MICROWAVE ENGINEERING

Lecture 27:
Microwave
Filters - part 1

MICROWAVE FILTERS

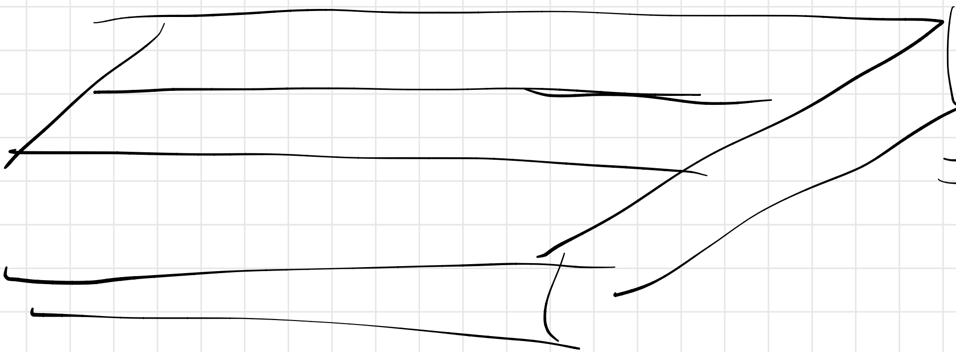
Typical Response :
- LOW-PASS
- HIGH-PASS
- BAND-PASS
- BAND-REJECT (STOP-BAND)

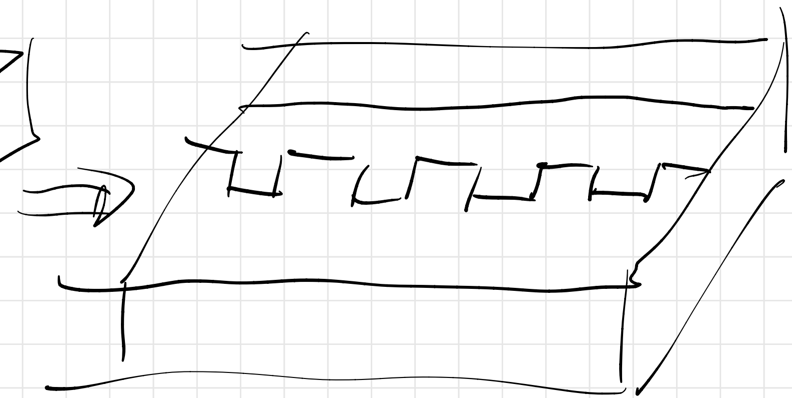
Filter is composed of WAVEGUIDE or transmission line periodically loaded with reactive elements

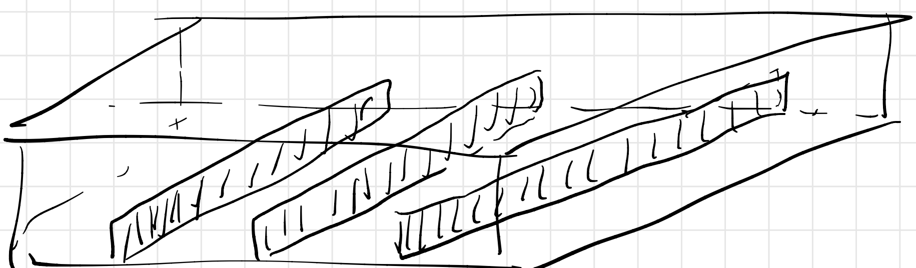
Methods for design → IMAGE PARAMETER METHOD

→ INSERTION LOSS METHOD

FILTERS ARE PERIODIC STRUCTURES







Analysis of infinite periodic structures

An infitely long periodic structure can be modeled through small cells of length d with suscaptance b which will be described with ABCD matrix:

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix}$$

$$\left[\frac{d}{2}\right][b]\left[\frac{d}{2}\right]$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\frac{\theta}{2} & j\sin\frac{\theta}{2} \\ j\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}}_{d/2 \text{ line}} \underbrace{\begin{bmatrix} 1 & 0 \\ jb & 1 \end{bmatrix}}_{\text{shunt susceptance}} \underbrace{\begin{bmatrix} \cos\frac{\theta}{2} & j\sin\frac{\theta}{2} \\ j\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}}_{d/2 \text{ line}}$$

$$= \begin{bmatrix} \left(\cos\theta - \frac{b}{2}\sin\theta\right) & j\left(\sin\theta + \frac{b}{2}\cos\theta - \frac{b}{2}\right) \\ j\left(\sin\theta + \frac{b}{2}\cos\theta + \frac{b}{2}\right) & \left(\cos\theta - \frac{b}{2}\sin\theta\right) \end{bmatrix}$$

