Pulse dispersion in optical fibers

D. Pagnoux

Whatever its origin, the **dispersion** is a linear phenomenon which results in a temporal broadening of a pulse during its propagation in a medium (with or without guiding).

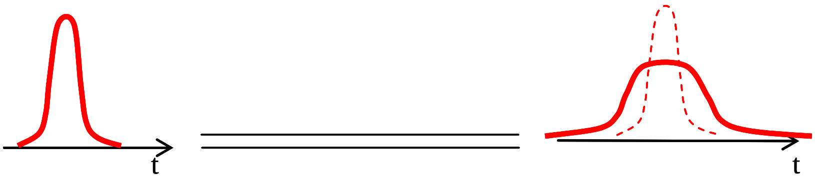


Figure 1 : schematic representation of usual effect of dispersion on the temporal shape of a pulse

In an optical fiber, dispersion may have three different causes :

- Group velocity of modes, around a given central wavelength, are different → **Intermodal dispersion** D_I (or simply "modal dispersion"), only in multimode fibers ;
- In a given mode, group velocities of wave packets centered around different wavlengths are different → **chromatic dispersion** D_C of the mode ;
- In a given mode, at a given wavelength, group velocities of the two orthogonal eigen polarizations are different → **polarization mode dispersion** (PMD)

In a multimode fiber, D_I, D_C and PMD can exist at the same time but, as PMD << D_C << D_I , we can neglect D_C and PMD → D_I only is taken into account.

In a single mode fiber, D_I=0. Most of the time (for example in a standard fibre @ 1.55µm) we have PMD << D_C and thus we can neglect PMD → only D_C is taken under consideration.

In a single mode fiber with an index profile specially designed for obtaining $D_C \sim 0$ → PMD must be taken into account (case of high bit rate telecom fibers).

I. Intermodale dispersion (in multimode fibers)

The energy of a pulse launched into a multimode fiber is split in all the transverse modes of this fiber (decomposition basis). The excitation coefficient of each mode is the normalized integral overlap between the spatial distribution of the incident beam and that of the mode. Each mode having its own group velocity, the energy will exit the fiber over time, according to the excited modes.

I.1 Approximate calculation of the temporal broadening τ of a pulse, in a step index fiber

We consider the simplified approach based on ray tracing (valid only if the core diameter is large compared to the wavelength → multimode fiber).

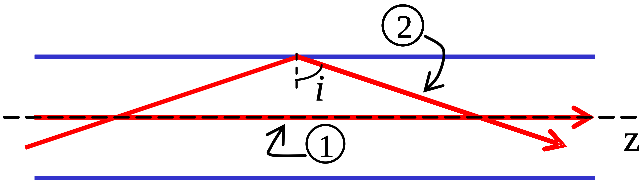


Figure 2 : schematic representation of the fastest ray and of the slowest ray in the core of a step index fiber.

Group propagation time (time of flight of a ray) : $t_g = \dfrac{L}{v_g}$ avec $v_g \approx \dfrac{c}{n_1}\sin i$

For the ray 1 → sin i = 1 → $v_g \approx \dfrac{c}{n_1}$ → $t_{g1} = \dfrac{L}{c}n_1$

For the ray 2 → $\sin i_{\min} = \dfrac{n_2}{n_1}$ → $v_g \approx \dfrac{c}{n_1}\dfrac{n_2}{n_1}$ → $t_{g2} = \dfrac{L}{c}\dfrac{n_1^2}{n_2}$

Maximal group delay (= maximal temporal broadening of the pulse, due to propagation in the multimode regime) : $\Delta t_g = \tau = t_{g2} - t_{g1} = \dfrac{L}{c}.n_1.\left(\dfrac{n_1}{n_2} - 1\right) = \dfrac{L}{c}n_1\left(\dfrac{n_1 - n_2}{n_2}\right)$

With n_2 close to n_1 we have $\Delta = \left(\dfrac{n_1^2 - n_2^2}{2n_1^2}\right) = \dfrac{(n_1+n_2)(n_1-n_2)}{2n_1^2} \approx \dfrac{2n_1(n_1-n_2)}{2n_1^2} \approx \dfrac{n_1 - n_2}{n_1}$

thus $\tau = \dfrac{L}{c}n_1\Delta$

By definition, the intermodal dispersion is : $D_I = \dfrac{\tau}{L} \approx \dfrac{n_1.\Delta}{c}$ (unit : ns/km)

The modulation bandwidth B after a propagation length L, is approximately given by B=1/τ (unit : MHz).

