1. In the Bohr's model of the H atom, give an expression for:
a. the electron speed as a function of the orbit radius.
b. the total energy as a function of the orbit radius.

(a)

We recall these two equations

- $r = r_0 n^2 \quad (1)$
- $v_R = \alpha c \frac{1}{n} \quad (2)$

Using (1) we get n in function of the radius

$$\frac{r}{r_0} = n^2 \longrightarrow n = \sqrt{\frac{r}{r_0}} \quad (3)$$

Now we substitute (3) in (1)

$$v_R = \alpha c \frac{1}{n} = \alpha c \sqrt{\frac{r_0}{r}} \longrightarrow \boxed{v_R = \alpha c \sqrt{\frac{r_0}{r}}} \quad (4)$$

(b)

We recall the next equation:

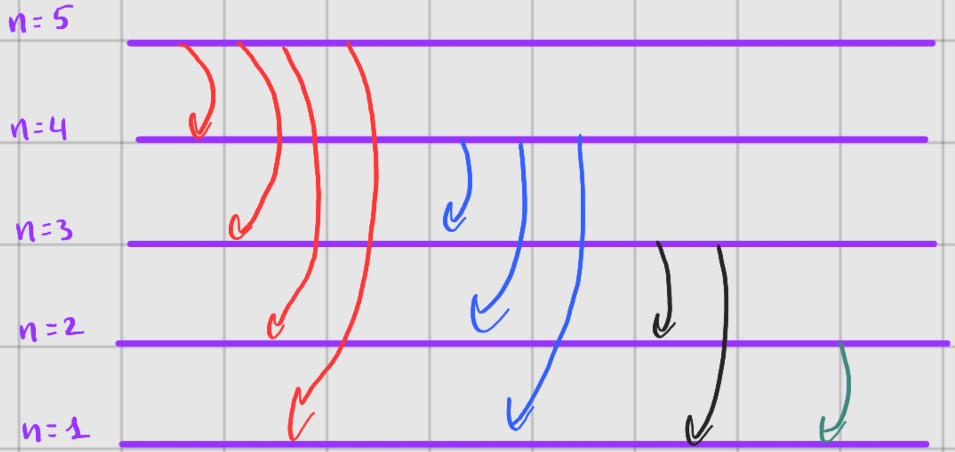
- $E = -\frac{R}{n^2} \quad (5)$

We substitute (3) in (5)

$$E = -\frac{R}{n^2} = -R \cdot \frac{r_0}{r} \longrightarrow \boxed{E = -R \frac{r_0}{r}} \quad (5)$$

2. How many different photons could be emitted upon a transition from the n=5 down to the fundamental n=1 of an H atom. Compute the exact frequency for one of the transitions.

There are 10 different photons that can be emitted when we start from level n=5 to the level n=1. We can see these transitions in the next graphic.



$\boxed{10 \text{ transitions}}$

In general: # transitions = $\frac{n(n-1)}{2}$

The frequency of transition $n=4$ to $n=2$

$$E_n = -\frac{R}{n^2}$$

$$\Rightarrow E_4 = -\frac{R}{16}$$

$$\Rightarrow E_2 = -\frac{R}{4}$$

$$E_4 - E_2 = h\nu_{42} \longrightarrow \nu_{42} = \frac{1}{h}(E_4 - E_2) = \frac{1}{h}\left[-\frac{R}{16} - \left(-\frac{R}{4}\right)\right] = \frac{3}{16}\frac{R}{h} =$$

$$= \frac{3}{16} \cdot \frac{13\,eV}{6{,}64\cdot 10^{-34}\,J\cdot s} \cdot \frac{1{,}602\cdot 10^{-19}\,J}{1\,eV} = 5{,}88\cdot 10^{14}\,s^{-1}$$

$$\boxed{\nu_{42} = 5{,}88\cdot 10^{14}\,Hz}$$

3. A commercial (green) laser pointer has a max/output power $P \leq 1\ mW$ and a beam spot $w_o = 1.1\ mm$. If $\lambda = 532\ nm$ is the light source wavelength, compute the (1.) pointer photon's flux and (2.) the number of photons emitted in 10 sec when you purposely cover half of the exit hole with your finger. (Neglect divergence).

