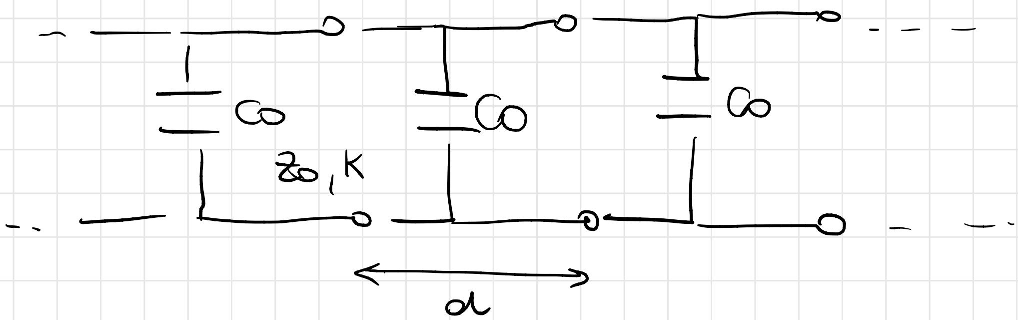
MICROWAVE ENGINEERING

Lecture 29:
Problems on periodic structures and Filters

Problem 1 Analysis of a periodic structure

Consider the following line:



$Z_0 = 50\ \Omega$

$d = 1$ cm

$C_0 = 2.666$ pF

Calculate the propagation constant, phase velocity and Bloch impedance at $f = 3$ GHz. Assume $K = K_0$.

At 3 GHz we have $K_0 d = \frac{2\pi f}{c} \cdot d = \frac{2\pi (3 \cdot 10^9)}{3 \cdot 10^8} \cdot 0.01 =$

$= 0.6283$ rad $= 36°$

Assuming to work in a passband mode, For an infinite structure

$$\boxed{\cos \beta d = \cos \theta - \frac{b}{2} \sin \theta}$$

$$\cos \beta d = \cos k_0 d - \frac{b}{2} \sin k_0 d$$

$$\frac{b}{2} = \frac{\omega C_0 Z_0}{2} =$$

$$= 1.256$$

$$\cos \beta d = \cos(0.6283) - \frac{b}{2} \sin(0.6283) = 0.07$$

$$\Downarrow$$

$$\beta d = 1.5$$

$$\beta = 1.5/0.01 = 150 \ \frac{rad}{m}$$

The phase velocity is:

$$V_p = \frac{k_o c}{\beta} = \frac{k_o d c}{\beta d} = \frac{0.6283}{1.5} \cdot c = \underbrace{0.42\,c}_{\text{slow wave regime}}$$

The Bloch Impedance is

$$Z_B = \frac{B Z_0}{\sqrt{A^2-1}}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta - \frac{b}{2}\sin\theta & j\left(\sin\theta + \frac{b}{2}\cos\theta - \frac{b}{2}\right) \\ j\left(\sin\theta + \frac{b}{2}\cos\theta + \frac{b}{2}\right) & \cos\theta - \frac{b}{2}\sin\theta \end{bmatrix}$$

$$\frac{b}{2} = 1.256$$

$$\theta = kod = 36^\circ$$

$$A = \cos\theta - \frac{b}{2}\sin\theta = 0.0707$$

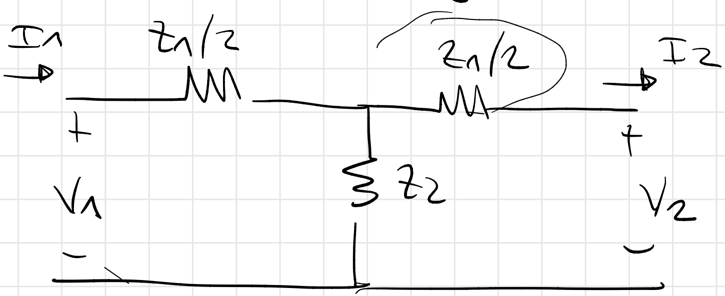
$$B = j\left(\sin\theta + \frac{b}{2}\cos\theta - \frac{b}{2}\right) = j\,0.3479$$

