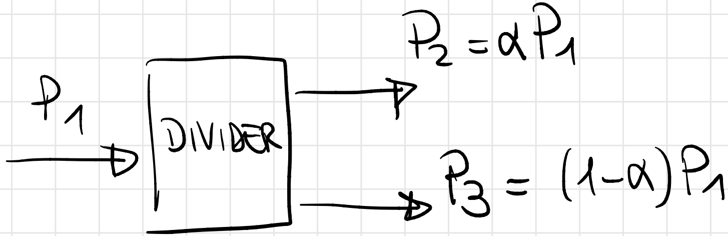
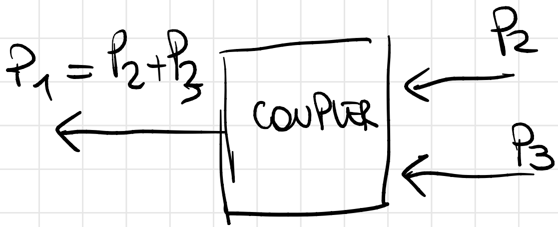
MICROWAVE ENGINEERING

Lecture 24:
Power dividers
and couplers



Either 3 or 4 ports

3 ports → T-junctions



4 ports → Directional couplers or hybrids

BASIC PROPERTIES

3 PORTS NETWORKS (T-JUNCTIONS)

If it's passive and there is no anisotropy → [S] is symmetrical
↓
NETWORK IS RECIPROCAL

NOTE: We cannot have a reciprocal, lossless and matched T-junction.

Demonstration: if $S_{ii}=0$ (all ports matched) and $S_{ij}=S_{ji}$

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

If the network is also lossless $\rightarrow$ $[S]$ must be unitary

$$\begin{cases} |S_{12}|^2 + |S_{13}|^2 = 1 \\ |S_{12}|^2 + |S_{23}|^2 = 1 \end{cases}$$

$$\left\lfloor\ |S_{13}|^2 + |S_{23}|^2 = 1\right.$$

and

$$\left[\begin{array}{l} S_{13}^* S_{23} = 0 \\ S_{23}^* S_{12} = 0 \\ S_{12}^* S_{13} = 0 \end{array}\right.$$

Both sets of equations cannot be satisfied

Example 1 — A CIRCULATOR

[NON RECIPROCAL
MATCHED
LOSSLESS

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

If lossless $[S]$ is unitary

$$S_{31}^* S_{32} = 0$$

$$S_{21}^* S_{23} = 0$$

$$S_{12}^* S_{13} = 0$$

$$|S_{12}|^2 + |S_{13}|^2 = 1$$

$$|S_{21}|^2 + |S_{23}|^2 = 1$$

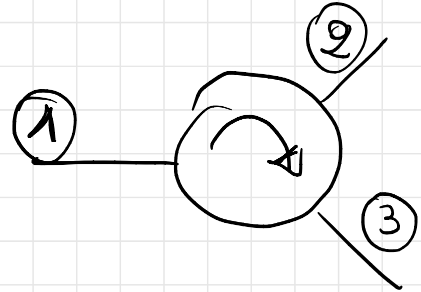
$$|S_{31}|^2 + |S_{32}|^2 = 1$$

① $\rightarrow$ $S_{12} = S_{23} = S_{31} = 0 \longrightarrow |S_{21}|^2 = |S_{32}|^2 = |S_{13}|^2 = 1$

② $\rightarrow$ $S_{21} = S_{32} = S_{13} = 0 \longrightarrow |S_{12}|^2 = |S_{23}|^2 = |S_{31}|^2 = 1$

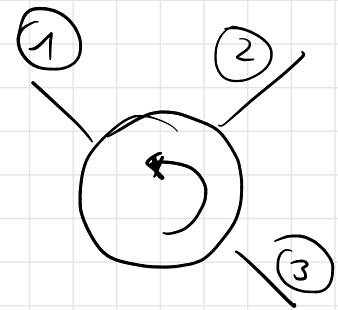
We can have 2 types of circulators:

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



or

$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



<u>Example 2</u>: Lossless, Reciprocal, with only 2 ports matched

<u>Example 3:</u> Lossy, Reciprocal, Matched at all ports (RESISTIVE DIVIDER)

