

Photonics / Nanophotonics

Lecture 25:
Problems on
Nanostructures



Problem 1 (Photonics AND Nanophotonics students)

Calculate the Bragg frequency and the gap-midgap ratio for the lowest bandgap of a 1D periodic structure comprising a stack of dielectric layers of equal optical thickness with $m_1 = 1.5$ and $m_2 = 3.5$ and $\Lambda = 2 \mu\text{m}$. Assume the structure is a quarter-wave stack.

$$\omega_{\text{Bragg}} = \frac{m_1 + m_2}{4m_1 m_2} \cdot \frac{2\pi c}{\Lambda} = \frac{1.5 + 3.5}{4 \cdot 1.5 \cdot 3.5} \cdot \frac{2\pi \cdot 10^8}{2 \cdot 10^{-6}} = 2.24 \cdot 10^{14} \frac{\text{rad}}{\text{s}}$$

$$\lambda_{\text{Bragg}} = \frac{2\pi c}{\omega} = 8.41 \mu\text{m}$$

If

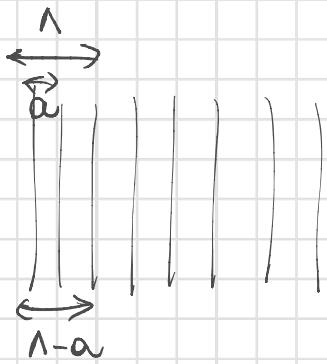
$\frac{\Delta \epsilon}{\epsilon} < 1$ then we can assume to have weak periodicity

$$\frac{\Delta E}{E} = \frac{4}{12.25} = \underline{\underline{0.32 < 1}}$$

\uparrow
 m_2^2

$$\Delta E = \Delta n^2$$

$$\frac{\Delta w}{w_{\text{Bragg}}} \approx \frac{\Delta m^2}{m^2} \cdot \frac{\sin \frac{\pi a}{\lambda}}{\pi}$$



The two layers comprising the stack are

$$L_1 = a$$

$$L_2 = \Lambda - a$$

$$m_1 L_1 = m_2 L_2 \quad \Rightarrow \quad m_1 a = m_2 (\Lambda - a) \Rightarrow$$

$$a = \frac{m_2 \lambda}{n_1 + m_2} = \frac{3.5}{5} 2 \cdot 10^{-6} = 1.4 \cdot 10^{-6} = L_1$$

$$L_2 = \Lambda - a = 0.6 \cdot 10^{-6} = 0.6 \mu\text{m}$$

$$\frac{\Delta \omega}{\omega_{\text{Bragg}}} \approx \frac{4}{12.25} \frac{\sin \left(\frac{\pi \cdot 1.4 \cdot 10^{-6}}{2 \cdot 10^{-6}} \right)}{\pi} = 0.0841$$

What changes if we consider $n_1 = 3.4$ and $n_2 = 3.6$

$$\omega_{\text{Bragg}} = \frac{n_1 + n_2}{6 n_1 n_2} \cdot \frac{2\pi c}{\Lambda} = \frac{7}{6 \cdot 3.4 \cdot 3.6} \cdot \frac{2 \cdot 10^8}{2 \cdot 10^{-6}} = 1.34 \cdot 10^{14} \frac{\text{rad}}{\text{s}}$$

$$\omega_{\text{Bragg}} = 14.03 \mu\text{m}$$

$$\varepsilon = 3.6^2 = 12.96 \quad \frac{\Delta \varepsilon}{\varepsilon} = 0.0031 \ll 1$$

$$\Delta \varepsilon = 0.04$$

$$\frac{\Delta \omega}{\omega_{\text{Bragg}}} = \frac{\Delta m^2}{M^2} \cdot \frac{\sin \pi \alpha / \lambda}{\pi} =$$

$$= \frac{0.04}{12.96} \cdot \frac{\sin \pi \frac{1.028 \cdot 10^{-6}}{2 \cdot 10^{-6}}}{\pi} =$$

$$= 9.81 \cdot 10^{-4}$$

$$L_2 \approx 1 \cdot 10^{-6}$$

$$a = \frac{m_2 \lambda}{m_1 + m_2} = 1.028 \cdot 10^{-6}$$

$$\approx \frac{\lambda}{2}$$

PROBLEM 2

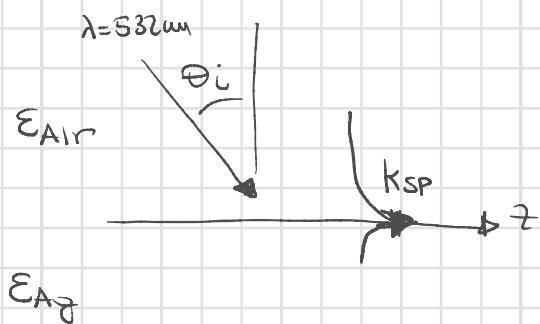
(Photonics and Nanophotonics students)

