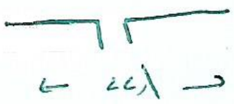
1 - Introduction

Def: The part of a transmitting or receiving system that is designed to radiate or to receive EM waves

Types

→ Electrically small antennas: Extend of antenna structure much less than λ

- Low directivity
- Low input resistance
  High input reactance
- Low radiation efficiency



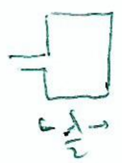
short dipole

small loop

→ Resonant antennas: They operate well at a single or selected narrow freq. bands.

- Low to moderate gain
- Real input impedance
- Narrow bandwidth

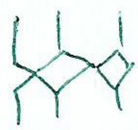
half wave dipole



Microstrip patch

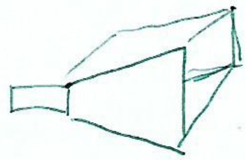
→ Broad band antennas: Pattern, gain & input impedance remain acceptable and are nearly constant over a wide freq. range

- Low to moderate constant gain
- Real input impedance
- Wide bandwidth



→ Aperture antennas: Have a physical aperture through which EM waves flow

- High gain, increases with freq
- Moderate bandwidth



2 - Fundamentals of EM and Radiation

→ Maxwell Eqs

→ Poynting theorem $\vec{S} = \frac{1}{2} \bar{E} \times \bar{H}^*$

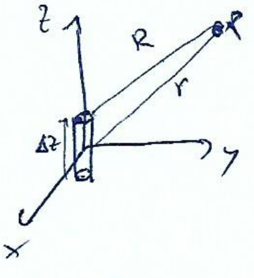
→ Plane wave $E_0 e^{-j\bar{k}\cdot\bar{r}}$

$H_0 e^{-j\bar{k}\cdot\bar{r}}$

•) **Radiation**

$$\vec{B} = \nabla \times \bar{A} \Rightarrow \vec{H} = \frac{1}{\mu} \nabla \times \vec{A}$$

•) **Ideal Dipole**



Fields: $\bar{E} = E_\theta \hat{\theta} = j\eta \frac{\beta}{4\pi} I \Delta z \frac{e^{-j\beta r}}{r} \sin\theta \, \hat{\theta} = E_\theta \hat{\theta}$

$\bar{H} = H_\phi \hat{\phi} = j \frac{\beta}{4\pi} I \Delta z \frac{e^{-j}}{r} \sin\theta \, \hat{\phi} = H_\phi \hat{\phi}$

Power: $P_{out} = \frac{\pi}{3} \eta I^2 \frac{\Delta z^2}{\lambda^2}$

3- Polarization of EM waves

Polarization refers to the orientation of the electric field vector.

Plane wave travelling in $+z$ direction: $E(z) = E_0 e^{-jkz} =$

$$= E_{0x} e^{-jkz} \hat{x} + E_{0y} e^{-jkz} \hat{y}$$

Final form:

$$\bar{E}(z) = \left(|E_{0x}| \hat{x} + |E_{0y}| e^{j\delta} \hat{y} \right) e^{-jkz} \qquad \delta = \varphi_y - \varphi_x$$

$$\bar{E}(x,t) = |E_{0x}| \cos(\omega t - kz) \hat{x} + |E_{0y}| \cos(\omega t - kz + \delta) \hat{y}$$

Linear Polarization: $\delta = 0$, $\delta = \pi$

RHCP $\delta = -\frac{\pi}{2}$ $|E_{0x}| = |E_{0y}| = |E_0|$ 

LHCP $\delta = \frac{\pi}{2}$ $|E_{0x}| = |E_{0y}| = |E_0|$ 

4 - Radiation Patterns and Antenna Parameters

Radiation Pattern:
- Decription of the angular variation of far field radiation level around the antenna.
- Vary inversely with distance
- Variation with angle depends on the antenna geometry
- Often field & power are normalised with respect their max. value

The radiation pattern is the variation of the radiated electric field over a sphere centered on the antenna

$r \to$ constant

$\varphi, \theta \to$ variation of the field

Normalized field pattern $|F(\theta,\varphi)|$ $\boxed{|F(\theta,\varphi)| = \frac{|E|}{|E|_{max}}}$

Normalised power pattern $P(\theta,\varphi)$: $\boxed{P(\theta,\varphi) = \frac{|E|^2}{|E|^2_{max}} = |F(\theta,\varphi)|^2}$

o) $\boxed{\text{3D Radiation Pattern}}$

Radiation pattern describes the variation of field magnitude or of power density ($\sim r^2$) as a function of the spherical cordinates (θ, φ)

Rad. pattern. always refer to far-field. (only in far-field angular distr. is indep. of the dist. from origin.)

