Erasmus+

EMIMEO : E(rasmus) Mundus on Innovative Microwave Electronics and Optics Master

Foundations of Electromagnetic Wave Propagation – 2nd part

Contributors:

Olivier Tantot

Guillaume Neveux

Serge Verdeyme

Université de Limoges

Università degli Studi di Brescia

Aston University

Universidad del País Vasco
Euskal Herriko Unibertsitatea
The University of the Basque Country

Universitatea Tehnică din Cluj-Napoca

Chapters:

2. Transmission line

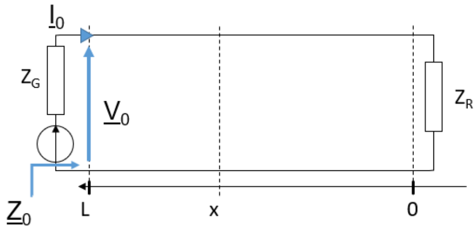
22. Theory of lines - line ended by a short-circuit

Short-circuited line $\underline{Z}_R$=0

simplified calculations if the origin is considered at the load

as

$\underline{V}(x = L) = \underline{V}_R = 0$ and $\underline{Z}(x = L) = \underline{Z}_R = 0$



then

$$\underline{V}(x) = \underline{Z}_C \underline{I}_R \mathrm{sh}\gamma x$$

$$\underline{I}(x) = \underline{I}_R \mathrm{ch}\gamma x$$

$$\underline{Z}(x) = \underline{Z}_C \mathrm{th}\gamma x$$

In the case of LWL:

$$\underline{V}(x) = j\underline{Z}_C \underline{I}_R \sin\beta x$$

$$\underline{I}(x) = \underline{I}_R \cos\beta x$$

$$\underline{Z}(x) = j\underline{Z}_C \mathrm{tg}\beta x$$

2. Transmission line

23. Theory of lines - line ended by a short-circuit

Short-circuited line $\underline{Z}_R$=0

In instantaneous values :

$$v(x,t) = Z_C I_R \sin\beta x \,.\, \cos(\omega t + \varphi + \frac{\pi}{2})$$

$$i(x,t) = I_R \cos\beta x \,.\, \cos(\omega t + \varphi)$$

$\underline{Z}_C$: is considered real, which is almost always the case

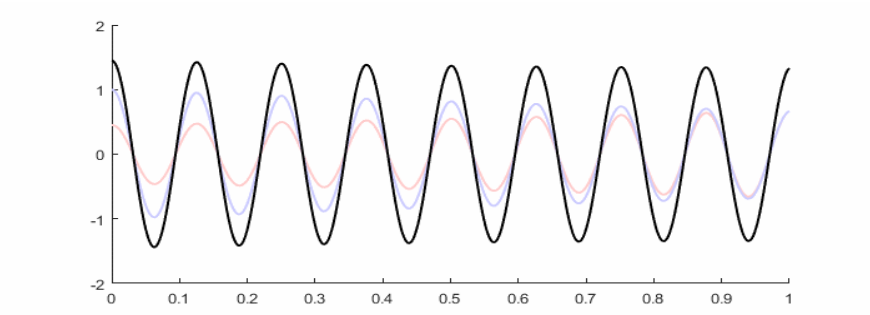
•..

•..

General case

Superposition of waves (V(x)= <u>Vi</u>.$e^{-\gamma x}$ +<u>Vr</u>.$e^{\gamma x}$) :

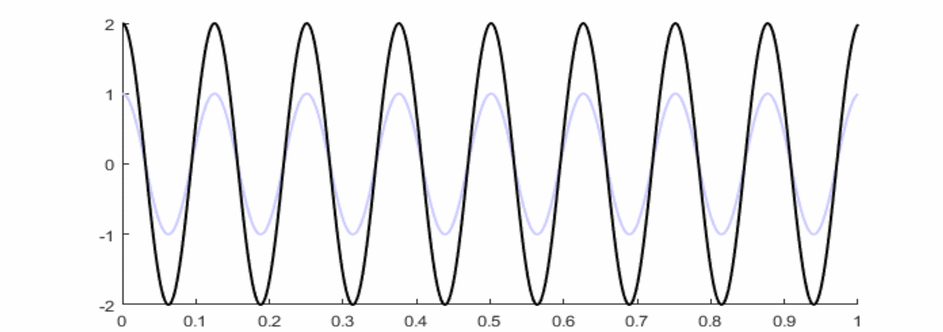
Semi-stationary wave



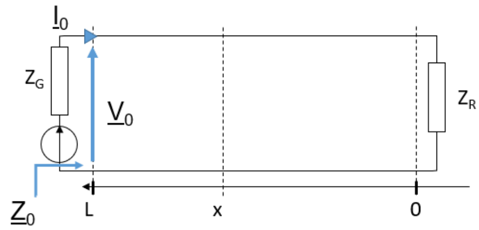
Case of the line without losses

Superposition of the waves (V(x)= <u>Vi</u>.$e^{-j\beta x}$ +<u>Vr</u>.$e^{j\beta x}$):

