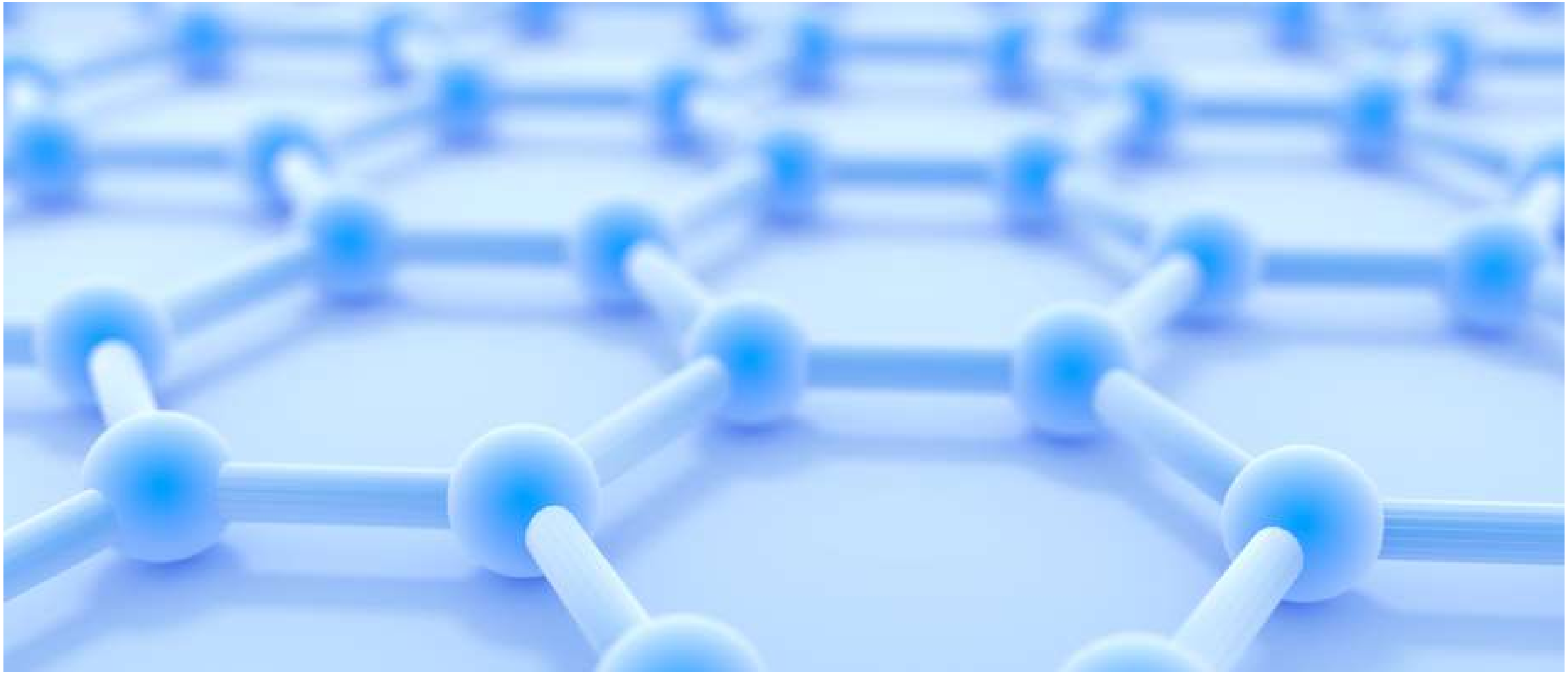

Photonics
aa 2021/2022

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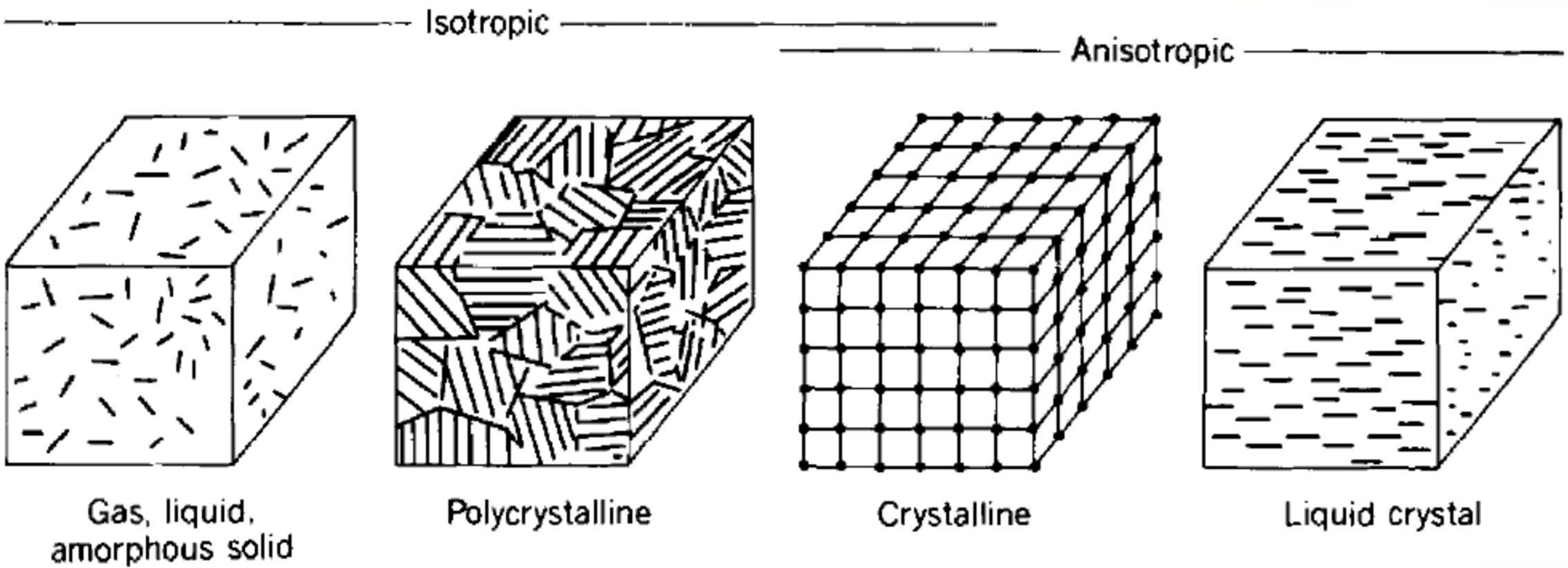
Polarization Optics:
Anisotropic media,
optical activity and magneto-optics

Anisotropic materials

A dielectric medium is said to be **anisotropic** if its macroscopic optical properties depend on direction.

