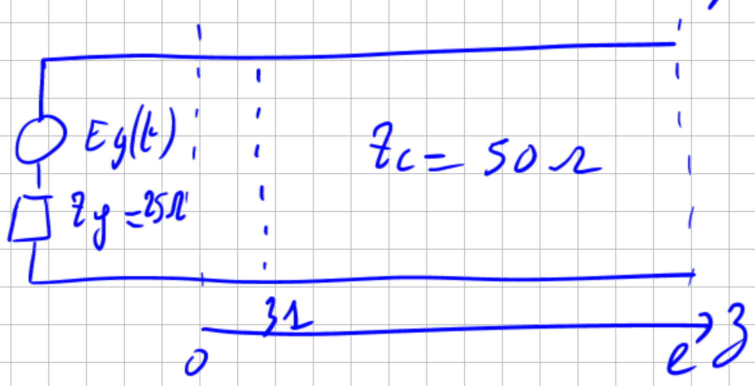
S. Verdeyme
Tutorial1
Sept 17th 2020

II

a)

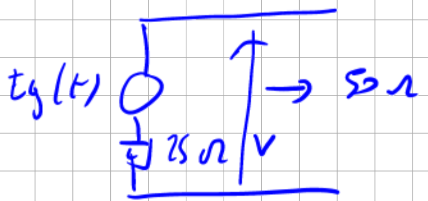


$$\rho_g = \frac{25-50}{25+50} = -\frac{1}{3}$$

$$\rho_L = 1$$

velocity of the wave : v

b) $t=0$, the generator delivers $\frac{\delta(t)\times 50}{50+25} = \frac{2}{3}\delta(t)$



$$V = \frac{50}{50+25} E_g(t)$$

at time $t_1 = \frac{z_1}{v}$ $v(z_1, t_1) = \frac{2}{3}\delta(t)$

at time $t = \frac{l}{v}$, the pulse is reflected by the load $\rho_L = 1$

at time $t_2 = \frac{2l - z_1}{v}$, $v(z_1, t_2) = \frac{2}{3}\delta(t-t_2)$

at time $t = \frac{2l}{v}$, this pulse is reflected by the generator $\rho_g = -\frac{1}{3}$

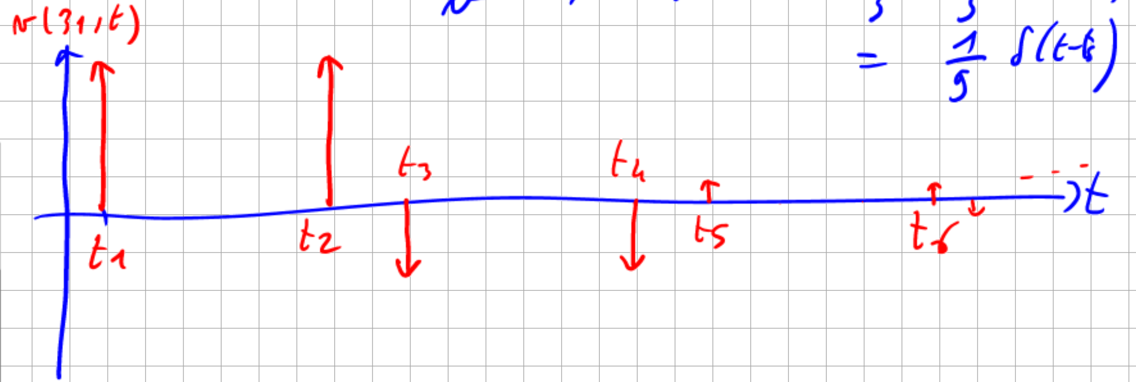
at time $t_3 = \frac{2l+z_1}{v}$, $v(z_1,t_3) = -\frac{1}{3}\times\frac{2}{3}\delta(t-t_3)$

