Spatial Optics

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Spatio temporal analogy

MONOCHROMATIC PLANE WAVE STRUCTURE

$$MPW(t,M) = e^{j(2\pi\upsilon t - \vec{k}.\overrightarrow{\mathbf{OM}})}$$

$$MPW(t,M) = e^{j2\pi\upsilon t}.\,e^{-j\vec{k}.\overrightarrow{\mathbf{OM}}}$$

Temporal

Spatial

Propagation along the Oz axis

$$MPW_{z=0}(t,x,y) = e^{j2\pi\upsilon t}.\,e^{-j(k_x.x+k_y.y)}$$

variable	t	x ; y
Frequency	ν	$N_x = \frac{k_x}{2\pi}\,;\,N_y = \frac{k_y}{2\pi}$

First analogy about the propagation

Temporal

Spatial

Monochromatic wave

$$MW_{z=0}(t) = e^{j2\pi\upsilon t}.$$

$$MW_{z\neq0}(t\ ) = e^{j2\pi\upsilon t}.\boxed{e^{-j\beta.z)}}$$

Plane wave

$$PW_{z=0}(x,y) = e^{-j(k_x.x+k_y.y)}$$

$$PW_{z\neq0}(x,y) = e^{-j(k_x.x+k_y.y)}.\boxed{e^{-j(k_z.z)}}$$

Transfert function

Analogy about the function decomposition : Fourier analysis

Temporal

Spatial

$$f(t) = \int_{-\infty}^{+\infty} \tilde{f}(\nu)\, e^{j2\pi\nu t} dt$$

FT^{-1}

$$f(x,y) = \int_{-\infty}^{+\infty} \tilde{f}(N_x, N_y) e^{-j(k_x.x+k_y.y)} dN_x dN_y$$

$$\tilde{f}(\nu) = \int_{-\infty}^{+\infty} f(t)\, e^{-j2\pi\nu t} dt$$

FT

$$\tilde{f}(N_x, N_y) = \int_{-\infty}^{+\infty} f(x,y)\, e^{j(k_x.x+k_y.y)} dxdy$$

1D

2D

!!signs

Analogy about general skills of the data processing

Applied to linear and translation invariant systems

Linearity

time invariance

$$in(t) \xrightarrow{\ast IR\ (impulse\ response)} out(t) = in \ast IR = FT^{-1}(\widetilde{in}.TF)$$

FT ↓ **LTI system Σ** ↑ FT^{-1}

$$\widetilde{in}(\upsilon) \xrightarrow{.TF} \widetilde{out}(\upsilon) = \widetilde{in}(\upsilon).TF$$

Linearity

space invariance

$$in(x,y) \xrightarrow{\ast PSF} out(x,y) = in \ast PSF = FT^{-1}(\widetilde{in}.TF)$$

FT ↓ **LTI system Σ** ↑ FT^{-1}

$$\widetilde{in}(N_x, N_y) \xrightarrow{.TF} \widetilde{out}(N_x, N_y) = \widetilde{in}(N_x, N_y).TF$$

Application to the diffraction grating formula

$$\sin(\theta) = \sin(\theta_0) + \frac{k\lambda}{a}$$