1. First we determine the intensity

$P = I \cdot S$ we assume that the spot has a circular shape

$S = \pi r^2$

$r = \frac{w_o}{2}$

$$I = \frac{P}{S} = \frac{P}{\pi (w_o/2)^2} = \frac{4P}{\pi w_o^2} = \frac{4 \cdot 1 \cdot 10^{-3}\ W}{\pi \cdot (1,1)^2\ 10^{-6}\ m^2} = 1052,66\ W/m^2$$

We calculate the flux like the number of photons per area per second

$$\phi = \frac{I}{h\nu} = \frac{I \cdot \lambda}{hc} = 1052,66 \frac{\cancel{J}}{s\,m^2} \cdot \frac{532 \cdot 10^{-9}\ \cancel{m}}{6,64 \cdot 10^{-34}\ \cancel{J} \cdot s \cdot 3 \cdot 10^{8}\ \frac{\cancel{m}}{\cancel{s}}} = 2,81 \cdot 10^{21} \frac{1}{m^2 s}$$

$$\boxed{\phi = 2,81 \cdot 10^{21}\ m^{-2} \cdot s^{-1}}$$

2. If we cover the half of area:

$$A' = A/2 = \frac{\pi}{8} w_o^2 = 0,475 \cdot 10^{-6}\ m^2$$

And the number of photons N:

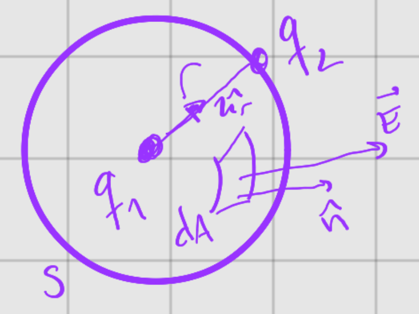
$$N = \phi \cdot A' \cdot t = 2,81 \cdot 10^{21} \frac{1}{m^2 s} \cdot 0,475 \cdot 10^{-6}\ m^2 \cdot 10\ s = 1,34 \cdot 10^{16}\ \text{photons}$$

$$\boxed{N = 1,34 \cdot 10^{16}\ \text{photons}}$$

4.

a. Give the potential energy of a charge q_2 located at r_2 due to the presence of a charge q_1 located at $\mathbf{r}_1$.
b. Derive the force exerted on the charge q_2 as due to the charge q_1.
c. Discuss the two charge possibilities.
d. Apply the above results to the case of an electron placed at a distance r from a nucleus of a H atom (Bohr's model).
e. Redo part d. for a nucleus of charge Ze.

(a) Using Gauss' Law for a q_1 charge



$$\oint_S \vec{E}\cdot d\vec{S} = \frac{q_{int}}{\varepsilon_0}$$

$$\oint_S \vec{E}\cdot\hat{n}\, ds = \frac{q_1}{\varepsilon_0} \rightarrow E\oint_S ds = \frac{q_1}{\varepsilon_0} \rightarrow E\, 4\pi r^2 = \frac{q_1}{\varepsilon_0}$$

$$\rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{u}_r$$

$\vec{E}\cdot\hat{n} = E_n \cos\theta = E_n = E$, $\theta = 0$

Scalar potential

$$\vec{E}(\vec{r}) = -\nabla\phi(\vec{r}) = -\frac{d}{dr}\phi(\vec{r})$$

$$\Rightarrow \int_\infty^{\phi(r)} d\phi(r) = -\int_\infty^r \vec{E}(\vec{r})\,dr \rightarrow \phi(r) - \underbrace{\phi(\infty)}_{0} = -\frac{q_1}{4\pi\varepsilon_0}\int_\infty^r r^{-2}\,dr$$

$$\Rightarrow \phi(r) = -\frac{q_1}{4\pi\varepsilon_0}(-1)\left[\frac{1}{r}\right]_\infty^r = \frac{q_1}{4\pi\varepsilon_0}\frac{1}{r}$$

Potential energy at q_2: $U = q_2\,\phi(r) \rightarrow \boxed{U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}}$

(b) For the force at q_2 due to q_1: $\vec{F}(\vec{r}) = q_2 \cdot \vec{E}(\vec{r})$

$$\boxed{\vec{F}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{u}_r}$$

ⓒ The two possible scenarios are

1) The charges repel each other

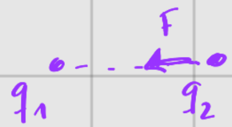
$q_1 = q_2 = e$ or $q_1 = q_2 = -e$

$$\vec{F}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\hat{u}_r$$

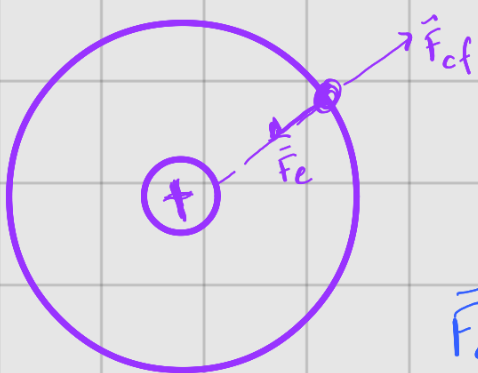
2) The charges attract each other

$q_1 = e$, $q_2 = -e$ or $q_1 = -e$, $q_2 = e$

$$\vec{F}(\vec{r}) = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\hat{u}_r$$



