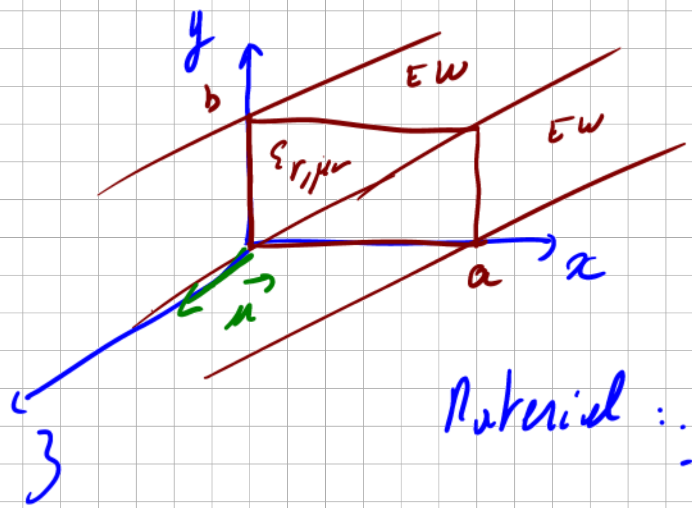
S. Verdeyme
Tutorial 9
Oct 4th 2021

Metallic rectangular waveguide

I Introduction



EW: electric wall

$\sigma \to \infty$

$\underline{a} > \underline{b}$

Material:
- linear
- isotropic
- homogeneous
- lossless
- nondispersive

In such waveguide, solutions
- TE modes ($E_z = 0$, $H_z \neq 0$)
- TM modes ($H_z = 0$, $E_z \neq 0$)

no TEM mode (only 1 conductor)

no hybrid mode ($E_z \neq 0$, $H_z \neq 0$)

see demonstration after

$\gamma = \alpha$ evanescent mode or $\gamma = j\beta$ propagating mode

II TE modes – $\gamma = j\beta$

2-1 Propagation equation

$$\begin{cases} \Delta_t H_z(x,y) + k_c^2 H_z(x,y) = 0 \\ \dfrac{\partial H_z(x,y)}{\partial n} = 0 \text{ on the EW} \end{cases}$$

$$\frac{\partial^2 H_z(x,y)}{\partial x^2} + \frac{\partial^2 H_z(x,y)}{\partial y^2} + k_c^2 H_z(x,y) = 0$$

$$k_c^2 = k_0^2 + \gamma^2$$
$$= k_0^2 - \beta^2$$

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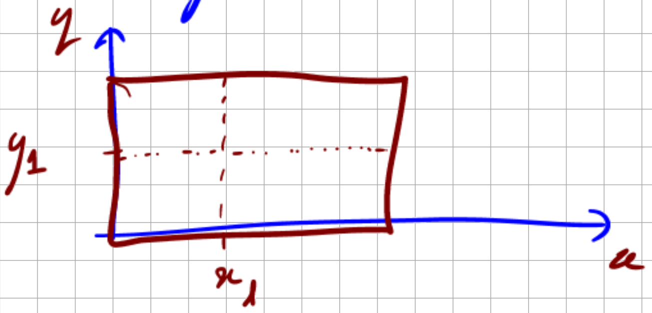
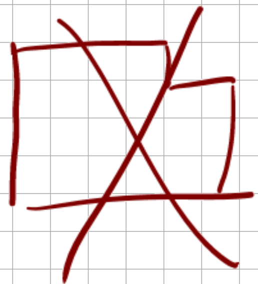
$$H_z(x,y) = f(x) \times g(y)$$





$$g(y)\frac{\partial^2 f(x)}{\partial x^2} + f(x)\frac{\partial^2 g(y)}{\partial y} + k_c^2 f(x)g(y) = 0$$

$$f(x) \neq 0 \text{ and } g(y) \neq 0 \quad (\vec{E} \neq 0, \vec{H} \neq 0)$$

$$\frac{1}{f(x)}\frac{\partial^2 f(x)}{\partial x^2} + \frac{1}{g(y)}\frac{\partial^2 g(y)}{\partial y^2} + \underbrace{k_c^2}_{} = 0$$

