ARRAY ANTENNAS

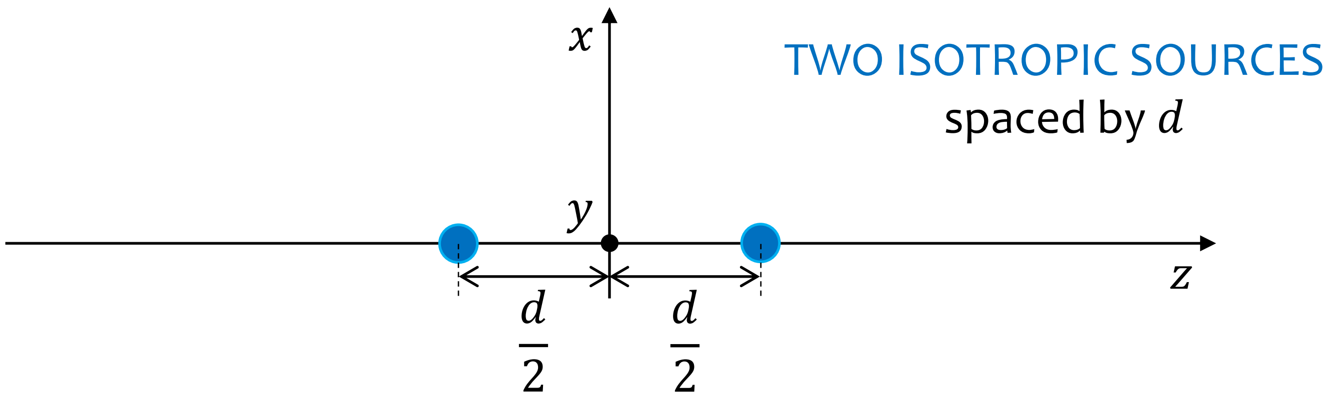
Combining the output of multiple antenna elements provides the possibility of changing the radiation pattern and a larger directivity can be achieved. A configuration of multiple radiating elements is referred to as an array antenna.

TWO-ELEMENT ARRAYS

In order to quickly introduce the arrays and explain how they can be used to control the radiation pattern, we discuss the most basic form of an array: the two-element array.

We begin the analysis by assuming that the two elementar radiators forming the array are isotropic sources: i.e. they transmit (and receive) equally in all directions in three dimensions. Although hypothetical, isotropic radiators provide a simple initial problem formulation. In the continuation of the lesson we will see that, if the isotropic sources are replaced by realistic antennas, the full pattern can be obtained by a simple multiplication process.

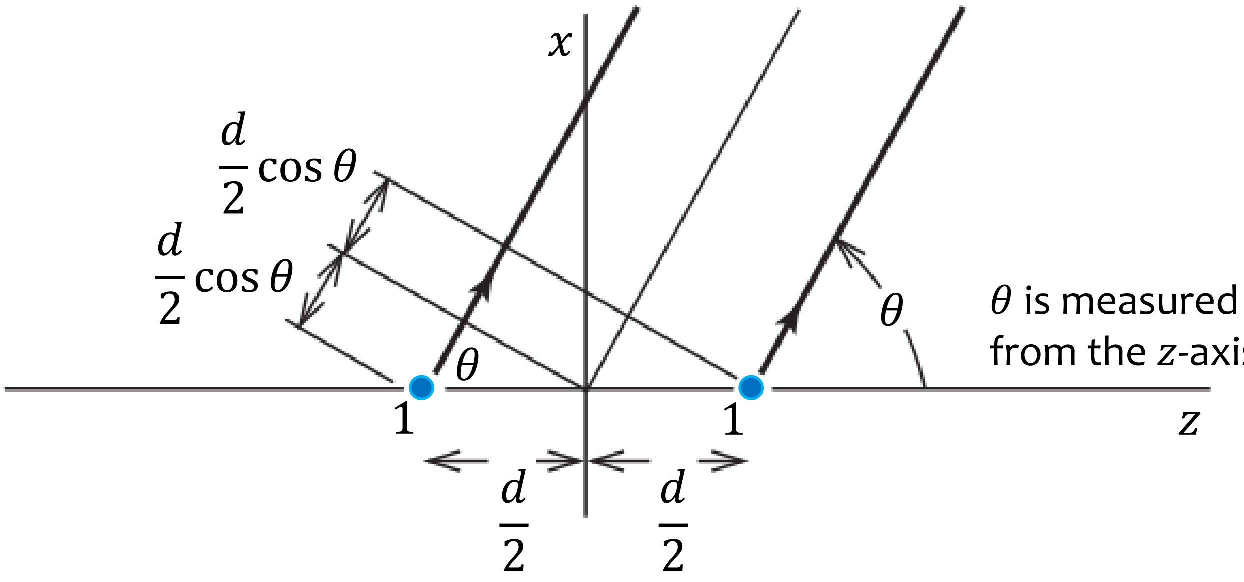
The pattern of an array of isotropic elements is called the array factor. The term factor indicates that in general one must multiply the array factor by the element pattern to obtain the full array pattern. As for any antenna, by reciprocity the obtained pattern is the same on reception.



First example

We consider two isotropic point sources with identical amplitude and phase currents, and spaced one-half wavelength apart $d = \lambda/2$.

We can calculate the **array factor** (AF) by summing the rays (the phasors) in the far-field, and phases corresponding to the path length differences must be accounted for.



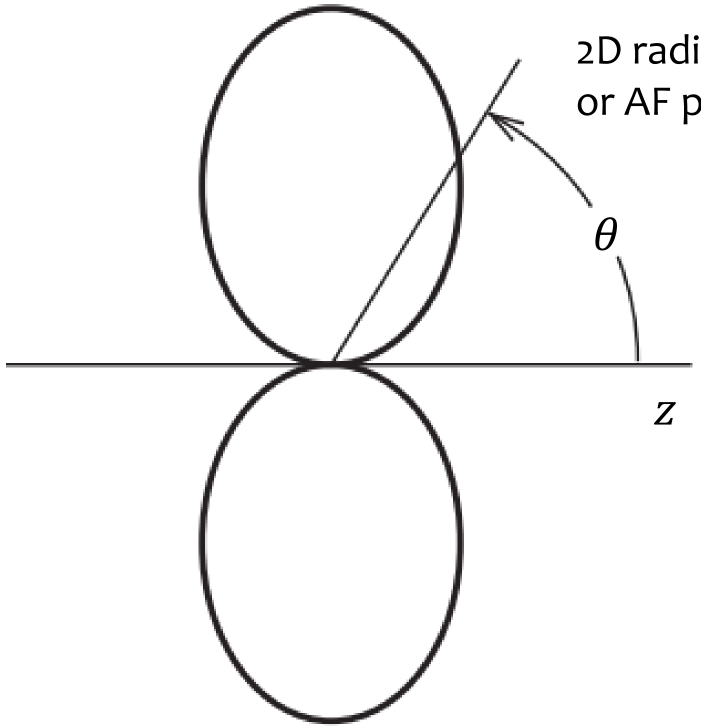
$$\mathrm{AF} = 1e^{-j\beta\frac{d}{2}\cos\theta} + 1e^{+j\beta\frac{d}{2}\cos\theta}$$

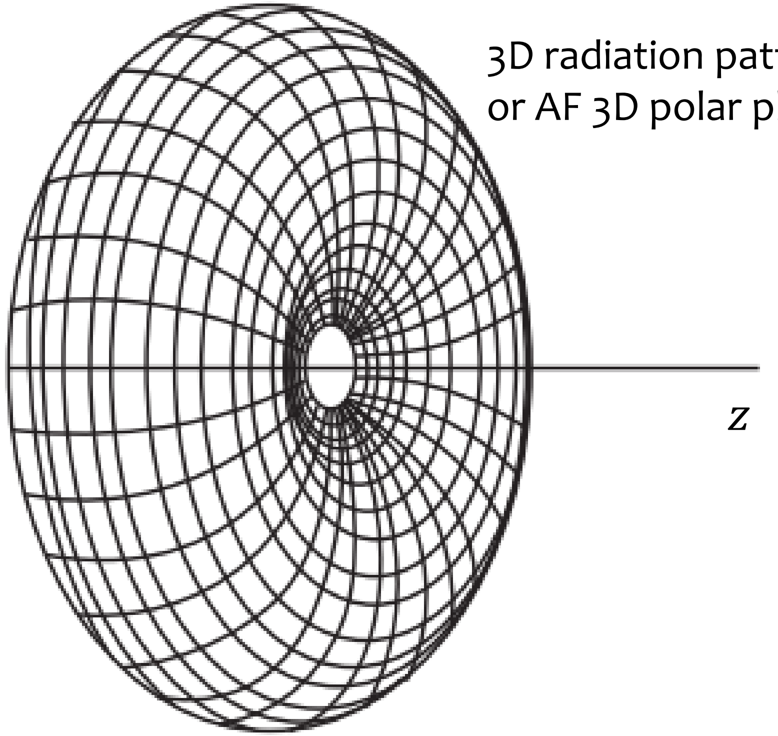
$$\mathrm{AF} = 2\cos\left(\beta\frac{d}{2}\cos\theta\right) = 2\cos\left(\frac{\pi}{2}\cos\theta\right)$$

