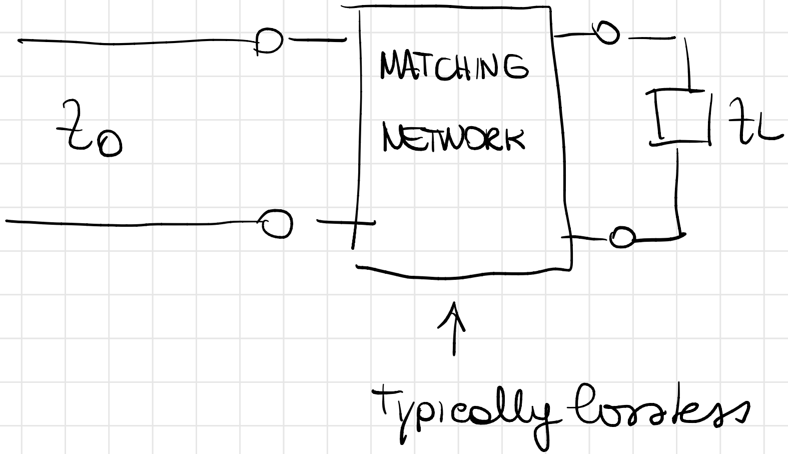
MICROWAVE ENGINEERING

Lecture 16:
Impedance
Matching and
Tuning



- Max power is delivered to the load (assuming the generator is matched)
- Power loss is minimized

- Improves signal-to-noise ratio
- Reduces amplitude and phase errors in power distribution networks.

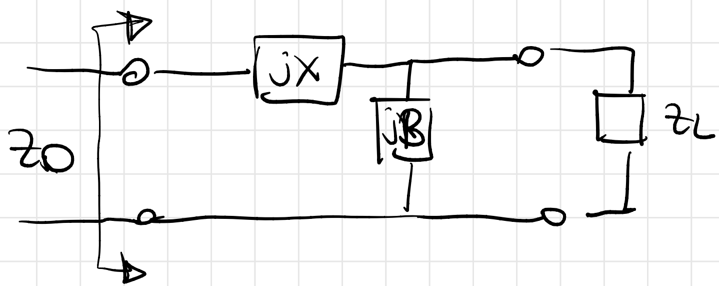
Matching network should be chosen to:

- MINIMIZE COMPLEXITY
- MATCHING REQUIRED BANDWIDTH
- IMPLEMENTATION REQUIREMENTS
- ADJUSTABILITY TO VARIABLE LOADS

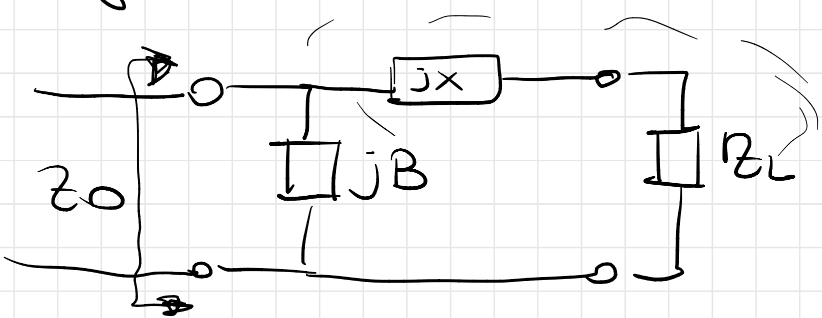
- MATCHING WITH LUMPED ELEMENTS

The simplest approach consists of an L-section

(1) if $z_\ell = \frac{Z_L}{Z_0}$ falls inside the $1+jX$ circle on the Smith's chart $\left(\frac{R_L}{Z_0} > 1\right)$ then the network is:



(2) if $z_e = \frac{Z_L}{Z_0}$ falls outside the $1+jX$ circle on Smith's chart $\left(\frac{R_L}{Z_0} < 1\right)$ then the matching network is:



LIMIT: jB and jX are actual inductors and capacitors.

