Set #7

20. Use the following property:

$$[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\hat{C} + \hat{B}[\hat{A}, \hat{C}]$$

to show that $[L_x, L_y] = i\hbar L_z$ e that $[L^2, L_x] = 0$.

21. Quantum numbers for macroscopic systems (*Bohr's Correspondence Principle: classical physics is recovered in the limit of large quantum #s*).

a. Estimate the angular momentum quantum number "n" for a Ferries wheel using the Bohr condition for the quantisation of the angular momentum. (Use your own parameters set for the estimate.)

b. The energy of a harmonic oscillator (i.e. mass and spring) are quantised via

$$E_n = \hbar\omega_o(n + \frac{1}{2})$$

where $\omega = \sqrt{k/m}$ is the natural frequency of oscillation. If you push down the back end of a car, it will spring back. Estimate the quantum number associated with this classical motion. (Use your own parameters set for the estimate).

22. How many possible orientations may the angular momentum of a rotating wheel (m=10 g, r=50 cm, T=1 sec) take?

How many possible angular momentum states do we span when turning the wheel's hub by 10 degrees. Compare with the above exercise (22) and briefly explain.

23. Show all possible orientations of the electron orbital angular momentum in the energy level n=3 (l=0,1,2). Give all the angles the orbital magnetic moment μ forms about a given direction (z).

- Use the following property:

$$[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\hat{C} + \hat{B}[\hat{A}, \hat{C}]$$

to show that $[L_x, L_y] = i\hbar L_z$ e that $[L^2, L_x] = 0$.

Commutator $[L_x, L_y] = ?$

$[\,,\,] \equiv L_x L_y - L_y L_x = (y p_z - z p_y)(z p_x - x p_z) - (z p_x - x p_z)(y p_z - z p_y)$

$= y p_x p_z z - \cancel{z^2 p_y p_x} - \cancel{x y p_z^2} + x p_y z p_z$

$- (y p_x z p_z - \cancel{x y p_z^2} - \cancel{z^2 p_x p_y} + x p_y p_z z)$

$= -y p_x [z, p_z] + x p_y [z, p_z]$

$= i\hbar \underbrace{(x p_y - y p_x)}_{L_z} \equiv i\hbar L_z$

- Independent Directions (x, y, z) always commute
- $[z, p_z] = i\hbar$ (in class)

Commutator $[L^2, L_x] = ?$

$[L^2, L_x] = [L_y^2, L_x] + [L_z^2, L_x]$

. Use: $[AB, C] = A[B, C] + [A, C]B$) (ops, commutators)

$= L_y [L_y, L_x] + [L_y, L_x] L_y +$

$L_z [L_z, L_x] + [L_z, L_x] L_z = -i\hbar L_y L_z - i\hbar L_z L_y$

$+ i\hbar L_z L_y + i\hbar L_y L_z$

Two by two have same order

$= 0$

Quantum numbers for macroscopic systems (*Bohr's Correspondence Principle: classical physics is recovered in the limit of large quantum #s*).

a. Estimate the angular momentum quantum number "n" for a Ferries wheel using the Bohr condition for the quantisation of the angular momentum. (Use your own parameters set for the estimate.)

b. The energy of a harmonic oscillator (i.e. mass and spring) are quantised via

$$E_n = \hbar\omega_o(n + \frac{1}{2})$$

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a) Ferry wheel

Take $R \simeq 5\,m$ $\quad m \simeq 2\times10^3\,kg$ $\quad T \simeq 20$ sec.

$v = \omega r = 2\pi R/T$ $\qquad L = mvR = 2\times10^3\,kg \,\dfrac{2\pi\,(5\,m)^2}{20\,\text{sec.}} = \underbrace{\left[\dfrac{kg\,m^2}{sec}\right]}_{J\cdot sec.} 1.57\times10^4$

Bohr's condition: $L = \hbar\cdot n$ or

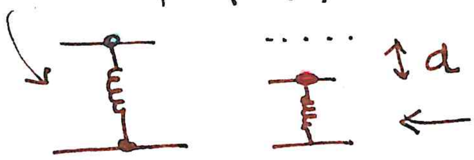
$1.57\times10^4\,\cancel{J\cdot sec.} = n\cdot 1.05\times10^{-34}\,\cancel{J\cdot s} \Rightarrow n \simeq 10^{38}$

