Digital Modulation and Channel Coding

Exam Guide

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Explanation and Introduction of this Document

I spent have spent a lot of time developing the template used to make this LaTeX document, I want others to benefit from this work so the source code for this template is available on GitHub [?].

1 Digital Modulation Questions

1.1 “Basic Theory” Questions

1.1.1 Complex Envelope

Define the concept of the ”complex envelope” and describe the major equations related to it.

ANSWER: The complex envelope is a mathematical representation that **simplifies the analysis of modulated signals by separating the slowly varying envelope from the rapidly oscillating carrier wave.** This separation is useful because it **allows for easier analysis and manipulation** of the modulated signal, particularly in the context of amplitude modulation (AM) and frequency modulation (FM) systems.
A modulated signal could be represented by the equation:

$$s(t) = A(t)cos(2\pi f_c t + \varphi(t)) \tag{1}$$

Where:

- $s(t)$ is the modulated signal.
- $A(t)$ is the time-varying amplitude (envelope).
- f_c is the carrier frequency.
- $\varphi(t)$ is the **phase modulation term,** , not to be confused with the instantaneous phase.

To analyse this signal using the complex envelope representation, we define the complex envelope $s_c(t)$ as follows:

$$s_c(t) = A(t)e^{j\phi(t)} \tag{2}$$

In this expression, ϕ, represents the *instantaneous* phase modulation as a function of time. The Euler's function term corresponds to the complex phasor that captures both the carrier frequency and phase of the transmitted signal. The full expression ultimately gives the amplitude, frequency, and phase information of the signal.

To fully flesh out this concept, let's examine the difference between AM, FM, and PM signals with regards to the complex envelope equation. Let's start by examining the parameter modulated in each technique:

- In AM: $A(t)$, The **amplitude**, information is encoded by changing signal amplitude.
- In PM: $\varphi(t)$, The **additive phase**, information is encoded by changing specifically this phase parameter.
- In FM: $f_c + f_\Delta(t)$, The **frequency**, information is encoded by changing specifically the frequency parameter.

Now, as for the instantaneous phase term, contained in the complex envelope representation. For PM and AM the complex envelope representation is quite simple, is basically the same as that shown above, they are fully demonstrated for posterity below.
However, for FM it is more complex; in order to have a signal which is continuous and differential-able, the instantaneous phase term, ϕ_{FM}, is the *integral of the frequency deviation function w.r.t. time.*

- In AM: $s_{c,AM}(t) = A(t)e^{j\phi AM(t)}$ with $\phi_{AM} = 2\pi f_c t$, i.e. simply the high freq, oscillating, carrier portion.
- In PM: $s_{c,PM}(t) = Ae^{j\phi PM(t)}$ with $\phi_{PM} = 2\pi f_c t + \varphi(t)$, i.e. the combination of the carrier and the additive phase, φ.
- In FM: $s_{c,FM}(t) = Ae^{j\phi FM(t)}$ with $\phi_{FM} = 2\pi \int_0^t (f_c + f_\Delta(\tau))\, \mathrm{d}\tau$, i.e. the comb. of f_c and the additive frequency term, $f_\Delta(\tau)$

Indicate the use of this concept (the complex envelope) in case of OFDM modulation.

ANSWER: This is a bit of a horrible question that belongs more in the realm of the *OFDM* section of questions in reality, but nonetheless here is the best approximation of a short answer.

Orthogonal Frequency Division Multiplexing (OFDM) is a modulation technique which involves dividing the available frequency bandwidth of the communication channel into multiple sub-carrier bandwidth frequencies.
To avoid any inter-band interference, sub-carriers are typically orthogonal to each other in the frequency domain.
Once split appropriately, the individual sub-carrier frequencies are *independently* modulated simultaneously using conventional modulation techniques (e.g. QAM).

The complex envelope representation of an OFDM signal is not quite the same as in traditional AM/PM/FM. However, it is still relevant in a more abstract sense to simplify the analysis of the modulation scheme.

To fully explore the relations, we shall split into some more individual details looking at specific aspects of OFDM:

1. Sub-Carriers and OFDM's Time-Domain Modulation Equation
    - As stated each sub-carrier is treated as if it were an individual, completely separate comm. channel.
    - The time domain representation of the combination of these channels is expressed by:

$$s_{OFDM}(t) = \sum_{n=0}^{N-1} S_n e^{j2\pi f_n t} \tag{3}$$

Where:
- $s_{OFDM}(t)$ is the entire OFDM system signal in the time domain.
- f_n is the frequency of each individual (the n^{th}) sub-carrier.
- N is the number of sub-carriers
- S_n is the individual sub-carrier's time domain signal, **this can be, and usually is, expressed as a complex envelope!**

Thus, the entire OFDM system's signal could be expressed by a, rather complex, complex envelope equation.

2 Individual Sub-Carrier Time-Domain Modulation Equation

- Each communication channel (sub-carrier) in OFDM will use a traditional modulation technique, like QAM.
- In QAM, each symbol represents a combination of amplitude levels and phase shifts, this can be expressed in complex envelope representation.
- Thus time domain representation of an individual sub-carrier is expressed by a complex envelope, the QAM one:

$$s_{ind}(t) = S_n e^{j2\pi f_n t} = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} A_{mn} A_{mn} cos(2\pi f_n t + \varphi_{mn}) p(t - mT_s)$$
$$\therefore s_{ind}(t) = s_c(t) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} A_{mn} e^{j\phi_{mn}} p(t - mT_s)$$

Where:

- M and N are the numbers of amplitude levels and phase shifts, respectively.
- A_{mn} is the amplitude for the m-th amplitude level and the n-th phase shift.
- φ_{mn} is the phase for the m-th amplitude level and the n-th phase shift.
- $p(t)$ is the pulse shaping function.
- T_s is the symbol period.
- **The $e^{j\phi_{mn}}$ term captures the phase information of the QAM signal.**
- **The amplitude levels and phase shifts are represented by A_{mn} and ϕ_{mn}.**
- **The pulse shaping function $p(t)$ is included to consider the shaping of the signal.**

3 OFDM Cyclic Prefix

- The cyclic prefix (CP) in OFDM systems added to mitigate the effects of multipath fading, it is essentially a replica of the end part of the OFDM symbol, added to the beginning of the signal its addition helps in maintaining orthogonality between sub-carriers.
- As it is a copy of the original signal, ti can be expressed in complex envelope form.

$$s_{\text{CP}}[n] = s[N - n], \; 0 \leq n < N_{\text{CP}} \tag{4}$$

1.1.2 The ”Nyquist pulse” and the roll-off parameter.

Question: Define the concept of the ”Nyquist pulse”, give the equations related to it, and the role of the roll-off parameter.

