100th ANNIVERSARY
哈爾濱工業大學
HARBIN INSTITUTE OF TECHNOLOGY
1920-2020

Lecture 11

Coulomb's Law

Date: 4/15/2025

Course Instructor:
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Introduction to Electromagnetism

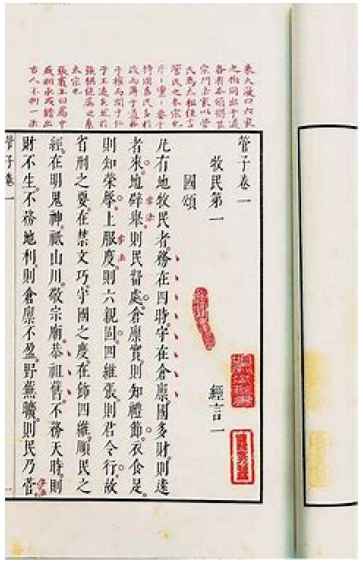
Chinese

- Documents suggest that magnetism was observed as early as 2000 BC

Greeks

- Electrical and magnetic phenomena as early as 700 BC
- Experiments with amber and magnetite

Attractive forces



1600

- William Gilbert showed electrification effects were not confined to just amber.
- The electrification effects were a general phenomena.

1785

- Charles Coulomb confirmed inverse square law form for electric forces

Introduction to Electromagnetism

1819

- Hans Oersted found a compass needle deflected when near a wire carrying an electric current.

1831

- Michael Faraday and Joseph Henry showed that when a wire is moved near a magnet, an electric current is produced in the wire.

1873

- James Clerk Maxwell used observations and other experimental facts as a basis for formulating the laws of electromagnetism.
    - Unified electricity and magnetism

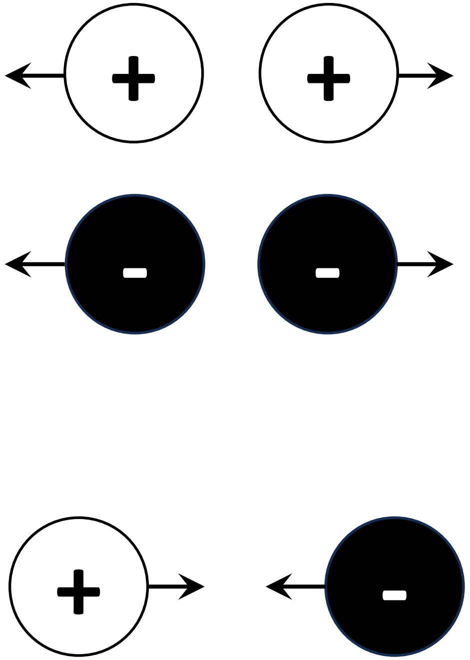
Starting from Microscopic View: Quantization of Charges

Basic models: point charges (just like the particle model in kinematics)

positive charge

negative charge

Key properties:



Same sign: **repel**

Opposite signs: **attract**

Starting from Microscopic View: Quantization of Charges

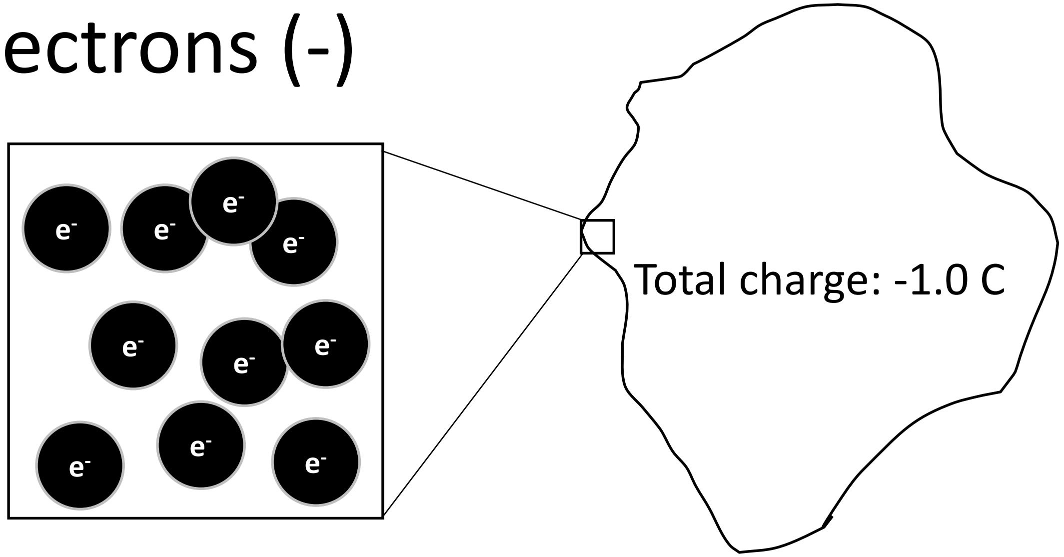
Unit for charges: coulomb (**C**), 1 C = 1 A·s

Carrier: protons/holes (+) and electrons (-)

$$q = \pm ne \qquad (n = 1,2,3,\cdots)$$

$$e = 1.602 \times 10^{-19}\,\mathrm{C}$$

Suppose we have a charged particle with any shape

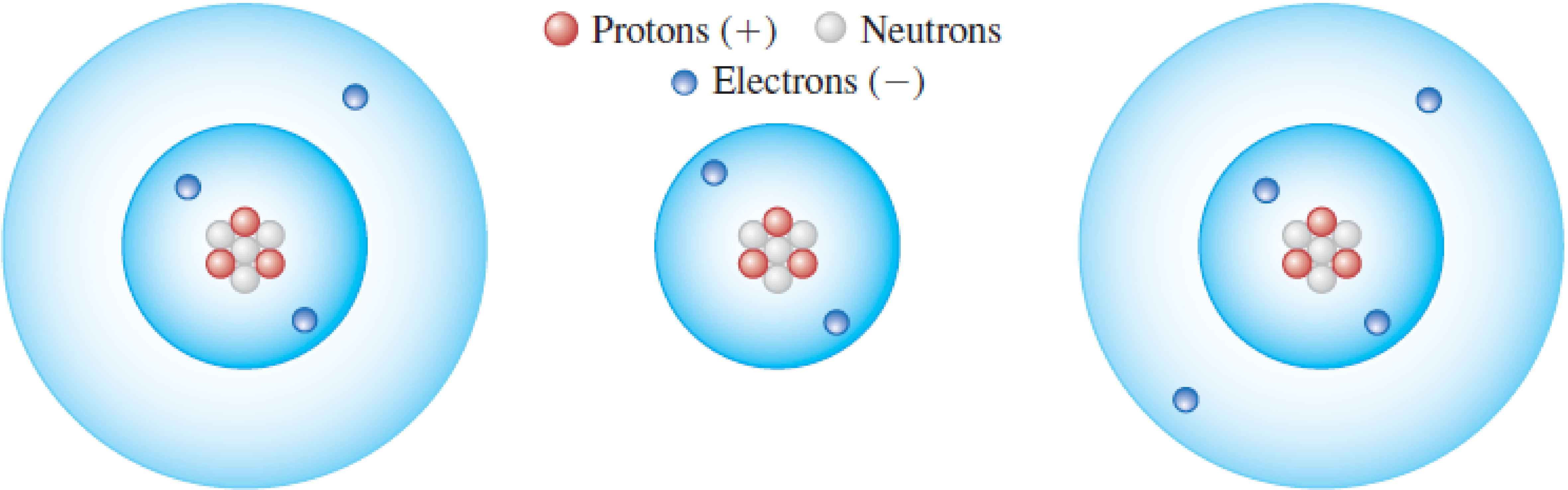


~6×10^{18} electrons

Mass of an electron: m_e=9.3 ×10^{-31} kg

Size of an electron: hard to define, but < 10^{-10} m in solids typically

More on Microscopic Charges



(a) Neutral lithium atom (Li):

3 protons (3+)
4 neutrons
3 electrons (3−)

Electrons equal protons:
Zero net charge

(b) Positive lithium ion (Li^+):

3 protons (3+)
4 neutrons
2 electrons (2−)

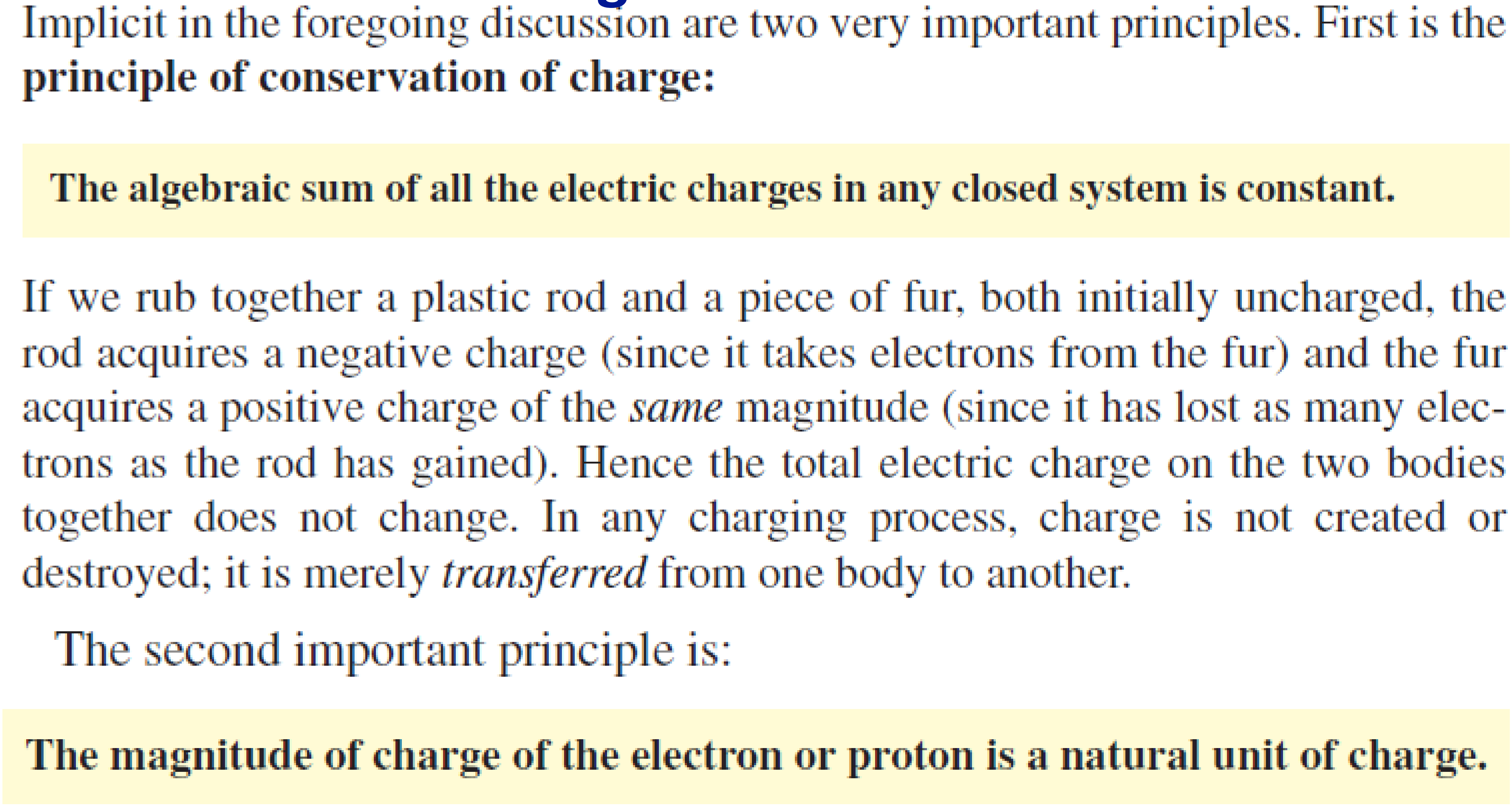
Fewer electrons than protons:
Positive net charge

(c) Negative lithium ion (Li^-):

3 protons (3+)
4 neutrons
4 electrons (4−)

More electrons than protons:
Negative net charge

Charge Conservation



Implicit in the foregoing discussion are two very important principles. First is the **principle of conservation of charge:**

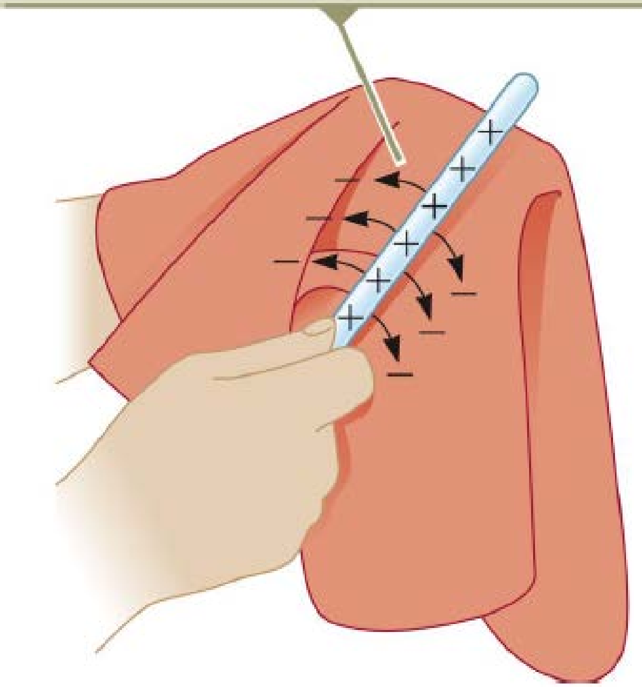
The algebraic sum of all the electric charges in any closed system is constant.

If we rub together a plastic rod and a piece of fur, both initially uncharged, the rod acquires a negative charge (since it takes electrons from the fur) and the fur acquires a positive charge of the *same* magnitude (since it has lost as many electrons as the rod has gained). Hence the total electric charge on the two bodies together does not change. In any charging process, charge is not created or destroyed; it is merely *transferred* from one body to another.

The second important principle is:

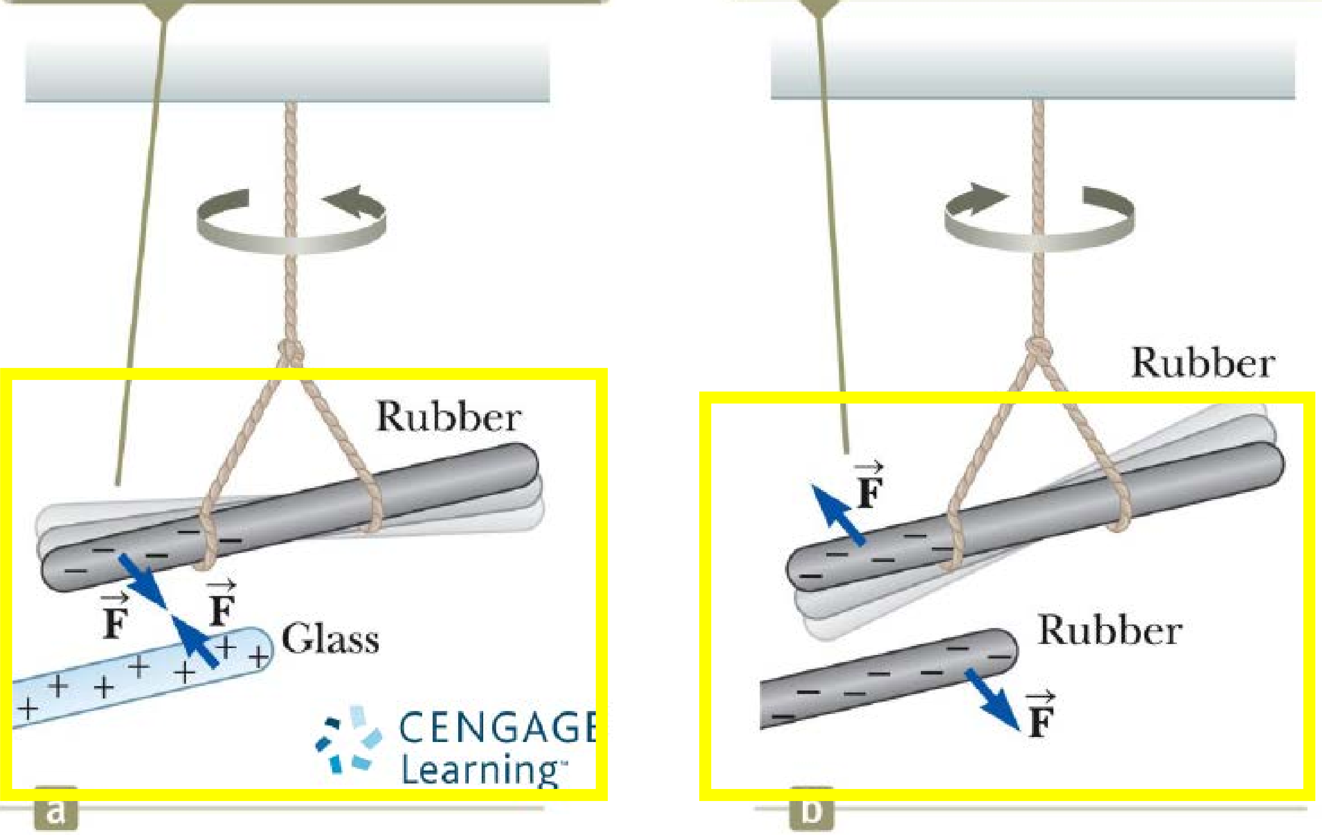
The magnitude of charge of the electron or proton is a natural unit of charge.

Qualitative Observation of Charge Interactions



Because of conservation of charge, each electron adds negative charge to the silk and an equal positive charge is left on the glass rod.

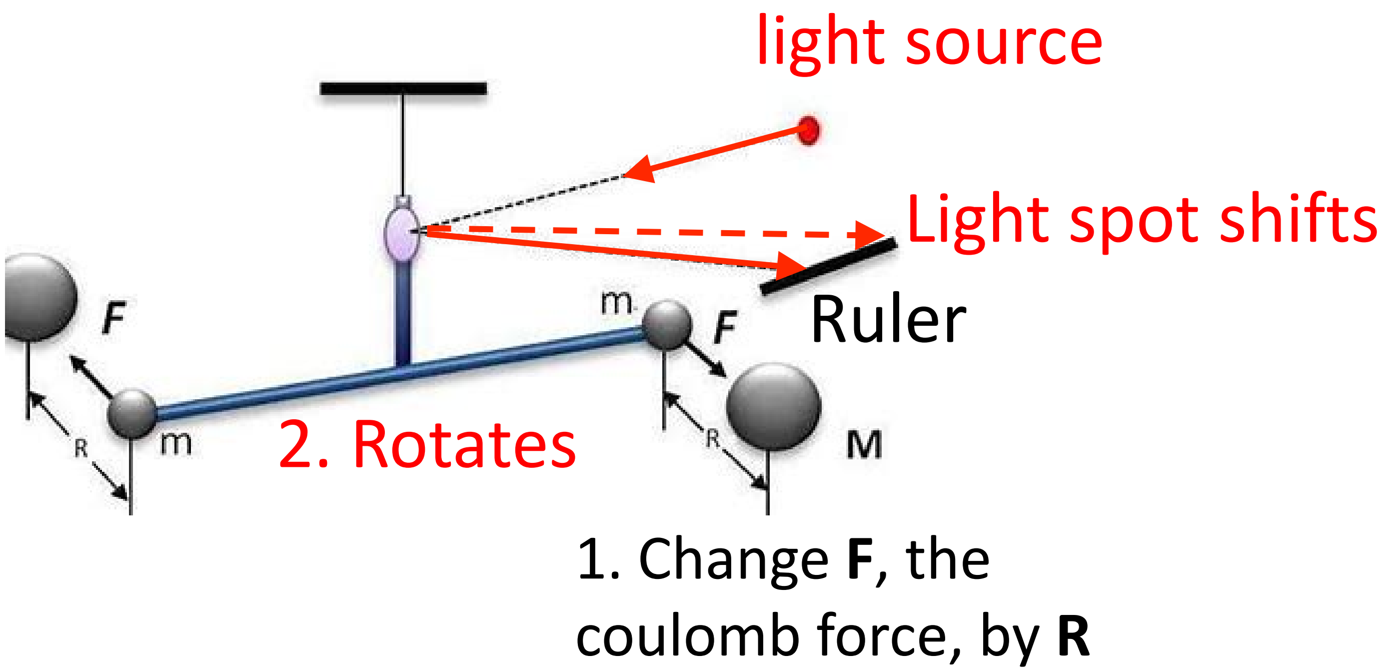
Transfers charges by rubbing (ignores why for now)



A negatively charged rubber rod suspended by a string is attracted to a positively charged glass rod.

A negatively charged rubber rod is repelled by another negatively charged rubber rod.

