# *College Algebra and Trigonometry*

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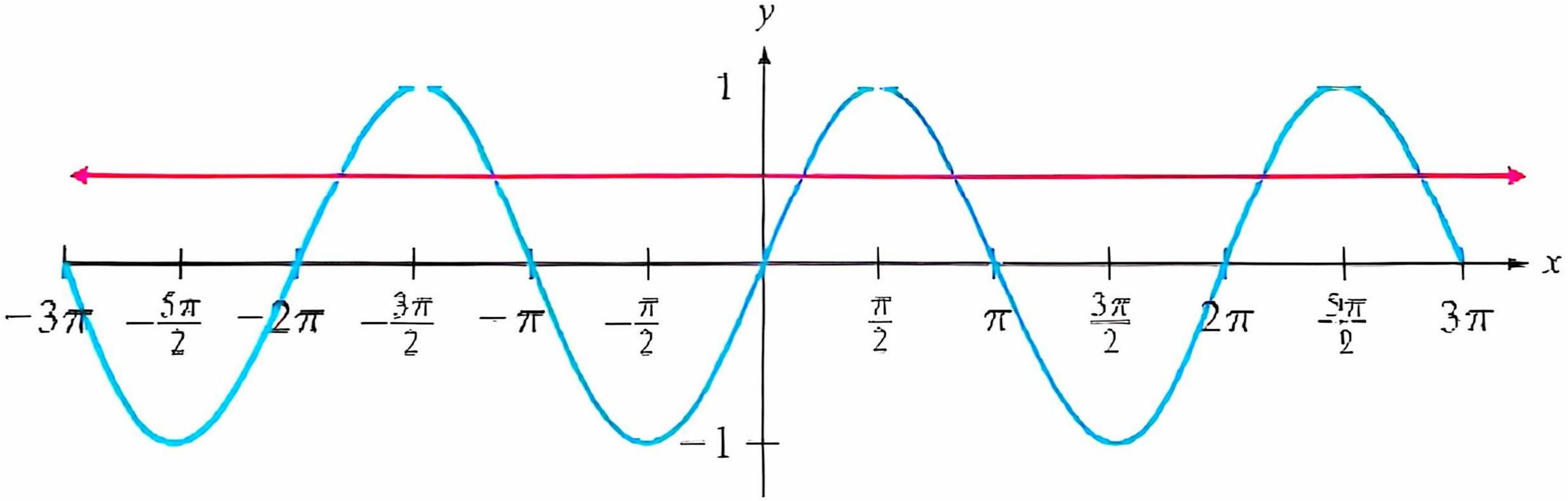
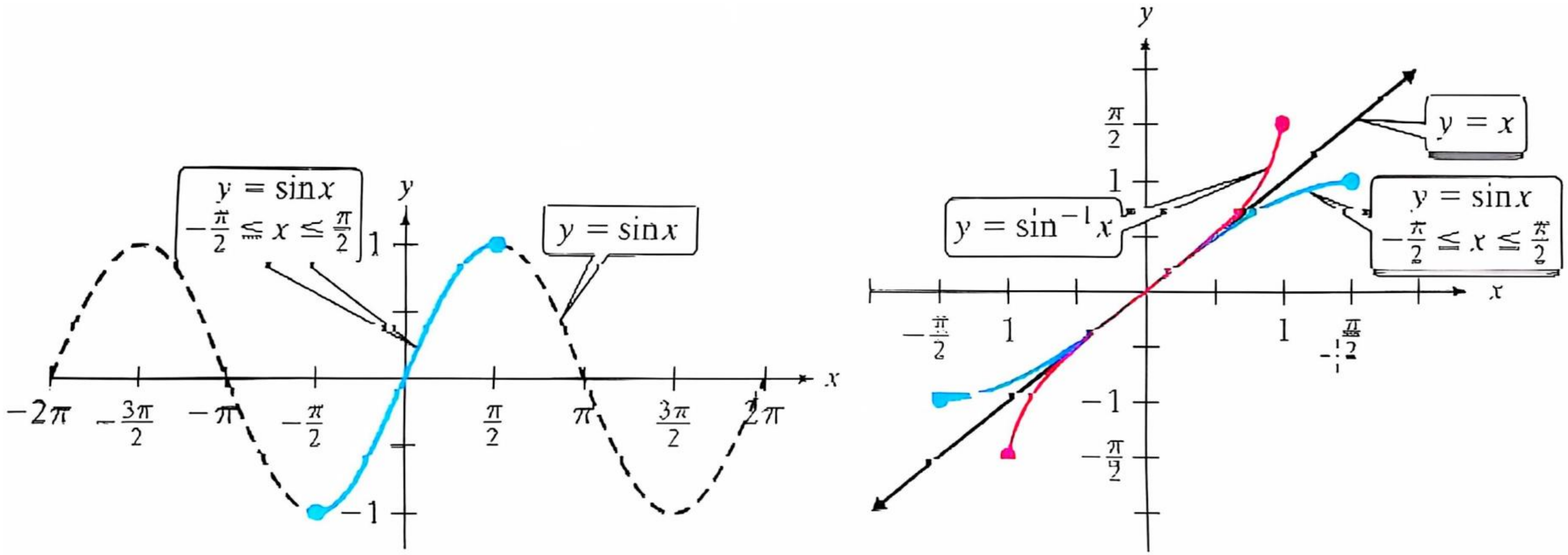
## ① Evaluate the Inverse Sine Function

