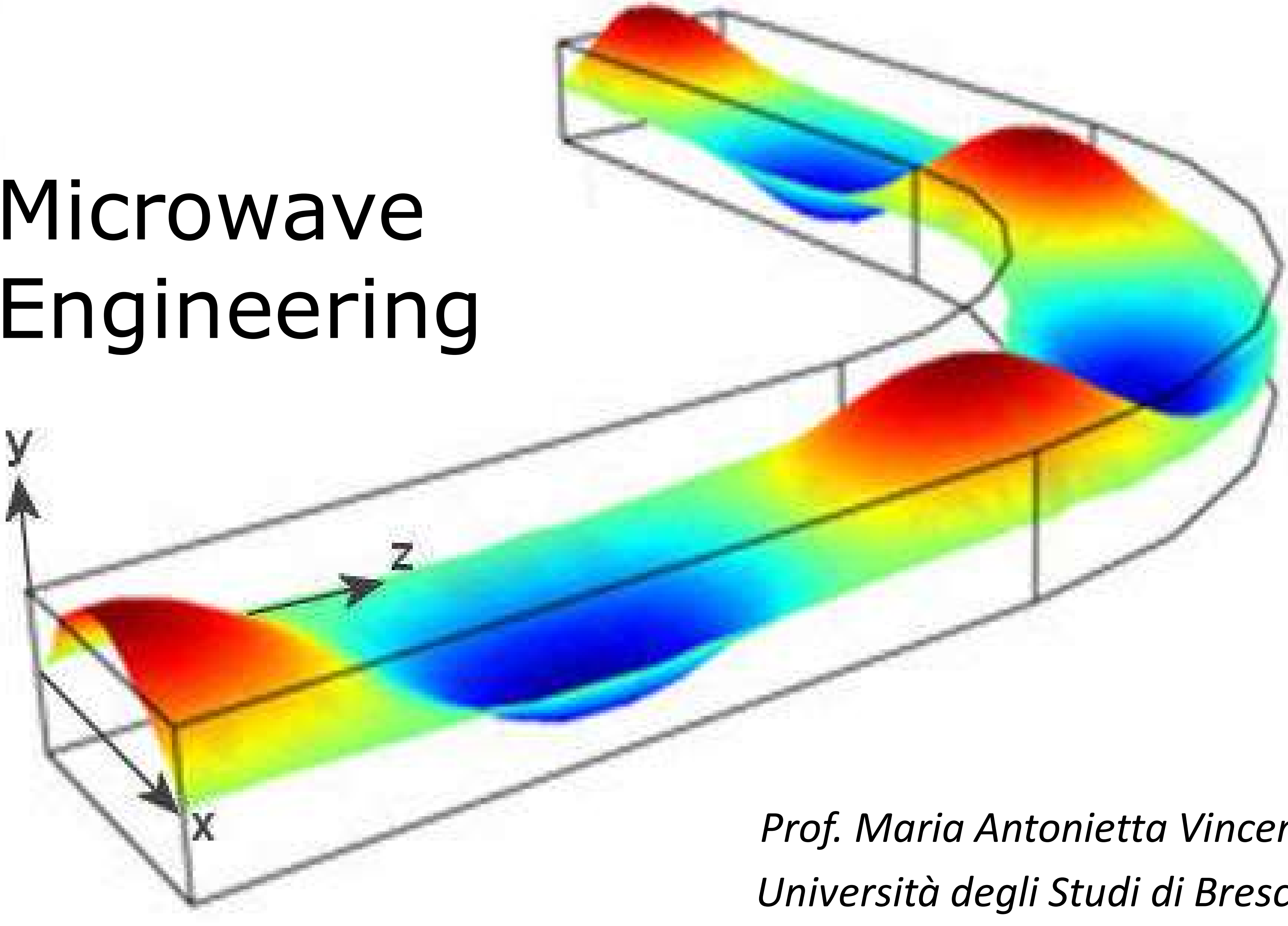
Microwave Engineering



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Energy and Power,
Polarization

Poynting Theorem

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"The power flowing out of a given volume V is equal to the time rate of the decrease in the energy stored within V minus the ohmic losses"

Let's start with time-harmonic Maxwell's equation for a linear, stationary, isotropic medium:

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} - \mathbf{M}_s$$ → 1. Scalar product of this equation by **H***

$$\nabla \times \mathbf{H}^* = -j\omega\varepsilon^*\mathbf{E}^* + \underbrace{\mathbf{J}_S^*}_{\text{Surface current}} + \underbrace{\mathbf{J}_C^*}_{\text{conduction current}}$$ → 2. Scalar product of this equation by **E**

