# CHAPTER 3
# Modal theory
# in cylindrical step index fiber

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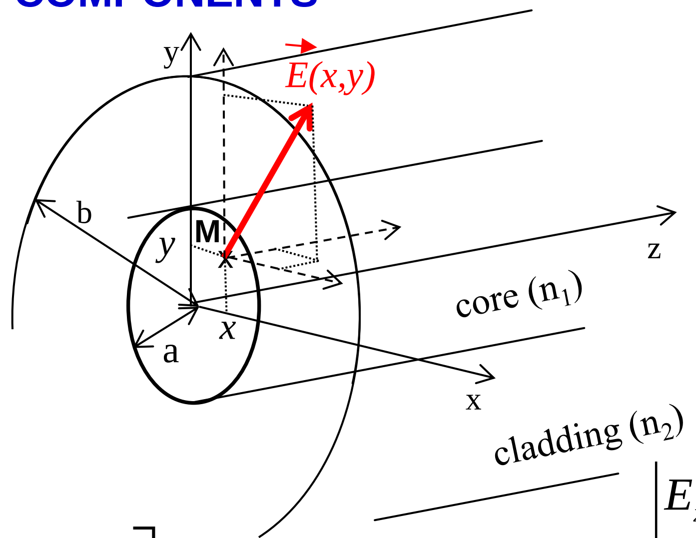
## CALCULATION OF THE FIELDS COMPONENTS




$$\vec{\mathcal{E}}(x,y,z) = \Re e\left[\vec{E}(x,y).e^{j(\omega t-\beta z)}\right]$$

**with** $$\vec{E}(x,y) = \left| \begin{array}{l} E_x(x,y).\vec{ex} \\ E_y(x,y).\vec{ey} \\ E_z(x,y).\vec{ez} \end{array} \right.$$

**→ we develop Maxwell equations *(pdf page 1)* :**

$$curl\vec{\mathcal{E}} = -\frac{\partial \vec{\mathcal{B}}}{\partial t} \qquad curl\vec{\mathcal{H}} = \varepsilon\frac{\partial \vec{\mathcal{E}}}{\partial t} \qquad div(\vec{\mathcal{D}}) = div(\varepsilon\vec{\mathcal{E}}) = \rho = 0$$

with $\frac{\partial}{\partial z}(\text{component}) = -j\beta.(\text{component})$ and $\frac{\partial}{\partial t}(\text{component}) = j\omega.(\text{component})$

## CALCULATION OF THE FIELDS COMPONENTS

$$\begin{cases} \dfrac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu H_x & (3) \\ -j\beta E_x - \dfrac{\partial E_z}{\partial x} = -j\omega\mu H_y & (4) \\ \dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = -j\omega\mu H_z & (5) \end{cases}$$

$$\begin{cases} \dfrac{\partial H_z}{\partial y} + j\beta H_y = +j\omega\varepsilon E_x & (6) \\ -j\beta H_x - \dfrac{\partial H_z}{\partial x} = +j\omega\varepsilon E_y & (7) \\ \dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = +j\omega\varepsilon E_z & (8) \end{cases}$$

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$$\left|\begin{array}{ll} \dfrac{\partial H_z}{\partial y} + j\beta H_y = +j\omega\varepsilon E_x & (6) \\ -j\beta H_x - \dfrac{\partial H_z}{\partial x} = +j\omega\varepsilon E_y & (7) \\ \dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = +j\omega\varepsilon E_z & (8) \end{array}\right.$$

**→ we write the transverse components Ex, Ey, Hx, et Hy versus the axial components Ez et Hz *(pdf page 2)* :**

$$E_x = \frac{-j}{\beta_t^2}\left[\beta\frac{\partial E_z}{\partial x} + \omega\mu\frac{\partial H_z}{\partial y}\right] \qquad H_x = \frac{-j}{\beta_t^2}\left[\beta\frac{\partial H_z}{\partial x} - \omega\varepsilon\frac{\partial E_z}{\partial y}\right]$$

$$E_y = \frac{-j}{\beta_t^2}\left[\beta\frac{\partial E_z}{\partial y} - \omega\mu\frac{\partial H_z}{\partial x}\right] \qquad H_y = \frac{-j}{\beta_t^2}\left[\beta\frac{\partial H_z}{\partial y} + \omega\varepsilon\frac{\partial E_z}{\partial x}\right]$$

**avec**

$$\beta_t^2 = k_0^2 n_i^2 - \beta^2$$

**et**

$$k_0 n_2 \le \beta \le k_0 n_1$$

## CALCULATION OF THE FIELDS COMPONENTS

→ **we can deduce, from the previous expressions, an equation which unknown factor is Ez**
→ **Helmoltz equation *(pdf page 3)* :**

$$\frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}+\beta_t^2 E_z=0$$

**(idem with Hz)**

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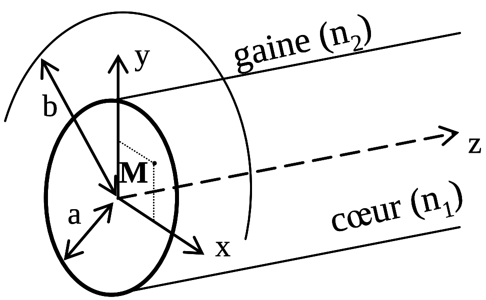
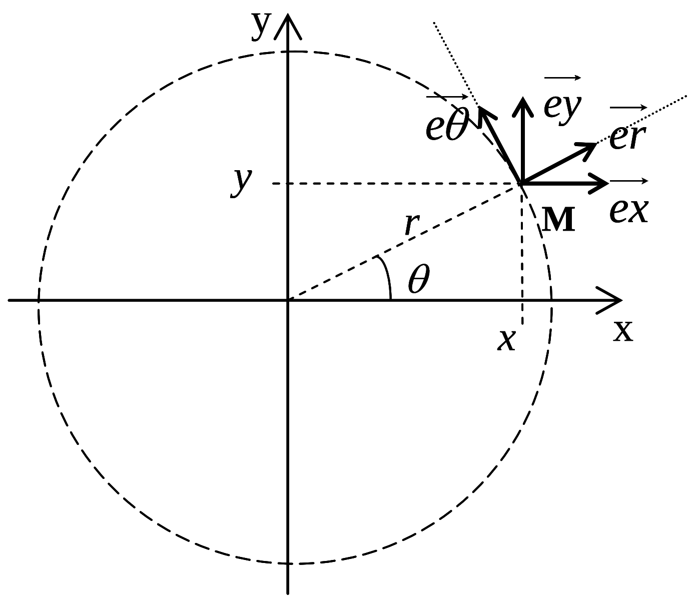
→ **fibre = cylindrical guide ==> change of coordinate system and et change of basis *(pdf page 3)* :**

$$(\vec{e_x}, \vec{e_y}, \vec{e_z}) \rightarrow (\vec{e_r}, \vec{e_\theta}, \vec{e_z})$$



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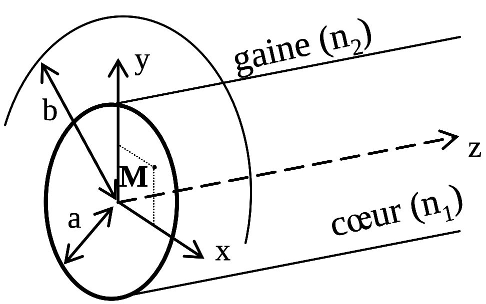


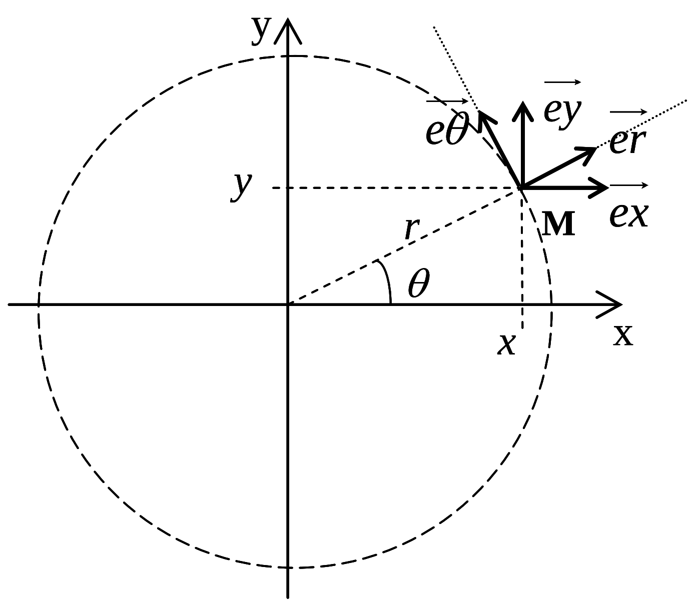


$$\vec{E}(x,y) = \left| \begin{array}{l} E_x(x,y).\overrightarrow{ex} \\ E_y(x,y).\overrightarrow{ey} \\ E_z(x,y).\overrightarrow{ez} \end{array} \right.$$

$$\vec{E}(r,\theta) = \left| \begin{array}{l} E_r(r,\theta).\overrightarrow{er} \\ E_\theta(r,\theta).\overrightarrow{e\theta} \\ E_z(r,\theta).\overrightarrow{ez} \end{array} \right.$$

## CALCULATION OF THE FIELDS COMPONENTS

→ **after some calculations, we find *(pdf page 4)* :**

$$E_r = -\frac{j}{\beta_t^2}\left(\beta\frac{\partial E_z}{\partial r} + \frac{\omega\mu}{r}\frac{\partial H_z}{\partial \theta}\right) \qquad H_r = -\frac{j}{\beta_t^2}\left(\beta\frac{\partial H_z}{\partial r} - \frac{\omega\varepsilon}{r}\frac{\partial E_z}{\partial \theta}\right)$$

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→ **and then we can deduce the Helmoltz equation *(pdf page 5)* :**

$$\boxed{\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E_z}{\partial\theta^2} + \beta_t^2 E_z = 0}$$

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→ **this equation can be solved by the method of separation of variables *(pdf page 5)* :**

$$E_z(r,\theta) = R_z(r).T_z(\theta)$$

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$$\underbrace{\frac{r^2}{R}\frac{\partial^2 R}{\partial r^2} + \frac{r}{R}\frac{\partial R}{\partial r} + r^2\beta_t^2}_{\mathbf{f(r)}} + \underbrace{\frac{1}{T}\frac{\partial^2 T}{\partial \theta^2}}_{\mathbf{g(\theta)}} = 0$$

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→ **this equation can be solved by the method of separation of variables** ***(pdf page 5)*** **:**

$$E_z(r,\theta) = R_z(r).T_z(\theta)$$

$$\underbrace{\frac{r^2}{R}\frac{\partial^2 R}{\partial r^2} + \frac{r}{R}\frac{\partial R}{\partial r} + r^2\beta_t^2}_{f(r)=+\nu^2} + \underbrace{\frac{1}{T}\frac{\partial^2 T}{\partial \theta^2}}_{g(\theta)=-\nu^2} = 0$$

## CALCULATION OF THE FIELDS COMPONENTS

$\boxed{g(\theta) = -\nu^2}$ ➔ $$\frac{1}{T}\frac{\partial^2 T}{\partial \theta^2} = -\nu^2$$ ➔ $$\frac{\partial^2 T}{\partial \theta^2} + \nu^2 T = 0$$ *(pdf page 6)*

if $\nu \neq 0$ $$\boxed{T(\theta) = \begin{cases} \cos(\nu\theta + \varphi_0) \\ \sin(\nu\theta + \varphi_0) \end{cases}}$$ **ν integer**

if $\nu = 0$ $$\boxed{T(\theta) = \text{constant}}$$

## CALCULATION OF THE FIELDS COMPONENTS

$\boxed{g(\theta) = -\nu^2}$ ➔ $$\frac{1}{T}\frac{\partial^2 T}{\partial \theta^2} = -\nu^2 \quad ➔ \quad \frac{\partial^2 T}{\partial \theta^2} + \nu^2 T = 0$$ *(pdf page 6)*

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$\boxed{f(r) = +\nu^2}$ ➔ $$\frac{\partial^2 R}{\partial r^2} + \frac{1}{R}\frac{\partial R}{\partial r} + \left(\beta_t^2 - \frac{\nu^2}{r^2}\right) R = 0$$ **Bessel equation**

$$R(r) = \begin{cases} AJ_\nu(\beta_t r) + A' N_\nu(\beta_t r) & \text{si } \beta_t \text{ is real} \quad -->\text{in the core} \\ CK_\nu(|\beta_t| r) + C' I_\nu(|\beta_t| r) & \text{si } \beta_t \text{ is imaginary} -->\text{i the cladding} \end{cases}$$

→ **taking into account the values of the functions for r = 0 and r = ∞ :**

$$\boxed{R(r) = \begin{cases} AJ_\nu(\beta_t r) & \text{if } \beta_t \text{ is real} \quad -->\text{in the core} \\ CK_\nu(|\beta_t| r) & \text{if } \beta_t \text{ is imaginary} -->\text{in the cladding} \end{cases}}$$

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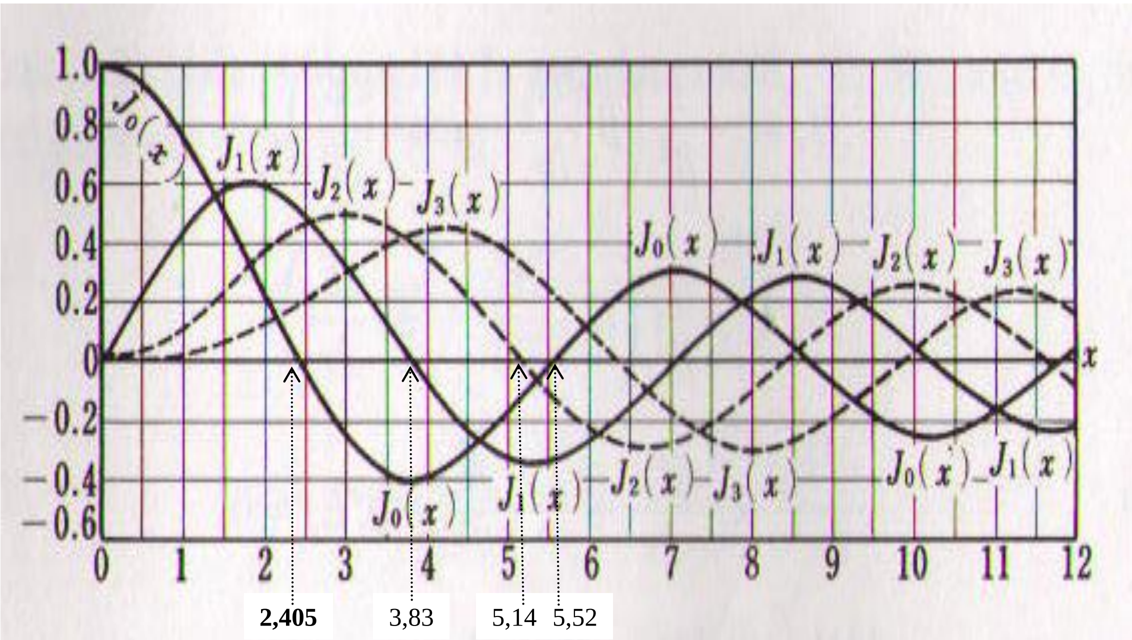


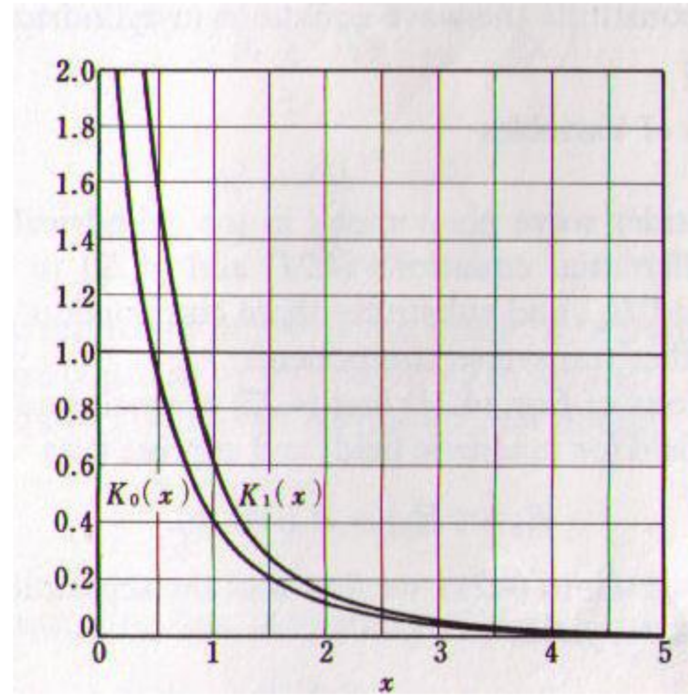


**First orders ν of the Bessel functions of first kind $J_\nu$ (ν = 0, 1, 2, 3)**

**First orders ν of the modified Bessel functions of first kind $K_\nu$ (ν = 0, 1)**

## CALCULATION OF THE FIELDS COMPONENTS

→ ***One can specify the expression of the axial components of the fields, $E_z$ and $H_z$ (pdf page 8)***

$$\begin{cases} E_z = AJ_\nu(\beta_{t1} r).\sin(\nu\theta) \quad --> \text{in the core} \\ \quad = CK_\nu(|\beta_{t2}| r).\sin(\nu\theta) --> \text{in the cladding} \end{cases}$$

$$\begin{cases} H_z = BJ_\nu(\beta_{t1} r).\cos\nu\theta \quad --> \text{in the core} \\ \quad = DK_\nu(|\beta_{t2}| r).\cos\nu\theta --> \text{in the cladding} \end{cases}$$

**<u>if ν = 0</u>: modes TE (Ez=0) or TM (Hz=0)**

**<u>if ν ≠ 0</u>: modes EH if *Hz* > *Ez***
**modes HE ifi *Ez* > *Hz* .**

→ ***We can now introduce the important following quantities: <u>u,w and V</u> (pdf page 9)***

