

Part 1

EX. 1

4 JULY 2018

1) (7,4) cyclic linear code

$$g(D) = D^3 + D^2 + 1$$

$$N=7$$

$$k=4$$

$$\# \text{ codewords} = 2^4 = 16$$

$$G = [I_n \mid P]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} (D^3+D^2+1) & g(D) \\ (D^2+D+1) & g(D) \\ (D+1) & g(D) \\ g(D) & \end{bmatrix} \quad \checkmark$$

0000 → 0000000
0001 → 0001101
0010 → 0010111
0011 → 0011010
0100 → 0100011
0101 → 0101110
0110 → 0110100
0111 → 0111001
1000 → 1000110
1001 → 1001011
1010 → 1010011
1011 → 1011100
1100 → 1100101
1101 → 1101000
1110 → 1110010
1111 → 1111111

$$\dim u = 3$$

$$H^+ = \begin{bmatrix} P \\ I_{N-k} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{z} = \underline{y} H^+ = \underbrace{\underline{c} H^+}_{=0} + \underline{e} H^+ = \underline{e H^+}$$

correct bit $\left\lfloor \frac{\dim u - 1}{2} \right\rfloor = 1$

detect $\dim u - 1 = 2$