Set #2

5. Refraction (Snell's law) is commonly understood within the context of classical electrodynamics in terns of "waves".

Should one adopt a description of electrodynamics in terms of "particles", could such refraction still be described in terms of "photons". Explain your reasoning.

6.

a. Compute the De Broglie wavelength (λ_{DB}) associated with a molecule of air ($\sim N_2$) in your room (STP).

Compute the average distance (d) between molecules (STP) and compare d and λ_{DB}.

b. Is this any different from the case discusse din class foe a conduction band electron (STP). Briefly explain.

7.

In a single-photon double-slit experiment the (single-photon "sp") source has been generated via a time-correlated photon parametric down-conversion process whereby a photon (pump) of energy $\hbar\omega_p$ is converted into a "pair" of photon each having energy $\hbar\omega_{sp}$.

a. If $\lambda_p = 355\ nm$ is the pump photon wavelength, compute the photon wavelength λ_{sp} of the down-converted single-photon impinging onto the double slit.

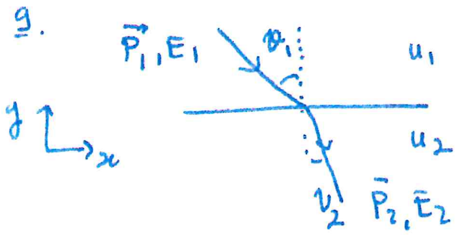
b. In a real experiment we need to place a "lens" between the image plane of the two slits and the farfield screen where the interference pattern is generated. Draw a scheme of the double slit interferometer showing all its components.

c. Derive an expression for the distance between the fringes in terms of the wavelength λ_{sp}, the lens focal length f and the distance d between the two slits.

d. Assuming that the center-to-center distance between the two slits is $d \simeq 500\ \mu m$, the focal length $f \simeq 500\ mm$, "estimate" the fringes distance. Compare your result with the observed fringe spacing that you may infer from the data collected (photon by photon) in the attached video.

✧ Refraction (Snell's law) is commonly understood within the context of classical electrodynamics in terns of "waves".
Could such a phenomenon still be described in terms of photons, should one adopt a description of electrodynamics in terms of "particles". Explain your reasoning.

9.



x : translational invariance

y: No translational invariance

x · component of momentum conserved

$$p_1 \cos\left(\frac{\pi}{2} - \vartheta_1\right) = p_2 \cos\left(\frac{\pi}{2} - \vartheta_2\right) \qquad (1)$$

Energy conserved:

$$E_1 = E_2$$

The refractive index $n = \frac{c|\vec{k}|}{\omega}$ experienced by a photon (p.le) of mom. $\vec{p}$ and En. E inside a medium is:

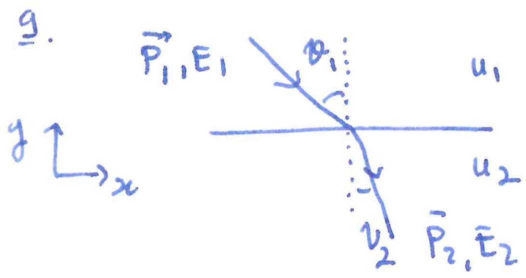
$$\boxed{n = \frac{c|\vec{k}|}{\omega} = \frac{c\,|\vec{p}|/\hbar}{E/\hbar} = \frac{c|\vec{p}|}{E}} \qquad (\vec{p} = \hbar\vec{k}\,,\; E = \hbar\omega)$$

$$n_1 = c\,\frac{|\vec{P}_1|}{E_1} \qquad n_2 = c\,\frac{|\vec{P}_2|}{E_2}$$

$$\frac{n_1}{n_2} = \frac{\cancel{c}\,|\vec{P}_1|/\cancel{E_1}}{\cancel{c}\,|\vec{P}_2|/\cancel{E_2}} = \frac{|\vec{P}_1|}{|\vec{P}_2|} = \frac{p_1}{p_2} \overset{(1)}{=} \frac{\cos\left(\frac{\pi}{2} - \vartheta_2\right)}{\cos\left(\frac{\pi}{2} - \vartheta_1\right)} = \frac{\sin\vartheta_2}{\sin\vartheta_1} \quad \text{or}$$

$$\boxed{n_1 \sin\vartheta_1 = n_2 \sin\vartheta_2} \qquad \text{(Snell's law)}$$

9.



