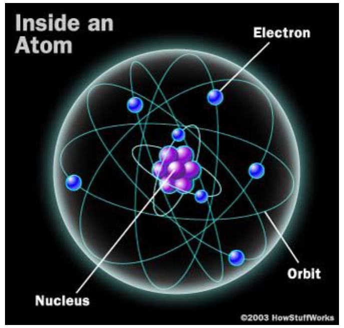
Introduction to lasers

Pr A. Desfarges-Berthelemot – Limoges University

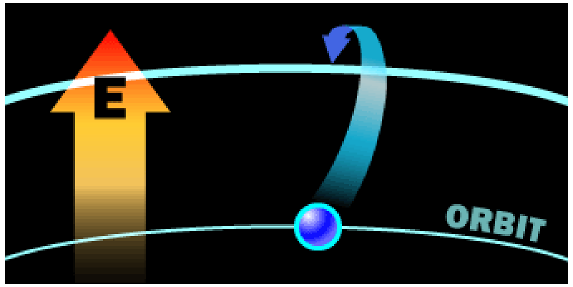
Chapter 2: Amplifier gain

I - Population inversion

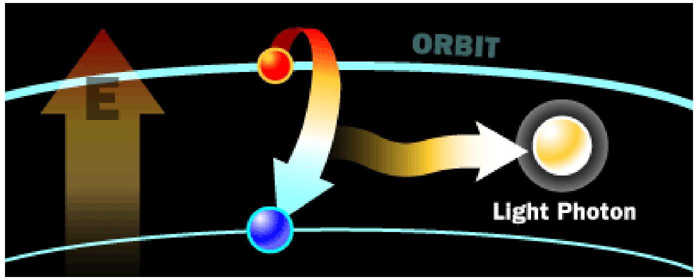
1. Generalities



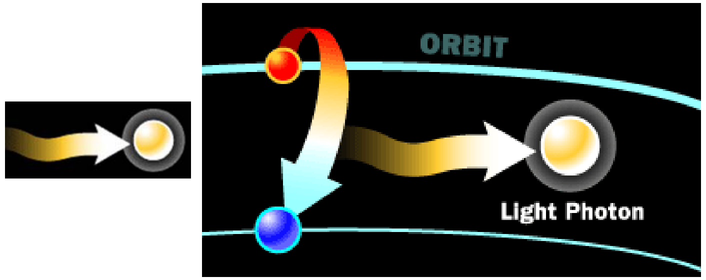
3 types of photon-atom interaction



Absorption

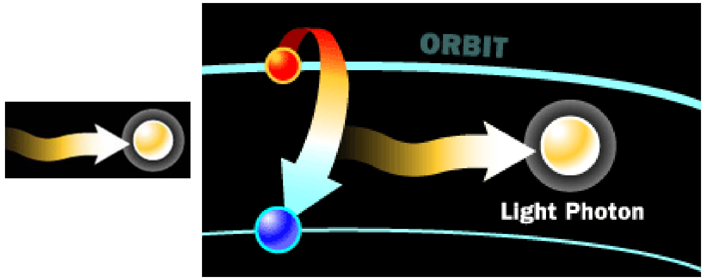


Spontaneous emission



Stimulated emission

→ **GAIN / AMPLIFICATION**



Stimulated emission

➡ **GAIN / AMPLIFICATION**

The emitted photon has the same:

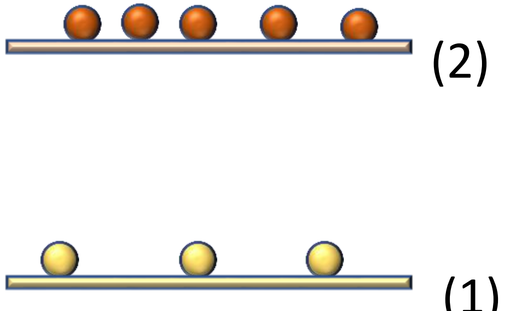
- frequency
- phase
- polarization
- direction of propagation

that the incident photon

Requirements :

- $E_{\text{incident photon}} = E_{\text{transition}}$
- population inversion ΔN :

Number of atoms/volume unit in the excited state > Number of atoms/volume unit in the fundamental state

