College Algebra and Trigonometry

Prof. Liang ZHENG

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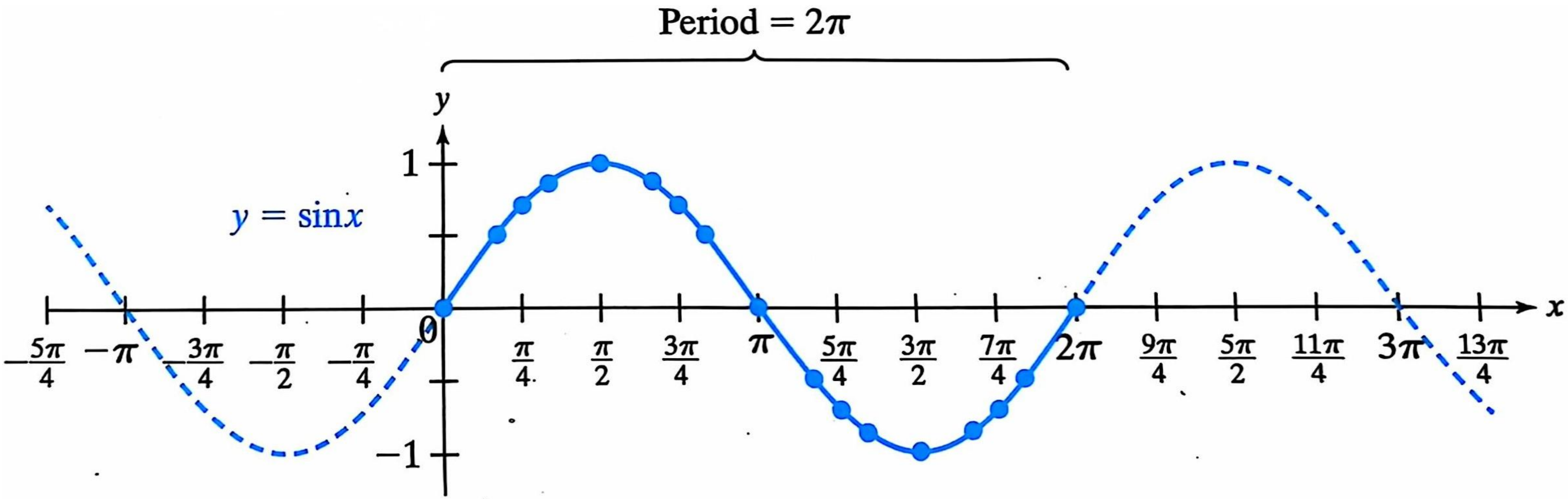
① Graph $y = sinx$

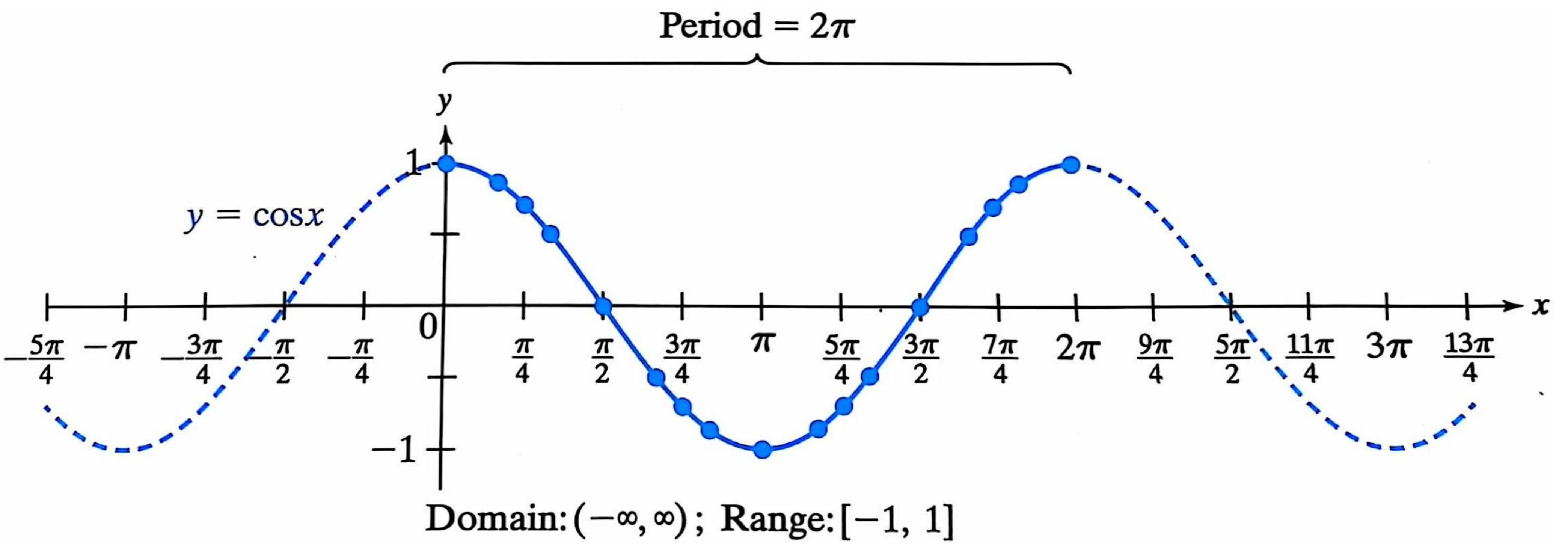
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{12}$	2π
sin$x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0



