5-November 2021

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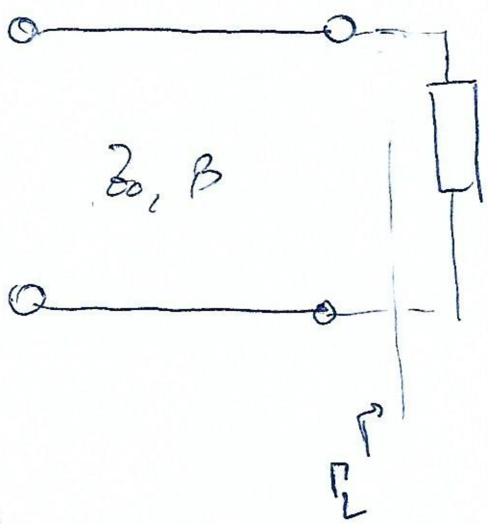


$Z_0 = 60\ \Omega$

$\Gamma_L = 0{,}4$

a)

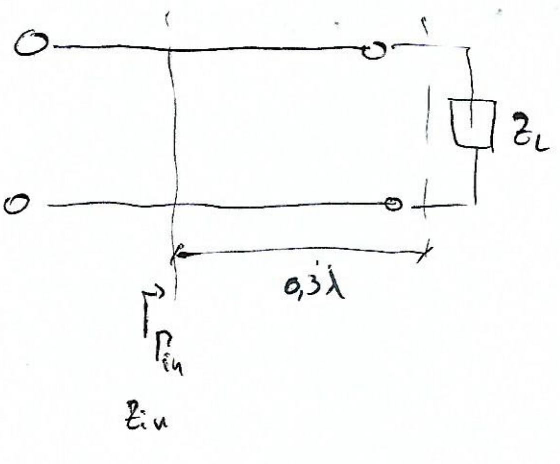
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \longrightarrow (Z_L + Z_0)\Gamma_L = Z_L - Z_0$$

$$Z_L - Z_L\Gamma_L = Z_0(1+\Gamma_L)$$

$$Z_L = Z_0 \frac{(1+\Gamma_L)}{1-\Gamma_L} = 60 \cdot \frac{1+0{,}4}{1-0{,}4} =$$

$Z_L = 140\ \Omega$

c)



$$\beta l = \frac{2\pi}{\lambda} \cdot 0{,}3\lambda = 0{,}6\pi$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} = 60 \cdot \frac{140 + j60 \cdot \tan(0{,}6\pi)}{60 + j140 \tan(0{,}6\pi)} =$$

$$= 60 \cdot \frac{[140 + j60 \cdot \tan(0{,}6\pi)] \cdot [60 - j140 \tan(0.6\pi)]}{60^2 + 140^2 \tan^2(0{,}6\pi)} =$$

$$\frac{60}{60^2 + 140^2 \tan^2(0{,}6\pi)} \cdot \left[140 \cdot 60 + j\left(-140^2 \tan(0{,}6\pi) + 60^2 \tan(0{,}6\pi)\right) + 140 \cdot 60 \cdot \tan^2(0{,}6\pi)\right] =$$

$$= \frac{60}{189253{,}8647} \cdot \left(87965{,}94202 + j\,49242{,}9360\right)$$

$$= 27{,}88823642 + j\,15{,}61170865$$

$$\boxed{Z_{in} = 27{,}88823642 + j\,15{,}61170865\ \Omega}$$

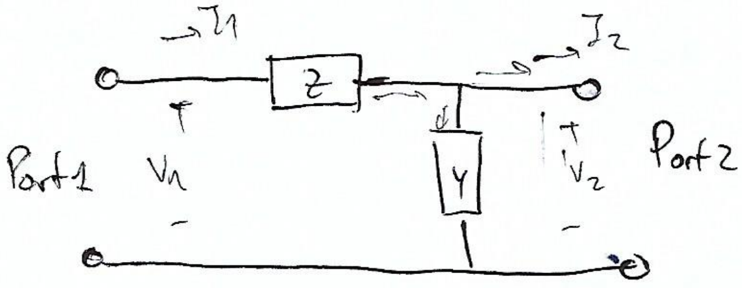
5)