**normalized transverse propagation constants**

**<u>In the core</u> (radius = a) :** $k = k_0 n_1$ **thus** $\beta_t = \beta_{t1} = \sqrt{k_0^2 n_1^2 - \beta^2}$ → $u = a\beta_{t1} = a\sqrt{k_0^2 n_1^2 - \beta^2}$

**<u>In the cladding</u> :** $k = k_0 n_2$ **thus** $|\beta_t| = |\beta_{t2}| = \sqrt{\beta^2 - k_0^2 n_2^2}$ → $w = a|\beta_{t2}| = a\sqrt{\beta^2 - k_0^2 n_2^2}$

$$\underline{u^2 + w^2} = a^2 k_0^2 (n_1^2 - n_2^2) = \left( \frac{2\pi}{\lambda_0} a\sqrt{n_1^2 - n_2^2} \right)^2 = \underline{V^2} \rightarrow \boxed{V = \frac{2\pi}{\lambda_0} a\sqrt{n_1^2 - n_2^2}}$$

**V : normalized spatial frequency**

## CALCULATION OF THE FIELDS COMPONENTS

→ ***Finally (pdf page 10)***

**if $\nu \neq 0$**

**In the core**

$$E_z = AJ_\nu\left(\frac{ur}{a}\right)\sin(\nu\theta)$$

$$E_r = \left[-A\frac{j\beta}{(u/a)}J_\nu^{'}\left(\frac{ur}{a}\right)+B\frac{j\omega\mu_0}{(u/a)^2}\frac{\nu}{r}J_\nu\left(\frac{ur}{a}\right)\right]\sin(\nu\theta)$$

$$E_\theta = \left[-A\frac{j\beta}{(u/a)^2}\frac{\nu}{r}J_\nu\left(\frac{ur}{a}\right)+B\frac{j\omega\mu_0}{(u/a)}J_\nu^{'}\left(\frac{ur}{a}\right)\right]\cos(\nu\theta)$$

$$H_z = BJ_\nu\left(\frac{ur}{a}\right)\cos(\nu\theta)$$

$$H_r = \left[A\frac{j\omega\varepsilon_1}{(u/a)^2}\frac{\nu}{r}J_\nu\left(\frac{ur}{a}\right)-B\frac{j\beta}{(u/a)}J_\nu^{'}\left(\frac{ur}{a}\right)\right]\cos(\nu\theta)$$

$$H_\theta = \left[-A\frac{j\omega\varepsilon_1}{(u/a)}J_\nu^{'}\left(\frac{ur}{a}\right)+B\frac{j\beta}{(u/a)^2}\frac{\nu}{r}J_\nu\left(\frac{ur}{a}\right)\right]\sin(\nu\theta)$$

**In the cladding**

$$E_z = CK_\nu\left(\frac{wr}{a}\right)\sin(\nu\theta)$$

$$E_r = \left[C\frac{j\beta}{(w/a)}K_\nu^{'}\left(\frac{wr}{a}\right)-D\frac{j\omega\mu_0}{(w/a)^2}\frac{\nu}{r}K_\nu\left(\frac{wr}{a}\right)\right]\sin(\nu\theta)$$

$$E_\theta = \left[C\frac{j\beta}{(w/a)^2}\frac{\nu}{r}K_\nu\left(\frac{wr}{a}\right)-D\frac{j\omega\mu_0}{(w/a)}K_\nu^{'}\left(\frac{wr}{a}\right)\right]\cos(\nu\theta)$$

$$H_z = DK_\nu\left(\frac{wr}{a}\right)\cos(\nu\theta)$$

$$H_r = \left[-C\frac{j\omega\varepsilon_2}{(w/a)^2}\frac{\nu}{r}K_\nu\left(\frac{wr}{a}\right)+D\frac{j\beta}{(w/a)}K_\nu^{'}\left(\frac{wr}{a}\right)\right]\cos(\nu\theta)$$

$$H_\theta = \left[C\frac{j\varepsilon_2}{(w/a)}K_\nu^{'}\left(\frac{wr}{a}\right)-D\frac{j\beta}{(w/a)^2}\frac{\nu}{r}K_\nu\left(\frac{wr}{a}\right)\right]\sin(\nu\theta)$$

## CALCULATION OF THE FIELDS COMPONENTS

→ *… and (pdf page 11)*

### if ν = 0

**In the core** | **In the cladding**

**TE modes : $E_z$=0 , $H_\theta$=0**

In the core:

$$E_\theta = B\frac{j\omega\mu_0}{(u/a)}J_0'\left(\frac{ur}{a}\right)$$

$$H_z = BJ_0\left(\frac{ur}{a}\right)$$

$$H_r = -B\frac{j\beta}{(u/a)}J_0'\left(\frac{ur}{a}\right)$$

In the cladding:

$$E_\theta = -D\frac{j\omega\mu_0}{(w/a)}K_0'\left(\frac{wr}{a}\right)$$

$$H_z = DK_0\left(\frac{wr}{a}\right)$$

$$H_r = D\frac{j\beta}{(w/a)}K_0'\left(\frac{wr}{a}\right)$$

**TM modes : $H_z$=0 , $E_\theta$=0**

In the core:

$$E_r = -A\frac{j\beta}{(u/a)}J_0'\left(\frac{ur}{a}\right)$$

$$E_z = AJ_0\left(\frac{ur}{a}\right)$$

$$H_\theta = -A\frac{j\omega\varepsilon_1}{(u/a)}J_0'\left(\frac{ur}{a}\right)$$

In the cladding:

$$E_r = C\frac{j\beta}{(w/a)}K_0'\left(\frac{wr}{a}\right)$$

$$E_z = CK_0\left(\frac{wr}{a}\right)$$

$$H_\theta = C\frac{j\varepsilon_2}{(w/a)}K_0'\left(\frac{wr}{a}\right)$$

## MODES CLASSIFICATION - DISPERSION CURVES

- ***continuity of the tangential components of E and H at the core-cladding interface (pdf page 12)***

$$\left|\begin{array}{l} E_z^{coeur}(\mathrm{r{=}a}) = E_z^{gaine}(\mathrm{r{=}a}) \\ H_z^{coeur}(\mathrm{r{=}a}) = H_z^{gaine}(\mathrm{r{=}a}) \\ E_\theta^{coeur}(\mathrm{r{=}a}) = E_\theta^{gaine}(\mathrm{r{=}a}) \\ H_\theta^{coeur}(\mathrm{r{=}a}) = H_\theta^{gaine}(\mathrm{r{=}a}) \end{array}\right.$$

## MODES CLASSIFICATION - DISPERSION CURVES

➢ ***continuity of the tangential components of E and H at the core-cladding interface (pdf page 12)***

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➢ ***dispersion equation (pdf page 12)***

$$\left|\begin{array}{l} \underbrace{\left[\frac{J_\nu'(u)}{uJ_\nu(u)} + \frac{K_\nu'(w)}{wK_\nu(w)}\right]}_{F1}\underbrace{\left[\frac{\varepsilon_1 J_\nu'(u)}{\varepsilon_2 uJ_\nu(u)} + \frac{K_\nu'(w)}{wK_\nu(w)}\right]}_{F2} = \nu^2\underbrace{\left(\frac{1}{u^2}+\frac{1}{w^2}\right)}_{F3}\underbrace{\left(\frac{\varepsilon_1}{\varepsilon_2}\frac{1}{u^2}+\frac{1}{w^2}\right)}_{F4} \\ \text{with}\quad u^2 + w^2 = V^2 = k_0^2 a^2\left(n_1^2 - n_2^2\right) = \text{cte (determined by the fiber and the working wavelength)} \end{array}\right.$$

**V varies => β changes accordingly → β =f(V) : dispersion curve of the considered mode**

## MODES CLASSIFICATION - DISPERSION CURVES

$$\underbrace{\left[\frac{J_\nu^{'}(u)}{uJ_\nu(u)}+\frac{K_\nu^{'}(w)}{wK_\nu(w)}\right]}_{F1}\underbrace{\left[\frac{\varepsilon_1 J_\nu^{'}(u)}{\varepsilon_2 uJ_\nu(u)}+\frac{K_\nu^{'}(w)}{wK_\nu(w)}\right]}_{F2}=\nu^2\underbrace{\left(\frac{1}{u^2}+\frac{1}{w^2}\right)}_{F3}\underbrace{\left(\frac{\varepsilon_1}{\varepsilon_2}\frac{1}{u^2}+\frac{1}{w^2}\right)}_{F4}$$

**if $\nu = 0$ → F1 = 0 : dispersion equation of TE modes**
**→ or F2 = 0 : dispersion equation of TM modes**

**if $\nu \neq 0$ → the entire equation must be solved : HE and EH modes**

## MODES CLASSIFICATION - DISPERSION CURVES

$$\underbrace{\left[\frac{J_\nu'(u)}{uJ_\nu(u)}+\frac{K_\nu'(w)}{wK_\nu(w)}\right]}_{F1}\underbrace{\left[\frac{\varepsilon_1 J_\nu'(u)}{\varepsilon_2 uJ_\nu(u)}+\frac{K_\nu'(w)}{wK_\nu(w)}\right]}_{F2}=\nu^2\underbrace{\left(\frac{1}{u^2}+\frac{1}{w^2}\right)}_{F3}\underbrace{\left(\frac{\varepsilon_1}{\varepsilon_2}\frac{1}{u^2}+\frac{1}{w^2}\right)}_{F4}$$

**if $\nu = 0$ → F1 = 0 : dispersion equation of TE modes**
**→ or F2 = 0 : dispersion equation of TM modes**

**if $\nu \neq 0$ → the entire equation must be solved : HE and EH modes**

- **designation of the electromagnetic modes *(pdf page 13)***

**$\nu = 0$ : $TE_{0l}$ and $TM_{0l}$**

**$\nu \neq 0$ : $EH_{\nu l}$ and $HE_{\nu l}$**

## MODES CLASSIFICATION - DISPERSION CURVES

- **weak guidance approximation *(pdf page 13)***

$$\Delta = \left(\frac{n_1^2 - n_2^2}{2n_1^2}\right) = \frac{ON^2}{2n_1^2}$$

**$\Delta$ : relative index difference**

**if $n_1$ close to $n_2$ such that $\Delta < 10^{-2}$ :**
**propagation in WEAK GUIDANCE conditions**

## MODES CLASSIFICATION - DISPERSION CURVES

➢ **weak guidance approximation *(pdf page 13)***

$$\boxed{\Delta = \left( \frac{n_1^2 - n_2^2}{2n_1^2} \right) = \frac{ON^2}{2n_1^2}}$$

**Δ : relative index difference**

**if $n_1$ is close to $n_2$ such that $\Delta < 10^{-2}$ :**
**propagation in WEAK GUIDANCE conditions**

**Thus, the dispersion equation becomes :** $F_1^2 = \nu^2 F_3^2 \Leftrightarrow F_1 = \pm \nu \, F_3$ ***(pdf page 14)***

**resulting in :**

$$\boxed{\frac{J_\nu^{'}(u)}{uJ_\nu(u)} + \frac{K_\nu^{'}(w)}{wK_\nu(w)} = \pm \nu \left( \frac{1}{u^2} + \frac{1}{w^2} \right)}$$

→ **if $\nu = 0$ : $TE_{0l}$ (exact) et $TM_{0l}$ (approximate)**

→ **if $\nu \neq 0$ signe + : $EH_{\nu l}$**
**signe - : $HE_{\nu l}$**

## MODES CLASSIFICATION - DISPERSION CURVES IN THE WEAK GUIDANCE APPROXIMATION

*(pdf page 15)*

→ **if $\nu = 0$ : TE or TM modes** ➔

$$u\frac{J_0(u)}{J_1(u)} = \frac{-wK_0(w)}{K_1(w)}$$

→ **if $\nu \neq 0$ signe + : EH modes** ➔

$$u\frac{J_\nu(u)}{J_{\nu+1}(u)} = \frac{-wK_\nu(w)}{K_{\nu+1}(w)}$$

→ **if $\nu \neq 0$ signe - : HE modes** ➔

$$u\frac{J_{\nu-2}(u)}{J_{\nu-1}(u)} = \frac{-wK_{\nu-2}(w)}{K_{\nu-1}(w)}$$

## MODES CLASSIFICATION - DISPERSION CURVES IN THE WEAK GUIDANCE APPROXIMATION

(pdf page 15)

→ if $\nu = 0$ : TE or TM modes → $$u\frac{J_0(u)}{J_1(u)} = \frac{-wK_0(w)}{K_1(w)}$$

→ if $\nu \neq 0$ signe + : EH modes → $$u\frac{J_\nu(u)}{J_{\nu+1}(u)} = \frac{-wK_\nu(w)}{K_{\nu+1}(w)}$$

→ if $\nu \neq 0$ signe - : HE modes → $$u\frac{J_{\nu-2}(u)}{J_{\nu-1}(u)} = \frac{-wK_{\nu-2}(w)}{K_{\nu-1}(w)}$$

→ if $\nu = 0$ : TE or TM modes — m=1

→ if $\nu \neq 0$ signe + : EH modes — m= $\nu$+1

→ if $\nu \neq 0$ signe - : HE modes — m= $\nu$-1

$$\boxed{u\frac{J_{m-1}(u)}{J_m(u)} = \frac{-wK_{m-1}(w)}{K_m(w)}}$$

**m integer $\geq 0$**

## MODES CLASSIFICATION - DISPERSION CURVES IN THE WEAK GUIDANCE APPROXIMATION

*(pdf page 15)*

→ if $\nu = 0$ : TE or TM modes → $$u\frac{J_0(u)}{J_1(u)} = \frac{-wK_0(w)}{K_1(w)}$$

→ if $\nu \neq 0$ signe + : EH modes → $$u\frac{J_\nu(u)}{J_{\nu+1}(u)} = \frac{-wK_\nu(w)}{K_{\nu+1}(w)}$$

→ if $\nu \neq 0$ signe - : HE modes → $$u\frac{J_{\nu-2}(u)}{J_{\nu-1}(u)} = \frac{-wK_{\nu-2}(w)}{K_{\nu-1}(w)}$$

→ if $\nu = 0$ : TE or TM modes — m=1

→ if $\nu \neq 0$ signe + : EH modes — m= $\nu$+1

→ if $\nu \neq 0$ signe - : HE modes — m= $\nu$-1

$$\boxed{u\frac{J_{m-1}(u)}{J_m(u)} = \frac{-wK_{m-1}(w)}{K_m(w)}}$$

**m integer ≥ 0**

$$\text{with} \quad u^2 + w^2 = V^2$$

## MODES CLASSIFICATION - DISPERSION CURVES
## IN THE WEAK GUIDANCE APPROXIMATION

***(pdf page 15)***

→ **if $\nu = 0$ : TE or TM modes** — **m=1**

→ **if $\nu \neq 0$ signe + : EH modes** — **m= $\nu$+1**, **$\nu$ = m-1**

→ **if $\nu \neq 0$ signe - : HE modes** — **m= $\nu$-1**, **$\nu$ = m+1**

$$\boxed{u\frac{J_{m-1}(u)}{J_m(u)} = \frac{-wK_{m-1}(w)}{K_m(w)}}$$

**m integer $\geq 0$**

## MODES CLASSIFICATION - DISPERSION CURVES IN THE WEAK GUIDANCE APPROXIMATION

*(pdf page 15)*

→ if $\nu = 0$ : TE or TM modes → m=1

→ if $\nu \neq 0$ signe + : EH modes → m= ν+1, $\nu = m-1$

→ if $\nu \neq 0$ signe - : HE modes → m= ν-1, $\nu = m+1$

$$\boxed{u\frac{J_{m-1}(u)}{J_m(u)} = \frac{-wK_{m-1}(w)}{K_m(w)}}$$

m integer $\geq 0$

**for a given value of m, same dispersion relationship for certain modes:**

if m=1 → $TE_{0,l}$, $TM_{0l}$ and $HE_{2,l}$ (→ degenerated modes) → $LP_{1,l}$ mode

if m>1 → $EH_{m-1,\,l}$, and $HE_{m+1,\,l}$ (→ degenerated modes) → $LP_{m,l}$ mode m>1

if m=0 → $HE_{1,l}$ mode only → $LP_{0,l}$ mode

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Université de Limoges

## LP MODES - CUTOFF FREQUENCY

- **distribution of the electric field in the LP modes** ***(pdf page 16)***

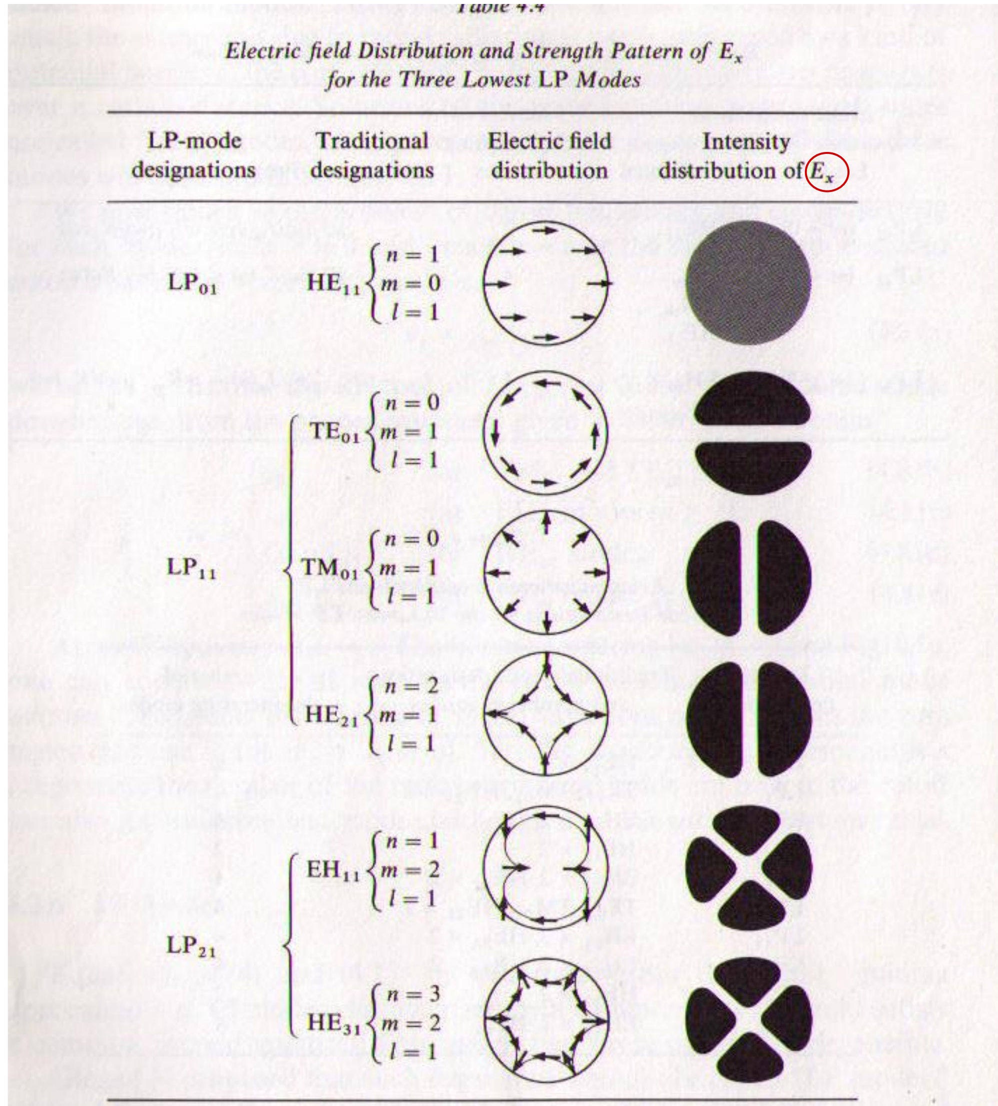


Table 4.4

*Electric field Distribution and Strength Pattern of $E_x$ for the Three Lowest LP Modes*

| LP-mode designations | Traditional designations | | Electric field distribution | Intensity distribution of $E_x$ |
|---|---|---|---|---|
| $LP_{01}$ | $HE_{11}$ | $n = 1$, $m = 0$, $l = 1$ | | |
| $LP_{11}$ | $TE_{01}$ | $n = 0$, $m = 1$, $l = 1$ | | |
| | $TM_{01}$ | $n = 0$, $m = 1$, $l = 1$ | | |
| | $HE_{21}$ | $n = 2$, $m = 1$, $l = 1$ | | |
| $LP_{21}$ | $EH_{11}$ | $n = 1$, $m = 2$, $l = 1$ | | |
| | $HE_{31}$ | $n = 3$, $m = 2$, $l = 1$ | | |