Trick: c.c. of the Ampere's law

3. Subtract

$$\mathbf{H}^* \cdot \nabla \times \mathbf{E} - \mathbf{E} \cdot \nabla \times \mathbf{H}^* = -j\omega\mu|\mathbf{H}|^2 + j\omega\varepsilon^*|\mathbf{E}|^2 - \left(\mathbf{E} \cdot \mathbf{J}_s^* + \mathbf{H}^* \cdot \mathbf{M}_s\right) - \sigma|\mathbf{E}|^2$$

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}^*) = \nabla \cdot \mathbf{S}$$

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Poynting Theorem

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}^*) = \mathbf{H}^* \cdot \nabla \times \mathbf{E} - \mathbf{E} \cdot \nabla \times \mathbf{H}^* = j\omega\left(\varepsilon^* |\mathbf{E}|^2 - \mu |\mathbf{H}|^2\right) - \left(\mathbf{E} \cdot \mathbf{J}_s^* + \mathbf{H}^* \cdot \mathbf{M}_s\right) - \sigma |\mathbf{E}|^2$$

Integrate over the volume each term and then apply the divergence theorem

$$\iiint_V \nabla \mathbf{F} dV = \oiint_S \left(\mathbf{F} \cdot \hat{n}\right) dS$$

$$\iiint_V \nabla \cdot (\mathbf{E} \times \mathbf{H}^*) dV = \oiint_S \mathbf{E} \times \mathbf{H}^* \cdot \hat{n} dS$$

$$= j\omega \iiint_V \left(\varepsilon^* |\mathbf{E}|^2 - \mu |\mathbf{H}|^2\right) dV - \iiint_V \left(\mathbf{E} \cdot \mathbf{J}_s^* + \mathbf{H}^* \cdot \mathbf{M}_s\right) dV - \sigma \iiint_V |\mathbf{E}|^2 dV$$

$$\boxed{\begin{aligned} \mu &= \mu' - j\mu'' \\ \varepsilon &= \varepsilon' - j\varepsilon'' \end{aligned}} \Rightarrow$$

$$-\frac{1}{2} \iiint_V \left(\mathbf{E} \cdot \mathbf{J}_s^* + \mathbf{H}^* \cdot \mathbf{M}_s\right) dV$$

$$= \frac{1}{2} \oiint_S \mathbf{E} \times \mathbf{H}^* \cdot \hat{n} dS + \frac{\sigma}{2} \iiint_V |\mathbf{E}|^2 dV + \frac{\omega}{2} \iiint_V \left(\varepsilon'' |\mathbf{E}|^2 + \mu'' |\mathbf{H}|^2\right) dV$$

$$+ \frac{j\omega}{2} \iiint_V \left(\mu' |\mathbf{H}|^2 - \varepsilon' |\mathbf{E}|^2\right) dV$$

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Interpretation of the Poynting Theorem

$$-\frac{1}{2}\iiint_V \left(\mathbf{E}\cdot\mathbf{J}_s^* + \mathbf{H}^*\cdot\mathbf{M}_s\right) dV = \frac{1}{2}\oiint_S \mathbf{E}\times\mathbf{H}^*\cdot\hat{n}\,dS + \frac{\sigma}{2}\iiint_V |\mathbf{E}|^2\, dV + \frac{\omega}{2}\iiint_V \left(\varepsilon''|\mathbf{E}|^2 + \mu''|\mathbf{H}|^2\right) dV + \frac{j\omega}{2}\iiint_V \left(\mu'|\mathbf{H}|^2 - \varepsilon'|\mathbf{E}|^2\right) dV$$

Power provided by the source

Poynting vector: Power flowing out of a closed surface

Power loss through Joule heating

Reactive power stored by the e.m. field in the volume

$$P_S = P_0 + P_L + P_R$$

In general:

$$P_R = 2j\omega\left(W_m - W_e\right)$$

$$\boxed{\begin{aligned} W_m &= \frac{1}{4}\mathrm{Re}\iiint_V \mathbf{H}\cdot\mathbf{B}^*dV \\ W_e &= \frac{1}{4}\mathrm{Re}\iiint_V \mathbf{E}\cdot\mathbf{D}^*dV \end{aligned}}$$

NOTE: They become the expressions in the Poynting theorem for real, constant and scalar ε and μ

