Set #1

1. In the Bohr's model of the H atom, give an expression for:
   a. the electron speed as a function of the orbit radius.
   b. the total energy as a function of the orbit radius.

2. How many different photons could be emitted upon a transition from the n=5 down to the fundamental n=1 of an H atom. Compute the exact frequency for one of the transitions.

3. A commercial (green) laser pointer has a max/output power $P \leq 1\ mW$ and a beam spot $w_o = 1.1\ mm$. If $\lambda = 532\ nm$ is the light source wavelength, compute the (1.) pointer photon's flux and (2.) the number of photons emitted in 10 sec when you purposely cover half of the exit hole with your finger. (Neglect divergence).

4.
   a. Give the potential energy of a charge q_2 located at $\mathbf{r}_2$ due to the presence of a charge q_1 located at $\mathbf{r}_1$.
   b. Derive the force exerted on the charge q_2 as due to the charge q_1.
   c. Discuss the two charge possibilities.
   d. Apply the above results to the case of an electron placed at a distance r from a nucleus of a H atom (Bohr's model).
   e. Redo part d. for a nucleus of charge Ze.

Extra.

In a photoelectric experiment Ca is used as photocathode and the following values of stopping potential V_s vs. wavelength λ are measured:

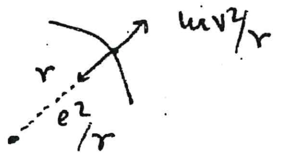
λ, Å	2536	3132	3650	4047
ν, Hz $\times 10^{15}$	1.18	0.958	0.822	0.741
V_s, V	1.95	0.98	0.50	0.14

Calculate the Planck constant $\hbar$ and the work function Φ.

1. In the Bohr's model of the hydrogen atom, give an expression of:

a. the electron speed as a function of the orbit radius.
b. the total energy as a function of the orbit radius.
c. the fraction of the potential energy that contributes to the total energy.

1. a.) In ogni punto dell'orbita forza di Coulomb deve essere uguale alla forza radiale: $\frac{e^2}{r^2} = \frac{mv^2}{r} \Rightarrow v^2 = \frac{e^2}{mr}$



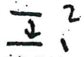
b. En. potenziale: $U = -\frac{e^2}{r}$ $\left(U = \frac{q_1 q_2}{r} \quad F = \frac{q_1 q_2}{r^2} \right)$

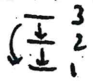
En. cinetica: $K = \frac{1}{2} m v^2 = \frac{1}{2} \cancel{m} \frac{e^2}{\cancel{m} r} = \frac{e^2}{2r}$

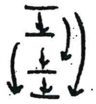
E = En. totale $= U + K = \frac{e^2}{2r} - \frac{e^2}{r} = -\frac{1}{2}\frac{e^2}{r}$

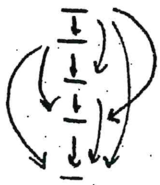
c. $E = \frac{1}{2}\left(-\frac{e^2}{r}\right)$

How many different photons could be emitted upon a transition from the n=5 down to the fundamental n=1 of an hydrogen atom. Show the frequencies.

Da $n=2 \rightarrow n=1$  un solo fotone

Da $n=3 \rightarrow n=1$  3 diversi fotoni

Da $n=4 \rightarrow n=1$  6 diversi fotoni

Da $n=5 \rightarrow n=1$  10 diversi fotoni

In generale, da n al fondamentale ($n=1$) uno ha $\boxed{\dfrac{n(n-1)}{2}}$ differenti fotoni emessi.

- Transizione $n=5 \rightarrow n=4$

$$E_n = -\frac{R}{n^2} \qquad E_5 - E_4 = -\frac{R}{25} + \frac{R}{16} = R\left(\frac{1}{16} - \frac{1}{25}\right) = 13.6\ eV \left(\frac{1}{16} - \frac{1}{25}\right) = 0.306\ eV$$

$$= h\nu = \frac{hc}{\lambda_{54}}$$

$$\lambda_{54}^{-1} = \frac{0.306\ \cancel{eV}}{12.4 \times 10^3\ \cancel{eV}\ \text{Å}} \Rightarrow \lambda_{54} \cong 40523\ \text{Å} \cong 4\ \mu m$$

Transizione $n=5 \rightarrow n=3$

$$E_5 - E_3 = R\left(\frac{1}{9} - \frac{1}{25}\right) = 0.967\ eV$$

$$= \frac{hc}{\lambda_{54}}$$

$$\lambda_{54} = hc / 0.967\ eV = \frac{12.4 \times 10^3\ \cancel{eV}\ \text{Å}}{0.967\ \cancel{eV}} = \ldots \cong 1.28\ \mu m$$

