

Optoelectronics

Key notes

4/ Light sources

1. Light sources

Selecting a proper light source is a fundamental aspect of an optoelectronic device. We can find multiple light sources available: traditional tungsten lamps, high pressure gas lamps, lasers, etc. All of them are diverse in the characteristics, so what should be our criteria in selecting them?

- Small size, so light can be easily injected in the fiber.
- Linear behaviour with current, the main goal is to translate electrical signals into light signals.
- Monochromatic, in order to match the wavelength with low dispersion and losses of the fiber
- Intense, so the losses over distance can be overcome
- Reliable, stable and cheap, these characteristics are always desirable.

Based on these criteria there is a good candidate light sources based in semiconductors from which there are two main groups:

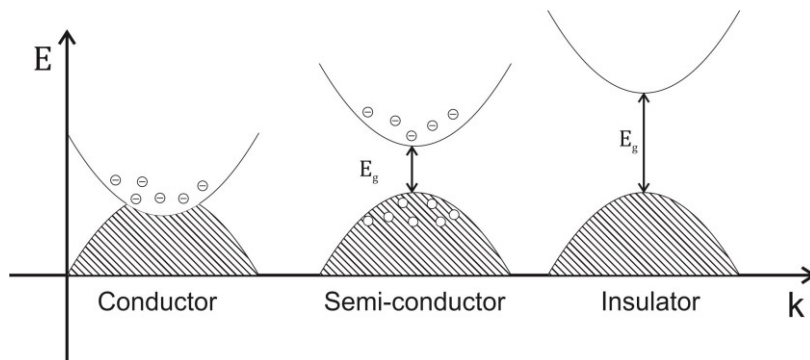
- a) Incoherent: light emitting diodes (LED's)
- b) Coherent: injection laser diodes (ILD's)

Both of them are fundamented in the same phenomenon, the emission of light by a p-n junction.

2. Semiconductors physics

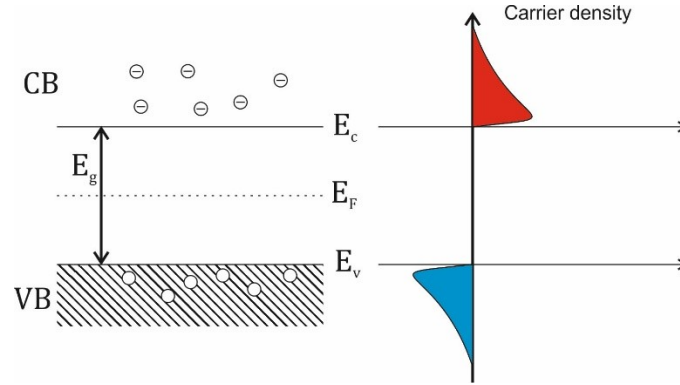
According to the band theory of solids, materials may be classified into three categories from the point of view of electrical conduction. These are (i) conductors, (ii) insulators, and (iii) semiconductors. This classification comes after solving the allowed levels of energy that the given material have, which are not only dependent in the material chemical composition but in the structure. From these energy levels, we can describe 3 regions:

- The valence band. Here lay all the energy levels that are occupied and allowed by the orbital in its less energetic level. At 0 degrees Kelvin all these levels are fully occupied by electrons. This level extends from the ground level $E = -\infty$ up to E_v
- The conduction band. This is the highest band of allowed energy levels inside the material, which extends from the vacuum level E_0 down to energy E_c
- The band gap, also called the forbidden gap. This region covers from the level limiting the conduction and the valence bands and it contains no energy levels.



The previous classification is based in this band gap.

- If there is no band gap or even the conduction and valence bands overlap, we have a *conductor*
- If the band gap energy is very high and therefore all the electrons will remain in the valence band, we have an *insulator*.
- If the band gap is small enough there is some mobility from both bands and we are in the case of a *semiconductor*.