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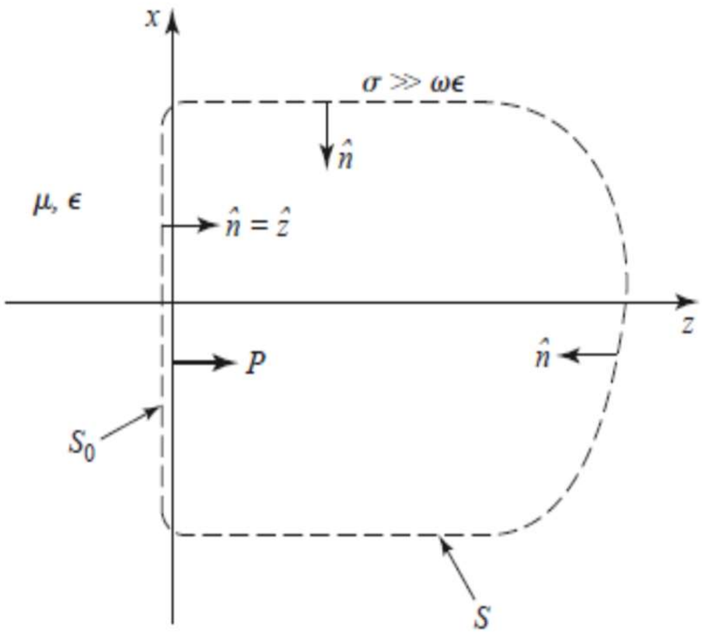
Power absorbed in a good conductor

The power lost in a good conductor can be calculated by only using the fields at its surface. The real average power entering the conductor volume defined by the surface S_0 and surface S is:



$$P_{av} = \frac{1}{2}\mathrm{Re}\oiint_{S_0+S} \mathbf{E}\times\mathbf{H}^* \cdot \hat{n}\,dS$$

The contribution from surface S can be zeroed by properly choosing such surface

$$P_{av} = \frac{1}{2}\mathrm{Re}\oiint_{S_0} \mathbf{E}\times\mathbf{H}^* \cdot \hat{z}\,dS$$

$$\hat{z}\cdot\left(\mathbf{E}\times\mathbf{H}^*\right) = \left(\hat{z}\times\mathbf{E}\right)\cdot\mathbf{H}^* = \eta\mathbf{H}\cdot\mathbf{H}^*$$

$$\mathbf{H} = \hat{n}\times\mathbf{E}/\eta$$

$$P_{av} = \frac{R_s}{2}\oiint_{S_0} \left|\mathbf{H}\right|^2 dS$$

Surface resistivity

$$\boxed{R_s = \mathrm{Re}(\eta) = \mathrm{Re}\left[(1+j)\sqrt{\frac{\omega\mu}{2\sigma}}\right] = \frac{1}{\sigma\delta_s}}$$

Polarization

POLARIZATION OF A PLANE WAVE

It refers to the orientation of the E field vector at a position r and a time t.

Light is naturally unpolarized, i.e., the polarization direction change randomly.
Light can be generated or modified in order to be polarized.
A POLARIZER is a device that transforms unpolarized light into polarized light.

A field that propagates in the *z* direction can be written as:

$$\bar{E} = \hat{x}E_x + \hat{y}E_y = \hat{x}E_{x0}e^{-jkz} + \hat{y}E_{y0}e^{-jkz}$$

where E_{x0} e E_{y0} are complex and can be written as $E_{x0} = a_x e^{j\varphi_x}, E_{y0} = a_y e^{j\varphi_y}$.

Polarization

We can rearrange the expression of E by highlighting the phase difference between the y and x components,

$$\bar{E} = \left(a_x\hat{x} + a_y\,\hat{y}e^{j\varphi}\right) e^{j\varphi_x}e^{-jkz}, \text{ with } \varphi = \varphi_y - \varphi_x, \text{ and set } \varphi_x = 0.$$

Therefore

$$\bar{E} = \left(a_x\hat{x} + a_y\,\hat{y}e^{j\varphi}\right)e^{-jkz}$$

The time domain expression of this field is:

$$\bar{e}(t,z) = \Re e\left\{\bar{E}e^{j\omega}\right\} = \hat{x}a_x\cos(\omega t - kz) + \hat{y}a_y\cos(\omega t - kz + \varphi)$$

The amplitude of $\bar{e}(t,z)$ is $|\bar{e}(t,z)| = \sqrt{a_x^2cos^2(\omega t - kz) + a_y^2cos^2(\omega t - kz + \varphi)}$.

The «direction» of $\bar{e}(t,z)$, i.e., the polarization, is defined in the x-y plane by the angle

$$\psi = arctg\left(\frac{e_y(z,t)}{e_x(z,t)}\right).$$

NOTE: amplitude and direction of the electric field vector are functions of time, even if the e-field stays always perpendicular to the propagation direction