$\theta = kd$ $\quad$ k propagation constant of the UNLOADED LINE

At the n^{th} cell we can also write:

$$V_{n+1} = V_n e^{-\gamma d} \leftarrow$$

$$I_{n+1} = I_n e^{-\gamma d} \leftarrow$$

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = \begin{bmatrix} V_{n+1} e^{\gamma d} \\ I_{n+1} e^{\gamma d} \end{bmatrix}$$

$$\begin{bmatrix} A - e^{\gamma d} & B \\ C & D - e^{\gamma d} \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = 0$$

The only non-trivial solution is:

$$AD + e^{2\gamma d} - (A+D)\, e^{\gamma d} - BC = 0$$

Since the network is reciprocal $AD - BC = 1$

$$\rightarrow \quad 1 + e^{2\gamma d} - (A+D)\, e^{\gamma d} = 0 \qquad \text{dividing by } e^{\gamma d}$$

$$e^{-\gamma d} + e^{\gamma d} = A + D$$

From ABCD matrix $\quad \dfrac{A+D}{2} = \cos\theta - \dfrac{b}{2}\sin\theta = \cosh \gamma d$

From matrix

We recall that $\gamma = \alpha + j\beta$

$$\cosh \gamma d = \cosh(\alpha d)\cos(\beta d) + j\sinh(\alpha d)\sin(\beta d) \quad (1)$$

$$= \underbrace{\cos\theta - \frac{b}{2}\sin\theta}_{\text{purely real}}$$

<u>CASE 1</u> $\alpha = 0 \quad \beta \neq 0$ NON ATTENUATING PROPAGATING WAVE (DEFINES A PASS-BAND)

Eq 1 reduces to

$$\cos\beta d = \cos\theta - \frac{b}{2}\sin\theta \Rightarrow$$ <u>solved for β</u>

CASE $\alpha \neq 0$, $\beta = 0, \pi$ NON-PROPAGATING, ATTENUATED WAVES

(Define Stopband)

Eq 1 reduces to

$$\cosh \alpha d = \left| \cos\theta - \frac{b}{2}\sin\theta \right| \geq 1$$

$$\Downarrow$$

1 solution ($\alpha > 0$) for positive travelling waves

The characteristic impedance of the cell is $Z_B = Z_0 \frac{V_{n+1}}{I_{n+1}}$

BLOCH IMPEDANCE

from the ABCD matrix

$$\begin{bmatrix} A - e^{\gamma d} & B \\ C & D - e^{\gamma d} \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = 0$$

$$(A - e^{\gamma d}) V_{n+1} + B I_{n+1} = 0$$

$$Z_B = Z_0 \frac{-B}{A - e^{\gamma d}}$$

$e^{\gamma d}$ can be expressed as a function of A and D

$$1 + e^{2\gamma d} - (A+D)\, e^{\gamma d} = 0 \qquad \text{if } x = e^{\gamma d}$$

$$e^{\gamma d} = \frac{(A+D) \pm \sqrt{(A+D)^2 - 4}}{2}$$

The Bloch impedance has 2 possible solutions:

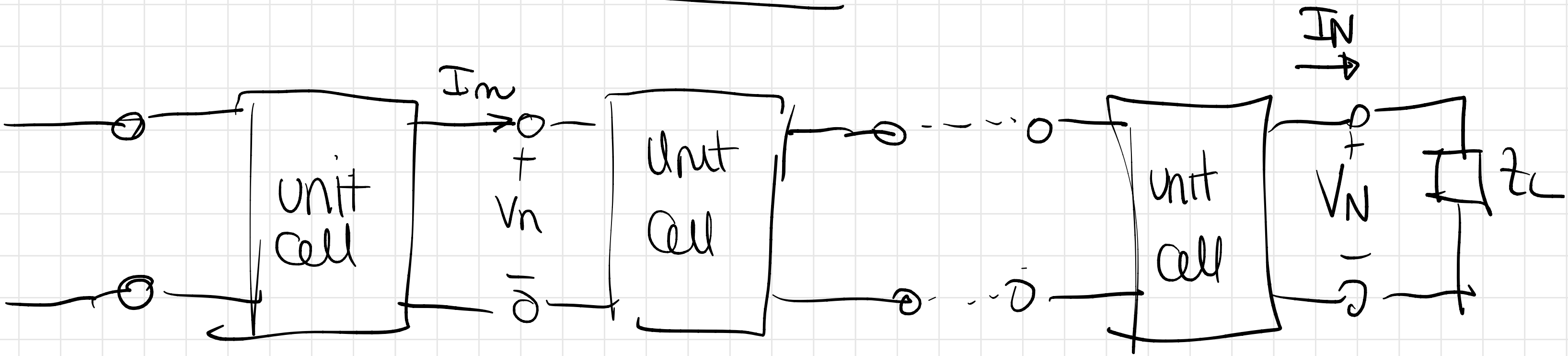
$$Z_B^{\pm} = \frac{-2BZ_0}{2A - A - D \mp \sqrt{(A+D)^2 - 4}}$$

in a symmetric cell the expression reduces to:

$$\boxed{Z_B^{\pm} = \frac{\pm B Z_0}{\sqrt{A^2 - 1}}}$$

($\pm$ indicate posive/negative traveling waves)

TERMINATED PERIODIC STRUCTURE



If we have no losses we can replace γd with $j\beta n d$

So at the n^{th} cell we have:

$$V_n = V_0^+ e^{-j\beta md} + V_0^- e^{j\beta nd}$$

$$I_n = I_0^+ e^{-j\beta nd} + I_0^- e^{j\beta nd} = \frac{V_0^+}{Z_B^+} e^{-j\beta nd} + \frac{V_0^-}{Z_B^-} e^{j\beta nd}$$

The incident and reflected voltages at the n^{th} cell

$$V_n^+ = V_0^+ e^{-j\beta nd}$$

$$V_n^- = V_0^- e^{j\beta nd}$$

$$V_n = V_n^+ + V_n^-$$

$$I_n = I_n^+ + I_n^- = \frac{V_n^+}{Z_B^+} + \frac{V_n^-}{Z_B^-}$$

At $n=N$

$$V_N = V_N^+ + V_N^- = Z_L I_N = Z_L \left(\frac{V_N^+}{Z_B^+} + \frac{V_N^-}{Z_B^-} \right)$$

The reflection coeff. at the load is:

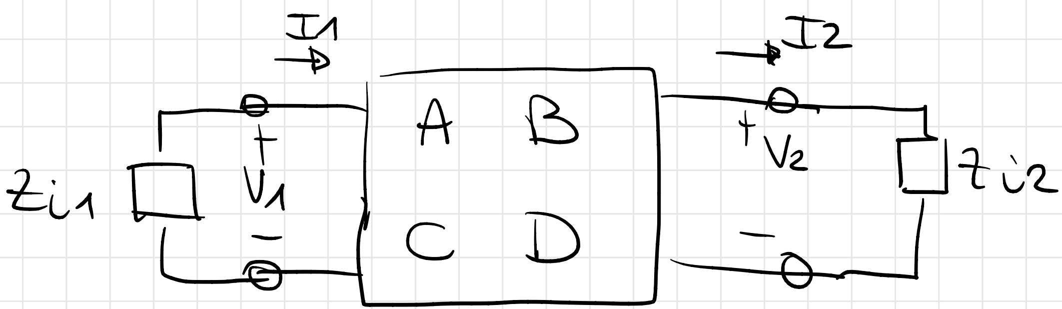
$$\Gamma = \frac{V_N^-}{V_N^+} = - \frac{\frac{Z_L}{Z_B^+} - 1}{\frac{Z_L}{Z_B^-} - 1}$$

If the cells are symmetric $(A=D)$ $\Rightarrow$ $Z_B^+ = -Z_B^- = Z_B$

so the reflection coeff. is:

$$\boxed{\Gamma = \frac{Z_L - Z_B}{Z_L + Z_B}}$$

IMAGE PARAMETER METHOD



$Z_{i,1}$ and Z_{i2} are the input impedances of ports 1 and 2 when they are terminated with Z_{i2}, and Z_{i1}

