Continuous Phase Modulation

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CPM modulation

Modulazione a FASE CONTINUA (CPM)

inviluppo costante → Amplif. in SATURAZIONE

(un valore di) occupata

questa banda ragionevole, dobbiamo impor

continuità della fase → correlazione

tra i simboli → ricev. (OTT.) complicato – (MLSE

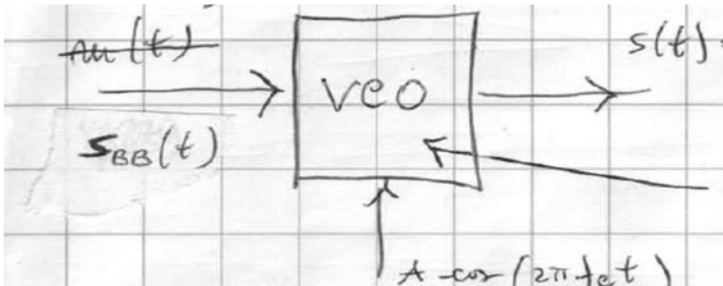
(Modello di principio:

(modulo in freq. una portante)

con un segnale PAM

$\theta_i(t)$

1) Constant envelope: amplifier in saturation (efficient use of Power) !!!
2) Phase Continuity: correlation between different symbols -> optimal receiver is very complicated;
3) Basic principle: FM modulation of a PAM modulating signal



$$s(t) = A\cos\big(\overbrace{2\pi f_c t + \varphi(t)}^{\theta_i(t)}\big)$$

$$\underbrace{S_{BB}(t)}\ \cancel{m(t)} = \sum_k a_k\, g(t-kT)$$

$$\varphi(t) = 2\pi h \int_{-\infty}^{t} \sum_k a_k\, g(t'-kT)\,dt' =$$

h= modulation index

$$\varphi(t) = 2\pi h \cdot \sum_k a_k\, q(t-kT)$$

CONVENZIONI: $a_k = \pm 1, \pm 3, \dots$, ed area di $g(t) = 1/2$ $\big(q(\infty) = 1/2\big)$

$$\left(q(t) = \int^{t} g(t')\,dt'\right)$$

$2\pi h \frac{1}{2} a_K = \pi h a_K \rightarrow$ è il contributo finale alla fase di a_K

h=modulation index

$$s(t) = \mathrm{Re}\left\{ \underbrace{A e^{j\varphi(t)}} e^{j2\pi f_c t} \right\} \rightarrow \tilde{s}(t) = A e^{j\varphi(t)}$$

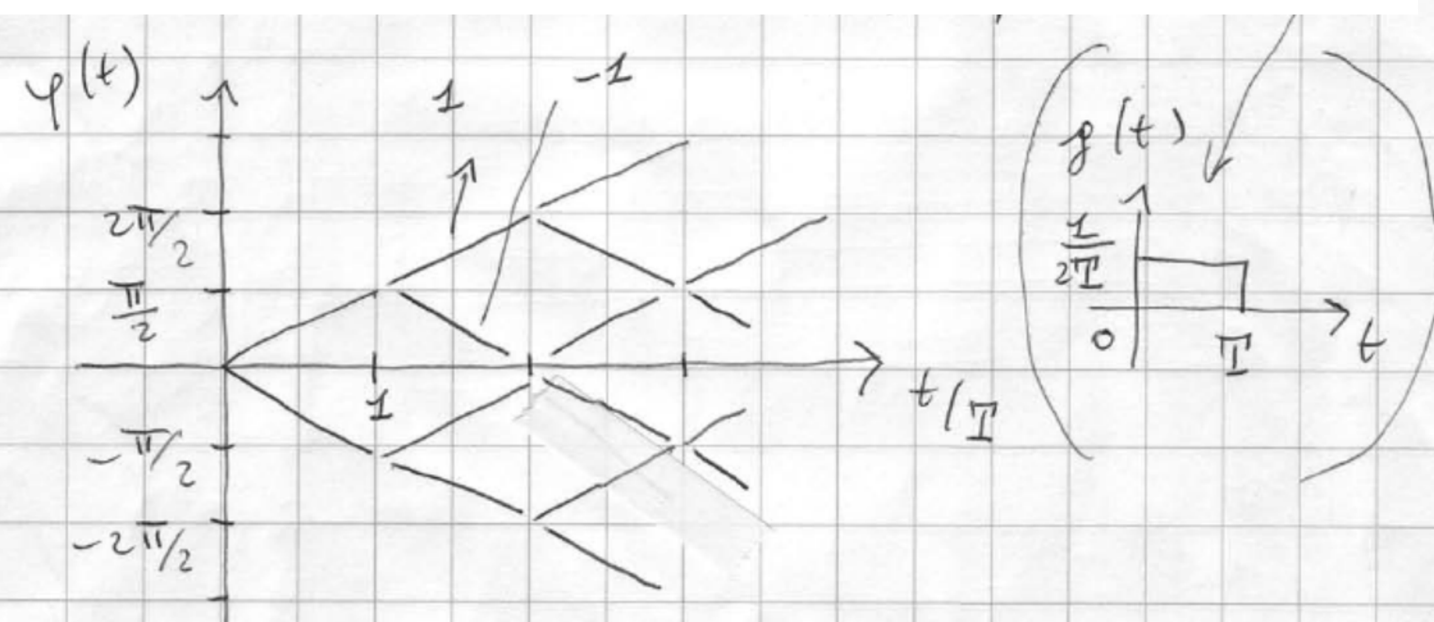
Se la durata di $g(t) \le T_{simb.}$ → modulazione CPM a risposta totale

Altrimenti (più intervalli) → risposta parziale !!!

If the duration of g(t) is < or = to Ts, then Total response CPM;
Otherwise, Partial response CPM.

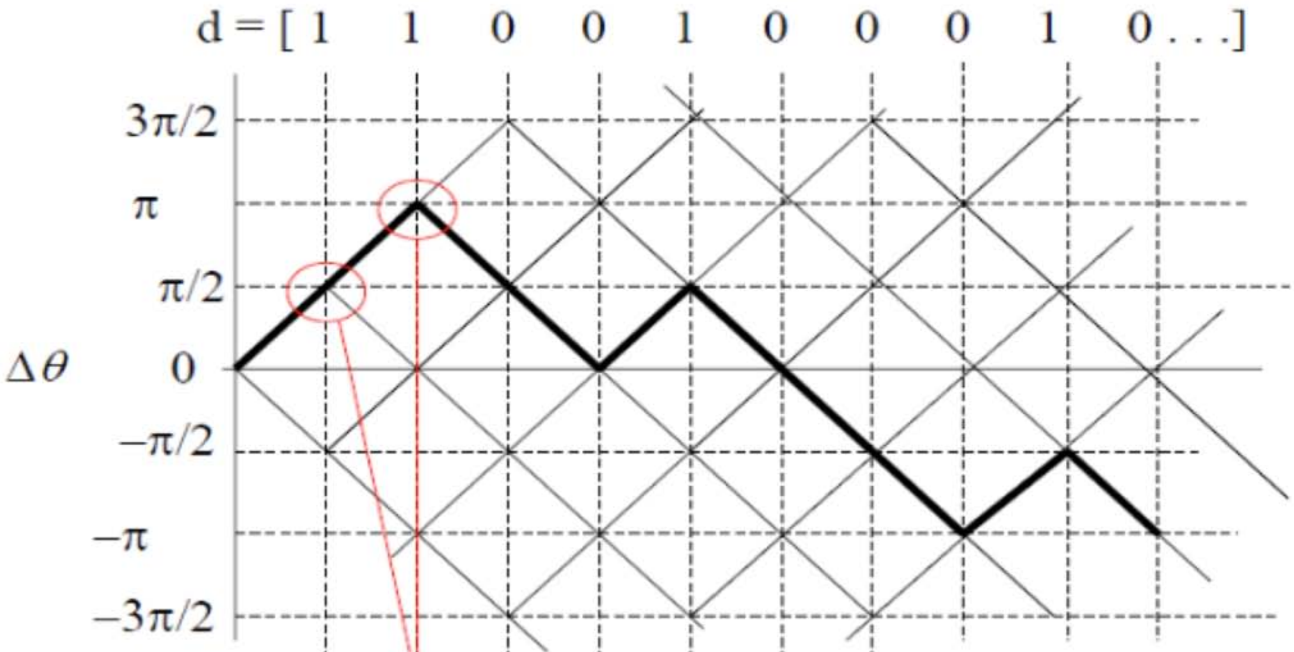
MSK modulation

$$h=\frac{1}{2} \rightarrow \pi h a_k = \frac{\pi}{2} a_k \; ; \; g(t)$$



* MSK Phase tree

MSK Trellis



Every bit interval there is either an upshift or a

Note phase remains continuous at bit changes
Only certain phase transitions are possible

