Optoelectronics

Key notes

1/ Geometrical optics and ray tracing

1. Light nature, duality wave-particle

Wave–particle duality is the concept in quantum mechanics that every particle or quantum entity may be described as either a particle or a wave. Light is not an exception to this rule and It has been the first discussed under this scope.

This debate has been a hot topic since the first studies of Newton after which, the light, has been understood as a current of particles until the first evidences of its wave nature by the end of the XVII century. It is with the discoveries and formalism of the Maxwell equations (1873) when in fact the evidence of the light being an electromagnetic wave.

Nevertheless, the wave conception of light does not provide the whole picture about its nature since multiple effects as emission or absorption cannot be fully explained from this point of view. It is only with the quantum formalism (1930) that both conceptions (wave and particle) can coexist.

Even the quantum formalism is the most complete and allows to fully explain mathematically light, here we are going to focus on the main effects and properties related to light from either a particle or a wave nature in order to simplify the study. The more relevant properties to our point of view (optoelectronics) are:

- Refraction
- Reflexion
- Chromatic dispersion
- Polarization
- Diffraction
- Interference

In this document there are covered the first

2. Refraction

Refraction is the redirection of a wave as it passes from one medium to another. The redirection can be caused by the wave's change in speed or by a change in the medium. This effect is the most common in daily life and explain a lot of optical effects.

The speed of light is a constant independently of the medium we are referenced to but that cannot be equally said by its phase. The phase of light is determined by its wavelength and the speed of the phase in this medium v , which is $v_0 = \frac{c_0}{\lambda}$ in vacuum. The relation between the phase velocity of a wave in a given medium and the velocity in vacuum is denoted as the refractive index

$$n = \frac{c}{v}$$

This magnitude is always bigger (or equal) to 1. The refractive index provides us a sense on how strong is the interaction between light and the medium it is traveling through, the stronger the interaction the higher the index (in a general case). It is also important to say that different wavelengths ,λ, will interact differently with a given material. In a general case this refractive index will be dependant in the wavelength of our light.

$$n = n(\lambda)$$

The way that the refractive index varies along the spectrum (wavelengths) separate materials in to groups: normal dispersion materials and anomalous dispersion materials. This variation is called chromatic dispersion $\frac{dn}{d\lambda}$

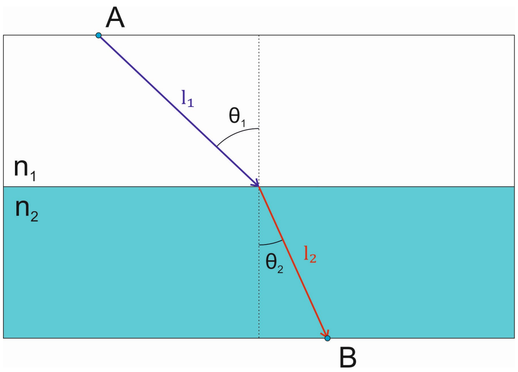
$$\frac{dn}{d\lambda} < 0 \quad \rightarrow \quad Normal\ dispersion$$
$$\frac{dn}{d\lambda} > 0 \quad \rightarrow \quad Anomalous\ dispersion$$

This variation in the phase velocity will produce a change in the direction of propagation of light when it crosses an interface between two mediums. This effect is refraction and it can be easily explained in two ways: The Fermat's principle or interface wavefront continuity. Both explanations provide the same solution which is the called Snell's law.

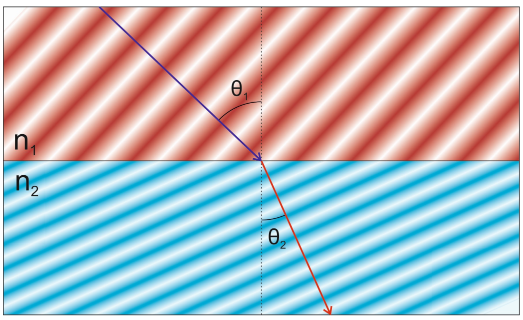
$$n_1 sin\theta_1 = n_2 sin\theta_2$$

In order to describe the problem in a general way we are going to consider a ray of light traveling in a medium of refractive index n_1. It will face a new medium with a refractive index n_2 under an angle to the normal θ_1. After crossing the interface, the light will change the direction of propagation and travel with an angle θ_2 (see figures for clarification).

Fermat's principle: This principle says that light travels between two points A and B using the path that requires the least time to travel. As the figure shows, we can describe our problem under this scope. The minimization of the path (l_1+l_2) will always results in the Snell's law.



