Quantum Technologies

Class Notes

Andrew Simon Wilson

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EMIMEO Programme

Module Coordinator: Prof. Artoni Maurizio

Author Details

Andrew Simon Wilson, BEng

Post-graduate Master's Student - EMIMEO Programme

andrew.wilson@protonmail.com andrew-simon-wilson AS-Wilson +44 7930 560 383

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Explanation and Introduction of this Document

I wrote this document for the students studying Quantum Technologies to have a nice set of notes, and correct reference code and graphs for the module. I hope that it is sufficient for this task and it helps all of your studies.
I spent have spent a lot of time developing the template used to make this LaTeX document, I want others to benefit from this work so the source code for this template is available on GitHub [1].

1 Introduction

1.1 Introduction

TODO - Not quite sure what should be put here.

1.2 Max Planck, The Concept of "Quanta", and Planck's Constant

Quantum theory and mechanics was initially developed and discovered by one man; Max Karl Ernst Ludwig Planck (23/04/1858 Kiel, Duchy of Horstein   04/10/1957, Göttingen, West Germany). Or, as he is more commonly known, simply ***Max Planck***.

He happened upon the principle phenomena involved in quantum physics during research on black body radiation. He published these papers from 1900 - 1901 and they earned him the Nobel prize for physics in 1918.
Planck's initial discovery is the foundation of all of quantum physics and is known as "Planck's Postulate", it states that all electromagnetic radiation is made of very small "particles" known as quanta. These quanta have an energy given by the quanta's frequency and a constant, known as "Planck's Constant":

$$E_h = h\nu = \frac{hC}{\lambda} \rightarrow \text{Energy of One Single Quanta} \tag{1}$$

So if one was to "send" 10 quanta the energy delivered will be 10 quanta, discrete and finite, and given by $10 \cdot h\nu = 10 \cdot E_h$. To be absolutely clear at a given frequency, ν, the amount of energy that can be sent will be integer multiples of Planck's equation, it will be *discrete*.
Quanta are also very commonly referred to by another name; *Photons*. Photons are what make up light, and the discoveries of Planck and others were incredibly important to forwarding science to what it is today. Another incredibly important discovery of quantum physics which is very commonly now is the dual wave-particle nature or behaviour of very small particles. But we shall discuss all of this in more detail photons, quanta and some of the very first implications of Planck's equation very soon.

1.3 Black Body Radiation

A black body is something which is in complete temperature equilibrium, i.e. it's temperature is not changing, it is emitting as much temperature as it is receiving. The earliest form, which likely would have provided the measurements that allowed Planck's to develop his theorem, is shown in Figure 1[2].

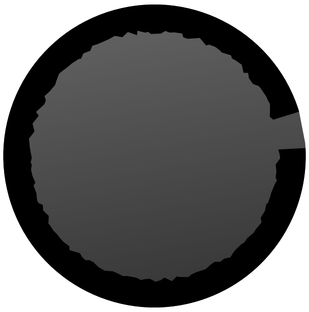

Figure 1: The Construction of an Ideal Black Body, a Platinum Cavity with a Small Hole.

The sun is (nearly) a black body and will absorb EM radiation (in the form of photons) and then emit radiation according to a curve, as shown in Figure 2. But, the interesting, and rather baffling, thing about black bodies is that they will emit photons even when in equilibrium and when no photon has impinged upon them!

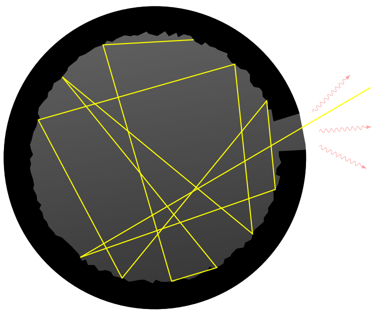

Figure 2: The Ideal Black Body Radiating after Receiving In-Falling Light (One Photon).

