

Photonics

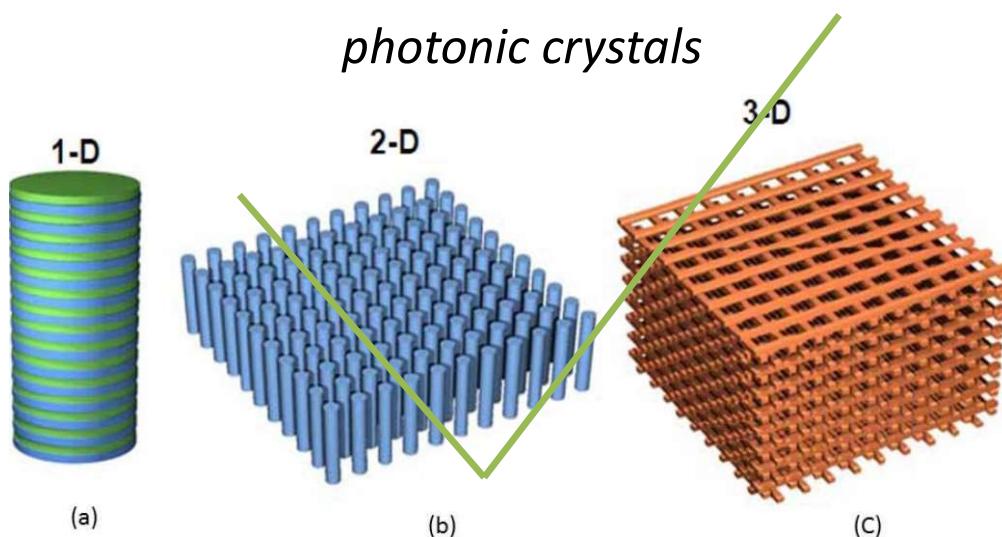
Prof. Maria Antonietta Vincenti
Università degli Studi di Brescia

Metamaterials



Introduction

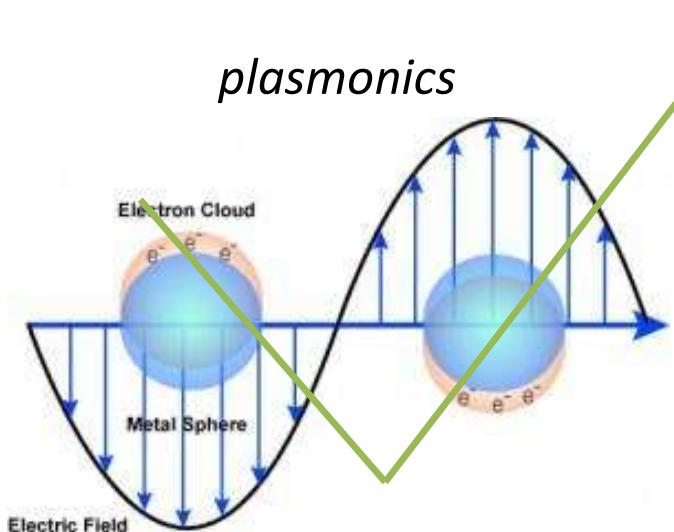
Scientists and researchers refuse to be confined by the materials refined from naturally occurring compounds and they are creating new materials or nanostructured materials that exhibit unusual electronic and optical properties.



(a)

(b)

(c)

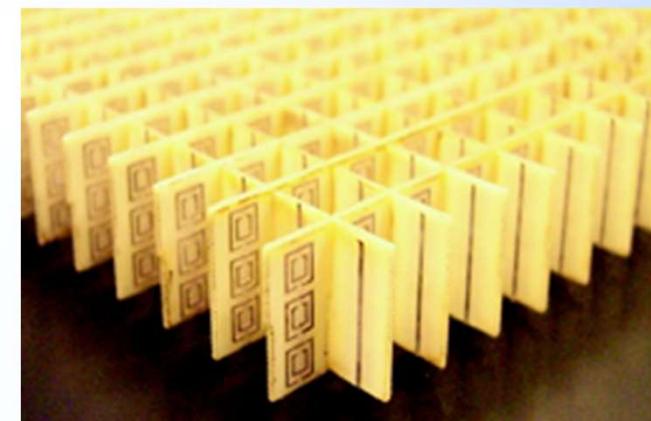


Electric Field

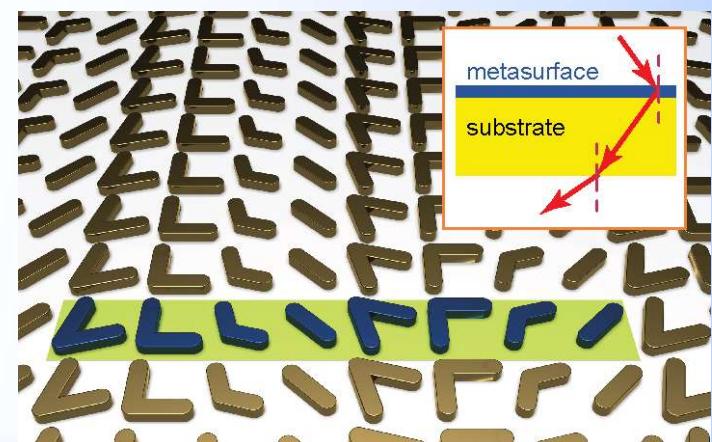
Electron Cloud

Metal Sphere

metamaterials



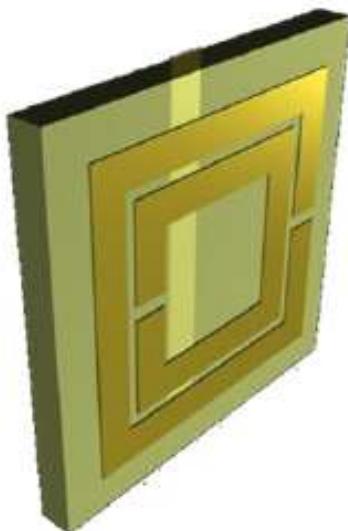
metasurfaces





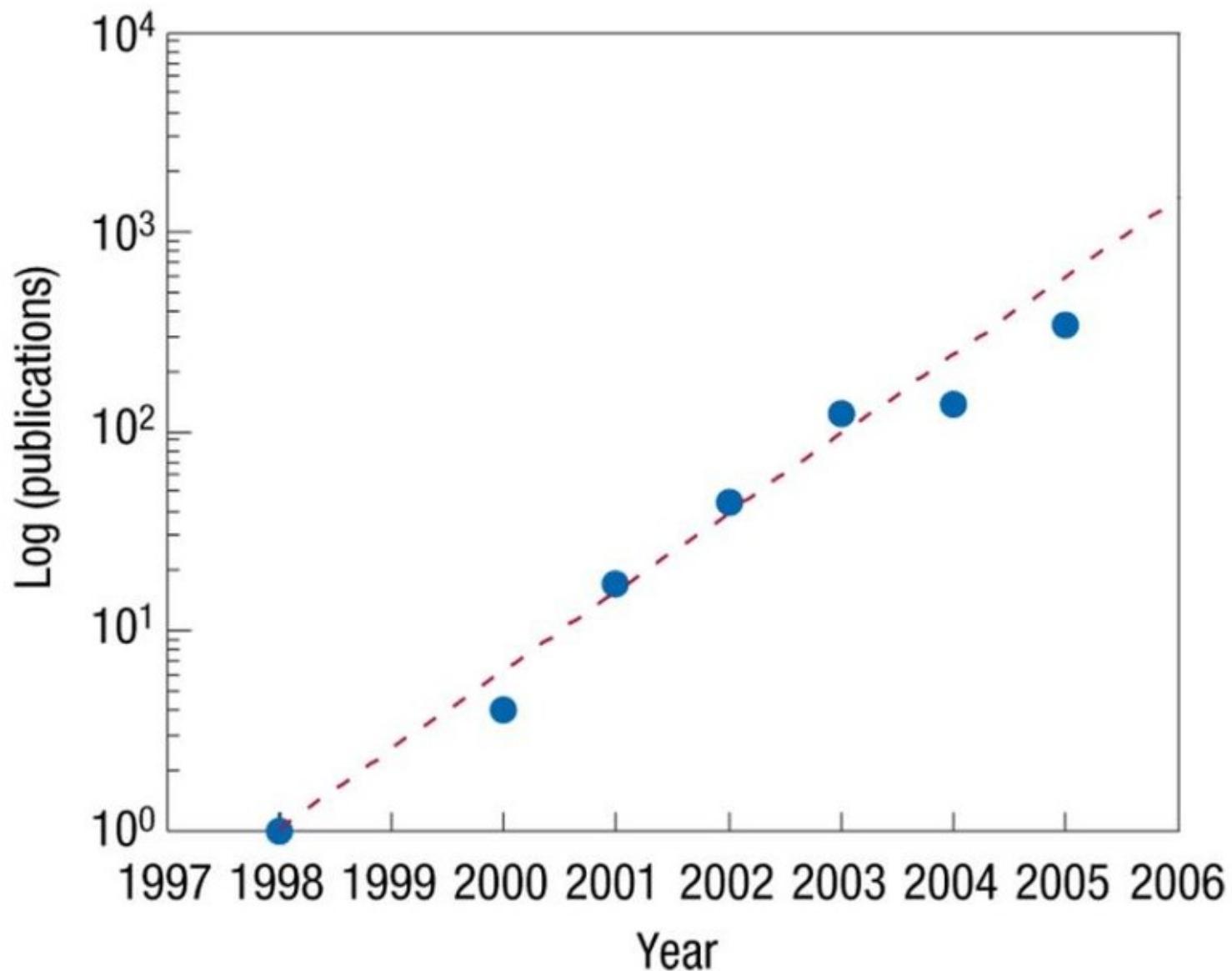
Introduction

WHAT ARE METAMATERIALS?



There is no universally accepted definition

- Engineered composites
- Structures whose properties are derived from their physical structure and not their chemistry
- Structures that exhibit properties not observed in nature
- Structures that exhibit properties not observed in their constituent materials
- Composite materials that are purposely engineered to provide material properties that are not otherwise attainable with ordinary materials.





METAMATERIALS CAN BE

RESONANT

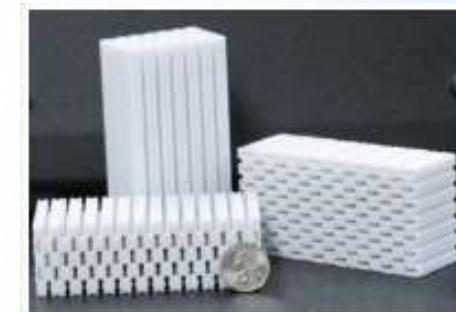


- Period is comparable to λ
- Oscillating currents emulate atomic resonances



- Frequency selective
- Sensitive to fabrication defects
- Can be designed at will to achieve certain properties

NON-RESONANT



- Period is much smaller than λ
- Nothing resonates or scatters from unit cells



- Greatest potential to be broadband
- Greatest potential for wide field-of-view
- Greatest tolerance to structural deformations
- Fewer “magical” properties than resonant metamaterials.



Why do we need to make artificial materials?

We have reached a stage when naturally available materials can no longer meet the demands of fast-developing fundamental science and applications.

Example:

The best resolution of a conventional microscope is on the order of 100nm because of the fundamental diffraction limit. Electron based microscopy allows to go under this limit but it works only for conducting materials.

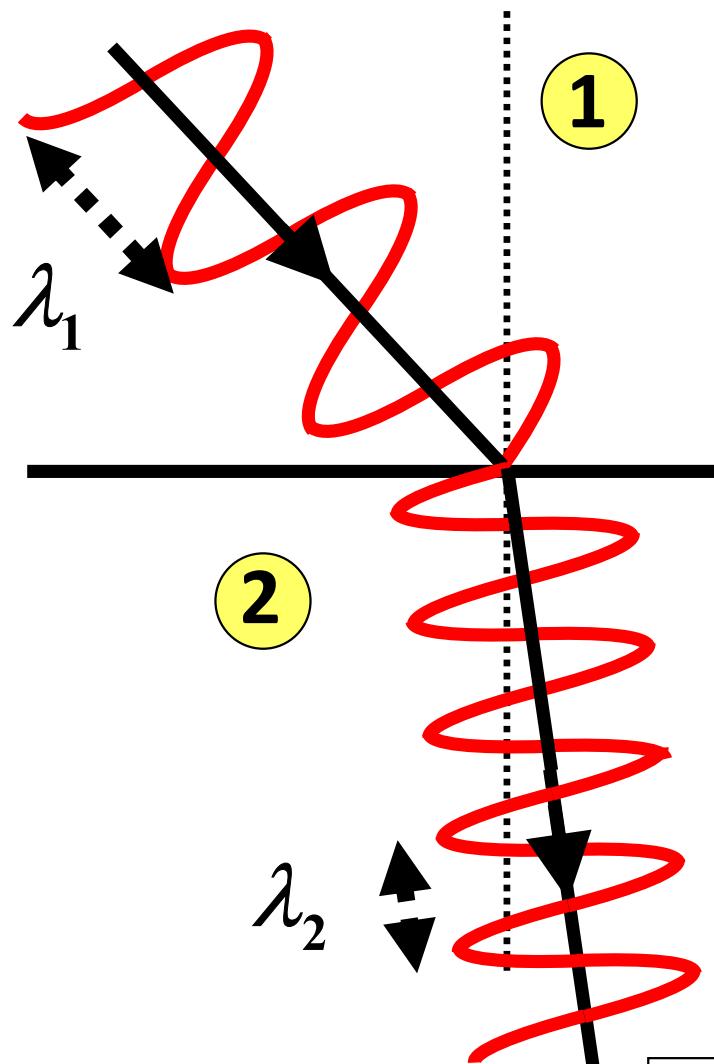


Can we develop an all-optical technique for nano-scale imaging

?



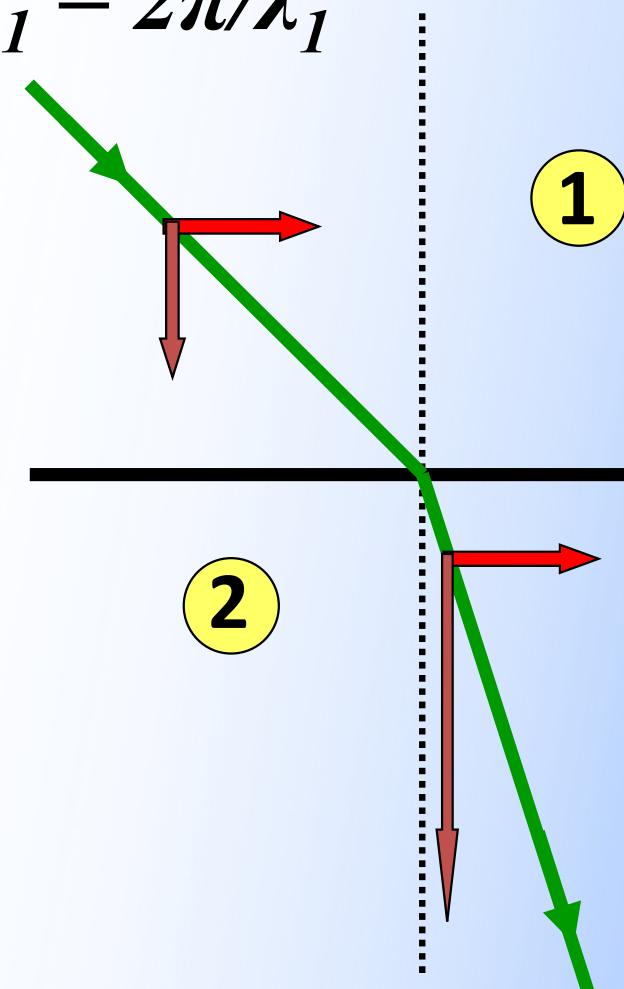
This is what we studied so far...



$$v_{p1} = \frac{c}{n_1}$$
$$v_{p2} = \frac{c}{n_2}$$

c = speed of light in vacuum
 $n_{1,2}$ = refractive indices

$$k_1 = 2\pi/\lambda_1$$



$$k_2 = 2\pi/\lambda_2$$



Definition of velocities

Phase Velocity:

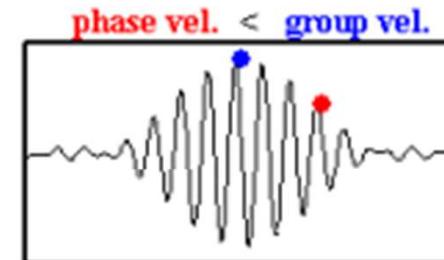
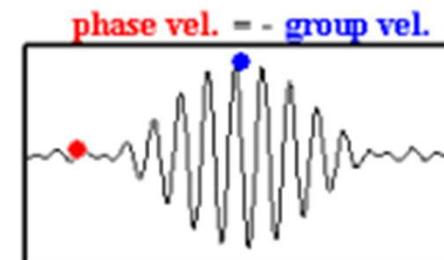
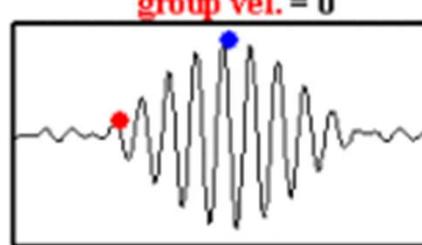
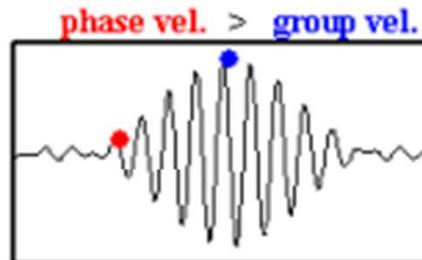
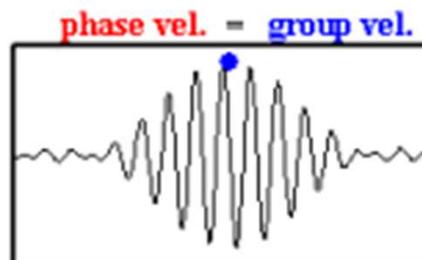
Velocity of the phase front of the wave

$$V_p = \frac{\omega}{k}$$

Group Velocity:

Velocity of the wave envelope

$$V_g = \frac{\partial \omega}{\partial k}$$





From EM theory we have ω, k, ϵ, c

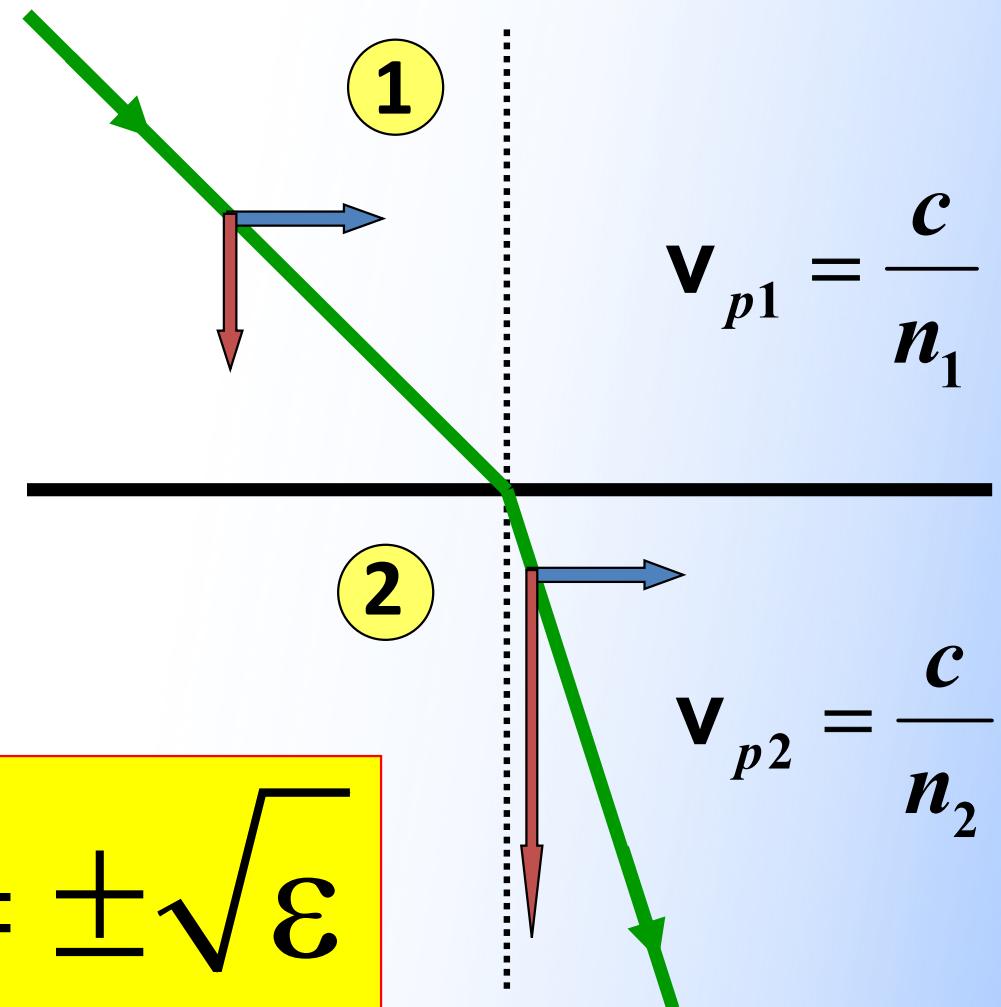
$$k^2 = \frac{\omega^2}{c^2} \epsilon$$

$$k = \pm \frac{\omega}{c} \sqrt{\epsilon}$$

But people called this refractive index as a way of indicating the bending

$$n = \pm \sqrt{\epsilon}$$

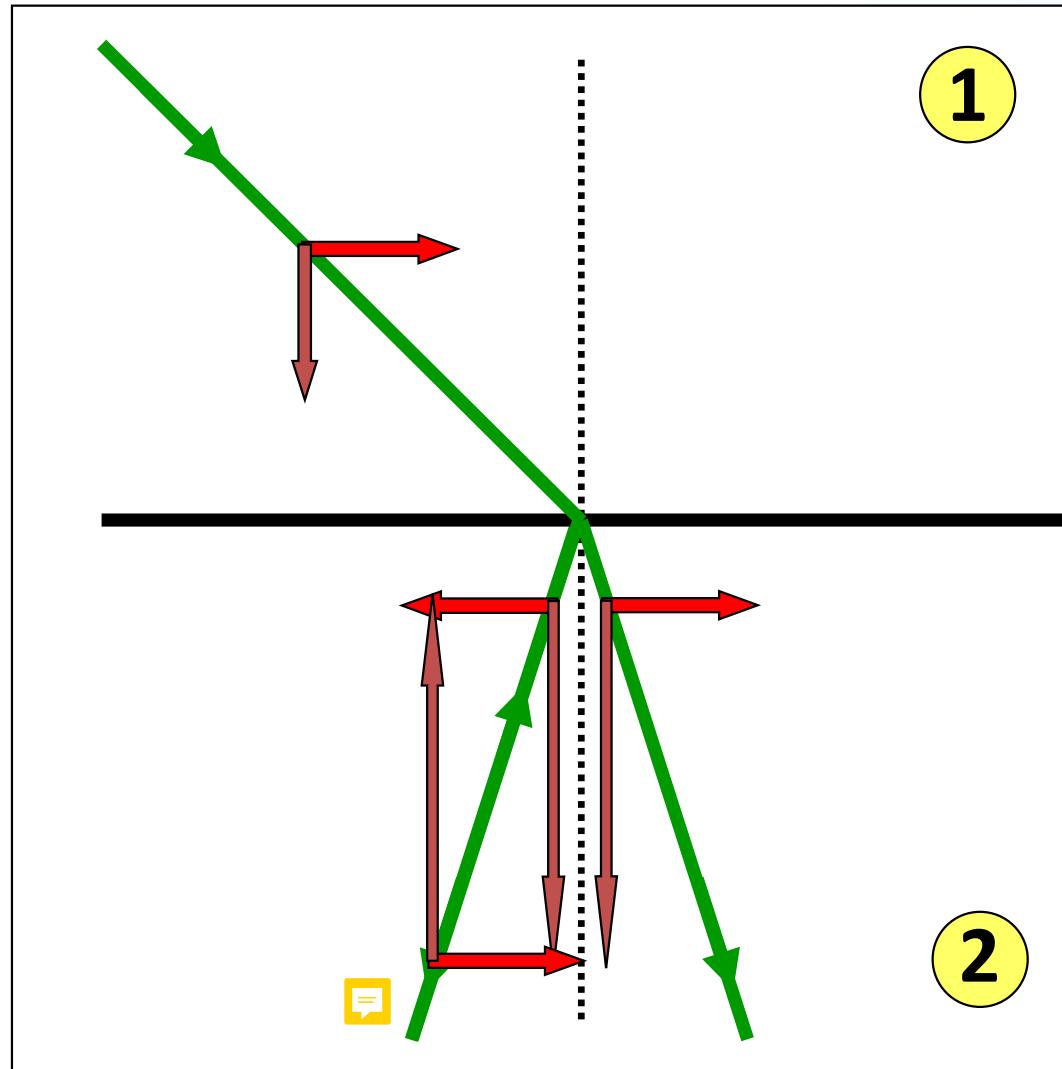
THE BLUE ARROWS MUST BE EQUAL



So, define the refractive index



Question:
is negative refraction possible?



YES! AS A BACKWARD WAVE



Question:
is negative refraction possible?

Can this be done simply by choosing...

$$n = +\sqrt{\epsilon}$$

OR

?

$$n = -\sqrt{\epsilon}$$



Is this refraction possible?

Well actually no...
we don't have to forget about magnetic permeability!

So, what should the index *really* be?

$$n = \pm\sqrt{\epsilon\mu}$$



UNIVERSITY
OF BRESCIA

Did anyone spot this
a long time ago?