Wavefront continuity: In the interface between mediums, the wave will change its velocity, so its frequency. As we must assure the continuity of the phase in the interface of both mediums, the direction of propagation must change to make both wavefronts match.



As said before, this both approaches to the problem provide the same solution, the Snell's law. This equation has an interesting limit case, the critical angle. Given two media with refractive index n_1 and n_2 so the second media has smaller refractive index $n_2 < n_1$, there is a maximum angle that can be transmitted to the second medium, the critical angle. In fact, at this angle, the light it not transmitted but travels along the interface between both.

$$\sin\theta_c = \frac{n_2}{n_1}\sin 90 = \frac{n_2}{n_1}$$

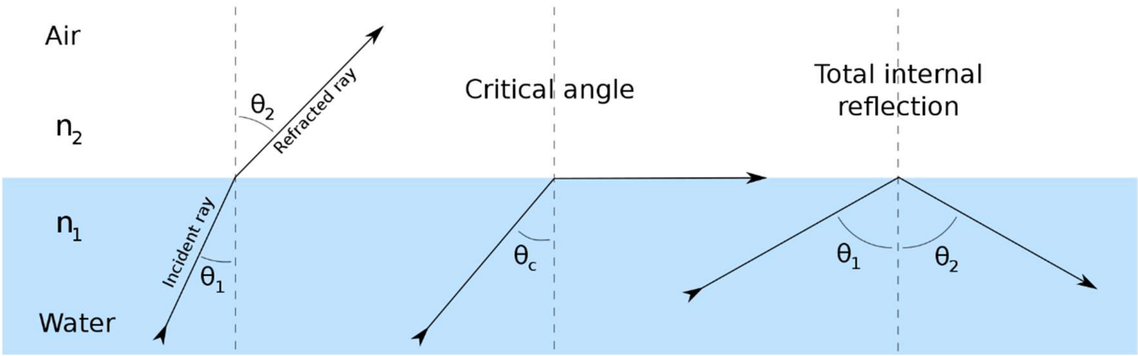
With any angle greater than the critical angle the light cannot cross between both mediums and so it will be reflected back to the first medium

3. Reflexion

Reflection is the case when the light cannot cross to the second medium, this can happen for several reasons, the second medium is metallic, the angel of incidence is greater than the critical or continuity reasons. In fact, light is usually both refracted and reflected as it will be explained in the polarization point.

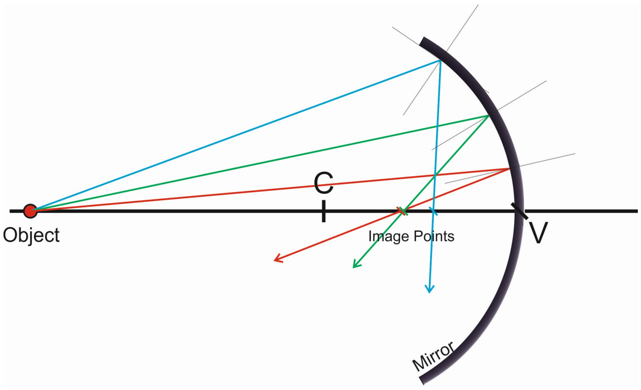
As the light is reflected in the interface, the direction of propagation will change again but in this case to be back in the same medium. In the same manner as we can demonstrate the Snell's law through the Fermat's principle, for the problem of reflection we arrive to the only solution that the angle of incidence θ_i and the angle of reflexion θ_r must be equal

$$\theta_i = \theta_r$$



4. Lens and mirrors

Image formation using lens and mirrors is based in these principles, we just need to consider extent sources (aka objects) and non-planar surfaces (in general spherical). The easiest example is considering a punctual object and a spherical mirror. The image of this punctual object is formed where all the reflected rays cross. As we can see in the following figure, not all the rays cross in the same spot so our image is going to be blur (this condition is called spherical aberration). To avoid this problem, we only need to consider small angles diverging from the axis so all the rays cross in (almost) the same point, this is the paraxial regime.