Thus, per unit of length : $B_L = 1/D_I$ (MHz.km)

I.2 Example

Let us consider a step index fiber, of length L= 3km with n_1 = 1.465 and n_2 =1.45 working around the operating wavelength λ_T.

$$\tau = 3 \text{ x } \frac{1}{3\ 10^5} \text{ x } 1.465 \text{ x } \left(\frac{1.465\text{-}1.45}{1.45}\right) = 1.52\ 10^{-7} s \quad = 152\ ns$$

$$D_I = \frac{152}{3} \approx 51 \text{ ns/km}$$

Bandwidth for this section of fiber : $B = \frac{1}{\tau} = \frac{1}{152\ 10^{-9}} = \ 6.58\ 10^6\text{Hz} \ = 6.58 \text{ MHz}$

The modulation bandwidth per unit of length is : $B_L = \frac{1}{D_I} = \frac{1}{51\ 10^{-9}} \approx\ 20\ 10^6\text{Hz.km} \approx\ 20 \text{ MHz.km}$

In a graded index fiber, it is possible to equalize the propagation times of certain rays (helical rays, meridian rays…) by adjusting the index profile, but it is not possible to equalize the propagation times of all the existing rays.

With an optimized profile (Gloge profile, very close to a parabolic index) we can reduce τ by a factor $1/\Delta \sim 1/10^{-2} \sim 100$.

II. Chromatic dispersion

II.1 Definitions

By definition, a pulse has a limited duration Δt (modulation by a pulse of a carrier wave centered at f_0 (or λ_0)). The width of its spectrum is Δf such that Δt.Δf = κ (κ≈ 1, depending on the temporal shape of the pulse).

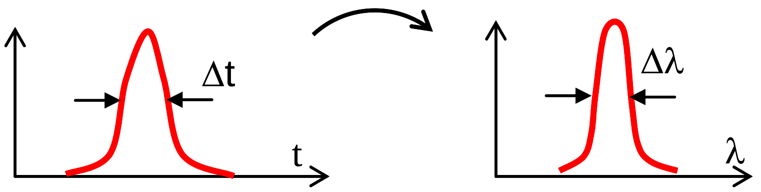


Figure 3 : temporal shape and spectrum of a light pulse

With $f = \dfrac{c}{\lambda}$ $\qquad df = -c\dfrac{d\lambda}{\lambda^2} \Rightarrow \Delta\lambda = \dfrac{\lambda^2}{c}\Delta f \approx \dfrac{\lambda^2}{c}\dfrac{1}{\Delta t}$

Example : if Δt=10ps and λ_0 = 1µm

$$\Delta\lambda = \frac{\left(10^{-6}\right)^2}{3.10^8} \text{x} \frac{1}{10.10^{-12}} = 3.10^{-10} m = 0.3 nm$$

The chromatic dispersion is due to two main causes :

* the refractive index of the material depends on λ
--> n= f(λ) : the material is dispersive

λ varies => n(λ) varies => $v_\varphi = \dfrac{c}{n(\lambda)}$ varies

=> $N_g = n(\lambda) - \lambda\dfrac{dn(\lambda)}{d\lambda}$ varies

=> $v_g = \dfrac{c}{N_g}$ varies with λ

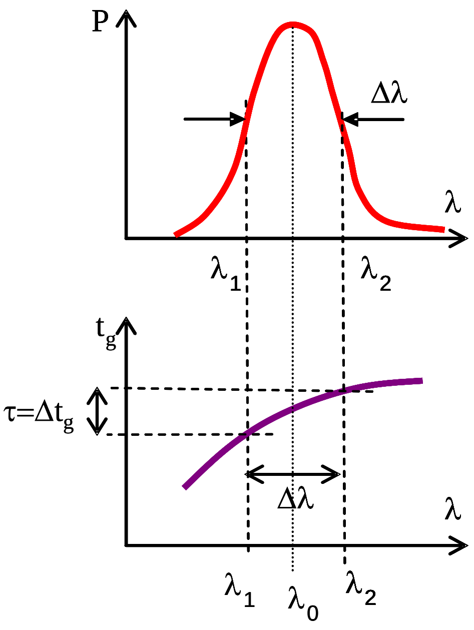
} **material dispersion (D_{mat})**

* when the wave is guided, even if the material is not dispersive
β=f(λ) varies $\Rightarrow v_g = \dfrac{d\omega}{d\beta}$ varies with λ

} **dispersion of the guide (D_{gui})**

In a first approximation, the chromatic dispersion is $D_c \approx D_{mat} + D_{gui}$

By definition $\boxed{D_c = \frac{\tau}{L.\Delta\lambda} \quad \text{unit : ps/(nm.km)} \qquad (1)}$ where τ is the temporal broadening of the pulse, L is the length of the fiber and Δλ is the spectral width of the pulse.



$$\tau = \left| t_g(\lambda_2) - t_g(\lambda_1) \right| \approx \frac{dt_g}{d\lambda} \Delta\lambda \qquad (2)$$

with t_g the group propagation time : $t_g = \frac{L}{v_g}$

Thus $\boxed{D_c = \frac{1}{L}\frac{dt_g}{d\lambda}}$ (s/m.m in the SI) (3)

<u>In practical applications, Dc is generally expressed in ps/(nm.km)</u>

<u>Figure 4 :</u> definition of the chromatic dispersion around λ_0

<u>Comments</u> :

➢ From relation (3), we can write:

$$D_c = \frac{1}{L}\frac{dt_g}{d\lambda} = \frac{1}{L}\frac{d}{d\lambda}\left(\frac{L}{v_g}\right) = \frac{d}{d\lambda}\left(\frac{d\beta}{d\omega}\right) = \frac{d}{d\omega}\left(\frac{d\beta}{d\omega}\right).\frac{d\omega}{d\lambda} = -\frac{2\pi c}{\lambda^2}\frac{d^2\beta}{d\omega^2} \qquad (4)$$

As pointed out in the chapter 2, the expression confirms that the chromatic dispersion of a mode can be deduced from the dispersion curve of this mode presented under the form β=f(ω) or, more difficult, from the dispersion curve presented under the form ω=f(β).

