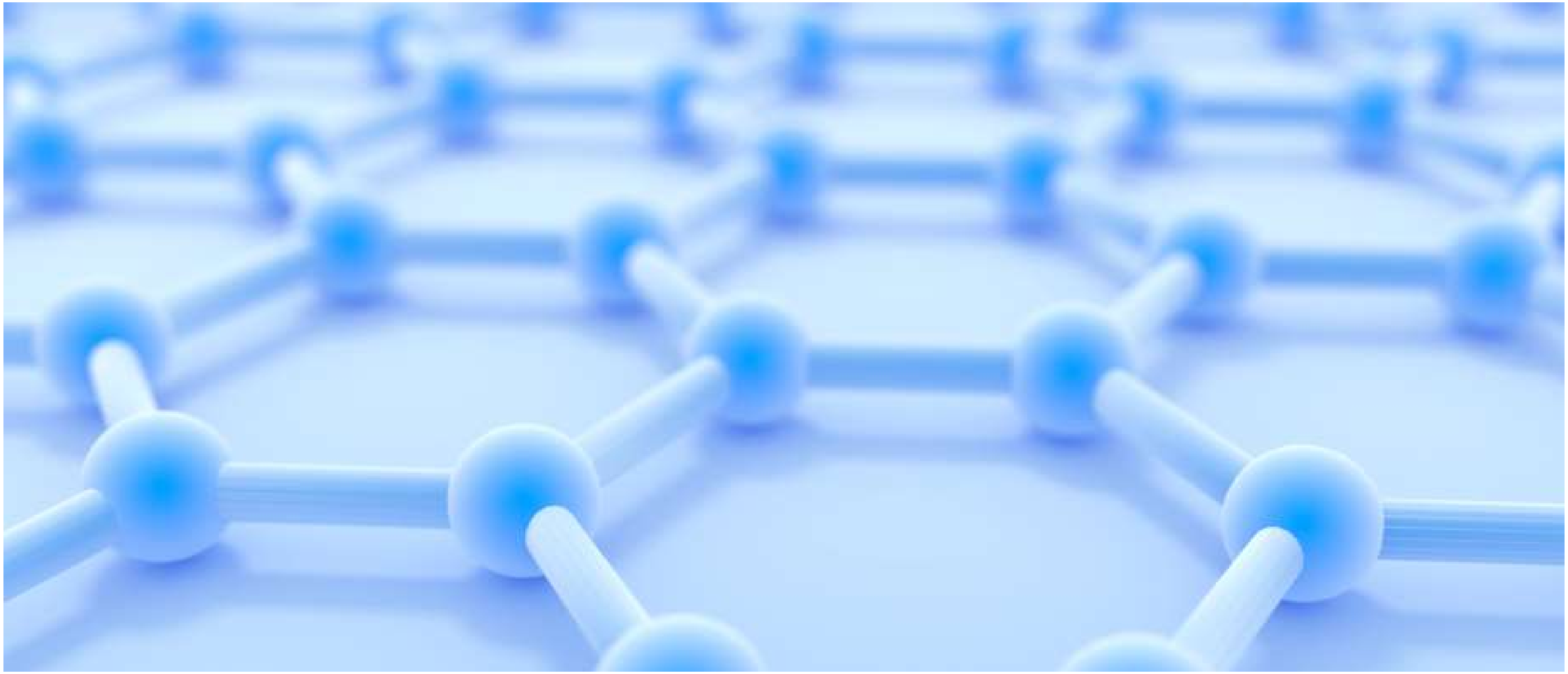

Photonics

aa 2021/2022

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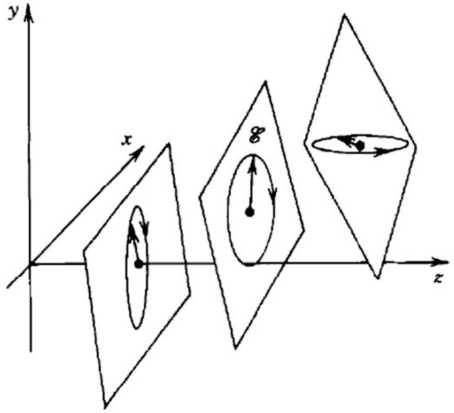
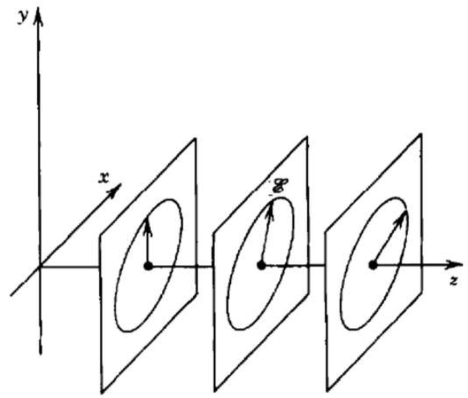
Polarization Optics:
Polarization of light

Polarization

Let’s consider a plane wave propagating in the z direction $\mathbf{k} = k_z\hat{z}$. The E field vector must be perpendicular to $\mathbf{k}$, therefore it lies in the x-y plane.





POLARIZATION

position of the tip of the E field vector in the plane xy (perpendicular to the propagation) at a position r and a time t.

Light is naturally unpolarized, i.e., the polarization direction changes randomly.

Light can be generated or modified in order to be polarized.
A POLARIZER is a device that transforms unpolarized light into polarized light.

Polarization

Why is polarization important?

- ✓ The amount of light reflected at the boundary between two media depends on the polarization of the incident wave;
- ✓ The amount of light absorbed by certain materials is polarization dependent;
- ✓ Light scattering from matter is generally polarization sensitive;
- ✓ The refractive index of anisotropic materials depends on polarization;
- ✓ The polarization plane of linearly polarized light can be rotated when passing through certain media (optically active media, liquid crystals, magnetic materials,..)

Polarization

If we consider a monochromatic plane wave traveling in the z direction, we can write the electric field in the following form:

$$\bar{E} = \hat{x}E_x + \hat{y}E_y = \hat{x}E_{x0}e^{-jkz} + \hat{y}E_{y0}e^{-jk}$$

where E_{x0} e E_{y0} are complex and can be written as $E_{x0} = a_x e^{j\varphi_x}, E_{y0} = a_y e^{j\varphi_y}$.

We can rearrange the expression of E by highlighting the phase difference between the y and x components,

$$\bar{E} = \left(a_x\hat{x} + a_y\,\hat{y}e^{j\varphi}\right)e^{j\varphi_x}e^{-jkz}\text{, with } \varphi = \varphi_y - \varphi_x\text{, and set } \varphi_x = 0.$$

Therefore

$$\bar{E} = \left(a_x\hat{x} + a_y\,\hat{y}e^{j\varphi}\right)e^{-jkz}$$

Polarization

$$\bar{E} = \left(a_x \hat{x} + a_y\, \hat{y} e^{j\varphi}\right) e^{-jk}$$

From this expression one can derive the real time expression of the field as:

$$\bar{e}(t,z) = \Re e\left\{\bar{E} e^{j\omega}\ \right\} = \hat{x} a_x \cos(\omega t - kz) + \hat{y} a_y \cos(\omega t - kz + \varphi)$$

In this expression we can identify two important quantities:

The **amplitude**: $\overline{|e}(t,z)| = \sqrt{a_x^2 cos^2(\omega t - kz) + a_y^2 cos^2(\omega t - kz + \varphi)}$.

