College Algebra and Trigonometry

Prof. Liang ZHENG

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① Recognize Basic Functions

Table 2-2 Basic Functions and Their Graphs

1. **Linear Functions:** $f(x) = mx + b$

   Constant Functions: $f(x) = b$

   Identity Function: $f(x) = x$

4. **Absolute Value Function:** $f(x) = |x|$

2. **Quadratic Function:** $f(x) = x^2$

   Square Root Function: $f(x) = \sqrt{x}$

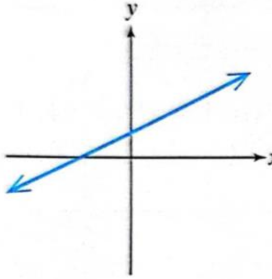
5. **Reciprocal Function:** $f(x) = \frac{1}{x}$

3. **Cubic Function:** $f(x) = x^3$

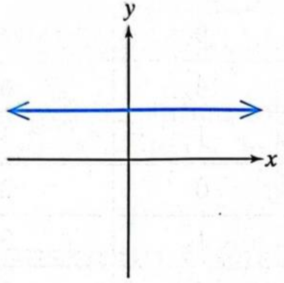
   Cubic Root Function: $f(x) = \sqrt[3]{x}$

Table 2-2 Basic Functions and Their Graphs

1. Linear functions $f(x) = mx + b$

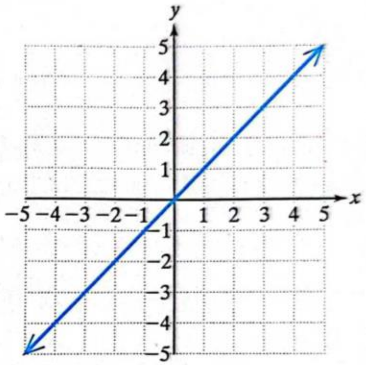


Constant functions $f(x) = b$



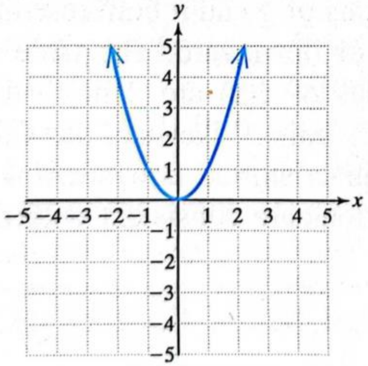
2. Identity function: $f(x) = x$

x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2



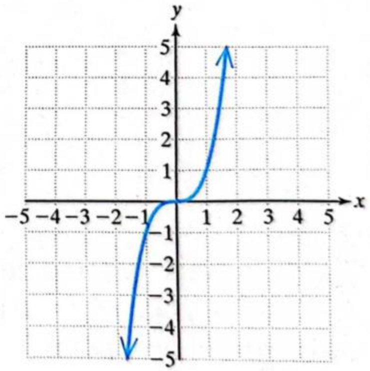
3. Quadratic function: $f(x) = x^2$ (graph is a parabola)

x	$f(x)$
−2	4
−1	1
0	0
1	1
2	4



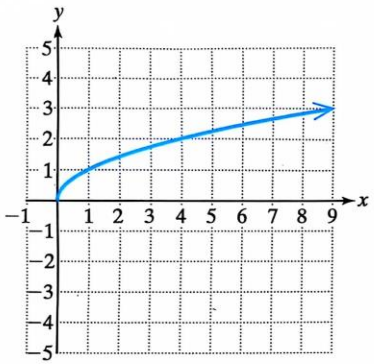
4. Cube function: $f(x) = x^3$

x	$f(x)$
−2	−8
−1	−1
0	0
1	1
2	8



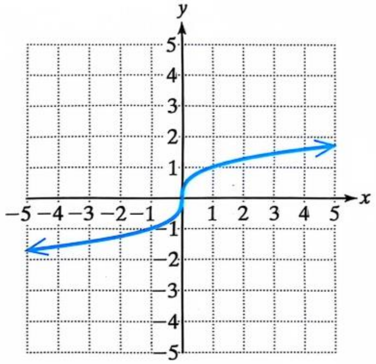
5. Square root function: $f(x) = \sqrt{x}$

x	$f(x)$
0	0
1	1
4	2
9	3
16	4



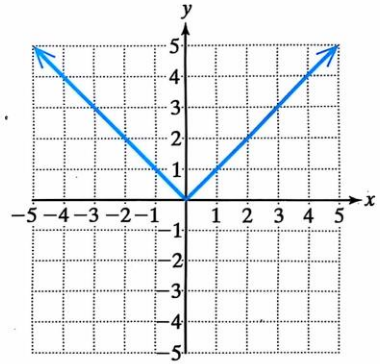
6. Cube root function: $f(x) = \sqrt[3]{x}$

