RADIATION PATTERNS AND ANTENNA PARAMETERS

The radiation pattern is a description of the angular variation of far-field radiation level around the antenna. Parameters can be defined to describe the performance of an antenna, when it is connected to a circuit and used to transmit or receive signals.

RADIATION PATTERNS

An antenna radiation pattern (or antenna pattern) is defined as a mathematical function or graphical representation of the radiation properties of the antenna as a function of spatial coordinates. In most cases, the radiation pattern is determined in the far-field region and is represented as a function of the directional coordinates. The radiation property of most concern is the two or three-dimensional spatial distribution of radiated intensity as a function of the observer’s position along a path or surface of constant radius.

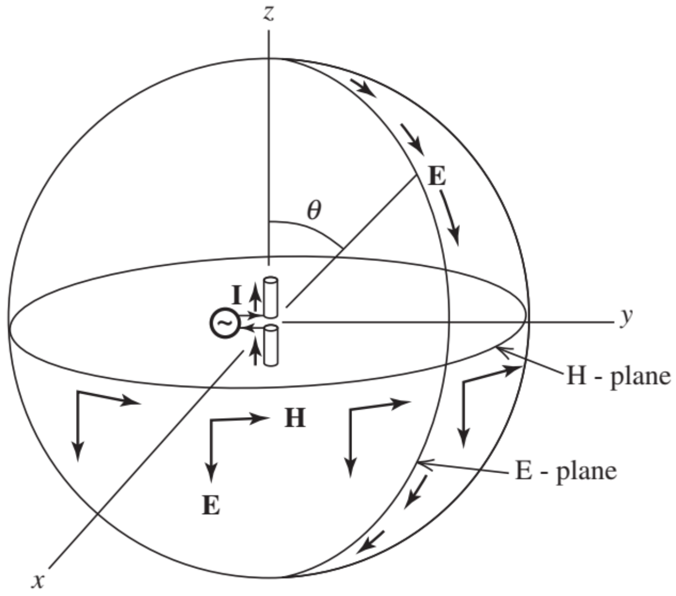
In general the radiation fields from a transmitting antenna vary inversely with distance; that is the radiated fields observed far from any antenna decay with distance as a spherical wave. The variation with observation angle however depends on the antenna geometry. In fact, the relationship of an antenna to its radiation pattern forms a large part of antenna investigations.

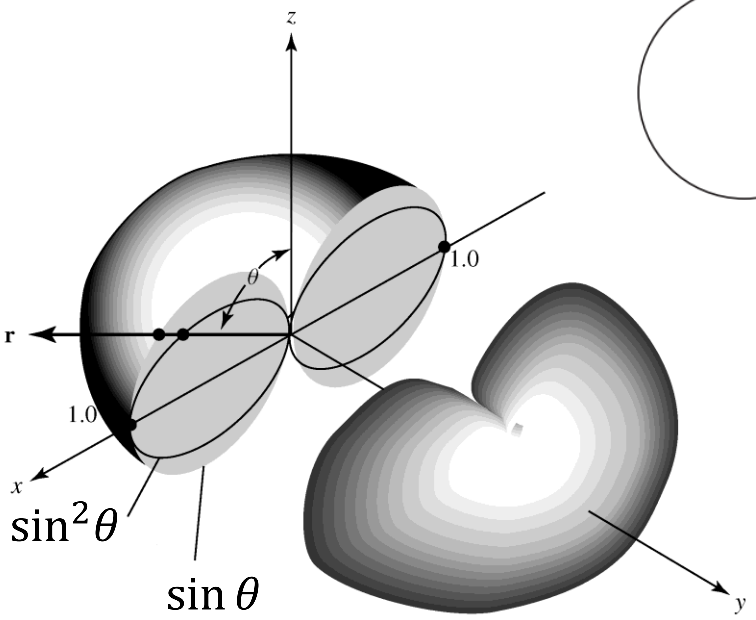
Often the field and power patterns are normalized with respect to their maximum value, yielding normalized field and power patterns. Also, the power pattern is usually plotted on a logarithmic scale in decibels (dB).

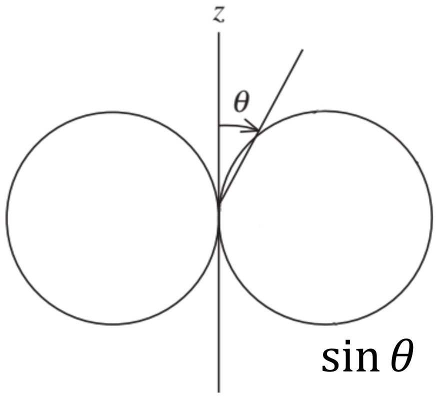
EXAMPLE: RADIATION PATTERN OF AN IDEAL DIPOLE

Let's consider a sphere whose center is the origin and having radius r. An electric field probe moved over the sphere surface and oriented parallel to E_θ will have an output proportional to $\sin\theta$ and, due to the axial symmetry, does not depend on φ: the radiation pattern looks like a doughnut ! It is customary to plot radiation patterns normalized to 1: the resulting patterns are unit-less and do not depend on the radius of the sphere r (which must be in the far-field), as one would expect as in the far-field the angular distribution of field does not depend on the distance from the radiator.

$$\mathbf{E} = j\eta \frac{\beta}{4\pi} I\Delta z \frac{e^{-j\beta r}}{r} \sin\theta \ \hat{\boldsymbol{\theta}} = E_\theta \hat{\boldsymbol{\theta}} \qquad \mathbf{H} = \frac{E_\theta}{\eta} \hat{\boldsymbol{\phi}}$$







The radiation pattern is the variation of the radiated electric field over a sphere centered on the antenna, r is constant and there are only θ and φ variations of the field. It is convenient to normalize the field expression such that its maximum value is unity. The normalized **magnitude field pattern** $|F(\theta,\varphi)|$ reads as

$$\boxed{|F(\theta,\varphi)| = \frac{|E|}{|E|_{MAX}}}$$

where $|E|$ is the magnitude of the electric field (and for linearly polarized fields $\mathbf{E}$ has only one component E).

The radiation properties can be described by using the normalized **power pattern** $P(\theta,\varphi)$, which gives the angular dependence of the power density (measured in W/m²) and it can be calculated by using the Poynting vector $\mathbf{S}$

$$\mathrm{Re}\{\mathbf{S}\cdot\hat{\mathbf{r}}\} = \frac{1}{2\eta}|E|^2 \qquad \boxed{P(\theta,\varphi) = \frac{|E|^2}{|E|^2_{MAX}} = |F(\theta,\varphi)|^2}$$

Frequently, patterns are plotted in decibels and magnitude patterns and power patterns are the same in dB

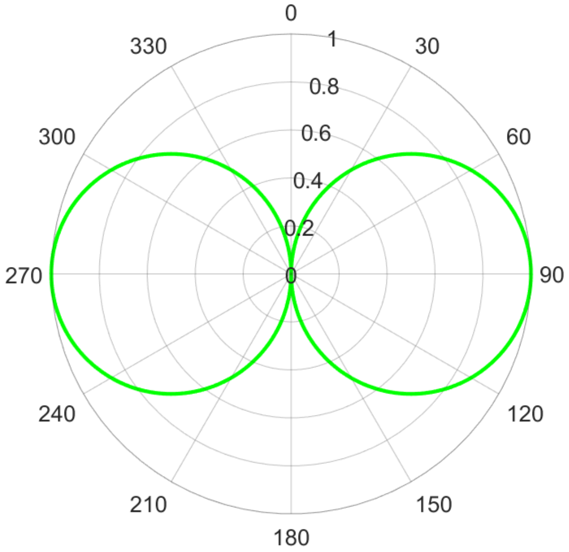
$$|F(\theta,\varphi)|_{dB} = 20\log|F(\theta,\varphi)| \qquad P(\theta,\varphi)_{dB} = 10\log P(\theta,\varphi) = 10\log|F(\theta,\varphi)|^2 = 20\log|F(\theta,\varphi)|$$

