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Lesson 11
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V First TE_{10} mode

$m=1 \quad n=0$

$$H_z(x,y) = H_0 \cos\frac{\pi}{a}x$$

$$f_{c_{TE_{10}}} = \frac{c}{2\sqrt{\varepsilon_r \mu_r}}\,\frac{1}{a}$$

$$h_z(x,y,z,t) = H_0 \cos\frac{m\pi}{a}x\,\cancel{\cos\frac{n\pi}{b}y}\,\cos(\omega t-\beta z)$$

$$h_x(x,y,z,t) = \frac{-\beta}{k_c^2} H_0 \frac{m\pi}{a}\sin\frac{m\pi}{a}x\,\cancel{\cos\frac{n\pi}{b}y}\,\sin(\omega t-\beta z)$$

$$h_y(x,y,z,t) = \frac{-\beta}{k_c^2} H_0 \frac{n\pi}{b}\cos\frac{m\pi}{a}x\,\sin\frac{n\pi}{b}y\,\sin(\omega t-\beta z) = 0$$

$$e_x(x,y,z,t) = -\frac{\omega\mu}{k_c^2} H_0 \frac{n\pi}{b}\cos\frac{m\pi}{a}x\,\sin\frac{n\pi}{b}y\,\sin(\omega t-\beta z) = 0$$

$$e_y(x,y,z,t) = +\frac{\omega\mu}{k_c^2} H_0 \frac{m\pi}{a}\sin\frac{m\pi}{a}x\,\cos\frac{n\pi}{b}y\,\sin(\omega t-\beta z)$$

$$h_z(x,y,z,t) = H_0\cos\frac{\pi}{a}x\,\cos(\omega t-\beta z)$$

$$h_x(x,y,z,t) = -\frac{\beta}{k_c^2} H_0 \frac{\pi}{a}\sin\frac{\pi}{a}x\,\sin(\omega t-\beta z)$$

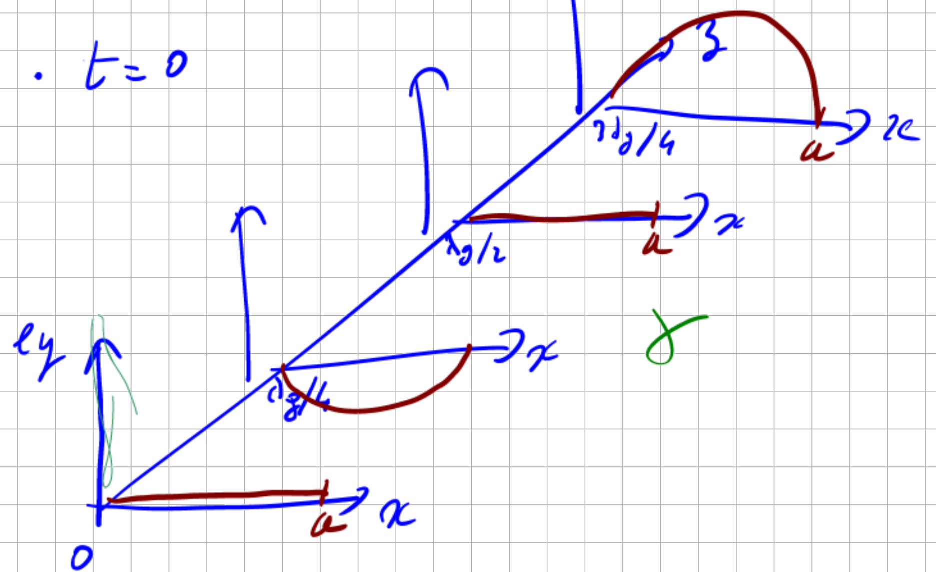
$$h_y(x,y,z,t) = 0$$

$$e_x(x,y,z,t) = 0$$

$$e_y(x,y,z,t) = \frac{\omega\mu}{k_c^2} H_0 \frac{\pi}{a}\sin\frac{\pi}{a}x\,\sin(\omega t-\beta z)$$