x	$f(x)$
−8	−2
−1	−1
0	0
1	1
8	2



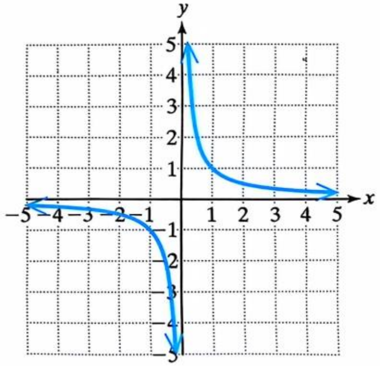
7. Absolute value function: $f(x) = |x|$

x	$f(x)$
−2	2
−1	1
0	0
1	1
2	2



8. Reciprocal function: $f(x) = \frac{1}{x}$

x	$f(x)$
−2	−½
−1	−1
−½	−2
½	2
1	1
2	½



② Apply Vertical and Horizontal Translations (Shifts)

Vertical Translations of Graphs:

Consider a function defined by $y = f(x)$. Let k be a positive real number.

- **The graph of $y = f(x) + k$ is the graph of $y = f(x)$ shifted k units upward.**
- **The graph of $y = f(x) - k$ is the graph of $y = f(x)$ shifted k units downward.**

e.g. $f(x) = x^2$ $g(x) = x^2 + 2$ $h(x) = x^2 - 2$

Example 1:

Use translations to graph the following functions.

a) $g(x) = |x| - 3$

b) $h(x) = x^3 + 2$

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Horizontal Translations of Graphs:

Consider a function defined by $y = f(x)$. Let k be a positive real number.

- **The graph of $y = f(x - h)$ is the graph of $y = f(x)$ shifted h units to the right.**
- **The graph of $y = f(x + h)$ is the graph of $y = f(x)$ shifted h units to the left.**

Example 2:

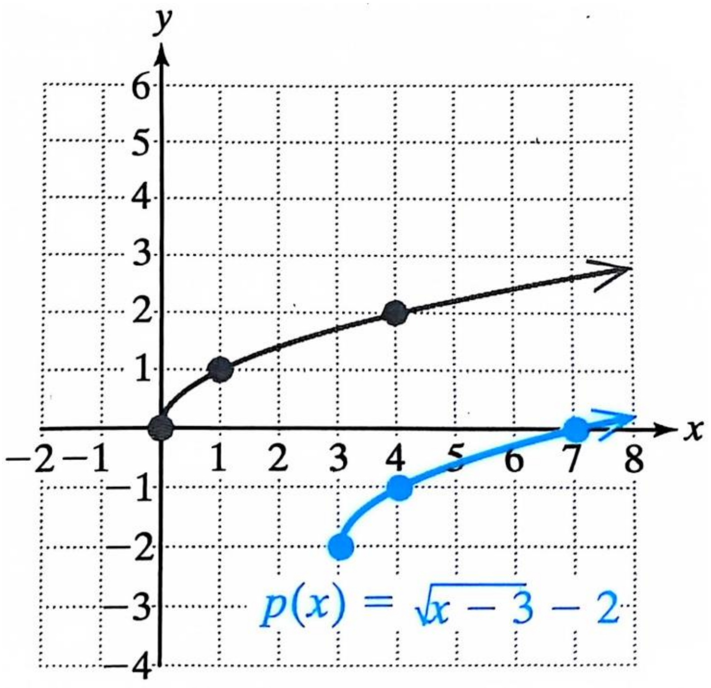
Translating a graph horizontally:

$$g(x) = (x + 3)^2$$

Example 3:

Translating a graph horizontally and vertically:

$$p(x) = \sqrt{x - 3} - 2$$



③ Apply Vertical and Horizontal Shrinking and Stretching

Vertical Shrinking and Stretching of Graphs:

Consider a function defined by $y = f(x)$. Let a be a positive real number.

- If $a > 1$, then the graph of $y = af(x)$ is the graph of $y = f(x)$ stretched vertically by a factor of a.
- If $0 < a < 1$, then the graph of $y = af(x)$ is the graph of $y = f(x)$ shrunk vertically by a factor of a.

Horizontal Shrinking and Stretching of Graphs:

Consider a function defined by $y = f(x)$. Let a be a positive real number.

- If $a > 1$, then the graph of $y = f(ax)$ is the graph of $y = f(x)$ shrunk horizontally by a factor of $1/a$.
- If $0 < a < 1$, then the graph of $y = af(x)$ is the graph of $y = f(x)$ stretched horizontally by a factor of $1/a$.

