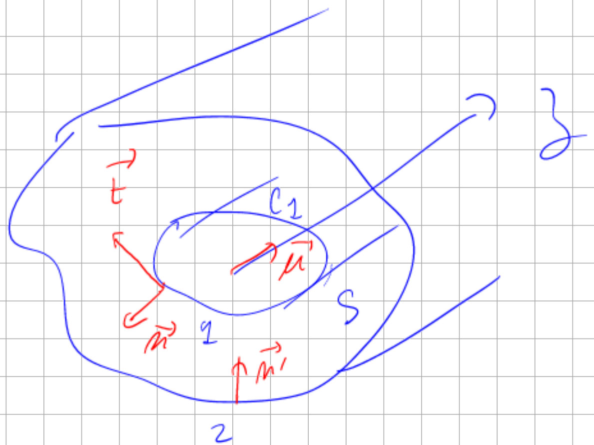
Lesson 4
Sept 15th 2020

IV V, I, Z_c in the line

$$\vec{J_s}(x,y) = \vec{n} \times \vec{H_t}(x,y)$$



$$\vec{J_s}(x,y) = \vec{n} \times \left[\frac{1}{Z}\left(\vec{u} \times \vec{E_t}(x,y)\right)\right]$$

$$= \frac{1}{Z}\left[\vec{u}\left(\vec{n}\cdot\vec{E_t}(x,y)\right) - \vec{E_t}(x,y)\underbrace{\left(\vec{n}\cdot\vec{u}\right)}_{0}\right]$$

$$= \frac{1}{Z}\left(\vec{n}\cdot\vec{E_t}(x,y)\right)\vec{u}$$

$$\vec{E_t}(x,y) = -\vec{\nabla}_t V(x,y) = -\frac{\partial V(x,y)}{\partial n}\vec{n} - \frac{\partial V(x,y)}{\partial t}\vec{t}$$

$$\vec{J_s}(x,y) = -\frac{1}{Z}\left(\vec{n}\cdot\left(\frac{\partial V(x,y)}{\partial n}\vec{n} + \frac{\partial V(x,y)}{\partial t}\vec{t}\right)\right)\vec{u}$$

$$\vec{J_s}(x,y) = -\frac{1}{Z}\frac{\partial V(x,y)}{\partial n}\vec{u}$$

I_1 current in conductor 1

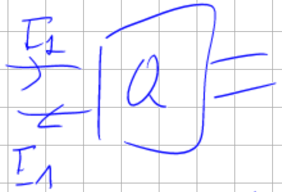
$$I_1 = \int_{C_1} \vec{J_{s1}}(x,y)\cdot\vec{u}\, dC_1$$

$$I_1 = \frac{1}{Z}\int_{C_1} \vec{n}\cdot\vec{E_t}(x,y)\, dC_1$$

I_2 current in conductor 2

$$I_2 = \frac{1}{Z}\int_{C_2} \vec{n}'\cdot\vec{E_t}(x,y)\, dC_2$$

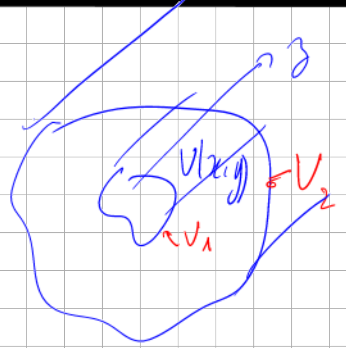
$$I_1 + I_2 = 0$$



$$I_1 + I_2 = \frac{1}{Z}\left[\int_{C_1} \vec{n}\cdot\vec{E_t}(x,y)\, dC_1 + \int_{C_2} \vec{n}'\cdot\vec{E_t}(x,y)\, dC_2\right]$$

