College Algebra and Trigonometry

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① Define Trigonometric Functions of Acute Angles

Definition of Trigonometric Functions of Acute Angles

Function Name	Definition	Example
sine	$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$	$\sin\theta = \dfrac{3\text{ ft}}{5\text{ ft}} = \dfrac{3}{5}$
cosine	$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$	$\cos\theta = \dfrac{4\text{ ft}}{5\text{ ft}} = \dfrac{4}{5}$
tangent	$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$	$\tan\theta = \dfrac{3\text{ ft}}{4\text{ ft}} = \dfrac{3}{4}$
cosecant	$\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$	$\csc\theta = \dfrac{5\text{ ft}}{3\text{ ft}} = \dfrac{5}{3}$
secant	$\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$	$\sec\theta = \dfrac{5\text{ ft}}{4\text{ ft}} = \dfrac{5}{4}$
cotangent	$\cot\theta = \dfrac{\text{adj}}{\text{opp}}$	$\cot\theta = \dfrac{4\text{ ft}}{3\text{ ft}} = \dfrac{4}{3}$

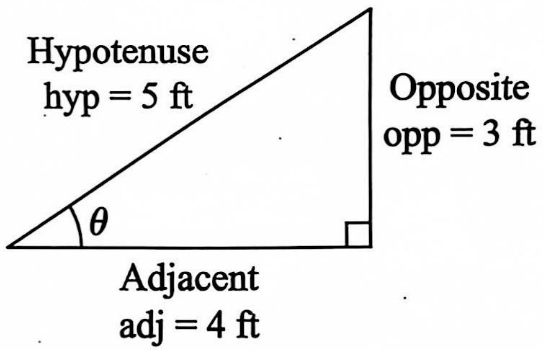
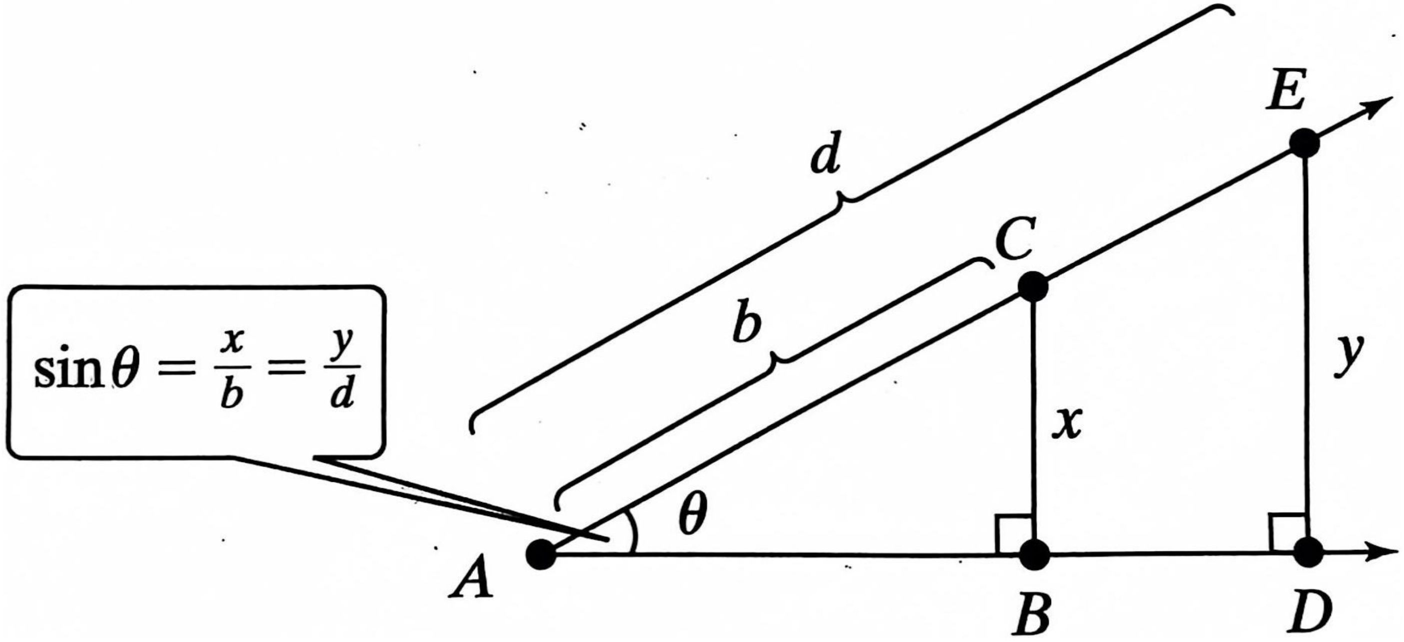


