Spatial Optics
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CH3 Fourier optics

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Free propagations and lenses

Free propagation

Linearity

space invariance

$$
\begin{array}{ccc}
in(x,y) & \xrightarrow{\;*\,PSF\;} & out(x,y) = in * PSF = FT^{-1}(\widetilde{in}.TF) \\
\downarrow \text{FT} & \textbf{LTI system } \Sigma & \uparrow \text{FT}^{-1} \\
\widetilde{in}(N_x,N_y) & \xrightarrow{\;.\text{TF}\;} & \widetilde{out}(N_x,N_y) = \widetilde{in}(N_x,N_y).\text{TF}
\end{array}
$$

Previous chapter

$$TF_z\,(N_x, N_y) = e^{-j\frac{2\pi}{\lambda}z}.\,e^{+j\pi\lambda z({N_x}^2+{N_y}^2)}$$

Point Spread function ?

Point Spread function of a free propagation over a distance z

$$PSF_z(x,y) = spheric\ wave$$

$$PSF_d(x,y) = Cte\,.\,e^{-j\pi(\frac{x^2+y^2}{\lambda d})} \quad \text{with } z = d$$

Convolution

$$f_d(x,y) = f_0(x,y) * e^{-j\pi(\frac{x^2+y^2}{\lambda d})}$$

$$f_d(x,y) = \int_{-\infty}^{+\infty} f_0(x_0,y_0))\ e^{-j\pi(\frac{(x-x_0)^2+(y-y_0)^2}{\lambda d})} dx_0 dy_0$$

$$f_d(x,y) = e^{-j\pi(\frac{x^2+y^2}{\lambda d})} \int_{-\infty}^{+\infty} f_0(x_0,y_0)\, e^{-j\pi(\frac{x_0^2+y_0^2}{\lambda d})} e^{j2\pi(\frac{xx_0+yy_0}{\lambda d})} dx_0 dy_0$$

3

Transmission of a lens (focal length f)

Linearity yes

space invariance !!!no!!!

$$f_{\Pi}(x,y) = f_{\Pi 0}(x,y).\tau(x,y) = f_{\Pi 0}(x,y)\, e^{j\pi\left(\frac{x^2+y^2}{\lambda f}\right)}$$

Lens

Free propagation distance d

$$f_{\Pi L-}(x,y) \xrightarrow{.\tau(x,y)} f_{\Pi L+}(x,y) \xrightarrow{*\,PSF} f_d(x,y)$$

$$f_{d=f}(x,y) = e^{-j\pi\left(\frac{x^2+y^2}{\lambda d}\right)} \int_{-\infty}^{+\infty} f_{\Pi L-}(x_0,y_0)\, \underbrace{e^{j\pi\left(\frac{x_0^2+y_0^2}{\lambda f}\right)}\, e^{-j\pi\left(\frac{x_0^2+y_0^2}{\lambda d}\right)}}\, e^{j2\pi\left(\frac{xx_0+yy_0}{\lambda d}\right)} dx_0 dy_0$$

If $d = f$ >>> simplification

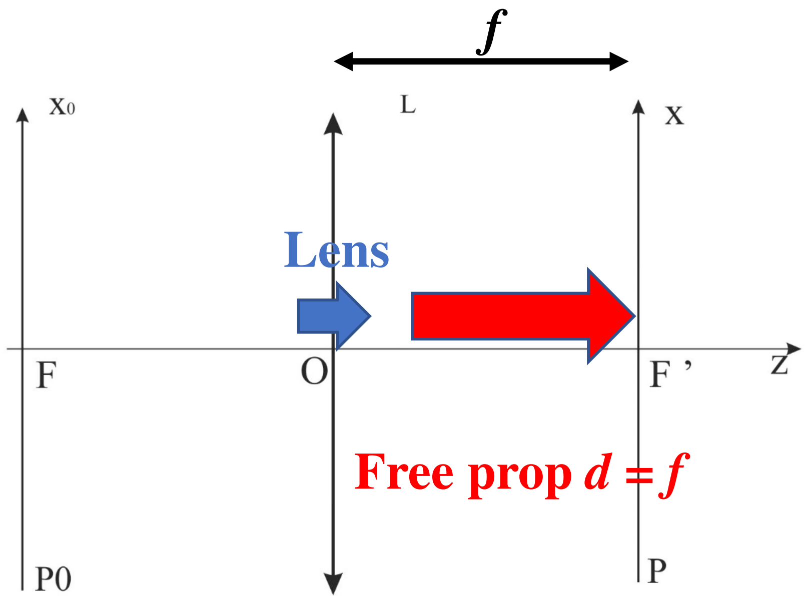
$$f_{d=f}(x,y) = e^{-j\pi\left(\frac{x^2+y^2}{\lambda d}\right)} \int_{-\infty}^{+\infty} f_{\Pi L-}(x_0,y_0)\, e^{j2\pi\left(\frac{xx_0+yy_0}{\lambda d}\right)} dx_0 dy_0$$

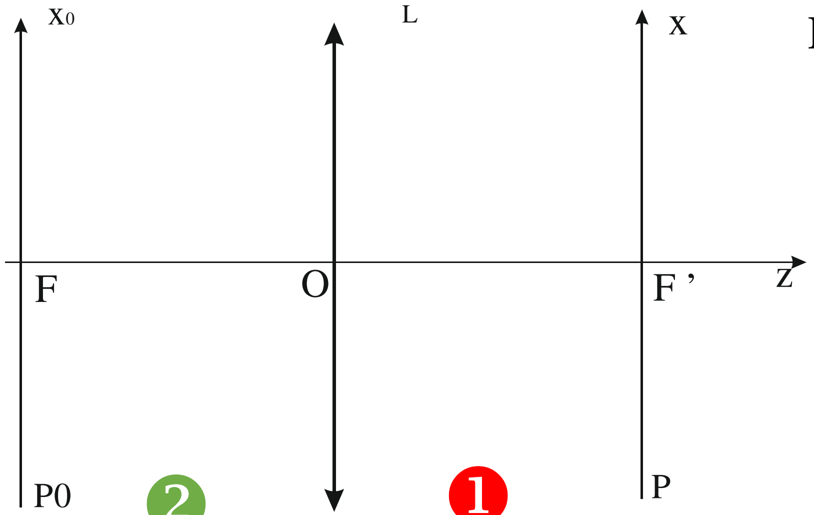
$$f_{d=f}(x,y) = e^{-j\pi\left(\frac{x^2+y^2}{\lambda f}\right)} \int_{-\infty}^{+\infty} f_{\Pi L-}(x_0,y_0)\, e^{j2\pi\left(\frac{xx_0+yy_0}{\lambda f}\right)} dx_0 dy_0$$

And denoting

$$N_x = \frac{x}{\lambda f} \text{ and } N_y = \frac{y}{\lambda f}$$

$$\boxed{f_{d=f}(x,y) = e^{-j\pi\lambda f(N_x^2+N_y^2)} FT(f_{\Pi L-})}$$





For a global scheme

propagation > lens > propagations

$$f_0 \xrightarrow{2} f_{\Pi 0} \xrightarrow{1} f_{d=f}$$

1 $$f_{d=f}(x,y) = e^{-j\pi\lambda f(N_x^2+N_y^2)} FT(f_{\Pi L-})$$

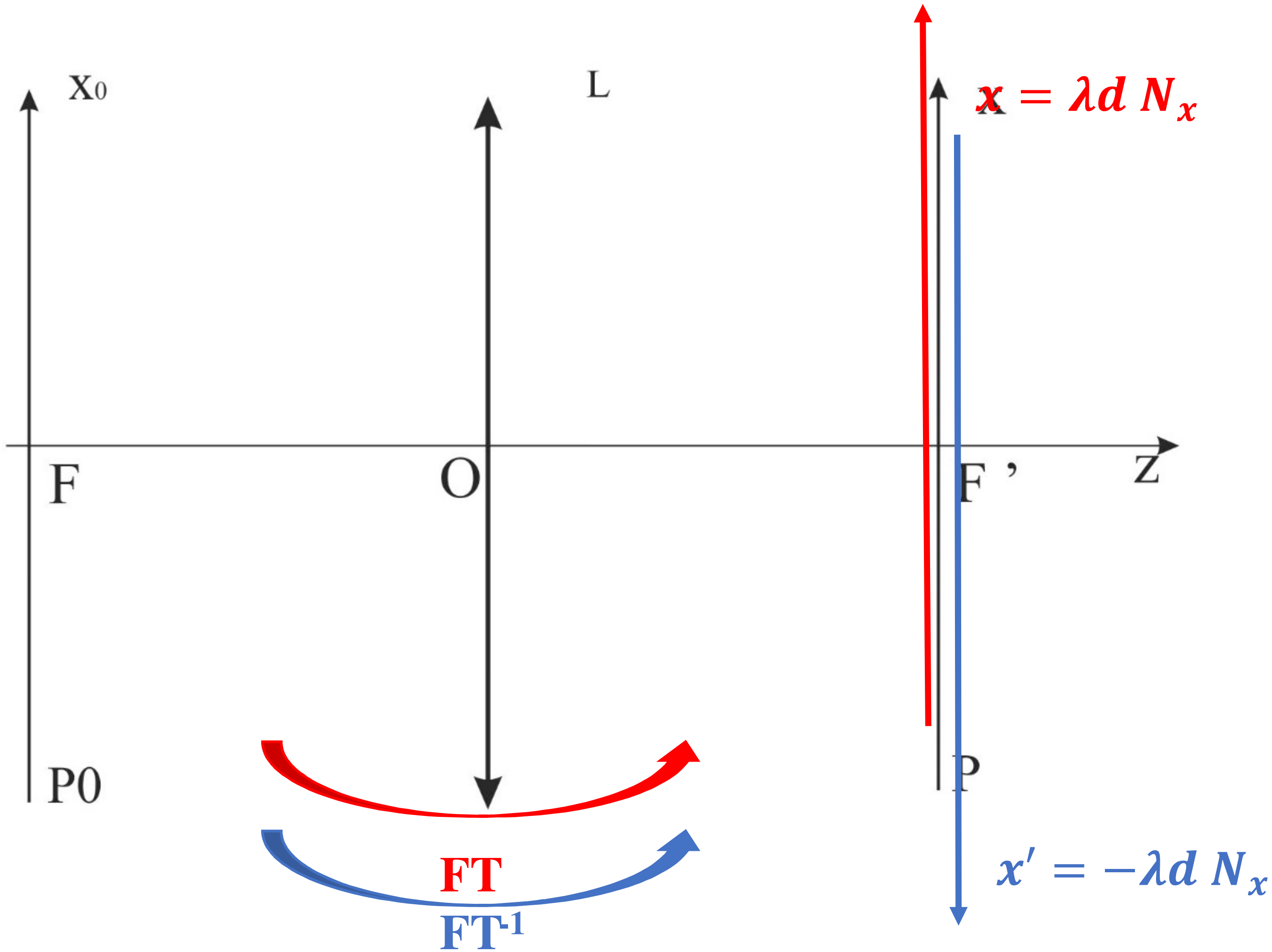
$$\tilde{f}_0 \xrightarrow{2} \boxed{FT(f_{\Pi L-}) = TF_{d_0}.FT(f_0)} \quad TF_{d_0}(N_x,N_y) = e^{-j\frac{2\pi}{\lambda}d_0}.e^{+j\pi\lambda d_0({N_x}^2+{N_y}^2)}$$