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{a + bi - Z_0}{a + bi + Z_0} = \frac{(a - Z_0) + bi}{(a+Z_0)^2 + b^2} \cdot (a + Z_0) - bi =$$

$$= \frac{(a - Z_0)(a + Z_0) - (a - Z_0)bi + (a + Z_0)bi + b^2}{(a+Z_0)^2 + b^2} =$$

$$= \frac{a^2 - Z_0^2 + jb\left[\cancel{-a} + Z_0 \cancel{+a} + Z_0\right] + b^2}{Den} = \frac{a^2 + b^2 - Z_0^2}{(a+Z_0)^2 + b^2} + j\,\frac{Z_0 b}{(a+Z_0)^2 + b^2}$$

2.



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad \begin{aligned} V_1 &= A V_2 + B I_2 \\ I_1 &= C V_2 + D I_2 \end{aligned}$$

- $A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$ $\qquad V_2 = \dfrac{\frac{1}{Y}}{Z+\frac{1}{Y}} V_1 = \dfrac{\frac{1}{Y}}{\frac{YZ+1}{Y}} V_1 = \dfrac{1}{YZ+1} V_1$

$$A = \frac{V_1}{\frac{V_1}{1+YZ}} \rightarrow \boxed{A = 1+YZ}$$

- $B = \left.\dfrac{V_1}{I_2}\right|_{V_2=0}$ $\qquad V_1 = Z \cdot I_1 \longrightarrow V_1 = Z \cdot I_2 \rightarrow \boxed{B = Z}$