θ is measured counter-clockwise from the z-axis

Normalizing the array factor for a maximum value of unity gives the normalized radiation pattern $|F(\theta,\varphi)|$

$$|F(\theta,\varphi)| = \left|\cos\left(\frac{\pi}{2}\cos\theta\right)\right|$$

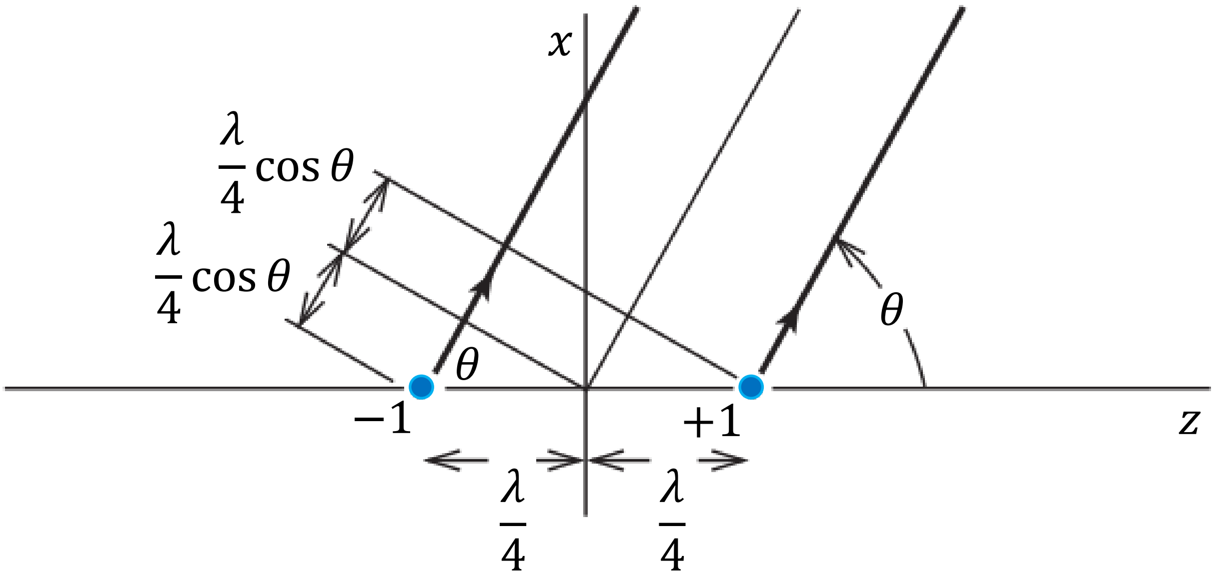




Second example

We consider two isotropic point sources with identical amplitude and opposite phase (i.e. with a phase difference of π), and spaced one-half wavelength apart $d = \lambda/2$.

The array factor (AF) is calculated by phasor addition in the far-field.



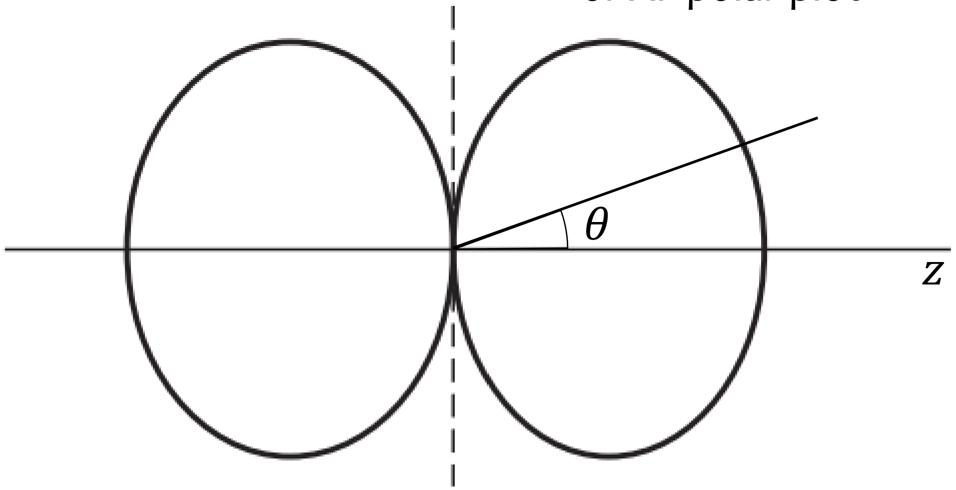
$$\mathrm{AF} = -1e^{-j\beta\frac{d}{2}\cos\theta} + 1e^{+j\beta\frac{d}{2}\cos\theta}$$

$$\mathrm{AF} = 2j\sin\left(\beta\frac{d}{2}\cos\theta\right) = 2j\sin\left(\frac{\pi}{2}\cos\theta\right)$$

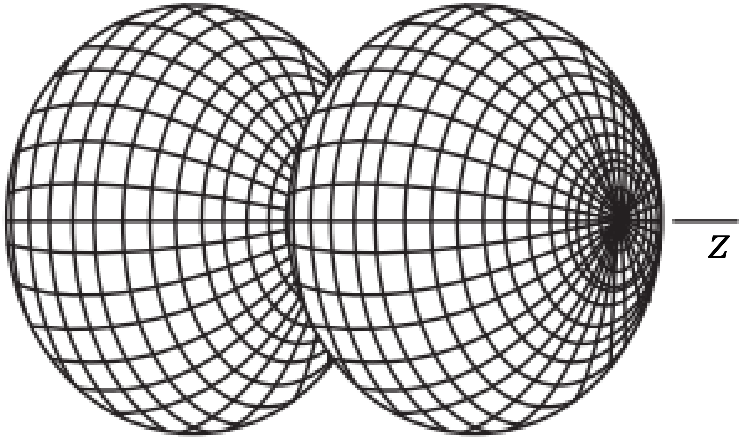
Normalizing the array factor for a maximum value of unity gives the normalized radiation pattern $|F(\theta, \varphi)|$

$$|F(\theta, \varphi)| = \left|\sin\left(\frac{\pi}{2}\cos\theta\right)\right|$$

2D radiation pattern
or AF polar plot



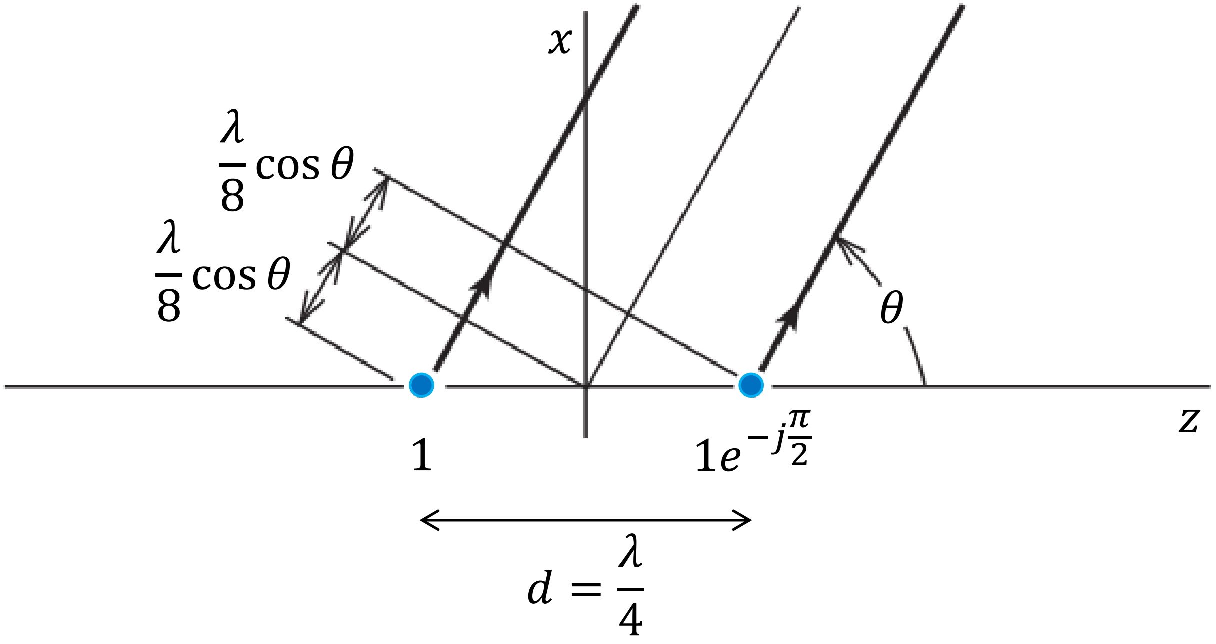
3D radiation pattern
or AF 3D polar plot



Third example

We consider two isotropic point sources with identical amplitude and 90° out of phase (the right point source lag the left by $\pi/2$ radians), and spaced a quarter-wavelength apart $d = \lambda/4$.

We can calculate the array factor (AF) by summing the rays in the far-field.