Well... nearly

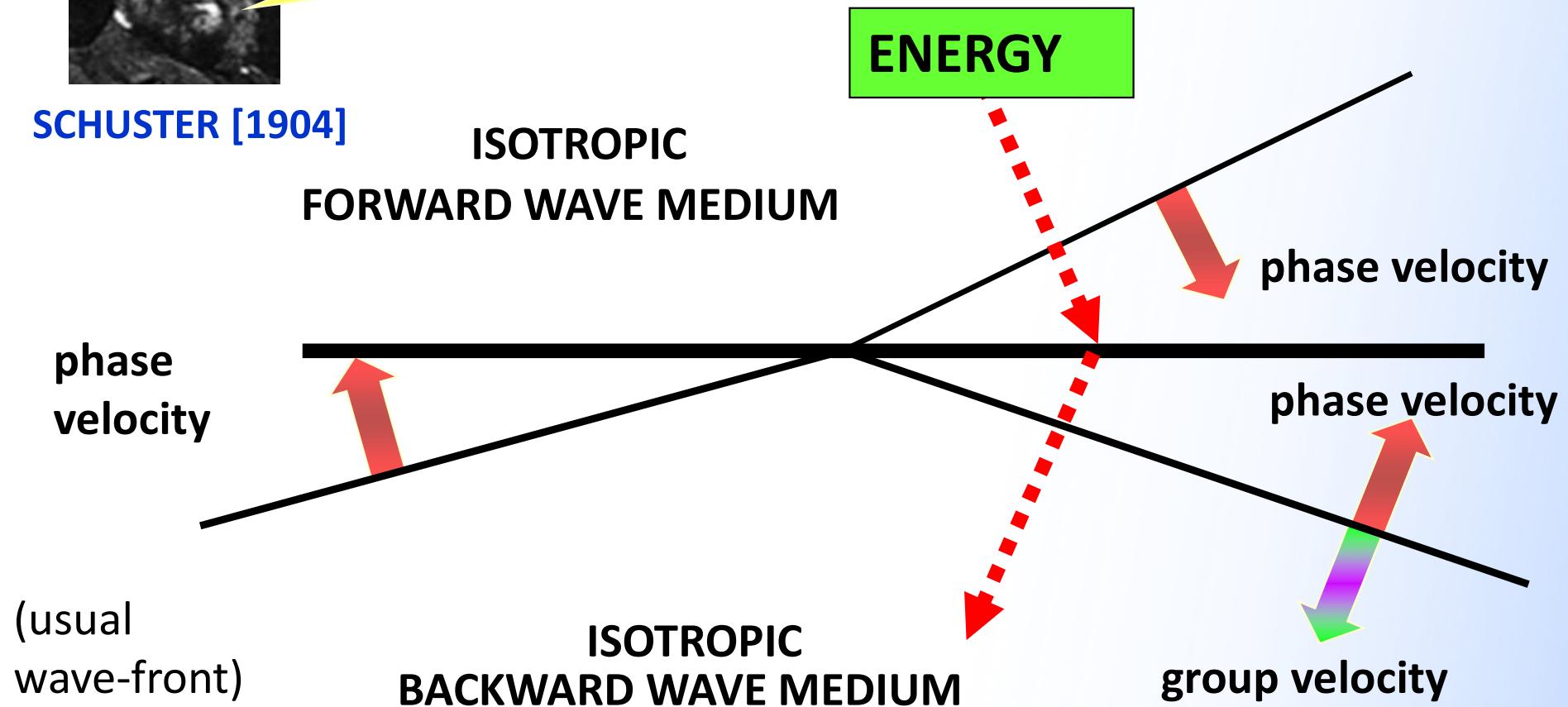


UNIVERSITY
OF BRESCIA



SCHUSTER [1904]

“a curious result follows”
“wave is bent over to other
side of the normal”



(usual
wave-front)

ISOTROPIC
BACKWARD WAVE MEDIUM

group velocity

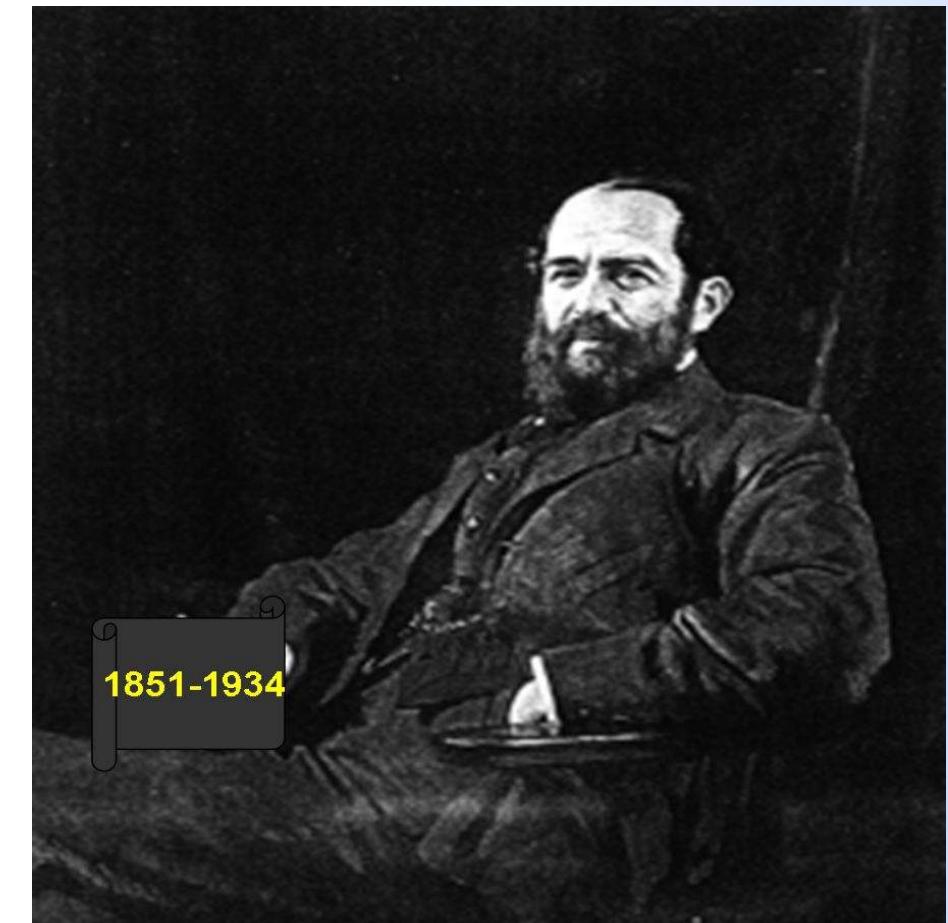
STILL μ WAS NOT CONSIDERED



Horace and Arthur 1904



Sir Horace Lamb (1849-1934)

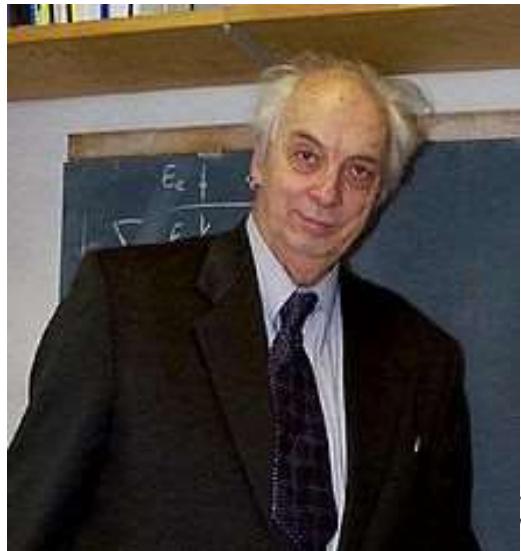


1851-1934

Sir Arthur Schuster (1851-1934)

❖ LAMB: *"It is hardly to be expected that the notion of negative group velocity will have any important physical application"*

❖ SCHUSTER: *"It is doubtful how far the results have any application"*



First paper about negative refraction

VOLUME 10, NUMBER 4

JANUARY-FEBRUARY 1968

538.30

THE ELECTRODYNAMICS OF SUBSTANCES WITH SIMULTANEOUSLY NEGATIVE
VALUES OF ϵ AND μ

V. G. VESELAGO

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Usp. Fiz. Nauk 92, 517-526 (July, 1964)

1. INTRODUCTION

THE dielectric constant ϵ and the magnetic permeability μ are the fundamental characteristic quantities which determine the propagation of electromagnetic waves in matter. This is due to the fact that they are the only parameters of the substance that appear in the dispersion equation

$$\left| \frac{\omega^2}{c^2} \epsilon_{ij} \mu_{ij} - k^2 \delta_{ij} + k_i k_j \right| = 0, \quad (1)$$

II. THE PROPAGATION OF WAVES IN A SUBSTANCE WITH $\epsilon < 0$ AND $\mu < 0$. "RIGHT-HANDED" AND "LEFT-HANDED" SUBSTANCES

To ascertain the electromagnetic laws essentially connected with the sign of ϵ and μ , we must turn to those relations in which ϵ and μ appear separately, and not in the form of their product, as in (1)-(3). These relations are primarily the Maxwell equations and the constitutive relations

$$\nabla \cdot \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t},$$



How do we get negative refraction?

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$$

For plane waves we have: $E(\omega)e^{-j(\omega t - \mathbf{k} \cdot \mathbf{r})}$, $B(\omega)e^{-j(\omega t - \mathbf{k} \cdot \mathbf{r})}$

$$\mathbf{B} = \mu(\omega)\mathbf{H}, \quad \mathbf{D} = \epsilon(\omega)\mathbf{E}$$

$$-j\mathbf{k} \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

$$-j\mathbf{k} \times \mathbf{H} = j\omega\epsilon\mathbf{E}$$

Apply $\mathbf{k} \times$ operator to first equation to obtain:

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) = (\omega\mu)\mathbf{k} \times \mathbf{H} = -(\omega\mu)(\omega\epsilon\mathbf{E})$$



How do we get negative refraction?

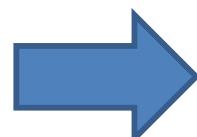
$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) = (\omega\mu) \mathbf{k} \times \mathbf{H} = -(\omega\mu)(\omega\epsilon\mathbf{E})$$

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) = \mathbf{k}(\mathbf{k} \cdot \mathbf{E}) - (\mathbf{k} \cdot \mathbf{k})\mathbf{E} = -(\omega\mu)(\omega\epsilon\mathbf{E})$$

$\nabla \bullet \mathbf{E} = 0$

A red diagonal line with an arrowhead at the start of the first term $\mathbf{k}(\mathbf{k} \cdot \mathbf{E})$ has the text $\nabla \bullet \mathbf{E} = 0$ written above it.

$$\mathbf{k} \cdot \mathbf{k} = \omega^2 \mu \epsilon$$



$$k = \pm \omega \sqrt{\mu \epsilon} = \pm \frac{\omega}{c} n$$

Refractive index can be either positive or negative



Right Handed and Left Handed Media



$\epsilon, \mu > 0$

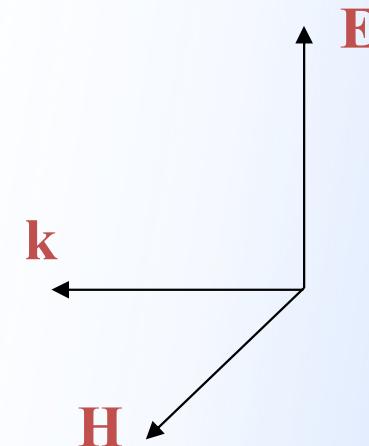
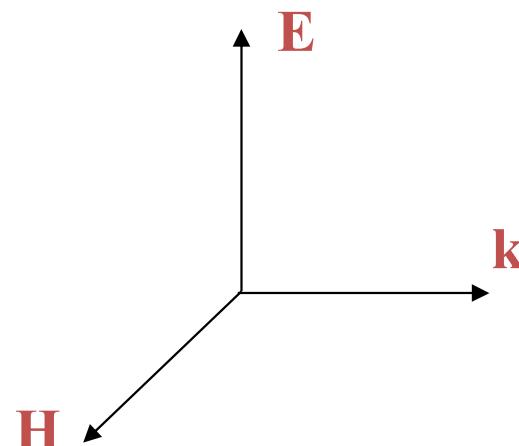
$$\mathbf{k} \times \mathbf{E} = \omega \mu \mathbf{H}$$

$$\mathbf{k} \times \mathbf{H} = -\omega \epsilon \mathbf{E}$$

$\epsilon, \mu < 0$

$$\mathbf{k} \times \mathbf{E} = -|\omega \mu| \mathbf{H}$$

$$\mathbf{k} \times \mathbf{H} = |\omega \epsilon| \mathbf{E}$$





Where is the energy flowing?

$$\mathbf{k} \times \mathbf{E} = \omega \mu \mathbf{H}$$

$$\mathbf{k} \times \mathbf{H} = -\omega \epsilon \mathbf{E}$$

$$\mathbf{E} \times (\mathbf{k} \times \mathbf{E}) = \omega \mu \mathbf{E} \times \mathbf{H} = \omega \mu \mathbf{S}$$

$$\mathbf{E} \times (\mathbf{k} \times \mathbf{E}) = (\mathbf{E} \cdot \mathbf{E}) \mathbf{k} - (\mathbf{E} \cdot \mathbf{k}) \mathbf{E}$$

$$\mathbf{S} = \frac{|\mathbf{E}|^2}{\omega \mu} \mathbf{k}$$

Energy flow is always a positive quantity



Where is the energy flowing?

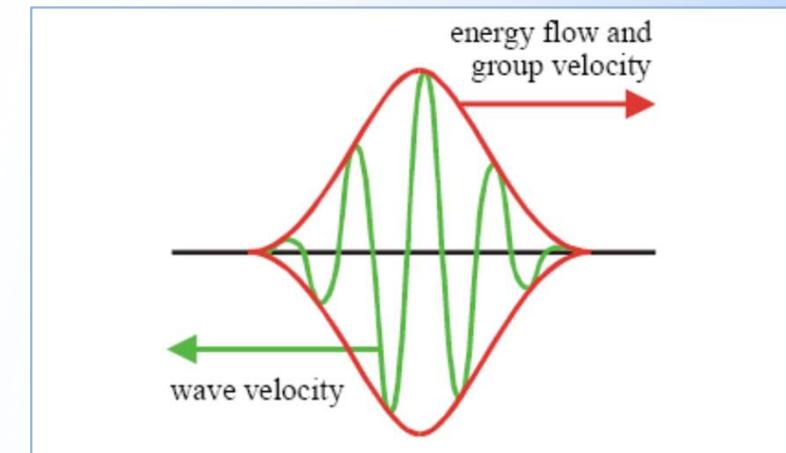
$$\mathbf{k} = \pm \frac{\omega}{c} \sqrt{\mu \epsilon} \hat{\mathbf{k}}$$

If both ϵ and μ are simultaneously negative, then the choice

$\mathbf{k} = -\frac{\omega}{c} n \hat{\mathbf{k}}$ is the only one that ensures positive energy flow.

\mathbf{S} and \mathbf{k} always have to point in the opposite direction if $\mu < 0$

$$\mathbf{S} = \frac{|E|^2}{\omega \mu} \mathbf{k}$$



Veselago: \mathbf{S} remains a right handed quantity, i.e., energy must flow away from the boundary and the source. The consequence of this is **forward propagation with negative phase velocity**.

In other words,
To obtain negative refraction we need to add
magnetic effects where none seems to exist!



We need to make
metamaterials

Supernatural or beyond Nature



*Can we have negative refraction without
having a negative index material?*

YES



Negative permittivity



Diffraction managing



Does negative permittivity exist in nature? YES, it does!

Metals below the plasma frequency
→ negative permittivity

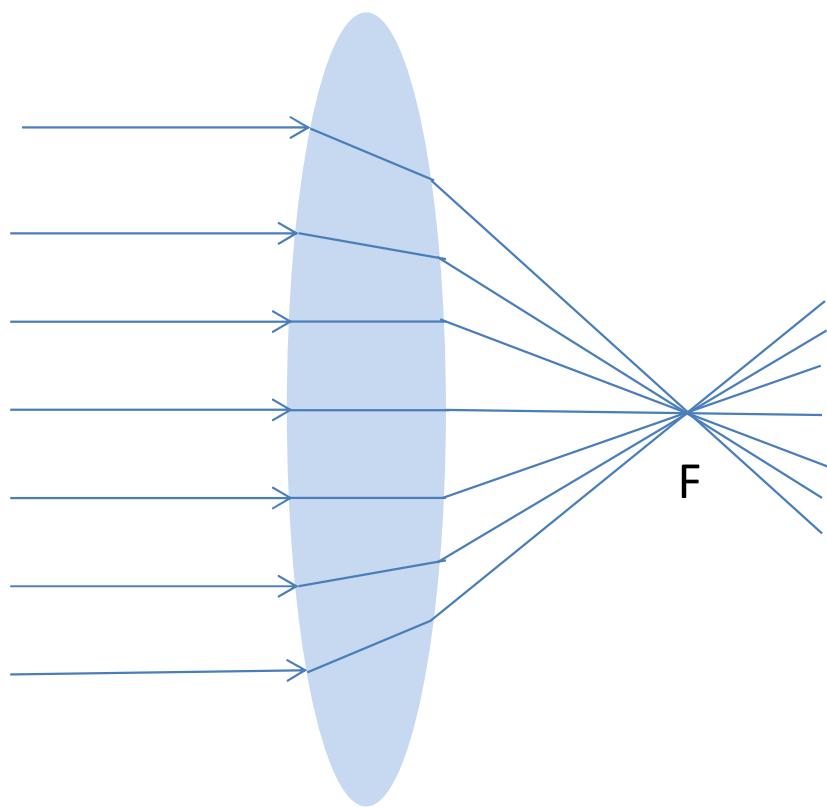
Prof. Sir J. B. Pendry, Imperial College in London (1999) showed that negative permittivity was sufficient to realize a perfect - flat - lens!



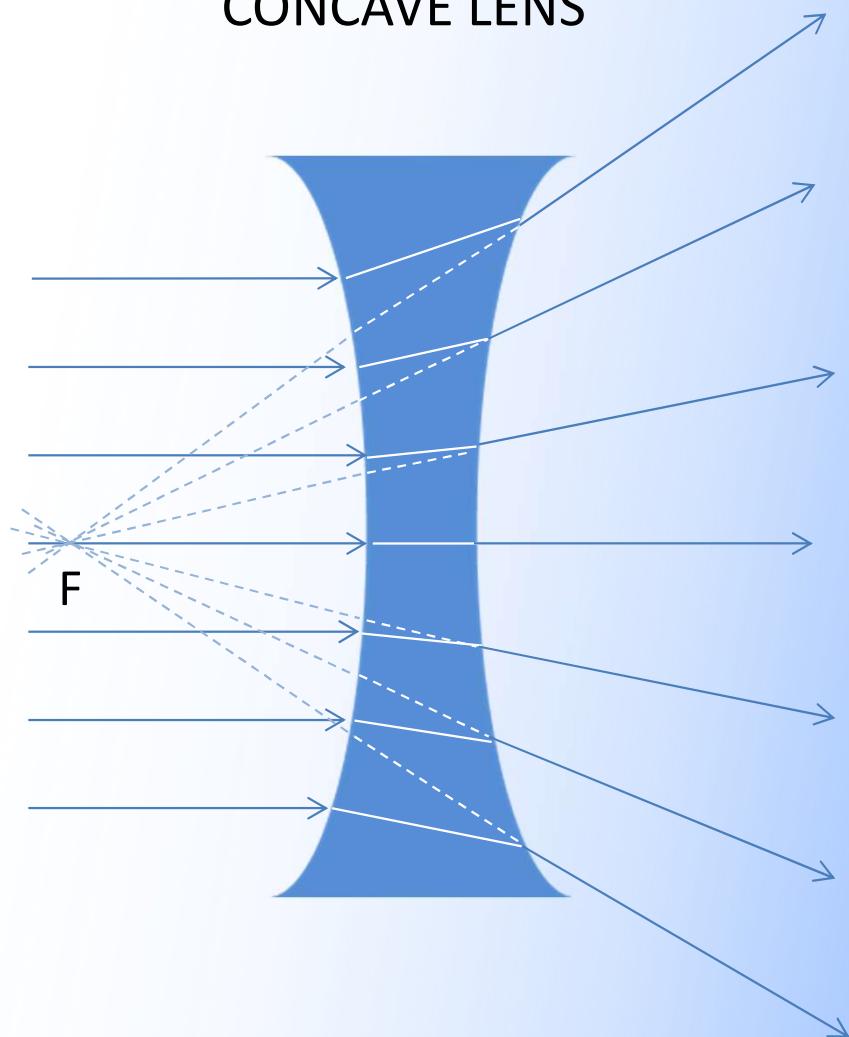


Conventional Lenses

CONVEX LENS



CONCAVE LENS





Conventional Imaging

- Spot size is limited to $\lambda/2NA$:
 $\lambda \rightarrow$ wavelength used in the imaging system
 $NA \rightarrow$ Numerical Aperture of the imaging system
($NA = n\sin\theta$)
- Diffraction limit:
Conventional imaging loses the information of the near-field

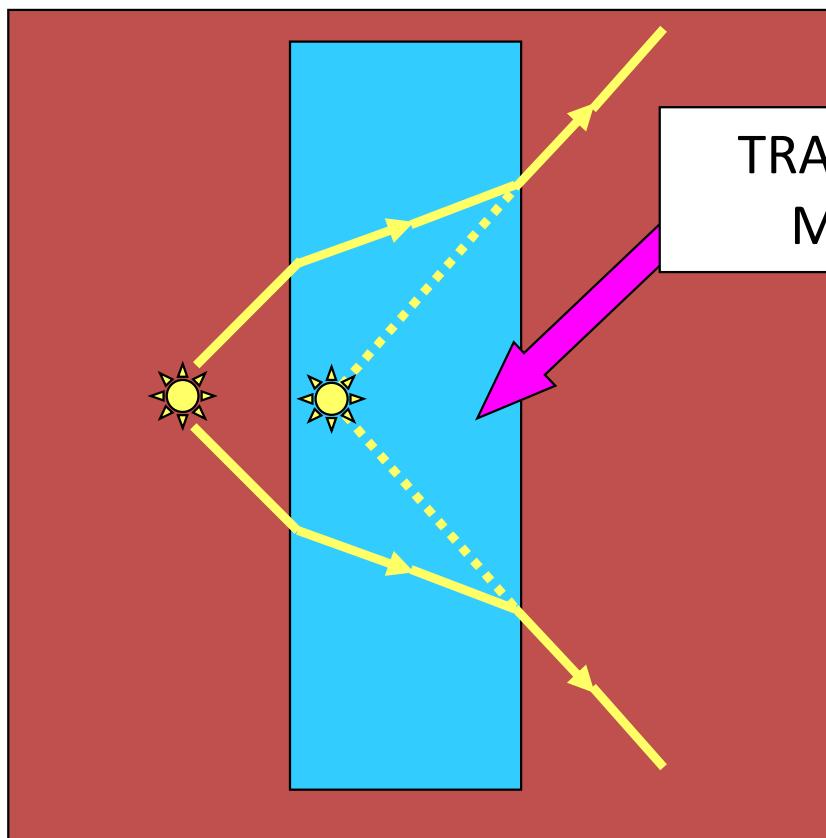


We can improve the resolution if we recover the information of the near-field



NORMAL:
POSITIVE

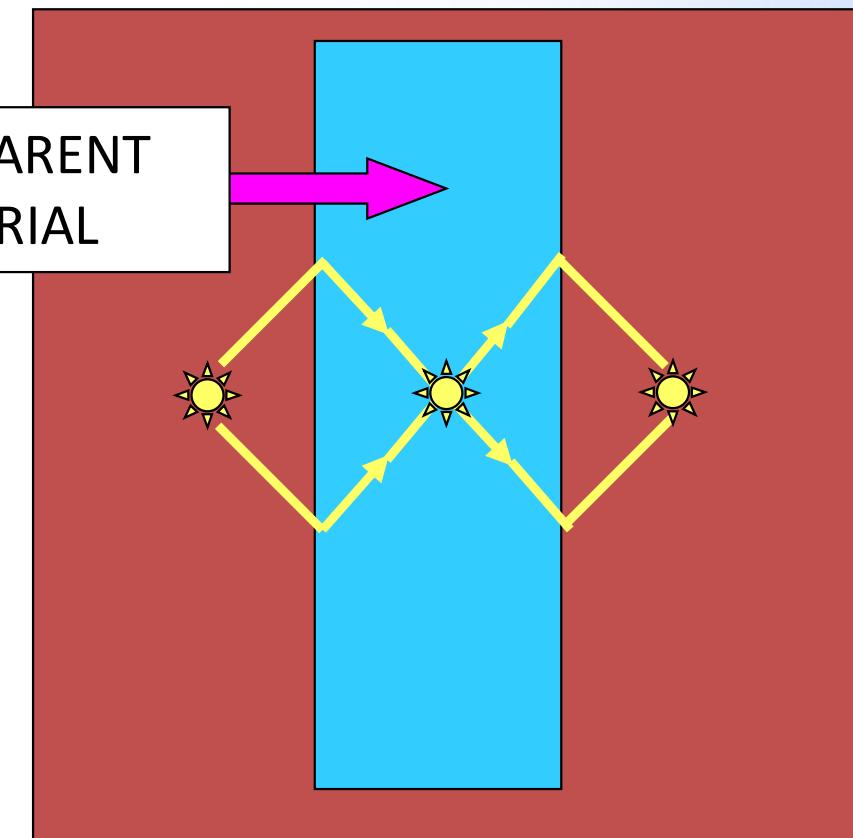
METAMATERIAL:
NEGATIVE



TRANSPARENT
MATERIAL

VIRTUAL IMAGE

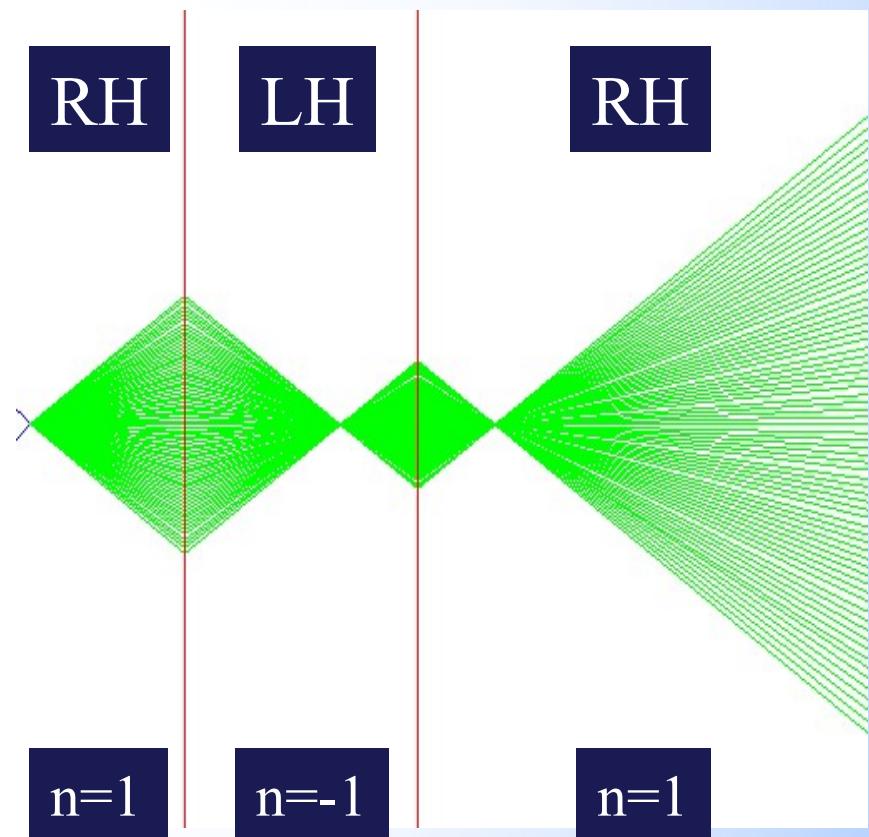
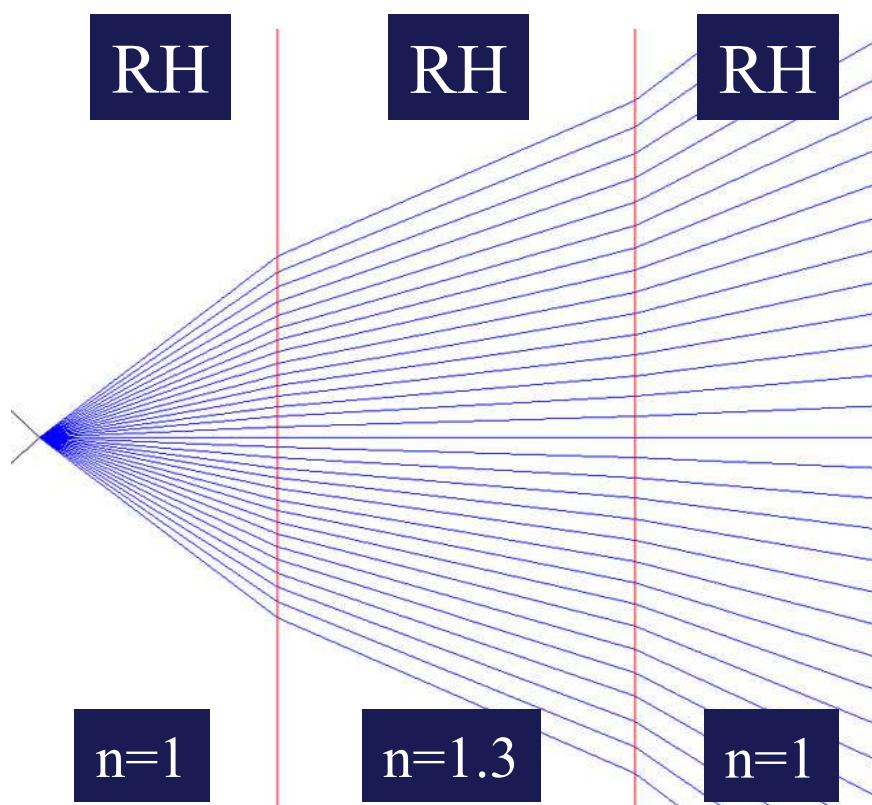
REAL IMAGE