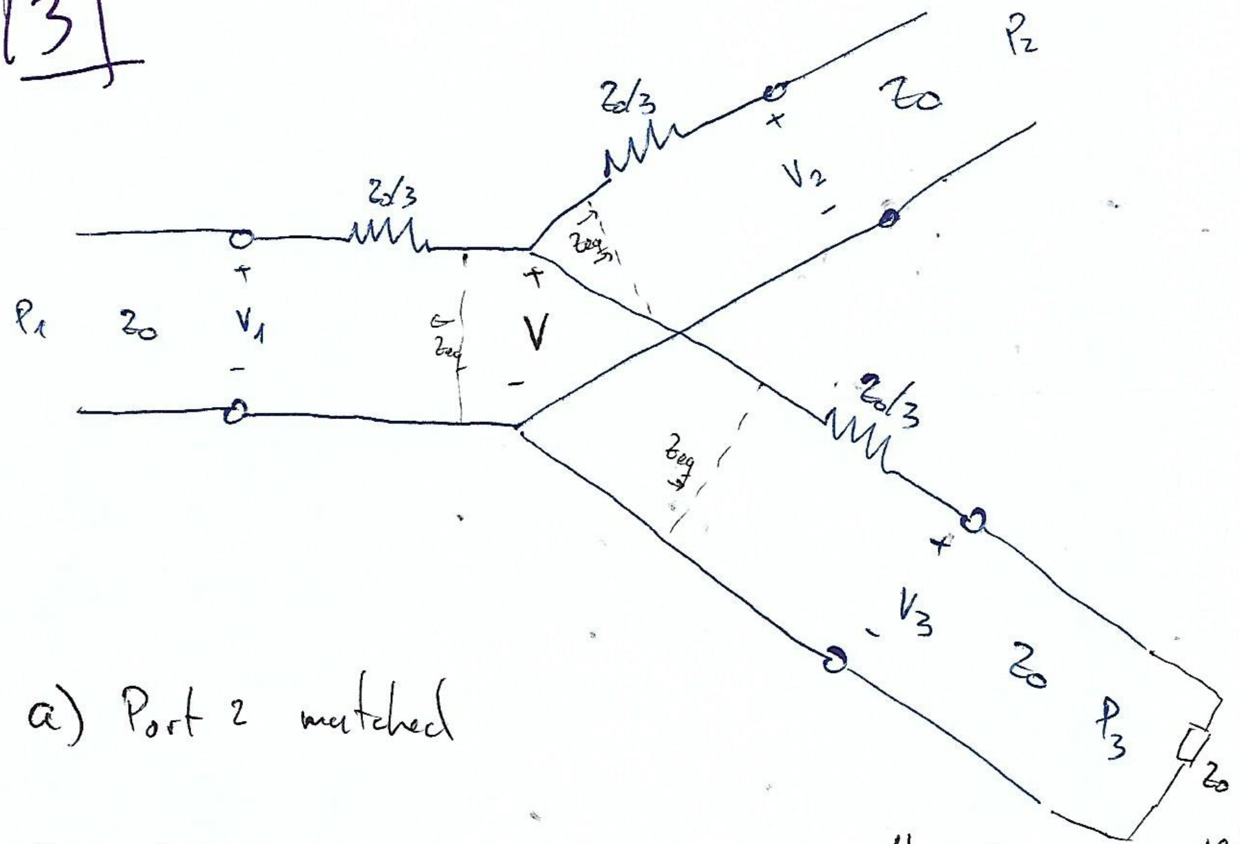
$$I_1 = I_2$$

- $C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0}$ $\qquad V_2 = \dfrac{1}{Y} I_1 \longrightarrow \boxed{C = Y}$

- $D = \left.\dfrac{I_1}{I_2}\right|_{V_2=0}$ $\qquad I_1 = I_2 \longrightarrow \boxed{D = 1}$

$$ABCD = \begin{bmatrix} 1+YZ & Z \\ Y & 1 \end{bmatrix}$$

$\boxed{3}$



a) Port 2 matched

$$Z_{eq} = \frac{Z_0}{3} + Z_0 = \frac{4}{3} Z_0$$

$$Z_{eq} \| Z_{eq} = \frac{\frac{4}{3} Z_0 \cdot \frac{4}{3} Z_0}{\frac{4}{3} Z_0 + \frac{4}{3} Z_0} = \frac{\frac{16 Z_0^{\cancel{2}}}{9}}{\frac{8 Z_0}{3}} = \frac{16 \cdot 3}{8 \cdot 9} Z_0 = \frac{2 \cdot \cancel{8} \cdot \cancel{3}}{\cancel{8} \cdot 3 \cdot \cancel{3}} Z_0 = \frac{2}{3} Z_0$$

$$V_3 = \frac{Z_0}{\frac{Z_0}{3} + Z_0} V = \frac{Z_0}{\frac{4}{3} Z_0} V = \frac{3}{4} V$$

$$V = \frac{Z_{eq} \| Z_{eq}}{Z_{eq} \| Z_{eq} + \frac{Z_0}{3}} V_1 = \frac{\frac{2}{3} Z_0}{\frac{2}{3} Z_0 + \frac{Z_0}{3}} V_1 = \frac{\frac{2 Z_0}{3}}{Z_0} V_1 = \frac{2}{3} V_1$$

$$\rightarrow V_3 = \frac{\cancel{3}}{4} \cdot \frac{2}{\cancel{3}} V_1 = \frac{1}{2} V_1$$

$$P_1 = \frac{1}{2} \frac{V_1^2}{Z_0}$$

$$P_3 = \frac{1}{2} \frac{V_3^2}{Z_0} = \frac{1}{2} \frac{\left(\frac{1}{2} V_1\right)^2}{Z_0} = \frac{1}{2} \frac{1}{2} \cdot \left(\frac{1}{2} \frac{V_1^2}{Z_0}\right) = \frac{1}{4} P_1 \Rightarrow \boxed{P_3 = 0{,}25\,W}$$

b) Port mismatch $\Gamma = 0{,}3$

Calculating load:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow \quad Z_L = Z_0 \frac{1+\Gamma_L}{1-\Gamma_L} = Z_0 \frac{1{,}3}{0{,}7} \Rightarrow \boxed{Z_L = 1{,}86\, Z_0}$$

bc everything else is matched.

