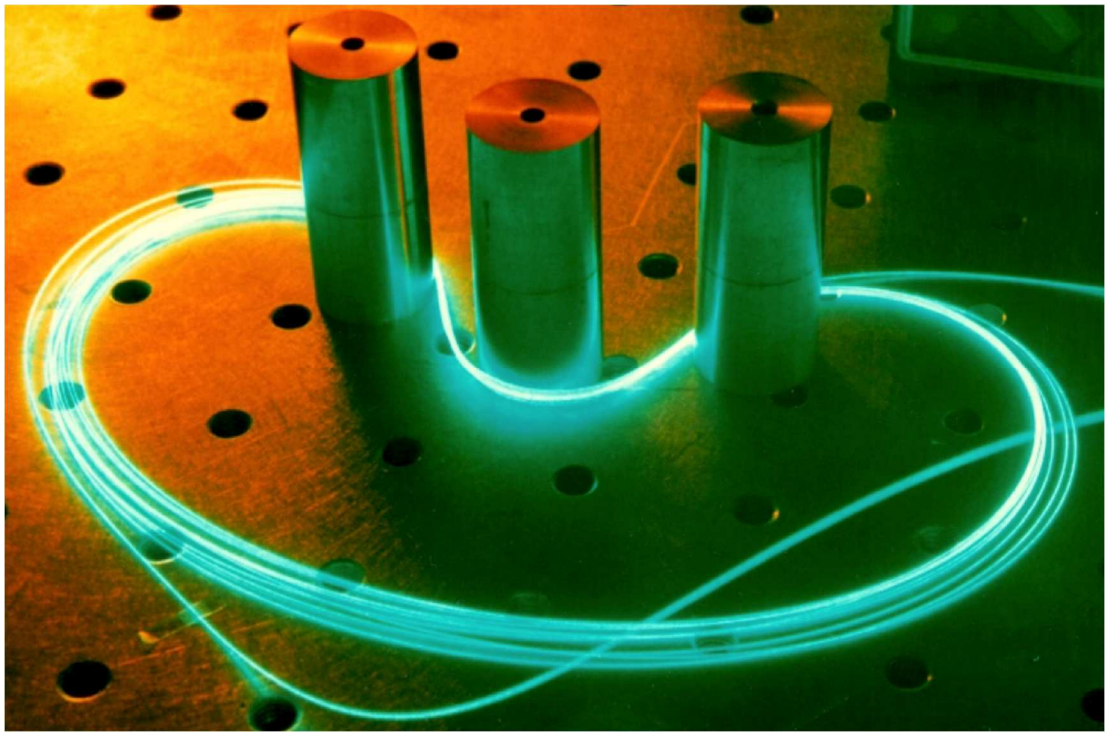
Fiber amplifiers and lasers



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An optical fibre can be turned into an active device quite easily. This part of the teaching unit presents:

- the physical principles at stake in fiber amplifiers and lasers

- how to use these physical laws to design telecom amplifiers or high-power amplifiers

- how this laboratory curiosity has evolved into a key-enabling technology for the 21st century.

Outline

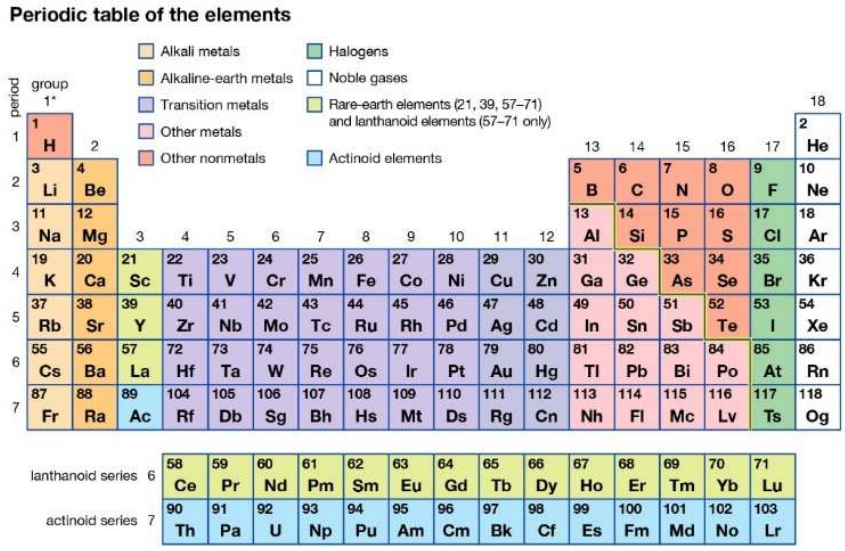
Introduction

Following the discovery by Elias Snitzer in 1961 of the laser action of neodymium in glass, the field of fiber lasers has known rapid and important developments. It has several goals: to obtain new laser transition wavelengths for specific applications (e.g. laser surgery, imaging, communication, atmospheric propagation); to achieve compact and efficient solid-state lasers sources with practical pump wavelengths; and to optimize the physical properties of laser materials for high power or high energy applications (e.g. thermonuclear fusion, machining, laser surgery, range finding, defense). The technology of fiber amplifiers and lasers was driven essentially by the need for optical amplifiers in lightwave communications in the mid-1980s but, since then, has largely spread out to various application fields.

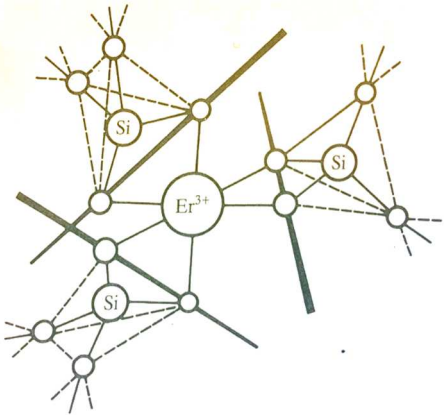
I Fabrication of silica-based RE-doped fibers

I.1 What is a lasing glass?

Glass is an inorganic product of fusion cooled to a rigid condition without crystallization. The structural organization of glass is well defined at the scale of a few atoms but is completely random, asymmetric and aperiodic at a larger scale. The glass lattice is built from basic structural units made of *network former* atoms. The most common is the silica tetrahedron $(SiO_4)^{2-}$. Other usual glass formers are GeO_2, P_2O_5, B_2O_3. The tetrahedron units are connected by their corners through oxygen atoms (bridging oxygens), these random connections form a *disordered* 3D lattice. Other compounds (Na^+, Al^{3+}) can be added to the glass as *network modifiers*. They facilitate the incorporation of lasing elements. Trivalent rare-earths (RE) (light green in the periodic table shown below) are the only ions for which laser oscillation was observed in a glass host, whether in bulk or fiber form. The most commonly used ions are Nd^{3+}, Yb^{3+}, Ho^{3+}, Er^{3+}, and Tm^{3+}, depending on the application. The structure of typical RE-doped silicate glass is shown in Fig. 1b.



(a)



(b)

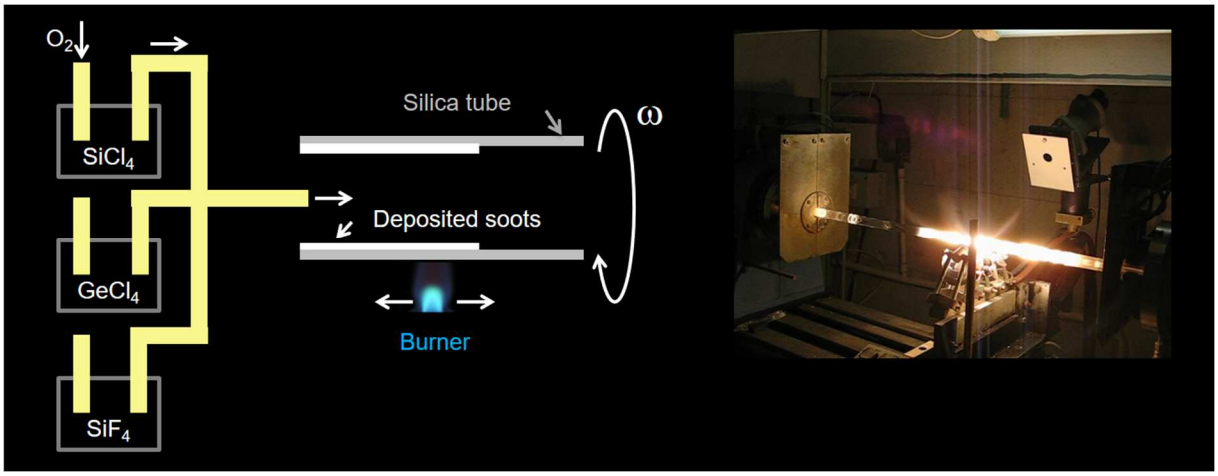
Fig. 1 (a) Periodic table. (b) Possible site for Er^{3+} in basic silicate glass.

