Semester S1 –Basics of active and non linear electronics

RF Power amplifiers (JM Nebus)

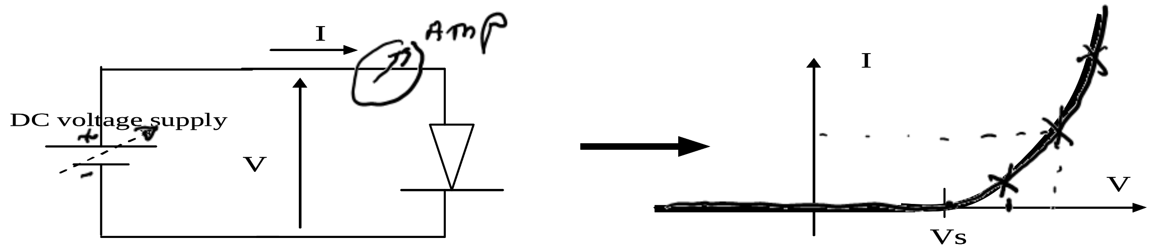
COURSE N° 2

Chapter I I : Large signal Analysis of Active circuits

I] Linear versus non linear behaviour of a device

Example of a two port device: The diode

First imagine that we measure the current of a diode versus voltage using DC generator, volt meter and ampere meter.



We can obtain, assuming that there is no temperature dependance (ideal case) the following equation which is the static I/V characteristic function of the diode.

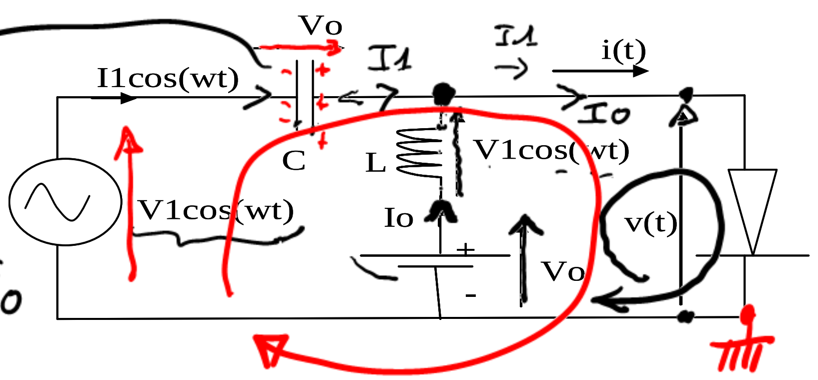
$$I = Is \cdot (e^{\alpha \cdot V} - 1) = Is \cdot e^{\alpha \cdot V}$$ When V>> Vs   Is and α are constants

Now we can drive this diode with both DC and sine wave voltages .

To do so , we need a biasing network including a self inductor L and a capacitor C .

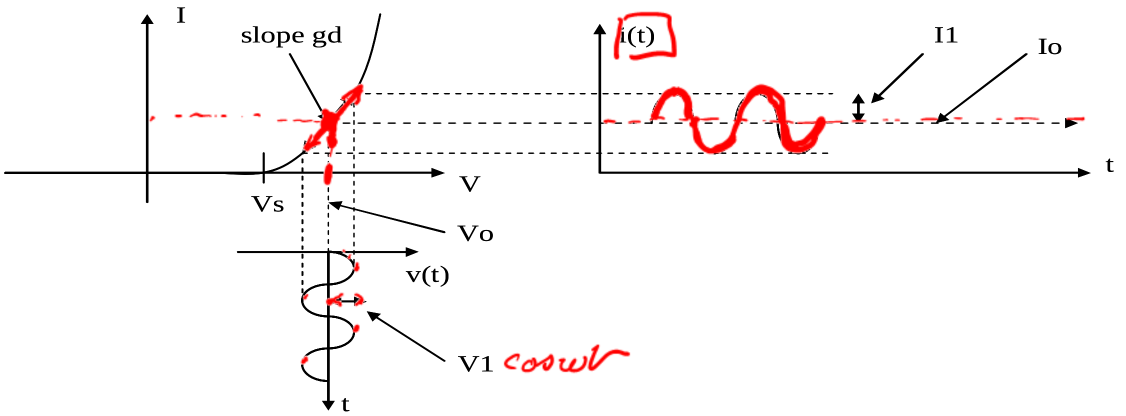
DC
$\omega = 0$
$|Z_c| = \frac{1}{C\omega} = \infty$
$L\omega = 0$
$\omega \to Z_c = \left|\frac{1}{C\omega}\right| = 0$

$v(t) = \underline{V_0} + \underline{v_1 \cos\omega t}$



- **Linear behavior**

If the sine wave signal is small , we have a linear behaviour .



We can write the equation that describes the behaviour of the device by using a Taylor serie expansion limited to the first order because ΔV is small (linear case – small signal)

$$I = F(V) = F(V_0 + \Delta V) = F(V_0) + \left.\frac{dF}{dV}\right|_{V0} . \Delta V$$

$\Delta V \longrightarrow v_1 \cos\omega t$

$$I = Is \cdot e^{\alpha(V_0 + \Delta V)} = Is(e^{\alpha(V_0)}) + \alpha Is(e^{\alpha(V_0)}) \cdot \Delta V$$

$$I(t) = I_0 + I_1 \cos\omega t$$

ΔV being equal to V_1.cos(wt) we have

$$\cancel{I = F(V) = F(V_0 + \Delta V) = F(V_0) + \frac{dF}{dV_{V0}} . \Delta V}$$

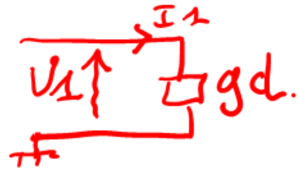
$$\cancel{I = Is \cdot e^{\alpha(V_0 + \Delta V)} = Is(e^{\alpha(V_0)}) + \alpha Is(e^{\alpha(V_0)}) \cdot \Delta V}$$

As

$$V = V_0 + \Delta V = V_0 + V_1\cos(wt)$$

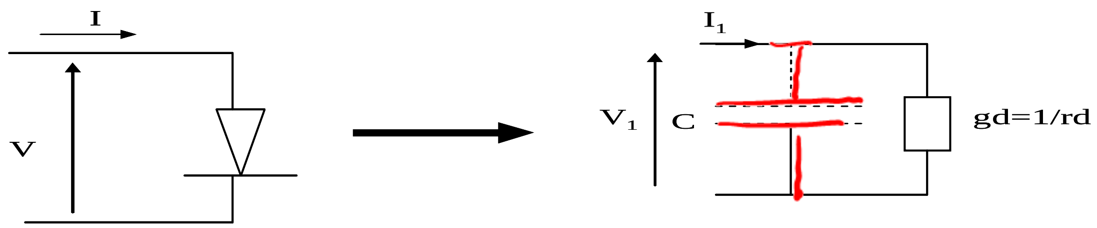
$$I(t) = Is(e^{\alpha(V_0)}) + \alpha Is(e^{\alpha(V_0)})\cdot V_1\cos(wt) = I_0 + I_1\cos(wt) = I_0 + g_d\cdot V_1\cos(wt)$$

