Lecture 12

Gauss's Law

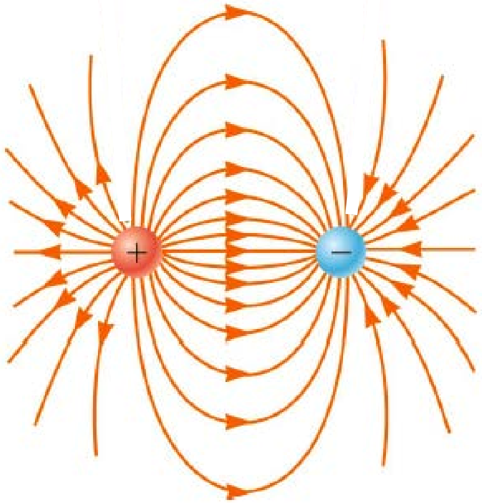
Date: 4/22/2025

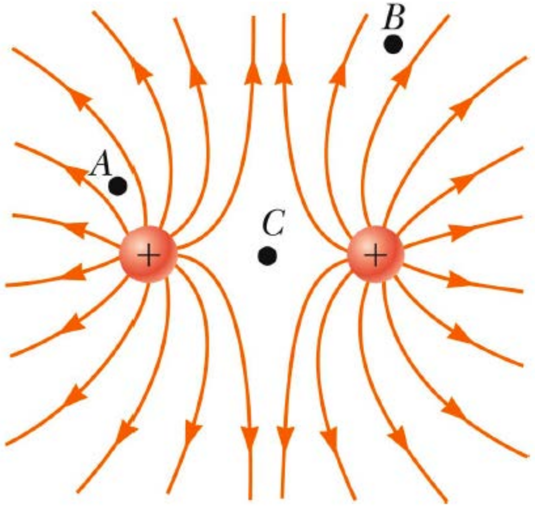
Course Instructor:
Jingtian Hu (胡竞天)

Introduction to Gauss's Law

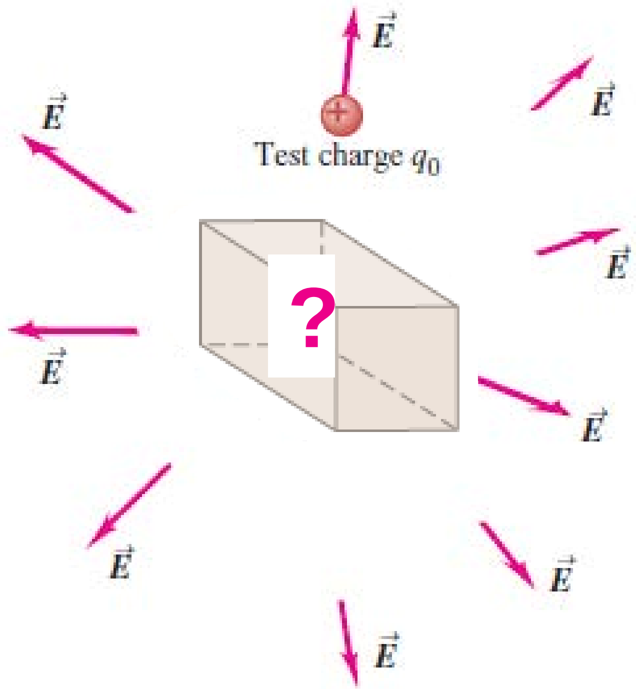
Coulomb's law: from charge distribution to electrical fields

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\vec{e}_r$$





What if we want to calculate charge distribution from electrical fields?



Gauss's law

Flux of Electric Fields

In a simplified scenario:

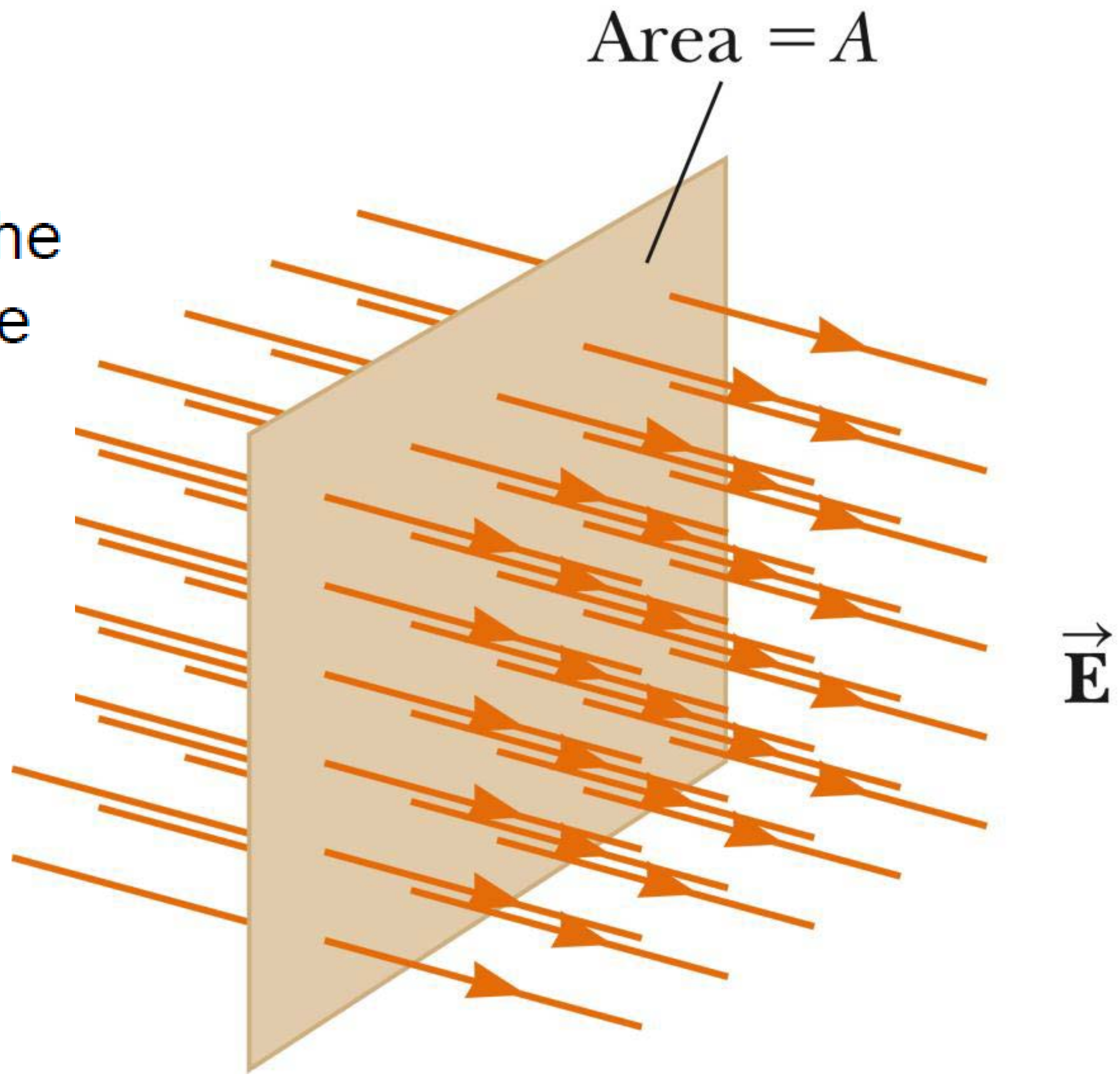
Electric flux is the product of the magnitude of the electric field and the surface area, *A*, perpendicular to the field.

$\Phi_E = EA$

Units: $N \cdot m^2 / C$

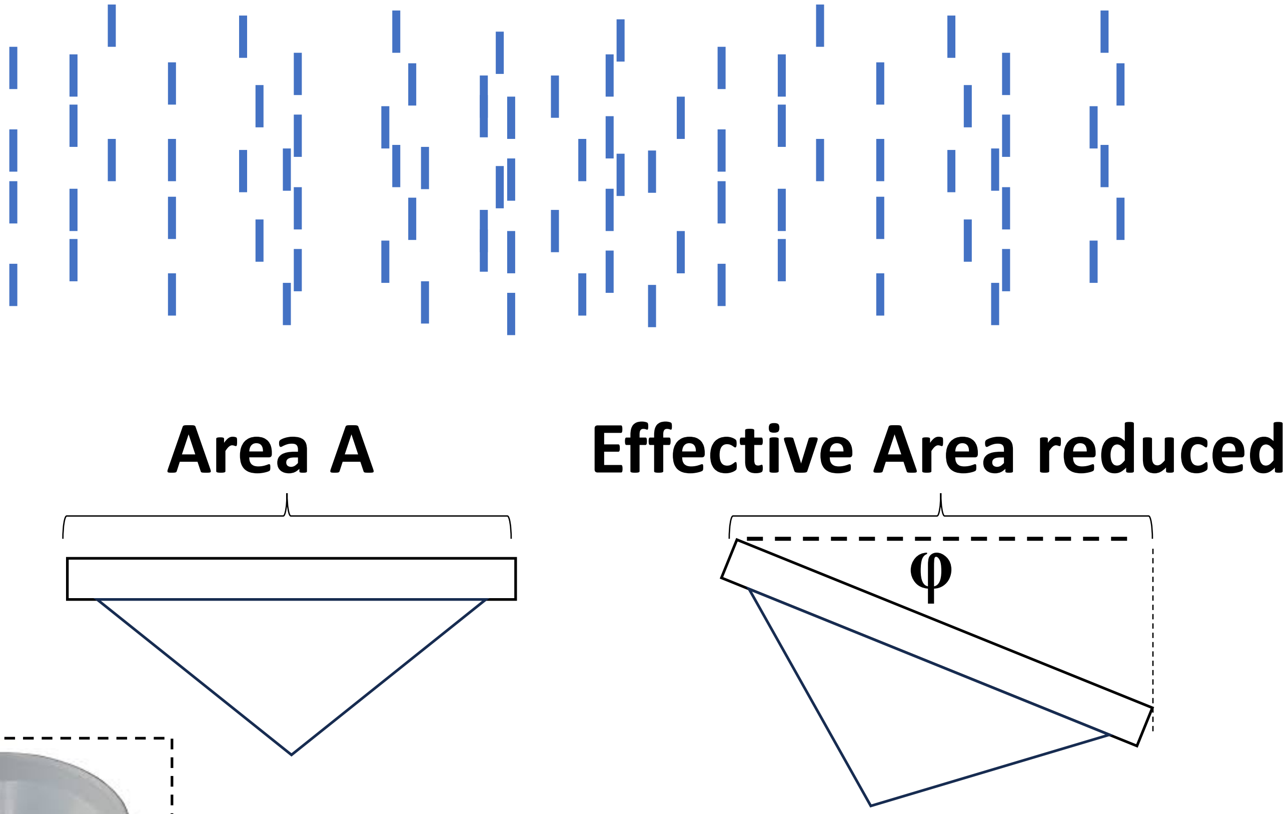
Assumes:

- Uniform electric field $\vec{E}$
- Flat surface A perpendicular to $\vec{E}$



Flux of Electric Fields

If you didn’t get what a flux is, think about rain falls:



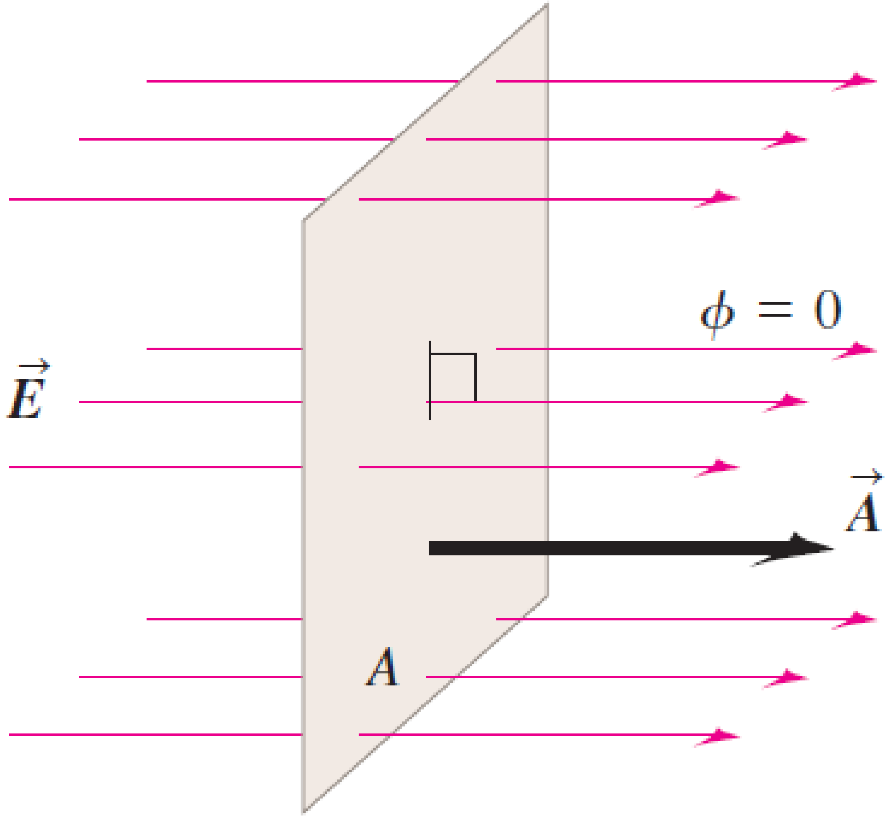
$A_{effective} = A\cos\varphi$

Flux of rain on the funnel: amount of rain collected per unit time

Flux of Electric Fields with Vectors

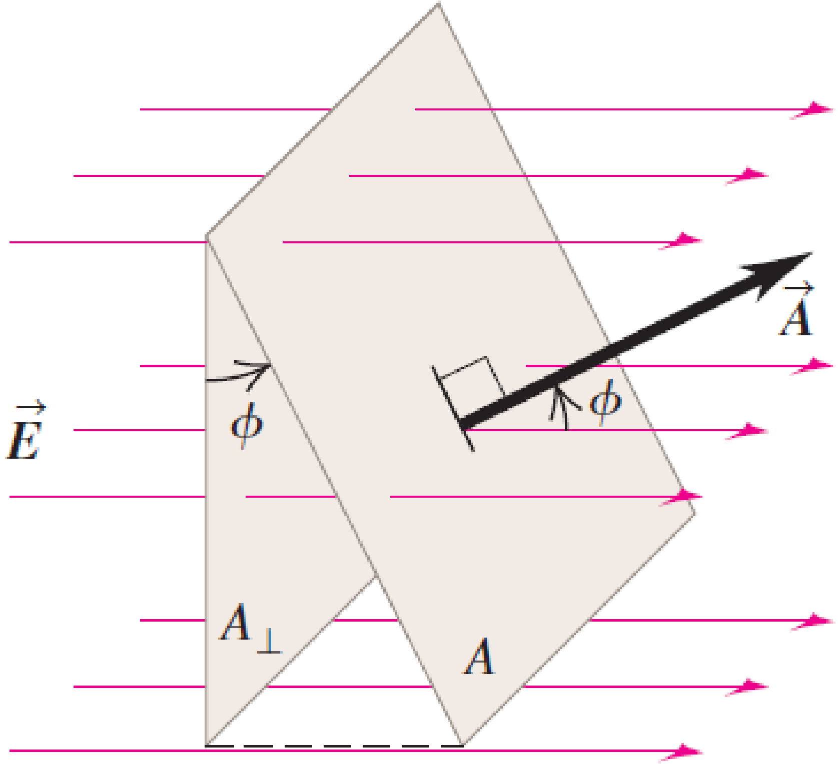
(a) Surface is face-on to electric field:

- $\vec{E}$ and $\vec{A}$ are parallel (the angle between $\vec{E}$ and $\vec{A}$ is $\phi = 0$).
- The flux $\boxed{\Phi_E = \vec{E} \cdot \vec{A}} = EA$.



