Semester S1 – Module 3

Module Fundamentals of coherent photonics

TUTORIAL

SPATIAL OPTICS_1

SPATIO TEMPORAL ANALYSIS

Exercise 1: Temporal signal versus spatial beam

A - Decomposition of a temporal signal into a sum of monochromatic signals

Note: A temporal signal is monochromatic when it is described by a sinusoidal function (from $t \to -\infty$ to $t \to +\infty$.

At any point M in space, the electric field E(t) of a light radiation is given by:

$$E(t) = a.cos^2(2\pi\nu_0 t).e^{-\pi t^2/\tau^2},$$

with a = constant (V/m), ν_0 = 0.3 10^{15}Hz, $\tau = 3/\nu_0$ (time constant (s)).

1) Plot E(t). Is this signal monochromatic?
2) Calculate its spectrum $\tilde{E}(\nu)$ and plot it with respect to the frequency ν.
3) Demonstrate that this spectrum is made of three components. Give their amplitudes, their central frequencies and their spectral width $\delta\nu$ (full width at $e^{-\pi}$ maximum).
4) How should the time constant be varied so that the three spectral components become strictly monochromatic?

B - Decomposition of a monochromatic wave into a sum of monochromatic plane waves

At a given time t, consider a monochromatic wave (λ_0 = 1µm) which propagates in the plane (x,z). Its spatial profile is described in the plane of abscissa z = 0 by the electric field:

$$E(x, z = 0) = a.cos^2(2\pi N_0 x).e^{-\pi x^2/L^2},$$

with a = constant (V/m), $N_0 = 10^3$ m, $L = 3/N_0$.

1) Plot E(x,z=0). Is the studied wave plane?
2) Calculate its spatial-frequency spectrum $\tilde{E}(\mathrm{Nx}, \mathrm{z} = 0)$ and plot it with respect to the spatial frequency Nx.
3) Demonstrate that this spatial-frequency spectrum is made of three components. Give their amplitudes, their central spatial frequencies and their width δN (full width at $e^{-\pi}$ maximum).
4) These three spectral components correspond to three monochromatic waves for which we will indicate:
   - the angle θ of their average direction of propagation with the z-axis,
   - their angle of divergence $\delta\theta$ de divergence (full width at $e^{-\pi}$ maximum).
5) Why can we say that each of these three components is not a plane wave? How should the length L be varied so that the three monochromatic waves become strictly plane?

Exercise 2: Spatial frequency Spectrum

We consider a 2-dimensional problem where Oz is the mean direction of propagation and x the transverse direction.

1) Give the expression of the field $E_0(x, z)$ corresponding to a plane wave of module a_0. The wavelength is λ_0 and the direction of propagation makes an angle θ_0 with the z-axis in the xOz plane. Use the complex form.
2) Give the expression of the field $E_0(x, 0)$ and identify the spatial frequency $N_{x0} = \frac{\sin\theta_0}{\lambda_0}$.
3) A diffraction grating of transmission $\tau(x) = \frac{1}{2}\left[1 + \cos 2\pi \frac{x}{a}\right]$ is in the plane of abscissa z = 0. Derive the expression of the field $E(x, 0)$ transmitted by the grating of pitch "a".

4) By analyzing the shape of the field $E(x, 0)$, show that there exists three orders of diffraction. Characterize these orders by the associated spatial frequencies $N_x = \frac{\sin\theta}{\lambda_0}$. θ is the angle of propagation after diffraction onto the grating. To do this, we will specify the relationship between N_x, N_{x0}, a.

5) Verify that the previous relationship corresponds to the grating equation.
