WIRE ANTENNAS

Wire antennas are still the most prevalent of all antenna forms; they are simple in concept, easy to construct and inexpensive. We discuss resonant antennas: at resonance the current distribution is a standing wave and the input reactance is zero.

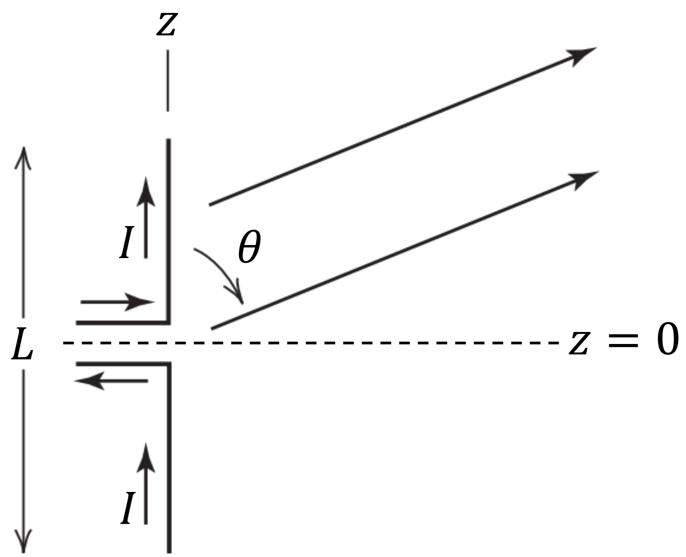
To obtain completely accurate solutions for wire antennas, the current on the wire must be solved for, subject to the boundary condition that the tangential electric field is zero along the wire. This approach gives rise to an integral equation, that is very difficult to solve in both exact and approximate form (but it can be solved by numerical methods for any shape of the wire).

Here we will use a much simpler but effective approach that involves an assumed form for the current distribution. The radiation integral may then be avaluated and thus also the pattern parameters. For dipoles the current must be zero at the ends and we assume that the current distribution is sinusoidal : this is a good approximation verified by measurements.

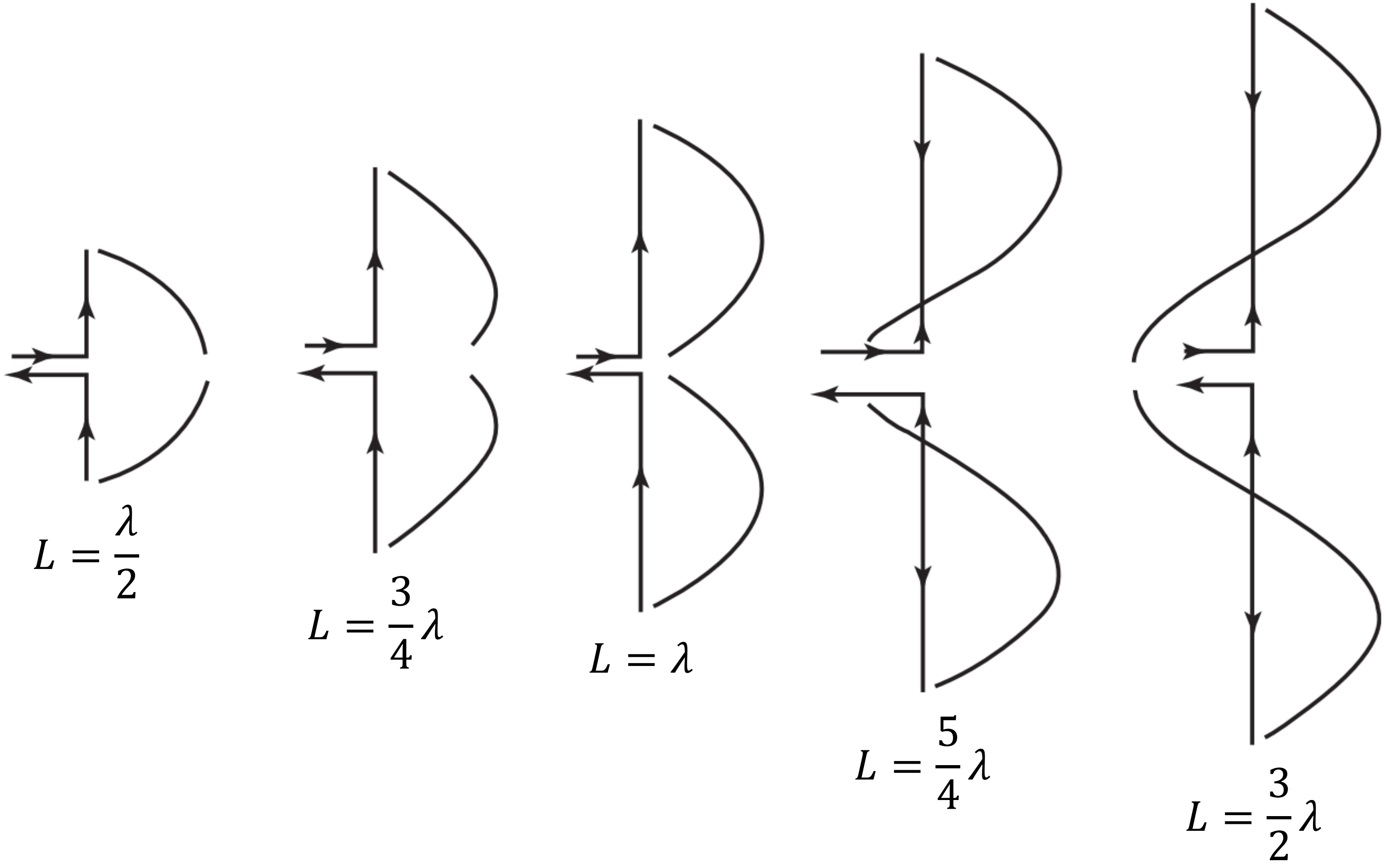
We assume that wire dipole is oriented along the z-axis, and that it is fed at the center from a balanced two-wire transmission line (like an ideal bifilar line), that is, the currents on each wire are equal in magnitude and opposite in direction.

$$I(z) = I_m \sin\left[\beta\left(\frac{L}{2} - |z|\right)\right] \qquad |z| \le \frac{L}{2}$$

$$\beta = \frac{2\pi}{\lambda}$$



In the following figure, current distributions on various dipoles are plotted together with the antennas used to generate them. The sinusoidal curves superimposed on the antennas indicate the intensity of the current on the wire: the value of the curve at point z is the current value $I(z)$ on the wire at same point z and the arrows indicate current directions.



We observe that for dipoles longer than one wavelength, the currents on the wire are not all in the same direction: we expect to see some large cancelling effects in the radiation pattern. The previous plots represent the maximum excitation state: in fact, a sinusoidal waveform generator of angular frequency $\omega = 2\pi c/\lambda$ is connected to the input terminals (by means of a transmission line) and the standing wave pattern of current at any instant of time is obtained by multiplying $I(z)$ by $\cos\omega t$.