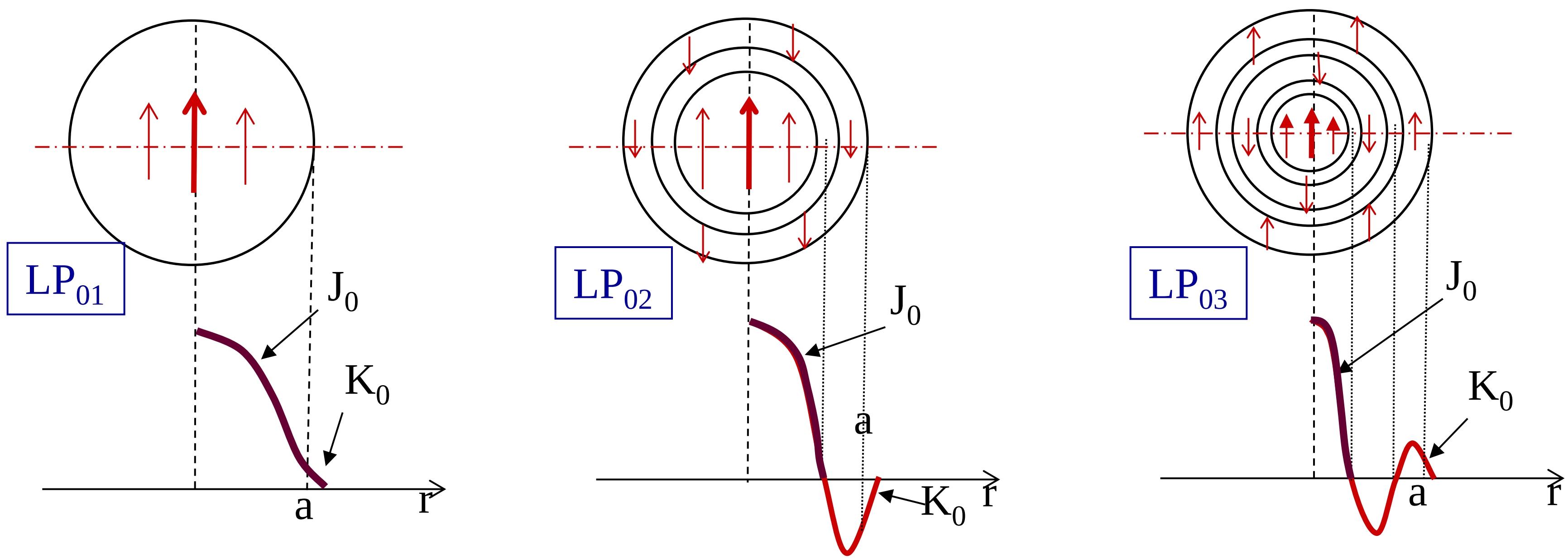
## LP MODES - CUTOFF FREQUENCY

➢ **distribution of the electric field in the LP$_{m,l}$ modes *(pdf page 16)***

→ **along a radius : following J$_m$function in the core and following K$_m$ function in the cladding**

→ **along a circle at a fixed distance from the center : following the cos (mθ+φ) er sin (mθ+φ) function**

**examples (with m=0) : → LP$_{0,l}$ modes**



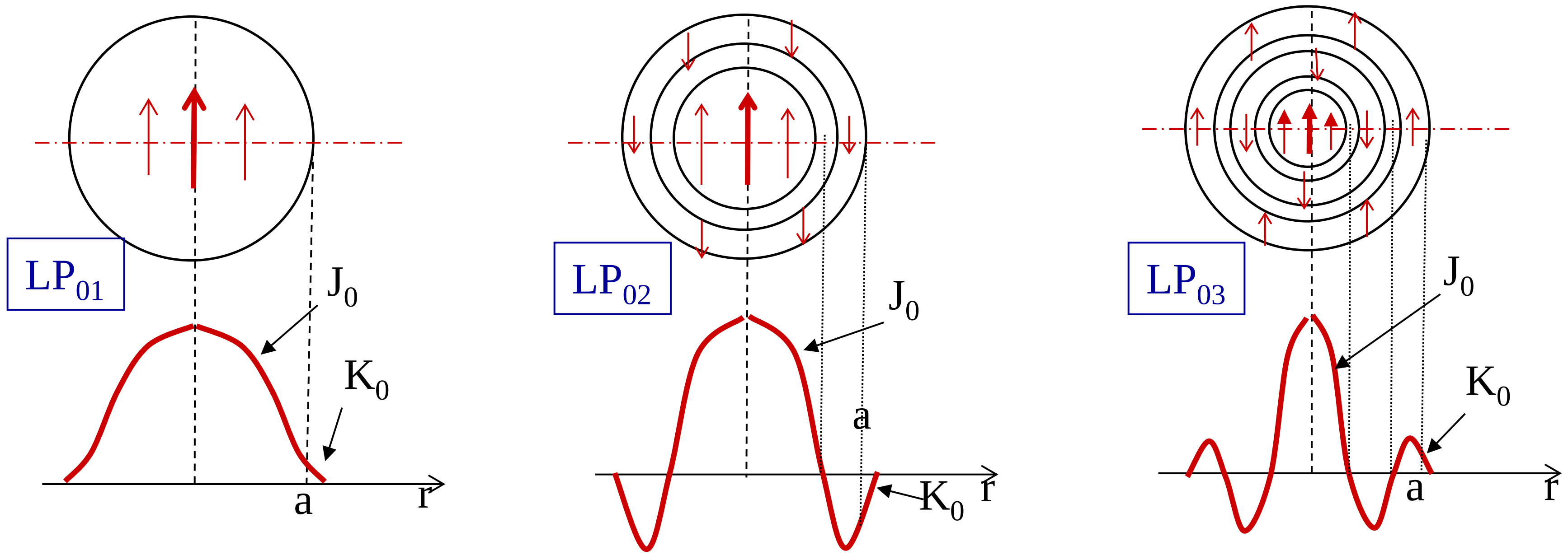
# LP MODES - CUTOFF FREQUENCY

➢ **distribution of the electric field in the $LP_{m,l}$ modes *(pdf page 16)***

→ **along a radius : following $J_m$function in the core and following $K_m$ function in the cladding**

→ **along a circle at a fixed distance from the center : following the cos (mθ+φ) er sin (mθ+φ) function**

**examples (with m=0) : → $LP_{0,l}$ modes**

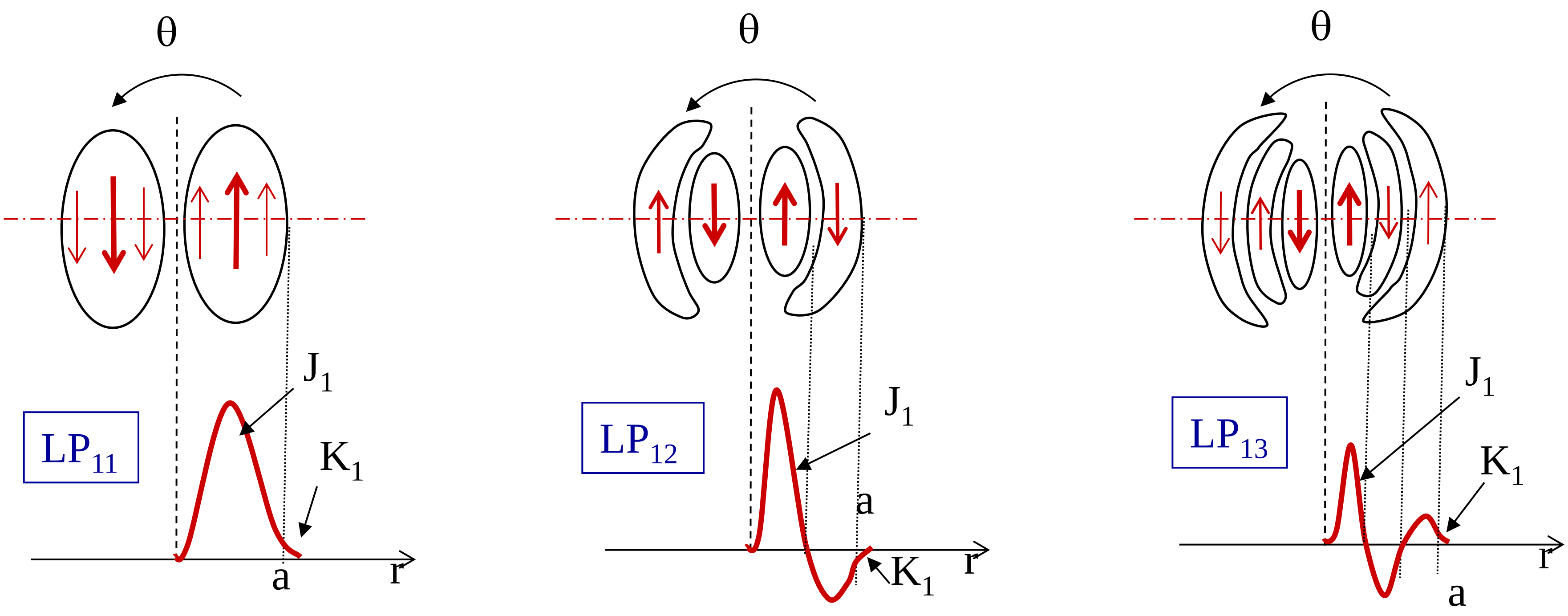



**<u>intensity distribution in</u> $LP_{0,l}$ modes → one central circular lobe surrounded by (*l*-1) rings**

# LP MODES - CUTOFF FREQUENCY

➢ **distribution of the electric field in the $LP_{m,l}$ modes** ***(pdf page 18)***

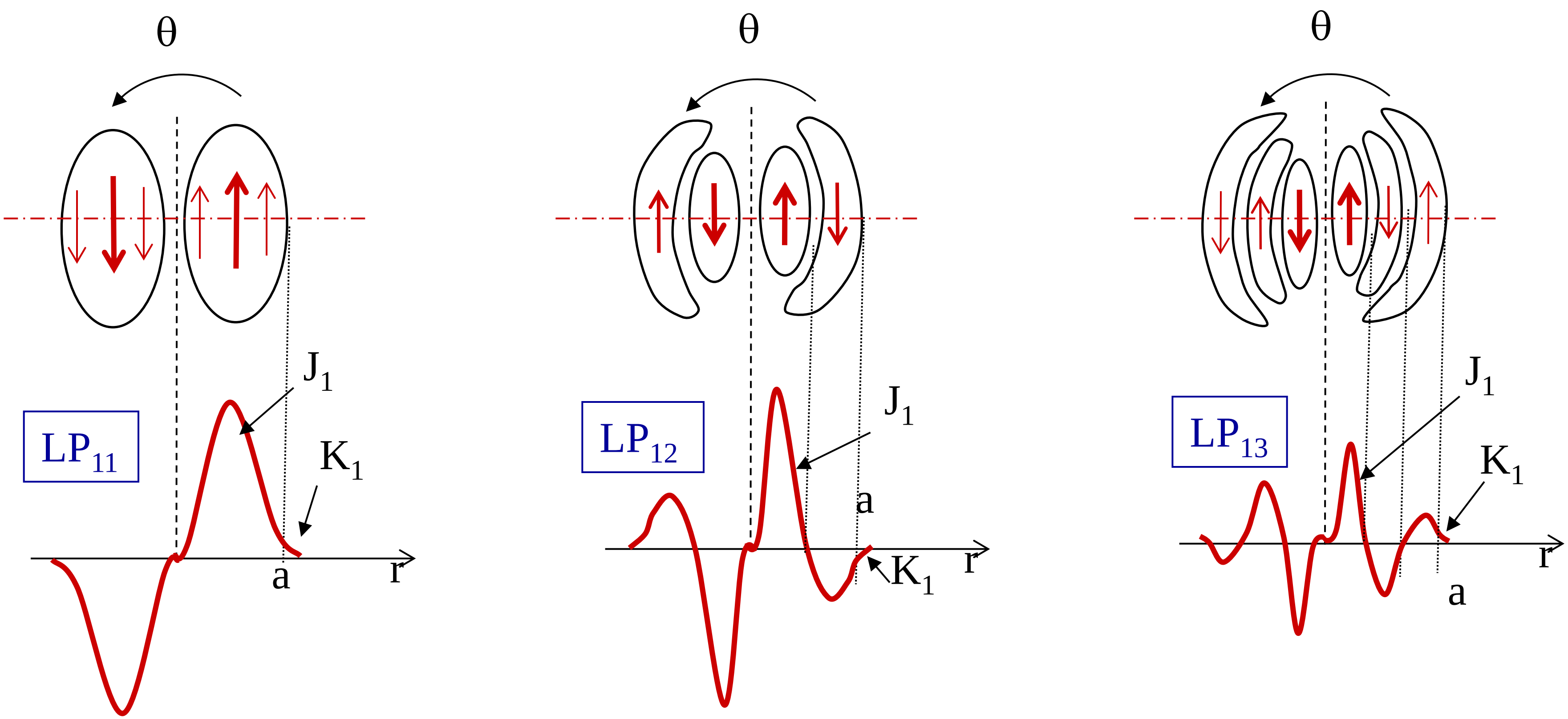
**other examples (with m=1) :**



## LP MODES - CUTOFF FREQUENCY

- **distribution of the electric field in the $LP_{m,l}$ modes *(pdf page 18)***

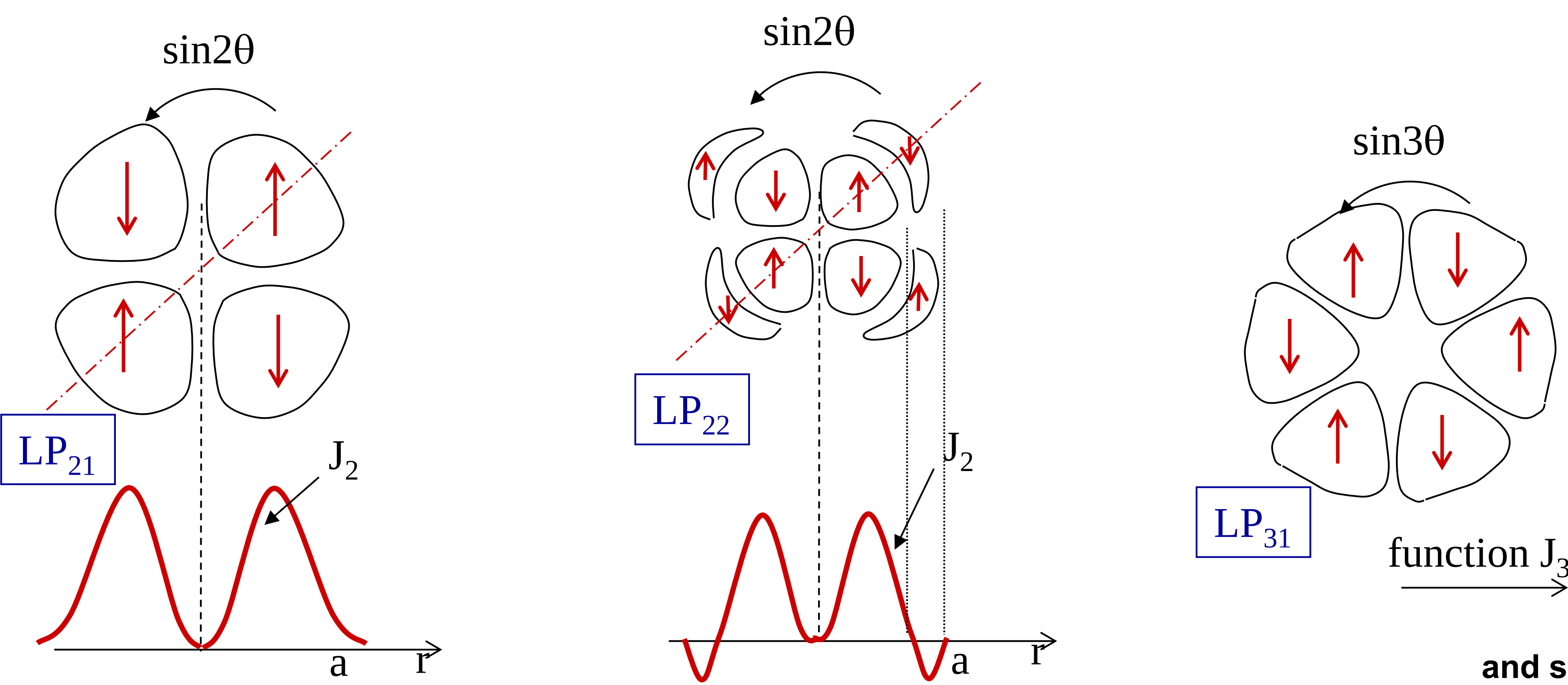
**other examples (with m=1) :**



## LP MODES - CUTOFF FREQUENCY

➢ **distribution of the electric field in the LP$_{m,l}$ modes** ***(pdf page 19)***