$$k_c^2 = \left(\frac{\pi}{a}\right)^2$$

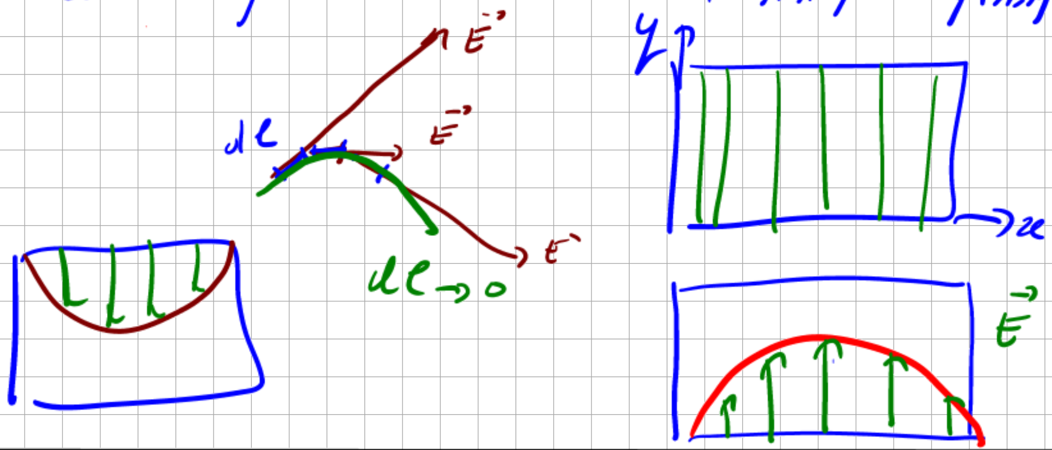
. $t=0$



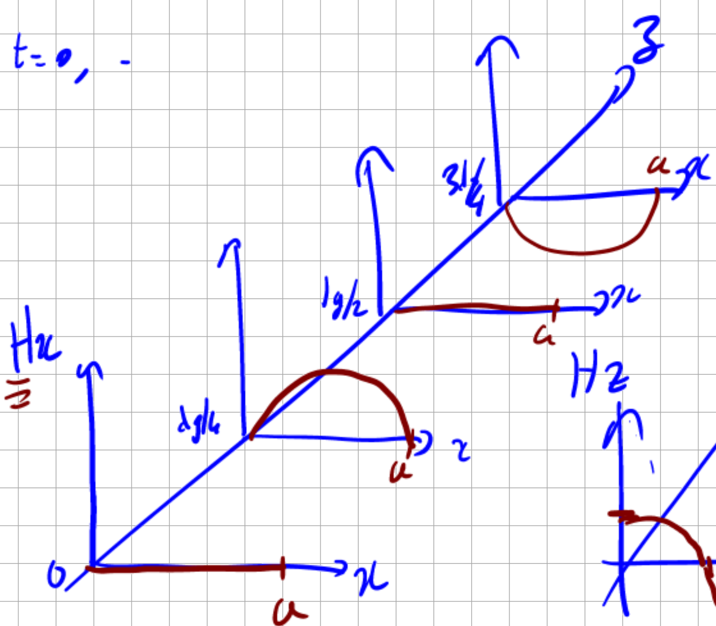
$$z = \frac{\lambda_g}{4} \qquad \beta z = \frac{2\pi}{\lambda_g}\,\frac{\lambda_g}{4} = \frac{\pi}{2}$$

