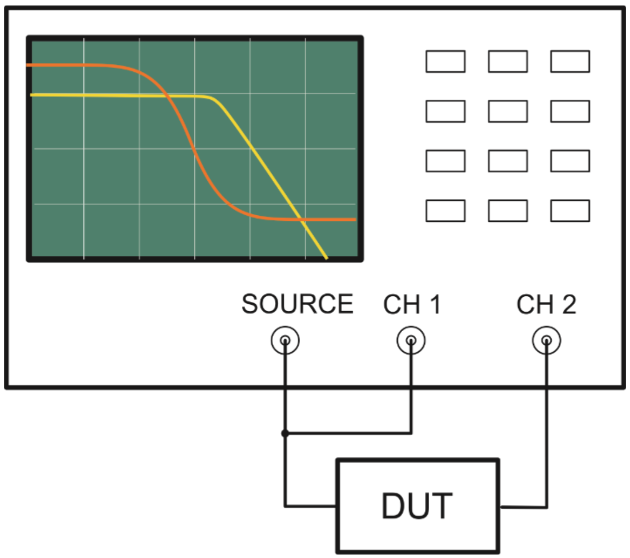
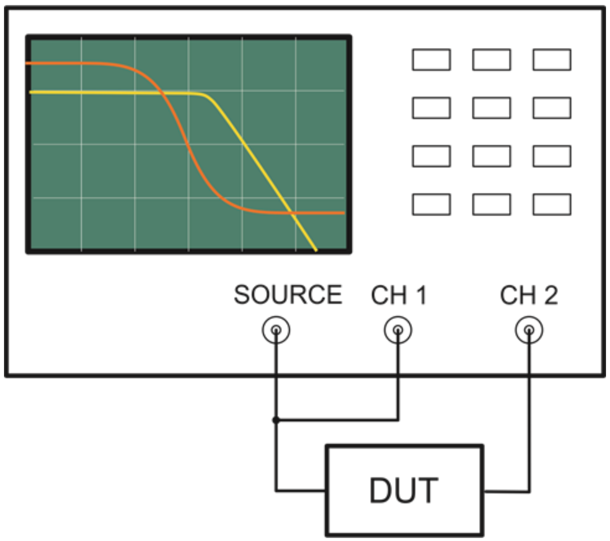
Network analysis



- commonly the measurement is carried out by supplying a sinusoid to the DUT input and by measuring its output
  - *see the use of the modified swept spectrum analyzer*
  - *Vector Network Analyzer*

- by using an FFT analyzer it is possible to use a faster technique for bandwidth limited analysis:
  - recalling that an FFT analyzer behaves like a bank-of-filter analyzer, if we supply the DUT with a signal producing the same power in each FFT bins, the FFT of the DUT response, appropriately scaled, will represent the transfer function of the DUT itself.

FFT network analyzer

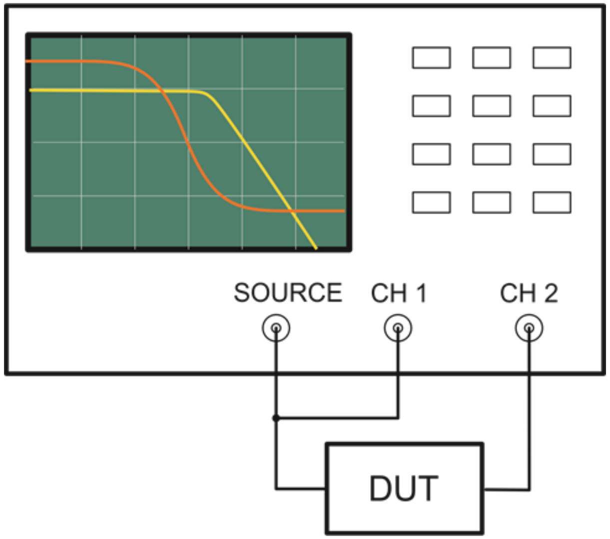


- internal source
  - PRN: pseudo random noise. A "white noise" over the bandwidth of interest, the signal can be synchronized with the time record, no leakage also without windowing

