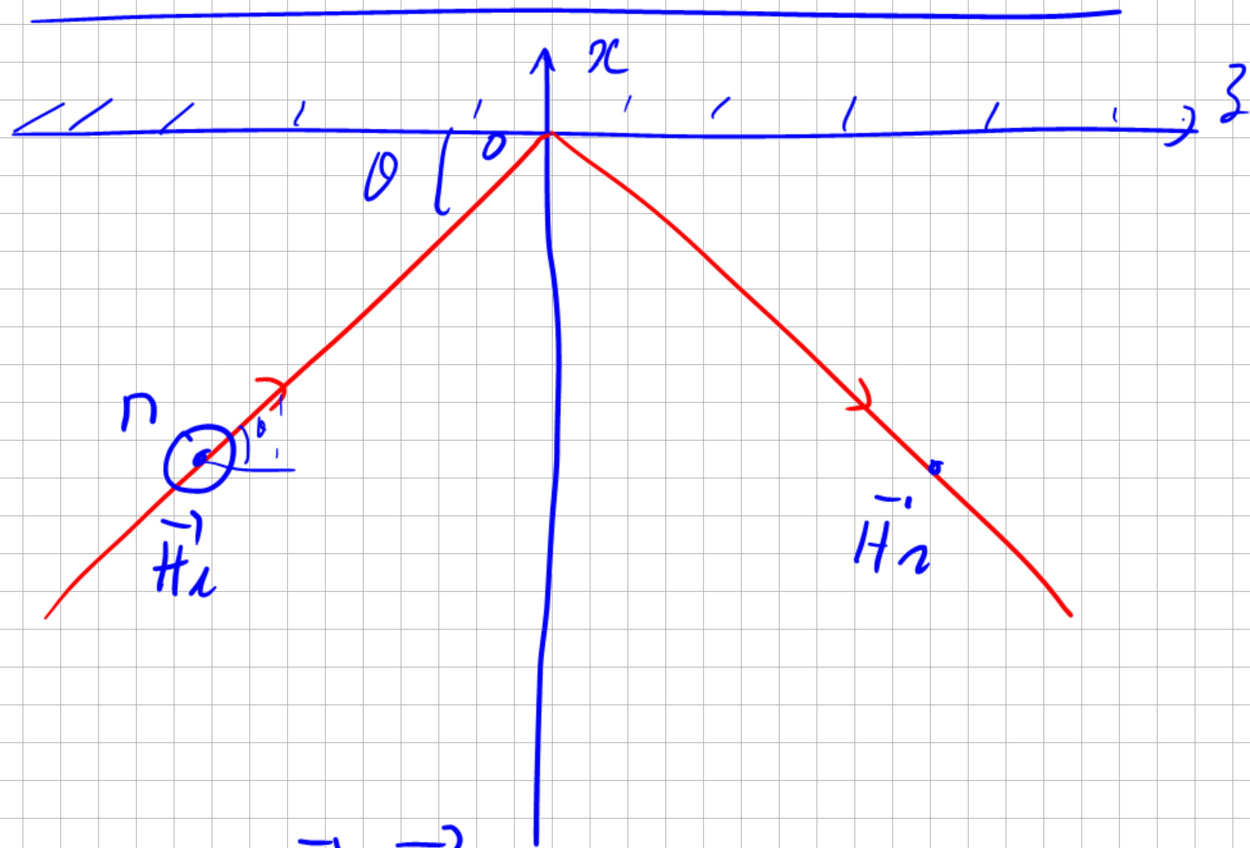
Correction examen 2022 - 2023

I



$$\vec{H_i} = H_0\, e^{-j\,\vec{k_i}\cdot\vec{On}}\, \vec{e_y}$$

$$\vec{k_i} = k\,\vec{u} = k\sin\theta\,\vec{e_x} + k\cos\theta\,\vec{e_z}$$

$$\vec{On} = x\,\vec{e_x} + z\,\vec{e_z}$$

$$\vec{k_i}\cdot\vec{On} = k\,x\sin\theta + k\,z\cos\theta$$

$$\vec{H_i} = H_0\, e^{-jk(x\sin\theta + z\cos\theta)}\, \vec{e_y}$$

$$\vec{E_i} = -Z_0\, \vec{u} \wedge \vec{H_i} \qquad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$= Z\left(\sin\theta\,\vec{e_x} + \cos\theta\,\vec{e_z}\right) \wedge H_i\,\vec{e_y}$$

$$\vec{E_i} = Z_0 \begin{vmatrix} +\cos\theta \; H_0 \, e^{-jk(x\sin\theta + z\cos\theta)} \\ 0 \\ +\sin\theta \; H_0 \, e^{-jk(x\sin\theta + z\cos\theta)} \end{vmatrix}$$