A large variety of glasses are suitable as hosts for RE. However, in device applications where the pump is a low-power laser diode, the laser must also be put into the form of an optical fiber. An optical fiber is a thin strand of glass guiding light by the principle of total internal reflection. Passive optical fibers are used to transmit information but also as lighting/collecting elements in endoscopes, for example. By incorporating RE inside the core of an optical fiber, a rare-earth-doped optical fiber can be fabricated and used as an active device, a laser oscillator or an amplifier. Several restrictions and constraints related to the fiber manufacturing technology led to the selection of only a few glasses: oxide-based glasses (e.g. aluminosilicate and germanosilicate glasses), fluoride-based glasses, and

chalcogenide glasses are nowadays used for the fabrication of RE-doped fibers. Oxide-based glasses are by far the most widespread glasses used for fiber amplifiers (telecom) and lasers.

I.2. Fabrication of rare-earth doped optical fibers

The first step in each fiber fabrication method is to realize the fiber preform. The preform can be viewed as a macroscopic replica of the fiber to be fabricated. To reduce the amount of impurities, the preform is composed of synthetized glasses. Modified chemical vapor deposition (MCVD) process is widespread for elaboration of fiber preforms. It is schematically depicted below.

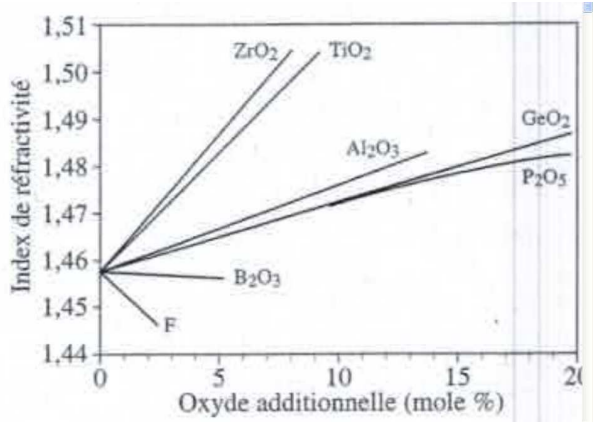


Halide vapors (e.g. $SiCl_4$) react with oxygen thanks to an oxygen-hydrogen flame according to the reaction below:

$$SiCl_4 + O_2 \rightarrow SiO_2 + 2Cl_2$$

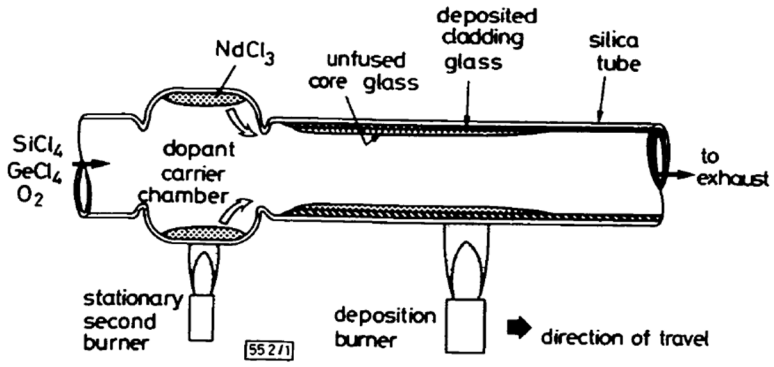
The resulting products are soots of SiO_2, which are deposited inside a rotating quartz tube. The porous SiO_2 layer on the inside of the deposition tube is subsequently dried by heating in a chlorine atmosphere, after which it is fused to form a clear nonporous layer. By incorporating doping elements such as GeO_2 and following the reaction $GeCl_4 + O_2 \rightarrow GeO_2 + 2Cl_2$

one can change the refractive index. The incorporation of GeO_2 in SiO_2 raises the refractive index. The variation of the refractive index versus the concentration of the doping oxide in the glass is shown below.



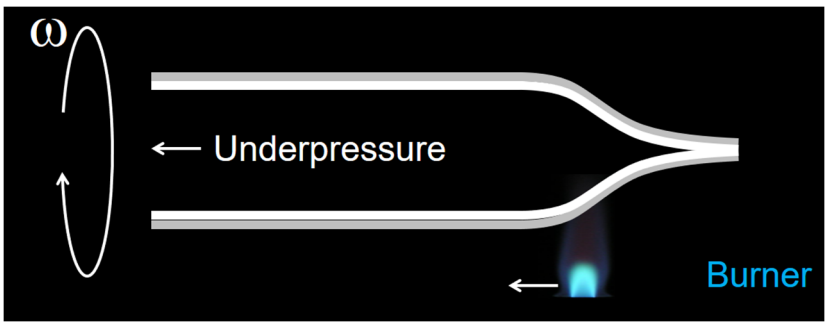
SiO_2, referred to as "fused silica", is used as the cladding glass while doped fused silica (e.g. GeO_2-doped SiO_2 glass) is used as the core material. This ensures light guidance in the core by total internal reflection at the core/cladding interface. The picture below shows an MCVD lathe.

The RE-doped fiber preform is fabricated using the same MCVD fabrication process, with a number of important modifications to permit the incorporation of further dopants into the core glass. Prior to deposition, a conventional deposition tube is prepared by inserting the required dopant $RECl_3 6H_2O$ into a dopant carrier chamber (figure below), where it is dehydrated by heating under a chlorine atmosphere.

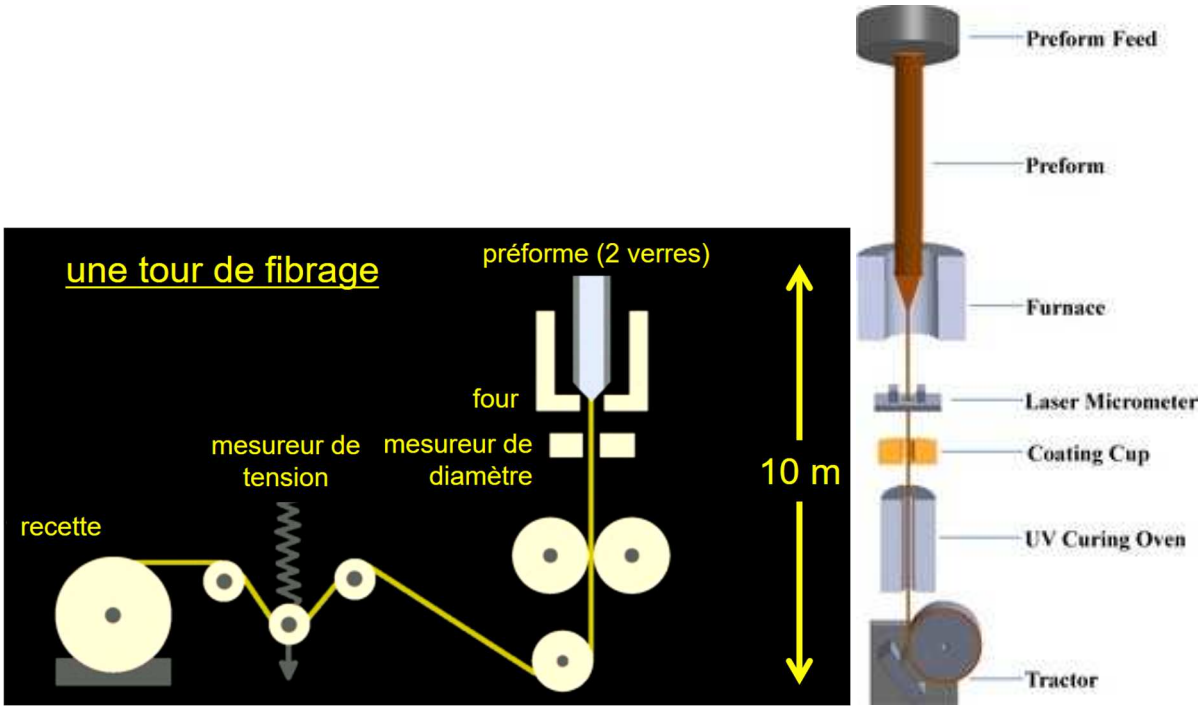


MCVD process for low vapor-pressure dopants. From J. E. Townsend, S. B. Poole and D. N. Payne, "Solution-doping technique for fabrication of rare-earth-doped optical fibres," in *Electronics Letters*, vol. 23, no. 7, pp. 329-331, 26 March 1987, doi: 10.1049/el:19870244.

The cladding glass is deposited in the usual manner. During the core deposition, however, the dopant carrier chamber is heated to around 1000°C by a stationary second burner to produce small quantities of $RECl_3$ vapor. The vapor is carried downstream by the reactant flow, where it is oxidized to RE_2O_3 in the hot zone formed by the deposition burner and incorporated into the core. The porous core layer on the inside of the deposition tube is subsequently dried by heating in a chlorine atmosphere, after which it is fused to form a clear nonporous layer. The tube is then collapsed to form a solid rod.



Next, the drawing process melts the preform (rod) inside a furnace and pulls from it a glass filament of reduced dimensions. Upon solidifying, this filament becomes the fiber. The process also includes the cladding of the fiber by a polymer jacket, which provides enhanced mechanical strength.



