Lecture 1 & 2

Planck Hypotesis:
- Energy of single photon. $E = h\nu$
- Energy of a beam: $E_{beam} = n^{o}_{photons}\, h\nu$

Photoelectric effect:

$$h\nu = E_K + \Phi$$

- $h\nu$: single photon energy
- E_K: kinetik en. of single e^-
- Φ: Work function: → The potential energy needed to extract one e^- from the metal

∘) There is a freq. threshold: ν_{Th}

$E_K = h\nu - \Phi \rightarrow E_K$ can't be negative

$h\nu - \Phi \geq 0 \Rightarrow \nu_{Th} = \frac{\Phi}{h}$

∘) To meassure the kinetik energy we calculate the potential energy needed to repeal the $e^- \rightarrow V_{stop}$

$$E_K = eV_{stop}$$

⊛ Flux Problem:

$I = \frac{P}{S}$, $E_{tot} = I \cdot S \cdot t$, $n^{o}_{photons} = \frac{E_{tot}}{h\nu}$

I: intensity or flux

Bohr's Model

$$L_0 = n\hbar$$

Quantized angular momentum

∘) Orbits: $F_{centrifuga} = F_{elec.}$ $\quad m\frac{v_R^2}{r} = \frac{e^2}{r^2} \Rightarrow r = \frac{\hbar^2}{me^2} n^2 = r_0 n^2$

∘) Velocity: $L_0 = m v_R \cdot r \Rightarrow v_R = \frac{e^2}{\hbar} \frac{1}{n} = \alpha c \frac{1}{n}$

∘) Energy: $E_{Tot} = E_K + E_{pot} \Rightarrow E = \frac{1}{2} m v_R^2 - \frac{e^2}{r} \Rightarrow E = -\frac{me^4}{2\hbar^2} \frac{1}{n^2} = -\frac{R}{n^2}$

∘) Energy Levels: $\Delta E = E_2 - E_1 = h\nu_{21}$

↳ ~~n^o rotations from one level to other (decay) $= \tau_{21} \cdot \nu_{21}$~~ ?

⊛ Angular momemt

$\vec{L} = m\vec{r} \times \vec{v}$

⊛ Velocidad angular

$\omega = \frac{v_R}{r}$

⊛ Forza centrifuga

$|F_{cf}| = m\omega^2 r$

Lecture 3 & 4

Wave Nature of Matter

↳ Momentum of a photon $\quad p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} = \hbar k$

$p = \frac{h}{\lambda} \quad , \quad p = \hbar k \qquad k = \frac{2\pi}{\lambda}$

⊛ From kinetik Energy

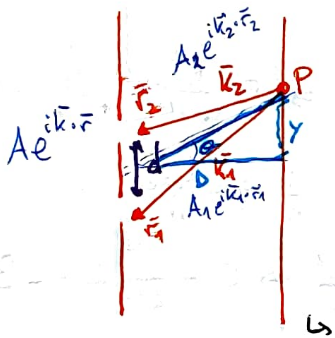
$$E = \frac{p^2}{2m}$$

↳ De Broglie wavelength

$$\lambda_{DB} = \frac{h}{p} = \frac{h}{m \cdot v}$$

$$\lambda_{DB} = \frac{h}{\sqrt{2mE}}$$

Particle Interferences



→ Two slits the same : $A_0 = A_1 = A_2$

→ screen far away $k = k_1 = k_2$

→ $\bar{r}_1 - \bar{r}_2 = \vec{d}$

Intesity in P $\quad I_p = |A|^2 \left| e^{i\bar{k}\cdot\bar{r}_1} + e^{i\bar{k}\cdot\bar{r}_2} \right|^2$

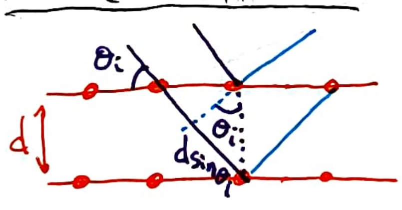
$$I_p = 2\,|A|^2 \left[1 + \cos\left(\bar{k} \cdot (\bar{r}_1 - \bar{r}_2) \right) \right]$$