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Linear Polarization

$$\varphi = 0 \text{ or } \varphi = \pi$$

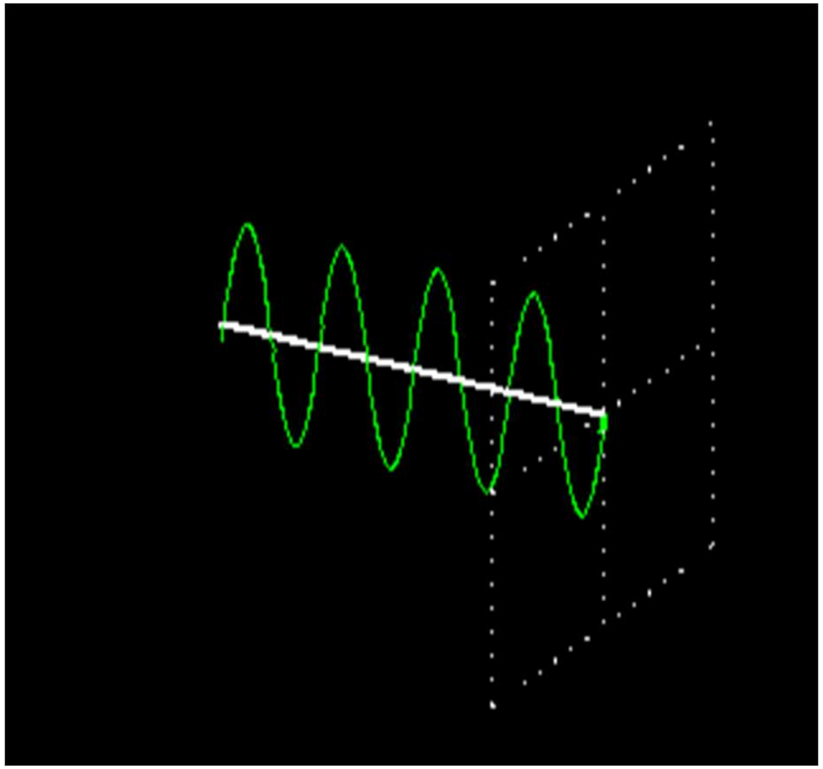
We fix z=0, the tip of vector $\bar{e}(t,0)$ moves in time along a line in the x-y plane:

$$\bar{e}(t,0) = (\hat{x}a_x + \hat{y}a_y)\cos(\omega t), \varphi = 0$$
$$\bar{e}(t,0) = (\hat{x}a_x - \hat{y}a_y)\cos(\omega t), \varphi = \pi$$

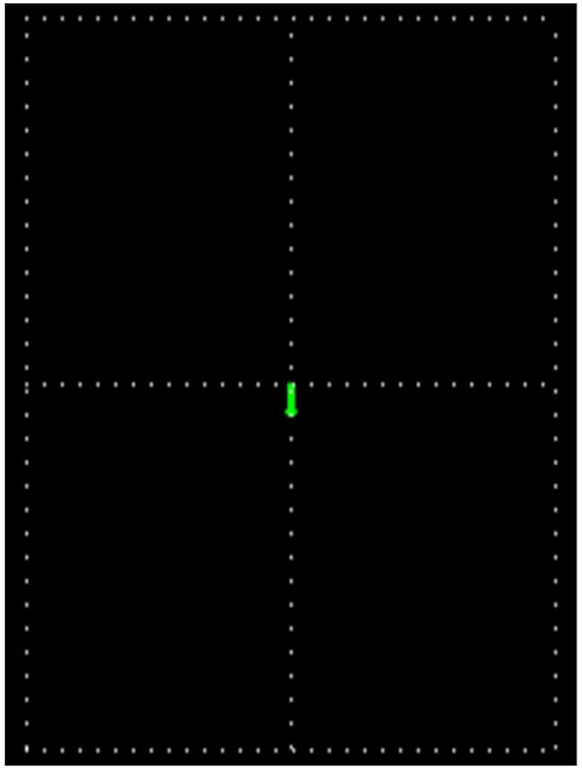
Example: for $\varphi = \pi$ the **polarization direction** at z=0 is $\psi = arctg\left(\frac{e_y(z,t)}{e_x(z,t)}\right) = arctg\left(\frac{-a_y}{a_x}\right)$ and **it is constant in time a space.**

Linear Polarization

E-field 3D view



E-field front view



Right Handed Circular Polarization

$$\varphi = -\pi/2\,,\, a_x = a_y = a$$

We fix z=0, the tip of vector $\bar{e}(t,0)$ moves in time:

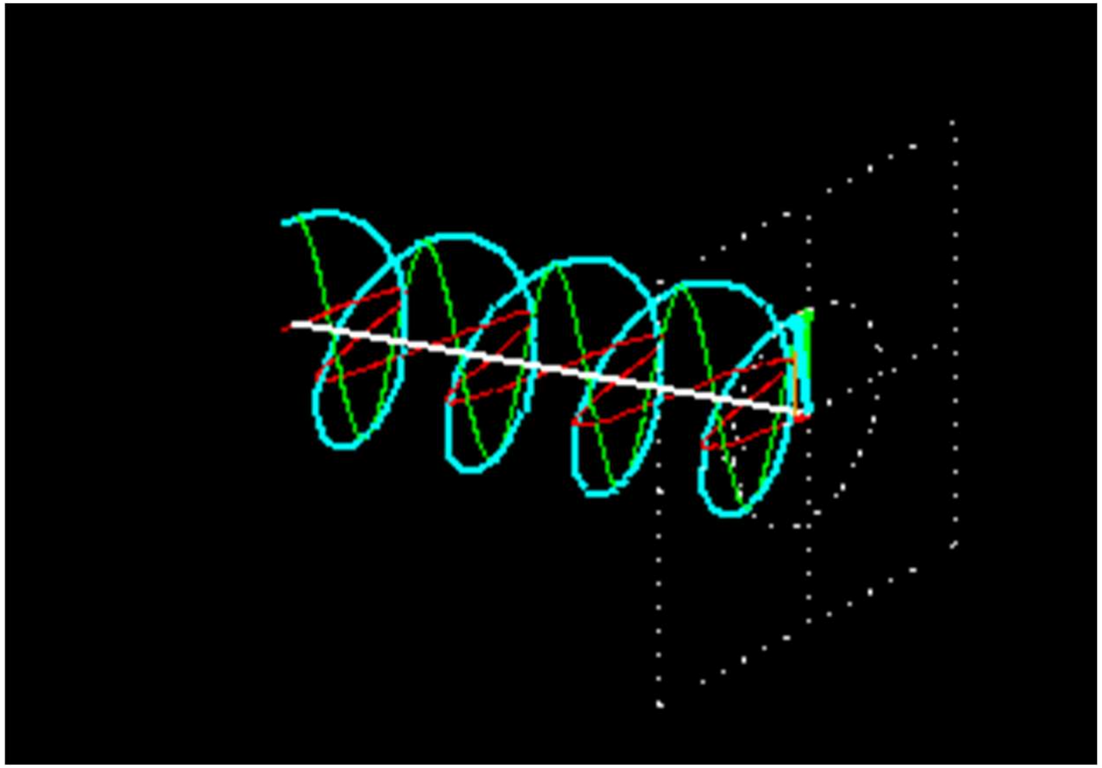
$$\bar{e} = \hat{x}a\cos(\omega t - kz) + \hat{y}a\cos(\omega t - kz - \pi/2) = \hat{x}a\cos(\omega t) + \hat{y}a sin(\omega t)$$

The **polarization direction** at z=0 is $\psi = arctg\left(\frac{e_y(z,t)}{e_x(z,t)}\right) = arctg\left(\frac{sin(\omega t)}{cos(\omega t)}\right) = \omega t$ and **it moves on a x-y plane circle counterclockwise with velocity ωt**

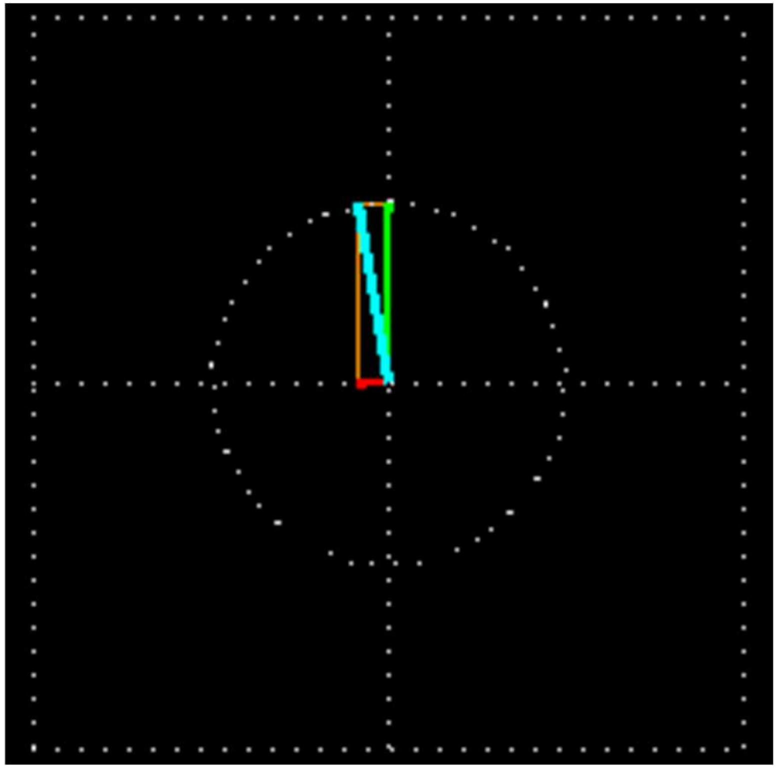
OBSERVATION: A right-circularly polarized wave is the superposition of two linearly-polarized waves, one polrized along x and the other polarized along pi/2 and phase-shifted by $-\pi/2$

Right Handed Circular Polarization

E-field 3D view



E-field front view



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Left Handed Circular Polarization

$$\varphi = \pi/2, a_x = a_y = a$$

We fix z=0, the end of vector $\bar{e}(t,0)$ moves in time in the x-y plane:

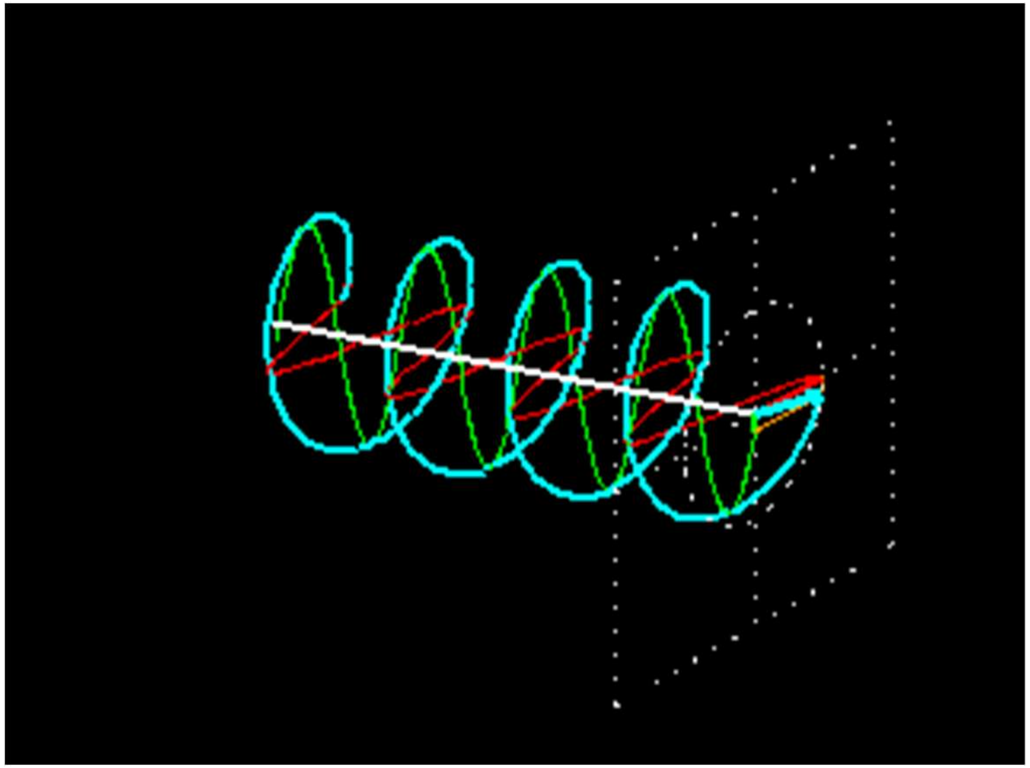
$$\bar{e} = \hat{x}a\cos(\omega t - kz) + \hat{y}a\cos(\omega t - kz + \pi/2) = \hat{x}a\cos(\omega t) - \hat{y}asin(\omega t)$$

The **polarization direction** at z=0 is $\psi = arctg\left(\frac{e_y(z,t)}{e_x(z,t)}\right) = arctg\left(\frac{-sin(\omega t)}{cos(\omega t)}\right)$ and **it moves on a x-y plane circle clockwise**

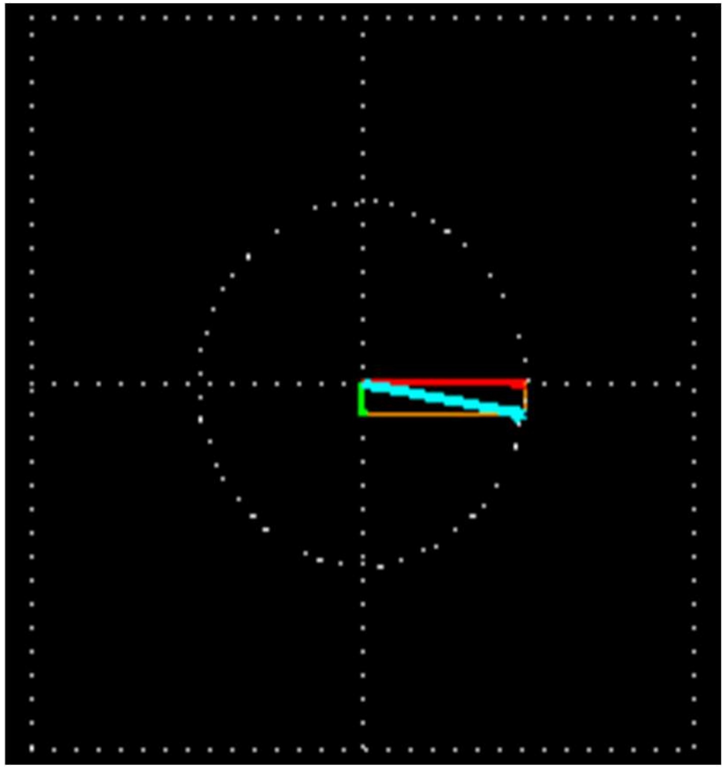
OBSERVATION: A left-circularly polarized wave is the superposition of two linearly-polarized waves, one polarized along x and the other polarized along pi/2 and phase-shifted by $\pi/2$