**still other examples (with m=2 or 3) :**

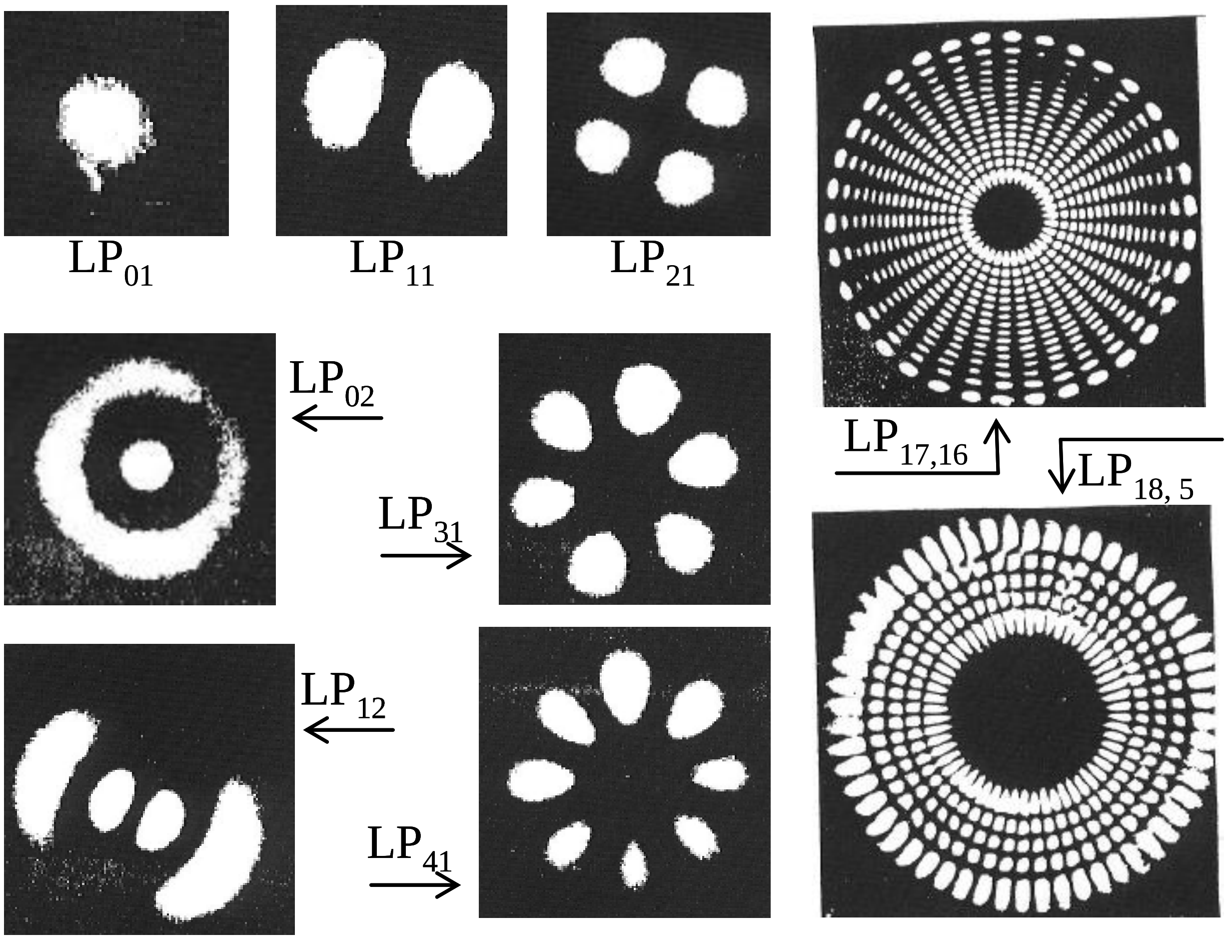



**and so on…**

**<u>intensity distribution in LP$_{m,l}$ modes (m≠0)</u> → pattern with *l* rings and 2m lobes in each ring**

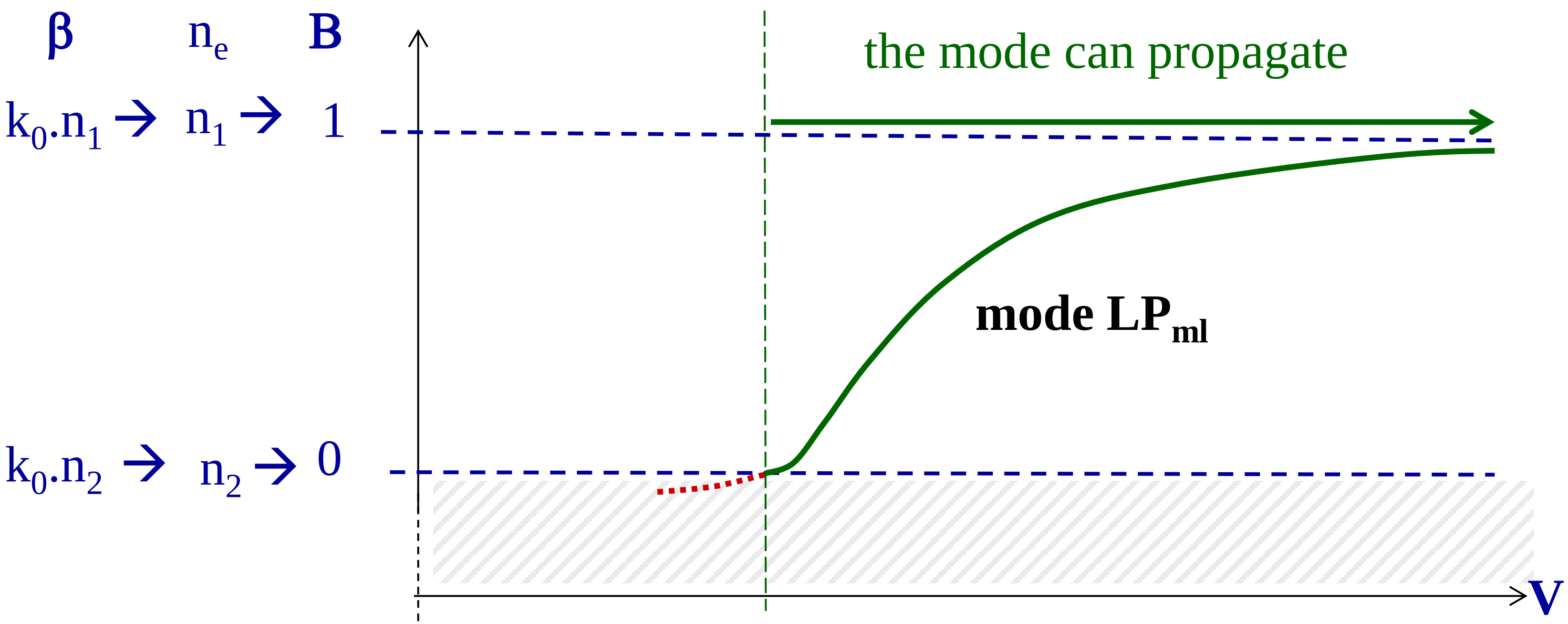
## LP MODES - CUTOFF FREQUENCY

➢ **distribution of the electric field in the $LP_{m,l}$ modes** ***(pdf page 19)***



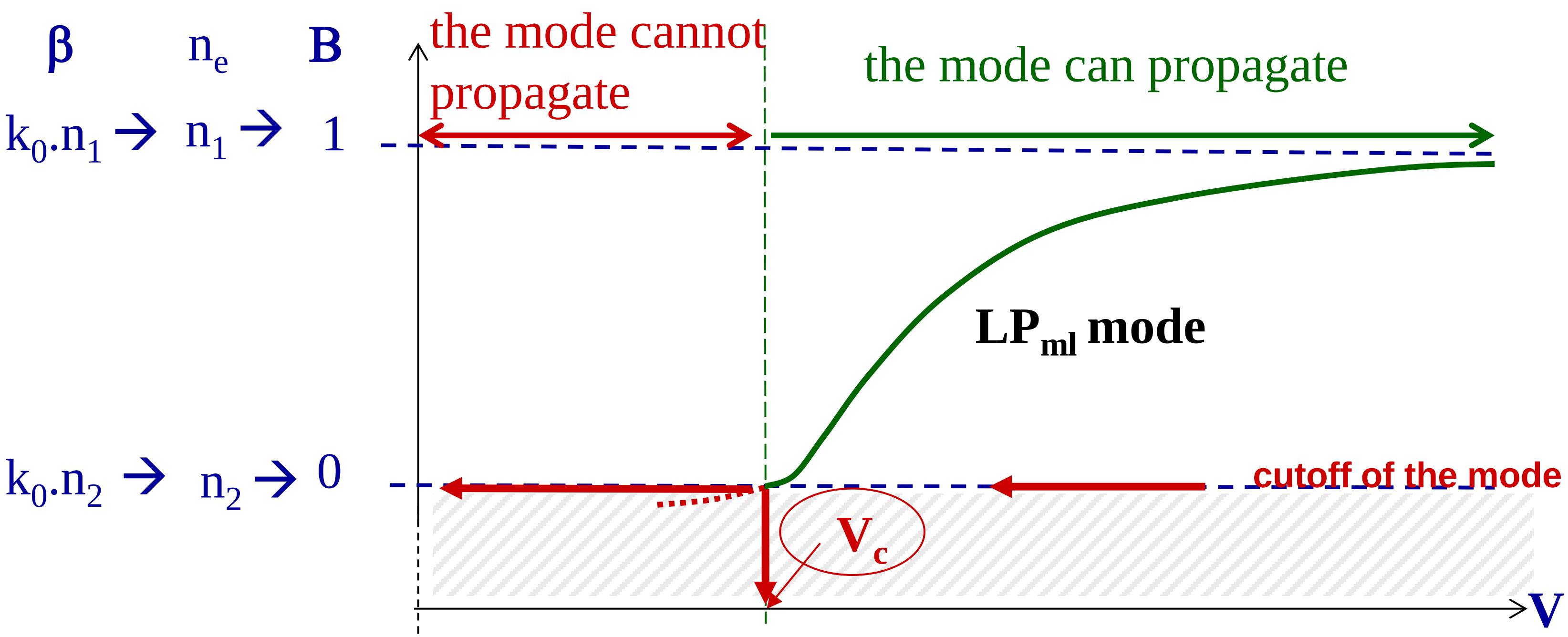
## LP MODES - CUTOFF FREQUENCY

➢ cutoff normalized spatiale frequency for LP modes *(pdf page 20)*



## LP MODES - CUTOFF FREQUENCY

➢ **cutoff normalized spatial frequency for LP modes *(pdf page 20)***




**propagation condition for the $LP_{ml}$ mode : $V > V_c(LP_{ml})$**

## LP MODES - CUTOFF FREQUENCY

➢ **cutoff normalized spatial frequency for LP modes *(pdf page 20)***

**At the cutoff of the mode :** $\beta = k_0 n_2$

$$w = a\left|\beta_{t2}\right| = a\sqrt{\beta^2 - k_0^2 n_2^2} = 0$$

$$u = V = V_c$$

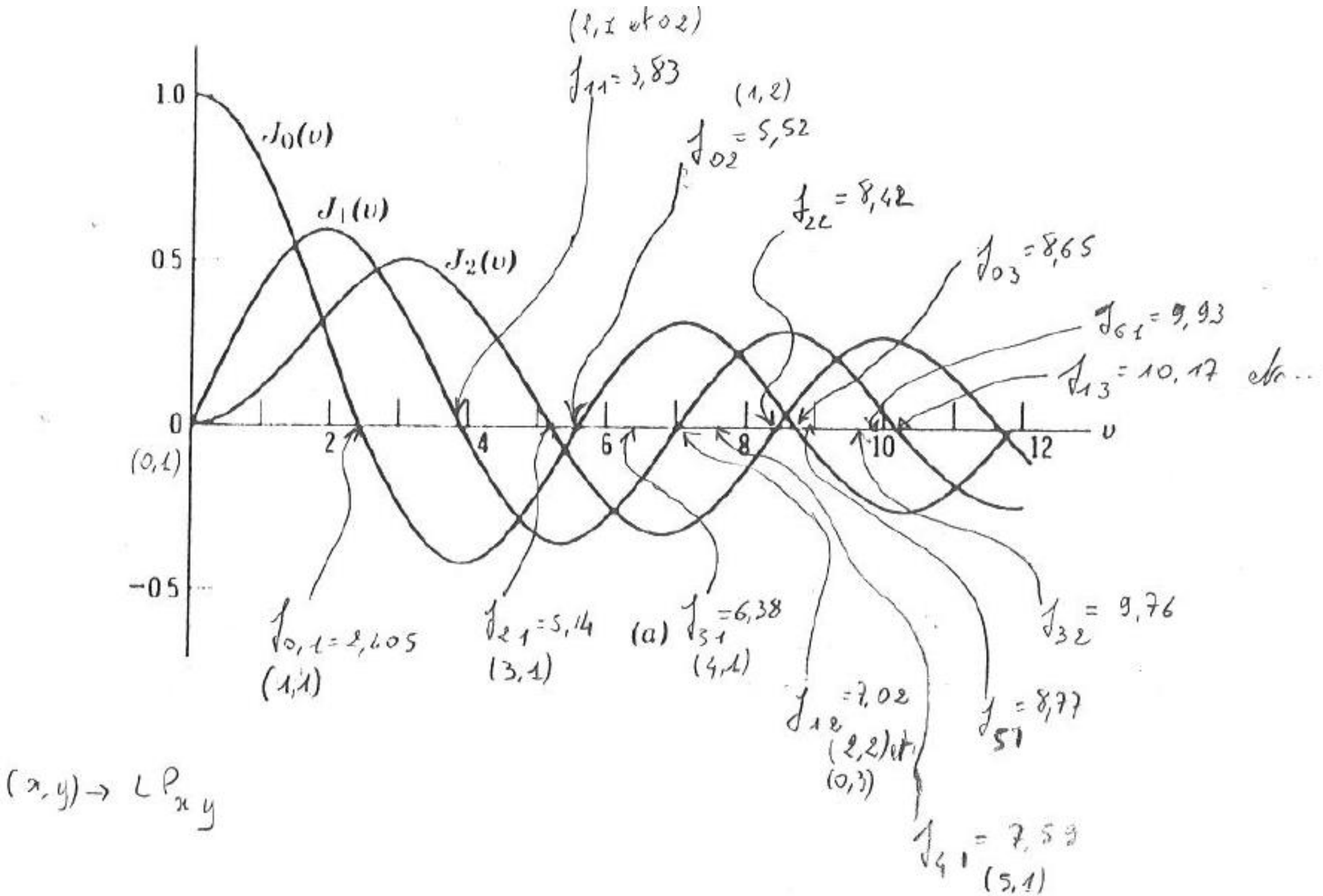
**and the dispersion equation** $u\dfrac{J_{m-1}(u)}{J_m(u)} = \dfrac{-wK_{m-1}(w)}{K_m(w)}$ **becomes** $\boxed{u\dfrac{J_{m-1}(u)}{J_m(u)} = 0 \quad \text{avec} \quad u = V_c}$