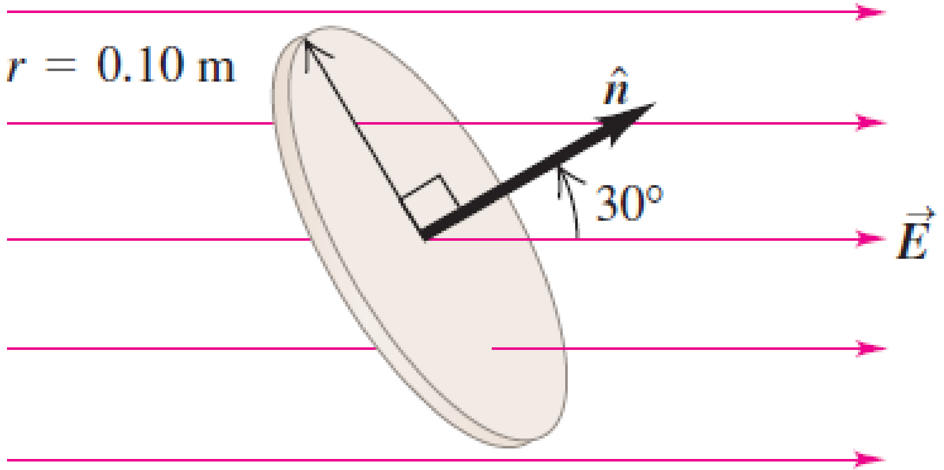
(b) Surface is tilted from a face-on orientation by an angle ϕ:

- The angle between $\vec{E}$ and $\boxed{\vec{A}}$ is ϕ.
- The flux $\boxed{\Phi_E = \vec{E} \cdot \vec{A}} = EA \cos \phi$.



Example Problem: Flux through a Disk

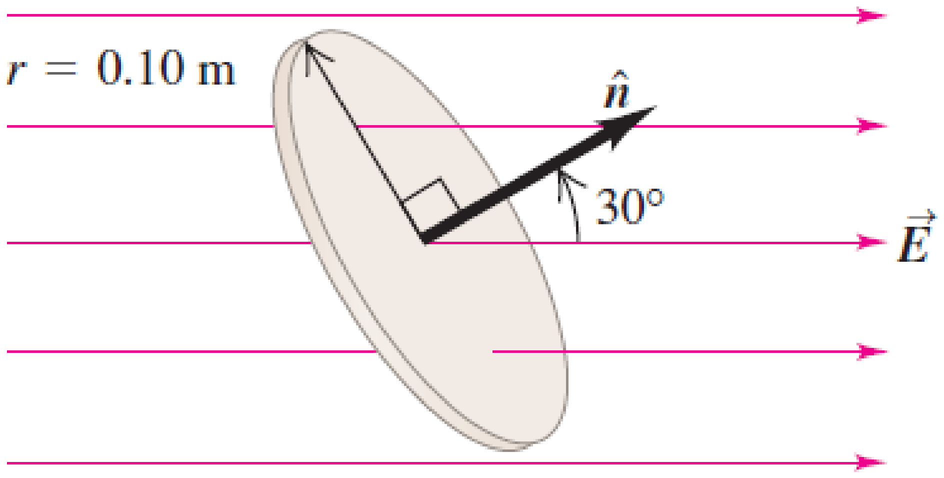
A disk of radius 0.10 m is oriented with its normal unit vector $\hat{n}$ at 30° to a uniform electric field $\vec{E}$ of magnitude 2.0×10^3 N/C (Fig. 22.7). (Since this isn't a closed surface, it has no "inside" or "outside." That's why we have to specify the direction of $\hat{n}$ in the figure.) (a) What is the electric flux through the disk? (b) What is the flux through the disk if it is turned so that $\hat{n}$ is perpendicular to $\vec{E}$? (c) What is the flux through the disk if $\hat{n}$ is parallel to $\vec{E}$?



Example Problem: Flux through a Disk

IDENTIFY and SET UP: This problem is about a flat surface in a uniform electric field, so we can apply the ideas of this section. We calculate the electric flux using Eq. (22.1).

$$\Phi_E = EA\cos\phi \qquad \text{(electric flux for uniform } \vec{E}\text{, flat surface)} \qquad (22.1)$$



Example Problem: Flux through a Disk

IDENTIFY and SET UP: This problem is about a flat surface in a uniform electric field, so we can apply the ideas of this section. We calculate the electric flux using Eq. (22.1).

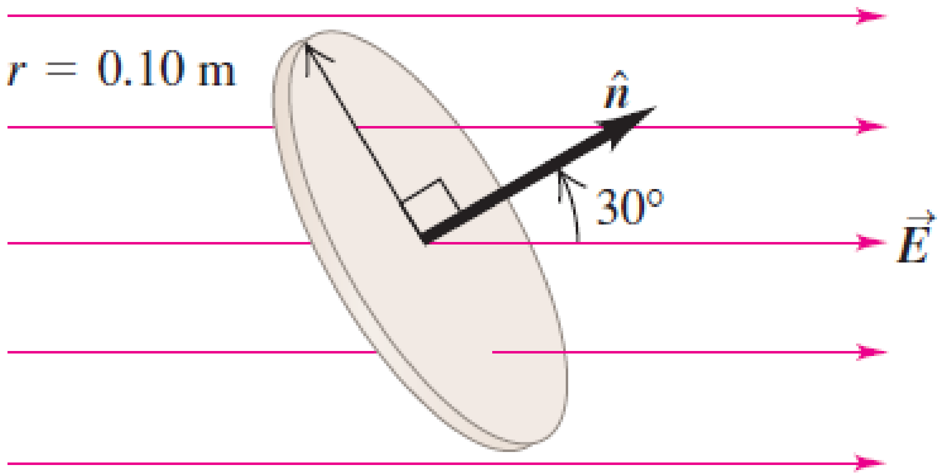
$$\Phi_E = EA\cos\phi \qquad \text{(electric flux for uniform } \vec{E}\text{, flat surface)} \qquad (22.1)$$

EXECUTE: (a) The area is $A = \pi(0.10\text{ m})^2 = 0.0314\text{ m}^2$ and the angle between $\vec{E}$ and $\vec{A} = A\hat{n}$ is $\phi = 30°$, so from Eq. (22.1),

$$\begin{aligned}\Phi_E &= EA\cos\phi = (2.0 \times 10^3\text{ N/C})(0.0314\text{ m}^2)(\cos 30°)\\ &= 54\text{ N}\cdot\text{m}^2/\text{C}\end{aligned}$$

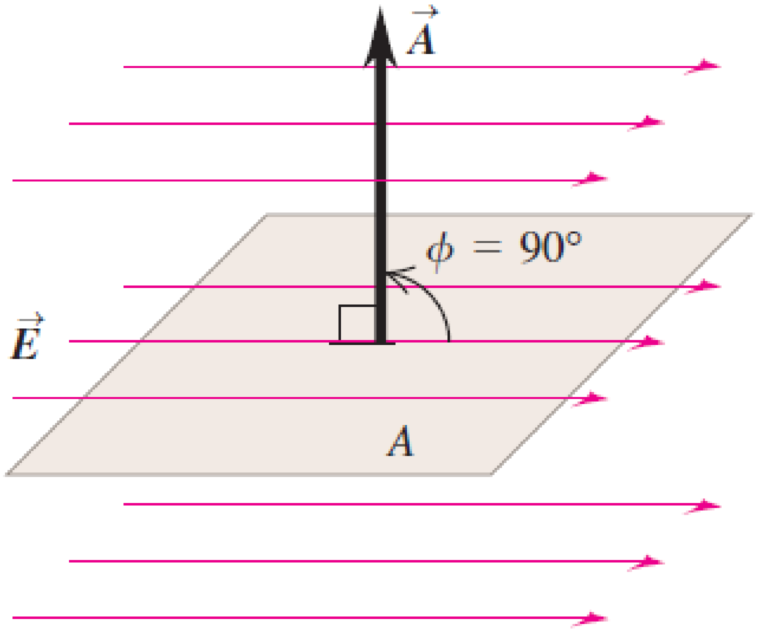
Example Problem: Flux through a Disk

(b) What is the flux through the disk if it is turned so that $\hat{\boldsymbol{n}}$ is perpendicular to $\vec{E}$? (c) What is the flux through the disk if $\hat{\boldsymbol{n}}$ is parallel to $\vec{E}$?



(c) Surface is edge-on to electric field:

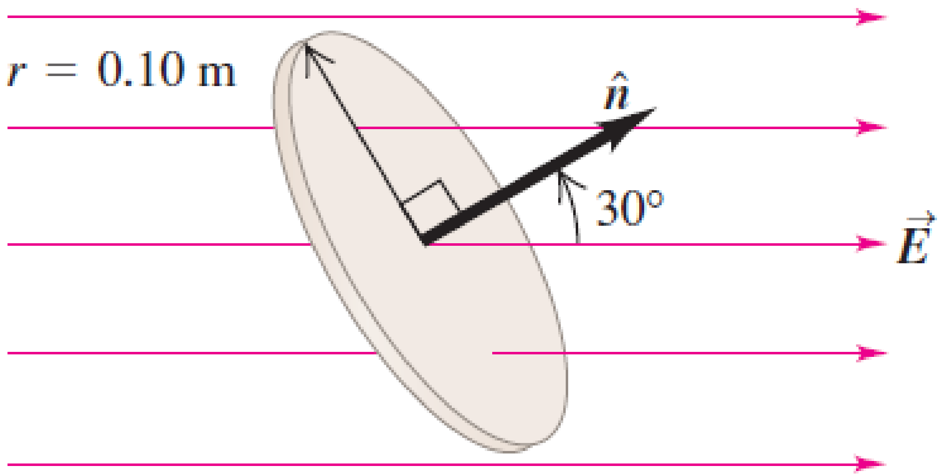
- $\vec{E}$ and $\vec{A}$ are perpendicular (the angle between $\vec{E}$ and $\vec{A}$ is $\phi = 90°$).
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos 90° = 0$.



(b) The normal to the disk is now perpendicular to $\vec{E}$, so $\phi = 90°$, $\cos \phi = 0$, and $\Phi_E = 0$.

Example Problem: Flux through a Disk

(b) What is the flux through the disk if it is turned so that $\hat{n}$ is perpendicular to $\vec{E}$? (c) What is the flux through the disk if $\hat{n}$ is parallel to $\vec{E}$?



(c) The normal to the disk is parallel to $\vec{E}$, so $\phi = 0$ and $\cos\phi = 1$:

$$\Phi_E = EA\cos\phi = (2.0 \times 10^3\ \mathrm{N/C})(0.0314\ \mathrm{m^2})(1)$$
$$= 63\ \mathrm{N \cdot m^2/C}$$

Generalization to Arbitrary *E* and *A*

What happens if the electric field $\vec{E}$ isn't uniform but varies from point to point over the area A? Or what if A is part of a curved surface? Then we divide A into many small elements dA, each of which has a unit vector $\hat{\boldsymbol{n}}$ perpendicular to it and a vector area $d\vec{A} = \hat{\boldsymbol{n}}\, dA$. We calculate the electric flux through each element and integrate the results to obtain the total flux:

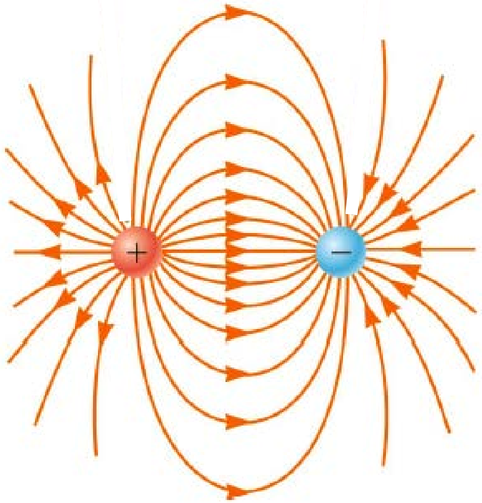
$$\Phi_E = \int E\cos\phi \, dA = \int E_{\perp}\, dA = \int \vec{E}\cdot d\vec{A} \qquad \text{(general definition of electric flux)} \qquad (22.5)$$

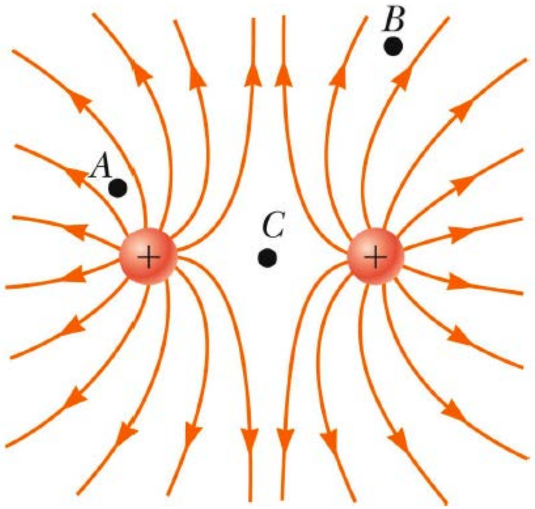
Will come back to this later, but why do we spend so much time discussing flux?

Again: to Understand Gauss's Law

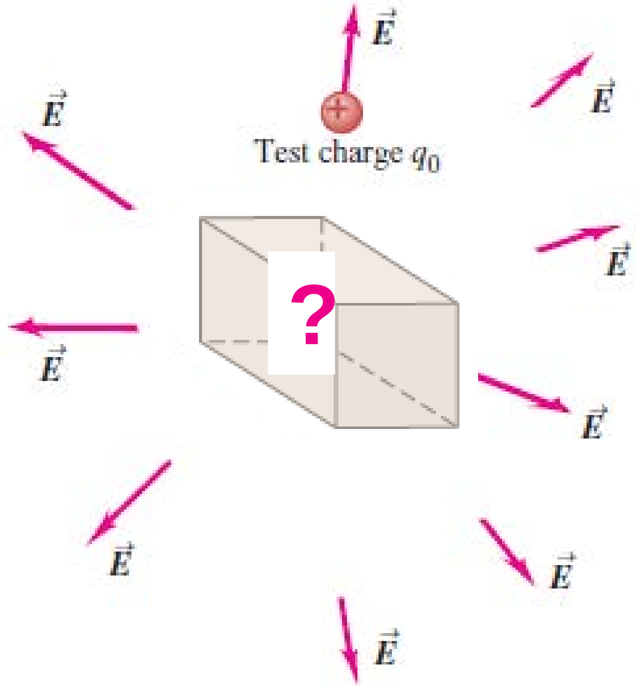
Coulomb's law: from charge distribution to electrical fields

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\vec{e}_r$$



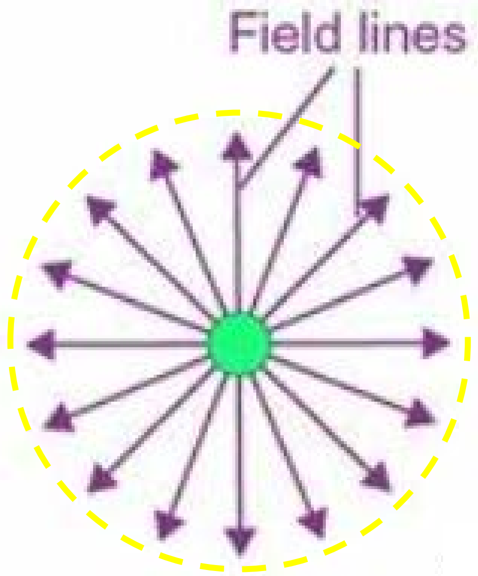


What if we want to calculate charge distribution from electrical fields?