Stationary wave



Short-circuited line $\underline{Z}_R$=0



Under these conditions the input impedance $\underline{Z}_G$ becomes:

$$\underline{Z}_G = \underline{Z}(L) = j\underline{Z}_C \mathrm{tg}\beta L = j\underline{Z}_C \mathrm{tg}\frac{2\pi}{\lambda}L$$

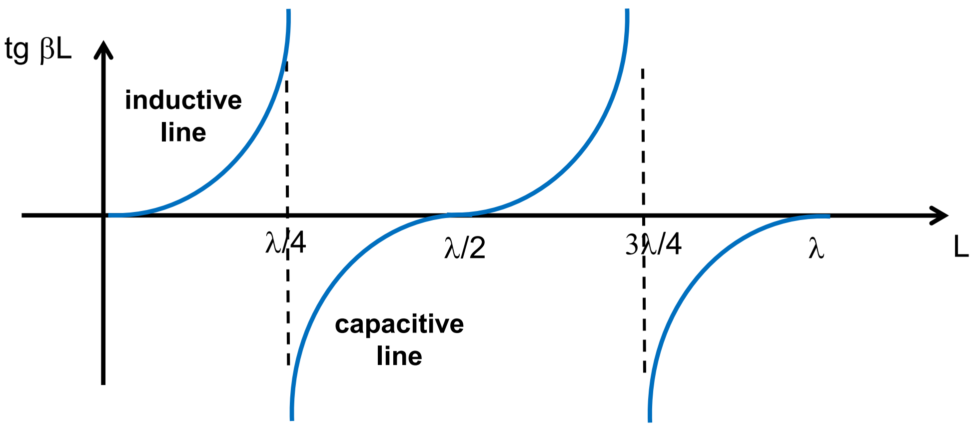
$\underline{Z}_C$: is considered real, which is almost always the case

Short-circuited $\underline{Z}_R$=0

Evolution of $\underline{Z}_G$ as a function of tg βL :

$$\underline{Z}_G = \underline{Z}(L) = j\underline{Z}_C \mathrm{tg}\beta L = j\underline{Z}_C \mathrm{tg}\frac{2\pi}{\lambda}L$$

Considering real $\underline{Z}_C$, which is almost always the case, we draw:



2. Transmission line

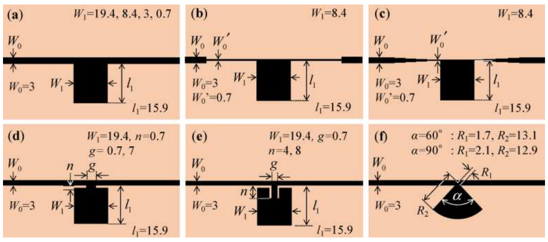
28. Theory of lines - line ended by a short-circuit

Short-circuited $\underline{Z}_R=\underline{0}$

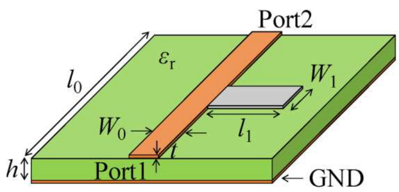
Evolution of $\underline{Z}_G$ as a function of tg βL :

………
………

The stub is a direct application of this property. It is a short-circuited line of variable length used to adjust its input impedance (to bring a given impedance in a plane)



[6]



[6]

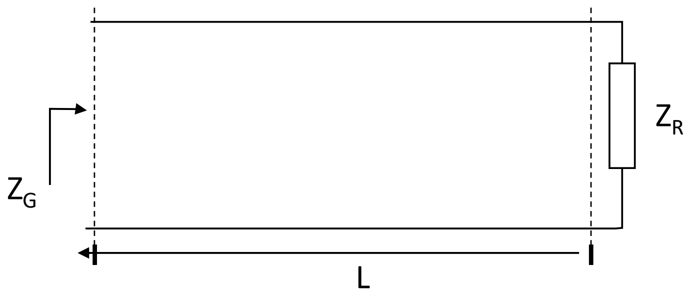
[6] Y. Kusama and R. Isozaki, "Compact and Broadband Microstrip Band-Stop Filters with Single Rectangular Stubs", Applied Sciences, vol. 9, n0 248, 2019, doi: 10.3390/app9020248

Erasmus+ E(rasmus) Mundus on Innovative Microwave Electronics and Optics

2. Transmission line

29. Theory of lines - line ended by a short-circuit

Short-circuited $\underline{Z}_R=\underline{0}$



Other remarks:

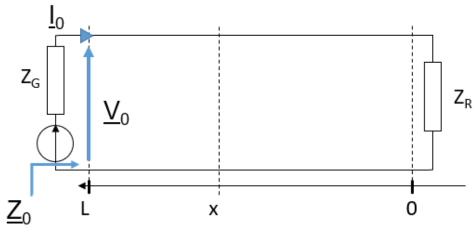
……………………………………………………………………………………………

➢ A quarter-wave line short-circuited at one end, brings back an open circuit at the other end (HF isolator)

