Basics of Active and Nonlinear HF Electronics – Tutorial

We consider a FET and its nonlinear drain current source I_{DS} modeled by the below equation as a function of its control voltages V_{GS} and V_{DS} :

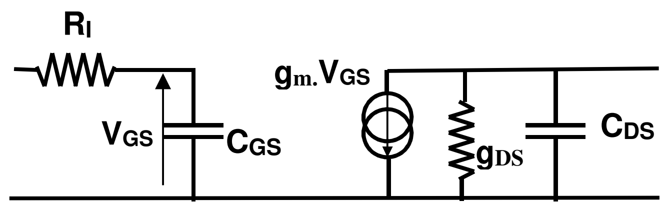
$$I_{DS} = I_{DSS}\left[1 - \frac{V_{GS}(t)}{V_P}\right]^2 (1 + K.V_{DS}(t))$$

The model is accurate in the variation range $-1.5V < V_{GS} < 0V$; $1V < V_{DS} < 7V$

with $I_{DSS} = 60$ mA; $V_P = -1.5$ V; $K = 0.05$; $C_{GS} = 0.5$ pF; $C_{DS} = 0.1$ pF; $R_I = 4\ \Omega$.

The FET is biased at $V_{GS0} = -0.75$ V and $V_{DS0} = 4$ V

The small signal linear model of the FET in these conditions is shown below:



Study of parallel-cells of FET

1) Draw the I_{DS}-V_{DS} curves in the variation range
2) Determine the values of transconductance g_m and drain conductance g_{DS} at the selected bias point.
3) Determine the small-signal linear model corresponding to n parallel-cells of the FET. Determine the variation laws (scaling rules) that give equivalent elements ($C_{GS}(n)$ …) of the parallel-cells as a function of the number of cells n and the equivalent elements ($C_{GS}(1)$ …) of a single-cell FET.
4) Determine the maximum power gain G_{MAX} for an ideal power matching at small-signal (linear) level. Determine its cutoff frequency f_C and its maximum frequency f_{MAX}.
5) After determining the optimum power load in large signal class A operation, determine the maximum gain G_{MAX} in this loading condition. Estimate the maximum output power value in class A. Compare these values to G_{MAX} and its associated maximum output power when the FET is ideally matched to give optimum gain in small signal operation.
6) Same questions 4) and 5) in the case of n-parallel FETs

$$I_{DS} = I_{DSS}\left[1-\frac{V_{GS}(t)}{V_P}\right]^2 (1+K.V_{DS}(t)) \quad *$$

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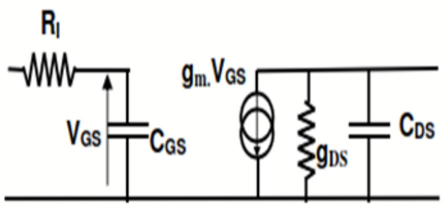
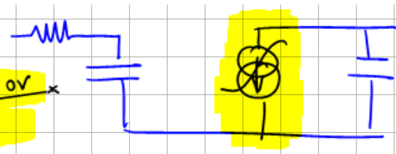
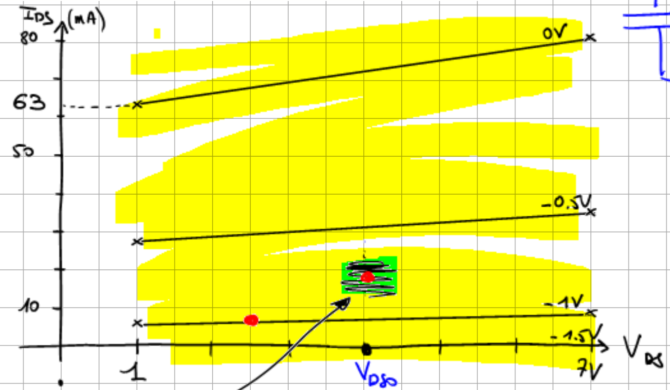
with $I_{DSS} = 60$ mA; $V_P = -1.5$ V; $K = 0.05$; $C_{GS} = 0.5$ pF; $C_{DS} = 0.1$ pF; $R_i = 4\ \Omega$.