If we go back to the example of the ideal dipole …

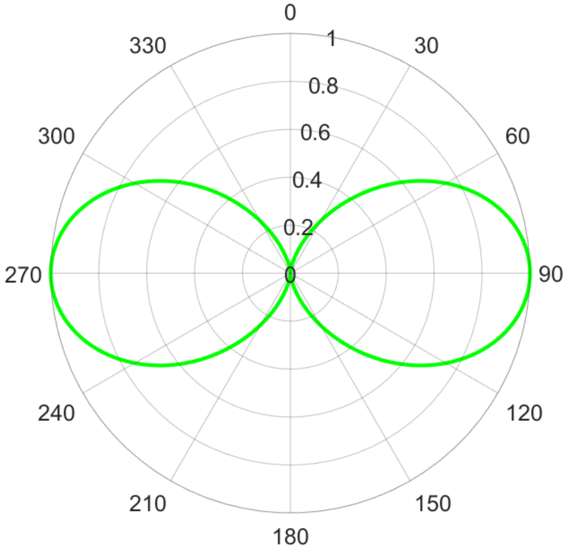
$$E = E_\theta = j\eta \frac{\beta}{4\pi} I\Delta z \frac{e^{-j\beta r}}{r} \sin\theta \qquad\qquad |E|_{MAX} = \eta \frac{\beta}{4\pi} I\Delta z \frac{1}{r}$$

$$|F(\theta,\varphi)| = \frac{|E|}{|E|_{MAX}} = |\sin\theta| \qquad P(\theta,\varphi) = \frac{|E|^2}{|E|_{MAX}^2} = |\sin\theta|^2 \qquad P(\theta,\varphi)_{dB} = 10\log P(\theta,\varphi) = 10\log|\sin\theta|^2$$

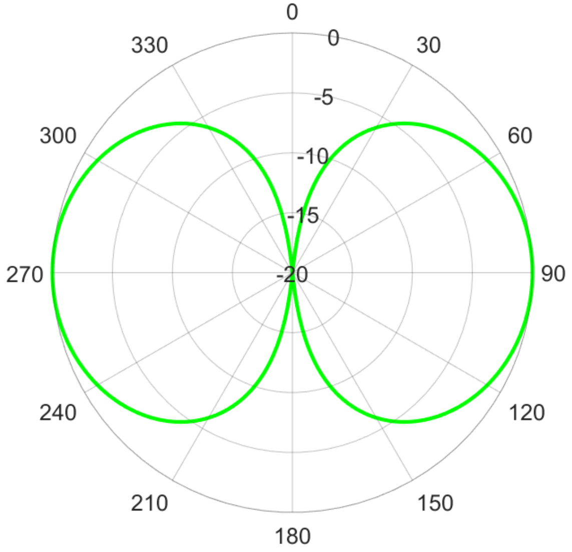
FIELD PATTERN



POWER PATTERN



POWER PATTERN (dB scale)



3D RADIATION PATTERN

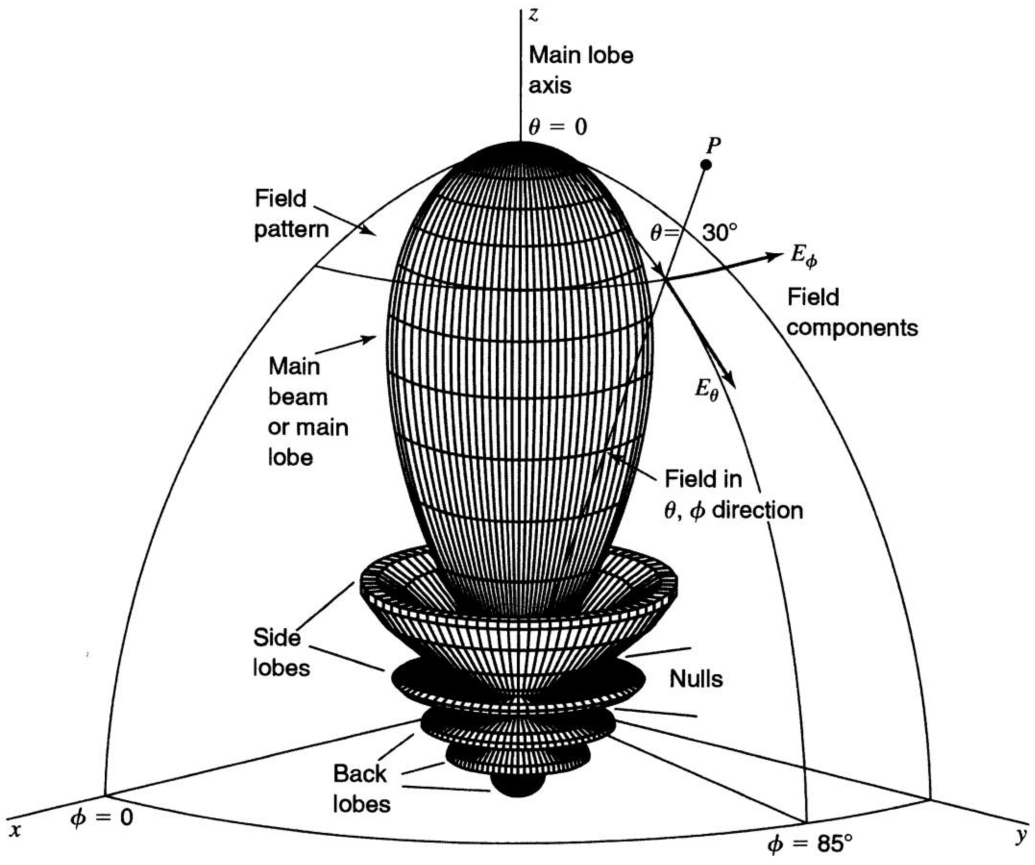
The radiation pattern describes the variation of field magnitude or of power density (proportional to field squared) as a function of the spherical coordinates (θ, φ).

Radiation patterns refer always to the far-field, as only in the far-field region the field angular distribution is independent of the distance from the origin.

The 3D pattern surface is obtained by setting the distance from the origin in the direction (θ, φ) to be proportional to the field magnitude or power density.

Usually, the patterns are normalized with respect to the maximum value.

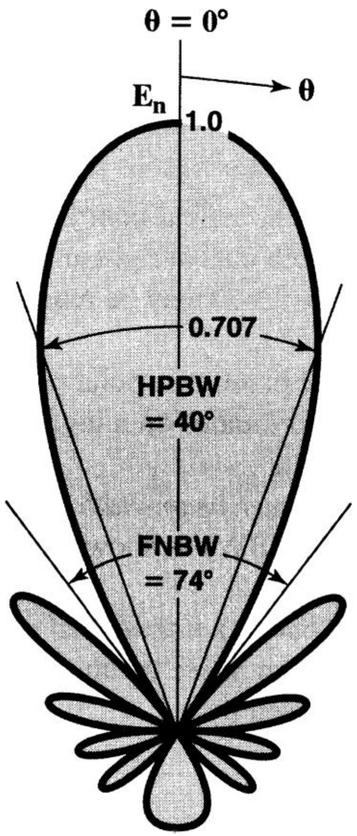
Most of the radiation is contained in a main lobe (or beam) accompanied by radiation also in minor lobes (side and back lobes). Between the lobes there are nulls where the field goes to zero. A radiation pattern could have more than one main lobe (those lobes have maximum normalized magnitude equal to 1).

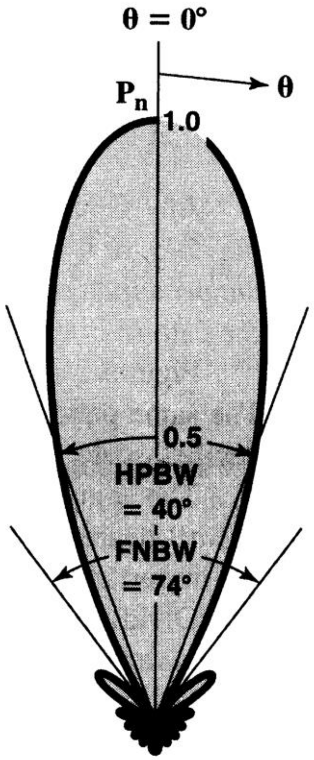


2D RADIATION PATTERN

A bidimensional pattern is obtained by considering the intersection of the 3D radiation pattern with a given plane; this plane can be described by θ =constant or φ =constant and contains the pattern maximum.

polar plot of field pattern (linear scale)