Quantitative Observation of Charge Interactions



light source

Light spot shifts

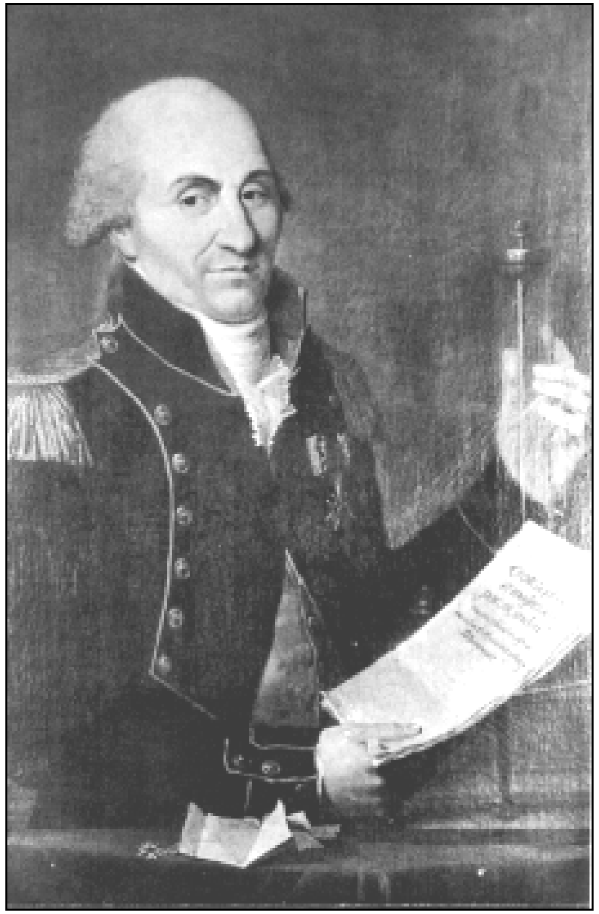
Ruler

2. Rotates

1. Change **F**, the coulomb force, by **R**

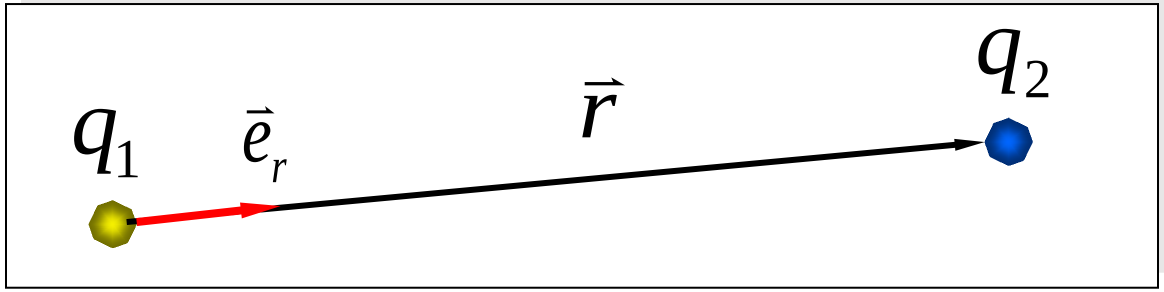
$$\boldsymbol{F} \sim constant \cdot \boldsymbol{R}^{-2}$$

Coulomb's law



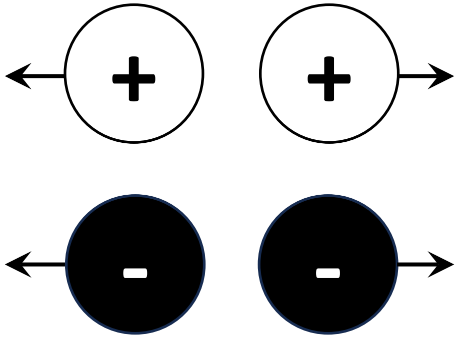
C. A. Coulomb 1736 –1806

Coulomb's Law

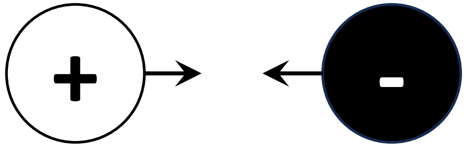


$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\vec{e}_r$$

Key properties:



Same sign: **repel**



Opposite signs: **attract**

Magnitude

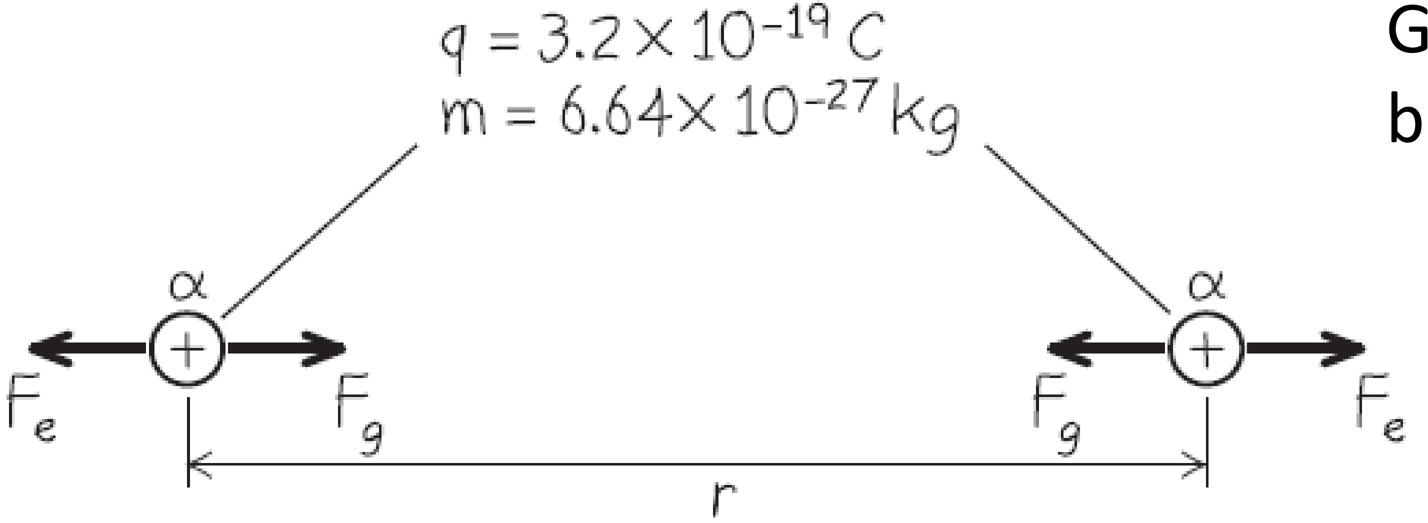
$$F = \boxed{\frac{1}{4\pi\varepsilon_0}}\frac{q_1 q_2}{r^2}$$

$$\boxed{\cong 8.988 \times 10^9\ \mathrm{N\cdot m^2/C^2}}$$

$$\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2} \quad \text{and} \quad \frac{1}{4\pi\epsilon_0} = k = 8.988 \times 10^9\ \mathrm{N\cdot m^2/C^2}$$

Example 21.1 Electric force vs gravitational force

An α particle (the nucleus of a helium atom) has mass $m = 6.64 \times 10^{-27}$ kg and charge $q = +2e = 3.2 \times 10^{-19}$ C. Compare the magnitude of the electric repulsion between two α ("alpha") particles with that of the gravitational attraction between them.



Given: gravitational forces exists between any two masses:

$$F_g = G\frac{m^2}{r^2}$$

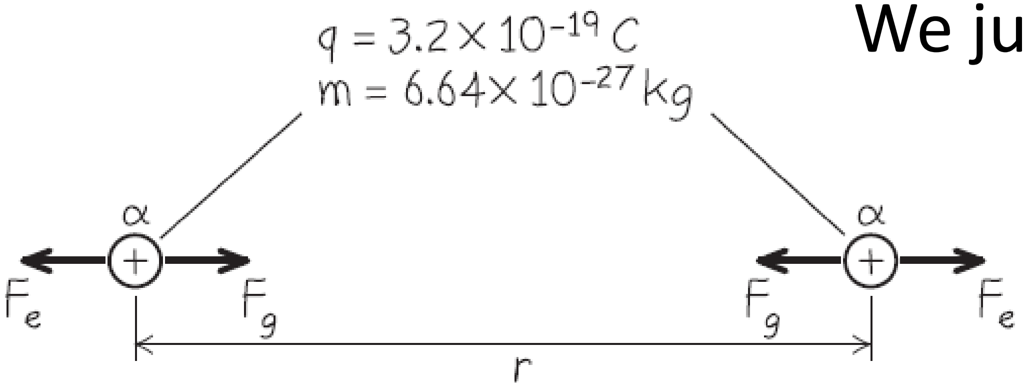
where G is a constant:

$6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$

Example 21.1 Electric force vs gravitational force

An α particle (the nucleus of a helium atom) has mass $m = 6.64 \times 10^{-27}$ kg and charge $q = +2e = 3.2 \times 10^{-19}$ C. Compare the magnitude of the electric repulsion between two α ("alpha") particles with that of the gravitational attraction between them.

We just learned: $F_e = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{r^2}$ for electric repulsion



$$\frac{F_e}{F_g} = \frac{1}{4\pi\epsilon_0 G}\frac{q^2}{m^2}$$

$$= \frac{9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}}{6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}}\frac{(3.2 \times 10^{-19}\ \mathrm{C})^2}{(6.64 \times 10^{-27}\ \mathrm{kg})^2}$$

$$= 3.1 \times 10^{35}$$

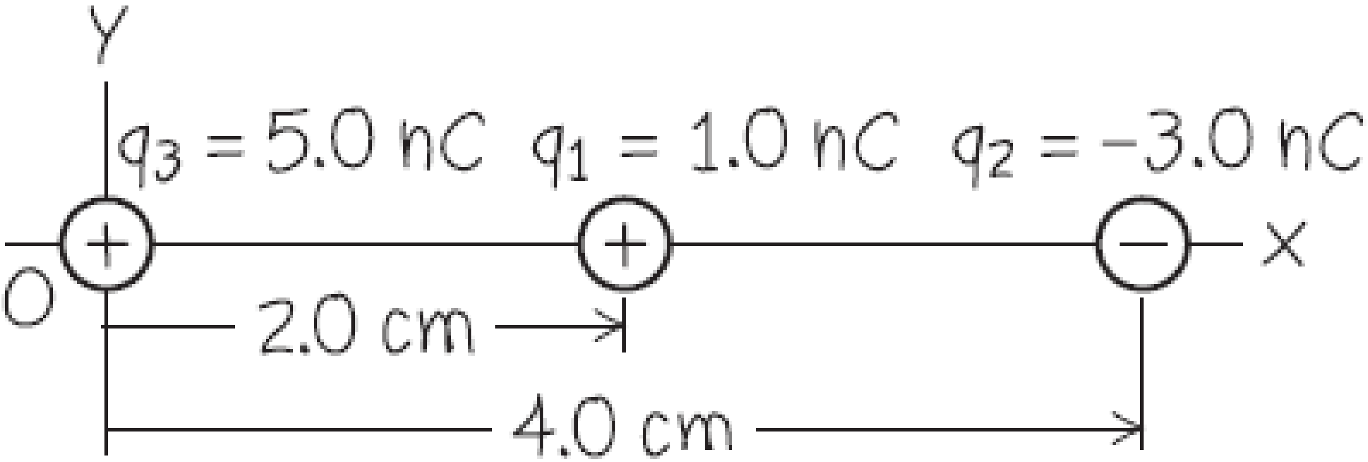
Is electric repulsion always much larger than gravity?

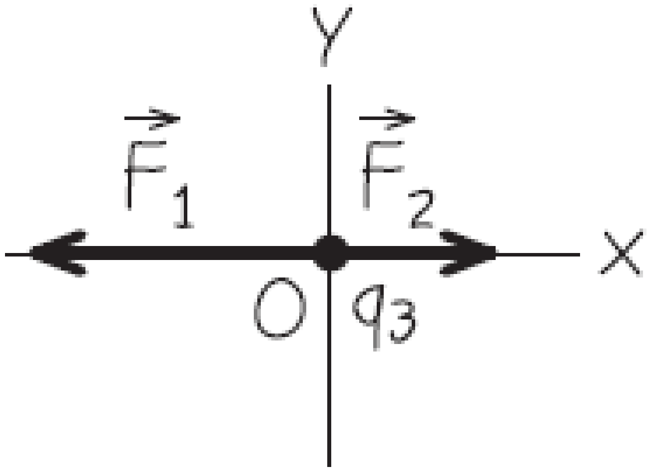
Superposition of Electrical Forces

Coulomb’s law as we have stated it describes only the interaction of two *point* charges. Experiments show that when two charges exert forces simultaneously on a third charge, the total force acting on that charge is the *vector sum* of the forces that the two charges would exert individually. This important property, called the **principle of superposition of forces,** holds for any number of charges. By using this principle, we can apply Coulomb’s law to *any* collection of charges. Two of the examples at the end of this section use the superposition principle. Strictly speaking, **Coulomb’s law as we have stated it should be used only for point charges *in vacuum*.** If matter is present in the space between the charges, the net force acting on each charge is altered because charges are induced in the molecules of the intervening material.

Example 21.3 Vector addition of electric forces on a line

Two point charges are located on the x-axis of a coordinate system: $q_1 = 1.0\ \text{nC}$ is at $x = +2.0\ \text{cm}$, and $q_2 = -3.0\ \text{nC}$ is at $x = +4.0\ \text{cm}$. What is the total electric force exerted by q_1 and q_2 on a charge $q_3 = 5.0\ \text{nC}$ at $x = 0$?





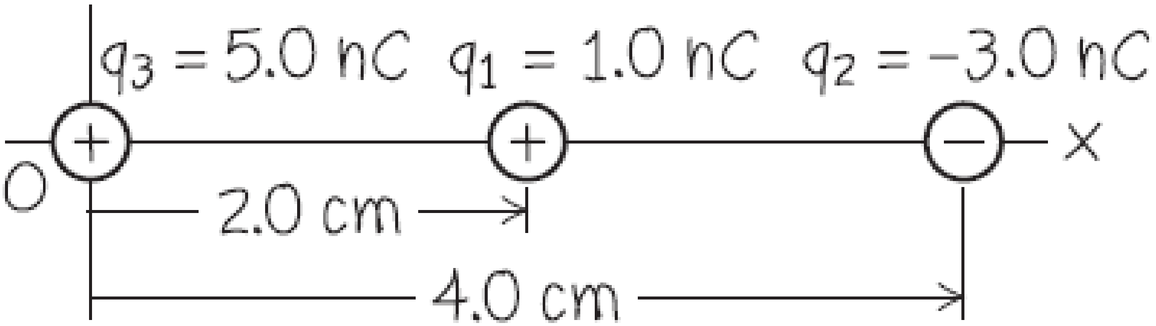
Again, Coulomb's Law:

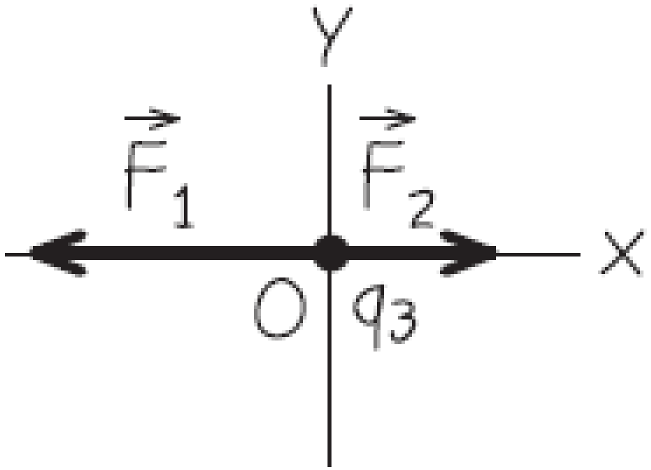
$$F_{1\ \text{on}\ 3} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.0 \times 10^{-9}\ \text{C})(5.0 \times 10^{-9}\ \text{C})}{(0.020\ \text{m})^2}$$

$$= 1.12 \times 10^{-4}\ \text{N} = 112\ \mu\text{N}$$

Example 21.3 Vector addition of electric forces on a line

Two point charges are located on the x-axis of a coordinate system: $q_1 = 1.0\ \text{nC}$ is at $x = +2.0\ \text{cm}$, and $q_2 = -3.0\ \text{nC}$ is at $x = +4.0\ \text{cm}$. What is the total electric force exerted by q_1 and q_2 on a charge $q_3 = 5.0\ \text{nC}$ at $x = 0$?



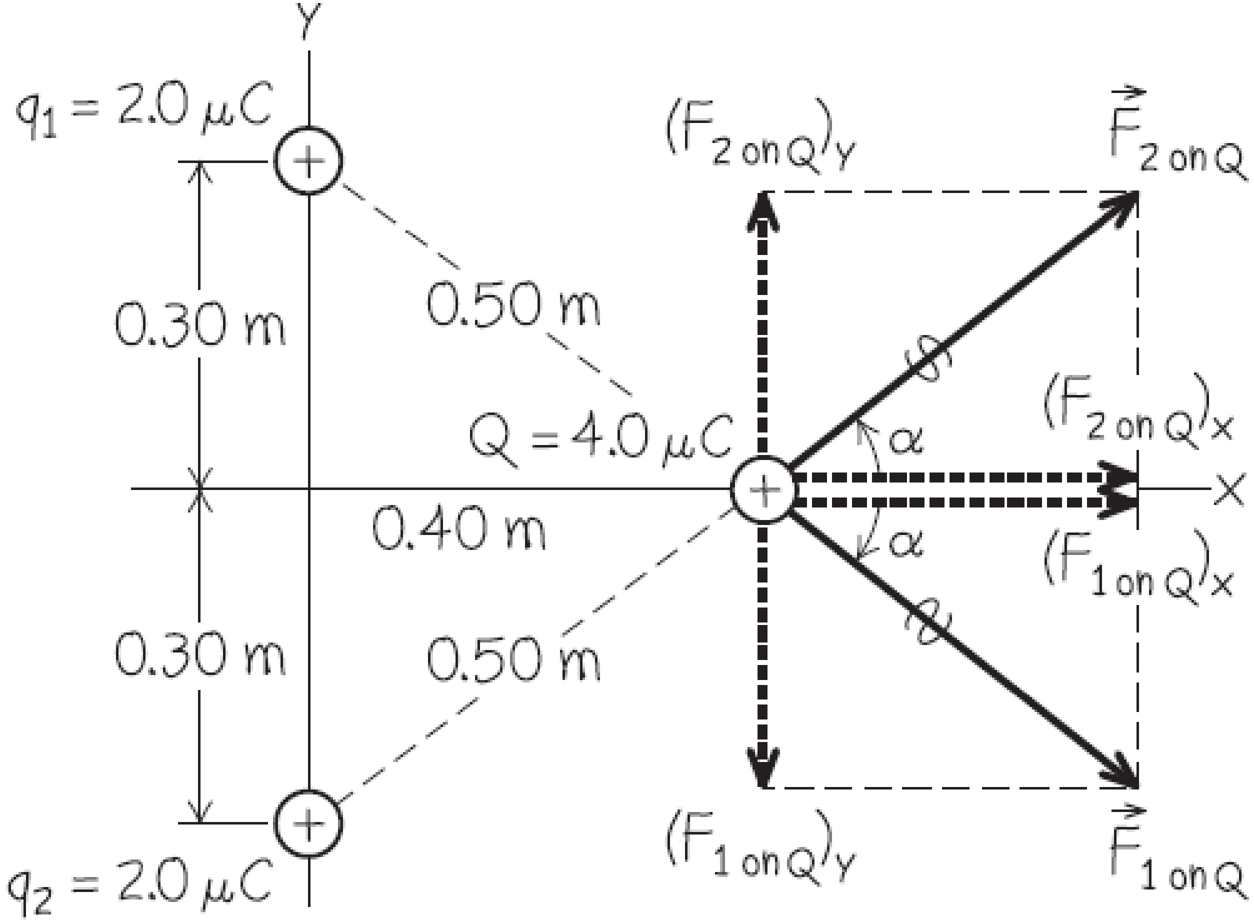


In the same way you can show that $F_{2\text{ on }3} = 84\ \mu\text{N}$.
$\vec{F}_{1\text{ on }3} = (-112\ \mu\text{N})\hat{\imath}$ and $\vec{F}_{2\text{ on }3} = (84\ \mu\text{N})\hat{\imath}$.

$$\vec{F}_3 = \vec{F}_{1\text{ on }3} + \vec{F}_{2\text{ on }3} = (-112\ \mu\text{N})\hat{\imath} + (84\ \mu\text{N})\hat{\imath} = (-28\ \mu\text{N})\hat{\imath}$$

Example 21.4 Vector addition of electric forces in a plane

Two equal positive charges $q_1 = q_2 = 2.0\ \mu C$ are located at $x = 0$, $y = 0.30$ m and $x = 0$, $y = -0.30$ m, respectively. What are the magnitude and direction of the total electric force that q_1 and q_2 exert on a third charge $Q = 4.0\ \mu C$ at $x = 0.40$ m, $y = 0$?



Electric Field: Introduction

The electric force is a field force.

Field forces can act through space.

- The effect is produced even with no physical contact between objects.

Faraday developed the concept of a field in terms of electric fields.

An **electric field** is said to exist in the region of space around a charged object.

