# MICROWAVE ENGINEERING

Lecture 23:
Problems on
Resonators

# 1 Half-wave Microstrip Resonator

Consider a microstrip resonator constructed from a $\lambda/2$ length of a 50Ω open-circuit microstrip line, whose width is $W = 0.508$ cm. The substrate is Teflon ($\varepsilon_r = 2.08$, $\tan\delta = 0.0004$) with a thickness 0.159 cm. The conductors are copper. Calculate the length of the line at 5 GHz and the Q of the resonator ignoring fringing fields at the end of the line. Let's assume the effective permittivity of the microstrip is $\varepsilon_e = 1.8$.

NOTE: For a microstrip resonator the attenuations in the conductor and in the dielectric are:

$$\alpha_c = R_s / Z_0 W \qquad \alpha_d = \frac{k_0 \varepsilon_r (\varepsilon_e - 1) \tan\delta}{2\sqrt{\varepsilon_e}\,(\varepsilon_r - 1)}$$

$$\sigma_{copper} = 5.813 \cdot 10^7 \ \frac{S}{m}$$

The resonant length of the line $l = \frac{\lambda}{2} = \frac{V_p}{2f} = \frac{c}{2f\sqrt{\varepsilon_e}} =$

$$= \frac{3 \cdot 10^8}{2(5 \cdot 10^9)\sqrt{1.8}} = 2.24 \text{ cm}$$

The propagation constant is: $\beta = \frac{2\pi f}{V_p} = \frac{2\pi f \sqrt{\varepsilon_e}}{c} =$

$$= \frac{2\pi (5 \cdot 10^9)\sqrt{1.8}}{3 \cdot 10^8} = 141 \frac{\text{rad}}{\text{m}}$$

The Q factor of a microstrip resonator $Q = \frac{\beta}{2\alpha}$

Attenuation from the conductor is

$$\alpha_c = \frac{R_s}{Z_0 W} = \frac{1.84 \cdot 10^{-2}}{50 \cdot 0.508 \cdot 10^{-2}} = 0.07 \ \frac{Np}{m}$$

$$R_s = \sqrt{\frac{\omega \mu_0}{2\sigma}} = \sqrt{\frac{2\pi f \mu_0}{2 \cdot \sigma}} = 1.84 \cdot 10^{-2} \ \Omega$$

Attenuation from the dielectric is :

$$\alpha_d = \frac{k_0 \varepsilon_r (\varepsilon_e - 1) \tan\delta}{2\sqrt{\varepsilon_e} \ (\varepsilon_r - 1)} = \frac{104.7 \cdot 2.08 \cdot 0.8 \cdot 0.0004}{2\sqrt{1.8} \ (1.08)} = 0.024 \ Np/m$$

$$\underline{Q} = \frac{\beta}{2\alpha} = \frac{151}{2(0.07 + 0.024)} = \underline{783}$$

② Design of a rectangular waveguide cavity

A rectangular waveguide cavity is made from a piece of copper WR-187 H-band waveguide with $a = 4.755$ cm and $b = 2.215$ cm. The cavity is filled with polyethylene ($\varepsilon_r = 2.25$, $\tan\delta = 0.0004$). If resonance occurs at $f = 5$ GHz, find the required length $d$ and resulting $Q$ for the $\ell = 1$ and $\ell = 2$ resonant modes.

($\sigma_{copper} = 5.813 \cdot 10^7$ S/m)

