# Optoelectronics

*Key notes*

2/ Light as wave. Polarization and interference

# 1. Light as wave

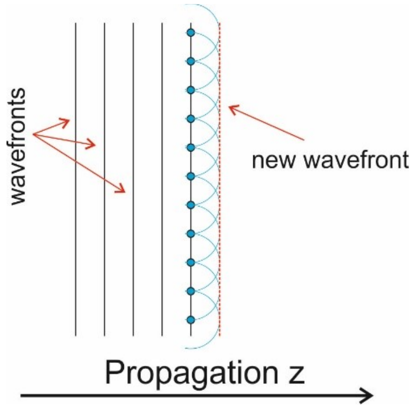
In the previous chapter, light was treated in a classical corpuscular way. Even some of the explanations could be justified using phase continuity, no formalism was introduced. There are some properties that are inherit from the wave nature of light such as:

- Polarization
- Diffraction
- Interference

In order to explain this phenomena, we are going to describe light as an electromagnetic wave

$$\varphi(\vec{r},t) = A(\vec{r})e^{i(\vec{k}\vec{r}-\omega t)}$$

Where $\vec{r}$ is the position vector of the wave, $\omega$ is the carrier frequency and $\vec{k}$ the propagation vector. In addition, for some explanation we are going to assume the Huygens principle that states that every point on a wavefront is itself the source of spherical wavelets, and the secondary wavelets emanating from different points mutually interfere. IN the next image there is an exemplification using a planar wave.




# 2. Polarization

When we talk about polarization we are considering the plane in which the electric field of the photons is oscillating. Each individual photon has its own electric field that can vibrate in any plane perpendicular to the propagation direction.

$$\vec{E} \cdot \vec{k} = 0$$

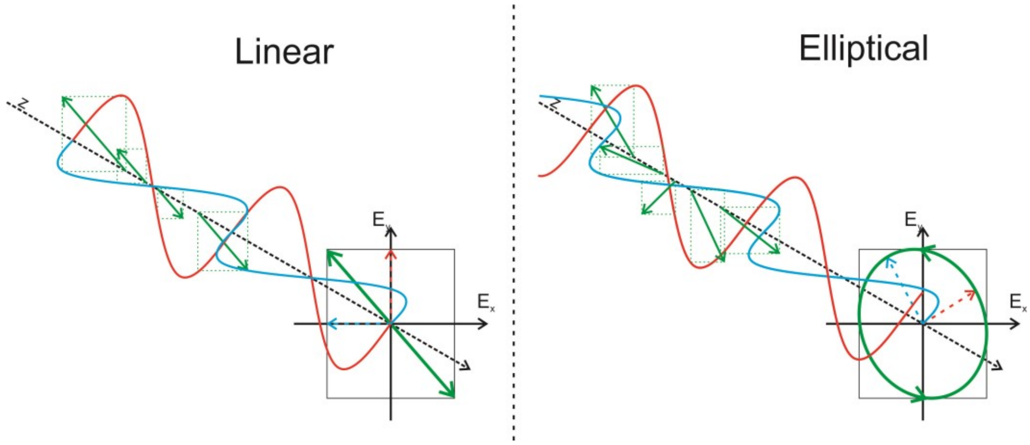
When the all the photons in the light beam vibrate in the same plane we can talk about polarization. If we consider propagation of light in the z axis, we can describe the electric field as:

$$\vec{E}(z,t) = \begin{bmatrix} A_x \\ A_y \end{bmatrix} e^{i(kz-\omega t)}$$

Where $A_{x/y}$ are the amplitudes over the designated x and y axes. Based on the values of the amplitude we can talk about three cases:

If the relative phase between both amplitudes is random ($A_i = Ae^{i\delta}$, no temporal coherence) the light is *unpolarised*.

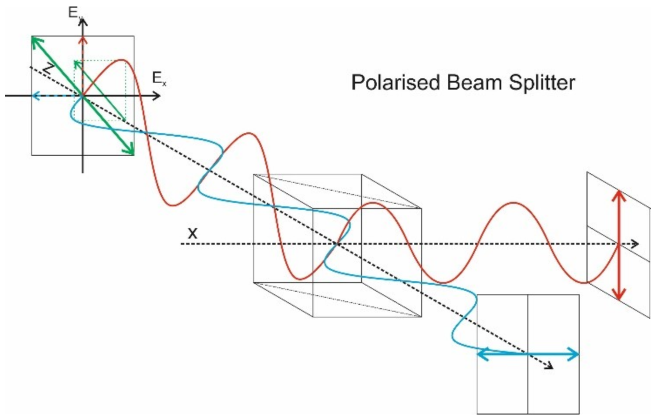
- If both amplitudes are in phase (they are real), the electric field vibrates in the same plane and we have *linear polarization*.
- If there exist a temporal delay between both components, $\frac{A_x}{A_y} \propto e^{i\phi}$, the plane of polarization changes along propagation and we have *elliptical polarization*.




Based on this description of the field it is possible to describe some optical components as matrix that vary the relative intensity and phase of each of the components. The more important to know are beam splitters and waveplates.