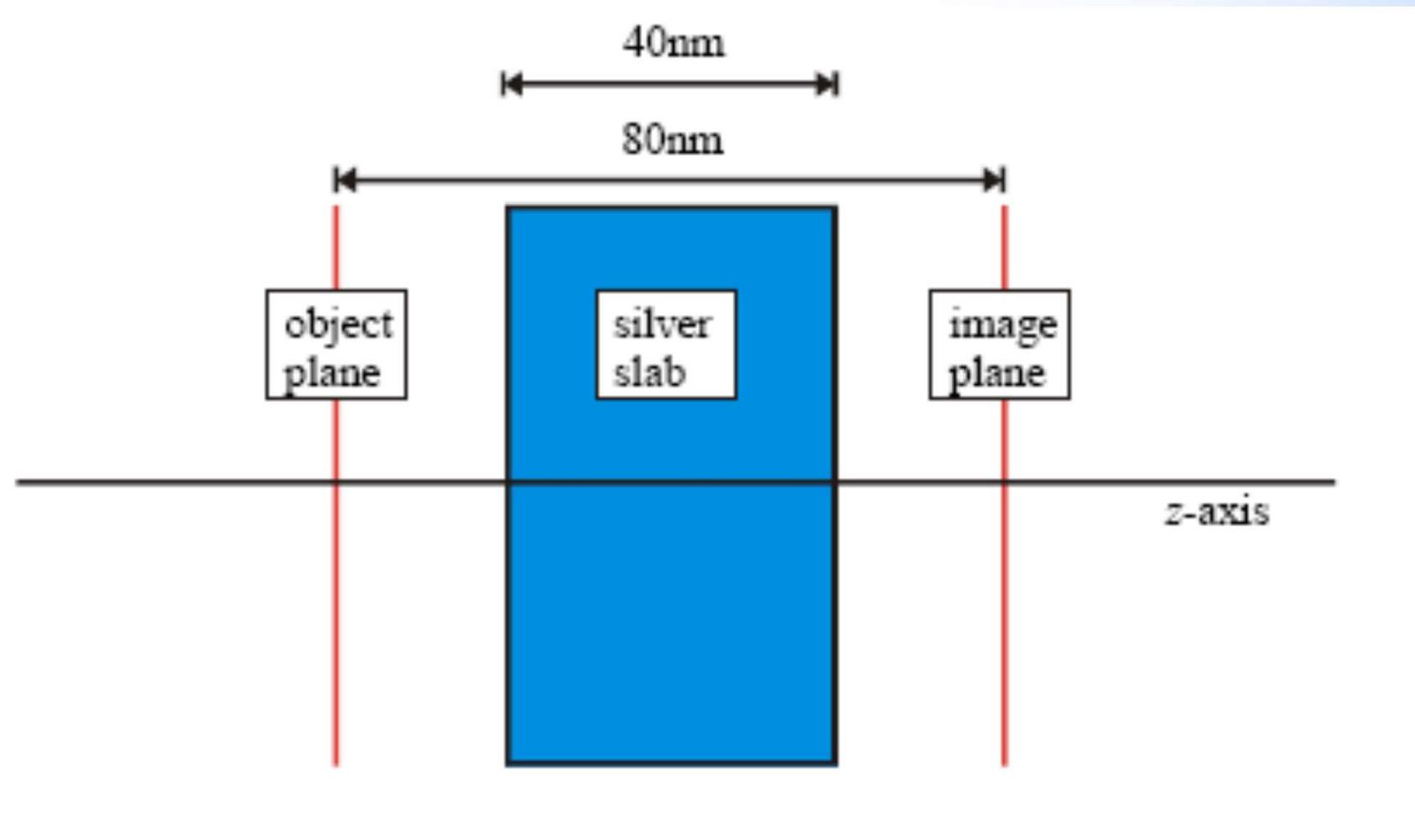
UNIVERSITY
OF BRESCIA

Flat Lenses



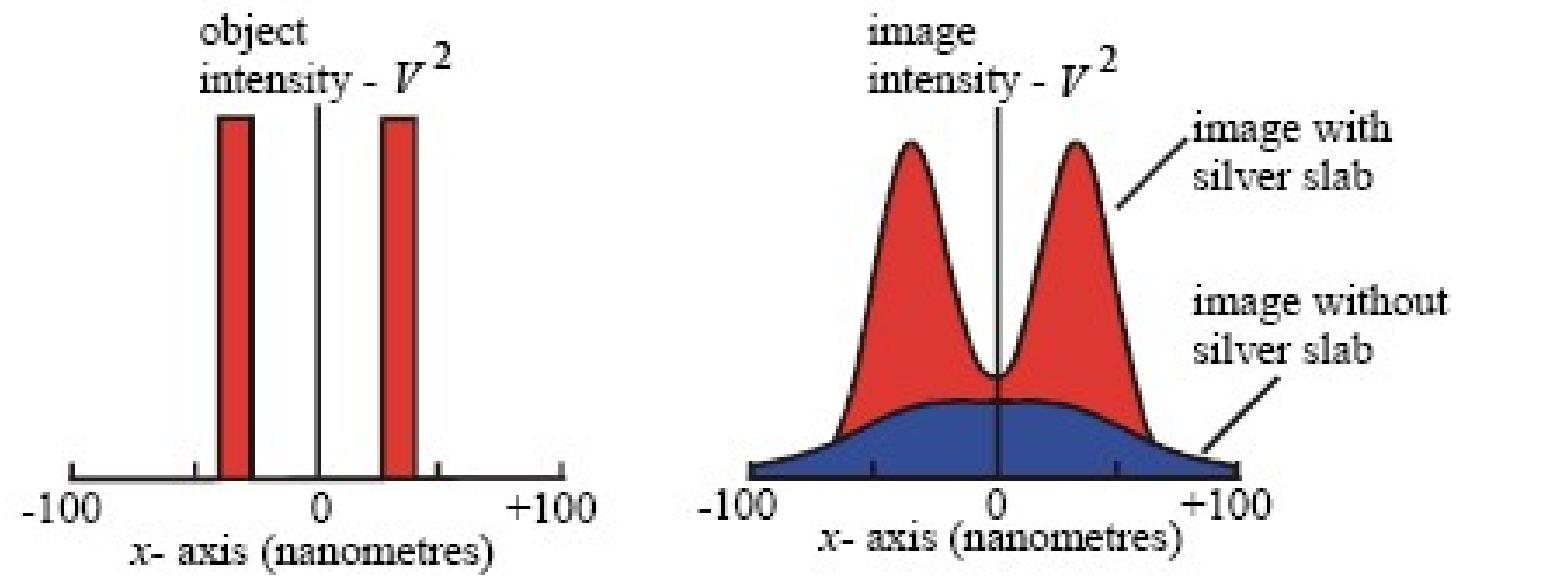


Flat lens with silver films





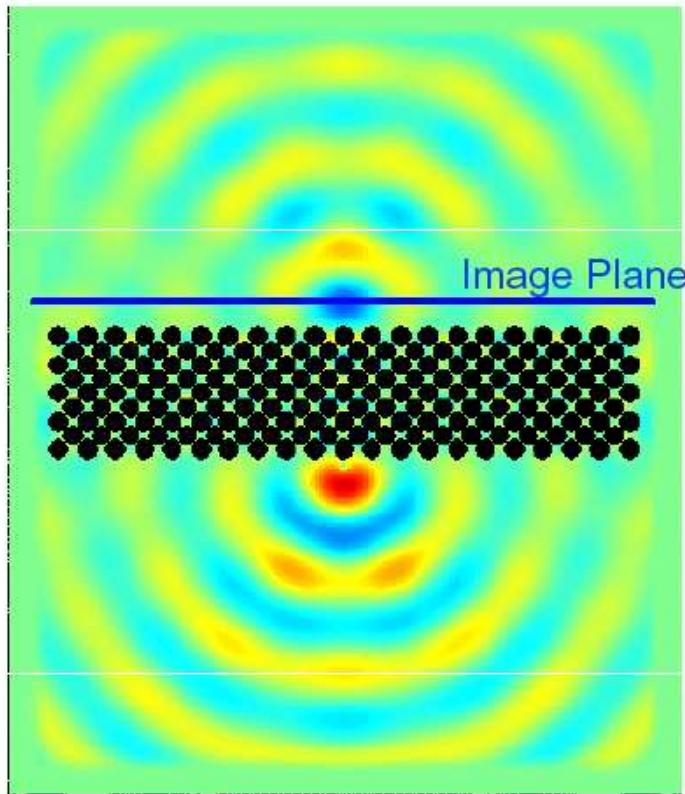
Super-resolution with silver slab



Losses limit the resolution in silver lenses

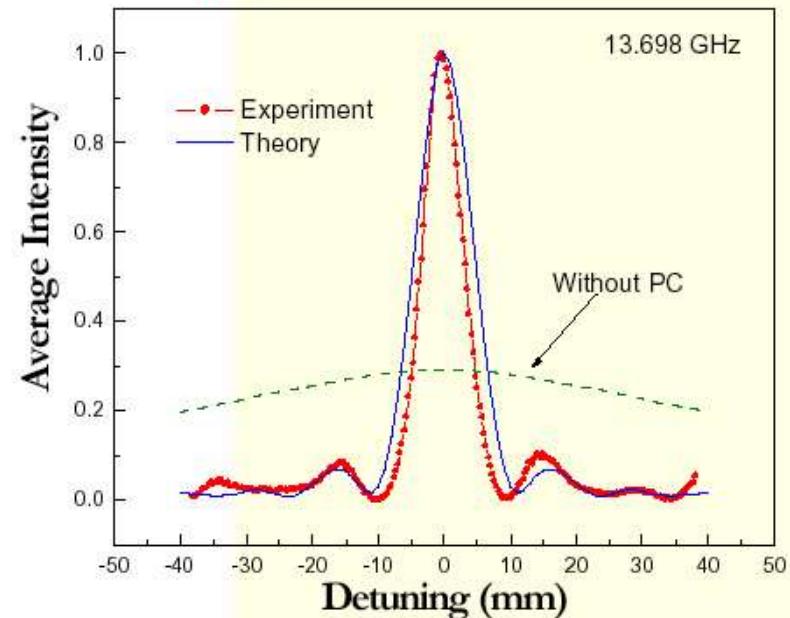


Photonic crystals



Electric field distribution after image formation

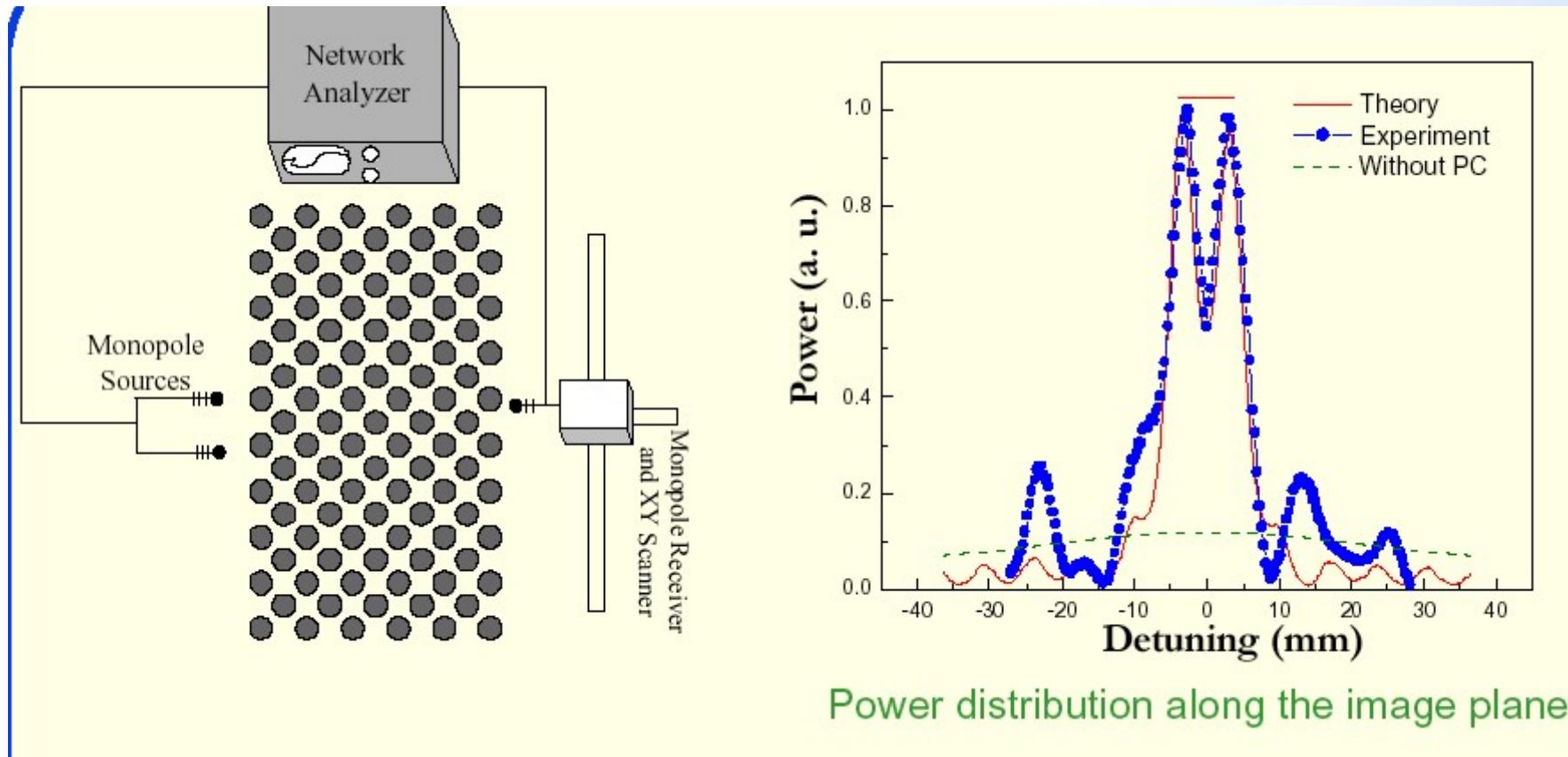
Time averaged intensity distribution along the image plane



NOTE: Photonic crystals cannot be classified as metamaterials! Although the period and the unit cell size are comparable to the wavelength, the size of the meta-atoms are of the order of the wavelength as well. PhC cannot be considered as effective media, therefore they cannot be classified as metamaterials.



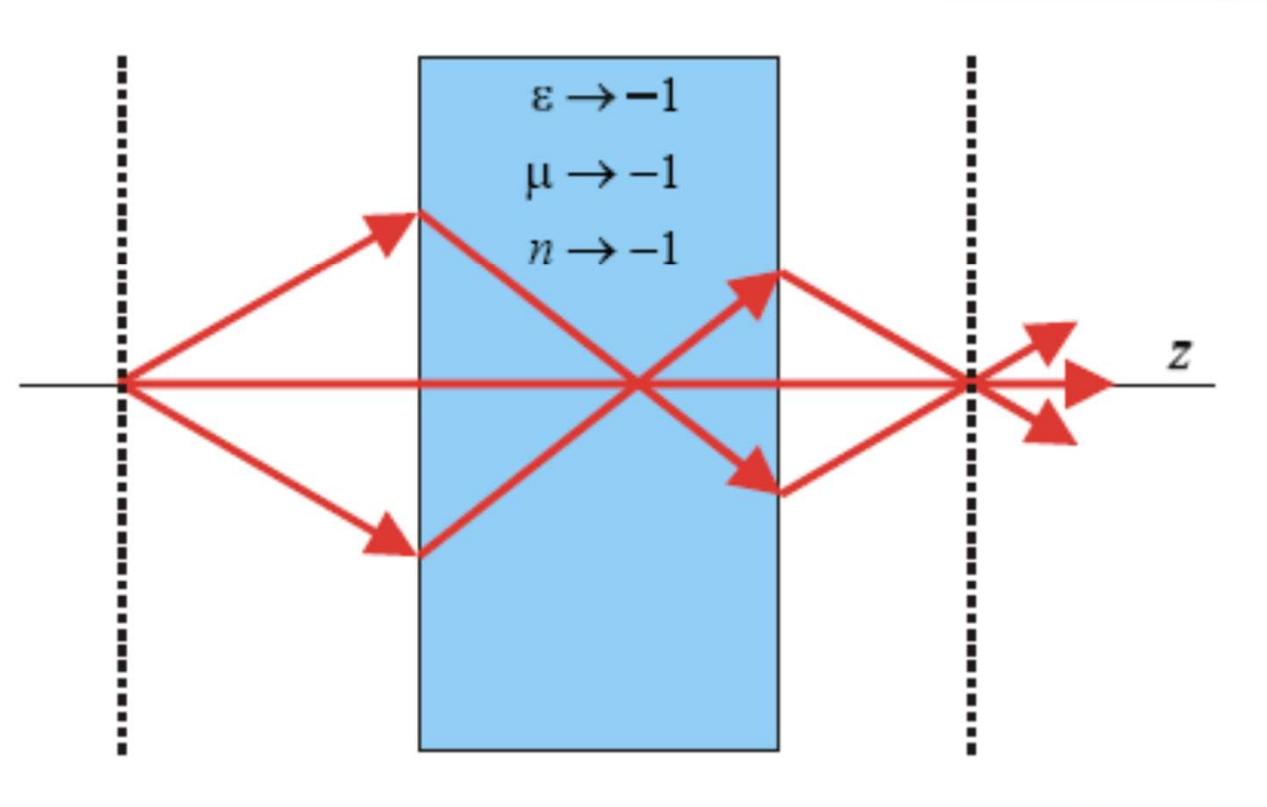
Super-resolution with photonic crystals



Separation between two sources is $\lambda/3$



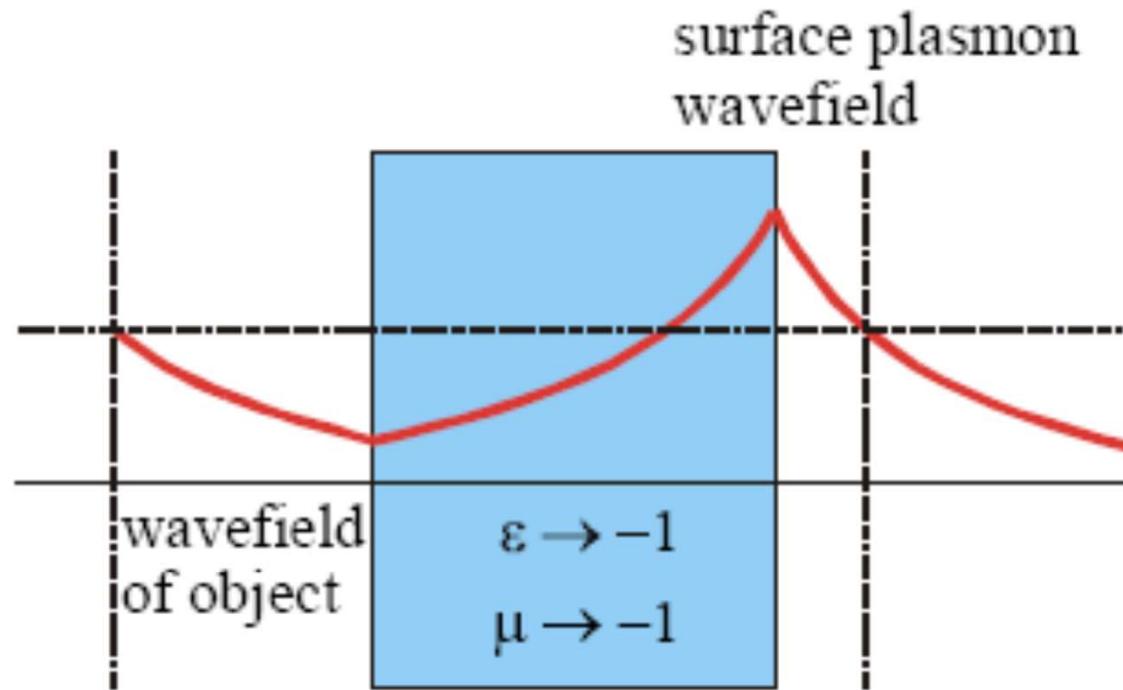
Super-resolution with negative index slab



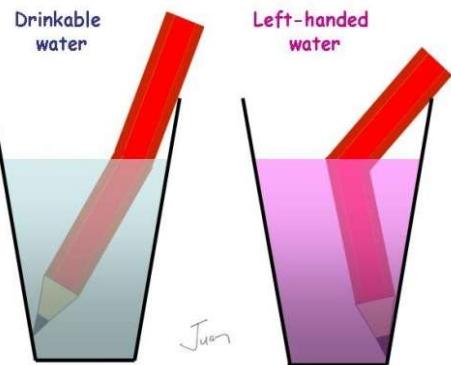
Negative Index Medium (NIM) must be impedance matched with the Positive Index Medium (PIM)



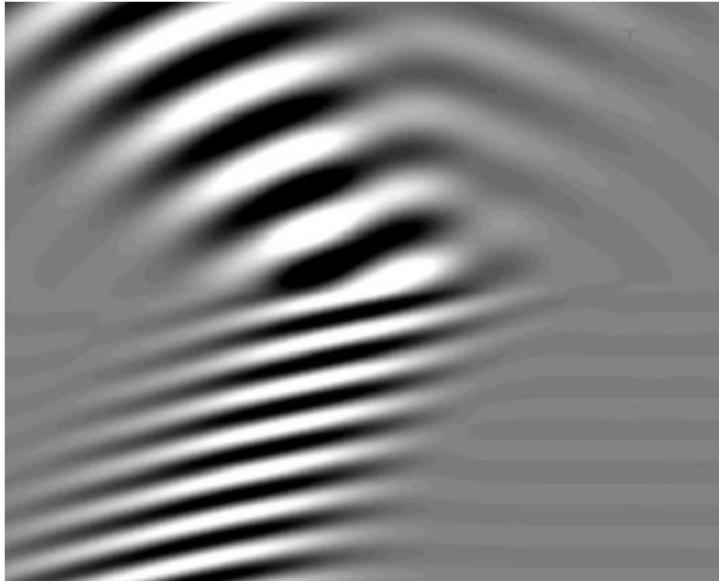
Super-resolution with negative index slab



This lens works with the excitation of surface plasmons and is able to transfer the information of the near field that is commonly lost in conventional imaging



Inverted Snell's law



I don't
believe it!
Light is
bending the
wrong way



EXTRA,
EXTRA!!!
Are school
text books
wrong?!?





Negative Refraction



(a)



(b)



(c)

- (a) Calculated ray-tracing image of a metal rod in an empty glass.
- (b) The glass is filled with normal water, ($n=1.2$) → ordinary refraction.
- (c) The water is replaced by "water" with a fictitious refractive index of $n=-1.2$.



