

6.1

$$16. (a) a^3 - b^3 \\ = (a-b)(a^2 + ab + b^2)$$

$$(b) \cos^3 x - \cot^3 x \\ = (\cos x - \cot x)(\cos^2 x + \cot^2 x + (\cos x)(\cot x))$$

$$24. \frac{\tan^2 x}{\sin x \sec x} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\sin x \cdot \frac{1}{\cos x}} = \sin x$$

$$33. \frac{8\sin^2 x + 2\sin x + 1}{\sin^2 x + \sin x} = \frac{8\sin^2 x \left(1 + \frac{1}{4\sin x} + \frac{1}{8\sin^2 x}\right)}{\sin^2 x \left(1 + \frac{1}{\sin x}\right)}$$

$$= 8\sin x + 1$$

$$38. \sin(-x) + \cot(-x) \cos(-x) = -\csc x$$

$$\sin(-x) + \frac{\cos(-x)}{\sin(-x)} - \cos(-x) = -\csc x$$

$$\frac{1}{\sin(-x)} = -\csc x$$

$$43. \frac{1}{1+\sin t} + \frac{1}{\sin t} = 2\sec^2 t$$

$$= \frac{2}{1-\sin^2 t} = \frac{2}{\cos^2 t} = 2\sec^2 t$$

$$48. \frac{\cos \theta}{\tan \theta + \sec \theta} = 7 - 3\sin \theta$$

$$\frac{\cos \theta}{\frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}} = \frac{\cos \theta}{\frac{\sin \theta + 1}{\cos \theta}} = \frac{\cos^2 \theta}{1 + \sin \theta} \\ = 1 - \sin \theta$$

$$53. \ln |\cot t| - \ln |\csc t| = \ln \left| \frac{\cot t}{\csc t} \right| \\ = \ln \left| \frac{\cot t}{\frac{1}{\sin t}} \right| \\ = \ln |\sin t|$$

$$58. \frac{1 + \tan^2 x}{\tan x} = \frac{1}{\sin x - \cos x}$$

$$66. \frac{\sqrt{(4\cos)^2 + 16^2}}{4 + \cos^2}$$

$$= 6\csc \theta$$

$$72. \frac{\tan x \sin x}{\tan x - \sin x} = \frac{\sin x}{1 - \cos x}$$

$$\frac{\sin^2 x}{\sin x - \cos x \sin^2 x} = \frac{\sin^2 x}{\sin x - \cos x \sin x} \\ = \frac{\sin^2 x}{\sin x(1 - \cos x)} \\ = \frac{\sin x}{1 - \cos x}$$

$$98. \sin x \cot x + \cos x \tan^2 x = \csc x$$

$$\sin x \frac{\cos x}{\sin x} + \cos x \frac{\sin^2 x}{\cos^2 x} =$$

$$\cos x + \frac{\sin^2 x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos x} \\ = \frac{1}{\cos x} \\ = \sec x$$

6.2

Ex
12) $\sin(-75^\circ) = \sin(30-45)$
 $= \sin(30)\cos(45) - \cos(30)\sin(45)$
 $= \frac{1}{2} \cdot \frac{\sqrt{2}}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$
 $= \frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{4}$
 $= \frac{\sqrt{2} - \sqrt{6}}{4}$

23.
 $\cos^2 \frac{\pi}{3} \cos \frac{7\pi}{6} - \sin^2 \frac{\pi}{3} \sin \frac{7\pi}{6} = \cos\left(\frac{2\pi}{3} + \frac{7\pi}{6}\right)$
 $= \cos\left(\frac{4\pi + 7\pi}{6}\right)$
 $= \cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$

30.
 $\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$
 $= \left(\frac{5}{9}\right)\left(-\frac{7}{9}\right) + \left(\frac{2}{3}\right)\left(\frac{15}{18}\right) = -\frac{5}{36} + \frac{3}{12}$
 $\sin \beta = 1 - \cos^2 \beta = \frac{1}{19}$
 $\cos \alpha = 1 - \sin^2 \alpha = 1 - \frac{4}{9}$

