Lecture 13

Electric Potential

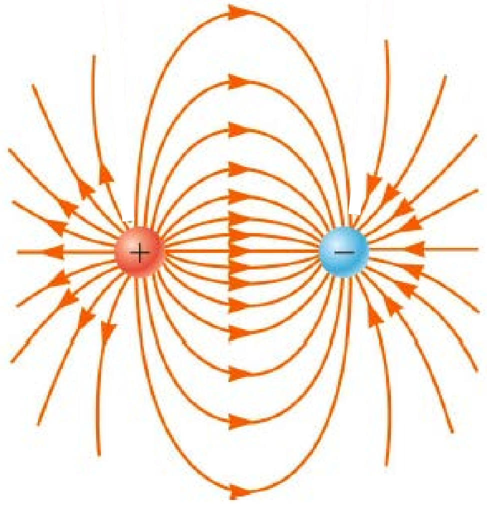
Date: 4/22/2025

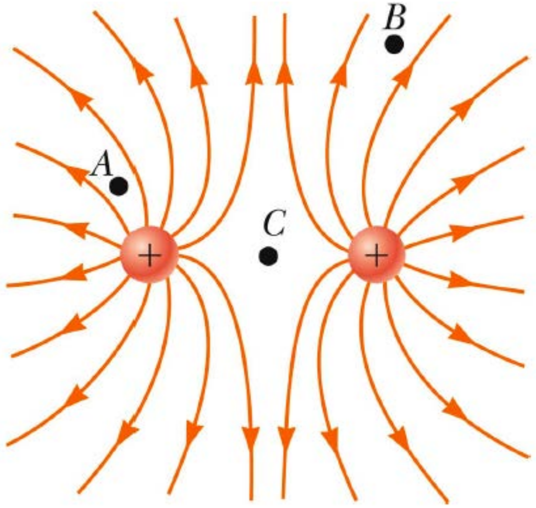
Course Instructor:
Jingtian Hu (胡竞天)

Previous Lecture: Gauss's Law

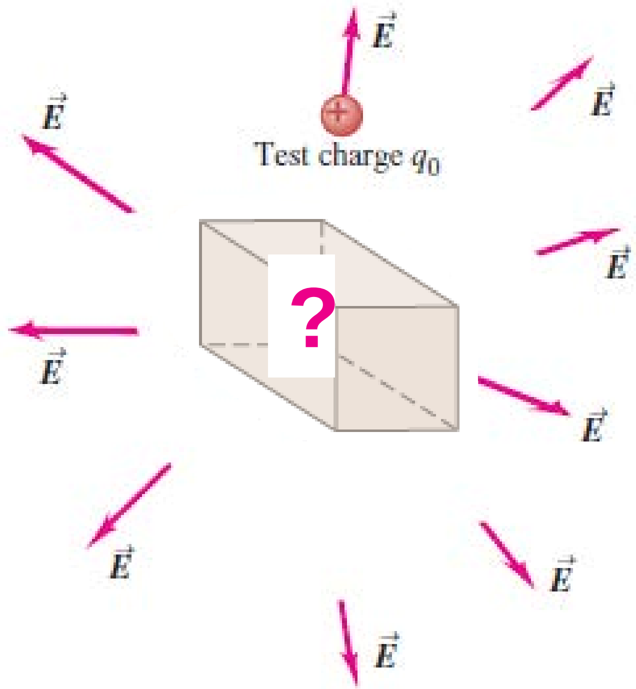
Coulomb's law: from charge distribution to electrical fields

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\vec{e}_r}$$





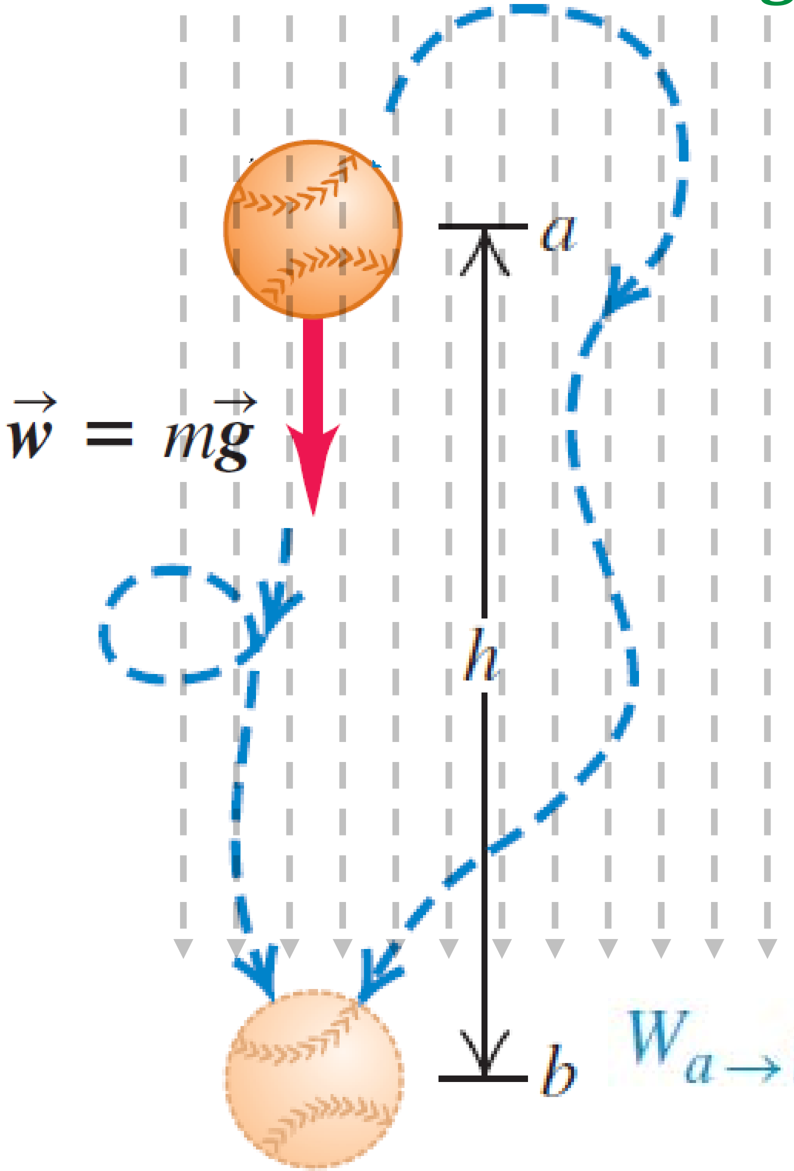
Calculate charge distribution from electrical fields?



What about energy of the system?

Gravitational vs Electrostatic Forces

Total mechanical energy is *conserved*

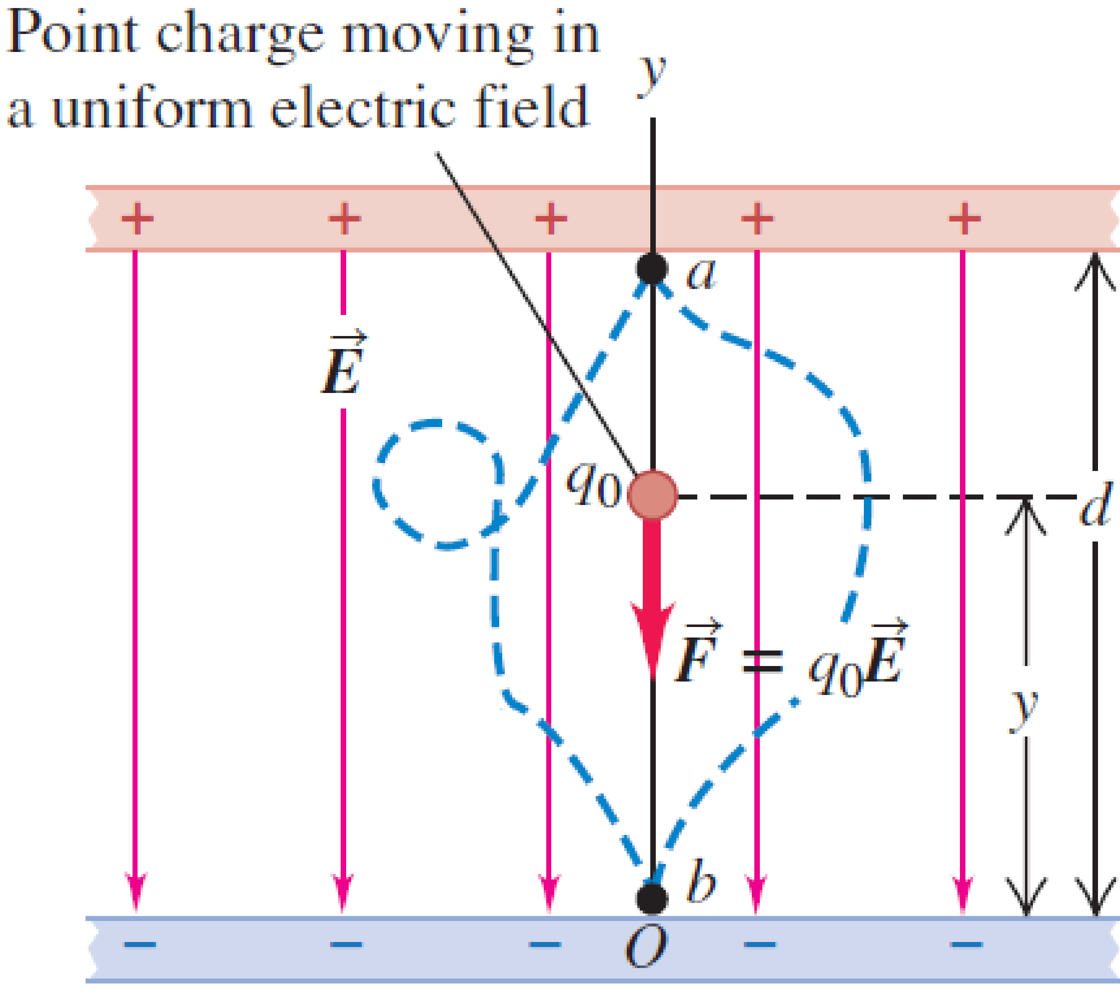


uniform gravitational field

The work done by the gravitational force is the same for any path from *a* to *b*:

$$W_{a \to b} = -\Delta U = \boxed{mg}\boxed{h}$$

Similarly, for a charge *q* in *E*



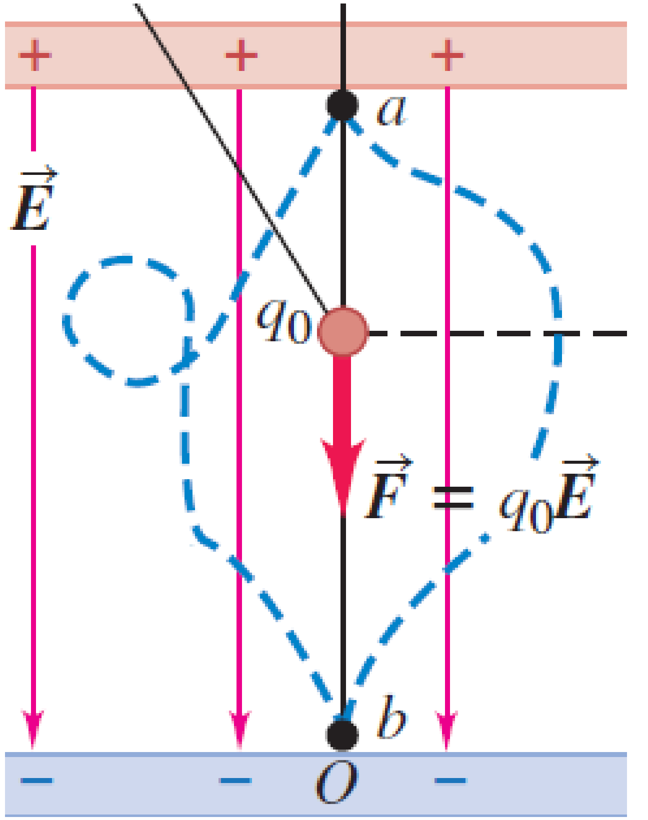
$$W_{a \to b} = -\Delta U = \boxed{q_0 E}\boxed{d}$$

Electric Potential Energy in a Uniform Field

A pair of charged parallel metal plates sets up a uniform electric field with magnitude *E*. The field exerts a downward force with magnitude: $F = q_0E$

So $W_{a\to b} = Fd = q_0Ed$

This work is positive, since the force is in the same direction as the net displacement of the test charge.



