GROUP VELOCITY DISPERSION (GVD)

- DISPERSION INDUCED PULSE BROADENING

The effects of GVD on optical pulses propagating in an optical fiber are studied by setting $\gamma = 0$ in the NLSE; $F(z,\tau)$ satisfies the equation:

$$i \frac{\partial F}{\partial z} = \frac{B_2}{2} \frac{\partial^2 F}{\partial t^2} \qquad (1)$$

$B'' = B_2$

Equation (1) is readily solved by using the Fourier transform. If $\hat{F}(z,\omega)$ is the Fourier transform of $F(z,t)$ such that

$$F(z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{F}(z,\omega)\, e^{-i\omega t}\, d\omega \qquad (2)$$

then it satisfies an ordinary differential equation

$$i\frac{\partial \hat{F}}{\partial z} = -\frac{1}{2}\beta_2 \omega^2 \hat{F} \qquad (3)$$

whose solution is given by

$$\underbrace{\hat{F}(z,\omega)}_{\text{OUT}} = \underbrace{\hat{F}(z=0,\omega)}_{\text{IN}} \overbrace{\boxed{\exp\left(\frac{i}{2}\beta_2 \omega^2 z\right)}}^{\text{CHANGES}} \qquad (4)$$

Equation 4 shows that GVD changes the phase of each spectral component of the pulse by an amount that depends on both the frequency ω and the propagated distance z.

By substituting Eq. (4) in Eq. (2) the general solution of Eq. (1) is given by

$$F(z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{F}(0,\omega)\exp\left(\frac{i}{2}\beta_2\omega^2 z - i\omega t\right)d\omega \quad (5)$$

where $\hat{F}(0,\omega)$ is the Fourer Tranform at the input.

$$\hat{F}(0,\omega) = \mathcal{J}\left(F(0,t)\right) \quad (6)$$

Eq (5) and Eq. (6) can be used for input pulses of arbitrary shapes

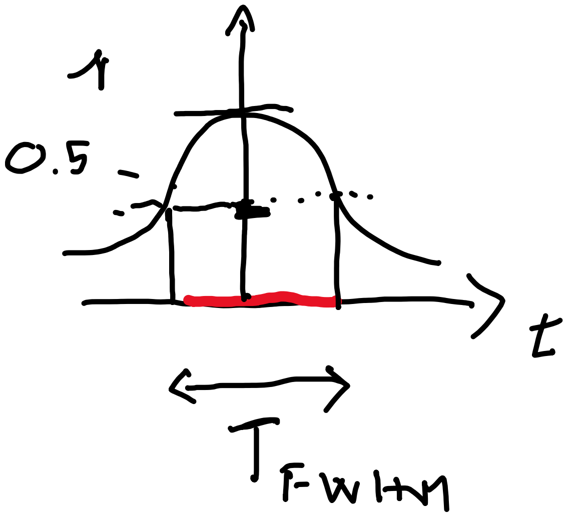
- Gaussian Pulses

As a first simple example, consider the case of a Gaussian pulse for which the input envelope is:

$$F(0,\tau) = \exp\left(-\frac{t^2}{2t_0^2}\right) \qquad (7)$$

where T_0 is the half-width (at $1/e$-intensity point). In practice, it is customary to use the full width at half maximum (FWHM) in place of T_0.

For a gaussian pulse we have



$$T_{FWHM} \simeq 1.665\, t_0 = 2(\ln 2)^{1/2}\, t_0 \qquad (8)$$

By using Eq. (5) and Eq. (7) and carrying out the integration, we have

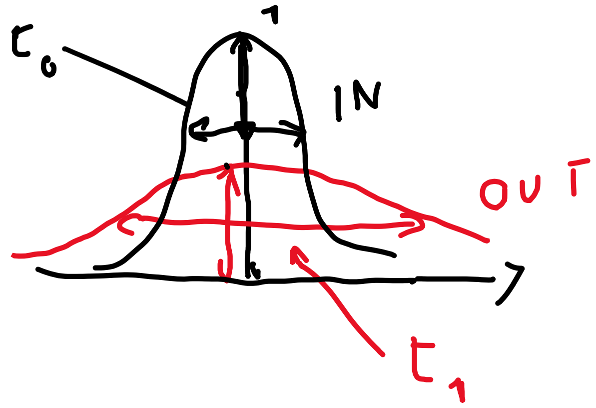
$$F(z,t) = \frac{t_0}{(t_0^2 - i\beta_2 z)^{1/2}} \cdot \exp\left(-\frac{t^2}{\underbrace{2(t_0^2 - i\beta_2 z)}_{E_1}}\right) \qquad (9)$$

Thus, a gaussian pulse maintains its shape on propagation but its width t_1 increases as

$$T_1(z) = t_0\left(1 + \left(z/L_D\right)^2\right)^{1/2} \tag{10}$$

where $L_D = t_0^2/|\beta_2|$.

Equation 10 shows how GVD broadens a gaussian pulse.



- temporal broadening
- amplitude reduction

(Energy conservation)

The extent of broadening is governed by the dispersion lenght L_D. For a given fiber length, short pulses broaden more because of a smaller dispersion lenght.

At $z = L_D$, a gaussian pulse broadens by a factor of $\sqrt{2}$.

A comparison of Eq (7) and Eq (9) shows that although the input pulse is unchirped (with no phase modulation), the transmitted pulse becomes chirped. This can be seen clearly by writing $F(z,t)$ in the form

$$F(z,t) = |F(z,t)| \exp[i\phi(z,t)] \tag{11}$$

where

$$\phi(z,t) = \frac{-\operatorname{sign}(\beta_2)(z/L_D)}{1+(z/L_D)^2}\frac{t^2}{t_0^2} + \frac{1}{2}\tan^{-1}\left(\frac{z}{L_D}\right) \tag{12}$$

The time dependence of the phase $\phi(z,t)$ implies that the instantaneous frequency differs across the pulse from the central frequency ω_0. The difference $\delta\omega$ is just the time derivative $-\partial\phi/\partial t$ and is given by

$$\partial\omega(T) = -\frac{\partial\phi}{\partial t} = \frac{\mathrm{sign}(\beta_2)\,(2z/L_D)\; t}{1 + (z/L_D)^2 \;\; T_0^2} \tag{13}$$

$\partial\omega$ is the chirp. The chirp $\partial\omega$ depends on the sign of β_2.

Dispersion-induced pulse broadening can be understood by recalling that different frequency components of a pulse travel at slightly different speeds along the fiber because of GVD. More specifically, red components travel faster than blue components in the normal GVD ($\beta_2 > 0$), while the opposite occurs in

The anomalous regime ($\beta_2 < 0$).

I underline that the pulse can maintain its width only if all spectral components arrive together.

Let's check now theoretical results with numerical simulations.