3D obtained from origin to direction (θ, φ)

Most of radiation contained in a main lobe
Also, there is rad. in minor lobes
Between the lobes nulls of the field that goes to zero

o) $\boxed{\text{2D Radiation Pattern}}$

Is obtained with the intersection between 3D rad. pat. and a given plane. $\begin{pmatrix} \theta = \text{const} \\ \text{or} \\ \varphi = \text{const} \end{pmatrix}$

c) $\boxed{\text{Parameters}}$

→ Half-power beamwidth (HPBW): In a plane containing the direction of the maximum of the main lobe (beam). The angular separation of the points where the main beam of the power pattern $\underline{P(\theta,\varphi)}$ equals to one-half (-3dBm) of max. value.

On magnitude $|F(\theta,\varphi)| \xrightarrow[\text{is}]{\text{value}} 1/\sqrt{2} \cong 0,707$

→ First-null beamwidth (FNBW)

The angular separation between the first nulls.

→ Isotropic pattern: Spherical radiation pattern due to antenna having equal rad. in all directions (ideal)

→ Omnidirectional pattern: In a selected plane it is a circular rad. pattern.

→ Directional pattern: Antenna radiates more efficiently in some directions than in others.

→ E-plane pattern: 2D rad. patt. in a plane containing $\bar{E}$ vector and direction of max. rad.

for linearly polarized far-field

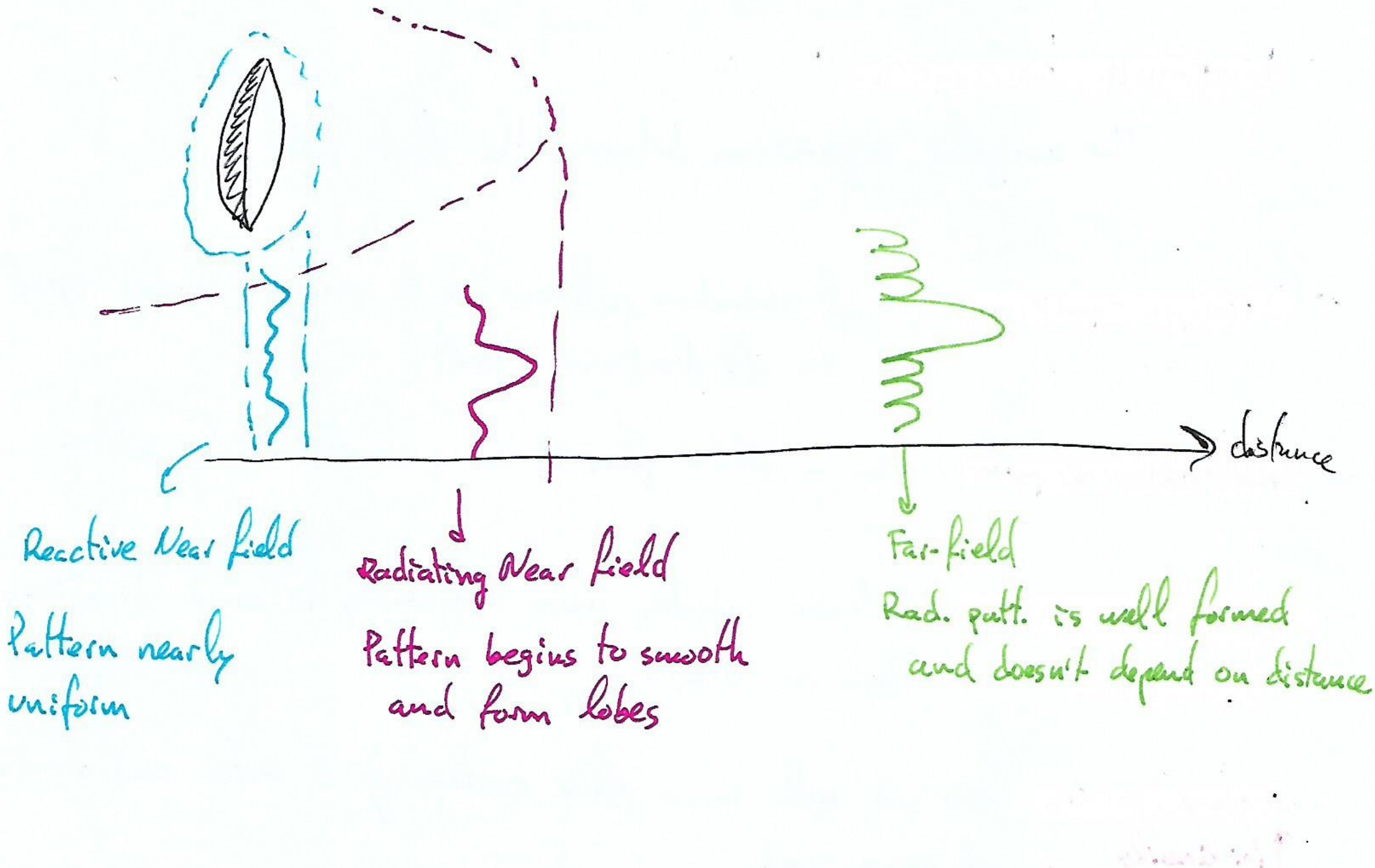
→ H-plane pattern: 2D rad. patt. in a plane containing $\bar{H}$ vector and direction of max. rad.