The far-field can be calculated starting from the magnetic vector potential, which is in the same direction as the wire

$$\mathbf{A} = \iiint_{v'} \mu \mathbf{J} \frac{e^{-j\beta R}}{4\pi R} dv' = \mu \int_{-\frac{L}{2}}^{+\frac{L}{2}} I(z')\hat{\mathbf{z}} \frac{e^{-j\beta R}}{4\pi R} dz' = \mu \frac{e^{-j\beta r}}{4\pi r} \int_{-\frac{L}{2}}^{+\frac{L}{2}} I(z') e^{j\beta \cos\theta z'} dz' \,\hat{\mathbf{z}} = A_z \hat{\mathbf{z}}$$

The problem of obtaining A_z is reduced to that of calculating the following integral

$$\int_{-\frac{L}{2}}^{+\frac{L}{2}} I(z') e^{j\beta \cos\theta z'} dz' = \int_{-\frac{L}{2}}^{0} I_m \sin\left[\beta\left(\frac{L}{2} + z'\right)\right] e^{j\beta \cos\theta z'} dz' + \int_{0}^{+\frac{L}{2}} I_m \sin\left[\beta\left(\frac{L}{2} - z'\right)\right] e^{j\beta \cos\theta z'} dz'$$

The previous integral can be evaluated by using the following indefinite integral

$$\int \sin(bz + c)e^{az} dz = \frac{e^{az}}{a^2 + b^2}[a \sin(bz + c) - b \cos(bz + c)]$$

and the result reads as

$$\int_{-\frac{L}{2}}^{+\frac{L}{2}} I(z')e^{j\beta \cos\theta z'} dz' = \frac{2I_m}{\beta} \frac{\cos\left(\frac{\beta L}{2}\cos\theta\right) - \cos\left(\frac{\beta L}{2}\right)}{\sin^2\theta}$$

E and **H** in the far-field can be obtained by using the general procedure valid for a linear current distribution

$$\mathbf{E} = -j\omega\left(A_\theta\widehat{\boldsymbol{\theta}} + A_\phi\widehat{\boldsymbol{\phi}}\right) = -j\omega A_\theta\widehat{\boldsymbol{\theta}} = j\omega \sin\theta\, A_z\widehat{\boldsymbol{\theta}} = j\omega \sin\theta\, \mu \frac{e^{-j\beta r}}{4\pi r}\frac{2I_m}{\beta}\frac{\cos\left(\frac{\beta L}{2}\cos\theta\right) - \cos\left(\frac{\beta L}{2}\right)}{\sin^2\theta}\widehat{\boldsymbol{\theta}}$$

$$\mathbf{H} = \frac{1}{\eta}\hat{\mathbf{r}} \times \mathbf{E} = -\frac{j\beta}{\mu}A_\theta\widehat{\boldsymbol{\phi}} = \frac{j\beta}{\mu}\sin\theta\, A_z\widehat{\boldsymbol{\phi}} = \frac{j\beta}{\mu}\sin\theta\, \mu\frac{e^{-j\beta r}}{4\pi r}\frac{2I_m}{\beta}\frac{\cos\left(\frac{\beta L}{2}\cos\theta\right) - \cos\left(\frac{\beta L}{2}\right)}{\sin^2\theta}\widehat{\boldsymbol{\varphi}}$$

The far-field is given by

$$\mathbf{E} = j\eta \frac{e^{-j\beta r}}{2\pi r} I_m \frac{\cos\left(\frac{\beta L}{2}\cos\theta\right) - \cos\left(\frac{\beta L}{2}\right)}{\sin\theta} \hat{\boldsymbol{\theta}}$$

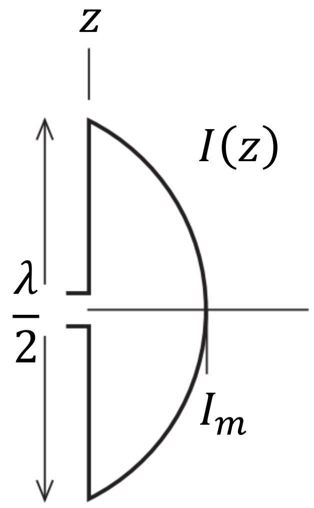
$$\mathbf{H} = j \frac{e^{-j\beta r}}{2\pi r} I_m \frac{\cos\left(\frac{\beta L}{2}\cos\theta\right) - \cos\left(\frac{\beta L}{2}\right)}{\sin\theta} \hat{\boldsymbol{\varphi}}$$

The radiated fields and the input impedance depends on $\beta L/2$, so on the ratio of the wire length to the wavelength L/λ.

HALF-WAVE DIPOLE

The straight wire length is $L = \lambda/2$ and the wire is fed in its center, where the current distribution reaches the maximum value

$$I(z) = I_m \sin\left[\beta\left(\frac{\lambda}{4} - |z|\right)\right] \qquad |z| \le \frac{\lambda}{4}$$



$\mathbf{E}$ and $\mathbf{H}$ in the far-field are

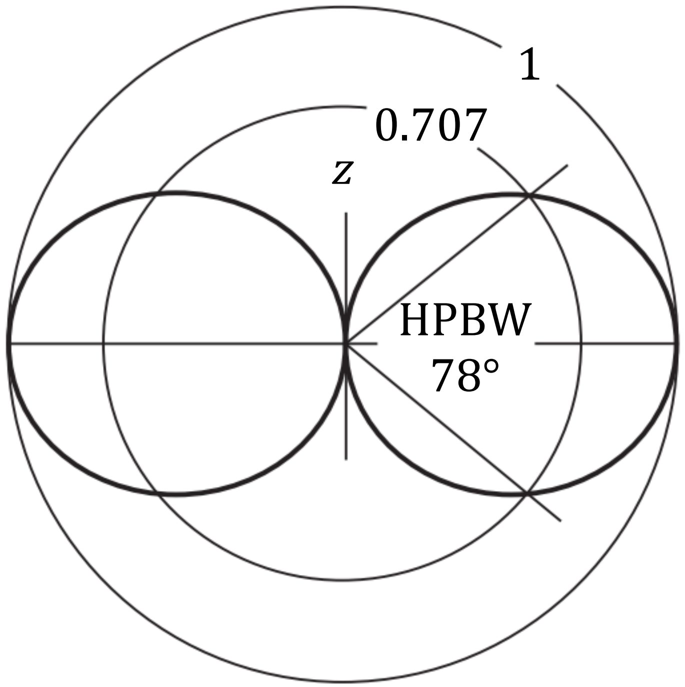
$$\mathbf{E} = j\eta \frac{e^{-j\beta r}}{2\pi r} I_m \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\,\hat{\boldsymbol{\theta}}$$

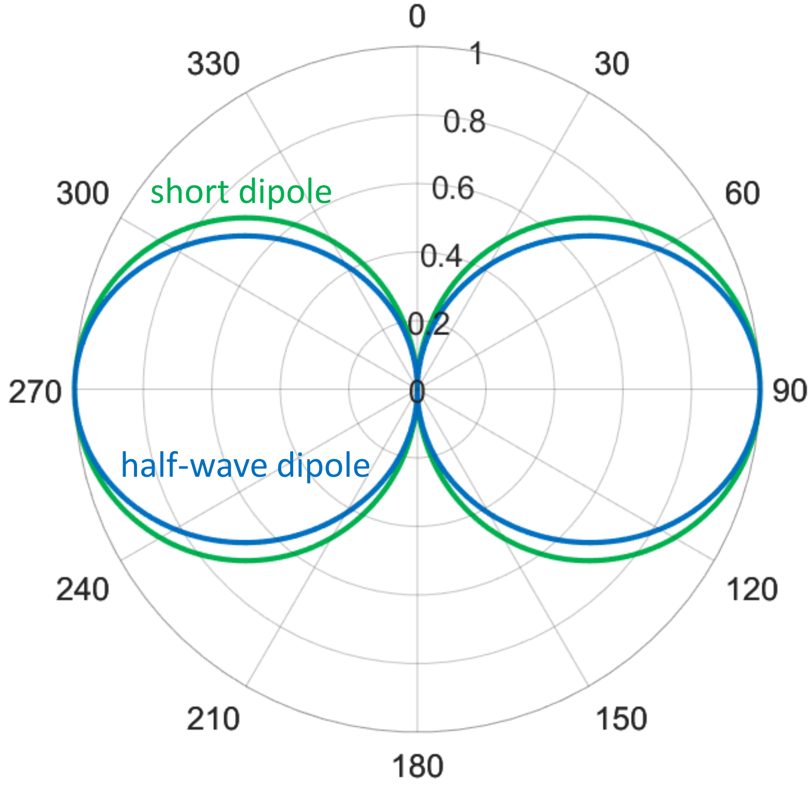
$$\mathbf{H} = j \frac{e^{-j\beta r}}{2\pi r} I_m \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\,\hat{\boldsymbol{\varphi}}$$