**Thus the cutoff normalized spatial frequencies of the LP$_{ml}$ modes are :** ***(voir pdf page 21)***

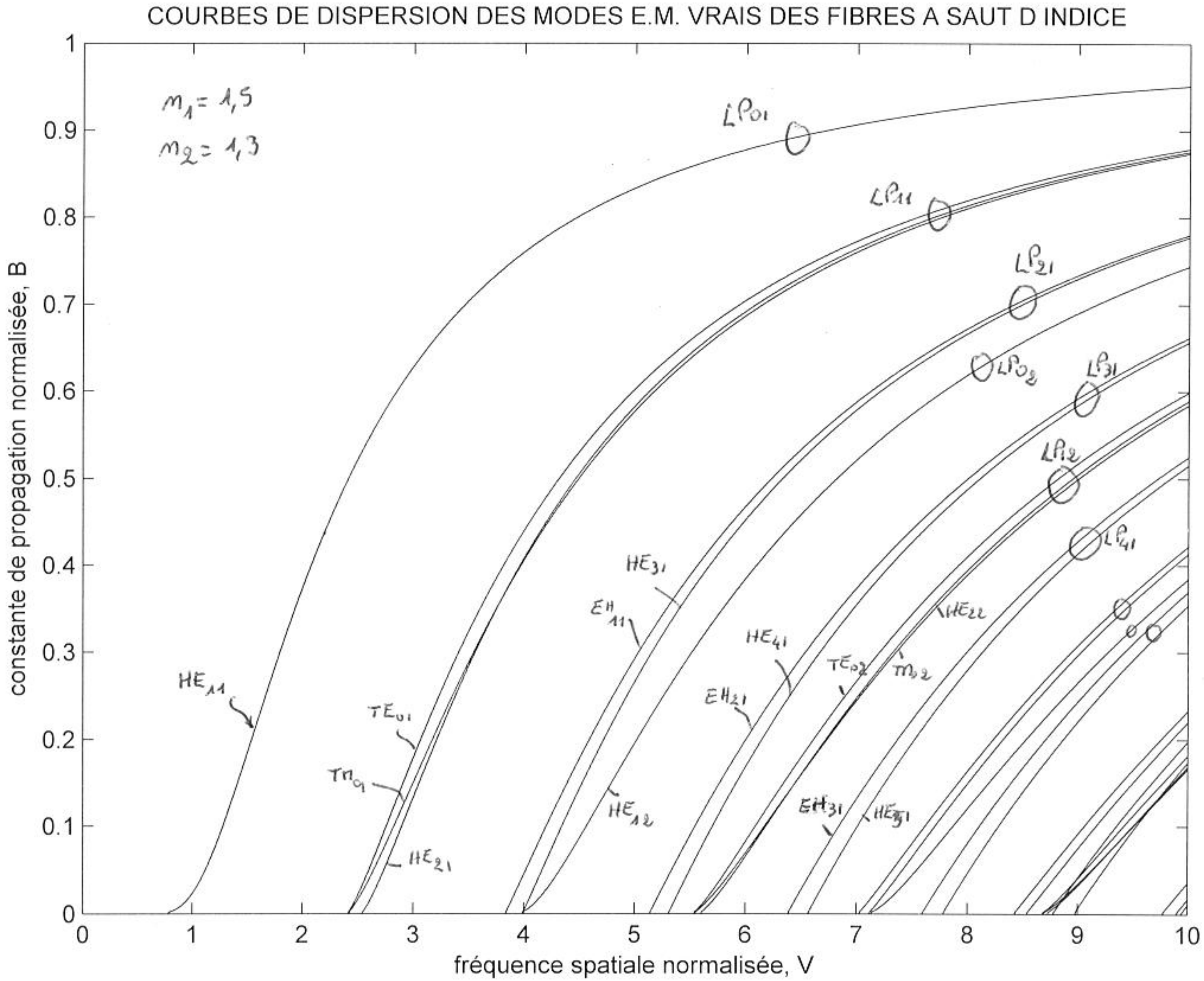
$$\begin{cases} m \neq 0 \qquad l \geq 1 \qquad V_c(LP_{m,l}) = j_{m-1,\,l} \\ \\ m = 0 \quad \begin{array}{ll} l = 1 & V_c(LP_{0,1}) = 0 \\ l > 1 & V_c\left(LP_{0,l}\right) = j_{1,\,l-1} \end{array} \end{cases}$$

## LP MODES - CUTOFF FREQUENCY

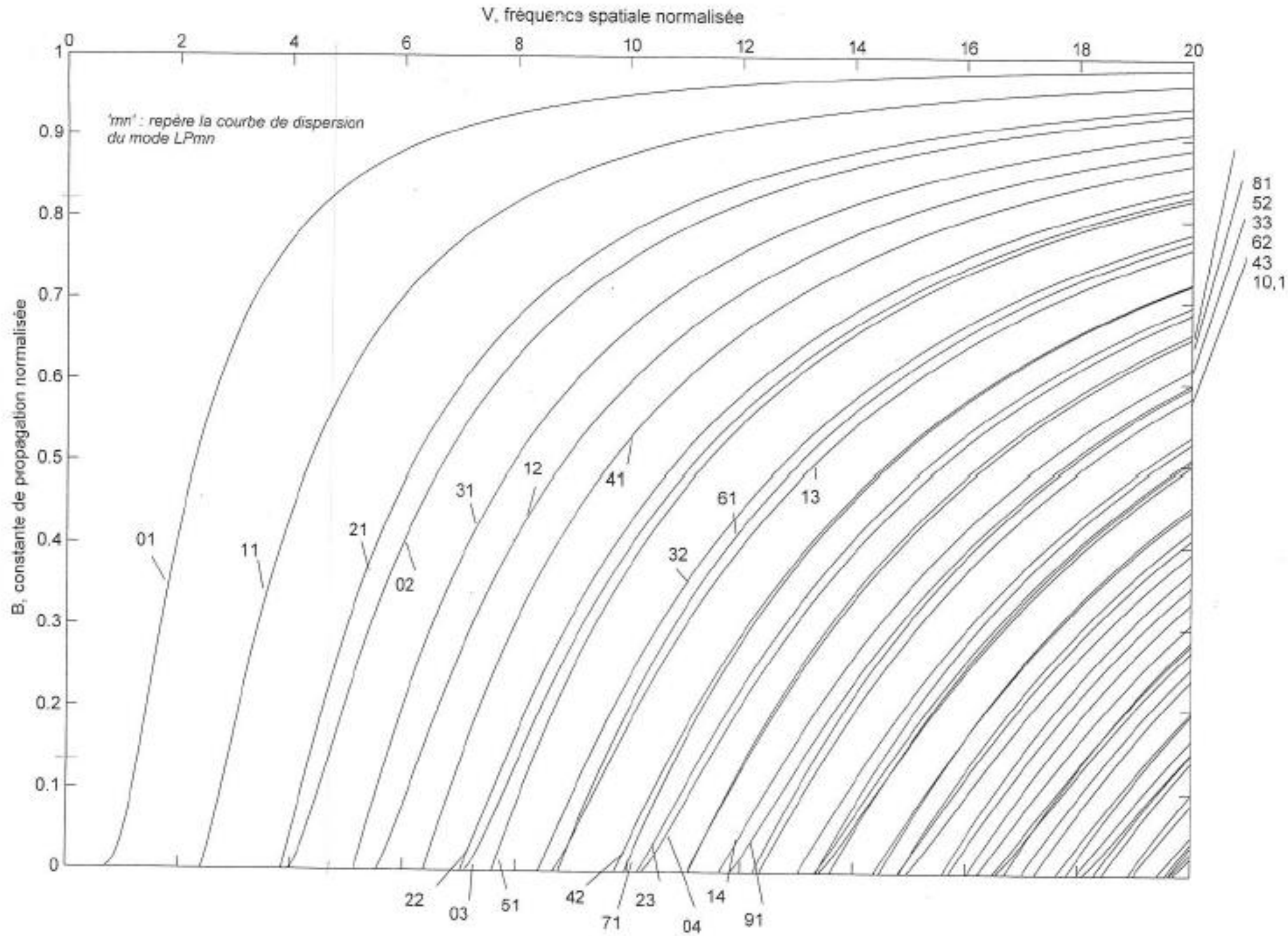
➢ **cutoff normalized spatial frequency for LP modes**



## DISPERSION CURVES (electromagnetic modes)



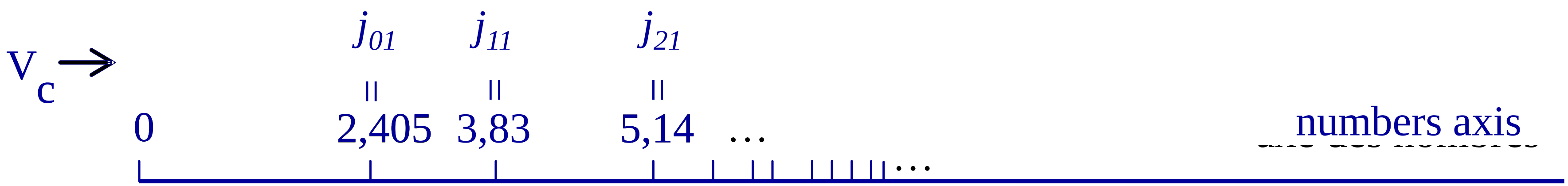
# DISPERSION CURVES (LP modes)




COURBES DE DISPERSION DES MODES LP DES FIBRES A SAUT D'INDICE

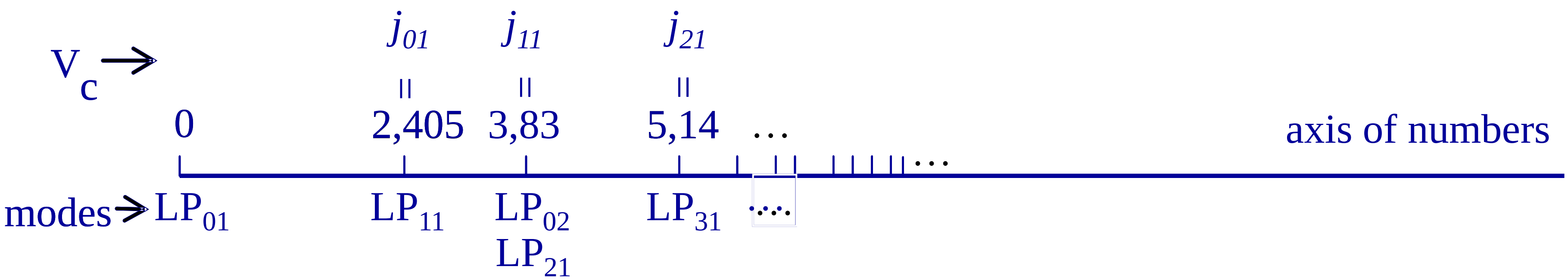
## LP MODES - CUTOFF NORMALIZED SPATIAL FREQUENCY

**reminder : propagation condition of the $LP_{ml}$ mode : $V > V_c(LP_{ml})$**



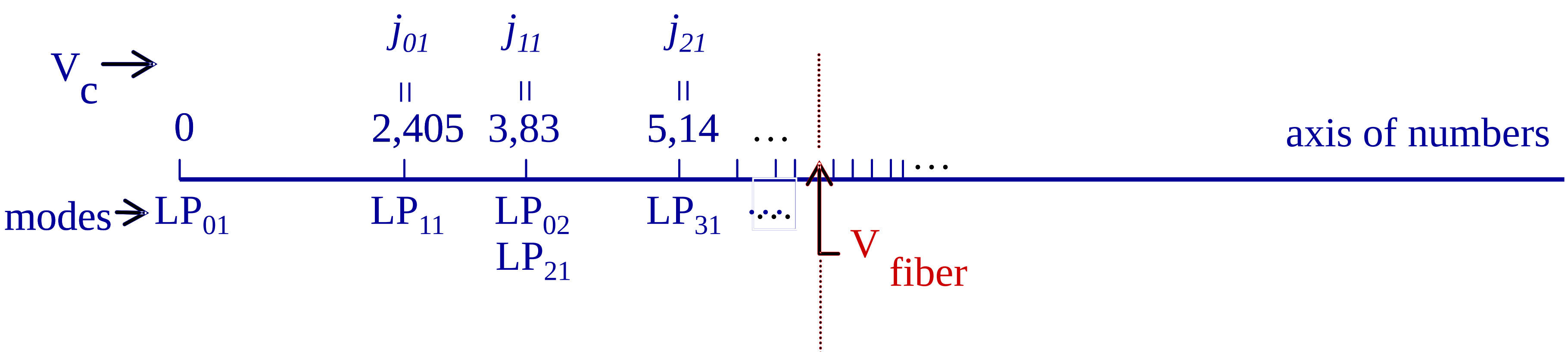
## LP MODES - CUTOFF NORMALIZED SPATIAL FREQUENCY

**reminder : propagation condition of the $LP_{ml}$ mode : $V > V_c(LP_{ml})$**



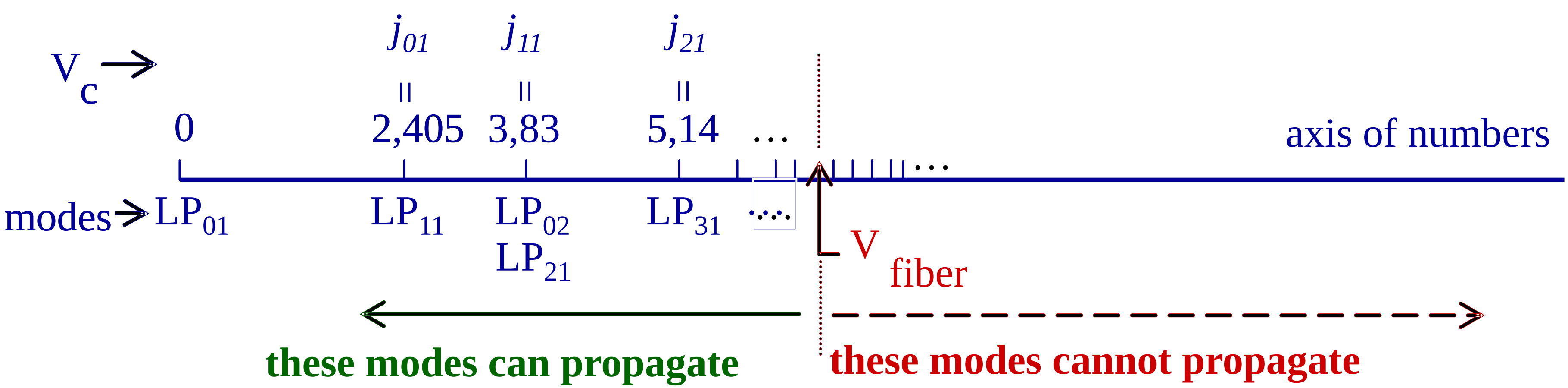
## LP MODES - CUTOFF NORMALIZED SPATIAL FREQUENCY

**reminder : propagation condition of the $LP_{ml}$ mode : $V > V_c(LP_{ml})$**



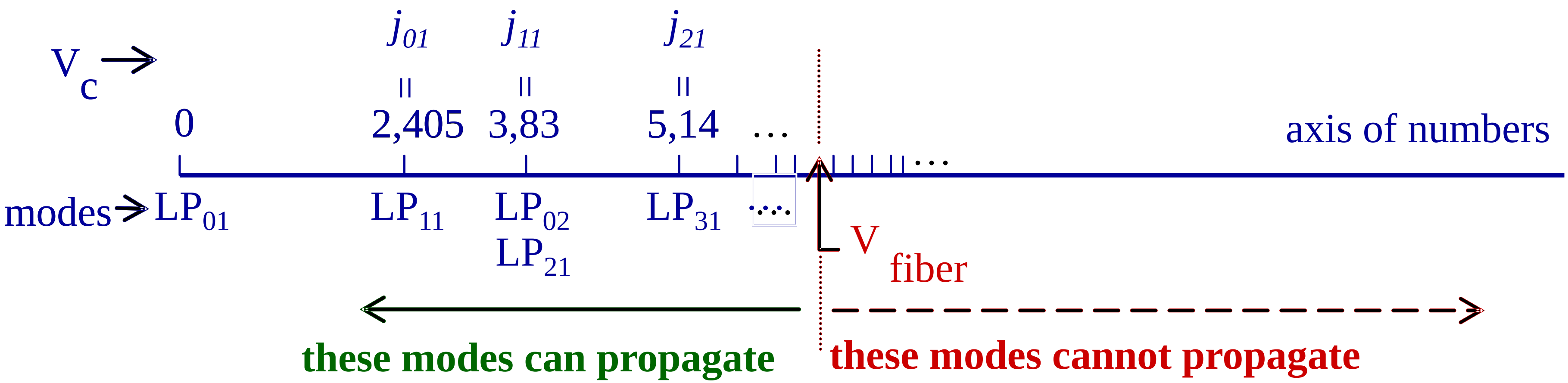
## LP MODES - CUTOFF NORMALIZED SPATIAL FREQUENCY

**reminder : propagation condition of the $LP_{ml}$ mode : $V > V_c(LP_{ml})$**



## LP MODES - CUTOFF NORMALIZED SPATIAL FREQUENCY

**reminder : propagation condition of the $LP_{ml}$ mode : $V > V_c(LP_{ml})$**



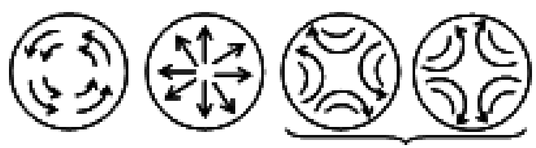

**The cutoff wavelength of the $LP_{ml}$ mode is $\lambda_c$, such that $V(\lambda_c) = V_c$**

$$\frac{2\pi}{\lambda_c}.a.ON = V_c \quad \Rightarrow \quad \boxed{\lambda_c = \frac{2\pi}{V_c}.a.ON}$$

**<u>possible propagation</u> of the X mode if $V_c(X) < V$** $\boxed{\Rightarrow \lambda < \lambda_c(X)}$

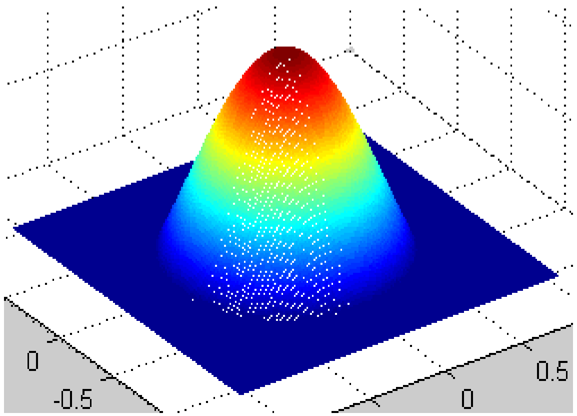
# LP MODES - CUTOFF NORMALIZED SPATIAL FREQUENCY

