$\eta = 120\pi$

Formulas

$$\Omega_A = \iint_{4\pi} |F(\theta,\varphi)|^2 \, d\Omega = \int_0^{2\pi}\int_0^{\pi} |F(\theta,\varphi)|^2 \sin\theta \, d\theta \, d\varphi$$

$$D = \frac{U_m}{U_{Ave}} = \frac{\frac{P}{\Omega_A}}{\frac{P}{4\pi}} = \frac{4\pi}{\Omega_A}$$

$$\cancel{|F(\theta,\varphi)|^2 = \frac{P(r,\theta,\varphi)}{\max\{P(r,\theta,\varphi)\}} = \frac{|E|^2}{|E_{max}|^2}} = \frac{S}{\max\{S\}}$$

$$S = \frac{P_{in} \, e_r D}{4\pi R^2} = \frac{P D}{4\pi R^2}$$

$$S = \frac{1}{2\eta}|E|^2$$

$$G = e_r D$$

$$G = \frac{4\pi}{\lambda^2} A_e$$

$$\frac{U_m}{R^2} = \max\left[\underbrace{\mathrm{Re}(\vec{S}\cdot\hat{r})}_{=S}\right]$$

$$P_i = P_{in} e_r$$

$$|F(\theta,\varphi)|^2 = \frac{P(r,\theta,\varphi)}{\max\{P(r,\theta,\varphi)\}} = \frac{|E|^2}{|E_{max}|^2} = \frac{S}{\max(S)}$$

$$P_R = \iint_{\Sigma} \bar{S}\, d\bar{s}$$

$$P_R = \frac{1}{2} R_R |I|^2$$

$$P_{max\ transfer} = \frac{1}{8}\frac{|V_A|^2}{R}$$

$$P = A_e S$$

$$V_A = E \cdot h$$

$$P_R = G_T G_R \left(\frac{\lambda}{4\pi R}\right)^2 P_T$$

Ideal dipole

Δz

$$P_R = \frac{\pi}{3}\eta \frac{I^2 \Delta z^2}{\lambda^2}$$

$$R_R = 80\pi^2 \left(\frac{\Delta z}{\lambda}\right)^2$$

$$D = \frac{3}{2}$$

monopole

$$P_R = \frac{P_R(\text{dipole})}{2}$$

$$R_R = \frac{R_R(\text{dipole})}{2}$$

$$D = 2D(\text{dipole})$$

$$\delta = \frac{1}{\sqrt{\pi \mu f \sigma}}$$

$$R_D = \frac{L}{2\pi a} R_s$$

$$R_s = \sqrt{\frac{\pi \mu f}{\sigma}}$$

$$\nabla \times E = -j\omega\mu H$$

Array antennas

$$\psi = \beta d \cos\theta + \alpha$$

$$\left|\frac{AF}{N}\right| = |F(\theta,\varphi)| = \frac{\sin(N\psi/2)}{\sin(\psi/2)}$$

max $\rightarrow$ $\psi = 0, \pm 2\pi, \pm 4\pi, \pm 6\pi$

Null $\rightarrow$ $\psi = \pm \frac{2k\pi}{N}$ $\quad k \neq N, 2N, 3N$

$$\int \cos^m\theta \sin\theta \, d\theta = \qquad \begin{array}{l} x = \cos\theta \\ d(\cos\theta) = -\sin\theta \, d\theta \end{array}$$

$$\int \sin^3 d\theta = \int (1-\cos^2\theta) \sin\theta \, d\theta = \qquad \begin{array}{l} \cos\theta = x \\ d(\cos\theta) = -\sin\theta \, d\theta = dx \end{array}$$

$$\int \sin^5 d\theta = \int [1-\cos^2\theta]^2 \sin\theta \, d\theta = \qquad \begin{array}{l} \cos\theta = x \\ d(\cos\theta) = -\sin\theta \, d\theta = dx \end{array}$$

Power radiated: $P = \iint\limits_{\Sigma} \bar{S}\cdot d\bar{s} = \iint\limits_{\Omega} |\bar{S}|\, \bar{r}\cdot\bar{r}\, r^2 \sin\theta\, d\theta\, d\varphi$

Power density: $S = \frac{1}{2\eta}|E|^2$

Magnitude field pattern: $|F(\theta,\varphi)| = \frac{|E|}{|E|_{max}}$

Power pattern: $P(\theta,\varphi) = |F(\theta,\varphi)|^2$

---

o) Friss $P_R(P_T)$

Transmitting antenna

$S = \frac{P_T}{4\pi R^2}$

$\hookrightarrow$ gain $S = G_T \frac{P_T}{4\pi R^2}$

Receiving antenna

$P_R = A_{eR} S.$

$A_{eR} = G_R \frac{\lambda^2}{4\pi}$

$\Rightarrow P_R = G_T G_R \left(\frac{\lambda}{4\pi R}\right)^2 P_T$

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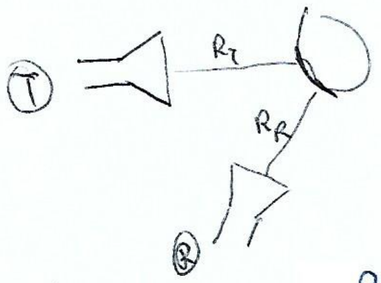
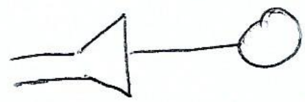
o) Radar eq. bistatic