The macroscopic properties are, however, dictated by the microscopic properties like shape and orientation of molecules and their organization in space:

- Gas, liquids and amorphous solids are isotropic because molecules are oriented randomly in space, so macroscopically, the material behave isotropically.
- On the other hand, if the molecules/particles have a preferred orientation, then the material shows anisotropy.



Anisotropic materials

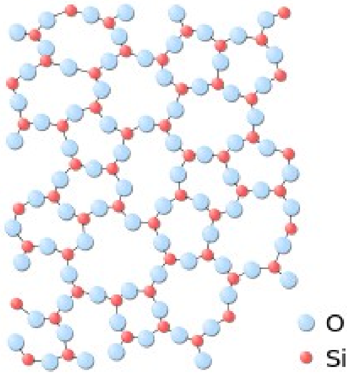
isotropic/anisotropic

response as a function of direction of propagation

Isotropic

Glass

$$\mathbf{D} = \varepsilon\mathbf{E}$$



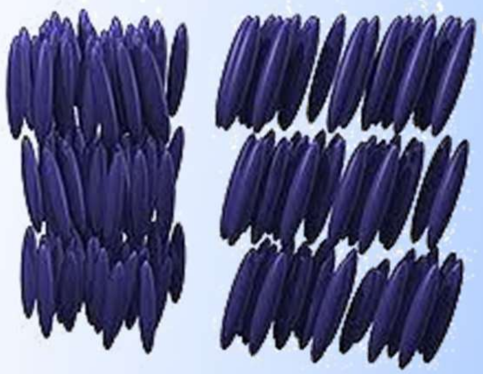
ε is a scalar

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

Anisotropic

Liquid crystal

$$\mathbf{D} = \boldsymbol{\varepsilon} \cdot \mathbf{E}$$



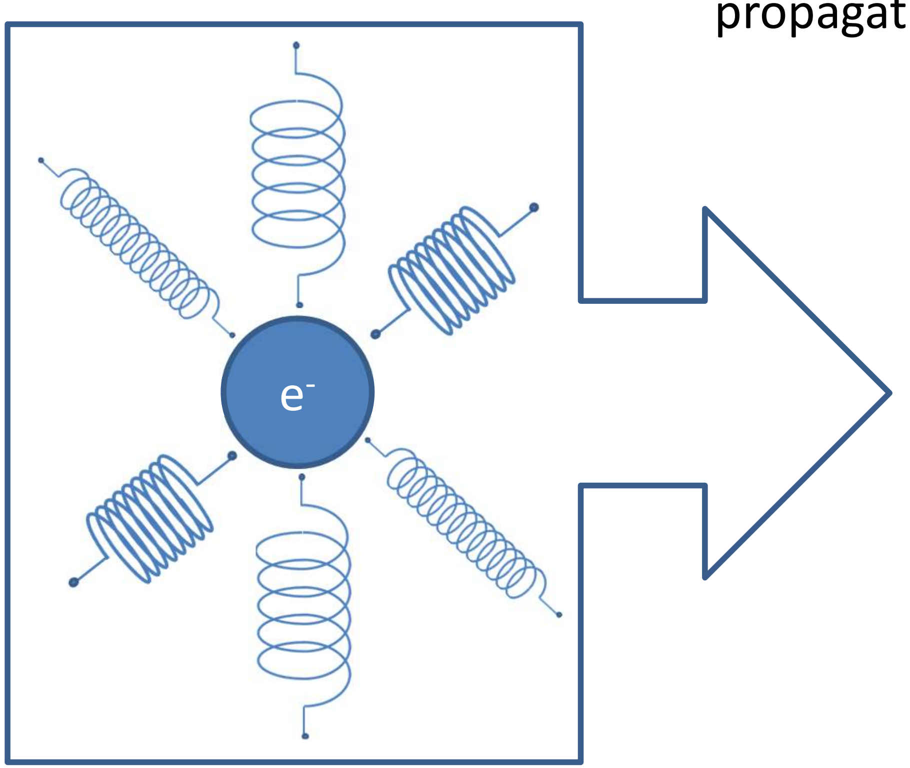
ε is a tensor

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

Anisotropic materials

What is the physical meaning of ***anisotropy***?

The **polarization** in a crystal by a given electric field **varies** depending on the direction of the applied field with respect to the crystal lattice.
This also implies that **phase velocity** can assume a **different value** depending on the direction of propagation and polarization.



Bound electron connected to a fictitious set of springs

The displacement of an electron under the action of an external field E depends on the direction of the field and its magnitude. Therefore, we can express the polarization as:

$$\mathbf{P} = \varepsilon_0 \boldsymbol{\chi} \mathbf{E}$$

or

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \varepsilon_0 \begin{pmatrix} \chi_{11} & \chi_{12} & \chi_{13} \\ \chi_{21} & \chi_{22} & \chi_{23} \\ \chi_{31} & \chi_{32} & \chi_{33} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

Principal axes

The elements of the permittivity tensor depend on how the coordinate system is chosen relative to the crystal structure. However, we can always find a coordinate system for which the off-diagonal elements ε_{ij} vanish. Under these circumstances we can write:

$$D_1 = \epsilon_1 E_1, \qquad D_2 = \epsilon_2 E_2, \qquad D_3 = \epsilon_3 E_3$$

Where $\epsilon_1 = \epsilon_{11},\ \epsilon_2 = \epsilon_{22},\ \epsilon_3 = \epsilon_{33}$.

In this specific case D and E are always parallel. This coordinate system defines the **PRINCIPAL AXES** of the crystal.

From now on we will always assume that our coordinate system x, y, z (or alternatively x_1, x_2, x_3) coincides with the principal axes of the crystal.

The **PRINCIPAL REFRACTIVE INDEXES** are:

$$n_1 = \sqrt{\epsilon_1/\epsilon_0},\ \ n_2 = \sqrt{\epsilon_2/\epsilon_0},\ \ n_3 = \sqrt{\epsilon_3/\epsilon_0}.$$

