Chapter 35

The Nature of Light and the Laws of Geometric Optics

CHAPTER OUTLINE

▲ *This photograph of a rainbow shows a distinct secondary rainbow with the colors reversed. The appearance of the rainbow depends on three optical phenomena discussed in this chapter—reflection, refraction, and dispersion. (Mark D. Phillips/Photo Researchers, Inc.)*

In this first chapter on optics, we begin by introducing two historical models for light and discussing early methods for measuring the speed of light. Next we study the fundamental phenomena of geometric optics—reflection of light from a surface and refraction as the light crosses the boundary between two media. We will also study the dispersion of light as it refracts into materials, resulting in visual displays such as the rainbow. Finally, we investigate the phenomenon of total internal reflection, which is the basis for the operation of optical fibers and the burgeoning technology of fiber optics.

35.1 The Nature of Light

Before the beginning of the nineteenth century, light was considered to be a stream of particles that either was emitted by the object being viewed or emanated from the eyes of the viewer. Newton, the chief architect of the particle theory of light, held that particles were emitted from a light source and that these particles stimulated the sense of sight upon entering the eye. Using this idea, he was able to explain reflection and refraction.

Most scientists accepted Newton's particle theory. During his lifetime, however, another theory was proposed—one that argued that light might be some sort of wave motion. In 1678, the Dutch physicist and astronomer Christian Huygens showed that a wave theory of light could also explain reflection and refraction.

In 1801, Thomas Young (1773–1829) provided the first clear demonstration of the wave nature of light. Young showed that, under appropriate conditions, light rays interfere with each other. Such behavior could not be explained at that time by a particle theory because there was no conceivable way in which two or more particles could come together and cancel one another. Additional developments during the nineteenth century led to the general acceptance of the wave theory of light, the most important resulting from the work of Maxwell, who in 1873 asserted that light was a form of high-frequency electromagnetic wave. As discussed in Chapter 34, Hertz provided experimental confirmation of Maxwell's theory in 1887 by producing and detecting electromagnetic waves.

Although the wave model and the classical theory of electricity and magnetism were able to explain most known properties of light, they could not explain some subsequent experiments. The most striking of these is the photoelectric effect, also discovered by Hertz: when light strikes a metal surface, electrons are sometimes ejected from the surface. As one example of the difficulties that arose, experiments showed that the kinetic energy of an ejected electron is independent of the light intensity. This finding contradicted the wave theory, which held that a more intense beam of light should add more energy to the electron. An explanation of the photoelectric effect was proposed by Einstein in 1905 in a theory that used the concept of quantization developed by Max Planck (1858–1947) in 1900. The quantization model assumes that the energy of a light wave is present in particles called *photons;* hence, the energy is said to be quantized. According to Einstein's theory, the energy of a photon is proportional to the frequency of the electromagnetic wave:

Energy of a photon

$$E = hf \tag{35.1}$$

where the constant of proportionality $h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s}$ is Planck's constant (see Section 11.6). We will study this theory in Chapter 40.

In view of these developments, light must be regarded as having a dual nature: **Light exhibits the characteristics of a wave in some situations and the characteristics of a particle in other situations.** Light is light, to be sure. However, the question "Is light a wave or a particle?" is inappropriate. Sometimes light acts like a wave, and at other times it acts like a particle. In the next few chapters, we investigate the wave nature of light.

35.2 Measurements of the Speed of Light

Light travels at such a high speed ($c = 3.00 \times 10^8$ m/s) that early attempts to measure its speed were unsuccessful. Galileo attempted to measure the speed of light by positioning two observers in towers separated by approximately 10 km. Each observer carried a shuttered lantern. One observer would open his lantern first, and then the other would open his lantern at the moment he saw the light from the first lantern. Galileo reasoned that, knowing the transit time of the light beams from one lantern to the other, he could obtain the speed. His results were inconclusive. Today, we realize (as Galileo concluded) that it is impossible to measure the speed of light in this manner because the transit time is so much less than the reaction time of the observers.

Roemer's Method

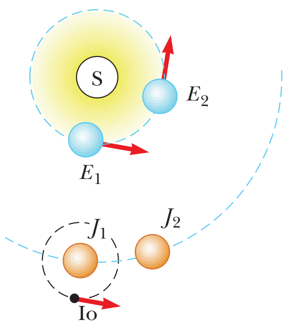


Figure 35.1 Roemer's method for measuring the speed of light. In the time interval during which the Earth travels 90° around the Sun (three months), Jupiter travels only about 7.5° (drawing not to scale).

In 1675, the Danish astronomer Ole Roemer (1644–1710) made the first successful estimate of the speed of light. Roemer's technique involved astronomical observations of one of the moons of Jupiter, Io, which has a period of revolution around Jupiter of approximately 42.5 h. The period of revolution of Jupiter around the Sun is about 12 yr; thus, as the Earth moves through 90° around the Sun, Jupiter revolves through only $(1/12)90° = 7.5°$ (Fig. 35.1).

An observer using the orbital motion of Io as a clock would expect the orbit to have a constant period. However, Roemer, after collecting data for more than a year, observed a systematic variation in Io's period. He found that the periods were longer than average when the Earth was receding from Jupiter and shorter than average when the Earth was approaching Jupiter. If Io had a constant period, Roemer should have seen it become eclipsed by Jupiter at a particular instant and should have been able to predict the time of the next eclipse. However, when he checked the time of the second eclipse as the Earth receded from Jupiter, he found that the eclipse was late. If the interval between his observations was three months, then the delay was approximately 600 s. Roemer attributed this variation in period to the fact that the distance between the Earth and Jupiter changed from one observation to the next. In three months (one quarter of the period of revolution of the Earth around the Sun), the light from Jupiter must travel an additional distance equal to the radius of the Earth's orbit.

Using Roemer's data, Huygens estimated the lower limit for the speed of light to be approximately 2.3×10^8 m/s. This experiment is important historically because it demonstrated that light does have a finite speed and gave an estimate of this speed.

Fizeau's Method

The first successful method for measuring the speed of light by means of purely terrestrial techniques was developed in 1849 by French physicist Armand H. L. Fizeau (1819–1896). Figure 35.2 represents a simplified diagram of Fizeau's apparatus. The basic procedure is to measure the total time interval during which light travels from some point to a distant mirror and back. If d is the distance between the light source (considered to be at the location of the wheel) and the mirror and if the time interval for one round trip is Δt, then the speed of light is $c = 2d/\Delta t$.

To measure the transit time, Fizeau used a rotating toothed wheel, which converts a continuous beam of light into a series of light pulses. The rotation of such a wheel controls what an observer at the light source sees. For example, if the pulse traveling toward the mirror and passing the opening at point A in Figure 35.2 should return to the wheel at the instant tooth B had rotated into position to cover the return path, the pulse would not reach the observer. At a greater rate of rotation, the opening at point C could move into position to allow the reflected pulse to reach the observer. Knowing the distance d, the number of teeth in the wheel, and the angular speed of the wheel, Fizeau arrived at a value of 3.1×10^8 m/s. Similar measurements made by subsequent investigators yielded more precise values for c, which led to the currently accepted value of $2.997\,9 \times 10^8$ m/s.

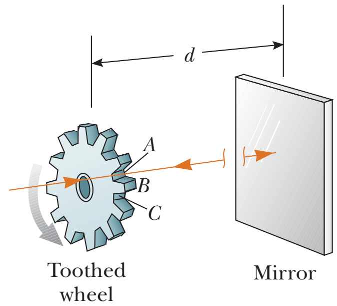


Figure 35.2 Fizeau's method for measuring the speed of light using a rotating toothed wheel. The light source is considered to be at the location of the wheel; thus, the distance d is known.

Example 35.1 Measuring the Speed of Light with Fizeau's Wheel

Assume that Fizeau's wheel has 360 teeth and is rotating at 27.5 rev/s when a pulse of light passing through opening A in Figure 35.2 is blocked by tooth B on its return. If the distance to the mirror is 7 500 m, what is the speed of light?

Solution The wheel has 360 teeth, and so it must have 360 openings. Therefore, because the light passes through opening A but is blocked by the tooth immediately adjacent to A, the wheel must rotate through an angular displacement of (1/720) rev in the time interval during which the light pulse makes its round trip. From the definition of angular speed, that time interval is

$$\Delta t = \frac{\Delta\theta}{\omega} = \frac{(1/720)\text{ rev}}{27.5\text{ rev/s}} = 5.05 \times 10^{-5}\text{ s}$$

Hence, the speed of light calculated from this data is

$$c = \frac{2d}{\Delta t} = \frac{2(7\,500\text{ m})}{5.05 \times 10^{-5}\text{ s}} = \boxed{2.97 \times 10^8\text{ m/s}}$$

35.3 The Ray Approximation in Geometric Optics

The field of **geometric optics** involves the study of the propagation of light, with the assumption that light travels in a fixed direction in a straight line as it passes through a uniform medium and changes its direction when it meets the surface of a different medium or if the optical properties of the medium are nonuniform in either space or time. As we study geometric optics here and in Chapter 36, we use what is called the **ray approximation.** To understand this approximation, first note that the rays of a given wave are straight lines perpendicular to the wave fronts as illustrated in Figure 35.3 for a plane wave. In the ray approximation, we assume that a wave moving through a medium travels in a straight line in the direction of its rays.

If the wave meets a barrier in which there is a circular opening whose diameter is much larger than the wavelength, as in Figure 35.4a, the wave emerging from the opening continues to move in a straight line (apart from some small edge effects); hence, the ray approximation is valid. If the diameter of the opening is on the order of the wavelength, as in Figure 35.4b, the waves spread out from the opening in all directions. This effect is called *diffraction* and will be studied in Chapter 37. Finally, if the opening is much smaller than the wavelength, the opening can be approximated as a point source of waves (Fig. 35.4c). Similar effects are seen when waves encounter an opaque object of dimension d. In this case, when $\lambda \ll d$, the object casts a sharp shadow.

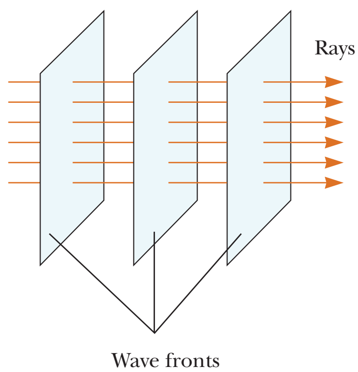
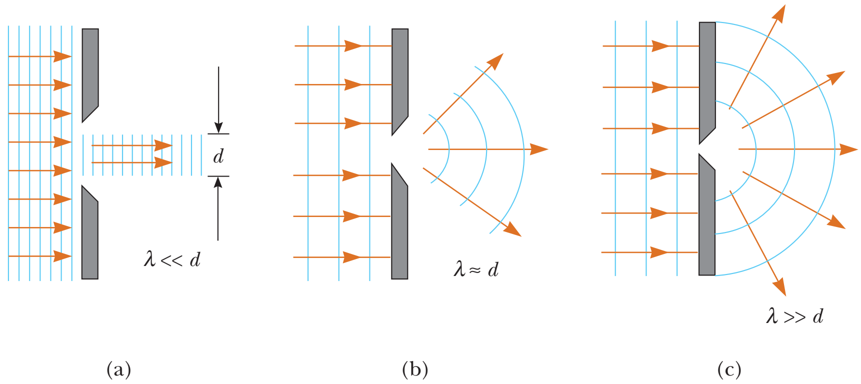


Figure 35.3 A plane wave propagating to the right. Note that the rays, which always point in the direction of the wave propagation, are straight lines perpendicular to the wave fronts.



Active Figure 35.4 A plane wave of wavelength λ is incident on a barrier in which there is an opening of diameter d. (a) When $\lambda \ll d$, the rays continue in a straight-line path, and the ray approximation remains valid. (b) When $\lambda \approx d$, the rays spread out after passing through the opening. (c) When $\lambda \gg d$, the opening behaves as a point source emitting spherical waves.

At the Active Figures link at http://www.pse6.com, you can adjust the size of the opening and observe the effect on the waves passing through.

The ray approximation and the assumption that $\lambda \ll d$ are used in this chapter and in Chapter 36, both of which deal with geometric optics. This approximation is very good for the study of mirrors, lenses, prisms, and associated optical instruments, such as telescopes, cameras, and eyeglasses.

35.4 Reflection

When a light ray traveling in one medium encounters a boundary with another medium, part of the incident light is reflected. Figure 35.5a shows several rays of a beam of light incident on a smooth, mirror-like, reflecting surface. The reflected rays are parallel to each other, as indicated in the figure. The direction of a reflected ray is in the plane perpendicular to the reflecting surface that contains the

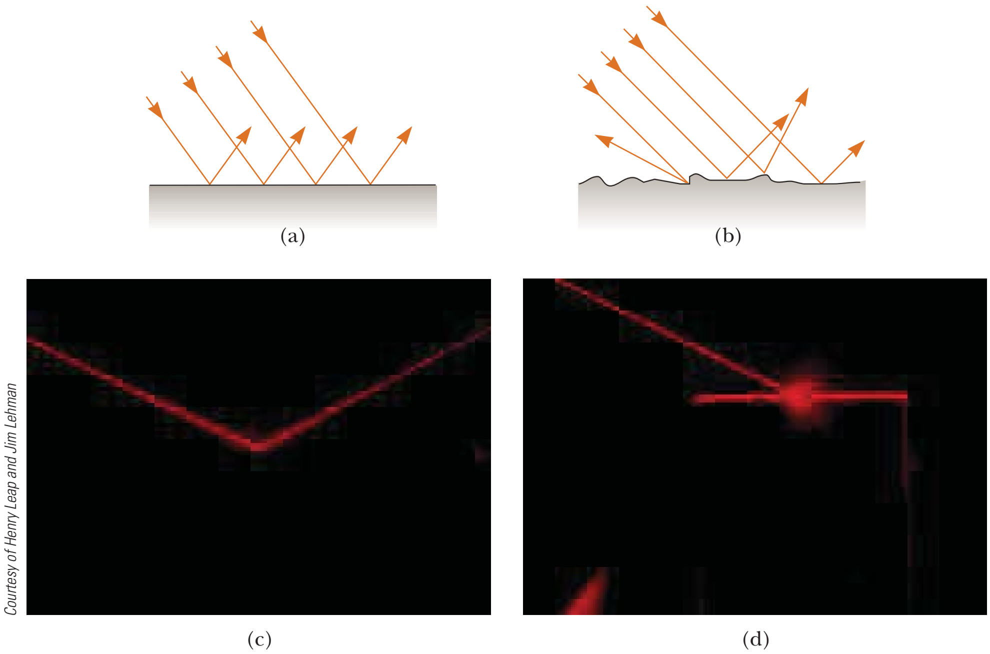
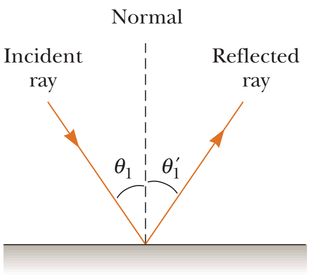


Figure 35.5 Schematic representation of (a) specular reflection, where the reflected rays are all parallel to each other, and (b) diffuse reflection, where the reflected rays travel in random directions. (c) and (d) Photographs of specular and diffuse reflection using laser light.



Active Figure 35.6 According to the law of reflection, $\theta'_1 = \theta_1$. The incident ray, the reflected ray, and the normal all lie in the same plane.

At the Active Figures link at http://www.pse6.com, vary the incident angle and see the effect on the reflected ray.

incident ray. Reflection of light from such a smooth surface is called **specular reflection.** If the reflecting surface is rough, as shown in Figure 35.5b, the surface reflects the rays not as a parallel set but in various directions. Reflection from any rough surface is known as **diffuse reflection.** A surface behaves as a smooth surface as long as the surface variations are much smaller than the wavelength of the incident light.

The difference between these two kinds of reflection explains why it is more difficult to see while driving on a rainy night. If the road is wet, the smooth surface of the water specularly reflects most of your headlight beams away from your car (and perhaps into the eyes of oncoming drivers). When the road is dry, its rough surface diffusely reflects part of your headlight beam back toward you, allowing you to see the highway more clearly. In this book, we concern ourselves only with specular reflection and use the term *reflection* to mean specular reflection.

Consider a light ray traveling in air and incident at an angle on a flat, smooth surface, as shown in Figure 35.6. The incident and reflected rays make angles θ_1 and θ'_1, respectively, where the angles are measured between the normal and the rays. (The normal is a line drawn perpendicular to the surface at the point where the incident ray strikes the surface.) Experiments and theory show that **the angle of reflection equals the angle of incidence:**

$$\theta'_1 = \theta_1 \quad (35.2)$$

Law of reflection

This relationship is called the **law of reflection.**

PITFALL PREVENTION

35.1 Subscript Notation

We use the subscript 1 to refer to parameters for the light in the initial medium. When light travels from one medium to another, we use the subscript 2 for the parameters associated with the light in the new medium. In the current discussion, the light stays in the same medium, so we only have to use the subscript 1.

Quick Quiz 35.1 In the movies, you sometimes see an actor looking in a mirror and you can see his face in the mirror. During the filming of this scene, what does the actor see in the mirror? (a) his face (b) your face (c) the director's face (d) the movie camera (e) impossible to determine

Example 35.2 The Double-Reflected Light Ray — Interactive

Two mirrors make an angle of 120° with each other, as illustrated in Figure 35.7a. A ray is incident on mirror M_1 at an angle of 65° to the normal. Find the direction of the ray after it is reflected from mirror M_2.

Solution Figure 35.7a helps conceptualize this situation. The incoming ray reflects from the first mirror, and the reflected ray is directed toward the second mirror. Thus, there is a second reflection from this latter mirror. Because the interactions with both mirrors are simple reflections, we categorize this problem as one that will require the law of reflection and some geometry. To analyze the problem, note that from the law of reflection, we know that the first reflected ray makes an angle of 65° with the normal. Thus, this ray makes an angle of $90° - 65° = 25°$ with the horizontal.

From the triangle made by the first reflected ray and the two mirrors, we see that the first reflected ray makes an angle of 35° with M_2 (because the sum of the interior angles of any triangle is 180°). Therefore, this ray makes an angle of 55° with the normal to M_2. From the law of reflection, the second reflected ray makes an angle of 55° with the normal to M_2.

To finalize the problem, let us explore variations in the angle between the mirrors as follows.

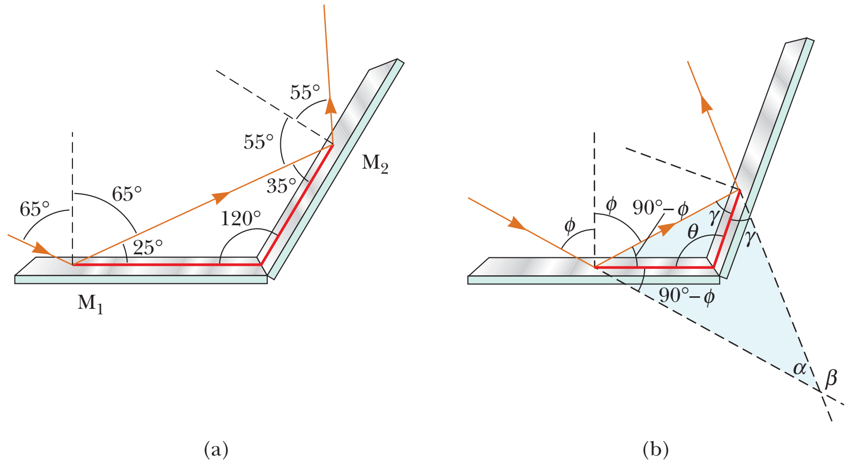


Figure 35.7 (Example 35.2) (a) Mirrors M_1 and M_2 make an angle of 120° with each other. (b) The geometry for an arbitrary mirror angle.

What If? **If the incoming and outgoing rays in Figure 35.7a are extended behind the mirror, they cross at an angle of 60°, so that the overall change in direction of the light ray is 120°. This is the same as the angle between the mirrors. What if the angle between the mirrors is changed? Is the overall change in the direction of the light ray always equal to the angle between the mirrors?**

Answer Making a general statement based on one data point is always a dangerous practice! Let us investigate the change in direction for a general situation. Figure 35.7b shows the mirrors at an arbitrary angle θ and the incoming light ray striking the mirror at an arbitrary angle ϕ with respect to the normal to the mirror surface. In accordance with the law of reflection and the sum of the interior angles of a triangle, the angle γ is $180° - (90° - \phi) - \theta = 90° + \phi - \theta$. Considering the triangle highlighted in blue in Figure 35.7b, we see that

$$\alpha + 2\gamma + 2(90° - \phi) = 180°$$

$$\alpha = 2(\phi - \gamma)$$

The change in direction of the light ray is angle β, which is $180° - \alpha$:

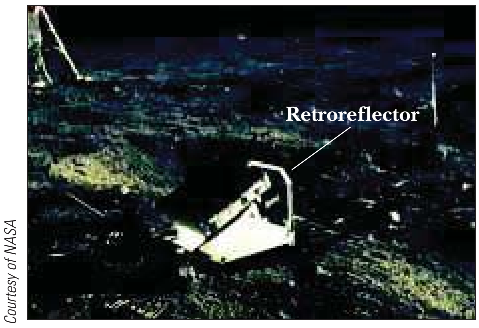
$$\begin{aligned}\beta &= 180° - \alpha = 180° - 2(\phi - \gamma)\\ &= 180° - 2[\phi - (90° + \phi - \theta)]\\ &= 360° - 2\theta\end{aligned}$$

Notice that β is not equal to θ. For $\theta = 120°$, we obtain $\beta = 120°$, which happens to be the same as the mirror angle. But this is true only for this special angle between the mirrors. For example, if $\theta = 90°$, we obtain $\beta = 180°$. In this case, the light is reflected straight back to its origin.

Investigate this reflection situation for various mirror angles at the Interactive Worked Example link at **http://www.pse6.com.**

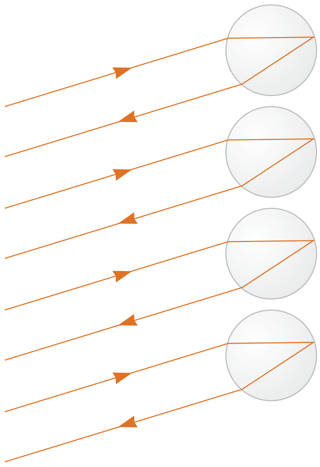
As discussed in the **What If?** section of the preceding example, if the angle between two mirrors is 90°, the reflected beam returns to the source parallel to its original path. This phenomenon, called *retroreflection*, has many practical applications. If a third mirror is placed perpendicular to the first two, so that the three form the corner of a cube, retroreflection works in three dimensions. In 1969, a panel of many small reflectors was placed on the Moon by the *Apollo 11* astronauts (Fig. 35.8a). A laser beam from the Earth is reflected directly back on itself and its transit time is measured. This information is used to determine the distance to the Moon with an uncertainty of 15 cm. (Imagine how difficult it would be to align a regular flat mirror so that the reflected laser beam would hit a particular location on the Earth!) A more everyday application is found in automobile taillights. Part of the plastic making up the taillight is formed into many tiny cube corners (Fig. 35.8b) so that headlight beams from cars approaching from the rear are reflected back to the drivers. Instead of cube corners, small spherical bumps are sometimes used (Fig. 35.8c). Tiny clear spheres are used in a coating material found on many road signs. Due to retroreflection from these spheres, the stop sign in Figure 35.8d appears much brighter than it would if it were simply a flat, shiny surface reflecting most of the light hitting it away from the highway.

Another practical application of the law of reflection is the digital projection of movies, television shows, and computer presentations. A digital projector makes use



(a)

(b)



(c)

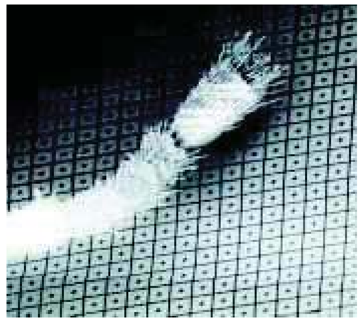
(d)

Figure 35.8 Applications of retroreflection. (a) This panel on the Moon reflects a laser beam directly back to its source on the Earth. (b) An automobile taillight has small retroreflectors that ensure that headlight beams are reflected back toward the car that sent them. (c) A light ray hitting a transparent sphere at the proper position is retroreflected. (d) This stop sign appears to glow in headlight beams because its surface is covered with a layer of many tiny retroreflecting spheres. What would you see if the sign had a mirror-like surface?

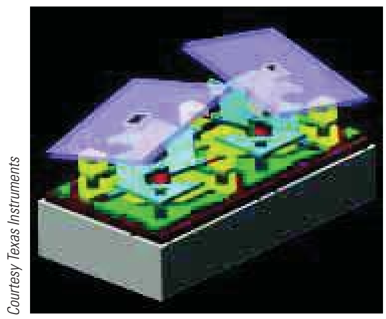
of an optical semiconductor chip called a *digital micromirror device.* This device contains an array of over one million tiny mirrors (Fig. 35.9a) that can be individually tilted by means of signals to an address electrode underneath the edge of the mirror. Each mirror corresponds to a pixel in the projected image. When the pixel corresponding to a given mirror is to be bright, the mirror is in the "on" position—oriented so as to reflect light from a source illuminating the array to the screen (Fig. 35.9b). When the pixel for this mirror is to be dark, the mirror is "off"—tilted so that the light is reflected away from the screen. The brightness of the pixel is determined by the total time interval during which the mirror is in the "on" position during the display of one image.

Digital movie projectors use three micromirror devices, one for each of the primary colors red, blue, and green, so that movies can be displayed with up to 35 trillion colors. Because information is stored as binary data, a digital movie does not degrade with time as does film. Furthermore, because the movie is entirely in the form of computer software, it can be delivered to theaters by means of satellites, optical discs, or optical fiber networks.

Several movies have been projected digitally to audiences and polls show that 85 percent of the viewers describe the image quality as "excellent." The first all-digital movie, from cinematography to post-production to projection, was *Star Wars Episode II: Attack of the Clones* in 2002.



(a)



(b)

Figure 35.9 (a) An array of mirrors on the surface of a digital micromirror device. Each mirror has an area of about 16 μm^2. To provide a sense of scale, the leg of an ant appears in the photograph. (b) A close-up view of two single micromirrors. The mirror on the left is "on" and the one on the right is "off."