$$k_c^2 = k_x^2 + k_y^2$$

$$\frac{1}{f(x)}\frac{\partial^2 f(x)}{\partial x^2} + k_x^2 = 0$$

$$\frac{1}{g(y)}\frac{\partial^2 g(y)}{\partial y^2} + k_y^2 = 0$$

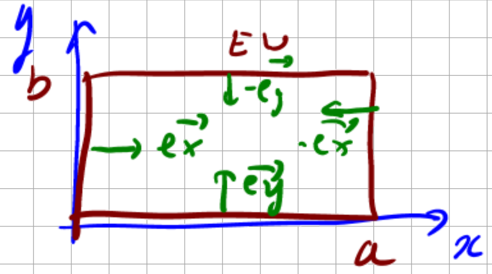
$$H_z(r,\theta) = f(r)\,g(\theta)$$

$$f(x) = A\cos(k_x x)x + B\sin(k_x x)x$$

$$g(y) = C\cos(k_y)y + D\sin(k_y)y$$

From boundary conditions:

$$\frac{\partial H_z(x,y)}{\partial n} = 0 \text{ on } K_c \text{ EW}$$



$x = 0 \qquad \frac{\partial H_z(x,y)}{\partial x} = 0$

$x = a \qquad \frac{\partial H_z(x,y)}{\partial x} = 0$

$y = 0 \qquad \frac{\partial H_z(x,y)}{\partial y} = 0$

$y = b \qquad \frac{\partial H_z(x,y)}{\partial y} = 0$

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$x=0 \quad g(y)\,\dfrac{\partial f(x)}{\partial x}=0 \Rightarrow \dfrac{\partial f(x)}{\partial x}=0$

$-A k_x \sin k_x x + B k_x \cos k_x x = 0$

$\Rightarrow B=0$

$x=a \quad f(x) = A\cos k_x x$

$\dfrac{\partial f(x)}{\partial x} = -A k_x \sin k_x x = 0$

$\sin k_x a = 0$

$k_x a = m\pi \quad , m = 0, 1, 2, 3 \ldots$

$\| \; k_x = \dfrac{m\pi}{a}$

$y=0 \quad \dfrac{\partial g(y)}{\partial y} = 0 \Rightarrow D = 0$

$g(y) = C\cos k_y y + D \sin k_y y$

$y=b \quad \sin k_y b = 0$

$\| \; k_y = \dfrac{n\pi}{b} \quad , n = 0, 1, 2, 3, \ldots$

$k_c^2 = k_x^2 + k_y^2$

$= \left(\dfrac{m\pi}{a}\right)^2 + \left(\dfrac{n\pi}{b}\right)^2$

$\| \; H_z(x,y) = \dfrac{AC}{H_0} \cos \dfrac{m\pi}{a} x \cos \dfrac{n\pi}{b} y$

2.2 - Cut-off frequencies

$k_c^2 = k_0^2 - \beta^2$

$= \left(\dfrac{m\pi}{a}\right)^2 + \left(\dfrac{n\pi}{b}\right)^2$

$= \left(\dfrac{\omega}{v}\right)^2 - \beta^2$

$\varepsilon_r = 1$

$a = 22.86 \text{ mm} \qquad \mu_r = 1$

$m = 1 \; , n = 0$

$\left(\dfrac{\pi}{a}\right)^2 = \left(\dfrac{\omega}{c}\right)^2 \varepsilon_r \mu_r - \beta^2$

$= \left(\dfrac{\pi}{0{,}02286}\right)^2 = 18\,886$

$\omega = 10 \text{ rd/s} \quad \left(\dfrac{10}{3\times 10^8}\right)^2 - \beta^2 = 18\,886$

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Cut of frequency is the lower frequency for wich β exists for given n, m

$f_{cTE_{nm}} \rightarrow \beta = 0$

$$k_c^2 = \left(\frac{\omega_{cTE_{nm}}}{c}\right)^2 \varepsilon_r \mu_r$$

$$f_{cTE_{nm}} = \frac{c}{2\pi\sqrt{\varepsilon_r \mu_r}} \sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}$$

