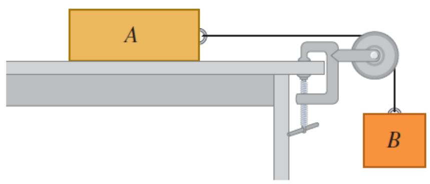
Problem Set 4 (Due 3/25/2025 before class)

Late homework will NOT be accepted, unless you have notified the course instructor 3 days BEFORE deadline.

Part I (60%)

5.34 •• Consider the system shown in Fig. E5.34. Block A weighs 45.0 N and block B weighs 25.0 N. Once block B is set into downward motion, it descends at a constant speed. (a) Calculate the coefficient of kinetic friction between block A and the tabletop. (b) A cat, also of weight 45.0 N, falls asleep on top of block A. If block B is now set into downward motion, what is its acceleration (magnitude and direction)?

Figure **E5.34**



IDENTIFY: Constant speed means zero acceleration for each block. If the block is moving, the friction force the tabletop exerts on it is kinetic friction. Apply $\Sigma\vec{F} = m\vec{a}$ to each block.

SET UP: The free-body diagrams and choice of coordinates for each block are given by Figure 5.34. $m_A = 4.59$ kg and $m_B = 2.55$ kg.

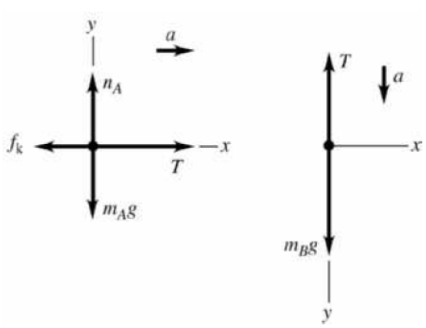
EXECUTE: **(a)** $\Sigma F_y = ma_y$ with $a_y = 0$ applied to block B gives $m_B g - T = 0$ and $T = 25.0$ N. $\Sigma F_x = ma_x$ with $a_x = 0$ applied to block A gives $T - f_k = 0$ and $f_k = 25.0$ N. $n_A = m_A g = 45.0$ N and

$$\mu_k = \frac{f_k}{n_A} = \frac{25.0\text{ N}}{45.0\text{ N}} = 0.556.$$

(b) Now let A be block A plus the cat, so $m_A = 9.18$ kg. $n_A = 90.0$ N and $f_k = \mu_k n = (0.556)(90.0\text{ N}) = 50.0$ N. $\sum F_x = ma_x$ for A gives $T - f_k = m_A a_x$. $\sum Fy = ma_y$ for block B gives $m_B g - T = m_B a_y$. a_x for A equals a_y for B, so adding the two equations gives

$m_B g - f_k = (m_A + m_B)a_y$ and $a_y = \dfrac{m_B g - f_k}{m_A + m_B} = \dfrac{25.0\text{ N} - 50.0\text{ N}}{9.18\text{ kg} + 2.55\text{ kg}} = -2.13\text{ m/s}^2$. The acceleration is upward and block B slows down.

EVALUATE: The equation $m_B g - f_k = (m_A + m_B)a_y$ has a simple interpretation. If both blocks are considered together then there are two external forces: $m_B g$ that acts to move the system one way and f_k that acts oppositely. The net force of $m_B g - f_k$ must accelerate a total mass of $m_A + m_B$.



5.45 •• A 1125-kg car and a 2250-kg pickup truck approach a curve on the expressway that has a radius of 225 m. (a) At what angle should the highway engineer bank this curve so that vehicles traveling at 65.0 mi/h can safely round it regardless of the condition of their tires? Should the heavy truck go slower than the lighter car? (b) As the car and truck round the curve at find the normal force on each one due to the highway surface.

IDENTIFY: We can use the analysis done in Example 5.22. As in that example, we assume friction is negligible.

SET UP: From Example 5.22, the banking angle β is given by $\tan\beta = \dfrac{v^2}{gR}$. Also, $n = mg/\cos\beta$.

$65.0\ \text{mi/h} = 29.1\ \text{m/s}$.

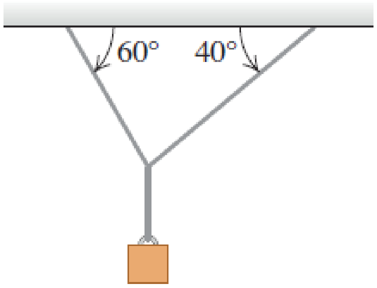
EXECUTE: **(a)** $\tan\beta = \dfrac{(29.1\ \text{m/s})^2}{(9.80\ \text{m/s}^2)(225\ \text{m})}$ and $\beta = 21.0°$. The expression for $\tan\beta$ does not involve the mass of the vehicle, so the truck and car should travel at the same speed.