$$\boxed{I1 = g_d\cdot V_1}$$



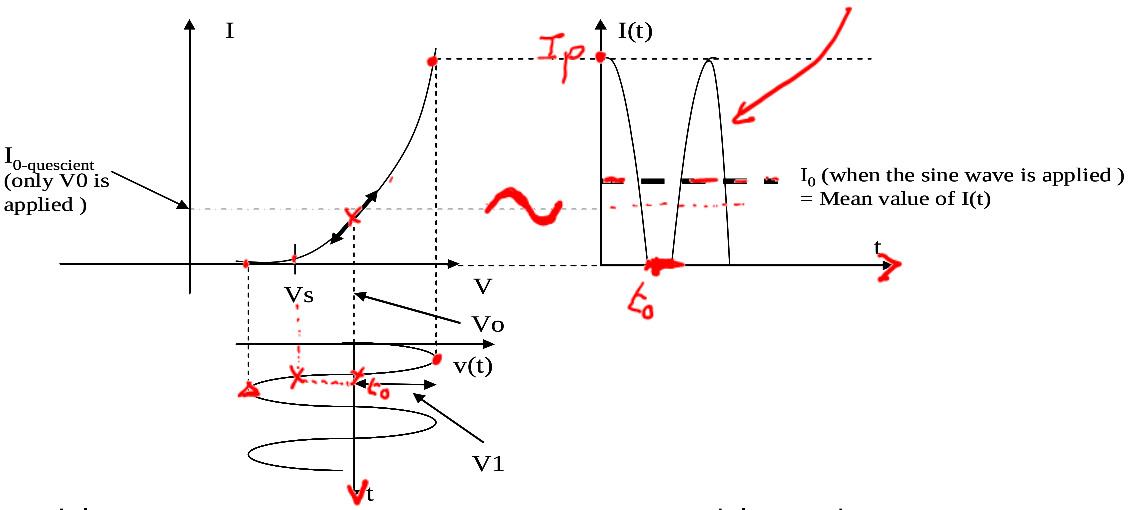
So we introduce the linear conductance of the diode g_d at the biasing voltage V_0 .

Note that we only consider here the conductive behaviour of the diode. If there is a phase shift between the voltage and current at F0 , a reactive part exists and there is a capacitance in parallel with the conductance in the equivalent model of the diode.



Linear and in the ON region

Now if the sine wave signal is large, we have a non linear behaviour and if we plot the time domain current and voltage waveforms we have.



If we want to calculate the current response we can write :

$$I(t) = Is \cdot (e^{\alpha \cdot (V_0 + V_1 \cos(wt))} - 1) = Is \cdot e^{\alpha \cdot (V_0 + V_1 \cos(wt))} = Is \cdot e^{\alpha \cdot V_0} \cdot e^{\alpha \cdot V_1 \cos(wt)}$$

→ Bessel Function.

To go further it is quite complicated:

Either we need Bessel functions to express the current as the sum of harmonic components

Or we can write a Taylor serie expansion which is now not limited to the first order

$(V_1 \cos wt)^2 = \frac{1}{2} + \frac{1}{2}\cos 2wt$

$$I = F(V) = F(V_0 + \Delta V) = F(V_0) + \underbrace{\frac{dF}{dV}\bigg|_{V0}}_{gd_1} \cdot \Delta V + \frac{1}{2}\underbrace{\frac{d^2F}{dV^2}\bigg|_{V0}}_{gd_2} \cdot \Delta V^2 + \frac{1}{3!}\underbrace{\frac{d^3F}{dV^3}\bigg|_{V0}}_{gd_3} \cdot \Delta V^3$$

$\rightarrow \frac{3}{4}\cos wt + \frac{1}{4}\cos 3wt$

With $\Delta V = V_1 \cos(wt)$; ΔV^2 will give second harmonic component cos(2wt)

and ΔV^3 will give third harmonic component cos(3wt)

$$I(t) = I_0 + I_1 \cdot \cos(wt) + I_2 \cdot \cos(2wt) + I_3 \cdot \cos(3wt)$$

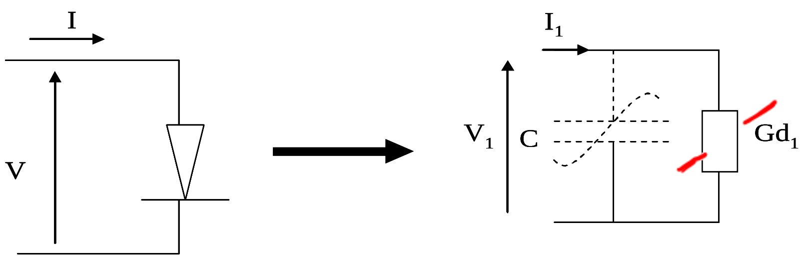
If we want to continue we need to use a computer and a simulation software like SPICE or ADS .

$Gd_1 \simeq gd$

We can introduce here the large signal conductance of the diode Gd_1 at the fundamental frequency which is a describing function of the diode for a sine wave voltage.

$$Gd_1 = \frac{I_1}{V_1}$$

We can consider in a similar way (but not developed here) that if we take into account the parallel capacitive effect at the fundamental frequency we have :