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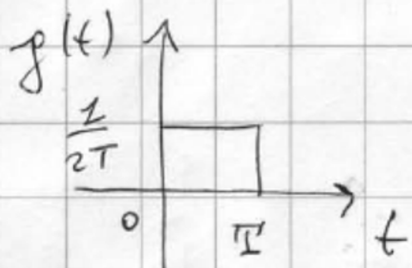
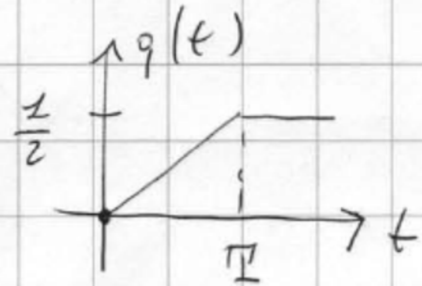
$\boxed{\text{MSK}}$

$$\varphi(t) = 2\pi \underset{1/2}{h} \int_{-\infty}^{t} \sum_{k} a_k \, g(t' - kT)\, dt' \qquad a_k = \pm 1$$

$$g \rightarrow \frac{1}{2T}\,\mathrm{rect}\left(\frac{t - T/2}{T}\right) \qquad\qquad T_S = T \ !!!$$

$$q(t) = \int_{-\infty}^{t} g(t')\,dt' = \frac{1}{2T}\, t\, \mathrm{rect}\left(\frac{t - T/2}{T}\right) + \frac{1}{2}\varepsilon(t - T)$$





$$\theta_i(t) = 2\pi f_c t + \frac{\pi}{2} \sum_{n=-\infty}^{k-1} a_n + \frac{\pi}{2} a_k \frac{t - kT}{T} \quad ; \quad t \in [kT, kT+T]$$

$$\theta_i(t) = 2\pi f_c t + \frac{\pi}{2}\sum_{n=-\infty}^{k-1} a_n + \frac{\pi}{2} a_k \frac{t-kT}{T} \quad ; \quad t \in [kT, kT+T]$$

$$\theta_i(t) = 2\pi\left(f_c + \frac{a_k}{4T}\right)t + \frac{\pi}{2}\sum_{n=-\infty}^{k-1} a_n - \frac{\pi}{2} k a_k$$

$$s(t) = A\cos\left(2\pi f_c t + \frac{2\pi}{4}\frac{a_k}{T} t + \frac{\pi}{2}\sum_{n=-\infty}^{k-1} a_n - \frac{\pi}{2} k a_k\right)$$

$$* \quad f_1 = f_c - \frac{1}{4T} \quad ; \quad f_2 = f_c + \frac{1}{4T}$$

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$$\Delta f = \frac{1}{2T} \rightarrow \qquad (\varphi_i = 0 \, !!!)$$

$$A \rightarrow \sqrt{\frac{2E_b}{T_b}}$$

$$s(t) = A \left[c_k \, h_a(t - k2T) \cos(2\pi f_c t) + \right.$$

$$\left. - d_k \, h_a(t - k2T - T) \sin(2\pi f_c t) \right] =$$

$*$ in $[kT, kT + T]$

* ck, dk, +/-1, indepentent, related to ak

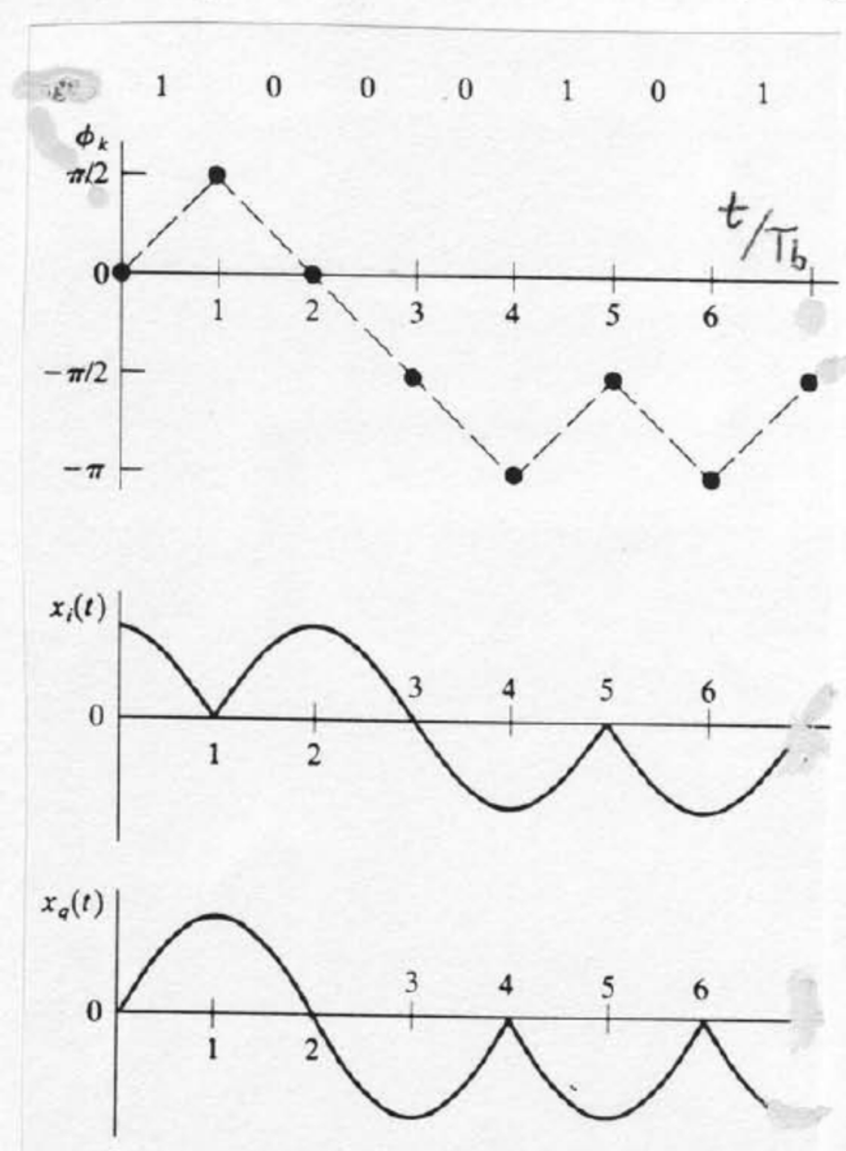
$$\begin{cases} h_a(t) = \cos\left(\frac{\pi t}{2T}\right), & -T \le t \le T \\ h_a(t-T) = \sin\left(\frac{\pi t}{2T}\right), & 0 \le t \le 2T \end{cases}$$