UNIVERSITY
OF BRESCIA

Negative index material properties

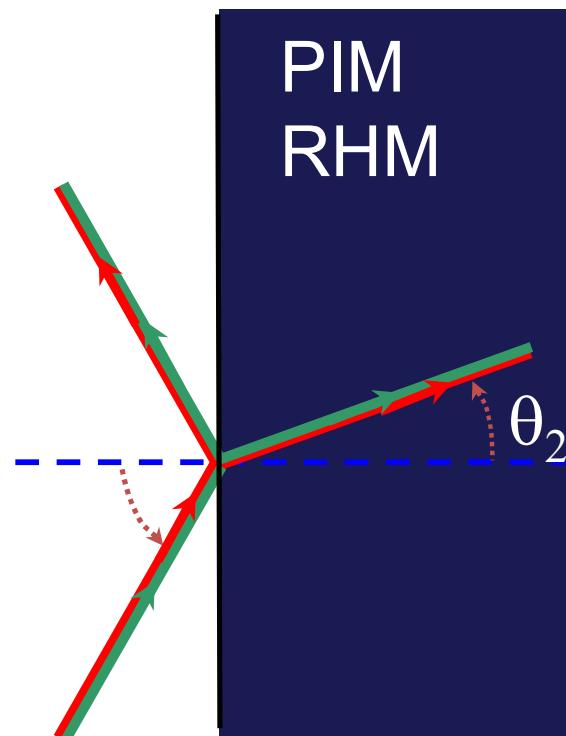


Reversal of Snell's law

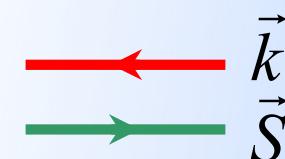
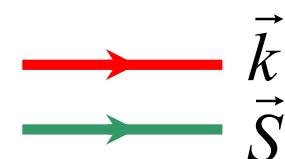
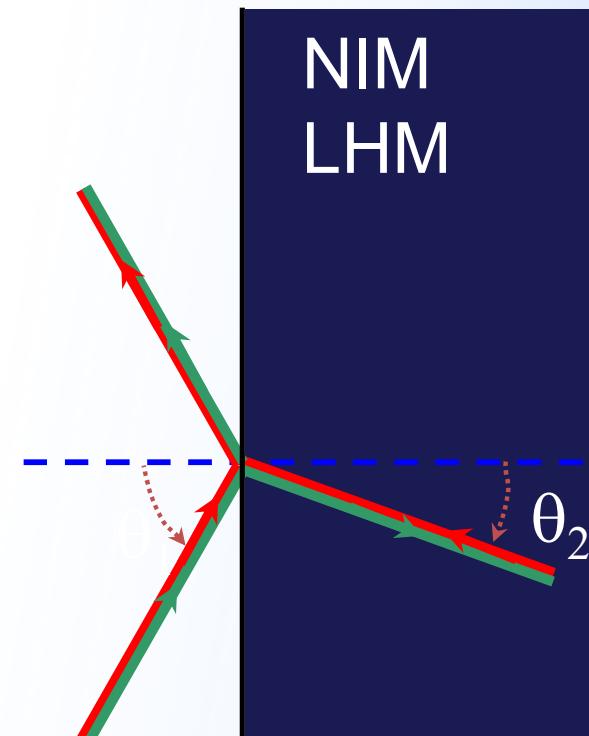
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$n_2 < 0 \Rightarrow \theta_2 < 0$$

PIM
RHM



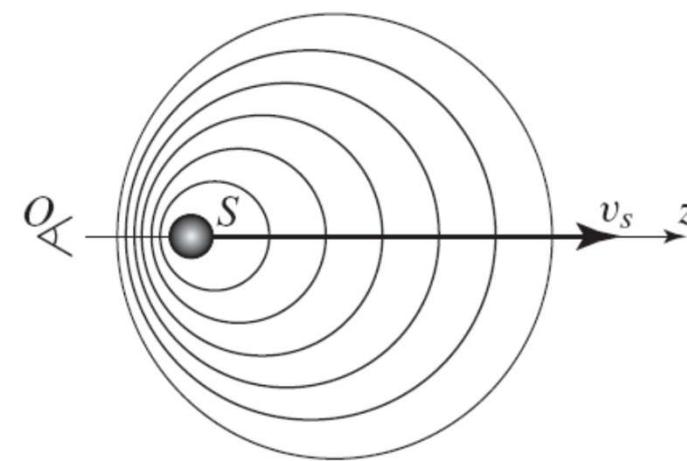
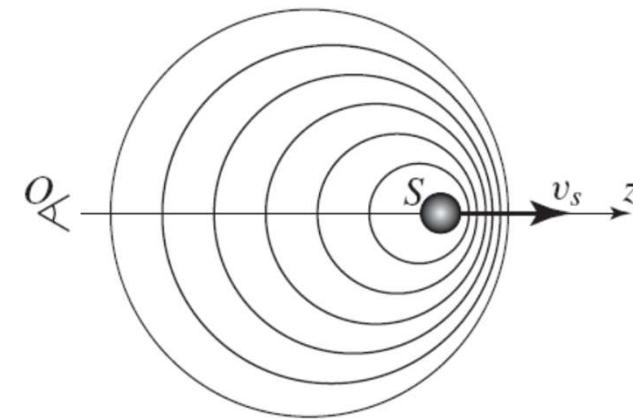
PIM
RHM





Reversal of Doppler effects

- Doppler effect in RH media
- Doppler effect in LH media





Dispersion in negative index media

Group Velocity sign depends on the material:

$$v_g = \frac{\partial \omega}{\partial k};$$

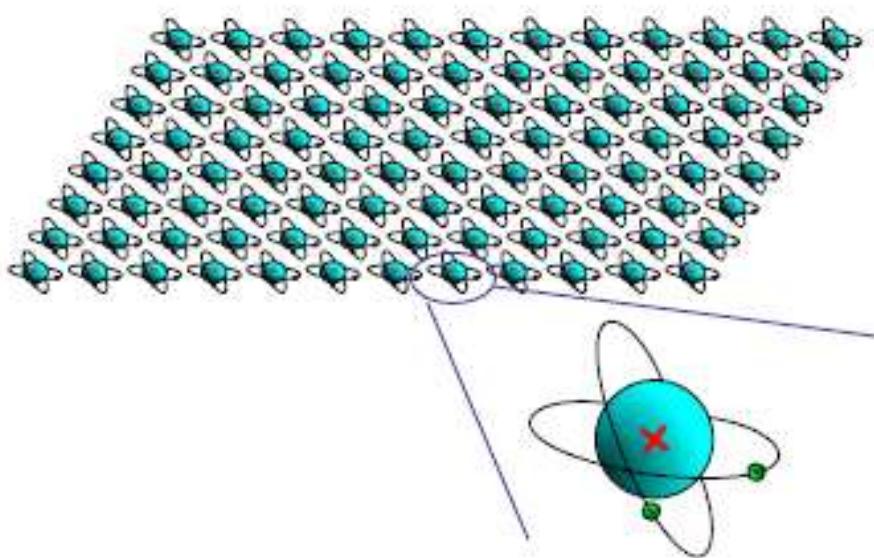
Negative phase Velocity:

$$v_p = \frac{\omega}{k} < 0$$

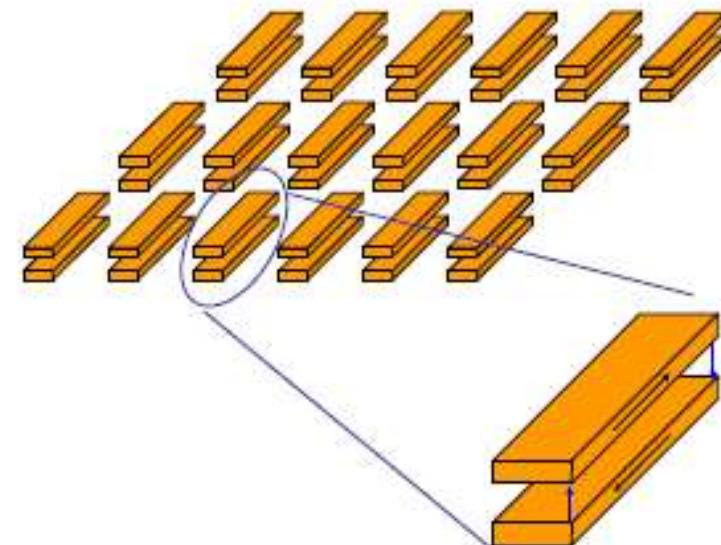


How to make metamaterials

A Metamaterial is an arrangement of artificial structural elements, designed to achieve advantageous and unusual electromagnetic properties. ---Metamorphose



A natural material with its atoms

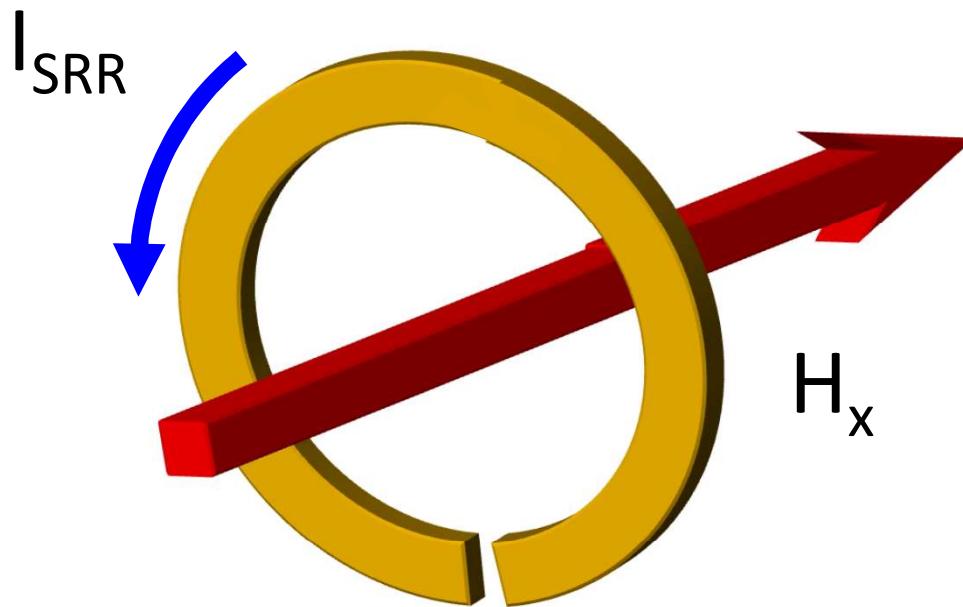


A metamaterial with artificially structured “atoms”

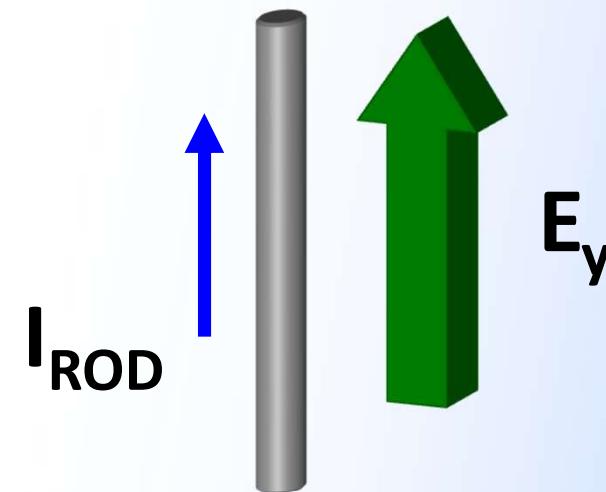


How to make metamaterials

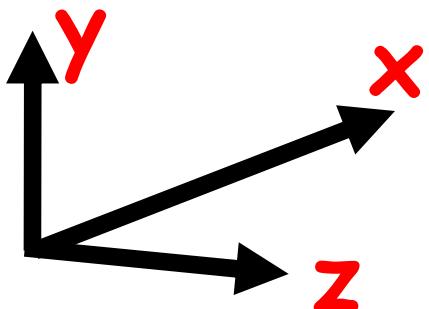
Embedding a metal split-ring and a metal rod creates left-handedness



Magnetic resonance
(Negative μ)



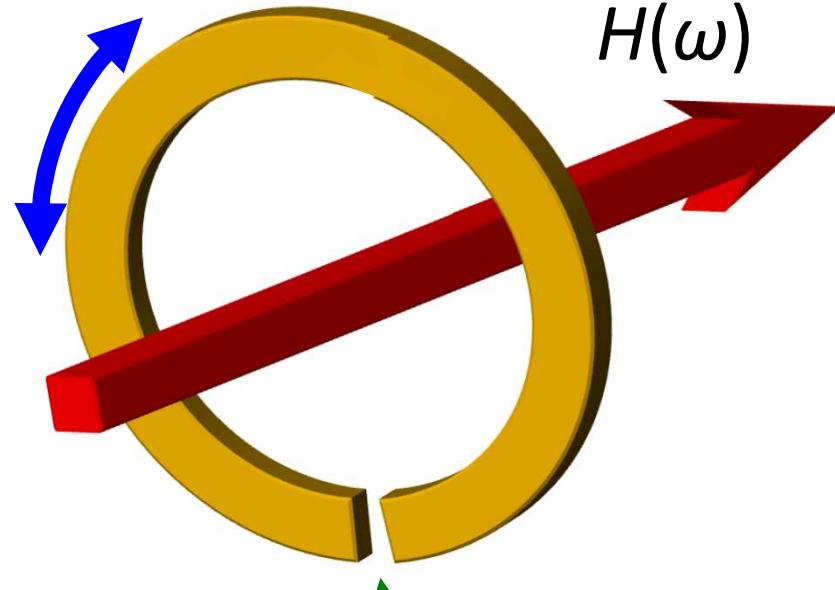
Electric resonance
(Negative ϵ)



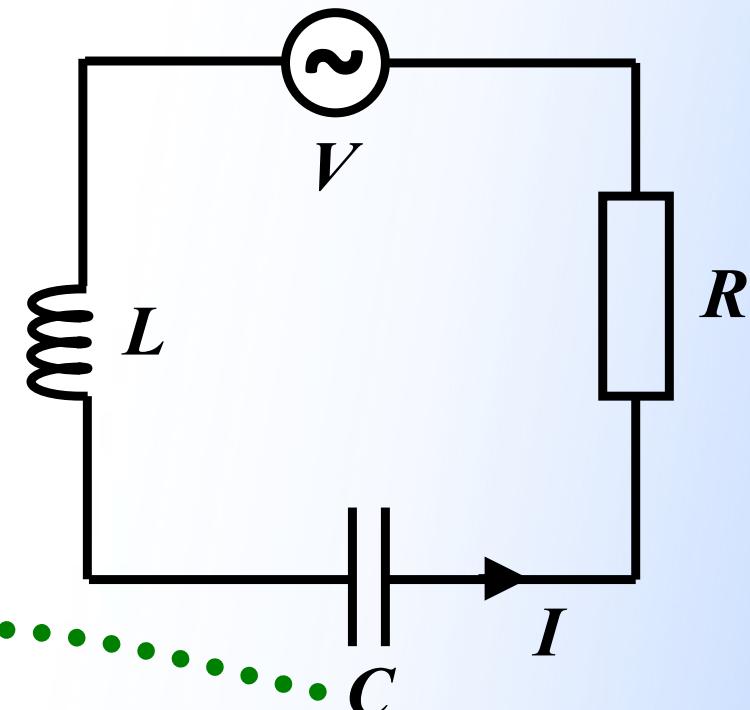


How to make a metamaterial

AC current



INDUCED E.M.F.



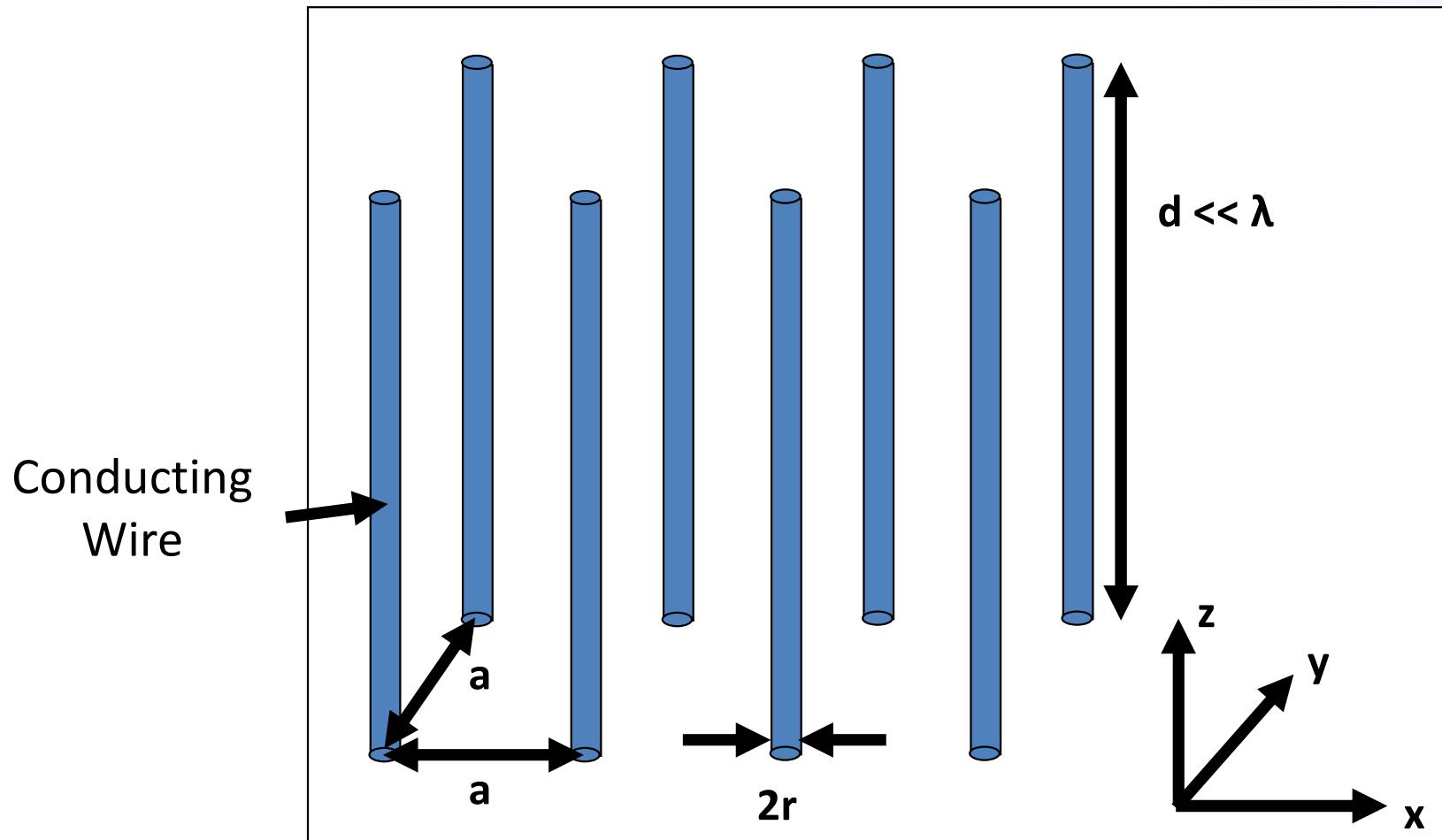
$$V = L \frac{dI}{dt} + IR + \frac{Q}{C}$$

$\rightarrow \mu(\omega)$



How to make a metamaterial

Short Wires Lead To $\epsilon(\omega)$



Infinite Periodic array of conducting wires



GHz range

First experimental realization

Experimental Realizations

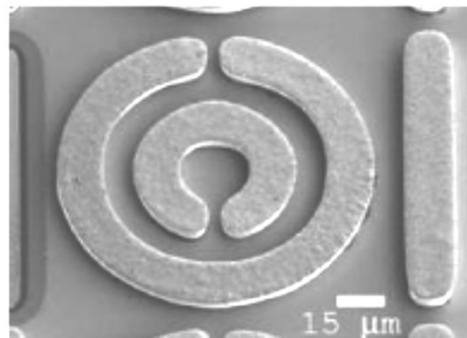


Smith et al: Phys. Rev. Lett. 84, 4184 (2000)

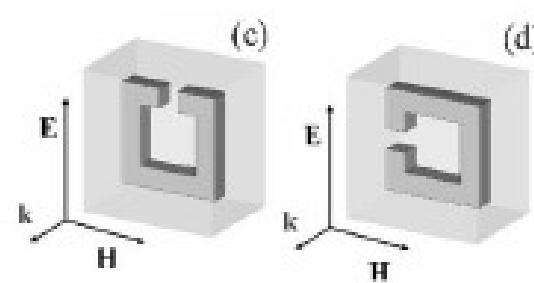


Experimental Realizations

1–2.7 THz

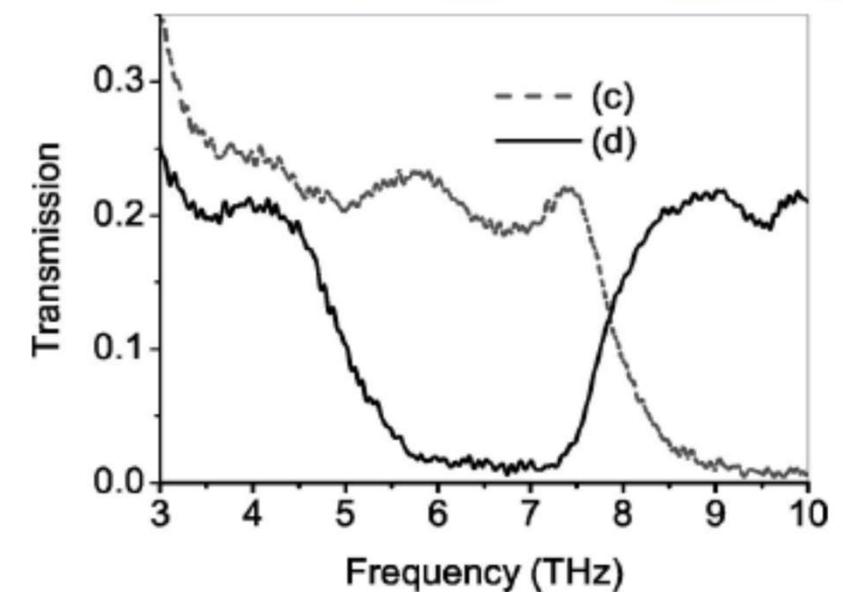


Moser et al. PRL 90 063901 (2005)



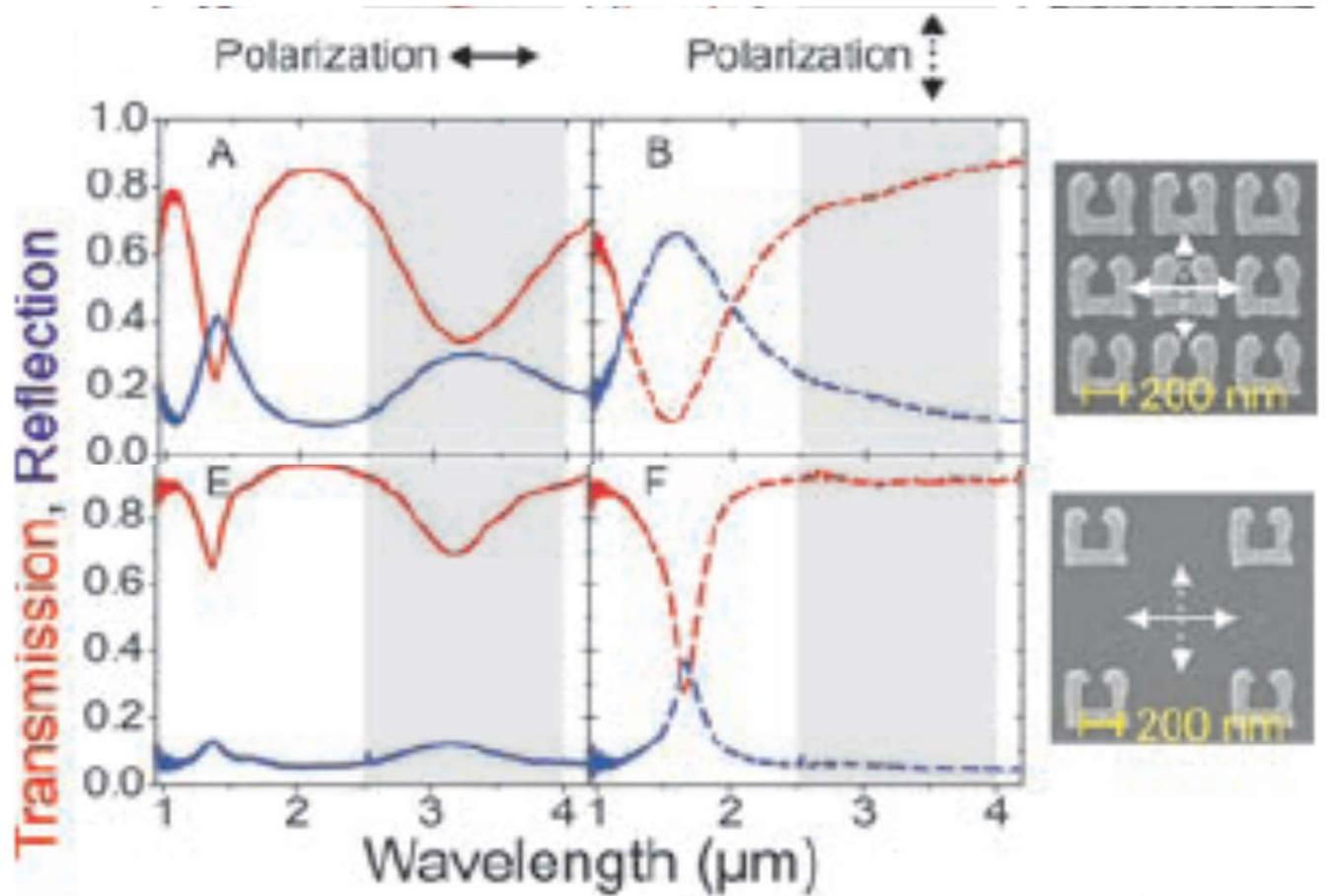
Katsarakis et al. Opt Lett 30(11) 1348 (2005)

