UNIVERSITÀ DEGLI STUDI DI BRESCIA

Photonics Final Test
June 18, 2021

Student ID:

Name:

Instructions: You have 1.5 hours to complete the test. Please write everything with blue or black ink pen so that all your work can be read easily. You can use your calculator. If you don't have a calculator, you can leave the formulas in expression forms and still get full score for the questions/exercises. Use of course notes or internet resources will invalidate the results of the test.
VERY IMPORTANT: Please WRITE YOUR FULL NAME AND STUDENT ID on this sheet and all your sheets where the problems are solved!

Questions:

1. A dentist observes and checks the patient's teeth using a mirror with an 8 cm radius. For the hole to be seen clearly by the doctor, what is the distance between the patient's teeth and the mirror? Motivate your answer.

A. less than 4 cm in front of a concave mirror
B. less than 4 cm in front of a convex mirror
C. more than 4 cm in front of the concave mirror
D. more than 4 cm in front of the convex mirror

2. A plane wave crosses a boundary, located at z=0, between a medium 1, which occupies the half space z<0 and it has refractive index n_1, and a medium 2, which occupies the z>0 half space and it has refractive index $n_2 = \frac{n_1}{3}$.

a. What is the value of reflectance R and transmittance T if the angle of incidence is equal to 0 (i.e., normal incidence)?
b. Suppose the plane wave crosses the boundary going from medium 1 to medium 2 at oblique incidence. What condition should the plane wave satisfy to observe a zero in reflectance? What is the value of Brewster's angle at this boundary?
c. Suppose the plane wave crosses the boundary going from medium 1 to medium 2 at oblique incidence. What is the value of critical angle at this boundary?
d. Total internal reflection occurs when light, crossing the boundary from medium 1 (z<0) to medium 2 (z>0), is obliquely incident above the critical angle. Explain why, under these circumstances, the electric field is not equal to zero in medium 2.

3. An object is placed on the left hand side of a slab made of a right-handed medium [Figure 1(a)] and a left-handed-medium [Figure 1(b)]. Using the ray theory, draw the refraction of light at each interface in the two scenarios. Under what circumstances one can achieve the refraction illustrated in Figure 1(b)?