$$f_{cTE_{10}} = \frac{3 \cdot 10^8}{2} \frac{1}{0{,}02286}$$

$$= 6.56 \text{ GHz}$$

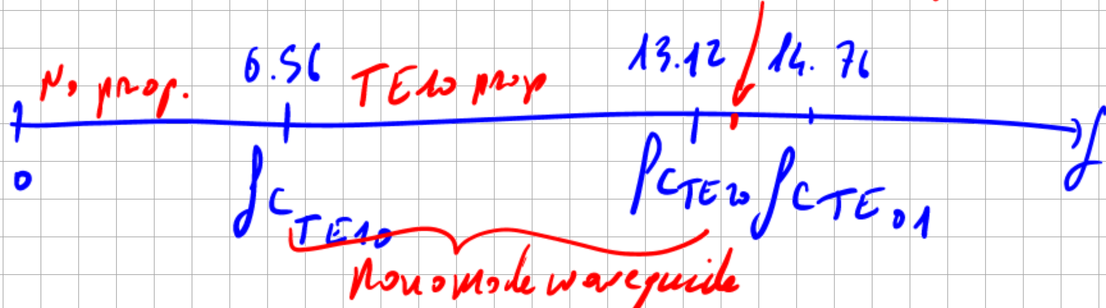
$$= \frac{c}{2a}$$

$b = 10.16$ mm

$$f_{cTE_{01}} = \frac{c}{2} \frac{1}{b} = 14.76 \text{ GHz}$$

(n, m)

TE10, TE20 prop.

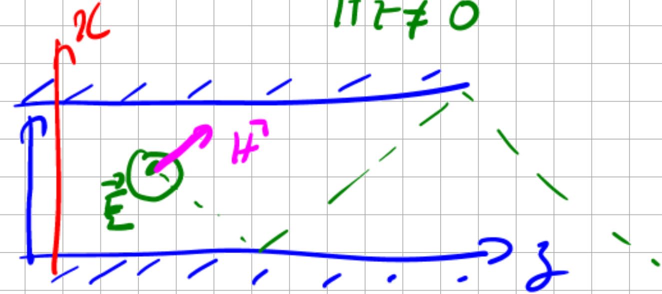


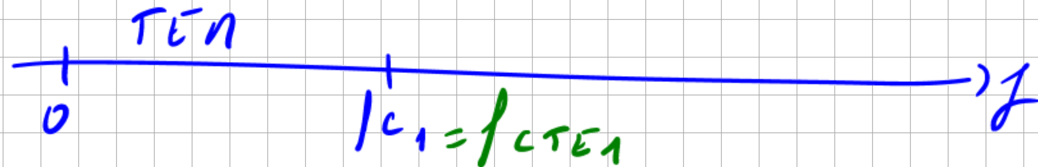
$f_{cTE_{20}} = 2 \times f_{cTE_{10}}$

Monomode waveguide $f_{cTE_{10}} < f < f_{cTE_{20}}$

Tutorial 3:

$$f_{c_1} = \frac{c}{4d}$$





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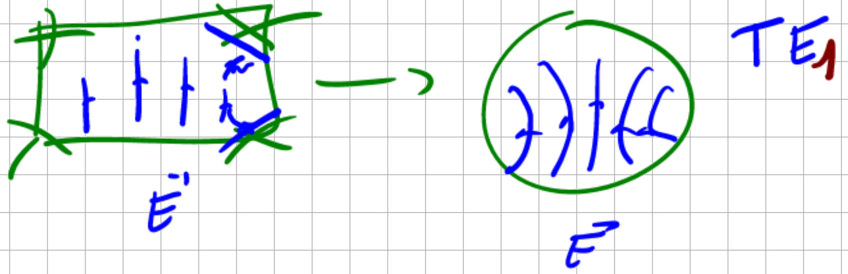