We consider this band gap to be small when the energy from thermal effects can promote electrons to the upper levels, $E_c - E_v = E_g \approx 1 - 10 \text{ eV}$. Under these circumstances we can define the density of states $g(E)$ (energy levels, per unit of energy per unit of volume) of our given material and the probability of finding an electron on these levels $f(E)$

$$g(E) = \left(\frac{4\pi}{h^3}\right) (2m_e)^{3/2} (E - E_c)^{1/2}$$

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$

Where k is the Boltzmann's constant and E_F is the energy of the Fermi level (a virtual level of reference between the conduction and the valence bands). If the band gap is small enough ($E - E_F > 4kT$) we can neglect the 1 in the denominator so:

$$f(E) = e^{-\frac{E-E_F}{kT}}$$

This is called the Boltzmann's approximation and it is valid for all the semiconductors. Based on these equations we can calculate the number of free electrons in the conduction band simply as the integral in all these levels.

$$n = \int_{E_c}^{E_0} n(E) dE = \int_{E_c}^{E_0} g(E) f(E) dE \approx \int_{E_c}^{\infty} g(E) f(E) dE$$

Using the previous equations, we arrive to:

$$n = N_c e^{-\frac{E_c-E_F}{kT}}$$

$$N_c = 2 \left(2\pi m_e \frac{kT}{h^2} \right)^{3/2}$$

Where N_c is the effective density of states in the conduction band. In the same way, these calculations can be done for the density of holes (carriers in the valence band). In these case, we

use the probability of not finding an electron $1 - f(E)$ which is the same as saying finding an unoccupied state, or a state populated by a hole.

$$p = \int_{-\infty}^{E_v} p(E) dE = \int_{-\infty}^{E_v} g(E) [1 - f(E)] dE$$

$$p = N_v e^{\frac{E_v - E_F}{kT}}$$

$$N_v = 2 \left(2\pi m_h \frac{kT}{h^2} \right)^{3/2}$$

At the equilibrium when there is no presence of dopants both p and n must be equal and so we can define the intrinsic carrier density (n_i) as:

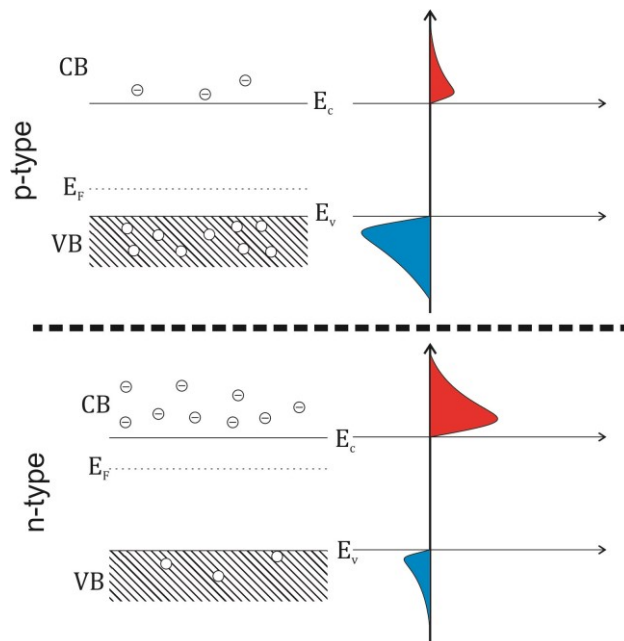
$$n = p = n_i$$

$$n_i^2 = np = N_v N_c e^{-\frac{E_c - E_F}{kT}} e^{\frac{E_v - E_F}{kT}} = N_v N_c e^{\frac{E_v - E_c}{kT}}$$