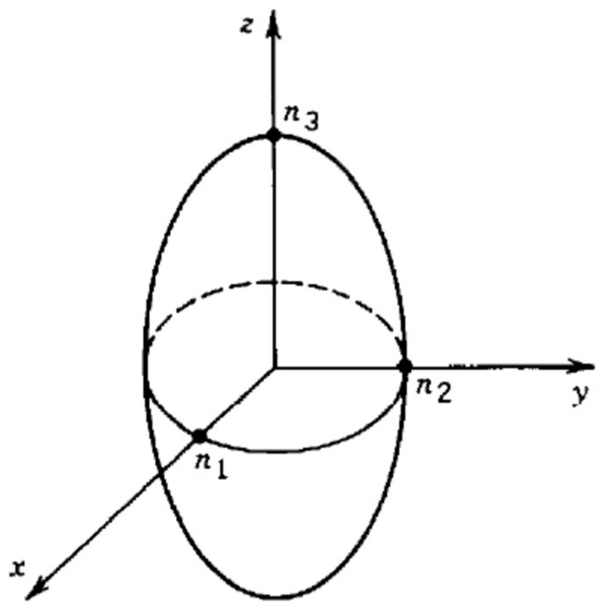
Index Ellipsoid

The **INDEX ELLIPSOID** or **OPTICAL INDICATRIX** is the quadratic representation of the electric impermeability tensor $\boldsymbol{\eta} = \epsilon_0 \boldsymbol{\epsilon}^{-1}$:

$$\sum_{ij} \eta_{ij} x_i x_j = 1, \qquad i,j = 1,2,3.$$

If the principal axes are used as the coordinate system, then we would obtain the following **index ellipsoid**:

$$\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} = 1.$$



NOTE: The shape of the ellipsoid depends on the type of crystal under investigation. For a uniaxial crystal, the index ellipsoid reduces to an ellipsoid of revolution, for an isotropic medium becomes a sphere.

Propagation in anisotropic materials

Light propagation in anisotropic materials can be complicated. One way to simplify this problem is to consider plane waves propagating along the principal axes of the crystal.

In this particular scenario, we have the so-called **NORMAL MODES:**
Let's assume that x-y-z is a coordinate system that overlaps with the principal axes and that we have a plane wave traveling in the z direction and polarized along x. Such wave will propagate with phase velocity c/n_1 and will not change its polarization while propagating.
The same considerations can be repeated for waves propagating along the other two axes and polarized on another principal axis.

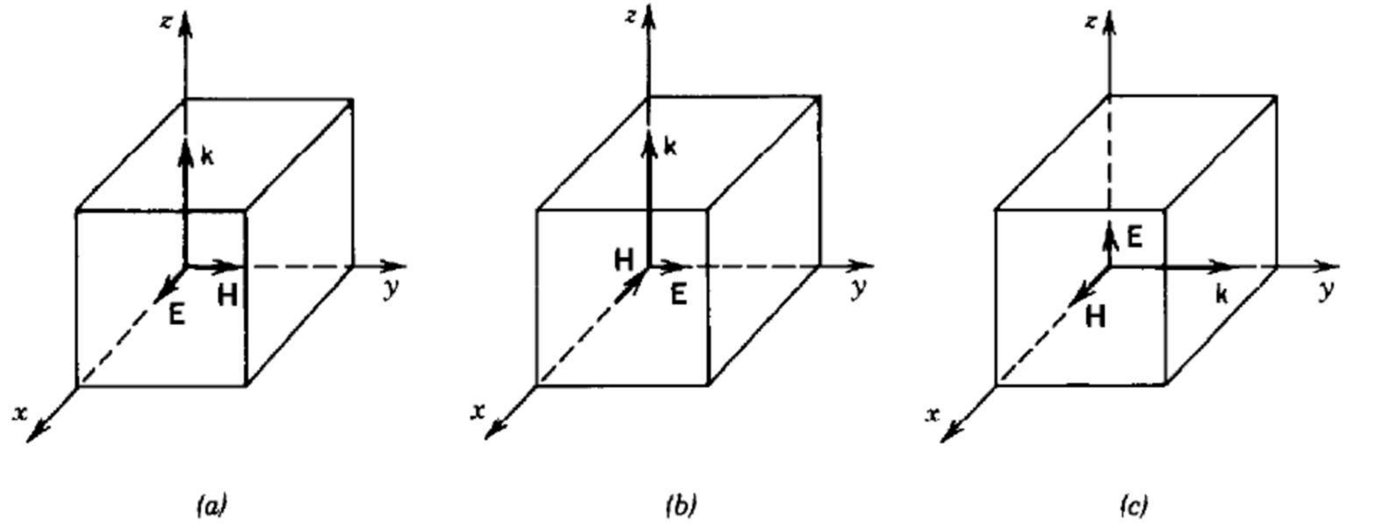


Figure 6.3-4 A wave traveling along a principal axis and polarized along another principal axis has a phase velocity c_o/n_1, c_o/n_2, or c_o/n_3, if the electric field vector points in the x, y, or z directions, respectively. (a) $k = n_1 k_o$; (b) $k = n_2 k_o$; (c) $k = n_3 k_o$.

Propagation in anisotropic materials

If our plane wave now still propagates along one principal axis, say z, but it is polarized in the x-y plane, the propagating mode should be studied as the superposition of normal modes, in this specific case two plane waves polarized along x and y.

The two components will experience different phase velocities and, therefore, accumulate a phase shift between the two components. The result of this phenomenon is that the recombination of the two normal modes will produce and elliptically polarized wave.

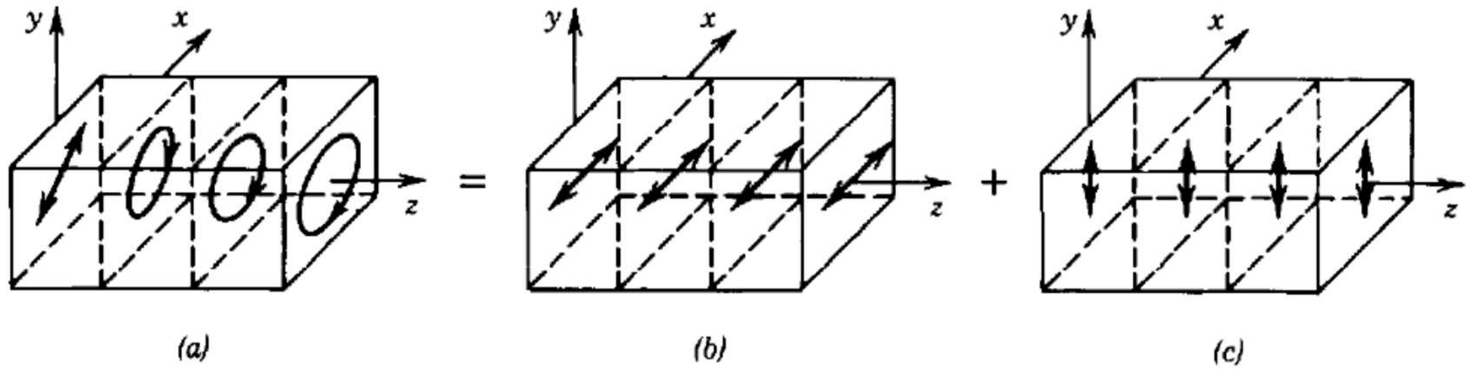


Figure 6.3-5 A linearly polarized wave at 45° in the $z = 0$ plane is analyzed as a superposition of two linearly polarized components in the x and y directions (normal modes), which travel at velocities c_o/n_1 and c_o/n_2. As a result of phase retardation, the wave is converted into an elliptically polarized wave.