The «**direction**» , i.e., the **polarization** is defined in the x-y plane by the angle:

$$\psi = arctg\left(\frac{e_y(z,t)}{e_x(z,t)}\right).$$

NOTE: amplitude and direction of the electric field vector are functions of time, even if the e-field stays always perpendicular to the propagation direction

Linear Polarization

$$\varphi = 0 \text{ or } \varphi = \pi$$

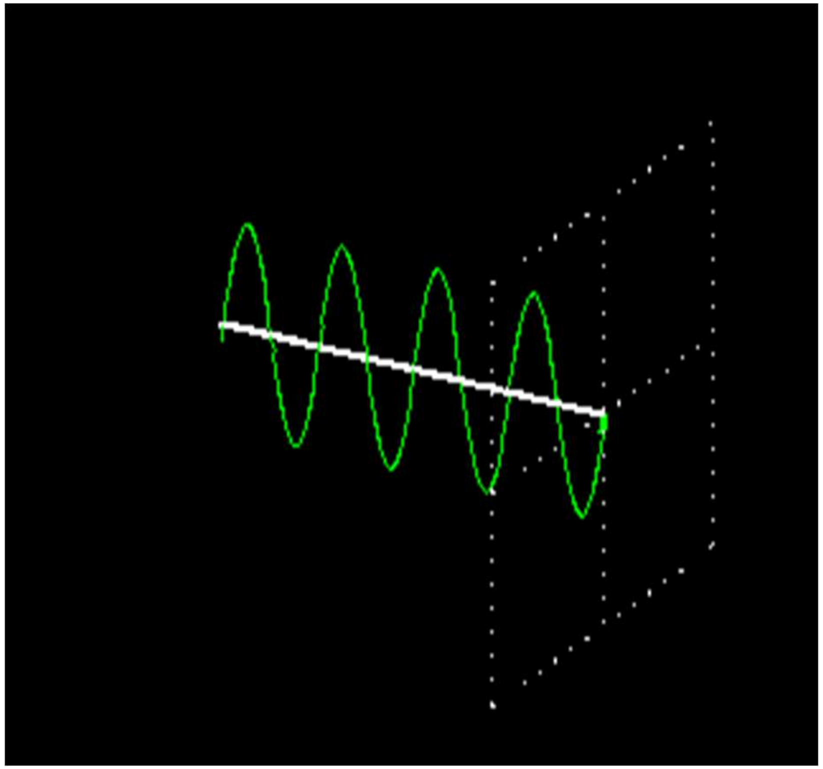
We fix z=0, the tip of vector $\bar{e}(t,0)$ moves in time along a line in the x-y plane:

$$\bar{e}(t,0) = \left(\hat{x}a_x + \hat{y}a_y\right)\cos(\omega t),\ \varphi = 0$$
$$\bar{e}(t,0) = \left(\hat{x}a_x - \hat{y}a_y\right)\cos(\omega t),\ \varphi = \pi$$

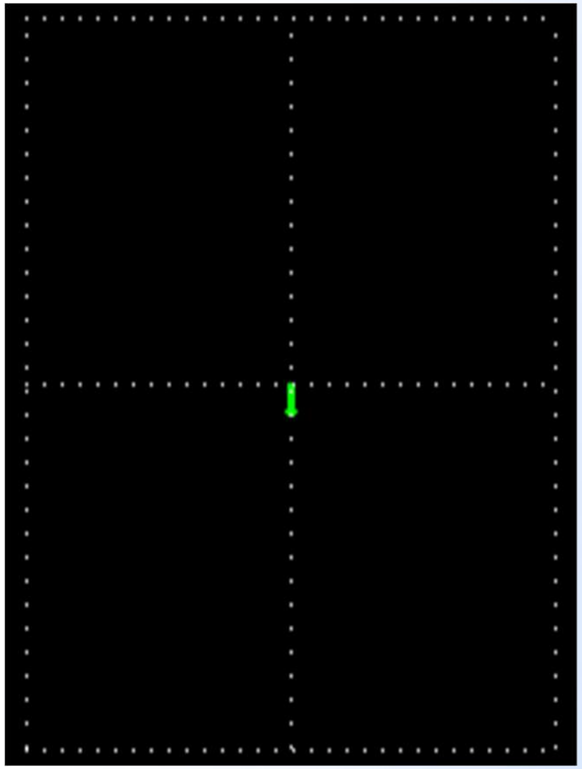
Example: for $\varphi = \pi$ the **polarization direction** at z=0 is $\psi = arctg\left(\frac{e_y(z,t)}{e_x(z,t)}\right) = arctg\left(\frac{-a_y}{a_x}\right)$ and **it is constant in time a space.**

Linear Polarization

E-field 3D view



E-field front view



Right Handed Circular Polarization

$$\varphi = -\pi/2, a_x = a_y = a$$

We fix z=0, the end of vector $\bar{e}(t,0)$ moves in time in the x-y plane:

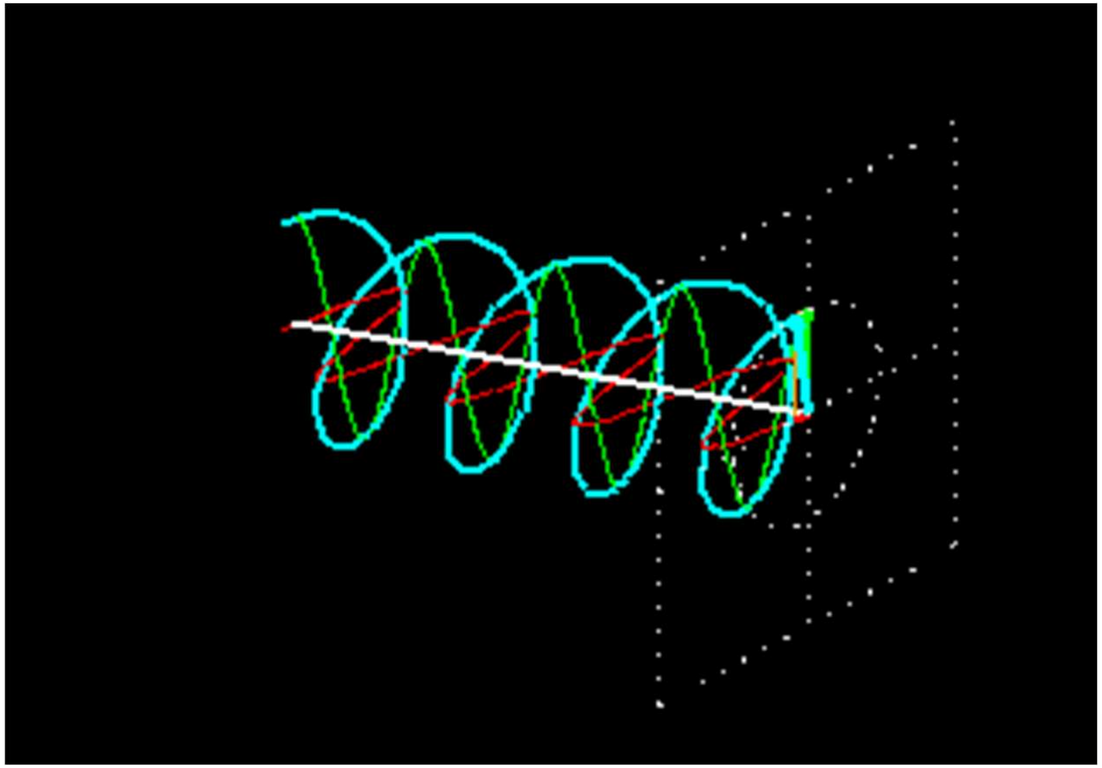
$$\bar{e} = \hat{x}a\cos(\omega t - kz) + \hat{y}a\cos(\omega t - kz - \pi/2) = \hat{x}a\cos(\omega t) + \hat{y}a sin(\omega t)$$

The **polarization direction** at z=0 is $\psi = arctg\left(\frac{e_y(z,t)}{e_x(z,t)}\right) = arctg\left(\frac{sin(\omega t)}{cos(\omega t)}\right)$ and **it moves on a x-y plane circle counterclockwise**

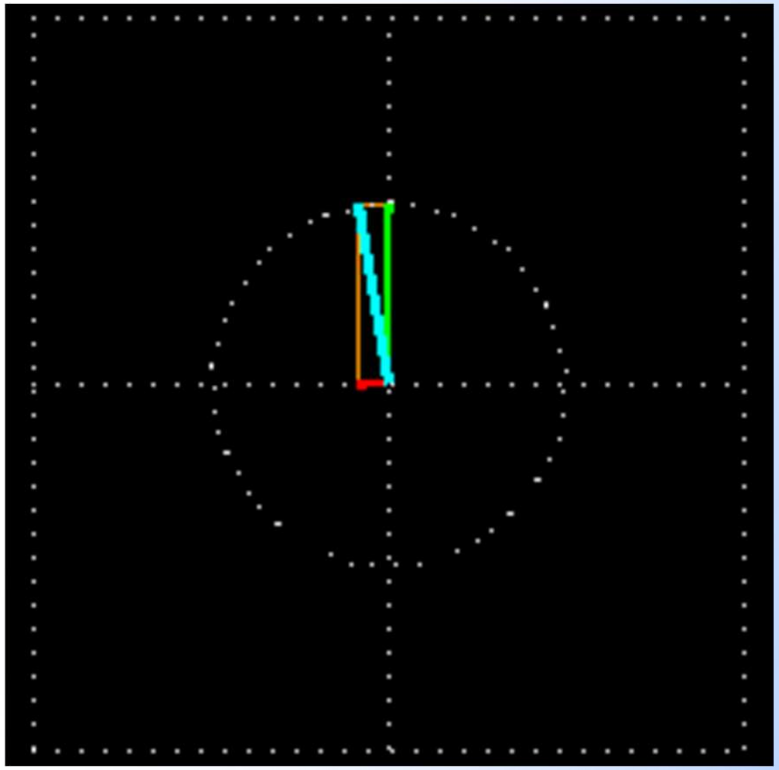
OBSERVATION: A left-circularly polarized wave is the superposition of two linearly-polarized waves, one polarized along x and the other polarized along pi/2 and phase-shifted by $-\pi/2$

