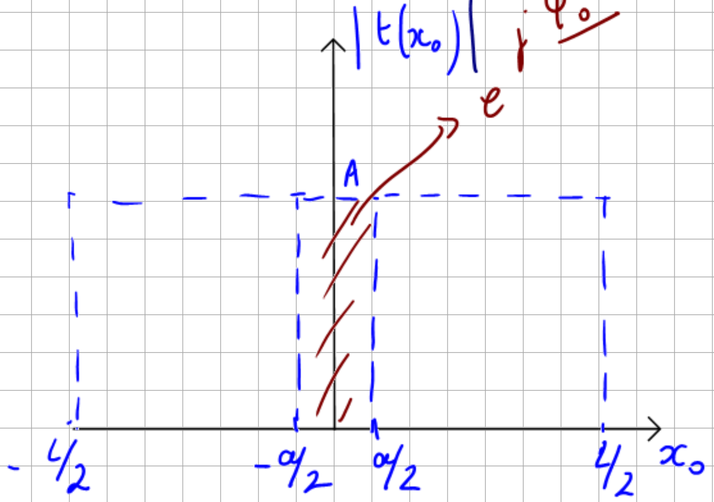
Tutorial 3 Spatial Filtering Fundamentals of photonics

I



$$t(x_0) = A\left(\mathrm{rect}_L(x_0) - \mathrm{rect}_a(x) + e^{j\varphi_0}\,\mathrm{rect}_a(x)\right)$$

$\varphi_0 \ll 1$ rad

$$t(x_0) = A\Big(\mathrm{rect}_L(x_0) - \cancel{\mathrm{rect}_a(x)} + \underbrace{(\cancel{1} + j\varphi_0)}_{\text{Taylor expansion of } e^{j\varphi_0}}\,\mathrm{rect}_a(x)\Big)$$

$$\boxed{t(x_0) = A\left(\mathrm{rect}_L(x_0) + j\varphi_0\,\mathrm{rect}_a(x)\right)}$$

Incident beam in $z = 0^-$ plane:

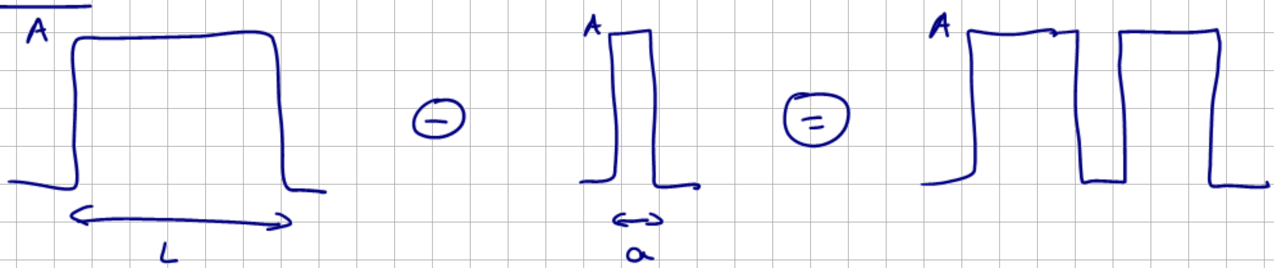
$$E_i(z=0^-) = 1 \cdot e^{-jk\underset{0}{\underset{\shortparallel}{z}}} = 1$$