- **Recall that a function must be one-to-one to have an inverse function.**
- **It is seen that any horizontal line between $-1 \le y \le 1$ intersects the graph of $y = \sin x$ infinitely many times, thus $y = \sin x$ is not a one-to-one function.**



- **However, suppose we restrict the domain of $y = \sin x$ to the interval $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, the graph of the restricted sine function is one-to-one and contains the entire range of y values $-1 \le y \le 1$.**
- **The inverse of this restricted sine function is called the inverse sine function and is denoted by $\sin^{-1}$ or arcsin. The graphs of $y = \sin x$ and $y = \sin^{-1} x$ are symmetric with respect to the line $y = x$ as expected.**



## The Inverse Sine Function

The **inverse sine function** (or arcsine) denoted by $\sin^{-1}$ or arcsin is the inverse of the restricted sine function $y = \sin x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Therefore,

$$y = \sin^{-1} x \quad \Leftrightarrow \quad \sin y = x$$

$$y = \arcsin x \quad \Leftrightarrow \quad \sin y = x$$

where $-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

- **$y = \sin^{-1} x$ is read as "$y$ equals the inverse sine of $x$", and $y = \arcsin x$ is read as "$y$ equals the arcsine of $x$".**
- **To evaluate $y = \sin^{-1} x$ or $y = \arcsin x$ means to find an angle y between $-\pi/2$ and $\pi/2$, inclusive, whose sine value is $x$.**

**Example 1:**

**Evaluate the following inverse sine functions.**

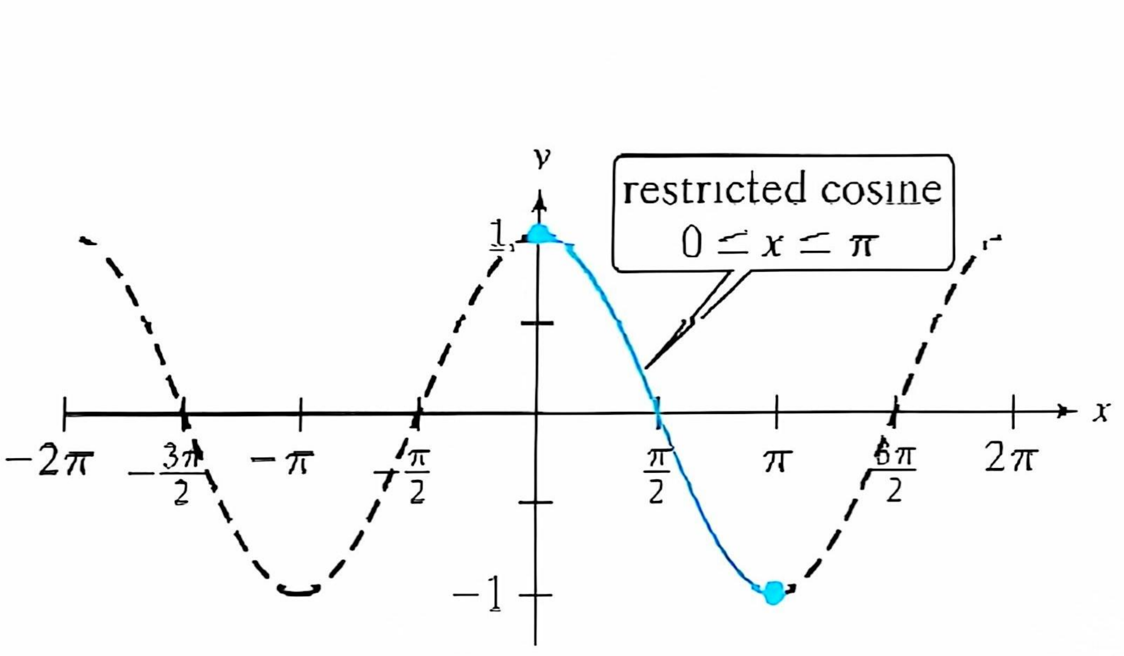
a) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

