Chirped Gaussian Pulses

For an initially unchirped Gaussian pulse, Eq (10) shows that dispersion-induced broadening of the pulse does not depend on the sign of the GVD parameter β_2.

This behavion changes if the Gaussia pulse has a initial frequency chirp. In the case of linearly chirped Gaussian pulses, the evelope of the input can be written as

$$F(0,t) = \exp\left(-\frac{(1+iC)}{2}\frac{t^2}{t_0^2}\right) \qquad (14)$$

where C is the chirp parameter.

By using Eq. (11) one finds that the istantaneous frequency increases linearly from the leading to the trailing edge for $C > 0$ while the opposite occurs for $C < 0$.

The numerical value of C can be estimated from the spectral width of the Gaussian pulse.

By substituting Eq (14) into Eq (6), $\tilde{F}(0,\omega)$

$$\tilde{F}(0,\omega) = \left(\frac{2\pi T_0^2}{1+iC}\right)^{1/2} \exp\left(-\frac{\omega^2 T_0^2}{2(1+iC)}\right) \qquad (15)$$

The spectral half-width (at 1/e-intensity point) is given by:

$$\Delta\omega = \left(1+C^2\right)^{1/2} / T_0 \qquad (16)$$

In The absence of CHIRP ($C = 0$), The spectral width is transfor limited, That is it satisfies The relation $\Delta W \, T_0 = 1$. The spectral width is enhanced by a factor of $(1 + C^2)^{1/2}$ in The presence of linear chirp. Eq. (16) can be used To estimate $|C|$ from experimental Traces of ΔW T_0.

To obtain The Transmitti Field, we can integrate with The result:

$$F(z,t) = \frac{t_0}{\left[t_0^2 - i\beta_2 z(1+iC)\right]^{1/2}} \exp\left(-\frac{(1+iC)t^2}{2\left[t_0^2 - i\beta_2 z(1+iC)\right]}\right) \quad (17)$$