The wave number $k$ is:

$$k = \frac{2\pi f \sqrt{\varepsilon_r}}{c} = 157.08 \text{ cm}^{-1}$$

For $m=1$, $n=0$ the length can be derived from the expression:

$$f = \frac{c}{2\pi\sqrt{\mu_r \varepsilon_r}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \Rightarrow$$

$$= \frac{c}{2\pi\sqrt{\mu_r \varepsilon_r}} \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

$$k = \frac{2\pi f \sqrt{\varepsilon_r}}{c} = \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

$$k^2 - \left(\frac{\pi}{a}\right)^2 = \left(\frac{l\pi}{d}\right)^2$$

$$d = \frac{l\pi}{\sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}} = \begin{cases} l=1 & d = 2.2\ \text{cm} \quad (101\ \text{mode}) \\ l=2 & d = 4.4\ \text{cm} \quad (102\ \text{mode}) \end{cases}$$

$$R_S = \sqrt{\frac{\omega \mu_0}{2\sigma}} = 1.84 \cdot 10^{-2}\ \Omega$$

$$\eta = \frac{377}{\sqrt{\varepsilon_r}} = 251.3\ \Omega$$

For the conductor :

$$Q_c = \frac{(kad)^3 b \eta}{2\pi^2 R_S} \frac{1}{(2l^2 a^3 b + 2bd^3 + l^2 a^3 d + ad^3)}$$

For $l=1 \Rightarrow Q_c = 8403$

$l=2 \Rightarrow Q_c = 11898$

For the dielectric :

$$Q_d = \frac{2\omega W_e}{P_d} = \frac{\varepsilon'}{\varepsilon''} = \frac{1}{\tan\delta} = \frac{1}{0.0004} = 2500$$

(for both $l=1$ and $l=2$)

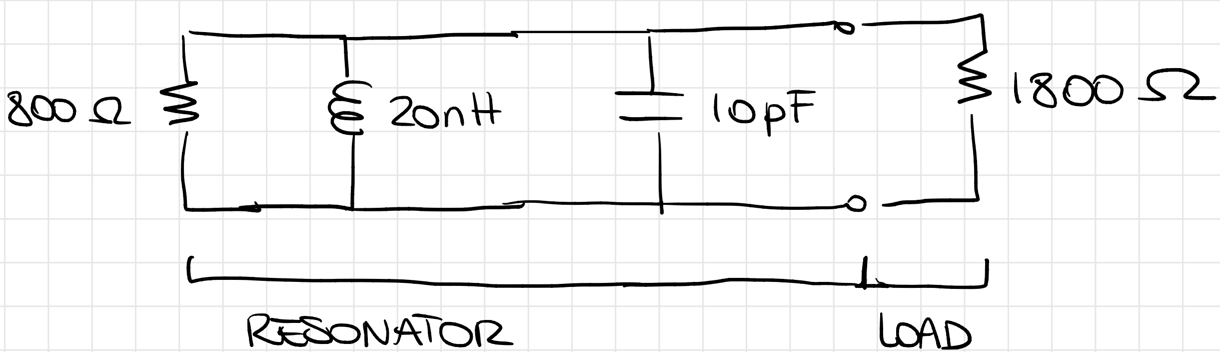
The total Q is:

$l=1 \Rightarrow$ $$Q_{l=1} = \left(\frac{1}{Q_c} + \frac{1}{Q_d}\right)^{-1} = \left(\frac{1}{8403} + \frac{1}{2500}\right)^{-1} = 1925$$

$l=2 \Rightarrow$ $$Q_{l=2} = \left(\frac{1}{11898} + \frac{1}{2500}\right)^{-1} = 3065$$

## (3) Resonant RLC circuit

Consider the following RLC resonant circuit. Calculate the resonant frequency of the circuit, the unloaded Q and the loaded Q.




$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{20\cdot 10^{-9}\cdot 10\cdot 10^{-12}}} = 355.88\ \text{MHz}$$

The unloaded Q is:

$$Q = \omega_0 RC = 2\pi f_0 RC = 2\pi \cdot 355.88 \cdot 10^6 \cdot 800 \cdot 10 \cdot 10^{-12} = 17.9$$