ANSWER: The Nyquist Pulse (NP), A.K.A. the “Nyquist filter” or “Nyquist pulse shaping”, is a concept/technique used to shape a transmitted signal in a way that satisfies the **Nyquist criterion** for **zero intersymbol interference (ISI)** in a communication system. It states that the transmitted pulses must be designed in such a way that the pulse shape and spacing allow for perfect recovery of the original symbols at the receiver without any interference between adjacent symbols.

The NP for a given modulation technique/communication system/channel is typically designed specifically for it, so as to ensure zero ISI in transmission, meaning that the received pulses do not overlap in time and cause interference.

The most common forms of the NP are defined in the **frequency domain as** the NP with a raised cosine (RC, NP-SC) or with a square root raised cosine (SRRC NP-SRRC). In the time domain however, one can define the NP as follows:

$$p(t) = \frac{sin(\pi t/T)}{\pi t/T} \cdot \frac{cos(\pi \alpha t/T)}{1 - (2\alpha t/T)^2} \tag{5}$$

Where:

- $p(t)$ is the pulse shape
- T is the symbol period (the time interval in which a single symbol is transmitted).
- α is the **roll-off factor**.

The **roll-off period**, α, determines the sharpness of the pulse’s frequency response and must be a value between 0 and 1, **with $\alpha = 0$ giving a rect pulse shape and $\alpha = 1$ giving a perfect sinc pulse shape.** A commonly used value for α is 0.5, which corresponds to a *raised cosine* pulse.

The change in the roll-off parameter gives a trade of in the performance of the system, a **higher roll-off factor** results in a **narrower bandwidth**, which can help in **conserving bandwidth but increasing ISI.** On the other hand, a **lower roll-off factor** increases the bandwidth, which can **reduce intersymbol interference but requires a larger occupied bandwidth.**

Quick note on the idea of a symbol: A symbol is the basic building blocks of digital communication systems,it can represent a single bit or a combination of bits, depending on the modulation scheme. For example, in binary modulation schemes like Binary Phase-Shift Keying (BPSK), each symbol represents one bit (0 or 1). In higher-order modulation schemes like Quadrature Amplitude Modulation (QAM), each symbol represents multiple bits.

Considering PAM base-band modulation, describe the Nyquist pulses and the role of the roll-off parameter.

ANSWER: As above, the Nyquist Pulse (NP) is a concept/technique used to shape a transmitted signal in a way that satisfies the **Nyquist criterion** for **zero intersymbol interference (ISI)**. PAM (Pulse Amplitude Modulation) is a modulation scheme where the amplitude of a pulse is varied to represent different symbols. as for NP in the context of PAM baseband modulation, first let's start with the basic, general definition of a digital pulse for transmitting information:

$$p(t) = \sum_{n=-\infty}^{\infty} A_n p_0(t - nT) \tag{6}$$

Thus, this is a discretised, time-series function, Where:

- A_n is the amplitude of the n^{th} pulse.
- p_0 is the basic pulse shape (often a rectangular pulse, but it can be other shapes).
- T is the pulse duration or the time between the start of successive pulses.

For a basic pulse shape, i.e. a rect, the roll-off parameter controls the sharpness of the pulse's frequency response, affecting the bandwidth and ISI of transmitted signals. In other words it is a trade-off between bandwidth efficiency and susceptibility to ISI.

As stated above, the raised cosine pulse is a commonly used Nyquist pulse shape in PAM, and could be given by something like:

$$p(t) = \frac{sin(\pi t/T)}{\pi t/T} \cdot cos(\pi \alpha t/T) \tag{7}$$

The fist term in the equation, $\frac{sin(\pi t/T)}{\pi t/T}$, is the sinc-like function, it is required as part of the Nyquist criterion which gives requirements for zero ISI.

The roll-off parameter, α adjusts the transition bandwidth of the pulse. A higher α results in a slower roll-off in the time domain, meaning the pulse occupies a larger bandwidth in the frequency domain, while a lower α leads to a faster roll-off in time-domain, reducing the bandwidth in frequency-domain but potentially increasing or causing ISI.

1.2 OFDM Questions

1.2.1 A Quick Intro/Reminder to/on OFDM

OFDM is a digital modulation technique widely used in modern wireless communication systems. It is particularly well-suited for high-data-rate transmission over frequency-selective fading channels. OFDM divides the available spectrum into multiple orthogonal subcarriers, allowing simultaneous transmission of data on each subcarrier. This parallel transmission provides robustness against frequency-selective fading and enables efficient spectrum utilization.
Key Equations and Functionality:
1. **Generation of OFDM Signal in Time Domain:**
The time-domain signal $x(t)$ in an OFDM system can be generated by the Inverse Fast Fourier Transform (IFFT) of the modulated data symbols:

$$x(t) = \text{IFFT}\left(X_k\right)$$

where X_k represents the complex modulation symbols for each subcarrier.
2. **Modulation and Demodulation:**
The modulation of data onto each subcarrier is typically done using complex modulation schemes such as Quadrature Amplitude Modulation (QAM) or Phase Shift Keying (PSK). The received signal is demodulated to recover the original data.
3. **Orthogonality of Subcarriers:**
The key feature of OFDM is the orthogonality between subcarriers. The frequency spacing between subcarriers is chosen such that the subcarriers are orthogonal to each other. This orthogonality minimizes interference between subcarriers.
4. **Frequency-Domain Representation:**
In the frequency domain, the OFDM signal is represented as a sum of individual subcarriers:

$$X_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n e^{-j2\pi kn/N}$$

where N is the number of subcarriers, x_n is the value of the time-domain signal at sample n, and k is the index of the subcarrier.
5. **Cyclic Prefix:**
A cyclic prefix is often added to the beginning of each OFDM symbol. This copy of the end part of the symbol provides a guard interval, helping to mitigate the effects of multipath interference.
6. **Equalization:**
Channel equalization is performed at the receiver to compensate for the effects of the channel. The receiver estimates the channel response and applies frequency-domain equalization.
7. **FFT Operation:**

At the receiver, a Fast Fourier Transform (FFT) is applied to the received signal to convert it back to the frequency domain. This process is crucial for demodulating the data from individual subcarriers.
Overall, OFDM provides a robust and efficient method for high-speed data transmission, especially in environments with challenging channel conditions. The orthogonality of subcarriers, the use of a cyclic prefix, and advanced signal processing techniques contribute to the success of OFDM in various communication standards such as Wi-Fi, LTE, and 5G.

1.2.2 OFDM Basic Idea Questions

Question: Describe the analytical expression of an OFDM symbol.

