CHAPTER 3
Modal theory
in cylindrical step index fiber

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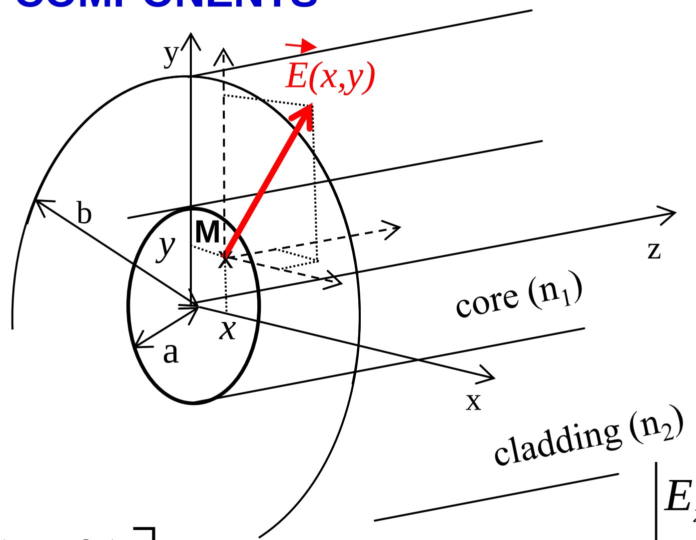
CALCULATION OF THE FIELDS COMPONENTS



$$\vec{\mathcal{E}}(x,y,z) = \Re e\left[\vec{E}(x,y).e^{j(\omega t-\beta z)}\right]$$

with $$\vec{E}(x,y) = \left| \begin{array}{l} E_x(x,y).\overrightarrow{ex} \\ E_y(x,y).\overrightarrow{ey} \\ E_z(x,y).\overrightarrow{ez} \end{array} \right.$$

→ we develop Maxwell equations *(pdf page 1)* :

$$curl\vec{\mathcal{E}} = -\frac{\partial \vec{\mathcal{B}}}{\partial t} \qquad curl\vec{\mathcal{H}} = \varepsilon\frac{\partial \vec{\mathcal{E}}}{\partial t} \qquad div(\vec{\mathcal{D}}) = div(\varepsilon\vec{\mathcal{E}}) = \rho = 0$$

with $\frac{\partial}{\partial z}(\text{component}) = -\text{j}\beta.(\text{component})$ and $\frac{\partial}{\partial t}(\text{component}) = \text{j}\omega.(\text{component})$

CALCULATION OF THE FIELDS COMPONENTS

$$\left|\begin{array}{ll} \dfrac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu H_x & (3) \\ -j\beta E_x - \dfrac{\partial E_z}{\partial x} = -j\omega\mu H_y & (4) \\ \dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = -j\omega\mu H_z & (5) \end{array}\right.$$

$$\left|\begin{array}{ll} \dfrac{\partial H_z}{\partial y} + j\beta H_y = +j\omega\varepsilon E_x & (6) \\ -j\beta H_x - \dfrac{\partial H_z}{\partial x} = +j\omega\varepsilon E_y & (7) \\ \dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = +j\omega\varepsilon E_z & (8) \end{array}\right.$$

CALCULATION OF THE FIELDS COMPONENTS

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$$\left|\begin{array}{ll} \dfrac{\partial H_z}{\partial y} + j\beta H_y = +j\omega\varepsilon E_x & (6) \\ -j\beta H_x - \dfrac{\partial H_z}{\partial x} = +j\omega\varepsilon E_y & (7) \\ \dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = +j\omega\varepsilon E_z & (8) \end{array}\right.$$

→ we write the transverse components Ex, Ey, Hx, et Hy versus the axial components Ez et Hz *(pdf page 2)* :

$$E_x = \frac{-j}{\beta_t^2}\left[\beta\frac{\partial E_z}{\partial x} + \omega\mu\frac{\partial H_z}{\partial y}\right]$$

$$E_y = \frac{-j}{\beta_t^2}\left[\beta\frac{\partial E_z}{\partial y} - \omega\mu\frac{\partial H_z}{\partial x}\right]$$

$$H_x = \frac{-j}{\beta_t^2}\left[\beta\frac{\partial H_z}{\partial x} - \omega\varepsilon\frac{\partial E_z}{\partial y}\right]$$

$$H_y = \frac{-j}{\beta_t^2}\left[\beta\frac{\partial H_z}{\partial y} + \omega\varepsilon\frac{\partial E_z}{\partial x}\right]$$

avec

$$\beta_t^2 = k_0^2 n_i^2 - \beta^2$$

et

$$k_0 n_2 \le \beta \le k_0 n_1$$

CALCULATION OF THE FIELDS COMPONENTS

→ **we can deduce, from the previous expressions, an equation which unknown factor is Ez**
→ **Helmoltz equation *(pdf page 3)* :**

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \beta_t^2 E_z = 0$$

(idem with Hz)

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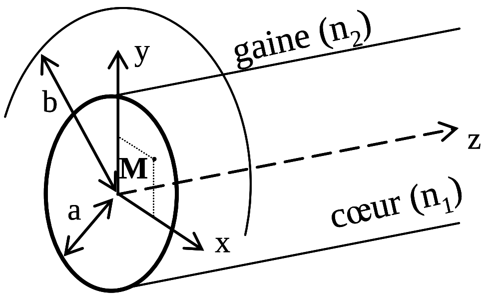
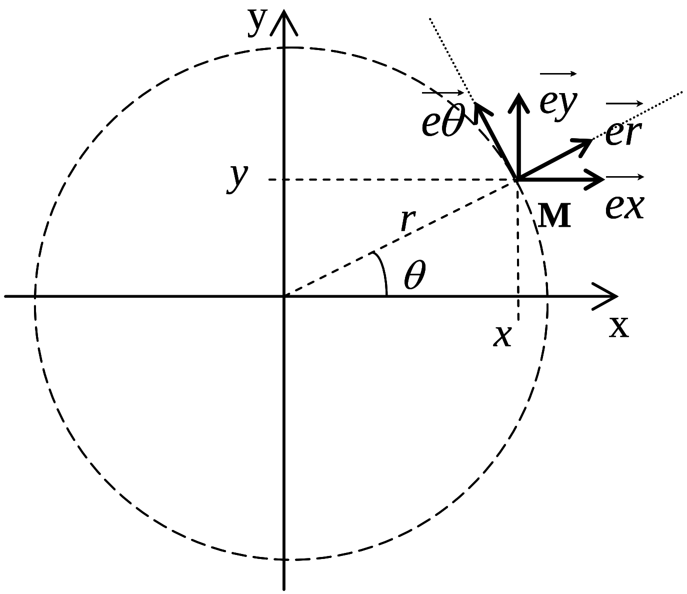
(idem with Hz)

→ **fibre = cylindrical guide ==> change of coordinate system and et change of basis *(pdf page 3)* :**

$$(\overrightarrow{e_x}, \overrightarrow{e_y}, \overrightarrow{e_z}) \rightarrow (\overrightarrow{e_r}, \overrightarrow{e_\theta}, \overrightarrow{e_z})$$





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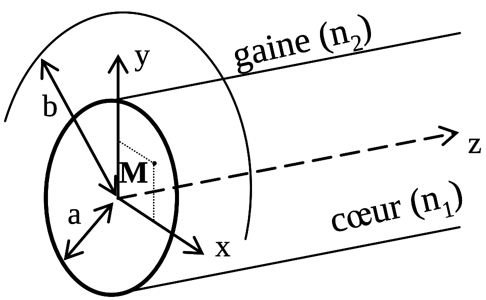

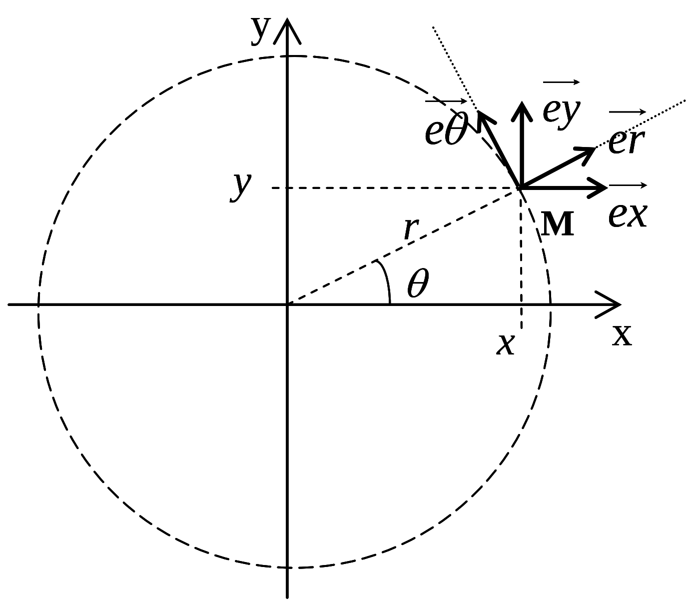

$$\vec{E}(x,y) = \left| \begin{array}{l} E_x(x,y).\overrightarrow{ex} \\ E_y(x,y).\overrightarrow{ey} \\ E_z(x,y).\overrightarrow{ez} \end{array} \right.$$

$$\vec{E}(r,\theta) = \left| \begin{array}{l} E_r(r,\theta).\overrightarrow{er} \\ E_\theta(r,\theta).\overrightarrow{e\theta} \\ E_z(r,\theta).\overrightarrow{ez} \end{array} \right.$$

CALCULATION OF THE FIELDS COMPONENTS

→ **after some calculations, we find *(pdf page 4)* :**

$$E_r = -\frac{j}{\beta_t^2}\left(\beta\frac{\partial E_z}{\partial r} + \frac{\omega\mu}{r}\frac{\partial H_z}{\partial \theta}\right) \qquad H_r = -\frac{j}{\beta_t^2}\left(\beta\frac{\partial H_z}{\partial r} - \frac{\omega\varepsilon}{r}\frac{\partial E_z}{\partial \theta}\right)$$

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→ **and then we can deduce the Helmoltz equation *(pdf page 5)* :**

$$\boxed{\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \theta^2} + \beta_t^2 E_z = 0}$$

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→ **this equation can be solved by the method of separation of variables *(pdf page 5)* :**

$$E_z(r,\theta) = R_z(r).T_z(\theta)$$

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$$\underbrace{\frac{r^2}{R}\frac{\partial^2 R}{\partial r^2} + \frac{r}{R}\frac{\partial R}{\partial r} + r^2\beta_t^2}_{\mathbf{f(r)}} + \underbrace{\frac{1}{T}\frac{\partial^2 T}{\partial \theta^2}}_{\mathbf{g(\theta)}} = 0$$

CALCULATION OF THE FIELDS COMPONENTS

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$$E_z(r,\theta) = R_z(r).T_z(\theta)$$

$$\underbrace{\frac{r^2}{R}\frac{\partial^2 R}{\partial r^2} + \frac{r}{R}\frac{\partial R}{\partial r} + r^2\beta_t^2}_{f(r)=+\nu^2} + \underbrace{\frac{1}{T}\frac{\partial^2 T}{\partial \theta^2}}_{g(\theta)=-\nu^2} = 0$$

f(r)= + ν^2 **g(θ)= - ν^2**

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CALCULATION OF THE FIELDS COMPONENTS

$\boxed{g(\theta) = -\nu^2}$ ➔ $$\frac{1}{T}\frac{\partial^2 T}{\partial \theta^2} = -\nu^2$$ ➔ $$\frac{\partial^2 T}{\partial \theta^2} + \nu^2 T = 0$$

(pdf page 6)

if $\nu \neq 0$

$$\boxed{T(\theta) = \begin{cases} \cos(\nu\theta + \varphi_0) \\ \sin(\nu\theta + \varphi_0) \end{cases}}$$

ν integer

if $\nu = 0$

$$\boxed{T(\theta) = \text{constant}}$$

CALCULATION OF THE FIELDS COMPONENTS

$\boxed{g(\theta) = -\nu^2}$ ➔ $$\frac{1}{T}\frac{\partial^2 T}{\partial \theta^2} = -\nu^2 \quad ➔ \quad \frac{\partial^2 T}{\partial \theta^2} + \nu^2 T = 0$$ ***(pdf page 6)***

if $\nu \neq 0$ $$\boxed{T(\theta) = \begin{cases} \cos(\nu\theta + \varphi_0) \\ \sin(\nu\theta + \varphi_0) \end{cases}}$$ **ν integer**

if $\nu = 0$ $$\boxed{T(\theta) = \text{constant}}$$

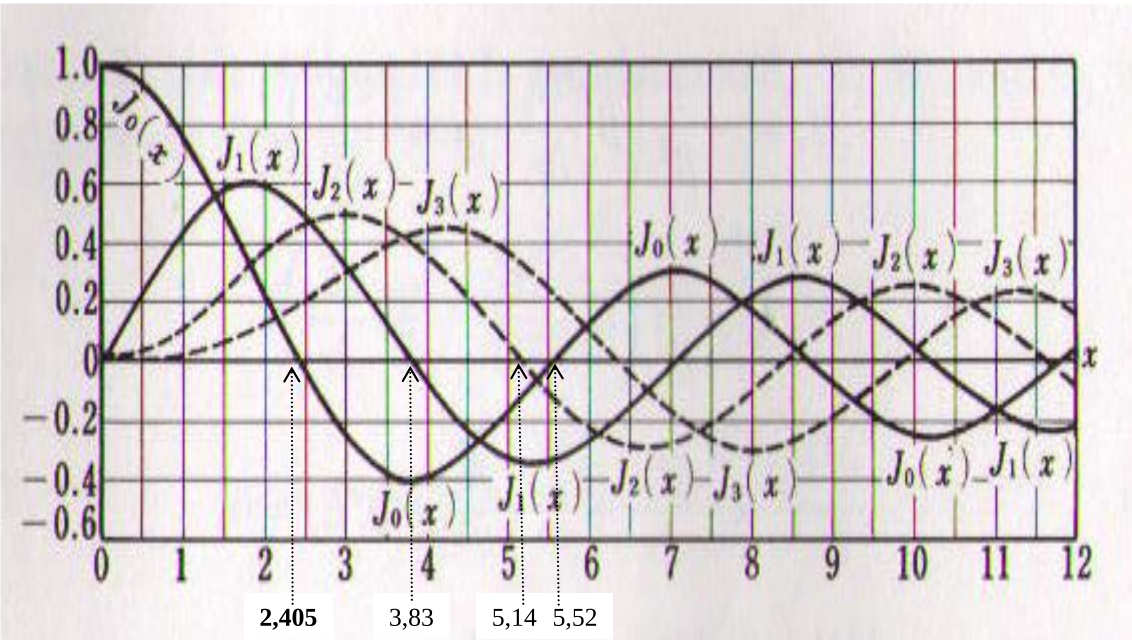
$\boxed{f(r) = +\nu^2}$ ➔ $$\frac{\partial^2 R}{\partial r^2} + \frac{1}{r}\frac{\partial R}{\partial r} + \left(\beta_t^2 - \frac{\nu^2}{r^2}\right) R = 0$$ **Bessel equation**

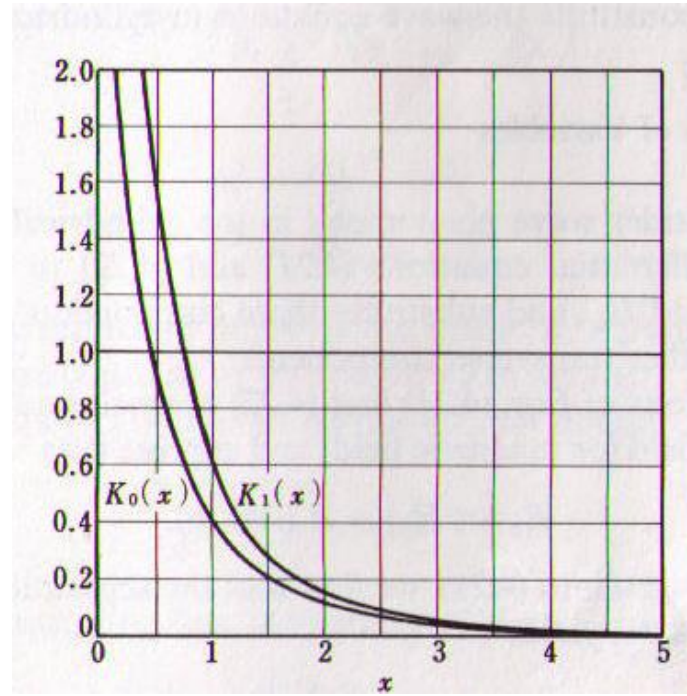
$$R(r) = \begin{cases} AJ_\nu(\beta_t r) + A' N_\nu(\beta_t r) & \text{si } \beta_t \text{ is real} \quad --> \text{in the core} \\ CK_\nu(|\beta_t| r) + C' I_\nu(|\beta_t| r) & \text{si } \beta_t \text{ is imaginary} --> \text{i the cladding} \end{cases}$$

→ **taking into account the values of the functions for r = 0 and r = ∞ :**

$$\boxed{R(r) = \begin{cases} AJ_\nu(\beta_t r) & \text{if } \beta_t \text{ is real} \quad --> \text{in the core} \\ CK_\nu(|\beta_t| r) & \text{if } \beta_t \text{ is imaginary} --> \text{in the cladding} \end{cases}}$$

CALCULATION OF THE FIELDS COMPONENTS





First orders ν of the Bessel functions of first kind J_ν (ν = 0, 1, 2, 3)

First orders ν of the modified Bessel functions of first kind K_ν (ν = 0, 1)

CALCULATION OF THE FIELDS COMPONENTS

→ ***One can specify the expression of the axial components of the fields, E_z and H_z (pdf page 8)***

$$\begin{cases} E_z = AJ_\nu(\beta_{t1} r).\sin(\nu\theta) \quad --> \text{in the core} \\ \quad = CK_\nu(|\beta_{t2}| r).\sin(\nu\theta) --> \text{in the cladding} \end{cases}$$

$$\begin{cases} H_z = BJ_\nu(\beta_{t1} r).\cos\nu\theta \quad --> \text{in the core} \\ \quad = DK_\nu(|\beta_{t2}| r).\cos\nu\theta --> \text{in the cladding} \end{cases}$$

if $\nu = 0$: **modes TE (Ez=0) or TM (Hz=0)**

if $\nu \neq 0$: **modes EH if *Hz > Ez***
modes HE ifi *Ez > Hz* .