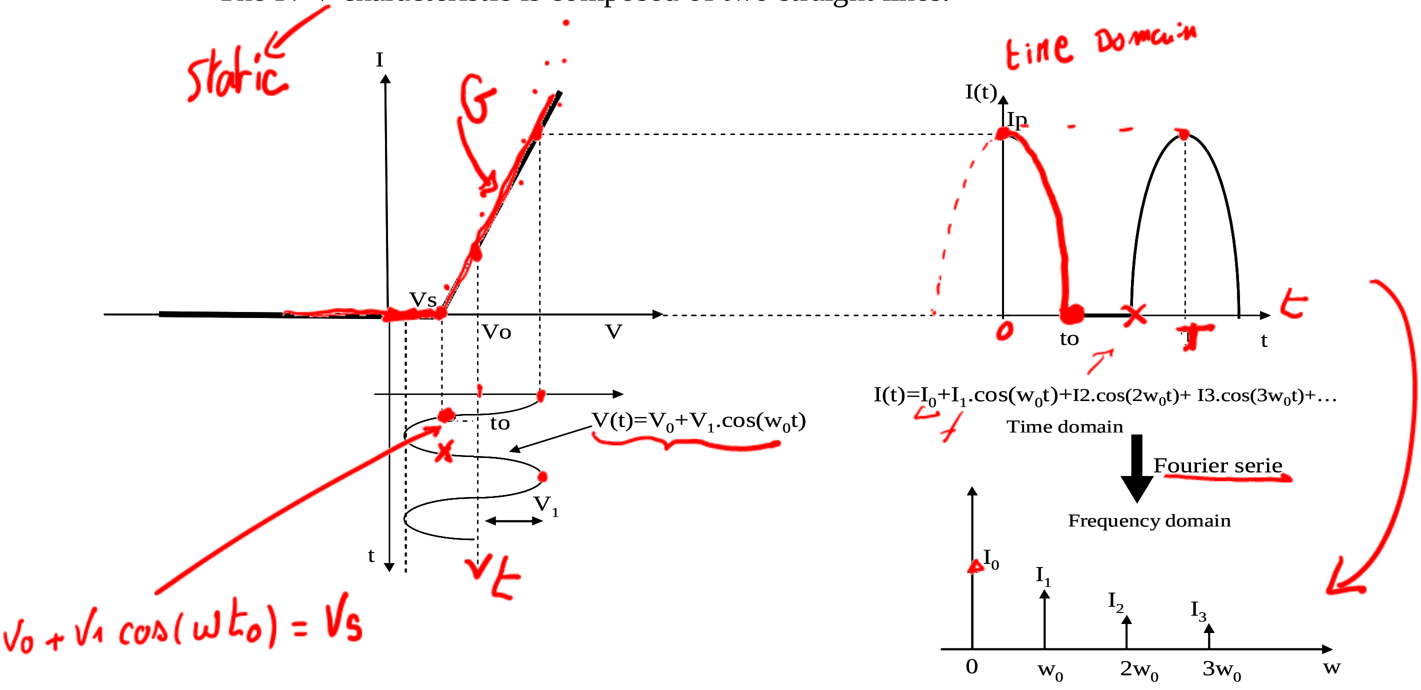


Non Linear in the ON region at the fundamental frequency

To continue with analytical calculation we will accept a reasonable assumption and consider the current versus voltage characteristic of the diode as a linear piecewise characteristic.

II) Simplified analytical approach and aperture angle notion

The I / V characteristic is composed of two straight lines.



using time variable t

$$I(t) = I_0 + I_1 \cos wt + I_2 \cos 2wt$$

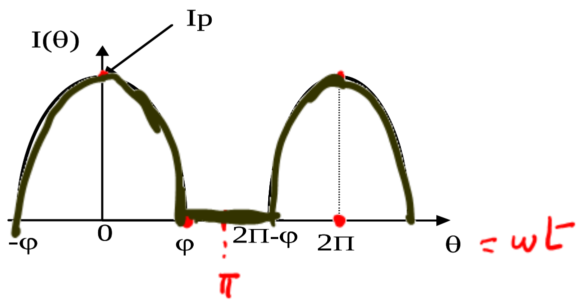
$$I_0 = \frac{1}{2T}\int_0^T I(t)\,dt \qquad I_1 = \frac{2}{2T}\int_0^T I(t)\cdot\cos wt\,dt.$$

$$I_2 = \frac{2}{2T}\int_0^T I(t)\cdot\cos 2wt\,dt.$$

The current time domain waveform is periodic and so it can be expressed in terms of Fourier serie expansion.

When $t=t_0$ $V(t_0)=Vs$ and When $t=0$ $I(t) = I_P$

Using a variable change $\theta=wt$ we can define an aperture angle $\varphi= wt_0$

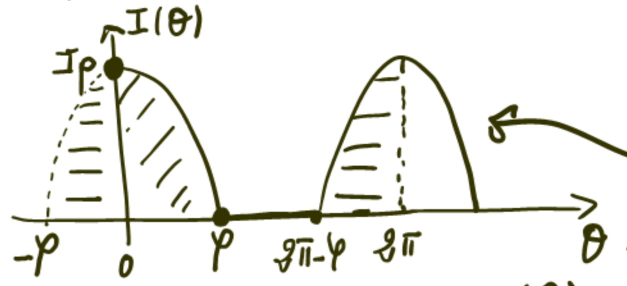


$$I(\theta) = I_0 + I_1 \cdot \cos(\theta) + I_2 \cdot \cos(2\theta) + I_3 \cdot \cos(3\theta)$$

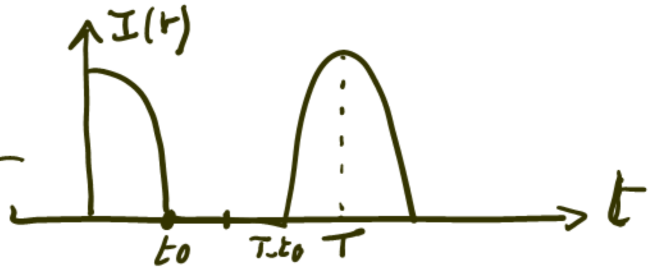
To calculate the values of I_0 , I_1 , I_2 , I_3 which are the DC and harmonic spectral components of the time domain current I(t) we need to do the following calculation :

$$I(\theta) = \frac{I_p}{1-\cos\varphi}(\cos\theta - \cos\varphi) \qquad 0 < \theta < \varphi \quad \text{and} \quad 2\pi-\varphi < \varphi < 2\pi$$

$$I(\theta) = 0 \qquad \varphi < \theta < 2\pi - \varphi$$



$$\theta = \omega t$$
$$\varphi = \omega t_0$$

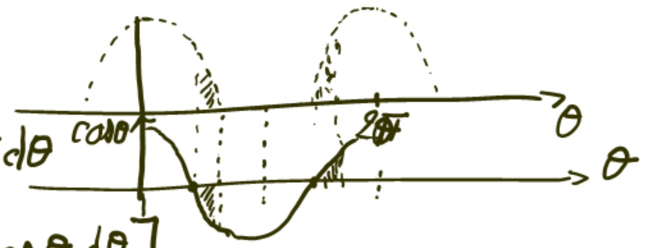


$$I(\theta) = I_0 + I_1 \cos(\theta) + I_2 \cos(2\theta) + \dots$$

$$I_0 = \frac{1}{2\pi}\int_0^{2\pi} I(\theta)\,d\theta = 2 \times \frac{1}{2\pi}\int_0^{\varphi} \frac{I_p}{1-\cos\varphi}(\cos\theta - \cos\varphi)\,d\theta$$

$$I_0 = \frac{1}{\pi}\frac{I_p}{1-\cos\varphi}\left\{\left[\sin\theta\right]_0^{\varphi} - \left[\theta\cos\varphi\right]_0^{\varphi}\right\} = \boxed{\frac{I_p\,(\sin\varphi - \varphi\cos\varphi)}{\pi(1-\cos\varphi)}}$$

$$I_1 = \frac{2}{2\pi}\int_0^{2\pi} \underbrace{I(\theta)\cdot\cos\theta}\,d\theta$$



$$I_1 = 2 \times \frac{2}{2\pi}\int_0^{\varphi} \frac{I_p}{1-\cos\varphi}(\cos\theta - \cos\varphi)\cdot\cos\theta\,d\theta$$

$$I_1 = \frac{2}{\pi}\frac{I_p}{1-\cos\varphi}\left[\int_0^{\varphi}\cos^2\theta\,d\theta - \int_0^{\varphi}\cos\varphi\cos\theta\,d\theta\right]$$

