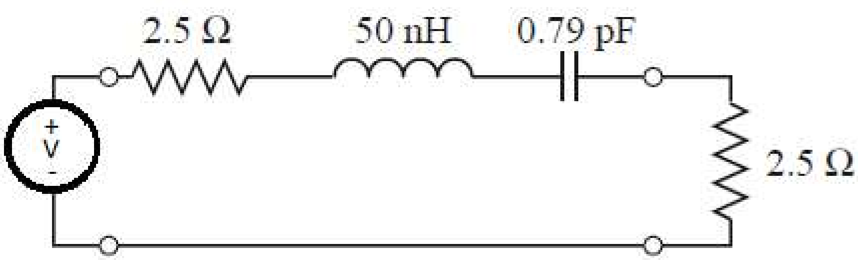
Microwave Engineering Final Test
July 17, 2020

Student ID:
Name:
Instructions: You have 1.5 hours to complete the test. Please write everything with blue or black ink pen so that all your work can be read easily. You can use your calculator. If you don't have a calculator, you can leave the formulas in expression forms and still get full score for the questions/exercises. Use of course notes or internet resources will invalidate the results of the test. Use of your cell phone is allowed only for scanning test and emailing the file at the end of the exam.
<u>VERY IMPORTANT: Please WRITE YOUR FULL NAME AND STUDENT ID on the first sheet you scan. If you forget to include your name, I will not be able to put your material on record and therefore the test will NOT BE VALID!</u>

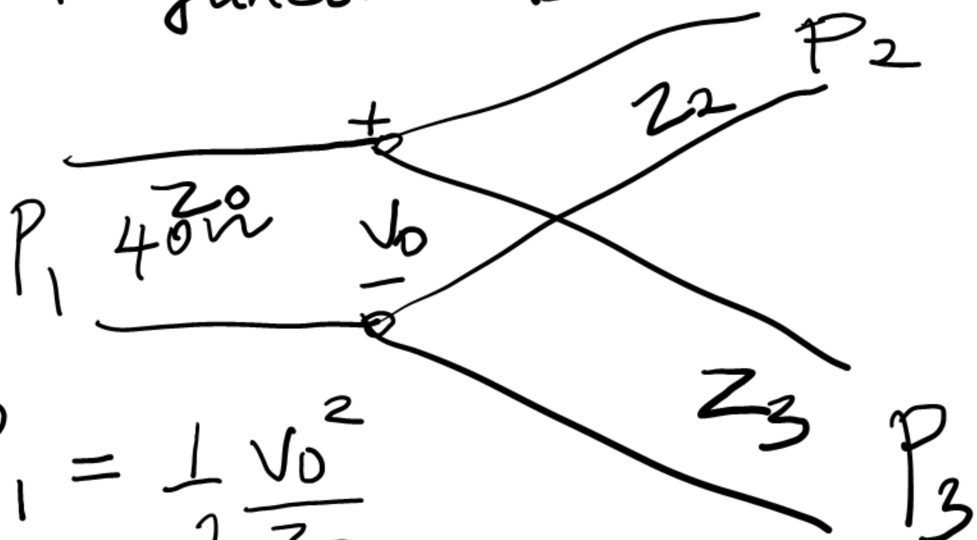
Questions:

1. A series RLC resonator with an external load is shown below. Find:
   a. The resonant frequency;
   b. The input impedance at the resonant frequency;
   c. The unloaded and the loaded Q at the resonant frequency.



2. Design a lossless T-junction divider with a 40 Ω source impedance to give a 4:1 power split. Design quarter-wave matching transformers to convert the impedances of the output lines to 40 Ω. Determine the magnitude of the scattering parameters for this circuit, using a 40 Ω characteristic impedance.

