ENIMEO - Fundamentals on coherent optics - propagation in optical fibers

Exercice 3 Correction

Part 1

1/ $B\varphi = \frac{\Delta\beta}{k_0} = \frac{|\beta x - \beta y|}{k_0}$ with $\beta_x = k_0 n_{ex}$ and $\beta_y = k_0 n_{ey}$

$\Rightarrow \beta\varphi = |n_{ex} - n_{ey}|$

2/ the phase shift after a propagation over a length z is

$\varphi_x = \beta_x z = k_0 n_{ex} z$ for the mode polarized along x $(HE_{11}x)$

$\varphi_y = \beta_y z = k_0 n_{ey} z$ " " " y $(HE_{11}y)$

The phase shift between the two is $\Delta\varphi = |\varphi_x - \varphi_y| = k_0 |n_{ex} - n_{ey}| z = k_0 B\varphi z$

$\Delta\varphi = 2\pi$ for $z = L_B \Rightarrow 2\pi = \frac{2\pi}{\lambda} B\varphi L_B \Rightarrow L_B = \frac{\lambda}{B\varphi}$

3/ $B\varphi = \frac{\lambda}{L_B} = \sigma\lambda^2 \Rightarrow \sigma = \frac{1}{\lambda L_B} \Rightarrow \sigma = \frac{1}{1.55\,10^{-6} \times 3\,10^{-3}} = 215\,10^{+6}\,m^{-2}$

$n_{ey} = n_{ex} \pm \beta\varphi$ with $B\varphi = \frac{\lambda}{L_B} = \frac{1.55\,10^{-6}}{3\,10^{-3}} = 0{,}517\,10^{-3}$

Thus n_e polar $\perp = 1.4455 + 0.517\,10^{-3} = \underline{1.44602}$

or n_e polar $\perp = 1.4455 - 0.517\,10^{-3} = \underline{1.44498}$

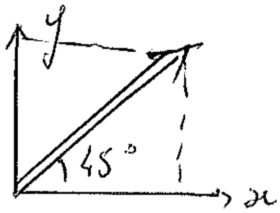
4/ $B_G = N_{gx} - N_{gy} = (n_{ex} - n_{ey}) - \lambda \frac{d(n_{ex} - n_{ey})}{d\lambda} = B\varphi - \lambda \frac{dB\varphi}{d\lambda}$

For this fiber $B_G = \sigma\lambda^2 - \lambda(2\sigma\lambda) = -\sigma\lambda^2 = \underline{-B\varphi}$

$B_G = (\lambda = 1.55\,\mu m) = -0.517\,10^{-3}$

5/ $N_{gy} = N_{gx} + |B_G| = 1.4722 + 0.517\,10^{-3} = \underline{1.47272}$

6/ The energy is equally coupled in the two polarization modes HE_{11x} and HE_{11y} (because launched polarization at 45° of the two



Polarization maintaining fiber ⇒ no exchange of energy between the two modes along the propagation

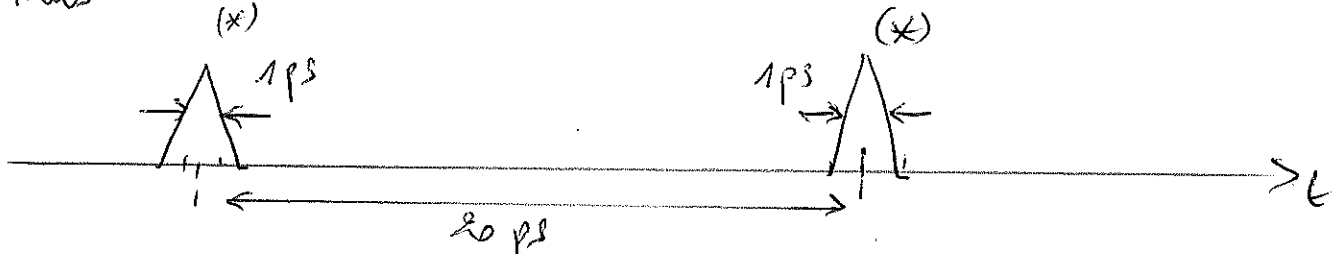
$$v_g = \frac{c}{N_g} = \frac{L}{t_g} \Rightarrow t_g = \frac{L N_g}{c}$$ (t_g = propagation time of a pulse)

for the mode polarized along x (HE_{11x}) → $t_{g_x} = \frac{10 \times 1.4722}{3\ 10^8}$
$= 49.07\ 10^{-9}\ s$

for the mode polarized along y ($HE_{11}y$) $t_{g_y} = \frac{10 \times 1.47272}{3\ 10^8}$
$= 49{,}09\ 10^{-9}\ s$

