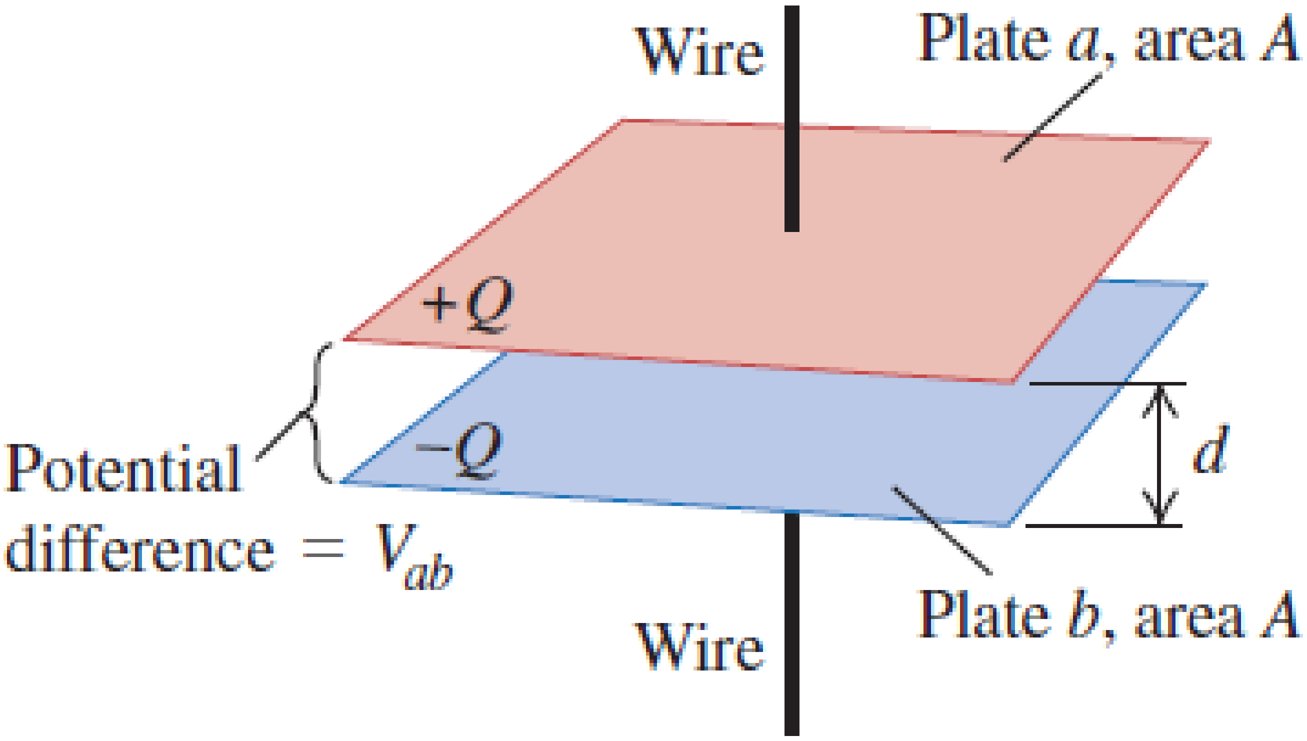
Lecture 15

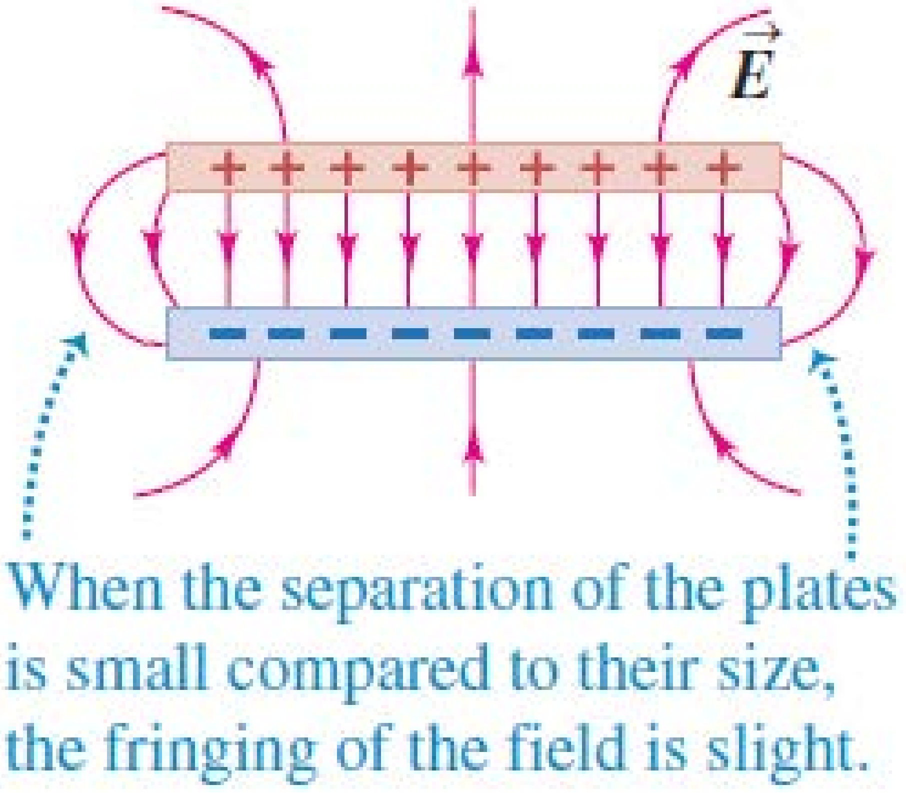
Current, Resistance, & Electromotive Force

Date: 5/29/2025

Course Instructor:
Jingtian Hu (胡竞天)

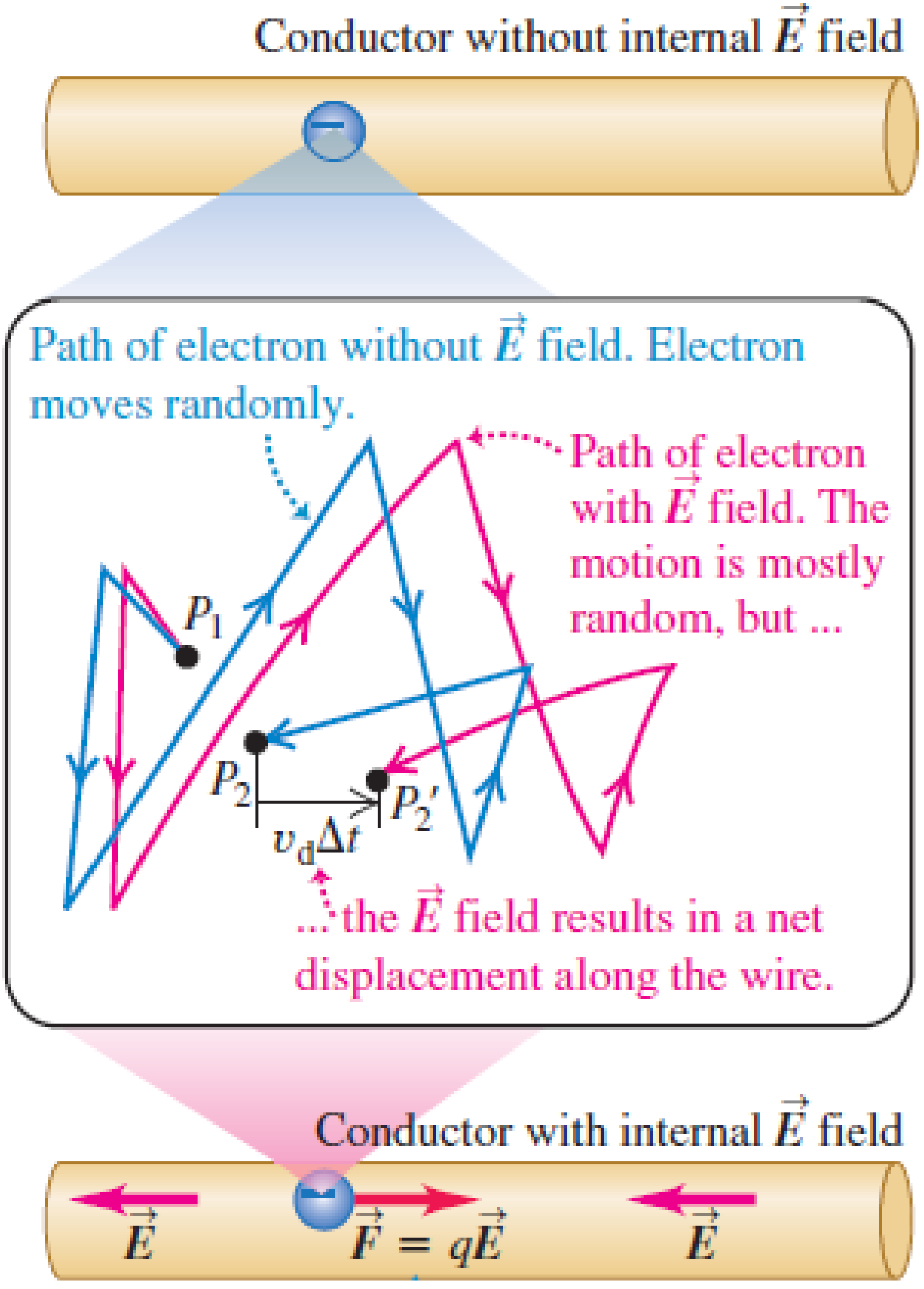
Previous Lecture: Capacitance





Current

A **current** is any motion (flow) of charge from one region to another.

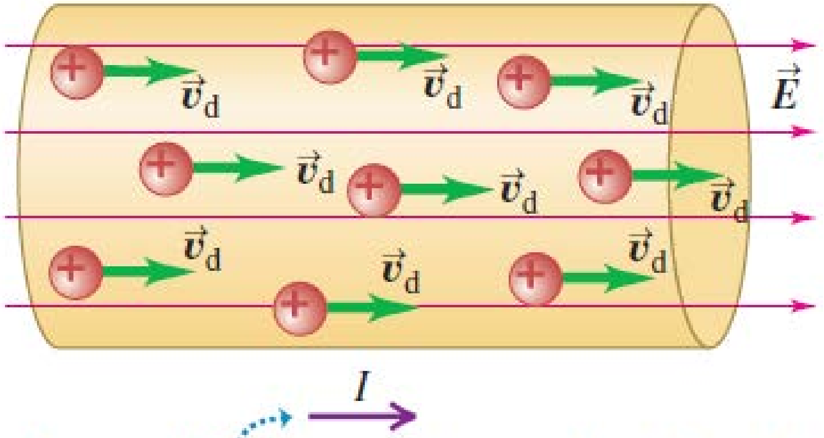


Under ***E***, charged particle (such as a free electron) is subjected to a force ***F*** = q***E*****.** If the charged particle were moving in *vacuum*, this steady force would cause a steady acceleration in the direction of ***F***

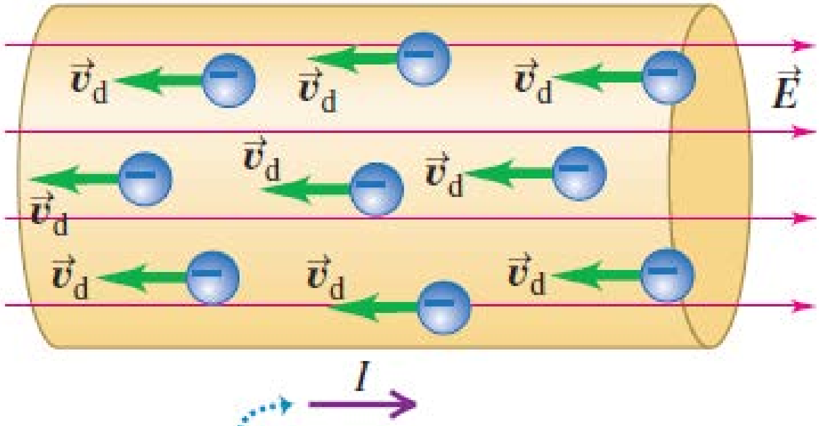
But a charged particle moving in a *conductor* undergoes frequent **collisions** with the ions of the material and their steady-state velocity is described in terms of the drift velocity $\boldsymbol{v}_d$

An electron has a negative charge q, so the force on it due to the $\vec{E}$ field is in the direction opposite to $\vec{E}$.

The Direction of Current Flow



A **conventional current** is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.