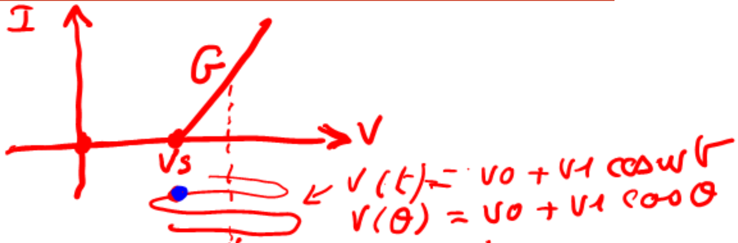
$$\cos^2\theta \rightarrow \frac{1}{2} + \frac{1}{2}\cos 2\theta$$

$$I_1 = \frac{2I_p}{\pi(1-\cos\varphi)}\left\{\left[\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right]_0^{\varphi} - \left[\cos\varphi\sin\theta\right]_0^{\varphi}\right\}$$

$$\sin 2\theta = 2\sin\theta\cdot\cos\theta$$

$$I_1 = \frac{I_p}{\pi(1-\cos\varphi)}\left[\varphi - \sin\varphi\cos\varphi\right] \longrightarrow \boxed{I_1 = \frac{I_p}{\pi(1-\cos\varphi)}(\varphi - \sin\varphi\cos\varphi)}$$

$$\begin{cases} I = G\cdot(V - V_s) & V > V_s \\ I = 0 \end{cases}$$



$$\begin{cases} I = G(V_0 + V_1\cos\theta - V_s) & V > V_s \\ I = 0 & V < V_s \end{cases} \rightarrow G V_1\left(\cos\theta - \left(\frac{V_s - V_0}{V_1}\right)\right)$$

$V_0 + V_1 \cos\varphi = V_s \rightarrow \cos\varphi = \frac{V_s - V_0}{V_1}$

$I(\theta) = G V_1(\cos\theta - \cos\varphi)$

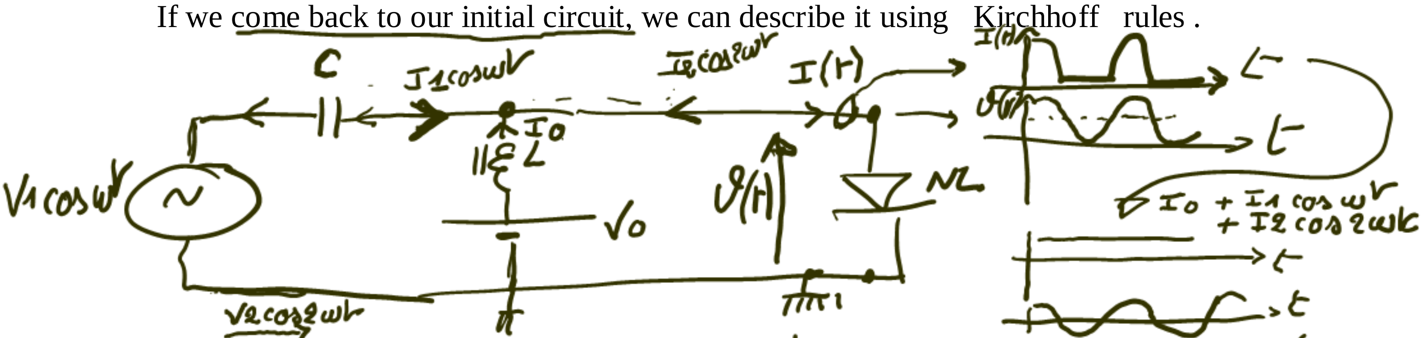
$\theta = 0 \quad I(\theta) = I_p$ ON TIME $-\varphi < \theta < \varphi$

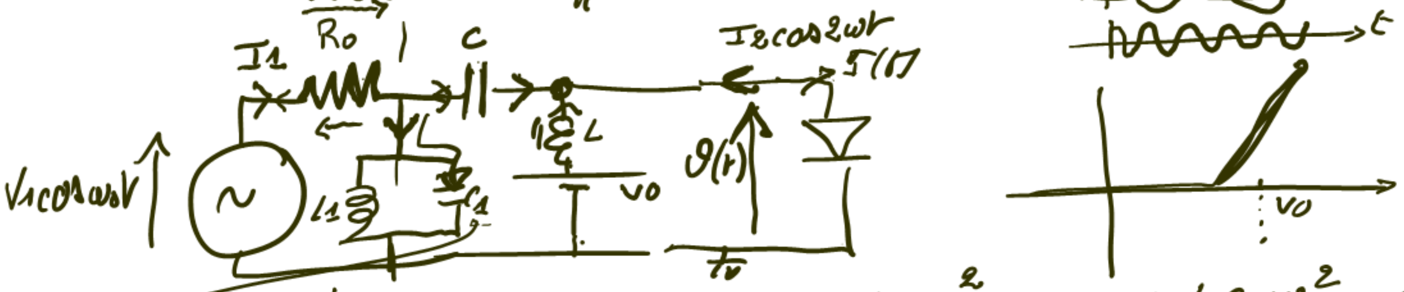
$I(0) = I_p = \underbrace{G V_1}(1 - \cos\varphi) \qquad G V_1 = \frac{I_p}{1 - \cos\varphi} \rightarrow \begin{cases} I(\theta) = I_p \frac{(\cos\theta - \cos\varphi)}{1 - \cos\varphi} \\ I(\theta) = 0 \quad \text{OFF TIME} \end{cases}$

So to summarise we will use the following relationships:

$$\begin{cases}
I_0 = \left(\frac{2}{2\pi}\right)\cdot\int_0^{\varphi}\left(\frac{Ip}{(1-\cos(\varphi))}\right)\cdot\left(\cos(\Theta) - \cos(\varphi)\right)\cdot d\Theta = \boxed{\frac{Ip\cdot(\sin(\varphi) - \varphi\cdot\cos(\varphi))}{\pi(1-\cos(\varphi))}} \\
I_1 = \left(\frac{4}{2\pi}\right)\cdot\int_0^{\varphi}\left(\frac{Ip}{(1-\cos(\varphi))}\right)\cdot\left(\cos(\Theta) - \cos(\varphi)\right)\cdot(\cos(\Theta))d\Theta = \boxed{\frac{Ip\cdot(\varphi - \sin(\varphi)\cdot\cos(\varphi))}{\pi(1-\cos(\varphi))}} \\
I_n = \left(\frac{4}{2\pi}\right)\cdot\int_0^{\varphi}\left(\frac{Ip}{(1-\cos(\varphi))}\right)\cdot\left(\cos(\Theta) - \cos(\varphi)\right)\cdot(\underbrace{\cos^n}(\Theta))d\Theta = \boxed{\frac{Ip\cdot(\cos(\varphi)\cdot\sin(n\varphi) - n\cdot\sin(\varphi)\cos(n\varphi))}{\pi\cdot n\cdot(n^2-1)(1-\cos(\varphi))}}
\end{cases}$$

If we come back to our initial circuit, we can describe it using Kirchhoff rules .





