Lecture 3 : DSSS (Direct Sequence Spread Spectrum)

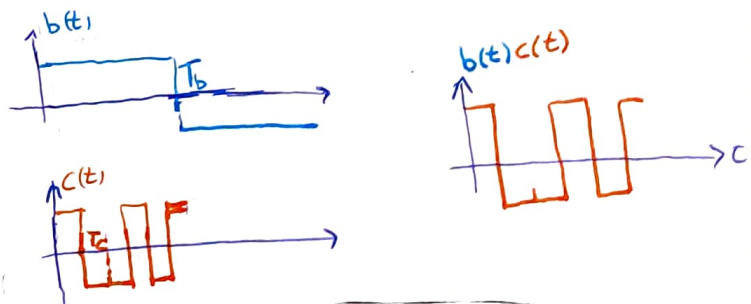
* It was introduced for military applications.
* The problem was due to jamming: when I transmit a signal if someone is able to detect the position in freq. of the signal he can send a disturbance and destroy the telecommunication

Basic Idea

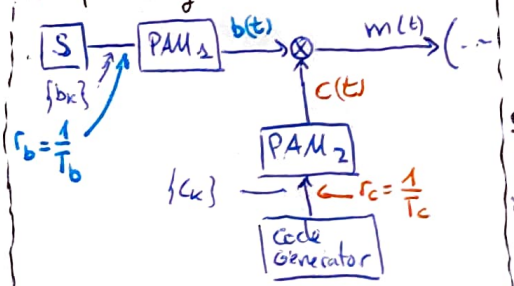
* To avoid the problem of jamming ⇒ Spread Spectrum Strategy
  ↳ I will take the spectrum of my signal and I will enlarge the bandwidth in order to reduce the power ⇒ Reducing the power makes more difficult to see if there is a communication
* The spreading is obtained using a code. I transmit my (spread) signal and at the receiver I should know the same code to come back to the original signal.
* It is an amplitude modulation and it needs a coherent receiver.
  ↳ It needs to know $c(t)$ and synchronization i.e. the timing

Spreading / Despreading

* We want to transmit $b(t)$ using binary-PAM with T_b time given to 1-bit
* Before the transmission we multiply $b(t)$ by a sort of a carrier $c(t)$ ⟶ $m(t) = b(t)c(t)$
  ↳ $c(t)$ is not a sinusoidal carrier, neither a square wave but a sort of PAM signal with a sequence of "1" and "0" (the code)
  ↳ Chip time T_c ⇒ smaller than T_b so spectrum asociated to T_c is wider than T_b
* We send $m(t)$ to the channel



* Spreading Block Diagram



* Bandwidth

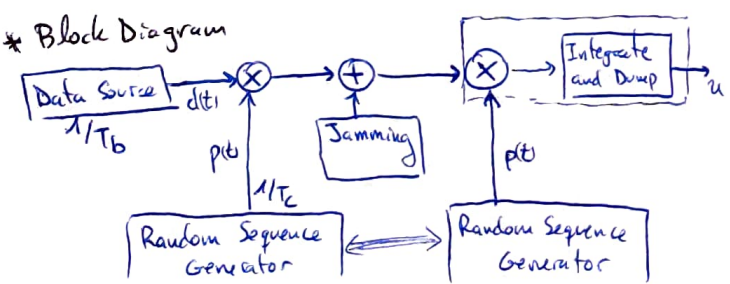
$$B_T \simeq \frac{1}{T_c} \gg \frac{1}{T_b}$$

spread!!

Spreading
$m(t) = b(t)\,c(t)$

Despreading
$m(t)\cdot c(t) = b(t)\,c(t)\cdot c(t) = b(t)$

* Block Diagram

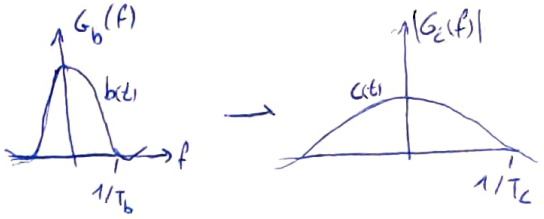


→ The signal is spread, transmitted despread

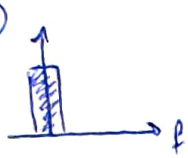
→ The jamming is only despread

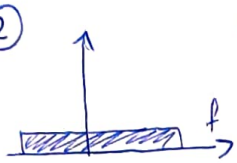
Processing Gain (G)

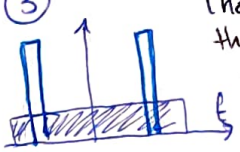
* It is the ratio between T_b and T_c → $G = \frac{T_b}{T_c}$
  It tells us how large is the spread, meaning how much the BW has been increased

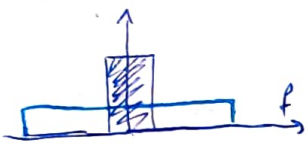


* Process.

