

# Exam 2021 - June - 18

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## Problem 1

$$R = 8 \text{ cm}, \quad \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

$z_1 \rightarrow$  object dist.

$z_2 \rightarrow$  image dist.

$$f = \frac{R}{2}$$

A) Concave mirror &  $z_1 < 4 \text{ cm}$

$$R = -8 \text{ cm} \rightarrow \frac{1}{f} = \frac{2}{-R} = \frac{2}{-(-8)} = \frac{1}{4} \quad \left| \begin{array}{l} z_1 < 4 \text{ cm} \\ \text{for instance } z_1 = 3 \text{ cm} \end{array} \right.$$

$$\frac{1}{z_2} = \frac{1}{4} - \frac{1}{3} \Rightarrow z_2 = -12 \text{ cm} \rightarrow \text{virtual image, } \underline{\text{not valid}}$$

B) Convex mirror &  $z_1 < 4 \text{ cm}$

$$R = 8 \text{ cm} \rightarrow \frac{1}{f} = -\frac{1}{4} \quad \left| \begin{array}{l} z_1 < 4 \text{ cm} \Rightarrow z_1 = 3 \text{ cm} \end{array} \right.$$

$$\frac{1}{z_2} = -\frac{1}{4} - \frac{1}{3} = -\frac{7}{12} \rightarrow z_2 = -\frac{12}{7} \text{ cm} \rightarrow \text{virtual, } \underline{\text{not valid}}$$

C) Concave mirror &  $z_1 > 4 \text{ cm}$

$$R = -8 \text{ cm} \rightarrow \frac{1}{f} = \frac{1}{4} \quad \left| \begin{array}{l} z_1 = 6 \text{ cm} \end{array} \right.$$

$$\frac{1}{z_2} = \frac{1}{4} - \frac{1}{6} = \frac{1}{20} \rightarrow \boxed{z_2 = 20 \text{ cm} \rightarrow \text{real, valid}}$$

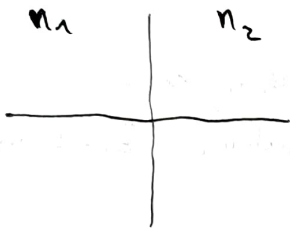
D) convex &  $z_1 > 4 \text{ cm}$

$$R = 8 \text{ cm} \rightarrow \frac{1}{f} = -\frac{1}{4} \quad \Bigg| \quad z_1 = 5 \text{ cm}$$

$$\frac{1}{z_2} = -\frac{1}{4} - \frac{1}{5} = \frac{-9}{20} \rightarrow z_2 = -\frac{20}{9} \text{ cm} \rightarrow \text{virtual, not valid}$$

Problem 2)

$$n_2 = \frac{n_1}{3}$$



$$\Rightarrow n_1 > n_2$$

a)

Calculate  $\theta_t$  with snell:

$$n_i \sin(\theta_i) = n_t \sin(\theta_t) \rightarrow \sin \theta_t = \frac{n_i}{n_t} \sin(\theta_i) \overset{\theta_i=0}{=} 0$$
$$\theta_t = 0$$

$$r = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)} = \frac{n_1 \cdot 1 - \frac{n_1}{3} \cdot 1}{n_1 \cdot 1 + \frac{n_1}{3} \cdot 1} = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{1}{2}$$

$$R = |r|^2 = \frac{1}{4} \rightarrow T = 1 - R = \frac{3}{4}$$

b)

(2)

To have zero reflectance.

$$r_{TE} = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

We cannot achieve zero reflectance with TE waves

$$r_{TM} = \frac{n_i \cos(\theta_t) - n_t \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

The condition is

$$n_i \cos \theta_t = n_t \cos \theta_i$$

for TM waves

$$\text{Brewster: } \theta_B = \arctan\left(\frac{n_2}{n_1}\right) = \arctan\left(\frac{1/3}{1}\right) = \arctan\left(\frac{1}{3}\right)$$

$$\boxed{\theta_B \approx 18.44^\circ}$$

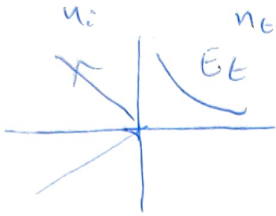
c) the critical angle is when we are travelling from medium 1 to medium 2 and:  $n_1 > n_2$  and  $\theta_c = \frac{\pi}{2}$

So;

$$n_1 \sin(\theta_i) = n_2 \sin(\theta_t) \underset{\theta_t = \frac{\pi}{2}}{=} n_2 \rightarrow \sin \theta_c = \frac{n_2}{n_1} = \frac{1/3}{1}$$

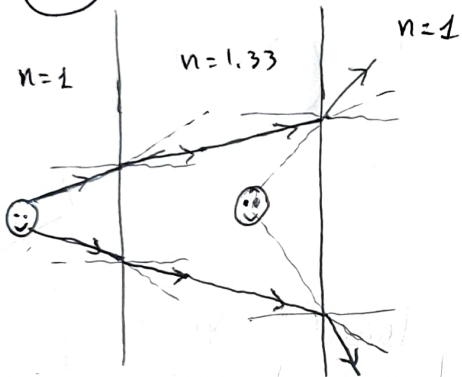
$$\boxed{\theta_c = 19.47^\circ}$$

d) Under these circumstances we have the evanescent wave that is different from zero.



### Problem 3/

1a)

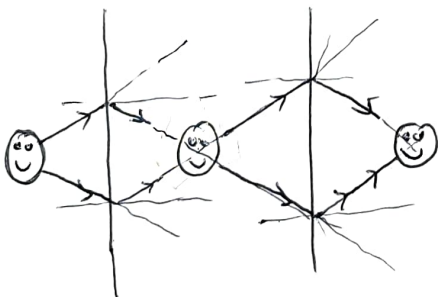


First: Going from  $n_i < n_t \Rightarrow \theta_i > \theta_t$

Second: Going from  $n_i > n_t \Rightarrow \theta_i < \theta_t$

Virtual image

1b)



First: Same angle as before but  $k$  opposite sign

Second: Same angle,  $k$  opposite sign

Real image.

To achieve this we need to have a medium with  $\epsilon < 0$  and  $\mu < 0$ .