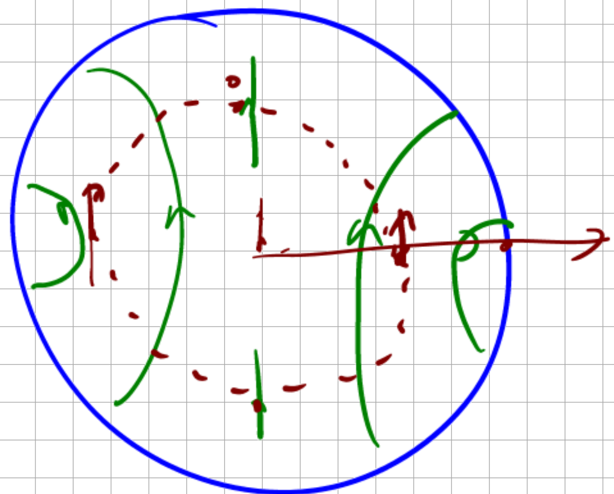

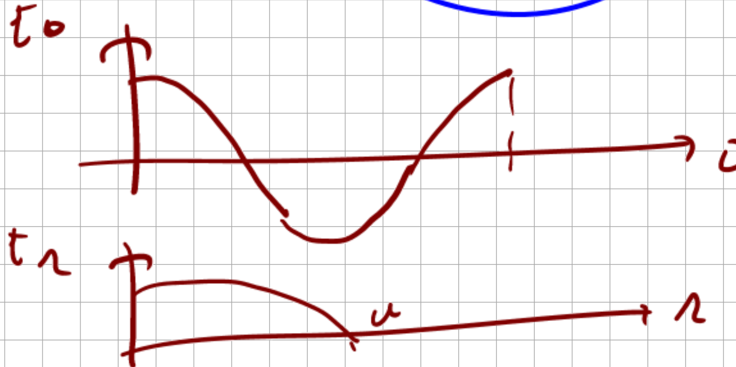

7.3 - other properties.

- wavelength

$$k_c^2 = k_0^2 - \beta^2$$

$$H_z(x,y) = H_0 \cos\frac{m\pi}{a}x \cos\frac{n\pi}{b}y$$

$$H_z(x,y,z) = H_0 \cos\frac{m\pi}{a}x \cos\frac{n\pi}{b}y \, e^{-j\beta z}$$

$$h_z(x,y,z,t) = \mathrm{Re}\left(H_z(x,y,z)\, e^{j\omega t}\right)$$

H_0 real

$$h_z(x,y,z,t) = H_0 \cos\frac{m\pi}{a}x \cos\frac{n\pi}{b}y \cos(\omega t - \beta z)$$

$\lambda_g = \frac{2\pi}{\beta}$ spatial period

guided wavelength λ_g

$\lambda_c = \frac{2\pi}{k_c}$ $\qquad k_c^2 = k_0^2 - \beta^2$

$\lambda_0 = \frac{2\pi}{k_0} = \frac{\omega}{v}$ $\qquad \frac{1}{\lambda_c^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_g^2}$

for $f = f_{cTE_{mn}}$, $\lambda_{g\,TE_{mn}} \to \infty$

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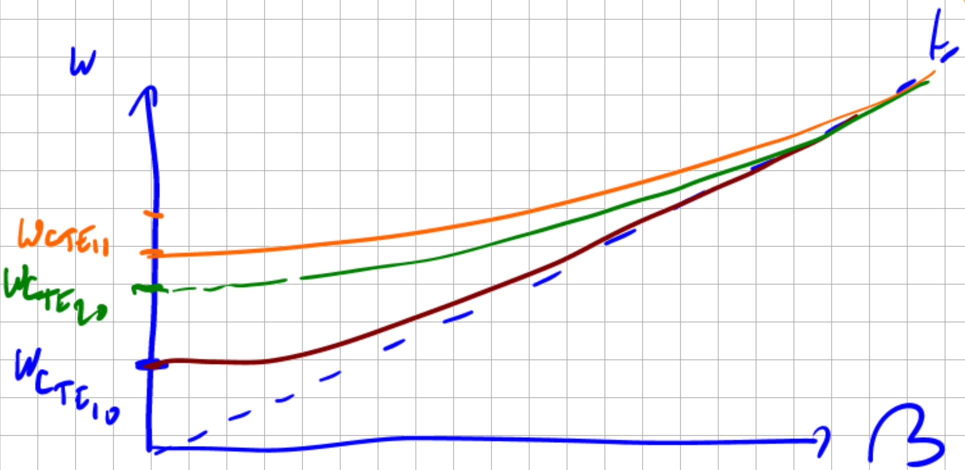
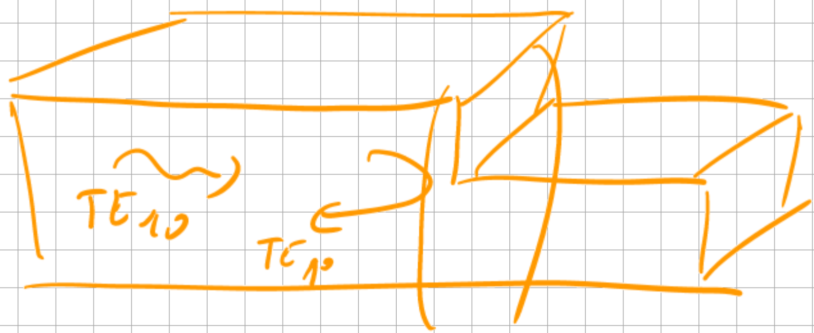
$\beta = k_0$





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