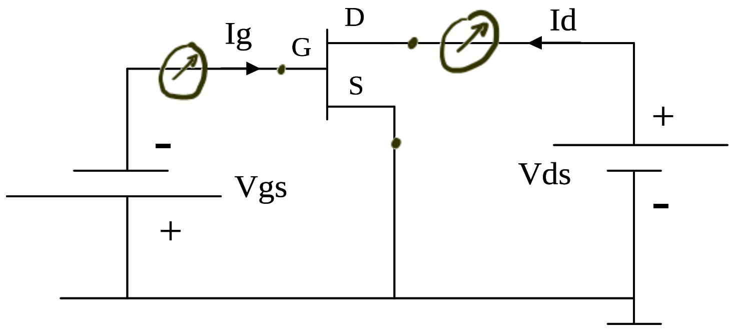
$|Z_{C_1}| = \frac{1}{C_1(2\omega)} = 0$

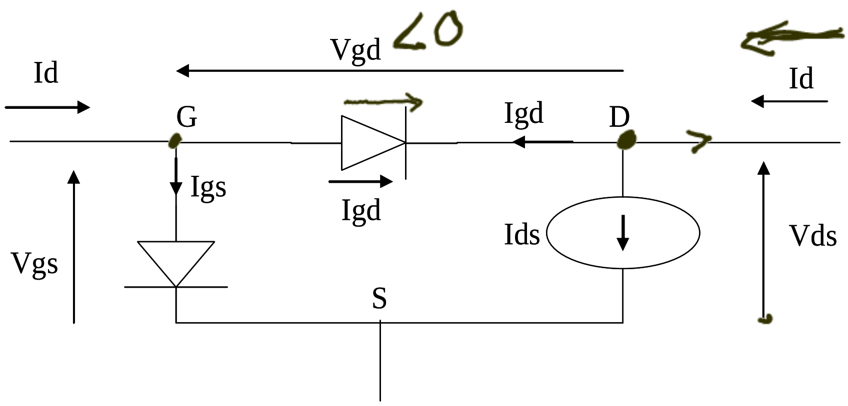
$Y = jC\omega + \frac{1}{jL\omega} = \frac{1 - LC\omega^2}{jL\omega}$

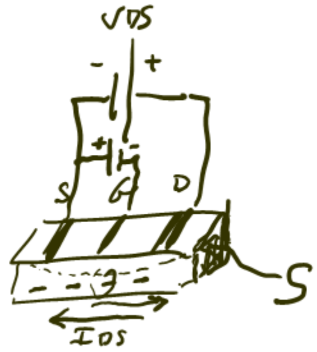
si $LC\omega^2 = 1$
$Y = 0$
$Z = \frac{1}{Y} = \infty$

III) Application to the field effect transistor (FET) - Operating classes

In a similar way, we can measure the static I/V characteristics of a FET using to DC voltage source Vgs and Vds







