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Problema 1

For concave mirrors R is negative: $R = -20\,cm$

4 times greater than the object.

- If we consider real image, the magnification is:

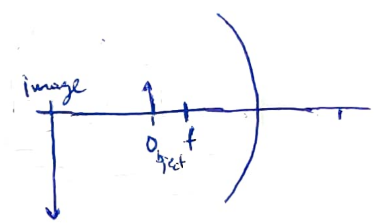
$$M = -\frac{z_2}{z_1} = -4 \Rightarrow z_2 = 4z_1$$

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{2}{-R} \Longrightarrow \frac{1}{z_1} + \frac{1}{4z_1} = \frac{2}{20} = \frac{1}{10}$$

$$\frac{5}{4z_1} = \frac{1}{10} \Rightarrow z_1 = \frac{5}{4} \cdot 10 \Rightarrow \boxed{z_1 = 12{,}5\,cm}$$

$$\Downarrow$$

$$\boxed{z_2 = 50\,cm}$$



- If we consider virtual image, the magnification is:

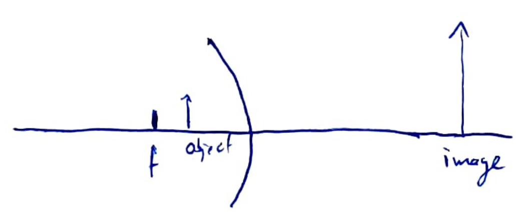
$$M = -\frac{z_2}{z_1} = 4 \longrightarrow z_2 = -4z_1$$

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{2}{-R} \longrightarrow \frac{1}{z_1} - \frac{1}{4z_1} = \frac{1}{10}$$

$$\frac{3}{4z_1} = \frac{1}{10} \longrightarrow z_1 = 10 \cdot \frac{3}{4} = \frac{15}{2} \Rightarrow \boxed{z_1 = 7{,}5\,cm}$$

$$\Downarrow$$

$$\boxed{z_2 = -30\,cm}$$



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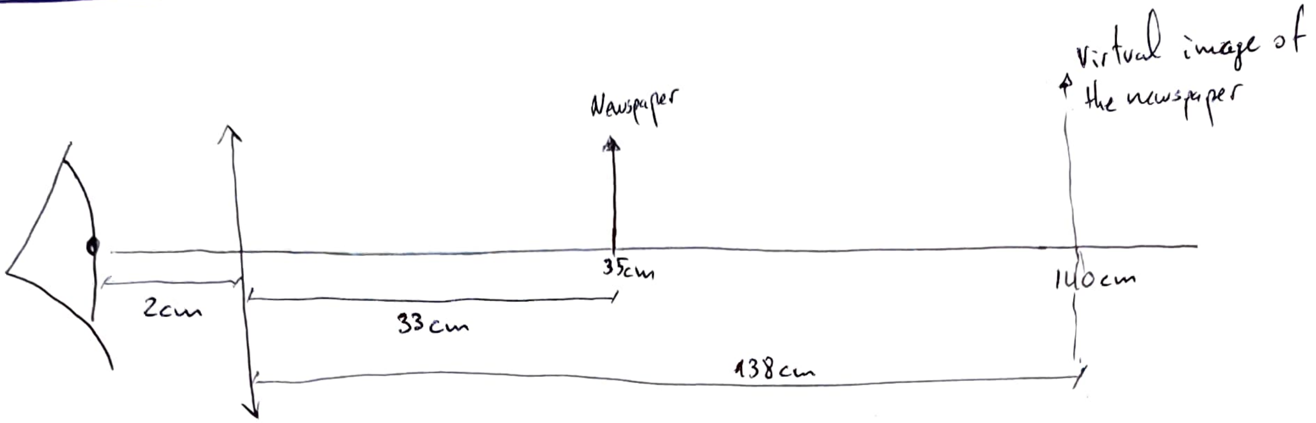
Problem 2