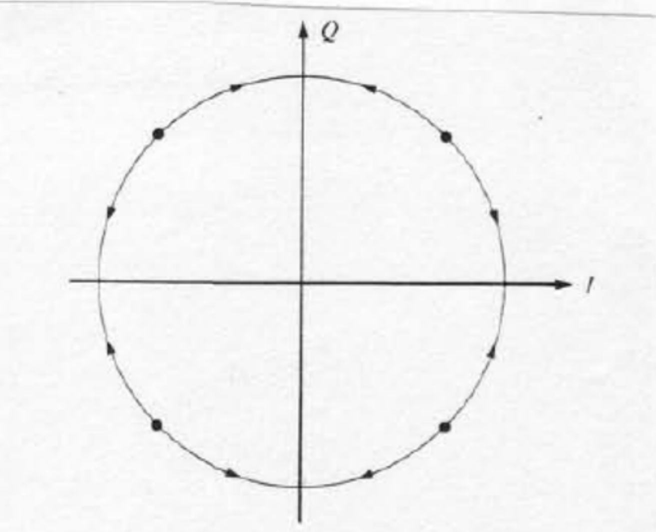
→ O-PSK with p(t)=sin(.)

$$\begin{cases} s_I(t) = \pm\sqrt{\frac{2E_b}{T_b}} \cos\left(\frac{\pi}{2T_b} t\right), & -T_b \le t \le T_b \\ s_Q(t) = \pm\sqrt{\cdot}\, \sin\left(\frac{\pi}{2T_b} t\right), & 0 \le t \le 2T_b \end{cases}$$

The value is related to ak …

Minimum FSK: MSK modulation.
QPSK with offset + sin(.) shaped pulses





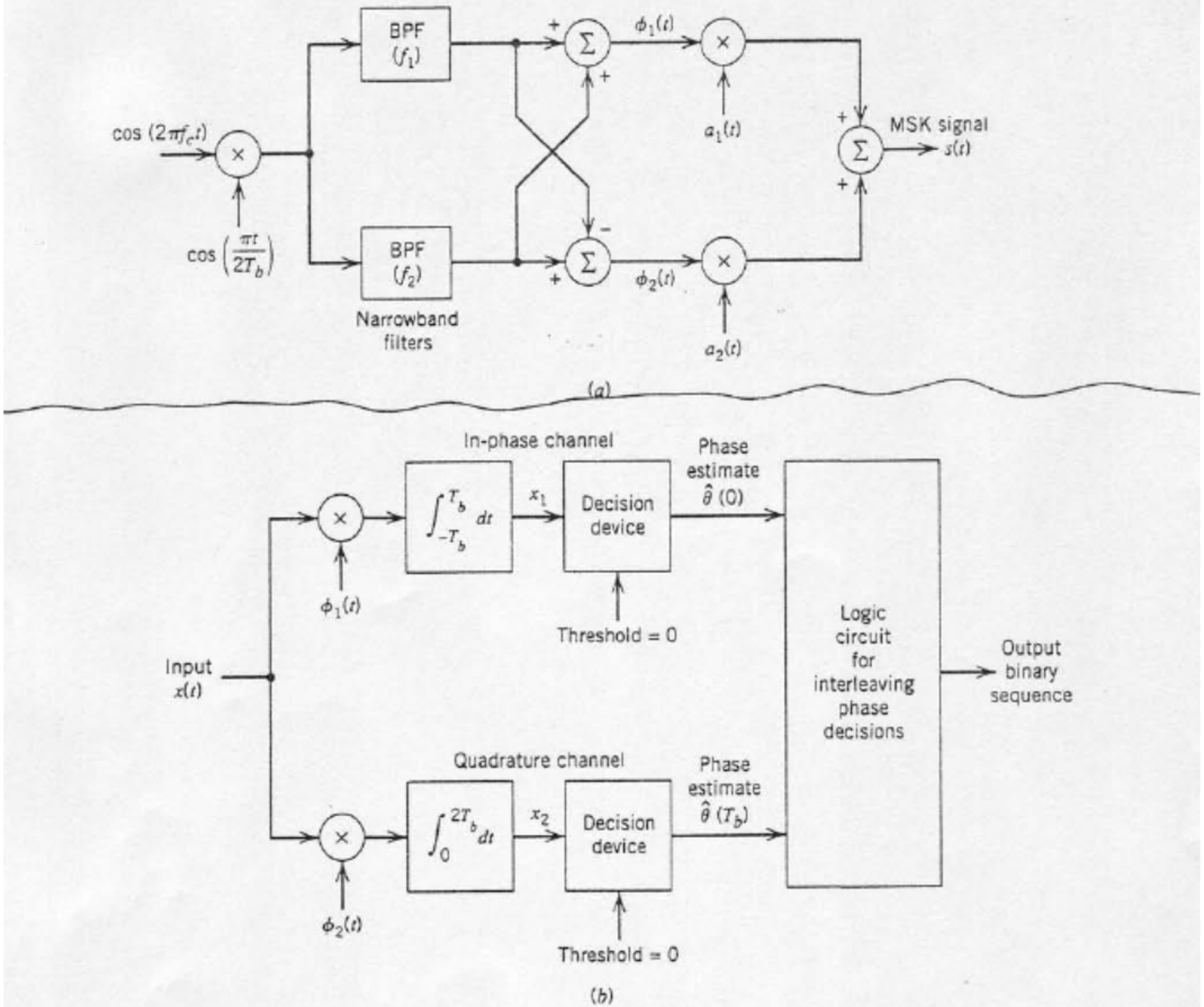


FIGURE 6.31 Block diagrams for (a) MSK transmitter and (b) coherent MSK receiver.

$$R.O.: \max\left[(r, s_i) - \frac{1}{2} \|s_i\|^2 \right]$$

MSK optimal receiver
(working with complex envelopes)

$$\mathrm{Re}\left\{ A \cdot \int_{-\infty}^{+\infty} \tilde{x}(t) \exp\left[-j\pi \sum_k a_k\, q(t - kT) \right] dt \right\} =$$

$z(t)$ $(*)$

Complex envelope of r(t): z(t)

$$Re\left\{A\cdot\int_{-\infty}^{+\infty}\cancel{\tilde{x}(t)}\,\exp\left[-j\pi\sum_k a_k\, q(t-kT)\right]dt\right\}=$$

$z(t)$ $(*)$

$$\varphi(t)=2\pi\cdot\frac{1}{2}\sum(\;)\rightsquigarrow\tilde{s}(t)=A\cdot e^{j\varphi(t)}$$

$$\left(\tilde{s}^*(t)=A\cdot e^{-j(\cdot)}\right)$$

$$\int s_1(t)\cdot s_2(t)\,dt=\frac{1}{2}\,Re\{\cdot\}$$

$$\left\{\int z_1(t)\,z_2^*(t)\,dt\right\}$$

$$\mathrm{Re}\left\{ A \cdot \int_{-\infty}^{+\infty} z(t)\, \exp\left[-j\pi \sum_k a_k\, q\,(t-kT) \right] dt \right\} =$$

$$s^*(t) = A\, e^{-j\varphi(t)}$$

$$\varphi(t) = 2\pi h \cdot \sum_k a_k\, q\,(t-kT)$$

$$\Big\downarrow \text{MSK}$$

$$\varphi(t) = \frac{\pi}{2} \sum_{-\infty}^{k-1} a_n + \frac{\pi}{2} a_k \cdot \frac{t-kT}{T} \quad ; \left(t \in [kT, kT+T] \right)$$

$$\mathrm{Re}\{\cdot\} = \sum_k \int^{kT+T} \mathrm{Re}\,(\cdot)\, dt = \sum_k \mathrm{Re}\left\{ e^{-j\frac{\pi}{2}\sum_{-\infty}^{k-1} a_n} \cdot \right.$$

$$\varphi(t) = \frac{\pi}{2}\sum_{-\infty}^{k-1} a_n + \frac{\pi}{2} a_k \cdot \frac{t-kT}{T} \;; \quad \left(t \in [kT, kT+T]\right)$$

$$\mathrm{Re}\{\cdot\} = \sum_k \int^{kT+T} \mathrm{Re}(\cdot)\, dt = \sum_k \mathrm{Re}\left\{ e^{-j\frac{\pi}{2}\sum_{-\infty}^{k-1} a_n} \cdot \int_{kT}^{kT+T} \left(z(t)\, e^{-j\frac{\pi}{2} a_k (t-kT)/T} \right) dt \right\}$$

$$\left(\int_{-\infty}^{+\infty} (\cdot) = \sum_k \int_{kT}^{(k+1)\cdot T} (\cdot)\, dt \right)$$

$$e^{-j\frac{\pi}{2}\sum_{-\infty}^{k-1} a_n} = \begin{cases} \pm 1 \\ \pm j \end{cases}$$

The associated optimal receiver structure is really very complex …

Quasi never used in practice …

MSK performances (there are many approximations …). N is the number of tr. symbols.