Let's find the values of B and X:

If our load is $Z_L = R_L + jX_L$ $\qquad z_L = \frac{Z_L}{Z_0}$

and if $\boxed{R_L > Z_0}$

$$Z_0 = jX + \frac{1}{jB + \frac{1}{R_L + jX_L}}$$

after rearranging

$$B = \frac{X_L \pm \sqrt{R_L/Z_0}\,\sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2}$$

always > 0

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{B R_L}$$

Two possible solutions can be found for X and B:

$X > 0 \rightarrow$ inductor $\quad B > 0 \rightarrow$ capacitor

$X < 0 \rightarrow$ capacitor $\quad B < 0 \rightarrow$ inductor

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If $\boxed{R_L < Z_0}$

$$\frac{1}{Z_0} = jB + \frac{1}{R_L + j(X + X_L)}$$

Solving for B and X

$$X = \pm\sqrt{R_L(Z_0 - R_L)} - X_L$$

$Z_0 > R_L$

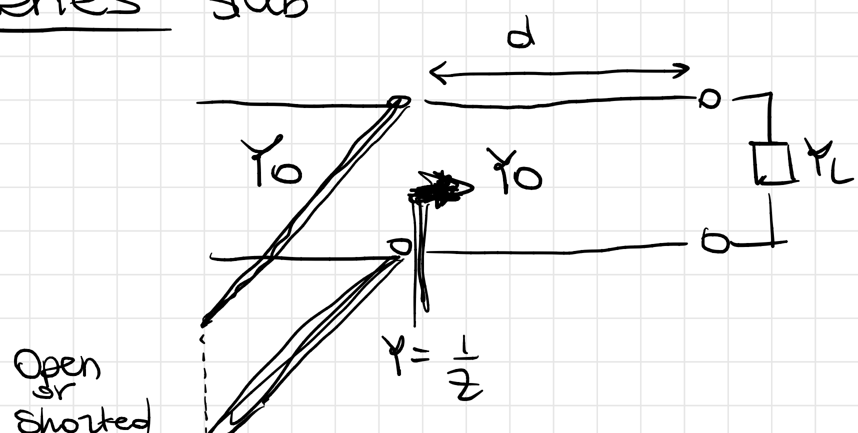
$\sqrt{} > 0$

$$B = \frac{\pm\sqrt{(Z_0 - R_L)/R_L}}{Z_0}$$

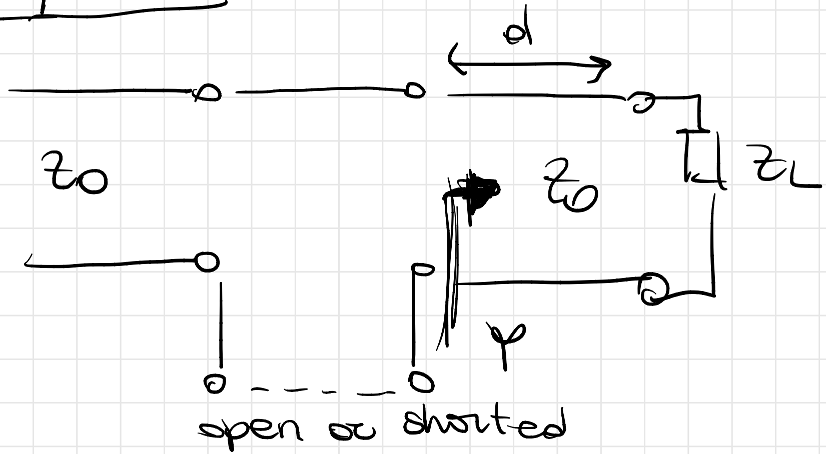
SINGLE STUB TUNING

very convenient and easy to fabricate.

series stub



parallel/shunt stub



ADJUSTABLE PARAMETERS :

- distance $\underline{d}$
- the value of susceptance or reactance of the stub.

- shunt-stub < open / shorted] we select d so that the admittance $Y = Y_0 + jB$ when seen from a distance d

- series - stub open / shorted] We select d so that the impedance seen is $Z = Z_0 \mp jX$

The stub reactance is chosen to cancel the imaginary part $\rightarrow$ $-jB$ or $-jX$