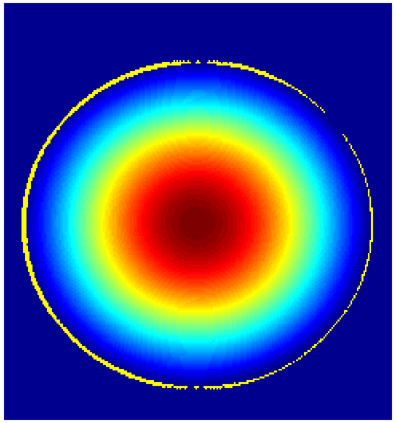
➢ **summary on the first $LP_{ml}$ modes** ***(pdf page 26)***

| Modes LP | $V_c$ | Modes dégénérés<br>((x) = nombre de polars) | Nombre de modes dégénérés |
|---|---|---|---|
| $LP_{01}$ | 0 | $HE_{11}$ (2) = $HE_{11x}$ et $HE_{11y}$ | 2 |
| $LP_{11}$ | 2,405 | $TE_{01}$ (1), $TM_{01}$ (1), et $HE_{21}$ (2)<br>$E_z = 0$ $E_\theta=0$<br><br>(lignes du champ électrique) | 4 |
| $LP_{21}$ | 3,83 | $EH_{11}$ (2) et $HE_{31}$ (2) | 4 |
| $LP_{02}$ | 3,83 | $HE_{12}$ (2) | 2 |
| $LP_{31}$ | 5,14 | $EH_{21}$ (2) et $HE_{41}$ (2) | 4 |
| $LP_{12}$ | 5,52 | $TE_{02}$ (1), $TM_{02}$ (1), et $HE_{22}$ (2) | 4 |
| $LP_{41}$ | 6,38 | $EH_{31}$ (2) et $HE_{51}$ (2) | 4 |
| $LP_{22}$ | …… | ………. | …. |
| etc….. | | ………. | …. |

## Step index fibers : LP modes of the lowest orders

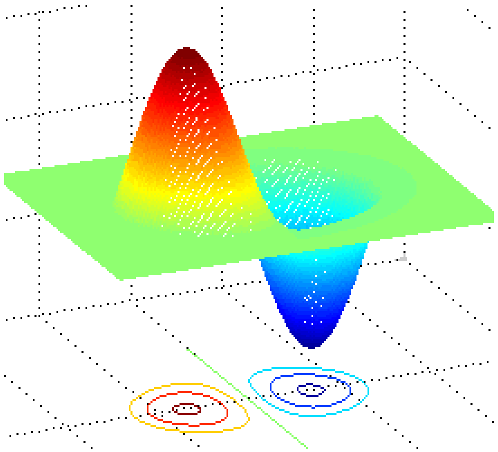
**LP $_{01}$ mode**

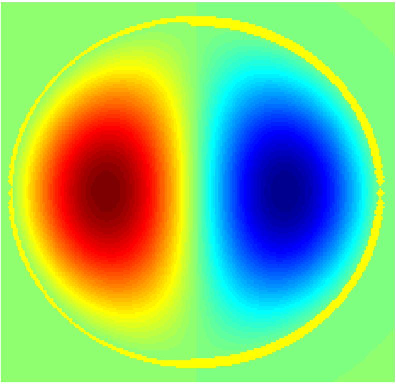


**$V_c = 0$**

**LP $_{11}$ mode**





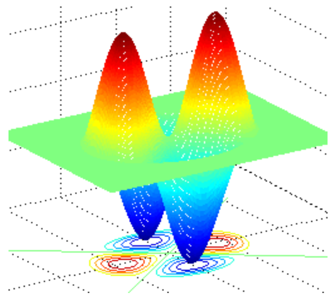
**$V_c = 2{,}405$**

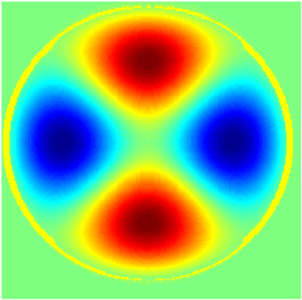
The yellow circle represents the boundary between the core and the cladding

*a = 40µm; NA =0,24*

## Step index fibers : LP modes of the lowest orders (cont'd))

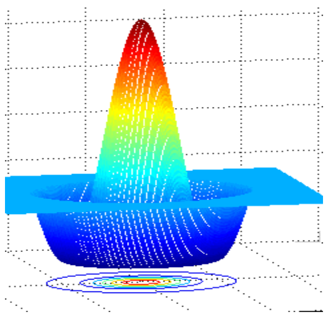
**LP $_{21}$ mode**

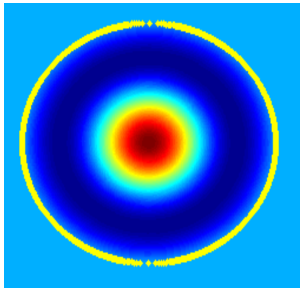




**$V_c$ = 3,83**

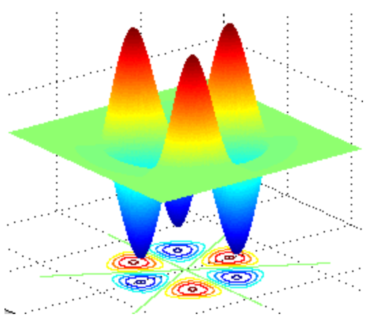
**LP $_{02}$ mode**

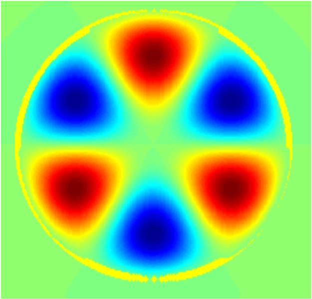




**$V_c$ = 3,83**

**LP $_{31}$ mode**





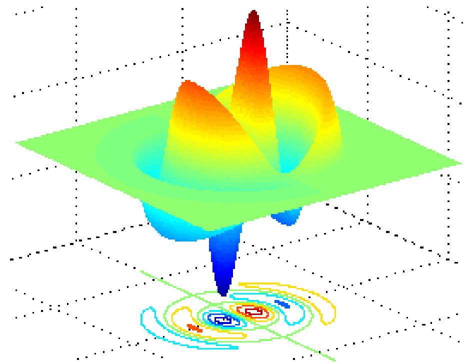
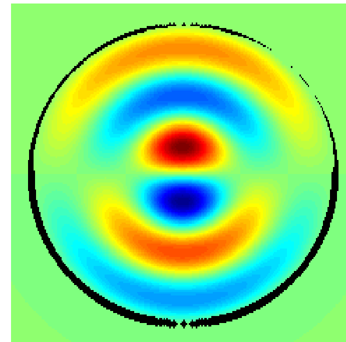
**$V_c$ = 5,14**

The yellow circle represents the boundary between the core and the cladding

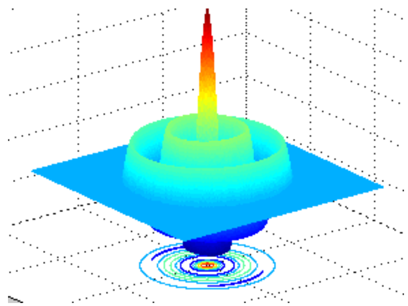
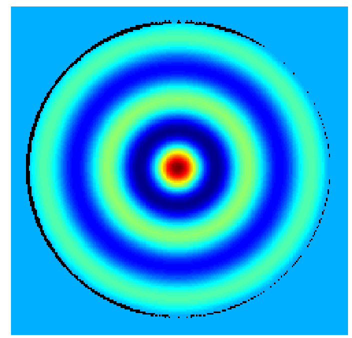
*a = 40µm; NA =0,24*

## Step index fibers : other LP modes

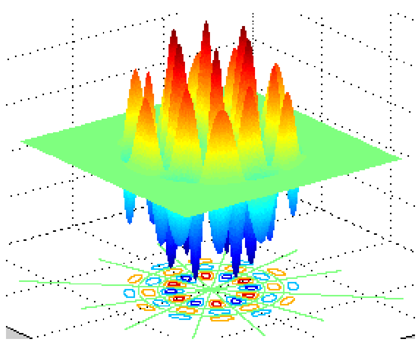
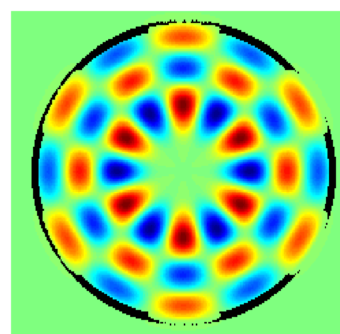
**LP $_{1,3}$ mode**

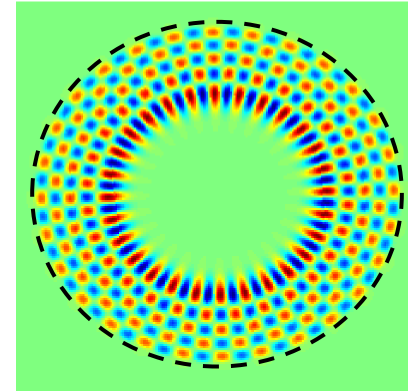
**LP $_{0,5}$ mode**

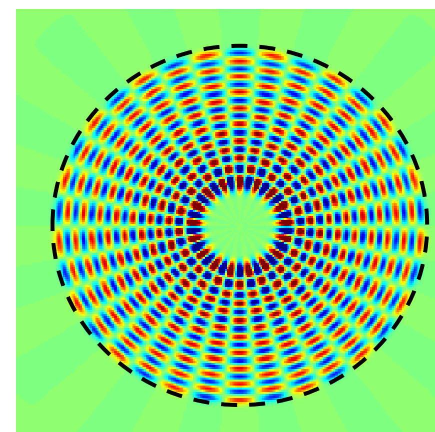
**LP $_{6,3}$ mode**

The black circle represents the boundary between the core and the cladding



**LP $_{28,5}$ mode**

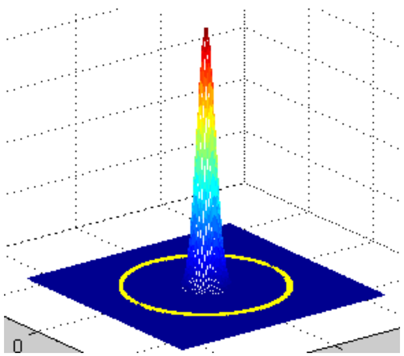


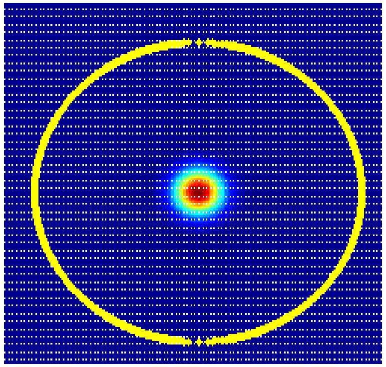
**LP $_{17,16}$ mode**

*a = 40µm; NA =0,24*

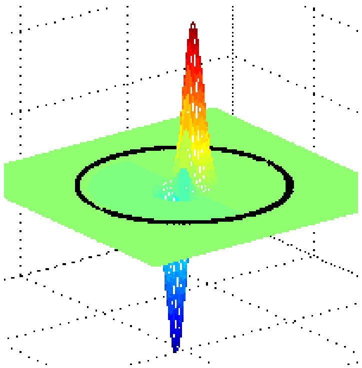
## Graded index fibers : LP modes of lowest order

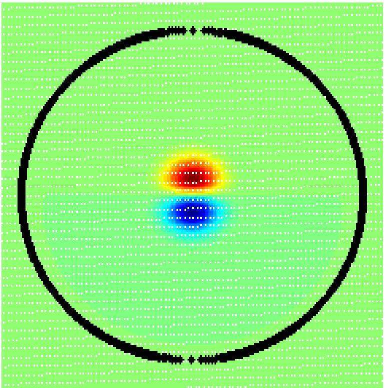
**LP $_{01}$ mode**



**LP $_{11}$ mode**



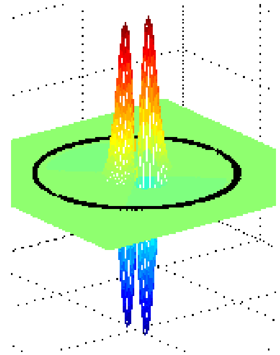


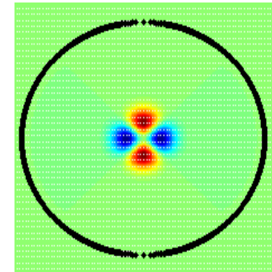
The yellow or black circles represent the boundary between the core and the cladding

*a = 40µm; NA =0,24*

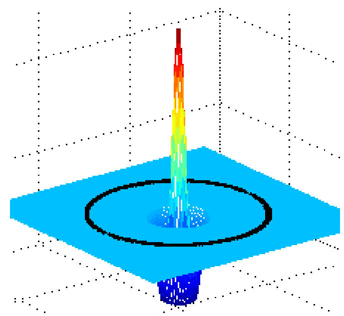
## Graded index fibers : LP modes of lowest order (cont'd)

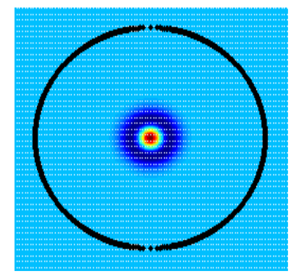
**$LP_{21}$ mode**



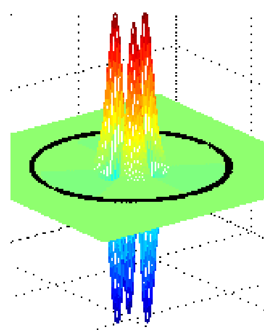


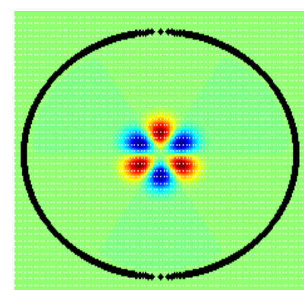
**$LP_{02}$ mode**





**$LP_{31}$ mode**



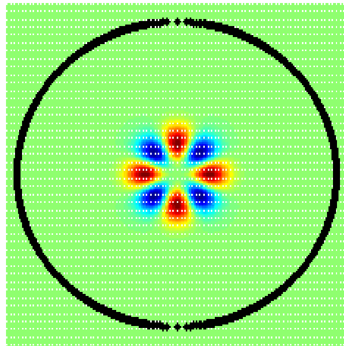


The black circle represents the boundary between the core and the cladding

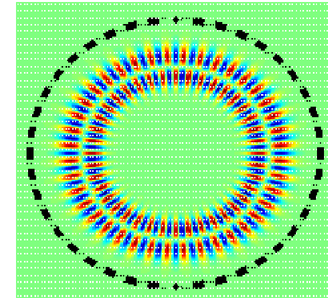
*a = 40µm; NA =0,24*

## Graded index fibers : other LP modes

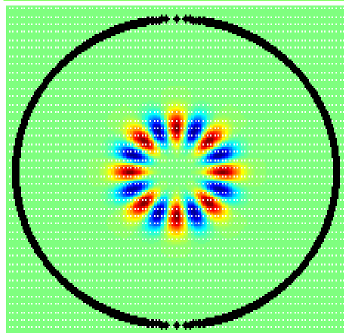
**LP $_{4,\,1}$ mode**



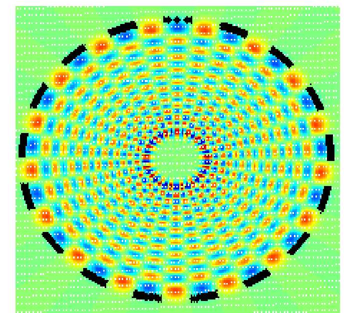
**LP $_{40,\,2}$ mode**



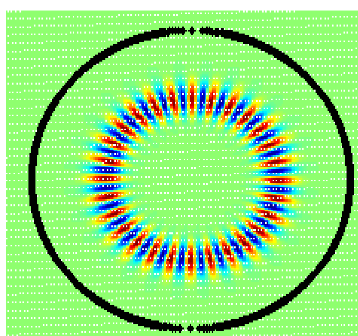
**LP $_{8,\,1}$ mode**



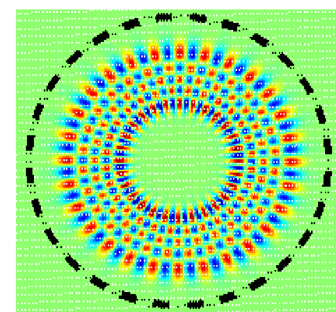
**LP $_{17,\,16}$ mode**



**LP $_{28,\,1}$ mode**



**LP $_{28,\,5}$ mode**

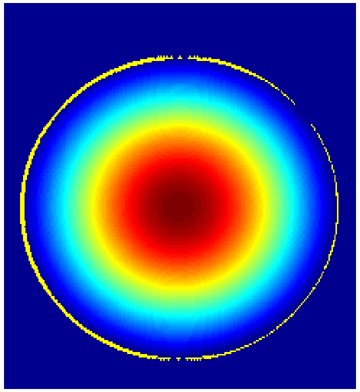
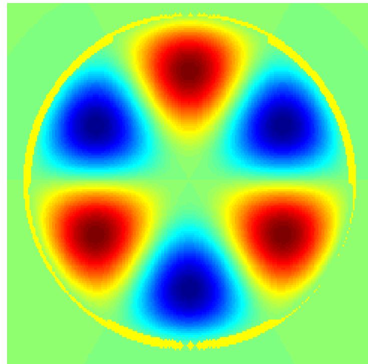
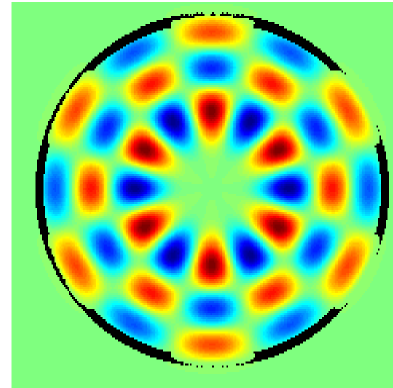
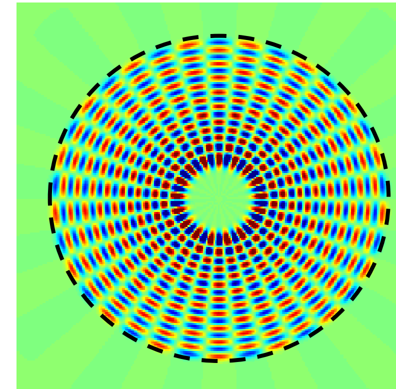


