Communication Theory 2 (6 CFU): 13th of June, 2013 [Prof. P. Migliorati, Univ. of Brescia]

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Answer to the questions according to the order assigned in the text.
Use also the sheets back.
Any "not given" answer will be taken into account (producing a penalty) in the overall evaluation.
Any "schematic and clearly described" answer will be appreciated.
The given answers should be reported in the original signed exam document. The minutes will not be taken into account in the final evaluation.

Questions

1. Block Codes
   - A systematic block code is described by the parity check matrix H indicated in Fig. 1. Determine and draw the generator matrix of this code.

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

   - Indicate the possible code-words. Is this a cyclic code ?
   - Consider now a block code with: $N = 48$, $K = 24$, $d = 12$. Determine the number of possible codewords, the probability of error (in case of hard and soft decision) and the minimum required bandwidth if the transmitted information bit-rate is 100 Mbit/s.

$H = [P^T \mid I_{N-K \times N-K}]$ $\quad G = [I_{K \times K} \mid P]$ sistematico

$$G = \begin{bmatrix} 1000110 \\ 0100011 \\ 0010111 \\ 0001101 \end{bmatrix}$$ V

$\overline{X} = \overline{m}\,\overline{\overline{G}}$

0000 → 0000000
0001 → 0001101
0010 → 0010111
0011 → 0011010
0100 → 0100011
0101 → 0101110
0110 → 0110100
0111 → 0111001
1000 → 1000110
1001 → 1001011
1010 → 1010001
1011 → 1011100
1100 → 1100101
1101 → 1101011
1110 → 1110010
1111 → 1111111 V

è ciclico se Resto $\left\{ \frac{g_{(D)} D^7+1}{g(D)} \right\} = 0$

D^7 | 1 | D^3+D^2+1

$D^7\ D^6\ D^4$ | $D^4+D^3+D^2+1$

$D^6\ D^4$ | 1

$D^5\ D^3$

$D^5\ D^4\ D^3$ | 1

$D^3\ D^2\ 1$

$D^3\ D^2\ 1$

0

$\Rightarrow$ il codice è ciclico ✓

- Le parole parities sono $2^K = 2^{24}$

SOFT DECISION $P(E) \overset{\sim}{=} Q\sqrt{\frac{2E_b}{N_0}\frac{K}{N}d} = Q\sqrt{\frac{2E_b}{N_0}\frac{24}{48}\cdot 12} = Q\sqrt{12\frac{E_b}{N_0}}$

HARD DECISION $P(E) \overset{\sim}{=} Q\sqrt{\frac{2E_b}{N_0}\frac{K}{N}(t+1)} = Q\sqrt{\frac{2E_b}{N_0}\frac{24}{48}(5+1)} = Q\sqrt{6\frac{E_b}{N_0}}$

$t = \frac{d-1}{2} = 5$

$B_{0_{MIN}} = \min\left\{\frac{1}{T_S 2}(1+\delta)\right\} = \frac{R_b}{2} = 50\ MHz$

$T_S = T_b$ Binaria

$B_c = \frac{N}{K} B_{0_{MIN}} = 100\ MHz$ ✓