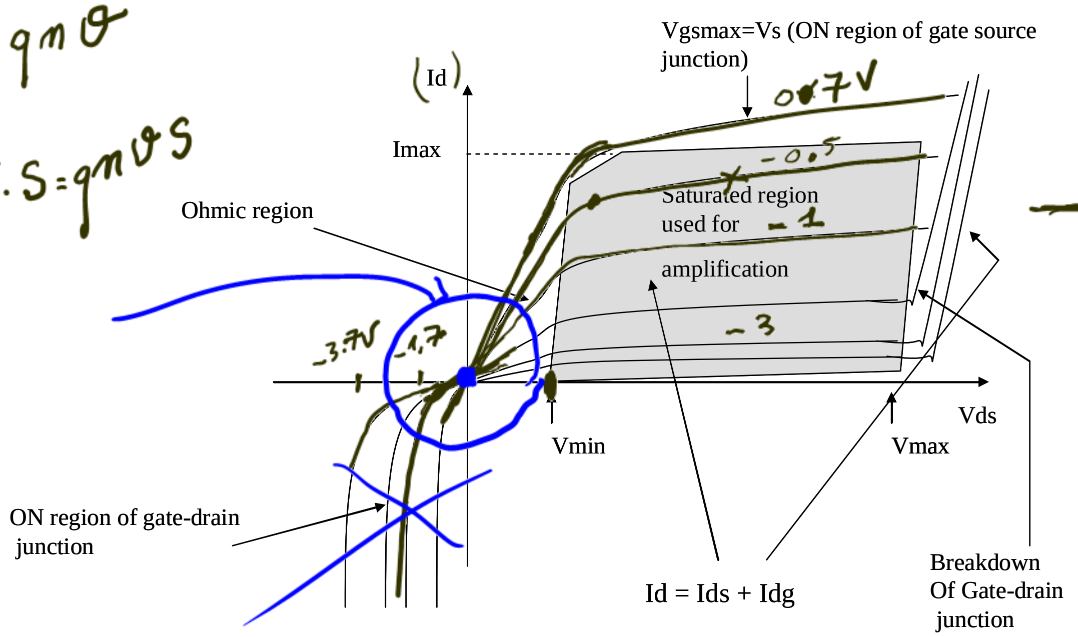
We obtain the following characteristic

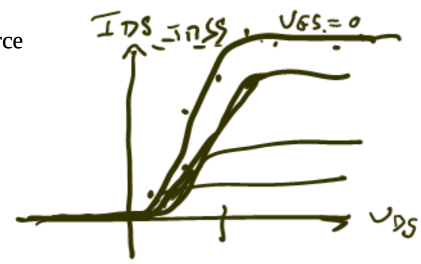
$J = qnv$

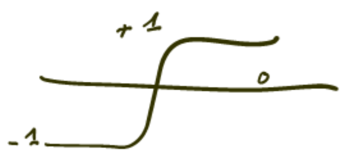
A/m^2

$I = J.S = qnvS$

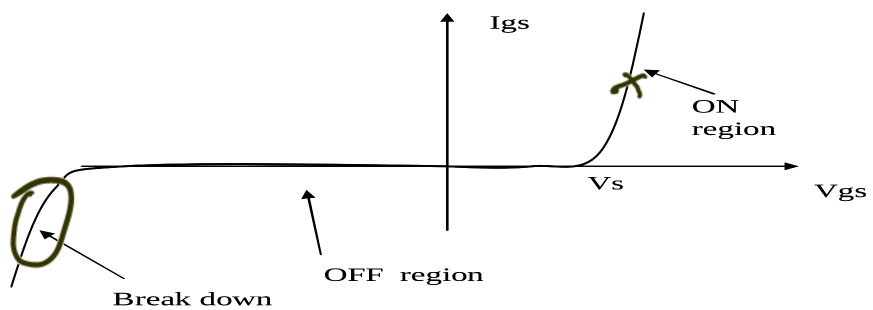
A



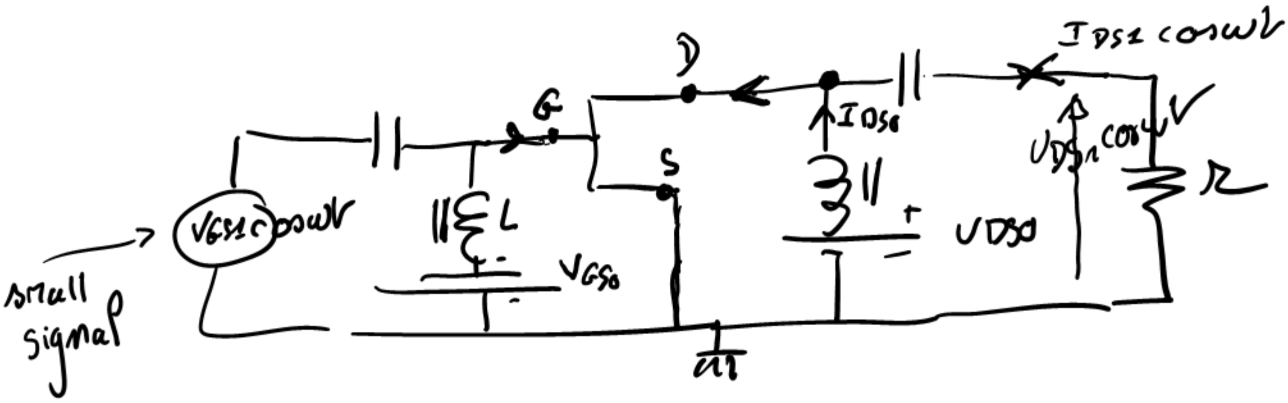




$I_{DS} = I_{DSS}\left(1 - \frac{v_{gs}}{v_p}\right)^2$

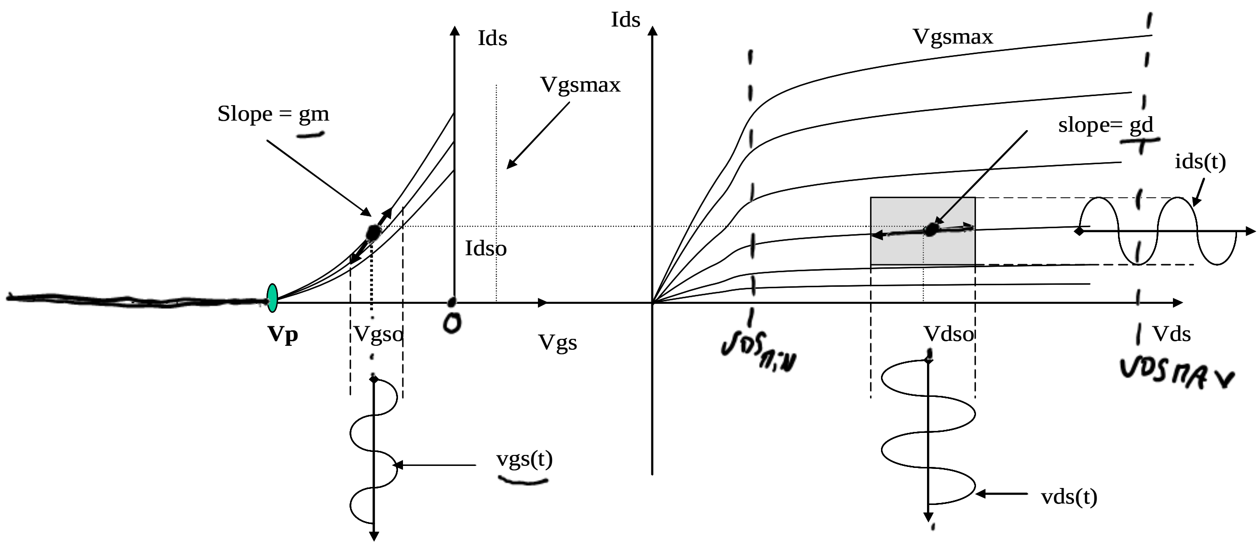


Let us consider now the following circuit:



As far as sinusoidal voltages vgs(t) and vds(t) remain small , the behavior of the transistor is linear and we can illustrated this linear behavior as following .

- **Linear behavior** (VGS0, VDS0)

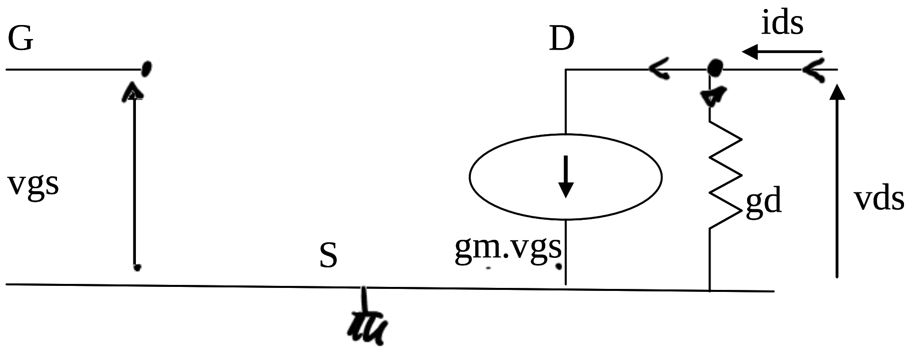


$$I_{ds} = F(V_{gs},V_{ds}) = F(V_{gs0}+\Delta V_{gs},V_{ds0}+\Delta V_{ds}) = F(V_{gs0},V_{ds0}) + \left.\frac{dF}{dV_{gs}}\right|_{V_{gs0},V_{ds0}} .\Delta V_{gs} + \left.\frac{dF}{dV_{ds}}\right|_{V_{gs0},V_{ds0}} .\Delta V_{ds}$$

$$I_{ds} = I_{ds0} + gm\cdot \Delta V_{gs} + gd\cdot \Delta V_{ds}$$

$$V_{gs}(t) = V_{gs0} + V_{gs1}\cdot \cos(wt)$$
$$V_{ds}(t) = V_{ds0} + V_{ds1}\cdot \cos(wt)$$
$$I_{ds}(t) = I_{ds0} + ids(t) = I_{ds0} + I_{ds1}\cos(wt) = I_{ds0} + gm\cdot V_{gs1}\cos(wt) + gd\cdot V_{ds1}\cos(wt)$$
$$I_{ds1} = gm\cdot V_{gs1} + gd\cdot V_{ds1}$$

So the small signal linear equivalent model of the transistor is :



We assume for the moment that internal capacitive effects are negligible .

(Cgs and Cds) present an infinite impedance (open circuit) at the operating frequency F0 .

