PROBLEM P1

THE POWER DENSITY RADIATED BY AN ANTENNA IS GIVEN BY

$$P(r,\theta,\varphi) \begin{cases} \frac{1}{r^2} P_0 \cos^M(\theta) & 0 \le \theta \le \pi/2 \quad \text{AND } M \text{ IS AN INTEGER} \\ 0 & \pi/2 < \theta \le \pi \end{cases}$$

CALCULATE THE ANTENNA DIRECTIVITY

SOLUTION

THE DIRECTIVITY IS $D = \frac{U_m}{U_{AVE}} = \frac{\frac{P}{\Omega_A}}{\frac{P}{4\pi}} = \frac{4\pi}{\Omega_A}$

AND THE BEAM SOLID ANGLE READS AS $\Omega_A = \iint_{4\pi} |F(\theta,\varphi)|^2 \, d\Omega$

SINCE FROM THE TEXT OF THE EXERCISE WE KNOW HOW THE POWER DENSITY IS RADIATED IT IS CONVENIENT TO CALCULATE THE DIRECTIVITY FROM THE RADIATION PATTERN $F(\theta,\varphi)$

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} |F(\theta,\varphi)|^2 \sin\theta \, d\theta \, d\varphi}$$

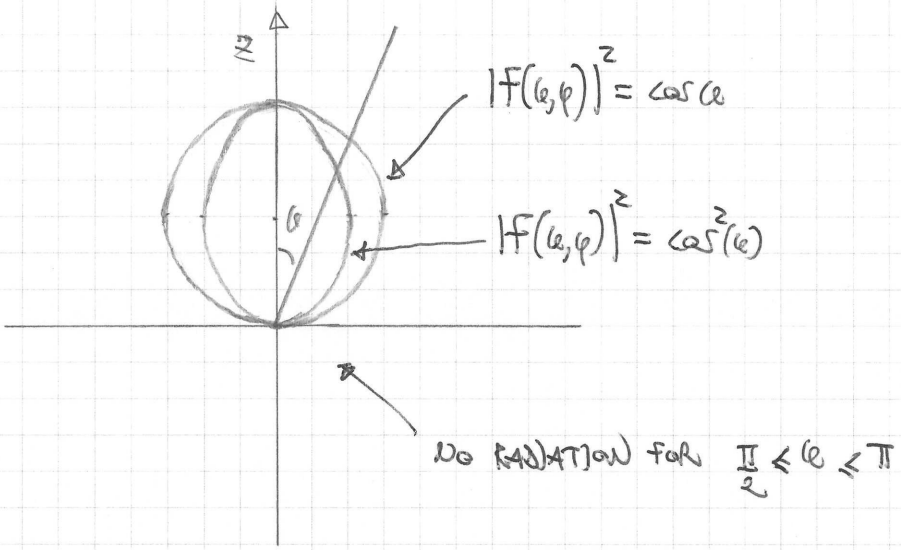
THE RADIATION PATTERN F IS NORMALIZED TO UNITY

$$F(\theta,\varphi) = \sqrt{\frac{P(r,\theta,\varphi)}{MAX\{P(r,\theta,\varphi)\}}} \Rightarrow |F(\theta,\varphi)|^2 = \frac{P(r,\theta,\varphi)}{MAX\{P(r,\theta,\varphi)\}}$$

SINCE $\max\{\cos^n(\theta)\} = 1$ WE OBTAIN THAT $\max\{P(r,\theta,\varphi)\} = \frac{1}{r^2} P_0$

THE NORMALIZED POWER PATTERN IS $|F(\theta,\varphi)|^2 = \cos^n(\theta)$ AND WE OBSERVE THAT $F(\theta,\varphi)$ IS INDEPENDENT OF φ

IT IS ALWAYS USEFUL AND ADVISABLE TO STUDY $|F|$ OR $|F|^2$ AND TO PLOT THE RADIATION PATTERN (EVEN IF THE TEXT OF THE PROBLEM DOES NOT EXPLICITLY ASK FOR IT)



THE PATTERN EXHIBITS AXIAL SYMMETRY AROUND THE z-AXIS

$$D = \frac{4\pi}{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \cos^n(\theta)\sin\theta\, d\theta\, d\varphi} = \frac{4\pi}{\int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} \cos^n(\theta)\sin\theta\, d\theta}$$

$$D = \frac{4\pi}{2\pi\int_0^{\frac{\pi}{2}} \cos^n(\theta)\sin\theta\, d\theta}$$

IN ORDER TO CALCULATE THE INTEGRAL WE CHANGE THE INTEGRATION VARIABLE

$$\cos\theta = x$$

$$d(\cos\theta) = -\sin\theta\, d\theta = dx$$

$$D = \frac{2}{-\int_1^0 x^n dx}$$

AND BY REVERSING THE LIMITS OF INTEGRATION

$$D = \frac{2}{\int_0^1 x^n dx}$$

$$D = \frac{2}{\left.\frac{x^{m+1}}{m+1}\right|_0^1} = 2(m+1)$$

AS EXPECTED FROM THE RADIATION PATTERN, AS THE INTEGER m INCREASES, THE DIRECTIVITY INCREASES