The FET is biased at $V_{GS0} = -0.75$ V and $V_{DS0} = 4$ V Bias point





V_{GS}	V_{DS}	$\left(1-\frac{V_{GS}}{V_P}\right)^2$	$1+K\,V_{DS}$	I_{DS}
0V	1V	1	1.05	63 mA
	7V		1.35	81 mA
-0.5V	1V	$\left(1-\frac{-0.5}{-1.5}\right)^2 = \frac{4}{9}$	1.05	28 mA
	7V		1.35	36 mA
-1V	1V	$\left(1-\frac{-1}{-1.5}\right)^2 = \frac{1}{9}$	1.05	7 mA
	7V		1.35	9 mA



$$f(x,y) = f(x_0,y_0) + df$$

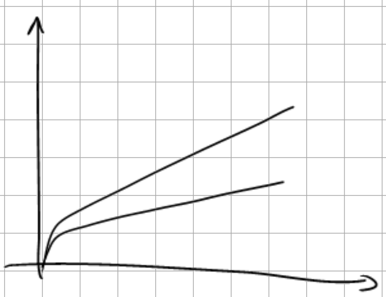
$$\left.\begin{array}{l} x = x_0 + dx \\ y = y_0 + dy \end{array}\right\} \qquad df = \left(\frac{\partial f}{\partial x}\right)_{(x_0,y_0)} dx + \left(\frac{\partial f}{\partial y}\right)_{(x_0,y_0)} dy$$

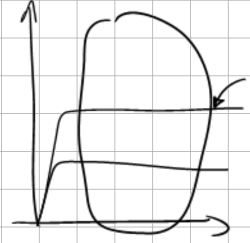
$$I_{DS} = I_{DSS}\left[1-\frac{V_{GS}(t)}{V_P}\right]^2 (1+K.V_{DS}(t))$$

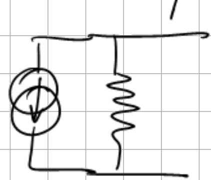
$$g_m(V_{GS},V_{DS}) = \left(\frac{\partial I_{DS}}{\partial V_{gs}}\right)_{(V_{gs0},V_{ds0})} = -2\,\frac{I_{DSS}}{V_P}\left(1-\frac{V_{GS_0}}{V_P}\right)\left(1+K\,V_{DS_0}\right) = -2\,\frac{60\text{ mA}}{-1.5}\left(1-\frac{-0.75}{-1.5}\right)\left(1+0.005\times 4\right) = 48\text{ m}\,\Omega^{-1}$$

$$g_{DS} = \left(\frac{\partial I_{DS}}{\partial V_{DS}}\right)_{(V_{gs0},V_{ds0})} = K\,I_{DSS}\left(1-\frac{V_{GS_0}}{V_P}\right)^2 = 0.005\times 60\text{ mA}\left(1-\frac{-0.75}{-1.5}\right)^2 = 0.75\text{ m}\,\Omega^{-1}$$

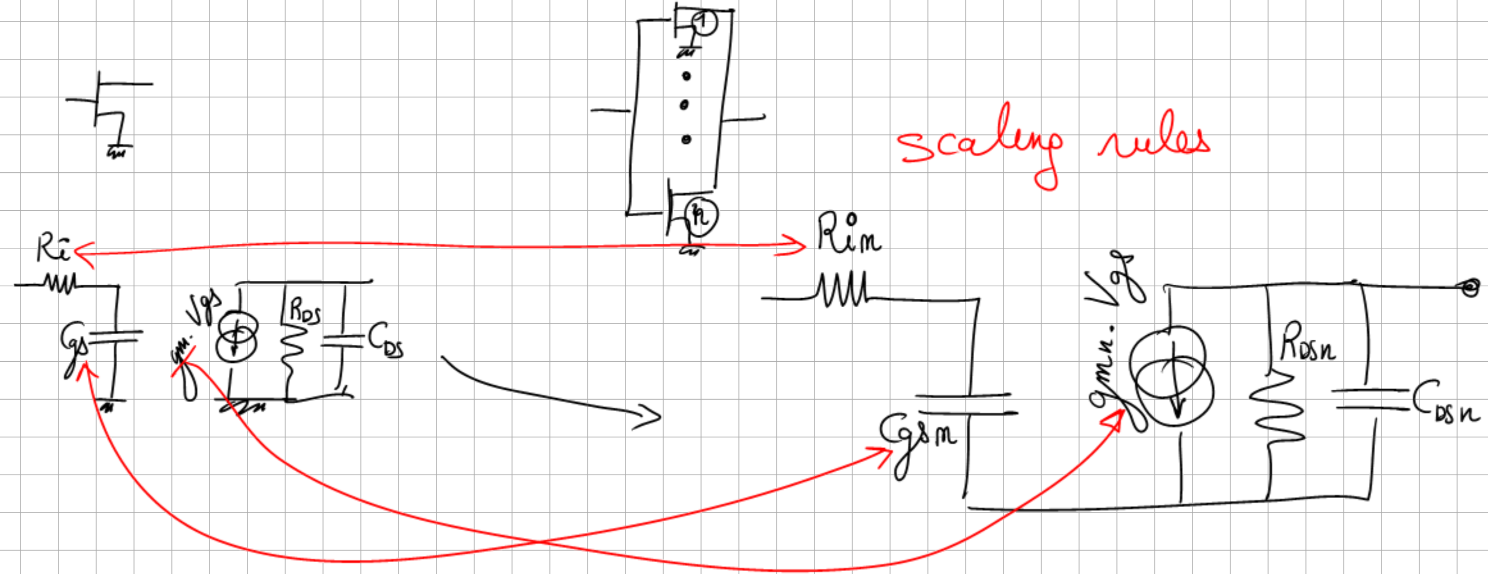
$$\boxed{R_{DS} = \frac{1}{g_{DS}} = 1.3\text{ k}\Omega}$$

