Basics of Active and Nonlinear HF Electronics – Tutorial

We consider a FET and its nonlinear drain current source I_{DS} modeled by the below equation as a function of its control voltages V_{GS} and V_{DS} :

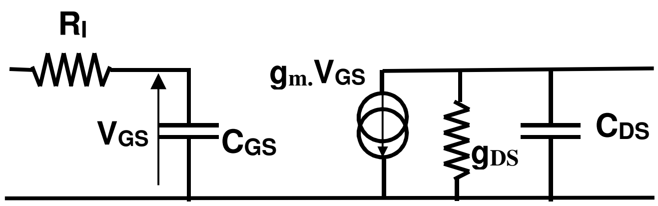
$$I_{DS} = I_{DSS}\left[1 - \frac{V_{GS}(t)}{V_P}\right]^2 (1 + K.V_{DS}(t))$$

The model is accurate in the variation range $-1.5V < V_{GS} < 0V$; $1V < V_{DS} < 7V$

with I_{DSS} = 60 mA; V_P = -1.5 V; K = 0.05; C_{GS} = 0.5 pF; C_{DS} = 0.1 pF; R_I= 4 Ω .

The FET is biased at V_{GS0} = - 0.75 V and V_{DS0} = 4 V

The small signal linear model of the FET in these conditions is shown below:



Study of parallel-cells of FET

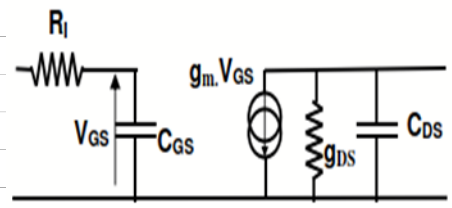
1) Draw the I_{DS}-V_{DS} curves in the variation range
2) Determine the values of transconductance g_m and drain conductance g_{DS} at the selected bias point.
3) Determine the small-signal linear model corresponding to n parallel-cells of the FET. Determine the ~~variation laws~~ scaling rules that give equivalent elements ($C_{GS}(n)$ …) of the parallel-cells as a function of the number of cells n and the equivalent elements ($C_{GS}(1)$ …) of a single-cell FET.
4) Determine the maximum power gain G_{MAX} for an ideal power matching at small-signal (linear) level. Determine its cutoff frequency f_C and its maximum frequency f_{MAX}.
5) After determining the optimum power load in large signal class A operation, determine the maximum gain G_{MAX} in this loading condition. Estimate the maximum output power value in class A. Compare these values to G_{MAX} and its associated maximum output power when the FET is ideally matched to give optimum gain in small signal operation.
6) Same questions 4) and 5) in the case of n-parallel FETs

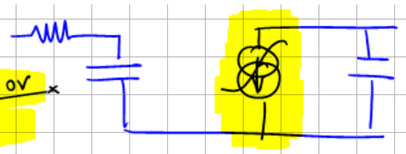
$$I_{DS} = I_{DSS}\left[1-\frac{V_{GS}(t)}{V_P}\right]^2 (1+K.V_{DS}(t)) \quad *$$

The model is accurate in the variation range $-1.5V < V_{GS} < 0V$; $1V < V_{DS} < 7V$

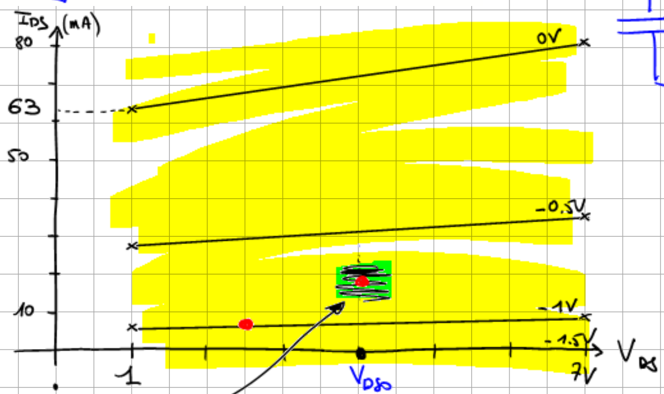
with $I_{DSS} = 60$ mA; $V_P = -1.5$ V; $K = 0.05$; $C_{GS} = 0.5$ pF; $C_{DS} = 0.1$ pF; $R_I = 4\ \Omega$.