35.5 Refraction

When a ray of light traveling through a transparent medium encounters a boundary leading into another transparent medium, as shown in Figure 35.10, part of the energy is reflected and part enters the second medium. The ray that enters the second medium is bent at the boundary and is said to be **refracted.** The incident ray, the reflected ray, and the refracted ray all lie in the same plane. The **angle of refraction,** θ_2 in Figure 35.10a, depends on the properties of the two media and on the angle of incidence through the relationship

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} = \text{constant} \tag{35.3}$$

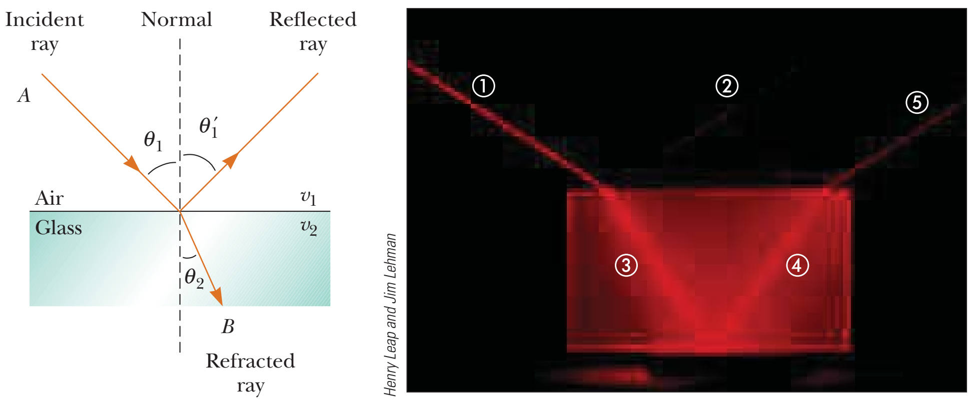
where v_1 is the speed of light in the first medium and v_2 is the speed of light in the second medium.

The path of a light ray through a refracting surface is reversible. For example, the ray shown in Figure 35.10a travels from point A to point B. If the ray originated at B, it would travel to the left along line BA to reach point A, and the reflected part would point downward and to the left in the glass.

Quick Quiz 35.2 If beam ① is the incoming beam in Figure 35.10b, which of the other four red lines are reflected beams and which are refracted beams?

From Equation 35.3, we can infer that when light moves from a material in which its speed is high to a material in which its speed is lower, as shown in Figure 35.11a, the angle of refraction θ_2 is less than the angle of incidence θ_1, and the ray is bent *toward* the normal. If the ray moves from a material in which light moves slowly to a material in which it moves more rapidly, as illustrated in Figure 35.11b, θ_2 is greater than θ_1, and the ray is bent *away* from the normal.

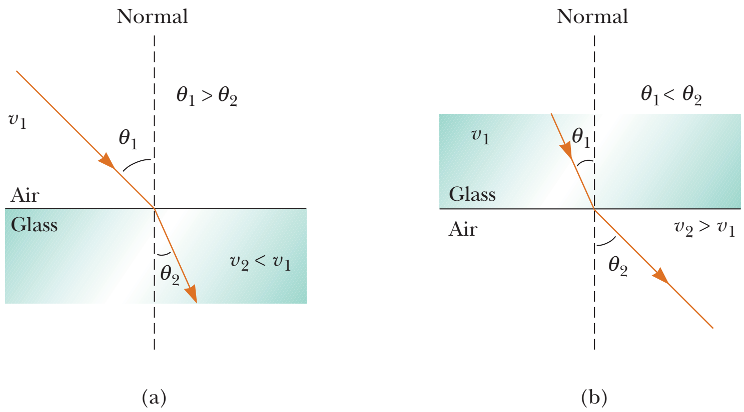
The behavior of light as it passes from air into another substance and then re-emerges into air is often a source of confusion to students. When light travels in air,



(a) (b)

Active Figure 35.10 (a) A ray obliquely incident on an air–glass interface. The refracted ray is bent toward the normal because $v_2 < v_1$. All rays and the normal lie in the same plane. (b) Light incident on the Lucite block bends both when it enters the block and when it leaves the block.

At the Active Figures link at http://www.pse6.com, vary the incident angle and see the effect on the reflected and refracted rays.



Active Figure 35.11 (a) When the light beam moves from air into glass, the light slows down on entering the glass and its path is bent toward the normal. (b) When the beam moves from glass into air, the light speeds up on entering the air and its path is bent away from the normal.

At the Active Figures link at http://www.pse6.com, light passes through three layers of material. You can vary the incident angle and see the effect on the refracted rays for a variety of values of the index of refraction (page 1104) of the three materials.

its speed is 3.00×10^8 m/s, but this speed is reduced to approximately 2×10^8 m/s when the light enters a block of glass. When the light re-emerges into air, its speed instantaneously increases to its original value of 3.00×10^8 m/s. This is far different from what happens, for example, when a bullet is fired through a block of wood. In this case, the speed of the bullet is reduced as it moves through the wood because some of its original energy is used to tear apart the wood fibers. When the bullet enters the air once again, it emerges at the speed it had just before leaving the block of wood.

To see why light behaves as it does, consider Figure 35.12, which represents a beam of light entering a piece of glass from the left. Once inside the glass, the light may encounter an electron bound to an atom, indicated as point A. Let us assume that light is absorbed by the atom; this causes the electron to oscillate (a detail represented by the double-headed vertical arrows). The oscillating electron then acts as an antenna and radiates the beam of light toward an atom at B, where the light is again absorbed. The details of these absorptions and radiations are best explained in terms of quantum mechanics (Chapter 42). For now, it is sufficient to think of light passing from one atom to another through the glass. Although light travels from one glass atom to another at 3.00×10^8 m/s, the absorption and radiation that take place cause the *average* light speed through the material to fall to about 2×10^8 m/s. Once the light emerges into the air, absorption and radiation cease and the speed of the light returns to the original value.

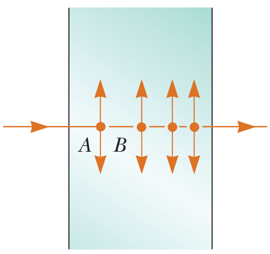


Figure 35.12 Light passing from one atom to another in a medium. The dots are electrons, and the vertical arrows represent their oscillations.

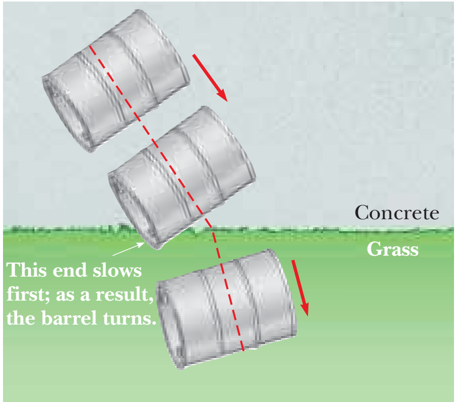


Figure 35.13 Overhead view of a barrel rolling from concrete onto grass.

A mechanical analog of refraction is shown in Figure 35.13. When the left end of the rolling barrel reaches the grass, it slows down, while the right end remains on the concrete and moves at its original speed. This difference in speeds causes the barrel to pivot, and this changes the direction of travel.

PITFALL PREVENTION

35.2 *n* Is Not an Integer Here

We have seen n used several times as an integer, such as in Chapter 18 to indicate the standing wave mode on a string or in an air column. The index of refraction n is *not* an integer.

Index of Refraction

In general, the speed of light in any material is *less* than its speed in vacuum. In fact, *light travels at its maximum speed in vacuum.* It is convenient to define the **index of refraction** n of a medium to be the ratio

Index of refraction

$$n \equiv \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v} \quad (35.4)$$

From this definition, we see that the index of refraction is a dimensionless number greater than unity because v is always less than c. Furthermore, n is equal to unity for vacuum. The indices of refraction for various substances are listed in Table 35.1.

As light travels from one medium to another, its frequency does not change but its wavelength does. To see why this is so, consider Figure 35.14. Waves pass an observer at point A in medium 1 with a certain frequency and are

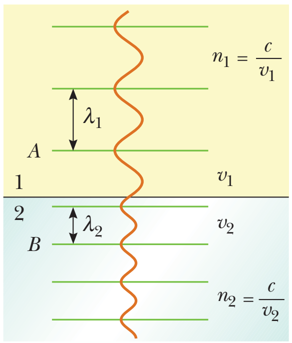


Figure 35.14 As a wave moves from medium 1 to medium 2, its wavelength changes but its frequency remains constant.

Table 35.1

Indices of Refraction[a]

Substance	Index of Refraction	Substance	Index of Refraction
Solids at 20°C		*Liquids at 20°C*	
Cubic zirconia	2.20	Benzene	1.501
Diamond (C)	2.419	Carbon disulfide	1.628
Fluorite (CaF_2)	1.434	Carbon tetrachloride	1.461
Fused quartz (SiO_2)	1.458	Ethyl alcohol	1.361
Gallium phosphide	3.50	Glycerin	1.473
Glass, crown	1.52	Water	1.333
Glass, flint	1.66		
Ice (H_2O)	1.309	*Gases at 0°C, 1 atm*	
Polystyrene	1.49	Air	1.000 293
Sodium chloride (NaCl)	1.544	Carbon dioxide	1.000 45

[a] All values are for light having a wavelength of 589 nm in vacuum.

incident on the boundary between medium 1 and medium 2. The frequency with which the waves pass an observer at point B in medium 2 must equal the frequency at which they pass point A. If this were not the case, then energy would be piling up at the boundary. Because there is no mechanism for this to occur, the frequency must be a constant as a light ray passes from one medium into another. Therefore, because the relationship $v = f\lambda$ (Eq. 16.12) must be valid in both media and because $f_1 = f_2 = f$, we see that

$$v_1 = f\lambda_1 \quad \text{and} \quad v_2 = f\lambda_2 \tag{35.5}$$

Because $v_1 \neq v_2$, it follows that $\lambda_1 \neq \lambda_2$.

We can obtain a relationship between index of refraction and wavelength by dividing the first Equation 35.5 by the second and then using Equation 35.4:

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1} \tag{35.6}$$

This gives

$$\lambda_1 n_1 = \lambda_2 n_2$$

If medium 1 is vacuum, or for all practical purposes air, then $n_1 = 1$. Hence, it follows from Equation 35.6 that the index of refraction of any medium can be expressed as the ratio

$$n = \frac{\lambda}{\lambda_n} \tag{35.7}$$

where λ is the wavelength of light in vacuum and λ_n is the wavelength of light in the medium whose index of refraction is n. From Equation 35.7, we see that because $n > 1$, $\lambda_n < \lambda$.

We are now in a position to express Equation 35.3 in an alternative form. If we replace the v_2/v_1 term in Equation 35.3 with n_1/n_2 from Equation 35.6, we obtain

Snell's law of refraction

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{35.8}$$

The experimental discovery of this relationship is usually credited to Willebrord Snell (1591–1627) and is therefore known as **Snell's law of refraction.** We shall examine this equation further in Sections 35.6 and 35.9.

PITFALL PREVENTION

35.3 An Inverse Relationship

The index of refraction is *inversely* proportional to the wave speed. As the wave speed v decreases, the index of refraction n increases. Thus, the higher the index of refraction of a material, the more it *slows down* light from its speed in vacuum. The more the light slows down, the more θ_2 differs from θ_1 in Equation 35.8.

Quick Quiz 35.3 Light passes from a material with index of refraction 1.3 into one with index of refraction 1.2. Compared to the incident ray, the refracted ray (a) bends toward the normal (b) is undeflected (c) bends away from the normal.

Quick Quiz 35.4 As light from the Sun enters the atmosphere, it refracts due to the small difference between the speeds of light in air and in vacuum. The *optical* length of the day is defined as the time interval between the instant when the top of the Sun is just visibly observed above the horizon to the instant at which the top of the Sun just disappears below the horizon. The *geometric* length of the day is defined as the time interval between the instant when a geometric straight line drawn from the observer to the top of the Sun just clears the horizon to the instant at which this line just dips below the horizon. Which is longer, (a) the optical length of a day, or (b) the geometric length of a day?

Example 35.3 An Index of Refraction Measurement

A beam of light of wavelength 550 nm traveling in air is incident on a slab of transparent material. The incident beam makes an angle of 40.0° with the normal, and the refracted beam makes an angle of 26.0° with the normal. Find the index of refraction of the material.

Solution Using Snell's law of refraction (Eq. 35.8) with these data, and taking $n_1 = 1.00$ for air, we have

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2} = (1.00)\frac{\sin 40.0°}{\sin 26.0°}$$

$$= \frac{0.643}{0.438} = \boxed{1.47}$$

From Table 35.1, we see that the material could be fused quartz.

Example 35.4 Angle of Refraction for Glass

A light ray of wavelength 589 nm traveling through air is incident on a smooth, flat slab of crown glass at an angle of 30.0° to the normal, as sketched in Figure 35.15. Find the angle of refraction.

Solution We rearrange Snell's law of refraction to obtain

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

From Table 35.1, we find that $n_1 = 1.00$ for air and $n_2 = 1.52$ for crown glass. Therefore,

$$\sin\theta_2 = \left(\frac{1.00}{1.52}\right)\sin 30.0° = 0.329$$

$$\theta_2 = \sin^{-1}(0.329) = \boxed{19.2°}$$

Because this is less than the incident angle of 30°, the refracted ray is bent toward the normal, as expected. Its change in direction is called the *angle of deviation* and is given by $\delta = |\theta_1 - \theta_2| = 30.0° - 19.2° = 10.8°$.

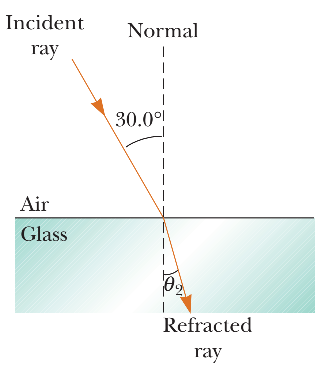


Figure 35.15 (Example 35.4) Refraction of light by glass.

Example 35.5 Laser Light in a Compact Disc

A laser in a compact disc player generates light that has a wavelength of 780 nm in air.

(A) Find the speed of this light once it enters the plastic of a compact disc ($n = 1.55$).

Solution We expect to find a value less than 3.00×10^8 m/s because $n > 1$. We can obtain the speed of light in the plastic by using Equation 35.4:

$$v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.55}$$

$$v = \boxed{1.94 \times 10^8 \text{ m/s}}$$

(B) What is the wavelength of this light in the plastic?

Solution We use Equation 35.7 to calculate the wavelength in plastic, noting that we are given the wavelength in air to be $\lambda = 780$ nm:

$$\lambda_n = \frac{\lambda}{n} = \frac{780 \text{ nm}}{1.55} = \boxed{503 \text{ nm}}$$

Example 35.6 Light Passing Through a Slab

Interactive

A light beam passes from medium 1 to medium 2, with the latter medium being a thick slab of material whose index of refraction is n_2 (Fig. 35.16a). Show that the emerging beam is parallel to the incident beam.

Solution First, let us apply Snell's law of refraction to the upper surface:

$$(1) \qquad \sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

Applying this law to the lower surface gives

$$(2) \qquad \sin\theta_3 = \frac{n_2}{n_1}\sin\theta_2$$

Substituting Equation (1) into Equation (2) gives

$$\sin\theta_3 = \frac{n_2}{n_1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin\theta_1$$

Therefore, $\theta_3 = \theta_1$, and the slab does not alter the direction of the beam. It does, however, offset the beam parallel to itself by the distance d shown in Figure 35.16a.

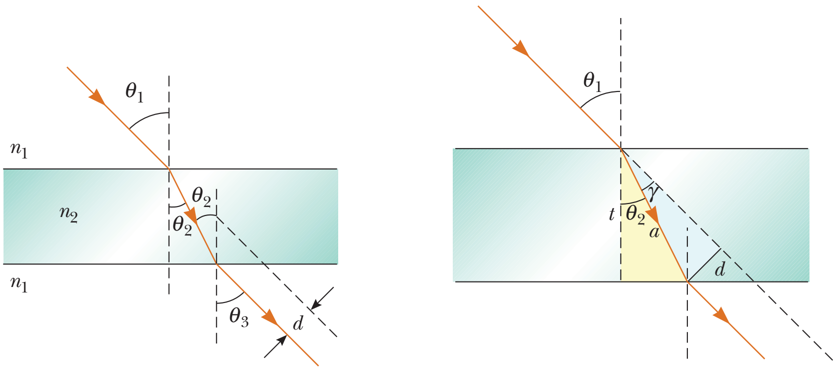


Figure 35.16 (Example 35.6) (a) When light passes through a flat slab of material, the emerging beam is parallel to the incident beam, and therefore $\theta_1 = \theta_3$. The dashed line drawn parallel to the ray coming out the bottom of the slab represents the path the light would take if the slab were not there. (b) A magnification of the area of the light path inside the slab.

What If? **What if the thickness *t* of the slab is doubled? Does the offset distance *d* also double?**

Answer Consider the magnification of the area of the light path within the slab in Figure 35.16b. The distance a is the hypotenuse of two right triangles. From the gold triangle, we see

$$a = \frac{t}{\cos\theta_2}$$

and from the blue triangle,

$$d = a\sin\gamma = a\sin(\theta_1 - \theta_2)$$

Combining these equations, we have

$$d = \frac{t}{\cos\theta_2}\sin(\theta_1 - \theta_2)$$

For a given incident angle θ_1, the refracted angle θ_2 is determined solely by the index of refraction, so the offset distance d is proportional to t. If the thickness doubles, so does the offset distance.

Explore refraction through slabs of various thicknesses at the Interactive Worked Example link at **http://www.pse6.com.**

35.6 Huygens's Principle

In this section, we develop the laws of reflection and refraction by using a geometric method proposed by Huygens in 1678. **Huygens's principle** is a geometric construction for using knowledge of an earlier wave front to determine the position of a new wave front at some instant. In Huygens's construction,

> all points on a given wave front are taken as point sources for the production of spherical secondary waves, called wavelets, which propagate outward through a medium with speeds characteristic of waves in that medium. After some time interval has passed, the new position of the wave front is the surface tangent to the wavelets.

First, consider a plane wave moving through free space, as shown in Figure 35.17a. At $t = 0$, the wave front is indicated by the plane labeled AA'. In Huygens's construction, each point on this wave front is considered a point source. For clarity, only three points on AA' are shown. With these points as sources for the wavelets, we draw circles, each of radius $c\,\Delta t$, where c is the speed of light in vacuum and Δt is some time interval during which the wave propagates. The surface drawn tangent to these wavelets is the plane BB', which is the wave front at a later time, and is parallel to AA'. In a similar manner, Figure 35.17b shows Huygens's construction for a spherical wave.

PITFALL PREVENTION

35.4 Of What Use Is Huygens's Principle?

At this point, the importance of Huygens's principle may not be evident. Predicting the position of a future wave front may not seem to be very critical. However, we will use Huygens's principle in later chapters to explain additional wave phenomena for light.

Christian Huygens

Dutch Physicist and Astronomer (1629–1695)

Huygens is best known for his contributions to the fields of optics and dynamics. To Huygens, light was a type of vibratory motion, spreading out and producing the sensation of light when impinging on the eye. On the basis of this theory, he deduced the laws of reflection and refraction and explained the phenomenon of double refraction. (*Courtesy of Rijksmuseum voor de Geschiedenis der Natuurwetenschappen and Niels Bohr Library.)*

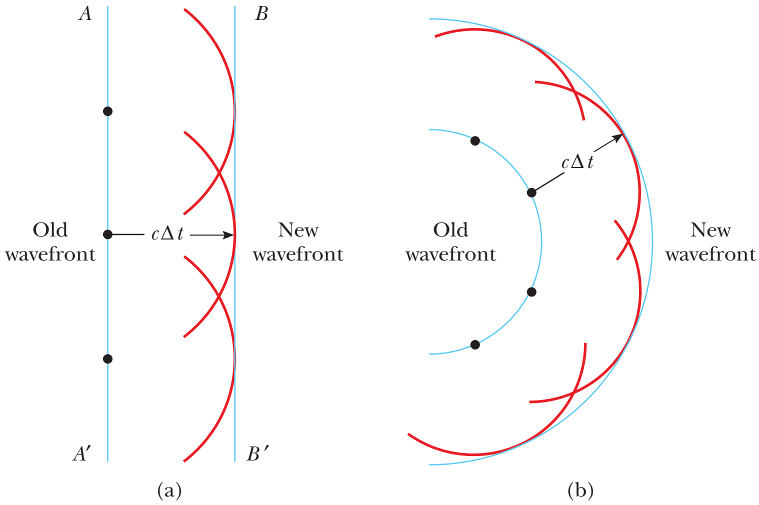


Figure 35.17 Huygens's construction for (a) a plane wave propagating to the right and (b) a spherical wave propagating to the right.

Huygens's Principle Applied to Reflection and Refraction

The laws of reflection and refraction were stated earlier in this chapter without proof. We now derive these laws, using Huygens's principle.

For the law of reflection, refer to Figure 35.18a. The line AB represents a wave front of the incident light just as ray 1 strikes the surface. At this instant, the wave at A sends out a Huygens wavelet (the circular arc centered on A) toward D. At the same time, the wave at B emits a Huygens wavelet (the circular arc centered on B) toward C. Figure 35.18a shows these wavelets after a time interval Δt, after which ray 2 strikes the surface. Because both rays 1 and 2 move with the same speed, we must have $AD = BC = c\,\Delta t$.

The remainder of our analysis depends on geometry, as summarized in Figure 35.18b, in which we isolate the triangles ABC and ADC. Note that these two triangles are congruent because they have the same hypotenuse AC and because $AD = BC$. From Figure 35.18b, we have

$$\cos \gamma = \frac{BC}{AC} \quad \text{and} \quad \cos \gamma' = \frac{AD}{AC}$$

where, comparing Figures 35.18a and 35.18b, we see that $\gamma = 90° - \theta_1$ and $\gamma' = 90° - \theta_1'$. Because $AD = BC$, we have

$$\cos \gamma = \cos \gamma'$$

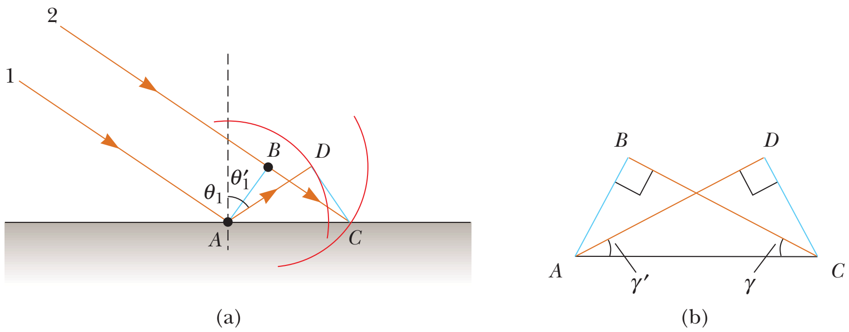


Figure 35.18 (a) Huygens's construction for proving the law of reflection. At the instant that ray 1 strikes the surface, it sends out a Huygens wavelet from A and ray 2 sends out a Huygens wavelet from B. We choose a radius of the wavelet to be $c\,\Delta t$, where Δt is the time interval for ray 2 to travel from B to C. (b) Triangle ADC is congruent to triangle ABC.

Therefore,

$$\gamma = \gamma'$$
$$90° - \theta_1 = 90° - \theta_1'$$

and

$$\theta_1 = \theta_1'$$

which is the law of reflection.

Now let us use Huygens's principle and Figure 35.19 to derive Snell's law of refraction. We focus our attention on the instant ray 1 strikes the surface and the subsequent time interval until ray 2 strikes the surface. During this time interval, the wave at A sends out a Huygens wavelet (the arc centered on A) toward D. In the same time interval, the wave at B sends out a Huygens wavelet (the arc centered on B) toward C. Because these two wavelets travel through different media, the radii of the wavelets are different. The radius of the wavelet from A is $AD = v_2\,\Delta t$, where v_2 is the wave speed in the second medium. The radius of the wavelet from B is $BC = v_1\,\Delta t$, where v_1 is the wave speed in the original medium.

From triangles ABC and ADC, we find that

$$\sin\theta_1 = \frac{BC}{AC} = \frac{v_1\,\Delta t}{AC} \qquad \text{and} \qquad \sin\theta_2 = \frac{AD}{AC} = \frac{v_2\,\Delta t}{AC}$$

If we divide the first equation by the second, we obtain

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

But from Equation 35.4 we know that $v_1 = c/n_1$ and $v_2 = c/n_2$. Therefore,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

which is Snell's law of refraction.

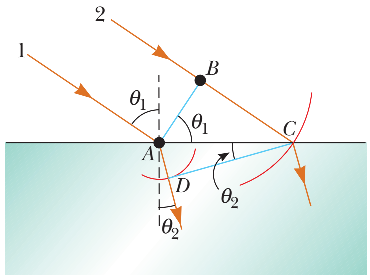


Figure 35.19 Huygens's construction for proving Snell's law of refraction. At the instant that ray 1 strikes the surface, it sends out a Huygens wavelet from A and ray 2 sends out a Huygens wavelet from B. The two wavelets have different radii because they travel in different media.

35.7 Dispersion and Prisms

An important property of the index of refraction n is that, for a given material, the index varies with the wavelength of the light passing through the material, as Figure 35.20 shows. This behavior is called **dispersion.** Because n is a function of wavelength, Snell's law of refraction indicates that light of different wavelengths is bent at different angles when incident on a refracting material.

As we see from Figure 35.20, the index of refraction generally decreases with increasing wavelength. This means that violet light bends more than red light does when passing into a refracting material. To understand the effects that dispersion can have on light, consider what happens when light strikes a prism, as shown in Figure 35.21. A ray of single-wavelength light incident on the prism from the left emerges refracted from its original direction of travel by an angle δ, called the **angle of deviation.**

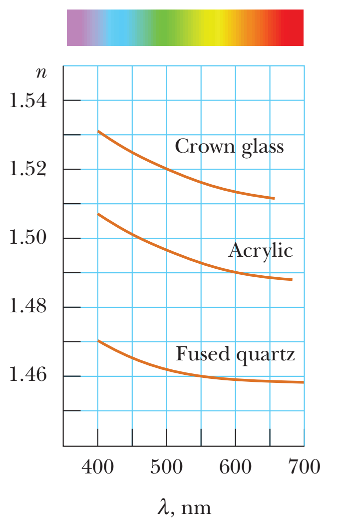


Figure 35.20 Variation of index of refraction with vacuum wavelength for three materials.

Now suppose that a beam of *white light* (a combination of all visible wavelengths) is incident on a prism, as illustrated in Figure 35.22. The rays that emerge spread out in a series of colors known as the **visible spectrum.** These colors, in order of decreasing wavelength, are red, orange, yellow, green, blue, and violet. Clearly, the angle of deviation δ depends on wavelength. Violet light deviates the most, red the least, and the remaining colors in the visible spectrum fall between these extremes. Newton showed that each color has a particular angle of deviation and that the colors can be recombined to form the original white light.

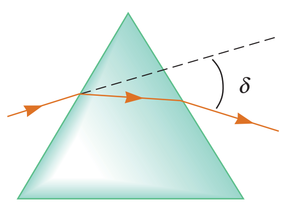
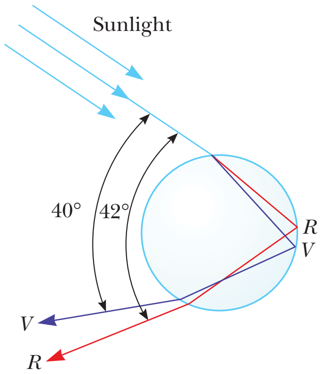


Figure 35.21 A prism refracts a single-wavelength light ray through an angle δ.