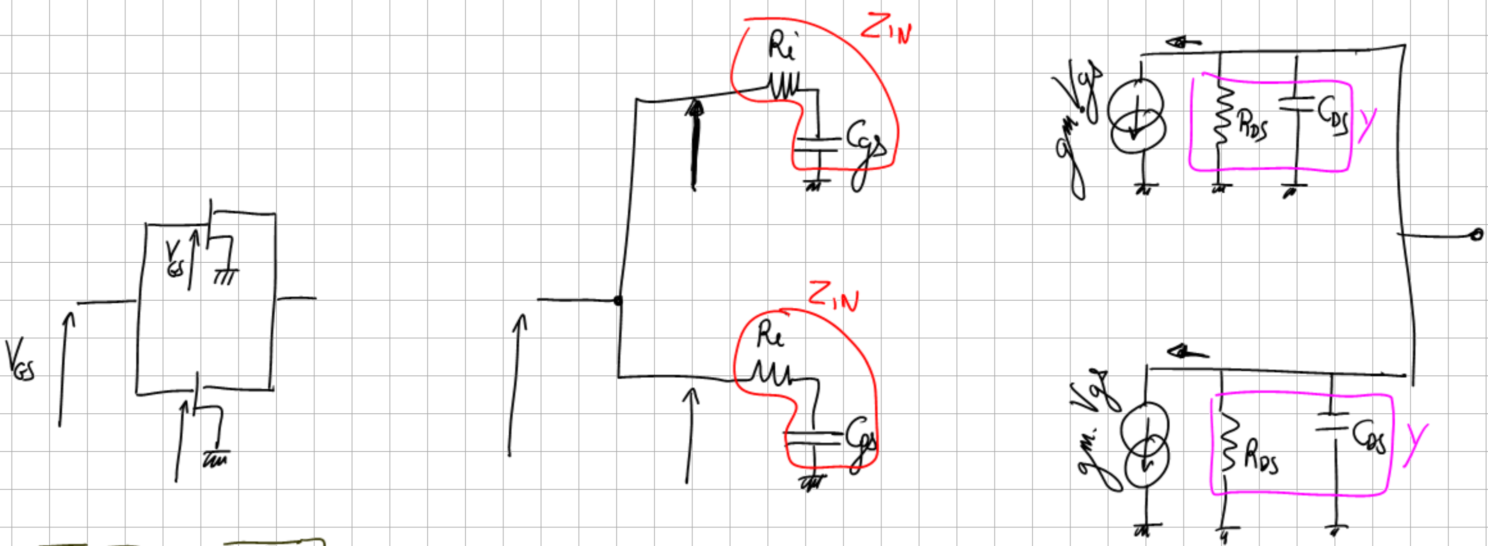


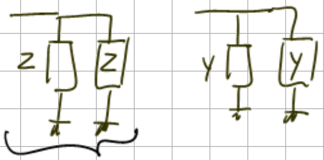


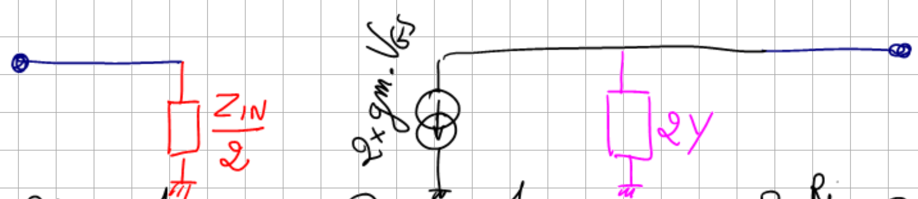


scaling rules



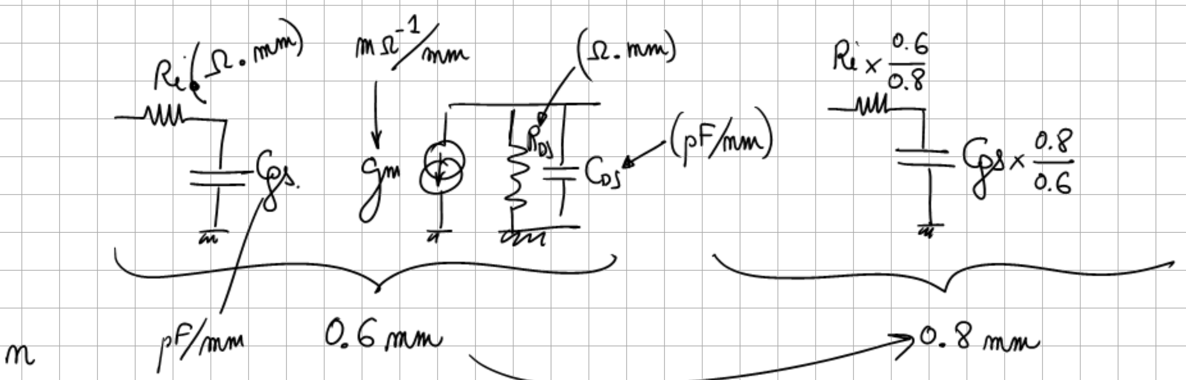






$$Z_{IN} = R_i + \frac{1}{jC_{GS}\omega} \longrightarrow \frac{Z_{IN}}{2} = \frac{R_i}{2} + \frac{1}{j2C_{GS}\omega} = \underbrace{R + \frac{1}{jC\omega}} \longrightarrow R = \frac{R_i}{2} \quad C = 2C_{GS}$$

$$\underbrace{Y = \frac{1}{R_{DS}} + jC_{DS}\omega} \longrightarrow 2Y = \frac{2}{R_{DS}} + j2C_{DS}\omega = \underbrace{\frac{1}{R_p}} + j\underbrace{C_p\omega} \longrightarrow R_p = \frac{R_{DS}}{2} \quad C_p = 2C_{DS}$$



Technology :
$$\left\{\begin{aligned} R_i &= 1\,\Omega.\mathrm{mm} \times \\ R_{DS} &= 125\,\Omega.\mathrm{mm} \times \\ C_{gs} &= 2\,\mathrm{pF/mm} \times \end{aligned}\right\}$$

$T_1 \to 8 \times 75\,\mu m = 0.6\,mm$

$$R_{i_1} = \frac{1\,\Omega.\mathrm{mm}}{0.6}$$

$$R_{DS1} = \frac{125}{0.6}$$

$$C_{gs_1} = 2\,\mathrm{pF} \times 0.6$$

$$\boxed{\begin{aligned} R_i,\ R_{DS} &\to \frac{R_i}{n},\ \frac{R_{DS}}{n} \\ g_m,\ C_{gs},\ C_{DS} &\to n\,g_m,\ n\,C_{gs},\ n\,C_{DS} \end{aligned}}$$

Scalingrules

4) Determine the maximum power gain G_{MAX} for an ideal power matching at small-signal (linear) level. Determine its cutoff frequency f_C and its maximum frequency f_{MAX}.

