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Problema 1

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

A 3 port network cannot be reciprocal, lossless and be matched at the same time.

$[S]$ is symmetrical because $S = S^t$

$[S]$ is matched because the reflexion coeffients are zero

$$S_{ii} = 0 \longrightarrow S_{11} = S_{22} = S_{33} = 0$$

So it has to be lossy

To be lossless: $[S]^t \cdot [S^*] = [U]$

$$\frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

a) Yes

b) No

c) Yes

d) To be isolated the two outputs (2 and):

The transmission coefficients S_{23} and S_{32} have to be zero and we see that:

$$S_{23} = S_{32} = 1$$

So it is not isolated

e) We can calculate [Z] matrix:

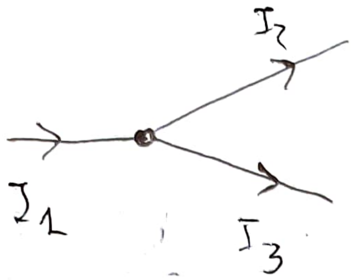
$$[U]+[S] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{bmatrix}$$

$$[U]-[S] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1/2 & -1/2 \\ -1/2 & 1 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix}$$

$$([U]-[S])^{-1} = \frac{2}{9}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

using wolfram alpha
and $([U]-[S])^{-1}$ is pseudo inverse
since $([U]-[S])$ is singular

$$[Z] = (U+S)(U-S)^{-1} = \frac{1}{9}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \rightarrow [V] = [Z][I]$$



$$\left.\begin{aligned} I_1 &= I_2 + I_3 \\ I_2 &= I_3 \end{aligned}\right\} \quad \begin{aligned} I_1 &= 2 I_2 \\ I_3 &= I_2 \end{aligned}$$

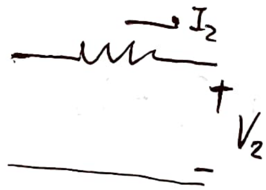
$$V_1 = \frac{1}{9}\left(2I_1 - I_2 - I_3\right) = \frac{2}{9} I_2$$

$$V_2 = \frac{1}{9}\left(-I_1 + 2I_2 - I_3\right) = -\frac{1}{9} I_2$$

$$V_3 = \frac{1}{9}\left(-2I_2 - I_2 + 2I_2\right) = -\frac{1}{9} I_2$$

$$V_2 = V_3 = -\frac{1}{9} I_2$$

Disclaimer: As I_2 is defined like



is in the opposite direction so we have to add a minus sign

$$V_1^+ = 10 \rightarrow I_2 = \frac{9}{2} V_1^+$$

$$V_2 = V_3 = -\frac{1}{9}(-1) I_2 = \frac{1}{\cancel{9}} \cdot \frac{\cancel{9}}{2} V_1^+ = \frac{10}{2}$$

$$\boxed{V_2 = V_3 = 5\,V}$$

f)

$$P_1 = \frac{1}{2} |a_1|^2 \left(1 - |\Gamma_1|^2\right) \qquad a_1 = \frac{V_1^+}{\sqrt{Z_0}}, \quad \Gamma_1 = \frac{b_1}{a_1} = S_{11} = 0$$

$$P_1 = \frac{1}{2} \frac{|V_1^+|^2}{Z_0} = \frac{1}{2} \frac{10^2}{50} = 1 \rightarrow \boxed{P_1 = 1W}$$

f) $P_2 = \frac{1}{2}|a_2|^2 \left(1-|\Gamma_2|\right)^2$

$P_3 = \frac{1}{2}|a_3|^2 \left(1-|\Gamma_3|\right)^2$

$\Bigg\} \quad P_2 = P_3 = \frac{1}{2}|a_2|^2 = \frac{1}{2}\frac{|V_2^+|^2}{Z_0} = \frac{5^2}{2\cdot 50}$

$$\boxed{P_2 = P_3 = \frac{1}{4}\,W}$$

$a_2 = \frac{V_2^+}{\sqrt{Z_0}}$

$a_3 = \frac{V_3^+}{\sqrt{Z_0}}$

$\Bigg\} \quad a_2 = a_3$

$\downarrow$

$V_2^+ = V_3^+$

$\Gamma_2 = \frac{b_2}{a_2} = S_{22} = 0$

$\Gamma_3 = \frac{b_3}{a_3} = S_{33} = 0$

g)

$P_1 = P_{Loss} + P_2 + P_3 \longrightarrow \quad P_{Loss} = P_1 - P_2 - P_3 = 1 - \frac{1}{4} - \frac{1}{4}$

$$\boxed{P_{Loss} = \frac{1}{2}\,W}$$

Problem 2

We have the next conditions

$$5\,GHz = f_{101} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{d}\right)^2}$$

$$6.5\,GHz = f_{102} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{2}{d}\right)^2}$$

$$7.2\,GHz = f_{011} = \frac{c}{2}\sqrt{\left(\frac{1}{b}\right)^2 + \left(\frac{1}{d}\right)^2}$$

