MICROWAVE ENGINEERING

Lecture 15:
Problems on
Waveguides

Problem 1 : consider a section of an air filled K-band waveguide ($\varepsilon_r = 1$). From the dimensions typically assigned for this band, determine the cutoff frequencies of the first two propagating modes.

The geometrical sizes from the table are 1.07 x 0.43 cm ($a > b$)

$a = 1.07$ cm

$b = 0.43$ cm

$$f_{c\,mn} = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

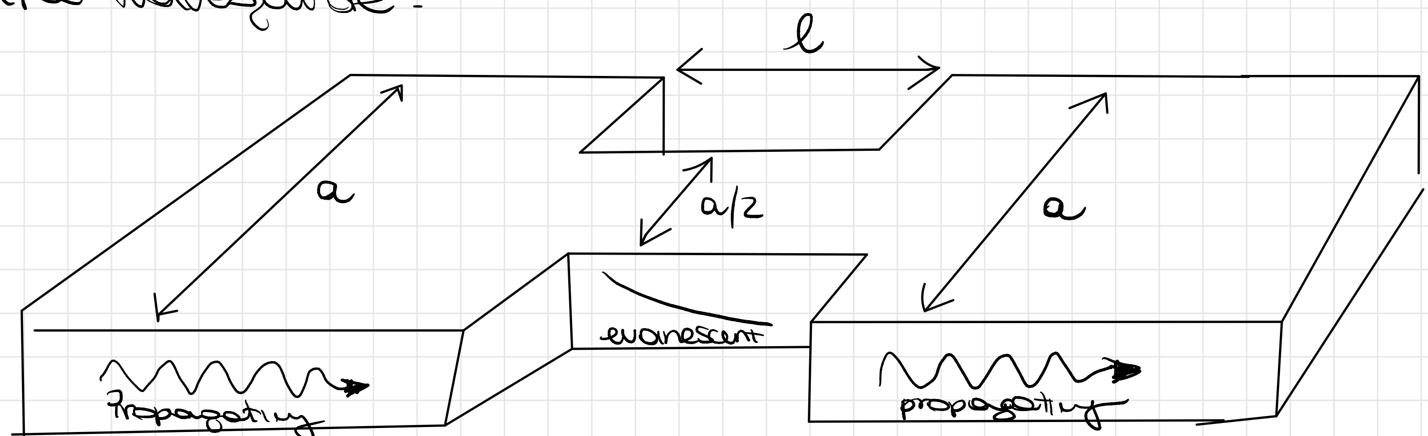
$$f_{c10} = \frac{1}{2\pi\sqrt{\mu_0\varepsilon_0}}\sqrt{\left(\frac{\pi}{a}\right)^2} = \frac{c}{2\pi}\cdot\frac{\pi}{a} = \frac{c}{2a} = \frac{3\cdot 10^8}{2\cdot 1.07\cdot 10^{-2}} = 1.4\cdot 10^{10}\ \text{Hz} = 14\ \text{GHz}$$

$$f_{c20} = \frac{1}{2\pi\sqrt{\mu_0\varepsilon_0}}\sqrt{\left(\frac{2\pi}{a}\right)^2} = \frac{c}{2\pi}\,\frac{2\pi}{a} = \frac{c}{a} = \frac{3\cdot 10^8}{1.07\cdot 10^{-2}} = 28\ \text{GHz}$$

$$f_{c01} = \frac{1}{2\pi\sqrt{\mu_0\varepsilon_0}}\sqrt{\left(\frac{\pi}{b}\right)^2} = \frac{c}{2\pi}\,\frac{\pi}{b} = \frac{c}{2b} = \frac{3\cdot 10^8}{2\cdot 0.43\cdot 10^{-2}} = 34.88\ \text{GHz}$$

$$f_{c11} = \frac{1}{2\pi\sqrt{\mu_0\varepsilon_0}}\sqrt{\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2} = 37.5\ \text{GHz}$$

<u>Problem 2</u> : An attenuator can be made using a section of waveguide operating below cutoff, as shown in picture. If $a = 2.286$ cm and the operating frequency is $f = 12$ GHz, determine the length of the cutoff section to achieve 100 dB attenuation. Ignore effects of reflection and discontinuities and assume only the lowest order TE mode is propagating in the waveguide.