……………………………………………………………………………………………

➢ A half-wave line short-circuited at one end, brings back a short-circuit at the other end

Open circuit line (infinite $\underline{Z}_R$)



- simplified calculations if the origin of the axis is considered at the load $\underline{I}(x=0)=\underline{I}_R=0$ et $\underline{Z}(x=0)=\underline{Z}_R=\infty$
- we write at th origin:

We then obtain:

$$\underline{V}(x)=\underline{V}_R \operatorname{ch}\gamma x$$
$$\underline{I}(x)=\frac{\underline{V}_R}{\underline{Z}_C}\operatorname{sh}\gamma x$$
$$\underline{Z}(x)=\frac{\underline{V}(x)}{\underline{I}(x)}=\frac{\underline{Z}_C}{\operatorname{th}\gamma x}$$

In the case of LWL :

$$\underline{V}(x)=\underline{V}_R\cos\beta x$$
$$\underline{I}(x)=j\frac{\underline{V}_R}{\underline{Z}_C}\sin\beta x$$
$$\underline{Z}(x)=\frac{\underline{V}(x)}{\underline{I}(x)}=-j\frac{\underline{Z}_C}{\operatorname{th}\beta x}$$

Open circuit line (infinite $\underline{Z}_R$)

Consider the voltage-current waves in space and time:

$$\begin{cases} \underline{v}(x,t)=V_R e^{j\varphi} e^{j\omega t}\cos\beta x \\ \underline{i}(x,t)=e^{j\frac{\pi}{2}}\dfrac{V_R}{Z_C} e^{j\varphi} e^{j\omega t}\sin\beta x \end{cases}$$

Either in instantaneous valuess ($\underline{Z}_C$*: est considered as real)*:

$$\begin{cases} v(x,t)=V_R\cos\beta x\,.\cos(\omega t+\varphi) \\ i(x,t)=\dfrac{V_R}{Z_C}\sin\beta x\,.\cos(\omega t+\varphi+\dfrac{\pi}{2}) \end{cases}$$

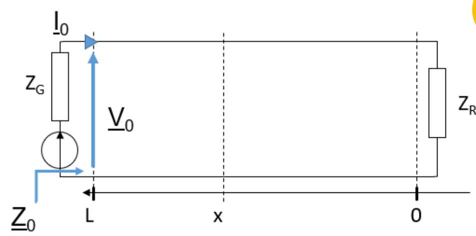
- ....
- ....

Quarter wave line. Half wave line

We have shown:

$$\underline{Z}_0 = \underline{Z}_C \frac{\underline{Z}_R + j\underline{Z}_C \mathrm{tg}\beta L}{\underline{Z}_C + j\underline{Z}_R \mathrm{th}\beta L}$$

For LWL



if $\beta L = \frac{(2k+1)\pi}{2} \Rightarrow L = (2k+1)\frac{\lambda}{4} \Rightarrow \mathrm{tg}\beta L \to \infty$

then ....

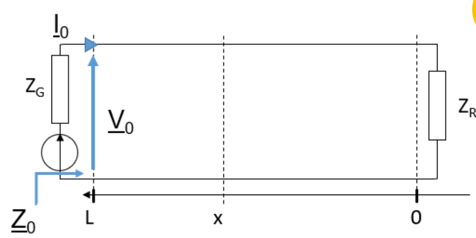
....

Quarter wave line. Half wave line

We have shown:

$$\underline{Z}_0 = \underline{Z}_C \frac{\underline{Z}_R + j\underline{Z}_C \mathrm{tg}\beta L}{\underline{Z}_C + j\underline{Z}_R \mathrm{th}\beta L}$$

For LWL



if $\beta L = k\pi \Rightarrow L = \frac{k\lambda}{2} \Rightarrow \mathrm{tg}\beta L \to 0$

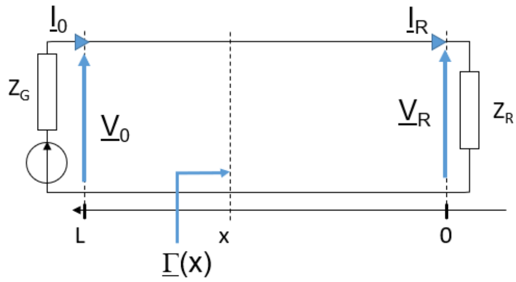
then $\underline{Z}_0 = \underline{Z}_R$

....