In this way, one obtains a rare-earth-doped fiber. The core only is doped with RE. The incorporation of RE does not modify the refractive index of the core. Therefore, it does not modify the electromagnetic modes guided in the fiber. However, it clearly modifies the way light will interact with matter.

Exercise series#1 Modal overlap

II Spectroscopy of rare-earth ions: Light-matter interactions

Consider an ensemble of N atoms forming a lasing medium. Each atom exhibits two energy levels such that the energy difference is $\Delta E = h\nu_{12} = hc/\lambda_{12}$. The concentration (in ions/m^3) of the upper level is N_2, while that of the lower level is N_1. At all times, $N = N_1 + N_2$.

We now consider photons travelling through this medium. There are three types of light-matter interaction: absorption, spontaneous emission and stimulated emission.

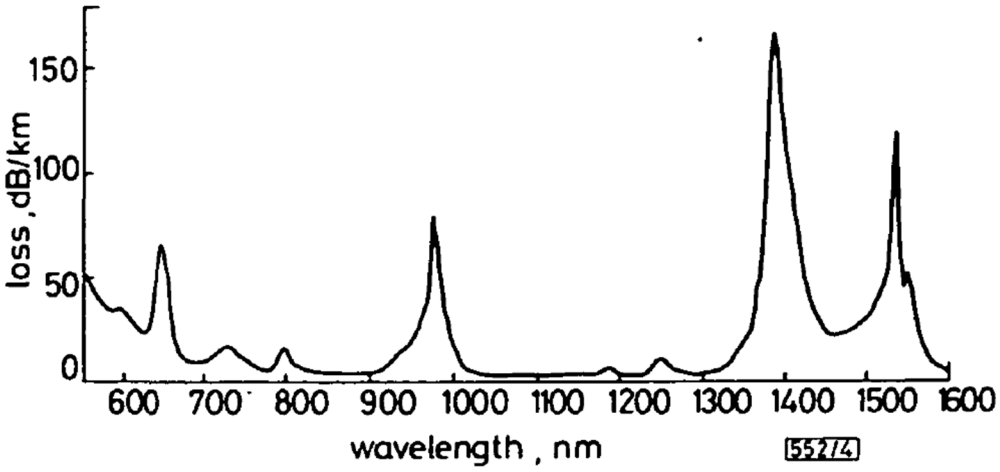
II.1 Absorption

If photons of frequency ν_{12} pass through the medium, there is a possibility of the light being absorbed by electrons in the ground state (1). Electrons are excited in the upper state (2). N_1 decreases. N_2 increases. This corresponds to a net loss in terms of photons.

In usual (undoped) silica fibers operated at near-infrared frequencies, absorption is very low, on the order of 0.2 dB.km^{-1} at 1550 nm, because (a) SiO_2 glass does not present absorption transitions at these frequencies and (b) no unwanted impurities, that could present absorption transitions at these frequencies, are introduced in the fiber preform during the MCVD process. After 15 (resp. 100) km of propagation, the input power P_{in} has been reduced by a factor of 2 (resp. 100). This very low attenuation level explains why the modern telecommunication systems are operated at 1550 nm.

In RE-doped fibers, however, the absorption is usually very high at specific wavelengths. As shown in Fig. 3a , the attenuation level of an Er^{3+}-doped fiber was measured to 70 dB/km at 650 nm, 80 dB/km at 980 nm, and 110 dB/km at 1530 nm for a low concentration of only 2 parts of Er^{3+} in 10^6.

As shown in the energy level diagram of Er-doped glass (Fig. 3b), the three absorption peaks can be attributed to three absorption transitions from the ground state. The 160 dB.km^{-1} peak at 1390 nm, however, does not correspond to any absorption transition in Er^{3+}. It is attributed to unwanted chemicals (hydroxyl group OH^-) resulting from presence of water in the fiber preform.



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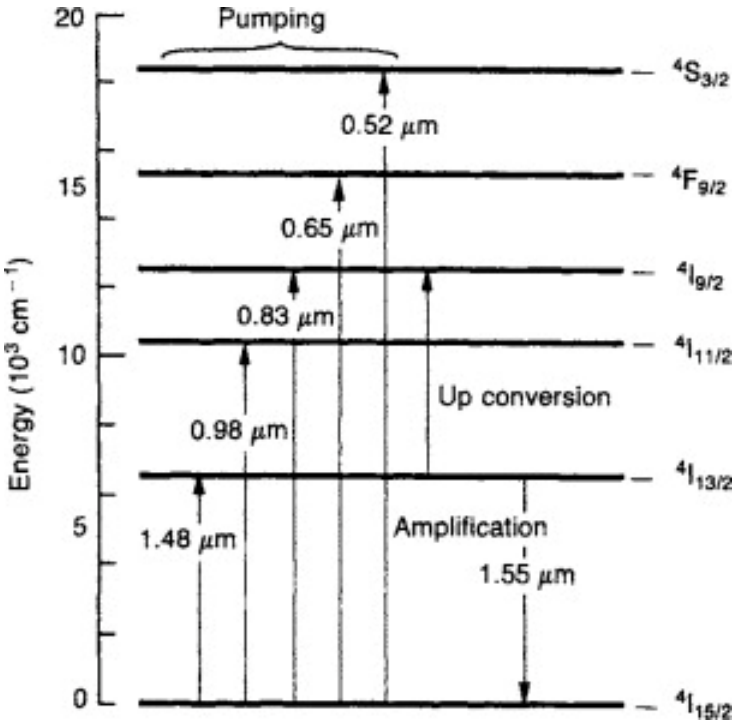


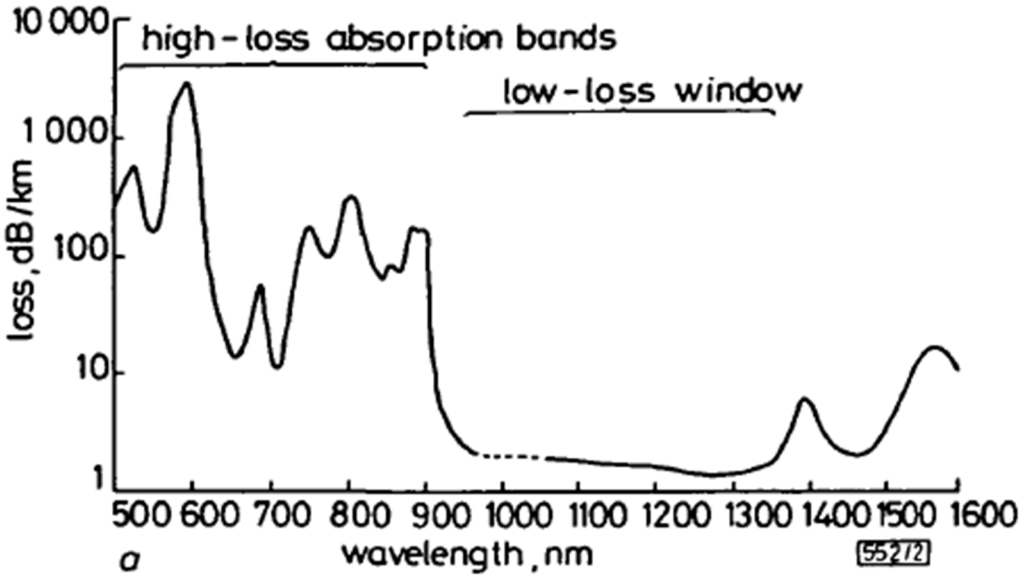
(b)

Fig.3 (a) Absorption spectrum of fiber containing 2 parts in 10^6 of Er^{3+} (from S. B. Poole and D. N. Payne, and M. E. Fermann "Fabrication of low-loss optical fibres containing rare-earth ions," in *Electronics Letters*, vol. 21, no. 17, pp. 737-738, 15 August 1985). **(b) Energy level diagram for Er^{3+}.**

II.2 Spontaneous emission

Now, suppose atoms are initially in state (2) because of photons' absorption for example. Spontaneous decay events to the lower state (1) may occur. The system releases energy in the form of a photon of frequency ν_{12}, which increases the population N_1 of the lower state. Note that the photons are emitted stochastically: there is no phase relationship between photons emitted from a group of atoms in the

upper level. Spontaneous emission is said to be incoherent. The fluorescence spectrum originates from spontaneous emission. As an example, Fig. 4 shows the absorption (a) and emission (b) spectra of Nd^{3+}. Of course, the fluorescence spectrum can be correlated to the laser transitions in the energy level diagram. Fig. 5 shows the laser transitions for trivalent RE in glass hosts and corresponding energy levels.