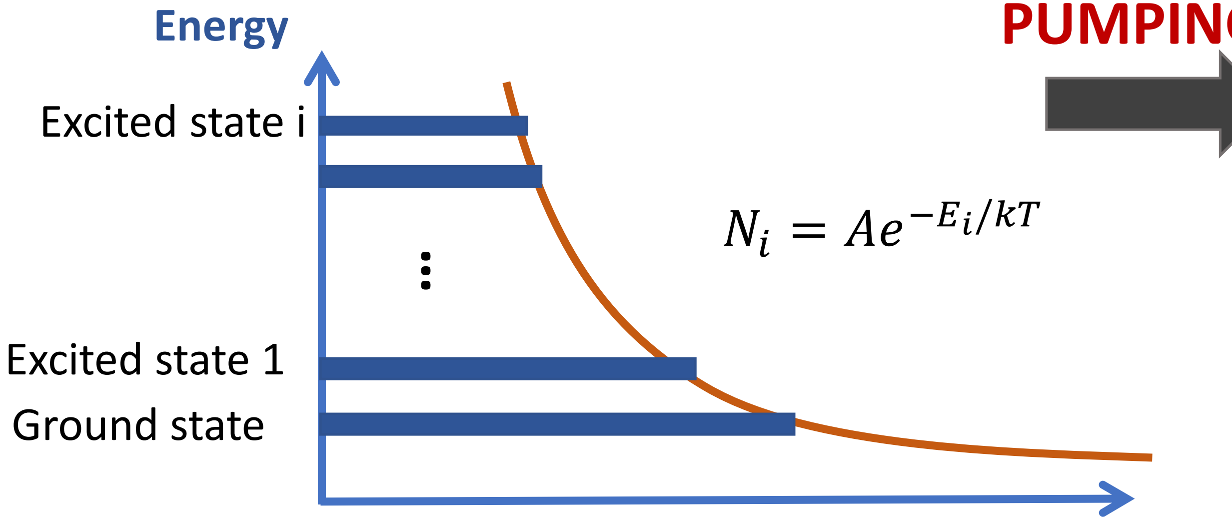


Laser transition

$\Delta N = N2 - N1 > 0$

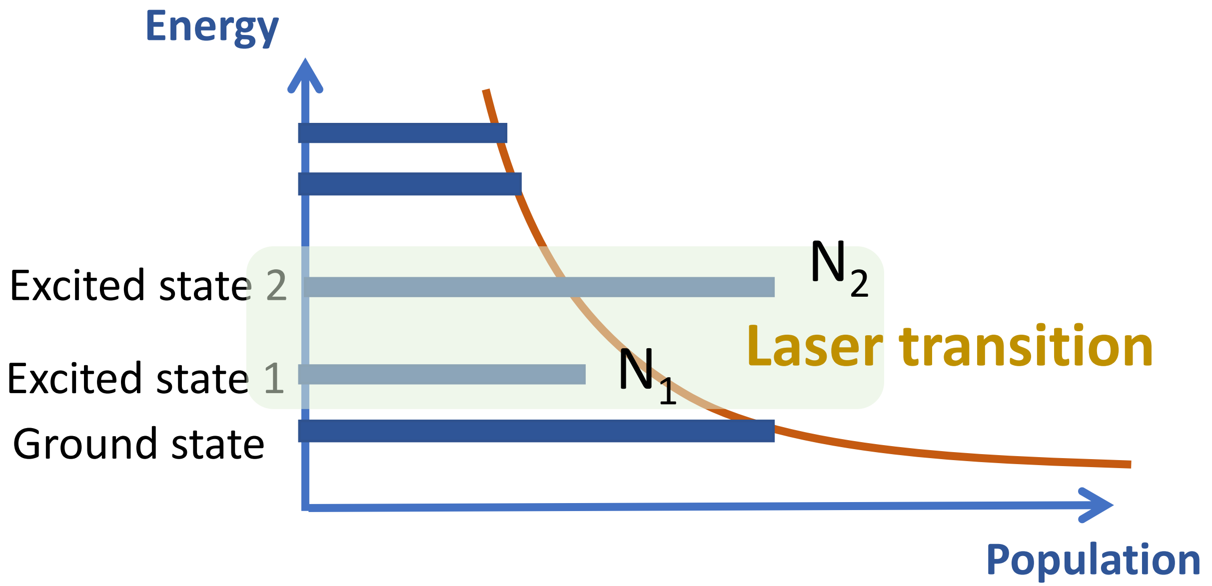
If ΔN > 0 : amplifying medium
If ΔN < 0 : absorbent medium
If ΔN = 0 : transparent medium