Domain: $(-\infty, \infty)$; Range: $[-1, 1]$

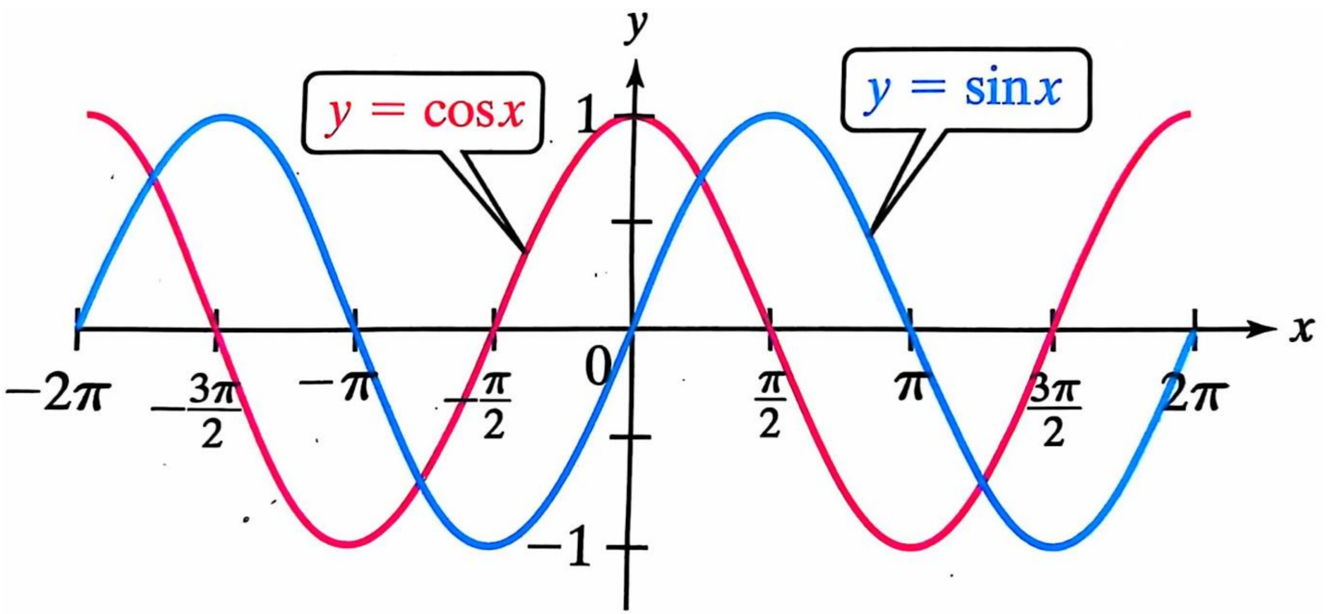
① Graph $y = cosx$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{12}$	2π
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1