en $x = 0$, $\vec{H_i} = \vec{H_r}$

$$\vec{H_r} = H_0 \, e^{-j \vec{k_r} \cdot \vec{OM}} \; \vec{e_y}$$

$$\vec{k_r} \cdot \vec{OM} = -kx\sin\theta + kz\cos\theta$$

$$\vec{H_r} = H_0 \, e^{-j(kz\cos\theta - kx\sin\theta)} \; \vec{e_y}$$

$$\vec{E_r} = -Z_0 \, \vec{u}' \wedge \vec{H_r}$$

$$= -Z_0 \left(-\sin\theta \, \vec{e_x} + \cos\theta \, \vec{e_z}\right) \wedge H_r \, \vec{e_y}$$

$$\vec{E_r} = +Z_0 \begin{vmatrix} +\cos\theta \; H_0 \, e^{-j(kz\cos\theta - kx\sin\theta)} \\ 0 \\ +\sin\theta \; H_0 \, e^{-j(kz\cos\theta - kx\sin\theta)} \end{vmatrix}$$

$$\vec{H_T} = \vec{H_i} + \vec{H_r}$$

$$= H_0 \left(e^{-jkx\sin\theta} + e^{jkx\sin\theta}\right) e^{-jkz\cos\theta}$$

$$= 2 H_0 \cos(kx\sin\theta) \, e^{-jkz\cos\theta}$$

$$\vec{E}_t = \left| \begin{array}{l} Z_0 \cos\theta\, H_0 \left(e^{-jkx\sin\theta} + e^{jkz\cos\theta}\right) \\ 0 \\ Z_0 \sin\theta H_0 \left(e^{jkx\sin\theta} - e^{-jkx\sin\theta}\right) e^{-jkz\cos\theta} \end{array} \right.$$

$$\vec{E}_t = \left| \begin{array}{l} 2 Z_0 \cos\theta\, H_0 \cos(kx\sin\theta) \\ 0 \\ 2j\, Z_0 \sin\theta\, H_0 \sin(kx\sin\theta)\, e^{-jkz\cos\theta} \end{array} \right.$$

1-1 en $x = -a$, $E_j = 0$

$\Rightarrow \sin(ka\sin\theta) = 0 \Rightarrow ka\sin\theta = m\pi$

$*m=1$ $\sin\theta = \frac{\pi}{ka} < 1$

$\Rightarrow ka > \pi$

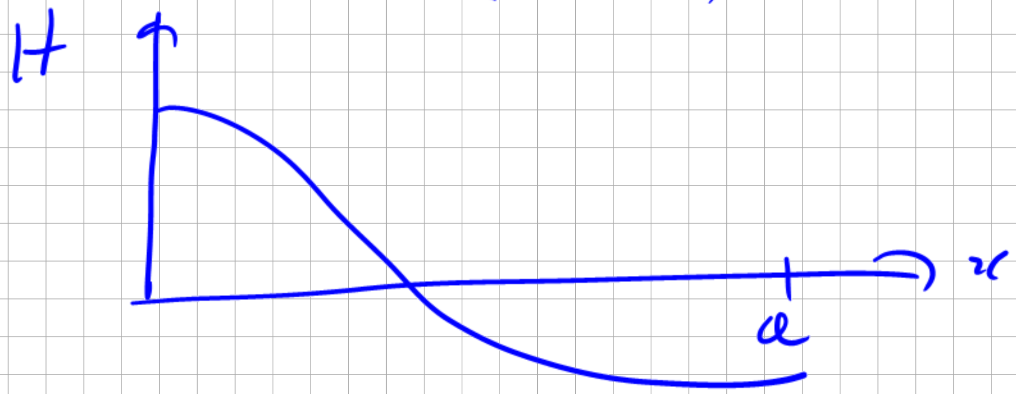
$\Rightarrow k > \frac{\pi}{a}$

$$\omega_{c1} = c\,\frac{\pi}{a}$$

$$f_{c1} = \frac{c}{2a}$$

$$\vec{H}_T = 2H_0 \cos\left(k\,x\sin\theta\right) e^{-jk_3\cos\theta}\,\vec{e}_y$$

$$= 2H_0 \cos\left(\frac{\pi}{a}x\right) e^{-jk_3\cos\theta}$$

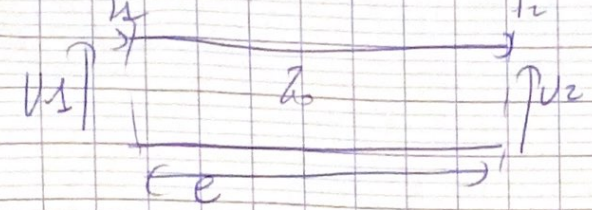


$$\beta^2 = (k\cos\theta)^2 = k^2 - (k\sin\theta)^2$$

$$\Rightarrow \quad \beta^2 = k^2 - \left(\frac{\pi}{a}\right)^2$$

$$\Rightarrow \quad \frac{2\pi}{\lambda_g} = k^2 - \left(\frac{\pi}{a}\right)^2$$

