College Algebra and Trigonometry

Prof. Liang ZHENG

Fall 2024

① Simplify Imaginary Numbers

The Imaginary Number *i* :

- $i^2 = -1$ **and** $i = \sqrt{-1}$
- **If *b* is a positive real number, then** $\sqrt{-b} = i\sqrt{b}$

Example 1: Write Imaginary Numbers / Simplify in terms of *i*

a) $\sqrt{-25}$ b) $\sqrt{-\frac{50}{9}}$ c) $\sqrt{-25}\cdot\sqrt{-9}$ d) $\frac{\sqrt{-50}}{\sqrt{-18}}$

② Write Complex Numbers in the form $a+bi$

Complex Number: $a+bi$

***a*: Real part** **b: Imaginary part**

Special Cases:

1) $a = 0$: pure imaginary **2) $b = 0$: real number**

Example 2: Write Complex Numbers in standard form

a) $3-\sqrt{-100}$

b) $\dfrac{-6+\sqrt{-18}}{3}$

③ Perform Operations on Complex Numbers

$$i^1 = i \qquad i^5 = i^4 \cdot i = i \qquad i^9 = i$$

$$i^2 = -1 \qquad i^6 = i^4 \cdot i^2 = 1 \cdot (-1) = -1 \qquad i^{10} = -1$$

$$i^3 = -i \qquad i^7 = i^4 \cdot i^3 = 1 \cdot (-i) = -i \qquad i^{11} = -i \qquad \cdots\cdots$$

$$i^4 = 1 \qquad i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1 \qquad i^{12} = 1$$

Example 3: Simplify Powers of *i*

a) i^{48} b) i^{50} c) i^{23} d) i^{-19}

Example 4: Add and Subtract Complex Numbers

a) $(5+2i)+(3-6i)-(-2+2i)$

b) Subtract $(\frac{1}{2}+\frac{2}{3}i)$ from $(\frac{3}{4}+\frac{5}{6}i)$

Example 5: Multiply and Divide Complex Numbers

a) $(5+2i)(1-2i)$

b) $(3+4i)(3-4i)$

c) $\dfrac{5+2i}{1-i}$

d) $(1-\sqrt{3}i)^{-1}$

Definition of a Quadratic Equation :

- **A quadratic equation in the variable is an equation of the form:**

$$ax^2 + bx + c = 0$$

① Solve by using the Zero product Property

Zero Product Property :

If $mn = 0$, then $m = 0$ or $n = 0$.

Example 1: Apply the Zero Product Property to solve the equations:

a) $x^2 - 8x = 0$

b) $2x(2x - 7) = -12.$

② Solve by using the Square Root Property

Square Root Property :

If $x^2 = k$, then $x = \sqrt{k}$ or $-\sqrt{k}$.

Example 2: Apply the Zero Product Property to solve:

a) $x^2 = 36$ **b) $9y^2 + 25 = 0$** **c) $(w - 3)^2 = 50$**

③ Complete the Square

Perfect Square Trinomial		Factored Form
$x^2+10x+25$	→	$(x+5)^2$
$t^2-8t+16$	→	$(t-4)^2$
$z^2+12z+36$	→	$(z+6)^2$

Example 3: **Complete the Square**

a) x^2+12x **b)** $x^2-\frac{3}{5}x$ **c)** $2x^2-9x$