Lecture 18

Magnetic Forces & Fields

Date:

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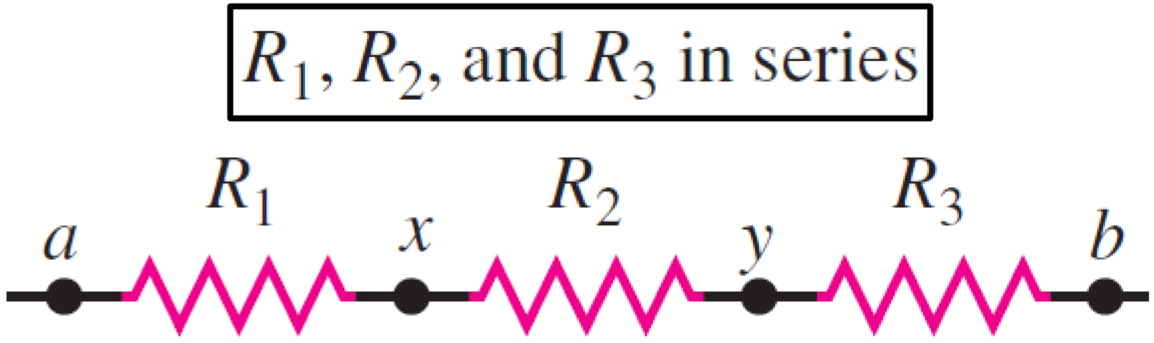
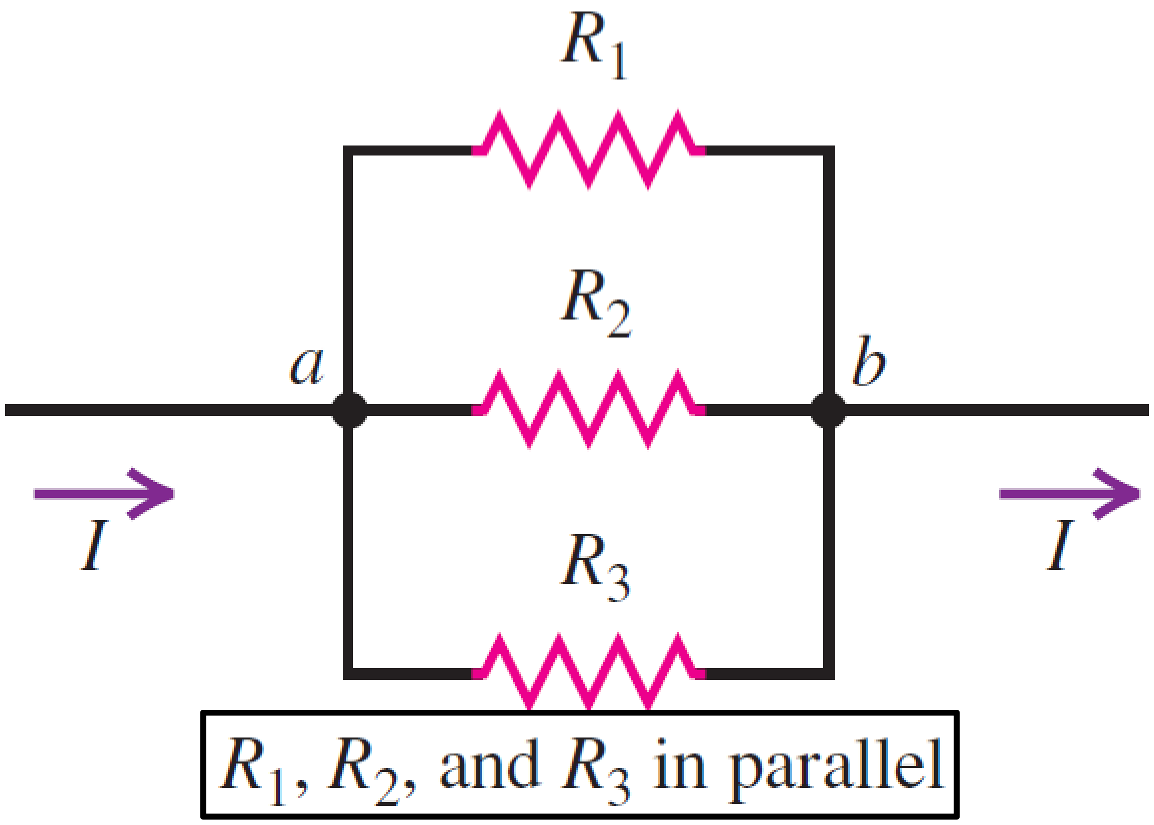
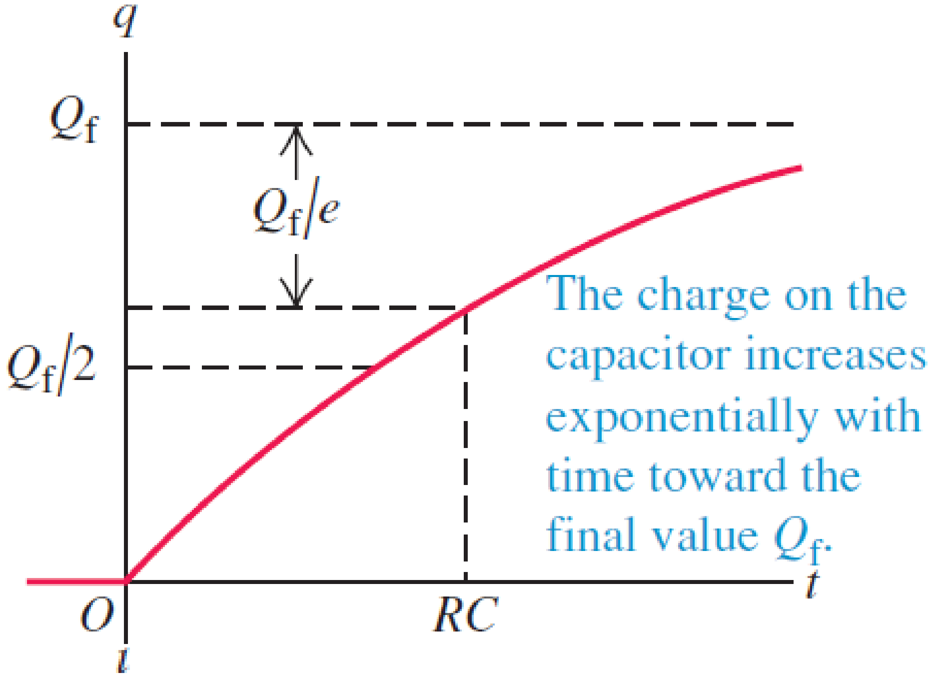
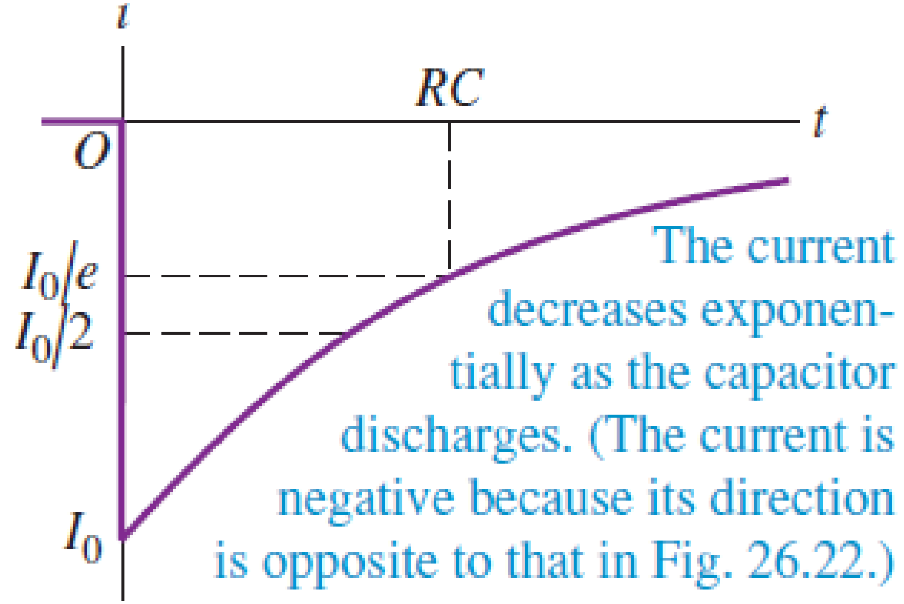
Previous Lecture: Direct Current Circuits



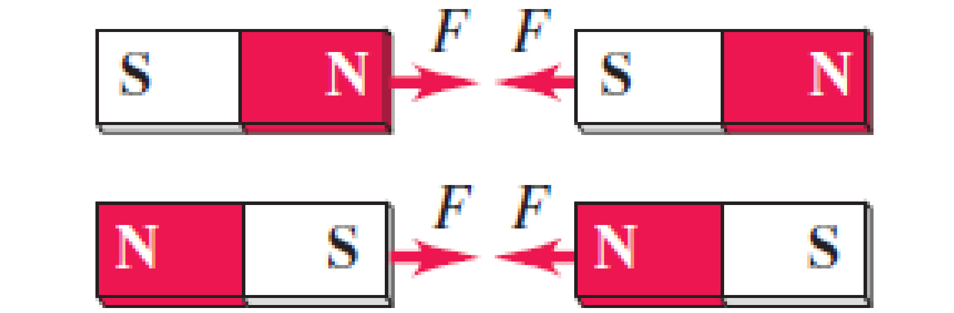




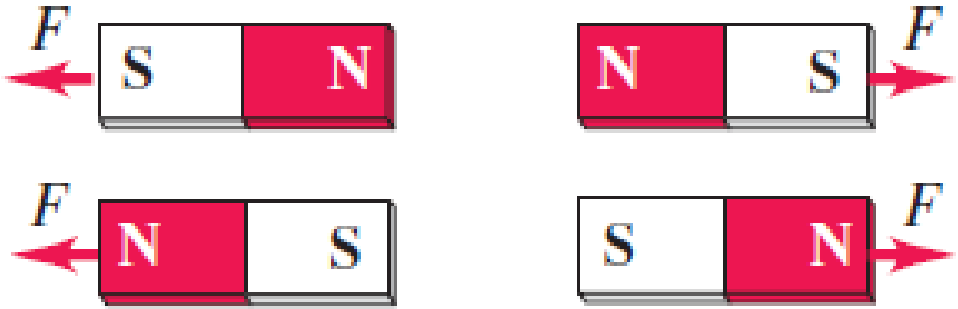


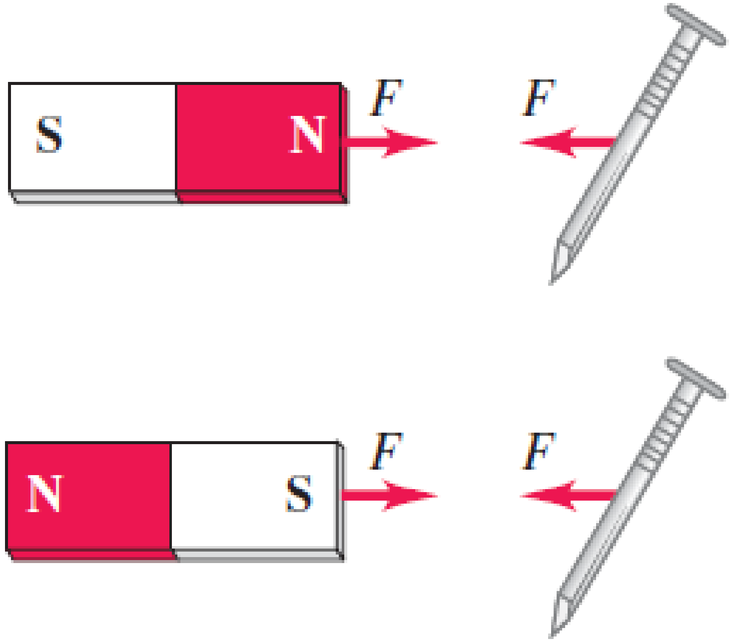
Magnetism

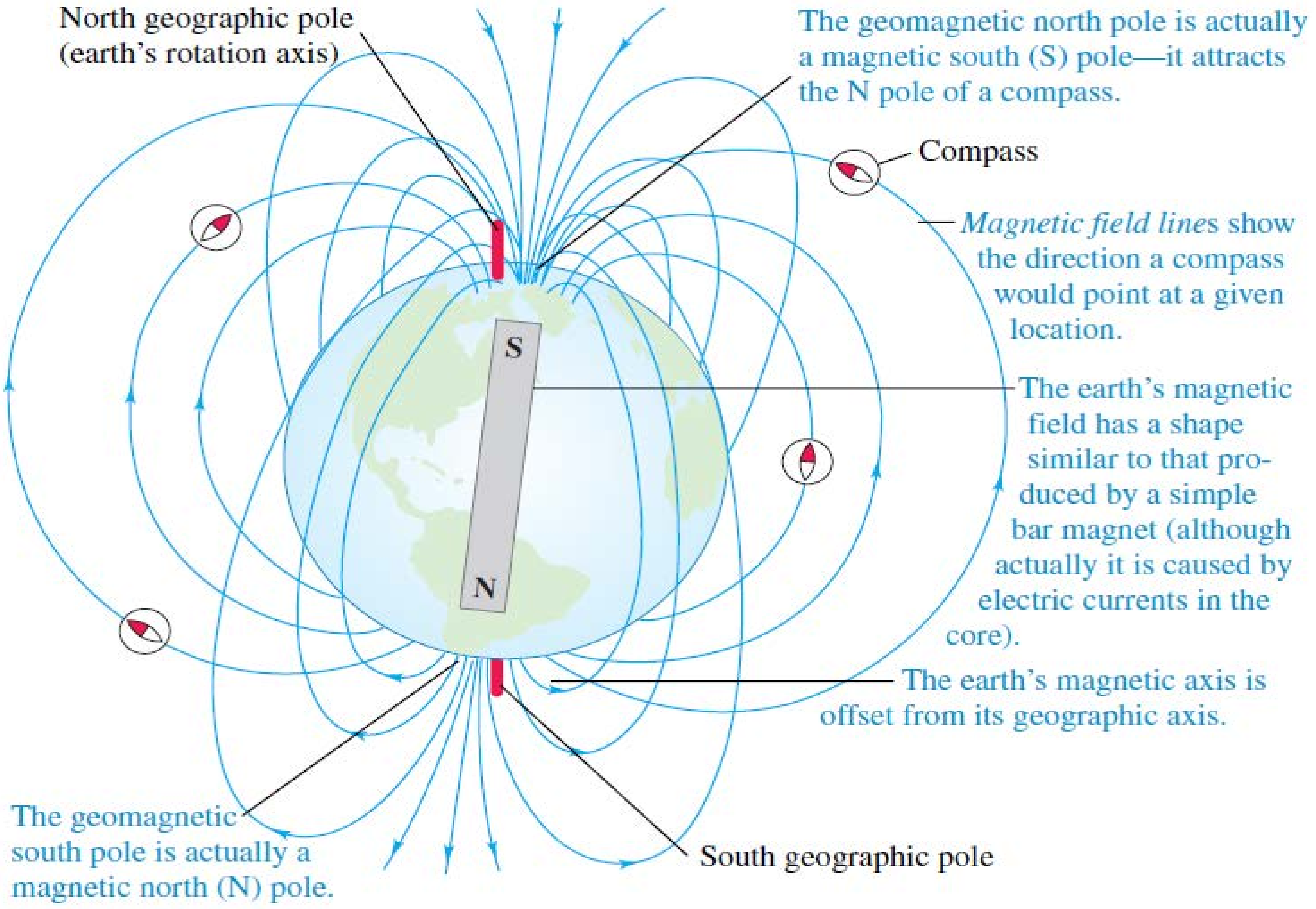
(a) Opposite poles attract.



(b) Like poles repel.





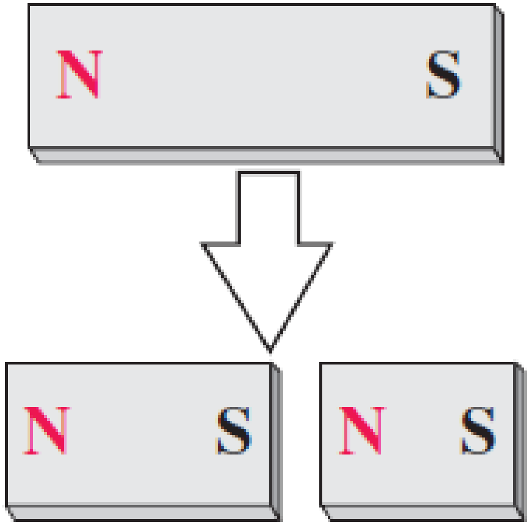


North geographic pole
(earth's rotation axis)
The geomagnetic north pole is actually a magnetic south (S) pole—it attracts the N pole of a compass.
Compass
Magnetic field lines show the direction a compass would point at a given location.
S
N
The earth's magnetic field has a shape similar to that produced by a simple bar magnet (although actually it is caused by electric currents in the core).
The earth's magnetic axis is offset from its geographic axis.
The geomagnetic south pole is actually a magnetic north (N) pole.
South geographic pole

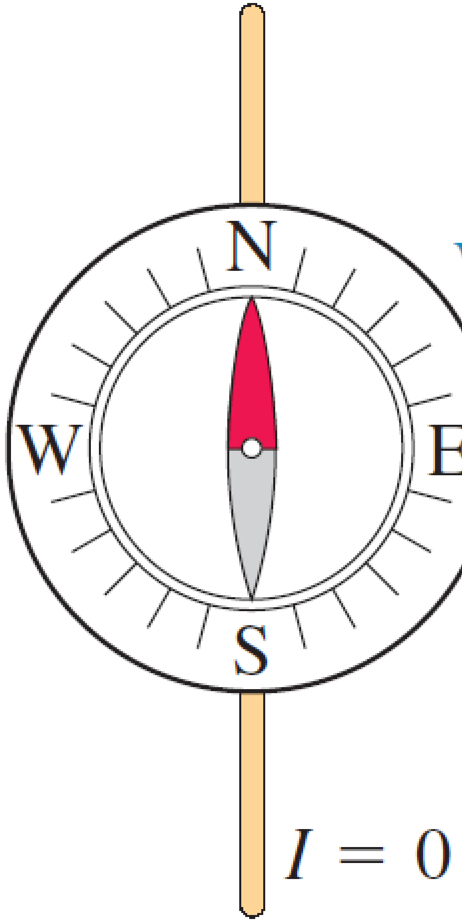
Magnets and Magnetic Fields

In contrast to electric charges, magnetic poles always come in pairs and can't be isolated.

Breaking a magnet in two ...