The normalized radiation pattern $|F(\theta,\varphi)|$ does not depend on φ due to axial symmetry of the problem, and it reads as

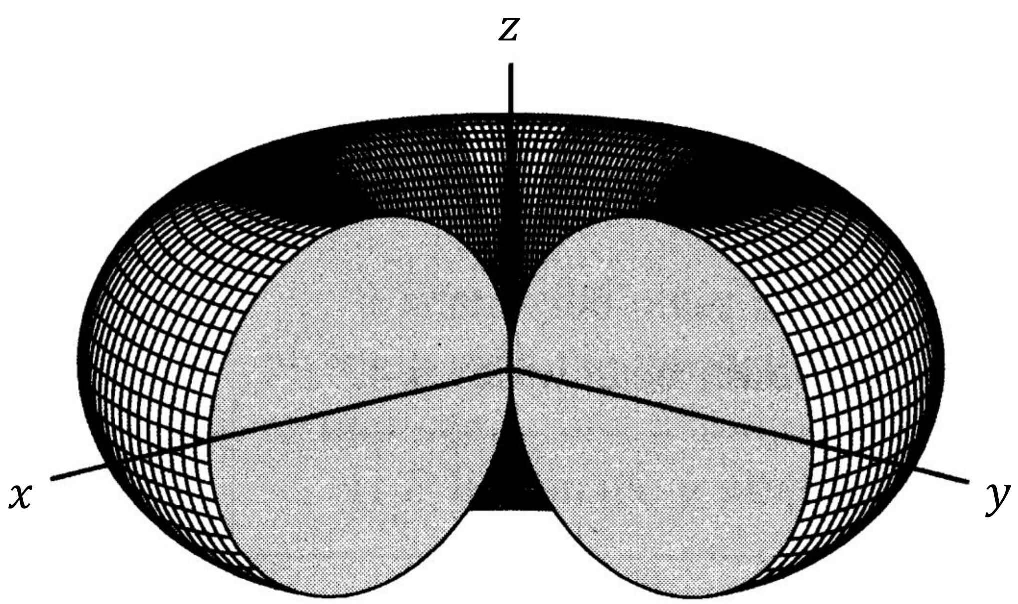
$$|F(\theta,\varphi)| = \left|\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right|$$

The pattern in the *E*-plane has the shape of the number eight , and in the *H*-plane is omnidirectional (i.e. it is a circle): the radiation pattern is very similar to the pattern of the short dipole, but the 3D "donut" is more flattened.





The half-power beamwidth (HPBW) in the E-plane is $78°$ and the directivity (which can ben calculated by numerical methods) is $D = 1.64$; the directivity is larger than for a short dipole as a consequence of the fact that the main lobe is narrower.



$$E_\theta \cong j60 \frac{e^{-j\beta r}}{r} I_m \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

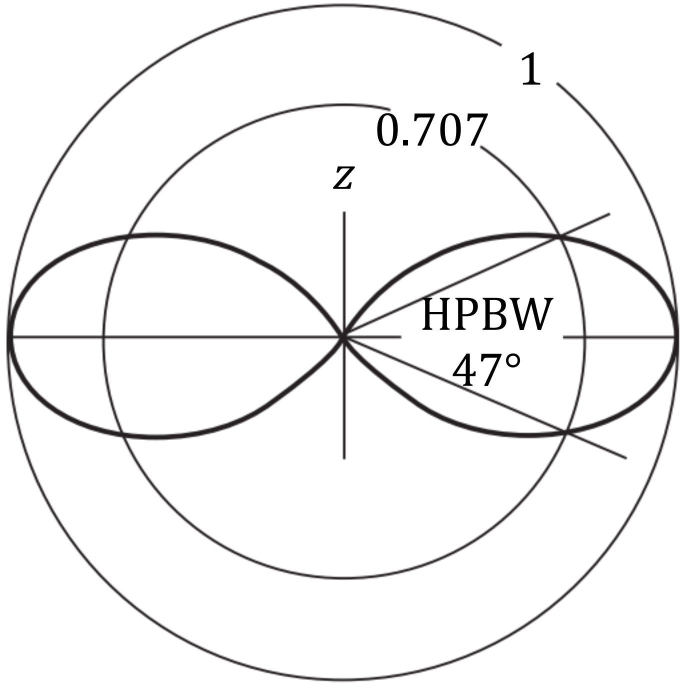
$$H_\varphi = \frac{E_\theta}{\eta} = j\frac{1}{2\pi}\frac{e^{-j\beta r}}{r} I_m \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

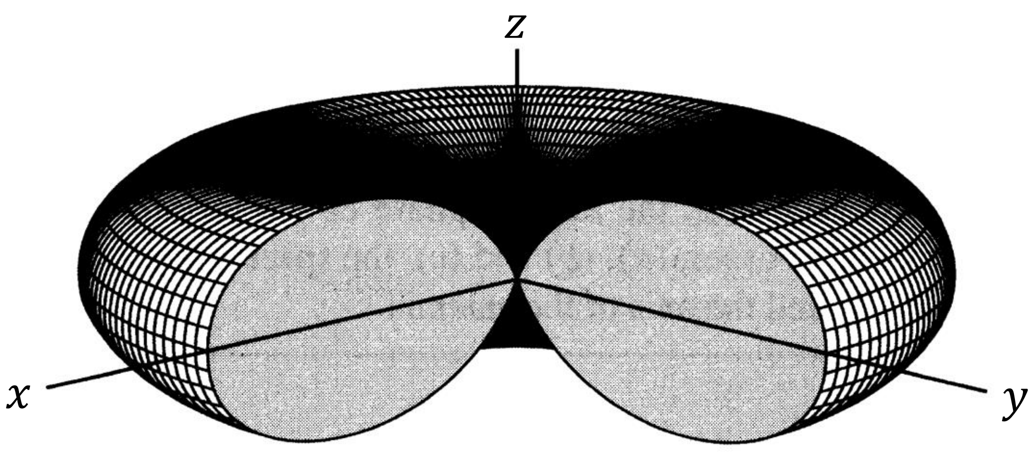
$$D = 1.64$$

For a full-wave dipole with $L = \lambda$, the normalized field pattern is

$$|F(\theta, \varphi)| = \frac{1}{2}\left|\frac{\cos(\pi \cos\theta) + 1}{\sin\theta}\right|$$