→ ***We can now introduce the important following quantities: u,w and V (pdf page 9)***

normalized transverse propagation constants

In the core (radius = a) : $k = k_0 n_1$ **thus** $\beta_t = \beta_{t1} = \sqrt{k_0^2 n_1^2 - \beta^2}$ → $u = a\beta_{t1} = a\sqrt{k_0^2 n_1^2 - \beta^2}$

In the cladding : $k = k_0 n_2$ **thus** $|\beta_t| = |\beta_{t2}| = \sqrt{\beta^2 - k_0^2 n_2^2}$ → $w = a|\beta_{t2}| = a\sqrt{\beta^2 - k_0^2 n_2^2}$

$$u^2 + w^2 = a^2 k_0^2 (n_1^2 - n_2^2) = \left(\frac{2\pi}{\lambda_0} a\sqrt{n_1^2 - n_2^2}\right)^2 = V^2 \rightarrow \boxed{V = \frac{2\pi}{\lambda_0} a\sqrt{n_1^2 - n_2^2}}$$

V : normalized spatial frequency

CALCULATION OF THE FIELDS COMPONENTS

→ ***Finally (pdf page 10)***

if $\nu \neq 0$

In the core

$$E_z = AJ_\nu\left(\frac{ur}{a}\right)\sin(\nu\theta)$$

$$E_r = \left[-A\frac{j\beta}{(u/a)}J_\nu'\left(\frac{ur}{a}\right) + B\frac{j\omega\mu_0}{(u/a)^2}\frac{\nu}{r}J_\nu\left(\frac{ur}{a}\right)\right]\sin(\nu\theta)$$

$$E_\theta = \left[-A\frac{j\beta}{(u/a)^2}\frac{\nu}{r}J_\nu\left(\frac{ur}{a}\right) + B\frac{j\omega\mu_0}{(u/a)}J_\nu'\left(\frac{ur}{a}\right)\right]\cos(\nu\theta)$$

$$H_z = BJ_\nu\left(\frac{ur}{a}\right)\cos(\nu\theta)$$

$$H_r = \left[A\frac{j\omega\varepsilon_1}{(u/a)^2}\frac{\nu}{r}J_\nu\left(\frac{ur}{a}\right) - B\frac{j\beta}{(u/a)}J_\nu'\left(\frac{ur}{a}\right)\right]\cos(\nu\theta)$$

$$H_\theta = \left[-A\frac{j\omega\varepsilon_1}{(u/a)}J_\nu'\left(\frac{ur}{a}\right) + B\frac{j\beta}{(u/a)^2}\frac{\nu}{r}J_\nu\left(\frac{ur}{a}\right)\right]\sin(\nu\theta)$$

In the cladding

$$E_z = CK_\nu\left(\frac{wr}{a}\right)\sin(\nu\theta)$$

$$E_r = \left[C\frac{j\beta}{(w/a)}K_\nu'\left(\frac{wr}{a}\right) - D\frac{j\omega\mu_0}{(w/a)^2}\frac{\nu}{r}K_\nu\left(\frac{wr}{a}\right)\right]\sin(\nu\theta)$$

$$E_\theta = \left[C\frac{j\beta}{(w/a)^2}\frac{\nu}{r}K_\nu\left(\frac{wr}{a}\right) - D\frac{j\omega\mu_0}{(w/a)}K_\nu'\left(\frac{wr}{a}\right)\right]\cos(\nu\theta)$$

$$H_z = DK_\nu\left(\frac{wr}{a}\right)\cos(\nu\theta)$$

$$H_r = \left[-C\frac{j\omega\varepsilon_2}{(w/a)^2}\frac{\nu}{r}K_\nu\left(\frac{wr}{a}\right) + D\frac{j\beta}{(w/a)}K_\nu'\left(\frac{wr}{a}\right)\right]\cos(\nu\theta)$$

$$H_\theta = \left[C\frac{j\varepsilon_2}{(w/a)}K_\nu'\left(\frac{wr}{a}\right) - D\frac{j\beta}{(w/a)^2}\frac{\nu}{r}K_\nu\left(\frac{wr}{a}\right)\right]\sin(\nu\theta)$$

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CALCULATION OF THE FIELDS COMPONENTS

→ *… and (pdf page 11)*

if ν = 0

In the core | **In the cladding**

TE modes : $E_z=0$, $H_\theta=0$

In the core:

$$E_\theta = B\frac{j\omega\mu_0}{(u/a)}J_0'\left(\frac{ur}{a}\right)$$

$$H_z = BJ_0\left(\frac{ur}{a}\right)$$

$$H_r = -B\frac{j\beta}{(u/a)}J_0'\left(\frac{ur}{a}\right)$$

In the cladding:

$$E_\theta = -D\frac{j\omega\mu_0}{(w/a)}K_0'\left(\frac{wr}{a}\right)$$

$$H_z = DK_0\left(\frac{wr}{a}\right)$$

$$H_r = D\frac{j\beta}{(w/a)}K_0'\left(\frac{wr}{a}\right)$$

TM modes : $H_z=0$, $E_\theta=0$

In the core:

$$E_r = -A\frac{j\beta}{(u/a)}J_0'\left(\frac{ur}{a}\right)$$

$$E_z = AJ_0\left(\frac{ur}{a}\right)$$

$$H_\theta = -A\frac{j\omega\varepsilon_1}{(u/a)}J_0'\left(\frac{ur}{a}\right)$$

In the cladding:

$$E_r = C\frac{j\beta}{(w/a)}K_0'\left(\frac{wr}{a}\right)$$

$$E_z = CK_0\left(\frac{wr}{a}\right)$$

$$H_\theta = C\frac{j\varepsilon_2}{(w/a)}K_0'\left(\frac{wr}{a}\right)$$

MODES CLASSIFICATION - DISPERSION CURVES

- ***continuity of the tangential components of E and H at the core-cladding interface (pdf page 12)***

$$\left|\begin{array}{l} E_z^{coeur}(\mathrm{r{=}a}) = E_z^{gaine}(\mathrm{r{=}a}) \\ H_z^{coeur}(\mathrm{r{=}a}) = H_z^{gaine}(\mathrm{r{=}a}) \\ E_\theta^{coeur}(\mathrm{r{=}a}) = E_\theta^{gaine}(\mathrm{r{=}a}) \\ H_\theta^{coeur}(\mathrm{r{=}a}) = H_\theta^{gaine}(\mathrm{r{=}a}) \end{array}\right.$$

MODES CLASSIFICATION - DISPERSION CURVES

- ***continuity of the tangential components of E and H at the core-cladding interface (pdf page 12)***

$$
\left|\begin{array}{l}
E_z^{coeur}(\mathrm{r{=}a}) = E_z^{gaine}(\mathrm{r{=}a}) \\
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E_\theta^{coeur}(\mathrm{r{=}a}) = E_\theta^{gaine}(\mathrm{r{=}a}) \\
H_\theta^{coeur}(\mathrm{r{=}a}) = H_\theta^{gaine}(\mathrm{r{=}a})
\end{array}\right.
$$

- ***dispersion equation (pdf page 12)***

$$
\underbrace{\left[\frac{J_\nu'(u)}{uJ_\nu(u)} + \frac{K_\nu'(w)}{wK_\nu(w)}\right]}_{F1}\underbrace{\left[\frac{\varepsilon_1 J_\nu'(u)}{\varepsilon_2 uJ_\nu(u)} + \frac{K_\nu'(w)}{wK_\nu(w)}\right]}_{F2} = \nu^2 \underbrace{\left(\frac{1}{u^2}+\frac{1}{w^2}\right)}_{F3}\underbrace{\left(\frac{\varepsilon_1}{\varepsilon_2}\frac{1}{u^2}+\frac{1}{w^2}\right)}_{F4}
$$

with $u^2 + w^2 = V^2 = k_0^2 a^2 \left(n_1^2 - n_2^2\right) = \text{cte}$ (determined by the fiber and the working wavelength)

V varies => β changes accordingly → β =f(V) : dispersion curve of the considered mode

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MODES CLASSIFICATION - DISPERSION CURVES

$$\underbrace{\left[\frac{J_\nu^{'}(u)}{uJ_\nu(u)}+\frac{K_\nu^{'}(w)}{wK_\nu(w)}\right]}_{F1}\underbrace{\left[\frac{\varepsilon_1 J_\nu^{'}(u)}{\varepsilon_2 uJ_\nu(u)}+\frac{K_\nu^{'}(w)}{wK_\nu(w)}\right]}_{F2}=\nu^2\underbrace{\left(\frac{1}{u^2}+\frac{1}{w^2}\right)}_{F3}\underbrace{\left(\frac{\varepsilon_1}{\varepsilon_2}\frac{1}{u^2}+\frac{1}{w^2}\right)}_{F4}$$

if $\nu = 0$ → F1 = 0 : dispersion equation of TE modes
→ or F2 = 0 : dispersion equation of TM modes

if $\nu \neq 0$ → the entire equation must be solved : HE and EH modes

MODES CLASSIFICATION - DISPERSION CURVES

$$\underbrace{\left[\frac{J_\nu^{'}(u)}{uJ_\nu(u)}+\frac{K_\nu^{'}(w)}{wK_\nu(w)}\right]}_{F1}\underbrace{\left[\frac{\varepsilon_1 J_\nu^{'}(u)}{\varepsilon_2 uJ_\nu(u)}+\frac{K_\nu^{'}(w)}{wK_\nu(w)}\right]}_{F2}=\nu^2\underbrace{\left(\frac{1}{u^2}+\frac{1}{w^2}\right)}_{F3}\underbrace{\left(\frac{\varepsilon_1}{\varepsilon_2}\frac{1}{u^2}+\frac{1}{w^2}\right)}_{F4}$$

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if $\nu \neq 0$ → the entire equation must be solved : HE and EH modes

- **designation of the electromagnetic modes *(pdf page 13)***

$\nu = 0$: TE_{0l} and TM_{0l}

$\nu \neq 0$: $EH_{\nu l}$ and $HE_{\nu l}$

MODES CLASSIFICATION - DISPERSION CURVES

- **weak guidance approximation *(pdf page 13)***

$$\Delta = \left(\frac{n_1^2 - n_2^2}{2n_1^2} \right) = \frac{ON^2}{2n_1^2}$$

Δ : relative index difference

if n_1 close to n_2 such that $\Delta < 10^{-2}$:
propagation in WEAK GUIDANCE conditions

MODES CLASSIFICATION - DISPERSION CURVES

➢ **weak guidance approximation *(pdf page 13)***

$$\boxed{\Delta = \left(\frac{n_1^2 - n_2^2}{2n_1^2}\right) = \frac{ON^2}{2n_1^2}}$$

Δ : relative index difference

if n_1 is close to n_2 such that $\Delta < 10^{-2}$:
propagation in WEAK GUIDANCE conditions

Thus, the dispersion equation becomes : $F_1^2 = \nu^2 F_3^2 \Leftrightarrow F_1 = \pm\nu\ F_3$ ***(pdf page 14)***

resulting in :

$$\boxed{\frac{J_\nu'(u)}{uJ_\nu(u)} + \frac{K_\nu'(w)}{wK_\nu(w)} = \pm\nu\left(\frac{1}{u^2} + \frac{1}{w^2}\right)}$$

→ **if $\nu = 0$: TE_{0l} (exact) et TM_{0l} (approximate)**

→ **if $\nu \neq 0$ signe + : $EH_{\nu l}$**
signe - : $HE_{\nu l}$

MODES CLASSIFICATION - DISPERSION CURVES IN THE WEAK GUIDANCE APPROXIMATION

(pdf page 15)

→ **if $\nu = 0$: TE or TM modes** ➔

$$u\frac{J_0(u)}{J_1(u)} = \frac{-wK_0(w)}{K_1(w)}$$

→ **if $\nu \neq 0$ signe + : EH modes** ➔

$$u\frac{J_\nu(u)}{J_{\nu+1}(u)} = \frac{-wK_\nu(w)}{K_{\nu+1}(w)}$$

→ **if $\nu \neq 0$ signe - : HE modes** ➔

$$u\frac{J_{\nu-2}(u)}{J_{\nu-1}(u)} = \frac{-wK_{\nu-2}(w)}{K_{\nu-1}(w)}$$

MODES CLASSIFICATION - DISPERSION CURVES IN THE WEAK GUIDANCE APPROXIMATION

(pdf page 15)

→ **if $\nu = 0$: TE or TM modes** → $$u\frac{J_0(u)}{J_1(u)} = \frac{-wK_0(w)}{K_1(w)}$$

→ **if $\nu \neq 0$ signe + : EH modes** → $$u\frac{J_\nu(u)}{J_{\nu+1}(u)} = \frac{-wK_\nu(w)}{K_{\nu+1}(w)}$$

→ **if $\nu \neq 0$ signe - : HE modes** → $$u\frac{J_{\nu-2}(u)}{J_{\nu-1}(u)} = \frac{-wK_{\nu-2}(w)}{K_{\nu-1}(w)}$$

→ **if $\nu = 0$: TE or TM modes** — **m=1**

→ **if $\nu \neq 0$ signe + : EH modes** — **m= ν+1**

→ **if $\nu \neq 0$ signe - : HE modes** — **m= ν-1**

$$\boxed{u\frac{J_{m-1}(u)}{J_m(u)} = \frac{-wK_{m-1}(w)}{K_m(w)}}$$

m integer ≥ 0

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Université de Limoges

MODES CLASSIFICATION - DISPERSION CURVES IN THE WEAK GUIDANCE APPROXIMATION

(pdf page 15)

→ if $\nu = 0$: TE or TM modes → $$u\frac{J_0(u)}{J_1(u)} = \frac{-wK_0(w)}{K_1(w)}$$

→ if $\nu \neq 0$ signe + : EH modes → $$u\frac{J_\nu(u)}{J_{\nu+1}(u)} = \frac{-wK_\nu(w)}{K_{\nu+1}(w)}$$

→ if $\nu \neq 0$ signe - : HE modes → $$u\frac{J_{\nu-2}(u)}{J_{\nu-1}(u)} = \frac{-wK_{\nu-2}(w)}{K_{\nu-1}(w)}$$

→ if $\nu = 0$: TE or TM modes — m=1

→ if $\nu \neq 0$ signe + : EH modes — m= ν+1

→ if $\nu \neq 0$ signe - : HE modes — m= ν-1

$$\boxed{u\frac{J_{m-1}(u)}{J_m(u)} = \frac{-wK_{m-1}(w)}{K_m(w)}}$$

m integer ≥ 0

$$\text{with} \quad u^2 + w^2 = V^2$$

MODES CLASSIFICATION - DISPERSION CURVES IN THE WEAK GUIDANCE APPROXIMATION

(pdf page 15)

→ **if $\nu = 0$: TE or TM modes** — m=1

→ **if $\nu \neq 0$ signe + : EH modes** — m= ν+1, ν = m-1

→ **if $\nu \neq 0$ signe - : HE modes** — m= ν-1, ν = m+1

$$\boxed{u\frac{J_{m-1}(u)}{J_m(u)} = \frac{-wK_{m-1}(w)}{K_m(w)}}$$

m integer ≥ 0

MODES CLASSIFICATION - DISPERSION CURVES IN THE WEAK GUIDANCE APPROXIMATION

(pdf page 15)

→ if $\nu = 0$: TE or TM modes — m=1

→ if $\nu \neq 0$ signe + : EH modes — m= ν+1, ν = m-1

→ if $\nu \neq 0$ signe - : HE modes — m= ν-1, ν = m+1

$$\boxed{u\frac{J_{m-1}(u)}{J_m(u)} = \frac{-wK_{m-1}(w)}{K_m(w)}}$$

m integer ≥ 0

for a given value of m, same dispersion relationship for certain modes:

if m=1 → $TE_{0,l}$, TM_{0l} and $HE_{2,l}$ (→ degenerated modes) ⇒ $LP_{1,l}$ mode

if m>1 → $EH_{m-1,\,l}$, and $HE_{m+1,\,l}$ (→ degenerated modes) ⇒ $LP_{m,l}$ mode m>1

if m=0 → $HE_{1,l}$ mode only ⇒ $LP_{0,l}$ mode

LP MODES - CUTOFF FREQUENCY

- **distribution of the electric field in the LP modes**
  (pdf page 16)