ⓓ Applying the results to Bohr's model



$$U_e(r) = -\frac{e^2}{4\pi\varepsilon_0}\cdot\frac{1}{r}$$

$$\vec{F}_e(r) = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r^2}$$

$L = n\hbar$

$L = m v r \rightarrow v = \frac{L}{mr}$

$$\vec{F}_{cf} = m\,w^2 r \overset{w = \frac{v}{r}}{=} m\left(\frac{v}{r}\right)^2 r \Rightarrow \vec{F}_{cf} = m\frac{v^2}{r}$$

Force balance $\rightarrow \sum F = 0 \rightarrow \vec{F}_e + \vec{F}_{cf} = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r^{\cancel{2}}} + m\frac{v^2}{\cancel{r}} = 0$

$$\Rightarrow -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{\cancel{r}} + \cancel{m}\frac{L^2}{m^{\cancel{2}} r^{\cancel{2}}} = 0 \Longrightarrow \frac{e^2}{4\pi\varepsilon_0} = \frac{L^2}{mr} \Rightarrow r = \frac{L^2}{e^2 m}4\pi\varepsilon_0$$

$$\Rightarrow r = \frac{n^2\hbar^2}{e^2 m}4\pi\varepsilon_0 = 4\pi\varepsilon_0 r_0 n^2 \Rightarrow r = 4\pi\varepsilon_0 r_0 n^2$$

Now we apply the expression of r in F_e and U_e

$$\boxed{\vec{F}_{e_n} = -\frac{e^2}{(4\pi\varepsilon_0)^3 r_0^2}\,\frac{1}{n^4}\,\vec{u}_r}$$

$$\boxed{U_{e_n} = -\frac{e^2}{(4\pi\varepsilon_0)^2 r_0}\,\frac{1}{n^2}}$$

e) When we us $q_1 = Ze$

$$r = 4\pi\varepsilon_0 \frac{r_0}{Z} n^2 \Rightarrow$$

$$\boxed{\vec{F}_{e_n} = -\frac{Z^3 e^2}{(4\pi\varepsilon_0)^3 r_0^2}\,\frac{1}{n^4}\,\vec{u}_r}$$

$$\boxed{U_{e_n} = -\frac{Z^2 e^2}{(4\pi\varepsilon_0)^2 r_0}\,\frac{1}{n^2}}$$

Extra.

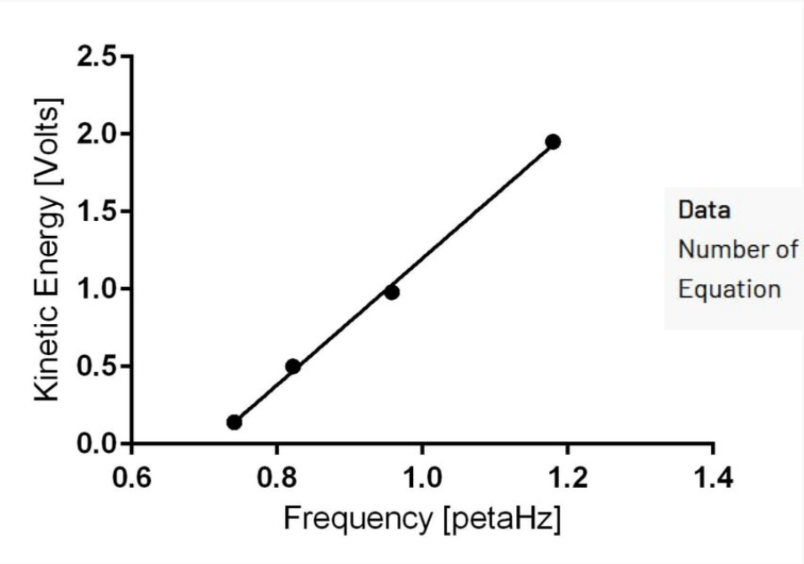
In a photoelectric experiment Ca is used as photocathode and the following values of stopping potential V_s vs. wavelength λ are measured:

λ, Å	2536	3132	3650	4047
ν, Hz $\times 10^{15}$	1.18	0.958	0.822	0.741
V_s, V	1.95	0.98	0.50	0.14

Calculate the Planck constant $\hbar$ and the work function Φ.

Using an online tool to compute the linear regresion we obtain:

Linear Regression



Data		
Number of XY pairs	4	
Equation	Y = 4.082*X - 2.885	

Now we have to compare with the next equation

$$E_K = h\nu - \phi$$

So, comparing both equations:

•) $h = 4{,}082 \cdot 10^{-15}\ V \cdot s = 4{,}082 \cdot 10^{-15} \frac{J}{C} \cdot s = \frac{4{,}082 \cdot 10^{-15}}{6{,}24 \cdot 10^{18}} [J \cdot s]$

$h = 6{,}54 \cdot 10^{-34}\ J \cdot s \longrightarrow \hbar = \frac{h}{2\pi} \Rightarrow \boxed{\hbar = 1{,}041 \cdot 10^{-34}\ J \cdot S}$

•) $\boxed{\phi = 2{,}885\ V}$