The half-power beamwidth is $47°$ and the directivity is $D = 2.5$

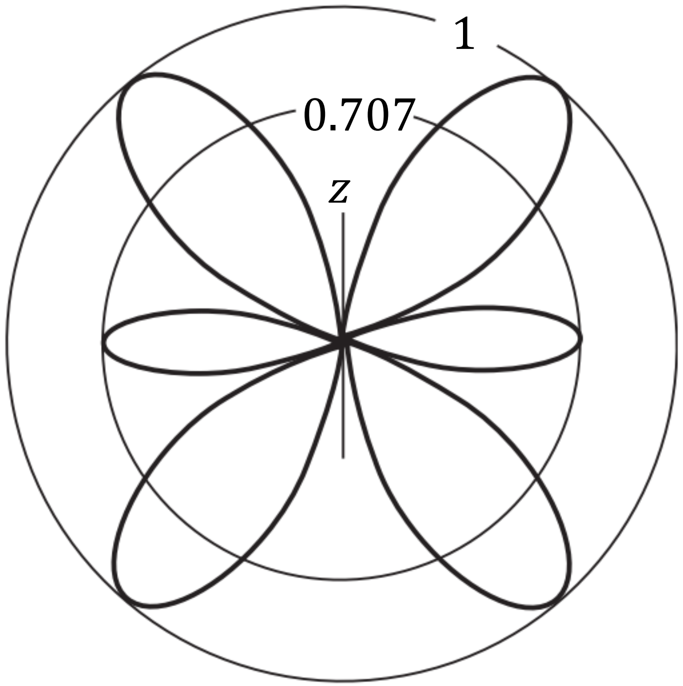


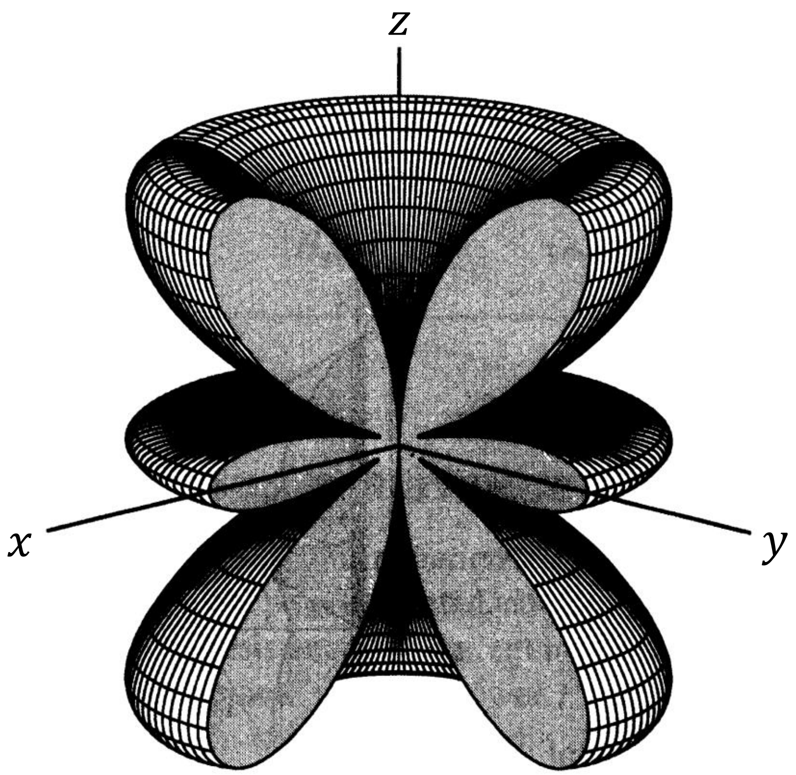


As a further example we can consider $L = 3\lambda/2$ and the resulting normalized field pattern is the following (0.7148 is the normalization factor)

$$|F(\theta,\varphi)| = 0.7148\left|\frac{\cos\left(\frac{3}{2}\pi\cos\theta\right)}{\sin\theta}\right|$$

The pattern has multiple lobe structure (and each lobe is bounded by null directions) due to the cancelling effect of oppositely directed currents on the wire





The radiation resistance is obtained by finding the radiated power (by integrating the flux of the Poynting vector through the surface of a sphere that encloses the antenna):

$$P_R = \iint_{\Sigma} \mathbf{S} \cdot d\mathbf{s} = \iint_{\Omega} \frac{1}{2\eta} |E_\theta|^2 \, r^2 \sin\theta \; d\theta \; d\phi = \frac{1}{2\eta} \int_0^{2\pi} \int_0^{\pi} |E_\theta|^2 r^2 \sin\theta \; d\theta d\varphi$$

$$P_R = \frac{1}{2\eta} \int_0^{2\pi} \int_0^{\pi} \eta^2 \frac{1}{(2\pi r)^2} I_m^2 \left[\frac{\cos\left(\frac{\beta L}{2}\cos\theta\right) - \cos\left(\frac{\beta L}{2}\right)}{\sin\theta} \right]^2 r^2 \sin\theta \; d\theta d\varphi$$

$$P_R = \frac{\eta}{4\pi} I_m^2 \int_0^{\pi} \frac{\left[\cos\left(\frac{\beta L}{2}\cos\theta\right) - \cos\left(\frac{\beta L}{2}\right)\right]^2}{\sin\theta} d\theta \cong 30 I_m^2 \int_0^{\pi} \frac{\left[\cos\left(\frac{\beta L}{2}\cos\theta\right) - \cos\left(\frac{\beta L}{2}\right)\right]^2}{\sin\theta} d\theta$$

The previous integral can be evaluated numerically and for the half-wave dipole we obtain

$$P_R = 30 I_m^2 1.22 = 36.6 I_m^2 \qquad\qquad R_R = \frac{2P_R}{I_m^2} = \cong 73\ \Omega$$

It can be proved that the half-wave dipole also has a reactive component, and the complete input impedance is

$$\boxed{Z_A = 73 + j42.5\ \Omega}$$

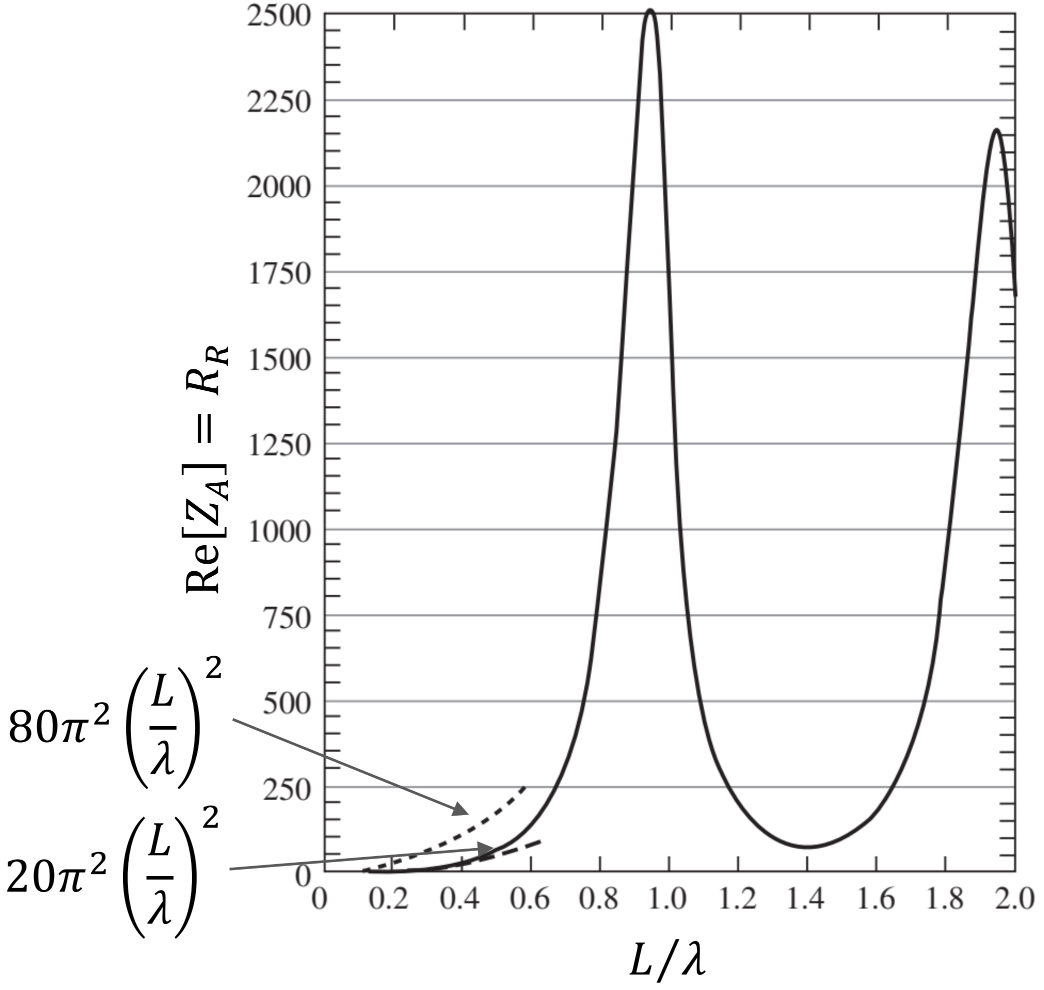
The complete input impedance of dipoles with finite wire diameter can be calculated by using a more complete theoretical analysis or by resorting to numerical methods.