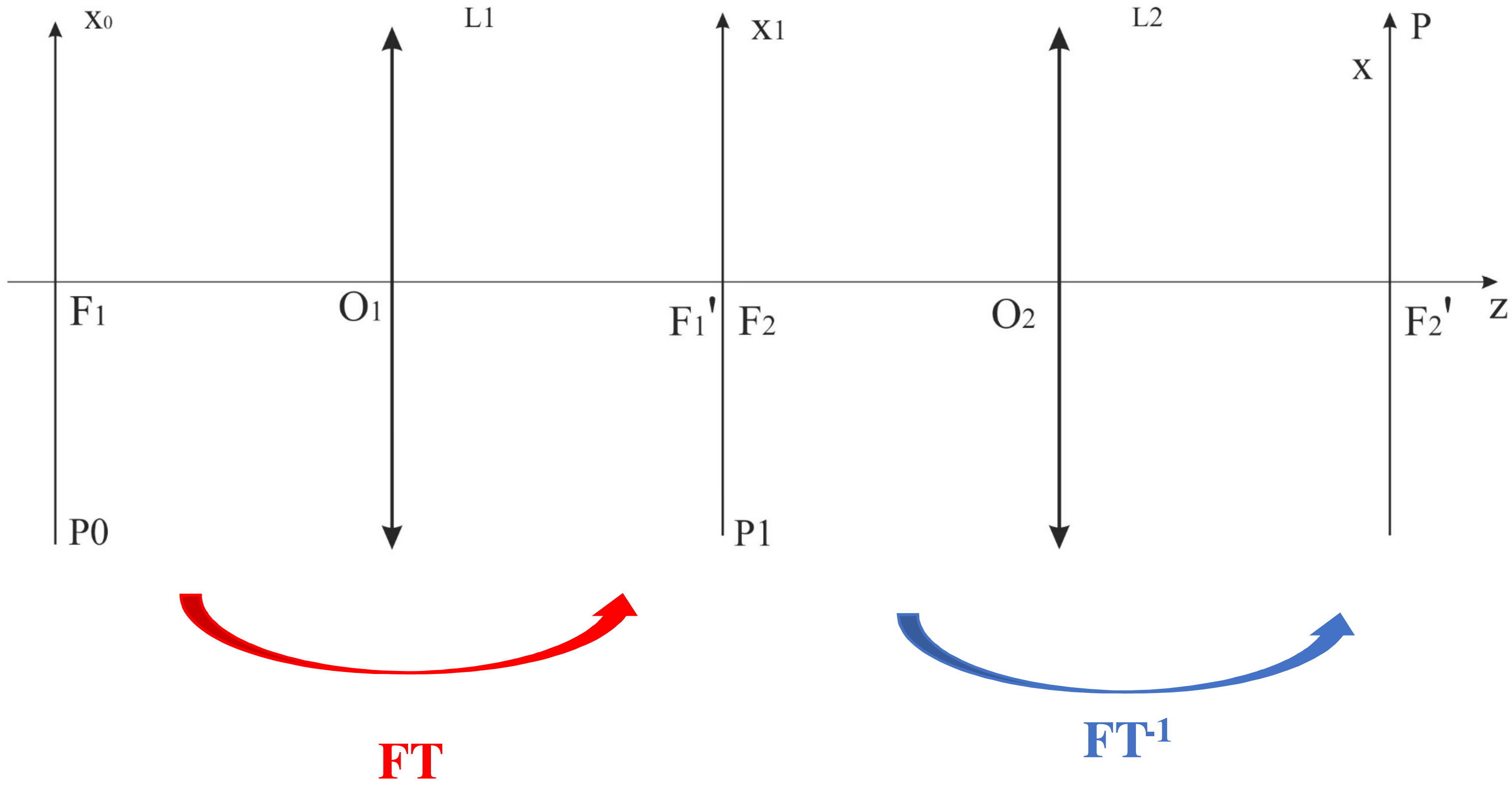
as

$$f_{d=f}(x,y) = e^{-j\pi\lambda f(N_x^2+N_y^2)} e^{+j\pi\lambda d_0(N_x^2+N_y^2)} FT(f_0)$$

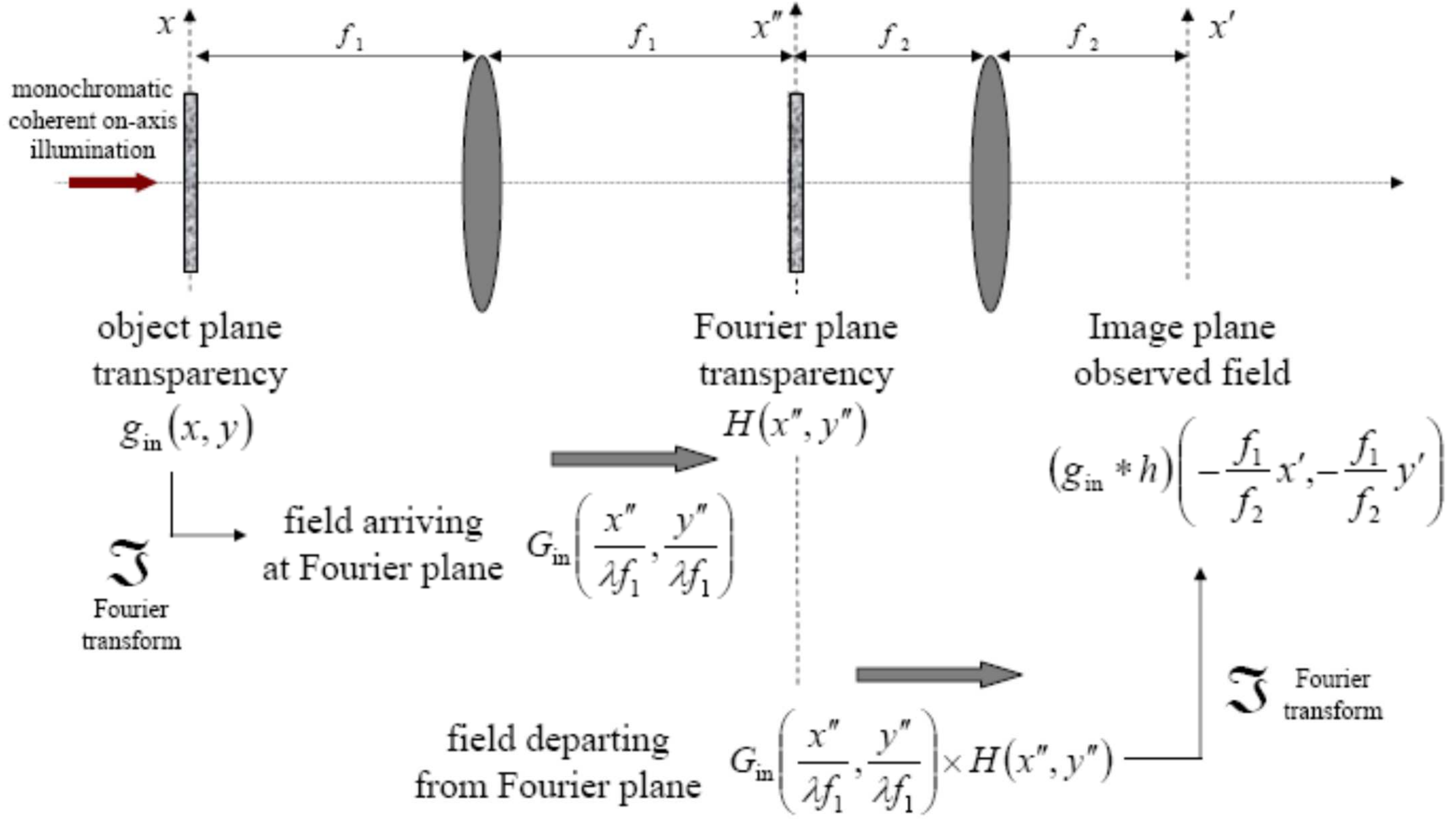
$$f_{d=f}(x,y) = \underbrace{e^{-j\pi\lambda f(N_x^2+N_y^2)}e^{+j\pi\lambda f(N_x^2+N_y^2)}}_{\text{If } d_0 = f} FT(f_0)$$