Gauss's law

Gauss’s Law Starting from a Point Charge

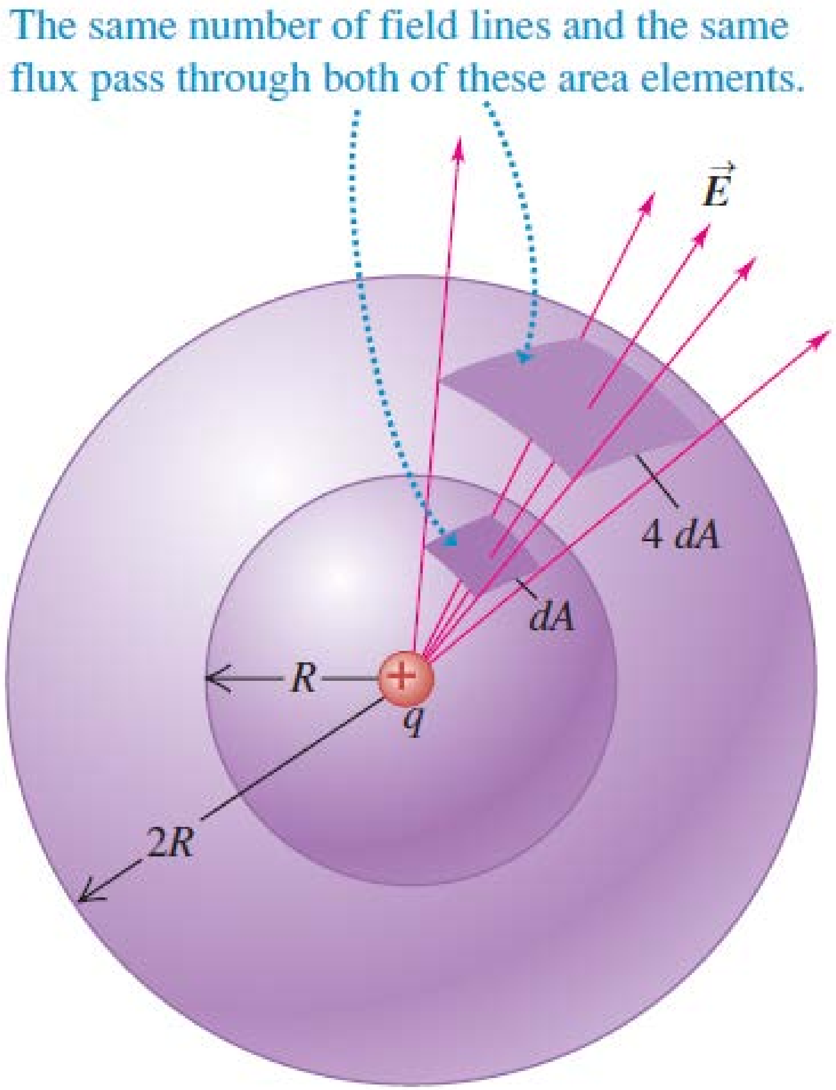


What is the flux of ***E*** from the point charge on the spherical surface $r = R$?

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$$

At each point on the surface, $\vec{E}$ is perpendicular to the surface, and its magnitude is the same at every point, just as in Example 22.3 (Section 22.2). The total electric flux is the product of the field magnitude E and the total area $A = 4\pi R^2$ of the sphere:

Gauss's Law Starting from a Point Charge



What is the flux of ***E*** from the point charge on the spherical surface *r = R*?

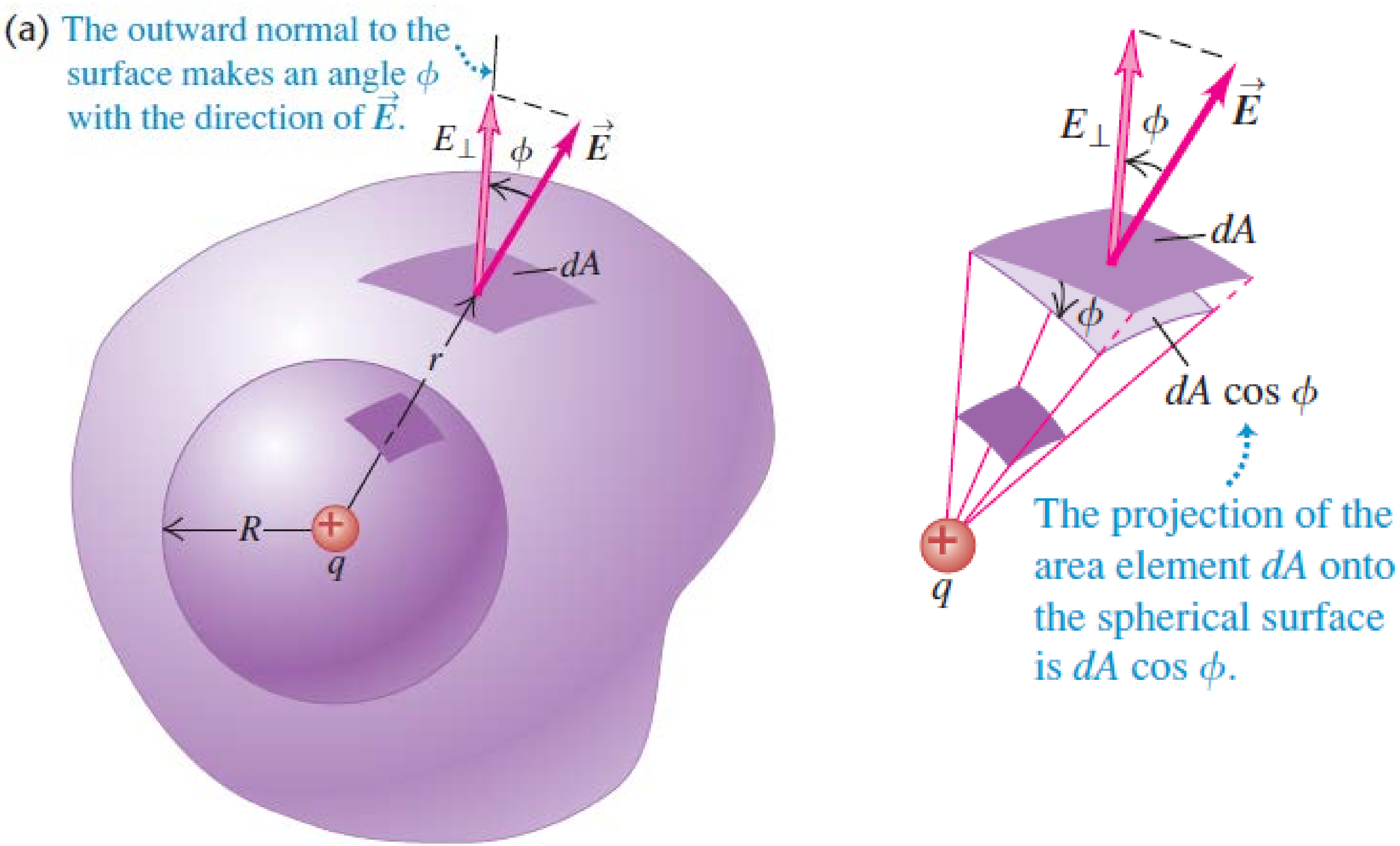
$$\Phi_E = EA = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}(4\pi R^2) = \frac{q}{\epsilon_0}$$

Independent of R! Is this a coincidence?

Short answer: No!

Some analysis: larger sphere means lower electrical field line density but larger area

Gauss's Law: Generalization to Arbitrary Surfaces



Gauss's Law: General Form

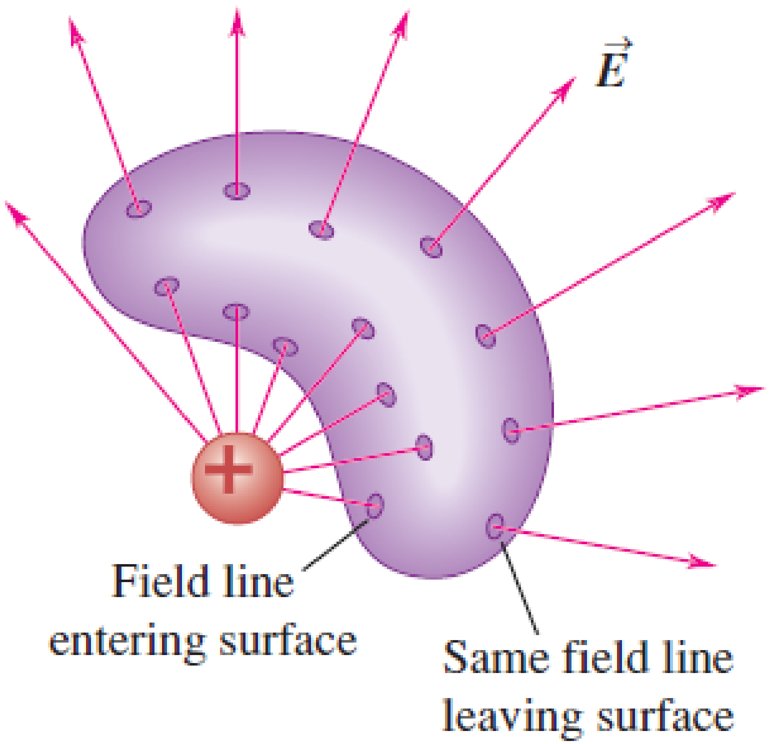
Thus the *total* electric flux through the irregular surface, given by any of the forms of Eq. (22.5), must be the same as the total flux through a sphere, which Eq. (22.6) shows is equal to q/ϵ_0. Thus, for the irregular surface,

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0} \tag{22.7}$$

Equation (22.7) holds for a surface of *any* shape or size, provided only that it is a *closed* surface enclosing the charge q. The circle on the integral sign reminds us that the integral is always taken over a *closed* surface.

Gauss's Law: General Form

22.13 A point charge *outside* a closed surface that encloses no charge. If an electric field line from the external charge enters the surface at one point, it must leave at another.



For a closed surface enclosing *no* charge,

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = 0$$

Electric field lines can begin or end inside a region of space only when there is charge in that region.

Gauss's Law: General Form

The total (resultant) electric field $\vec{E}$ at any point is the vector sum of the $\vec{E}$ fields of the individual charges. Let Q_{encl} be the *total* charge enclosed by the surface: $Q_{\text{encl}} = q_1 + q_2 + q_3 + \cdots$

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \qquad \text{(Gauss's law)} \qquad (22.8)$$

The total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by ϵ_0.

What does Gauss's law do for us?

Derive Q from measurements of *E* (quantitatively)

Gauss's Law: General Form

The total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by ϵ_0.

CAUTION **Gaussian surfaces are imaginary** Remember that the closed surface in Gauss's law is *imaginary;* there need not be any material object at the position of the surface. We often refer to a closed surface used in Gauss's law as a **Gaussian surface.**

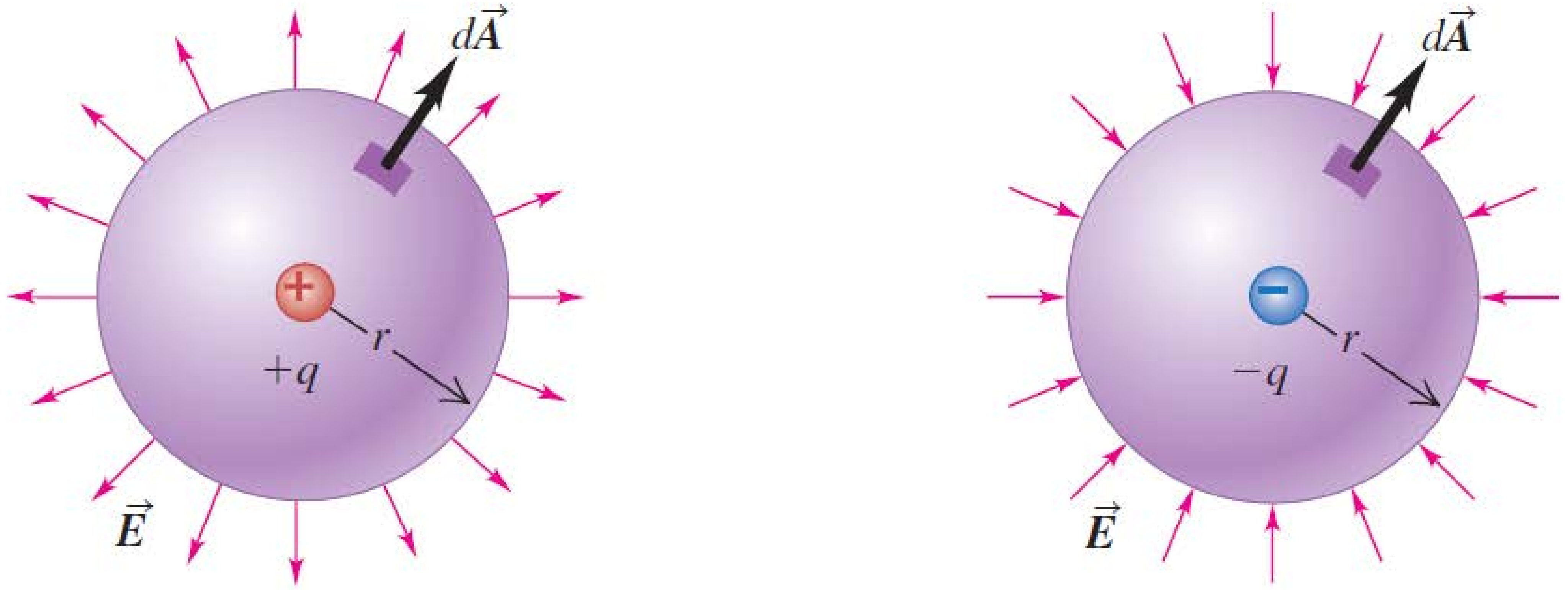
Using the definition of Q_{encl} and the various ways to express electric flux given in Eq. (22.5), we can express Gauss's law in the following equivalent forms:

$$\Phi_E = \oint E \cos\phi \, dA = \oint E_{\perp} \, dA = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \quad \text{(various forms of Gauss's law)} \qquad (22.9)$$

Gauss's Law: General Form

(a) Gaussian surface around positive charge: positive (outward) flux

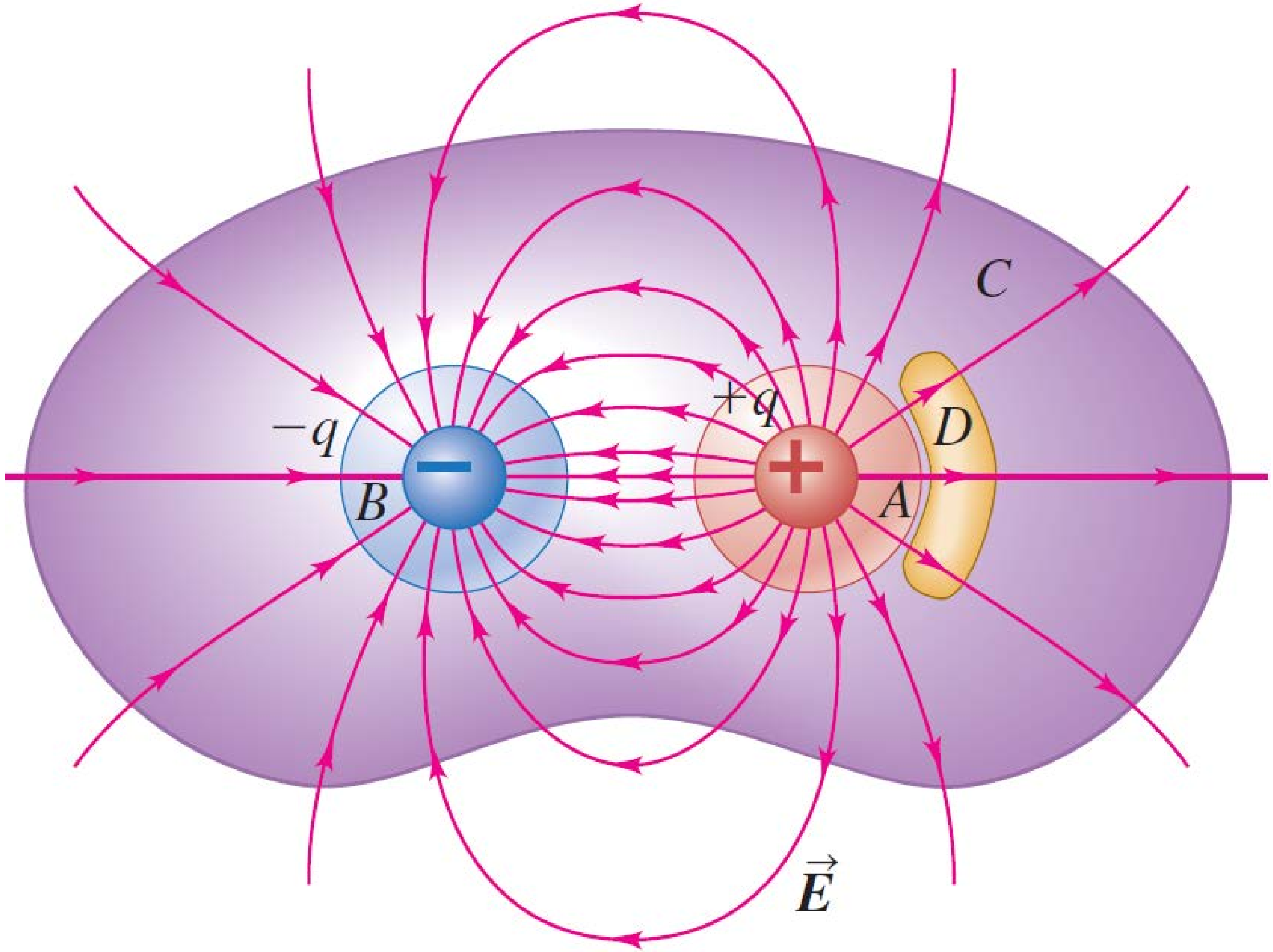
(b) Gaussian surface around negative charge: negative (inward) flux



$$\Phi_E = \oint E_{\perp}\, dA = \oint \left(\frac{\pm q}{4\pi\epsilon_0 r^2}\right) dA = \frac{\pm q}{4\pi\epsilon_0 r^2} \oint dA = \frac{\pm\ q}{4\pi\epsilon_0 r^2}\, 4\pi r^2 = \frac{\pm q}{\epsilon_0}$$