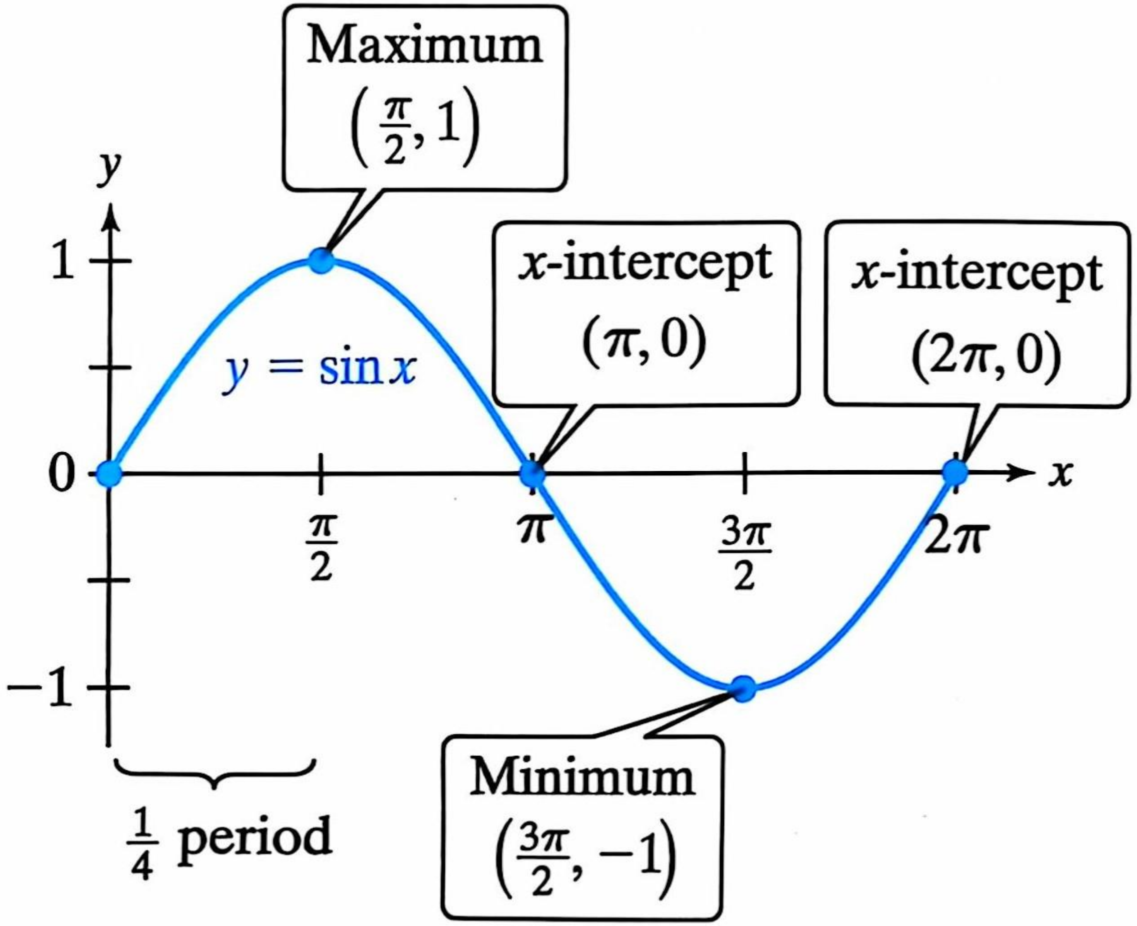
Characteristics of the Graphs of $y = \sin x$ and $y = \cos x$

