Esempio: $\underbrace{G}_{\text{SIST.}}$ ottenuta da $g(D)$

$$\begin{cases} g(D) = D^4 + D + 1 \longrightarrow (15, 11) \\ G = ? \end{cases}$$

$$G = \left| \begin{array}{ccccccccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ & 1 & & & & & & & & & & \\ & & 1 & & & & & & & & & \\ & & & 1 & & & & & & & & \\ & & & & 1 & & & 0 & & & & ? \\ & & & & & 1 & & & & & & \\ & & & & & & 1 & & & & & \\ & & & & & & & 1 & & & & \\ & & 0 & & & & & & 1 & & & \\ & & & & & & & & & 1 & & \\ & & & & & & & & & & 1 & \end{array} \right|$$

11   11   4

$$x^{(i)}(D) = x(D)\, D^{i} \bullet \mathrm{mod}\left(D^{15}+1\right)$$

$$x(D) \longrightarrow 00000 \cdots 1 \longrightarrow 1 \cdot D^0$$

$$x(D) \cdot D^4 = D^4$$

$$
\begin{array}{r|l}
D^4 & D^4+D+1 \\
\hline
D^4+0+0+D+1 & 1 \\
\hline
D+1 &
\end{array}
$$

$$\underbrace{D^4 = g(D)\cdot(1)}+D+1$$

D^4+D+1 è PAROLA del codice !!!

è l'ultima riga di G !!!

0 - - - - - - 1 | 0 0 1 1

ripetiamo per D^5:

$$D^5 = D\cdot g(D) + D^2 + D \longrightarrow D^5+D^2+D = D\cdot(D^4+D+1)$$

$$\underset{D^5}{\overset{\uparrow}{0 - - - - - 1}}\ 0 \mid 0\ \underset{D^2}{1}\ \underset{D}{1}\ 0 \quad (\leftarrow +1)$$

$$D^6 = D^2 \cdot g(D) + D^3 + D^2$$

$$D^6 + D^3 + D^2 = D^2 \cdot g(D)$$!!! +2

0 -- 1 0 0 | 1 1 0 0

$$D^7 = (D^3+1)\, g(D) + D^3 + D + 1$$

$$D^7 + D^3 + D + 1 = (D^3+1)\, g(D)$$

-- 0 1 0 0 0 | 1 0 1 1

$$D^8 = (D^4 + D + 1)\, g(D) + D^2 + 1$$

$D^8 + D^2 + 1 \longrightarrow$ -- 0 1 0 0 0 0 | 0 1 0 1

D^9

-- 0 1 0 0 0 0 0 | 1 0 1 0

↑

9

$$D^{10} = \underbrace{\left(D^6 + D^3 + D^2 + 1\right) g(D) + D^2 + D + 1}$$

$$\underbrace{D^{10} + D^2 + D + 1}$$

0 1 0 0 0 0 0 0 | 0 1 1 1

↑ 10

+1

←

D^{11}

— — 1 — — | 1 1 1 0

↑ 11

$$D^{12} = \underbrace{\left(D^8 + D^5 + D^4 + D^2 + 1\right) g(D) + D^3 + D^2 + D + 1}$$

~~ 1 ~~ | 1 1 1 1

↑ 12

$$D^{13} = \left(D^9 + D^6 + D^5 + D^3 + D + 1\right) \underbrace{g(D) + D^3 + D^2 + 1}$$

0 . 1 — 0 | 1 1 0 1

↑ 13

~~ | 1 0 0 1

$$D^{14} = \left(D^{10} + D^7 + D^6 + D^4 + D^2 + D + 1\right) g(D) + D^3 + 1$$

5/

Problem 8.4:

(a) The generator polynomial for the (15,11) Hamming code is given as $g(p) = p^4 + p + 1$. We will express the powers p^l as : $p^l = Q_l(p)g(p) + R_l(p)$ $l = 4, 5, ...14$, and the polynomial $R_l(p)$ will give the parity matrix $\mathbf{P}$, so that $\mathbf{G}$ will be $\mathbf{G} = [\mathbf{I}_{11}|\mathbf{P}]$. We have :

$$
\begin{aligned}
p^4 &= g(p) + p + 1 \\
p^5 &= pg(p) + p^2 + p \\
p^6 &= p^2 g(p) + p^3 + p^2 \\
p^7 &= (p^3 + 1)g(p) + p^3 + p + 1 \\
p^8 &= (p^4 + p + 1)g(p) + p^2 + 1 \\
p^9 &= (p^5 + p^2 + p)g(p) + p^3 + p \\
p^{10} &= (p^6 + p^3 + p^2 + 1)g(p) + p^2 + p + 1 \\
p^{11} &= (p^7 + p^4 + p^3 + p)g(p) + p^3 + p^2 + p \\
p^{12} &= (p^8 + p^5 + p^4 + p^2 + 1)g(p) + p^3 + p^2 + p + 1 \\
p^{13} &= (p^9 + p^6 + p^5 + p^3 + p + 1)g(p) + p^3 + p^2 + 1 \\
p^{14} &= (p^{10} + p^7 + p^6 + p^4 + p^2 + p + 1)g(p) + p^3 + 1
\end{aligned}
$$

Using $R_l(p)$ (with $l = 4$ corresponding to the last row of $\mathbf{G}$,... $l = 14$ corresponding to the first row) for the parity matrix $\mathbf{P}$ we obtain :

$$
\mathbf{G} = \left[\begin{array}{ccccccccccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1
\end{array}\right]
\begin{array}{l}
10 \quad 8 \\ \\ \\ \\ \\ \\ \\ \\ +2 \\ +1 \\ \\
\end{array}
$$

(b) In order to obtain the generator polynomial for the dual code, we first factor $p^{15} + 1$ into : $p^{15+1} = g(p)h(p)$ to obtain the parity polynomial $h(p) = (p^{15} + 1)/g(p) = p^{11} + p^8 + p^7 + p^5 + p^3 + p^2 + p + 1$. Then, the generator polynomial for the dual code is given by :

$$p^{11}h(p^{-1}) = 1 + p^3 + p^4 + p^6 + p^8 + p^9 + p^{10} + p^{11}$$