EMIMEO : E(rasmus) Mundus on Innovative Microwave Electronics and Optics Master

Foundations of Electromagnetic Wave Propagation – 2nd part

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Chapters:

Main goal : to study the propagation lines « TEM lines »

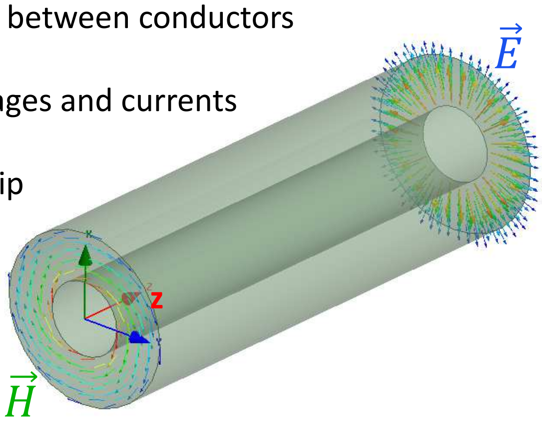
TEM lines (Transverse Electric and Magnetic):

- Electric and magnetic fields : contained in the plane which is perpendicular to the conductor (no longitudinal component of the field, $H_z = 0$, $E_z = 0$)

- ……………………………………………………………:
  depends on the dimensions and of the dielectric between conductors

- Allows to determine along the conductors, voltages and currents

- applications : coaxial lines, bifilar lines, microstrip or coplanar line and stripline



Modelization method [1]:

- Application of the "voltage-current" concept

- TEM line assimilated to a circuit of elements in networks

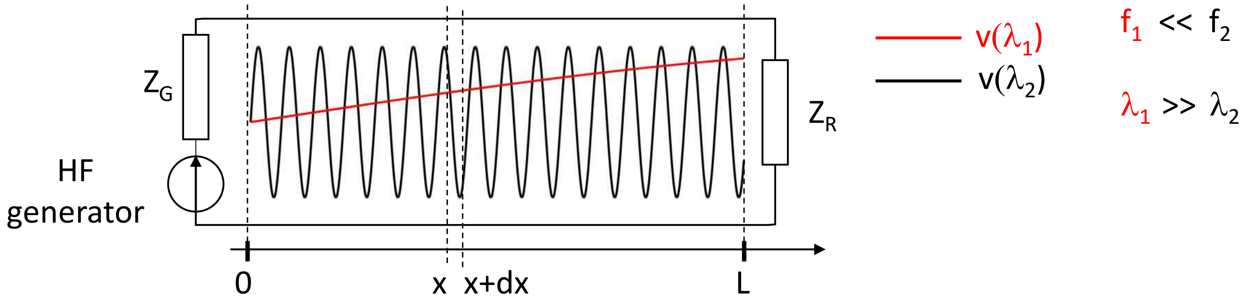
- length L
- powered by a HF generator with internal impedance Z_G at one end
- Closed by a termination (load impedance) Z_R at the other end

Transmission line schematic:

[1] David M. Pozar, Microwave Engineering, Third Edition, John Wiley & Sons Inc.

Modelization method:

Transmission line diagram:



in HF, L >> λ (wavelength λ =c/f)

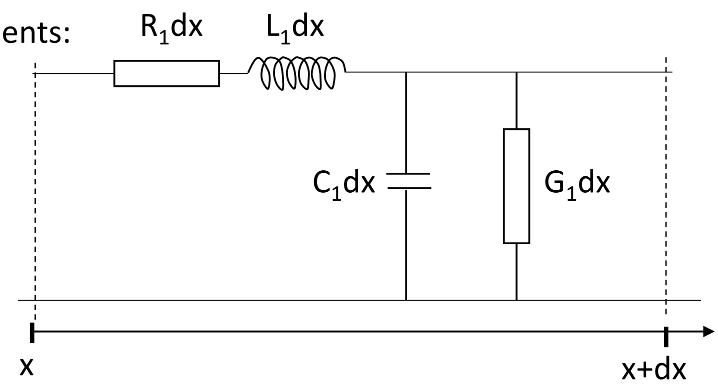
- ……

- LF case (L << λ) is different: quasi-stationary state approximation is applied

Modelization method:

- TEM line assimilated to a circuit of elements in networks: ………………………………

Decomposition of the line in identical elements:



- of a length dx << λ

- quadripoles with localized constants: R_1, L_1, G_1, C_1 ……………………………… of the line

