Tutorial 3 :

$\Delta t = 136\ fs$

$\Delta\nu = Rep = 470\ MHz$

$\lambda_0 = 859\ nm$

$P_{out} = 25\ mW$

1) → Course

2) → Course

3)

$$B \underset{\uparrow}{=} 1/\Delta t \Rightarrow B = \frac{1}{136\ 10^{-15}} = 7.3\ 10^{12}\ Hz$$

Mode lock regime

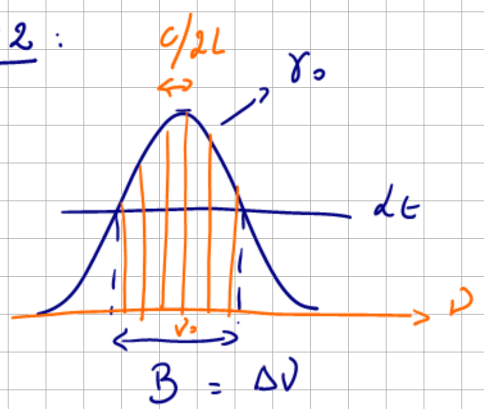
$$\nu = \frac{c}{\lambda} \Rightarrow \Delta\nu = B = \frac{c}{\lambda_0^2}\Delta\lambda \Rightarrow \Delta\lambda = \frac{\lambda_0^2}{c}.B = 18\ nm.$$

$$Rep = \frac{c}{2[L]} \Rightarrow [L] = \frac{c}{2\,Rep} = \frac{3.10^8}{2.\ 470\ 10^6} = 32\ cm.$$

$$N = \frac{B}{Rep} = 15\ 532$$

$$\hat{P} = \frac{\bar{P}}{\Delta t.\ Rep} = 390\ W.$$

Ex2 :



1) a) $M \simeq \frac{B}{\frac{c}{2L}} = \text{Integer part}\left(\frac{B}{\frac{c}{2L}}\right) + 1$

$$\nu_m = \nu_0 + m\frac{c}{2L}$$

1.b)

$$E(t,z) = \sum_m A_m.\, e^{j\left(2\pi\nu_m t - \frac{2\pi\nu_m}{c} z\right)} \qquad \varphi_m = 0$$

$$k_m = \frac{2\pi}{\lambda_m} = \frac{2\pi\nu_m}{c}$$

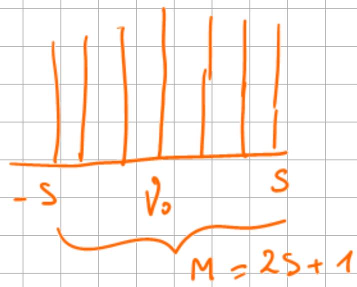
$$E(z,t) = \sum_m A_m \, e^{j 2\pi \nu_m (t - z/c)}$$

1.c) $$E(z,t) = A(t - z/c)\, e^{j 2\pi \nu_0 (t - z/c)}$$

$$\nu_m = \nu_0 + m \cdot \frac{c}{2L}$$

$$E(z,t) = \sum_{m=-S}^{+S} A_m \, e^{j 2\pi (\nu_0 + m \frac{c}{2L})(t - z/c)} \qquad \frac{c}{2L} = 1/T \qquad T = \frac{2L}{c}$$

$$= \underbrace{\sum_{-S}^{+S} A_m \, e^{j \frac{2\pi m}{T} (t - z/c)}}_{A(t - z/c)} \cdot e^{j 2\pi \nu_0 (t - z/c)}$$



2.a) $$X = \frac{2\pi}{T}(t - z/c) \qquad A_m = A$$

$$A(t - z/c) = A \sum_{-S}^{+S} e^{j X m} = A \cdot \left(e^{-jSX} + \cdots + e^{+jSX} \right) =$$

$$= A e^{-jSX} \underbrace{\left(1 + \underbrace{e^{jX}}_{q} + \cdots + e^{j 2SX} \right)}_{Q}$$

