Fundamentals of Coherent Photonics
Linear Propagation in Optical Wave-guides - Notes & Tutorials

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19th September, 2021

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Introduction

What is the rational behind this document, why would I make it?
Put simply, I like to have a permanent and archive-able copy of my work and for it to be in a presentable format. This results in easy revision, ensures I have fully explored the topic and questions, and provides an easy to read way for professors to check my understanding and to give others help with the topic.
I spent have spent a lot of time developing the template used to make this LaTeX document, I want others to benefit from this work so the source code for this template is available on GitHub [1].

1 Class Notes, Study Notes, and Revision

The propagation constant, β

Firstly, please note this is sometimes (extremely confusingly) referred to as the wavenumber in some textbooks (and vice-versa, the wavenumber is called the propagation constant). However, in the case of these notes this will always be the propagation constant (with the wavenumber being something else entirely, described in these notes elsewhere, and denoted k). measured in m^{-1}
Denoted β
each mode will result in a different solution to it's propagation constant (sometimes called a wavenumber in textbooks),

$$\beta =$$

Where:

β is something

The wavenumber, k

$$k = \frac{\omega}{c} = \frac{2\pi\nu}{c}$$
$$= \frac{2\pi}{\lambda} = nk_0$$

Where:

ω (greek letter "omega") is the angular, optical frequency in Radians/Second

ν (greek letter "nu") is the periodic frequency in Hertz

c is the speed

λ (greek letter "lambda") is the wavelength of the light in metres

The "V" Parameter, "Fibre" Parameter or Normalised Spatial Frequency, V

and governs the number of modes able

$$V = \frac{2\pi a}{\lambda}\sqrt{n_{core}^2 - n_{clad}^2}$$

Where:

λ is the wavelength of the light propagating down the fibre

n_{core} is the refractive index of the core's material (unitless)

n_{clad} is the refractive index of the cladding's material (unitless)

V is the normalised spatial frequency, the fibre parameter, or the "V" parameter

Where:

$\bar{\Theta}_c$ is the critical angle for rays to make total internal reflections at the core-cladding boundary (Radians)

2 Tutorials

2.1 Tutorial One

2.1.1 Question One

Step index fibres are constituted by a cylindrical core refractive index $n_1(\lambda)$, surrounded by a cladding refractive index $n_2(\lambda)$. Most of the time, the cladding is masde of pure silica and the core is made of silica doped with germanium.

Part A

What is the meaning of the expression "Step Index"?

"Step Index" refers to the refractive index profile of the core and the difference between the core and cladding refractive indices (usually denoted n_1 and n_2 respectively). In a step index fibre the index of the core remains constant through it's radius and then the refractive index value "steps" immediately at the cladding. Figure 1 shows the cross section of a step index fibre (on the left of the image) and a graph plotting refractive index versus cross-sectional radius, the x-axis represents the refractive index as it varies against the radius of the cross section (represented by the y-axis).

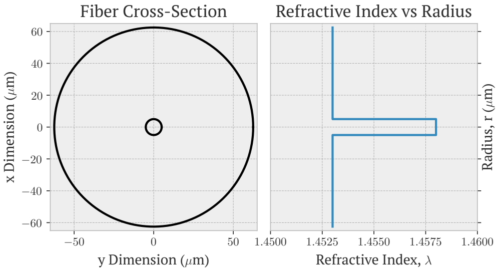

Figure 1: An image demonstrating the meaning of a "step index" fibre: on the left is the cross-section of the fibre, and the right shows the refractive index vs radius

Part B

What is the role of germanium in the core?

Germanium is one of a myriad of available materials that are used to change the refractive index of silica to desirable values. Making the silica in the core of an optical fibre slightly impure results in an increase of the core's refractive index which results in light being able to propagate through the core due to the difference between the core and cladding refractive indices.

Part C

In what range of wavelengths are optical fibres used for telecommunication systems? Why?

Simple / Short Answer:
The range of wavelengths used in optical communications is between 850nm - 1675nm. The reason for this particular range is due to the suitably low attenuation and dispersion characteristics at these wavelengths in silica fibre.

Detailed / Long Answer:
For a number of reason, there are a lot of nuances in the answer to this question. The first optical fibres operated in a band with a centre wavelength around 850-870nm [2][3] where efficient enough light sources and detectors were available, and only for short distance as the loss was 4 dB/km. [3]

However, since the development of these original telecommunications systems optical fibre technologies have advanced significantly and there is a range of frequencies which are considered to be within the practical application window for modern optical fibres. Sources differ and as such listed below are the frequencies reported to be used (and which sources report them):

- 1265nm - 1675nm (EMIMEO course notes: Linear Propagation in Optical Fibres, Chapter One)
- 1260nm - 1675nm [2]
- 1250nm - 1650nm: Additionally specified that most modern telecommunications are only conducted on the C-Band (1530-1565nm), L-Band (1565-1625nm), with the use of Raman amplification this is extended to the E&S bands (1360-1463nm, 1460-1530nm), and originally the O-Band was used (1260-1360nm) [3]