- This charged object is the **source charge.**

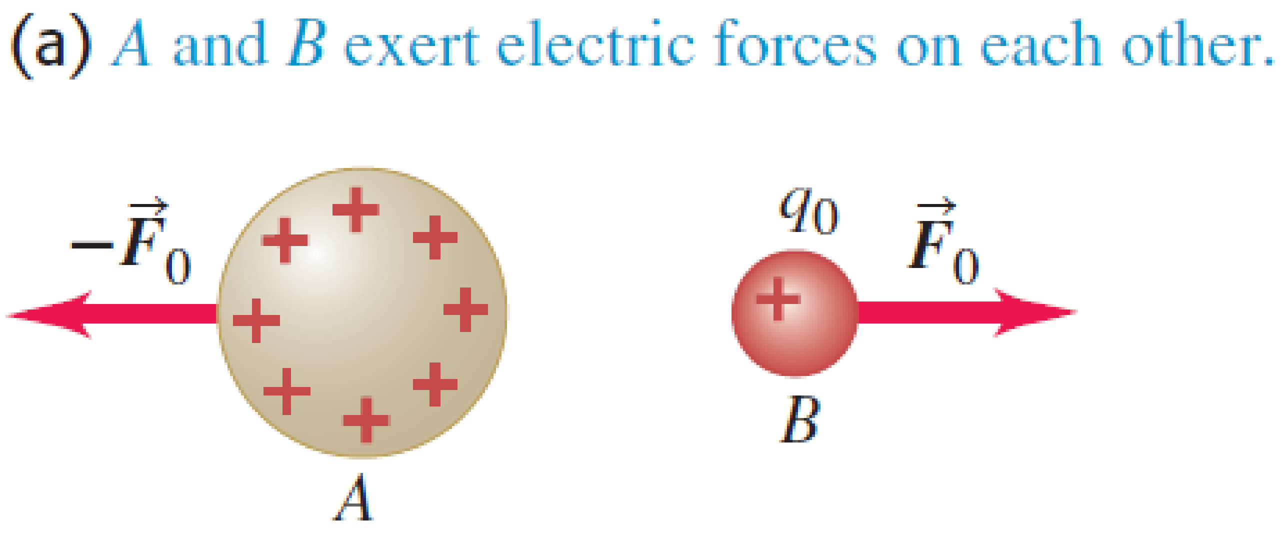
When another charged object, the **test charge**, enters this electric field, an electric force acts on it.

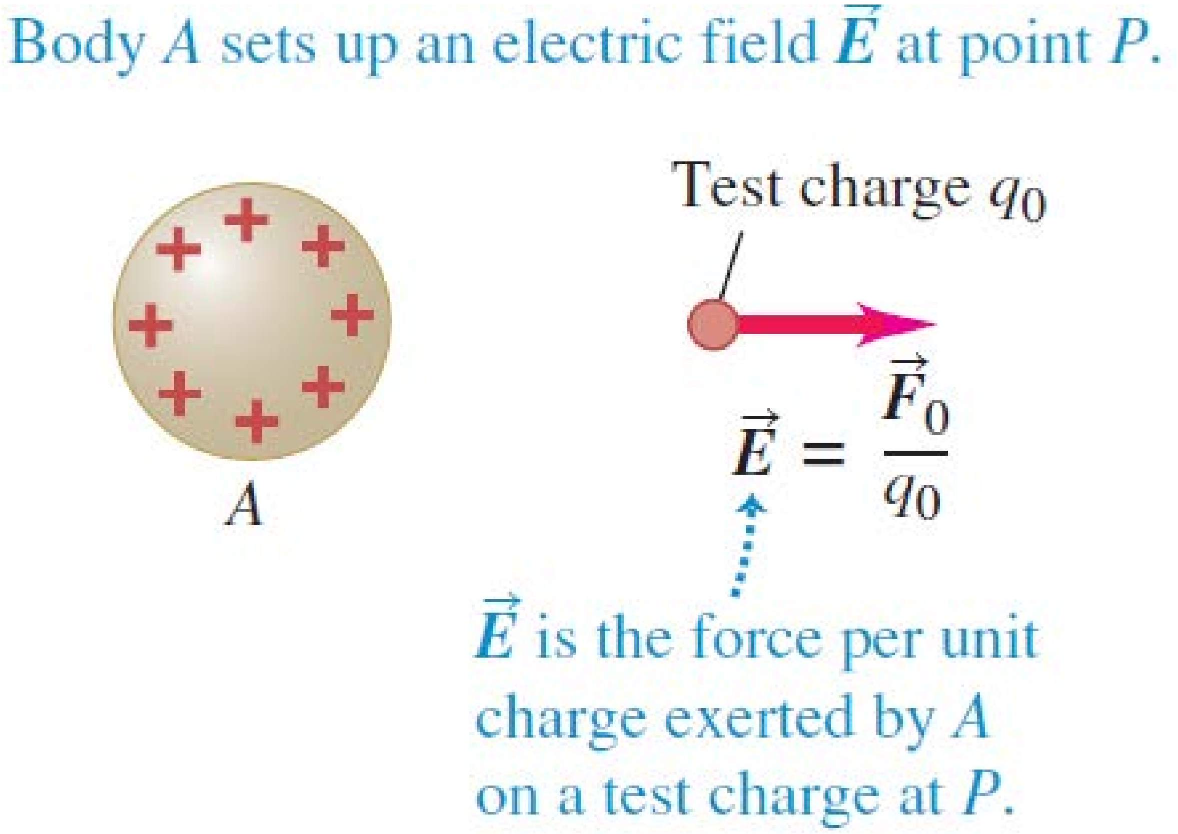
The electric field is defined as the electric force on the test charge per unit charge.

The electric field vector, $\vec{\mathbf{E}}$, at a point in space is defined as the electric force acting on a positive test charge, q_o, placed at that point divided by the test charge:

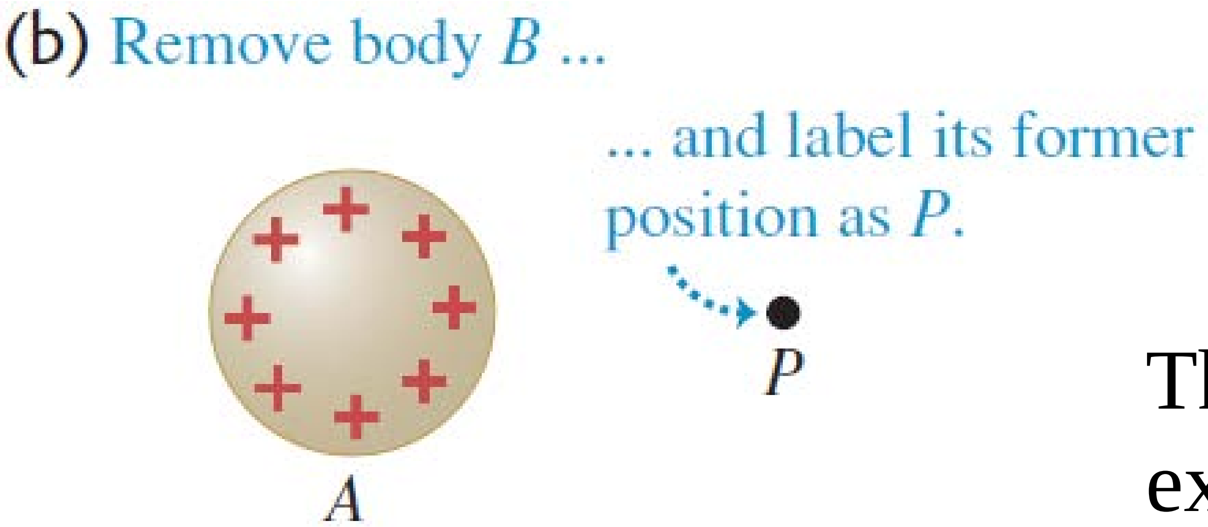
$$\vec{\mathbf{E}} \equiv \frac{\vec{\mathbf{F}}}{q_o}$$

Electric Field: Introduction


(a) A and B exert electric forces on each other.


Body A sets up an electric field $\vec{E}$ at point P.

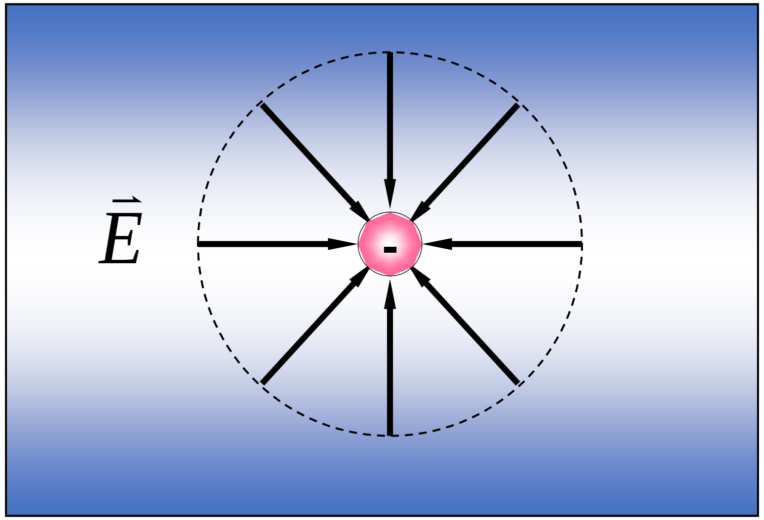
$\vec{E}$ is the force per unit charge exerted by A on a test charge at P.

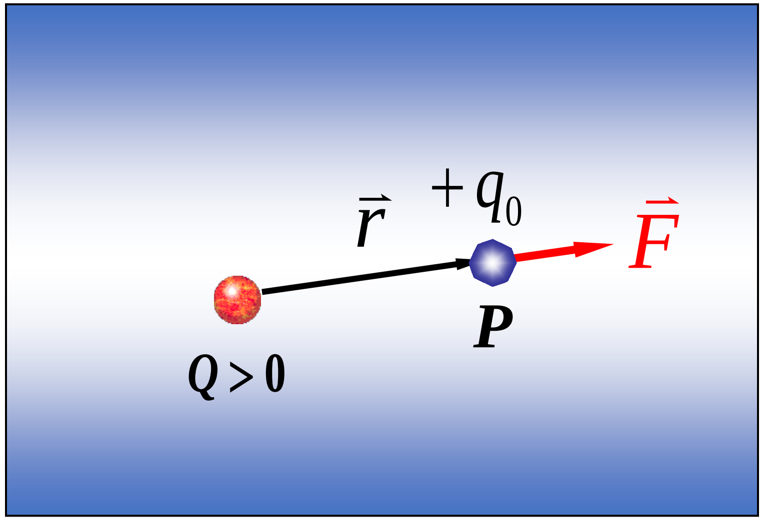

(b) Remove body B ...

The electric force on a charged body is exerted by the electric field created by *other* charged bodies.

Example: Electric Field from a Point Charge

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{r^2}\vec{e}_r \qquad \vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\vec{e}_r$$





This is the source charge Q
But how do we measure E?

Test charge q_0,
$q_0 << Q$

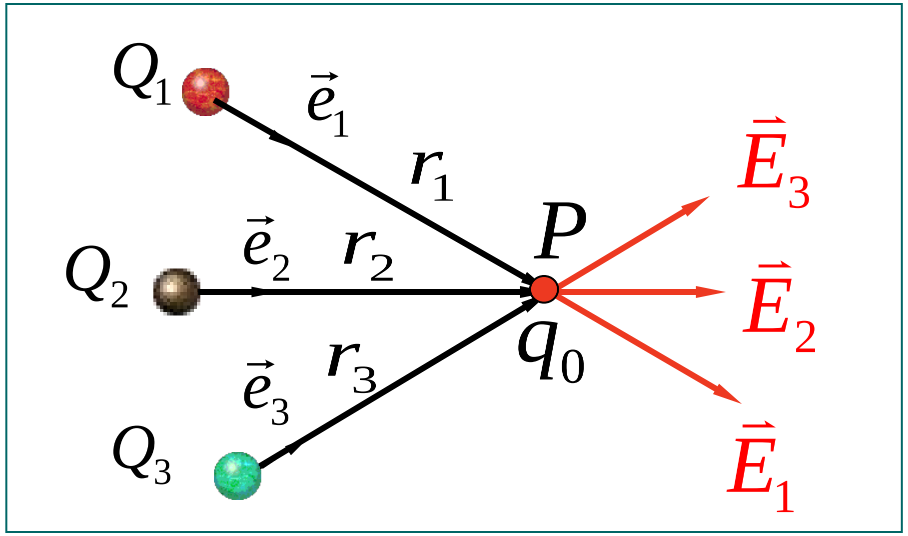
Superposition Principles of Electrical Forces

$$\vec{F}_i = \frac{1}{4\pi\varepsilon_0}\frac{q_0 Q_i}{r_i^2}\vec{e}_i$$

$$\vec{F} = \sum_i \vec{F}_i$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \sum_i \frac{\vec{F}_i}{q_0}$$

$$\boxed{\vec{E} = \sum_i \vec{E}_i = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{Q_i}{r_i^2}\vec{e}_i}$$



Electric Field from Arbitrarily-Shaped Volumes

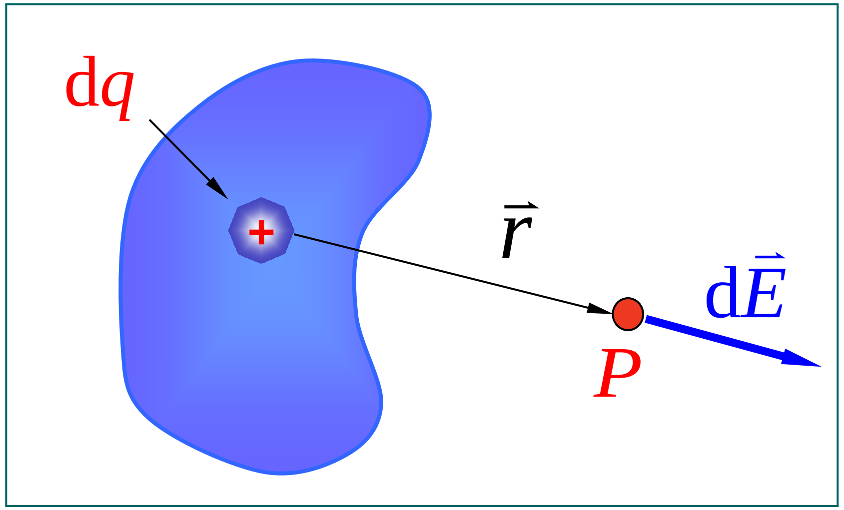
$$\mathrm{d}\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{d}q}{r^2}\vec{e}_r \qquad \vec{E} = \int \mathrm{d}\vec{E} = \int \frac{1}{4\pi\varepsilon_0}\frac{\vec{e}_r}{r^2}\mathrm{d}q$$

Volume charge density ρ

$$\mathrm{d}q = \rho \mathrm{d}V$$

- $\rho \equiv Q / V$ with units C/m^3

$$\boxed{\vec{E} = \int_V \frac{1}{4\pi\varepsilon_0}\frac{\rho\vec{e}_r}{r^2}\mathrm{d}V}$$



Electric Field from Arbitrarily-Shaped Surfaces

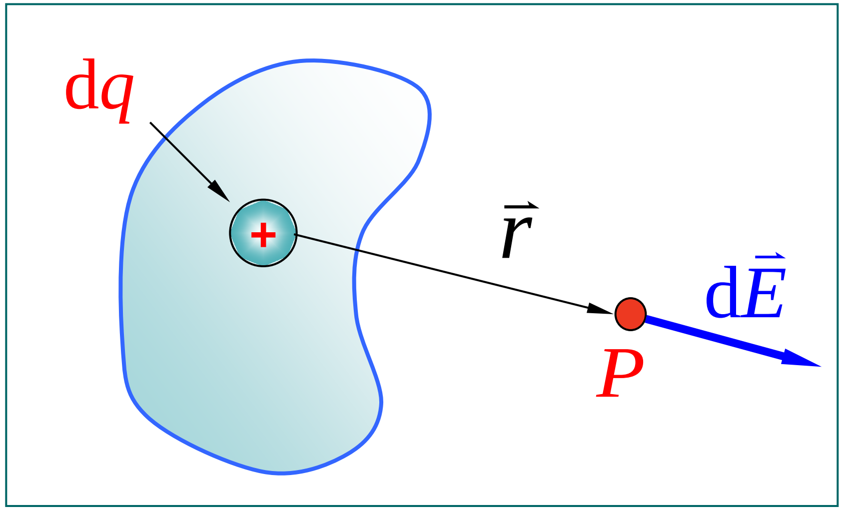
$$\mathrm{d}\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{d}q}{r^2}\vec{e}_r \qquad \vec{E} = \int \mathrm{d}\vec{E} = \int \frac{1}{4\pi\varepsilon_0}\frac{\vec{e}_r}{r^2}\mathrm{d}q$$

Surface charge density σ

$$\mathrm{d}q = \sigma \mathrm{d}S$$

- $\sigma \equiv Q / A$ with units C/m²

$$\boxed{\vec{E} = \int_S \frac{1}{4\pi\varepsilon_0}\frac{\sigma\vec{e}_r}{r^2}\mathrm{d}S}$$



Electric Field from Arbitrarily-Shaped Lines

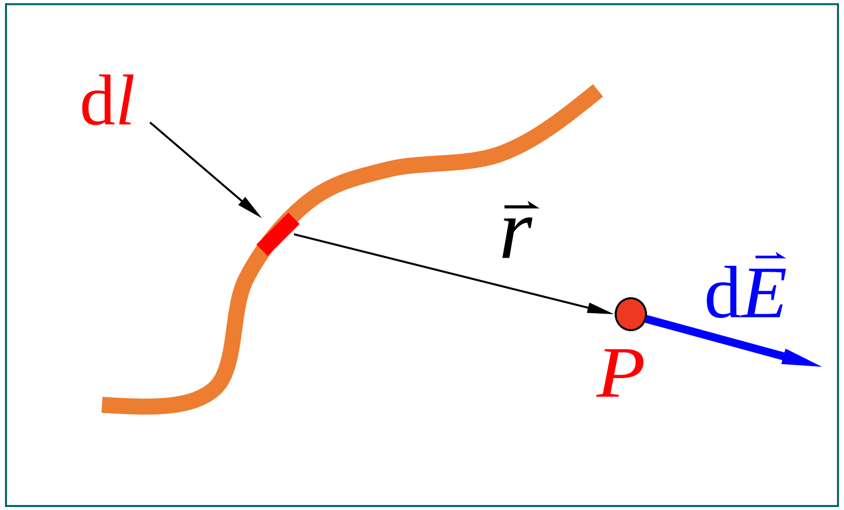
$$\mathrm{d}\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{d}q}{r^2}\vec{e}_r \qquad \vec{E} = \int \mathrm{d}\vec{E} = \int \frac{1}{4\pi\varepsilon_0}\frac{\vec{e}_r}{r^2}\mathrm{d}q$$

Linear charge density λ

$$\mathrm{d}q = \lambda \mathrm{d}l$$

- $\lambda \equiv Q / \ell$ with units C/m

$$\boxed{\vec{E} = \int_l \frac{1}{4\pi\varepsilon_0}\frac{\lambda\vec{e}_r}{r^2}\mathrm{d}l}$$

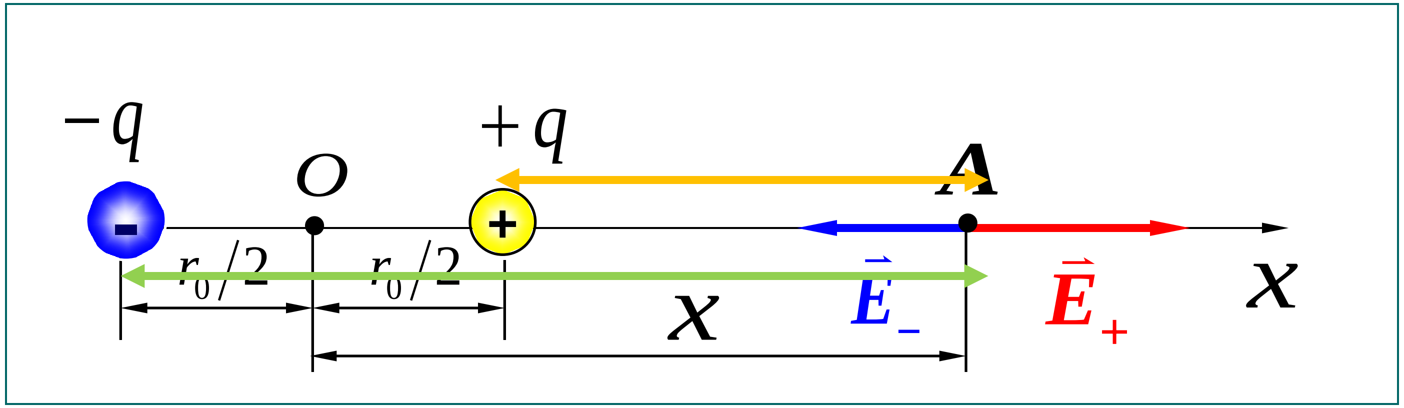


Electrical Fields from Dipoles: A Special Case

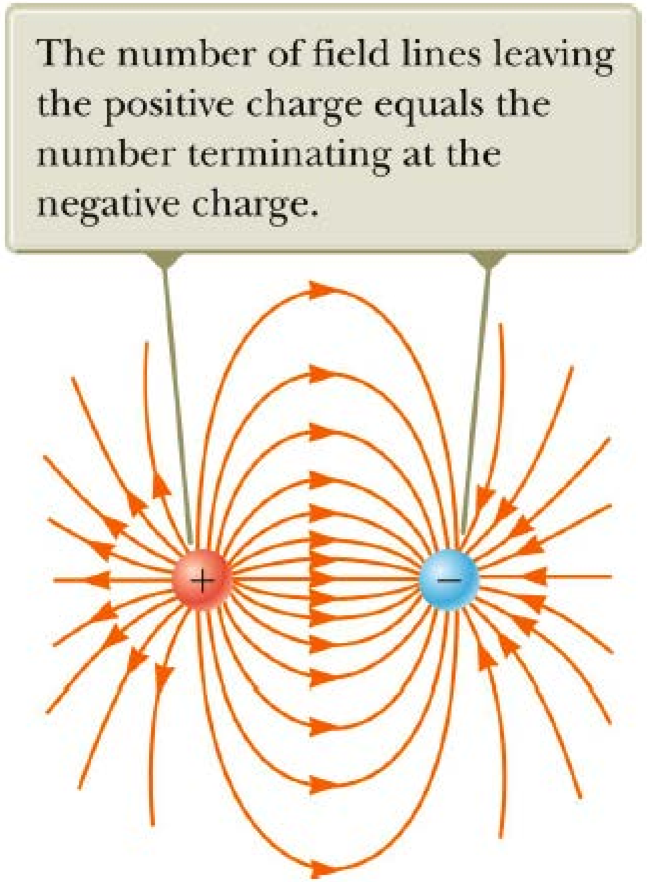
Problem statement: **Calculate electric field along the dipole axis**

$$\vec{E}_{+} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\boxed{(x - r_0/2)}^2}\vec{i} \qquad \vec{E}_{-} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{\boxed{(x + r_0/2)}^2}\vec{i}$$

$$\vec{E} = \vec{E}_{+} + \vec{E}_{-} = \frac{q}{4\,\pi\varepsilon_0}\left[\frac{2xr_0}{(x^2 - r_0^2/4)^2}\right]\vec{i}$$

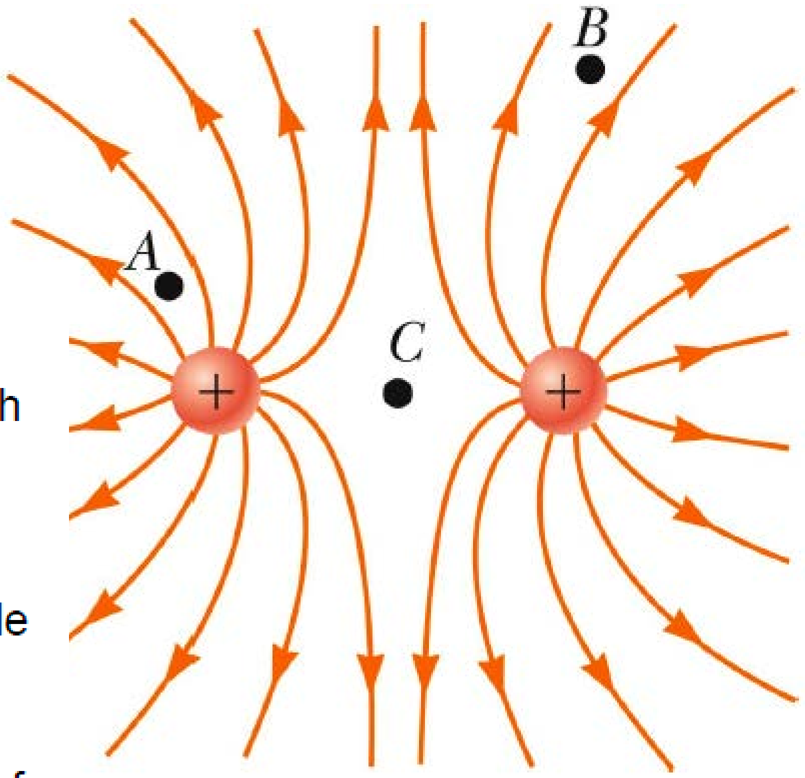


Electrical Fields from Dipole and Like Charges


The number of field lines leaving the positive charge equals the number terminating at the negative charge.