(b) For the car, $n_{\text{car}} = \dfrac{(1125\ \text{kg})(9.80\ \text{m/s}^2)}{\cos 21.0°} = 1.18\times10^4\ \text{N}$ and $n_{\text{truck}} = 2n_{\text{car}} = 2.36\times10^4\ \text{N}$, since $m_{\text{truck}} = 2m_{\text{car}}$.

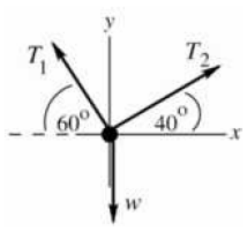
EVALUATE: The vertical component of the normal force must equal the weight of the vehicle, so the normal force is proportional to m.

5.57 ••• Two ropes are connected to a steel cable that supports a hanging weight as shown in Fig. P5.57. (a) Draw a free-body diagram showing all of the forces acting at the knot that connects the two ropes to the steel cable. Based on your force diagram, which of the two ropes will have the greater tension? (b) If the maximum tension either rope can sustain without breaking is 5000 N, determine the maximum value of the hanging weight that these ropes can safely support. You can ignore the weight of the ropes and the steel cable.

Figure **P5.57**



IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to the knot.
SET UP: $a = 0$. Use coordinates with axes that are horizontal and vertical.
EXECUTE: **(a)** The free-body diagram for the knot is sketched in Figure 5.57.
T_1 is more vertical so supports more of the weight and is larger. You can also see this from $\Sigma F_x = ma_x$:
$T_2 \cos 40° - T_1 \cos 60° = 0$. $T_2 \cos 40° - T_1 \cos 60° = 0$.
(b) T_1 is larger so set $T_1 = 5000$ N. Then $T_2 = T_1/1.532 = 3263.5$ N. $\Sigma F_y = ma_y$ gives
$T_1 \sin 60° + T_2 \sin 40° = w$ and $w = 6400$ N.
EVALUATE: The sum of the vertical components of the two tensions equals the weight of the suspended object. The sum of the tensions is greater than the weight.

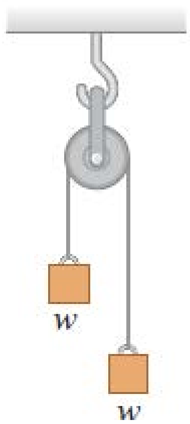


Part II (40%)

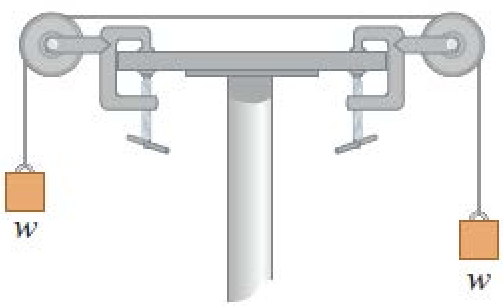
For problems 1-3. In the following figures each of the suspended blocks has weight w. The pulleys are frictionless and the ropes have negligible weight. Calculate, in each case, the tension T in the rope in terms of the weight w. In each case, include the free-body diagram or diagrams you used to determine the answer.

1. One block of weight *w* hanging through a pulley

2. Two blocks of weight *w* hanging through a pulley



3. Two blocks of weight *w* hanging through two pulleys



Answer:

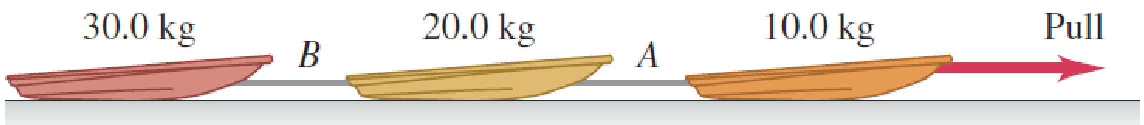
IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to each weight.

SET UP: Two forces act on each mass: w down and $T(= w)$ up.

EXECUTE: In all cases, each string is supporting a weight w against gravity, and the tension in each string is w.

EVALUATE: The tension is the same in all three cases.

For Problems 4-5. Three sleds are being pulled horizontally on frictionless horizontal ice using horizontal ropes (see Figure below). The pull is of magnitude 125 N.