Reflection coefficient

the origin of the axis is considered at the charge

We define the reflection coefficient at a point as the ratio of the reflected wave to the incident wave:



given:

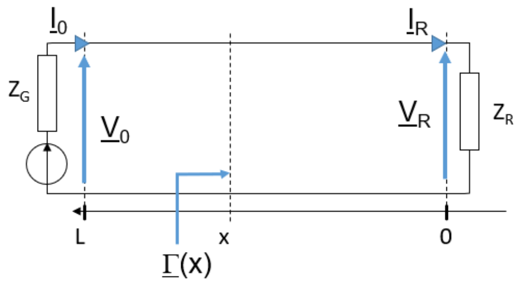
....

with: $\underline{V}_r = \frac{1}{2}(\underline{V}_R - \underline{Z}_C \underline{I}_R)$ $\underline{V}_i = \frac{1}{2}(\underline{V}_R + \underline{Z}_C \underline{I}_R)$

Reflection coefficient

the origin of the axis is considered at the charge

We define the reflection coefficient at a point as the ratio of the reflected wave to the incident wave:



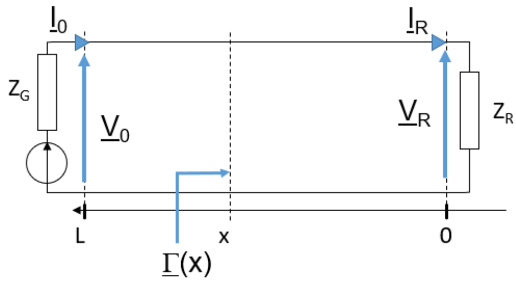
then: $\underline{\Gamma}(x) = \frac{(\underline{V}_R - \underline{Z}_C \underline{I}_R)e^{-2\gamma x}}{\underline{V}_R + \underline{Z}_C \underline{I}_R}$

therefore: $\underline{\Gamma}(x) = \frac{(\underline{Z}_R - \underline{Z}_C)e^{-2\gamma x}}{\underline{Z}_R + \underline{Z}_C}$

36. Theory of lines – reflection coefficient

Reflection coefficient

the origin of the axis is considered at the charge



At the origin, therefore at the level of the load, the reflection coefficient is equal to:

$\boxed{}$

Therefore: $\underline{\Gamma}_R = \Gamma_R e^{j\theta_R}$ ← Phase shift term introduced by the reflection

Erasmus+ E(rasmus) Mundus on Innovative Microwave Electronics and Optics

37. Theory of lines – reflection coefficient in particular case

Reflection coefficient

the origin of the axis is considered at the charge

$$\underline{\Gamma}_R = \frac{\underline{Z}_R - \underline{Z}_C}{\underline{Z}_R + \underline{Z}_C}$$

Interesting cases:

Line with arbitrary reflection coefficient

the origin of the axis is considered at the charge

given: $$\underline{V}(x) = \underline{V}_i e^{\gamma x} + \underline{V}_r e^{-\gamma x}$$

$$\underline{V}(x) = \underline{V}_i e^{\gamma x}\left(1 + \frac{\underline{V}_r}{\underline{V}_i} e^{-2\gamma x}\right)$$

then: $$\underline{V}(x) = \underline{V}_i e^{\gamma x}\left(1 + \underline{\Gamma}_R e^{-2\gamma x}\right)$$

Line with arbitrary reflection coefficient

the origin of the axis is considered at the charge

given: $$\underline{I}(x) = \underline{I}_i e^{\gamma x} + \underline{I}_r e^{-\gamma x}$$

$$\underline{I}(x) = \underline{I}_i e^{\gamma x}\left(1 + \frac{\underline{I}_r}{\underline{I}_i} e^{-2\gamma x}\right)$$

with $$\frac{\underline{V}_i}{\underline{I}_i} = \underline{Z}_C = -\frac{\underline{V}_r}{\underline{I}_r}$$

Then: $$\underline{I}(x) = \underline{I}_i e^{\gamma x}\left(1 - \underline{\Gamma}_R e^{-2\gamma x}\right)$$

Line with arbitrary reflection coefficient

the origin of the axis is considered at the charge

given: $$\underline{Z}(x) = \frac{\underline{V}(x)}{\underline{I}(x)} = \frac{\underline{V}_i e^{\gamma x}(1+\underline{\Gamma}_R e^{-2\gamma x})}{\underline{I}_i e^{\gamma x}(1-\underline{\Gamma}_R e^{-2\gamma x})}$$

then: $$\boxed{\underline{Z}(x) = \underline{Z}_C \frac{1+\underline{\Gamma}_R e^{-2\gamma x}}{1-\underline{\Gamma}_R e^{-2\gamma x}}}$$

Line with arbitrary reflection coefficient

the origin of the axis is considered at the charge

We consider the line without losses, we obtain then:

$$\begin{cases} |\underline{V}(x)| = |\underline{V}_i e^{j\beta x}|\,|(1+\Gamma_R e^{j(\theta_R - 2\beta x)})| \\ |\underline{I}(x)| = |\underline{I}_i e^{j\beta x}|\,|(1-\Gamma_R e^{j(\theta_R - 2\beta x)})| \\ |\underline{Z}(x)| = |\underline{Z}_C| \dfrac{|1+\Gamma_R e^{j(\theta_R - 2\beta x)}|}{|1-\Gamma_R e^{j(\theta_R - 2\beta x)}|} \end{cases}$$

Line with arbitrary reflection coefficient

the origin of the axis is considered at the charge

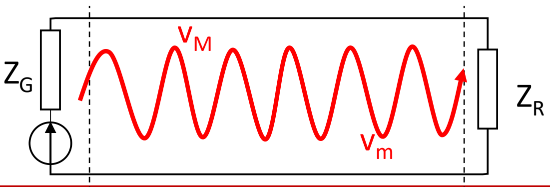
We are interested in the amplitudes:

……………………………….