42.
 $\sin\left(\frac{\pi}{2} + x\right) = \cos x$
 $= \sin \frac{\pi}{2} \cos x + \cos \frac{\pi}{2} \sin x$
 $= \cos x + 0 \cdot \sin x$
 $= \cos x$

56 $\frac{\sin(\alpha - \beta)}{\sin(\alpha + \beta)} = \frac{\tan \alpha - \tan \beta}{\tan \alpha + \tan \beta}$

$$= \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\sin \alpha \cos \beta + \cos \alpha \sin \beta} \cdot \frac{1}{\cos \alpha \cos \beta}$$

$$= \frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta} = \frac{\tan \alpha - \tan \beta}{\tan \alpha + \tan \beta}$$

63
a) $8 \sin \alpha - 6 \cos \alpha$ $\sqrt{8^2 + 6^2} = 10$
 $\cos \alpha = \frac{8}{10}$ $\sin \alpha = \frac{6}{10}$

6.3

$$8) \sin u = \frac{4}{7} \quad \tan u > 0$$

$$\sin^2 u + \cos^2 u = 1$$

$$\frac{16}{49} + \cos^2 u = 1$$

$$\cos u = \frac{\sqrt{33}}{7}$$

$$\tan u = \frac{\frac{4}{7}}{\frac{\sqrt{33}}{7}} = \frac{4\sqrt{33}}{33}$$

$$15. \frac{2 \tan 67.5^\circ}{1 - \tan^2 67.5^\circ} \quad \tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\tan 135^\circ = -1$$

$$19. \cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} =$$

$$\cos(2u) = \cos^2 u - \sin^2 u$$

$$\cos\left(\frac{\pi}{4}\right) - \sin^2\left(\frac{\pi}{8}\right) = \cos\left(2 \cdot \frac{\pi}{8}\right) = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$21. \frac{\sin 2u}{1 - \cos^2 u} = \cot u$$

$$\sin 2u = 2 \sin u \cos u \quad \cos^2 u = 1 - \sin^2 u$$

$$\frac{2 \sin u \cos u}{1 - (1 - \sin^2 u)} = \frac{\sin 2u}{1 - \cos^2 u} = \cot u$$

35.

$$\sin^2 a$$

$$\sin^2 a = 1 - \cos^2 a$$

$$= (\sin a)^2$$

$$= (1 - \cos^2 a)^2 = 1 - 2 \cos^2 a + \cos^4 a$$

$$43. \tan 75^\circ = \tan(45^\circ + 30^\circ)$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$$

$$= 2$$

$$5. \sin a = \frac{32}{65} \quad \frac{\pi}{2} < a < \pi$$

$$\left(\frac{32}{65}\right)^2 + \cos^2 a = 1$$

$$\cos a = \frac{-56}{65}$$

$$\tan a = \frac{-32}{56}$$

$$\sec a = \frac{-65}{56}$$

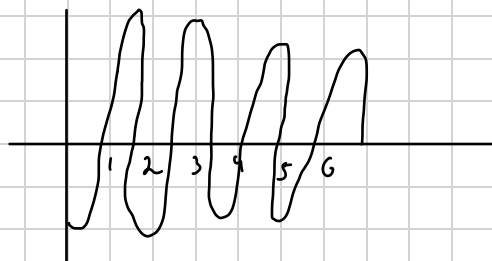
$$\csc a = \frac{65}{32}$$

$$55) 2 \sin^2 \frac{\alpha}{2} + 4 \cos^2 \frac{\alpha}{2} - 3 = \cos \alpha$$

$$2 \sin^2 \left(\frac{\alpha}{2}\right) + 4 \cos^2 \left(\frac{\alpha}{2}\right) - 3 = \cos \alpha$$

$$72) y = -2 \sin ha \cos ha = -\sin 2a$$

b)



40.