Transizione $n=5 \rightarrow n=2$

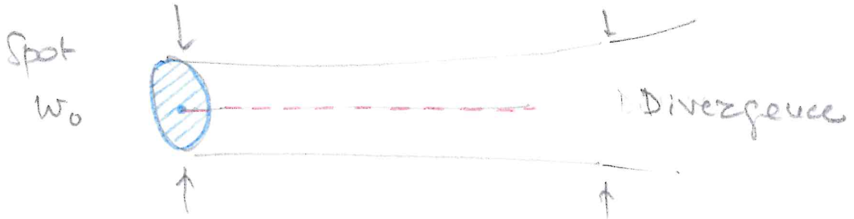
$$E_5 - E_2 = R\left(\frac{1}{4} - \frac{1}{25}\right) = 2.85\ eV$$

$$= \frac{hc}{\lambda_{52}}$$

$$\lambda_{52} = hc / 2.85\ eV = \frac{12.4 \times 10^3\ eV\ \text{Å}}{} = \ldots \simeq 0.43\ \mu m$$

A commercial (green) laser pointer has a max/output power $P \leq 1\ mW$ and a beam spot $w_o = 1.1\ mm$. If $\lambda = 532\ nm$ is the light source wavelength, compute the (1.) pointer photon's flux and (2.) the number of photons emitted in 10 sec when you purposely cover half of the exit hole with your finger. (Neglect divergence).

1.) $\text{Intensity} = \dfrac{\text{Energy}}{\text{surface} \times \text{time}} = \dfrac{\text{Power}}{\text{surface}} = \dfrac{1\,mW}{\pi \left(\frac{1.1\,mm}{2}\right)^2} = \dfrac{10^{-3}\,J}{sec\left(\frac{1.1}{2}\right)^2 \pi\ 10^{-6}\,m^2}$



$= 1.05 \times 10^{3} \left(\dfrac{J}{m^2\,sec}\right)$

(Flux)

2.) Energy across $\frac{1}{2}$ beam spot in 10 secs: $1.05 \times 10^{3} \dfrac{J}{m^2\,sec} \times \underbrace{\dfrac{1}{2}\left(\dfrac{1.1}{2}\,mm\right)^2 \pi}_{\frac{1}{2}\text{ beam spot surface}} \times 10\ sec$

$= [J]\ \dfrac{1.05 \times 10^{3}}{\cancel{m^2}\ \cancel{sec}}\ \dfrac{1}{2}\left(\dfrac{1.1}{2}\right)^2 \pi \times 10^{-6}\ \cancel{m^2} \times 10\ \cancel{sec}$

$\cong 5 \times 10^{-3}\ (J)$

Energy of 1 photon: $E = h\nu = hc/\lambda = \dfrac{12.41 \times 10^{3}\,eV \cdot \overset{\circ}{A}}{532\ nm} \cong [eV]\ 2.3 \times 10^{-3}$

$\overset{\circ}{A} \to 10^{-10}\,m$, $nm \to 10^{-9}\,m$, $eV \to 1.6 \times 10^{-19}\,J$

$\cong [J]\ 3.7 \times 10^{-22}$

$\text{Avg}\ \#\ \text{photons} = \dfrac{5 \times 10^{-3}\ \cancel{(J)}}{3.7 \times 10^{-22}\ \cancel{(J)}} \cong 10^{19}$

5. a. Give the potential energy of a charge q_2 located at $\mathbf{r}_2$ due to the presence of a charge q_1 located at $\mathbf{r}_1$.
b. Derive the force exerted on the charge q_2 as due to the charge q_1.
c. Discuss the two charge possibilities.
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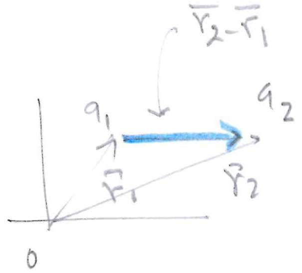
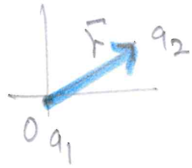
a.)



$$\phi = \frac{q_1 q_2}{|\bar{r}_1-\bar{r}_2|} \qquad \{q_1, q_2\} = \text{charges}$$

Translation: $\bar{r}_1 = 0 \qquad \bar{r}_2 \equiv \bar{r} \qquad \phi = \dfrac{q_1 q_2}{|\bar{r}|}$