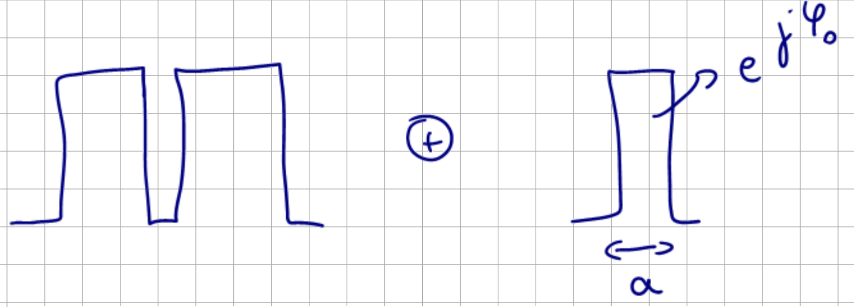
After the pupil in $z = 0^+$

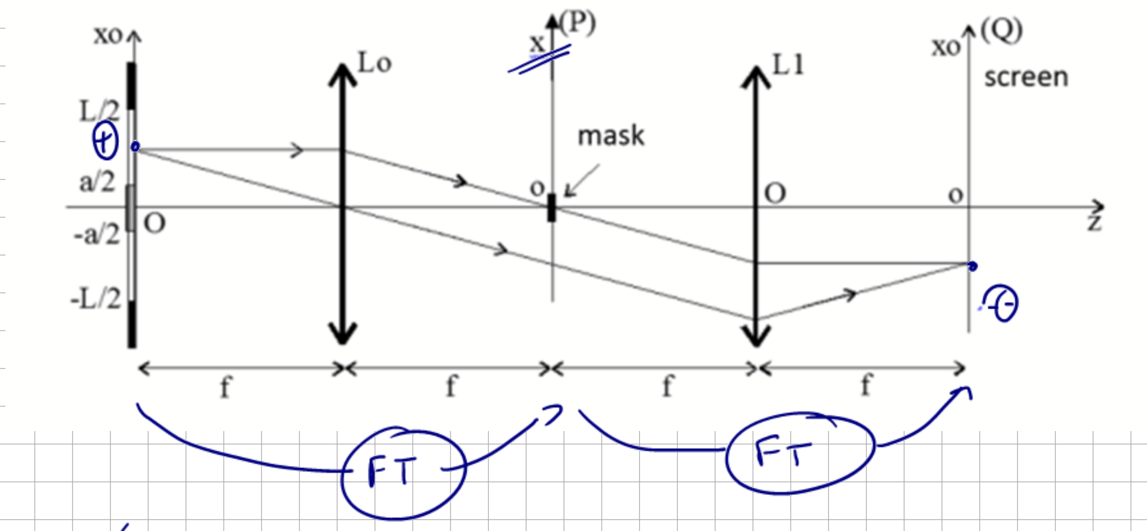
$$E_t = E_i \cdot t(x_0) = t(x_0) = \mathrm{rect}_L(x_0) + j\varphi_0\,\mathrm{rect}_a(x_0)$$

↳ after the mask

Comment:







$$FT\left(FT\, f(x_0)\right) = f(-x_0)$$

$$\mathrm{rect}_L(x_0) \xrightarrow{FT}$$

$$FT\left(\mathrm{rect}_L(x_0)\right) = \int_{-\infty}^{+\infty} \mathrm{rect}_L(x_0)\, e^{j 2\pi N_x x_0}\, dx_0$$

$$FT\left(\mathrm{rect}_L(x_0)\right) = L\, \mathrm{sinc}\left(\pi N_x L\right)$$

$$N_x = \frac{x}{\lambda f} \quad (\text{see lecture})$$

$$E_p(x) = \left(FT\left(t(x_0)\right)\right)_{x/\lambda f} \rightarrow \text{Field in (P) plane.}$$

$$t(x_0) = A\left(\mathrm{rect}_L(x_0) + j\varphi_0\, \mathrm{rect}_a(x_0)\right)$$

