PROBLEM A5

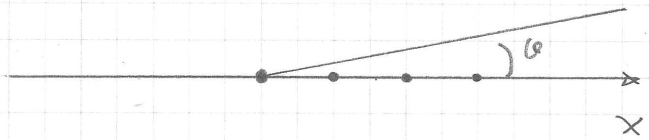
DESIGN A UNIFORM LINEAR ARRAY OF ISOTROPIC RADIATORS SUCH THAT

1) THE MAXIMUM OF THE ANTENNA FACTOR IS 4 AND IT IS IN A DIRECTION FORMING AN ANGLE OF 60° WITH THE DIRECTION OF ALIGNMENT

2) THE DIRECTION OF ALIGNMENT IS ONE OF THE TWO NULL DIRECTIONS BOUNDING THE MAIN LOBE

SOLUTION

$|AF|_{MAX} = 4$ SINCE $|AF| = \left| \dfrac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \right|$ $|AF|_{MAX} = N \Rightarrow N = 4$



THERE IS A MAXIMUM FOR $\theta = 60^\circ$ $\left(\frac{\pi}{3} \text{ RADIAN}\right)$

$$\psi = 0 \qquad \psi = \frac{2\pi d}{\lambda}\cos\theta + \alpha = 0 \qquad \frac{\pi d}{\lambda} + \alpha = 0$$

THERE IS A NULL FOR $\theta = 0$

$$\psi = \pm\frac{2\pi}{N} = \pm\frac{\pi}{2} \qquad \psi = \frac{2\pi d}{\lambda}\cos\theta + \alpha = \pm\frac{\pi}{2}$$

$$\frac{\pi d}{\lambda} + \frac{\alpha}{2} = \pm\frac{\pi}{4}$$

WE HAVE TWO UNKNOWNS AND TWO EQUATIONS AND THEREFORE WE CAN FIND THE SOLUTION

$$\begin{cases} \pi\dfrac{d}{\lambda} + \alpha = 0 \\ \pi\dfrac{d}{\lambda} + \dfrac{\alpha}{2} = \pm\dfrac{\pi}{4} \end{cases}$$

LET'S START BY CONSIDERING THE SIGN +

$$\begin{cases} \pi \dfrac{d}{\lambda} + \alpha = 0 \\ \pi \dfrac{d}{\lambda} + \dfrac{\alpha}{2} = + \dfrac{\pi}{4} \end{cases}$$

IF WE SUBTRACT THE SECOND EQUATION FROM THE FIRST WE OBTAIN $\dfrac{\alpha}{2} = -\dfrac{\pi}{4}$ $\qquad \alpha = -\dfrac{\pi}{2}$

$$\frac{d}{\lambda} = \frac{1}{2} \qquad d = \frac{\lambda}{2}$$

IF WE ASSUME THE SIGN −

$$\begin{cases} \pi \dfrac{d}{\lambda} + \alpha = 0 \\ \pi \dfrac{d}{\lambda} + \dfrac{\alpha}{2} = -\dfrac{\pi}{4} \end{cases}$$

$\dfrac{\alpha}{2} = \dfrac{\pi}{4}$ $\qquad \alpha = \dfrac{\pi}{2}$ $\qquad \dfrac{d}{\lambda} = -\dfrac{1}{2}$ A DISTANCE CANNOT BE NEGATIVE

THE PARAMETERS OF THE ARRAY ARE $N = 4$ $\quad d = \dfrac{\lambda}{2}$ $\quad \alpha = -\dfrac{\pi}{2}$

WE CAN PLOT THE TOTAL RADIATION PATTERN

- MAXIMA $\qquad \psi = 0, \pm 2\pi, \pm 4\pi, \dots$

$\psi = 0$ $\qquad \psi = \dfrac{2\pi d}{\lambda} \cos\theta - \dfrac{\pi}{2} = \pi \cos\theta - \dfrac{\pi}{2} = 0$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \pm 60°$$

$\psi = \pm 2\pi$ $\qquad \psi = \dfrac{2\pi d}{\lambda} \cos\theta - \dfrac{\pi}{2} = \pi \left(\cos\theta - \dfrac{1}{2} \right) = \pm 2\pi$