Left Handed Circular Polarization

E-field 3D view



E-field front view



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RHP + LHP

What happens if we sum a right- and a left-handed circularly polarized waves of the same amplitude a?

$$\bar{e} = \underbrace{\hat{x}a\cos(\omega t - kz) + \hat{y}a\cos(\omega t - kz + \pi/2)}_{\text{LHP}} + \underbrace{\hat{x}a\cos(\omega t - kz) + \hat{y}a\cos(\omega t - kz - \pi/2)}_{\text{RHP}}$$

$$\bar{e} = \hat{x}2a\cos(\omega t)$$

LINEARLY POLARIZED WAVE

Key Takeaways *EM Waves can be linearly, circularly, or elliptically polarized. A circularly polarized wave can be represented as a sum of two linearly polarized waves having phase shift. A linearly polarized wave can be represented as a sum of two circularly polarized waves. In the general case, waves are elliptically polarized.*

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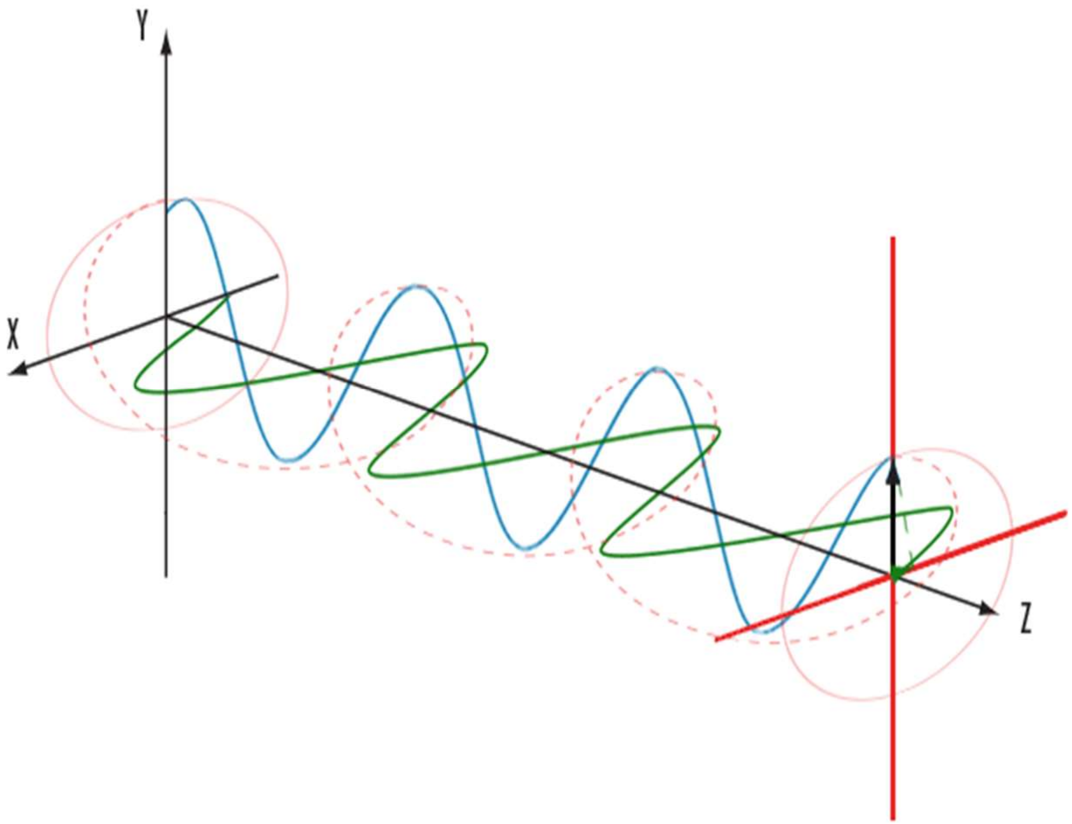
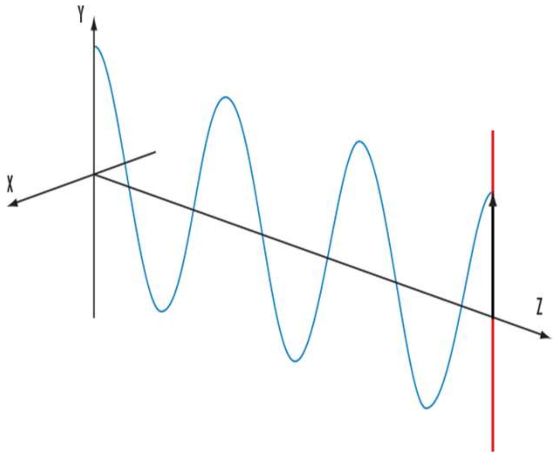
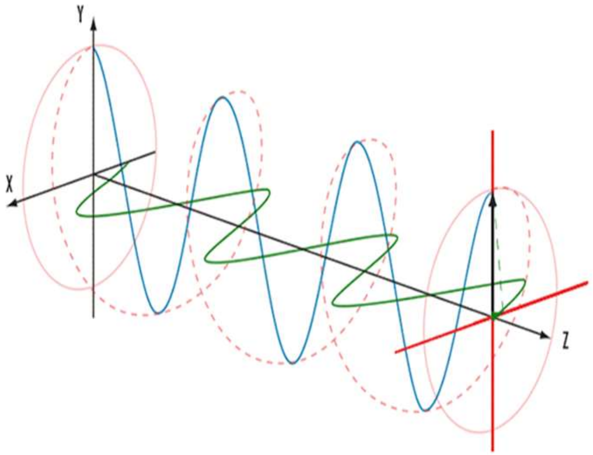
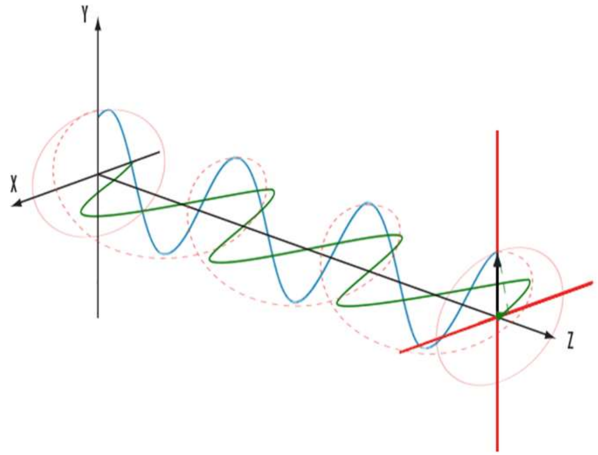
Elliptical Polarization