MAX voltage $$\left|\underline{V}_M\right| = \left|\underline{V}_i\right|\left|(1+\Gamma_R)\right|$$

MIN current $$\left|\underline{I}_m\right| = \left|\underline{I}_i\right|(1-\Gamma_R)$$

MAX impedance $$\left|\underline{Z}_M\right| = \left|\frac{\underline{V}_M}{\underline{I}_m}\right|$$



Line with arbitrary reflection coefficient

the origin of the axis is considered at the charge

We are interested in the amplitudes:

…………………………………….

MIN voltage $$\left|\underline{V}_m\right| = \left|\underline{V}_i\right|(1-\Gamma_R)$$

MAX current $$\left|\underline{I}_M\right| = \left|\underline{I}_i\right|(1+\Gamma_R)$$

MIN impedance $$\left|\underline{Z}_m\right| = \left|\frac{\underline{V}_m}{\underline{I}_M}\right|$$

Line with arbitrary reflection coefficient

Two consecutive maxima or minima of voltage or current are separated by:

$$(\theta_R - 2\beta x_n) - (\theta_R - 2\beta x_{n+1}) = 2\beta(x_n - x_{n+1})$$

$$2n\pi - 2(n+1)\pi = -2\pi$$

………………………………………………………

Line with arbitrary reflection coefficient

Voltage Standing Wave Ratio is defined as thge term VSWR or s:

………………………………………………………………

Γ dB	|Γ|	s
0	1	∞
-5	0,562	3,6
-10	0,316	1,9
-20	0,1	1,22
-26,4	0,048	1,1
-30	0,032	1,07
-32,3	0,024	1,05
-40	0,01	1,02
-80	0,0001	1,0002

and as $0 \leq \Gamma_R \leq 1$ $1 \leq s \leq \infty$

LWL

$$\alpha = 0$$

$$\beta = \omega.\sqrt{L.C}$$

$$v_\varphi = \frac{1}{\sqrt{L.C}}$$

Neither α nor v_φ depend on f
➔ **No distortion**

Low losses

$$\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right)$$

$$\beta \cong \omega.\sqrt{L.C}$$

$$v_\varphi \cong \frac{1}{\sqrt{L.C}}$$

Neither α nor v_φ depend on f
➔ **No distortion**

Losses of any kind

Heaviside **GL=RC**

Heaviside
➔ loss minimization
➔ α as low as possible
(but neither null nor very weak)

$$\alpha = \sqrt{R.G}$$

$$\beta = \omega.\sqrt{L.C}$$

$$v_\varphi = \frac{1}{\sqrt{L.C}}$$

Neither α nor v_φ depend on f
➔ **No distortion**

Losses of any kind

Low loss

$$\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right)$$

$$\beta \cong \omega.\sqrt{L.C}$$

$$v_\varphi \cong \frac{1}{\sqrt{L.C}}$$

Neither α nor v_φ depend on f
➔ **No distortion**

Heaviside **GL=RC**

Heaviside
➔ loss minimization
➔ α as low as possible
(but neither null nor very weak)

$$\alpha = \sqrt{R.G}$$

$$\beta = \omega.\sqrt{L.C}$$

$$v_\varphi = \frac{1}{\sqrt{L.C}}$$

Neither α nor v_φ depend on f
➔ **No distortion**

~~Heaviside~~

α and v_φ depend on f !!
➔ **Distortion !!**

α = α(f)
Amplitude distortion
➔ slope correction filter

$v_\varphi = v_\varphi(f)$
Phase distortion
➔ tend towards Heaviside conditions
(➔ L++)

- pupinization
- krarupization

2. Transmission line

48. Theory of lines - review

Amplitude distortion (when α DOES NOT depend on f)