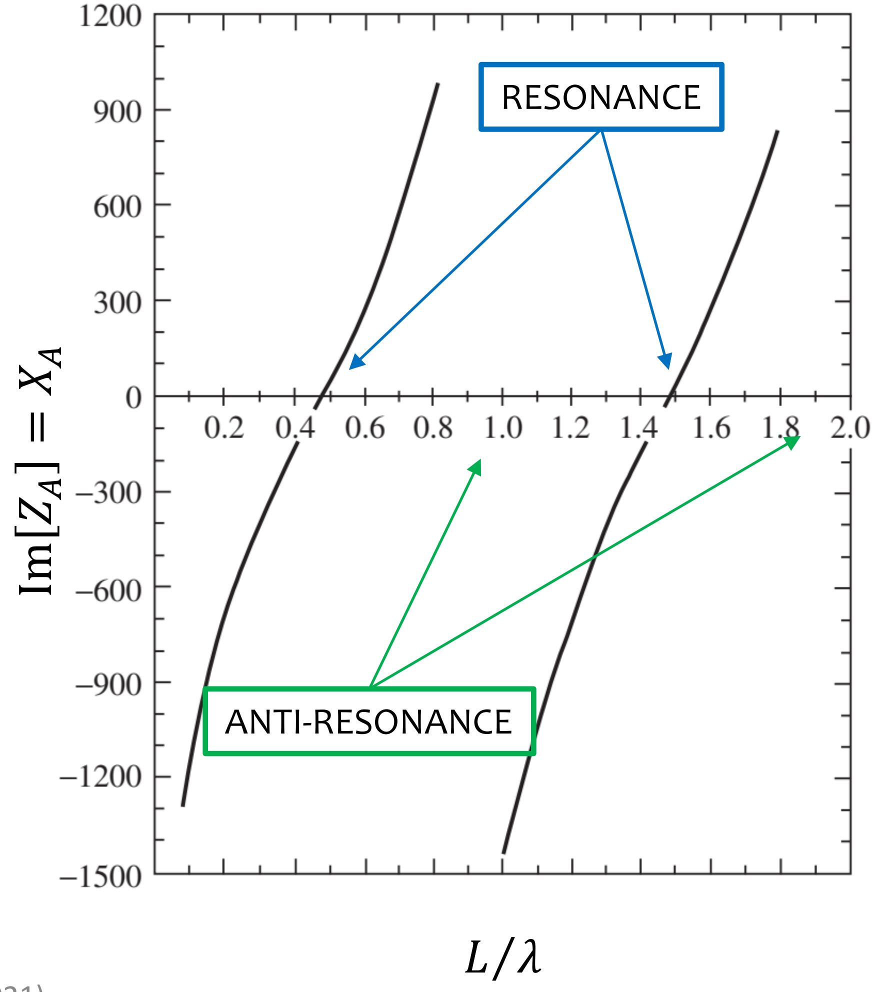
We observe that from the radiated power we can also obtain the directivity

$$D = \frac{4\pi U_m}{P_R} = \frac{4\pi \frac{1}{2\eta} \frac{\eta^2}{(2\pi)^2} I_m^2}{36.6 I_m^2} = \frac{\frac{\eta}{2\pi}}{36.6} \cong 1.64$$

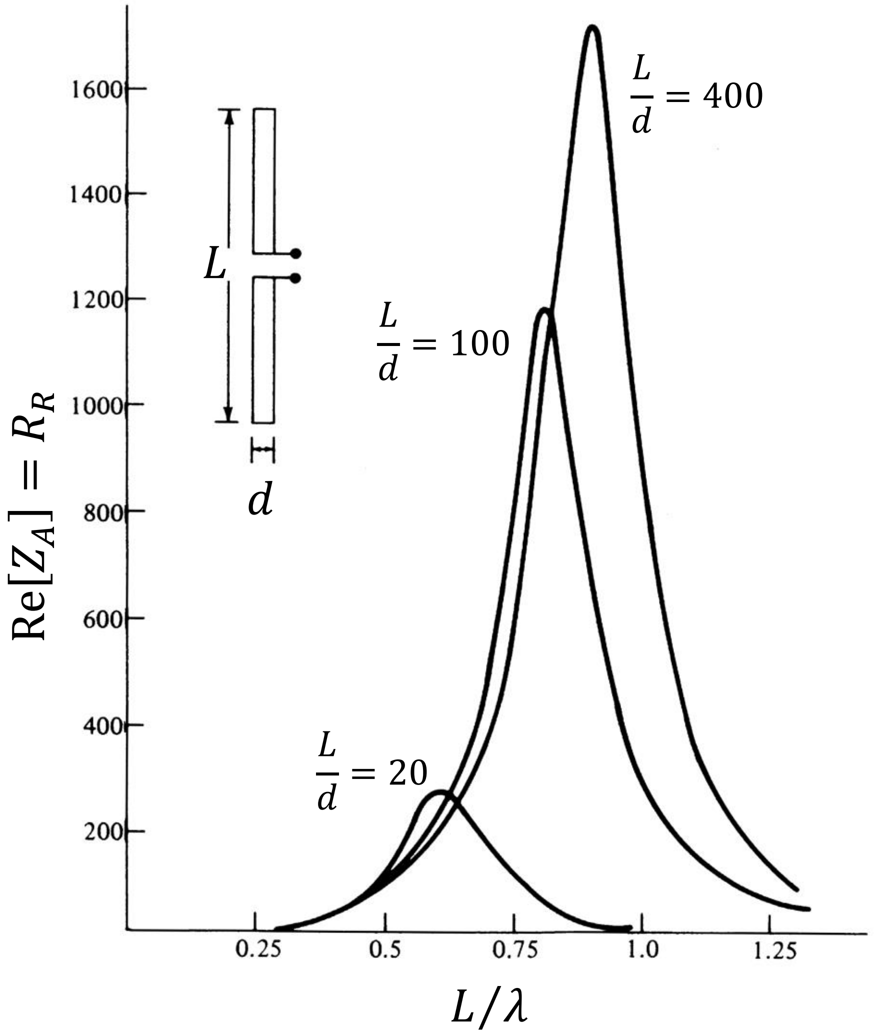
INPUT IMPEDANCE Z_A OF A CENTER-FED WIRE DIPOLE

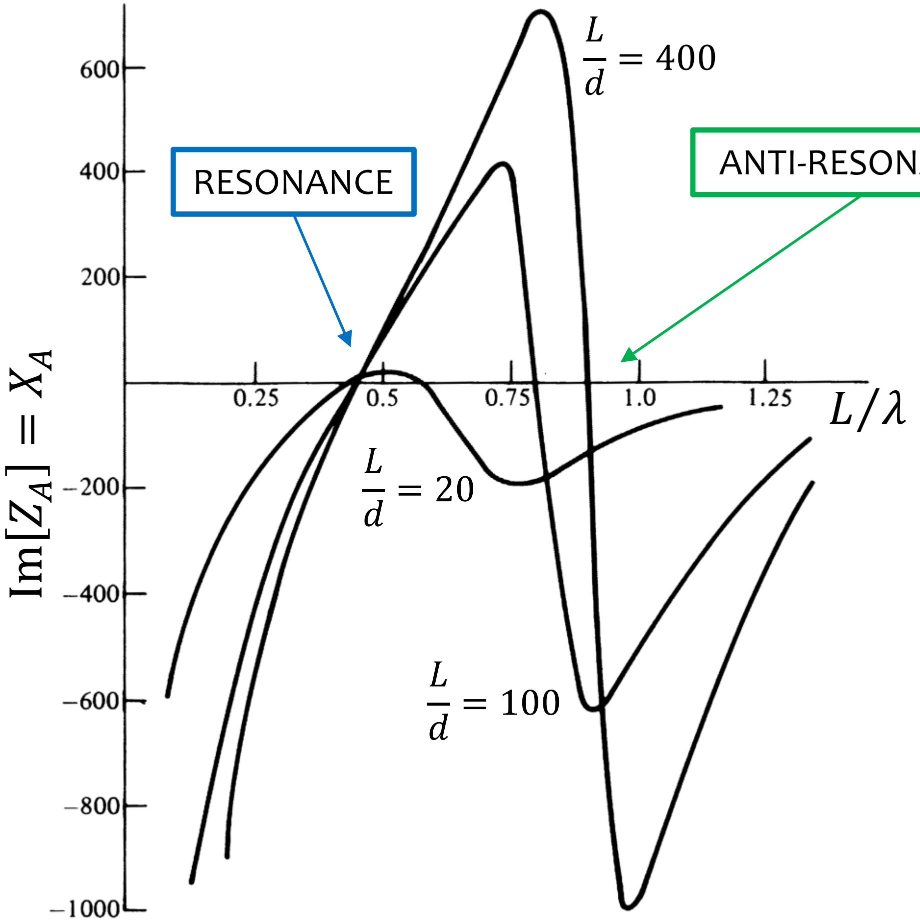
In this example the radius of the wire is $a = 0.0005\,\lambda$. **The wire is at resonance when $\mathbf{Im}[Z_A] = X_A = 0$ and it is at anti-resonance when $|X_A| = \infty$. In order to obtain $X_A = 0$ the dipole must be shortened to $L = 0.48\lambda$, the correponding value of the radiation resistance decreases to $R_R \cong 65\ \Omega$**