From at least around the mid-1800s scientists had been trying to describe the spectrum of black body radiation, the curve of radiated power versus wavelength (or the spectrum) is shown in Figure 2. They could quite well describe and model the higher wavelengths of this curve (and the very low wavelengths) but they couldn't yet find a way to describe the "middle" portion, roughly around the wavelengths of visible light and IR radiation.

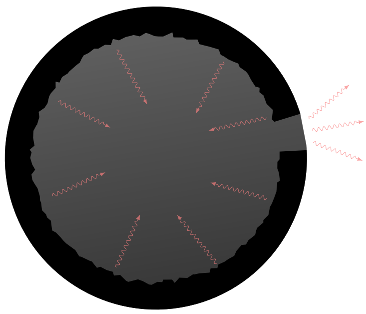

Figure 3: The Ideal Black Body still Radiating without In-Falling Light.

The classical curve of black body radiation is given by the Rayleigh-Jeans equation, two very important figures in the field of optics and physics. They could only describe the upper wavelengths of black body radiation, that would soon change.

It is said that Planck's mentor (in a story that must have occurred a hundred times before in science) told him it was not possible to describe the black body phenomena and that the limits of science had been reached. He ultimately revolutionised this field of study, based on the experimental results observed by others he formulated Planck's Law for black body radiation, given by Equation 2.

$$q_\lambda = \frac{2\pi c^2 h \lambda^{-5}}{e^{\frac{ch}{k_B \lambda T}} - 1} \tag{2}$$

Where:

λ = Wavelength

k_B = Boltzmann's Constant

c = Celerity, Speed of Light in a Vacuum

q_λ = Energy Flux

h = Planck's Constant

T = Temperature

This equation perfectly described the emission curve of a black body based on temperature, shown below in Figure 2. Planck had unwittingly stumbled upon the basis of one of the most incredible fields of study in physics, and arguably one of the most important. He would go on to use this to develop quantum theory and the rest is history!

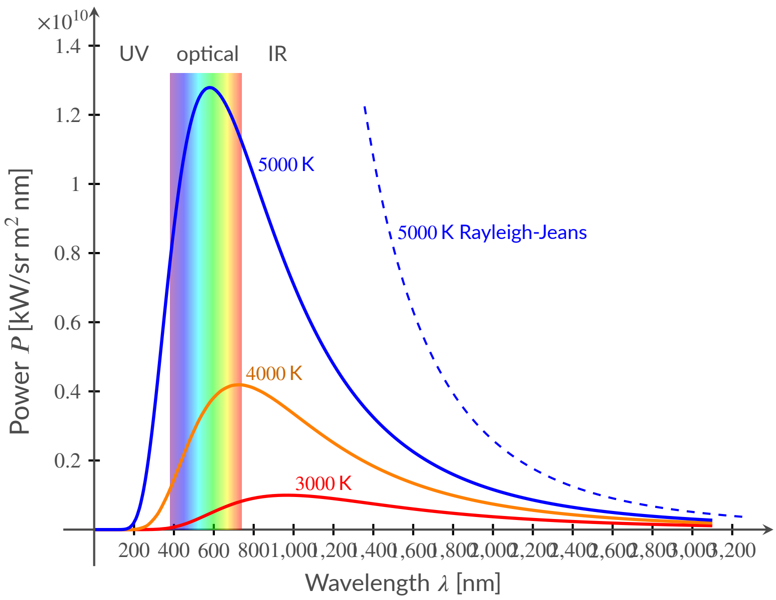

Figure 2: Black Body Radiation Curve at Different Temperatures, the Classical Model (Rayleigh-Jones Curve) is Marked with a Dashed Line.

2 The Photoelectric Effect

In 1905 Albert Einstein would use the findings of Planck's papers to finally describe the results of an experiment which had, until then, not been fully understood. The resulting effect was named the "Photoelectric Effect" [3], it states "that electrons are emitted when electromagnetic radiation, such as light, hits a material". Electrons emitted in this way are known as "Photoelectrons" and this discovery was another key step in the development of quantum theory. It still finds uses in practical applications for chemistry and solid-state electronics today.