o) $\boxed{\text{Far-field Regions}}$

By convention the distance where the value of r which the path length deviation due to the parallel ray assumption is $\frac{\lambda}{16}$

→ Large antennas: Diameter of the length $D > 2.5\lambda$
Far-field distance: $r_{ff} = 2D^2/\lambda$

→ Small antennas: Diameter of the length $D << \lambda$
Far-field distance: $r_{ff} = 3\lambda$



o) Basic Parameters of Antennas

Directivity How much it concentrates energy in one direction in preference to radiation in other directions

Rad intensity → $U(\theta,\varphi) = \mathrm{Re}\{\vec{S}\} \cdot r^2 \hat{r}$

$$U(\theta,\varphi) = \underset{\text{max}}{U_m} \, |F(\theta,\varphi)|^2$$

$$U_{ave} = \frac{1}{4\pi} \iint_{4\pi} U(\theta,\varphi)\, d\Omega = \frac{P}{4\pi}$$

The directivity is defined as the ratio of the radiation intensity in a certain direction to the average radiation intensity

Reference direction usually is max rad.

$$\boxed{D = \frac{U_m}{U_{ave}} = \frac{U_m}{P/4\pi}}$$

beam solid angle

$$\Omega_A = \iint_{4\pi} |F(\theta,\varphi)|^2 d\Omega = \int_0^{2\pi}\int_0^{\pi} |F(\theta,\varphi)|^2 \sin\theta \, d\theta \, d\varphi$$

Also:

$$D = \frac{U_m / r^2}{U_{ave}/r^2} = \frac{\max\left[\mathrm{Re}(\vec{S}\cdot\hat{r})\right]}{P/4\pi r^2}$$

ratio of max to average intensities [W/sr]

ratio of max. to average power densities [W/m²]

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\iint_{4\pi} |F(\theta,\varphi)|^2 d\Omega}$$

→ Gain

(1) Ratio of the radiation intensity in the max. direction to the radiation intensity that would be obtained if the power fed to the antenna were radiated isotropically

(2) Ratio of the power density in the max. direction (@ distance r) to the power density that would be obtained at the same distance if the power fed to the antenna were radiated isotropically

$$\boxed{G = \frac{U_m}{P_{in}/4\pi} = \frac{U_m/r^2}{P_{in}/(4\pi r^2)}}$$

As there is some ohmic losses the radiated power P is not the same as the input power Pin.

Radiation Efficiency:

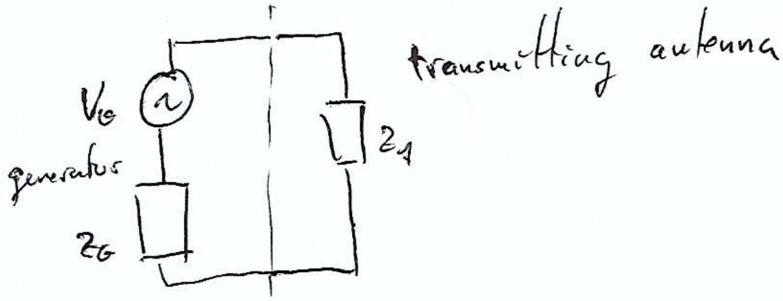
$$\boxed{e_r = \frac{P}{P_{in}}} \qquad 0 \leq e_r \leq 1$$

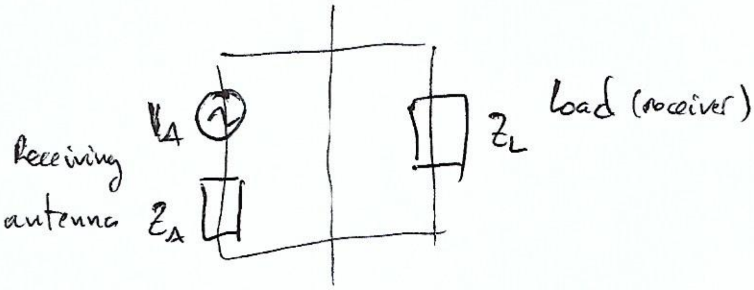
$$\boxed{G = e_r D}$$

→ Antenna Impedance

The input impedance can be affected by objects that are nearby

$$Z_A = R_A + j X_A$$





Antenna resistance: two terms: Radiation resistance R_R & loss (ohmic) resistance R_D

$$R_A = R_R + R_D$$

$$P_R = \frac{1}{2} R_R |I|^2$$

$$P_D = \frac{1}{2} R_D |I|^2$$

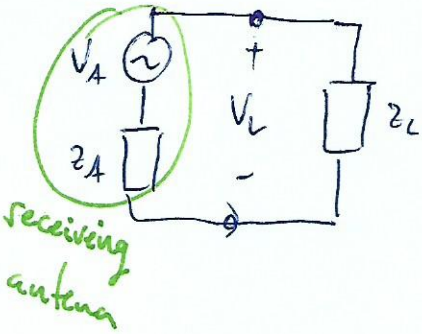
$$e_r = \frac{P_R}{\underset{P_R + P_D}{\underset{\shortparallel}{P_{in}}}} = \frac{P_R}{P_R + P_D} = \frac{R_R}{R_R + R_D}$$

5 - Antennas in Communication Systems

Antenna is the interface between the radio-freq. circuits of the transmitter or receiver and free space.