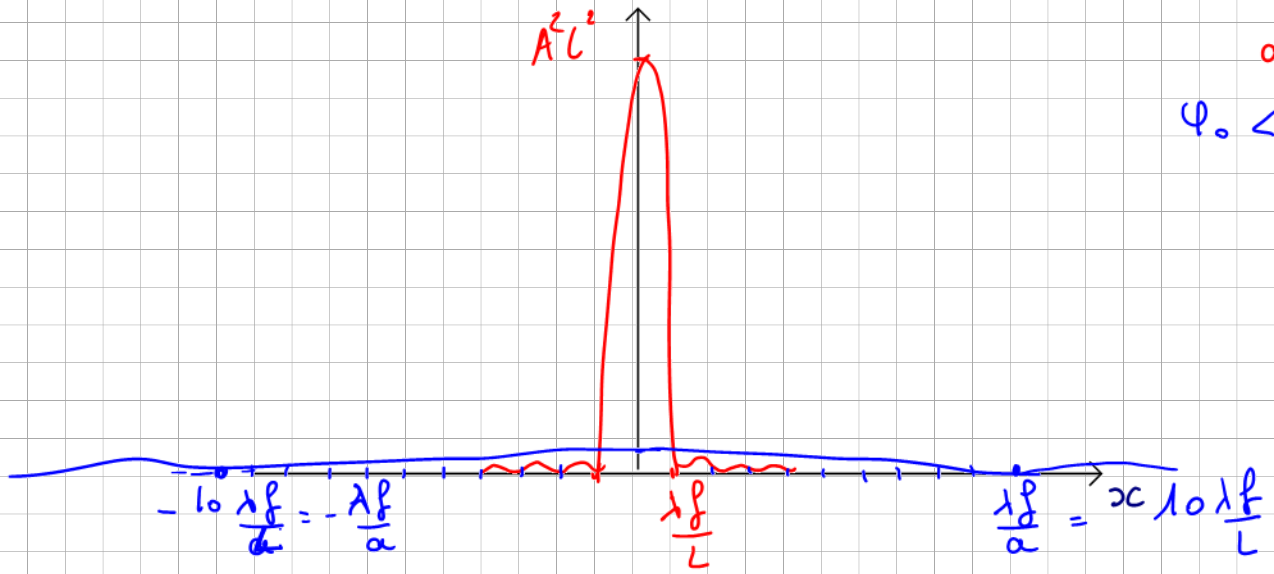
$$E_p(x) = A\left(L\, \mathrm{sinc}\left(\pi N_x L\right) + j\varphi_0\, a\, \mathrm{sinc}\left(\pi N_x a\right)\right)$$

$$I_p(x) = \left|E_p\right|^2 = A^2\left(L^2 \mathrm{sinc}^2\left(\pi N_x L\right) + \varphi_0^2 a^2 \mathrm{sinc}^2\left(\pi N_x a\right)\right)$$

$$\boxed{I_p(x) = A^2\left(L^2 \mathrm{sinc}^2\left(\pi \frac{xL}{\lambda f}\right) + \varphi_0^2 a^2 \mathrm{sinc}^2\left(\pi \frac{xa}{\lambda f}\right)\right)}$$

$a = L/10$

$\varphi_0 << 1$ rad



3) The mask removes the red sinc coming from the large slit of length L

The image of the large slit is black in the Q plane.

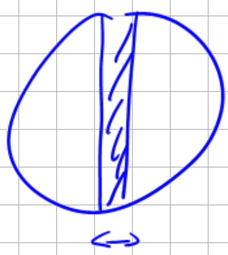
The major part of the energy coming from the phase-shifted part of size a is kept

Then in the Q plane, the phase-shifted part appears bright on a dark background

without filtering :