$$= \frac{1}{Z}\iint_S \underbrace{\vec{\nabla}_t\cdot\vec{E_t}(x,y)}_{0}\, dS$$

$$= 0$$



$$V = V_1 - V_2$$

$$I = I_1$$

Characteristic impedance Z_c

$$Z_c = \frac{V}{I}$$

$$V' = V_2 - V_1 \qquad Z_c = \frac{V'}{I'}$$

$$I' = I_2$$

$$V(z) = V e^{-j\beta z}$$

$$I(z) = I e^{-j\beta z}$$

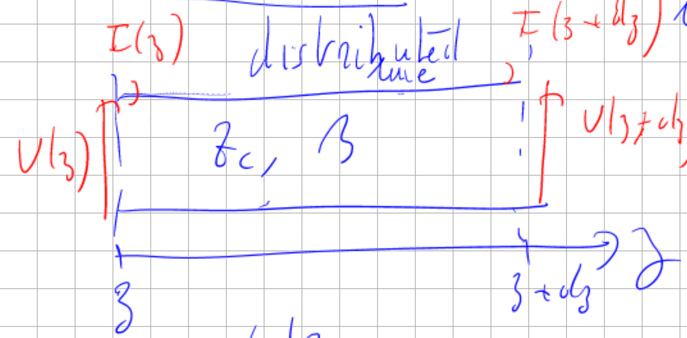
$$\frac{\partial V(z)}{\partial z} = -j\beta V e^{-j\beta z} = -j\beta Z_c \underbrace{I e^{-j\beta z}}_{I(z)}$$

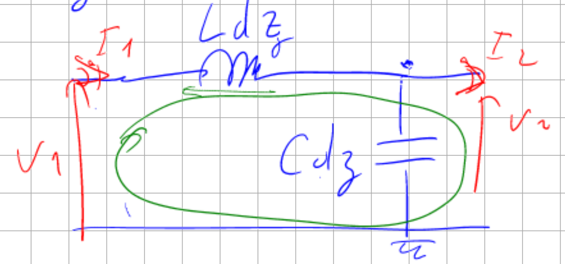
$$-\frac{\partial V(z)}{\partial z} = j\beta Z_c I(z) \qquad (1)$$

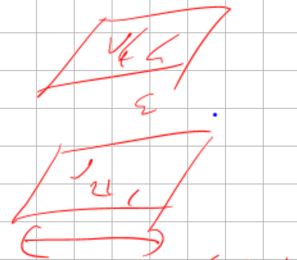
$$\frac{\partial I(z)}{\partial z} = -j\beta I e^{-j\beta z} = -j\beta \frac{V}{Z_c} e^{-j\beta z}$$

$$-\frac{\partial I(z)}{\partial z} = j\frac{\beta}{Z_c} V(z) \qquad (2)$$

V Equivalent lumped element circuit of the TEM line

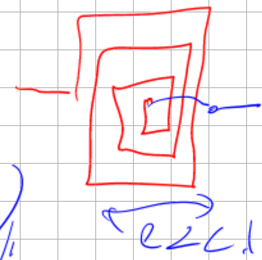






L and C are lumped elements

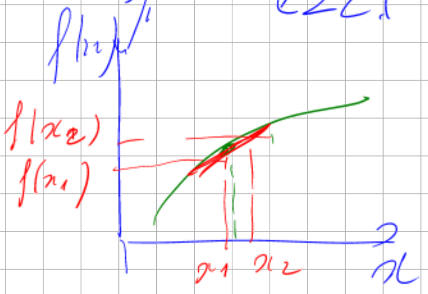
$$V_1 = jL\,dz\,\omega I_1 + V_2$$



$$V(z) = jL\,dz\,\omega I(z) + V(z+dz)$$

$$\frac{V(z) - V(z+dz)}{dz} = -\frac{\partial V(z)}{\partial z}$$

$$\frac{\partial f(x)}{\partial x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \text{ when } x_2 - x_1 \to 0$$