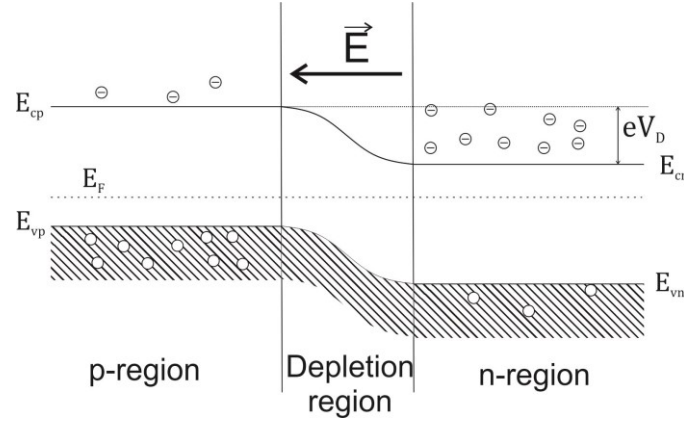
$$n_i = \sqrt{N_v N_c} e^{-\frac{E_g}{2kT}}$$

3. The p-n junction at equilibrium

In the previous section, the analysis of a homogenous perfect semiconductor was done but in a general case this is not going to be the situation. Normally, our material will have some impurities or dopants that will disturb the energy structure band. Based on the nature of these impurities we will see a variation in the number of free electrons (negative carriers) available. If the number of negative carriers (electrons) is increased by the dopants (donor) we talk about an n-type semiconductor and if it decreases (acceptor) we have a p-type semiconductor. These dopants are creating an intermediate level in the band gap that is able to promote electrons to the conduction band (donor level) or that takes some electrons from the valence band (acceptor level). As a consequence of this, the Fermi level is displaced towards the higher concentration of carriers.



The so called p-n junction is a material consisting in to semiconductors (one p and one n) fusion abruptly. This scenario creates a system of energy levels consequence of the dopants and acceptor on both sides of the interphase. As the electric charge concentration is different, and electric field appears (and so a potential V_D) in the intermediate region producing a diffusion of carriers between the p and n regions. This intermediate zone is called the depletion region.



At the equilibrium, we can describe the electron density in the p-region as:

$$n_{p0} = N_c e^{-\frac{E_{cp} - E_{Fp}}{kT}}$$

And for the n-region

$$n_{n0} = N_c e^{-\frac{E_{cn} - E_{Fn}}{kT}}$$

As the Fermi level remains constant in the whole system, the relation between both is:

$$\frac{n_{p0}}{n_{n0}} = e^{\frac{E_{cn} - E_{cp}}{kT}}$$

So the potential between the p-region and the n-region is:

$$eV_D = E_{cp} - E_{cn} = kT \ln \left(\frac{n_{n0}}{n_{p0}} \right)$$

To calculate the amount of carriers in each zone we need to consider that the concentration of dopants is small enough so the electron and hole densities remain independent of it.

$$n_{n0} p_{n0} = n_{p0} p_{p0} = n_i^2$$

And so for each region the donors (electron carriers) or acceptors (hole carriers) are no longer strongly dependant by temperature but dopant concentration (N_a acceptor concentration and N_d donor concentration).

$$p_{p0} \approx N_a \quad // \quad n_{n0} \approx N_d$$

Leading to:

$$n_{p0} = \frac{n_i^2}{p_{p0}} = \frac{n_i^2}{N_a}$$

$$p_{n0} = \frac{n_i^2}{n_{n0}} = \frac{n_i^2}{N_d}$$

Substituting these equalities in the hole and electron concentrations we arrive to:

$$n_{p0} = N_d e^{\frac{-eV_D}{kT}}$$

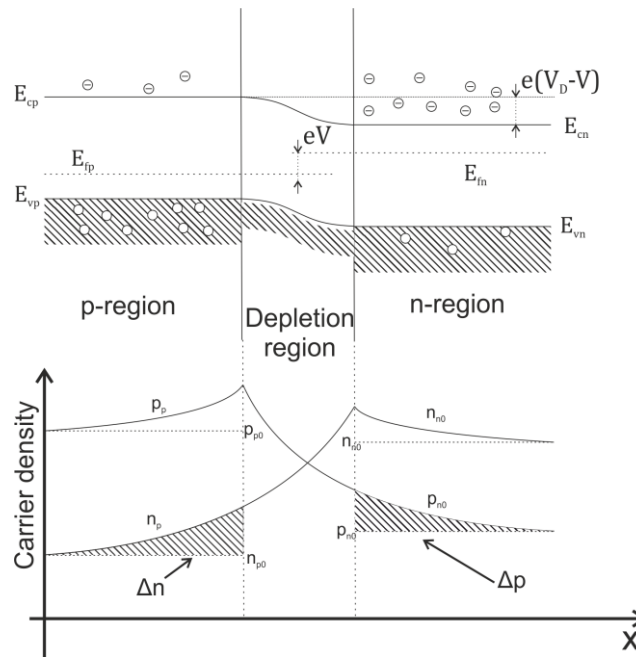
$$p_{n0} = N_a e^{\frac{-eV_D}{kT}}$$

Where

$$V_D = \frac{kT}{e} \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

4. The p-n junction with external voltage

The previously analysed case is the equilibrium state, no external influence. When a potential between both ends of the semiconductor is applied the carriers' densities change. The depletion region is the more sensitive part of the junction. The external voltage will introduce an extra potential difference in this region and so modifying the characteristics and the densities. The carriers will move from the p region to the n region (for electrons, and vice-versa for holes) creating a current that tries compensates the external potential difference.



Now, the potential well has changes and the densities for the new minor carriers can be written in different forms:

$$n_p = n_{n0} e^{\frac{-e(V_D - V)}{kT}} = N_d e^{\frac{-e(V_D - V)}{kT}} = n_{p0} e^{\frac{eV}{kT}}$$

$$p_n = p_{p0} e^{\frac{-e(V_D - V)}{kT}} = N_a e^{\frac{-e(V_D - V)}{kT}} = p_{n0} e^{\frac{eV}{kT}}$$

Basically we have created a deviation from the equilibrium in which there is an excess concentration of the minority carriers and majority carriers independently of the region.

$$\Delta p = p - p_{n0} = n - n_{n0}$$

$$\Delta n = n - n_{p0} = p - p_{p0}$$

5. Carriers' lifetime, diffusion length and injection efficiency

In this situation (p-n junction under an external voltage) there is a constant recombination of the carriers that move from both sides of the junction. In the depletion region they face the opposite carrier and a recombination of both takes place, annihilating both carriers and realising energy (that can be thermal, a high energy photon, a phonon or a combination of them). The involvement of a phonon will be defined by the structure of the material, if the minimum energy level of the conduction band has the same momentum as the higher level of the valence band we talk about direct band gap. In the opposite case a phonon is required and we have indirect band gap. Of course, for fabricating LEDs, direct band gaps are highly preferred.

The rate of recombination in a given material is proportional to the excess carrier density itself. Taking the n region, this recombination is expressed as:

$$-\frac{d\Delta p}{dt} = \frac{\Delta p}{\tau_h}$$

$$\Delta p(t) = \Delta p(0)e^{-t/\tau_h}$$

Where τ_h is the mean lifetime of the excess minority holes. The same expression is obtained for electrons

$$\Delta n(t) = \Delta n(0)e^{-t/\tau_e}$$

This mean life is a description of the time that a carrier will travel in the semiconductor before it recombines, in different materials the distance that this carrier will travel may vary as a function of the mobility of such carrier $\mu_{e/h}$ (defines by the material and the structure). In the same sense that we have define the rate of recombination, we can arrive to the flux of minority carriers using the diffusion coefficient:

$$D_{h/e} = \frac{\mu_{h/e}kT}{e}$$

Following a similar approach as before we obtain that its variation over the distance is also a function of the lifetime.

$$\Delta p(x) = \Delta p(0)e^{-\frac{x}{\sqrt{D_h\tau_h}}} = \Delta p(0)e^{-\frac{x}{L_h}}$$

Where we have introduced the diffusion length of the minority carriers $L_{h/e} = \sqrt{D_{h/e}\tau_{h/e}}$