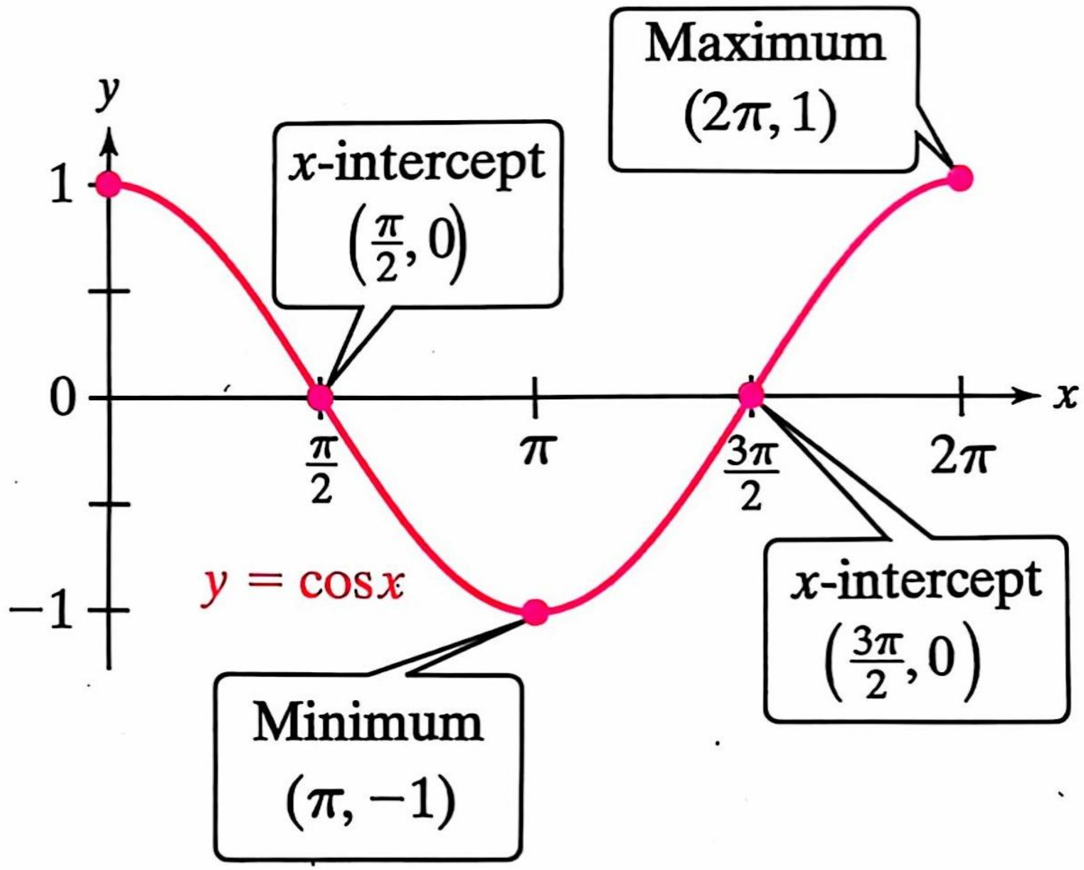
- The domain is $(-\infty, \infty)$.
- The range is $[-1, 1]$.
- The period is 2π.
- The graph of $y = \sin x$ is symmetric with respect to the origin. $y = \sin x$ is an odd function.
- The graph of $y = \cos x$ is symmetric with respect to the y-axis. $y = \cos x$ is an even function.
- The graphs of $y = \sin x$ and $y = \cos x$ differ by a horizontal shift of $\frac{\pi}{2}$.



② Graph $y = A\sin x$ and $y = A\cos x$

Key Points of $y = \sin x$ and $y = \cos x$:

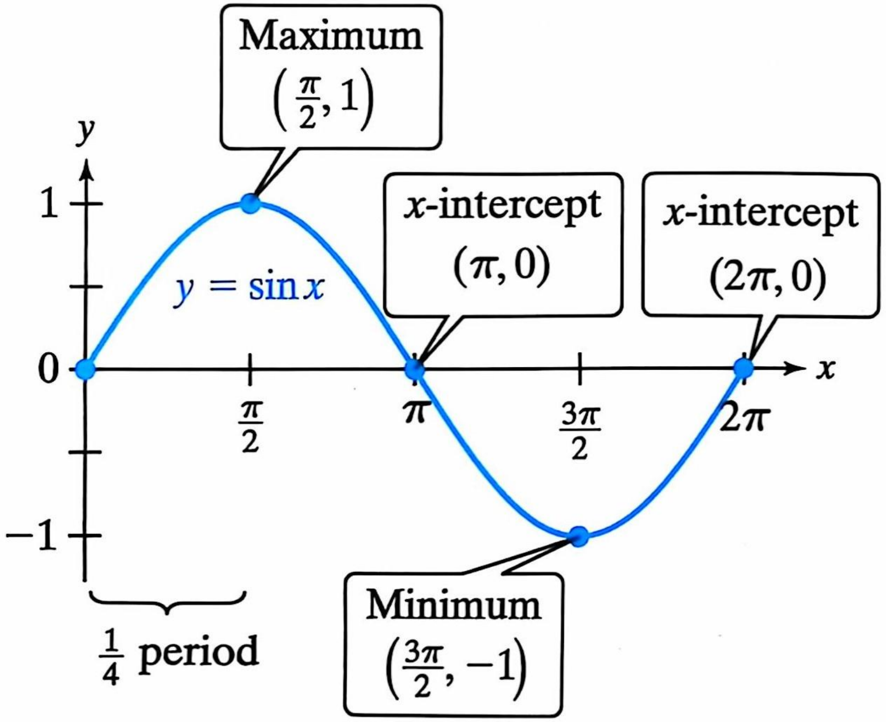


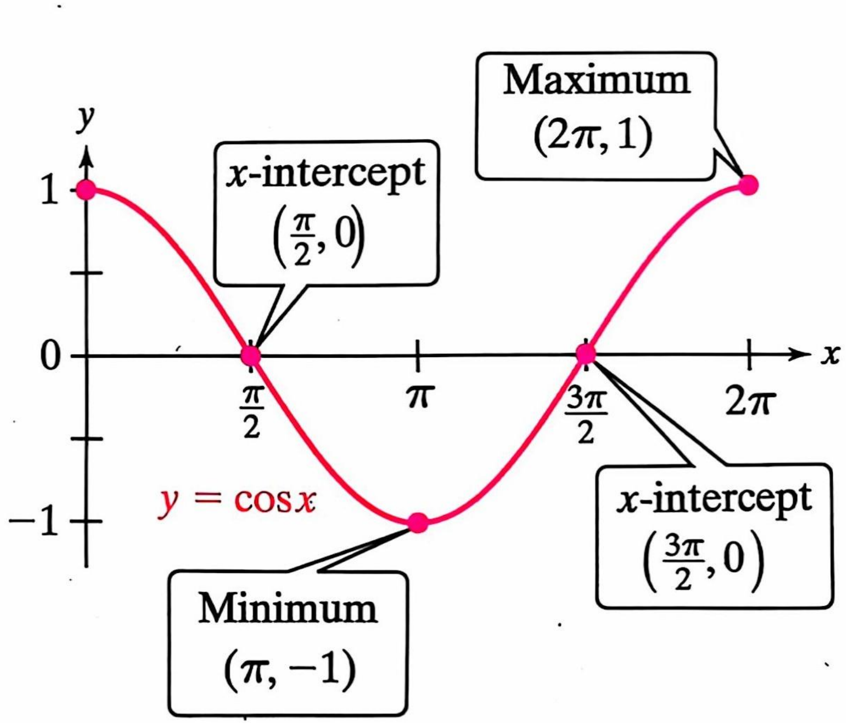


Example 1:

Graph the function and identify the key points on one full period.

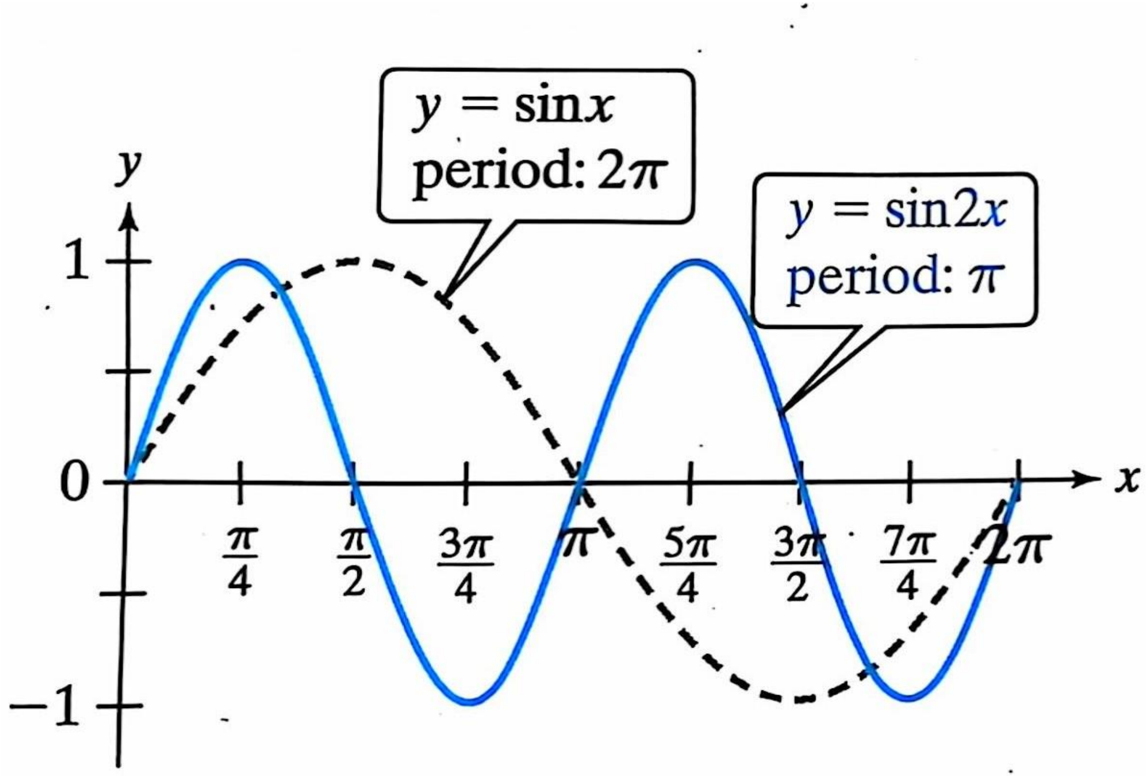
a) $y = 3\sin x$ b) $y = -\frac{1}{2}\cos x$