Effective Length and Effective Area

Receiving antenna converts the power density arriving to a current on the connected circuit. We can assume that is a plane wave and receiving antena is equivalent to a generator like:



$$\boxed{V_A = E_i \cdot h}$$

$h \rightarrow$ effective length (or height) [m]

Receiving antenna can also be seen as converting incident power density ($S = |S|$ in [W/m²]) into received power by the collecting area of antenna called $\rightarrow$ effective area (or effective aperture): A_e [m²]

$$\boxed{P_{rm} = A_e S}$$

8

Some more relations

$$P_{rm} = A_e \frac{1}{2\eta} |E|^2$$

$$h = 2\sqrt{\frac{R_A A_e}{\eta}}$$

$$G = \frac{4\pi}{\lambda^2} A_e$$

- For a fixed λ. As the maximum antenna effective area increases (ie increasing antenna physical size). the beam solid angle decreases that means power is more concentrated in angular space.

c) Friis Transmission Formula

We want to know the power transfer.

Consider transmitting and receiving antenna separated by a distance R

calculate power received as a function of power transmitted

$P_R = f(P_T)$

1 → If transmitting antenna isotropic: $S = \frac{P_T}{4\pi R^2}$

↳ transmitter has gain (G_T): $S = G_T \frac{P_T}{4\pi R^2}$

2 → Receiving antenna has a effective area A_{eR} $\quad P_R = A_{eR} S = A_{eR} \frac{P_T}{4\pi R^2}$

↳ Effective area is proportional to gain: $A_{eR} = G_R \frac{\lambda^2}{4\pi}$

$$\boxed{P_R = G_T G_R \left(\frac{\lambda}{4\pi R}\right)^2 P_T}$$ Friis formula

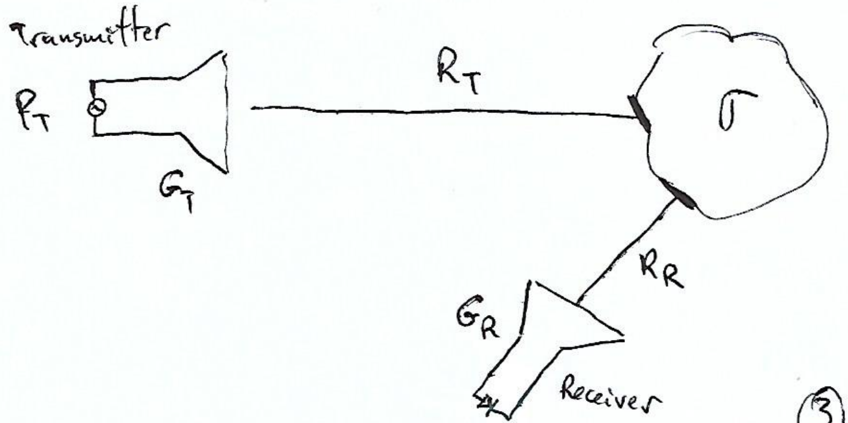
If antennas not aligned:

$G_T = G_T(\theta_T, \varphi_T)$

$G_R = G_R(\theta_R, \varphi_R)$

o) Radar Equation

General case



(1) Power density incident on radar target:

$$S_i = G_T \cdot \frac{P_T}{4\pi R_T^2}$$

(2) Power density received is the scattered by the target.

$$S_R = \frac{P_i}{4\pi R_R^2}$$

(3) The relation between P_i & S_i:

$$\boxed{P_i = \underbrace{\sigma}_{\text{cross section}} S_i}$$

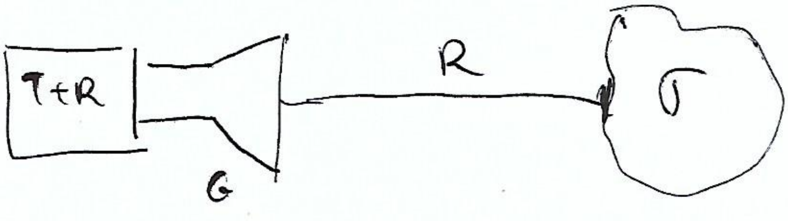
(4) Finally, the power at the receiver: $P_R = A_{eR} S_R = G_R \frac{\lambda^2}{4\pi} S_R$:

$$\boxed{P_R = \sigma G_T G_R \frac{\lambda^2}{(4\pi)^3 R_T^2 R_R^2} P_T}$$

Radar equation

Radar cross section [m²]
the Area intercepting that amount of power P_i which, when scattered isotropically produces at receiving antenna a power density S_R which is equal to that scattered by the actual target