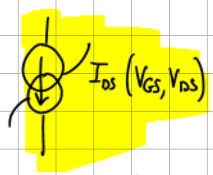
The FET is biased at $V_{GS0} = -0.75$ V and $V_{DS0} = 4$ V Bias point

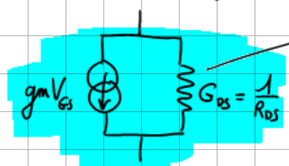




V_{GS}	V_{DS}	$\left(1-\frac{V_{GS}}{V_P}\right)^2$	$1+K\,V_{DS}$	I_{DS}
0V	1V	1	1.05	63 mA
	7V		1.35	81 mA
-0.5V	1V	$\left(1-\frac{-0.5}{-1.5}\right)^2 = \frac{4}{9}$	1.05	28 mA
	7V		1.35	36 mA
-1V	1V	$\left(1-\frac{-1}{-1.5}\right)^2 = \frac{1}{9}$	1.05	7 mA
	7V		1.35	9 mA







$$f(x,y) = f(x_0,y_0) + df$$

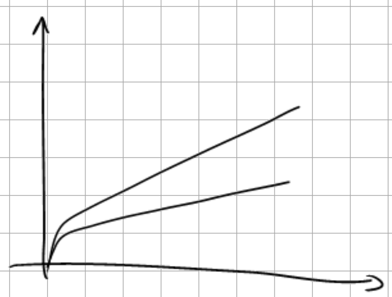
$$\left.\begin{aligned} x &= x_0 + dx \\ y &= y_0 + dy \end{aligned}\right\} \qquad df = \left(\frac{\partial f}{\partial x}\right)_{(x_0,y_0)} dx + \left(\frac{\partial f}{\partial y}\right)_{(x_0,y_0)} dy$$

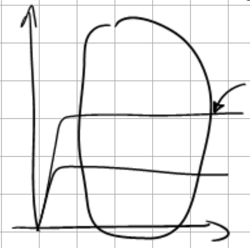
$$I_{DS} = I_{DSS}\left[1-\frac{V_{GS}(t)}{V_P}\right]^2 (1+K.V_{DS}(t))$$

$$g_m(V_{GS},V_{DS}) = \left(\frac{\partial I_{DS}}{\partial V_{gs}}\right)_{(V_{gs0},V_{ds0})} = -2\frac{I_{DSS}}{V_P}\left(1-\frac{V_{GS_0}}{V_P}\right)\left(1+K\,V_{DS_0}\right) = -2\,\frac{60\text{ mA}}{-1.5}\left(1-\frac{-0.75}{-1.5}\right)\left(1+0.005\times 4\right) = 48\ m\ \Omega^{-1}$$

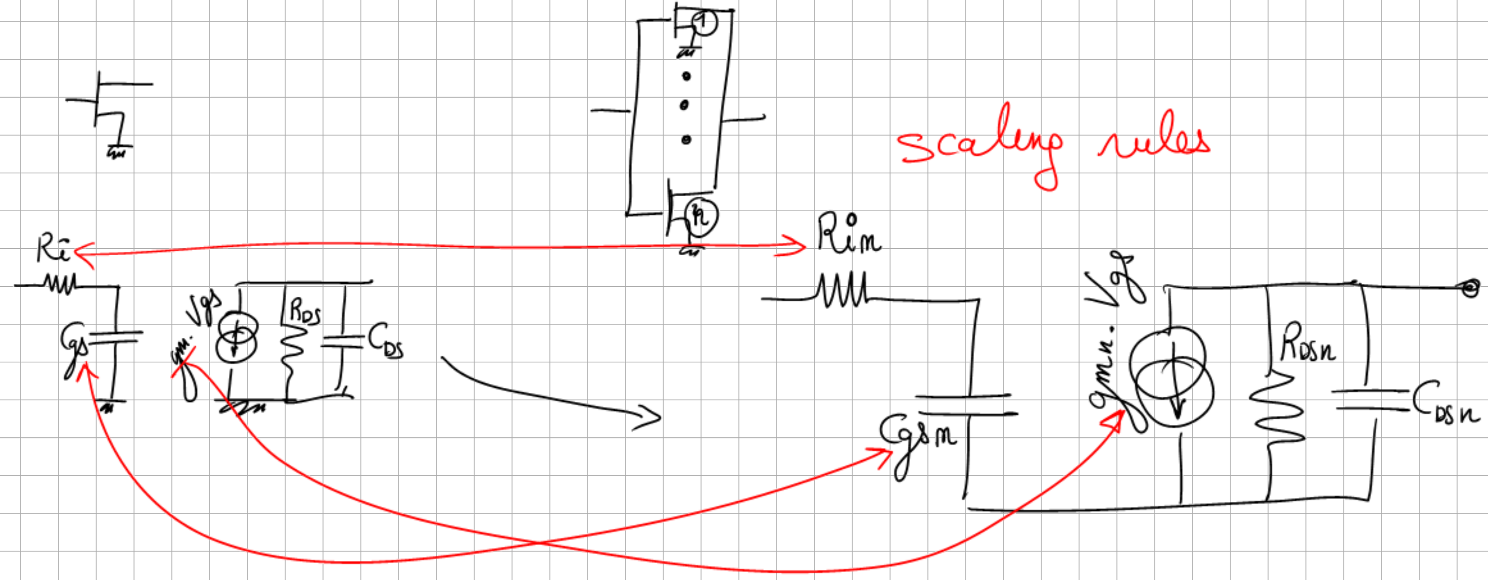
$$g_{DS} = \left(\frac{\partial I_{DS}}{\partial V_{DS}}\right)_{(V_{gs0},V_{ds0})} = K\,I_{DSS}\left(1-\frac{V_{GS_0}}{V_P}\right)^2 = 0.005\times 60\text{ mA}\left(1-\frac{-0.75}{-1.5}\right)^2 = 0.75\ m\,\Omega^{-1}$$