FOUR PORTS NETWORKS (DIRECTIONAL COUPLERS)

if reciprocal and matched

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$\begin{cases} S_{13}^* S_{23} + S_{14}^* S_{24} = 0 & \leftarrow \text{multiply by } S_{24}^* \\ S_{14}^* S_{13} + S_{24}^* S_{23} = 0 & \leftarrow \text{multiply by } S_{13}^* \end{cases}$$

Subtracting the first eq. from the second

$$-S_{24}^{*} S_{13}^{*} S_{23} + S_{14}^{*} |S_{24}|^2 = 0$$

$$S_{14}^{*} |S_{13}|^2 + S_{24}^{*} S_{23} S_{13}^{*} = 0$$

$$\boxed{S_{14}^{*} \left(|S_{13}|^2 - |S_{24}|^2 \right) = 0} \quad 1$$

Then we have

$$S_{12}^{*} S_{23} + S_{14}^{*} S_{34} = 0 \qquad \leftarrow \text{multiply by } S_{12}$$

$$S_{14}^{*} S_{12} + S_{34}^{*} S_{23} = 0 \qquad \leftarrow \text{“} \quad \text{“} \quad S_{34}$$

$$|S_{12}|^2 S_{23} + S_{12} S_{14}^* S_{34} = 0$$

$$- S_{14}^* S_{34} S_{12} + |S_{34}|^2 S_{23} = 0$$

$$\boxed{S_{23} \left(|S_{12}|^2 - |S_{34}|^2 \right) = 0} \quad ②$$

If ① and ② have to be satisfied $\Rightarrow$ $S_{14} = S_{23} = 0$

From which result is:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

DIRECTIONAL COUPLER

For the coupler to be lossless

$$\left.\begin{aligned} |S_{12}|^2 + |S_{13}|^2 &= 1 \\ |S_{12}|^2 + |S_{24}|^2 &= 1 \\ |S_{13}|^2 + |S_{34}|^2 &= 1 \\ |S_{24}|^2 + |S_{34}|^2 &= 1 \end{aligned}\right] \Rightarrow \begin{array}{c} |S_{13}| = |S_{24}| \\ \& \\ |S_{12}| = |S_{34}| \end{array}$$

If we choose a phase reference for 3 of the 4 ports:

$$\begin{cases} S_{12} = S_{34} = \alpha \\ S_{13} = \beta e^{j\theta} \\ S_{24} = \beta e^{j\phi} \end{cases}$$

α, β real

θ, ϕ constants

$$[S] = \begin{bmatrix} 0 & \alpha & \beta e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta e^{j\phi} \\ \beta e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta e^{j\phi} & \alpha & 0 \end{bmatrix}$$

$$S_{12}^{*} S_{13} + S_{24}^{*} S_{34} = 0$$

$$\cancel{\alpha}\cancel{\beta} e^{j\theta} + \cancel{\beta} e^{j\phi} \cancel{\alpha} = 0$$

$$\boxed{\theta + \phi = \pi \pm 2n\pi}$$

So we can have the following couplers:

1. The symmetrical coupler: $\theta = \phi = \frac{\pi}{2}$ (equal phase)

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

② The Antisymmetrical coupler : $\theta = 0 \quad \phi = \pi$

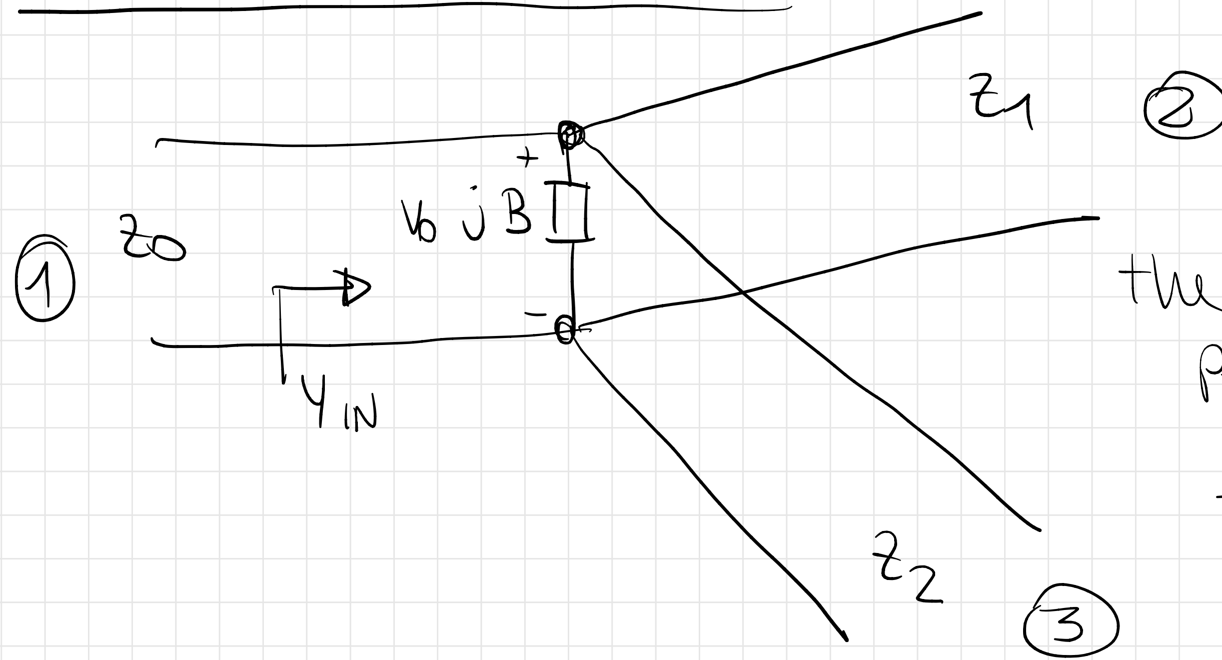
(phases are 180° apart)

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

<u>NOTE</u> : α and β are NOT INDEPENDENT

$$\boxed{|\alpha|^2 + \beta^2 = 1}$$

LOSSLESS POWER DIVIDER



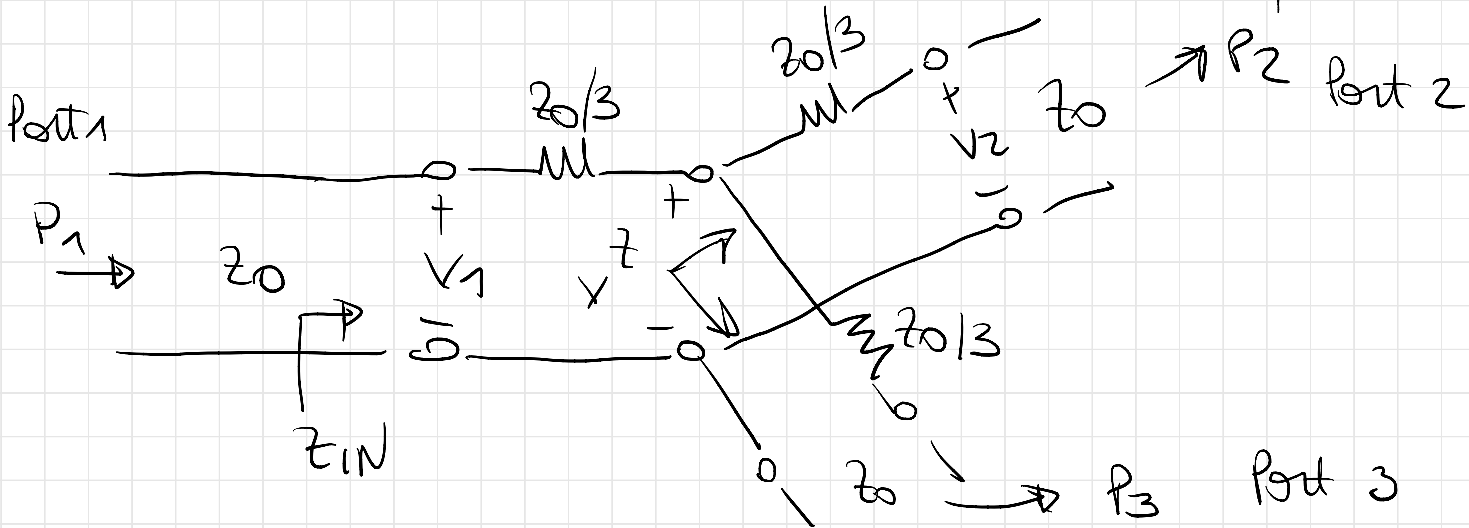
the TWO output ports are not **matched**

$$Y_{IN} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

For example : if we assume $Z_0 = 50\,\Omega$ and we want a 3dB (equal split) power divider then (if $B=0$) $\rightarrow$ $Z_1 = Z_2 = \underline{100\,\Omega}$