Now we have another Z_{eq} in the port 2 Z_{eq2}

$$Z_{eq_2} = \frac{Z_0}{3} + 1{,}86\, Z_0 = \frac{1+3\cdot 1{,}86}{3} Z_0 = 2{,}20\, Z_0$$

$$V_3 = \frac{Z_0}{\frac{Z_0}{3} + \frac{Z_0}{3}} V = \frac{3}{4} V = 0{,}535\, V_1$$

$$V = \frac{Z_{eq} \parallel Z_{eq_2}}{Z_{eq} \parallel Z_{eq_2} + Z_0/3} V_1 = \frac{0{,}83\, Z_0}{(0{,}83 + \frac{1}{3}) Z_0} V_1 = 0{,}7135\, V_1$$

$$Z_{eq} \parallel Z_{eq2} = \frac{\frac{4}{3} Z_0 \cdot 2{,}20\, Z_0}{(\frac{4}{3} + 2{,}20) Z_0} = \frac{\frac{4\cdot 2{,}20\, Z_0^{\cancel{2}}}{\cancel{3}}}{\frac{(4+3\cdot 2{,}20)\cancel{Z_0}}{\cancel{3}}} = \frac{8{,}8}{10{,}6} Z_0 = 0{,}83\, Z_0$$

$$P_3 = \frac{1}{2} \frac{V_3^2}{Z_0} = \frac{1}{2} \frac{(0{,}535 \cdot V_1)^2}{Z_0} = 0{,}535^2 \cdot \frac{1}{2} \frac{V_1^2}{Z_0} = 0{,}535^2 P_1 = 0{,}2862$$

$$P_1 = \frac{1}{2} \frac{V_1^2}{Z_0}$$

Exam 17 - July - 2020

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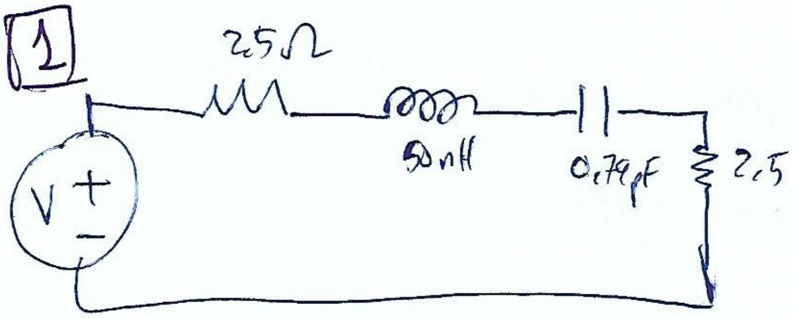


a)

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{50\cdot10^{-9}\cdot 0{,}79\cdot10^{-12}}} = \cancel{5{,}03\ GHz}\ 0{,}8\ GHz$$

b)

$$Z_{in} = R + j\omega L + \frac{1}{j\omega C} = R = 2{,}5\,\Omega$$

c)

Unloaded: $$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{2{,}5}\sqrt{\frac{50\cdot10^{-9}}{0{,}79\cdot10^{-12}}} = 100{,}63$$

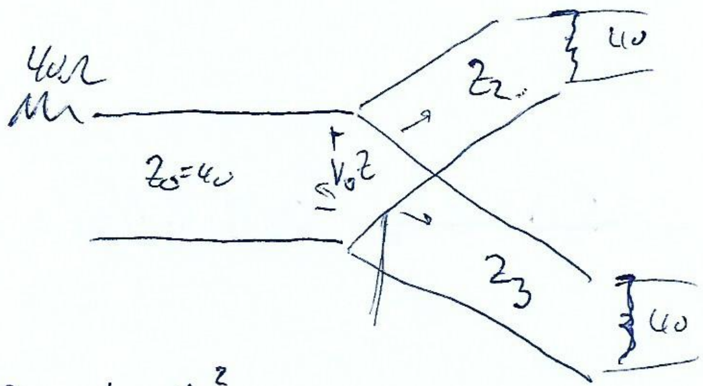
$$Q_E = \frac{\omega_0 L}{R_L} = \frac{2\pi\, 0{,}8\cdot10^{9}\cdot 50\cdot10^{-9}}{2{,}5} = 100{,}53$$