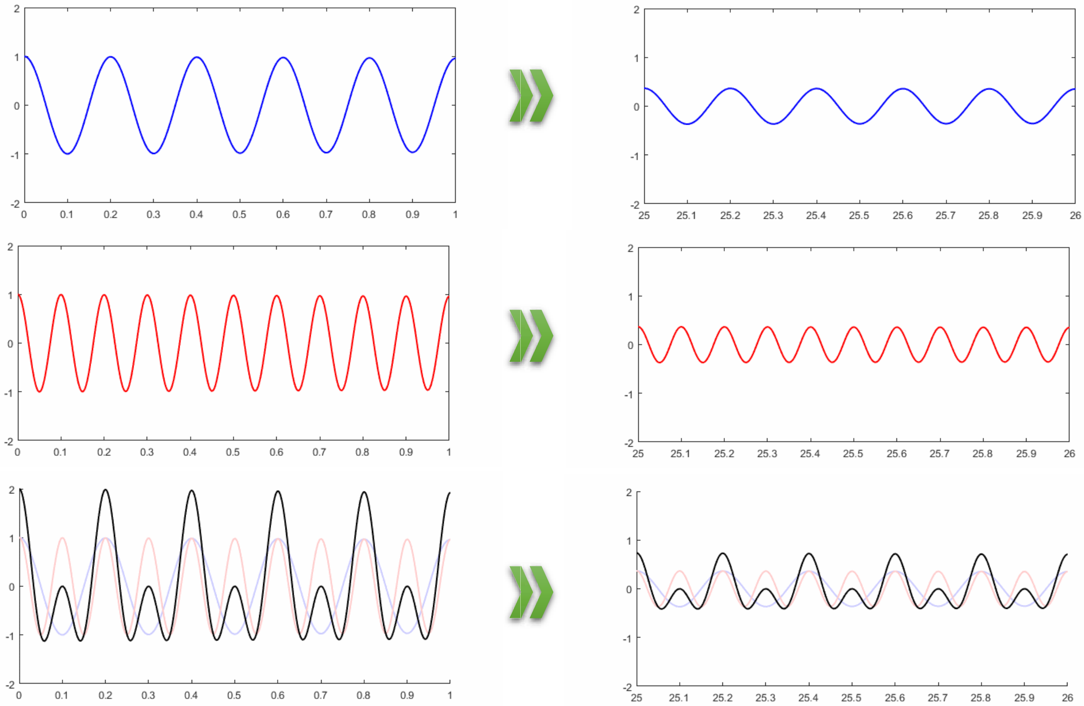
input

output

f1

f2

Superposition of signals



α = const

$V\varphi$ = const

2. Transmission line

49. Theory of lines - review

Amplitude distortion (when α depend on f

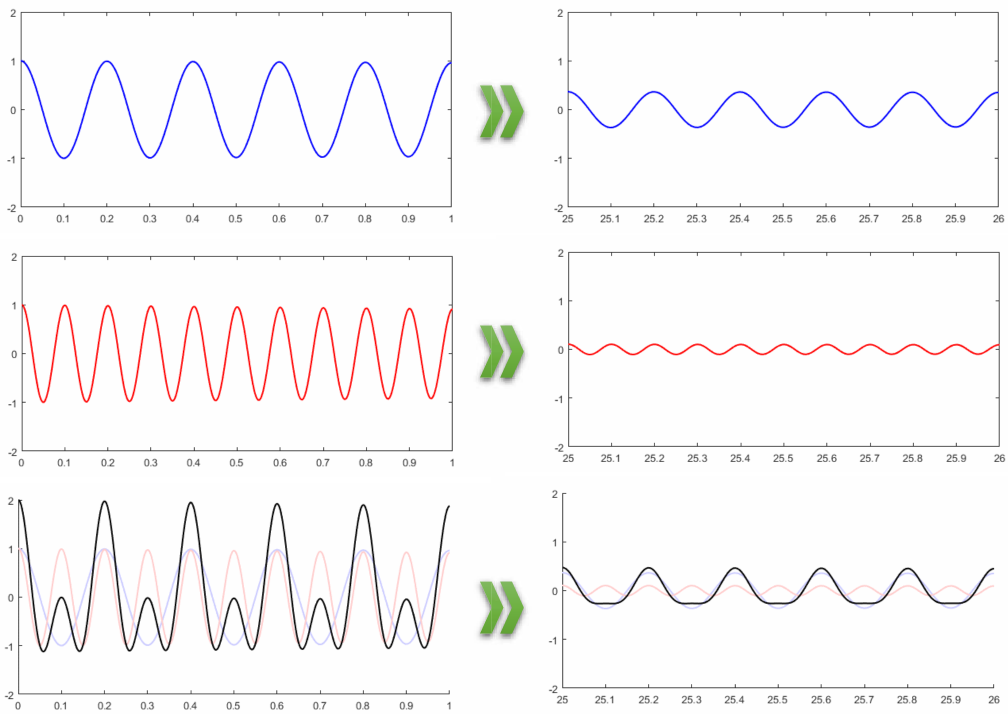
input

output

f1

f2

Superposition of signals



~~α = const~~

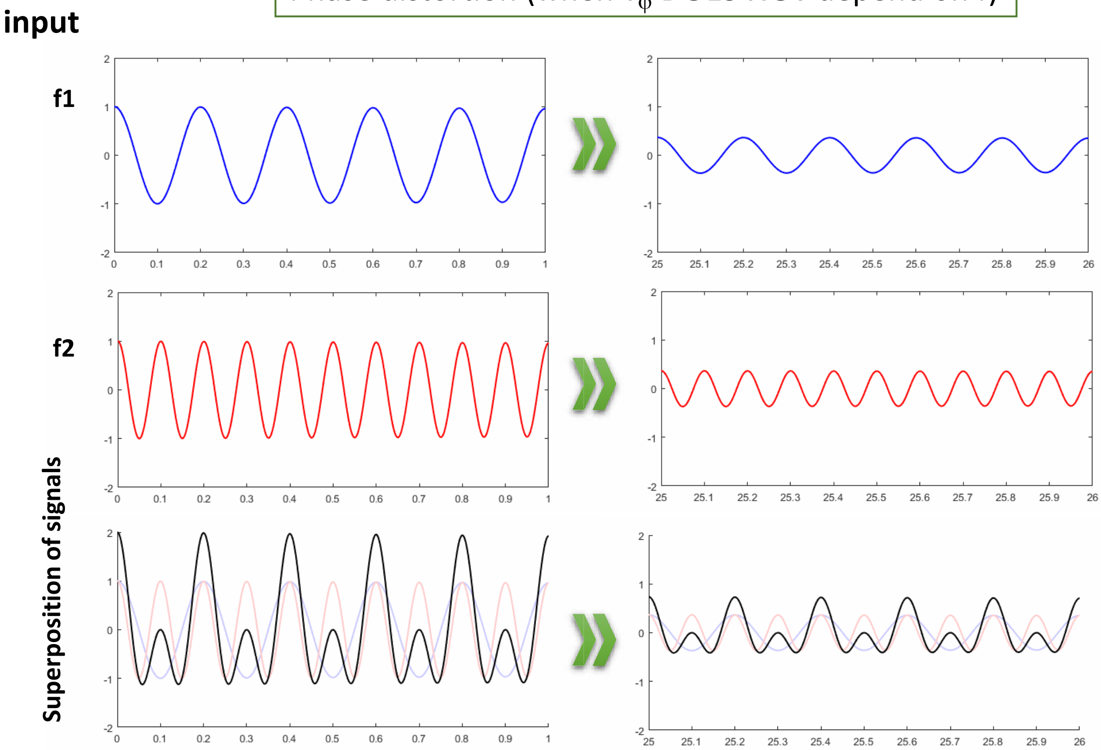
$\alpha 2 = 3 * \alpha 1$