If transmitter and receiver are the same antenna



$$\boxed{P_R = \sigma G^2 \frac{\lambda^2}{(4\pi)^3 R^4} P_T}$$

•) Effective Isotropicaly Radiated Power

It is the amount of power emitted from an isotropic antenna to obtain the same power density in the direction of the actual antenna pattern peak with gain G_T

Other words: To obtain the same radiation intensity produced
↳ by the directional antenna in its pattern max direction an isotopic antenna would require an input power G_T times greater

$$\boxed{EIRP = G_T P_T}$$

6 - Small dipoles and Small loops

$$\int_0^{\pi} \sin^3\theta \, d\theta = \frac{4}{3}$$

$$dA = r^2 \sin\theta \, d\theta \, d\phi$$

$$d\Omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\phi$$

a) Ideal dipole

In far field

$\beta r >> 1$

$$\bar{E} = j\eta \frac{\beta}{4\pi} I \Delta z \frac{e^{-j\beta r}}{r} \sin\theta \, \hat{\theta} = E_\theta \hat{\theta}$$

$$\bar{H} = j \frac{\beta}{4\pi} I \Delta z \frac{e^{-j\beta r}}{r} \sin\theta \, \hat{\phi} = H_\phi \hat{\phi}$$

Real radiated power

$$P_R = \iint_{\Sigma} \bar{S} \cdot d\bar{s} = \iint_{\Omega} |\bar{S}| \, \hat{r} \cdot \hat{r} \, r^2 \sin\theta \, d\theta \, d\phi = \frac{1}{2} \left(\frac{I \Delta z}{4\pi}\right)^2 \eta \beta^2 \int_0^{2\pi} d\phi \int_0^{\pi} \sin^3\theta \, d\theta =$$

$$P_R = \frac{\pi}{3} \eta \frac{I^2 \Delta z^2}{\lambda^2}$$

Radiation resistance

$$R_R = \frac{2 P_R}{I^2} = \frac{2\pi}{3} \eta \left(\frac{\Delta z}{\lambda}\right)^2 \simeq 80 \pi^2 \left(\frac{\Delta z}{\lambda}\right)^2$$

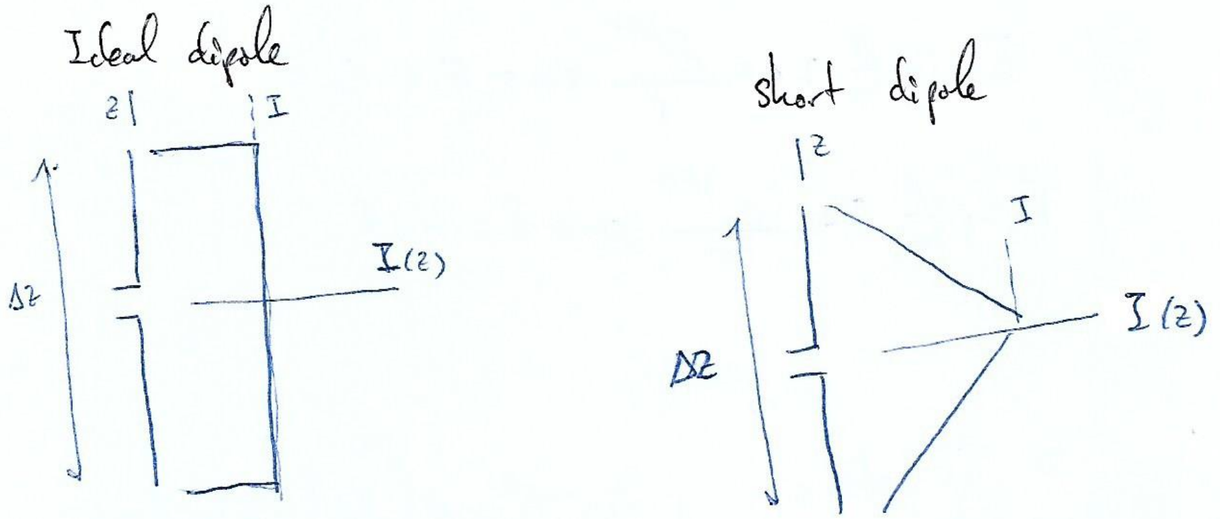
Directivity

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\iint_{4\pi} |F(\theta,\varphi)|^2 d\Omega} = \frac{4\pi}{\int_0^{2\pi} \int_0^{\pi} |F(\theta,\varphi)|^2 \sin\theta \, d\theta \, d\varphi} = \frac{3}{2}$$

$$|F(\theta,\varphi)| = |\sin\theta|$$

o) $\boxed{\text{Short Dipole}}$

Center-fed wire dipole of length $\Delta z << \lambda$ supporting a current distribution which is triangular



What changes is the factor: $I\Delta z \longrightarrow \frac{I\Delta z}{2}$

Far-field fields:

$$\bar{E} = j\eta \frac{\beta}{4\pi} \frac{1}{2} I\Delta z \frac{e^{-j\beta r}}{r} \sin\theta \, \hat{\theta}$$

$$\bar{H} = j \frac{\beta}{4\pi} \frac{1}{2} I\Delta z \frac{e^{-j\beta r}}{r} \sin\theta \, \hat{\phi}$$