The input impedance depend on the length to diameter ratio of the wire (L/d). Starting from the values of Z_A it is possible to verify that the thicker the wire, the wider is the wire antenna working bandwidth.

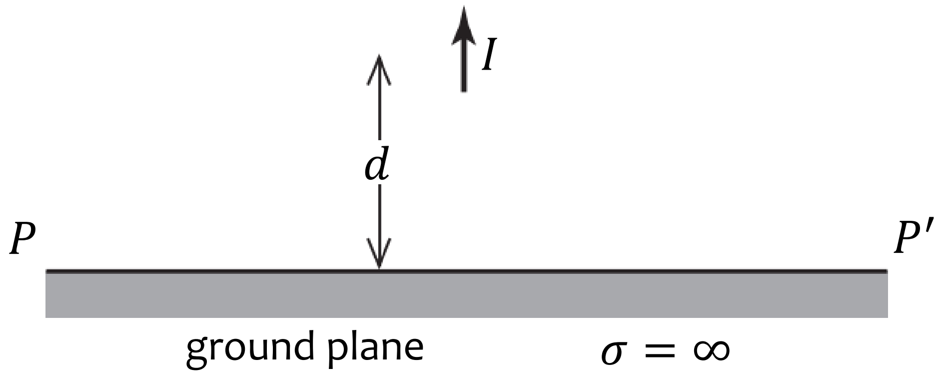


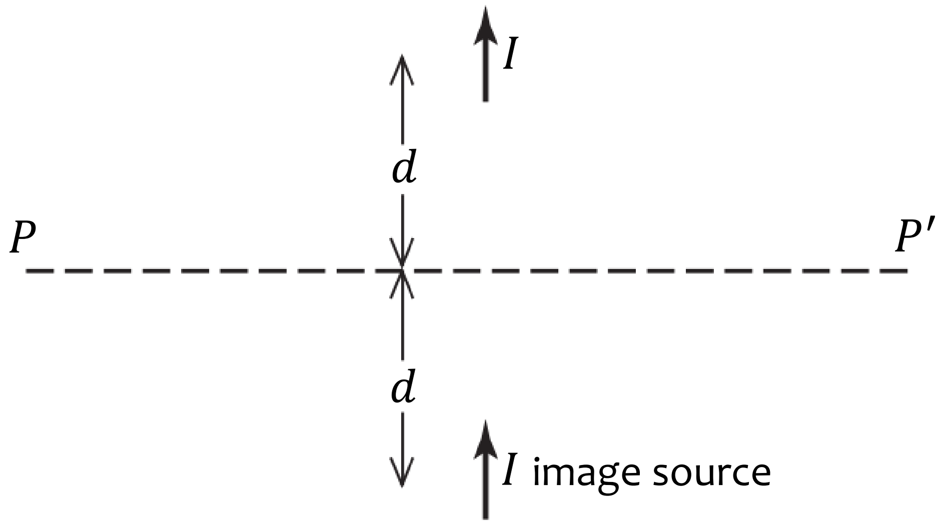


MONOPOLE

Let’s consider a dipole at a distance d from a perfect ground plane and oriented perpendicular to the ground plane (in the chosen reference system the plane is PP'). Clearly, we are interested in knowing the electromagnetic field only in the upper half-space.

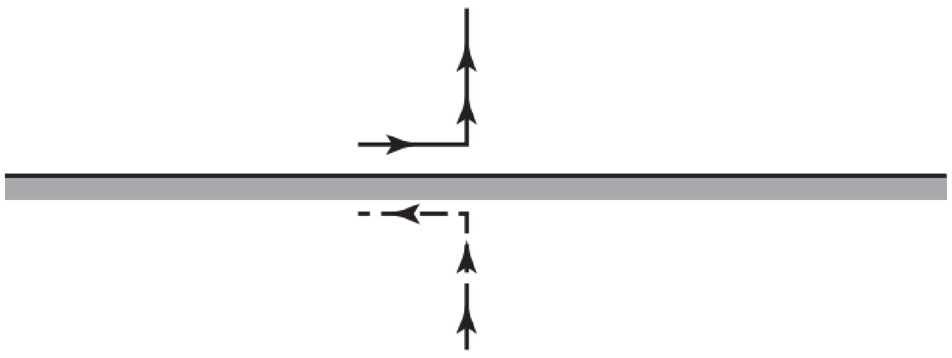
The same electromagnetic field in the upper half-space is obtained if the ground plane is removed and an image source is added in the lower half-space (that source is oriented as the dipole and the distance between the image and the plane is d). In fact, it can be proved that the equivalent system with the image source satisfies the boundary condition of zero tangential electric field along plane PP': therefore, since the source configuration above the plane and the boundary conditions were not altered, the fields above the plane PP' are identical.

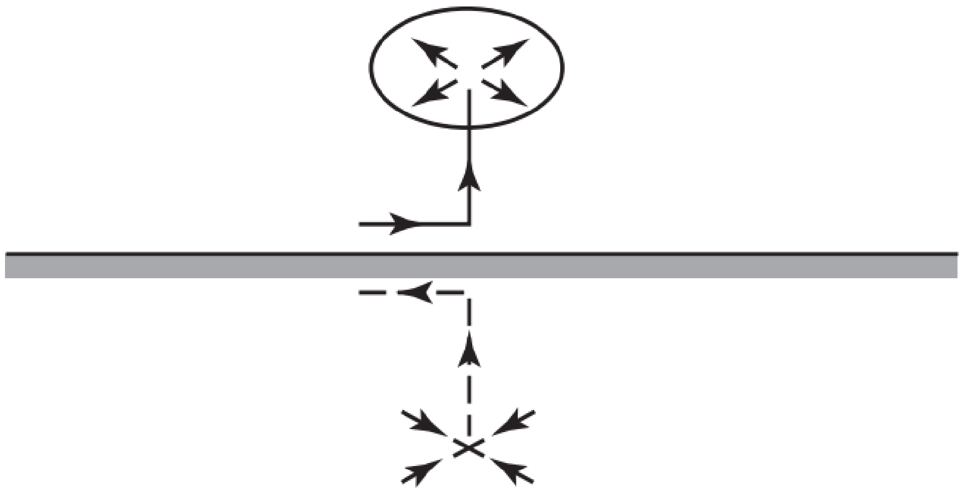




A monopole is a dipole that has been divided in half at its center point and fed against a ground plane.

Since the currents above and below the cutting plane are oriented in the same direction and have the same magnitude, the introduction of the ground plane does not change the field distribution in the upper half-space.





The currents and charges on a monopole are the same as on the upper half of its dipole counterpart, but the terminal voltage is only half that of the dipole. The voltage is half because the gap width of the input terminal (or port) is half that of the dipole, and the same electric field over half the distance gives half the voltage.