4. Find the acceleration of the system
5. Find the tension in ropes A and B.

IDENTIFY: Apply Newton's second law to the three sleds taken together as a composite object and to each individual sled. All three sleds have the same horizontal acceleration a.

SET UP: The free-body diagram for the three sleds taken as a composite object is given in Figure 5.14a and for each individual sled in Figure 5.14b–d. Let $+x$ be to the right, in the direction of the acceleration. $m_{\text{tot}} = 60.0$ kg.

EXECUTE: **(a)** $\Sigma F_x = ma_x$ for the three sleds as a composite object gives $P = m_{\text{tot}}a$ and

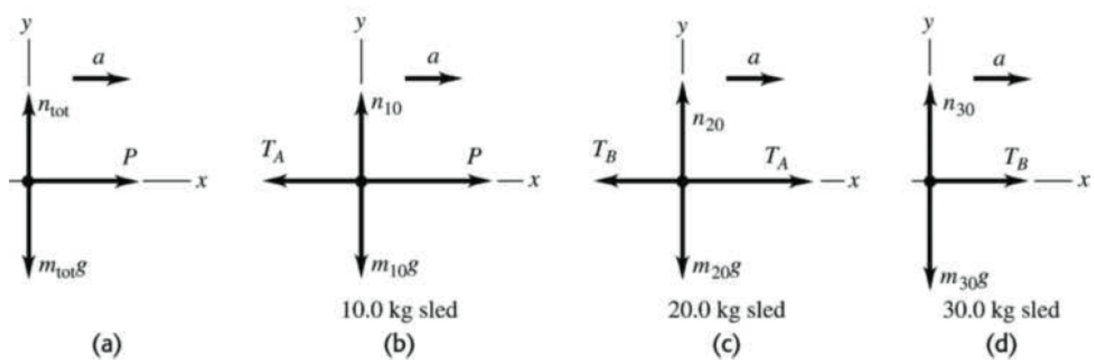
$$a = \frac{P}{m_{\text{tot}}} = \frac{125\text{ N}}{60.0\text{ kg}} = 2.08\text{ m/s}^2.$$

(b) $\Sigma F_x = ma_x$ applied to the 10.0 kg sled gives $P - T_A = m_{10}a$ and

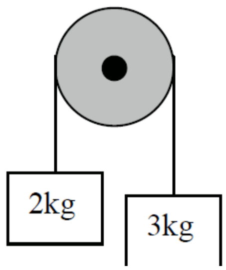
$T_A = P - m_{10}a = 125\text{ N} - (10.0\text{ kg})(2.08\text{ m/s}^2) = 104\text{ N}$. $\Sigma F_x = ma_x$ applied to the 30.0 kg sled gives

$T_B = m_{30}a = (30.0\text{ kg})(2.08\text{ m/s}^2) = 62.4\text{ N}$.

EVALUATE: If we apply $\Sigma F_x = ma_x$ to the 20.0 kg sled and calculate a from T_A and T_B found in part (b), we get $T_A - T_B = m_{20}a$. $a = \dfrac{T_A - T_B}{m_{20}} = \dfrac{104\text{ N} - 62.4\text{ N}}{20.0\text{ kg}} = 2.08\text{ m/s}^2$, which agrees with the value we calculated in part (a).

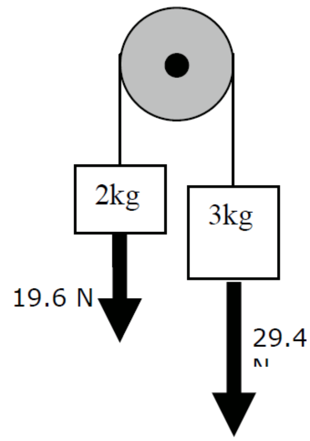


6. Two masses are hung over a mass-less and friction-less pulley as shown. What is the resulting acceleration of the system?



Known: Mass m_1= 2 kg
Mass m_2=3 kg

Unknown: Acceleration a =? m/s^2



Define: Consider the two masses as forces pulling in opposite directions due to the pulley. Find the net force for this situation. We will arbitrarily consider the direction of the 3kg mass as positive. Find the weight of each and subtract.

$$F_{net} = m_2g - m_1g \quad = (m_2 - m_1)\, g$$

However, when a mass is used in the next step, the entire mass of the system since the whole system will be moving.

$$F_{net} = ma$$

$$a = \frac{F_{net}}{m_1 + m_2} = \frac{(m_1 - m_2)g}{m_1 + m_2}$$

Output: $a = \dfrac{(m_2 - m_1)g}{m_1 + m_2} = \dfrac{(3 - 2)9.8\ m/s^2}{3 + 2} = \dfrac{9.8\ m/s^2}{5} = 2.0\ m/s^2$