RESISTIVE POWER DIVIDER

Lossy, reciprocal, matched at all ports!



$$Z = Z_0/3 + Z_0 = \frac{4}{3} Z_0$$

$$Z_{IN} = \frac{Z_0}{3} + \underset{Z // Z}{\frac{2}{3} Z_0} = Z_0$$ ← matched
To feed the line

$\Downarrow$

$$S_{11} = S_{22} = S_{33} = 0$$

We can calculate

$$V = V_1 \frac{2 Z_0/3}{\frac{Z_0}{3} + \frac{2}{3} Z_0} = \frac{2}{3} V_1$$

The output voltages are

$$V_2 = V_3 = V \frac{Z_0}{Z_0 + \frac{Z_0}{3}} = \frac{3}{4} V = \frac{1}{2} V_1$$

It follows:

$$S_{21} = S_{31} = S_{23} = \frac{1}{2}$$ (-6dB below the input power)

Because the network is also reciprocal

$$[S] = \frac{1}{2} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

The power at the input:

$$P_1 = P_{IN} = \frac{1}{2}\frac{V_1^2}{Z_0}$$

The output Power is

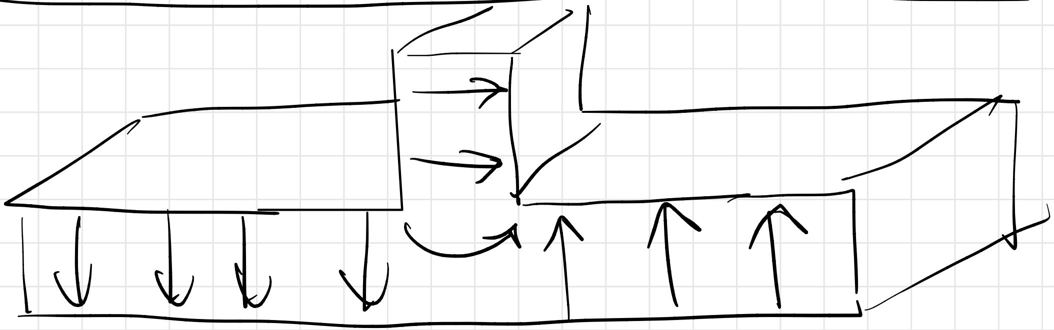
$$P_2 = P_3 = \frac{1}{2}\frac{\left(\frac{1}{2}V_1\right)^2}{Z_0} = \frac{1}{8}\frac{V_1^2}{Z_0} = \frac{1}{4}P_{IN}$$

$$P_2 + P_3 = \frac{1}{2}P_{IN}$$

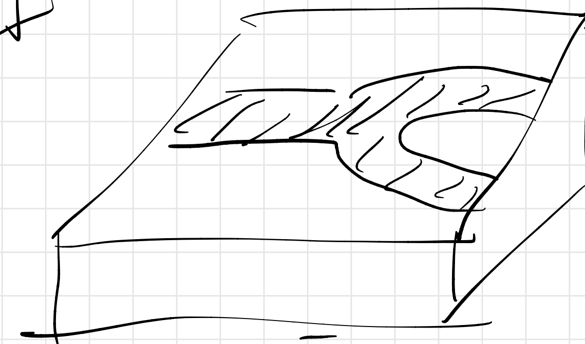
Half of the P_{IN} is lost in the resistors