➢ Warning : out of the guided optics community, people are used to consider the limited development of the phase term around the central pulsation ω_0 :

$$\varphi(\omega) = \beta L = \beta(\omega_0).L + L.\frac{d\beta}{d\omega}(\omega-\omega_0) + \frac{L}{2}\frac{d^2\beta}{d\omega^2}(\omega-\omega_0)^2 + ...$$

where $L.\frac{d\beta}{d\omega} = \frac{L}{v_g} = t_g$ represents the propagation time of a pulse having its spectrum centered at ω_0, over the length L, and $\frac{d^2\beta}{d\omega^2}$ is improperly called "group velocity dispersion" (the factor $L.\frac{d^2\beta}{d\omega^2}$ more precisely represents a dispersion of the group propagation time). Thus, we should not confuse this "group velocity dispersion" with the chromatic dispersion defined in (3) and (4). In particular, their signs are opposite: a positive group velocity dispersion (so-called "normal dispersion" occuring at short wavelengths (see below)) corresponds to a negative chromatic dispersion.

II.2 Expressions of the group propagation time t_g and of the temporal broadening τ.

Let us first remind the following relations: $v_g = \frac{d\omega}{d\beta}$ et $t_g = \frac{L}{v_g} = L\frac{d\beta}{d\omega} = \frac{L}{c}\frac{d\beta}{dk_0}$ (since $\omega = k_0 c$) (5)

$t_g = \frac{L}{v_g} =$ and $v_g = \frac{c}{N_g} \Rightarrow t_g = \frac{L}{c.N_g} = \frac{L}{c}\left(n_e - \lambda\frac{dn_e}{d\lambda}\right)$

Expression of t_g as a function of λ :

- If light propagates in free space, in a medium of index $n \rightarrow n_e = n$

  thus $t_g = \frac{L}{c}\left(n - \lambda\frac{dn}{d\lambda}\right)$

- If propagating light is guided → we must use $t_g = \frac{L}{c}\frac{d\beta}{dk_0}$ with

- $\frac{k_0}{dk_0} = -\frac{\lambda}{d\lambda} \Rightarrow dk_0 = -k_0\frac{d\lambda}{\lambda} = -\frac{2\pi d\lambda}{\lambda^2}$ thus $t_g = -\frac{L}{c}\frac{\lambda^2}{2\pi}\frac{d\beta}{d\lambda}$ (6)

The temporal broadening of a pulse having a spectral width equal to $\Delta\lambda$ is given by (2) : $\tau \approx \frac{dt_g}{d\lambda}\Delta\lambda$.

Using(6), we can deduce : $\tau = -\frac{L.\Delta\lambda}{2\pi.c}.\frac{d}{d\lambda}\left(\lambda^2\frac{d\beta}{d\lambda}\right) = -\frac{L.\Delta\lambda}{2\pi.c}.\left(2\lambda\frac{d\beta}{d\lambda} + \lambda^2\frac{d^2\beta}{d\lambda^2}\right)$ (7)

Using the data extracted from the normalized dispersion curve of a mode B= f(V) and by denormalizing, we can plot the curve β=f(λ), then calculate the first and second derivatives, respectively $\frac{d\beta}{d\lambda}$ and $\frac{d^2\beta}{d\lambda^2}$. Finally, from (7), we can find $D_c = -\frac{1}{2\pi .c}.\left(2\lambda\frac{d\beta}{d\lambda} + \lambda^2\frac{d^2\beta}{d\lambda^2}\right)$.

<u>Comment</u> : this expression takes into account both the dispersion of the guide AND the material dispersion. The material dispersion is taken in consideration when B is denormalized at each wavelength, because the indices of the core and of the cladding (respectively $n_1$and n_2) which occur in the relation between B and β are in fact two functions of the wavelength ($n_1(\lambda)$ and $n_2(\lambda)$) given by the Sellmeier relations.

