College Algebra and Trigonometry

Prof. Liang ZHENG

Fall 2024

Ch 3 Polynomial and Rational Functions

① Graph a Quadratic Function written in Vertex Form

Quadratic Function : **A function defined by $f(x) = ax^2 + bx + c$ $(a \neq 0)$.**

It can also be expressed in the vertex form as $f(x) = a(x-h)^2 + k$

The graph of $f(x)$ is a parabola with vertex (h, k).

Vertex Form

$$f(x) = ax^2 + bx + c = a(x-h)^2 + k$$

$$h = -\frac{b}{2a} \qquad k = \frac{4ac - b^2}{4a}$$

Example 1:

Given $f(x) = -2(x-1)^2 + 8$.

a) Determine the graph opens upward or downward.

b) Identify the vertex.

c) Determine the x-intercept(s) and y-intercept.

d) Sketch the function.

e) Determine the axis of symmetry.

f) Determine the maximum or minimum value of $f(x)$.

g) Write the domain and range in interval notation.

② Write $f(x) = ax^2 + bx + c$ in Vertex Form.

Example 2:

Find the vertex and axis of symmetry of the quadratic functions:

a) $f(x) = 3x^2 + 12x - 15$.

b) $f(x) = -x^2 + 6x + 16$.

c) $f(x) = \frac{1}{2}x^2 - \frac{1}{3}x + \frac{1}{6}$.

③ Solve Applications Involving Quadratic Functions

Example 3:

A stone is thrown from a 35-m cliff at an initial speed of 60 m/s at a angle of 30° from the horizontal. The height of the stone can be modeled by $h(t) = -5t^2 + 30t + 35$ in meters and t is the time in seconds after the stone is released.

a) Determine the time at which the stone is at its maximum height.

b) Determine the time at which the stone will hit the ground.

Example 4:

A rectangular parking lot is to be constructed against a long and solid wall. The developer has purchased 200 m of fencing.

a) Determine the dimensions for the parking lot that would maximize its area.

b) What is the maximum area of the parking lot.