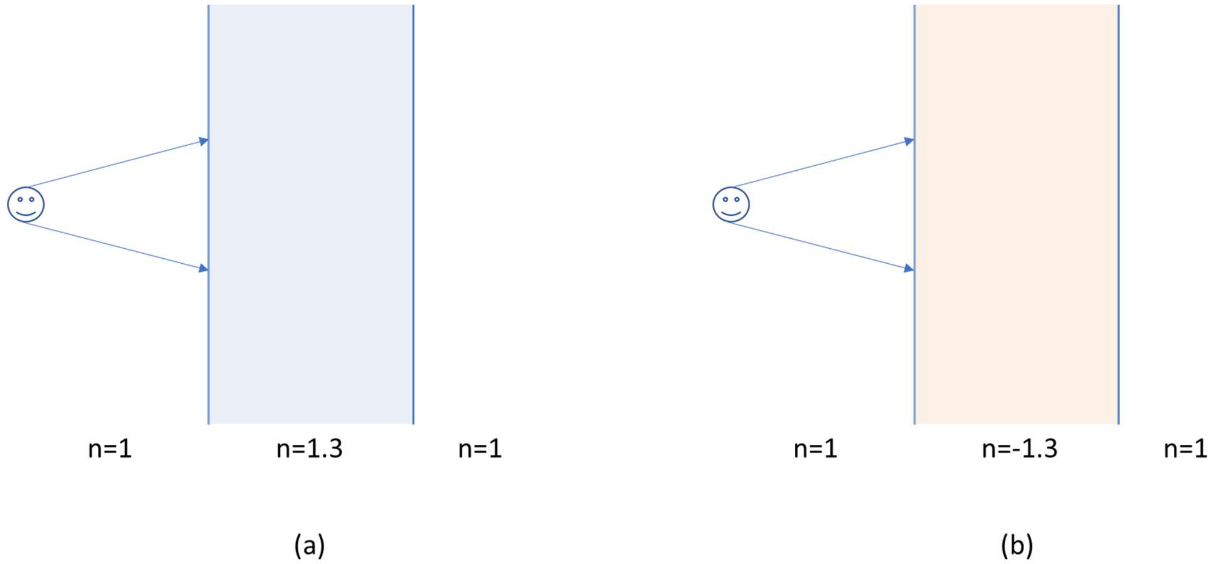


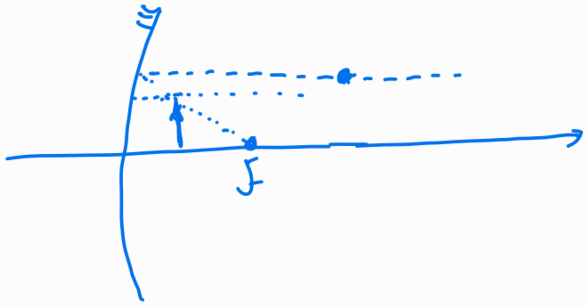
Figure 1

1) $f = -\dfrac{R}{2} \rightarrow f = \pm 4\,cm$ $\begin{cases} f = 4\,cm \text{ (CONCAVE)} & R<0 \\ f = -4\,cm \text{ (CONVEX)} & R>0 \end{cases}$

BEST OPTION! → MAGNIFIED AND STRAIGHT IMAGE!

A. CONCAVE $\quad f = 4\,cm$

$R = -8\,cm \quad z_1 = 2\,cm$



$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

$$\frac{1}{2} + \frac{1}{z_2} = \frac{1}{4}$$

$$\frac{1}{z_2} = -\frac{1}{4} \rightarrow z_2 = -4\,cm$$

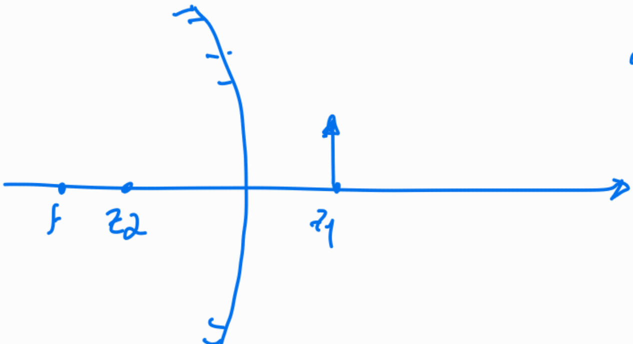
Virtual image

$$M = -\frac{z_2}{z_1} = \frac{4}{2} = 2$$

→ MAGNIFIED

B. CONVEX

$R = 8\,cm \quad f = -4\,cm$



$$\frac{1}{2} + \frac{1}{z_2} = -\frac{1}{4}$$

$$\frac{1}{z_2} = -\frac{1}{4} - \frac{1}{2} \rightarrow \frac{1}{z_2} = \frac{-3}{4}$$

$$\boxed{z_2 = -\frac{4}{3}\,cm}$$ Virtual

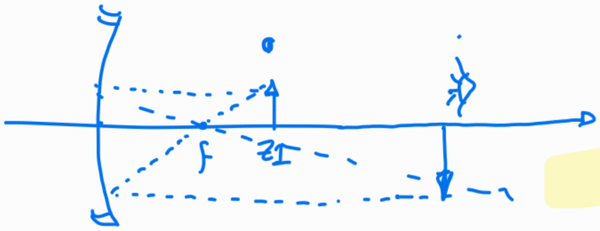
$$M = -\left(\frac{-\frac{4}{3}}{2}\right) = \frac{2}{3}$$

→ REDUCED

C. CONCAVE

$R = -8\,cm \quad f = 4\,cm$

$z_1 = 6\,cm$



$$\frac{1}{6} + \frac{1}{z_2} = \frac{1}{4}$$

$$\frac{1}{z_2} = \frac{1}{4} - \frac{1}{6} \rightarrow \frac{1}{z_2} = \frac{3-2}{12}$$

$$\boxed{z_2 = 12\,cm}$$ real

$M < 0$

→ INVERTED

D. CONVEX

$R = 8\,cm \quad f = -4\,cm$

$z_2 = 6\,cm$

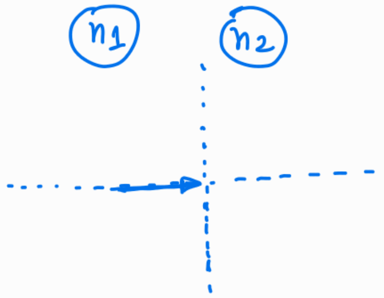
$$\frac{1}{z_2} = -\frac{1}{4} - \frac{1}{6} \qquad \frac{1}{z_2} = \frac{-5}{12}$$

$z_2 = -\frac{12}{5}\,cm$

Imaginary.

$$M = -\frac{\left(\frac{-12}{5}\right)}{6} = \frac{2}{5} \rightarrow \text{REDUCED}$$

2)