Example 4:

Stretch or shrink a graph of function vertically:

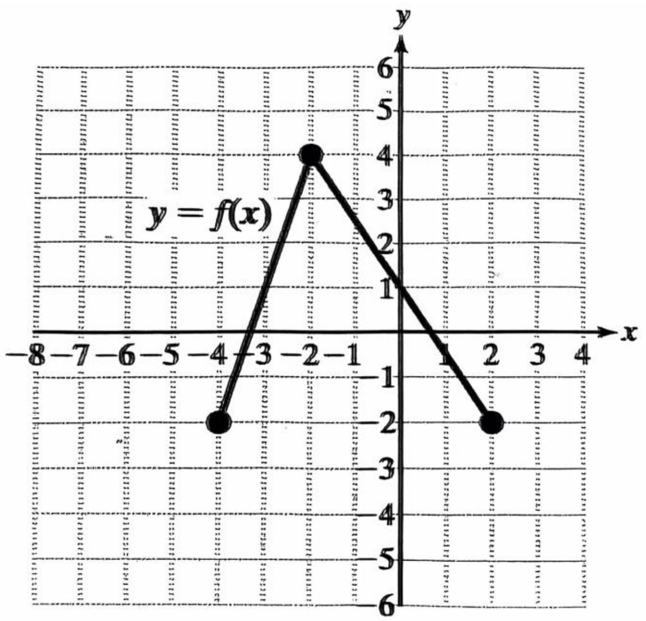
a) $f(x) = |x|$ b) $g(x) = 2|x|$ c) $f(x) = |x|/2$

Example 5:

Stretch or shrink a graph of function horizontally.

The graph of $y = f(x)$ is shown. Graph:

a) $y = f(2x)$ b) $y = f(x/2)$



④ Apply Reflections across the *x*- and *y*- axes.

Reflections across the *x*- and *y*- axes:

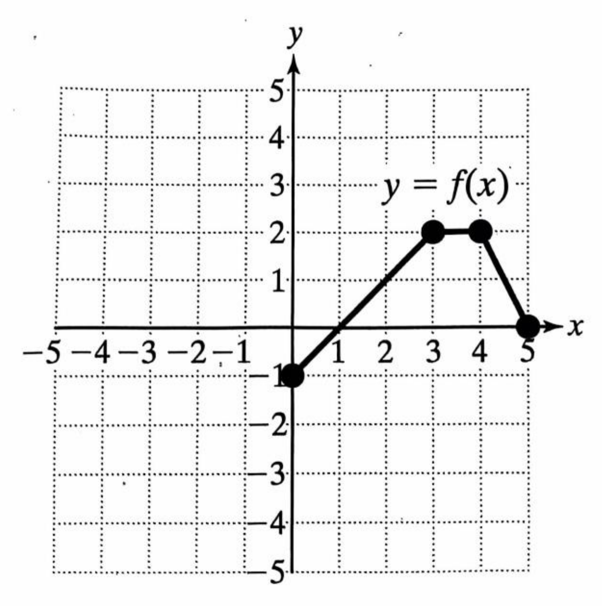
Consider a function defined by $y = f(x)$.

- **The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected across the *x*-axis.**
- **The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected across the *y*-axis.**

Example 6:

The graph of $y = f(x)$ is shown. Graph:

a) $y = -f(x)$ **b) $y = f(-x)$**



⑤ Summarize Transformations of Graphs.

Transformations of Functions

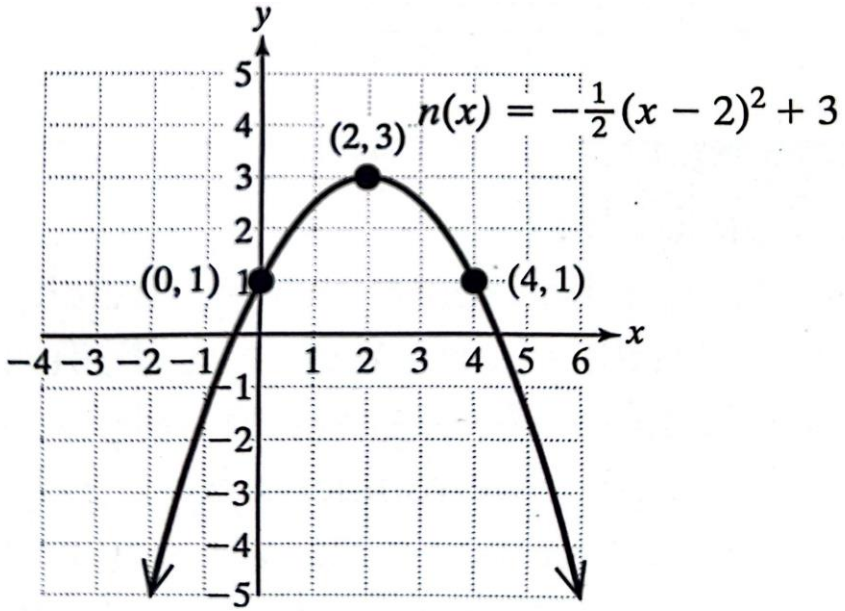
Consider a function defined by $y = f(x)$. If h, k, and a represent positive real numbers, then the graphs of the following functions are related to $y = f(x)$ as follows.