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Energy conserved:

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wave of freq. ω and mom. $\vec{p} = \hbar \vec{k}$: $\omega = \dfrac{c|\vec{k}|}{n}$ (n = refr. index)

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a. Compute the De Broglie wavelength (λ_{DB}) associated with a molecule of air ($\sim N_2$) in your room (STP). Compute the average distance (d) between molecules (STP) and compare d and λ_{DB}.

b. Is this any different from the case discussed in class for a conduction band electron (STP)? Briefly explain.

a) Kinetic En. (Thermal) $E_k = \frac{p^2}{2m} = \underbrace{3\times\frac{1}{2} k_B T}_{\text{equipartition}}$ or $p = \sqrt{3 m k_B T}$

$$\lambda_{DB} = \frac{h}{p} = \frac{hc}{\sqrt{3mc^2 k_B T}}$$

air ($\sim N_2$) N atomic mass $= 14\ u$ $\Rightarrow$ $N_2 = 28\ u$ $T = 300°K$ (STP)

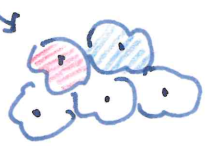
$$\lambda_{DB} = \frac{12.41\times10^3\ eV\cdot Å}{\sqrt{3\times 28\times 931\times10^6\ eV\times 8.6\times10^{-5}\ \frac{eV}{°K}\cdot 300°K}} = [Å]\ 0.27$$

For an ideal gas: $PV = \underbrace{n}_{\#\text{ moles}} RT = \overbrace{N}^{\#\text{ p.les}} k_B T$ or $\frac{N}{V} = \frac{p}{k_B T}$ (p ← pressure, T ← temp.)

P_{atm} (STP) $= 10^5\ \frac{Nw}{m^2}$ or $\frac{N}{V} = \frac{10^5\ Nw/m^2}{8.6\times10^{-5}\ \frac{eV}{°K}\cdot 300°K} =$

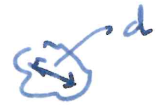
$$= \frac{10^5\ Nw/m^2}{8.5\times10^{-5}\times1.6\times10^{-19}\ \underbrace{J}_{Nw\cdot m}\cdot 300} = \left[\frac{\#}{m^3}\right]\ 2.4\times10^{25}$$

$\frac{V}{N}$ = vol. occupied by each p.le on avg

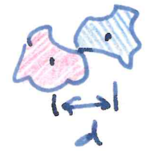


$= 4.12\times10^{-26}\ m^3$

$\left(\frac{V}{N}\right)^{1/3} \approx$ distance over which the mol. lives on avg.



