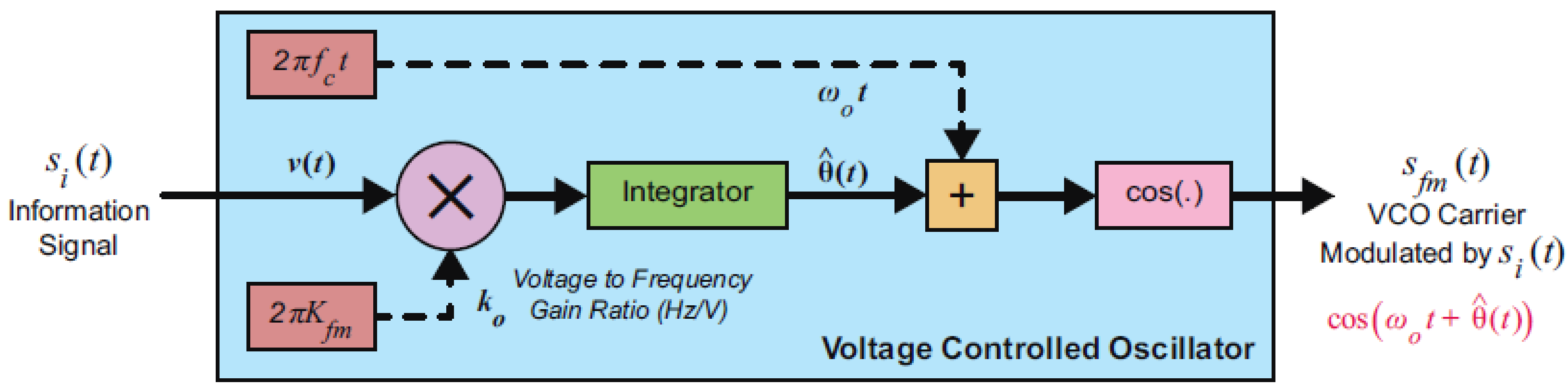
Digital Systems for Telecommunications

Using the Software Defined Radio
for implementing simple analog modulations
The FM case.

FM Modulation

- One of the simplest forms of analogue FM modulator is a controlled oscillator, having quiescent frequency fc and amplitude Ac
  - the phase (and therefore effectively the frequency) of the output changes in response to amplitude variations of an input control signal
  - the information is multiplied by k0=*2πKfm*, representing the modulation constant
  - the product is then integrated (changing its phase by 90 degrees)

$$s_{fm}(t) = A_c \cos\Big(\omega_c t + \underbrace{\theta_{fm}(t)}_{\dot{\theta}(t)}\Big) = A_c \cos\Big(\omega_c t + \underbrace{2\pi K_{fm}}_{k_o} \times \int_{-\infty}^{t} s_i(t)\, dt\Big)$$



FM modulation with a sine wave

- The Frequency Deviation Δ*f*, and the Modulation Index β*fm* are introduced; let's consider simple cos(.) modulating signal

$$\theta_{fm}(t) = 2\pi K_{fm} A_i \times \int_{-\infty}^{t} \cos(\omega_i t)\, dt$$

