Igor Lopez Gonzalez

Problem 1

a)

We have the expression: $\vec{B} = \frac{1}{c}\, \hat{k} \times \vec{E}$

with: $\vec{B} = \mu \vec{H}$

$$\vec{H} = \frac{1}{\mu c}\, \hat{k} \times \vec{E} \underset{\mu=\mu_0}{=} \frac{1}{\mu_0 c} E_0 \cos(\omega t - kz) \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} =$$

$$= \frac{1}{\mu_0 c} E_0 \cos(\omega t - kz)\,(-\hat{e}_x) = -\frac{1}{\mu_0}\sqrt{\varepsilon \mu}\; E_0 \cos(\omega t - kz)\, \hat{e}_x =$$

$$\underset{\varepsilon=\varepsilon_r\varepsilon_0}{=} -\sqrt{\frac{\varepsilon_0\, \varepsilon_r}{\mu_0}}\; E_0 \cos(\omega t - kz)\, \hat{e}_x = -\frac{\sqrt{\varepsilon_r}}{\eta_0} E_0 \cos(\omega t - kz)\, \hat{e}_x =$$

$$= -\frac{\sqrt{2{,}55}}{120\pi}\; 30 \cos(\omega t - kz)\, \hat{e}_x$$

$$\boxed{\vec{H} = -0{,}13 \cos(\omega t - kz)\, \hat{e}_x}$$

b)

For the phase velocity we have: $V_p = \frac{\omega}{\beta}$

$$V_p = \frac{\omega}{\beta} = \frac{2\pi f}{\frac{2\pi}{\lambda_n}} = \underbrace{\lambda_n f}_{\substack{\lambda_n = \frac{\lambda}{n} \\ n = \sqrt{\varepsilon_r \mu_r}}} = \underbrace{\frac{\lambda}{\sqrt{\varepsilon_r \mu_r}} f}_{\mu_r = 1} = \frac{c_0}{\sqrt{\varepsilon_r}} = \frac{3 \cdot 10^8}{\sqrt{2{,}35}}$$

$$\boxed{V_p = 1{,}89 \cdot 10^8 \ m/s}$$

c)

We use: $V_p = \frac{\Delta z}{\Delta t}$ and $\Delta \varphi = \Delta t \cdot \omega$

$$\Delta \varphi = \omega \cdot \frac{\Delta z}{V_p} = \omega \cdot \frac{z_2 - z_1}{V_p} = 2\pi f \cdot \frac{1{,}7 - 0{,}5}{V_p} = 2\pi \cdot 2{,}4 \cdot 10^9 \ \frac{1{,}2}{1{,}89 \cdot 10^8}$$

$$\boxed{\Delta \varphi = 95.37 \ rad}$$

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Problem 2

a)

$$E_1 = 2\cos(\omega t - kz)\,\hat{e}_x + 2\sin(\omega t - kz)\,\hat{e}_y =$$

$$= 2\cos(\omega t - kz)\,\hat{e}_x + 2\cos\left(\omega t - kz - \frac{\pi}{2}\right)\hat{e}_y$$

We have:

$$\left.\begin{aligned} a_x = a_y = 2 \\ \varphi = -\frac{\pi}{2} \end{aligned}\right\}$$ Right Handed Circular Polarization

$$\bar{E}_2 = 2\cos(\omega t + kz)\,\hat{e}_x + 2\sin(\omega t + kz)\,\hat{e}_y =$$

$$= 2\cos(\omega t + kz)\,\hat{e}_x + 2\cos\left(\omega t + kz - \frac{\pi}{2}\right)\hat{e}_y$$

Because it is in the opposite direction we have:

$$\left.\begin{aligned} a_x = a_y = 2 \\ \varphi = +\frac{\pi}{2} \end{aligned}\right\}$$ Left Handed Circular Polarization

b)

$$E_1 = 2\cos(\omega t - kz)\,\hat{e}_x - 2\sin(\omega t - kz)\,\hat{e}_y =$$

$$= 2\cos(\omega t - kz)\,\hat{e}_x + 2\sin(\omega t - kz + \pi)\,\hat{e}_y =$$

$$= 2\cos(\omega t - kz)\,\hat{e}_x + 2\cos\left(\omega t - kz + \pi - \frac{\pi}{2}\right)\hat{e}_y =$$

$$= 2\cos(\omega t - kz)\,\hat{e}_x + 2\cos\left(\omega t - kz + \frac{\pi}{2}\right)\hat{e}_y$$

So we have:

$$\left.\begin{array}{l} a_x = a_y = 2 \\ \varphi = \frac{\pi}{2} \end{array}\right\}$$ Left Handed Circular Polarization

$$E_2 = 2\cos(\omega t + kz)\,\hat{e}_x - 2\sin(\omega t + kz)\,\hat{e}_y =$$ ↳ using the same calculation as the previous case.

$$= 2\cos(\omega t + kz)\,\hat{e}_x + 2\cos\left(\omega t + kz + \frac{\pi}{2}\right)\hat{e}_y$$

This wave is traveling in the opposite direction, so:

$$\left.\begin{array}{l} a_x = a_y = 2 \\ \varphi = -\frac{\pi}{2} \end{array}\right\}$$ Right Handed Circular Polarization

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Problem 3

We know $\theta_c = 45°C$ $\longrightarrow$ $\theta_c = \arcsin\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$

$$\frac{\pi}{4} = \arcsin\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\sin\frac{\pi}{4} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \longrightarrow \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{\sqrt{2}}{2}$$

We have too: $\theta_B = \text{arctg}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$

$$\theta_B = \text{arctg}\left(\frac{\sqrt{2}}{2}\right) \longrightarrow \boxed{\theta_B = 0{,}615 \text{ rad}}$$