NOTE: An anisotropic crystal effectively behaves as a wave retarder!

Propagation in anisotropic materials

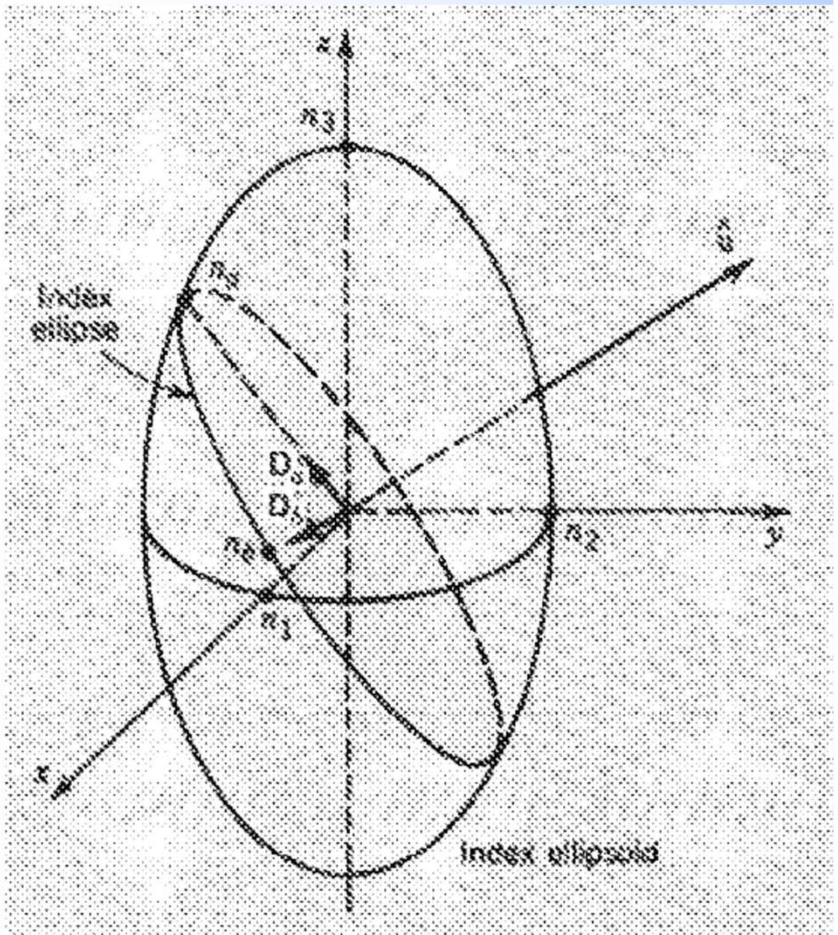
If we consider the general case of a plane wave traveling in an anisotropic crystal in an arbitrary direction defined by the unit vector $\hat{\boldsymbol{u}}$, we can find that the two normal modes are linearly polarized waves. The refractive indices n_a and n_b and the direction of polarization of these modes can be determined by using the index ellipsoid.

1. Draw a plane passing through the origin of the index ellipsoid normal to $\hat{u}$. The intersection of the plane with the ellipsoid is called **index ellipse**;

2**.** The half-lengths of the major and minor axes od the Index ellipse are the **refractive indices n_a and n_b of the two normal modes**.

3. The direction of the major and minor axes of the index Ellipse are the directions of the vectors $\mathbf{D_a}$ and $\mathbf{D_b}$ for the Normal modes. These directions are orthogonal.

4. The directions of the electric field vectors $\mathbf{E_a}$ and $\mathbf{E_b}$ may Be determined from the expression $\mathbf{D} = \boldsymbol{\varepsilon} \cdot \mathbf{E}$



Propagation in anisotropic materials

SPECIAL CASE: UNIAXIAL CRYSTALS

In uniaxial crystals (n_1=n_2=n_0 and n_3=n_e) the index ellipsoid is an ellipsoid of revolution. For a wave whose direction of travel $\hat{\boldsymbol{u}}$ forms an angle θ with the optic axis, the index ellipse has half-lengths n_0 and n(θ) where:

$$\frac{1}{n^2(\theta)} = \frac{\cos^2(\theta)}{n_0^2} + \frac{\sin^2(\theta)}{n_e^2}$$

According to this equation we can say that regardless the angle θ we will have two normal modes:

- an **ordinary wave** that travels according to the ordinary refractive index n_0 ;

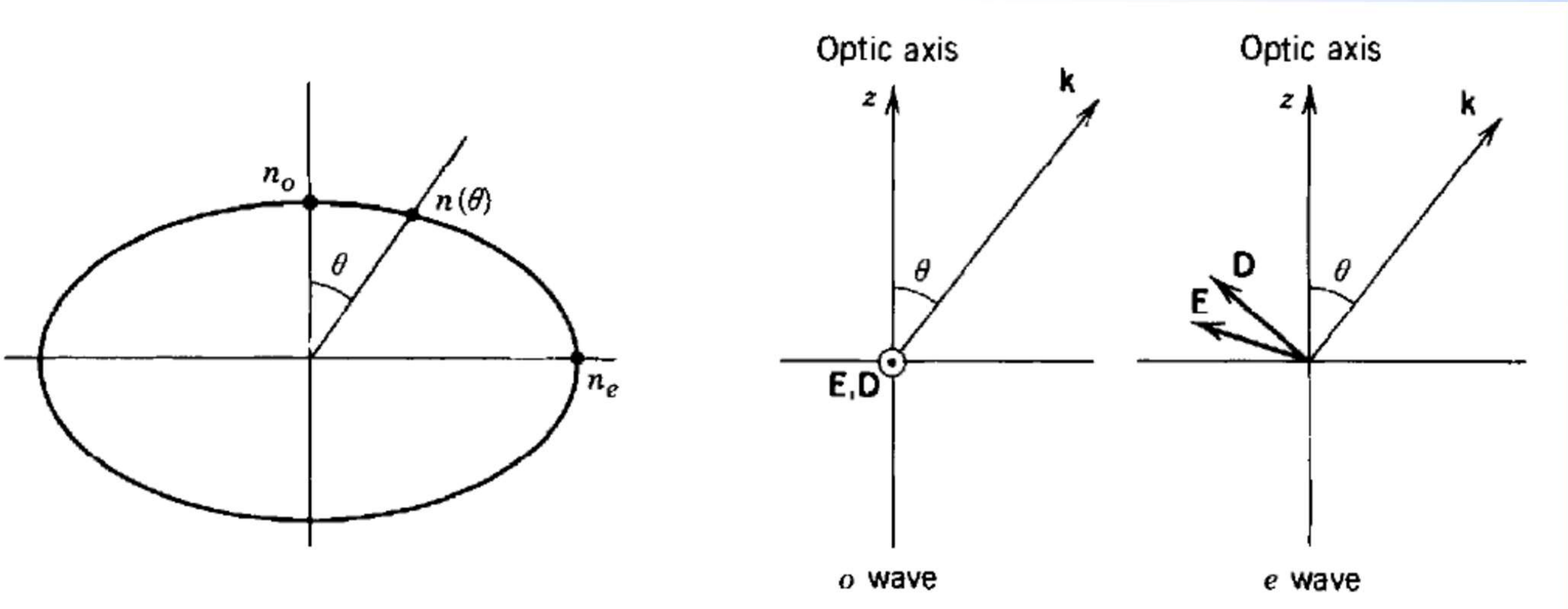
- an **extraordinary wave** that travels according to the refractive index n(θ): it can be seen from the equation that when θ changes from 0° to 90° the extraordinary index changes from n_0 to n_e.