Right Handed Circular Polarization

E-field 3D view



E-field front view



Left Handed Circular Polarization

$$\varphi = \pi/2\,, a_x = a_y = a$$

We fix z=0, the tip of vector $\bar{e}(t,0)$ moves in time:

$$\bar{e} = \hat{x}a\cos(\omega t - kz) + \hat{y}a\cos(\omega t - kz + \pi/2) = \hat{x}a\cos(\omega t) - \hat{y}asin(\omega t)$$

The **polarization direction** at z=0 is $\psi = arctg\left(\frac{e_y(z,t)}{e_x(z,t)}\right) = arctg\left(\frac{-sin(\omega t)}{cos(\omega t)}\right)$ and **it moves on a x-y plane circle clockwise**

OBSERVATION: A right(left)-circularly polarized wave is the superposition of two linearly-polarized waves, one polarized along x and the other polarized along pi/2 and phase-shifted by –(+)$\pi/2$

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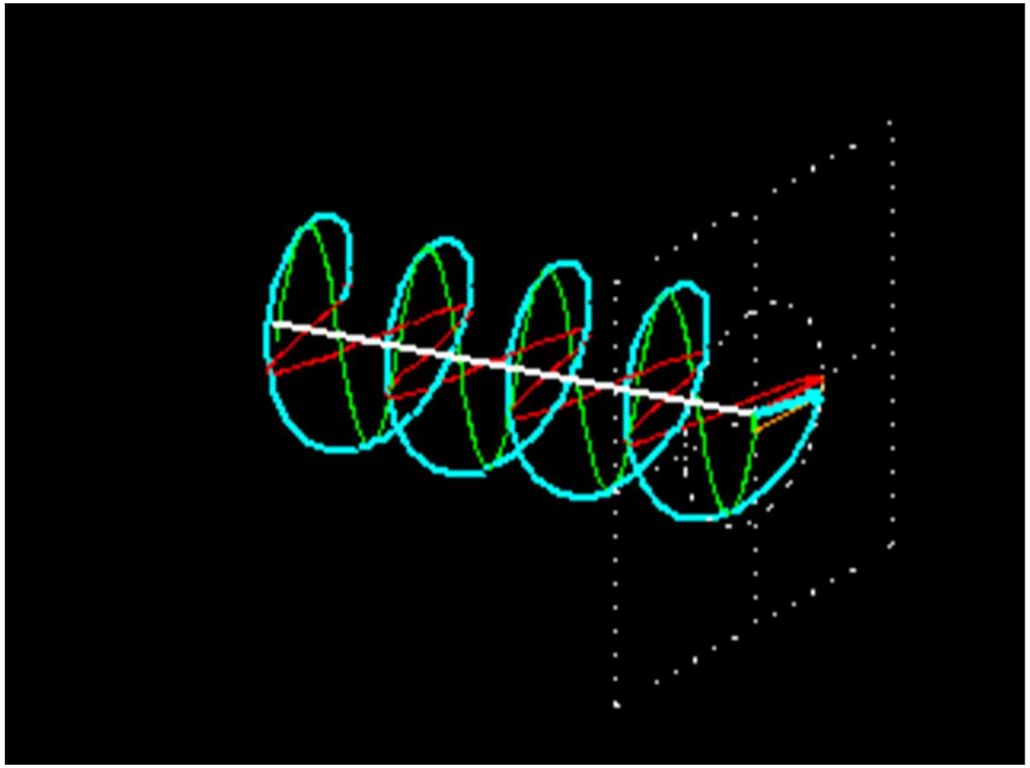
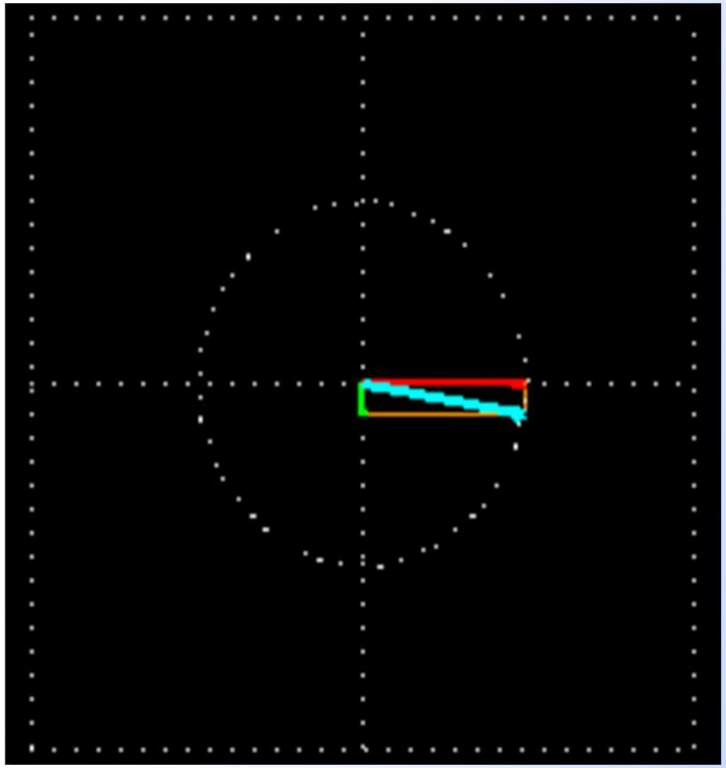
Left Handed Circular Polarization

E-field 3D view



E-field front view



RHP + LHP

What happens if we sum a right- and a left-handed circularly polarized waves of the same amplitude?

$$\bar{e} = \underbrace{\hat{x}a\cos(\omega t - kz) + \hat{y}a\cos(\omega t - kz + \pi/2)}_{\text{LHP}} + \underbrace{\hat{x}a\cos(\omega t - kz) + \hat{y}a\cos(\omega t - kz - \pi/2)}_{\text{RHP}}$$

$$\bar{e} = \hat{x}2a\cos(\omega t)$$

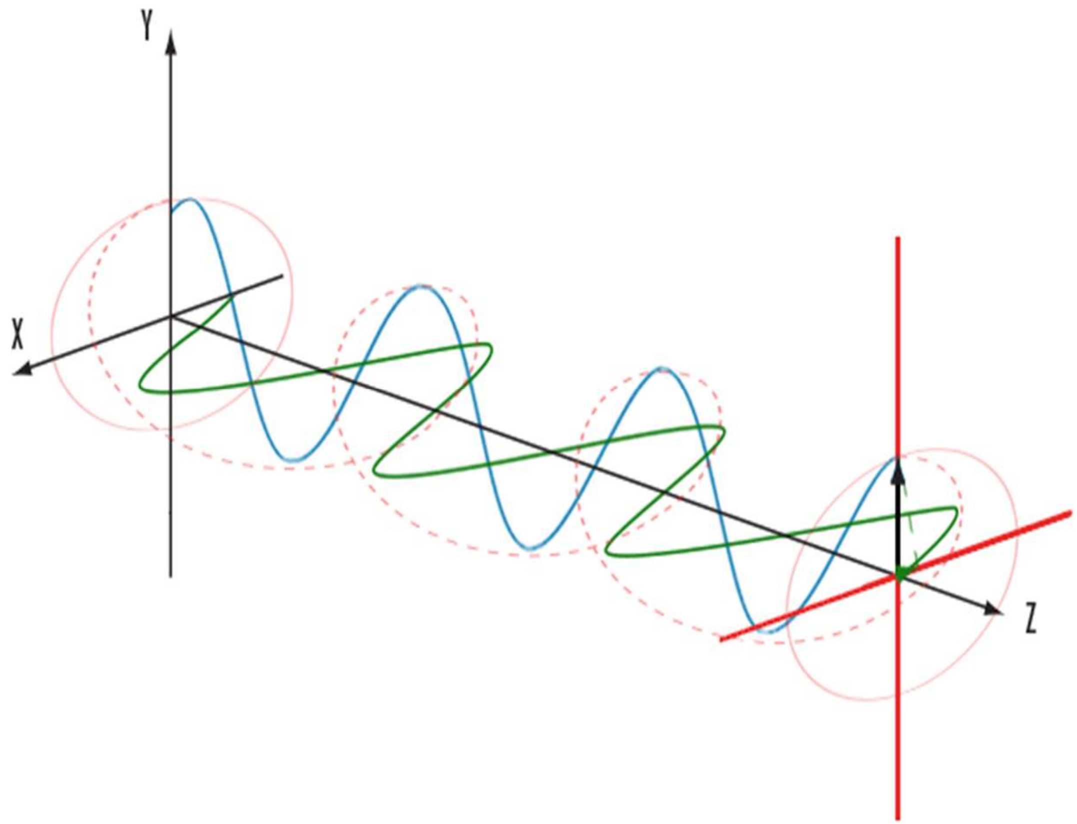
LINEARLY POLARIZED WAVE

Key Takeaways *EM Waves can be linearly, circularly, or elliptically polarized. A circularly polarized wave can be represented as a sum of two linearly polarized waves having phase shift. A linearly polarized wave can be represented as a sum of two circularly polarized waves. In the general case, waves are elliptically polarized.*