The dispersion of light into a spectrum is demonstrated most vividly in nature by the formation of a rainbow, which is often seen by an observer positioned between the Sun

Figure 35.22 White light enters a glass prism at the upper left. A reflected beam of light comes out of the prism just below the incoming beam. The beam moving toward the lower right shows distinct colors. Different colors are refracted at different angles because the index of refraction of the glass depends on wavelength. Violet light deviates the most; red light deviates the least.



Active Figure 35.23 Path of sunlight through a spherical raindrop. Light following this path contributes to the visible rainbow.

At the Active Figures link at http://www.pse6.com, you can vary the point at which the sunlight enters the raindrop to verify that the angles shown are the maximum angles.

PITFALL PREVENTION

35.5 A Rainbow of Many Light Rays

Pictorial representations such as Figure 35.23 are subject to misinterpretation. The figure shows one ray of light entering the raindrop and undergoing reflection and refraction, exiting the raindrop in a range of 40° to 42° from the entering ray. This might be interpreted incorrectly as meaning that *all* light entering the raindrop exits in this small range of angles. In reality, light exits the raindrop over a much larger range of angles, from 0° to 42°. A careful analysis of the reflection and refraction from the spherical raindrop shows that the range of 40° to 42° is where the *highest-intensity light* exits the raindrop.

and a rain shower. To understand how a rainbow is formed, consider Figure 35.23. A ray of sunlight (which is white light) passing overhead strikes a drop of water in the atmosphere and is refracted and reflected as follows: It is first refracted at the front surface of the drop, with the violet light deviating the most and the red light the least. At the back surface of the drop, the light is reflected and returns to the front surface, where it again undergoes refraction as it moves from water into air. The rays leave the drop such that the angle between the incident white light and the most intense returning violet ray is 40° and the angle between the white light and the most intense returning red ray is 42°. This small angular difference between the returning rays causes us to see a colored bow.

Now suppose that an observer is viewing a rainbow, as shown in Figure 35.24. If a raindrop high in the sky is being observed, the most intense red light returning from the drop can reach the observer because it is deviated the most, but the most intense violet light passes over the observer because it is deviated the least. Hence, the observer sees this drop as being red. Similarly, a drop lower in the sky would direct the most intense violet light toward the observer and appears to be violet. (The most intense red light from this drop would pass below the eye of the observer and not be

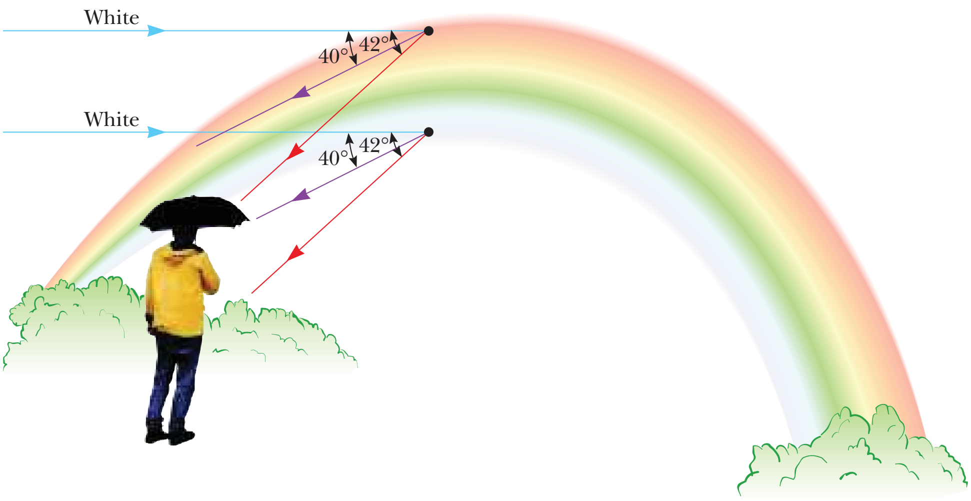


Figure 35.24 The formation of a rainbow seen by an observer standing with the Sun behind his back.

seen.) The most intense light from other colors of the spectrum would reach the observer from raindrops lying between these two extreme positions.

The opening photograph for this chapter shows a *double rainbow*. The secondary rainbow is fainter than the primary rainbow and the colors are reversed. The secondary rainbow arises from light that makes two reflections from the interior surface before exiting the raindrop. In the laboratory, rainbows have been observed in which the light makes over 30 reflections before exiting the water drop. Because each reflection involves some loss of light due to refraction out of the water drop, the intensity of these higher-order rainbows is small compared to the intensity of the primary rainbow.

Quick Quiz 35.5 Lenses in a camera use refraction to form an image on a film. Ideally, you want all the colors in the light from the object being photographed to be refracted by the same amount. Of the materials shown in Figure 35.20, which would you choose for a camera lens? (a) crown glass (b) acrylic (c) fused quartz (d) impossible to determine

Example 35.7 Measuring *n* Using a Prism

Although we do not prove it here, the minimum angle of deviation δ_{min} for a prism occurs when the angle of incidence θ_1 is such that the refracted ray inside the prism makes the same angle with the normal to the two prism faces,[1] as shown in Figure 35.25. Obtain an expression for the index of refraction of the prism material.

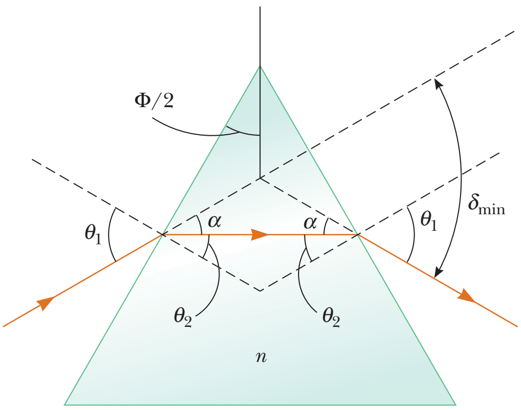


Figure 35.25 (Example 35.7) A light ray passing through a prism at the minimum angle of deviation δ_{min}.

Solution Using the geometry shown in Figure 35.25, we find that $\theta_2 = \Phi/2$, where Φ is the apex angle and

$$\theta_1 = \theta_2 + \alpha = \frac{\Phi}{2} + \frac{\delta_{min}}{2} = \frac{\Phi + \delta_{min}}{2}$$

From Snell's law of refraction, with $n_1 = 1$ because medium 1 is air, we have

$$\sin \theta_1 = n \sin \theta_2$$

$$\sin\left(\frac{\Phi + \delta_{min}}{2}\right) = n \sin(\Phi/2)$$

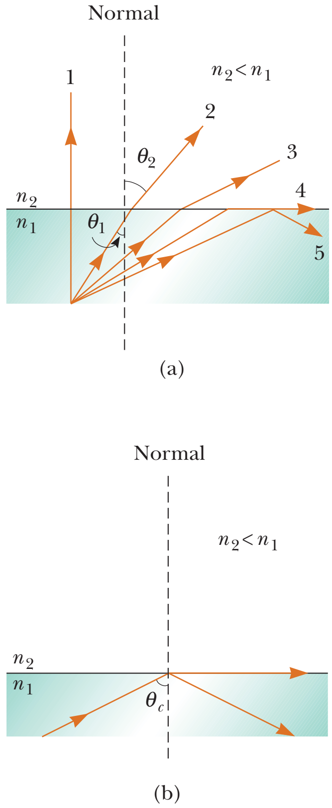
$$\boxed{n = \frac{\sin\left(\frac{\Phi + \delta_{min}}{2}\right)}{\sin(\Phi/2)}} \quad (35.9)$$

Hence, knowing the apex angle Φ of the prism and measuring δ_{min}, we can calculate the index of refraction of the prism material. Furthermore, we can use a hollow prism to determine the values of n for various liquids filling the prism.

35.8 Total Internal Reflection

An interesting effect called **total internal reflection** can occur when light is directed from a medium having a given index of refraction toward one having a lower index of refraction. Consider a light beam traveling in medium 1 and meeting the boundary between medium 1 and medium 2, where n_1 is greater than n_2 (Fig. 35.26a). Various possible directions of the beam are indicated by rays 1 through 5. The refracted rays are bent away from the normal because n_1 is greater than n_2. At some particular angle of incidence θ_c, called the **critical angle,** the refracted light ray moves parallel to the boundary so that $\theta_2 = 90°$ (Fig. 35.26b).

[1] The details of this proof are available in texts on optics.



Active Figure 35.26 (a) Rays travel from a medium of index of refraction n_1 into a medium of index of refraction n_2, where $n_2 < n_1$. As the angle of incidence θ_1 increases, the angle of refraction θ_2 increases until θ_2 is 90° (ray 4). For even larger angles of incidence, total internal reflection occurs (ray 5). (b) The angle of incidence producing an angle of refraction equal to 90° is the critical angle θ_c. At this angle of incidence, all of the energy of the incident light is reflected.

At the Active Figures link at http://www.pse6.com, you can vary the incident angle and see the effect on the refracted ray and the distribution of incident energy between the reflected and refracted rays.

For angles of incidence greater than θ_c, the beam is entirely reflected at the boundary, as shown by ray 5 in Figure 35.26a. This ray is reflected at the boundary as it strikes the surface. This ray and all those like it obey the law of reflection; that is, for these rays, the angle of incidence equals the angle of reflection.

We can use Snell's law of refraction to find the critical angle. When $\theta_1 = \theta_c$, $\theta_2 = 90°$ and Equation 35.8 gives

$$n_1 \sin \theta_c = n_2 \sin 90° = n_2$$

Critical angle for total internal reflection

$$\sin \theta_c = \frac{n_2}{n_1} \qquad (\text{for } n_1 > n_2) \tag{35.10}$$

This equation can be used only when n_1 is greater than n_2. That is, **total internal reflection occurs only when light is directed from a medium of a given index of refraction toward a medium of lower index of refraction.** If n_1 were less than n_2, Equation 35.10 would give $\sin \theta_c > 1$; this is a meaningless result because the sine of an angle can never be greater than unity.

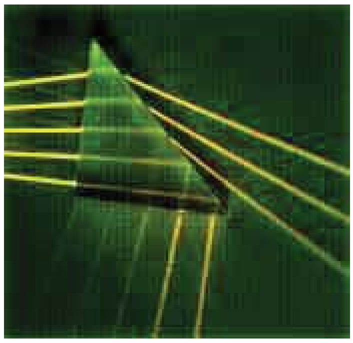
The critical angle for total internal reflection is small when n_1 is considerably greater than n_2. For example, the critical angle for a diamond in air is 24°. Any ray inside the diamond that approaches the surface at an angle greater than this is completely reflected back into the crystal. This property, combined with proper faceting, causes diamonds to sparkle. The angles of the facets are cut so that light is "caught" inside the crystal through multiple internal reflections. These multiple reflections give the light a long path through the medium, and substantial dispersion of colors occurs. By the time the light exits through the top surface of the crystal, the rays associated with different colors have been fairly widely separated from one another.

Cubic zirconia also has a high index of refraction and can be made to sparkle very much like a genuine diamond. If a suspect jewel is immersed in corn syrup, the difference in n for the cubic zirconia and that for the syrup is small, and the critical angle is therefore great. This means that more rays escape sooner, and as a result the sparkle completely disappears. A real diamond does not lose all of its sparkle when placed in corn syrup.

Quick Quiz 35.6 In Figure 35.27, five light rays enter a glass prism from the left. How many of these rays undergo total internal reflection at the slanted surface of the prism? (a) 1 (b) 2 (c) 3 (d) 4 (e) 5.

Quick Quiz 35.7 Suppose that the prism in Figure 35.27 can be rotated in the plane of the paper. In order for *all five* rays to experience total internal reflection from the slanted surface, should the prism be rotated (a) clockwise or (b) counterclockwise?

Quick Quiz 35.8 A beam of white light is incident on a crown glass–air interface as shown in Figure 35.26a. The incoming beam is rotated clockwise, so that the incident angle θ increases. Because of dispersion in the glass, some colors of light experience total internal reflection (ray 4 in Figure 35.26a) before other colors, so that the beam refracting out of the glass is no longer white. The last color to refract out of the upper surface is (a) violet (b) green (c) red (d) impossible to determine.



Courtesy of Henry Leap and Jim Lehman

Figure 35.27 (Quick Quiz 35.6 and 35.7) Five nonparallel light rays enter a glass prism from the left.

Example 35.8 A View from the Fish's Eye

Find the critical angle for an air–water boundary. (The index of refraction of water is 1.33.)

Solution We can use Figure 35.26 to solve this problem, with the air above the water having index of refraction n_2 and the water having index of refraction n_1. Applying Equation 35.10, we find that

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{1}{1.33} = 0.752$$

$$\theta_c = \boxed{48.8°}$$

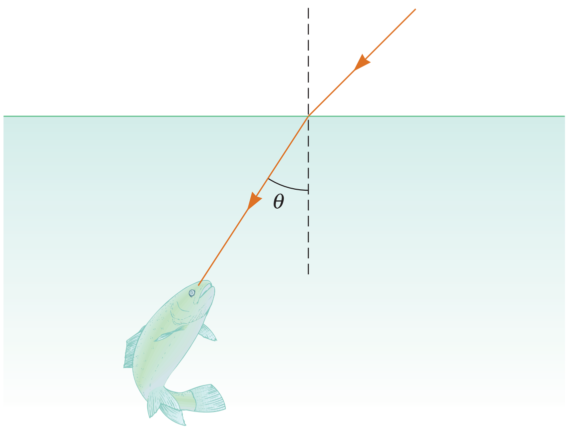
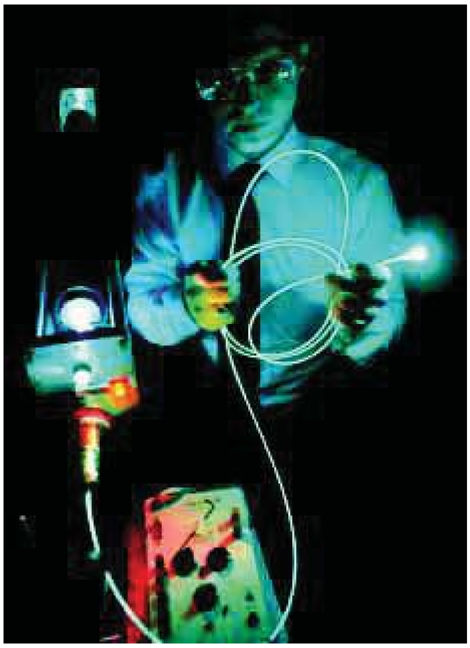


Figure 35.28 (Example 35.8) **What If?** A fish looks upward toward the water surface.

What If? **What if a fish in a still pond looks upward toward the water's surface at different angles relative to the surface, as in Figure 35.28? What does it see?**

Answer Because the path of a light ray is reversible, light traveling from medium 2 into medium 1 in Figure 35.26a follows the paths shown, but in the *opposite* direction. A fish looking upward toward the water surface, as in Figure 35.28, can see out of the water if it looks toward the surface at an angle less than the critical angle. Thus, for example, when the fish's line of vision makes an angle of 40° with the normal to the surface, light from above the water reaches the fish's eye. At 48.8°, the critical angle for water, the light has to skim along the water's surface before being refracted to the fish's eye; at this angle, the fish can in principle see the whole shore of the pond. At angles greater than the critical angle, the light reaching the fish comes by means of internal reflection at the surface. Thus, at 60°, the fish sees a reflection of the bottom of the pond.



(*Left*) Strands of glass optical fibers are used to carry voice, video, and data signals in telecommunication networks. (*Right*) A bundle of optical fibers is illuminated by a laser.

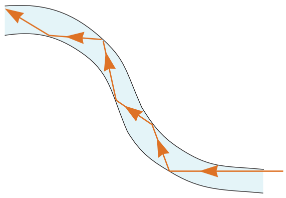


Figure 35.29 Light travels in a curved transparent rod by multiple internal reflections.

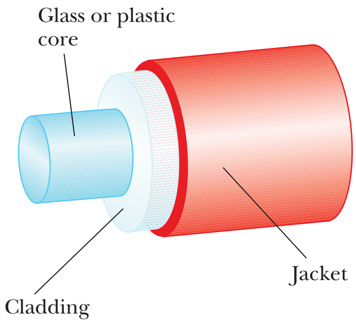


Figure 35.30 The construction of an optical fiber. Light travels in the core, which is surrounded by a cladding and a protective jacket.

Optical Fibers

Another interesting application of total internal reflection is the use of glass or transparent plastic rods to "pipe" light from one place to another. As indicated in Figure 35.29, light is confined to traveling within a rod, even around curves, as the result of successive total internal reflections. Such a light pipe is flexible if thin fibers are used rather than thick rods. A flexible light pipe is called an **optical fiber.** If a bundle of parallel fibers is used to construct an optical transmission line, images can be transferred from one point to another. This technique is used in a sizable industry known as *fiber optics*.

A practical optical fiber consists of a transparent core surrounded by a *cladding*, a material that has a lower index of refraction than the core. The combination may be surrounded by a plastic *jacket* to prevent mechanical damage. Figure 35.30 shows a cutaway view of this construction. Because the index of refraction of the cladding is less than that of the core, light traveling in the core experiences total internal reflection if it arrives at the interface between the core and the cladding at an angle of incidence that exceeds the critical angle. In this case, light "bounces" along the core of the optical fiber, losing very little of its intensity as it travels.

Any loss in intensity in an optical fiber is due essentially to reflections from the two ends and absorption by the fiber material. Optical fiber devices are particularly useful for viewing an object at an inaccessible location. For example, physicians often use such devices to examine internal organs of the body or to perform surgery without making large incisions. Optical fiber cables are replacing copper wiring and coaxial cables for telecommunications because the fibers can carry a much greater volume of telephone calls or other forms of communication than electrical wires can.

35.9 Fermat's Principle

Pierre de Fermat (1601–1665) developed a general principle that can be used to determine the path that light follows as it travels from one point to another. **Fermat's principle** states that **when a light ray travels between any two points, its path is**

the one that requires the smallest time interval. An obvious consequence of this principle is that the paths of light rays traveling in a homogeneous medium are straight lines because a straight line is the shortest distance between two points.

Let us illustrate how Fermat's principle can be used to derive Snell's law of refraction. Suppose that a light ray is to travel from point P in medium 1 to point Q in medium 2 (Fig. 35.31), where P and Q are at perpendicular distances a and b, respectively, from the interface. The speed of light is c/n_1 in medium 1 and c/n_2 in medium 2. Using the geometry of Figure 35.31, and assuming that light leaves P at $t = 0$, we see that the time at which the ray arrives at Q is

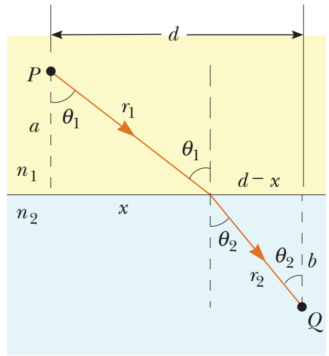


Figure 35.31 Geometry for deriving Snell's law of refraction using Fermat's principle.

$$t = \frac{r_1}{v_1} + \frac{r_2}{v_2} = \frac{\sqrt{a^2 + x^2}}{c/n_1} + \frac{\sqrt{b^2 + (d - x)^2}}{c/n_2} \qquad (35.11)$$

To obtain the value of x for which t has its minimum value, we take the derivative of t with respect to x and set the derivative equal to zero:

$$\begin{aligned}\frac{dt}{dx} &= \frac{n_1}{c}\frac{d}{dx}\sqrt{a^2 + x^2} + \frac{n_2}{c}\frac{d}{dx}\sqrt{b^2 + (d - x)^2} \\ &= \frac{n_1}{c}\left(\tfrac{1}{2}\right)\frac{2x}{(a^2 + x^2)^{1/2}} + \frac{n_2}{c}\left(\tfrac{1}{2}\right)\frac{2(d - x)(-1)}{[b^2 + (d - x)^2]^{1/2}} \\ &= \frac{n_1 x}{c(a^2 + x^2)^{1/2}} - \frac{n_2(d - x)}{c[b^2 + (d - x)^2]^{1/2}} = 0\end{aligned}$$

or

$$\frac{n_1 x}{(a^2 + x^2)^{1/2}} = \frac{n_2(d - x)}{[b^2 + (d - x)^2]^{1/2}} \qquad (35.12)$$

From Figure 35.31,

$$\sin\theta_1 = \frac{x}{(a^2 + x^2)^{1/2}} \qquad \sin\theta_2 = \frac{d - x}{[b^2 + (d - x)^2]^{1/2}}$$

Substituting these expressions into Equation 35.12, we find that

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

which is Snell's law of refraction.

This situation is equivalent to the problem of deciding where a lifeguard who can run faster than he can swim should enter the water to help a swimmer in distress. If he enters the water too directly (in other words, at a very small value of θ_1 in Figure 35.31), the distance x is smaller than the value of x that gives the minimum value of the time interval needed for the guard to move from the starting point on the sand to the swimmer. As a result, he spends too little time running and too much time swimming. The guard's optimum location for entering the water so that he can reach the swimmer in the shortest time is at that interface point that gives the value of x that satisfies Equation 35.12.

It is a simple matter to use a similar procedure to derive the law of reflection (see Problem 65).

SUMMARY

Take a practice test for this chapter by clicking on the Practice Test link at http://www.pse6.com.

In geometric optics, we use the **ray approximation,** in which a wave travels through a uniform medium in straight lines in the direction of the rays.

The **law of reflection** states that for a light ray traveling in air and incident on a smooth surface, the angle of reflection θ_1' equals the angle of incidence θ_1:

$$\theta_1' = \theta_1 \qquad (35.2)$$

Light crossing a boundary as it travels from medium 1 to medium 2 is **refracted,** or bent. The angle of refraction θ_2 is defined by the relationship

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} = \text{constant} \tag{35.3}$$

The **index of refraction** n of a medium is defined by the ratio

$$n \equiv \frac{c}{v} \tag{35.4}$$

where c is the speed of light in a vacuum and v is the speed of light in the medium. In general, n varies with wavelength and is given by

$$n = \frac{\lambda}{\lambda_n} \tag{35.7}$$

where λ is the vacuum wavelength and λ_n is the wavelength in the medium. As light travels from one medium to another, its frequency remains the same.

Snell's law of refraction states that

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \tag{35.8}$$

where n_1 and n_2 are the indices of refraction in the two media. The incident ray, the reflected ray, the refracted ray, and the normal to the surface all lie in the same plane.

Total internal reflection occurs when light travels from a medium of high index of refraction to one of lower index of refraction. The **critical angle** θ_c for which total internal reflection occurs at an interface is given by

$$\sin \theta_c = \frac{n_2}{n_1} \qquad (\text{for } n_1 > n_2) \tag{35.10}$$

QUESTIONS

1. Light of wavelength λ is incident on a slit of width d. Under what conditions is the ray approximation valid? Under what circumstances does the slit produce enough diffraction to make the ray approximation invalid?

2. Why do astronomers looking at distant galaxies talk about looking backward in time?

3. A solar eclipse occurs when the Moon passes between the Earth and the Sun. Use a diagram to show why some areas of the Earth see a total eclipse, other areas see a partial eclipse, and most areas see no eclipse.

4. The display windows of some department stores are slanted slightly inward at the bottom. This is to decrease the glare from streetlights or the Sun, which would make it difficult for shoppers to see the display inside. Sketch a light ray reflecting from such a window to show how this technique works.

5. You take a child for walks around the neighborhood. She loves to listen to echoes from houses when she shouts or when you clap loudly. A house with a large flat front wall can produce an echo if you stand straight in front of it and reasonably far away. Draw a bird's-eye view of the situation to explain the production of the echo. Shade in the area where you can stand to hear the echo. **What If?** The child helps you to discover that a house with an L-shaped floor plan can produce echoes if you are standing in a wider range of locations. You can be standing at any reasonably distant location from which you can see the inside corner. Explain the echo in this case and draw another diagram for comparison. **What If?** What if the two wings of the house are not perpendicular? Will you and the child, standing close together, hear echoes? **What If?** What if a rectangular house and its garage have a breezeway between them, so that their perpendicular walls do not meet in an inside corner? Will this structure produce strong echoes for people in a wide range of locations? Explain your answers with diagrams.

6. The F-117A stealth fighter (Figure Q35.6) is specifically designed to be a *non*-retroreflector of radar. What aspects of its design help accomplish this? *Suggestion:* Answer the previous question as preparation for this one. Note that the bottom of the plane is flat and that all of the flat exterior panels meet at odd angles.

Courtesy of U.S. Air Force, Langley Air Force Base

Figure Q35.6

7. Sound waves have much in common with light waves, including the properties of reflection and refraction. Give examples of these phenomena for sound waves.
8. Does a light ray traveling from one medium into another always bend toward the normal, as shown in Figure 35.10a? Explain.
9. As light travels from one medium to another, does the wavelength of the light change? Does the frequency change? Does the speed change? Explain.
10. A laser beam passing through a nonhomogeneous sugar solution follows a curved path. Explain.
11. A laser beam with vacuum wavelength 632.8 nm is incident from air onto a block of Lucite as shown in Figure 35.10b. The line of sight of the photograph is perpendicular to the plane in which the light moves. Find the speed, frequency, and wavelength of the light in the Lucite.
12. Suppose blue light were used instead of red light in the experiment shown in Figure 35.10b. Would the refracted beam be bent at a larger or smaller angle?
13. The level of water in a clear, colorless glass is easily observed with the naked eye. The level of liquid helium in a clear glass vessel is extremely difficult to see with the naked eye. Explain.
14. In Example 35.6 we saw that light entering a slab with parallel sides will emerge offset, but still parallel to the incoming beam. Our assumption was that the index of refraction of the material did not vary with wavelength. If the slab were made of crown glass (see Fig. 35.20), what would the outgoing beam look like?
15. Explain why a diamond sparkles more than a glass crystal of the same shape and size.
16. Explain why an oar partially in the water appears bent.
17. Total internal reflection is applied in the periscope of a submarine to let the user "see around corners." In this device, two prisms are arranged as shown in Figure Q35.17, so that an incident beam of light follows the path shown. Parallel tilted silvered mirrors could be used, but glass prisms with no silvered surfaces give higher light throughput. Propose a reason for the higher efficiency.