$n_2 = \frac{n_1}{3}$

$\theta_i = 0^\circ$

$n_1 \sin\theta_i = n_2 \sin\theta_2$

$n_1 \sin 0^\circ = \frac{n_1}{3} \sin\theta_2$

$\theta_2 = a\sin(3 \sin 0^\circ)$

$\boxed{\theta_2 = 0^\circ}$

a) $r_{TE} = \dfrac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}$

$$R = r_{TE}^2 = \left(\frac{n_1 \cdot 1 - \frac{1}{3} n_1 \cdot 1}{n_1 \cdot 1 + \frac{1}{3} n_1 \cdot 1}\right)^2 = \left(\frac{1 - \frac{1}{3}}{1 + \frac{1}{3}}\right)^2 = \left(\frac{\frac{2}{3}}{\frac{4}{3}}\right)^2 \Rightarrow \boxed{R = \frac{1}{4}}$$

$$T = 1 - R = \frac{3}{4}$$

$$\tau_{TE} = \frac{2 \, n_1 \cos\theta_i}{n_2 \cos\theta_t + n_1 \cos\theta_i} = \frac{2}{\frac{1}{3} + 1} = \frac{2}{\frac{4}{3}} = \frac{6}{4} = \frac{3}{2}$$

$$T = |\tau|^2 \frac{n_2 \cos\theta_t}{n_1 \cos\theta_i} = \frac{9}{4} \cdot \frac{1/3}{1} = \frac{3}{4}$$

b) Reflectance $= 0$ ONLY FOR TM WAVES WITH $n_1 > n_2$.

FOR TE: $r_{TE} = \dfrac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} = 0$, can't be achieved!

$n_1 \cos\theta_i = n_2 \cos\theta_t$

$\boxed{\cos\theta_t = 3 \cos\theta_i}$

FOR TM: $r_{TM} = \dfrac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} = 0$

$n_1 \cos\theta_t = n_2 \cos\theta_i$

$n_1 \sin\theta_i = n_2 \sin\theta_t$

$$\theta_B = a\tan\left(\frac{n_2}{n_1}\right) = a\tan\left(\frac{1}{3}\right) = 18{,}43^\circ$$

c) θ_c occurs when $\theta_T = 90°$

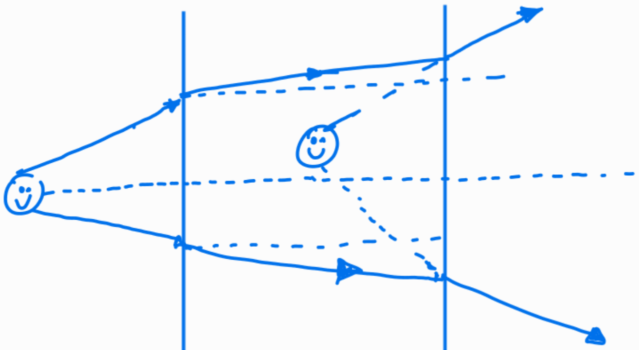
From Snell's law: $n_1 \sin\theta_i = n_2 \sin\theta_T$

$$\sin\theta_i = \frac{n_2}{n_1}$$

$$\theta_i = \operatorname{asin}\left(\frac{1}{3}\right) \rightarrow \theta_i = 19,47°$$

d) Evanescent transmittance.

3) a)

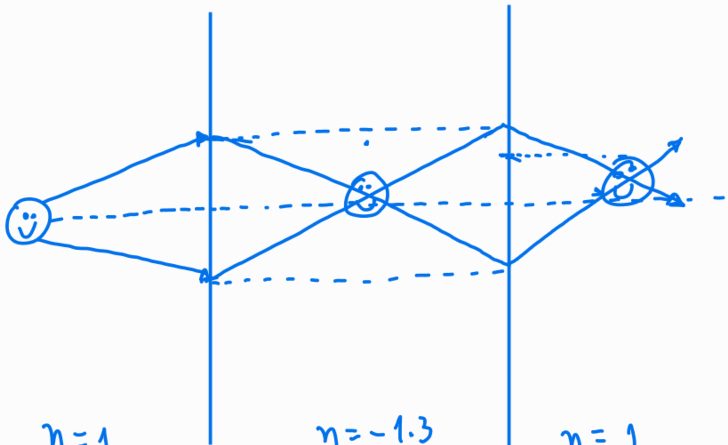


$n=1$ ① $n=1.3$ ② $n=1$

① $\sin\theta_T = \frac{1}{1.3}\sin\theta_i$ $\boxed{\theta_T < \theta_i}$

② $\sin\theta_T = \frac{1.3}{1}\sin\theta_i$ $\boxed{\theta_T > \theta_i}$

b)



$n=1$ $n=-1.3$ $n=1$

$n < 0 \rightarrow$ Metamaterial

$\varepsilon, \mu < 0.$