2) T-Junction Divider



$$P_1 = \frac{1}{2}\frac{V_0^2}{Z_0}$$

4 : 1 Power split

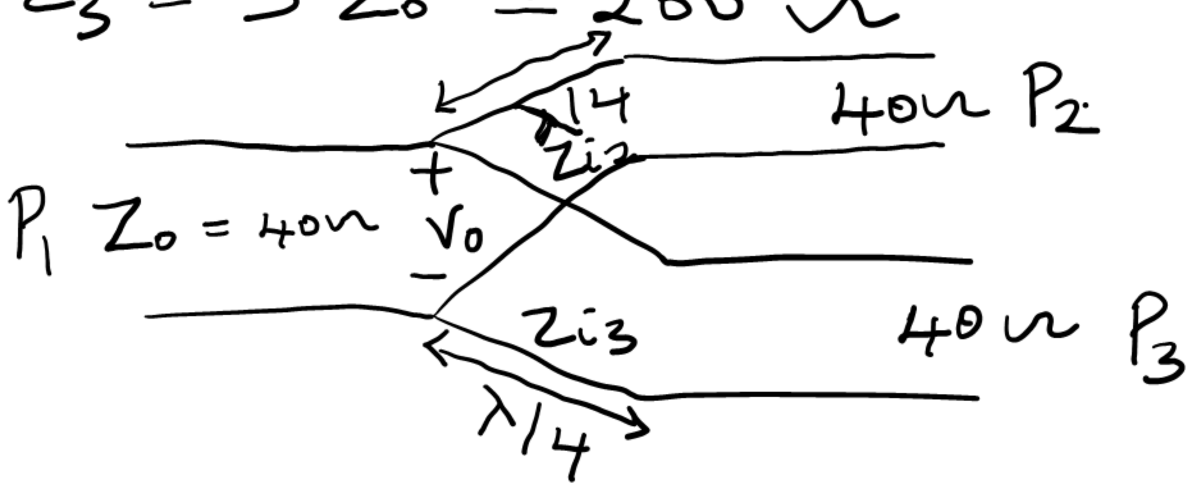
$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_2} = \frac{4}{5}P_1 = \frac{4}{10}\frac{V_0^2}{Z_0}$$

$$\frac{1}{2}\frac{V_0^2}{Z_2} = \frac{4}{10}\frac{V_0^2}{Z_0} \Rightarrow Z_2 = \frac{10 V_0^2 Z_0}{2 \cdot 4 \cdot V_0^2}$$

$$Z_2 = \frac{5}{4} Z_0 = 50\Omega$$

$$P_3 = \frac{1}{2}\frac{V_0^2}{Z_3} = \frac{1}{5}P_1 = \frac{1}{10}\frac{V_0^2}{Z_0}$$

$$Z_3 = 5 Z_0 = 200\ \Omega$$



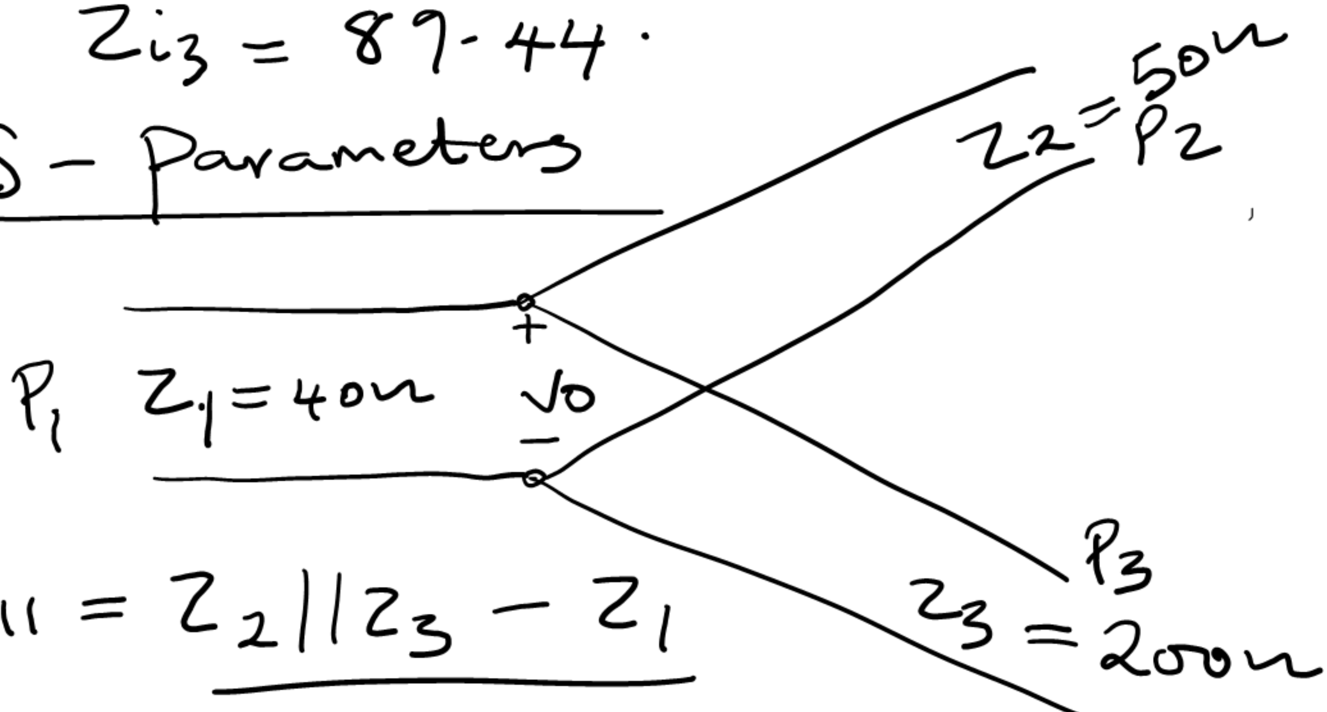
$$Z_{i2} = \sqrt{Z_2 \cdot 40} = \sqrt{50 \cdot 40}$$