The work done by E is independent of the path:

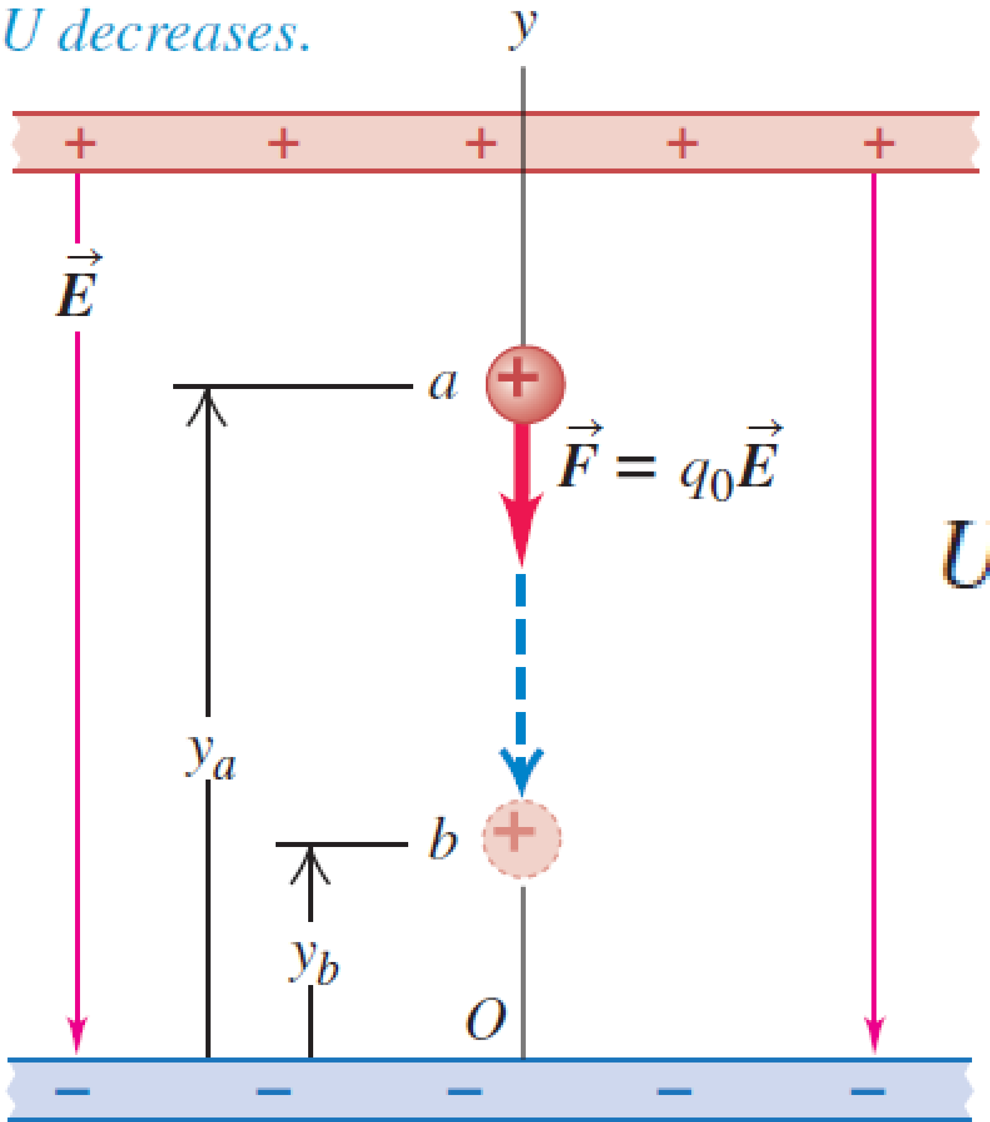
We can represent this work W_{ab} with a *potential-energy* function *U* again: $U = q_0Ey$

When the test charge moves from height y_a to height y_b the work done on the charge by the field is given by:

$$W_{a\to b} = -\Delta U = -(U_b - U_a) = -(q_0Ey_b - q_0Ey_a) = q_0E(y_a - y_b)$$

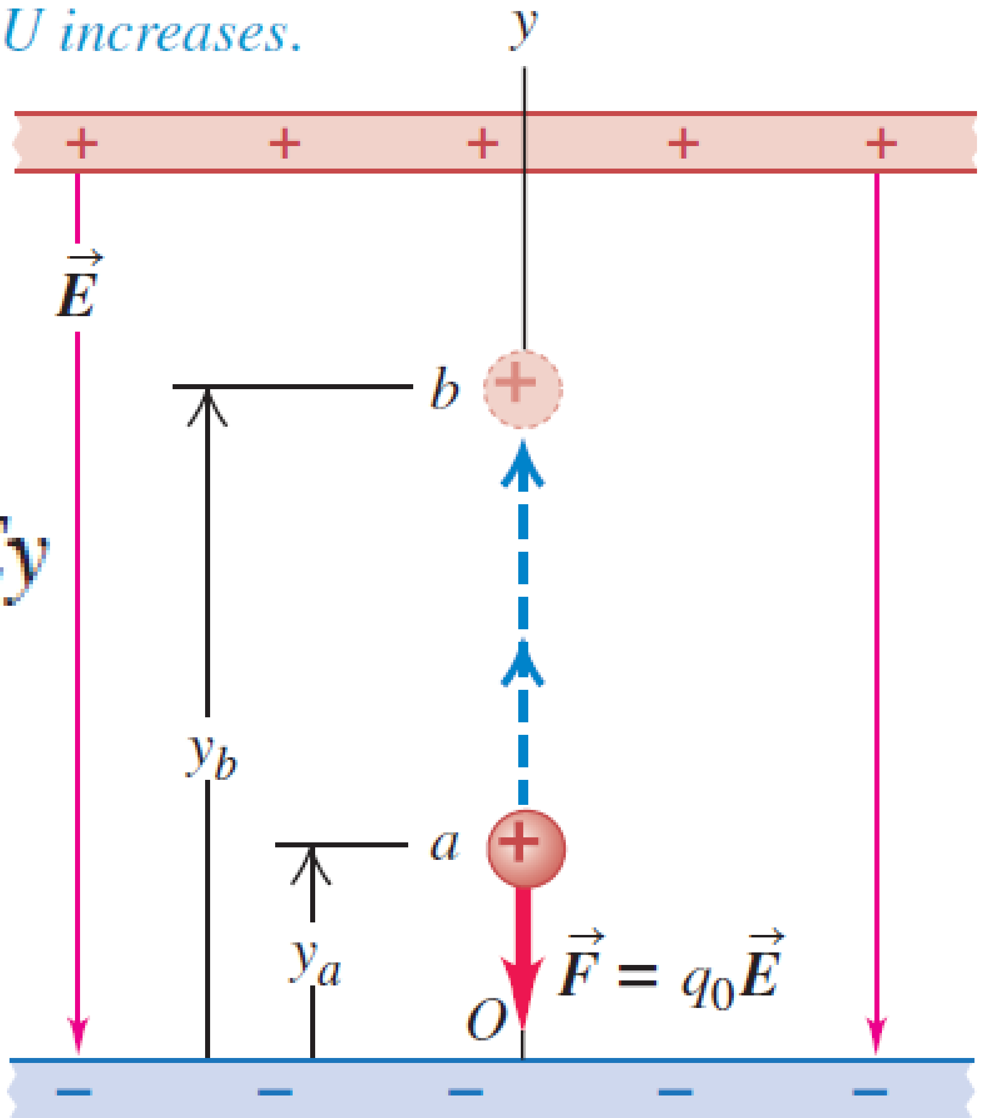
Electric Potential Energy in a Uniform Field

(a) Positive charge moves in the direction of $\vec{E}$:
- Field does *positive* work on charge.
- *U decreases.*



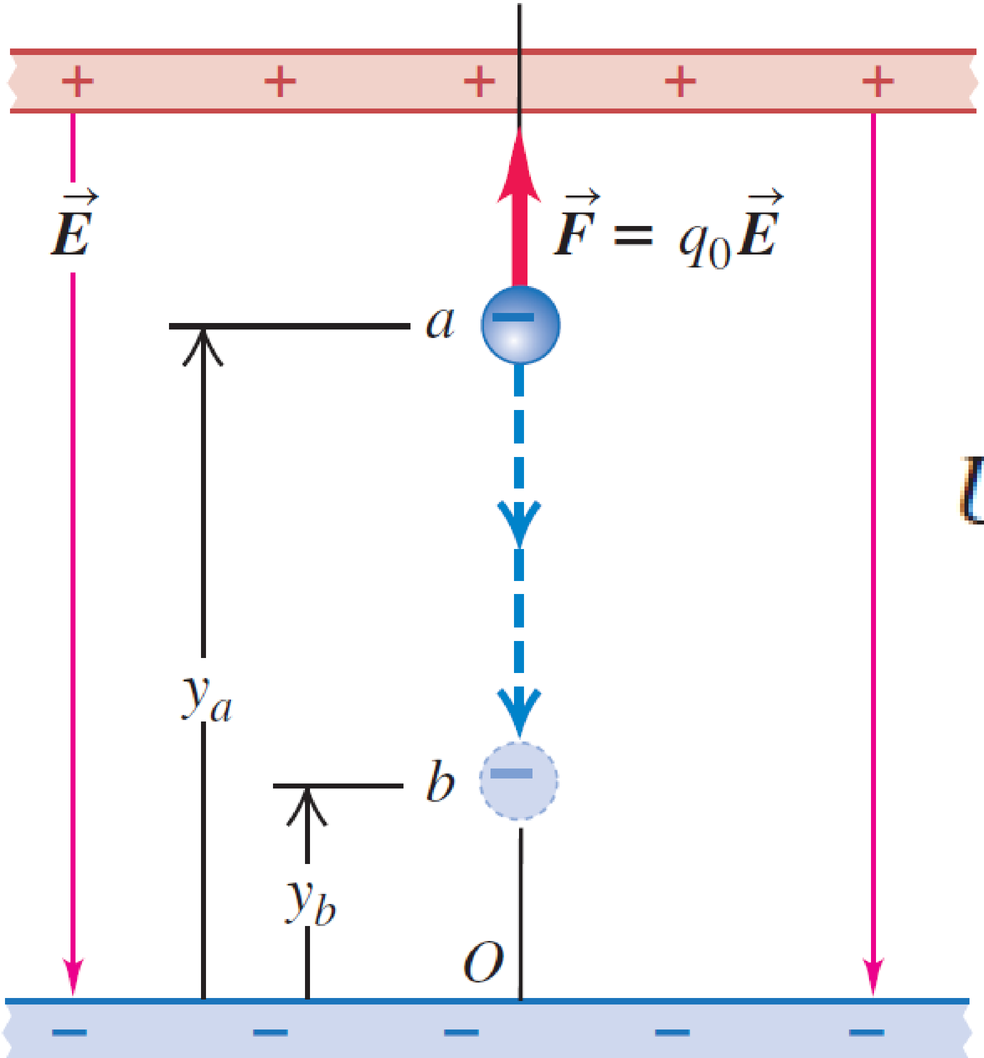
$$U = q_0Ey$$

(b) Positive charge moves opposite $\vec{E}$:
- Field does *negative* work on charge.
- *U increases.*