Steady state: populations governed by Boltzmann statistics



$$N_i = Ae^{-E_i/kT}$$

PUMPING



ΔN = N1 – N2 > 0 : Amplifying medium

ΔN<0→ Absorbent medium

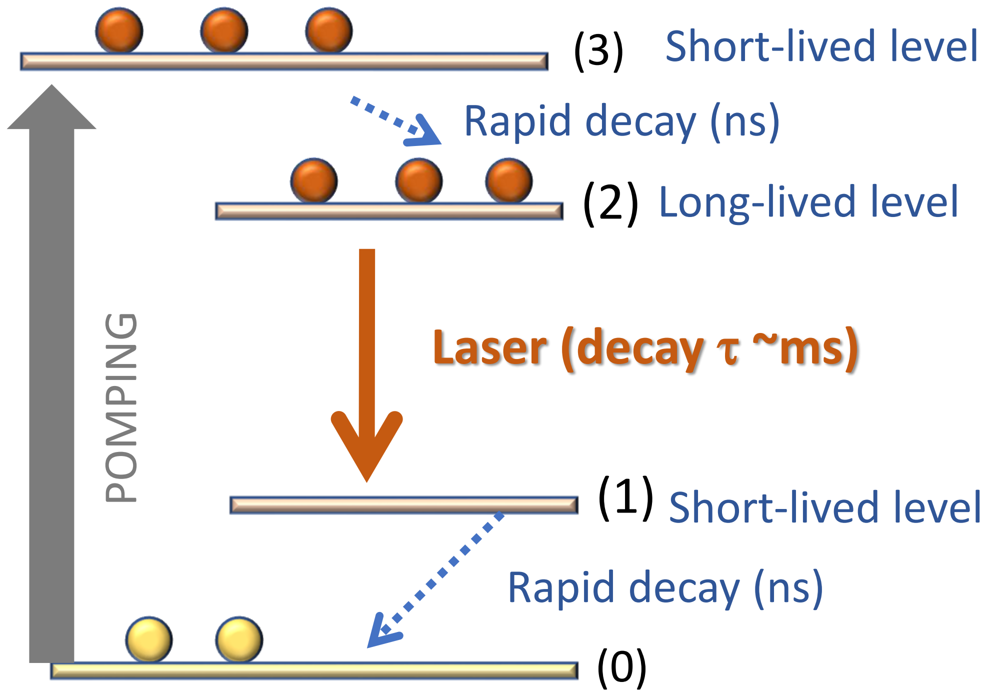
- **W = probability density (s^{-1}) that an unexcited atom absorb one single photon**
- **W = $\sigma(\nu).\phi$**

$\sigma(\nu)$: transition cross section at the frequency ν i.e. transition probability between two energy levels

ϕ: photon-flux density (photons /cm^2.s) = I/hν and $I(z) = \frac{\varepsilon_0 c}{2}.|E(z)|^2$

W: probability density of both stimulated emission and absorption

2. Four-level pumping scheme



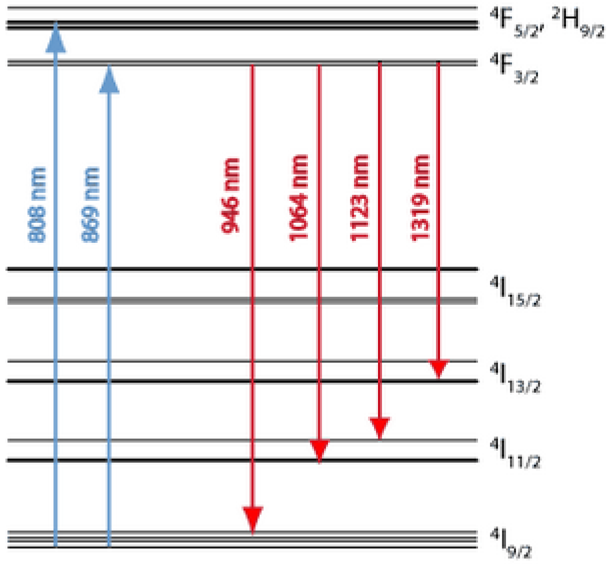


Figure 1: Energy level structure and common pump and laser transitions of the trivalent neodymium ion in Nd^{3+}:YAG.