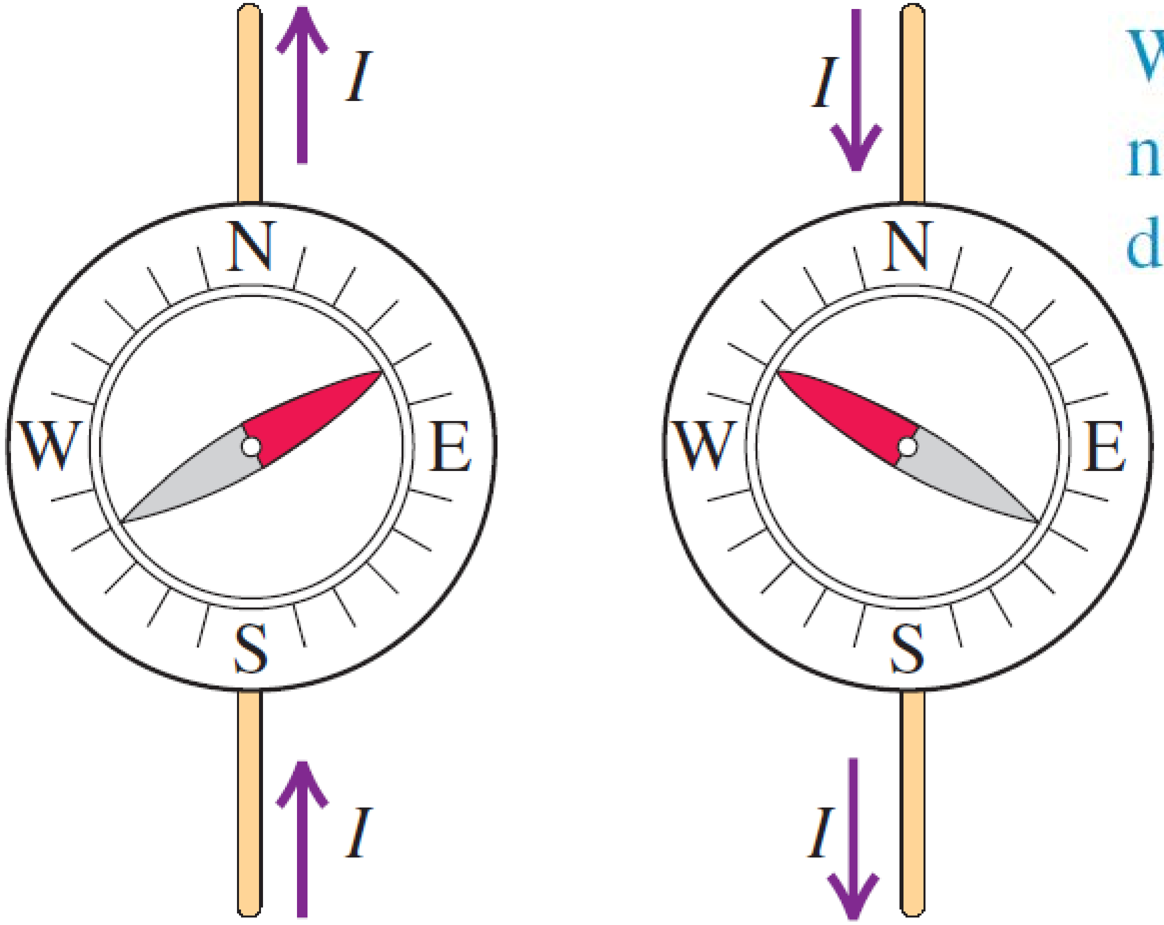


... yields two magnets, not two isolated poles.



When the wire carries no current, the compass needle points north.

Magnets and Magnetic Fields



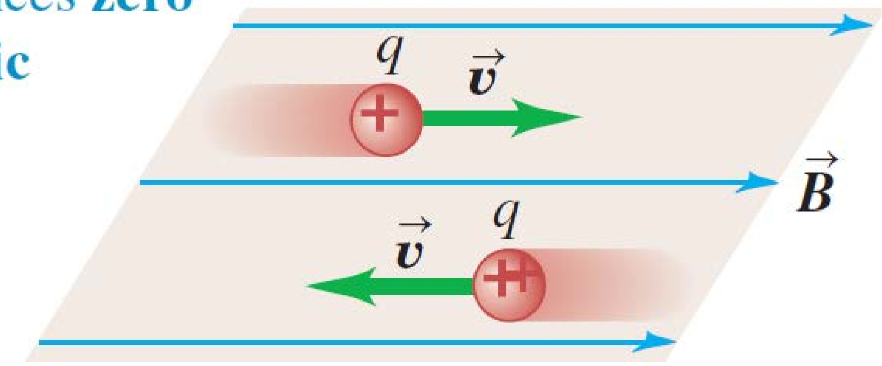
When the wire carries a current, the compass needle deflects. The direction of deflection depends on the direction of the current.

Magnetic interactions similar to Coulomb's Law:

- A moving charge or a current creates a **magnetic field** in the surrounding space (in addition to its *electric* field).
- The magnetic field exerts a force F on any other moving charge or current that is present in the field.

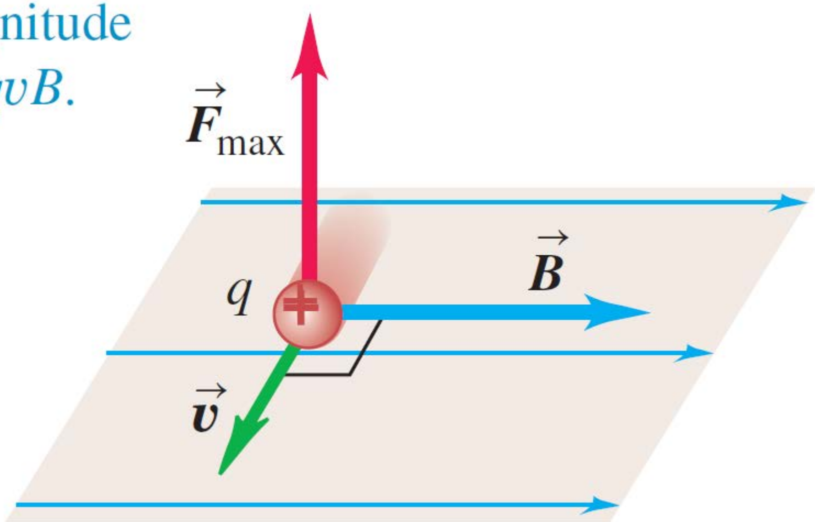
Magnetic Forces on Moving Charges

A charge moving **parallel** to a magnetic field experiences **zero magnetic force.**



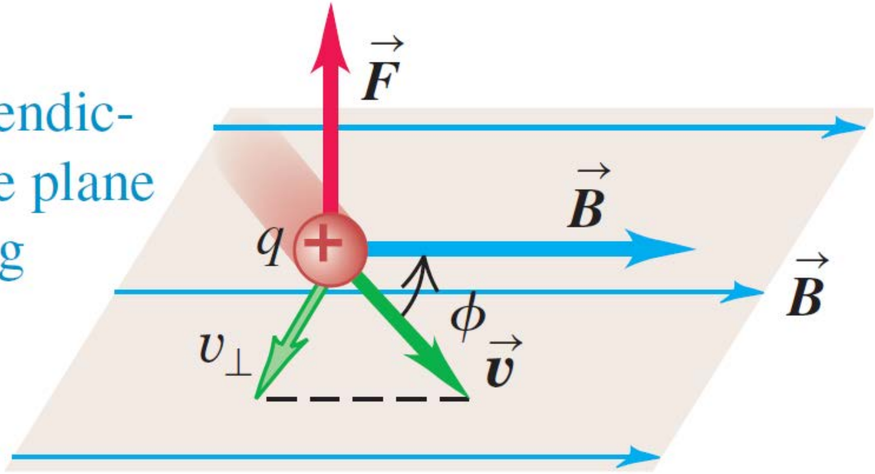
A charge moving **perpendicular** to a magnetic field experiences a maximal magnetic force with magnitude

$F_{max} = qvB$.



A charge moving at an angle ϕ to a magnetic field experiences a magnetic force with magnitude $F = |q|v_{\perp}B = |q|vB \sin\phi$.

$\vec{F}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.



Magnetic forces:

- Magnitude proportional to q
- Magnitude proportional to B
- Depends on the particle's velocity v
- Direction perpendicular to B

Magnetic Forces on Moving Charges

$$F = |q|v_{\perp}B = |q|vB\sin\phi$$

To complete the description, we use the same right-hand rule that we used to define the vector product in Section 1.10. (It would be a good idea to review that section before you go on.) Draw the vectors $\vec{\boldsymbol{v}}$ and $\vec{\boldsymbol{B}}$ with their tails together, as in Fig. 27.7a. Imagine turning $\vec{\boldsymbol{v}}$ until it points in the direction of $\vec{\boldsymbol{B}}$ (turning through the smaller of the two possible angles). Wrap the fingers of your right hand around the line perpendicular to the plane of $\vec{\boldsymbol{v}}$ and $\vec{\boldsymbol{B}}$ so that they curl around with the sense of rotation from $\vec{\boldsymbol{v}}$ to $\vec{\boldsymbol{B}}$. Your thumb then points in the direction of the force $\vec{\boldsymbol{F}}$ on a *positive* charge.

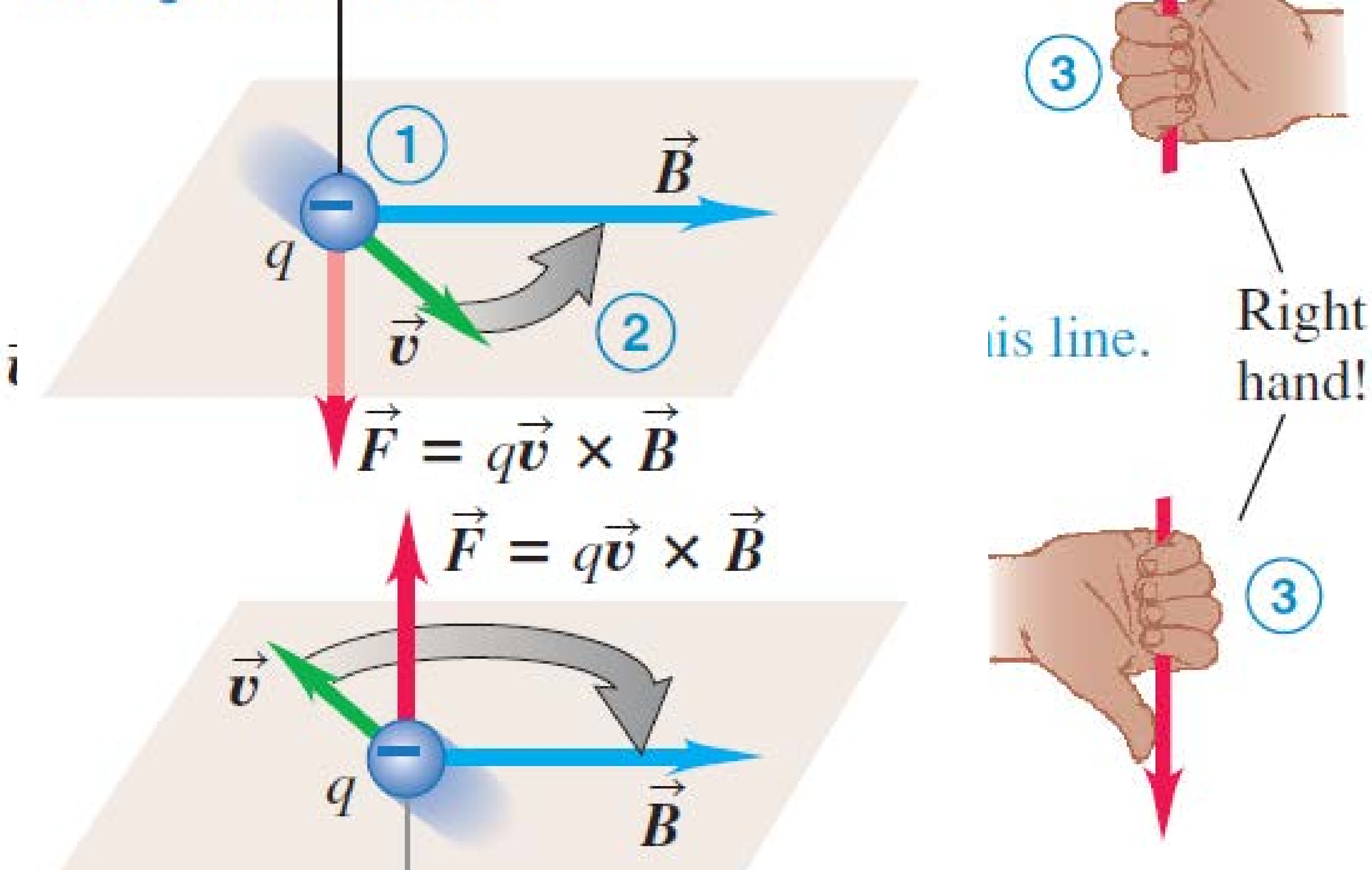
$$\vec{\boldsymbol{F}} = q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} \qquad \text{(magnetic force on a moving charged particle)}$$

Magnetic Forces on Moving Charges

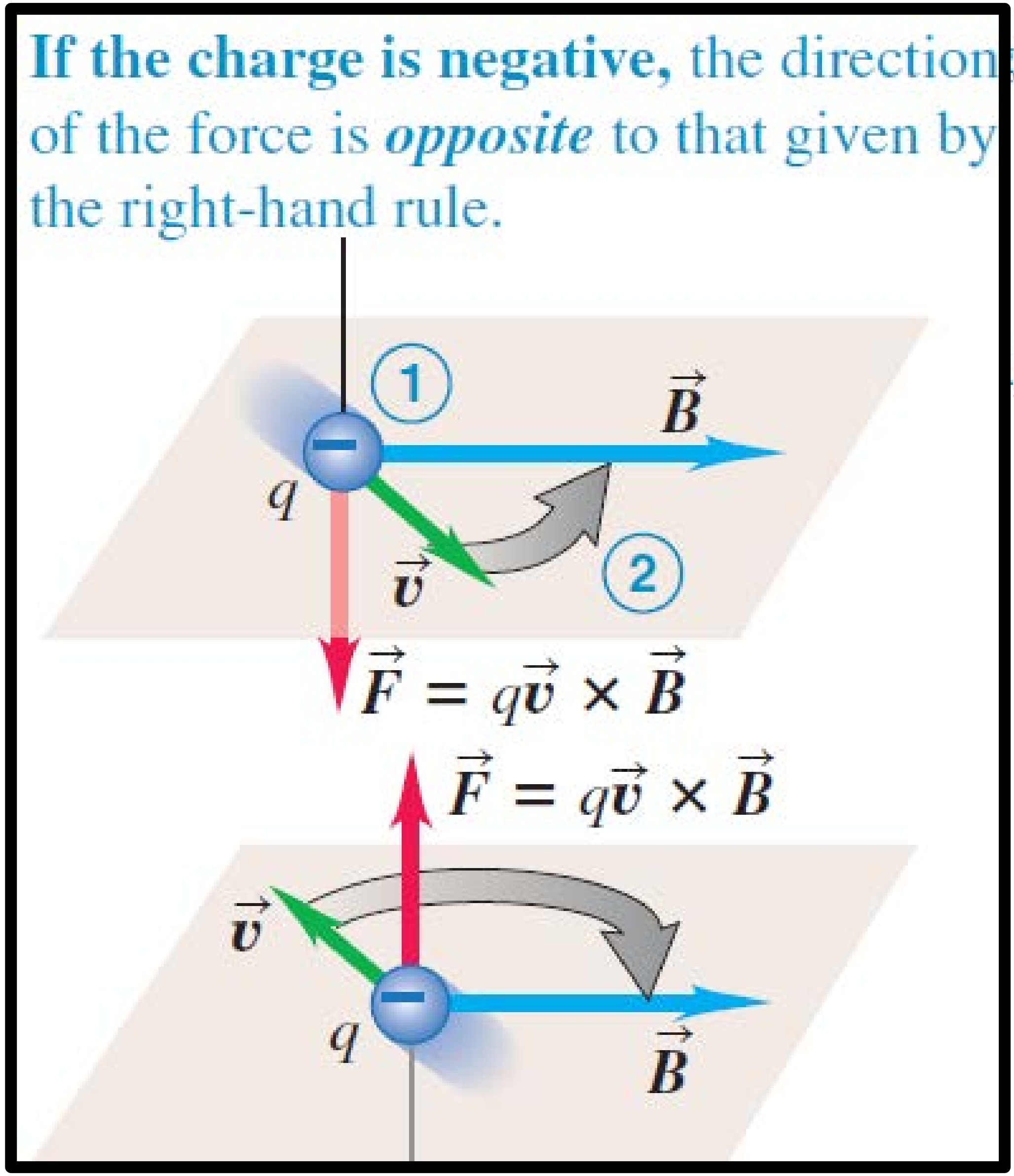
Right-hand rule for the direction of magnetic force on a **positive** charge moving in a magnetic field:

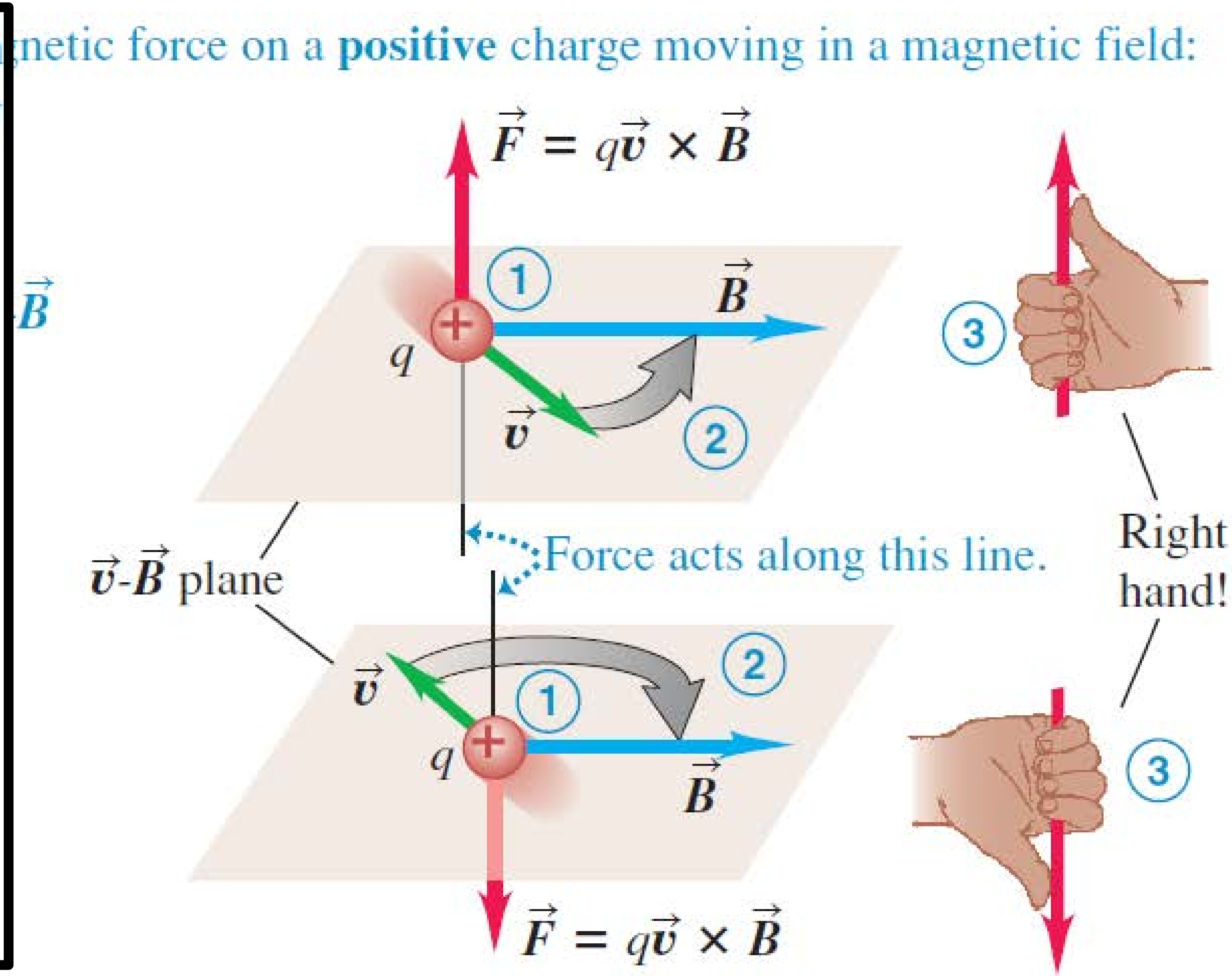
1. Place the $\vec{v}$ and $\vec{B}$ vectors tail to tail.
2. Imagine turning $\vec{v}$ toward $\vec{B}$ in the $\vec{v}$-$\vec{B}$ plane (through the smaller angle).
3. The force acts along a line perpendicular to the $\vec{v}$-$\vec{B}$ plane. Curl the fingers of your *right hand* around this line in the same direction you rotated $\vec{v}$. Your thumb now points in the direction the force acts.

