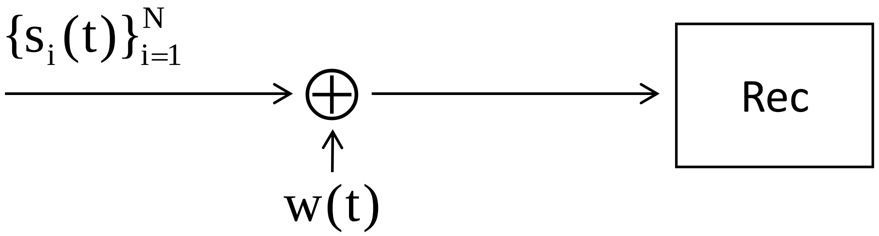
Signal spaces

Introduction



Where **N** is the dimension of the vector space

$$s_i(t) = \sum_{k=1}^{N} s_{i,k} \cdot \phi(t) \qquad s_{i,k} = \int_0^T s_i(t) \cdot \phi(t)dt$$

$$\langle s_i(t), s_k(t) \rangle = \int_0^T s_i(t) \cdot s_k(t)dt = \sum_{k=1}^{N} s_{i,k} s_{j,k}$$

$$|\langle s_i, s_j \rangle| \le \|s_i\| \cdot \|s_j\|$$

Schwarz inequality

$$\langle \phi_i(t), \phi_k(t) \rangle = \delta_{i,k} = \begin{cases} 0; & k \neq i \\ 1; & k = i \end{cases}$$

$$\big| \|s_i\| - \|s_j\| \big| \le \|s_i + s_j\| \le \|s_i\| + \|s_j\|$$

Triangular inequality

Inner product as Dirac's delta

AWGN

AWGN Noise in $(0, T]$

$$n(t) = \sum_k n_k \phi_k(t) \qquad n_k = \int_0^T n(t)\phi_k(t)dt$$

$$E[n_k n_j] = \begin{cases} 0 & k \neq j \\ N_0/2 & k = j \end{cases} \qquad E[n_k] = 0 \qquad E[(n(t) - \sum n_k \phi_k(t))^2] = 0$$

PASS-BAND Signal (1/2)

$$s(t) = \mathrm{Re}\{z(t) \cdot e^{j2\pi ft}\} = \frac{z(t)}{2} e^{j2\pi ft} + \frac{\overline{z(t)}}{2} e^{-j2\pi ft} = |z(t)| \cos(2\pi ft + \arg(z(t))$$

$$z(f) = 2 \leq (f + f_0) U(f + f_0)$$

Where U(f) is unity step function

If $s(t) = A(t) \cos(2\pi f_0 t + \varphi(t))$

With A and phi slowly change with respect to f0 (low frequency functions)

$$\downarrow$$

$$z(t) = A(t) e^{j\varphi(t)}$$

$$\downarrow$$

$$z(t) = x(t) + j \cdot y(t) \longrightarrow s(t) = x(t) \cos(2\pi f_0 t) - y(t) \sin(2\pi f_0 t)$$

Phase and quadrature components

PASS-BAND Signal (2/2)

$$s_1(t)s_2(t) = \frac{1}{2}\mathrm{Re}\{z_1(t)\overline{z_2(t)}\} + \frac{1}{2}\mathrm{Re}\{z_1(t)z_2(t)e^{j4\pi f_0 t}\}$$

Energy

$$\int s_1(t)s_2(t)dt = \frac{1}{2}\mathrm{Re}\{\int z_1(t)\overline{z_2(t)}dt\} \qquad \int s^2(t)dt = \frac{1}{2}\int |z(t)|^2 dt$$

$$\int A(t)\cos(2\pi f_0 t + \varphi_1(t))\cos(2\pi f_0 t + \varphi_2(t))dt = \frac{1}{2}\int A(t)\cos(\varphi_1(t) - \varphi_2(t))dt$$

$$\int A^2(t)\cos^2(2\pi f_0 t + \varphi(t))dt = \frac{1}{2}\int A^2(t)dt$$

PASS-BAND Functions

$$\phi_k(t) = \mathrm{Re}\{z_k(t)e^{j2\pi f_0 t}\} = A(t)\cos(2\pi f_0 + \varphi(t))$$

If we consider $jz_k(t) \rightarrow \phi_{k'}(t) = \mathrm{Re}\{jz_k(t)e^{j2\pi f_0 t}\} = -\mathrm{Im}\{z_k(t)e^{j2\pi f_0 t}\} = -A_k(t)\sin(2\pi f_0 t + \varphi_k(t))$

Also $\phi_k(t) \perp \phi_{k'}(t)$

If we have signals with components along k and k' axis, we obtain:

$$s_{ik}\phi_k(t) + s_{ik'}\phi_{k'}(t) = \mathrm{Re}\{(s_{ik} + js_{ik'})z_k(t)e^{j2\pi f_0 t}\}$$

QAM

$$s_i(t) = a \cdot g(t)\cos(2\pi f_0 t) - b \cdot g(t)\sin(2\pi f_0 t)$$ with $a = \pm 1, \pm 3, \ldots.$

In general we have:

$$s_i(t) = \mathrm{Re}\{\sum_k d_k g(t - kT) e^{j2\pi f_0 t}\} = \sum_k a_k g(t - kT)\cos(j2\pi f_0 t) - \sum_k b_k g(t - kT)\sin(j2\pi f_0 t)$$