Turbo Codes: an Introduction

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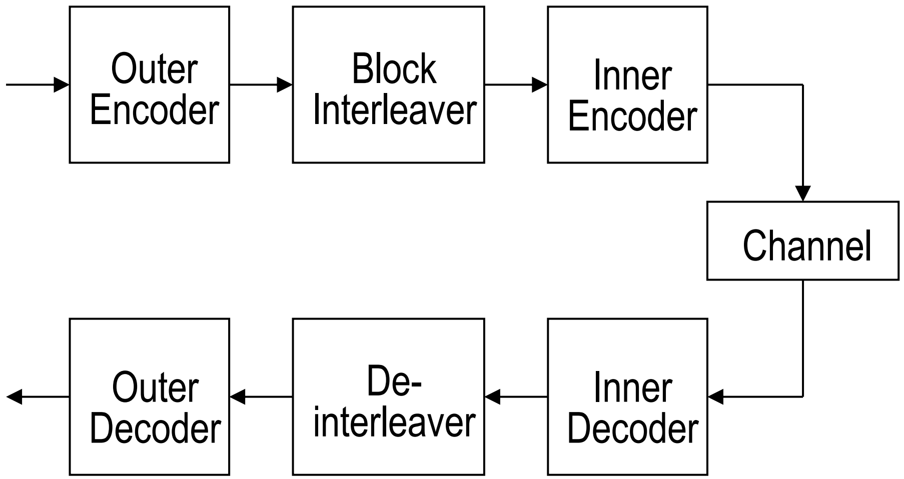
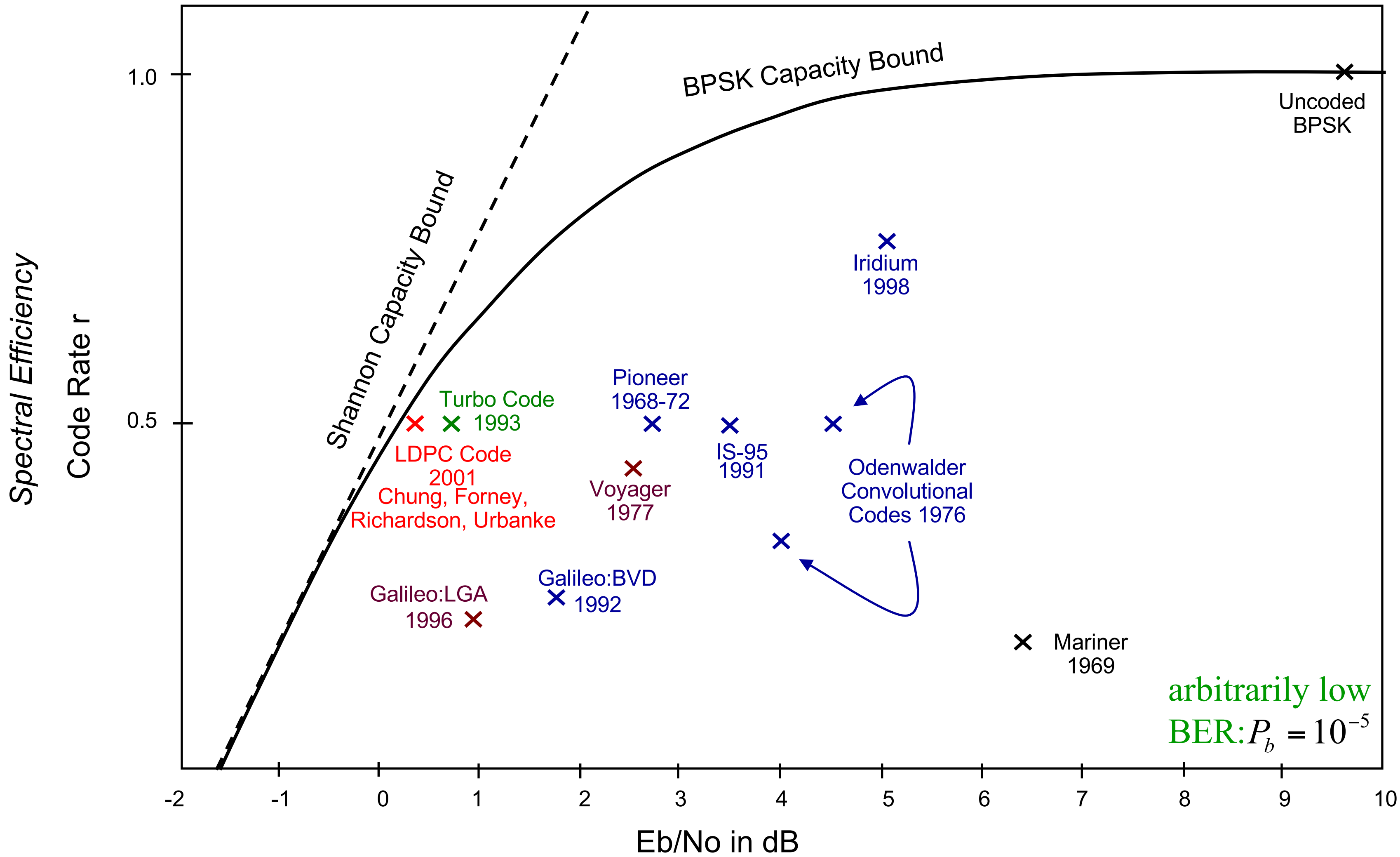
Standard (classic-style) Concatenated Coding (Series) 2



- Turbo codes (1993)
- LDPC (1962, and then (rediscovered !!!) 1999)

Power Efficiency of Standard Binary Channel Codes



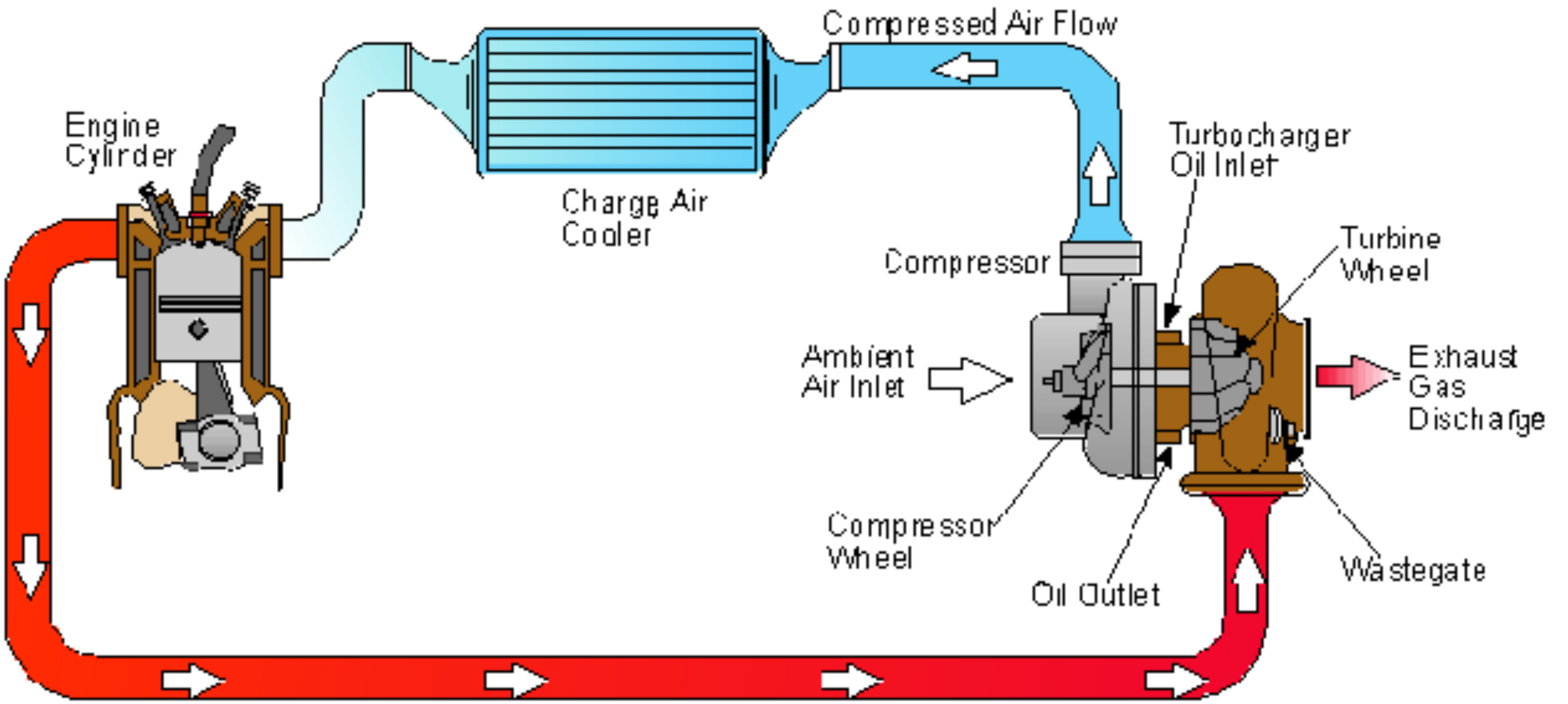
Turbo Codes

- Background
  - Proposed by Berrou, Glavieux, …, in ICC 1993
  - Performance within 0.5 dB from channel capacity !!!