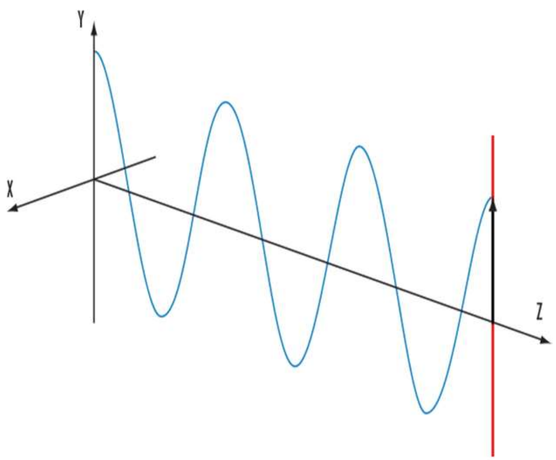
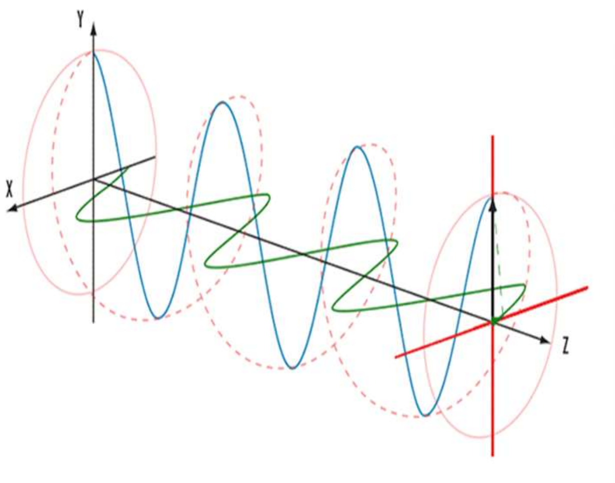
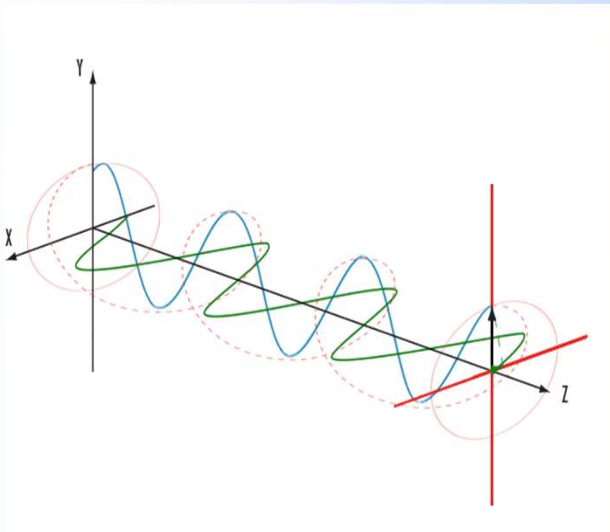
Elliptical Polarization

$$a_x \neq a_y, \varphi \neq 0, \pm\pi, \pm\pi/2$$

The end of the E field vector describes an ellipse and the polarization is said elliptical



Polarization Summary

LINEAR	CIRCULAR	ELLIPTICAL
		
$\varphi = 0$ or $\varphi = \pi$	$\varphi = \mp\pi/2$, $a_x = a_y = a$	All other cases

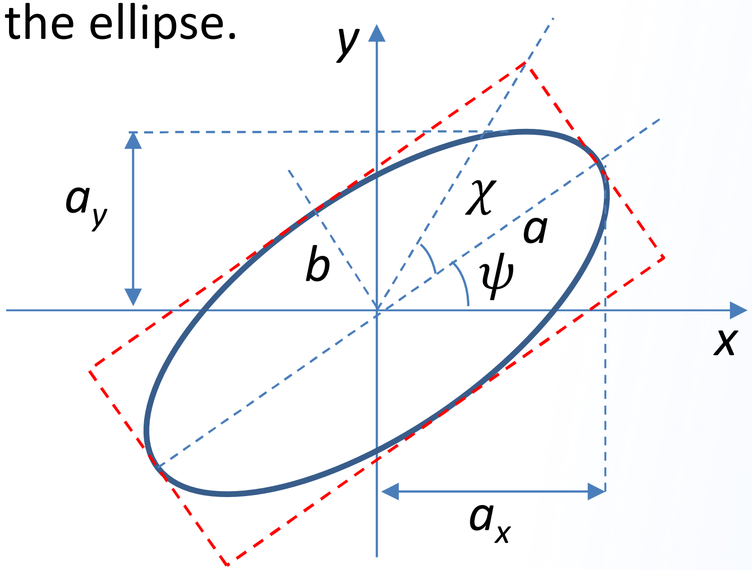
Poincarè Sphere

The state of polarization of a light wave can be described by two real parameters:

- **MAGNITUDE RATIO:** $R = \frac{a_y}{a_x}$
- **PHASE DIFFERENCE:** $\varphi = \varphi_y - \varphi_x$

These two quantities are sometimes combined into a single number known as the **complex polarization ratio** $Rexp(j\varphi)$.

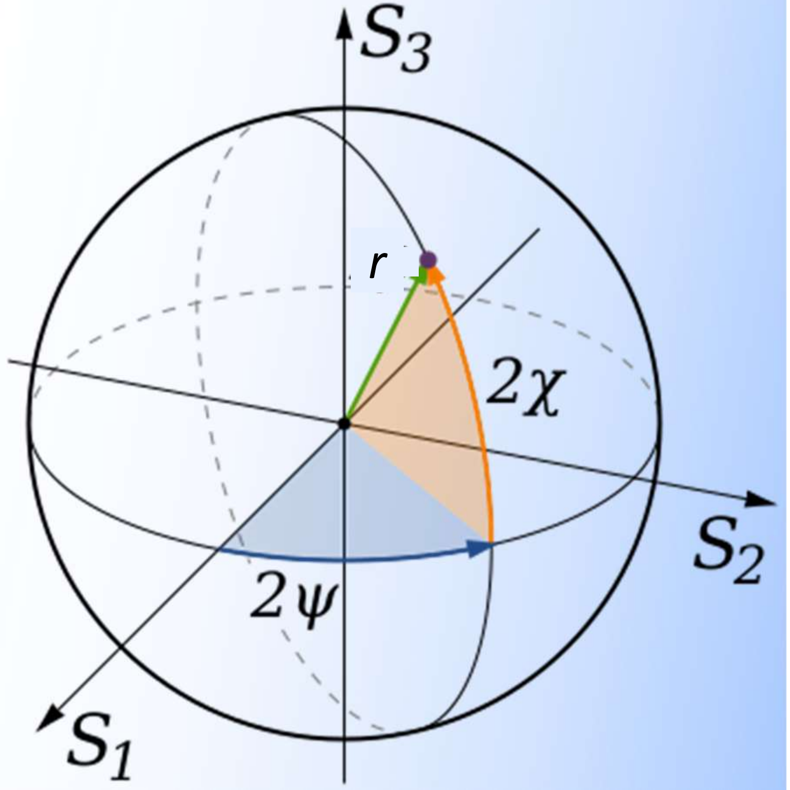
As an alternative, polarization state can be characterized by two angles ψ, that defines the direction of the major axis, and χ, that represents the ellipticity of the wave, namely the ratio of the minor to major axes of the ellipse.



Poincarè Sphere

The Poincarè sphere is a geometrical construct in which the state of polarization is represented by a point on the surface of a sphere of unit radius. Coordinates on the sphere are defined as follows:

- $r = 1$;
- $\theta = 90° - 2\chi$;
- $\phi = 2\psi$.