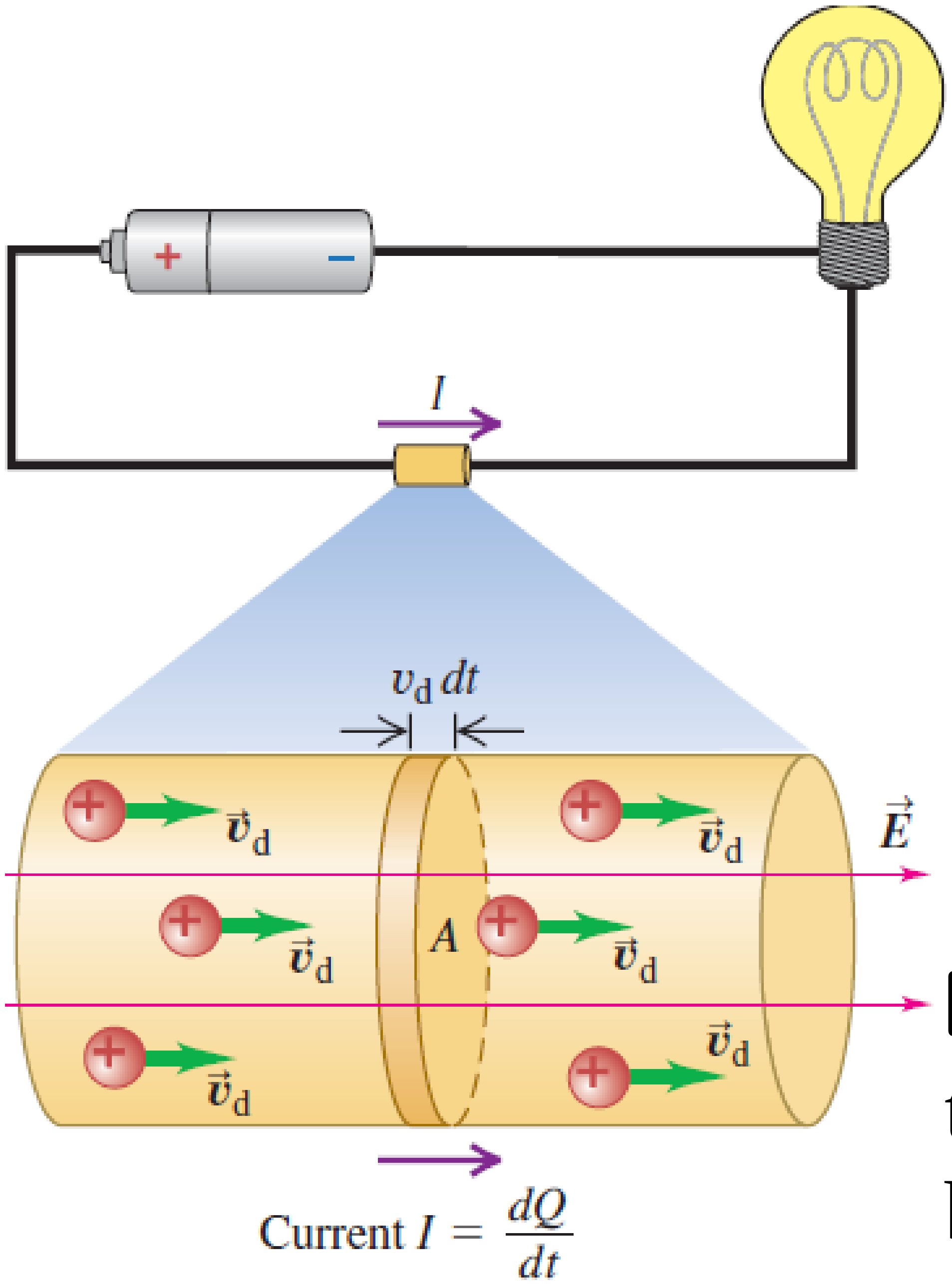


In a metallic conductor, the moving charges are electrons — but the *current* still points in the direction positive charges would flow.

Current carrying materials (***carriers***):

- Metals: moving charges are always (negative) electrons,
- **Ionized gas (plasma) or an ionic solution**: both electrons and positively charged ions
- Semiconductors such as germanium (Ge) or silicon (Si): partly by electrons and partly by motion of vacancies, also known as ***holes***

The Direction of Current Flow

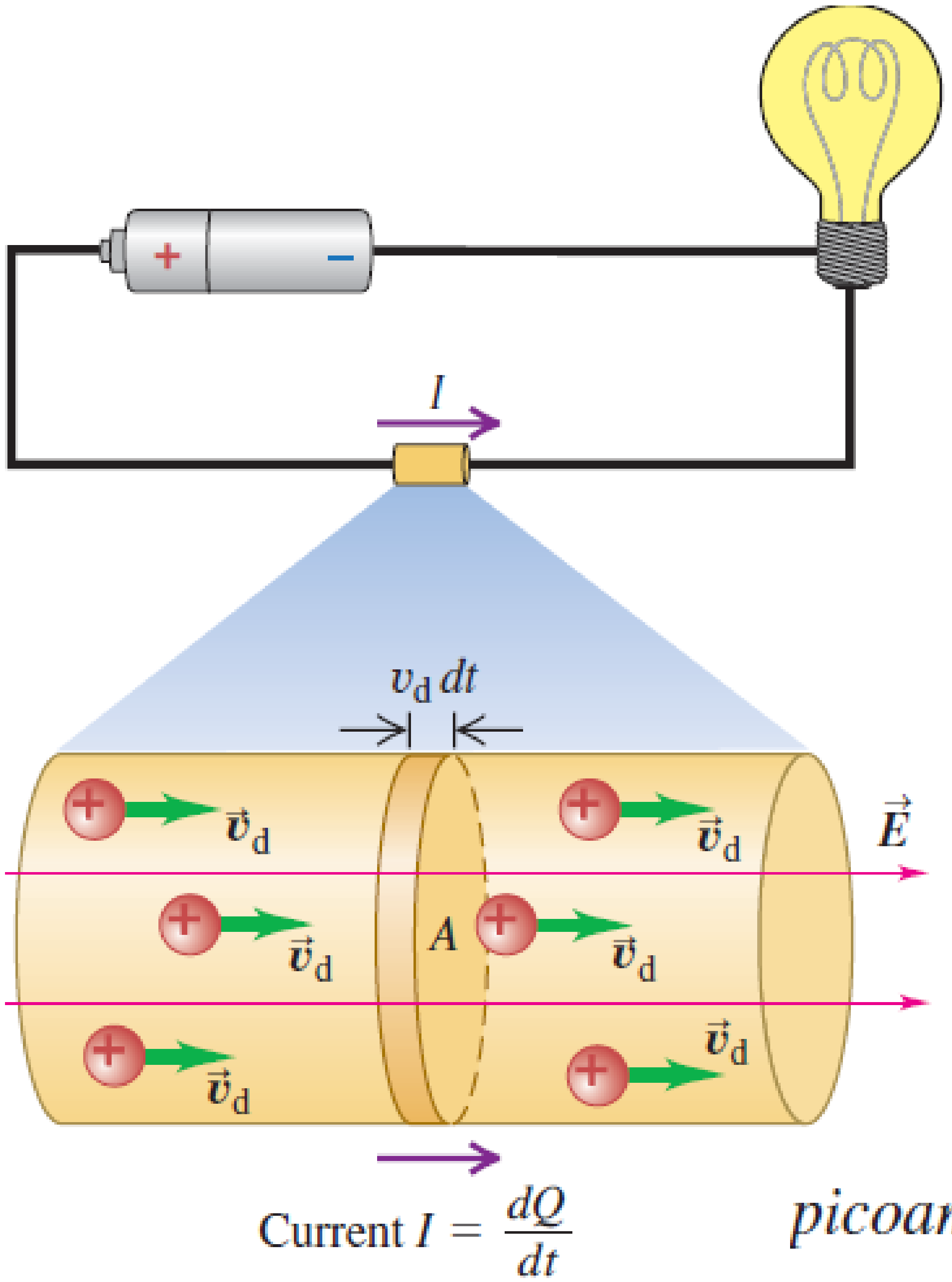


Consider the moving charges to be *positive*, so they are moving in the same direction as the current. We define the **current** through the cross-sectional area ***A*** to be ***the net charge flowing through the area per unit time***. Thus, if a net charge ***dQ*** flows through an area in a time , the current through the area is:

$$I = \frac{dQ}{dt} \qquad \text{(definition of current)}$$

CAUTION Current is not a vector Although we refer to the *direction* of a current, current as defined by the equation is ***not*** a vector quantity.

The Direction of Current Flow



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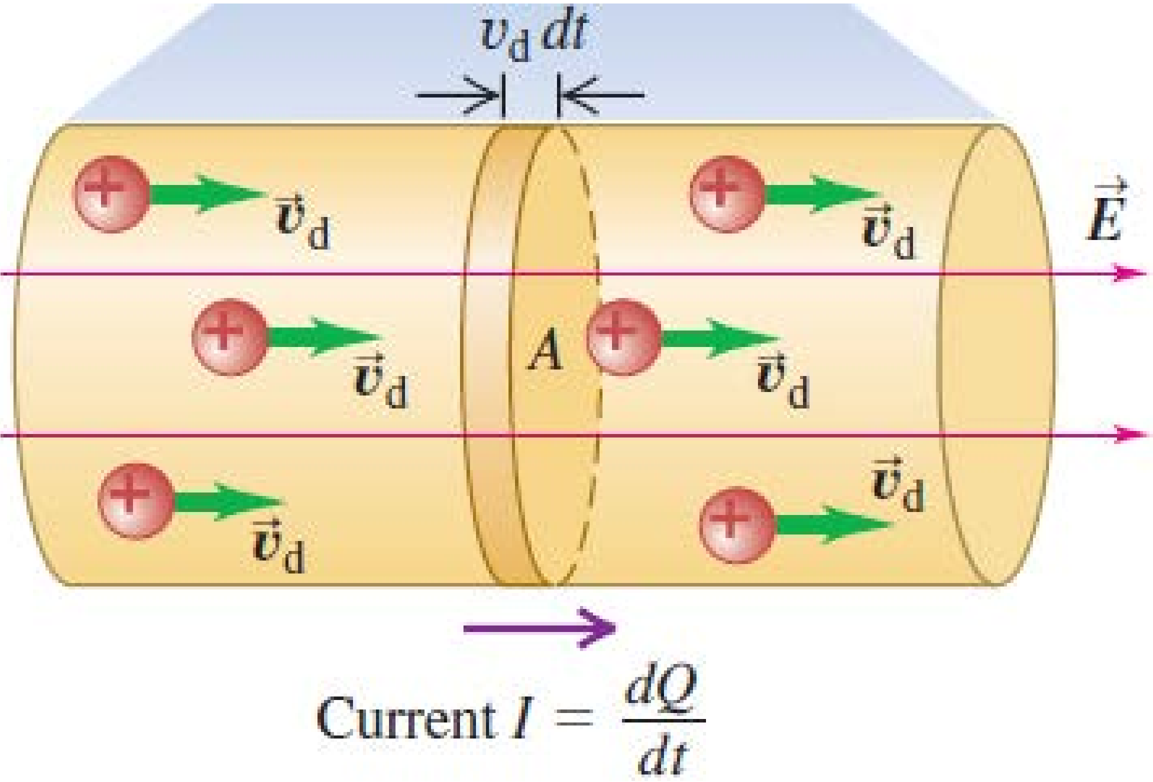
SI unit: **ampere (*A*);** one ampere is defined to be *one coulomb per second*

picoamperes $(1\ \text{pA} = 10^{-12}\ \text{A})$ *nanoamperes* $(1\ \text{nA} = 10^{-9}\ \text{A})$

Current, Drift Velocity, and Current Density

Now we will derive some properties of current based on ***a microscopic scheme***, frequently used in *solid-state physics* or *semiconductor physics*.



- n: the concentration of charge carriers (unit m^{-3})
- v_d: drift velocity of the moving charge carriers
- $v_d\,dt$: The distance that an average charge moves
- $Av_d\,dt$: volume of this section
- $nAv_d\,dt$: number of charge carriers
- $qnAv_d\,dt$: total charge flowed

Current, Drift Velocity, and Current Density

$$dQ = q(nAv_d\,dt) = nqv_dA\,dt$$

and the current is

$$I = \frac{dQ}{dt} = nqv_dA$$

The current *per unit cross-sectional area* is called the **current density** J:

$$J = \frac{I}{A} = nqv_d$$

The units of current density are amperes per square meter A/m^2

Current density is a vector, but current is not. ***J*** describes how charges flow at a certain point, and the vector's direction tells you about the direction of the flow Current describes how charges flow through an extended object such as a wire.