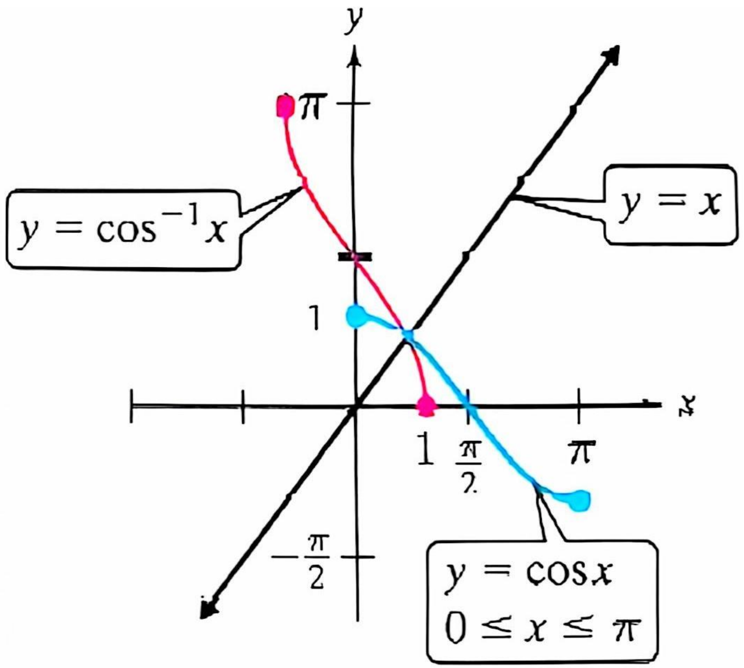
b) $\arcsin\frac{1}{2}$

c) $\sin^{-1}2$

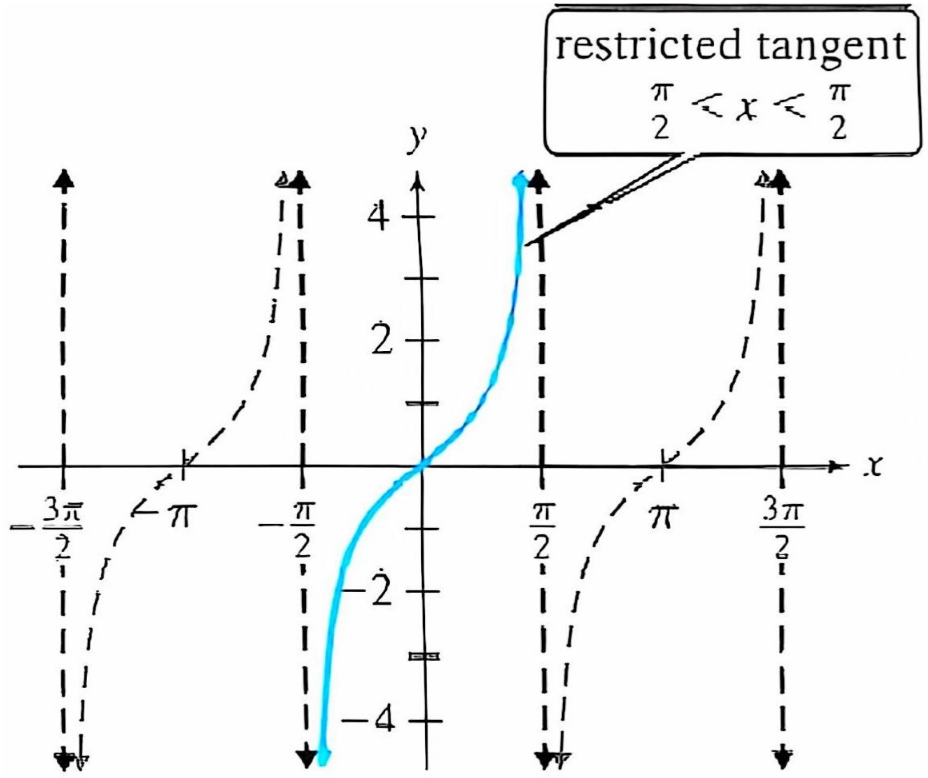
## ② Evaluate the Inverse Cosine and Tangent Functions

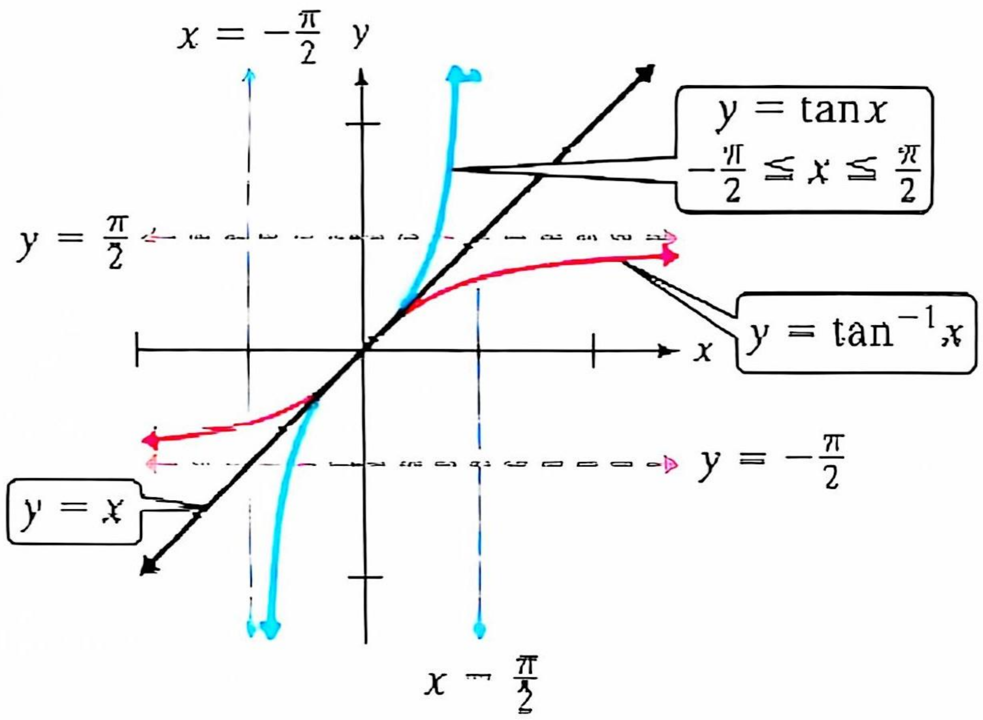
- **The inverse cosine and tangent functions are defined in similar ways. The domain of $y = \cos x$ and $y = \tan x$ must each be restricted to create one-to-one function on an interval containing all values in the range.**
- **The restricted cosine function is defined on $0 \le x \le \pi$. The graph of $\cos^{-1} x$ or $\arccos x$ is shown as below:**