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$$d^2 = \|s_1\|^2 + \|s_2\|^2 - 2(s_1, s_2)$$

$\varphi_1(t)$ $\varphi_2(t)$

$$\left(N \cdot T = \int_0^{NT} 1 \cdot dt \ !!! \right)$$

$$d^2 = N \cdot \frac{A^2}{2} T - 2A^2 \int_0^{NT} \left[\cos\left(2\pi f_0 t + \varphi_1(t)\right) \cdot \cos\left(2\pi f_0 t + \varphi_2(t)\right) \right] dt =$$

$$= A^2 \cdot \int_0^{NT} \left(1 - \cos \Delta\varphi(t)\right) dt = \frac{2E_s}{T} \cdot \int_0^{NT} \left(1 - \cos \Delta\varphi(t)\right) dt$$

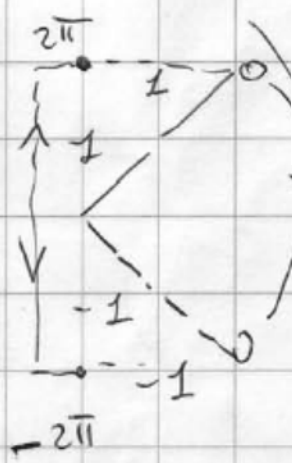
(*)

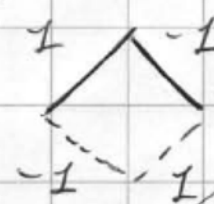
$E_s = E_b \log_2 M = E_b$

Minimum distance sequences differ in two consecutive bits:

1, -1 e -1, 1 oppure

1, 1 e -1, -1





$(=2T)$

$$d^2 = 4E_s = 4E_b = \frac{2E_s}{T}\cdot\int_0^{2T}(\,)\,dt = 4E_s$$

$$P(E) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Partial response CPM: g(t) duration greater than T (usually LT)

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* $g(t)$ ha durata $> T$!!!

$$g(t) \neq 0, \ \forall t \in (0, LT)$$

$\left(2\pi h \, q(\infty)\right)$!!

$$\varphi(t) = \pi h \sum_{n=-\infty}^{k-L} a_n + 2\pi h \cdot \sum_{n=k-L+1}^{k-1} a_n \, q(t-nT) + 2\pi h \, a_k \, q(t-kT) =$$

$$\left[\begin{array}{l} \varphi(t) = 2\pi h \cdot \sum_n a_n \, q(t-nT) \\ q(\infty) = 1/2 \end{array} \right]$$

Many terms of memory …
receiver (optimal) very, very complicated (complexity exponential with L)

RICEVITORI SEMPLIFICATI

*) q(t) approx. in ric. con uno di durata minore ...

↳ si riduce L e quindi la complessità (n° di STATI) del ric.

**) Approx. il segnale come un PAM ...
(simile all'MSK ...)

↳ in teoria: (per mod. BINARIE)
Il CPM è una somma di 2^{L-1} PAM !!!
(- con pot. decrescente all'aumentare dell'ordine del coeff. considerato)

***) Approx. del segnale come una sinusoide nell T (in realtà scelta in un insieme di possibili sinusoidi ...)
(sin. non ortogonali !!!)

Simplified receivers (non optimal):
*) approximation of q(t) at the receiver …
There is a reduction of L, and so of the receiver memory and compexity;

**) The CPM signal is interpreted as a sum of PAM (2^{L-1}), and only the first component are used … (as MSK);

***) The CPM signal is approximated as a sum of sinusoids … and then the receiver is set accordingly …

Tamed Frequency Modulation, A Novel Method to Achieve Spectrum Economy in Digital Transmission

Abstract—This paper describes a new type of frequency modulation, called Tamed Frequency Modulation (TFM), for digital transmission. The desired constraint of a constant envelope signal is combined with a maximum of spectrum economy which is of great importance, particularly in radio channels. The out-of-band radiation is substantially less as compared with other known constant envelope modulation techniques. With synchronous detection, a penalty of only 1 dB in error performance is encountered as compared with four-phase modulation. The idea behind TFM is the proper control of the frequency of the transmitter oscillator, such that the phase of the modulated signal becomes a smooth function of time with correlative properties. Simple and flexible implementation schemes are described.

7.6 La modulazione TFM

Una modulazione CPM con spettro particolarmente compatto, proposta infatti originariamente per le comunicazioni radiomobili, è la *Tamed Frequency Modulation* (TFM), introdotta in modo euristico come variante dell'MSK. Ha traiettorie di fase molto addolcite, che alla fine di ogni simbolo portano la fase ad assumere valori sempre multipli $\pi/4$. La modulazione è binaria, con $h = 1/2$. La trasformata di Fourier della forma d'onda modulante in frequenza, centrata sull'asse dei tempi, è

$$G(f) = \frac{1}{2}\cos^2 \pi f T \frac{\pi f T}{\sin \pi f T} \tag{7.12}$$

per $|f| \leq 1/4T$ e nulla altrove. La forma d'onda modulante in fase $q(t)$ è mostrata in fig. 7.3. La durata è teoricamente infinita, ma in pratica si può ritenere[7] $L = 3$ o 4.

TFM: h=1/2, L=3 or 4 (approx.), g(t); evolution of MSK;
Very soft phase transition (with multiples of pi/4), and so, good power spectrum;
d^2=1.59 (d^2(MSK)=2, d^2(GMSK)=1.79);
h0(t): 98% (GMSK: h0(t): 99.6 %)