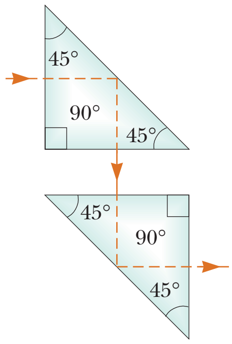


Figure Q35.17

18. Under certain circumstances, sound can be heard over extremely great distances. This frequently happens over a body of water, where the air near the water surface is cooler than the air higher up. Explain how the refraction of sound waves in such a situation could increase the distance over which the sound can be heard.
19. When two colors of light (X and Y) are sent through a glass prism, X is bent more than Y. Which color travels more slowly in the prism?
20. Retroreflection by transparent spheres, mentioned in Section 35.4 in the text, can be observed with dewdrops. To do so, look at the shadow of your head where it falls on dewy grass. Compare your observations to the reactions of two other people: The Renaissance artist Benvenuto Cellini described the phenomenon and his reaction in his *Autobiography*, at the end of Part One. The American philosopher Henry David Thoreau did the same in *Walden*, "Baker Farm," paragraph two. Try to find a person you know who has seen the halo—what did they think?
21. Why does the arc of a rainbow appear with red on top and violet on the bottom?

22. How is it possible that a complete circle of a rainbow can sometimes be seen from an airplane? With a stepladder, a lawn sprinkler, and a sunny day, how can you show the complete circle to children?

23. Is it possible to have total internal reflection for light incident from air on water? Explain.

24. Under what conditions is a mirage formed? On a hot day, what are we seeing when we observe "water on the road"?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 35.1 The Nature of Light
Section 35.2 Measurements of the Speed of Light

1. The *Apollo 11* astronauts set up a panel of efficient corner-cube retroreflectors on the Moon's surface. The speed of light can be found by measuring the time interval required for a laser beam to travel from Earth, reflect from the panel, and return to Earth. If this interval is measured to be 2.51 s, what is the measured speed of light? Take the center-to-center distance from Earth to Moon to be 3.84×10^8 m, and do not ignore the sizes of the Earth and Moon.

2. As a result of his observations, Roemer concluded that eclipses of Io by Jupiter were delayed by 22 min during a 6 month period as the Earth moved from the point in its orbit where it is closest to Jupiter to the diametrically opposite point where it is farthest from Jupiter. Using 1.50×10^8 km as the average radius of the Earth's orbit around the Sun, calculate the speed of light from these data.

3. In an experiment to measure the speed of light using the apparatus of Fizeau (see Fig. 35.2), the distance between light source and mirror was 11.45 km and the wheel had 720 notches. The experimentally determined value of c was 2.998×10^8 m/s. Calculate the minimum angular speed of the wheel for this experiment.

4. Figure P35.4 shows an apparatus used to measure the speed distribution of gas molecules. It consists of two slotted rotating disks separated by a distance d, with the slots displaced by the angle θ. Suppose the speed of light is measured by sending a light beam from the left through this apparatus. (a) Show that a light beam will be seen in the detector (that is, will make it through both slots) only if its speed is given by $c = \omega d/\theta$, where ω is the angular speed of the disks and θ is measured in radians. (b) What is the measured speed of light if the distance between the two slotted rotating disks is 2.50 m, the slot in the second disk is displaced 1/60 of one degree from the slot in the first disk, and the disks are rotating at 5 555 rev/s?

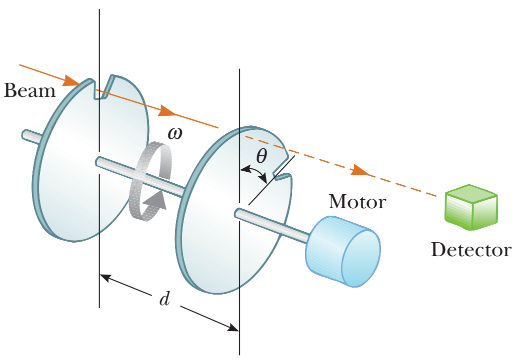


Figure P35.4

Section 35.3 The Ray Approximation in Geometric Optics
Section 35.4 Reflection
Section 35.5 Refraction

Note: You may look up indices of refraction in Table 35.1.

5. A dance hall is built without pillars and with a horizontal ceiling 7.20 m above the floor. A mirror is fastened flat against one section of the ceiling. Following an earthquake, the mirror is in place and unbroken. An engineer makes a quick check of whether the ceiling is sagging by directing a vertical beam of laser light up at the mirror and observing its reflection on the floor. (a) Show that if the mirror has rotated to make an angle ϕ with the horizontal, the normal to the mirror makes an angle ϕ with the vertical. (b) Show that the reflected laser light makes an angle 2ϕ with the vertical. (c) If the reflected laser light makes a spot on the floor 1.40 cm away from the point vertically below the laser, find the angle ϕ.

6. The two mirrors illustrated in Figure P35.6 meet at a right angle. The beam of light in the vertical plane P strikes mirror 1 as shown. (a) Determine the distance the

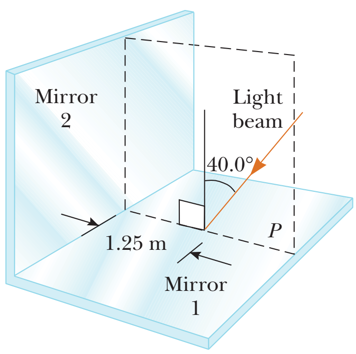


Figure P35.6

reflected light beam travels before striking mirror 2. (b) In what direction does the light beam travel after being reflected from mirror 2?

7. Two flat rectangular mirrors, both perpendicular to a horizontal sheet of paper, are set edge to edge with their reflecting surfaces perpendicular to each other. (a) A light ray in the plane of the paper strikes one of the mirrors at an arbitrary angle of incidence θ_1. Prove that the final direction of the ray, after reflection from both mirrors, is opposite to its initial direction. In a clothing store, such a pair of mirrors shows you an image of yourself as others see you, with no apparent right–left reversal. (b) **What If?** Now assume that the paper is replaced with a third flat mirror, touching edges with the other two and perpendicular to both. The set of three mirrors is called a *corner-cube reflector.* A ray of light is incident from any direction within the octant of space bounded by the reflecting surfaces. Argue that the ray will reflect once from each mirror and that its final direction will be opposite to its original direction. The *Apollo 11* astronauts placed a panel of corner cube retroreflectors on the Moon. Analysis of timing data taken with it reveals that the radius of the Moon's orbit is increasing at the rate of 3.8 cm/yr as it loses kinetic energy because of tidal friction.

8. How many times will the incident beam shown in Figure P35.8 be reflected by each of the parallel mirrors?

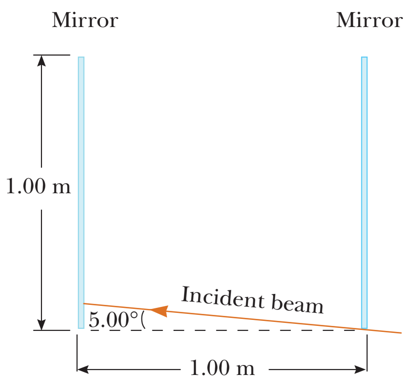


Figure P35.8

9. The distance of a lightbulb from a large plane mirror is twice the distance of a person from the plane mirror. Light from the bulb reaches the person by two paths. It travels to the mirror at an angle of incidence θ, and reflects from the mirror to the person. It also travels directly to the person without reflecting off the mirror. The total distance traveled by the light in the first case is twice the distance traveled by the light in the second case. Find the value of the angle θ.

10. A narrow beam of sodium yellow light, with wavelength 589 nm in vacuum, is incident from air onto a smooth water surface at an angle of incidence of 35.0°. Determine the angle of refraction and the wavelength of the light in water.

11. *Compare this problem with the preceding problem.* A plane sound wave in air at 20°C, with wavelength 589 mm, is incident on a smooth surface of water at 25°C, at an angle of incidence of 3.50°. Determine the angle of refraction for the sound wave and the wavelength of the sound in water.

12. The wavelength of red helium–neon laser light in air is 632.8 nm. (a) What is its frequency? (b) What is its wavelength in glass that has an index of refraction of 1.50? (c) What is its speed in the glass?

13. An underwater scuba diver sees the Sun at an apparent angle of 45.0° above the horizon. What is the actual elevation angle of the Sun above the horizon?

14. A ray of light is incident on a flat surface of a block of crown glass that is surrounded by water. The angle of refraction is 19.6°. Find the angle of reflection.

15. A laser beam is incident at an angle of 30.0° from the vertical onto a solution of corn syrup in water. If the beam is refracted to 19.24° from the vertical, (a) what is the index of refraction of the syrup solution? Suppose the light is red, with vacuum wavelength 632.8 nm. Find its (b) wavelength, (c) frequency, and (d) speed in the solution.

16. Find the speed of light in (a) flint glass, (b) water, and (c) cubic zirconia.

17. A light ray initially in water enters a transparent substance at an angle of incidence of 37.0°, and the transmitted ray is refracted at an angle of 25.0°. Calculate the speed of light in the transparent substance.

18. An opaque cylindrical tank with an open top has a diameter of 3.00 m and is completely filled with water. When the afternoon Sun reaches an angle of 28.0° above the horizon, sunlight ceases to illuminate any part of the bottom of the tank. How deep is the tank?

19. A ray of light strikes a flat block of glass ($n = 1.50$) of thickness 2.00 cm at an angle of 30.0° with the normal. Trace the light beam through the glass, and find the angles of incidence and refraction at each surface.

20. Unpolarized light in vacuum is incident onto a sheet of glass with index of refraction n. The reflected and refracted rays are perpendicular to each other. Find the angle of incidence. This angle is called *Brewster's angle* or the *polarizing angle.* In this situation the reflected light is linearly polarized, with its electric field restricted to be perpendicular to the plane containing the rays and the normal.

21. When the light illustrated in Figure P35.21 passes through the glass block, it is shifted laterally by the distance d. Taking $n = 1.50$, find the value of d.

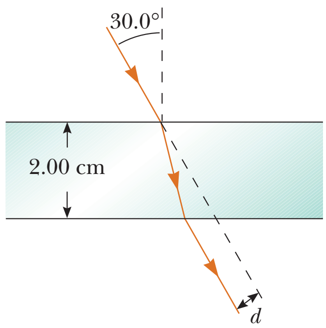


Figure P35.21 Problems 21 and 22.

22. Find the time interval required for the light to pass through the glass block described in the previous problem.

23. The light beam shown in Figure P35.23 makes an angle of 20.0° with the normal line NN' in the linseed oil. Determine the angles θ and θ'. (The index of refraction of linseed oil is 1.48.)

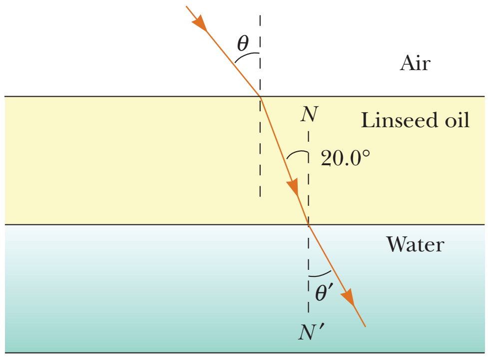


Figure P35.23

24. Three sheets of plastic have unknown indices of refraction. Sheet 1 is placed on top of sheet 2, and a laser beam is directed onto the sheets from above so that it strikes the interface at an angle of 26.5° with the normal. The refracted beam in sheet 2 makes an angle of 31.7° with the normal. The experiment is repeated with sheet 3 on top of sheet 2, and, with the same angle of incidence, the refracted beam makes an angle of 36.7° with the normal. If the experiment is repeated again with sheet 1 on top of sheet 3, what is the expected angle of refraction in sheet 3? Assume the same angle of incidence.

25. When you look through a window, by how much time is the light you see delayed by having to go through glass instead of air? Make an order-of-magnitude estimate on the basis of data you specify. By how many wavelengths is it delayed?

26. Light passes from air into flint glass. (a) What angle of incidence must the light have if the component of its velocity perpendicular to the interface is to remain constant? (b) **What If?** Can the component of velocity parallel to the interface remain constant during refraction?

27. The reflecting surfaces of two intersecting flat mirrors are at an angle θ ($0° < \theta < 90°$), as shown in Figure P35.27. For a light ray that strikes the horizontal mirror, show that the emerging ray will intersect the incident ray at an angle $\beta = 180° - 2\theta$.

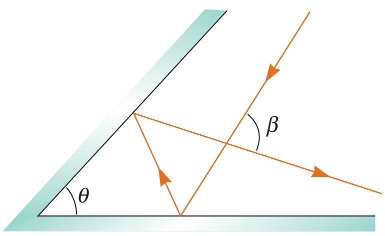


Figure P35.27

Section 35.6 Huygens's Principle

28. The speed of a water wave is described by $v = \sqrt{gd}$, where d is the water depth, assumed to be small compared to the wavelength. Because their speed changes, water waves refract when moving into a region of different depth. Sketch a map of an ocean beach on the eastern side of a landmass. Show contour lines of constant depth under water, assuming reasonably uniform slope. (a) Suppose that waves approach the coast from a storm far away to the north-northeast. Demonstrate that the waves will move nearly perpendicular to the shoreline when they reach the beach. (b) Sketch a map of a coastline with alternating bays and headlands, as suggested in Figure P35.28. Again make a reasonable guess about the shape of contour lines of constant depth. Suppose that waves approach the coast, carrying energy with uniform density along originally straight wavefronts. Show that the energy reaching the coast is concentrated at the headlands and has lower intensity in the bays.

Ray Atkeson/Image Archive

Figure P35.28

Section 35.7 Dispersion and Prisms

Note: The apex angle of a prism is the angle between the surface at which light enters the prism and the second surface the light encounters.

29. A narrow white light beam is incident on a block of fused quartz at an angle of 30.0°. Find the angular width of the light beam inside the quartz.

30. Light of wavelength 700 nm is incident on the face of a fused quartz prism at an angle of 75.0° (with respect to the normal to the surface). The apex angle of the prism is 60.0°. Use the value of n from Figure 35.20 and calculate the angle (a) of refraction at this first surface, (b) of incidence at the second surface, (c) of refraction at the second surface, and (d) between the incident and emerging rays.

$\boxed{31.}$ A prism that has an apex angle of 50.0° is made of cubic zirconia, with $n = 2.20$. What is its angle of minimum deviation?

32. A triangular glass prism with apex angle 60.0° has an index of refraction of 1.50. (a) Show that if its angle of incidence on the first surface is $\theta_1 = 48.6°$, light will pass symmetrically through the prism, as shown in

Figure 35.25. (b) Find the angle of deviation $\delta_{\min}$ for $\theta_1 = 48.6°$. (c) **What If?** Find the angle of deviation if the angle of incidence on the first surface is 45.6°. (d) Find the angle of deviation if $\theta_1 = 51.6°$.

33. A triangular glass prism with apex angle $\Phi = 60.0°$ has an index of refraction $n = 1.50$ (Fig. P35.33). What is the smallest angle of incidence θ_1 for which a light ray can emerge from the other side?

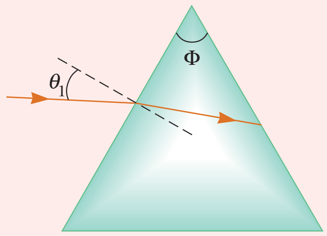


Figure P35.33 Problems 33 and 34.

34. A triangular glass prism with apex angle Φ has index of refraction n. (See Fig. P35.33.) What is the smallest angle of incidence θ_1 for which a light ray can emerge from the other side?

35. The index of refraction for violet light in silica flint glass is 1.66, and that for red light is 1.62. What is the angular dispersion of visible light passing through a prism of apex angle 60.0° if the angle of incidence is 50.0°? (See Fig. P35.35.)

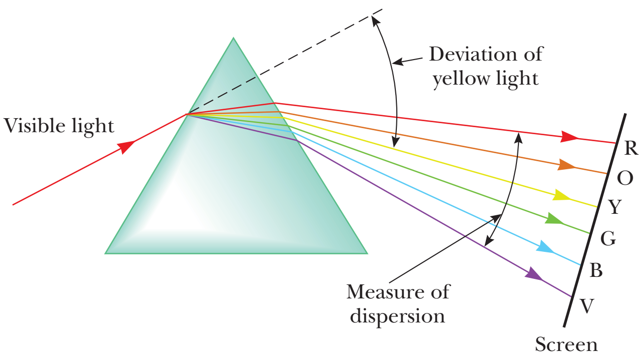


Figure P35.35

Section 35.8 Total Internal Reflection

36. For 589-nm light, calculate the critical angle for the following materials surrounded by air: (a) diamond, (b) flint glass, and (c) ice.

37. Repeat Problem 36 when the materials are surrounded by water.

38. Determine the maximum angle θ for which the light rays incident on the end of the pipe in Figure P35.38 are subject to total internal reflection along the walls of the pipe. Assume that the pipe has an index of refraction of 1.36 and the outside medium is air.

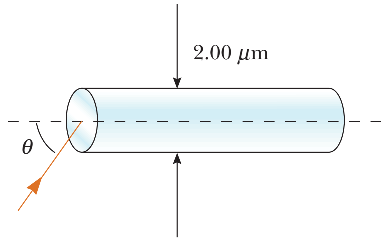


Figure P35.38

39. Consider a common mirage formed by super-heated air just above a roadway. A truck driver whose eyes are 2.00 m above the road, where $n = 1.000\,3$, looks forward. She perceives the illusion of a patch of water ahead on the road, where her line of sight makes an angle of 1.20° below the horizontal. Find the index of refraction of the air just above the road surface. (*Suggestion:* Treat this as a problem in total internal reflection.)

40. An optical fiber has index of refraction n and diameter d. It is surrounded by air. Light is sent into the fiber along its axis, as shown in Figure P35.40. (a) Find the smallest outside radius R permitted for a bend in the fiber if no light is to escape. (b) **What If?** Does the result for part (a) predict reasonable behavior as d approaches zero? As n increases? As n approaches 1? (c) Evaluate R assuming the fiber diameter is 100 μm and its index of refraction is 1.40.

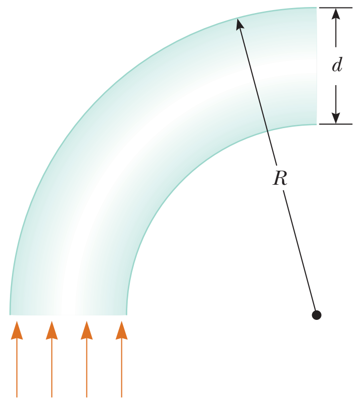


Figure P35.40

41. A large Lucite cube ($n = 1.59$) has a small air bubble (a defect in the casting process) below one surface. When a penny (diameter 1.90 cm) is placed directly over the bubble on the outside of the cube, the bubble cannot be seen by looking down into the cube at any angle. However, when a dime (diameter 1.75 cm) is placed directly over it, the bubble can be seen by looking down into the cube. What is the range of the possible depths of the air bubble beneath the surface?

42. A room contains air in which the speed of sound is 343 m/s. The walls of the room are made of concrete, in which the speed of sound is 1 850 m/s. (a) Find the critical angle for total internal reflection of sound at the concrete–air boundary. (b) In which medium must the sound be traveling in order to undergo total internal

reflection? (c) "A bare concrete wall is a highly efficient mirror for sound." Give evidence for or against this statement.

43. In about 1965, engineers at the Toro Company invented a gasoline gauge for small engines, diagrammed in Figure P35.43. The gauge has no moving parts. It consists of a flat slab of transparent plastic fitting vertically into a slot in the cap on the gas tank. None of the plastic has a reflective coating. The plastic projects from the horizontal top down nearly to the bottom of the opaque tank. Its lower edge is cut with facets making angles of 45° with the horizontal. A lawnmower operator looks down from above and sees a boundary between bright and dark on the gauge. The location of the boundary, across the width of the plastic, indicates the quantity of gasoline in the tank. Explain how the gauge works. Explain the design requirements, if any, for the index of refraction of the plastic.

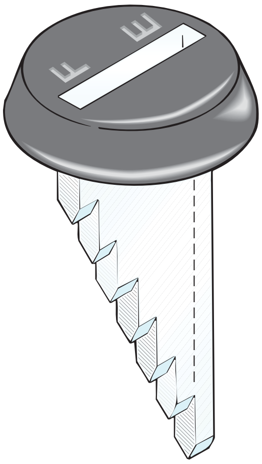


Figure P35.43

Section 35.9 Fermat's Principle

44. The shoreline of a lake runs east and west. A swimmer gets into trouble 20.0 m out from shore and 26.0 m to the east of a lifeguard, whose station is 16.0 m in from the shoreline. The lifeguard takes negligible time to accelerate. He can run at 7.00 m/s and swim at 1.40 m/s. To reach the swimmer as quickly as possible, in what direction should the lifeguard start running? You will need to solve a transcendental equation numerically.

Additional Problems

45. Figure P35.45 shows a desk ornament globe containing a photograph. The flat photograph is in air, inside a vertical slot located behind a water-filled compartment having the shape of one half of a cylinder. Suppose you are looking at the center of the photograph and then rotate the globe about a vertical axis. You find that the center of the photograph disappears when you rotate the globe beyond a certain maximum angle (Fig. P35.45b). Account for this phenomenon and calculate the maximum angle. Briefly describe what you would see when you turn the globe beyond this angle.

Courtesy Edwin Lo

(a) (b)

Figure P35.45

46. A light ray enters the atmosphere of a planet where it descends vertically to the surface a distance h below. The index of refraction where the light enters the atmosphere is 1.000, and it increases linearly to the surface where it has the value n. (a) How long does it take the ray to traverse this path? (b) Compare this to the time interval required in the absence of an atmosphere.

47. A narrow beam of light is incident from air onto the surface of glass with index of refraction 1.56. Find the angle of incidence for which the corresponding angle of refraction is half the angle of incidence. (*Suggestion:* You might want to use the trigonometric identity $\sin 2\theta = 2 \sin \theta \cos \theta$.)

48. (a) Consider a horizontal interface between air above and glass of index 1.55 below. Draw a light ray incident from the air at angle of incidence 30.0°. Determine the angles of the reflected and refracted rays and show them on the diagram. (b) **What If?** Suppose instead that the light ray is incident from the glass at angle of incidence 30.0°. Determine the angles of the reflected and refracted rays and show all three rays on a new diagram. (c) For rays incident from the air onto the air–glass surface, determine and tabulate the angles of reflection and refraction for all the angles of incidence at 10.0° intervals from 0° to 90.0°. (d) Do the same for light rays coming up to the interface through the glass.

$\boxed{49.}$ A small underwater pool light is 1.00 m below the surface. The light emerging from the water forms a circle on the water surface. What is the diameter of this circle?

50. One technique for measuring the angle of a prism is shown in Figure P35.50. A parallel beam of light is directed on the angle so that parts of the beam reflect from opposite sides. Show that the angular separation of the two reflected beams is given by $B = 2A$.

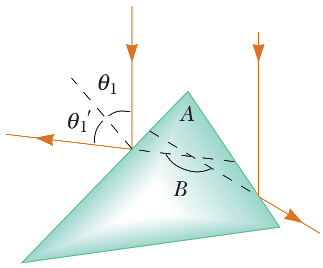


Figure P35.50

51. The walls of a prison cell are perpendicular to the four cardinal compass directions. On the first day of spring, light from the rising Sun enters a rectangular window in

the eastern wall. The light traverses 2.37 m horizontally to shine perpendicularly on the wall opposite the window. A young prisoner observes the patch of light moving across this western wall and for the first time forms his own understanding of the rotation of the Earth. (a) With what speed does the illuminated rectangle move? (b) The prisoner holds a small square mirror flat against the wall at one corner of the rectangle of light. The mirror reflects light back to a spot on the eastern wall close beside the window. How fast does the smaller square of light move across that wall? (c) Seen from a latitude of 40.0° north, the rising Sun moves through the sky along a line making a 50.0° angle with the southeastern horizon. In what direction does the rectangular patch of light on the western wall of the prisoner's cell move? (d) In what direction does the smaller square of light on the eastern wall move?

52. Figure P35.52 shows a top view of a square enclosure. The inner surfaces are plane mirrors. A ray of light enters a small hole in the center of one mirror. (a) At what angle θ must the ray enter in order to exit through the hole after being reflected once by each of the other three mirrors? (b) **What If?** Are there other values of θ for which the ray can exit after multiple reflections? If so, make a sketch of one of the ray's paths.

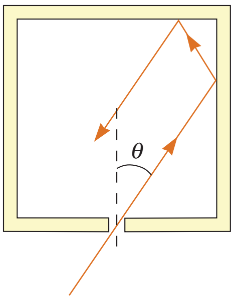


Figure P35.52

53. A hiker stands on an isolated mountain peak near sunset and observes a rainbow caused by water droplets in the air 8.00 km away. The valley is 2.00 km below the mountain peak and entirely flat. What fraction of the complete circular arc of the rainbow is visible to the hiker? (See Fig. 35.24.)

54. A 4.00-m-long pole stands vertically in a lake having a depth of 2.00 m. The Sun is 40.0° above the horizontal. Determine the length of the pole's shadow on the bottom of the lake. Take the index of refraction for water to be 1.33.

55. A laser beam strikes one end of a slab of material, as shown in Figure P35.55. The index of refraction of the slab is 1.48. Determine the number of internal reflections of the beam before it emerges from the opposite end of the slab.

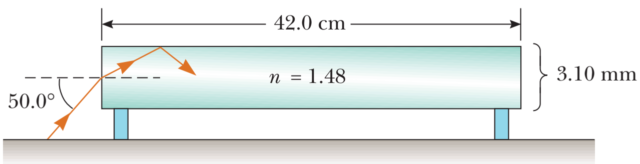


Figure P35.55

56. When light is incident normally on the interface between two transparent optical media, the intensity of the reflected light is given by the expression

$$S_1' = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2 S_1$$

In this equation S_1 represents the average magnitude of the Poynting vector in the incident light (the incident intensity), S_1' is the reflected intensity, and n_1 and n_2 are the refractive indices of the two media. (a) What fraction of the incident intensity is reflected for 589-nm light normally incident on an interface between air and crown glass? (b) **What If?** Does it matter in part (a) whether the light is in the air or in the glass as it strikes the interface?

57. Refer to Problem 56 for its description of the reflected intensity of light normally incident on an interface between two transparent media. (a) When light is normally incident on an interface between vacuum and a transparent medium of index n, show that the intensity S_2 of the transmitted light is given by $S_2/S_1 = 4n/(n+1)^2$. (b) Light travels perpendicularly through a diamond slab, surrounded by air, with parallel surfaces of entry and exit. Apply the transmission fraction in part (a) to find the approximate overall transmission through the slab of diamond, as a percentage. Ignore light reflected back and forth within the slab.

58. **What If?** This problem builds upon the results of Problems 56 and 57. Light travels perpendicularly through a diamond slab, surrounded by air, with parallel surfaces of entry and exit. The intensity of the transmitted light is what fraction of the incident intensity? Include the effects of light reflected back and forth inside the slab.