$$\frac{1}{Q} = \frac{1}{Q_U} + \frac{1}{Q_E} \Rightarrow Q = \left(\frac{1}{Q_U} + \frac{1}{Q_E}\right)^{-1} = \cancel{86{,}81}\ 50{,}29$$

[2]

Lossless T-junction 40Ω impedance source

4:1 power split



$$P_1 = \frac{1}{2}\frac{V_0^2}{Z_0}$$

$$\frac{P_2}{P_3} = 4 \rightarrow P_2 = 4P_3$$

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_2} =$$

Lossless: $P_1 = P_2 + P_3 = 4P_3 + P_3 = 5P_3$

$$P_3 = \frac{1}{2}\frac{V_0^2}{Z_3}$$

$$\cancel{\frac{1}{2}}\frac{\cancel{V_0^2}}{Z_0} = 5\cdot\cancel{\frac{1}{2}}\frac{\cancel{V_0^2}}{Z_3} \Rightarrow \boxed{Z_3 = 5Z_0} \boxed{= 200\Omega}$$

$$P_2 = 4P_3 \rightarrow \frac{1}{\cancel{2}}\frac{\cancel{V_0^2}}{Z_2} = \frac{4}{\cancel{2}}\frac{\cancel{V_0^2}}{Z_3}$$

$$Z_2 = \frac{Z_3}{4} = \frac{5Z_0}{4} \rightarrow \boxed{Z_2 = 50\Omega}$$

Quarter λ/4

$$Z_{in} = \frac{Z_0^2}{Z_L} \rightarrow$$

$$Z_{02} = \sqrt{Z_{in_2} Z_L} = \sqrt{50\cdot 40} = 44{,}72\ \Omega$$

$$Z_{03} = \sqrt{Z_{in_3}\cdot Z_L} = \sqrt{200\cdot 40} = 89{,}44\ \Omega$$

②

The reflex coef:

from Z_2 we see: $40 \parallel 200 = 33{,}33\,\Omega \rightarrow S_{22} = \rho_2 = \dfrac{33{,}33-40}{33{,}33+40} =$

$S_{22} = -0{,}09$

from Z_3 we see: $40 \parallel 50 = 22{,}22\,\Omega$

$\hookrightarrow S_{33} = \dfrac{22{,}22-40}{22{,}22+40} = -0{,}285$

$$|S_{21}| = |S_{12}| = \sqrt{\frac{P_2}{P_1}} = \sqrt{\frac{\dfrac{\cancel{V_0^2}\cdot 4}{\cancel{2}\cdot 5\,\cancel{Z_0}}}{\dfrac{\cancel{V_0^2}}{\cancel{2Z_0}}}} = \sqrt{\frac{4}{5}} = 0{,}89$$

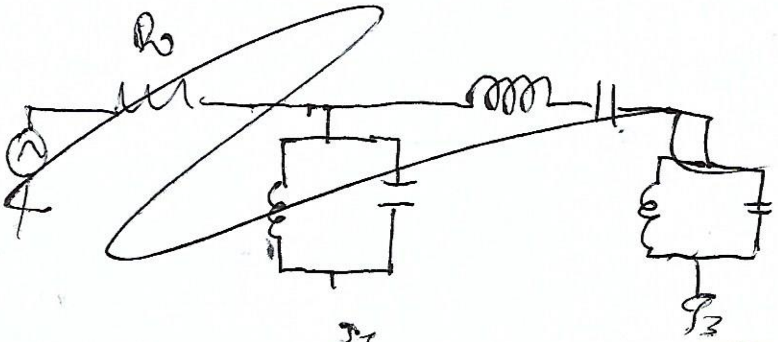
$$|S_{31}| = |S_{13}| = \sqrt{\frac{P_3}{P_1}} = \sqrt{\frac{\dfrac{1\,\cancel{V_0^2}}{\cancel{2}\cdot 5\,\cancel{Z_0}}}{\dfrac{\cancel{V_0^2}}{\cancel{2Z_0}}}} = \sqrt{\frac{1}{5}} = 0{,}45$$