q(t) for TFM

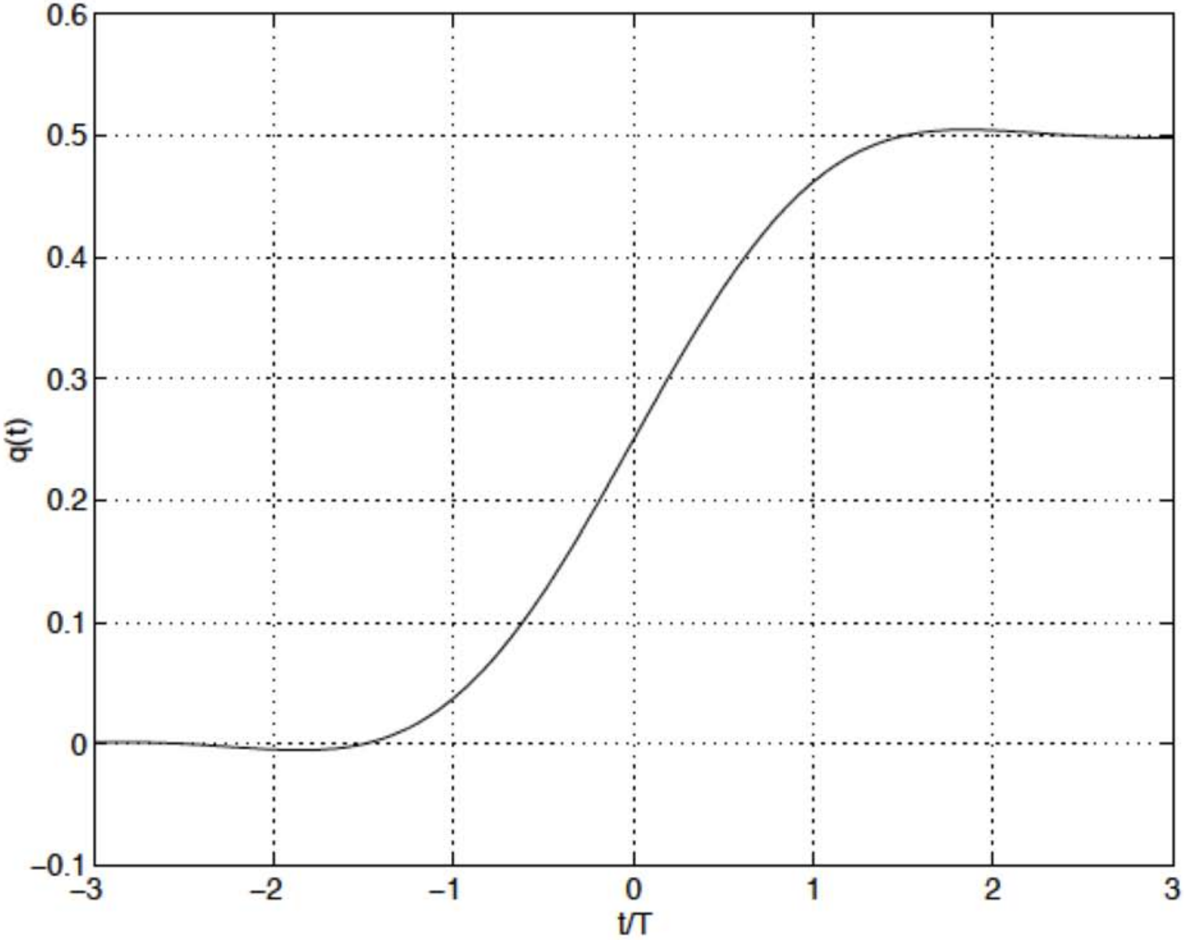


Fig. 7.3 - Forma d'onda modulante in fase (TFM)

TFM phase tree

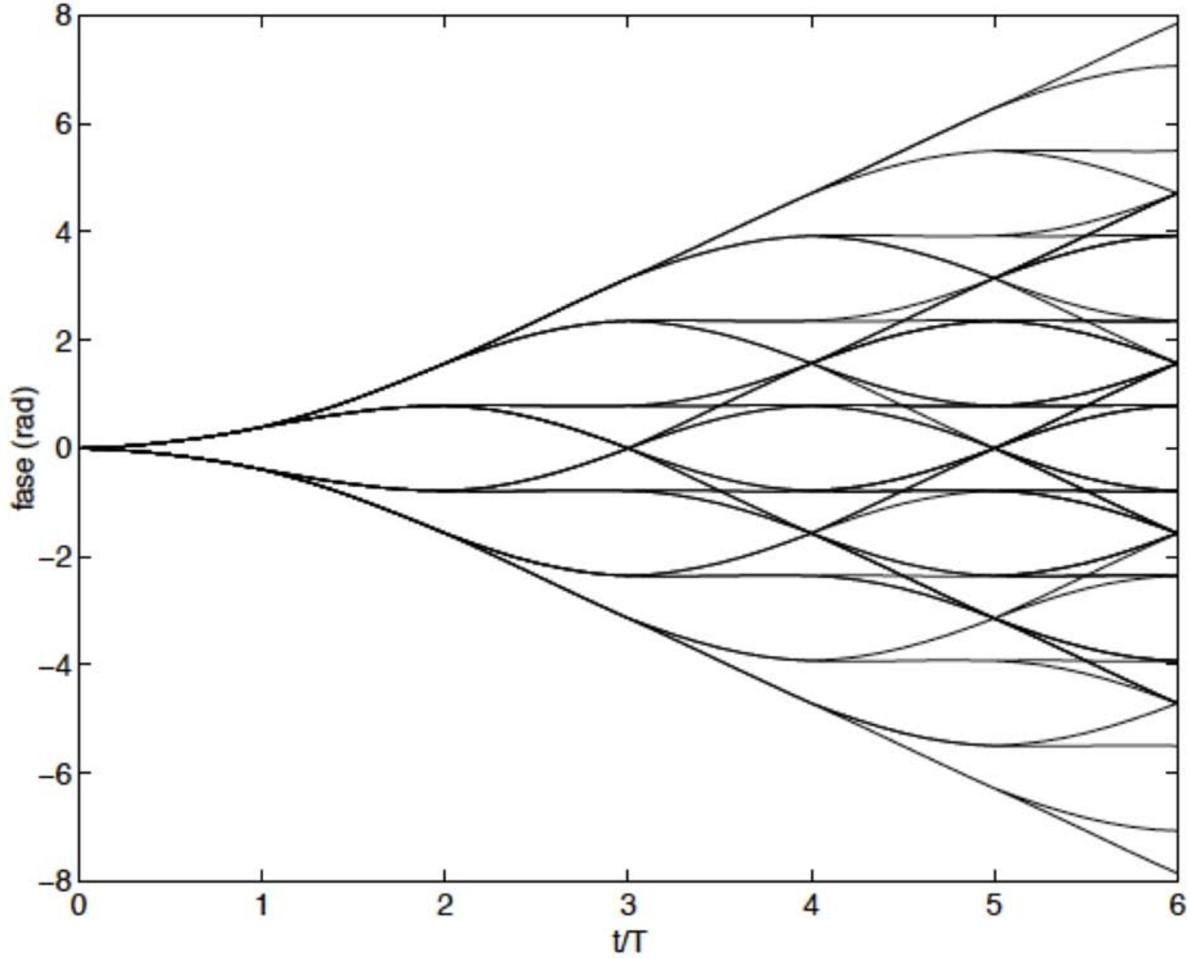


Fig. 7.4 - Albero delle fasi (TFM)