The black circle represents the boundary between the core and the cladding

*a = 40µm; NA =0,24*

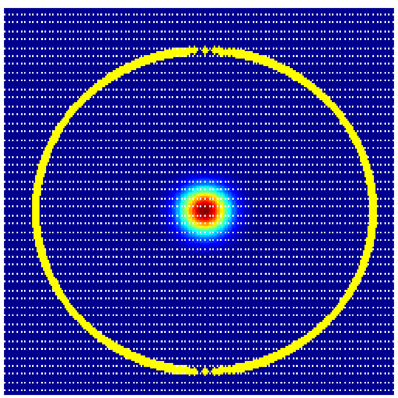
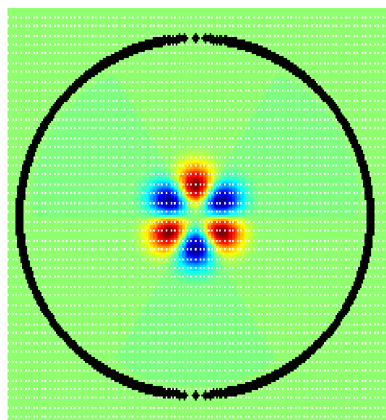
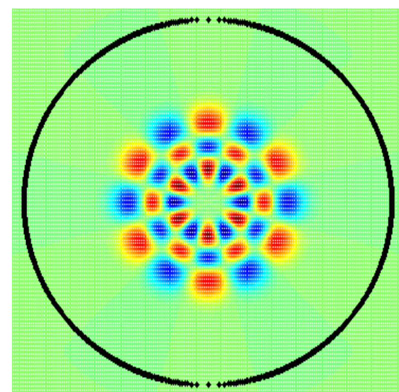
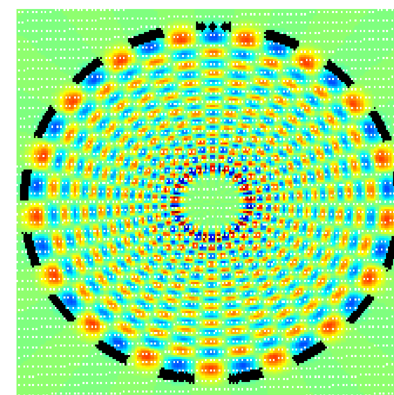
## Comparison of modes of step index fibers vs modes of graded index fibers

**step index fiber**

**$LP_{01}$ mode** **$LP_{3,1}$ mode** **$LP_{6,3}$ mode** **$LP_{17,16}$ mode**

**graded index fiber**

The yellow or black circles represent the boundary between the core and the cladding

*a = 40µm; NA =0,24*

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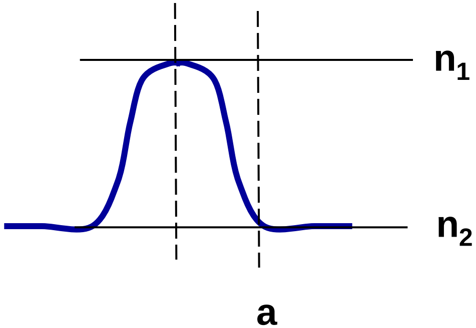
## NUMBER OF MODES ABLE TO PROPAGATE IN A MULTIMODE FIBER (*pdf page 26*)

**index profile given by :**

$$\begin{cases} n_{core} = n_1(r) = n_1\left[1-2\Delta\left(\dfrac{r}{a}\right)^g\right]^{1/2} & \mathrm{r} \le \mathrm{a} \\ n_{cladding} = n_2 & \mathrm{r} \ge \mathrm{a} \end{cases}$$




**number of EM modes :** $\mathcal{N}_{EM} = \dfrac{V^2}{2}\dfrac{g}{g+2}$

**number of LP modes :** $\mathcal{N}_{LP} = \dfrac{\mathcal{N}_{EM}}{4} = \dfrac{V^2}{8}\dfrac{g}{g+2}$

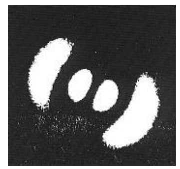
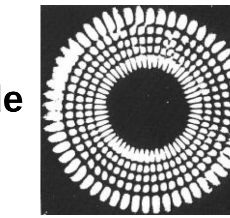
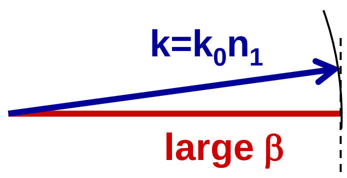
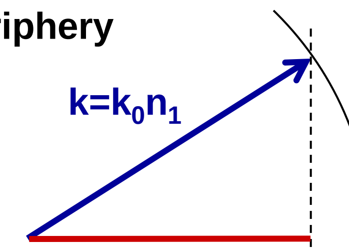
→ **fiber with a parabolic index profile ➔ g = 2** $\mathcal{N}_{EM} = \dfrac{V^2}{4}$ **and** $\mathcal{N}_{LP} = \dfrac{V^2}{16}$

→ **fiber with a step index profile ➔ g = ∞** $\mathcal{N}_{EM} = \dfrac{V^2}{2}$ **and** $\mathcal{N}_{LP} = \dfrac{V^2}{8}$

**example : step index fiber, with NA = 0.2, a = 25µm, λ=0.85µm ➔ V = 37 ➔** $\mathcal{N}_{LP} \sim 170$

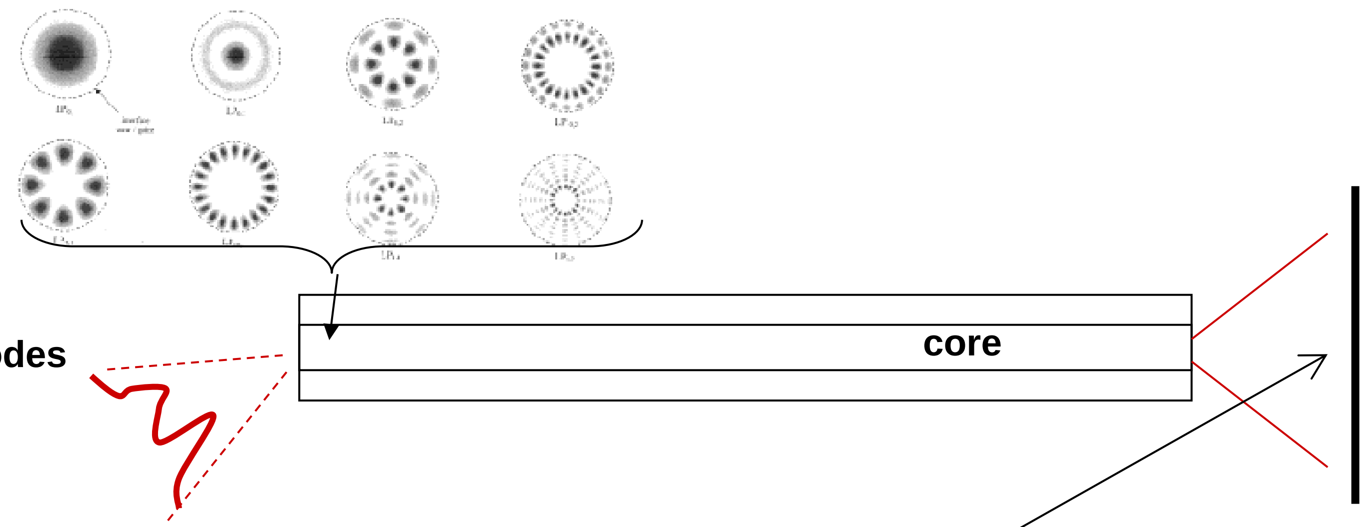
## WHAT IS THE "ORDER" OF A MODE IN A MULTIMODE FIBER ? *(pdf page 27)*

**order of the $LP_{m\,l}$ mode : $M = 2l + m - 1$**

| **M small** | **M large** |
|---|---|
| → simple pattern of the mode<br>→ (low number of lobes)  | → complexe pattern of the mode<br>(large number of lobes)  |
| → **low order mode** | → **high order mode** |
| → energy rather in the center | → energy rather at the periphery |
| → $\beta$ close to $k_0n_1$  <br>(mode associated to very inclined rays) | → $\beta$ close to $k_0n_2$  <br>(mode associated to little inclined rays) |
| → low $v_\varphi$ and<br>→ large $v_g$ | → large $v_\varphi$ and<br>→ low $v_g$ |

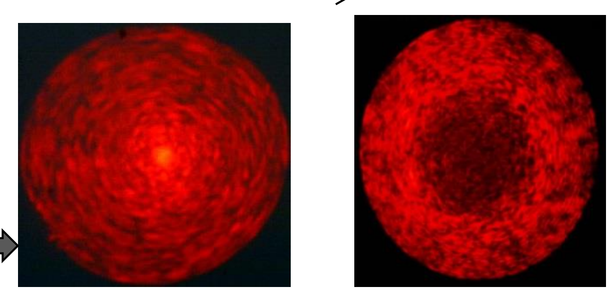
## OVERLAP OF MODES, COUPLING, SPECKLE *(pdf page 28)*

**Excitation of modes in the core :**




**Spatial overlap of guided modes : "speckle" →**

high density of low order modes



high density of high order modes

"Speckle"

**Changes in the speckle along the propagation are due to :**
**→ changes in the relative phase shifts between the modes**
**→mode coupling occuring in axially non uniforme or perturbated guides (along z)**

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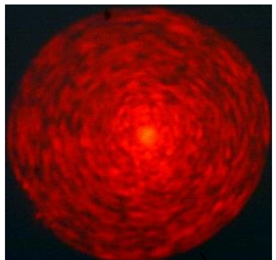
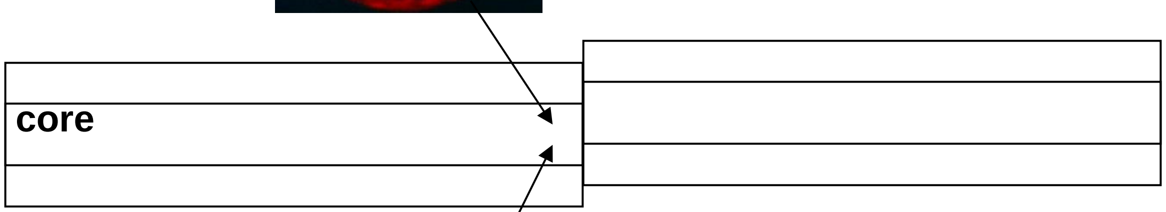
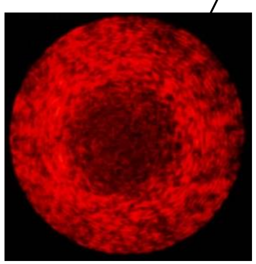
## SENSITIVITY OF PERFORMANCES OF SOME COMPONENTS TO THE "MODAL POPULATION" *(pdf page 28)*

→ **case of a misaligned connector**

**if**  **➔ loss =** $\alpha_1$



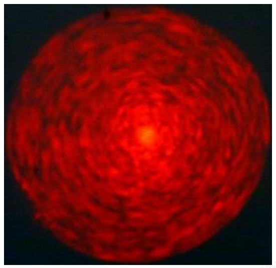

**with** $\alpha_2 > \alpha_1$

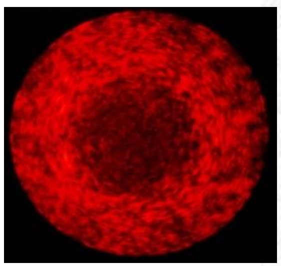
**if**  **➔ loss =** $\alpha_2$

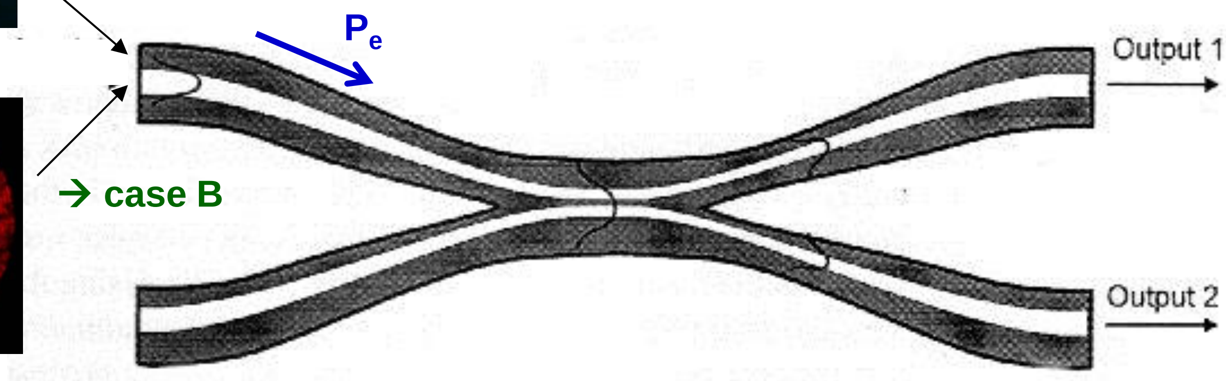
**+ possible significant change in the modal population at the junction !**

## SENSITIVITY OF PERFORMANCES OF SOME COMPONENTS TO THE "MODAL POPULATION" *(pdf page 28)*

→ <u>case of a fused coupler</u>

if  → case A

if  → case B




$P_{S1}$

$P_{S2}$

coupling ratio : $\dfrac{P_{S1}}{P_{S1}+P_{S2}}(caseA) > \dfrac{P_{S1}}{P_{S1}+P_{S2}}(caseB)$

excess loss (dB) : $10\log\dfrac{P_e}{P_{S1}+P_{S2}}(caseB) > 10\log\dfrac{P_e}{P_{S1}+P_{S2}}(caseA)$

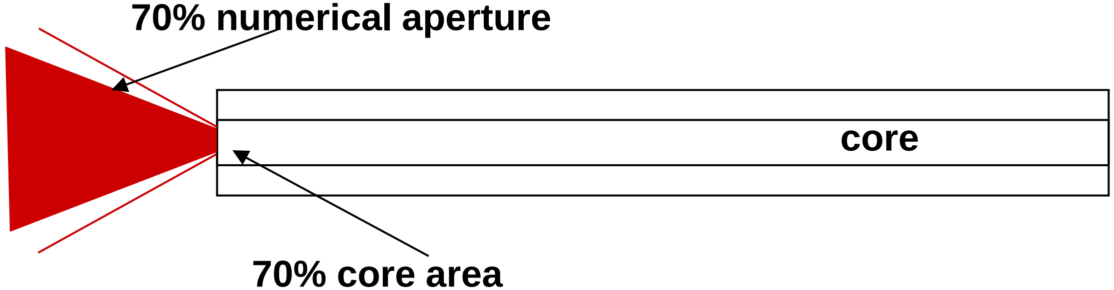
**+ possible significant change in the modal population in the coupler !**

## SENSITIVITY OF PERFORMANCES OF SOME COMPONENTS TO THE "MODAL POPULATION" *(pdf page 28)*

➔ **necessity of characterizing (and using) the components in the conditions of "equilibrium mode distribution " allowing steady state propagation**

**"Equilibrium mode distribution" : modal population which is overall invariant along the fiber**

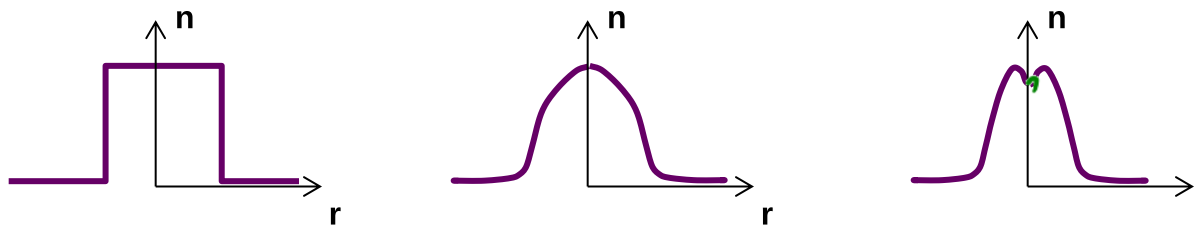
**how to obtain it ? → use of a "mode scrambler"**
**→ or**