•) Constructive Interference : $d \sin\theta = m\lambda$

•) Destructive Interference: $d \sin\theta = (m + \frac{1}{2})\lambda$

↳ $\tan\theta \simeq \sin\theta \simeq \theta \simeq \frac{y}{D}$

Particle Diffraction



Bragg

$$2d \sin\theta_i = n\lambda$$

n → diffraction order

Lecture 5, 6, 7, 8 & 9

Schrödinger Equation

•) General: $i\hbar\,\partial_t \Psi(\bar{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\bar{r},t)\right]\Psi(\bar{r},t)$

↳ If $V(\bar{r},t) = V(\bar{r})$ (conservative) $\Rightarrow$ $\Psi(\bar{r},t) = \Psi(\bar{r})\, e^{-i\frac{E}{\hbar}t}$

↳ S.E. indep of time: $\nabla^2\Psi(\bar{r}) + \frac{2m}{\hbar^2}\left(E - V(\bar{r})\right)\Psi(\bar{r}) = 0$
(Stationary State)

↳ We obtain eigenvalue and eigenfunction: $\{E, \Psi_E(\bar{r})\}$

↳ General solution: $\Psi(\bar{r},t) = \sum_{\{E\}} C_E\, \Psi_E(\bar{r})\, e^{-i\frac{E}{\hbar}t}$

•) Eigenfunctions properties: $\Psi_E(\bar{r})$

↳ $\Psi_E(\bar{r})$: continous, differenciable, finite $|\bar{r}| \to \infty$

↳ Orthonomality: $\int \Psi^*_{E_1}(\bar{r})\, \Psi_{E_2}(\bar{r})\, d\bar{r} = \begin{cases} \delta_{E_1,E_2} & \to \text{if discrete} \\ \delta(E_1 - E_2) & \to \text{if continous} \end{cases}$

$\delta = \begin{cases} 1 \to E_1 = E_2 \\ 0 \to E_1 \neq E_2 \end{cases}$

$\delta(x-\alpha) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ip(x-\alpha)}\,dp$

$\delta(\alpha x) = \frac{1}{|\alpha|}\delta(x)$

$\int_{-\infty}^{\infty} f(t)\,\delta(t-T) = f(T)$

↳ $\int_{-\infty}^{\infty}\delta(x)\,dx = 1$

•) Wavefunctions properties: $\Psi(\bar{r},t)$

↳ Square-integrable : $|\Psi(\bar{r},t)|^2$ debe existir

↳ Prob. of finding the part. in $\bar{r}$ and $\bar{r}+d\bar{r}$

↳ $\int |\Psi(\bar{r})|^2\, d\bar{r} = 1$

↳ Observables

$$\langle \hat{O}(t)\rangle = \int \Psi^*(\bar{r},t)\,\hat{O}\,\Psi(\bar{r},t)\,d\bar{r}$$

↳ $\hat{p} = -i\hbar\nabla$

$$\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{e}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \theta}\hat{e}_\theta + \frac{1}{\rho\sin\theta}\frac{\partial f}{\partial \varphi}\hat{e}_\varphi$$

Free Particle

↳ $V=0 \Rightarrow \frac{d^2}{dx^2}\psi(x) + \frac{2mE}{\hbar^2}\psi(x) = 0 \xrightarrow{\text{solutions}} \psi(x) = Ae^{\pm ikx}$

2 set of sols. $\begin{cases} E, \psi^+ \\ E, \psi^- \end{cases}$

The solution into the eq. gives us the eigenvalues E $\quad E = \frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m}$

We have two states $\pm ikx$!

General solution → The superposition (normalised) $\Rightarrow \psi_K = \frac{1}{\sqrt{2\pi}}\left(e^{ikx} + e^{-ikx}\right) \cdot e^{-i\frac{E(k)}{\hbar}t}$

↳ The solution for free particle are plane waves but it is not totally true instead we have wavepacket (superposition of plane waves with a weight)

$$\psi(x,t) = \int_{-\infty}^{+\infty} W(k)\, e^{ikx - i\frac{E}{\hbar}t}\, dk$$

Infinite Potential Well

↳ $V = \begin{cases} \infty & x<0,\ x>L \\ 0 & 0 \leq x \leq L \end{cases} \Rightarrow$

outside: The prob of finding the particle is zero $\psi(x) = 0$

Inside: $V=0 \rightarrow \frac{d^2}{dx^2}\psi(x) + \frac{2m}{\hbar^2}E\,\psi(x) = 0$ + B.C $\psi(0) = \psi(L) = 0$