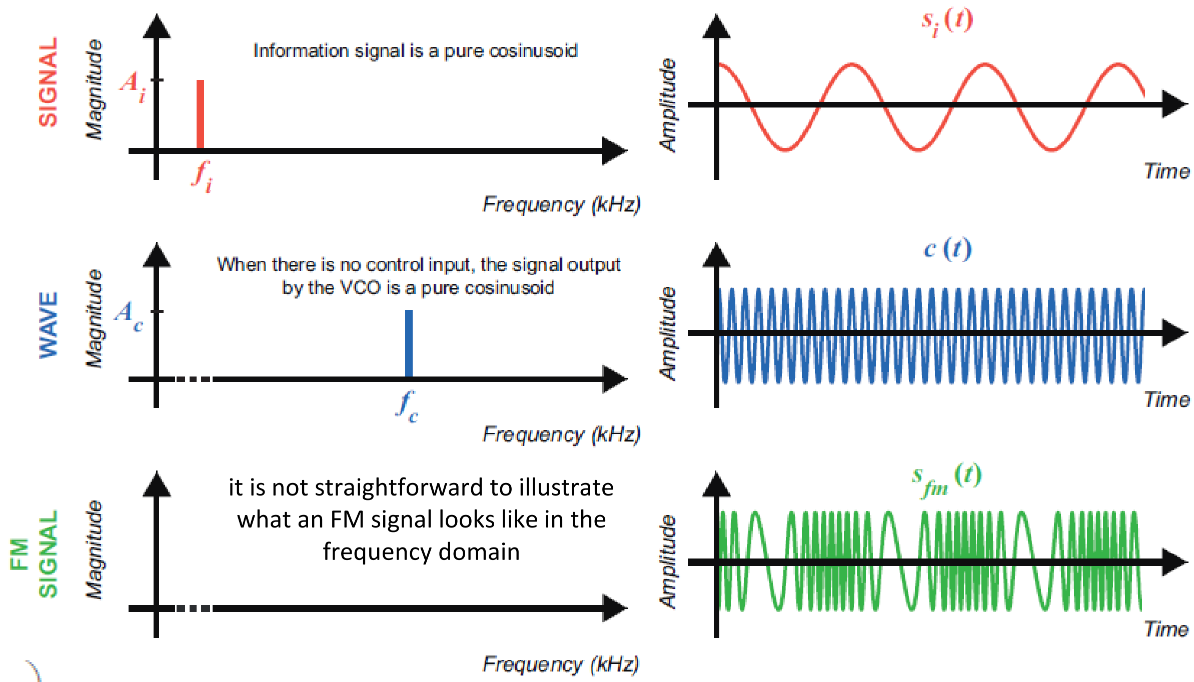
$$= 2\pi K_{fm} A_i \times \frac{\sin(\omega_i t)}{\omega_i}$$

$$= \frac{K_{fm} A_i}{f_i} \sin(\omega_i t)$$

$$= \frac{\Delta f}{f_i} \sin(\omega_i t)$$

$$= \beta_{fm} \sin(\omega_i t)$$

$$s_{fm}(t) = A_c \cos\left(\omega_c t + \beta_{fm} \sin(\omega_i t) \right)$$



FM Signal Bandwidth

- Frequency modulation is either considered to be a *Narrowband* or a *Wideband* process; and the value of β*fm* is what determines this.
  - If β*fm* << 1, it is considered to be *Narrowband FM* (NFM)
  - If β*fm* >> 1 it is *Wideband FM* (WFM)

$$s_{fm}(t) = A_c \cos\left(\omega_c t + \beta_{fm} \sin(\omega_i t) \right) = A_c \cos(\omega_c t) \cos\left(\beta_{fm} \sin(\omega_i t) \right) - A_c \sin(\omega_c t) \sin\left(\beta_{fm} \sin(\omega_i t) \right)$$

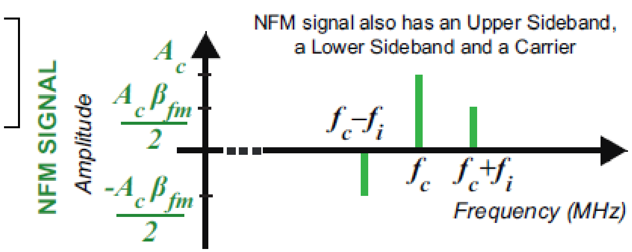
- NFM resembles somehow AM:

$$\cos\left(\beta_{fm} \sin(\omega_i t) \right) \approx 1$$

$$\sin\left(\beta_{fm} \sin(\omega_i t) \right) \approx \beta_{fm} \sin(\omega_i t)$$

$$s_{fm-nfm}(t) = A_c \cos(\omega_c t) - A_c \sin(\omega_c t)\, \beta_{fm} \sin(\omega_i t)$$

$$= A_c \left[\cos(\omega_c t) + \frac{\beta_{fm}}{2} \cos(\omega_c + \omega_i) t - \frac{\beta_{fm}}{2} \cos(\omega_c - \omega_i) t \right]$$

