ENIMEO

Fundamentals on coherent optics : Linear propagation in optical waveguides exercice 5 correction

Part 1

1/ numerical aperture $NA = \sqrt{n_1^2 - n_2^2} \Rightarrow n_1 = \sqrt{NA^2 + n_2^2}$ with $NA = 0.11$

@ λ_p , $n_2(\lambda_p) = 1.45 \Rightarrow n_1(\lambda_p) = \sqrt{0.11^2 + 1.45^2} = 1.454$

@ λ_m , $n_2(\lambda_m) = 1.461 \Rightarrow n_1(\lambda_m) = \sqrt{0.11^2 + 1.461^2} = 1.465$

2/ Weak guidance approximation (WGA) suitable if $\Delta = \frac{NA^2}{2 n_1^2} < 10^{-2}$

Over the range $[\lambda_m, \lambda_p]$, we can calculate $\Delta_{max} = \left.\frac{NA^2}{2 n_1^2}\right|_{min}$

$$\Delta_{max} = \frac{0.11^2}{2 \times 1.454^2} \simeq 2.9\ 10^{-3} < 10^{-2} \Rightarrow \text{WGA suitable}$$

This means that we are allowed to consider that the TE, TM, EH and HE electromagnetic modes are degenerated into families of modes called "LP modes" (for "linearly polarized modes"). In a LP mode, the lines of the electric field are rectilinear and parallel to each other (i.e. they are oriented in the same given direction) all over the mode cross section,

3/ When perturbations are applied to the fiber, the speckle at the output varies if several modes are guided (due to mode coupling and phase shift changes between the modes).

If the spatial intensity distribution does not change, this means that only one mode propagates. This mode is the fundamental mode (LP_{01} mode).

$\Rightarrow$ the fiber works in the single mode regime $\Rightarrow V(\lambda p) < 2.405$

$$V(\lambda_p) < 2.405 \Rightarrow \frac{2\pi}{\lambda_p} a\, NA < 2.405 \Leftrightarrow a < \frac{2.405\, \lambda_p}{2\pi\, NA}$$

numerical application : $\underline{\underline{a <}} \dfrac{2.405 \times 1.064}{2\pi \times 0.11} = \underline{\underline{3.7\ \mu m}}$

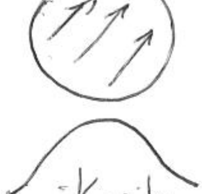
4/ $\nu_S = \nu_m - 5R$ with $\nu_m = \dfrac{c}{\lambda_m}$ and $\nu_S = \dfrac{c}{\lambda_S}$

thus $\dfrac{c}{\lambda_S} = \dfrac{c}{\lambda_m} - 5R \Leftrightarrow \lambda_S = \dfrac{c}{\dfrac{c}{\lambda_m} - 5R} = \dfrac{\lambda_m}{1 - 5R\dfrac{\lambda_m}{c}}$

$$= \frac{532\ 10^{-9}}{1 - 5 \times 13\ 10^{12}\ \dfrac{532\ 10^{-9}}{3\ 10^{8}}} = 601\ 10^{-9}\ m$$
$$= 601\ nm$$

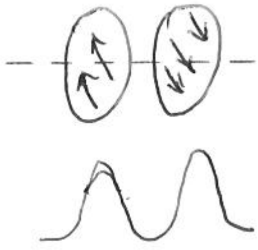
5/ D_1 is the intensity distribution in the LP_{02} mode ($V_c = 3.83$)
D_2 " " in the LP_{21} mode ($V_c = 3.83$)
↑ see provided tables and dispersion curves

This means that, at λ_S, we have $V > 3.83$
All the modes with a cutoff frequency lower than V can propagate.
The mode with $V_c < 3.83$ can propagate. they are:
LP_{01} mode ($V_c = 0$) and LP_{11} mode ($V_c = 2.405$)
(see the provided information: dispersion curves and table of zeros of Bessel functions)