- Principal (New) Characteristics
  - Parallel (instead of series) concatenated coding
  - Recursive systematic convolutional encoders
  - Pseudo-random interleaving
  - Iterative (SISO) decoding (as the turbo engine …)

Turbo principle

- Turbo Engine …



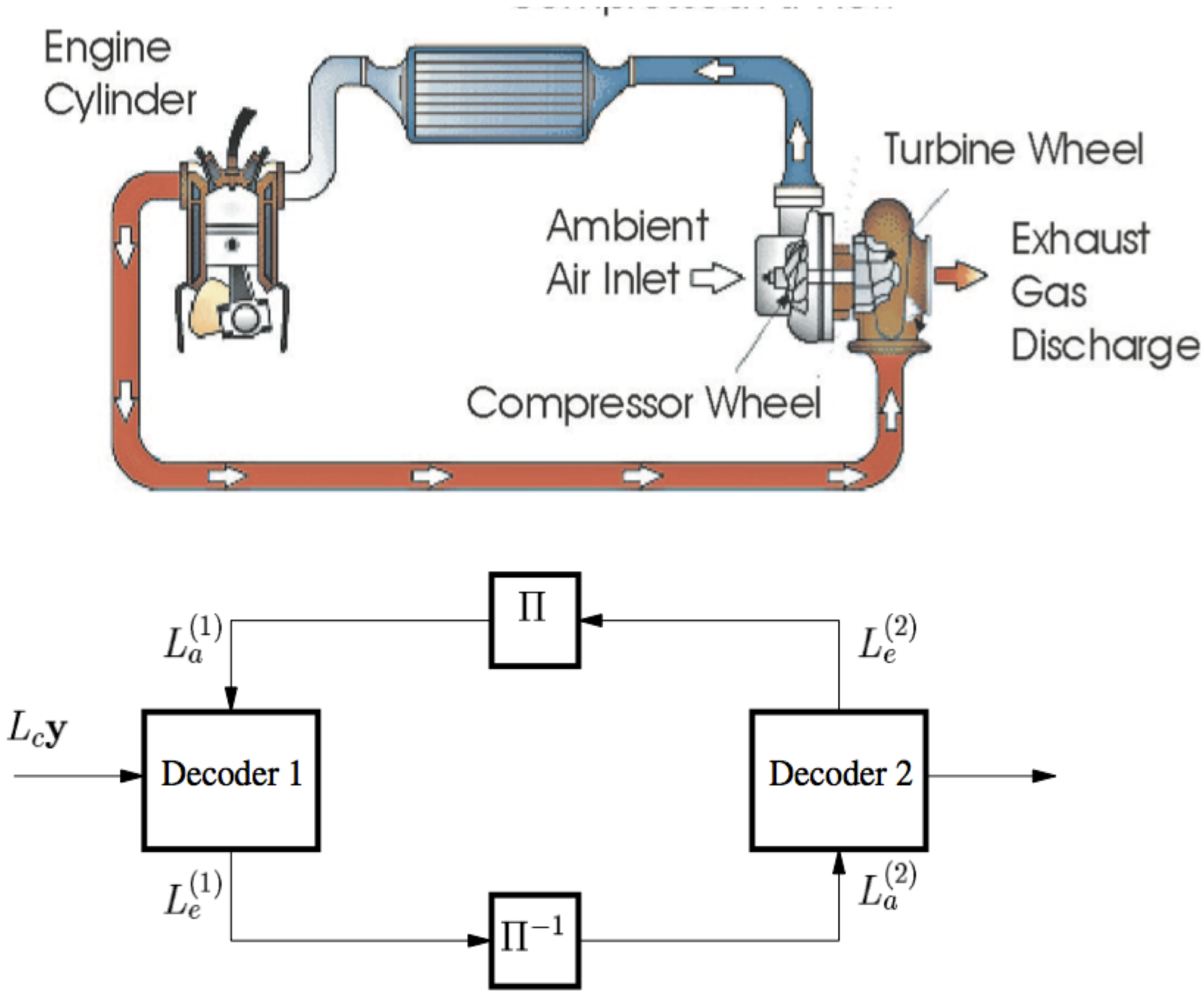
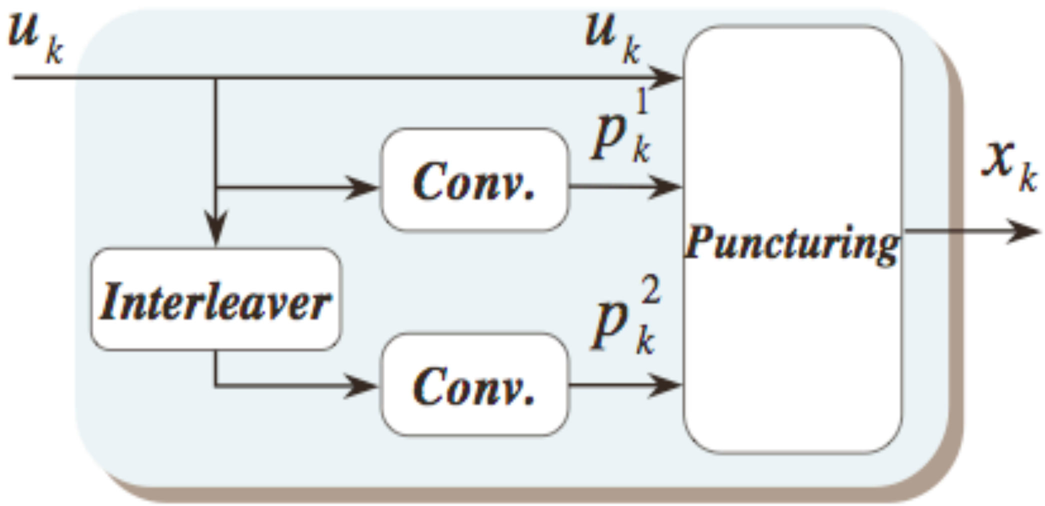
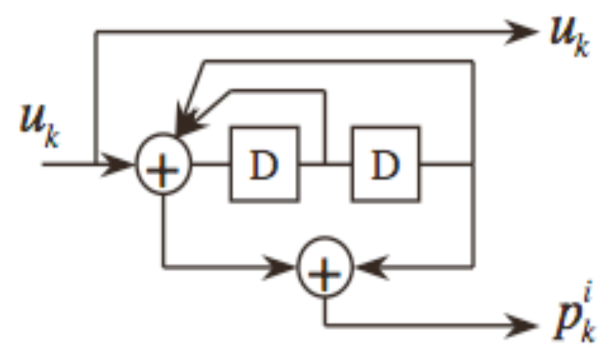


Figure 2: A mechanical turbo engine and a turbo decoder.

Turbo Encoder



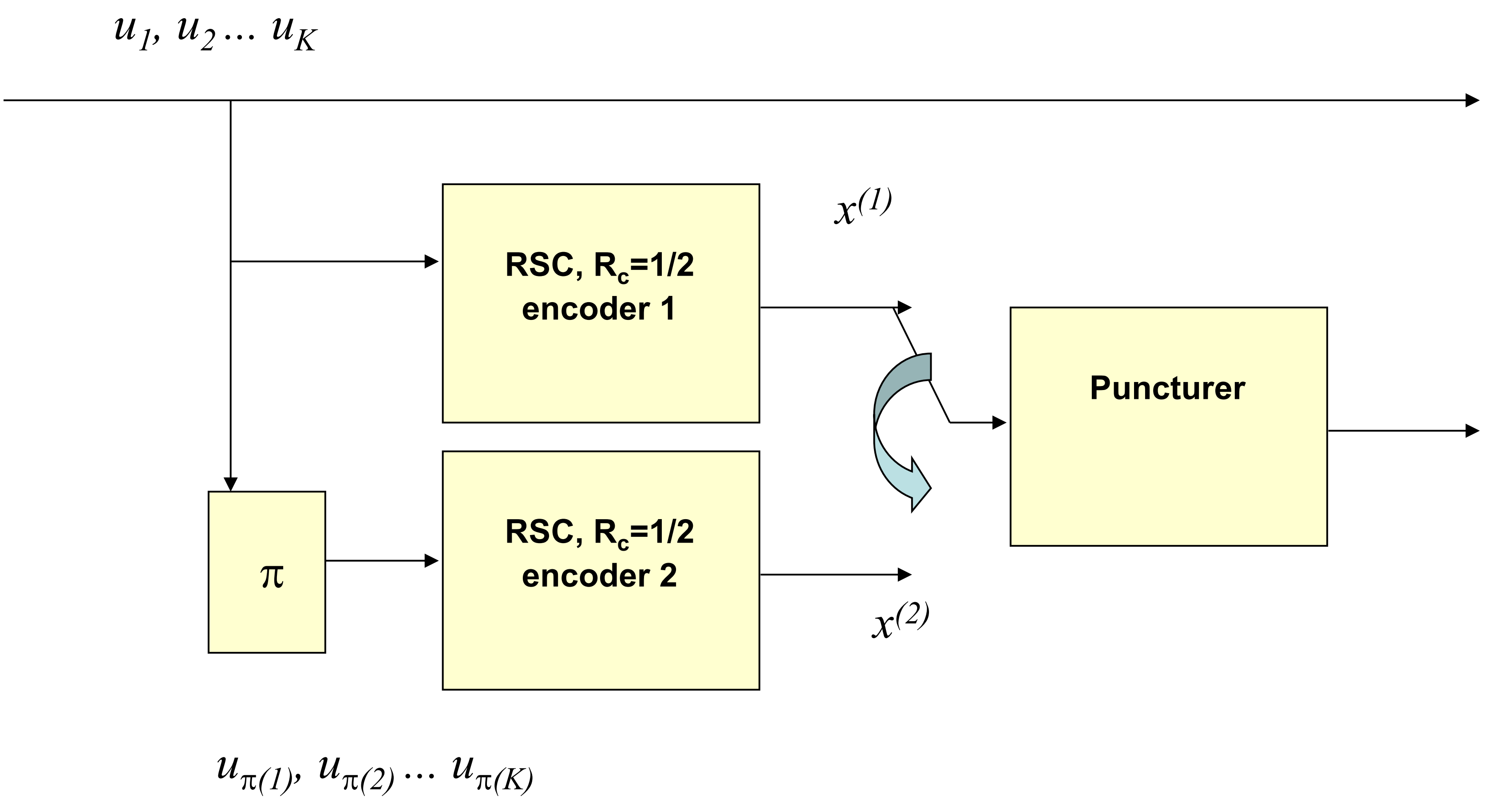
Conv. Convolutional encoder



Convolutional encoder

- **Recursive systematic convolutional (RSC)** encoders
- **Interleaver** can break up problematic input patterns

Parallel Concatenation, with Inter-leaver



SISO Decoder (1)

Soft decoder (Soft-In-Soft-Out):

Every binary symbols is described by a unique value: (+1/-1), *Algebric value* or *Log-Likelihood-Ratio* (LLR):

$$L(u) = \log\left(\frac{P(u=+1)}{P(u=-1)}\right)$$

At the input of the decoder, from the AWGN channel, we receive y instead of u:

$$L_c(u) = \log\left(\frac{P(u=+1\mid y)}{P(u=-1\mid y)}\right) = \log\left(\frac{P(y\mid u=+1)}{P(y\mid u=-1)}\right) = \frac{2}{\sigma^2}\,y$$

To evaluate the "a posteriori" algebric value, we use the BCJR algorithm (MAP, Maximum-a-Posteriori)