II.3 <u>Study of the dispersions considered separately</u>

II.3.1 <u>Material dispersion</u>

<u>Hypotheses</u> : 1. Plane wave → propagation of non guided light (i.e. no guided wave)

2. the index of the medium (the core) depends on λ -> $n_1(\lambda)$ → dispersive medium

3. we consider the temporal broadening of a pulse having a spectral width equal to Δλ

$$t_g = t_{mat} = \frac{L}{v_g} = \frac{L}{c}N_g = \frac{L}{c}\left(n_1 - \lambda\frac{dn_1}{d\lambda}\right)$$

$$\tau_{mat} = \Delta t_g = \frac{dt_{mat}}{d\lambda}\Delta\lambda = \frac{L\Delta\lambda}{c}\frac{d}{d\lambda}\left(n_1 - \lambda\frac{dn_1}{d\lambda}\right)$$

$$= \frac{L}{c}\Delta\lambda.\left(\frac{dn_1}{d\lambda} - \left(1x\frac{dn_1}{d\lambda} + \lambda\frac{d^2n_1}{d\lambda^2}\right)\right)$$

$$= -\frac{\lambda L}{c}\Delta\lambda.\frac{d^2n_1}{d\lambda^2}$$ (λ being the wavelength in the vaccuum)

D_c dispersion due to the material $= D_{mat} = \frac{\tau_{mat}}{L.\Delta\lambda} = -\frac{\lambda}{c}\frac{d^2n_1}{d\lambda^2}$ (8)

