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Lesson 5
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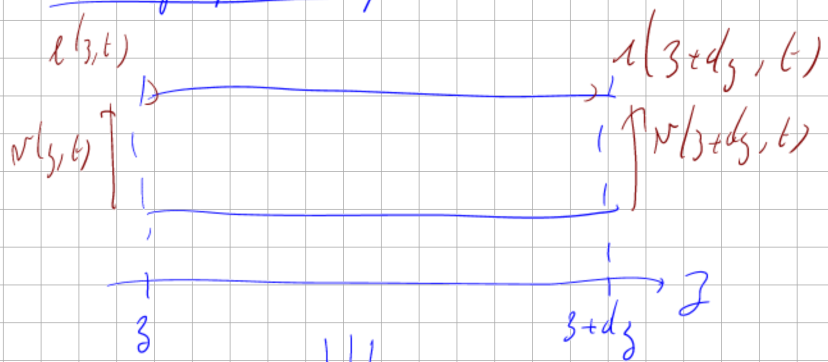
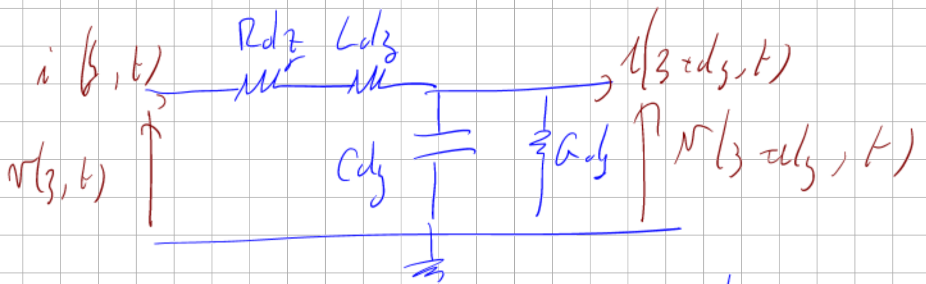
Time domain and frequency domain transmission along a TEM line

I Telegraphist equation



$dz << \lambda$



$$v(z,t) = R\,dz\, i(z,t) + L\,dz \frac{\partial i(z,t)}{\partial t} + v(z+dz,t)$$

$$-\frac{\partial v(z,t)}{\partial z} = R\, i(z,t) + L \frac{\partial i(z,t)}{\partial t}$$

$$-\frac{\partial i(z,t)}{\partial z} = G\, v(z,t) + C \frac{\partial v(z,t)}{\partial t}$$

$$\begin{cases} \dfrac{\partial^2 v(z,t)}{\partial z^2} = LC \dfrac{\partial^2 v(z,t)}{\partial t^2} + (LG+RC)\dfrac{\partial v(z,t)}{\partial t} + RG\, v(z,t) \\ \dfrac{\partial^2 i(z,t)}{\partial z^2} = LC \dfrac{\partial^2 i(z,t)}{\partial t^2} + (LG+RC)\dfrac{\partial i(z,t)}{\partial t} + RG\, i(z,t) \end{cases}$$

$$v(z,t) \xrightarrow{\mathcal{L}} V(z,p) = \int_0^{+\infty} v(z,t)\, e^{-pt}\, dt$$

harmonic $p = j\omega$

$$\frac{\partial v(z,t)}{\partial t} \xrightarrow{\mathcal{L}} p\, V(z,p) \quad \text{if } v(z,0^+) = 0$$

$$\frac{\partial^2 v(z,t)}{\partial t^2} \xrightarrow{\mathcal{L}} p^2\, V(z,p)$$

$$\frac{\partial^2 V(z,p)}{\partial z^2} = \left(LCp^2 + (LG+RC)p + RG\right) V(z,p)$$

$$= \underbrace{(Lp+R)(Cp+G)}_{\gamma^2(p)}\, V(z,p)$$

$$\frac{\partial^2 V(z,p)}{\partial z^2} = \gamma^2(p)\, V(z,p)$$

$$\gamma(p) = \sqrt{(R+Lp)(G+Cp)}$$

propagation constant

$$V(z,p) = V_1(p)\, e^{-\gamma(p) z} + V_2(p)\, e^{+\gamma(p) z}$$

harmonic domain : $p = j\omega$

$$v(z,t) = \Re e\left(V(z)\, e^{j\omega t}\right)$$

$$V(z) = V_1 e^{-\gamma z} + V_2 e^{+\gamma z}$$

$$\gamma = \sqrt{(R+jL\omega)(G+jC\omega)} = \alpha + j\beta$$

$$V_1 = |V_1| e^{j\varphi_1} \qquad V_2 = |V_2| e^{j\varphi_2}$$

$$v(z,t) = \Re e\left(|V_1| e^{j\varphi_1} e^{-\alpha z} e^{-j\beta z} e^{j\omega t} + |V_2| e^{j\varphi_2} e^{\alpha z} e^{j\beta z} e^{j\omega t}\right)$$

incident wave

$$v(z,t) = |V_1|\, e^{-\alpha z} \cos(\omega t - \beta z + \varphi_1) + |V_2| e^{\alpha z} \cos(\omega t + \beta z + \varphi_2)$$

reflected wave