$\bar{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$

↳ position vector

b.)
$$\bar{F} = -\bar{\nabla}\phi$$
$$= -\bar{\nabla}\left(\frac{q_1 q_2}{|\bar{r}|}\right) = -q_1 q_2\, \nabla\left(\frac{1}{|\bar{r}|}\right)$$

$$\nabla \frac{1}{|\bar{r}|} = \nabla \frac{1}{\sqrt{x^2+y^2+z^2}} = \hat{i}\frac{\partial}{\partial x}\frac{1}{\sqrt{\ }} + \hat{j}\frac{\partial}{\partial y}\frac{1}{\sqrt{\ }} + \hat{k}\frac{\partial}{\partial z}\frac{1}{\sqrt{\ }}$$

$$\frac{\partial}{\partial x}(x^2+y^2+z^2)^{-1/2} = -\frac{1}{2}\frac{2x}{(x^2+\dots)^{3/2}} = -\frac{x}{|\bar{r}|^3}\,\hat{i}$$

Similarly for $\partial y, \partial z$:

General case

$$\bar{F} = -q_1 q_2\left(-\frac{x\hat{i}}{|\bar{r}|^3} - \frac{y\hat{j}}{|\bar{r}|^3} - \frac{z\hat{k}}{|\bar{r}|^3}\right) = +q_1 q_2\left(\frac{\bar{r}}{|\bar{r}|^3}\right) \qquad \left[+q_1 q_2 \frac{\bar{r}_2-\bar{r}_1}{|\bar{r}_2-\bar{r}_1|}\right]$$

(Attractive)

c.) If $q_1 = +|e| \quad q_2 = -|e| \qquad \bar{F}_{12} = -|e|^2 \dfrac{\bar{r}}{|\bar{r}|^3}$ force is opposite to $\bar{r}$

If $q_1 = +|e| \quad q_2 = +|e| \qquad \bar{F}_{12} = +|e|^2 \dfrac{\bar{r}}{|\bar{r}|^3}$ " " along $\bar{r}$

(Repulsive)

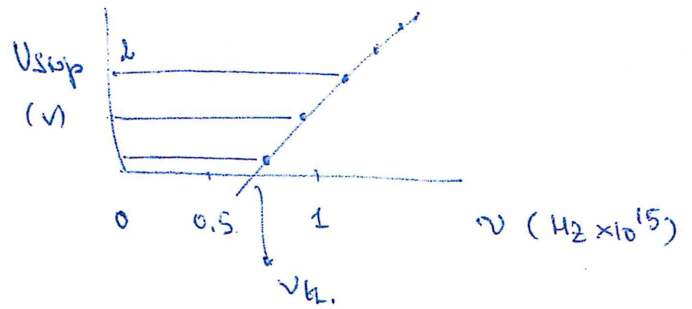
d.) $\phi = -\dfrac{|e|^2}{|\bar{r}|} \qquad \bar{F} = -|e|^2 \dfrac{\bar{r}}{|\bar{r}|^3}$ (Attractive)

e.) $q_1 = +Z|e| \quad q_2 = -|e| \qquad \phi = -\dfrac{Z|e|^2}{|\bar{r}|} \qquad \bar{F} = -Z|e|^2 \dfrac{\bar{r}}{|\bar{r}|^3}$

In a photoelectric experiment Ca is used as photocathode, and the following values of stopping potential vs. wavelength are measured:

λ, Å	2536	3132	3650	4047
ν, Hz $\times 10^{15}$	1.18	0.958	0.822	0.741
V_s, V	1.95	0.98	0.50	0.14

Calculate the Planck constant $\hbar$ and the work function Φ.



λ	2536	3650	4047	(Å)
ν	1.18	0.82	0.74	(10^{15} Hz)
V_s	1.95	0.50	0.14	(V)

$$\left.\begin{array}{l} h\nu = E + \phi \\ E = e\,V_{stop} \end{array}\right\} \quad e V_{stop} = h\nu - \phi \qquad V_{stop} = \left(\frac{h}{e}\right)\nu - \frac{\phi}{e}$$

$$\text{Slope} = \frac{(1.95 - 0.14)\,\text{V}}{(1.18 - 0.74)\,10^{15}\,\text{Hz}} \equiv \frac{h}{e}$$