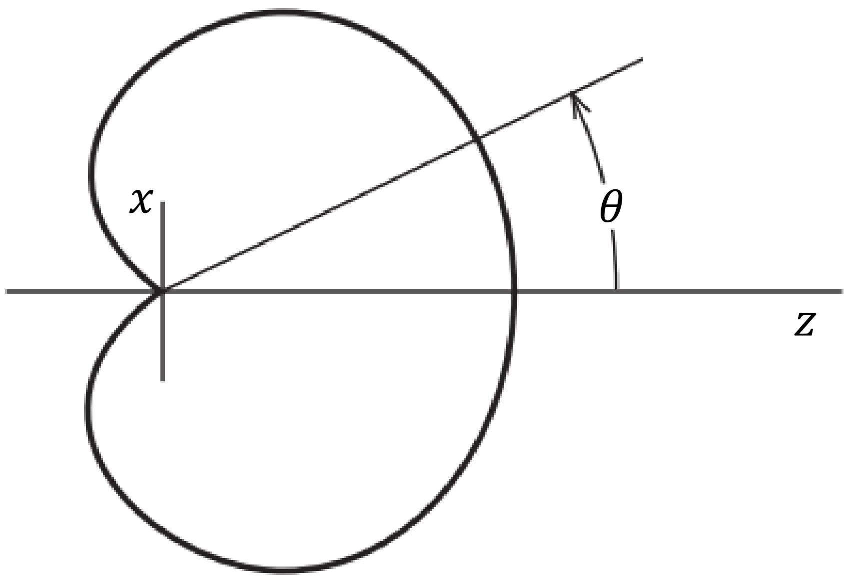
$$
\begin{aligned}
\mathrm{AF} &= 1e^{-j\beta\frac{d}{2}\cos\theta} + 1e^{-j\frac{\pi}{2}}e^{+j\beta\frac{d}{2}\cos\theta} \\
&= e^{-j\frac{\pi}{4}}\left(e^{+j\frac{\pi}{4}}e^{-j\beta\frac{d}{2}\cos\theta} + e^{-j\frac{\pi}{4}}e^{+j\beta\frac{d}{2}\cos\theta}\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{AF} &= 2e^{-j\frac{\pi}{4}}\cos\left(\beta\frac{d}{2}\cos\theta - \frac{\pi}{4}\right) \\
&= 2e^{-j\frac{\pi}{4}}\cos\left(\frac{\pi}{4}\cos\theta - \frac{\pi}{4}\right)
\end{aligned}
$$

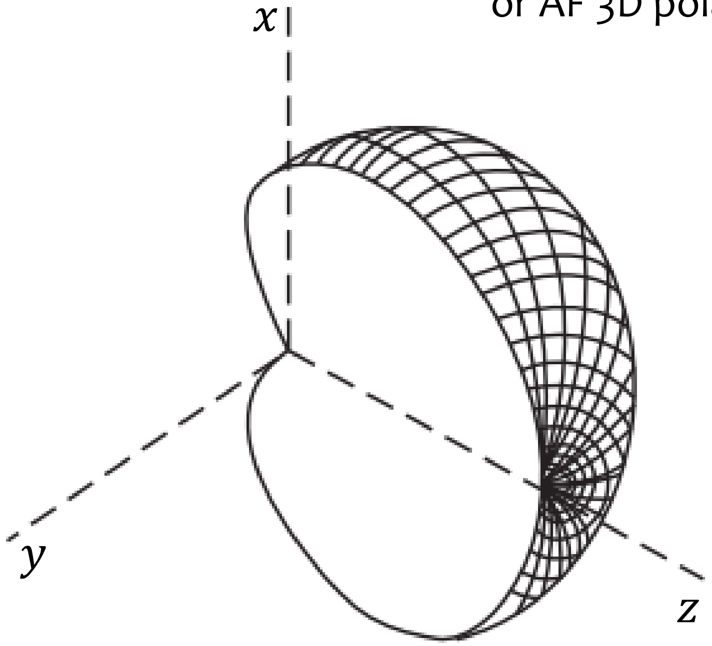
Normalizing the array factor for a maximum value of unity gives the normalized radiation pattern $|F(\theta, \varphi)|$

$$|F(\theta, \varphi)| = \left|\cos\left[\frac{\pi}{4}(\cos\theta - 1)\right]\right|$$

2D radiation pattern
or AF polar plot



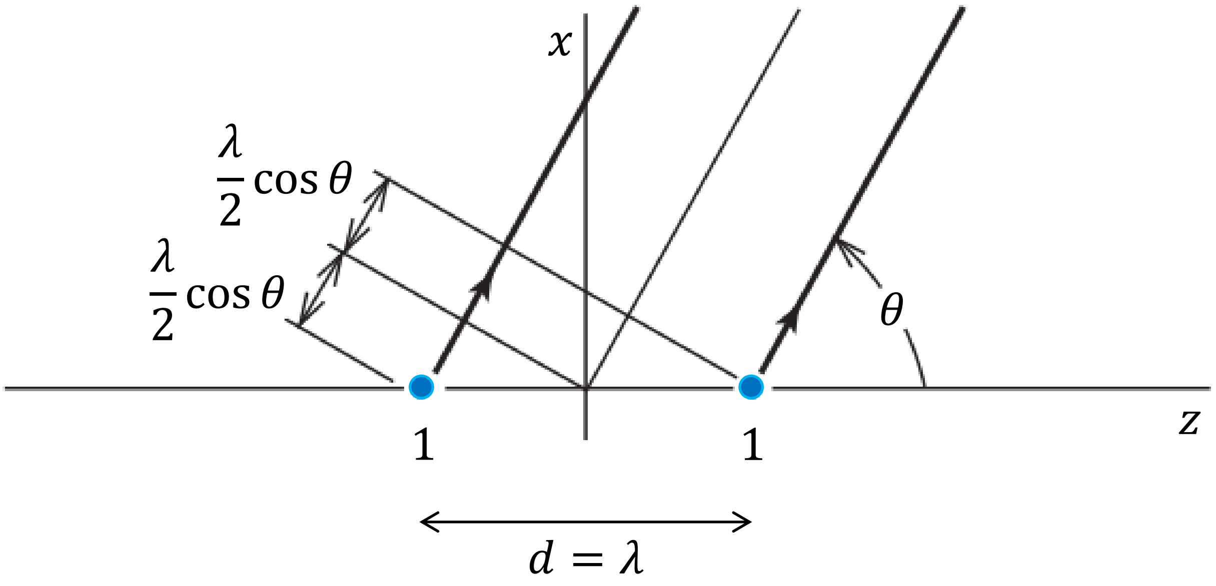
3D radiation pattern
or AF 3D polar plot



Fourth example

We consider two isotropic point sources with identical amplitude and phase (they are in-phase), and spaced one wavelength apart $d = \lambda$.

We can calculate the array factor (AF) by summing the rays in the far-field.

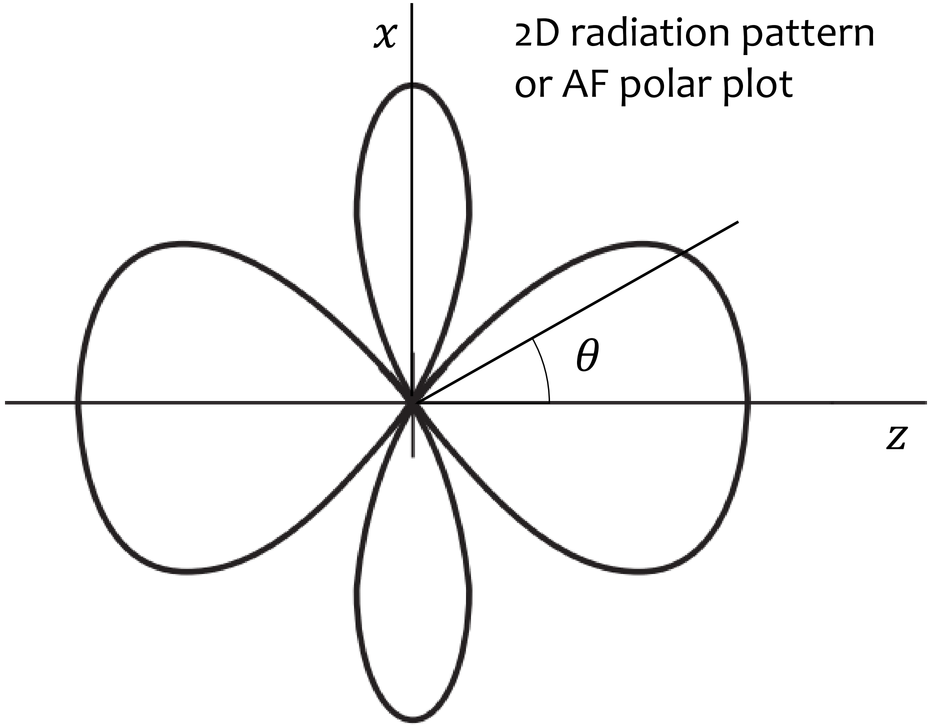


$$\mathrm{AF} = 1e^{-j\beta\frac{d}{2}\cos\theta} + 1e^{+j\beta\frac{d}{2}\cos\theta}$$

