About Sellmeier relation and the calculation of the material dispersion :

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See : https://en.wikipedia.org/wiki/Sellmeier_equation

See also : https://www.rp-photonics.com/sellmeier_formula.html

Sellmeier equation

From Wikipedia, the free encyclopedia

The **Sellmeier equation** is an empirical relationship between refractive index and wavelength for a particular transparent medium. The equation is used to determine the dispersion of light in the medium.

It was first proposed in 1872 by Wilhelm Sellmeier and was a development of the work of Augustin Cauchy on Cauchy's equation for modelling dispersion.[1]

Contents [hide]

The equation [edit]

In its original and the most general form, the Sellmeier equation is given as

$$n^2(\lambda) = 1 + \sum_i \frac{B_i \lambda^2}{\lambda^2 - C_i},$$

where n is the refractive index, λ is the wavelength, and B_i and C_i are experimentally determined *Sellmeier coefficients*. These coefficients are usually quoted for λ in micrometres. Note that this λ is the vacuum wavelength, not that in the material itself, which is λ/n. A different form of the equation is sometimes used for certain types of materials, e.g. crystals.

Each term of the sum representing an absorption resonance of strength B_i at a wavelength $\sqrt{C_i}$. For example, the coefficients for BK7 below correspond to two absorption resonances in the ultraviolet, and one in the mid-infrared region. Close to each absorption peak, the equation gives non-physical values of $n^2 = \pm\infty$, and in these wavelength regions a more precise model of dispersion such as Helmholtz's must be used.

If all terms are specified for a material, at long wavelengths far from the absorption peaks the value of n tends to

$$n \approx \sqrt{1+\sum_i B_i} \approx \sqrt{\varepsilon_r},$$

where ε_r is the relative dielectric constant of the medium.

For characterization of glasses the equation consisting of three terms is commonly used:[2][3]

$$n^2(\lambda) = 1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3},$$

As an example, the coefficients for a common borosilicate crown glass known as *BK7* are shown below:

Coefficient	Value
B_1	1.03961212
B_2	0.231792344
B_3	1.01046945
C_1	6.00069867×10^{-3} μm^2
C_2	2.00179144×10^{-2} μm^2
C_3	1.03560653×10^{2} μm^2

The Sellmeier coefficients for many common optical materials can be found in the online database of RefractiveIndex.info.

For common optical glasses, the refractive index calculated with the three-term Sellmeier equation deviates from the actual refractive index by less than 5×10^{-6} over the wavelengths' range[4] of 365 nm to 2.3 μm, which is of the order of the homogeneity of a glass sample.[5] Additional terms are sometimes added to make the calculation even more precise.

Sometimes the Sellmeier equation is used in two-term form:[6]

$$n^2(\lambda) = A + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2}.$$

Here the coefficient A is an approximation of the short-wavelength (e.g., ultraviolet) absorption contributions to the refractive index at longer wavelengths. Other variants of the Sellmeier equation exist that can account for a material's refractive index change due to temperature, pressure, and other parameters.

Coefficients [edit]

Table of coefficients of Sellmeier equation[7]

Material	B_1	B_2	B_3	C_1, μm^2	C_2, μm^2	C_3, μm^2
borosilicate crown glass (known as *BK7*)	1.03961212	0.231792344	1.01046945	6.00069867×10^{-3}	2.00179144×10^{-2}	103.560653
sapphire (for ordinary wave)	1.43134930	0.65054713	5.3414021	5.2799261×10^{-3}	1.42382647×10^{-2}	325.017834
sapphire (for extraordinary wave)	1.5039759	0.55069141	6.5927379	5.48041129×10^{-3}	1.47994281×10^{-2}	402.89514
fused silica	0.696166300	0.407942600	0.897479400	4.67914826×10^{-3}	1.35120631×10^{-2}	97.9340025
Magnesium fluoride	0.48755108	0.39875031	2.3120353	0.001882178	0.008951888	566.13559

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For pure silica (SiO_2), which is the material used for manufacturing most of the optical fibers, the Sellmeier relation is

$$n^2 - 1 = \frac{0.6961663\lambda^2}{\lambda^2 - 0.0684043^2} + \frac{0.4079426\lambda^2}{\lambda^2 - 0.1162414^2} + \frac{0.8974794\lambda^2}{\lambda^2 - 9.896161^2}$$

Let us note that :

1- the above relation is also called "dispersion formula".
2- The phase velocity is $v_\varphi = \frac{c}{n}$. In guided optics, $v_\varphi = \frac{\omega}{\beta}$
3- The group index is $n_g = n - \lambda \frac{dn}{d\lambda}$ and the group velocity is $v_g = \frac{c}{n_g}$. In guided optics, $v_g = \frac{d\omega}{d\beta}$
4- the derivative dn/dλ is often called "chromatic dispersion".
5- Furthermore, as shown in the provided written course of chapter 4, the quantity $\frac{d^2\beta}{d\omega^2} = \frac{d(\frac{1}{v_g})}{d\omega}$ is called "group velocity dispersion = GVD", while it is rather a quantity corresponding to a group time delay dispersion (the group time delay t_g over a length of propagation L being related to v_g by $v_g = \frac{L}{t_g} => \ t_g = \frac{L}{v_g}$).

   From the above relations, one can easily show that $\frac{d^2\beta}{d\omega^2} = \frac{1}{c}\frac{dn_g}{d\omega} = \frac{\lambda^3}{2\pi c^2}\frac{d^2n}{d\lambda^2}$. This quantity can be expressed in s^2/m or in fs^2/mm
6- Finally, the chromatic material dispersion is $D = D_{mat} = \frac{1}{L}\frac{dt_g}{d\lambda} = -\frac{\lambda}{c}\frac{d^2n}{d\lambda^2}$. It is expressed in s/(m.m) or ps/(nm.km).
7- From 5 and 6, we easily see that the relation between the GVD $\frac{d^2\beta}{d\omega^2}$ and the chromatic dispersion D is : $\frac{d^2\beta}{d\omega^2} = -\frac{\lambda^2}{2\pi c}.D$

On the following internet site, one can easily find the curve n = f(λ) for different materials, and in particular for pure silica ：

https://refractiveindex.info/?shelf=main&book=SiO2&page=Malitson

On this page, one can obtain the refractive index at a given wavelength, the derivative dn/dλ, the group index, the GVD and the chromatic dispersion, at this wavelength.