h0(t) for TFM

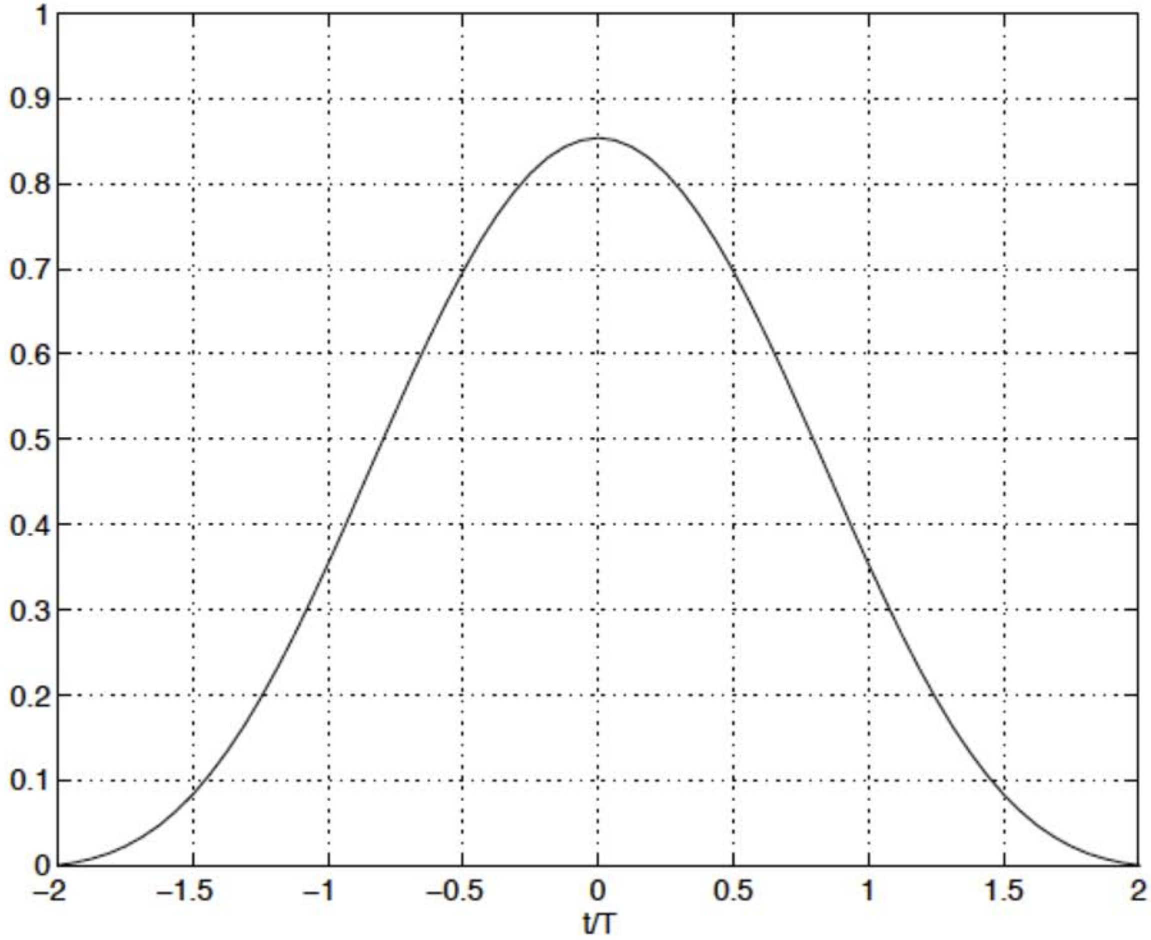


Fig. 7.5 - Forma d'onda elementare $h_0(t)$ (TFM)

CONCLUSION

In this paper we have described a novel and promising type of frequency modulation, named Tamed Frequency Modulation, for digital transmission. A very low out-of-band radiation is obtained as compared with other constant-envelope modulation techniques. In this way the severe constraints of the radio field can be met with the receiving filter shown in Fig. 12 and the PSDF of TFM in Fig. 3. For example, with a radio channel spacing of 25 kHz and a required data rate of 16 kbits/s the power radiated into the adjacent channel can be 85 dB lower than the power radiated into the wanted channel.

The detection quality is almost the same as for four-phase modulation.

Finally, it is shown that the implementation of the TFM transmitter and receiver can be relatively simple.

GMSK Modulation for Digital Mobile Radio Telephony

KAZUAKI MUROTA, MEMBER, IEEE, AND KENKICHI HIRADE, MEMBER, IEEE

Abstract—This paper is concerned with digital modulation for future mobile radio telephone services. First, the specific requirements on the digital modulation for mobile radio use are described. Then, premodulation Gaussian filtered minimum shift keying (GMSK) with coherent detection is proposed as an effective digital modulation for the present purpose, and its fundamental properties are clarified with the aid of machine computation. The constitution of modulator and demodulator is then discussed from the viewpoints of mobile radio applications. The superiority of this modulation is supported by some experimental test results.

GMSK: Gaussian Minimum Shift Keying

$\boxed{\text{GMSK}}$

$h = 1/2$, con impulsi rett-modulanti, addolciti da un filtro gaussiano

Agendo sulla banda del filtro gaussiano si può ottenere un compromesso tra prestazioni (d_{min}) e banda occupata dal segnale modulato

Con $B\,(\text{a } 3\,dB) = \frac{0{,}2}{T} \rightsquigarrow$ circa TFM

Con $B = \infty \rightarrow$ MSK

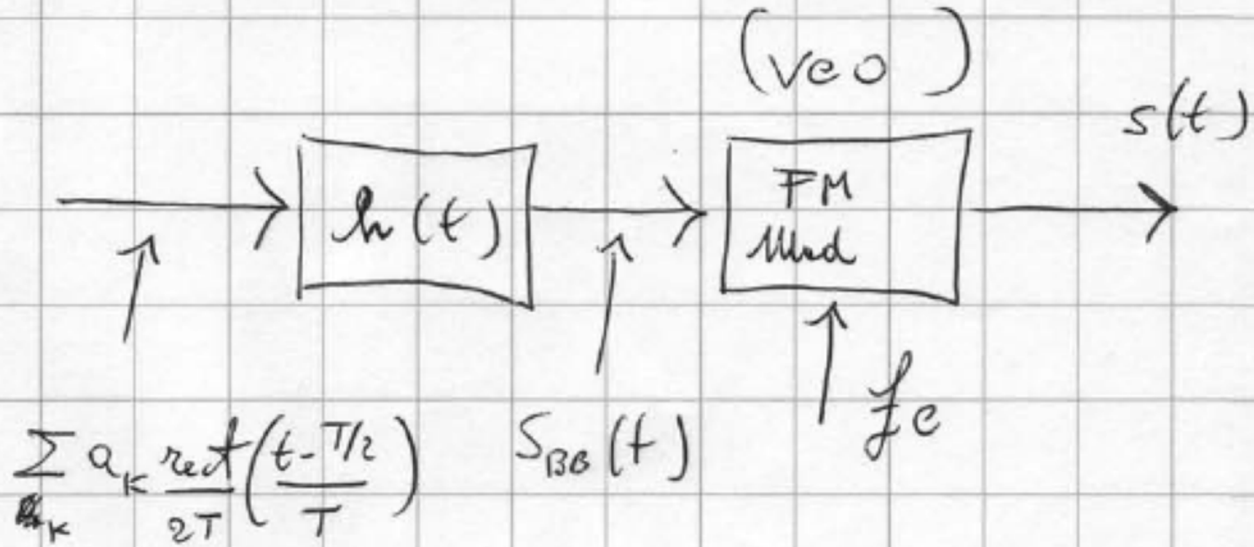
$\ast$ GSM $\rightarrow B = 0{,}3/T$

GMSK:
h=1/2, p=rect(.) filtered by a gaussian signal, with adjustable 3 dB Bandwidt (B).

B=0.2/T quasi TFM.

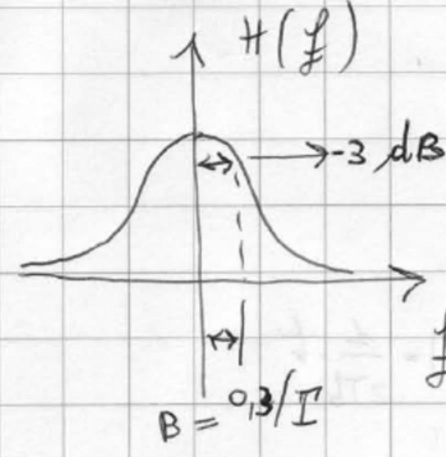
B=infty, MSK.