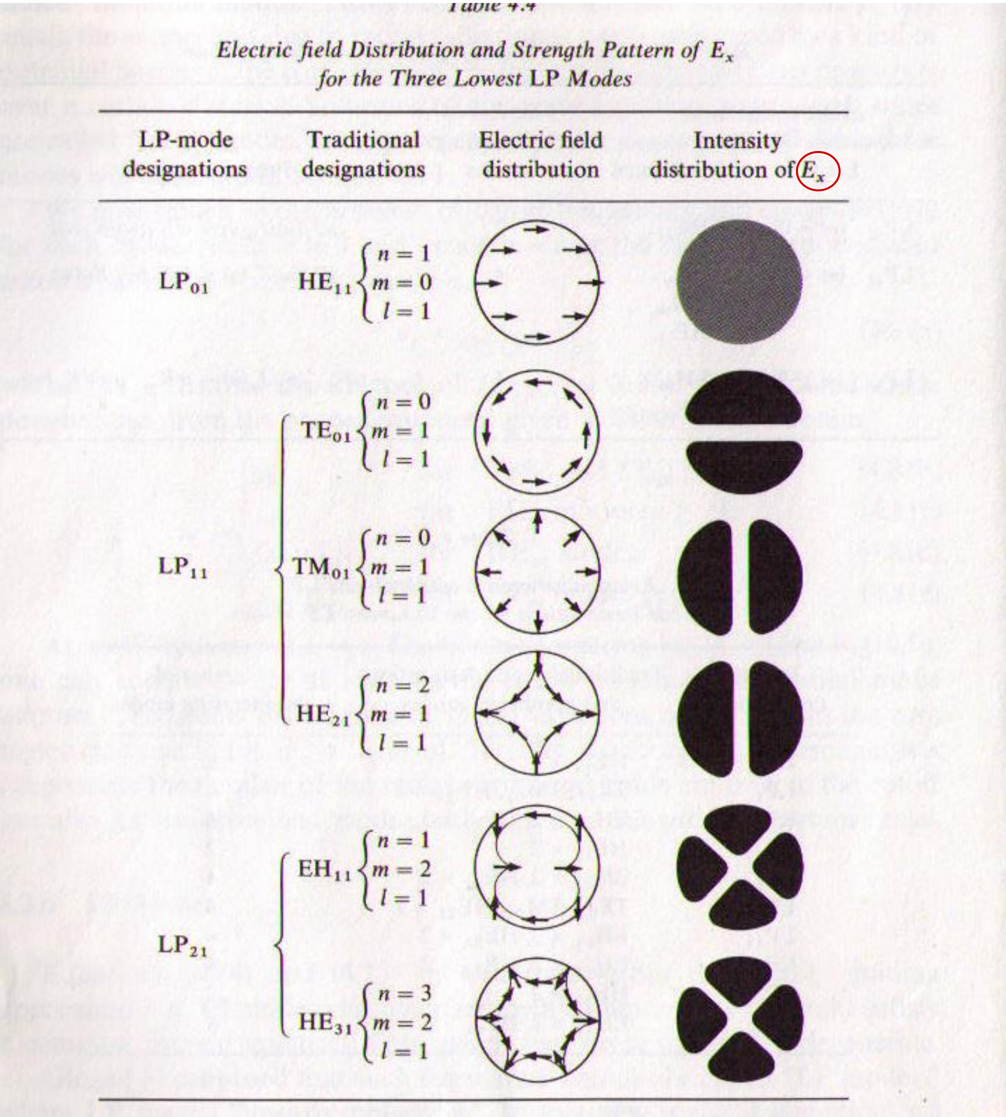


Table 4.4

Electric field Distribution and Strength Pattern of E_x for the Three Lowest LP Modes

LP-mode designations	Traditional designations	Electric field distribution	Intensity distribution of E_x
LP_{01}	HE_{11} $\{n=1, m=0, l=1$		
LP_{11}	TE_{01} $\{n=0, m=1, l=1$		
	TM_{01} $\{n=0, m=1, l=1$		
	HE_{21} $\{n=2, m=1, l=1$		
LP_{21}	EH_{11} $\{n=1, m=2, l=1$		
	HE_{31} $\{n=3, m=2, l=1$		

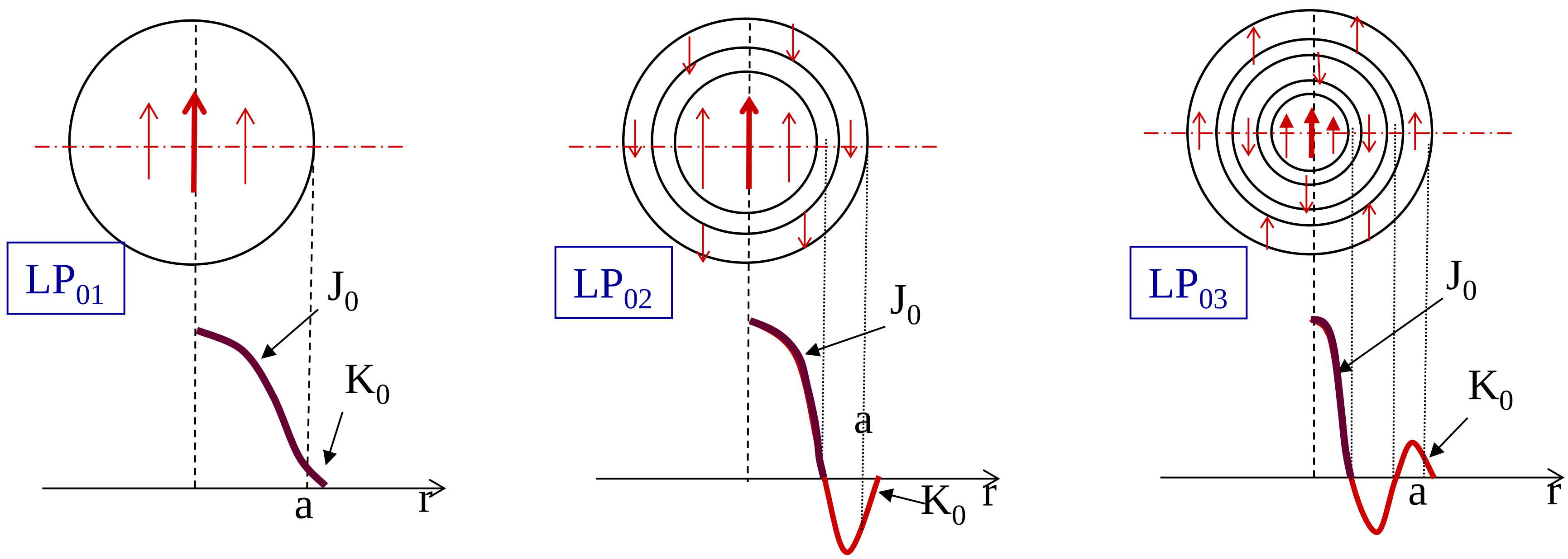
LP MODES - CUTOFF FREQUENCY

- **distribution of the electric field in the LP$_{m,l}$ modes *(pdf page 16)***

→ along a radius : following J$_m$function in the core and following K$_m$ function in the cladding

→ along a circle at a fixed distance from the center : following the cos (mθ+φ) er sin (mθ+φ) function

examples (with m=0) : → LP$_{0,l}$ modes



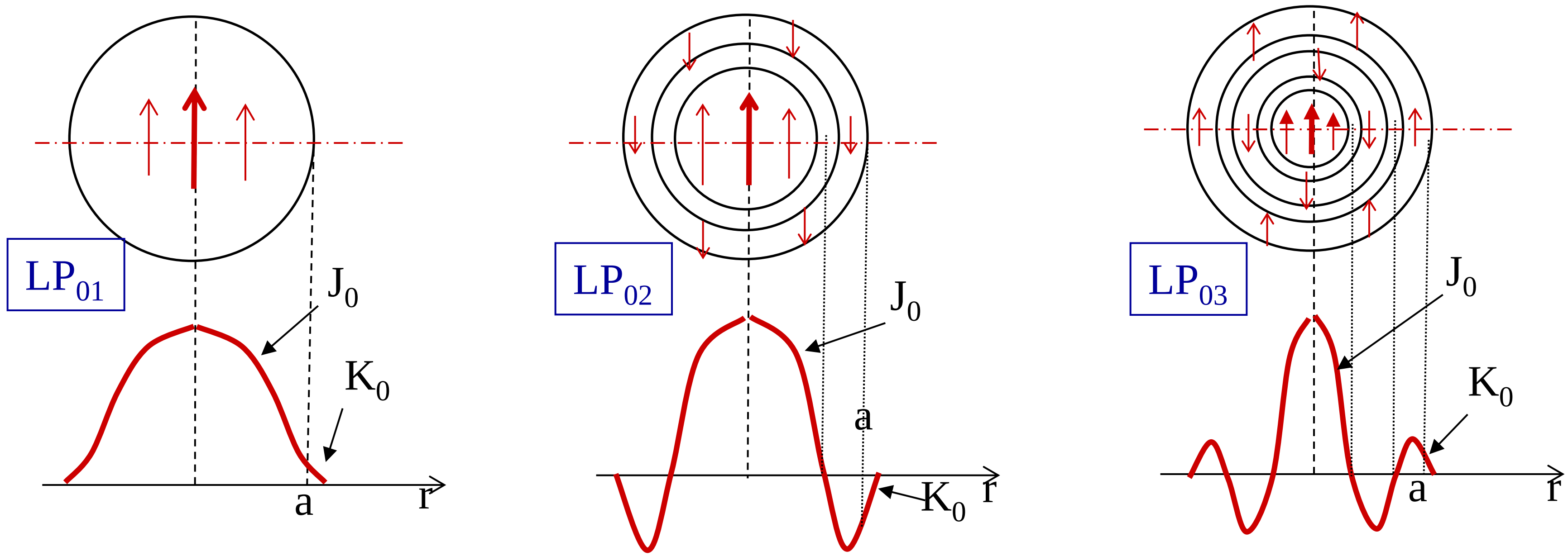
LP MODES - CUTOFF FREQUENCY

- **distribution of the electric field in the $LP_{m,l}$ modes *(pdf page 16)***

→ along a radius : following J_m function in the core and following K_m function in the cladding

→ along a circle at a fixed distance from the center : following the cos (mθ+φ) er sin (mθ+φ) function

examples (with m=0) : → $LP_{0,l}$ modes

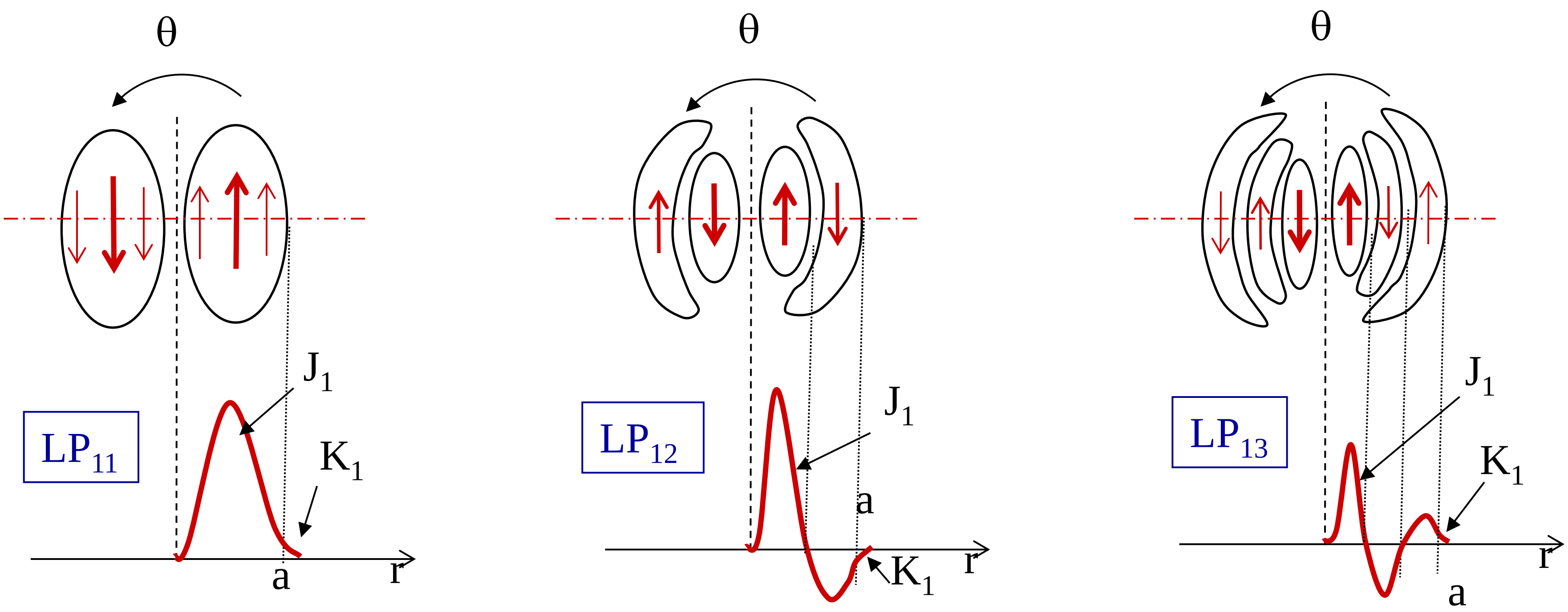


<u>intensity distribution in</u> $LP_{0,l}$ modes → one central circular lobe surrounded by (l-1) rings

LP MODES - CUTOFF FREQUENCY

➤ **distribution of the electric field in the $LP_{m,l}$ modes** ***(pdf page 18)***

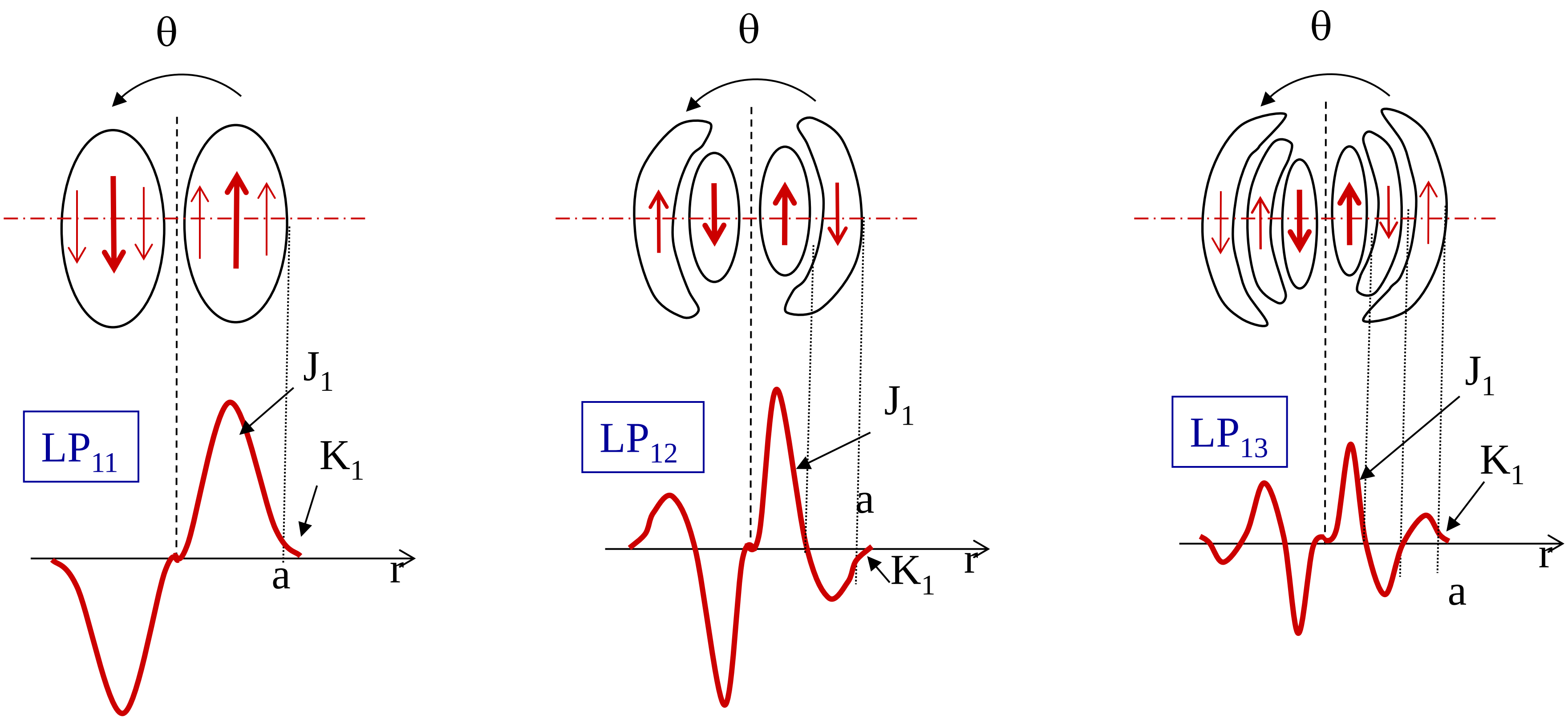
other examples (with m=1) :