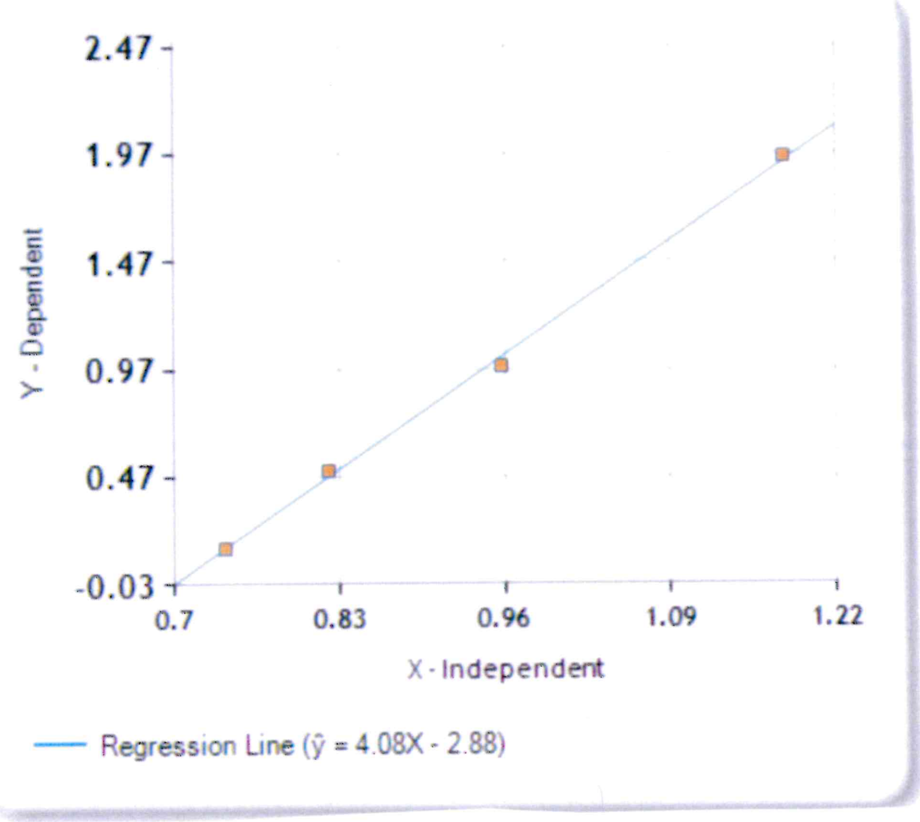
$$h = \frac{e\,(1.95-0.14)\,\text{V}}{(1.18-0.74)\,10^{15}\,\text{Hz}} = \frac{(1.6\times10^{-19}\,\text{C})(1.81\,\text{V})}{0.44\times10^{15}\,\text{Hz}} \cong 6.59\times10^{-34}\ \text{J/sec}^{-1}$$

"Planck's const."

$$\text{Se } V_{stop} = 0 \Rightarrow \frac{\phi}{e} = \frac{h}{e}\nu_{th} \qquad \nu_{th} \simeq 0.7\times10^{15}\,\text{Hz}$$

$$\phi \cong 6.6\times10^{-34}\,\text{J}/\cancel{\text{Hz}} \times 0.7\times10^{15}\,\cancel{\text{Hz}} = 4.62\times10^{-19}\,\text{J} \cong 2.9\ \text{eV}$$

Using best fit:



$$V \qquad\qquad [\nu]\times 10^{15}\ sec^{-1}$$

$$\left.\begin{aligned} E_k &= eV_{stop} \\ &= h\nu - \Phi \end{aligned}\right] \quad V_{stop} = +\frac{h}{e}\cdot\nu - \frac{\Phi}{e} \equiv \left[\frac{h}{e}10^{15}\right][\nu] - \frac{\Phi}{e}$$

Then:

$$\frac{h}{e}\cdot 10^{15} = 4.08\left[\frac{V}{sec^{-1}}\right] \Rightarrow h = 4.08\times10^{-15}\, e\left[\frac{V}{sec^{-1}}\right] = 4.08\times10^{-15}\times 1.6\times10^{-19}\ \frac{C\cdot V}{sec^{-1}}$$

$$= 6.54\times10^{-34}\ J.sec.$$

$$\frac{\Phi}{e} = 2.8\ V \Rightarrow \Phi = 2.8\,eV$$

$$2.8\ V\cdot 1.6\times10^{-19}\,C = 4.5\times10^{-19}\ J$$

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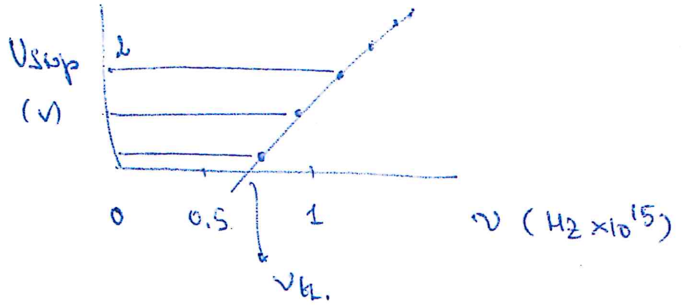