$$\boxed{f_{d=f}(x,y) = FT(f_0)}$$

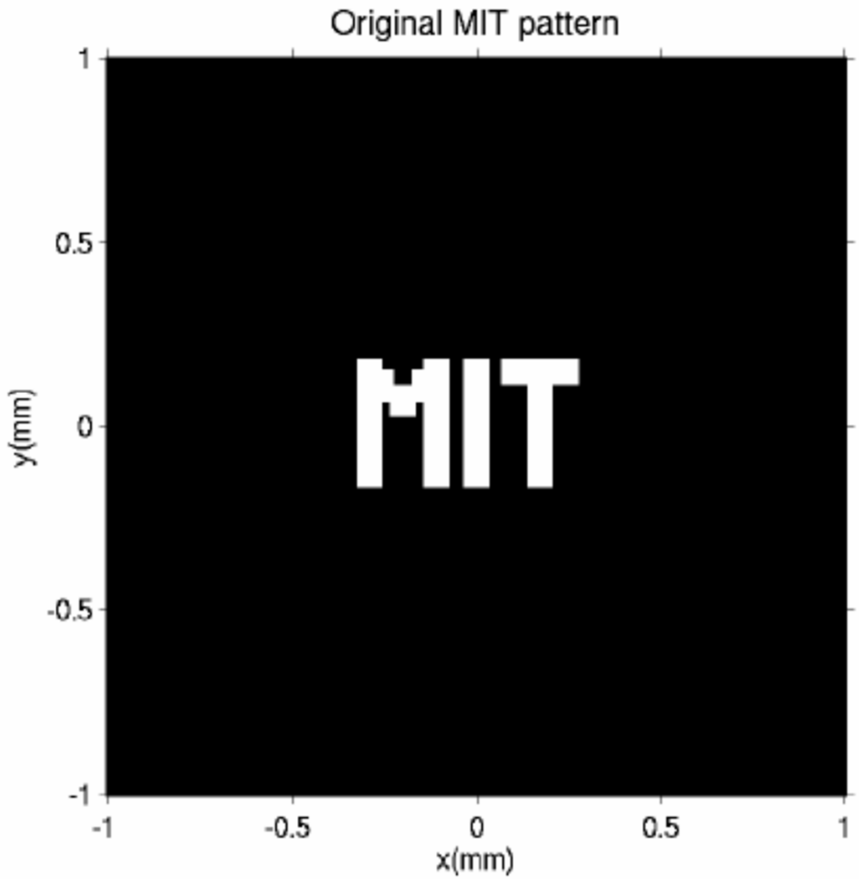



x0
L1
x1
L2
P
x
F1
O1
F1'
F2
O2
F2'
z
P0
P1
FT
FT-1

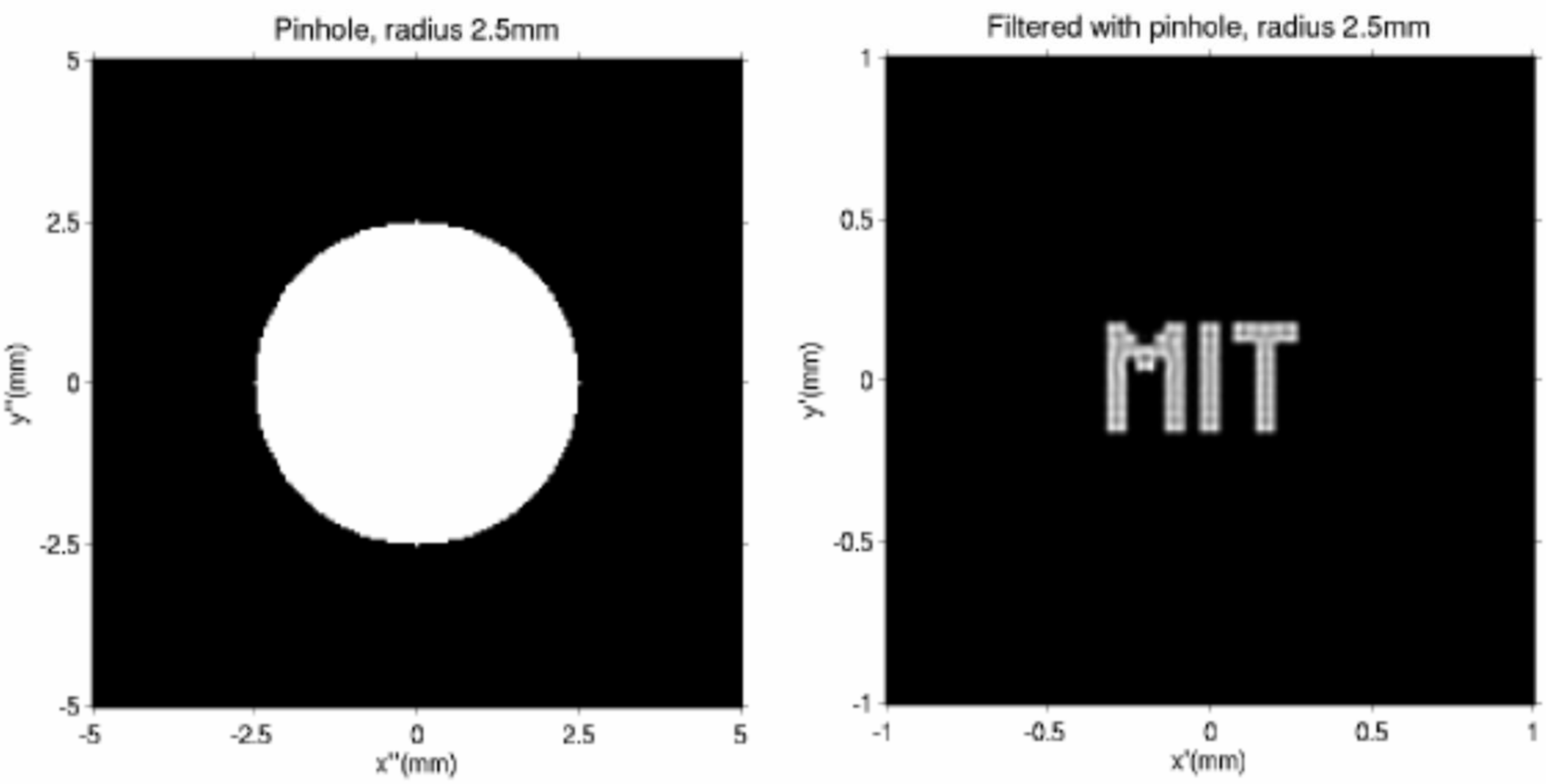
Spatial filtering with the 4F system



Examples: the amplitude MIT pattern



Weak low–pass filtering



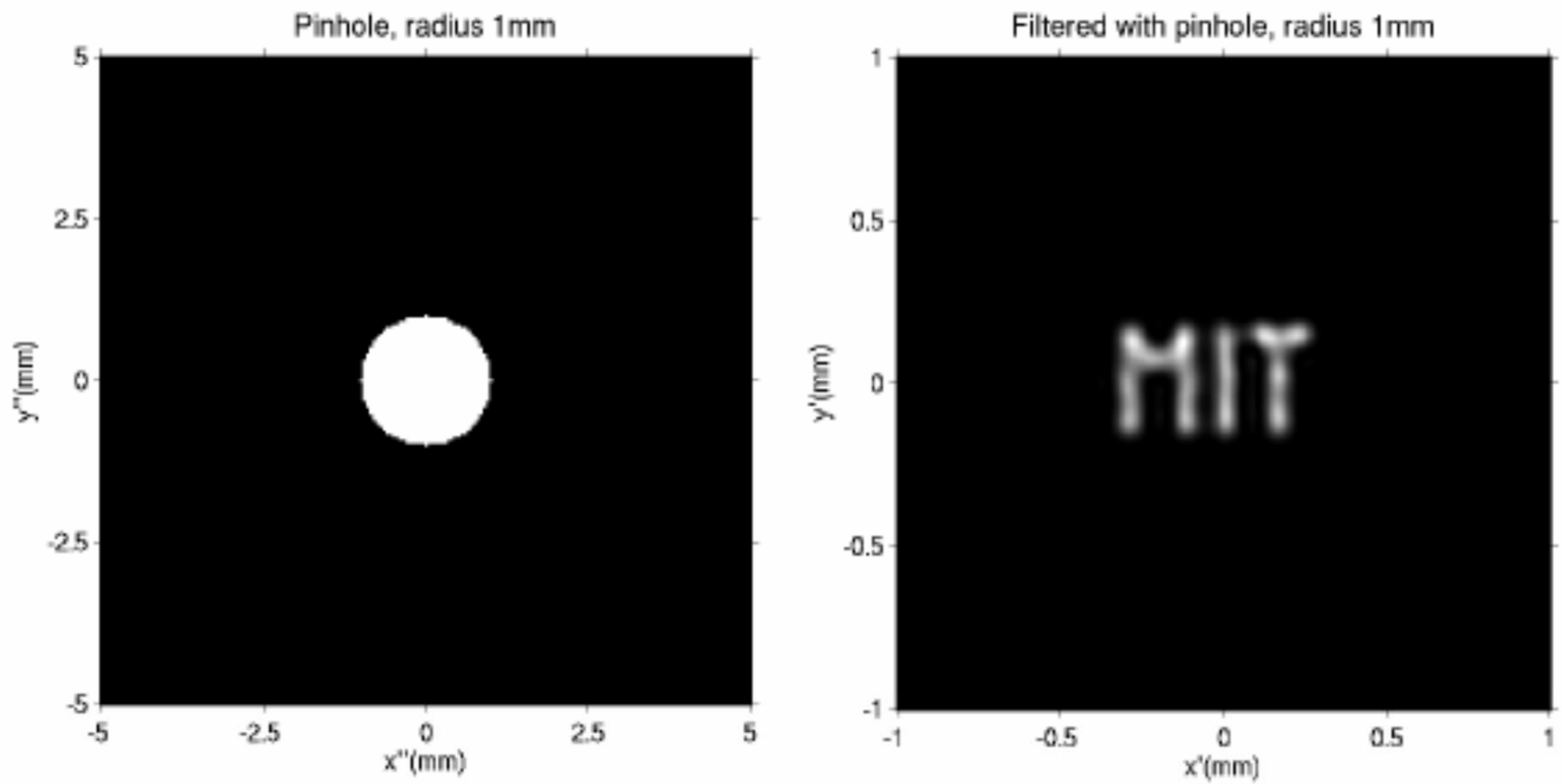
f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