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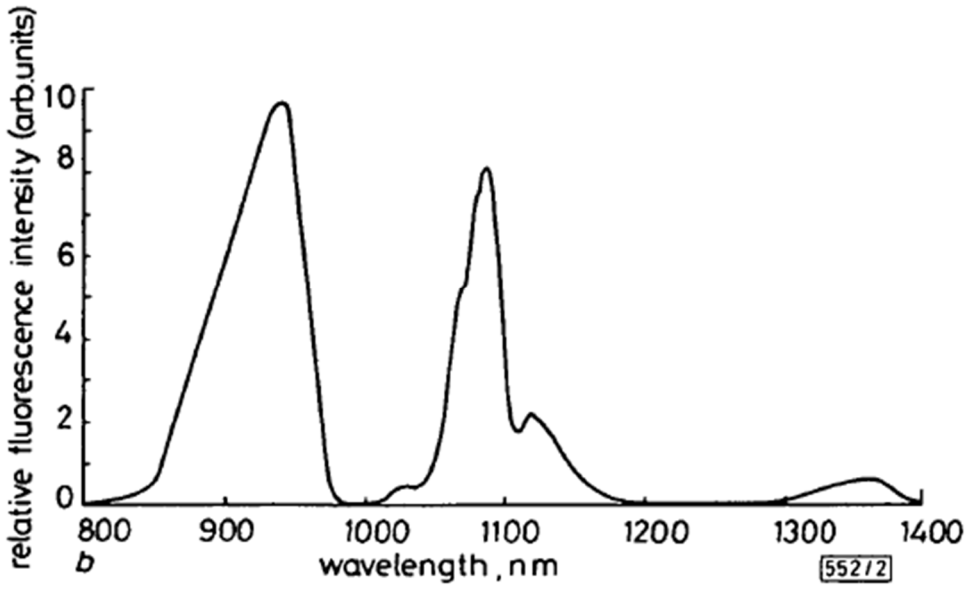


(b)

Fig. 4 (a) Absorption spectrum of fiber containing 30 parts of Nd^{3+} in 10^6 parts. (b) Fluorescence spectrum of fiber containing 300 parts of Nd^{3+} in 10^6 parts.

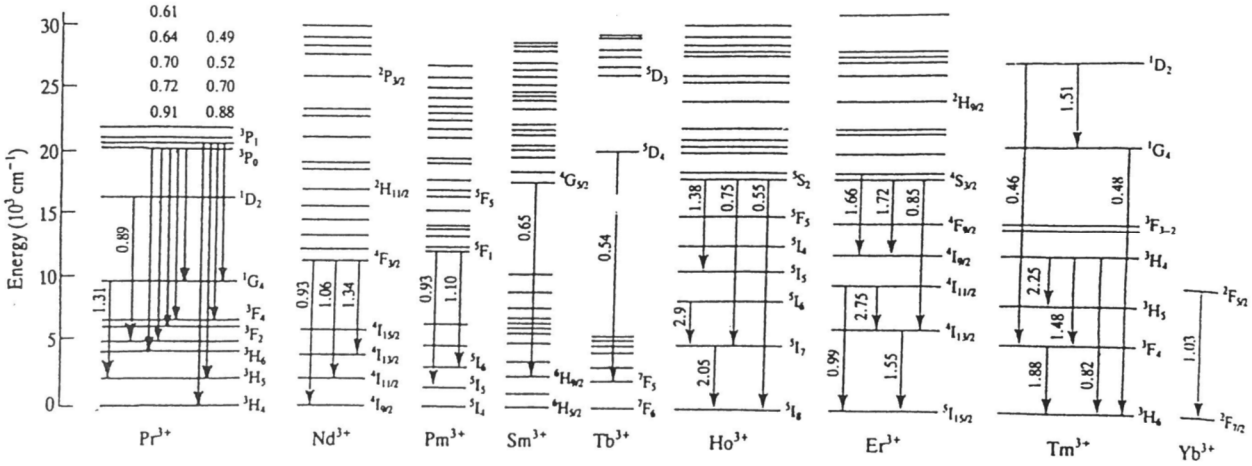


Fig. 5 The laser transitions for trivalent RE in glass hosts and corresponding energy levels.

It is clear from Fig. 4b and Fig. 5 that the emission peaks obtained at 930 nm and 1060 nm when Nd^{3+} is pumped at 808 nm can be attributed to the laser transitions $^4F_{3/2} \rightarrow {}^4I_{9/2}$ and $^4F_{3/2} \rightarrow {}^4I_{11/2}$, respectively.

II.3 Stimulated emission

Stimulated emission occurs when an atom in the upper level is de-excited by a nearby photon that has the correct frequency ν_{12}, matching the energy gap. In this case the relaxation to the lower state is accompanied with the emission of a photon with frequency ν_{12}, as well. Since the original photon is not absorbed (the energy level is already occupied), this interaction results in the production of a twin photon. The process of stimulated emission is coherent and is at the core of any laser. **LASER means light amplification by stimulated emission of radiation.**

Exercise series#2 Spectroscopy of rare earth ions

III Population inversion

III.1 Two-level laser system

At thermal equilibrium, $\frac{N_2}{N_1} = exp\left(-\frac{E_2-E_1}{kT}\right)$

At T = 300 K, kT = 4.14×10^{-21} J = 0.026 eV

At λ = 1 µm, wavelength corresponding to the lasing transition, $\Delta E = hc/\lambda$ = 1.24 eV >> kT

$$\frac{N_2}{N_1} \rightarrow 0^+$$

There are no atoms in the upper level.

$T \rightarrow \infty$, $1/kT \rightarrow 0$,

$$\frac{N_2}{N_1} = 1$$

N_2 never exceeds N_1. So, absorption dominates.

In a two-level system, the probability of a photon causing stimulated emission is the same as that of a photon being absorbed. **There is no net gain.**

If $N_2 = N_1$ the material is transparent. In general $N_1 > N_2$ then absorption dominates.

III.2 Three-level laser system

An extra level is required to ensure that $N_2 > N_1$.

Consider the same group of N atoms but able to exist in either state (1), or state (2) or state (3). Here $E_3 > E_2 > E_1$. Populations are N_3, N_2, N_1 such that, at all times, $N = N_3 + N_2 + N_1$.

The atoms in level (1) are now subject to light at frequency ν_{13}. The process of absorption will populate level (3). This process is called **pumping**. However, level (3) is not the excited state for the laser transition. So, the excited atoms must de-excite quickly to level (2). The energy released in this process may be emitted in the form of a photon via spontaneous emission (radiative transition) or in the form of a lattice vibration (a phonon) (non-radiative transition in terms of light) when the energy gap is small. At this point, atoms in level (2) can either decay spontaneously to level (1) or via stimulated emission if photons with frequency ν_{12} pass through the medium in due time. If the lifetime of level (2)

$\tau_{21} = 1/A_{21}$, where A_{21} is Einstein's coefficient A, is much longer than the lifetime of the non-radiative transition 3 → 2, population of level (3) can be considered as essentially zero, while population of level (2) will be high. If $N_2 > N/2$, then $N_2 > N_1$, net gain can be obtained. This condition is called **population inversion**.

In such a 3-level system, the lower level of the laser transition is the ground level, which is naturally highly populated. Since more than 50% of the atoms must be in the upper state, very strong pumping rate is needed in 3-level laser systems. Er^{3+}, emitting at 1.55 µm when pumped at 0.98 µm, is an example of such a 3-level laser system.

III.3 Four-level laser system

The energy level diagram of a 4-level laser system, Nd^{3+} pumped at 808 m and emitting at 1064 nm, is shown in Fig. 6. The pumping transition excites atoms in the ground level to the pump level. From level 4, atoms decay rapidly to level 3. Since the lifetime of the laser transition is much longer than that of level 4, a population accumulates in level 3. These atoms may relax to level 2, the lower level of the laser transition, via spontaneous or stimulated emission of photons. If the lower level of the laser transition has a short lifetime, much shorter than that of the upper level, the population N_2 is negligible. $N_2 = N_4 = 0$. Then, as long as $N_3 > 0$, population inversion is achieved. This is possible because the lower level of the laser transition is an excited state from which atoms quickly relax to the ground state in a non-radiative (fast) fashion.