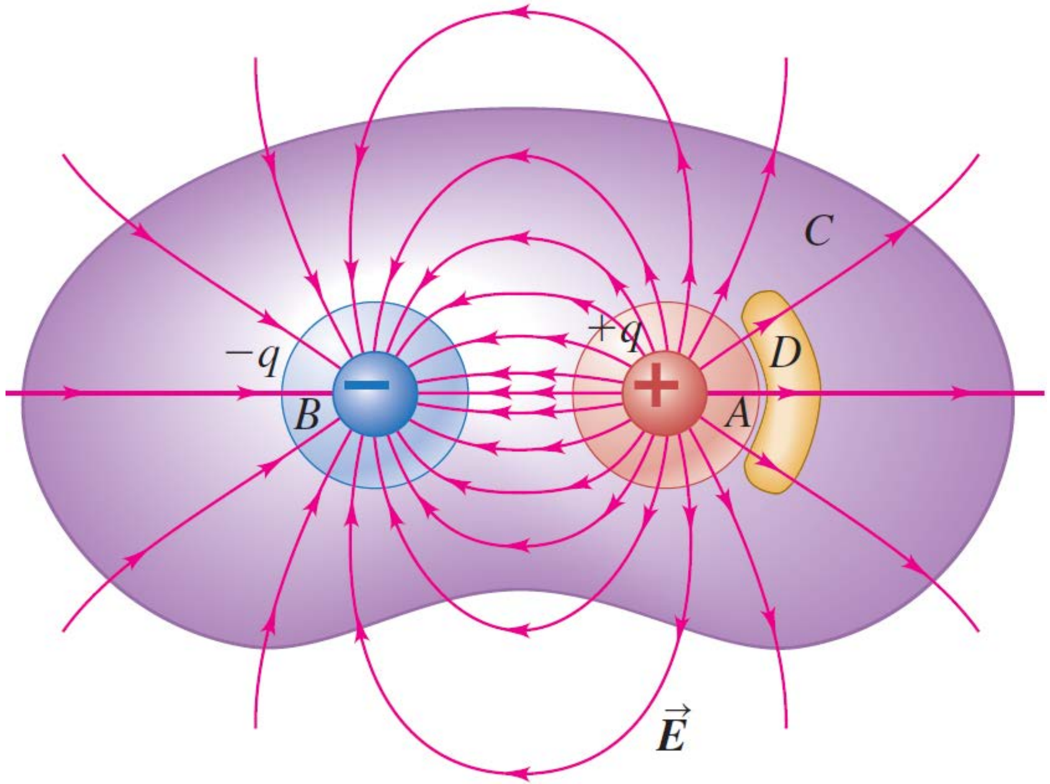
Conceptual Example 22.4 Electric flux and enclosed charge

Figure 22.15 shows the field produced by two point charges $+q$ and $-q$ (an electric dipole). Find the electric flux through each of the closed surfaces A, B, C, and D.



Conceptual Example 22.4 Electric flux and enclosed charge

Fig. 22.15, surface A (shown in red) encloses the positive charge, so $Q_{encl} = +q$; surface B (in blue) encloses the negative charge, so $Q_{encl} = -q$; surface C (in purple) encloses *both* charges, so $Q_{encl} = +q + (-q) = 0$; and surface D (in yellow) encloses no charges, so $Q_{encl} = 0$. Hence, without having to do any integration, we have $\Phi_{EA} = +q/\epsilon_0$, $\Phi_{EB} = -q/\epsilon_0$, and $\Phi_{EC} = \Phi_{ED} = 0$.



Example 22.5 Field of a charged conducting sphere

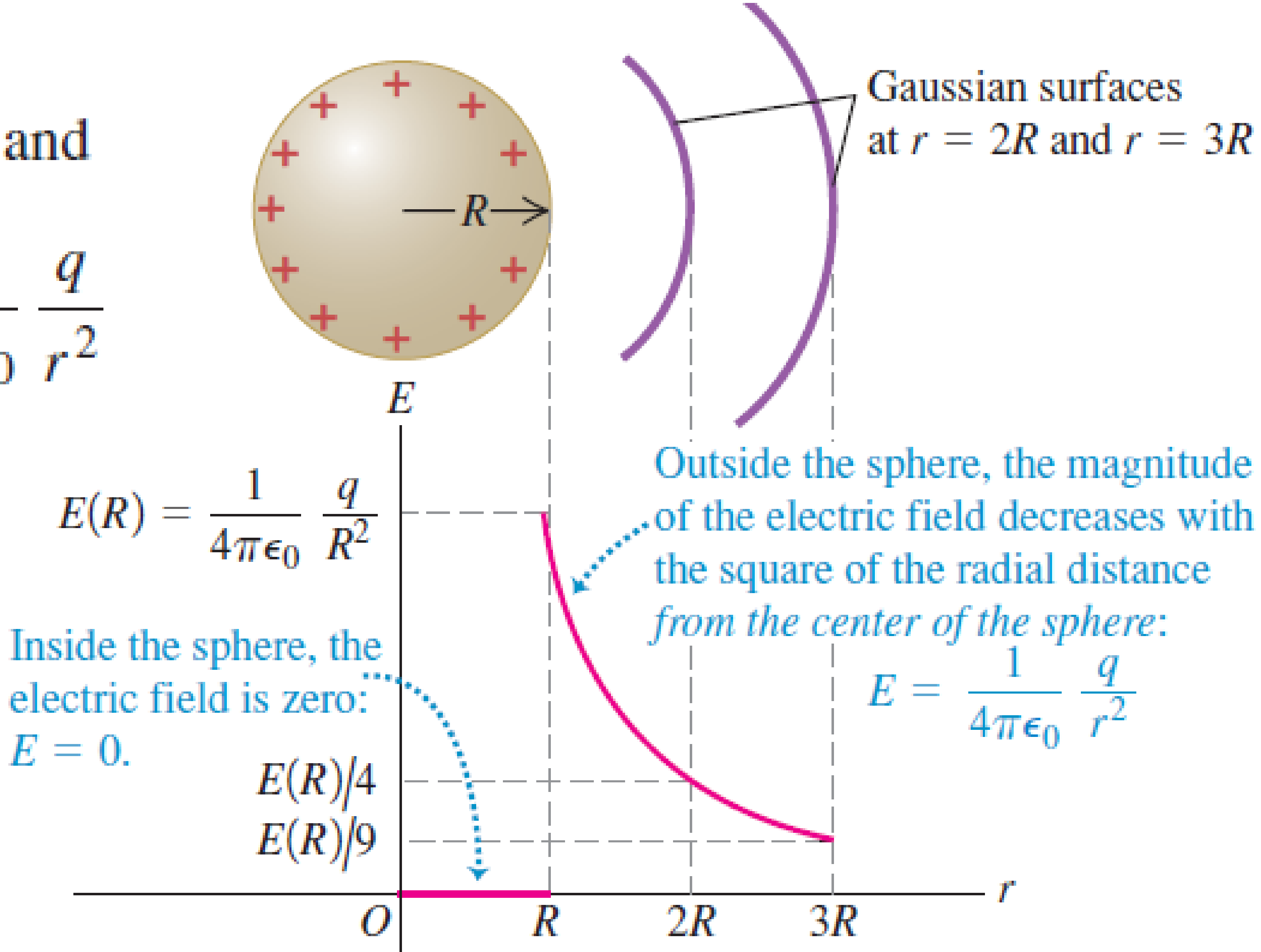
We place a total positive charge q on a solid conducting sphere with radius R (Fig. 22.18). Find $\vec{E}$ at any point inside or outside the sphere.

$$E(4\pi r^2) = \frac{q}{\epsilon_0} \quad \text{and}$$

(outside a charged conducting sphere) $$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

Just outside the surface of the sphere

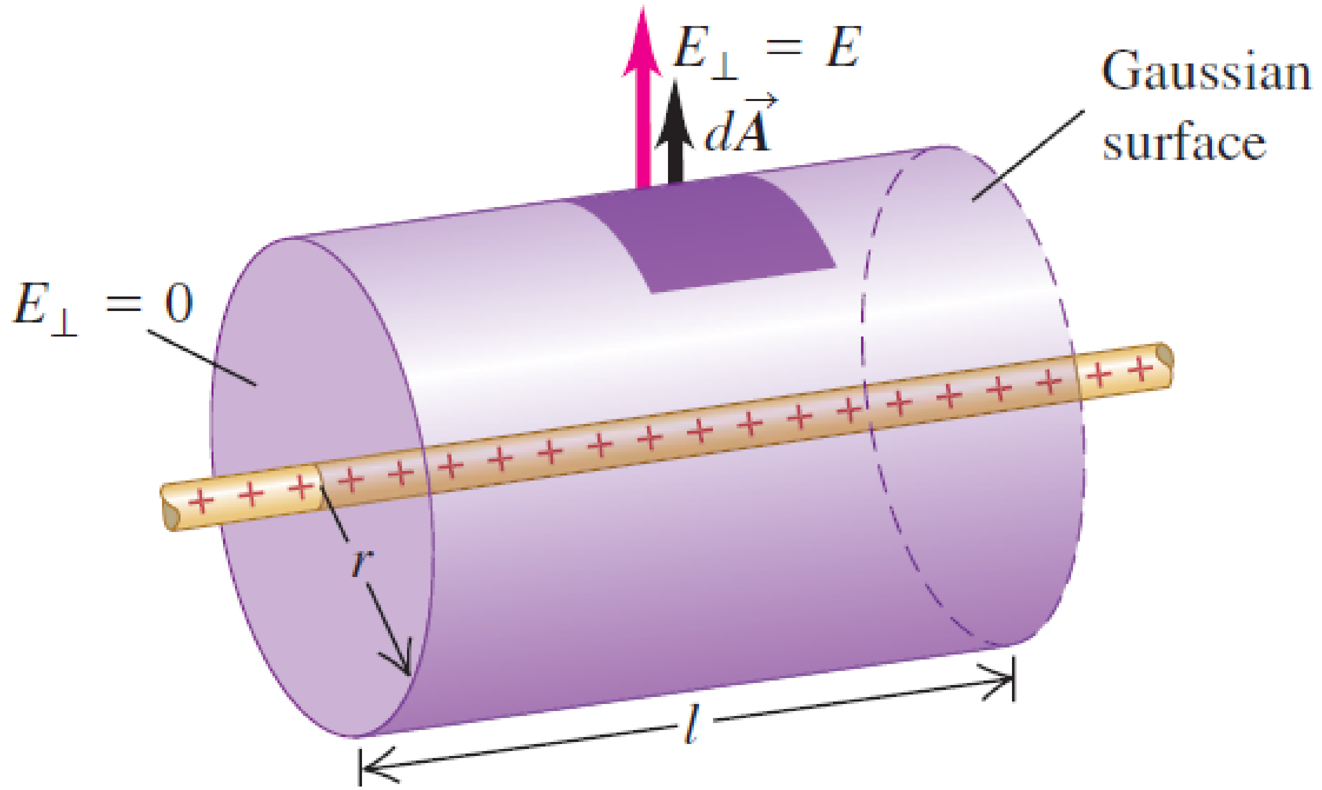
$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$$



Example 22.6 Field of a uniform line charge

Electric charge is distributed uniformly along an infinitely long, thin wire. The charge per unit length is λ (assumed positive). Find the electric field using Gauss's law.

22.19 A coaxial cylindrical Gaussian surface is used to find the electric field outside an infinitely long, charged wire.



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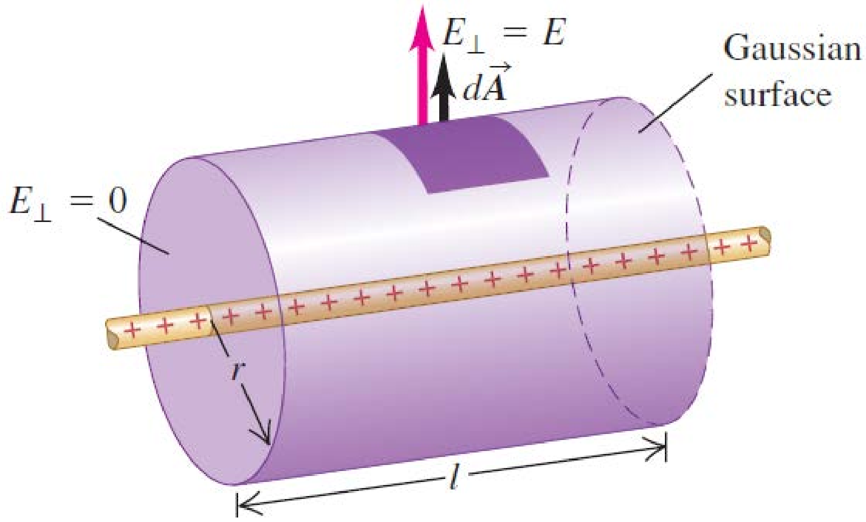


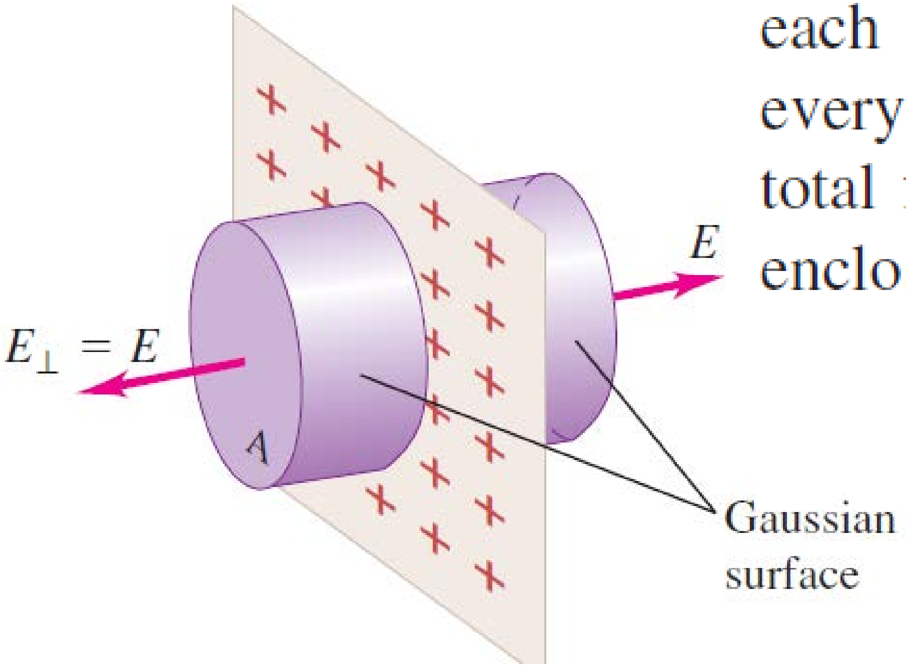
$\vec{E} \cdot \hat{n} = E_\perp = -E$ everywhere.) The area of the cylindrical surface is $2\pi rl$, so the flux through it—and hence the total flux Φ_E through the Gaussian surface—is $EA = 2\pi rlE$. The total enclosed charge is $Q_{encl} = \lambda l$, and so from Gauss's law, Eq. (22.8),

$$\Phi_E = 2\pi rlE = \frac{\lambda l}{\epsilon_0} \quad \text{and}$$

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r} \qquad \text{(field of an infinite line of charge)}$$