The monopole has an input impedance that is therefore half that of its dipole counterpart.

$$Z_{A,\text{monopole}} = \frac{V_{A,\text{monopole}}}{I_{A,\text{monopole}}} = \frac{\frac{1}{2}V_{A,\text{dipole}}}{I_{A,\text{dipole}}} = \frac{1}{2}Z_{A,\text{dipole}}$$

This result does also coincide with what is expected for the radiation resistance: the field radiated by a monopole only extend over a hemisphere, and so power radiated is only half that of a dipole with the same current.

$$R_{R,\text{monopole}} = \frac{2P_{R,\text{monopole}}}{\left|I_{A,\text{monopole}}\right|^2} = \frac{P_{R,\text{dipole}}}{\left|I_{A,\text{dipole}}\right|^2} = \frac{1}{2}R_{R,\text{dipole}}$$

The radiation resistance of a short monopole of length h (and the length of the short dipole is $\Delta z = 2h$) is

$$R_R = 40\pi^2\left(\frac{h}{\lambda}\right)^2$$

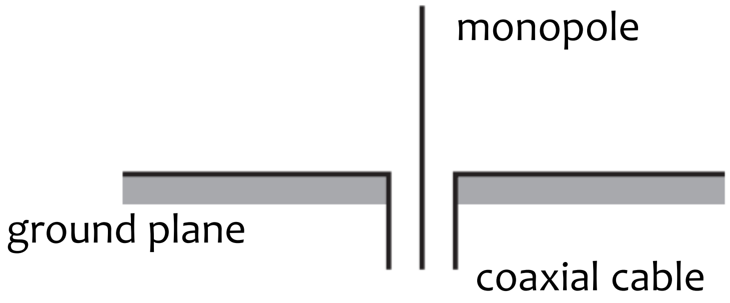
The input impedance of a quarter-wave monopole obtained by halving a half-wave monopole is

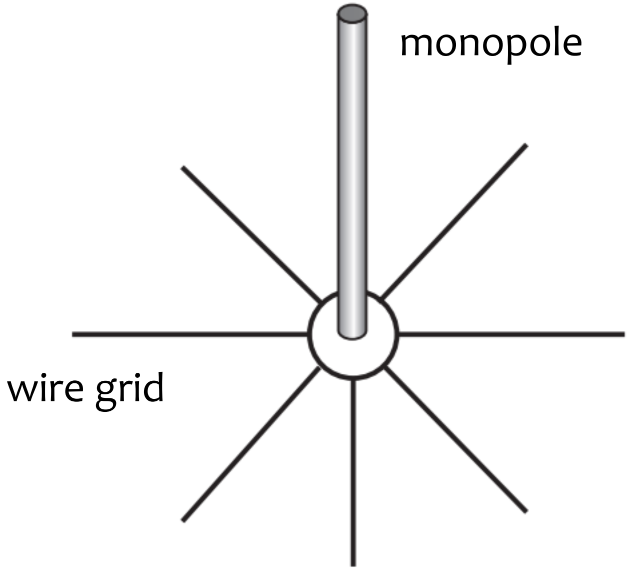
$$Z_A = \frac{1}{2}(73 + j42.5) = 36.5 + j21.25\ \Omega$$

A monopole fed against a perfect ground plane radiates one-half of the total power of the corresponding dipole because the power is distributed in the same fashion but only over half as much space. As a result, the beam solid angle of a monopole is one-half that of the dipole, leading to a doubling of the directivity

$$D_{\mathrm{monopole}} = \frac{4\pi}{\Omega_{A,\mathrm{monopole}}} = \frac{4\pi}{\frac{1}{2}\Omega_{A,\mathrm{dipole}}} = 2D_{\mathrm{dipole}}$$

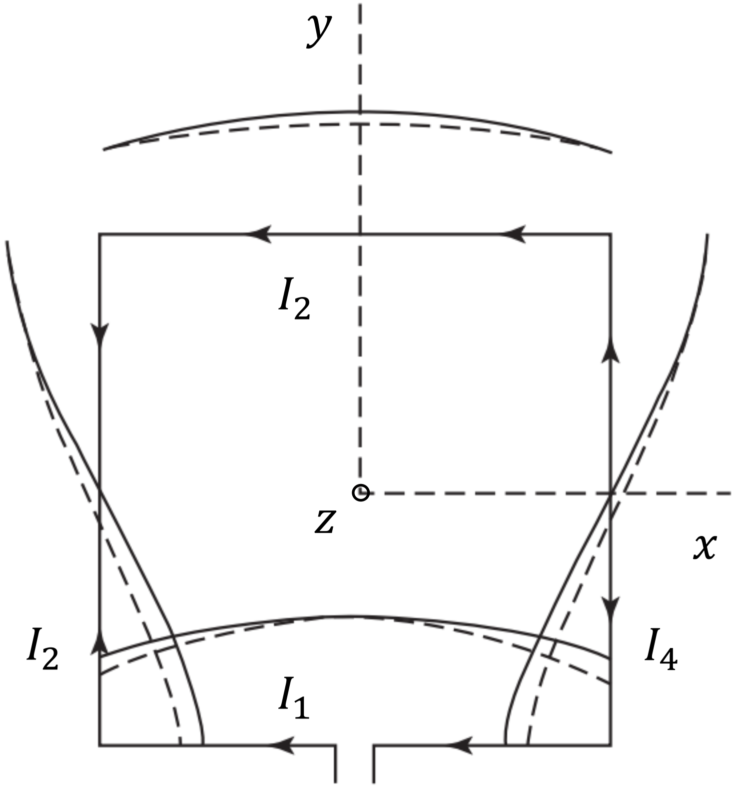
Monopole can also be fed from a coaxial cable behind the ground plane and the perfect ground plane can be replaced by a wire grid or even by a curved metallic surface.





LARGE LOOP ANTENNAS

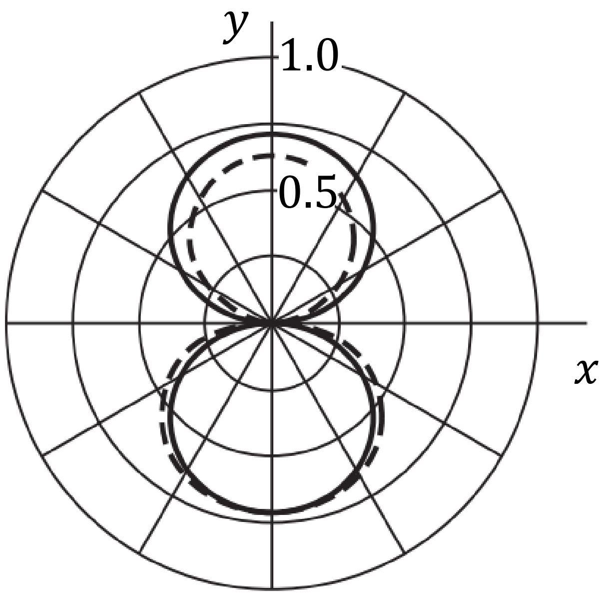
A loop antenna is considered large when its perimeter L_p is much larger than a tenth of a wavelength, and these loops can have either a circular or square shape. As a consequence, large loops have a current amplitude and phase that vary with position around the loop, causing the impedance and pattern to depend on loop size (this is not surprising since it is also observed in straight wire dipoles). There is a radiation pattern maximum perpendicular to the plane of the loop if its perimeter is about a wavelength or more.