- allows to model the voltage-current wave variations in space and time along the line

Modelization method:

- TEM line assimilated to a circuit of elements in networks: R_1, L_1 ,G_1, C_1

R_1 : ………………………………………………………………………………… (Ω/m)

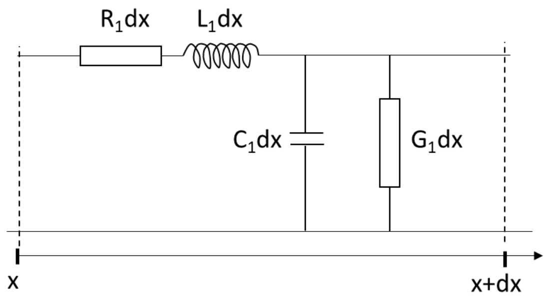
L_1 : …………………………………………………………………………………(H/m)

C_1 :…………………………………………………………………………………

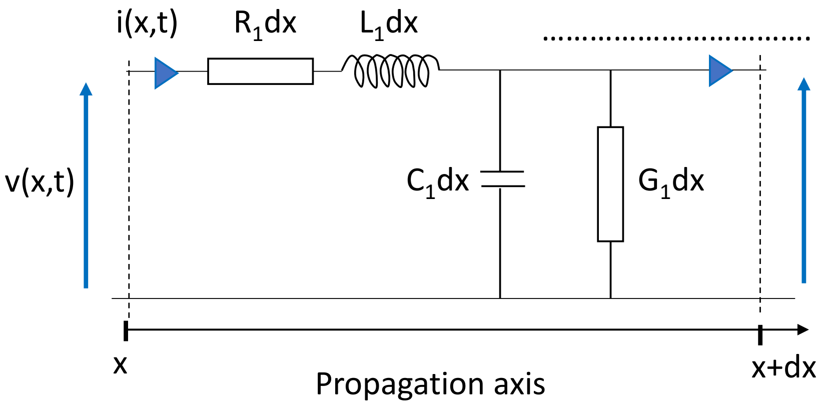
………………………………. (F/m)

G_1 : …………………………………………………………………………………

…………………… (S/m)



General case : lines with losses
- study of the element included between x and x+dx



Propagation effect neglected dt = 0 for dx << λ

………………………………

From Kirchoff's laws (law of meshes and knots), we obtain [1]:

$$-L_1C_1\frac{\partial^2 v}{\partial t^2}+\frac{\partial^2 v}{\partial x^2}=(R_1C_1+G_1L_1)\frac{\partial v}{\partial t}+R_1C_1v \qquad -L_1C_1\frac{\partial^2 i}{\partial t^2}+\frac{\partial^2 i}{\partial x^2}=(R_1C_1+G_1L_1)\frac{\partial i}{\partial t}+R_1C_1i$$

Telegraphers' equations

2. Transmission line

7. Theory of lines - model

In the sinusoidal regime, we use the complex notation to :

- dissociate the temporal contribution from the geometrical one
- Solve the propagation equations

$$\mathrm{v}(x,t) = R_e\left(\underline{v}(x,t)\right) = \mathrm{V}(x)cos\left(\omega t + \varphi_v(x)\right)$$

$$\mathrm{i}(x,t) = R_e\left(\underline{i}(x,t)\right) = \mathrm{I}(x)cos\left(\omega t + \varphi_i(x)\right)$$

with $\underline{v}(x,t) = \underline{V}(x)e^{j\omega t}$ and ……………………………

$\underline{i}(x,t) = \underline{I}(x)e^{j\omega t}$ and ……………………………

$V(x)$ and $I(x)$ which are the real magnitudes (modulus) of v and i

$\varphi_v(x)$ and $\varphi_i(x)$ which are the phases of v and i

2. Transmission line

8. Theory of lines – voltage and current propagation equations

$$-\mathrm{L}_1\mathrm{C}_1\frac{\partial^2 \mathrm{v}}{\partial \mathrm{t}^2} + \frac{\partial^2 \mathrm{v}}{\partial \mathrm{x}^2} = (\mathrm{R}_1\mathrm{C}_1 + \mathrm{G}_1\mathrm{L}_1)\frac{\partial \mathrm{v}}{\partial \mathrm{t}} + \mathrm{R}_1\mathrm{C}_1\mathrm{v}$$

$$-\mathrm{L}_1\mathrm{C}_1\frac{\partial^2 \mathrm{i}}{\partial \mathrm{t}^2} + \frac{\partial^2 \mathrm{i}}{\partial \mathrm{x}^2} = (\mathrm{R}_1\mathrm{C}_1 + \mathrm{G}_1\mathrm{L}_1)\frac{\partial \mathrm{i}}{\partial \mathrm{t}} + \mathrm{R}_1\mathrm{C}_1\mathrm{i}$$

