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### **Exercise 1**

Consider a plane wave normally incident (from glass) on a half space of copper. If the wave frequency f=4 GHz, compute the propagation constant, the intrinsic impedance, and the skin depth for the conductor. Also compute the reflection and transmission coefficients.

For copper, use:  $\sigma = 6x10^7$  S/m,  $\mu = \mu_0$ ,  $\epsilon = \epsilon_0$ ; for glass use  $\sigma = 0$  S/m,  $\mu = \mu_0$ ,  $\epsilon = 2.25$   $\epsilon_0$ .

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#### Exercise 2

A microwave antenna feed network operating at 5 GHz requires a  $50\Omega$  printed transmission line that is  $16\lambda$  long. Possible choices are:

- 1)copper microstrip, with d=0.16 cm,  $\varepsilon_r$ =2.20, and  $\tan \delta$ = 0.001, or
- 2)copper stripline, with b=0.32 cm,  $\varepsilon_r$ =2.20, t=0.01 mm, and tan $\delta$  =0.001. Which line should be used if attenuation is to be minimized? In both cases give the attenuation constant in dB/m.
- •Background formulas for microstrip lines:

The effective dielectric constant of a microstrip line is given approximately by

$$\epsilon_{e} = \frac{\epsilon_{r} + 1}{2} + \frac{\epsilon_{r} - 1}{2} \frac{1}{\sqrt{1 + 12d/W}}.$$
(3.195)

The effective dielectric constant can be interpreted as the dielectric constant of a homogeneous medium that equivalently replaces the air and dielectric regions of the microstrip line, as shown in Figure 3.26. The phase velocity and propagation constant are then given by (3.193) and (3.194).

Given the dimensions of the microstrip line, the characteristic impedance can be calculated as

$$Z_{0} = \begin{cases} \frac{60}{\sqrt{\epsilon_{e}}} \ln \left( \frac{8d}{W} + \frac{W}{4d} \right) & \text{for } W/d \leq 1 \\ \frac{120\pi}{\sqrt{\epsilon_{e}} \left[ W/d + 1.393 + 0.667 \ln \left( W/d + 1.444 \right) \right]} & \text{for } W/d \geq 1. \end{cases}$$
(3.196)

For a given characteristic impedance  $Z_0$  and dielectric constant  $\epsilon_r$ , the W/d ratio can be found as

$$\frac{W}{d} = \begin{cases}
\frac{8e^{A}}{e^{2A} - 2} & \text{for } W/d < 2 \\
\frac{2}{\pi} \left[ B - 1 - \ln(2B - 1) + \frac{\epsilon_{r} - 1}{2\epsilon_{r}} \left\{ \ln(B - 1) + 0.39 - \frac{0.61}{\epsilon_{r}} \right\} \right] & \text{for } W/d > 2, \\
(3.197)
\end{cases}$$

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#### Exercise 2 (continued)

where

$$A = \frac{Z_0}{60} \sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1} \left( 0.23 + \frac{0.11}{\epsilon_r} \right)$$
$$B = \frac{377\pi}{2Z_0 \sqrt{\epsilon_r}}.$$

Considering a microstrip line as a quasi-TEM line, we can determine the attenuation due to dielectric loss as

$$\alpha_d = \frac{k_0 \epsilon_r (\epsilon_e - 1) \tan \delta}{2 \sqrt{\epsilon_e} (\epsilon_r - 1)} \text{ Np/m}, \qquad (3.198)$$

where  $\tan \delta$  is the loss tangent of the dielectric. This result is derived from (3.30) by multiplying by a "filling factor,"

$$\frac{\epsilon_r(\epsilon_e-1)}{\epsilon_e(\epsilon_r-1)},$$

which accounts for the fact that the fields around the microstrip line are partly in air (loss-less) and partly in the dielectric (lossy). The attenuation due to conductor loss is given approximately by [8]

$$\alpha_c = \frac{R_s}{Z_0 W} \text{Np/m}, \qquad (3.199)$$

where  $R_s = \sqrt{\omega \mu_0/2\sigma}$  is the surface resistivity of the conductor. For most microstrip substrates, conductor loss is more significant than dielectric loss; exceptions may occur, however, with some semiconductor substrates.

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#### Exercise 2 (continued)

• Background formulas for strip lines:

When designing stripline circuits one usually needs to find the strip width, given the characteristic impedance (and height b and relative permittivity  $\epsilon_r$ ), which requires the inverse of the formulas in (3.179). Such formulas have been derived as

$$\frac{W}{b} = \begin{cases} x & \text{for } \sqrt{\epsilon_r} Z_0 < 120 \ \Omega \\ 0.85 - \sqrt{0.6 - x} & \text{for } \sqrt{\epsilon_r} Z_0 > 120 \ \Omega, \end{cases}$$
(3.180a)

where

$$x = \frac{30\pi}{\sqrt{\epsilon_r} Z_0} - 0.441. \tag{3.180b}$$

Since stripline is a TEM line, the attenuation due to dielectric loss is of the same form as that for other TEM lines and is given in (3.30). The attenuation due to conductor loss can be found by the perturbation method or Wheeler's incremental inductance rule. An approximate result is

$$\alpha_{c} = \begin{cases} \frac{2.7 \times 10^{-3} R_{s} \epsilon_{r} Z_{0}}{30\pi (b - t)} A & \text{for } \sqrt{\epsilon_{r}} Z_{0} < 120 \Omega \\ \frac{0.16 R_{s}}{Z_{0} b} B & \text{for } \sqrt{\epsilon_{r}} Z_{0} > 120 \Omega \end{cases}$$
(3.181)

with

$$A = 1 + \frac{2W}{b - t} + \frac{1}{\pi} \frac{b + t}{b - t} \ln \left( \frac{2b - t}{t} \right),$$

$$B = 1 + \frac{b}{(0.5W + 0.7t)} \left( 0.5 + \frac{0.414t}{W} + \frac{1}{2\pi} \ln \frac{4\pi W}{t} \right),$$

where t is the thickness of the strip.

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## **Exercise 3**

Consider a lossless two-port network.

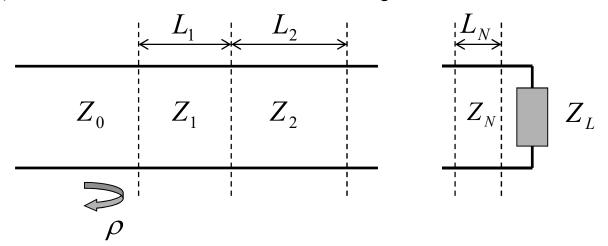
- (a) If the network is reciprocal, show that  $|S_{21}|^2 = 1 |S_{11}|^2$ .
- (b) If the network is nonreciprocal, show that it is impossible to have unidirectional transmission, where  $S_{12} = 0$  and  $S_{21} \neq 0$ .

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### **Exercise 4**

a) Consider the transmission line described in the figure.



Explain how to compute the reflection coefficient using the theory of small reflections.

b) Using  $Z_0$ =50  $\Omega$ ,  $Z_L$ =90  $\Omega$  and  $Z_1$ =75  $\Omega$  (with  $L_1$ =20 cm), and assuming a phase velocity equal to the phase velocity in vacuum, compute the reflection coefficient at input for an operating frequency f=3 GHz.