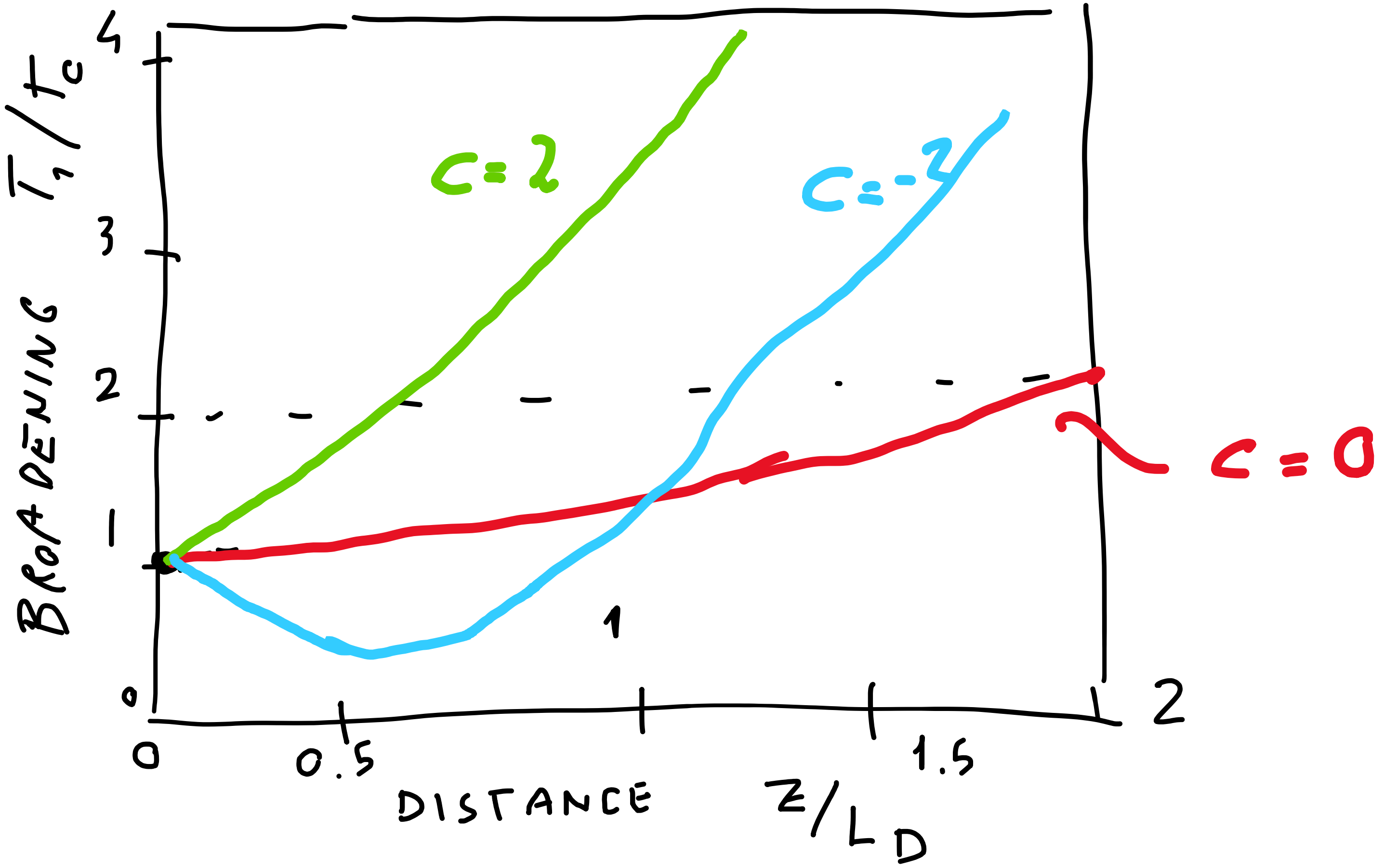
Thus, even a chirped Gaussian pulse maintains its gaussian shape on propagation. The width T_1 after propagating a distance z is related to t_0 by the relation

$$\frac{T_1}{t_0} = \left[\left(1 + \frac{C\beta_2 z}{t_0^2}\right)^2 + \left(\frac{\beta_2 z}{t_0^2}\right)^2\right]^{1/2} \quad (18)$$

This equation shows that broadening depends on the relative signs of β_2 and C.

- A gaussian pulse broadens monotonically with z if $\boxed{\beta_2 C > 0}$
- A gaussian pulse goes through an initial narrowing stage with z if $\boxed{\beta_2 C < 0}$

you can plot expression (18)



$\beta_2 > 0$

As to the figure above, for $\beta_2 < 0$, the same curves are obtained if the sign of C is revered.

In the case $\beta_2 C < 0$, the pulse duration becomes minimum at a distance

$$z_{min} = \frac{|C|}{1+C^2} L_D \qquad (19)$$

The minimum value of the pulse width a $z = z_{min}$ is

$$t_1^{min} = \frac{t_0}{(1+C^2)^{1/2}} \qquad (20)$$

By using Eq (16) and (10) one finds that at $z = z_{min}$ the pulse is Fourier transfor limited, that is $\Delta\omega\, T_1^{min} = 1$.

Initial narrowing of the pulse for the case $\beta_2 C < 0$ can be undestood by ref. to Eq (13), which shows the dispersion-induced chirp on an initially unchirped gaussian pulse. When the pulse is initially chirped and the condition $\beta_2 C < 0$ is satisfied, the dispersion induced chirp is in opposite direction to that of the initial chirp. As a result the net chirp is reduced, leading to pulse narrowing. The minimum pulse duration occurs at a point at which the two chirps cancel each other. With a further increase in the propagation distance, the dispersion induced chirp starts to dominate over the initial chirp, and the pulse begins to broaden.