Calculate the phase velocity of a surface plasmon propagating on a flat silver-air interface at $\lambda = 532 \text{ nm}$ and compare it to the phase velocity of light in air.

Assume the dielectric permittivity of silver at $\lambda = 532 \text{ nm}$ is

$$\epsilon_{\text{Ag}} = -9.3 - j0.87 \quad \text{whereas the permittivity of air is } \epsilon_{\text{Air}} = 1.$$

Determine the decay length of the surface plasmon along the propagation direction.



$$k_{\text{SP}} = k_0 n_{\text{SP}}$$

Since we can satisfy
 $|\epsilon'| \gg |\epsilon''|$

$$n_{SP} = \sqrt{\frac{\epsilon'_{Ag} \epsilon_{Air}}{\epsilon'_{Ag} + \epsilon_{Air}}} = \sqrt{\frac{-9.3}{-8.3}} = 1.0585$$

$$n_{SP} > n_{Air}$$

The phase velocity of the SP is : $v_{p_{SP}} = \frac{c}{n_{SP}} = \frac{3 \cdot 10^8}{1.0585} = 2.83 \cdot 10^8 \frac{m}{s}$

$$< v_{p_{Air}} = c$$

The propagation constant of the SP is a complex quantity

$$k_{SP} = \frac{\omega}{c} n_{SP} = \frac{\omega}{c} \sqrt{\frac{\epsilon'_{Ag} \epsilon_{Air}}{\epsilon'_{Ag} + \epsilon_{Air}}} = k_{SP}^I - j k_{SP}^{II}$$

The propagation length (decay length)

$$L_{SP} = \frac{1}{2 \kappa_{SP}''} = \frac{C}{\omega} \frac{\sqrt{|\epsilon_m'|} (|\epsilon_m'| - \epsilon_d)^{3/2}}{\epsilon_m'' \epsilon_d^{3/2}} =$$

1

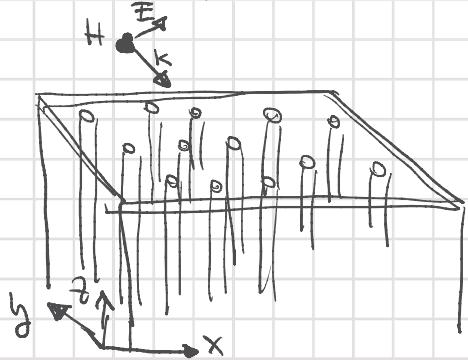
$1/m_{SP}''$

$$= \frac{1}{2} \frac{\lambda}{2\pi} \frac{1}{m_{SP}''} = 7.1 \cdot 10^{-6} \text{ m}$$

$\underline{\underline{1}}$

Problem 3 (Photonics and Nanophotonics students)

Consider a wave medium with metal fill factor $f = 0.1$. Assume for metal a complex, frequency dependent dielectric constant that can be modeled with a single Drude oscillator with $\omega_p = 2\pi \cdot 2.18 \cdot 10^{15} \text{ s}^{-1}$ and damping $\gamma = 2\pi \cdot 4.35 \cdot 10^{12} \text{ s}^{-1}$ and for the dielectric a dispersion-free permittivity $\epsilon_d = 2.25$. Can this structure be considered a hyperbolic metamaterial for a TM-polarized wave at 500 nm?



$$\omega_p = 2\pi \cdot 2.18 \cdot 10^{15} \text{ s}^{-1}$$

$$\gamma = 2\pi \cdot 4.35 \cdot 10^{12} \text{ s}^{-1}$$

$$\epsilon_d = 2.25$$

$$\begin{aligned}\varepsilon_m = \varepsilon_i &= 1 - \frac{\omega_p^2}{\omega^2 - j\omega\gamma} = 1 - \frac{(2\pi \cdot 2.18 \cdot 10^{15})^2}{\left(\frac{2\pi C}{500 \cdot 10^{-9}}\right)^2 - j\left(\frac{2\pi C}{500 \cdot 10^{-9}}\right)(2\pi \cdot 4.35 \cdot 10^{12})} \\ &= \underline{-12.2 - j 0.09}\end{aligned}$$

We can assume the wire medium to be an effective uniaxial medium

$$\bar{\varepsilon} = \varepsilon_{\perp} (\hat{x}\hat{x} + \hat{y}\hat{y}) + \varepsilon_{\parallel} \hat{z}\hat{z}$$

↗ parallel to the
wire axis

$$\langle D_{\parallel} \rangle = \frac{\varepsilon_0 \varepsilon_{\parallel} \langle E_{\parallel} \rangle}{\varepsilon_0 f \varepsilon_i E_{\parallel,i} + \varepsilon_0 (1-f) \varepsilon_h E_{\parallel,h}}$$

$$\boxed{\epsilon_{\parallel} = f \epsilon_i + (1-f) \epsilon_h}$$

ϵ_{\perp} can be calculated looking at a section in the xy plane.

$$L_x = L_y = \frac{1}{2}$$

$$\boxed{\epsilon_{\perp} = \epsilon_h \left(1 + 2f \frac{\epsilon_i - \epsilon_h}{\epsilon_i + \epsilon_h - f(\epsilon_i - \epsilon_h)} \right)}$$

When $f = 0.1 \rightarrow$

$$\boxed{\begin{aligned} \epsilon_{\parallel} &= 0.805 - j0.0096 \\ \epsilon_{\perp} &= 3.01 - j0.0027 \end{aligned}}$$

Our wire medium

$$[\epsilon] = \begin{bmatrix} \epsilon_{\perp} & 0 & 0 \\ 0 & \epsilon_{\perp} & 0 \\ 0 & 0 & \epsilon_{\parallel} \end{bmatrix}$$

The dispersion relation assuming \mathbf{k} in the xz plane

$$\boxed{\frac{k_x^2}{\epsilon_{||}} + \frac{k_z^2}{\epsilon_{\perp}} = \left(\frac{\omega}{c}\right)^2}$$

for $f=0.1$ $\underline{\epsilon_{||} \epsilon_{\perp} > 0} \Rightarrow$ THE MEDIUM IS NOT HYPERBOLIC

Problem 5

Photonics and Nanophotonics Students

A camera lens ($n_{\text{ge}} = 1.55$) is coated with a cryolite film ($n_{\text{CR}} = 1.3$) as an anti-reflection coating. a) What is the optimal film thickness for incident green light ($\lambda_0 = 532 \text{ nm}$)?

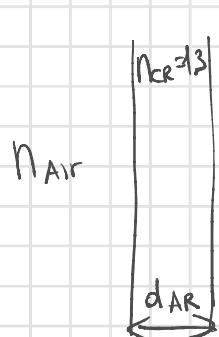
b) Under this scenario and with the calculated thickness, are we able to completely suppress reflection? Assume the medium outside the camera is air ($n_{\text{Air}} = 1$) and $\lambda = 532 \text{ nm}$.

a) Optimal thickness for anti-reflection coating

$$d_{\text{AR}} = \frac{\lambda_{\text{eff}}}{4} \quad \lambda_{\text{eff}} = \frac{\lambda_0}{n_{\text{CR}}} = \frac{532 \cdot 10^{-9}}{1.3} = 409.23 \cdot 10^{-9}$$

$$d_{\text{AR}} = \frac{409.23 \cdot 10^{-9}}{4} = 102.3 \text{ nm}$$

(b)



$$n_{\text{GL}} = 1.55$$

for $R = 0$

$$\underline{n_{\text{coating}} = \sqrt{n_{\text{air}} n_{\text{GL}}}}$$

this is
not satisfied

So the right medium for an AR coating for the cover in air

$$M_x = \sqrt{1.55} = 1.245$$

$$d_x = \frac{\lambda}{4} = \frac{532 \cdot 10^{-9}}{M_x} = \frac{532 \cdot 10^{-9}}{1.245} = 106.8 \text{ nm}$$