↳ Solution: Infinite set of solutions: $\left\{\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right),\ E_n = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}n\right)^2\right\}$ $\quad K_n = \frac{n\pi}{L}$

↳ Orthonormality: $\int \psi_m^*(x)\,\psi_n(x)\,dx = \delta_{mn}$

↳ Pauli's exclusion principle: There only can be 2 e^- (diff spin) at the same energy level

$N^o_{e^-} = 2n_F \Rightarrow E_F = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\cdot\frac{N}{2}\right)^2 \Rightarrow K_F = \frac{\pi}{2L}N \Rightarrow \underset{\text{velocity}}{V_F} = \frac{P_F}{m} = \frac{\hbar k_F}{m}$

3D Confined Particle

↳ Solution: $\left\{\psi_{n_x,n_y,n_z}(\vec{r}) = \left(\sqrt{\frac{2}{L}}\right)^3 \sin(k_x x)\sin(k_y y)\sin(k_z z),\ E_{n_x,n_y,n_z} = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2\left(n_x^2 + n_y^2 + n_z^2\right)\right\}$

↳ Conditions

$K_x = \frac{\pi}{L}n_x$

$k_y = \frac{\pi}{L}n_y$

$K_z = \frac{\pi}{L}n_z$

↳ Quantum Degeneracy

i.e. ψ_{213} and ψ_{123} have the same Energy $E_{213} = E_{123}$ but not the same prob. distribution

↳ Fermi energy level

$\frac{1}{8}$ x Volume sphere radious n_F

2 possible e^- in each energy level

$N^o_{e^-} = 2 \cdot \frac{1}{8}\frac{4\pi}{3}n_F^3$

$n_F = \left(\frac{3N}{\pi}\right)^{1/3}$, $\quad E_F = \frac{\hbar^2}{2m}K_F^{2/3}$

$K_F = \left(3\pi^2\frac{N}{L^3}\right)^{1/3}$, $P_F = \hbar K_F$, $V_F = \frac{P_F}{m}$

•) Finite Asymmetrical Well

$$V(x) = \begin{cases} V_1 & x<0 \\ 0 & 0 \le x \le a \\ V_2 & x>a \end{cases} \overset{\text{Solution}}{\Longrightarrow} \psi(x) \begin{cases} B_1\, e^{+K_1 x} \\ C(k)\, \sin[kx+\delta(k)] \\ A_2\, e^{-K_2 x} \end{cases}$$

$$K_1 = \frac{\sqrt{2m(V_1-E)}}{\hbar} \qquad k = \frac{\sqrt{2mE}}{\hbar} \qquad K_2 = \frac{\sqrt{2m(V_2-E)}}{\hbar}$$

↳ orthogonality

$$|C(k)|^2 = \frac{4K}{2ka + \sin 2\delta - \sin 2ka + 2\delta}$$

↳ Continuity

$$\begin{cases} \delta(k) = \pi n_1 + \sin^{-1}\left(\frac{\hbar k}{\sqrt{2mV_1}}\right) \\ \delta(k) = \pi n_2 + \sin^{-1}\left(\frac{\hbar k}{\sqrt{2mV_2}}\right) - ka \end{cases}$$

$$\hookrightarrow ka = -\sin^{-1}\left(\frac{\hbar k}{\sqrt{2mV_1}}\right) - \sin^{-1}\left(\frac{\hbar k}{\sqrt{2mV_2}}\right) - \pi(n_2 - n_1)$$

↳ When $V = V_1 = V_2$

$$ka = -2a \sin^{-1}\left(\frac{\hbar k}{\sqrt{2mV}}\right) + \pi n \xrightarrow{E \ll V} k_n = \frac{\pi}{a}\left[1 - \frac{\hbar\sqrt{2}}{a\sqrt{mV}}\right] n \qquad E_n = \frac{\hbar^2}{2m} k_n^2$$

$$E_n = \underbrace{\frac{\hbar^2 \pi^2}{2ma^2}}_{E_1} \underbrace{\left(1 - \frac{\hbar\sqrt{2}}{a\sqrt{mV}}\right)^2}_{\text{correction}} n^2$$

↳ There is a small prob. of finding part. outside the well (tunnelling).

