College Algebra and Trigonometry

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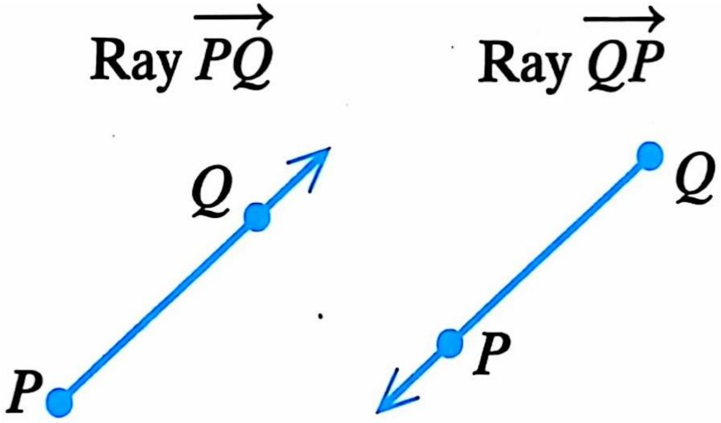
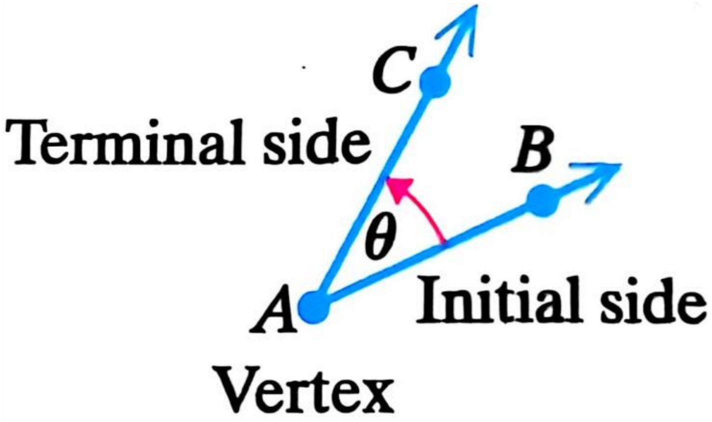
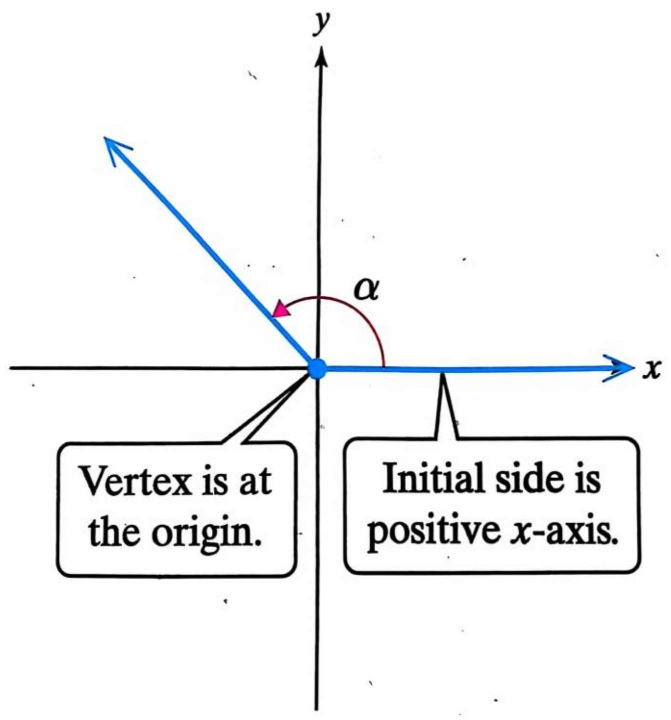
① Find Degree Measure

- A **ray** is a part of a line that consists of an endpoint and all points on the line to one side of the endpoint.
- An **angle** is formed by rotating a ray about its endpoint. The starting position of the ray is called the **initial side** of the angle, and the final position of the ray is called the **terminal side** of the angle. The common endpoint is called the **vertex** of the angle which is often denoted by a capital letter such as A.