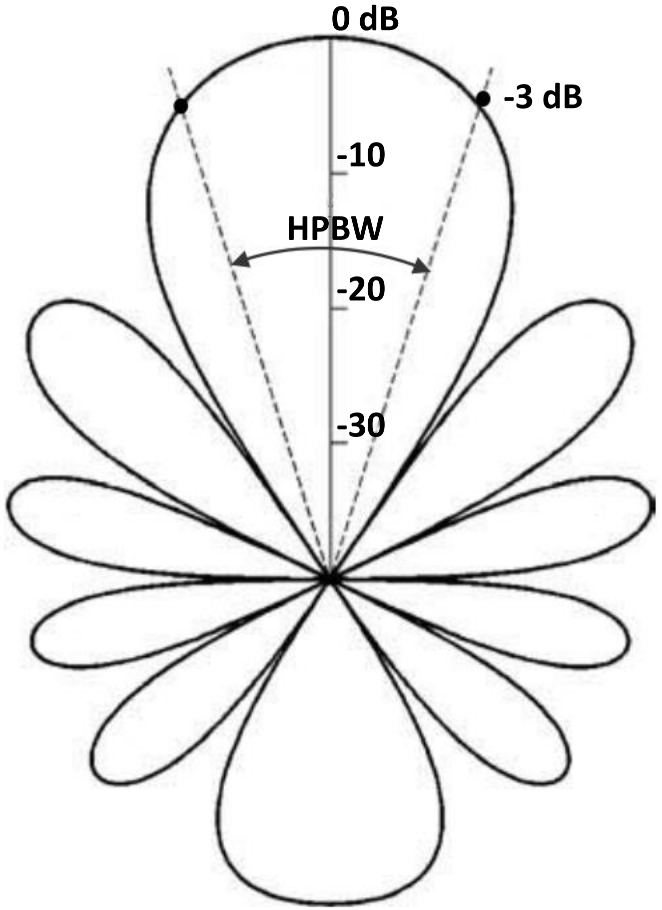




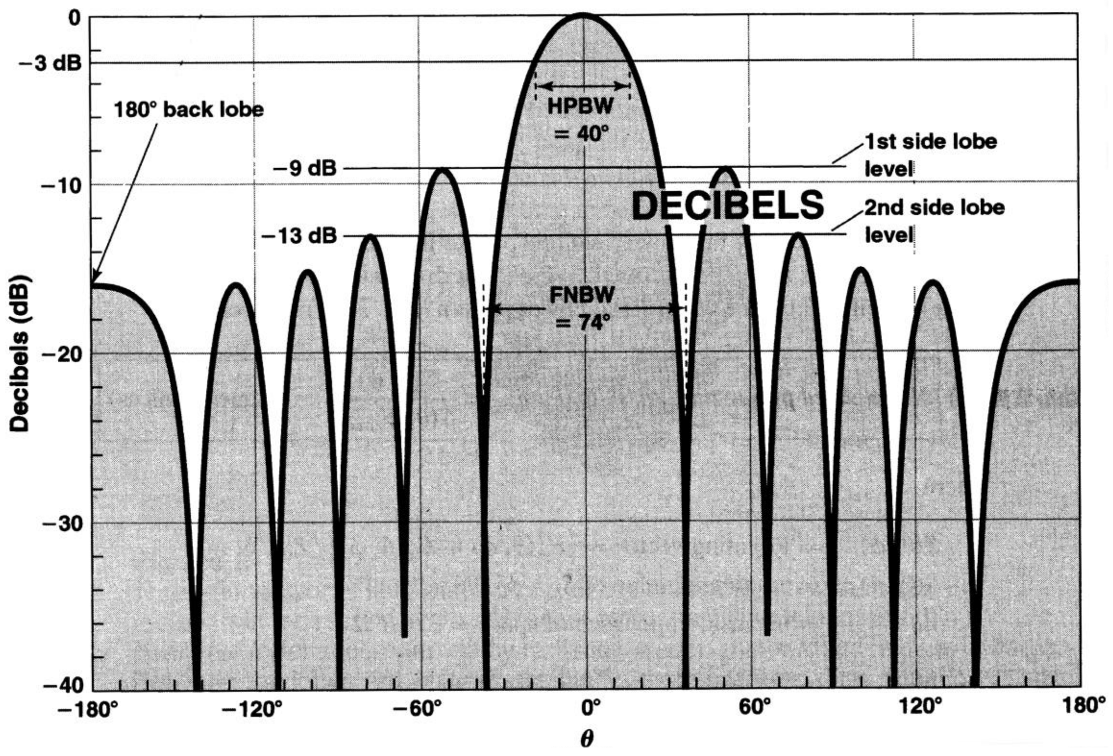
polar plot of power pattern (linear scale)

Power patterns are usually plotted on a logarithmic (decibel, dB) scale: this scale is desirable because a logarithmic scale can accentuate in more details those parts of the pattern that have very low values (as the secondary lobes).

polar plot of power pattern (dB scale)



Cartesian plot of power pattern (dB scale)



Two parameters can be used to measure the width of the main lobe:

Half-power beamwidth (HPBW): in a plane containing the direction of the maximum of the main lobe (beam), it is the angular separation of the points where the main beam of the power pattern $P(\theta,\varphi)$ equals one-half (-3 dB) of the maximum value; on the magnitude field pattern $|F(\theta,\varphi)|$ these points correspond to the value $1/\sqrt{2} \cong 0.707$.

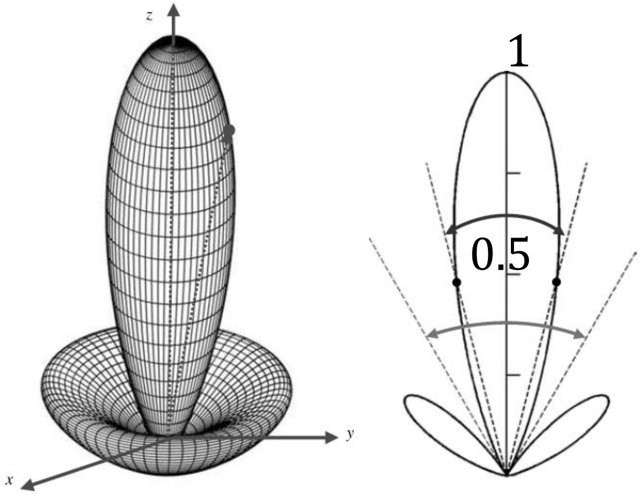
First-null beamwidth (FNBW); in a plane containing the direction of the maximum of the main lobe, it is the angular separation between the first nulls (i.e. the nulls delimiting the main beam).

Example 1: ideal dipole, $P(\theta,\varphi) = \sin^2\theta$

HPBW$=135° - 45° = 90°$

FNBW$=180° - 0° = 180°$

Example 2: $P(\theta,\varphi) = \begin{cases} \cos^2\theta\cos^2 3\theta & 0 \le |\theta| < \pi/2 \\ 0 & \pi/2 \le |\theta| \le \pi \end{cases}$



HPBW $\cong 28.7°$

FNBW $= 60°$

Isotropic pattern: it is a spherical radiation pattern due to an antenna having equal radiation in all directions; it is an ideal concept and it is not physically achievable.

Omnidirectional pattern: in a selected plane it is a circular radiation pattern. Several radiators (short dipoles, monopoles, resonant wires,...) can exhibit omnidirectional patterns.

Directional pattern: the antenna radiates much more efficiently in some directions than in others. In practice, an antenna is considered directional when it is more directive than a half-wavelength dipole.

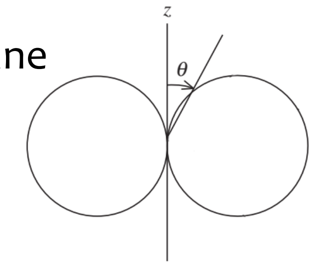
If the antenna far-field is linearly polarized two **principal plane patterns** can be defined.

E-plane pattern: it is the 2D radiation pattern in a plane containing the $\mathbf{E}$ vector and the direction of maximum radiation.