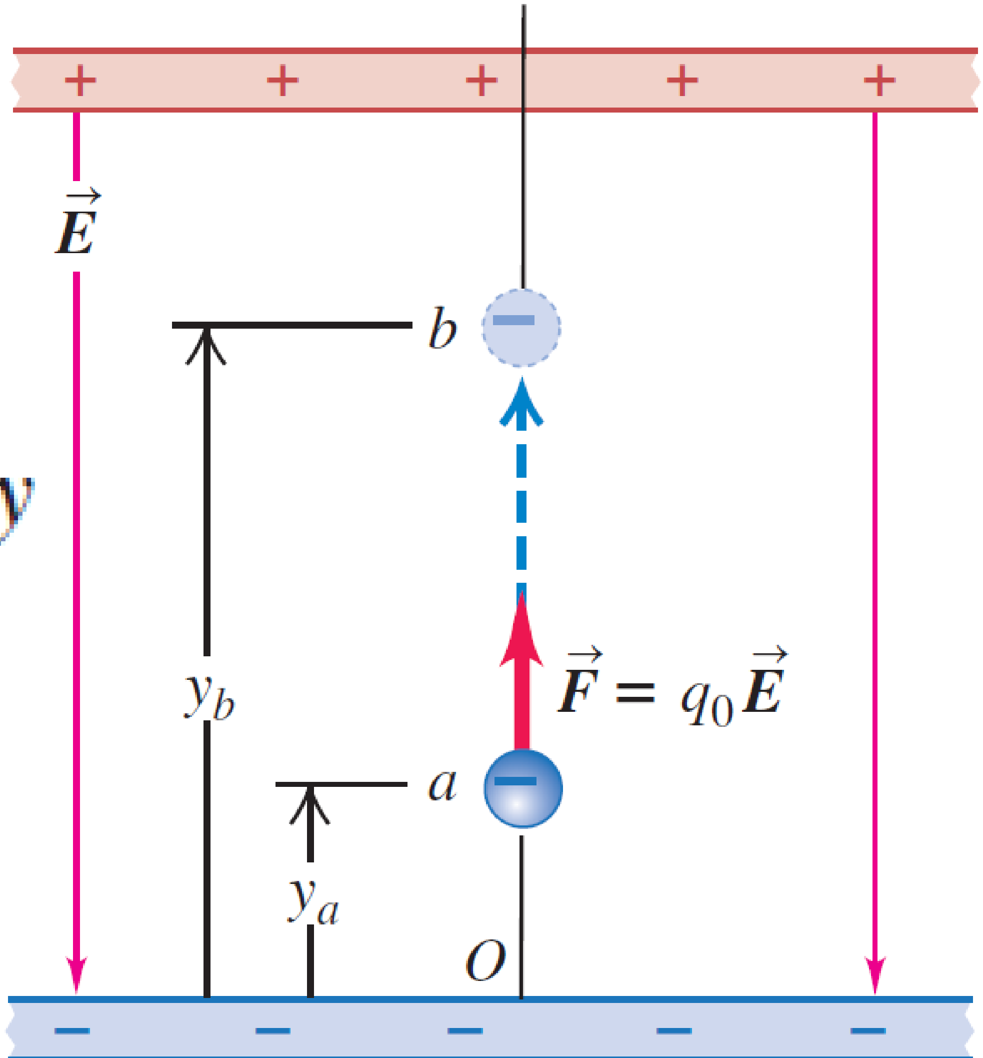
Electric Potential Energy in a Uniform Field

(a) Negative charge moves in the direction of $\vec{E}$:
- Field does *negative* work on charge.
- *U increases.*



$$U = q_0Ey$$

(b) Negative charge moves opposite $\vec{E}$:
- Field does *positive* work on charge.
- *U decreases.*



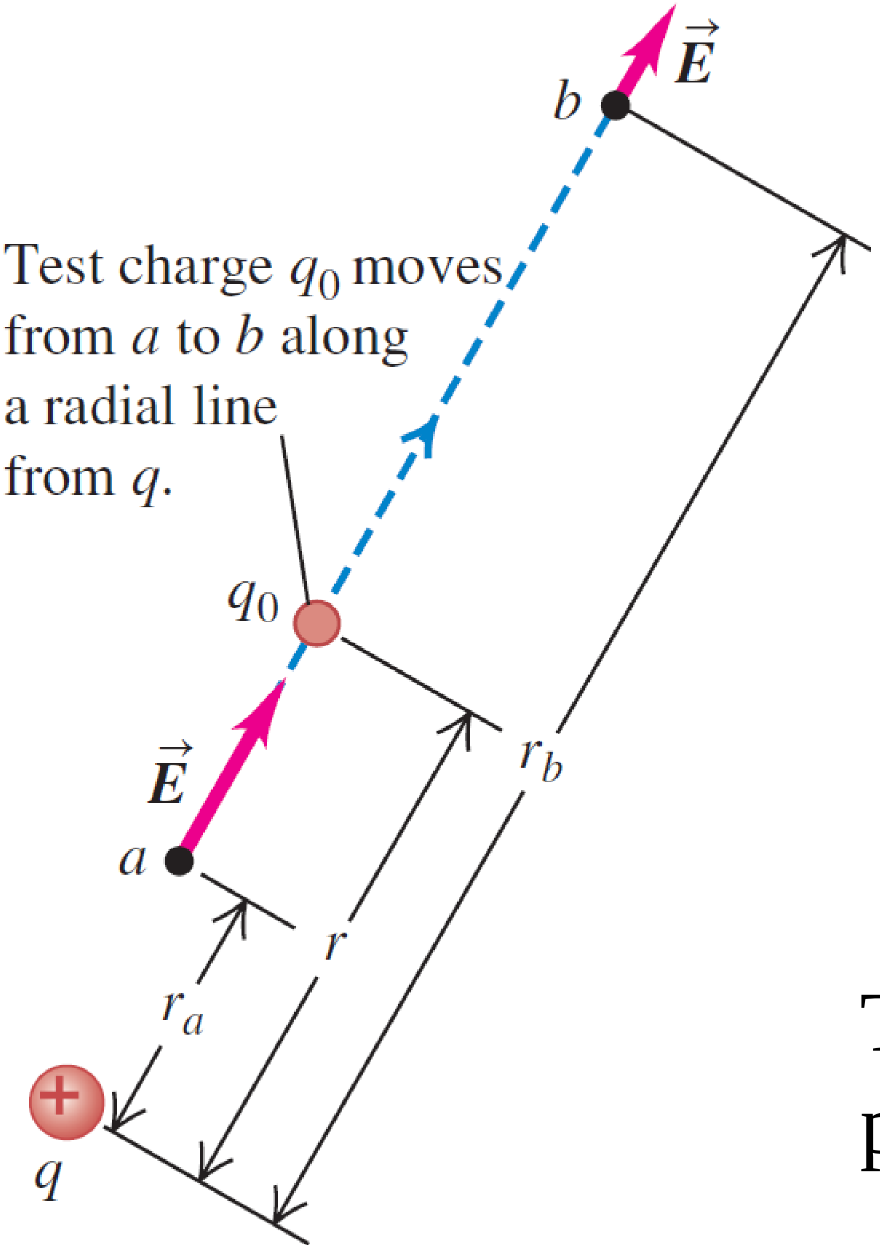
Electric Potential Energy in a Uniform Field

CAUTION: Electric potential energy

The relationship between electric potential energy change and motion in an electric field is an important one that we'll use often, but that takes some effort to truly understand.

We will derive it at the end of this chapter

Electric Potential Energy of Point Charges



To derive **potential energy**, we always start from **forces** that do the **work**!

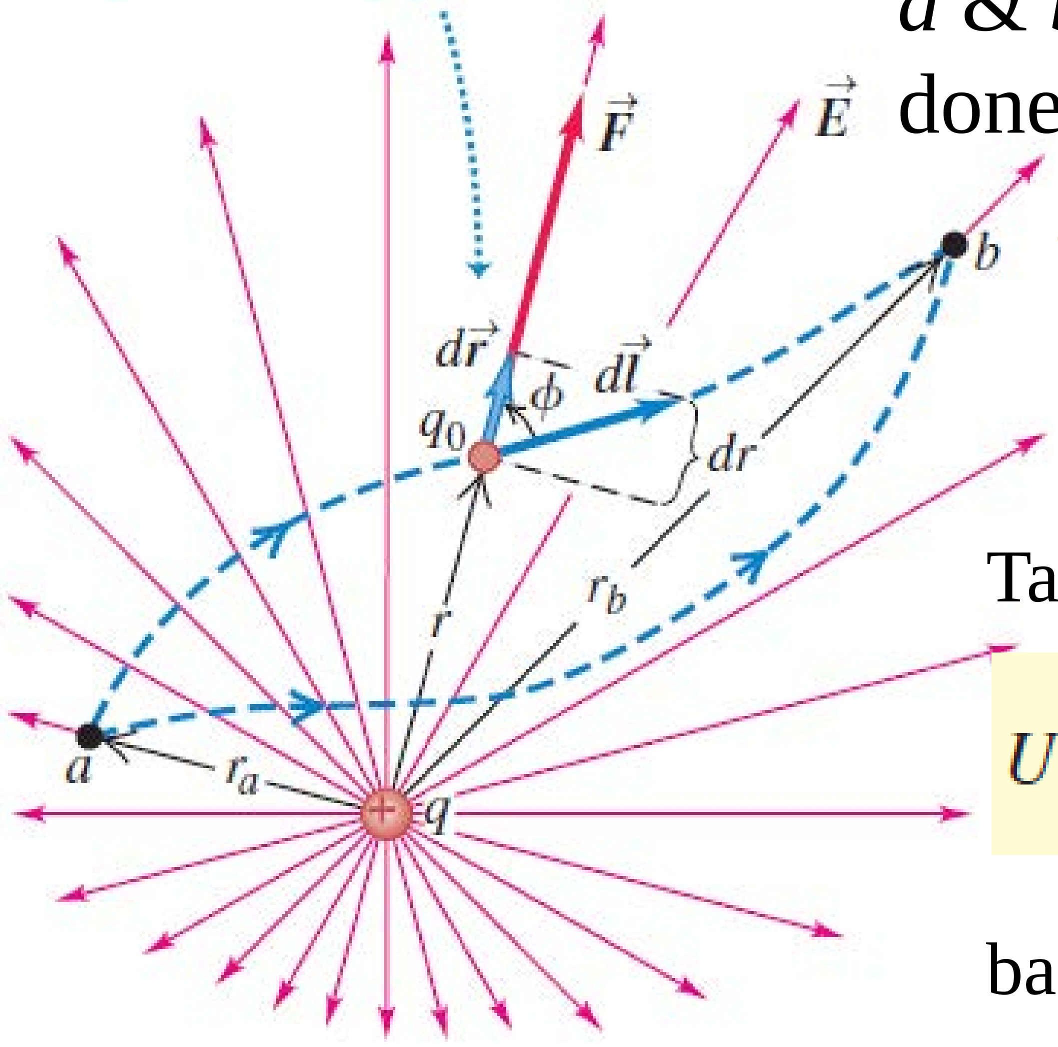
$$F_r = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}$$

$$W_{a\to b} = \int_{r_a}^{r_b} F_r\,dr$$

$$= \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}dr = \frac{qq_0}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

The work done by the electric force for this particular path depends **only on the endpoints**

Electric Potential Energy of Point Charges



Now consider a general displacement in which *a* & *b* do *not* lie on the same radial line. Work done on during this displacement is given by:

$$W_{a\to b} = \int_{r_a}^{r_b} F\cos\phi\, dl = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\cos\phi\, dl$$

radial component of the displacement $\boxed{\cos\phi\, dl = dr}$

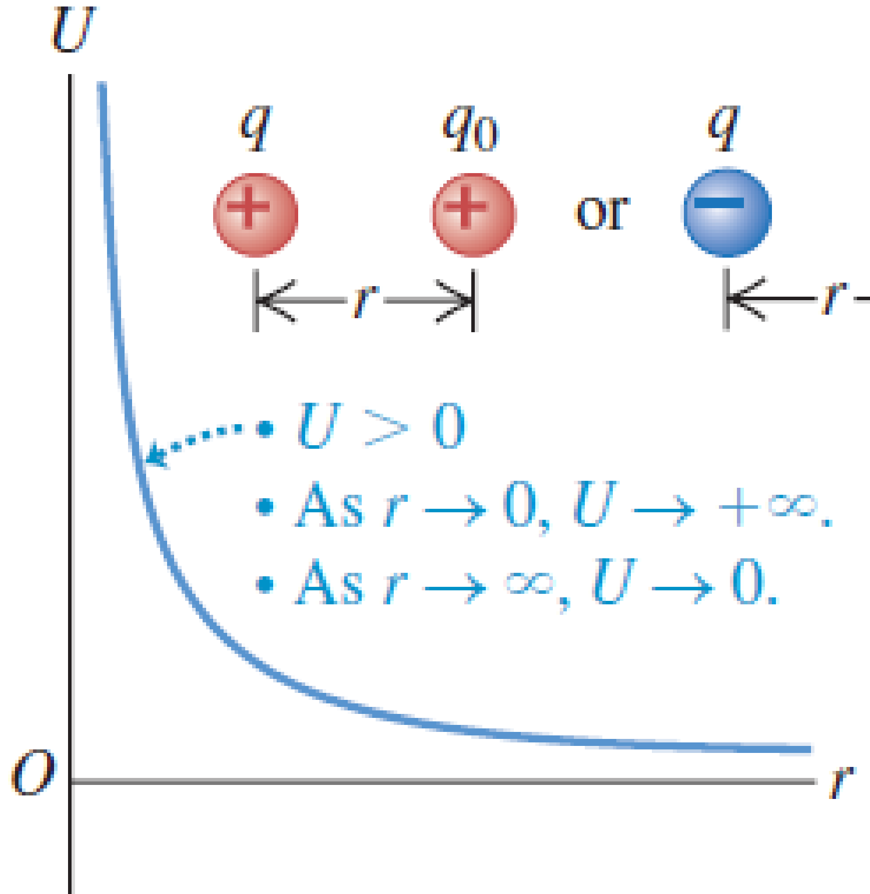
Taking the energy at infinity to be $r_a = 0$, and $r_b = r$:

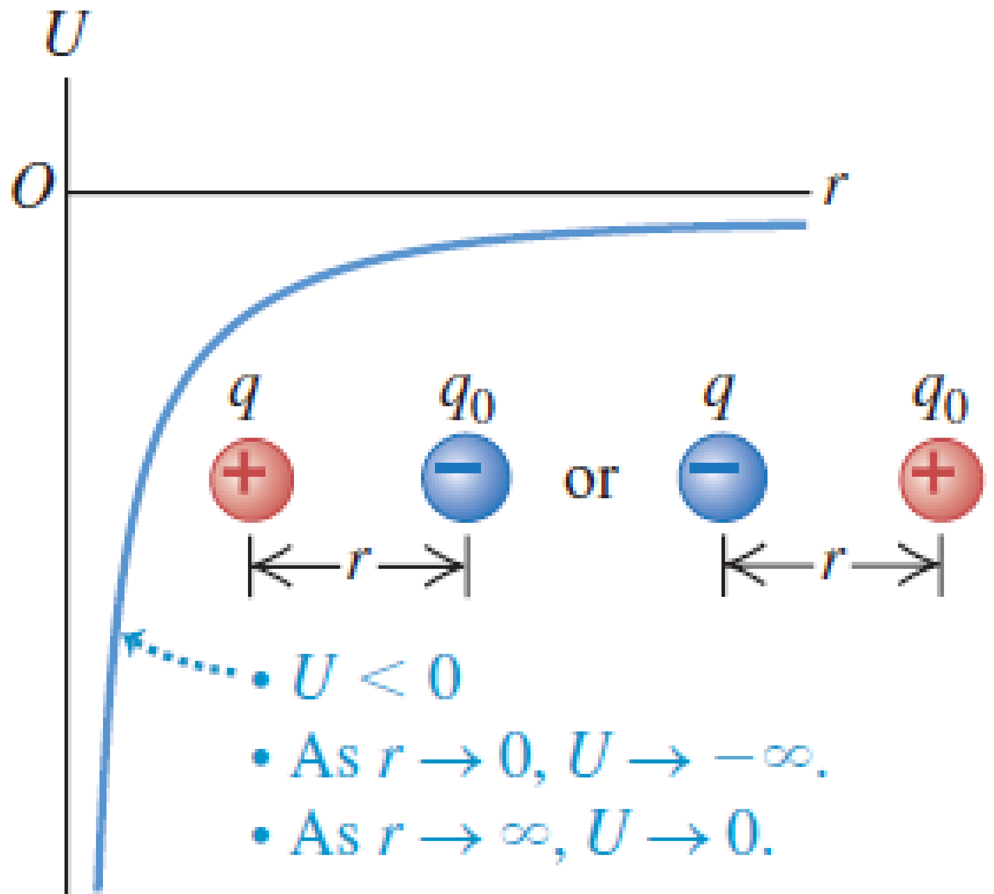
$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$$ (electric potential energy of two point charges q and q_0)

based on the work-energy theorem

Electric Potential Energy of Point Charges

Electric potential energy vs. electric force Don't confuse equation for the potential energy of two point charges with the radial component of the electric force that one charge exerts on the other. Potential energy ***U* is proportional to 1/*r*** while the force component ***F*$_r$ is proportional to 1/*r*^{2}**





Example 23.1 Conservation of energy

A positron (the electron's antiparticle) has mass 9.11×10^{-31} kg and charge $q_0 = +e = +1.60 \times 10^{-19}$ C. Suppose a positron moves in the vicinity of an α (alpha) particle, which has charge $q = +2e = 3.20 \times 10^{-19}$ C and mass 6.64×10^{-27} kg. The α particle's mass is more than 7000 times that of the positron, so we assume that the α particle remains at rest. When the positron is 1.00×10^{-10} m from the α particle, it is moving directly away from the α particle at 3.00×10^{6} m/s. (a) What is the positron's speed when the particles are 2.00×10^{-10} m apart? (b) What is the positron's speed when it is very far from the α particle? (c) Suppose the initial conditions are the same but the moving particle is an electron (with the same mass as the positron but charge $q_0 = -e$). Describe the subsequent motion.

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Both particles have positive charge, so the positron speeds up as it moves away from the α particle. From the energy conservation equation, the final kinetic energy is: $K_b = \frac{1}{2}mv_b^2 = K_a + U_a - U_b$

$$K_a = \frac{1}{2}mv_a^2 = \frac{1}{2}(9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^{6}\ \text{m/s})^2$$

$$= 4.10 \times 10^{-18}\ \text{J}$$

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$$= 4.10 \times 10^{-18}\ \text{J}$$

$$U_a = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r_a} = (9.0 \times 10^{9}\ \text{N} \cdot \text{m}^2/\text{C}^2)\ \frac{(3.20 \times 10^{-19}\ \text{C})(1.60 \times 10^{-19}\ \text{C})}{1.00 \times 10^{-10}\ \text{m}}$$

$$= 4.61 \times 10^{-18}\ \text{J}$$

$$U_b = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r_b} = 2.30 \times 10^{-18}\ \text{J}$$

Example 23.1 Conservation of energy

Both particles have positive charge, so the positron speeds up as it moves away from the α particle. From the energy conservation equation, the final kinetic energy is: $K_b = \frac{1}{2}mv_b^2 = K_a + U_a - U_b$

$$U_a = 4.61 \times 10^{-18}\ \text{J} \qquad U_b = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r_b} = 2.30 \times 10^{-18}\ \text{J}$$

Hence the positron kinetic energy and speed at $r = r_b$ are

$$K_b = \frac{1}{2}mv_b^2 = 4.10 \times 10^{-18}\,\text{J} + 4.61 \times 10^{-18}\,\text{J} - 2.30 \times 10^{-18}\,\text{J}$$

$$= 6.41 \times 10^{-18}\,\text{J}$$

$$v_b = \sqrt{\frac{2K_b}{m}} = \sqrt{\frac{2(6.41 \times 10^{-18}\ \text{J})}{9.11 \times 10^{-31}\ \text{kg}}} = 3.8 \times 10^{6}\ \text{m/s}$$

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$$K_c = K_a + U_a - U_c = 4.10 \times 10^{-18}\,\text{J} + 4.61 \times 10^{-18}\,\text{J} - 0$$
$$= 8.71 \times 10^{-18}\,\text{J}$$

When the positron and particle are very far, the final potential energy U_c approaches 0! $K_c = K_a + U_a - U_c = 4.10 \times 10^{-18}\,\text{J} + 4.61 \times 10^{-18}\,\text{J} - 0$

Example 23.1 Conservation of energy

When the positron and particle are very far, the final potential energy U_c approaches 0!

$$K_c = K_a + U_a - U_c = 4.10 \times 10^{-18}\,\text{J} + 4.61 \times 10^{-18}\,\text{J} - 0$$
$$= 8.71 \times 10^{-18}\,\text{J}$$

$$v_c = \sqrt{\frac{2K_c}{m}} = \sqrt{\frac{2(8.71 \times 10^{-18}\ \text{J})}{9.11 \times 10^{-31}\ \text{kg}}} = 4.4 \times 10^{6}\ \text{m/s}$$

(c) The electron and α particle have opposite charges, so the force is attractive and the electron slows down as it moves away. Changing the moving particle's sign from +*e* to -*e* means that the initial potential energy is now $U_a = -4.61\times10^{-18}$ J which makes the total mechanical energy *negative*:

$$K_a + U_a = (4.10 \times 10^{-18}\,\text{J}) - (4.61 \times 10^{-18}\,\text{J})$$
$$= -0.51 \times 10^{-18}\,\text{J}$$

Example 23.1 Conservation of energy

The total mechanical energy would have to be positive for the electron to move infinitely far away from the α particle. Like a rock thrown upward at low speed from the earth's surface, it will reach a maximum separation r_d from the α particle before reversing direction. At this point its speed and its kinetic energy are zero, so at separation r_d we have:

$$U_d = K_a + U_a - K_d = (-0.51 \times 10^{-18}\ \text{J}) - 0$$

$$U_d = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r_d} = -0.51 \times 10^{-18}\ \text{J}$$

$$r_d = \frac{1}{U_d}\frac{qq_0}{4\pi\epsilon_0} = \frac{(9.0 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)}{-0.51 \times 10^{-18}\ \text{J}}(3.20 \times 10^{-19}\ \text{C})(-1.60 \times 10^{-19}\text{C})$$

$$= 9.0 \times 10^{-10}\ \text{m}$$

Example 23.1 Conservation of energy

$$r_d = \frac{1}{U_d}\frac{qq_0}{4\pi\epsilon_0} = \frac{(9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})}{-0.51 \times 10^{-18}\ \mathrm{J}}(3.20 \times 10^{-19}\ \mathrm{C})(-1.60 \times 10^{-19}\mathrm{C})$$

$$= 9.0 \times 10^{-10}\ \mathrm{m}$$

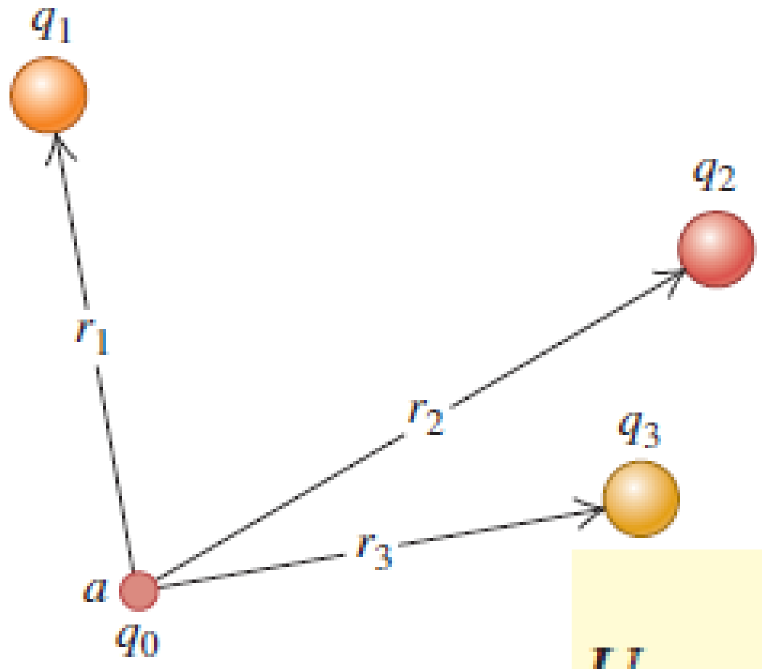
For $r_b = 2.00 \times 10^{-10}$ m we have $U_b = -2.30 \times 10^{-18}$ J, so the electron kinetic energy and speed at this point are

$$K_b = \tfrac{1}{2} m v_b{}^2 = 4.10 \times 10^{-18}\ \mathrm{J} + (-4.61 \times 10^{-18}\ \mathrm{J})$$

$$- (-2.30 \times 10^{-18}\ \mathrm{J}) = 1.79 \times 10^{-18}\ \mathrm{J}$$

$$v_b = \sqrt{\frac{2K_b}{m}} = \sqrt{\frac{2(1.79 \times 10^{-18}\ \mathrm{J})}{9.11 \times 10^{-31}\ \mathrm{kg}}} = 2.0 \times 10^6\ \mathrm{m/s}$$

Potential Energy with Several Point Charges



23.8 The potential energy associated with a charge q_0 at point a depends on the other charges q_1, q_2, and q_3 and on their distances r_1, r_2, and r_3 from point a.