$B_1 = C(k) \sin\delta(k) \rightarrow$ continuity at $x=0$

$$\text{Prob} = \int_{-d}^{0} |\psi(x,t)|^2 dx = |B_1|^2 \int_{-d}^{0} e^{2K_1 x} dx = |B_1|^2 \frac{1 - e^{2K_1 d}}{2K_1}$$

•) Parabollic Well

↳ Potential $V(x) = \frac{1}{2} K x^2 \Rightarrow$ S.E. $\left[\frac{d^2}{d\bar{x}^2} + (2\bar{E} - \bar{x}^2)\right]\psi(\bar{x}) = 0 \xrightarrow{\text{try}} \psi(\bar{x}) = e^{-\frac{\bar{x}^2}{2}} U(\bar{x})$

↳ $\omega_0 = \sqrt{\frac{K}{m}}$ ↳ $\bar{x} = \frac{x}{\sqrt{\hbar/m\omega_0}}$ ↳ $\bar{E} = \frac{E}{\hbar\omega_0}$

↳ Solutions

$$\left\{ \psi_n(x) = A_n \underbrace{e^{-\frac{m\omega_0}{2\hbar}x^2}}_{\text{gaussian}} \underbrace{H_n\left(\sqrt{\frac{m\omega_0}{\hbar}}\, x\right)}_{\text{hermite polynomial}}, \quad E_n = \hbar\omega_0\left(n + \frac{1}{2}\right) \right\}$$

↳ energy equally spaced

•) Heisenberg

$$\Delta p(t) = \sqrt{\langle p^2(t)\rangle - \langle p(t)\rangle^2}$$

$$\Delta x(t) = \sqrt{\langle x^2(t)\rangle - \langle x(t)\rangle^2}$$

$$\Delta p\, \Delta x \le \frac{\hbar}{2}$$

Lecture 10, 11, 12, 13 & 14

∘) Inverse Barrier

↳ $V(x) = \begin{cases} V & 0 \le x \le a \\ 0 & x<0,\ x>a \end{cases} \xrightarrow{\text{Solutions}} \psi(x) = \begin{cases} A_1 e^{ikx} + B_1 e^{-ikx} \\ A e^{ik_1 x} + B e^{-ik_1 x} \\ A_2 e^{ikx} \end{cases}$ $\quad k = \frac{\sqrt{2mE}}{\hbar}$ $\quad k_1 = \frac{\sqrt{2m(V-E)}}{\hbar}$

↳ Using continuity get transmission

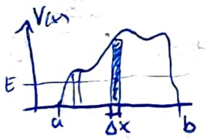
$T = \left|\frac{A_2}{A_1}\right|^2 = \left[1 + \frac{V^2}{4E(V-E)} \sinh^2(k_1 a)\right]^{-1}$ (probability) $\qquad R = \left|\frac{B_1}{A_1}\right|^2 \quad 1 + \frac{4E(V-E)}{V^2} \cdot \frac{1}{\sinh^2(k_1 a)}$

$T + R = 1$

↳ If $E>V$ but just above the barrier there is still reflection ⇒ Because e^- is not a ball!!

$a k_1 = a \frac{\sqrt{2m(E-V)}}{\hbar} i$, $\quad \frac{\sinh(ix)}{i} = \sin x$ $\Big\}$ ⇒ $R \neq 0$ → Transmission resonance to avoid reflection $\quad k_1 a = \pi n$

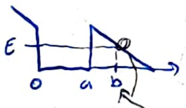
∘) Generic Barrier

↳ 

↳ $V(x)$ is big ⇒ $V - E \gg \frac{\hbar^2}{2ma^2}$ ⇒ $k_1 a \gg 1$ ⇒ $\sinh^2(k_1 a) = \left(\frac{e^{k_1 a} - e^{-k_1 a}}{2}\right)^2 \simeq \frac{e^{2k_1 a}}{4}$

⇒ $T \simeq T_0 e^{-2k_1 a} \xrightarrow[\text{for each } x:\ a \to \Delta x]{} P_{Tot} = \exp\left\{-2\sqrt{\frac{2m}{\hbar^2}} \int_a^b \sqrt{V(x) - E}\, dx\right\}$

∘) Potential Well in Electric Field

↳ 

$V(x) = \begin{cases} V_0 - |e|\varepsilon(x-a) & x>a \\ 0 & 0 \le x \le a \\ V_0 + |e|\varepsilon(x-a) & x<0 \end{cases}$

