SYNTHESIS OF GMSK USING AMPLITUDE MODULATED PULSES

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ABSTRACT

We present a QAM approximation scheme for GMSK. First, we show the representation of GMSK by a sum of Pulse Amplitude Modulation signals (PAM) and then we find an approximation using only one of the PAM signals. This PAM can be split up into two independent signals driven by the same BPSK. Thus we get a two diversity channel approximation out of one GMSK signal. We use the Cross Relation Method to estimate the channel.

1. INTRODUCTION

All blind equalization methods need a linear representation of the desired signal. So we introduce a linear approximation for GMSK, which allows the application of blind or semi-blind equalization methods. The method is a combination of the publications of Z. Ding [1] and P. A. Laurent [2]. From Laurent we use the PAM-representation of the CPFSK signal. So we split up GMSK into eight different PAM signals. But we use only one of them, because of its domination. This PAM is in fact a QAM, because the modulating sequence has symbols taken out of $\{-1, -j, +1, +j\}$. This single QAM signal can be divided into two diversity channels using the derotation scheme of Ding. For the channel estimation we use the Cross Relation Method introduced by G. Xu, H. Liu, L. Tong, and Th. Kailath [3], which was improved by M. Zoltowsk, D. Tseng, and T. Thomas [4] by using Basis Functions.

2. LINEAR DECOMPOSITION OF GMSK SIGNALS

MSK can be interpreted as an offset QPSK [8]. Our goal is to find a representation of the received GMSK signal $x(t)$, which consists of the superposition of pulses $h(t)$ modulated by the transmitted symbol sequence $\{a_k\}$.

$$x(t) = \sum_{n=-\infty}^{\infty} a_n h(t - nT) + w(t), \quad a_n \in \mathcal{A}$$

where T is the symbol period, $w(t)$ is the noise, and $\mathcal{A}$ is the input constellation set.

We start with the nonlinear baseband signal of GMSK.

$$s(t) = \exp\left[j\frac{\pi}{2} \sum_{k=-\infty}^{\infty} \alpha_k \psi(t - kT) \right]$$

where $\alpha_k \in \{-1, +1\}$ is the transmitted binary data.

The continuous phase modulation (CPM) pulse $\psi(t)$ is the integration of the output of the gaussian lowpass filter.

The pulse response of the gaussian lowpass filter is the convolution of the gaussian pulse

$$g(t) = B\sqrt{\frac{2\pi}{\ln 2}} \exp\left[-\frac{2\pi^2 B^2 t^2}{\ln 2} \right]$$

and the rectangular NRZ pulse

$$\mathrm{rect}(t) = \begin{cases} 1/T, & |t| \le T/2 \\ 0, & |t| > T/2 \end{cases}$$

This leads to pulse response of the gaussian filter [6]

$$q(t) = \frac{1}{2}\left[\mathrm{erf}\left(\gamma\left(t + T/2\right)\right) - \mathrm{erf}\left(\gamma\left(t - T/2\right)\right) \right],$$

where

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp\left(-u^2\right) du \quad \text{and} \quad \gamma = \sqrt{\frac{2}{\ln 2}}\pi B.$$

The integration of $q(t)$ gives us the CPM pulse

$$\psi(t) = \int_{-\infty}^{t} q(\tau - 2T) d\tau$$

$$= \frac{T}{2} + \sum_{i \in \{-1/2, +1/2\}} i \Bigg[(t - (2-i)T) \,\mathrm{erf}\left(\gamma\left(t - (2-i)T\right)\right) + \frac{1}{\gamma\sqrt{\pi}} \exp\left(-\gamma^2 \left(t - (2-i)T\right)^2\right) \Bigg].$$

Note that we set the GMSK parameter $BT = 0.3$ (like in GSM systems). Hence we set the time-shift of $q(t)$ in the integral to $2T$, because we get an approximately causal pulse $\psi(t)$ then. If one wants to use other values for BT, one has to consider, that $q(t)$ gets broader for smaller BTs and for higher BTs $q(t)$ will be narrower. So, it may be necessary to change the time-shift in the formula for $\psi(t)$ for $BT \neq 0.3$. Figure 1 shows pulse response of the gaussian filter and the CPM pulseform.

Since $\psi(0) < 10^{-4}$ we can make the good approximation

$$\psi(t) \approx \begin{cases} 0, & t < 0 \\ 1, & t > 4T. \end{cases}$$

Again, if one uses $BT \neq 0.3$, it may be necessary to change the upper time-value for this approximation (e.g. $t > 5T$ for smaller BTs).