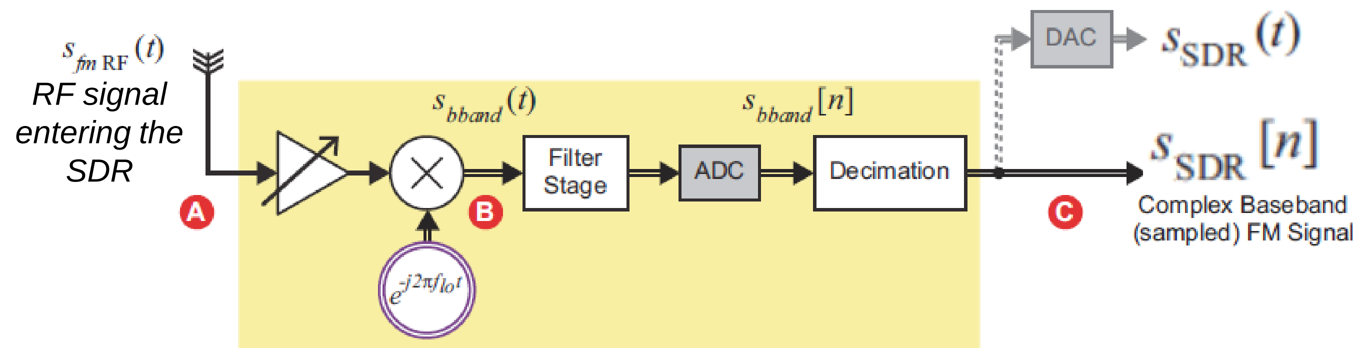


- For WFM the Carson's Rule is used:

$$B = 2(\beta_{fm} + 1) f_i = 2\left(\frac{\Delta f}{f_i} + 1 \right) f_i = 2(\Delta f + f_i) \text{ Hz}$$

Receiving FM using SDR

- What happens when a FM signal is acquired by the SDR?



$$s_{bband}(t) = s_{fm\mathrm{RF}}(t)e^{-j\omega_{lo}t} = s_{fm\mathrm{RF}}(t)\times\left(\cos(\omega_{lo}t) - j\sin(\omega_{lo}t)\right) = A_c\cos\left(\omega_c t + \theta_{fm}(t)\right)\times\left(\cos(\omega_{lo}t) - j\sin(\omega_{lo}t)\right)$$

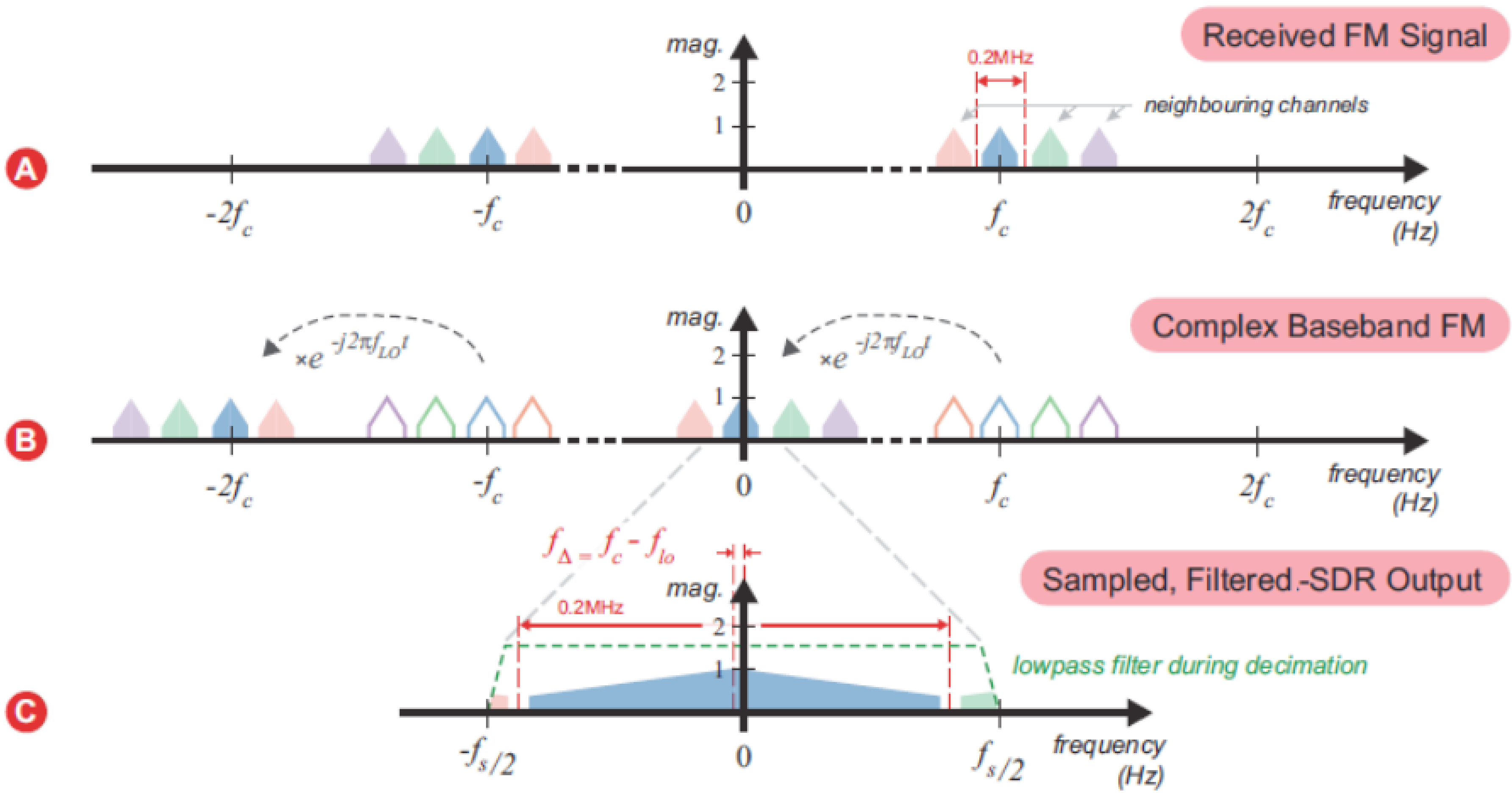
$$= \frac{A_c}{2}\left[\underbrace{\cos\left(\omega_c t + \theta_{fm}(t) - \omega_{lo}t\right)} + \underbrace{\cancel{\cos\left(\omega_c t + \theta_{fm}(t) + \omega_{lo}t\right)}}\right] - j\frac{A_c}{2}\left[\underbrace{\cancel{\sin\left(\omega_c t + \theta_{fm}(t) + \omega_{lo}t\right)}}_{\text{high freq components}} - \underbrace{\sin\left(\omega_c t + \theta_{fm}(t) - \omega_{lo}t\right)}_{\text{baseband components}}\right]$$