- **The restricted tangent function is defined on $-\frac{\pi}{2} < x < \frac{\pi}{2}$. The graph of $\tan^{-1} x$ or arctan$x$ is shown as below.**
- **The restricted tangent function has vertical asymptotes at $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$. The inverse tangent function has horizontal asymptotes at $y = -\frac{\pi}{2}$ and $y = \frac{\pi}{2}$.**



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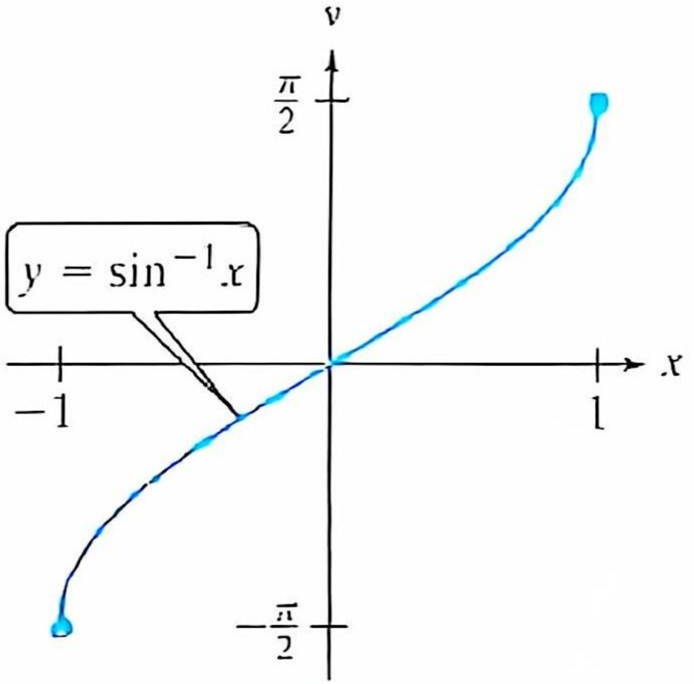
## Inverse Trigonometric Functions

| Restricted Function | Inverse Function | Graph |
|---|---|---|
| $y = \sin x$<br>$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$<br>$-1 \le y \le 1$ | The **inverse sine function** (or arcsine), denoted by $\sin^{-1}$ or arcsin, is defined by<br>$y = \sin^{-1} x \quad \Leftrightarrow \quad \sin y = x$<br>$y = \arcsin x \quad \Leftrightarrow \quad \sin y = x$<br>$-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ |  |

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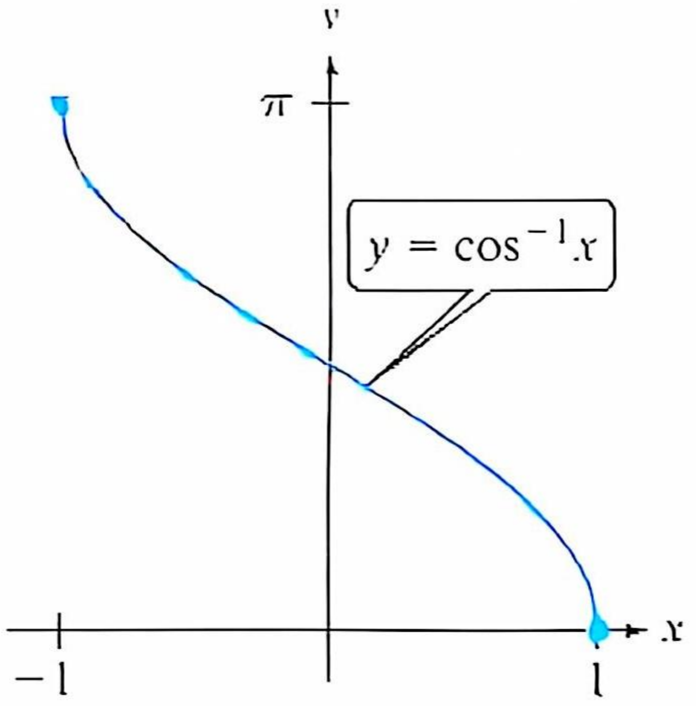
### Inverse Trigonometric Functions

| Restricted Function | Inverse Function | Graph |
| --- | --- | --- |
| $y = \cos x$<br>$0 \le x \le \pi$<br>$-1 \le y \le 1$ | The **inverse cosine function** (or arccosine), denoted by $\cos^{-1}$ or arccos, is defined by<br>$y = \cos^{-1} x \iff \cos y = x$<br>$y = \arccos x \iff \cos y = x$<br>$-1 \le x \le 1$ and $0 \le y \le \pi$ |  |

