MICROWAVE ENGINEERING

Lecture 10:
Transmission
lines problems

1. A transmission line has the following per unit length parameters:

$$L = 0.2 \ \frac{\mu H}{m}$$

$$C = 30 \ \frac{pF}{m}$$

$$R = 5 \ \frac{\Omega}{m}$$

$$G = 0.01 \ \frac{S}{m}$$

Calculate the propagation constant and characteristic impedance of this line for $f = 500$ MHz. Recalculate those quantities in absence of losses.

The propagation constant is:

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\omega = 2\pi f = 2\pi \cdot 500 \cdot 10^6 = 3.14 \cdot 10^9 \ \frac{rad}{s}$$

$$\gamma = \sqrt{\left[5 + j \cdot 3.14 \cdot 10^9 \cdot 0.2 \cdot 10^{-6}\right]\left[0.01 + j\, 3.14 \cdot 10^9 \cdot 30 \cdot 10^{-12}\right]} =$$

$$= \sqrt{\left[5 + j\, 628\right]\left[0.01 + j\, 0.094\right]} = 0.43 + j\, 7.69 \ \frac{np}{m}, \frac{rad}{m}$$

The characteristic impedance of the line are:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = 81 + j\, 3.97 \ \Omega$$

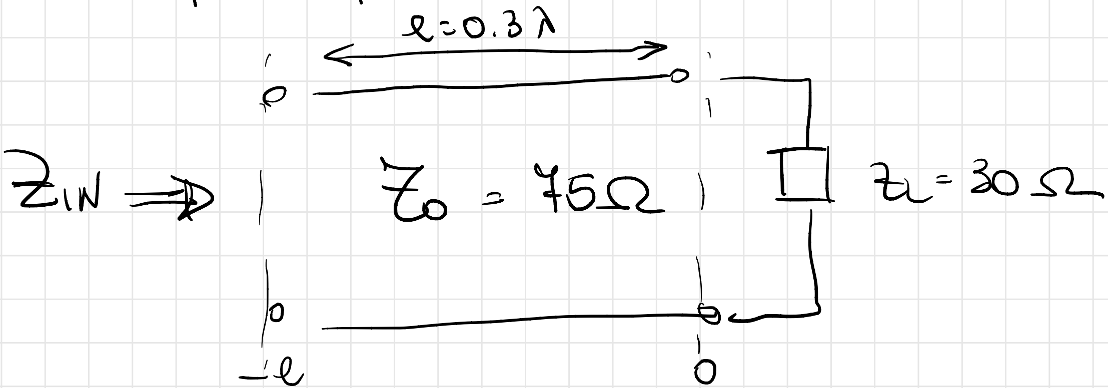
If we suppose losses are absent ($R = G = 0$) then we get:

$$\beta = \omega\sqrt{LC} = 7.69 \ \frac{rad}{m}$$

$$Z_0 = \sqrt{\frac{L}{C}} = 81.65\,\Omega$$

2 A lossless transmission line of electrical length 0.3λ is terminated with a load impedance as in figure. Find:

- the reflection coefficient at the load;
- the SWR on the line;
- the reflection coefficient at the input of the line;
- the input impedance on the line.



The reflection coefficient at the load:

$$\Gamma_L = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30-75}{30+75} = \frac{-45}{105} = -0.43$$

The standing wave ratio is:

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.43}{1-0.43} = 2.5$$

The reflection at the input of the line is:

$$\Gamma_{IN} = \Gamma_L \, e^{-2j\beta \ell} = \qquad \beta = \frac{2\pi}{\lambda} \qquad \ell = 0.3\lambda$$

$$= -0.43 \, e^{-j\,0.6\pi} = 0.13 + j0.4 \qquad \beta\ell = \frac{2\pi}{\cancel{\lambda}} \cdot 0.3\cancel{\lambda} = 0.6\pi$$

The input impedance is:

$$Z_{IN} = Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell} = 75 \frac{30 + j75 \tan(0.6\pi)}{75 + j30 \tan(0.6\pi)} =$$

$$= 124.9 + j\,77 \quad \Omega$$

(3) A lossless transmission line is terminated with 100Ω load. If the SWR = 1.5 find the two possible characteristic impedance of the line.