Transformation	Effect on the Graph of f	Changes to Points on f
Vertical translation (shift) $y = f(x) + k$ $y = f(x) - k$	Shift upward k units Shift downward k units	Replace (x, y) by $(x, y + k)$. Replace (x, y) by $(x, y - k)$.
Horizontal translation (shift) $y = f(x - h)$ $y = f(x + h)$	Shift to the right h units Shift to the left h units	Replace (x, y) by $(x + h, y)$. Replace (x, y) by $(x - h, y)$.
Vertical stretch/shrink $y = a[f(x)]$	Vertical stretch (if $a > 1$) Vertical shrink (if $0 < a < 1$) Graph is stretched/shrunk vertically by a factor of a.	Replace (x, y) by (x, ay).
Horizontal stretch/shrink $y = f(a \cdot x)$	Horizontal shrink (if $a > 1$) Horizontal stretch (if $0 < a < 1$) Graph is shrunk/stretched horizontally by a factor of $\frac{1}{a}$.	Replace (x, y) by $\left(\frac{x}{a}, y\right)$.
Reflection $y = -f(x)$ $y = f(-x)$	Reflection across the x-axis Reflection across the y-axis	Replace (x, y) by $(x, -y)$. Replace (x, y) by $(-x, y)$.

Example 7:

Use Multiple Transformations to graph the function:

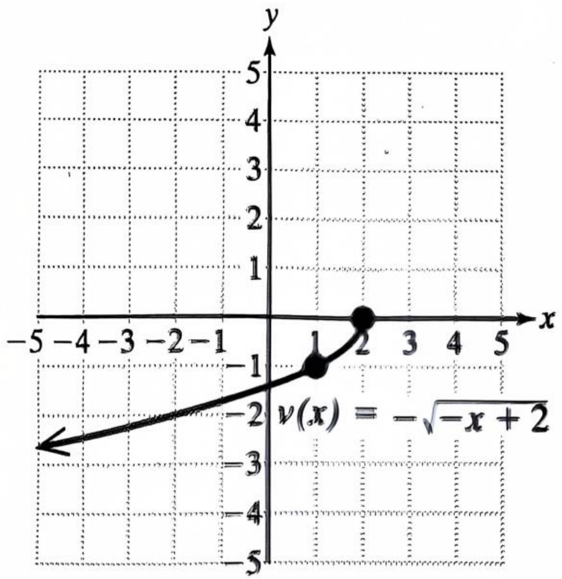
$$m(x) = -\frac{1}{2}(x-2)^2 + 3$$



Example 8:

Use Multiple Transformations to graph the function:

$$n(x) = -\sqrt{-x+2}$$



① Test for Symmetry

Test for Symmetry:

Consider an equation in the variables x and y.

- **The graph of the equation is symmetric with respect to y-axis if substituting $-x$ for x in the equation results in an equivalent equation.**
- **The graph of the equation is symmetric with respect to x-axis if substituting $-y$ for y in the equation results in an equivalent equation.**
- **The graph of the equation is symmetric with respect to the origin if substituting $-x$ for x and $-y$ for y in the equation results in an equivalent equation.**

Example 1:

Determine whether the graph is symmetric with respect to *x*-axis, *y*-axis, or the origin:

a) $y = |x|$ **b)** $x = y^2 - 4$

Example 2:

Determine whether the graph is symmetric with respect to *x*-axis, *y*-axis, or the origin:

$$x^2 + y^2 = 9$$

② Identify Even and Odd Functions.