$\cos\theta = \dfrac{1}{2} \pm 2$ $\qquad$ THERE IS NO SOLUTION

THE ONLY TWO MAXIMUM DIRECTIONS ARE $\theta = \pm 60°$

- NULL DIRECTIONS $\quad \psi = \pm \frac{2k\pi}{N} \quad k \neq N, k \neq 2N, \ldots$

$k=1 \quad \psi = \pm \frac{2\pi}{4} = \pm \frac{\pi}{2} \quad \psi = \pi\left(\cos\theta - \frac{1}{2}\right) = \pm \frac{\pi}{2}$

$\cos\theta = \frac{1}{2} \pm \frac{1}{2}$

$\cos\theta = 1 \quad \theta = 0^\circ$

$\cos\theta = 0 \quad \theta = \pm 90^\circ$

$k=2 \quad \psi = \pm \frac{4\pi}{4} = \pm \pi \quad \psi = \pi\left(\cos\theta - \frac{1}{2}\right) = \pm \pi$

$\cos\theta = \frac{1}{2} \pm 1$

$\cos\theta = \frac{3}{2}$ NO SOLUTION

$\cos\theta = -\frac{1}{2} \quad \theta = \pm 120^\circ$

$k=3 \quad \psi = \pm \frac{6\pi}{4} = \pm \frac{3}{2}\pi \quad \psi = \pi\left(\cos\theta - \frac{1}{2}\right) = \pm \frac{3}{2}\pi$

$\cos\theta = \frac{1}{2} \pm \frac{3}{2}$

$\cos\theta = 2$ NO SOLUTION

$\cos\theta = -1 \quad \theta = 180^\circ$

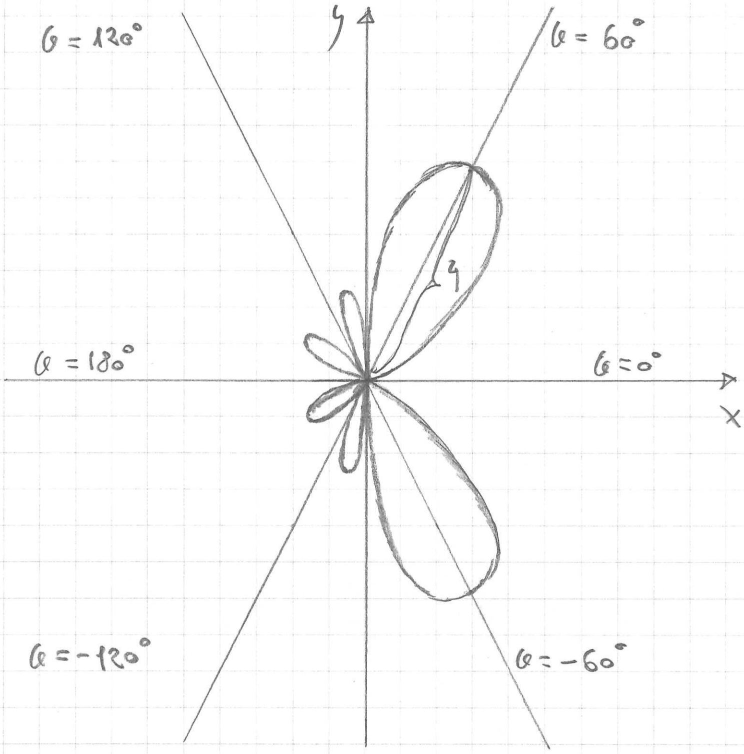
$k=5 \quad \psi = \pm \frac{10\pi}{4} = \pm \frac{5}{2}\pi \quad \psi = \pi\left(\cos\theta - \frac{1}{2}\right) = \pm \frac{5}{2}\pi$

$\cos\theta = \frac{1}{2} \pm \frac{5}{2}$ NO SOLUTION

THE FULL LIST OF THE RADIATION PATTERN NULL DIRECTIONS IS

$\theta = 0^\circ, \pm 90^\circ, \pm 120^\circ, 180^\circ$

$N\,|F(\theta,\varphi)|$



THE LOWER PART OF THE RADIATION PATTERN IS A MIRRORED COPY OF THE UPPER PART

THERE ARE TWO MAIN LOBES AND 4 SECONDARY LOBES