CALCULUS

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① Derivative of the Sine Function

- **The derivative of the sine function is the cosine function.**

$$\frac{d}{dx}(sinx) = cosx$$

Proof： Apply the Sum-to-Product Formula

$$sinx - siny = 2sin\frac{x-y}{2}cos\frac{x+y}{2}$$

Example 1

Find derivatives of the sine function involving differences, products, and quotients.

(a) $y = x^2 - \sin x$ (b) $y = x^2 \sin x$ (c) $y = \frac{\sin x}{x}$

② Derivative of the Cosine Function

- **The derivative of the cosine function is the negative sine function.**

$$\frac{d}{dx}(cosx) = -sinx$$

Proof : Apply the Sum-to-Product Formula

$$cosx - cosy = -2sin\frac{x+y}{2}sin\frac{x-y}{2}$$

Example 2 Find derivatives of the cosine function in combinations with other functions.

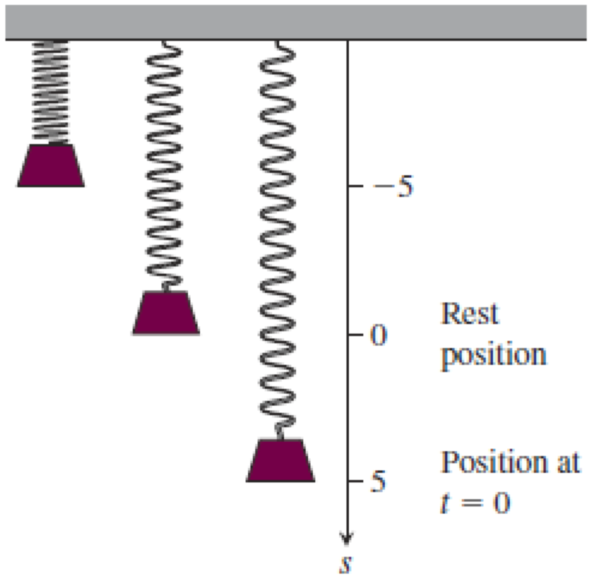
(a) $y = 5x + \cos x$ (b) $y = \sin x\cos x$ (c) $y = \frac{\cos x}{1-\sin x}$

③ Simple Harmonic Motion

- Simple harmonic motion models the motion of an object or weight bobbing freely up and down on the end of a spring, with no resistance. The motion is periodic and repeats indefinitely, so we represent it using trigonometric functions.
- The next example models the motion with no opposing forces (such as friction).

Example 3

A weight hanging from a spring is stretched down 5 units beyond its rest position and released at time $t = 0$ to bob up and down. Its position at any later time t is $s = 5\cos t$. What are its velocity and acceleration at time t?



④ Derivatives of the Other Basic Trigonometric Functions

$$\frac{d}{dx}(tanx) = \frac{d}{dx}\left(\frac{sinx}{cosx}\right) = sec^2x$$

$$\frac{d}{dx}(cotx) = \frac{d}{dx}\left(\frac{cosx}{sinx}\right) = -csc^2x$$

$$\frac{d}{dx}(secx) = \frac{d}{dx}\left(\frac{1}{cosx}\right) = secxtanx$$

$$\frac{d}{dx}(cscx) = \frac{d}{dx}\left(\frac{1}{sinx}\right) = -cscxcotx$$

Example 4 Find dy/dx if

(a) $y = xsecx + 2\sqrt{x}$ (b) $y = \dfrac{sinx}{1 + cosx}$

Example 5 Find y'' if

(a) $y = \tan x$ (b) $y = \cot x$

Example 6 Find y' and y'' for the following functions:

(a) $y = sin\theta \cos\theta$ (b) $y = x^2 sinx$

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- The differentiability of the trigonometric functions throughout their domains implies their continuity at every point in their domains (Theorem 1, Section 3.2).
- So we can calculate limits of algebraic combinations and compositions of trigonometric functions by direct substitution.

Example 7 Calculate the following limits by direct substitution:

$$(a)\quad \lim_{x\to 0}\frac{\sqrt{2+secx}}{\cos(\pi - tanx)} \qquad\qquad (b)\quad \lim_{x\to 0} sec\left[cosx+\pi\tan\left(\frac{\pi}{4secx}\right)-1\right]$$

Example 8 Find the following trigonometric limits.

$$(a)\quad \lim_{\theta\to 0}\cos\left(\frac{\pi\theta}{sin\theta}\right) \qquad\qquad (b)\quad \lim_{\theta\to\frac{\pi}{6}}\frac{sin\theta-\frac{1}{2}}{\theta-\frac{\pi}{6}}$$