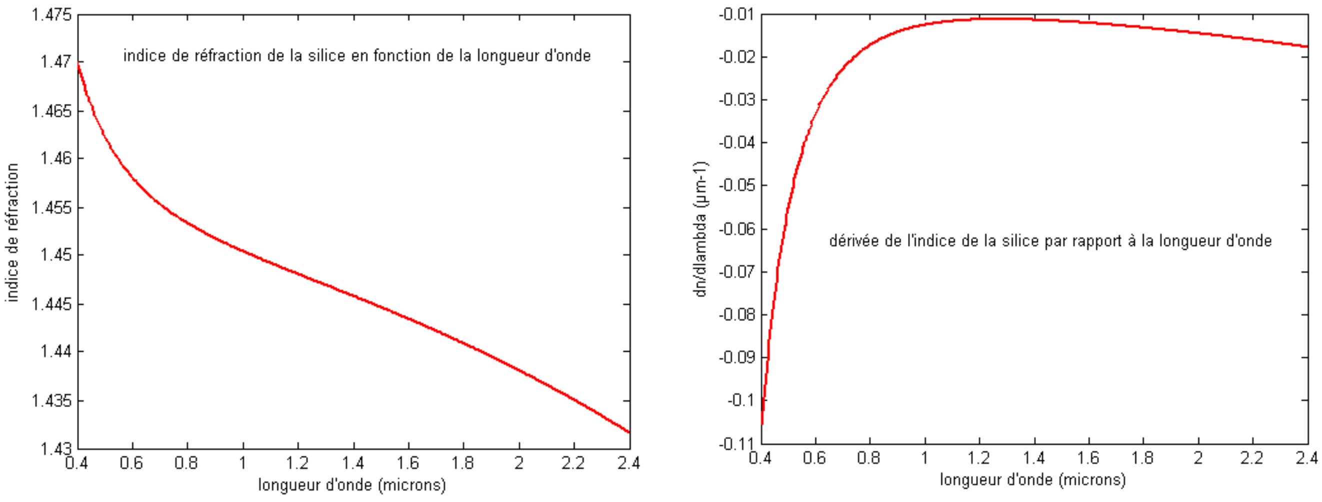


Figure 5 : Curves n=f(λ) and dn/dλ=f(λ) in pure silica

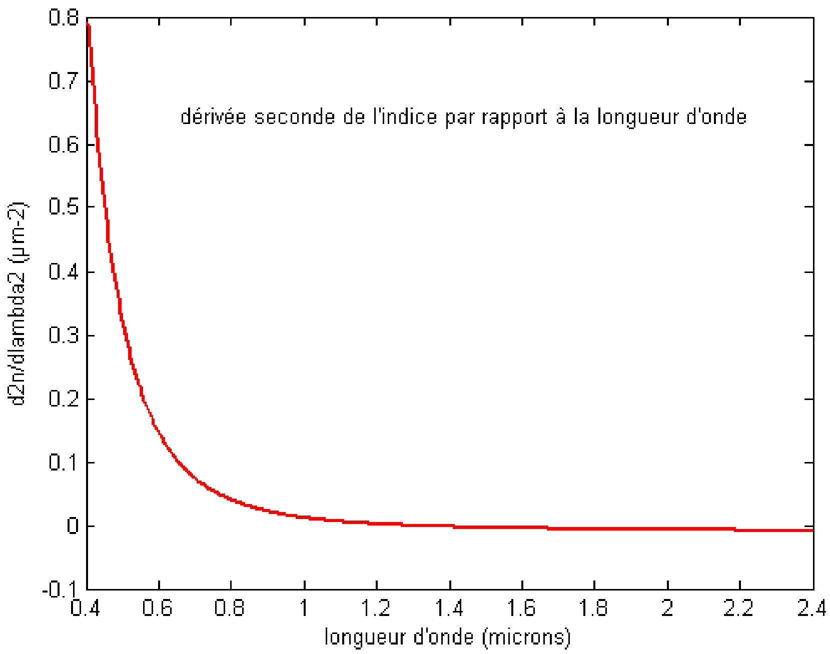


Figure 6 : Curve $d^2n/d\lambda^2 = f(\lambda)$ in pure silica

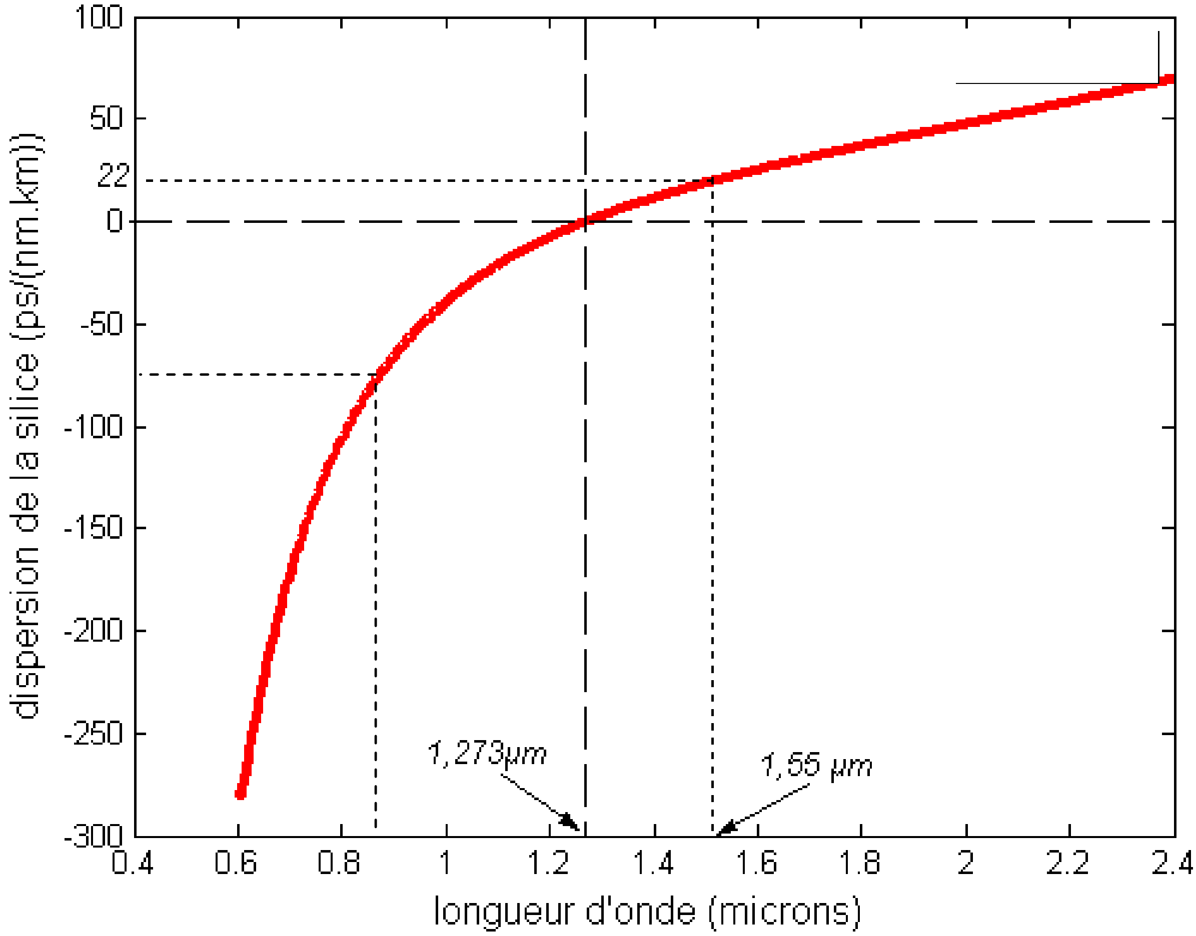


Figure 7 : Material dispersion in pure silica, versus wavelength

Exemple : Let us consider a single mode fiber of length L=1km, in which light from a LED with $\Delta\lambda$=40nm around λ_0=850nm is launched.

D_{mat} = -75ps/(nm.km). $\tau_{mat} = L.\Delta\lambda.|D_{mat}| = 1\text{x}40\text{x}75 = 3000\,ps = 3ns$

This temporal broadening is due to material dispersion, regardless of the guiding phenomenon.

II.3. 2 Dispersion of the guide

Hypothesis : non dispersive medium $\Rightarrow \dfrac{dn_1}{d\lambda}=0$; $\dfrac{dn_2}{d\lambda}=0$; $\dfrac{d\Delta}{d\lambda}=0$

since $k_0=\dfrac{2\pi}{\lambda} \quad \Rightarrow \dfrac{dn_1}{dk_0}=\dfrac{dn_2}{dk_0}=\dfrac{d\Delta}{d\lambda}=0$

From (5), the group propagation time is $t_g=t_{gui}=\dfrac{L}{c}\dfrac{d\beta}{dk_0}$

Our goal is to express t_{gui} as a function of B and V in order to be able to find t_{gui} from the normalized dispersion curve B=f(V)

We first express β as a function of B. (Later, we'll express k_0 as a function of V).