①  This is the useful signal. The signal we want to transmit

②  We apply the spreading and the original signal is enlarged

③  There is a jamming that is usually narrow-band

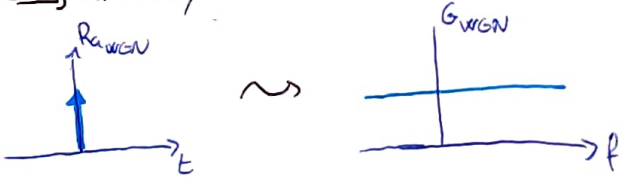
④  At the receiver we do despreading, that is the multiplication by C(t).
→ Original signal come back to the original spread
→ The jamming due to despreading is enlarged in the BW

⑤
→ We apply LPF that filters the contribution of the jamming
→ Processing Gain says how large is the spreading and how efective is the operation against interferences
→ Increase G ⇒ increase robustness against jamming ⇒ but I need more BW

⑥ In the case of wide-band interferences what happens is the same because the operation of despreading is only related to the original signal (the jamming is not recomputed

Code Sequences

* It is important to have a strategy to generate code sequences because the code should be different from one transmission to another.
* A good charateristic of the code sequences ⇒ Similarity to white noise:
  ↳ Autocorrelation (R_a) very impulsive in time
  ⇒ constant power density



* The option: Pseudorandom Sequences Generators
  ↳ Easy to be generated
  ↳ Have randomness properties
  ↳ Have long periods
  ↳ Difficult to reconstruct from a short segment

Maximal Length Sequences (m-sequences)

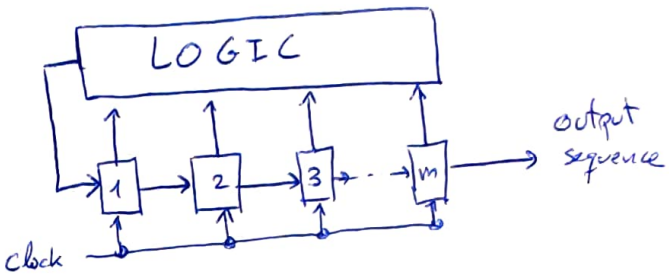
* A m-sequence is a sequence in which the period is the maximum that is possible
We will generate a periodic sequence ⇒ but with very long period

a) Block diagram of a generator

↳ We have a shift register with m-bits, each clock time the bits move to the right

↳ We have a logic part which creates a feedback that is a linear combination of the bit

↳ There is an initial setting different from zero sequence



↳ At each clock time, the last bit is pushed out from the memory (generic sequence)
There are only 2^m possible states, but we cannot use zero ⇒ $N_{max} = 2^m - 1$

b) Properties of the m-sequences

RUN_1 RUN_2 RUN_3 RUN_4
0 0 1 1 1 0 1

Lengths: $RUN_1 = 2$ $RUN_3 = 1$
$RUN_2 = 3$ $RUN_4 = 1$

* RUN: A set of same bits:
If the length of the RUN is very long ⇒ Indication that the sequence is not random

① Balance Property: The average value is close to zero ⇒ More or less the same amount of "1" and "0"

→ The sequence is odd (zero seq. not used) so number of "1" is greater than number of "0" but only in one point

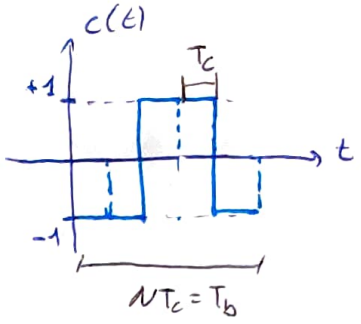
② Property: $\frac{1}{2}$ of the RUN show a length equal to 1
$\frac{1}{4}$ of the RUN show a length equal to 2
$\frac{1}{8}$ of the RUN show a length equal to 3
⋮

If the RUN increases is very unlikely to have it.