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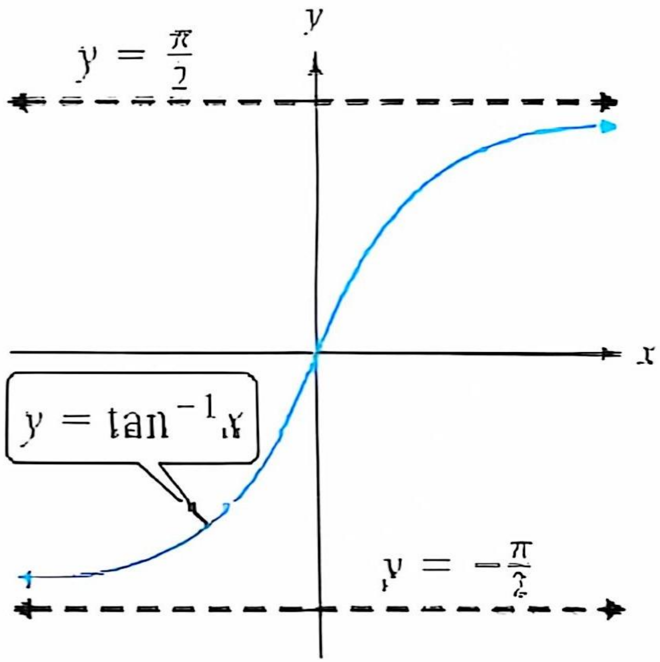
### Inverse Trigonometric Functions

| **Restricted Function** | **Inverse Function** | **Graph** |
| --- | --- | --- |
| $y = \tan x$<br>$-\frac{\pi}{2} < x < \frac{\pi}{2}$<br>$y \in \mathbb{R}$<br>Vertical asymptotes:<br>$x = -\frac{\pi}{2}, x = \frac{\pi}{2}$ | The **inverse tangent function** (or arctangent), denoted by $\tan^{-1}$ or arctan, is defined by<br>$y = \tan^{-1} x \quad \Leftrightarrow \quad \tan y = x$<br>$y = \arctan x \quad \Leftrightarrow \quad \tan y = x$<br>$x \in \mathbb{R}$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$<br>Horizontal asymptotes:<br>$y = -\frac{\pi}{2}, y = \frac{\pi}{2}$ |  |