Starting from $B=\dfrac{\beta^2-k_0^2n_2^2}{k_0^2\left(n_1^2-n_2^2\right)}$ (see chapter 3) with $\beta=k_0n_e$

$$\rightarrow \quad B=\frac{n_e^2-n_2^2}{n_1^2-n_2^2}=\frac{\left(n_e-n_2\right).\left(n_e+n_2\right)}{\left(n_1-n_2\right).\left(n_1+n_2\right)} \qquad n_2\le n_e\le n_1$$

In the weakly guidance approximation, n_1 is close to n_2 and thus $n_1+n_2\approx 2n_1$ et $n_1+n_e\approx 2n_1$

$$B\approx\frac{n_e-n_2}{n_1-n_2}=\frac{\beta/k_0-n_2}{n_1-n_2} \Leftrightarrow \beta\approx\left[B\left(n_1-n_2\right)+n_2\right].k_0$$

If we make use of $\Delta=\dfrac{n_1^2-n_2^2}{2n_1^2}=\dfrac{\left(n_1+n_2\right)\left(n_1-n_2\right)}{2n_1^2}\approx\dfrac{2n_1\left(n_1-n_2\right)}{2n_1^2}\approx\dfrac{n_1-n_2}{n_1}$ (see chap. 3)

thus $n_1-n_2\approx n_1\Delta$ and β becomes: $\beta=k_0\left[n_2+n_1\Delta B\right]$ (9)

By reporting (9) in (5) we obtain: $t_{gui}=\dfrac{L}{c}\dfrac{d}{dk_0}\left[k_0n_2+n_1\Delta k_0B\right]$

$$=\frac{L}{c}\left[n_2+n_1\Delta\frac{d\left(k_0B\right)}{dk_0}\right] \qquad (10)$$

Now, we can introduce $V=k_0a\sqrt{n_1^2-n_2^2}=k_0.\underbrace{a.n_1\sqrt{2\Delta}}_{A}$ => $k_0=\dfrac{V}{A}$ (with A = constant)

In (10) : $$\frac{d\left(k_0 B\right)}{dk_0}=\frac{d\left(\frac{V}{A}B\right)}{d\left(\frac{V}{A}\right)}=\frac{d\left(VB\right)}{dV}$$

and (10) becomes: $$t_{gui}=\frac{L}{c}\left[n_2+n_1\Delta\frac{d\left(VB\right)}{dV}\right] \qquad (11)$$

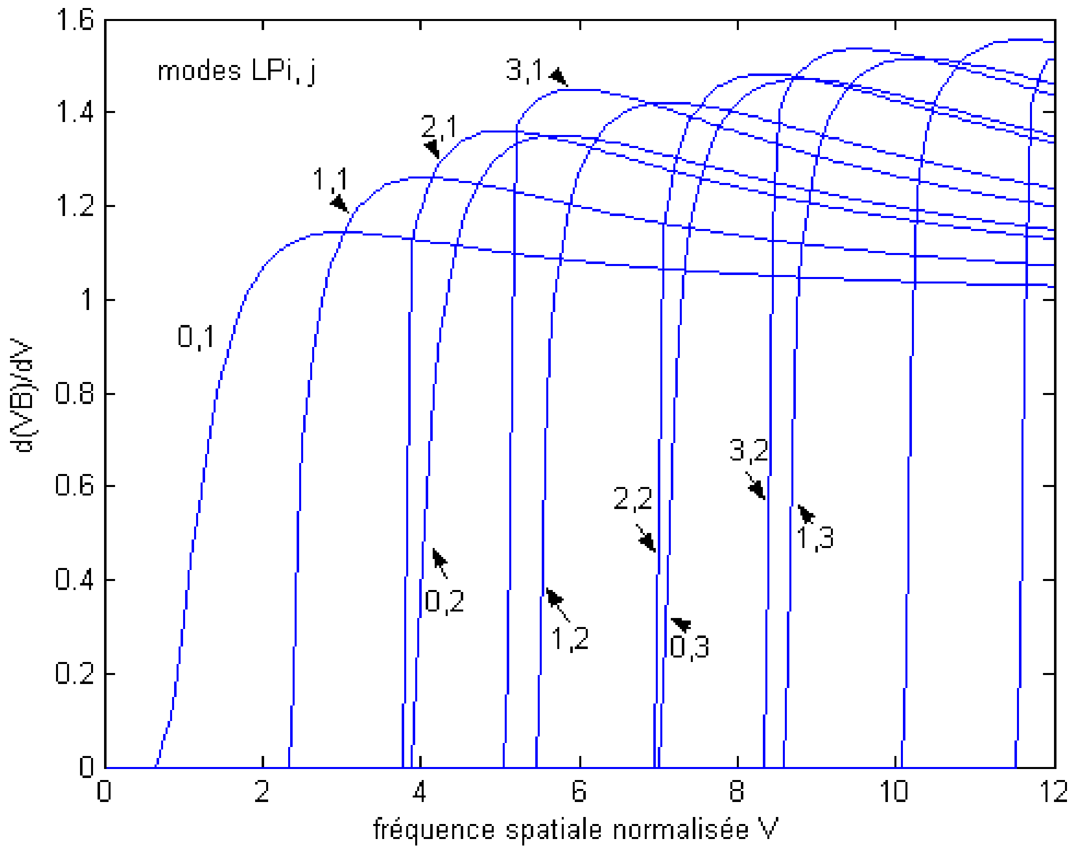


Figure 8 : Curves of d(VB)/dV versus V for different LP modes

Note : on the dispersion curve, we can see that, for large values of V, B tends toward 1 (almost constant), thus VB $\rightarrow$ V , et $\frac{d\left(VB\right)}{dV}\rightarrow 1$, which can be seen on the curve $\frac{d\left(VB\right)}{dV}=f\left(V\right)$

