Tutorial on S parameters – Issue 2

We consider an octopole (figure 1), geometrically symmetrical with respect to the planes P_1 et P_2, composed of 4 lossless lines of length $\lambda_0/4$. Their characteristic admittances are noted Y_1 and Y_2. They are normalized with respect to $Z_0 = 50\ \Omega$.

$y_1 = Y_1 . Z_0$ $\qquad\qquad$ $y_2 = Y_2 . Z_0$

The device is realized with microstrip technology with different widths w_1, w_2. [S] parameter calculations will be performed at the junctions boundaries, the length of the access lines, of characteristic impedance Z_0 not being considered in the analysis (ℓ negligible).

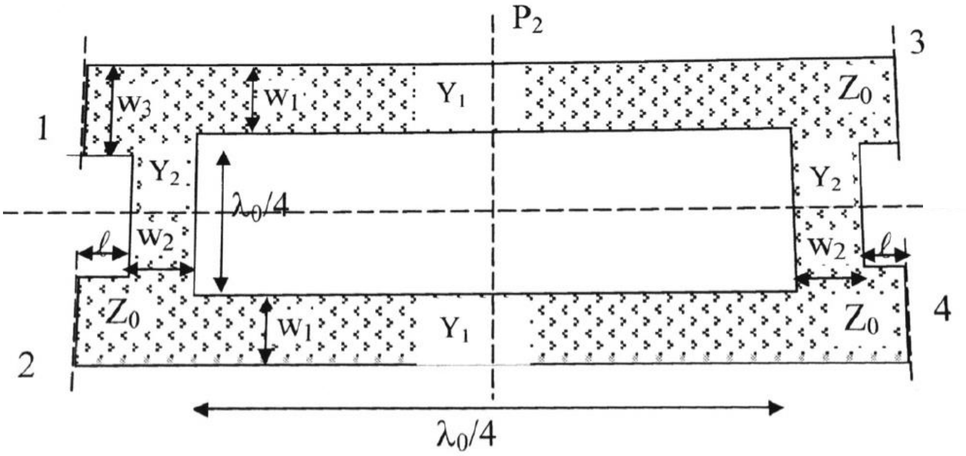


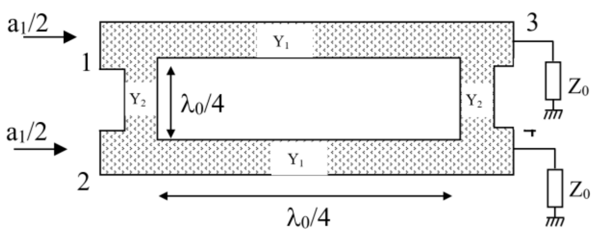
Figure 1

Matrix [S] and chain matrix [A] calculations are performed at $f_0 = v/\lambda_0$,where v is the phase velocity of the wave in the different media.

To calculate the [S] parameters of the octopole, it is interesting to decompose it into even and odd modes.

1) Excitation in even mode

a) Show that, for even-mode excitation, the behavior of the circuit is not disturbed if it is treated as follows (Figure 3):



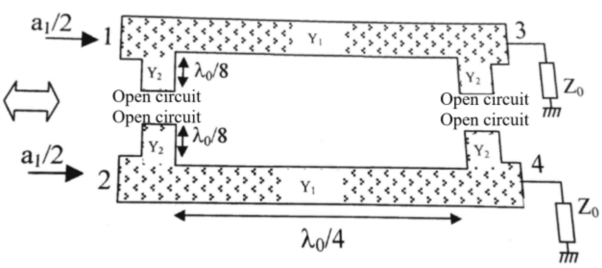


Figure 2 $\qquad\qquad$ Figure 3

b) Calculate the chain matrix[A] of a section of a characteristic admittance line section Y_1 and length ℓ (figure 4).