Same radiation pattern and Directivity

$$|F(\theta,\varphi)| = |\sin\theta|$$

$$D = \frac{3}{2}$$

Power radiated

→ magnitude is the half so power is the quarter

$$P_r = \frac{1}{4} \frac{\pi}{3} \eta \frac{I^2 \Delta z^2}{\lambda^2}$$

Radiation resistance

$$R_R = \frac{1}{2} \frac{\pi}{3} \eta \frac{\Delta z^2}{\lambda^2} \simeq 20 \pi^2 \left(\frac{\Delta z}{\lambda}\right)^2$$

Dissipation Resistance R_D

skin depth: $\boxed{\delta = \dfrac{1}{\sqrt{\pi \mu f \sigma}}}$

R_D : $\boxed{R_D = \dfrac{L}{2\pi a \delta} \dfrac{1}{\sigma} = \dfrac{L}{2\pi a} \sqrt{\dfrac{\pi \mu f}{\sigma}}}$

$$R_D = \frac{L}{2\pi a} R_S$$

R_S surface resist.

$$R_S = \sqrt{\frac{\pi \mu f}{\sigma}}$$

R_D for short dipole $\longrightarrow$ $\frac{1}{3}$ of R_D of ideal dipole

o) $\boxed{\text{Small Loop}}$

→ Can be proved that the radiation fields of small loops are independent of the shape of the loop and only depend on the area of the loop.

→ Same rad. pattern as ideal dipole.

But loop electric field is horizontally polarized : $E = E_\varphi \hat{\varphi}$

Dipole electric field is vertically polarized : $\bar{E} = E_\theta \hat{\theta}$

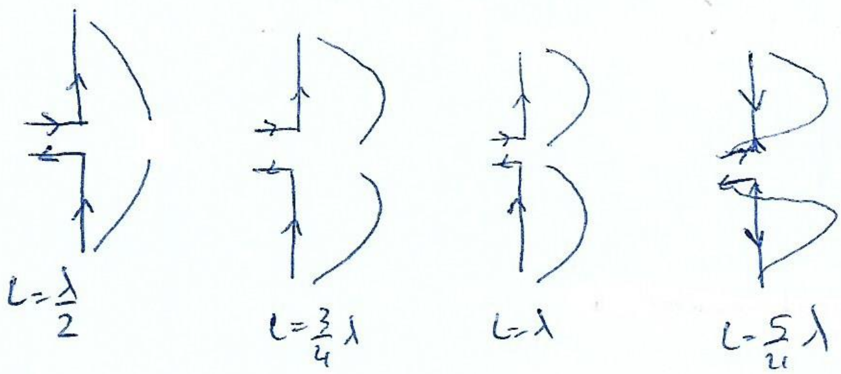
o) Null in perpendicular to the plane of the loop

Max. rad. in the plane of the loop (pattern is omnidirectional)

7 - Wire Antennas

Simple in concept, easy to construct and cheap.

Some current distributions depending on the length of the wire



When $L > \lambda$ we see that the current on the wire are not in the same direction, we can expect cancelling effects in the radiation pattern.

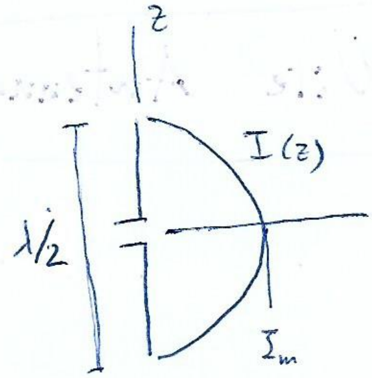
→ Fields E & B
→ Far-field

Radiating fields and the input impedance depends on $\frac{\beta L}{2}$ → so ratio $\frac{\text{wire length}}{\text{wavelength}} = \frac{L}{\lambda}$

•) Half-Wave Dipole

Wire length: $L = \lambda/2$

The wire is fed in its center



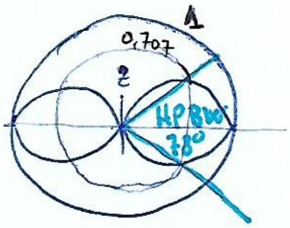
$\bar{E}$ & $\bar{H}$ far-fields

→ Normalised rad. pattern $|F(\theta,\varphi)|$ doesn't depend on φ due to axial symmetry

$$|F(\theta,\varphi)| = \left|\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right|$$

→ Pattern in E-plane shape of number 8

Pattern in H-plane is omnidirectional → it is a circle



HPBW = 78°

D = 1,64

→ Radiated power: $P_R = 36{,}6\, I_m^2$

→ Radiation resistance: $R_e \simeq 73\ \Omega$

→ Complete input: $Z_A = 73 + j\,42{,}5\ \Omega$

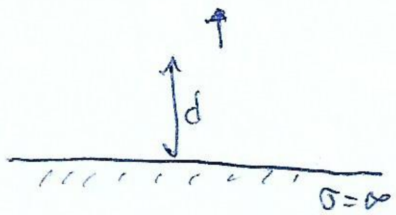
@ resonance if $\mathrm{Im}[Z_A] = X_A = 0$