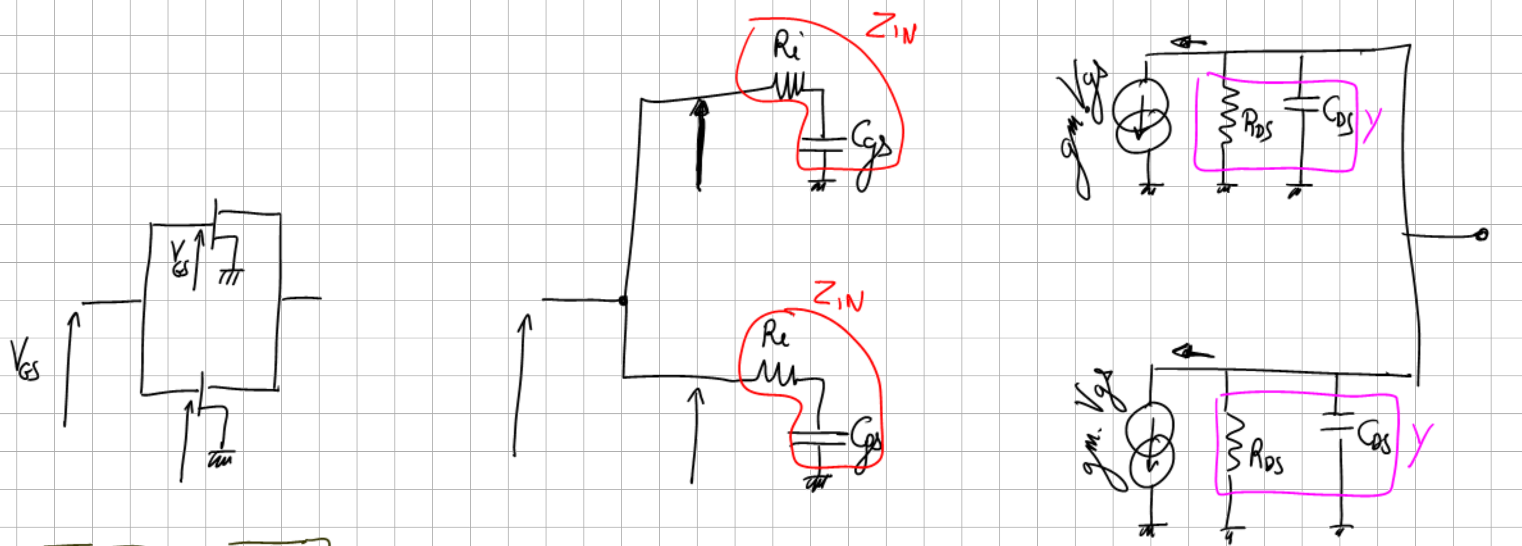
$$\boxed{R_{DS} = \frac{1}{g_{DS}} = 1.3\ k\Omega}$$

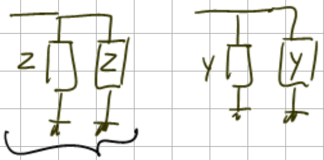


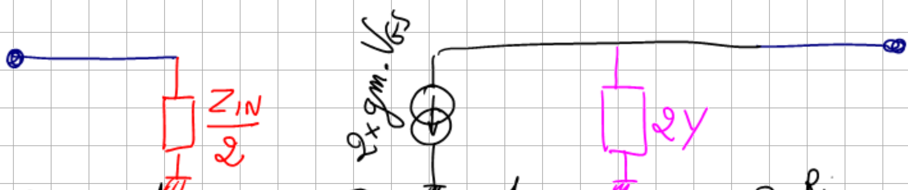


scaling rules



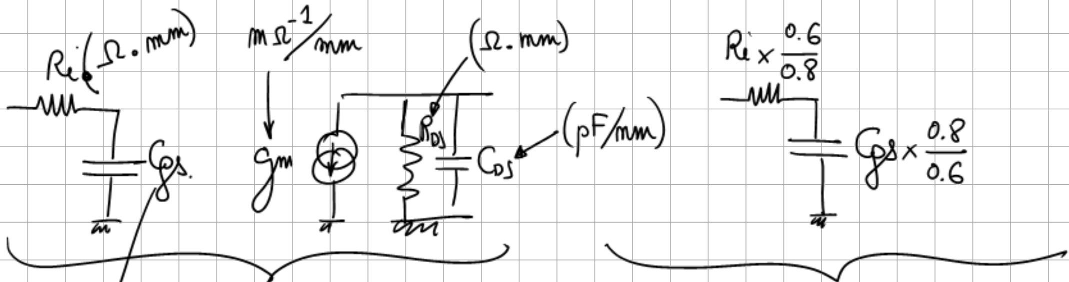






$$Z_{IN} = R_i + \frac{1}{jC_{GS}\omega} \longrightarrow \frac{Z_{IN}}{2} = \frac{R_i}{2} + \frac{1}{j2C_{GS}\omega} = \underbrace{R + \frac{1}{jC\omega}} \longrightarrow R = \frac{R_i}{2} \quad C = 2C_{GS}$$