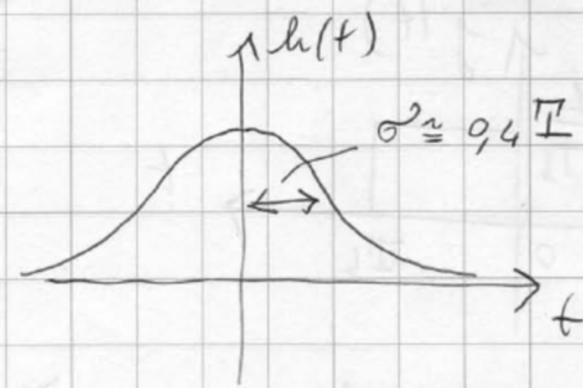
In 3G GSM mobile systems: B=0.3/T



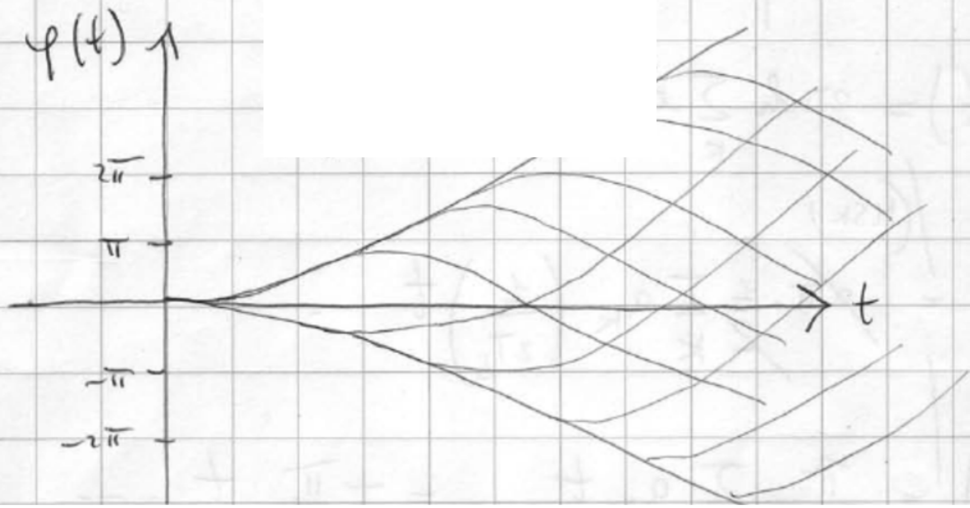
$$H(f) = \exp\left\{ -\frac{f^2}{B^2} \frac{\ln 2}{2} \right\}$$

(Banda a 3 dB del filtro)

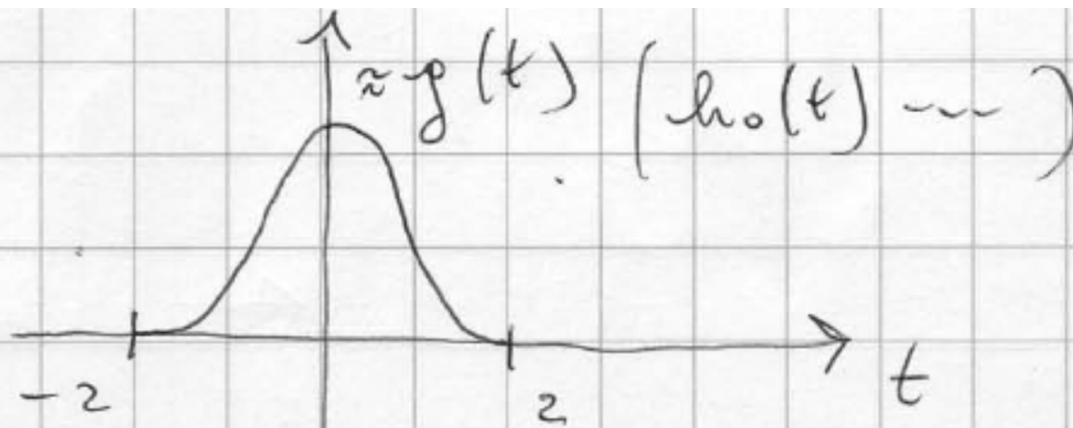




$$* \quad g(t)=\frac{1}{2T}\mathrm{rect}\left(\frac{t-T/2}{T}\right) * h(t)$$



$d^2 = 1{,}79 \cdot (2E_b) \longrightarrow (d^2 = 4E_b$ nel MSK$)$



Receiver Similar to MSK, so, quite simple !!!

h0(t) for GMSK

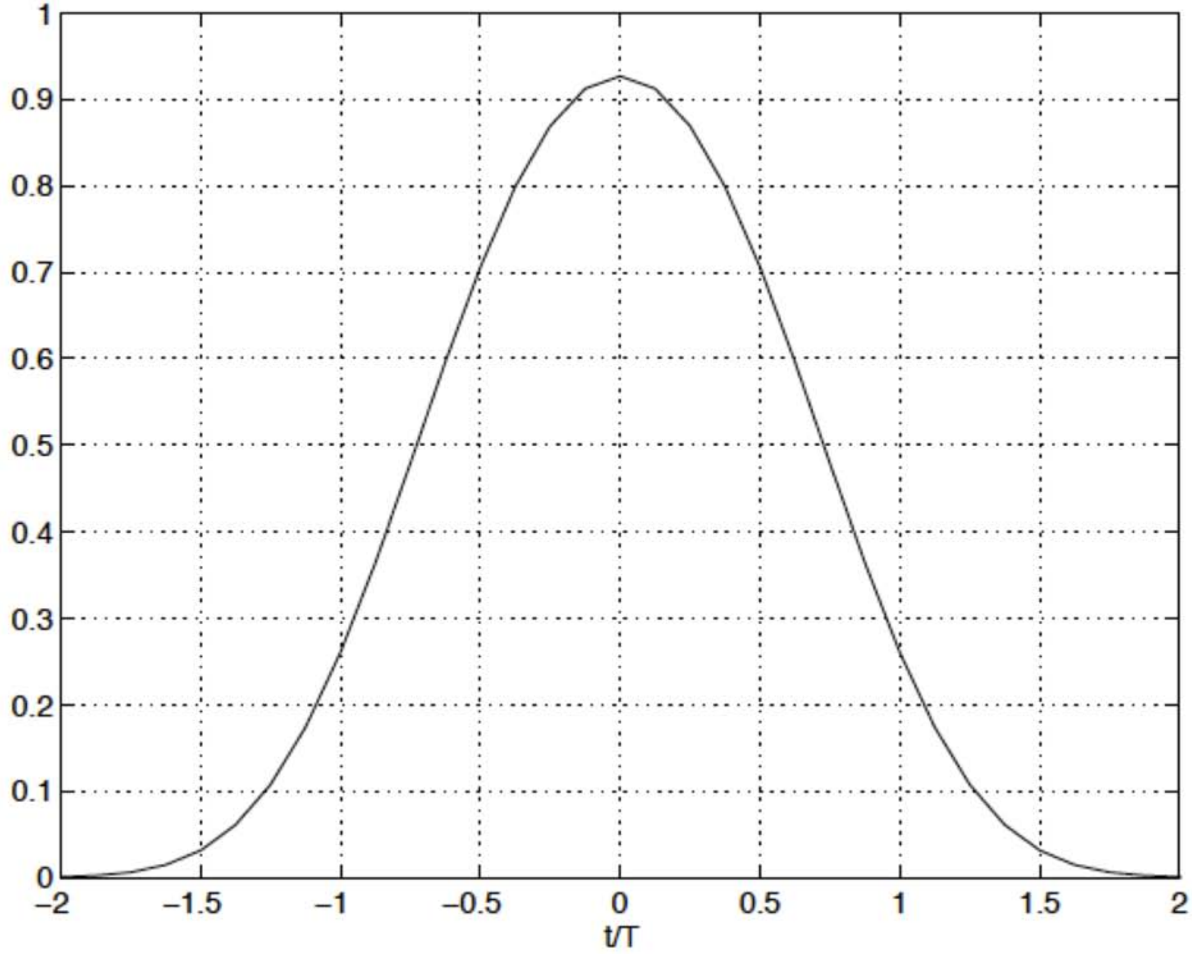


Fig. 7.6 - Forma d'onda elementare $h_0(t)$ (GMSK; $BT = 0.3$)