Even and Odd Functions:

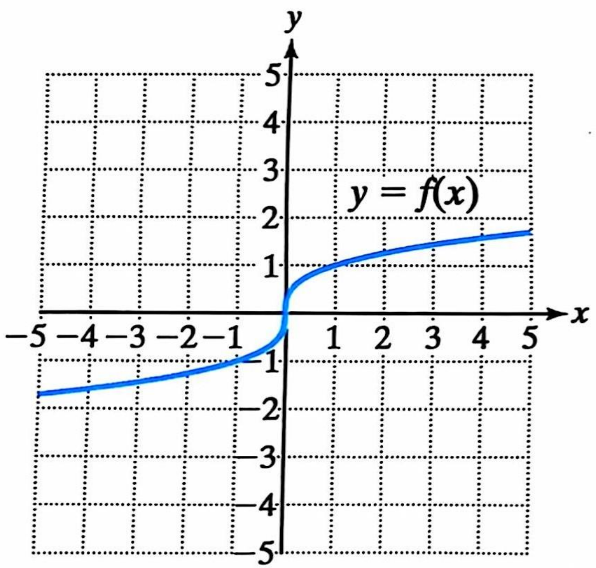
Consider an equation in the variables x and y.

- **A function is an even function if $f(-x) = f(x)$ for all x in the domain of f. The graph of an even function is symmetric with respect to y-axis.**
- **A function is an odd function if $f(-x) = -f(x)$ for all x in the domain of f. The graph of an odd function is symmetric with respect to *the origin*.**

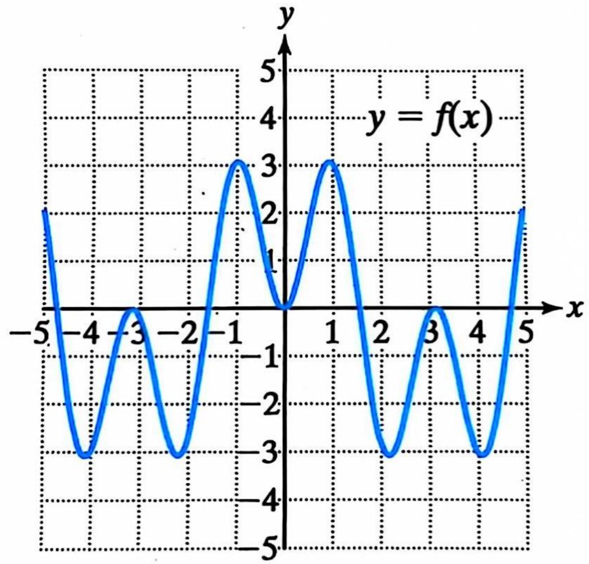
Example 3:

Determine whether the function is even, odd, or neither.

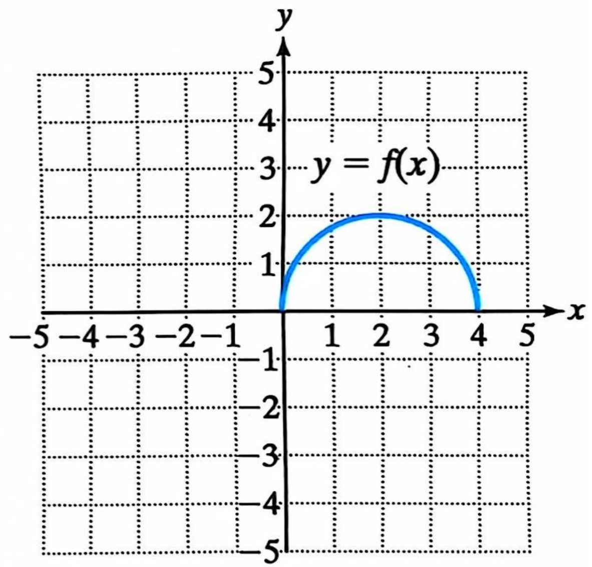
a.



b.



c.



Example 4:

Determine whether the function is even, odd, or neither.