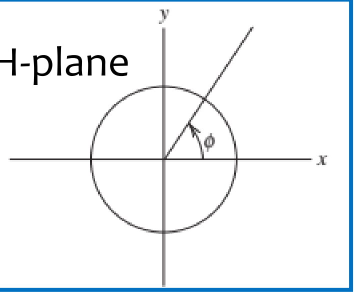
H-plane pattern: it is the 2D radiation pattern in a plane containing the $\mathbf{H}$ vector and the direction of maximum radiation.

principal plane patterns of the ideal dipole

E-plane

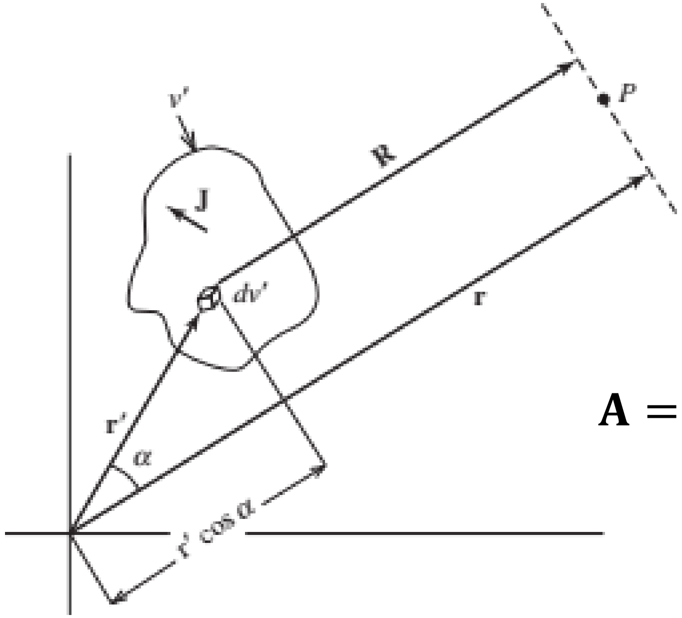


H-plane



FAR-FIELD REGIONS

In the far-field of an antenna, the fields exhibit local plane wave behaviour (field components are transverse to the radial direction) and have $1/r$ magnitude dependence. The observation point is in the far-field when the rays coming from the current source can be assumed as parallel and we can write the distance from a generic point of the source in the following way (and we have already used this formula):



$$R = r - r' \cos\alpha = r - r' \frac{\mathbf{r} \cdot \mathbf{r}'}{rr'} = r - \hat{\mathbf{r}} \cdot \mathbf{r}'$$

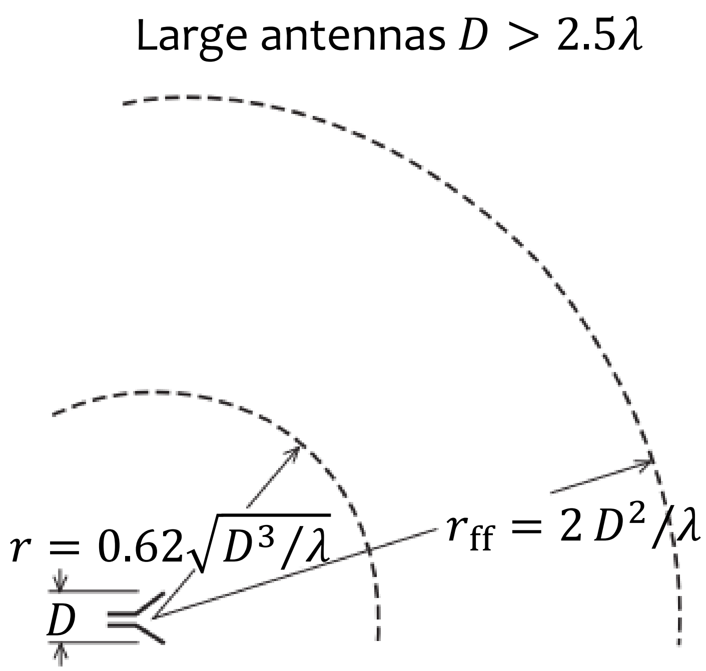
$$\mathbf{A} = \iiint_{v'} \mu \mathbf{J} \frac{e^{-j\beta R}}{4\pi R} dv' \cong \int_{v'} \mu \mathbf{J} \frac{e^{-j\beta(r - \hat{\mathbf{r}} \cdot \mathbf{r}')}}{4\pi\, r} dv' = \mu \frac{e^{-j\beta r}}{4\pi\, r} \int_{v'} \mathbf{J} e^{j\beta \hat{\mathbf{r}} \cdot \mathbf{r}'} dv'$$

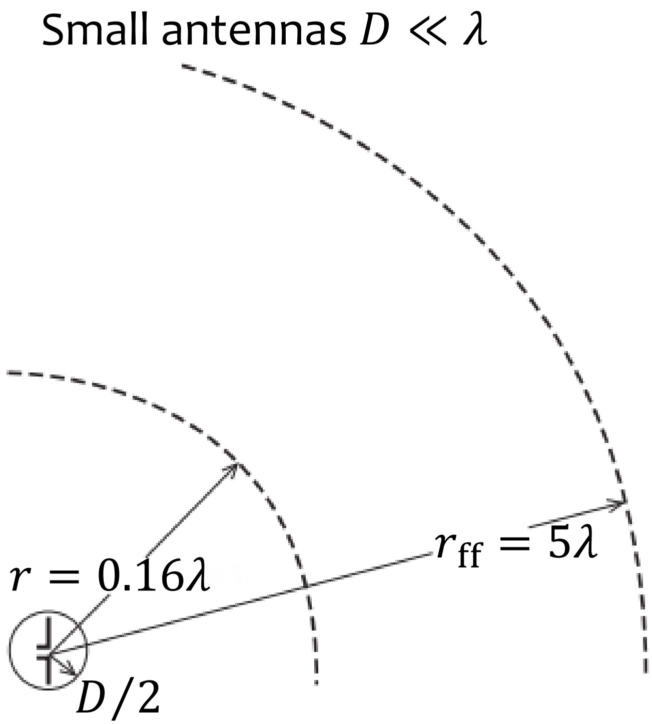
By convention, the distance where the far-field begins is taken to be that value of r for which the path length deviation due to the parallel ray assumption is a sixteenth of a wavelength, and this corresponds to a phase error of $\beta\Delta r = 2\pi/\lambda \cdot \lambda/16 = \pi/8$

- Large antennas (the diameter or length fulfills $D > 2.5\lambda$): the far-field distance is $r_{\text{ff}} = 2\,D^2/\lambda$
- Small antennas (the diameter or length fulfills $D \ll \lambda$): the far-field distance is $r_{\text{ff}} = 5\lambda$

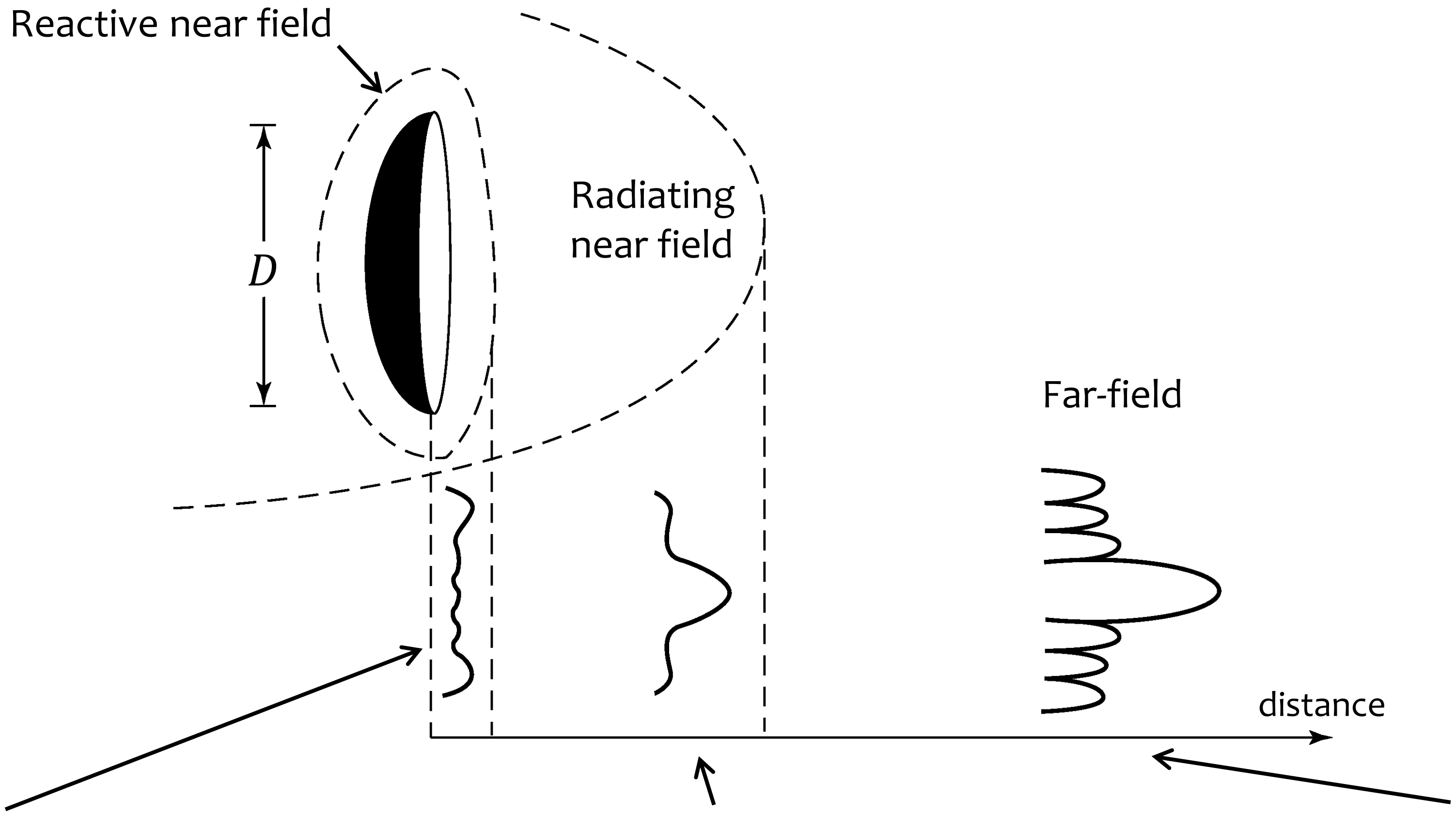
In all the cases, the far-field region must satisfy the most restrictive of these conditions: $r > 2\,D^2/\lambda$, $r > 5D$, $r > 1.6\lambda$