- **Non- Linear behavior**

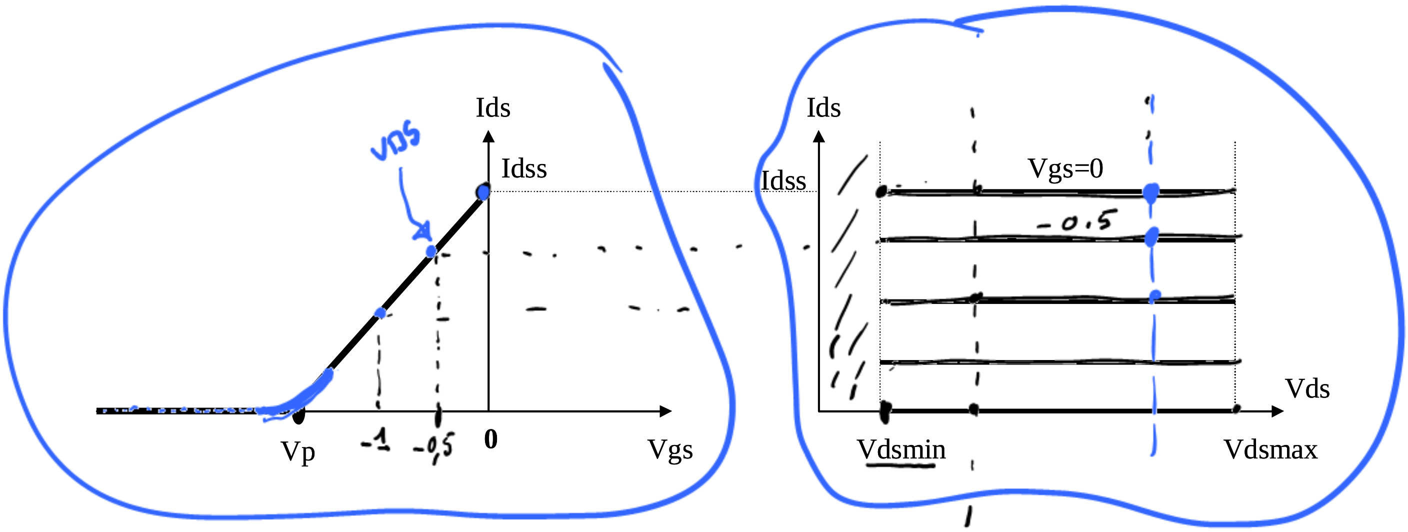
If the magnitude of the sinusoidal signal increases we cannot use a first order Taylor expansion and higher order must be taken into account .

$$I_{ds}=F(V_{gs},V_{ds})=F(V_{gs0}+\Delta V_{gs},V_{ds0}+\Delta V_{ds})=F(V_{gs0},V_{ds0})+\left.\frac{dF}{dV_{gs}}\right|_{V_{gs0},V_{ds0}}.\Delta V_{gs}+\left.\frac{dF}{dV_{ds}}\right|_{V_{gs0},V_{ds0}}.\Delta V_{ds}$$

$$+\underbrace{\frac{1}{2}\left.\frac{d^2F}{dV_{gs}^{\;2}}\right|_{V_{gs0},V_{ds0}}}_{g_{m_2}}.\left(\Delta V_{gs}\right)^2+\underbrace{\frac{1}{2}\left.\frac{dF}{dV_{ds}}\right|_{V_{gs0},V_{ds0}}}_{g_{d2}}.\left(\Delta V_{ds}\right)^2+\underbrace{\frac{2\;d^2F}{dV_{gs}dV_{ds}}}_{g_m g_d}\Big|_{V_{gs0},V_{ds0}}.\Delta V_{gs}.\Delta V_{ds}+\dots\dots\dots\dots$$

Which is quite complicated, so we will proceed to some simplifications

We make a simplification of the I/V characteristic of the transistor usable only in the saturated region. So we consider the following linear piecewise characteristic.

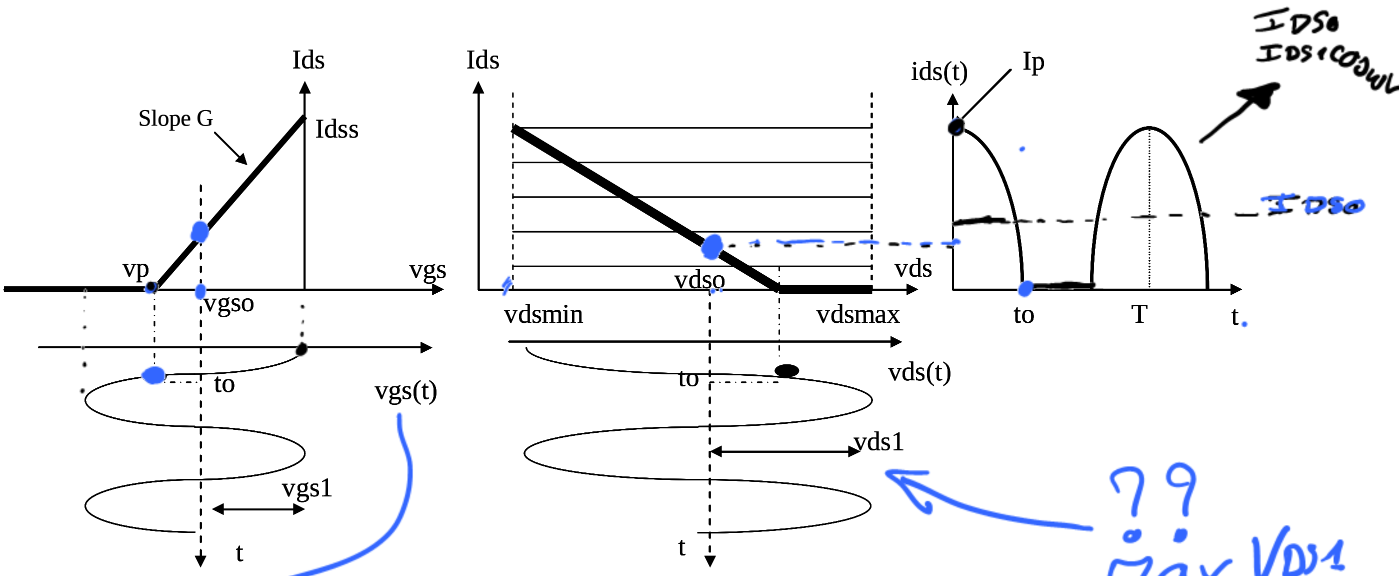


VGS < 0

VDS

VDSmin < VDS < VDSmax

We consider also a large input sinusoidal voltage Vgs(t) as represented below.