a) $f(x) = -2x^4 + 5|x|$ b) $g(x) = 4x^3 - x$ c) $h(x) = 2x^2 + x$

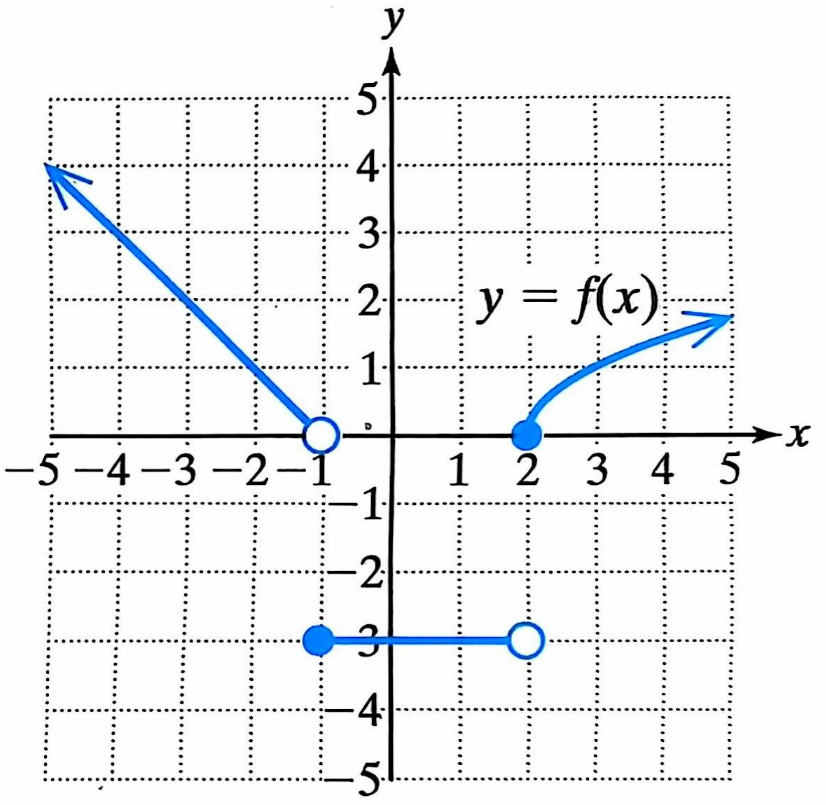
③ Graph Piecewise-Defined Functions.

Example 5: Interpreting a piecewise-defined function.

Evaluate the function for the given values of *x*.

$$f(x) = \begin{cases} -x-1 & \text{for} \quad x < -1 \\ -3 & \text{for} \quad -1 \le x < 2 \\ \sqrt{x-2} & \text{for} \quad x \ge 2 \end{cases}$$

a) $f(-3)$ b) $f(-1)$ c) $f(2)$ d) $f(6)$



Example 6:

Graph the function defined by:

$$f(x) = \begin{cases} -3x & \text{for} \quad x < 1 \\ -3 & \text{for} \quad x \geq 1 \end{cases}$$

Example 7:

Graph the function defined by:

$$f(x) = \begin{cases} x + 3 & \text{for} \quad x < -1 \\ x^2 & \text{for} \quad -1 \leq x < 2 \end{cases}$$

Step Functions: a special category of piecewise-defined functions.

The graph of a step function is a series of discontinuous "steps".

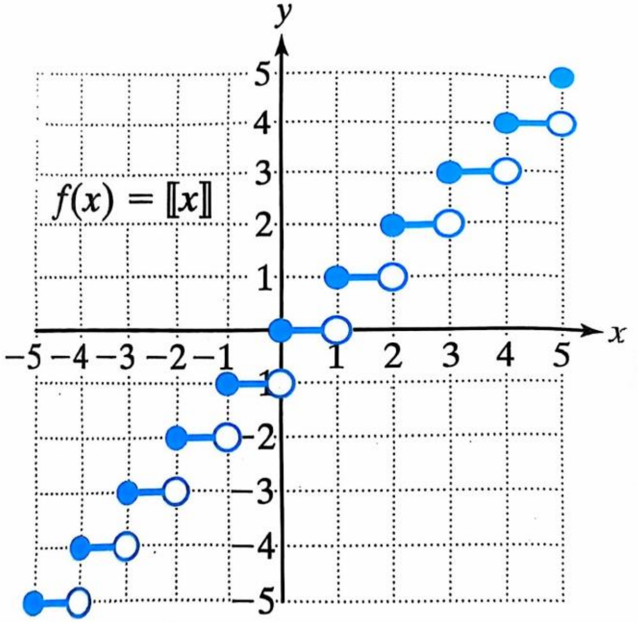
One important step function is called the **greatest integer function or floor function**, defined by: $f(x) = [x]$

where $[x]$ is the greatest integer less than or equal to x.

The operation $[x]$ may also be denoted as int(x) or floor(x).

Example 8:

Graph the function defined by: $f(x) = [x]$

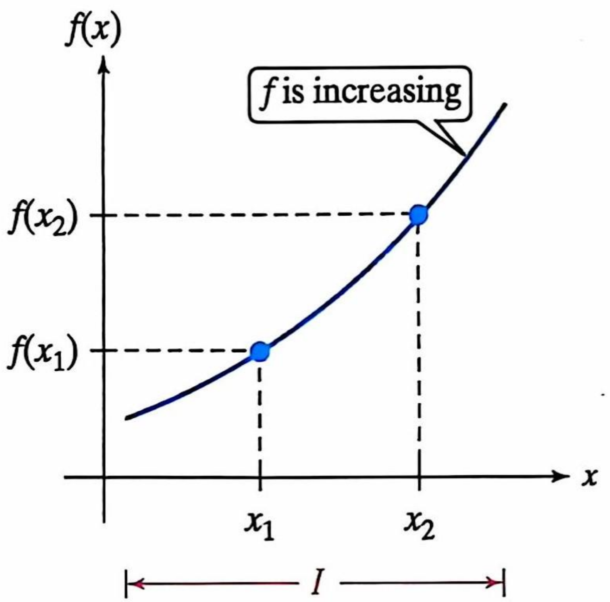


④ Investigate Increasing, Decreasing, and Constant Behavior of a Function.