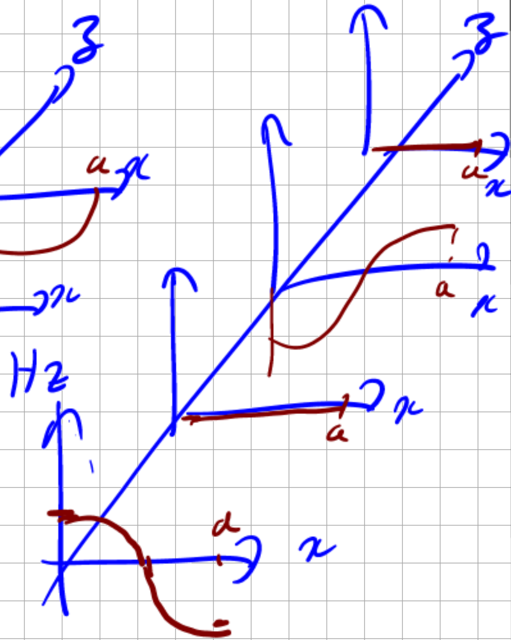
Electric field lines:

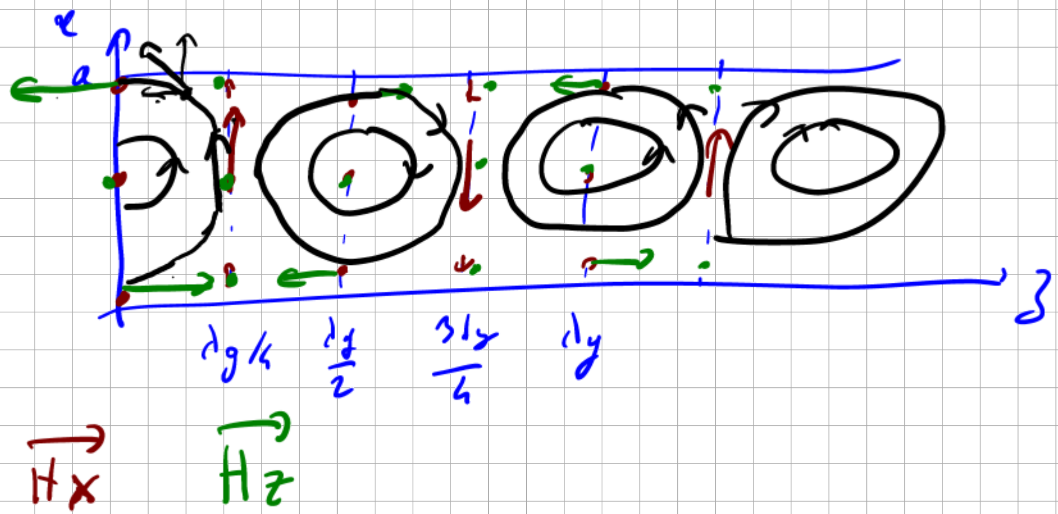
$$\vec{E}(x,y,z,t) = \vec{e}_y(x,y,z,t)$$