This is very important because, as we have discussed before, the electric field only appears in the limit between the n-region and p-region (the so called depletion region), which basically means that the current in our semiconductor is only to the diffusion of holes and electrons. First we are going to consider this current density for holes in the n-region:

$$J_h = -eD_h \left. \frac{d\Delta p(x)}{dx} \right|_0 = e \frac{D_h}{L_h} \Delta p(0) = e \frac{D_h}{L_h} (p_n - p_{n0})$$

This carrier density was already calculated as a function of the potential applied V . Also, an analogue analysis can be done for the electron current density.

$$J_h = e \frac{D_h}{L_h} p_{n0} [e^{\frac{eV}{kT}} - 1]$$

$$J_e = e \frac{D_e}{L_e} n_{p0} [e^{\frac{eV}{kT}} - 1]$$

The addition of both current densities is the total density that can only reach a saturation value of J_s

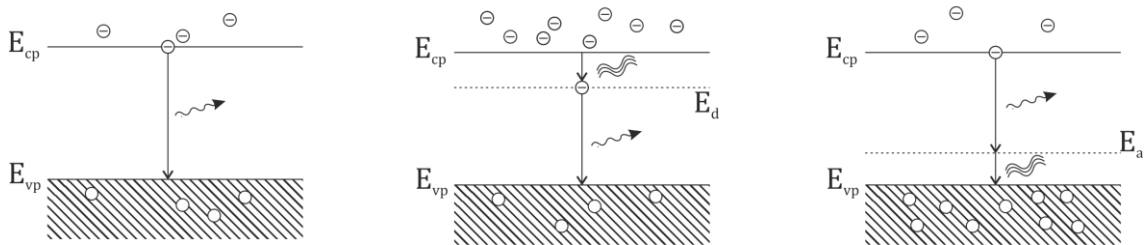
$$J = J_h + J_e = e \left(\frac{D_h}{L_h} p_{n0} + \frac{D_e}{L_e} n_{p0} \right) [e^{\frac{eV}{kT}} - 1] = J_s [e^{\frac{eV}{kT}} - 1]$$

A special and interesting case is that in which one of the regions is heavily doped causing that the current is mainly carried by either holes or electrons. If the heavily doped region is p, the holes recombine with electrons creating the afore mentioned recombination radiation. The efficiency of this process is called injection efficiency η_{inj}

$$\eta_{inj} = \frac{J_h}{J_h + J_e} = \frac{1}{1 + \frac{J_e}{J_h}} = \left(1 + \left(\frac{D_e}{D_h} \right) \left(\frac{L_h}{L_e} \right) \left(\frac{N_a}{N_d} \right) \right)^{-1}$$

6. Emission spectrum

As discussed before, when a potential difference is applied in a p-n junction, an excess of minority carriers will raise and try to move to the opposite side of the junction resulting in a recombination with the majority carriers. From this recombination, some energy will be emitted. The most probable transitions are the ones shown in the image.

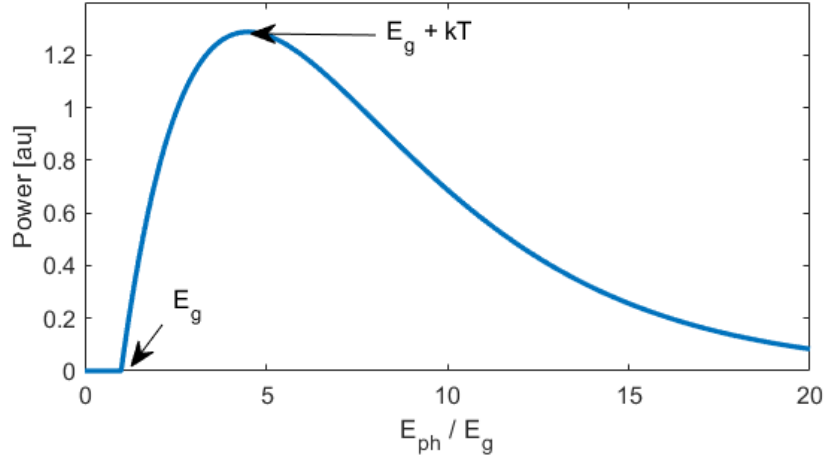


There must be noted that the last two cases involve the emission of low energy photons (thermal) and they are a lot less frequent as the energy levels from the acceptors and the donor are less dense. We are going to consider that only the first case happens, so the energy of a photon emitted in the recombination must be as low as the energy gap