TE_{10} mode propagates in the first section

The wavenumber associated to the operating frequency is

$$k = \frac{2\pi f}{c} = \frac{2\pi \cdot 12 \cdot 10^9}{3 \cdot 10^8} = 251.3 \ m^{-1}$$

The cutoff wavenumber for TE_{10} is:

$$k_c = \sqrt{\left(\frac{\pi}{a/2}\right)^2} = \frac{2\pi}{a} = 274.85 \ m^{-1}$$

$k_c > k \Rightarrow$ the prop. constant will be purely imaginary in the center region

$$\beta = \sqrt{k^2 - k_c^2} \Rightarrow \alpha = \sqrt{k_c^2 - k^2} = 111.3 \ Np/m$$

To get 100 dB attenuation

$$-100\,dB = 20\log_{10} e^{-\alpha \ell} \Rightarrow 10^{-5} = e^{-\alpha \ell}$$

$$-100 = 20\log_{10} x$$

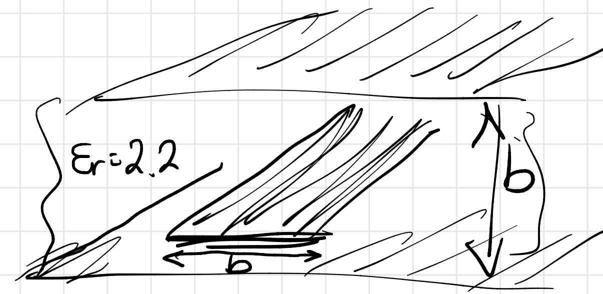
$$x = 10^{-5}$$

$$\ln 10^{-5} = -\alpha \ell \Rightarrow \ell = \frac{\ln 10^{-5}}{-\alpha} = \frac{-11.5}{-1.113} = 10.3\ \text{cm}$$

Problem 3: Design a stripline transmission line for a 70 Ω characteristic impedance. The ground plane separation is 0.316 cm and the dielectric constant of the filling material is $\varepsilon_r = 2.2$. What is the guide wavelength on the transmission line if $f = 3\,GHz$?

$$\frac{W}{b} = \begin{cases} x & \sqrt{\varepsilon_r}\, Z_0 \leq 120 \\ 0.85 - \sqrt{0.6 - x} & \sqrt{\varepsilon_r}\, Z_0 > 120 \end{cases}$$

$$x = \frac{30\pi}{\sqrt{\varepsilon_r}\, Z_0} - 0.441$$



The effective impedance is $\sqrt{\varepsilon_r} Z_0 = \sqrt{2.2} \cdot 70 = 103.8 < 120\,\Omega$.

$$\frac{W}{b} = x = \frac{30\pi}{\sqrt{\varepsilon_r}\, Z_0} - 0.441 \Rightarrow W = b\left(\frac{30\pi}{\sqrt{\varepsilon_r}\, Z_0} - 0.441\right) =$$

$$= 0.316 \cdot 10^{-2} \left(\frac{30\pi}{103.8} - 0.441\right) = 0.147 \text{ cm}$$

the wavelength in the waveguide is

$$\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_r}} = \frac{c}{\sqrt{\varepsilon_r}\, f} = \frac{3 \cdot 10^8}{\sqrt{2.2} \cdot 3 \cdot 10^9} = \frac{0.1}{\sqrt{2.2}} = 6.74 \text{ cm}$$

Problem 4 : Consider a circular waveguide with $a = 0.8$ cm, filled with a dielectric material with $\varepsilon_r = 2.3$.
Calculate the cutoff frequencies and identify the first four propagating modes.