Regardless of which exact values are stated in the various sources (they are all roughly the same), specifying a range of realistic wavelengths for the propagation of light down an optical fibre is necessary for a number of reasons, these being:

The attenuation (a coefficient given the symbol α and measured in dB/Km) is at it's lowest across this range of values, allowing propagation of light without over-degradation of signal. Across the range of frequencies the attenuation is in the range of 0.3 - 1 dB/Km with a significant dip at 1550nm). And;

The dispersion characteristics (D_λ, ps/Km-nm), which is the the mount of spread (change in period) of a pulse as it propagates down a fibre, are most desirable in this range. Dispersion is considered to be chromatic dispersion (dispersion of light differs with wavelength) which is made of two components material dispersion and waveguide dispersion (with the latter held as being larger than the former [2]). The equations used to calculate dispersion is shown in Equations 1 & 2:

$$\sigma_\tau = |D| \cdot \sigma_\lambda \cdot L \tag{1}$$

$$D = D_\lambda + D_w \tag{2}$$

Where:

σ_τ is the spectral change of pulse width

σ_λ is the spectrum of the original pulse

L is the length of the fiber (Km)

D is the total dispersion coefficient (ps/Km-nm)

D_λ is the material dispersion (ps/Km-nm), and;

D_w is the waveguide dispersion (ps/Km-nm)

2.1.2 Question Two

A manufacturer of silica fibres receives an order for a step index fibre which must fulfil the three following conditions:

C1: *The numerical aperture, which is assumed to be independent of the wavelength, must be 0.12 (i.e.* $NA = 0.12$*)*

C2: *The fibre must be single mode @* $\lambda = 800nm$

C3: *The fibre must be able to guide at least two LP modes @* $\lambda = 750nm$

This manufacturer has 5 different fibres described in the Table 1:

Fibre Property	***Fibre 1***	***Fibre 2***	***Fibre 3***	***Fibre 4***	***Fibre 5***
Index, n_1 ***(@ 800nm)***	*1.456*	*1.456*	*1.458*	*1.458*	*1.46*
Core diameter (μm ***)***	*5*	*6*	*5*	*6*	*5*

Table 1: *Manufacturers' fibre properties*

Part A & B

Part a - Verify that the weak guidance approximation can be used for these fibres. What kind of transverse modes can be considered in this case?

Part b - What are the two fibres that fulfil the condition C1?

Using Equations 3 - 5 and assuming n_{clad} is approximately 1.453 the values for the Numerical Aperture (NA), the Fraction Refractive Index Difference (Δ), and Normalised Spatial Frequency (V) can be calculated as shown in Table 2.

$$NA = Sin(\Theta_a) = \sqrt{n_{core}^2 - n_{clad}^2} \quad (3)$$

$$\Delta = \frac{n_{core}^2 - n_{clad}^2}{2 \cdot n_{core}^2} \quad (4)$$

$$V = \frac{2\pi a}{\lambda}\sqrt{n_{core}^2 - n_{clad}^2} \quad (5)$$

Where:

λ is the wavelength of the light propagating down the fibre

Δ is the fractional refractive index difference (unitless)

Θ_a is the acceptance angle for rays incident from air into the fibre (Radians)

NA is the Numerical Aperture and describes the light-gathering capacity of the fibre (unitless)

n_{core} is the refractive index of the core's material (unitless)

n_{clad} is the refractive index of the cladding's material (unitless)

V is the normalised spatial frequency, the fibre parameter, or the "V" parameter and governs the number of modes able to propagate at a given wavelength

Fibre Property	***Fibre 1***	***Fibre 2***	***Fibre 3***	***Fibre 4***	***Fibre 5***
Index, n_1 (@ 800nm)	*1.456*	*1.456*	*1.458*	*1.458*	*1.46*
Core diameter (μm)	*5*	*6*	*5*	*6*	*5*
Numerical Aperture, NA	*0.09*	*0.09*	*0.12*	*0.12*	*0.14*
Fractional refractive index difference, Δ	*0.00206*	*0.00206*	*0.00342*	*0.00342*	*0.00478*
Normalised spatial frequency, V, (@ $\lambda = 800$nm)	*1.77*	*2.12*	*2.36*	*2.83*	*2.75*

Table 2: *Results of the calculations on the fibres*

The weak guidance approximation / limit states that as long as the refractive index of the core (n_{core}) is close enough to that of the cladding (n_{clad}) such that the fractional refractive index difference (Δ) is less that 0.001 ($\Delta \leq 10^{-2}$), this means that from the values calculated in Table 2 the weak guidance approximation may be used with all the fibres.
As the weak guidance approximation applies, only Linearly Polarized (LP) modes can propagate.

Finally, only two fibres will meet the requirement of condition C1 (The numerical aperture must be 0.12 i.e. $NA = 0.12$), namely fibres 2 & 3

Part C

Determine the limit values of the diameter of the core (maximal value and minimal value) imposed by the conditions C2 and C3. Deduce that the Fibre 3 only fulfils all the conditions required by the customer.