The charges are equal and opposite.

The number of field lines leaving the positive charge equals the number of lines terminating on the negative charge.



The charges are equal and positive.

The same number of lines leave each charge since they are equal in magnitude.

At a great distance, the field is approximately equal to that of a single charge of $2q$.

Since there are no negative charges available, the field lines end infinitely far away.

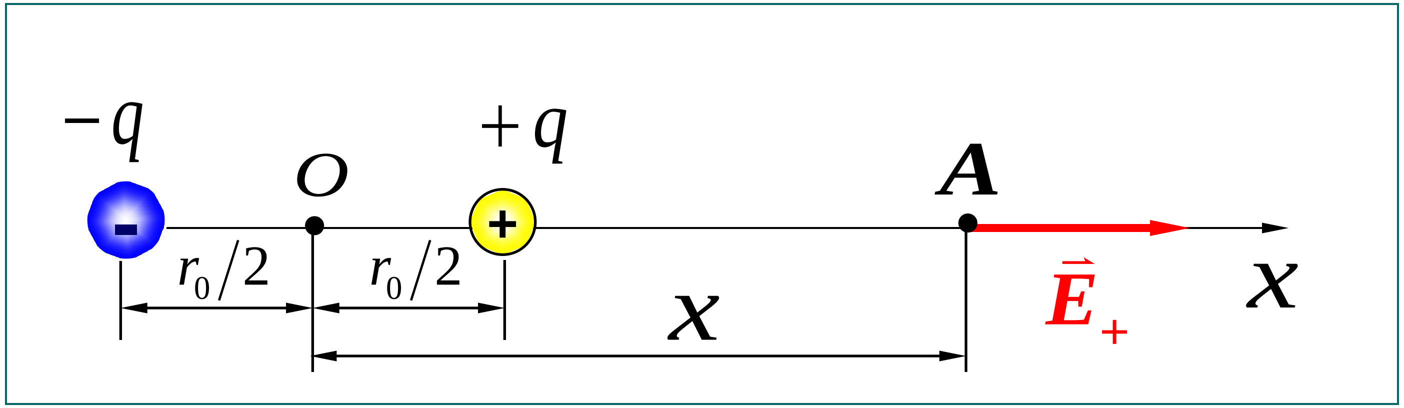
Quick question: what if the charges are reversed?

Electrical Fields from Dipoles: A Special Case

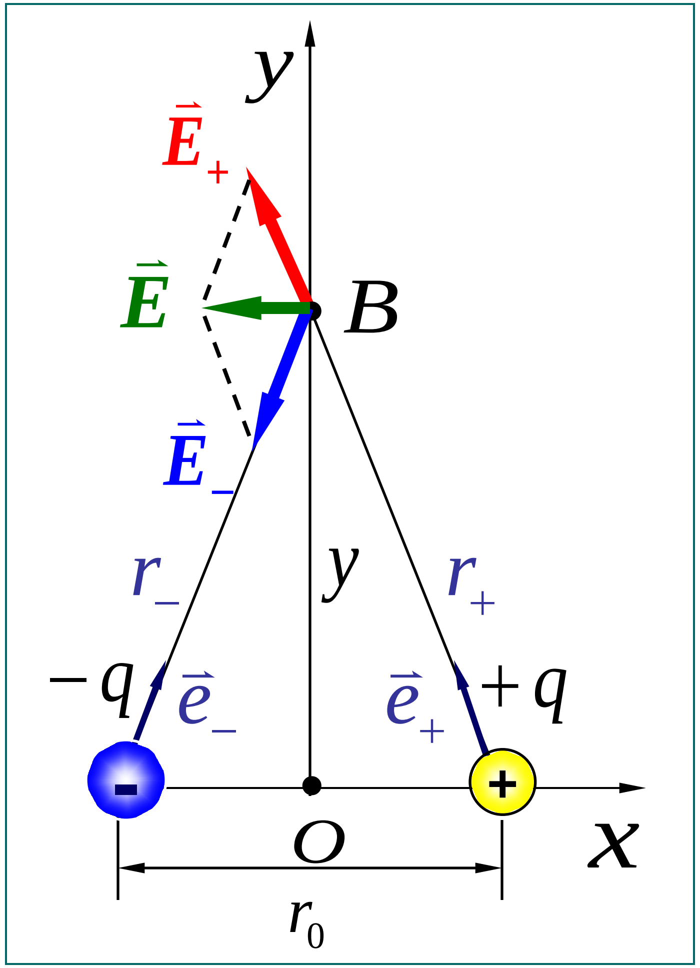
$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{2xr_0}{(x^2 - r_0^2/4)^2}\right]\vec{i}$$

$$x >> r_0$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2r_0q}{x^3}\vec{i} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{x^3}$$



Electrical Fields from Dipoles: Special Case II



$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+^2}\vec{e}_+$$

$$\vec{E}_- = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r_-^2}\vec{e}_-$$

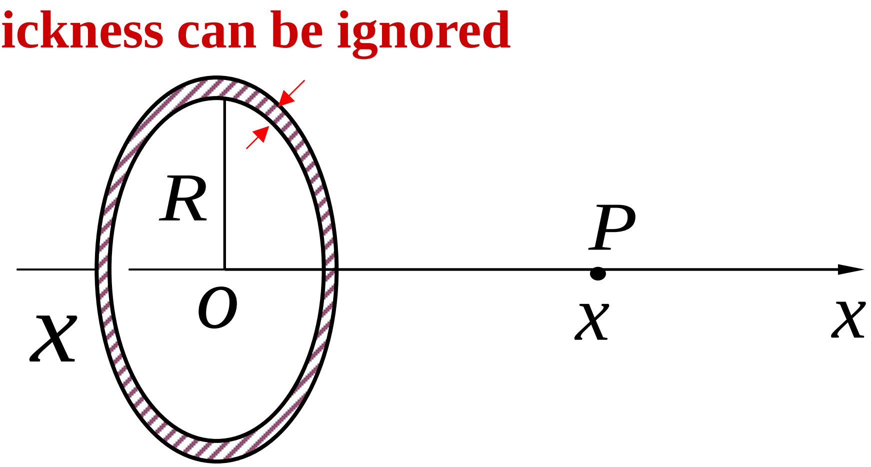
$$r_+ = r_- = r = \sqrt{y^2 + (\frac{r_0}{2})^2}$$

$$\vec{E} = \vec{E}_+ + \vec{E}_- = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}$$

$$y >> r_0 \quad \vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{y^3}$$

Electrical Fields from a Ring

Problem statement: **Calculate electric field along the axis of a ring with a total charge of q, uniformly distributed**



Electrical Fields from a Ring

Solution: $\lambda = \frac{q}{2\pi R}$ $\quad$ $dq = \lambda dl$ $\quad$ $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{r^2}$

$$d\vec{E} = d\vec{E}_x + d\vec{E}_\perp$$

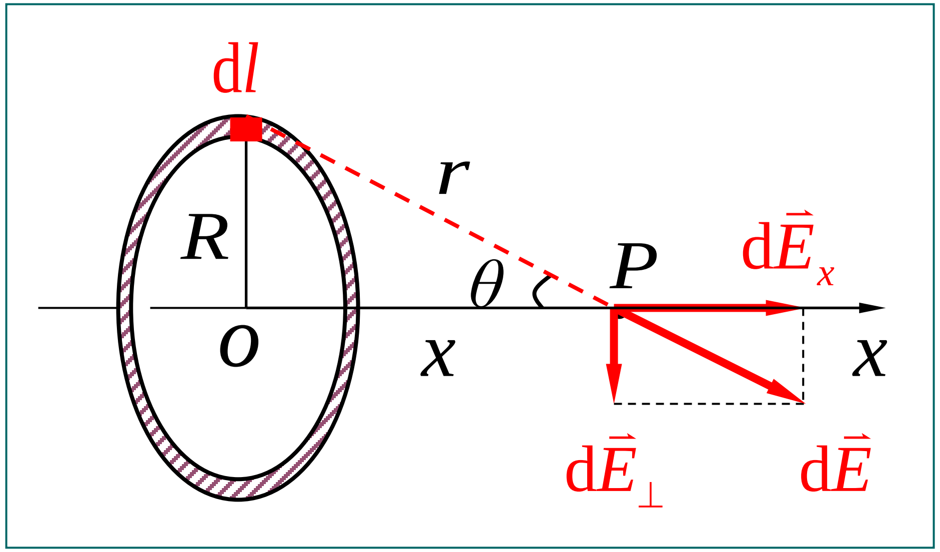
Since $E_\perp = \int_l dE_\perp = 0$

so $E = \int_l dE_x = \int_l dE\cos\theta$

$$= \int \frac{\lambda dl}{4\pi\varepsilon_0 r^2}\cdot\frac{x}{r}$$

$$= \frac{\lambda x}{4\pi\varepsilon_0 r^3}\int_0^{2\pi R} dl$$

$$= \frac{qx}{4\pi\varepsilon_0 (x^2+R^2)^{3/2}}$$



Electrical Fields from a Ring

Observations

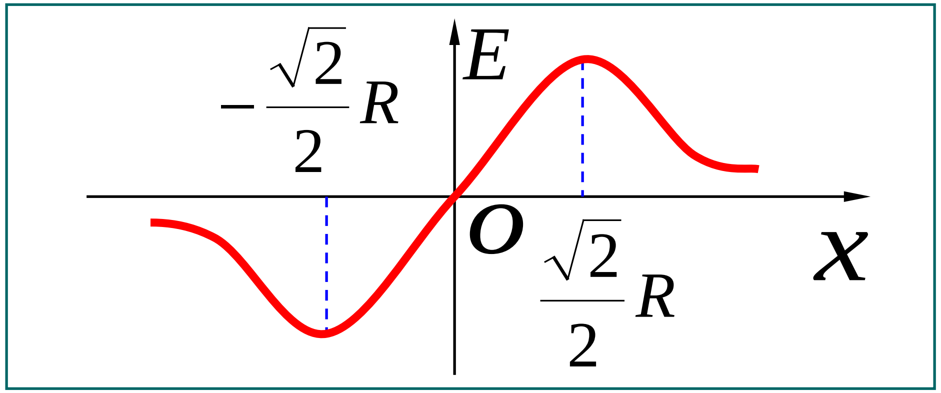
(1) $x >> R$

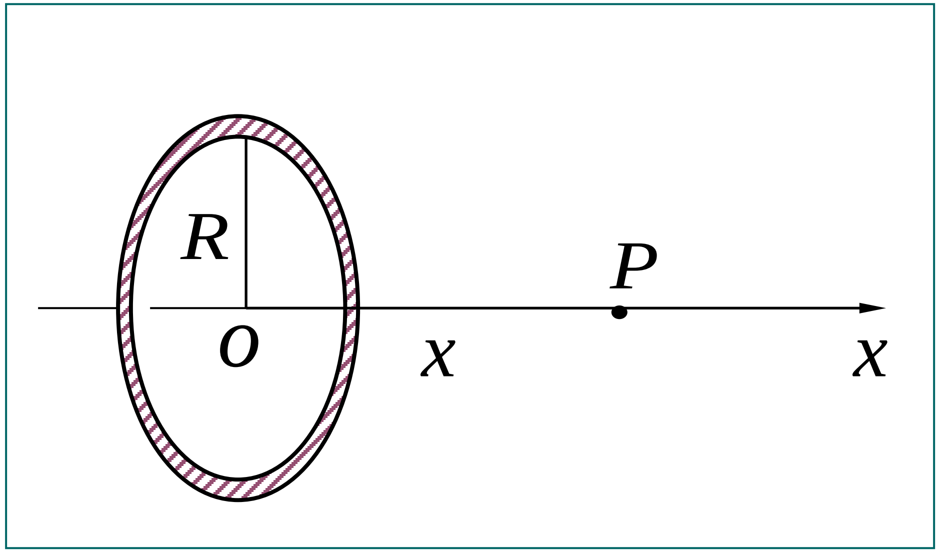
(2) $x = 0$

$E_o = 0$

(3) $\frac{\mathrm{d}E}{\mathrm{d}x} = 0$

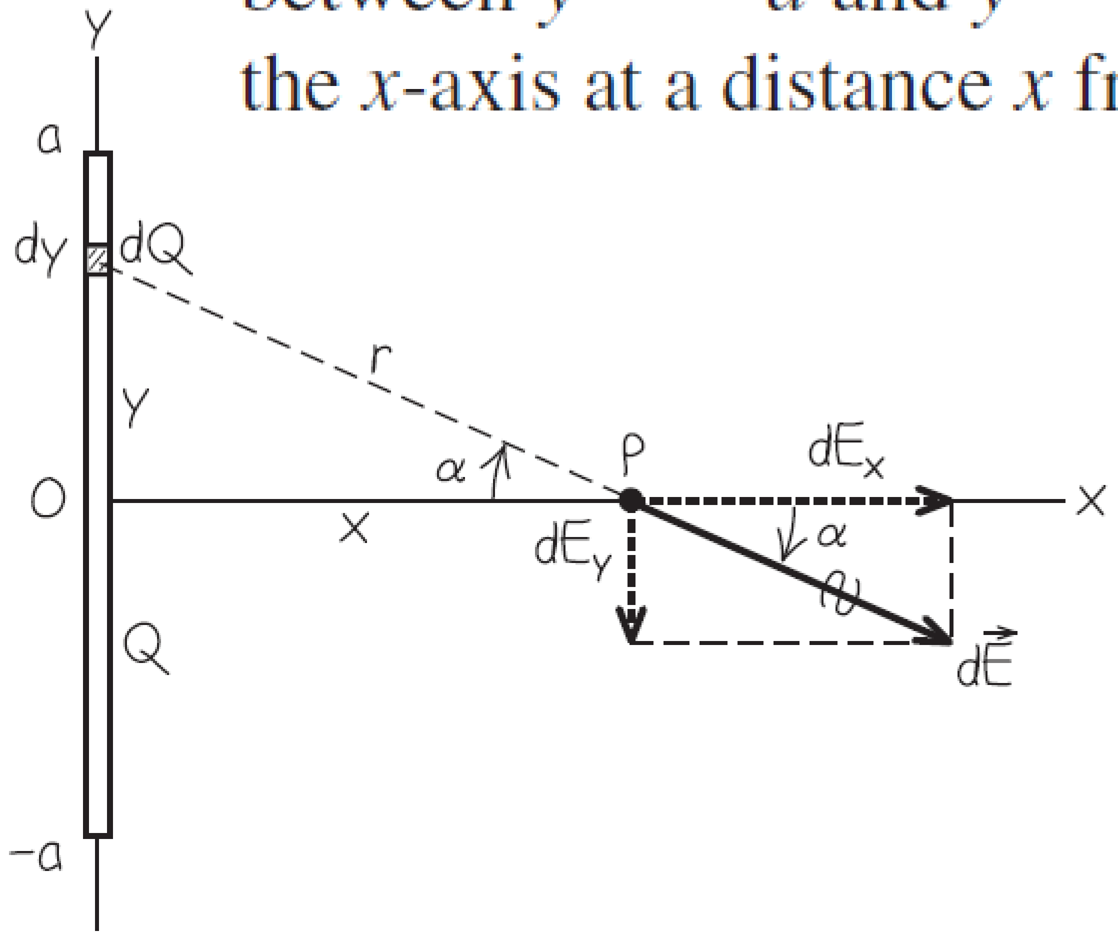
$x = \pm \frac{\sqrt{2}}{2} R$



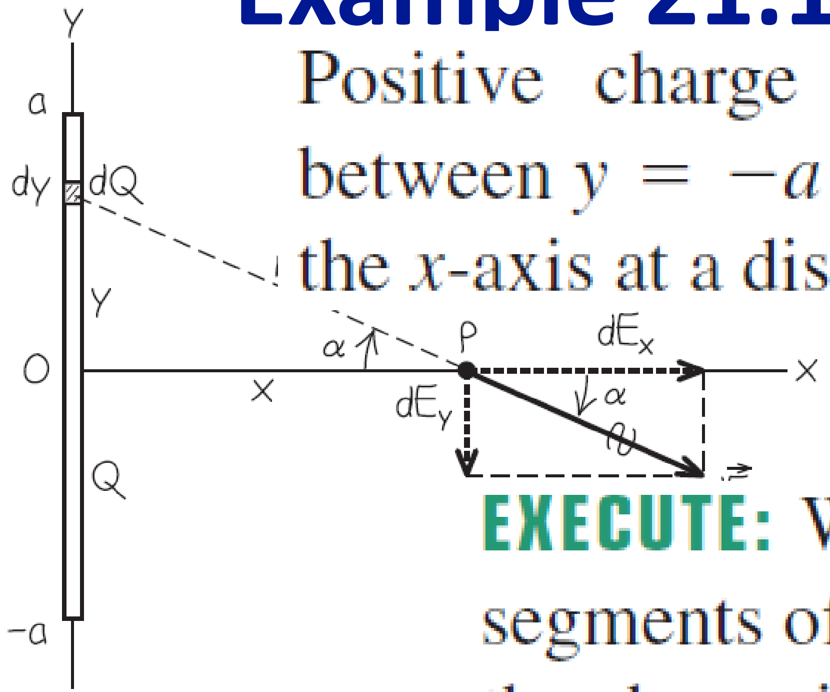


Example 21.10 Field of a charged line segment

Positive charge Q is distributed uniformly along the y-axis between $y = -a$ and $y = +a$. Find the electric field at point P on the x-axis at a distance x from the origin.



Example 21.10 Field of a charged line segment



Positive charge Q is distributed uniformly along the y-axis between $y = -a$ and $y = +a$. Find the electric field at point P on the x-axis at a distance x from the origin.

EXECUTE: We divide the line charge of length $2a$ into infinitesimal segments of length dy. The linear charge density is $\lambda = Q/2a$, and the charge in a segment is $dQ = \lambda\, dy = (Q/2a)dy$. The distance r from a segment at height y to the field point P is $r = (x^2 + y^2)^{1/2}$, so the magnitude of the field at P due to the segment at height y is

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{dy}{(x^2 + y^2)}$$

Example 21.10 Field of a charged line segment

Positive charge Q is distributed uniformly along the y-axis between $y = -a$ and $y = +a$. Find the electric field at point P on the x-axis at a distance x from the origin.