ANSWER: Orthogonal Frequency Division Multiplexing (OFDM) is a modulation scheme widely used in modern communication systems. An OFDM symbol can be described analytically in the time domain as follows:
The time-domain representation of an OFDM symbol is given by the Inverse Discrete Fourier Transform (IDFT) of the modulated data symbols. The basic expression for an OFDM symbol $x(t)$ with N subcarriers can be written as:

$$x(t) = \sum_{k=0}^{N-1} X_k e^{j2\pi f_k t}$$

where: - X_k is the complex modulation symbol for the k-th subcarrier, - f_k is the frequency of the k-th subcarrier, - j is the imaginary unit.
In the frequency domain, the OFDM symbol can be represented as a set of subcarriers. The complex baseband signal X_k for each subcarrier can be obtained by the Discrete Fourier Transform (DFT) of the time-domain symbol. The expression for X_k is given by:

$$X_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n e^{-j2\pi kn/N}$$

where: - x_n is the value of the time-domain signal at sample n.
The above equations highlight the key principles of OFDM, where data is modulated onto multiple orthogonal subcarriers, and the time-domain signal is the sum of these modulated subcarriers. The orthogonality between subcarriers is a crucial aspect of OFDM that allows for efficient spectrum utilization and simplifies the equalization process.

Question: Describe the block diagram of an OFDM encoder and decoder.

ANSWER: The block diagram of an OFDM (Orthogonal Frequency Division Multiplexing) system typically involves two main components: the OFDM encoder (transmitter) and the OFDM decoder (receiver). Let's break down each of them:

OFDM Encoder (Transmitter):
The OFDM encoder is responsible for converting input data into the OFDM symbol, modulating it onto subcarriers, and preparing it for transmission.
1. **Serial to Parallel Conversion:** - Input data is often in a serial format. The first block converts this serial data stream into parallel data streams.

$$x_1, x_2, \ldots, x_N$$

2. **Mapping to Subcarriers:** - Each parallel data stream is mapped to a specific subcarrier. This mapping is typically done using complex modulation schemes like Quadrature Amplitude Modulation (QAM) or Phase Shift Keying (PSK).

$$X_1, X_2, \ldots, X_N$$

3. **IFFT (Inverse Fast Fourier Transform):** - The parallel data streams are then fed into an IFFT block to convert them from the frequency domain to the time domain. This process generates the time-domain OFDM symbol.

$$x(t) = \text{IFFT}(X_1, X_2, \ldots, X_N)$$

4. **Cyclic Prefix Addition:** - A cyclic prefix is often added to the beginning of the OFDM symbol to mitigate the effects of multipath interference. This involves copying the end part of the symbol and adding it to the beginning.

$$x_{\text{with prefix}}(t)$$

OFDM Decoder (Receiver):
The OFDM decoder is responsible for recovering the original data from the received OFDM symbol.
1. **Cyclic Prefix Removal:** - The received signal is first processed to remove the cyclic prefix.

$$x(t)$$

2. **FFT (Fast Fourier Transform):** - The time-domain signal is then passed through an FFT block to convert it from the time domain back to the frequency domain.

$$X_1, X_2, \ldots, X_N$$

3. **Subcarrier Demapping:** - The data on each subcarrier is demapped, reversing the process of

modulation.

$$x_1, x_2, \dots, x_N$$

4. **Parallel to Serial Conversion:** - Finally, the parallel data streams are converted back to a serial format.

$$\text{Output Data Stream}$$

The above steps represent a high-level overview of the OFDM encoding and decoding processes. Real-world implementations may include additional steps for synchronization, channel estimation, and error correction.

1.2.3 Channel Equalization, Cyclic Prefix, and Pilot Carriers Questions

Question: Describe the channel equalization procedure performed in the OFDM modulation systems.

ANSWER: In OFDM (Orthogonal Frequency Division Multiplexing) systems, channel equalization is a crucial step in the receiver to compensate for the effects of the channel and improve the overall performance of the communication system. The channel introduces distortions, such as frequency-selective fading and intersymbol interference (ISI), which can degrade the received signal. Channel equalization aims to mitigate these effects and recover the transmitted data accurately. Here's a general procedure for channel equalization in OFDM systems:
1. Channel Estimation:
Before equalization, the receiver needs to estimate the characteristics of the channel. This involves obtaining information about the channel's frequency response, which may vary across different subcarriers in an OFDM system. Channel estimation is typically performed using known training symbols that are embedded in the transmitted signal.
2. Frequency-Domain Equalization:
Once the channel characteristics are estimated, frequency-domain equalization is applied. This involves dividing the received signal by the estimated frequency response of the channel. The equalized frequency-domain symbols are then obtained by dividing the received symbols by the corresponding channel frequency response:

$$Y_k^{\text{equalized}} = \frac{Y_k}{H_k}$$

where: - Y_k is the received symbol on the k-th subcarrier, - H_k is the estimated channel frequency response for the k-th subcarrier, - $Y_k^{\text{equalized}}$ is the equalized symbol on the k-th subcarrier.
3. Time-Domain Equalization:
After frequency-domain equalization, the signal is transformed back to the time domain using an Inverse Fast Fourier Transform (IFFT). This helps in mitigating intersymbol interference (ISI) caused by the channel:

$$y(t) = \text{IFFT}(Y_k^{\text{equalized}})$$

4. Decision Feedback Equalization (Optional):
In some cases, decision feedback equalization can be employed to further improve performance. This technique uses feedback from the detected symbols to refine the equalization process.
5. Demodulation and Decoding:
Finally, the equalized signal is demodulated, and the data is decoded to recover the original transmitted information.
It's important to note that channel conditions may vary over time, so adaptive algorithms are often used for channel estimation and equalization to continuously update the equalization parameters based on the changing channel characteristics. Advanced equalization techniques, such as Maximum Likelihood Sequence Estimation (MLSE) or iterative algorithms, may also be employed for improved performance in challenging channel conditions.

Question: Describe in some detail the channel equalization procedure used in the OFDM modulation systems, indicating the role of the cyclic prefix and of the pilot carriers.