↳ Fermi energy

$E = E_F$

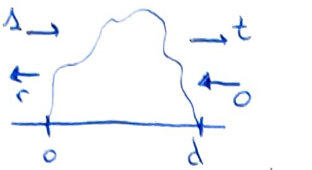
$\phi = V_0 - E_F$

Tunnelling at integral$_{a,b}$: $a \to \frac{V_0 - E}{|e|\varepsilon} + a = b$ $\qquad V_0 - E = \overbrace{d}^{b-a}|e|\varepsilon$

$P_T = \exp\left\{-\frac{4}{3|e|\varepsilon}\sqrt{\frac{2m}{\hbar^2}}(V_0 - E)^{3/2}\right\} \simeq \exp\left\{-\frac{4}{3}\sqrt{\frac{2m|e|\varepsilon}{\hbar^2}}\, d^{3/2}\right\}$

↳ STM

•) Transfer Matrix

↳ 

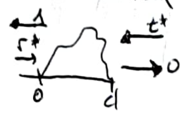
$\bar{u}(d) = M\,\bar{u}(0)$

moving dir.

$\bar{u}(0) = \begin{pmatrix} 1 \\ r \end{pmatrix}$ $\bar{u}(d) = \begin{pmatrix} t \\ 0 \end{pmatrix}$

moving dir.

↳ Time reversal



$\bar{u}(d) = M\,\bar{u}(0)$

$\bar{u}(d) = \begin{pmatrix} 0 \\ t^* \end{pmatrix}$ $\bar{u}(0) = \begin{pmatrix} r^* \\ 1 \end{pmatrix}$

↳ The matrix has to fullfill both time normal and time reversal

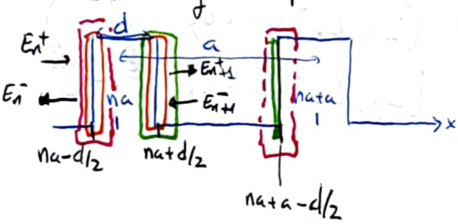
$$M = \frac{1}{1-|r|^2}\begin{pmatrix} t & -tr^* \\ -t^*r & t^* \end{pmatrix} \qquad \det(M) = \frac{|t|^2}{1-|r|^2}$$

⊛ No absorption: $M = 1$

⊛ Absorption: $M \neq 1$

•) Bloch Modes in a Periodic Structure

↳ First using transfer matrix



$K = \frac{\omega}{c}$

Barrier: $\begin{pmatrix} E^+_{n+1} \\ E^-_{n+1} \end{pmatrix}\Bigg|_{x=na+\frac{d}{2}} = \overbrace{\frac{1}{1-|r|^2}\begin{pmatrix} t & -tr^* \\ -t^*r & t^* \end{pmatrix}}^{M_1}\begin{pmatrix} E^+_n \\ E^-_n \end{pmatrix}\Bigg|_{x=na-\frac{d}{2}}$

Travel to next Barrier: $\begin{pmatrix} E^+_{n+1} \\ E^-_{n+1} \end{pmatrix}\Bigg|_{x=na+a-\frac{d}{2}} = \underbrace{\begin{pmatrix} e^{ik(a-d)} & 0 \\ 0 & e^{ik(a-d)} \end{pmatrix}}_{M_2}\begin{pmatrix} E^+_n \\ E^-_n \end{pmatrix}\Bigg|_{x=na+\frac{d}{2}}$

⊛ Resultant chain matrix:

$$\begin{pmatrix} E^+_{n+1} \\ E^-_{n+1} \end{pmatrix}\Bigg|_{x=na+a-\frac{d}{2}} = M_2 M_1 \begin{pmatrix} E^+_n \\ E^-_n \end{pmatrix}\Bigg|_{x=na-\frac{d}{2}}$$

↳ Bloch Theorem:

$$E_{n+1}(x+a) = e^{ika} E_n(x)$$

$$\Downarrow$$

$$\begin{pmatrix} E^+_{n+1} \\ E^-_{n+1} \end{pmatrix}\Bigg|_{x=na+a-\frac{d}{2}} = \underbrace{\begin{pmatrix} e^{ika} & 0 \\ 0 & e^{ika} \end{pmatrix}}_{M_3}\begin{pmatrix} E^+_n \\ E^-_n \end{pmatrix}\Bigg|_{x=na-\frac{d}{2}}$$

$$\Rightarrow \underbrace{\left[M_2 M_1 - M_3\right]}_{M}\begin{pmatrix} E^+_n \\ E^-_n \end{pmatrix}\Bigg|_{x=na-\frac{d}{2}} = 0$$