**Fine modal characterization of multimode components : "selective excitation" of modes**

## PROPAGATION IN SINGLE MODE FIBERS (V < 2,405) : SCALAR APPROACH ($LP_{01}$ MODE)

**index profiles:**

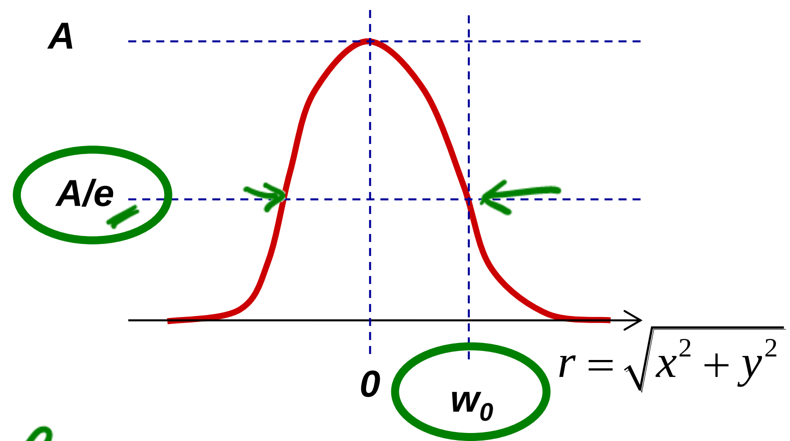



**step index** **graded index** **typical real profile**

**if 1,2 < V < 4**

**whatever the index profile**
**$LP_{01}$ mode ~ gaussian mode**

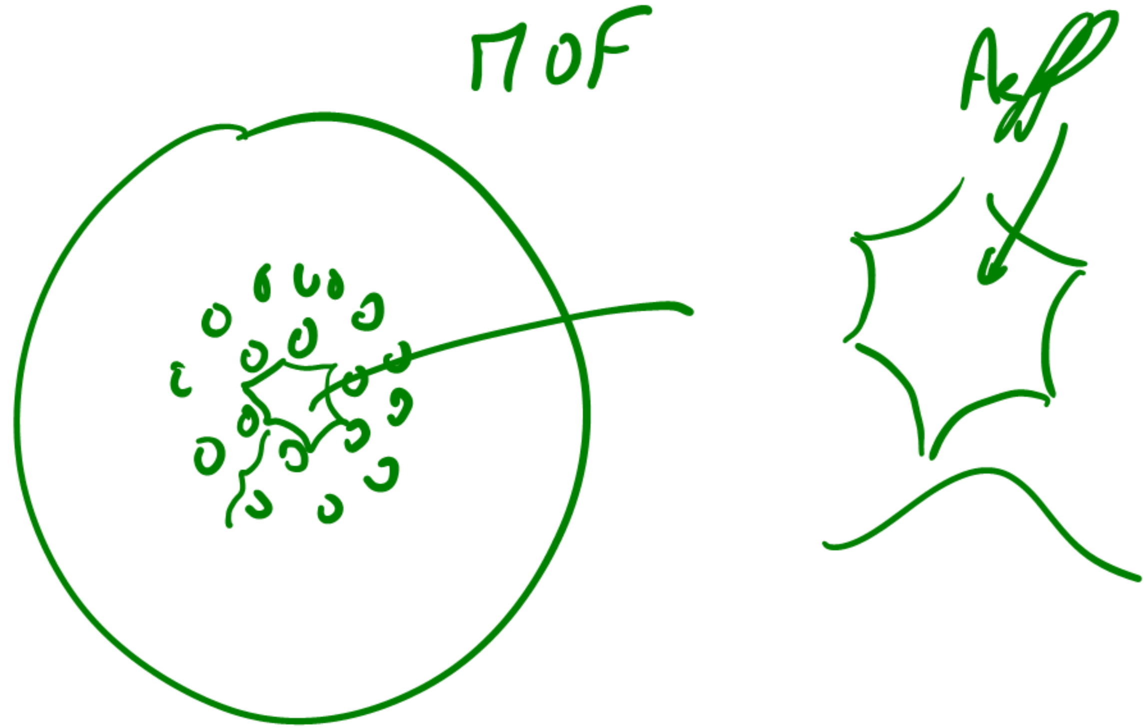
$\psi_{LP01}(x,y)$




$\ln e = 1$

$$\psi_{LP01}(x,y) \approx A.\exp\left(-\frac{x^2+y^2}{w_0^2}\right)$$

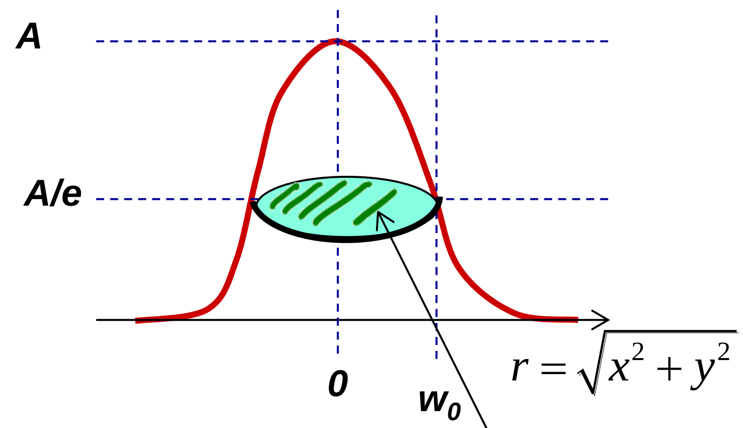
**mode field radius (MFR)**


MOF
Aeff

## PROPAGATION IN SINGLE MODE FIBERS (V < 2,405) : SCALAR APPROACH ($LP_{01}$ MODE)

$$\psi_{LP01}(x,y) \approx A.\exp\left(-\frac{x^2+y^2}{w_0^2}\right)$$

$\psi_{LP01}(x,y)$




**mode field radius (MFR)**

$$w_0 = a\left(0{,}65+\frac{1{,}619}{V^{3/2}}+\frac{2{,}879}{V^6}\right)$$

$1{,}2 < V < 4$

→ **In single mode fibers, loss at misaligned splices depend on $w_0$**

**General expression of the "effective area" of a mode:**

$$A_{eff} = \frac{\left|\int |\psi|^2\, dS\right|^2}{\int |\psi|^4\, dS}$$

**For a gaussian mode:** $A_{eff} = \pi.w_0^2$

→ **In single mode fibers, non linear effects (Kerr, Raman, Brillouin…) depend on $A_{eff}$**

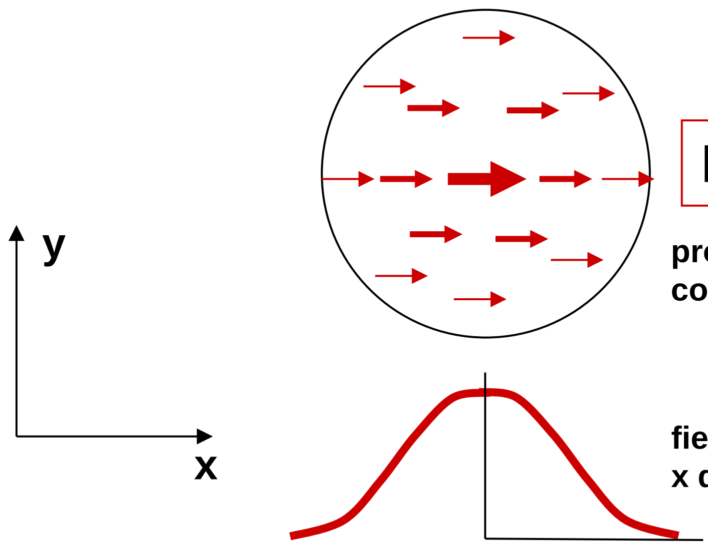
## PROPAGATION IN SINGLE MODE FIBERS (V < 2.405) : VECTORIAL APPROACH ($HE_{11}$ MODES)

### 2 expressions for the field of the $HE_{11}$ mode

$LP_{01} \rightarrow HE_{11}$

$$\begin{cases} E_x = \begin{cases} E_0 \dfrac{J_0\left(\frac{ur}{a}\right)}{J_0(u)} & r \le a \\ E_0 \dfrac{K_0\left(\frac{wr}{a}\right)}{K_0(w)} & r \ge a \end{cases} \\ E_y \simeq 0 \end{cases}$$

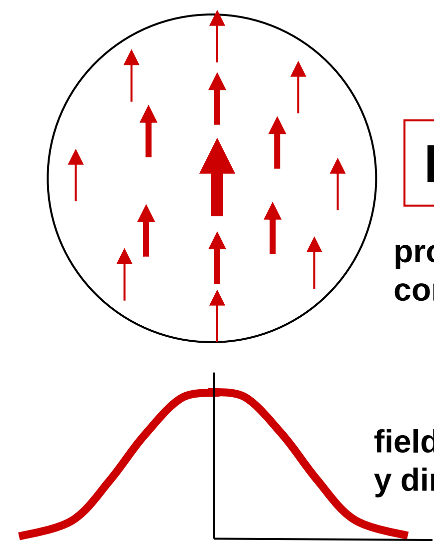
$$\begin{cases} E_x \simeq 0 \\ E_y = \begin{cases} E_0 \dfrac{J_0\left(\frac{ur}{a}\right)}{J_0(u)} & r \le a \\ E_0 \dfrac{K_0\left(\frac{wr}{a}\right)}{K_0(w)} & r \ge a \end{cases} \end{cases}$$




$HE_{11x}$

propagation constant $\beta_x$ //

field in the x direction



$HE_{11y}$

propagation constant $\beta_y$ //

field in the y direction

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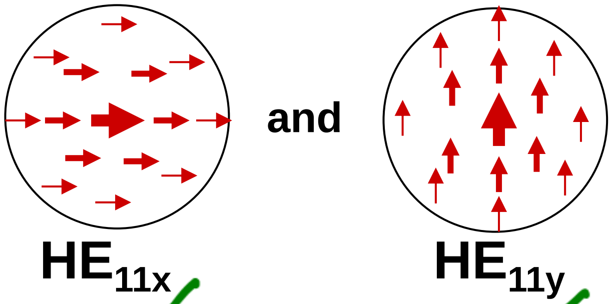
## PROPAGATION IN SINGLE MODE FIBERS (V < 2,405) : VECTORIAL APPROACH ($HE_{11}$ MODES)

### polarization states of light in the core

if and are excited





$$\vec{E} = E_x \cos(\omega t - \beta_x z).\vec{ex} + E_y \cos\left[\omega t - \beta_y z\right].\vec{ey}$$

$$\vec{E} = E_x \cos(\omega t - \beta_x z).\vec{ex} + E_y \cos\left[\omega t - \beta_x z + \underbrace{\overbrace{(\beta_x - \beta_y)}^{\delta\beta} z}_{\varphi = \delta\beta.z}\right].\vec{ey}$$

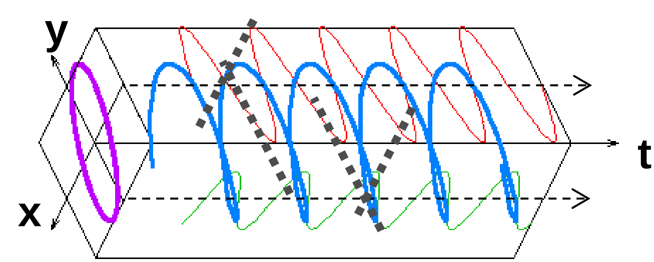
$\beta_x$-$\beta_y$= $\delta\beta$ : linear birefringence of the fiber

**for a given z :**

* if $\varphi = \delta\beta.z = 0$ : linear polarization

* if $\varphi = \delta\beta.z = \pi/2$ and $E_x= E_y$ : circular polarization

* general case (any $\varphi$ and/or $E_x \neq E_y$ : elliptical polarization



## PROPAGATION IN SINGLE MODE FIBERS (V < 2,405) : VECTORIAL APPROACH ($HE_{11}$ MODES)

### phase effects of the birefringence of the fiber (1) :

phase birefringence

$\beta_x = k_0 n_{ex}$

$\beta_y = k_0 n_{ey}$

$$\beta_x - \beta_y = \delta\beta = \frac{2\pi}{\lambda}(n_{ex} - n_{ey})$$

$$\vec{E} = E_x \cos(\omega t - \beta_x z).e\vec{x} + E_y \cos\left[\omega t - \beta_x z + \underbrace{(\beta_x - \beta_y)z}_{\varphi}\right].e\vec{y}$$

$B_\varphi = \delta\beta / k_0 = | n_{ex} - n_{ey} |$    $B_\varphi$ : normalized phase birefringence

* if $n_{ex} \neq n_{ey}$ : $v_{\varphi x} = c/n_{ex} \neq v_{\varphi y} = c/n_{ey}$ → phase velocities of $HE_{11x}$ and $HE_{11y}$ are different

* along z, the phase shift φ between Ex and Ey increases (→ polarization state changes : see previous and next slides)

* the phase shift φ increases by $\Delta\varphi = 2\pi$ over a length $L_b$ (i.e. between z and z+ $L_b$)

$B_\varphi \nearrow$

$L_b$ : beat length of the fiber

$L_B \searrow$

$$\Delta\varphi = 2\pi \quad \text{and} \quad \Delta\varphi = \varphi(z + L_B) - \varphi(z) = \frac{2\pi}{\lambda_0}\left|n_{ex} - n_{ey}\right| L_b \quad \Rightarrow \quad L_b = \frac{\lambda_0}{\left|n_{ex} - n_{ey}\right|} = \frac{\lambda_0}{B_\varphi}$$

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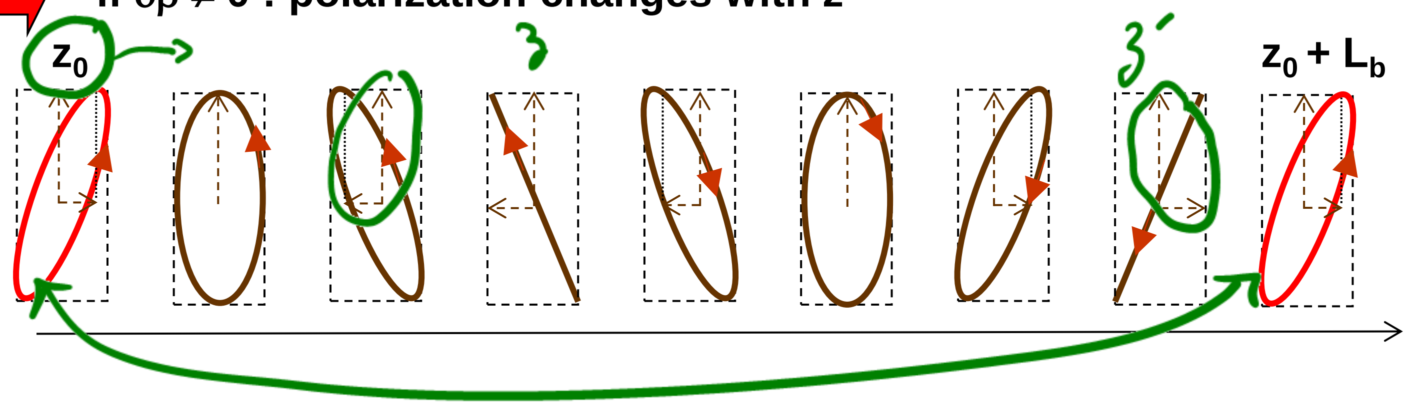
## PROPAGATION IN SINGLE MODE FIBERS (V < 2,405) : VECTORIAL APPROACH ($HE_{11}$ MODES)

### phase effects of the birefringence of the fiber (2) :

$$\beta_x - \beta_y = \delta\beta = \frac{2\pi}{\lambda}(n_{ex} - n_{ey})$$

$$\vec{E} = E_x \cos(\omega t - \beta_x z).\vec{ex} + E_y \cos\left[\omega t - \beta_x z + \overbrace{(\beta_x - \beta_y)}^{\delta\beta} z\right].\vec{ey}$$

**if $\delta\beta \neq 0$ : polarization changes with z**




$\delta\beta . L_b = 2\pi$

$L_b$ = beat length

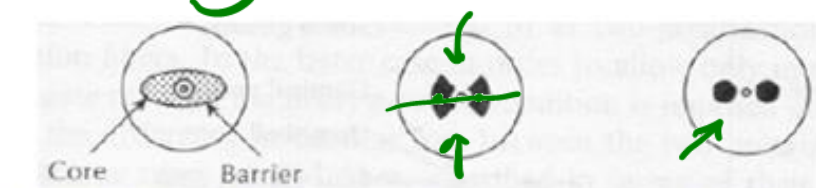
$\beta_x \neq \beta_y$

$L_B$

**In order to maintain a linear polarization in a single mode fiber :**

- **excite only one mode ($HE_{11x}$ or $HE_{11y}$)**
- **avoid mode coupling → $L_b$ as short as few mm → high $\delta\beta$ required → highly birefringent fibers**

$L_B \downarrow \Rightarrow \delta\beta \uparrow$




Elliptical stress-cladding bow-tie PANDA

"polarization maintaining fibers or PM fibers"

## PROPAGATION IN SINGLE MODE FIBERS (V < 2,405) : VECTORIAL APPROACH ($HE_{11}$ MODES)

### group effects of the birefringence of the fiber :

$$\beta_x - \beta_y = \delta\beta = \frac{2\pi}{\lambda}(n_{ex} - n_{ey})$$

$$\vec{E} = E_x \cos(\omega t - \beta_x z).\vec{ex} + E_y \cos\left[\omega t - \beta_x z + \underbrace{(\beta_x - \beta_y)}z\right].\vec{ey}$$

If $n_{ex} \neq n_{ey}$ , then $N_{ex} \neq N_{ey}$ and $v_{gx} = \dfrac{c}{N_{ex}} \neq v_{gy} = \dfrac{c}{N_{ey}}$

$N = n_e - \lambda \dfrac{dn_e}{d\lambda}$

→ **group velocities of pulses propagating in $HE_{11x}$ and $HE_{11y}$ are different**

→ **<u>polarization mode dispersion (PMD)</u>**

$B_G = N_{ex} - N_{ey}$

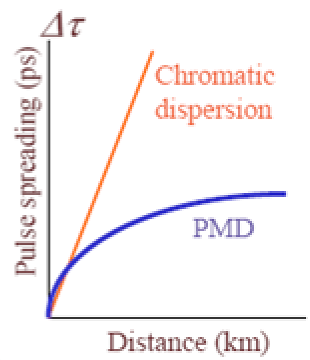
$B_G$ : **<u>normalized group birefringence</u>**

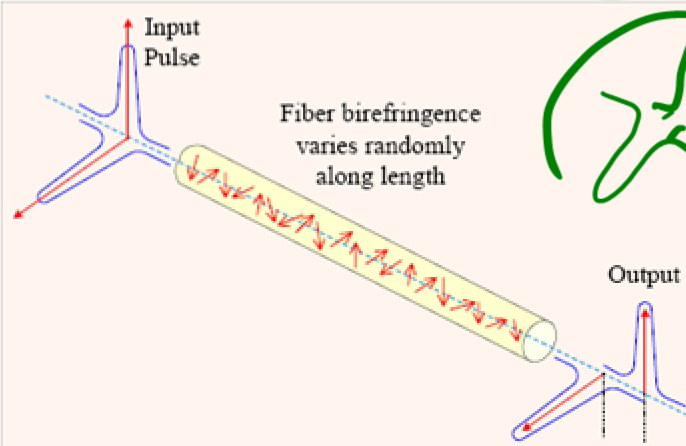
$$\Delta t = |t_{gx} - t_{gy}| = \left|\frac{z}{v_{gx}} - \frac{z}{v_{gy}}\right|$$

$$= |N_{gx} - N_{gy}|\frac{z}{c} = B_G \cdot \frac{z}{c}$$

$\Delta\tau = \Delta t$

due to mode coupling along the fiber (if not PM fiber) →




Pulse spreading caused by polarization dispersion:

$\Delta\tau = D_{PMD}\sqrt{L}$  $D_{PMD} \sim 0.05$ to 1 ps/km$^{0.5}$

If R=40Gbps, L=100→ $D_{PMD}$ <0.25 ps/km$^{0.5}$ to have $\Delta\tau$ < 2.5ps

**End of chapter 3**