6 THz





Experimental Realizations



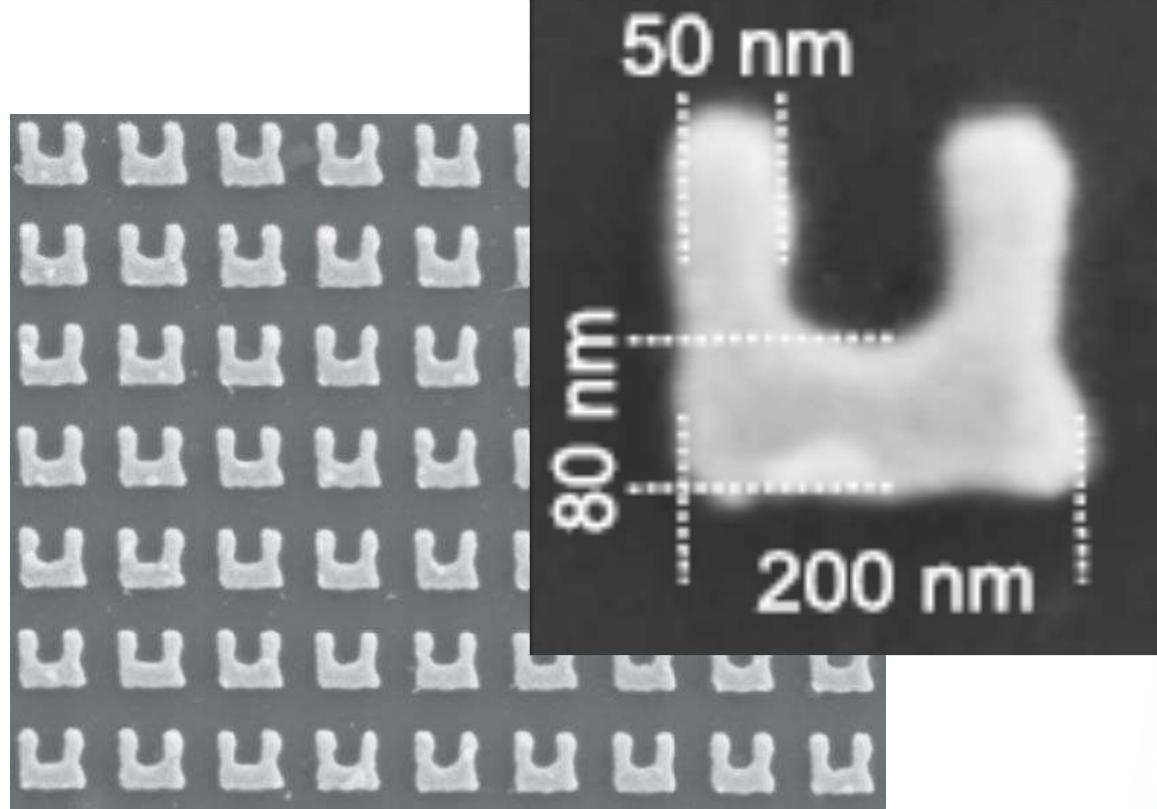
100 THz

Linden et al. SCIENCE 306 1351 (2004)

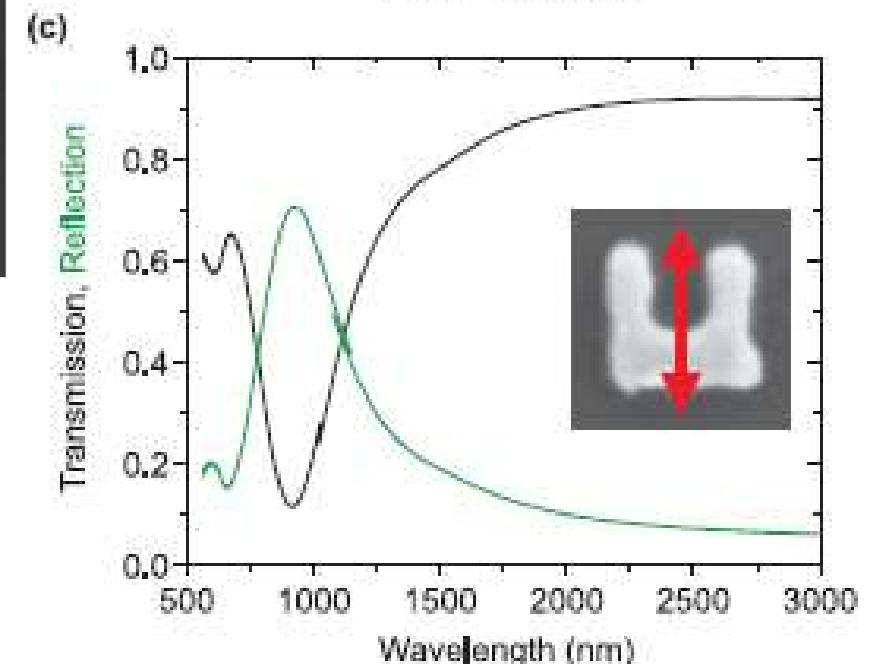
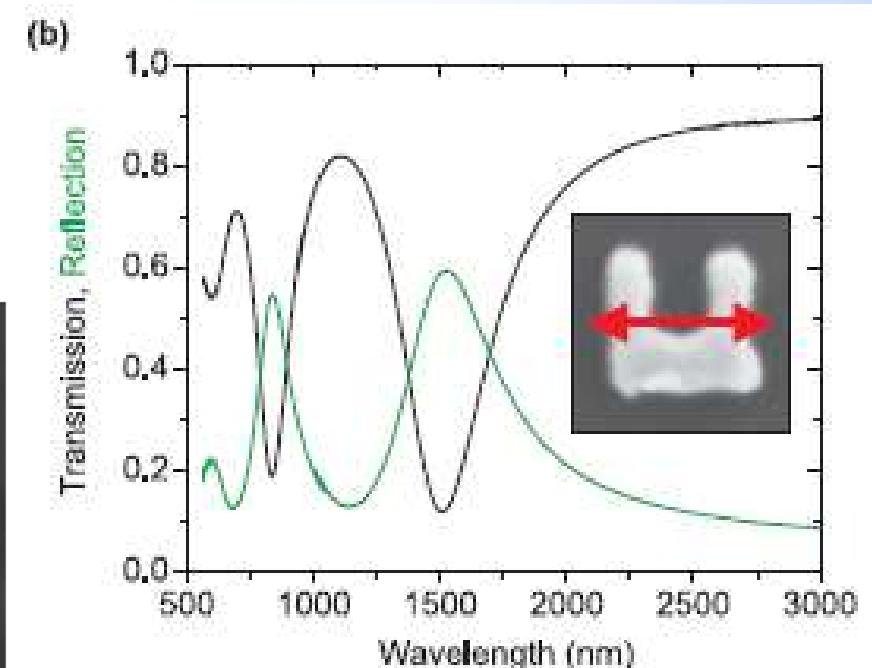


Experimental Realizations

Magnetic resonances at
1.5 μm and 800 nm

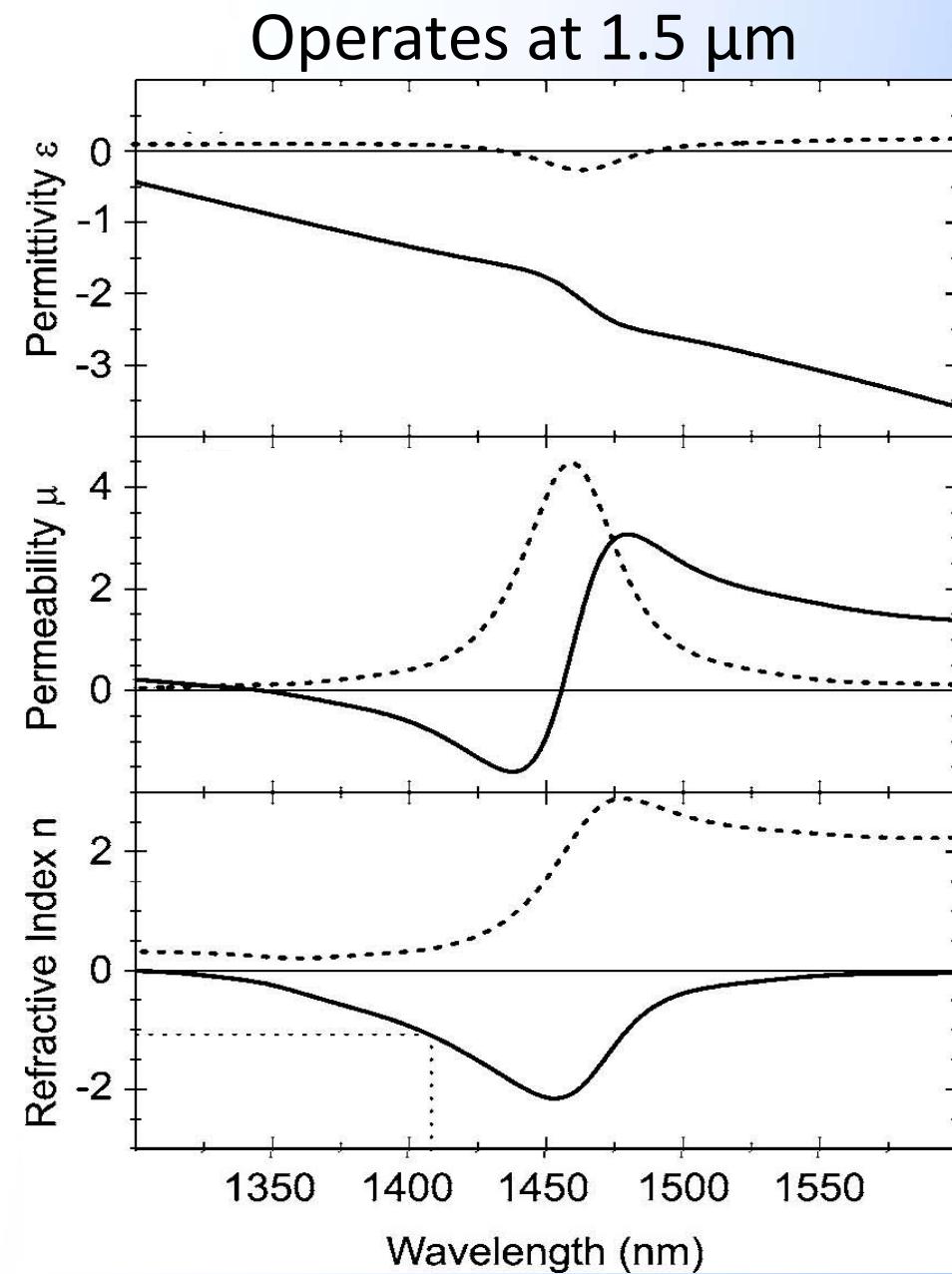
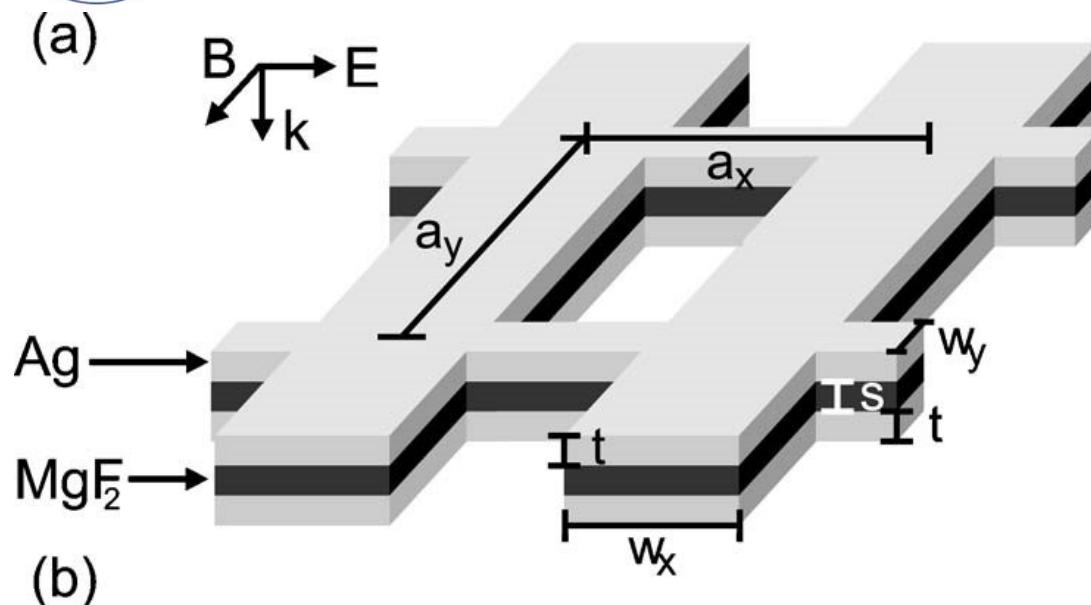


C. Enkrich et al., Phys. Rev. Lett. 95, 203901 (2005)



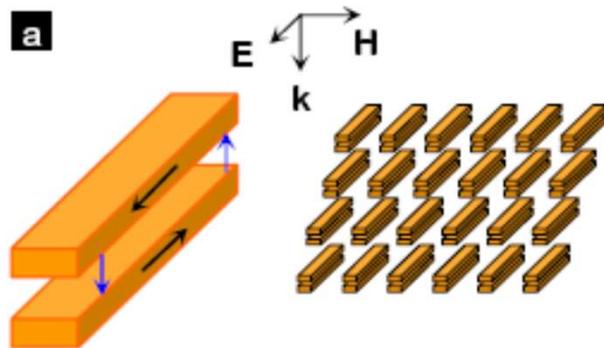


Experimental Realizations

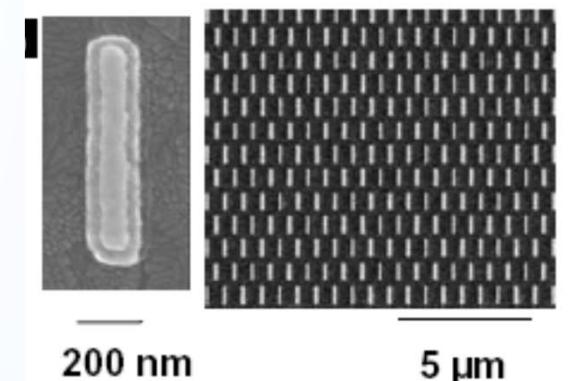




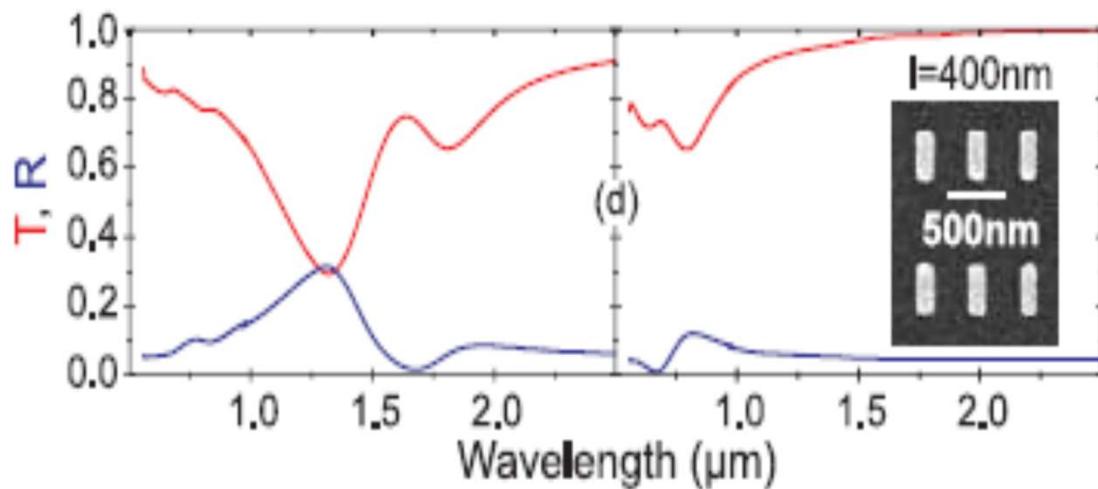
Experimental Realizations



$$\text{Re}(n) = -0.3$$
$$\lambda = 1.5 \mu\text{m}$$



V. M. Shalaev et al., Opt. Lett. 30, 3356-3358 (2005)



$1.3 \mu\text{m}$

Negative permeability

G. Dolling et al., Opt. Lett. 30, 3198-3200 (2005)



Fabrication process

TRANSPARENT ELECTRODE

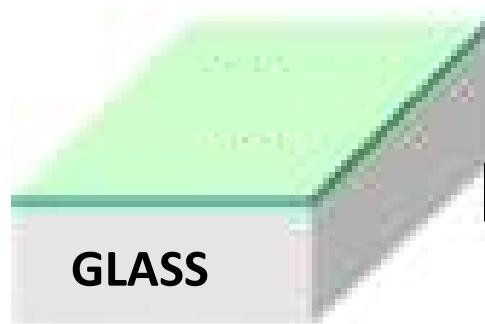
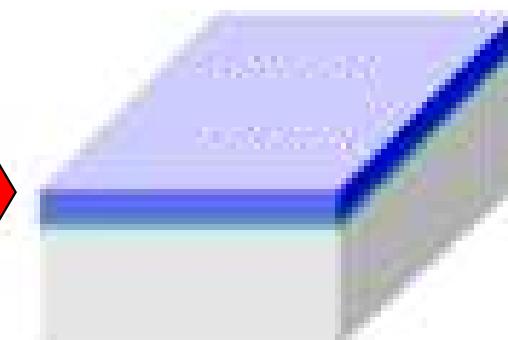
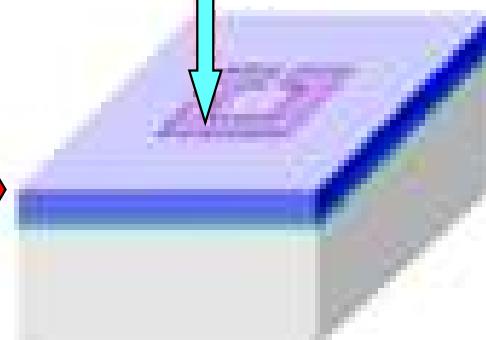


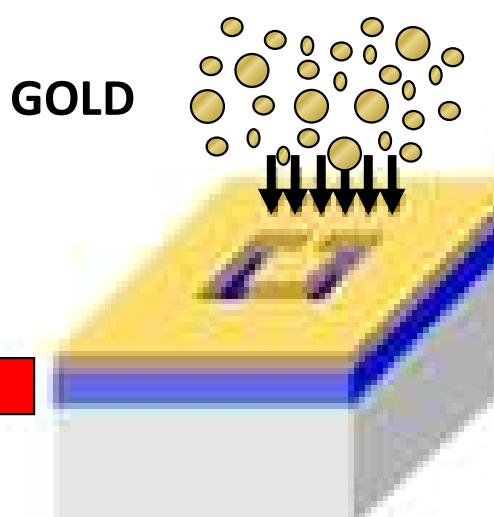
PHOTO-RESIST



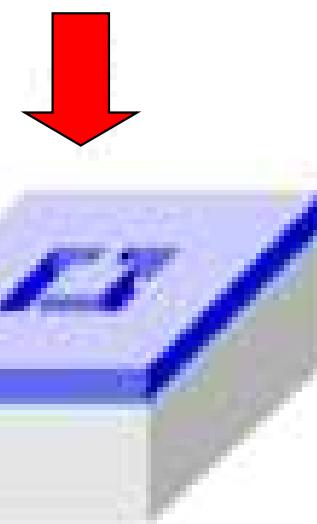
ELECTRONS



SPIN COATING



E- BEAM WRITING



LIFT OFF

GOLD EVAPORATION

DEVELOPMENT



UNIVERSITY
OF BRESCIA

The quest for invisibility



Is Harry Potter's cloak really that magical?



Invisibility in Nature

Chameleon Camouflage





Invisibility to Radars

Stealth fighter

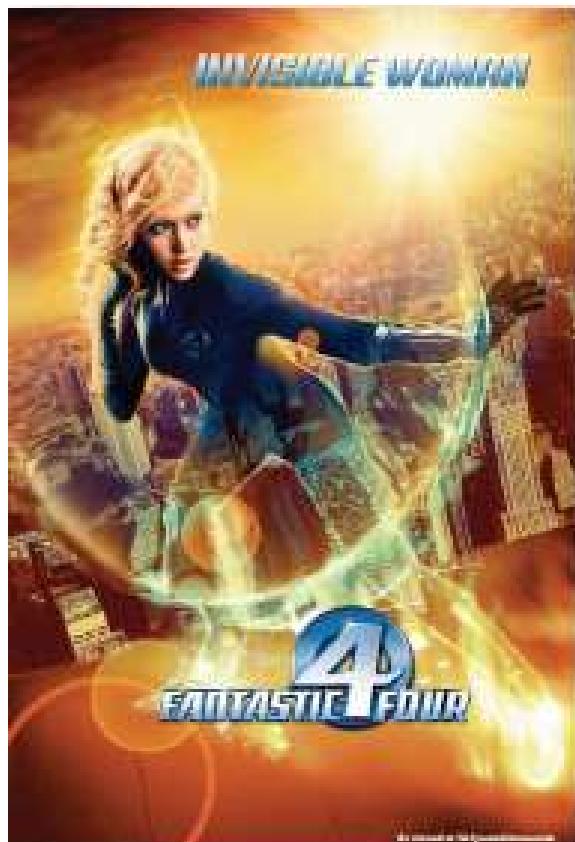


Very small radar cross section: shape and absorbing paint



Invisibility in Hollywood

Fantastic 4: The Invisible Woman Lee and Kirby (1961)



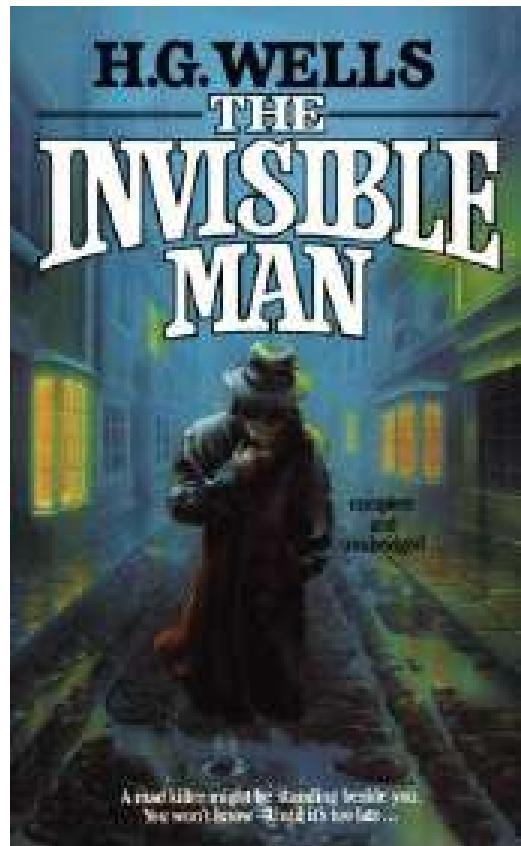
"... she achieves these feats by bending all wavelengths of light in the vicinity around herself ... without causing any visible distortion." -- Introduction from Wikipedia



Invisibility in Hollywood

The Invisible Man

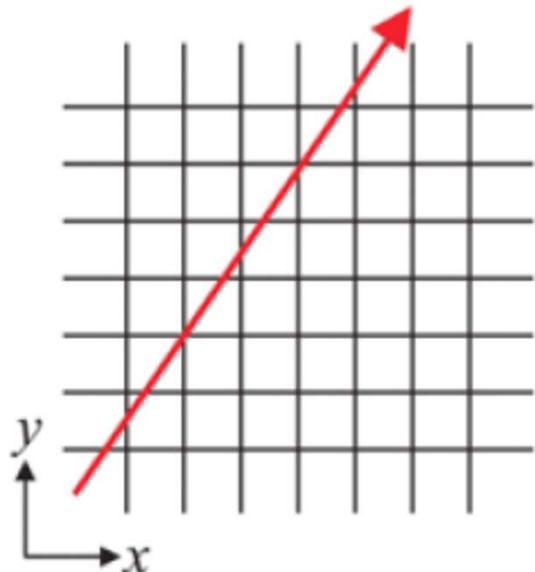
H.G. Wells (1897)



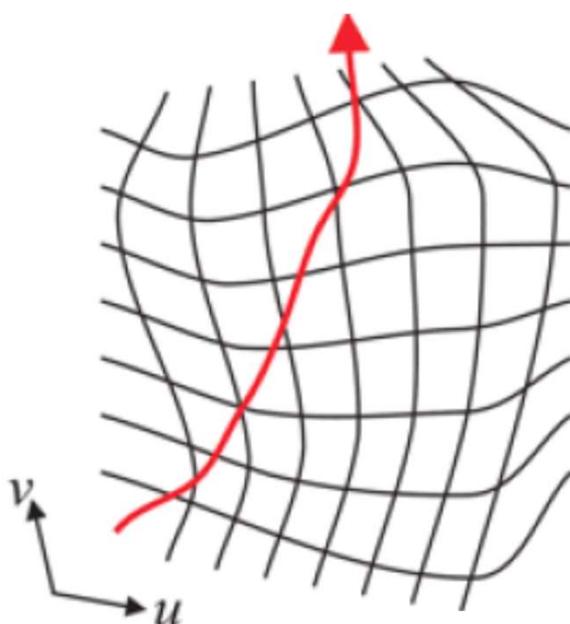
"... it was an idea ... to lower the refractive index of a substance, solid or liquid, to that of air — so far as all practical purposes are concerned." -- Chapter 19
"Certain First Principles"



Transformation of Maxwell's equations

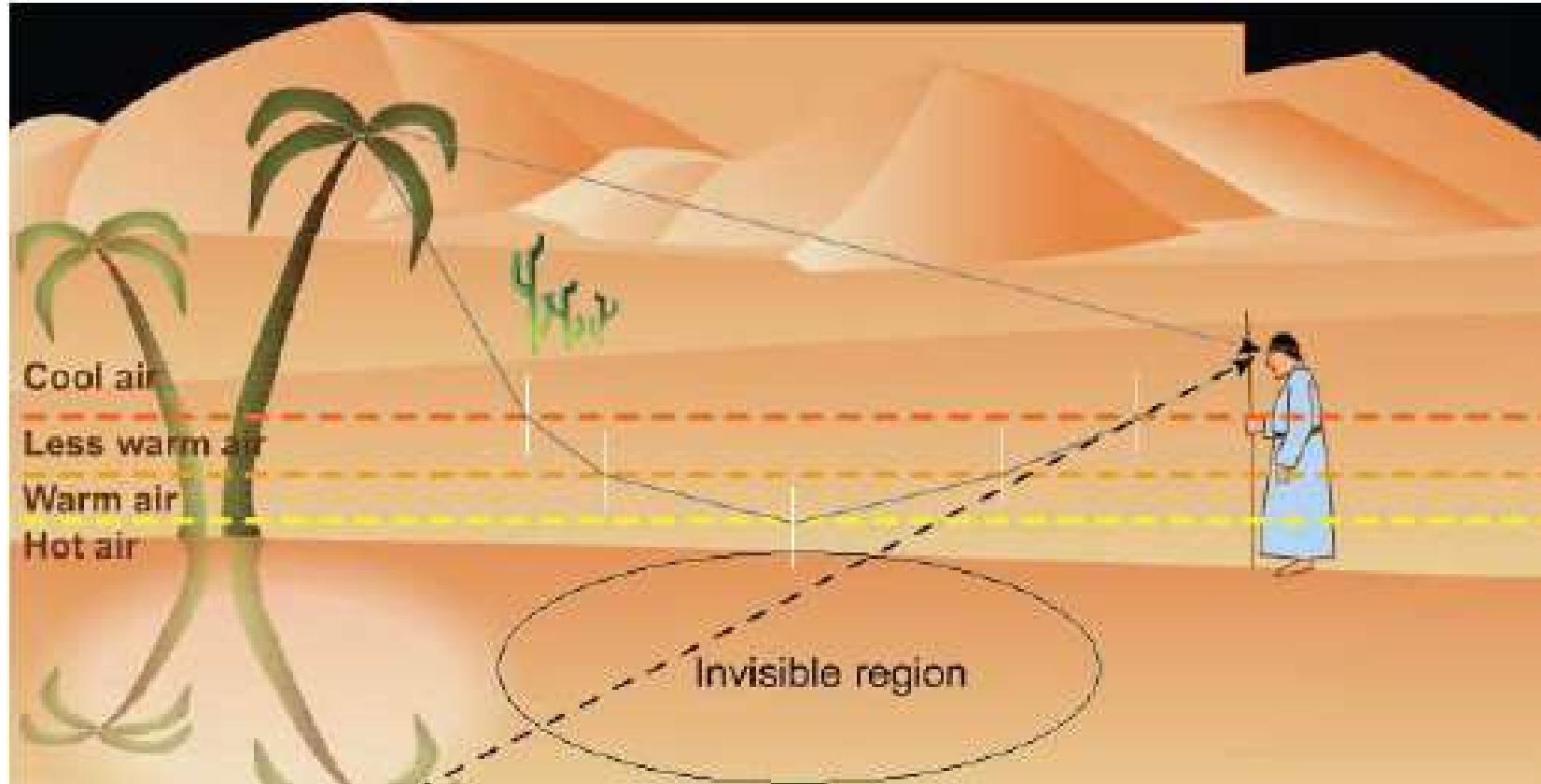


Straight field line in Cartesian coordinates



Cloaking can be realized by properly designing ϵ and μ tensors in the space to achieve the desired distortion and avoid a particular region in space.

