Semester S1

Basics of active and non linear electronics

RF Power amplifiers (JM Nebus)

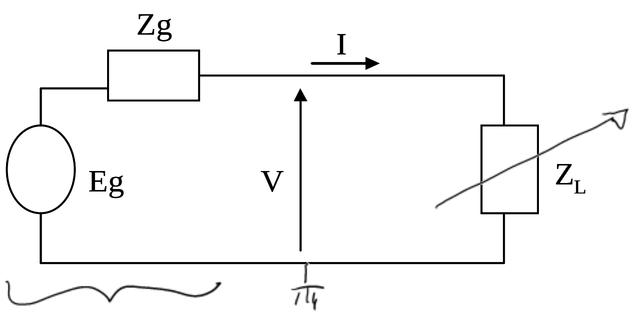
COURSE N° 4

Chapter IV

Power matching of microwave transistors using distributed components

I] Power matching conditions

For the simple following circuit



We have the following relationships

$$V = \frac{Z_L I}{Z_L + Z_G} \qquad I = \frac{E_G}{Z_L + Z_G} \qquad Z_L = R_L + jX_L \qquad Z_G = R_G + jX_G$$

$$P_L = \frac{1}{2}\mathrm{Re}(V.I^*) = \frac{1}{2}\frac{R_L.E_G{}^2}{(R_L + R_G)^2 + (X_L + X_G)^2}$$

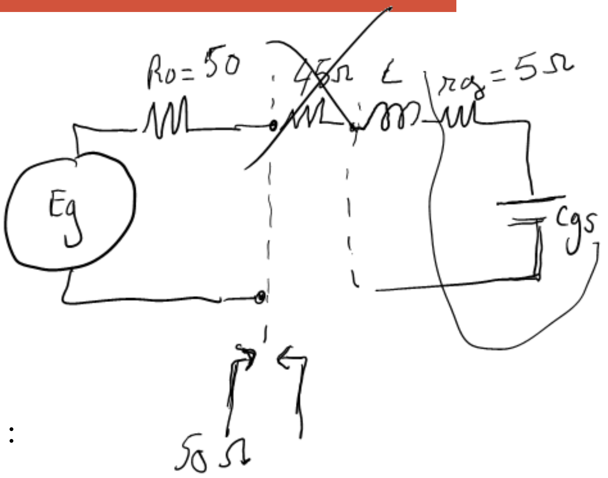
Max

If

$$\frac{\partial P_L}{\partial R_L} = 0 \qquad \frac{\partial P_L}{\partial X_L} = 0$$

We have the maximum (available power) absorbed by Z_L

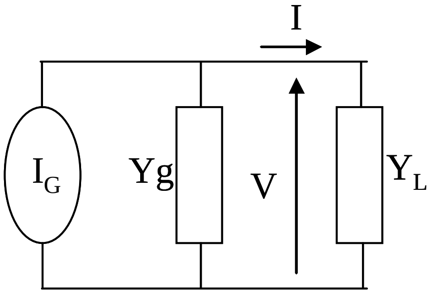
These derivatives are equal to 0 when $R_L = R_G$ and $X_L = -X_G$ so $\boxed{Z_L = Z_G^*}$



And

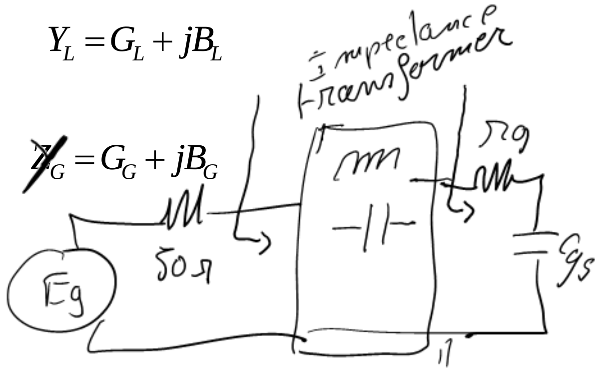
$$\boxed{P_{L-\max} = \underbrace{P_{available}} = \frac{1}{8}\cdot\frac{|E_G|^2}{R_G}}$$