Figure 21.24 shows that the x- and y-components of this field are $dE_x = dE \cos \alpha$ and $dE_y = -dE \sin \alpha$, where $\cos \alpha = x/r$ and $\sin \alpha = y/r$. Hence

$$dE_x = \frac{1}{4\pi\epsilon_0} \frac{Q}{2a} \frac{x\,dy}{(x^2 + y^2)^{3/2}}$$

$$dE_y = \frac{1}{4\pi\epsilon_0} \frac{Q}{2a} \frac{y\,dy}{(x^2 + y^2)^{3/2}}$$

Example 21.10 Field of a charged line segment

Positive charge Q is distributed uniformly along the y-axis between $y = -a$ and $y = +a$. Find the electric field at point P on the x-axis at a distance x from the origin.

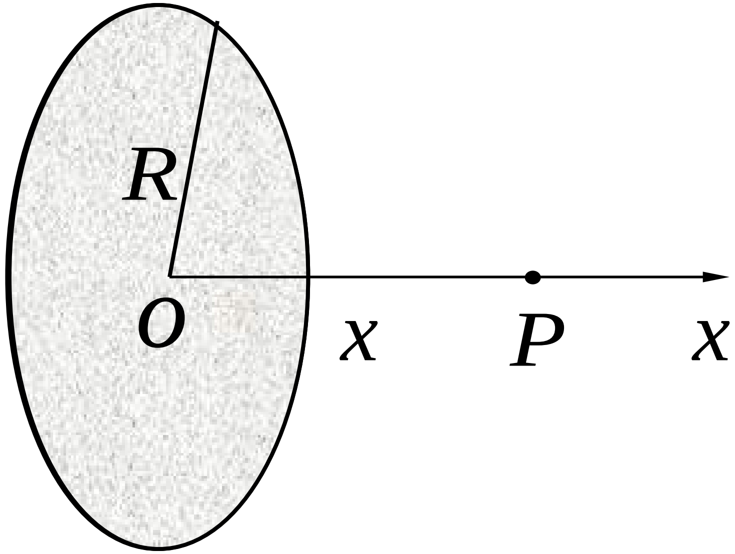
To find the total field at P, we must sum the fields from all segments along the line—that is, we must integrate from $y = -a$ to $y = +a$. You should work out the details of the integration (a table of integrals will help). The results are

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{+a}\frac{x\,dy}{(x^2 + y^2)^{3/2}} = \frac{Q}{4\pi\epsilon_0}\frac{1}{x\sqrt{x^2 + a^2}}$$

$$E_y = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{+a}\frac{y\,dy}{(x^2 + y^2)^{3/2}} = 0$$

Example 21.11 Field of a uniformly charged disk

A nonconducting disk of radius R has a uniform positive surface charge density σ. Find the electric field at a point along the axis of the disk a distance x from its center. Assume that x is positive.



Charge density is uniformly (σ),
what is the total ***E*** along *x*

Electrical Fields from a Disk

Recall for a ring of total charge q:

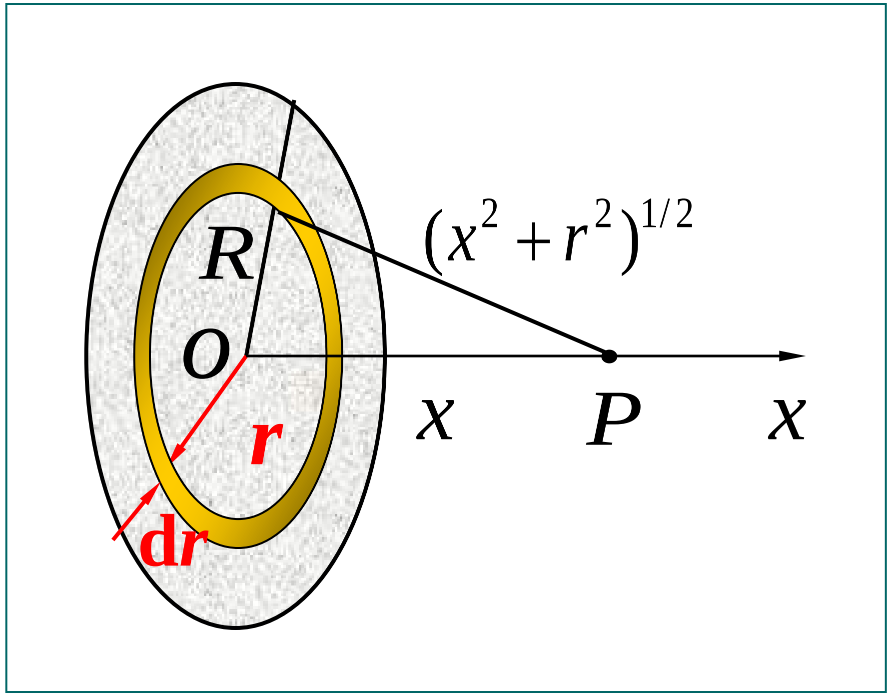
$$\mathbf{E} = \frac{qx}{4\pi\varepsilon_0(x^2+R^2)^{3/2}}$$

Solution: $\sigma = q/\pi R^2 \qquad \mathrm{d}q = \sigma 2\pi r\mathrm{d}r$

$$\mathrm{d}E_x = \frac{x\mathrm{d}q}{4\pi\varepsilon_0(x^2+r^2)^{3/2}} = \frac{\sigma}{2\varepsilon_0}\frac{xr\mathrm{d}r}{(x^2+r^2)^{3/2}}$$

$$E = \int \mathrm{d}E_x$$

$$= \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{\sqrt{x^2}} - \frac{1}{\sqrt{x^2+R^2}}\right)$$



Electrical Fields from a Disk

Observations

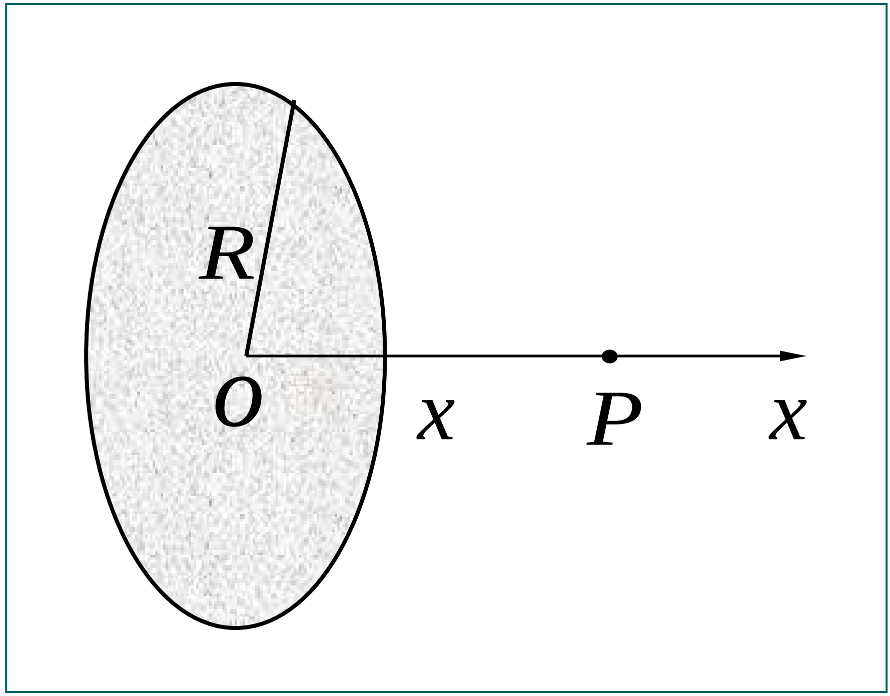
$$E = \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{\sqrt{x^2}} - \frac{1}{\sqrt{x^2+R^2}}\right)$$

$$x << R$$

$$E \approx \frac{\sigma}{2\varepsilon_0}$$

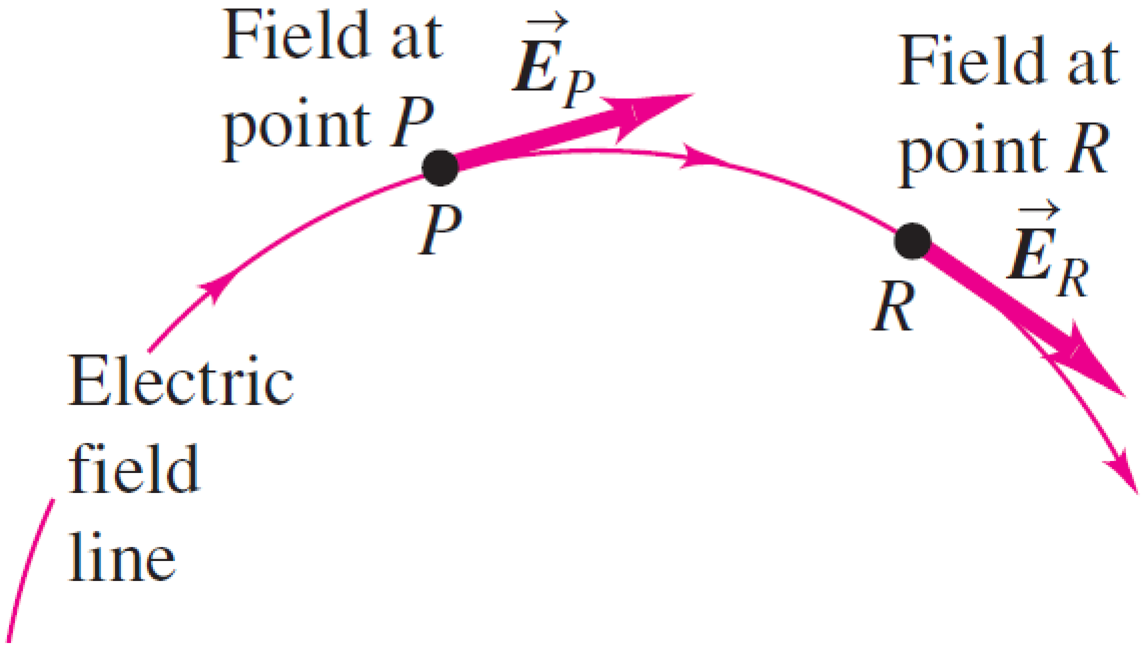
$$x >> R$$

$$E \approx \frac{q}{4\pi\varepsilon_0 x^2}$$



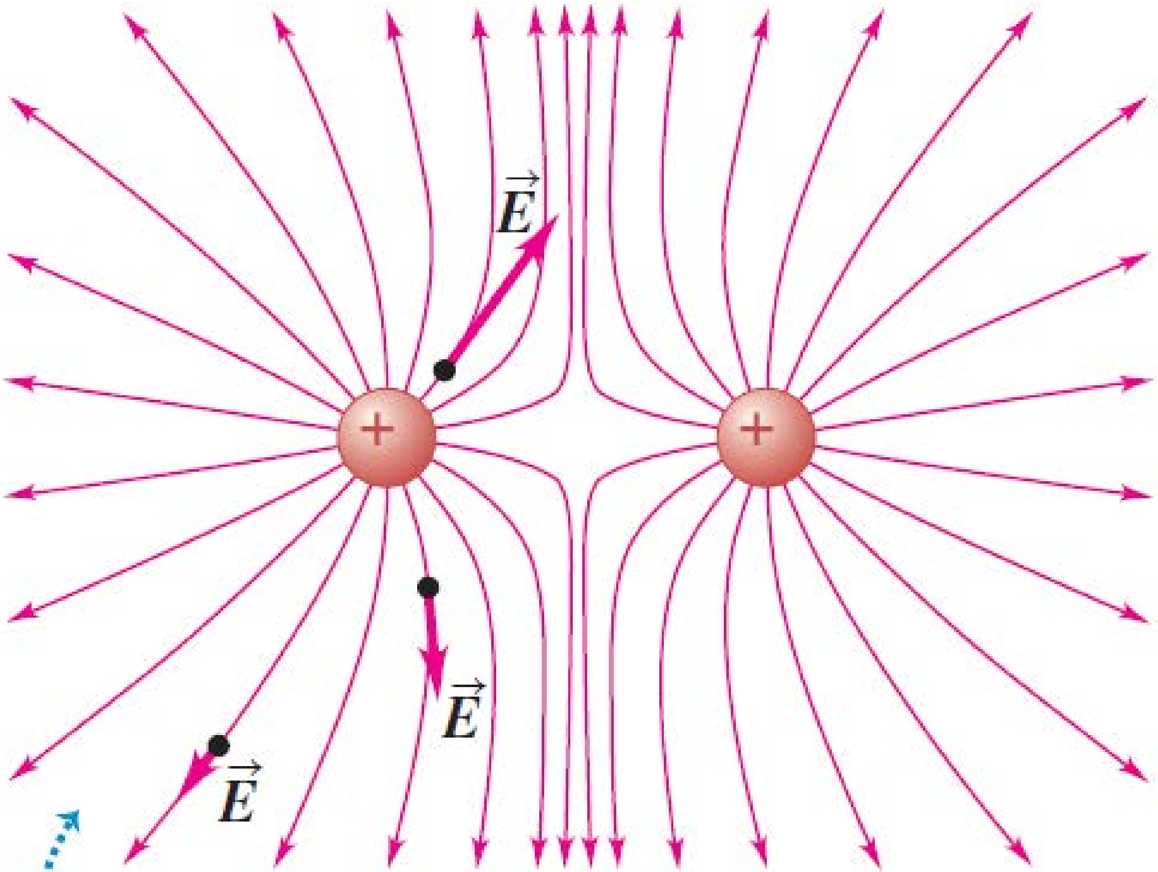
Electric Field Lines

21.27 The direction of the electric field at any point is tangent to the field line through that point.



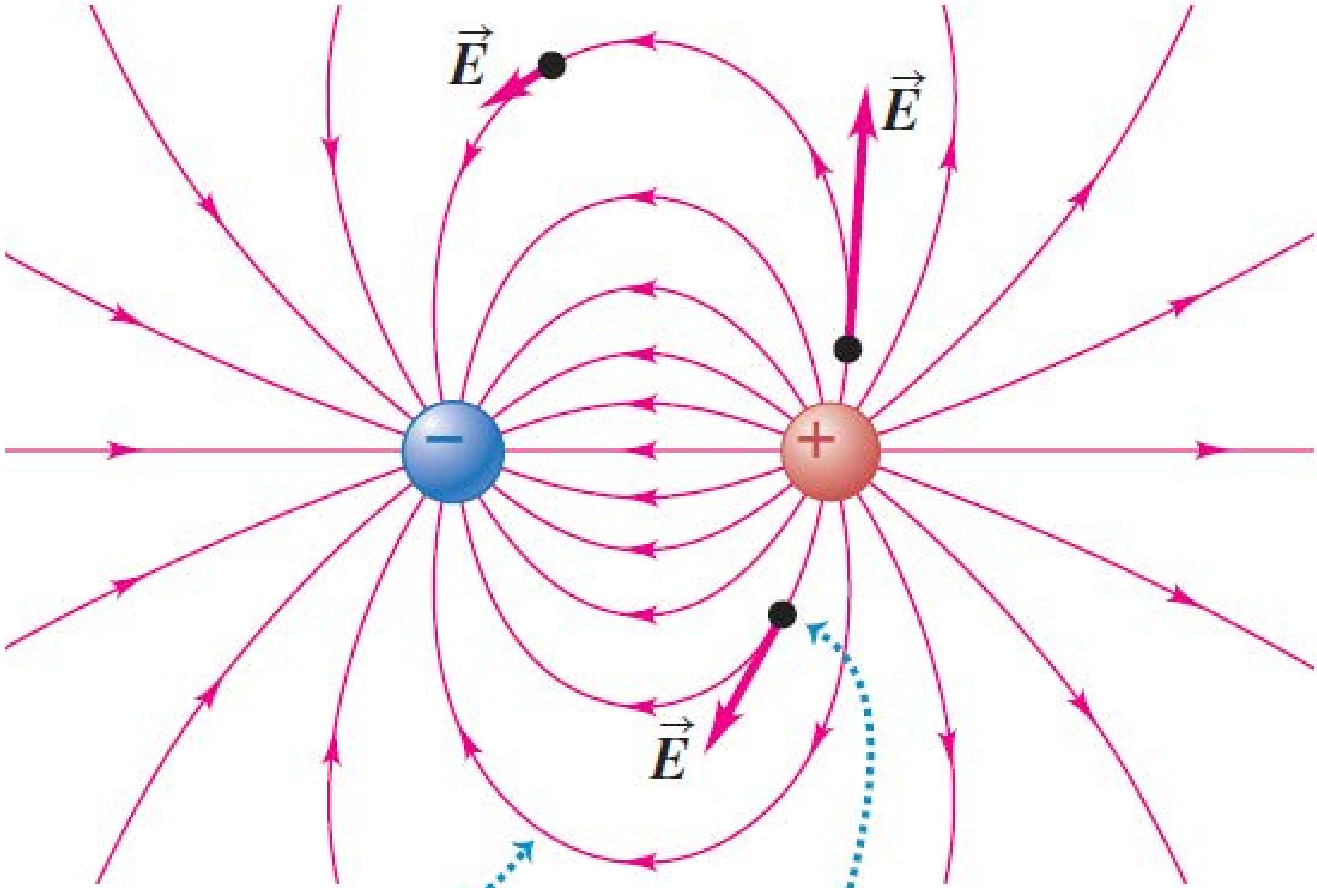
Electric Field Lines

(c) Two equal positive charges



Field lines are close together where the field is strong, farther apart where it is weaker.

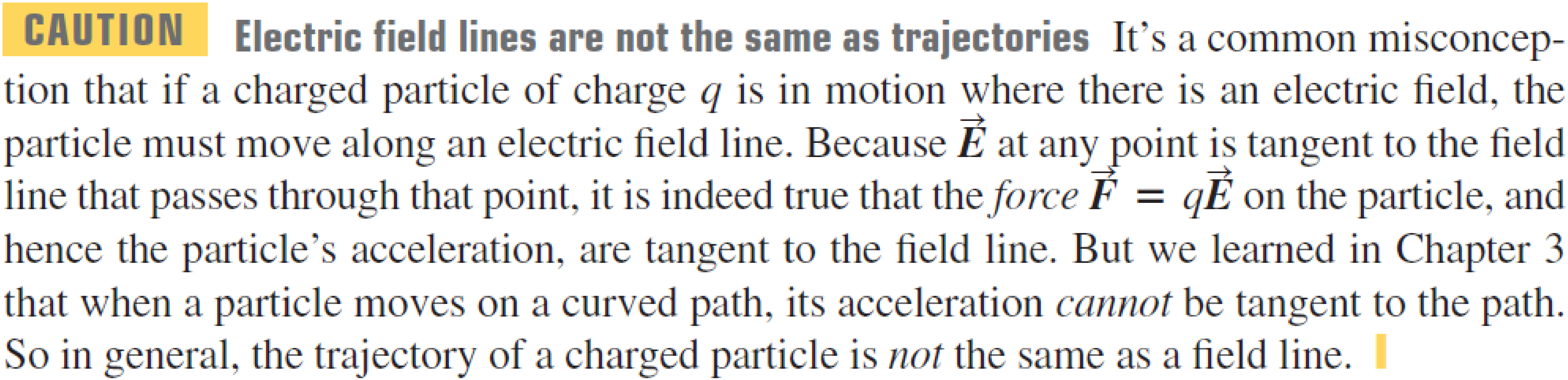
b) Two equal and opposite charges (a dipole)



ways point
) charges
and toward (−) charges.

At each point in space, the electric field vector is *tangent* to the field line passing through that point.

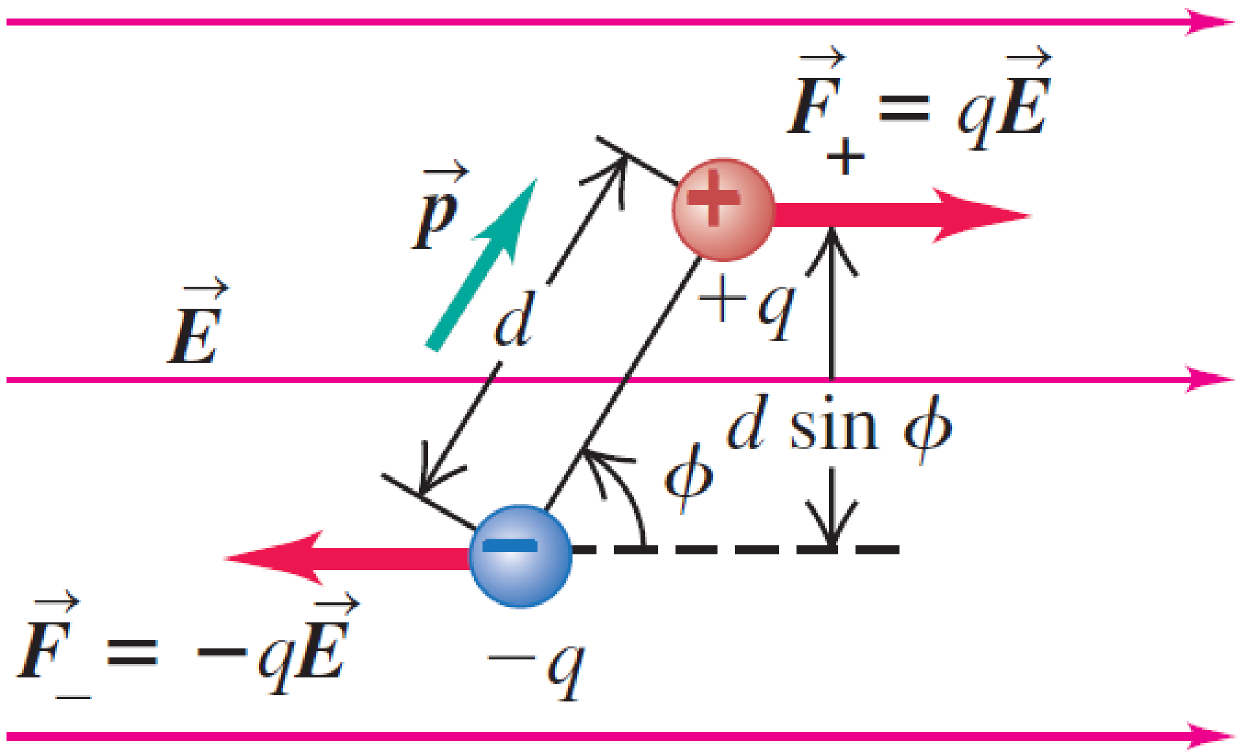
Electric Field Lines



CAUTION **Electric field lines are not the same as trajectories** It's a common misconception that if a charged particle of charge q is in motion where there is an electric field, the particle must move along an electric field line. Because $\vec{E}$ at any point is tangent to the field line that passes through that point, it is indeed true that the *force* $\vec{F} = q\vec{E}$ on the particle, and hence the particle's acceleration, are tangent to the field line. But we learned in Chapter 3 that when a particle moves on a curved path, its acceleration *cannot* be tangent to the path. So in general, the trajectory of a charged particle is *not* the same as a field line.