Distorted field line in distorted coordinates



The bending of light due to the gradient in refractive index in a desert mirage



How do we design ε and μ tensors at will?



We need metamaterials!



Spherical Coordinates

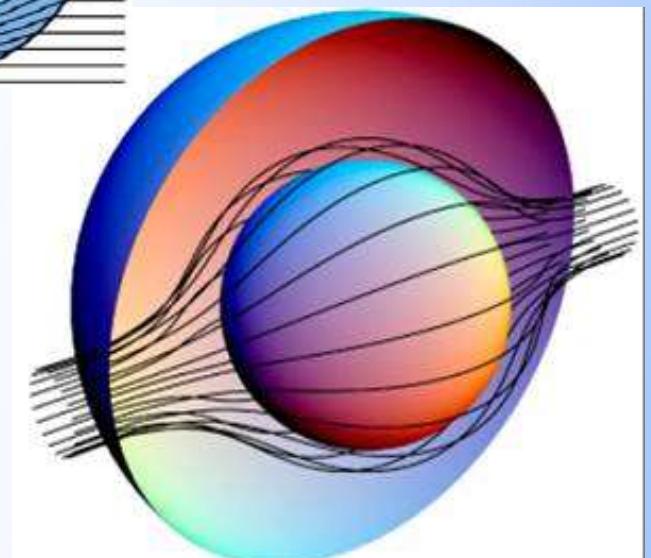
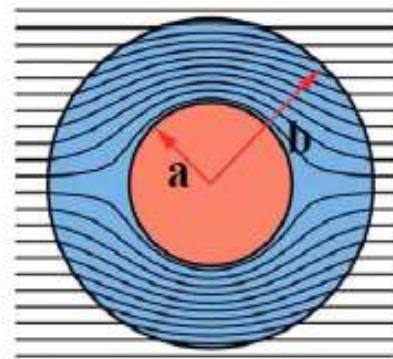
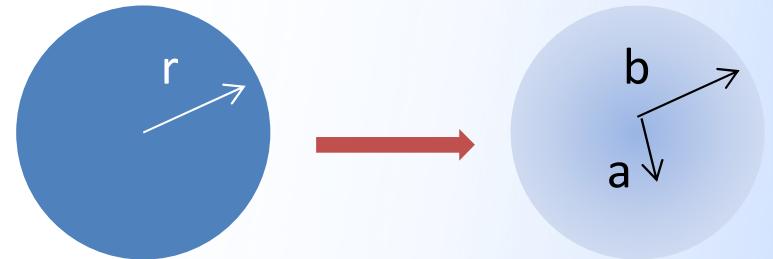
$$0 < r < b \longrightarrow a < r' < b$$



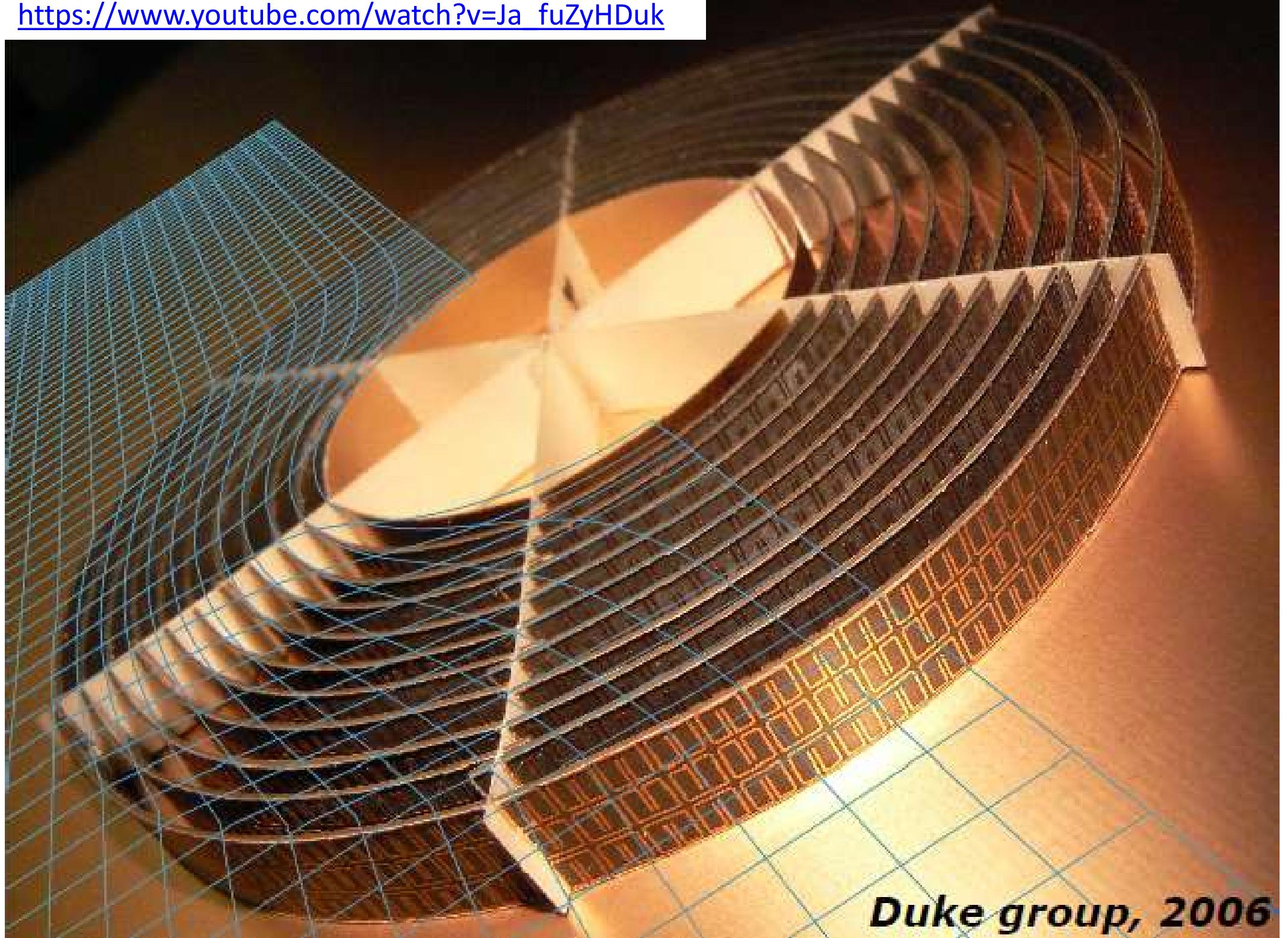
$$r' = \frac{b-a}{b}r + a \quad \theta' = \theta \quad \varphi' = \varphi$$



$$\left\{ \begin{array}{l} \varepsilon'_r = \mu'_r = \frac{b}{b-a} \frac{(r-a)^2}{r^2} \\ \varepsilon'_\theta = \mu'_\theta = \frac{b}{b-a} \\ \varepsilon'_\varphi = \mu'_\varphi = \frac{b}{b-a} \end{array} \right.$$



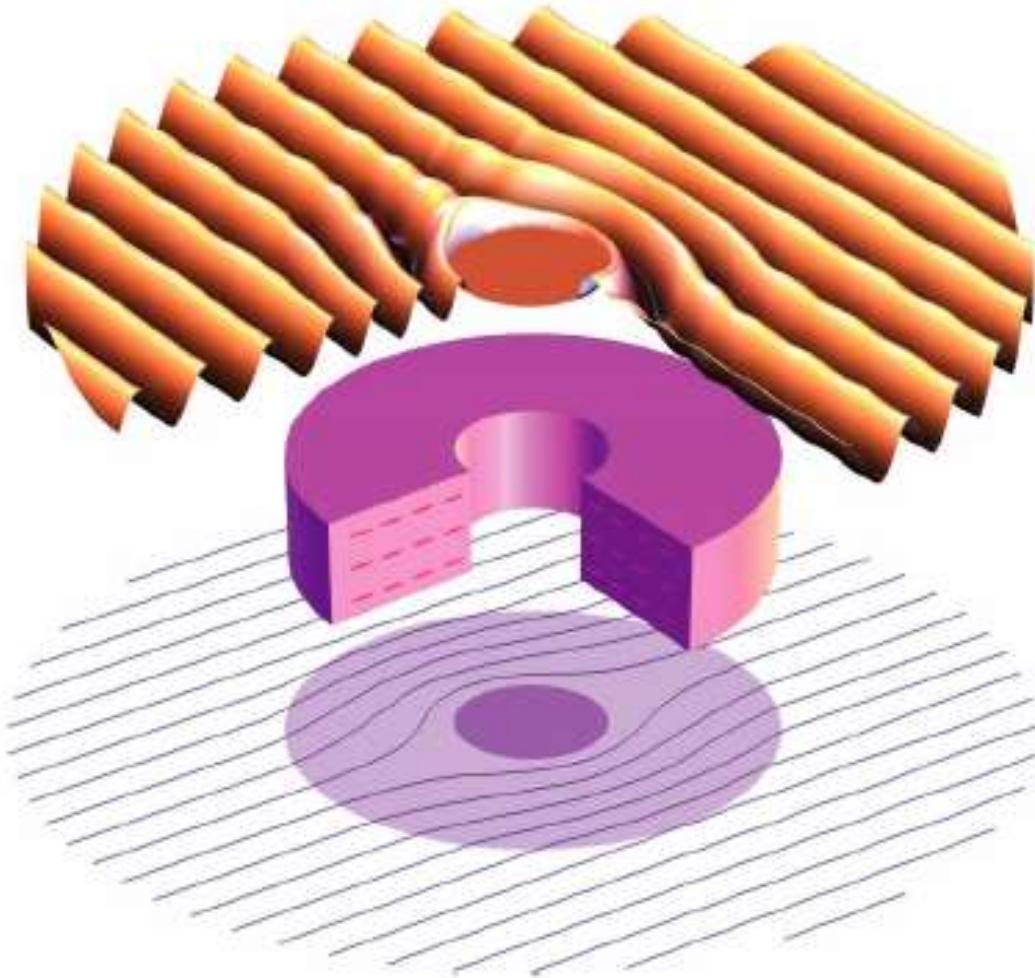
https://www.youtube.com/watch?v=Ja_fuZyHDuk



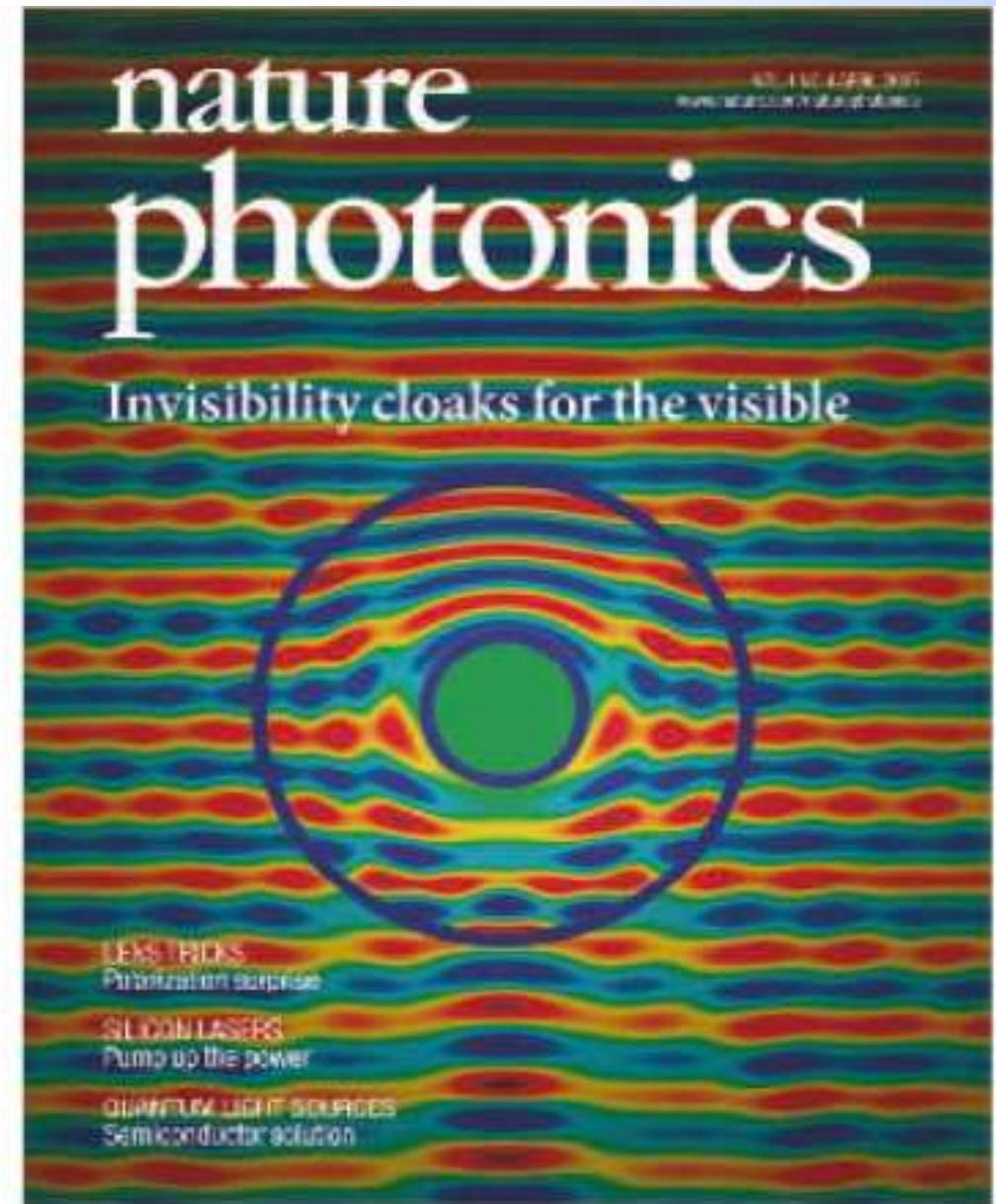
Duke group, 2006



UNIVERSITY
OF BRESCIA



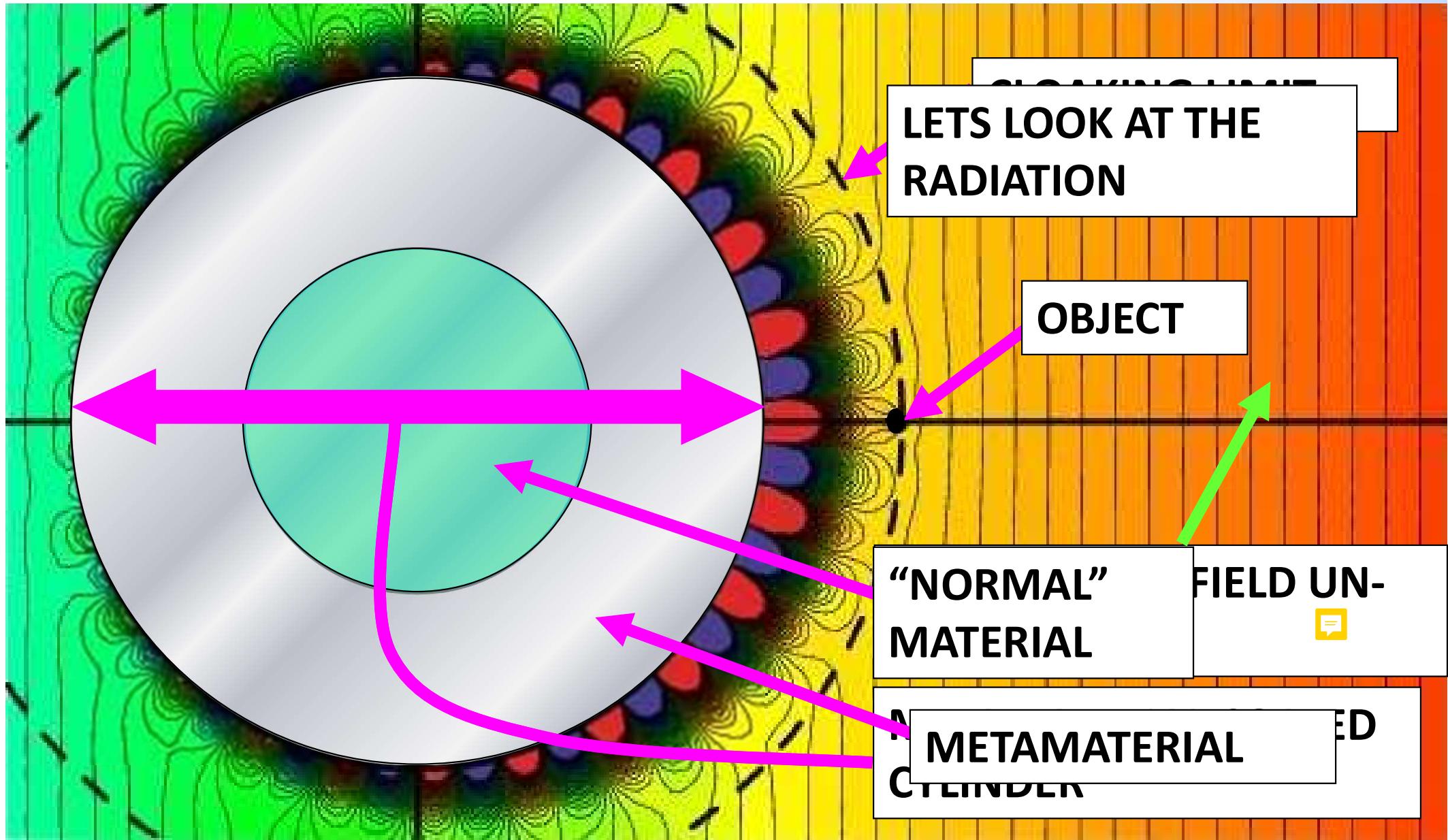
Invisibility Cloak



Cover article of Nature Photonics (April, 2007)



Does the object need
to be inside the cloak?





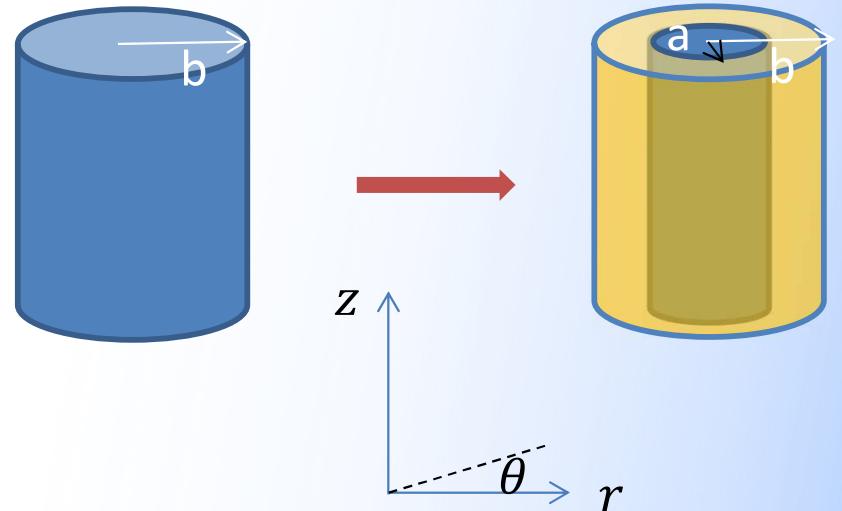
Invisibility Cloak

Cylindrical Coordinates

$$0 < r < b \longrightarrow a < r' < b$$



$$r' = \frac{b-a}{b}r + a \quad \theta' = \theta \quad z' = z$$



$$\left\{ \begin{array}{l} \epsilon'_r = \mu'_r = \frac{r-a}{r} \\ \epsilon'_\theta = \mu'_\theta = \frac{r}{r-a} \\ \epsilon'_\varphi = \mu'_\varphi = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r} \end{array} \right.$$

TE incidence



$$\left\{ \begin{array}{l} \epsilon'_z = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r} \\ \mu'_\theta = \frac{r}{r-a} \\ \mu'_r = \frac{r-a}{r} \end{array} \right. \text{ To maintain the dispersion relation only}$$

$$\left\{ \begin{array}{l} \mu'_\theta \epsilon'_z = \text{const} \\ \mu'_r \epsilon'_z = \text{const} \end{array} \right.$$

$$\left\{ \begin{array}{l} \epsilon'_z = \left(\frac{b}{b-a}\right)^2 \\ \mu'_\theta = 1 \\ \mu'_r = \left(\frac{r-a}{r}\right)^2 \end{array} \right.$$

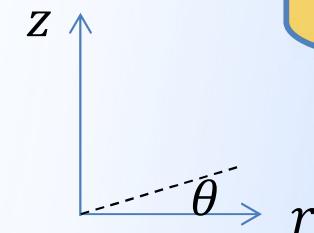
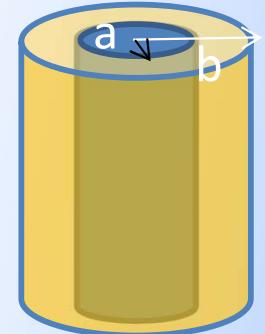
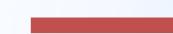
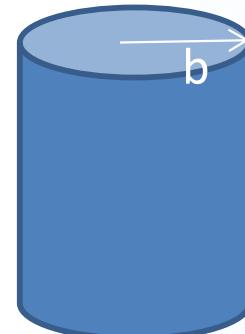


Cylindrical Coordinates

$$0 < r < b \longrightarrow a < r' < b$$



$$r' = \frac{b-a}{b}r + a \quad \theta' = \theta \quad z' = z$$



$$\left\{ \begin{array}{l} \epsilon'_r = \mu'_r = \frac{r-a}{r} \\ \epsilon'_\theta = \mu'_\theta = \frac{r}{r-a} \\ \epsilon'_\varphi = \mu'_\varphi = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r} \end{array} \right.$$

TM incidence



$$\left\{ \begin{array}{l} \mu'_z = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r} \\ \epsilon'_\theta = \frac{r}{r-a} \\ \epsilon'_r = \frac{r-a}{r} \end{array} \right.$$

To maintain the dispersion relation

$$\left\{ \begin{array}{l} \mu'_z \epsilon'_\theta = \text{const} \\ \mu'_z \epsilon'_r = \text{const} \end{array} \right.$$

$$\left\{ \begin{array}{l} \mu'_z = 1 \\ \epsilon'_\theta = \left(\frac{b}{b-a}\right)^2 \\ \epsilon'_r = \left(\frac{b}{b-a}\right)^2 \left(\frac{r-a}{r}\right)^2 \end{array} \right.$$