$z_1 = 33\ cm$

$z_2 = -138\ cm$

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} \Rightarrow \frac{1}{33} - \frac{1}{138} = \frac{1}{f} \Rightarrow \frac{1518}{35} = \frac{1}{f}$$

$$\boxed{f = 43{,}37\ cm}$$

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Problem 3

Starting with the formula: $R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right]$

$$R_1 = z_1\left[1 + \left(\frac{z_0}{z_1}\right)^2\right] \Rightarrow \left(\frac{z_0}{z_1}\right)^2 = \frac{R_1}{z_1} - 1 \Rightarrow z_0^2 = z_1^2\left(\frac{R_1}{z_1} - 1\right) = z_1 R_1 - z_1^2$$

$$R_2 = z_2\left[1 + \left(\frac{z_0}{z_2}\right)^2\right] \Rightarrow z_0^2 = z_2 R_2 - z_2^2$$

Using both equations we get z_1

$$z_1 R_1 - z_1^2 = z_2 R_2 - z_2^2 \quad \& \quad z_2 = z_1 + d$$

$$z_1 R_1 - z_1^2 = (z_1 + d) R_2 - (z_1 + d)^2$$

$$z_1 R_1 - z_1^2 = z_1 R_2 + d R_2 - z_1^2 - d^2 - 2 z_1 d$$

$$z_1 (R_1 - R_2 + 2d) = d R_2 - d^2$$

$$\boxed{z_1 = \frac{d(R_2 - d)}{R_1 - R_2 + 2d}}$$

Now for z_2:

$$z_2 = z_1 + d = \frac{d R_2 - d^2}{R_1 - R_2 + 2d} + \frac{R_1 d - R_2 d + 2d^2}{(R_1 - R_2 + 2d)} = \frac{R_1 d + d^2}{R_1 - R_2 + 2d}$$

$$\boxed{z_2 = \frac{d(R_1 + d)}{R_1 - R_2 + 2d}}$$

For z_0 we obtain:

$$z_0^2 = z_1 R_1 - z_1^2 = z_1 (R_1 - z_1) = \frac{d(R_2-d)}{R_1-R_2+2d}\left[R_1 - \frac{d(R_2-d)}{R_1-R_2+2d}\right]$$

$$\boxed{z_0 = \sqrt{\frac{d(R_2-d)}{R_1-R_2+2d}\left[R_1 - \frac{d(R_2-d)}{R_1-R_2+2d}\right]}}$$

Finally the waist:

$$W(z) = W_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2} = \sqrt{\frac{\lambda z_0}{\pi}\left[1 + \left(\frac{z}{z_0}\right)^2\right]}$$

$$\boxed{W(z) = \sqrt{\frac{\lambda}{\pi}\sqrt{\frac{d(R_2-d)}{R_1-R_2+2d}\left[R_1 - \frac{d(R_2-d)}{R_1-R_2+2d}\right]} \cdot \left[1 + z^2 \cdot \frac{1}{\frac{d(R_2-d)}{R_1-R_2+2d}\left[R_1 - \frac{d(R_2-d)}{R_1-R_2+2d}\right]}\right]}}$$

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Problem 4

a)

Our equation: $\sin\theta_q = \sin\theta_i + q\dfrac{\lambda}{\Lambda}$

$q = 2$, $\lambda = 600\,nm$, $\theta_q = 33°$, $\theta_i° = 0$

$$\sin(33°) = 0 + 2\cdot\frac{600\cdot10^{-9}}{\Lambda} \Rightarrow \Lambda = \frac{2\cdot600\cdot10^{-9}}{\sin(33°)}$$

$$\Lambda = 2{,}203\,\mu m/slit = 2203\,nm/slit$$

If the grating wide is 4,4cm:

Number of slits: $N = \dfrac{\text{wide grating}}{\text{length/slit}} = \dfrac{4{,}4\cdot10^{-2}}{2203\cdot10^{-9}} \Rightarrow \boxed{N = 19970 \text{ slits}}$

b)

Rayleigth criterion $\Delta(\sin\theta)_{min} = \dfrac{\lambda}{Nd}$

and: $\Delta(\lambda)_{min} = \dfrac{\lambda}{qN}$

$$\Delta(\lambda)_{min} = \frac{600\cdot10^{-9}}{2\cdot19970} = 1{,}502\cdot10^{-11} \longrightarrow \boxed{\Delta(\lambda)_{min} \simeq 0{,}015\cdot nm}$$