Propagation in anisotropic materials

IMPORTANT

The vector **D** of the **ordinary wave** is **normal** to the plane defined by the optic axis (z) and the direction of propagation **k** and the vectors **E** and **D** are **parallel**.

The **extraordinary wave**, on the other hand, has a vector **D** that is **normal** to **k** and lies in the k-z plane, but it is **not parallel to E**.



Dispersion Relation

For ordinary non absorbing crystals the χ tensor is symmetric so there always exists a set of coordinate axes, called principal axes, such that the χ tensor assumes the diagonal form:

$$\boldsymbol{\chi} = \begin{pmatrix} \chi_{11} & 0 & 0 \\ 0 & \chi_{22} & 0 \\ 0 & 0 & \chi_{33} \end{pmatrix}$$

For such materials, the plane wave equation can be written in the following form:

$$\nabla \times \left(\nabla \times \mathbf{E} \right) + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\frac{1}{c^2} \boldsymbol{\chi} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

It then follows that the crystal can sustain a monochromatic plane wave in the usual form $e^{-j(\omega t - \mathbf{kr})}$ provided the propagation vector **k** satisfies the equation:

$$\mathbf{k} \times \left(\mathbf{k} \times \mathbf{E} \right) + \frac{\omega^2}{c^2} \mathbf{E} = -\frac{\omega^2}{c^2} \boldsymbol{\chi} \mathbf{E}$$

Dispersion Relation

The equation is equivalent to the following three equations:

$$\left(-k_y^2 - k_z^2 + \frac{\omega^2}{c^2}\right)E_x + k_x k_y E_y + k_x k_z E_z = -\frac{\omega^2}{c^2}\chi_{11}E_x$$

$$k_y k_x E_x + \left(-k_x^2 - k_z^2 + \frac{\omega^2}{c^2}\right)E_y + k_y k_z E_z = -\frac{\omega^2}{c^2}\chi_{22}E_y$$

$$k_z k_x E_x + k_z k_y E_y + \left(-k_x^2 - k_y^2 + \frac{\omega^2}{c^2}\right)E_z = -\frac{\omega^2}{c^2}\chi_{33}E_z$$

For simplicity let's now suppose we have a wave propagating along the principal axis *x*. In this case $k_x = k$ and $k_y = k_z = 0$ so that the three equations reduce to:

$$\frac{\omega^2}{c^2}E_x = -\frac{\omega^2}{c^2}\chi_{11}E_x \quad \Longrightarrow \quad E_x = 0 \quad \Longrightarrow$$ E field transverse to the *x* axis

$$\left(-k^2 + \frac{\omega^2}{c^2}\right)E_y = -\frac{\omega^2}{c^2}\chi_{22}E_y \quad \Longrightarrow \quad \text{If } E_y \neq 0 \quad \Longrightarrow \quad k = \frac{\omega}{c}\sqrt{1+\chi_{22}} = \frac{\omega}{c}\sqrt{K_{22}}$$

$$\left(-k^2 + \frac{\omega^2}{c^2}\right)E_z = -\frac{\omega^2}{c^2}\chi_{33}E_z \quad \Longrightarrow \quad \text{If } E_z \neq 0 \quad \Longrightarrow \quad k = \frac{\omega}{c}\sqrt{1+\chi_{33}} = \frac{\omega}{c}\sqrt{K_{33}}$$

Dispersion Relation

In an anisotropic medium the *phase velocity* ω/k can assume different values according to the propagation direction

More generally we can show that for any direction of the propagation vector **k**, there are two possible values of the phase velocity. If we define the *three principal indices of refraction*:

$$\boxed{\begin{aligned} n_1 &= \sqrt{1+\chi_{11}} = \sqrt{K_{11}} \\ n_2 &= \sqrt{1+\chi_{22}} = \sqrt{K_{22}} \\ n_3 &= \sqrt{1+\chi_{33}} = \sqrt{K_{33}} \end{aligned}}$$

And then solving the system for a non-trivial solution for E_x, E_y and E_z:

$$\left(-k_y^2 - k_z^2 + \frac{\omega^2}{c^2}\right)E_x + k_x k_y E_y + k_x k_z E_z = -\frac{\omega^2}{c^2}\chi_{11}E_x$$

$$k_y k_x E_x + \left(-k_x^2 - k_z^2 + \frac{\omega^2}{c^2}\right)E_y + k_y k_z E_z = -\frac{\omega^2}{c^2}\chi_{22}E_y$$

$$k_z k_x E_x + k_z k_y E_y + \left(-k_x^2 - k_y^2 + \frac{\omega^2}{c^2}\right)E_z = -\frac{\omega^2}{c^2}\chi_{33}E_z$$