For a current generator we have the same property :



$$Y_L = G_L + jB_L$$

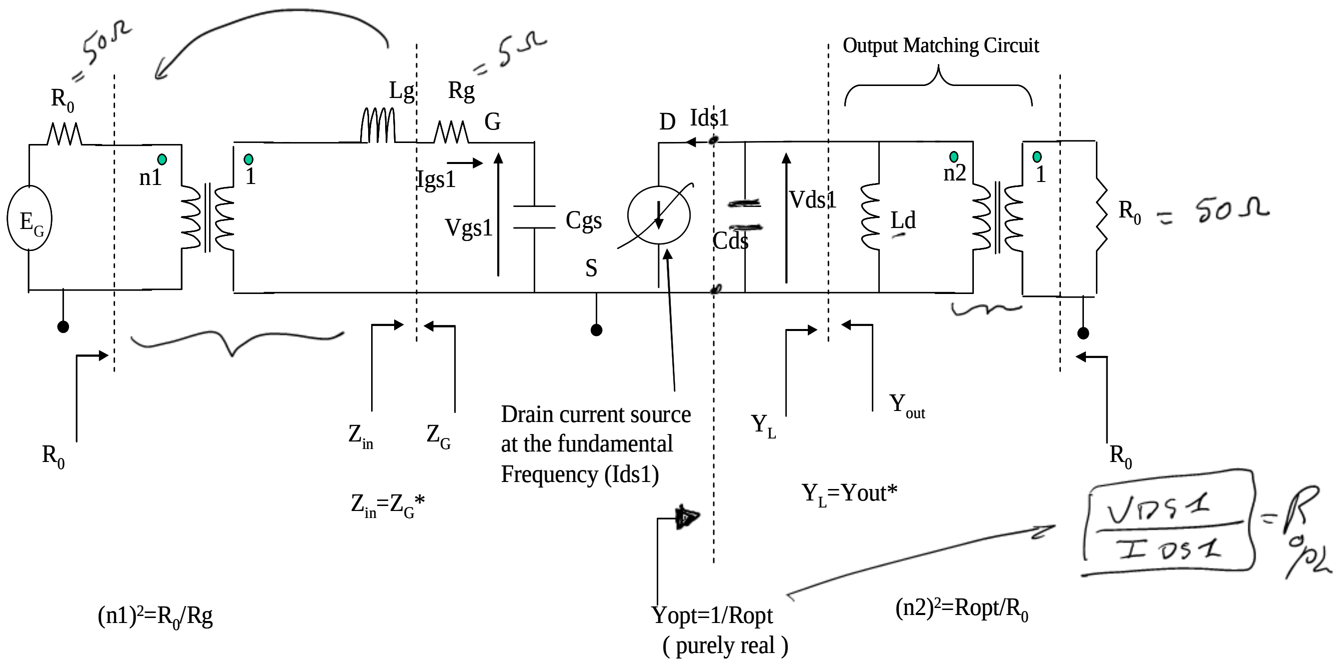
$$\cancel{Y}_G = G_G + jB_G$$



$$P_{L-\max} = P_{available} = \frac{1}{8}\cdot\frac{|I_G|^2}{G_G}$$

II] Application to transistor's input and output matching

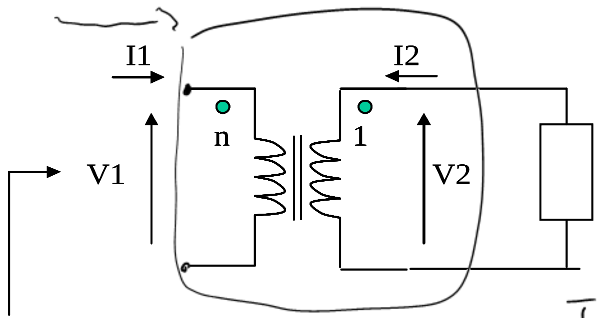
(with ideal impedance transformers)