LP_{01} mode

intensity



LP_{11} mode

intensity along - - - -

6/ LP_{02} and LP_{21} modes propagate @ $\lambda_S \Rightarrow V(\lambda_S) > V_c|_{LP_{02}} = V_c|_{LP_{21}} = 3.83$

$$\Rightarrow \frac{2\pi}{\lambda_S} a\, NA > 3.83 \Leftrightarrow a > \frac{3.83\, \lambda_S}{2\pi\, NA}$$

numerical application : $\underline{\underline{a >}} \dfrac{3.83 \times 0.601}{2\pi \times 0.11} = 3.33\ \mu m > \underline{\underline{3.3\ \mu m}}$

Finally, with 3/ we can write $3.3\,\mu m < a < 3.7\,\mu m$

$$a = 3.5 \pm 0.2\,\mu m$$

7/ $V(\lambda_m) = \frac{2\pi}{\lambda_m}\, a\, NA = \frac{2\pi}{0.532} \times 3.5 \times 0.11 = 4.55$

For the LP_{12} mode shown in D_3 we have $V_c|_{LP_{12}} = 5.52$
(see the tables and dispersion curves). Thus $V(\lambda_m) < V_c|_{LP_{12}}$
This means that the LP_{12} mode cannot propagate @ λ_m in this fiber.

8/ $V(\lambda_m) = 4.55$ $\quad V(\lambda_p) = \frac{2\pi}{\lambda_p}\, a\, NA = \frac{2\pi}{\lambda_m}\, a\, NA\, \frac{\lambda_m}{\lambda_p}$

$$= V(\lambda_m)\, \frac{\lambda_p/2}{\lambda_p} = 4.55 \times \frac{1}{2} = 2.27$$

On the dispersion curve of LP_{01} mode we read:
for $V(\lambda_m) = 4.55 \rightarrow B = 0.82$; for $V(\lambda_p) = 2.27$ $\quad B = 0.45$

Because $B = \frac{n_e^2 - n_2^2}{\underbrace{(n_1^2 - n_2^2)}_{NA}} \Rightarrow n_e = \sqrt{B.NA^2 + n_2^2}$

numerical application @ λ_m $\quad n_e(\lambda_m) = \sqrt{0.82 \times 0.11^2 + 1.461^2} = 1.4644$

$$v_\varphi = \frac{c}{n_e(\lambda_m)} = \frac{3\,10^8}{1.4644} = 2.049\,10^8\, m\, s^{-1}$$

@ λ_p $\quad n_e = \sqrt{0.45 \times 0.11^2 + 1.45^2} = 1.4519$

$$v_\varphi = \frac{c}{n_e(\lambda_p)} = \frac{3\,10^8}{1.4519} = 2.066\,10^8\, m\, s^{-1}$$

9/ The field from fiber F2, launched in F_1, is perfectly Gaussian.
It is an even field. Among the modes able to propagate in F_1 @ λ_m
(i.e. LP_{01}, LP_{11}, LP_{02} and LP_{21}), it cannot excite odd modes
because the integral overlap is 0. Thus, it cannot excite the

LP_{11} mode nor the LP_{21} mode. It can only excite the LP_{01} mode and the LP_{02} mode.

At the output of fiber F_1, we will observe a superimposition of LP_{01} and LP_{02} modes which is a pattern with a circular symmetry

Part 2

1/ $V(\lambda_p) = \frac{2\pi}{\lambda_p} a \, NA = \frac{2\pi}{1.064} \times 3.5 \times 0.11 = 2.27$ (already calculated)

$$NA = \sqrt{n_1^2 - n_2^2} \Rightarrow \underline{\underline{n_1}} = \sqrt{n_2^2 + NA^2} = \sqrt{1.45 + 0.11^2} = \underline{\underline{1.454 \text{ @ } \lambda_p}}$$

2/ Modal dispersion occurs in fibers when several modes can propagate. In fiber F_1 @ λ_p, as $V(\lambda_p) = 2.27 < 2.405$, only the fundamental mode (LP_{01} mode) can propagate.