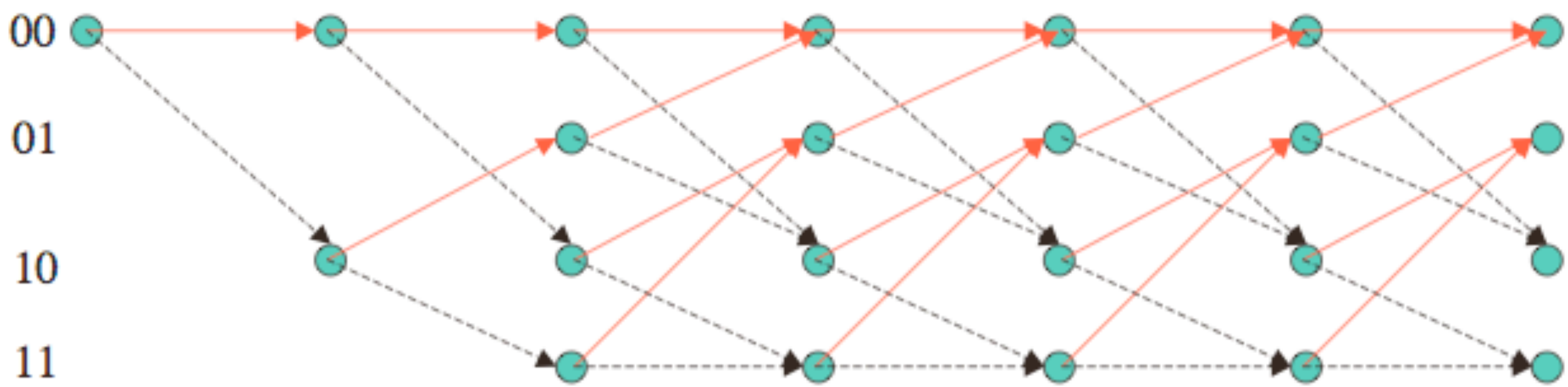
SISO iterative decoding …

$$L_p(u_t) = \log\left(\frac{P[y_t \mid u_t = +1]}{P[y_t \mid u_t = -1]}\right) + \log\left(\frac{P[u_t = +1]}{P[u_t = -1]}\right) + \log\left(\frac{\sum_{m',m:u=+1} \alpha_{t-1}(m') P[y_t^{(p)} \mid x_t^{(p)}(m',m)] \beta_t(m)}{\sum_{m',m:u=-1} \alpha_{t-1}(m') P[y_t^{(p)} \mid x_t^{(p)}(m',m)] \beta_t(m)}\right)$$

$$L_p(u_t) = \quad L_c(u_t) \quad + \quad L_a(u_t) \quad + \quad L_e(u_t)$$

A Posteriori Probabilities

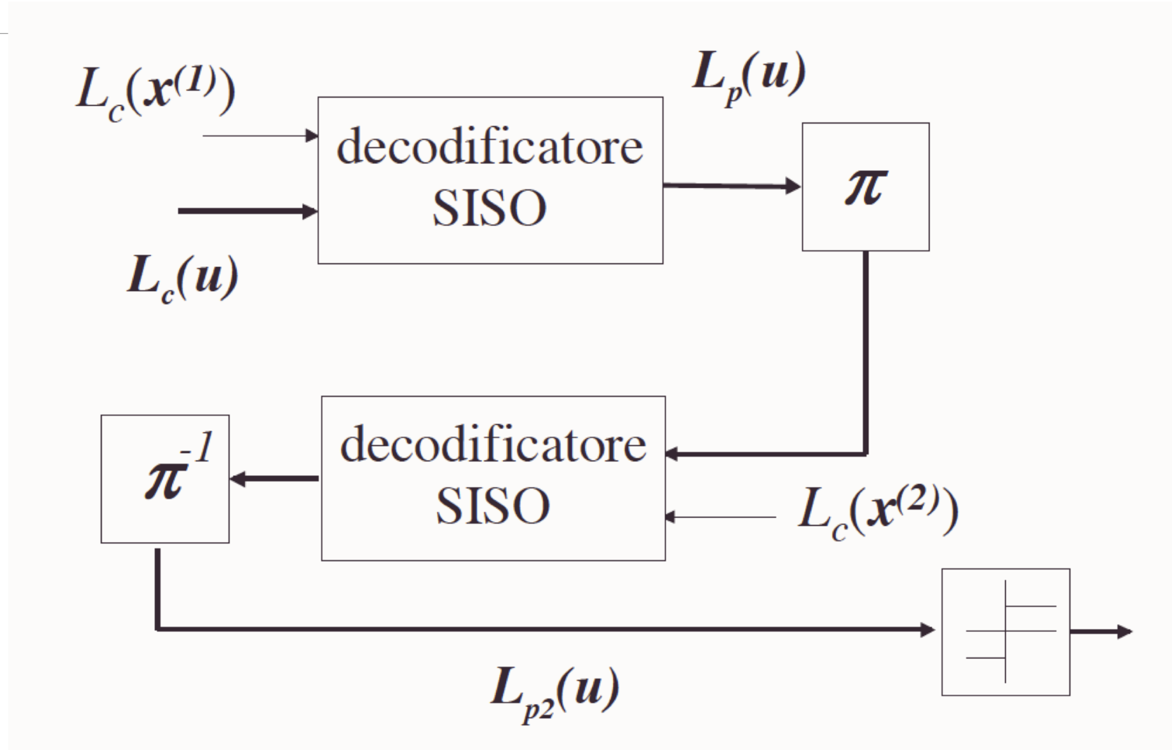
- BCJR algorithm finds the probabilities of the bits



Log likelihood ratio $L(u_k) = \log\left(\frac{P(u_k = +1|\mathbf{y})}{P(u_k = -1|\mathbf{y})}\right)$

Hard Decision: $\hat{u}_k = \mathrm{sgn}[L(u_k)]$

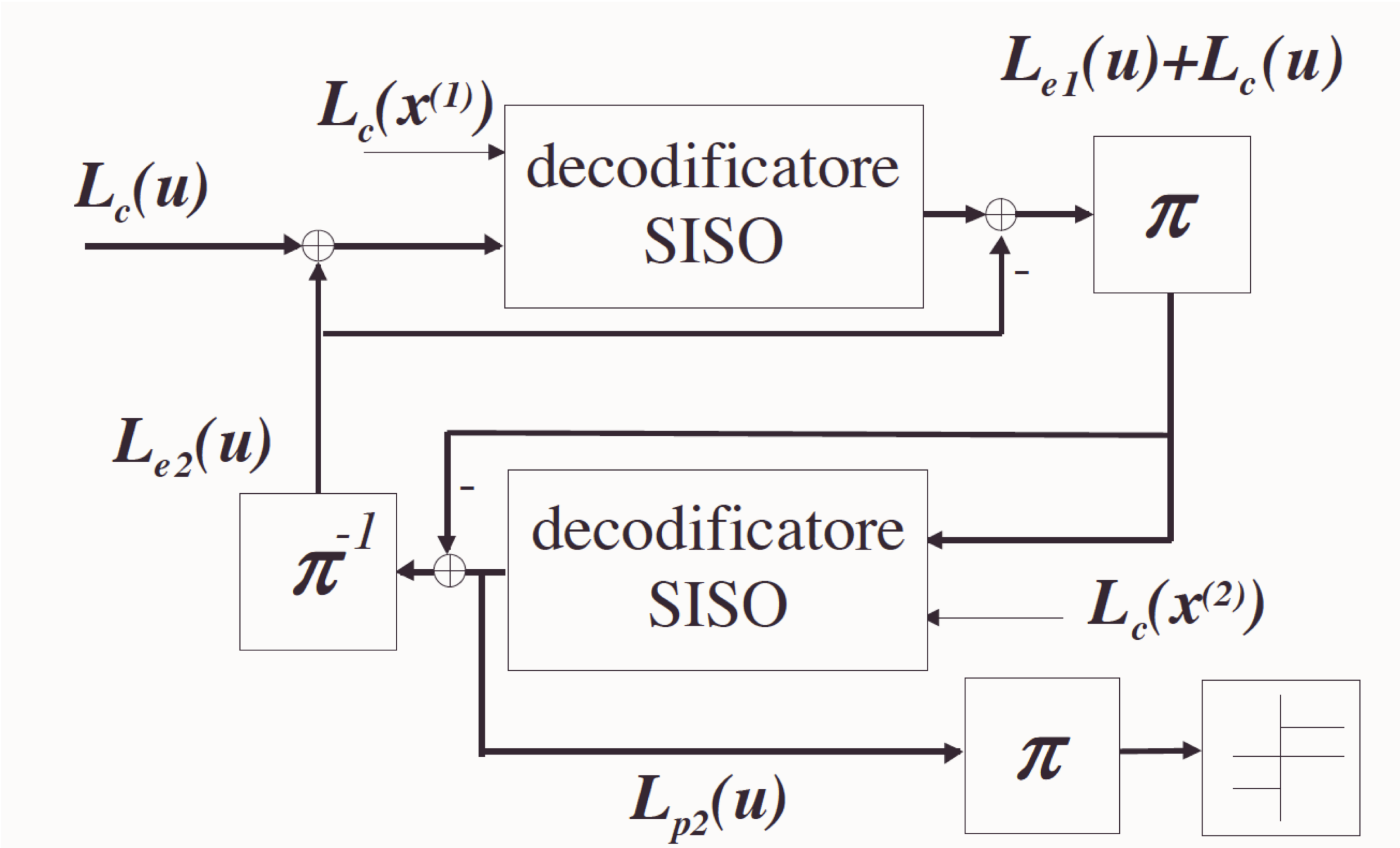
SISO decoding



- The first decoder decides considering the signal observation from the channel, whereas the second decoder get helps from the information that arrives also from the first decoder …
- New estimation … more affordable ?

!!! YES, but only if we consider the really new (The “extrinsic” !!!) information … !!!

