College Algebra and Trigonometry

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6.1 Fundamental Trigonometric Identities

6.2 Sum and Difference Formula

6.3 Double-Angle, Power-Reducing, and Half-Angle Formulas

6.4 Product-to-Sum and Sum-to-Product Formulas

6.5 Trigonometric Equations

① Simplify Trigonometric Expressions

Fundamental Identities

Reciprocal Identities	$\csc x = \dfrac{1}{\sin x}$	$\sec x = \dfrac{1}{\cos x}$	$\cot x = \dfrac{1}{\tan x}$
	$\sin x = \dfrac{1}{\csc x}$	$\cos x = \dfrac{1}{\sec x}$	$\tan x = \dfrac{1}{\cot x}$
Quotient Identities	$\tan x = \dfrac{\sin x}{\cos x}$	$\cot x = \dfrac{\cos x}{\sin x}$	
Pythagorean Identities	$\sin^2 x + \cos^2 x = 1$	$\tan^2 x + 1 = \sec^2 x$	$1 + \cot^2 x = \csc^2 x$
Even and Odd Identities	$\sin(-x) = -\sin x$	$\cos(-x) = \cos x$	
	$\csc(-x) = -\csc x$	$\sec(-x) = \sec x$	
	$\tan(-x) = -\tan x$	$\cot(-x) = -\cot x$	

Example 1:

Simplify the expression. Write the final form with no fractions.

$$sec^2x\ cotx\ cosx$$

Example 2:

Simplify the expression. Write the final form with no fractions.

$$\frac{cos\theta}{1+sin\theta}+tan\theta$$

Example 3:

Simplify the expression. Write the final form with no fractions.

$$\frac{tan^2t-1}{tantsint+sint}$$

② Verify Trigonometric Identities

Identity:

An equation is called an identity if it is true for all the values of the variable in the domain.

Example 4:

Verify the equation is an identity.

$$\frac{\cos(-x)\tan(-x)}{\sin x} = -1$$

Example 5:

Verify the equation is an identity.

$$\frac{1}{1-\cos\theta}-\frac{1}{1+\cos\theta}=2\cot\theta\csc\theta$$

Example 6:

Verify the equation is an identity.

$$\frac{1-\sin t}{1+\sin t}=(\sec t-\tan t)^2$$

Example 7:

Verify the equation is an identity.

$$\ln|\sin x| + \ln|\sec x| = \ln|\tan x|$$

Example 8:

Verify the equation is an identity.

$$\frac{\sin x}{1 + \sin x} = \frac{\csc x - 1}{\cot^2 x}$$

③ Write an Algebraic Expression as a Trigonometric Expression

Example 9:

Write the expression $\sqrt{x^2+9}$ as a function of θ, where $0<\theta<\frac{\pi}{2}$, by making the substitution $x=3tan\theta$.

Skill Practice:

Write the expression $\sqrt{x^2-25}$ as a function of θ, where $0<\theta<\frac{\pi}{2}$, by making the substitution $x=5sec\theta$.