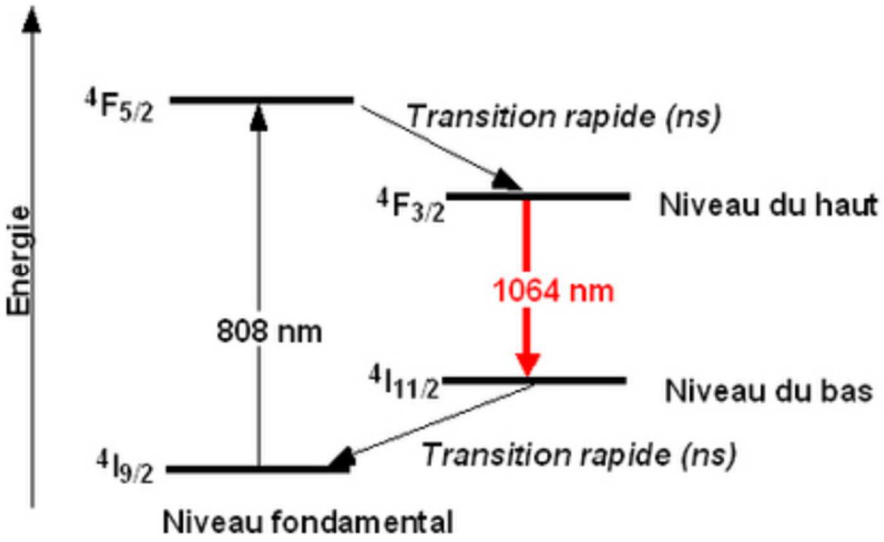
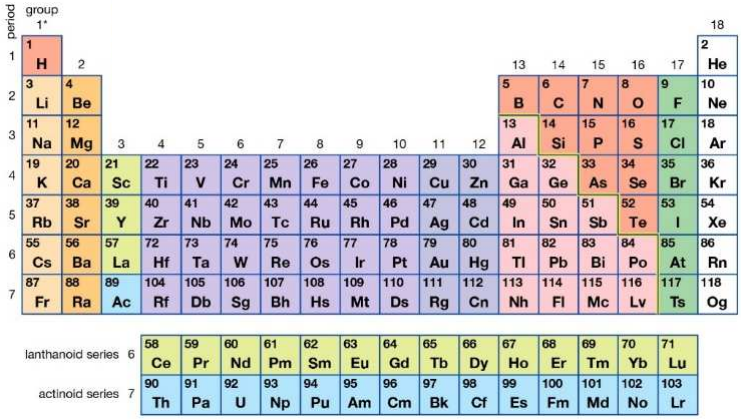


Fig. 6 Energy level diagram of Nd^{3+} acting as a 4-level laser system.

Quiz on Lectures 1-3



1. **What is the structure of the material composing optical fibers?**
   a. Crystalline
   b. Amorphous
   c. Gaseous
2. **What is the most common glass used for fabricating optical fibers?**
   a. GeO_2
   b. P_2O_5
   c. SiO_2
3. **What does GeO_2 doping do?**
   a. It raises the index of refraction.
   b. It decreases the index of refraction.
   c. It provides laser action.
4. **In the periodic table of the elements what is generic name for rare earths?**
   a. Rare gases
   b. Actinoids
   c. Lanthanoids
5. **The core of an amplifying fiber is composed of:**
   a. SiO_2 only
   b. SiO_2, GeO_2 and one rare-earth
   c. SiO_2 and GeO_2
   d. GeO_2 and one rare-earth
6. **In rare-earths diluted in glasses the absorption spectrum is composed of:**
   a. A single Dirac peak
   b. A single broadband peak
   c. A succession of Dirac peaks
   d. A succession of broadband peaks
7. **The properties of photons emitted spontaneously are predictable.**
   a. True
   b. False
8. **What does the acronym LASER mean?**
   a. Light absorbed spontaneously followed by emitted radiation
   b. Light amplification by spontaneous emission of radiation
   c. Light amplification by stimulated emission of radiation
9. **What is the shape of the spatial (transverse) distribution of electromagnetic energy in a step-index fiber?**
   a. Lorentzian-like
   b. Gaussian-like
   c. Step-like
   d. Dirac
   e. Bell shaped
10. **What is the definition of the overlap factor?**
    a. The fraction of EM energy overlapping with the core
    b. The fraction of EM energy interacting with the rare-earths
    c. The fraction of EM energy at the pump wavelength interacting with the EM energy at the laser wavelength
    d. The fraction of light overlapping with the rare-earth doped area

11. What are the limiting values of the overlap factor?

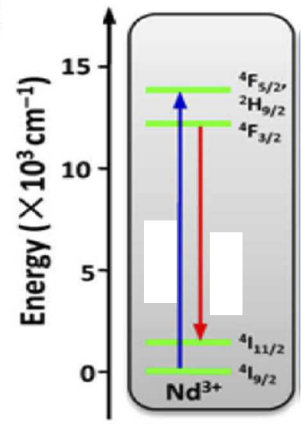
a. $+\infty$ and $-\infty$
b. 0 and 2π
c. 0 and 1
d. –1 and +1

12. What is the formula to convert between the linear wavenumber $\tilde{\nu}$ expressed in cm^{-1} and the wavelength λ expressed in µm?

a. $\tilde{\nu} = \lambda^{-1}$
b. $\tilde{\nu} = 10^{4}\lambda^{-1}$
c. $\tilde{\nu} = 10^{-4}\lambda^{-1}$

13. Using the Boltzmann law $\frac{N_2}{N_1} = exp\left(-\frac{E_2-E_1}{kT}\right)$, show that $N_2 < N_1$ in a two-level laser system.

14. Using the energy diagram below, calculate the wavelengths of the absorption and emission transitions in Nd^{3+}.



15. Calculate the energy in eV of the laser photon.

E.M.I.M.E.O. consortium. This learning Programme has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

IV Rate equations

Applying a signal to a collection of atoms with the frequency ω tuned near one of the atomic transition frequencies ω_a, will cause the population $N_1(t)$ and $N_2(t)$ in the collection of atoms to begin changing slowly because of *stimulated transitions between the two levels E_1 and E_2*. The rate of changes of the populations are given by the atomic rate equations, which contain both stimulated terms (absorption and emission) or relaxation (or energy-decay) terms. These atomic rate equations are of great value in analyzing pumping and population inversion in laser systems. Solution of the rate equations for strong applied signals also lead to population saturation effects, which are of very great importance in understanding the large-signal saturation behavior of laser amplifiers and the power output of laser oscillators.

IV.1. Spontaneous energy decay or relaxation

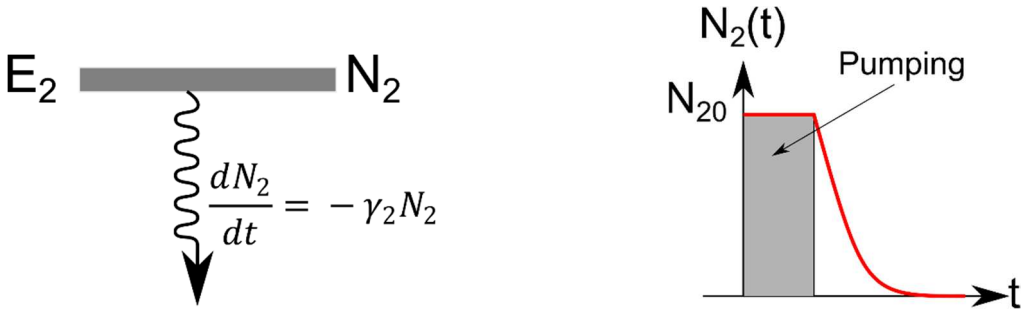
Suppose that a certain number N_2 of atoms have been pumped into some upper level E_2. These atoms will then spontaneously drop down or relax to lower energy levels, giving up their excess internal energy in the process. The rate at which atoms spontaneously decay or relax downward from any upper level E_2 is given by a *spontaneous energy-decay rate*, often called γ_2, times the instantaneous number of atoms in the level, or:

$$\frac{dN_2}{dt} = -\gamma_2 N_2(t) = -\frac{N_2(t)}{\tau_2} \qquad \text{(IV.1)}$$

If an initial number of atoms N_{20} are pumped into the level at t = 0 by a short intense pumping pulse, and the pumping process is turned off, the number of atoms in the upper level will decay exponentially in the form:

$$N_2(t) = N_{20}e^{-\gamma_2 t} = N_{20}e^{-\frac{t}{\tau_2}} \qquad \text{(IV.2)}$$

where $\tau_2 = 1/\gamma_2$ is the lifetime of the upper level E_2 for energy decay to *all lower levels*.



Spontaneous energy decay when pumping is turned off.

IV.2. Stimulated transition rates

Suppose now an optical signal applied to these atoms to cause stimulated transitions. The strength of the signal is characterized by its intensity I (in W per m^2), or by the magnitude of its E field (or H field). It can also be characterized by the number of photons n(t) per unit volume in the applied signal. n(t) is the electromagnetic energy density of the applied signal divided by the quantum energy unit hν.

Such a signal will cause upward transitions at a rate proportional to the applied signal intensity times the number of atoms in the starting level. The population of the upper level will then change according to:

$$\frac{dN_2}{dt} = Kn(t)N_1(t) \qquad \text{(IV.3)}$$

The same applied signal will also cause any atoms initially in the upper level to begin making downward transition at a rate which is proportionnal to the population of the upper level times the number of photons:

$$\frac{dN_2}{dt} = -Kn(t)N_2(t) \qquad \text{(IV.4)}$$

The constant K measures the strength of the stimulated response on that particular atomic transition.

So, in total, the rate equation for the atomic population is given by:

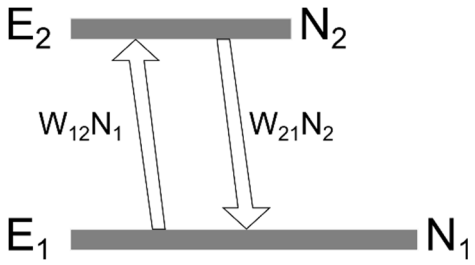
$$\frac{dN_2}{dt} = Kn(t)N_1(t) - Kn(t)N_2(t) - \gamma_2 N_2(t) \qquad \text{(IV.5)}$$

where n(t) is directly proportional to the applied signal intensity or power density.