$$= -\frac{1}{3}\delta(t-t_3)$$

at time $t_4 = \frac{4l - z_1}{v}$, $v(z_1, t_4) = -\frac{1}{3}\delta(t-t_4)$

at time $t_5 = \frac{4l+z_1}{v}$, $v(z_1,t_5) = -\frac{1}{3}\times -\frac{1}{3}\delta(t-t_5)$

$$= \frac{1}{9}\delta(t-t_5)$$



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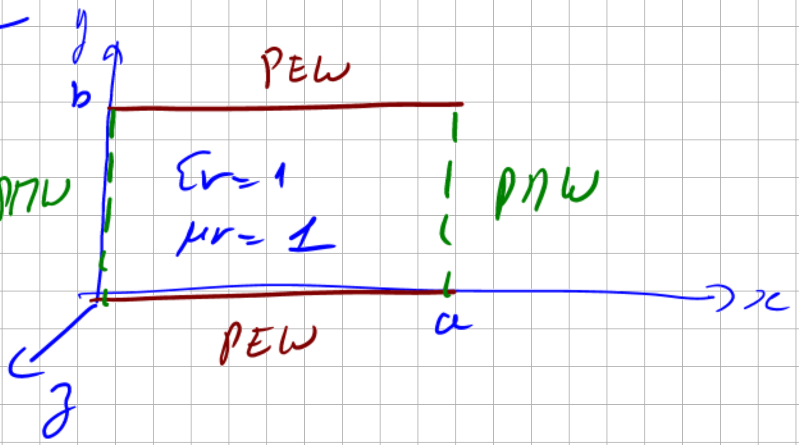
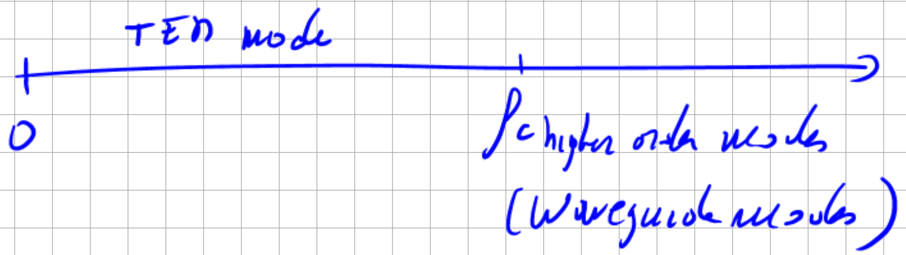


1) - 2 conductors
- homogeneous dielectric material between the 2 conductors

$\Rightarrow$ TEM exists.



2) TE mode:

a)

$$\Delta_t H_z(x,y) + k_c^2 H_z(x,y) = 0$$

$$k_c^2 = k_0^2 - \beta^2 \qquad (\gamma = j\beta)$$

$$\frac{\partial H_z(x,y)}{\partial y} = 0 \text{ for } y = 0 \text{ and } y = b$$

$$H_z(x,y) = 0 \text{ for } x = 0 \text{ and } x = a$$

$$H_z(x,y) = f(x) \times g(y)$$

$$\rightarrow \frac{\partial^2 f(x)}{\partial x^2} + k_x^2 f(x) = 0$$

$$\frac{\partial^2 g(y)}{\partial y^2} + k_y^2 g(y) = 0$$

$$k_c^2 = k_x^2 + k_y^2$$

$\rightarrow$ Solution of propagation equation:

$$H_z(x,y) = (A\cos k_x x + B\sin k_x x) \times (C\cos k_y y + D\sin k_y y)$$

$H_z(x,y) = 0$ for $x = 0 \Rightarrow A = 0$

$H_z(x,y) = 0$ for $x = a \Rightarrow k_x = \frac{n\pi}{a}, n \in \mathbb{N}$

$\frac{\partial H_z(x,y)}{\partial y} = 0$ for $y = 0 \Rightarrow D = 0$

$\frac{\partial H_z(x,y)}{\partial y} = 0$ for $y = b \Rightarrow k_y = \frac{m\pi}{b}, m \in \mathbb{N}$