⊛ The determinant of M gives us the equation of the structure

$$0 = \frac{1-|r|^2}{|t|^2} e^{2ika} - 2\,\mathrm{Re}\left[\frac{e^{ik(a-d)}}{t^*}\right] e^{ika} + 1$$

⊛ Bloch theorem for e^-

$$\psi(x+a) = e^{ika}\psi(x)$$

Lecture 15, 16, 17 & 18

•) Angular Momentum

↳ Operators: Compute $\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$ and $p_x = -i\hbar\,\partial_x$

$$\hat{L}_x = y p_z - z p_y = -i\hbar\,(y\partial_z - z\partial_y)$$

$$\hat{L}_y = z p_x - x p_z = -i\hbar\,(z\partial_x - x\partial_z)$$

$$\hat{L}_z = x p_y - y p_x = -i\hbar\,(x\partial_y - y\partial_x)$$

$$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$$

↳ Theorem:

- If operator commute we can always find a common set of eigenfunctions that describes both.
- I cannot measure simultaneusly two observables if they don't commute

$$[L^2, L_x] = [L^2, L_y] = [L^2, L_z] = 0 \rightarrow \text{Common base}$$

↳ Commutators and properties

- $[A,B] = A\cdot B - BA$
- $[A,B] = -[B,A]$
- $[A,BC] = [A,B]C + B[A,C]$
- $[A+B,C] = [A,C] + [B,C]$
- $[A,B+C] = [A,B] + [A,C]$

↳ Commutators Results

$[\hat{p}, \hat{E}_k] = 0$ $\qquad [\hat{L}_x, L_y] = i\hbar L_z$

$[\hat{x}, \hat{p}_x] = i\hbar$ $\qquad [L_z, L_x] = i\hbar L_y$

$[L_y, L_z] = i\hbar L_x$

•) Orbital equation

$$\begin{cases} L^2\,\varphi_{\ell m} = \hbar^2\,\ell(\ell+1)\,\varphi_{\ell m} \\ L_z\,\varphi_{\ell m} = \hbar\, m_{\ell m}\,\varphi_{\ell m} \end{cases}$$

$\ell = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$

$m_\ell = -\ell, \ldots, -1, 0, 1, \ldots, \ell$

$$L = \hbar\sqrt{\ell(\ell+1)}$$

↳ Orbital magnetic momentum

$a = \pi r^2 \hat{a}$

$I = q\nu$

$$\vec{\mu} = I a \hat{a} = q\nu\pi r^2 \hat{a} = -|e|\nu\pi r^2 \hat{a}$$

$$\vec{L} = \vec{r} \times \vec{p} = mrv = mr\omega r = 2\pi m \nu r^2 \hat{a}$$

$$\vec{\mu} = -\frac{|e|}{2m}\vec{L} \longrightarrow |\vec{\mu}| = \mu_B\sqrt{\ell(\ell+1)}$$

↳ Zeeman (apply Mag field)

$$\text{Energy} = E_0 - \underbrace{\vec{\mu}\cdot\vec{B}}_{\mu_z} = E_0 + \mu_B B\, m_\ell$$

•) SPIN

$$\begin{cases} S^2\,\chi_{s,m_s} = \hbar^2 s(s+1)\,\chi_{s,m_s} \\ S_z\,\chi_{s,m_s} = \hbar\, m_s\,\chi_{s,m_s} \end{cases}$$

$$s = \frac{1}{2} \Rightarrow \begin{array}{l} m_s = -\frac{1}{2} \\ m_s = +\frac{1}{2} \end{array}$$

