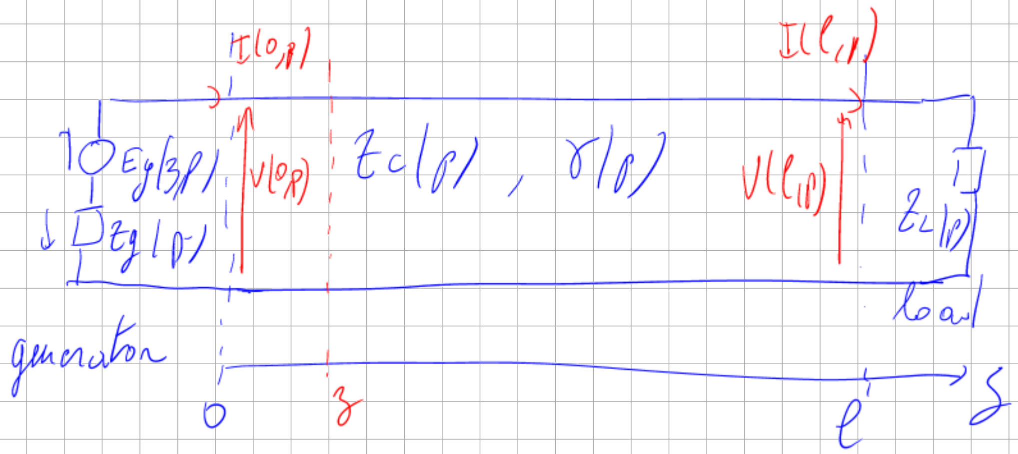
S. Verdeyme
Lesson 7
Sept 24th, 2021

III Expression of $V(z,p)$, $I(z,p)$ as a function of the generator, the line and the load characteristics



lossless line $\gamma(p) = a\,p = \frac{p}{v}$

$$v = \frac{1}{\sqrt{LC}}$$

$$\begin{cases} V(z,p) = V_i(p)\, e^{-\gamma(p)z} + V_r(p)\, e^{+\gamma(p)z} \\ I(z,p) = \dfrac{1}{Z_c(p)}\left(V_i(p)\, e^{-\gamma(p)z} - V_r(p)\, e^{+\gamma(p)z}\right) \end{cases}$$

$$V_i(p),\ V_r(p) = f\big(E_g(z,p),\ Z_g(z,p),\ Z_c(p),\ \gamma(p),\ Z_L(p)\big)$$

* $z = 0$

$$E_g(p) = Z_g(p)\, I(0,p) + V(0,p)$$

$$E_g(p) = \frac{Z_g(p)}{Z_c(p)}\big(V_i(p) - V_r(p)\big) + V_i(p) + V_r(p)$$

$$= V_i(p)\left(\frac{Z_g(p)+Z_c(p)}{Z_c(p)}\right) - V_r(p)\left(\frac{Z_g(p)-Z_c(p)}{Z_c(p)}\right)$$

$$\frac{Z_c(p)\, E_g(p)}{Z_g(p)+Z_c(p)} = V_i(p) - \underbrace{\frac{Z_g(p)-Z_c(p)}{Z_g(p)+Z_c(p)}}_{\Gamma_g(p)} V_r(p)$$

$\Gamma_g(p)$ reflection coefficient on the generator.

* $\Gamma(l,p) = \Gamma_L(p)$ reflection coef on the load

$$= \frac{Z_L(p) - Z_c(p)}{Z_L(p) + Z_c(p)} = \frac{V_r(p)}{V_i(p)}\, e^{+2\gamma(p)l}$$

$$V_r(p) = \Gamma_L(p)\, V_i(p)\, e^{-2\gamma(p)\ell}$$

$$\frac{Z_c(p)}{Z_g(p)+Z_c(p)} E_g(p) = V_i(p)\left[1 - \Gamma_g(p)\Gamma_L(p)\, e^{-2\gamma(p)\ell}\right]$$

$$V_i(p) = \frac{Z_c(p)}{Z_g(p)+Z_c(p)} E_g(p)\, \frac{1}{1-\Gamma_g(p)\Gamma_L(p)\, e^{-2\gamma(p)\ell}}$$

$$V_r(p) = \frac{Z_c(p)}{Z_g(p)+Z_c(p)} E_g(p)\, \frac{\Gamma_L(p)\, e^{-2\gamma(p)\ell}}{1-\Gamma_g(p)\Gamma_L(p)\, e^{-2\gamma(p)\ell}}$$

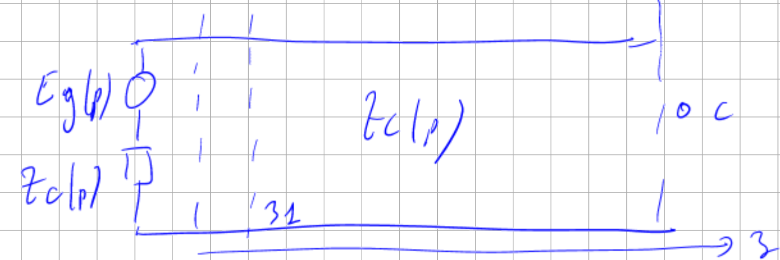
$$V(z,p) = \frac{Z_c(p)}{Z_g(p)+Z_c(p)} E_g(p)\, \frac{e^{-\gamma(p)z} + \Gamma_L(p)\, e^{-\gamma(p)(2\ell-z)}}{1-\Gamma_g(p)\Gamma_L(p)\, e^{-2\gamma(p)\ell}}$$

$$I(z,p) = \frac{1}{Z_g(p)+Z_c(p)} E_g(p)\, \frac{e^{-\gamma(p)z} - \Gamma_L(p)\, e^{-\gamma(p)(2\ell-z)}}{1-\Gamma_g(p)\Gamma_L(p)\, e^{-2\gamma(p)\ell}}$$

$$V(z,p),\ I(z,p) \xrightarrow{\mathcal{L}^{-1}} v(z,t),\ i(z,t)$$

difficult to do

Tutorial 2



$$Z_g(p) = Z_c(p) \qquad Z_L(p) \to \infty$$