LP MODES - CUTOFF FREQUENCY

➢ **distribution of the electric field in the $LP_{m,l}$ modes *(pdf page 18)***

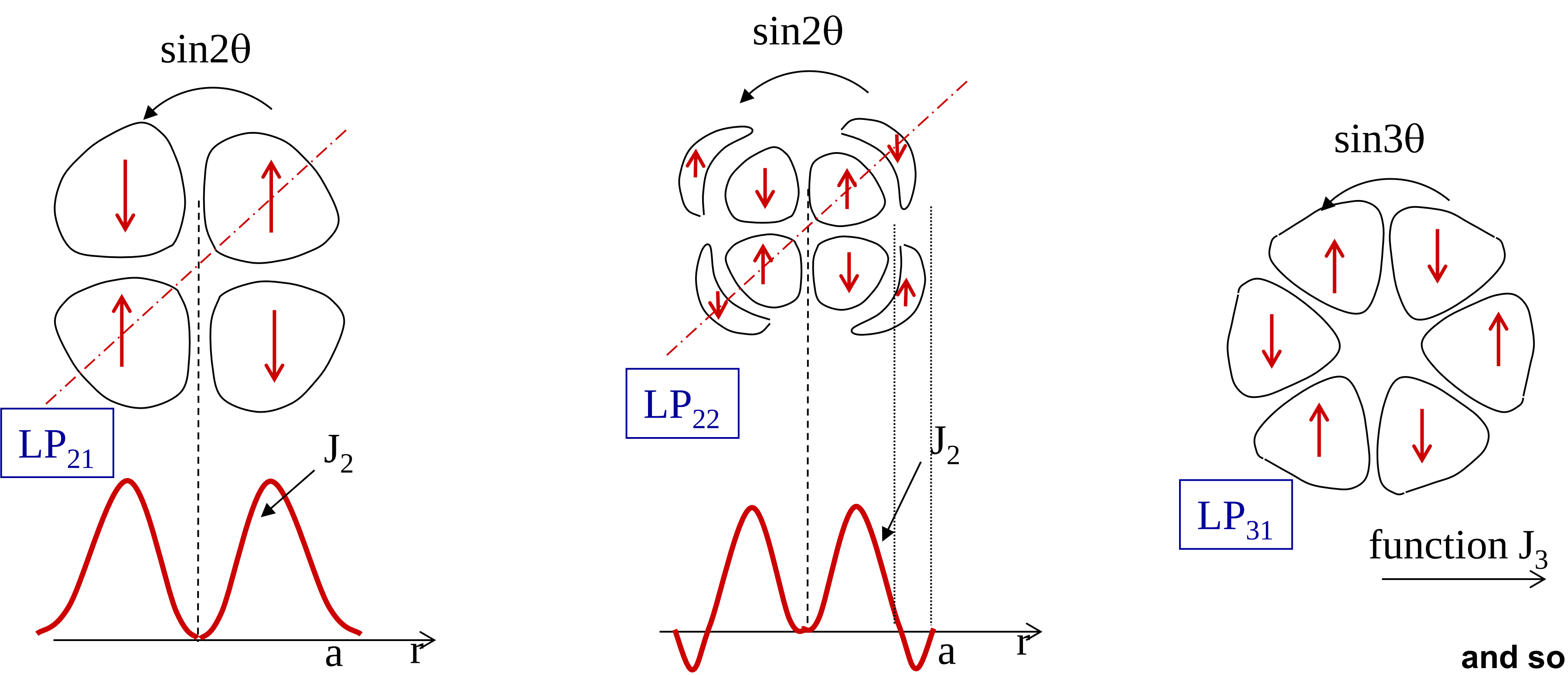
other examples (with m=1) :



LP MODES - CUTOFF FREQUENCY

➢ **distribution of the electric field in the $LP_{m,l}$ modes *(pdf page 19)***

still other examples (with m=2 or 3) :



and so on…

<u>intensity distribution in $LP_{m,l}$ modes (m≠0)</u> → pattern with *l* rings and 2m lobes in each ring

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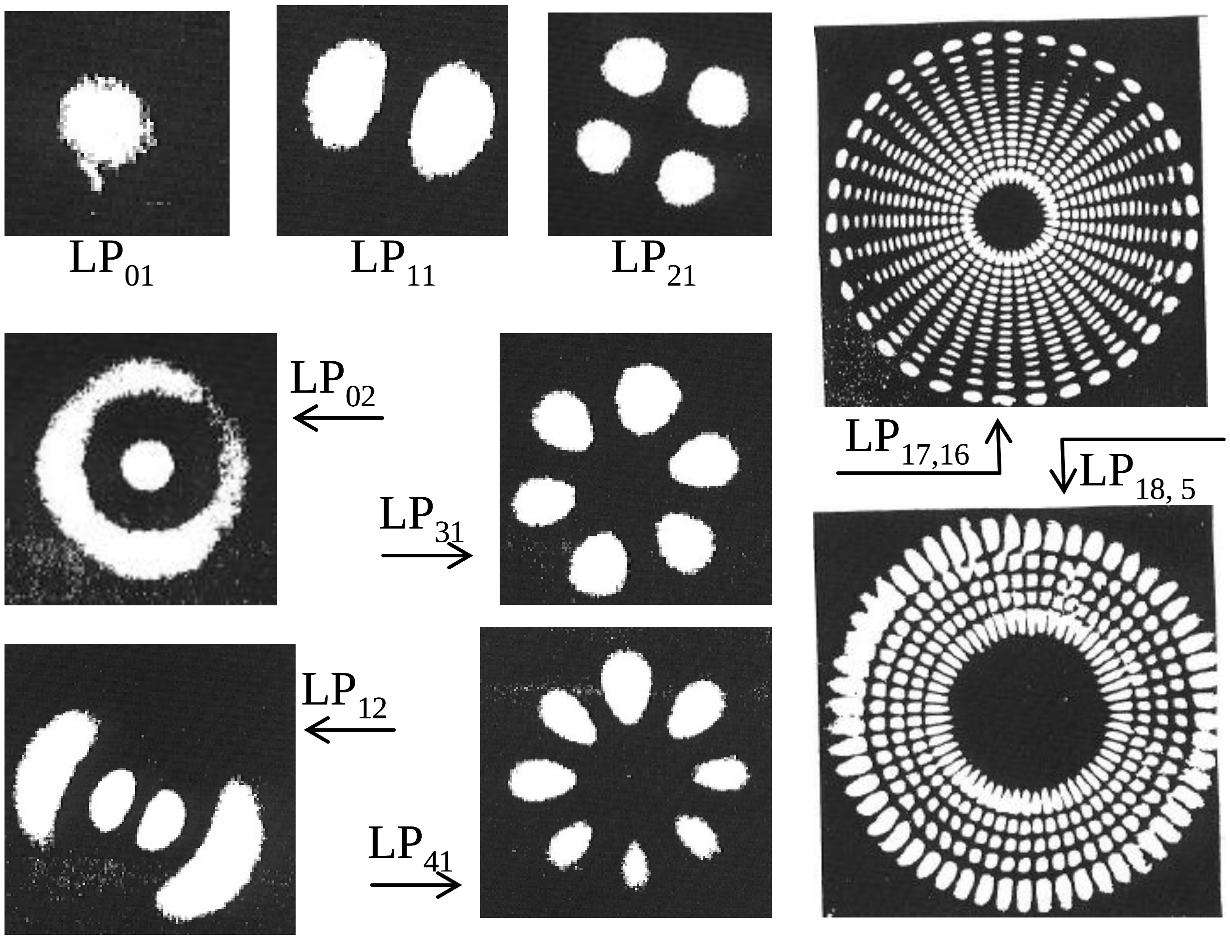
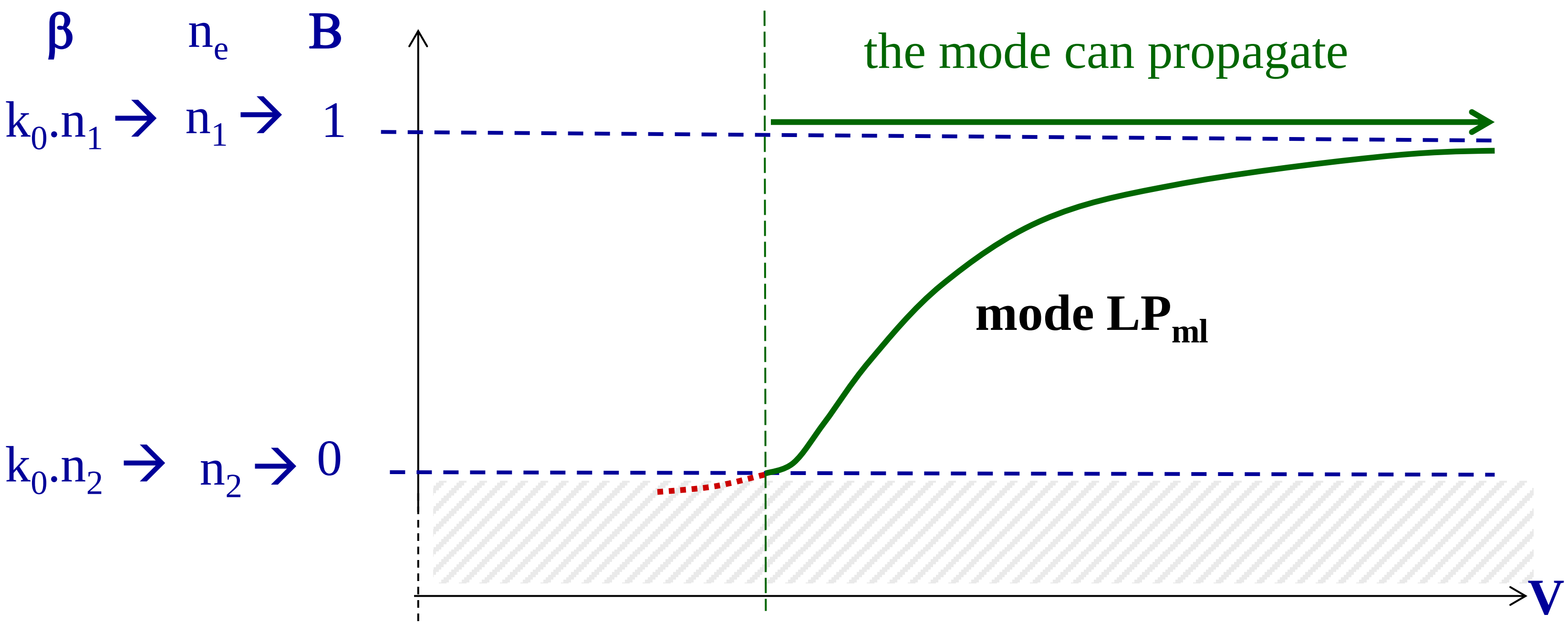
LP MODES - CUTOFF FREQUENCY

➢ **distribution of the electric field in the $LP_{m,l}$ modes** ***(pdf page 19)***



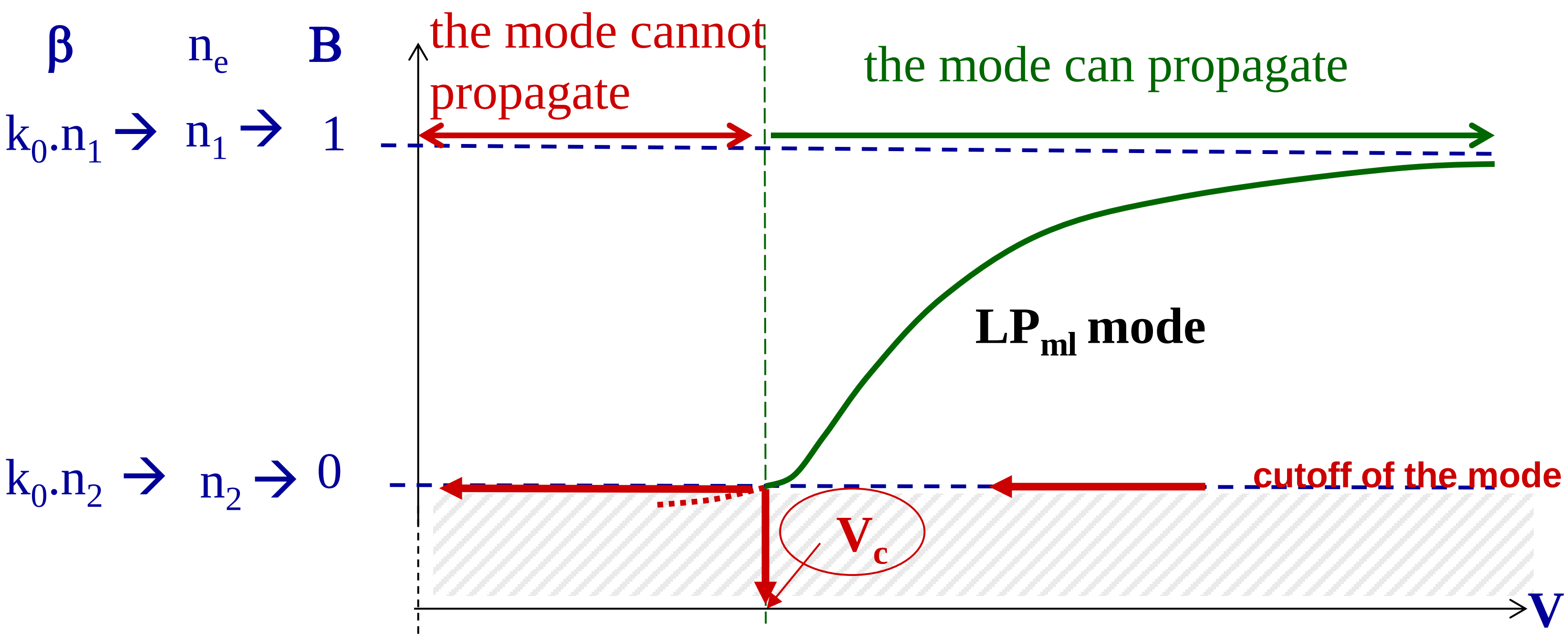
LP MODES - CUTOFF FREQUENCY

- **cutoff normalized spatiale frequency for LP modes** ***(pdf page 20)***



LP MODES - CUTOFF FREQUENCY

- **cutoff normalized spatial frequency for LP modes *(pdf page 20)***



propagation condition for the LP_{ml} mode : $V > V_c(LP_{ml})$

LP MODES - CUTOFF FREQUENCY

➢ **cutoff normalized spatial frequency for LP modes** ***(pdf page 20)***

At the cutoff of the mode : $\beta = k_0 n_2$

$$w = a\left|\beta_{t2}\right| = a\sqrt{\beta^2 - k_0^2 n_2^2} = 0$$

$$u = V = V_c$$

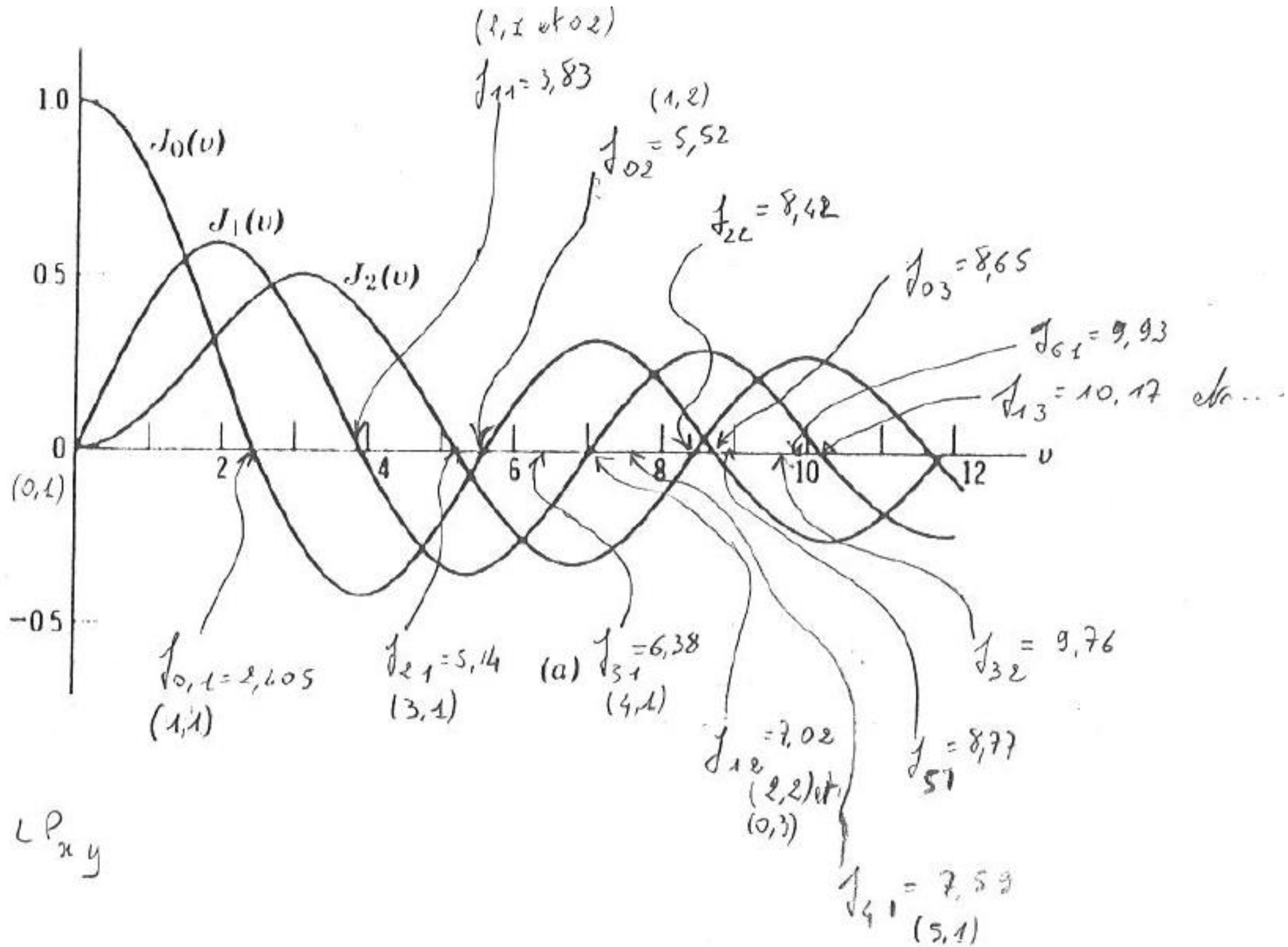
and the dispersion equation $u\dfrac{J_{m-1}(u)}{J_m(u)} = \dfrac{-wK_{m-1}(w)}{K_m(w)}$ **becomes** $\boxed{u\dfrac{J_{m-1}(u)}{J_m(u)} = 0 \quad \text{avec} \quad u = V_c}$