@ anti-resonance if $|X_A| = \infty$

The thicker the wire, the wider is the "wire antenna" working bandwidth

• $\boxed{\text{Monopoles}}$

Consider a dipole at distance d from a perfect ground plane.
(Image method)



A monopole is a dipole that has been divided in half at its center point and fed against a ground plane

→ Input impedance: Half of dipole $Z_{A_{monopole}} = \frac{1}{2} Z_{A_{dipole}}$

→ Radiation resistance: $R_{R_{monopole}} = \frac{1}{2} R_{R_{dipole}}$

→ Directivity: $D_{monopole} = 2 D_{dipole}$

o) Large Loop Antennas

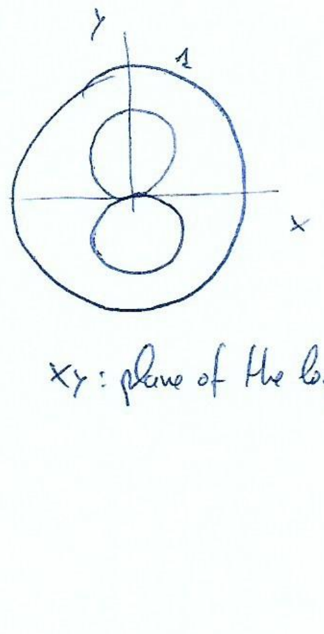
A loop antenna is considered large when its perimeter L_p is much larger than a $\frac{\lambda}{10}$.

Current and phase vary with position around the loop causing the impedance and pattern to depend on loop size.

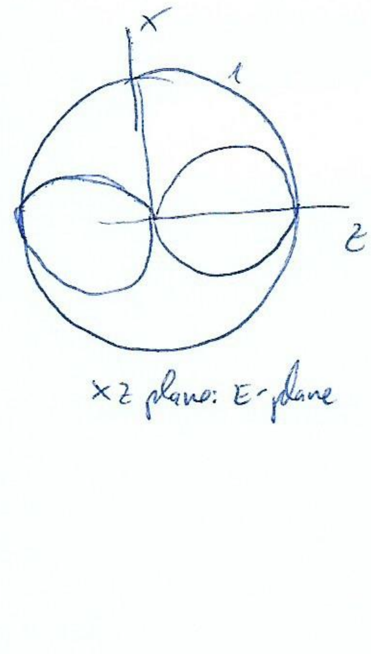
↳ <u>One-wavelength square loop</u>

Each side is $\lambda/4$

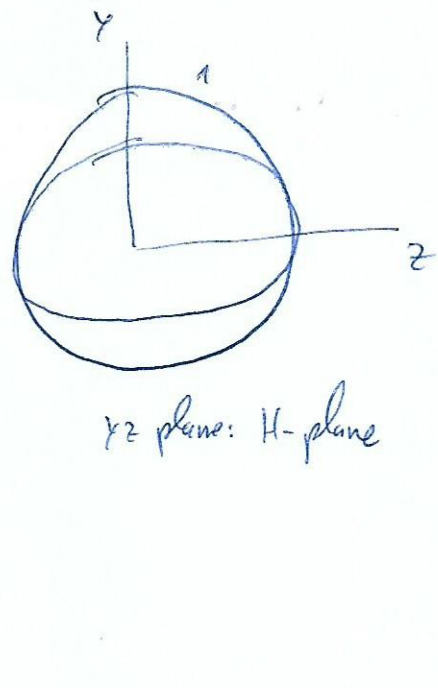
Radiation max at normal to the plane of the loop



xy: plane of the loop



xz plane: E-plane



yz plane: H-plane

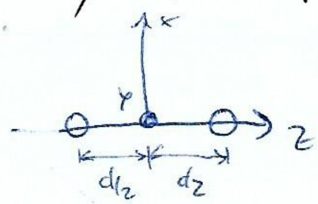
8 - Array Antennas

Combining the output of multiple antenna elements : provides the possibility of changing the rad. pattern and a larger directivity can be achieved.

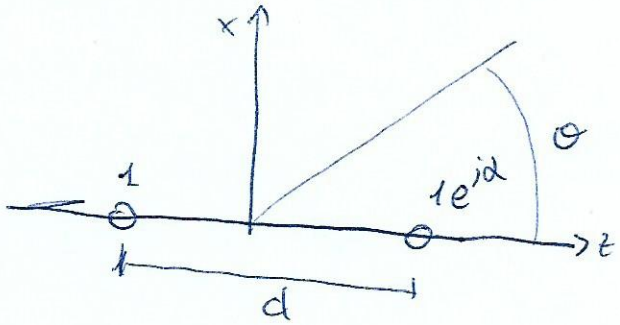
a) Two-Element Array

→ First consideration: Two elementar radiators forming the array are isotropic sources.