$$Z_{i1} = \sqrt{\frac{AB}{CD}}$$

$$Z_{i2} = \sqrt{\frac{BD}{AC}}$$ and $$Z_{i2} = \frac{D}{A} Z_{i1}$$

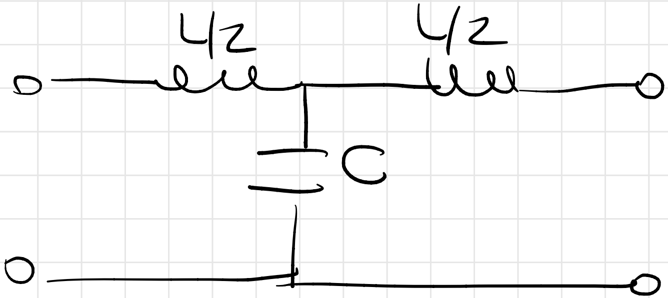
if the network is symmetric $Z_{i2} = Z_{i1}$

$$e^{-\gamma} = \sqrt{AD} = \sqrt{BC}$$

$\downarrow$

$\cosh \gamma$

CONSTANT-K FILTER SECTIONS (CONSTANT-K LOW-PASS PROTOTYPE)



works as a low-pass filter

For a generic T-network $$Z_{iT} = \sqrt{Z_1 Z_2}\sqrt{1+\frac{Z_1}{4Z_2}}$$

$$\boxed{\begin{aligned} Z_1 &= j\omega L \\ Z_2 &= \frac{1}{j\omega C} \end{aligned}}$$

$$Z_{iT} = \sqrt{\frac{L}{C}}\sqrt{1-\frac{\omega^2 LC}{4}}$$

If the cut-off frequency is $\omega_C = \frac{2}{\sqrt{LC}}$ and $R_0 = \sqrt{\frac{L}{C}} = K =$ constant

then

$$Z_{i\Gamma_1} = R_0 \sqrt{1 - \frac{\omega^2}{\omega_C^2}}$$

If $\omega = 0 \Rightarrow Z_{1T} = R_0$

For a generic T-network

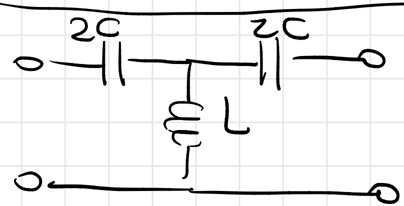
$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1}{Z_2} + \frac{Z_1^2}{4Z_2^2}} = 1 - \frac{2\omega^2}{\omega_C^2} + \frac{2\omega}{\omega_C}\sqrt{\frac{\omega^2}{\omega_C^2} - 1}$$

We can identify two frequency regions

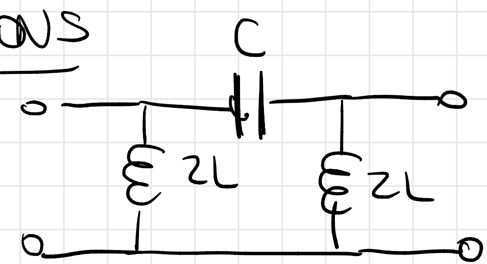
1. $\omega < \omega_c$: PASSBAND
   Z_π IS REAL
   γ IS IMAGINARY $\begin{pmatrix} \alpha = 0 \\ \beta \neq 0 \end{pmatrix}$

2. $\omega > \omega_c$: STOPBAND
   Z_π IS IMAGINARY
   γ IS REAL $\begin{pmatrix} \alpha \neq 0 \\ \beta = 0 \end{pmatrix}$

