Name: Nguyen Hoang Nam

Matricola: 726015

Part 1: Error Control Code

1) Generator matrix: ???

$$G = \begin{bmatrix} 0&1&1&0&1&1&0&1&1 \\ 1&1&0&1&1&0&1&1&0 \end{bmatrix} = \begin{bmatrix} 1&0&1&1&0&1&1&0&1 \\ 0&1&1&0&1&1&0&1&1 \end{bmatrix} \Rightarrow N=9, K=2$$

$\Rightarrow$ Possible code word:

Input	Output
00	000000000
01	011011011
10	101101101
11	110110110

$\Rightarrow$ All code are transpose of original code 011011011

$\Rightarrow$ This is a cyclic code

Generator polinominal: $g(D) = D^7 + D^6 + D^4 + D^3 + D + 1$

$= (D^6 + D^3 + 1)(D+1) = D^7 + D^4 + D + D^6 + D^3 + 1$

$d_{min} = \min(W_H) = 6$

* Parity check matrix: $P = \begin{bmatrix} 1&1&0&1&1&0&1 \\ 1&0&1&1&0&1&1 \end{bmatrix}$

$$\Rightarrow H^T = \begin{bmatrix} 1&1&0&1&1&0&1 \\ 1&0&1&1&0&1&1 \\ 1&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0 \\ 0&0&1&0&0&0&0 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&1&0&0 \\ 0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&1 \end{bmatrix} \Rightarrow \text{Syndrome} \begin{cases} x_1 + x_2 + x_3 = 0 \\ x_1 + x_4 = 0 \\ x_2 + x_5 = 0 \\ x_1 + x_2 + x_6 = 0 \\ x_1 + x_7 = 0 \\ x_2 + x_8 = 0 \\ x_1 + x_2 + x_9 = 0 \end{cases}$$

$M_S = ???$

Possible 1-bit error in $x_1 \Rightarrow$ Syndrome = 1101101

Possible 2-bit error in $x_4, x_5 \Rightarrow$ Syndrome = 0110000

Possible 3-bit error in $x_1, x_4, x_5 \Rightarrow$ Syndrome = 1011101

* Consider $N = 48$, $K = 24$, $d = 12$ $\Rightarrow t = 5$; $R = 0.5$

$\Rightarrow$ Possible code word $= 2^K = 2^{24}$

Probability of error for hard decision:

$$P(E) = \sum_{i=t+1}^{N} \binom{N}{i} p^i (1-p)^{N-i} \qquad t = \left\lfloor \frac{d-1}{2} \right\rfloor = 5 \qquad p \approx Q\left(\sqrt{\frac{2E_b}{N_0} R}\right);\ R = \frac{K}{N} = \frac{1}{2}$$

$$= \sum_{i=6}^{48} \binom{48}{i} p^i (1-p)^{48-i} \qquad \Rightarrow p \approx Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Where $p = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$

Minimum band width

$$BW_{min} = \frac{B_R}{2R} = \frac{10 \times 10^6}{2 \times 0.5} = 10 \times 10^6 (Hz) = 10 (MHz)$$