The region interior to the far-field, called the near field, is divided into the reactive near field where the reactive fields dominate over the radiation and the radiating near field where the radiation fields dominate over the reactive fields.





The field pattern of an antenna, as the observation distance is varied from the reactive near field to the far-field, changes in shape because of variations of the fields (both magnitude and phase). <u>The radiation patterns can be calculated or measured only at distances where the far-field conditions are satisfied.</u>



In the reactive near field region the pattern is nearly uniform

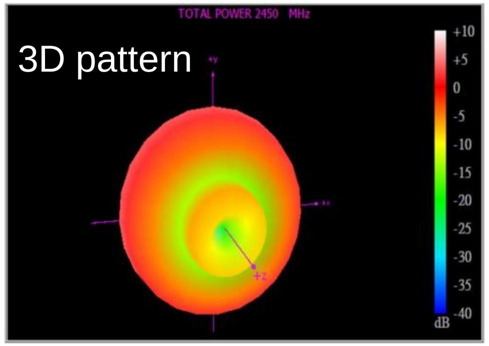
… by increasing the distance the pattern begins to smooth and forms lobes

in the far-field the radiation pattern is well formed and it does not depend on distance

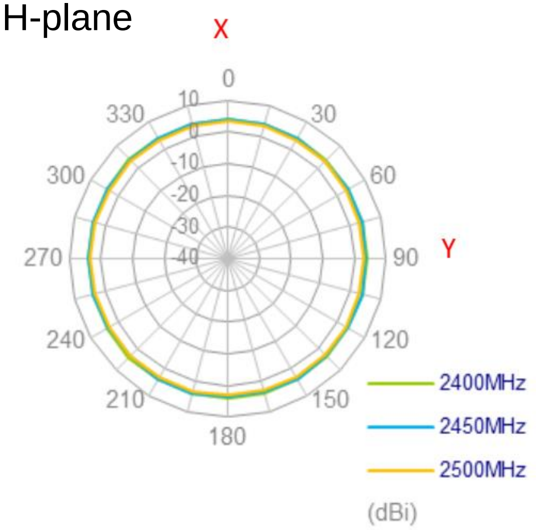
EXAMPLES of patterns reported in the data-sheet of off-the-shelf antennas

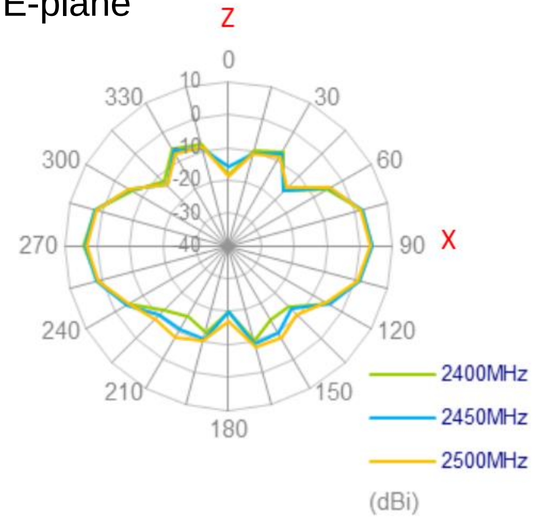
WiFi dipole antenna
model GW.34.5153 by Taoglas

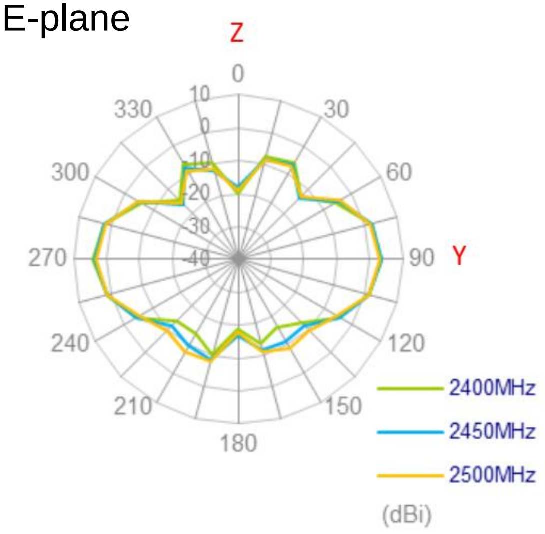
measurement setup



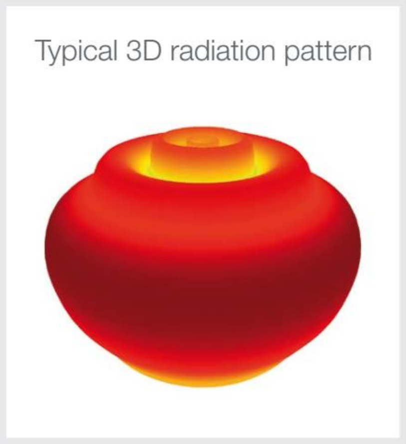
XY Plane	XZ Plane	YZ Plane
H-plane	E-plane	E-plane