$\simeq$ avg distance between two contiguous mol.s



$\simeq 34\ Å$

Avg. distance (d) between mol.s is much larger than their λ_{DB}

(classical limit) $(d \gg \lambda_{DB})$

b) In the CB electrons discussed in class we were in the opposite limit

(quantum limit) $(d \ll \lambda_{DB})$

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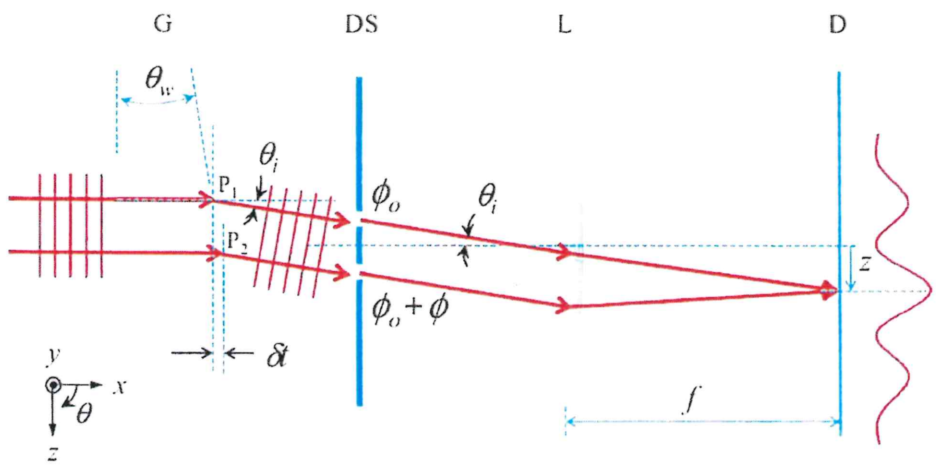
a.) Energy Conservation: $h\nu_p = 2 \times h\nu_{sp} \Rightarrow \nu_{sp} = \frac{1}{2}\nu_p$

$\frac{c}{\lambda_{sp}} = \frac{1}{2}\frac{c}{\lambda_p} \Rightarrow \lambda_{sp} = 2\lambda_p = 710\,nm,$

b).

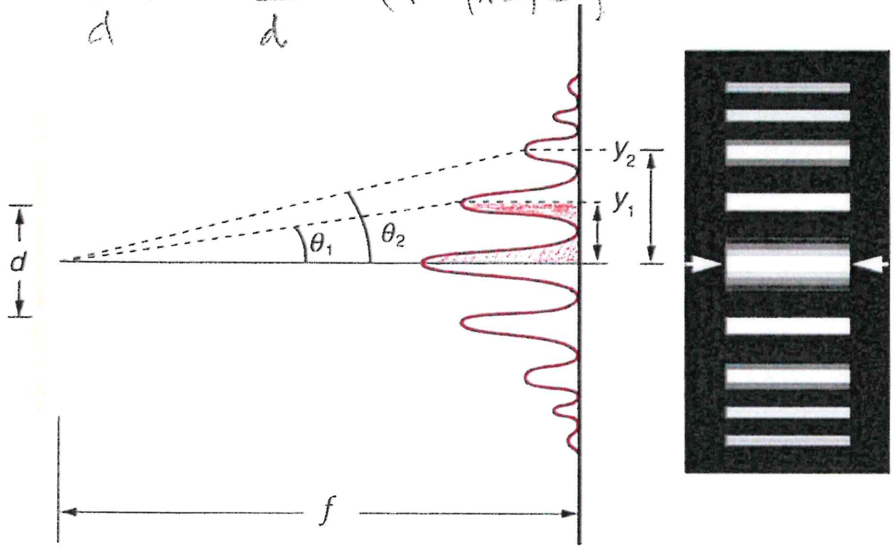


c.) Max interference takes place where: $d \sin\theta = n \cdot \lambda_{sp}$ integer

When $n=1$ Distance between peaks is y_1 and angle $\theta = \theta_1$

Now: $y_1 = f \cdot \mathrm{Tg}\,\theta_1 \simeq f \cdot \sin\theta_1$ small angles