Figure 5-15

- **It is very important to note that the values of the trigonometric functions depend only on the measure of the angle, not the size of the triangle.**
- **Triangles ΔABC and ΔADE are similar triangles with common angle θ, thus:**

$$\sin\theta = \frac{x}{b} = \frac{y}{d}$$



② Evaluate Trigonometric Functions of Acute Angles

Example 1:

Suppose that a right triangle has legs of length 5 cm and 12 cm. Evaluate the six trigonometric functions of the smaller angle.

Example 2:

Suppose that $\cos\theta = \frac{\sqrt{5}}{3}$ **for the acute angle** θ**. Evaluate** $\tan\theta$**.**

③ Determine Trigonometric Function Values for Special Angles

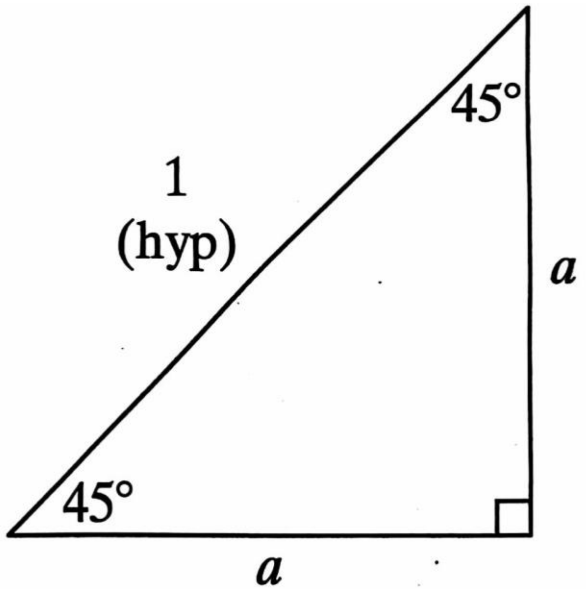
An isosceles right triangle is a right triangle in which two legs are of equal length. Two acute angles in this triangle have equal measures of 45° or π/4.

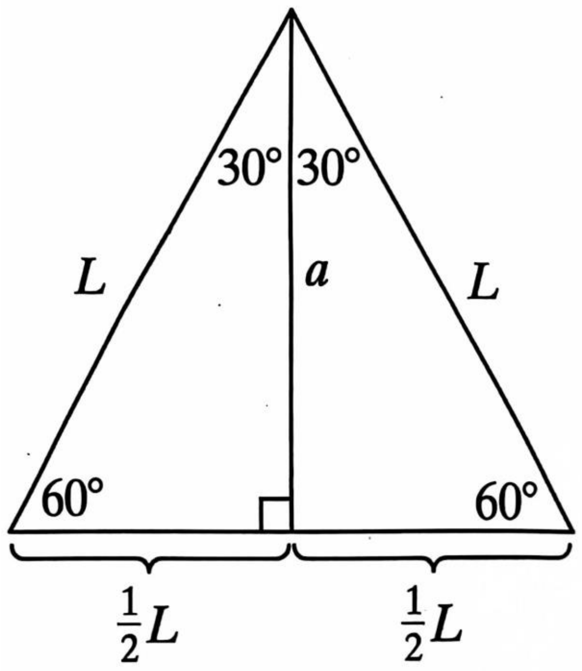
Example 3:

Evaluate sin45°, cos45°, tan45°.

