OPTICAL SOLITONS

A fascinating manifestation of the fiber nonlinearity occurs through optical solitons, formed as a result of the interplay between the dispersive and nonlinear effects. The word soliton refers to special kinds of wave packets that can propagate undistorted over long distances. Solitons have been discovered in many branches of physics (i.e. water waves, plasma physics, accustics, BEC, ... where the wave propagation is described by a NLSE).

NLSE is a universal model in physics.

In the context of optical fibers, not only are solitons of fundamental interest but they have also found practical applications in the field of fiber-optic communications.

Our lecture is devoted to the study of pulse propagation in optical fibers in the regime in which both the GVD and the SPM are equally important and must be considered simultaneously.

To be honest, the history of solitons dates back to 1834, the year in which Scott Russel obseved a heap of water in a canal that propagated undistorted over several kilometers. I want to show a quote from his report published in 1844:

I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped—not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation.

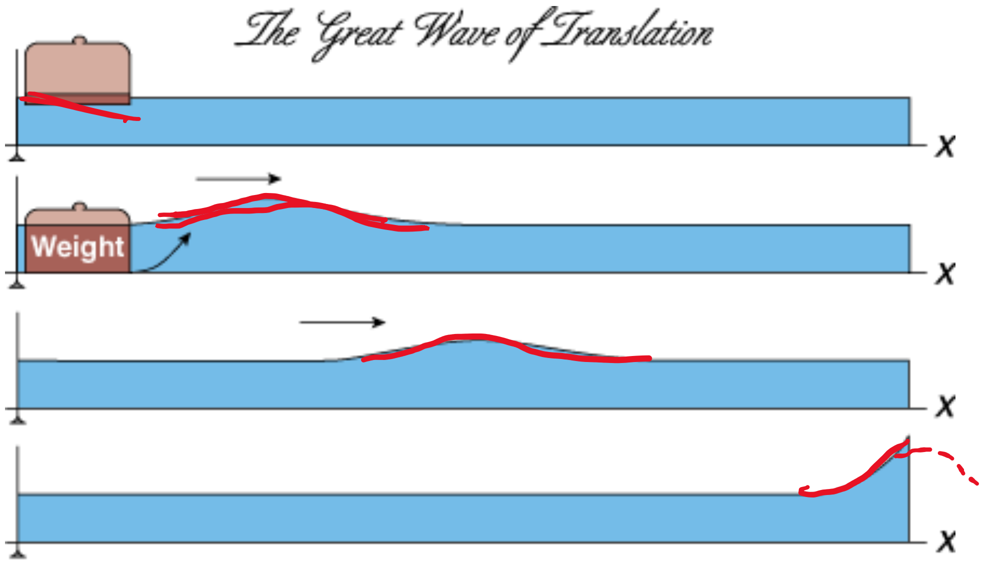


Illustration adapted from a copperplate etching by J. Scott Russell depicting the 30-foot tank he built in his back garden in 1834. [Courtesy Chris Eilbeck, Heriot-Watt University, Edinburgh]

Such waves were later called solitary waves or solitons. The term soliton was coined in 1965 to reflect the particlelike nature of those solitary waves that remained intact even after mutual collisions.

Since then, solitons have been demonstrated and studied in many branches of physics, in particular optics.

In the context of optical fibers, the use of solitons for optical communications was first suggested in 1973, and exploited in real systems by the year 1999.

The starting point is the NLSE

$$i \frac{\partial F}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 F}{\partial t^2} + \gamma |F|^2 F = 0$$

we consider, anomalous dispersion regime $\beta_2 = -1$, $\gamma = 1$. The NLSE takes the standard form:

$$i \frac{\partial F}{\partial z} + \frac{1}{2} \frac{\partial^2 F}{\partial t^2} + |F|^2 F = 0$$

FUNDAMENTAL SOLITONS described by

$$F(z,t) = \eta \, \mathrm{sech}(\eta t) \exp(i \eta^2 z/2)$$

The parameter η determines not only the soliton amplitude but also its temporal width.

The canonical form of the fundamental soliton is obtaine by choosing $\eta = 1$. With this choice, it becomes :

$$\underline{F(z,t) = \operatorname{sech}(t) \exp(i z/2)}$$

One can verify by direct substitution in the NLSE that this expression is indeed a solution of the NLSE.

$$i \frac{\partial F}{\partial z} = i \operatorname{sech}(t) \cdot \frac{i}{2} \exp(i z/2) = -\frac{1}{2} \operatorname{sech}(t) \exp(i z/2)$$

$$+\frac{1}{2}\frac{\partial^2 F}{\partial t^2} = \frac{1}{2}\frac{\partial}{\partial t}\left(\frac{\partial F}{\partial t}\right) = \frac{1}{2}\exp\left(i\,z/2\right)\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}\operatorname{sech}(t)\right) =$$

$$= \frac{1}{2}\exp\left(i\,z/2\right)\left(\operatorname{sech}(t) - 2\operatorname{sech}^3(t)\right)$$

$$+|F|^2 F = \operatorname{sech}^2(t)\cdot\operatorname{sech}(t)\exp\left(i\,z/2\right).$$

$$i\frac{\partial F}{\partial z} + \frac{1}{2}\frac{\partial^2 F}{\partial t^2} + |F|^2 F = 0$$

$$-\frac{1}{2}\cancel{\operatorname{sech}(t)}\,e^{i z/2} + \frac{1}{2}\left[\cancel{\operatorname{sech}(t)} - \cancel{2\operatorname{sech}^3(t)}\right]e^{i z/2} + \cancel{\operatorname{sech}^3(t)}\,e^{i z/2} = 0$$

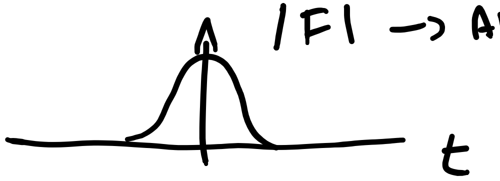
$$0 = 0$$

The soliton expression can be calculated analytically by using the IST (Inverse Scattering Method), or can be obtained by solving the NLSE directly. This is out of the scope of my lectures.

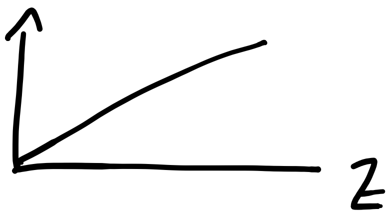
study numerically the dynamics of soliton in amplitude and phase

$|F| \to$ AMPLITUDE

sech(t)



$\measuredangle F$



$e^{i z/2}$ — PHASE