Example 22.7 Field of a uniform line charge

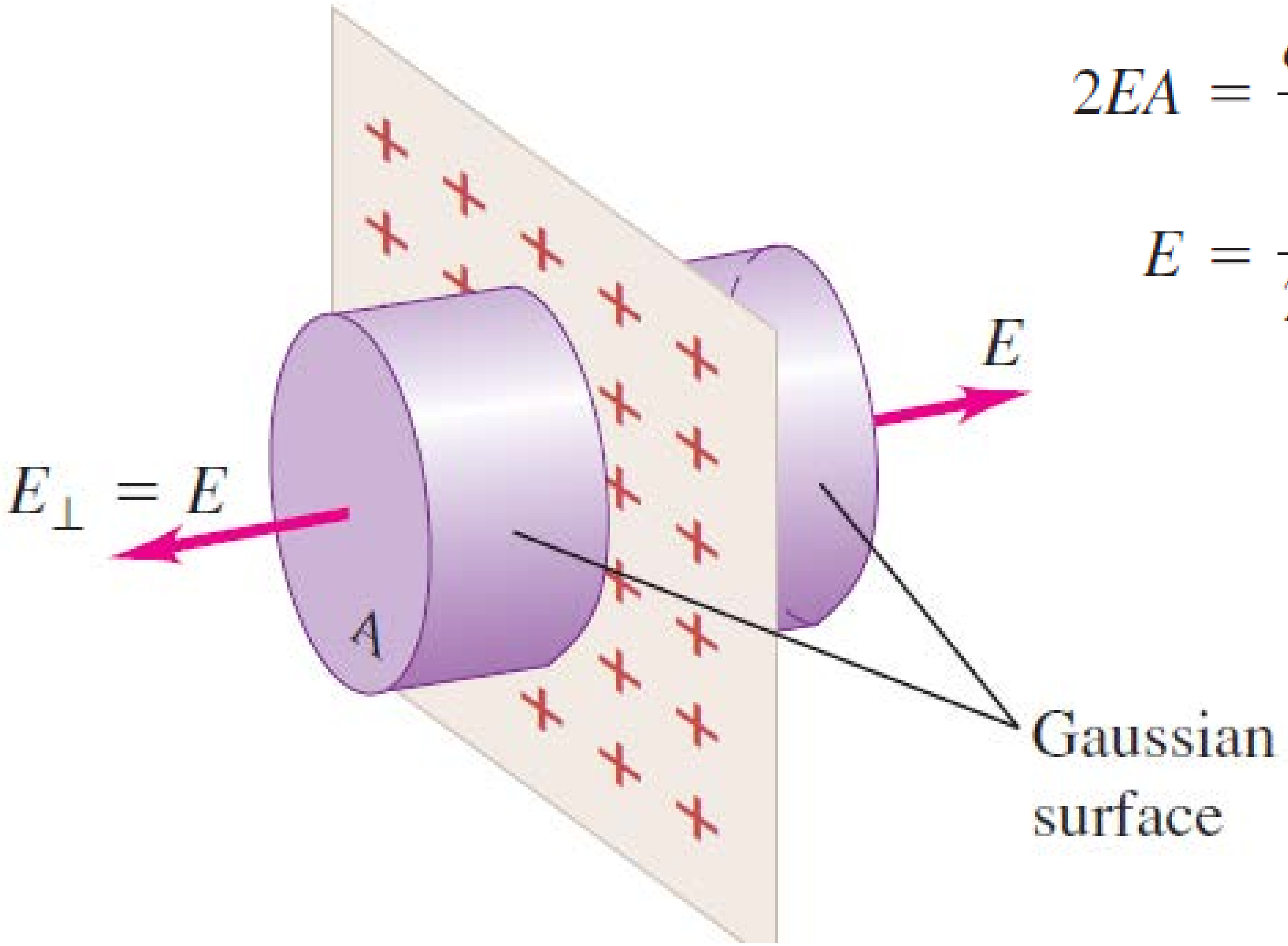
Use Gauss's law to find the electric field E caused by a thin, flat, infinite sheet with a uniform positive surface charge density σ.



EXECUTE: The flux through the cylindrical part of our Gaussian surface is zero because $\vec{E} \cdot \hat{n} = 0$ everywhere. The flux through each flat end of the surface is $+EA$ because $\vec{E} \cdot \hat{n} = E_\perp = E$ everywhere, so the total flux through both ends—and hence the total flux Φ_E through the Gaussian surface—is $+2EA$. The total enclosed charge is $Q_{encl} = \sigma A$, and so from Gauss's law,

Example 22.7 Field of a uniform line charge

Use Gauss's law to find the electric field E caused by a thin, flat, infinite sheet with a uniform positive surface charge density σ.

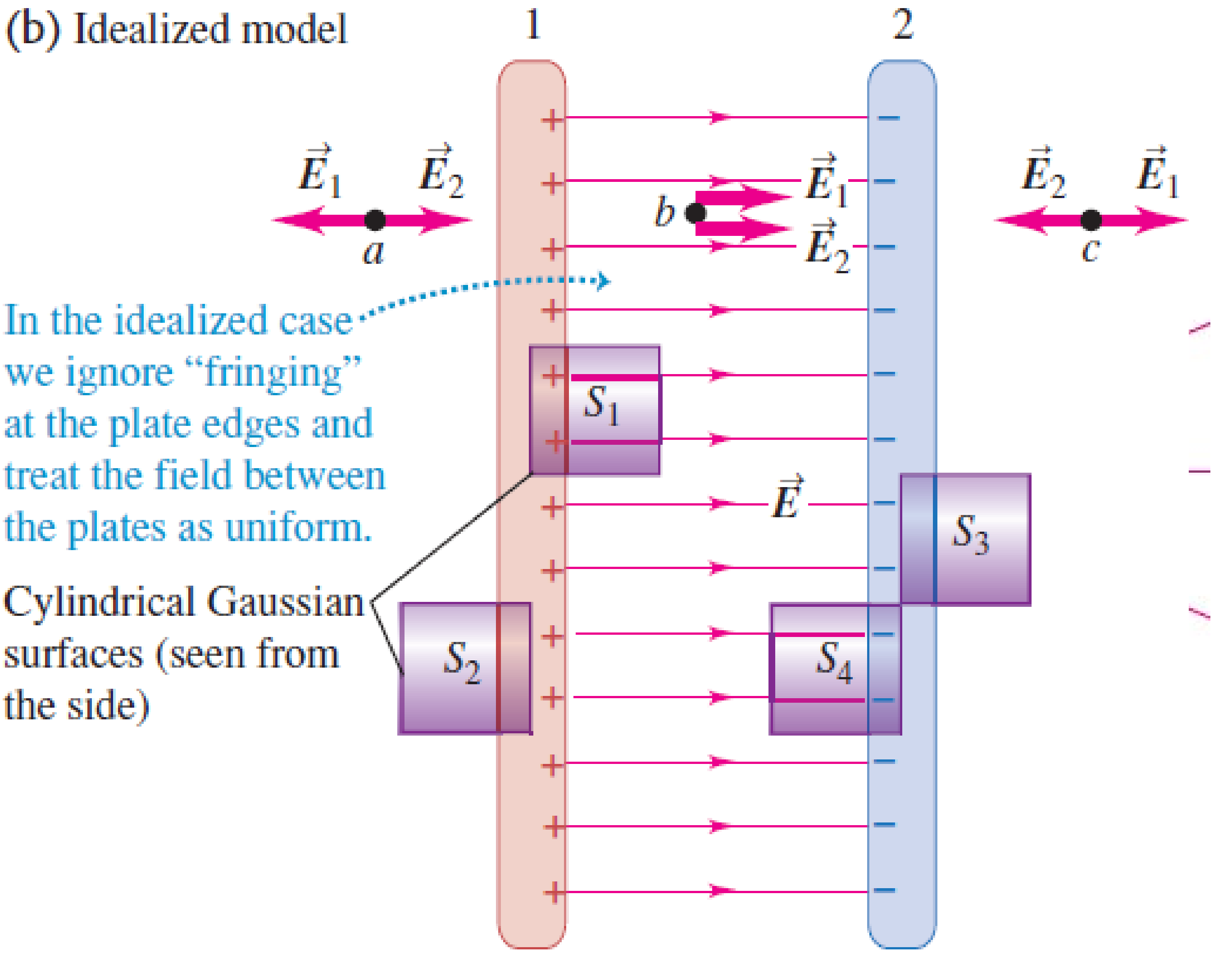


$$2EA = \frac{\sigma A}{\epsilon_0} \quad \text{and}$$

$$E = \frac{\sigma}{2\epsilon_0} \qquad \text{(field of an infinite sheet of charge)}$$

Example 22.8 Field between oppositely charged parallel conducting plates

Two large plane parallel conducting plates are given charges of equal magnitude and opposite sign; the surface charge densities are $+\sigma$ and $-\sigma$. Find the electric field in the region between the plates.

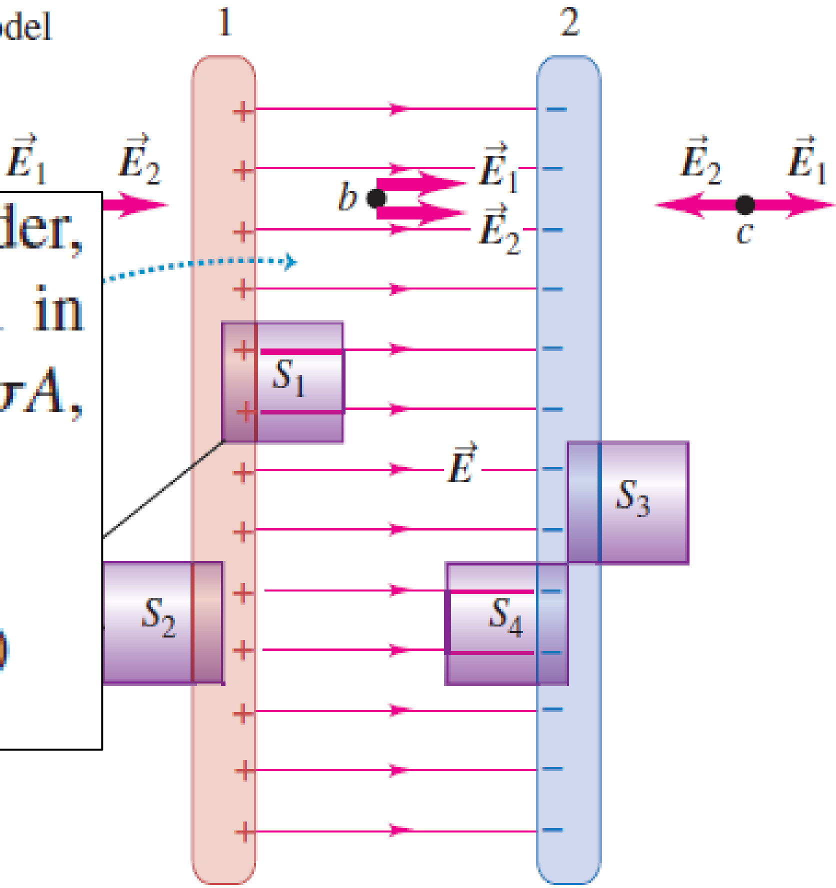


we can assume that the field is uniform in the interior region between the plates, as in Fig. 22.21b, and that the charges are distributed uniformly over the opposing surfaces. To exploit this symmetry, we can use the shaded Gaussian surfaces and These surfaces are cylinders with flat ends of area A; one end of each surface lies *within* a plate

Example 22.8 Field between oppositely charged parallel conducting plates

Two large plane parallel conducting plates are given charges of equal magnitude and opposite sign; the surface charge densities are $+\sigma$ and $-\sigma$. Find the electric field in the region between the plates.

(b) Idealized model

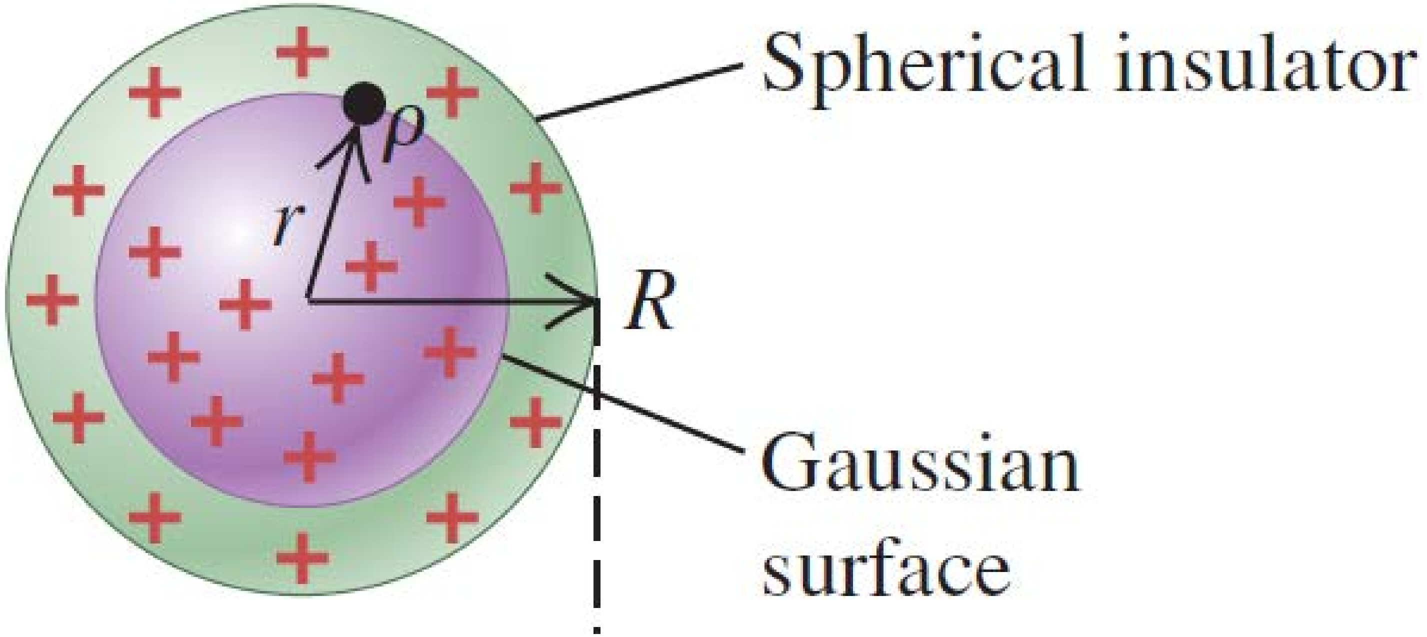


surface. There is no flux through the side walls of the cylinder, since these walls are parallel to $\vec{E}$. So the total flux integral in Gauss's law is EA. The net charge enclosed by the cylinder is σA, so Eq. (22.8) yields $EA = \sigma A/\epsilon_0$; we then have

$$E = \frac{\sigma}{\epsilon_0} \text{ (field between oppositely charged conducting plates)}$$

Example 22.9 Field of a uniformly charged sphere

Positive electric charge Q is distributed uniformly *throughout the volume* of an *insulating* sphere with radius R. Find the magnitude of the electric field at a point P a distance r from the center of the sphere.