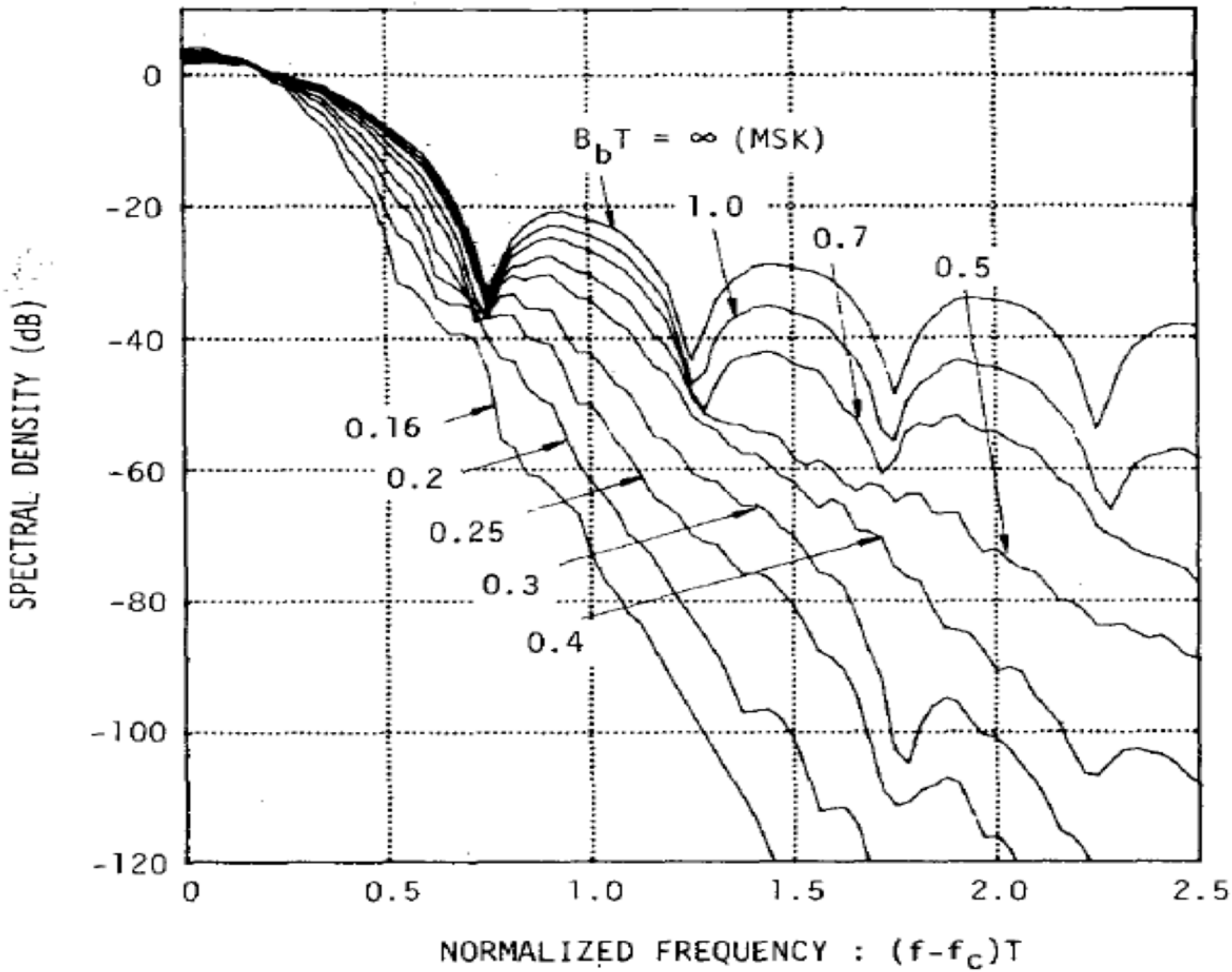


Fig. 2. Power spectra of GMSK.

TFM and GMK power spectrum

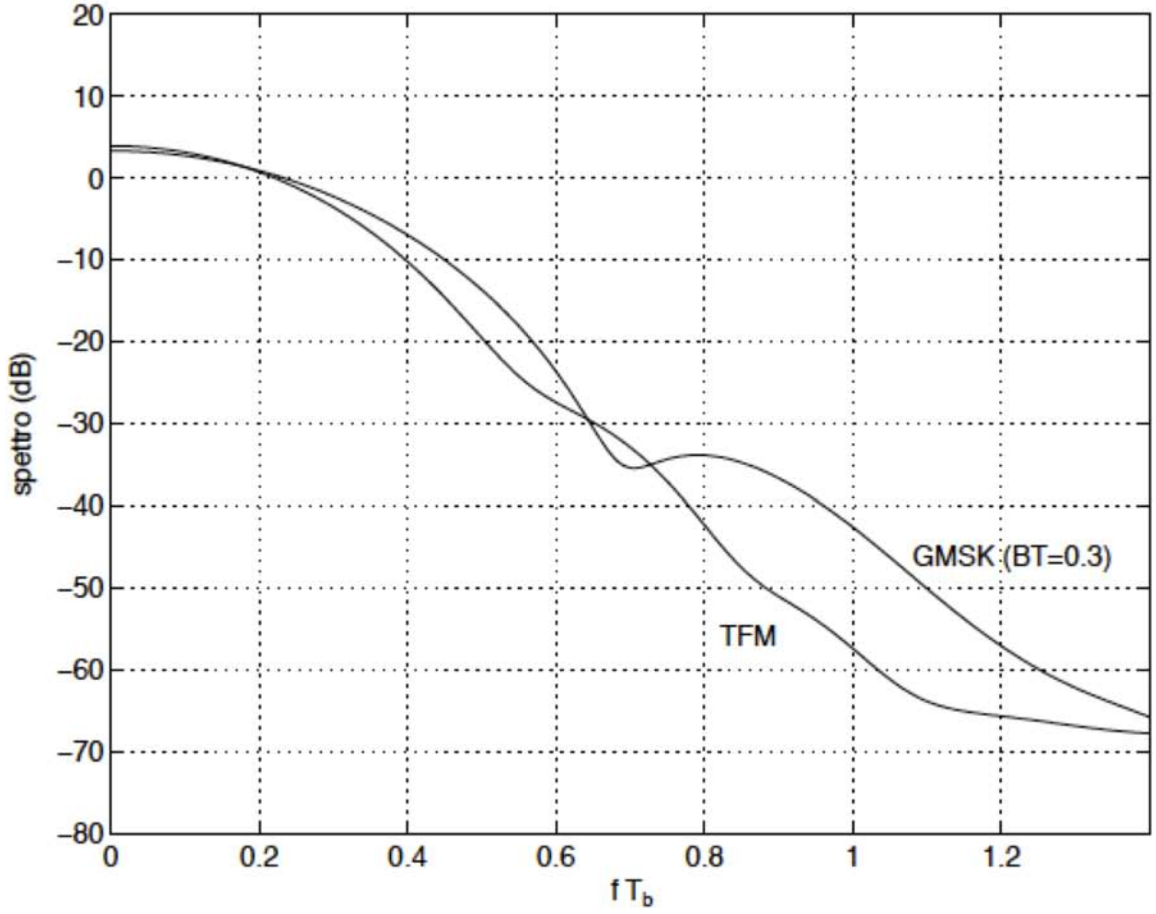


Fig. 7.9 - Spettro della modulazione TFM e GMSK ($BT = 0.3$)

V. CONCLUSION

As an effective digital modulation for mobile radio use, premodulation Gaussian-filtered minimum shift keying (GMSK) modulation with coherent detection has been proposed. The fundamental properties have been analyzed with the aid of machine computation. The constitution of modulator and demodulator has also been discussed. The superiority of this modulation has been supported by experimental results.