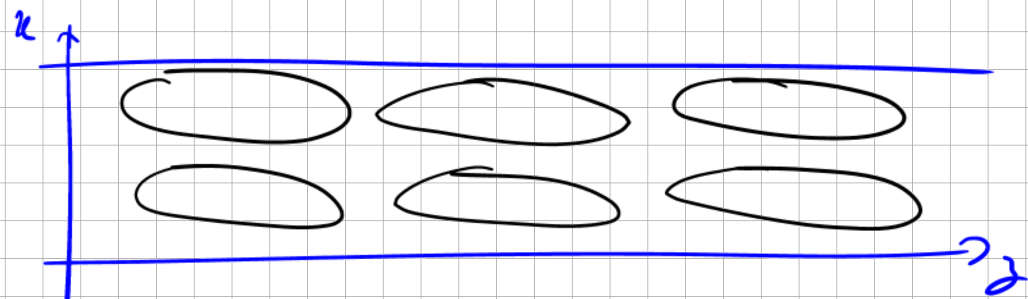
t=0,



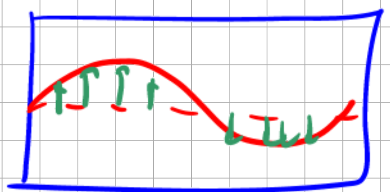




TE 20

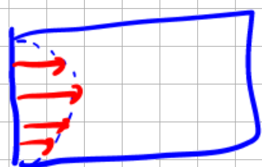


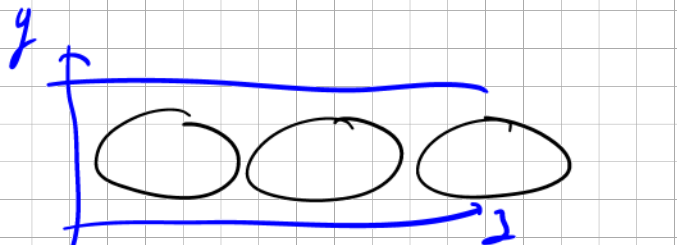
TE 10



Magnetic field lines :

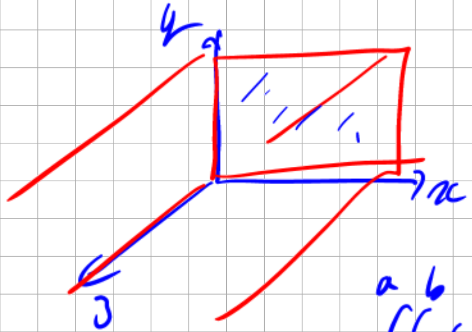
TE 01





VI Power and losses in waveguide

5.1 Transmitted power



no losses

$$\overline{P} = \frac{1}{2}\mathrm{Re}\int_0^a\int_0^b \left(\vec{E}t(x,y) \times \vec{H}t^*(x,y)\right)\cdot\vec{u}\,dx\,dy$$

$$0 \quad f_{cTE_{10}} \quad f_{cTE_{20}} \quad f_1 \quad f_{cTM_{11}} \quad f$$

at f_1 $\quad \vec{E}t(x,y) = a_1\,\vec{E}t_{TE_{10}}(x,y) + a_2\,\vec{E}t_{TE_{20}}(x,y) + a_3\,\vec{E}t_{TM_{11}}(x,y) + \dots$

$$\vec{H}t(x,y) = a_1'\,\vec{H}t_{TE_{10}}(x,y) + a_2' \dots$$

$$\overline{P} = \frac{1}{2}\mathrm{Re}\int_0^a\int_0^b \left(a_1 a_1'^*\,\vec{E}t_{TE_{10}}(x,y) \times \vec{H}t^*_{TE_{10}}\right) + a_1 a_2'^*\,\vec{E}t_{TE_{10}}(x,y) \times \vec{H}t^*_{TE_{20}}(x,y) + \dots$$

Orthogonality of modes.

$$\int_0^a\int_0^b \left(\vec{E}t_{mode\,1} \times \vec{H}t^*_{mode\,2}\right)\cdot\vec{u}\,dS = 0$$

$$\overline{P} = \sum_1 \overline{P}_{mode}$$

at f_1

$$\overline{P}_{TM_{11}} = \frac{1}{2}\mathrm{Re}\int_0^a\int_0^b \left(\vec{E}t(x,y) \times \vec{H}t^*(x,y)\right)\cdot\vec{u}\,dS$$