in Q plane uniform intensity

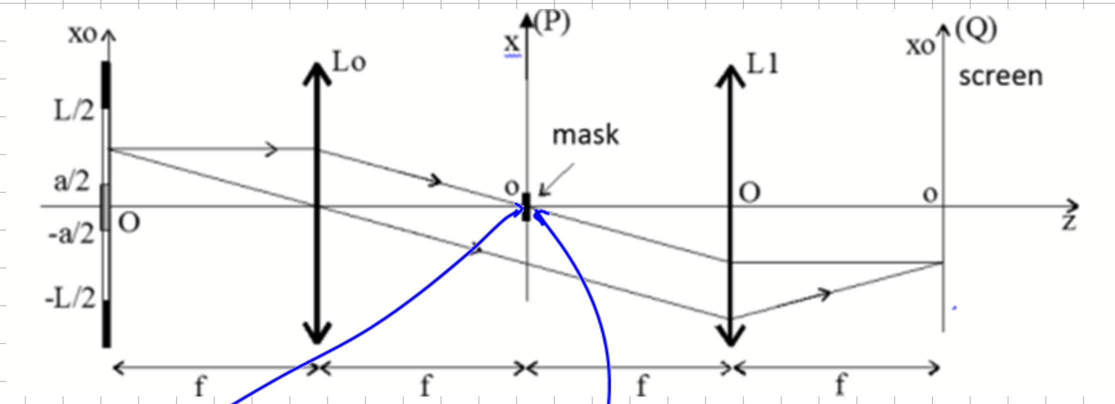
with filtering



4) $T(x_0) = a\, e^{j\varphi(x_0)}$

a) $T(x_0) = a\left(1 + j\varphi(x_0)\right)$ $\qquad \varphi(x_0) << 1$ rad

b) In the P plane



$$\underline{E_P(x)} = FT\left(T(x_0)\right) = a\left(\delta\left(\frac{x}{\lambda f}\right) + j\,\tilde{\varphi}\left(\frac{x}{\lambda f}\right)\right)$$

$$N_x = \frac{x}{\lambda f}$$

c) After the mask

$$\boxed{T_M = 1 - \delta\left(x/\lambda f\right)}$$

$$\underline{E'_P(x)} = E_P(x) \cdot T_M(x) = a\,j\,\tilde{\varphi}\left(\frac{x}{\lambda f}\right)$$

↳ Transmission of the mask

In the Q phane

$$E_Q(x_0) = FT\left(E'_P(x)\right) = a\,j\,\varphi(-x_0)$$

$$I_Q(x_0) = a^2\,\varphi^2(-x_0)$$

$$I_Q \,\%\, \varphi^2(-x_0)$$

The phase structure becomes a bright image on dark background.

II Phase contrast.

In the P Plane:

$$E_P(x) = a\left(\delta\left(\frac{x}{\lambda f}\right) + j\,\tilde{\varphi}\left(\frac{x}{\lambda f}\right)\right)$$

After filtering:

$$e^{j\pi/2}$$

$$E'_P(x) = a\left(j\,\delta\left(\frac{x}{\lambda f}\right) + j\,\tilde{\varphi}\left(\frac{x}{\lambda f}\right)\right)$$

$$= a\,j\left(\delta\left(\frac{x}{\lambda f}\right) + \tilde{\varphi}\left(\frac{x}{\lambda f}\right)\right)$$

In the Q plane:

$$E_Q(x_0) = a\,j\left(1 + \varphi(-x_0)\right)$$

$$I_\varphi(x_0) = a^2 \left(1 + \varphi(-x_0)\right)^2$$
$$= a^2 \left(1 + 2\varphi(-x_0) + \underbrace{\cancel{\varphi^2(-x_0)}}_{\text{small quantity}}\right)$$

$$I_Q(x) = a^2\left(1 + 2\varphi(-x_0)\right)$$

$\hookrightarrow$ linear form $\Rightarrow$ better dynamic

To increase the contrast, see question 2.

2) $E_P(x) = a\left(\delta\left(\frac{x}{\lambda f}\right) + j\,\tilde{\varphi}\left(\frac{x}{\lambda f}\right)\right)$

After filtering:

$$E'_P(x) = a\left(j\,b\,\delta\left(\frac{x}{\lambda f}\right) + j\,\tilde{\varphi}\left(\frac{x}{\lambda f}\right)\right)$$
$$= a j \left(b\,\delta\left(\frac{x}{\lambda f}\right) + \tilde{\varphi}\left(\frac{x}{\lambda f}\right)\right)$$

In the Q plane:

$$E_Q(x_0) = a j \left(b + \varphi(-x_0)\right)$$
$$I_Q(x_0) = a^2\left(b + \varphi(-x_0)\right)^2$$
$$= a^2\left(b^2 + 2b\varphi(-x_0) + \cancel{\varphi^2(-x_0)}\right)$$

$b^2 < 1 \Rightarrow$ better contrast!