$V\varphi$ = const

2. Transmission line

50. Theory of lines - review

Phase distortion (when v_φ **DOES NOT** depend on f)



α = const

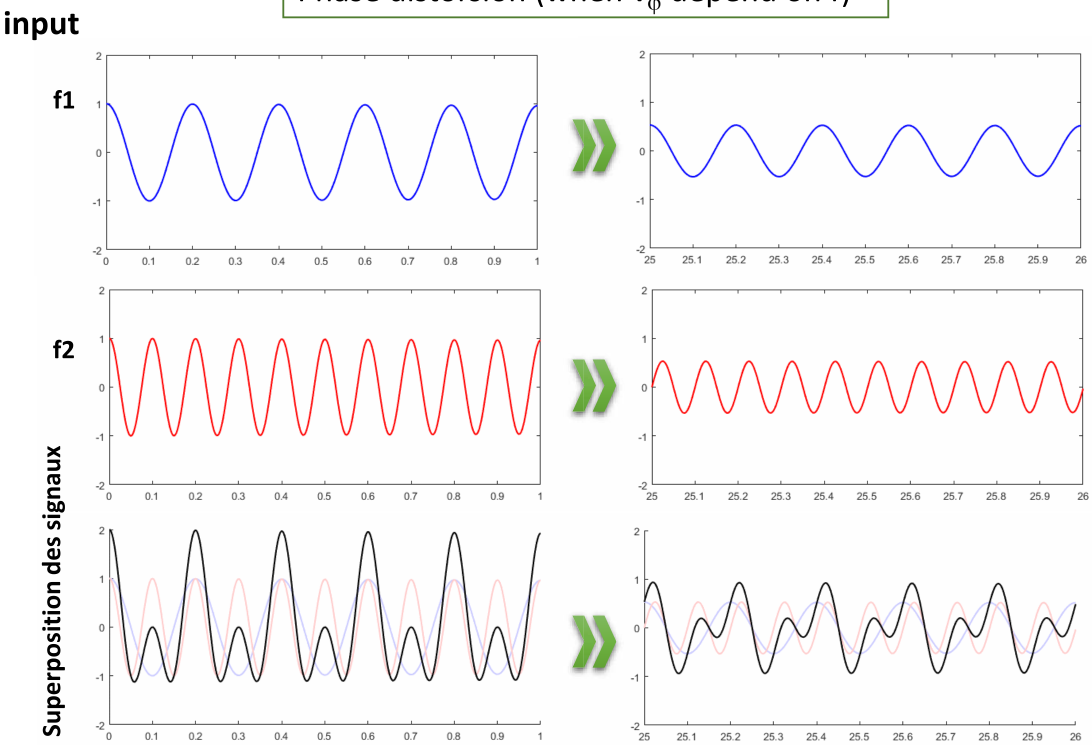
$V\varphi$ = const

Erasmus+ E(rasmus) Mundus on Innovative Microwave Electronics and Optics

2. Transmission line

51. Theory of lines - review

Phase distorsion (when v_φ depend on f)