Current, Drift Velocity, and Current Density

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Example 25.1 Current density and drift velocity in a wire

An 18-gauge copper wire (the size usually used for lamp cords), with a diameter of 1.02 mm carries a constant current of 1.67 *A* to a 200-W lamp. The free-electron density in the wire is 8.5×10^{28} per cubic meter. Find (a) the current density and (b) the drift speed.

$$A = \frac{\pi d^2}{4} = \frac{\pi(1.02 \times 10^{-3}\ \mathrm{m})^2}{4} = 8.17 \times 10^{-7}\ \mathrm{m}^2$$

The magnitude of the current density is then

$$J = \frac{I}{A} = \frac{1.67\ \mathrm{A}}{8.17 \times 10^{-7}\ \mathrm{m}^2} = 2.04 \times 10^{6}\ \mathrm{A/m}^2$$

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(b) From Eq. (25.3) for the drift velocity magnitude v_d, we find

$$v_d = \frac{J}{n|q|} = \frac{2.04 \times 10^6 \text{ A/m}^2}{(8.5 \times 10^{28} \text{ m}^{-3})|-1.60 \times 10^{-19} \text{ C}|}$$

$$= 1.5 \times 10^{-4} \text{ m/s} = 0.15 \text{ mm/s}$$

Resistivity

The current density $\boldsymbol{J}$ in a conductor depends on the electric field $\boldsymbol{E}$ and on the properties of the material. In general, this dependence can be quite complex. But for some materials, especially metals, at a given temperature, is nearly *directly proportional* to $\boldsymbol{E}$ and the ratio of the magnitudes of and is constant, the resistivity ρ (Ohm's Law):

$$\rho = \frac{E}{J} \qquad \text{(definition of resistivity)}$$

1 V/A is called one *ohm*

Ω

Unit: $\Omega \cdot \mathrm{m} = (\mathrm{V/m})/(\mathrm{A/m^2}) = \mathrm{V \cdot m/A}$

Ohm's Law is an *idealized model* that describes the behavior of some materials quite well but **is not a general description** of *all* matter.

The reciprocal of resistivity is **conductivity.** Unit: $(\Omega \cdot \mathrm{m})^{-1}$

Resistivity

- *Semiconductors* have resistivities intermediate between metals and insulators.
- A material that obeys Ohm's law reasonably well is called an ohmic conductor or a linear conductor
- Many materials show deviate from Ohm's-law behavior; they are nonohmic, or nonlinear

	Substance	$\rho\ (\Omega \cdot m)$	Substance	$\rho\ (\Omega \cdot m)$
Conductors			**Semiconductors**	
Metals	Silver	1.47×10^{-8}	Pure carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}	Pure germanium	0.60
	Gold	2.44×10^{-8}	Pure silicon	2300
	Aluminum	2.75×10^{-8}	**Insulators**	
	Tungsten	5.25×10^{-8}	Amber	5×10^{14}
	Steel	20×10^{-8}	Glass	10^{10}–10^{14}
	Lead	22×10^{-8}	Lucite	$>10^{13}$
	Mercury	95×10^{-8}	Mica	10^{11}–10^{15}
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}	Quartz (fused)	75×10^{16}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulfur	10^{15}
	Nichrome	100×10^{-8}	Teflon	$>10^{13}$
			Wood	10^{8}–10^{11}

Resistivity and Temperature

The resistivity of a *metals* nearly always ***increases*** with temperature

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)] \quad \text{(temperature dependence of resistivity)} \qquad (25.6)$$

where ρ_0 is the resistivity at a reference temperature T_0 (often taken as 0 ºC or 20 ºC) and $\rho(T)$ is the resistivity at temperature , which may be higher or lower than T_0. The factor α is called the **temperature coefficient of resistivity**

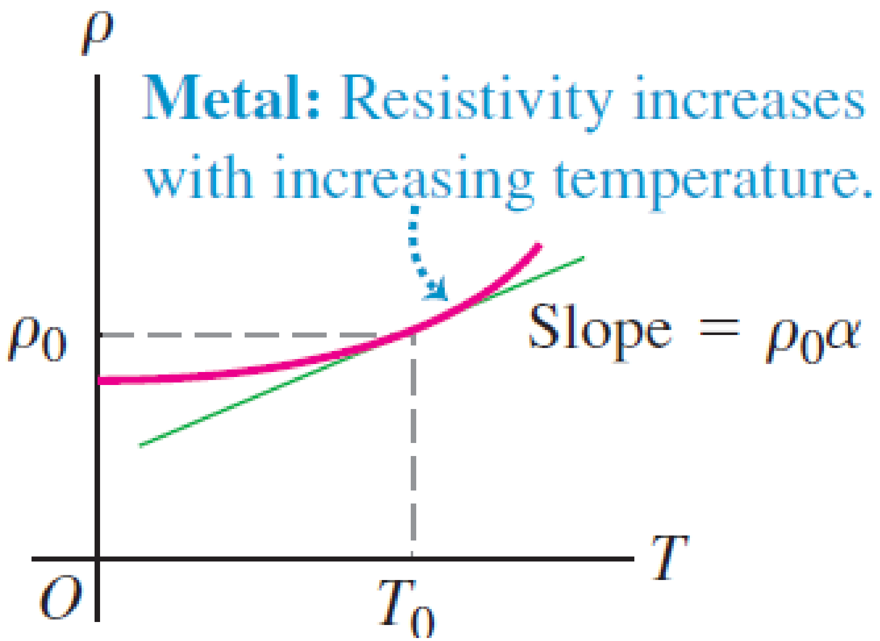
Table 25.2 Temperature Coefficients of Resistivity (Approximate Values Near Room Temperature)

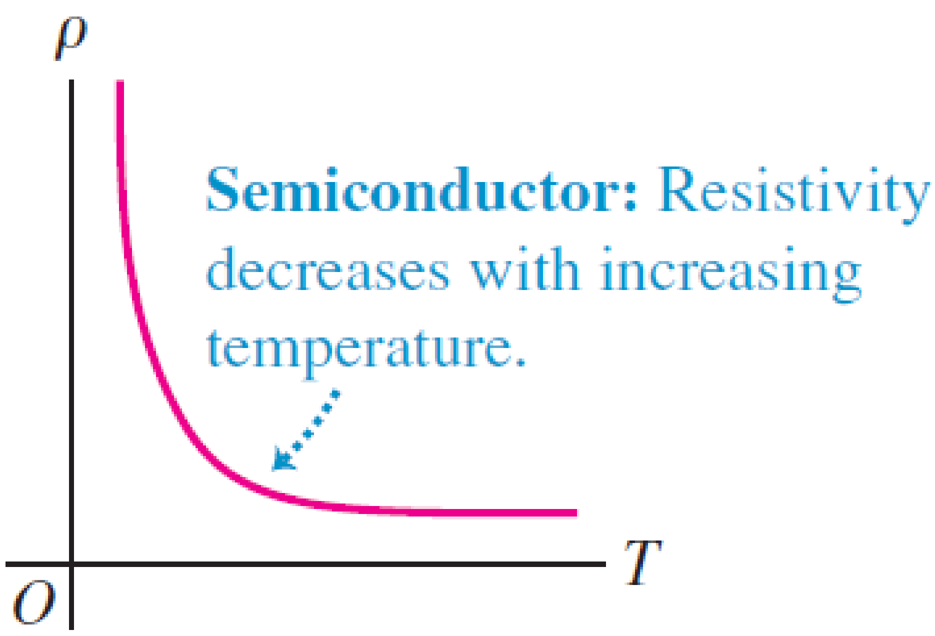
Material	$\alpha\ [(^\circ\mathrm{C})^{-1}]$	Material	$\alpha\ [(^\circ\mathrm{C})^{-1}]$
Aluminum	0.0039	Lead	0.0043
Brass	0.0020	Manganin	0.00000
Carbon (graphite)	−0.0005	Mercury	0.00088
Constantan	0.00001	Nichrome	0.0004
Copper	0.00393	Silver	0.0038
Iron	0.0050	Tungsten	0.0045

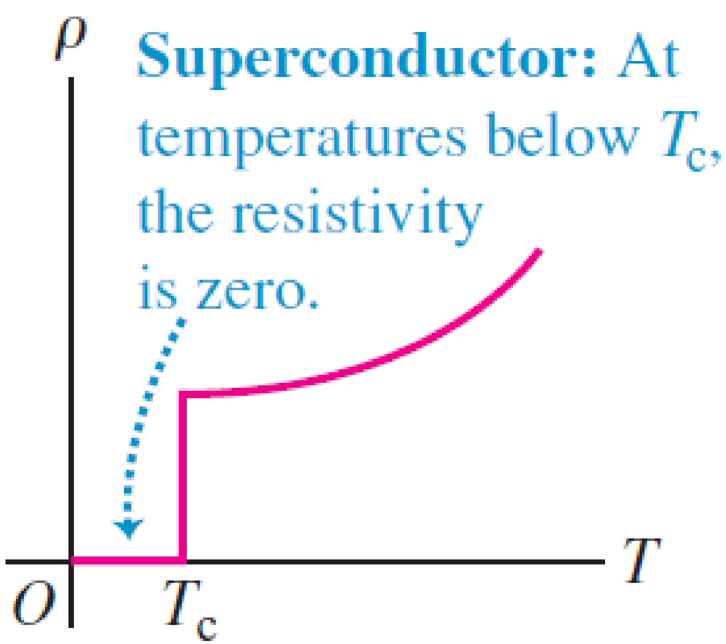
Resistivity and Temperature

The resistivity of graphite (a nonmetal) and semiconductors *decreases* with increasing temperature, since at higher temperatures, more electrons are “shaken loose” from the atoms and become mobile.

Superconductivity: At below 4.2 K (around -269 ºC) the resistivity of mercury suddenly dropped to zero







Resistance

Ohm's law is not convenient to use

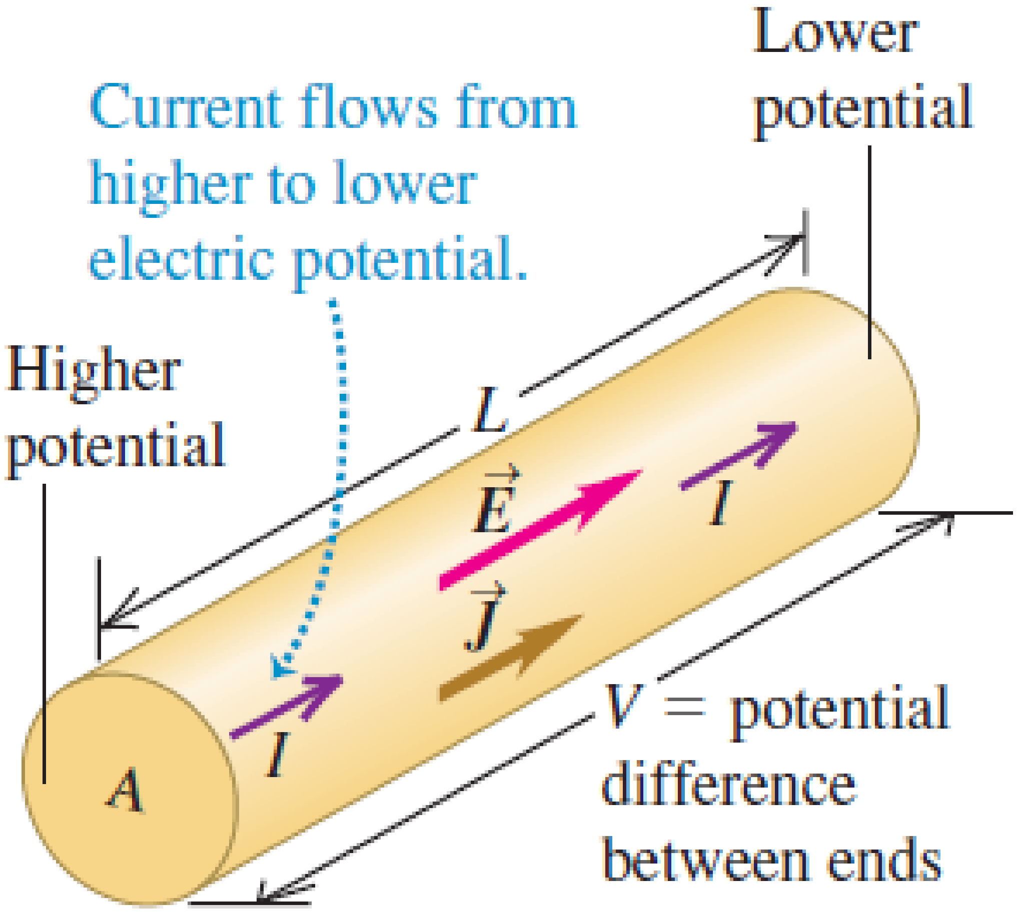
$$\vec{E} = \rho\vec{J}$$

E and ***J*** cannot be measured readily

$$\frac{V}{L} = \frac{\rho I}{A} \quad \text{or} \quad V = \frac{\rho L}{A} I$$

The ratio of ***V*** to ***I*** for a particular conductor is called its **resistance *R***

$$R = \frac{V}{I}$$



A conductor with uniform cross section. The current density is uniform over any cross section, and the electric field is constant along the length.