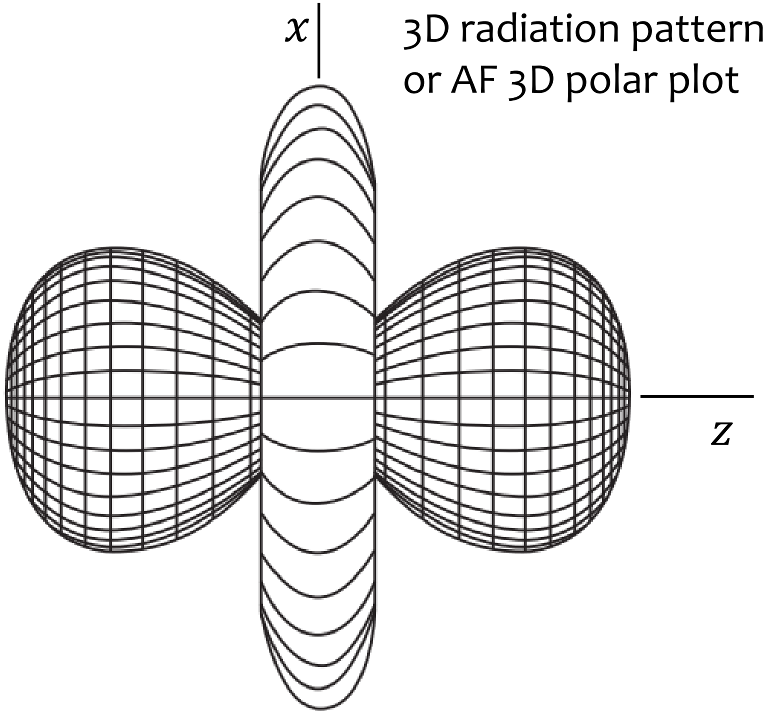
$$\mathrm{AF} = 2\cos\left(\beta\frac{d}{2}\cos\theta\right) = 2\cos(\pi\cos\theta)$$

Normalizing the array factor for a maximum value of unity gives the normalized radiation pattern $|F(\theta, \varphi)|$

$$|F(\theta, \varphi)| = |\cos(\pi \cos \theta)|$$

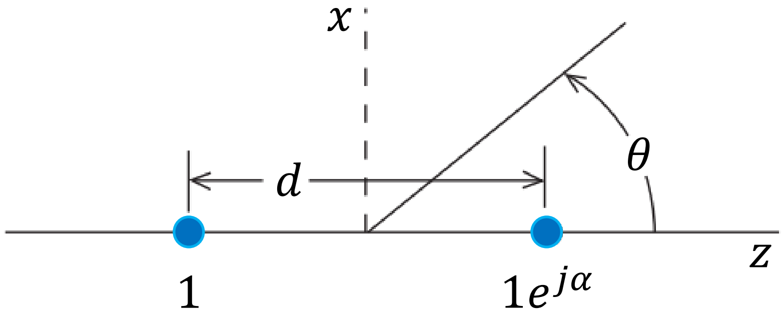


2D radiation pattern
or AF polar plot



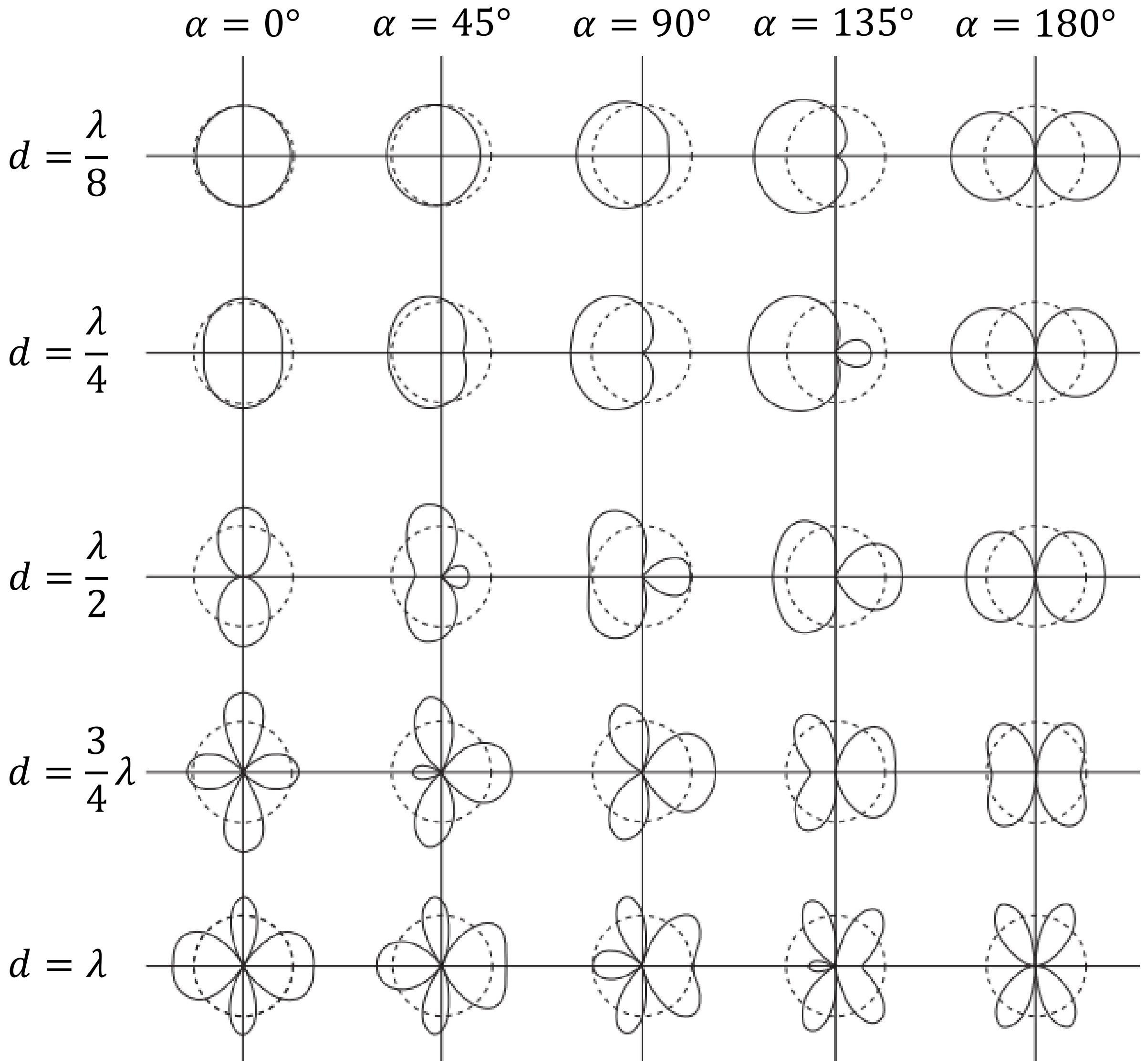
3D radiation pattern
or AF 3D polar plot

The figure on the right shows the polar plots of array factors for various combinations of element spacing d and excitation phase α. The absolute field strength has been preserved to allow comparison; also shown is a unit circle representing the radiation from a hypothetical isotropic point source with the same input current. We emphasize that phase control can significantly change the shape of the array factor.



$$\mathrm{AF} = 2e^{j\frac{\alpha}{2}} \cos\left(\beta \frac{d}{2} \cos\theta + \frac{\alpha}{2}\right)$$

$$F(\theta, \varphi) = \left|\cos\left(\beta \frac{d}{2} \cos\theta + \frac{\alpha}{2}\right)\right|$$



N-ELEMENT LINEAR ARRAYS

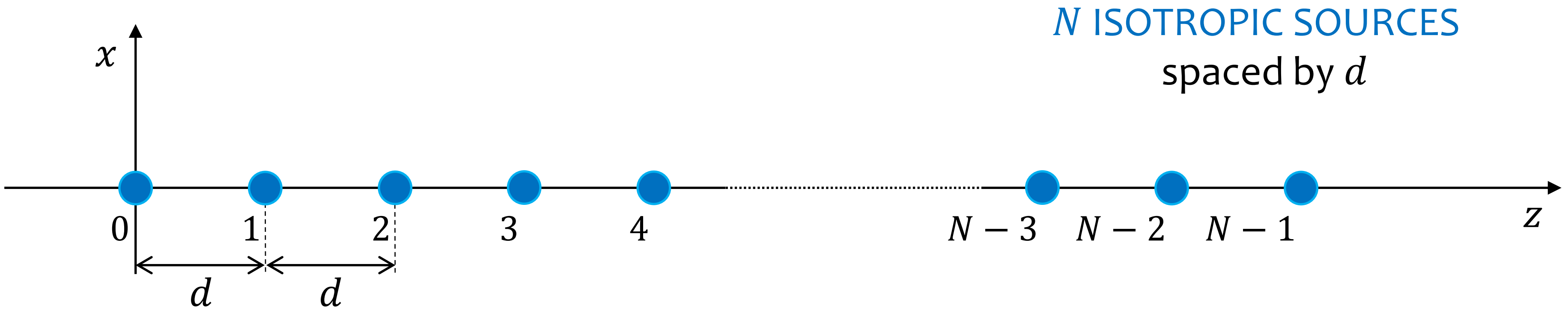
Arrays are popular because of the ability to shape the pattern through spacing and excitation adjustments along with the unique capability of scanning the pattern in angular space by adjusting the phases.