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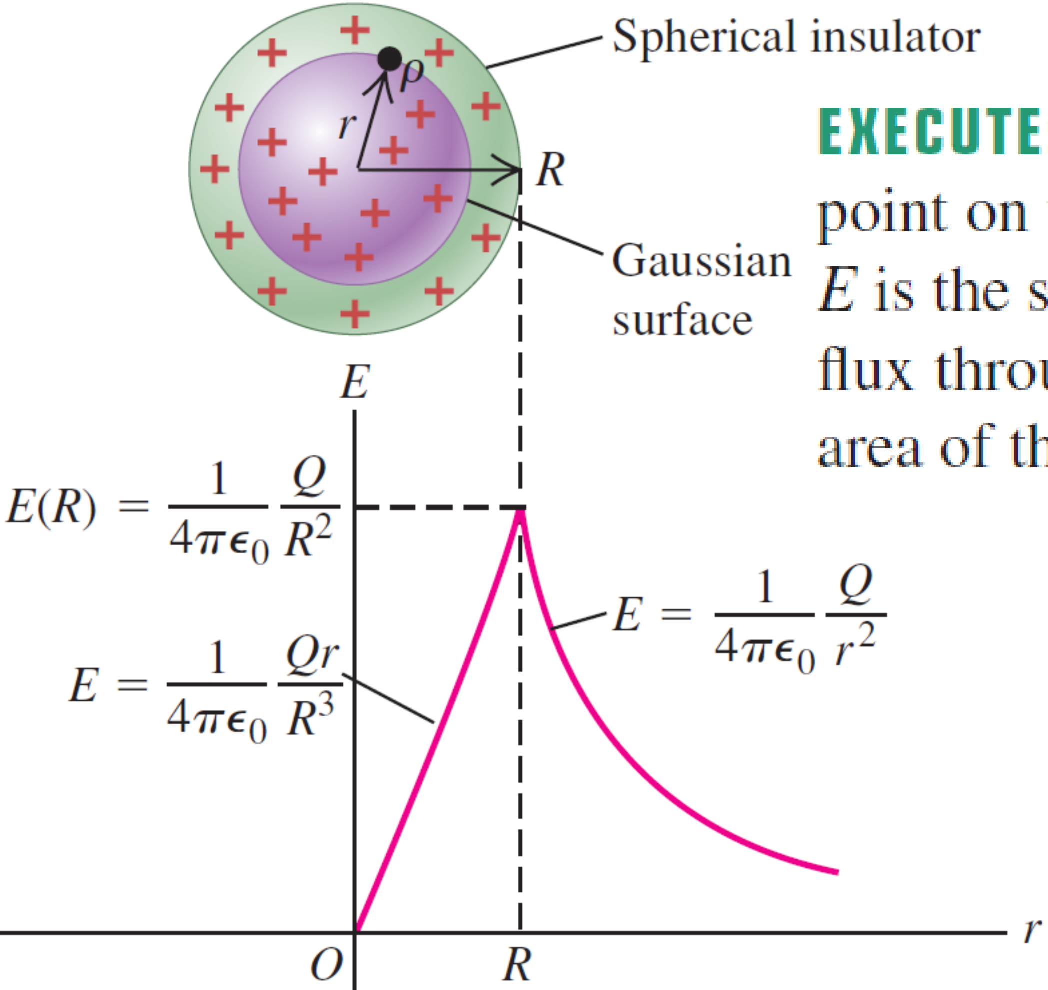
Positive electric charge Q is distributed uniformly *throughout the volume* of an *insulating* sphere with radius R. Find the magnitude of the electric field at a point P a distance r from the center of the sphere.



EXECUTE: From symmetry, the direction of $\vec{E}$ is radial at every point on the Gaussian surface, so $E_\perp = E$ and the field magnitude E is the same at every point on the surface. Hence the total electric flux through the Gaussian surface is the product of E and the total area of the surface $A = 4\pi r^2$—that is, $\Phi_E = 4\pi r^2 E$.

$$\rho = \frac{Q}{4\pi R^3/3}$$

Example 22.9 Field of a uniformly charged sphere

Positive electric charge Q is distributed uniformly *throughout the volume* of an *insulating* sphere with radius R. Find the magnitude of the electric field at a point P a distance r from the center of the sphere.

The volume V_{encl} enclosed by the Gaussian surface is $\frac{4}{3}\pi r^3$, so the total charge Q_{encl} enclosed by that surface is

$$Q_{\text{encl}} = \rho V_{\text{encl}} = \left(\frac{Q}{4\pi R^3/3}\right)\left(\tfrac{4}{3}\pi r^3\right) = Q\frac{r^3}{R^3}$$

Then Gauss's law, Eq. (22.8), becomes

$$4\pi r^2 E = \frac{Q}{\epsilon_0}\frac{r^3}{R^3} \quad \text{or}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3} \quad \text{(field inside a uniformly charged sphere)}$$

Example 22.9 Field of a uniformly charged sphere

Positive electric charge Q is distributed uniformly *throughout the volume* of an *insulating* sphere with radius R. Find the magnitude of the electric field at a point P a distance r from the center of the sphere.

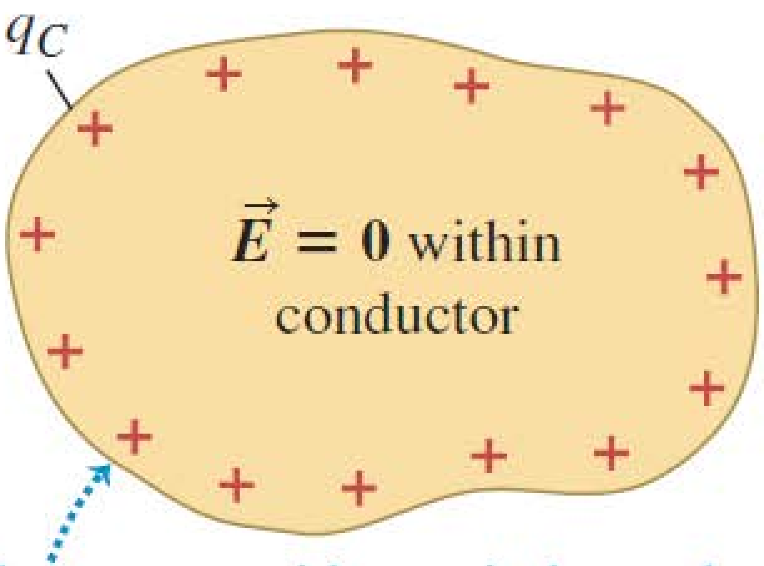
To find E *outside* the sphere, we take $r > R$. This surface encloses the entire charged sphere, so $Q_{\text{encl}} = Q$, and Gauss's law gives

$$4\pi r^2 E = \frac{Q}{\epsilon_0} \quad \text{or}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \quad \text{(field outside a uniformly charged sphere)}$$

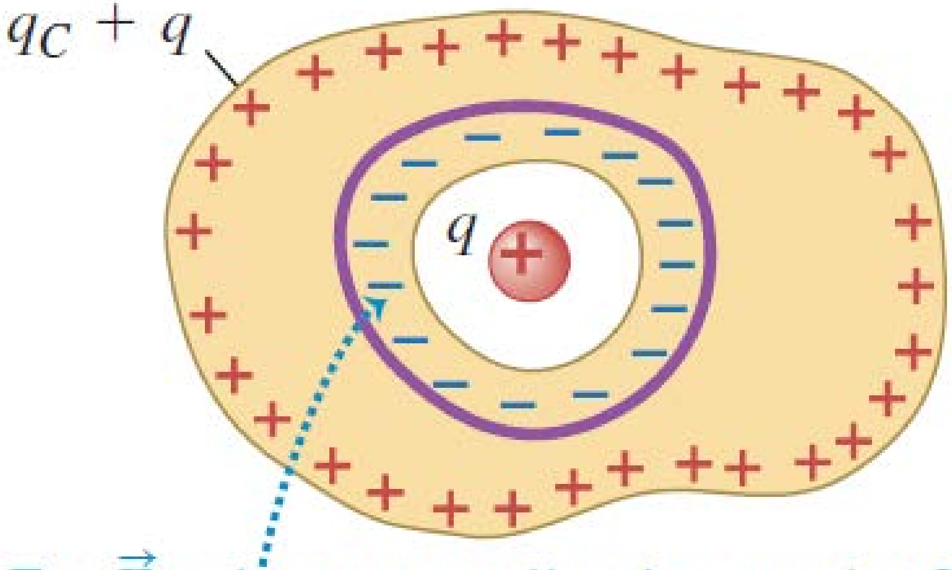
Charges on Conductors

(a) Solid conductor with charge q_C



The charge q_C resides entirely on the surface of the conductor. The situation is electrostatic, so $\vec{E} = \mathbf{0}$ within the conductor.

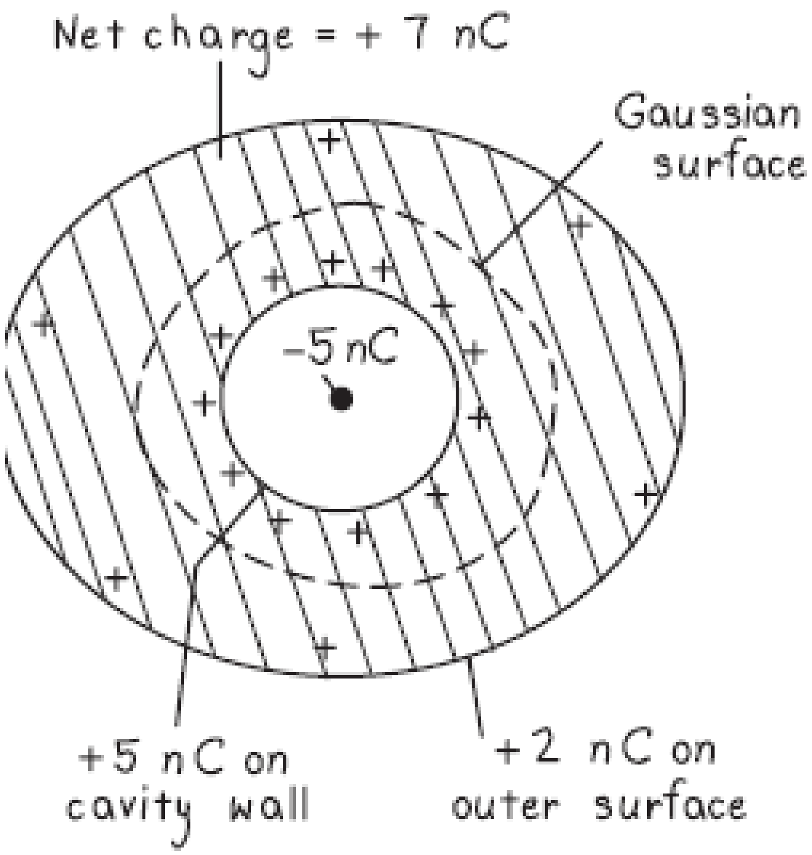
(c) An isolated charge q placed in the cavity



For $\vec{E}$ to be zero at all points on the Gaussian surface, the surface of the cavity must have a total charge $-q$.

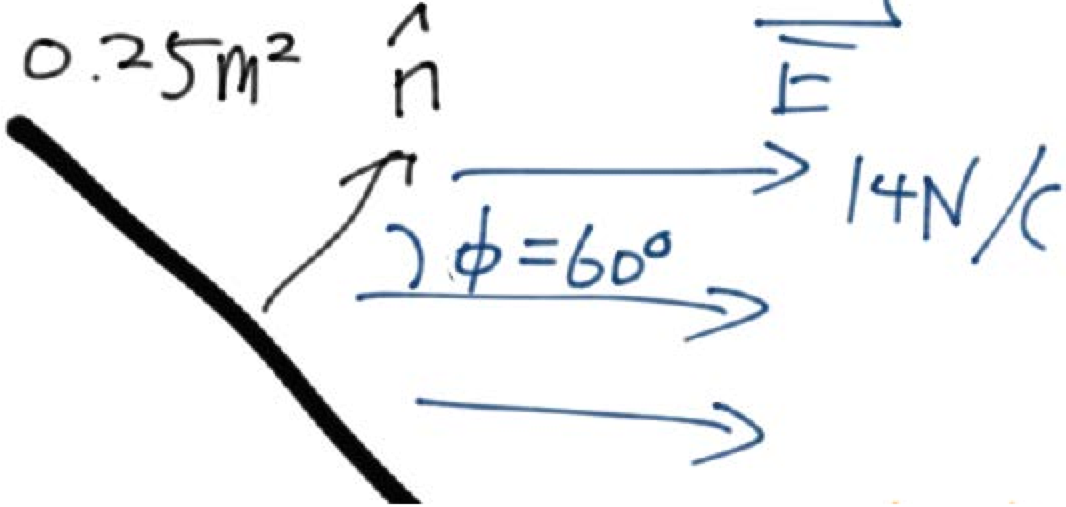
Conceptual Example 22.11 A conductor with a cavity

A solid conductor with a cavity carries a total charge of +7 nC. Within the cavity, insulated from the conductor, is a point charge of -5 nC. How much charge is on each surface (inner and outer) of the conductor?



There is zero electric field inside the bulk conductor and hence zero flux through the Gaussian surface shown, so the charge on the cavity wall must be the opposite of the point charge.

22.1 • A flat sheet of paper of area 0.250 m^2 is oriented so that the normal to the sheet is at an angle of 60° to a uniform electric field of magnitude 14 N/C. (a) Find the magnitude of the electric flux through the sheet. (b) Does the answer to part (a) depend on the shape of the sheet? Why or why not? (c) For what angle ϕ between the normal to the sheet and the electric field is the magnitude of the flux through the sheet (i) largest and (ii) smallest? Explain your answers.