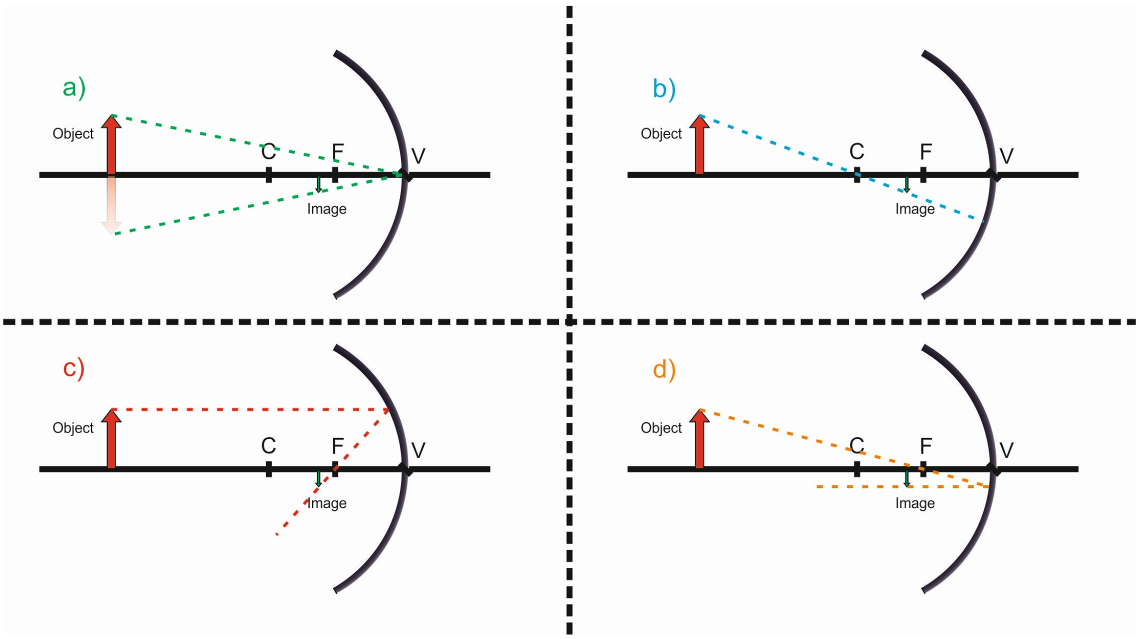


Under the paraxial regime, we can trace two rays and whatever their directions are and still obtain the same point, the image point. There is a particular point characterized by the surface called the focal point. This position in the axis is where all the rays coming parallel to the axis will intersect after the reflection. Its position is determined by the radius of curvature

$$f = \frac{R}{2}$$

There are several rays that we can trace in order to obtain the position of the image and we can use whatever we desire. For practical reasons, there is a set of 4 rays commonly used, the so called main rays or principal rays

a) The ray that goes to the intersection of the axis and the surface. This ray is going to be reflected symmetrically to the axis.
b) The rays that crosses the centre of curvature. This is going to be perpendicular to the surface so it is going to be reflected in the same direction
c) The ray parallel to the axis. This ray is going to cross the focal point
d) The ray that crosses the focal point. This ray is going to be reflected parallel to the axis



Using two of this rays we can obtain by raytracing the image position and size. In general, this image is not going to be as the same size as the original object, it is going to be reduced/augmented and can also be flipped. The relation between the height of the object (h) and the image (h') is the magnification factor.

$$M = \frac{h'}{h}$$

As general criteria we consider that the origin of coordinates is in the surface of the mirror where it crosses our axis (V). Given this origin of coordinates, all the positions are determined as positive (right from V) or negative (left from V). Using this reference, the reflexion condition and the paraxial approximation we can obtain the analytical expression that relates the object (p) and image (q) positions.

$$\frac{1}{q} + \frac{1}{p} = \frac{2}{R}$$

Exactly the same analysis can be done for refraction, the only difference is applying the Snell's law instead of the refraction condition. In a given case with two media with indexes 1 and 2 and a spherical interface of radius R. This leads to:

$$\frac{n_2}{q} - \frac{n_1}{p} = \frac{(n_2 - n_1)}{R}$$

With a magnification factor:

$$M = \frac{h'}{h} = \frac{n_1}{n_2}\frac{q}{p}$$

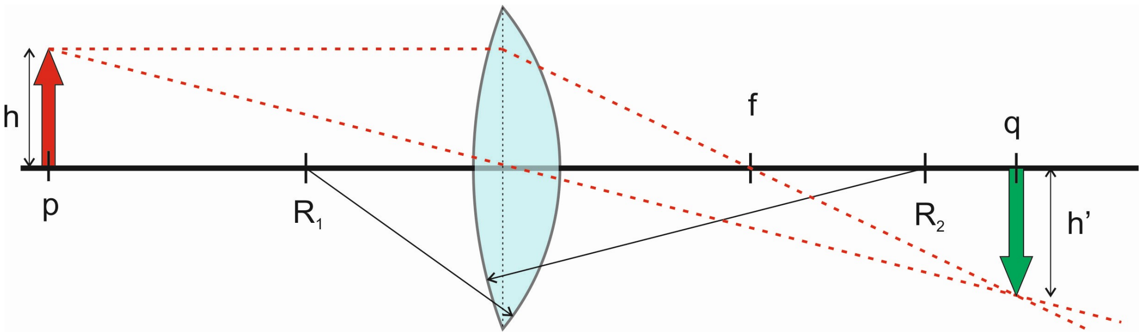
One can easily see that using a mirror is a particular case where $n_1 = -n_2 = 1$.

Using this equations, we can describe the image formation due to a lens, which is nothing more than two spherical surfaces where the first medium and the last are air.

$$\frac{1}{q} - \frac{1}{p} = \frac{1}{f}$$

Being the focal of the lens:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

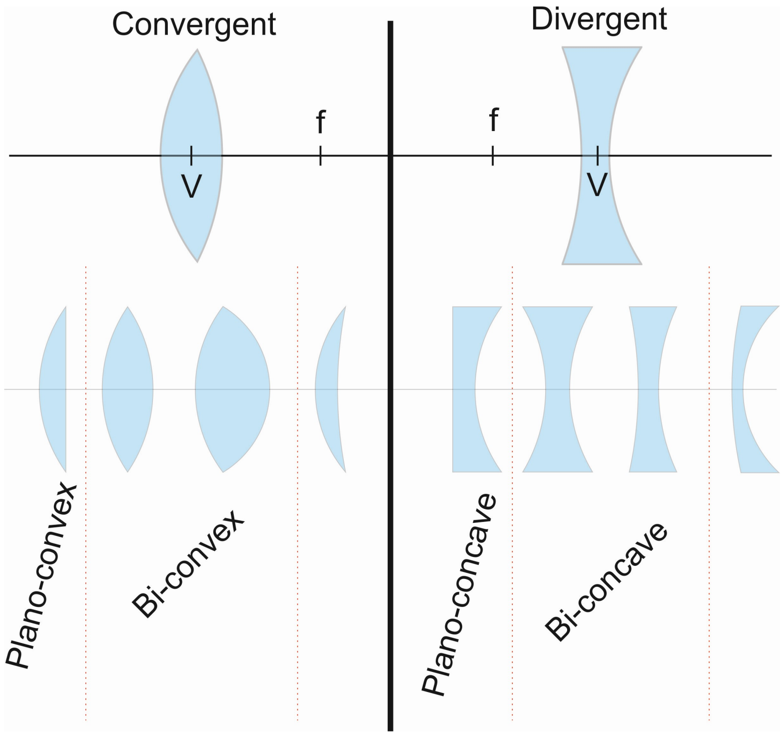


Common lens are made of silica so their refractive index $n = 1.45$

5. Types of lens and Images

Finally, Let's make a quick review on the type of lenses that we can obtain varying the values of R_1 and R_2. Basically there exists two types of lens, convergent and divergent, whose names indicates if the angle of divergence of the beam is going to increase or decrease after the lens.

In terms of values, convergent lens are denoted by positive focal and divergent lenses are noted by negative focals.



In the same sense it is possible to classify the image created by a lens or a mirror based on the position of the image.

- If the image position is negative (before the lens), the image is virtual
- If the image position is positive (after the lens), the image is real

The difference between a real and a virtual image is the fact that it is not possible to observe the virtual image using a plane as it should be placed in between the lens and the object. Also the magnifying factor provides information

- If $M > 1$, the image is augmented.
- If $1 > M > 0$, the image is reduced
- If $0 > M > -1$, the image is reduced and flipped (respect the axis)
- If $M < -1$, the image is augmented and flipped

Recommended bibliography:

- Eugene Hecht, << Optics >>, 2017 3^{rd} edition Pearson Education, ISBN 9780133977226

- R. A. Serway and J. W. Jewett, << Physics for Scientists & Engineers>>, 6^{th} edition Thomson_Brooks, ISBN 9789386650672 (Chapter 35-36)

- Sears and Zemansky, << University Physics>>, 12^{th} edition 2009 Pearson Education, ISBN 978-607-442-288-7 (Vol. 2, Chapters 33-34)