With this approximation we meet the condition of [2], that the phase shift function is zero for times less than zero and reaches the maximum after a finite interval.

Now let us observe the time period $[nT, (n+1)T)$. The phase contributions of the bits $\alpha_{n+1}, \alpha_{n+2}, \ldots$ are still zero and the phase shifts caused by the bits $\alpha_{n-4}, \alpha_{n-5}, \ldots$ are constant. So we can separate the constant terms.

$$s(t) = \exp\left[j\frac{\pi}{2} \sum_{k=-\infty}^{\infty} \alpha_k \psi(t - kT) \right]$$

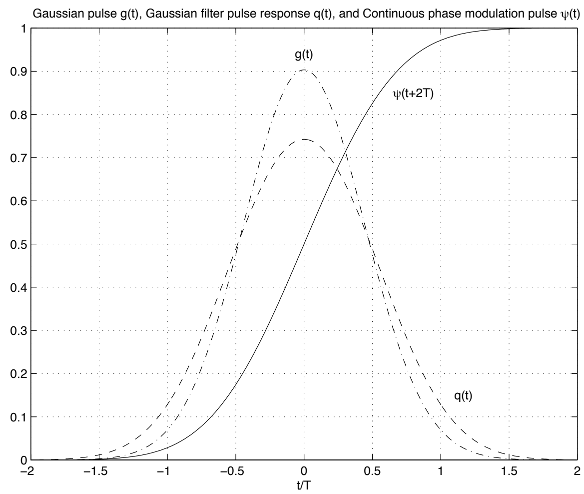

Figure 1. CPM pulse $\psi(t)$.

$$\approx \exp\left(j\frac{\pi}{2}\sum_{k=-\infty}^{n-4}\alpha_k\right)\prod_{k=n-3}^{n}\exp\left[j\frac{\pi}{2}\alpha_k\psi(t-kT)\right],$$
$$t\in[nT,(n+1)T)$$

Because the first part is constant we examine the factors of the product. Using Euler's formula and since $\alpha_k \in \{-1,+1\}$, we get

$$\exp\left[j\frac{\pi}{2}\alpha_k\psi(t-kT)\right] =$$
$$\cos\left[\frac{\pi}{2}\psi(t-kT)\right]+j\alpha_k\sin\left[\frac{\pi}{2}\psi(t-kT)\right],$$
$$n-3\le k\le n,\quad nT\le t<(n+1)T$$

We eliminate the cos with the following equation

$$\cos\left(\frac{\pi}{2}\psi\right)=\cos\left(-\frac{\pi}{2}\psi\right)=\sin\left(\frac{\pi}{2}-\frac{\pi}{2}\psi\right)$$

Now we have a formulation with two sines. To be able to describe $s(t)$ with one single function we introduce the generalized phase pulse function [2]

$$\beta(t)=\begin{cases}\sin\left(\frac{\pi}{2}\psi(t)\right), & t<4T\\ \sin\left(\frac{\pi}{2}-\frac{\pi}{2}\psi(t-4T)\right), & t\ge 4T.\end{cases}$$

The usage of $\beta(t)$ leads to a new expression for the baseband signal:

$$s(t)\approx\exp\left(j\frac{\pi}{2}\sum_{k=-\infty}^{n-4}\alpha_k\right)\times$$
$$\prod_{k=n-3}^{n}\left[\beta(t-kT+4T)+j\alpha_k\beta(t-kT)\right],$$
$$t\in[nT,(n+1)T)$$

To get a compact formulation we use the abbreviation

$$S_m(t)=\beta(t+mT),$$

thus

$$s(t)\approx\exp\left(j\frac{\pi}{2}\sum_{k=-\infty}^{n-4}\alpha_k\right)\times$$
$$\prod_{k=0}^{3}\left[S_{4+k}(t-nT)+j\alpha_{n-k}S_k(t-nT)\right],$$
$$t\in[nT,(n+1)T).$$

This equation is still only valid for a finite time interval. The right side of this equation consists of $2^4=16$ terms, but only 8 of them contain α_n. These 8 functions in time are distinct. They form the pulses of our PAM representation of GMSK ($h_k(t)$) and are called the signal components [2]. The others ($c_k(t)$) are only time-shifted versions of these signal components. The following list shows the structure of $h_k(t)$ and $c_k(t)$ and their duration. It can be seen, that $c_0(t), c_1(t), c_3(t)$, and $c_7(t)$ are equal $h_0(t)$ just shifted by $T, 2T, 3T$, and $4T$, respectively. $c_2(t)$ and $c_5(t)$ are related with $h_1(t)$ in the same manner, $c_4(t)$ with $h_2(t)$, and $c_6(t)$ with $h_3(t)$.