Thus the cutoff normalized spatial frequencies of the $\mathbf{LP_{m\mathit{l}}}$ modes are : ***(voir pdf page 21)***

$$\begin{cases} m \neq 0 \qquad l \geq 1 \qquad V_c(LP_{m,l}) = j_{m-1,\,l} \\ m = 0 \qquad \begin{array}{ll} l = 1 & V_c(LP_{0,1}) = 0 \\ l > 1 & V_c\left(LP_{0,l}\right) = j_{1,\,l-1} \end{array} \end{cases}$$

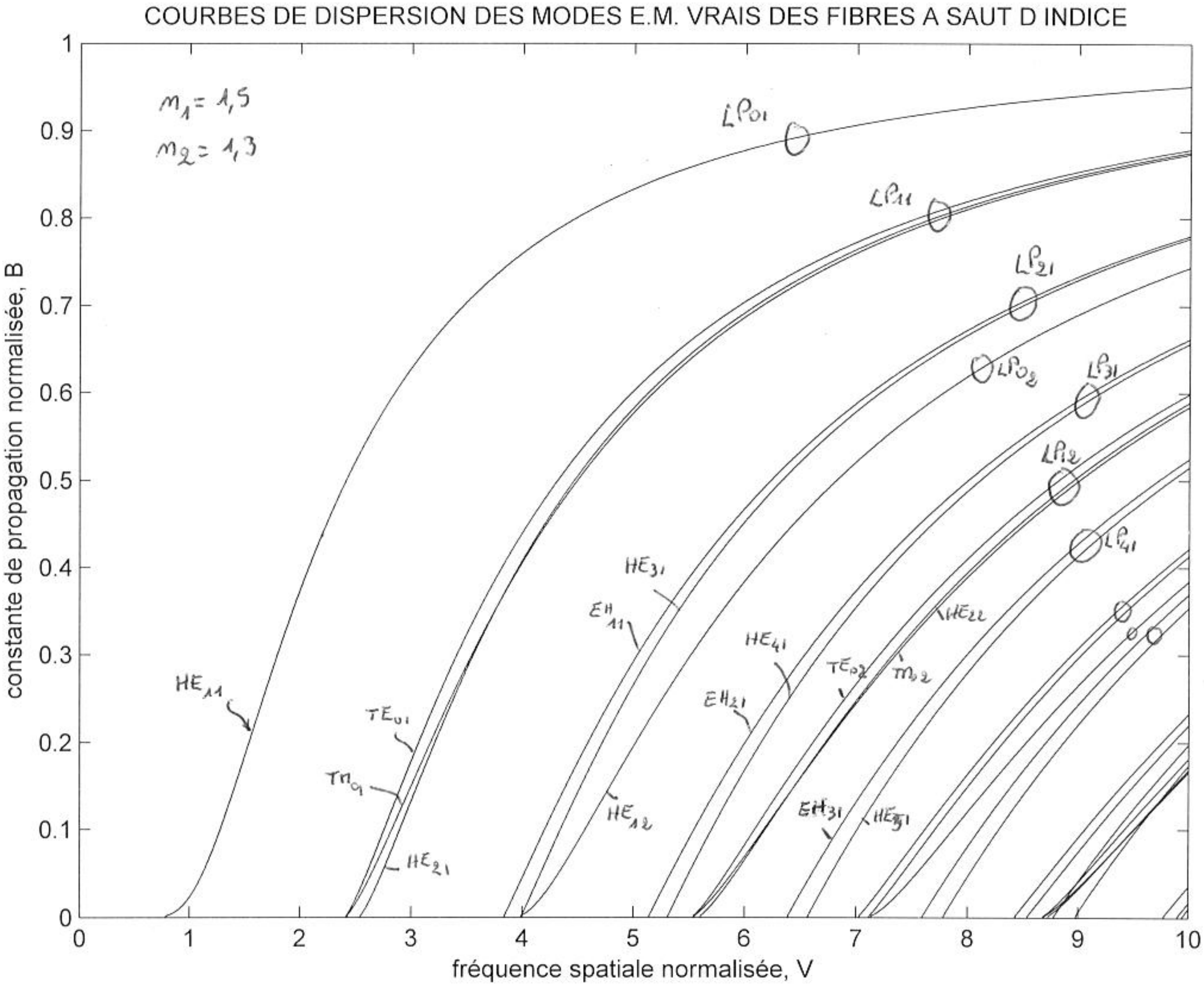
LP MODES - CUTOFF FREQUENCY

➢ **cutoff normalized spatial frequency for LP modes**

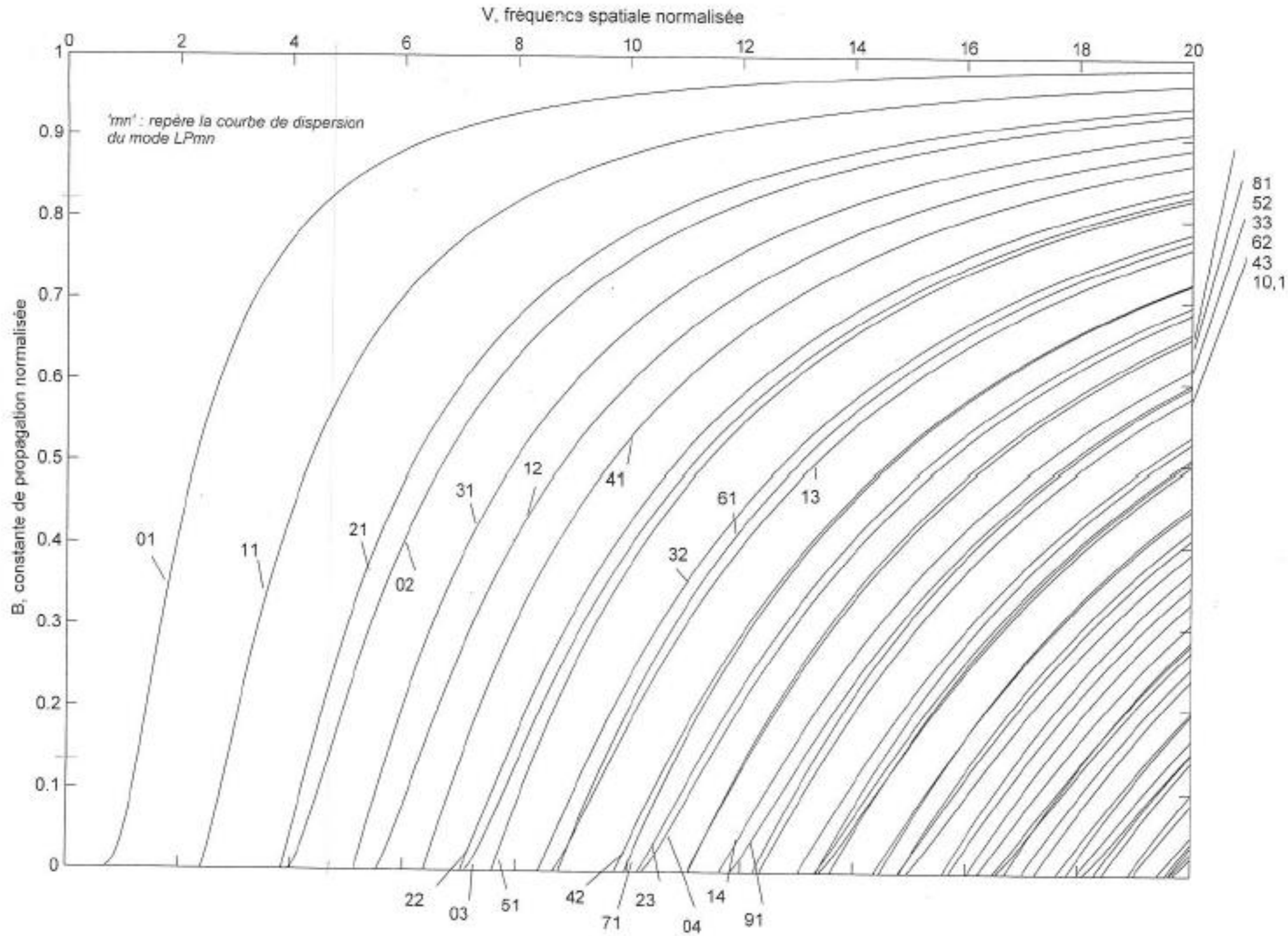


$(x, y) \rightarrow LP_{xy}$

DISPERSION CURVES (electromagnetic modes)



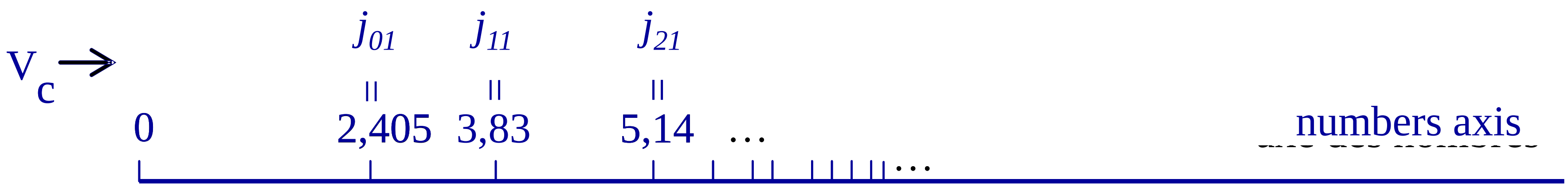
DISPERSION CURVES (LP modes)



COURBES DE DISPERSION DES MODES LP DES FIBRES A SAUT D'INDICE

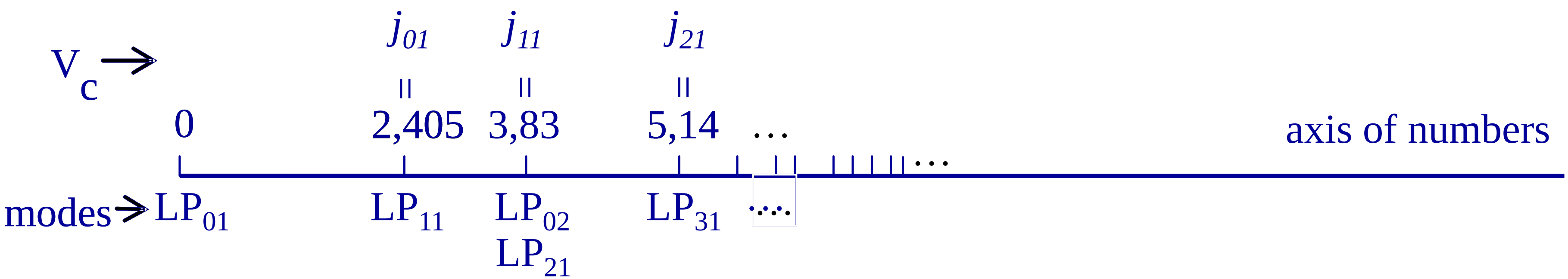
LP MODES - CUTOFF NORMALIZED SPATIAL FREQUENCY

reminder : propagation condition of the LP_{ml} mode : $V > V_c(LP_{ml})$



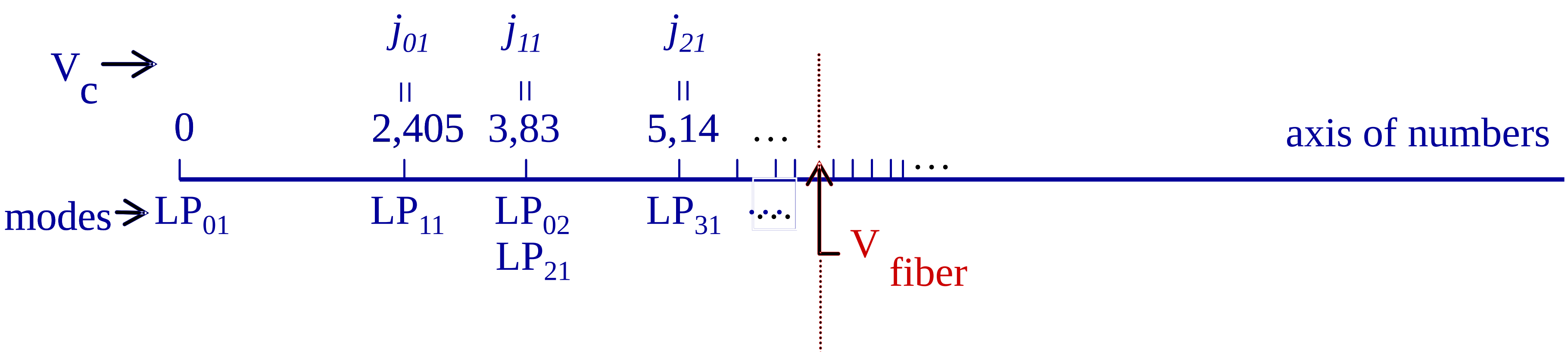
LP MODES - CUTOFF NORMALIZED SPATIAL FREQUENCY

reminder : propagation condition of the LP_{ml} mode : $V > V_c(LP_{ml})$



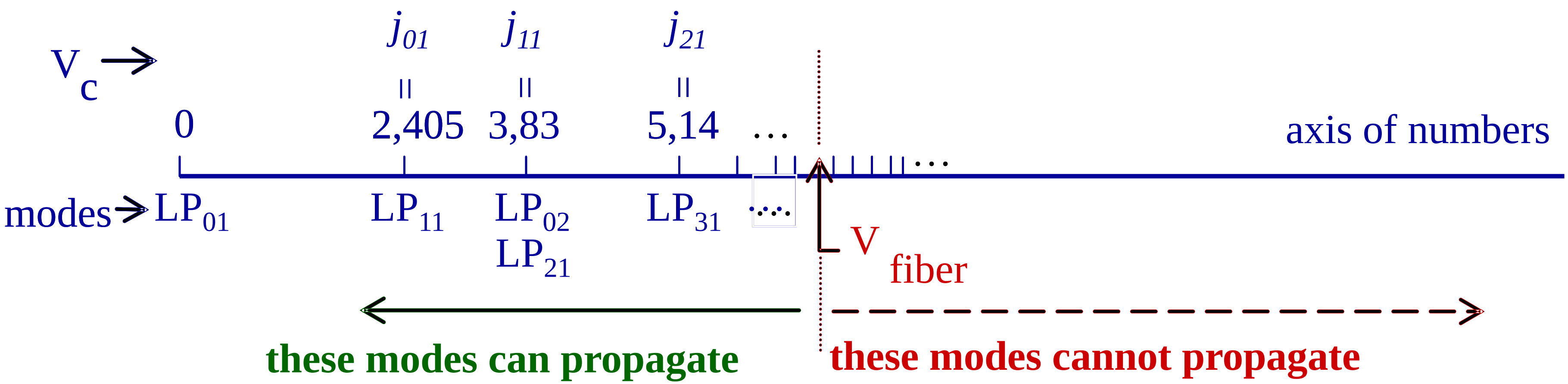
LP MODES - CUTOFF NORMALIZED SPATIAL FREQUENCY

reminder : propagation condition of the LP_{ml} mode : $V > V_c(LP_{ml})$



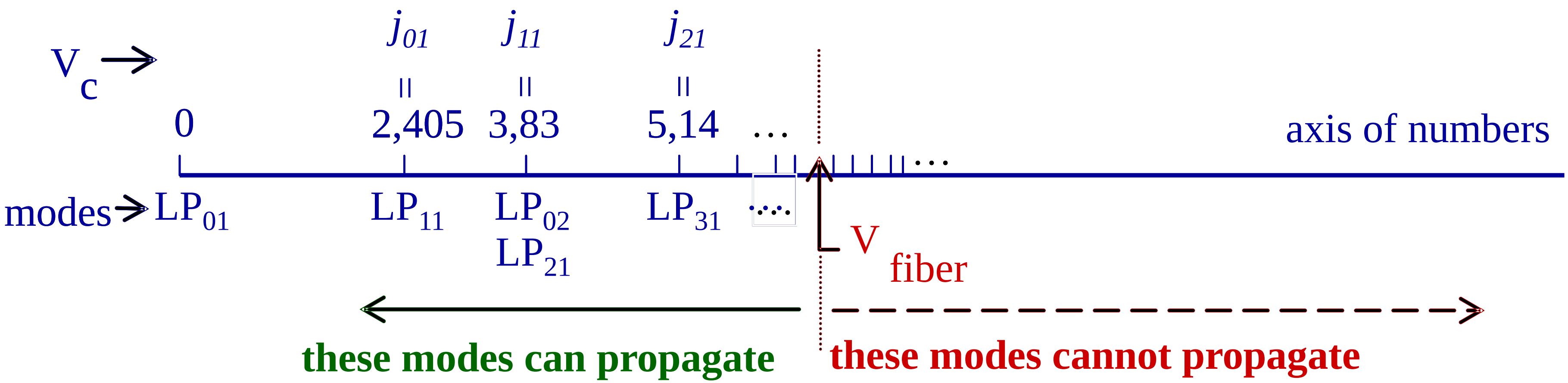
LP MODES - CUTOFF NORMALIZED SPATIAL FREQUENCY

reminder : propagation condition of the LP_{ml} mode : $V > V_c(LP_{ml})$



LP MODES - CUTOFF NORMALIZED SPATIAL FREQUENCY

reminder : propagation condition of the LP_{ml} mode : $V > V_c(LP_{ml})$



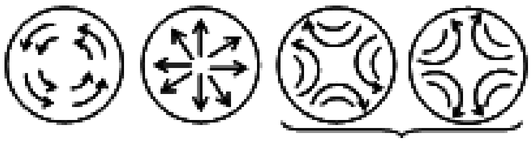
The cutoff wavelength of the LP_{ml} mode is λ_c, such that $V(\lambda_c) = V_c$

$$\frac{2\pi}{\lambda_c}.a.ON = V_c \quad \Rightarrow \quad \boxed{\lambda_c = \frac{2\pi}{V_c}.a.ON}$$

