4. Matching circuits with lumped elements

RF circuits on silicon rarely use matching networks like stubs, or transformers with transmission lines to save space. For example, a quarter of a wavelength is 2.5 cm long at 3 GHz, and a quarter-wave stub will take up far too much space on a chip of just a few square millimeters. Thus, the localized elements that we studied in the previous course are systematically being used to realize impedance matching of the active elements (Transistors, Diodes ... etc).
The reduced footprint also leads to minimizing the number of elements to be used and this course will be limited to the simplest circuit design techniques, with only a self and a capacitor.

RLC circuit transformers

RLC circuits are the most used circuits in RF circuits. Indeed, they make it possible to produce matching circuits, resonators for oscillators, or frequency selective filters.

1.1.2. Series Circuit

The simplest circuit is shown below :

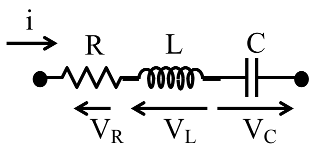

Figure 18 - Circuit R-L-C série

In this circuits, the voltage, V_R, V_L, V_C accross the different elements can be written as a function of i, the current flowing in the circuit.

$$V_R = R.i$$
$$V_L = jL\omega.i$$
$$V_C = \frac{j}{C\omega}.i$$

This circuit resonates when the inductor voltage V_L is the opposite of the capacitor voltage V_C. Cette égalité peut s'écrire de la façon suivante :

$$L\omega = \frac{1}{C\omega}$$

The frequency for which this happens, ω_0, deduced from the previous equation:

$$\omega_0^2 = \frac{1}{LC}$$

at resonance, $V_C + V_L = 0$, and i can be written as:

$$i = \frac{V_R}{R}$$

The voltage accross the inductor can be written as:

$$V_L = jL\omega_0 . i$$

and:

$$V_L = jL\omega_0 . \frac{V_R}{R}$$

The ratio between the modulus of voltages V_L and V_R is called Q factor (*coefficient de surtension in French)*, Q_0, of the resonator. Q_0 can be written as:

$$Q_0 = \frac{V_L}{V_R} = \frac{L\omega_0}{R}$$

This circuit Q-factor can be written as (also):

$$Q_0 = \frac{V_C}{V_R} = \frac{1}{RC\omega_0}$$

Q-factor can also be written as (also, again):

$$Q_0 = \frac{\sqrt{L/C}}{R}$$

This expression is the ratio of the resonator impedance ($\sqrt{L/C}$) and the resistor R, and can be computed independently of ω_0.
This impedance ratio will be useful in the following computations.

1.1.3. Parallel circuit

An RLC circuit can also be done by connecting inductors, capacitors and resistor in series. A very similar analysis can be conducted analyzing currents in each branch.

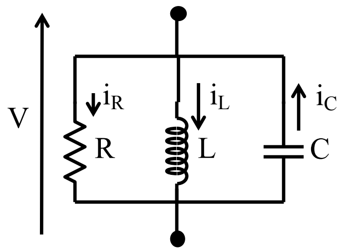

Figure 19 - Circuit R-L-C parallèle

In this circuit, the currents, i_R, i_L, i_C crossing each element can be written as a function of V the voltage on the circuit.

$$i_R = \frac{V}{R}$$
$$i_L = j\frac{V}{L\omega}$$
$$i_C = jC\omega.V$$

The circuit resonates when the current across the inductor, i_L, cancels i_C the current accross the capacitor. This leads to the same equation as previously:

$$L\omega = \frac{1}{C\omega}$$

The frequency for which this equation is true, ω_0, can be deduced from the previous equation:

$$\omega_0^2 = \frac{1}{LC}$$

at resonance, $i_C + i_L = 0$, and the current in the resistor i_R can be written as follows:

$$V = R.i_R$$

The current across the capacitor can be written as:

$$i_C = jC\omega_0 V$$

then

$$i_C = jC\omega_0.R.i_R$$

The ratio between i_c and i_R is the Q of the resonator Q_0, and this coefficient is the ratio between branch *currents*:

$$Q_0 = \frac{i_C}{i_R} = RC\omega_0$$

Q_0 can be written as:

$$Q_0 = \frac{i_L}{i_R} = \frac{R}{L\omega_0}$$

This can also be written as:

$$Q_0 = \frac{R}{\sqrt{L/C}}$$

This expression is the ratio between the impedance of the resonator ($1/\sqrt{\frac{L}{C}}$) and the resistor R, and it can be computed without determining ω_0.

1.1.4. Modified parallel resonant circuit

These basic circuits can be slightly modified in order to analyze matching conventional circuits. Parallel circuit elements, are L_P, R_P, Q_P. The modified circuit elements are L_S, R_S, Q_S.