The equations of the telegraphers in sinusoidal regime become:

$$\frac{\partial^2 \underline{V}(\mathrm{x})}{\partial \mathrm{x}^2} = (\mathrm{R}_1 + j\mathrm{L}_1\omega)(\mathrm{G}_1 + j\mathrm{C}_1\omega)\underline{V}(x) \text{ and } \frac{\partial^2 \underline{I}(\mathrm{x})}{\partial \mathrm{x}^2} = (\mathrm{R}_1 + j\mathrm{L}_1\omega)(\mathrm{G}_1 + j\mathrm{C}_1\omega)\underline{I}(x)$$

we set that ……………………………………

The equations become: …………………………………… and ……………………………………

Equations for voltage and current propagation along the line

Equations for voltage and current propagation along the line

$$\frac{\partial^2 \underline{V}(\mathrm{x})}{\partial \mathrm{x}^2} = \gamma^2 \underline{V}(x) \quad \text{and} \quad \frac{\partial^2 \underline{I}(\mathrm{x})}{\partial \mathrm{x}^2} = \gamma^2 \underline{I}(x)$$

Linear propagation constant, ……………………………………. of the line

$$\boxed{\gamma = \sqrt{(\mathrm{R}_1 + j\mathrm{L}_1\omega)(\mathrm{G}_1 + j\mathrm{C}_1\omega)}}$$

The solutions of the two propagation equations are:

……………………………………………………… and ………………………………………………………

The current can also be expressed from the voltage:

$$\boxed{\underline{I}(\mathrm{x}) = \frac{\underline{\underline{V_i}}}{Z_c} e^{-\gamma x} - \frac{\underline{\underline{V_r}}}{Z_c} e^{\gamma x}}$$

We then deduce the following relations:

$$\frac{\underline{\underline{V_i}}}{\underline{I_i}} = -\frac{\underline{\underline{V_r}}}{\underline{V_r}} = Z_c$$

with ………………………………. where Z_c is the characteristic impedance of the 2nd …………………………………… of the line

Evidence of the superposition of two waves:

we set:

$$\gamma = \sqrt{(R_1 + j\omega L_1)(G_1 + jC_1\omega)} = \alpha + j\beta$$

α : linear attenuation coefficient
β : linear coefficient of phase shift
positive constants (physically)

Let's write again v(x,t) :

$$\underline{v}(x,t) = \underline{V}(x)e^{j\omega t}$$

let's have: $$\underline{v}(x,t) = \underline{V}_i e^{-\gamma x} e^{j\omega t} + \underline{V}_r e^{+\gamma x} e^{j\omega t}$$

Evidence of the superposition of two waves:

Taking into account the linear coefficients α and β , we obtain:

α is positive or null and x > 0

$$\underline{v}(x,t) = \underbrace{V_i e^{-\alpha x} e^{j(\text{-}\beta x+\omega t+\varphi_i)}}_{} + \underbrace{V_r e^{\alpha x} e^{j(\beta x+\omega t+\varphi_r)}}_{}$$

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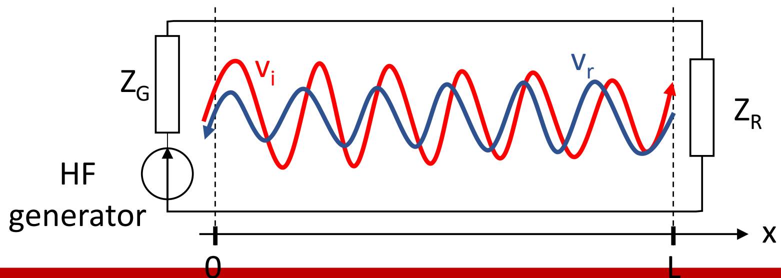


Wave characteristics:

Consider the incident wave: $\underline{v}_i(x,t) = V_i e^{-\alpha x} e^{j(\omega t - \beta x + \varphi_i)}$