<u>possible propagation</u> of the X mode if $V_c(X) < V$ $\boxed{\Rightarrow \lambda < \lambda_c(X)}$

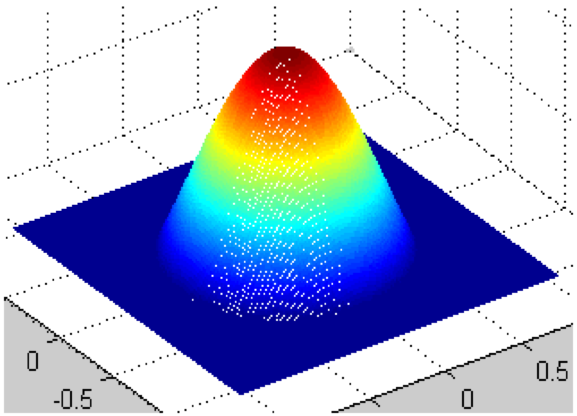
LP MODES - CUTOFF NORMALIZED SPATIAL FREQUENCY

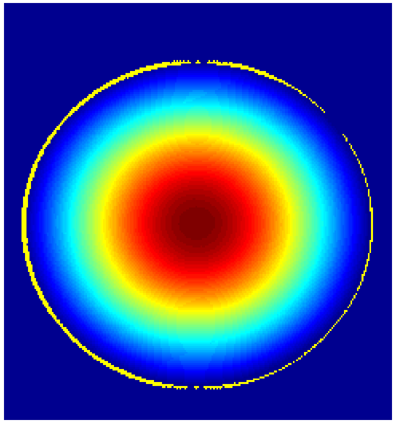
➢ **summary on the first LP_{ml} modes** ***(pdf page 26)***

Modes LP	V_c	Modes dégénérés ((x) = nombre de polars)	Nombre de modes dégénérés
LP_{01}	0	HE_{11} (2) = HE_{11x} et HE_{11y}	2
LP_{11}	2,405	TE_{01} (1), TM_{01} (1), et HE_{21} (2) $E_z = 0$ $E_\theta=0$  (lignes du champ électrique)	4
LP_{21}	3,83	EH_{11} (2) et HE_{31} (2)	4
LP_{02}	3,83	HE_{12} (2)	2
LP_{31}	5,14	EH_{21} (2) et HE_{41} (2)	4
LP_{12}	5,52	TE_{02} (1), TM_{02} (1), et HE_{22} (2)	4
LP_{41}	6,38	EH_{31} (2) et HE_{51} (2)	4
LP_{22}	……	……….	….
etc…..		……….	….

Step index fibers : LP modes of the lowest orders

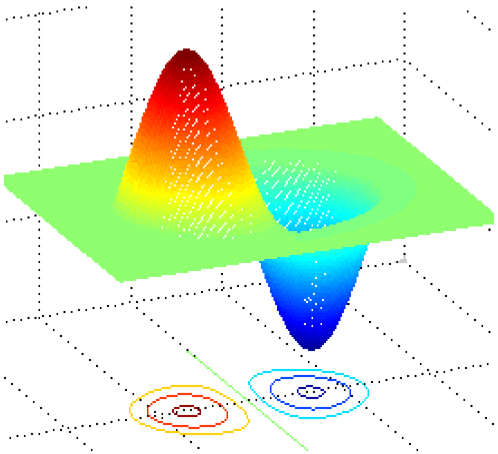
LP$_{01}$ mode

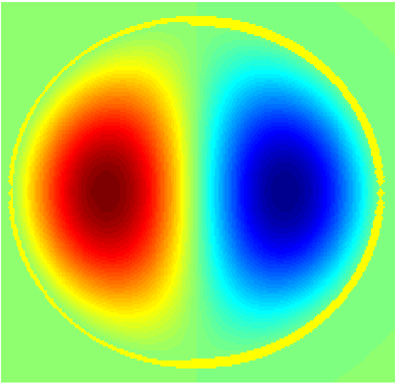




$V_c = 0$

LP$_{11}$ mode





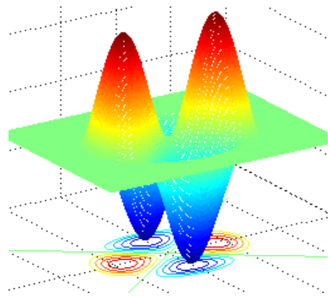
$V_c = 2{,}405$

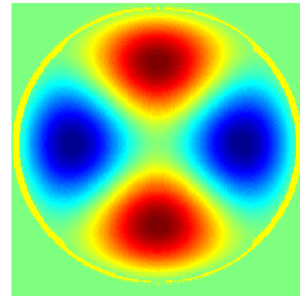
The yellow circle represents the boundary between the core and the cladding

a = 40µm; NA =0,24

Step index fibers : LP modes of the lowest orders (cont'd))

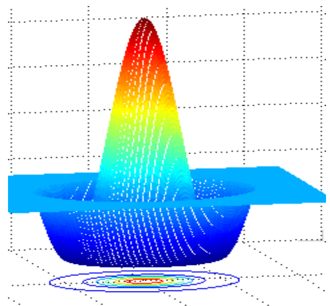
LP $_{21}$ mode

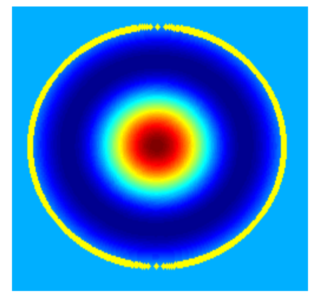




V_c = 3,83

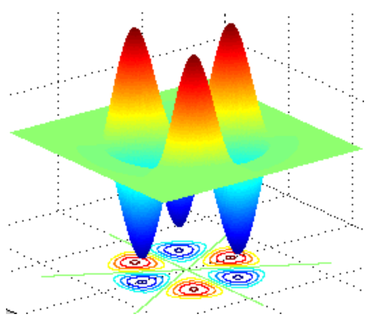
LP $_{02}$ mode

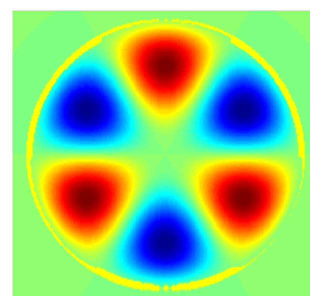




V_c = 3,83

LP $_{31}$ mode





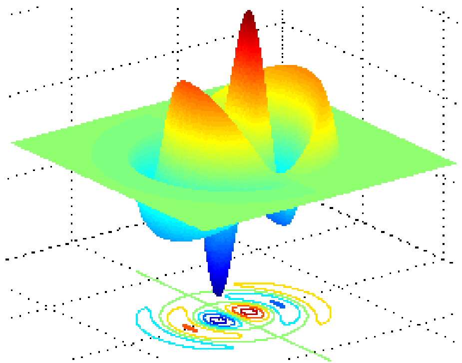
V_c = 5,14

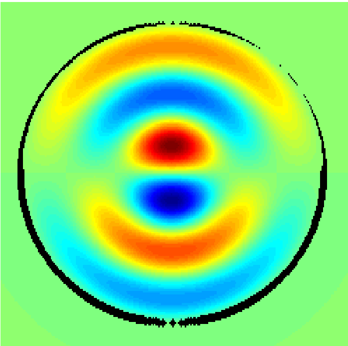
The yellow circle represents the boundary between the core and the cladding

a = 40µm; NA =0,24

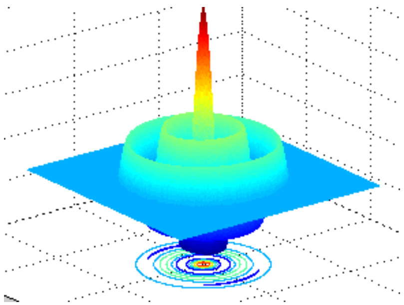
Step index fibers : other LP modes

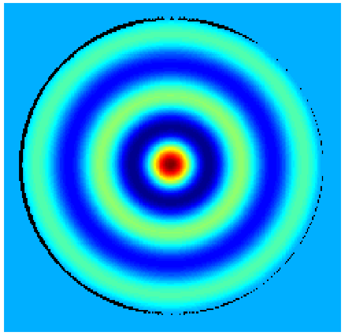
LP $_{1,3}$ mode



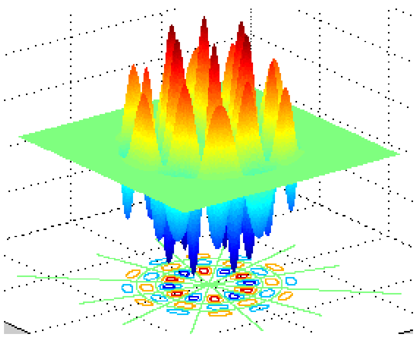


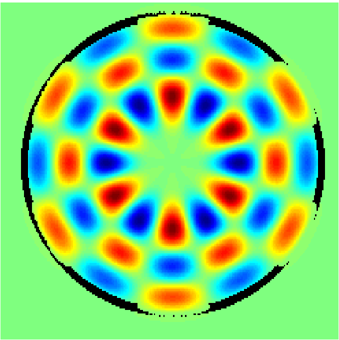
LP $_{0,5}$ mode



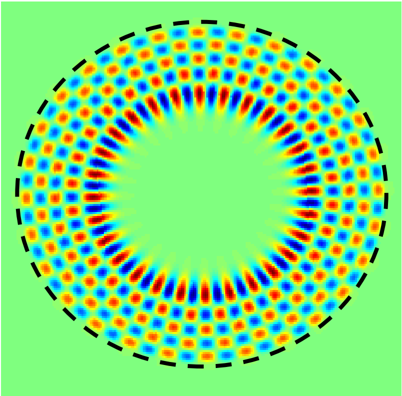


LP $_{6,3}$ mode

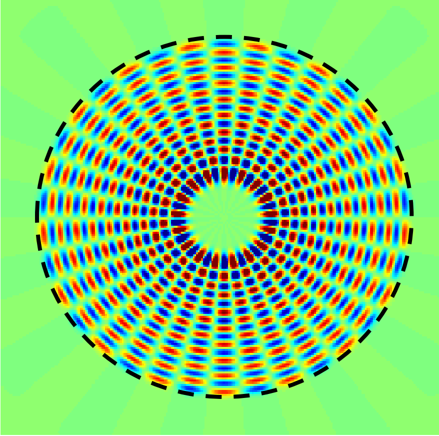




The black circle represents the boundary between the core and the cladding



LP $_{28,5}$ mode

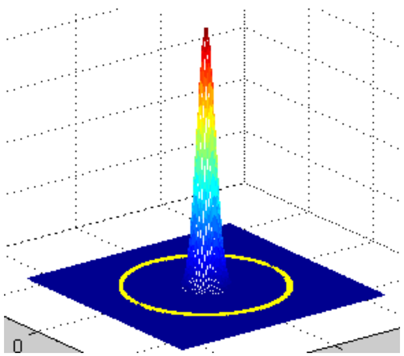


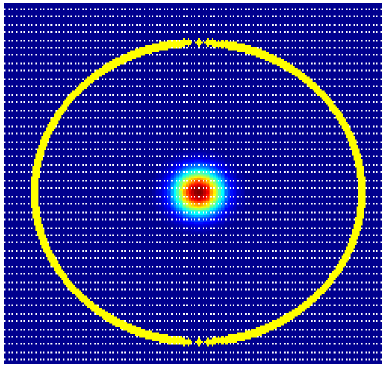
LP $_{17,16}$ mode

a = 40µm; NA =0,24

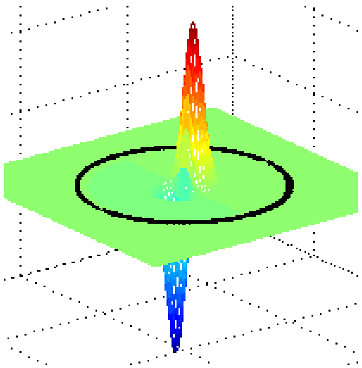
Graded index fibers : LP modes of lowest order

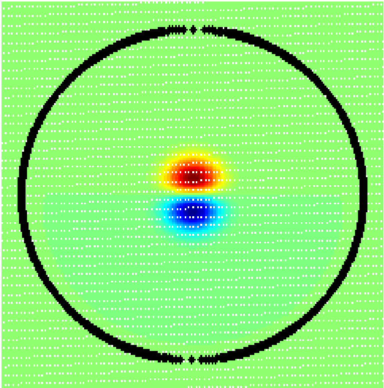
LP $_{01}$ mode





LP $_{11}$ mode



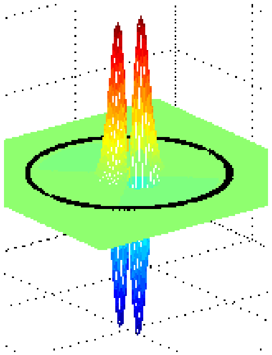


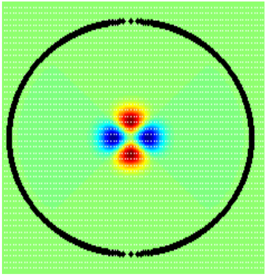
The yellow or black circles represent the boundary between the core and the cladding

a = 40µm; NA =0,24

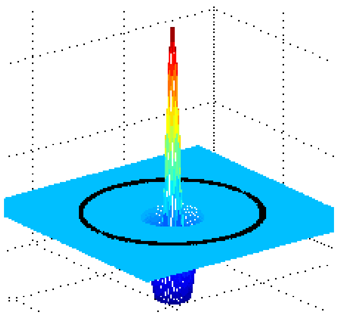
Graded index fibers : LP modes of lowest order (cont'd)

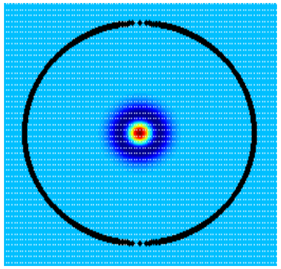
LP $_{21}$ mode



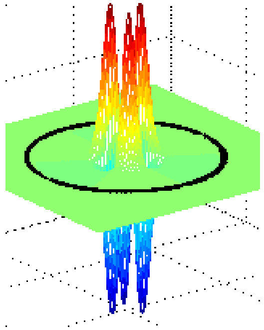


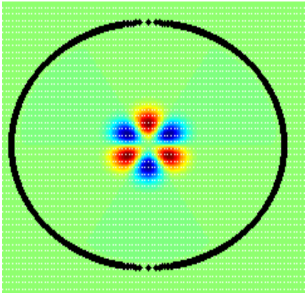
LP $_{02}$ mode





LP $_{31}$ mode





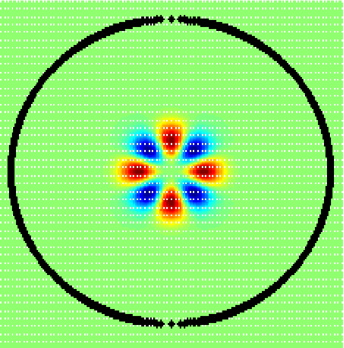
The black circle represents the boundary between the core and the cladding

a = 40µm; NA =0,24

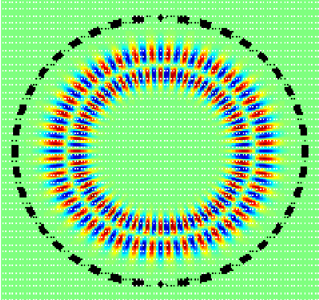
With the support of the Erasmus+ Programme of the European Union

Graded index fibers : other LP modes

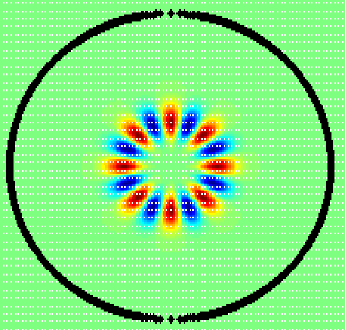
LP $_{4,\,1}$ mode



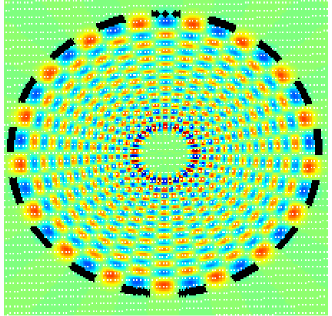
LP $_{40,\,2}$ mode



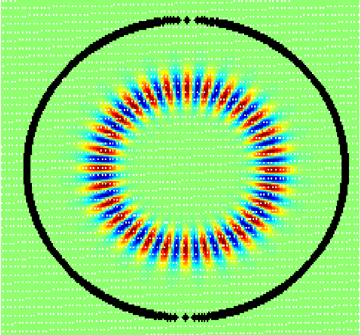
LP $_{8,\,1}$ mode



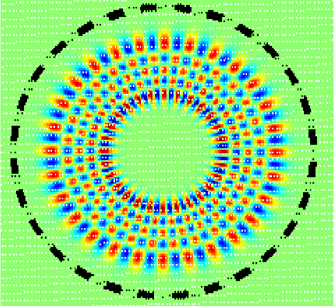
LP $_{17,\,16}$ mode



LP $_{28,\,1}$ mode



LP $_{28,\,5}$ mode

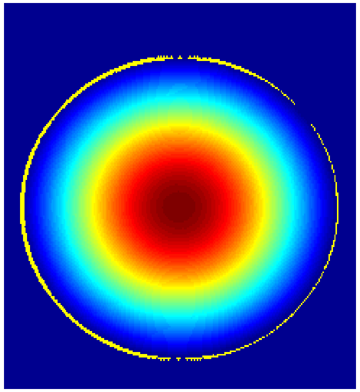


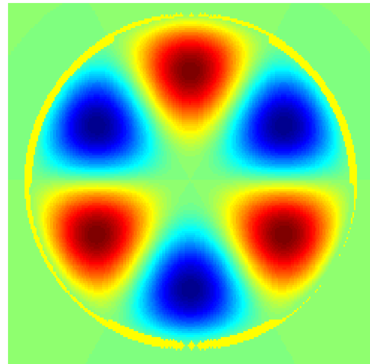
The black circle represents the boundary between the core and the cladding