If the charge is negative, the direction of the force is ***opposite*** to that given by the right-hand rule.

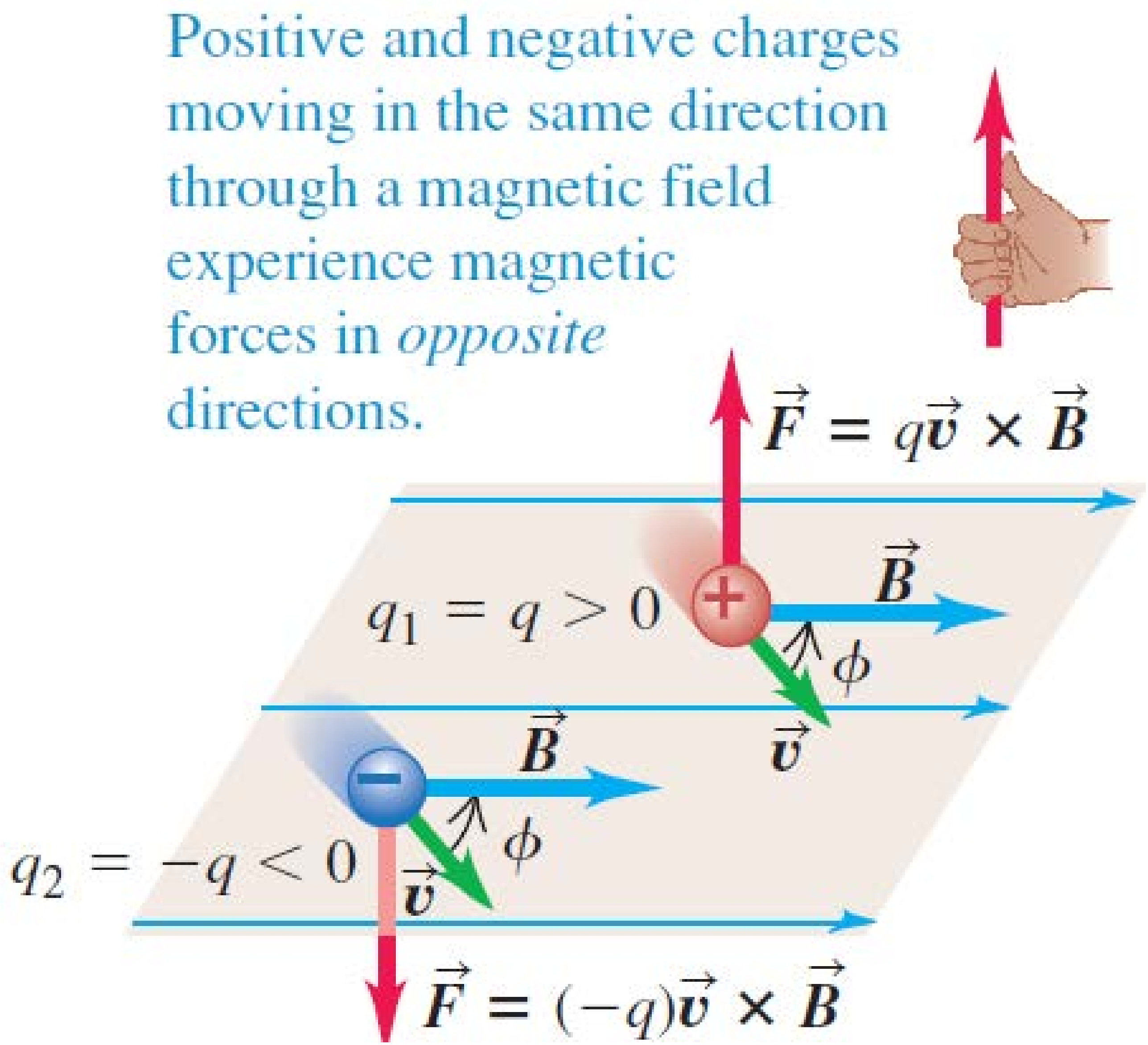


Magnetic Forces on Moving Charges





Magnetic Forces on Moving Charges



SI unit of B is equivalent to $1\ \mathrm{N \cdot s/C \cdot m}$

$$1\ \text{tesla} = 1\ \mathrm{T} = 1\ \mathrm{N/A \cdot m}$$

Measuring Magnetic Fields with Test Charges

To explore an unknown magnetic field, we can measure the magnitude and direction of the force on a *moving* test charge and then use Eq. (27.2) to determine B.

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \text{(magnetic force on a moving charged particle)}$$

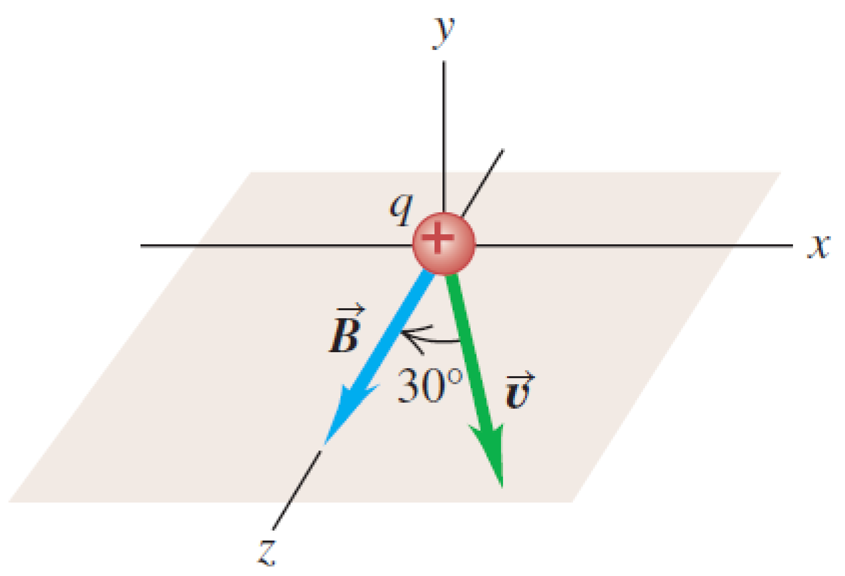
When a charged particle moves through a region of space where *both* electric and magnetic fields are present, both fields exert forces on the particle. The total force is the vector sum of the electric and magnetic forces:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Example 27.1 Magnetic force on a proton

A beam of protons ($q = 1.6 \times 10^{-19}$ C) moves at 3.0×10^5 m/s through a uniform 2.0-T magnetic field directed along the positive z-axis, as in Fig. 27.10. The velocity of each proton lies in the xz-plane and is directed at 30° to the $+z$-axis. Find the force on a proton.

Directions of $\vec{v}$ and $\vec{B}$ for a proton in a magnetic field.



Example 27.1 Magnetic force on a proton

EVALUATE: We check our result by evaluating the force using vector language and Eq. (27.2). We have

$$\vec{v} = (3.0 \times 10^5 \text{ m/s})(\sin 30^\circ)\hat{\imath} + (3.0 \times 10^5 \text{ m/s})(\cos 30^\circ)\hat{k}$$

$$\vec{B} = (2.0 \text{ T})\hat{k}$$

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$= (1.6 \times 10^{-19} \text{ C})(3.0 \times 10^5 \text{ m/s})(2.0 \text{ T})$$

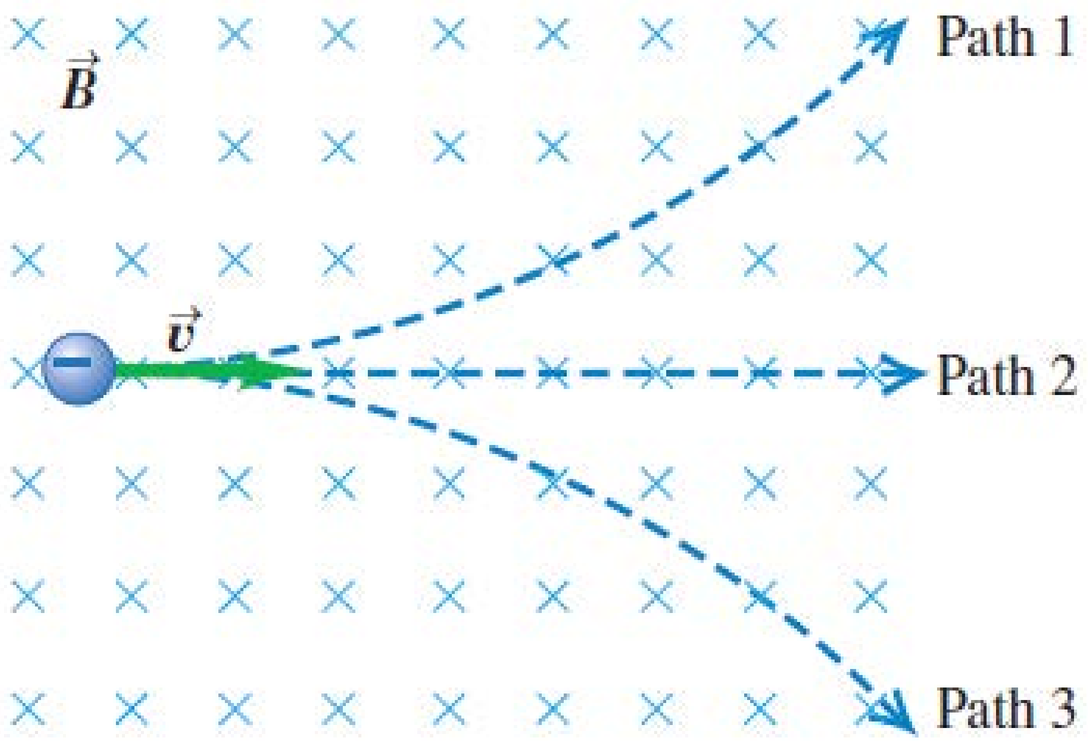
$$\times (\sin 30^\circ\hat{\imath} + \cos 30^\circ\hat{k}) \times \hat{k}$$

$$= (-4.8 \times 10^{-14} \text{ N})\hat{\jmath}$$

(Recall that $\hat{\imath} \times \hat{k} = -\hat{\jmath}$ and $\hat{k} \times \hat{k} = \mathbf{0}$.) We again find that the force is in the negative y-direction with magnitude 4.8×10^{-14} N.

Test Your Understanding of Section

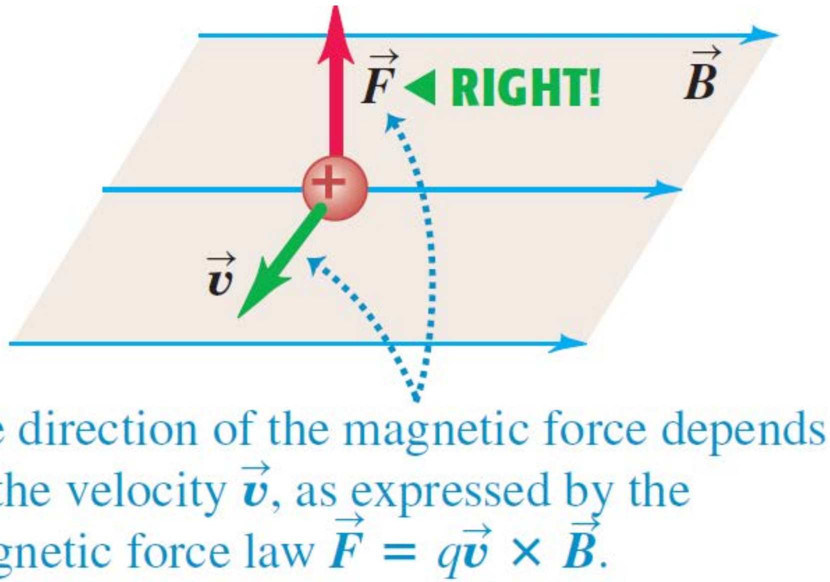
Test Your Understanding of Section 27.2 The figure at right shows a uniform magnetic field $\vec{B}$ directed into the plane of the paper (shown by the blue $\times$'s). A particle with a negative charge moves in the plane. Which of the three paths—1, 2, or 3—does the particle follow?

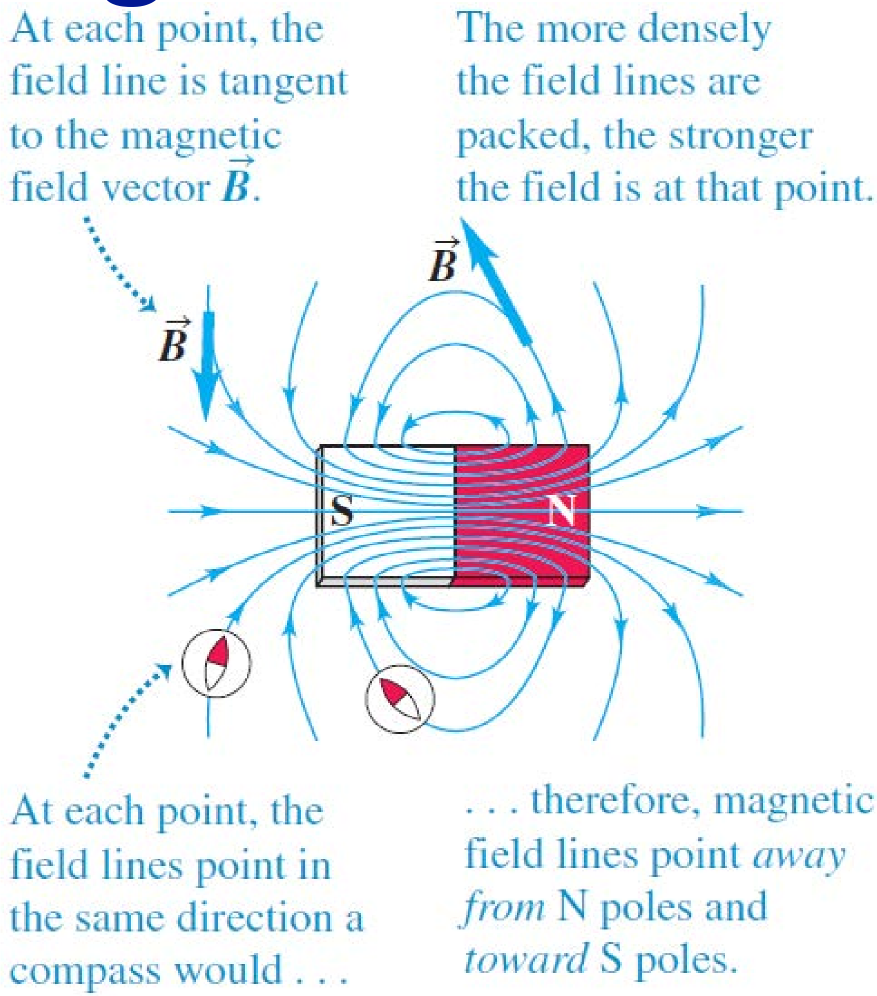


$$\vec{F} = q\vec{v} \times \vec{B}$$

Magnetic Field Lines and Magnetic Flux

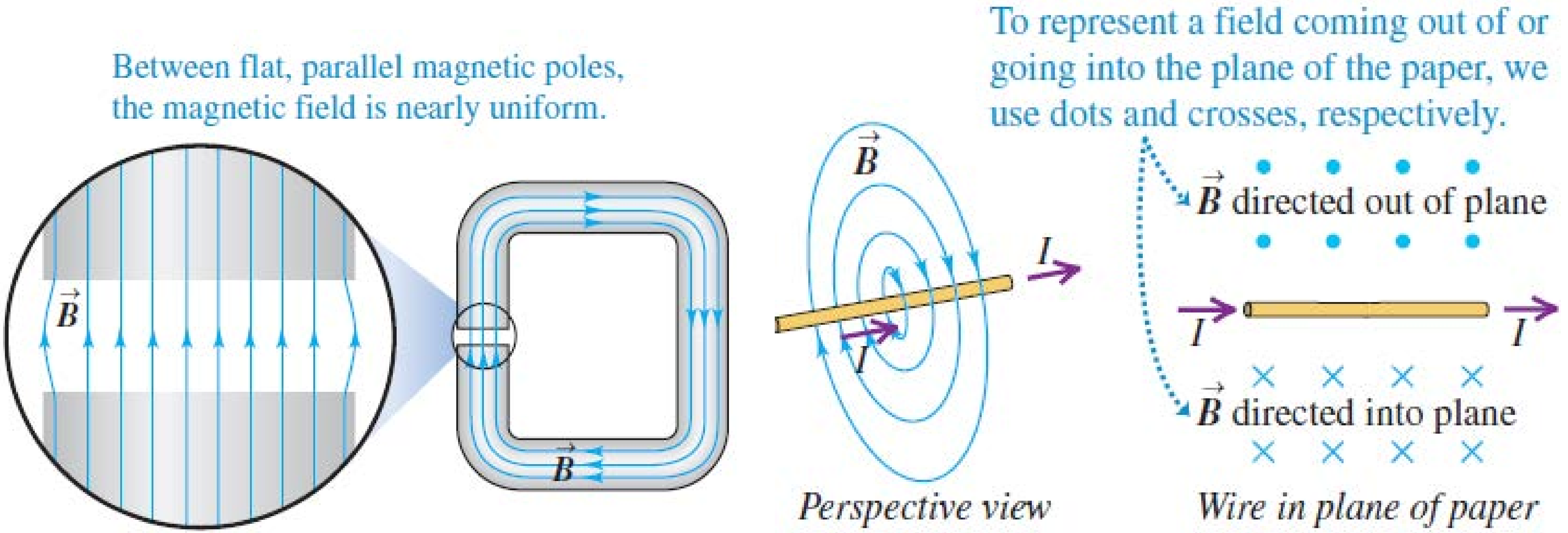
We can represent any magnetic field by **magnetic field lines,** just as we did for the earth's magnetic field.