2.1 The Photoelectric Experiment

The diagram of the photoelectric experiment is shown in Figure 3, it consists of an ammeter, controlled DC voltage source, a collector, a photocathode, and a light source emitting monochromatic light.
If the DC voltage is kept constant and lower frequency light, of frequency v_1, is emitted towards the photocathode (as in Figure 3) there is no current measured at the ammeter, no mater how long the light is shone or how intensely.
However if light of a higher frequency, v_2, is shone at the photocathode (as in Figure 4), the ammeter begins to immediately show current!

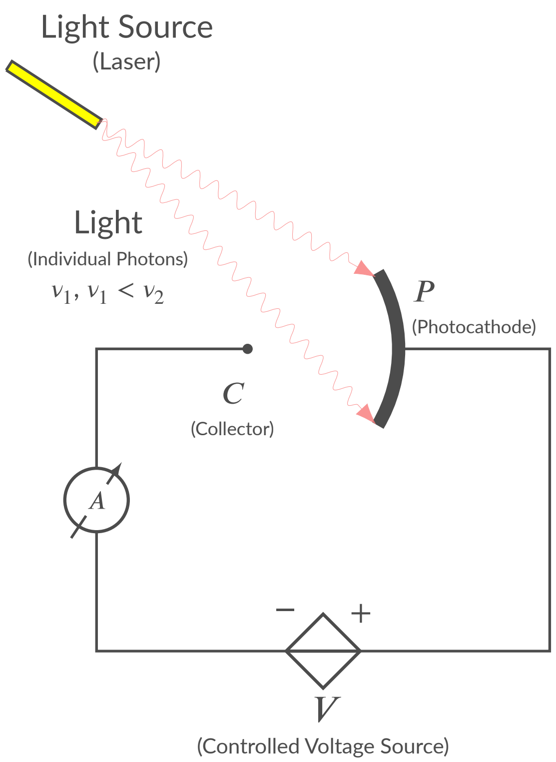

Figure 3: A Diagram of the Photoelectric Experiment. Lower frequency light is being emitted and there is no activity in the experiment.

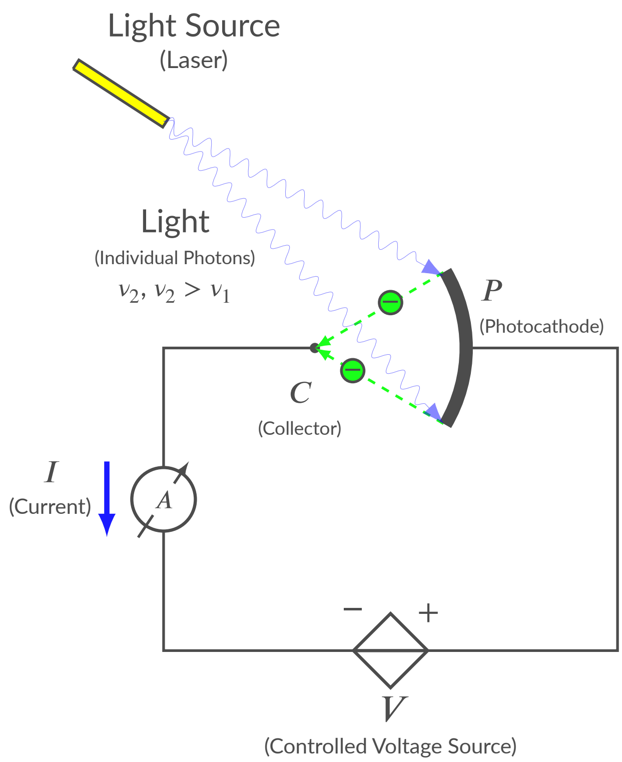

Figure 4: A Diagram of the Photoelectric Experiment. Higher frequency light is being emitted and electrons are passing across to the collector, and current is measured at the ammeter.