Also: $L = \hbar\sqrt{\ell(\ell+1)}$ or

$1.57\times10^4\,\cancel{J\cdot s} = 1.05\times10^{-34}\,\cancel{J\cdot s}\sqrt{\ell(\ell+1)}$ or

$\ell(\ell+1) = 2.2\times10^{76}$ $\qquad \ell^2 \simeq 2.2\times10^{76} \cong 1.5\times10^{38}$ (ok) b.)

man + spring eq. position



$\updownarrow d \sim 10\ cm$

At the max stretching position the two forces are equal (magnitude)

$\uparrow kd$ (spring force)

$\downarrow mg$ (gravity: m = body)

$\downarrow$

80 kg

① $k = \frac{mg}{d}$ ② $\omega_0 = \sqrt{k/m}$: $\omega_0 = \sqrt{\frac{mg}{d \cdot m}} = \sqrt{g/d} = \sqrt{\frac{9.8\ m/s^2}{10\ cm}} = \ldots = 2\pi \times 1.58\ \frac{rad}{sec}$

$\hbar\omega_0 = 6.6 \times 10^{-34}\ J \cdot sec \cdot 1.58\ \frac{1}{sec}$

$\cong 10^{-33}\ J$

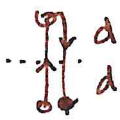
If there is no dissipation energy is conserved $\frac{1}{2} m v_{max}^2 = mgd = E$

$E = 80\ kg \cdot 9.8\ \frac{m}{s^2} \cdot 10^{-2}\ m = 7.84\ J$

The system osc/w: $E = \hbar\omega(n + \frac{1}{2})$ (h.o.) $n + \frac{1}{2} = \frac{E}{\hbar\omega_0} \cong 10^{34}$

or $n \simeq 10^{34}$ (huge!!)

Check estimate:



period $T_0 = 2\pi/\omega_0 = \frac{1}{1.58}$ sec.

Vel. $\sim \frac{4d}{T_0} = \frac{40\ cm}{\frac{1}{1.58}\ sec} = 0.4 \times 1.58\ m/sec.$

$E \cong \frac{1}{2} m v_{el}^2 = \frac{1}{2} 80\ kg \left(0.4 \times 1.58\ \frac{m}{s}\right)^2 =$

$= 15.9\ J \neq 7.84\ J$ because

vel → sort of a "mean" vel. (uniform motion)

- How many possible orientations may the angular momentum of a rotating wheel (m=10 g, r=50 cm, T=1 sec) take? How many possible angular momentum states do we span when turning the wheel's axis by 10 degrees.

Ang. Mom. of the wheel: $L = mvr = m\omega r^2$ $\qquad \omega T = 2\pi$

$$= \underbrace{10^{-3}\,\text{kg}\ 10}_{m}\ \underbrace{\frac{2\pi}{1\,\text{sec.}}}_{\omega}\ (50\times10^{-2}\,\text{m})^2 = 1.57\times10^{-2}\ \frac{\text{kg m}^2}{\text{s}}$$

$$\equiv 1.57\times10^{-2}\ \frac{\text{kg m}^2}{\text{s}} \times \frac{\hbar}{\hbar} = \frac{1.57\times10^{-2}\ \text{J}\cdot\text{s}}{6.6\times10^{-34}\ \text{J}\cdot\text{s}}\ \hbar$$

$$\cong 2.38\times10^{31}\ \hbar$$

- From Q.M. the ang. mom. eigenvalues are: $\hbar\sqrt{\ell(\ell+1)}$ (ORBITAL)

So for our wheel we have:

$$2.38\times10^{31}\ \hbar = \hbar\sqrt{\ell(\ell+1)} \quad \text{or} \quad \boxed{\ell \simeq 10^{31}}$$

Big integer !!

- From. Q.M. all possible orientations are:

$\hbar m_\ell$ with. $\qquad -\ell \le m_\ell \le \ell$

z

Largest projection $\hbar\ell = \hbar\sqrt{\ell(\ell+1)}\cos\vartheta_0$

Projection at 10° $\hbar\ell_{10} = \hbar\sqrt{\ell(\ell+1)}\cos\vartheta_1$

$\llcorner \vartheta_1 10^\circ$

$$\vartheta_0 = \cos^{-1}\frac{\ell}{\sqrt{\ell(\ell+1)}} \simeq 0 \qquad \ell_{10} = \sqrt{\ell(\ell+1)}\cos10^\circ \simeq 10^{31}\times\cos10^\circ$$

When wheel axis goes from $\vartheta_0 = 0 \to \vartheta_1 = 10^\circ$ ℓ goes from $10^{31} \to 10^{31}\cos10^\circ$

Spanned ang. mom. $10^{31} - 10^{31}\cos10^\circ = 1.5\times10^{29}$!!