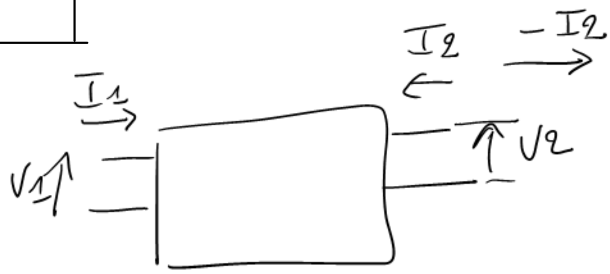
The ideal impedance transformer is represented as following

$\frac{1}{2} V_1 I_1 = -\frac{1}{2} V_2 I_2$

lossless.



Z2= - V2 / I2

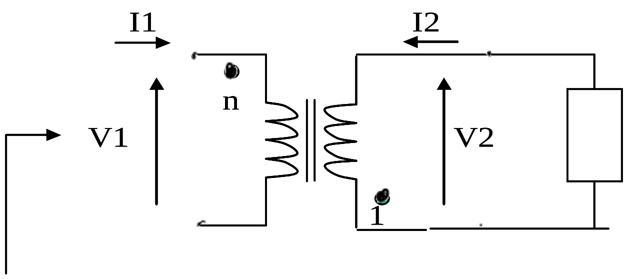


Z1= (V1/ I1) = n².Z2

And the chain (ABCD) matrix is :

$$\boxed{\begin{pmatrix} V1 \\ I1 \end{pmatrix} = \begin{pmatrix} n & 0 \\ 0 & \frac{1}{n} \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix}}$$

We can have in a same manner an impedance transformer with 180° out of phase between input and output voltages . In this case we have the following symbolic representation and the associated chain matrix



Z2= - V2 / I2

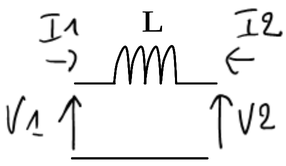
Z1= (V1/ I1) = n².Z2

$$\begin{pmatrix} V1 \\ I1 \end{pmatrix} = \begin{pmatrix} -n & 0 \\ 0 & -\frac{1}{n} \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix}$$

III] Impedance transformer with lumped capacitors and inductors

The chain matrix of a serie inductor is

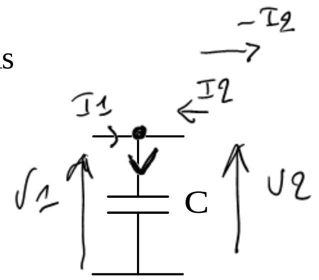
$$\begin{pmatrix} V1 \\ I1 \end{pmatrix} = \begin{pmatrix} 1 & jLw \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix}$$



$V1 = V2 + jL\omega I1$
$V1 = V2 - jL\omega I2$
$I1 = -I2$

The chain matrix of a parallel capacitor is

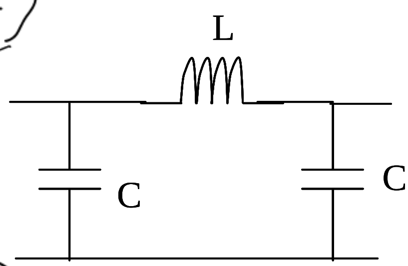
$$\begin{pmatrix} V1 \\ I1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ jCw & 1 \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix}$$



$V1 = V2$
$I1 = -I2 + jC\omega V2$

Doing the matrix product for cascaded components the chain matrix of the following circuit is : $\begin{pmatrix} 1 & 0 \\ jC\omega & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & jL\omega \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ jC\omega & 1 \end{pmatrix}$

$$\begin{pmatrix} V1 \\ I1 \end{pmatrix} = \begin{pmatrix} 1-LCw^2 & jLw \\ jCw(1-LCw^2)+jCw & 1-LCw^2 \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix}$$