Example 4:

Evaluate sin60°, cos60°, tan60°.





Trigonometric Function Values of Special Angles

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$30° = \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$45° = \frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60° = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

Example 5:

Simplify: a) $\tan 60° - \tan 30°$ b) $2\sin\frac{\pi}{3}\cos\frac{\pi}{3}$

④ Use Fundamental Identities

Reciprocal and Quotient Identities

$\csc\theta = \dfrac{1}{\sin\theta}$ or $\sin\theta = \dfrac{1}{\csc\theta}$	$\sin\theta$ and $\csc\theta$ are reciprocals.
$\sec\theta = \dfrac{1}{\cos\theta}$ or $\cos\theta = \dfrac{1}{\sec\theta}$	$\cos\theta$ and $\sec\theta$ are reciprocals.
$\cot\theta = \dfrac{1}{\tan\theta}$ or $\tan\theta = \dfrac{1}{\cot\theta}$	$\tan\theta$ and $\cot\theta$ are reciprocals.
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$\tan\theta$ is the ratio of $\sin\theta$ and $\cos\theta$.
$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$	$\cot\theta$ is the ratio of $\cos\theta$ and $\sin\theta$.

Example 6:

Given $\sin\alpha = 8/17$ and $\cos\alpha = 15/17$, find the values of the other trigonometric functions of α.

Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $\cot^2\theta + 1 = \csc^2\theta$

Proof:

Example 7:

Given that $tan\theta = \frac{12}{5}$ for an acute angle θ. Find the values of $\sec\theta$ and $\csc\theta$.

Cofunctions:

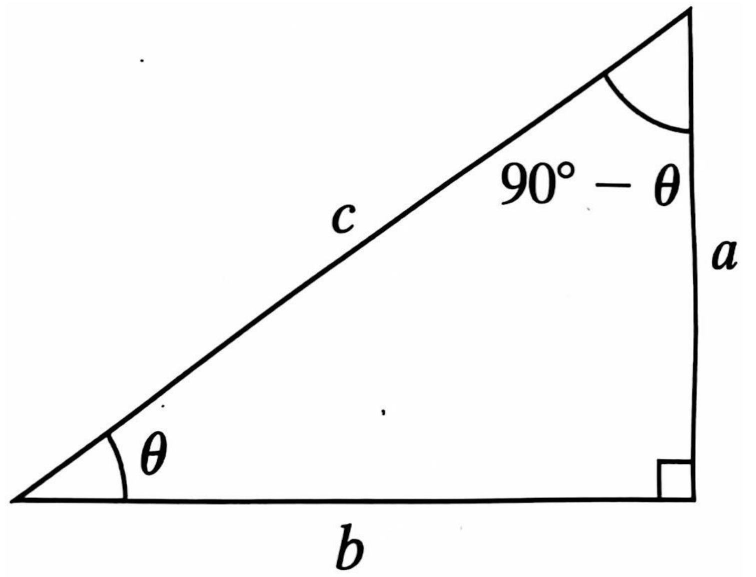
For an acute angle θ, two trigonometric functions f and g are cofunctions if

$$f(\theta) = g(90° - \theta) \quad \text{and} \quad g(\theta) = f(90° - \theta)$$

$$\sin\theta = \cos(90° - \theta) = \frac{a}{c}$$

$$\tan\theta = \cot(90° - \theta) = \frac{a}{b}$$

$$\sec\theta = \csc(90° - \theta) = \frac{c}{b}$$



Cofunction Identities

Cofunctions of complementary angles are equal.	
$\sin\theta = \cos(90° - \theta)$ $\quad \cos\theta = \sin(90° - \theta)$ $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ $\quad \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$	Sine and cosine are cofunctions.
$\tan\theta = \cot(90° - \theta)$ $\quad \cot\theta = \tan(90° - \theta)$ $\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$ $\quad \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$	Tangent and cotangent are cofunctions.
$\sec\theta = \csc(90° - \theta)$ $\quad \csc\theta = \sec(90° - \theta)$ $\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$ $\quad \csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$	Secant and cosecant are cofunctions.