Moderate low–pass filtering

(*aka* blurring)

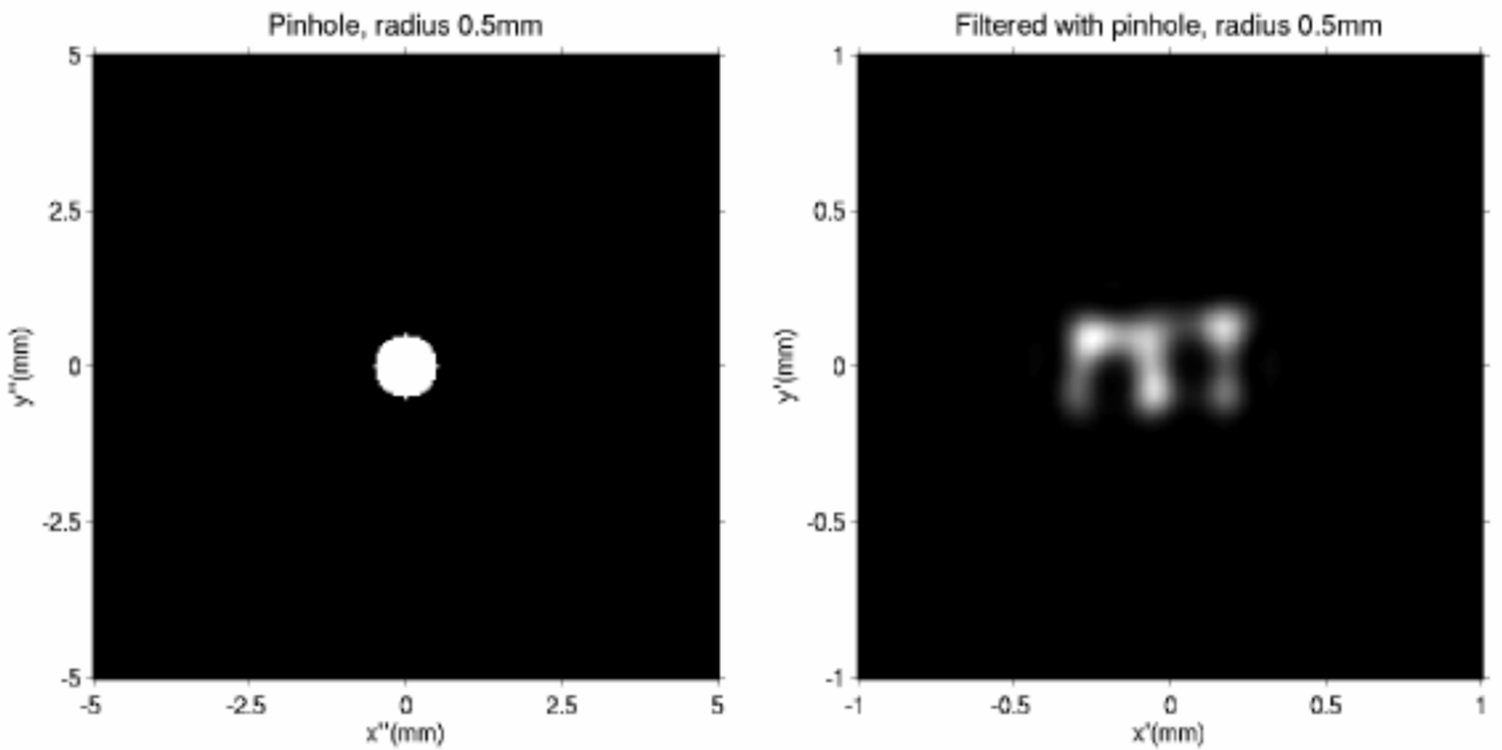


f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

Strong low–pass filtering

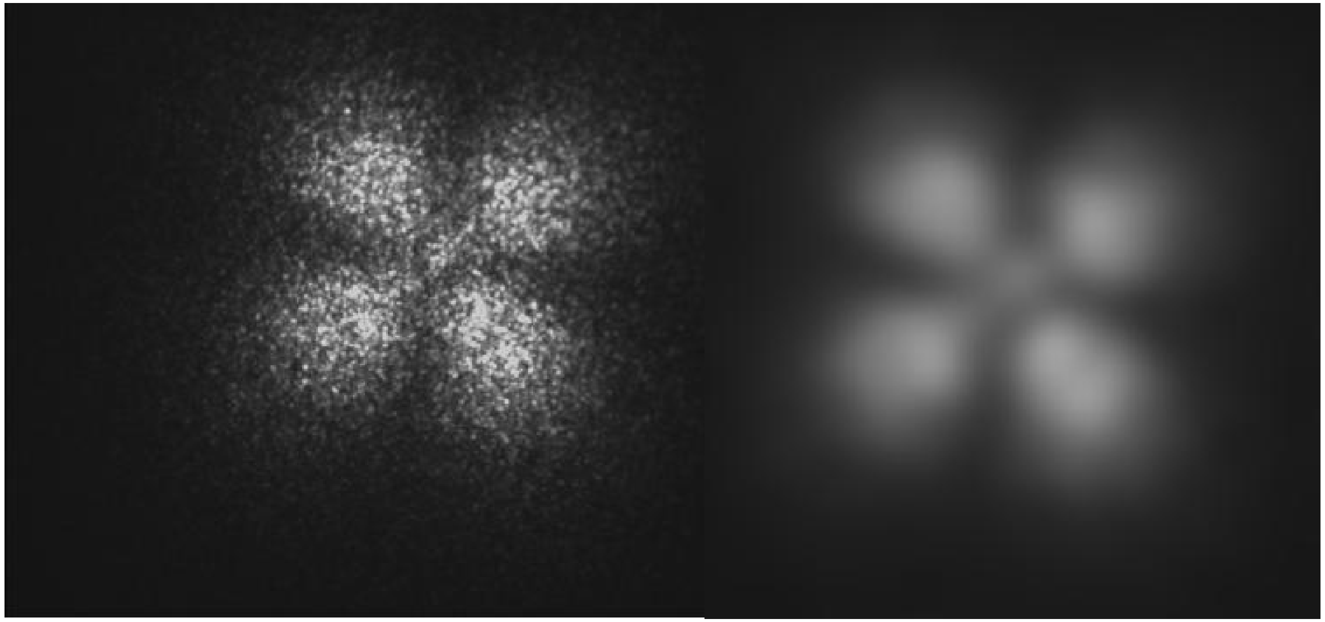


f_1=20cm
λ=0.5μm

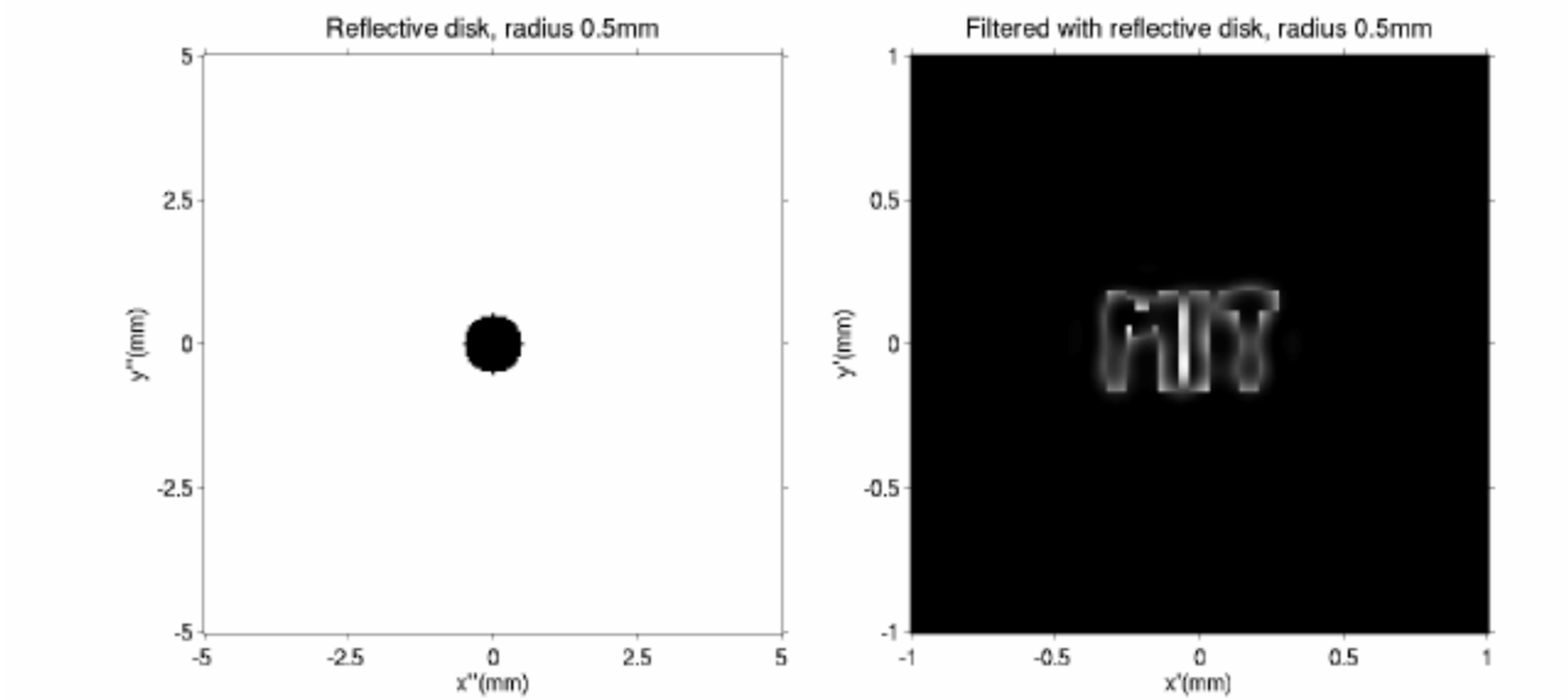
Fourier filter

Intensity @ image plane

Filtrage des bruits (par exemple
lorsque le signal intéressant