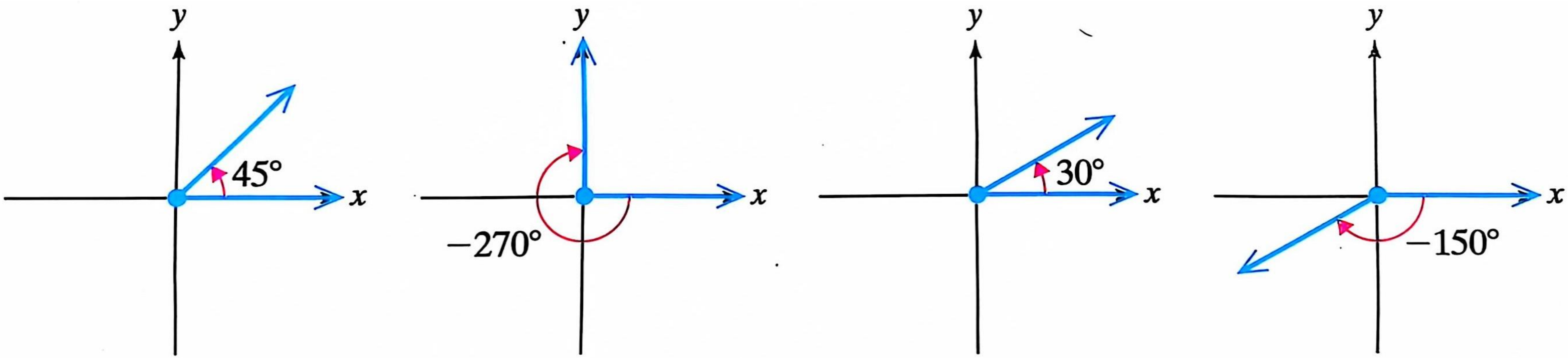




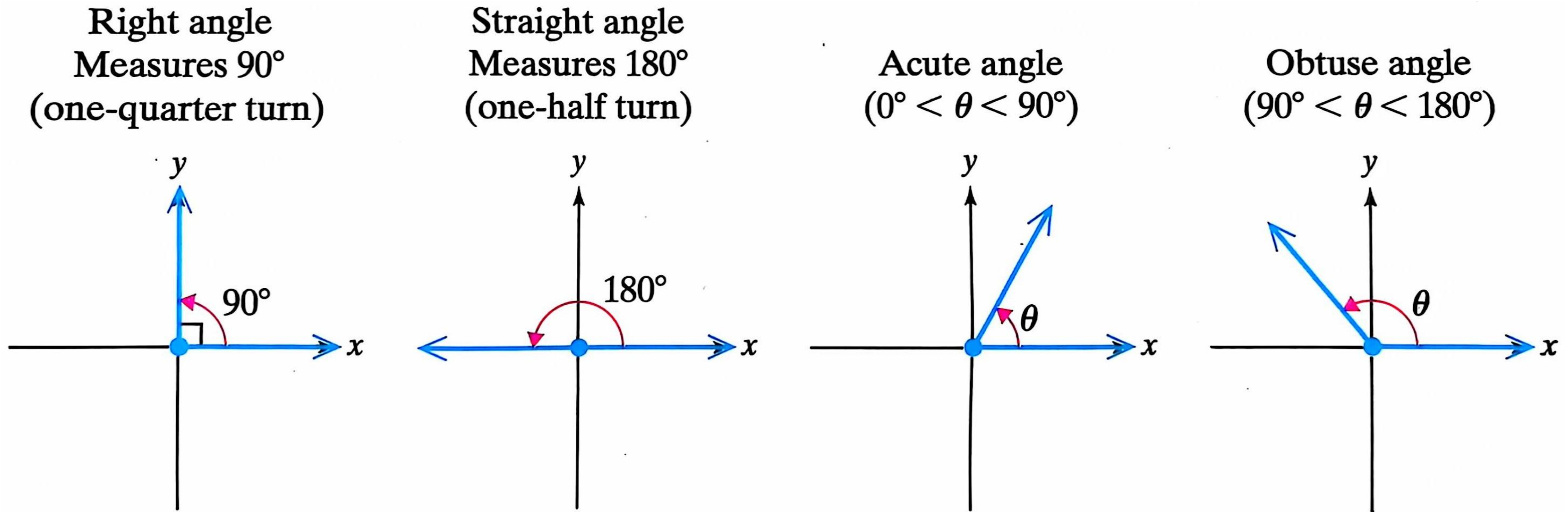
- **Angle A in the previous figure can be denoted by $\angle A$ or by $\angle BAC$ or by $\angle CAB$.**
- **Greek letters such as α, β, θ, and γ, are often used to denote angles.**
- **An angle is in standard position if its vertex is at the origin in the *xy*-plane, and its initial side is the positive *x*-axis.**



- **The measure of an angle quantifies the direction and amount of rotation from the initial side to the terminal side. The measure of an angle is positive if the rotation is counterclockwise, and the measure of an angle is negative if the rotation is clockwise. One unit with which to measure an angle is the degree. One full rotation of a ray about its endpoint is 360 degrees, denoted by 360°.**



- **Some key terms associated with the measure of an angle.**



- **If the sum of the measures of two angles is 90°, they are called complementary.**

  If the sum of the measures of two angles is 180°, they are called supplementary.