$$\underbrace{Y = \frac{1}{R_{DS}} + jC_{DS}\omega} \longrightarrow 2Y = \frac{2}{R_{DS}} + j2C_{DS}\omega = \underbrace{\frac{1}{R_p}} + \underbrace{jC_p\omega} \longrightarrow R_p = \frac{R_{DS}}{2} \quad C_p = 2C_{DS}$$



n pF/mm 0.6 mm → 0.8 mm

Technology :
$$\left\{\begin{aligned} R_i &= 1\,\Omega.mm \\ R_{DS} &= 125\,\Omega.mm \\ C_{GS} &= 2\,pF/mm \end{aligned}\right.$$

$T_1 \to 8\times 75\,\mu m = 0.6\,mm$

$$R_{i_1} = \frac{1\,\Omega.mm}{0.6}$$
$$R_{DS1} = \frac{125}{0.6}$$
$$C_{GS_1} = 2\,pF\times 0.6$$

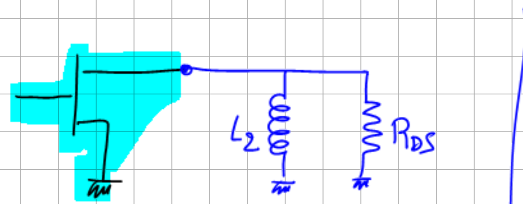
$$\boxed{\begin{aligned} R_i,\ R_{DS} &\rightarrow \frac{R_i}{n},\ \frac{R_{DS}}{n} \\ g_m,\ C_{GS},\ C_{DS} &\rightarrow n\,g_m,\ n\,C_{GS},\ n\,C_{DS} \end{aligned}}$$

Scalingrules

4) Determine the maximum power gain G_{MAX} for an ideal power matching at small-signal (linear) level. Determine its cutoff frequency f_C and its maximum frequency f_{MAX}.

5) After determining the optimum power load in large signal class A operation, determine the maximum gain G_{MAX} in this loading condition. Estimate the maximum output power value in class A. Compare these values to G_{MAX} and its associated maximum output power when the FET is ideally matched to give optimum gain in small signal operation.

6) Same questions 4) and 5) in the case of n-parallel FETs



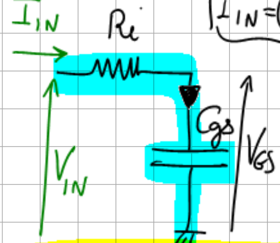
optimum loading for G_{max} @ f_0

$$\underbrace{Y_{C_{DS}} = jC_{DS}\omega_0 + Y_{L_2} = \frac{-j}{L_2\omega_0} = 0}$$

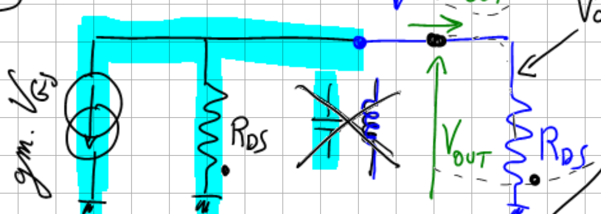
$$C_{DS}\,\omega_0 = \frac{1}{L_2\,\omega_0}$$

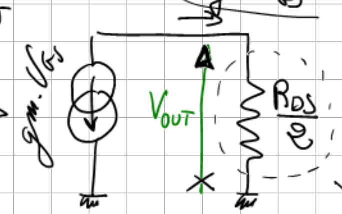
$$\boxed{L_2 = \frac{1}{C_{DS}\,\omega_0^2}}$$

$$\boxed{I_{IN} = (jC_{GS}\omega)V_{GS}}$$



$V_{OUT} = R_{DS}\,I_{OUT}$





$$P_{IN} = \frac{1}{2}Re\left(V_{IN}I_{IN}^*\right) = \frac{1}{2}Re\left(\underbrace{\left[R_i + \cancel{\frac{1}{jC_{GS}\omega}}\right]I_{IN}}_{V_{IN}} \times \underbrace{I_{IN}^*}_{|I_{IN}|^2}\right) = \frac{1}{2}R_i|I_{IN}|^2 = \frac{1}{2}R_i|jC_{GS}\omega V_{GS}|^2$$
$$= \frac{1}{2}R_i C_{GS}^2\omega^2|V_{GS}|^2$$