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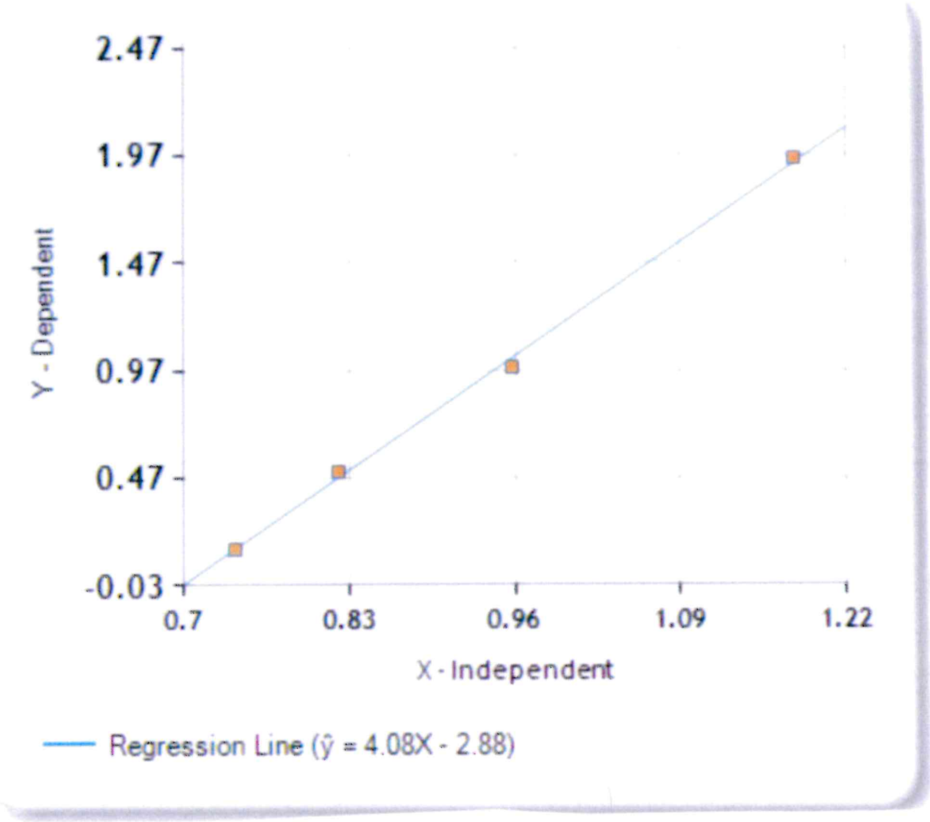
$$\text{Se } V_{stop} = 0 \Rightarrow \frac{\phi}{e} = \frac{h}{e}\nu_{th} \qquad \nu_{th} \simeq 0.7\times10^{15}\,\text{Hz}$$

$$\phi \cong 6.6\times10^{-34}\,\text{J}/\cancel{\text{Hz}} \times 0.7\times10^{15}\,\cancel{\text{Hz}} = 4.62\times10^{-19}\,\text{J} \cong 2.9\ \text{eV}$$

using best fit:



$$E_k = e\,\overset{V}{V_{stop}} = h\nu - \Phi \quad \Big] \quad V_{stop} = +\frac{h}{e}\cdot\underset{}{\overset{[\nu]\times 10^{15}\ sec^{-1}}{\nu}} - \frac{\Phi}{e} \equiv \left[\frac{h}{e}\,10^{15}\right][\nu] - \frac{\Phi}{e}$$

Then:

$$\frac{h}{e}\cdot 10^{15} = 4.08\left[\frac{V}{sec^{-1}}\right] \Rightarrow h = 4.08\times 10^{-15}\, e\left[\frac{V}{sec^{-1}}\right] = 4.08\times 10^{-15} \times 1.6\times 10^{-19}\ \frac{C\cdot V}{sec^{-1}}$$

$$= 6.54\times 10^{-34}\ J\cdot sec.$$

$$\frac{\Phi}{e} = 2.8\ V \Rightarrow \Phi = 2.8\ eV$$

$$2.8\ V\cdot 1.6\times 10^{-19}\ C = 4.5\times 10^{-19}\ J$$