$$-\frac{\partial V(z)}{\partial z} = j L\omega\, I(z) \qquad (3)$$

$$I(z) = I(z+dz) + j\,\underline{C\,dz\,\omega\, V(z+dz)}$$

$$V(z+dz) = \frac{\partial V(z)}{\partial z}dz + V(z)$$

$$jC\omega\, dz\left(\frac{\partial V(z)}{\partial z}dz + V(z)\right) \# jC\omega V(z)dz$$

$$\cancel{dz^2} \Rightarrow \frac{I(z)-I(z+dz)}{dz} = jC\omega V(z)$$

$$I(z) = I(z+dz) + jC\,dz\,\omega V(z)$$

$$-\frac{\partial I(z)}{\partial z} = jC\omega V(z) \qquad (4)$$

$$-\frac{\partial V(z)}{\partial z} = j\beta Z_c I(z) \quad (1)$$

(1) and (3) $\Rightarrow$ $\beta Z_c = L\omega$

$\gamma = j\beta = j\frac{\omega}{v}$ $\qquad \frac{\omega}{v} Z_c = L\omega$

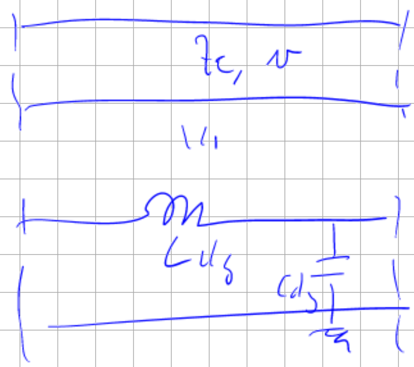
$$L = \frac{Z_c}{v} \qquad H/m$$

(2) and (4)

$$-\frac{\partial I(z)}{\partial z} = j\frac{\beta}{Z_c}V(z) \qquad (2)$$

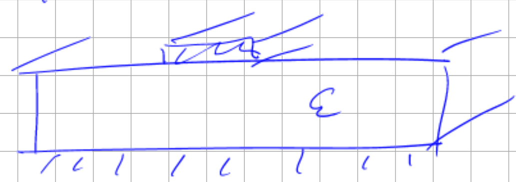
$$C\omega = \frac{\beta}{Z_c}$$

$$\Rightarrow C = \frac{1}{Z_c v} \qquad F/m$$



VI Losses in TEM transmission lines

quasi TEM mode $\qquad \mu = 1$



– Metallic losses on conductors :

$$\overline{P_{met}} = \frac{1}{2} R_s \iint_{S_{met}} |H_t(x,y)|^2 \, dS_{met}$$

$$R_s = \frac{1}{\sigma \delta}$$

σ metal conductivity (S/m)