!!!

$$V_{gs}(\theta)=V_{gs0}+V_{gs1}\cdot\cos(\theta)$$

$$\boxed{V_p=V_{gs0}+V_{gs1}\cdot\cos(\varphi)}$$

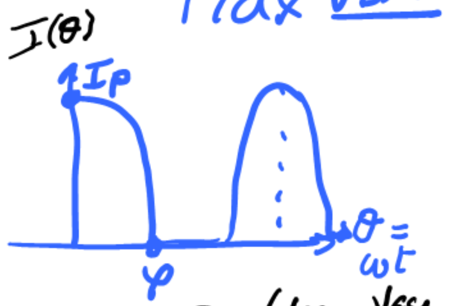
$$\cos(\varphi)=\frac{V_p-V_{gs0}}{V_{gs1}}$$

$\theta = \omega t$

$\varphi = \omega t_o$

$\varphi = (\mathrm{Arccos}(\varphi))^{-1}$

↳ $\sin\varphi$



$I(\theta) = G \times (V_{GSo} + V_{GS1}\cos\theta - V_p)$

$I(\theta) = G V_{GS1}\left(\cos\theta - \frac{V_p - V_{GSo}}{V_{GS1}}\right)$

$I(\theta) = G V_{GS1}(\cos\theta - \cos\varphi)$

And For θ=0

$$\begin{cases} I_p = G\cdot(V_{gs1}+V_{gs0}-V_p) = G\cdot V_{gs1}\left(1+\dfrac{V_{gs0}-V_p}{V_{gs1}}\right) = G\cdot V_{gs1}(1-\cos(\varphi)) \\ G\cdot V_{gs1} = \dfrac{I_p}{1-\cos(\varphi)} \end{cases}$$

The current waveform is similar to the one described previously for the diode . The only difference is that the voltage threshold Vp is here a negative value . For the diode we have considered a positive voltage threshold Vs .

So the harmonic components are Ids_0 , Ids_1 , Ids_n are obtained using the same formula .

$$Ids(\theta) = Ids_0 + Ids_1 \cdot \cos(\theta) + Ids_2 \cdot \cos(2\theta) + Ids_3 \cdot \cos(3\theta)$$

Ip , φ

$$Ids_o = \frac{Ip \cdot (\sin(\varphi) - \varphi \cdot \cos(\varphi))}{\pi(1 - \cos(\varphi))}$$

$$Ids_1 = \frac{Ip \cdot (\varphi - \sin(\varphi) \cdot \cos(\varphi))}{\pi(1 - \cos(\varphi)}$$

$$Ids_n = \frac{Ip \cdot (\cos(\varphi) \cdot \sin(n\varphi) - n \cdot \sin(\varphi)\cos(n\varphi))}{\pi \cdot n \cdot (n^2 - 1)(1 - \cos(\varphi)}$$

At the transistor output we want two main things

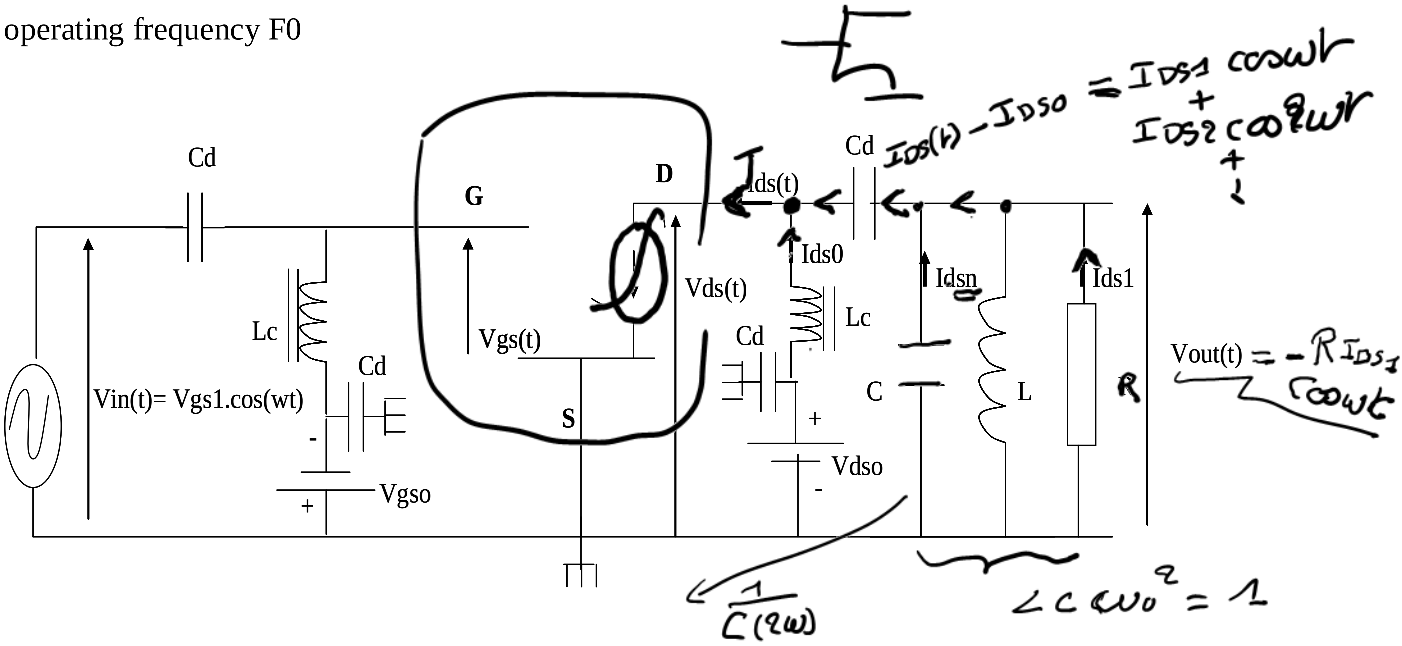
- First, we want to reject harmonic components

- Secondly we want to get the maximum magnitude of the drain voltage at the fundamental frequency in order to have the maximum output RF power level at the fundamental operating frequency.

To do that we choose to have

$$\begin{cases} Vds_0 = \dfrac{Vds_{max} + Vds_{min}}{2} \\ Vds_1 = \dfrac{Vds_{max} - Vds_{min}}{2} \\ I_p = I_{dss} \end{cases}$$

And we use a resonant parallel circuit having its center frequency at the fundamental operating frequency F0



So given a value of the aperture angle φ which defines the operating class of the transistor

And also knowing the value of Ip which depends on the input voltage Vgs(t) at t=0 we have :

$$V_{gs0} + V_{gs1}\cos(\varphi) = Vp$$

$$\varphi = A\mathrm{rccos}\left(\frac{Vp - V_{gs0}}{V_{gs1}}\right)$$

$$I_{ds0} = \frac{Ip\cdot(\sin(\varphi) - \varphi\cdot\cos(\varphi))}{\pi(1-\cos(\varphi)}$$

$$I_{ds1} = \frac{Ip\cdot(\varphi - \sin(\varphi)\cdot\cos(\varphi))}{\pi(1-\cos(\varphi)}$$

$$P_{dc} = V_{ds0}\cdot I_{ds0}$$

$$P_{out} = \frac{1}{2}\cdot V_{ds1}\cdot I_{ds1}$$

$$\eta_d = \frac{P_{out}}{P_{dc}}$$

$$R_L = \frac{V_{ds1}}{I_{ds1}}$$

P_{dc} is the DC consumption , P_{out} is the output RF power ,η_d is the drain efficiency and R_L is the load resistance required to have the maximum output RF power .

Furthermore, we need to have the resonance of the output parallel circuit at the fundamental frequency $LCw_0^{\ 2} = 1$