→ The pattern of an array of isotropic elements is called the array factor.



→ There are different examples to calculate AF (array factor)



$$AF = 2e^{j\frac{\alpha}{2}} \cos\left(\beta \frac{d}{2} \cos\theta + \frac{\alpha}{2}\right)$$

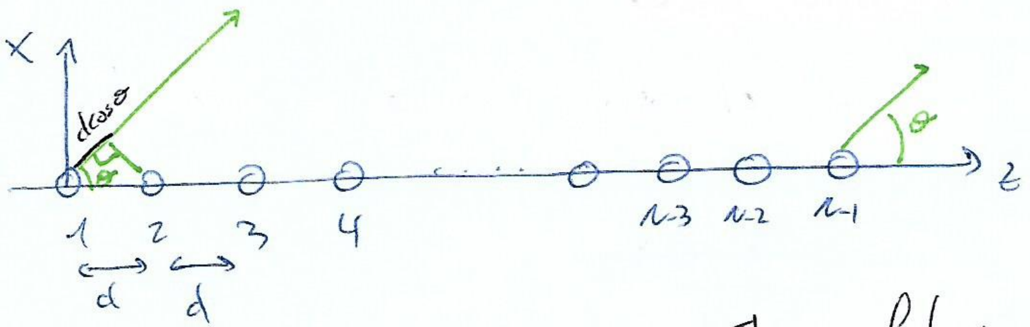
$$F(\theta,\varphi) = \left|\cos\left(\beta \frac{d}{2} \cos\theta + \frac{\alpha}{2}\right)\right|$$

N - Element Linear Arrays

→ Ability to shape the pattern through spacing and excitation adjustments.

→ Transverse dimension can be very large (compared to λ)
Able to get extremely high directivity (and gain)

→ The total pattern of the array is the product of the element pattern and array factor.



N isotropic sources

$$AF = \sum_{n=0}^{N-1} A_n e^{jn\underbrace{(\beta d\cos\theta + \alpha)}_{\psi}}$$

The array factor is periodic in variable ψ with periodic 2π

Visible region: $-\beta d + \alpha \leq \psi \leq \beta d + \alpha$

$\frac{d}{\lambda}$ determines how much of the array factor appears in the visible region

→ If only one period appears in visible region $\Rightarrow 2\beta d = 2\pi \Rightarrow d = \frac{\lambda}{2}$

→ Spaces larger than $\frac{\lambda}{2}$ more than one period will be visible and there could be more than one main lobe in the visible region → grating lobes

→ If equal magnitude $A_0 = A_1 = \dots A_{N-1} \underset{\text{geometric series}}{\Rightarrow} AF = A_0 e^{j(N-1)\frac{\psi}{2}} \dfrac{\sin(N\frac{\psi}{2})}{\sin(\frac{\psi}{2})}$

Some results: for normalised array factor for N element uniformly excited

$$\frac{AF}{N} = \frac{\sin\left(N\frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$$

$$\psi = \beta d\cos\theta + \alpha$$

$$|F(\theta,\varphi)| = \frac{\sin(N\psi/2)}{\sin|\psi/2|}$$

→ Symmetric around $\psi=0$ (even)

→ Max $= N$ for $\psi = 0, \pm 2\pi, \pm 4\pi$

→ Null: $\psi_{null} = \pm 2k\pi/N$, $k \neq N, 2N, 3N$

→ main lobes width $\frac{2\pi}{N}$ in var. ψ

minor lobes width: $\frac{2\pi}{N}$ in ψ

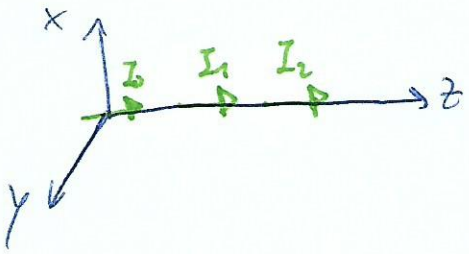
→ Number of lobes $N-1$: 1 main and $N-2$ side

→ See examples

c) The complete array pattern and pattern multiplication

First, we replace the isotropic point sources for lined currents

Array → collinear



$$AF = \sum_{n=0}^{N-1} I_n e^{j\beta n d\cos\theta}$$

The field pattern of an array consisting of similar elements is the product of the pattern of one of them and the pattern of array of isotropic point sources with same locations, relative amplitudes, and phases

$$|F_{array}(\theta,\varphi)| = |F_{element}(\theta,\varphi)| \frac{|AF(\theta)|}{|AF(\theta)|_{max}}$$

if elements equally spaced and uniformly excited

$$|F_{array}(\theta,\varphi)| = |F_{element}(\theta,\varphi)| \frac{\left|\sin\left(N\frac{\beta d\cos\theta+\alpha}{2}\right)\right|}{\left|N\sin\left(\frac{\beta d\cos\theta+\alpha}{2}\right)\right|}$$