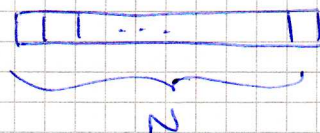


HARD ERROR PROBABILITY



← t = ERROR CORRECTING CAPABILITY

* we have errors if we have $t+1, t+2, \dots$ errors

IF we assume ERRORS ARE INDEPENDENT EVENTS → let's model them as a BINOMIAL DISTRIBUTION

$$E = Q \left(\sqrt{\frac{2E_b}{N_0} \cdot \frac{k}{N} (t+1)} \right) \rightarrow \text{RATE is used as a PENALTY ("wasted" energy)}$$

↑
Hamming distance

$$P_w(E) = \underbrace{\binom{N}{t+1}}_{\text{wrong bits}} E^{t+1} \underbrace{(1-E)^{N-(t+1)}}_{\text{correct bits}} \quad // \quad t+1 \text{ errors}$$

$$P(E) \leq \sum_{h=t+1}^N \binom{N}{h} E^h (1-E)^{N-h} \quad \text{PROB. OF ERROR (WORD)}$$

(union bound)

$$P_b(E) = \left(\frac{2t+1}{N} \right) P_w(E) \quad \text{PROB. OF ERROR (BIT)}$$

↑ min. distance
↓ length

SOFT ERROR PROBABILITY

We need intuition on vector space ...

we can say $Q \left(\sqrt{\frac{2E_b}{N_0} \cdot \frac{k}{N} \cdot d^*} \right) \rightarrow \text{Euclidean distance}$

... very difficult to estimate ...

Let's compare soft-hard decision (using only the 1st term)

$$P_H(E) = Q \left(\sqrt{\frac{2E_b}{N_0} \cdot \frac{k}{N} (t+1)} \right) \quad P_S(E) = Q \left(\sqrt{\frac{2E_b}{N_0} \cdot \frac{k}{N} d^*} \right)$$

factor 2 is very effective since Q is exponential! → $2t+1$

if $d_{\min} \cdot R > 1 \rightarrow$ we have a GAIN
otherwise (< 1) → code isn't efficient

} $d_{\min} \cdot R = \underline{\underline{\text{GAIN}}}$