**Example 2:**

**Find the exact values.**

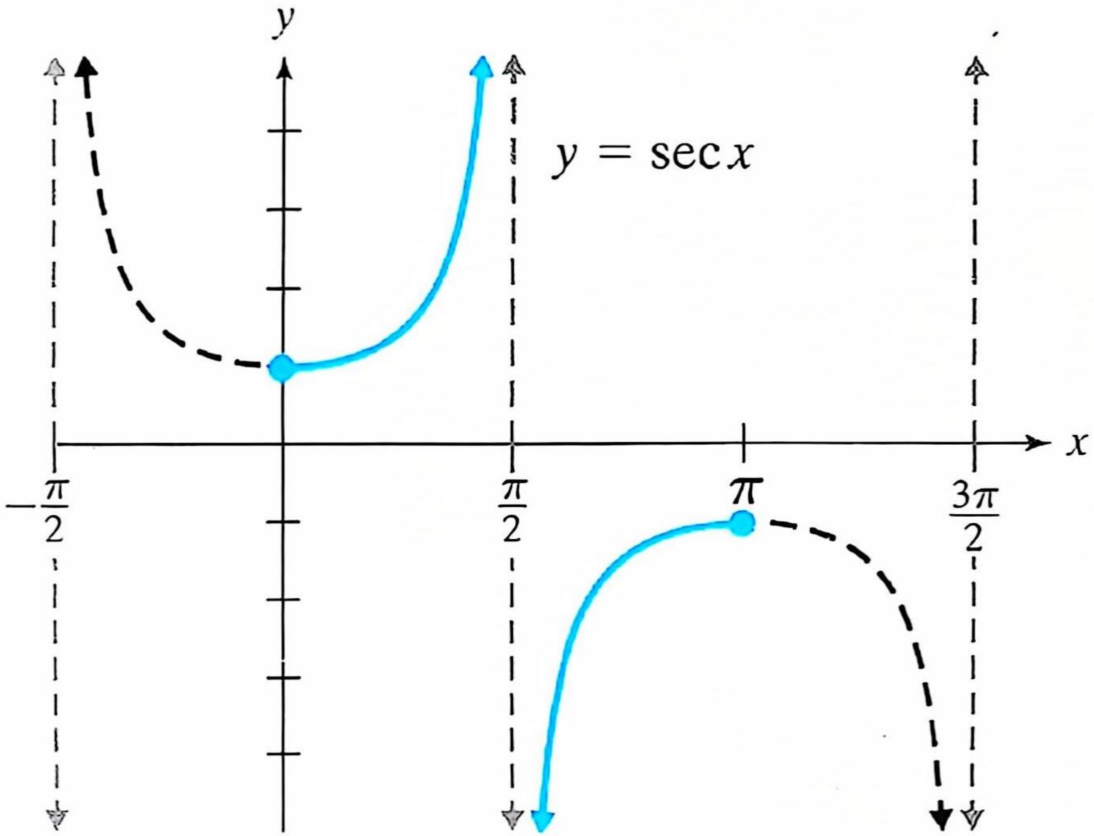
a) $\cos^{-1}\left(-\frac{1}{2}\right)$

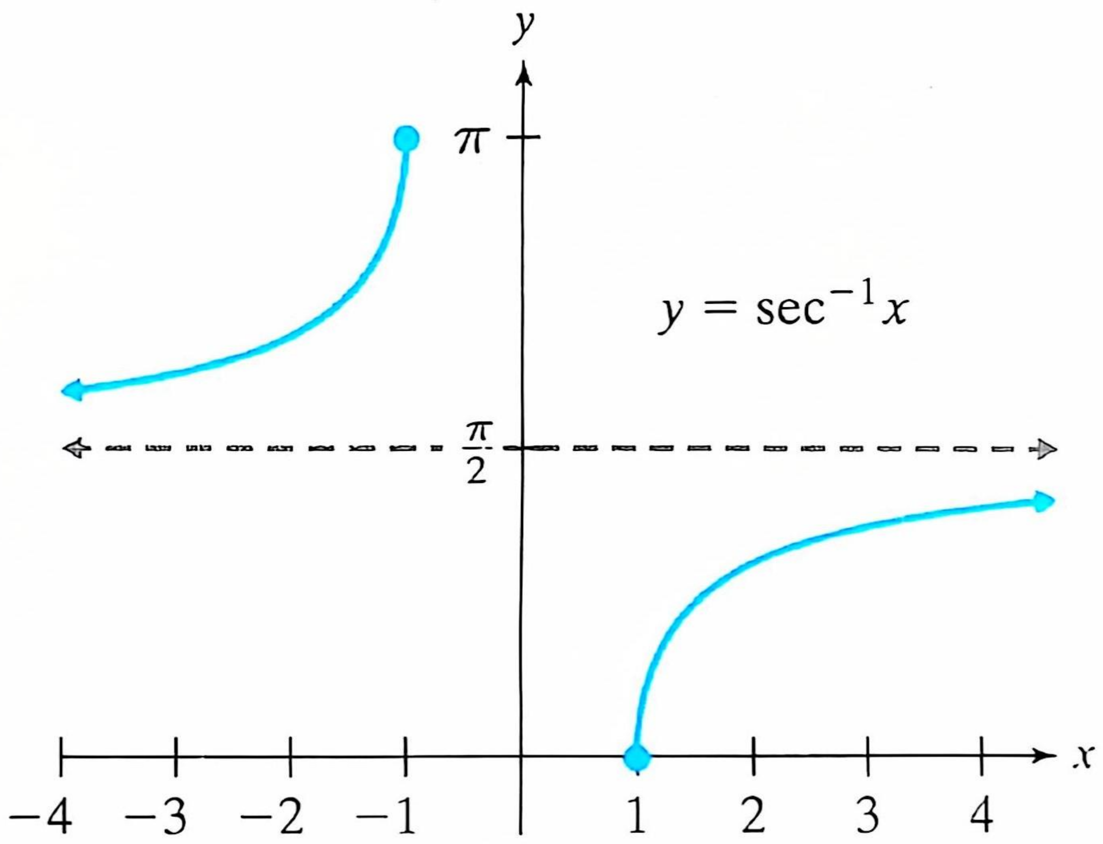
b) $\tan^{-1}\sqrt{3}$

c) $arctan(-1)$

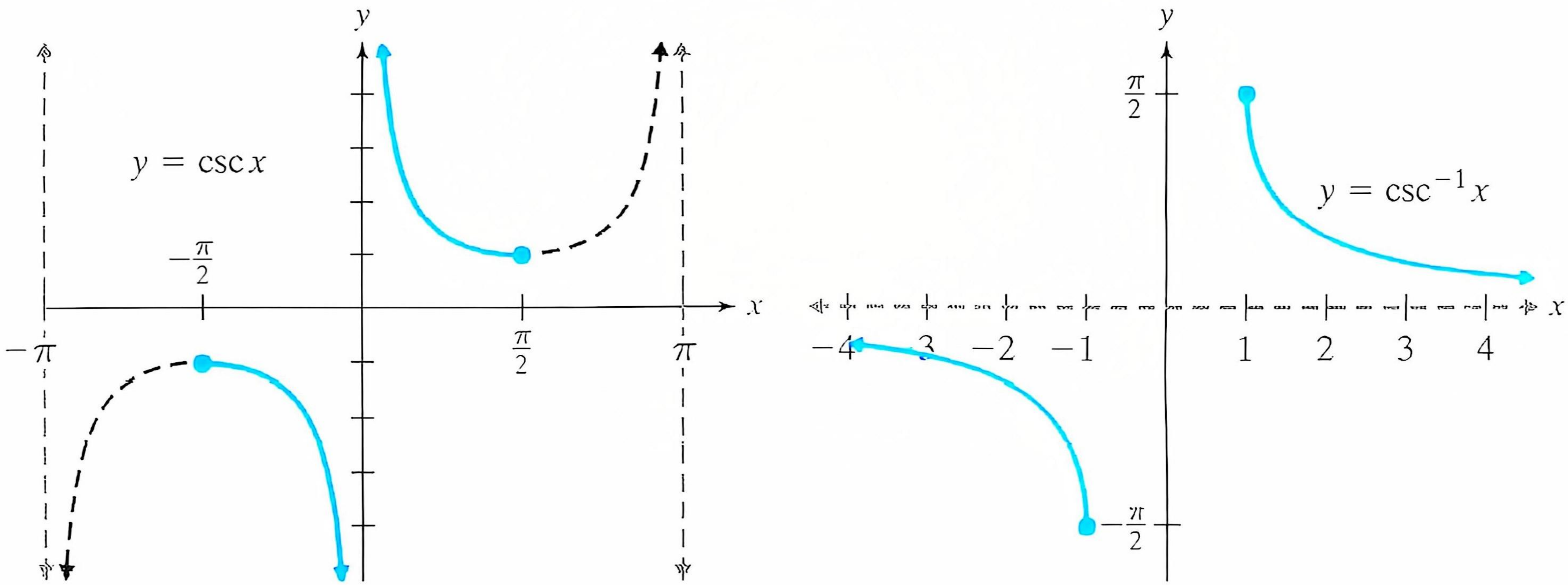
## ③ Evaluate the Inverse Secant, Cosecant, and Cotangent Functions

$$y = \sec^{-1} x \quad \Leftrightarrow \quad \sec y = x, \text{ where } |x| \geq 1 \text{ and } 0 \leq y < \frac{\pi}{2} \text{ or } \frac{\pi}{2} < y \leq \pi.$$