Examples:

1. Points on the equator ($\chi = 0°$) represent linear polarizations.
2. Points on the equator ($\chi = 0°$) with $\phi = 0°$ or $\phi = 180°$ represents linear polarizations along x and y axes.
3. North and south pole ($\chi = \mp 90°$) represent respectively RH and LH circularly polarized light.

An animated representation of the Poincarè Sphere can be found here: https://www.youtube.com/watch?v=G-yBPDYJdi0

Stokes Parameters

The two parameters R and φ describe the state of polarization but contain no information about the intensity of the wave.
Another representation that contains those information is the **Stokes vector**.
This is a set of four real numbers (S_0, S_1, S_2, S_3) called the **Stokes parameters**.

The first of these parameters is proportional to the optical intensity: $S_0 = a_x^2 + a_y^2$.
The other three parameters represent the Cartesian coordinates of the point on the Poincarè Sphere.

- $S_1 = S_0 \cos 2\chi cos2\psi;$
- $S_2 = S_0 \cos 2\chi sin2\psi;$
- $S_3 = S_0 sin\, 2\chi;$

The Stokes parameters can be also related to the field parameters and between themselves as follows:

- $S_0 = a_x^2 + a_y^2;$
- $S_1 = a_x^2 - a_y^2;$
- $S_2 = 2a_x a_y cos\varphi;$
- $S_3 = 2a_x a_y sin\varphi;$

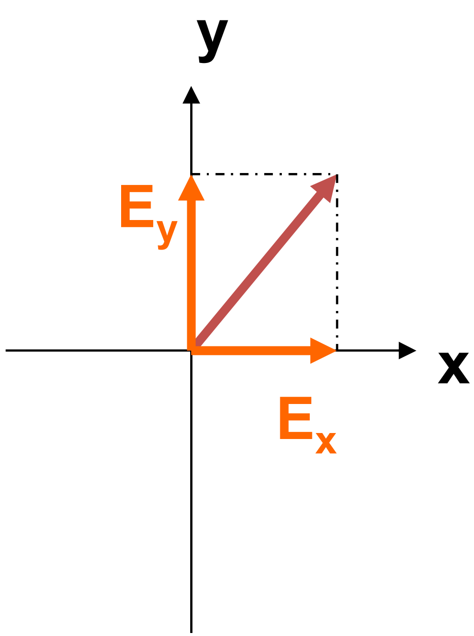
- $S_1^2 + S_2^2 + S_3^2 = S_0^2;$

Jones Matrix description

Another possible representation to describe the state of polarization of an optical field is the Jones vector representation.

A monochromatic plane wave can be represented through the x and y components of the electric field vector. If we consider the E-vector as shown in picture:

$$\mathbf{E} = \left(a_x \hat{x} + a_y\, \hat{y} e^{j\varphi}\right)$$



The state of polarization of light is determined by

- the relative amplitudes and,
- the relative phases ($\varphi = \varphi_y - \varphi_x$) of these components

The complex amplitude is written as a two-element matrix, the Jones vector

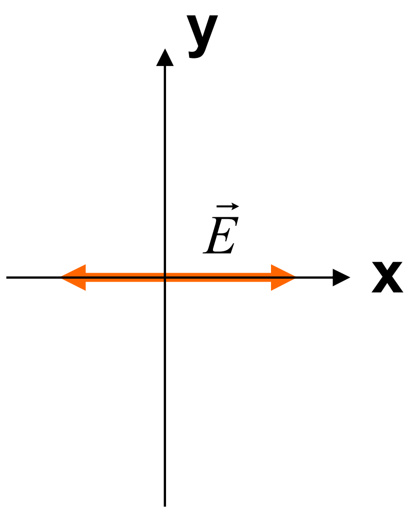
Jones vector representation

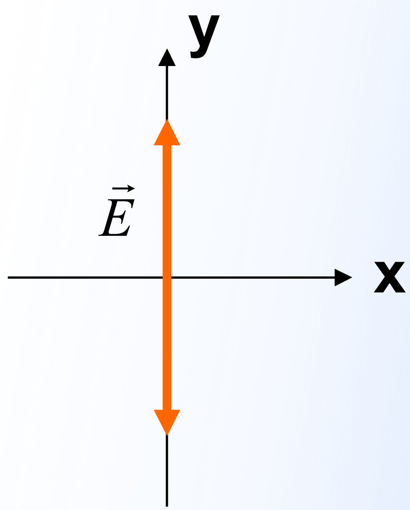
$$\boxed{\mathbf{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} a_x \\ a_y e^{j\varphi} \end{bmatrix}}$$

Jones Matrix description

Horizontally or Vertically polarized light

The electric field oscillations are only along the x- or y-axis. The arrows indicate the sense of movement as the beam approaches you





A normalized form of the Jones vector is obtained by dividing by the square root of the sum of the squares of the two moduli, i.e. $\sqrt{|E_{0x}|^2 + |E_{0y}|^2}$

The Jones vector for horizontally (x) polarized light is:

$$\boxed{A\begin{bmatrix}1\\0\end{bmatrix}}$$

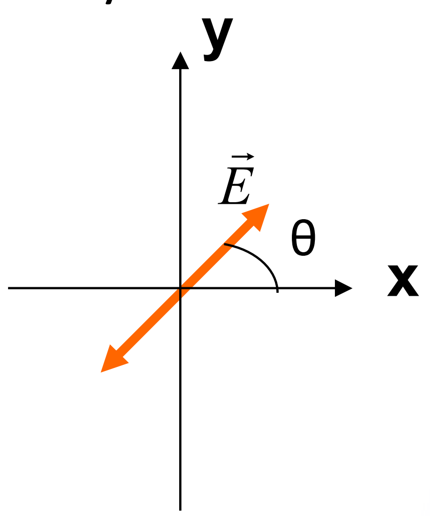
The Jones vector for vertically (y) polarized light is:

$$\boxed{A\begin{bmatrix}0\\1\end{bmatrix}}$$

Jones Matrix description

Linearly polarized light

The electric field oscillations are in the x-y plane. The arrows indicate the sense of movement as the beam approaches you



The Jones vector for linearly polarized light is:

$$\boxed{A\begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix}}$$

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Jones Matrix description

Circularly polarized light

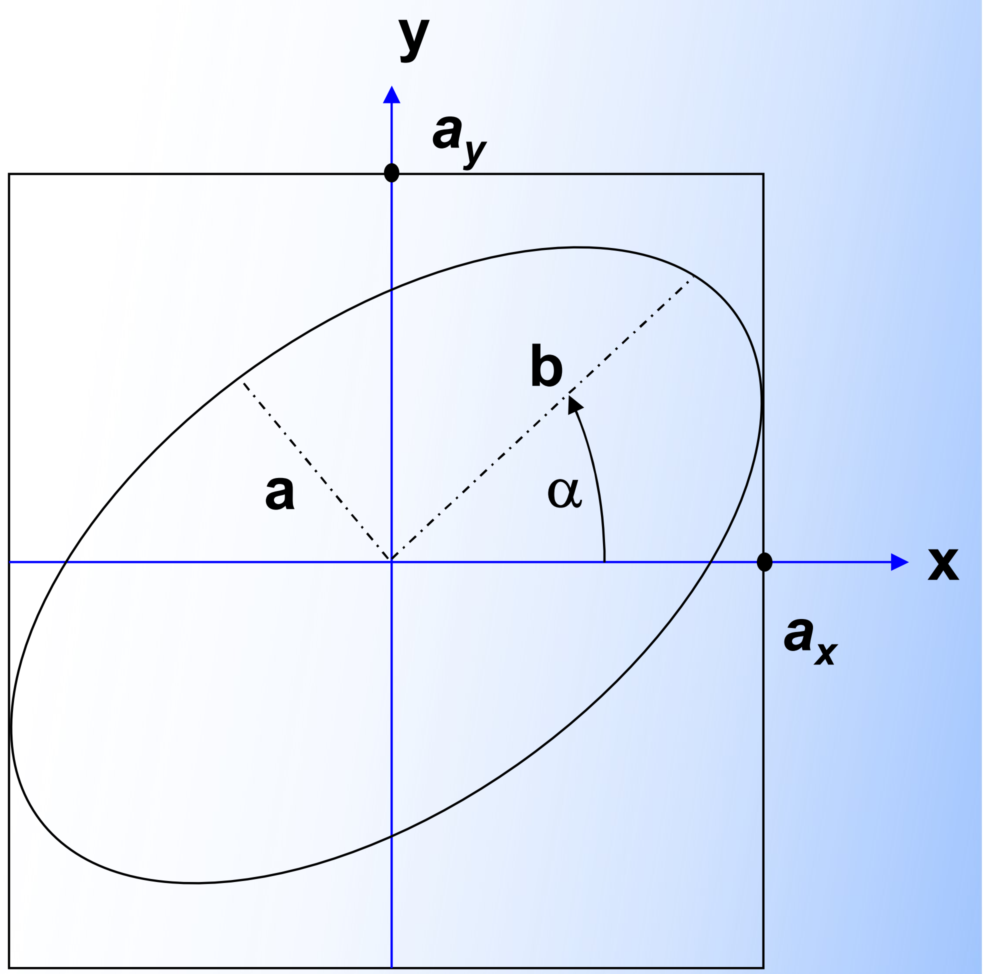
- The Jones vector for this case is $\boxed{A\begin{bmatrix}1\\-j\end{bmatrix}}$
- This vector represents circularly polarized light, where the end of the vector E rotates counterclockwise, viewed head-on
- This mode is called right-circularly polarized light
- What is the corresponding vector for left-circularly polarized light?
- Can we find what is the sum of a right-circularly polarized light and a left-circularly polarized light with Jones Matrix?