We know that $$SWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$1+|\Gamma| = SWR - SWR|\Gamma| \Rightarrow |\Gamma| + |\Gamma| SWR = SWR - 1$$

$$\Rightarrow \quad |\Gamma| = \frac{SWR-1}{SWR+1} = \frac{0.5}{2.5} = 0.2$$

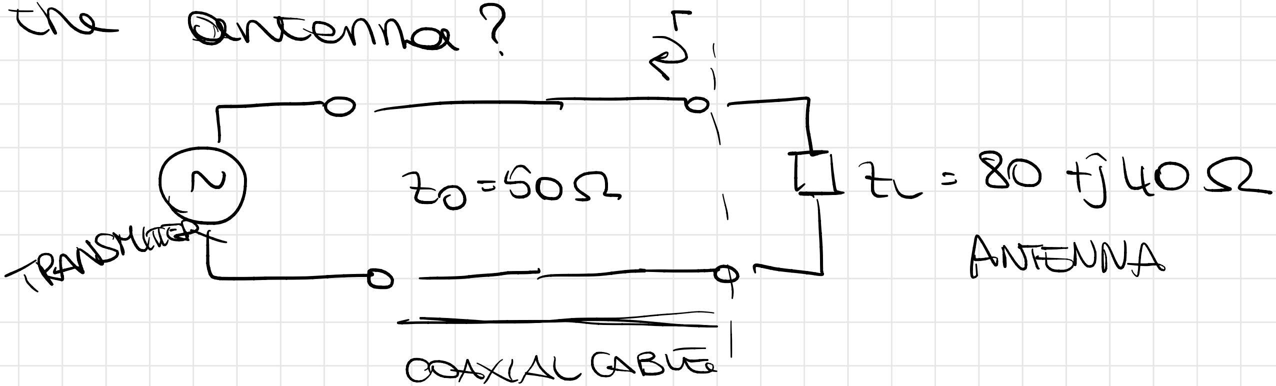
$$\boxed{\Gamma \begin{cases} \Gamma^{+} = 0.2 \\ \Gamma^{-} = -0.2 \end{cases}}$$

We also know that $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$$\begin{cases} \Gamma^+ = \dfrac{Z_L - Z_0}{Z_L + Z_0} = 0.2 \\[2ex] \Gamma^- = \dfrac{Z_L - Z_0}{Z_L + Z_0} = -0.2 \end{cases}$$

$$\begin{cases} Z_0 = Z_L \dfrac{1-\Gamma^+}{1+\Gamma^+} = 100 \dfrac{1-0.2}{1+0.2} = 66.7\,\Omega \\[2ex] Z_0 = Z_L \dfrac{1-\Gamma^-}{1+\Gamma^-} = 100 \dfrac{1+0.2}{1-0.2} = 150\,\Omega \end{cases}$$

(4) A radio transmitter is connected to an antenna having an impedance $Z_L = 80 + j40\,\Omega$ with a $50\,\Omega$ coaxial cable. If the $50\,\Omega$ transmitter can deliver 30 W when connected to a $50\,\Omega$ load, how much power is delivered to the antenna?



$P_{IN} = 30\,W$

The reflection coefficient at the load is:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{80 + j40 - 50}{80 + j40 + 50} = \frac{30 + j40}{130 + j40} =$$

$$= 0.29 + j0.2$$

$$|\Gamma|^2 = 0.135$$

$$P_{LOAD} = \underline{P_{IN}} - P_{REF} = \frac{|V_0^+|^2}{2Z_0} - |\Gamma|^2 \frac{|V_0^+|^2}{2Z_0} =$$

$$= \frac{|V_0^+|^2}{2Z_0}\left(1 - |\Gamma|^2\right) = P_{IN}\left(1 - |\Gamma|^2\right) =$$

$$30\left(1 - 0.135\right) = \underline{25.94\,W}$$