Algebraic sum (*not* a vector sum):

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

- It follows that for **every electric field** due to a **static charge distribution**, the **force** exerted by that field is **conservative.**
- U is defined to be **zero** when **all** the **distances** are infinite

Potential Energy with Several Point Charges

The previous equation gives the potential energy associated with the presence of the test charge q_0 in the field $\boldsymbol{E}$ produced by q_1, q_2, q_3, … But there is also potential energy involved in **assembling these charges**. The *total* potential energy U is the sum of the potential energies of interaction for each pair of charges is:

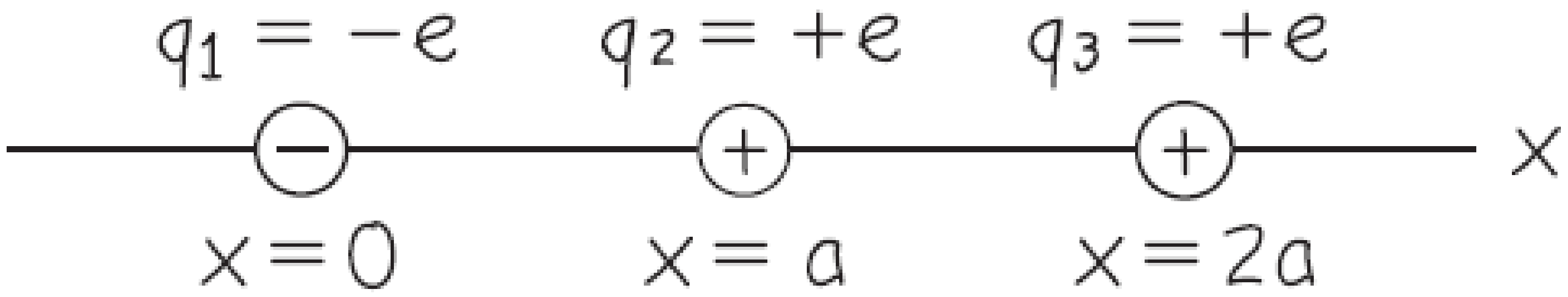
$$U = \frac{1}{4\pi\epsilon_0} \sum_{i<j} \frac{q_i q_j}{r_{ij}}$$

This sum extends over all *pairs* of charges: the $i < j$ condition avoids double counting!

Example 23.2 A system of point charges

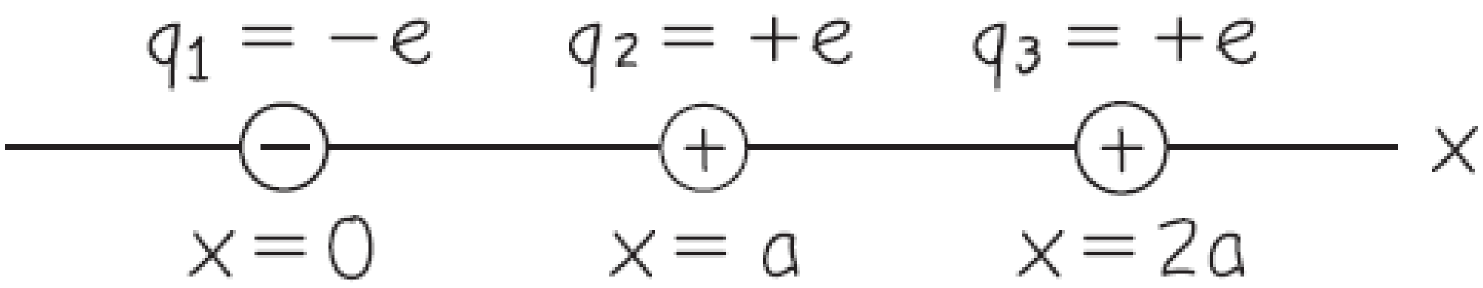
Two point charges are located on the x-axis, $q_1 = -e$ at $x = 0$ and $q_2 = +e$ at $x = a$. (a) Find the work that must be done by an external force to bring a third point charge $q_3 = +e$ from infinity to $x = 2a$. (b) Find the total potential energy of the system of three charges.

23.10 Our sketch of the situation after the third charge has been brought in from infinity.



Example 23.2 A system of point charges

23.10 Our sketch of the situation after the third charge has been brought in from infinity.



The work W equals the difference between (i) the potential energy U associated with q_3 when it is at $x = 2a$ and (ii) the potential energy when it is infinitely far away. The (ii) of these is zero, so the **work required is equal to *U***.

$$W = U = \frac{q_3}{4\pi\epsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right) = \frac{+e}{4\pi\epsilon_0}\left(\frac{-e}{2a} + \frac{+e}{a}\right) = \frac{+e^2}{8\pi\epsilon_0 a}$$

Positive as expected: we need some efforts to move the third charge closer to q_2

Example 23.2 A system of point charges

(b) Find the total potential energy of the system of three Charges. **Which one to use?**

$$U = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}+\cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

Energy of **all pairs** of charges

Energy of a **single charge** in a potential created by multiple charges

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}}+\frac{q_1 q_3}{r_{13}}+\frac{q_2 q_3}{r_{23}}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{(-e)(e)}{a}+\frac{(-e)(e)}{2a}+\frac{(e)(e)}{a}\right] = \frac{-e^2}{8\pi\epsilon_0 a}$$

Electric Potential

Previously: potential energy U associated with a *test charge* in E. Concept of **electric potential**, often called simply **potential**: describe this potential energy on a "*per unit charge*" basis.

Potential is *potential energy per unit charge*: $V = \dfrac{U}{q_0}$ or $U = q_0 V$

Unit: $1\ \mathrm{V} = 1\ \text{volt} = 1\ \mathrm{J/C} = 1\ \text{joule/coulomb}$

Think about the work needed to move a charge from *a* to *b*

$$\frac{W_{a\to b}}{q_0} = -\frac{\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right) = -(V_b - V_a) = V_a - V_b$$

Electric Potential

V_{ab} the potential of **a** with respect to **b** equals the **work done by the electric force** when a **UNIT** charge moves from *a* to *b*

Recall: $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{qq_0}{r}$ (electric potential energy of two point charges q and q_0)

So: $V = \dfrac{U}{q_0} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}$ (potential due to a point charge)

Example 23.8 A charged conducting sphere

A solid conducting sphere of radius has a total charge *q*. Find the electric potential everywhere, both outside and inside the sphere

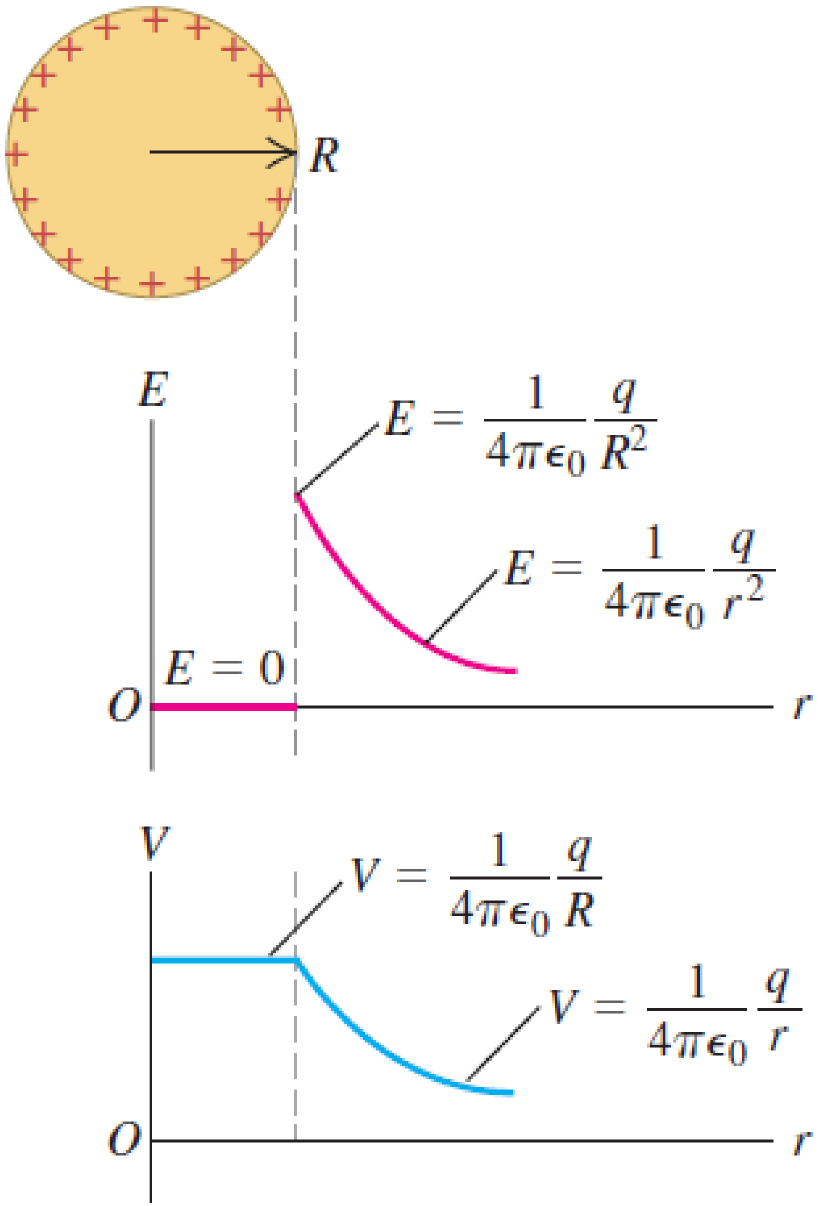
Solution: We used Gauss's law to find the electric *field* at all points for this charge distribution. We can use that result to determine the potential.

As usual, we take V = 0 at infinity. Then the potential at a point outside the sphere at a distance from its center is the same as that due to a point charge at the center:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

Inside the sphere, ***E*** is zero everywhere. Hence **no work is done** on a test charge that moves from any point to any other point. **V is a constant**!

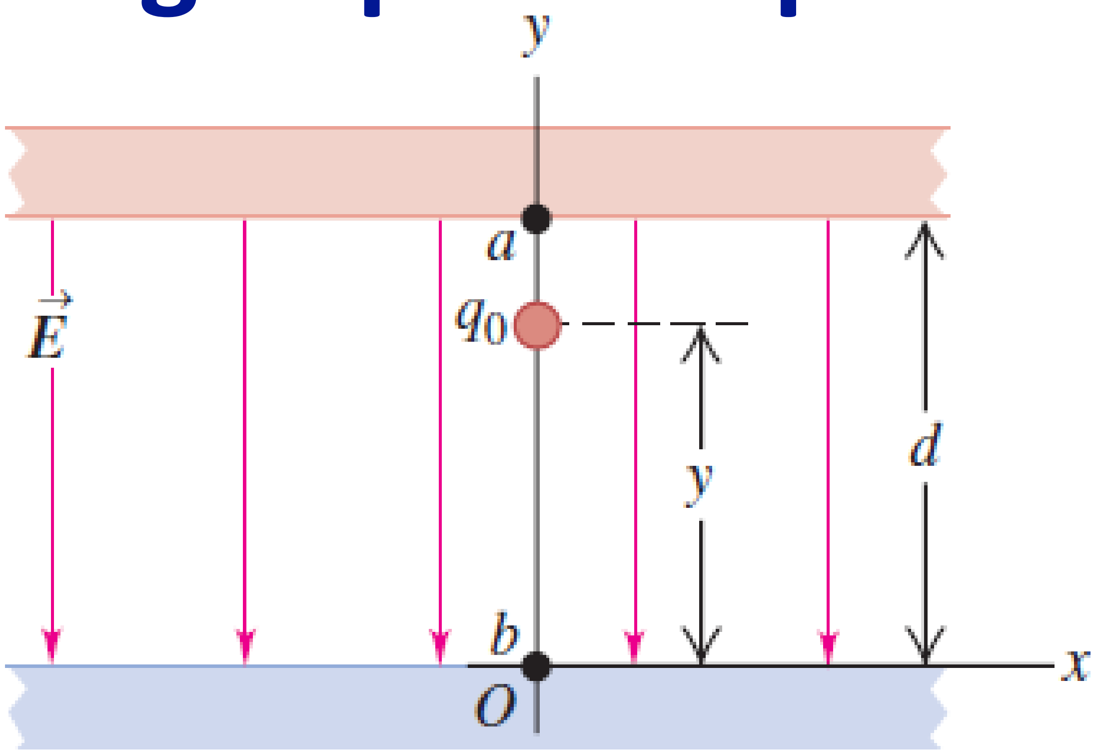
Example 23.8 A charged conducting sphere