59. The light beam in Figure P35.59 strikes surface 2 at the critical angle. Determine the angle of incidence θ_1.

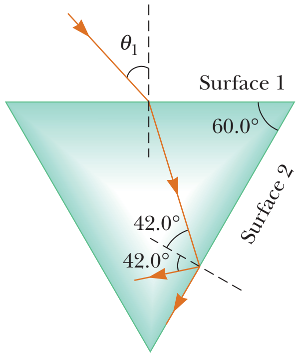


Figure P35.59

60. Builders use a leveling instrument with the beam from a fixed helium–neon laser reflecting in a horizontal plane from a small flat mirror mounted on an accurately vertical rotating shaft. The light is sufficiently bright and the rotation rate is sufficiently high that the reflected light appears as a horizontal line wherever it falls on a wall. (a) Assume the mirror is at the center of a circular grain elevator of radius R. The mirror spins with constant angular velocity ω_m. Find the speed of the spot of laser light on the wall. (b) **What If?** Assume the spinning mirror is at a perpendicular distance d from point O on a flat vertical wall. When the spot of laser light on the wall is at distance x from point O, what is its speed?

61. A light ray of wavelength 589 nm is incident at an angle θ on the top surface of a block of polystyrene, as shown in Figure P35.61. (a) Find the maximum value of θ for which the refracted ray undergoes total internal reflection at the left vertical face of the block. **What If?** Repeat the calculation for the case in which the polystyrene block is immersed in (b) water and (c) carbon disulfide.

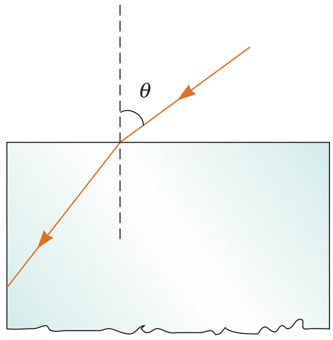


Figure P35.61

62. Refer to Quick Quiz 35.4. By how much does the duration of an optical day exceed that of a geometric day? Model the Earth's atmosphere as uniform, with index of refraction 1.000 293, a sharply defined upper surface, and depth 8 614 m. Assume that the observer is at the Earth's equator, so that the apparent path of the rising and setting Sun is perpendicular to the horizon.

63. A shallow glass dish is 4.00 cm wide at the bottom, as shown in Figure P35.63. When an observer's eye is placed as shown, the observer sees the edge of the bottom of the empty dish. When this dish is filled with water, the observer sees the center of the bottom of the dish. Find the height of the dish.

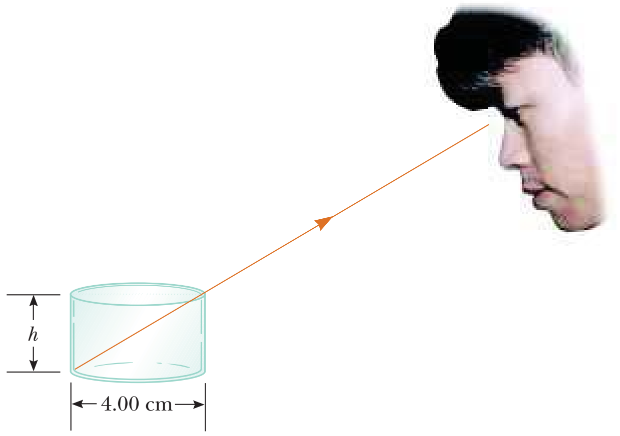


Figure P35.63

64. A ray of light passes from air into water. For its deviation angle $\delta = |\theta_1 - \theta_2|$ to be 10.0°, what must be its angle of incidence?

65. Derive the law of reflection (Eq. 35.2) from Fermat's principle. (See the procedure outlined in Section 35.9 for the derivation of the law of refraction from Fermat's principle.)

66. A material having an index of refraction n is surrounded by a vacuum and is in the shape of a quarter circle of radius R (Fig. P35.66). A light ray parallel to the base of the material is incident from the left at a distance L above the base and emerges from the material at the angle θ. Determine an expression for θ.

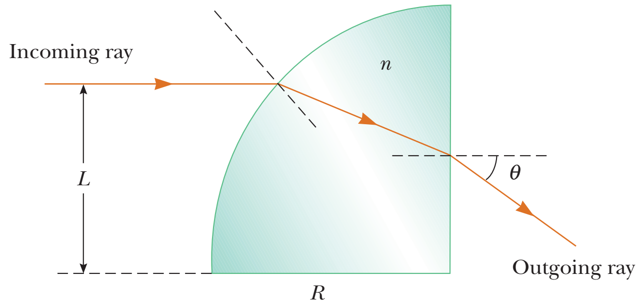


Figure P35.66

67. A transparent cylinder of radius $R = 2.00$ m has a mirrored surface on its right half, as shown in Figure P35.67. A light ray traveling in air is incident on the left side of the cylinder. The incident light ray and exiting light ray are parallel and $d = 2.00$ m. Determine the index of refraction of the material.

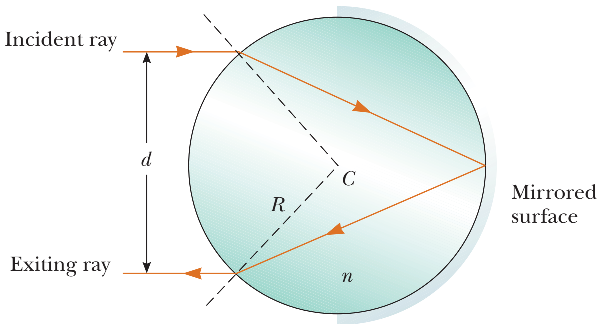


Figure P35.67

68. Suppose that a luminous sphere of radius R_1 (such as the Sun) is surrounded by a uniform atmosphere of radius R_2 and index of refraction n. When the sphere is viewed from a location far away in vacuum, what is its apparent radius? You will need to distinguish between the two cases (a) $R_2 > nR_1$ and (b) $R_2 < nR_1$.

69. A. H. Pfund's method for measuring the index of refraction of glass is illustrated in Figure P35.69. One face of a slab of thickness t is painted white, and a small hole scraped clear at point P serves as a source of diverging rays when the slab is

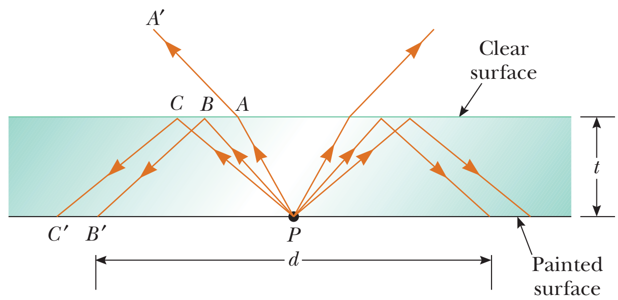


Figure P35.69

illuminated from below. Ray PBB' strikes the clear surface at the critical angle and is totally reflected, as are rays such as PCC'. Rays such as PAA' emerge from the clear surface. On the painted surface there appears a dark circle of diameter d, surrounded by an illuminated region, or halo. (a) Derive an equation for n in terms of the measured quantities d and t. (b) What is the diameter of the dark circle if $n = 1.52$ for a slab 0.600 cm thick? (c) If white light is used, the critical angle depends on color caused by dispersion. Is the inner edge of the white halo tinged with red light or violet light? Explain.

70. A light ray traveling in air is incident on one face of a right-angle prism of index of refraction $n = 1.50$ as shown in Figure P35.70, and the ray follows the path shown in the figure. Assuming $\theta = 60.0°$ and the base of the prism is mirrored, determine the angle ϕ made by the outgoing ray with the normal to the right face of the prism.

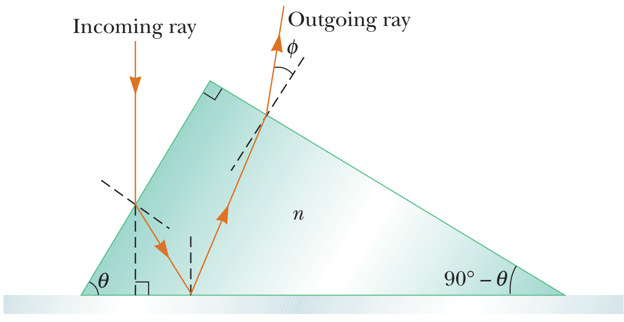


Figure P35.70

71. A light ray enters a rectangular block of plastic at an angle $\theta_1 = 45.0°$ and emerges at an angle $\theta_2 = 76.0°$, as shown in Figure P35.71. (a) Determine the index of refraction of the plastic. (b) If the light ray enters the plastic at a point $L = 50.0$ cm from the bottom edge, how long does it take the light ray to travel through the plastic?

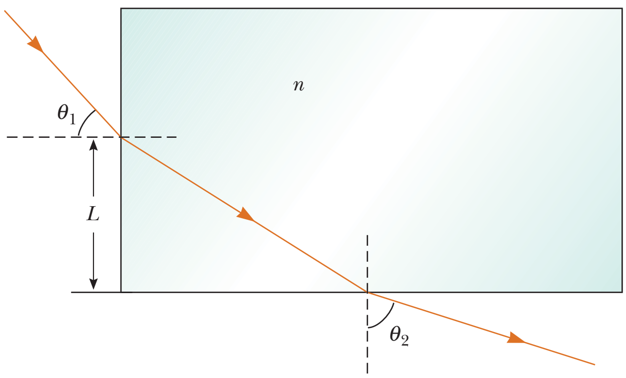


Figure P35.71

72. Students allow a narrow beam of laser light to strike a water surface. They arrange to measure the angle of refraction for selected angles of incidence and record the data shown in the accompanying table. Use the data to verify Snell's law of refraction by plotting the sine of the angle of incidence versus the sine of the angle of refraction. Use the resulting plot to deduce the index of refraction of water.

Angle of Incidence (degrees)	Angle of Refraction (degrees)
10.0	7.5
20.0	15.1
30.0	22.3
40.0	28.7
50.0	35.2
60.0	40.3
70.0	45.3
80.0	47.7

Answers to Quick Quizzes

35.1 (d). The light rays from the actor's face must reflect from the mirror and into the camera. If these light rays are reversed, light from the camera reflects from the mirror into the eyes of the actor.

35.2 Beams ② and ④ are reflected; beams ③ and ⑤ are refracted.

35.3 (c). Because the light is entering a material in which the index of refraction is lower, the speed of light is higher and the light bends away from the normal.

35.4 (a). Due to the refraction of light by air, light rays from the Sun deviate slightly downward toward the surface of the Earth as the light enters the atmosphere. Thus, in the morning, light rays from the upper edge of the Sun arrive at your eyes before the geometric line from your eyes to the top of the Sun clears the horizon. In the evening, light rays from the top of the Sun continue to arrive at your eyes even after the geometric line from your eyes to the top of the Sun dips below the horizon.

35.5 (c). An ideal camera lens would have an index of refraction that does not vary with wavelength so that all colors would be bent through the same angle by the lens. Of the three choices, fused quartz has the least variation in n across the visible spectrum.

35.6 (b). The two bright rays exiting the bottom of the prism on the right in Figure 35.27 result from total internal reflection at the right face of the prism. Notice that there is no refracted light exiting the slanted side for these rays. The light from the other three rays is divided into reflected and refracted parts.

35.7 (b). Counterclockwise rotation of the prism will cause the rays to strike the slanted side of the prism at a larger angle. When all five rays strike at an angle larger than the critical angle, they will all undergo total internal reflection.

35.8 (c). When the outgoing beam approaches the direction parallel to the straight side, the incident angle is approaching the critical angle for total internal reflection. The index of refraction for light at the violet end of the visible spectrum is larger than that at the red end. Thus, as the outgoing beam approaches the straight side, the violet light experiences total internal reflection first, followed by the other colors. The red light is the last to experience total internal reflection.

Chapter 36

Image Formation

CHAPTER OUTLINE

▲ *The light rays coming from the leaves in the background of this scene did not form a focused image on the film of the camera that took this photograph. Consequently, the background appears very blurry. Light rays passing though the raindrop, however, have been altered so as to form a focused image of the background leaves on the film. In this chapter, we investigate the formation of images as light rays reflect from mirrors and refract through lenses. (Don Hammond/CORBIS)*

This chapter is concerned with the images that result when light rays encounter flat and curved surfaces. We find that images can be formed either by reflection or by refraction and that we can design mirrors and lenses to form images with desired characteristics. We continue to use the ray approximation and to assume that light travels in straight lines. Both of these steps lead to valid predictions in the field called *geometric optics.* In subsequent chapters, we shall concern ourselves with interference and diffraction effects—the objects of study in the field of *wave optics.*

36.1 Images Formed by Flat Mirrors

We begin by considering the simplest possible mirror, the flat mirror. Consider a point source of light placed at O in Figure 36.1, a distance p in front of a flat mirror. The distance p is called the **object distance.** Light rays leave the source and are reflected from the mirror. Upon reflection, the rays continue to diverge (spread apart). The dashed lines in Figure 36.1 are extensions of the diverging rays back to a point of intersection at I. The diverging rays appear to the viewer to come from the point I behind the mirror. Point I is called the **image** of the object at O. Regardless of the system under study, we always locate images by extending diverging rays back to a point at which they intersect. **Images are located either at a point from which rays of light *actually* diverge or at a point from which they *appear* to diverge.** Because the rays in Figure 36.1 appear to originate at I, which is a distance q behind the mirror, this is the location of the image. The distance q is called the **image distance.**

Images are classified as **real** or **virtual. A real image is formed when light rays pass through and diverge from the image point; a virtual image is formed when the light rays do not pass through the image point but only appear to diverge from that point.** The image formed by the mirror in Figure 36.1 is virtual. The image of an object seen in a flat mirror is *always* virtual. Real images can be displayed on a

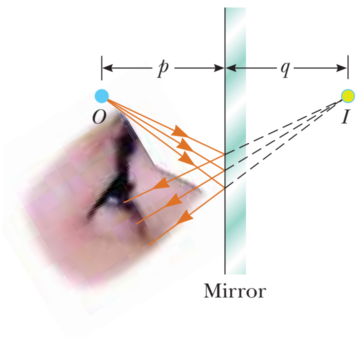
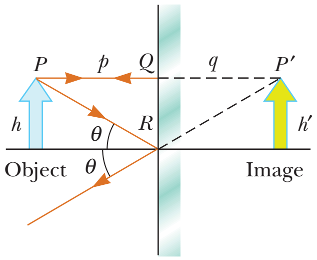


Figure 36.1 An image formed by reflection from a flat mirror. The image point I is located behind the mirror a perpendicular distance q from the mirror (the image distance). The image distance has the same magnitude as the object distance p.



Active Figure 36.2 A geometric construction that is used to locate the image of an object placed in front of a flat mirror. Because the triangles PQR and $P'QR$ are congruent, $|p| = |q|$ and $h = h'$.

At the Active Figures link at http://www.pse6.com, you can move the object and see the effect on the image.

screen (as at a movie), but virtual images cannot be displayed on a screen. We shall see an example of a real image in Section 36.2.

We can use the simple geometry in Figure 36.2 to examine the properties of the images of extended objects formed by flat mirrors. Even though there are an infinite number of choices of direction in which light rays could leave each point on the object, we need to choose only two rays to determine where an image is formed. One of those rays starts at P, follows a horizontal path to the mirror, and reflects back on itself. The second ray follows the oblique path PR and reflects as shown, according to the law of reflection. An observer in front of the mirror would trace the two reflected rays back to the point at which they appear to have originated, which is point P' behind the mirror. A continuation of this process for points other than P on the object would result in a virtual image (represented by a yellow arrow) behind the mirror. Because triangles PQR and $P'QR$ are congruent, $PQ = P'Q$. We conclude that **the image formed by an object placed in front of a flat mirror is as far behind the mirror as the object is in front of the mirror.**

Geometry also reveals that the object height h equals the image height h'. Let us define **lateral magnification** M of an image as follows:

Lateral magnification

$$M \equiv \frac{\text{Image height}}{\text{Object height}} = \frac{h'}{h} \qquad (36.1)$$

This is a general definition of the lateral magnification for an image from any type of mirror. (This equation is also valid for images formed by lenses, which we study in Section 36.4.) For a flat mirror, $M = 1$ for any image because $h' = h$.

Finally, note that a flat mirror produces an image that has an *apparent* left–right reversal. You can see this reversal by standing in front of a mirror and raising your right hand, as shown in Figure 36.3. The image you see raises its left hand. Likewise, your hair appears to be parted on the side opposite your real part, and a mole on your right cheek appears to be on your left cheek.

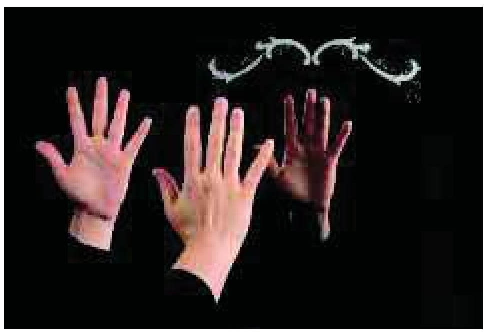
This reversal is not *actually* a left–right reversal. Imagine, for example, lying on your left side on the floor, with your body parallel to the mirror surface. Now your head is on the left and your feet are on the right. If you shake your feet, the image does not shake its head! If you raise your right hand, however, the image again raises its left hand. Thus, the mirror again appears to produce a left–right reversal but in the up–down direction!

The reversal is actually a *front–back reversal,* caused by the light rays going forward toward the mirror and then reflecting back from it. An interesting exercise is to stand in front of a mirror while holding an overhead transparency in front of you so that you can read the writing on the transparency. You will also be able to read the writing on the image of the transparency. You may have had a similar experience if you have attached a transparent decal with words on it to the rear window of your car. If the

PITFALL PREVENTION

36.1 Magnification Does Not Necessarily Imply Enlargement

For optical elements other than flat mirrors, the magnification defined in Equation 36.1 can result in a number with magnitude larger *or* smaller than 1. Thus, despite the cultural usage of the word *magnification* to mean *enlargement,* the image could be smaller than the object.



George Semple

Figure 36.3 The image in the mirror of a person's right hand is reversed front to back. This makes the right hand appear to be a left hand. Notice that the thumb is on the left side of both real hands and on the left side of the image. That the thumb is not on the right side of the image indicates that there is no left-to-right reversal.

decal can be read from outside the car, you can also read it when looking into your rearview mirror from inside the car.

We conclude that the image that is formed by a flat mirror has the following properties.

- The image is as far behind the mirror as the object is in front.
- The image is unmagnified, virtual, and upright. (By upright we mean that, if the object arrow points upward as in Figure 36.2, so does the image arrow.)
- The image has front–back reversal.

Quick Quiz 36.1 In the overhead view of Figure 36.4, the image of the stone seen by observer 1 is at *C*. At which of the five points *A*, *B*, *C*, *D*, or *E* does observer 2 see the image?

Figure 36.4 (Quick Quiz 36.1) Where does observer 2 see the image of the stone?

Quick Quiz 36.2 You are standing about 2 m away from a mirror. The mirror has water spots on its surface. True or false: It is possible for you to see the water spots and your image both in focus at the same time.

Conceptual Example 36.1 Multiple Images Formed by Two Mirrors

Two flat mirrors are perpendicular to each other, as in Figure 36.5, and an object is placed at point *O*. In this situation, multiple images are formed. Locate the positions of these images.

Solution The image of the object is at I_1 in mirror 1 and at I_2 in mirror 2. In addition, a third image is formed at I_3. This third image is the image of I_1 in mirror 2 or, equivalently, the image of I_2 in mirror 1. That is, the image at I_1 (or I_2) serves as the object for I_3. Note that to form this image at I_3, the rays reflect twice after leaving the object at *O*.

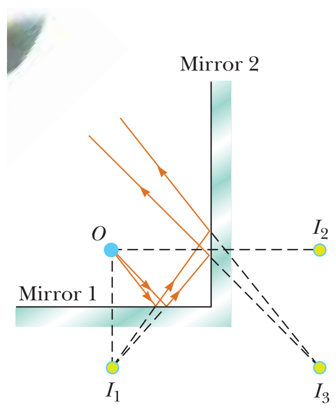


Figure 36.5 (Conceptual Example 36.1) When an object is placed in front of two mutually perpendicular mirrors as shown, three images are formed.

Conceptual Example 36.2 The Levitated Professor

The professor in the box shown in Figure 36.6 appears to be balancing himself on a few fingers, with his feet off the floor. He can maintain this position for a long time, and he appears to defy gravity. How was this illusion created?

Solution This is one of many magicians' optical illusions that make use of a mirror. The box in which the professor stands is a cubical frame that contains a flat vertical mirror positioned in a diagonal plane of the frame. The professor straddles the mirror so that one foot, which you see, is in front of the mirror, and the other foot, which you cannot see, is behind the mirror. When he raises the foot in front of the mirror, the reflection of that foot also rises, so he appears to float in air.

Courtesy of Henry Leap and Jim Lehman

Figure 36.6 (Conceptual Example 36.2) An optical illusion.

Conceptual Example 36.3 The Tilting Rearview Mirror

Most rearview mirrors in cars have a day setting and a night setting. The night setting greatly diminishes the intensity of the image in order that lights from trailing vehicles do not blind the driver. How does such a mirror work?

Solution Figure 36.7 shows a cross-sectional view of a rearview mirror for each setting. The unit consists of a reflective coating on the back of a wedge of glass. In the day setting (Fig. 36.7a), the light from an object behind the car strikes the glass wedge at point 1. Most of the light enters the wedge, refracting as it crosses the front surface, and reflects from the back surface to return to the front surface, where it is refracted again as it re-enters the air as ray B (for *bright*). In addition, a small portion of the light is reflected at the front surface of the glass, as indicated by ray D (for *dim*).

This dim reflected light is responsible for the image that is observed when the mirror is in the night setting (Fig. 36.7b). In this case, the wedge is rotated so that the path followed by the bright light (ray B) does not lead to the eye. Instead, the dim light reflected from the front surface of the wedge travels to the eye, and the brightness of trailing headlights does not become a hazard.

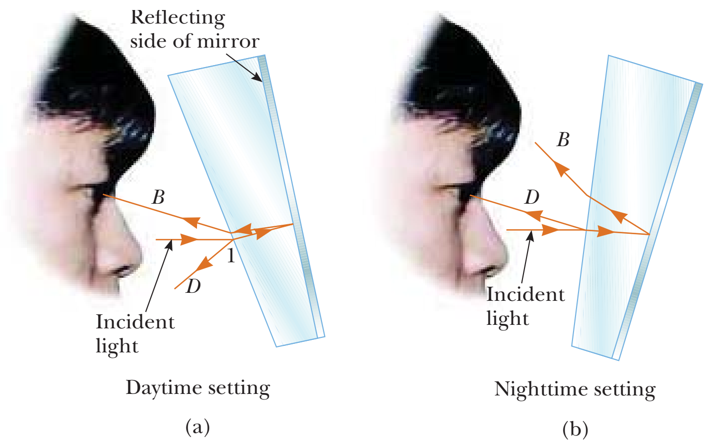
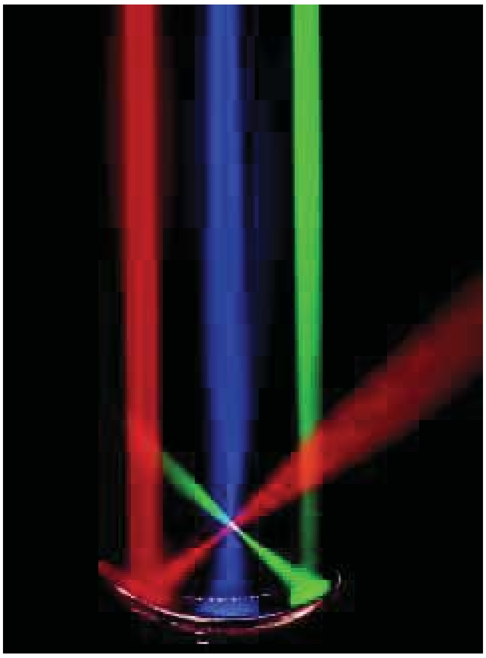


Figure 36.7 (Conceptual Example 36.3) Cross-sectional views of a rearview mirror. (a) With the day setting, the silvered back surface of the mirror reflects a bright ray B into the driver's eyes. (b) With the night setting, the glass of the unsilvered front surface of the mirror reflects a dim ray D into the driver's eyes.



Ken Kay/Fundamental Photographs

Figure 36.8 Red, blue, and green light rays are reflected by a curved mirror. Note that the three colored beams meet at a point.

36.2 Images Formed by Spherical Mirrors

Concave Mirrors

A **spherical mirror,** as its name implies, has the shape of a section of a sphere. This type of mirror focuses incoming parallel rays to a point, as demonstrated by the colored light rays in Figure 36.8. Figure 36.9a shows a cross section of a spherical mirror, with its surface represented by the solid, curved black line. (The blue band represents the structural support for the mirrored surface, such as a curved piece of glass on which the silvered surface is deposited.) Such a mirror, in which light is reflected from the inner, concave surface, is called a **concave mirror.** The mirror has a radius of curvature R, and its center of curvature is point C. Point V is the center of the spherical section, and a line through C and V is called the **principal axis** of the mirror.

Now consider a point source of light placed at point O in Figure 36.9b, where O is any point on the principal axis to the left of C. Two diverging rays that originate at O are shown. After reflecting from the mirror, these rays converge and cross at the image point I. They then continue to diverge from I as if an object were there. As a result, at point I we have a real image of the light source at O.

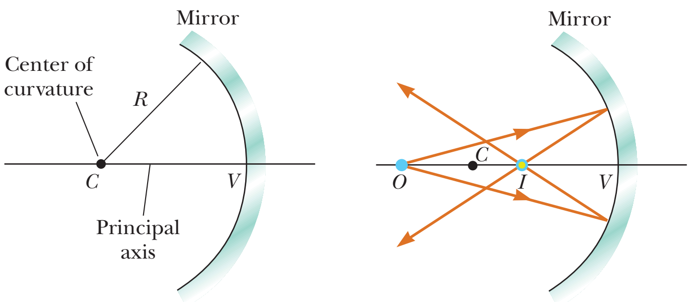


Figure 36.9 (a) A concave mirror of radius R. The center of curvature C is located on the principal axis. (b) A point object placed at O in front of a concave spherical mirror of radius R, where O is any point on the principal axis farther than R from the mirror surface, forms a real image at I. If the rays diverge from O at small angles, they all reflect through the same image point.

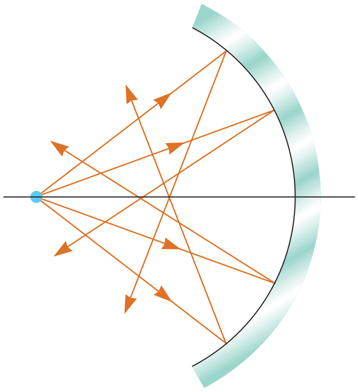


Figure 36.10 Rays diverging from the object at large angles from the principal axis reflect from a spherical concave mirror to intersect the principal axis at different points, resulting in a blurred image. This condition is called *spherical aberration*.