If we have the relationship

$\boxed{LCw^2 = 1}$

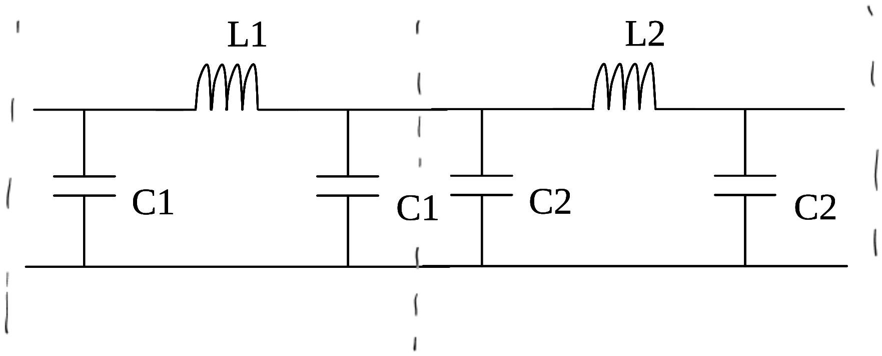
$\begin{pmatrix} 0 & jL\omega \\ jC\omega & 0 \end{pmatrix}$

$LC\omega^2 = 1$
$\omega = \dfrac{1}{\sqrt{LC}}$

The matrix becomes

$$\begin{pmatrix} V1 \\ I1 \end{pmatrix} = \begin{pmatrix} 0 & jLw \\ jCw & 0 \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix} = \begin{pmatrix} 0 & j\sqrt{\frac{L}{C}} \\ j\sqrt{\frac{C}{L}} & 0 \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix}$$

And if we connect two different cells , we obtain an equivalent impedance transformer :



With $L_1C_1w^2 = 1$ and $L_2C_2w^2 = 1$ and $L_2 \neq L_1$ $C_2 \neq C_1$

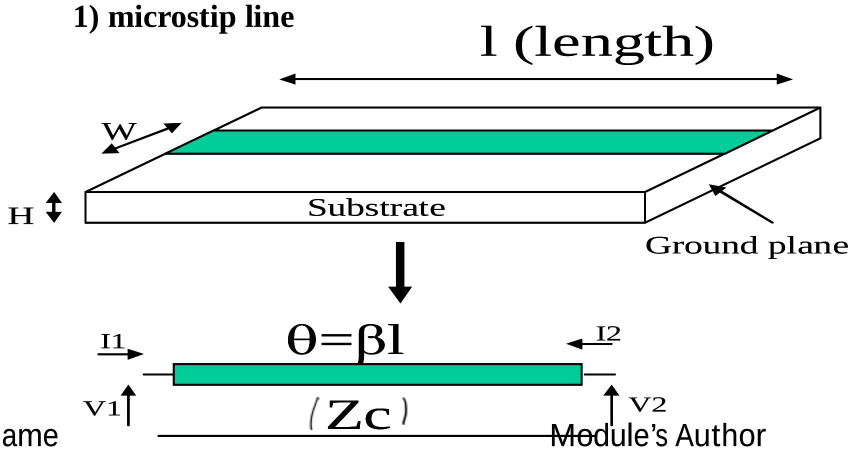
And also $\sqrt{\frac{L_1}{C_1}} \neq \sqrt{\frac{L_2}{C_2}}$

$$\begin{pmatrix} 0 & j\sqrt{\frac{L1}{C1}} \\ j\sqrt{\frac{C1}{L1}} & 0 \end{pmatrix}\begin{pmatrix} 0 & j\sqrt{\frac{L2}{C2}} \\ j\sqrt{\frac{C2}{L2}} & 0 \end{pmatrix}$$

$$\begin{pmatrix} V1 \\ I1 \end{pmatrix} = \begin{pmatrix} -\sqrt{\frac{L_1C_2}{L_2C_1}} & 0 \\ 0 & -\sqrt{\frac{L_2C_1}{L_1C_2}} \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix} = \begin{pmatrix} -n & 0 \\ 0 & -\frac{1}{n} \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix}$$