A solid conducting sphere of radius has a total charge q. Find the electric potential everywhere, both outside and inside the sphere

Example 23.9 Oppositely charged parallel plates

Find the potential at any height between the two oppositely charged parallel plates discussed in Section 23.1



EXECUTE: The potential $V(y)$ at coordinate y is the potential energy per unit charge:

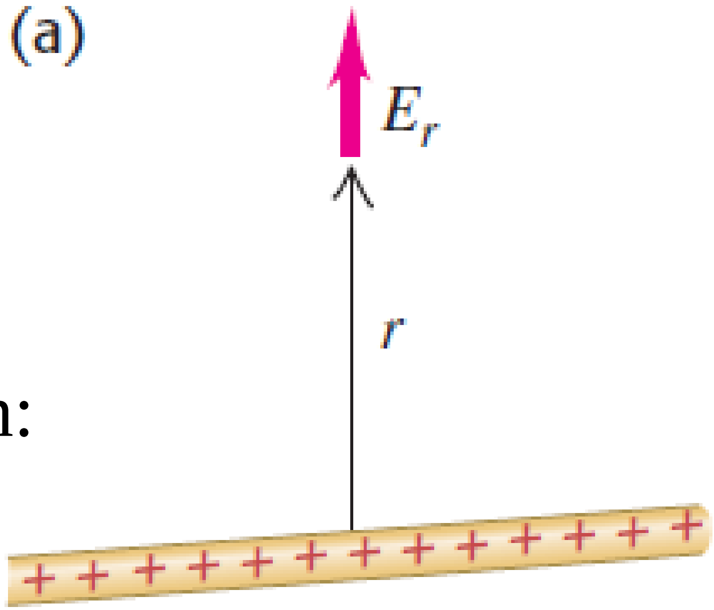
$$V(y) = \frac{U(y)}{q_0} = \frac{q_0 E y}{q_0} = Ey$$

CAUTION "0 potential" is arbitrary

Example 23.10 An infinite line charge

Find the potential at a distance from a very long line of charge with linear charge density (charge per unit length) λ.

Recall: radial distance from a long straight-line charge (Fig. 23.19a) has only a radial component given by $E_r = \lambda/2\pi\epsilon_0 r$



We use this expression to find the potential by the relation:
Electric field -- Force – Work -- Potential Energy -- Potential

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E_r\,dr = \frac{\lambda}{2\pi\epsilon_0}\int_{r_a}^{r_b}\frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_b}{r_a}$$

Example 23.10 An infinite line charge

If we take point b at infinity and set $V_b = 0$, we find that V_a is *infinite* for any finite distance r_a from the line charge: $V_a = (\lambda/2\pi\epsilon_0)\ln(\infty/r_a) = \infty$. This is *not* a useful way to define V for this problem! The difficulty is that the charge distribution itself extends to infinity.

Instead, as recommended in Problem-Solving Strategy 23.1, we set $V_b = 0$ at point b at an arbitrary but *finite* radial distance r_0. Then the potential $V = V_a$ at point a at a radial distance r is given by $V - 0 = (\lambda/2\pi\epsilon_0)\ln(r_0/r)$, or

$$V = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_0}{r}$$

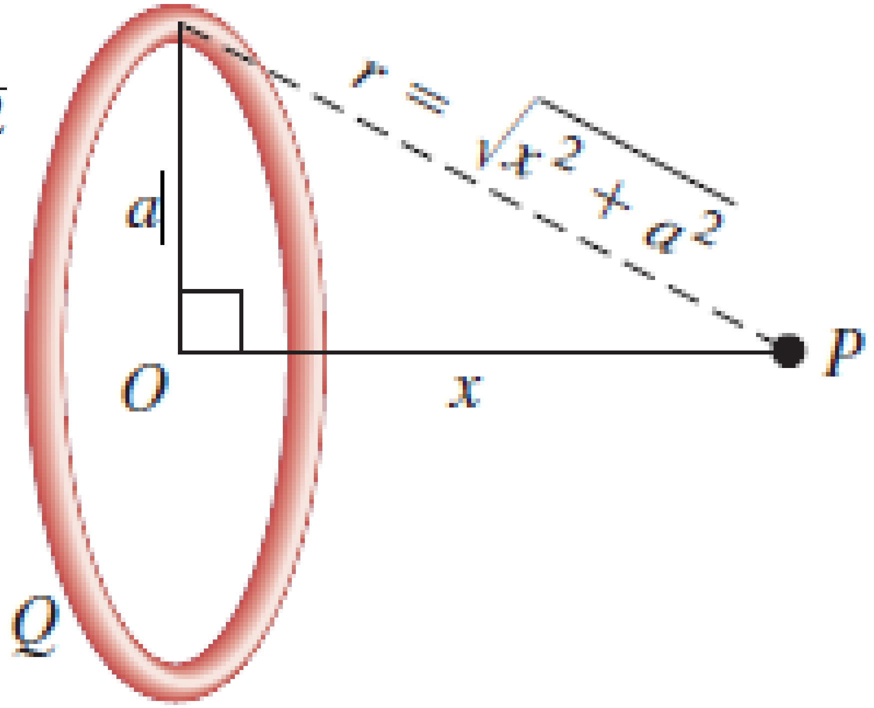
Example 23.11 A ring of charge

Electric charge Q is distributed uniformly around a thin ring of radius a (Fig. 23.20). Find the potential at a point ***P*** on the ring axis at a distance from the center of the ring.

Sounds difficult? But all parts of the ring (and therefore all elements of the charge distribution) are at **the same distance** from P: $r = \sqrt{x^2 + a^2}$

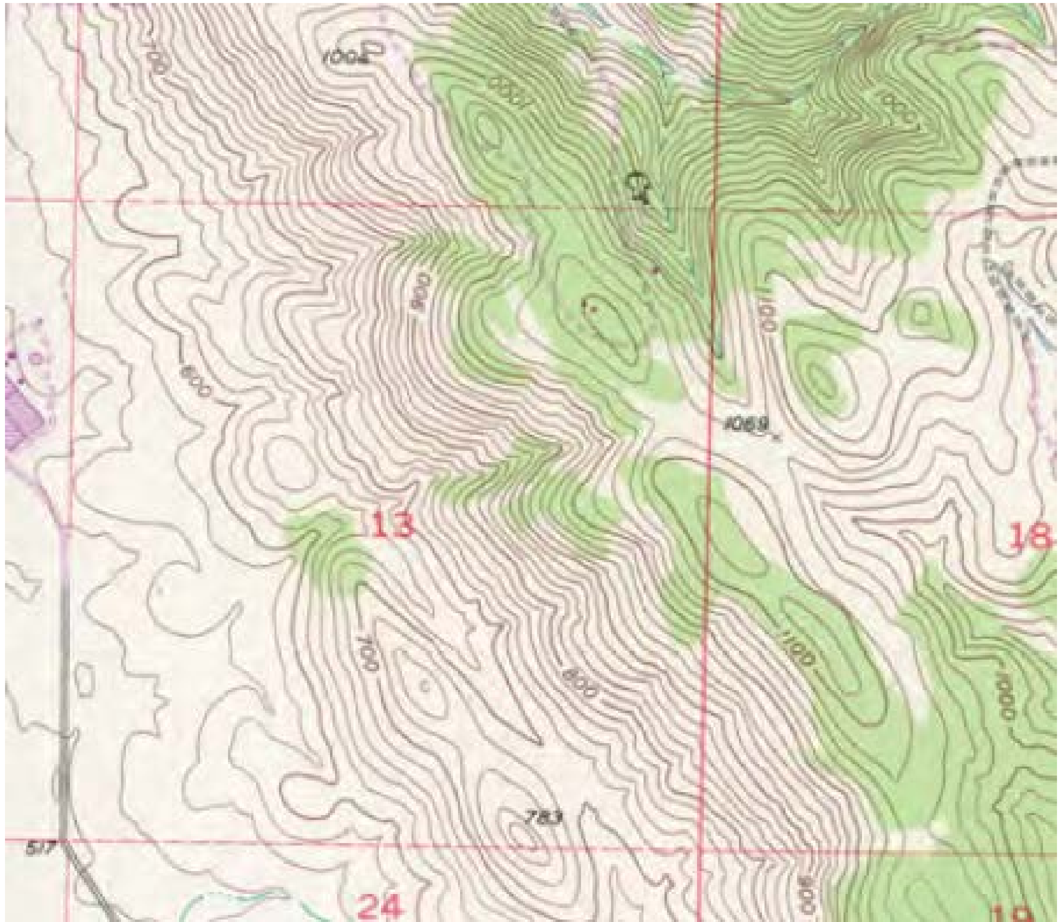
$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

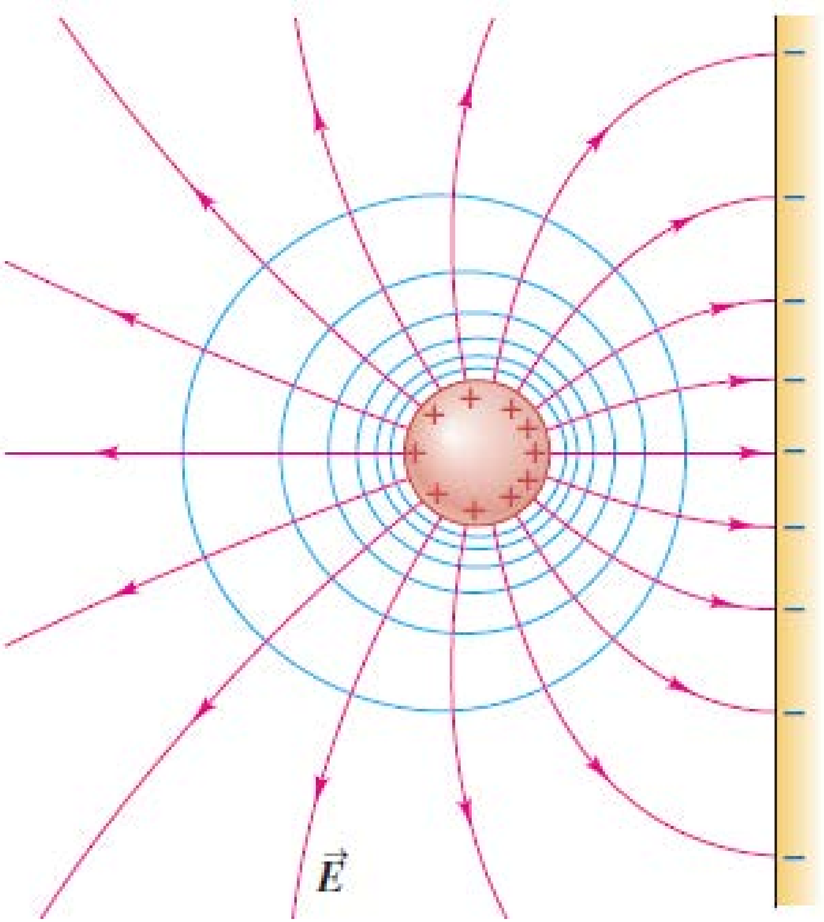
$$= \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2 + a^2}}\int dq = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2 + a^2}}$$



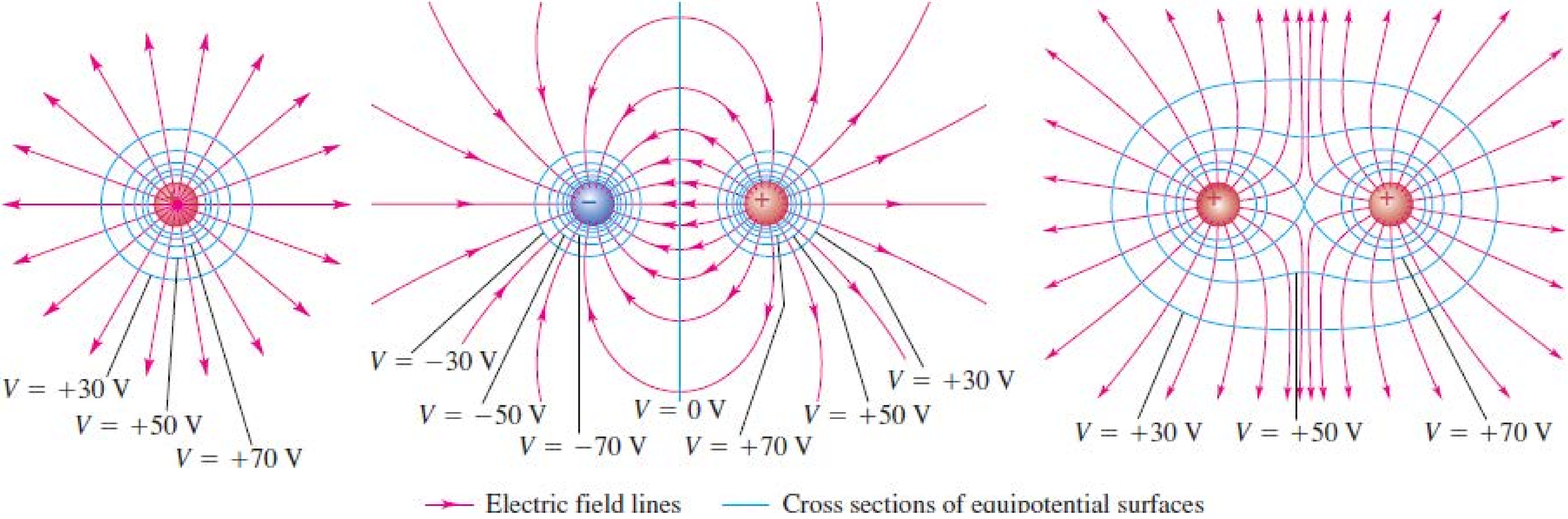
Equipotential Surfaces (Won't Test, but…)

By analogy to contour lines on a topographic map, an **equipotential surface** is a three-dimensional surface on which the *electric potential V* is the same at every point.