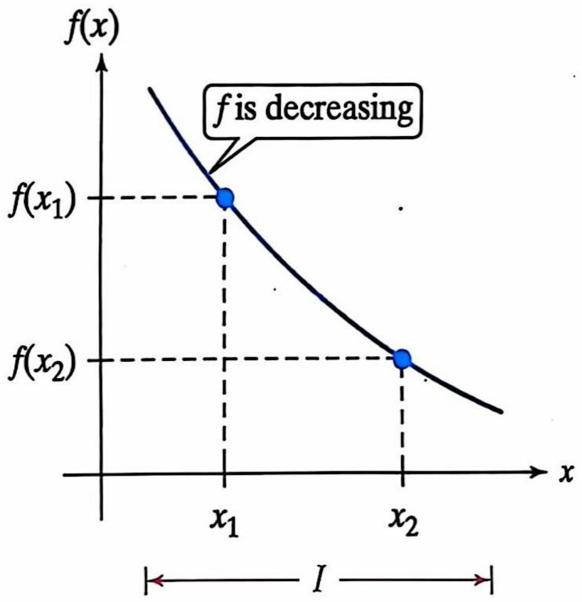
Intervals of Increasing, Decreasing, and Constant Behavior

Suppose that I is an interval contained within the domain of a function f.

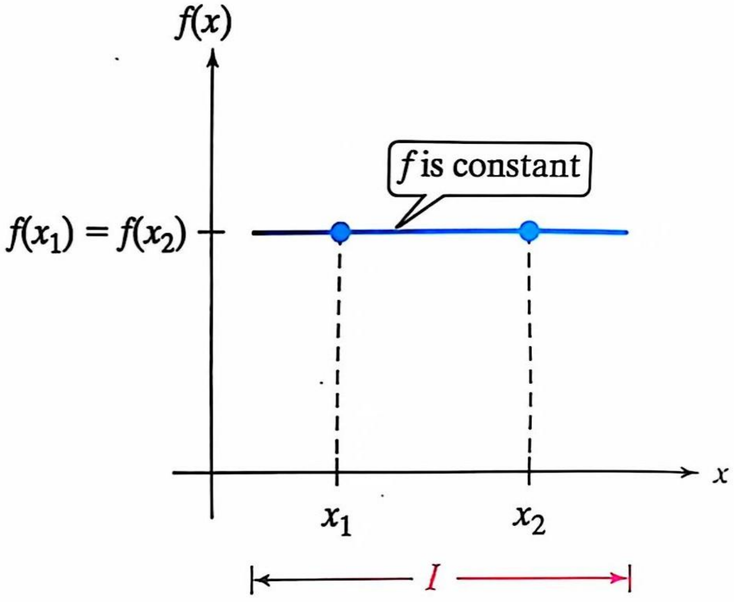
- f is increasing on I if $f(x_1) < f(x_2)$ for all $x_1 < x_2$ on I.
- f is decreasing on I if $f(x_1) > f(x_2)$ for all $x_1 < x_2$ on I.
- f is constant on I if $f(x_1) = f(x_2)$ for all x_1 and x_2 on I.