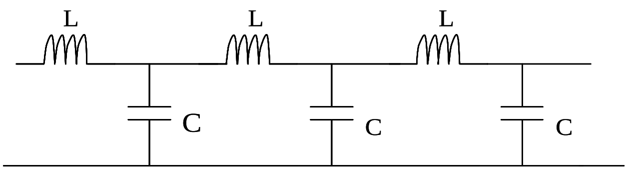
III] Matching circuits with distributed components (microstip line)

1) microstip line



The permittivity of the substrate is $\varepsilon=\varepsilon_0 \cdot \varepsilon_r$

The equivalent model of a lossless line is :



Here L is in henry/meter and C in farad /meter

The characteristic impedance of the line is defined as

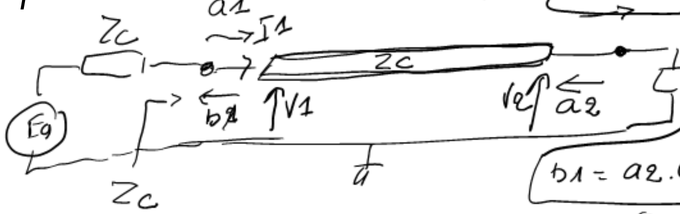
$$Zc=\sqrt{\frac{L}{C}}$$

The guided wavelength is

$$\lambda=\frac{\lambda_0}{\sqrt{\varepsilon_r}} \qquad \lambda_0=\frac{c}{f}=\frac{3\cdot 10^8}{f} \qquad \omega=2\pi f$$

(1) $\boxed{b_2=a_1 \cdot e^{-j\theta}}$

The input / output phase shift is



$\boxed{b_1=a_2 \cdot e^{-j\theta}}$ (2)

$$\theta=\beta.l=\frac{2\pi\cdot l}{\lambda}=\omega\cdot\sqrt{LC}$$

Kurokawa

As voltages and currents are not accurately measured directly , Kurokawa invented the power wave concept :

He defined a linear combination of voltage and current as following

$$a_1=\frac{V_1+R_0\cdot I_1}{2\sqrt{R_0}} \qquad b_1=\frac{V_1-R_0\cdot I_1}{2\sqrt{R_0}} \qquad b_2=\frac{V_2-R_0\cdot I_2}{2\sqrt{R_0}} \qquad a_2=\frac{V_2+R_0\cdot I_2}{2\sqrt{R_0}}$$

$|a_1|^2 \rightarrow W$ $\quad \sqrt{W}$ $\quad$ 3 $\quad$ 4 $\quad$ 5 $\quad$ 6

Equation I

The unit of the magnitude of these power waves is

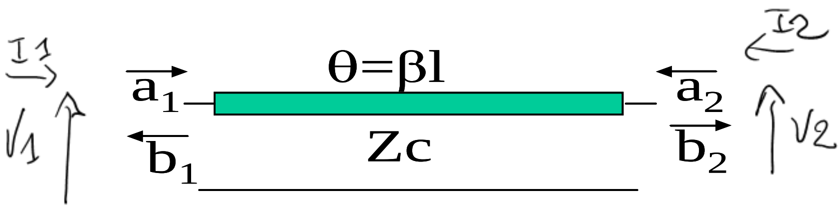
$$\sqrt{Watt}$$

So that their squares are expressed in Watt and are measurable which makes them very useful in microwave measurements .

We have the following relationships

$$P_1 = \frac{1}{2} \mathrm{R}_{eal}(V_1 \cdot I_1^*) = \frac{1}{2}\left(|a_1|^2 - |b_1|^2\right) \qquad P_2 = -\frac{1}{2} \mathrm{R}_{eal}(V_2 \cdot I_2^*) = \frac{1}{2}\left(|b_2|^2 - |a_2|^2\right)$$

For a lossless line we have



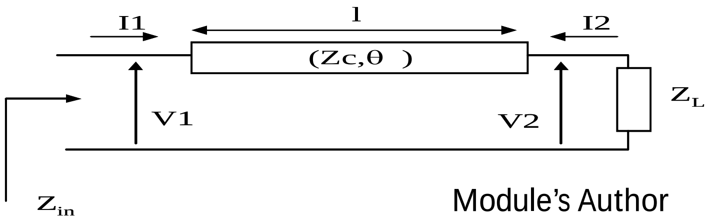
$$b_2 = a_1 \cdot e^{-j\theta} \qquad b_1 = a_2 \cdot e^{-j\theta}$$