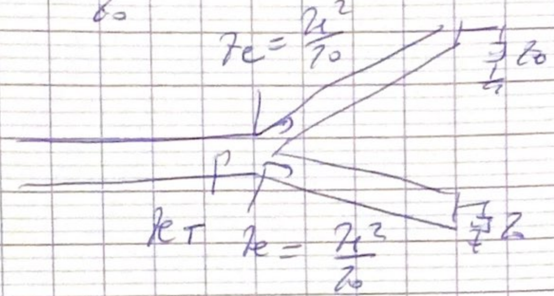
II 2-1



$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} \cos\beta\ell & jZ_0 \sin\beta\ell \\ \frac{1}{Z_0}\sin\beta\ell & \cos\beta\ell \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix}$$

2-2- a) $Z_e = \dfrac{Z_1^2}{Z_0}$

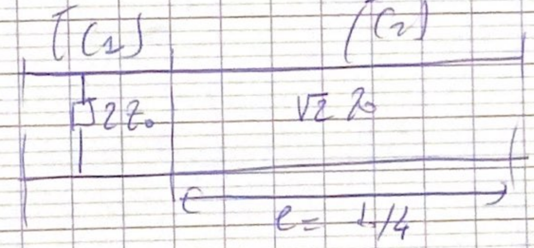
b)



$$Z_{eT} = \frac{Z_1^2}{2Z_0}$$

c) $Z_{eT} = Z_0 \Rightarrow Z_1 = \sqrt{2}\, Z_0$

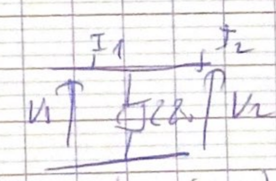
2-3



1 Computation of C_1:

$V_1 = V_2$

$I_1 = I_2 + \dfrac{V_2}{2Z_0}$

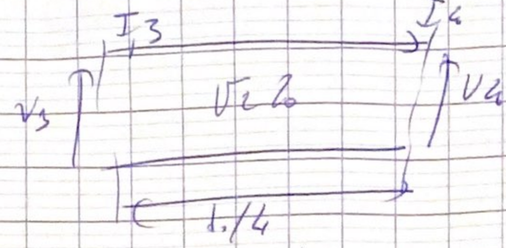


$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{2Z_0} & 1 \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix}$$

Normalization (Z_0):

$$\begin{pmatrix} v_1 \\ i_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} v_2 \\ i_2 \end{pmatrix}$$

Computation of C_2 : $\beta l = \frac{\pi}{2}$



$$\begin{pmatrix} V_3 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 & j\sqrt{2} Z_0 \\ \frac{1}{\sqrt{2} Z_0} & 0 \end{pmatrix} \begin{pmatrix} V_4 \\ I_4 \end{pmatrix}$$

$$\begin{pmatrix} v_3 \\ i_3 \end{pmatrix} = \begin{pmatrix} 0 & j\sqrt{2} \\ \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} v_4 \\ i_4 \end{pmatrix}$$

ii Computation of $[C_T] = [C_1] \times [C_2]$:

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} 0 & j\sqrt{2} \\ \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} v_4 \\ i_4 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & j\sqrt{2} \\ \frac{1}{\sqrt{2}} & j\frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} v_4 \\ i_4 \end{pmatrix}$$

$$a_1 + b_1 = j\sqrt{2}\,(b_2 - a_2)$$
$$a_1 - b_1 = \frac{1}{\sqrt{2}}(b_2 + a_2) + j\frac{\sqrt{2}}{2}(b_2 - a_2)$$
$$= b_2\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) + a_2\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)$$
$$= j\sqrt{2}\, b_2$$

$\Rightarrow \quad b_2 = -j\frac{\sqrt{2}}{2} a_1 + \frac{1}{2} a_2 \quad , \quad b_1 = -j\frac{\sqrt{2}}{2} a_2 .$

2.4 $S_{21} = -j\frac{\sqrt{2}}{2}$ $\quad |S_{21}|^2 = \frac{1}{2}$

$S_{22} = \frac{1}{2}$ $\quad |S_{22}|^2 = \frac{1}{4}$

Lossless network: $|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 = 1$

$\Rightarrow |S_{31}|^2 = \frac{1}{2}$

$|S_{21}|^2 + |S_{22}|^2 + |S_{32}|^2 = 1$

$\Rightarrow |S_{32}|^2 = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \neq 0$