In real instantaneous real expression $v_i(x,t) = V_i e^{-\alpha x} \cos(\omega t - \beta x + \varphi_i)$

Note the double space-time periodicity

At a point on the line, the voltage is a sinusoidal function of periodicity in time:

$$T = \frac{2\pi}{\omega}$$

x fixed,
TEMPORAL PERIOD

At a given time, the voltage is a sinusoidal function of x of periodicity in space:

$$\lambda = \frac{2\pi}{\beta}$$

t fixed,
SPATIAL PERIOD

Wave characteristics:

Phase velocity is defined as the speed at which the wave moves at constant phase:

Given the phase: $\Phi = \omega t - \beta x + \varphi_i$

Its total differential is written (for constant phase) $d\Phi = \omega dt - \beta dx = 0$

then : $\omega dt = \beta dx$ therefore: $v_\varphi = \frac{\omega}{\beta} = \frac{dx}{dt}$ PHASE VELOCITY

Remark :

- the study of the reflected wave leads to the same results
- V_i and V_r are called AMORTED PROGRESSIVE WAVES
- ………………………………………………………………………………………………………
………………………………………………………………………

Paramètre de propagation

$$\gamma = \alpha + j\beta$$

α ....

β ....

➢ Particular case : line without loss (LWL), then $R_1 = 0$ and $G_1 = 0$

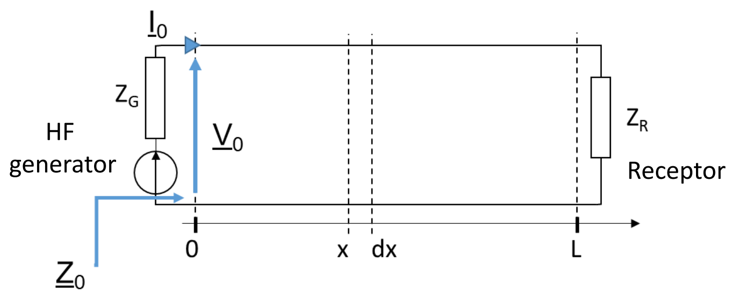
$$\gamma = \sqrt{(j\omega L_1)(jC_1\omega)} = j\omega\sqrt{L_1C_1} = \alpha + j\beta$$

so : $\beta = \omega\sqrt{L_1C_1}$ $\alpha = 0$ **Wave propagation without attenuation**

and : $v = \dfrac{1}{\sqrt{L_1C_1}}$ ....

Voltage and current along the line (with losses)

from the previous equations



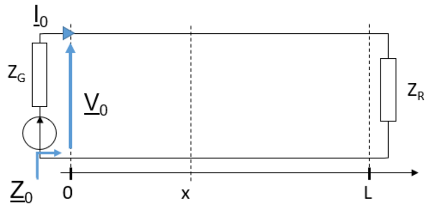
$$\underline{V}(x) = \underline{V}_0 \operatorname{ch}\gamma x - \underline{Z}_C \underline{I}_0 \operatorname{sh}\gamma x$$

$$\underline{I}(x) = -\frac{\underline{V}_0}{\underline{Z}_C} \operatorname{sh}\gamma x + \underline{I}_0 \operatorname{ch}\gamma x$$

$$\underline{Z}(x) = \frac{\underline{V}(x)}{\underline{I}(x)} = \underline{Z}_C \frac{\underline{Z}_0 - \underline{Z}_C \operatorname{th}\gamma x}{\underline{Z}_C - \underline{Z}_0 \operatorname{th}\gamma x}$$

2. Transmission line — 17. Theory of lines - voltage current and impedance along a line

Particular case: line without losses (LWL), $\alpha = 0$ and $\gamma = j\beta$



$$\underline{V}(x) = \underline{V}_0 \cos\beta x - j\underline{Z}_C \underline{I}_0 \sin\beta x$$

$$\underline{I}(x) = -j\frac{\underline{V}_0}{\underline{Z}_C}\sin\beta x + \underline{I}_0 \cos\beta x$$

$$\underline{Z}(x) = \underline{Z}_C \frac{\underline{Z}_0 - j\underline{Z}_C \mathrm{tg}\beta x}{\underline{Z}_C - j\underline{Z}_0 \mathrm{tg}\beta x}$$

with:

$$\mathrm{ch}\gamma x = \mathrm{ch} j\beta x = \cos\beta x$$
$$\mathrm{sh}\gamma x = \mathrm{sh} j\beta x = j\sin\beta x$$
$$\mathrm{th}\gamma x = \mathrm{th} j\beta x = j\mathrm{tg}\beta x$$