Dispersion Relation

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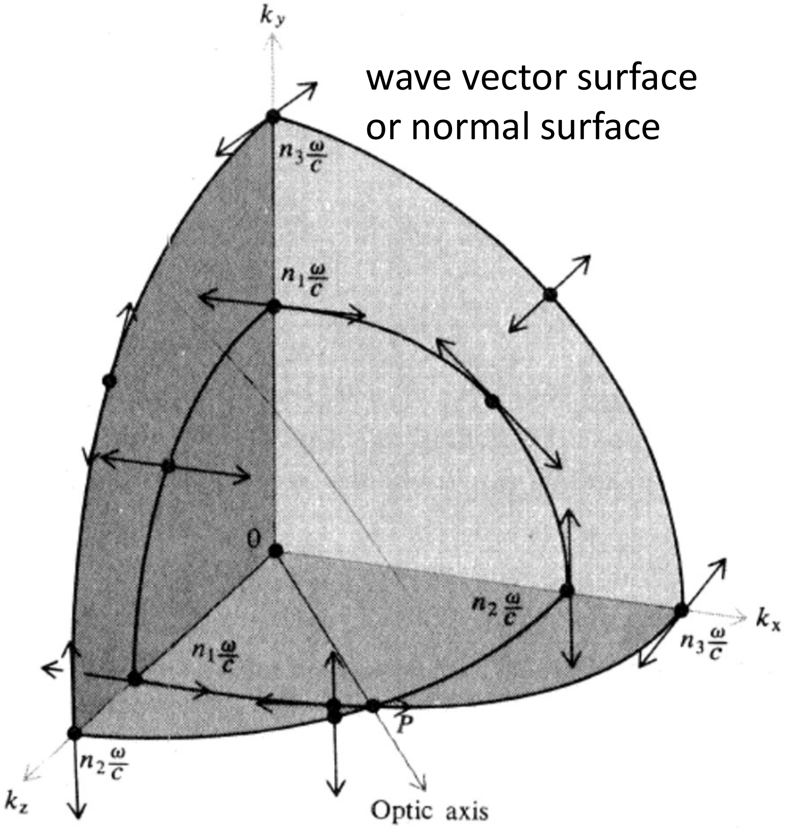
We should solve:

$$\begin{vmatrix} (n_1\,\omega/c)^2 - k_y^2 - k_z^2 & k_x k_y & k_x k_z \\ k_y k_x & (n_2\,\omega/c)^2 - k_x^2 - k_z^2 & k_y k_z \\ k_z k_x & k_z k_y & (n_3\,\omega/c)^2 - k_x^2 - k_y^2 \end{vmatrix} = 0$$



Depending on the plane of propagation the determinant assumes a different form. For example, if we consider k_z=0 the determinant is the product of two factors. Setting them at zero in turn gives the following equations:

$$\begin{cases} k_x^2 + k_y^2 = (n_3\,\omega/c)^2 & \text{circle} \\ \dfrac{k_x^2}{(n_2\,\omega/c)^2} + \dfrac{k_y^2}{(n_1\,\omega/c)^2} = 1 & \text{ellipse} \end{cases}$$

Dispersion relation

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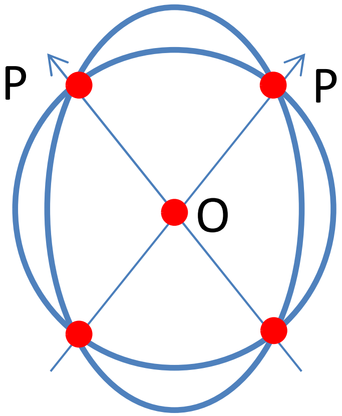
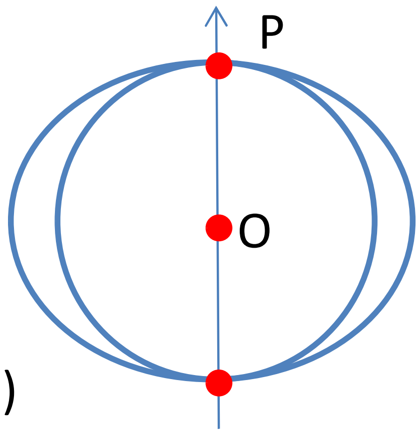
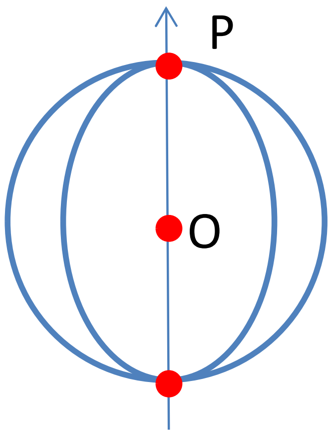
Anisotropic materials: Type of crystals



(a)



(b)



(c)

We have seen that when unpolarized light or light with arbitrary polarization propagates through crystals, it can be considered as the superposition of two independent waves (normal modes), polarized orthogonally to each other and propagating with two different phase velocities.

Points P in the wave vector surface represent the only direction for which the two values of k are equal, and they correspond to the *optical axes* of the crystal.

We can classify anisotropic materials in the following cathegories:

BIAXIAL CRYSTALS (a): - different principal indices n_1, n_2 and n_3
- 2 optic axes

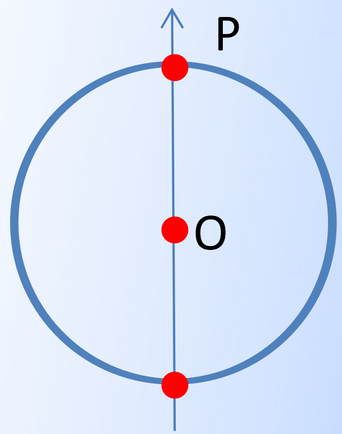
UNIAXIAL CRYSTALS (b) and (c): - two of the principal indices are equal
- 1 optic axis

ISOTROPIC CRYSTALS (see next slide): - all principal indices are equal
- 1 optic axis