The transverse dimension of arrays can be very large (compared to the wavelength) and array antennas are capable of extremely high directivities (and gains).

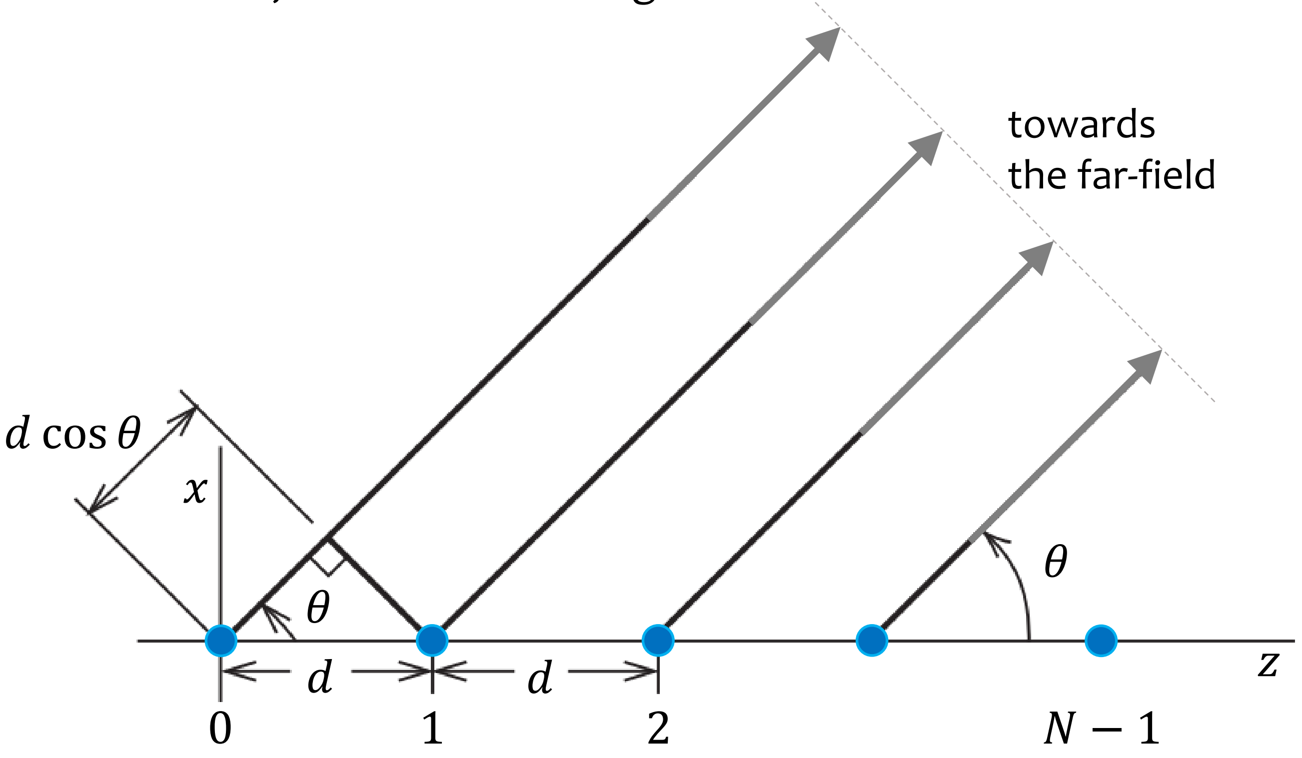
We continue the development of arrays, beginning with linear arrays of multiple equally spaced isotropic elements. We will analyse how the pattern of individual elements is included.

The basic array antenna model consists of two parts: the pattern of one of the elements by itself (the element pattern) and the pattern of the array with the actual elements replaced by isotropic point sources (the array factor). **The total pattern of the array is the product of the element pattern and array factor.**

In a linear array the radiating elements are placed along a line; in our study those elements are identical, equally spaced and fed by currents having the same magnitude.



We consider linear arrays with isotropic elements equally spaced along the z-axis. In order to obtain the array factor we sum the phasors in the far-field, where the electromagnetic field can be locally approximated by a plane wave. The array has N radiators (named $0,1,2,\cdots N-2, N-1$) and the first element is located at the origin $z=0$. The phase of the radiator 0 is set to zero for convenience: the wave emitted by radiator 1 travels a distance of $d\cos\theta$ less than the wave mitted by radiator 0 and thus the spatial phase delay is $-(-\beta d\cos\theta) = \beta d\cos\theta$. In fact, across the array the phase of each element leads its nearest neighbor on the left by the same amount of $\beta d\cos\theta$. We also assume that the relative phase between adjacent transmitting elements is the same α.



$$\mathrm{AF} = \sum_{n=0}^{N-1} a_n e^{j\beta n d\cos\theta} \qquad a_n = A_n e^{jn\alpha}$$

$$\mathrm{AF} = \sum_{n=0}^{N-1} A_n e^{jn(\beta d\cos\theta+\alpha)}$$

The antenna factor can be rearranged as a function of a new variable ψ and it is a Fourier series:

$$\mathrm{AF} = \sum_{n=0}^{N-1} A_n e^{jn\psi} \qquad \psi = \beta d \cos\theta + \alpha = \frac{2\pi}{\lambda} d \cos\theta + \alpha$$

The array factor is periodic in the variable ψ with period 2π.

It is worth observing that the array factor of a linear array along the z-axis is a function of θ but not of φ (although the element pattern may be φ-dependent): it has rotational symmetry around z.

Since in our problem $0 \le \theta \le \pi$ (and $-1 \le \cos\theta \le 1$) we immediately get the following expression for the visible region of ψ

$$-\beta d + \alpha \le \psi \le \beta d + \alpha$$

We emphasize that the ratio of the element spacing to the wavelength d/λ determines how much of the array factor appears in the visible region.

Suppose that exactly one period appears in the visible region: $2\beta d = 2\pi$ which implies $d = \lambda/2$. Exactly one period of the array factor appears in the visible region when the element spacing is one-half wavelength. For spacings larger than a half-wavelength, more than one period will be visible and there may be more than one main lobe in the visible region: additional main lobes are called grating lobes.

A very important special case is that of the uniformly excited array: an array whose elements emit fields with the same magnitudes and this is easily achieved by feeding the elements by using identical current magnitudes

$$A_0 = A_1 = A_2 = \cdots = A_{N-1} \qquad\qquad \mathrm{AF} = A_0 \sum_{n=0}^{N-1} e^{jn(\beta d \cos\theta + \alpha)} = A_0 \sum_{n=0}^{N-1} e^{jn\psi}$$

We realize that the last formula is a geometric series and can be rewritten in a compact form

$$\mathrm{AF} = A_0 \sum_{n=0}^{N-1} e^{jn\psi} = A_0 \frac{1 - e^{jN\psi}}{1 - e^{j\psi}} = A_0 e^{j(N-1)\frac{\psi}{2}} \frac{e^{-jN\frac{\psi}{2}} + e^{jN\frac{\psi}{2}}}{e^{-j\frac{\psi}{2}} + e^{j\frac{\psi}{2}}} = A_0 e^{j(N-1)\frac{\psi}{2}} \frac{\sin\left(N\frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$$

The factor $e^{j(N-1)\frac{\psi}{2}}$ can be neglected since we are interested in the magnitude of the far field, and we could also observe that the maximum of the array factor is $A_0 N$ and it can be convenient to normalize the antenna factor (i.e. we have to divide it by $A_0 N$).

The normalized array factor for an N element, uniformly excited ($a_n = A_0 e^{jn\alpha}$), equally spaced linear array reads as

$$\boxed{\frac{\mathrm{AF}}{N} = \frac{\sin\left(N\frac{\psi}{2}\right)}{N\sin\left(\frac{\psi}{2}\right)}} \qquad \boxed{\psi = \beta d\cos\theta + \alpha} \qquad \boxed{|F(\theta,\varphi)| = \left|\frac{\sin\left(N\frac{\psi}{2}\right)}{N\sin\left(\frac{\psi}{2}\right)}\right|}$$

The magnitude of the normalized array factor is the radiation pattern of the corresponding array of isotropic radiators.