$\gamma = \alpha \qquad f_1 < f_{TM_{11}}$

$\overline{P}_{TM_{11}} = 0$

because $\vec{E}t = f \dots$

$\vec{H}t = \alpha \dots$

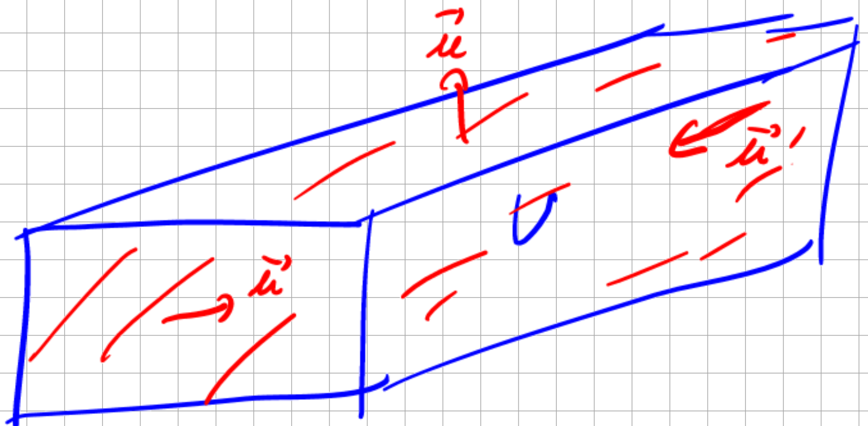
$$\frac{1}{2}\iint_S \vec{E}t \times \vec{H}t^* \cdot \vec{u}\,dS = j\,B.$$

$$\overline{P} = \sum_1^N \overline{P}_n \qquad N \text{ propagative mode}$$

$$\frac{1}{2}\iint_S \left(\vec{E}_t \wedge \vec{H}_t^*\right)\cdot\vec{u}\, dS = 2 j\omega\left(\overline{W_m}-\overline{W_e}\right)$$

$$\overline{W_e} = \frac{1}{4}\varepsilon' \iiint |\vec{E}|^2 dV$$

$$\overline{W_m} = \frac{1}{4}\mu' \iiint |\vec{H}|^2 dV$$



Propagative mode : $2j\omega(\overline{W_m}-\overline{W_e}) = 0$

$\Rightarrow \overline{W_e} = \overline{W_m}$

Evanescent mode ?

$\overline{W_m} \neq \overline{W_e}$

$\overline{W_m} > \overline{W_e}$ inductive effect (L)

$\overline{W_e} > \overline{W_m}$ (C)

5-2- Losses in waveguide

Dielectric losses :

$$P_{diel} = \frac{1}{2}\omega\,\varepsilon'' \iiint_V |\underline{E}_0|^2 dV$$

$$\varepsilon = \varepsilon' - j\varepsilon''$$

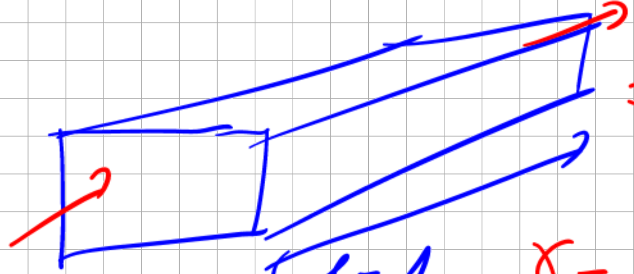
hyp : we consider that for small perturbation (low losses), $\vec{E}$ and $\vec{H}$ are not modified.

metallic losses   finite conductivity σ.

$$\overline{P_{met}} = \frac{1}{2} R_s \iint_{S_{met}} |H|^2 \, dS_{met}$$

$$R_s = \frac{1}{\sigma \delta} \quad , \quad \delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$

$$\overline{P_{tot}} = \overline{P_{diel}} + \overline{P_{met}}$$



$$\overline{P_i} - \overline{P_{tot}} = \overline{P_i} e^{-2\alpha \times 1}$$

$\overline{P_i}$

$l = 1$

$$\gamma = \alpha + j\beta$$

$$k_0^2 = k_c^2 + \beta^2$$

$$\overline{P_i} e^{-2\alpha} \# P_i (1 - 2\alpha) , \qquad \alpha \ll 1$$

$$\overline{P_i}(1 - 2\alpha) = \overline{P_i} - \overline{P_{tot}}$$

$$\alpha = \frac{\overline{P_{tot}}}{2 \overline{P_i}}$$