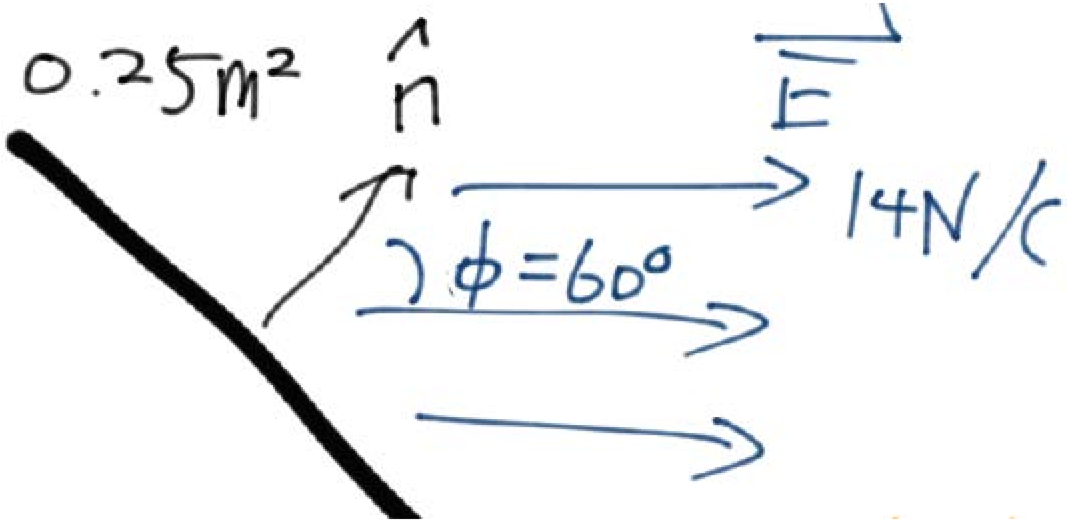


$$\vec{dA} = \hat{n}\, dA$$

$$\Phi = \int \vec{E} \cdot d\vec{A}$$

$$= \int \vec{E} \cdot \hat{n}\, dA$$

independent of A



(C) $\Phi = \int \vec{E} \cdot \hat{n} \cdot dA$

$= \vec{E} \cdot \hat{n} \cdot A$

$= EA \cdot \cos\phi$

$\Phi_{max} = EA \qquad \phi = 0 \text{ or } 180^\circ$

$\Phi_{min} = 0 \qquad \phi = 90$

$\vec{dA} = \hat{n}\, dA$

$\Phi = \int \vec{E} \cdot d\vec{A}$

$= \int \vec{E} \cdot \hat{n}\, dA$

$= \int |\vec{E}| \cdot \cos\phi\, dA$

$= \frac{E}{2} \int dA$

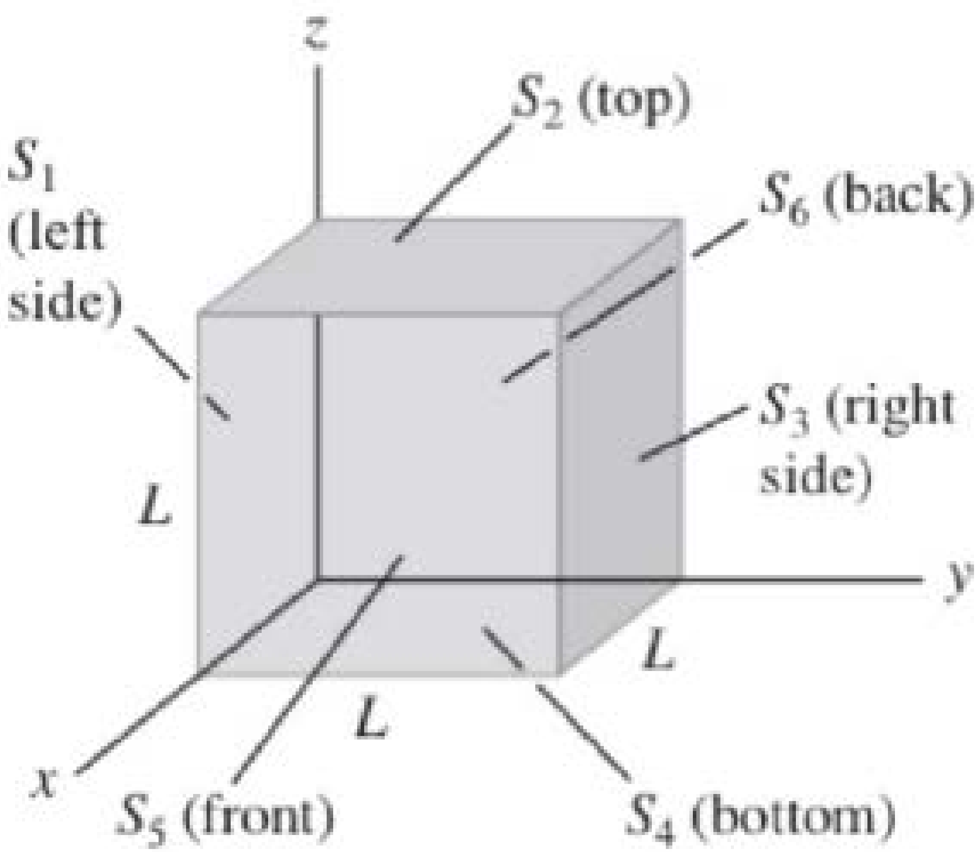
$= \frac{EA}{2}$

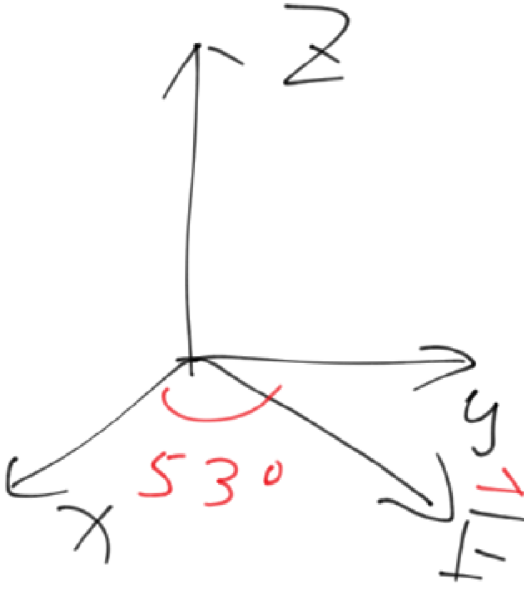
$= 14N/C \cdot 0.25\, m^2/2$

$= 1.75\ N \cdot m^2/C$

22.6 • The cube in Fig. E22.6 has sides of length $L = 10.0$ cm. The electric field is uniform, has magnitude $E = 4.00 \times 10^3$ N/C, and is parallel to the xy-plane at an angle of 53.1° measured from the $+x$-axis toward the $+y$-axis. (a) What is the electric flux through each of the six cube faces S_1, S_2, S_3, S_4, S_5, and S_6? (b) What is the total electric flux through all faces of the cube?

Figure **E22.6**

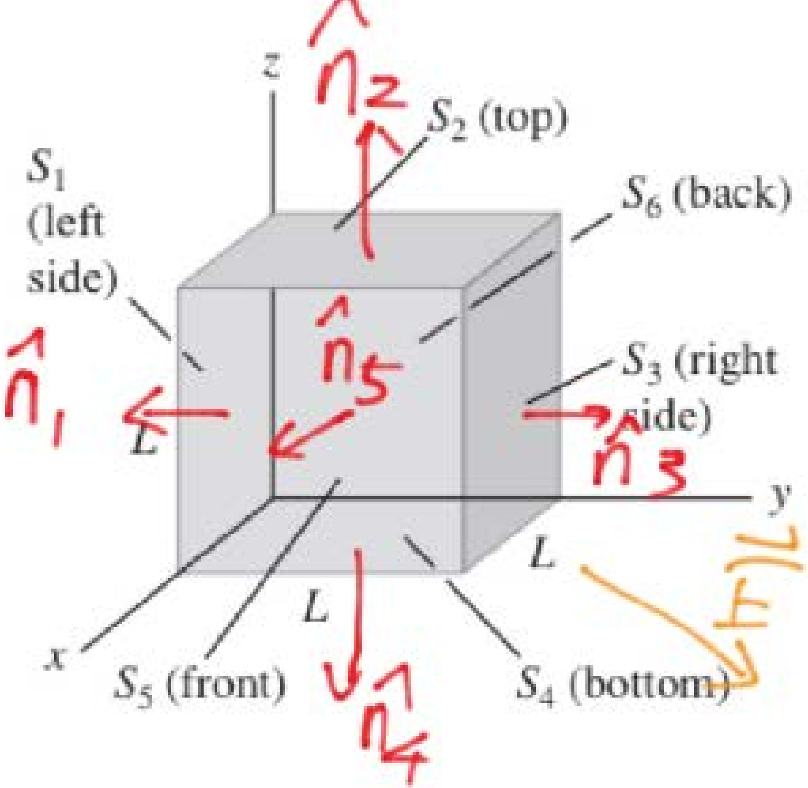




Convention: $\hat{n}$ points outward

22.6 • The cube in Fig. E22.6 has sides of length L =10.0 cm. The electric field is uniform, has magnitude $E = 4.00 \times 10^3$ N/C, and is parallel to the xy-plane at an angle of 53.1° measured from the $+x$-axis toward the $+y$-axis. (a) What is the electric flux through each of the six cube faces S_1, S_2, S_3, S_4, S_5, and S_6? (b) What is the total electric flux through all faces of the cube?

Figure **E22.6**



$\hat{n}_1 = -\hat{j} = (0,-1,0)$

$\hat{n}_2 = \hat{k} = (0,0,1)$

$\hat{n}_3 = \hat{j} = (0,1,0)$

$\hat{n}_4 = -\hat{k} = (0,0,-1)$

$\hat{n}_5 = \hat{i} = (1,0,0)$

$\hat{n}_6 = -\hat{i} = (-1,0,0)$

$\hat{E} = 0.6E\,\hat{i} + 0.8E\,\hat{j}$

$$\Phi = \int \vec{E}\cdot d\vec{A} = \underline{\vec{E}\cdot\vec{A} = \vec{E}\cdot\hat{n}A}$$

flat surface

uniform $\vec{E}$

$$\Phi_i = \vec{E}\cdot\hat{n}_i\cdot L^2$$

$$\Phi_1 = (0.6E\,\hat{i} + 0.8E\,\hat{j})\cdot(-\hat{j})\cdot L^2 = -0.8EL^2$$

$$\Phi_2 = (0.6E\,\hat{i} + 0.8E\,\hat{j})\cdot\hat{k} = 0 \quad \text{same for } \hat{n}_4$$

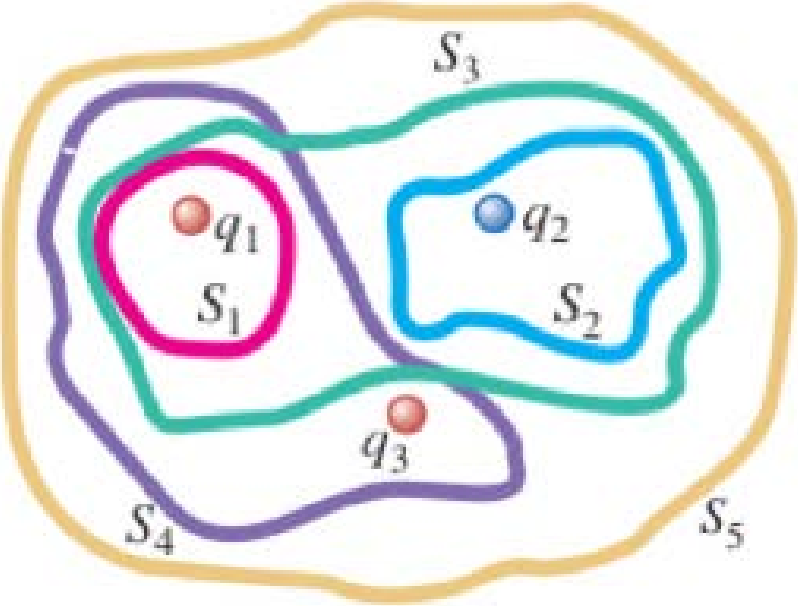
$$\Phi_3 = -\Phi_1 = 0.8EL^2$$

$$\Phi_5 = 0.6EL^2$$

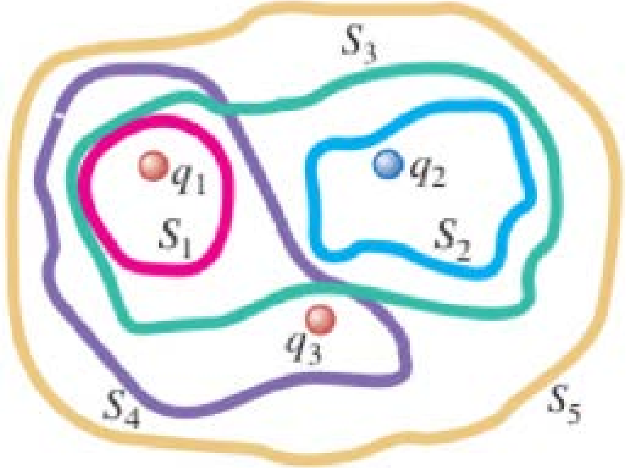
$$\Phi_6 = -0.6EL^2$$

$$\sum_{i=1,2\ldots,6} \Phi_i = 0.$$

22.8 • The three small spheres shown in Fig. E22.8 carry charges $q_1 = 4.00$ nC, $q_2 = -7.80$ nC, and $q_3 = 2.40$ nC. Find the net electric flux through each of the following closed surfaces shown in cross section in the figure: (a) S_1; (b) S_2; (c) S_3; (d) S_4; (e) S_5. (f) Do your answers to parts (a)–(e) depend on how the charge is distributed over each small sphere? Why or why not?



Surface	What it encloses
S_1	q_1
S_2	q_2
S_3	q_1 and q_2
S_4	q_1 and q_3
S_5	q_1 and q_2 and q_3



Surface	What it encloses
S_1	q_1
S_2	q_2
S_3	q_1 and q_2
S_4	q_1 and q_3
S_5	q_1 and q_2 and q_3

$$\Phi_{E,1} = \frac{q_1}{\epsilon_0}$$

$$= \frac{4.0\times10^{-9}\,C}{8.854\times10^{-12}\,C^2/(N\cdot m^2)}$$

$$= 451.8\ N\cdot m^2/C$$

$$\Phi_{E,2} = \frac{q_2}{\epsilon_0}$$

$$= \frac{-7.8\times10^{-9}\,C}{8.854\times10^{-12}\,C^2/(N\cdot m^2)}$$

$$= -881.\ N\cdot m^2/C$$

$$\Phi_{E,3} = \frac{q_1+q_2}{\epsilon_0}$$

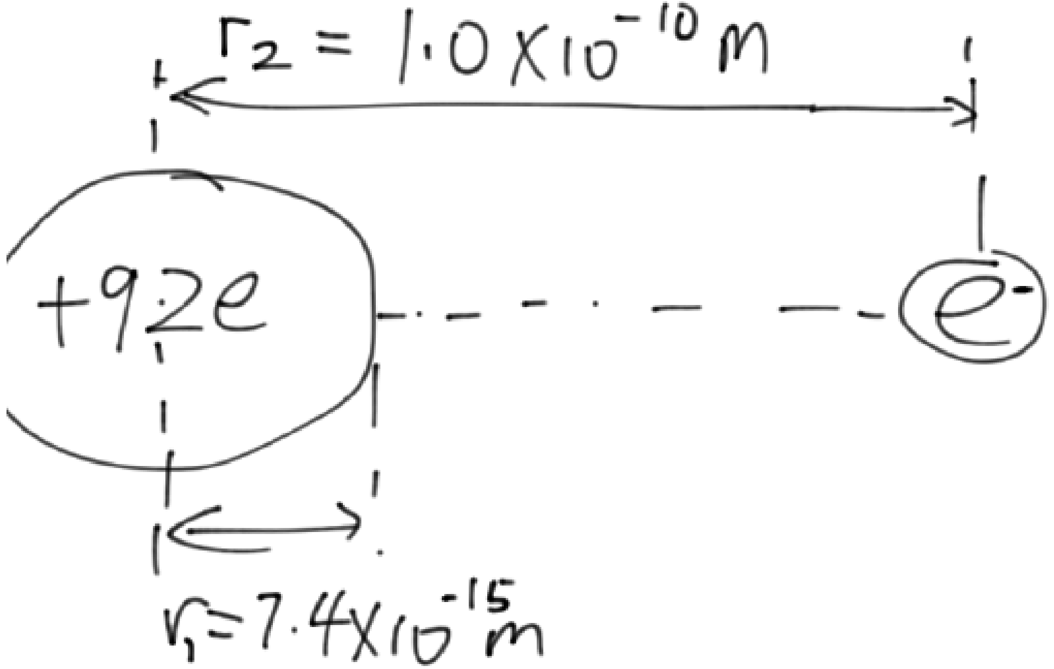
$$= \frac{(4.0-7.8)\times10^{-9}\,C}{8.854\times10^{-12}\,C^2/(m^2\cdot N)}$$

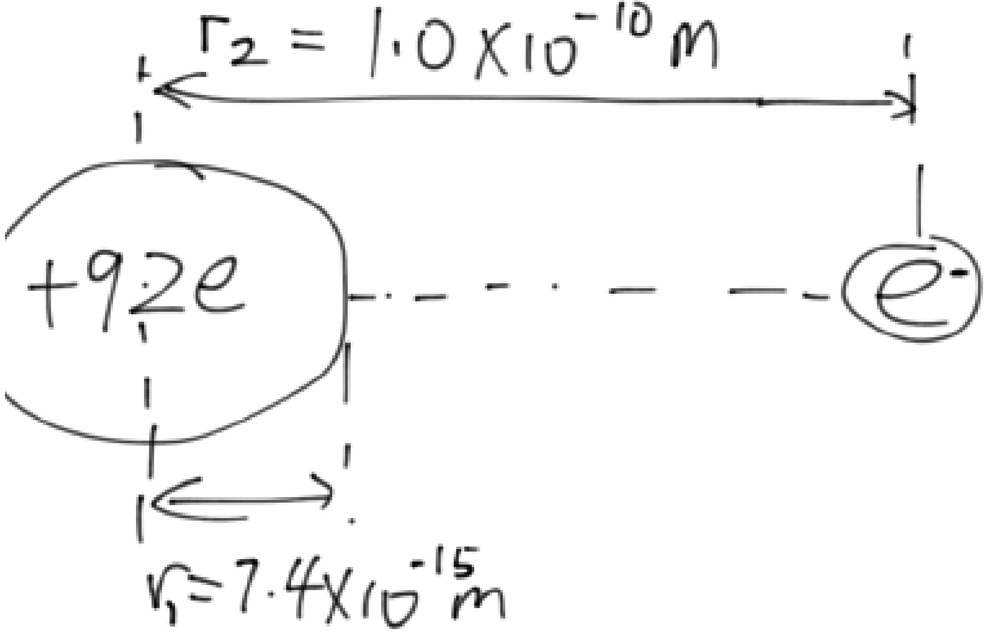
$$= -429.2\ N\cdot m^2/C$$

$$\Phi_{E,4} = \frac{q_1+q_3}{\epsilon_0} = \frac{(4.0+2.4)\times10^{-9}\,C}{8.854\times10^{-12}\,C^2/(N\cdot m^2)} = 722.8\ N\cdot m^2/C$$

$$\Phi_{E,5} = \frac{q_1+q_2+q_3}{\epsilon_0} = \frac{(4.0-7.8+2.4)\times10^{-9}\,C}{8.854\times10^{-12}\,C^2/(N\cdot m^2)} = -158.1\ N\cdot m^2/C$$

22.12 • **Electric Fields in an Atom.** The nuclei of large atoms, such as uranium, with 92 protons, can be modeled as spherically symmetric spheres of charge. The radius of the uranium nucleus is approximately 7.4×10^{-15} m. (a) What is the electric field this nucleus produces just outside its surface? (b) What magnitude of electric field does it produce at the distance of the electrons, which is about 1.0×10^{-10} m? (c) The electrons can be modeled as forming a uniform shell of negative charge. What net electric field do they produce at the location of the nucleus?