o) Radar monostatic



$G_T = G_R = G$

$R_T = R_R = R$

Power dens incident : $S_i = G_T \frac{P_T}{4\pi R_T^2}$

Power dens. scatt : $S_R = \frac{P_i}{4\pi R_R^2}$

Cross sect : $P_i = S_i \cdot \sigma$

Power receiv : $P_R = A_e S_R$

$\left.\right\}$ $P_R = \sigma G_T G_R \frac{\lambda^2}{(4\pi)^3 R_T^2 R_R^2} P_T$

o) Linear pol. time domain

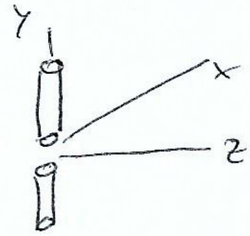
$$\mathcal{E}(z,t) = |E_{ox}| \cos(\omega t - kz)\,\hat{x} + |E_{oy}| \cos(\omega t - kz + \varphi)\,\hat{y}$$

with $\varphi = 0, \pi$, $\varphi = \varphi_y - \varphi_x$

phasor dom.

$$E(z) = \left(|E_{ox}|\,\hat{x} + |E_{oy}| e^{j\varphi}\,\hat{y}\right) e^{-jkz}$$ E_{ox}, E_{oy} complex

Source emitting Linear → Ideal dipole



o) Circular polarization time

We have $|E_{ox}| = |E_{oy}| = |E_0|$

and $\varphi = \pm \pi/2$

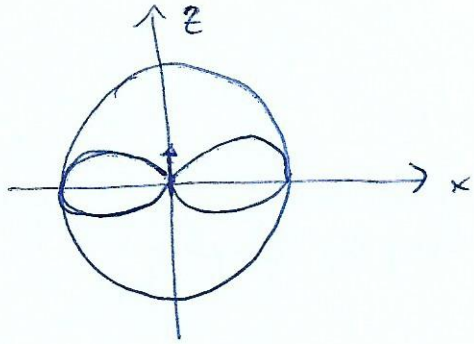
$$\mathcal{E}(z,t) = |E_0| \cos(\omega t - kz)\,\hat{x} + |E_0| \cos(\omega t - kz + \varphi)\,\hat{y}$$

phasor

$$E(z) = |E_0| \left(\hat{x} + e^{j\varphi}\,\hat{y}\right) e^{-jkz}$$

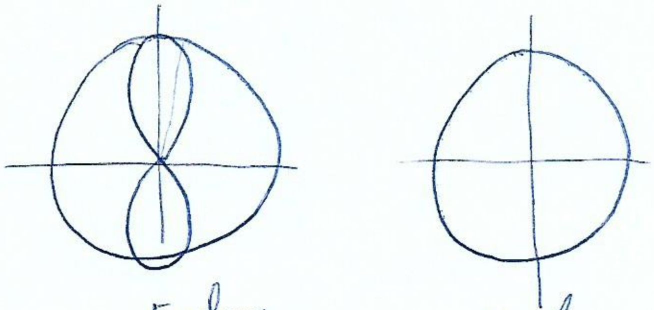
Source emitting: Crossed ideal dipole with $\frac{\lambda}{4}$ delay

a) Radiation pattern short dipole E-plane



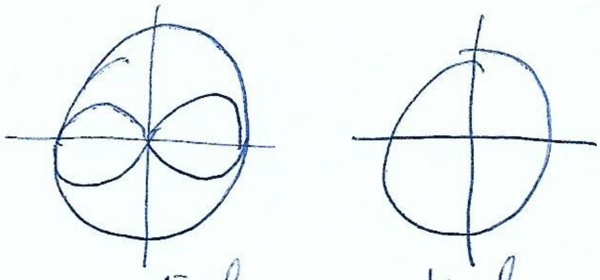
factor $I\Delta z \rightarrow \frac{I\Delta z}{2}$