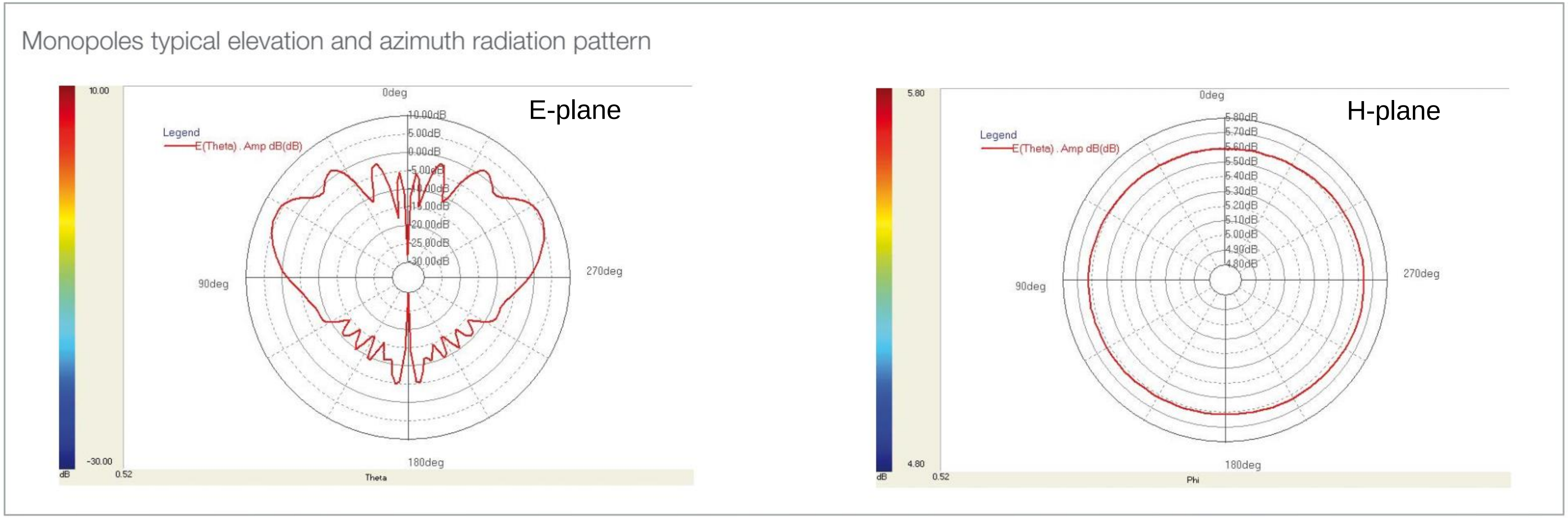




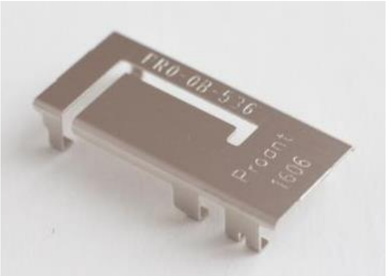


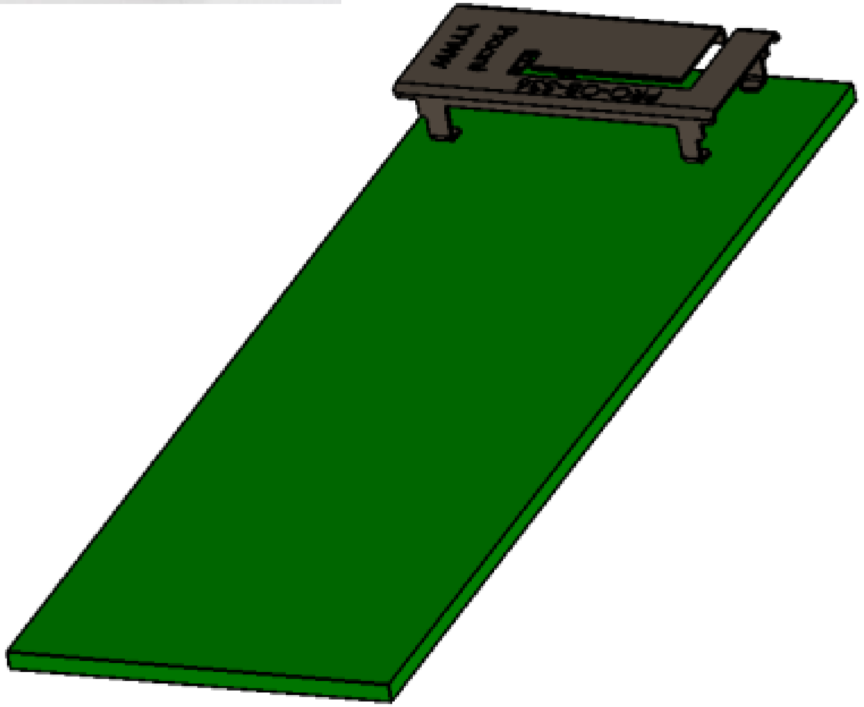
Monopole by MVG

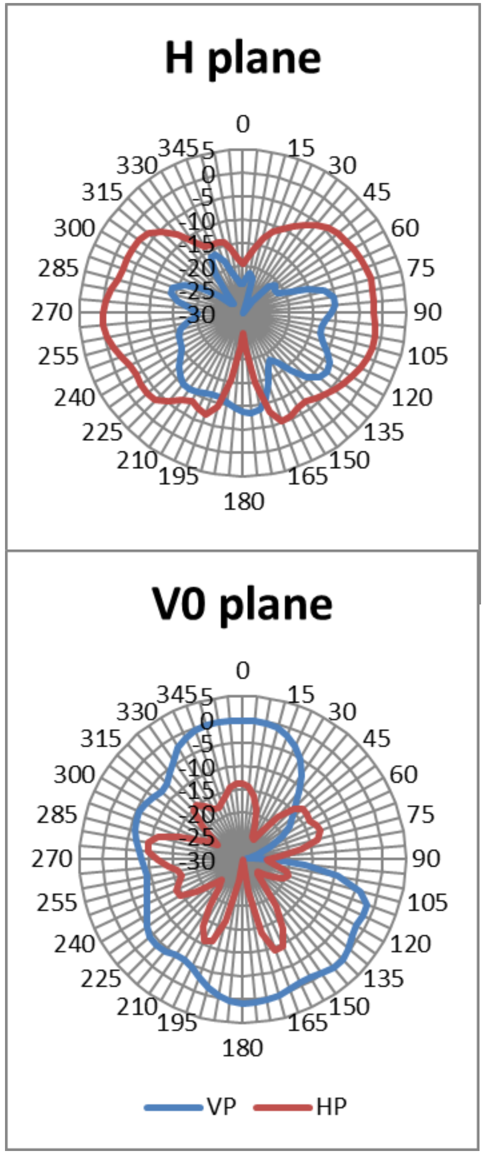


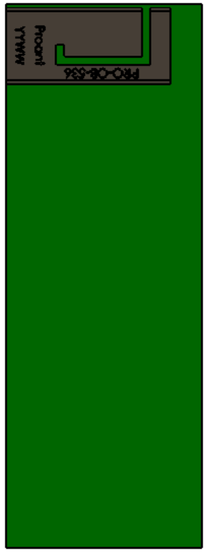


WLAN antenna (Onboard SMD) by proAnt



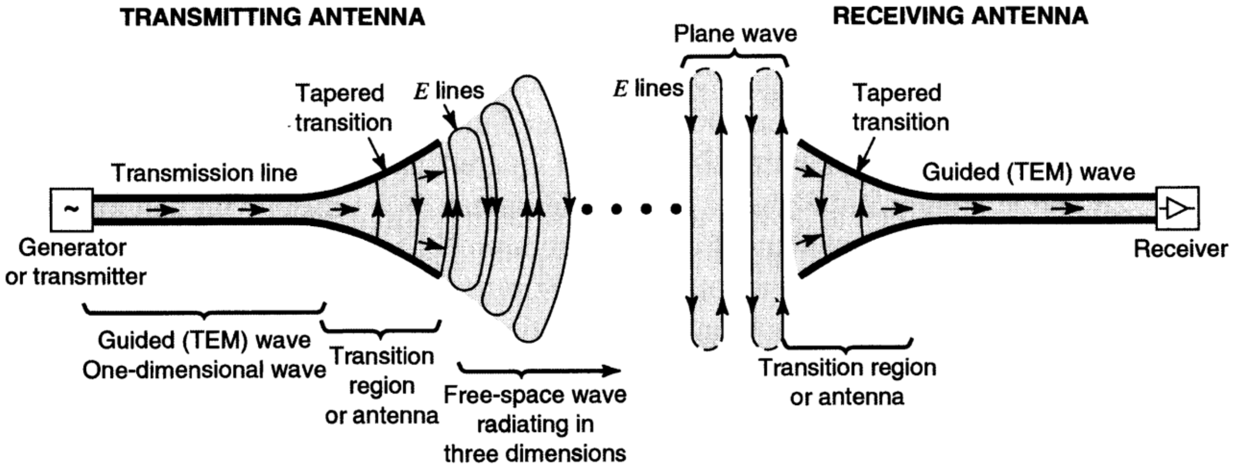






BASIC PARAMETERS OF ANTENNAS

An antenna is a transducer between a guided wave and a free-space wave (or vice versa).