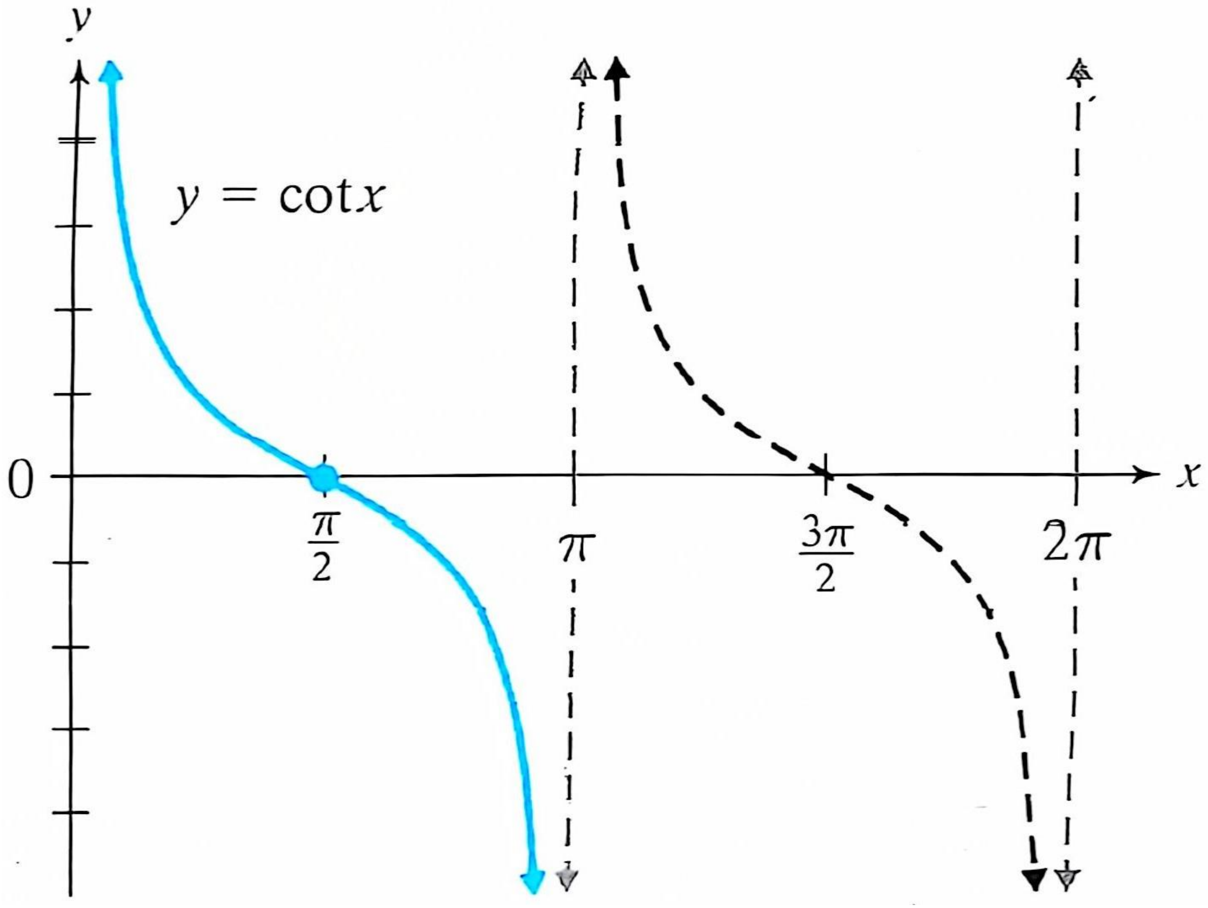


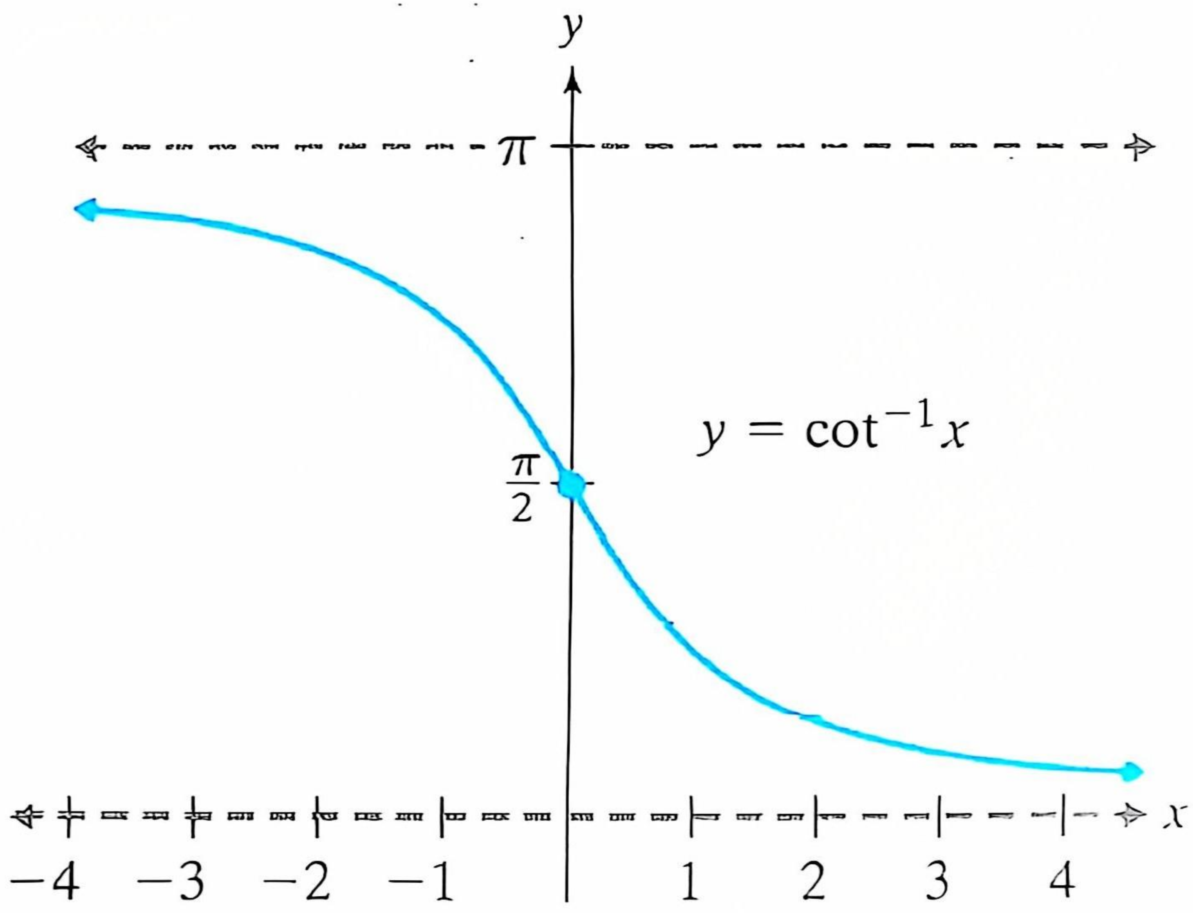



$$y = \csc^{-1} x \quad \Leftrightarrow \quad \csc y = x, \text{ where } |x| \geq 1 \text{ and } -\frac{\pi}{2} \leq y < 0 \text{ or } 0 < y \leq \frac{\pi}{2}.$$



$y = \cot^{-1} x \quad \Leftrightarrow \quad \cot y = x$, where $x$ is any real number and $0 < y < \pi$.



**Example 5:**

**Find the exact values.**

a) $\csc^{-1} 2$

b) $sec^{-1}\sqrt{2}$

c) $\cot^{-1} 1$