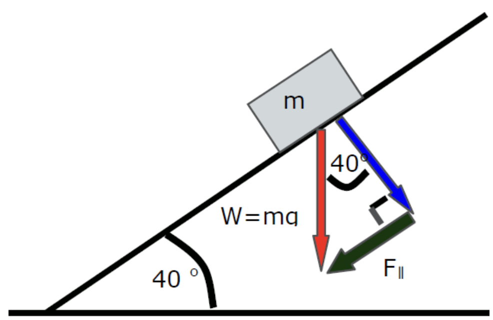
7. A box slides down a frictionless inclined plane. It makes an angle of 40 degrees with the horizontal. What is the acceleration of the box as it slides down the incline?

Known : Angle of incline 40°

Unknown :Acceleration, a =?m/s^2

Define: The component of the weight that pulls the box down the incline is the parallel force, $F_{\parallel}$
To find $F_{\parallel}$, use

$\sin\theta = \dfrac{opp}{hyp}$



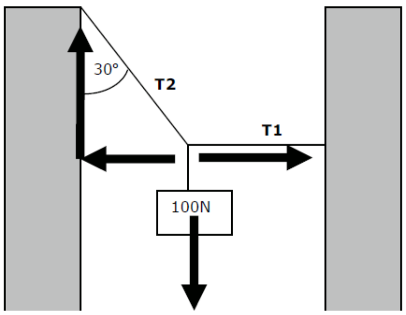
The opposite side is our $F_{\parallel}$ in this case,

$F_{\parallel} = \sin\theta(\text{weight})$
Weight $= mg$
$F_{net} = ma$, which is equal to $F_{\parallel}$
$F_{net} = \sin 40(mg)$
$ma = \sin 40(mg)$
Notice the mass cancels out leaving our acceleration.

Output : $a = \sin 40(g) = 0.643 \times 9.8\ m/s^2 = 6.3\ m/s^2 = 6.3\ m/s/s$
Substantiate: Units are correct, sig figs are correct and magnitude is reasonable.

8. Consider a 100 N weight hanging from two cables attached to walls as shown.

Calculate the tension/force in the second cable, T2



Solution:

Consider the tension in T2 as horizontal and vertical components. Consider the right triangle formed where the vertical component is 100N to balance the hanging weight. Find the ypotenuse which is the force on T2. cos (30) = 100N/T2

Solving for T2 gives 115N.

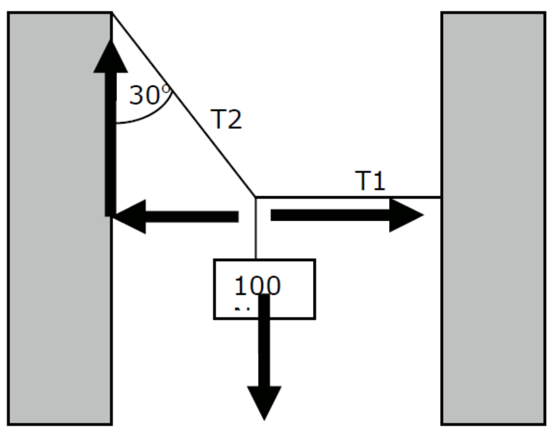
Known: Weight =100 kg
Angle of cable = 30.0°

Unknown: Tension in cable T2=? N

Define: The weight is in static equilibrium
$F_{net} = 0$ N

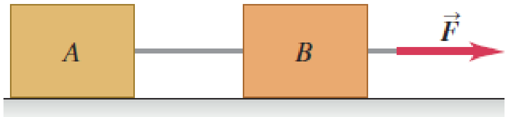
The vertical component of the tension, T2, is equal to the weight.

Calculate T2 using:
$$\cos\theta = \frac{adj}{hyp} = \frac{W}{T2},\ T2 = \frac{W}{\cos\theta}$$



Output: $T2 = \frac{100\ N}{\cos 30} = \frac{100\ N}{0.866} = 115\ N$

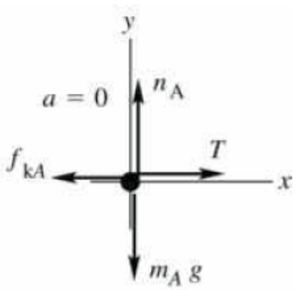
9. Two crates connected by a rope lie on a horizontal surface (Fig. E5.35). Crate A has mass m_A and crate B has mass m_B The coefficient of kinetic friction between each crate and the surface is μ_k The crates are pulled to the right at constant velocity by a horizontal force $\boldsymbol{F}$. In terms of μ_k, m_A and m_B, calculate the tension in the rope connecting the blocks. Include the free-body diagram or diagrams you used to determine each answer.