- If ωΔ = ωc – ω*lo*=0 $\quad = \dfrac{A_c}{2}e^{j\theta_{fm}(t)} = \dfrac{A_c}{2}\angle\theta_{fm}(t)$
- If ωΔ = ωc – ω*lo*

$$= \frac{A_c}{2}\left[\cos\left(\omega_\Delta t + \theta_{fm}(t)\right) + j\sin\left(\omega_\Delta t + \theta_{fm}(t)\right)\right] = \frac{A_c}{2}e^{j\left(\omega_\Delta t + \theta_{fm}(t)\right)} = \frac{A_c}{2}e^{j\left(\omega_\Delta t + 2\pi K_{fm}\times\int_{-\infty}^{t} s_i(t)dt\right)}$$

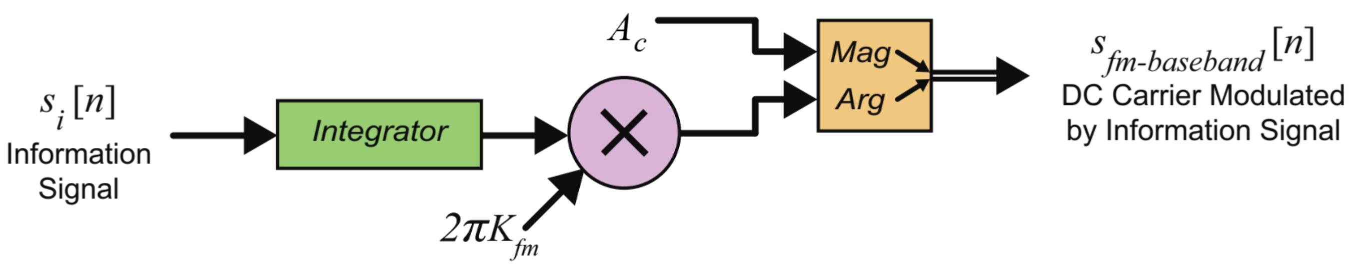
$$= \frac{A_c}{2}\left[\cos\left(\omega_\Delta t + 2\pi K_{fm}\times\int_{-\infty}^{t} s_i(t)dt\right) + j\sin\left(\omega_\Delta t + 2\pi K_{fm}\times\int_{-\infty}^{t} s_i(t)dt\right)\right]$$