2. Transmission line — 18. Theory of lines - voltage current and impedance along a line

Input impedance (x=0) dof the line as a function of the load impedance $Z(x=L)=Z_R$

from :
$$\underline{Z}_R = \underline{Z}(x = L) = \underline{Z}_C \frac{\underline{Z}_0 - \underline{Z}_C \mathrm{th}\gamma L}{\underline{Z}_C - \underline{Z}_0 \mathrm{th}\gamma L}$$

We deduce the input impedance of the line:

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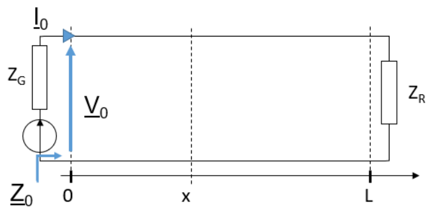

Line terminated by $\underline{Z}_R = Z_C$

we write: $$\underline{Z}_0 = \underline{Z}_C \frac{\underline{Z}_C + \underline{Z}_C \text{th}\gamma L}{\underline{Z}_C + \underline{Z}_C \text{th}\gamma L}$$ then: $$\underline{Z}_0 = \underline{Z}_C$$

Everything happens as if the generator was directly closed on Z_C

Using the expression for $\underline{Z}(x)$, on obtient : $$\underline{Z}(x) = \underline{Z}_C$$

At any point on the line, the impedance is equal to $\underline{Z}_C$

Line terminated by $\underline{Z}_R = Z_C$

with : $$\underline{V}(x) = \underline{V}_0 e^{-\gamma x}$$ $$\underline{I}(x) = \underline{I}_0 e^{-\gamma x}$$

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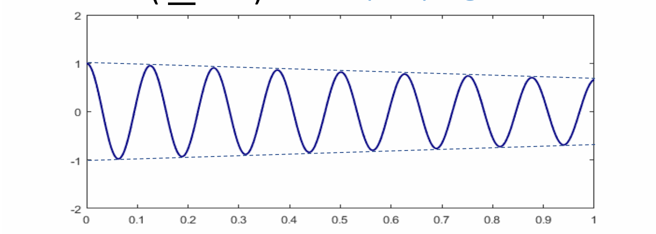
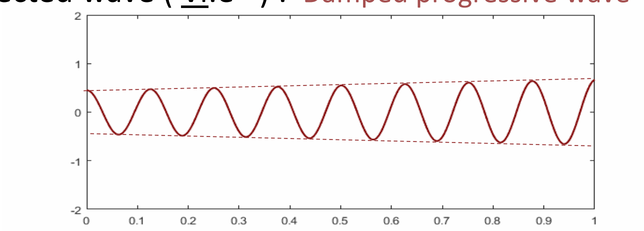
General case

Incident wave ($\underline{V}i.e^{-\gamma x}$) : Damped progressive wave

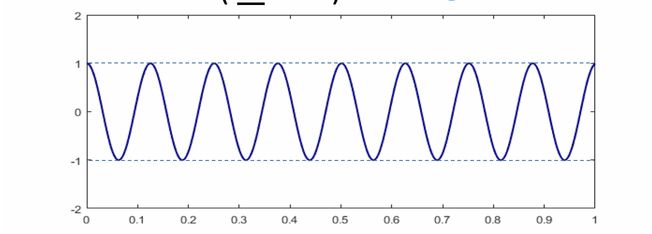


Reflected wave ($\underline{V}r.e^{\gamma x}$) : Damped progressive wave

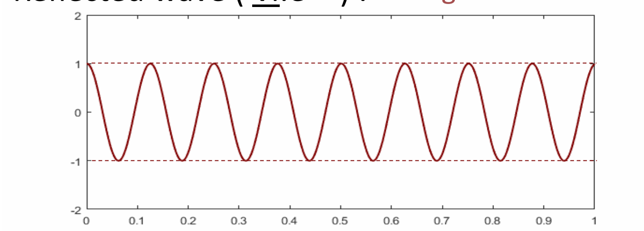


Case of the line without losses

Incident wave ($\underline{V}i.e^{-j\beta x}$) : Progressive wave



Reflected wave ($\underline{V}r.e^{j\beta x}$) : Progressive wave



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