$$E_{ph} = \frac{hc}{\lambda} \geq E_g$$

Through a simplified calculus we can obtain that the power radiated as a function of this energy is

$$P = \alpha (E_{ph} - E_g) e^{-\frac{(E_{ph} - E_g)}{kT}}$$



Where α is a proportionality constant. Although this equation is far from reality (it is usually a more symmetrical bell) it provides a good approximation to obtain the peak energy of emission λ_{peak} , the spectral width $\Delta\lambda$ and the relative spectral width γ

$$\lambda_{peak} = \frac{hc}{E_{peak}} = \frac{hc}{(E_g + kT)}$$

$$\Delta\lambda = -\left(\frac{hc}{E_{peak}^2}\right)\Delta E_{ph} \approx -\left(\frac{hc}{E_{peak}^2}\right)2.4kT$$

$$\gamma = \frac{\Delta\lambda}{\lambda_{peak}} \approx \frac{2.4kT}{E_{peak}}$$

7. Quantum efficiencies

Finally, in order to describe how is the efficiency of any semiconductor we must separate the losses intrinsic to the material characteristic and internal structure and the ones that appear due to the system and how the LED was designed. The efficiency due to the internal causes is the internal quantum efficiency η_{int} and the efficiency of the design of the LED is the external quantum efficiency η_{ext}

$$\eta = \eta_{int}\eta_{ext}$$

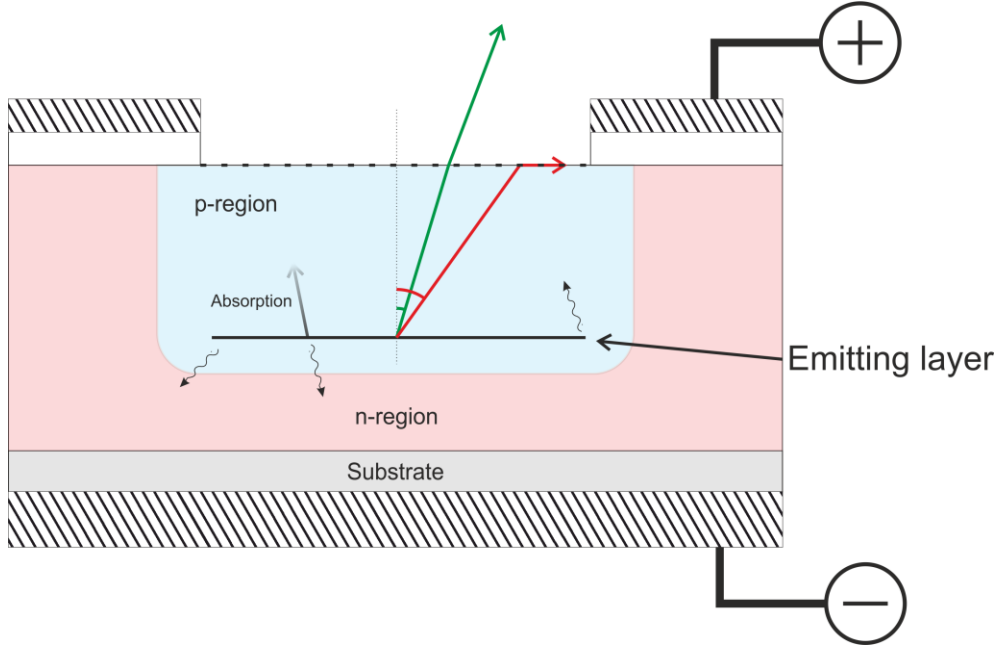
In a previous section we have talked about the lifetime of our carriers, which describes the probability if a carrier to recombine over time. Also, we have mentioned that this recombination may only emit a photon (radiative recombination, τ_{rr}) or relax by thermal emission (non-radiative recombination, τ_{nr}). The probability of this to effects is related and they add to obtain the total lifetime