Receiving FM using SDR



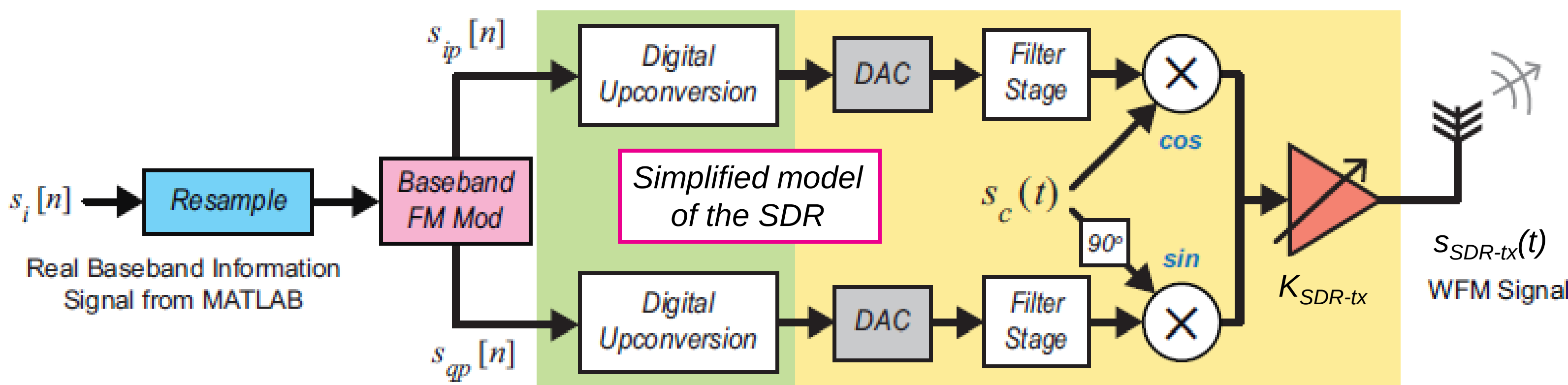
Transmitting FM using SDR

- We have previously seen that the FM-BB signal is $= A_c\, e^{j\theta_{fm}(t)} = A_c \angle \theta_{fm}(t)$



$$\theta_{fm}(t) = 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t)\,dt$$

- A high level block diagram of the processes required to implement this modulator/ transmitter is shown here:



Transmitting FM using SDR

- Let's consider this BB signal

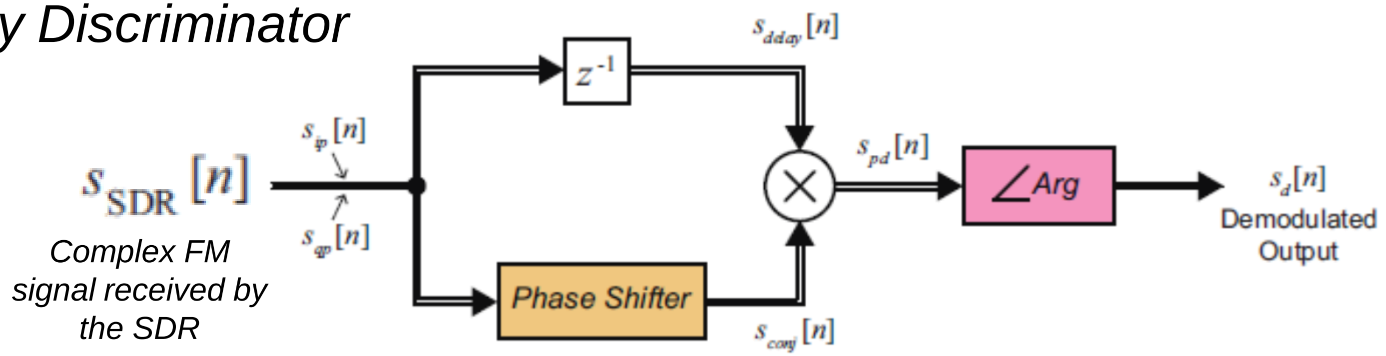
$$s_{fm-baseband}(t) = A_c e^{-j\theta_{fm}(t)} = A_c \angle -\theta_{fm}(t) = A_c \angle -2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t)dt$$