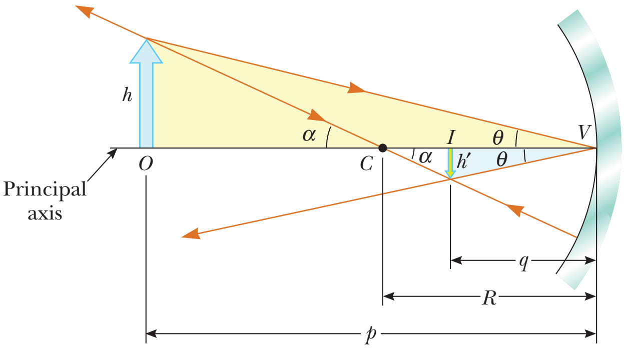


Figure 36.11 The image formed by a spherical concave mirror when the object O lies outside the center of curvature C. This geometric construction is used to derive Equation 36.4.

We shall consider in this section only rays that diverge from the object and make a small angle with the principal axis. Such rays are called **paraxial rays.** All paraxial rays reflect through the image point, as shown in Figure 36.9b. Rays that are far from the principal axis, such as those shown in Figure 36.10, converge to other points on the principal axis, producing a blurred image. This effect, which is called **spherical aberration,** is present to some extent for any spherical mirror and is discussed in Section 36.5.

We can use Figure 36.11 to calculate the image distance q from a knowledge of the object distance p and radius of curvature R. By convention, these distances are measured from point V. Figure 36.11 shows two rays leaving the tip of the object. One of these rays passes through the center of curvature C of the mirror, hitting the mirror perpendicular to the mirror surface and reflecting back on itself. The second ray strikes the mirror at its center (point V) and reflects as shown, obeying the law of reflection. The image of the tip of the arrow is located at the point where these two rays intersect. From the gold right triangle in Figure 36.11, we see that $\tan\theta = h/p$, and from the blue right triangle we see that $\tan\theta = -h'/q$. The negative sign is introduced because the image is inverted, so h' is taken to be negative. Thus, from Equation 36.1 and these results, we find that the magnification of the image is

$$M = \frac{h'}{h} = -\frac{q}{p} \tag{36.2}$$

We also note from the two triangles in Figure 36.11 that have α as one angle that

$$\tan\alpha = \frac{h}{p-R} \quad \text{and} \quad \tan\alpha = -\frac{h'}{R-q}$$

from which we find that

$$\frac{h'}{h} = -\frac{R-q}{p-R} \tag{36.3}$$

If we compare Equations 36.2 and 36.3, we see that

$$\frac{R-q}{p-R} = \frac{q}{p}$$

Simple algebra reduces this to

Mirror equation in terms of radius of curvature

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} \tag{36.4}$$

This expression is called the **mirror equation.**

If the object is very far from the mirror—that is, if p is so much greater than R that p can be said to approach infinity—then $1/p \approx 0$, and we see from Equation 36.4 that $q \approx R/2$. That is, when the object is very far from the mirror, the image point is halfway between the center of curvature and the center point on the mirror, as shown in Figure 36.12a. The incoming rays from the object are essentially parallel

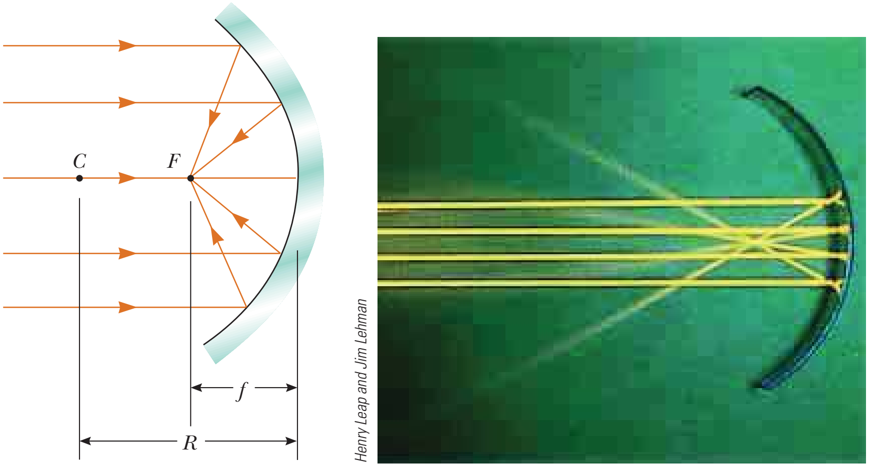


Figure 36.12 (a) Light rays from a distant object ($p \rightarrow \infty$) reflect from a concave mirror through the focal point F. In this case, the image distance $q \approx R/2 = f$, where f is the focal length of the mirror. (b) Reflection of parallel rays from a concave mirror.

in this figure because the source is assumed to be very far from the mirror. We call the image point in this special case the **focal point** F and the image distance the **focal length** f, where

$$f = \frac{R}{2} \tag{36.5}$$

Focal length

In Figure 36.8, the colored beams are traveling parallel to the principal axis and the mirror reflects all three beams to the focal point. Notice that the point at which the three beams intersect and the colors add is white.

Focal length is a parameter particular to a given mirror and therefore can be used to compare one mirror with another. The mirror equation can be expressed in terms of the focal length:

$$\boxed{\frac{1}{p} + \frac{1}{q} = \frac{1}{f}} \tag{36.6}$$

Mirror equation in terms of focal length

Notice that the focal length of a mirror depends only on the curvature of the mirror and not on the material from which the mirror is made. This is because the formation of the image results from rays reflected from the surface of the material. The situation is different for lenses; in that case the light actually passes through the material and the focal length depends on the type of material from which the lens is made.

Courtesy of Thomson Consumer Electronics

A satellite-dish antenna is a concave reflector for television signals from a satellite in orbit around the Earth. The signals are carried by microwaves that, because the satellite is so far away, are parallel when they arrive at the dish. These waves reflect from the dish and are focused on the receiver at the focal point of the dish.

PITFALL PREVENTION

36.2 The *Focal* Point Is Not the *Focus* Point

The focal point *is usually not* the point at which the light rays focus to form an image. The focal point is determined solely by the curvature of the mirror—it does not depend on the location of the object at all. In general, an image forms at a point different from the focal point of a mirror (or a lens). The *only* exception is when the object is located infinitely far away from the mirror.

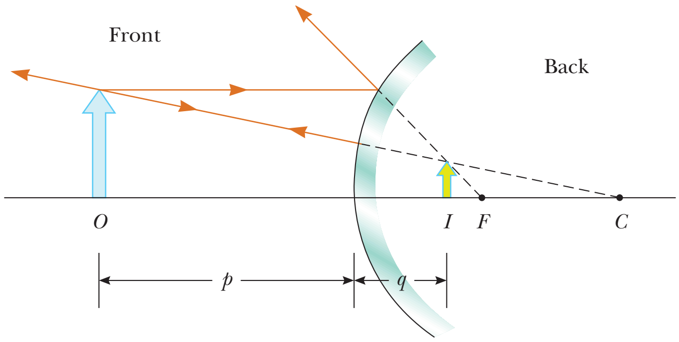


Figure 36.13 Formation of an image by a spherical convex mirror. The image formed by the real object is virtual and upright.

Convex Mirrors

Figure 36.13 shows the formation of an image by a **convex mirror**—that is, one silvered so that light is reflected from the outer, convex surface. This is sometimes called a **diverging mirror** because the rays from any point on an object diverge after reflection as though they were coming from some point behind the mirror. The image in Figure 36.13 is virtual because the reflected rays only appear to originate at the image point, as indicated by the dashed lines. Furthermore, the image is always upright and smaller than the object. This type of mirror is often used in stores to foil shoplifters. A single mirror can be used to survey a large field of view because it forms a smaller image of the interior of the store.

We do not derive any equations for convex spherical mirrors because we can use Equations 36.2, 36.4, and 36.6 for either concave or convex mirrors if we adhere to the following procedure. Let us refer to the region in which light rays move toward the mirror as the *front side* of the mirror, and the other side as the *back side*. For example, in Figures 36.11 and 36.13, the side to the left of the mirrors is the front side, and the side to the right of the mirrors is the back side. Figure 36.14 states the sign conventions for object and image distances, and Table 36.1 summarizes the sign conventions for all quantities.

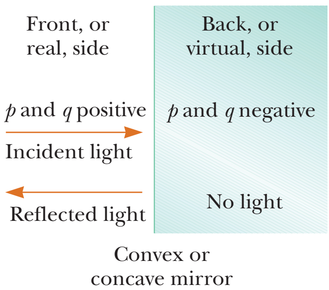


Figure 36.14 Signs of p and q for convex and concave mirrors.

Ray Diagrams for Mirrors

The positions and sizes of images formed by mirrors can be conveniently determined with *ray diagrams*. These graphical constructions reveal the nature of the image and can be used to check results calculated from the mirror and magnification equations. To draw a ray diagram, we need to know the position of the object and the locations of the mirror's focal point and center of curvature. We then draw three principal rays to locate the image, as shown by the examples in Figure 36.15.

Table 36.1

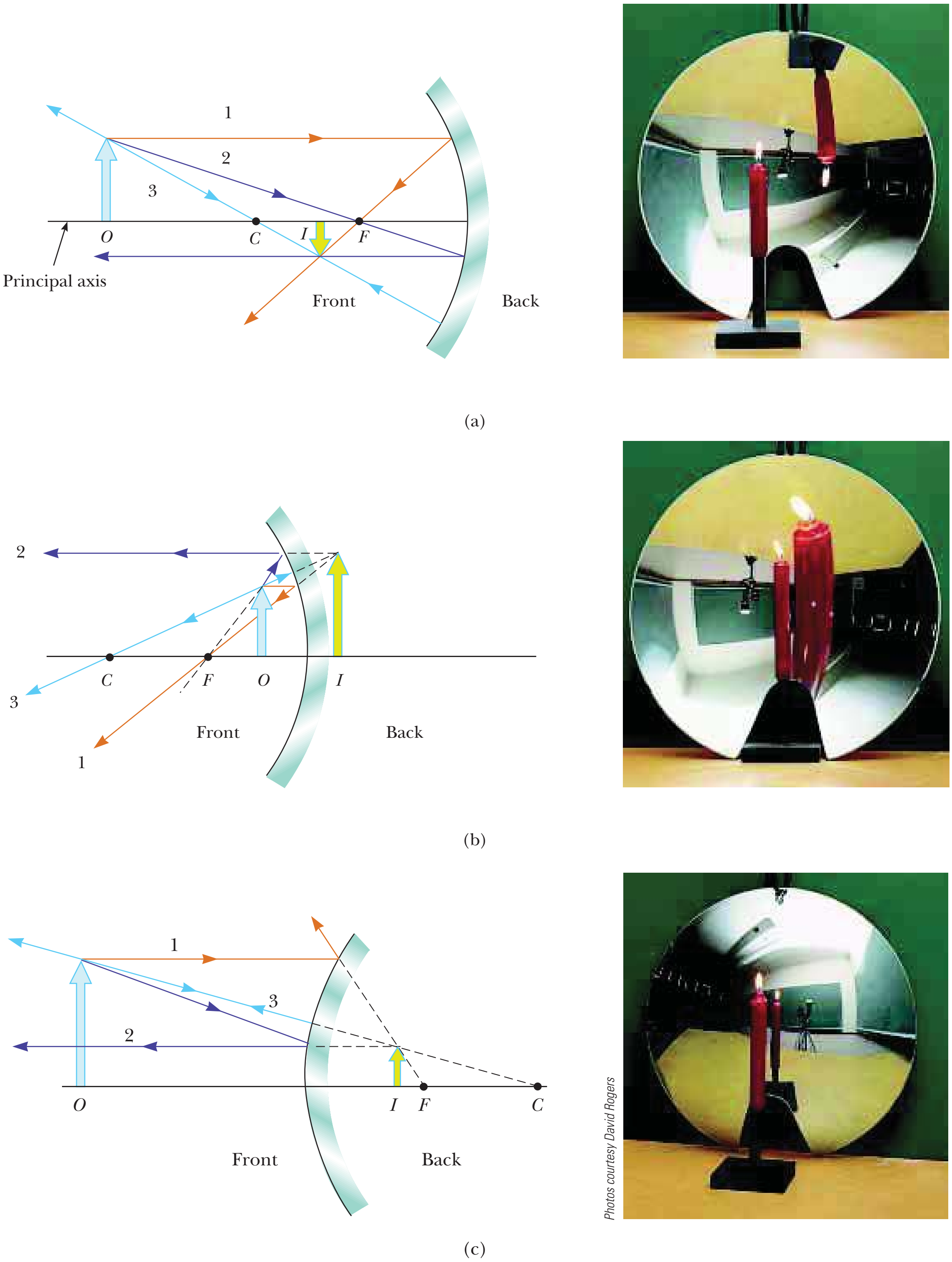
Sign Conventions for Mirrors

Quantity	Positive When	Negative When
Object location (p)	Object is in front of mirror (real object)	Object is in back of mirror (virtual object)
Image location (q)	Image is in front of mirror (real image)	Image is in back of mirror (virtual image)
Image height (h')	Image is upright	Image is inverted
Focal length (f) and radius (R)	Mirror is concave	Mirror is convex
Magnification (M)	Image is upright	Image is inverted

PITFALL PREVENTION

36.3 Watch Your Signs

Success in working mirror problems (as well as problems involving refracting surfaces and thin lenses) is largely determined by proper sign choices when substituting into the equations. The best way to become adept at this is to work a multitude of problems on your own. Watching your instructor or reading the example problems is no substitute for practice.



Photos courtesy David Rogers

Active Figure 36.15 Ray diagrams for spherical mirrors, along with corresponding photographs of the images of candles. (a) When the object is located so that the center of curvature lies between the object and a concave mirror surface, the image is real, inverted, and reduced in size. (b) When the object is located between the focal point and a concave mirror surface, the image is virtual, upright, and enlarged. (c) When the object is in front of a convex mirror, the image is virtual, upright, and reduced in size.

At the Active Figures link at http://www.pse6.com, you can move the objects and change the focal length of the mirrors to see the effect on the images.

PITFALL PREVENTION

36.4 We Are *Choosing* a Small Number of Rays

A *huge* number of light rays leave each point on an object (and pass through each point on an image). In a principal-ray diagram, which displays the characteristics of the image, we choose only a few rays that follow simply stated rules. Locating the image by calculation complements the diagram.

These rays all start from the same object point and are drawn as follows. We may choose any point on the object; here, we choose the top of the object for simplicity. For concave mirrors (see Figs. 36.15a and 36.15b), we draw the following three principal rays:

- Ray 1 is drawn from the top of the object parallel to the principal axis and is reflected through the focal point F.
- Ray 2 is drawn from the top of the object through the focal point and is reflected parallel to the principal axis.
- Ray 3 is drawn from the top of the object through the center of curvature C and is reflected back on itself.

The intersection of any two of these rays locates the image. The third ray serves as a check of the construction. The image point obtained in this fashion must always agree with the value of q calculated from the mirror equation. With concave mirrors, note what happens as the object is moved closer to the mirror. The real, inverted image in Figure 36.15a moves to the left as the object approaches the focal point. When the object is at the focal point, the image is infinitely far to the left. However, when the object lies between the focal point and the mirror surface, as shown in Figure 36.15b, the image is virtual, upright, and enlarged. This latter situation applies when you use a shaving mirror or a makeup mirror, both of which are concave. Your face is closer to the mirror than the focal point, and you see an upright, enlarged image of your face.

For convex mirrors (see Fig. 36.15c), we draw the following three principal rays:

- Ray 1 is drawn from the top of the object parallel to the principal axis and is reflected *away from* the focal point F.
- Ray 2 is drawn from the top of the object toward the focal point on the back side of the mirror and is reflected parallel to the principal axis.
- Ray 3 is drawn from the top of the object toward the center of curvature C on the back side of the mirror and is reflected back on itself.

In a convex mirror, the image of an object is always virtual, upright, and reduced in size as shown in Figure 36.15c. In this case, as the object distance decreases, the virtual image increases in size and moves away from the focal point toward the mirror as the object approaches the mirror. You should construct other diagrams to verify how image position varies with object position.

NASA

Figure 36.16 (Quick Quiz 36.4) What type of mirror is this?

Quick Quiz 36.3 You wish to reflect sunlight from a mirror onto some paper under a pile of wood in order to start a fire. Which would be the best choice for the type of mirror? (a) flat (b) concave (c) convex.

Quick Quiz 36.4 Consider the image in the mirror in Figure 36.16. Based on the appearance of this image, you would conclude that (a) the mirror is concave and the image is real. (b) the mirror is concave and the image is virtual. (c) the mirror is convex and the image is real. (d) the mirror is convex and the image is virtual.

Example 36.4 The Image formed by a Concave Mirror **Interactive**

Assume that a certain spherical mirror has a focal length of $+10.0$ cm. Locate and describe the image for object distances of

(A) 25.0 cm,

(B) 10.0 cm, and

(C) 5.00 cm.

Solution Because the focal length is positive, we know that this is a concave mirror (see Table 36.1).

(A) This situation is analogous to that in Figure 36.15a; hence, we expect the image to be real. We find the image distance by using Equation 36.6:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{25.0 \text{ cm}} + \frac{1}{q} = \frac{1}{10.0 \text{ cm}}$$

$$q = \boxed{16.7 \text{ cm}}$$

The magnification of the image is given by Equation 36.2:

$$M = -\frac{q}{p} = -\frac{16.7 \text{ cm}}{25.0 \text{ cm}} = -0.668$$

The fact that the absolute value of M is less than unity tells us that the image is smaller than the object, and the negative sign for M tells us that the image is inverted. Because q is positive, the image is located on the front side of the mirror and is real.

(B) When the object distance is 10.0 cm, the object is located at the focal point. Now we find that

$$\frac{1}{10.0 \text{ cm}} + \frac{1}{q} = \frac{1}{10.0 \text{ cm}}$$

$$q = \boxed{\infty}$$

which means that rays originating from an object positioned at the focal point of a mirror are reflected so that the image is formed at an infinite distance from the mirror; that is, the rays travel parallel to one another after reflection. This is the situation in a flashlight, where the bulb filament is placed at the focal point of a reflector, producing a parallel beam of light.

(C) When the object is at $p = 5.00$ cm, it lies halfway between the focal point and the mirror surface, as shown in Figure 36.15b. Thus, we expect a magnified, virtual, upright image. In this case, the mirror equation gives

$$\frac{1}{5.00 \text{ cm}} + \frac{1}{q} = \frac{1}{10.0 \text{ cm}}$$

$$q = \boxed{-10.0 \text{ cm}}$$

The image is virtual because it is located behind the mirror, as expected. The magnification of the image is

$$M = -\frac{q}{p} = -\left(\frac{-10.0 \text{ cm}}{5.00 \text{ cm}}\right) = +2.00$$

The image is twice as large as the object, and the positive sign for M indicates that the image is upright (see Fig. 36.15b).

What If? Suppose you set up the candle and mirror apparatus illustrated in Figure 36.15a and described in part (A) of the example. While adjusting the apparatus, you accidentally strike the candle with your elbow so that it begins to slide toward the mirror at velocity v_p. How fast does the image of the candle move?

Answer We solve the mirror equation, Equation 36.6, for q:

$$q = \frac{fp}{p-f}$$

Differentiating this equation with respect to time gives us the velocity of the image $v_q = dq/dt$:

$$v_q = \frac{dq}{dt} = \frac{d}{dt}\left(\frac{fp}{p-f}\right) = -\frac{f^2}{(p-f)^2}\frac{dp}{dt} = -\frac{f^2 v_p}{(p-f)^2}$$

For the object position of 25.0 cm in part (A), the velocity of the image is

$$v_q = -\frac{f^2 v_p}{(p-f)^2} = -\frac{(10.0 \text{ cm})^2 v_p}{(25.0 \text{ cm} - 10.0 \text{ cm})^2} = -0.444 v_p$$

Thus, the speed of the image is less than that of the object in this case.

We can see two interesting behaviors of this function for v_q. First, note that the velocity is negative regardless of the value of p or f. Thus, if the object moves toward the mirror, the image moves toward the left in Figure 36.15 without regard for the side of the focal point at which the object is located or whether the mirror is concave or convex. Second, in the limit of $p \rightarrow 0$, the velocity v_q approaches $-v_p$. As the object moves very close to the mirror, the mirror looks like a plane mirror, the image is as far behind the mirror as the object is in front, and both the object and the image move with the same speed.

Investigate the image formed for various object positions and mirror focal lengths at the Interactive Worked Example link at ***http://www.pse6.com.***

Example 36.5 The Image from a Convex Mirror **Interactive**

An anti-shoplifting mirror, as shown in Figure 36.17, shows an image of a woman who is located 3.0 m from the mirror. The focal length of the mirror is -0.25 m. Find

(A) the position of her image and

(B) the magnification of the image.

Solution (A) This situation is depicted in Figure 36.15c. We should expect to find an upright, reduced, virtual image. To find the image position, we use Equation 36.6:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{-0.25\text{ m}}$$

$$\frac{1}{q} = \frac{1}{-0.25\text{ m}} - \frac{1}{3.0\text{ m}}$$

$$q = \boxed{-0.23\text{ m}}$$

The negative value of q indicates that her image is virtual, or behind the mirror, as shown in Figure 36.15c.

(B) The magnification of the image is

$$M = -\frac{q}{p} = -\left(\frac{-0.23\text{ m}}{3.0\text{ m}}\right) = \boxed{+0.077}$$

The image is much smaller than the woman, and it is upright because M is positive.

Figure 36.17 (Example 36.5) Convex mirrors, often used for security in department stores, provide wide-angle viewing.

Investigate the image formed for various object positions and mirror focal lengths at the Interactive Worked Example link at http://www.pse6.com.

36.3 Images Formed by Refraction

In this section we describe how images are formed when light rays are refracted at the boundary between two transparent materials. Consider two transparent media having indices of refraction n_1 and n_2, where the boundary between the two media is a spherical surface of radius R (Fig. 36.18). We assume that the object at O is in the medium for which the index of refraction is n_1. Let us consider the paraxial rays leaving O. As we shall see, all such rays are refracted at the spherical surface and focus at a single point I, the image point.

Figure 36.19 shows a single ray leaving point O and refracting to point I. Snell's law of refraction applied to this ray gives

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Because θ_1 and θ_2 are assumed to be small, we can use the small-angle approximation $\sin \theta \approx \theta$ (with angles in radians) and say that

$$n_1\theta_1 = n_2\theta_2$$

Now we use the fact that an exterior angle of any triangle equals the sum of the two opposite interior angles. Applying this rule to triangles OPC and PIC in Figure 36.19 gives

$$\theta_1 = \alpha + \beta$$

$$\beta = \theta_2 + \gamma$$

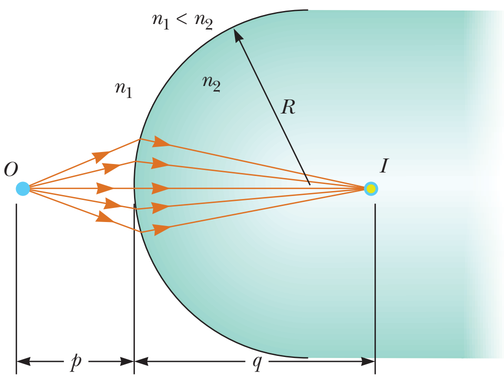


Figure 36.18 An image formed by refraction at a spherical surface. Rays making small angles with the principal axis diverge from a point object at O and are refracted through the image point I.

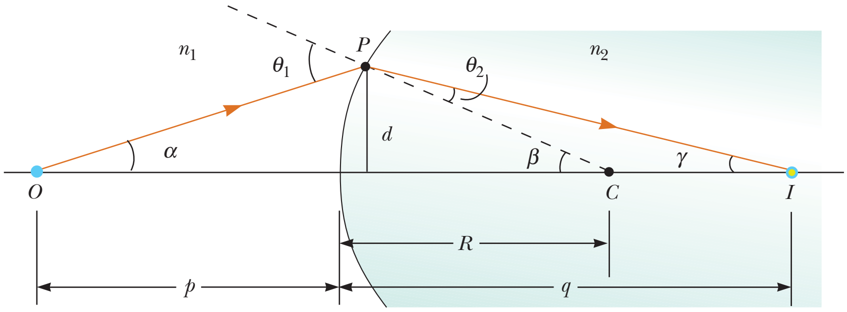


Figure 36.19 Geometry used to derive Equation 36.8, assuming that $n_1 < n_2$.

If we combine all three expressions and eliminate θ_1 and θ_2, we find that

$$n_1\alpha + n_2\gamma = (n_2 - n_1)\beta \qquad (36.7)$$

From Figure 36.19, we see three right triangles that have a common vertical leg of length d. For paraxial rays (unlike the relatively large-angle ray shown in Fig. 36.19), the horizontal legs of these triangles are approximately p for the triangle containing angle α, R for the triangle containing angle β, and q for the triangle containing angle γ. In the small-angle approximation, $\tan\theta \approx \theta$, so we can write the approximate relationships from these triangles as follows:

$$\tan\alpha \approx \alpha \approx \frac{d}{p} \qquad \tan\beta \approx \beta \approx \frac{d}{R} \qquad \tan\gamma \approx \gamma \approx \frac{d}{q}$$

We substitute these expressions into Equation 36.7 and divide through by d to give

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \qquad (36.8)$$

Relation between object and image distance for a refracting surface

For a fixed object distance p, the image distance q is independent of the angle that the ray makes with the axis. This result tells us that all paraxial rays focus at the same point I.

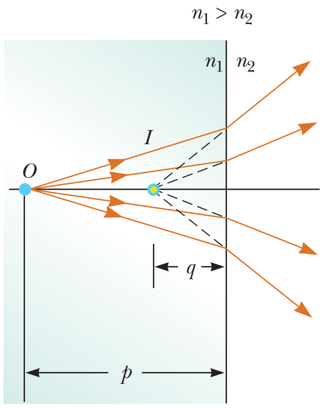
As with mirrors, we must use a sign convention if we are to apply this equation to a variety of cases. We define the side of the surface in which light rays originate as the front side. The other side is called the back side. Real images are formed by refraction in back of the surface, in contrast with mirrors, where real images are formed in front of the reflecting surface. Because of the difference in location of real images, the refraction sign conventions for q and R are opposite the reflection sign conventions. For example, q and R are both positive in Figure 36.19. The sign conventions for spherical refracting surfaces are summarized in Table 36.2.

We derived Equation 36.8 from an assumption that $n_1 < n_2$ in Figure 36.19. This assumption is not necessary, however. Equation 36.8 is valid regardless of which index of refraction is greater.

Table 36.2

Sign Conventions for Refracting Surfaces

Quantity	Positive When	Negative When
Object location (p)	Object is in front of surface (real object)	Object is in back of surface (virtual object)
Image location (q)	Image is in back of surface (real image)	Image is in front of surface (virtual image)
Image height (h')	Image is upright	Image is inverted
Radius (R)	Center of curvature is in back of surface	Center of curvature is in front of surface



Active Figure 36.20 The image formed by a flat refracting surface is virtual and on the same side of the surface as the object. All rays are assumed to be paraxial.

At the Active Figures link at http://www.pse6.com, you can move the object to see the effect on the location of the image.