$$Z_B = \frac{BZ_0}{\sqrt{A^2-1}} = \frac{j0.3479\cdot 50}{\underbrace{\sqrt{0.0707^2-1}}} =$$

$$= \frac{j0.3479\cdot 50}{j\left(\sqrt{1-0.0707^2}\right)} = 17.4\,\Omega$$

Problem 2

Extract the image impedance and propagation constant of the following T-network:



$$Z_{iT} = \sqrt{\frac{AB}{CD}}$$

$$e^{\gamma} = \sqrt{AD} + \sqrt{BC}$$

Let's calculate the ABCD parameters for the network:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 + \frac{Z_1}{2Z_2}$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = Z_1 + \frac{Z_1^2}{4Z_2}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{Z_2}$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = 1 + \frac{Z_1}{2Z_2}$$

$$z_\pi = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{\cancel{\left(1+\frac{z_1}{2z_2}\right)}\left(z_1+\frac{z_1^2}{4z_2}\right)}{\frac{1}{z_2}\cancel{\left(1+\frac{z_1}{2z_2}\right)}}} =$$

$$= \sqrt{z_2 z_1 + \frac{z_1^2}{4}} = \sqrt{z_1 z_2}\sqrt{1+\frac{z_1}{4z_2}}$$

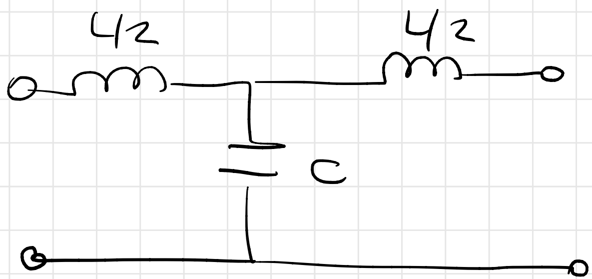
$$e^\gamma = \sqrt{AD} + \sqrt{BC} = 1 + \frac{z_1}{2z_2} + \sqrt{\frac{1}{z_2}\left(z_1+\frac{z_1^2}{4z_2}\right)} =$$

$$= 1 + \frac{z_1}{2z_2} + \sqrt{\frac{z_1}{z_2} + \frac{z_1^2}{4z_2^2}}$$

Problem 3 Low-pass composite filter design

Design a low-pass composite filter with a cutoff frequency at $f_c = 2\,MHz$ and impedance $R_0 = 75\,\Omega$. Consider the infinite attenuation pole $f_\infty = 2.05\,MHz$.

The constant-k T section circuit



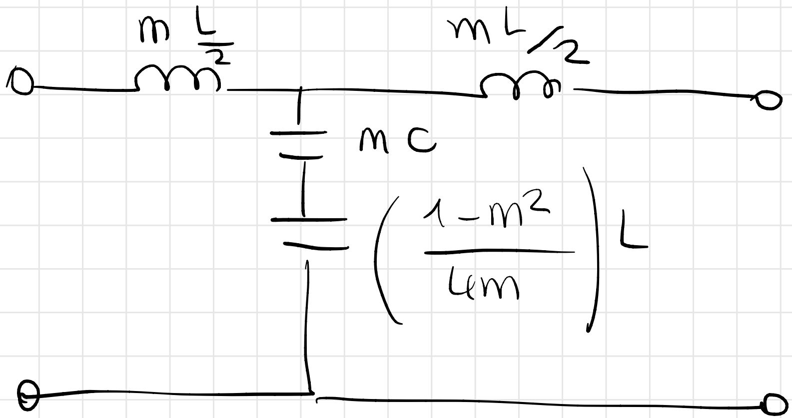
$$R_0 = \sqrt{\frac{L}{C}}$$

$$\omega_c = \frac{2}{\sqrt{LC}}$$

$$L = \frac{2R_0}{\omega_c} = \frac{2R_0}{2\pi f_c} = \frac{2 \cdot 75}{2\pi \cdot 2 \cdot 10^6} = 11.93\,\mu H$$

$$C = \frac{2}{w_c R_o} = \frac{2}{2\pi f_c R_o} = \frac{\cancel{2}}{\cancel{2}\pi\, 2 \cdot 10^6 \cdot 75} = 2.12 \text{ nF}$$