Resistance

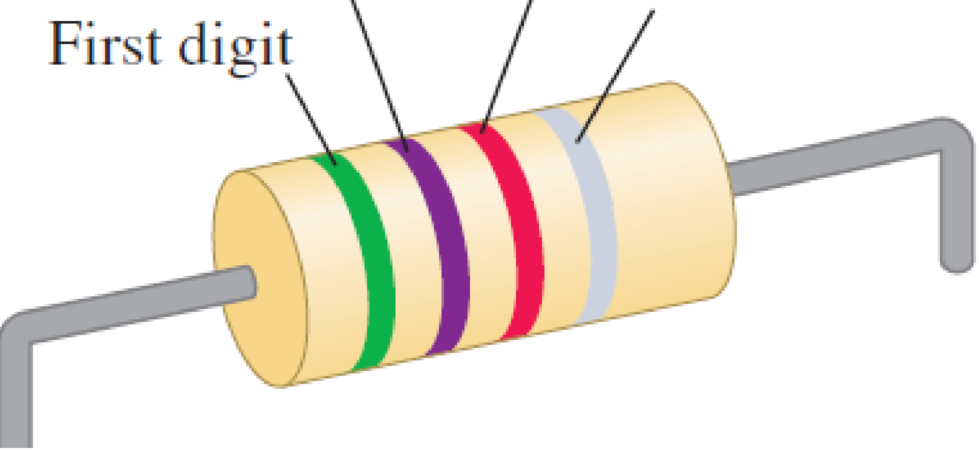
Resistance R of a particular conductor is related to the resistivity ρ of its material by

$$R = \frac{\rho L}{A} \quad \text{(relationship between resistance and resistivity)}$$

If ρ is constant, as is the case for ohmic materials, then so is R.

$$V = IR \quad \text{(relationship among voltage, current, and resistance)}$$

This resistor has a resistance of 5.7 kΩ with a precision of 10%.



Color	Value as Digit	Value as Multiplier
Black	0	1
Brown	1	10
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5

Resistance: Temperature Dependence

Because the **resistivity** of a material varies with temperature, the **resistance** of a conductor also varies with T. ***Approximately linear***:

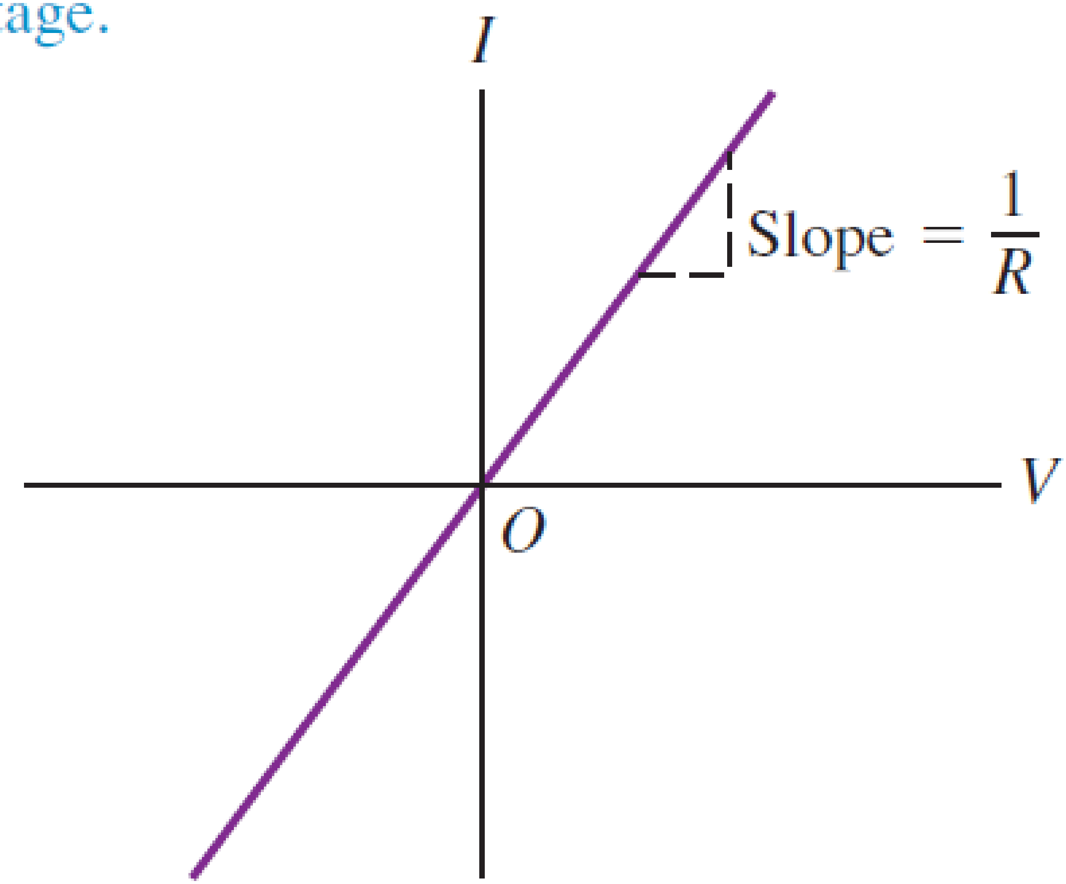
$$R(T) = R_0[1 + \alpha(T - T_0)]$$

The *change* in resistance resulting from a temperature change T-T_0 is:

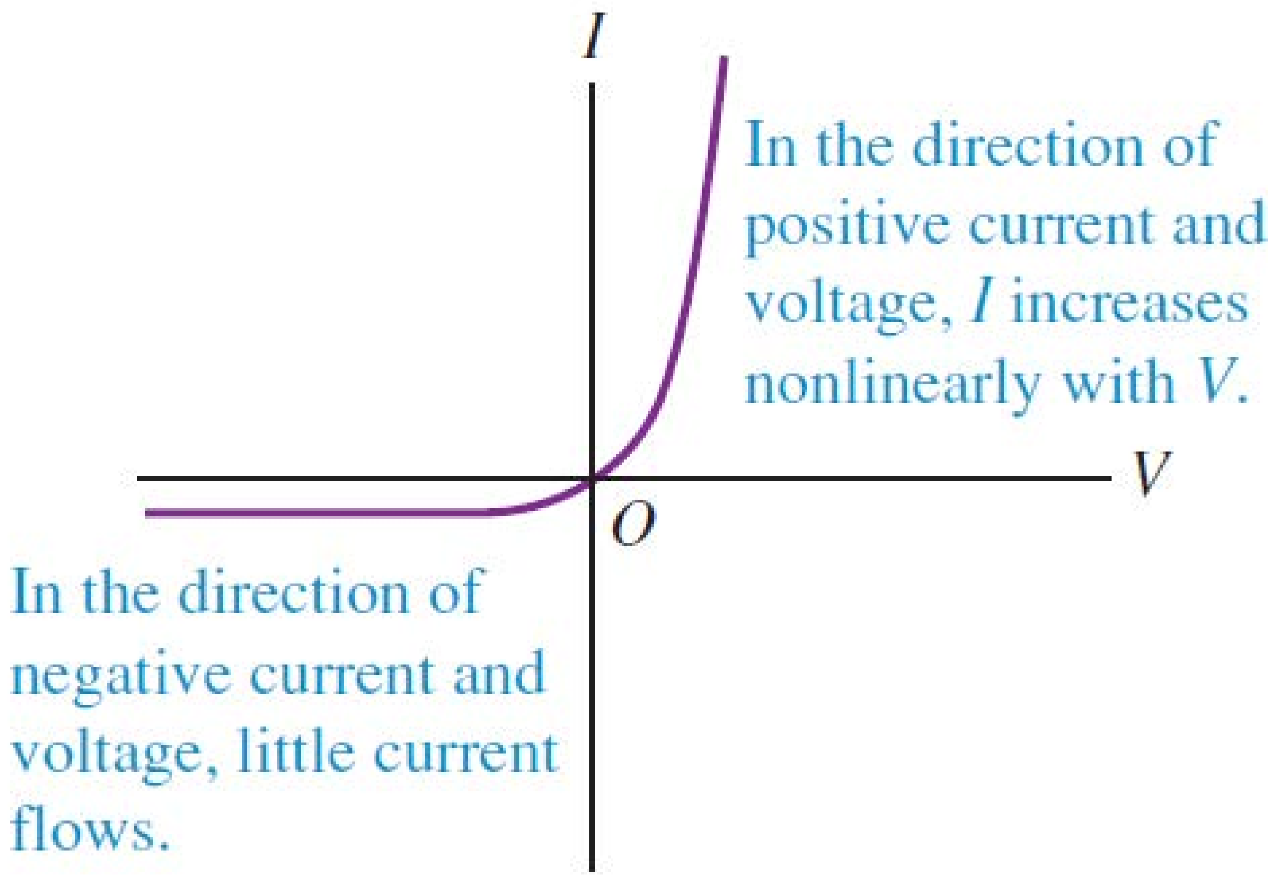
$$R_0\alpha(T - T_0)$$

Resistance: Metals vs Semiconductor Diode

Ohmic resistor (e.g., typical metal wire): At a given temperature, current is proportional to voltage.



Semiconductor diode: a nonohmic resistor



Example 25.2 Electric field, potential difference, and resistance in a wire

The 18-gauge copper wire of Example 25.1 has a cross-sectional area of 8.20×10^{-7} m^2. It carries a current of 1.67 A. Find (a) the electric-field magnitude in the wire; (b) the potential difference between two points in the wire 50.0 m apart; (c) the resistance of a 50.0-m length of this wire.

From Table 25.1, $\rho = 1.72 \times 10^{-8}\ \Omega \cdot \mathrm{m}$

(a) $$E = \rho J = \frac{\rho I}{A} = \frac{(1.72 \times 10^{-8}\ \Omega \cdot \mathrm{m})(1.67\ \mathrm{A})}{8.20 \times 10^{-7}\ \mathrm{m}^2} = 0.0350\ \mathrm{V/m}$$

(b) The potential difference is (by definition):

$$V = EL = (0.0350\ \mathrm{V/m})(50.0\ \mathrm{m}) = 1.75\ \mathrm{V}$$

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(**c**) The resistance of 50.0 m of this wire is:

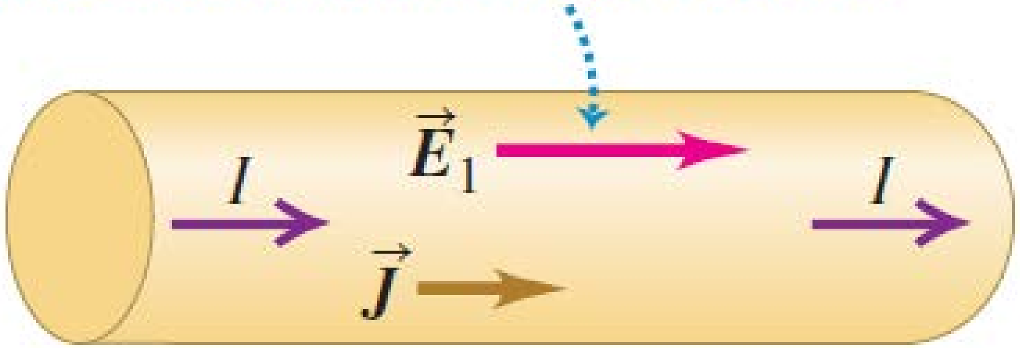
$$R = \frac{\rho L}{A} = \frac{(1.72 \times 10^{-8}\ \Omega \cdot \mathrm{m})(50.0\ \mathrm{m})}{8.20 \times 10^{-7}\ \mathrm{m}^2} = 1.05\ \Omega$$

Alternatively, we can find *R* by:

$$R = \frac{V}{I} = \frac{1.75\ \mathrm{V}}{1.67\ \mathrm{A}} = 1.05\ \Omega$$

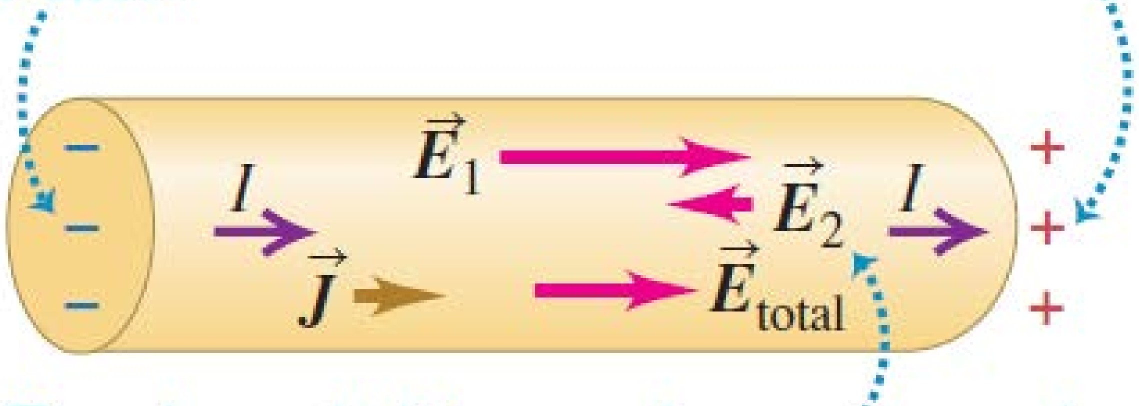
Electromotive Force and Circuits

(a) An electric field $\vec{E}_1$ produced inside an isolated conductor causes a current.