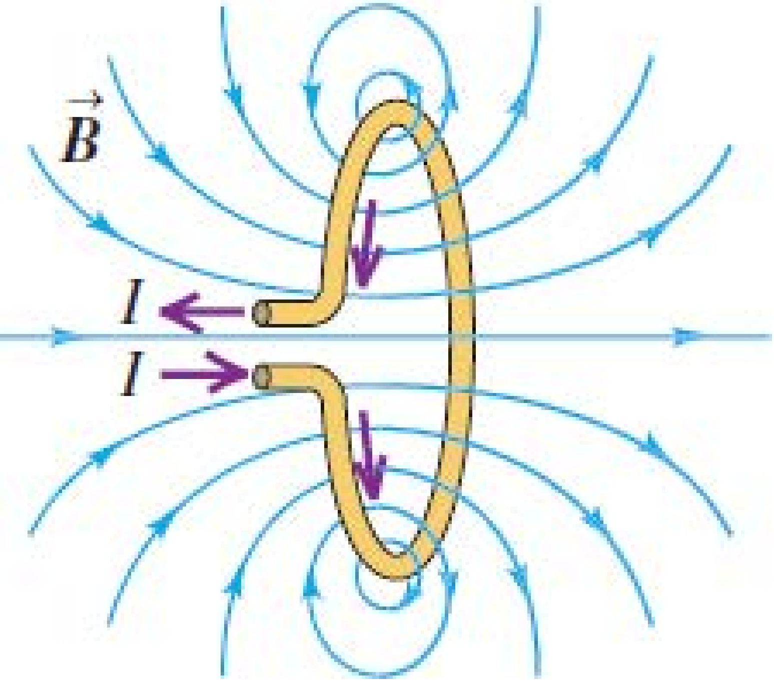


CAUTION **Magnetic field lines are not "lines of force"** Magnetic field lines are sometimes called "magnetic lines of force," but that's not a good name for them; unlike electric field lines, they *do not* point in the direction of the force on a charge

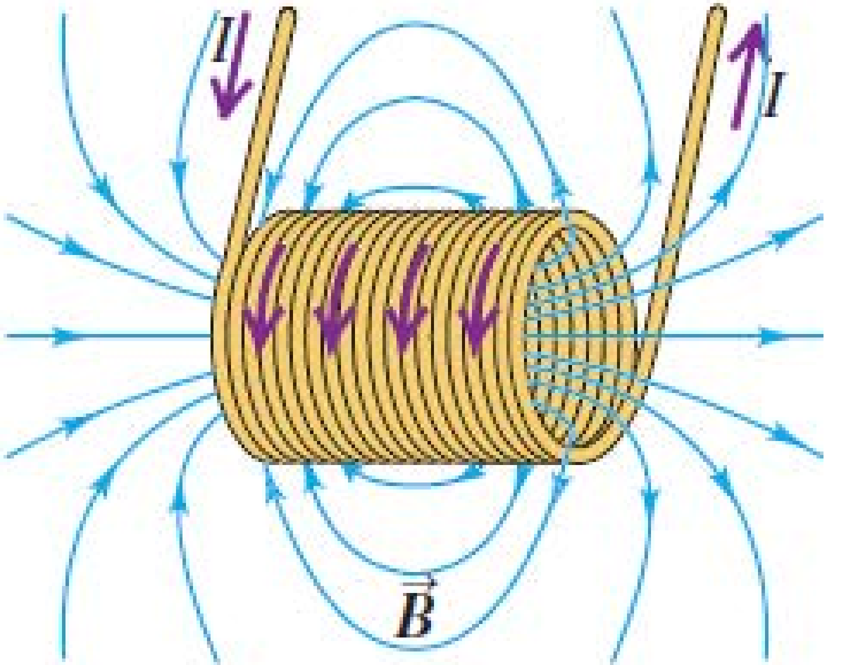
Magnetic Flux and Gauss's Law for Magnetism



Magnetic Flux and Gauss's Law for Magnetism

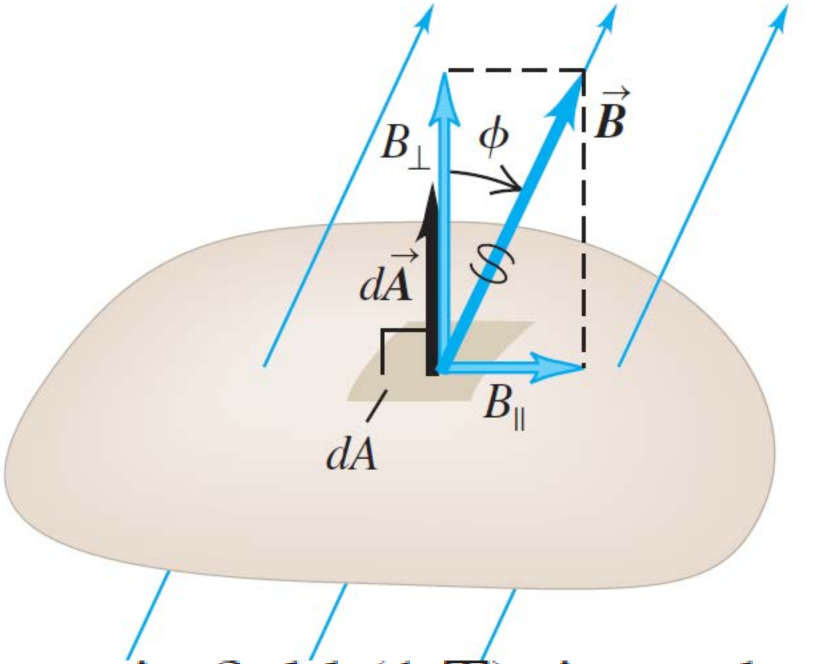


Notice that the field of the loop and, especially, that of the coil look like the field of a bar magnet (see Fig. 27.11).



Magnetic Flux and Gauss's Law for Magnetism

$$d\Phi_B = B_\perp\, dA = B\cos\phi\, dA = \vec{B}\cdot d\vec{A}$$



$$\Phi_B = \int B_\perp\, dA = \int B\cos\phi\, dA = \int \vec{B}\cdot d\vec{A}$$

The SI unit of magnetic flux is equal to the unit of magnetic field (1 T) times the unit of area (1 m^2). This unit is called the **weber** (1 Wb), in honor of the German physicist Wilhelm Weber (1804–1891):

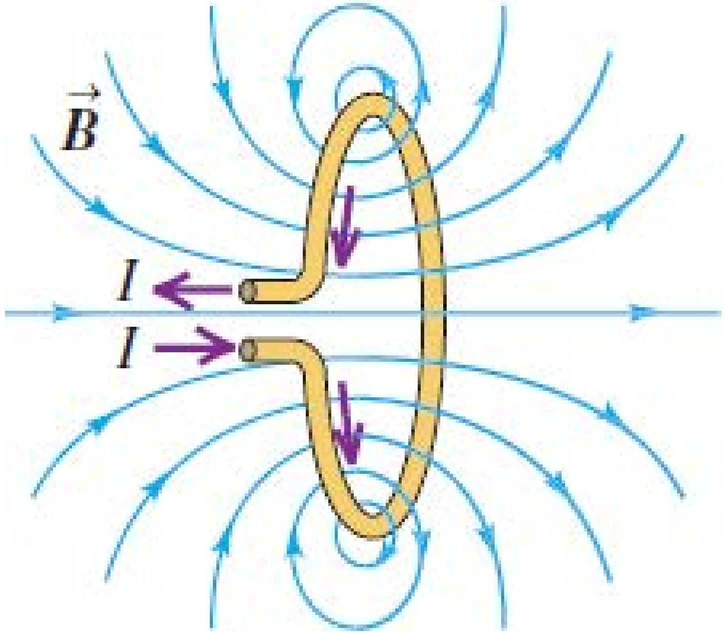
$$1\ \text{Wb} = 1\ \text{T}\cdot\text{m}^2$$

The total magnetic flux through a closed surface is always zero.

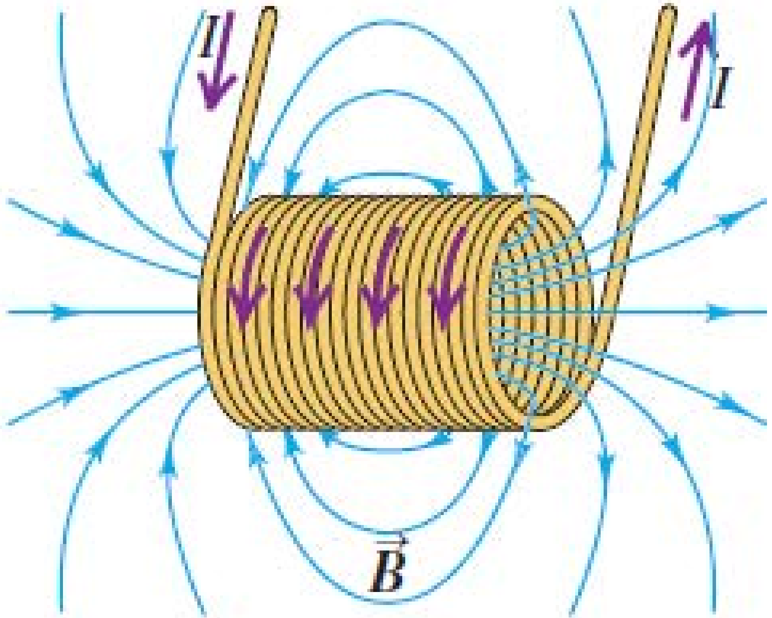
$$\oint \vec{B}\cdot d\vec{A} = 0$$

Magnetic Flux and Gauss's Law for Magnetism

CAUTION Magnetic field lines have no ends Unlike electric field lines that begin and end on electric charges, magnetic field lines *never* have end points; such a point would indicate the presence of a monopole. You might be tempted to draw magnetic field lines that begin at the north pole of a magnet and end at a south pole. But as Fig. 27.11 shows, the field lines of a magnet actually continue through the interior of the magnet. Like all other magnetic field lines, they form closed loops



Notice that the field of the loop and, especially, that of the coil look like the field of a bar magnet (see Fig. 27.11).



Magnetic Flux and Gauss's Law for Magnetism

For Gauss's law, which always deals with *closed* surfaces, the vector area element $d\vec{A}$ in Eq. (27.6) always points *out of* the surface. However, some applications of *magnetic* flux involve an *open* surface with a boundary line; there is then an ambiguity of sign in Eq. (27.6) because of the two possible choices of direction for $d\vec{A}$. In these cases we choose one of the two sides of the surface to be the "positive" side and use that choice consistently.

If the element of area dA in Eq. (27.5) is at right angles to the field lines, then $B_\perp = B$; calling the area $dA_\perp$, we have

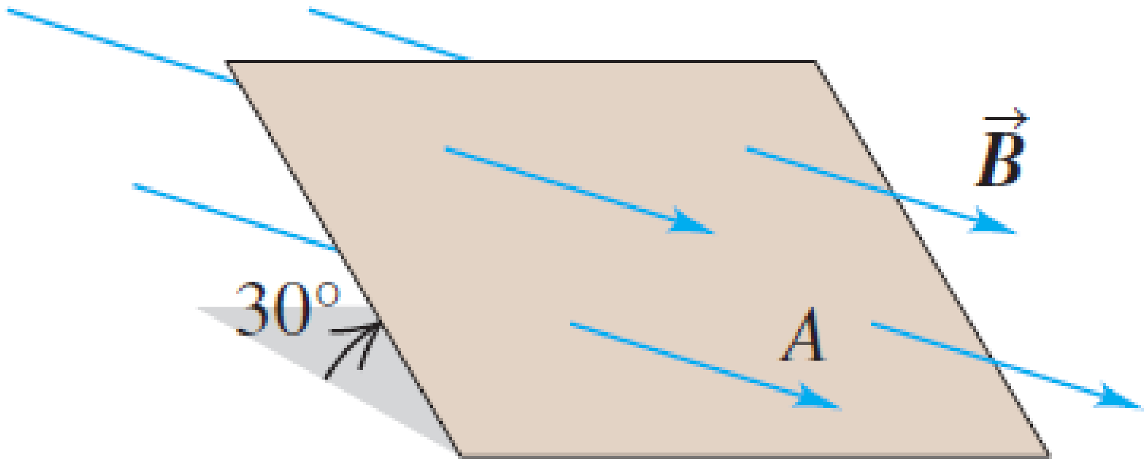
$$B = \frac{d\Phi_B}{dA_\perp} \tag{27.9}$$

That is, the magnitude of magnetic field is equal to *flux per unit area* across an area at right angles to the magnetic field. For this reason, magnetic field $\vec{B}$ is sometimes called **magnetic flux density.**

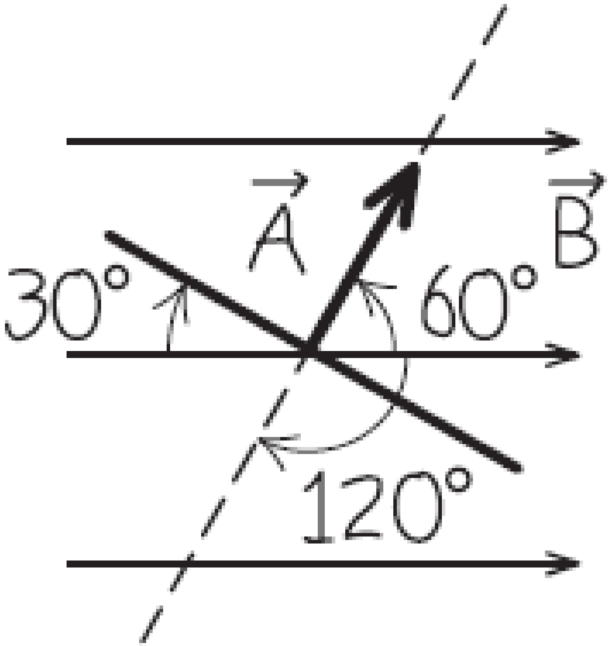
Example 27.2 Magnetic flux calculations

Figure 27.16a is a perspective view of a flat surface with area 3.0 cm^2 in a uniform magnetic field $\vec{B}$. The magnetic flux through this surface is +0.90 mWb. Find the magnitude of the magnetic field and the direction of the area vector $\vec{A}$.

(a) Perspective view



Our sketch of the problem (edge-on view)



Example 27.2 Magnetic flux calculations

Figure 27.16a is a perspective view of a flat surface with area 3.0 cm^2 in a uniform magnetic field $\vec{B}$. The magnetic flux through this surface is +0.90 mWb. Find the magnitude of the magnetic field and the direction of the area vector $\vec{A}$.

EXECUTE: The area A is 3.0×10^{-4} m^2; the direction of $\vec{A}$ is perpendicular to the surface, so ϕ could be either 60° or 120°. But Φ_B, B, and A are all positive, so $\cos\phi$ must also be positive. This rules out 120°, so $\phi = 60°$ (Fig. 27.16b). Hence we find

$$B = \frac{\Phi_B}{A \cos \phi} = \frac{0.90 \times 10^{-3}\ \text{Wb}}{(3.0 \times 10^{-4}\ \text{m}^2)(\cos 60°)} = 6.0\ \text{T}$$

Motion of Charged Particles in a Magnetic Field

Motion of a charged particle under the action of a magnetic field alone is always motion with constant speed.

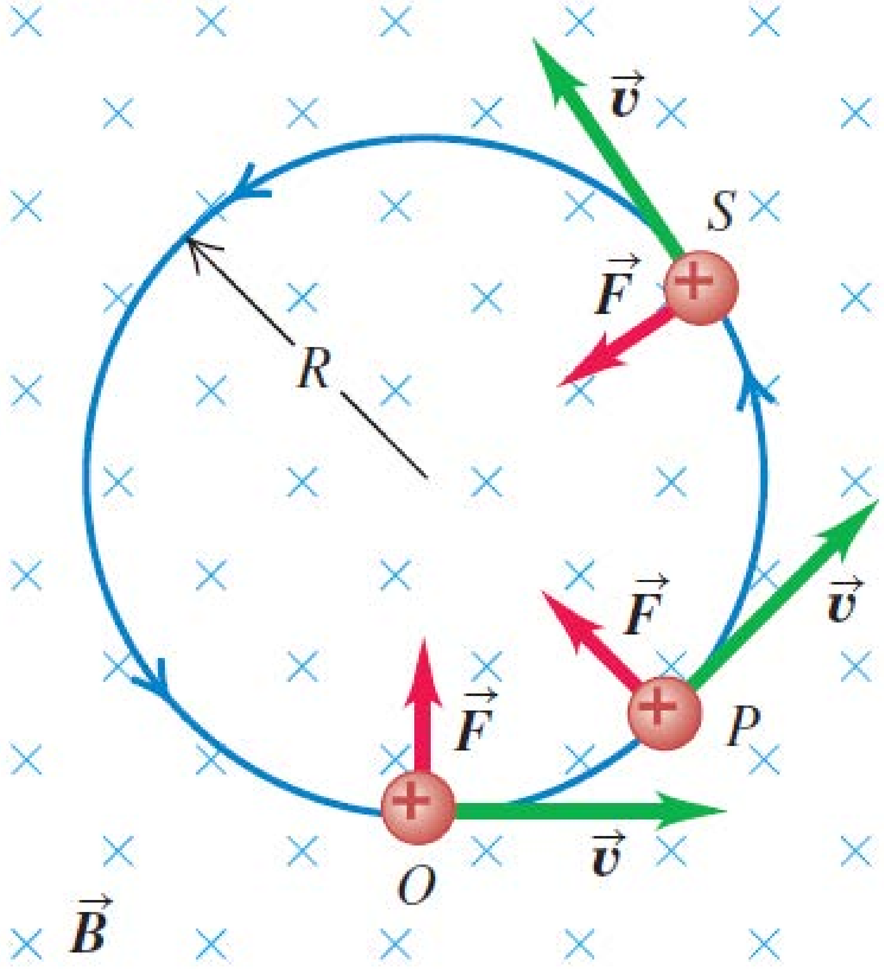
$$F = |q|vB = m\frac{v^2}{R}$$

$$\boxed{R = \frac{mv}{|q|B}}$$

The angular speed of the particle:

$$\omega = \frac{v}{R} = v\frac{|q|B}{mv} = \frac{|q|B}{m}$$

A charge moving at right angles to a uniform $\vec{B}$ field moves in a circle at constant speed because $\vec{F}$ and $\vec{v}$ are always perpendicular to each other.



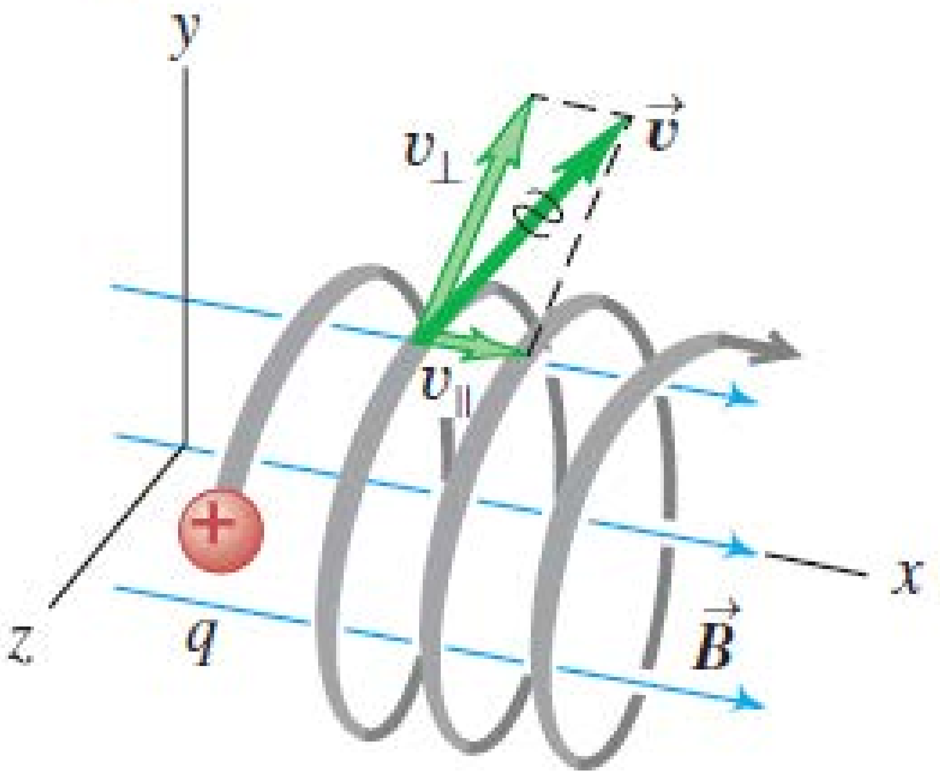
Motion of Charged Particles in a Magnetic Field

The number of revolutions per unit time is $f = \omega/2\pi$. This frequency f is independent of the radius R of the path. It is called the **cyclotron frequency;** in a particle accelerator called a *cyclotron*, particles moving in nearly circular paths are given a boost twice each revolution, increasing their energy and their orbital radii but not their angular speed or frequency. Similarly, one type of *magnetron*, a common source of microwave radiation for microwave ovens and radar systems, emits radiation with a frequency equal to the frequency of circular motion of electrons in a vacuum chamber between the poles of a magnet.