- **A degree can be divided into 60 equal parts called minutes (min or $'$), and each minute can be divided into 60 equal parts called seconds (sec or $''$).**
- $1\ \text{min} = \left(\frac{1}{60}\right)^{\circ}$ **or** $1' = \left(\frac{1}{60}\right)^{\circ}$
- $1\ \text{sec} = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^{\circ}$ **or** $1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^{\circ}$
- **For example, 60 degrees, 30 minutes, 15 seconds is denoted as $60°30'15''$.**

Example 1:

Convert $81°30'36''$ to decimal degrees.

Example 2:

Convert $159.26°$ to degree, minute, second form.

Skill practice:

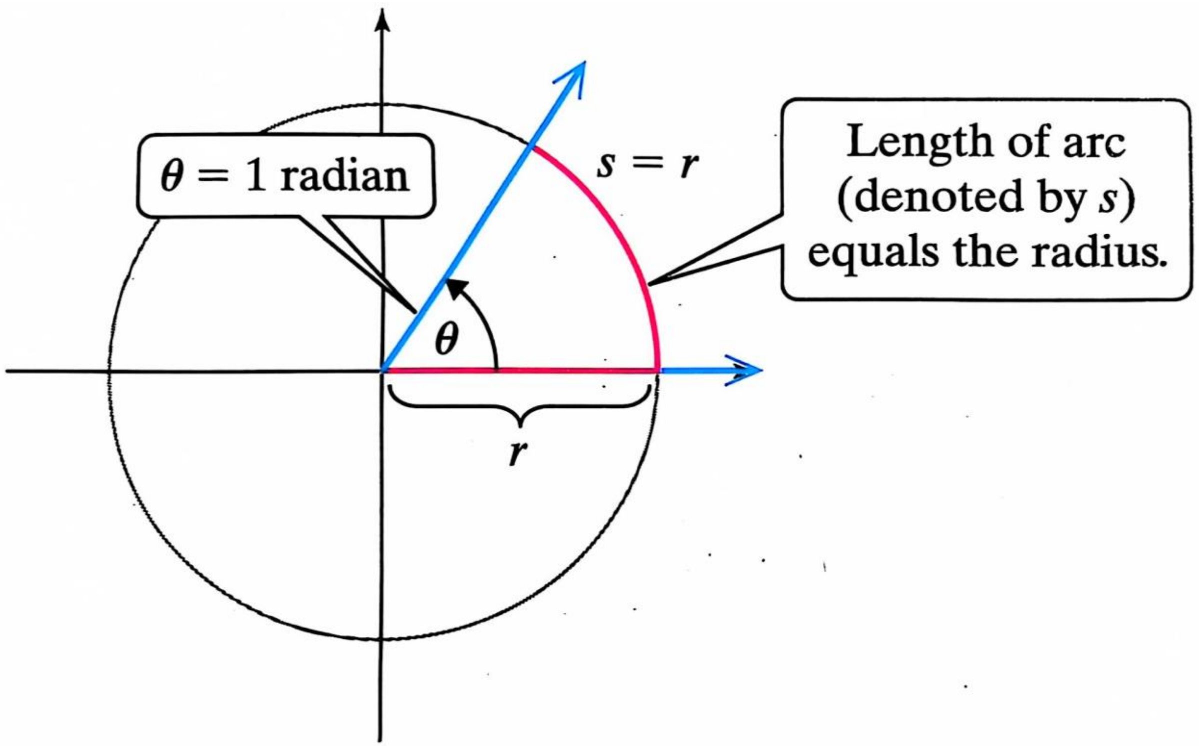
a) **Convert $135°27'18''$ to decimal degrees.**

b) **Convert $88.48°$ to degree, minute, second form.**

② Find Radian Measure

Definition of One Radian:

A central angle that intercepts an arc on the circle with length equal to the radius of the circle has a measure of 1 radian (1 rad or 1).



Definition of Radian Measure of an Angle:

The radian measure of a central angle θ subtended by an arc of length s on a circle of radius r is given by $\theta = \frac{s}{r}$.

- The angular measure of one full rotation is 2π (rad), or 360° .
- The angular measure of one half rotation is π (rad), or 180° .
- The angular measure of one quarter rotation is $\pi/2$ (rad), or 90° .