Three section bandstop

0,5 eq-ripp $\qquad Z_0 = 75\,\Omega$

BW = 10%

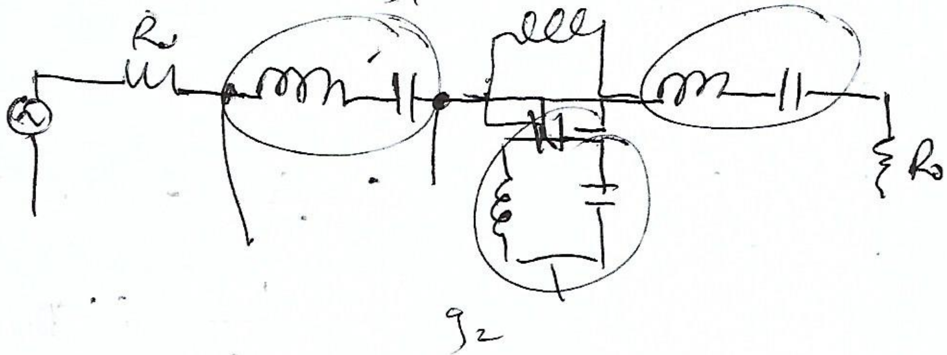
$f_c = 3\ GHz$



$g_1 = 1{,}5963$

$g_2 = 1{,}0967$

$g_3 = 1{,}5963$



$$L_1' = \frac{R_0}{\omega_0 g_1 \Delta} = 16{,}62\ nH$$

$$C_1' = \frac{g_1 \Delta}{\omega_0 R_0} = 0{,}169\ pF$$

$$L_2' = \frac{g_2 \Delta R_0}{\omega_0} = 0{,}29\ nH$$

$$C_2' = \frac{1}{\omega_0 g_2 \Delta R_0} = 10{,}61\ pF$$

Low pass filter

$f_c = 2\,GHz$

$Z_0 = 50$

$IL = 15\,dB$ at $3\,GHz \rightarrow N=5$

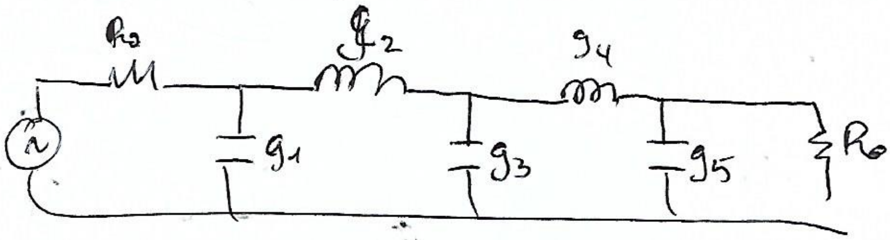
$g_1 = 0,6180$

$g_2 = 1,618$

$g_3 = 2$

$g_4 = 1,618$

$g_5 = 0,6180$



$C_1' = \frac{C_1}{\omega_c R} = 0,9835\,pF$

$L_2' = \frac{L_2 R}{\omega_c} = 6,438\,nH$

High pass filter

3dB eq. ripple

$Z_0 = 75$

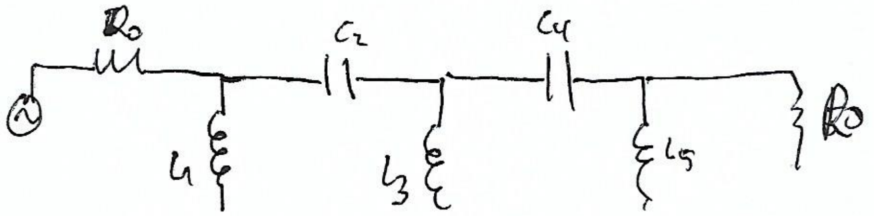
$f_c = 3\,GHz$