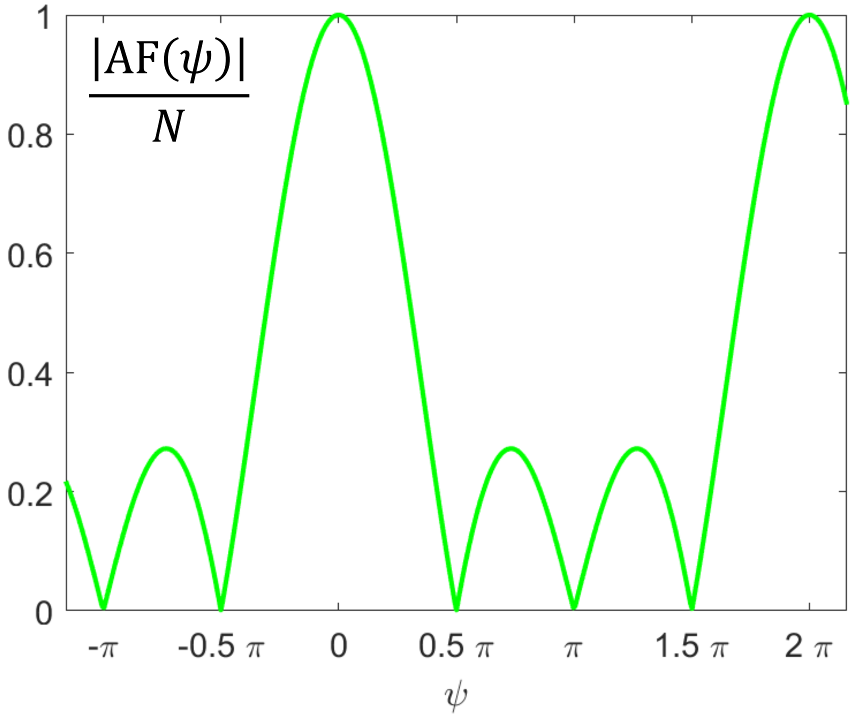
- The array factor magnitude has a period of 2π in the variable ψ.
- The array factor is symmetric around $\psi = 0$ (it is an even function).
- The array factor maximum is N, and this value is achieved for $\psi_{MAX} = 0, \pm 2\pi, \pm 4\pi, \pm 6\pi, \cdots$
- The null directions are $\psi_{NULL} = \pm 2k\,\pi/N$ where k is an integer (with $k \neq N, k \neq 2N, k \neq 3N, \cdots$).
- The main lobes are of width $4\pi/N$ in the variable ψ, whereas for the minor lobes the width is $2\pi/N$.
- The number of lobes in one period equals $N-1$: one main lobe and $N-2$ side lobes.

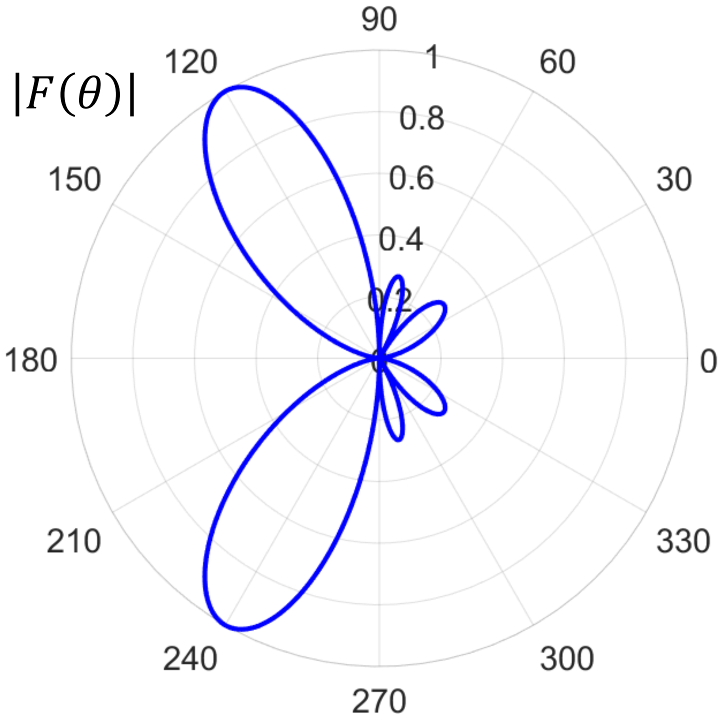
First example

4-element linear array, uniformly excited and equally spaced with the following parameters: spacing $d = \lambda/2$, and interelement phasing $\alpha = \pi/2$.

$$\frac{|\mathrm{AF}|}{N} = |F(\theta,\varphi)| = \left|\frac{\sin\left(4\frac{\psi}{2}\right)}{4\sin\left(\frac{\psi}{2}\right)}\right|$$

$$\psi = \beta d\cos\theta + \alpha = \pi\cos\theta + \frac{\pi}{2}$$

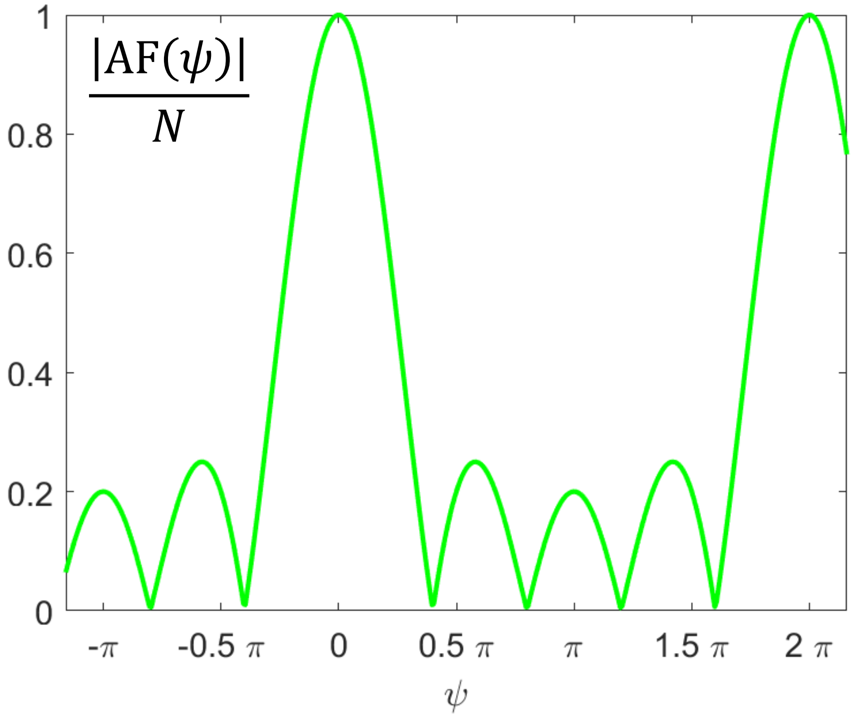


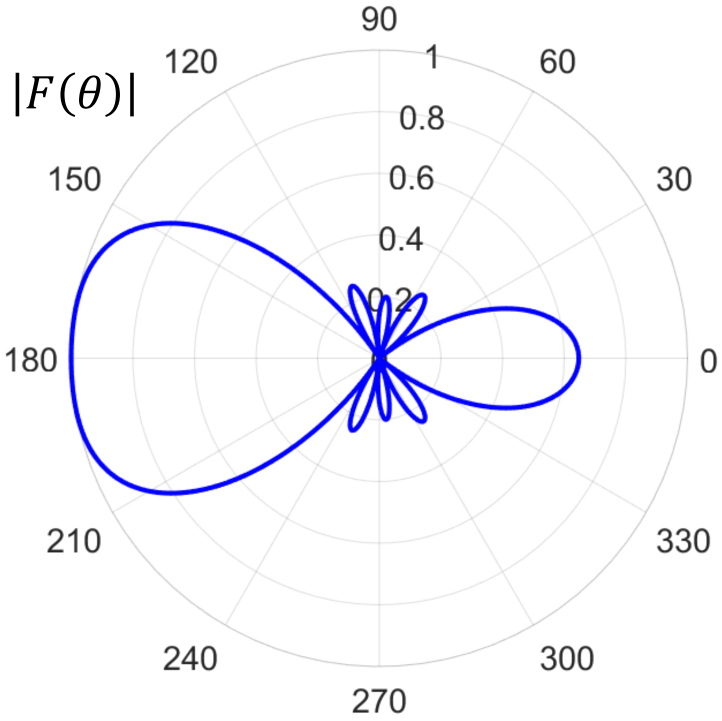


Second example

5-element linear array, uniformly excited and equally spaced with the following parameters: spacing $d = 0.45\lambda$, and interelement phasing $\alpha = 0.9\pi$.

$$\frac{|\mathrm{AF}|}{N} = |F(\theta,\varphi)| = \left|\frac{\sin\left(5\,\frac{\psi}{2}\right)}{5\sin\left(\frac{\psi}{2}\right)}\right| \qquad \psi = \beta d\cos\theta + \alpha = 0.9\,\pi\cos\theta + 0.9\,\pi$$