est dans les basses fréquences)



Moderate high–pass filtering



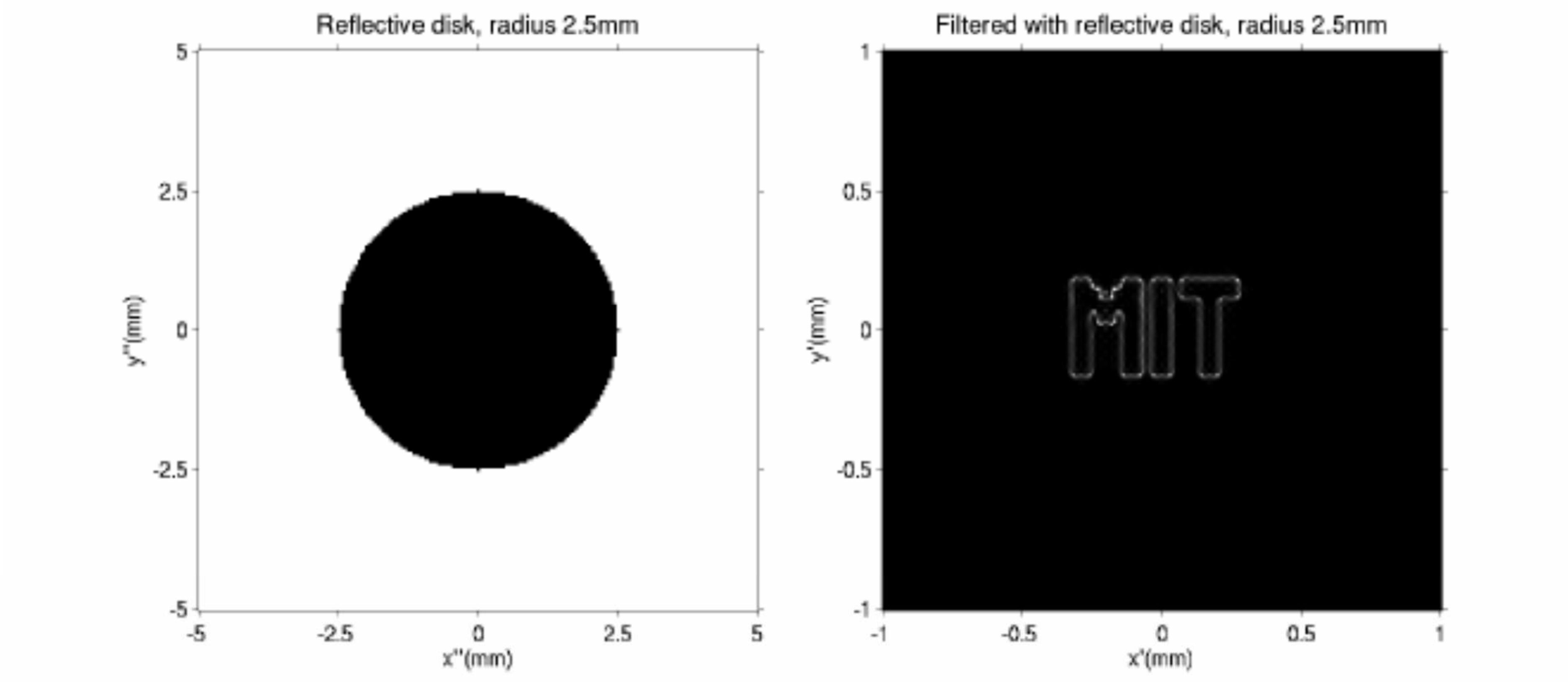
f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

Strong high–pass filtering

(*aka* edge enhancement)

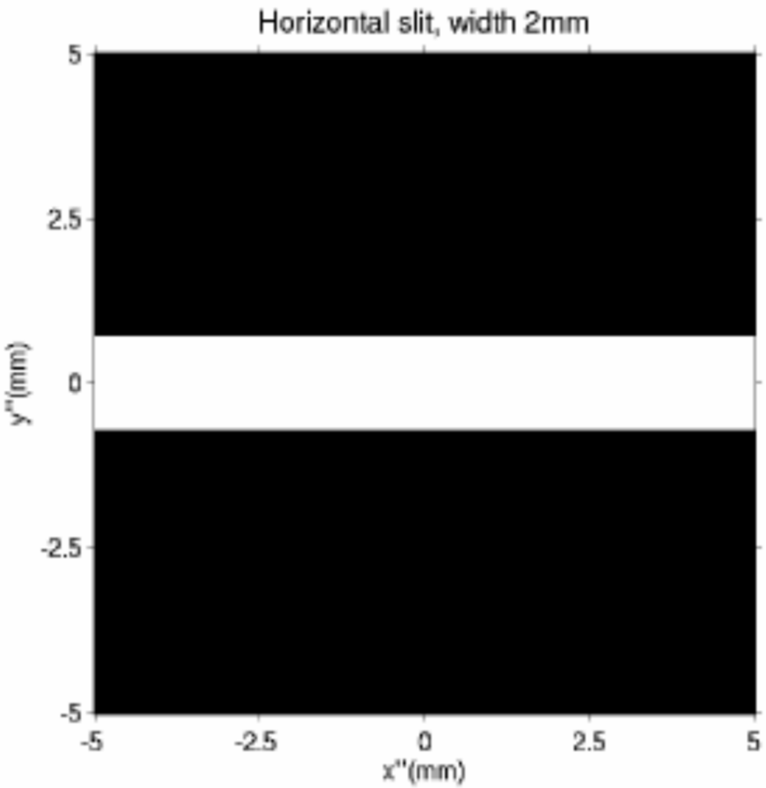


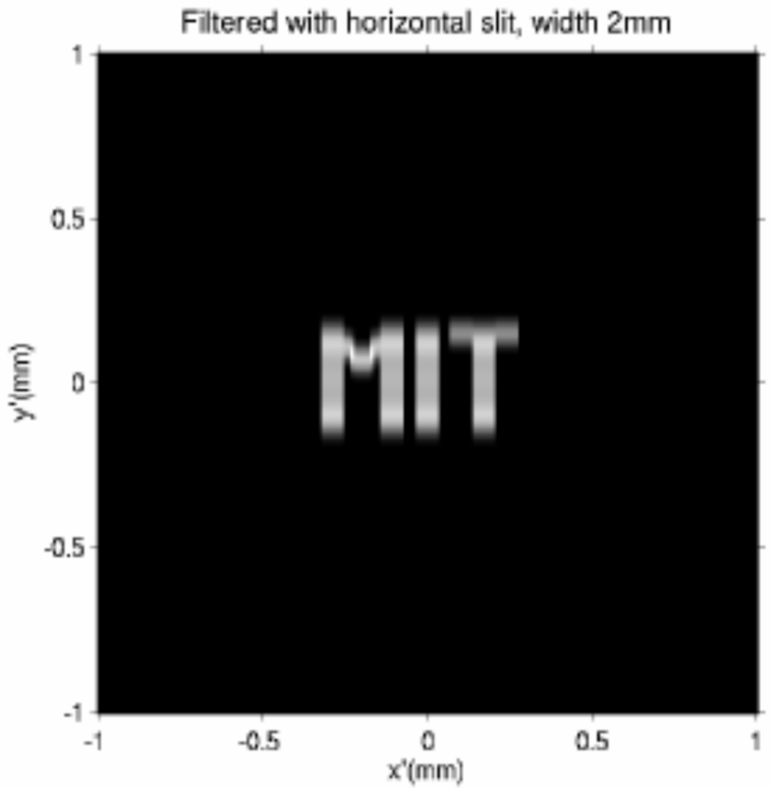
f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