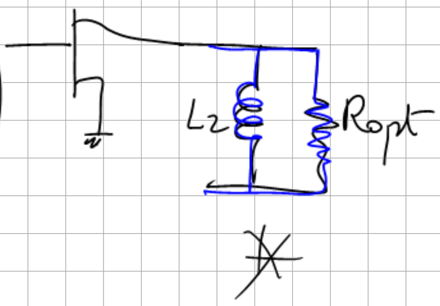
$$P_{OUT} = \frac{1}{2}Re\left(V_{OUT}I_{OUT}^*\right) = \frac{1}{2}Re\left(V_{OUT}\frac{V_{OUT}^*}{R_{DS}}\right) = \frac{1}{2}\frac{|V_{OUT}|^2}{R_{DS}} = \frac{1}{2}\frac{g_m^2 R_{DS}^2}{4\,R_{DS}}|V_{GS}|^2$$
$$= \frac{1}{8}g_m^2 R_{DS}|V_{GS}|^2$$

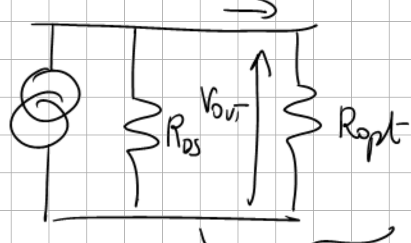
V, I, Z: $\boxed{V = Z\,I}$ $\Rightarrow$ $\boxed{I = Y\,V}$

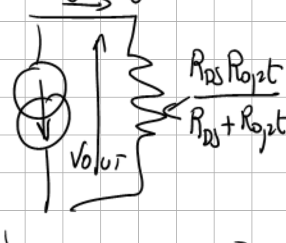
$$\boxed{G_{max} = \frac{P_{OUT}}{P_{IN}} = \frac{g_m^2 R_{DS}}{4R_i C_{GS}^2\omega^2}}$$

$$V_{OUT} = -\frac{g_m R_{DS}}{2}V_{GS}$$

POWER MATCHING



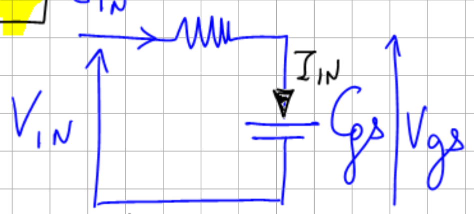




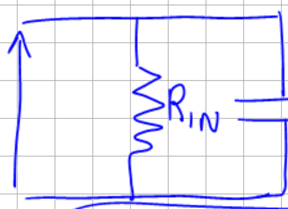
$$I_{OUT} = \frac{V_{OUT}}{R_{opt}} \qquad V_{OUT} = \frac{R_{DS}R_{opt}}{R_{DS}+R_{opt}}(-g_m V_{gs})$$

$$\boxed{G_P(R_{opt})} \qquad G_{max} = G_P(R_{DS})$$

INPUT EQUIVALENT

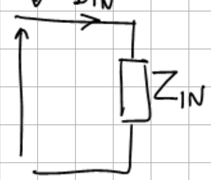


$\equiv$ Gω



$$\boxed{R_{IN} = \frac{1}{R_i\, C_{gs}^2 \omega^2}}$$

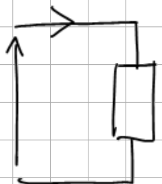
$$V_{IN} = Z_{IN} \times I_{IN} \Leftarrow$$



$$P_{IN} = \frac{1}{2}\,\mathrm{Re}\left(V_{IN} I_{IN}^*\right) = \frac{1}{2}\,\mathrm{Re}\left(Z_{IN} \underbrace{I_{IN} I_{IN}^*}_{\underbrace{|I_{IN}|^2}_{\text{real}}}\right)$$

$$= \frac{1}{2}\underbrace{\mathrm{Re}(Z_{IN})}_{R_i} \times \underbrace{|I_{IN}|^2}_{|jC_{gs}\omega \times V_{gs}|^2}$$

$$\boxed{P_{IN} = \frac{1}{2} R_i\, C_{gs}^2 \omega^2 |V_{gs}|^2}$$



$$Y_{IN} = \frac{1}{R_{IN}} + jC_{gs}\omega$$

$$I_{IN} = Y_{IN} \times V_{IN}$$

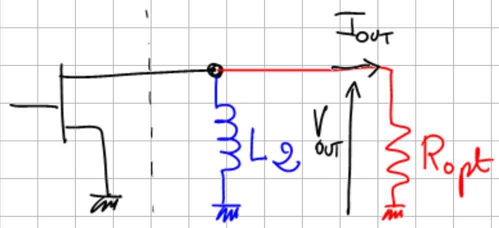
$$P_{IN} = \frac{1}{2}\,\mathrm{Re}\left(V_{IN} I_{IN}^*\right) = \frac{1}{2}\,\mathrm{Re}\left(\underbrace{V_{IN} Y_{IN}^* V_{IN}^*}_{|V_{IN}|^2}\right)$$

$$= \frac{1}{2}\,\mathrm{Re}(Y_{IN}^*) \times |V_{GS}|^2$$

$$= \frac{1}{2}\,\frac{1}{R_{IN}} \times |V_{GS}|^2$$

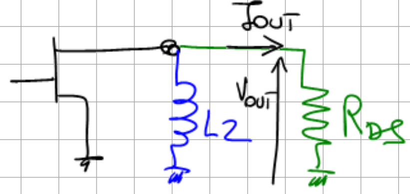
$$\boxed{P_{IN} = \frac{1}{2} R_i\, C_{gs}^2 \omega^2 |V_{GS}|^2}$$