SISO iterative decoding …

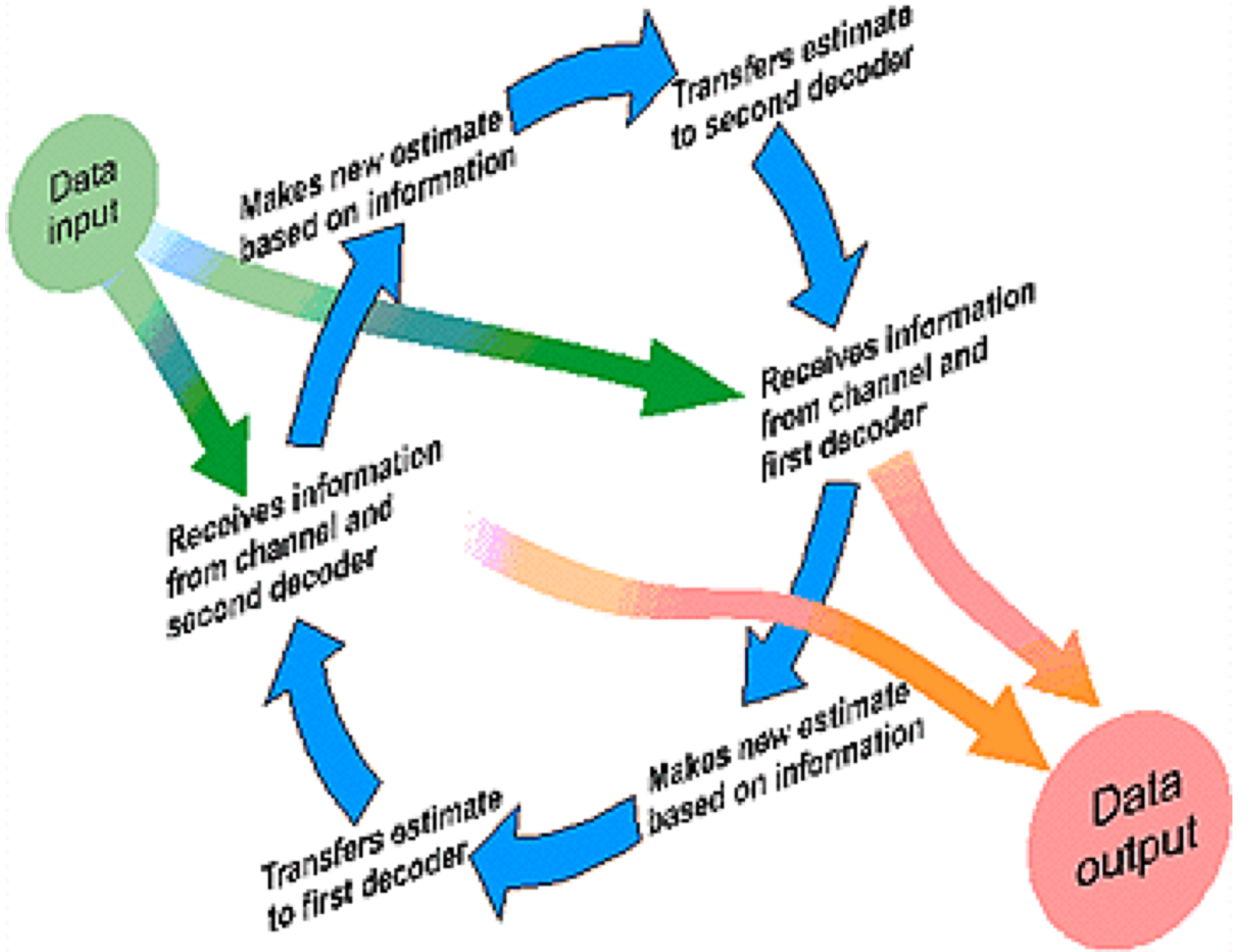


Extrinsic Information

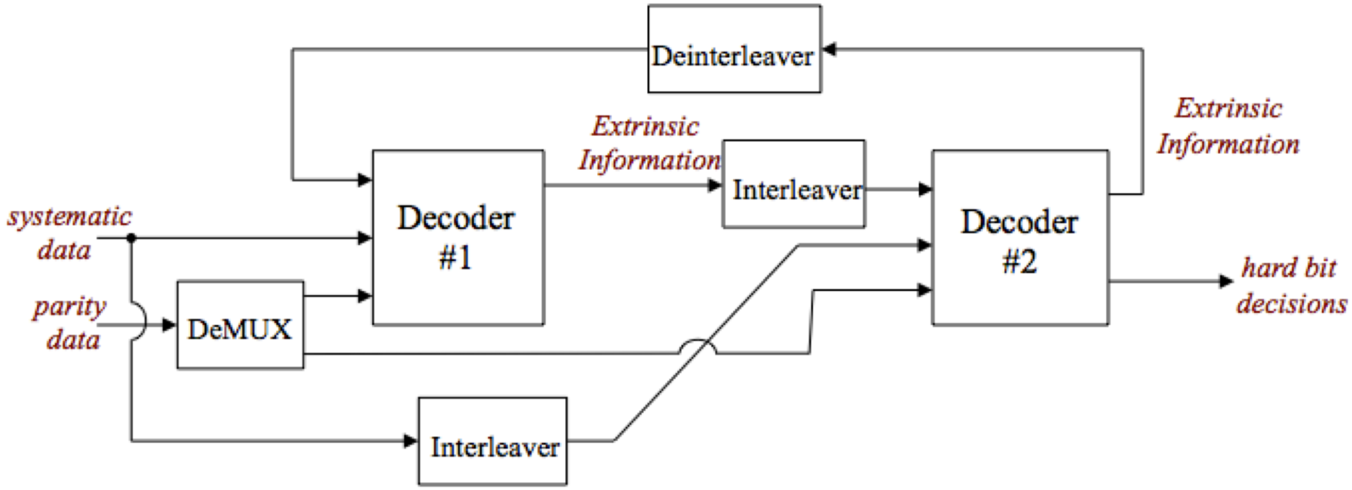
- The extrinsic information is found by subtracting the corresponding input from the LLR output, i.e.

- It is necessary to subtract the information that is already available at the other decoder in order to prevent "positive feedback".
- The extrinsic information is the amount of new information gained by the current decoder step.

Iterative Decoding



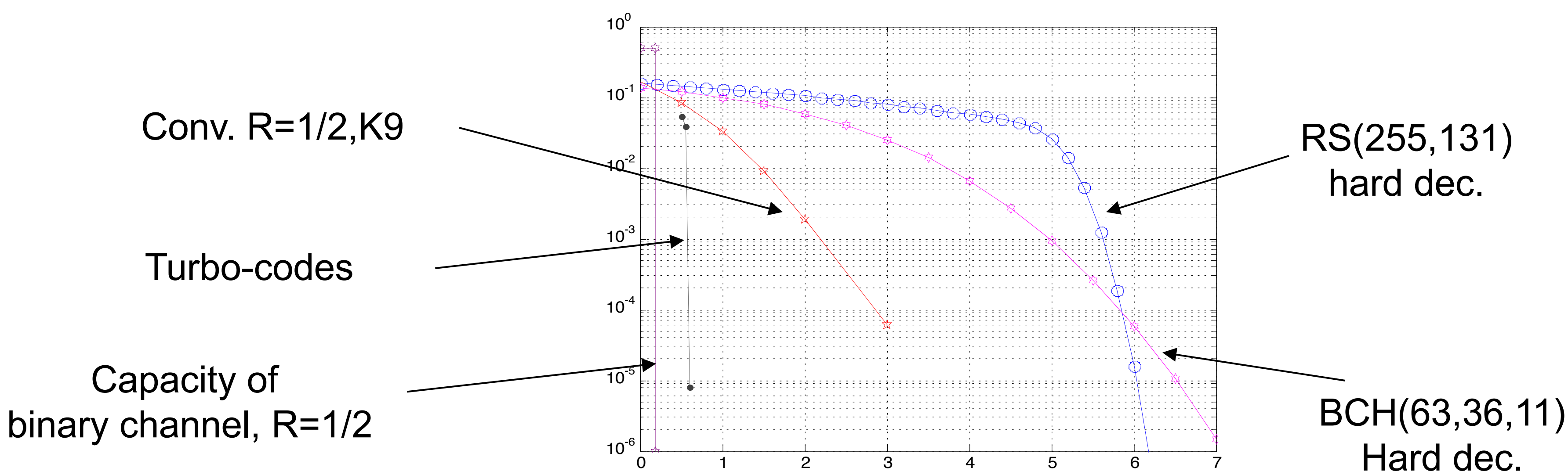
Iterative Decoding



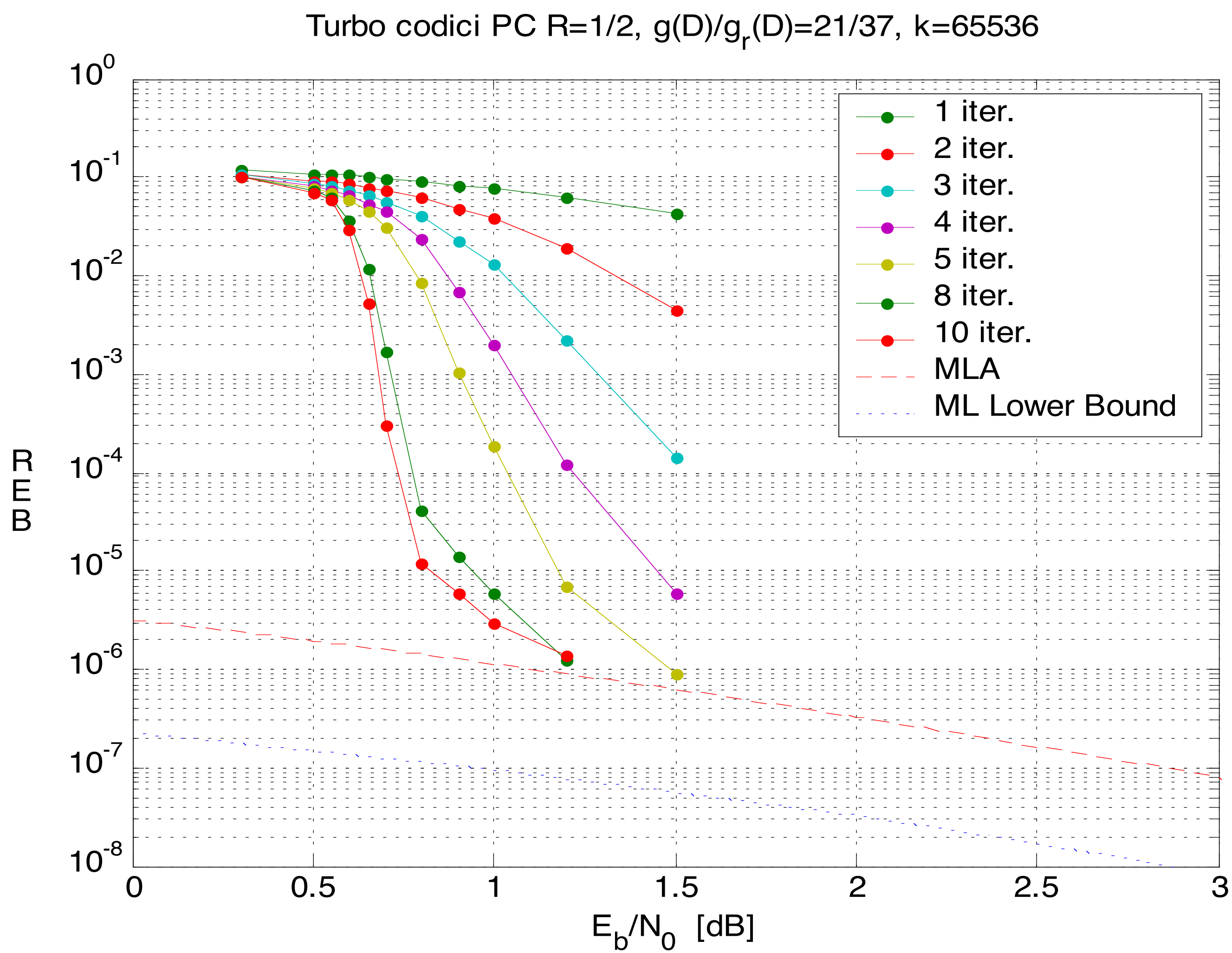
- There is one decoder for each elementary encoder.
  - Estimates the *a posteriori probability* (APP) of each data bit.
  - **Extrinsic Information** is derived from the APP.
- The ***Extrinsic Information*** is used as ***a priori*** information by the other decoder.
- Decoding continues for a set number of iterations.
  - Obeys law of diminishing returns