For all $x_1 < x_2$ on I, $f(x_1) < f(x_2)$



For all $x_1 < x_2$ on I, $f(x_1) > f(x_2)$



For all x_1 and x_2 on I, $f(x_1) = f(x_2)$

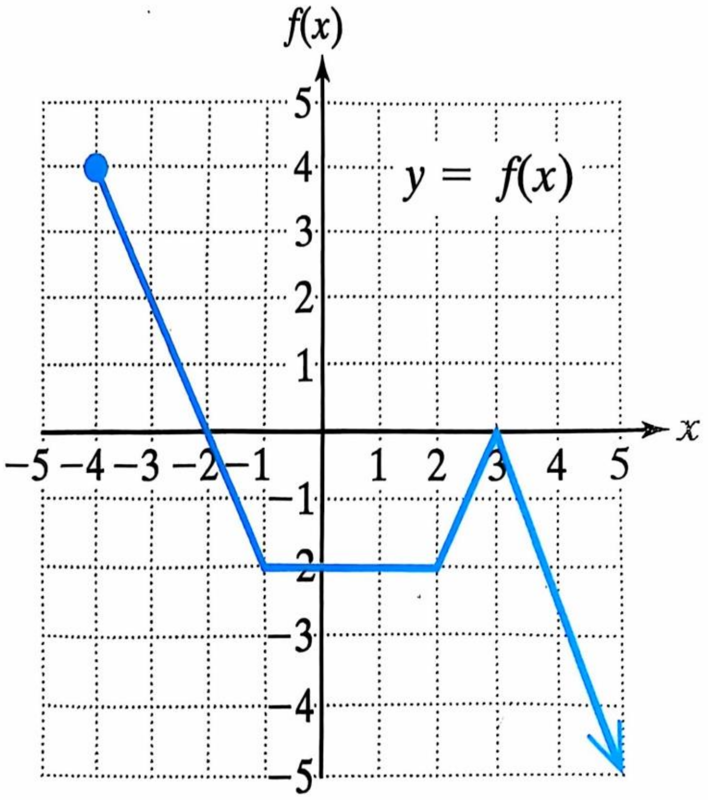
Example 10:

Use interval notation to write the intervals over which f is:

a) Increasing

b) Decreasing

c) Constant

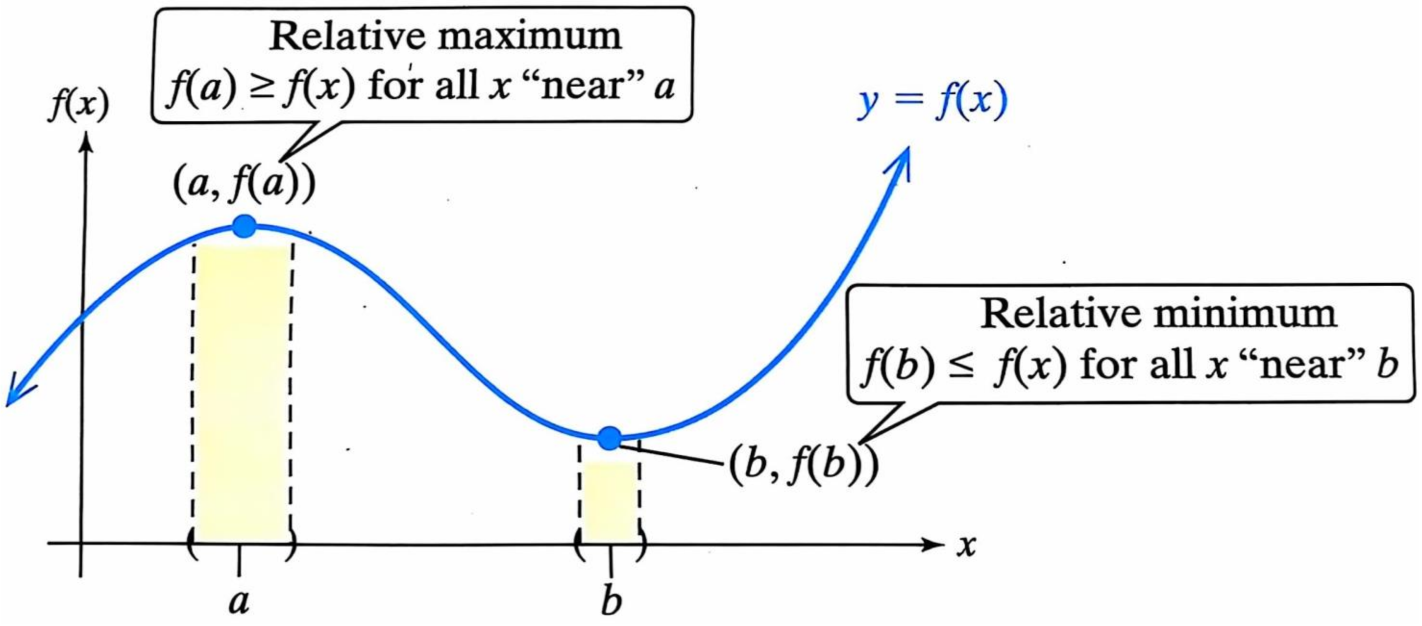


⑤ Determine Relative Minima and Maxima of a Function.

Relative/Local Minimum and Maximum Values:

- $f(a)$ is a **relative/local maximum** of f if there exists an open interval containing a such that $f(a) \geq f(x)$ for all x in the interval.
- $f(b)$ is a **relative/local minimum** of f if there exists an open interval containing b such that $f(b) \leq f(x)$ for all x in the interval.

Note: An **open** interval is an interval in which the endpoints are **not** included.



Example 11: Find the Relative/Local Maxima and Minima

For the graph of $y = g(x)$ shown,

a. Determine the location and value of any relative maxima.

b. Determine the location and value of any relative minima.