The experimental results described by the photoelectric effect inherently disagree with, and were unexplained by, classical electromagnetics which predicts that *continuous* light waves transfer energy to electrons, which would then be emitted when they accumulate enough energy. An alteration in the intensity of light would theoretically (according to classical EM) change the kinetic energy of the emitted electrons, with sufficiently dim light resulting in a delayed emission. The experimental results instead showed that electrons are dislodged **only** when the light exceeds a certain frequency - regardless of the light's intensity or duration of exposure.

2.2 The Photoelectric Equation

So, as previously stated, in 1905 Albert Einstein used the findings of Planck to finally describe the results of the photoelectric experiment. He made an approximation, and stated the one quanta is absorbed by one electron (this is an approximation that he made to simplify his calculations, it is not true in every case). Then he stated that before the quanta is absorbed by the electron it's total energy is given by Equation 3. This is a simple, linear equation, rearranged in eq. 4 it states what the *kinetic* energy of the quanta is before interacting with the electron.

$$h\nu = E_k + \Phi \tag{3}$$

$$E_k = h\nu - \Phi \tag{4}$$

Where:

E_k = Kinetic Energy

Φ = Potential Energy (work function)

ν = Frequency

h = Planck's Constant

Equation 4 provides a clearer explanation for why we were not propagating any photons across the gap when the frequency of the light was too low, we had to be over a threshold frequency, ν_{th}, in order to overcome the potential!
Figure 5 shows this rather simple relationship. There are also some other insights to be gleamed here, namely, that E_K cannot be negative, that the work function is clearly material dependent, and that we can state a relationship to calculate the threshold frequency, given in Equation 5:

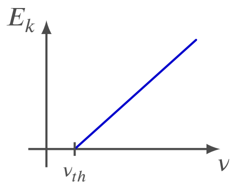

Figure 5: Something...

$$\begin{aligned} h\nu &\geq 0 \\ \therefore h\nu &\geq \Phi \\ \implies \nu &\geq \frac{\Phi}{h} \\ \therefore \nu_{th} &= \frac{\Phi}{h} \end{aligned} \tag{5}$$

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11 Appendix

11.1 Constants & Relevant Definitions

11.1.1 Constants

Table 1: *Important constants involved in Quantum Mechanics*

Symbol/Definition	*Name/info*	*Value*
$\mathbf{c}$	*Speed of Light in Vacuum* [4]	2.998×10^{8} *metres/second (m/s)*
$\mathbf{e}$	*Charge of an Electron* [5]	-1.602×10^{-19} *Coulomb (C)*
$\mathbf{h}$	*Planck's Constant* [6]	6.626×10^{-34} *Joule·second (J·s)* $= 4.136 \times 10^{-15}$ *eV·second (eV·s)*
$\hbar = \frac{\mathbf{h}}{2\pi}$	*The reduced Planck constant, Planck's constant in terms of Radians instead of Hertz.* [7]	1.055×10^{-34} *Joule·second (J·s)* $= 0.658 \times 10^{-15}$ *eV·second (eV·s)*
$\mathbf{k_e} = \frac{\mathbf{1}}{4\pi\epsilon_0}$	*Coulomb's Constant, the Electric Force Constant, or the Electrostatic Constant.* [8]	$8.988 \times 10^{9}\ \frac{Newton \cdot metre^2}{Coulomb^2} \left(\frac{N \cdot m^2}{C^2}\right)$
$\mathbf{N_A}$	*Avogadro's Constant* [9]	6.022×10^{23} $mole^{-1}$ *or* $\frac{1}{mole}$
$\mathbf{G}$	*Gravitational Constant* [10]	$6.672 \times 10^{-11}\ \frac{metre^3}{Kilogram \cdot second^2} \left(\frac{m^3}{Kg \cdot s^2}\right)$ $= 6.672 \times 10^{-8}\ \frac{centimetre^3}{gram \cdot second^2} \left(\frac{cm^3}{g \cdot s^2}\right)$
$\mathbf{k_B} = \frac{\mathbf{R}}{\mathbf{N_A}}$ $\left(\frac{Molar\ Gas\ Constant}{Number\ of\ Molecules}\right)$	*Boltzmann's Constant, this relates the relative kinetic energy of particles in a gas with the thermodynamic temperature of the gas.* [11]	1.38×10^{-23} *Joule·Kelvin (J·K)* $= 8.617 \times 10^{-5}$ *eV·Kelvin (eV·K)*
$\mathbf{hc}$	*Planck's Constant · Speed of Light in Vacuum*	$19.865 \cdot 10^{-26}$ *Joules·metre (J·m)* $12.41 \cdot 10^{3}$ *electronvolt·Angstrom (eV·Å)* 1241 *Mega-electronvolt·femto-metre (MeV·fm)*