Performance …

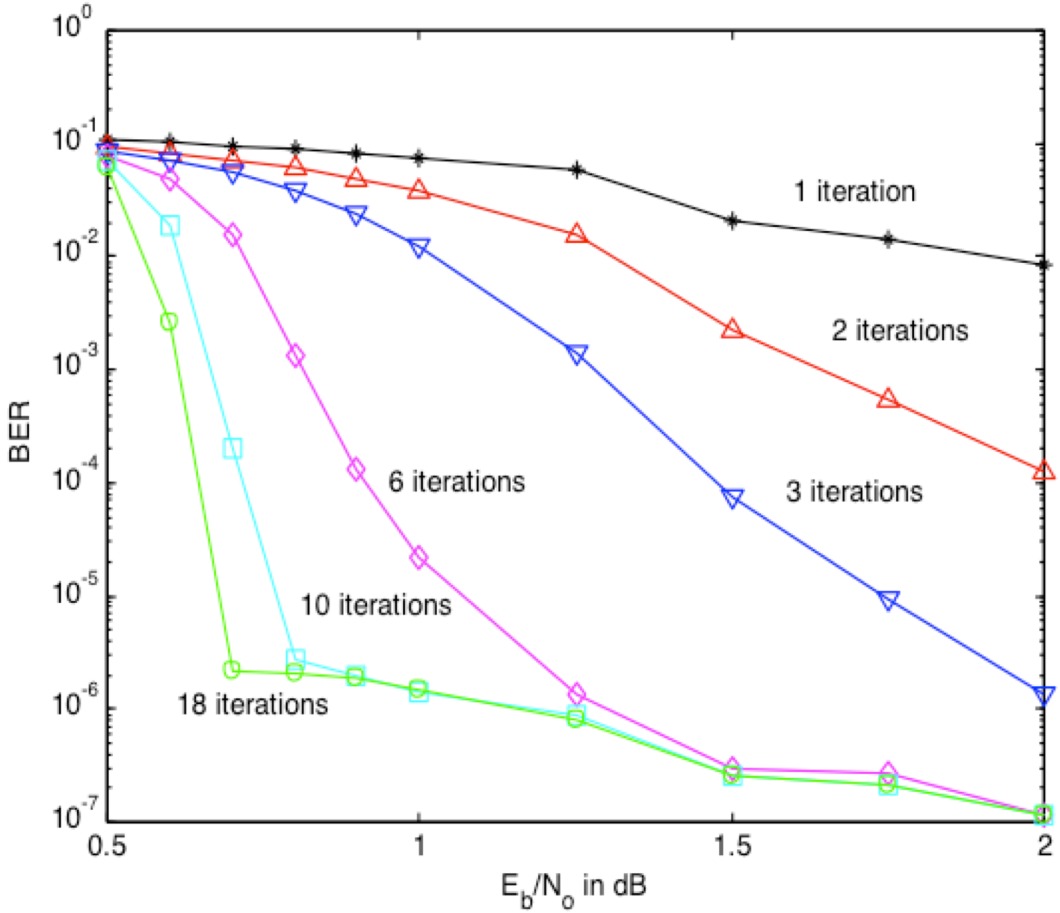
- Shannon: random codes and large blocks (many bits) !!!
- Also very Important: soft decoding [therefore, convolutional codes (???, e.g.: LDPC)]



Iterations: Waterfall (Cliff) and Error Floor

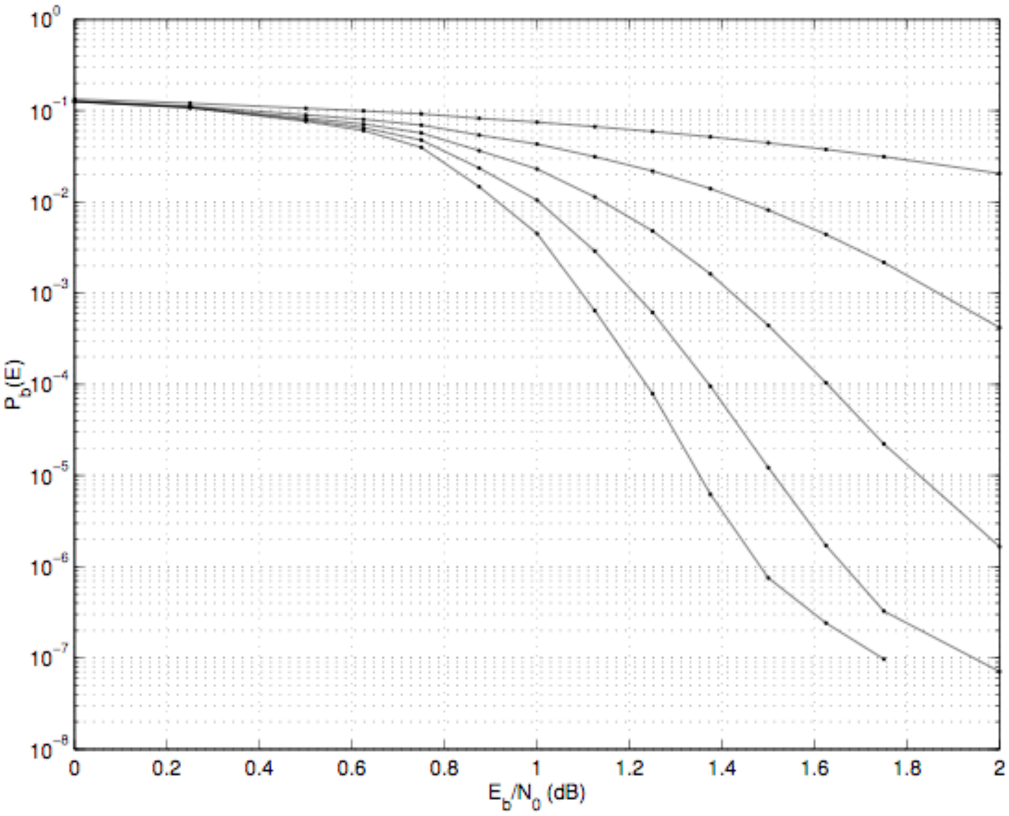


Performance as a Function of Number of Iterations



- $K_c = 5$
- $r = 1/2$
- $K = 65{,}536$
- Log-MAP algorithm
- AWGN

Iterations



10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$P_b(E)$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
E_b/N_0 (dB)