∘) $\frac{4}{c^2} f_{101}^2 = \frac{1}{a^2} + \frac{1}{d^2} \longrightarrow \frac{1}{a^2} = \frac{4}{c^2} f_{101}^2 - \frac{1}{d^2}$

∘) $\frac{4}{c^2} f_{102}^2 = \frac{1}{a^2} + \frac{4}{d^2} \Rightarrow \frac{1}{a^2} = \frac{4}{c^2} f_{102}^2 - \frac{4}{d^2}$

$$\frac{4}{c^2} f_{101}^2 - \frac{1}{d^2} = \frac{4}{c^2} f_{102}^2 - \frac{4}{d^2}$$

$$\frac{4}{c^2} f_{101}^2 - \frac{4}{c^2} f_{102}^2 = \frac{1}{d^2} - \frac{4}{d^2}$$

$$\frac{1}{d^2} = \frac{4}{c^2}\left(f_{102}^2 - f_{101}^2\right) \longrightarrow \boxed{d = 0,0361\,m}$$

∘) $\boxed{a = 0,0539\,m}$

•) $\frac{1}{b^2} = \frac{4}{c^2} \cdot f_{011}^2 - \frac{1}{d^2}$

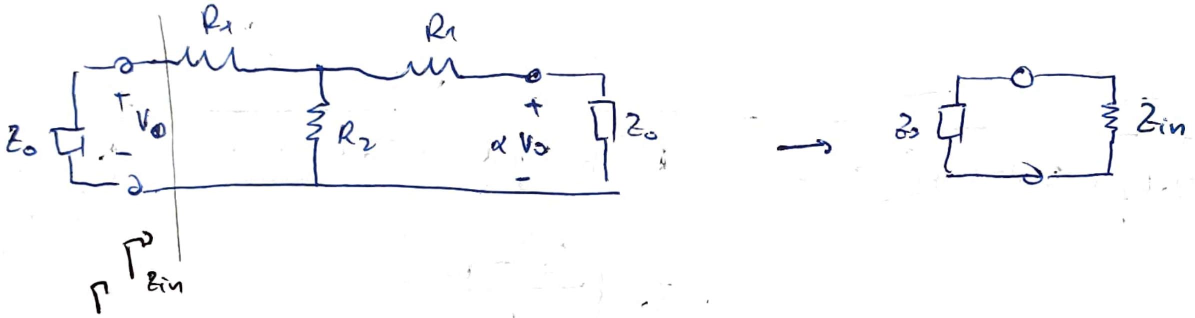
$\boxed{b = 0,0255 m}$

Problem 3

Attenvator [S] matrix

$$[S] = \begin{bmatrix} 0 & \alpha \\ \alpha & 0 \end{bmatrix}$$

For T configuration



$$Z_{in} = R_1 + R_2 \,//\, R_1 + Z_0 = R_1 + \frac{R_2(R_1+Z_0)}{Z_0+R_1+R_2}$$

$$\Gamma = S_{11} = 0 = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \Rightarrow 0 \quad Z_{in} - Z_0 = 0$$

$$R_1(Z_0 + R_1 + R_2) + R_2(R_1 + Z_0) - Z_0(Z_0 + R_1 + R_2) = 0$$

$$R_1^2 + R_1R_2 + \cancel{R_1Z_0} + R_1R_2 + \cancel{R_2Z_0} - Z_0^2 \cancel{- R_1Z_0} \cancel{- R_2Z_0} = 0$$

$$2R_1R_2 = Z_0^2 - R_1^2 \longrightarrow \boxed{R_2 = \frac{Z_0^2 - R_1^2}{2R_1}}$$

$$S_{21} = \alpha = \frac{Z_0}{Z_0 + R_1} \cdot \frac{R_2 // (R_1 + Z_0)}{R_2 // (R_1 + Z_0) + R_1} =$$

$$= \frac{Z_0}{Z_0 + R_1} \cdot \frac{R_2 (R_1 + Z_0)}{Z_0 + R_1 + R_2} \cdot \frac{1}{\frac{R_2 (R_1 + Z_0)}{Z_0 + R_1 + R_2} + R_1} =$$

$$= \frac{Z_0}{(\cancel{Z_0 + R_1})} \cdot \frac{R_2 (\cancel{R_1 + Z_0})}{(\cancel{Z_0 + R_1 + R_2})} \cdot \frac{(\cancel{Z_0 + R_1 + R_2})}{R_2 (R_1 + Z_0) + R_1 (Z_0 + R_1 + R_2)} =$$

$$= \frac{Z_0 R_2}{R_1 R_2 + Z_0 R_2 + Z_0 R_1 + R_1^2 + R_1 R_2} = \frac{Z_0 R_2}{R_1^2 + 2 R_1 R_2 + Z_0 R_1 + Z_0 R_2} =$$

previous cond

$$= \frac{Z_0 R_2}{Z_0^2 + Z_0 (R_1 + R_2)} = \frac{R_2}{Z_0 + R_1 + R_2}$$

$$\alpha (Z_0 + R_1 + R_2) = R_2$$

$$\alpha (Z_0 + R_1) = R_2 (1 - \alpha)$$

$$R_1 = \frac{R_2}{\alpha} (1 - \alpha) - Z_0 = \frac{Z_0^2 - R_1^2}{2 R_1 \alpha} (1 - \alpha) - Z_0$$