Table 1: *Important constants involved in Quantum Mechanics* (Continued)

$\hbar c$	*Normalised Planck's Constant · Speed of Light in Vacuum*	$3.165 \cdot 10^{-26}$ *Joules·metre (J·m)* 1973 *electronvolt·Angstrom (eV·Å)* 197.3 *Mega-electronvolt·femto-metre (MeV·fm)*
$k_c e^2$	*Coulomb's Constant·energy*^{2}	1.44 *Mega-electronvolt·femto-metre (MeV·fm)*
$\frac{k_c e^2}{\hbar c}$	*The Fine-Structure Constant* [12]	$\frac{1}{137}$
$\mu_B = \frac{e\hbar}{2m_e}$	*The Bohr Magneton* [13]	9.27×10^{-24} *Joule/Tesla (J/T)* 5.79×10^{-5} *electronvolt/Tesla (eV/T)*

11.1.2 Relevant Classical Definitions

TODO

Force Moving on a Charge	Electric Field of a Charge
Magnetic Field of a Current	Induced Electromotive Force
Energy Density in the Field	

Table 2: *Important Definitions Involved in Classical Physics that will be Relevant for Quantum Physics.*

11.2 Units Involved and Some Important Starting Equations

Table 3: *Important Units Involved in Classical Physics that will be Relevant for Quantum Physics.*

Measurement/Info	***Abbreviation***	***SI Unit (& Other Common/Useful Units)***
Distance	s	*metres (m)*
Mass	m	*kilograms (kg)*
Time	t	*second (s)*
Velocity	v	*metres/Second (m/s)*
Momentum	p	$\frac{\text{kilogram}\cdot\text{metres}}{\text{second}} \left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)$
Force	F	*Newtons (N),* $\frac{\text{kilogram}\cdot\text{metres}}{\text{second}^2} \left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right)$
Energy, Work Done	W, E	*Joules (J), Newton metres (Nm)*
Power	P	*Watts (W),* $\frac{\text{Joules}}{\text{second}} \left(\frac{\text{J}}{\text{s}}\right)$
Electric Charge	q	*Coulombs (C), Ampere·seconds (A·s)*
Electric Charge Density	ρ	$\frac{\text{Coulomb}}{\text{metre}^3} \left(\frac{\text{C}}{\text{m}^3}\right)$
Electric Potential	φ	*Volts (V),* $\frac{\text{Joules}}{\text{Coulomb}} \left(\frac{\text{J}}{\text{C}}\right)$
Electric Field	$\vec{E}$	$\frac{\text{Volts}}{\text{metre}} \left(\frac{\text{V}}{\text{m}}\right), \frac{\text{Newtons}}{\text{Coulomb}} \left(\frac{\text{N}}{\text{C}}\right)$
Electric Current	I	*Amperes (A),* $\frac{\text{Coulomb}}{\text{second}} \left(\frac{\text{C}}{\text{s}}\right)$
Electric Current Density	$\vec{J}$	$\frac{\text{Amperes}}{\text{metre}^2} \left(\frac{\text{A}}{\text{m}^2}\right)$

Table 3: *Important Units Involved in Classical Physics that will be Relevant for Quantum Physics.* (Continued)