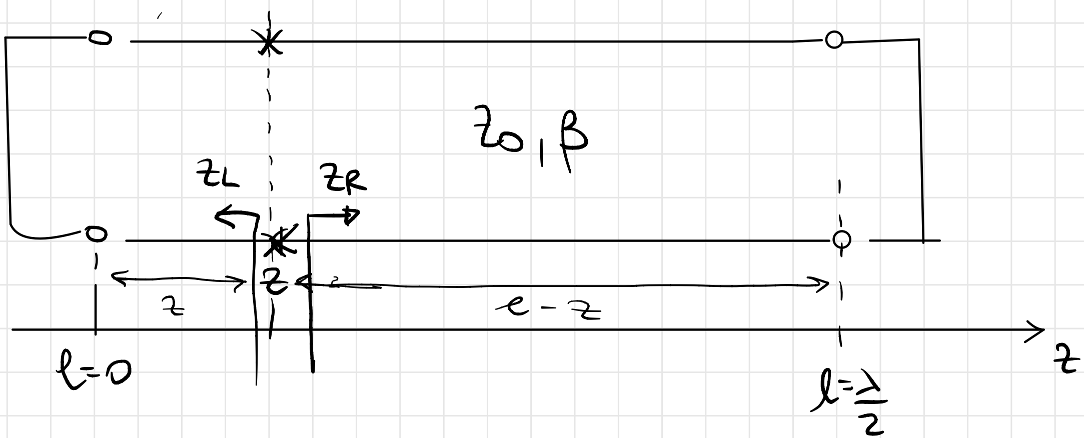
The external Q factor:

$$Q_e = \frac{R_L}{\omega_0 L} = \frac{1800}{2\pi f_0 \cdot 20 \cdot 10^{-9}} = 40.24$$

The loaded Q factor is:

$$Q_L = \frac{1}{\frac{1}{Q} + \frac{1}{Q_e}} = \frac{1}{\frac{1}{17.9} + \frac{1}{40.24}} = 12.38$$

(4) Consider the resonator in the drawing ($\lambda/2$ line shortened at both ends. Calculate, at an arbitrary point on the line $z$ the impedances $Z_L$ and $Z_R$ (left and right) and show that $Z_L = Z_R^*$.




$$Z_L = j Z_0 \tan \beta z$$

$$Z_R = j Z_0 \tan \beta (\ell - z)$$

We know that $l = \frac{\lambda}{2} \Rightarrow \beta l = \pi$

$$Z_R = j Z_0 \tan(\underbrace{\beta l}_{\pi} - \beta z) = j Z_0 \tan(\pi - \beta z) =$$

$$= -j Z_0 \tan(\beta z) = Z_L^*$$

⑤ An air-filled, brass-plated rectangular waveguide cavity has dimensions:

$$a = 4\ \text{cm}$$

$$b = 2\ \text{cm}$$

$$d = 5\ \text{cm}$$

Find the resonant frequency and Q of the $TE_{101}$ and $TE_{102}$ modes.

$$\left(\sigma_{BRASS} = 2.56 \cdot 10^{7}\ \frac{S}{m}\right)$$

The resonance frequency

$$f_{mnl} = \frac{c}{2\pi\sqrt{\underbrace{\mu_r\varepsilon_r}_{1}}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

Therefore

$$f_{101} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2} = 4.802 \text{ GHz}$$

$$f_{102} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{2\pi}{d}\right)^2} = 7.075 \text{ GHz}$$

We then calculate the resistivity value at each freq.

at 4.802 GHz $\Rightarrow$ $R_{s_{101}} = \sqrt{\frac{\omega\mu_0}{2\sigma}} = 0.0272\,\Omega$

at 7.075 GHz $\Rightarrow$ $R_{s_{102}} = \sqrt{\frac{\omega\mu_0}{2\sigma}} = 0.033\,\Omega$

The wavenumbers of the two modes are

$$\Rightarrow k_{101} = \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2} = 100.58\ m^{-1}$$

$$k_{102} = \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{2\pi}{d}\right)^2} = 148.18\ m^{-1}$$

$$Q_c = \frac{(kad)^3\, b\eta}{2\pi^2 R_s} \frac{1}{\left(2l^2a^3b + 2bd^3 + l^2a^3d + ad^3\right)} \qquad \eta = \eta_0$$

So we have:

$$Q_{101} = 7253$$

$$Q_{102} = 9119$$