Equipotential Surfaces (Won’t Test, but…)



Field lines and equipotential surfaces are always mutually perpendicular. In general, field lines are curves, and equipotentials are curved surfaces. For the special case of a *uniform* field, in which the field lines are straight, parallel, and equally spaced, the equipotentials are parallel *planes* perpendicular to the field lines.

Equipotential Surfaces (Won't Test, but…)

CAUTION E need not be constant over an equipotential surface. On a given equipotential surface, the potential V has the same value at every point.

- **When all charges are at rest, the surface of a conductor is always an equipotential surface.**
- **When all charges are at rest, the electric field just outside a conductor must be perpendicular to the surface at every point.**
- **When all charges are at rest, the entire solid volume of a conductor is at the same potential**

Equipotential surfaces vs. Gaussian surfaces Don't confuse equipotential surfaces with the Gaussian surfaces we encountered in Chapter 22. Gaussian surfaces have relevance only when we are using Gauss's law, and we can choose *any* Gaussian surface that's convenient. We are *not* free to choose the shape of equipotential surfaces; the shape is determined by the charge distribution.

Potential Gradient

Electric field and potential are closely related. Equation (23.17), restated here, expresses one aspect of that relationship:

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$$

But by definition: $$V_a - V_b = \int_b^a dV = -\int_a^b dV$$

So: $$-\int_a^b dV = \int_a^b \vec{E} \cdot d\vec{l}$$

Potential Gradient

So: $$-\int_a^b dV = \int_a^b \vec{E}\cdot d\vec{l} \quad \text{or} \quad -dV = \vec{E}\cdot d\vec{l}$$

Or in xyz components: $$-dV = E_x dx + E_y dy + E_z dz$$

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z} \qquad \text{(components of } \vec{E} \text{ in terms of } V\text{)} \qquad (23.19)$$

This is consistent with the units of electric field being V/m. In terms of unit vectors we can write $\vec{E}$ as

$$\vec{E} = -\left(\hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) \qquad (\vec{E} \text{ in terms of } V) \qquad (23.20)$$

Potential Gradient

$$\vec{E} = -\left(\hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) \qquad (\vec{E} \text{ in terms of } V) \qquad (23.20)$$

In vector notation the following operation is called the **gradient** of the function f:

$$\vec{\nabla} f = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) f \qquad (23.21)$$

The operator denoted by the symbol $\vec{\nabla}$ is called "grad" or "del." Thus in vector notation,

$$\vec{E} = -\vec{\nabla} V \qquad (23.22)$$

This is read "$\vec{E}$ is the negative of the gradient of V" or "$\vec{E}$ equals negative grad V." The quantity $\vec{\nabla} V$ is called the *potential gradient*.

Potential Gradient (in Radial Components)

The operator denoted by the symbol $\vec{\nabla}$ is called "grad" or "del." Thus in vector notation,

$$\vec{E} = -\vec{\nabla}V \tag{23.22}$$

This is read "$\vec{E}$ is the negative of the gradient of V" or "$\vec{E}$ equals negative grad V." The quantity $\vec{\nabla}V$ is called the *potential gradient.*

If $\vec{E}$ is radial with respect to a point or an axis and r is the distance from the point or the axis, the relationship corresponding to Eqs. (23.19) is

$$E_r = -\frac{\partial V}{\partial r} \qquad \text{(radial electric field)} \tag{23.23}$$

Example 23.13 Potential of a point charge

The potential at a radial distance from a point charge q is $V = q/4\pi\epsilon_0 r$ Find the vector electric field from this expression for V.

$$E_r = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{1}{4\pi\epsilon_0}\frac{q}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

vector electric field is

$$\vec{\boldsymbol{E}} = \hat{\boldsymbol{r}}E_r = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\boldsymbol{r}}$$

Summary

$$W_{a\to b} = U_a - U_b \qquad (23.2)$$

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r} \qquad (23.9)$$

(two point charges)

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \qquad (23.10)$$

(q_0 in presence of other point charges)

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z} \qquad (23.19)$$

$$\vec{E} = -\left(\hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) \qquad (23.20)$$

(vector form)

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad (23.14)$$

(due to a point charge)

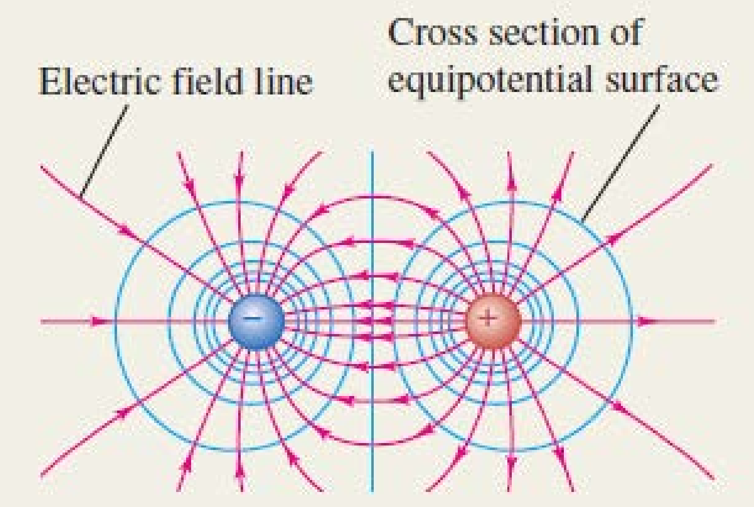
$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \qquad (23.15)$$

(due to a collection of point charges)

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \qquad (23.16)$$

(due to a charge distribution)

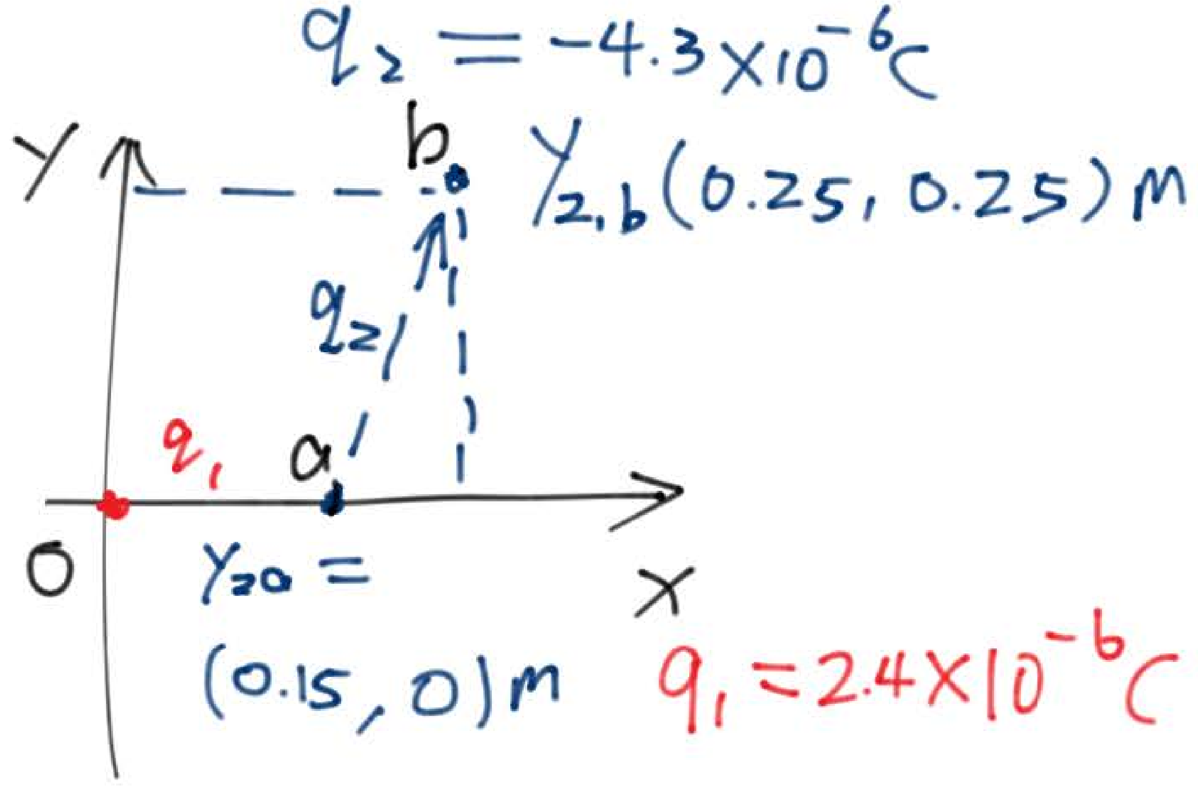
$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\cos\phi\, dl \qquad (23.17)$$



23.1 •• A point charge $q_1 = +2.40\ \mu C$ is held stationary at the origin. A second point charge $q_2 = -4.30\ \mu C$ moves from the point $x = 0.150$ m, $y = 0$ to the point $x = 0.250$ m, $y = 0.250$ m. How much work is done by the electric force on q_2?

$W = -\Delta U_2$

If Force is external to the system work done ON q_2 is equal to its potential energy change (gravity, ...)



Otherwise Work done on q_2 = − potential energy change

Still remember energy conservation?

We want to know W

$$-W = \Delta U_2 = U_{2b} - U_{2a} \qquad U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

$$= \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r_b} - \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_a}$$

$$= \frac{q_1 q_2}{4\pi\epsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) \qquad r_b = \sqrt{(0.25\,m)^2 + (0.25\,m)^2} = 0.354\,m$$

$$= 9\times10^{9}\,N\cdot m^2/C^2 \cdot 2.4\times10^{-6}C\cdot(-4.3\times10^{-6}C)\left(\frac{1}{0.354\,m} - \frac{1}{0.15\,m}\right)$$

$$= 0.356\,J \qquad W = -0.356\,J \quad \text{check!}$$

When we calculate the change of potential energy due to a force, we need to ask

gravity (weight) $\longleftrightarrow U_{grav}$

(same) spring $\longleftrightarrow U_{elastic}$

Electrostatic $\longleftrightarrow U_{elec}$

For these internal forces

$$\Delta U = -W$$

$$\Delta U = -W$$

since the system (for positive W) is using its energy to do work

For this problem: $+q$ and $-q$ attracts each other, so you need an external force to drag q_2 further (from $a \to b$)

That force does $+$ work and inputs energy to the system so ΔU is $+$

W is negative

23.3 •• Energy of the Nucleus. How much work is needed to assemble an atomic nucleus containing three protons (such as Be) if we model it as an equilateral triangle of side 2.00×10^{-15} m with a proton at each vertex? Assume the protons started from very far away.