ANSWER: In OFDM (Orthogonal Frequency Division Multiplexing) systems, channel equalization is crucial for mitigating the effects of the channel, including frequency-selective fading and intersymbol interference (ISI). The cyclic prefix and pilot carriers play significant roles in the channel equalization process. Here's a more detailed description of the channel equalization procedure in OFDM systems, incorporating these elements:
1. **Cyclic Prefix (CP) and Guard Interval:** - The cyclic prefix is a copy of the end part of the OFDM symbol that is added to the beginning. It helps combat ISI by providing a guard interval between symbols, allowing time for multipath signals to settle before the next symbol arrives. - The guard interval is also known as the cyclic extension. The CP assists in transforming the frequency-selective channel into a flat fading channel, simplifying equalization.
2. **Pilot Carriers:** - Pilot carriers are known symbols inserted into the OFDM symbol and transmitted over each OFDM symbol. These symbols are typically known to both the transmitter and receiver and are used for channel estimation. - The positions of the pilot carriers are predefined, and their known values allow the receiver to estimate the channel response at those specific frequencies.
3. **Channel Estimation:** - During the training or known pilot symbols, the receiver measures the received signal on the pilot carriers and compares it to the known transmitted values. This process helps estimate the frequency response of the channel at those pilot carrier frequencies. - The estimated channel response is then interpolated to obtain frequency response estimates for all subcarriers.
4. **Frequency-Domain Equalization:** - The frequency-domain equalization involves dividing the received signal in the frequency domain by the estimated channel frequency response for each subcarrier. The equalized frequency-domain symbols are obtained by dividing the received symbols by the corresponding

channel frequency response:

$$Y_k^{\text{equalized}} = \frac{Y_k}{H_k}$$

where: - Y_k is the received symbol on the k-th subcarrier, - H_k is the estimated channel frequency response for the k-th subcarrier, - $Y_k^{\text{equalized}}$ is the equalized symbol on the k-th subcarrier.
5. **Inverse Fast Fourier Transform (IFFT):** - After frequency-domain equalization, the signal is transformed back to the time domain using an Inverse Fast Fourier Transform (IFFT). This helps mitigate ISI caused by the channel:

$$y(t) = \text{IFFT}(Y_k^{\text{equalized}})$$

6. **Decision Feedback Equalization (Optional):** - Decision feedback equalization can be employed, where feedback from the detected symbols is used to refine the equalization process further.
7. **Demodulation and Decoding:** - The equalized signal is then demodulated, and the data is decoded to recover the original transmitted information.
By incorporating the cyclic prefix and pilot carriers, OFDM systems enhance their resilience to channel impairments, simplify the equalization process, and enable accurate estimation of the channel frequency response for effective equalization.

1.2.4 OFDM Advantages and Disadvantages Questions

Question: Discuss about the main advantages and problems of the OFDM modulation systems, motivating the answers.

ANSWER: Advantages of OFDM (Orthogonal Frequency Division Multiplexing) Modulation Systems:
1. **Spectral Efficiency:** - One of the primary advantages of OFDM is its high spectral efficiency. By dividing the available spectrum into multiple orthogonal subcarriers, OFDM enables efficient use of the frequency domain, allowing for high data rates.
2. **Resistance to Frequency Selective Fading:** - OFDM is well-suited for environments with frequency-selective fading and multipath interference. The use of multiple subcarriers helps mitigate the effects of fading on specific frequencies, improving the overall system performance.
3. **Robustness to Interference:** - OFDM exhibits robustness to narrowband interference. Since interference affects only a subset of the subcarriers, the overall impact on the system is minimized. This makes OFDM suitable for environments with various types of interference.
4. **Cyclic Prefix for Channel Equalization:** - The cyclic prefix aids in channel equalization by providing a guard interval between symbols. This mitigates intersymbol interference and simplifies the equalization process, particularly in the presence of multipath propagation.
5. **Flexibility in Resource Allocation:** - OFDM allows for flexible resource allocation, enabling adaptive modulation and coding schemes for different subcarriers based on channel conditions. This adaptability

enhances system performance under varying channel conditions.
6. **Ease of Implementation:** - OFDM is well-suited for implementation using Fast Fourier Transform (FFT) algorithms, making it computationally efficient. This ease of implementation contributes to its widespread adoption in various communication standards.
Problems and Challenges of OFDM Modulation Systems:
1. **High Peak-to-Average Power Ratio (PAPR):** - OFDM signals often exhibit a high Peak-to-Average Power Ratio, which can lead to inefficiencies in power amplification. This requires additional measures, such as peak reduction techniques, to avoid signal distortion and improve power efficiency.
2. **Sensitivity to Frequency Offset:** - OFDM is sensitive to carrier frequency offsets, which can result from oscillator inaccuracies. Frequency synchronization becomes crucial to avoid degradation in performance due to frequency offset.
3. **Impulse Noise Sensitivity:** - OFDM systems may be sensitive to impulse noise, which can lead to errors in the received signal. Robust error correction and detection mechanisms are necessary to handle such noise.
4. **Complex Receiver Design:** - The receiver in an OFDM system is often complex, requiring synchronization and channel estimation mechanisms. This complexity can increase the cost of implementation and pose challenges in real-time processing.
5. **Channel Estimation Errors:** - Accurate channel estimation is essential for effective equalization. Errors in channel estimation, especially in fast-changing channels, can lead to performance degradation. Sophisticated algorithms are required to mitigate these errors.
6. **Limited Performance in Highly Mobile Environments:** - In highly mobile environments, rapid changes in channel conditions can pose challenges for OFDM systems. Doppler shifts and time-varying channels may require additional techniques to maintain reliable communication.
In summary, while OFDM offers several advantages, it also faces challenges related to power efficiency, sensitivity to frequency offsets, and the complexity of receiver design. Addressing these challenges often involves the use of advanced signal processing techniques and error mitigation strategies. Despite these issues, OFDM remains a widely adopted modulation scheme in many wireless communication standards due to its overall efficiency and adaptability.

1.3 DSSS/CDMA Questions

1.3.1 DSSS Basic Ideas Questions

Introduction to Direct Sequence Spread Spectrum (DSSS):
Direct Sequence Spread Spectrum (DSSS) is a digital modulation technique commonly used in wireless communication systems. The fundamental principle of DSSS involves spreading the information signal over a much wider bandwidth than the minimum required for transmission. This spreading is achieved by modulating the information signal with a pseudorandom noise sequence known as the spreading code. The spreading code effectively ”spreads” the signal in the frequency domain, making it appear as noise to systems that are not equipped with the correct code. At the receiver, the original signal is recovered by despreadng, which involves multiplying the received signal by the same spreading code used at the transmitter. The use of spreading codes provides several advantages, including resistance to interference and improved security.
Key Equations and Functions:
1. **Spread Signal (Transmitter Side):** The transmitted signal in DSSS ($x(t)$) is the product of the information signal ($s(t)$) and the spreading code ($c(t)$):

$$x(t) = s(t) \cdot c(t)$$

Here, $x(t)$ is the spread signal, $s(t)$ is the original information signal, and $c(t)$ is the spreading code.
2. **Received Signal (Channel Model):** In the presence of noise ($n(t)$) and channel effects, the received signal ($r(t)$) can be expressed as:

$$r(t) = h(t) \cdot x(t) + n(t)$$

where $h(t)$ represents the channel impulse response.
3. **Despreading (Receiver Side):** To recover the original information signal ($\hat{s}(t)$), the received signal is multiplied by the same spreading code used at the transmitter:

$$\hat{s}(t) = \int_{-\infty}^{\infty} r(t) \cdot c(t)\, dt$$

This process effectively correlates the received signal with the spreading code, isolating the original information signal.
4. **Processing Gain:** The effectiveness of DSSS is often quantified by the processing gain (G), which is the ratio of the spread bandwidth to the information bandwidth:

$$G = \frac{B_{\text{spread}}}{B_{\text{info}}}$$

Here, B_{spread} is the bandwidth of the spread signal, and B_{info} is the bandwidth of the original information signal.