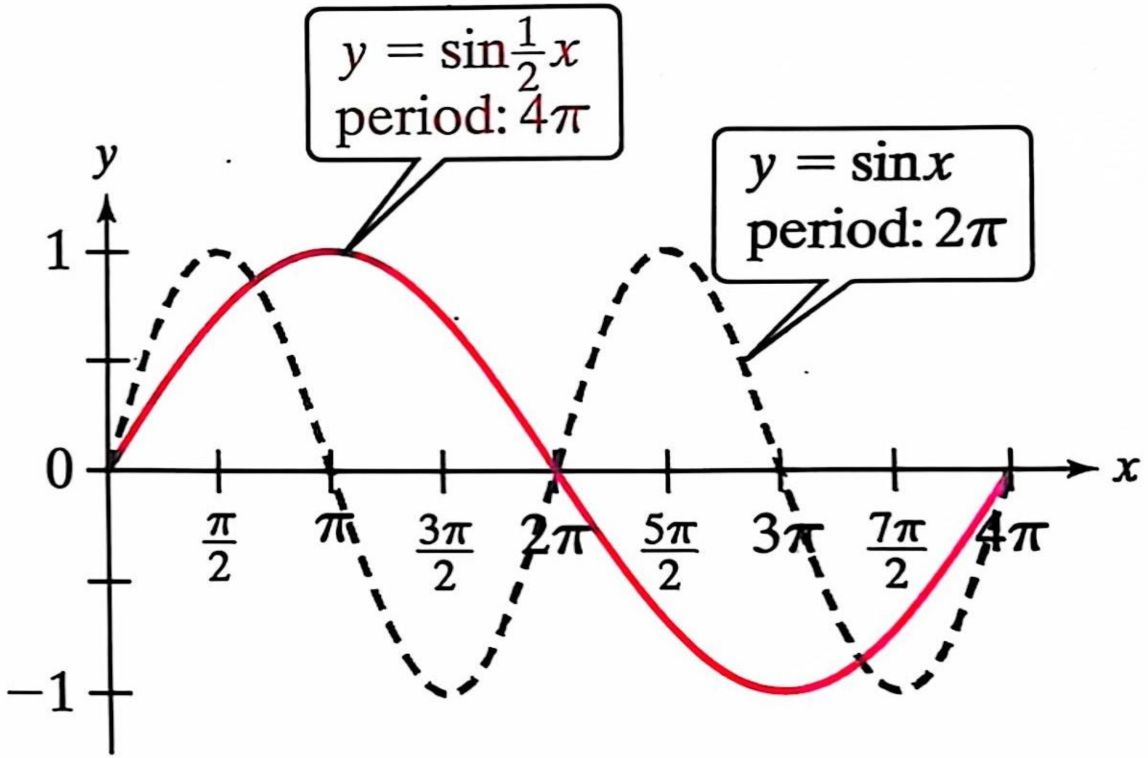




③ Graph $y = A\sin Bx$ and $y = A\cos Bx$

Recall from Section 2.6 that the graph of $y = f(Bx)$ is the graph of $y = f(x)$ with a horizontal shrink or stretch.





Amplitude and Period of the Sine and Cosine functions:

For $y = A\sin Bx$ and $y = A\sin Bx$ and $B > 0$, the amplitude and period are

$$\text{Amplitude} = |A| \quad \text{and} \quad \text{Period} = \frac{2\pi}{B}$$

Example 2:

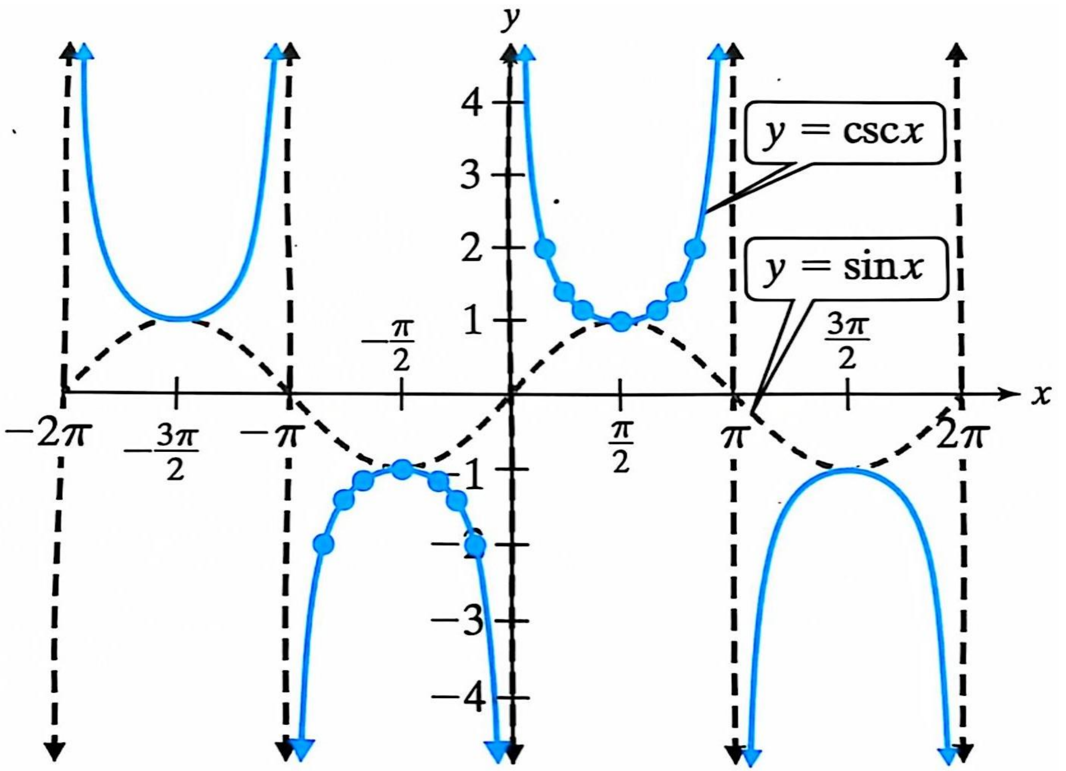
Given $f(x) = 4\sin 3x$

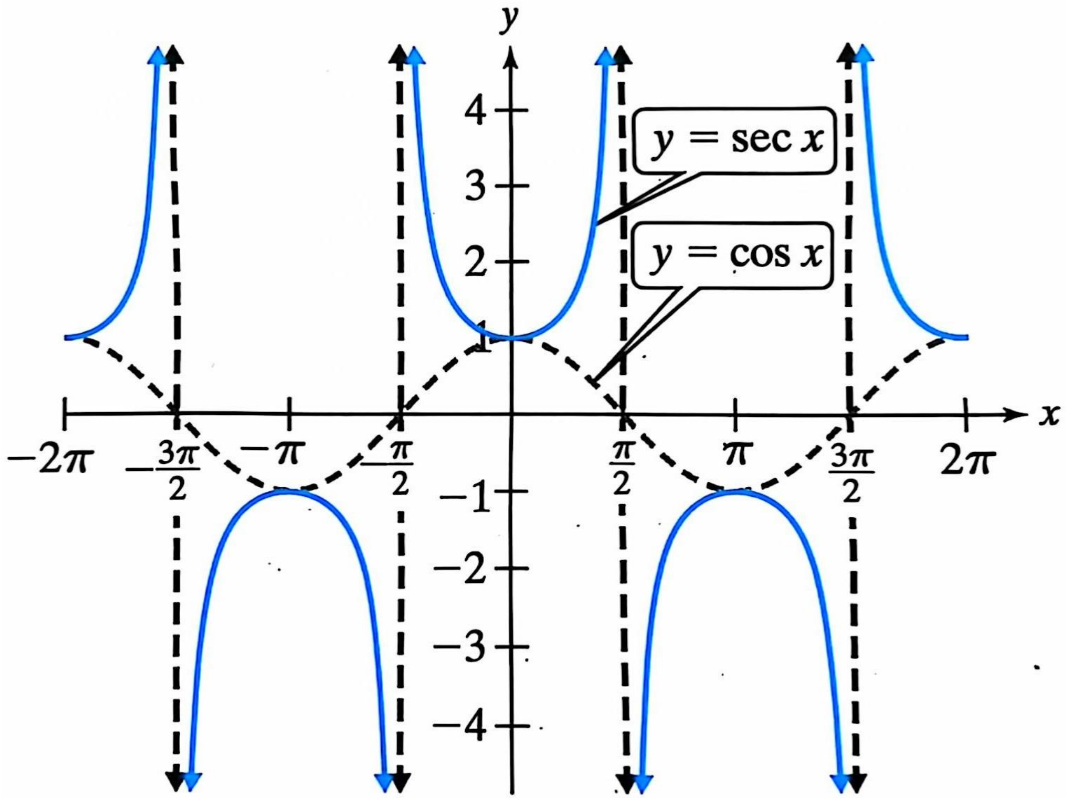
a) Identify the amplitude and period.

b) Graph the function and identify the key points on one full period.