The effect of the Interleaver

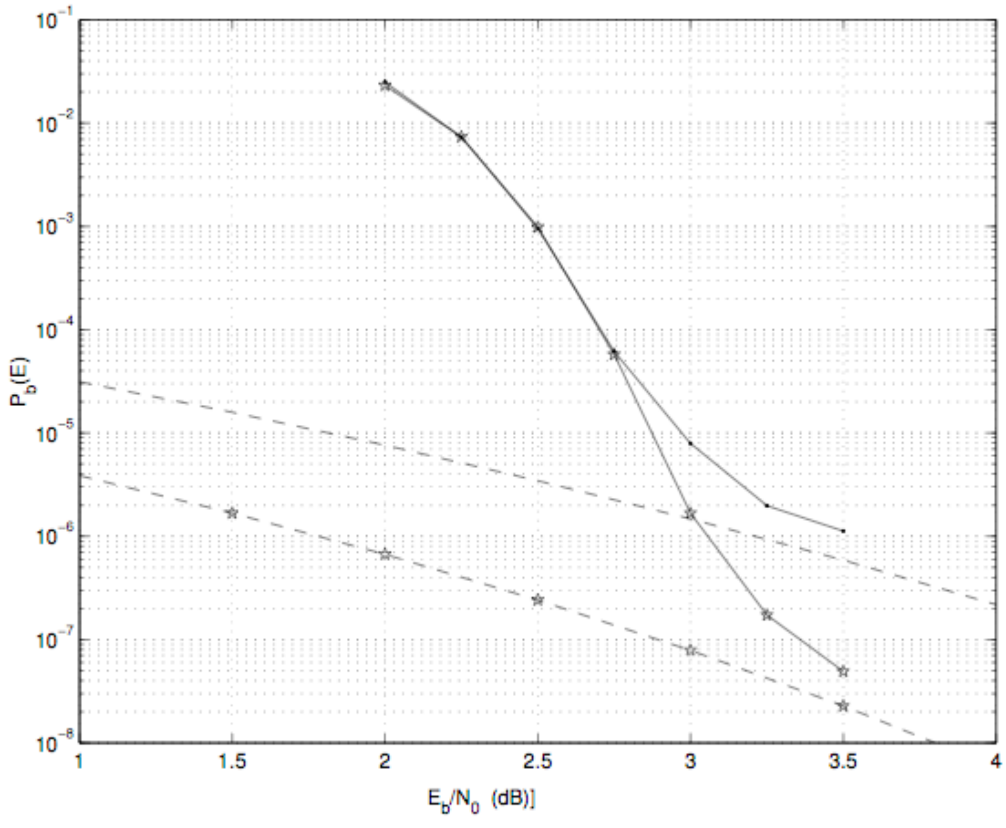
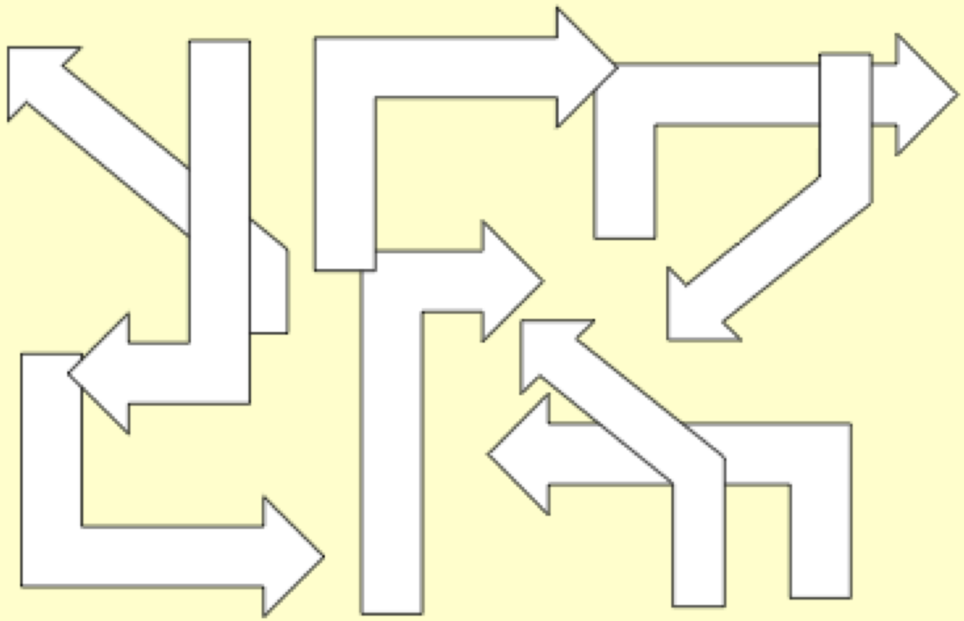


Figura 7.2: Prestazioni di uno stesso turbo codice con due diversi *interleaver*

The effect of the Interleaver

Permutation

In conclusion :

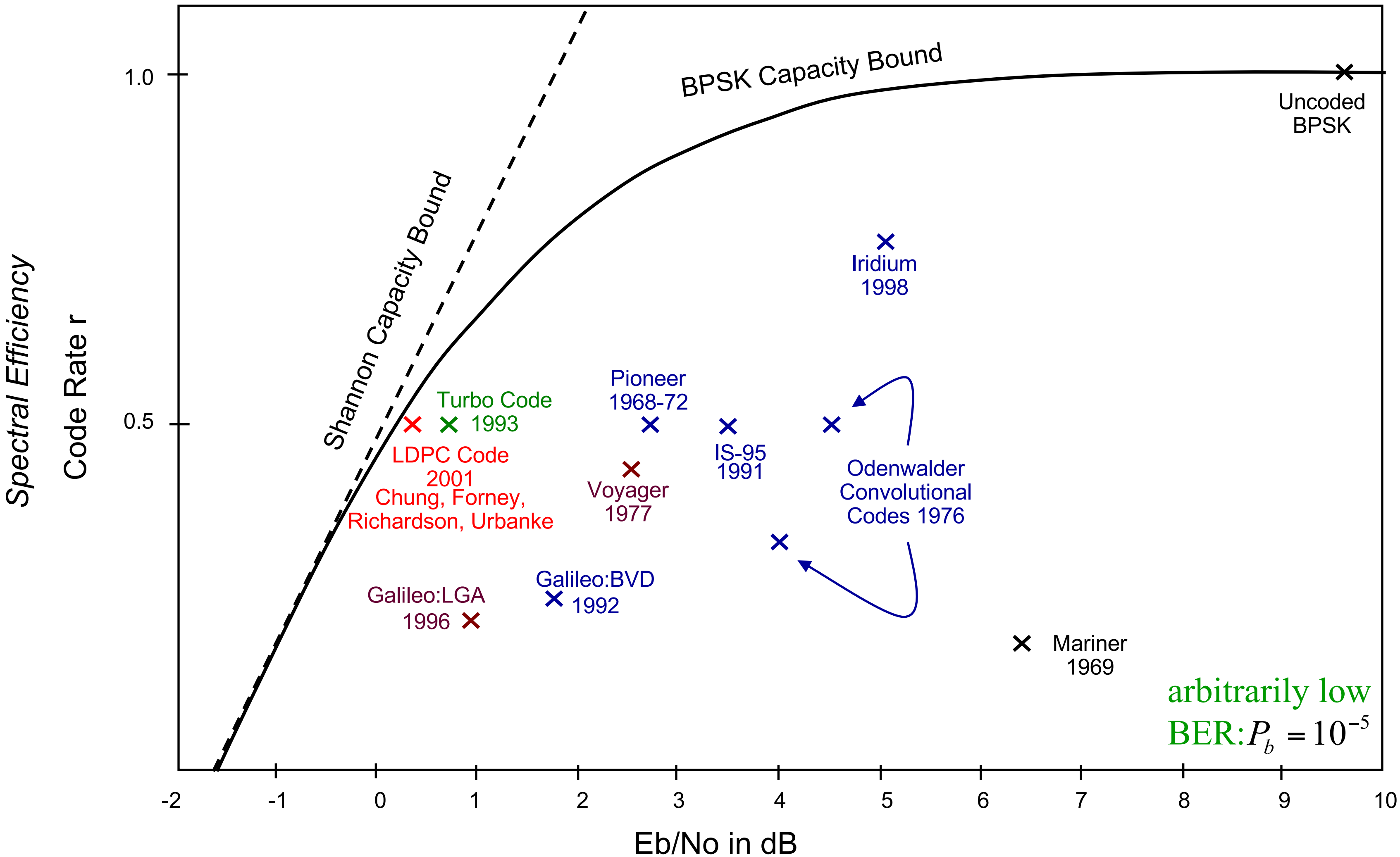


No ideal permutation for the time being (does it exist ?)

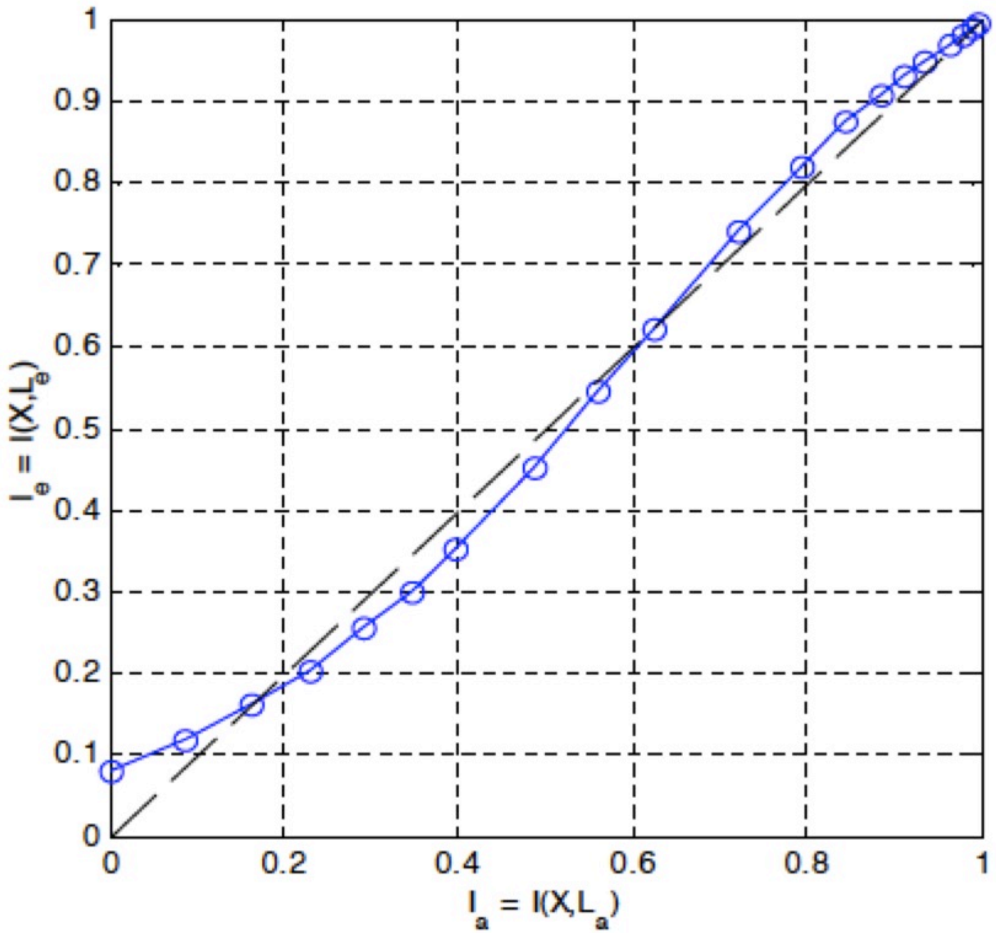
Turbo code performance

- Coding dilemma:
  - "All codes are good, except those that we can think of."
- Random coding argument:
  - Truly random codes approach capacity, but are not feasible.
  - Turbo codes appear random, yet have enough structure to allow practical decoding.
- Distance spectrum argument:
  - Traditional code design focused on maximizing the ***minimum distance***.
    - d_{min} determines performance at high SNR
  - With turbo codes, the goal is to reduce the ***multiplicity*** of low weight code words.
    - Even with small d_{min}, remarkable performance can be achieved at low SNR.