From the graph showing the dispersion curves of LP modes (made using the bessel functions) it can be seen that in order to be single mode @ 800nm the V parameter of any fibre at this wavelength must be less than 2.405. In this same vein of thinking in order to propagate more than one mode @ 750nm the V parameter at this wavelength must be more than 2.405.

These new conditions are stated mathematically in Equation 6, additionally, rearranging Equation 5 to find suitable radius' will result in Equation 7, and solving for our two conditions will give us a range of core radius' equal to Equation 8 ($4.78 <$ diameter, ø < 5.1).

$$V = \begin{cases} < 2.405 & \text{when } \lambda = 800nm,\ NA = 0.12 \\ > 2.405 & \text{when } \lambda = 750nm,\ NA = 0.12 \end{cases} \tag{6}$$

$$a = \frac{V \cdot \lambda}{2\pi \cdot NA} \tag{7}$$

From the new conditions and equations:

$$a < \frac{2.405 \cdot \lambda_{800}}{2\pi \cdot NA} \implies a < \frac{2.405 \cdot (800 \times 10^{-9})}{2\pi \cdot 0.12}$$

$$a > \frac{2.405 \cdot \lambda_{750}}{2\pi \cdot NA} \implies a < \frac{2.405 \cdot (750 \times 10^{-9})}{2\pi \cdot 0.12}$$

$$\therefore 2.39\mu m < \text{a} < 2.55\mu m$$

$$\therefore 4.78\mu m < \text{ø} < 5.1\mu m \tag{8}$$

Finally, from the previous part of the question we know that the only fibres which fulfil condition C1 are fibres 3 and 4 (thus they will satisfy the NA requirement, which was assumed in the equations). Also, since we have just calculated that fibre 4 does not meet the radius requirements inferred by conditions C1 and C2, it can be stated that only fibre 3 meets all three conditions.

2.1.3 Question Three

A blue light beam from an argon laser emitting @ $\lambda_A = 457nm$ is launched in a piece of Fibre 3.

Part A

What are the LP modes able to propagate in the fibre at this wavelength?

Calculating in much the same way as before, and fully demonstrated in Equation 9, the V parameter of fibre 3 at the wavelength of 457nm is 4.12. Consulting the dispersion curve

$$V = \frac{2\pi(2.5 \times 10^{-6})}{(457 \times 10^{-9})}\sqrt{1.458^2 - 1.453^2} = 4.12 \tag{9}$$

Part B

Sketch a schematic representation of the energy distribution in each of these modes.

Part C

With any injection conditions, what can we observe on a screen set in front of the output face of the fibre?

Part D

Fibre 3 is spliced to a fibre F_M, single mode @ λ_A and one measures the power at the output of this second fibre. What do we note if we handle the Fibre 3? Justify your answer. What do we note if we handle the Fibre F_M, without touching Fibre 3? Justify your answer.

2.1.4 Question Four

We now work with the Fibre 3 only (Fibre F_M is removed), @ $\lambda_T = 800nm$.

Part A

Using the provided information, evaluate the propagation constant, β, of the fundamental LP mode @ λ_T.

Part B

Deduce the phase velocity of a continuous wave carried by this fundamental mode.

Part C

Why is the velocity of a pulse propagating in the fibre lower than this phase velocity?

2.1.5 Question Five

In fact, the core of the fibre is elliptical, the axis of the ellipse being oriented along two perpendicular directions x and y. The modes HE_{11x} and HE_{11y} composing the LP_{01} mode, respectively polarized along x and y, are no longer degenerated.

Part A

What does this expression means: "the modes HE_{11x} and HE_{11y} are no longer degenerated"?

Part B

At the wavelength λ_T, the effective indices of the two modes are $n_{ex} = 1.45549$ for the HE_{11x} mode and $n_{ex} = 1.45551$ for the HE_{11y} mode. Show that, at this wavelength, the two modes in phase periodically along their propagation, every 4 cm (the wave's spatial period).

Part C

What precaution should we take to ensure that a linearly polarized wave, launched at the input of a few meter long piece of fibre, remains linearly polarized at the output?

2.2 Tutorial Two

2.2.1 Question One

Step index fibres are constituted by a cylindrical core refractive index $n_1(\lambda)$, surrounded by a cladding refractive index $n_2(\lambda)$. Most of the time, the cladding is masde of pure silica and the core is made of silica doped with germanium.

References

[1] M. Jennings and A. Wilson. Academic report template. [Online]. Available: https://github.com/mjennings061/Academic-Report-Template

[2] B. E. A. Saleh and M. C. Teich, *Fundamentals of Photonics*, 3rd ed. John Wiley & Sons Inc, 10475 Crosspoint Boulevard, Indianapolis, IN 46256: Wiley, 2019, pp. 1224–1284.

[3] L. Thévenaz, *Advanced Fiber Optics*, 1st ed. EPFL Press, Presses Polytechniques et Universitaires, Romandes, EPFL Post Office Box 119, CH-1015 Lausanne, Switzerland: EPFL Press, 2011, pp. 1–27.