For a given mode, what is the value of the pulse broadening τ_{gui} due to the dispersion of the guide ?

$$\tau_{gui} = \Delta t_g = \frac{dt_{gui}}{d\lambda}\Delta\lambda$$

$$\tau_{gui} = \frac{d}{d\lambda}\left[\frac{L}{c}\left(n_2 + n_1\Delta\frac{d(VB)}{dV}\right)\right]\Delta\lambda = \frac{L}{c}n_1\Delta\frac{d}{d\lambda}\left(\frac{d(VB)}{dV}\right)\Delta\lambda \qquad (12)$$

$$= \frac{L}{c}n_1\Delta\frac{d}{dV}\left(\frac{d(VB)}{dV}\right).\frac{dV}{d\lambda}.\Delta\lambda$$

however $\frac{dV}{d\lambda} = -\frac{2\pi}{\lambda^2} a\ ON = -\frac{V}{\lambda}$ thus : $\tau_{gui} = -\frac{L}{c}n_1\Delta\frac{V}{\lambda}\frac{d^2(VB)}{dV^2}\Delta\lambda$

$$\boxed{D_c \text{ dispersion due to the guiding phenomenon: } D_{gui} = \frac{\tau_{gui}}{L.\Delta\lambda} = -\frac{n_1\Delta}{c\lambda}V\frac{d^2(VB)}{dV^2}} \qquad (13)$$

For a fiber operating in the single mode regime, we study the curve $V\frac{d^2(VB)}{dV^2} = f(V)$, with $0 \le V \le 2.405$, for the LP_{01} mode (see Figure 9):

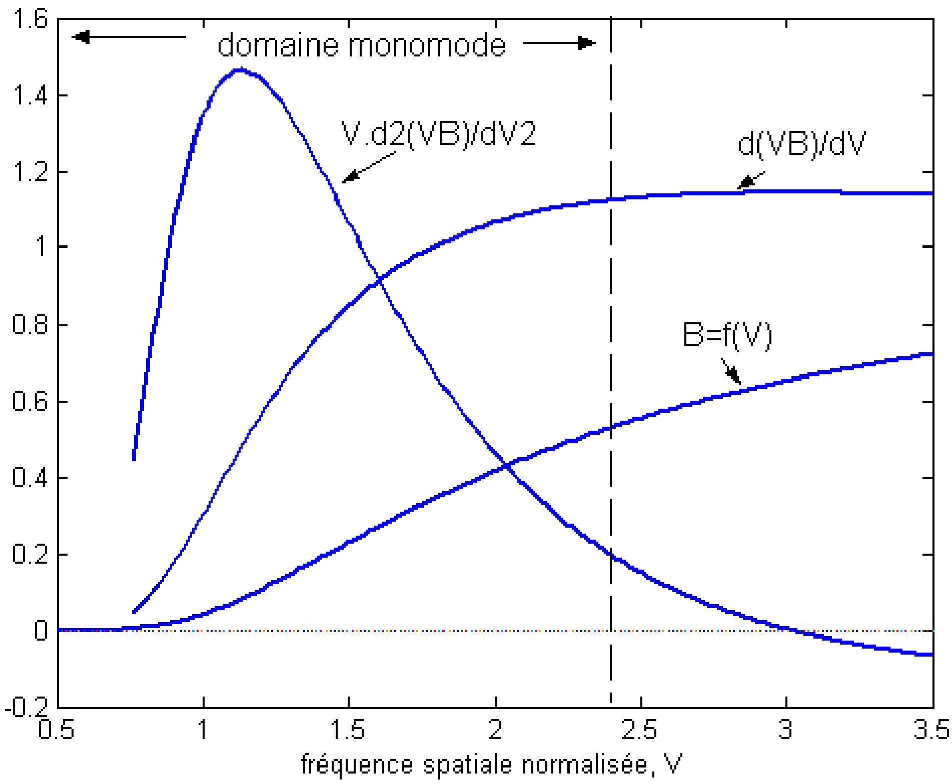


Figure 9 : Curves B=f(V), d(VB)/dV and V $d^2(VB)/dV^2$ as a function of V for the fundamental mode