Neglecting spontaneous emission (IV.5) becomes:

$$\frac{dN_2}{dt} = -\frac{dN_1}{dt} = W_{12}N_1 - W_{21}N_2 \qquad \text{(IV.6)}$$

The quantities W_{12} and W_{21} in units of [s^{-1}] are referred to as the *upward* and *downward stimulated-transition probabilities* per atom per unit time, produced by the applied signal acting on the lower-level and upper-level atoms, respectively.



Upward and downward stimulated transitions between two energy levels.

with

$$W_{12} = W_{21} = \frac{3}{4\pi\tau h\Delta\omega}\frac{\lambda^3}{}\frac{\varepsilon|\tilde{E}|^2}{1+\frac{4(\omega-\omega_a)^2}{\Delta\omega^2}} \qquad \text{(IV.7)}$$

Here, we can say that ***<u>the applied signal gives</u>*** *each atom in the lower level E_1 a probability W_{12} per unit time of making a stimulated transition to the upper level, absorbing a quantum of energy in the process. Similarly, the applied signal gives each atom in the upper level an equal probability W_{21} per unit time of making a transition downward to the lower level, giving up one quantum of energy in the process.*

Exercise series#3 Rate equations

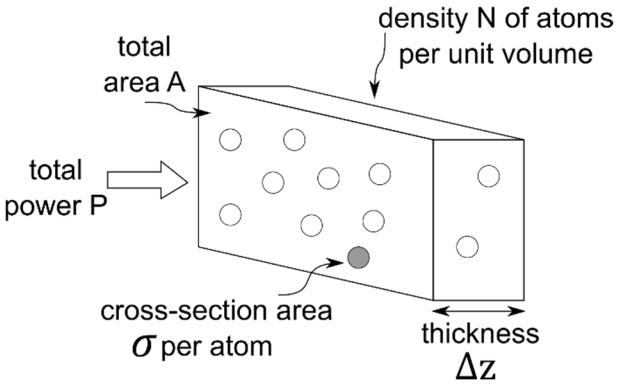
V Stimulated transition cross-sections

Suppose a small black particle with a capture area (or cross-section) σ illuminated by an optical wave having intensity or power per unit area I = P/A. The net power ΔP_{abs} absorbed by this object from the wave will then be its capture area times the incident power per unit area:

$$\Delta P_{abs} = \sigma \times \frac{P}{A} = \sigma I$$

V.1. Power equation

Consider next a thin slab of thickness Δz and transverse area A containing *densities* of N_1 and N_2 of atoms in the lower and upper energy levels. Each atom in the lower level has an absorption cross-section of σ_{12} and each atom in the upper level has an emission cross-section of σ_{21}.



The total number of atoms in the slab will be $N_1A\Delta z$. Then the total absorbing area S_{abs} that results from all lower-level atoms will be $N_1A\Delta z \times \sigma_{12}$. The absorbed power by the atoms in the slab from an incident wave carrying a total power P distributed over the area A will then be:

$$\Delta P_{abs} = P \times \frac{S_{abs}}{A} = P \times \frac{N_1 A \Delta z \sigma_{12}}{A} = N_1 \sigma_{12} \times P \Delta z$$

Similarly, the total emitting area that results from all upper-level atoms will be $N_2A\Delta z \times \sigma_{21}$ and the emitted power will be:

$$\Delta P_{em} = P \times \frac{S_{em}}{A} = P \times \frac{N_2 A \Delta z \sigma_{21}}{A} = N_2 \sigma_{21} \times P \Delta z$$

So that the net absorbed power will be:

$$\Delta P = (N_2\sigma_{21} - N_1\sigma_{12}) \times P\Delta z$$

The net growth or decay with distance caused by an atomic transition for a wave carrying power P or intensity *I* through an atomic medium can then be written as:

$$\frac{dP}{dz} = -(N_1\sigma_{12} - N_2\sigma_{21}) \times P$$

$$\frac{dI}{dz} = -(N_1\sigma_{12} - N_2\sigma_{21}) \times I$$

V.2. Amplification coefficients

The growth or decay rate for a wave passing through an absorbing or amplifying medium may also be written as

$$I(z) = I(z_0)e^{-2\alpha_m(z-z_0)}$$

which corresponds to the differential equation

$$\frac{dI}{dz} = -2\alpha_m I$$

Hence, the attenuation coefficient $2\alpha_m$ is given by

$$\boldsymbol{2\alpha_m} = N_1\sigma_{12} - N_2\sigma_{21} = -(N_2 - N_1)\sigma = \boldsymbol{-\Delta N\sigma}$$

$2\alpha_m > 0$ if $N_1\sigma_{12} > N_2\sigma_{21}$ or equivalently if $\Delta N\sigma < 0$ (no inversion) → exponential decay

$2\alpha_m < 0$ if $N_1\sigma_{12} < N_2\sigma_{21}$ or equivalently if $\Delta N\sigma > 0$ (inversion) → exponential growth

V.3. Relation between transition rates W_{ij} and stimulated transition cross-sections

Consider absorption in a 2-level laser system.

$$\frac{dN_1}{dt} = -W_{12}N_1$$

For fixed z, the variation is proportional to:

- the density of electrons in the lower state N_1
- the absorption cross—section σ_{12}
- the number of photons N_{ph} per unit of time per surface area, also called the photon flux Φ

$$\frac{dN_1}{dt} = -N_1\sigma_{12}\Phi$$

where

$$\Phi = \frac{Energy\ carried\ by\ the\ wave}{Energy\ of\ a\ photon} \times \frac{1}{unit\ of\ time} \times \frac{1}{surface\ area}$$

$$\Phi = \frac{P \times dt}{h\nu} \times \frac{1}{dt} \times \frac{1}{A} = \frac{P}{Ah\nu}$$

Hence,

$$W_{12} = \sigma_{12}\Phi = \sigma_{12}\frac{P}{Ah\nu} = \sigma_{12}\frac{I}{h\nu}$$

$$[\mathrm{m}^2] \times \frac{[\mathrm{W}]}{[\mathrm{m}^2][\mathrm{J}]} = [\mathrm{m}^2] \times \frac{[\mathrm{W}]}{[\mathrm{m}^2][\mathrm{W.s}]} = [\mathrm{s}^{-1}]$$

For a laser ion in a certain electronic state, the rate of transitions (in events per second) is given as the corresponding cross-section times the photon flux (in photons per square meter and second).

VI Saturation effects in laser amplifiers

VI.1. Saturation intensities in laser materials

The amplification coefficient for a signal wave passing through a laser amplifier is proportional to the population difference. At the same time, for a strong enough input signal the stimulated transition rate may become large enough to saturate the population difference and thus reduce the gain coefficient seen by the signal. This process is commonly referred to as saturation of the gain (or absorption) coefficient by the applied signal. Saturation behavior in a laser amplifier can be expected therefore whenever the signal strength becomes strong enough for the signal itself to reduce the growth rate (or attenuation rate).

$$\frac{dI}{dz} = \pm 2\alpha_m I = \pm \Delta N \sigma I$$

See tutorial "rate equations", exercise IV

$$\Delta N = \frac{\Delta N_0}{1 + \tau_{\text{eff}} W_{sig}}$$

This formula can be also expressed in terms of intensities:

$$\Delta N = \frac{\Delta N_0}{1 + \frac{I}{I_{sat}}}$$

where I_{sat} is the saturation intensity or the value of signal intensity passing through the medium that will saturate the gain (or loss) coefficient to half its small-signal or unsaturated value.

$$I_{sat} = \frac{h\nu}{\sigma_{sig}\tau_{\text{eff}}}$$

From this formula, the saturation intensity is inversely proportional to the transition cross section σ, that is, the larger the cross section, the easier the transition is to saturate. The saturation intensity is also inversely proportional to the recovery time since the longer the recovery time (the slower the recovery rate) the easier the transition is to saturate.

Example: Consider a 4-level laser medium such as Yb^{3+}. In 4-level laser system, the saturation intensity does not depend on how the pumping intensity is applied to the laser medium.

$$I_{sat} = \frac{h\nu}{\sigma_{21}\tau_{\text{eff}}} = \frac{hc}{\lambda\sigma\tau} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{1.05 \times 10^{-6} \times 5 \times 10^{-25} \times 10^{-3}} = 3.8 \times 10^{8}\ \text{W/m}^2$$

For a standard step-index fiber with A = 100 µm², this corresponds to P_{sat} = 38 mW.

Example: Consider a 3-level laser medium such as Er^{3+}. In 3-level laser system Wp appears in the formula for I_{sat}.

$$\tau_{\text{eff}} = \frac{2\,\tau_{21}}{1 + \tau_{21} W_p}$$

But, if W_p is on the order of $1/\tau_{21}$ or $W_p = \gamma_{21}$ then τ_{eff} reduces to τ_{21}.

$$I_{sat} = \frac{h\nu}{\sigma_{21}\tau_{\text{eff}}} = \frac{hc}{\lambda\sigma\tau} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{1.55 \times 10^{-6} \times 5 \times 10^{-25} \times 10^{-2}} = 2.56 \times 10^{7}\ \text{W/m}^2$$

For a standard step-index fiber with A = 100 µm², this corresponds to P_{sat} = 2.56 mW.

