MICROWAVE ENGINEERING

Lecture 6:

Problems on

Plane Waves, Polarization

Transmission, Reflection

PROBLEM 1 - PLANE WAVES

A plane wave propagating in a lossless dielectric has an electric field:

$$\boxed{E_x = E_0 \cos(\underbrace{151 \cdot 10^{10}}_{\omega} t - \underbrace{61.6}_{k} z)} \quad [V/m]$$

Determine:

- wavelength
- phase velocity
- wave impedance
- dielectric constant of the medium

The lossless medium is not magnetic.

Generally we can write

$$E_x = E_0 \cos(\omega t - \underline{k}z)$$

Wavelength is defined as:

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{61.6} = 0.102 \text{ m}$$

Phase velocity is: $V_p = \dfrac{\omega}{k} = \dfrac{151 \cdot 10^{10}}{61.6} = 2.45 \cdot 10^8$ m/s

$$V_p = \frac{c}{n}$$

The dielectric constant is $\underline{\varepsilon_r} = n^2 = \left(\dfrac{c}{V_p}\right)^2 = \left(\dfrac{3 \cdot \cancel{10^8}}{2.45 \cdot \cancel{10^8}}\right)^2 = \underline{1.5}$

The wave impedance

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{1.5}} = 307.8\ \Omega$$

PROBLEM 2 - PLANE WAVES

A 10 MHz uniform plane wave is traveling in a non-magnetic medium with $\mu = \mu_0$ and $\varepsilon_r = 9$. Find:

a) the phase velocity

b) the wavenumber

c) the wavelength in the medium

d) the impedance of the medium

The frequency is $f = 10 \cdot 10^6$ Hz which corresponds to an angular frequency $\omega = 2\pi f = 6.28 \cdot 10^7 \ \frac{rad}{s}$

a)
The phase velocity is $V_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \cdot 10^8}{\sqrt{9}} = 10^8 \ m/s$

$V_p = \frac{\omega}{k}$

b)
The wave number is defined as $\rightarrow$ $k = \frac{\omega}{V_p} = \frac{6.28 \cdot 10^7}{10^8} =$

$= 0.628 \ \frac{rad}{m}$

c) The wavelength in the medium is

$$\lambda = \frac{2\pi}{k} = \frac{6.28}{0.628} = 10 m$$

d) The impedance of the medium is

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{3} = 125.67 \ \Omega$$

PROBLEM 3 - POLARIZATION

The electric field of a plane wave is given by

$$E(z,t) = \hat{x}\, 3 \cos(\omega t - kz) + \hat{y}\, 4 \cos(\omega t - kz) \ \frac{V}{m}$$

Determine:

a) The polarization state

b) The modulus of E (amplitude)

c) the polarization direction

---

a) $a_x \neq a_y$ and $\varphi = 0$ $\Rightarrow$ linearly polarized field

b) The amplitude

$$|E| = \sqrt{3^2 \cos^2(\omega t - kz) + 4^2 \cos^2(\omega t - kz)} = 5 \cos(\omega t - kz) \frac{V}{m}$$

c) The polarization direction is defined as

$$\psi = \text{atan}\left(\frac{e_y}{e_x}\right) = \text{atan}\left(\frac{4}{3}\right) = 53.1^\circ$$

PROBLEM 4 - POLARIZATION

Show that a linearly polarized plane wave of the form $\bar{E} = E_0(\hat{x} + 2\hat{y})e^{-jk_0z}$ can be represented as the sum of a RHCP and a LHCP wave.

We need to express the sum of two generic RHCP and LHCP waves.

$$\bar{E} = \underbrace{A(\hat{x} - j\hat{y})e^{-jk_0z}}_{RHCP} + \underbrace{B(\hat{x} + j\hat{y})e^{-jk_0z}}_{LHCP}$$

We need to impose:

$$\begin{array}{ll} \hat{x} & \\ \hat{y} & \end{array} \begin{cases} A + B = E_0 \\ -jA + jB = 2E_0 \end{cases}$$

$$\begin{cases} A + B = E_0 \\ -jA + jB = 2A + 2B \end{cases}$$

$$\begin{cases} // \\ A(2+j) = B(j-2) \end{cases}$$

$$\begin{cases} // \\ A = B\left(\frac{j-2}{2+j}\right) \end{cases}$$

$$\begin{cases} B\left(\dfrac{j-2}{j+2}\right) + B = E_0 \\ A = B\left(\dfrac{j-2}{2+j}\right) \end{cases}$$

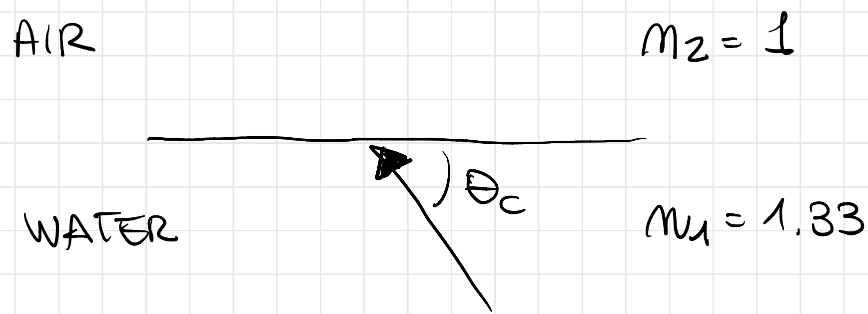
$$\begin{cases} jB - 2\cancel{B} + jB \cancel{+2B} = E_0(2+j) \\ \quad " \end{cases}$$

$$\begin{cases} B = E_0\left(\dfrac{2+j}{2j}\right) \\ A = \left(\dfrac{j-2}{\cancel{j+2}}\right) E_0 \left(\dfrac{\cancel{2+j}}{2j}\right) = E_0\left(\dfrac{j-2}{2j}\right) \end{cases}$$

$$\begin{cases} B = E_0\left(\dfrac{1}{2} - j\right) \\ A = E_0\left(\dfrac{1}{2} + j\right) \end{cases}$$