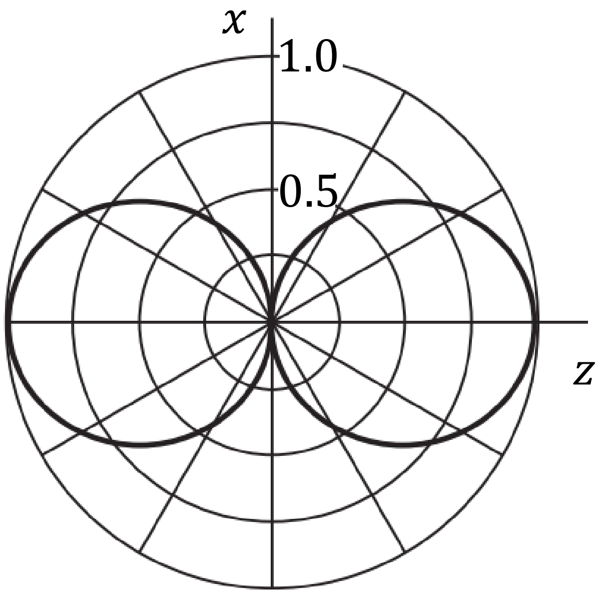
One-wavelength square loop: each side is of length $\lambda/4$.
The solid curve is the sinusoidal current distribution, the dashed curve is the current magnitude obtained from more exact numerical methods.
Starting from this current, the vector magnetic potential can be calculated and explicit expressions for the electric and magnetic field in the far-field are obtained.
Those mathematical expressions are involved and are not reported here, but it is interesting to observe that in the far-field the electric field has both θ-directed and φ-directed components.

One-wavelength square loop: radiation is maximum normal to the plane of the loop (along the z-axis) and in that direction is polarized parallel to the loop side containing the feed (x-direction). In the plane of the loop, there is a null in the direction parallel to the side containing the feed point (x-direction) and a lobe peak perpendicular to the null direction (y-direction). We emphasize that these results are quite different from the small (infinitesimal loop), which has a null perpendicular to the plane of the loop, and maximum radiation in the plane of the loop where the pattern is omnidirectional.

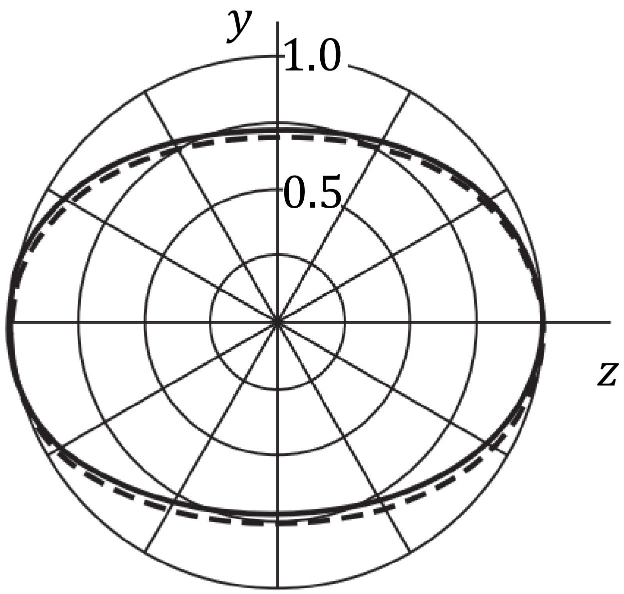
xy plane: plane of the loop



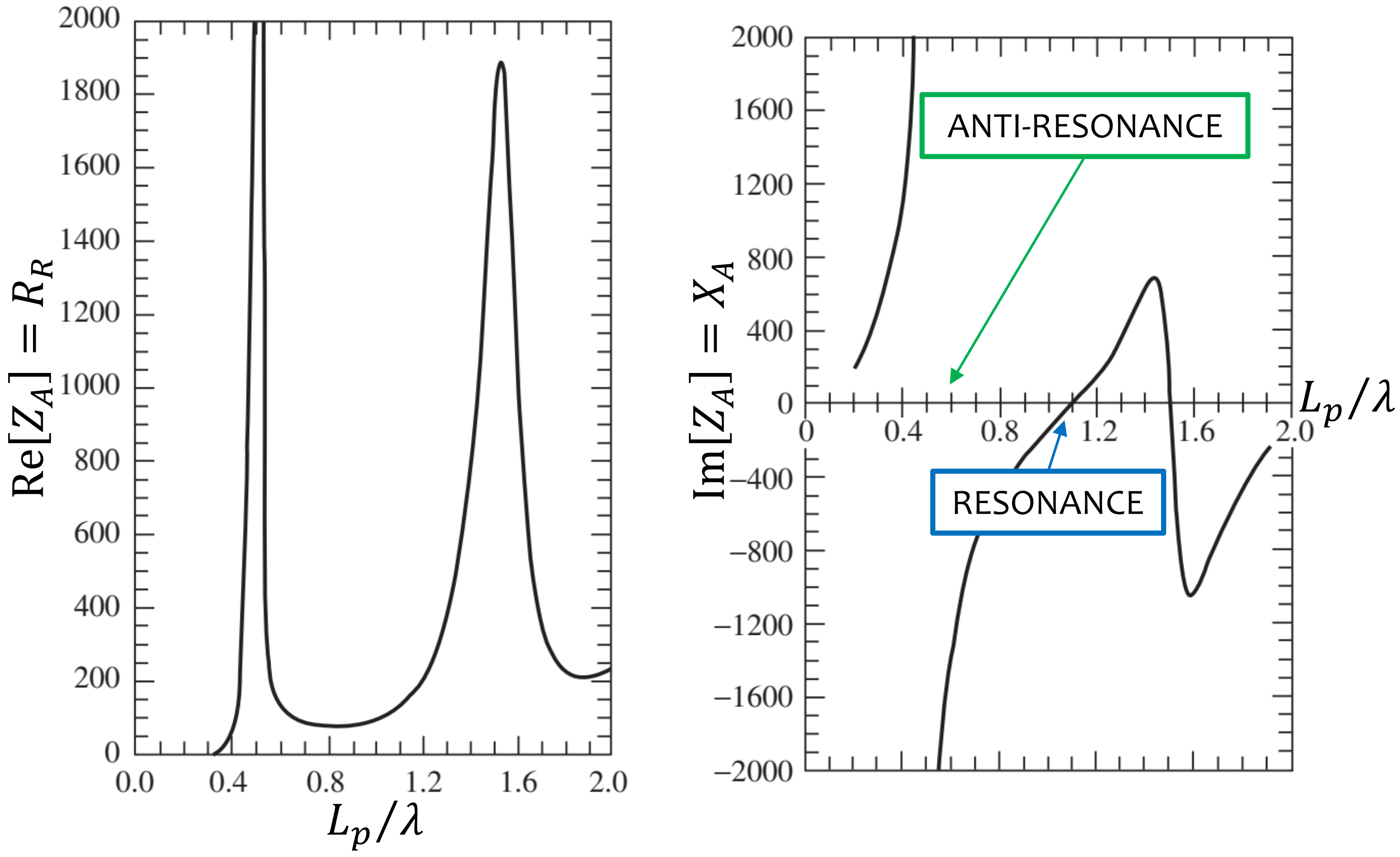
xz plane: E-plane



yz plane: H-plane



The impedance of the square loop antenna can be calculated by resorting to numerical methods. The reactance is inductive below a perimeter of about 0.4λ after which it becomes capacitive and alternates between the two for larger perimeters. This behavior is similar to the impedance for a dipole except the dipole begins as capacitive for short lengths. Also note in comparing the loop to the dipole that for increasing loop perimeter anti-resonance (i.e. large reactance) is encountered first, followed by resonance (zero reactance), whereas the dipole has the reverse sequence. For a one-wavelength perimeter the input impedance is $Z_A = 102 - j42\ \Omega$; exact resonance occurs at about $L_p = 1.1\lambda$ where $Z_A = 133\ \Omega$ and at that resonance the directivity is $D = 2.2$.



FIRST EXPERIMENTAL SETUP PROVING ELECTROMAGNETIC PROPAGATION

The dipole and the loop developed by H. Hertz were the first antennas (in 1886). The transmitting antenna was a loaded half-wave dipole and the receiving antenna was a half-wave loop; these antennas operated at the frequency of $37.5\ \mathrm{MHz}$ ($\lambda = 8\ m$). The induction coil produced sparks at the gap of the dipole, resulting in sparks at the gap of the loop at a distance of several meters in Hertz's laboratory: this was the first evidence of electromagnetic propagation!