In summary, DSSS involves spreading the information signal over a wide bandwidth using a pseudorandom spreading code. This spreading provides benefits such as increased resistance to interference, improved security, and robustness in challenging communication environments. The key equations describe the modulation, channel model, despreading, and the processing gain associated with DSSS.

Code Division Multiple Access (CDMA):

Code Division Multiple Access (CDMA) is a digital cellular technology that allows multiple users to share the same frequency band simultaneously. CDMA uses a spread spectrum technique, where each user is assigned a unique code to differentiate their signals. This enables multiple users to transmit and receive data concurrently without interference.

Basic Idea:

In CDMA, each user's signal is spread over a wide frequency band using a unique code. This spreading process makes CDMA signals appear as noise to systems using different codes, allowing multiple signals to coexist in the same frequency band.

Mathematical Representation:

Let $s_i(t)$ be the signal for user i, and $c_i(t)$ be the unique code assigned to that user. The transmitted signal for user i is given by the product of the data signal and the unique code:

$$x_i(t) = s_i(t) \cdot c_i(t)$$

The total transmitted signal in a CDMA system with N users is the sum of the individual user signals:

$$x(t) = \sum_{i=1}^{N} s_i(t) \cdot c_i(t)$$

Signal Reception:

At the receiver, the received signal $r(t)$ is multiplied by the code assigned to the desired user j and integrated over the symbol duration:

$$\text{Received Signal for User } j : \quad y_j = \int_{T_s} r(t) \cdot c_j(t)\, dt$$

where: - T_s is the symbol duration.

The received signal is then compared to a threshold to determine the presence or absence of the signal.

Orthogonality:

One key feature of CDMA is the use of orthogonal codes. If $c_i(t)$ and $c_j(t)$ are the codes for users i and j, the cross-correlation between these codes is ideally zero:

$$\int_{T_S} c_i(t) \cdot c_j(t)\, dt = 0$$

This orthogonality property minimizes interference between users.
Advantages:
- **Soft Capacity:** CDMA allows for a large number of users to share the same frequency band.
- **Security:** The use of unique codes for each user provides a level of security as signals from other users appear as noise.
- **Robustness to Fading:** CDMA is resilient to multipath fading, enhancing the reliability of wireless communication.
In summary, CDMA is a multiple access technique that uses unique codes to enable multiple users to share the same frequency band simultaneously. The orthogonality of the codes minimizes interference, making CDMA suitable for cellular communication systems.

Question: Describe the basic ideas used in the CDMA systems, giving an idea about its performance.

ANSWER: Certainly! Code Division Multiple Access (CDMA) is a digital cellular technology that allows multiple users to share the same frequency band simultaneously. The basic idea behind CDMA is to assign a unique code to each user, and these codes are used to separate and identify different users in the same frequency band.
Let's denote $x_i(t)$ as the transmitted signal for the i-th user, and $c_i(t)$ as the unique code assigned to that user. The transmitted signal for the i-th user can be expressed as:

$$x_i(t) = c_i(t) \cdot s_i(t)$$

where $s_i(t)$ is the information signal for the i-th user.
All users in the system transmit at the same time and on the same frequency, but each user's signal is spread over the entire bandwidth using the unique code assigned to them. The received signal at the base station can be expressed as the sum of the transmitted signals from all users:

$$r(t) = \sum_i x_i(t) = \sum_i c_i(t) \cdot s_i(t)$$

The receiver uses the knowledge of the codes to separate the signals of different users. To extract the i-th user's signal, the received signal is multiplied by the corresponding code, and then integrated over time:

$$\hat{s}_i(t) = \int_{-\infty}^{\infty} r(t) \cdot c_i(t)\, dt$$

CDMA has some key advantages:

1. **Capacity**: CDMA allows multiple users to share the same frequency band simultaneously, leading to increased system capacity.
2. **Robustness**: CDMA is resistant to interference because each user's signal is spread over the entire bandwidth, and interference from other users appears as background noise.
3. **Security**: The use of unique codes for each user provides a level of security, as unauthorized users without the correct code will have difficulty decoding the transmitted signals.
The performance of CDMA is influenced by factors such as the signal-to-noise ratio (SNR), the number of users, and the quality of the codes used. As more users are added to the system, the capacity increases, but the SNR requirements also increase. CDMA systems often use techniques like power control to manage the interference and maintain a balance between system capacity and performance.

Question: Define and describe the basic properties of the m-sequences. Why are this seq. used in the DSSS modulation?

ANSWER: M-sequences, or maximal-length sequences, are a type of pseudorandom binary sequence with certain unique properties. They are extensively used in Direct Sequence Spread Spectrum (DSSS) modulation, a technique commonly employed in wireless communication systems. Here are the basic properties of m-sequences and an explanation of their use in DSSS modulation:
Basic Properties of M-sequences:
1. **Maximum Length:** M-sequences are binary sequences that achieve the maximum possible length for a register of a given size. In other words, an m-sequence generated by a shift register of n stages has a length of $2^n - 1$. This property means that the sequence repeats after $2^n - 1$ bits, exhibiting a pseudo-random behavior.
2. **Balanced Autocorrelation:** The autocorrelation function of an m-sequence is balanced, except for a peak at zero shift. This balanced autocorrelation property is crucial for spread spectrum applications, as it helps to minimize interference between different users or signals.
3. **White Noise-Like Spectrum:** The power spectrum of an m-sequence is flat over a wide range of frequencies, making it similar to white noise. This property is desirable in spread spectrum systems because it allows signals to be spread across a broad frequency band, making them less susceptible to interference and more robust against channel impairments.
Use in DSSS Modulation:
DSSS is a spread spectrum modulation technique where the data signal is multiplied (or modulated) by a pseudorandom spreading sequence, typically an m-sequence. The key reasons for using m-sequences in DSSS modulation are:
1. **Increased Bandwidth:** By multiplying the data signal with a pseudorandom spreading sequence, the bandwidth of the transmitted signal is increased. This wide bandwidth helps in achieving resistance against narrowband interference and frequency-selective fading.
2. **Interference Rejection:** The balanced autocorrelation property of m-sequences ensures that the correlation between different sequences (used by different users or in different time instances) is low, leading

to effective rejection of interference. This is crucial in environments where multiple users share the same frequency band.
3. **Improved Security:** The pseudo-random nature of m-sequences provides a level of security for the communication system. Without knowledge of the spreading sequence, it is challenging for unauthorized receivers to demodulate and recover the original data.
In summary, m-sequences in DSSS modulation contribute to increased bandwidth, interference rejection, and enhanced security, making them a suitable choice for spread spectrum communication systems.