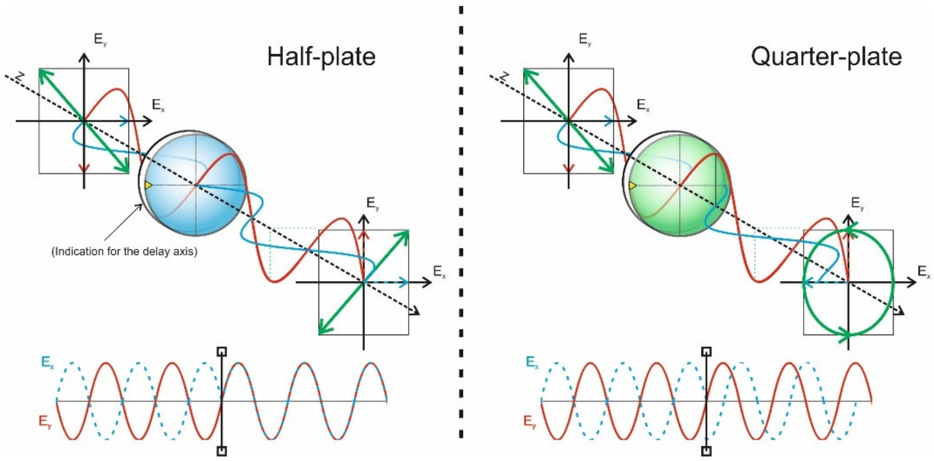
- A *beam splitter* is a component that divides a given beam in two components. Besides, this division can be based on amplitude or in the polarization (polarised beam splitter or commonly cube beam splitter). They can be described as:

$$BS = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$$




- *Waveplates* are optical components that introduce a delay $e^{i\phi}$ between the components of the field. The two more common used are half lambda waveplates ($\phi = \pi$), which transforms a polarization into its orthogonal; and quarter lambda waveplates ($\phi = \frac{\pi}{2}$) which transform linear polarization into elliptical. They can be described as:

$$WP = \begin{bmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{bmatrix}$$




It is important to remember that these components can be rotated respect to the axis of polarization that we have chosen, in order to implement this rotation, we just need to apply a rotation matrix before the component.

$$RT = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

## 3. Fresnel coefficients and Brewster angle

Previously we have seen that light can be reflected or refracted when facing an interface between two different media. Now that we know how to describe the polarization of light we also must ensure continuity in the momentum, which will make different frontier conditions for the electric field that is beating perpendicular to the interface and parallel to it. This condition is introduce considering an incident field, a reflected field and a transmitted field that must fulfil

$$\overrightarrow{E_\iota} + \overrightarrow{E_r} = \vec{E}_t$$

As mention before, we chose the components of the electric field as parallel $A_{||}$ or perpendicular $A_\perp$ to the interface. Applying the proper conditions and using the Maxell equations we can arrive the Fresnel coefficients. These coefficients describe how each component of the field is going to be divided between reflected and transmitted fields.

$$r_\perp = \frac{n_1 cos\theta_1 - n_2 cos\theta_2}{n_1 cos\theta_1 + n_2 cos\theta_2} = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}$$

$$t_\perp = \frac{2n_1 cos\theta_1}{n_1 cos\theta_1 + n_2 cos\theta_2}$$

$$r_{||} = \frac{n_2 cos\theta_1 - n_1 cos\theta_2}{n_1 cos\theta_2 + n_2 cos\theta_1} = -\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)}$$

$$r_{||} = \frac{2n_1 cos\theta_1}{n_1 cos\theta_2 + n_2 cos\theta_1}$$

With this analysis we observe that in a general case the beam is going to be both reflected and refracted but there exists a special case in which the perpendicular component is not reflected, *the Brewster angle*. The denominator in the $r_\perp$ is a $tan$ function that can take the value infinity when the condition $\theta_1 + \theta_2 = \frac{\pi}{2}$. Using the Snell's law and the complementarity of angles ($\sin\frac{\pi}{2} - \theta = \cos\theta$) we can find the condition of the Brewster angle.

$$\theta_B = \arctan\frac{n_2}{n_1}$$

## 4. Interference

In practice we do not measure the electric field of light, but intensity. Intensity is nothing more than the average power transfer over one cycle of the wave. We can define it in different equivalent ways:

$$I = < E|E > = E \cdot E^* = |E|^2 = A^2$$

Where $E = Ae^{i\phi}$ is the electric field at a given point in the space and $E^* = Ae^{-i\phi}$ its conjugate. It is worth to note that this field can be the summation of different fields at that given point which will produce an intensity different that the one produced by the addition of both sources.

$$E = E_1 + E_2$$

$$I = |E_1|^2 + |E_2|^2 + 2 < E_1|E_2 >$$

This last term is the interference between both fields due to the phase between them. When this term is positive we have constructive interference and when it is destructive we have destructive interference. In both cases, the fields $E_1$ and $E_2$ must be coherent between them, if not the interference term will be very fast oscillating leading to an average of 0, therefore not producing interference at all.

To describe properly the effect, we need to apply the Huygens principle mentioned at the beginning, so we consider that every point in a wave front of any shape acts as new spherical source that preserves the amplitude and the phase of that point. The expression of a spherical source is:

$$E(r,t) = \frac{A}{r}e^{i(kr-\omega t)}$$

Where the position $r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$ centred at the source point $r_0 = (x_0, y_0, z_0)$. One easy way to implement this is the single slit diffraction experiment, that shows the nature of light in contraposition to the classical corpuscular description.

*The diffraction experiment is proposed as an exercise

*Recommended bibliography:*


- Eugene Hecht, << Optics >>, 2017 3rd edition Pearson Education, ISBN 9780133977226

- R. A. Serway and J. W. Jewett, << Physics for Scientists & Engineers>>, 6th edition Thomson_Brooks, ISBN 9789386650672 (Chapter 37-38)

- Sears and Zemansky, << University Physics>>, 12th edition 2009 Pearson Education, ISBN 978-607-442-288-7 (Vol. 2, Chapters 35-36)