https://www.rp-photonics.com/yag_lasers.html

Narrow bandwidth: 120GHz, 0.4nm
→ without tunability

$$\boxed{\Delta N_O = \frac{\tau N_a W_P}{(1+\tau W_P)}}$$

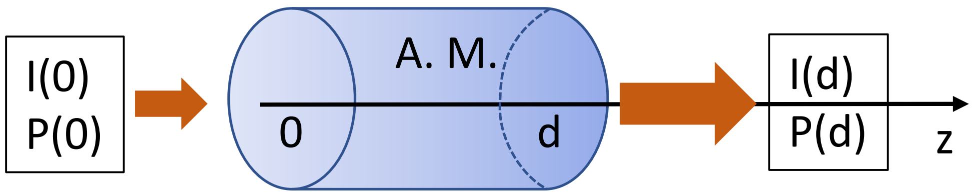
- Population inversion (without signal)
- Na = N_1+N_2: Total number of atoms per volume unit
- W_P: pumping rate (s^{-1}), transition probability between levels (0) and (3)

II - Small-signal gain

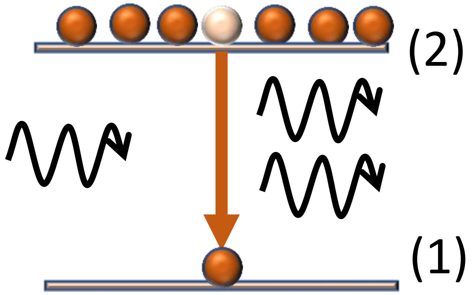
→ Link between gain and population inversion

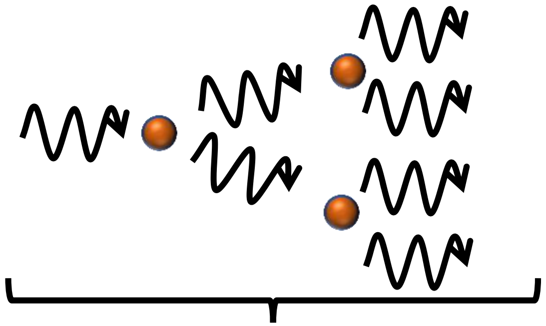
Concept of gain

A monochromatic beam of frequency $\nu_L = \frac{E_1 - E_2}{h}$ is illuminating an amplifying medium under pumping and population inversion



$$\boldsymbol{G_0 = \frac{P(d)}{P(0)}}$$

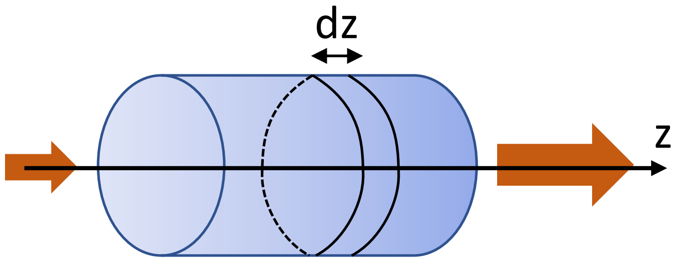




Gain

The intensity I(z) increases as the length of propagation inside the amplifying medium increases