$$R_1^2 = \frac{(Z_0^2 - R_1^2)(1 - \alpha)}{2\alpha} - R_1 Z_0 = \frac{1}{2\alpha} \left[Z_0^2 - Z_0^2 \alpha - R_1^2 + R_1^2 \alpha \right]$$

$$2\alpha R_1^2 = Z_0^2 (1 - \alpha) + R_1^2 (\alpha - 1) \longrightarrow R_1^2 (2\alpha - \alpha + 1) = Z_0^2 (1 - \alpha)$$

$$R_1^2 = Z_0^2 \, \frac{1-\alpha}{\alpha+1}$$

$$R_1 = Z_0 \sqrt{\frac{1-\alpha}{1+\alpha}}$$

So we get:

$$\boxed{R_1 = Z_0 \sqrt{\frac{1-\alpha}{1+\alpha}} \qquad R_2 = \frac{Z_0^2 - R_1^2}{2R_1}}$$

$Z_0 = 50\,\Omega$

- $\alpha = -3\,dB \rightarrow \alpha = 0,708$

  $R_1 = 20,67\,\Omega$

  $R_2 = 50,14\,\Omega$

- $\alpha = -10\,dB \rightarrow \alpha = 0,316$

  $R_1 = 36,05\,\Omega$

  $R_2 = 16,66\,\Omega$

- $\alpha = -20\,dB \rightarrow \alpha = 0.1$

  $R_1 = 45,23\,\Omega$

  $R_2 = 5,022\,\Omega$

For π configuration

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = 0 \implies R_1^2 R_2 - 2 R_1 Z_0^2 - R_2 Z_0^2 = 0$$

$$R_2 = \frac{2 R_1 Z_0^2}{R_1^2 - Z_0^2}$$

$$S_{21} = \alpha \quad \frac{\frac{R_1 Z_0}{R_1 + Z_0}}{\frac{R_1 \cdot Z_0}{R_1 + Z_0} + R_2} = \frac{\frac{Z_0 R_1}{\cancel{Z_0 + R_1}}}{\frac{Z_0 R_1 + R_1 R_2 + Z_0 R_2}{\cancel{Z_0 + R_1}}} = \frac{Z_0 R_1}{Z_0 R_1 + R_1 R_2 + Z_0 R_2} =$$

$$= \frac{Z_0 R_1}{Z_0 R_1 + R_2 (R_1 + Z_0)}$$

$$\alpha Z_0 R_1 + \alpha R_2 (R_1 + Z_0) = Z_0 R_1$$

$$R_2 \alpha (Z_0 + R_1) = Z_0 R_1 (1 - \alpha)$$

$$R_2 = \frac{Z_0 R_1 (1 - \alpha)}{\alpha (Z_0 + R_1)}$$

$$\implies \frac{2 \cancel{R_1} Z_0^{\cancel{2}}}{(\cancel{Z_0 + R_1})(R_1 - Z_0)} = \frac{\cancel{Z_0} \cancel{R_1} (1 - \alpha)}{\alpha \, (\cancel{Z_0 + R_1})} \implies \frac{2 \alpha}{1 - \alpha} + Z_0 = R_1$$

So:

$$R_1 = Z_0 + \frac{2\alpha}{1-\alpha} \qquad R_2 = \frac{2 R_1 Z_0}{R_1^2 - Z_0^2}$$

∘) $\alpha = -3\,dB = 0{,}708$

$R_1 = 54{,}85\ \Omega$

$R_2 = 16{,}37\ k\Omega$

∘) $\alpha = -10\,dB = 0{,}316$

$R_1 = 50{,}92\ \Omega$

$R_2 = 78{,}3\ k\Omega$

∘) $\alpha = -20\,dB = 0{,}1$

$R_1 = 50{,}22\ \Omega$

$R_2 = 320\ k\Omega$

Problem 4

- First: Constant-K T section

$$L = \frac{R_0}{2\omega_c} = \frac{R_0}{4\pi f_c} = \frac{75}{4\pi \cdot 50 \cdot 10^6} = 119,4 \text{ nH}$$

$$C = \frac{1}{2\omega_c R_0} = \frac{1}{4\pi 50 \cdot 10^6 \cdot 75} = 21,22 \text{ pF}$$

- Second: m-derived T section

$$m = \sqrt{1 - \left(\frac{\omega_{\infty}}{\omega_c}\right)^2} = 0,28$$

$$\frac{2C}{m} = 151,6 \text{ pF}$$

$$\frac{4mC}{1-m^2} = 25,8 \text{ pF}$$

$$\frac{L}{m} = 426,4 \text{ nH}$$

- Third: Bisected-π matching section

$m = 0,6$

$$\frac{2L}{m} = 398 \text{ nH}$$

$$\frac{2mC}{1-m^2} = 39,8 \text{ pF}$$

$$\frac{2C}{m} = 70,73 \text{ pF}$$