Equation II

Combining equations I and II we can obtain the chain matrix of the line :

$$\boxed{\begin{pmatrix} V1 \\ I1 \end{pmatrix} = \begin{pmatrix} \cos(\theta) & jZc \cdot \sin(\theta) \\ \dfrac{j\sin(\theta)}{Zc} & \cos(\theta) \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix}}$$

And if we connect the line to a load impedance Z_L we have at the input of the line an impedance Z_1



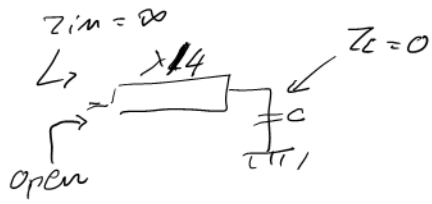
$$\boxed{Z_{in} = \frac{V_1}{I_1} = Z_c \cdot \frac{Z_L + jZ_c \tan(\theta)}{Z_C + jZ_L \tan(\theta)}}$$

2) Few specific examples

a) Impedance Inverter

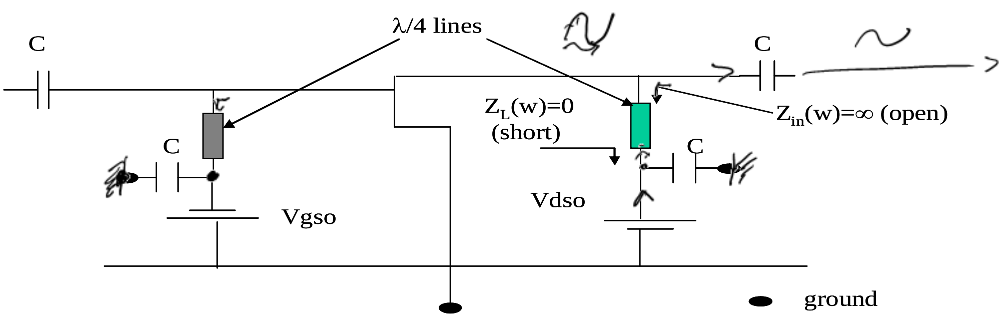
$$l = \frac{\lambda}{4} \qquad \theta = \frac{\pi}{2} \qquad Z_{in} = \frac{Z_c{}^2}{Z_L}$$

$$\boxed{\begin{pmatrix} V1 \\ I1 \end{pmatrix} = \begin{pmatrix} 0 & jZc \\ \frac{j}{Zc} & 0 \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix}}$$

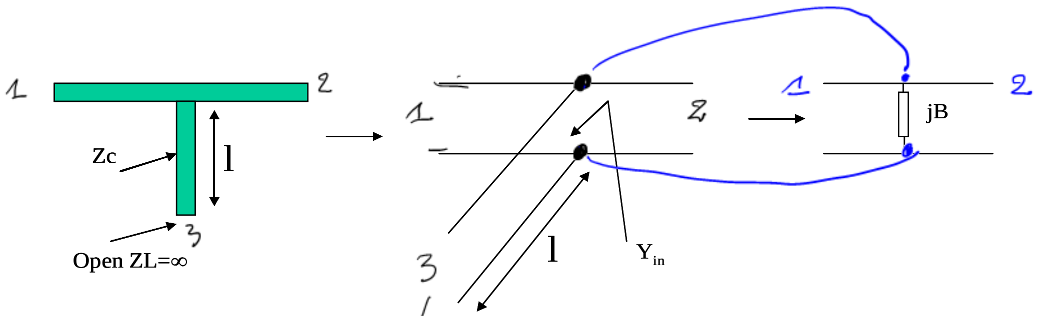


If $Z_L = 0$ (short circuit) → $Z_{in} = \infty$ (open circuit) and vice versa

This can be advantageously used for the design of transistor's bias circuits .