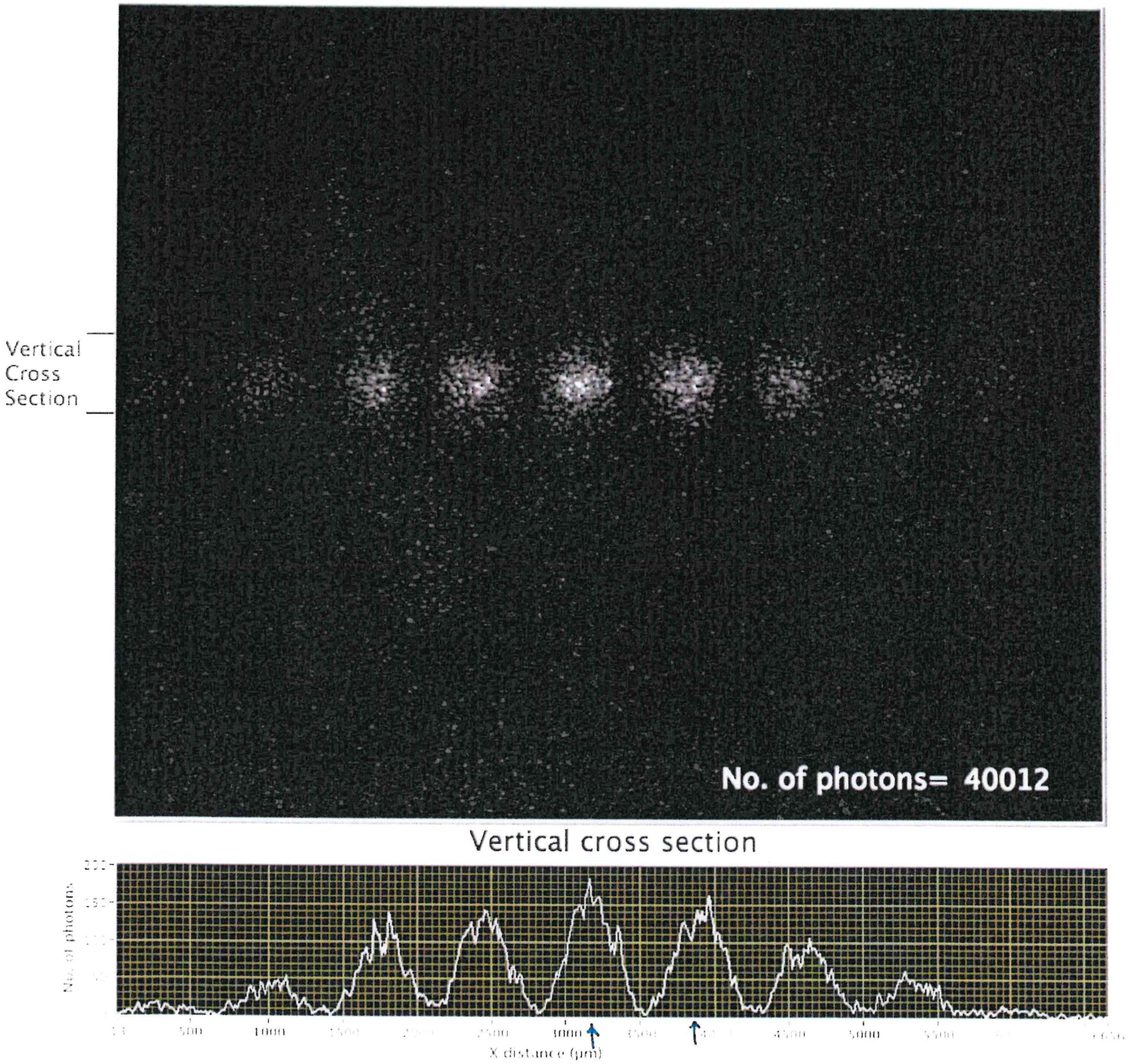
$\simeq f \cdot \frac{1 \cdot \lambda_{sp}}{d} = \frac{f \cdot \lambda_{sp}}{d}$ (1st fringe)



d.) $y_1 = \frac{f \cdot \lambda_{sp}}{d} = \frac{\cancel{500} \times 10^{-3}\,\mathrm{m} \cdot 710 \times 10^{-9}\,\mathrm{m}}{\cancel{500} \times 10^{-6}\,\mathrm{m}} = [\mathrm{m}]\ 7.1 \times 10^{-4} = 710\ \mu\mathrm{m}$

Video: distance is $\sim$ 3850 μm $-$ 3150 μm $= 700\ \mu$m

Young's double slit with a coherent source
photon by photon



~3150 ~3850