$$Z_{i2} = 44.72\Omega$$

$$Z_{i3} = \sqrt{Z_3 \cdot 40} = \sqrt{200 \cdot 40}$$

$$Z_{i3} = 89.44.$$

S - Parameters



$$S_{11} = \frac{Z_2 \| Z_3 - Z_1}{Z_2 \| Z_3 + Z_1}$$

$$= \frac{40 - 40}{40 + 40} = 0$$

$$S_{22} = \frac{Z_1 \| Z_3 - Z_2}{Z_1 \| Z_3 + Z_2}$$

$$= \frac{33.33 - 50}{33.33 + 50}$$

$$= -0.2$$

$$S_{33} = \frac{Z_1 \parallel Z_2 - Z_3}{Z_1 \parallel Z_2 + Z_3}$$

$$= \frac{22.22 - 200}{22.22 + 200}$$

$$= -0.8$$

$$S_{21} = S_{12} = \sqrt{\frac{P_2}{P_1}} = \sqrt{\frac{4}{5}} = 0.894$$

$$S_{31} = S_{13} = \sqrt{\frac{P_3}{P_1}} = \sqrt{\frac{1}{5}} = 0.447$$

Since the circuit is Lossless

$$|S_{21}|^2 + |S_{22}|^2 + |S_{23}|^2 = 1$$

$$0.894^2 + 0.2^2 + |S_{23}|^2 = 1$$

$$S_{23} = S_{32} = \sqrt{1 - 0.894^2 - 0.2^2}$$

$$= 0.4$$

$$|S| = \begin{bmatrix} 0 & 0.894 & 0.447 \\ 0.894 & 0.2 & 0.4 \\ 0.447 & 0.4 & 0.8 \end{bmatrix} \checkmark$$

1). $\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$

$$f_0 = \frac{1}{2\pi\sqrt{50\times10^{-9}\times0.79\times10^{-12}}}$$

$$f_0 = 0.8\,GHz$$

b). Input Impedance at the resonance frequency.

$$Z_{IN} = R + j\omega L + \frac{1}{j\omega C}$$

$$P = \frac{1}{2} I^2 Z_{IN} = P_{loss} + 2j\omega(W_m - W_e)$$

At resonance $W_m = W_e$

$P = \frac{1}{2} I^2 R$ Purely Real

$$Z_{IN} = R = 2.5\,\Omega$$

c). Unloaded Q

$$Q = \frac{\omega_0 L}{R} = \frac{2\times\pi f_0\times 50\times10^{-9}}{2.5}$$

$$Q = 100.53$$

External Q

$$Q_E = \frac{\omega_0 L}{R_L} = \frac{2 \times \pi \times f_0 \times 50 \times 10^{-9}}{2.5}$$

$$Q_E = 100.53$$

$$Q_L = \left(\frac{1}{Q} + \frac{1}{Q_E}\right)^{-1}$$

$$Q_L = \left(\frac{1}{100.53} + \frac{1}{100.53}\right)^{-1}$$

$$\underline{\underline{Q_L = 50.265}}$$

3. Design a four-section *bandpass* lumped-element filter having a maximally flat group delay response. The fractional bandwidth should be 5% with a center frequency of 2 GHz. Scale the elements of the filter considering the impedance is $Z_0 = 50\ \Omega$.
   HINT: Use the tables below for the *low-pass* prototype design and prototype filter transformation.