Power Efficiency of Standard Binary Channel Codes



EXtrinsic Information Transfer (EXIT) Chart



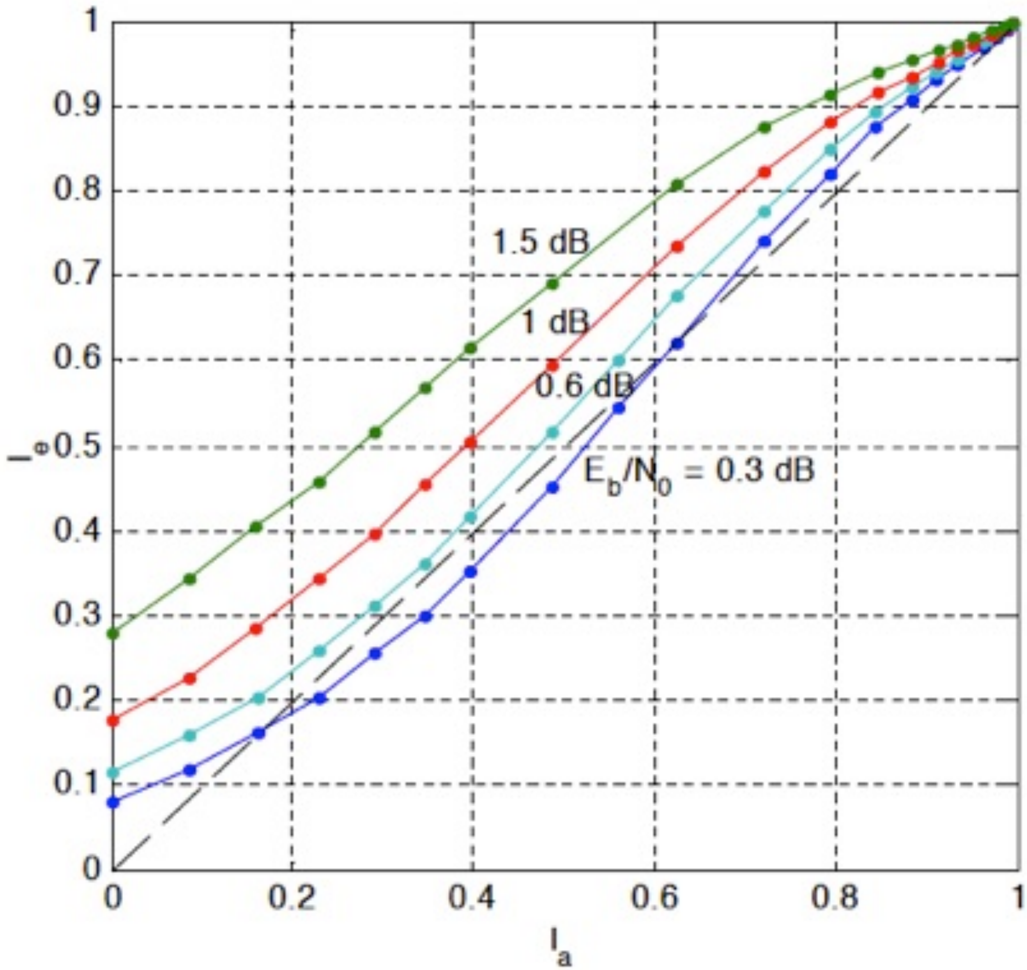
Ia = Mutual Information at the INPUT of the SISO decoder

Ie = Mutual Information at the OUTPUT of the SISO decoder

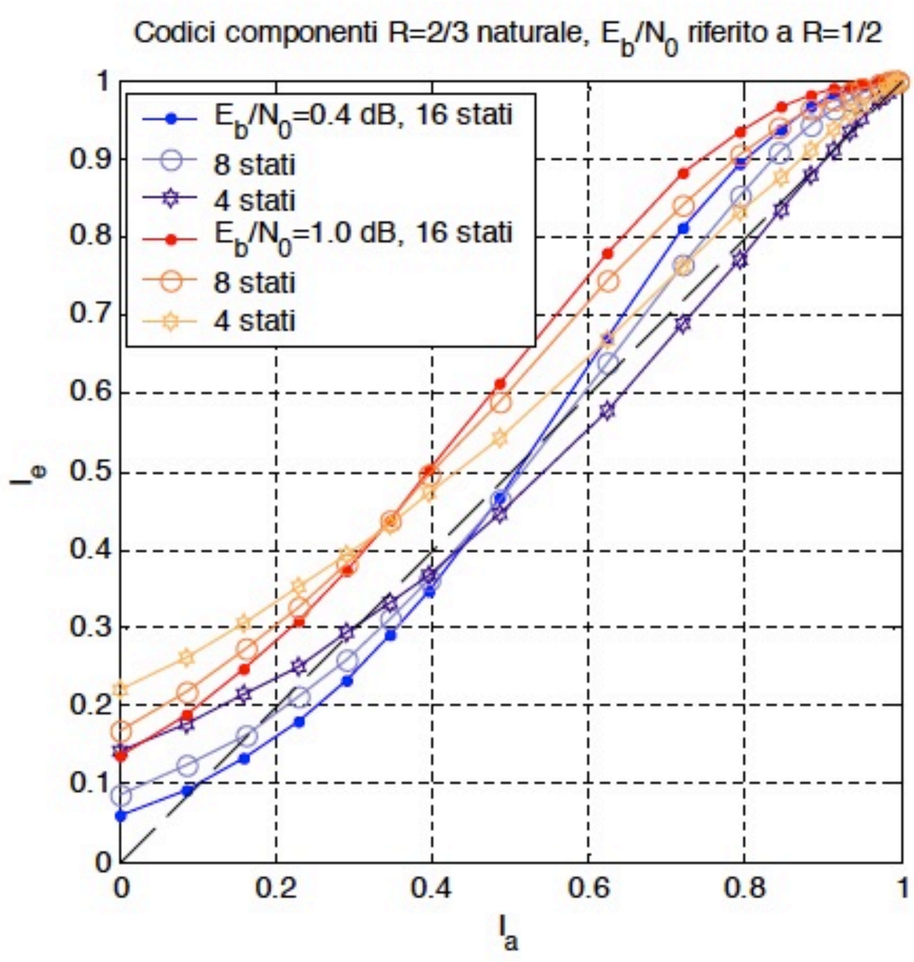
(X is the original information)

[tBr01] S. ten Brink, "Convergence behaviour of iteratively decoded parallel concatenated codes", IEEE Trans. on Comm., vol. 49, No. 10, Oct. 2001, pp. 1727-1737

EXIT (2): SNR



EXIT (3): Memory (Number of States)



EXIT Chart Analysis of Turbo Codes

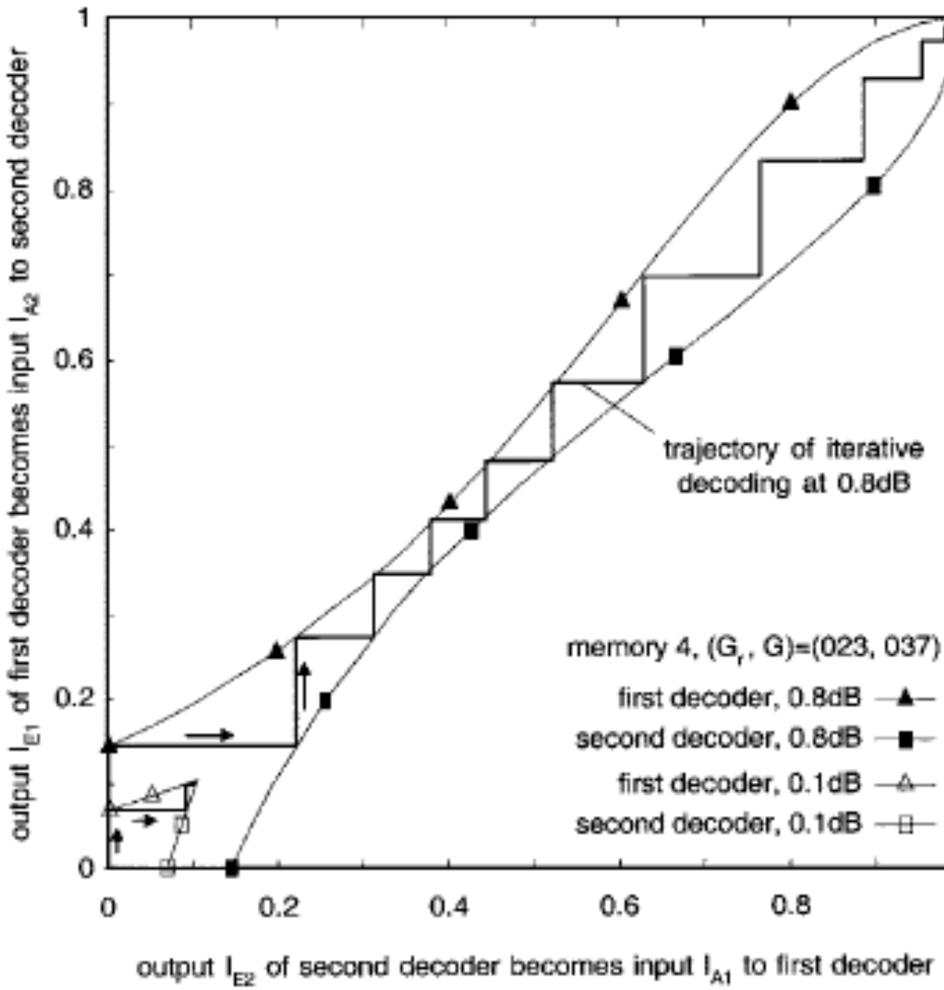
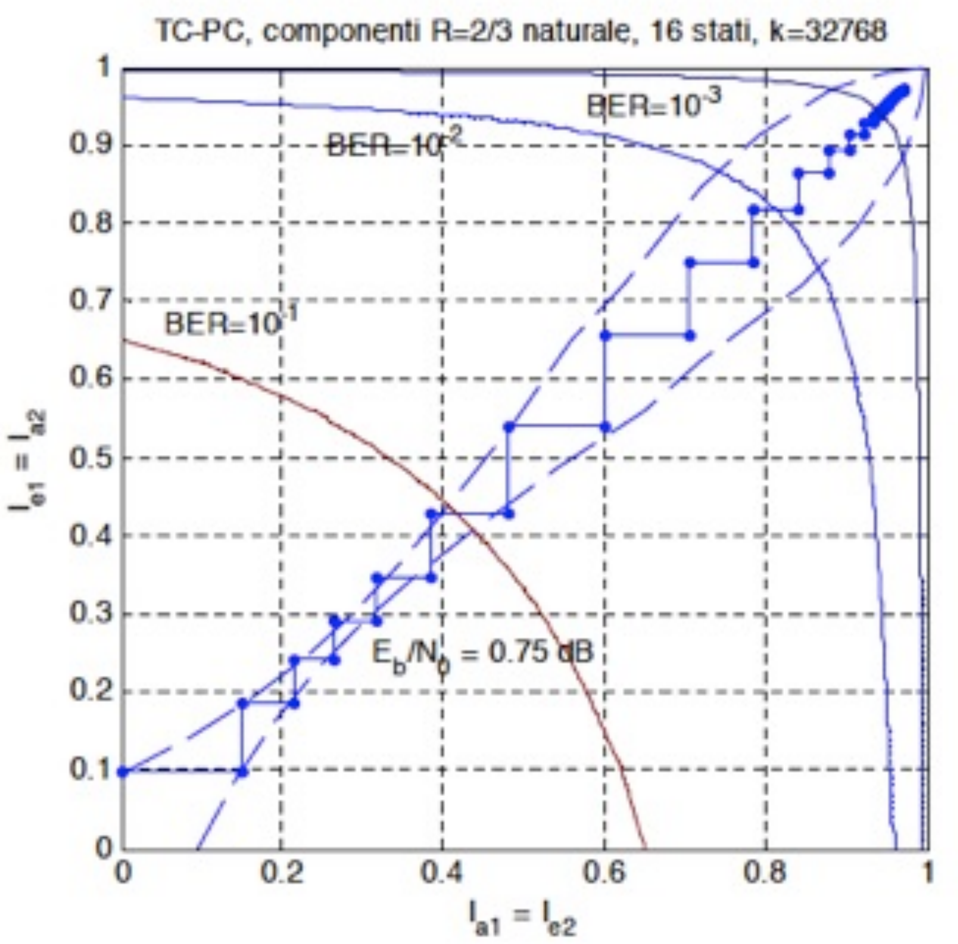