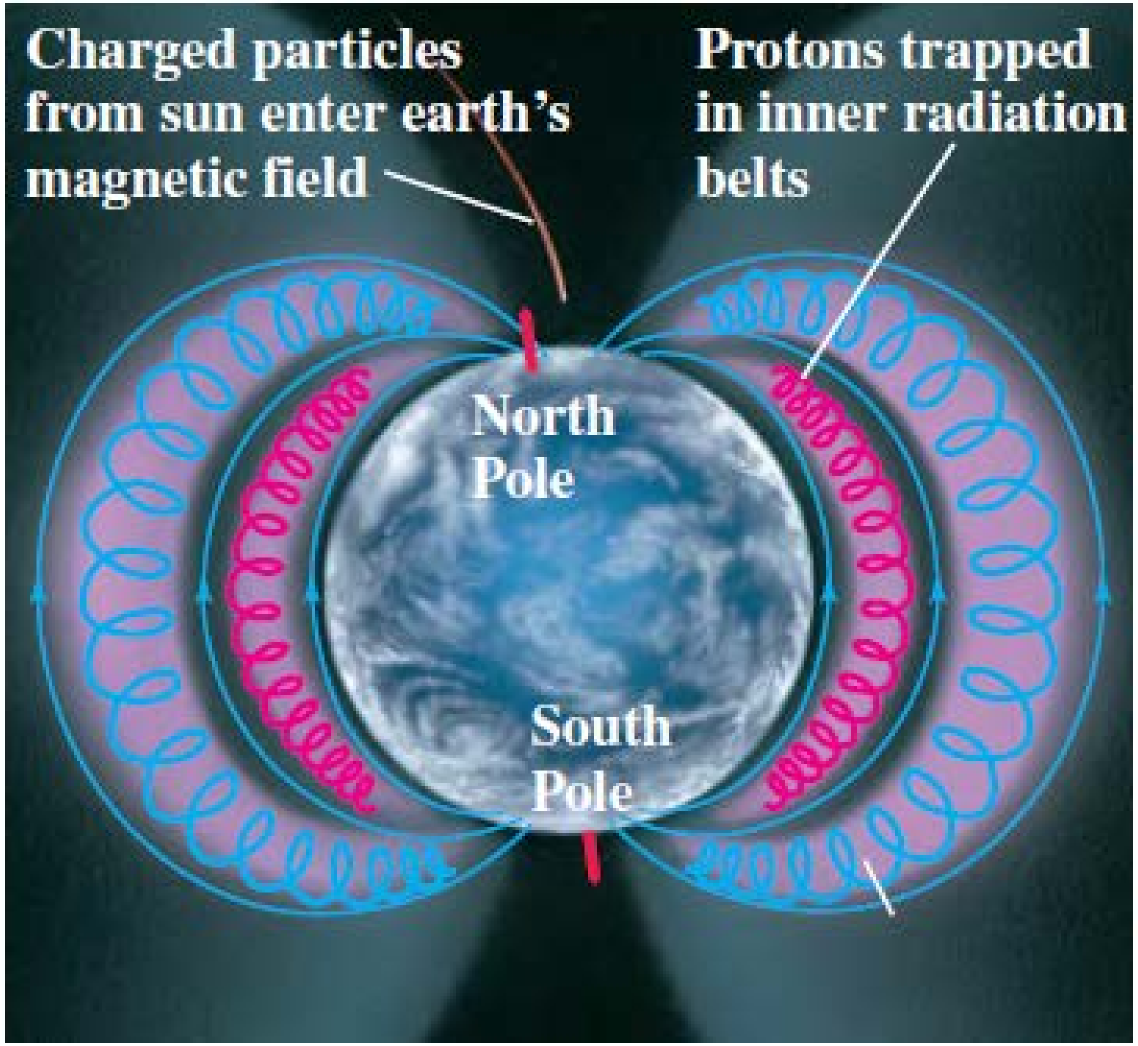
Motion of Charged Particles in a Magnetic Field

If the direction of the initial velocity is *not* perpendicular to the field, the velocity *component* parallel to the field is constant because there is no force parallel to the field. Then the particle moves in a helix (Fig. 27.18). The radius of the helix is given by Eq. (27.11), where is now the component of velocity perpendicular to the B field.

This particle's motion has components both parallel ($v_{\parallel}$) and perpendicular ($v_{\perp}$) to the magnetic field, so it moves in a helical path.



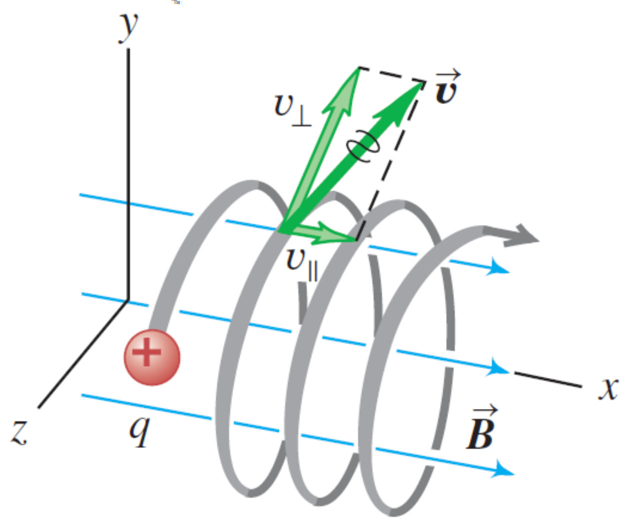
Motion of Charged Particles in a Magnetic Field



Aurora formation

Helical particle motion in a magnetic field

In a situation like that shown in Fig. 27.18, the charged particle is a proton ($q = 1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) and the uniform, 0.500-T magnetic field is directed along the x-axis. At $t = 0$ the proton has velocity components $v_x = 1.50 \times 10^5$ m/s, $v_y = 0$, and $v_z = 2.00 \times 10^5$ m/s. Only the magnetic force acts on the proton. (a) At $t = 0$, find the force on the proton and its acceleration. (b) Find the radius of the resulting helical path, the angular speed of the proton, and the *pitch* of the helix (the distance traveled along the helix axis per revolution).



Helical particle motion in a magnetic field

EXECUTE: (a) With $\vec{B} = B\hat{\imath}$ and $\vec{v} = v_x\hat{\imath} + v_z\hat{k}$, Eq. (27.2) yields

$$\vec{F} = q\vec{v} \times \vec{B} = q(v_x\hat{\imath} + v_z\hat{k}) \times B\hat{\imath} = qv_zB\hat{\jmath}$$
$$= (1.60 \times 10^{-19}\ \text{C})(2.00 \times 10^5\ \text{m/s})(0.500\ \text{T})\hat{\jmath}$$
$$= (1.60 \times 10^{-14}\ \text{N})\hat{\jmath}$$

(Recall that that $\hat{\imath} \times \hat{\imath} = \mathbf{0}$ and $\hat{k} \times \hat{\imath} = \hat{\jmath}$.) The resulting acceleration is

$$\vec{a} = \frac{\vec{F}}{m} = \frac{1.60 \times 10^{-14}\ \text{N}}{1.67 \times 10^{-27}\ \text{kg}}\hat{\jmath} = (9.58 \times 10^{12}\ \text{m/s}^2)\hat{\jmath}$$

Helical particle motion in a magnetic field

(b) Since $v_y = 0$, the component of velocity perpendicular to $\vec{B}$ is v_z; then from Eq. (27.11),

$$R = \frac{mv_z}{|q|B} = \frac{(1.67 \times 10^{-27}\ \text{kg})(2.00 \times 10^5\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(0.500\ \text{T})}$$
$$= 4.18 \times 10^{-3}\ \text{m} = 4.18\ \text{mm}$$

From Eq. (27.12) the angular speed is

$$\omega = \frac{|q|B}{m} = \frac{(1.60 \times 10^{-19}\ \text{C})(0.500\ \text{T})}{1.67 \times 10^{-27}\ \text{kg}} = 4.79 \times 10^7\ \text{rad/s}$$

Helical particle motion in a magnetic field

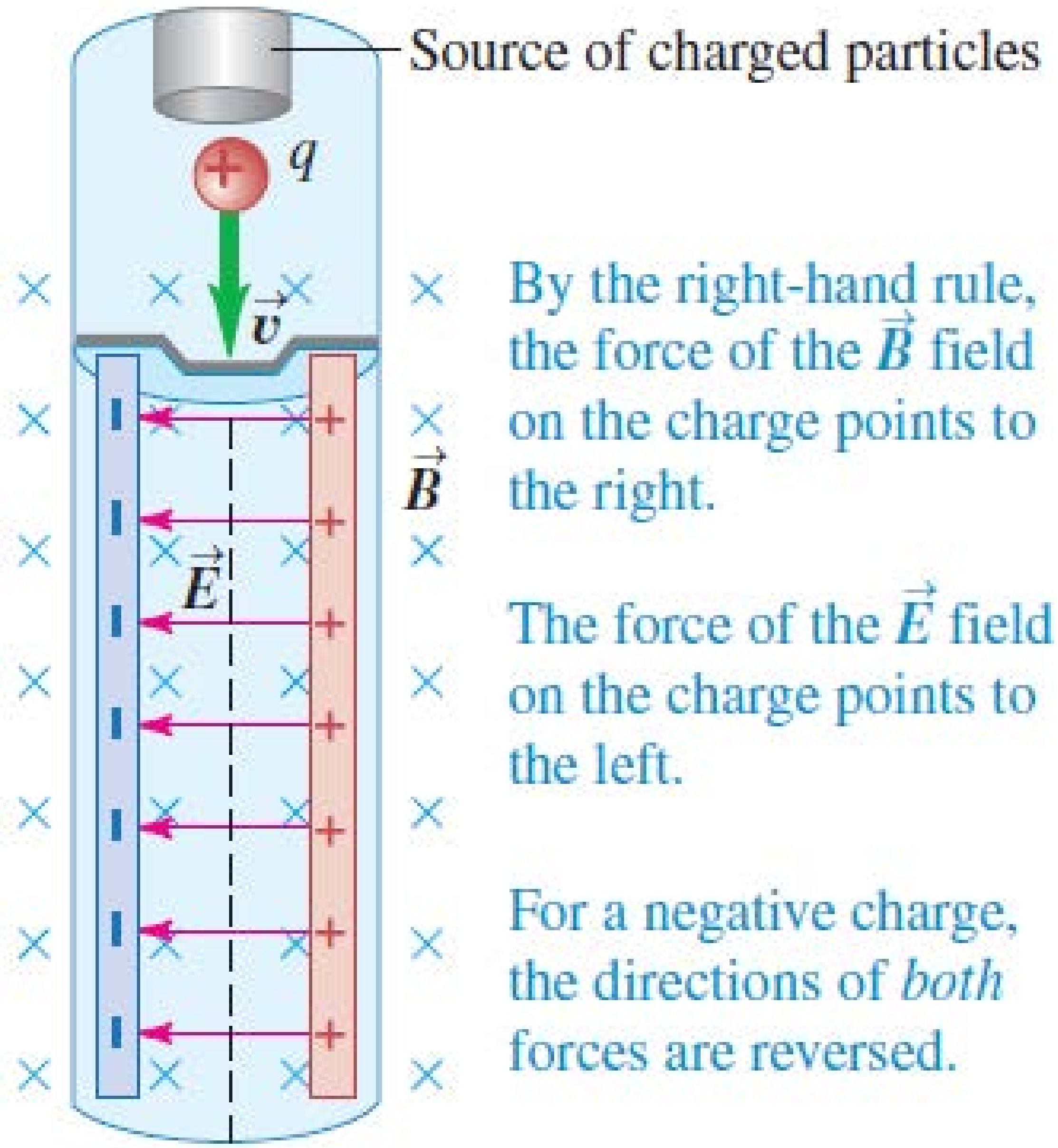
From Eq. (27.12) the angular speed is

$$\omega = \frac{|q|B}{m} = \frac{(1.60 \times 10^{-19}\ \text{C})(0.500\ \text{T})}{1.67 \times 10^{-27}\ \text{kg}} = 4.79 \times 10^{7}\ \text{rad/s}$$

The period is $T = 2\pi/\omega = 2\pi/(4.79 \times 10^{7}\ \text{s}^{-1}) = 1.31 \times 10^{-7}\ \text{s}$. The pitch is the distance traveled along the x-axis in this time, or

$$\begin{aligned} v_x T &= (1.50 \times 10^{5}\ \text{m/s})(1.31 \times 10^{-7}\ \text{s}) \\ &= 0.0197\ \text{m} = 19.7\ \text{mm} \end{aligned}$$

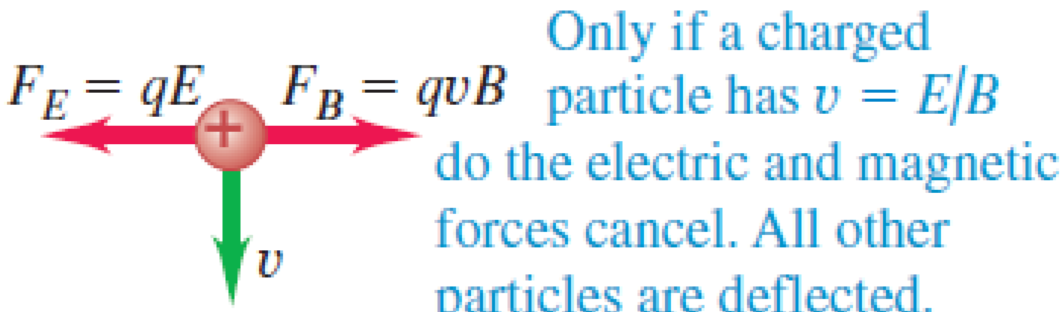
Applications of Motion of Charged Particles



Particles of a specific speed can be selected from the beam using an arrangement of electric and magnetic fields called a *velocity selector* for:

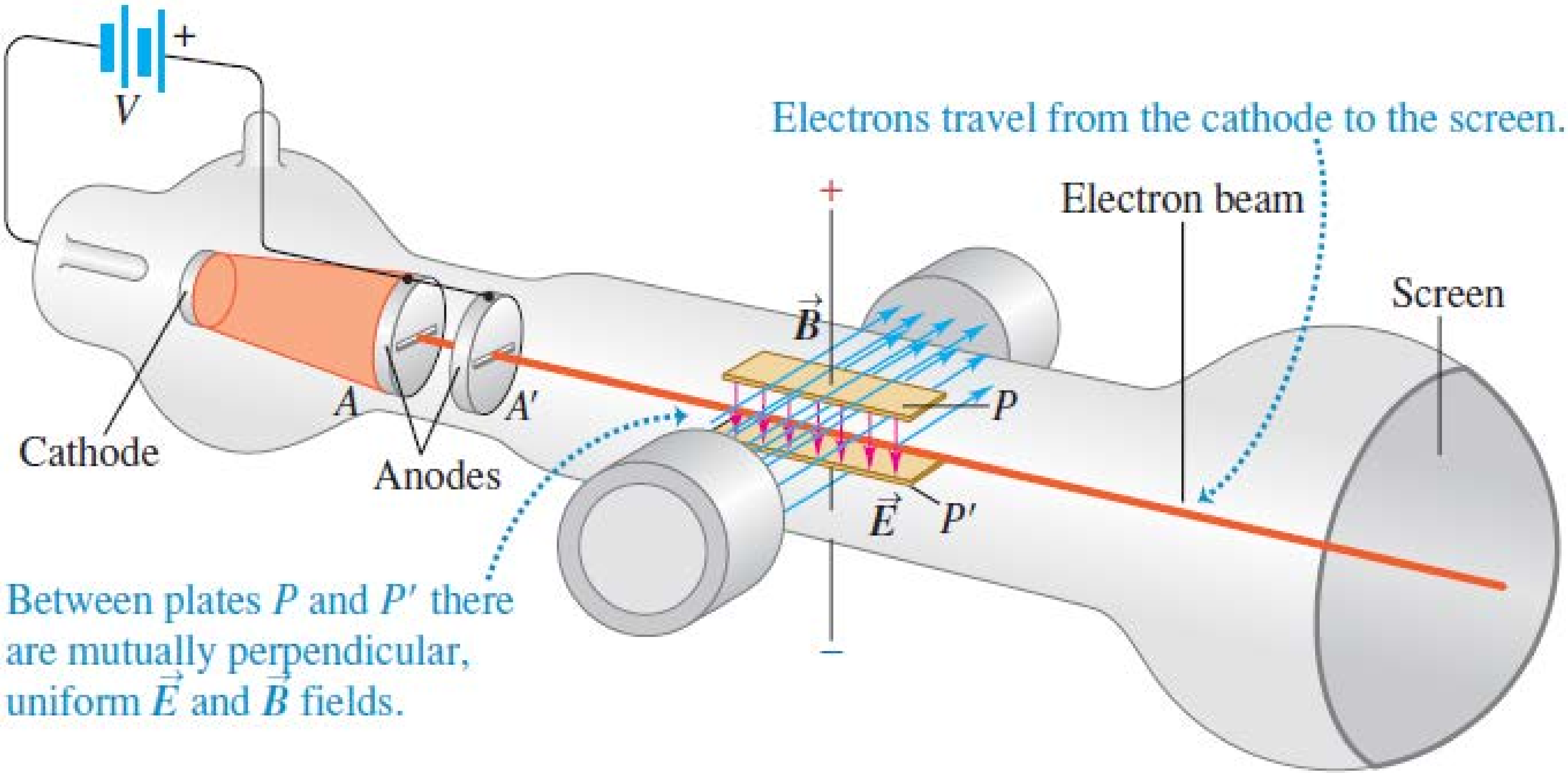
$$v = \frac{E}{B}$$

(b) Free-body diagram for a positive particle



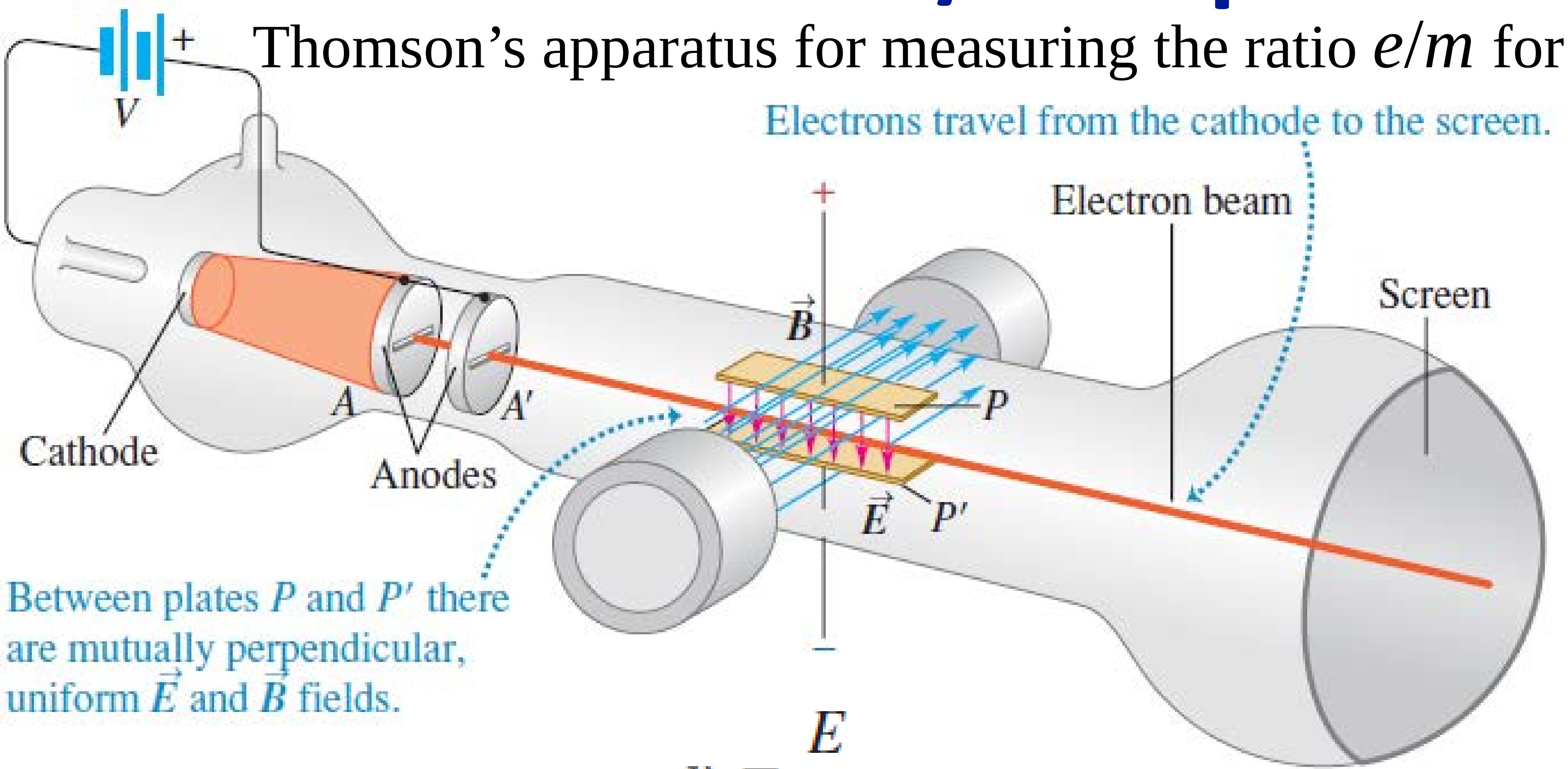
Thomson’s e/m Experiment

Thomson’s apparatus for measuring the ratio e/m for the electron



Thomson's e/m Experiment

Thomson's apparatus for measuring the ratio e/m for the electron



$$v = \frac{E}{B}$$

$$\frac{1}{2}mv^2 = eV \quad \text{or} \quad v = \sqrt{\frac{2eV}{m}}$$

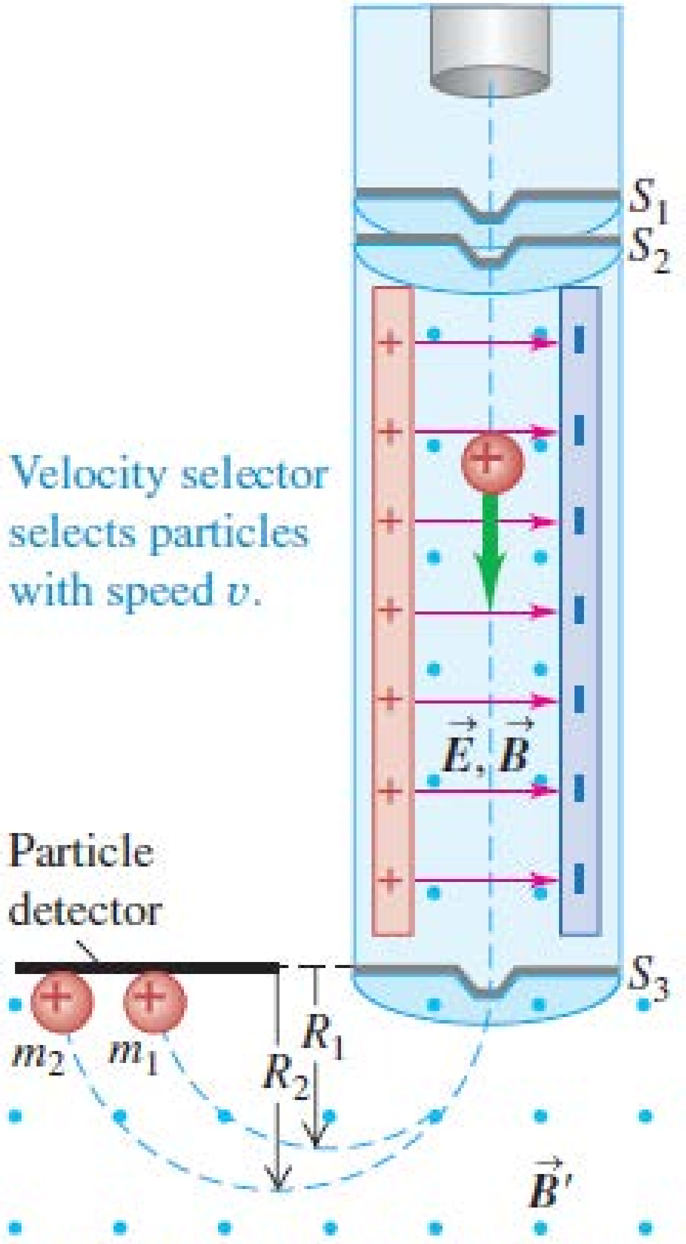
$$\frac{E}{B} = \sqrt{\frac{2eV}{m}} \quad \text{so} \quad \frac{e}{m} = \frac{E^2}{2VB^2}$$

Mass Spectrometers

called **mass spectrometers.** A variation built by Bainbridge is shown in Fig. 27.24. Positive ions from a source pass through the slits S_1 and S_2, forming a narrow beam. Then the ions pass through a velocity selector with crossed $\vec{E}$ and $\vec{B}$ fields, as we have described, to block all ions except those with speeds v equal to E/B. Finally, the ions pass into a region with a magnetic field $\vec{B}'$ perpendicular to the figure, where they move in circular arcs with radius R determined by Eq. (27.11): $R = mv/qB'$. Ions with different masses strike the detector (in Bainbridge's design, a photographic plate) at different points, and the values of R can be measured. We assume that each ion has lost one electron, so the net charge of each ion is just $+e$. With everything known in this equation except m, we can compute the mass m of the ion.

$$v = \frac{E}{B}$$

$$\boxed{R = \frac{mv}{|q|B}}$$



Magnetic field separates particles by mass; the greater a particle's mass, the larger is the radius of its path.

An e/m demonstration experiment

You set out to reproduce Thomson's e/m experiment with an accelerating potential of 150 V and a deflecting electric field of magnitude 6.0×10^6 N/C. (a) At what fraction of the speed of light do the electrons move? (b) What magnetic-field magnitude will yield zero beam deflection? (c) With this magnetic field, how will the electron beam behave if you increase the accelerating potential above 150 V?

EXECUTE: (a) From Eq. (27.14), the electron speed v is

$$v = \sqrt{2(e/m)V} = \sqrt{2(1.76 \times 10^{11}\ \text{C/kg})(150\ \text{V})}$$
$$= 7.27 \times 10^6\ \text{m/s} = 0.024c$$

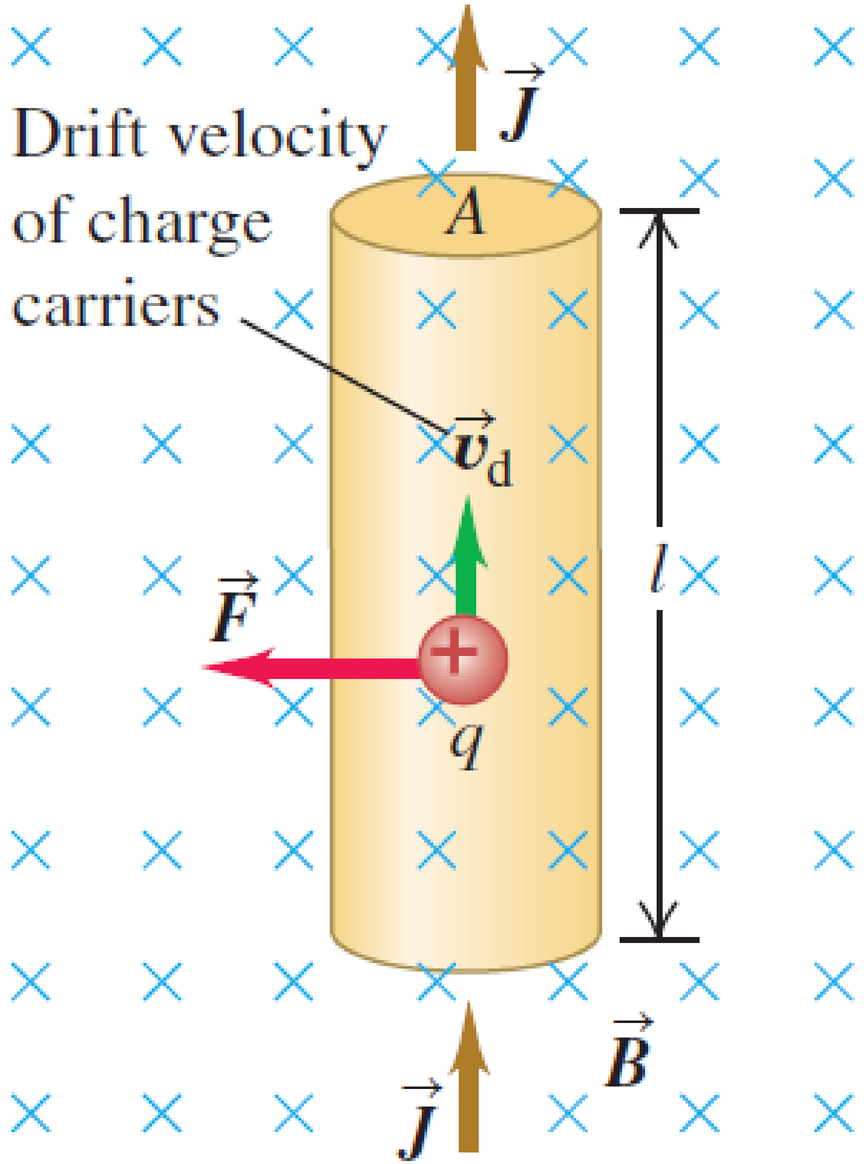
An e/m demonstration experiment

(b) From Eq. (27.13), the required field strength is

$$B = \frac{E}{v} = \frac{6.00 \times 10^6\ \mathrm{N/C}}{7.27 \times 10^6\ \mathrm{m/s}} = 0.83\ \mathrm{T}$$

(c) Increasing the accelerating potential V increases the electron speed v. In Fig. 27.23 this doesn't change the upward electric force eE, but it increases the downward magnetic force evB. Therefore the electron beam will turn *downward* and will hit the end of the tube below the undeflected position.

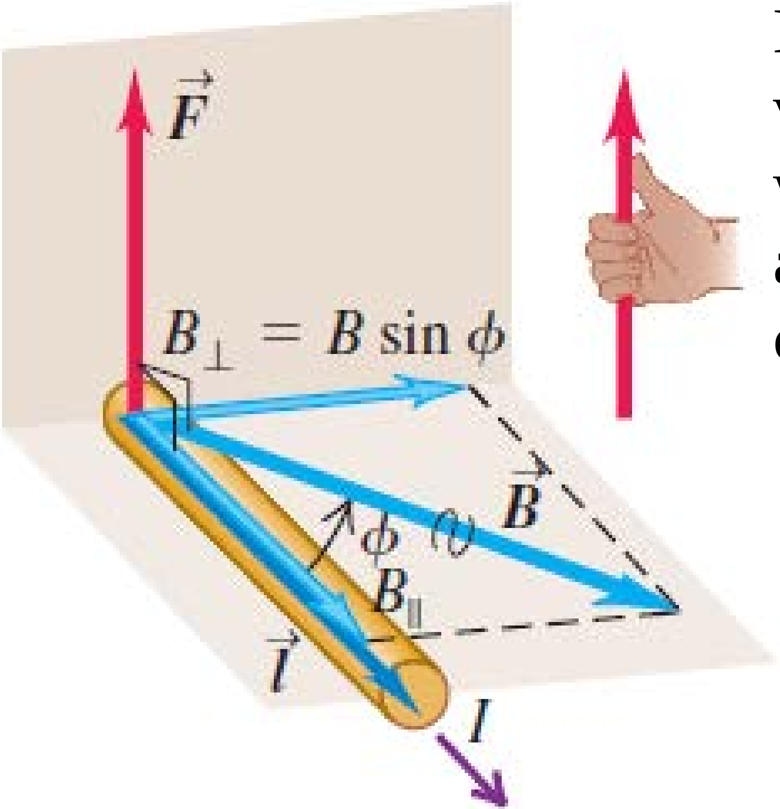
Magnetic Force on a Current-Carrying Conductor



Force $\vec{F}$ on a straight wire carrying a positive current and oriented at an angle ϕ to a magnetic field $\vec{B}$:

- Magnitude is $F = IlB_{\perp} = IlB \sin\phi$.
- Direction of $\vec{F}$ is given by the right-hand rule.

$$\vec{F} = q\vec{v}_d \times \vec{B}$$



Number of charges per unit volume is n; a segment with length l has volume Al and contains a number of charges equal to nAl

$$F = (nAl)(qv_dB)$$
$$= (nqv_dA)(lB)$$

Magnetic Force on a Current-Carrying Conductor

$$F = IlB$$

$$F = IlB_{\perp} = IlB\sin\phi$$

$$\vec{F} = I\vec{l} \times \vec{B} \qquad \text{(magnetic force on a straight wire segment)} \qquad (27.19)$$

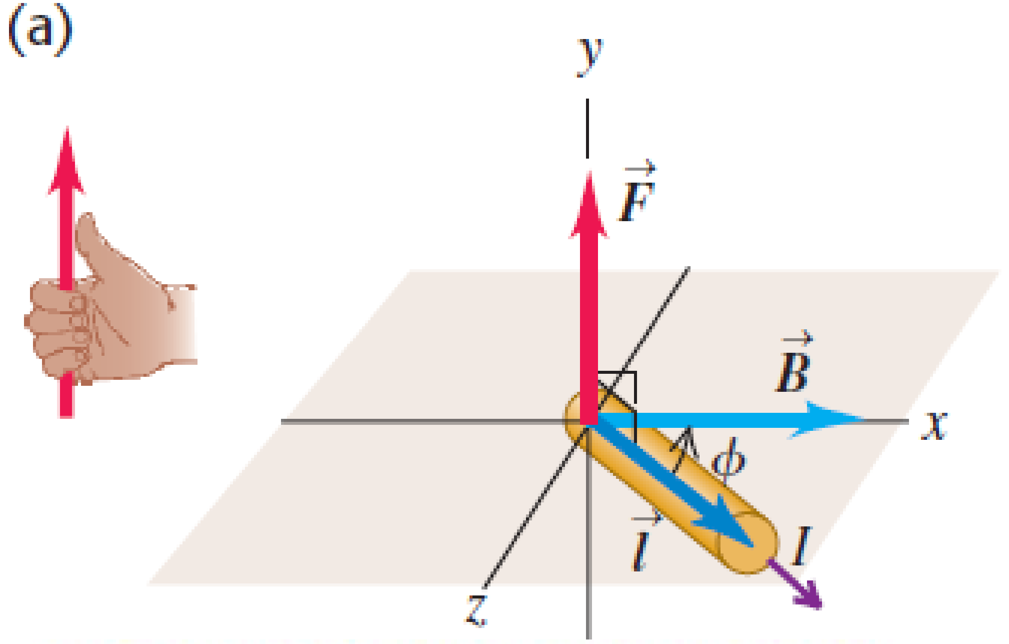
Figure 27.27 illustrates the directions of $\vec{B}$, $\vec{l}$, and $\vec{F}$ for several cases.

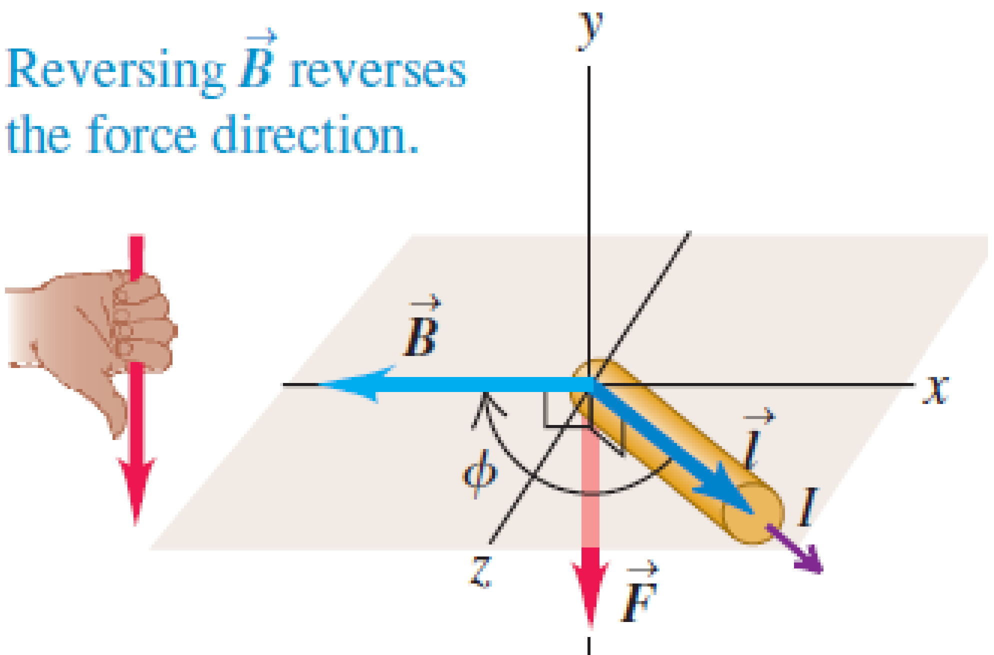
If the conductor is not straight, we can divide it into infinitesimal segments $d\vec{l}$. The force $d\vec{F}$ on each segment is

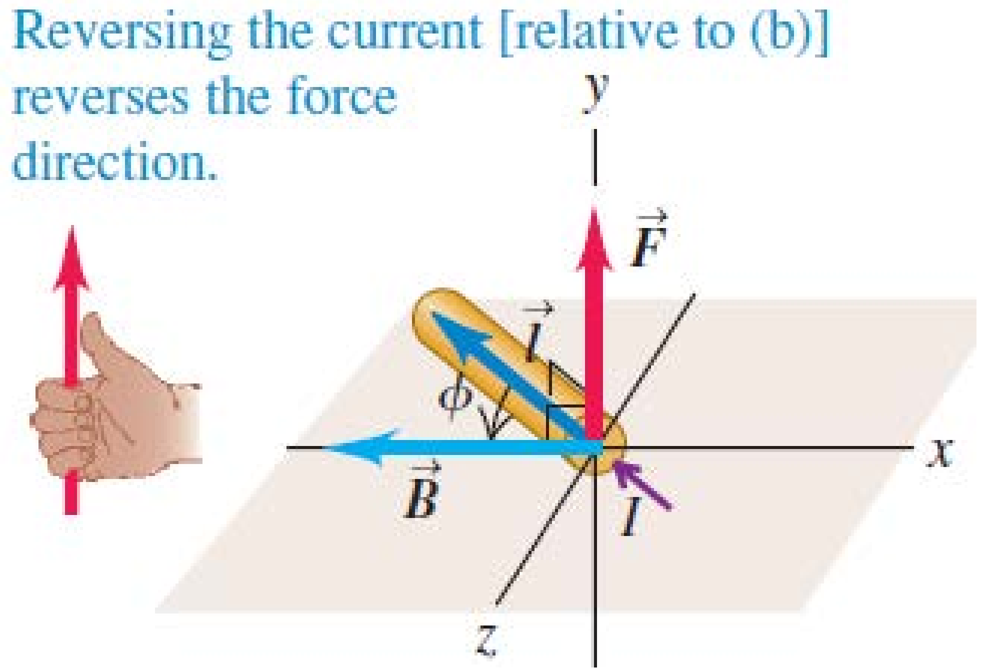
$$d\vec{F} = I\,d\vec{l} \times \vec{B} \qquad \text{(magnetic force on an infinitesimal wire section)} \qquad (27.20)$$