30dB at $2\,GHz \rightarrow N=5$

$g_1 = 3,4817$ $\quad g_4 = 0,7618$

$g_2 = 0,7618$ $\quad g_5 = 3,4817$

$g_3 = 4,5381$



$L_1 = \frac{R_0}{\omega_c g_1} = 1,143\,nH$ $\qquad L_3 = 0,877\,nH$ $\qquad L_5 = \frac{R_0}{\omega_c g_5} = 1,143\,nH$

$C_2 = \frac{1}{\omega_c R_0 g_2} = 0,929\,pF$ $\qquad C_4 = \frac{1}{\omega_c R_0 g_4} = 0,929\,pF$

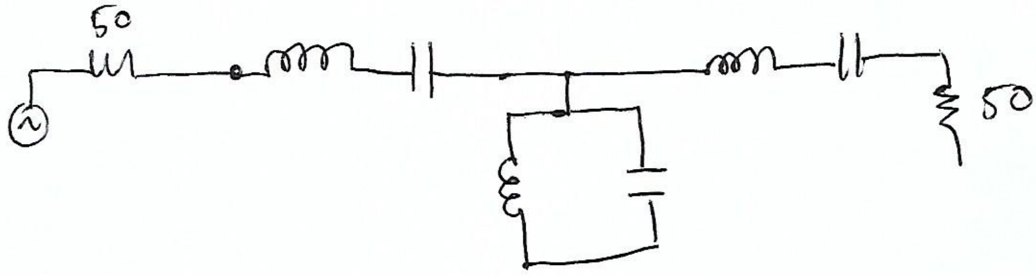
Band pass filter:

$$\frac{R_0 L}{\omega_0 \Delta} =$$

- 0,5 dB eq. ripple
- $N=3$
- $f_0 = 1\,GHz$

$BW = 10\%$ $\qquad \frac{\Delta}{R_0 \omega_0 L}$

$Z_0 = 50$



$g_1 = 1{,}5963$

$g_2 = 1{,}0967$

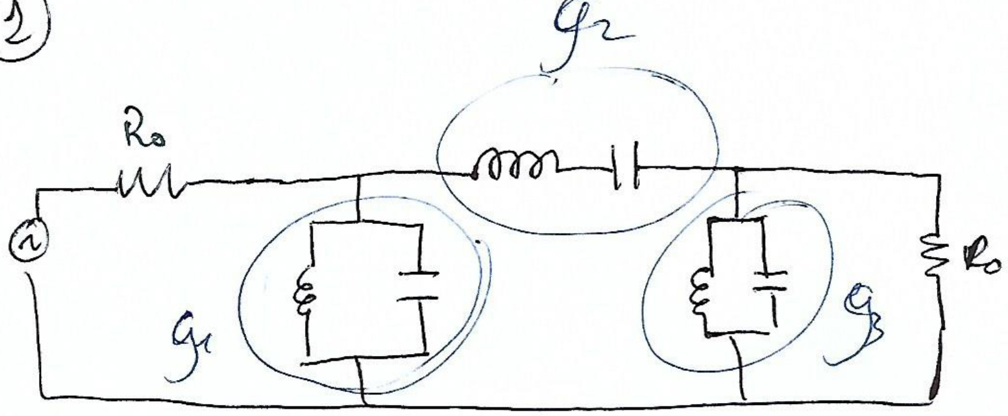
$g_3 = 1{,}5963$

$$L_1' = \frac{\Delta R}{\omega_0 C_1} = 0{,}5\,nH$$

$$C_1' = \frac{C}{\omega_0 \Delta R} = \cancel{1{,}6\,nF}\ 50\,pF$$

Bandpass

①



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