Jones Matrix description

In general, the Jones vector for the arbitrary case of elliptical polarization ($\delta \neq m\pi$; $\delta \neq (m+1/2)\pi$) is:

$$\mathbf{E} = \begin{bmatrix} a_x \\ a_y e^{j\varphi} \end{bmatrix} = \begin{bmatrix} A \\ B\left(\cos\varphi + j\sin\varphi\right) \end{bmatrix}$$

$$\tan 2\alpha = \frac{2a_x a_y \cos\varphi}{a_x^2 - a_y^2}$$



Orthogonal polarizations

Two polarization states represented by the Jones vectors J_1 and J_2 are said to be **orthogonal** if the inner product between J_1 and J_2 is zero:

$$(J_1, J_2) = E_{1,x}E_{2,x}^* + E_{1,y}E_{2,y}^*$$

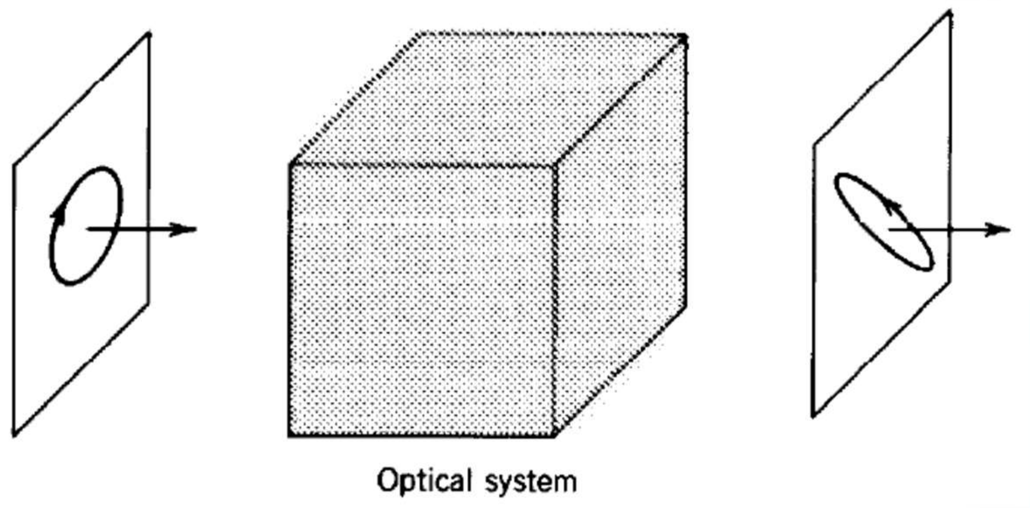
Where $E_{1,x/y}$ are the elements of J_1 and $E_{2,x/y}$ are the elements of J_2.

Two examples of orthogonal states are linear polarized light in the x and y direction or RHCP and LHCP.

Optical Elements: Jones Matrix description

Let's consider the transmission of a plane wave of arbitrary polarization through an optical system that maintains the plane-wave nature of the wave but alters its polarization.

The system is assumed to be linear, so that the principle of superposition can be applied.



Jones matrices can be used to describe optical elements and relate Jones vector of the incident wave to the Jones vector of the exiting wave.

Optical Elements: Jones Matrix description

If the vector of the incident light is $J_1 = \begin{bmatrix} A \\ B \end{bmatrix}$ and the vector of the emerging light is $J_2 = \begin{bmatrix} A' \\ B' \end{bmatrix}$ then one can write:

$$J_2 = TJ_1$$

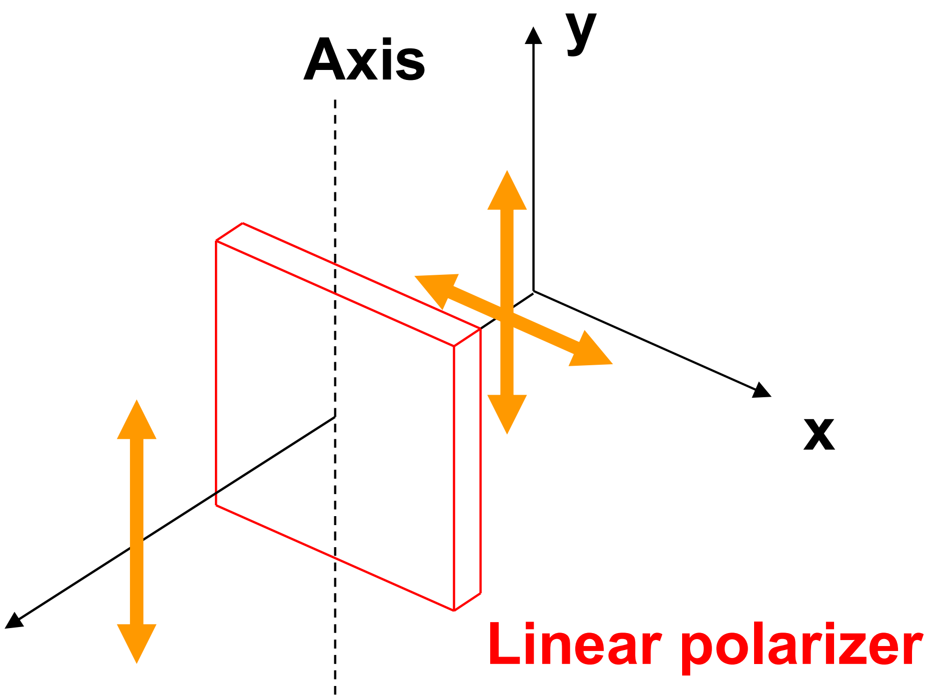
Jones matrix of the optical element

We can also write:

$$\begin{bmatrix} A' \\ B' \end{bmatrix} = T \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

Optical Elements: Linear Polarizer

A generic optical element can be represented as a (Jones) matrix. For example, the matrix descriptions of linear polarizers are:



$$M_{horizontal} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M_{vertical} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

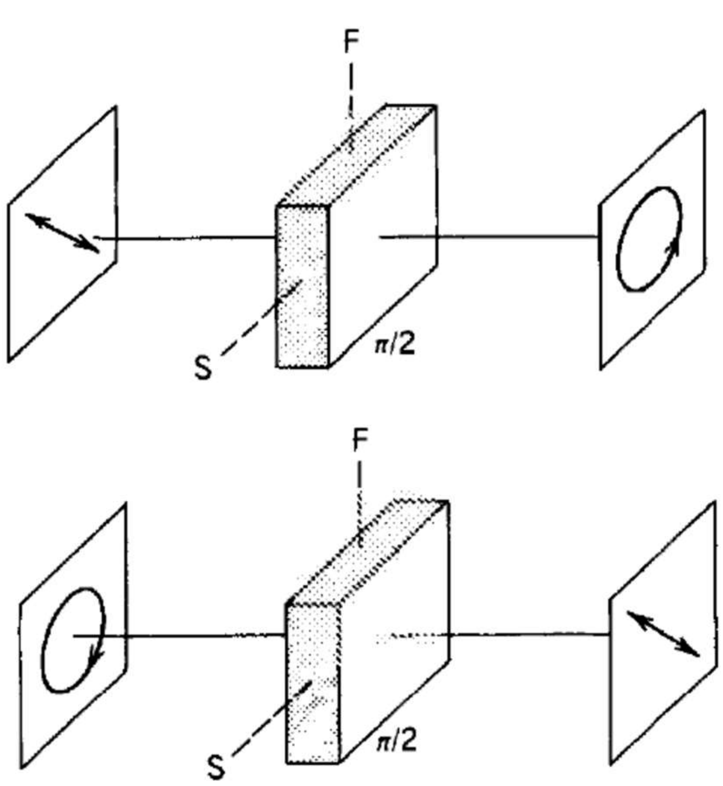
$$M_{\pm 45°} = \begin{bmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{bmatrix}$$