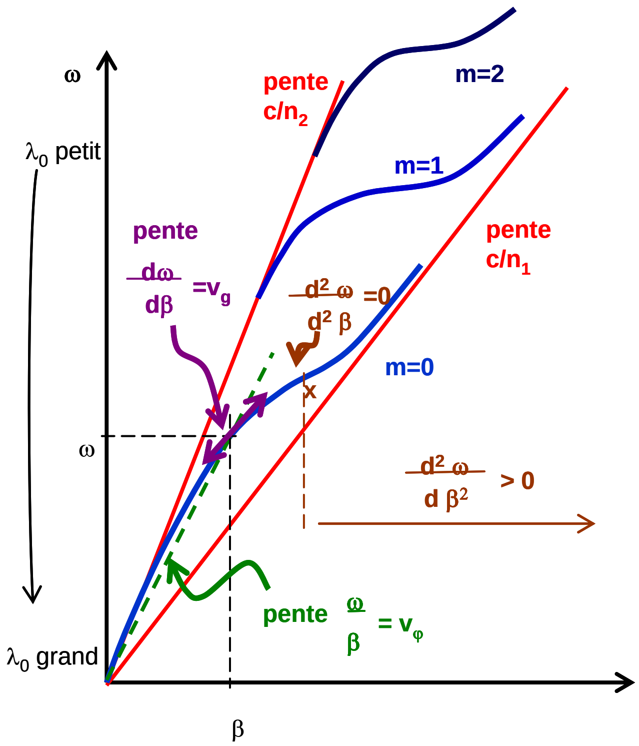
When considering the dispersion curves, we can note that, when V becomes lower than 2 (λ ↑), B rapidly decreases, thus n_e tends towards n_2. ==> the mode spreads far in the cladding ==> attenuation.

In order to work with reduced attenuation, we are going to limit the domain of V so that $2 \leq V \leq 2{,}405$ for which we have $0.2 \leq V\dfrac{d^2(VB)}{dV^2} \leq 0.45$. For example, for V=2.2 $V\dfrac{d^2(VB)}{dV^2} \approx 0{,}3$

For 2.4 ≤ V ≤ 2.8, the fiber is a two mode fiber but the LP_{11} mode is a leaky mode (very attenuated) and it is sometimes possible to work in thes conditions of quasi single mode regime.

<u>Comments</u> :

- In the single mode regime, for a step index fiber, we ALWAYS have $V\dfrac{d^2(VB)}{dV^2} > 0$ (see fig 9), i.e. the dispersion of the guide is always NEGATIVE.
- This is in good accordance with the dispersion curves presented in chapter 2.



Indeed, in single mode regime → long wavelengths → ω small → $\dfrac{d^2\omega}{d\beta^2} < 0$

→ $\dfrac{d^2\beta}{d\omega^2} > 0$ → $D_g < 0$: QED (here, we do not take the material dispersion into consideration)

<u>Numerical application</u> : λ= 1.55µm ON = 0.105 n_1 = 1.46 a = 4.5µm

thus $\Delta = \dfrac{ON^2}{2n_1^2} = 2.83.10^{-3}$ $V = \dfrac{2\pi}{\lambda} aON = 1.91$

on the curve of fig 9, we read: $V\dfrac{d^2(VB)}{dV^2} \approx 0,45$

$$D_{gui} = -\frac{1.46 \text{ x } 2.83.10^{-3}}{3.10^{8} \text{ x } 1.55.10^{-6}} \text{ x } 0.6 = -5 10^{-6} s/(m.m) = -5 ps/(nm.km)$$

At λ=1.55µm, $D_{mat} \approx$ 22 ps/(nm.km).

Thus, with $D_c \approx D_{mat} + D_{gui}$ we find : $D_c \approx$ -5 + 22 = 17 ps/(nm.km).

In a standard single mode fiber with a small core, (cutoff wavelength λ_c close to 1.2µm), we find :

D_{gui} (1.3µm) ~ -3 ps/(nm.km) and D_{mat} (1.3µm) ~ +3 ps/(nm.km) → $D_c \approx D_{mat} + D_{gui}$ cancel at 1.3µm.

<u>For λ < 1.3µm, D_c<0 → "normal" dispersion</u> (because it was historically observed first)

<u>And for λ > 1.3µm, D_c>0 → "anomalous" dispersion</u> (historically observed later, but is not fundamentally <u>"anomalous"</u>).

Discussion :

- D_{mat} depends on the material and we cannot significantly modify it : for this, manufacturers should strongly modify the concentration of dopants in the silica → modification of the indices but also decrease of the transparency…).
- One can modify D_{gui} by modifying the guiding conditions, i.e. by changing the index profile. But one can show that, in most of the cases, D_{gui} remains < 0 (i.e. the guiding phenomenon imposes that wave packets centered at long wavelengths propagate faster than wave packets centered at shorter wavelengths).
- Thus, by adjusting D_{gui}, we can cancel the chromatic dispersion at wavelengths for which D_{mat}>0, i.e. for λ> 1,27µm. → It is possible to obtain D_c = 0 at λ=1.55µm (dispersion shifted fiber = DSF) → It is also possible to obtain D_c = 0 at λ=1.3µm AND at λ=1.55µm, with small D_c between the two (dispersion flattened fiber =DFF).

- To cancel the chromatic dispersion at λ<1.27 µm, it is necessary to get D_{gui} >0 → Bragg fibers, small core microstructured fibers, photonic band gap (PBG) fibers… see subsequent courses on non conventionnal optical fibers.