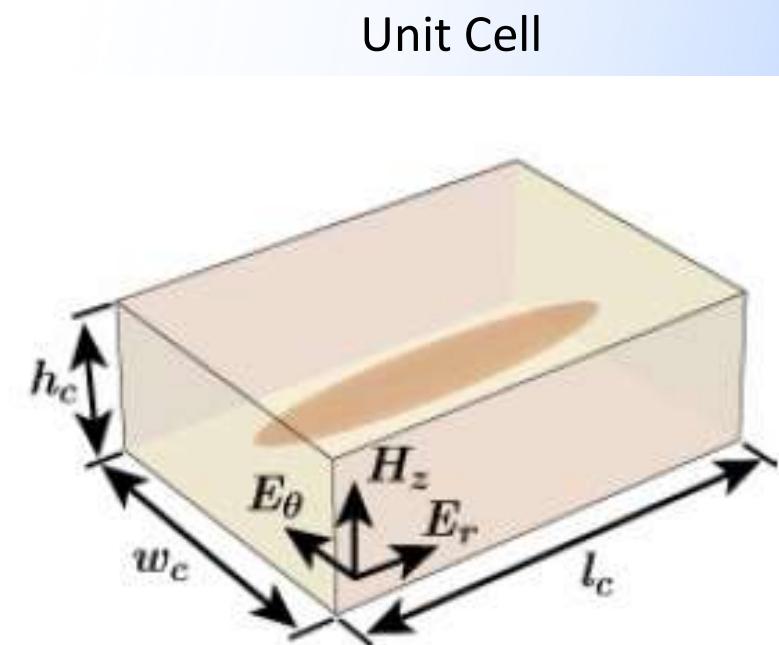
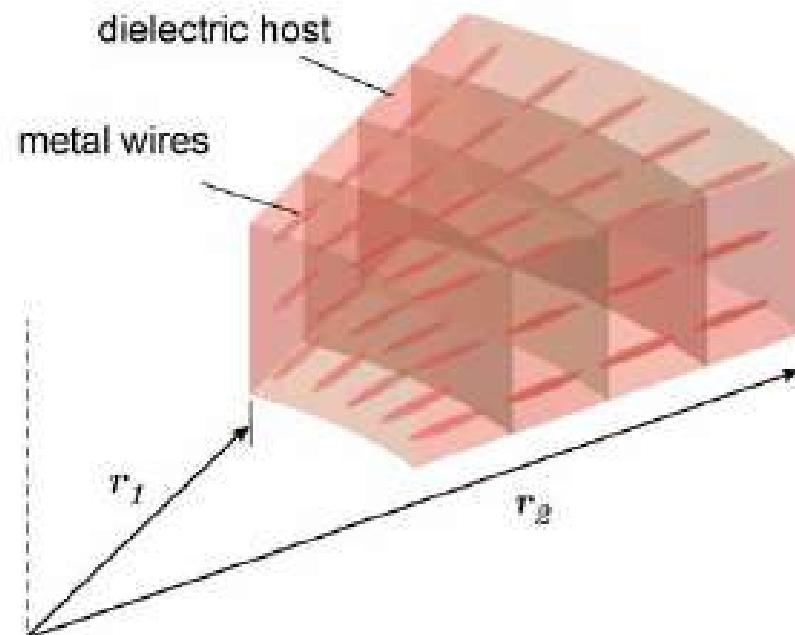
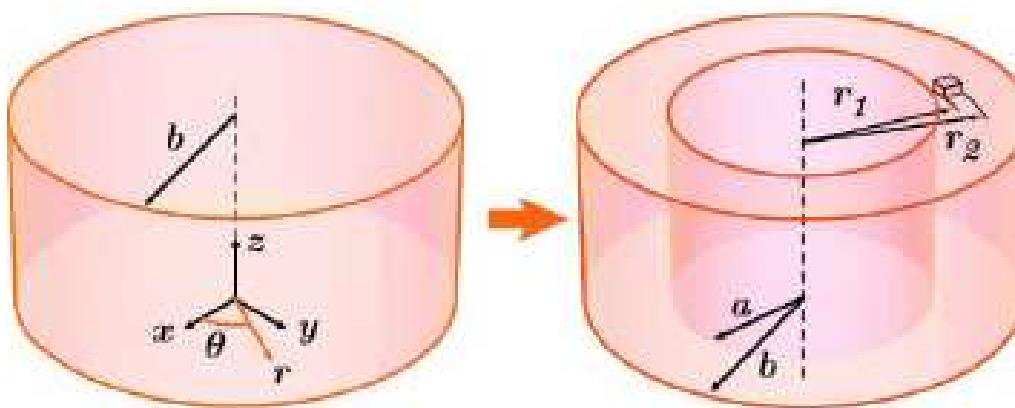
$$\left\{ \begin{array}{l} \mu'_z = 1 \longrightarrow \boxed{\text{NO MAGNETISM REQUIRED}} \\ \\ \varepsilon'_{\theta} = \left(\frac{b}{b-a} \right)^2 \longrightarrow \boxed{\text{PERMITTIVITY} > 1} \\ \\ \varepsilon'_r = \left(\frac{b}{b-a} \right)^2 \left(\frac{r-a}{r} \right)^2 \longrightarrow \boxed{\text{GRADIENT PERMITTIVITY IN } r \text{ DIRECTION.}} \\ \text{PERMITTIVITY VARIES FROM 0 TO 1} \end{array} \right.$$

- The magnetic cloak shows zero reflection
- Non-magnetic cloak → non-zero reflection
- Non-magnetic nature removes the challenging issue of the design
- The key to the implementation: radial distribution



Non magnetic Cloak: Round Brush

ϵ_r realized by metal wires of sub-wavelength size in radial direction



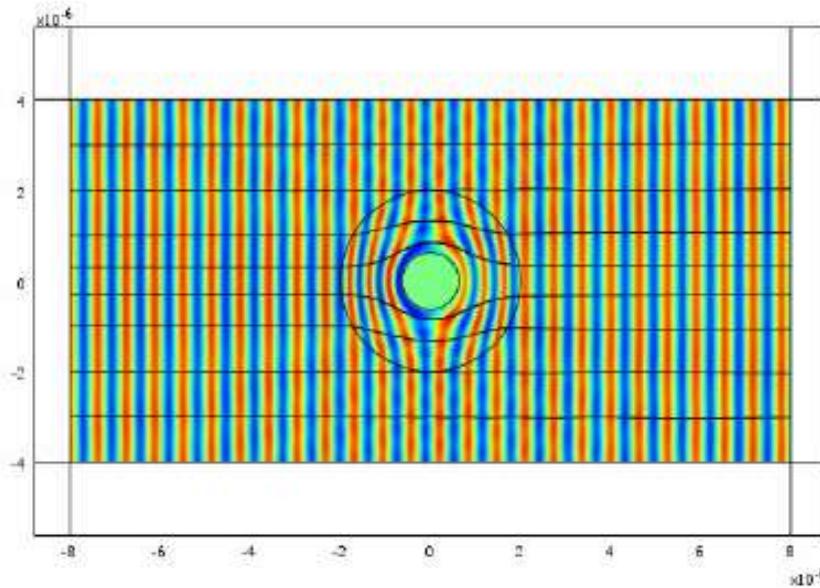
**Flexible control of ϵ_r ;
Negligible perturbation in ϵ_θ**

Cai, et al.,
Nature Photonics, 1, 224 (2007)

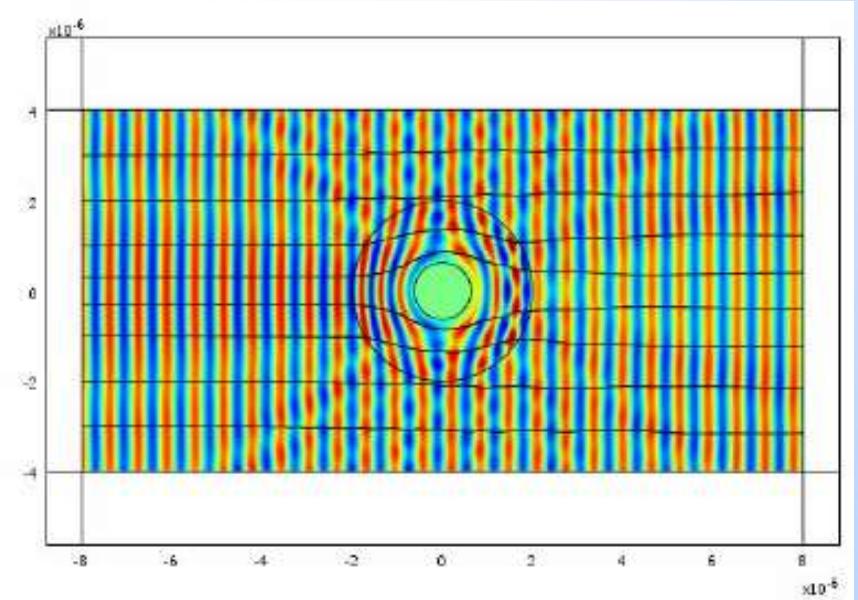


Magnetic vs Non Magnetic Cloak

Ideal Cloak



Non Magnetic Cloak



$$Z|_{r=b} = \sqrt{\frac{\mu_z}{\epsilon_\theta}}|_{r=b} = 1$$

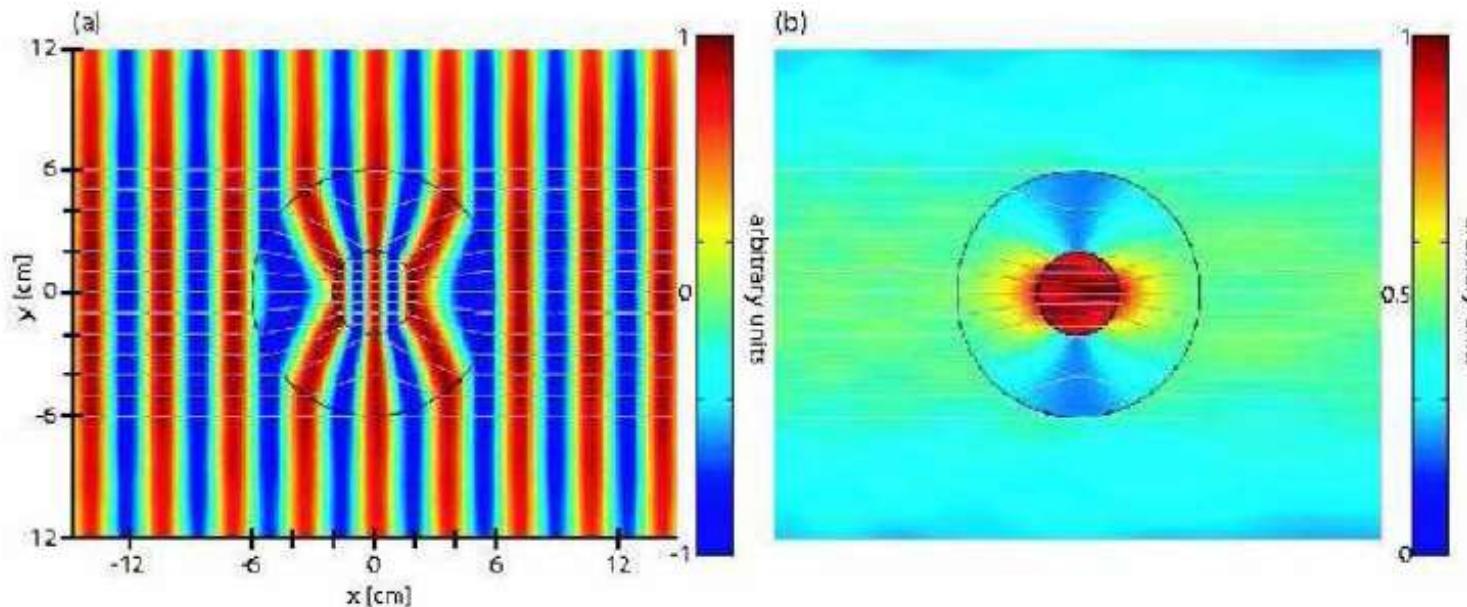
Perfectly matched impedance results in zero scattering

$$Z|_{r=b} = \sqrt{\frac{\mu_z}{\epsilon_\theta}}|_{r=b} = 1 - \frac{a}{b}$$

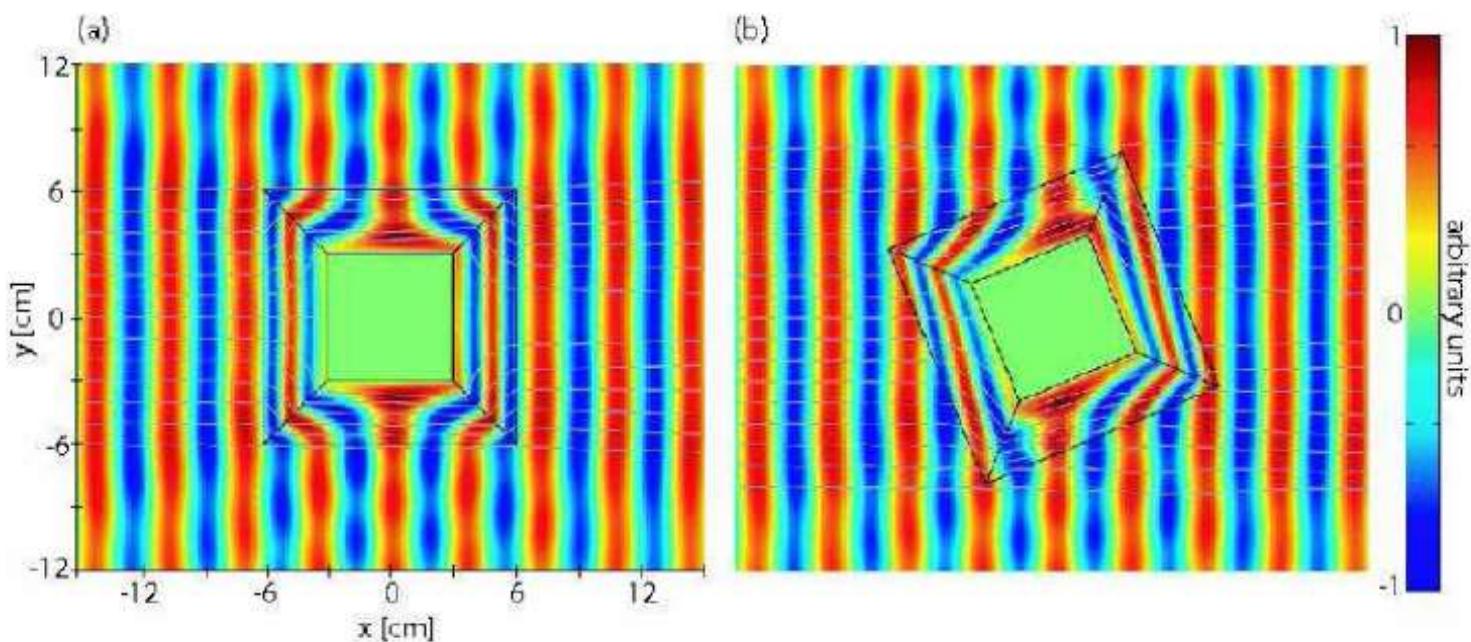
Detrimental scattering due to impedance mismatch



Engineered Optical Meta-Spaces



Optical
Concentrator



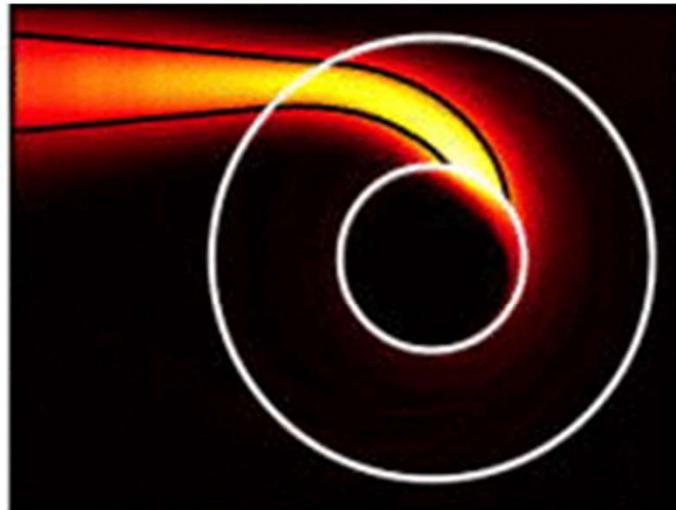
Square Cloak

*M. Rahm et al.,
Photronics and
Nanostructures -
Fundamentals and
Applications 6, 87-95
(2008)*



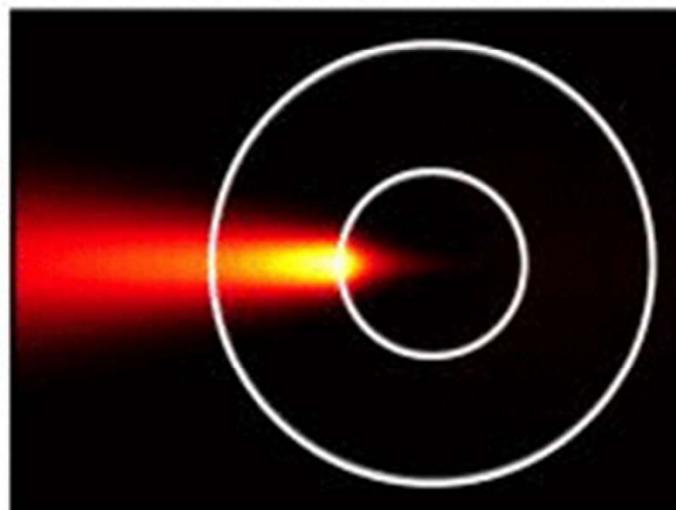
Engineered Optical Meta-Spaces

(a)



**Metamaterial
Black hole**

(b)

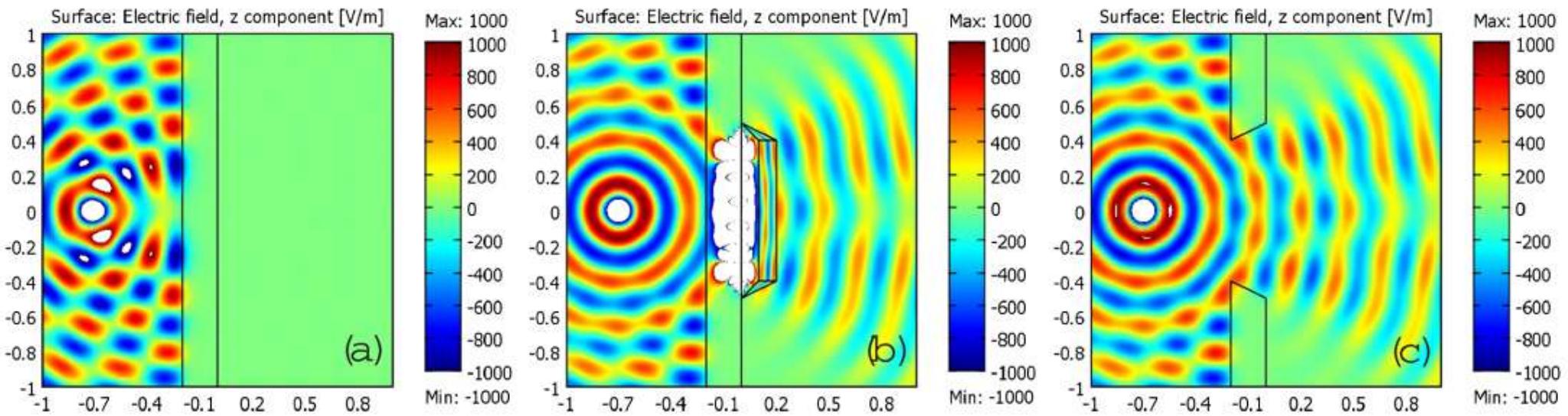
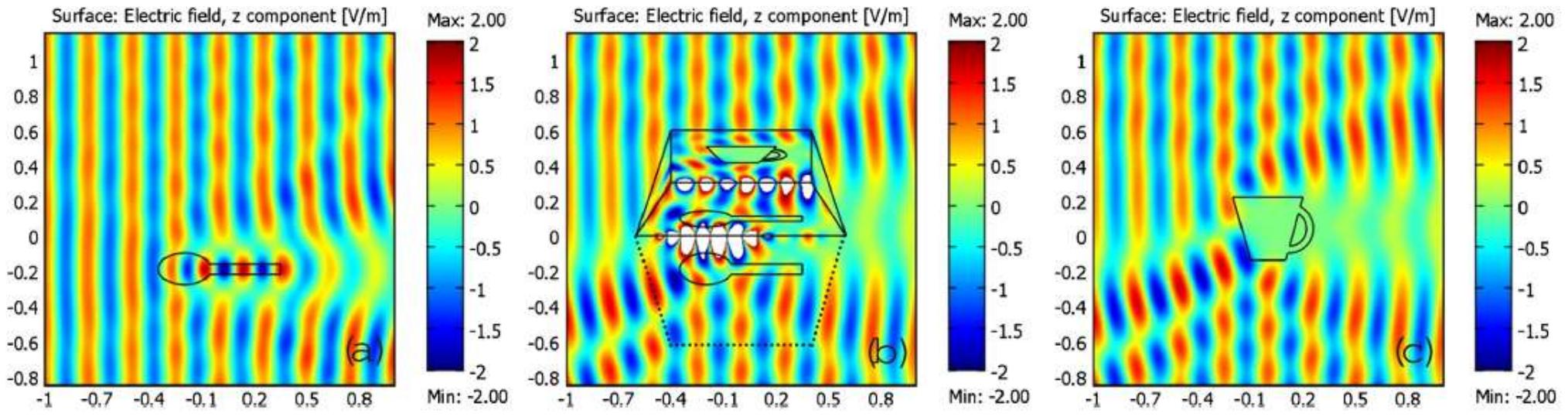


*E. Narimanov, A. Kildishev,
“Optical black hole:
Broadband omnidirectional light absorber,”
Applied Physics Letters. 95. 041106 (2009).*



Engineered Optical Meta-Spaces

Illusion Optics

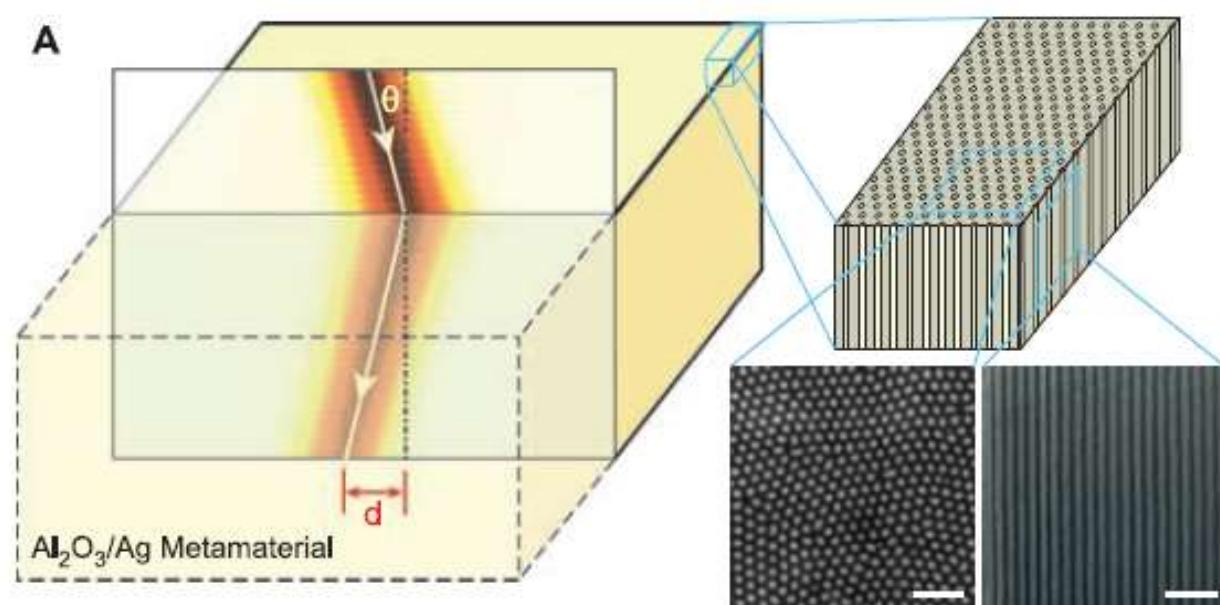




Hyperbolic Metamaterials

Hyperbolic materials, also called indefinite materials, are strongly anisotropic, artificial media. The term **indefinite** material refers to a medium whose permittivity and permeability along the principal axes are of opposite signs, resulting a strong anisotropy.

Anisotropic permittivity or permeability can enable negative refraction!



Yao et al, Science 321, (2008)



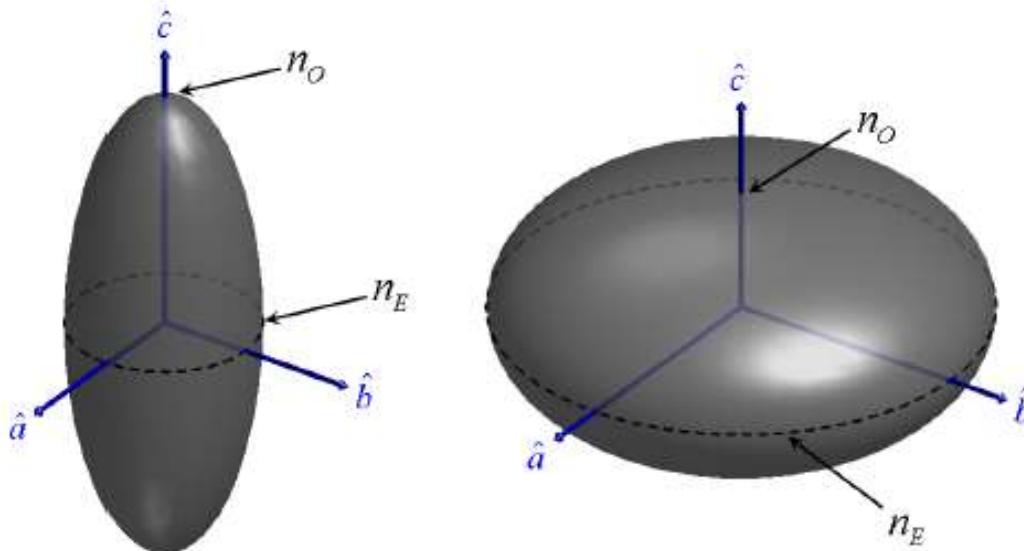
Dispersion surfaces of uniaxial crystals

Dielectric Tensor

$$\begin{bmatrix} \epsilon_0 & 0 & 0 \\ 0 & \epsilon_0 & 0 \\ 0 & 0 & \epsilon_e \end{bmatrix}$$

Dispersion Relation (ellipsoids)

$$\frac{k_x^2 + k_y^2}{\epsilon_e} + \frac{k_z^2}{\epsilon_o} = \left(\frac{\omega}{c_0} \right)^2$$



These ellipsoids are closed surfaces.
Possible values of k are limited.



Dispersion surfaces of hyperbolic metamaterials

If the tensor elements ε_o and ε_e have opposite sign then:

$$\varepsilon_o \varepsilon_e < 0$$

The dispersion relation now describes open surfaces - hyperbolas - that extend to infinity.

$$\frac{k_x^2 + k_y^2}{\varepsilon_e} + \frac{k_z^2}{\varepsilon_o} = \left(\frac{\omega}{c_0} \right)^2$$

Values of k in hyperbolic materials can go to infinity (in absence of losses!). This means there can exist propagating waves in hyperbolic materials with extremely large values of k . For a given frequency, these will have extremely small wavelength. These waves would be cutoff, or evanescent, in ordinary materials.