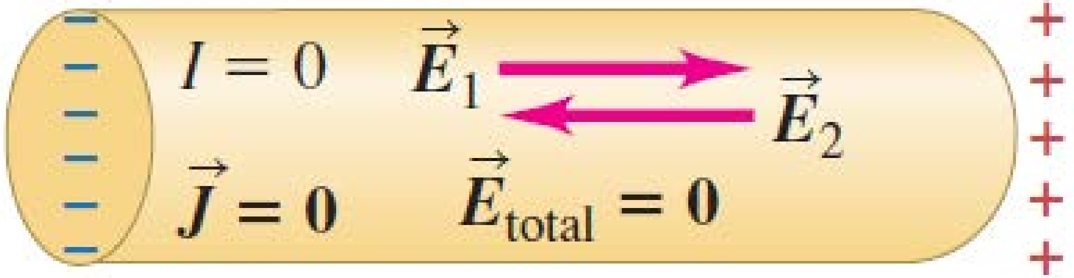
For a conductor to have a steady current, it must be part of a path that forms a ***closed*** loop or **complete circuit, otherwise…**

(b) The current causes charge to build up at the ends.

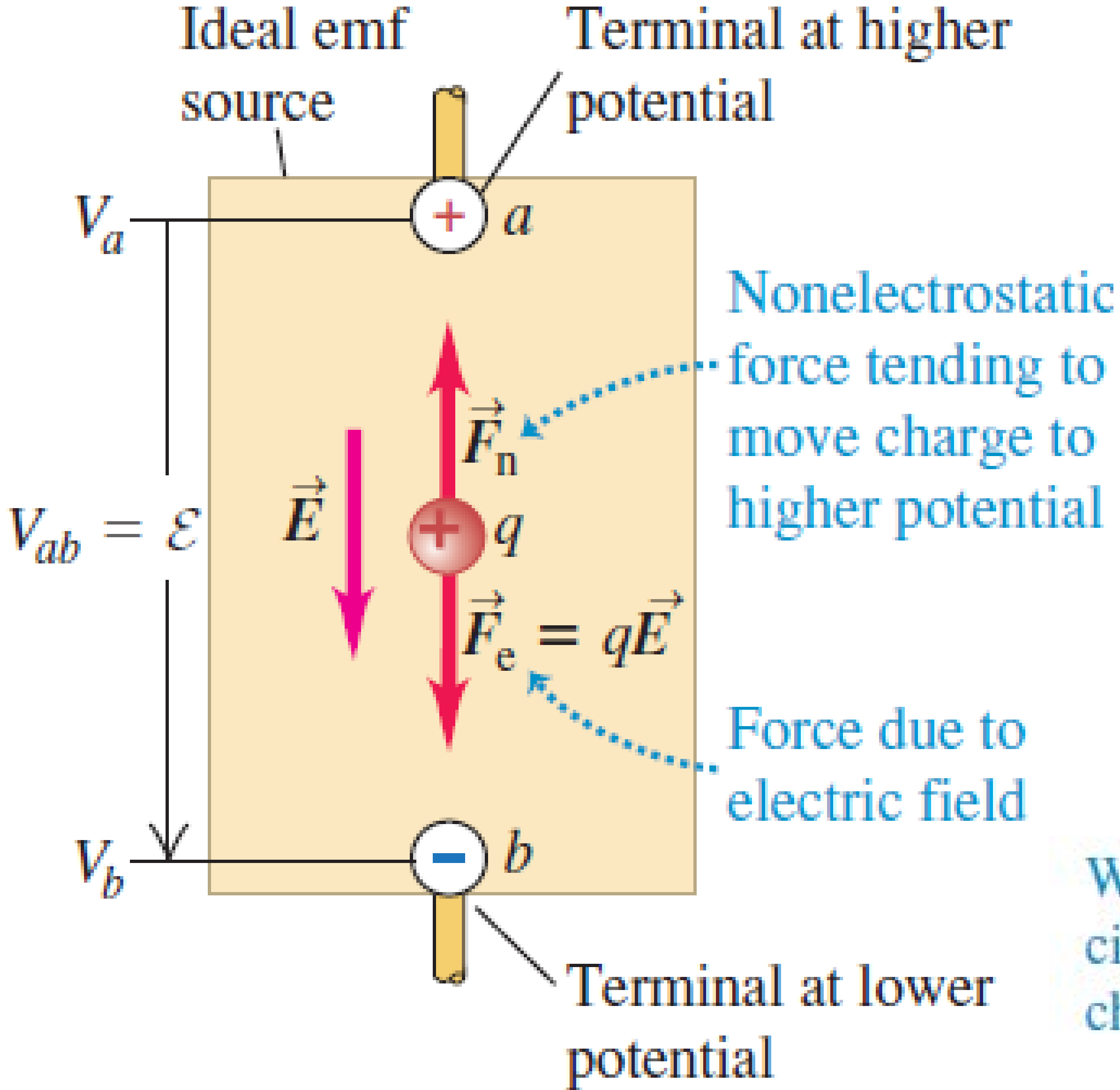


The charge buildup produces an opposing field $\vec{E}_2$, thus reducing the current.

(c) After a very short time $\vec{E}_2$ has the same magnitude as $\vec{E}_1$; then the total field is $\vec{E}_{total} = 0$ and the current stops completely.



Electromotive Force and Circuits



Electromotive force (abbreviated **emf**)

- Terminal *a*, marked + is maintained at *higher* potential than terminal b, marked -.
- *This is a poor term because* ***emf is not a force****, but like* ***potential***.

When the emf source is not part of a closed circuit, $F_n = F_e$ and there is no net motion of charge between the terminals.

Internal Resistance

Potential difference across a real source in a circuit is *not* equal to emf

- Charge moving through any real source encounters *resistance*
- *Called* the **internal resistance** of the source, denoted by r
- Actual voltage that can be used by the circuit is V_{ab}

$$V_{ab} = \mathcal{E} - Ir \qquad \text{(terminal voltage, source with internal resistance)}$$

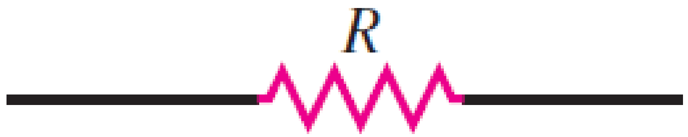
V_{ab}, called the **terminal voltage,** is less than the emf $\mathcal{E}$ because of the term Ir representing the potential drop across the internal resistance r.

Terminal voltage equals the emf only if no current is flowing through the source

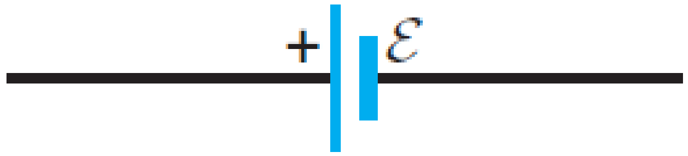
$$\mathcal{E} - Ir = IR \quad \text{or} \quad I = \frac{\mathcal{E}}{R + r} \qquad \text{(current, source with internal resistance)}$$

Symbols for Circuit Diagrams

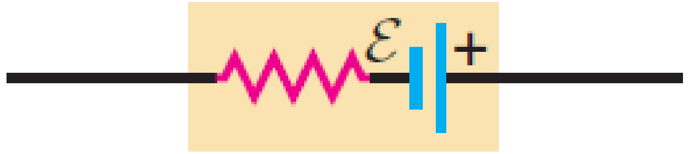
Conductor with negligible resistance



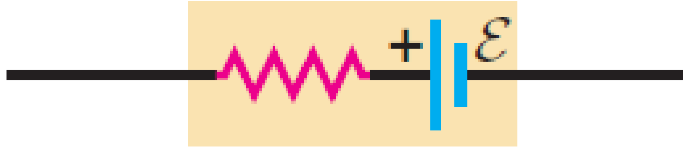
Resistor



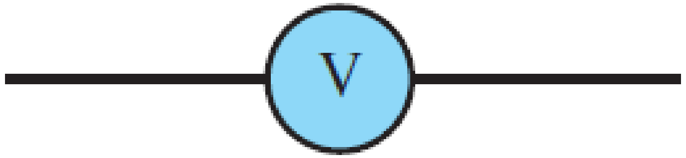
Source of emf (longer vertical line always represents the positive terminal, usually the terminal with higher potential)



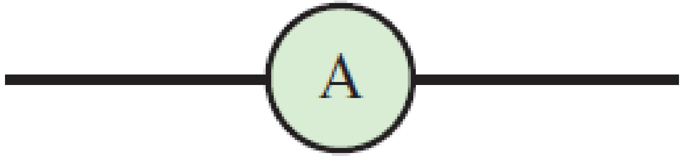
or



Source of emf with internal resistance r (r can be placed on either side)



Voltmeter (measures potential difference between its terminals)



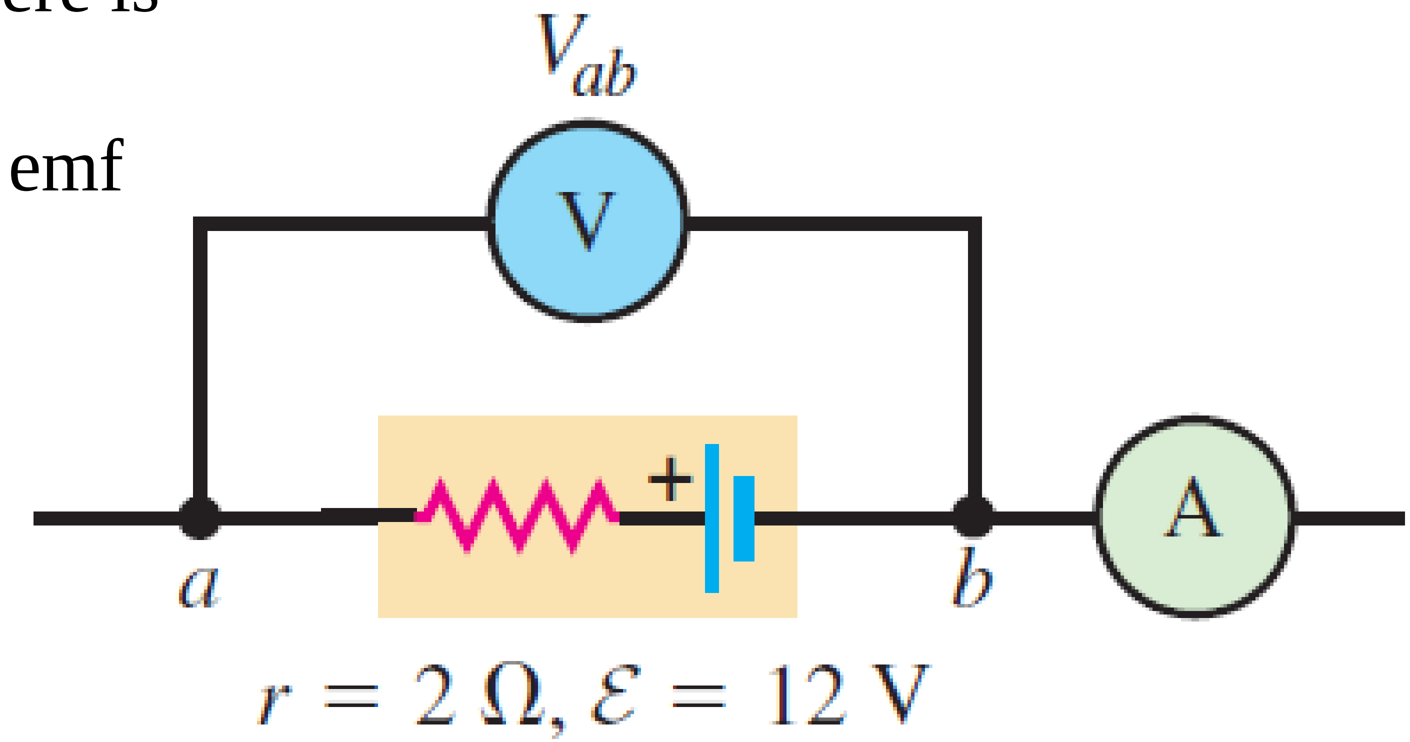
Ammeter (measures current through it)

Conceptual Example 25.4 A source in an open circuit

The wires to the left of a and b to the right of the ammeter A are not connected to anything. Determine the respective readings V_{ab} and I of the idealized voltmeter V and the idealized ammeter A.

- There is *zero* current because there is no complete circuit.
- Terminal voltage is the same as emf (12 V) because the current is 0

$$V_{ab} = \mathcal{E} - Ir$$



Example 25.5 A source in a complete circuit

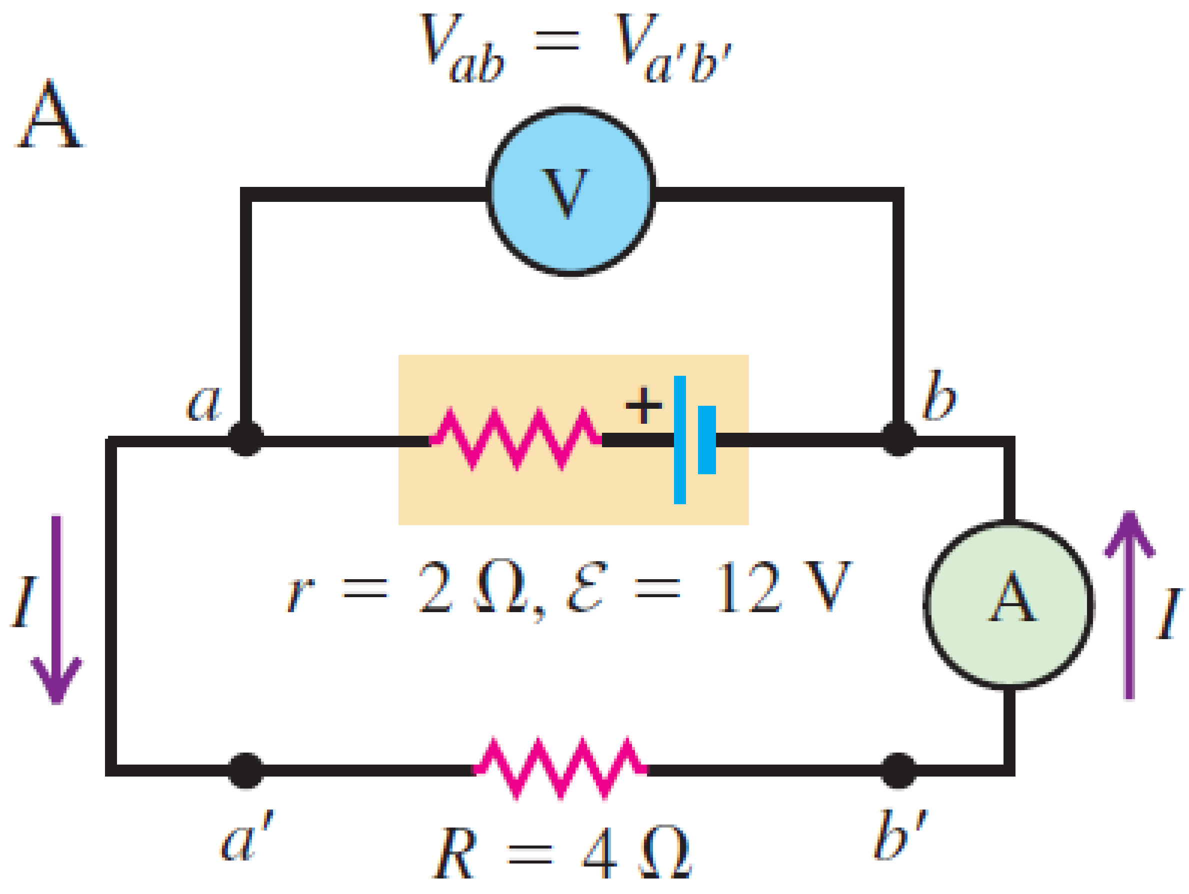
We add a 4-Ω resistor to the battery in the previous example
What are the voltmeter and ammeter readings V_{ab} and I now?

Solution: The ideal ammeter has zero resistance, so the total resistance external to the source is R = 4 Ω. The reading of the ammeter is then:

$$I = \frac{\mathcal{E}}{R + r} = \frac{12\text{ V}}{4\ \Omega + 2\ \Omega} = 2\text{ A}$$

The reading of the voltmeter is the voltage across R:

$$V_{a'b'} = IR = (2\text{ A})(4\ \Omega) = 8\text{ V}$$

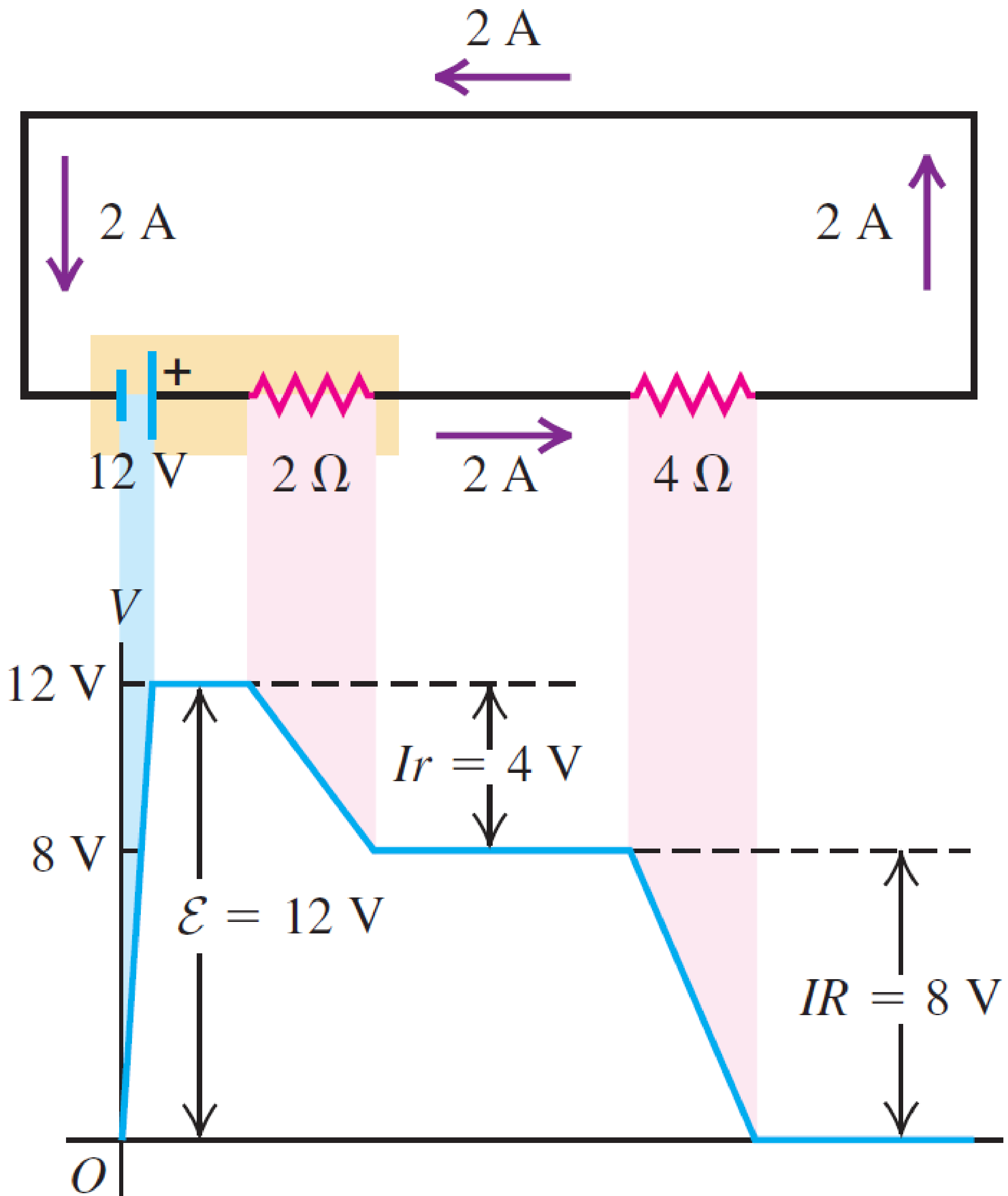


Potential Changes Around a Circuit

The net change in potential energy for a charge making a round trip around a complete circuit must be **zero**. Net change in potential around the circuit must also be **zero:**

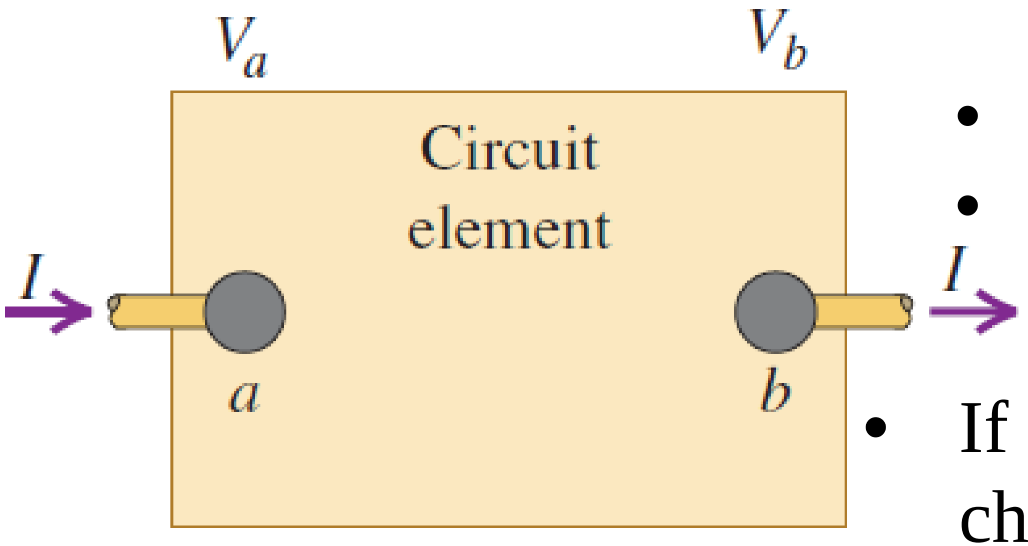
$$\mathcal{E} - Ir - IR = 0$$

- Potential increases by ε going from – to + side of the source
- Potential drops by *IR* across any resistor



Energy and Power in Electric Circuits

$$P = (V_a - V_b)I = V_{ab}I.$$



- When $V_{ab} > 0$: potential energy decreases as charge "falls" from V_a to lower potential V_b
- The moving charges don't gain kinetic energy
- QV_{ab} represents energy transferred into the circuit element – like ***heat*** generated by the resistor

- If the current is I, then in an interval dt an amount of charge $dQ = Idt$ passes through the element.

- The potential energy change for this amount of charge is $V_{ab}dQ = V_{ab}I\,dt$

Dividing this expression by dt, we obtain the *rate* at which energy is transferred either into or out of the circuit element – the *power*, denoted by P as

$$P = V_{ab}I \quad \text{(rate at which energy is delivered to or extracted from a circuit element)}$$

$$(1\ \mathrm{J/C})(1\ \mathrm{C/s}) = 1\ \mathrm{J/s} = 1\ \mathrm{W}$$

Power Input to a Pure Resistance

Current enters the higher-potential terminal of the device, and equation above represents the rate of transfer of electric potential energy *into* the circuit element

$$P = V_{ab} I = I^2 R = \frac{{V_{ab}}^2}{R} \qquad \text{(power delivered to a resistor)}$$

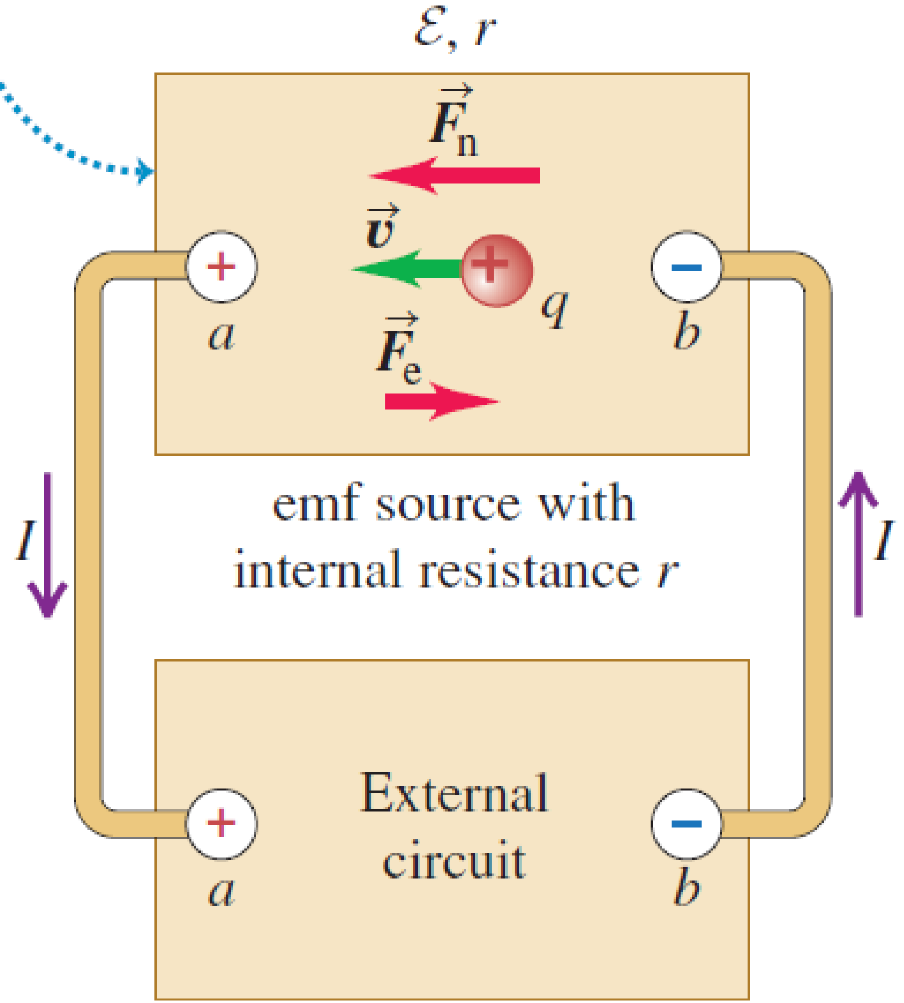
What becomes of this energy? The moving charges collide with atoms in the resistor and transfer some of their energy to these atoms, increasing the ***internal energy*** of the material.

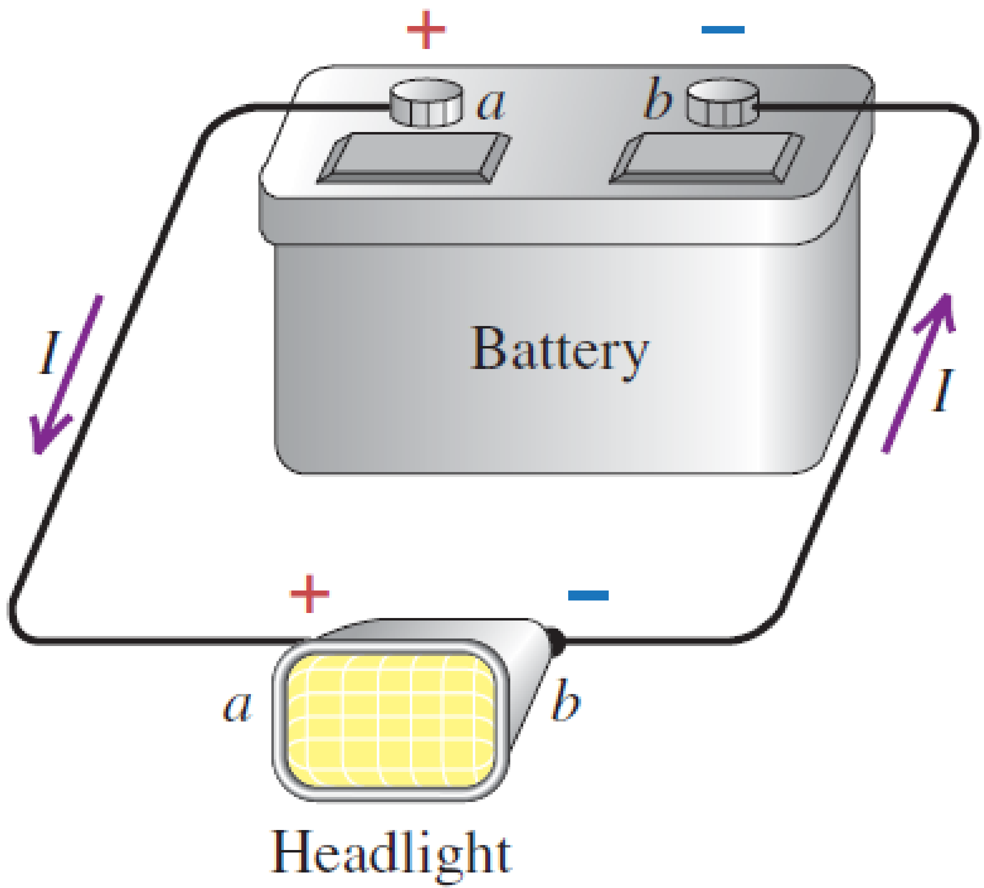
Power Output of a Source

- The emf source converts nonelectrical to electrical energy at a rate $\mathcal{E}I$.
- Its internal resistance *dissipates* energy at a rate I^2r.
- The difference $\mathcal{E}I - I^2r$ is its power output.

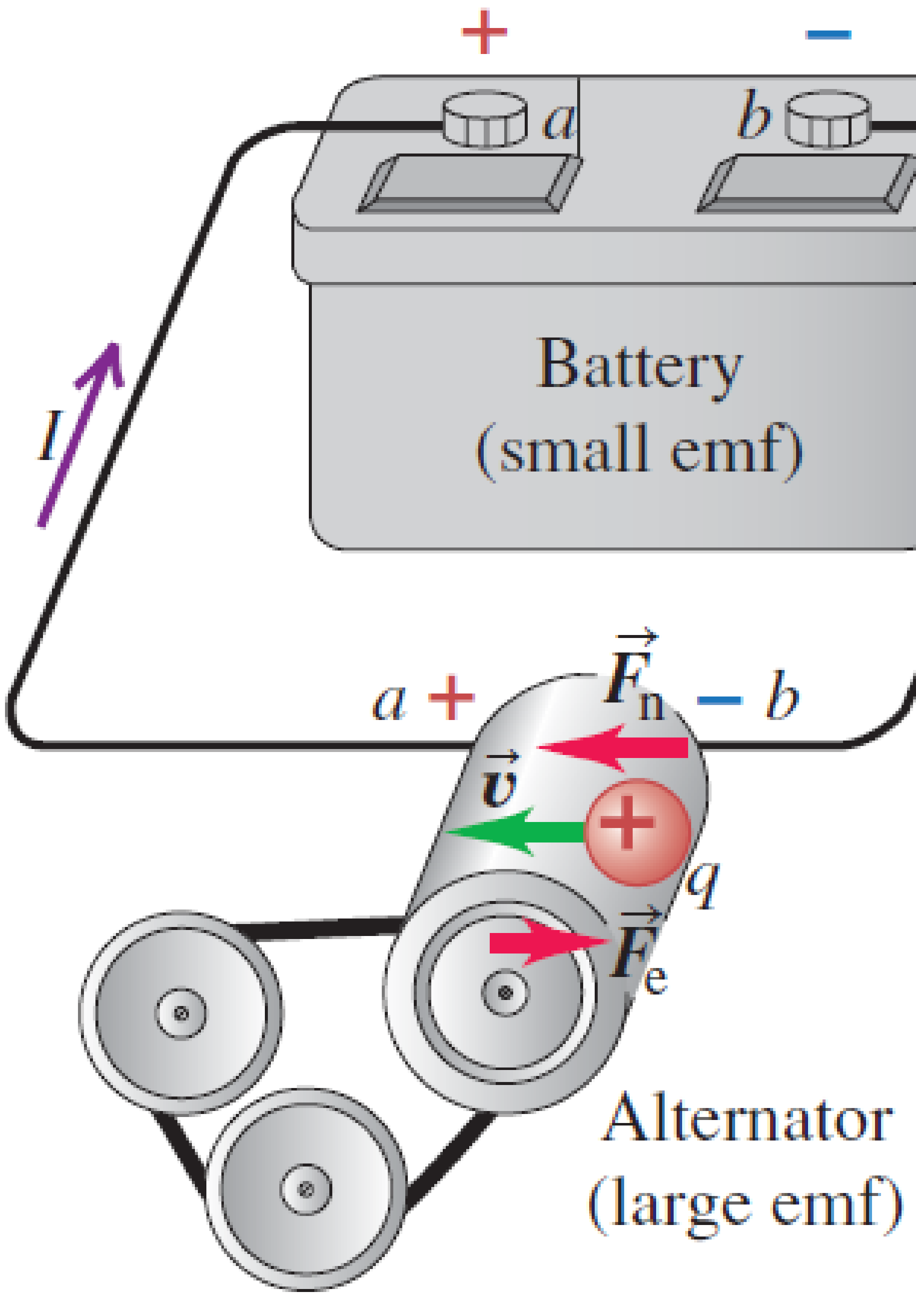
$$P = V_{ab}I \quad \text{where} \quad V_{ab} = \mathcal{E} - Ir$$

$$\text{so} \quad P = V_{ab}I = \mathcal{E}I - I^2r$$





Power Input to a Source



When two sources are connected in a simple loop circuit, the source with the larger emf delivers energy to the other.

- Lower source is pushing current backward through the upper source

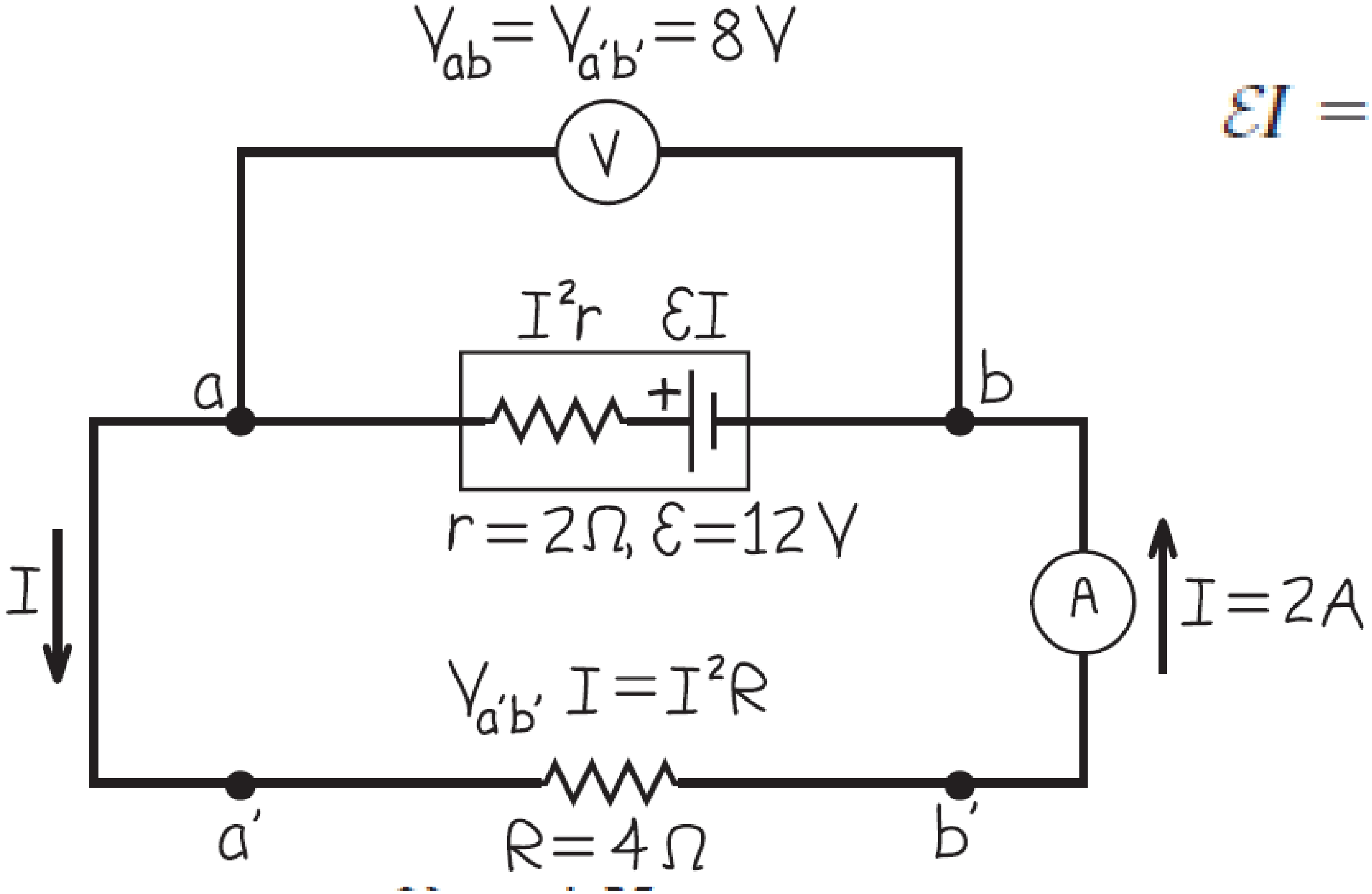
$$V_{ab} = \mathcal{E} + Ir$$

$$P = V_{ab}I = \mathcal{E}I + I^2r$$

is the total electrical power *input* to the upper source

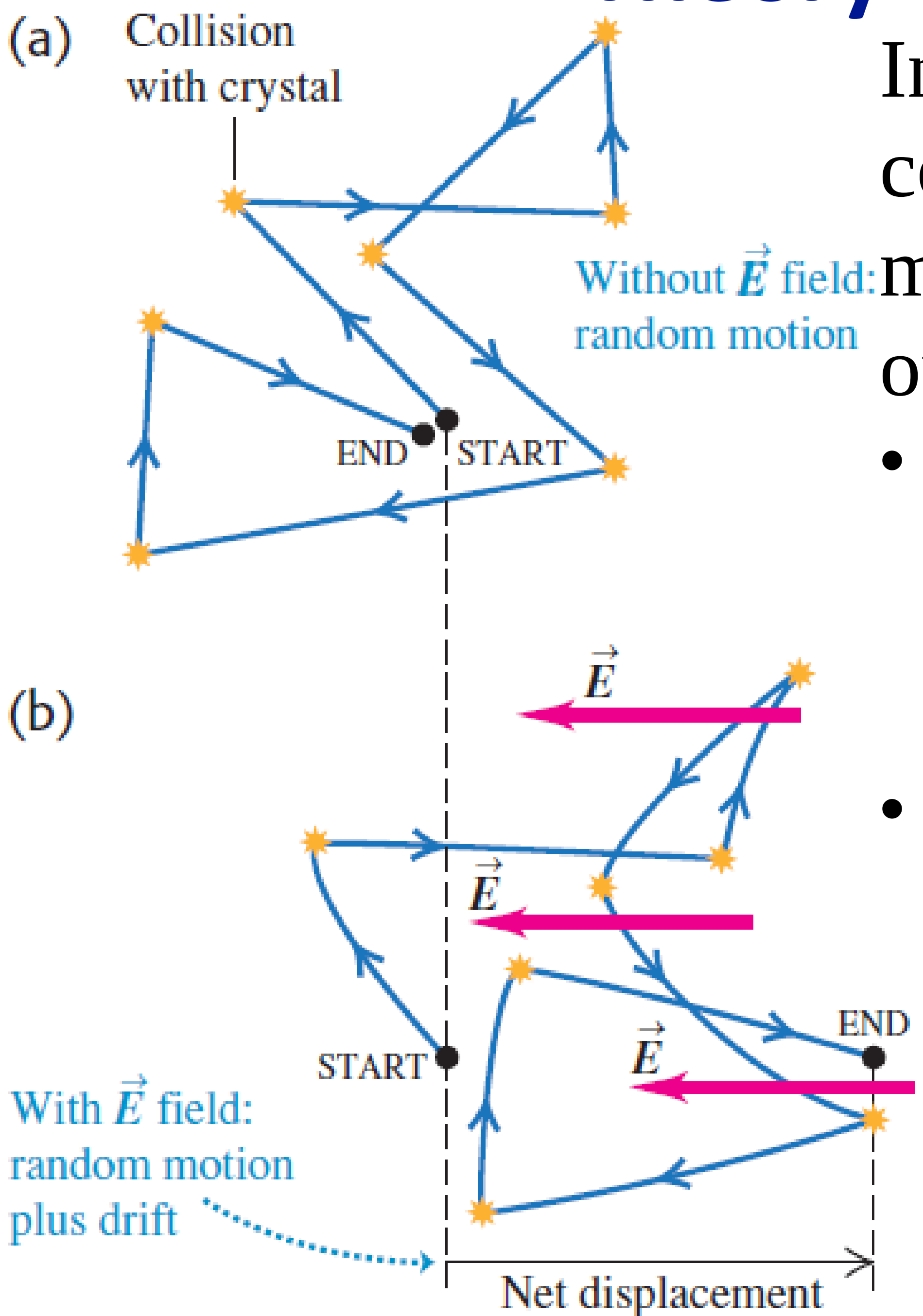
Example 25.8 Power input/output in a complete circuit

For the circuit below, find the rates of energy conversion (chemical to electrical) and energy dissipation in the battery, the rate of energy dissipation in the 4-Ω resistor, and the battery's net power output.



$$\mathcal{E}I = (12\text{ V})(2\text{ A}) = 24\text{ W}$$

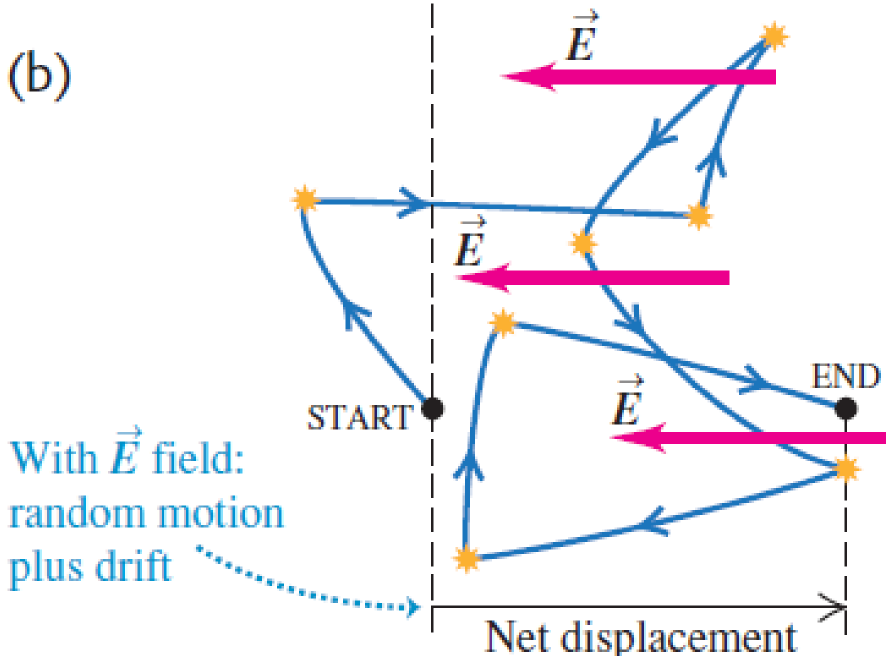
Theory of Metallic Conduction

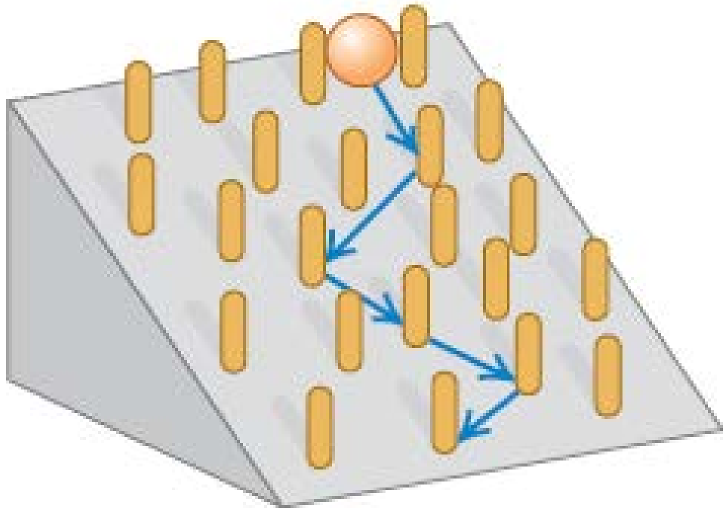


In the simplest microscopic model of conduction in a metal, each atom in the metallic crystal gives up one or more of its outer electrons.

- If there is no electric field, the electrons move in straight lines between collisions, the directions of their velocities are random, and on average they never get anywhere.
- But if an electric field is present, the paths curve slightly because of the acceleration caused by electric-field forces.

Theory of Metallic Conduction (not to be tested)





The average speed of random motion is of the order of 10^6 m/s while the average drift speed is *much* slower, of the order of 10^{-4} m/s.

The average time between collisions is called the mean free time, denoted by $\boldsymbol{\tau}$

$$\vec{J} = nq\vec{v}_{\mathrm{d}}$$

where n is the number of free electrons per unit volume, $q = -e$ is the charge of each, and $\boldsymbol{v}_{\mathrm{d}}$ is their average drift velocity.

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$$

where m is the electron mass.

Theory of Metallic Conduction (not to be tested)

We wait for a time τ, the average time between collisions, and then "turn on" the collisions. An electron that has velocity $\vec{v}_0$ at time $t = 0$ has a velocity at time $t = \tau$ equal to

$$\vec{v} = \vec{v}_0 + \vec{a}\tau$$

The velocity $\vec{v}_{av}$ of an *average* electron at this time is the sum of the averages of the two terms on the right. As we have pointed out, the initial velocity $\vec{v}_0$ is zero for an average electron, so

$$\vec{v}_{av} = \vec{a}\tau = \frac{q\tau}{m}\vec{E} \qquad (25.23)$$

Theory of Metallic Conduction (not to be tested)

After time $t = \tau$, the tendency of the collisions to decrease the velocity of an average electron (by means of randomizing collisions) just balances the tendency of the $\vec{E}$ field to increase this velocity. Thus the velocity of an average electron, given by Eq. (25.23), is maintained over time and is equal to the drift velocity $\vec{v}_d$:

$$\vec{v}_d = \frac{q\tau}{m}\vec{E}$$

Now we substitute this equation for the drift velocity $\vec{v}_d$ into Eq. (25.22):

$$\vec{J} = nq\vec{v}_d = \frac{nq^2\tau}{m}\vec{E}$$

Theory of Metallic Conduction (not to be tested)

Comparing this with Eq. (25.21), which we can rewrite as $\vec{J} = \vec{E}/\rho$, and substituting $q = -e$ for an electron, we see that the resistivity ρ is given by

$$\rho = \frac{m}{ne^2\tau} \qquad (25.24)$$

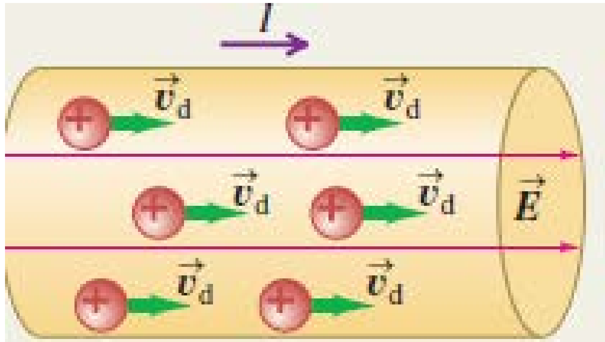
If n and τ are independent of $\vec{E}$, then the resistivity is independent of $\vec{E}$ and the conducting material obeys Ohm's law.

Turning the interactions on one at a time may seem artificial. But the derivation would come out the same if each electron had its own clock and the $t = 0$ times were different for different electrons. If τ is the average time between collisions, then $\vec{v}_d$ is still the average electron drift velocity, even though the motions of the various electrons aren't actually correlated in the way we postulated.

Summary

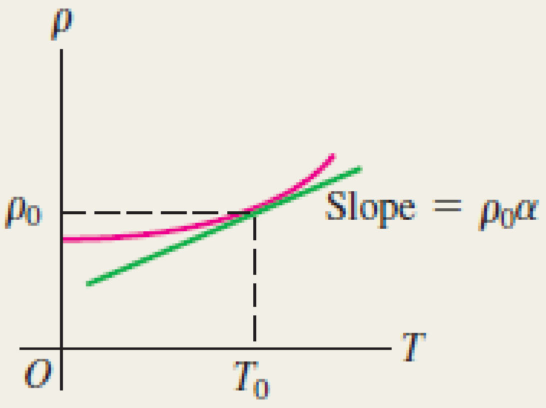
$$I = \frac{dQ}{dt} = n|q|v_d A$$

$$\vec{J} = nq\vec{v}_d$$



$$\rho = \frac{E}{J}$$

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$



Metal: ρ increases with increasing T.

$$V = IR$$

$$R = \frac{\rho L}{A}$$

$$V_{ab} = \mathcal{E} - Ir$$

(source with internal resistance)

$$P = V_{ab} I$$

(general circuit element)

$$P = V_{ab}I = I^2R = \frac{V_{ab}^2}{R}$$

(power into a resistor)