- Show all possible orientations of the electron <u>orbital</u> angular momentum in the energy level n=3 (l=0,1,2). Give all the angles that the orbital magnetic moment forms about a given direction (z).

$n=3 \quad l=0,1,2$ (possibili valori di l)

$L = \hbar\sqrt{l(l+1)} \Rightarrow L=0 \quad L=\sqrt{2}\,\hbar \quad L=\sqrt{6}\,\hbar$ (possibili valori di L)

$|\vec{\mu}| = \frac{|e|}{2m} L \Rightarrow |\vec{\mu}_1| = \frac{\hbar|e|}{2m}\, 0$

$|\vec{\mu}_2| = \frac{\hbar|e|}{2m}\sqrt{2} = \mu_B\sqrt{2} = 5.79\times10^{-5}\, eV/T\ \sqrt{2} =$

$|\vec{\mu}_3| = \;..\; = \mu_B\sqrt{6} = 5.79\times10^{-5}\, eV/T\ \sqrt{6}$

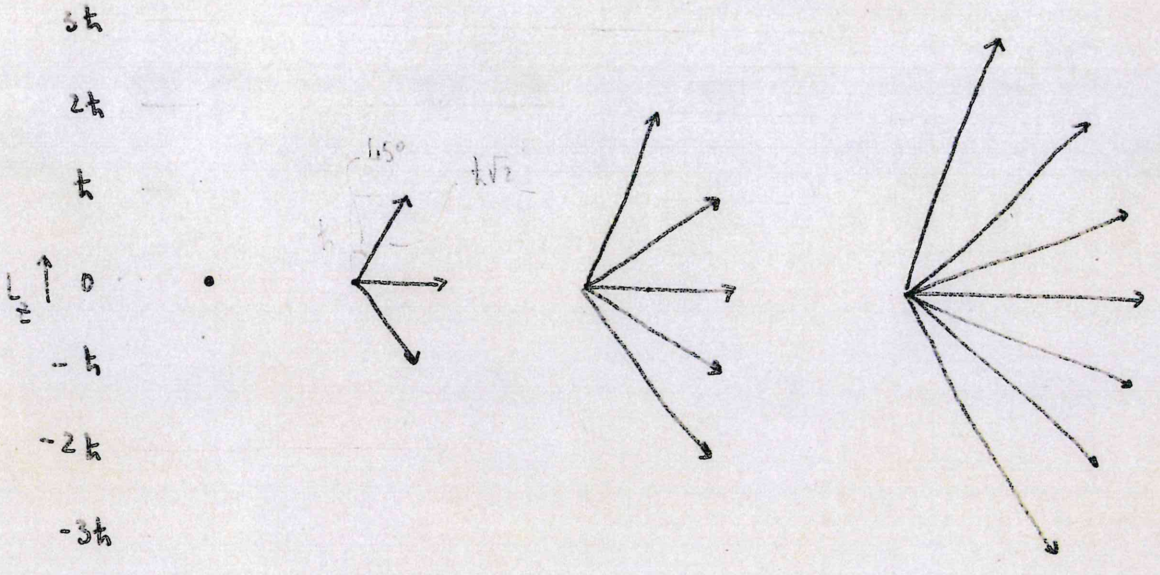
Tre possibili valori del mom. magnetico associato al moto orbitale.

Bohr prevede solo un mom. magnetico per n=3: $(L=\hbar n)$

$|\vec{\mu}| = \frac{\hbar e}{2m} n \rightarrow \mu_B\, 3$ (diverso dai 3 sopra)

Possibili orientazioni di $\vec{L}$ lungo una data direzione (z) sono date dalle possibili componenti di $\vec{L}$ lungo z.

L_z può assumere i valori: $\hbar m_l$ con $m_l = -l, -l+1, \ldots 0 \ldots l-1, l$.



$l=0$	$l=1$	$l=2$	$l=3$
$L=0$	$L=\hbar\sqrt{2}$	$L=\hbar\sqrt{6}$	$L=\hbar\sqrt{12}$

Per ogni l, i vettori mom. ang. L hanno tutti stessa lunghezza (non