Electric Dipoles

An **electric dipole** is a pair of point charges with equal magnitude and opposite sign (a positive charge *q* and a negative charge -*q*) separated by a distance *d*.



The forces and on the two charges both have magnitude , but their directions are opposite, and they add to zero.

Net force on an electric dipole in a uniform external electric field is zero.

Electric Dipoles

However, the two forces don't act along the same line, so their *torques* don't add to zero. We calculate torques with respect to the center of the dipole. Let the angle between the electric field $\vec{E}$ and the dipole axis be ϕ; then the lever arm for both $\vec{F}_+$ and $\vec{F}_-$ is $(d/2)\sin\phi$. The torque of $\vec{F}_+$ and the torque of $\vec{F}_-$ both have the same magnitude of $(qE)(d/2)\sin\phi$, and both torques tend to rotate the dipole clockwise (that is, $\vec{\tau}$ is directed into the page in Fig. 21.31). Hence the magnitude of the net torque is twice the magnitude of either individual torque:

$$\tau = (qE)(d\sin\phi) \tag{21.13}$$

where $d \sin \phi$ is the perpendicular distance between the lines of action of the two forces.

Electric Dipoles

The product of the charge q and the separation d is the magnitude of a quantity called the **electric dipole moment,** denoted by p:

$$p = qd \qquad \text{(magnitude of electric dipole moment)} \qquad (21.14)$$

The units of p are charge times distance $(\text{C} \cdot \text{m})$. For example, the magnitude of the electric dipole moment of a water molecule is $p = 6.13 \times 10^{-30}\ \text{C} \cdot \text{m}$.

CAUTION **The symbol p has multiple meanings** Be careful not to confuse dipole moment with momentum or pressure. There aren't as many letters in the alphabet as there are physical quantities, so some letters are used several times. The context usually makes it clear what we mean, but be careful.

$$\tau = pE \sin\phi \qquad \text{(magnitude of the torque on an electric dipole)} \qquad (21.15)$$

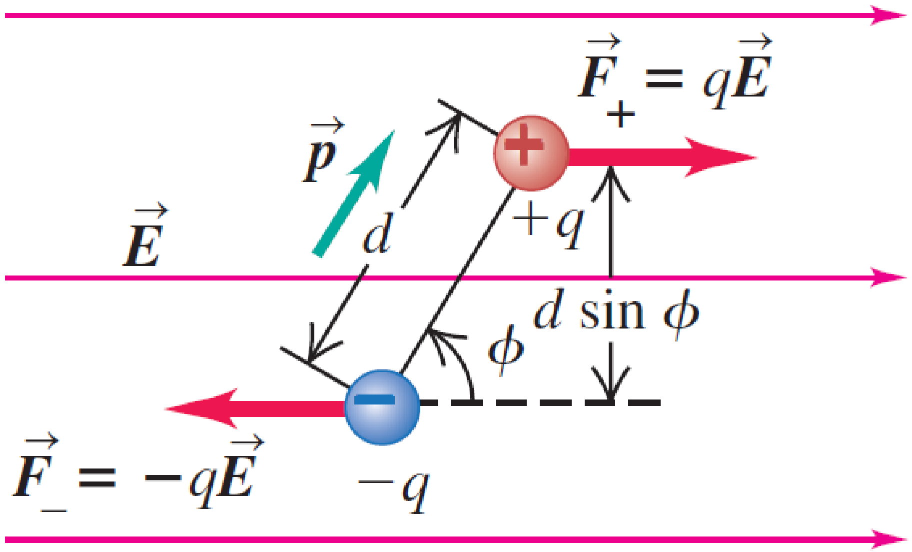
$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{p}} \times \vec{\boldsymbol{E}} \qquad \text{(torque on an electric dipole, in vector form)}$$

Potential Energy of an Electric Dipole

When a dipole changes direction in an electric field, the electric-field torque does *work* on it, with a corresponding change in potential energy. The work dW done by a torque τ during an infinitesimal displacement $d\phi$ is given by Eq. (10.19): $dW = \tau\, d\phi$. Because the torque is in the direction of decreasing ϕ, we must write the torque as $\tau = -pE \sin\phi$, and

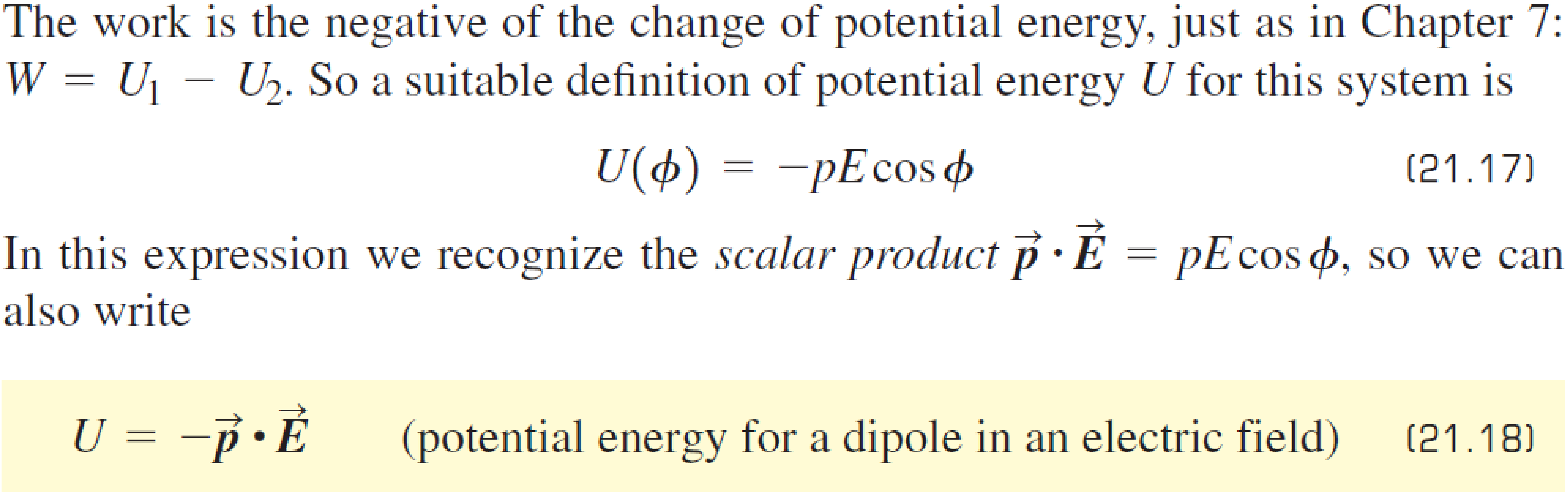
$$dW = \tau\, d\phi = -pE \sin\phi\, d\phi$$

In a finite displacement from ϕ_1 to ϕ_2 the total work



$$W = \int_{\phi_1}^{\phi_2} (-pE \sin\phi)\, d\phi$$
$$= pE \cos\phi_2 - pE \cos\phi_1$$

Potential Energy of an Electric Dipole



The work is the negative of the change of potential energy, just as in Chapter 7: $W = U_1 - U_2$. So a suitable definition of potential energy U for this system is

$$U(\phi) = -pE\cos\phi \qquad (21.17)$$

In this expression we recognize the *scalar product* $\vec{\boldsymbol{p}} \cdot \vec{\boldsymbol{E}} = pE\cos\phi$, so we can also write

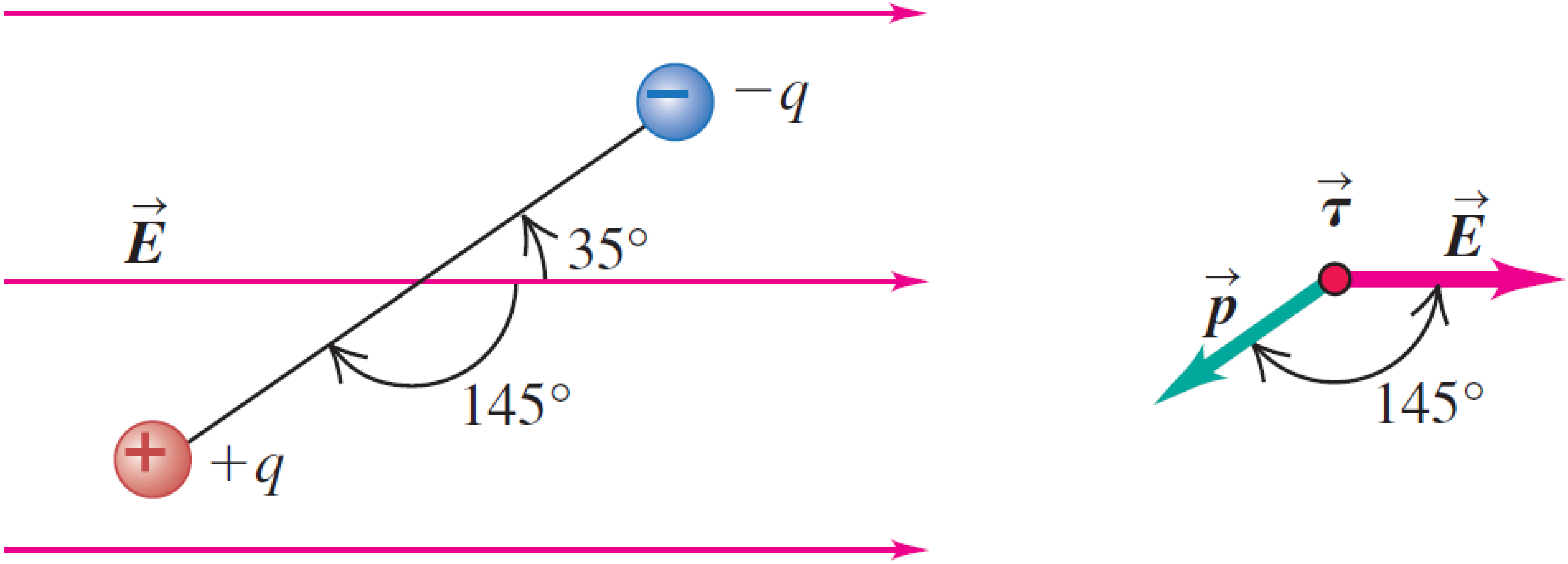
$$U = -\vec{\boldsymbol{p}} \cdot \vec{\boldsymbol{E}} \qquad \text{(potential energy for a dipole in an electric field)} \qquad (21.18)$$

Example 21.13 Force and torque on an electric dipole

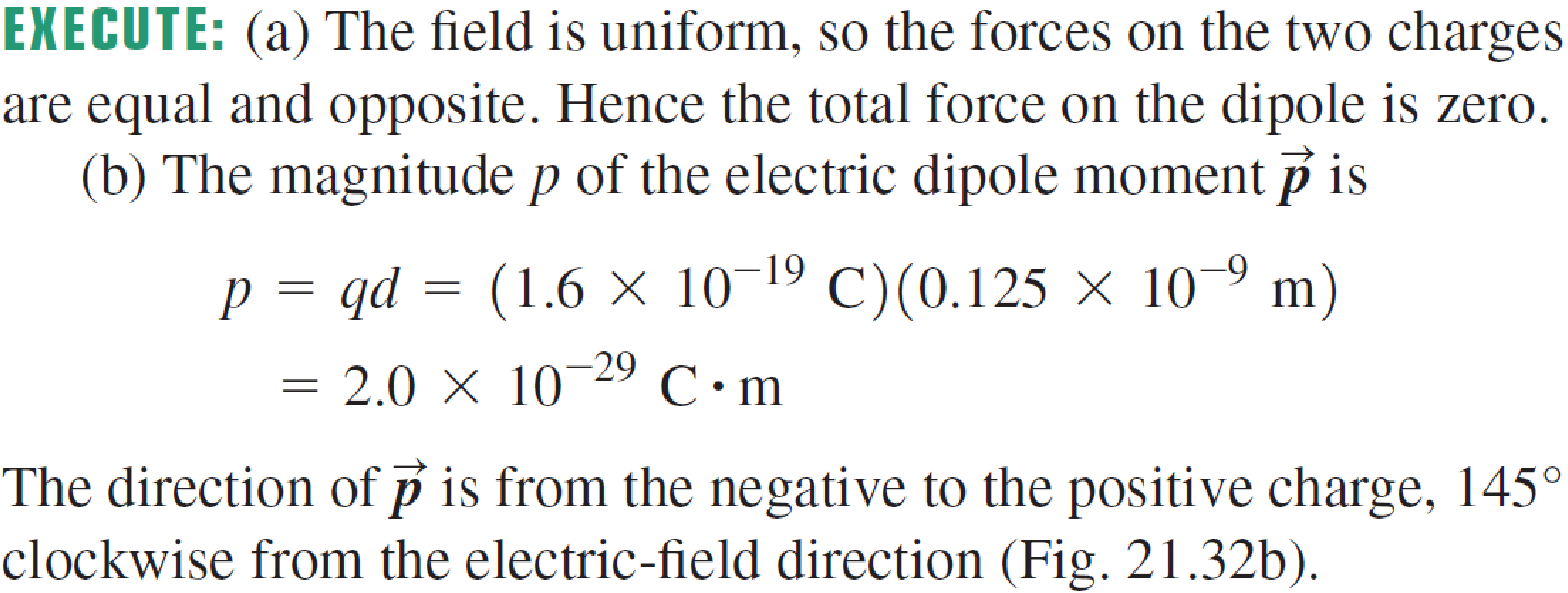
Figure 21.32a shows an electric dipole in a uniform electric field of magnitude 5.0×10^5 N/C that is directed parallel to the plane of the figure. The charges are $\pm 1.6 \times 10^{-19}$ C; both lie in the plane and are separated by 0.125 nm $= 0.125 \times 10^{-9}$ m. Find (a) the net force exerted by the field on the dipole; (b) the magnitude and direction of the electric dipole moment; (c) the magnitude and direction of the torque; (d) the potential energy of the system in the position shown.

Example 21.13 Force and torque on an electric dipole

Figure 21.32a shows an electric dipole in a uniform electric field of magnitude 5.0×10^5 N/C that is directed parallel to the plane of the figure. The charges are $\pm 1.6 \times 10^{-19}$ C; both lie in the plane and are separated by 0.125 nm $= 0.125 \times 10^{-9}$ m. Find (a) the net force exerted by the field on the dipole; (b) the magnitude and



Example 21.13 Force and torque on an electric dipole



EXECUTE: (a) The field is uniform, so the forces on the two charges are equal and opposite. Hence the total force on the dipole is zero.

(b) The magnitude p of the electric dipole moment $\vec{p}$ is

$$p = qd = (1.6 \times 10^{-19}\ \mathrm{C})(0.125 \times 10^{-9}\ \mathrm{m})$$
$$= 2.0 \times 10^{-29}\ \mathrm{C \cdot m}$$

The direction of $\vec{p}$ is from the negative to the positive charge, 145° clockwise from the electric-field direction (Fig. 21.32b).

Example 21.13 Force and torque on an electric dipole

(c) The magnitude of the torque is

$$\tau = pE\sin\phi = (2.0 \times 10^{-29}\ \mathrm{C})(5.0 \times 10^{5}\ \mathrm{N/C})(\sin 145°)$$

$$= 5.7 \times 10^{-24}\ \mathrm{N \cdot m}$$

From the right-hand rule for vector products (see Section 1.10), the direction of the torque $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{p}} \times \vec{\boldsymbol{E}}$ is out of the page. This corresponds to a counterclockwise torque that tends to align $\vec{\boldsymbol{p}}$ with $\vec{\boldsymbol{E}}$.

(d) The potential energy

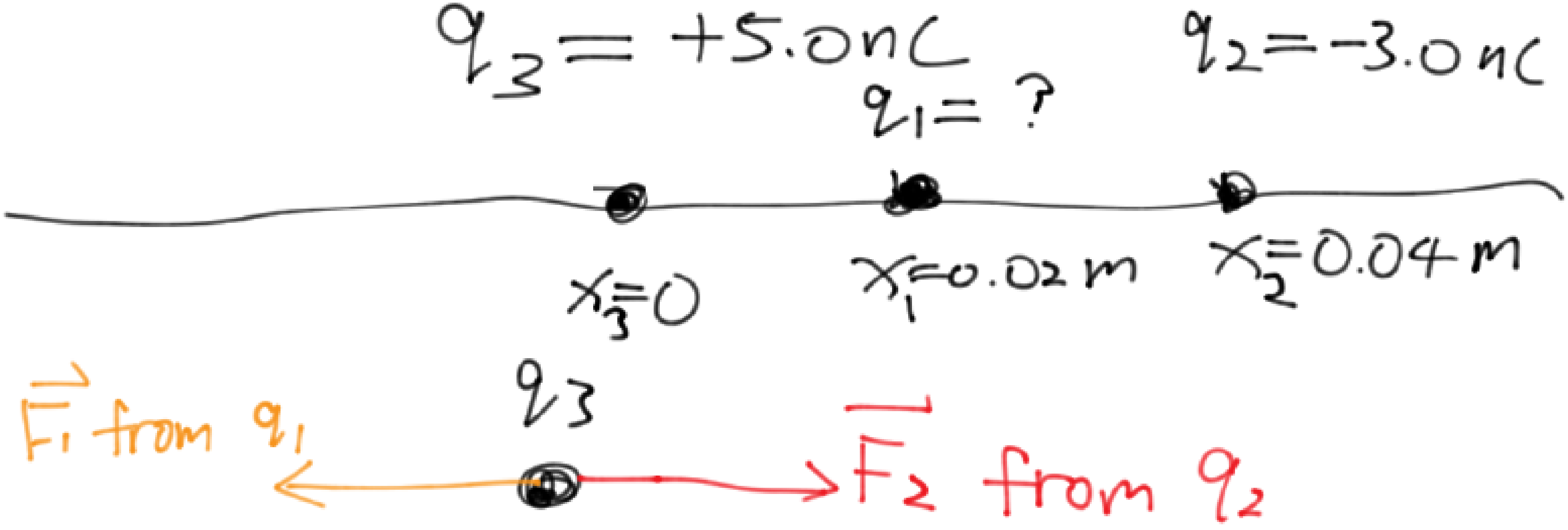
$$U = -pE\cos\phi$$

$$= -(2.0 \times 10^{-29}\ \mathrm{C \cdot m})(5.0 \times 10^{5}\ \mathrm{N/C})(\cos\ 145°)$$

$$= 8.2 \times 10^{-24}\ \mathrm{J}$$

Review Exercises

21.15 •• Three point charges are arranged on a line. Charge $q_3 = +5.00$ nC and is at the origin. Charge $q_2 = -3.00$ nC and is at $x = +4.00$ cm. Charge q_1 is at $x = +2.00$ cm. What is q_1 (magnitude and sign) if the net force on q_3 is zero?