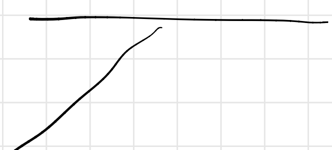
PROBLEM 5 - REFLECTION AND TRANSMISSION

In the visible part of the electromagnetic spectrum, the index of refraction of water is 1.33. What is the critical angle for light waves generated by an upward looking underwater light source?



$$k_1 \sin\theta_i = k_2 \underbrace{\sin\theta_t}_{1}$$

at $\theta_c \rightarrow \theta_t = 90°$



$$\cancel{\omega\sqrt{\mu_0}}\,\sqrt{\varepsilon_1}\,\sin\theta_c = \cancel{\omega\sqrt{\mu_0}}\,\sqrt{\underset{=1}{\varepsilon_2}}$$

$$\theta_c = \operatorname{asin}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \operatorname{asin}\left(\frac{1}{1.33}\right) = 48.8^\circ$$

PROBLEMS - REFLECTION AND TRANSMISSION

A plane wave is perpendicularly polarized and incident from air on a plane soil surface located at $z=0$. The electric field of the incident wave is

$$E^i = \hat{y}\, 100 \cos(\omega t - \pi x - 1.73\pi z) \quad \frac{V}{m}$$

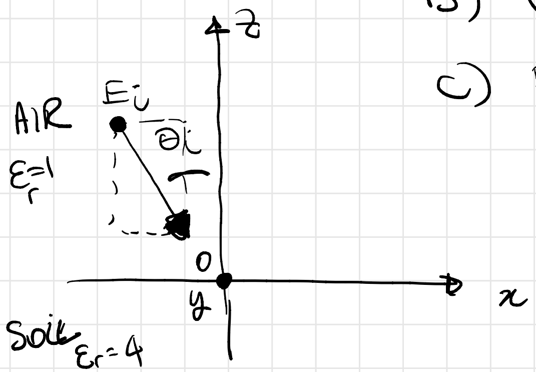
The soil is assumed to be a lossless dielectric with $\varepsilon_r = 4$

Determine:

a) k_1, k_2 and the incidence angle θ_i

b) the angle of transmission in the soil θ_t

c) Reflection and transmission coefficients



In the phasor form we have

$$\underline{E}^i = \hat{y}\, 100\, e^{-j\pi x - j 1.73\pi z} = \hat{y}\, 100\, e^{-j k_1 x_i}$$

$$k_1 x_i = \pi x + 1.73\pi z$$

$$k_1 x_i = k_1 x \sin\theta_i + k_1 z \cos\theta_i$$

$$* \begin{cases} k_1 \sin\theta_i = \pi \\ k_1 \cos\theta_i = 1.73\pi \end{cases}$$

$$\begin{cases} k_1^2 \sin^2\theta_i = \pi^2 \\ k_1^2 \cos^2\theta_i = (1.73)^2 \pi^2 \end{cases}$$ ← Squaring and adding

$$k_1^2 = \pi^2 + (1.73)^2 \pi^2 \Rightarrow \underline{k_1} = \sqrt{\pi^2 + 1.73^2 \pi^2} \simeq \underline{2\pi} \frac{\text{rad}}{\text{m}}$$

If we divide the first expression by the second we ~~x~~ system

$$\tan \theta_i = \frac{\pi}{1.73\pi}$$

$$\underline{\underline{\theta_i}} = \operatorname{atan}\left(\frac{1}{1.73}\right) = \underline{\underline{30^\circ}}$$

The wavelength in air : $\lambda_1 = \frac{2\pi}{k_1} = 1\ m$

The wavelengt in the soie is:

$$\lambda_2 = \frac{\lambda_1}{\sqrt{\varepsilon_r}} = \frac{1}{2} = 0.5\ m$$

it follows that

$$k_2 = \frac{2\pi}{\lambda_2} = \frac{2\pi}{0.5} = 4\pi\ rad/m$$

Since $\theta_i = 30°$ we use snell's law to calculate θ_t

$$k_1 \sin\theta_i = k_2 \sin\theta_t$$

$$\theta_t = \arcsin\left(\frac{k_1}{k_2}\sin\theta_i\right) = \arcsin\left(\frac{2\pi}{4\pi}\cdot\sin 30°\right)$$

$$= 14.5°$$

$$\eta_1 = \eta_0 = 377\,\Omega$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_r}} = 188.5\,\Omega$$

$$\Gamma = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = -0.38$$

$$T = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = 0{,}62$$