Think in steps:

① Nothing:

$U = 0$

② 1 charge

q

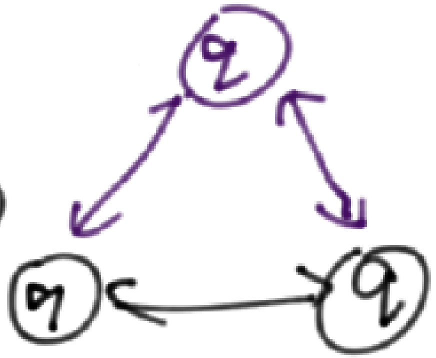
$U = 0$

③ second charge

$q \xleftrightarrow{r} q$

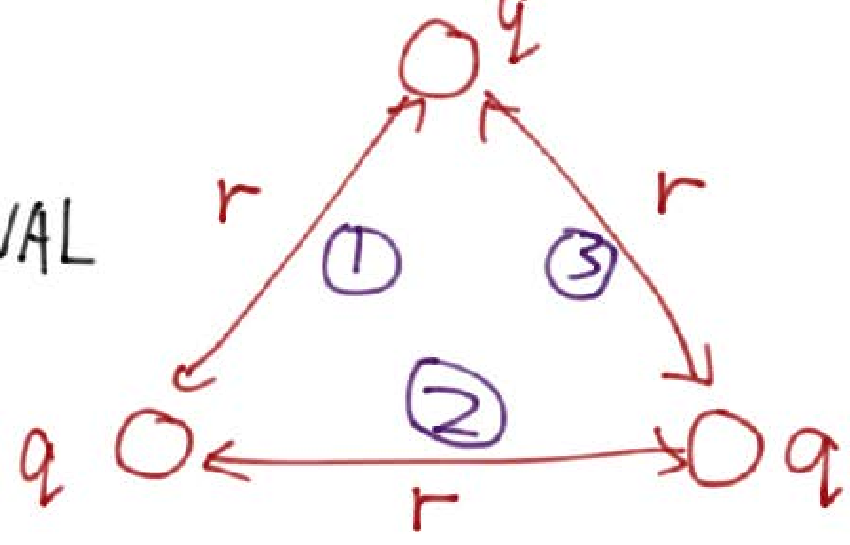
$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{r}$

④



23.3 •• Energy of the Nucleus. How much work is needed to assemble an atomic nucleus containing three protons (such as Be) if we model it as an equilateral triangle of side 2.00×10^{-15} m with a proton at each vertex? Assume the protons started from very far away.

Charge: 1.6×10^{-19} C



Here the work is EXTERNAL

$W = \Delta U = U_2$

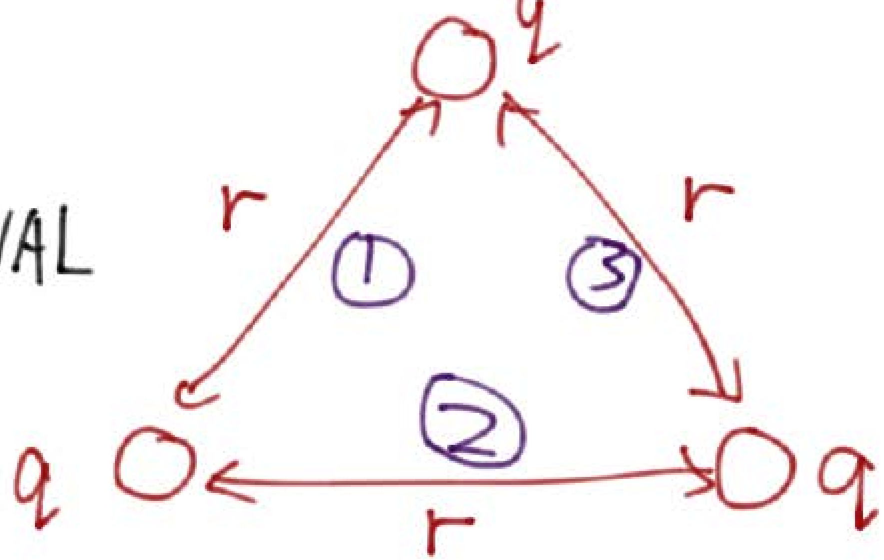
$= U_2 - U_1$

Assembled

$r' \sim \infty$

$U_1 = 0$

Charge: $1.6\times10^{-19}\,C$



Here the work is EXTERNAL

$W = \Delta U = U_2$

$= U_2 - U_1$

Assembled

$r \sim \infty$

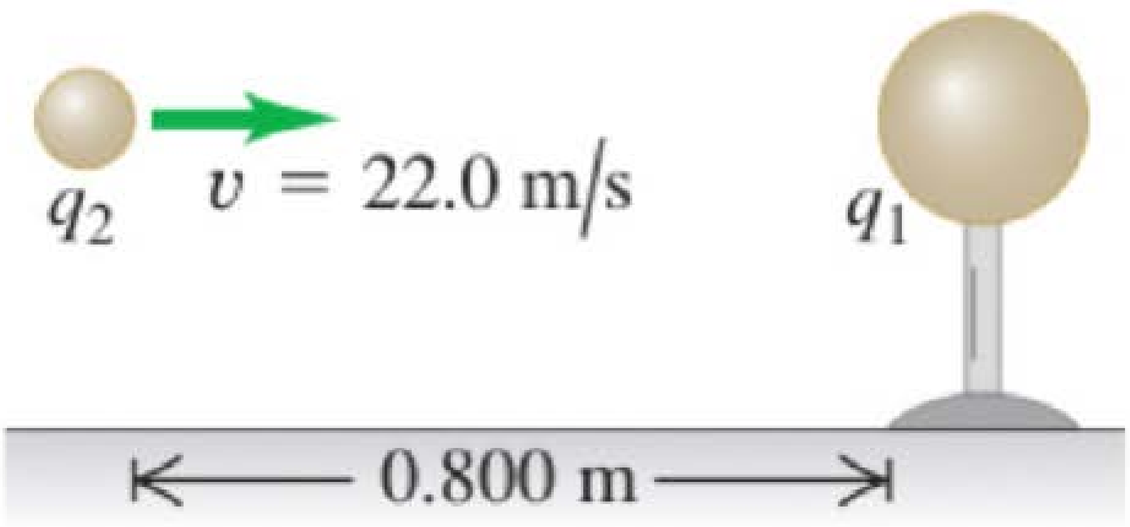
$U_1 = 0$

$$U_2 = \frac{1}{4\pi\epsilon_0}\cdot\frac{q^2}{r}\cdot 3$$

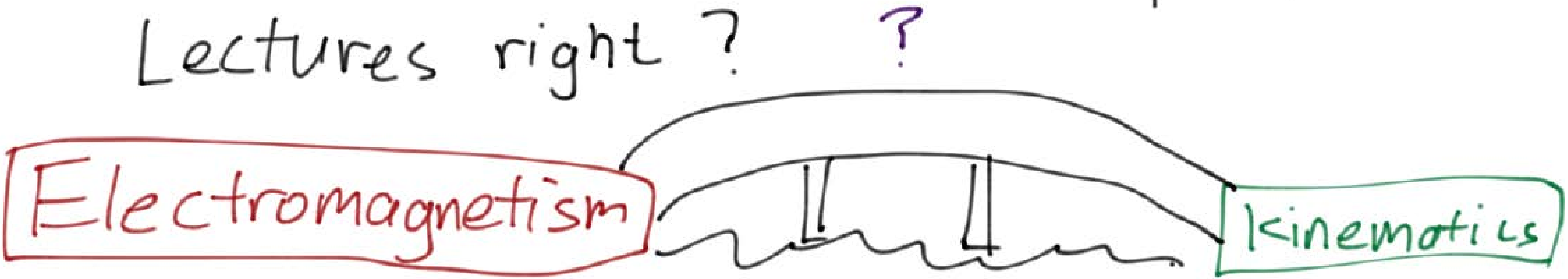
$$= 9\times10^{9}\,N\cdot m^2/C^2\cdot\frac{(1.6\times10^{-19}\,C)^2}{2\times10^{-15}\,m}\cdot 3 = 3.46\times10^{-13}\,J$$

23.5 •• A small metal sphere, carrying a net charge of $q_1 = -2.80\ \mu C$, is held in a stationary position by insulating supports. A second small metal sphere, with a net charge of $q_2 = -7.80\ \mu C$ and mass 1.50 g, is projected toward q_1. When the two spheres are 0.800 m apart, q_2 is moving toward q_1 with speed 22.0 m/s (Fig. E23.5). Assume that the two spheres can be treated as point charges. You can ignore the force of gravity. (a) What is the speed of q_2 when the spheres are 0.400 m apart? (b) How close does q_2 get to q_1?

Figure **E23.5**



Involves something from the previous Lectures right?



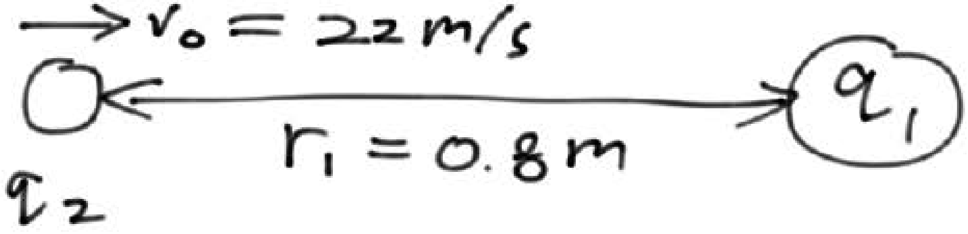
Energy conservation!

As the q_2 approaches q_1, q_2 decelerate due to electrostatic (repulsive) forces.

Kinetic Energy $\longrightarrow$ Potential Energy

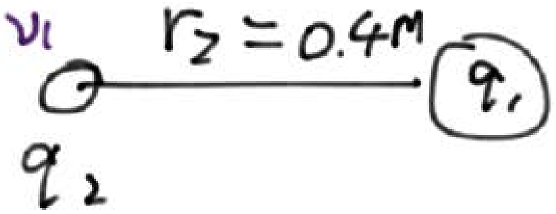
(a)

So at $t=0$ $\quad t=0$



$K_0 = \frac{1}{2} m v_0^2$

$U_0 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r_1}$ $\quad t = t_1$



At t_1 $\quad K_1 = \frac{1}{2} m v_1^2$ to be determined

$U_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_2}$ Known $\quad$ Only 1 variable

We need only 1 equation to connect them

$U_0 + K_0 = U_1 + K_1$ $\quad$ Energy Conserv.

$$\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_1} + \frac{1}{2} m v_0^2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_2} + \frac{1}{2} m v_1^2$$

May help to simplify:

$$\frac{q_1 q_2}{4\pi\epsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{1}{2}m\left(v_1^2-v_0^2\right)$$

$$v_1^2 = v_0^2 + \frac{q_1 q_2}{4\pi\epsilon_0}\cdot\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\frac{2}{m}$$

$$v_1 = \sqrt{(22\,m/s)^2 + 9\times10^9\,N\cdot m^2/C^2\cdot(-2.8\times10^{-6}C)(-7.8\times10^{-6}C)\cdot\left(\frac{1}{0.8m}-\frac{1}{0.4m}\right)\frac{2}{1.5\times10^{-3}kg}} = \sqrt{484-327.6}\ m/s$$

$$= 12.5\ m/s$$

(b) $U_0 + K_0 = U_2 + K_2$ How close?

$$\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_1} + \frac{1}{2}mv_0^2 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_2} + \frac{1}{2}m\underset{0}{\underbrace{v_2^2}}$$

$$\frac{1}{r_1} + \frac{\frac{1}{2}mv_0^2}{\frac{1}{4\pi\epsilon_0}q_1 q_2} = \frac{1}{r_2}$$

$$r_2 = \left(\frac{1}{r_1} + \frac{\frac{1}{2}mv_0^2}{\frac{1}{4\pi\epsilon_0}q_1 q_2}\right)^{-1}$$

$$\frac{1}{r_1} + \frac{\frac{1}{2}mv_0^2}{\frac{1}{4\pi\epsilon_0}q_1q_2} = \frac{1}{r_2}$$

$$r_2 = \left(\frac{1}{r_1} + \frac{\frac{1}{2}mv_0^2}{\frac{1}{4\pi\epsilon_0}q_1q_2} \right)^{-1}$$

$$= \left(\frac{1}{0.8m} + \frac{0.5 \cdot 1.5\times10^{-3}kg \cdot (22\,m/s)^2}{9\times10^9 N\cdot m^2/C^2 \cdot (-2.8\times10^{-6}C)(-7.8\times10^{-6}C)} \right)^{-1}$$

$$= \left(\frac{1}{0.8m} + 1.85\ m^{-1} \right)^{-1}$$

$$= 0.32\ m$$