We will then add a sharp cut-off section :



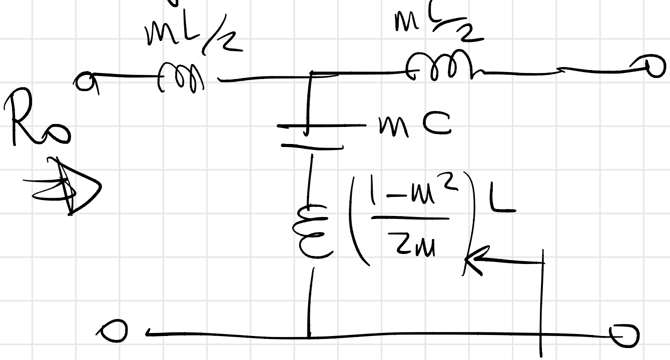
$$m = \sqrt{1 - \left(\frac{w_c}{w_\infty}\right)^2} = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2} = 0.2195$$

$$\frac{mL}{2} = 1.31\ \mu H$$

$$mC = 465.8\ pF$$

$$\frac{1-m^2}{4m} L = 12.94\ \mu H$$

Finally we add a matching section:



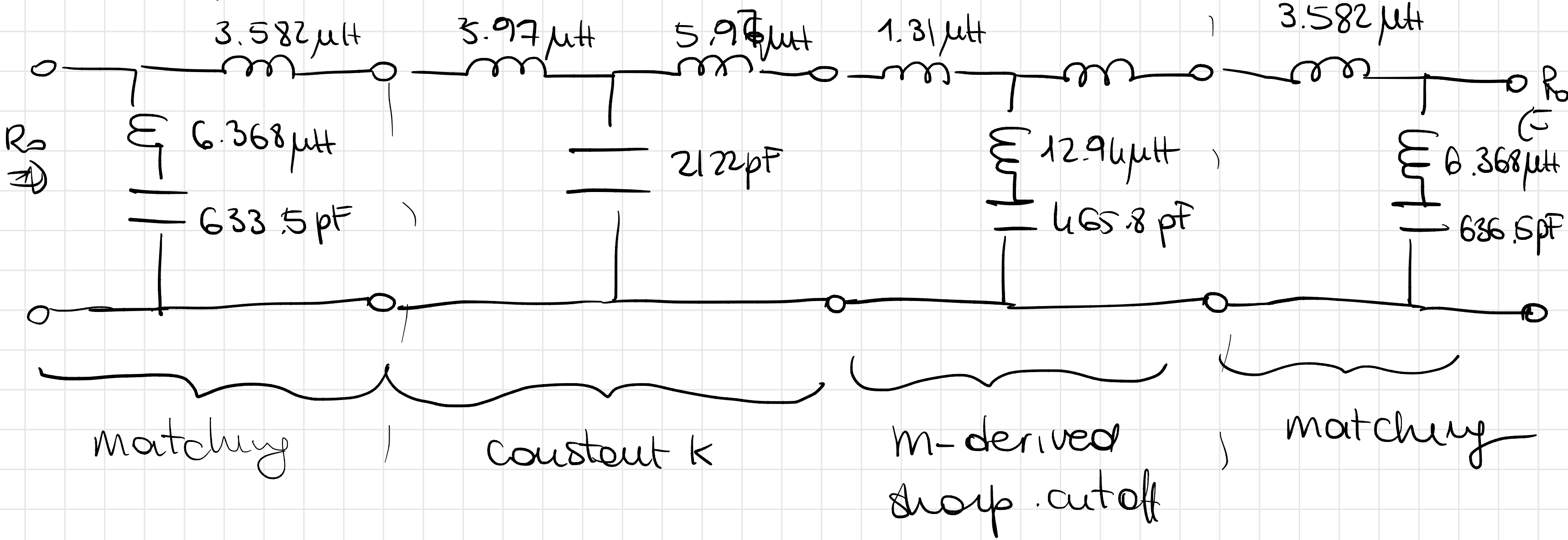
$$m = 0.6$$

$$m\frac{L}{2} = 3.582\ \mu H$$

$$mC = 636.5\ pF$$

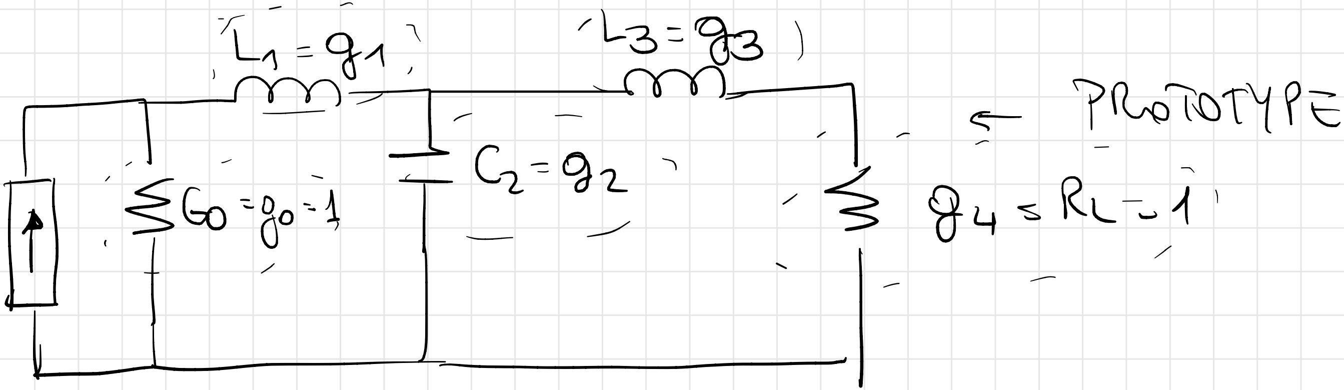
$$\frac{1-m^2}{2m} L = 6.368\ \mu H$$

The composite filter then is:



Problem 4 Bandpass filter design

Design a bandpass filter having a 0.5 dB equal ripple response with $N=3$. The center frequency is 1GHz, the bandwidth is 10% and the impedance is 50 Ω.



$g_1 = L_1 = 1.5963$

$g_2 = C_2 = 1.0967$

$g_3 = L_3 = 1.5963$

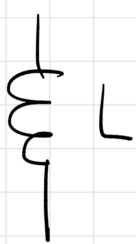
$g_4 = R_L = 1$

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0} = 0.1$$

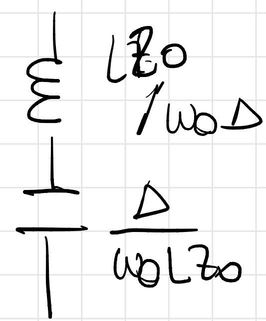
$$\omega_0 = 1 \cdot 10^{9} \text{ Hz}$$