Parameters are useful to describe the performance of an antenna to be connected to a transmitter or a receiver:

- Directivity, gain and efficiency
- Radiation resistance and input impedance

DIRECTIVITY

One very important quantitative description of an antenna is how much it concentrates energy in one direction in preference to radiation in other directions: this characteristic of an antenna is called directivity. Let us begin by recalling that the real power radiated by an antenna reads as

$$P = \mathrm{Re}\left\{\iint \frac{1}{2}(\mathbf{E}\times\mathbf{H}^*)\cdot d\mathbf{s}\right\} = \frac{1}{2}\mathrm{Re}\left\{\int_0^{2\pi}\int_0^{\pi}\left(E_\theta H_\varphi^* - E_\varphi H_\theta^*\right) r^2 \sin\theta\, d\theta\, d\varphi\right\}$$

$$P = \frac{1}{2\eta}\int_0^{2\pi}\int_0^{\pi}\left(|E_\theta|^2 + \left|E_\varphi\right|^2\right) r^2 \sin\theta\, d\theta\, d\varphi = \frac{1}{2\eta}\int_0^{2\pi}\int_0^{\pi}|\mathbf{E}|^2\, r^2 \sin\theta\, d\theta\, d\varphi$$

Since for any antenna the magnitude of the radiation in the far-fied decreases as $1/r$, it is convenient to introduce the radiation intensity $U(\theta,\varphi)$

$$U(\theta,\varphi) = \mathrm{Re}\{\boldsymbol{S}\}\cdot r^2\hat{\mathbf{r}}$$

it is the power radiated in a given direction per unit solid angle (and its unit of measurement is Watt per steradian: W/sr). An advantage of using U is that it is independent of distance.

From the definition of magnitude field pattern $|F(\theta,\varphi)|$ and radiation intensity $U(\theta,\varphi)$, and if the maximum of $U(\theta,\varphi)$ is U_m, we observe that

$$U(\theta,\varphi) = U_m|F(\theta,\varphi)|^2$$

It follows that the total radiated power is obtained by integrating the radiation intensity over all angles around the antenna (i.e. over the full solid angle 4π)

$$P = \int_0^{2\pi}\int_0^{\pi}\frac{U(\theta,\varphi)}{r^2}r^2\sin\theta\,d\theta\,d\varphi = \iint_{4\pi}U(\theta,\varphi)d\Omega = U_m\iint_{4\pi}|F(\theta,\varphi)|^2 d\Omega$$

Since $U(\theta,\varphi)$ is integrated over the full solid angle, the average intensity U_{ave} (average power per steradian) is defined as

$$U_{ave} = \frac{1}{4\pi}\iint_{4\pi}U(\theta,\varphi)d\Omega = \frac{P}{4\pi}$$

and obviously U_{ave} equals the radiation intensity that an isotropic source with the same power P would radiate.

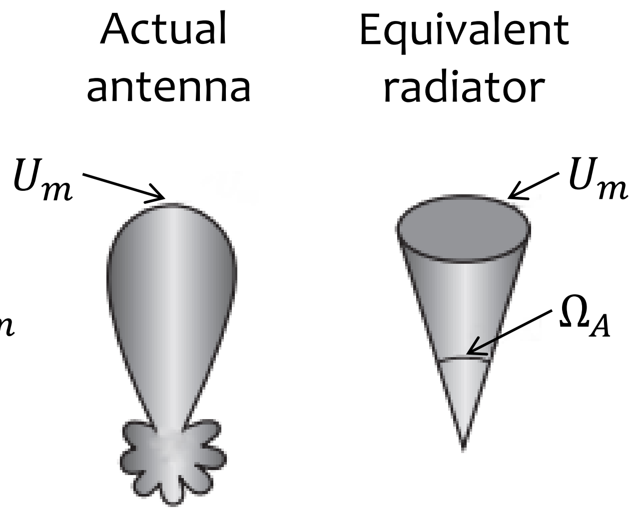
We can also define **beam solid angle** Ω_A as

$$\boxed{\Omega_A = \iint_{4\pi} |F(\theta,\varphi)|^2 d\Omega = \int_0^{2\pi}\int_0^{\pi} |F(\theta,\varphi)|^2 \sin\theta\, d\theta d\varphi}$$

$$P = U_m \Omega_A$$

Beam solid angle Ω_A of antenna is the solid angle through which all the power would flow if its radiation intensity were constant and equal to the maximum radiation intensity U_m for all angles within Ω_A.

The maximum radiation intensity is U_m and the total radiated power is P



All radiation is concentrated into a cone of solid angle Ω_A, inside this cone the radiation intensity is constant and equals the maximum radiation U_m of the actual antenna, and the total radiated power is $P = U_m \Omega_A$

As an example, let us consider the ideal dipole

$$U(\theta,\varphi)=\frac{1}{2}\left(\frac{I\Delta z}{4\pi}\right)^2\eta\beta^2\sin^2\theta=U_m\sin^2\theta$$

$$U_{ave}=\frac{P}{4\pi}=\frac{\frac{1}{2}\left(\frac{I\Delta z}{4\pi}\right)^2\eta\beta^2 2\pi\frac{4}{3}}{4\pi}=\frac{2}{3}U_m$$

and we observe that $U_m=1.5U_{ave}$ which means that in the direction of maximum radiation the intensity is 50% more than which would occur from an isotropic source radiating the same total power.

The beam solid angle Ω_A is readily calculated

$$\Omega_A=\int_0^{2\pi}\int_0^{\pi}\sin^2\theta\sin\theta\,d\theta d\varphi=\frac{8}{3}\pi$$

Directivity D is defined as the ratio of the radiation intensity in a certain direction to the average radiation intensity. The reference direction is usually taken to be that of the maximum radiation and, unless otherwise stated, directivity will be assumed to be this maximum value:

$$\boxed{D = \frac{U_m}{U_{ave}} = \frac{U_m}{P/4\pi}}$$

If both the numerator and the denominator are divided by r^2 the previous equation becomes a ratio of power densities (W/m²) at an artbitrary distance r from the antenna (and the result does not depend on r):

$$\boxed{D = \frac{U_m/r^2}{U_{ave}/r^2} = \frac{\max[\mathrm{Re}(\mathbf{S} \cdot \hat{\mathbf{r}})]}{P/4\pi r^2}}$$

Thus directivity has two interpretations: 1) ratio of maximum to average radiation intensities (which have units of W/sr); 2) ratio of maximum to average power densities (which have units of W/m²).

Since $P = U_m \Omega_A$ the directivity is entirely determined by the pattern shape:

$$\boxed{D = \frac{U_m}{U_{ave}} = \frac{P/\Omega_A}{P/4\pi} = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\iint_{4\pi} |F(\theta,\varphi)|^2 d\Omega}}$$

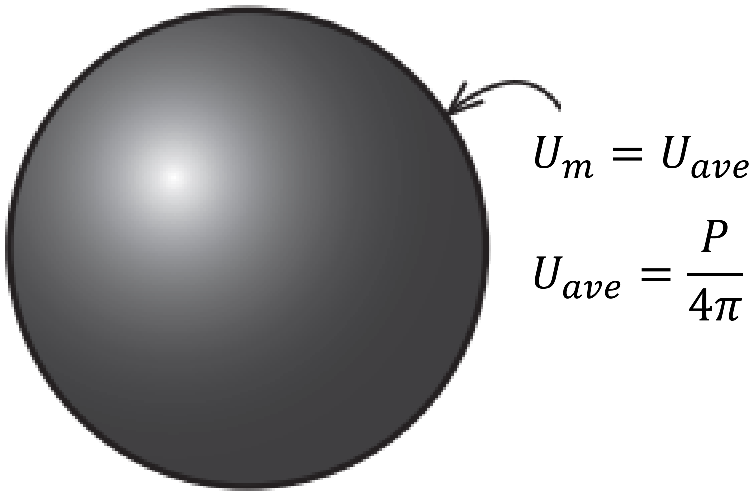
As example, we calculate the directivity of the ideal dipole both by using the definition of directivity and by resorting to the beam solid angle

$$D = \frac{U_m}{U_{ave}} = \frac{U_m}{\frac{2}{3} U_m} = \frac{3}{2}$$

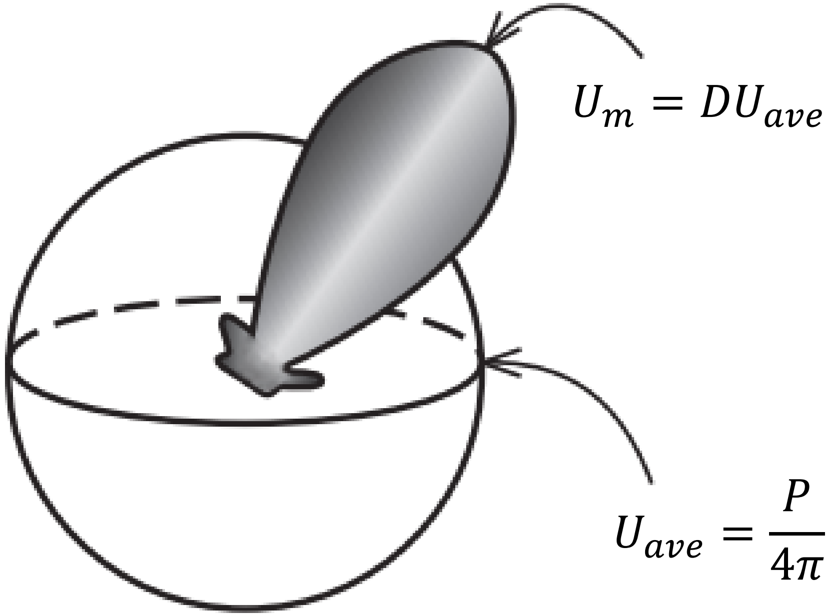
$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\iint_{4\pi} |F(\theta,\varphi)|^2 d\Omega} = \frac{4\pi}{\int_0^{2\pi} \int_0^{\pi} |F(\theta,\varphi)|^2 \sin\theta \, d\theta d\varphi} = \frac{4\pi}{\int_0^{2\pi} \int_0^{\pi} \sin^2\theta \sin\theta \, d\theta d\varphi} = \frac{4\pi}{\frac{8}{3}\pi} = \frac{3}{2}$$