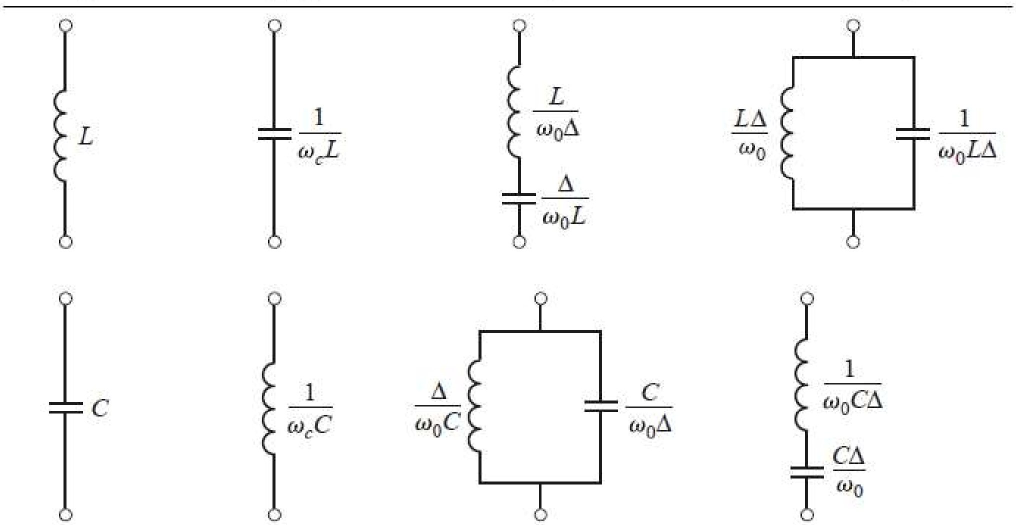
TABLE 8.5 Element Values for Maximally Flat Time Delay Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

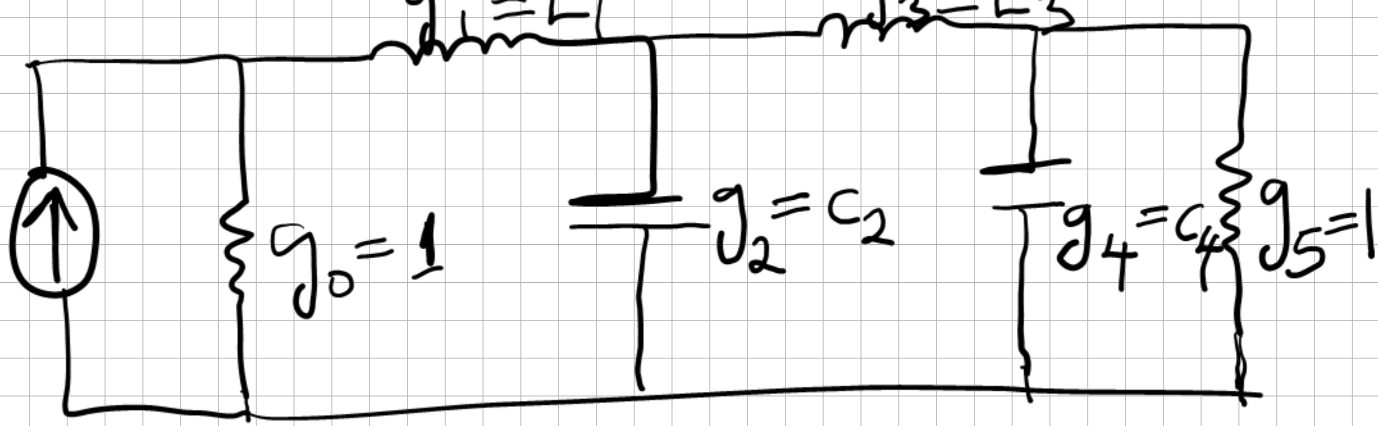
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.5774	0.4226	1.0000								
3	1.2550	0.5528	0.1922	1.0000							
4	1.0598	0.5116	0.3181	0.1104	1.0000						
5	0.9303	0.4577	0.3312	0.2090	0.0718	1.0000					
6	0.8377	0.4116	0.3158	0.2364	0.1480	0.0505	1.0000				
7	0.7677	0.3744	0.2944	0.2378	0.1778	0.1104	0.0375	1.0000			
8	0.7125	0.3446	0.2735	0.2297	0.1867	0.1387	0.0855	0.0289	1.0000		
9	0.6678	0.3203	0.2547	0.2184	0.1859	0.1506	0.1111	0.0682	0.0230	1.0000	
10	0.6305	0.3002	0.2384	0.2066	0.1808	0.1539	0.1240	0.0911	0.0557	0.0187	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

TABLE 8.6 Summary of Prototype Filter Transformations $\left(\Delta = \dfrac{\omega_2 - \omega_1}{\omega_0}\right)$