$\Rightarrow$ single mode regime $\Rightarrow$ no modal dispersion.

3/ Chromatic dispersion $D_c \simeq$ material dispersion (D_m) + dispersion of the guide (D_g)

4/ We know that $v_g = \frac{L}{t_g} = \frac{c}{N_g} \Rightarrow t_g = \frac{L}{c} N_g$ (1)

By definition $D_m = \frac{\Delta\tau_m}{L\,\Delta\lambda}$ (2) where $\Delta\tau_m$ is the time broadening of a pulse a spectral width $\Delta\lambda$ over a length L (see the provided relations)

$\Delta\tau_m = \frac{dt_g}{d\lambda}\Delta\lambda$ with (1) $\Delta\tau_m = \frac{L}{c}\Delta\lambda \frac{d(N_g)}{d\lambda}$ (3)

with (3), (2) becomes $D_m = \frac{1}{c}\frac{dN_g}{d\lambda} = \frac{1}{c}\frac{d}{d\lambda}\left(n_1 - \lambda\frac{dn_1}{d\lambda}\right)$

$$= \frac{1}{c}\left[\frac{dn_1}{d\lambda} - \left(\frac{dn_1}{d\lambda} + \lambda\frac{d^2n_1}{d\lambda^2}\right)\right] = -\frac{\lambda}{c}\frac{d^2n_1}{d\lambda^2}$$

5/ Dispersion of the guide D_g

for $V = 2.27$ we read on the provided curves $\frac{d^2(VB)}{dV^2} \simeq 0.1$

thus $V \frac{d^2(VB)}{dV^2} = 2.27 \times 0.1 = 0.227$

$$-\frac{n_1 \Delta}{c \lambda} = -\frac{n_1}{c\lambda} \frac{NA^2}{2n_1^2} = -\frac{NA^2}{2 c \lambda n_1} = -\frac{0.11^2}{2 \times 3\,10^8 \times 1.064\,10^{-6} \times 1.454} = -13\,10^{-6}\,s\,m^{-2}$$

$$\underline{D_g} = -\frac{n_1 \Delta}{c \lambda} V \frac{d^2(VB)}{dV^2} = (-13\,10^{-6}) \times 0.227 = -2.96\,10^{-6}\,s\,m^{-2}$$
$$= \underline{-2.96\ ps/(nm.km)}$$

Dispersion of the material (material dispersion D_m)

@ $\lambda_p = 1.064\,\mu m$ we read on the curve $\frac{d^2 n_1}{d\lambda^2} \simeq 0.008\,\mu m^{-2}$
$$= 0.008\,10^{12}\,m^{-2}$$
$$= 8\,10^9\,m^{-2}$$

$$\underline{D_m} = -\frac{\lambda}{c} \frac{d^2 n_1}{d\lambda^2} = -\frac{1.064\,10^{-6}}{3\,10^8} \times 8\,10^9 = -2.84\,10^{-5}\,s\,m^{-2}$$
$$= -28.4\,10^{-6}\,s\,m^{-2}$$
$$= \underline{-28.4\ ps/(nm.km)}$$

Chromatic Dispersion D_c

$$\underline{D_c} \simeq D_m + D_g = -2.96 - 28.4 = -31.36 \simeq \underline{-31.4\ ps/(nm.km)}$$

6/ By definition $D_c \simeq \frac{t_{g_2} - t_{g_1}}{L(\lambda_2 - \lambda_1)}$ Here we have $D_c < 0$

As $\lambda_1 < \lambda_2 \Rightarrow \lambda_2 - \lambda_1 > 0$ Thus $t_{g_2} < t_{g_1}$ for having $D_c < 0$

This means that the wavepacket P_2 will exit the fiber before the wavepacket P_1 (<u>P_2 will exit first</u>)