Review Exercises

$$\vec{F_2} = \frac{q_2 . q_3}{|x_2 - x_3|^2} \text{constant} \cdot \underline{(-\hat{i})}$$

q_2 is on the right side of q_3

$$\vec{F_1} = \frac{q_3 q_1}{|x_1 - x_3|^2} \cdot \text{constant} \, (-\hat{i})$$

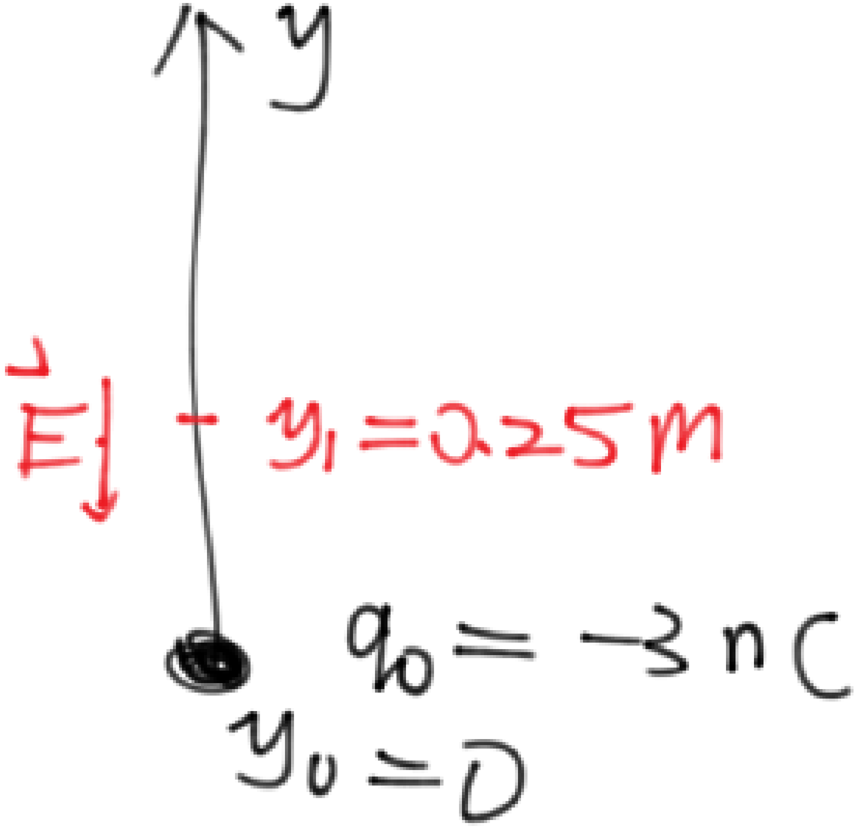
$$\vec{F_1} + \vec{F_2} = 0$$

$$c \cdot \cancel{(-\hat{i})} \frac{q_2 \cancel{q_3}}{|x_2 - \underset{\overset{\shortparallel}{0}}{x_3}|^2} + c \cancel{(-\hat{i})} \frac{\cancel{q_3} q_1}{|x_1 - \underset{\overset{\shortparallel}{0}}{x_3}|^2} = 0$$

$$q_1 = -\frac{|x_1|^2}{|x_2|^2} q_2 = 0.75 \, nC$$

Review Exercises

21.26 • A particle has charge −3.00 nC. (a) Find the magnitude and direction of the electric field due to this particle at a point 0.250 m directly above it. (b) At what distance from this particle does its electric field have a magnitude of 12.0 N/C?



(a) $$E = \frac{1}{4\pi\varepsilon_0}\frac{q_0}{r^2}\cdot\hat{r}$$

$$= 9.0\times10^{9} N\cdot m^2/C^2 \cdot \frac{-3\times10^{-9}C}{(0.25m)^2}\cdot\hat{j}$$

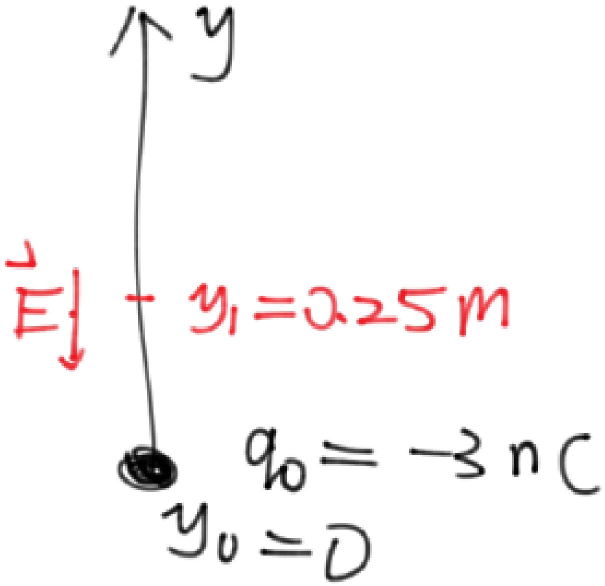
$$= -432\ N/C\ \hat{j}$$

Review Exercises

0.250 m directly above it. (b) At what distance from this particle does its electric field have a magnitude of 12.0 N/C?

$$(b)\quad |E(r=\overset{\overset{0.25\,m}{\uparrow}}{r_1})| = 432\ N/C = \frac{\text{constant}}{r_1^2} \quad (1)$$

$$|E(r=\underset{?}{r_2})| = 12\ N/C = \frac{\text{constant}}{r_2^2} \quad (2)$$



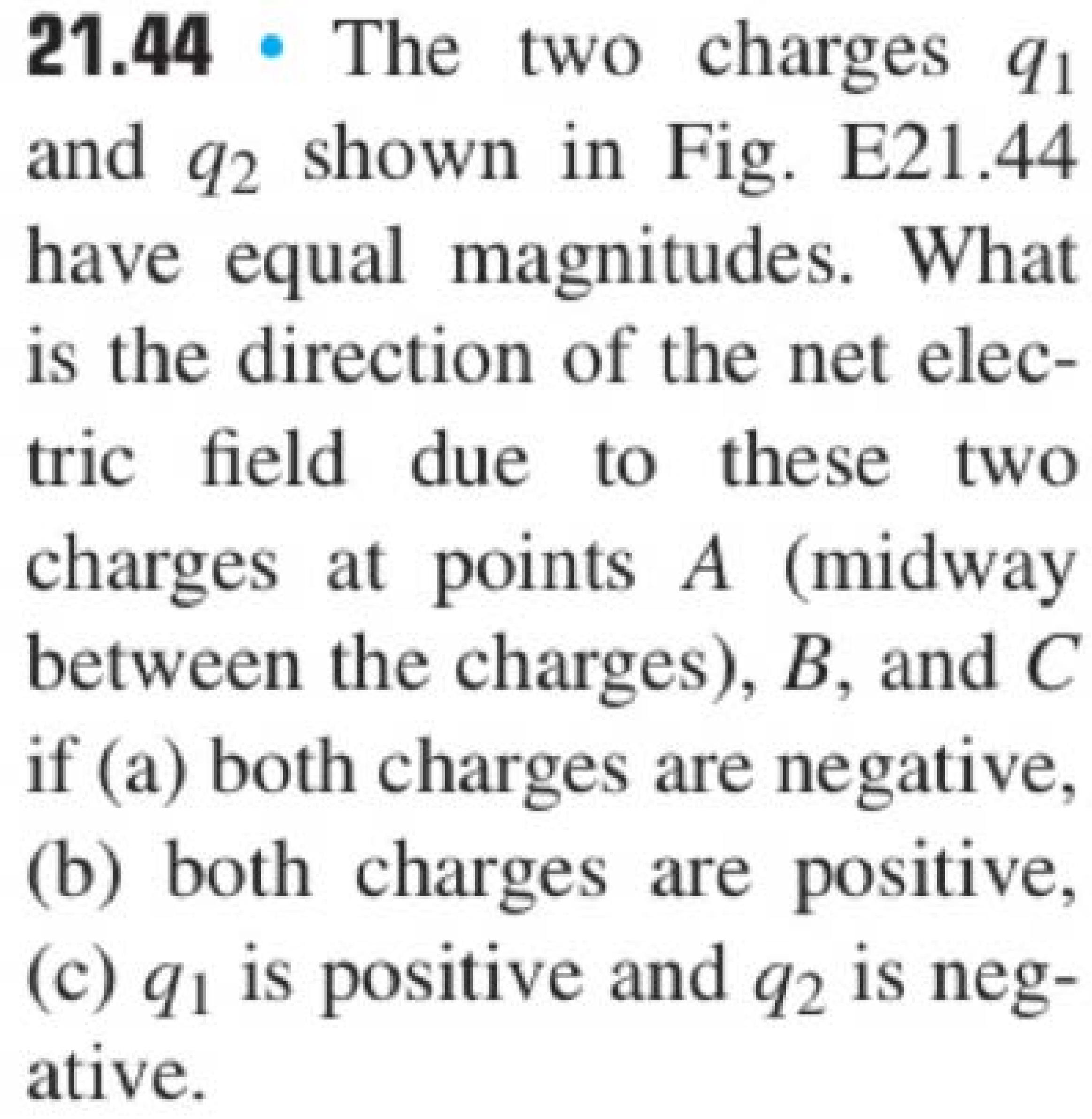
So: (1)/(2) gives

$$\frac{432\ \cancel{N/C}}{12\ \cancel{N/C}} = \frac{\cancel{C}/r_1^2}{\cancel{C}/r_2^2} = \frac{r_2^2}{r_1^2}$$

$$r_2^2 = 36 r_1^2 \quad \text{but} \quad r_1, r_2 > 0$$

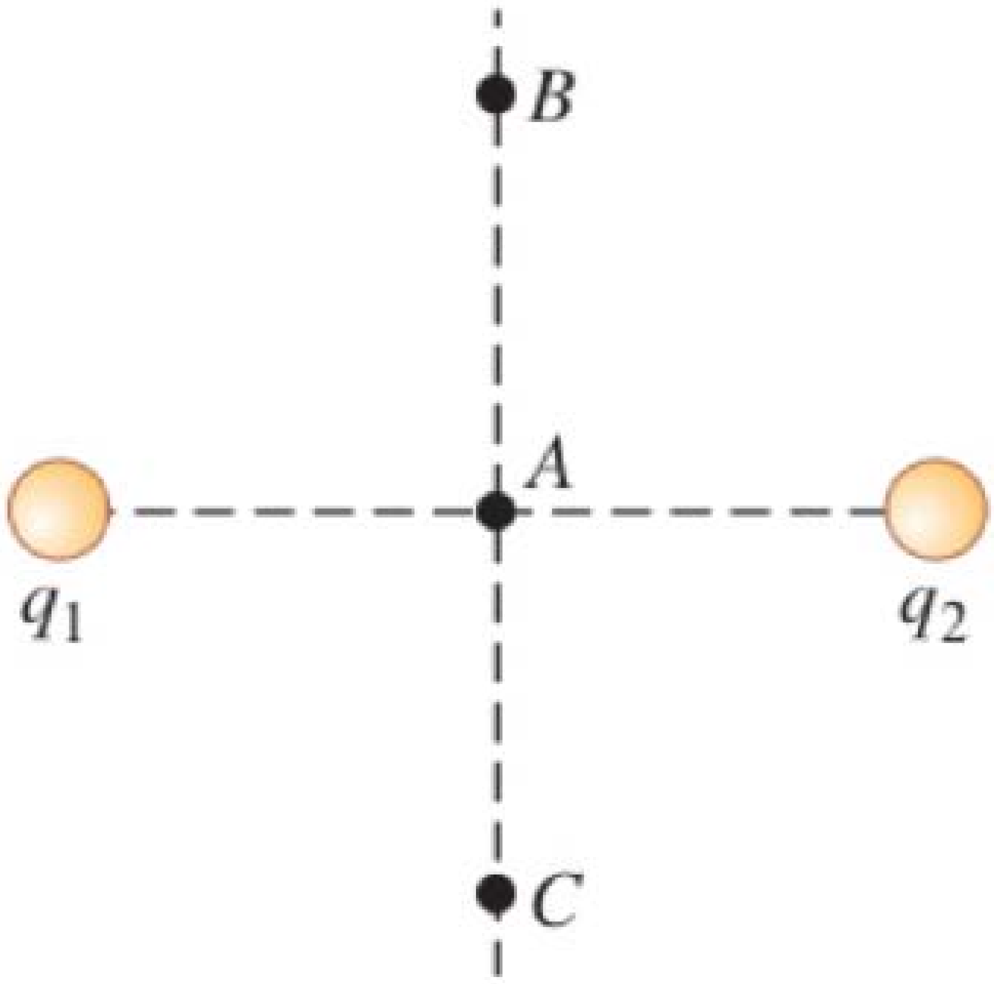
$$r_2 = 6 r_1 = 1.5\,m$$

Review Exercises

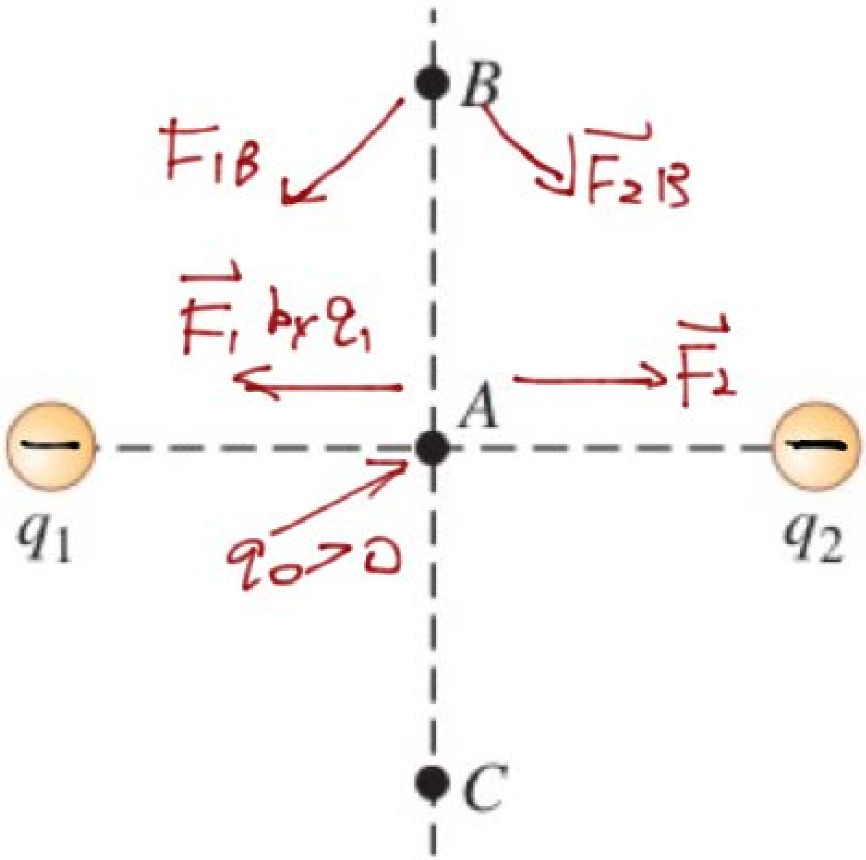


21.44 • The two charges q_1 and q_2 shown in Fig. E21.44 have equal magnitudes. What is the direction of the net electric field due to these two charges at points A (midway between the charges), B, and C if (a) both charges are negative, (b) both charges are positive, (c) q_1 is positive and q_2 is negative.

Figure **E21.44**



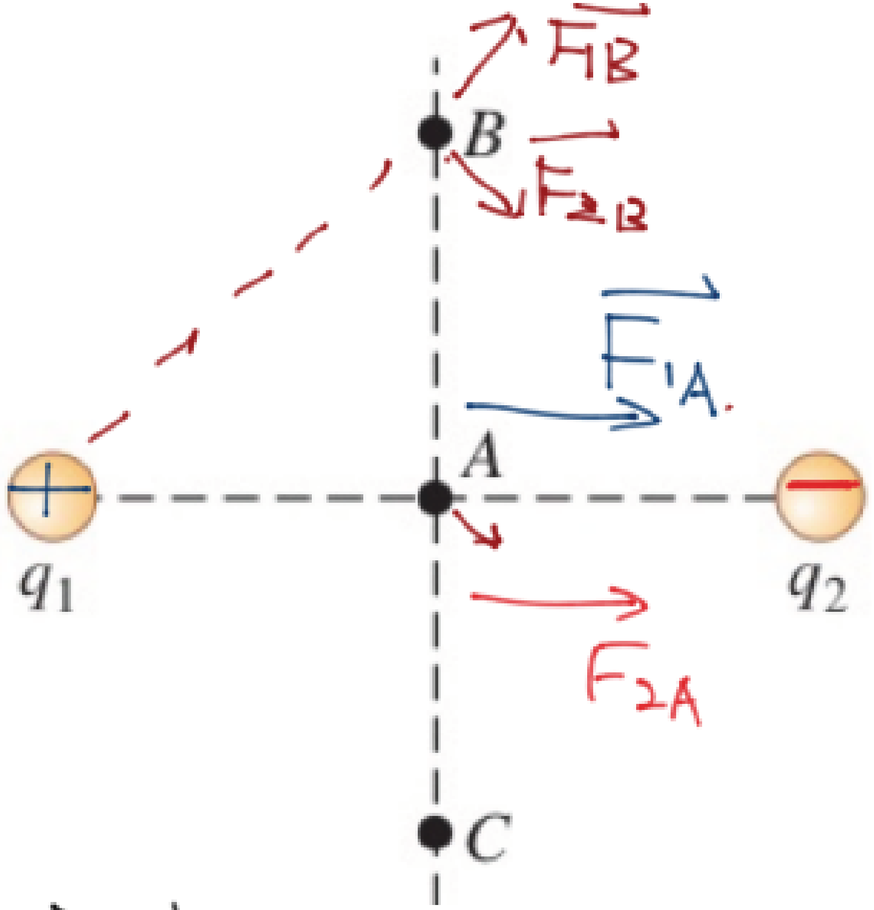
(a) $q_1, q_2 < 0$. Use a small test charge $|q_0| << |q_1|, |q_2|$. $q_0 > 0$
Place it at A, B, C



Because $|q_1| = |q_2|$
and distances to A
are the same
$|\vec{F_1}| = |\vec{F_2}|$ but in
opposite direction
Net force $= \vec{F_1} + \vec{F_2} = 0$
$\vec{E}_{total} = \frac{F_{total}}{q} = 0$ at A

$\vec{F}_{1B} = F_{1Bx}\hat{i} + F_{1By}\hat{j}$
$\vec{F}_{2B} = F_{2Bx}\hat{i} + F_{2By}\hat{j}$
$F_{1Bx} = -F_{2Bx}$

so $\vec{F}_{B,total} = 2F_{1By}\hat{j}$
$F_{1By} < 0$. $\vec{E}_B$ in $-\hat{j}$ direction



(c) $q_1>0$ $q_2<0$

$$\vec{F}_{A.total} = |\vec{F}_{1A}|\hat{i} + |F_{2A}|\hat{i}$$

in $+\vec{x}$ direction

$$\vec{E}_{A\,total} = \frac{F_{A total}}{q_0}$$

in $+x$ direction

Again:

$$\begin{cases} F_{1Bx} = F_{2Bx} \\ F_{1By} = -F_{2By} \end{cases}$$

$$\vec{F}_{1B} = F_{1Bx}\cdot\hat{i} + F_{1By}\cdot\hat{j}$$

$$\vec{F}_{2B} = F_{2Bx}\cdot\hat{i} + F_{2By}\hat{j}.$$

$$\vec{F_{1B}} = F_{1Bx}\cdot\hat{i} + F_{1By}\cdot\hat{j}$$

$$\begin{cases} F_{1Bx} = F_{2Bx} \\ F_{1By} = -F_{2By} \end{cases}$$

$$\vec{F_{2B}} = F_{2Bx}\cdot\hat{i} + F_{2By}\,\hat{j}.$$

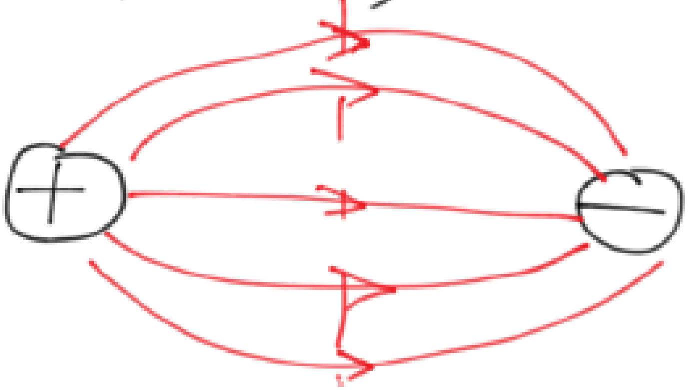
So $\vec{F}_{B.total} = \vec{F_{1B}} + \vec{F_{2B}}$

$= 2F_{1Bx}\,\hat{i}$

$$\vec{E_B} = \frac{\vec{F_B}}{q}$$

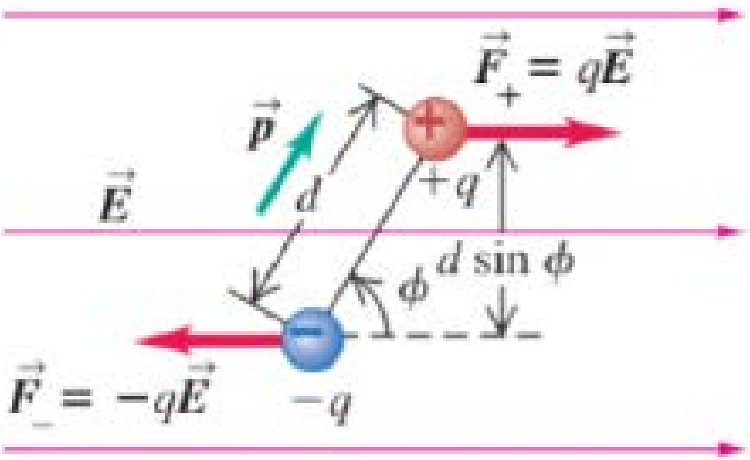
Also in +x

same for (c)



21.57 • Point charges $q_1 = -4.5$ nC and $q_2 = +4.5$ nC are separated by 3.1 mm, forming an electric dipole. (a) Find the electric dipole moment (magnitude and direction). (b) The charges are in a uniform electric field whose direction makes an angle of 36.9° with the line connecting the charges. What is the magnitude of this field if the torque exerted on the dipole has magnitude 7.2×10^{-9} N · m?

$$p = qd$$



$$\tau = (qE)(d \sin \phi)$$

(a): $p = qd$

$= 4.5nC \times 3.1mm$

$= 4.5 \times 10^{-9} C \times 3.1 \times 10^{-3} m$

$= 1.395 \times 10^{-11} C \cdot m$

21.57 • Point charges $q_1 = -4.5$ nC and $q_2 = +4.5$ nC are separated by 3.1 mm, forming an electric dipole. (a) Find the electric dipole moment (magnitude and direction). (b) The charges are in a uniform electric field whose direction makes an angle of 36.9° with the line connecting the charges. What is the magnitude of this field if the torque exerted on the dipole has magnitude 7.2×10^{-9} N · m?