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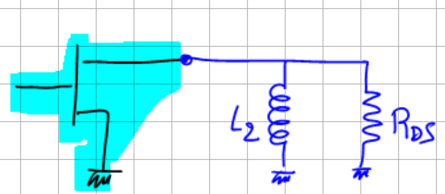


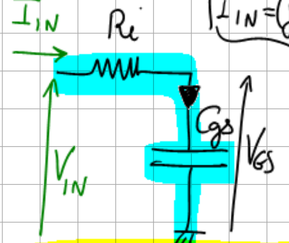
optimum loading for Gmax @ f_0

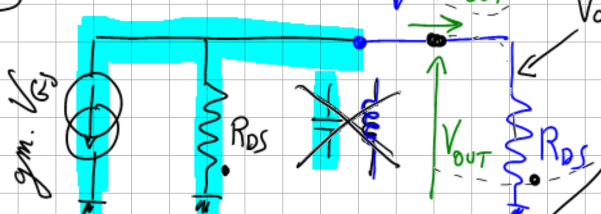
$$Y_{C_{DS}} = j\,C_{DS}\,\omega_0 + Y_{L_2} = \frac{-j}{L_2\,\omega_0} = 0$$

$$C_{DS}\,\omega_0 = \frac{1}{L_2\,\omega_0}$$

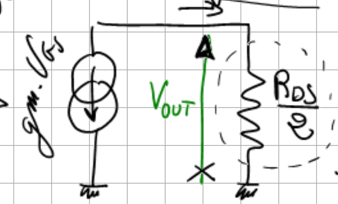
$$\boxed{L_2 = \frac{1}{C_{DS}\,\omega_0^2}}$$

$\boxed{I_{IN} = (j\,C_{gs}\,\omega)\,V_{gs}}$





$V_{OUT} = R_{DS}\,I_{OUT}$

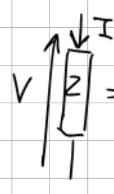


$$P_{IN} = \frac{1}{2}\mathrm{Re}\left(V_{IN}\,I_{IN}^*\right) = \frac{1}{2}\mathrm{Re}\left(\underbrace{\left[R_i + \cancel{\frac{1}{j C_{gs}\omega}}\right] I_{IN}}_{V_{IN}} \times I_{IN}^*\right) = \frac{1}{2} R_i\,|I_{IN}|^2 = \frac{1}{2} R_i\,|j\,C_{gs}\,\omega\,V_{gs}|^2$$

$$= \frac{1}{2} R_i\,C_{gs}^2\,\omega^2\,|V_{gs}|^2$$

$$P_{OUT} = \frac{1}{2}\mathrm{Re}\left(V_{OUT}\,I_{OUT}^*\right) = \frac{1}{2}\mathrm{Re}\left(V_{OUT}\,\frac{V_{OUT}^*}{R_{DS}}\right) = \frac{1}{2}\,\frac{|V_{OUT}|^2}{R_{DS}} = \frac{1}{2}\,\frac{g_m^2\,R_{DS}^2}{4\,R_{DS}}\,|V_{GS}|^2$$

$$= \frac{1}{8}\,g_m^2\,R_{DS}\,|V_{GS}|^2$$

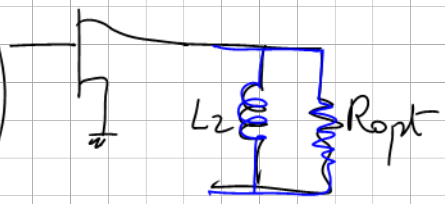


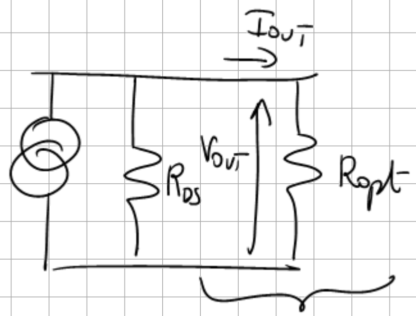
$\boxed{V = Z\,I} \Rightarrow \boxed{I = Y\,V}$

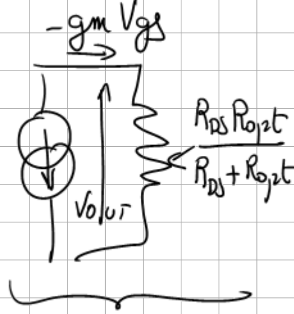
$$V_{OUT} = -\frac{g_m\,R_{DS}}{2}\,V_{GS}$$

$$\boxed{G_{max} = \frac{P_{OUT}}{P_{IN}} = \frac{g_m^2\,R_{DS}}{4\,R_i\,C_{gs}^2\,\omega^2}}$$

POWER MATCHING







$$I_{OUT} = \frac{V_{OUT}}{R_{opt}}$$

$$V_{OUT} = \frac{R_{DS} R_{opt}}{R_{DS} + R_{opt}} \left(-g_m V_{gs}\right)$$