Resistance	R	*Ohm* (Ω), $\frac{\textit{Volts}}{\textit{Ampere}}$ $\left(\frac{V}{A}\right)$
Resistivity	ρ	*Ohm·metre* ($\Omega\cdot m$)
Magnetic Flux Density	$\vec{B}$	*Tesla* (*T*), $\frac{\textit{Newtons}}{\textit{Ampere·metre}}$ $\left(\frac{N}{A\cdot m}\right)$
Magnetic Field Strength	$\vec{H}$	$\frac{\textit{Amperes}}{\textit{metre}}$ $\left(\frac{A}{m}\right)$
Magnetic Flux	$\vec{\Phi}$	*Weber* (*W*), *Tesla·metre*^{2} ($T\cdot m^2$)
Capacitance	C	*Farads* (*F*), $\frac{\textit{seconds}}{\textit{Ohm}}$ $\left(\frac{s}{\Omega}\right)$
Inductance	L	*Henries* (*H*), *Ohm·seconds* ($\Omega\cdot s$)

11.3 Conversions

1 electronvolt (eV)	1.602×10^{-19} Joules (J)
1 Angstrom (Å)	10×10^{-10} metres (m)
1 Ohm (Ω)	1.13×10^{-12} $\frac{\text{seconds}}{\text{centimetre}}$ $\left(\frac{\text{s}}{\text{cm}}\right)$
1 Farad (F)	9×10^{8} metres (m)
1 Henry (H)	1.13×10^{-12} $\frac{\text{seconds}^2}{\text{centimetre}}$ $\left(\frac{\text{s}^2}{\text{cm}}\right)$

Table 4: *Some Conversions for Quantum Mechanics*

11.4 Properties of Elemental Particles

Electron Properties [5]		
Property	***Abbreviation***	***Value***
Mass at rest	$\mathbf{m_e}$	9.109×10^{-31} *kilogram (kg)* 9.109×10^{-28} *gram (g)*
Charge	$\mathbf{q_e}, \mathbf{e^-}$	-1 *elementary charge (e)* -1.602×10^{-19} *Coulombs (C)*
Energy	$\mathbf{E_e = m_e c^2}$	0.511 *Mega electronvolt (MeV)*
Intrinsic Magnetic Moment	$\mu_\mathbf{e}$	-9.285×10^{-24} *Joule/Tesla (J/T)* -1.001 Bohr Magneton (μ_B)
Spin	$\mathbf{S_e}$	$\pm\frac{1}{2}$

Table 5: *Important Properties of the Electron for Quantum Mechanics*

Proton Properties [14]		
Property	***Abbreviation***	***Value***
Mass at rest	$\mathbf{m_p}$	1.673×10^{-27} *kilogram (kg)* 1.673×10^{-24} *gram (g)*
Charge	$\mathbf{q_p}, \mathbf{e^+}$	$+1$ *elementary charge (e)* $+1.602 \times 10^{-19}$ *Coulombs (C)*
Energy	$\mathbf{E_p = m_p c^2}$	938.3 *Mega electronvolt (MeV)*
Intrinsic Magnetic Moment	$\mu_\mathbf{p}$	$+1.411 \times 10^{-26}$ *Joule/Tesla (J/T)* $+1.521 \times 10^{-3}$ Bohr Magneton (μ_B)
Spin	$\mathbf{S_p}$	$\pm\frac{1}{2}$

Table 6: *Important Properties of the Proton for Quantum Mechanics*

Properties of Elemental Particles Cont...

Neutron Properties [15]		
Property	***Abbreviation***	***Value***
Mass at rest	$\mathbf{m_n}$	1.675×10^{-27} *kilogram (kg)* 1.675×10^{-24} *gram (g)*
Charge	$\mathbf{q_n}$	≈ 0 *elementary charge (e)* $(-2 \pm 8) \times 10^{-22}$ *e*
Energy	$\mathbf{E_n = m_n c^2}$	939.6 *Mega electronvolt (MeV)*
Intrinsic Magnetic Moment	$\boldsymbol{\mu_n}$	≈ 0 *Joule/Tesla (J/T)*
Spin	$\mathbf{S_n}$	$\pm\frac{1}{2}$

Table 7: *Important Properties of the Neutron for Quantum Mechanics*

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