Low-pass	High-pass	Bandpass	Bandstop



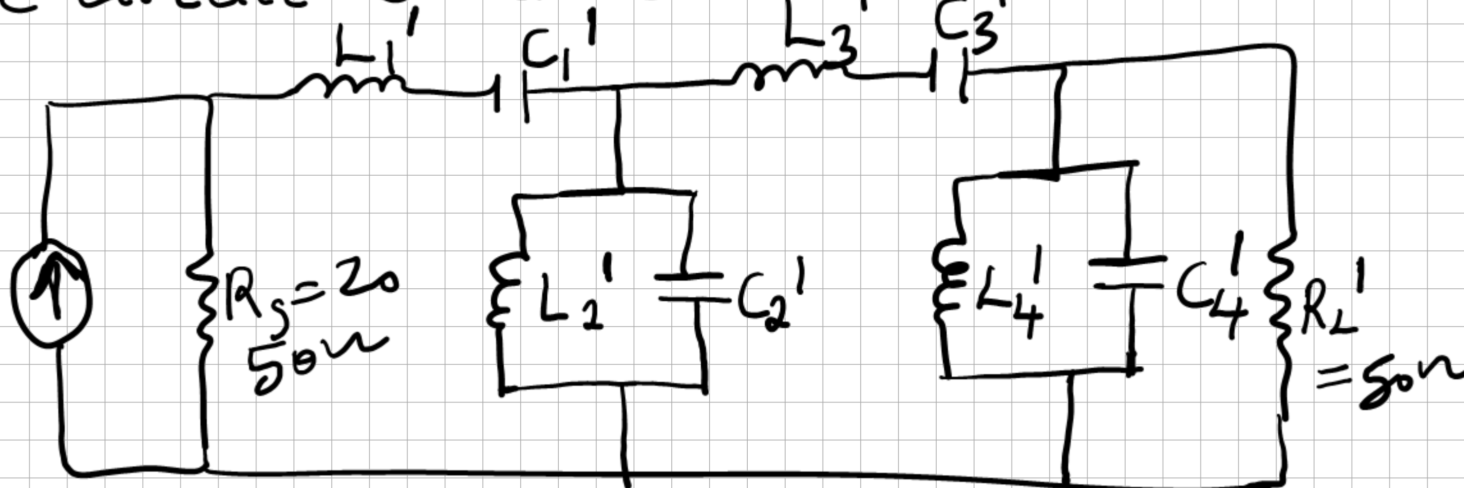


$N = 4$, $g_0 = 1$, $g_1 = 1.0598$, $g_2 = 0.5116$

$g_3 = 0.3181$ $g_4 = 0.1104$, $g_5 = 1$

Bandwidth = 5%, freq = 2 GHz, $Z_0 = 50\Omega$

The circuit becomes:



$$R_L' = g_5 \cdot Z_0 = 50\Omega$$

$$L_1' = \frac{L_1 \cdot Z_0}{\omega_0 \Delta} \quad C_1' = \frac{\Delta}{Z_0 \omega_0 L}$$

$$L_1' = \frac{1.0598 \times 50}{2 \times \pi \times 2 \times 10^9 \times 0.05} = 84.3\,nH$$

$$C_1' = \frac{0.05}{50 \times 2 \times \pi \times 2 \times 10^9 \times 1.0598} = 0.075 PF$$

$$L_2' = \frac{\Delta \cdot Z_0}{\omega_0 C_2} = \frac{0.05 \times 50}{2 \times \pi \times 2 \times 10^9 \times 0.5116} = 0.39\, nH$$

$$C_2' = \frac{C_2}{Z_0 \cdot \omega_0 \Delta} = \frac{0.5116}{50 \times 2 \times \pi \times 2 \times 10^9 \times 0.05}$$

$$= 16.28 PF$$

$$L_3' = \frac{L_3 Z_0}{\omega_0 \Delta} = \frac{0.3181 \times 50}{2 \times \pi \times 2 \times 10^9 \times 0.05}$$

$$= 25.3 nH$$

$$C_3' = \frac{\Delta}{Z_0 \omega_0 L_3} = \frac{0.05}{50 \times 2 \times \pi \times 2 \times 10^9 \times 0.3181}$$

$$= 0.25 PF$$

$$L_4' = \frac{\Delta Z_0}{\omega_0 C_4} = \frac{0.05 \times 50}{2 \times \pi \times 2 \times 10^9 \times 0.1104}$$

$$= 1.8 nH$$

$$C_4' = \frac{C_4}{Z_0 \omega_0 \Delta} = \frac{0.1104}{50 \times 2 \times \pi \times 2 \times 10^9 \times 0.05}$$

$$= \underline{3.5 PF}$$