Set #5

14.

Work out the reflection coefficient R for an electron reflecting off a potential (barrier) V, following the same steps as done in class for the transmission coefficient T.

a) Give a concise expression for incidence below barrier (E<V)
b) Give the value of R and T when E=V.
c) Give a concise expression for incidence above barrier (E>V)
   Provide an estimate for E=5 μeV, V=4.9 μeV and a=1 μm.
d) What does it mean that electrons are reflected back when passing above the barrier?
e) Find the energies at which electrons passing above the barrier are NOT at all reflected.

15.

Compute the *reflection (R) & transmission (T)* coefficients for a single free electron impinging on a barrier V through the "*current probability density*":

$$\vec{j}(\vec{r},t) = \frac{i\hbar}{2m}\left[\Psi(\vec{r},t)\,\vec{\nabla}\,\Psi^*(\vec{r},t) - \Psi^*(\vec{r},t)\,\vec{\nabla}\,\Psi(\vec{r},t)\right]$$

16.

a. Use the appropriate numerics (Matlab, Mathematica, Python, etc) and "Check" that the polynomials $u_n(x)$ (in class) are solutions of the of the Hermite equation under the condition that the Hermite equation's coefficient be even integer "$2n$".

b. Use the appropriate numerics (Matlab, Mathematica, Python, etc) and "check" that the eigen-functions $\psi_n(x)$ are an orthogonal set of eigen-functions.

Extra.

For an asymmetric 1D barrier the continuity at the well boundary at x=a yields the following eigenvalue equation:

$$\cot\left[ka + \delta(k)\right] = -\sqrt{\frac{2mV_2}{\hbar^2 k^2} - 1}$$

Check that the the correct solution for the energy has the following form (- sign):

$$ka + \delta(k) = -sin^{-1}\frac{\hbar k}{\sqrt{2mV_2}} + \pi n_2$$