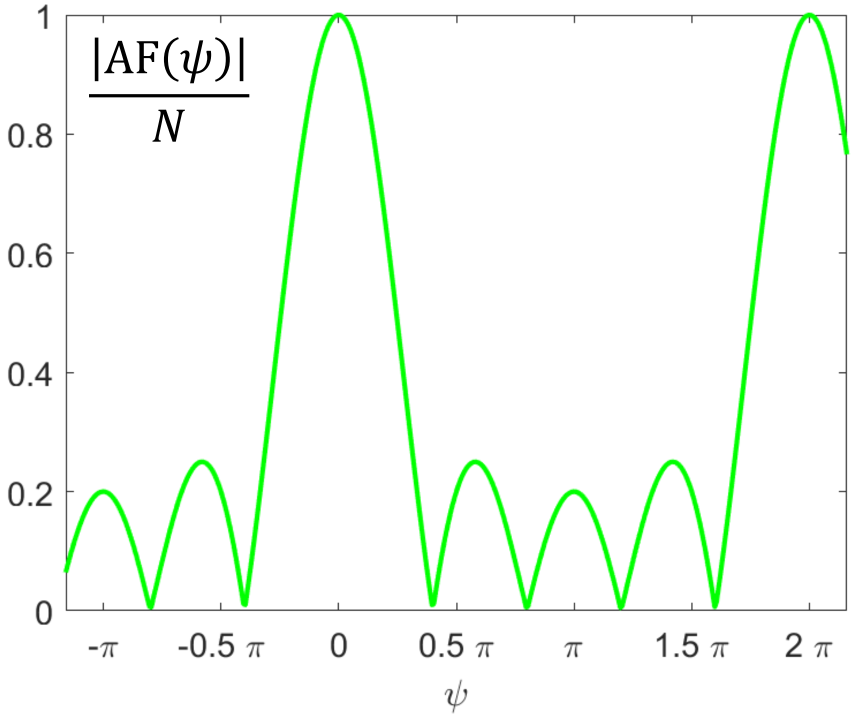


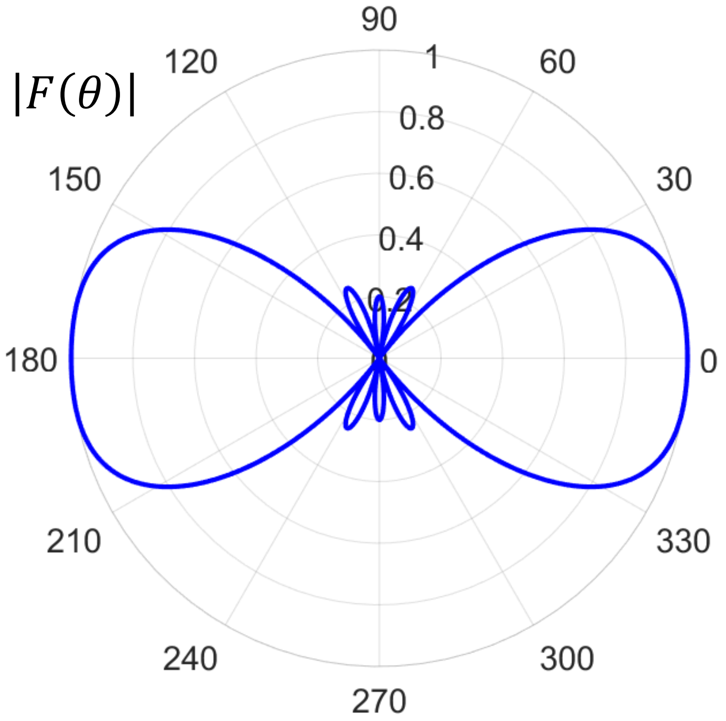


Third example

5-element linear array, uniformly excited and equally spaced with the following parameters: spacing $d = \lambda/2$, and interelement phasing $\alpha = \pi$.

$$\frac{|\mathrm{AF}|}{N} = |F(\theta,\varphi)| = \left|\frac{\sin\left(5\,\frac{\psi}{2}\right)}{5\sin\left(\frac{\psi}{2}\right)}\right| \qquad \psi = \beta d\cos\theta + \alpha = \pi\cos\theta + \pi$$

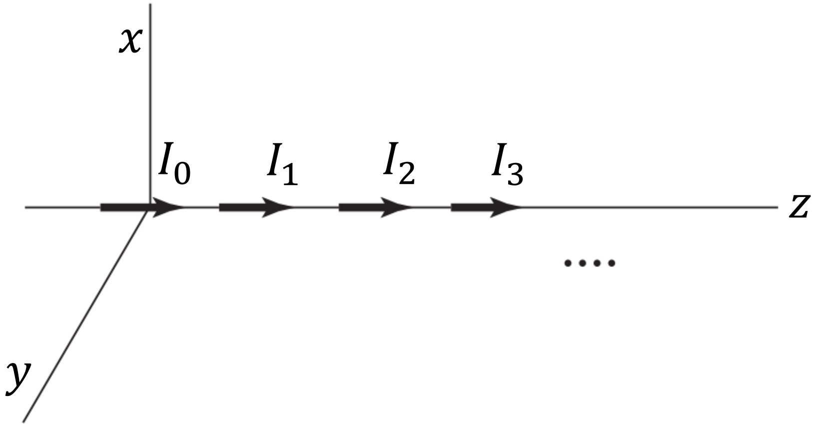




THE COMPLETE ARRAY PATTERN AND PATTERN MULTIPLICATION

So far, we have assumed that all elements in the array are isotropic point sources. Here we learn how to include element effects and explain the pattern multiplication which gives the complete array pattern as the product of the element pattern and the array factor.

When the elements of an array are placed along a line and the currents in each element also flow in the direction of that line, the array is said to be collinear. As an example of a collinear array, suppose we have N z-directed short dipoles, the elements are equally spaced a distance d apart and have currents $I_0, I_1, \cdots I_{N-1}$.



Since the currents are z-directed the magnetic vector potential is z-directed, as well. By summing all the contributions we obtain

$$A_z = \mu \frac{e^{-j\beta r}}{4\pi r} \Delta z \sum_{n=0}^{N-1} I_n e^{j\beta n d \cos\theta}$$

The only component of the electric field in the far-field is

$$E_\theta = j\omega \sin\theta\, A_z = j\omega\mu \frac{e^{-j\beta r}}{4\pi r} \Delta z \sin\theta \sum_{n=0}^{N-1} I_n e^{j\beta n d \cos\theta}$$

The antenna factor AF is easily identified in the previous expression

$$\mathrm{AF} = \sum_{n=0}^{N-1} I_n e^{j\beta n d \cos\theta}$$

and the function $\sin\theta$ accounts for the pattern of each element of the array: this factoring process holds in general whenever the elements have the same pattern and are similarly oriented (and we will consider only this case).

Let's consider a slightly more complicated case: we have N identical line-source elements forming a collinear array along the z-axis. The nth element is centered at z_n and has a current distribution i_n; moreover, we assume that each element has the same normalized current distribution over its length L: $i_n(z') = I_n i(z' - z_n)$, for $z_n - L/2 \leq z' \leq z_n + L/2$. The vector potential is then

$$A_z = \mu \frac{e^{-j\beta r}}{4\pi r} \sum_{n=0}^{N-1} \int_{z_n - L/2}^{z_n + L/2} I_n i(z' - z_n) e^{j\beta z' \cos\theta} dz'$$

The far-field electric field is

$$E_\theta = j\omega\mu \frac{e^{-j\beta r}}{4\pi r} \sum_{n=0}^{N-1} T_n(\theta) \qquad T_n(\theta) = \sin\theta \int_{z_n - L/2}^{z_n + L/2} I_n i(z' - z_n) e^{j\beta z' \cos\theta} dz'$$

and the integral can be rearranged by changing the integration variable $\tau = z' - z_n$, $d\tau = dz'$

$$T_n(\theta) = \sin\theta \left[\int_{-L/2}^{+L/2} I_n i(\tau) e^{j\beta\tau' \cos\theta} d\tau' \right] I_n e^{j\beta z_n \cos\theta}$$

And finally the total electric field is

$$E_\theta = j\omega\mu \frac{e^{-j\beta r}}{4\pi r} \sum_{n=0}^{N-1} T_n(\theta) = j\omega\mu \frac{e^{-j\beta r}}{4\pi r} \sin\theta \left[\int_{-L/2}^{+L/2} I_n i(\tau) e^{j\beta\tau' \cos\theta} d\tau' \right] \sum_{n=0}^{N-1} I_n e^{j\beta z_n \cos\theta}$$

The following factor

$$f(\theta) = \sin\theta \left[\int_{-L/2}^{+L/2} I_n i(\tau) e^{j\beta\tau' \cos\theta} d\tau' \right]$$

when normalized is the element radiation pattern and the following sum is the antenna factor

$$\mathrm{AF} = \sum_{n=0}^{N-1} I_n e^{j\beta z_n \cos\theta}$$

We observed that if the elements are equally spaced $z_n = nd$ and uniformly excited $I_n = I_0 e^{jn\alpha}$

$$AF = \sum_{n=0}^{N-1} I_n e^{j\beta z_n \cos\theta} = I_0 \sum_{n=0}^{N-1} e^{j\beta nd\cos\theta + jn\alpha} = I_0 e^{j(N-1)\frac{\beta d\cos\theta+\alpha}{2}} \frac{\sin\left(N\frac{\beta d\cos\theta+\alpha}{2}\right)}{\sin\left(\frac{\beta d\cos\theta+\alpha}{2}\right)}$$

The processing of factoring the pattern of an array into an element pattern and an array factor is referred to as the principle of pattern multiplication. It is stated as follows: **the field pattern of an array consisting of similar elements is the product of the pattern of one of the elements (the element pattern) and the pattern of an array of isotropic point sources with the same locations, relative amplitudes, and phases as the original array (the array factor).**

$$|F_{ARRAY}(\theta,\varphi)| = |F_{ELEMENT}(\theta,\varphi)| \frac{|AF(\theta)|}{|AF(\theta)|_{MAX}}$$