Let us consider an incremental cylinder of length dz and unit area



$\phi(z)$: photon flux density **entering** the cylinder

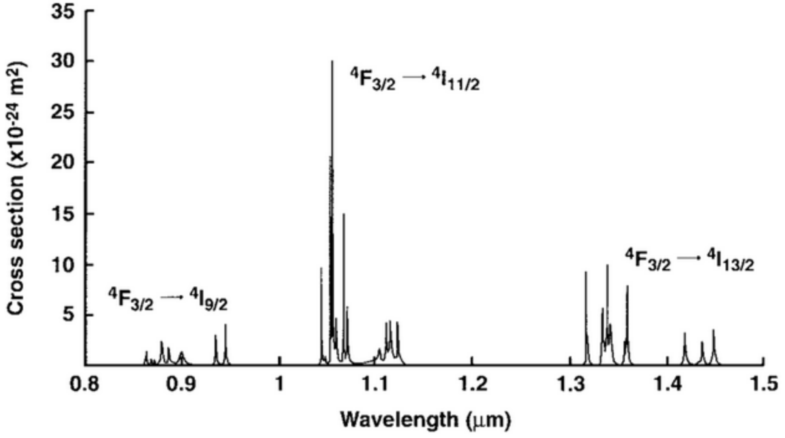
$\phi(z+dz) = \phi(z) + d\phi$: photon flux density **exiting** the cylinder

To complete

Comment:

γ_0 is a function of the frequency ν: $\gamma_0(\nu) = \Delta N_0 \sigma(\nu)$

To complete



Emission cross section of a Nd/YAG crystal

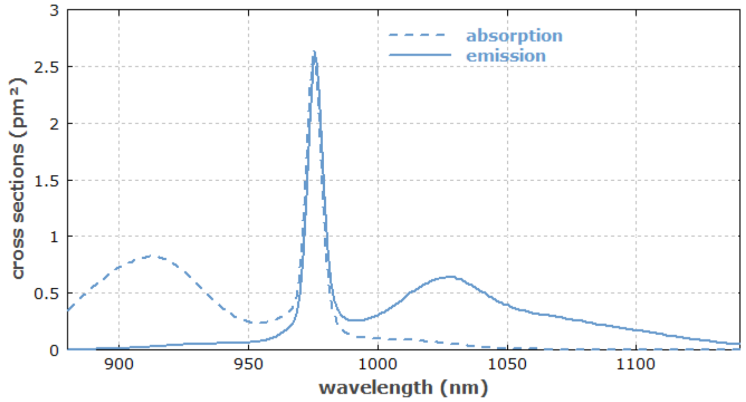


Figure 1: Effective absorption and emission cross sections of ytterbium-doped germanosilicate glass, as used in the cores of ytterbium-doped fibers. (Data from spectroscopic measurements by R. Paschotta)

https://www.rp-photonics.com/transition_cross_sections.html

Gain of amplification: To complete

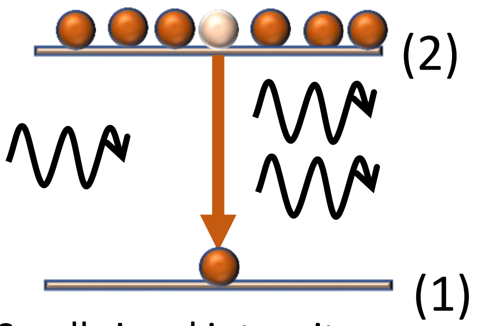
$$G_0 = \frac{P(d)}{P(0)} = e^{\gamma_0 d} \; with \; \gamma_0 = \sigma.\Delta N_0$$

III - Gain saturation

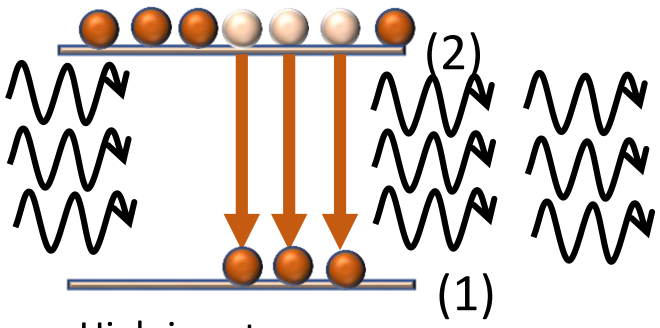
To complete

$$\Delta N(P) = \frac{\Delta N_0}{1 + P/P_{sat}}$$

P: signal power
Psat: saturation power



Small signal intensity
Unmodified population inversion



High input power
Reduced population inversion