- As a consequence, the SDR-TX signal is

$$\begin{aligned}
s_{\mathrm{SDR}-tx}(t) &= K_{\mathrm{SDR}\text{-}tx}\left[\Re e\left(s_{fm-baseband}(t)\right)\cos(\omega_c t) + \Im m\left(s_{fm-baseband}(t)\right)\sin(\omega_c t)\right] \\
&= K_{\mathrm{SDR}\text{-}tx}\left[A_c\cos\left(-\theta_{fm}(t)\right)\cos(\omega_c t) + A_c\sin\left(-\theta_{fm}(t)\right)\sin(\omega_c t)\right]. \\
&= \frac{A_c K_{\mathrm{SDR}\text{-}tx}}{2}\left[\cos\left(\omega_c t + \theta_{fm}(t)\right) + \cancel{\cos\left(\omega_c t - \theta_{fm}(t)\right)} + \cos\left(\omega_c t + \theta_{fm}(t)\right) - \cancel{\cos\left(\omega_c t - \theta_{fm}(t)\right)}\right] \\
&= \frac{A_c K_{\mathrm{SDR}\text{-}tx}}{2}\left[2\cos\left(\omega_c t + \theta_{fm}(t)\right) + 0\right] = A_c K_{\mathrm{SDR}\text{-}tx}\cos\left(\omega_c t + \theta_{fm}(t)\right) \\
&= A_c K_{\mathrm{SDR}\text{-}tx}\cos\left(\omega_c t + 2\pi K_{fm} \times \int_{-\infty}^{t} s_i(t)dt\right)
\end{aligned}$$

Non-Coherent FM Demodulation: The Discriminator

- A simple non-coherent FM demodulator is the *Complex Delay Line Frequency Discriminator*



- Expressing the BB signal in exponential form, we have: $= \frac{A_c}{2} e^{j\left(\omega_\Delta t + \theta_{fm}(t)\right)}$
- This complex signal is input to two parallel blocks.
  - One takes the conjugate of the signal to change its phase, and the other adds a time delay to the signal to retard it

$$s_{conj}(t) = \frac{A_c}{2} e^{-j\left(\omega_\Delta t + \theta_{fm}(t)\right)} \qquad s_{delay}(t) = \frac{A_c}{2} e^{j\left(\omega_\Delta [t-\tau] + \theta_{fm}(t-\tau)\right)}$$

  - These signals are then mixed together in a process called *phase detection*

$$s_{pd}(t) = s_{conj}(t) \times s_{delay}(t) = \frac{A_c^2}{4} e^{-j\left[\left(\omega_\Delta t + \theta_{fm}(t)\right) - \left(\omega_\Delta [t-\tau] + \theta_{fm}(t-\tau)\right)\right]}$$

Non-Coherent FM Demodulation: The Discriminator

- To extract the information signal, we simply take the argument:

$$s_d(t) = \angle s_{pd}(t) = -\left[\left(\omega_\Delta t + \theta_{fm}(t)\right) - \left(\omega_\Delta[t-\tau] + \theta_{fm}(t-\tau)\right)\right]$$

$$= -\left[\left(\omega_\Delta t - \omega_\Delta[t-\tau]\right) + \left(\theta_{fm}(t) - \theta_{fm}(t-\tau)\right)\right]$$

- When τ is a very small value, it resembles a differentiation operation:

$$s_d(t) \approx -\left[\frac{d}{dt}(\omega_\Delta t) + \frac{d}{dt}\left(\theta_{fm}(t)\right)\right] = -\left[\omega_\Delta + \theta_{fm}'(t)\right] = -\left[\omega_\Delta + 2\pi K_{fm} s_i(t)\right]$$

In the discrete domain: $s_d[n] = \angle\left\{\left(s_{ip}[n] - s_{qp}[n]\right) \times \left(s_{ip}[n-1] + s_{qp}[n-1]\right)\right\}$

- The argument of the phase detected signal varies in proportion to the original information signal, *si(t)*, with a DC offset!
  - The demodulated signal is then finally filtered to remove noise