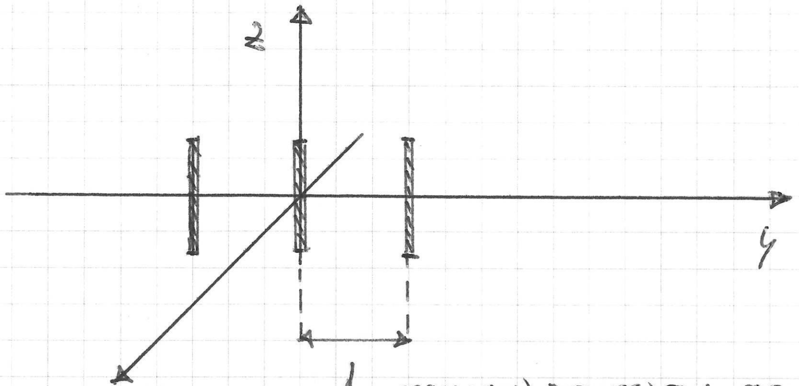
PROBLEM A1

LET US CONSIDER A LINEAR UNIFORM ARRAY CONSISTING OF THREE SHORT DIPOLES

THOSE RADIATORS ARE ARRANGED AS IN THE FOLLOWING FIGURE



THE WORKING FREQUENCY IS $f = 2{,}44$ GHz AND THE SEPARATION BETWEEN TWO ADJACENT ELEMENTS IS $d = \frac{2}{3}\lambda$

FIND MAXIMUM AND NULL DIRECTIONS IN THE (x,y) PLANE AND PLOT THE RADIATION PATTERN IN THE SAME PLANE FOR THE FOLLOWING TWO CASES

1) THE DIPOLE ARE IN PHASE $(\alpha = 0)$

2) THE PHASE DIFFERENCE BETWEEN ADJACENT ELEMENTS IS $\alpha = \frac{2\pi}{\lambda} d$

SOLUTION

1) WAVELENGTH $\lambda = \frac{c}{f} = \frac{3\,10^8}{2{,}44\,10^9} = 0{,}123 \text{ m} = 12{,}3 \text{ cm}$

THE SEPARATION IS $d = \frac{2}{3}\lambda = 8{,}2$ cm

LET'S RECAP THE IMPORTANT FORMULAS WE NEED TO STUDY A UNIFORM LINEAR ARRAY

$$\psi = \beta d \cos\theta + \alpha = \frac{2\pi}{\lambda} d \cos\theta + \alpha \qquad \left|\frac{AF}{N}\right| = |F(\theta,\varphi)| = \left|\frac{\sin\left(N\frac{\psi}{2}\right)}{N\sin\left(\frac{\psi}{2}\right)}\right|$$

THE RADIATION PATTERN OF THE SHORT DIPOLE IN THE (x,y) PLANE IS A CIRCLE :

THE RADIATION PATTERN OF THIS ARRAY IS THE SAME AS THE NORMALIZED ARRAY FACTOR $\left|\frac{AF}{N}\right| = |f|$

- MAXIMUM DIRECTIONS

THE MAXIMA OF $\left|\dfrac{\sin\left(N\frac{\psi}{2}\right)}{N\sin\left(\frac{\psi}{2}\right)}\right|$ ARE OBTAINED FOR $\psi = 0, \pm 2\pi, \pm 4\pi, \pm 6\pi, \ldots$

$\psi = 0 \Rightarrow \psi = \frac{2\pi}{\lambda} d\cos\varphi + \alpha = 0$

$d = \frac{2}{3}\lambda,\ \alpha = 0 \Rightarrow \psi = \frac{4}{3}\pi\cos\varphi = 0 \Rightarrow \varphi = \pm\frac{\pi}{2}$

WE HAVE JUST FOUND TWO MAXIMUM DIRECTIONS $\varphi = \pm\frac{\pi}{2}$

$\psi = \pm 2\pi \qquad \psi = \frac{2\pi}{\lambda} d\cos\varphi + \alpha = \pm 2\pi$

$d = \frac{2}{3}\lambda,\ \alpha = 0 \Rightarrow \psi = \frac{4}{3}\pi\cos\varphi = \pm 2\pi$

$\Rightarrow \cos\varphi = \pm\frac{3}{2}$ THERE IS NO SOLUTION !

$\psi = \pm 4\pi \qquad \psi = \frac{2\pi}{\lambda} d\cos\varphi + \alpha = \pm 4\pi$

$\psi = \frac{4\pi}{3}\cos\varphi = \pm 4\pi$

$\Rightarrow \cos\varphi = \pm 3$ THERE IS NO SOLUTION

WE CAN STOP HERE BECAUSE WE ARE SURE WE HAVE FOUND ALL THE SOLUTIONS

- NULL DIRECTIONS

THE ZEROS OF $\left|\dfrac{\sin\left(N\frac{\psi}{2}\right)}{N\sin\left(\frac{\psi}{2}\right)}\right|$ ARE OBTAINED FOR $N\frac{\psi}{2} = \pm k\pi$

WHERE k IS AN INTEGER

NULL DIRECTIONS ARE GIVEN BY $\varphi = \pm 2k\frac{\pi}{N}$ AND $k \neq N, k \neq 2N, \ldots$

(IF WE HAD $k = N$, $\varphi = \pm 2\pi$ BUT THIS IS A MAXIMUM CONDITION)

LET'S APPLY THIS RULE TO FIND ALL ZEROS (WHICH MEANS ALL NULL DIRECTIONS)

$k = 1$ $\quad \varphi = \pm \frac{2\pi}{N} = \pm \frac{2\pi}{3}$ $\qquad \varphi = \frac{2\pi}{\lambda} d\cos\theta + \alpha = \frac{4}{3}\pi\cos\theta = \pm\frac{2}{3}\pi$

$\frac{4}{3}\pi\cos\theta = \pm\frac{2}{3}\pi \qquad \cos\theta = \pm\frac{1}{2}$

$\cos\theta = \frac{1}{2} \qquad \theta = \pm\frac{\pi}{3} \quad (\pm 60^\circ)$

$\cos\theta = -\frac{1}{2} \qquad \theta = \pm\frac{2}{3}\pi \quad (\pm 120^\circ)$

WE GET FOUR NULL DIRECTIONS $\theta = \pm\frac{\pi}{3}, \pm\frac{2}{3}\pi$ $(\pm 60^\circ, \pm 120^\circ)$

$k = 2$ $\quad \varphi = \pm \frac{4\pi}{N} = \pm \frac{4\pi}{3}$ $\qquad \varphi = \frac{4}{3}\pi\cos\theta = \pm\frac{4}{3}\pi$

$\cos\theta = \pm 1$

$\cos\theta = 1 \qquad \theta = 0 \quad (0^\circ)$

$\cos\theta = -1 \qquad \theta = \pi \quad (180^\circ)$

WE GET TWO ADDITIONAL NULL DIRECTIONS $\theta = 0, \pi$ $(0^\circ, 180^\circ)$

$k = 3$ $\quad \varphi = \pm \frac{6\pi}{N} = \pm 2\pi$ $\qquad \varphi = \frac{4}{3}\pi\cos\theta = \pm 2\pi$

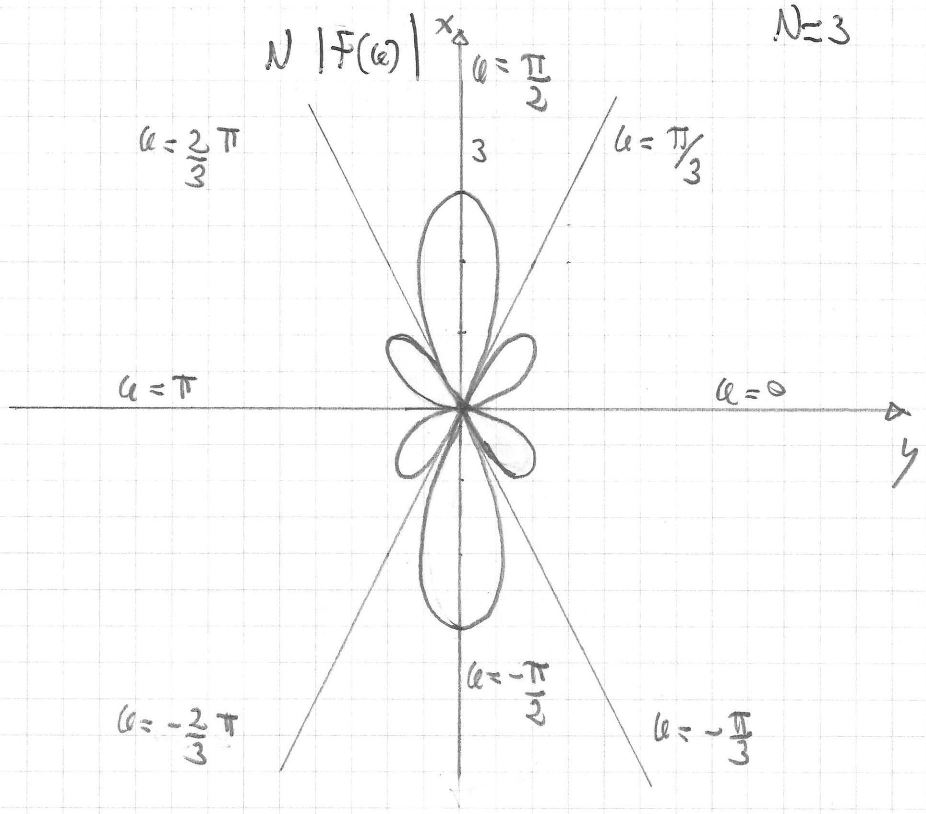
$\cos\theta = \pm\frac{3}{2}$ THERE IS NO SOLUTION

$k = 4$ $\quad \varphi = \pm \frac{8\pi}{N} = \pm \frac{8}{3}\pi$ $\qquad \varphi = \frac{4}{3}\pi\cos\theta = \pm\frac{8}{3}\pi$

$\cos\theta = \pm 2$ NO SOLUTION

WE CAN STOP AT $k = 3$ BECAUSE WE KNOW WE FOUND ALL THE ZEROS

WE HAVE ALL THE ESSENTIAL INFORMATION TO QUALITATIVELY PLOT THE RADIATION PATTERN



THERE MUST BE SYMMETRY WITH RESPECT TO THE y AXIS

2) THE PHASE DIFFERENCE BETWEEN ADJACENT ELEMENTS IS $\alpha = \frac{2\pi}{\lambda} d$

$$\alpha = \frac{2\pi}{\lambda} d = \frac{2\pi}{\lambda}\frac{2}{3}\lambda = \frac{4}{3}\pi \qquad \left(d = \frac{2}{3}\lambda\right)$$

- MAXIMUM DIRECTIONS

THE MAXIMA OF $\left|\frac{\sin\left(N\frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right|$ ARE OBTAINED FOR $\psi = 0, \pm 2\pi, \pm 4\pi, \ldots$

$$\psi = 0 \Rightarrow \psi = \frac{2\pi}{\lambda} d \cos\theta + \alpha = \frac{2\pi}{\lambda} d \cos\theta + \frac{2\pi}{\lambda} d = 0$$

$$\frac{2\pi}{\lambda} d (\cos\theta + 1) = 0 \Rightarrow \cos\theta = -1$$

$$\theta = \pi \quad (180^\circ)$$

$$\psi = \pm 2\pi \Rightarrow \psi = \frac{2\pi}{\lambda} d \cos\theta + \frac{2\pi}{\lambda} d = \pm 2\pi$$

$$\frac{2\pi}{\lambda} d (\cos\theta + 1) = \pm 2\pi \Rightarrow \cos\theta + 1 = \pm\frac{\lambda}{d} = \pm\frac{3}{2}$$

$$\cos\theta = -1 \pm \frac{3}{2}$$

$$\cos\theta = -1 + \frac{3}{2} = \frac{1}{2} \qquad \theta = \pm\frac{\pi}{3} \quad (\pm 60^\circ)$$

$\cos\theta = -1 - \frac{3}{2}$ NO SOLUTION

$$\psi = \pm 4\pi \Rightarrow \psi = \frac{2\pi}{\lambda} d \cos\theta + \frac{2\pi}{\lambda} d = \pm 4\pi$$

$$\frac{2\pi}{\lambda} d (\cos\theta + 1) = \pm 4\pi \Rightarrow \cos\theta + 1 = \pm\frac{2\lambda}{d} = \pm 3$$

$$\cos\theta = -1 \pm 3$$

$\cos\theta = 2$ NO SOLUTION

$\cos\theta = -4$ NO SOLUTION

WE FOUND ALL THE MAXIMA $\theta = \pi, \pm\frac{\pi}{3}$ $(\theta = 180^\circ, \pm 60^\circ)$

- NULL DIRECTIONS

THE ZEROS OF $\left| \dfrac{\sin\left(N\frac{\varphi}{2}\right)}{\sin\left(\frac{\varphi}{2}\right)} \right|$ ARE OBTAINED FOR $N\dfrac{\varphi}{2} = \pm k\pi$

k IS AN INTEGER AND $k \neq N$ $k \neq 2N, \dots$

IN ORDER TO FIND THE NULL DIRECTIONS WE NEED TO FIND THE SOLUTIONS OF

$$\varphi = \pm 2k\frac{\pi}{N} \quad k \text{ INTEGER} \quad k \neq N \quad k \neq 2N \quad k \neq 3N \dots$$

(k MUST BE DIFFERENT FROM A MULTIPLE OF N)

$k=1$ $\quad \varphi = \pm \dfrac{2\pi}{N} = \pm \dfrac{2\pi}{3}$

$$\varphi = \frac{2\pi}{\lambda} d\cos\theta + \frac{2\pi d}{\lambda} = \frac{2\pi}{\lambda} d\left(\cos\theta + 1\right) = \frac{4}{3}\pi\left(\cos\theta + 1\right) = \pm \frac{2}{3}\pi$$

$$\frac{4}{3}\pi\left(\cos\theta + 1\right) = \pm \frac{2}{3}\pi \qquad \cos\theta + 1 = \pm \frac{1}{2}$$

$$\cos\theta + 1 = \frac{1}{2} \qquad \cos\theta = -\frac{1}{2} \qquad \theta = \pm \frac{2}{3}\pi \quad \left(\theta = \pm 120^\circ\right)$$

$\cos\theta + 1 = -\frac{1}{2}$ $\quad \cos\theta = -\frac{3}{2}$ NO SOLUTION

$k=2$ $\quad \varphi = \pm \dfrac{4\pi}{N} = \pm \dfrac{4\pi}{3}$

$$\varphi = \frac{2\pi}{\lambda} d\left(\cos\theta + 1\right) = \frac{4\pi}{3}\left(\cos\theta + 1\right) = \pm \frac{4\pi}{3}$$

$$\cos\theta + 1 = \pm 1$$

$$\cos\theta + 1 = 1 \qquad \cos\theta = 0 \qquad \theta = \pm \frac{\pi}{2} \quad \left(\theta = \pm 90^\circ\right)$$

$\cos\theta + 1 = -1$ $\quad \cos\theta = -2$ NO SOLUTION

$k=3$ $\quad \varphi = \pm \dfrac{6\pi}{N} = \pm 2\pi$ WRONG BECAUSE $N = 3$

$$\varphi = \frac{2\pi}{\lambda} d\left(\cos\theta + 1\right) = \frac{4\pi}{3}\left(\cos\theta + 1\right) = \pm 2\pi$$

$\cos\theta + 1 = \pm\frac{3}{2}$ ← I USED THE SAME EQUATION FOR THE MAXIMA

$k=4$ $\qquad \psi = \pm \frac{8\pi}{N} = \pm \frac{8}{3}\pi$

$$\psi = \frac{2\pi}{\lambda} d (\cos\theta + 1) = \frac{4\pi}{3} (\cos\theta + 1) = \pm \frac{8}{3}\pi$$

$\cos\theta + 1 = \pm 2$

$\cos\theta = -1+2 = 1 \qquad \theta = 0 \qquad (\theta = 0^\circ)$

$\cos\theta = -1-2 = -3$ NO SOLUTION

$k=5$ $\qquad \psi = \pm \frac{10\pi}{N} = \pm \frac{10}{3}\pi$

$$\psi = \frac{2\pi}{\lambda} d (\cos\theta + 1) = \frac{4\pi}{3} (\cos\theta + 1) = \pm \frac{10}{3}\pi$$

$\cos\theta + 1 = \pm \frac{5}{2}$ NO SOLUTION

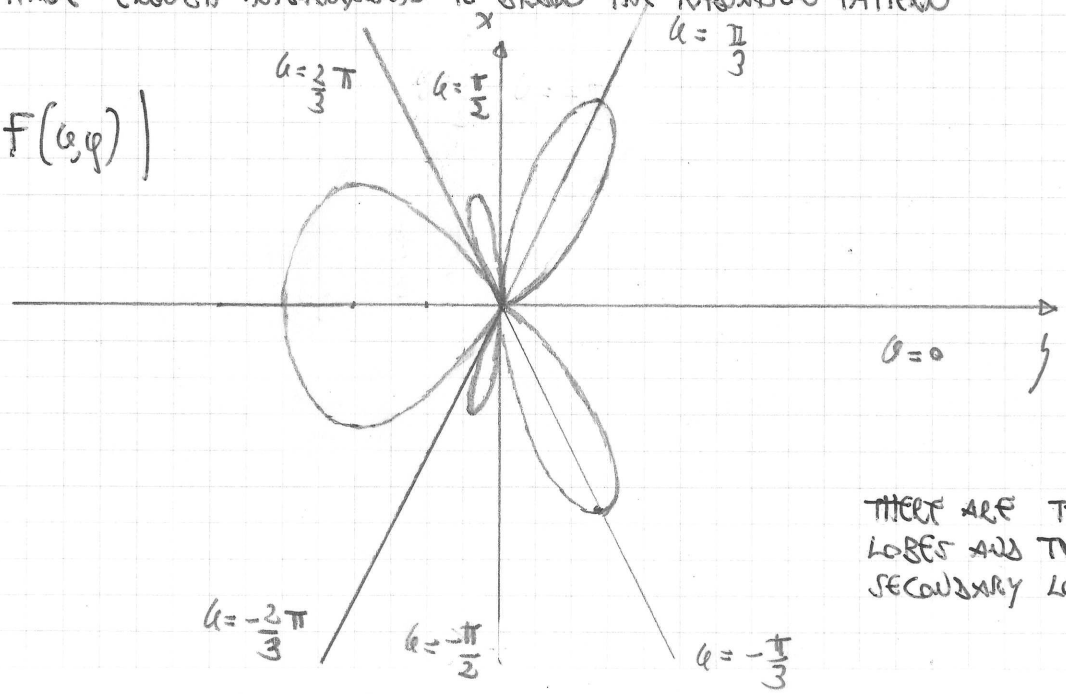
WE FOUND ALL THE NULL DIRECTIONS

$$\theta = \pm \frac{2}{3}\pi, \pm \frac{\pi}{2}, 0 \qquad (\theta = \pm 120^\circ, \pm 90^\circ, 0^\circ)$$

AND THE MAXIMA ARE OBTAINED FOR $\theta = \pi, \pm \frac{\pi}{3}$ $(\theta = 180^\circ, \pm 60^\circ)$

WE HAVE ENOUGH INFORMATION TO DRAW THE RADIATION PATTERN

$N\,|F(\theta,\varphi)|$



THERE ARE THREE MAIN LOBES AND TWO SECONDARY LOBES