INPUT POWER



$\downarrow$

$GP(R_{opt})$

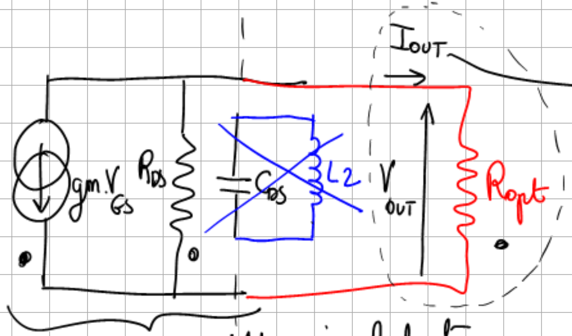


$\downarrow$

$G_{max} = GP(R_{DS})$

In both cases,

$$\boxed{L_2 = \frac{1}{C_{DS}\,\omega_0^2}}$$



$$I_{OUT} = \frac{V_{OUT}}{R_{opt}}$$

equivalent to

$$(R_{opt}//R_{DS}) = \frac{R_{opt}R_{DS}}{R_{opt}+R_{DS}}$$

$$V_{OUT} = \left(\frac{R_{opt}R_{DS}}{R_{opt}+R_{DS}}\right)\left(-g_m V_{GS}\right)$$

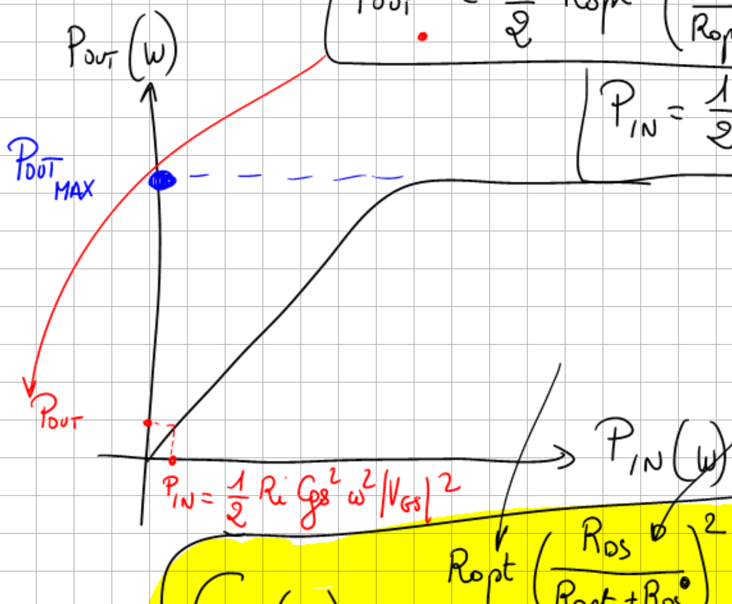
$$P_{OUT} = \frac{1}{2}\,\mathrm{Re}\left(V_{OUT}\,I_{OUT}^*\right) =$$

$$= \frac{1}{2}\,\mathrm{Re}\left(V_{OUT}\,\frac{V_{OUT}^*}{R_{opt}^*}\right)$$

$$= \frac{1}{2}\,\frac{|V_{OUT}|^2}{R_{opt}} = \frac{1}{2}\,\frac{\left(\frac{R_{opt}R_{DS}}{R_{opt}+R_{DS}}\right)^2 g_m^2\,|V_{GS}|^2}{R_{opt}}$$

$$\boxed{P_{OUT} = \frac{1}{2}\,R_{opt}\left(\frac{R_{DS}}{R_{opt}+R_{DS}}\right)^2 g_m^2\,|V_{GS}|^2}$$