Flat Refracting Surfaces

If a refracting surface is flat, then R is infinite and Equation 36.8 reduces to

$$\frac{n_1}{p} = -\frac{n_2}{q}$$

$$q = -\frac{n_2}{n_1}p \tag{36.9}$$

From this expression we see that the sign of q is opposite that of p. Thus, according to Table 36.2, **the image formed by a flat refracting surface is on the same side of the surface as the object.** This is illustrated in Figure 36.20 for the situation in which the object is in the medium of index n_1 and n_1 is greater than n_2. In this case, a virtual image is formed between the object and the surface. If n_1 is less than n_2, the rays in the back side diverge from each other at lesser angles than those in Figure 36.20. As a result, the virtual image is formed to the left of the object.

Quick Quiz 36.5 In Figure 36.18, what happens to the image point I as the object point O is moved to the right from very far away to very close to the refracting surface? (a) It is always to the right of the surface. (b) It is always to the left of the surface. (c) It starts off to the left and at some position of O, I moves to the right of the surface. (d) It starts off to the right and at some position of O, I moves to the left of the surface.

Quick Quiz 36.6 In Figure 36.20, what happens to the image point I as the object point O moves toward the right-hand surface of the material of index of refraction n_1? (a) It always remains between O and the surface, arriving at the surface just as O does. (b) It moves toward the surface more slowly than O so that eventually O passes I. (c) It approaches the surface and then moves to the right of the surface.

Conceptual Example 36.6 Let's Go Scuba Diving!

It is well known that objects viewed under water with the naked eye appear blurred and out of focus. However, a scuba diver using a mask has a clear view of underwater objects. Explain how this works, using the facts that the indices of refraction of the cornea, water, and air are 1.376, 1.333, and 1.00029, respectively.

Solution Because the cornea and water have almost identical indices of refraction, very little refraction occurs when a person under water views objects with the naked eye. In this case, light rays from an object focus behind the retina, resulting in a blurred image. When a mask is used, the air space between the eye and the mask surface provides the normal amount of refraction at the eye–air interface, and the light from the object focuses on the retina.

Example 36.7 Gaze into the Crystal Ball

A set of coins is embedded in a spherical plastic paperweight having a radius of 3.0 cm. The index of refraction of the plastic is $n_1 = 1.50$. One coin is located 2.0 cm from the edge of the sphere (Fig. 36.21). Find the position of the image of the coin.

Solution Because $n_1 > n_2$, where $n_2 = 1.00$ is the index of refraction for air, the rays originating from the coin are refracted away from the normal at the surface and diverge outward. Hence, the image is formed inside the paperweight and is *virtual*. Applying Equation 36.8 and noting

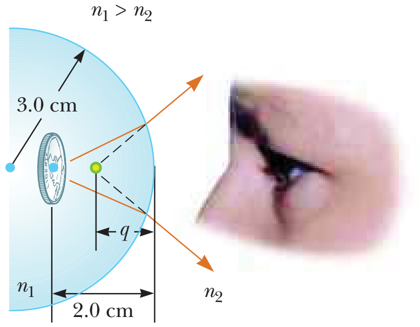


Figure 36.21 (Example 36.7) Light rays from a coin embedded in a plastic sphere form a virtual image between the surface of the object and the sphere surface. Because the object is inside the sphere, the front of the refracting surface is the *interior* of the sphere.

from Table 36.2 that R is negative, we obtain

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

$$\frac{1.50}{2.0\text{ cm}} + \frac{1}{q} = \frac{1.00 - 1.50}{-3.0\text{ cm}}$$

$$q = \boxed{-1.7\text{ cm}}$$

The negative sign for q indicates that the image is in front of the surface—in other words, in the same medium as the object, as shown in Figure 36.21. Being in the same medium as the object, the image must be virtual. (See Table 36.2.) The coin appears to be closer to the paperweight surface than it actually is.

Example 36.8 The One That Got Away

A small fish is swimming at a depth d below the surface of a pond (Fig. 36.22). What is the apparent depth of the fish, as viewed from directly overhead?

Solution Because the refracting surface is flat, R is infinite. Hence, we can use Equation 36.9 to determine the location of the image with $p = d$. Using the indices of refraction given in Figure 36.22, we obtain

$$q = -\frac{n_2}{n_1}p = -\frac{1.00}{1.33}d = \boxed{-0.752d}$$

Because q is negative, the image is virtual, as indicated by the dashed lines in Figure 36.22. The apparent depth is approximately three-fourths the actual depth.

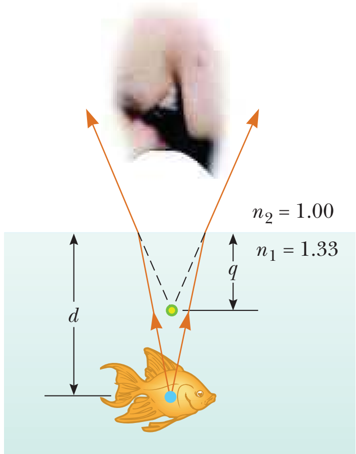


Figure 36.22 (Example 36.8) The apparent depth q of the fish is less than the true depth d. All rays are assumed to be paraxial.

What If? **What if you look more carefully at the fish and measure its apparent *height*, from its upper fin to its lower fin? Is the apparent height h' of the fish different from the actual height h?**

Answer Because all points on the fish appear to be fractionally closer to the observer, we would predict that the height would be smaller. If we let the distance d in Figure 36.22 be measured to the top fin and the distance to the bottom fin be $d + h$, then the images of the top and bottom of the fish are located at

$$q_{\text{top}} = -0.752d$$
$$q_{\text{bottom}} = -0.752(d + h)$$

The apparent height h' of the fish is

$$h' = q_{\text{top}} - q_{\text{bottom}} = -0.752d - [-0.752(d + h)]$$
$$= 0.752h$$

and the fish appears to be approximately three-fourths its actual height.

36.4 Thin Lenses

Lenses are commonly used to form images by refraction in optical instruments, such as cameras, telescopes, and microscopes. We can use what we just learned about images formed by refracting surfaces to help us locate the image formed by a lens. We recognize that light passing through a lens experiences refraction at two surfaces. The development we shall follow is based on the notion that **the image**

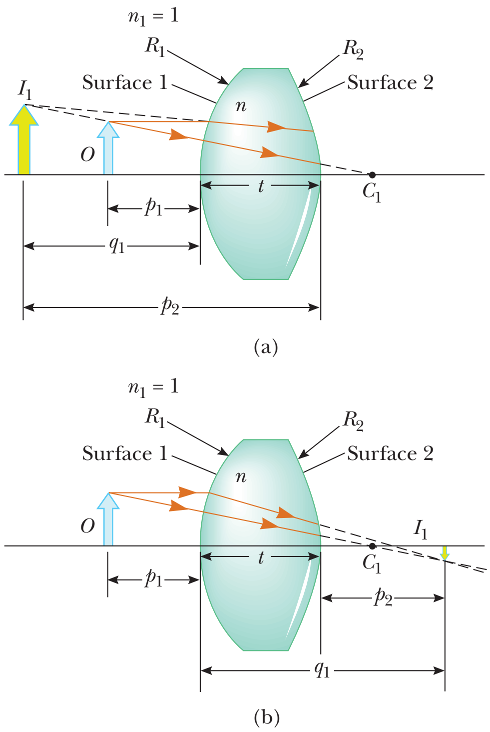


Figure 36.23 To locate the image formed by a lens, we use the virtual image at I_1 formed by surface 1 as the object for the image formed by surface 2. The point C_1 is the center of curvature of surface 1. (a) The image due to surface 1 is virtual so that I_1 is to the left of the surface. (b) The image due to surface 1 is real so that I_1 is to the right of the surface.

formed by one refracting surface serves as the object for the second surface. We shall analyze a thick lens first and then let the thickness of the lens be approximately zero.

Consider a lens having an index of refraction n and two spherical surfaces with radii of curvature R_1 and R_2, as in Figure 36.23. (Note that R_1 is the radius of curvature of the lens surface that the light from the object reaches first and that R_2 is the radius of curvature of the other surface of the lens.) An object is placed at point O at a distance p_1 in front of surface 1.

Let us begin with the image formed by surface 1. Using Equation 36.8 and assuming that $n_1 = 1$ because the lens is surrounded by air, we find that the image I_1 formed by surface 1 satisfies the equation

$$\frac{1}{p_1} + \frac{n}{q_1} = \frac{n-1}{R_1} \tag{36.10}$$

where q_1 is the position of the image due to surface 1. If the image due to surface 1 is virtual (Fig. 36.23a), q_1 is negative, and it is positive if the image is real (Fig. 36.23b).

Now we apply Equation 36.8 to surface 2, taking $n_1 = n$ and $n_2 = 1$. (We make this switch in index because the light rays approaching surface 2 are *in the material of the lens,* and this material has index n.) Taking p_2 as the object distance for surface 2 and q_2 as the image distance gives

$$\frac{n}{p_2} + \frac{1}{q_2} = \frac{1-n}{R_2} \tag{36.11}$$

We now introduce mathematically the fact that the image formed by the first surface acts as the object for the second surface. We do this by noting from Figure 36.23 that p_2, measured from surface 2, is related to q_1 as follows:

Virtual image from surface 1 (Fig. 36.23a): $\quad p_2 = -q_1 + t \quad$ (q_1 is negative)

Real image from surface 1 (Fig. 36.23b): $\quad p_2 = -q_1 + t \quad$ (q_1 is positive)

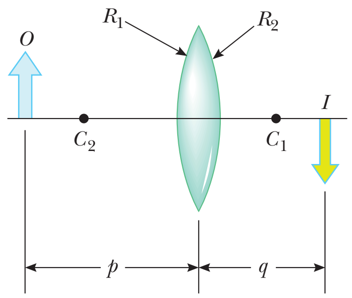


Figure 36.24 Simplified geometry for a thin lens.

where t is the thickness of the lens. For a *thin* lens (one whose thickness is small compared to the radii of curvature), we can neglect t. In this approximation, we see that $p_2 = -q_1$ for either type of image from surface 1. (If the image from surface 1 is real, the image acts as a virtual object, so p_2 is negative.) Hence, Equation 36.11 becomes

$$-\frac{n}{q_1} + \frac{1}{q_2} = \frac{1-n}{R_2} \tag{36.12}$$

Adding Equations 36.10 and 36.12, we find that

$$\frac{1}{p_1} + \frac{1}{q_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{36.13}$$

For a thin lens, we can omit the subscripts on p_1 and q_2 in Equation 36.13 and call the object distance p and the image distance q, as in Figure 36.24. Hence, we can write Equation 36.13 in the form

$$\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{36.14}$$

This expression relates the image distance q of the image formed by a thin lens to the object distance p and to the lens properties (index of refraction and radii of curvature). It is valid only for paraxial rays and only when the lens thickness is much less than R_1 and R_2.

The **focal length** f of a thin lens is the image distance that corresponds to an infinite object distance, just as with mirrors. Letting p approach ∞ and q approach f in Equation 36.14, we see that the inverse of the focal length for a thin lens is

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{36.15}$$

Lens makers' equation

This relationship is called the **lens makers' equation** because it can be used to determine the values of R_1 and R_2 that are needed for a given index of refraction and a desired focal length f. Conversely, if the index of refraction and the radii of curvature of a lens are given, this equation enables a calculation of the focal length. If the lens is immersed in something other than air, this same equation can be used, with n interpreted as the *ratio* of the index of refraction of the lens material to that of the surrounding fluid.

Using Equation 36.15, we can write Equation 36.14 in a form identical to Equation 36.6 for mirrors:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \tag{36.16}$$

Thin lens equation

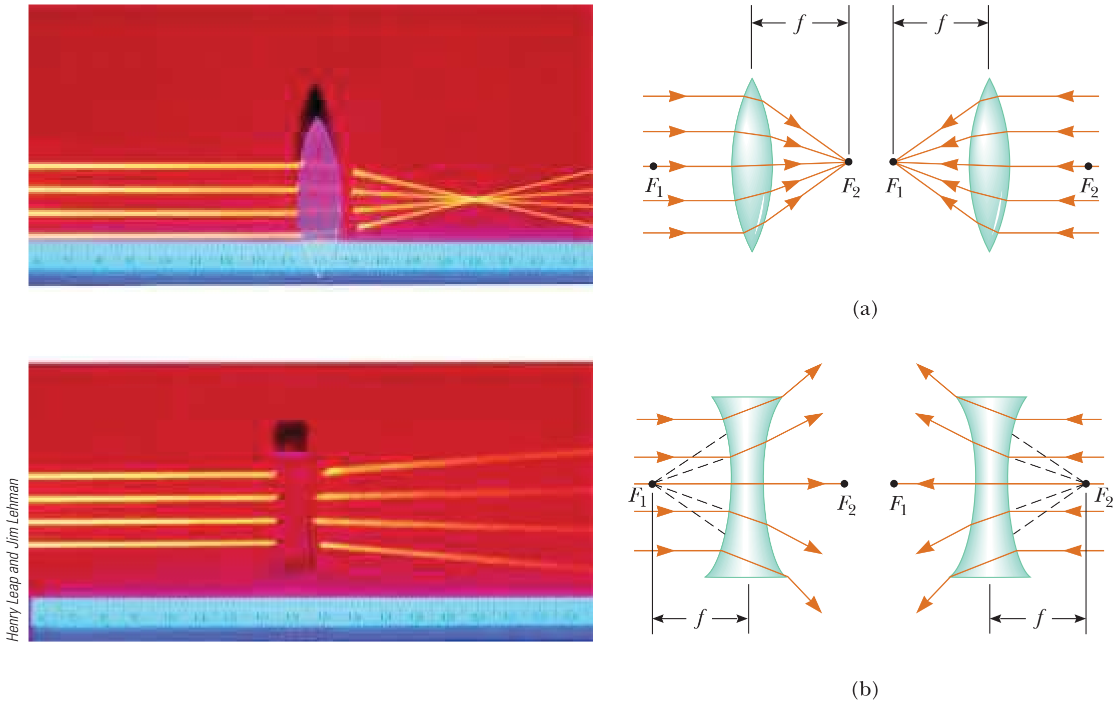


Figure 36.25 (*Left*) Effects of a converging lens (top) and a diverging lens (bottom) on parallel rays. (*Right*) Parallel light rays pass through (a) a converging lens and (b) a diverging lens. The focal length is the same for light rays passing through a given lens in either direction. Both focal points F_1 and F_2 are the same distance from the lens.

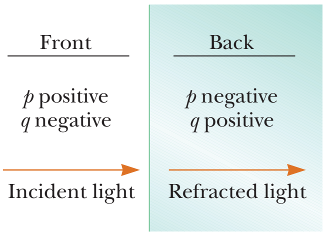


Figure 36.26 A diagram for obtaining the signs of p and q for a thin lens. (This diagram also applies to a refracting surface.)

PITFALL PREVENTION

36.5 A Lens Has Two Focal Points but Only One Focal Length

A lens has a focal point on each side, front and back. However, there is only one focal length—each of the two focal points is located the same distance from the lens (Fig, 36.25). This can be seen mathematically by interchanging R_1 and R_2 in Equation 36.15 (and changing the signs of the radii because back and front have been interchanged). As a result, the lens forms an image of an object at the same point if it is turned around. In practice this might not happen, because real lenses are not infinitesimally thin.

This equation, called the **thin lens equation,** can be used to relate the image distance and object distance for a thin lens.

Because light can travel in either direction through a lens, each lens has two focal points, one for light rays passing through in one direction and one for rays passing through in the other direction. This is illustrated in Figure 36.25 for a biconvex lens (two convex surfaces, resulting in a converging lens) and a biconcave lens (two concave surfaces, resulting in a diverging lens).

Figure 36.26 is useful for obtaining the signs of p and q, and Table 36.3 gives the sign conventions for thin lenses. Note that these sign conventions are the same as those for refracting surfaces (see Table 36.2). Applying these rules to a biconvex lens, we see that when $p > f$, the quantities p, q, and R_1 are positive, and R_2 is negative. Therefore, p, q, and f are all positive when a converging lens forms a real image of an object. For a biconcave lens, p and R_2 are positive and q and R_1 are negative, with the result that f is negative.

Table 36.3

Sign Conventions for Thin Lenses

Quantity	Positive When	Negative When
Object location (p)	Object is in front of lens (real object)	Object is in back of lens (virtual object)
Image location (q)	Image is in back of lens (real image)	Image is in front of lens (virtual image)
Image height (h')	Image is upright	Image is inverted
R_1 and R_2	Center of curvature is in back of lens	Center of curvature is in front of lens
Focal length (f)	Converging lens	Diverging lens

Various lens shapes are shown in Figure 36.27. Note that a converging lens is thicker at the center than at the edge, whereas a diverging lens is thinner at the center than at the edge.

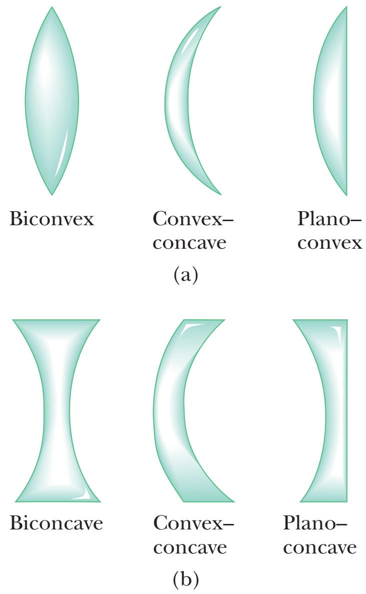


Figure 36.27 Various lens shapes. (a) Converging lenses have a positive focal length and are thickest at the middle. (b) Diverging lenses have a negative focal length and are thickest at the edges.

Magnification of Images

Consider a thin lens through which light rays from an object pass. As with mirrors (Eq. 36.2), we could analyze a geometric construction to show that the lateral magnification of the image is

$$M = \frac{h'}{h} = -\frac{q}{p}$$

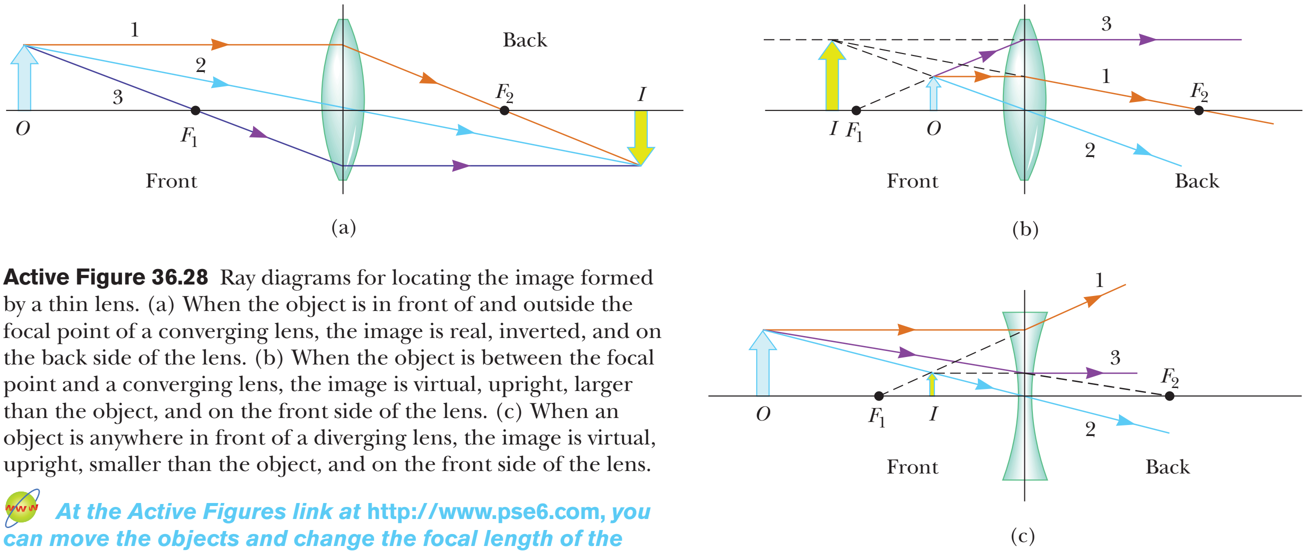
From this expression, it follows that when M is positive, the image is upright and on the same side of the lens as the object. When M is negative, the image is inverted and on the side of the lens opposite the object.

Ray Diagrams for Thin Lenses

Ray diagrams are convenient for locating the images formed by thin lenses or systems of lenses. They also help clarify our sign conventions. Figure 36.28 shows such diagrams for three single-lens situations.

To locate the image of a *converging* lens (Fig. 36.28a and b), the following three rays are drawn from the top of the object:

- Ray 1 is drawn parallel to the principal axis. After being refracted by the lens, this ray passes through the focal point on the back side of the lens.
- Ray 2 is drawn through the center of the lens and continues in a straight line.
- Ray 3 is drawn through the focal point on the front side of the lens (or as if coming from the focal point if $p < f$) and emerges from the lens parallel to the principal axis.



Active Figure 36.28 Ray diagrams for locating the image formed by a thin lens. (a) When the object is in front of and outside the focal point of a converging lens, the image is real, inverted, and on the back side of the lens. (b) When the object is between the focal point and a converging lens, the image is virtual, upright, larger than the object, and on the front side of the lens. (c) When an object is anywhere in front of a diverging lens, the image is virtual, upright, smaller than the object, and on the front side of the lens.

At the Active Figures link at http://www.pse6.com, you can move the objects and change the focal length of the lenses to see the effect on the images.

To locate the image of a *diverging* lens (Fig. 36.28c), the following three rays are drawn from the top of the object:

- Ray 1 is drawn parallel to the principal axis. After being refracted by the lens, this ray emerges directed away from the focal point on the front side of the lens.
- Ray 2 is drawn through the center of the lens and continues in a straight line.
- Ray 3 is drawn in the direction toward the focal point on the back side of the lens and emerges from the lens parallel to the principal axis.

For the converging lens in Figure 36.28a, where the object is to the left of the focal point ($p > f$), the image is real and inverted. When the object is between the focal point and the lens ($p < f$), as in Figure 36.28b, the image is virtual and upright. For a diverging lens (see Fig. 36.28c), the image is always virtual and upright, regardless of where the object is placed. These geometric constructions are reasonably accurate only if the distance between the rays and the principal axis is much less than the radii of the lens surfaces.

Note that refraction occurs only at the surfaces of the lens. A certain lens design takes advantage of this fact to produce the *Fresnel lens,* a powerful lens without great thickness. Because only the surface curvature is important in the refracting qualities of the lens, material in the middle of a Fresnel lens is removed, as shown in the cross sections of lenses in Figure 36.29. Because the edges of the curved segments cause some distortion, Fresnel lenses are usually used only in situations in which image quality is less important than reduction of weight. A classroom overhead projector often uses a Fresnel lens; the circular edges between segments of the lens can be seen by looking closely at the light projected onto a screen.

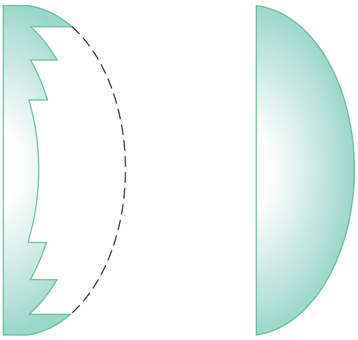


Figure 36.29 The Fresnel lens on the left has the same focal length as the thick lens on the right but is made of much less glass.

Quick Quiz 36.7 What is the focal length of a pane of window glass? (a) zero (b) infinity (c) the thickness of the glass (d) impossible to determine

Quick Quiz 36.8 Diving masks often have a lens built into the glass for divers who do not have perfect vision. This allows the individual to dive without the necessity for glasses, because the lenses in the faceplate perform the necessary refraction to provide clear vision. The proper design allows the diver to see clearly with the mask on *both* under water and in the open air. Normal eyeglasses have lenses that are curved on both the front and back surfaces. The lenses in a diving mask should be curved (a) only on the front surface (b) only on the back surface (c) on both the front and back surfaces.

Example 36.9 Images Formed by a Converging Lens

Interactive

A converging lens of focal length 10.0 cm forms images of objects placed

(A) 30.0 cm,

(B) 10.0 cm, and

(C) 5.00 cm from the lens.

In each case, construct a ray diagram, find the image distance and describe the image.

Solution

(A) First we construct a ray diagram as shown in Figure 36.30a. The diagram shows that we should expect a real, inverted, smaller image to be formed on the back side of the lens. The thin lens equation, Equation 36.16, can be used to find the image distance:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

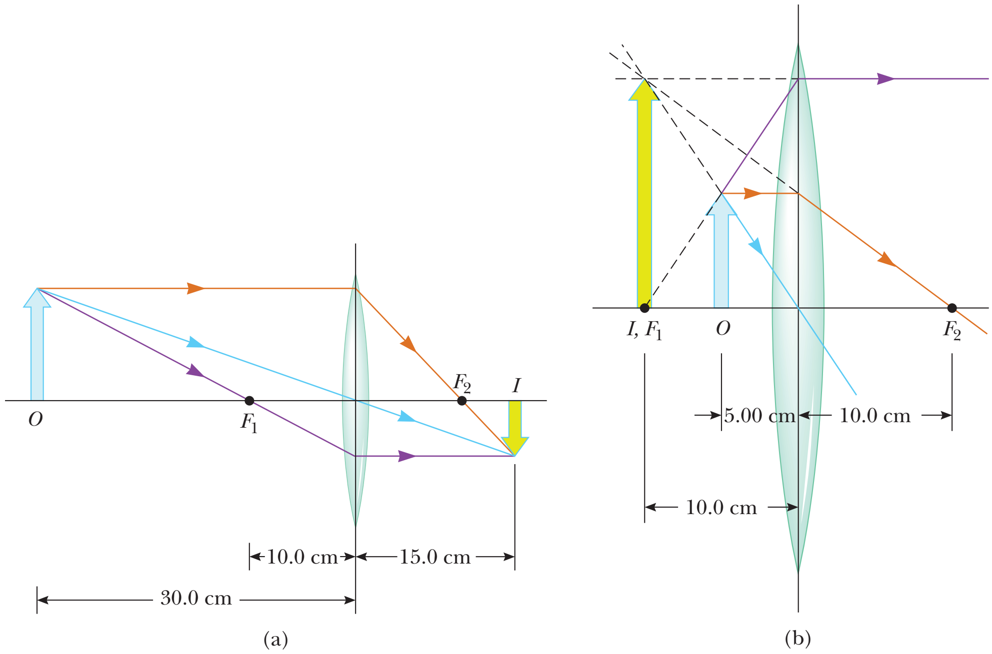


Figure 36.30 (Example 36.9) An image is formed by a converging lens. (a) The object is farther from the lens than the focal point. (b) The object is closer to the lens than the focal point.

$$\frac{1}{30.0 \text{ cm}} + \frac{1}{q} = \frac{1}{10.0 \text{ cm}}$$

$$q = \boxed{+15.0 \text{ cm}}$$

The positive sign for the image distance tells us that the image is indeed real and on the back side of the lens. The magnification of the image is

$$M = -\frac{q}{p} = -\frac{15.0 \text{ cm}}{30.0 \text{ cm}} = \boxed{-0.500}$$

Thus, the image is reduced in height by one half, and the negative sign for M tells us that the image is inverted.