Magnetic Force on a Current-Carrying Conductor

CAUTION Current is not a vector Recall from Section 25.1 that the current is not a vector. The direction of current flow is described by dl not I If the conductor is curved, the current is the same at all points along its length, but dl changes direction so that it is always tangent to the conductor. I

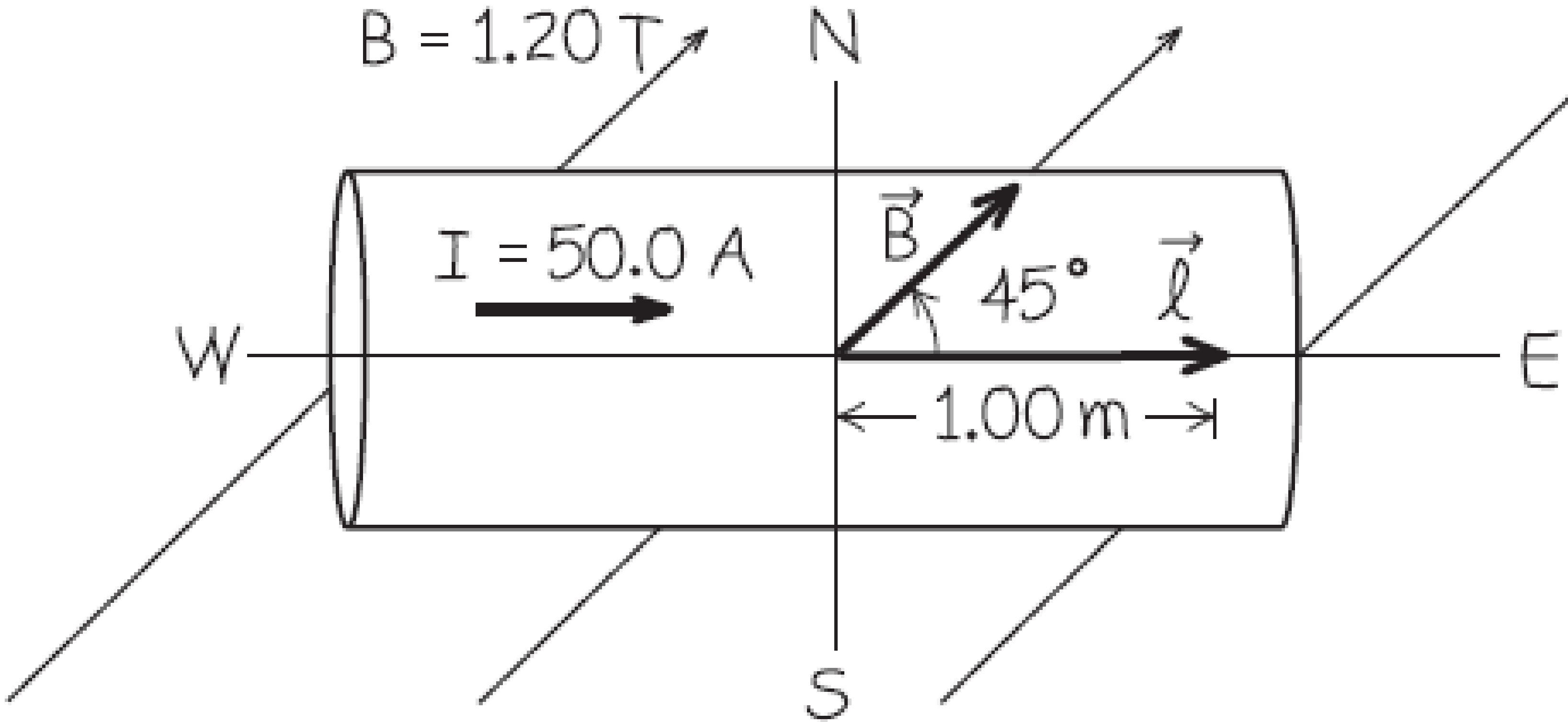






Example 27.7 Magnetic force on a straight conductor

A straight horizontal copper rod carries a current of 50.0 A from west to east in a region between the poles of a large electromagnet. In this region there is a horizontal magnetic field toward the northeast (that is, 45° north of east) with magnitude 1.20 T. (a) Find the magnitude and direction of the force on a 1.00-m section of rod. (b) While keeping the rod horizontal, how should it be oriented to maximize the magnitude of the force? What is the force magnitude in this case?



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EXECUTE: (a) The angle ϕ between the directions of current and field is 45°. From Eq. (27.18) we obtain

$$F = IlB\sin\phi = (50.0\text{ A})(1.00\text{ m})(1.20\text{ T})(\sin 45°) = 42.4\text{ N}$$

The direction of the force is perpendicular to the plane of the current and the field, both of which lie in the horizontal plane. Thus the force must be vertical; the right-hand rule shows that it is vertically *upward* (out of the plane of the figure).

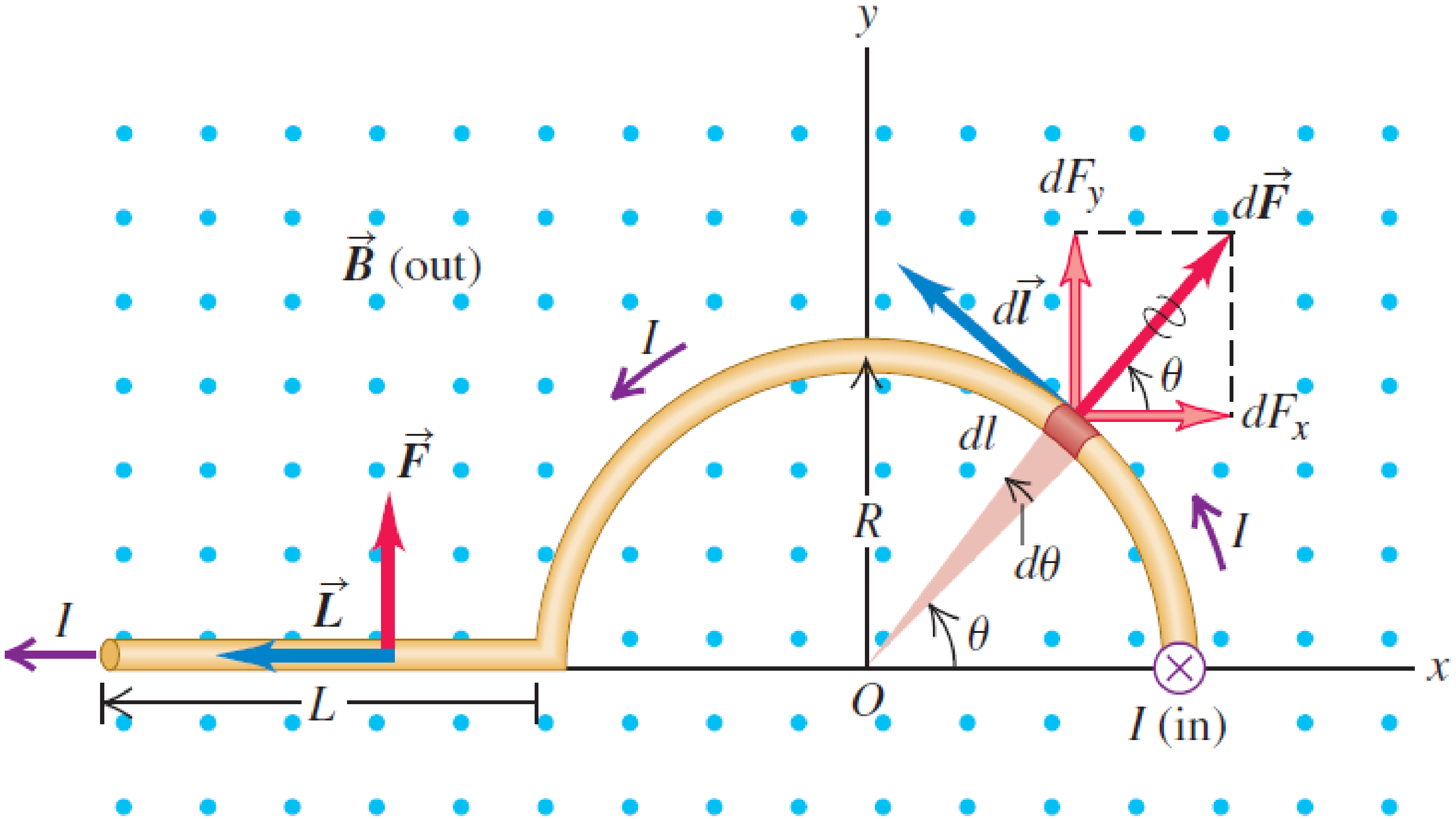
Example 27.7 Magnetic force on a straight conductor

A straight horizontal copper rod carries a current of 50.0 A from west to east in a region between the poles of a large electromagnet. In this region there is a horizontal magnetic field toward the northeast (that is, 45° north of east) with magnitude 1.20 T. (b) While keeping the rod horizontal, how should it be oriented to maximize the magnitude of the force? What is the force magnitude in this case?

(b) From $F = IlB\sin\phi$, F is maximum for $\phi = 90°$, so that $\vec{l}$ and $\vec{B}$ are perpendicular. To keep $\vec{F} = I\vec{l} \times \vec{B}$ upward, we rotate the rod *clockwise* by 45° from its orientation in Fig. 27.29, so that the current runs toward the southeast. Then $F = IlB = (50.0\ \text{A})(1.00\ \text{m})(1.20\ \text{T}) = 60.0\ \text{N}$.

Example 27.8 Magnetic force on a curved conductor

In Fig. 27.30 the magnetic field B is uniform and perpendicular to the plane of the figure, pointing out of the page. The conductor, carrying current I to the left, has three segments: (1) a straight segment with length L perpendicular to the plane of the figure, (2) a semicircle with radius , and (3) another straight segment with length parallel to the x-axis. Find the total magnetic force on this conductor.



Example 27.8 Magnetic force on a curved conductor

EXECUTE: For segment (1), $\vec{L} = -L\hat{k}$. Hence from Eq. (27.19), $\vec{F}_1 = I\vec{L} \times \vec{B} = \mathbf{0}$. For segment (3), $\vec{L} = -L\hat{\imath}$, so $\vec{F}_3 = I\vec{L} \times \vec{B} = I(-L\hat{\imath}) \times (B\hat{k}) = ILB\hat{\jmath}$.

For the curved segment (2), Fig. 27.20 shows a segment $d\vec{l}$ with length $dl = R\,d\theta$, at angle θ. The right-hand rule shows that the direction of $d\vec{l} \times \vec{B}$ is radially outward from the center; make sure you can verify this. Because $d\vec{l}$ and $\vec{B}$ are perpendicular, the magnitude dF_2 of the force on the segment $d\vec{l}$ is just $dF_2 = I\,dl\,B = I(R\,d\theta)B$. The components of the force on this segment are

$$dF_{2x} = IR\,d\theta\,B\cos\theta \qquad dF_{2y} = IR\,d\theta\,B\sin\theta$$

Example 27.8 Magnetic force on a curved conductor

To find the components of the total force, we integrate these expressions with respect to θ from $\theta = 0$ to $\theta = \pi$ to take in the whole semicircle. The results are

$$F_{2x} = IRB \int_0^{\pi} \cos\theta \, d\theta = 0$$

$$F_{2y} = IRB \int_0^{\pi} \sin\theta \, d\theta = 2IRB$$

Hence $\vec{F}_2 = 2IRB\hat{\boldsymbol{j}}$. Finally, adding the forces on all three segments, we find that the total force is in the positive y-direction:

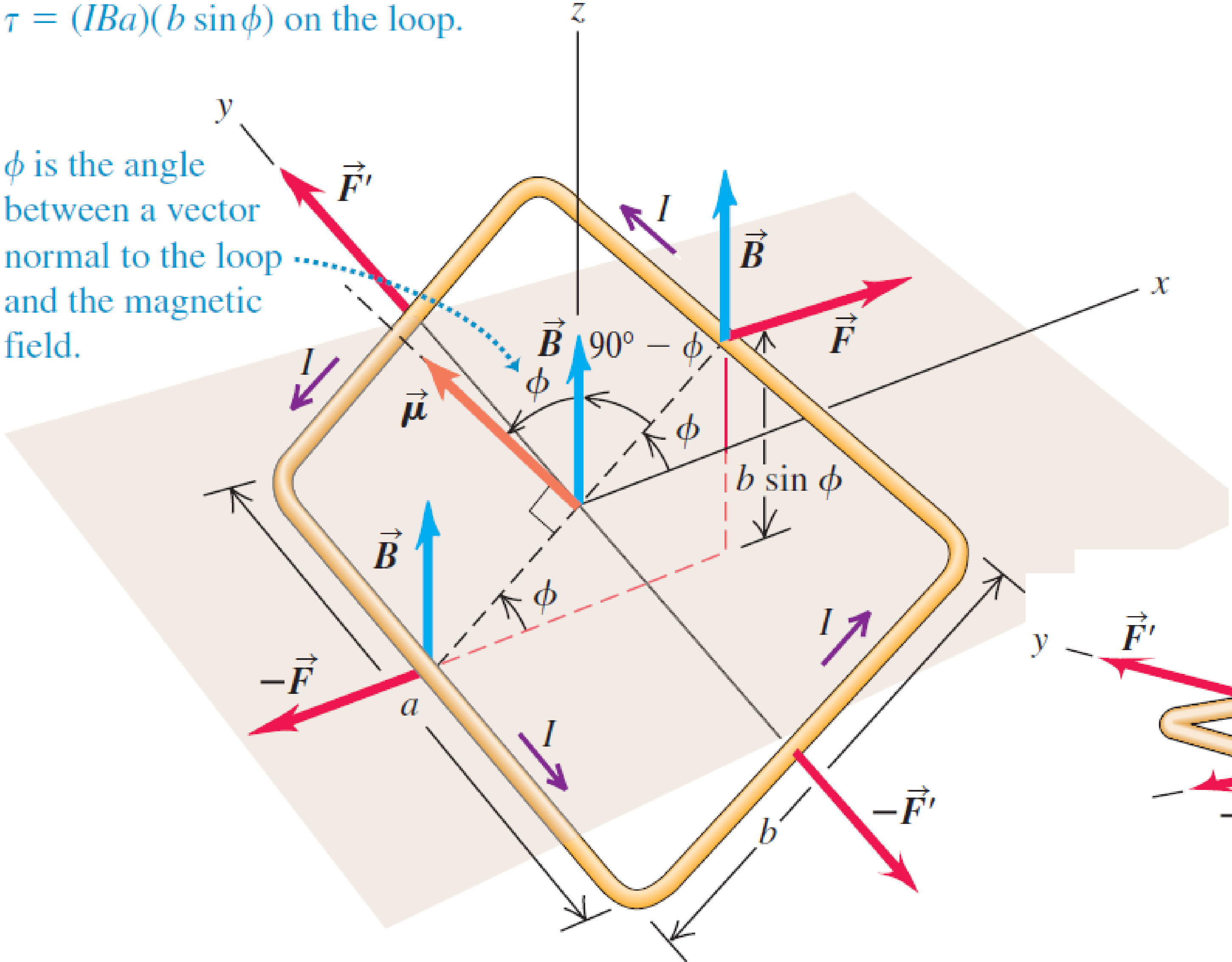
$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 + 2IRB\hat{\boldsymbol{j}} + ILB\hat{\boldsymbol{j}} = IB(2R + L)\hat{\boldsymbol{j}}$$

Force and Torque on a Current Loop

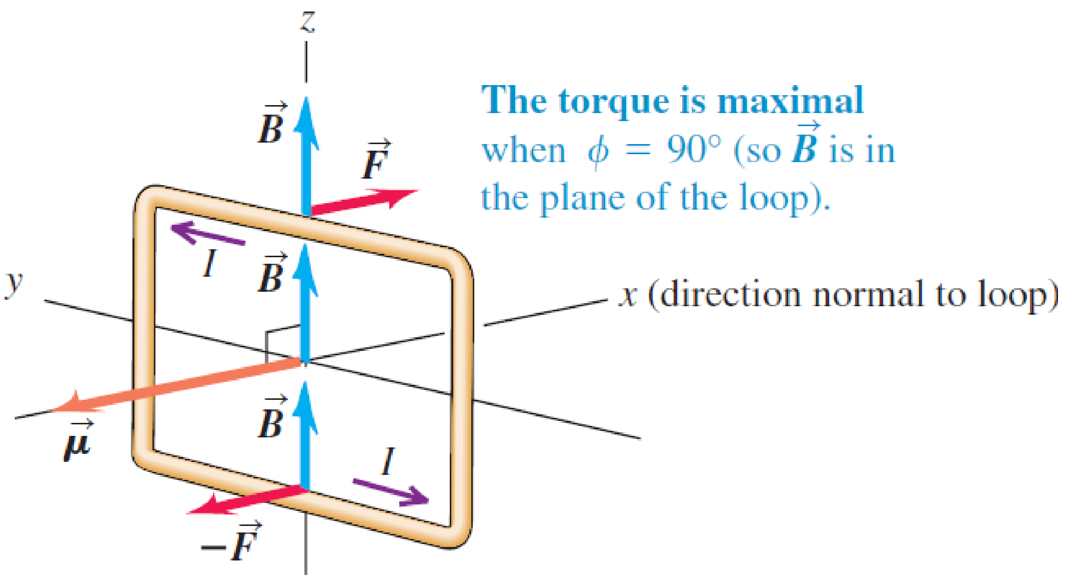
The two pairs of forces acting on the loop cancel, so no net force acts on the loop.

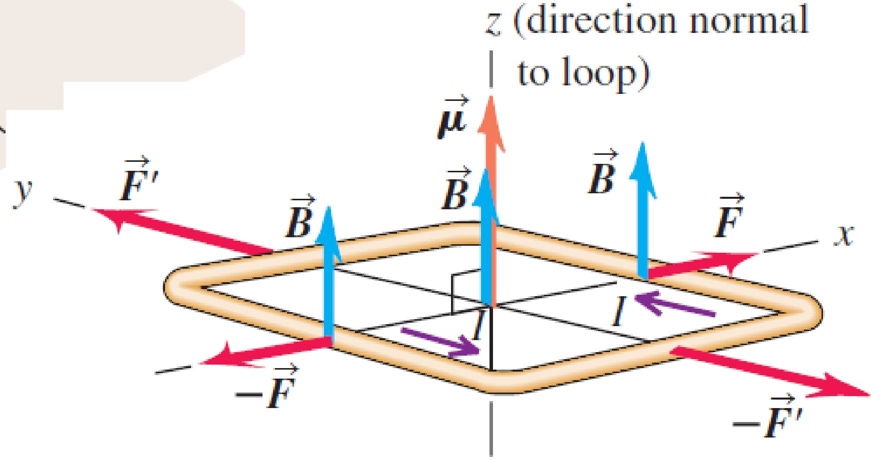
However, the forces on the a sides of the loop ($\vec{F}$ and $-\vec{F}$) produce a torque $\tau = (IBa)(b \sin\phi)$ on the loop.

ϕ is the angle between a vector normal to the loop and the magnetic field.



The torque is maximal when $\phi = 90°$ (so $\vec{B}$ is in the plane of the loop).





The torque is zero when $\phi = 0°$ (as shown here) or $\phi = 180°$. In both cases, $\vec{B}$ is perpendicular to the plane of the loop.

The loop is in stable equilibrium when $\phi = 0$; it is in unstable equilibrium when $\phi = 180°$.