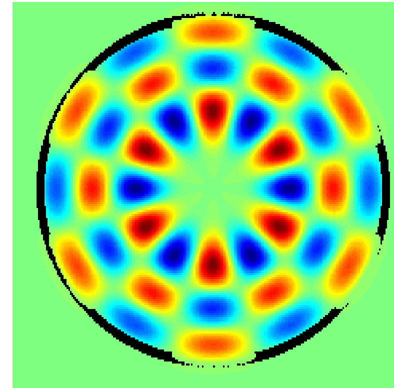
a = 40µm; NA =0,24

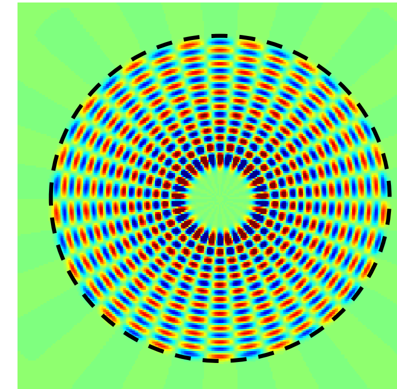
Comparison of modes of step index fibers vs modes of graded index fibers

step index fiber



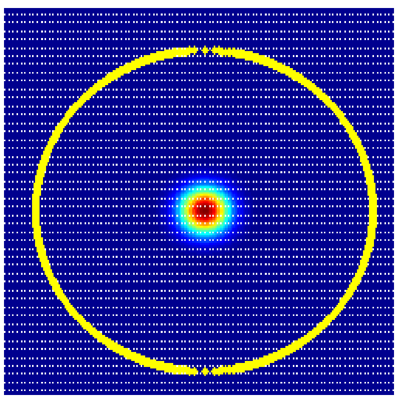


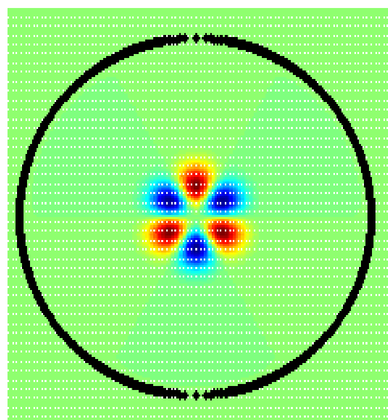


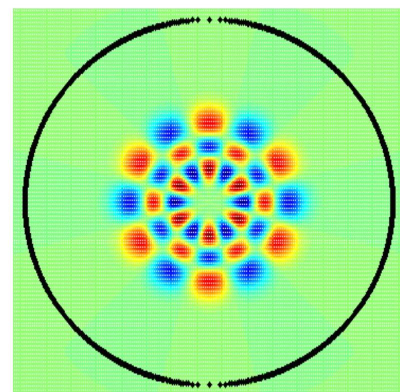


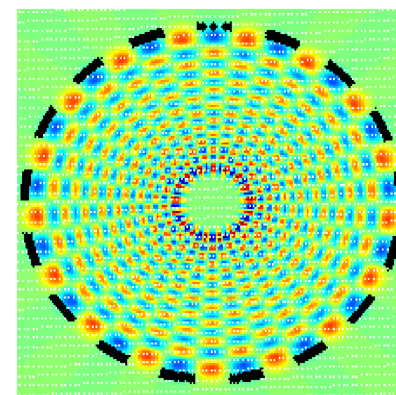
LP $_{01}$ mode | **LP $_{3,1}$ mode** | **LP $_{6,3}$ mode** | **LP $_{17,16}$ mode**

graded index fiber









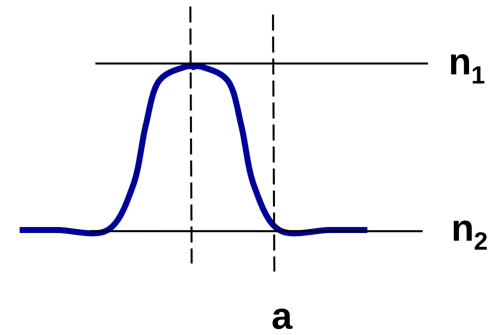
The yellow or black circles represent the boundary between the core and the cladding

a = 40µm; NA =0,24

NUMBER OF MODES ABLE TO PROPAGATE IN A MULTIMODE FIBER (*pdf page 26*)

index profile given by :

$$\begin{cases} n_{core} = n_1(r) = n_1\left[1-2\Delta\left(\dfrac{r}{a}\right)^g\right]^{1/2} & \mathrm{r} \le \mathrm{a} \\ n_{cladding} = n_2 & \mathrm{r} \ge \mathrm{a} \end{cases}$$



number of EM modes : $\mathcal{N}_{EM} = \dfrac{V^2}{2}\dfrac{g}{g+2}$

number of LP modes : $\mathcal{N}_{LP} = \dfrac{\mathcal{N}_{EM}}{4} = \dfrac{V^2}{8}\dfrac{g}{g+2}$

→ **fiber with a parabolic index profile ➔ g = 2** $\mathcal{N}_{EM} = \dfrac{V^2}{4}$ **and** $\mathcal{N}_{LP} = \dfrac{V^2}{16}$

→ **fiber with a step index profile ➔ g = ∞** $\mathcal{N}_{EM} = \dfrac{V^2}{2}$ **and** $\mathcal{N}_{LP} = \dfrac{V^2}{8}$

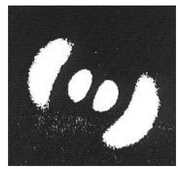
example : step index fiber, with NA = 0.2, a = 25µm, λ=0.85µm ➔ V = 37 ➔ $\mathcal{N}_{LP} \sim 170$

WHAT IS THE "ORDER" OF A MODE IN A MULTIMODE FIBER ? *(pdf page 27)*

order of the $LP_{m\,l}$ mode : $M = 2l + m - 1$

M small

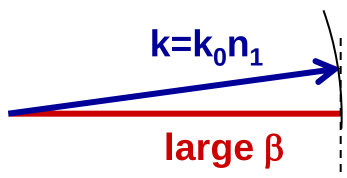
→ simple pattern of the mode
→ (low number of lobes)



→ **low order mode**

→ energy rather in the center

→ β close to k_0n_1

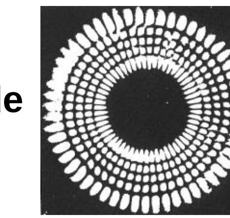


(mode associated to very inclined rays)

→ low v_φ and
→ large v_g

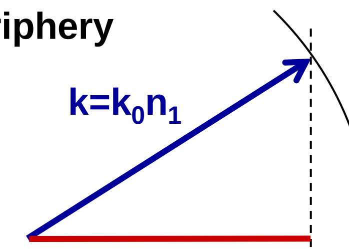
M large

→ complexe pattern of the mode
(large number of lobes)



→ **high order mode**

→ energy rather at the periphery



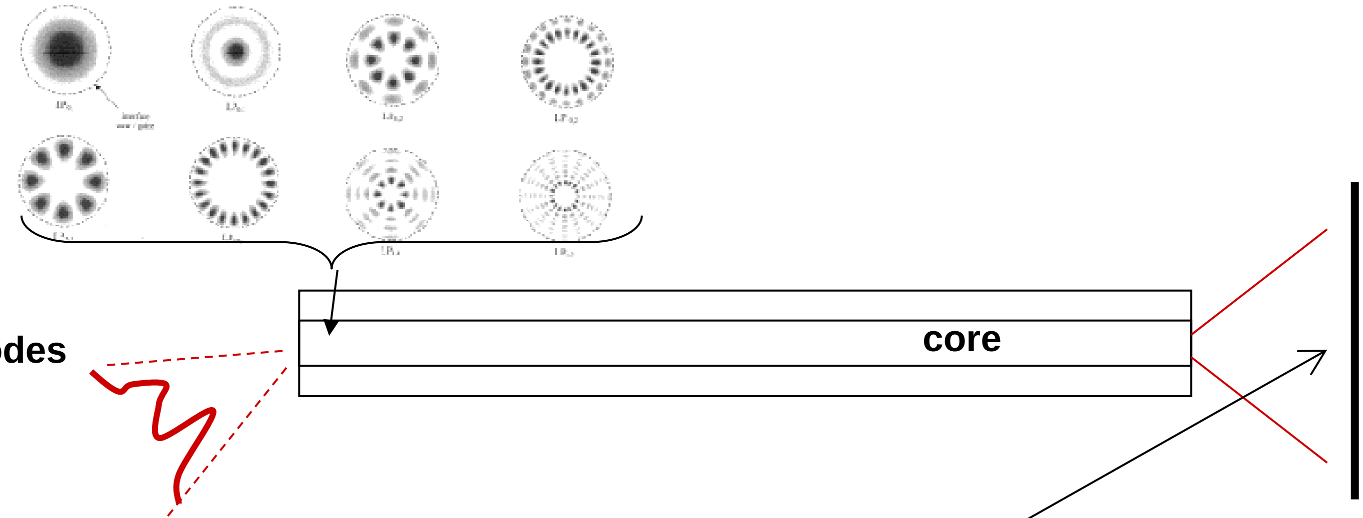
→ β close to k_0n_2

(mode associated to little inclined rays)

→ large v_φ and
→ low v_g

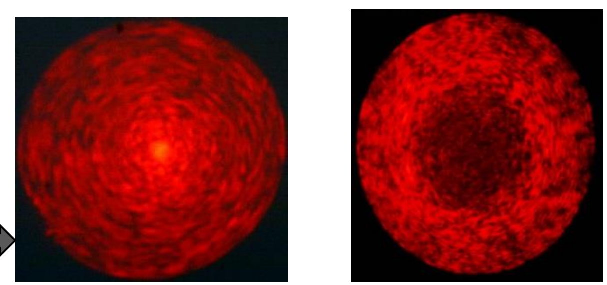
OVERLAP OF MODES, COUPLING, SPECKLE *(pdf page 28)*

Excitation of modes in the core :



Spatial overlap of guided modes : "speckle" →

high density of low order modes



high density of high order modes

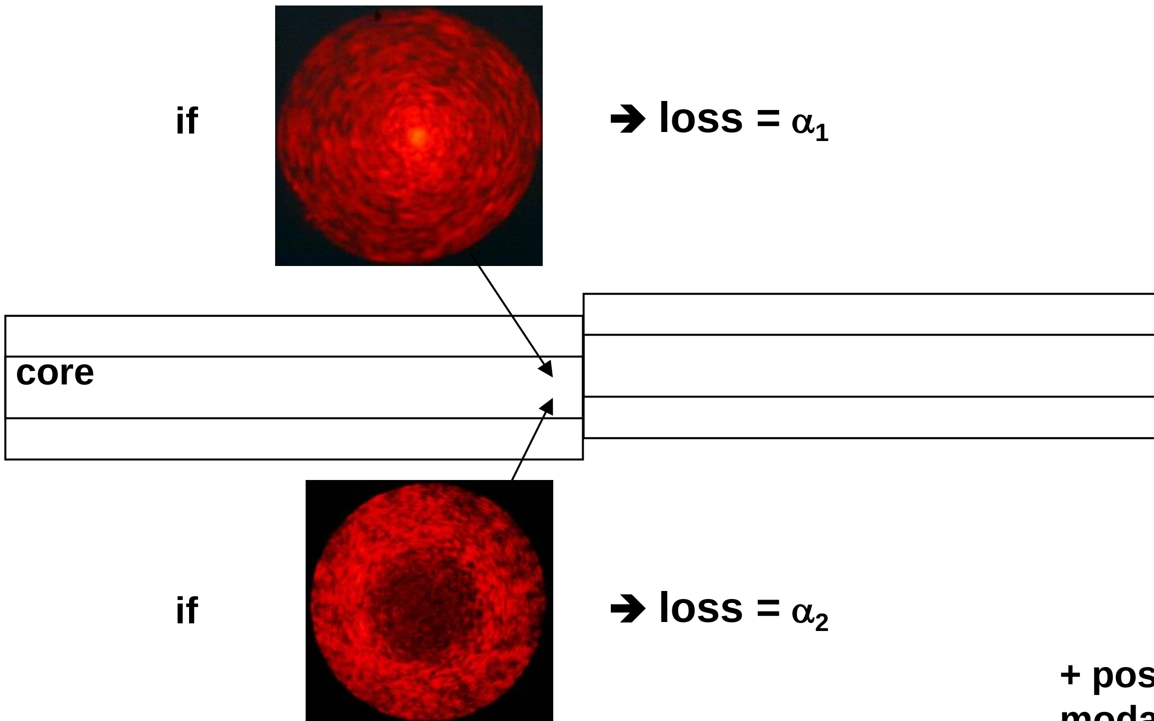
"Speckle"

Changes in the speckle along the propagation are due to :
→ changes in the relative phase shifts between the modes
→mode coupling occuring in axially non uniforme or perturbated guides (along z)

SENSITIVITY OF PERFORMANCES OF SOME COMPONENTS TO THE "MODAL POPULATION" *(pdf page 28)*

→ <u>case of a misaligned connector</u>

if ➔ **loss =** α_1



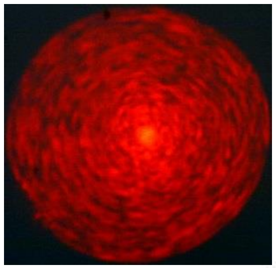
if ➔ **loss =** α_2

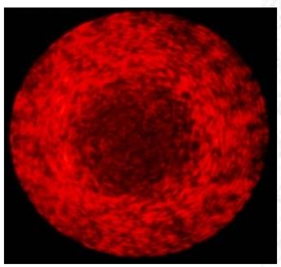
with $\alpha_2 > \alpha_1$

+ possible significant change in the modal population at the junction !

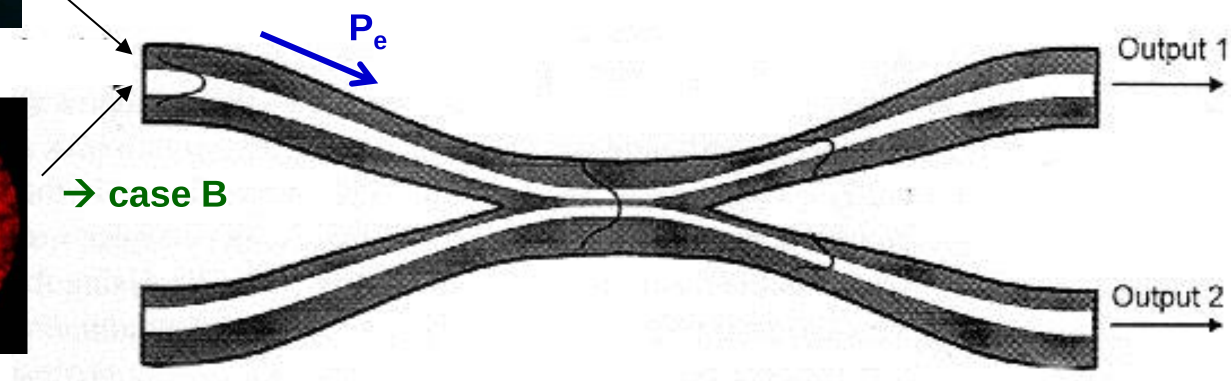
SENSITIVITY OF PERFORMANCES OF SOME COMPONENTS TO THE "MODAL POPULATION" *(pdf page 28)*

→ <u>case of a fused coupler</u>

if  → **case A**

if  → **case B**

P_e



P_{S1}

P_{S2}

$$\text{coupling ratio : } \frac{P_{S1}}{P_{S1}+P_{S2}}(caseA) > \frac{P_{S1}}{P_{S1}+P_{S2}}(caseB)$$

$$\text{excess loss (dB) : } 10\log\frac{P_e}{P_{S1}+P_{S2}}(caseB) > 10\log\frac{P_e}{P_{S1}+P_{S2}}(caseA)$$

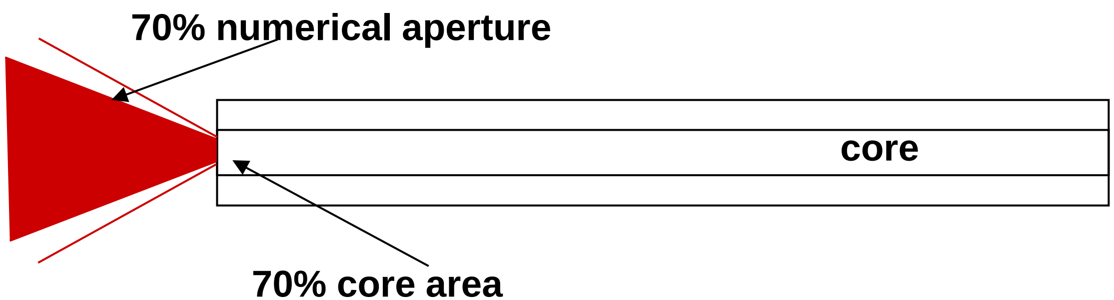
+ possible significant change in the modal population in the coupler !