(B) No calculation is necessary for this case because we know that, when the object is placed at the focal point, the image is formed at infinity. This is readily verified by substituting $p = 10.0$ cm into the thin lens equation.

(C) We now move inside the focal point. The ray diagram in Figure 36.30b shows that in this case the lens acts as a magnifying glass; that is, the image is magnified, upright, on the same side of the lens as the object, and virtual. Because the object distance is 5.00 cm, the thin lens equation gives

$$\frac{1}{5.00 \text{ cm}} + \frac{1}{q} = \frac{1}{10.0 \text{ cm}}$$

$$q = \boxed{-10.0 \text{ cm}}$$

and the magnification of the image is

$$M = -\frac{q}{p} = -\left(\frac{-10.0 \text{ cm}}{5.00 \text{ cm}}\right) = \boxed{+2.00}$$

The negative image distance tells us that the image is virtual and formed on the side of the lens from which the light is incident, the front side. The image is enlarged, and the positive sign for M tells us that the image is upright.

What If? What if the object moves right up to the lens surface, so that $p \to 0$? Where is the image?

Answer In this case, because $p \ll R$, where R is either of the radii of the surfaces of the lens, the curvature of the lens can be ignored and it should appear to have the same effect as a plane piece of material. This would suggest that the image is just on the front side of the lens, at $q = 0$. We can verify this mathematically by rearranging the thin lens equation:

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

If we let $p \to 0$, the second term on the right becomes very large compared to the first and we can neglect $1/f$. The equation becomes

$$\frac{1}{q} = -\frac{1}{p}$$

$$q = -p = 0$$

Thus, q is on the front side of the lens (because it has the opposite sign as p), and just at the lens surface.

Investigate the image formed for various object positions and lens focal lengths at the Interactive Worked Example link at **http://www.pse6.com.**

Example 36.10 The Case of a Diverging Lens

Interactive

Repeat Example 36.9 for a *diverging* lens of focal length 10.0 cm.

Solution

(A) We begin by constructing a ray diagram as in Figure 36.31a taking the object distance to be 30.0 cm. The diagram shows that we should expect an image that is virtual, smaller than the object, and upright. Let us now apply the thin lens equation with p = 30.0 cm:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{30.0 \text{ cm}} + \frac{1}{q} = \frac{1}{-10.0 \text{ cm}}$$

$$q = \boxed{-7.50 \text{ cm}}$$

The magnification of the image is

$$M = -\frac{q}{p} = -\left(\frac{-7.50 \text{ cm}}{30.0 \text{ cm}}\right) = \boxed{+0.250}$$

This result confirms that the image is virtual, smaller than the object, and upright.

(B) When the object is at the focal point, the ray diagram appears as in Figure 36.31b. In the thin lens equation, using p = 10.0 cm, we have

$$\frac{1}{10.0 \text{ cm}} + \frac{1}{q} = \frac{1}{-10.0 \text{ cm}}$$

$$q = \boxed{-5.00 \text{ cm}}$$

The magnification of the image is

$$M = -\frac{q}{p} = -\left(\frac{-5.00 \text{ cm}}{10.0 \text{ cm}}\right) = \boxed{+0.500}$$

Notice the difference between this situation and that for a converging lens. For a diverging lens, an object at the focal point does not produce an image infinitely far away.

(C) When the object is inside the focal point, at p = 5.00 cm, the ray diagram in Figure 36.31c shows that we expect a virtual image that is smaller than the object and upright. In

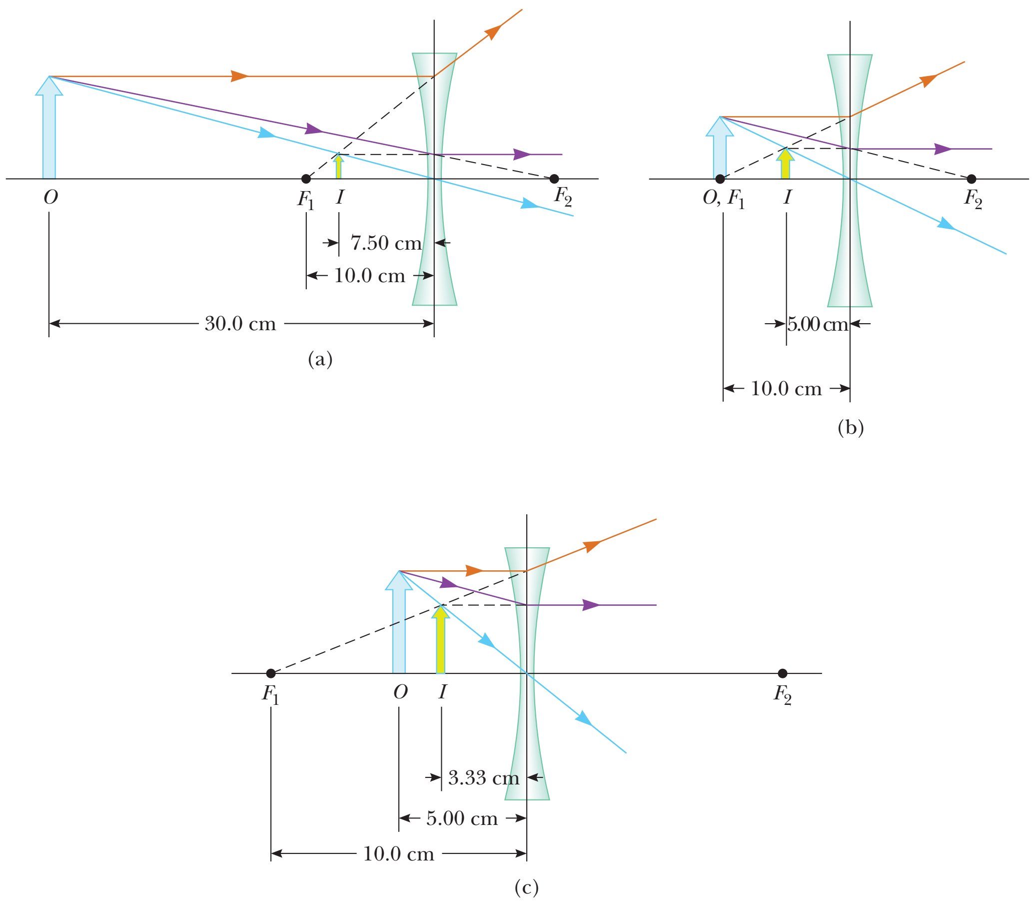


Figure 36.31 (Example 36.10) An image is formed by a diverging lens. (a) The object is farther from the lens than the focal point. (b) The object is at the focal point. (c) The object is closer to the lens than the focal point.

this case, the thin lens equation gives

$$\frac{1}{5.00 \text{ cm}} + \frac{1}{q} = \frac{1}{-10.0 \text{ cm}}$$

$$q = \boxed{-3.33 \text{ cm}}$$

and the magnification of the image is

$$M = -\left(\frac{-3.33 \text{ cm}}{5.00 \text{ cm}}\right) = \boxed{+0.667}$$

This confirms that the image is virtual, smaller than the object, and upright.

Investigate the image formed for various object positions and lens focal lengths at the Interactive Worked Example link at http://www.pse6.com.

Example 36.11 A Lens Under Water

A converging glass lens ($n = 1.52$) has a focal length of 40.0 cm in air. Find its focal length when it is immersed in water, which has an index of refraction of 1.33.

Solution We can use the lens makers' equation (Eq. 36.15) in both cases, noting that R_1 and R_2 remain the same in air and water:

$$\frac{1}{f_{\text{air}}} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_{\text{water}}} = (n' - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where n' is the ratio of the index of refraction of glass to that of water: $n' = 1.52/1.33 = 1.14$. Dividing the first equation by the second gives

$$\frac{f_{\text{water}}}{f_{\text{air}}} = \frac{n-1}{n'-1} = \frac{1.52 - 1}{1.14 - 1} = 3.71$$

Because $f_{\text{air}} = 40.0$ cm, we find that

$$f_{\text{water}} = 3.71 f_{\text{air}} = 3.71(40.0 \text{ cm}) = \boxed{148 \text{ cm}}$$

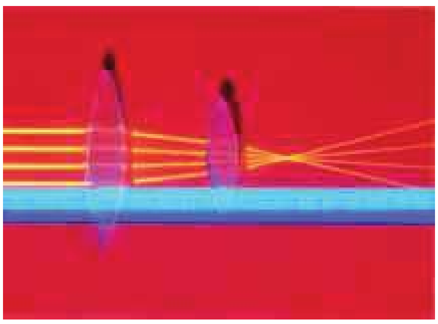
The focal length of any lens is increased by a factor $(n - 1)/(n' - 1)$ when the lens is immersed in a fluid, where n' is the ratio of the index of refraction n of the lens material to that of the fluid.

Combination of Thin Lenses

If two thin lenses are used to form an image, the system can be treated in the following manner. First, the image formed by the first lens is located as if the second lens were not present. Then a ray diagram is drawn for the second lens, with the image formed by the first lens now serving as the object for the second lens. The second image formed is the final image of the system. If the image formed by the first lens lies on the back side of the second lens, then that image is treated as a **virtual object** for the second lens (that is, in the thin lens equation, p is negative). The same procedure can be extended to a system of three or more lenses. Because the magnification due to the second lens is performed on the magnified image due to the first lens, the overall magnification of the image due to the combination of lenses is the product of the individual magnifications.

Let us consider the special case of a system of two lenses of focal lengths f_1 and f_2 in contact with each other. If $p_1 = p$ is the object distance for the combination, application of the thin lens equation (Eq. 36.16) to the first lens gives

$$\frac{1}{p} + \frac{1}{q_1} = \frac{1}{f_1}$$



Henry Leap and Jim Lehman

Light from a distant object is brought into focus by two converging lenses.

where q_1 is the image distance for the first lens. Treating this image as the object for the second lens, we see that the object distance for the second lens must be $p_2 = -q_1$. (The distances are the same because the lenses are in contact and assumed to be infinitesimally thin. The object distance is negative because the object is virtual.) Therefore, for the second lens,

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$$

$$-\frac{1}{q_1} + \frac{1}{q} = \frac{1}{f_2}$$

where $q = q_2$ is the final image distance from the second lens, which is the image distance for the combination. Adding the equations for the two lenses eliminates q_1 and gives

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_1} + \frac{1}{f_2}$$

If we consider replacing the combination with a single lens that will form an image at the same location, we see that its focal length is related to the individual focal lengths by

Focal length for a combination of two thin lenses in contact

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \qquad (36.17)$$

Therefore, **two thin lenses in contact with each other are equivalent to a single thin lens having a focal length given by Equation 36.17.**

Example 36.12 Where Is the Final Image? Interactive

Two thin converging lenses of focal lengths $f_1 = 10.0$ cm and $f_2 = 20.0$ cm are separated by 20.0 cm, as illustrated in Figure 36.32a. An object is placed 30.0 cm to the left of lens 1. Find the position and the magnification of the final image.

Solution Conceptualize by imagining light rays passing through the first lens and forming a real image (because $p > f$) in the absence of the second lens. Figure 36.32b shows these light rays forming the inverted image I_1. Once the light rays converge to the image point, they do not stop. They continue through the image point and interact with the second lens. The rays leaving the image point behave in the same way as the rays leaving an object. Thus, the image of the first lens serves as the object of the second lens. We categorize this problem as one in which we apply the thin lens equation, but in stepwise fashion to the two lenses.

To analyze the problem, we first draw a ray diagram (Figure 36.32b) showing where the image from the first lens falls and how it acts as the object for the second lens. The location of the image formed by lens 1 is found from the thin lens equation:

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f}$$

$$\frac{1}{30.0 \text{ cm}} + \frac{1}{q_1} = \frac{1}{10.0 \text{ cm}}$$

$$q_1 = \boxed{+15.0 \text{ cm}}$$

The magnification of this image is

$$M_1 = -\frac{q_1}{p_1} = -\frac{15.0 \text{ cm}}{30.0 \text{ cm}} = \boxed{-0.500}$$

The image formed by this lens acts as the object for the second lens. Thus, the object distance for the second lens is 20.0 cm − 15.0 cm = 5.00 cm. We again apply the thin lens equation to find the location of the final image:

$$\frac{1}{5.00 \text{ cm}} + \frac{1}{q_2} = \frac{1}{20.0 \text{ cm}}$$

$$q_2 = \boxed{-6.67 \text{ cm}}$$

The magnification of the second image is

$$M_2 = -\frac{q_2}{p_2} = -\frac{(-6.67 \text{ cm})}{5.00 \text{ cm}} = \boxed{+1.33}$$

Thus, the overall magnification of the system is

$$M = M_1 M_2 = (-0.500)(1.33) = \boxed{-0.667}$$

To finalize the problem, note that the negative sign on the overall magnification indicates that the final image is inverted with respect to the initial object. The fact that the absolute value of the magnification is less than one tells us that the final image is smaller than the object. The fact that q_2 is negative tells us that the final image is on the front, or left, side of lens 2. All of these conclusions are consistent with the ray diagram in Figure 36.32b.

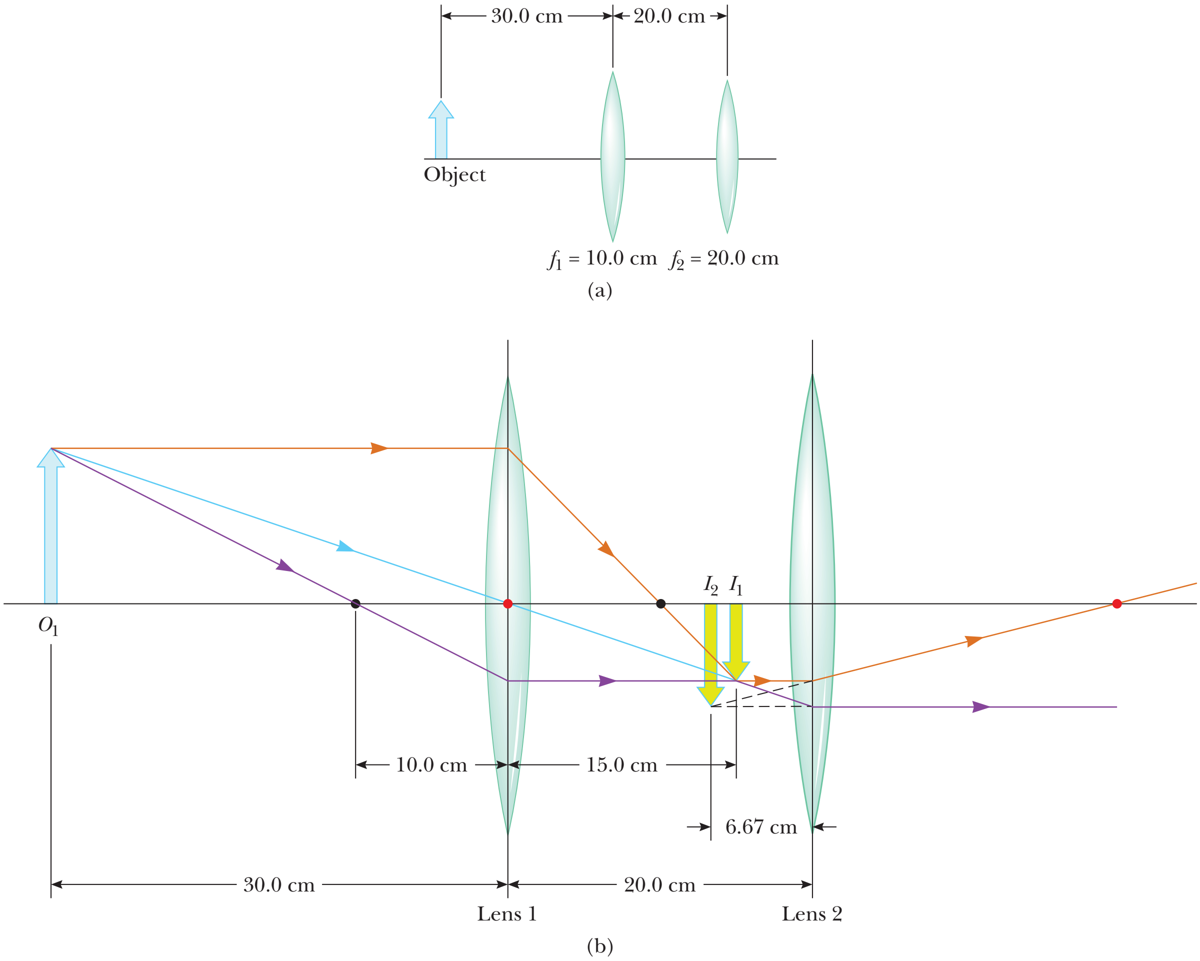


Figure 36.32 (Example 36.12) (a) A combination of two converging lenses. (b) The ray diagram showing the location of the final image due to the combination of lenses. The black dots are the focal points of lens 1 while the red dots are the focal points of lens 2.

What If? Suppose we want to create an upright image with this system of two lenses. How must the second lens be moved in order to achieve this?

Answer Because the object is farther from the first lens than the focal length of that lens, we know that the first image is inverted. Consequently, we need the second lens to invert the image once again so that the final image is upright. An inverted image is only formed by a converging lens if the object is outside the focal point. Thus, the image due to the first lens must be to the left of the focal point of the second lens in Figure 36.32b. To make this happen, we must move the second lens at least as far away from the first lens as the sum $q_1 + f_2 = 15.0 \text{ cm} + 20.0 \text{ cm} = 35.0 \text{ cm}$.

Investigate the image formed by a combination of two lenses at the Interactive Worked Example link at **http://www.pse6.com.**

Conceptual Example 36.13 Watch Your *p*'s and *q*'s!

Use a spreadsheet or a similar tool to create two graphs of image distance as a function of object distance—one for a lens for which the focal length is 10 cm and one for a lens for which the focal length is $-$ 10 cm.

Solution The graphs are shown in Figure 36.33. In each graph, a gap occurs where $p = f$, which we shall discuss. Note the similarity in the shapes—a result of the fact that image and object distances for both lenses are related according to the same equation—the thin lens equation.

The curve in the upper right portion of the $f = +10$ cm graph corresponds to an object located on the *front* side of a lens, which we have drawn as the left side of the lens in our previous diagrams. When the object is at positive infinity, a real image forms at the focal point on the back side (the positive side) of the lens, $q = f$. (The incoming rays are parallel in this case.) As the object moves closer to the lens, the image

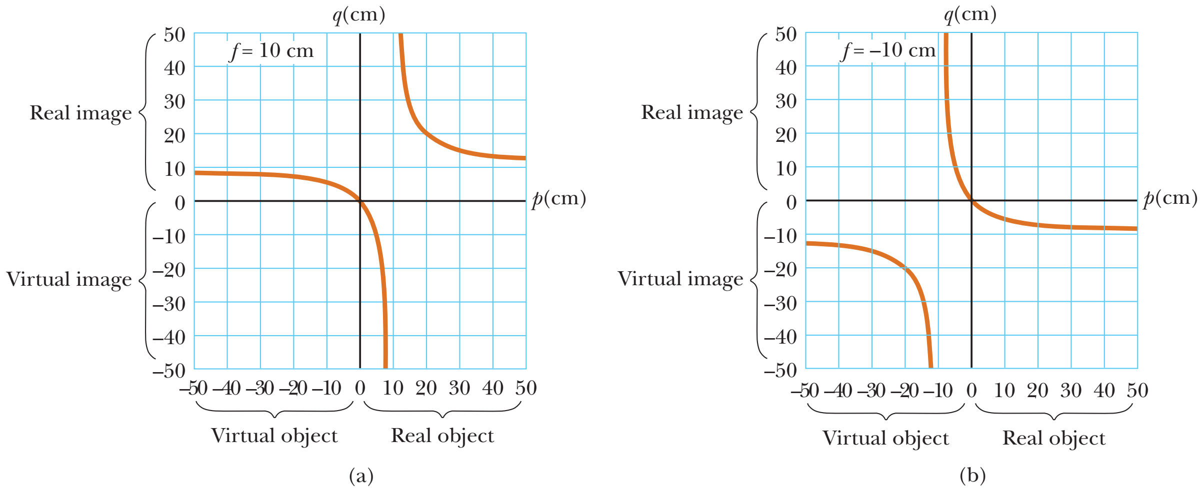


Figure 36.33 (Conceptual Example 36.13) (a) Image position as a function of object position for a lens having a focal length of + 10 cm. (b) Image position as a function of object position for a lens having a focal length of − 10 cm.

moves farther from the lens, corresponding to the upward path of the curve. This continues until the object is located at the focal point on the near side of the lens. At this point, the rays leaving the lens are parallel, making the image infinitely far away. This is described in the graph by the asymptotic approach of the curve to the line $p = f = 10$ cm.

As the object moves inside the focal point, the image becomes virtual and located near $q = -\infty$. We are now following the curve in the lower left portion of Figure 36.33a. As the object moves closer to the lens, the virtual image also moves closer to the lens. As $p \rightarrow 0$, the image distance q also approaches 0. Now imagine that we bring the object to the back side of the lens, where $p < 0$. The object is now a virtual object, so it must have been formed by some other lens. For all locations of the virtual object, the image distance is positive and less than the focal length. The final image is real, and its position approaches the focal point as p becomes more and more negative.

The $f = -10$ cm graph shows that a distant real object forms an image at the focal point on the front side of the lens. As the object approaches the lens, the image remains virtual and moves closer to the lens. But as we continue toward the left end of the p axis, the object becomes virtual. As the position of this virtual object approaches the focal point, the image recedes toward infinity. As we pass the focal point, the image shifts from a location at positive infinity to one at negative infinity. Finally, as the virtual object continues moving away from the lens, the image is virtual, starts moving in from negative infinity, and approaches the focal point.

36.5 Lens Aberrations

Our analysis of mirrors and lenses assumes that rays make small angles with the principal axis and that the lenses are thin. In this simple model, all rays leaving a point source focus at a single point, producing a sharp image. Clearly, this is not always true. When the approximations used in this analysis do not hold, imperfect images are formed.

A precise analysis of image formation requires tracing each ray, using Snell's law at each refracting surface and the law of reflection at each reflecting surface. This procedure shows that the rays from a point object do not focus at a single point, with the result that the image is blurred. The departures of actual images from the ideal predicted by our simplified model are called **aberrations.**

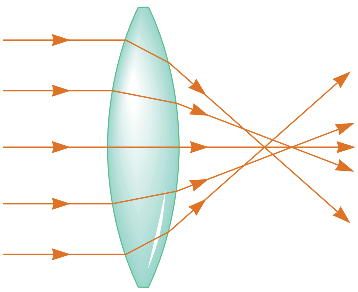


Figure 36.34 Spherical aberration caused by a converging lens. Does a diverging lens cause spherical aberration?

Spherical Aberrations

Spherical aberrations occur because the focal points of rays far from the principal axis of a spherical lens (or mirror) are different from the focal points of rays of the same wavelength passing near the axis. Figure 36.34 illustrates spherical aberration for parallel rays passing through a converging lens. Rays passing through points near the center of the lens are imaged farther from the lens than rays passing through points near the edges. Figure 36.10 earlier in the chapter showed a similar situation for a spherical mirror.

Many cameras have an adjustable aperture to control light intensity and reduce spherical aberration. (An aperture is an opening that controls the amount of light passing through the lens.) Sharper images are produced as the aperture size is reduced because with a small aperture only the central portion of the lens is exposed to the light; as a result, a greater percentage of the rays are paraxial. At the same time, however, less light passes through the lens. To compensate for this lower light intensity, a longer exposure time is used.

In the case of mirrors, spherical aberration can be minimized through the use of a parabolic reflecting surface rather than a spherical surface. Parabolic surfaces are not used often, however, because those with high-quality optics are very expensive to make. Parallel light rays incident on a parabolic surface focus at a common point, regardless of their distance from the principal axis. Parabolic reflecting surfaces are used in many astronomical telescopes to enhance image quality.

Chromatic Aberrations

The fact that different wavelengths of light refracted by a lens focus at different points gives rise to chromatic aberrations. In Chapter 35 we described how the index of refraction of a material varies with wavelength. For instance, when white light passes through a lens, violet rays are refracted more than red rays (Fig. 36.35). From this we see that the focal length of a lens is greater for red light than for violet light. Other wavelengths (not shown in Fig. 36.35) have focal points intermediate between those of red and violet.

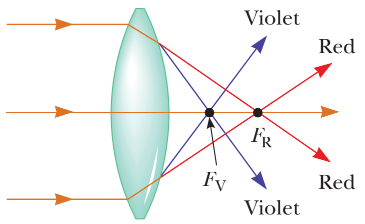


Figure 36.35 Chromatic aberration caused by a converging lens. Rays of different wavelengths focus at different points.

Chromatic aberration for a diverging lens also results in a shorter focal length for violet light than for red light, but on the front side of the lens. Chromatic aberration can be greatly reduced by combining a converging lens made of one type of glass and a diverging lens made of another type of glass.

Quick Quiz 36.9 A curved mirrored surface can have (a) spherical aberration but not chromatic aberration (b) chromatic aberration but not spherical aberration (c) both spherical aberration and chromatic aberration.

36.6 The Camera

The photographic **camera** is a simple optical instrument whose essential features are shown in Figure 36.36. It consists of a light-tight chamber, a converging lens that produces a real image, and a film behind the lens to receive the image. One focuses the camera by varying the distance between lens and film. This is accomplished with an adjustable bellows in antique cameras and with some other mechanical arrangement in contemporary models. For proper focusing—which is necessary for the formation of sharp images—the lens-to-film distance depends on the object distance as well as on the focal length of the lens.

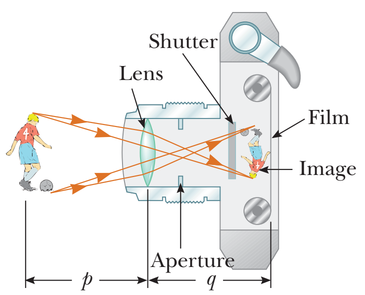


Figure 36.36 Cross-sectional view of a simple camera. Note that in reality, $p \gg q$.

The shutter, positioned behind the lens, is a mechanical device that is opened for selected time intervals, called *exposure times*. One can photograph moving objects by using short exposure times or photograph dark scenes (with low light levels) by using long exposure times. If this adjustment were not available, it would be impossible to take stop-action photographs. For example, a rapidly moving vehicle could move enough in the time interval during which the shutter is open to produce a blurred image. Another major cause of blurred images is the movement of the camera while the shutter is open. To prevent such movement, either short exposure times or a tripod should be used, even for stationary objects. Typical shutter speeds (that is, exposure times) are (1/30)s, (1/60)s, (1/125)s, and (1/250)s. For handheld cameras,