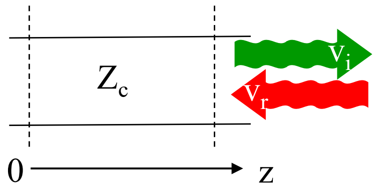
α = const

~~$V\varphi$ = const~~

$V\varphi 1 = 1.001 * V\varphi 2$

2h

For a line with a characteristic impedance Z_c , excited by a sinusoidal wave of pulsation ω, the solutions of telegraphers equations take the form:



$$\underline{V}(z,t) = \underline{V}_i e^{-\alpha z}.e^{-j\beta z}.e^{j\omega t} + \underline{V}_r e^{\alpha z}.e^{j\beta z}.e^{j\omega t}$$

$$\underline{I}(z,t) = \frac{\underline{V}_i}{Z_c} e^{-\alpha z}.e^{-j\beta z}.e^{j\omega t} - \frac{\underline{V}_r}{Z_c} e^{\alpha z}.e^{j\beta z}.e^{j\omega t}$$

The wave is the sum of :

- an incident wave of complex amplitude $\underline{V}_i$
- a reflected wave of complex amplitude $\underline{V}_r$

γ wave propagation constant

$\gamma = \alpha + j\beta$

α : attenuation constant per unit length

β : phase constant per unit length

In the case of Loss-Less transmission Line (LLL), α = 0.

α in Np/m or dB/m with 1 dB = 0.1151 Np (Nepers)

β in rad/m

Incident and reflected waves have a double periodicity

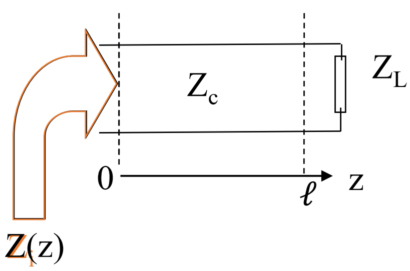
- temporal $\omega = \frac{2\pi}{T}$
- spatial $\beta = \frac{2\pi}{\lambda}$

Caracteristic impedance Z_c :

In the case of a lossless line (LLL), Z_c is real

Impédance at a point on the line :

$$Z(z) = \frac{V(z)}{I(z)} = Z_c \frac{\underline{V}_i e^{-\alpha z}.e^{-j\beta z} + \underline{V}_r e^{\alpha z}.e^{j\beta z}}{\underline{V}_i e^{-\alpha z}.e^{-j\beta z} - \underline{V}_r e^{\alpha z}.e^{j\beta z}}$$



Input impedance of a line of length ℓ loaded by Z_L

$$Z_i = Z_c \frac{Z_L + Z_c \operatorname{th} \gamma\ell}{Z_c + Z_L \operatorname{th} \gamma\ell}$$

for LLL :

α = 0

$$Z_i = Z_c \frac{Z_L + j\, Z_c \tan \beta\ell}{Z_c + j\, Z_L \tan \beta\ell}$$

Reflection coefficient of a LLL

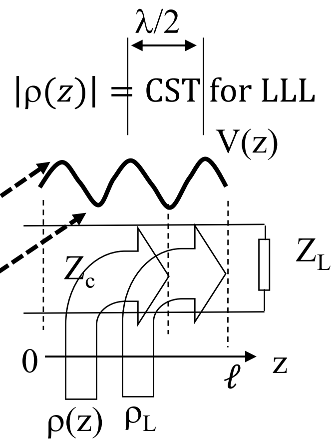
$$\rho(z) = \frac{\text{onde réfléchie}}{\text{onde incidente}} = \frac{\underline{V}_r.e^{j\beta z}}{\underline{V}_i.e^{-j\beta z}} = -\frac{\underline{I}_r e^{\alpha z}.e^{j\beta z}}{\underline{I}_i e^{-\alpha z}.e^{-j\beta z}}$$

Load reflection coefficient :

$$\rho_L = \rho(z=l) = \frac{Z_L - Z_c}{Z_L + Z_c}$$

Voltage Standing Wave Ratio :

$$VSWR = s = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1+|\rho_L|}{1-|\rho_L|}$$



References:

[1] David M. Pozar, Microwave Engineering, Third Edition, John Wiley & Sons Inc.; (ISBN 0-471-17096-8)
[2] Jia-Sheng Hong, M. J. Lancaster, Microstrip Filters for RF/Microwave Applications, John Wiley & Sons Inc. (ISBN: 0-471-38877-7)
[3] G. Ghione, M. Pirola, Microwave Electronics, Cambridge University Press (ISBN 978-1-107-17027-8)
[4] Richard Collier, Transmission Lines, Cambridge University Press (ISBN 978-1-107-02600-1)
[5] V. Teppati, A. Ferrero, M. Sayed, Modern RF and Microwave Measurement Techniques, Cambridge University Press (ISBN 978-1-107-03641-3)
[6] Y. Kusama and R. Isozaki, "Compact and Broadband Microstrip Band-Stop Filters with Single Rectangular Stubs", Applied Sciences, vol. 9, n0 248, 2019,