$$P_{LOSS} = P_{IN} - P_2 - P_3 = \frac{1}{2}P_{IN}$$

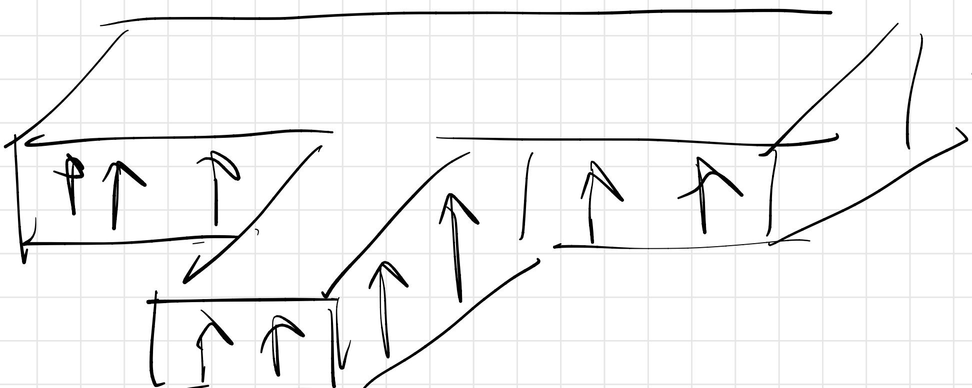
T-Junction practical implementation



E-plane waveguide

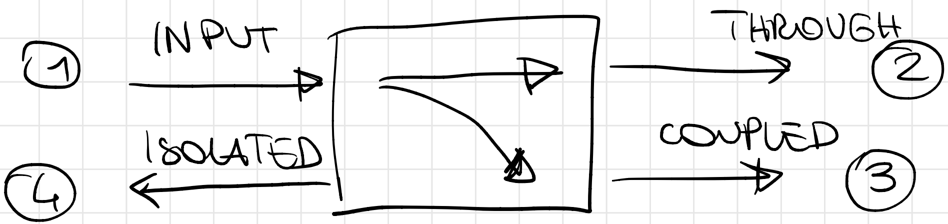


microstrip T-junction

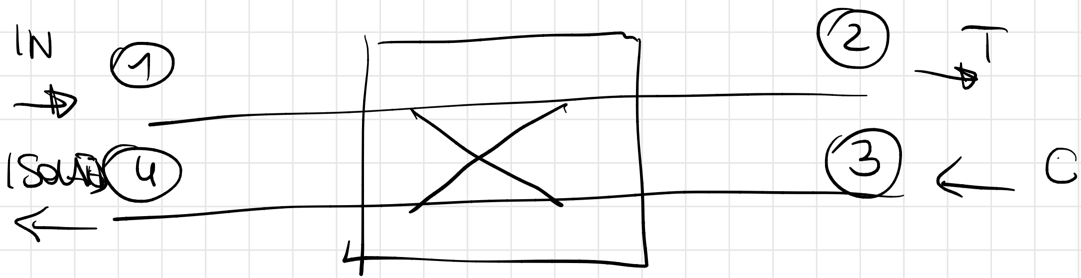


H-plane waveguide

How does a directional coupler work?



or



power goes in at port 1 and exits at port 3 with coupling factor $\boxed{|S_{13}|^2 = \beta^2}$

The rest of power goes to port 2 with coeff.

$$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$$

In our ideal directional coupler NO POWER GOES TO PORT 4

Fundamental parameters:

$$\text{COUPLING} = C = 10 \log \frac{P_1}{P_3} = -20 \log \beta \quad dB$$

$$\text{DIRECTIVITY} = D = 10 \log \frac{P_3}{P_4} = 20 \log \frac{\beta}{|S_{14}|} \quad dB$$

$$\text{ISOLATION} = I = 10 \log \frac{P_1}{P_4} = -20 \log |S_{14}| \quad dB$$

$$I = D + C$$

The ideal coupler will have $D = I = \infty$

α and β would depend on C only.

Special couplers $\left(C = 3\,dB \rightarrow \alpha = \beta = \frac{1}{\sqrt{2}} \right)$

HYBRID COUPLERS

(1) Quadrature hybrid — 90° phase shift between ports 2 and 3 when the input is at port 1.

SYMMETRICAL

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

② Magic-T Hybrid (rat-race hybrid)

ANTI SYMMETRICAL

180° phase shift between ports 2 and 3 when input is at port 4

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$