The proposed transformation is shown below:

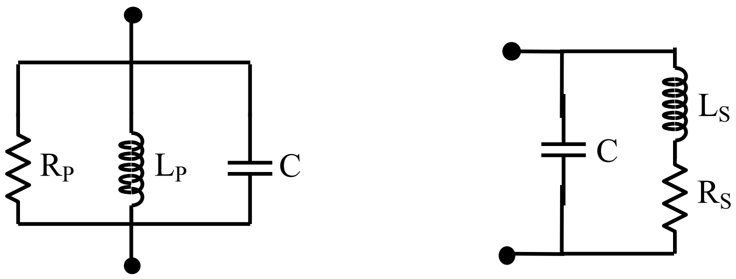

Figure 20 - Circuit R-L-C parallèle (gauche) et modifié (droite)

One can find an equivalence between the two circuits by computing the impedance of the inductor in parallel with the resistance (Z_P) or in series (Z_S) for both cases:

$$Z_P = \frac{jR_pL_P\omega_0}{R_P + jL_P\omega_0}$$

$$Z_P = jR_PL_P\omega_0.\frac{R_P - jL_P\omega_0}{R_P^2 + L_P^2\omega_0^2} = \frac{jR_P^2L_P\omega_0 + R_PL_P^2\omega_0^2}{R_P^2 + L_P^2\omega_0^2}$$

$$Z_S = jL_S\omega_0 + R_S$$

The equivalence is true if both Q_P and Q_S are equal.

$$Q_P = \frac{R_P}{L_P\omega_0} \rightarrow Q_P^2 = \frac{R_P^2}{(L_P\omega_0)^2}$$

$$(L_P\omega_0)^2 = \frac{R_P^2}{Q^2}$$

If the real parts of both impedances are equal and $Q_P = Q_S = Q$, one can write:

$$R_S = \frac{R_P(L_P\omega_0)^2}{R_P^2 + (L_P\omega_0)^2}$$

$$R_S = \frac{R_P(\frac{R_P^2}{Q^2})}{R_P^2 + \frac{R_P^2}{Q^2}}$$

$$R_S = R_P\frac{1}{Q^2 + 1}$$

then

$$R_P = R_S(1 + Q^2)$$

This circuit behaves like a resonating transformer with a transformation ratio of $(1 + Q^2)$.

One can also write that the imaginary parts are equal, leading to:

$$L_P = L_S(\frac{Q^2 + 1}{Q^2})$$

in practice, if $Q \gg 1, L_P \approx L_S$

in a similar manner:

$$C_P = C_S(\frac{Q^2 + 1}{Q^2})$$

in practice, if $Q \gg 1, C_P \approx C_S$

One can finally extract the input impedances using the following equation:

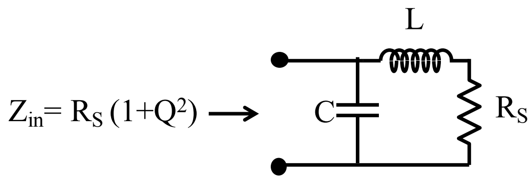

Figure 21 - Impédance d'entrée du circuit résonant modifié

One should note that $Z_{in} > R_s$ and that this resonating transformer allows *increasing* impédance compared to the one at the output.

1.1.5. Modified series circuit

One can follow the same priciple for a series resonant circuit. In this case the starting point is the following:

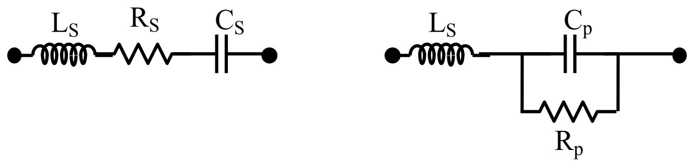

Figure 22 – Circuit R-L-C série (à gauche) en circuit modifié (à droite)

One can find an equivalence between the two circuits impedance (Z_S) or in parallel (Z_P). The previous relationship can be used to determine the equivalence between R_P/R_S

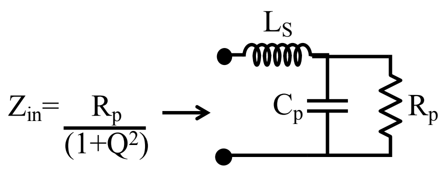

Figure 23 – Impédance d'entrée du circuit résonant modifié

One should note that $Z_{in} < R_s$ and that this resonant transformer allows *decreasing* the impedance seen from the input of the circuit.

In the case of both circuits, if $Q \gg 1$, then the transformation ratios can be simplified as Q^2, in the case of the first circuit in Figure 22, and $1/Q^2$in the case of the circuit in Figure 23.

We can then write that the transformation ratio is, $\frac{R_P}{R_S} = Q^2$ and therefore:

$$Q \approx \sqrt{\frac{R_P}{R_S}}$$

One can also note that:

$$R_P R_S \approx \frac{L_S}{C} \approx Z_0^2$$

Where Z_0 is the characteristic impedance of the L-C circuit. The same equation is used for the quarter wavelength line transformer, which is equivalent to the proposed circuit.