Force and Torque on a Current Loop

The force $\vec{F}$ on the right side of the loop (length a) is to the right, in the $+x$-direction as shown. On this side, $\vec{B}$ is perpendicular to the current direction, and the force on this side has magnitude

$$F = IaB \tag{27.21}$$

A force $-\vec{F}$ with the same magnitude but opposite direction acts on the opposite side of the loop, as shown in the figure.

The sides with length b make an angle $(90° - \phi)$ with the direction of $\vec{B}$. The forces on these sides are the vectors $\vec{F}'$ and $-\vec{F}'$; their magnitude F' is given by

$$F' = IbB\sin(90° - \phi) = IbB\cos\phi$$

The lines of action of both forces lie along the y-axis.

The *total* force on the loop is zero because the forces on opposite sides cancel out in pairs.

The net force on a current loop in a uniform magnetic field is zero. However, the net torque is not in general equal to zero.

Force and Torque on a Current Loop

(You may find it helpful at this point to review the discussion of torque in Section 10.1.) The two forces $\vec{F}'$ and $-\vec{F}'$ in Fig. 27.31a lie along the same line and so give rise to zero net torque with respect to any point. The two forces $\vec{F}$ and $-\vec{F}$ lie along different lines, and each gives rise to a torque about the y-axis. According to the right-hand rule for determining the direction of torques, the vector torques due to $\vec{F}$ and $-\vec{F}$ are both in the $+y$-direction; hence the net vector torque $\vec{\tau}$ is in the $+y$-direction as well. The moment arm for each of these forces (equal to the perpendicular distance from the rotation axis to the line of action of the force) is $(b/2)\sin\phi$, so the torque due to each force has magnitude $F(b/2)\sin\phi$. If we use Eq. (27.21) for F, the magnitude of the net torque is

$$\tau = 2F(b/2)\sin\phi = (IBa)(b\sin\phi) \qquad (27.22)$$

The torque is greatest when $\phi = 90°$, $\vec{B}$ is in the plane of the loop, and the normal to this plane is perpendicular to $\vec{B}$ (Fig. 27.31b). The torque is zero when ϕ is $0°$ or $180°$ and the normal to the loop is parallel or antiparallel to the field (Fig.

Force and Torque on a Current Loop

27.31c). The value $\phi = 0°$ is a stable equilibrium position because the torque is zero there, and when the loop is rotated slightly from this position, the resulting torque tends to rotate it back toward $\phi = 0°$. The position $\phi = 180°$ is an *unstable* equilibrium position; if displaced slightly from this position, the loop tends to move farther away from $\phi = 180°$. Figure 27.31 shows rotation about the y-axis, but because the net force on the loop is zero, Eq. (27.22) for the torque is valid for *any* choice of axis.

The area A of the loop is equal to ab, so we can rewrite Eq. (27.22) as

$$\tau = IBA\sin\phi \qquad \text{(magnitude of torque on a current loop)} \qquad (27.23)$$

The product IA is called the **magnetic dipole moment** or **magnetic moment** of the loop, for which we use the symbol μ (the Greek letter mu):

$$\mu = IA \qquad (27.24)$$

Force and Torque on a Current Loop

It is analogous to the electric dipole moment introduced in Section 21.7. In terms of $\boldsymbol{\mu}$, the magnitude of the torque on a current loop is

$$\tau = \mu B \sin\phi \tag{27.25}$$

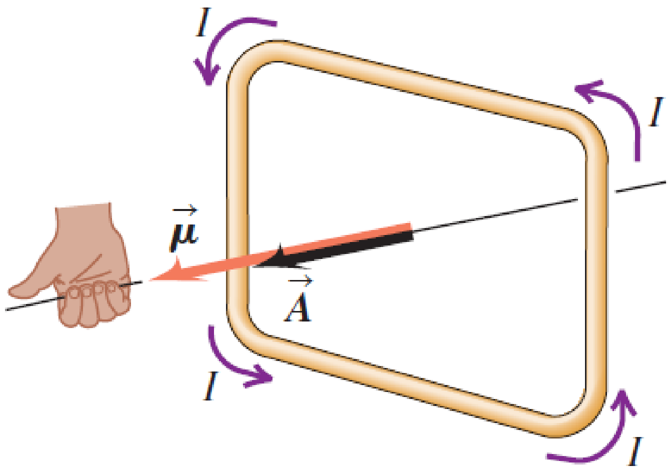
where ϕ is the angle between the normal to the loop (the direction of the vector area $\vec{A}$) and $\vec{\boldsymbol{B}}$. The torque tends to rotate the loop in the direction of *decreasing* ϕ—that is, toward its stable equilibrium position in which the loop lies in the *xy*-plane perpendicular to the direction of the field $\vec{\boldsymbol{B}}$ (Fig. 27.31c). A current loop, or any other body that experiences a magnetic torque given by Eq. (27.25), is also called a **magnetic dipole.**

Magnetic Torque: Vector Form

Finally, we can express this interaction in terms of the torque vector $\vec{\boldsymbol{\tau}}$, which we used for *electric*-dipole interactions in Section 21.7. From Eq. (27.25) the magnitude of $\vec{\boldsymbol{\tau}}$ is equal to the magnitude of $\vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$, and reference to Fig. 27.31 shows that the directions are also the same. So we have

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}} \qquad \text{(vector torque on a current loop)} \qquad (27.26)$$

This result is directly analogous to the result we found in Section 21.7 for the torque exerted by an *electric* field $\vec{\boldsymbol{E}}$ on an *electric* dipole with dipole moment $\vec{\boldsymbol{p}}$.



Potential Energy for a Magnetic Dipole

When a magnetic dipole changes orientation in a magnetic field, the field does work on it. In an infinitesimal angular displacement $d\phi$, the work dW is given by $\tau\, d\phi$, and there is a corresponding change in potential energy. As the above discussion suggests, the potential energy is least when $\vec{\boldsymbol{\mu}}$ and $\vec{\boldsymbol{B}}$ are parallel and greatest when they are antiparallel. To find an expression for the potential energy U as a function of orientation, we can make use of the beautiful symmetry between the electric and magnetic dipole interactions. The torque on an *electric* dipole in an *electric* field is $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{p}} \times \vec{\boldsymbol{E}}$; we found in Section 21.7 that the corresponding potential energy is $U = -\vec{\boldsymbol{p}} \cdot \vec{\boldsymbol{E}}$. The torque on a *magnetic* dipole in a *magnetic* field is $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$, so we can conclude immediately that the corresponding potential energy is

$$U = -\vec{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{B}} = -\mu B \cos\phi \qquad \text{(potential energy for a magnetic dipole)} \quad (27.27)$$

Magnetic Torque: Loops and Coils

An arrangement of particular interest is the **solenoid,** a helical winding of wire, such as a coil wound on a circular cylinder (Fig. 27.34). If the windings are closely spaced, the solenoid can be approximated by a number of circular loops lying in planes at right angles to its long axis. The total torque on a solenoid in a magnetic field is simply the sum of the torques on the individual turns. For a solenoid with N turns in a uniform field B, the magnetic moment is $\mu = NIA$ and

$$\tau = NIAB \sin\phi \tag{27.28}$$

where ϕ is the angle between the axis of the solenoid and the direction of the field. The magnetic moment vector $\vec{\boldsymbol{\mu}}$ is along the solenoid axis. The torque is greatest when the solenoid axis is perpendicular to the magnetic field and zero when they are parallel. The effect of this torque is to tend to rotate the solenoid into a position where its axis is parallel to the field. Solenoids are also useful as *sources* of magnetic field, as we'll discuss in Chapter 28.

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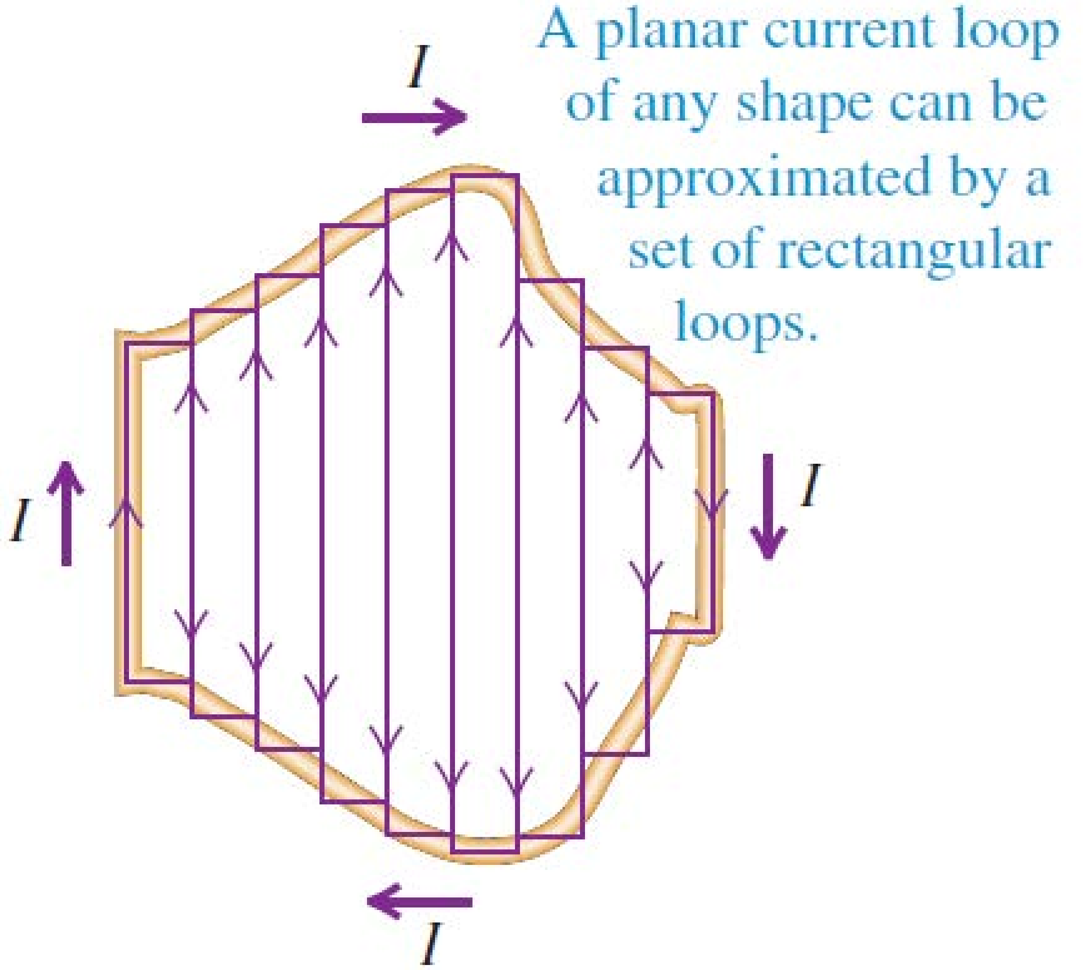
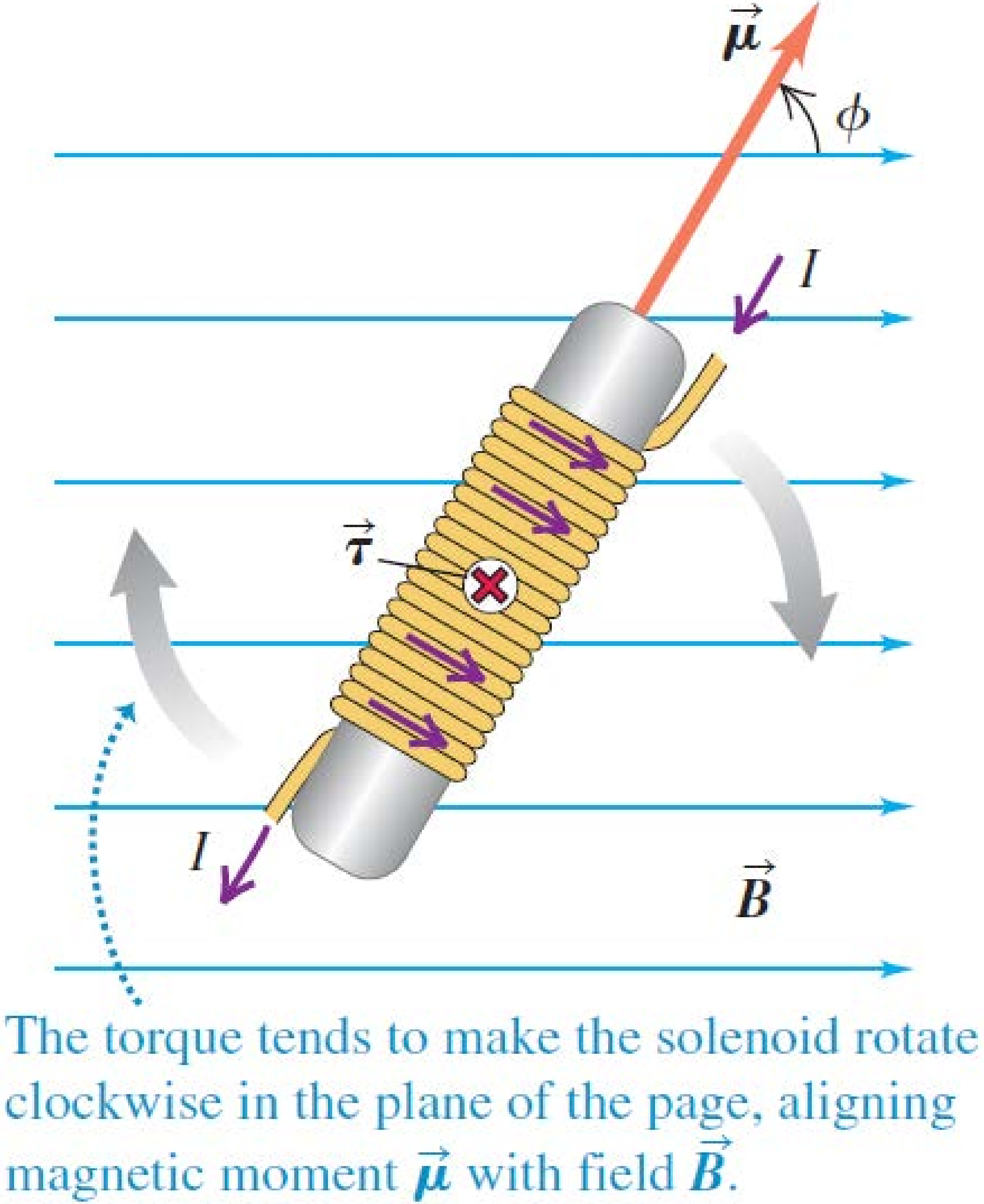
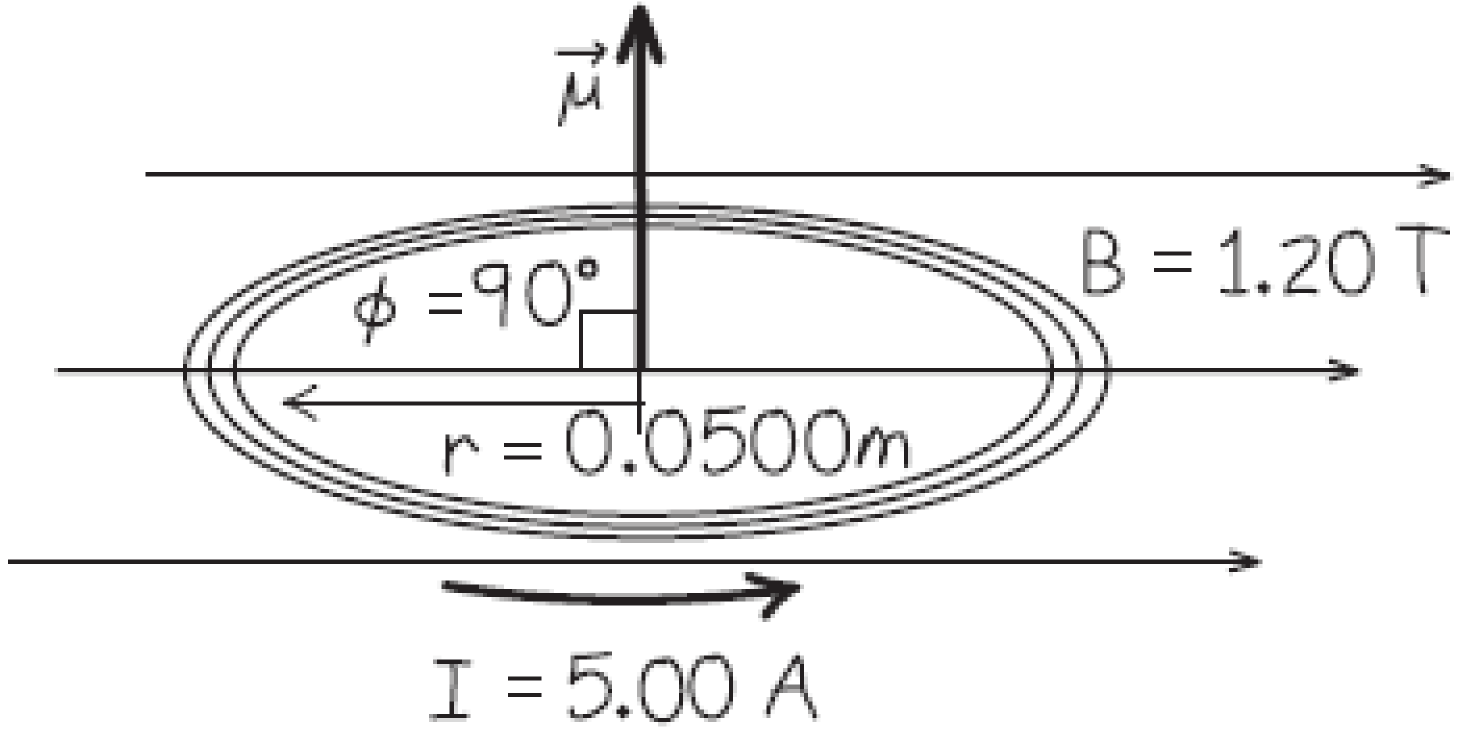
Magnetic Torque: Loops and Coils





Example 27.9 Magnetic torque on a circular coil

A circular coil 0.0500 m in radius, with 30 turns of wire, lies in a horizontal plane. It carries a counterclockwise (as viewed from above) current of 5.00 A. The coil is in a uniform 1.20-T magnetic field directed toward the right. Find the magnitudes of the magnetic moment and the torque on the coil.



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EXECUTE: The area of the coil is $A = \pi r^2$. From Eq. (27.24), the total magnetic moment of all 30 turns is

$$\mu_{\text{total}} = NIA = 30(5.00\ \text{A})\pi(0.0500\ \text{m})^2 = 1.18\ \text{A}\cdot\text{m}^2$$

The angle ϕ between the direction of $\vec{B}$ and the direction of $\vec{\mu}$ (which is along the normal to the plane of the coil) is 90°. From Eq. (27.25), the torque on the coil is

$$\begin{aligned}\tau &= \mu_{\text{total}} B \sin\phi = (1.18\ \text{A}\cdot\text{m}^2)(1.20\ \text{T})(\sin 90^\circ)\\ &= 1.41\ \text{N}\cdot\text{m}\end{aligned}$$

Example 27.10 Potential energy for a coil in a magnetic field

If the coil in Example 27.9 rotates from its initial orientation to one in which its magnetic moment $\vec{\boldsymbol{\mu}}$ is parallel to $\vec{\boldsymbol{B}}$, what is the change in potential energy?

$$\begin{aligned}
\Delta U &= U_2 - U_1 = -\mu B \cos\phi_2 - (-\mu B \cos\phi_1) \\
&= -\mu B(\cos\phi_2 - \cos\phi_1) \\
&= -(1.18\ \mathrm{A \cdot m^2})(1.20\ \mathrm{T})(\cos 0° - \cos 90°) = -1.41\ \mathrm{J}
\end{aligned}$$

Summary