b) Parallel line (open Stub $Z_L=\infty$)



↳ Port open $Z_L = \infty$

$$\boxed{Z_{in} = \frac{V_1}{I_1} = Z_c \cdot \frac{Z_L + jZ_c\tan(\theta)}{Z_C + jZ_L\tan(\theta)}}$$

$\longrightarrow$ $Y_{in} = \frac{1}{Z_{in}} = \frac{j\,tg(\theta)}{Z_c} = \boxed{jB}$

If ZL =∞

$$Z_{in} = -jZ_C \cdot \tan^{-1}(\theta) \qquad\qquad Y_{in} = \frac{1}{Z_{in}} = \frac{j}{Z_C}\cdot\tan(\theta) = jB$$

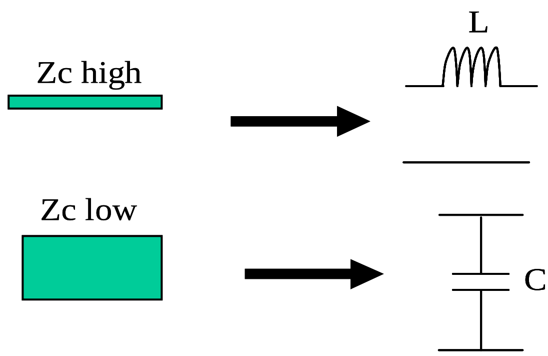
Depending on the sign of tan(θ) , Y_{in} is corresponding to a parallel inductance or a parallel capacitance and can be used for narrowband transistor matching.

c) Small length lines

θ small means cos(θ) ≅ 1 and sin(θ) ≅ θ

With high characteristic impedance Zc

$$\begin{pmatrix} 1 & jZ_c\theta \\ \frac{j\theta}{Z_c} & 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & jL\omega \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ jC\omega & 1 \end{pmatrix}$$

For Zc high

$$\begin{pmatrix} V1 \\ I1 \end{pmatrix} = \begin{pmatrix} \cos(\theta) & jZc\cdot\sin(\theta) \\ \dfrac{j\sin(\theta)}{Zc} & \cos(\theta) \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix} = \begin{pmatrix} 1 & jZc\theta \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix} = \begin{pmatrix} 1 & jLw \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix}$$

With :

$$\boxed{L = \frac{Zc\cdot\beta l}{w}}$$

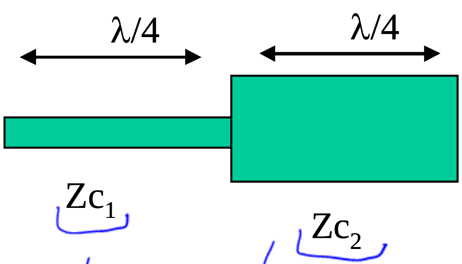
For Zc low

$$\begin{pmatrix} V1 \\ I1 \end{pmatrix} = \begin{pmatrix} \cos(\theta) & jZc\cdot\sin(\theta) \\ \dfrac{j\sin(\theta)}{Zc} & \cos(\theta) \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \dfrac{j\theta}{Zc} & 1 \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ jCw & 1 \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix}$$

With

$$\boxed{C = \frac{\beta l}{Zc\cdot w}}$$

d) Impedance transformer



$$\begin{pmatrix} V1 \\ I1 \end{pmatrix} = \begin{pmatrix} 0 & jZc_1 \\ \dfrac{j}{Zc_1} & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & jZc_2 \\ \dfrac{j}{Zc_2} & 0 \end{pmatrix} \begin{pmatrix} V2 \\ -I2 \end{pmatrix} = \begin{pmatrix} -\dfrac{Zc_1}{Zc_2} & 0 \\ 0 & -\dfrac{Zc_2}{Zc_1} \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix} = \begin{pmatrix} -n & 0 \\ 0 & -\dfrac{1}{n} \end{pmatrix} \cdot \begin{pmatrix} V2 \\ -I2 \end{pmatrix}$$

IV] Basic Power amplifier Architecture