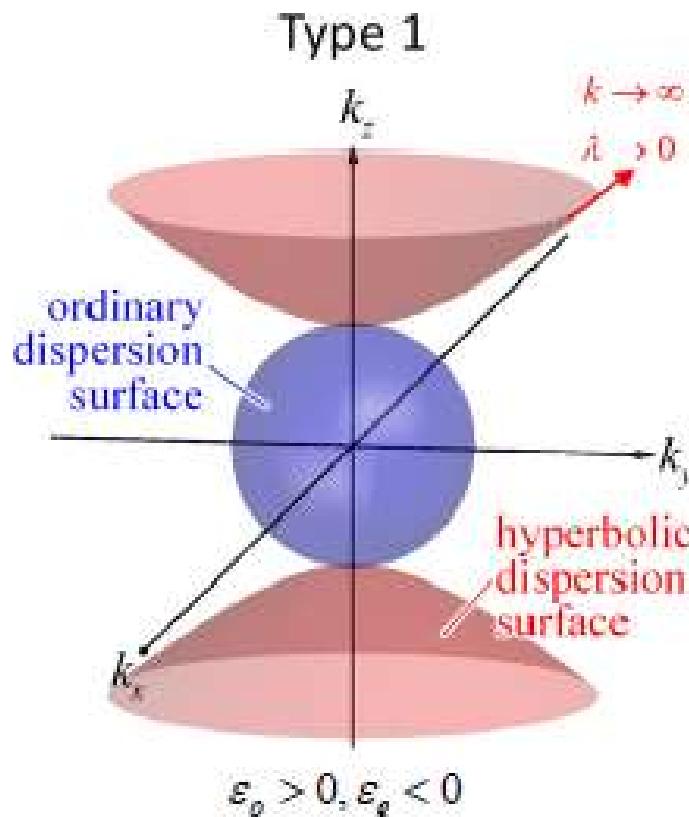
$$|k| = \frac{2\pi}{\lambda}, \quad k \rightarrow 0 \Rightarrow \lambda \rightarrow 0$$



Hyperbolic metamaterials

Type 1

$$\begin{bmatrix} -\varepsilon_a & 0 & 0 \\ 0 & \varepsilon_b & 0 \\ 0 & 0 & \varepsilon_c \end{bmatrix} \text{ or } \begin{bmatrix} \varepsilon_a & 0 & 0 \\ 0 & -\varepsilon_b & 0 \\ 0 & 0 & \varepsilon_c \end{bmatrix} \text{ or } \begin{bmatrix} \varepsilon_a & 0 & 0 \\ 0 & \varepsilon_b & 0 \\ 0 & 0 & -\varepsilon_c \end{bmatrix}$$



Low loss, good impedance match to air because they are predominantly dielectric like.

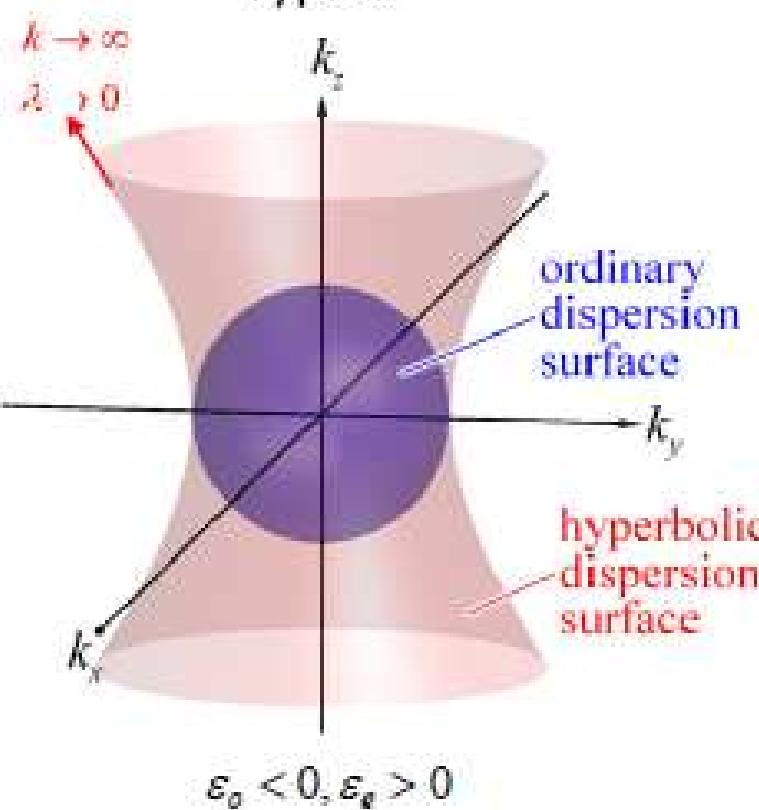


Hyperbolic metamaterials

Type 2

$$\begin{bmatrix} -\varepsilon_a & 0 & 0 \\ 0 & -\varepsilon_b & 0 \\ 0 & 0 & \varepsilon_c \end{bmatrix} \text{ or } \begin{bmatrix} -\varepsilon_a & 0 & 0 \\ 0 & \varepsilon_b & 0 \\ 0 & 0 & -\varepsilon_c \end{bmatrix} \text{ or } \begin{bmatrix} \varepsilon_a & 0 & 0 \\ 0 & -\varepsilon_b & 0 \\ 0 & 0 & -\varepsilon_c \end{bmatrix}$$

Type 2



High loss, high impedance mismatch because they are predominantly metal like.



How to make Hyperbolic metamaterials

Example: We know that in metals the dielectric constant is negative below the plasma frequency. In hyperbolic metamaterials, not all of the tensor elements are negative. This means we have to break the continuity of the metal in the direction of positive tensor elements. So, any structure where metal is continuous on some directions but not others will be hyperbolic.



Layers of metal-dielectric



Hyper lens



Multilayer fishnet



Array of nanorods



Array of metal-dielectric nanopyramids



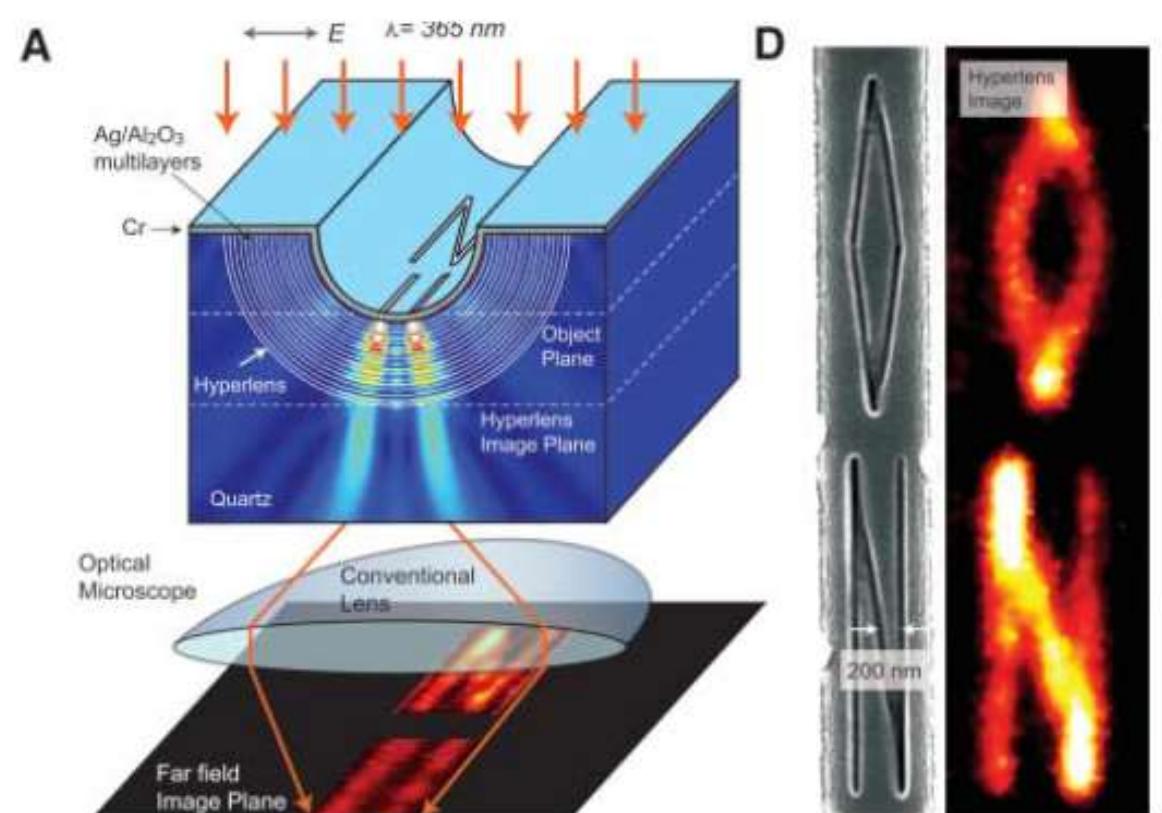
Graphene metamaterial

A. Poddubny, I. Iorsh, P. Belov, Y. Kivshar, *Nature Photonics*, vol. 7, pp. 958-967, 2013.



Hyperbolic metamaterials applications

- ✓ Negative Refraction (from dispersion, not Negative Refractive Index)
- ✓ Lensing
- ✓ Spontaneous emission engineering (Purcell-factor enhancement)
- ✓ Minaturization
- ✓ Tunable active devices
- ✓ Heat transfer engineering



Liu et al, *Science*. 315 1686 (2007)



Why going 2D?

- ✓ Planar technology is central to Integrated Circuit technology ;
- ✓ 2D metasurfaces have better chance to have major technological impact because they are easier to fabricate than their 3D counterparts;

What can we do with Metasurfaces?

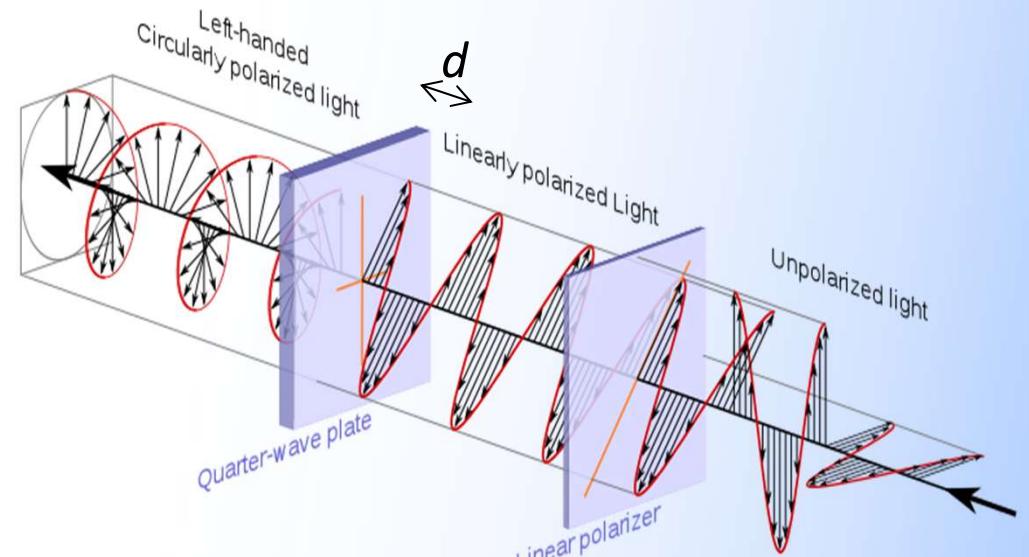
- ✓ Metasurfaces allow for local phase and amplitude control of light along the surface using optical antennas;
- ✓ New class of flat, compact and broadband components:(lenses, polarizers, etc.), beyond conventional diffractive optics ;
- ✓ Optical phased arrays for high speed wavefront control.



Conventional optical components rely on propagation effects



Camera lens



Quarter waveplate

Can we introduce an abrupt phase jump in the path?



How do we introduce an abrupt phase jump in the propagation?

We can use subwavelength optical resonators properly organized on a metasurface with suitably designed geometry in order to scatter light with the desired phase distribution.

What are the requirements for the metasurface elements?

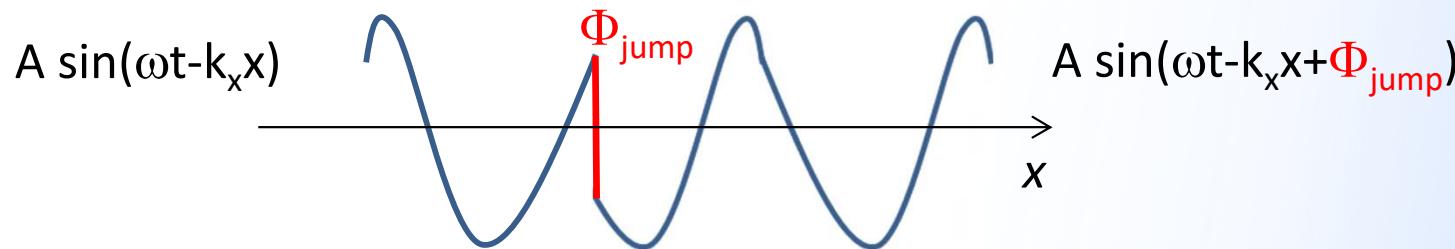
- Deep sub-wavelength thickness
- Sub-wavelength separation between the elements
- 2π phase coverage
- High and uniform scattering amplitude

What optical components can be fabricated with a metasurface?

- Lenses and Axiicons
- Polarizers
- Vortex plates
- Optical phased arrays

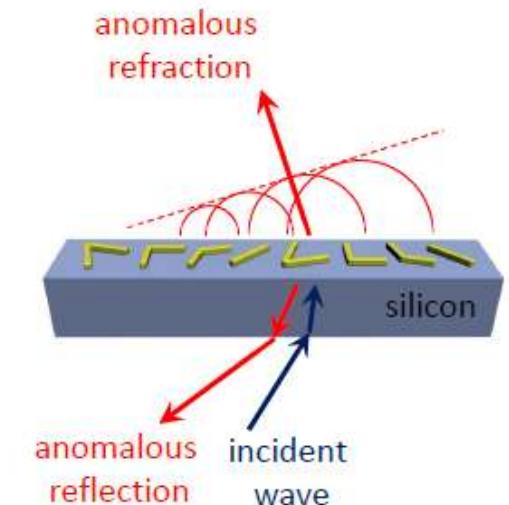
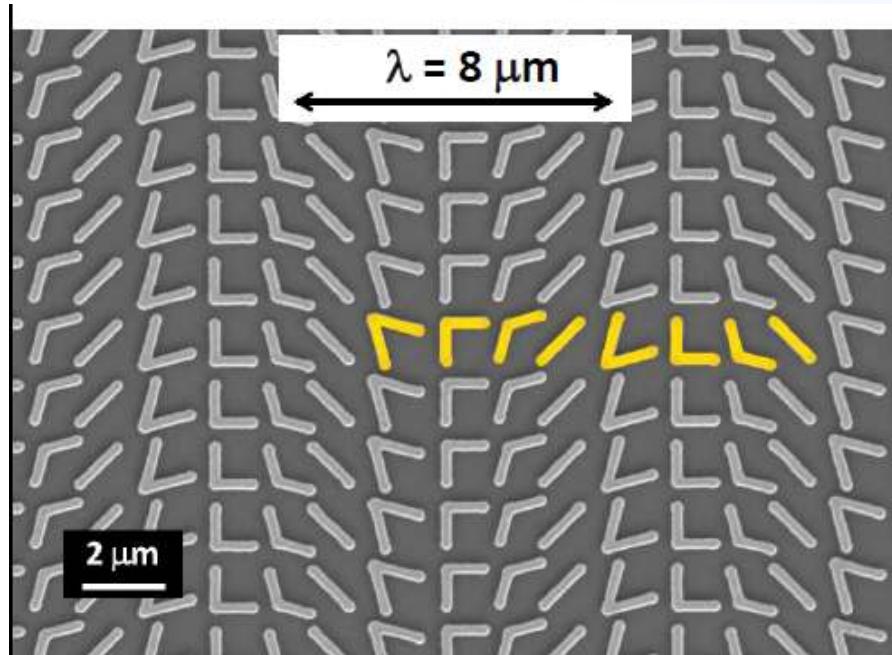


Light propagation and phase discontinuities



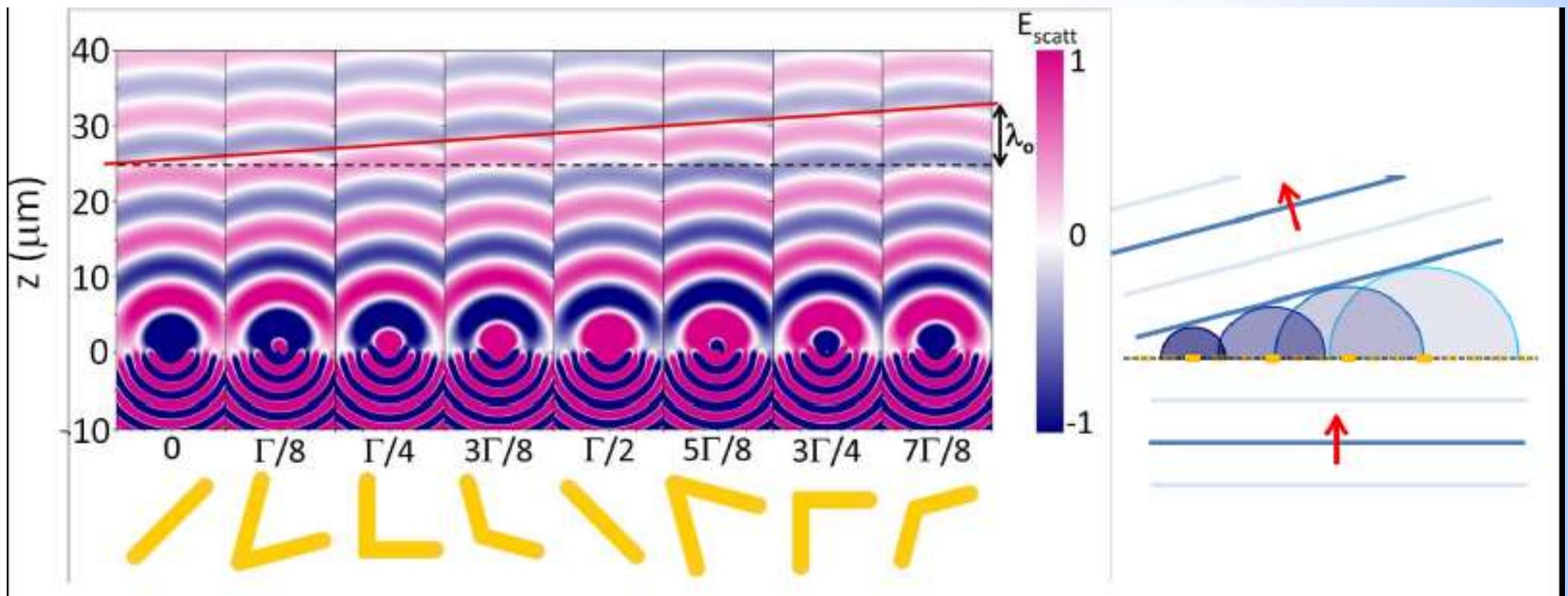
If we create a spatial distribution of different phase discontinuities along the entire interface we can make any desired wave front !

How?
Optically thin array of
sub-wavelength spaced
antennas





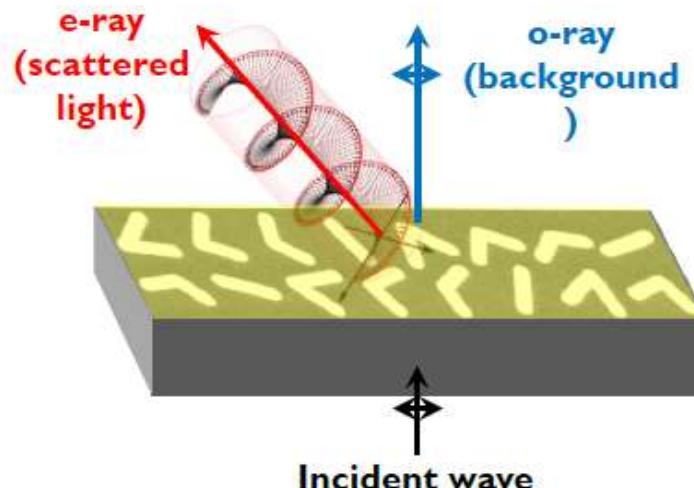
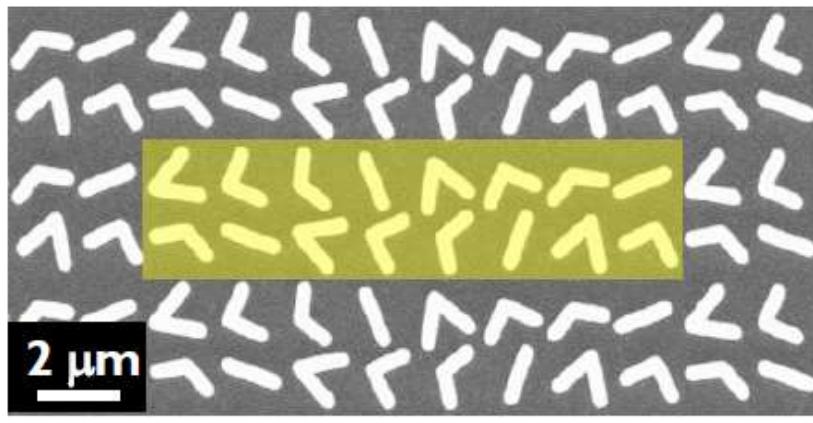
The antenna operate as secondary scatterers with a tailorable phase response, re-directing a normally-incident beam away from the normal



N. Yu *et al.*,
Science 334, 333 (2011)

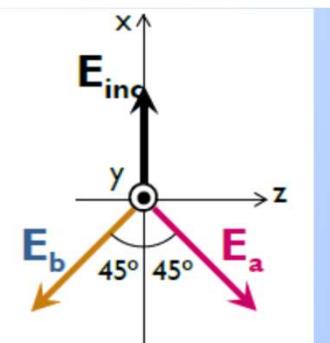
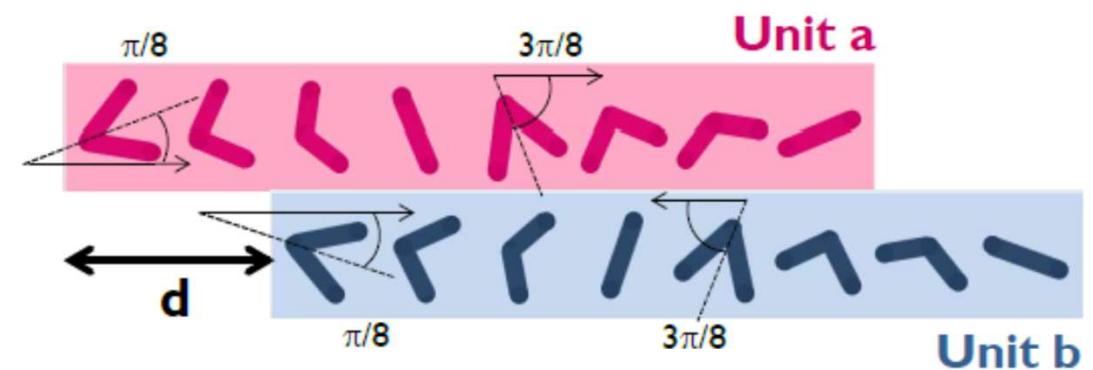


Quarter wave plate



Design based on
phased optical antennas

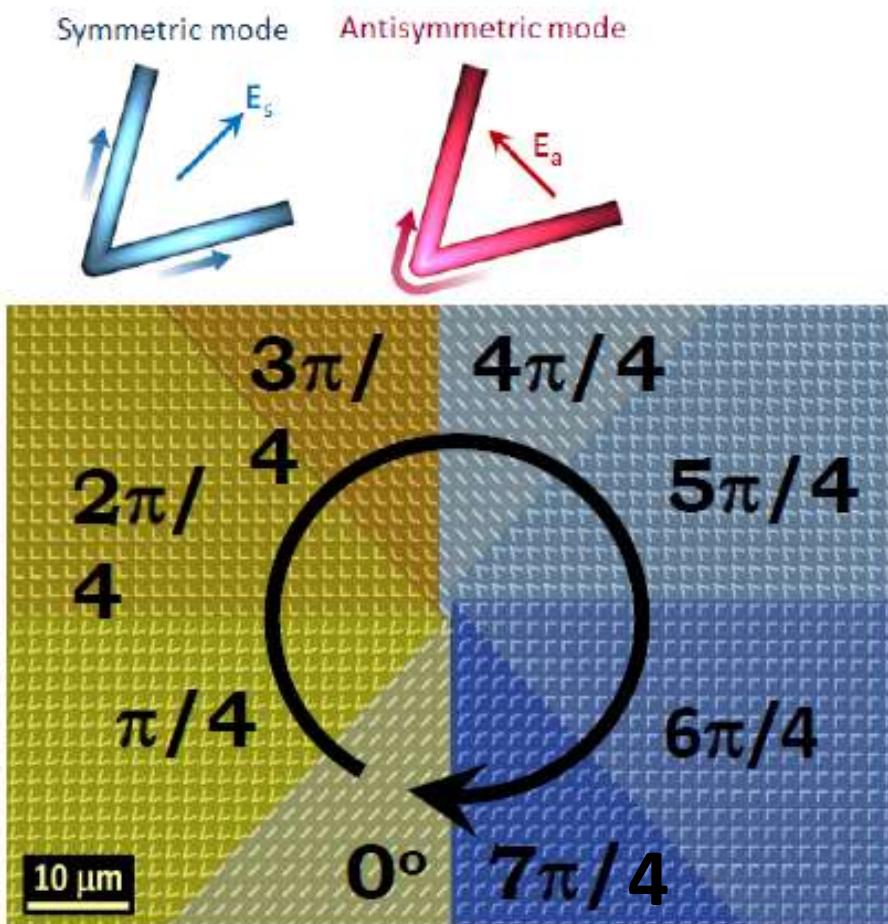
Wavefront reconfiguration



The two rows of antennas in the unit cell
create the two orthogonal polarized
components with 90 degree phase difference



Vortex plates



NOTE:

- ✓ Angular arrangements of antennas create optical vortices
- ✓ Radial arrangements create flat lenses