Ideal dipole



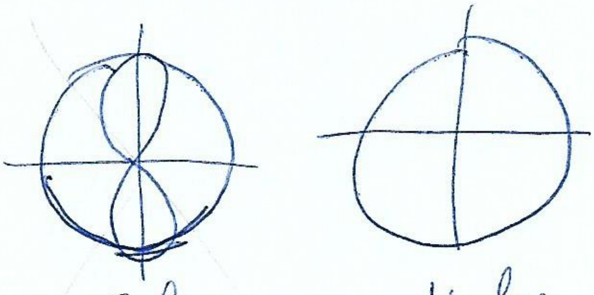
E-plane H-plane

Small loop



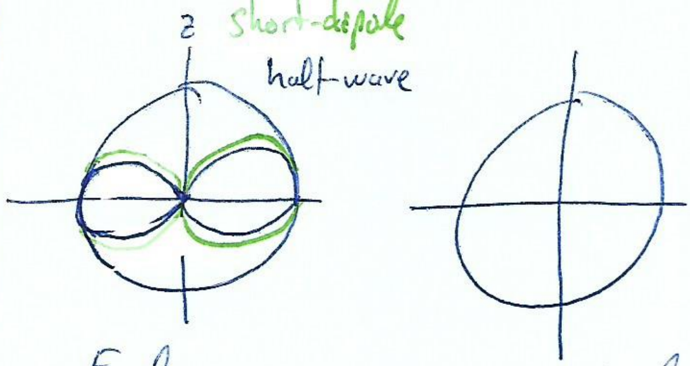
E-plane H-plane

Short dipole



E-plane H-plane

$\frac{\lambda}{2}$ dipole



E-plane H-plane

Linear polarization

$$E(z) = \left(|E_{0x}|\,\hat{x} + |E_{0y}|\,\hat{y}\right) e^{-jkz}$$

$$\nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ |E_{0x}| e^{-jkz} & |E_{0y}| e^{-jkz} & 0 \end{vmatrix} =$$

$$= \hat{x}(-1)\partial_z\left(|E_{0y}|\, e^{-jkz}\right) + \hat{y}\,\partial_z\left(|E_{0x}|\, e^{-jkz}\right) + \hat{z}\left[\underbrace{\cancel{\partial_x\left(|E_{0y}|\, e^{-jkz}\right)}}_{0} - \underbrace{\cancel{\partial_y\left(|E_{0x}|\, e^{-jkz}\right)}}_{0}\right] =$$

$$= +jk\,|E_{0y}|\, e^{-jkz}\,\hat{x} - jk\,|E_{0x}|\, e^{-jkz}\,\hat{y} =$$

$$= jk\left(|E_{0y}|\,\hat{x} - |E_{0x}|\,\hat{y}\right) e^{-jkz}$$

$$H = -\frac{1}{j\cancel{\omega}\mu}\, j\frac{\cancel{\omega}}{c}\left[|E_{0y}|\,\hat{x} - |E_{0x}|\,\hat{y}\right] e^{-jkz} = -\frac{1}{\eta}\left[|E_{0y}|\,\hat{x} - |E_{0x}|\,\hat{y}\right] e^{-jkz} =$$

$$= -\frac{1}{\eta}\left[|E_{0y}|\,\hat{x} + |E_{0x}|\, e^{j\pi}\,\hat{y}\right] e^{-jkz}$$

Circular polarization

E left hand $\rightarrow \varphi = \frac{\pi}{2}$ & $|E_{0x}| = |E_{0y}| = |E_0|$

$$E(z) = \left(|E_0|\,\hat{x} + |E_0|\,e^{j\pi/2}\right) e^{-jkz} = |E_0|\left(\hat{x} + e^{j\pi/2}\hat{y}\right) e^{-jkz}$$

$$\nabla \times \bar{E} = -j\omega\mu\bar{H}$$

$$\nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ |E_0|\,e^{-jkz} & |E_0|\,e^{j\pi/2}e^{-jkz} & 0 \end{vmatrix} =$$

$$= -\hat{x}\,\partial_z\left(|E_0|\,e^{j\pi/2}e^{-jkz}\right) + \hat{y}\,\partial_z\left(|E_{0y}|\,e^{-jkz}\right) =$$

$$= +\hat{x}\,jk|E_0|\,e^{j\pi/2}e^{-jkz} - \hat{y}\,jk|E_0|\,e^{-jkz} = jk|E_0|\left(\hat{x}\,e^{j\pi/2} - \hat{y}\right)e^{-jkz}$$

$$H = -\frac{1}{j\omega\mu}\cdot\nabla\times E = -\frac{1}{\cancel{j}\cancel{\omega}\mu}\cdot\cancel{j}\frac{\cancel{\omega}}{c}|E_0|\left(\hat{x}\,e^{j\pi/2} - \hat{y}\right)e^{-jkz} =$$

$$= -\frac{|E_0|}{\eta}\cdot\left(\hat{x}\,e^{j\pi/2} - \hat{y}\right)e^{-jkz} = -\frac{|E_0|}{\eta}\left(\hat{x}\,e^{j\pi/2} + \hat{y}\,e^{j\pi}\right)e^{-jkz} =$$

$$= -\frac{|E_0|}{\eta}\,e^{j\pi/2}\left[\hat{x} + e^{j\pi/2}\hat{y}\right]e^{-jkz} = \boxed{-j\,\frac{|E_0|}{\eta}\left[\hat{x} + e^{j\pi/2}\hat{y}\right]e^{-jkz}}$$