SHUNT STUB MATCHING

$$Z_L = \frac{1}{Y_L} = R_L + jX_L$$

$$Z = Z_0 \frac{(R_L + jX_L) + jZ_0 \tan \beta d}{Z_0 + j(R_L + jX_L)\tan \beta d}$$

$$Y = \frac{1}{Z} = G + jB$$

if $t = \tan \beta d$

(1) $$G = \frac{R_L(1+t^2)}{R_L^2 + (X_L + Z_0 t)^2}$$

(2) $$B = \frac{R_L^2 t - (Z_0 - X_L t)(X_L + Z_0 t)}{Z_0 [R_L^2 + (X_L + Z_0 t)^2]}$$

d has to be chosen so that $G = Y_0 = \frac{1}{Z_0}$

$$Z_0 (R_L - Z_0) t^2 - 2 X_L Z_0 t + (R_L Z_0 - R_L^2 - X_L^2) = 0$$

Solving for t:

$$t = \frac{X_L \pm \sqrt{R_L\left[(Z_0 - R_L)^2 + X_L^2\right]/Z_0}}{R_L - Z_0}$$

$$\boxed{\text{If } R_L \neq Z_0}$$

$\hookrightarrow$ d is calculated

If $\boxed{R_L = Z_0}$ $\quad t = -\dfrac{X_L}{2Z_0}$

The principal solutions are:

$$\boxed{\frac{d}{\lambda} = \begin{cases} \dfrac{1}{2\pi} \tan^{-1} \underline{t} & t \geq 0 \\[2ex] \dfrac{1}{2\pi}\left(\pi + \tan^{-1} \underline{t}\right) & t < 0 \end{cases}}$$

The stub susceptance $\underline{\underline{B_s = -B}}$

If the stub is open-circuit:

$$\boxed{\frac{l_o}{\lambda} = \frac{1}{2\pi} \tan^{-1}\left(\frac{B_s}{Y_0}\right) = -\frac{1}{2\pi} \tan^{-1}\left(\frac{B}{Y_0}\right)}$$

If the stub is short-circuited

$$\boxed{\frac{l_s}{\lambda} = \frac{-1}{2\pi} \tan^{-1}\left(\frac{Y_0}{B_s}\right) = \frac{1}{2\pi} \tan^{-1}\left(\frac{Y_0}{B}\right)}$$

If l_o and l_s are negative $\rightarrow$ we add $\frac{\lambda}{2}$ line to make the distance or length positive

SERIES - STUB

If load admittance is $Y_L = \frac{1}{Z_L} = G_L + jB_L$

then after a distance d

$$Y = Y_0 \frac{(G_L + jB_L) + jY_0 \tan\beta d}{Y_0 + j(G_L + jB_L)\tan\beta d} \qquad t = \tan\beta d$$

$$z = R + jX = \frac{1}{Y}$$

$$R = \frac{G_L(1+t^2)}{G_L^2 + (B_L + Y_0 t)^2} \leftarrow$$

$$X = \frac{G_L^2 t - (Y_0 - tB_L)(B_L + tY_0)}{Y_0 \left[G_L^2 + (B_L + Y_0 t)^2 \right]}$$

d is chosen so that $R = Z_0 = \frac{1}{Y_0}$

it follows:

$$\rightarrow Y_0 (G_L - Y_0) t^2 - 2 B_L Y_0 t + (G_L Y_0 - G_L^2 - B_L^2) = 0$$

$$\underline{t} = \frac{B_L \pm \sqrt{G_L \left[(Y_0 - G_L)^2 + B_L^2 \right] / Y_0}}{G_L - Y_0} \qquad G_L \neq Y_0$$

if $G_L = Y_0 \Rightarrow t = -\dfrac{B_L}{2Y_0}$

the principal solutions are

$$\boxed{\frac{d}{\lambda} = \begin{cases} \dfrac{1}{2\pi}\tan^{-1} t & t \geq 0 \\[2ex] \dfrac{1}{2\pi}\left(\pi + \tan^{-1} t\right) & t < 0 \end{cases}}$$

Imposing the susceptance of the stub cancels X then ($X_s = -X$)

<u>short-circuit</u>

$$\boxed{\frac{l_s}{\lambda} = \frac{1}{2\pi}\tan^{-1}\left(\frac{X_s}{Z_0}\right) = \frac{-1}{2\pi}\tan^{-1}\left(\frac{X}{Z_0}\right)}$$

open-circuit

$$\boxed{\frac{l_o}{\lambda} = -\frac{1}{2\pi}\tan^{-1}\left(\frac{Z_o}{X_s}\right) = \frac{1}{2\pi}\tan^{-1}\left(\frac{Z_o}{X}\right)}$$