$$\sin(5a) = 16 \sin^5(a) - 20 \sin^3(a) + 5 \sin(a)$$

6.4

$$9) \cos(-\alpha) \sin 2\alpha = 2 \sin \alpha \cos^2(\alpha) = 2 \sin \alpha \cdot \frac{1 + \cos(2\alpha)}{2}$$

$$= \sin(\alpha) + \sin \alpha \cos(2\alpha)$$

$$12) \sin 47.5^\circ \cos 37.5^\circ = \frac{\sqrt{3} + \sqrt{2}}{4}$$

$$16) \cos(A) \cos(B) = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$A = (\alpha + \gamma) \quad B = (\alpha - \gamma)$$

$$= \frac{1}{2} [\cos(2\alpha) + \cos(2\gamma)] = \frac{1}{2} [\cos^2(\alpha) - \sin^2(\alpha) + \cos^2(\gamma) - \sin^2(\gamma)]$$

not valid

$$22. \quad A = 40^\circ$$

$$B = 17^\circ$$

$$\cos 2(2^\circ) - \cos(17^\circ) = -2 \sin\left(\frac{20^\circ + 17^\circ}{2}\right) \sin\left(\frac{20^\circ - 17^\circ}{2}\right)$$

$$= -2 \sin(17^\circ) \sin(3^\circ)$$

$$26. \quad \boxed{\frac{\sqrt{6}}{2}}$$

$$28. \quad \frac{\cos 2\alpha - \cos 3\alpha}{\sin 2\alpha - \sin 3\alpha} = -\tan 2\alpha$$

$$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$= -2 \sin\left(\frac{4\alpha}{2}\right) \sin\left(\frac{-2\alpha}{2}\right) = -2 \sin(2\alpha) \sin(-\alpha) = 2 \sin(2\alpha) \sin(\alpha)$$

$$\frac{\cos(2\alpha) - \cos(3\alpha)}{\sin(2\alpha) - \sin(3\alpha)} = \frac{\sin 2\alpha}{-\cos(2\alpha)} = -\tan(2\alpha)$$

6.5

17. a. $\alpha = \frac{3\pi}{4}$ $\alpha = \frac{7\pi}{4}$

b. $3 \tan(\alpha) + 1 = -4 - 2 \tan \alpha$

$3 \tan \alpha + 2 \tan \alpha = -5$

$5 \tan \alpha = -5$

$\tan \alpha = -1$

$\alpha = \frac{3\pi}{4} + n\pi$

$\tan \alpha = -1$

20. a. $\alpha \in [0, 2\pi)$ b. $\sin 2\alpha = 1$ c. $\sin \frac{\alpha}{2} + 2K\pi$

a. $\alpha = \frac{3\pi}{2}$ $2\alpha = \frac{3\pi}{2} + 2K\pi$ c. $\frac{\alpha}{2} = \frac{3\pi}{2} + 2K\pi$

$\alpha = \frac{3\pi}{2}, \frac{7\pi}{2}$

24. $5 + \sin 3\alpha = 1$

b. $\alpha = \frac{\pi}{2}, \frac{3\pi}{2}$

a. $\alpha = \frac{\pi}{2} + 2K\pi$ $K \in \mathbb{Z}$

31. a. $\alpha = \frac{\pi}{4} + K\pi$

b. $\alpha = \frac{\pi}{4}$ $K=0$

$\alpha = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$ $K=1$

35. $\Delta \epsilon^2 v - 2 = 0$

$\Delta \epsilon^2 v = 2$

$\Delta \epsilon^2 v = 1 + \tan^2 v$

$\tan v = 1$

$\tan v = 1$ $v = \frac{\pi}{4}, \frac{5\pi}{4}$

$\tan v = -1$ $v = \frac{3\pi}{4}, \frac{7\pi}{4}$

41. $\alpha = \frac{\pi}{3}, \frac{5\pi}{3}$ $\alpha = \frac{2\pi}{3}, \frac{4\pi}{3}$

53. $1 - \sin \alpha = \cos \alpha$

$1^2 = \sin^2 \alpha + \cos^2 \alpha$

$1 = \sin^2(\alpha) + 2 \sin(\alpha) \cos(\alpha) + \cos^2(\alpha)$

$0 = 2 \sin(\alpha) \cos(\alpha)$

$\alpha = 0$	$1 = 1$
$\alpha = \pi$	$1 = -1 \quad \times$
$\alpha = \frac{\pi}{2}$	$0 = 0$
$\alpha = \frac{3\pi}{2}$	$2 = 0 \quad \times$

$\alpha = 0, \frac{\pi}{2}$