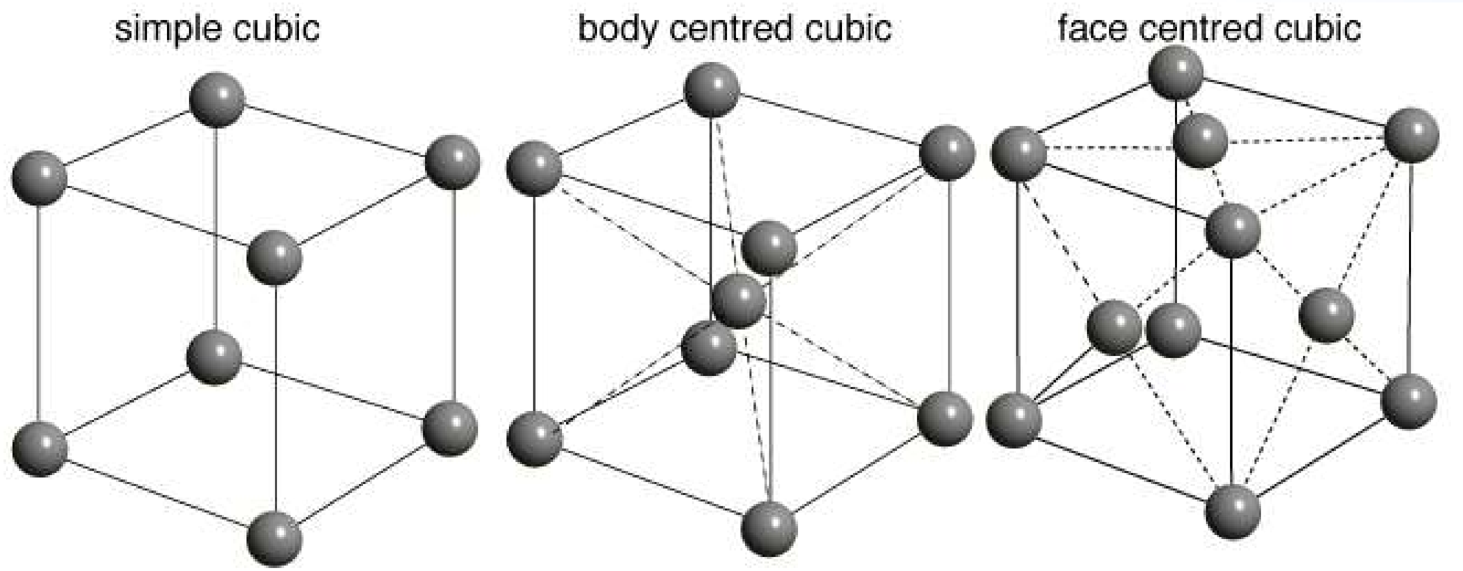
Anisotropic materials: Type of crystals

ISOTROPIC CRYSTALS

$$\boldsymbol{\chi} = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}, \quad \chi_{11} = \chi_{22} = \chi_{33} = a, \quad n = \sqrt{1+a}$$



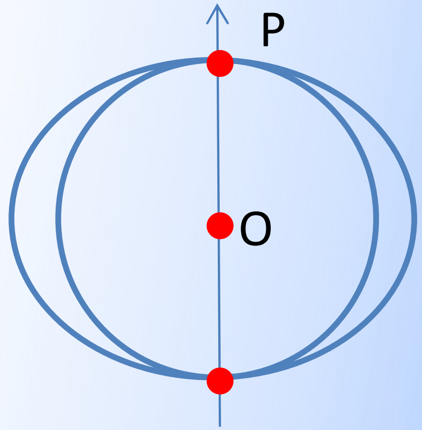
CUBIC



Anisotropic materials: Type of crystals

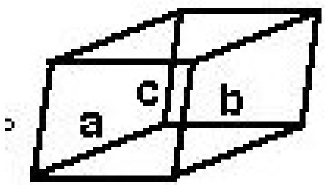
UNIAXIAL CRYSTALS

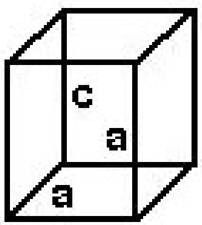
$$\boldsymbol{\chi} = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{pmatrix}, \quad \chi_{11} = \chi_{22} = a,\ \chi_{33} = b, \quad n_o = \sqrt{1+a},\ n_e = \sqrt{1+b}$$

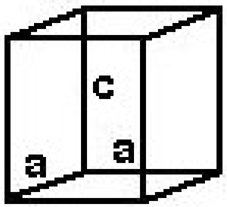


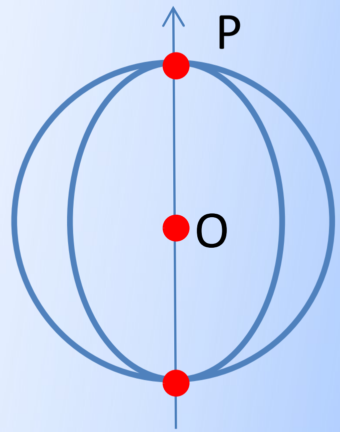
$n_o < n_e$

TRIGONAL **TETRAGONAL** **HEXAGONAL**







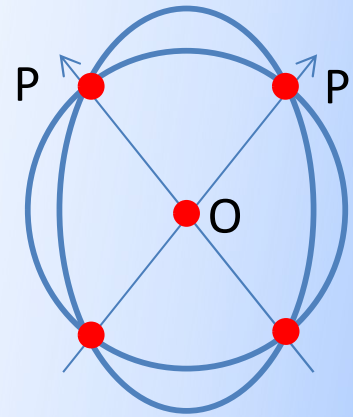


$n_o > n_e$

Anisotropic materials: Type of crystals

BIAXIAL CRYSTALS

$$\boldsymbol{\chi} = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}, \quad \chi_{11} = a,\ \chi_{22} = b,\ \chi_{33} = c, \quad n_1 = \sqrt{1+a},\ n_2 = \sqrt{1+b},\ n_3 = \sqrt{1+c}$$



TRICLINIC

MONOCLINIC

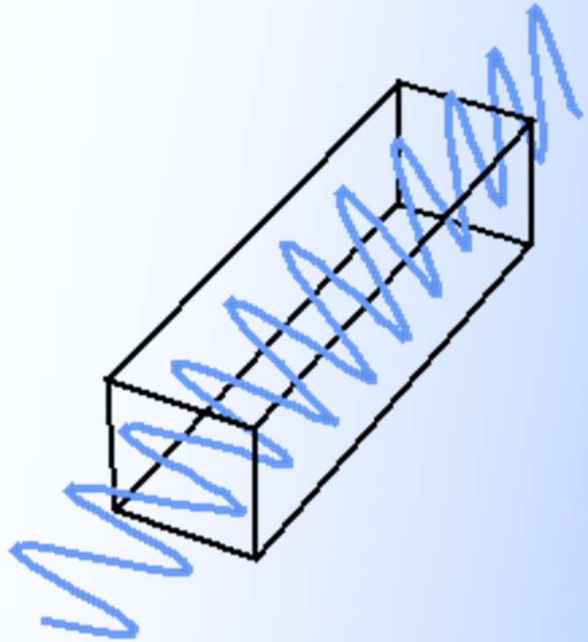
ORTHOROMBIC

Optical Activity

Certain materials naturally act as polarization rotators, a property known as **optical activity.**

Their normal modes are waves that are **circularly polarized**, rather than linearly polarized: waves with right and left circular polarizations travel at different wave velocities.

The **rotatory power** (rotation angle per unit length) of the optically active medium is:

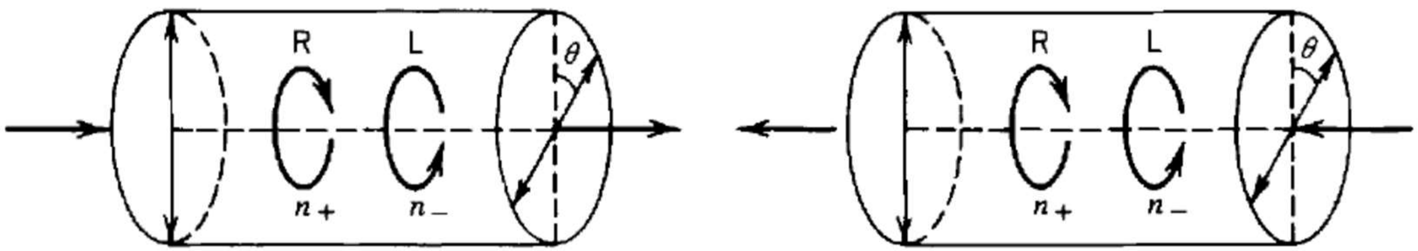


$$\boldsymbol{\rho = \pi/\lambda_0(n_- - n_+)}$$

The direction in which the polarization plane rotates is the same as that of the circularly polarized component with the greater phase velocity (smaller refractive index).