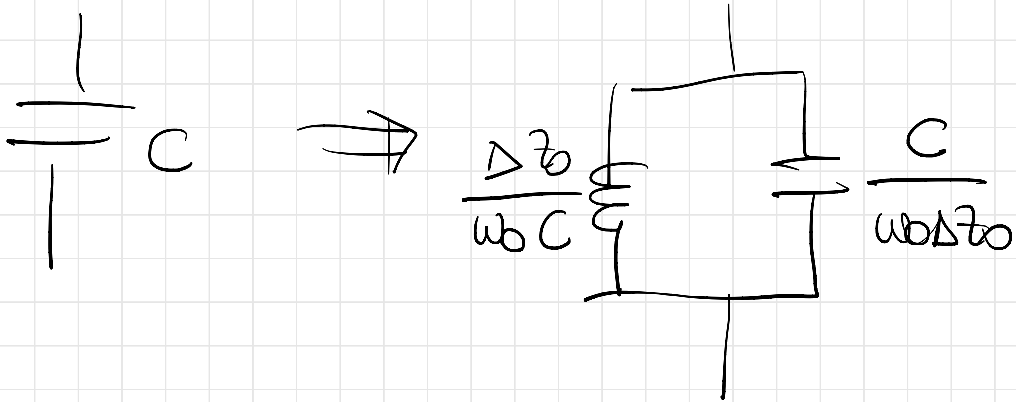
$$Z_0 = 50\ \Omega$$

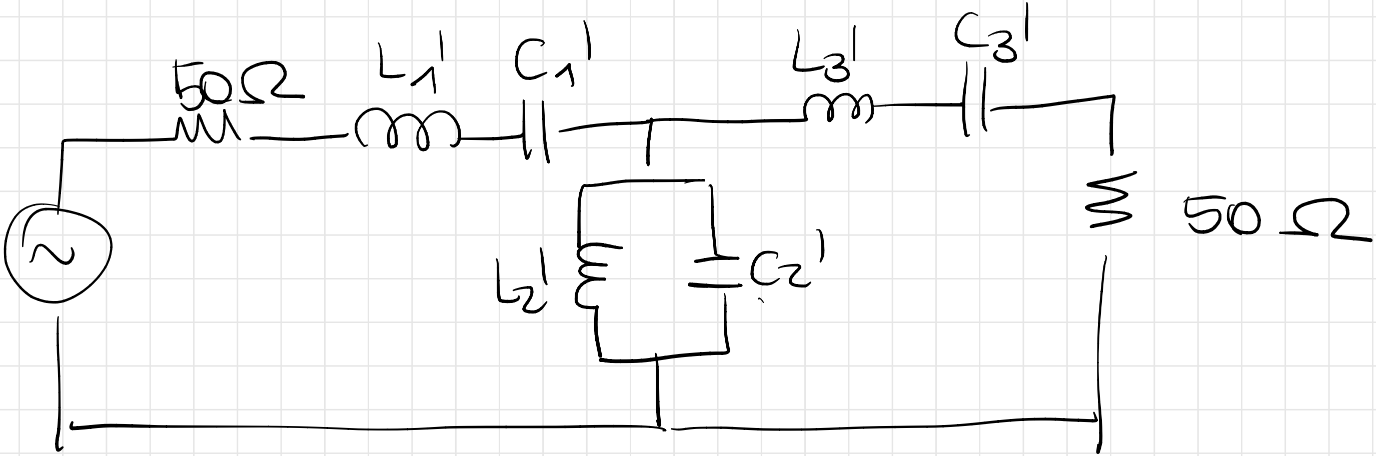
We can scale the low-pass prototype elements into a bandpass elements values (scaling also for impedance)



$\Longrightarrow$




C
$\frac{\Delta z_0}{\omega_0 C}$
$\frac{C}{\omega_0 \Delta z_0}$


50Ω
L_1'
C_1'
L_3'
C_3'
L_2'
C_2'
50 Ω

$$L_1' = \frac{L_1 Z_0}{\omega_0 \Delta} = 127 \text{ nH}$$

$$C_1' = \frac{\Delta}{\omega_0 L_1 Z_0} = 0.199 \text{ pF}$$

$$L_2' = \frac{\Delta Z_0}{\omega_0 C_2} = 0.726 \text{ nH}$$

$$C_2' = \frac{C_2}{\omega_0 \Delta Z_0} = 34.91 \text{ pF}$$

$$L_3' = \frac{L_3 Z_0}{\omega_0 \Delta} = 127 \text{ nH}$$

$$C_3' = \frac{\Delta}{\omega_0 L_3 Z_0} = 0.199 \text{ pF}$$