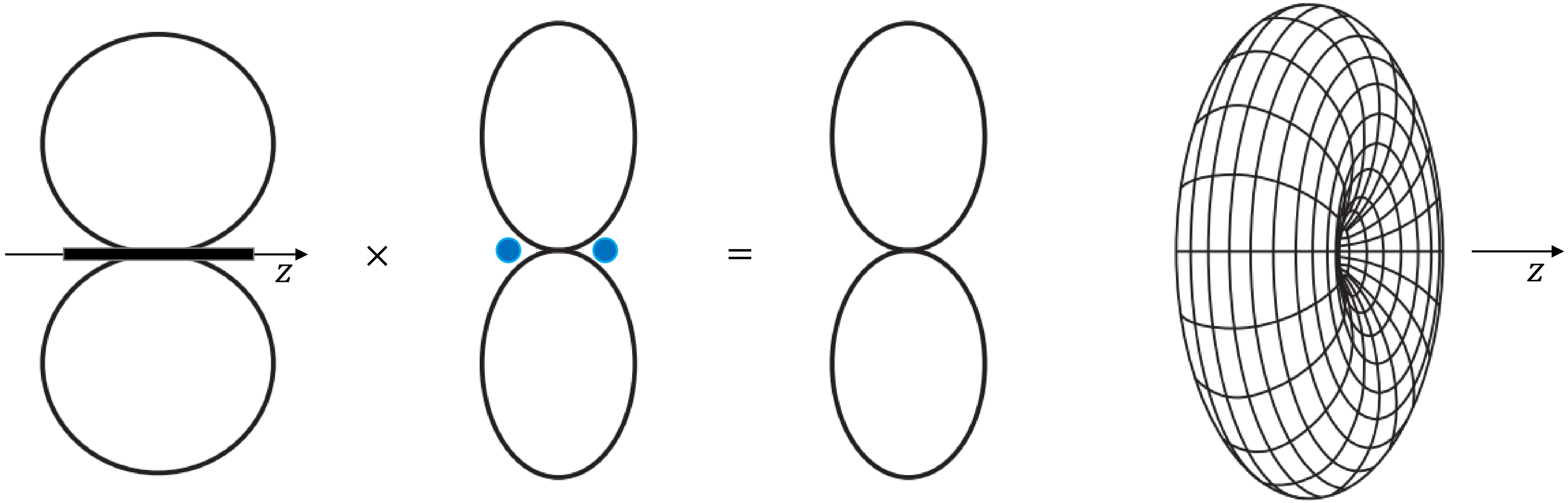
and if the elements are equally spaced and uniformly excited

$$\boxed{|F_{ARRAY}(\theta,\varphi)| = |F_{ELEMENT}(\theta,\varphi)| \frac{\left|\sin\left(N\frac{\beta d\cos\theta+\alpha}{2}\right)\right|}{\left|N\sin\left(\frac{\beta d\cos\theta+\alpha}{2}\right)\right|}}$$

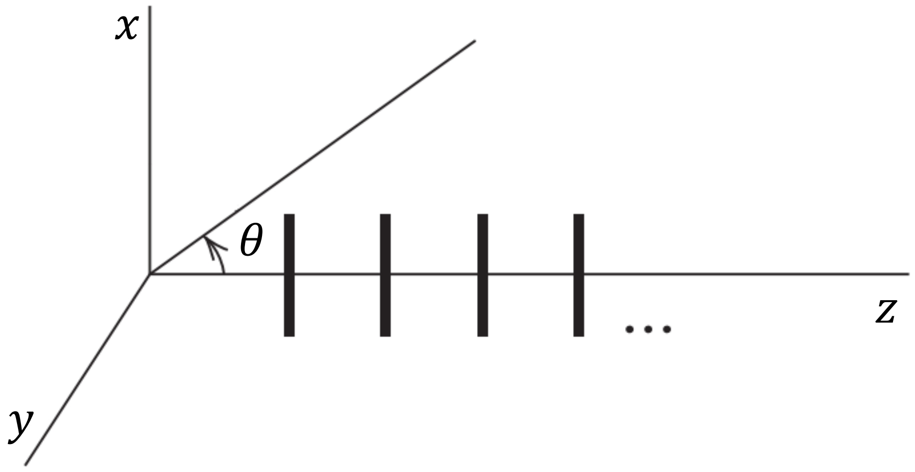
First example

To illustrate pattern multiplication, consider two collinear short dipoles spaced a half-wavelength apart and equally excited with $\alpha = 0$. The element pattern is $\sin\theta$ for an element along the z-axis and the array factor was found to be $\mathrm{AF} = \cos[(\pi/2)\cos\theta]$, the total pattern is then

$$|F_{ARRAY}(\theta,\varphi)| = \sin\theta\cos[(\pi/2)\cos\theta]$$



Array that have parallel elements, as in the following figure, have more complicated pattern expressions because the axis of symmetry of the array (z-axis) is no longer aligned with the axis of symmetry of the elements (x-axis), as for collinear array. So the pattern will be a function of both θ and φ rather than just θ.

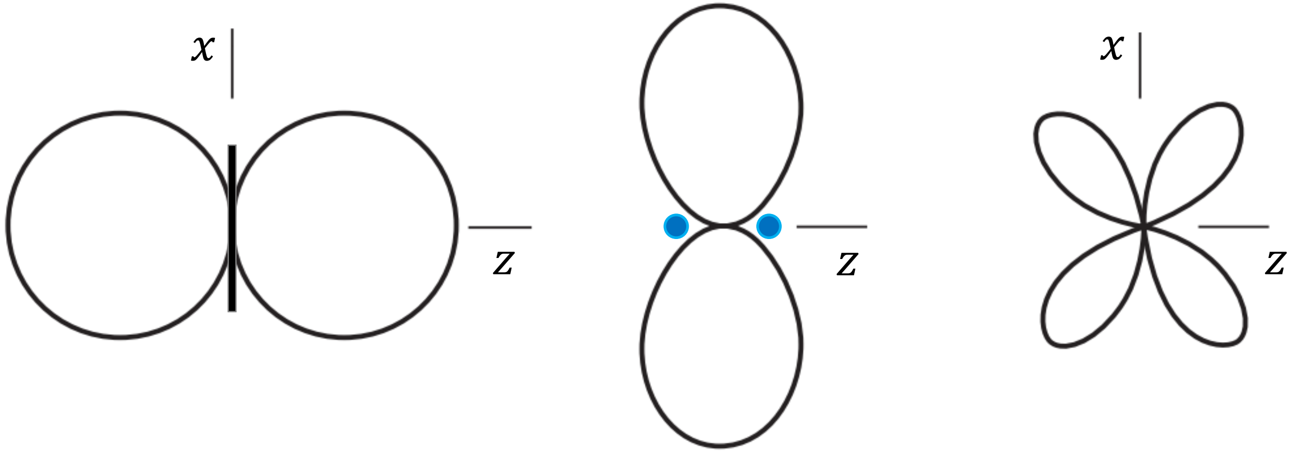


In order to simplify the problem, we can study the radiation pattern only in the principal planes.

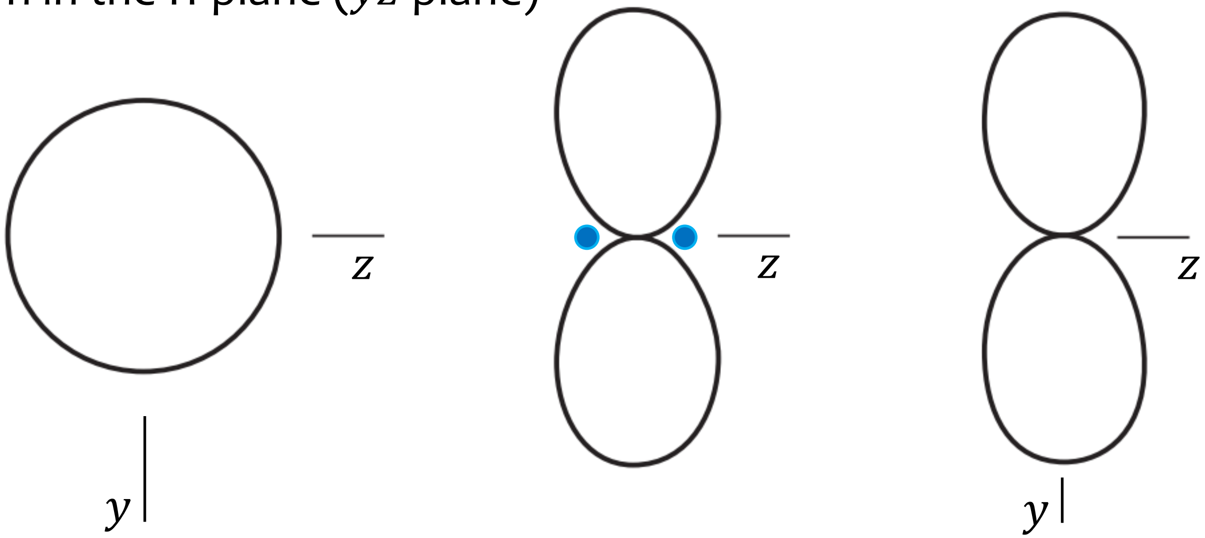
Second example

We study the radiation pattern of an array of two parallel short dipoles which are perpendicular to the alignment direction z; the spacing is $d = \lambda/2$, and the dipoles are fed by in-phase currents, $I_0 = I_1, \alpha = 0$

2D pattern in the E-plane (xz plane)



2D pattern in the H-plane (yz plane)



3D pattern