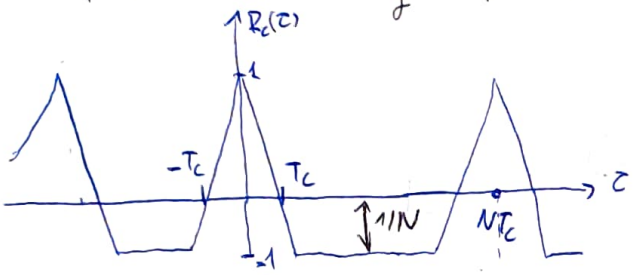
③ Autocorrelation function ⇒ Can be determined

* $T_b = N T_c$, $c(t) \in \{+1, -1\}$: code signal (generated)

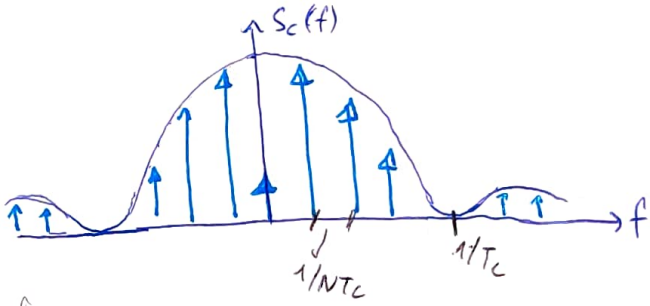
$$R_c(\tau) = \frac{1}{T_b} \int_{-T_b/2}^{T_b/2} c(t) \cdot c(t-\tau)\, dt = \begin{cases} 1 - \frac{N+1}{N T_c} |\tau| , & |\tau| \le T_c \\ -\frac{1}{N} & \text{otherwise} \end{cases}$$



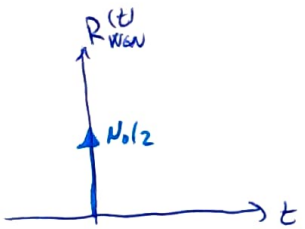
* The autocorrelation is just compare one c(t) with the shifted version of $c(t) \rightsquigarrow c(t-\tau)$
  ↳ If I am considering only one rect ⇒ The autocorrelation function is a triangle