Optical Elements: Wave retarders

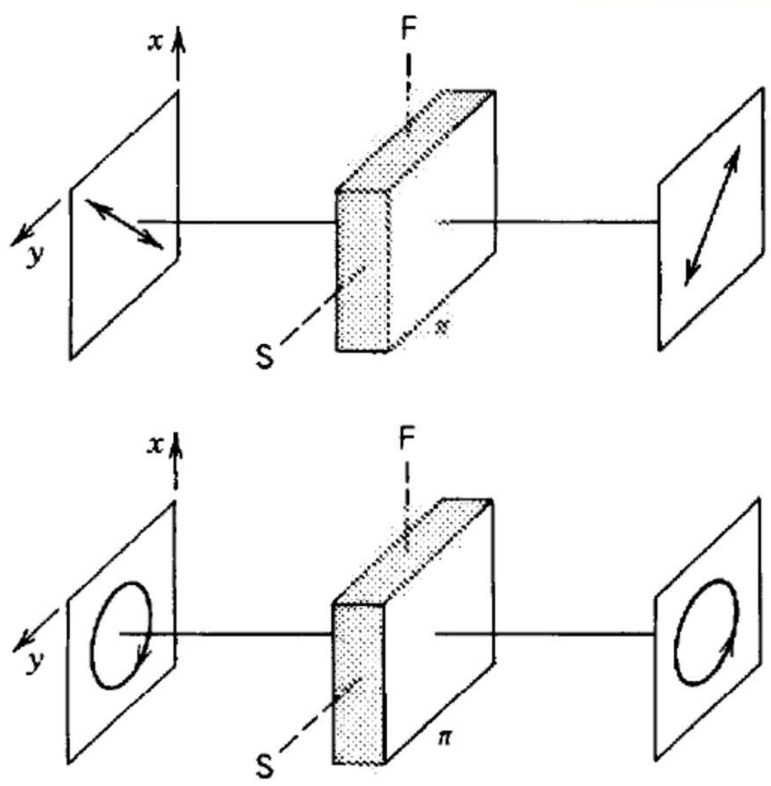
The matrix description of a wave retarder is:

$$\boxed{M_{retarder} = \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Gamma} \end{bmatrix}}$$

This optical element delays the y component of the incoming wave by a phase Γ. The x and y axes are called the fast and slow axis of the retarder, respectively.



Quarter-wave retarders



Half-wave retarders

Optical Elements: Polarization rotators

The matrix description of a polarization rotator is:

$$\boxed{M_{rotator} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}$$

This optical element converts a linearly polarized wave $\begin{bmatrix} cos\theta_1 \\ sin\theta_1 \end{bmatrix}$ into another linearly polarized wave $\begin{bmatrix} cos\theta_2 \\ sin\theta_2 \end{bmatrix}$, where $\theta_2 = \theta_1 + \theta$. In other words, it rotates the plane of polarization by an angle θ.

NOTE: The Jones matrix of a generic waveplate rotated by an angle θ is:

$$M_{rotatedwaveplate} = M_{rotator}(\theta) M_{waveplate} M_{rotator}(-\theta) = \begin{bmatrix} \cos\left(\Gamma/2\right) + j\sin\left(\Gamma/2\right)\cos(2\theta) & j\sin\left(\Gamma/2\right)\sin(2\theta) \\ j\sin\left(\Gamma/2\right)\sin(2\theta) & \cos\left(\Gamma/2\right) - j\sin\left(\Gamma/2\right)\cos(2\theta) \end{bmatrix}$$

Optical Elements: Jones Matrix description

Quarter-wave plate (with fast axis vertical)

$$M_{QW} = \begin{bmatrix} 1 & 0 \\ 0 & j \end{bmatrix}$$

Quarter-wave plate (with fast axis horizontal)

$$M_{QW} = \begin{bmatrix} 1 & 0 \\ 0 & -j \end{bmatrix}$$

Half-wave plate

$$M_{HW} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Isotropic Phase retarder

$$M_{PR} = \begin{bmatrix} e^{-j\Phi} & 0 \\ 0 & e^{-j\Phi} \end{bmatrix}$$

Circular Polarizer (Right)

$$M_{RHP} = \frac{1}{2}\begin{bmatrix} 1 & -j \\ j & 1 \end{bmatrix}$$

Circular Polarizer (Left)

$$M_{LHP} = \frac{1}{2}\begin{bmatrix} 1 & j \\ -j & 1 \end{bmatrix}$$

Cascaded elements and Normal Modes

A system composed by **multiple optical elements** can be described by using conventional matrix multiplication formulas.

Example: A system composed of an element with Jones matrix T_1 followed by another element with Jones matrix T_2 , is equivalent to a single system with Jones matrix:

$$\boldsymbol{T} = \boldsymbol{T_2 T_1}$$

The **normal modes** of a polarization system are the states of polarization that are not changed when the wave is transmitted through the system. These states have Jones vectors satisfying

$$\boldsymbol{TJ} = \boldsymbol{\mu J}$$

Where $\boldsymbol{\mu}$ is a constant. The normal modes are therefore the eigenvectors of the Jones matrix T and the values of $\boldsymbol{\mu}$ are the corresponding eigenvalues.

NOTE: Since T is a 2x2 matrix, there are only two independent normal modes.

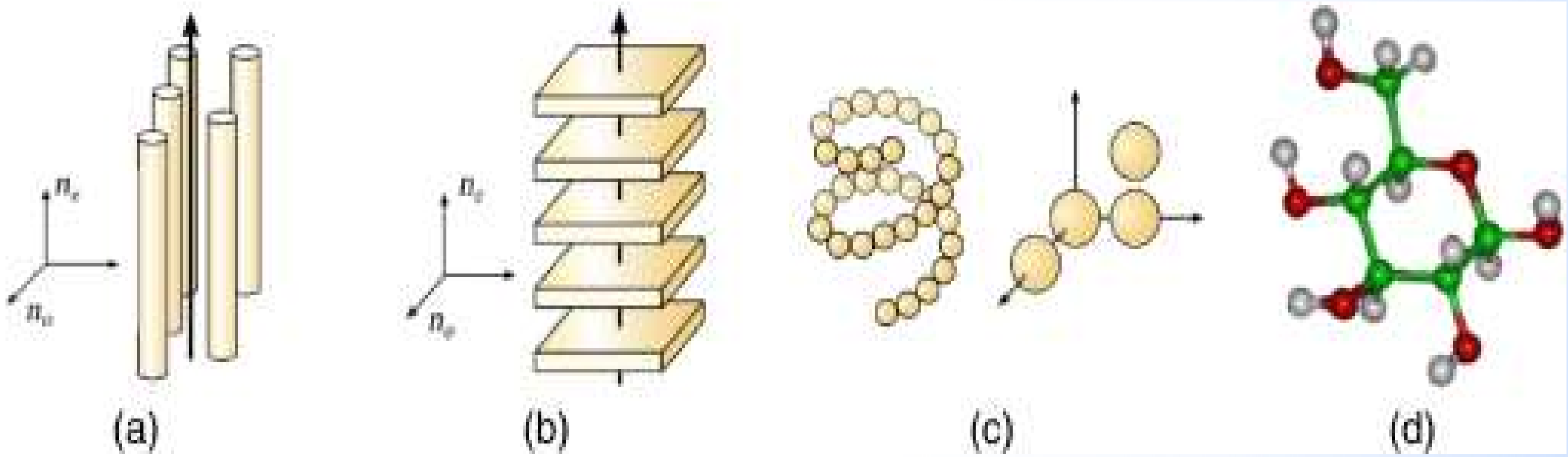
NOTE 2: If the matrix T is Hermitian ($T_{12}=T_{21}^*$) the normal modes are orthogonal, i.e., (J1,J2)=0

Stokes and Jones vectors

State of polarization	Stokes vectors	Jones vectors
Horizontal $\mathscr{P}$-state	$\begin{bmatrix}1\\1\\0\\0\end{bmatrix}$	$\begin{bmatrix}1\\0\end{bmatrix}$
Vertical $\mathscr{P}$-state	$\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$	$\begin{bmatrix}0\\1\end{bmatrix}$
$\mathscr{P}$-state at $+45°$	$\begin{bmatrix}1\\0\\1\\0\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$
$\mathscr{P}$-state at $-45°$	$\begin{bmatrix}1\\0\\-1\\0\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$
$\mathscr{R}$-state	$\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$
$\mathscr{L}$-state	$\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}$