1-dimensional blurring



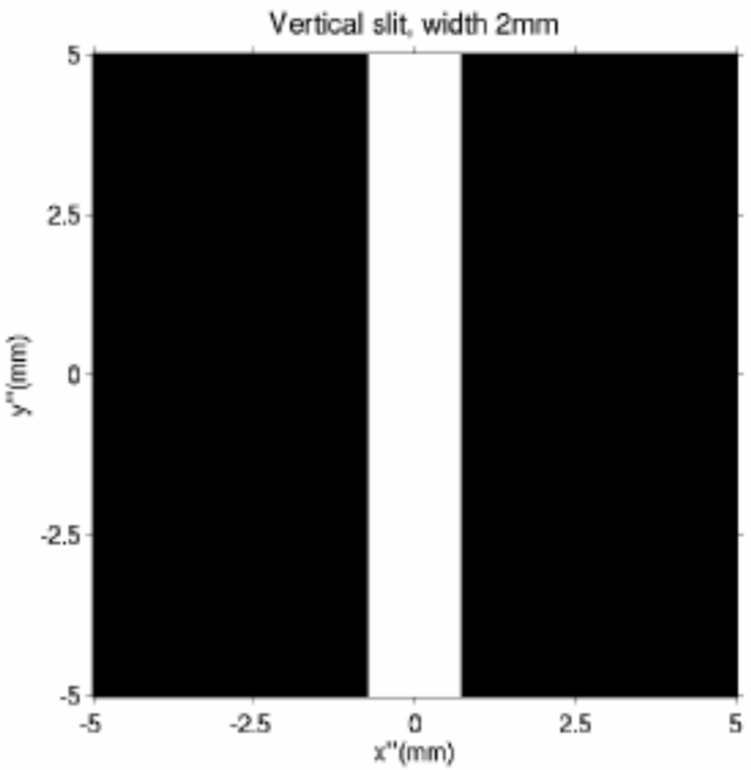


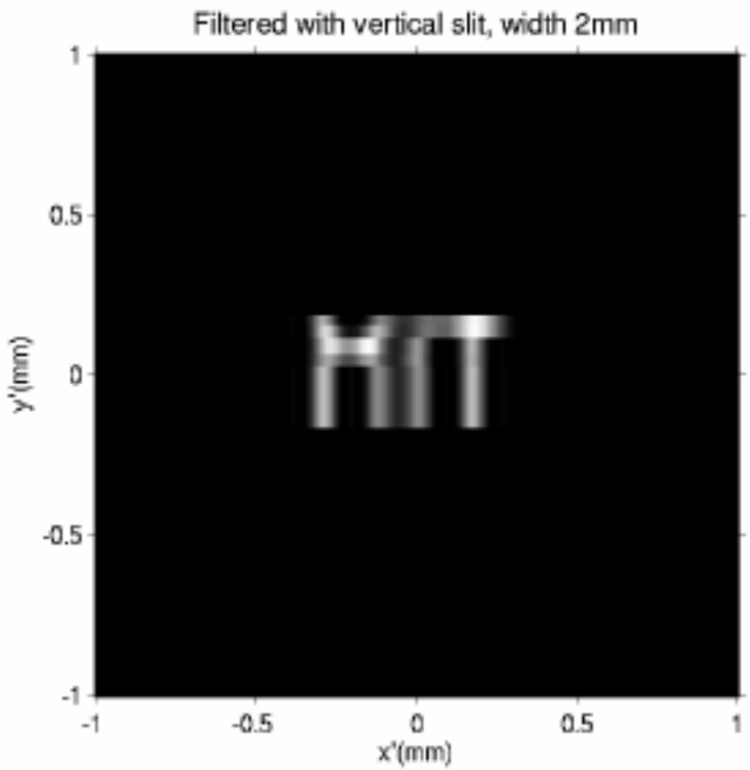
f_1=20cm
λ=0.5µm

Fourier filter

Intensity @ image plane

1-dimensional blurring



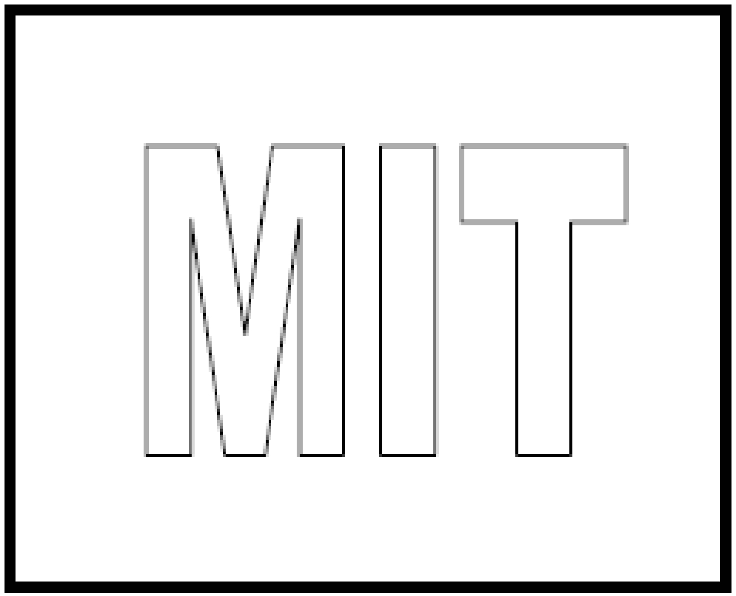


f_1=20cm
λ=0.5μm

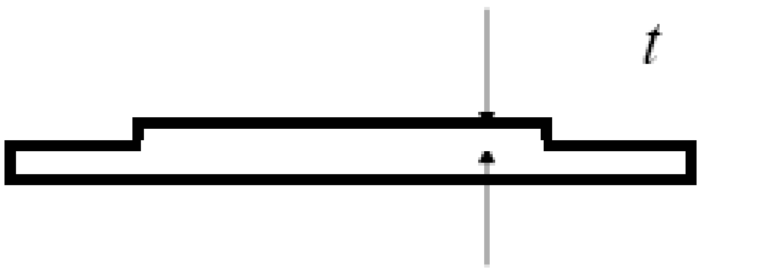
Fourier filter

Intensity @ image plane

Phase objects



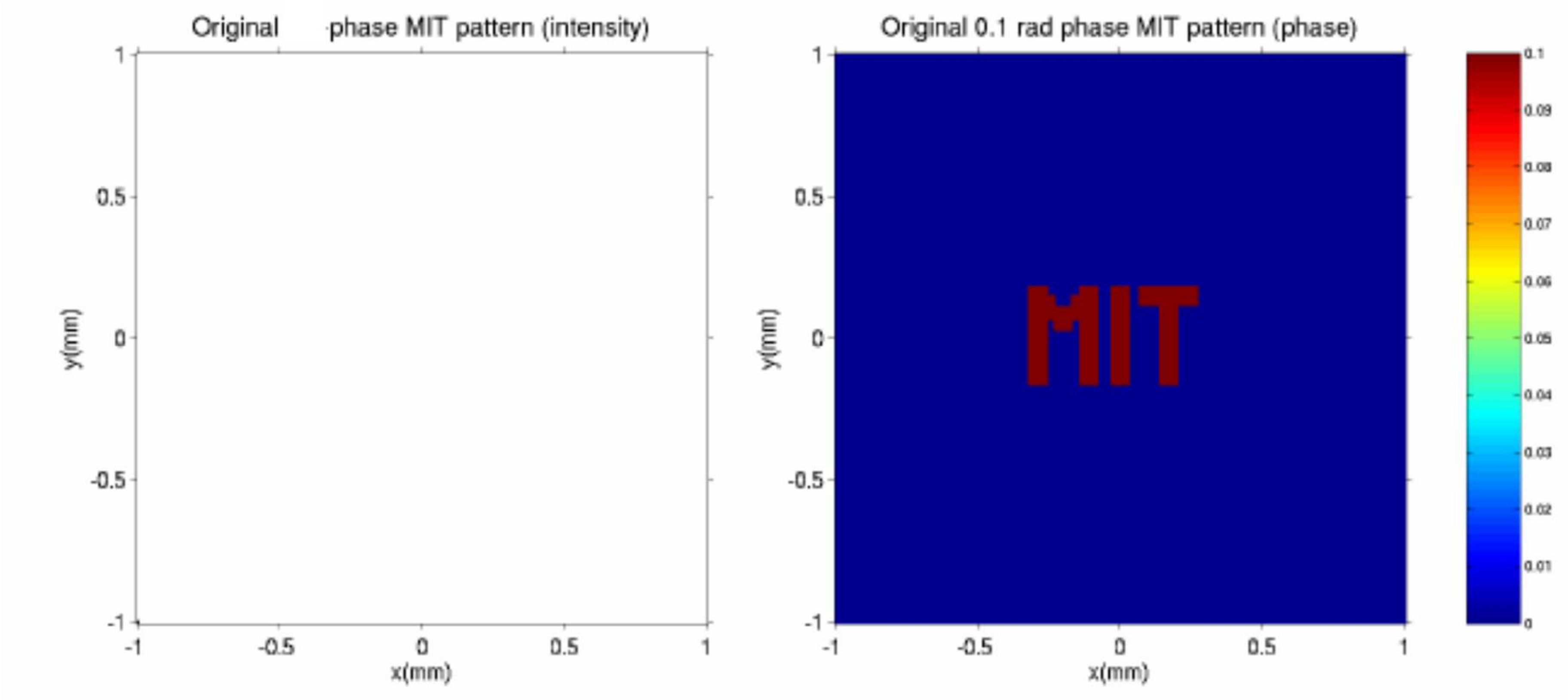
glass plate
(transparent)



protruding part
phase-shifts
coherent illumination
by amount $\varphi=2\pi(n-1)t/\lambda$

Often useful in imaging biological objects (cells, etc.)