$$\tau = (qE)(d\sin\phi)$$

$\phi = 36.9^\circ$ so $\cos\phi = 0.8$

$\sin\phi = 0.6$

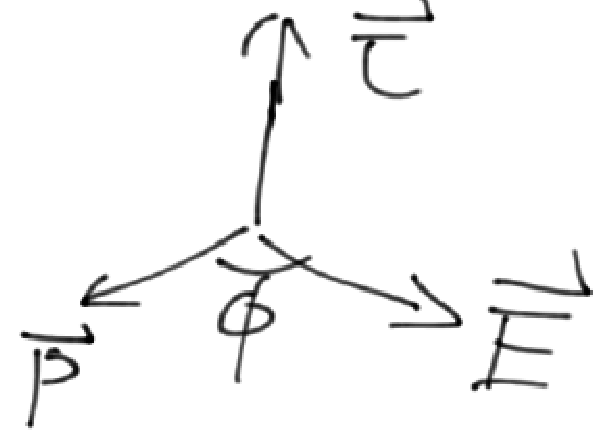
$$\tau = 7.2\times10^{-9}\,N\cdot m = \underbrace{q\,\underset{?}{E}\cdot d}_{\simeq 1.4\times10^{-11}\,C\cdot m}\ \underbrace{\sin\phi}_{0.6}$$

$$E = \frac{\tau}{qd\cdot\sin\phi} = \frac{7.2\times10^{-9}\,N\cdot m}{1.395\times10^{-11}\,C\cdot m\cdot 0.6}$$

$$\simeq 8.6\times10^{2}\,N/m$$

21.59 • Torque on a Dipole. An electric dipole with dipole moment $\vec{p}$ is in a uniform electric field $\vec{E}$. (a) Find the orientations of the dipole for which the torque on the dipole is zero. (b) Which of the orientations in part (a) is stable, and which is unstable? (*Hint:* Consider a small displacement away from the equilibrium position and see what happens.) (c) Show that for the stable orientation in part (b), the dipole's own electric field tends to oppose the external field.

$$\vec{\tau} = \vec{p} \times \vec{E}$$ (torque on an electric dipole, in vector form)

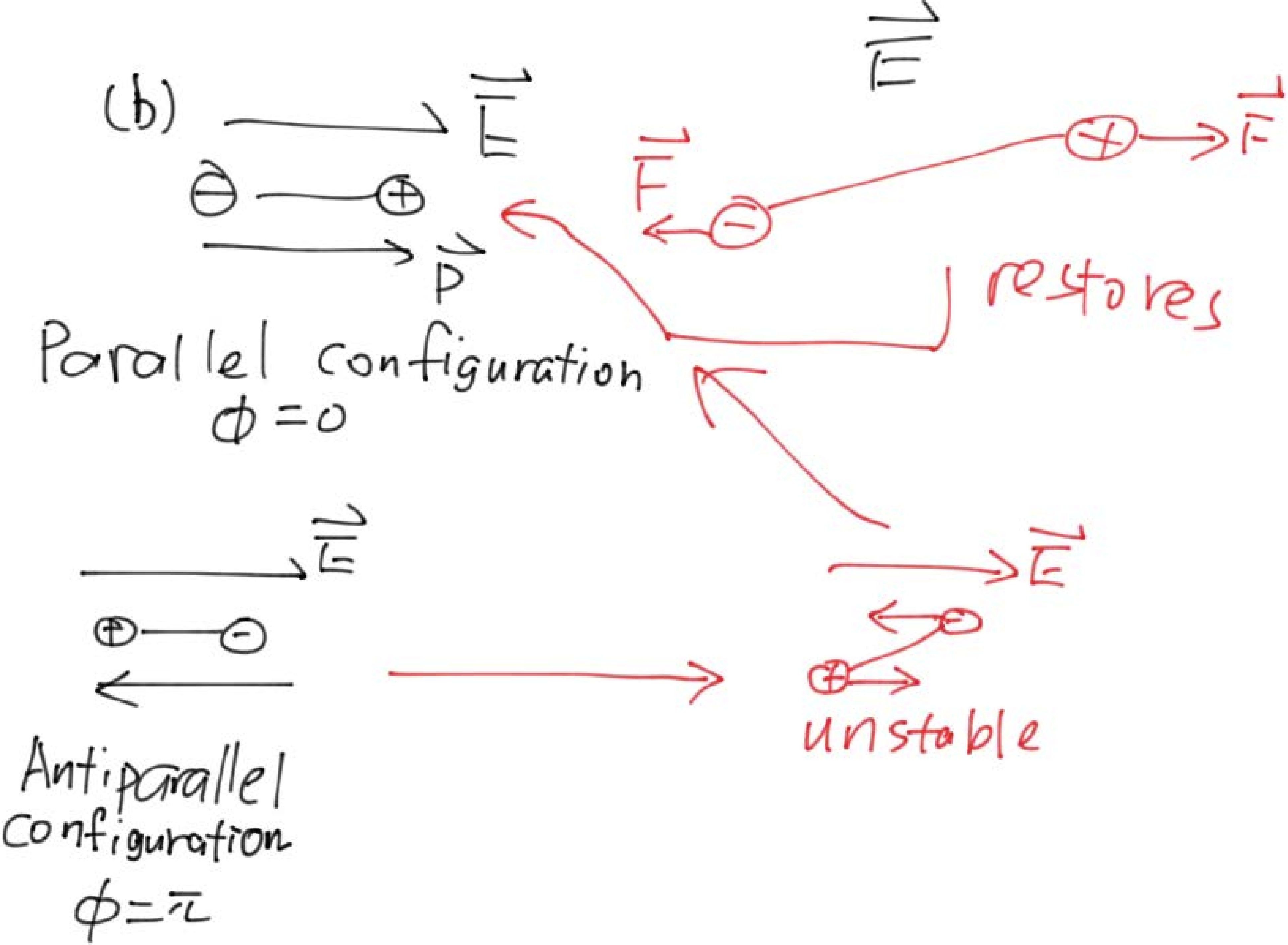


right hand rule

$$\tau = p \cdot E \cdot \sin\phi$$

$$= 0 \text{ when}$$

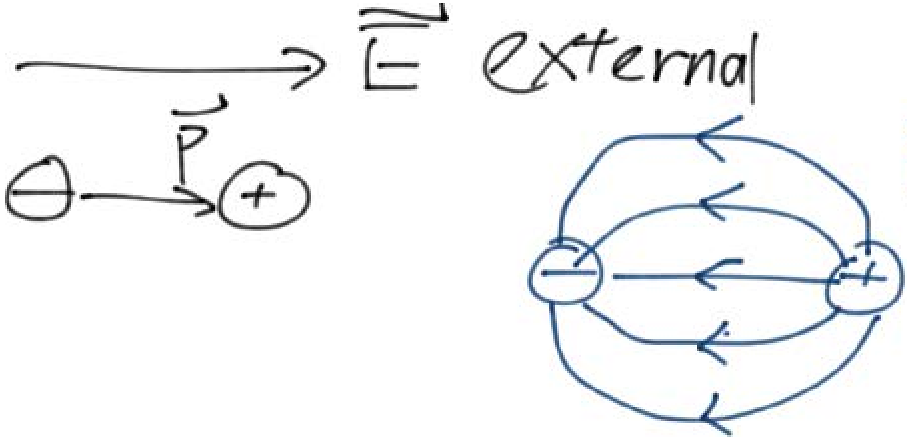
$$\phi = 0 \text{ or } \pi$$


(b)
$\vec{E}$
$\vec{p}$
Parallel configuration
$\phi = 0$
$\vec{E}$
$\vec{F}$
$\vec{F}$
restores
$\vec{E}$
Antiparallel configuration
$\phi = \pi$
$\vec{E}$
unstable

unstable

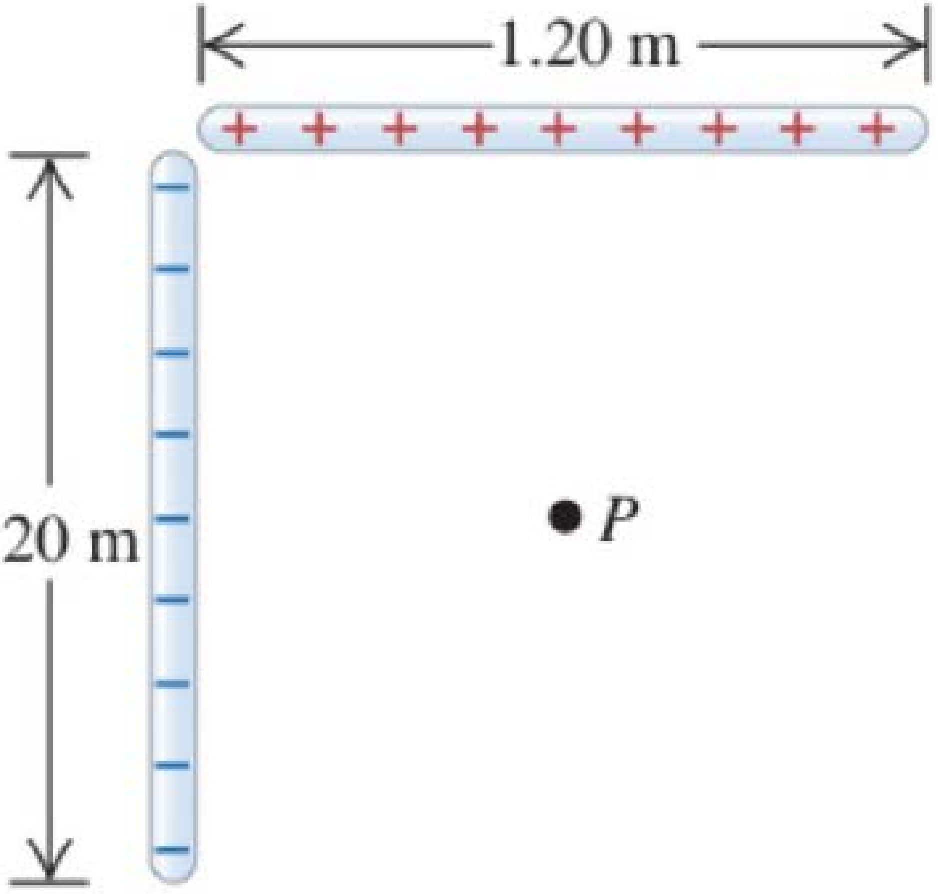
Antiparallel configuration

$\phi = \pi$



$\vec{E}_{internal}$ counters $\vec{E}_{external}$

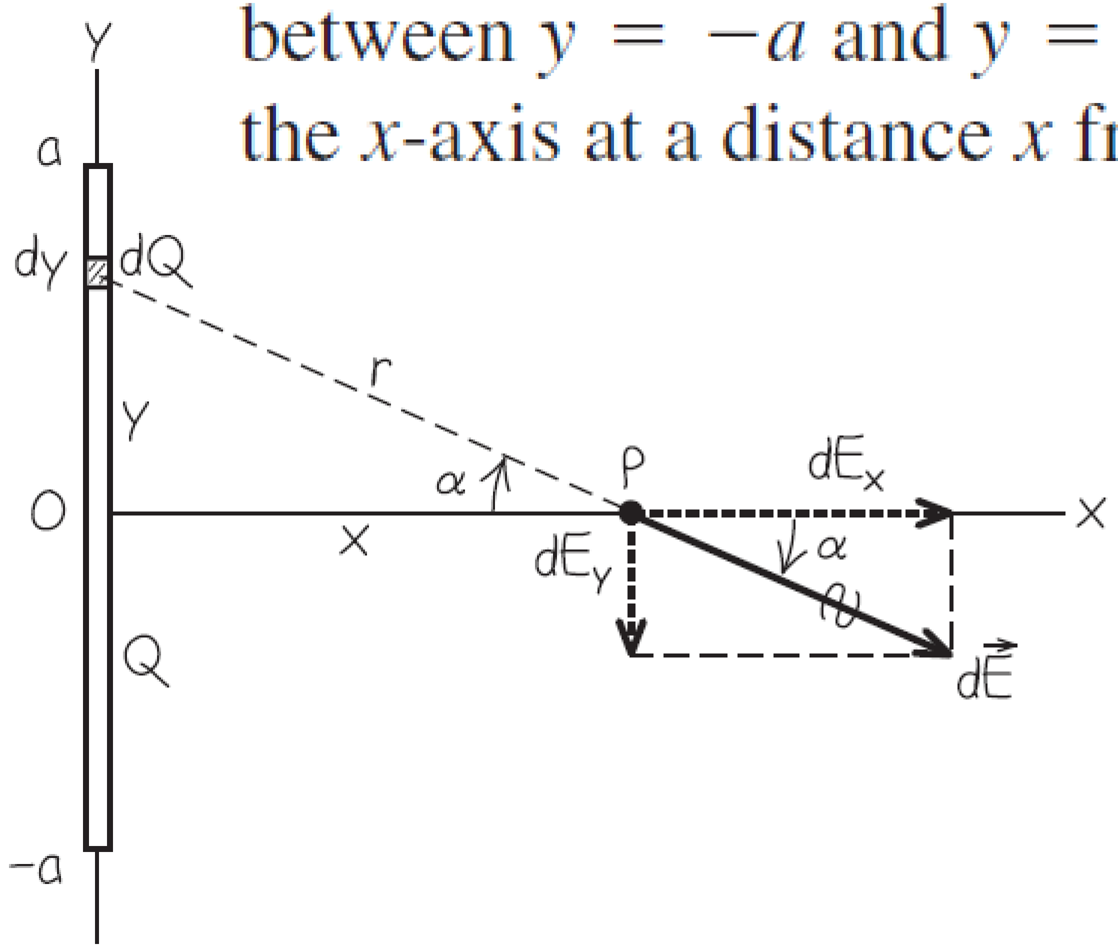
21.99 •• Two 1.20-m nonconducting wires meet at a right angle. One segment carries $+2.50\ \mu C$ of charge distributed uniformly along its length, and the other carries $-2.50\ \mu C$ distributed uniformly along it, as shown in Fig. P21.99.



(a) Find the magnitude and direction of the electric field these wires produce at point P, which is 60.0 cm from each wire. (b) If an electron is released at P, what are the magnitude and direction of the net force that these wires exert on it?

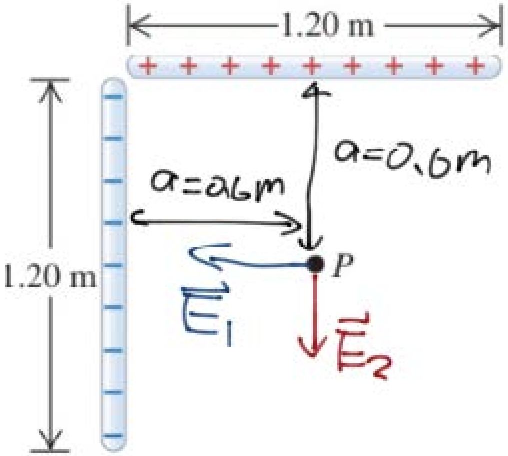
Recall the Example 21.10

Positive charge Q is distributed uniformly along the y-axis between $y = -a$ and $y = +a$. Find the electric field at point P on the x-axis at a distance x from the origin.



$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2 + a^2}}\hat{\imath}$$

$$|\vec{E_1}| = |\vec{E_2}| = \frac{1}{4\pi\epsilon_0}\frac{Q}{a\sqrt{2a^2}} = \frac{1}{4\sqrt{2}\pi\epsilon_0}\frac{Q}{a^2}$$



$$\vec{E}_{total} = \vec{E_1}+\vec{E_2} = \frac{1}{4\sqrt{2}\pi\epsilon_0}\frac{Q}{a^2}\cdot(-\hat{i}-\hat{j})$$

$|\vec{E}_{total}| = \frac{1}{4\pi\epsilon_0}\frac{Q}{a^2}$ in $(-\hat{i}-\hat{j})$ direction

for electron $q=-1.6\times10^{-19}C$. direction $\hat{i}+\hat{j}$

$$F = |\vec{E}_{total}||q| = \frac{1}{4\pi\epsilon_0}\frac{|Qq|}{a^2}.$$

$$= 9\times10^{9}\,N\,m^2/c^2\,\frac{2.5\times10^{-6}C\times1.6\times10^{-19}C}{0.6^2 m^2}$$

$$= 10^{-14}\,N$$

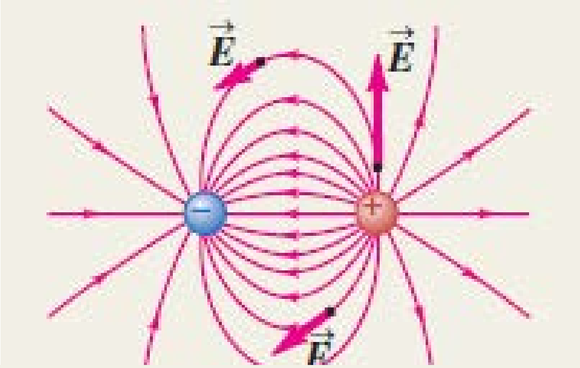
Summary

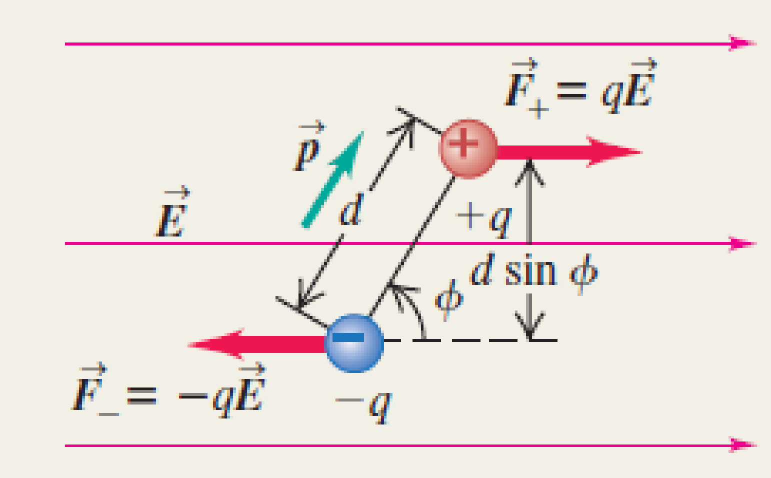
$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$$

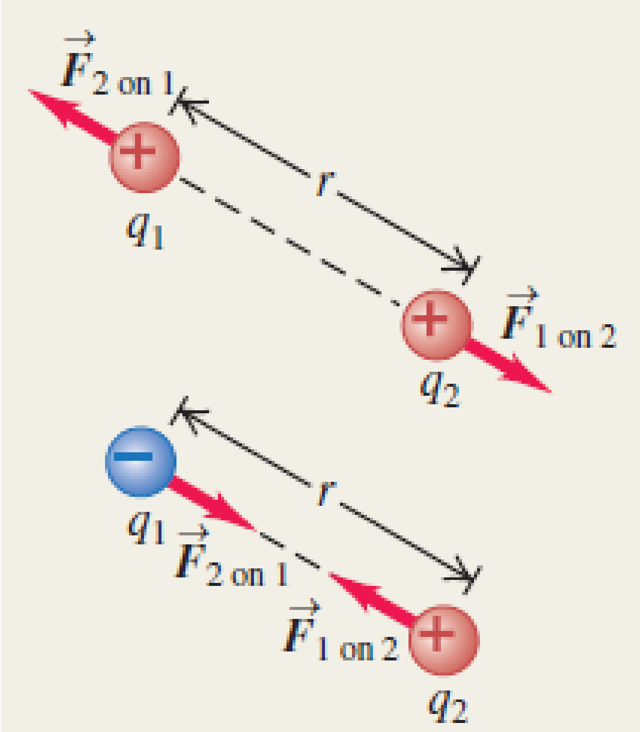
$$\frac{1}{4\pi\epsilon_0} = 8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

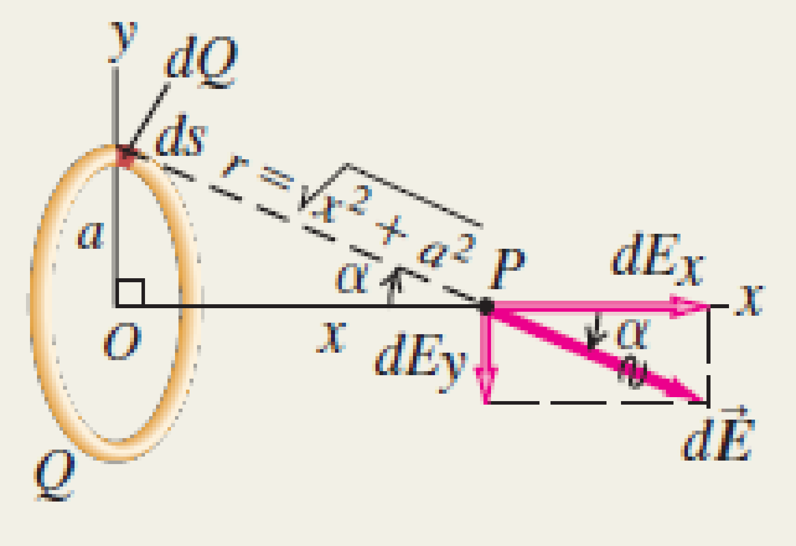
$$\vec{E} = \frac{\vec{F}_0}{q_0}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$









$$\tau = pE \sin\phi$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$