1.3.2 Multi-Path Fading and Rake Receiver Questions

Question: Describe why and when a DSSS modulation system is robust against multi-path fading.

ANSWER: Direct Sequence Spread Spectrum (DSSS) modulation systems exhibit robustness against multi-path fading due to the inherent characteristics of spread spectrum techniques. Here's an explanation of why and when DSSS modulation is robust against multi-path fading:
Why DSSS is Robust Against Multi-Path Fading:
1. **Frequency Diversity:** DSSS spreads the signal over a wide bandwidth using a pseudorandom spreading sequence. The use of a broad frequency band provides frequency diversity, meaning that even if certain frequency components of the signal experience fading due to multi-path propagation, other frequency components may still be intact. This diversity helps mitigate the impact of selective fading.
2. **Processing Gain:** DSSS systems achieve "processing gain" by spreading the signal over a much wider bandwidth than the information bandwidth. The processing gain is directly related to the ratio of the total bandwidth to the information bandwidth. This gain helps in recovering the original signal even in the presence of noise and fading. The processing gain improves the system's performance in challenging channel conditions, including those with multi-path fading.
3. **Low Probability of Intercept:** The pseudo-random nature of the spreading sequence in DSSS results in a low probability of intercept by unintended receivers. This characteristic makes DSSS signals less susceptible to interference and allows them to maintain performance even in environments with multi-path fading.
When DSSS is Robust Against Multi-Path Fading:
1. **Wide Bandwidth:** DSSS is most effective in scenarios where a wide bandwidth is available. The use of a wide bandwidth allows the spread spectrum signal to occupy multiple frequency components, increasing the chances that at least some of these components will not be affected by fading.
2. **Multipath Delay Spread:** DSSS is particularly robust in environments with moderate to long multipath delay spread. In such cases, the spreading sequence helps combat the effects of time dispersion by spreading the signal over a longer duration. This reduces the impact of intersymbol interference caused by delayed signal components arriving at different times.
3. **Coding Techniques:** Combining DSSS with error-correcting coding techniques further enhances its robustness. Error-correcting codes can mitigate the impact of fading-induced errors, improving the overall reliability of the communication system.
In summary, DSSS modulation systems are robust against multi-path fading due to their use of wide

bandwidth, processing gain, frequency diversity, and resistance to interception. These characteristics make DSSS a suitable choice for communication in environments where multi-path fading is a significant challenge.

Question: Describe the basic idea of the Rake Receiver, indicating also why this is working properly in the case of DSSS modulation.

ANSWER: The Rake Receiver is a receiver structure designed to combat the effects of multipath fading in wireless communication systems. It is particularly effective in scenarios where Direct Sequence Spread Spectrum (DSSS) modulation is employed. The basic idea behind the Rake Receiver is to exploit the diversity provided by multipath channels by combining the signals from different paths.
Basic Idea of the Rake Receiver:
1. **Multipath Fading:** In a wireless communication environment, the transmitted signal often reaches the receiver via multiple paths due to reflections, diffractions, and scattering. Each of these paths may experience different attenuation and phase shifts, leading to variations in the received signal strength over time.
2. **Combining Paths:** The Rake Receiver consists of multiple ”fingers” or branches, each corresponding to a different path that the signal might take to reach the receiver. Each finger is essentially a demodulator that processes a delayed version of the received signal, aligning it with the different propagation delays of the multipath components.
3. **Maximal Ratio Combining (MRC):** The Rake Receiver employs a technique called Maximal Ratio Combining (MRC) to combine the signals from different fingers. MRC assigns weights to each finger based on the received signal strength, taking into account the varying signal-to-noise ratios of different paths. This weighting maximizes the signal power and minimizes the effect of noise and interference.
4. **Improving Signal Quality:** By combining the contributions from different paths, the Rake Receiver effectively improves the overall signal quality. This is especially crucial in scenarios where some paths may experience fading or interference, as the contributions from other, less affected paths help mitigate these effects.
Why Rake Receiver Works Well with DSSS Modulation:
1. **Spread Spectrum and Processing Gain:** DSSS modulation provides processing gain by spreading the signal over a wide bandwidth. This characteristic enhances the ability of the Rake Receiver to distinguish and combine signals arriving via different paths. The spread spectrum nature of DSSS allows for effective exploitation of diversity.
2. **Resilience to Frequency-Selective Fading:** In multipath environments, different paths may experience frequency-selective fading, where certain frequency components are more attenuated than others. DSSS, with its wide bandwidth, helps to combat frequency-selective fading, and the Rake Receiver leverages this characteristic to combine the diverse signals.
3. **Time Dispersion Mitigation:** Multipath propagation can cause time dispersion, leading to intersymbol interference. The Rake Receiver, by processing signals from different paths independently, effectively mitigates the impact of time dispersion associated with multipath fading.
In summary, the Rake Receiver is well-suited for DSSS modulation because it leverages the diversity provided by multipath channels. By combining signals from different paths, the Rake Receiver improves the robustness

and reliability of the communication system, particularly in environments where multipath fading is a significant factor.

1.4 CPM Questions

Continuous Phase Modulation (CPM):
Continuous Phase Modulation is a type of modulation where the phase of the carrier signal is varied continuously in proportion to the message signal, as opposed to discrete changes as in phase-shift keying (PSK). One of the well-known CPM schemes is the Minimum Shift Keying (MSK).
Key Equations:
1. **Phase Modulation (PM):**

$$s(t) = A_c \cos\left(2\pi f_c t + k_p m(t)\right)$$

where: - $s(t)$ is the modulated signal. - A_c is the carrier amplitude. - f_c is the carrier frequency. - k_p is the phase sensitivity factor. - $m(t)$ is the message signal.
2. **Minimum Shift Keying (MSK):**

$$s(t) = A_c \cos\left(2\pi f_c t + \frac{\pi}{2} \sum_{n=0}^{T_b} \left[1 - 2m(t - nT_b)\right]\right)$$

where: - T_b is the bit duration.
3. **Phase Deviation ($\Delta\phi$) in MSK:**

$$\Delta\phi = \frac{\pi}{2}$$

MSK maintains a constant phase deviation of $\frac{\pi}{2}$ to achieve constant-envelope modulation.
Explanation:
In CPM, the phase of the carrier is directly proportional to the instantaneous amplitude of the message signal. MSK is a particular form of CPM that ensures constant phase deviation, leading to constant-envelope modulation. This constant-envelope property makes MSK particularly attractive for power-efficient communication.
The key idea in CPM is to achieve a smooth transition in phase, which results in better spectral efficiency and lower out-of-band interference compared to other modulation schemes like PSK. The continuous variation of phase helps in achieving a more bandwidth-efficient transmission.
Continuous Phase Modulation is widely used in various communication systems, especially in situations where spectral efficiency and power efficiency are crucial design factors.