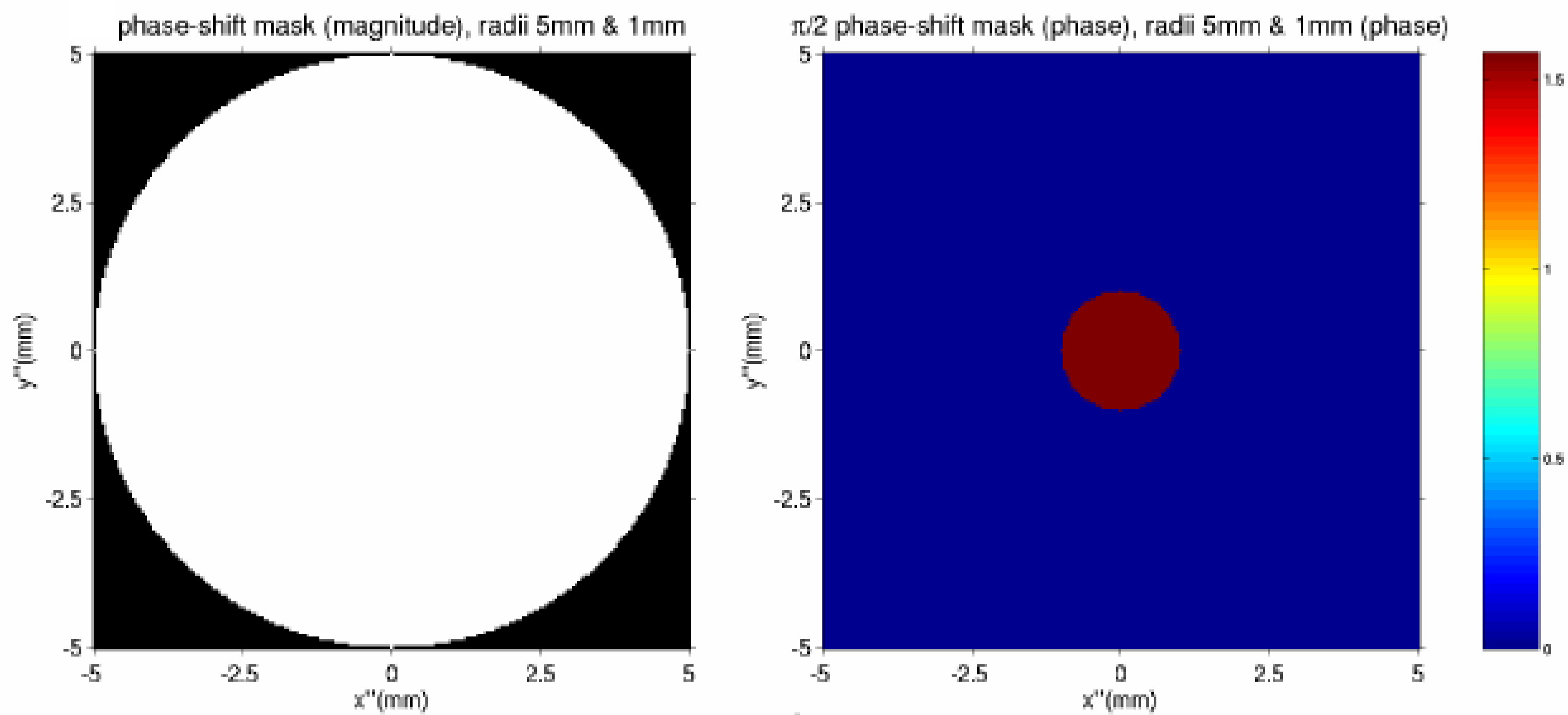
Viewing phase objects

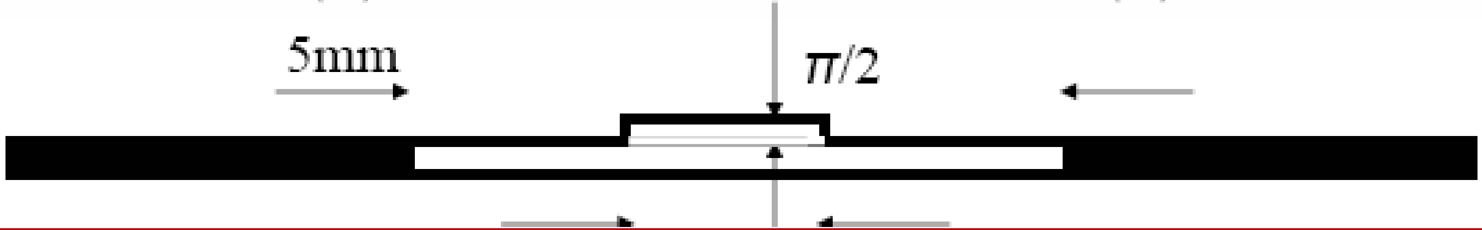


Intensity
(object is invisible)

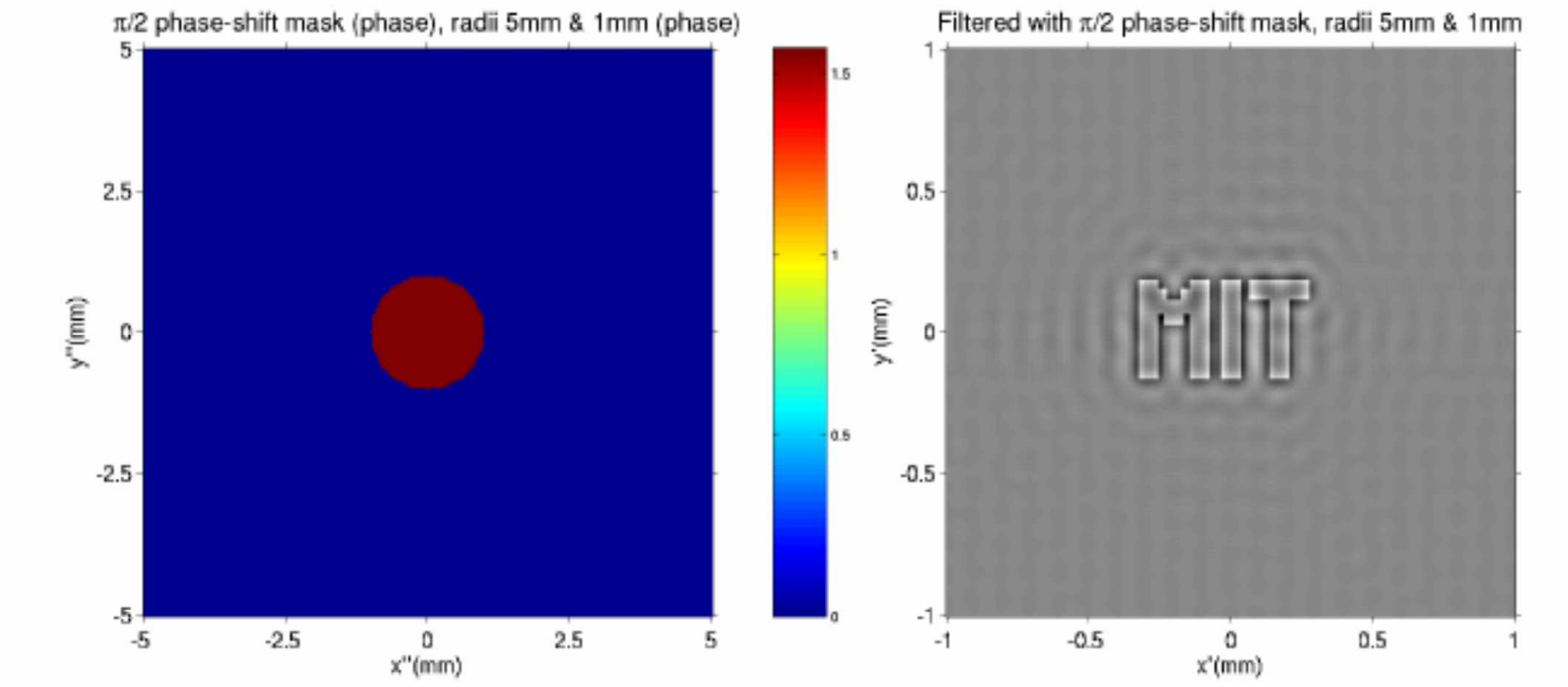
Amplitude
(need interferometer)

