

College Algebra and Trigonometry

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More Equations and Applications

① Solve Polynomial Equations

Example 1:

$$4x^3 + 12x^2 - 9x - 27 = 0$$

Example 2:

$$y^5 = 27y^2$$

② Solve Rational Equations

Example 3:

$$\frac{2x}{x-4} - \frac{3}{x+2} = \frac{x^2 + 14}{x^2 - 2x - 8}$$

Example 4:

$$\frac{18}{m^2 - 3m} + 2 = \frac{6}{m - 3}$$

③ Solve Absolute Value Equations

Let k represent a positive real number.

1. $|u| = k$ is equivalent to $u = k$ or $u = -k$.
2. $|u| = 0$ is equivalent to $u = 0$.
3. $|u| = -k$ has no solution.
4. $|u| = |w|$ is equivalent to $u = w$ or $u = -w$.

Example 5:

a) $2|3 - 2t| = 6$

b) $2 = |7w - 3| + 8$

Example 6:

a) $|2x - 5| = |x + 1|$

b) $|3 - y| = |y - 3|$

④ Solve Radical Equations and Equations with Rational Exponents

An equation with one or more radicals containing a variable is called a **radical equation**. For example:

$$\sqrt[3]{x} = 5$$

Extraneous Solutions

a) $\sqrt{x} = -10$

b) $\sqrt{x + 10} - 4 = x$

Solve Radical Equations

Example 7:

$$\sqrt{3m + 1} - \sqrt{m - 1} = 2$$

Example 8:

$$2(t + 1)^{2/3} = 8$$

⑤ Solve Equations in Quadratic Form

Equation in Quadratic Form

$$\left(2 + \frac{3}{x}\right)^2 - \left(2 + \frac{3}{x}\right) - 12 = 0$$

$$2w^{2/3} - 3w^{1/3} - 20 = 0$$

$$\xrightarrow{u = 2 + 3/x}$$

$$\xrightarrow{u = w^{1/3}}$$

New Equation

$$u^2 - u - 12 = 0$$

$$2u^2 - 3u - 20 = 0$$

Example 9:

a) $(2x^2 - 3)^2 - 12(2x^2 - 3) + 35 = 0$

b) $2w^{2/3} = 3w^{1/3} + 20$

Mixed Exercises

Example 10:

$$y + 4\sqrt{y} = 21$$

Example 11:

$$\sqrt{x + \sqrt{x + 2}} = 3$$

Properties of Inequality

Let a , b and c represent real numbers.

1. If $x < a$, then $a > x$.
2. If $a < b$, and $b < c$, then $a < c$.
3. If $a < b$, and $c < d$, then $a + b < c + d$.
4. If $a < b$, then $a + c < b + c$ and $a - c < b - c$.
5. If c is positive and $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
6. If c is negative and $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

These statements are also true for the symbols $>$, \leq , and \geq .

① Solve Linear Inequalities in One Variable

Example 1:

$$-6y + 3 < 21$$

Example 2:

$$\frac{x+1}{3} - \frac{2x-4}{6} \leq -\frac{x}{2}$$

② Solve Compound Linear Inequalities

Example 3:

$$\frac{y-1}{2} \leq 3 \quad \text{and} \quad \frac{2y-3}{5} \geq 1$$

Example 4:

$$5 < -2x + 7 \leq 11$$

③ Solve Absolute Value Inequalities

Properties involving Absolute Value Inequalities

Let k be a positive real number ($k > 0$).

1. $|u| < k$ is equivalent to $-k < u < k$.
2. $|u| > k$ is equivalent to $u < -k$ or $u > k$.

These statements are also true for the symbols \leq , and \geq .

Solve Absolute Value Inequalities

Example 5:

$$2|x - 6| - 3 \leq 7$$

Example 6:

$$5 < |2x - 3| \leq 9$$

④ Solve Applications of Inequalities

Example 7

Emily has test scores of 80, 84, 88, 96, and 98. Her score for online homework is 100 and is weighed as one test grade. She still needs to take the final exam which counts as two test grades.

What score does she need on the final exam to earn an “A” in the class (it means having an average of at least 90)?

Example 8

Suppose that a machine is calibrated to dispense 800 ml of orange juice into a plastic bottle, with a measurement error of no more than 5 ml. Let x represent the actual amount of orange juice poured into the bottle.

- a) Write an absolute value inequality that represents the interval in which to estimate x .
- b) Solve the inequality.