If the radiated power were distributed isotropically over all the space (i.e. the antenna exhibits an isotropic pattern), the radiation intensity would have a maximum value equal to its average value and the directivity would be unity $D = 1$. For a generic radiator, the radiation intensity in the maximum direction is $U_m = DU_{ave}$ and the average radiation intensity is $U_{ave} = P/4\pi$: in other words, the intensity (and the power density) radiated in the maximum direction is increased by a factor of D over what it would be if the same power had been isotropically radiated.

Isotropic radiator antenna $D = 1$



Actual antenna $D > 1$



GAIN

When an antenna is used in a system (say as transmitting or receiving antenna) we are interested in how efficiently the antenna transforms available power P_{in} at its input terminals into radiated power P, as well as its directive properties.

Gain of an antenna can be defined in two equivalent ways:

1) Gain is the ratio of the radiation intensity in the maximum direction to the radiation intensity that would be obtained if the power fed to the antenna were radiated isotropically.

2) Gain is the ratio of the power density in the maximum direction (and at a distance r) to the power density that would be obtained at the same distance if the power fed to the antenna were radiated isotropically.

$$\boxed{G = \frac{U_m}{P_{in}/4\pi} = \frac{U_m/r^2}{P_{in}/(4\pi r^2)}}$$

Gain can also be defined for any direction and can be written as $G(\theta,\varphi) = G|F(\theta,\varphi)|^2$. If no direction is specified it is assumed to be maximum gain (as in the two definitions written above).

In principle, the input power fed to the antenna P_{in} is not equal to the radiated power P because some of the input power is lost on the antenna due to the ohmic losses: it is an obvious consequence of the finite conductivity of real metals and of the losses of any dielectric. This definition does not include losses due to mismatches of impedance or polarization. In other words, the portion of input power that does not appear as radiated power is absorbed on the antenna and this prompts us to define **radiation efficiency** e_r as

$$\boxed{e_r = \frac{P}{P_{in}}}$$

Radiation efficiency is bounded as $0 \leq e_r \leq 1$, and by comparing the definitions of gain and directivity we can observe that

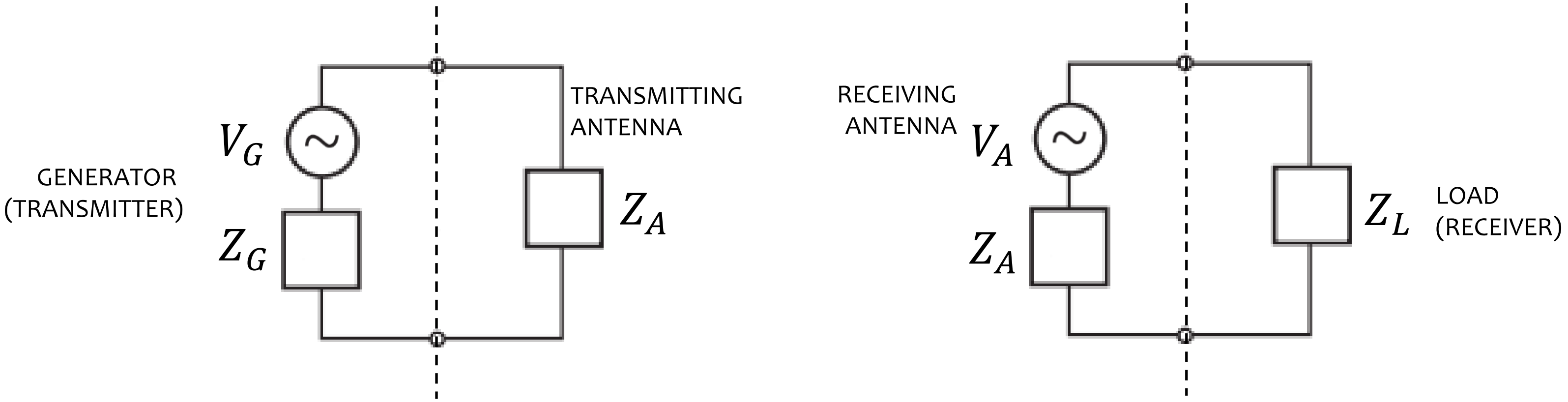
$$G = \frac{U_m}{P_{in}/4\pi} = \frac{U_m}{(P/e_r)/4\pi} = e_r \frac{U_m}{P/4\pi} \quad \Longrightarrow \quad \boxed{G = e_r D}$$

Due to the losses gain of an antenna is always smaller than its directivity, but in many cases we can assume that the losses are so small that the efficiency is unity $e_r = 1$.

ANTENNA IMPEDANCE

The antenna is an interface between the electromagnetic field (and the radiated waves) and the connecting circuit hardware. The antenna input terminal (port) forms the interface point and the circuit parameter of impedance is used to characterize the input to the antenna. The input impedance on an antenna can be affected by objects that are nearby, but the discussion here assumes an isolated antenna. As with conventional circuits, antenna impedance Z_A is composed of real R_A and imaginary X_A parts:

$$\boxed{Z_A = R_A + jX_A}$$



It must be underlined that, as consequence of reciprocity of Maxwell's equations, <u>the impedance of an antenna is identical for receiving and transmitting operation</u>.

From the Poynting theorem we can evaluate the complex power flowing through the antenna input port:

$$P_A = \frac{1}{2} V I^* = \frac{1}{2} Z_A |I|^2 = \frac{1}{2} R_A |I|^2 + j \frac{1}{2} X_A |I|^2 = P_R + P_D + j2\omega(W_m - W_e)$$

where P_R is the radiated power and P_D is the power dissipated in the metallic and dielectric components of the antenna: we can consider the antenna resistance as composed of two terms: the **radiation resistance** R_R and the **loss (or ohmic) resistance** R_D

$$\mathrm{Re}\{P_A\} = P_{in} = P_R + P_D = \frac{1}{2} R_R |I|^2 + \frac{1}{2} R_D |I|^2$$

$$\boxed{P_R = \frac{1}{2} R_R |I|^2} \qquad \boxed{P_D = \frac{1}{2} R_D |I|^2} \qquad \boxed{R_A = R_R + R_D}$$

The radiation efficiency of an antenna (i.e the ratio of the radiated power to the input power) can be expressed in terms of the antenna resistance:

$$e_r = \frac{P}{P_{in}} = \frac{P_R}{P_{in}} = \frac{\frac{1}{2} R_R |I|^2}{\frac{1}{2} R_R |I|^2 + \frac{1}{2} R_D |I|^2} = \frac{R_R}{R_R + R_D}$$

As an example, we can consider an ideal dipole

$$P_R = \frac{\pi}{3}\eta\frac{|I|^2\Delta z^2}{\lambda^2} \qquad\qquad R_R = \frac{2P_R}{|I|^2} = 2\frac{\pi}{3}\eta\frac{\Delta z^2}{\lambda^2} \cong 80\pi^2\left(\frac{\Delta z}{\lambda}\right)^2$$

and by assuming that the wire is made of a perfect electric conductor, in absence of ohmic losses, we infer that $R_D = 0$ and $e_r = 1$. We must observe that the radiation resistance of an ideal dipole is very small when compared to a reference impedance of $50\ \Omega$: for instance, if $\Delta z = 0.01\lambda$ then $R_R = 0.079\ \Omega$.