A laser amplifier will become saturated and give little or no additional gain when the signal passing through the laser material becomes on the order of the saturation intensity.

Similarly, the power level in a laser oscillator, at least under steady-state conditions, is going to build up to at most a few times the saturation intensity, at which point the gain in the laser medium will be saturated down to equal the losses in the laser cavity.

The saturation intensity is thus a very important measure of the amount of power per unit cross-sectional area that can be extracted from a practical laser system.

VI.2. Homogeneous saturation in laser amplifiers

$$\frac{dI}{I} = +2\alpha_m dz = \frac{\Delta N_0 \sigma dz}{1 + \frac{I}{I_{sat}}} = \frac{2\alpha_{m0} dz}{1 + \frac{I}{I_{sat}}}$$

where $2\alpha_{m0} = \Delta N_0 \sigma$ is the unsaturated gain coefficient and I_{sat} is the saturation intensity of the laser material. We cannot integrate this equation to obtain an overall value of $G = \exp(2\alpha_m L)$ since α_m is a

function of the intensity I and hence of the distance z along the amplifier. However if we assume an intensity I_{out} at the output end of the amplifier and an intensity I_{in} at the input end of the amplifier, this equation can be written in the form:

$$\frac{dI}{I}\left[1+\frac{I}{I_{sat}}\right]=2\alpha_{m0}dz$$

and integrated:

$$\int_{I=I_{in}}^{I=I_{out}}\left[\frac{1}{I}+\frac{1}{I_{sat}}\right]dI=2\alpha_{m0}\int_{z=0}^{z=L}dz$$

This integration yields:

$$[\ln(I)]_{I_{in}}^{I_{out}}+\frac{1}{I_{sat}}[I]_{I_{in}}^{I_{out}}=2\alpha_{m0}L$$

$$\ln\left(\frac{I_{out}}{I_{in}}\right)+\frac{I_{out}-I_{in}}{I_{sat}}=2\alpha_{m0}L$$

$$e^{\ln\left(\frac{I_{out}}{I_{in}}\right)+\frac{I_{out}-I_{in}}{I_{sat}}}=e^{2\alpha_{m0}L}$$

$$\frac{I_{out}}{I_{in}}.e^{\frac{I_{out}-I_{in}}{I_{sat}}}=G_0$$

with $G_0=\exp(2\alpha_{m0}L)$ the small-signal gain.

The overall gain of the amplifier $G\equiv\frac{I_{out}}{I_{in}}$ takes on the form:

$$G=G_0\times\exp\left[-\frac{I_{out}-I_{in}}{I_{sat}}\right]$$

This equation says that the overall gain G is reduced below the unsaturated gain G_0 by a factor that depends exponentially on the extracted intensity $I_{out}-I_{in}$ relative to the saturation intensity I_{sat}.

From this equation it is possible to find I_{in}/I_{sat} and deduce I_{out}/I_{sat} = G I_{in}/I_{sat}:

$$\frac{G}{G_0}=\exp\left[-\frac{I_{out}-I_{in}}{I_{sat}}\right]=\exp\left[-\frac{I_{out}}{I_{sat}}+\frac{I_{in}}{I_{sat}}\right]$$

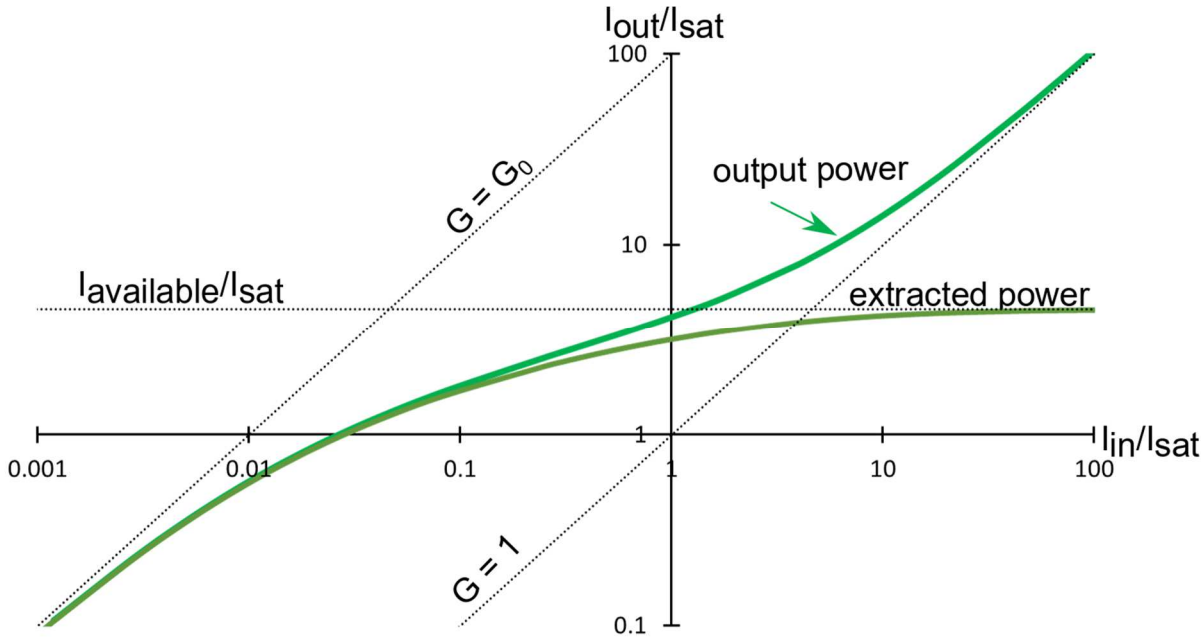
$$\ln\left(\frac{G}{G_0}\right)=\frac{I_{in}}{I_{sat}}(1-G)$$

$$\ln\left(\frac{G_0}{G}\right) = \frac{I_{in}}{I_{sat}}(G-1)$$

$$\frac{I_{in}}{I_{sat}} = \frac{1}{G-1}\ln\left(\frac{G_0}{G}\right)$$

$$\frac{I_{out}}{I_{sat}} = \frac{G}{G-1}\ln\left(\frac{G_0}{G}\right)$$

The amplifier output-versus-input curve for a value of the small-signal gain G_0 = 100 is given below:



Amplifier output-versus-input curve for a value of the small-signal gain G_0 = 100

To draw the output power curve:

First define G_0 (here 100).

Define G from G_0 to 1 by steps.

Calculate $\frac{I_{in}}{I_{sat}} = \frac{1}{G-1}\ln\left(\frac{G_0}{G}\right)$.

Calculate $\frac{I_{out}}{I_{sat}} = \frac{G}{G-1}\ln\left(\frac{G_0}{G}\right)$

To draw the asymptotes:

Calculate the power given by $\frac{I_{in}}{I_{sat}} \times G_0$. This is the asymptote on the left.

Draw the $\frac{I_{in}}{I_{sat}} = \frac{I_{in}}{I_{sat}}$ curve obtained for G = 1. This is the asymptote on the right.

How much power per unit cross-section area can be extracted from such an amplifier at different input signal levels?

$$I_{extract} = I_{out} - I_{in} = \frac{I_{out}}{I_{sat}} I_{sat} - \frac{I_{in}}{I_{sat}} I_{sat} = (G-1) \times \frac{1}{G-1} \ln\left(\frac{G_0}{G}\right) \times I_{sat} = \ln\left(\frac{G_0}{G}\right) \times I_{sat}$$

This value (actually $\frac{I_{extract}}{I_{sat}}$) is illustrated as a saturating curve in the previous figure.

At low intensity and high gain (G_0), $I_{extract}$ and I_{out} are essentially the same.

However, as the input intensity I_{in} increases to I_{sat} the extracted intensity approaches a limiting value, which is the maximum power available than can be extracted from the laser medium. As shown in the figure, this *maximum available power from the amplifier* (per unit area) is given by the limit of $I_{extract}$ when *G* approaches unity:

$$\lim_{G \to 1} \left(\ln\left(\frac{G_0}{G}\right) \times I_{sat} \right) = \ln G_0 \times I_{sat}$$

This is the maximum power per unit area that is available in the laser medium to be given to the input signal. It is given by the horizontal asymptote in the figure. It is limited by the value of the small-signal gain $G_0 = \exp(2\alpha_{m0}L) = \exp(\Delta N_0 \sigma L)$.