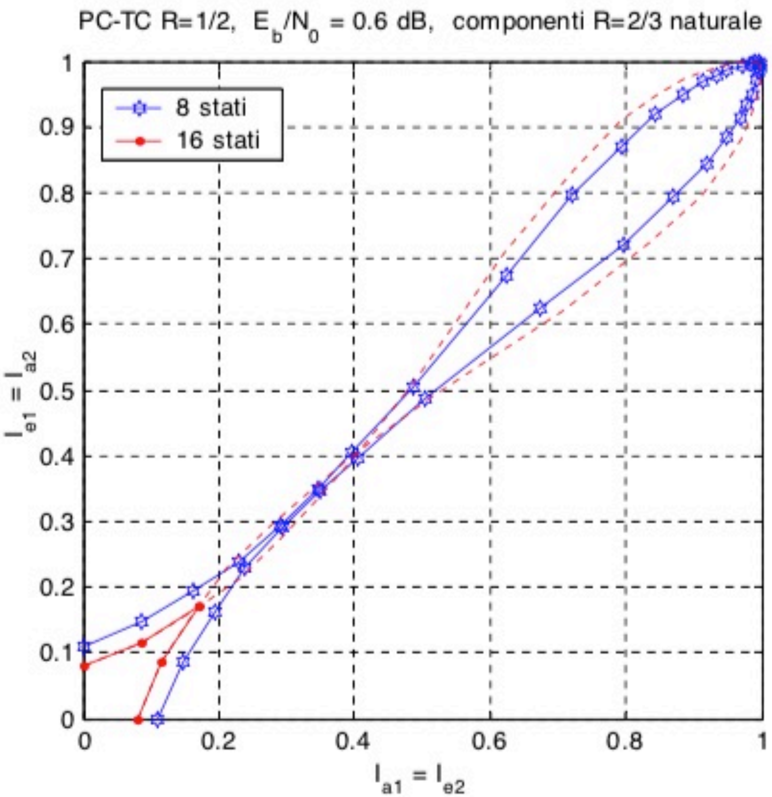
Fig. 5. Simulated trajectories of iterative decoding at $E_b/N_0 = 0.1$ dB and 0.8 dB (symmetric PCC rate 1/2, interleaver size 60 000 systematic bits).

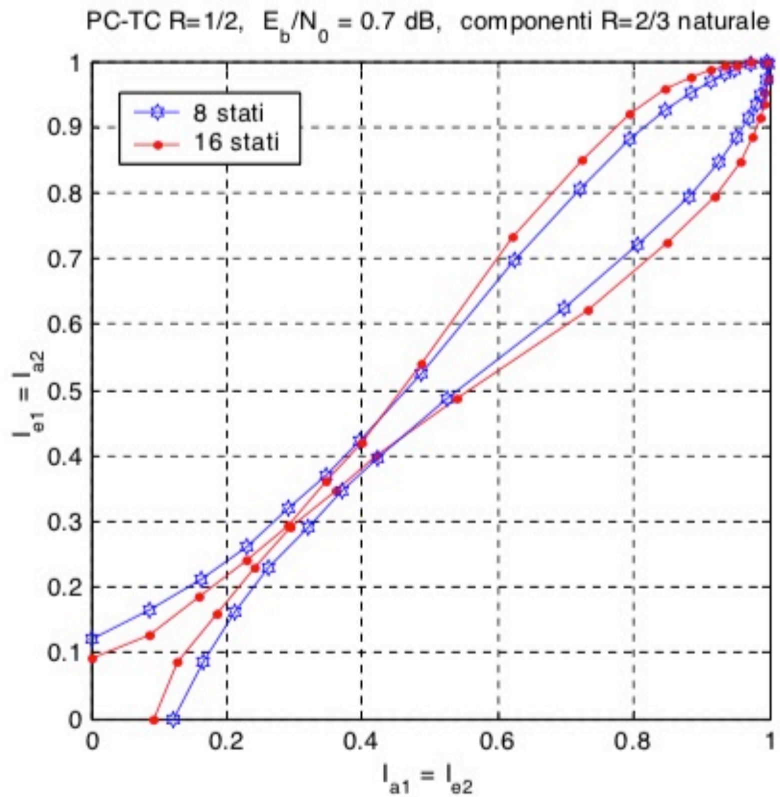
- PCCC (turbo) codes can be analyzed with an EXIT chart by plotting the mutual information transfer characteristics of the two decoders.
- Figure is from: S. ten Brink, "Convergence Behavior of Iteratively Decoded Parallel Concatenated Codes," IEEE Trans. Commun., Oct. 2001.



TC-PC, componenti R=2/3 naturale, 16 stati, k=32768
BER=10^-3
BER=10^-2
BER=10^-1
E_b/N_0 = 0.75 dB
I_e1 = I_a2
I_a1 = I_e2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

EXIT: Tunnel


PC-TC R=1/2, E_b/N_0 = 0.6 dB, componenti R=2/3 naturale
8 stati
16 stati
$I_{e1} = I_{a2}$
$I_{a1} = I_{e2}$


PC-TC R=1/2, E_b/N_0 = 0.7 dB, componenti R=2/3 naturale
8 stati
16 stati
$I_{e1} = I_{a2}$
$I_{a1} = I_{e2}$

EXIT: Tunnel

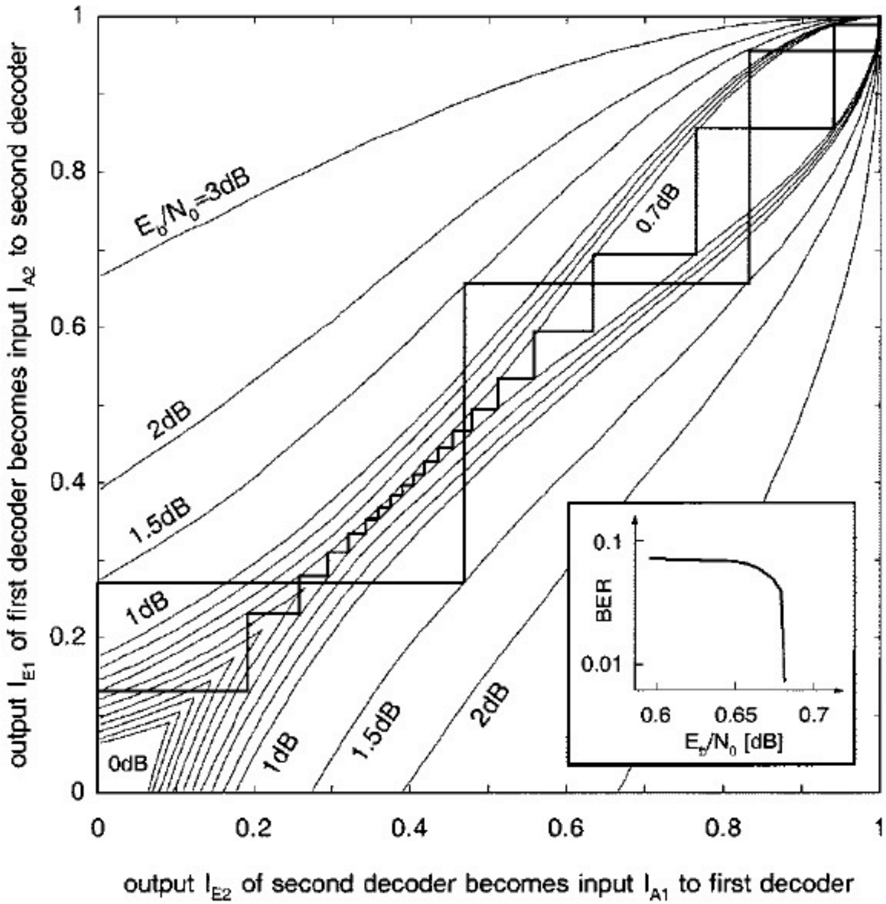


Fig. 6. EXIT chart with transfer characteristics for a set of E_b/N_0-values; two decoding trajectories at 0.7 dB and 1.5 dB (code parameters as in Fig. 5, PCC rate 1/2); interleaver size 10^6 bits.

Conclusion : End of Search

- Turbo codes achieved the theorical limits with small gap
- Give rise to new codes : Low Density Parity Check (LDPC)
- Need
  - Improvements in decoding delay

Conclusions

- It is now possible to closely approach the Shannon limit by using Turbo and LDPC codes.
- Binary capacity approaching codes can be combined with higher order modulation.
- These code are making their way into standards
  - Binary turbo: UMTS, cdma2000
  - Duo-binary turbo: DVB-RCS, 802.16
  - LDPC: DVB-S2 standard.
- Software for simulating turbo and LDPC codes can be found at: www.iterativesolutions.com


"That's
all
folks!"