Example 8:

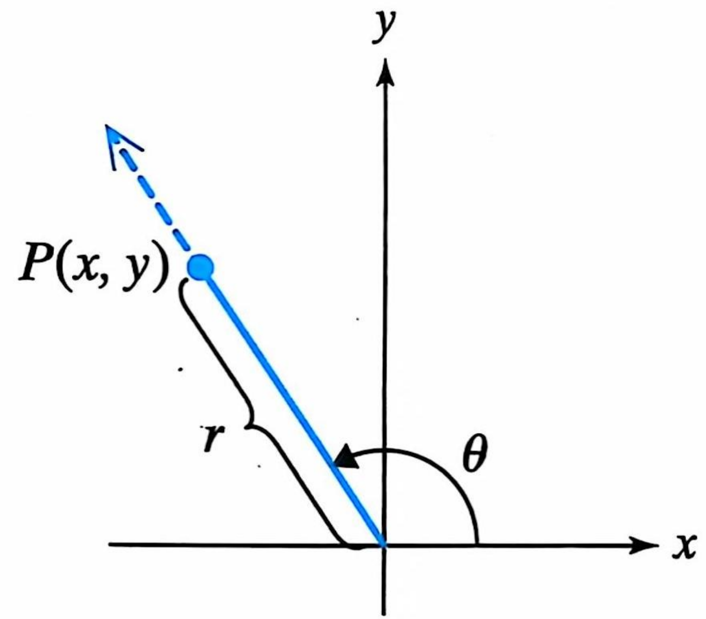
For each function value, find a cofunction with the same value.

a) $\boldsymbol{cot15° = 2 + \sqrt{3}}$ $\qquad$ b) $\boldsymbol{cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}}$

① Evaluate Trigonometric Functions of Any Angle

Trigonometric Functions of Any Angle

Let θ be an angle in standard position with point $P(x, y)$ on the terminal side, and let $r = \sqrt{x^2 + y^2} \neq 0$ represent the distance from $P(x, y)$ to $(0, 0)$. Then,

$\sin\theta = \dfrac{y}{r}$	$\csc\theta = \dfrac{r}{y}\ (y \neq 0)$	
$\cos\theta = \dfrac{x}{r}$	$\sec\theta = \dfrac{r}{x}\ (x \neq 0)$	
$\tan\theta = \dfrac{y}{x}\ (x \neq 0)$	$\cot\theta = \dfrac{x}{y}\ (y \neq 0)$	

Example 1:

Let $P(-2, -5)$ be a point on the terminal side of angle θ drawn in standard position. Find the values of the six trigonometric functions of angle θ.

Example 2:

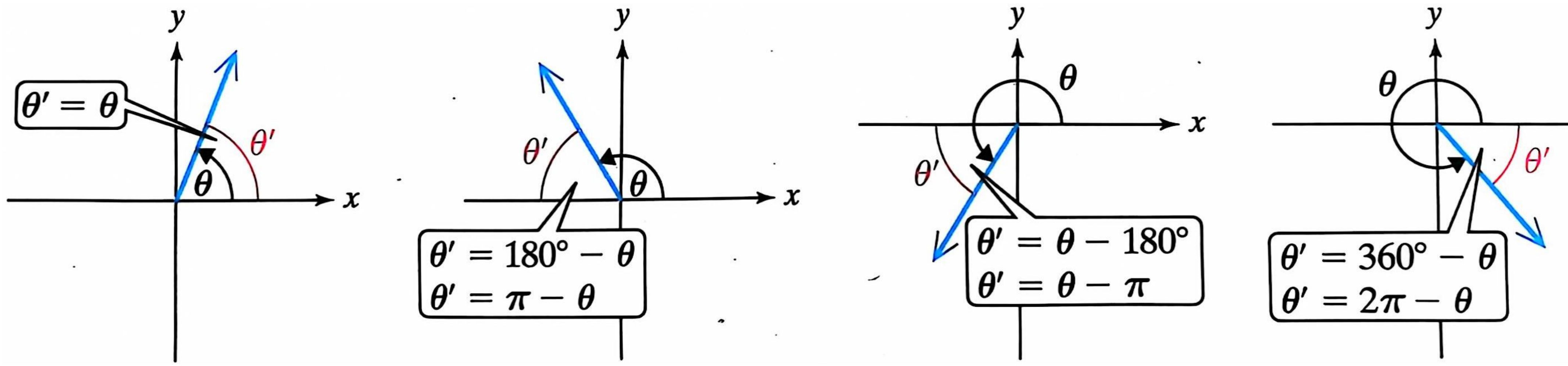
Find the values of sine, cosine, and tangent for the given angles.

a) $\theta = \frac{3\pi}{2}$ **b)** $\theta = 180°$

② Determine Reference Angles

Definition of a Reference Angle:

Let θ be an angle in standard position. The reference angle for θ is the acute angle θ' formed by the terminal side of θ and the horizontal axis.



Example 3:

Find the reference angle θ'.

a) $\theta = 315°$ **b)** $\theta = -\frac{13\pi}{12}$ **c)** $\theta = 3.5$ **d)** $\theta = \frac{25\pi}{4}$

③ Evaluate Trigonometric Functions Using Reference Angles

To find the value of a trigonometric function of a given angle θ:

1. **Determine the function value of the reference angle θ'.**
2. **Affix the appropriate sign based on the quadrant in which θ lies.**

Trigonometric Function Values of Special Angles

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$30^\circ = \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$45^\circ = \frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60^\circ = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

Example 4: Using Reference Angles to Evaluate Functions:

a) $\sin\frac{4\pi}{3}$ b) $\tan(-225°)$ c) $\sec\frac{11\pi}{6}$

Example 5:

Evaluate: a) $\sec\frac{9\pi}{2}$ b) $\sin(-510°)$

Example 6 & 7:

6) Given $\sin\theta = -\frac{4}{7}$ and $\cos\theta > 0$, find $\cos\theta$ and $\tan\theta$.

7) Given $\cos\theta = -\frac{3}{5}$ for θ in Quadrant II, find $\sin\theta$ and $\tan\theta$.

④ Identify the Domains of Trigonometric Functions

Function	Domain
sin θ	$(-\infty, \infty)$
cos θ	$(-\infty, \infty)$
$tan\theta = \frac{sin\theta}{cos\theta}$	$cos\theta \neq 0 \quad \Rightarrow \quad \theta \neq \frac{(2n+1)\pi}{2}$ for all integers n
$cot\theta = \frac{cos\theta}{sin\theta}$	$sin\theta \neq 0 \quad \Rightarrow \quad \theta \neq n\pi$ for all integers n
$sec\theta = \frac{1}{cos\theta}$	$cos\theta \neq 0 \quad \Rightarrow \quad \theta \neq \frac{(2n+1)\pi}{2}$ for all integers n
$csc\theta = \frac{1}{sin\theta}$	$sin\theta \neq 0 \quad \Rightarrow \quad \theta \neq n\pi$ for all integers n

Example 8:

Evaluate: a) $\boldsymbol{tan\frac{3\pi}{2}}$ b) **sec(–180°)** c) $\boldsymbol{cot(-\frac{\pi}{2})}$