$$\boxed{P_{IN} = \frac{1}{2}\,R_i\,C_{gs}^2\,\omega^2\,|V_{GS}|^2}$$



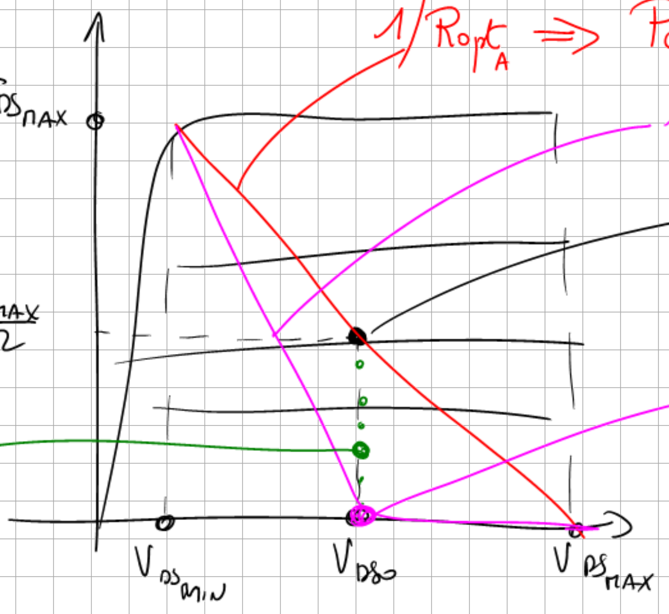
$$\boxed{G_P(R_{opt}) = 4\,\frac{R_{opt}R_{DS}}{(R_{opt}+R_{DS})^2}\,G_{max}}$$

$$\boxed{G_P(R_{opt}) = \frac{R_{opt}\left(\frac{R_{DS}}{R_{opt}+R_{DS}}\right)^2 g_m^2}{R_i\,C_{gs}^2\,\omega^2}}$$

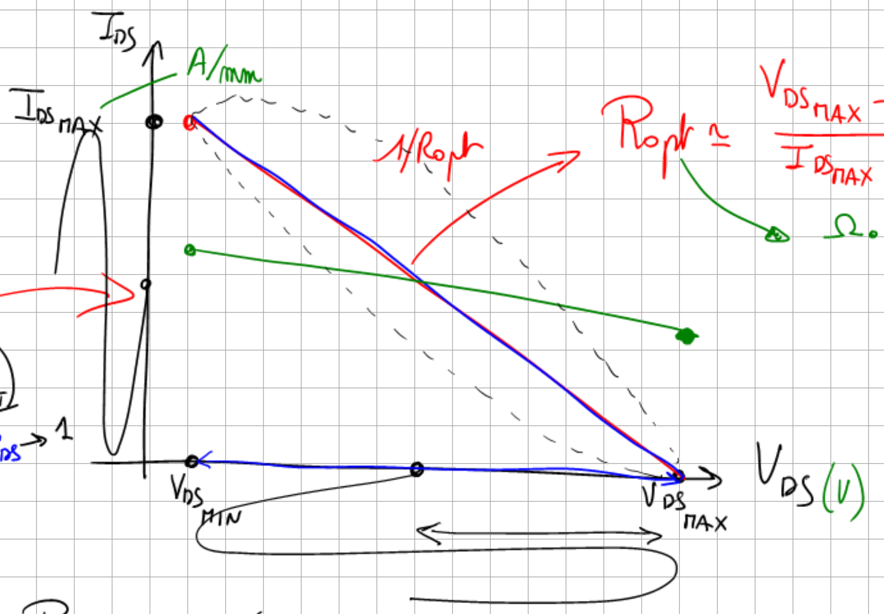
$$G_{max} = GP(R_{DS}) = \frac{1}{4}\,\frac{R_{DS}\,g_m^2}{R_i\,C_{gs}^2\,\omega^2}$$

Power Density = PD = 1W/mm → $P_{OUT_{MAX}}$
(technology)

$P_{OUT_{MAX_B}} \simeq P_{OUT_{MAX_A}}$



$P_{OUT_{MAX_{AB}}} \simeq 1,1\, P_{OUT_{MAX_A}}$ Class AB



$$R_{opt} \simeq \frac{V_{DS_{MAX}} - V_{DS_{MIN}}}{I_{DS_{MAX}} - 0} \quad \frac{V}{(A/mm)} \rightarrow \Omega . mm$$

$$P_{OUT} = \frac{1}{2} Re\left(V_{OUT}\, I_{OUT}^{*}\right)$$

$$= \frac{1}{2} |V_{OUT}|\,|I_{OUT}| \underbrace{\cos\left(\varphi_{V_{OUT}} - \varphi_{I_{OUT}}\right)}_{L2 \,//\, C_{DS} \rightarrow 1}$$

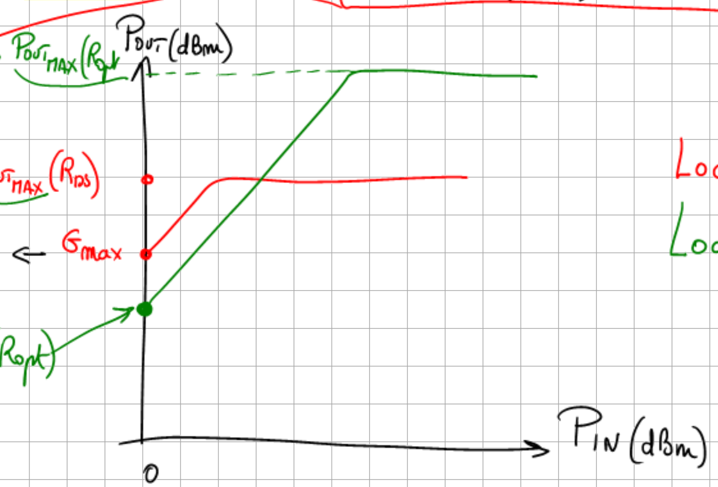
$$P_{OUT_{MAX}} = \frac{1}{2}\, \frac{V_{DS_{MAX}} - V_{DS_{MIN}}}{2} \times \frac{I_{DS_{MAX}}}{2}$$