$$Q = 1 + q + \cdots q^{2SX}$$

$$qQ = q + \cdots q^{\underbrace{(2S+1)}_{M}X}$$

$$Q - qQ = 1 - q^{MX}$$

$$\boxed{Q = \frac{1 - q^{MX}}{1 - q}}$$

$$A(t - z/c) = A\, e^{-jSX} \cdot \frac{1 - e^{jXM}}{1 - e^{jX}} = A\, e^{-jSX} \frac{e^{j\frac{XM}{2}} \cdot \left(e^{-j\frac{XM}{2}} - e^{j\frac{XM}{2}} \right)}{e^{j\frac{X}{2}} \left(e^{-j\frac{X}{2}} - e^{j\frac{X}{2}} \right)}$$

$$A(t - z/c) = A\, \cancel{e^{-jSX}} \, e^{j\frac{X}{2}(\cancel{M} - 1)} \cdot \frac{\sin \frac{MX}{2}}{\sin \frac{X}{2}}$$

$M = 2S + 1 \Rightarrow \frac{M-1}{2} = S$ $\qquad X = \frac{2\pi}{T}(t - z/c)$

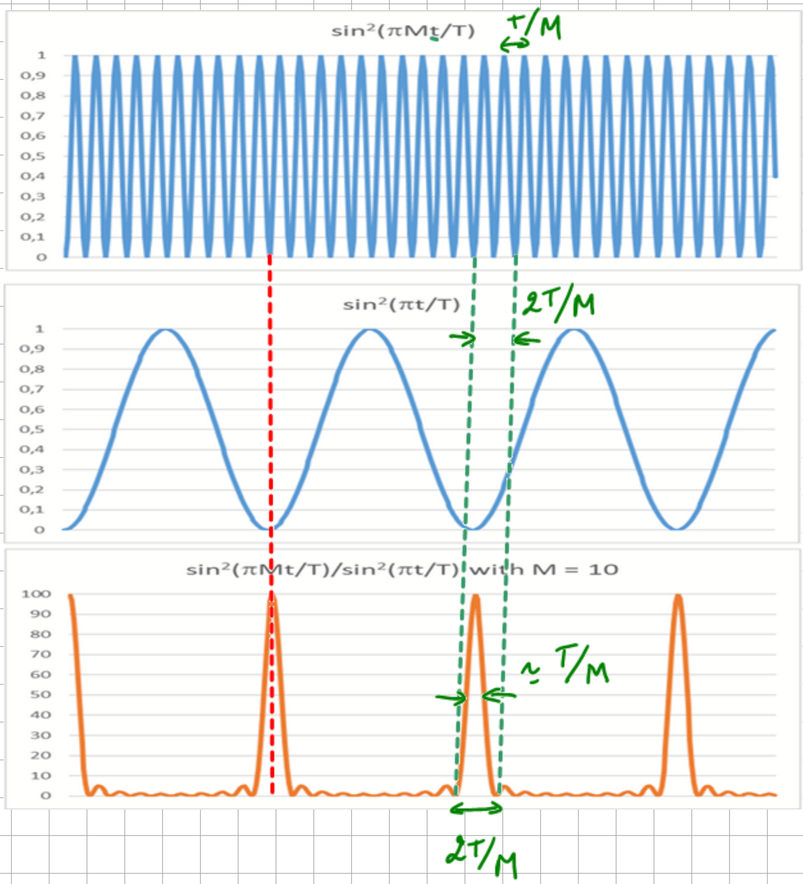
$$\boxed{A(t - z/c) = A \cdot \frac{\sin M\frac{\pi(t - z/c)}{T}}{\sin \frac{\pi(t - z/c)}{T}}}$$

2.b) $I(t, z) = \left| E(M,t) \right|^2 = A^2 \dfrac{\sin^2 \frac{M\pi(t - z/c)}{T}}{\sin^2 \frac{\pi(t - z/c)}{T}}$

$$\boxed{I(t, z=0) = A^2 \cdot \frac{\sin^2 M\pi t/T}{\sin^2 \pi t/T}}$$

$\sin^2 M\pi t/T \longrightarrow$ period T/M

$\sin^2 \pi t/T \longrightarrow T$



Pulse duration $\frac{T}{M} = \Delta t$

$T = \frac{2L}{c}$

$M \simeq \frac{B}{\frac{c}{2L}}$

$\boxed{\Delta t = \frac{1}{B}}$

NA : $\Delta t = 8$ ps.