SELF PHASE MODULATION (SPM)

An interesting manifestation of the intensity dependence of the refractive index in nonlinear optical media occurs through self-phase modulation (SPM). A general description of SPM in optical fibers requirer numerical solutions of the pulse propagation equation (NLSE).

A simplification occurs if the effect of GVD on SPM is negligible so that the B_2 term can be set to zero in the NLSE.

NLSE
$$j\frac{\partial F(z,t)}{\partial z} - \frac{B_2}{2}\frac{\partial^2 F(z,t)}{\partial t^2} + \gamma |F(z,t)|^2 F(z,t) = 0$$

In the case $B_2 = 0$, or in the limit $B_2 \to 0$, we have
$$j\frac{\partial F(z,t)}{\partial z} + \gamma |F(z,t)|^2 F(z,t) = 0$$

or

$$\frac{\partial F}{\partial z} = i \gamma |F|^2 F \qquad (1) \qquad F = F(z,t)$$

We can define a nonlinear lengh L_{LN} for (1)

$$L_{NL} = 1/\gamma \qquad (2)$$

where γ is the effective nonlinear coefficient, related to $\chi^{(3)}$ nonlinearity.

P.S. $L_{NL} = 1/\gamma P_0$, where P_0 is peak power of the pulse.
For sake of simplicity i set $P_0 = 1$.

Eq. (1) can be solved substituting $F = V \exp(i\phi_{NL})$ and equating the real and imaginary parts so that:

$$\frac{\partial V}{\partial z} = 0 \qquad \frac{\partial \phi_{NL}}{\partial z} = \gamma V^2 \qquad (3)$$

The amplitude V does not change along the fiber of length L.

The phase equation can be integrated analytically to obtain the general solution:

$$F(z,t) = F(z=0,t)\exp[i\phi_{NL}(z,t)] \tag{4}$$

where $F(0,t)$ is the envelope at $z=0$ and

$$\phi_{NL}(z,t) = |F(0,t)|^2 \gamma z \tag{5}$$

Equation (4) shows that SPM gives rise to an intensity-dependent phase shift but the pulse hape remains unaffeted in the temporal domain.

The nonlinear phase shift ϕ_{NL} in Eq (5) increases with fiber length z ($z = L$).

The maximum phase shift ϕ_{MAX} occurs at the pulse center located a $t = 0$.
with F normalized such that $|F(0,0)| = 1$ (f.i. considering gaussian pulses $e^{-t^2/2t_0^2}$), it is given by

$$\phi_{MAX} = \gamma L \qquad (7)$$

The physical meaning of the nonlinear length L_{NL} is clear from Eq. (7), it is the effective propagation distance at which $\phi_{MAX} = 1$.

The SPM-induced spectral broadening is a consequence of the time dependence of ϕ_{NL}.

This can be understood by noting that a temporally varying phase implies that the instantaneous optical frequency differs across the pulse from its central value ω_0.

The difference $\delta\omega$ is given by

$$\delta\omega(t) = \pm^{*} \frac{\partial \phi_{NL}}{\partial t} = \pm^{*} \gamma L \frac{\partial}{\partial t} |F(0,t)|^2 \quad (8)$$

The time dependence of $\delta\omega$ is referred to as frequency chirping. The chirped induced by SPM increases in magnitude with the propagated distance L.

* $\pm$ signs are due to the choice of the carrier factor $\exp(i(\pm)\omega_0 t + i\beta_0 z)$

In other words, new frequency components are generated continuously as the pulse propagates down the fiber. The SPM-generated frequency components broaden the spectrum over its initial width @ $z = 0$.

The extent of spectral broadening depends on the pulse shape. Consider, as an example, the case of a super-gaussian pulse. The SPM induced chirp $\delta\omega(\tau)$ is:

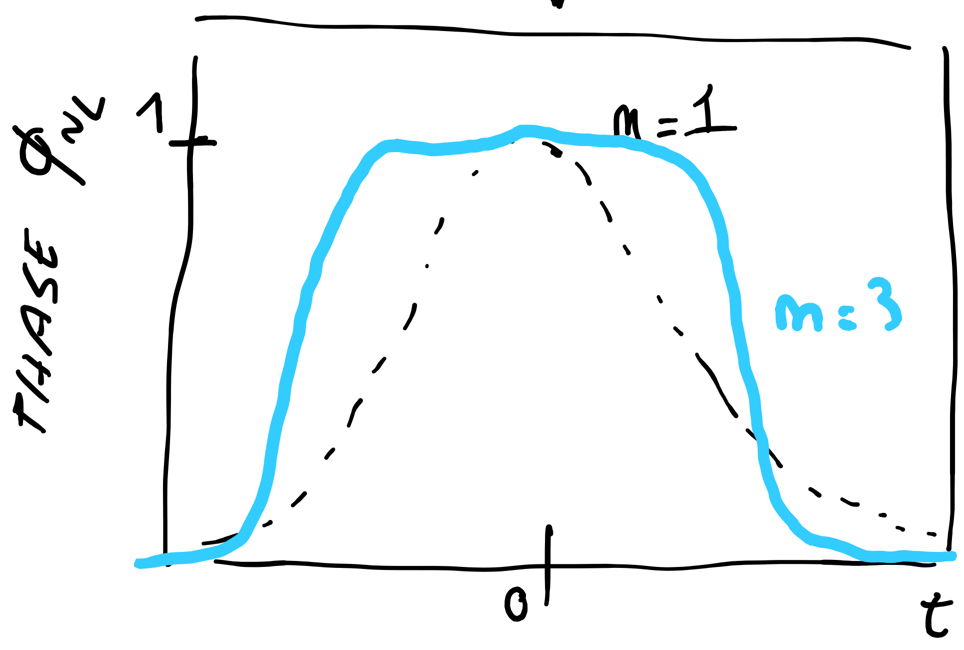
$$\delta\omega(t) = \frac{2m}{t_0}\,\gamma L \left(\frac{t}{t_0}\right)^{2m-1} \exp\left(-\left(\frac{t}{t_0}\right)^{2m}\right)$$

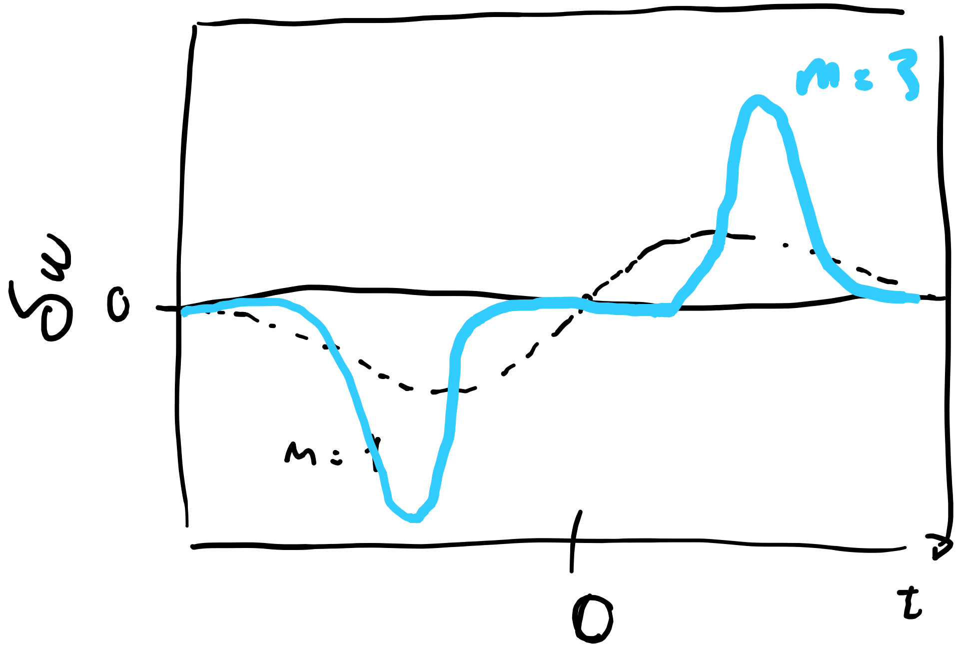
where for $m=1$ we have gaussia pulses.

For larger values of m, The pulse becomes nearly rectangular with increasingly steeper edges.

We can consider and plot The variation of The nonlinear phase shift ϕ_{NL} and The induced Frequency chirp $\delta\omega$ in The cases of gaussian $(m=1)$

and a supergaussia pulse ($m = 3$). As ϕ_{NL} is directy proportional to $|F(0,t)|^2$ in Eq. (5) its temporal variation is identical to that of the pulse intensity $I = |F|^2$.





The Temporal variation of the induced chirp $\delta\omega$ has interesting features. First, $\delta\omega$ is negative near the leading edge (red shift) and becomes positive near the trailing edge (blue shift) of the pulse. Second, the chirp is linear and positive over a large central region of the gaussian pulse. Third, the chirp is larger for pulses with steeper and trailing edges.

SPM induces NONLINEAR PHASE SHIFT.

...

CHANGES IN PULSE SPECTRA ?? (NEXT LECTURE).