HIGH-PASS CONSTANT-K SECTIONS



or

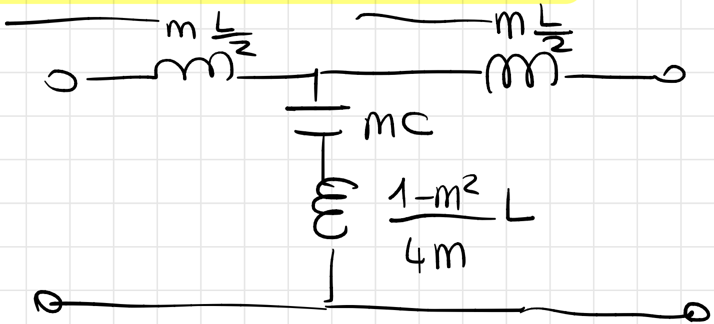


$$R_0 = \sqrt{\frac{L}{C}}$$

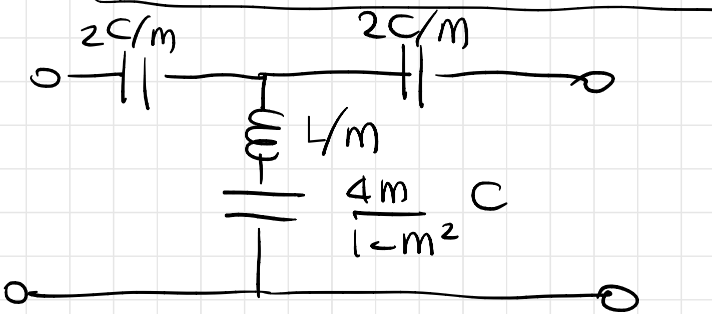
$$\omega_c = \frac{1}{2\sqrt{LC}}$$

M-DERIVED FILTER SECTIONS

LOW-PASS T-SECTION



HIGH-PASS T-SECTION



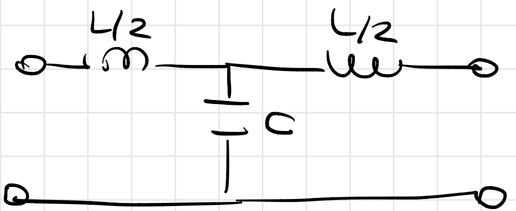
COMPOSITE FILTER :

Combining k-constant and m-derived sections we can realize the filters of desired properties

SUMMARY FOR DESIGN

LOW-PASS

Constant-k T section

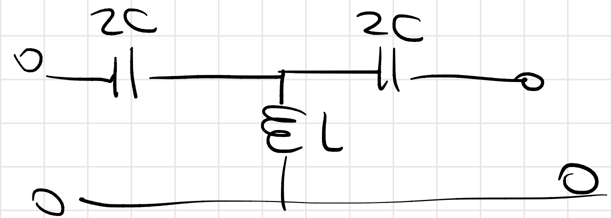


$$R_0 = \sqrt{L/C} \qquad L = \frac{2R_0}{\omega_c}$$

$$\omega_c = \frac{2}{\sqrt{LC}} \qquad C = \frac{2}{\omega_c R_0}$$

HIGH-PASS

Constant-k T-section

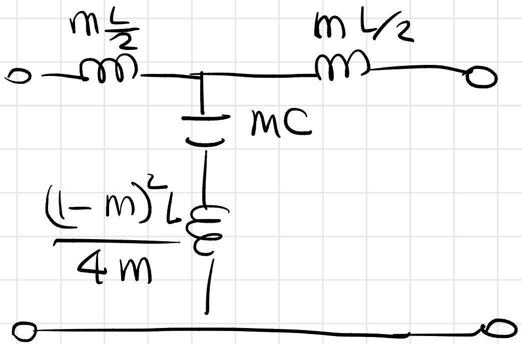


$$R_0 = \sqrt{L/C} \qquad L = \frac{R_0}{2\omega_c}$$

$$\omega_c = \frac{1}{2}\sqrt{LC} \qquad C = \frac{1}{2\omega_c} R_0$$

LOW-PASS

m-derived T-section

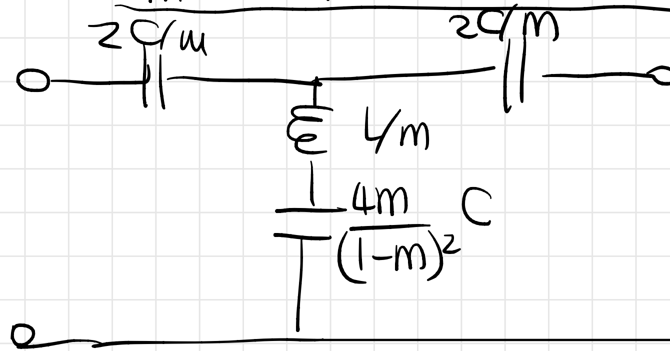


L, C as in constant-K

$$m = \begin{cases} \sqrt{1-\left(\omega_c/\omega_\infty\right)^2} & \text{sharp cut-off} \\ 0.6 & \text{for matching} \end{cases}$$

HIGH-PASS

m-derived T-section



L, C same as in constant-K

$$m = \begin{cases} \sqrt{1-\left(\omega_\infty/\omega_c\right)^2} & \text{for sharp cut-off} \\ 0.6 & \text{for matching} \end{cases}$$