For circular waveguides:

TE $\Rightarrow$ $$f_{c\,nm} = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{p'_{nm}}{2\pi a\sqrt{\mu\varepsilon}}$$

TM $\Rightarrow$ $$f_{c\,nm} = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{p_{nm}}{2\pi a\sqrt{\mu\varepsilon}}$$

$TE_{01} \Rightarrow$

$$f_{c_{TE01}} = \frac{3.832}{2\pi \cdot 0.8 \cdot 10^{-2} \sqrt{\mu_0 \varepsilon_0 \varepsilon_r}} = \frac{3.832\, c}{2\pi \cdot 0.8 \cdot 10^{-2} \sqrt{\varepsilon_r}} =$$

$$= 15 \text{ GHz}$$

$TE_{11} \Rightarrow$

$$f_{c_{TE11}} = \frac{1.841\, c}{2\pi \cdot 0.8 \cdot 10^{-2} \sqrt{\varepsilon_r}} = 7.24 \text{ GHz}$$

$TE_{21} \Rightarrow$

$$f_{c_{TE21}} = \frac{3.054\, c}{2\pi \cdot 0.8 \cdot 10^{-2} \sqrt{\varepsilon_r}} = 12.02 \text{ GHz}$$

$TM_{01} \Rightarrow f_{c\,TM_{01}} = \dfrac{2.405\,c}{2\pi a\sqrt{\varepsilon_r}} = 9.46\ \text{GHz}$

TM_{11}
TE_{01} $\Rightarrow f_{c\,TM_{11}} = \dfrac{3.832\,c}{2\pi a\sqrt{\varepsilon_r}} = 15\ \text{GHz}$

$TM_{21} \Rightarrow f_{c\,TM_{21}} = \dfrac{5.135\,c}{2\pi a\sqrt{\varepsilon_r}} = 20.2\ \text{GHz}$

$$\boxed{TE_{11} \rightarrow TM_{01} \rightarrow \begin{matrix} TM_{11} \\ TE_{01} \end{matrix} \rightarrow TE_{21} \rightarrow TM_{21}}$$

Problem 5 An X-band waveguide filled with TEFLON is operating at 9.5 GHz. Calculate the speed of light in this material phase and group velocity in the waveguide. Permittivity of teflon is $\varepsilon_r = 2.08$.

From table we get $a = 2.286$ cm
$b = 1,016$ cm

The speed of light in teflon is $\frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \cdot 10^8}{\sqrt{2.08}} = 2.08 \cdot 10^8$ m/s

Wavenumber: $k = \frac{2\pi f \sqrt{\varepsilon_r}}{c} = \frac{2\pi \cdot 9.5 \cdot 10^9 \sqrt{2.08}}{3 \cdot 10^8} = 287 \ m^{-1}$

The lowest order mode has $k_c = \sqrt{\left(\frac{\pi}{a}\right)^2} = \frac{\pi}{2.286 \cdot 10^{-2}} = 137.42 \ m^{-1}$

The propagation constant is:

$$\beta = \sqrt{k^2 - k_c^2} \cong 252 \ m^{-1}$$

The phase velocity is: $V_p = \dfrac{\omega}{\beta} = \dfrac{2\pi f}{\beta} = \dfrac{2\pi \cdot 9.5 \cdot 10^9}{252} = 2.37 \cdot 10^8$ m/s

The group velocity is:

$$V_g = \left(\frac{\partial \beta}{\partial \omega}\right)^{-1} = \left(\frac{\partial \beta}{\partial k} \frac{\partial k}{\partial \omega}\right)^{-1} = \left(\frac{k}{\beta}\sqrt{\mu\varepsilon}\right)^{-1} = \frac{\beta}{k\sqrt{\mu\varepsilon}} =$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$\frac{\partial k}{\partial \omega} = \sqrt{\mu\varepsilon}$$

$$= \frac{252 \cdot c}{k\sqrt{\varepsilon_r}} = 1.83 \cdot 10^8 \ m/s$$

$\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}$

$$\frac{\partial \beta}{\partial k} = \frac{\partial}{\partial k}\sqrt{k^2 - k_c^2} = \frac{1}{\cancel{2}}\underbrace{\left(k^2 - k_c^2\right)^{-\frac{1}{2}}}_{\beta} \cdot \cancel{2}k = \frac{k}{\beta}$$

$$\boxed{v_g < \frac{c}{\sqrt{\varepsilon_r}} < v_p}$$