Zernicke phase-shift mask





Imaging with Zernicke mask

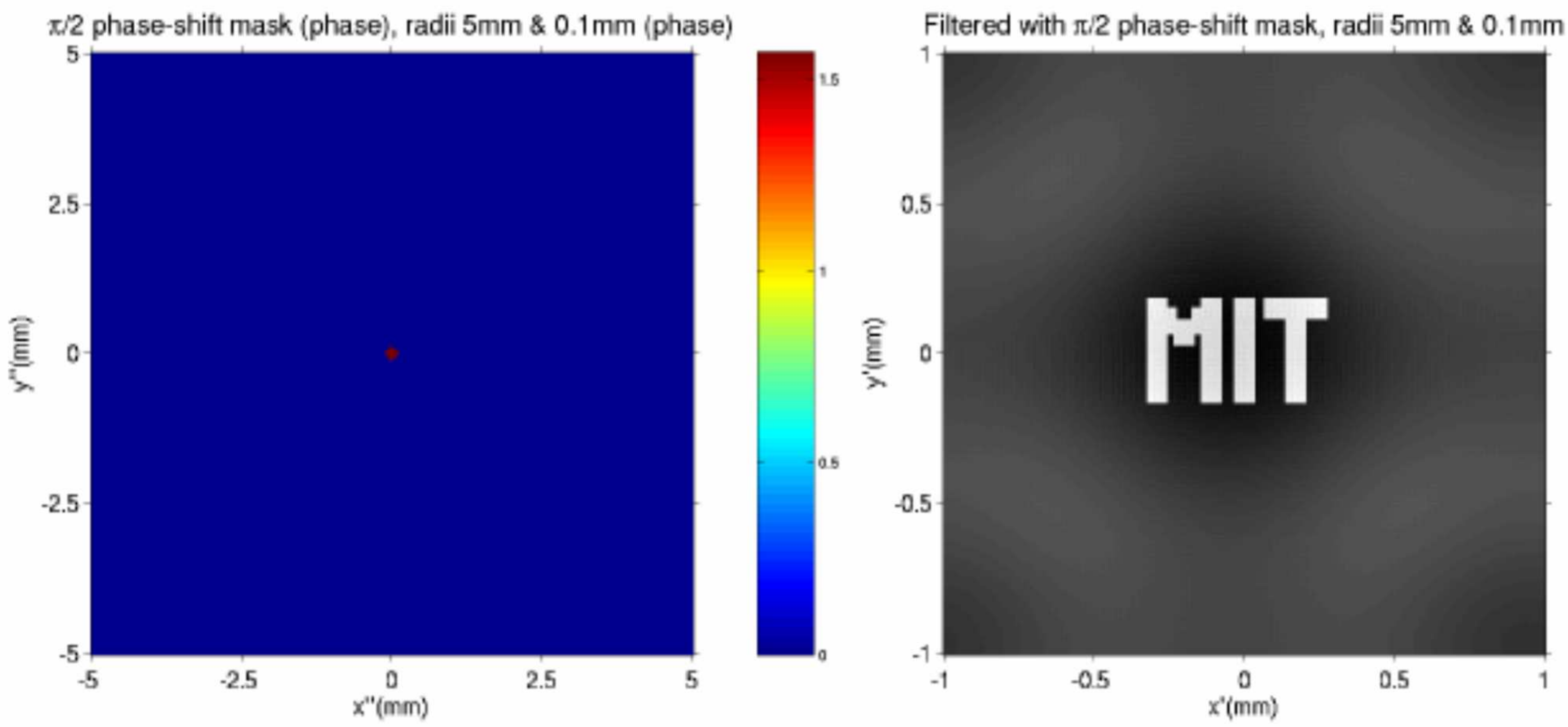


f_1=20cm
λ=0.5μm

Fourier filter

Intensity @ image plane

Imaging with Zernicke mask

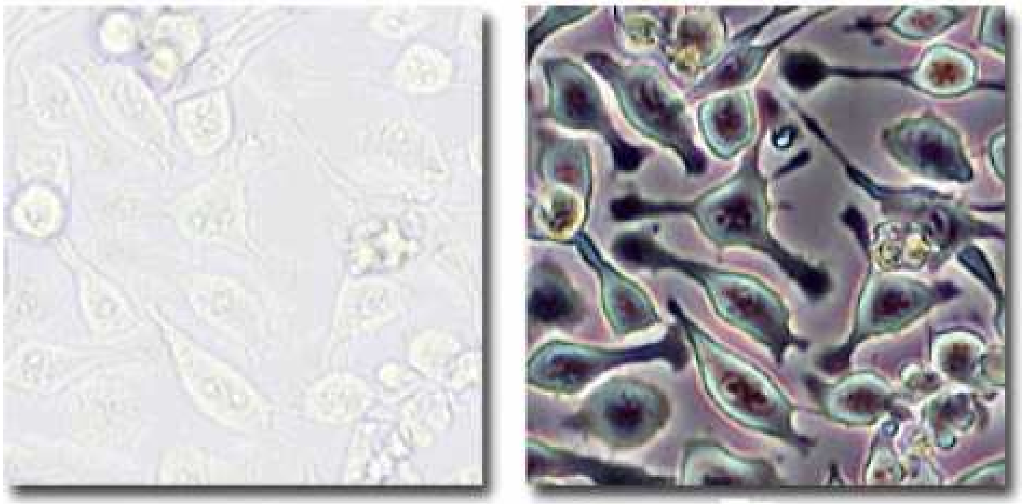


f_1=20cm
λ=0.5µm

Fourier filter

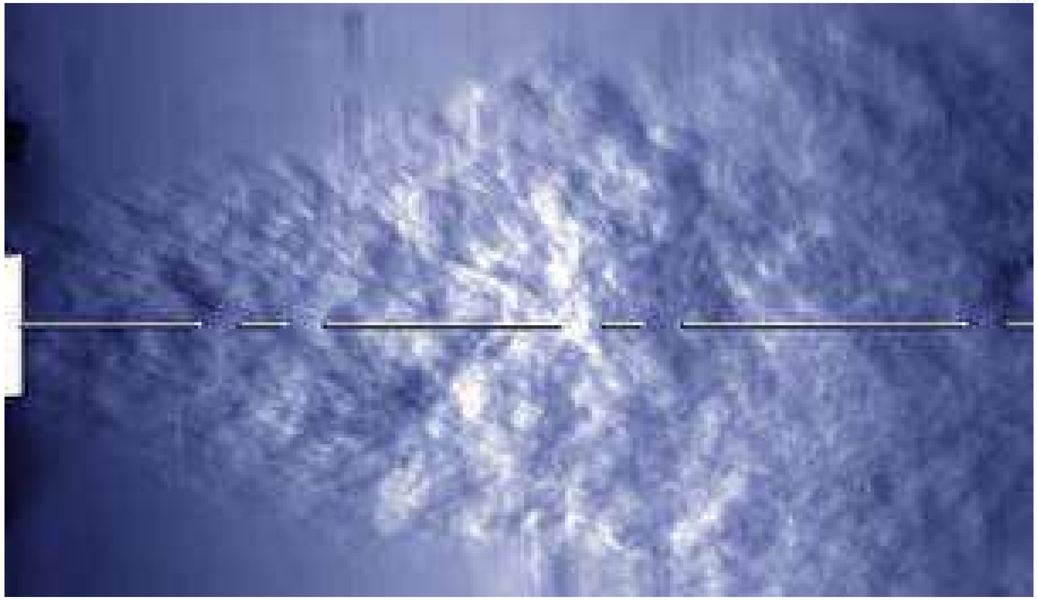
Intensity @ image plane

microscopy



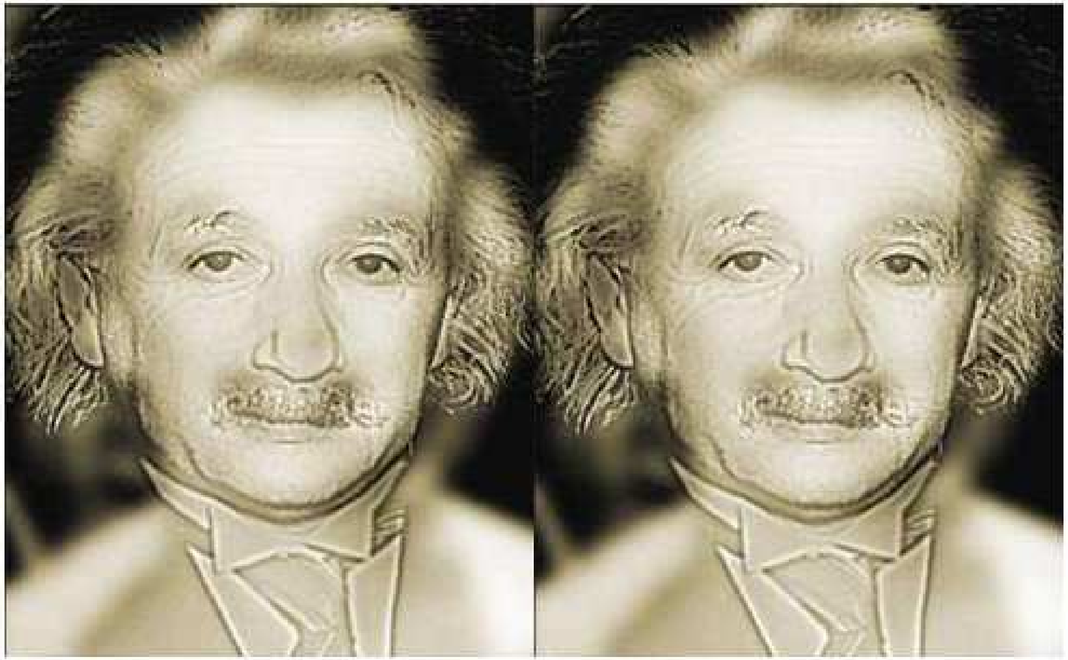
Fond clair Contraste de phase

Fluid mechanics



Application to incoherent beams

**Abraham Lincoln,
par Salvador Dali**



he/she « Marylin Einstein »