(a) With spherical symmetry:

$$\Phi_E = \oint \vec{E}\cdot\vec{n}\cdot dA$$
$$= 4\pi r^2 E(r)$$

So $E(r) = \dfrac{\Phi_E}{4\pi r^2}$

But $\Phi_E = \dfrac{q_{enclosed}}{\epsilon_0}$

So $E(r) = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_{enclosed}}{r^2}$ outside the charge ($r > r_1$)

$e = 1.6\times10^{-19}$ C

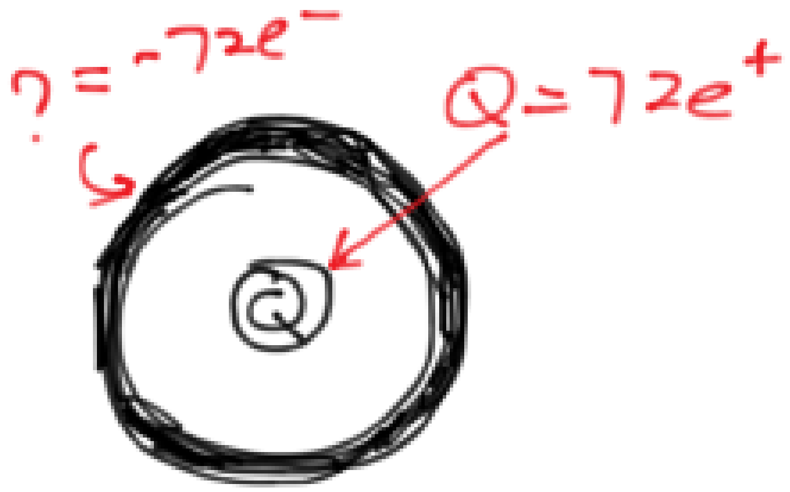
$Q = 92e = 1.6\times10^{-19}\,C \times 92 = 1.47\times10^{-17}$ C

$$E(r_1) = 9\times10^9\ N\cdot m^2/C^2 \cdot \frac{1.47\times10^{-17}C}{(7.4\times10^{-15}m)^2} = 2.42\times10^{21}\ N/C$$

$$E(r_2) = 9\times10^9\ Nm^2/C^2\ \frac{1.47\times10^{-17}c}{(10^{-10}m)^2} = 1.323\times10^{13}\ N/c$$

is about 1.0×10^{-10} m? (c) The electrons can be modeled as forming a uniform shell of negative charge. What net electric field do they produce at the location of the nucleus?

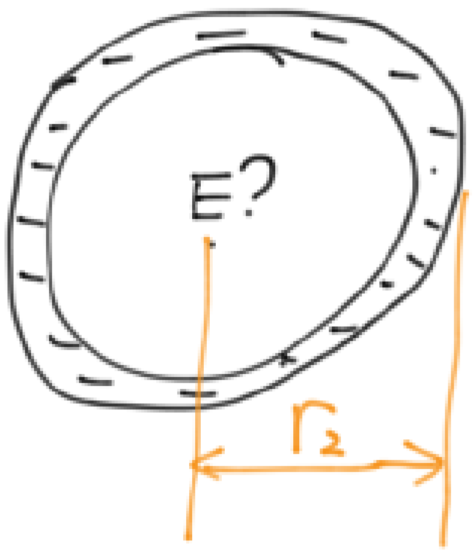
Analyze the problem: we have a core – shell structure



$\oint_{shell} dq = -Q$ for atoms

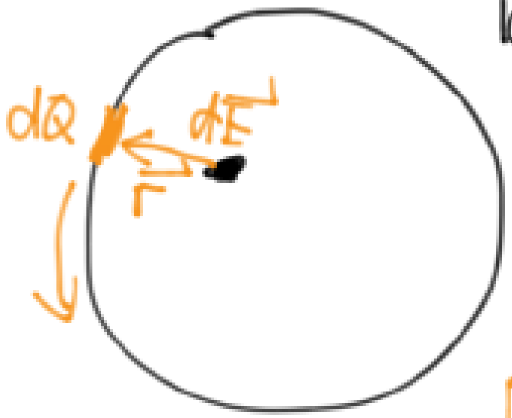
neutral, otherwise ions

We only worry about $\vec{E}$ contributed by the shell



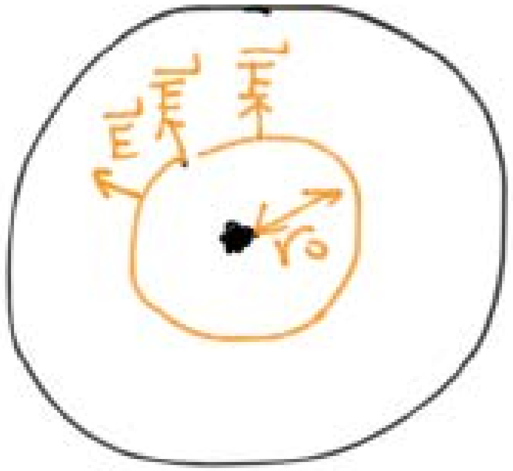
At the center $\vec{E} = 0$

Brute force methods with coulomb law



$$\vec{E} = \oint d\vec{E}$$
$$= \oint \frac{1}{4\pi\epsilon_0} \frac{dQ}{|\vec{r}|^2}$$

For those who loves calculus



Trick: Gauss's law + symmetry

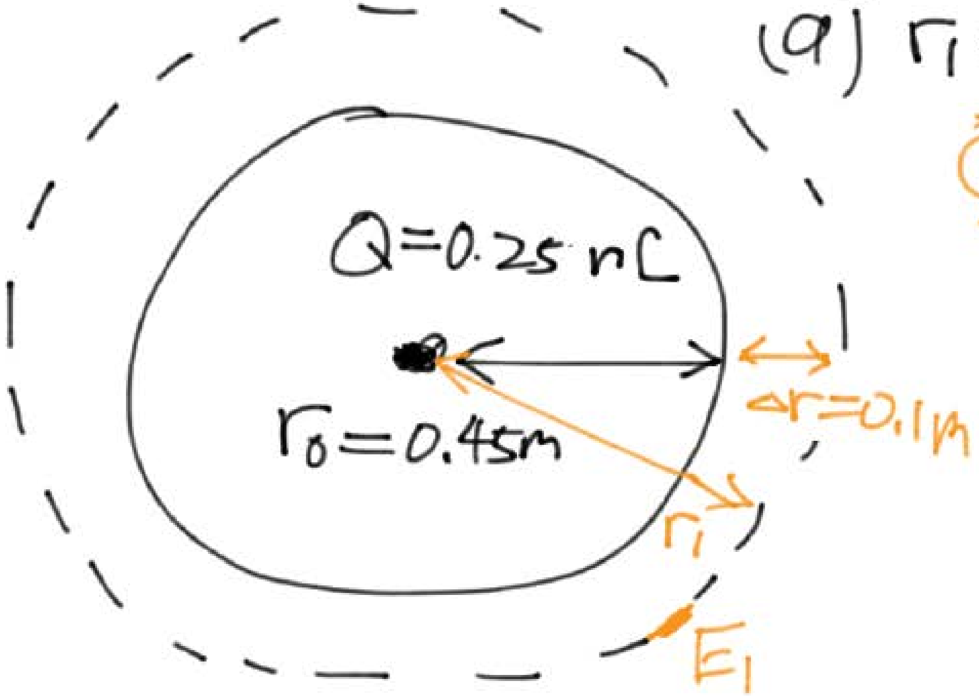
On a $r = r_0$ surface, $|\vec{E}|$ stays constant

$$\Phi = \underbrace{\frac{Q_{enclosed}}{\epsilon_0}}_{0} = \oint \vec{E}\cdot\vec{n}\cdot dA = 4\pi r_0^2 \cdot \underbrace{|\vec{E}|}_{0}$$

regardless of r_0
as long as $r_0 < r_2$

22.14 •• A solid metal sphere with radius 0.450 m carries a net charge of 0.250 nC. Find the magnitude of the electric field (a) at a point 0.100 m outside the surface of the sphere and (b) at a point inside the sphere, 0.100 m below the surface.

Draw the scheme to begin with.



(a) $r_1 = \Delta r + r_0 = 0.55m$

$$\Phi_1 = 4\pi r_1^2 E_1 = \frac{Q}{\epsilon_0}$$

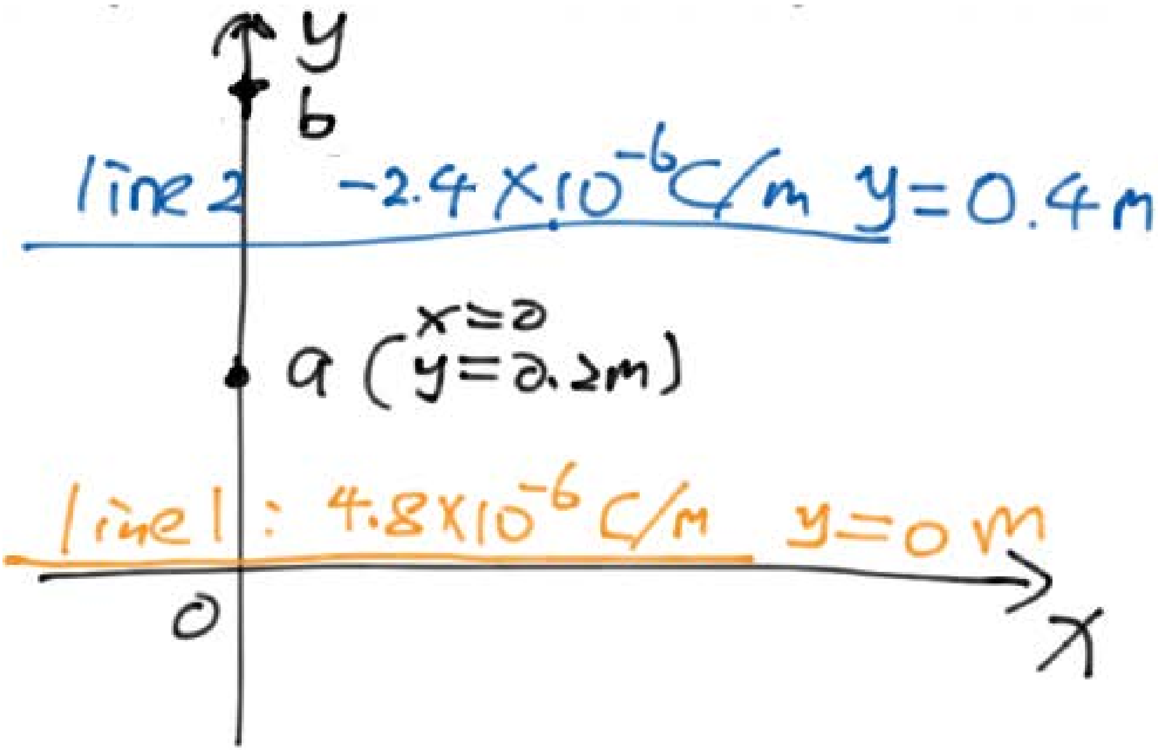
$$E_1 = \frac{Q}{4\pi\epsilon_0 r^2}$$

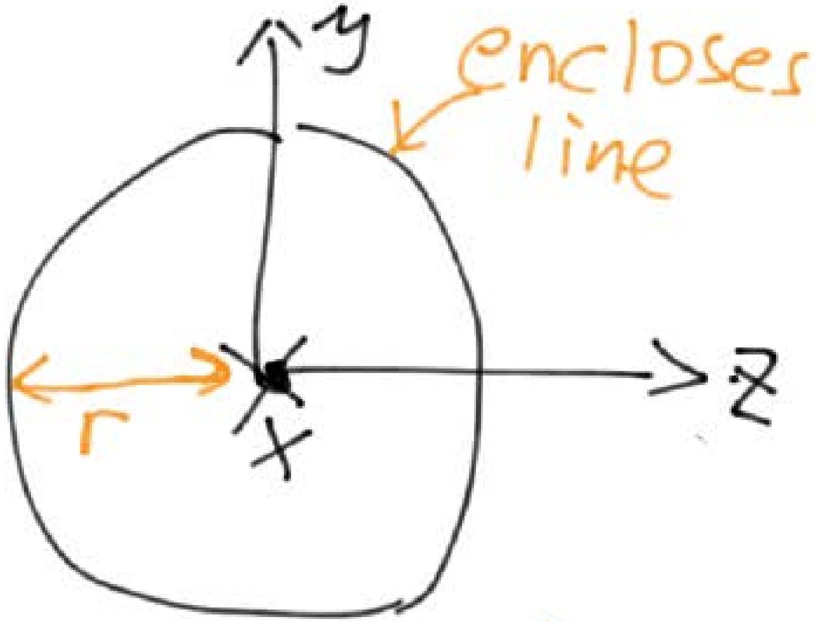
$$= 9\times10^9\,N{\cdot}m^2/C^2 \frac{2.5\times10^{-9}\,C}{(0.55m)^2}$$

$$= 74.4\ N/C$$

(b) Inside a metal $E = 0$

22.19 •• A very long uniform line of charge has charge per unit length 4.80 μC/m and lies along the x-axis. A second long uniform line of charge has charge per unit length -2.40 μC/m and is parallel to the x-axis at $y = 0.400$ m. What is the net electric field (magnitude and direction) at the following points on the y-axis: (a) $y = 0.200$ m and (b) $y = 0.600$ m?





Imagine an infinite long cylinder around x

$$\frac{Q}{\epsilon_0} = E \cdot A \Rightarrow \frac{\sigma}{\epsilon_0} \cdot L = E \cdot 4\pi r L$$

$Q = \sigma \cdot L$ $\quad$ $4\pi r \cdot L$

$$E = \frac{\sigma}{4\pi\epsilon_0 r}$$

$-2.4\,\mu C/m$ for line 2

$4.8\,\mu C/m$ for line 1

For (a) $r_{a1} = 0.2m$, $r_{a2} = 0.2m$ $\Rightarrow r_a$

$$E_a = \frac{\sigma_1}{4\pi\epsilon_0 r_a} + \frac{\sigma_2}{4\pi\epsilon_0 r_a} = \frac{\sigma_1 + \sigma_2}{4\pi\epsilon_0 r_a}$$

$$E_b = \frac{\sigma_1}{4\pi\epsilon_0 r_{b1}} + \frac{\sigma_2}{4\pi\epsilon_0 r_{b2}} \qquad r_{b1} = 3r_{b2}$$