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College Algebra and Trigonometry

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① Apply the Double-Angle Formulas

$$\sin 2\theta = \sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos(\theta+\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$$

$$= \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \tan(\theta+\theta) = \frac{\tan\theta + \tan\theta}{1-\tan\theta\tan\theta} = \frac{2\tan\theta}{1-\tan^2\theta}$$

Example 1:

Given $\sin\theta = 3/5$ for θ in Quadrant II, find the exact values of :

a) $\sin 2\theta$ b) $\cos 2\theta$ c) $\tan 2\theta$

The double-angle formula can be used with angles other than 2θ :

$$\sin 4\theta = 2\sin 2\theta\cos 2\theta \qquad \cos 6x = \cos^2 3x - \sin^2 3x$$

$$\sin(\alpha - 1) = 2\sin\frac{\alpha-1}{2}\cos\frac{\alpha-1}{2}$$

$$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$$

Example 2:

Verify the identity: $\sin 3x = 3\sin x - 4\sin^3 x$

② Apply the Power-Reducing Formulas

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

Proof.

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

Example 3:

Write $\sin^4 x + \cos^2 x$ in terms of first powers of cosine. **Range = ?**

③ Apply the Half-Angle Formulas

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}} \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$$

Proof: ("tan" will be proved in Example 5)

Example 4:

Use the half-angle formula to find the exact values of

a) sin67.5° **b) cos112.5°**

Example 5:

Show that:

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$$

Example 6:

If $\sin\alpha = -\frac{4}{5}$ **and** $\pi < \alpha < \frac{3\pi}{2}$, **find the exact values:**

a) $\sin\frac{\alpha}{2}$ b) $\cos\frac{\alpha}{2}$ c) $\tan\frac{\alpha}{2}$

① Apply the Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$

Example 1:

Write the product as a sum or difference.

a) $\sin 7x \cos 3x$ b) $\sin(-x) \sin 4x$

Example 2:

Use the product-to-sum formula to find the exact value of:

$$\sin 15^\circ \cos 75^\circ$$

Note:

$$\sin 15^\circ \cos 75^\circ = \sin 15^\circ \sin 15^\circ = \sin^2 15^\circ = \frac{1 - \cos 30^\circ}{2}$$

Skill Practice:

Use the product-to-sum formula to find the exact value of:

$$\cos 15^\circ \sin 75^\circ$$

② Apply the Sum-to-Product Formulas

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

Example 3:

Write each expression as a product.

a) $\sin 7\theta + \sin 3\theta$ **b)** $\cos\alpha - \cos 3\alpha$

Example 4:

Use a sum-to-product formula to find the exact value of

$$\cos 255° + \cos 195°$$

Example 5:

Verify the identity:

$$\frac{\sin 5x + \sin x}{\cos 5x + \cos x} = \tan 3x$$

① Solve Trigonometric Equations in Linear Form

Example 1:

Solve $2tanx = \sqrt{3} - tanx$

a) Over $[0, 2\pi)$.

b) Over the set of real numbers.

Skill Practice:

Solve $3cosx = 2\sqrt{2} - cosx$

a) Over $[0, 2\pi)$.

b) Over the set of real numbers.

② Solve Trigonometric Equations Involving Multiple or Compound Angles

Example 2:

$$2sin2x - \sqrt{3} = 0$$

a) Write the solution set for the general solution.

b) Write the solution set on the interval [0, 2π).

Example 3:

$$sin\frac{x}{2} - 1 = 0$$

a) Write the solution set for the general solution.

b) Write the solution set on the interval [0, 2π).

Example 4:

$$\sin\left(x-\frac{\pi}{3}\right)+\frac{\sqrt{2}}{2}=0$$

a) Write the solution set for the general solution.

b) Write the solution set on the interval [0, 2π).

Skill Practice:

$$\cot\left(x-\frac{\pi}{4}\right)=-1$$

a) Write the solution set for the general solution.

b) Write the solution set on the interval [0, 2π).

③ Solve Higher-degree Trigonometric Equations

Example 5:

Solve the equation on the interval [0, 2π):

$$2sin^2x + 7sinx - 4 = 0$$

Example 6:

Solve the equation on the interval [0, 2π):

$$cot^2x - 3 = 0$$

Example 7:

Solve the equation on the interval [0, 2π):

$$tanx\ sin^2x = tanx$$

④ Use Identities to Solve Trigonometric Equations

Example 8:

Solve the equation on the interval [0, 2π):

$$sec^2x - tanx = 1$$

Example 9:

Solve the equation on the interval [0, 2π):

$$cos3x + cosx = 0$$

Example 10:

Solve the equation on the interval [0, 2π):

$$sinx + 1 = cosx$$