Then $H_z(x,y) = H_0 \sin \frac{n\pi}{a} x \cos \frac{m\pi}{b} y$

The first mode is the TE10 mode

$$H_z(x,y) = H_0 \sin \frac{\pi}{a} x$$

Cut-off frequency:

$$f_{c_{TE_{10}}} = \frac{c}{2\pi} \frac{\pi}{a} = \frac{c}{2a}$$

b) $k_c^2 = k_0^2 - \beta^2$

$$\left(\frac{\pi}{a}\right)^2 = \left(\frac{\omega}{v}\right)^2 - \beta^2$$

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c$$

phase velocity v_φ:

$$v_\varphi = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\left(\frac{\omega}{v}\right)^2 - \left(\frac{\pi}{a}\right)^2}}$$

group velocity v_g

$$v_g = \frac{d\omega}{d\beta}$$

$$\omega = v \sqrt{\left(\frac{\pi}{a}\right)^2 + \beta^2}$$

$$v_g = v \frac{\beta}{\sqrt{\left(\frac{\pi}{a}\right)^2 + \beta^2}}$$

$$= v^2 \frac{\beta}{\omega}$$

c) $\vec{E}_t(x,y) = \frac{j\omega\mu}{k_c^2} \left(\vec{u} \wedge \vec{\nabla}_t H_z(x,y)\right)$

$$= \frac{j\omega\mu}{k_c^2} H_0 \frac{\pi}{a} \cos \frac{\pi}{a} x \ \vec{e_y}$$

$$E_y(x,y) = \frac{j\omega\mu}{k_c} H_0 \cos \frac{\pi}{a} x, \quad E_x(x,y) = 0$$

$$\vec{H}_t(x,y) = \frac{-j\beta}{k_c^2}\,\vec{\nabla}_t\,H_z(x,y)$$

$$= \frac{-j\beta}{k_c^2}\,\frac{\pi}{a}\,H_0 \cos\frac{\pi}{a}x \quad \vec{e_x}$$

$$H_x(x,y) = \frac{-j\beta}{k_c}\,H_0 \cos\frac{\pi}{a}x\,,\quad H_y(x,y) = 0$$

d) $W = 2\,W_e$

$$= 2 \times \frac{1}{4}\iiint |\vec{E}_t|\,dV$$

$$= \frac{1}{2}\varepsilon_0 \int_0^a\int_0^b\int_0^1 \left(\frac{\omega\mu}{k_c}\right)^2 |H_0|^2 \cos^2\frac{\pi}{a}x\,dx\,dy\,dz$$

$$= \left(\frac{\omega\mu}{k_c}\right)^2 |H_0|^2\,\frac{ab}{4}\,\varepsilon_0$$

$$P = \frac{1}{2}\,\mathrm{Re}\int_0^a\int_0^b \left(\vec{E}_t \wedge \vec{H}_t^*\right)\cdot\vec{u}\,dx\,dy$$

$$= \frac{1}{2}\int_0^a\int_0^b \left(\frac{\omega\mu}{k_c}\right)\frac{\beta}{k_c}|H_0|^2 \cos^2\frac{\pi}{a}x\,dx\,dy$$

$$= \frac{\omega\mu\beta}{k_c^2}\,|H_0|^2\,\frac{ab}{4}$$

$$\frac{P}{W} = \frac{\omega\mu\beta}{\cancel{k_c^2}}\,\frac{\cancel{ab}}{\cancel{4}}\,\frac{\cancel{4}\,\cancel{k_c^2}}{\omega^2\mu^2\,|H_0|^2\,\cancel{ab}\,\varepsilon_0}$$

$$= \frac{\beta}{\omega\mu\varepsilon}$$

$$= v^2\,\frac{\beta}{\omega} = v_g$$

v_g is the velocity of the energy.