$$\frac{1}{\tau} = \frac{1}{\tau_{rr}} + \frac{1}{\tau_{nr}}$$

In order to determine how efficient is a semiconductor emitting light we define the internal quantum efficiency as:

$$\eta_{int} = \frac{1/\tau_{rr}}{1/\tau} = \frac{1}{1 + \frac{\tau_{rr}}{\tau_{nr}}}$$

As it was said, the external quantum efficiency is impossible to generalize as it completely relies in the design and engineering process. As an example we are going to analyse the external efficiency of a surface emitting diode as the figure shows.



In this case we can consider that the light is emitted in a thin layer between the p and n regions (depletion region). The emission of a layer can be expressed as a Lambertian body considering both faces of the layer

$$\Phi_s^{total} = \int_s P d\Omega = 2 \left(\int_0^{\pi/2} P_0 2\pi \cos \theta \sin \theta d\theta \right) = 2\pi P_0$$

But only a fraction of this total flux will be useful. There are several factors that will limit this type of LED.

- Direction of the light. All the light emitted by the bot face of the layer will be absorbed and thermalized by the semiconductor, which already cuts by half the total flux, but also there will be limitations in the top layer. The light that tries to escape the diode by the top layer will encounter an interface between the semiconductor and the air. As the refractive index of the semiconductor is going to be a lot higher, there is a critical angle at which the light cannot longer cross. Thus, there are a factor F that determines the optical power that can be collected by the surface semiconductor-air.

$$F = \frac{\Phi_s}{\Phi_s^{total}} = \left(\frac{1}{2\pi P_0} \right) \int_0^{\theta_c} P_0 2\pi \cos \theta \sin \theta d\theta = \frac{1}{2} \sin^2 \theta_c$$

- Fresnel reflexion. As there is an interphase between to mediums, some fraction is going to be reflected and some transmitted. Under normal incidence and considering random polarization, the Fresnel coefficient is:

$$r = \left(\frac{n_s - n_a}{n_s + n_a} \right)^2$$

Where n_s is the index of the semiconductor and n_a is the index of the air. Even it is not completely true, we can consider θ_c to be very small so all the angles are approximately normal incidence. Besides, this is the reflectance, the transmittance is its complementary $t = (1 - r)$

- Absorption. There is a small distance between the emitting layer and the top surface that light must travel through. Even if this layer of semiconductor is very thin, the absorbance is extremely high as the energy gap can easily absorb our emitted photons, making the absorption not negligible. This last factor can be described as:

$$T = 1 - e^{-\alpha d} = 1 - a_s$$

Summarizing all these factors and including the internal efficiency, the efficiency of this LED is:

$$\eta = \eta_{int}\eta_{ext} = \eta_{int}FtT$$

$$\eta = \eta_{int} \left(\frac{\sin^2 \theta_c}{2} \right) (1 - r)(1 - a_s) = \eta_{int} \frac{2n_a^3}{n_s(n_s + n_a)^2}$$

Recommended bibliography:

- Anil K. Maini, << Laseres and Optoelectronics: Fundamentals, Devices and Applications>>, 2013 John Wiley and Sons Ltd, ISBN: 978-1-118-45887-7, Chapter (5, 9, 10)
- John P. Dakin and Robert G.W. Brown, << Handbook of Optoelectronics, Second Edition: Concepts, Devices and Techniques – Volume One>>, 2018 Taylor & Francis Group, ISBN: 978-1-4822-4178-5, Chapter (10,11,12,19)