↳ Spin magnetic momentum

$$\vec{\mu} = -\frac{|e|}{m}\vec{S}$$

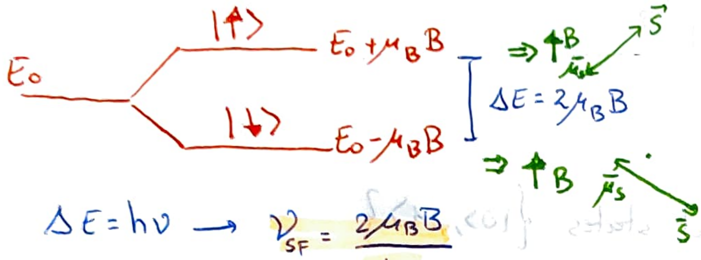
↳ Full wavefunction: in potential well

$$\Psi = A\sqrt{\frac{2}{L}}\,\sin\left(\frac{n\pi}{L}x\right)\varphi_{\ell,m_\ell}\,\chi_{s,m_s}$$

•) Spin Flip

↳ Constant mag. field $\vec{B} = B\vec{k}$

$E = E_0 - \vec{\mu}\cdot\vec{B} = E_0 - \mu_z B = E_0 + \mu_B 2 m_s B$

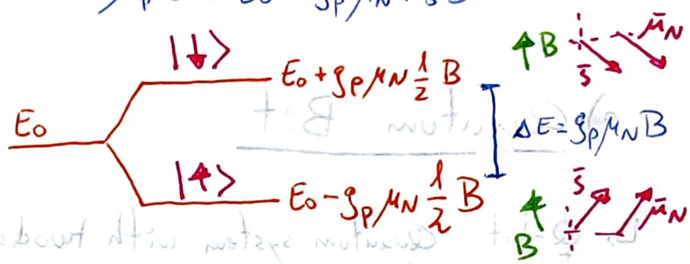


$\Delta E = h\nu \longrightarrow \nu_{SF} = \frac{2\mu_B B}{h}$

•) Nuclear Magnetic Resonance

↳ Protons: $l=0$, $s=\frac{1}{2}$ $\quad \vec{\mu} = +g_p \frac{|e|}{2m_p}\vec{s} = +g_p \mu_N \vec{s}$

$E = E_0 - \vec{\mu}_p\cdot\vec{B} = E_0 - g_p \mu_N m_s B$



$\Delta E = h\nu \Rightarrow \nu_{SF_p} = \frac{g_p \mu_N B}{h}$

Lecture 19 & 20

o) Quantum Bit

↳ q-bit = Quantum system with 2 distinct states $\{|0\rangle, |1\rangle\}$

$$a_0|\psi_0(x)\rangle + a_1|\psi_1(x)\rangle = a_0|0\rangle + a_1|1\rangle$$

↳ Operators:

$$I = |0\rangle\langle 0| + |1\rangle\langle 1|$$

$$\sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|$$

$$\sigma_y = i\left(|1\rangle\langle 0| - |0\rangle\langle 1|\right)$$

$$\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

↳ Commutators:

$$[\sigma_x, \sigma_y] = 2i\sigma_z$$

↳ Eigenstates:

$$\sigma_x \rightarrow \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

$$\sigma_y \rightarrow \frac{1}{\sqrt{2}}(|0\rangle \pm i|1\rangle)$$

$$\sigma_z \rightarrow |0\rangle, |1\rangle$$

↳ Matrix form

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\psi = a_0|0\rangle + a_1|1\rangle = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}$$

⊛ Calculation $\sigma_x[a_0|0\rangle + a_1|1\rangle] = a_0|1\rangle + a_1|0\rangle$

$$\sigma_x: \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_0 \end{pmatrix}$$

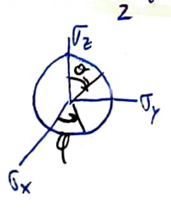
$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \langle 0|\sigma_x|0\rangle = (\sigma_x)_{00}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ 1 & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

↳ Bloch Sphere:

$$\psi = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$



↳ Entanglement:

⊛ The superposition state is said to be entangled when you can't split the superposition into the product of two independent states

$$|\psi\rangle \neq |\lambda\rangle_a \otimes |\phi\rangle_b$$

$$|\psi\rangle = \alpha|\phi_1\rangle_a|\phi_2\rangle_b + \beta|\phi_2\rangle_a|\phi_1\rangle_b$$

$\{|\phi_1\rangle_a, |\phi_2\rangle_a\} \rightarrow 2 \perp$ states part. a

$\{|\phi_1\rangle_b, |\phi_2\rangle_b\} \rightarrow 2 \perp$ states part b

↳ No clonning theory