Solution:

IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to each crate. The rope exerts force T to the right on crate A and force T to the left on crate B. The target variables are the forces T and F. Constant v implies $a = 0$.

SET UP: The free-body diagram for A is sketched in Figure 5.35a



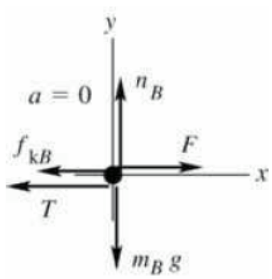
EXECUTE:

$$\Sigma F_y = ma_y$$
$$n_A - m_A g = 0$$
$$n_A = m_A g$$
$$f_{kA} = \mu_k n_A = \mu_k m_A g$$

Figure 5.35a

$$\Sigma F_x = ma_x$$
$$T - f_{kA} = 0$$
$$T = \mu_k m_A g$$

The free-body diagram for B is sketched in Figure 5.35b.



EXECUTE:

$$\Sigma F_y = ma_y$$
$$n_B - m_B g = 0$$
$$n_B = m_B g$$
$$f_{kB} = \mu_k n_B = \mu_k m_B g$$

$$\Sigma F_x = ma_x$$
$$F - T - f_{kB} = 0$$
$$F = T + \mu_k m_B g$$

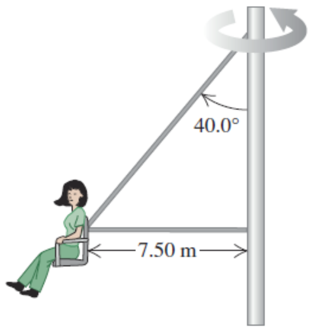
Use the first equation to replace T

$$F = \mu_k m_A g + \mu_k m_B g.$$

(a) $F = \mu_k (m_A + m_B) g$

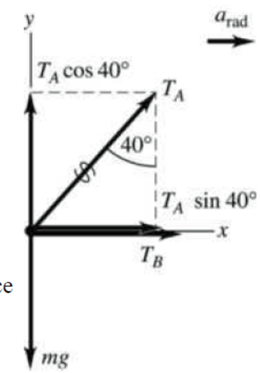
(b) $T = \mu_k m_A g$

10. In the swing, the seat is connected to two cables as shown in the figure, one of which is **horizontal**. The seat swings in a **horizontal** circle at a rate of 32.0 rpm (revolutions / min). If the seat weighs 255 N and an 825-N person is sitting in it, find the tension in each cable.



IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to the composite object of the person plus seat. This object moves in a horizontal circle and has acceleration a_{rad}, directed toward the center of the circle.

SET UP: The free-body diagram for the composite object is given in Figure 5.47. Let $+x$ be to the right, in the direction of $\vec{a}_{\text{rad}}$. Let $+y$ be upward. The radius of the circular path is $R = 7.50\text{ m}$. The total mass is $(255\text{ N} + 825\text{ N})/(9.80\text{ m/s}^2) = 110.2\text{ kg}$. Since the rotation rate is $32.0\text{ rev/min} = 0.5333\text{ rev/s}$, the period T is $\dfrac{1}{0.5333\text{ rev/s}} = 1.875\text{ s}$.



EXECUTE: $\Sigma F_y = ma_y$ gives $T_A \cos 40.0° - mg = 0$ and $T_A = \dfrac{mg}{\cos 40.0°} = \dfrac{255\text{ N} + 825\text{ N}}{\cos 40.0°} = 1410\text{ N}$.

$\Sigma F_x = ma_x$ gives $T_A \sin 40.0° + T_B = ma_{\text{rad}}$ and

$$T_B = m\frac{4\pi^2 R}{T^2} - T_A \sin 40.0° = (110.2\text{ kg})\frac{4\pi^2(7.50\text{ m})}{(1.875\text{ s})^2} - (1410\text{ N})\sin 40.0° = 8370\text{ N}$$

The tension in the horizontal cable is 8370 N and the tension in the other cable is 1410 N.

EVALUATE: The weight of the composite object is 1080 N. The tension in cable A is larger than this since its vertical component must equal the weight. $ma_{\text{rad}} = 9280\text{ N}$. The tension in cable B is less than this because part of the required inward force comes from a component of the tension in cable A.