$$\underbrace{P_{OUT_{MAX_A}}}_{W/mm} = \frac{1}{8} \underbrace{\left(V_{DS_{MAX}} - V_{DS_{MIN}}\right)}_{V} \underbrace{I_{DS_{MAX}}}_{A/mm}$$ ♡

$$\frac{V_{DS_{MAX}} - V_{DS_{MIN}}}{R_{opt}}$$

$$\boxed{P_{OUT_{MAX}} = \frac{1}{8}\, \frac{\left(V_{DS_{MAX}} - V_{DS_{MIN}}\right)^2}{R_{opt}}}$$ ♡

$$P_{OUT_{MAX}}(R_{DS}) = \frac{1}{8}\, \frac{\left(V_{DS_{MAX}} - V_{DS_{MIN}}\right)^2}{R_{DS}}$$



$\frac{R_{DS}}{m}$ $(m\, g_m)^2$

$$G_{max} = \frac{1}{4}\, \frac{R_{DS}\, g_m^2}{R_i\, C_{gs}^2\, \omega^2}$$

$\frac{R_i}{m}$ $(m\, C_{gs})^2$

$G_p(R_{opt})$

Load = R_{DS}

Load = R_{opt}

numerical example

Technology $\rightarrow$ $PD_0 = 1W/mm \longleftrightarrow R_{opt} = 10\Omega . mm$

$R_{DS} = 150\Omega . mm$

$G_{max_0} = 20dB = 100 \rightarrow G_p(R_{opt})$

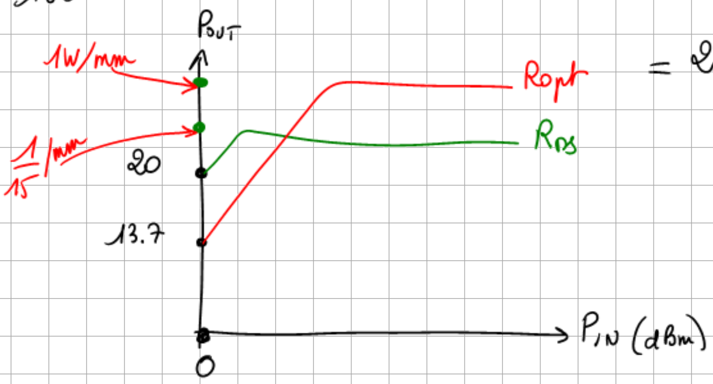
$$\boxed{PD_{(R_{DS})} = \underbrace{\frac{R_{opt}}{R_{DS}}}_{<1} \underbrace{PD_{(R_{opt})}}}$$

$$P_{D(R_{DS})} = \frac{10}{150} 1W/mm =$$

$$G_p(R_{opt}) = 4 \frac{R_{opt} R_{DS}}{(R_{opt}+R_{DS})^2} G_{max}$$

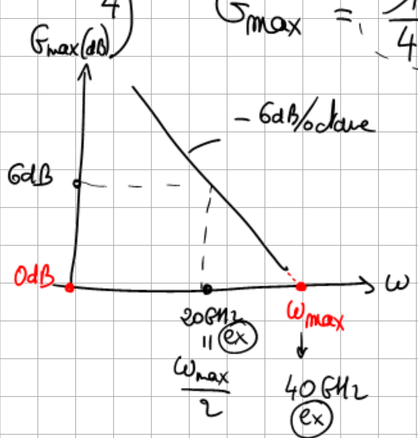
$$= 4 \times \frac{10 \times 150}{(10+150)^2} \times 100$$

$$= 23.4 = 13.7dB$$



Numerical values of tutorial

4) $$G_{max} = \frac{1}{4} \frac{R_{DS}\, gm^2}{R_i\, C_{gs}^2\, \omega^2}$$



$$f_c = \frac{gm}{2\pi C_{gs}} = \frac{48 . 10^{-3}}{2\pi\ 0{,}5\ 10^{-12}} = 15\ GHz$$

$$G_{max}(\omega_{max}) \triangleq 1$$

$$\omega_{max} = \frac{1}{2} \sqrt{\frac{R_{DS}}{R_i}}\ \frac{gm}{C_{gs}}$$

$$\boxed{f_{max} = \frac{gm}{4\pi C_{gs}} \sqrt{\frac{R_{DS}}{R_i}}} = \frac{48\ 10^{-3}}{4\pi\ 0{,}5\ 10^{-12}} \sqrt{\frac{1300}{4}}$$

$$= 137\ GHz$$

$R_{opt} =$