π (rad) = 180° $\Rightarrow$ 1 (rad) = $\left(\frac{180}{\pi}\right)^\circ \approx 57.3^\circ$ and $1^\circ = \frac{\pi}{180}$ rad ≈ 0.0175 rad

Example 3:

Convert from degrees to radians:

a) $210°$ b) $-135°$

Example 4:

Convert from radians to degrees:

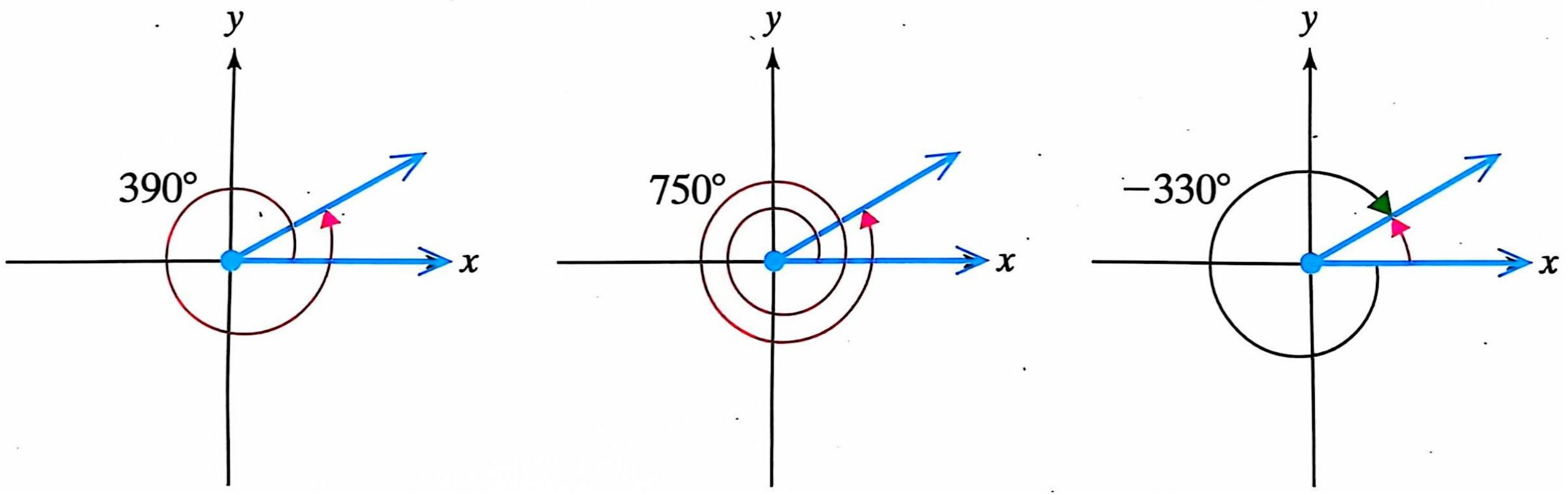
a) $\frac{\pi}{12}$ b) $-\frac{4\pi}{3}$

③ Determine Coterminal Angles

Coterminal Angles:

Two angles in standard position with the same initial side and same terminal side are called coterminal angles. Two angles in standard position are coterminal if their measures differ by a multiple of 360° or 2π.

$30° + (1)(360°) = 390°$ $\quad$ $30° + 2(360°) = 750°$ $\quad$ $30° + (-1)(360°) = -330°$



Example 5:

Find an angle coterminal to θ between 0° and 360°.

a) $\theta = 960°$ **b)** $\theta = -225°$

Example 6:

Find an angle coterminal to θ on the interval $[0, 2\pi]$.

a) $\theta = -\dfrac{5\pi}{6}$ **b)** $\theta = \dfrac{13\pi}{2}$

④ Compute Arc Length of a Sector of a Circle

Arc Length

Given a circle of radius *r*, the length *s* of an arc intercepted by a central angle θ (in radian) is given by

$$s = r\theta$$

Example 7:

Find the length of the arc made by an angle of 120° on a circle of radius 15 cm. Give the exact arc length and round to the nearest tenth of a centimeter.

⑤ Compute Linear and Angular Speed

Linear and Angular Speed

If a point on a circle of radius *r* moves through an angle θ radians in time *t*, the angular and linear speeds of the point are

Angular speed: $$\omega = \frac{\theta}{t}$$

Linear speed: $$v = r\omega = \frac{r\theta}{t}$$

Example 8:

A ceiling fan rotates at 90 rpm (revolutions per minute). For a point at the tip of a 1-meter blade, Find the angular and linear speeds. Round to the nearest whole unit.

⑥ Compute the Area of a Sector of a Circle

Area of a Sector

The area A of a sector of a circle of radius r with a central angle θ (in radian) is given by

$$\boldsymbol{A = \frac{1}{2}r^2\theta}$$

Example 9:

A crop sprinkler rotates through an angle of 150° and spray a distance of 30 m. Find the amount of area watered and round to the nearest whole unit.