$$\begin{aligned}
h_0(t)&=S_0(t)S_1(t)S_2(t)S_3(t) && \ldots\ 5T\\
h_1(t)&=S_0(t)S_5(t)S_2(t)S_3(t) && \ldots\ 3T\\
h_2(t)&=S_0(t)S_1(t)S_6(t)S_3(t) && \ldots\ 2T\\
h_3(t)&=S_0(t)S_5(t)S_6(t)S_3(t) && \ldots\ 2T\\
h_4(t)&=S_0(t)S_1(t)S_2(t)S_7(t) && \ldots\ T\\
h_5(t)&=S_0(t)S_5(t)S_2(t)S_7(t) && \ldots\ T\\
h_6(t)&=S_0(t)S_1(t)S_6(t)S_7(t) && \ldots\ T\\
h_7(t)&=S_0(t)S_5(t)S_6(t)S_7(t) && \ldots\ T\\
c_0(t)&=S_4(t)S_1(t)S_2(t)S_3(t) && \ldots\ 5T\\
c_1(t)&=S_4(t)S_5(t)S_2(t)S_3(t) && \ldots\ 5T\\
c_2(t)&=S_4(t)S_1(t)S_6(t)S_3(t) && \ldots\ 3T\\
c_3(t)&=S_4(t)S_5(t)S_6(t)S_3(t) && \ldots\ 5T\\
c_4(t)&=S_4(t)S_1(t)S_2(t)S_7(t) && \ldots\ 2T\\
c_5(t)&=S_4(t)S_5(t)S_2(t)S_7(t) && \ldots\ 3T\\
c_6(t)&=S_4(t)S_1(t)S_6(t)S_7(t) && \ldots\ 2T\\
c_7(t)&=S_4(t)S_5(t)S_6(t)S_7(t) && \ldots\ 5T
\end{aligned}$$

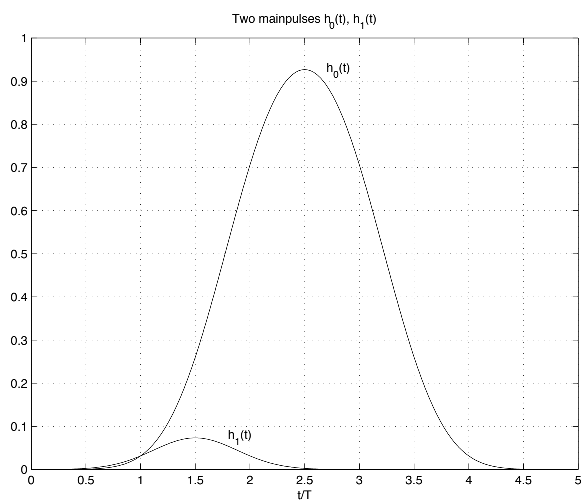

Figure 2. Two main pulses $h_0(t)$ and $h_1(t)$

Figures 2 and 3 show the pulses. Note that it seems that $h_2(t)$ and $h_3(t)$ have a longer non-zero duration than estimated (see Figure 3). That is correct, because we made the approximation, that $\psi(t)=0$ for $t<0$, and the pulses were plotted using the accurate representation of $\psi(t)$.

We observe that $h_0(t)$ has a duration of $5T$, hence it gives contributions to the next four time intervals, while $h_1(t)$ influences the next two intervals, and $h_2(t)$ and $h_3(t)$ the following one. That's why we get eight time-shifted versions of our signal components, which are the remainder of the time intervals before. With that knowledge we can generalize our representation of the CPM signal for all $t\in(-\infty,+\infty)$ by cancelling the eight pulses which do not take into account α_n from each time interval $[nT,(n+1)T)$, because they have already been included in the time intervals before, and then sum over all time intervals. Thus we have

$$s(t)=\sum_{n=-\infty}^{\infty}\sum_{k=0}^{7}\exp\left(j\frac{\pi}{2}A_{k,n}\right)h_k(t-nT)$$