Question: Describe the analytical expression of the modulated signal in case of MSK modulation

ANSWER: Certainly! In Minimum Shift Keying (MSK), the modulated signal is characterized by a continuous phase shift that changes linearly with time. The analytical expression for the MSK-modulated signal can be written as follows:
The MSK-modulated signal is given by:

$$s(t) = A_c \cos\left(2\pi f_c t + \frac{\pi}{2}\sum_{n=0}^{T_b}\left[1 - 2m(t - nT_b)\right]\right)$$

Here, the parameters are defined as:
- $s(t)$ is the modulated signal. - A_c is the carrier amplitude. - f_c is the carrier frequency. - t is time. - T_b is the bit duration. - $m(t)$ is the binary message signal, taking values of +1 or -1.
The term $\frac{\pi}{2}\sum_{n=0}^{T_b}\left[1 - 2m(t - nT_b)\right]$ represents the phase modulation component. In MSK, the phase of the carrier signal changes linearly with time, and the factor of $\frac{\pi}{2}$ ensures that the phase deviation is constant and equal to $\frac{\pi}{2}$.
This linear phase change over time is what distinguishes MSK from other modulation schemes and provides it with constant-envelope properties, making it particularly useful in applications where power efficiency is critical.

Question: Draw the phase-tree in the case of MSK modulation.

ANSWER: In the context of Minimum Shift Keying (MSK) modulation, the concept of a ”phase tree” is often used to visualize the continuous phase transitions that occur in the modulation process. The phase tree helps in understanding how the phase of the modulated signal evolves over time.
Phase Tree in MSK:
MSK uses a phase modulation scheme that ensures a constant phase deviation of $\frac{\pi}{2}$ radians, leading to a constant-envelope signal. The phase tree illustrates the phase transitions for each symbol in the MSK signal.
1. **Binary Phase Shifts:** - MSK is a type of continuous phase modulation where the phase of the carrier signal shifts in response to the binary message signal. - The binary message signal ($m(t)$) typically takes values of +1 or -1.
2. **Linear Phase Transitions:** - The phase of the carrier signal changes linearly with time. - For each symbol duration (T_b), the phase experiences a linear transition.
3. **Constant Phase Deviation:** - The key feature of MSK is that the phase deviation remains constant at $\frac{\pi}{2}$ radians. - This constant phase deviation is essential for maintaining a constant envelope.
Visualization:
A phase tree is often represented graphically, with branches showing the continuous phase transitions for different symbols. At each branch point, the phase undergoes a linear change, and the separation between branches represents the phase deviation.
Mathematically:
The phase tree can be expressed mathematically by considering the cumulative phase shift over time. For MSK, the cumulative phase shift for a given symbol period T_b is $\frac{\pi}{2}$.

$$\text{Cumulative Phase Shift} = \frac{\pi}{2}$$

This cumulative phase shift is distributed over the duration of the symbol, resulting in linear phase transitions. In summary, the phase tree in MSK provides a visual representation of the continuous phase transitions that occur in the modulation process. It helps in understanding how the phase of the carrier signal evolves over time, ensuring constant phase deviation and a constant envelope in the MSK-modulated signal.

Question: Determine the analytical expression of the likelihood function that should be maximized by the optimal receiver in the case of MSK modulation.

ANSWER: The likelihood function for MSK modulation is derived based on the statistical properties of the received signal. The optimal receiver aims to maximize the likelihood function to make decisions about the transmitted symbols. In MSK, the likelihood function is typically derived under the assumption of additive white Gaussian noise (AWGN) and is based on the received phase.
Assuming a binary message signal $m(t)$ that takes values of +1 or -1, the received MSK-modulated signal $r(t)$ with additive white Gaussian noise $n(t)$ is given by:

$$r(t) = A_c \cos\left(2\pi f_c t + \frac{\pi}{2}\sum_{n=0}^{T_b}\left[1 - 2m(t - nT_b)\right] + \phi\right) + n(t)$$

where: - A_c is the carrier amplitude. - f_c is the carrier frequency. - T_b is the bit duration. - $m(t)$ is the binary message signal. - ϕ is the initial phase. - $n(t)$ is the additive white Gaussian noise.
The likelihood function $L(m(t)|r(t))$ is the probability of observing the received signal $r(t)$ given the transmitted message $m(t)$. It is often expressed in terms of the conditional probability density function (PDF) of the received signal given the transmitted symbol.
For AWGN, the likelihood function is often proportional to the exponential of the negative squared error between the received and expected signals. The likelihood function $L(m(t)|r(t))$ can be expressed as:

$$L(m(t)|r(t)) \propto \exp\left(-\frac{|r(t) - s(t)|^2}{2\sigma^2}\right)$$

where: - $s(t)$ is the expected signal (the MSK-modulated signal without noise). - σ^2 is the noise variance.
The optimal receiver aims to maximize this likelihood function over all possible values of the transmitted symbol sequence $m(t)$. In practice, for mathematical convenience, it is often more common to work with the log-likelihood function, which simplifies calculations:

$$\ln L(m(t)|r(t)) \propto -\frac{|r(t) - s(t)|^2}{2\sigma^2}$$

Maximizing the log-likelihood function is equivalent to maximizing the likelihood function, and this process leads to the optimal receiver decision rule for MSK modulation in the presence of AWGN.