$$a_x \neq a_y, \varphi \neq 0, \pm\pi, \pm\pi/2$$

The end of the E field vector describes an ellipse and the polarization is said elliptical

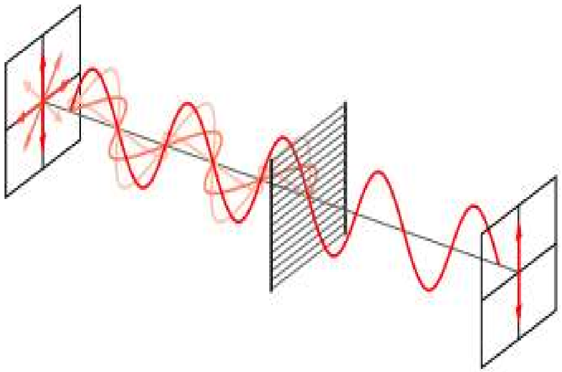


Polarization Summary

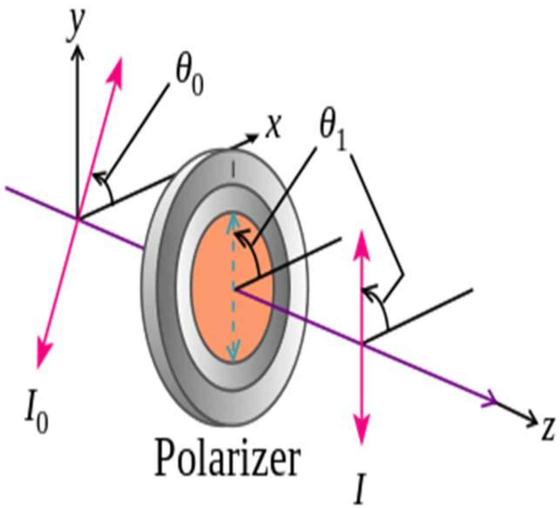
LINEAR	CIRCULAR	ELLIPTICAL
		
$\varphi = 0$ or $\varphi = \pi$	$\varphi = \pi/2$, $a_x = a_y = a$	All other cases

Polarizer

A wire grid polarizer



A polarizer is a filter that allows transmission only for a specific polarization. Example: a linear polarizer can be made by a *wire grid*. Light with electric field component parallel to the wires is absorbed/reflected by the filter, while the electric field component perpendicular to the grid passes through.



If linearly polarized light with intensity I_0 passes through a perfect linear polarizer, then the intensity of the light transmitted through the polarizer is $\boldsymbol{I = I_0 \cos^2(\theta_0 - \theta_1)}$, where $\theta_0 - \theta_1$ is the difference between light polarization angle and the polarizer axis angle.