- chirp sine: a sum of sinusoids having frequencies centered with the FFT bins

- white noise (truly random): the generator cannot be synchronized with the time record

FFT network analyzer

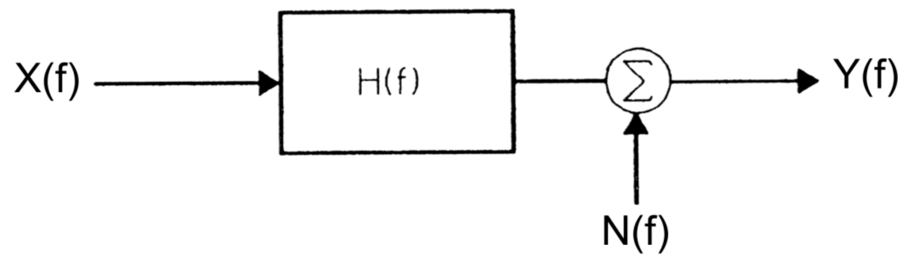


- for a given value of the peak to peak voltage of the stimulus signal, the chirp sine guaranties a better S/N ratio of the measurement

- the white noise generator is better suited for studying non perfectly linear systems: by averaging successive measurements it is possible to reduce the effect of the signal distortions introduced by the (small) non-linearities.

Transfer function measurement (1)

- model of the measurement in presence of noise:



$$Y(f) = X(f) \cdot H(f) + N(f)$$

- by computing the transfer function as the simple ratio between $Y(f)$ and $X(f)$ we introduce errors directly depending on the noise level:

$$\frac{Y(f)}{X(f)} = H(f) + \frac{N(f)}{X(f)}$$

- we can try to compute the energy spectral density of the output signal as:

$$G_{yy}(f) = Y(f) \cdot Y^*(f) = \big(X(f) \cdot H(f) + N(f)\big)\big(X^*(f) \cdot H^*(f) + N^*(f)\big) =$$

$$= G_{xx}(f)|H(f)|^2 + |N(f)|^2 + \underbrace{X(f) \cdot H(f) \cdot N^*(f) + X^*(f) \cdot H^*(f) \cdot N(f)}_{by\ averaging\ in\ time\ these\ terms\ approaches\ zero}$$

Transfer function measurement (2)

- and then averaging in time, reminding $N(t)$ is uncorrelated with $X(t)$, we have: $\frac{\overline{G_{yy}}}{\overline{G_{xx}}} = |H(f)|^2 + \frac{\overline{|N(f)|}^2}{\overline{G_{xx}}}$

  it is evident the error affecting the measurement of $H(f)$

- instead, calculating the cross spectrum between X and Y and taking the time averaging we obtain:

$$\overline{G_{yx}(f)} = \overline{Y(f) \cdot X^*(f)} = \overline{\big(X(f)H(f) + N(f)\big) \cdot X^*(f)} = \overline{G_{xx} \cdot H(f)} + \overline{N(f) \cdot X^*(f)}$$

$$\Rightarrow \quad \frac{\overline{G_{yx}(f)}}{\overline{G_{xx}(f)}} = H(f) + \underbrace{\frac{N(f) \cdot X^*(f)}{G_{xx}(f)}}_{term\ approaches\ zero}$$

the effect of the noise becomes now negligible

Correlation measurements

- given the functions $x(t)$ and $y(t)$, the correlation function $R_{xy}(\tau)$ is defined as:

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\cdot y(t+\tau)dt$$

- the correlation theorem states that:

$$R_{xy}(\tau) = F^{-1}[X(f)\cdot Y^*(f)]$$

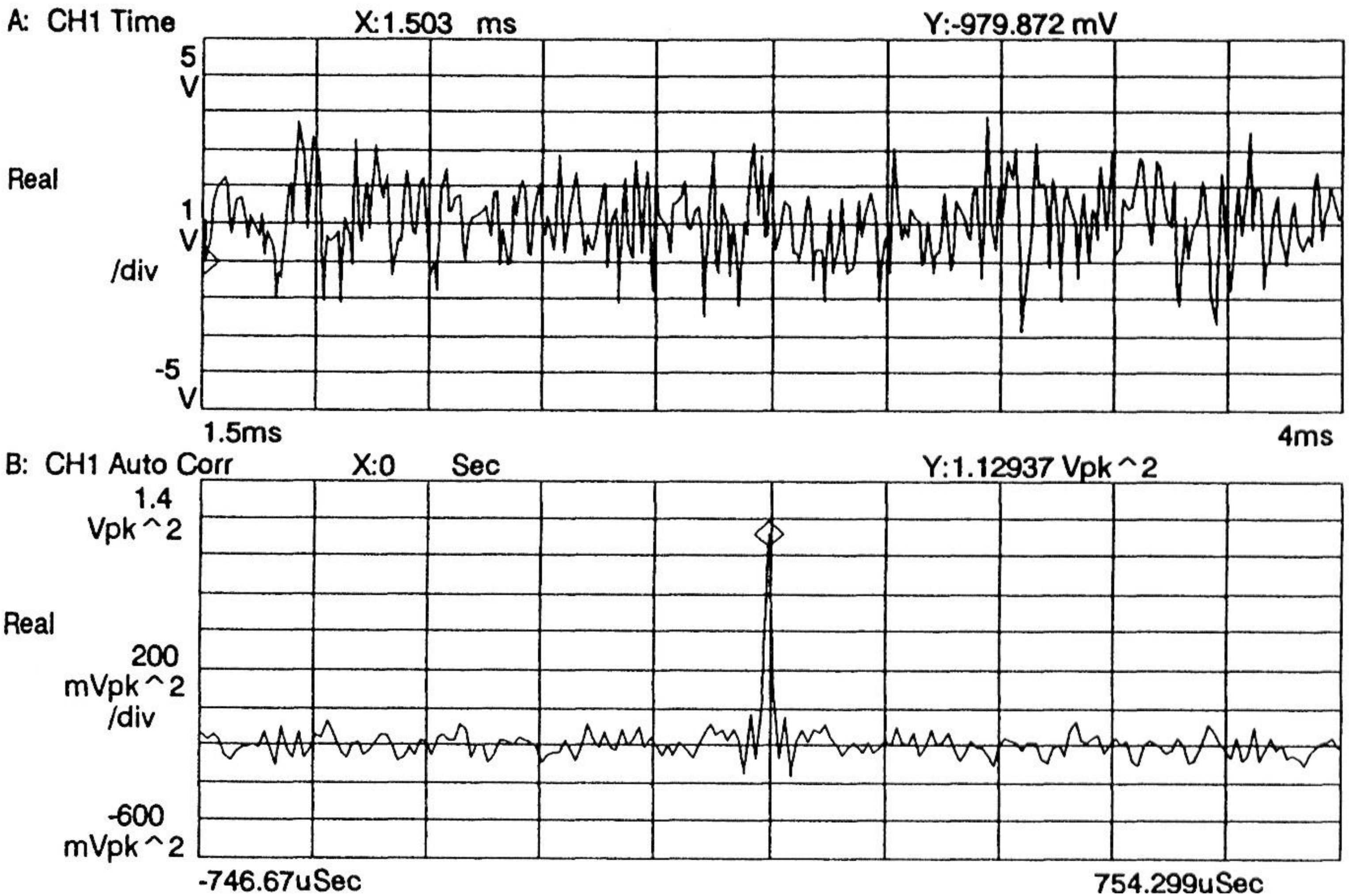
- if $x(t)$ and $y(t)$ become the same signal we will obtain the auto-correlation function:

$$R_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\cdot x(t+\tau)dt$$

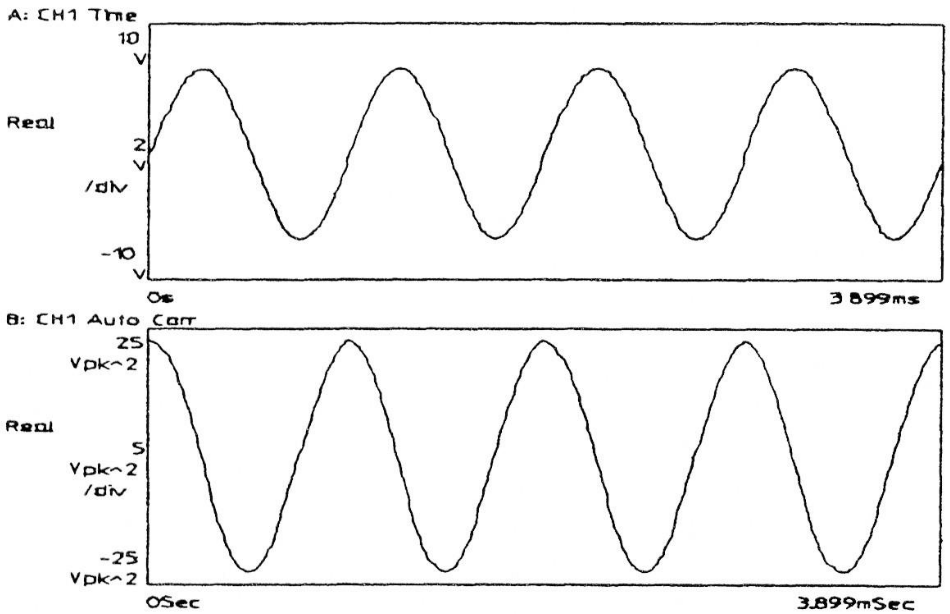
- the correlation theorem now states that:

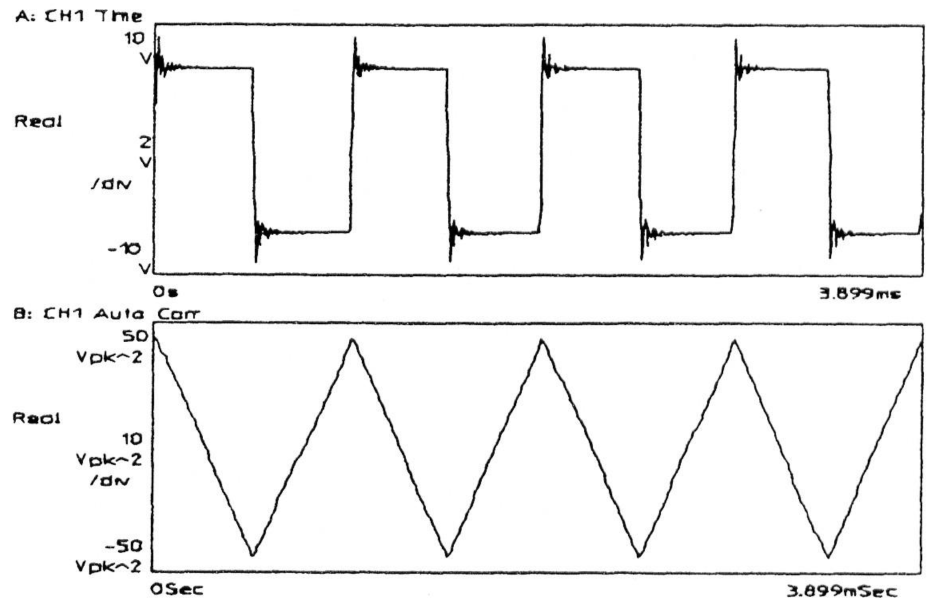
$$R_{xx}(\tau) = F^{-1}[X(f)\cdot X^*(f)]$$

Auto-correlation: random noise



Auto-correlation: periodic signal





Auto-correlation: noisy periodic signal