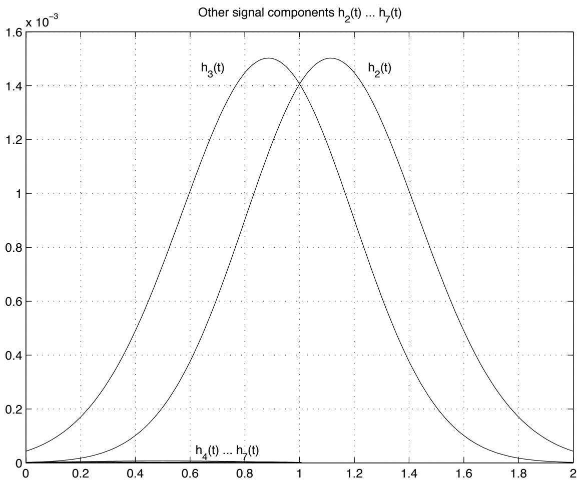

Figure 3. The other pulses $h_2(t), ..., h_7(t)$

with

$$
\begin{aligned}
A_{0,n} &= \textstyle\sum_{m=-\infty}^{n} \alpha_m \\
A_{1,n} &= \alpha_n + \textstyle\sum_{m=-\infty}^{n-2} \alpha_m \\
A_{2,n} &= \alpha_n + \alpha_{n-1} + \textstyle\sum_{m=-\infty}^{n-3} \alpha_m \\
A_{3,n} &= \alpha_n + \textstyle\sum_{m=-\infty}^{n-3} \alpha_m \\
A_{4,n} &= \alpha_n + \alpha_{n-1} + \alpha_{n-2} + \textstyle\sum_{m=-\infty}^{n-4} \alpha_m \\
A_{5,n} &= \alpha_n + \alpha_{n-2} + \textstyle\sum_{m=-\infty}^{n-4} \alpha_m \\
A_{6,n} &= \alpha_n + \alpha_{n-1} + \textstyle\sum_{m=-\infty}^{n-4} \alpha_m \\
A_{7,n} &= \alpha_n + \textstyle\sum_{m=-\infty}^{n-4} \alpha_m
\end{aligned}
$$

which can easily be proven.

3. APPROXIMATION WITH ONE PAM

If we compare Figure 2 with Figure 3, then we see that we can neglect the pulses $h_2(t), ..., h_7(t)$ because they are insignificant [5]. However, also $h_1(t)$ is negligible because its energy is less than 0.5% of signal energy. Thus the only signal component left is $h_0(t)$, which includes more than 99.5% of signal energy.

$$s(t) \approx \sum_{n=-\infty}^{\infty} a_n h_0(t - nT)$$

with

$$a_n = \exp\left(j\frac{\pi}{2} A_{0,n}\right) = \exp\left(j\frac{\pi}{2} \sum_{m=-\infty}^{n} \alpha_m\right)$$

$$= j\alpha_n \exp\left(j\frac{\pi}{2} \sum_{m=-\infty}^{n-1} \alpha_m\right) = j\alpha_n a_{n-1}$$

Figure 4 depicts the power spectrum of $s(t)$ and its components $h_0(t), h_1(t), h_2(t)$, and $h_3(t)$. We see again, that $h_0(t)$ is a proper approximation of $s(t)$. Note that if we consider the approximation error as additive noise the maximum SNR can't exceed 24dB even for noiseless channels.

4. DEROTATION OF QAM TO GET TWO DIVERSE CHANNELS

SOS and HOS based SIMO blind equalization algorithms need at least two diverse channels. One possibility is the usage of more than one physical antenna. But this means increasing costs. Another way is to sample with a higher rate, but since GMSK has little excess bandwidth, oversampling won't lead to good channel diversity. We use the cheapest choice, we take advantage of the

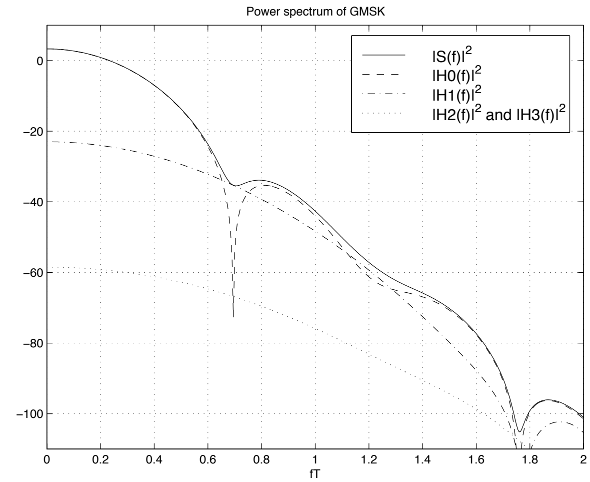

Figure 4. Power spectrum of $s(t)$

signal structure. The received GMSK signal is approximately

$$x(t) = \sum_{n=-\infty}^{\infty} a_n h(t - nT) + w(t)$$

with

$$h(t) = h_c(t) * h_0(t)$$

where $w(t)$ is the channel noise and $h_c(t)$ is the physical channel impulse response. For the baud-rate-sampled and therefore discrete received signal we get the following expression:

$$x_i = \sum_{n=-\infty}^{\infty} a_n h_{i-n} + w_i.$$

Remember that we modulated $h_0(t - nT)$ with the sequence

$$a_n = j\alpha_n a_{n-1}.$$

Let us introduce the derotated sequence [1]

$$\tilde{a}_n = j^{-n} a_n = \alpha_n \tilde{a}_{n-1}.$$

If we assume, without loss of generality, that $\tilde{a}_{n-1} \in \{-1, +1\}$, then also $\tilde{a}_n \in \{-1, +1\}$.

Now we insert our new sequence in the approximation of the received and sampled GMSK signal. This leads to

$$x_i = \sum_{n=-\infty}^{\infty} h_n j^{i-n} \tilde{a}_{i-n} + w_i$$

Similarly to $\tilde{a}_n$ we define a derotated received signal

$$\tilde{x}_i = j^{-i} x_i = \sum_{n=-\infty}^{\infty} h_n j^{-n} \tilde{a}_{i-n} + j^{-i} w_i.$$

As we stated before is $\tilde{a}_n \in \{-1, +1\}$. So $\tilde{a}_n$ is a real sequence, thus we can get two diverse channels just by splitting up $\tilde{x}_i$ in real- and imaginary-parts.

$$x_{1,i} = \Re\{\tilde{x}_i\} = \sum_{n=-\infty}^{\infty} \Re\{j^{-n} h_n\} \tilde{a}_{i-n} + \Re\{j^{-i} w_i\}$$

$$x_{2,i} = \Im\{\tilde{x}_i\} = \sum_{n=-\infty}^{\infty} \Im\{j^{-n} h_n\} \tilde{a}_{i-n} + \Im\{j^{-i} w_i\}$$

We get two different channels driven by the same BPSK sequence $\tilde{a}_n$.

$$h_{1,i} = \Re\{j^{-n}h_n\} \quad h_{2,i} = \Im\{j^{-n}h_n\}$$

So we can apply algorithms based on SIMO models, without using extra antennas or oversampling.

Many SOS methods require the non-existence of common zeros of the z-transforms of $h_{1,i}$ and $h_{2,i}$. In [1] it is shown, that $h_{1,i}$ and $h_{2,i}$ only share common zero(s), when the transfer function

$$H(z) = \sum_{n=0}^{L} h_n z^{-n}$$

has some zero(s) symmetric to the imaginary axis (or one or more on the imaginary axis).

5. CHANNEL ESTIMATION

After we have achieved a linear approximation for the GMSK signal, we can exploit this linear representation for blind channel estimation. The received signal after matched filtering at the i-th antenna, $i = 1, \ldots, M$, may be expressed as

$$y_i(t) = \sum_{m=0}^{N_b-1} a_m h_i(t - mT_0) + n_i(t), \quad i = 1, \ldots, M,$$

where a_m is the cumulative bit-sequence ($a_m = j\alpha_m a_{m-1}$), N_b is the number of transmitted bits, and $n_i(t)$ is the colored noise (after matched filtering). $h_i(t)$ is the "channel" impulse response, which is the convolution of the RF-channel impulse response $h_{RF_i}(t)$ and the autocorrelation $\varphi_{h_0}(t)$ of $h_0(t)$ since we use the suboptimal demodulator with the matched filter $h_0(-t)$.

$$h_i(t) = \varphi_{h_0}(t) * h_{RF_i}(t),$$

where "$*$" denotes the linear convolution. The RF multipath propagation channel $h_{RF_i}(t)$ is approximated by a sum of dominant specular paths, therefore

$$h_{RF_i}(t) = \sum_{k=1}^{p} g_{ik}\delta(t - \tau_k),$$

where p is the number of multipaths, g_{ik} and τ_k are the complex gain and delay, respectively, and $\delta(t)$ denotes the Dirac delta function.

6. CONCLUSION

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