Question: Describe the analytical expression of the modulated signal in case of MSK modulation, and the relationship with binary FSK

ANSWER: In Minimum Shift Keying (MSK) modulation, the modulated signal is characterized by a continuous phase shift that changes linearly with time. The analytical expression for the MSK-modulated signal can be written as follows:
The MSK-modulated signal is given by:

$$s(t) = A_c \cos\left(2\pi f_c t + \frac{\pi}{2}\sum_{n=0}^{T_b}\left[1 - 2m(t - nT_b)\right]\right)$$

Here: - $s(t)$ is the modulated signal. - A_c is the carrier amplitude. - f_c is the carrier frequency. - t is time. - T_b is the bit duration. - $m(t)$ is the binary message signal, taking values of +1 or -1.
The term $\frac{\pi}{2}\sum_{n=0}^{T_b}\left[1 - 2m(t - nT_b)\right]$ represents the phase modulation component. In MSK, the phase of the carrier signal changes linearly with time, and the factor of $\frac{\pi}{2}$ ensures that the phase deviation is constant and equal to $\frac{\pi}{2}$.
Now, let's discuss the relationship between MSK and Binary Frequency Shift Keying (FSK):
1. **Frequency Shift Keying (FSK):** In binary FSK, the frequency of the carrier signal is shifted between two predetermined frequencies to represent binary symbols. The frequency shift occurs instantaneously at the symbol boundaries. The expression for binary FSK can be written as:

$$s_{\text{FSK}}(t) = A_c \cos\left(2\pi f_1 t\right) \quad \text{or} \quad A_c \cos\left(2\pi f_2 t\right)$$

where f_1 and f_2 are the two carrier frequencies.
2. **Relationship with MSK:** MSK can be viewed as a special case of FSK where the frequency shift is constrained to be half of the bit rate, i.e., the carrier frequency transitions at a rate of $1/(2T_b)$. In fact, MSK is often referred to as a form of continuous-phase FSK. The relationship can be expressed mathematically as follows:

$$s_{\text{MSK}}(t) = A_c \cos\left(2\pi f_c t + \phi\right)$$

where f_c is the carrier frequency, and ϕ is a constant phase offset.
In summary, while MSK and binary FSK share similarities, MSK is distinguished by its continuous phase modulation, which results in constant envelope properties. MSK can be seen as a type of FSK where the frequency transitions occur smoothly and continuously, providing advantages in terms of spectral efficiency and power efficiency.

2 Channel Coding Questions

2.1 Block Codes - Questions

2.1.1 TBD

Question: A (7,4) cyclic linear block code is described by the generator polinomial g(D) = D3 + D2 + 1. Indicate the possible code-words. Determine the generator matrix of this code, in its systematic shape.

ANSWER:

Question: Indicate the values assumed by the "sindrome" associated to a possible single bit error, a possible two bits error, a possible three bits error.

ANSWER:

Question: Consider the Hamming code with N = 127. Determine the error probability in case of both hard (use the more precise estimation) and soft decoding.

ANSWER: A block code with N = 7 is characterized by the generator polynomial: gD)= (D + 1)(D3+D+1). Determine the minimum distance of this code. Is this code a cyclic code? Determine the number of possible codewords, the error probability in case of both hard (use the more precise estimation) and soft decoding.
A (7,4) linear block code is described by the generator polinomial g(D) = D3+D+1. Indicate the values assumed by the "sindrome" associated to a possible single bit error, a possible two bits error, a possible three bits error. In case of 1 error, how many different sindromes are possible ?
A block code is described by the parity check matrix indicated in Fig. 1. – Indicate the possible code-words. Is this a cyclic code ? – What is the probability of error in case of hard and soft decision ? – What is the minimum required bandwidth (in case of binary modulation) if the information bit-rate is equal to 10 Mbit/s.
1011100 1110010 0111001
Consider the Hamming code with N = 127. Determine the error probability in case of both hard (use the more precise estimation) and soft decoding. • Design a (6, 2) cyclic code by choosing the shortest possible generator polynomial1. Determine the generator matrix G (in the systematic form) for this code and find all the possible codewords. How many errors can be corrected by this code ?
A block code is characterized by the Generator matrix given in Figure 1. Determine the possible codewords. Determine the parity check matrix, and indicate the values assumed by one of the sindromes associated to a possible single bit error, a possible two bits error, and a possible 3 bits error. How many different distinct (not the same) sindromes are possible, in case of one error ?
011011011 110110110
Consider a block code with N = 48, K = 24, d = 12. Determine the number of possible codewords, the probability of error (hard decision, using the more precise approximation), and the minimum bandwidth required to transmit 10 Mbit/sec.

2.2 Convolutional Codes - Questions

2.2.1 Consider a convolutional code with R = 1/2, and octal generators (5, 2).

Question: Determine and draw the trellis diagram of the code.

ANSWER:

Question: Determine the code word associated to the information sequence: 010101100

ANSWER:

Question: Determine the bit-error probability (considering at least 3 non zero terms in the union bound), and the minimal bandwidth required in case of an information bit-rate equal to 10 Mbit/sec.

ANSWER:

2.2.2 Consider a convolutional code with R = 1/3, and octal generators (1, 3, 2).

Question: Determine and draw the tree and the state diagrams of the code.

ANSWER:

Question: Determine the code word associated to the information sequence: 010101100.

ANSWER:

Question: Determine the bit-error probability (considering at least 3 non zero terms in the union bound), and the minimal bandwidth required in case of an information bit-rate equal to 10 Mbit/sec.

ANSWER: Consider a convolutional code with R = 1/2, and octal generators (7,5). Draw the tree diagram of the code. Determine the code word associated to the information sequence 010101100, and the min imum bandwidth required in case of an information bit-rate equal to 10 Mbit/sec. .Determine and draw the trellis diagram of the code.

2.3 Cyclic Codes - Questions

2.3.1 TBD

Question:

ANSWER:

Question:

ANSWER:

2.4 BCH Codes - Questions

2.4.1 Consider the BCH code of length N = 31 and generator polynomial (in octal description) 107657. In this code there is 1 word composed by all zeros, 155 words with 7 ones, 465 with 8 ones, 5208 with 11 ones, ...

Question: What is the generator polynomial of this code (g(D)= ...) ? Determine the number of possible codewords, and the probability of error (in case of hard and soft decision).

ANSWER:

Question: The code is extended adding a final parity check bit (imposing an even number of "1"). (a) Determine the new probability of error (in case of hard and soft decision). (b) What is the minimum required bandwidth (in case of binary modulation) if the information bit-rate is equal to 1 Mbit/s.

ANSWER:

Question: Consider the following possible codewords. (c) 00000000000001000111110101111001 is a valid codeword ? (d) 00000000000001111010111110001001 is a valid codeword ?

ANSWER:

2.5 Turbo/LDPC Code Questions

2.5.1 TBD

Question: Describe the curves that represents the performance (P(E) as a function of Eb/No) of a turbo code, indicating the role of the iterations and of the interleaver.

ANSWER:

Question: Describe the basic idea of the bit-flipping algorithm for the hard decoding of an LDCP code.

ANSWER:

Question: Describe the basic idea of the tanner graphs and the bit-flipping algorithm for the decoding of an LDCP code.

ANSWER:

Question: Describe the basic idea of the Tanner graphs, using a simple example.

ANSWER:

Question: Describe the basic idea and motivation of the EXIT charts.

ANSWER:

Question: Describe the basic idea of the tanner graphs and the bit-flipping algorithm for the decoding of an LDCP code.

ANSWER: