Introduction to lasers

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Chapter 2: Amplifier gain

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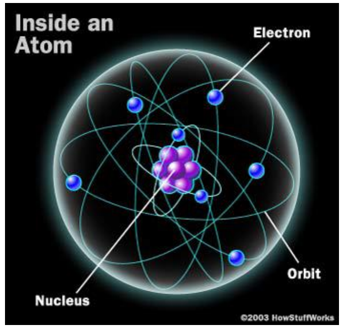
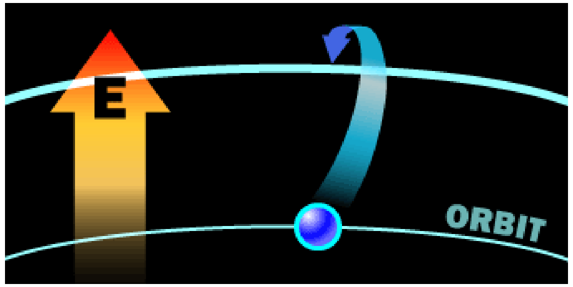
I - Population inversion

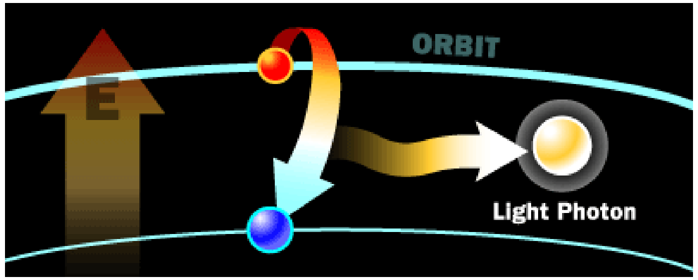
1. Generalities



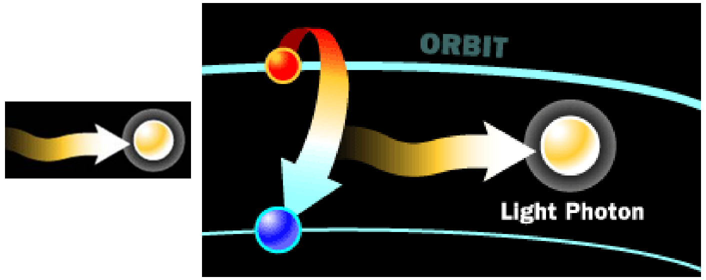
3 types of photon-atom interaction



Absorption



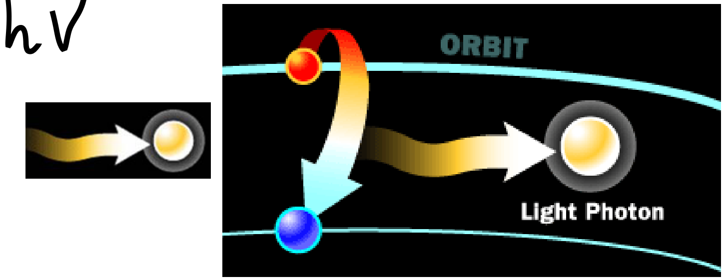
Spontaneous emission



Stimulated emission

→ **GAIN / AMPLIFICATION**

$h\nu$



E_2

E_1

Stimulated emission

➡ **GAIN / AMPLIFICATION**

The emitted photon has the same:

- frequency ν
- phase
- polarization
- direction of propagation

that the incident photon

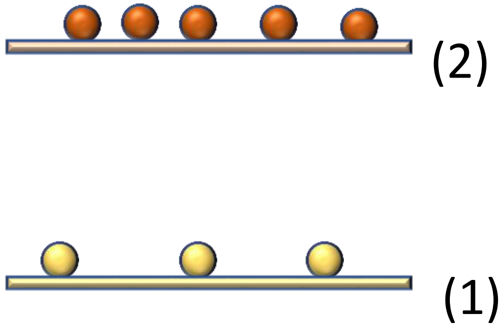
$$\nu = \frac{E_2 - E_1}{h}$$

$$\nu = \frac{c}{\lambda}$$

Requirements :

- $E_{\text{incident photon}} = E_{\text{transition}}$
- population inversion ΔN :

Number of atoms/volume unit in the excited state > Number of atoms/volume unit in the fundamental state



$\Delta N = N2 - N1 > 0$

Laser transition

If $\Delta N > 0$: amplifying medium
If $\Delta N < 0$: absorbent medium
If $\Delta N = 0$: transparent medium

Signal aborded

AM

(ν)

$E_1 \to E_2$

$h\nu$

(2)

(1)

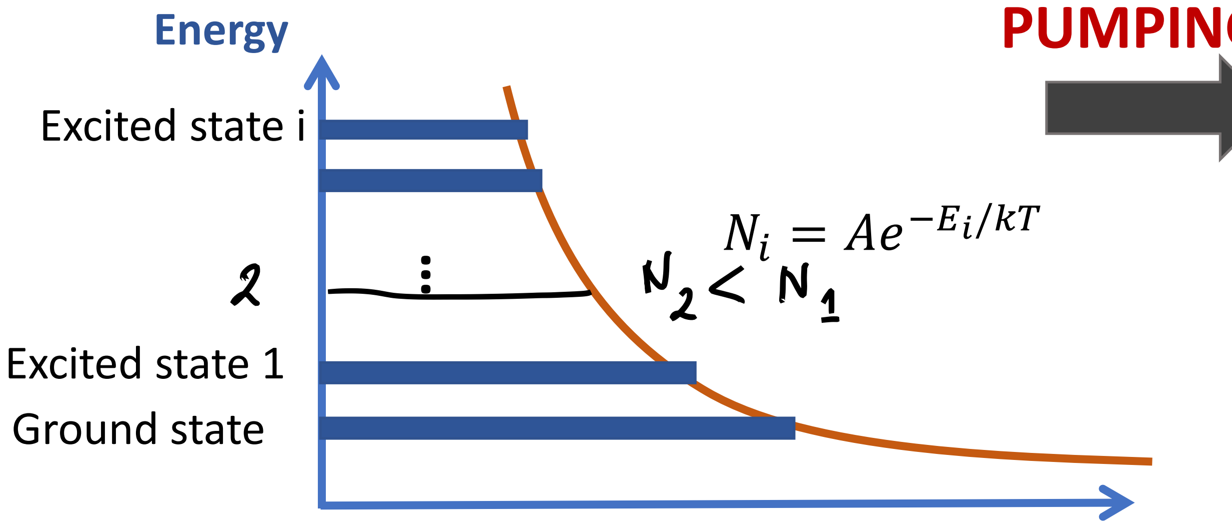
absorption $N_2 < N_1$

$h\nu$

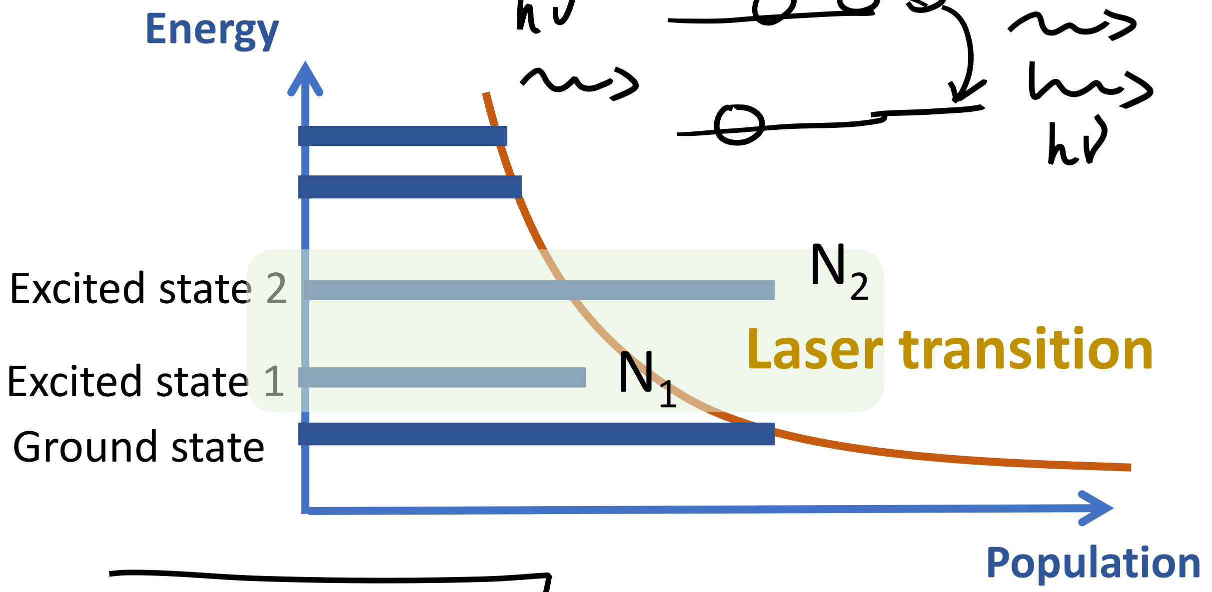
$h\nu$

$h\nu$

Steady state: populations governed by Boltzmann statistics

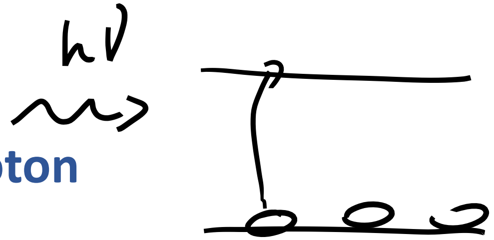


PUMPING



$\boxed{\Delta N = N2 - N1 > 0}$: Amplifying medium

$\Delta N<0 \rightarrow$ Absorbent medium



- **W = probability density (s^{-1}) that an unexcited atom absorb one single photon**
- **W = σ(ν).ϕ** → $cm^{-2}.s^{-1}$

s^{-1}

σ(ν): transition cross section at the frequency ν i.e. transition probability between two energy levels

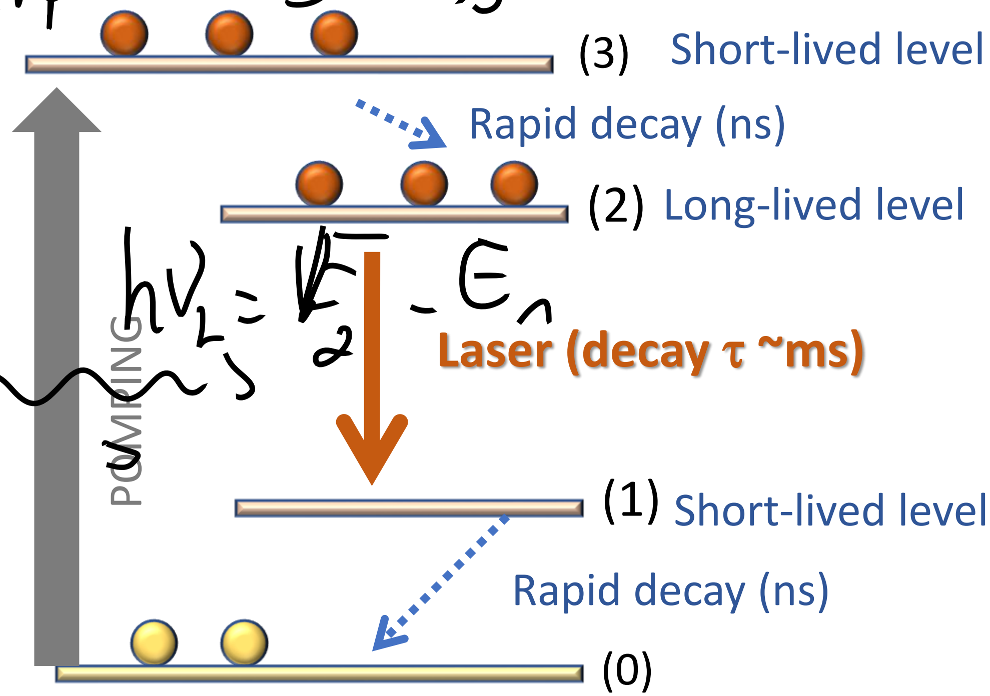
ϕ: photon-flux density (photons /cm^2.s) = I/hν and $I(z) = \frac{\varepsilon_0 c}{2}.|E(z)|^2$

cm^2

W: probability density of both stimulated emission and absorption

2. Four-level pumping scheme

$h\nu_p = E_3 - E_0$



$h\nu_L = E_2 - E_1$

$\nu_p > \nu_L \Rightarrow \lambda_p < \lambda_L$

Easier to get population inversion than for 3-level pumping syst because (1) is not the fundamental level used by the pump.

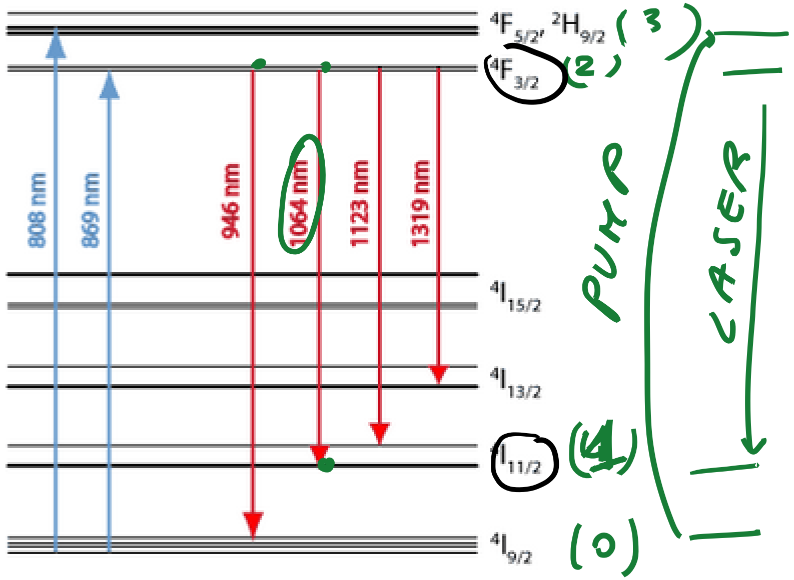


Figure 1: Energy level structure and common pump and laser transitions of the trivalent neodymium ion in Nd^{3+}:YAG.

https://www.rp-photonics.com/yag_lasers.html

Narrow bandwidth: 120GHz, 0.4nm
→ without tunability

$$\boxed{\Delta N_O = \frac{\tau N_a W_P}{(1+\tau W_P)}}$$

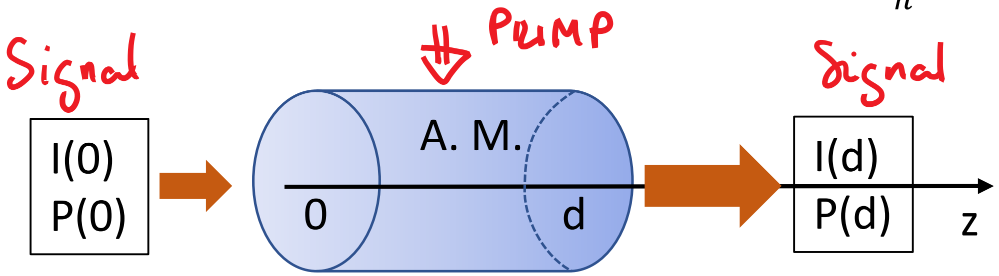
- Population inversion (without signal)
- Na = N_1+N_2: Total number of atoms per volume unit
- W_P: pumping rate (s^{-1}), transition probability between levels (0) and (3)

II - Small-signal gain

→ Link between gain and population inversion

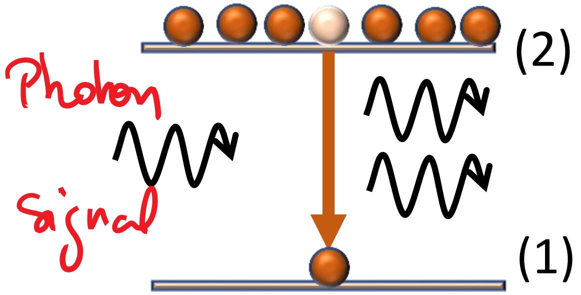
Concept of gain

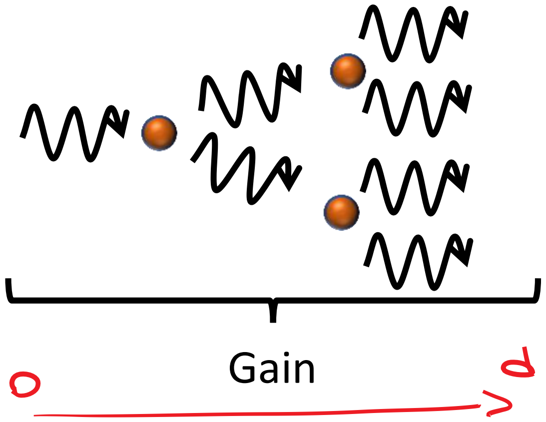
A monochromatic beam of frequency $\nu_L = \frac{E_2 - E_1}{h}$ is illuminating an amplifying medium under pumping and population inversion



$$G_0 = \frac{\boldsymbol{P(d)}}{\boldsymbol{P(0)}}$$

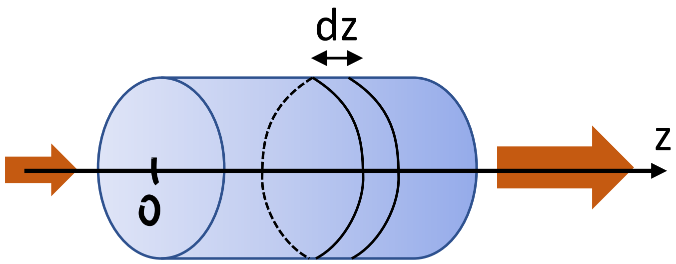
Note : $I = \frac{P}{S}$ ($W\,m^{-2}$)
S → cross section of the beam in the A.M.





The intensity I(z) increases as the length of propagation inside the amplifying medium increases

Let us consider an incremental cylinder of length dz and unit area



$\phi(z)$: photon flux density **entering** the cylinder $(m^{-2} s^{-1})$

$\phi(z+dz) = \phi(z) + d\phi$: photon flux density **exiting** the cylinder

To complete

$d\phi$ = number of photons gained by unit time and unit volume $(W \cdot \Delta N_0)$ · length of propagation (dz)

$d\phi \rightarrow m^{-2} s^{-1}$; $W \rightarrow s^{-1}$; $\Delta N_0 \rightarrow m^{-3}$; $dz \rightarrow m$

$$d\phi = W \Delta N_0 \, dz = \sigma \phi \Delta N_0 \, dz$$

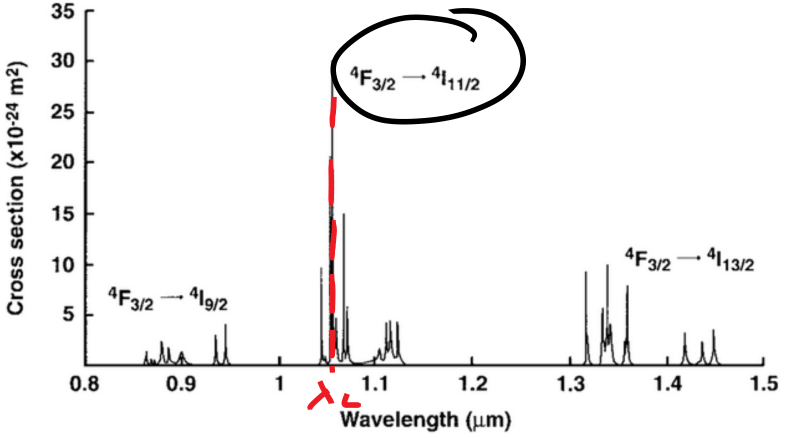
$$\frac{d\phi}{\phi} = \underbrace{\sigma \Delta N_0}_{\gamma_0} \, dz$$

γ_0 = small signal gain coefficient per unit length of the A.M.

Comment:

γ_0 is a function of the frequency ν: $\gamma_0(\nu) = \Delta N_0 \sigma(\nu)$

To complete



Emission cross section of a Nd/YAG crystal

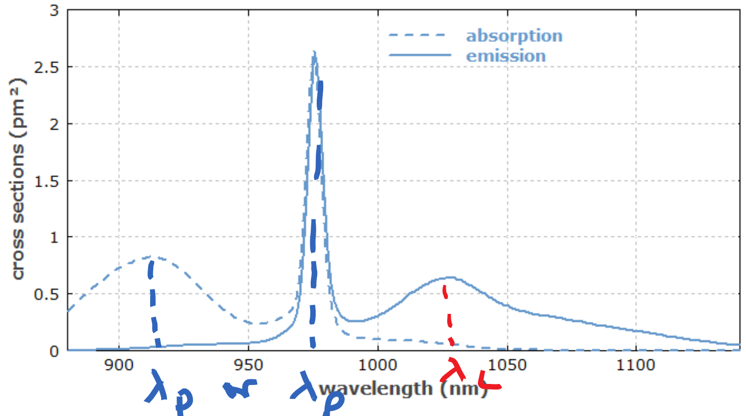


Figure 1: Effective absorption and emission cross sections of ytterbium-doped germanosilicate glass, as used in the cores of ytterbium-doped fibers. (Data from spectroscopic measurements by R. Paschotta)

https://www.rp-photonics.com/transition_cross_sections.html

at 980nm → pump absorption is more efficient than at 920nm but need to cool the pump laser diode to ensure that the wavelength does not shift out of the peak of the absorption cross section.

γ_0 will change with Ppump because $\Delta N_0 = f(P_{pump})$ See Tutorial 1

In the following, for the calculus of the gain, $\nu = \nu_L$ (fixed)

$\Rightarrow \gamma_0(\nu_L) = \Delta N_0 \sigma(\nu_L)$

Gain of amplification: To complete

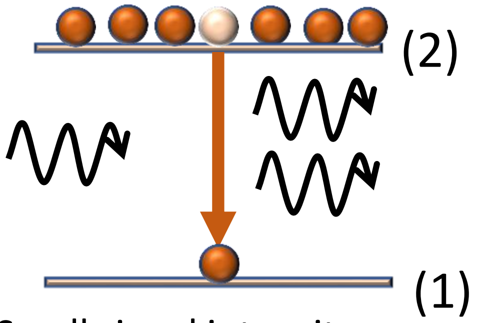
$$G_0 = \frac{P(d)}{P(0)} = e^{\gamma_0 d} \; with \; \gamma_0 = \sigma.\Delta N_0$$

III - Gain saturation

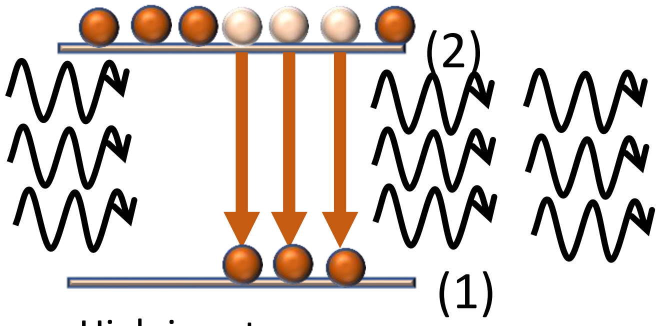
To complete

$$\Delta N(P) = \frac{\Delta N_0}{1 + P/_{Psat}}$$

P: signal power
Psat: saturation power



Small signal intensity
Unmodified population inversion



High input power
Reduced population inversion