Using earlier formulas

$$G_0 = \exp(2\alpha_{m0}L);\ 2\alpha_{m0} = \Delta N_0 \sigma;\ I_{sat} = \frac{h\nu}{\sigma_{sig}\tau_{\text{eff}}}$$

$I_{available}$ becomes:

$$I_{available} = \Delta N_0 \sigma L \frac{h\nu}{\sigma \tau_{\text{eff}}} = \Delta N_0 L \frac{h\nu}{\tau_{\text{eff}}}$$

Since I = P/A, I/L = P/V then:

$$\frac{I_{available}}{L} = \frac{\Delta N_0 h\nu}{\tau_{\text{eff}}} = \frac{P_{available}}{V}$$

This equation says that *the maximum power per unit volume that can be obtained from the laser medium is given by the initial small-signal inversion energy stored in the medium* $\Delta N_0 h\nu$ *times an effective recovery rate* $1/\tau_{\text{eff}}$. ***In other words, we can extract the initial inversion energy*** $\boldsymbol{\Delta N_0 h\nu}$ ***every*** $\boldsymbol{\tau_{\text{eff}}}$ ***only.***

VII Linewidth broadening mechanisms and effective transition cross-sections

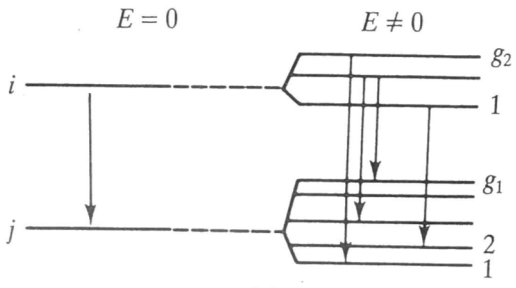
The probability of stimulated transition depends on the frequency and follows a lorentzian lineshape:

$$W_{12} = W_{21} = \frac{3}{4\pi\tau h\Delta\omega}\frac{\lambda^3}{}\frac{\varepsilon|\tilde{E}|^2}{1+\frac{4(\omega-\omega_a)^2}{\Delta\omega^2}}$$

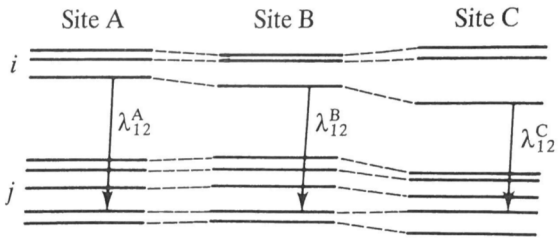
Actually, in disordered solids such as glasses, the lineshape will be modified and the linewidth will be broadened by a number of physical mechanisms.

Stark splitting plays a crucial role in linewidth broadening. The Stark effect is induced by the local crystalline field (also called the ligand field) surrounding the rare earth ion (Fig. 7a). It results in lifting the degeneracy of the two levels involved in the transition. Because of Stark splitting, there actually exists 8×7 possible transitions between the Stark manifolds.

Moreover, since in a glass host the ligand field randomly varies from site to site, the Stark splitting between energy levels is also random. This produces site-to-site variations in transition wavelengths and causes some sort of inhomogeneous line broadening (Fig. 7b).



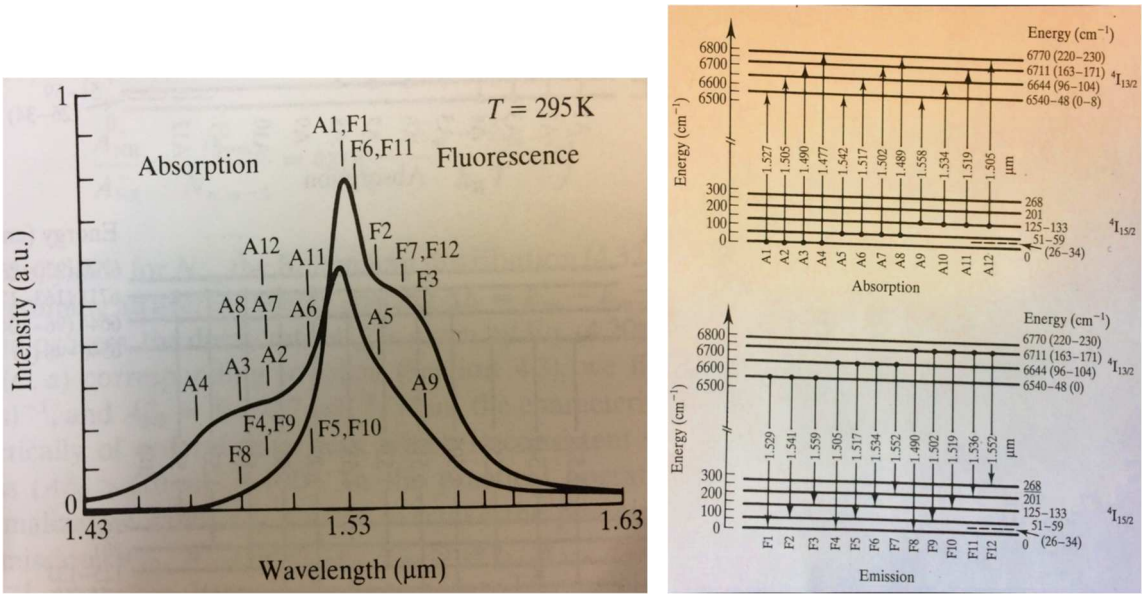
(a)



(b)

Fig. (a) Effect of Stark splitting of energy levels with degeneracies g_1 and g_2, caused by crystalline electric field E, with possible laser transitions. (b) Effect of inhomogeneous broadening, where random field variations from site to site cause changes in the Stark splitting and center wavelength of laser transitions.

Fig. 8a shows the absorption and emission spectra of aluminosilicate Er-doped fiber at room temperature, showing positions of Stark transitions. These spectra correspond to the complex energy level diagram shown in Fig. 8b. The Stark sublevels have small energy gaps and are therefore strongly coupled by the effect of fast thermalization. It is very difficult to identify the various combinations. Here, the combinations are deduced from a study at 4K, where phonon coupling is much weaker. The overall transition therefore appears as homogeneously broadened.



(a) **(b)**

Fig. (a) Absorption and fluorescence spectra of aluminosilicate Er-doped fiber showing structures changes and positions of Stark transitions. (b) Stark transitions corresponding to absorption and emission spectra shown in (a).

The effects of broadening mechanisms (Stark splitting, random distribution of sites and fast thermalization) are all captured in the *effective* emission and absorption cross-sections $\sigma_{em}(\lambda)$ and $\sigma_{abs}(\lambda)$. In this case, we consider the rate of optical transitions per active ion starting from a certain Stark level manifold and terminating in another Stark manifold. An example is given in Fig. 9 for the Ytterbium-doped aluminosilicate glass used in the cores of ytterbium-doped fibers.

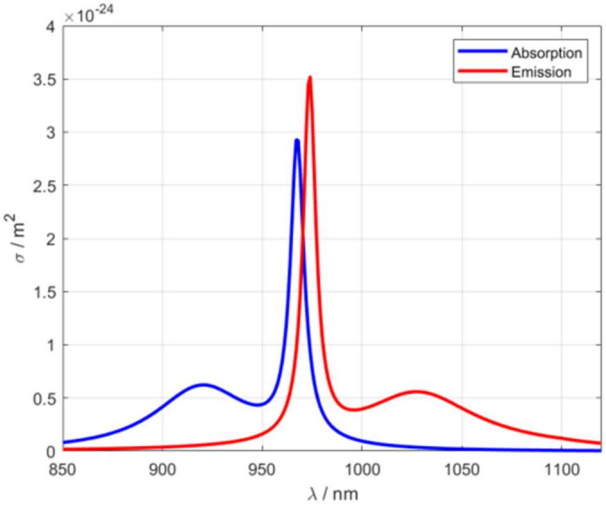


Fig. Absorption and emission cross-sections of Ytterbium-doped aluminosilicate glass as used in the cores of ytterbium-doped fibers, at room temperature.

Here, the strongest transition is that between the lowest-lying energy levels in both manifolds; it is seen as the "zero-phonon line" at ≈ 975 nm. (By definition, the zero-phonon line is the line corresponding to the transition not involving any phonons in both manifolds. That is the transition between the lowest sub-levels of both manifolds. There can thus be only one such line.) The (weaker) absorption at shorter wavelengths (e.g. 920 nm) is due to transitions to higher-lying sublevel in the upper manifold, which involves the emission of one or several phonons during subsequent thermalization. Similarly, the emission at longer wavelength (e.g. 1040 nm) is related to transitions to higher-lying levels of the ground-state manifold, and involves phonon emission during thermalization in the ground-state manifold.

Cross sections are usually directly obtained from absorption and emission measurements, and the knowledge of sublevel positions and cross sections for the contributing transitions (Fig. 8) is not required. For example, the measured absorption spectrum of an electronically non-excited sample, such as the one shown in Fig. 3a, reveals the absorption cross sections for transitions from the ground-state manifold to higher-lying Stark level manifolds.

If absorbing atoms or ions with an absorption cross-section σ_{abs} are distributed with a number density N in a medium, this leads to an intensity absorption coefficient α of the medium which is the product of N and σ_{abs}. For fibers with an undoped cladding, it is necessary to include also an overlap factor Γ (see lesson 1). In an analogous fashion, the gain coefficient for atoms or ions with a given emission cross-section σ_{em} can be calculated.