If $\boldsymbol{n_+ < n_-, \rho}$ is positive and the rotation is in the same direction as the electric field vector of the RHCP. These materials are called **DEXTROROTATORY**.

If $\boldsymbol{n_+ > n_-, \rho}$ is negative.
These materials are called **LEVOROTATORY**.



Optical Activity

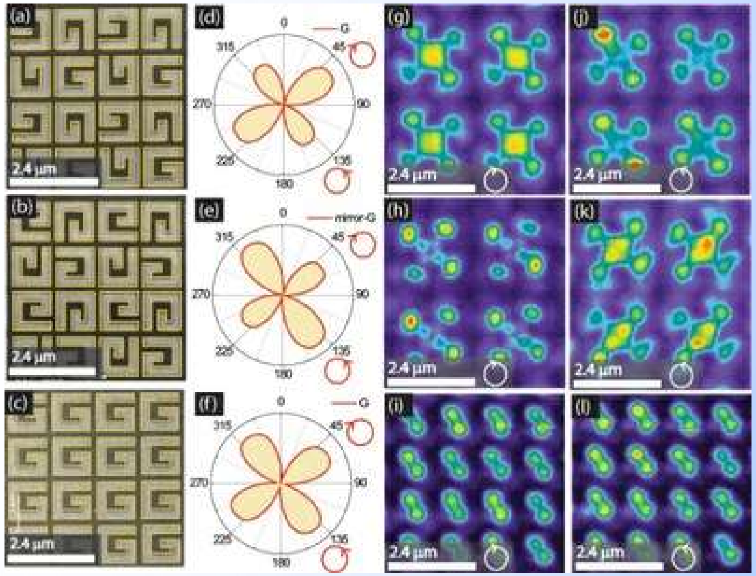
Optical activity typically occurs in asymmetric structures that have an intrinsically helical structure.

For example, if linearly polarized light passes through a sample containing ***chiral centers***, the plane of polarization rotates. Chiral media respond differently to left and right circularly polarized light – this property is called *circular dichroism.*

In an optically active medium the constitutive relations are:

$$\boldsymbol{D} = \epsilon \boldsymbol{E} + j\epsilon_0 \boldsymbol{G} \times \boldsymbol{E}$$

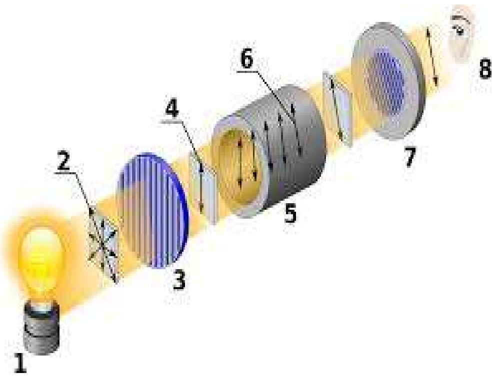
Where $\boldsymbol{G} = \boldsymbol{\xi k}$ is the **gyration vector.**



Chiral Nanostructure

Optical activity is measured with an instrument called polarimeter:

Polarimeter



1. Light source; 2. unpolarized light; 3. fixed polarizer; 4. polarized light; 5. optically-active sample; 6 polarization rotation; 7. analyzer; 8. sensor/eye

Magneto-Optics: The Faraday effect

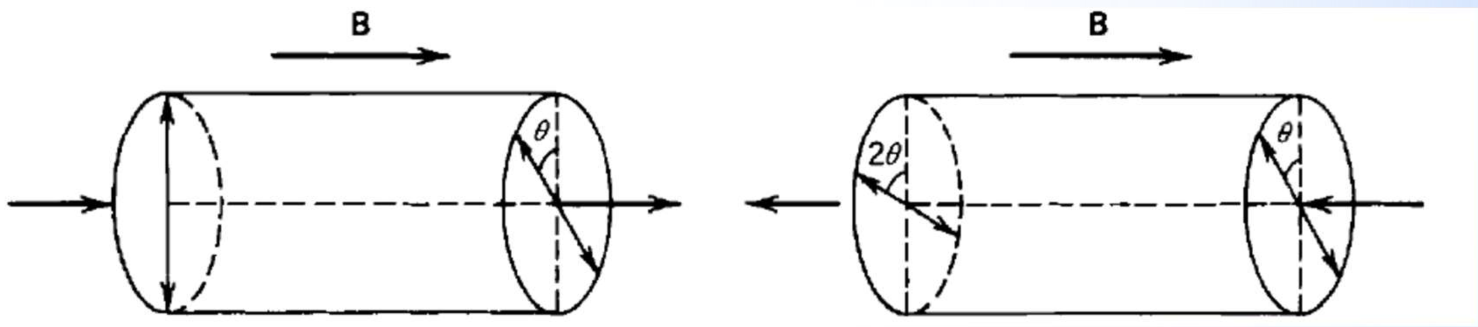
Many materials act as polarization rotators in the presence of a static magnetic field, a property known as **Faraday effect.**

The amount of rotation is proportional to the thickness of the material and the rotatory power ρ is proportional to the component of the magnetic flux density in the direction of the wave propagation:

$$\rho = \mathfrak{v}\boldsymbol{B}$$

Where $\mathfrak{v}$ is the **Verdet constant**.

Differently from optical activity the sense of rotation does not reverse with the reversal of the direction of propagation of the wave.



Magneto-Optics: The Faraday effect

In magneto-optic materials the electric permittivity tensor is altered by the application of a static magnetic field so that the constitutive relations are:

$$\boldsymbol{D} = \epsilon \boldsymbol{E} + j\epsilon_0 \boldsymbol{G} \times \boldsymbol{E}$$

Where $\boldsymbol{G} = \gamma \boldsymbol{B} = \gamma \mu \boldsymbol{H}$. The constant γ is the **magnetogyration coefficient.**

Magneto-optics materials are typically used as isolators since their rotation properties do not change with the direction of propagation (**G** does not depend on **k** as in optically active materials).

The Verdet constant is also a function of the wavelength:

$$\mathfrak{v} \approx -\frac{\pi\gamma}{\lambda_0 n}$$