$\sigma = \frac{1}{\rho}$ ρ resistivity

$\left(R = \rho \frac{\ell}{S} \right)$

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$ skin depth penetration

air metal

$\sigma \to \infty$

σ

H_0

$\vec{H}$

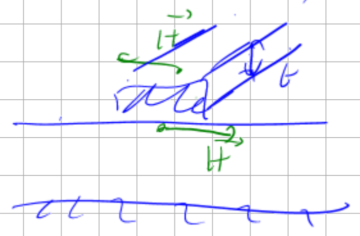
$\frac{H_0}{e}$

δ

z

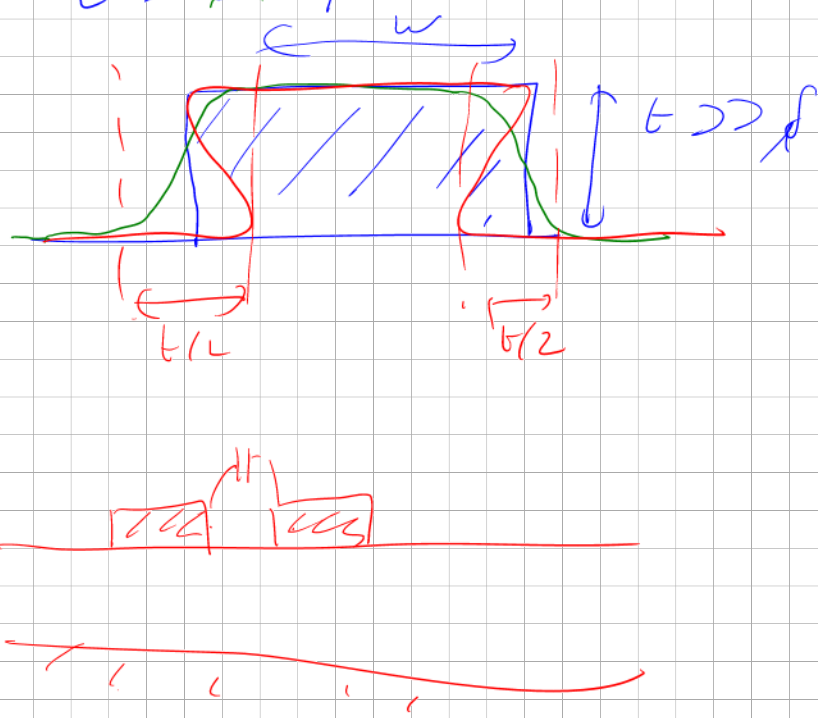
$$H(z) = H_0 \, e^{-\frac{z}{\delta}} \cos(\omega t - \beta z)$$

$\delta \sim 1 \mu m$



$t = 0.5 \mu m$

$t >> \delta$



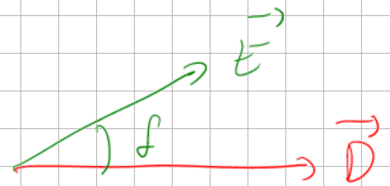
We consider that $\vec{E}_t(x,y)$, $\vec{H}_t(x,y)$ are not modified by the losses

* dielectric losses in the substrate

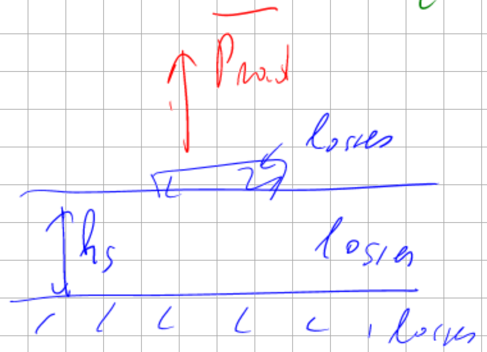
$$\varepsilon = \varepsilon' - j\,\varepsilon''$$

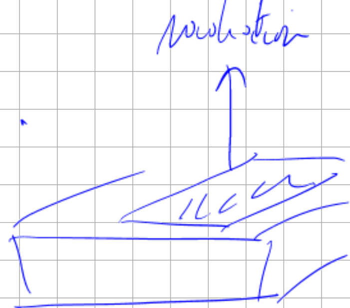
$$\overline{P}_{diel} = \frac{1}{2}\,\omega\,\varepsilon'' \iiint_V |E_t(x,y)|^2\,dV$$

$$\vec{D} = \varepsilon\,\vec{E}$$



$$\mathrm{tg}\,\delta = \frac{\varepsilon''}{\varepsilon'} \quad \text{loss tangent}$$





$$h_s \ll \lambda \Rightarrow \overline{P}_{rad} < 5\%\ \overline{P}_{losses}$$

$$\overline{P}_{met} = \frac{1}{2}\,R_s \iint_{S_{met}} |H_t|^2\,dS_{met} = \frac{1}{2}\,R\,|I|^2$$

$$\overline{P}_{diel} = \frac{1}{2}\,\omega\,\varepsilon'' \iiint_V |E_t|^2\,dV = \frac{1}{2}\,G\,|V|^2$$