$\Delta t_g = t_{g_y} - t_{g_x} = 0{,}02\ 10^{-9}\ s = 20\ ps$

thus



(*) no temporal broadening versus the input pulse because the chromatic dispersion is neglected → same temporal width as the input pulse

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Part 2

1/ a/ $v_g = \frac{L}{t} = \frac{100\ 10^3}{0,491317\ 10^{-3}} = 2,035346\ 10^8\ m\ s^{-1}$

b/ $v_g = \frac{c}{n_g} \Rightarrow n_g = \frac{c}{v_g} = 1,47395 \quad (\pm 10^{-5})$

2/ a/ see lecture or page 4/4

b/ $n_g = n_e - \lambda \frac{dn_e}{d\lambda} = A_2\lambda^2 + A_1\lambda + A_0 - \lambda(2A_2\lambda + A_1) = -A_2\lambda^2 + A_0$

c/ $-A_2\lambda^2 + A_0 = n_g \Rightarrow A_2 = \frac{A_0 - n_g}{\lambda^2 (\mu m)} = -16,45\ 10^{-4}\ \mu m^{-2}$

3/ $D_c = \frac{1}{c}\frac{d}{d\lambda}\left(n_e - \lambda\frac{dn_e}{d\lambda}\right) = \frac{1}{c}\left(\frac{dn_e}{d\lambda} - \left(\frac{dn_e}{d\lambda} + \lambda\frac{d^2n_e}{d\lambda^2}\right)\right) = -\frac{\lambda}{c}\frac{d^2n_e}{d\lambda^2}$

b/ $D_c = \frac{1}{c}\frac{dn_g}{d\lambda} = \frac{1}{c}\left(-2A_2\lambda\right) = \frac{-2A_2\lambda}{c} = \frac{+2\times 16,45\ 10^{-4}\times 1,55}{3\ 10^8}\ \frac{\mu m^{-1}}{m.s^{-1}}$

$= 17\ 10^{-12}\ \frac{\mu m^{-1}}{(m.s^{-1})} = 17\ 10^{-12}\ \frac{s}{m.\mu m}$

$= 17\ \frac{ps}{(nm.km)}$

4/ a/ $D_c \neq D_m$ because $D_c = D_m + D_g$ where D_g is the dispersion of the guide and most of the time $D_m \neq 0$

b/ To preserve the transparency of silica, the manufacturers cannot significantly change its composition (dopants) $\Rightarrow$ they cannot change D_m. On the contrary, they can change D_g by changing the index profile or by changing the guiding principle: microstructured fibers, Bragg fibers, hollow core fibers...

5/ $D_c = \frac{\tau}{L\sigma}$ with $\sigma = 0,3\,nm$; $D_c = 17\ ps/nm.km$; $\tau = 150\times 50\% = 75\,ps$

$L = \frac{\tau}{D_c\sigma} = \frac{75}{17\times 0,3} = 14,7\ km$

Ex 3 2/a)

$$v_g = \frac{c}{n_g} \quad \text{and} \quad v_g = \frac{d\omega}{d\beta}$$

$$\Rightarrow \quad n_g = \frac{c}{v_g} = c\,\frac{d\beta}{d\omega} \qquad \text{with } \beta = k_0 n_e \text{ and } k_0 = \frac{\omega}{c}$$

$$n_g = c\,\frac{d}{d\omega}(k_0 n_e) = c\,\frac{d}{d\omega}\left(\frac{\omega n_e}{c}\right) = \frac{d(\omega n_e)}{d\omega}$$

$$= n_e + \omega\,\frac{dn_e}{d\omega} \qquad \text{with } \omega = k_0 c = \frac{2\pi c}{\lambda_0}$$

$$n_g = n_e + \frac{2\pi c}{\lambda_0} \times \frac{d\,n_e}{d\lambda_0} \times \frac{d\lambda_0}{d\omega}$$

$$\lambda_0 = \frac{2\pi c}{\omega} \qquad d\lambda_0 = -2\pi c\,\frac{d\omega}{\omega^2}$$

$$\Rightarrow \frac{d\lambda_0}{d\omega} = -\frac{2\pi c}{\omega^2} = -2\pi c\left(\frac{\lambda_0}{2\pi c}\right)^2 = \frac{-\lambda_0^2}{2\pi c}$$

$$n_g = n_e + \frac{2\pi c}{\lambda_0} \times \frac{-\lambda_0^2}{2\pi c}\,\frac{dn_e}{d\lambda_0}$$

$$= n_e - \lambda_0\,\frac{dn_e}{d\lambda}$$