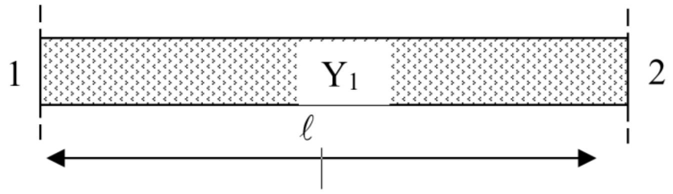


Figure 4

c) We give the input impedance Z_i of a line of characteristic impedance Z_0, of length ℓ loaded at its end by Z_L:

$$Z_i = Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}$$

Give the chain matrix of the quadripole presented in figure 5 :

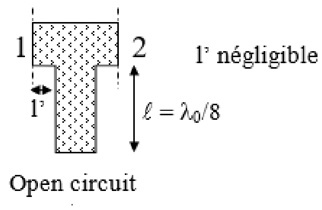


Figure 5

d) Give the chain matrix of the quadripole presented in figure 3 and deduce the normalized chain matrix with respect to Z_0 = 50 Ω.

e) The conversion relationships between the normalized chain matrix $[C_N]$ and matrix [S] of a quadripole are as follows:

$$[S] = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \qquad [C_N] = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$S_{11} = \frac{a + b - c - d}{a + b + c + d} \qquad S_{12} = \frac{2}{a + b + c + d} \qquad S_{21} = \frac{2.(a.d - b.c)}{a + b + c + d}$$

$$S_{22} = \frac{-a + b - c + d}{a + b + c + d}$$

Show that the matrix $[S]^P$ of the quadripole presented in figure 3 is written:

$$S_{11}^{P} = \frac{\dfrac{j}{y_1} - j\left(y_1 - \dfrac{y_2^2}{y_1}\right)}{-2\dfrac{y_2}{y_1} + \dfrac{j}{y_1} + j\left(y_1 - \dfrac{y_2^2}{y_1}\right)} \qquad S_{21}^{P} = \frac{2}{-2\dfrac{y_2}{y_1} + \dfrac{j}{y_1} + j\left(y_1 - \dfrac{y_2^2}{y_1}\right)}$$

$$S_{12}^{P} = S_{21}^{P} \qquad S_{11}^{P} = S_{22}^{P}$$

2) Excitation in odd mode

a) Show that, for odd-mode excitation, the octupole works as shown figure 7.

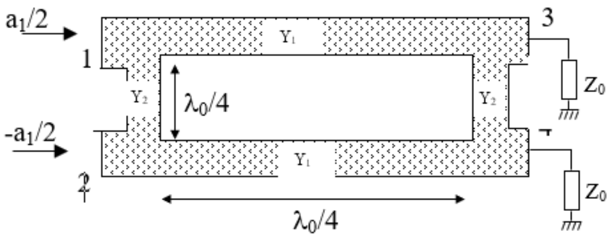


Figure 6

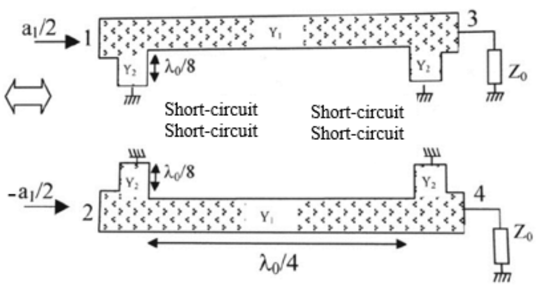


Figure 7

b) Repeat the procedure in part 1-b) to calculate the parameters $[S]^{i}$ of the quadripole presented in figure 7.

Show that:

$$S_{11}^{i} = \frac{\dfrac{j}{y_1} - j\left(y_1 - \dfrac{y_2^2}{y_1}\right)}{2\dfrac{y_2}{y_1} + j\left(\dfrac{1}{y_1} + y_1 - \dfrac{y_2^2}{y_1}\right)} \qquad S_{21}^{i} = \frac{2}{2\dfrac{y_2}{y_1} + j\left(\dfrac{1}{y_1} + y_1 - \dfrac{y_2^2}{y_1}\right)} \qquad S_{12}^{i} = S_{21}^{i}$$

$$S_{11}^{i} = S_{22}^{i}$$

3) Give the relationships between the matrix [S] of the octopole and the matrix $[S]^{i}$ and $[S]^{P}$.

4) Give a condition, relationship between y_1 and y_2, so that is matching at the 4 accesses of the octopole. Show then that S_{21} is equal to 0.

5) Give the conditions on y_1 and y_2 to obtain, at f_0, $|S_{31}| = |S_{41}| = -3$dB.

Have you designed a good 3 dB coupler?