SENSITIVITY OF PERFORMANCES OF SOME COMPONENTS TO THE "MODAL POPULATION" *(pdf page 28)*

➔ **necessity of characterizing (and using) the components in the conditions of "equilibrium mode distribution " allowing steady state propagation**

"Equilibrium mode distribution" : modal population which is overall invariant along the fiber

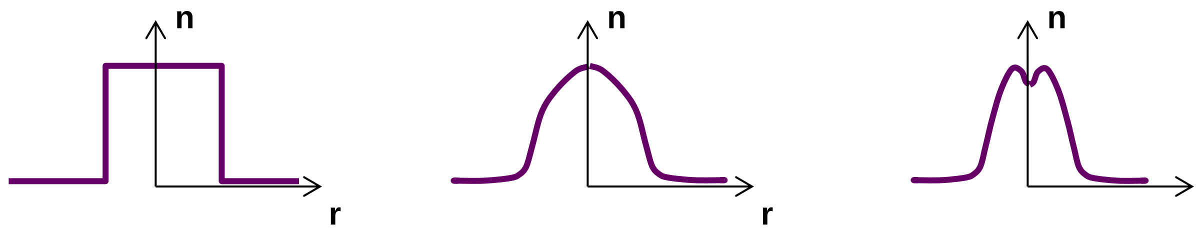
how to obtain it ? → use of a "mode scrambler"
→ or



Fine modal characterization of multimode components : "selective excitation" of modes

PROPAGATION IN SINGLE MODE FIBERS (V < 2,405) : SCALAR APPROACH (LP_{01} MODE)

index profiles:

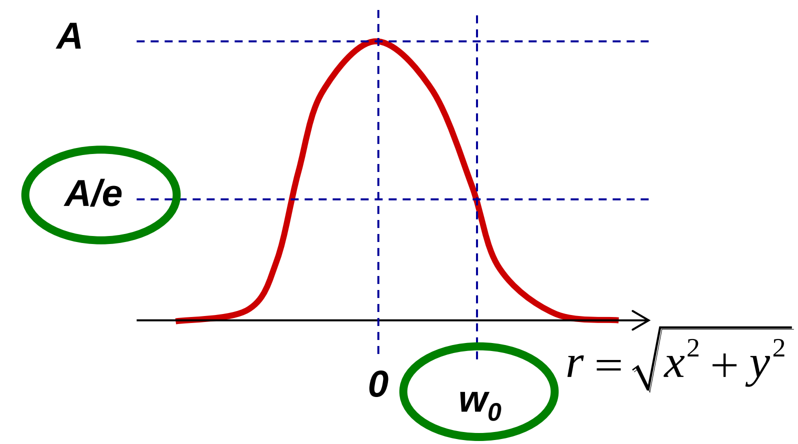


step index **graded index** **typical real profile**

if 1,2 < V < 4

whatever the index profile
LP_{01} mode ~ gaussian mode

$$\psi_{LP01}(x,y)$$



$$\psi_{LP01}(x,y) \approx A.\exp\left(-\frac{x^2+y^2}{w_0^2}\right)$$

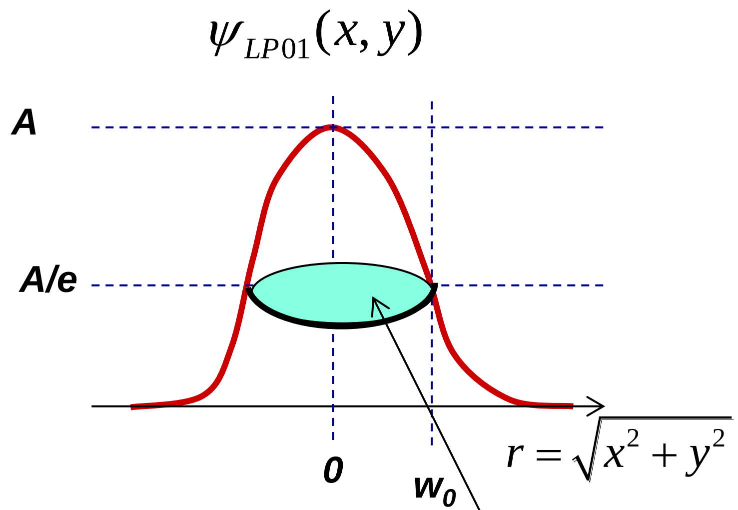
mode field radius (MFR)

PROPAGATION IN SINGLE MODE FIBERS (V < 2,405) : SCALAR APPROACH (LP_{01} MODE)

$$\psi_{LP01}(x,y) \approx A.\exp\left(-\frac{x^2+y^2}{w_0^2}\right)$$

mode field radius (MFR)

$$w_0 = a\left(0,65 + \frac{1,619}{V^{3/2}} + \frac{2,879}{V^6}\right)$$



→ In single mode fibers, loss at misaligned splices depend on w_0

General expression of the "effective area" of a mode:

$$A_{eff} = \frac{\left|\int |\psi|^2 \, dS\right|^2}{\int |\psi|^4 \, dS}$$

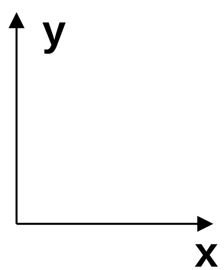
For a gaussian mode: $A_{eff} = \pi . w_0^2$

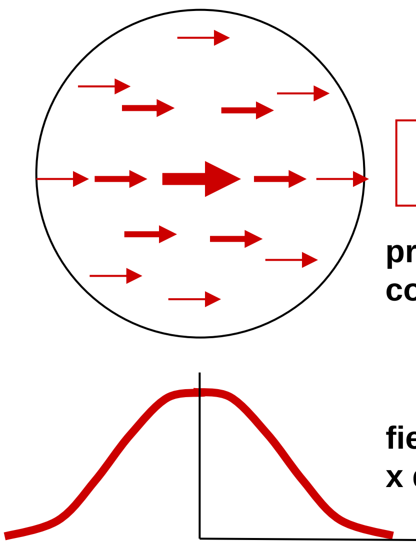
→ In single mode fibers, non linear effects (Kerr, Raman, Brillouin…) depend on A_{eff}

PROPAGATION IN SINGLE MODE FIBERS (V < 2,405) : VECTORIAL APPROACH (HE_{11} MODES)

2 expressions for the field of the HE_{11} mode

$$\begin{cases} E_x = \begin{cases} E_0 \dfrac{J_0\left(\dfrac{ur}{a}\right)}{J_0(u)} & \mathrm{r} \le \mathrm{a} \\ E_0 \dfrac{K_0\left(\dfrac{wr}{a}\right)}{K_0(w)} & \mathrm{r} \ge \mathrm{a} \end{cases} \\ E_y \simeq 0 \end{cases} \qquad \begin{cases} E_x \simeq 0 \\ E_y = \begin{cases} E_0 \dfrac{J_0\left(\dfrac{ur}{a}\right)}{J_0(u)} & \mathrm{r} \le \mathrm{a} \\ E_0 \dfrac{K_0\left(\dfrac{wr}{a}\right)}{K_0(w)} & \mathrm{r} \ge \mathrm{a} \end{cases} \end{cases}$$

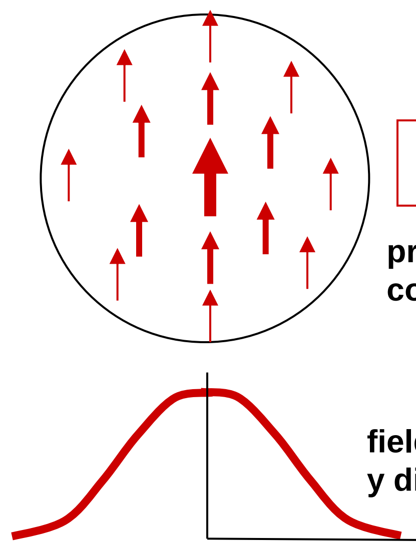




HE_{11x}

propagation constant β_x

field in the x direction



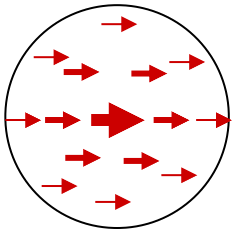
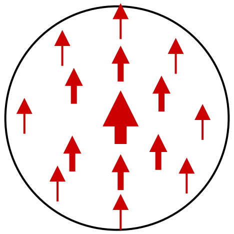
HE_{11y}

propagation constant β_y

field in the y direction

PROPAGATION IN SINGLE MODE FIBERS (V < 2,405) : VECTORIAL APPROACH (HE_{11} MODES)

polarization states of light in the core

if  and  are excited

$$\vec{E} = E_x \cos(\omega t - \beta_x z).\vec{ex} + E_y \cos\left[\omega t - \beta_y z\right].\vec{ey}$$

$$\Rightarrow \vec{E} = E_x \cos(\omega t - \beta_x z).\vec{ex} + E_y \cos\left[\omega t - \beta_x z + \underbrace{\overbrace{(\beta_x - \beta_y)}^{\delta\beta} z}_{\varphi = \delta\beta.z}\right].\vec{ey}$$

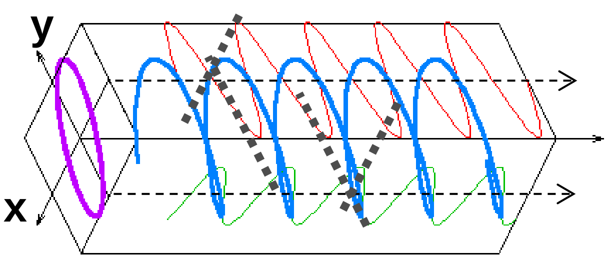
$\beta_x - \beta_y = \delta\beta$: linear birefringence of the fiber

for a given z :

* if $\varphi = \delta\beta.z = 0$: linear polarization

* if $\varphi = \delta\beta.z = \pi/2$ and $E_x = E_y$: circular polarization

* general case (any φ and/or $E_x \neq E_y$: elliptical polarization



PROPAGATION IN SINGLE MODE FIBERS (V < 2,405) : VECTORIAL APPROACH (HE_{11} MODES)

phase effects of the birefringence of the fiber (1) :

$$\vec{E} = E_x \cos(\omega t - \beta_x z).\vec{ex} + E_y \cos\left[\omega t - \beta_x z + \overbrace{(\beta_x - \beta_y)}^{\beta_x - \beta_y = \delta\beta = \frac{2\pi}{\lambda}(n_{ex} - n_{ey})} z\right].\vec{ey}$$

$B_\varphi = \delta\beta / k_0 = | \, n_{ex} - n_{ey} \, |$ B_φ : <u>normalized phase birefringence</u>

* if $n_{ex} \neq n_{ey}$: $v_{\varphi x} = c/n_{ex} \neq v_{\varphi y} = c/n_{ey}$ → phase velocities of HE_{11x} and HE_{11y} are different

* along z, the phase shift φ between Ex and Ey increases (→ polarization state changes : see previous and next slides)

* the phase shift φ increases by Δφ = 2π over a length L_b (i.e. between z and z+ L_b)

L_b : <u>beat length of the fiber</u>

$$\Delta\varphi = 2\pi \quad \text{and} \quad \Delta\varphi = \varphi(z + L_B) - \varphi(z) = \frac{2\pi}{\lambda_0}\left|n_{ex} - n_{ey}\right| L_b \quad \Rightarrow \quad L_b = \frac{\lambda_0}{\left|n_{ex} - n_{ey}\right|} = \frac{\lambda_0}{B_\varphi}$$

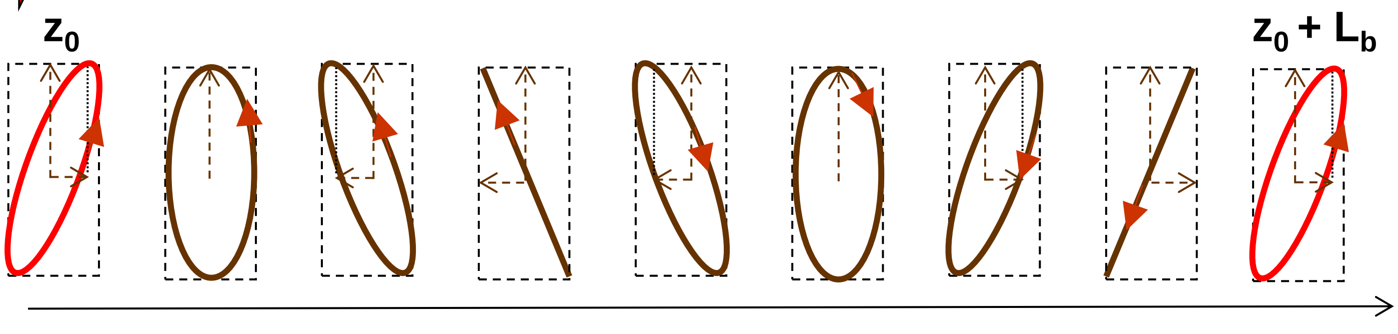
PROPAGATION IN SINGLE MODE FIBERS (V < 2,405) : VECTORIAL APPROACH (HE_{11} MODES)

phase effects of the birefringence of the fiber (2) :

$$\beta_x - \beta_y = \delta\beta = \frac{2\pi}{\lambda}(n_{ex} - n_{ey})$$

$$\vec{E} = E_x \cos(\omega t - \beta_x z).\vec{ex} + E_y \cos\left[\omega t - \beta_x z + \overbrace{(\beta_x - \beta_y)}^{\delta\beta} z\right].\vec{ey}$$

if $\delta\beta \neq 0$: polarization changes with z

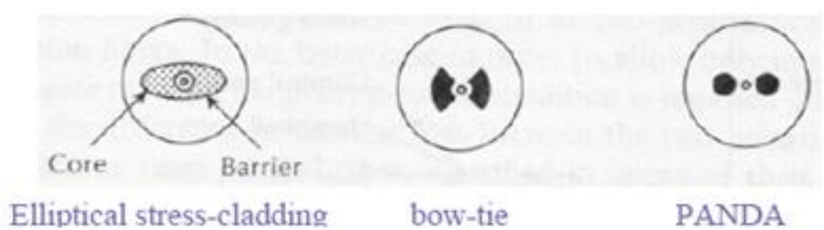


$\delta\beta . L_b = 2\pi$

L_b = beat length

In order to maintain a linear polarization in a single mode fiber :

- **excite only one mode (HE_{11x} or HE_{11y})**
- **avoid mode coupling → L_b as short as few mm → high $\delta\beta$ required → highly birefringent fibers**



"polarization maintaining fibers or PM fibers"

PROPAGATION IN SINGLE MODE FIBERS (V < 2,405) : VECTORIAL APPROACH (HE_{11} MODES)

group effects of the birefringence of the fiber :

$$\vec{E} = E_x \cos(\omega t - \beta_x z).\vec{ex} + E_y \cos\left[\omega t - \beta_x z + \overbrace{(\beta_x - \beta_y)}^{\beta_x-\beta_y = \delta\beta = \frac{2\pi}{\lambda}(n_{ex} - n_{ey})} z\right].\vec{ey}$$

If $n_{ex} \neq n_{ey}$, then $N_{ex} \neq N_{ey}$ and $v_{gx} = \dfrac{c}{N_{ex}} \neq v_{gy} = \dfrac{c}{N_{ey}}$

→group velocities of pulses propagating in HE_{11x} and HE_{11y} are different

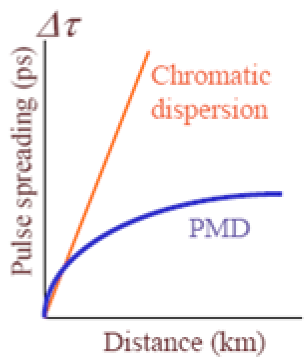
→ <u>polarization mode dispersion (PMD)</u>

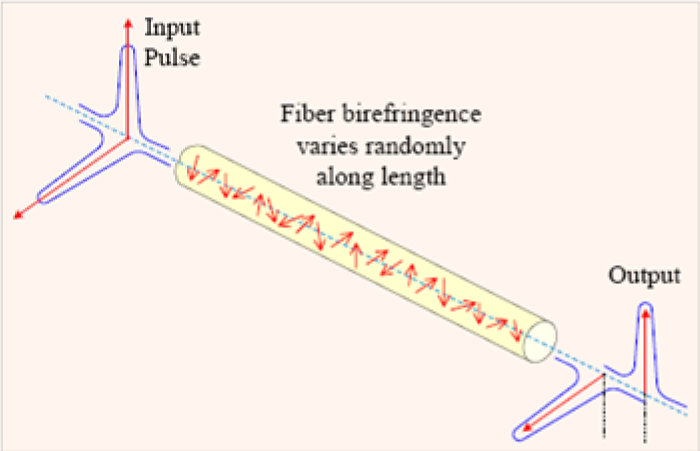
$B_G = N_{ex} – N_{ey}$

B_G : <u>normalized group birefringence</u>

$$\Delta t = \left|t_{gx} - t_{gy}\right| = \left|\frac{z}{v_{gx}} - \frac{z}{v_{gy}}\right|$$

$$= \left|N_{gx} - N_{gy}\right|\frac{z}{c} = B_G.\frac{z}{c}$$





Pulse spreading caused by polarization dispersion:

due to mode coupling along the fiber (if not PM fiber) → $\Delta\tau = D_{PMD}\sqrt{L}$ $D_{PMD} \sim 0.05$ to 1 ps/km$^{0.5}$

If R=40Gbps, L=100→ D_{PMD} <0.25 ps/km$^{0.5}$ to have $\Delta\tau$ < 2.5ps

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End of chapter 3

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