Power Spectral Density

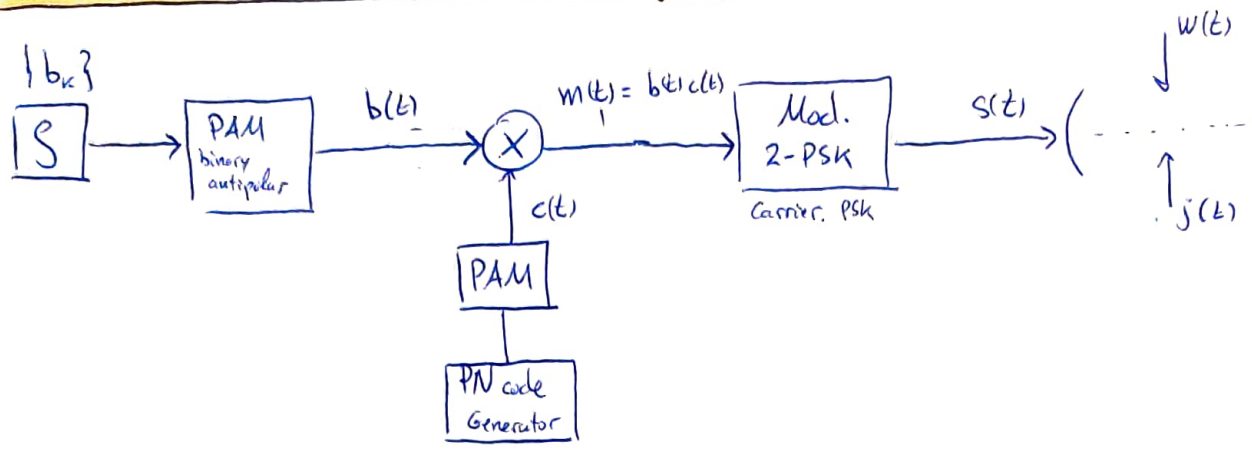


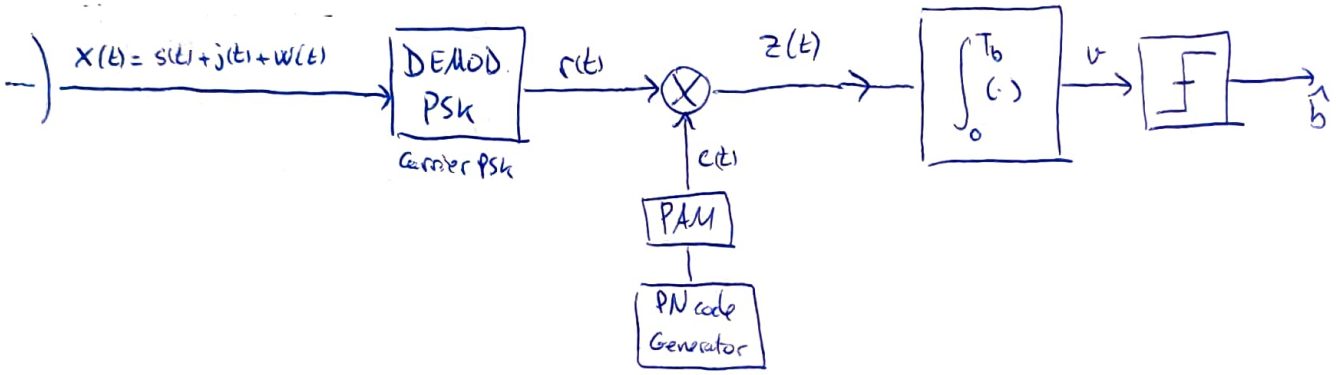
* Difference between this autocorrelation and the autocorrelation of the noise



If N is very big it will be more similar to the behaviour of the noise

Transmitter and Receiver Block Diagram





Error Probability

The result is: $P(E) = Q\left(\sqrt{\frac{E_b}{J T_c/2}}\right)$

$J \rightarrow$ power of the interference
$\hookrightarrow$ energy: $J \cdot T_b$
$\Rightarrow J T_c = J \frac{T_B}{G}$

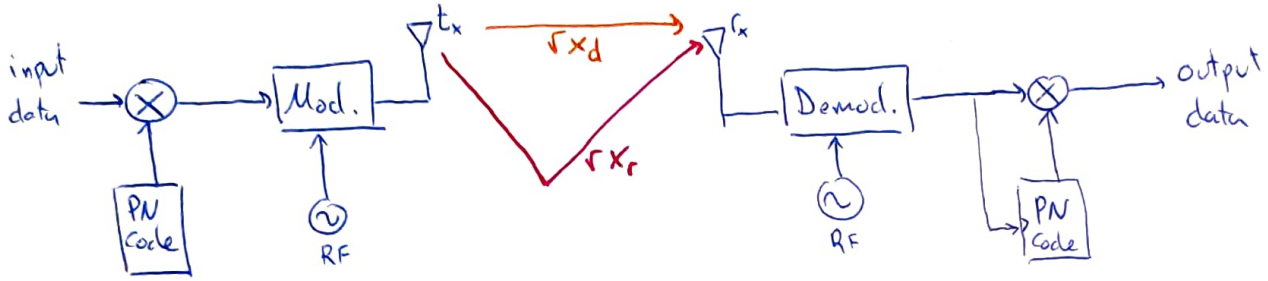
Comparing with the antipolar modulation: $P(E) = Q\left(\sqrt{\frac{E_b}{N_0/2}}\right)$

$\Rightarrow$ So, the disturbance is reduced by the processing gain G.
But increasing G $\Rightarrow$ increase BW

Robustness Against Multipath Fading

* Remember: Multipath fading makes the receiver to receive many different signals

* To make a calculation easier we suppose only two paths: the direct one and a non-direct one



→ We send: $m(t) = b(t)\,c(t)$

→ We receive: $r(t) = m(t) + \alpha_1 m(t-\tau) + \dots$

* An important operation at the receiver is the evalution of the correlation between the signal and the possible signal ⟹ We have an integral of the received signal multiplied by the code.

$$V = \int_0^{T_b} r(t)\cdot c(t)\,dt = \int_0^{T_b} [m(t) + \alpha_1 m(t-\tau)]\,c(t)\,dt = \int_0^{T_b} b(t)\cdot \underbrace{c^2(t)}_{1}\,dt + \int_0^{T_b} \alpha_1\, b(t-\tau)\,\underbrace{c(t-\tau)\,c(t)}_{\text{Autocorrelation of } c(t) \Rightarrow R_c(\tau)}\,dt$$

($b(t) = \pm 1$, $b(t-\tau) = \pm 1$)

* If $T_c < \tau_{min}$ ⟹ Autocorrelation is very small
  ↳ Attenuated by $1/N$
  ⟹ The contribution of the interference is very small ⟹ Robust against multipath fading
  ↳ In order to be robust we need and have a good autocorrelation function

Frequency Hoping Spread Spectrum

* Another class of Spread Spectrum
* The idea: Enlarge the spectrum by changing continously the freq. of the carrier. (Jumping in freqs.)
  ↳ The code establishes the pattern of frequencies ($f_1 \to f_7 \to f_{11} \to f_0 \to \dots$)
* You have a big spectrum because we are changing a lot of freqs. very quickly. Knowing the pattern of freqs., we are able to demodulate the signal.