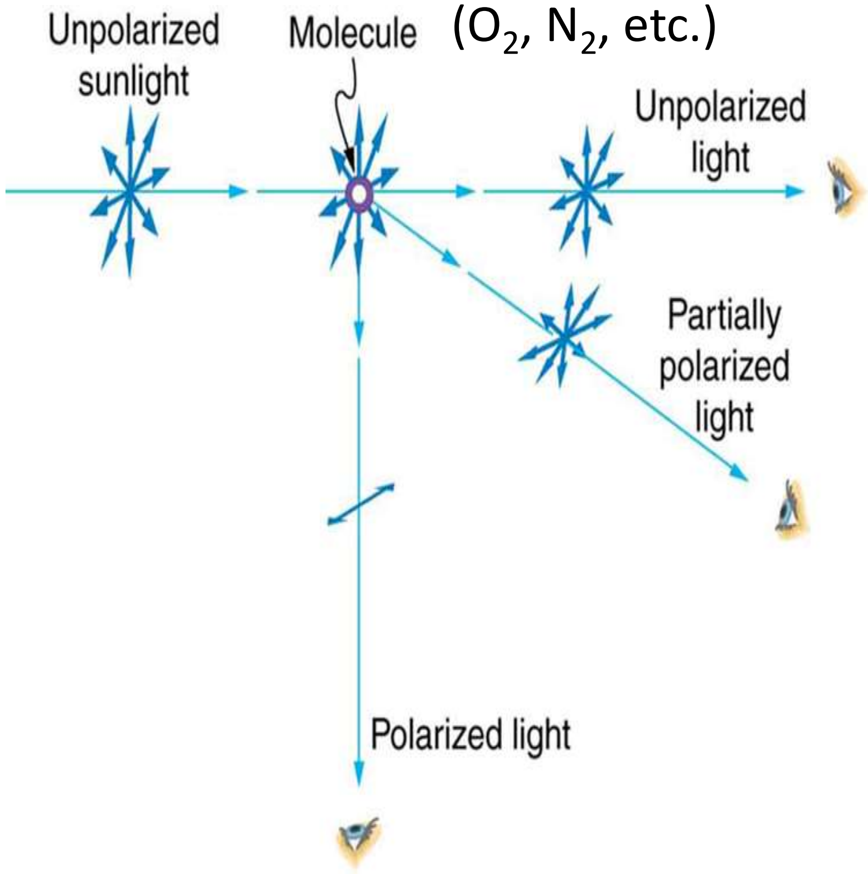
Light Polarization in Nature

Light polarization is exploited in biophotonics to enhance contrast in imaging. Some tissues or molecules are more sensitive to specific polarization orientations. In other words, they act as anisotropic media (ε is a tensor rather than a scalar and $\boldsymbol{D} = \boldsymbol{\varepsilon} \cdot \boldsymbol{E}$), and they respond differently to light depending on its polarization. For example, a birefringent medium shows two different refractive indices (an ordinary index n_o and an extraordinary index n_e) for 2 perpendicular orientations (see cases a and b below). For example, collagen fibers can be considered birefringent media.



Examples of structurally anisotropic models of tissues and tissue components: (a) system of long dielectric cylinders; (b) system of dielectric plates; (c) chiral aggregates of particles (e.g., proteins); and (d) glucose (or any chiral molecule) in tissues.

Polarization by scattering



Sunlight, as many other natural light sources, is unpolarized. However polarized light does exist in nature.

Example: sunlight scattered from the sky or sunlight reflected by surfaces. Light is a transverse wave, so the electrons of air molecules vibrate perpendicularly to the direction of traveling. The electrons reradiate acting like small antennae thus producing scattering. Since they are oscillating perpendicular to the direction of the light ray, they produce EM radiation that is polarized perpendicular to the direction of the ray.

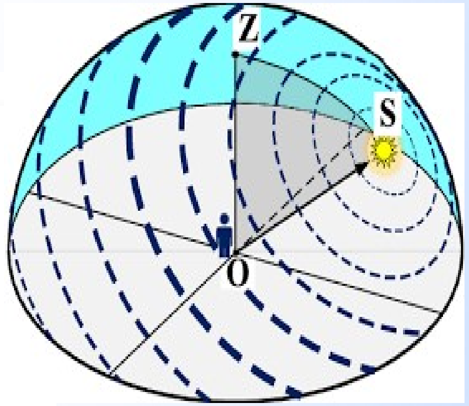
This scattered light is vertically polarized when viewed from an angle of 90° relative to the sun.

The plane of polarization of the scattered light is perpendicular to the plane defined by the incident and the scattered ray. Light that is scattered at an angle of 0° is nonpolarized and at intermediate angles it is partially polarized.

Polarization by scattering

- We cannot detect the polarization of light very well, but some animals can see polarized light (due to anisotropy in the eyes, e.g., microvilli)
- Many insects, octopi and mantis shrimps

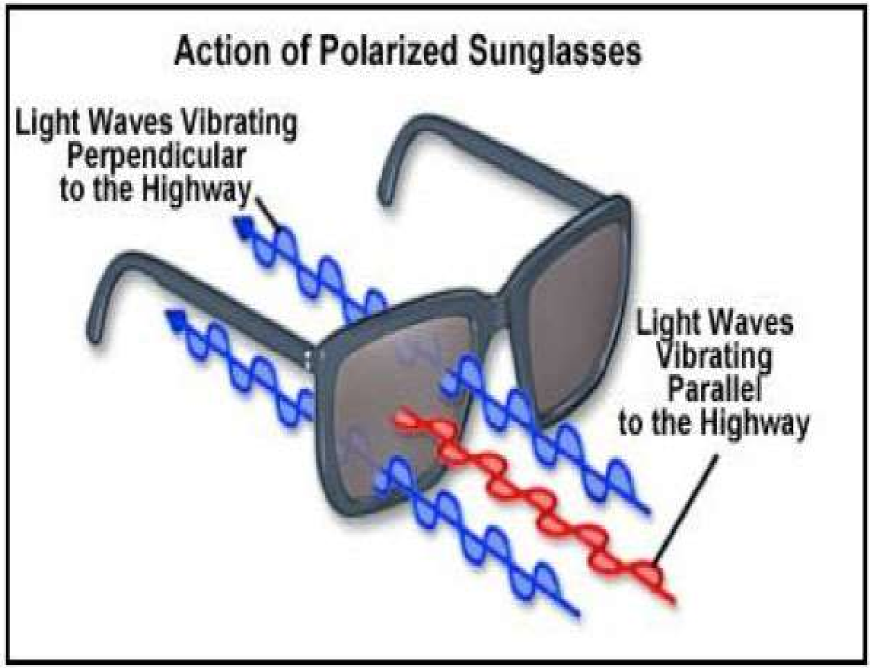
Polarized light in the sky

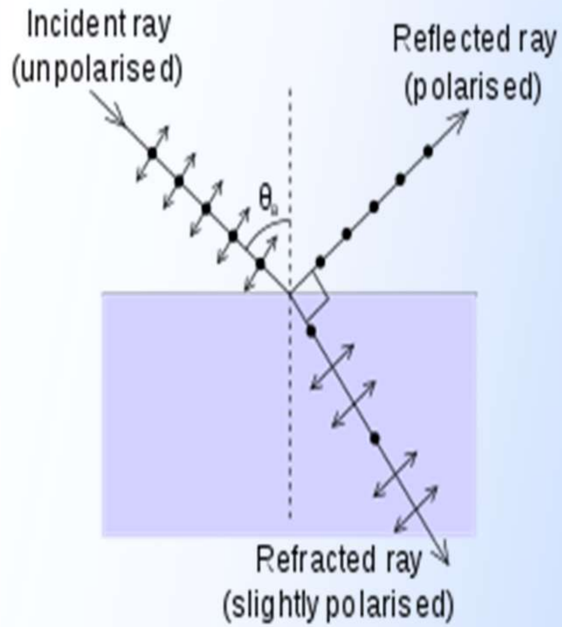


Bees' eyes are polarization sensitive. The use **polarization patterns in the sky** for navigation

Polarization by reflection

Polarized sunglasses





Polarized sunglasses use the fact that sunlight reflected from surfaces is (partially) polarized. In a large range of angles around the Brewster's angle (we will soon define this angle), the reflection of *p*-polarized light is much lower than *s*-polarized light. Thus, **if the sun is low in the sky, reflected light is mostly *s*-polarized**. Polarizing sunglasses use a polarizing material such as polaroid to block horizontally-polarized light, thus absorbing reflections from horizontal surfaces. The effect is strongest with smooth surfaces (which provide specular reflection) such as windows and water, but reflections from roads and the ground are also reduced.