① Graph $y = \csc x$ and $y = \sec x$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\csc x$	undefined	$\sqrt{2}$	1	$\sqrt{2}$	undefined	$-\sqrt{2}$	-1	$-\sqrt{2}$	undefined
$\sec x$	1	$\sqrt{2}$	undefined	$\sqrt{2}$	-1	$-\sqrt{2}$	undefined	$\sqrt{2}$	1



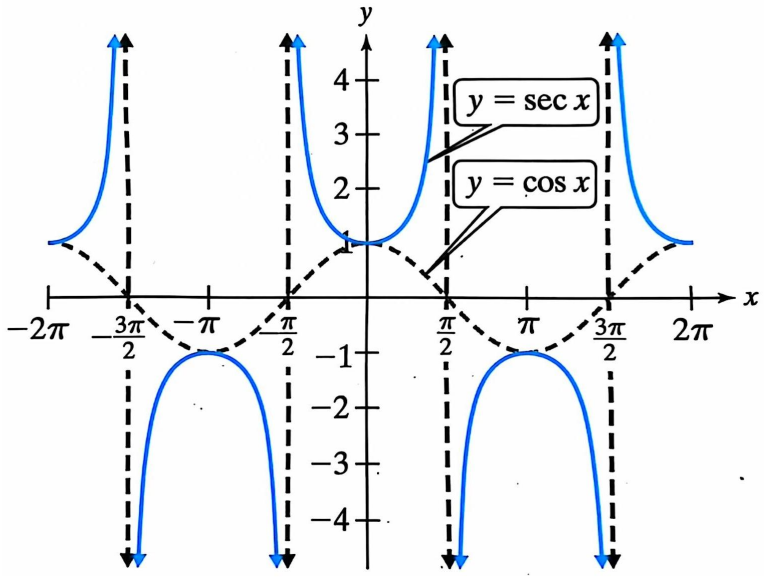


Graphs of the Cosecant and Secant Functions

	$y = \csc x$	$y = \sec x$
Domain	$\{x \mid x \neq n\pi$ for all integers $n\}$	$\{x \mid x \neq \frac{(2n+1)\pi}{2}$ for all integers $n\}$
Range	$\{y \mid y \leq -1$ or $y \geq 1\}$	$\{y \mid y \leq -1$ or $y \geq 1\}$
Amplitude	None ($y = \csc x$ increases and decreases without bound)	None ($y = \sec x$ increases and decreases without bound)
Period	2π	2π
Vertical Asymptotes	$x = n\pi$ (multiples of π)	$x = \frac{(2n+1)\pi}{2}$ (odd mulitples of $\frac{\pi}{2}$)
Symmetry	Origin (The cosecant function is an odd function.)	y-axis (The secant function is an even function.)

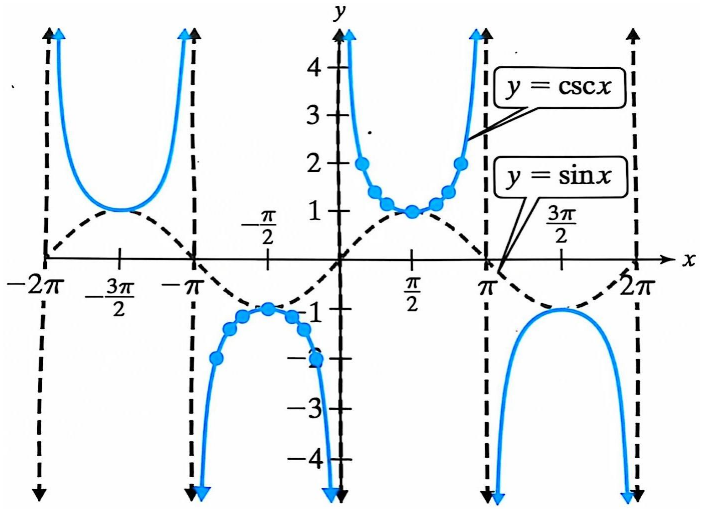
Example 1:

Graph $\quad y = 3\sec x$



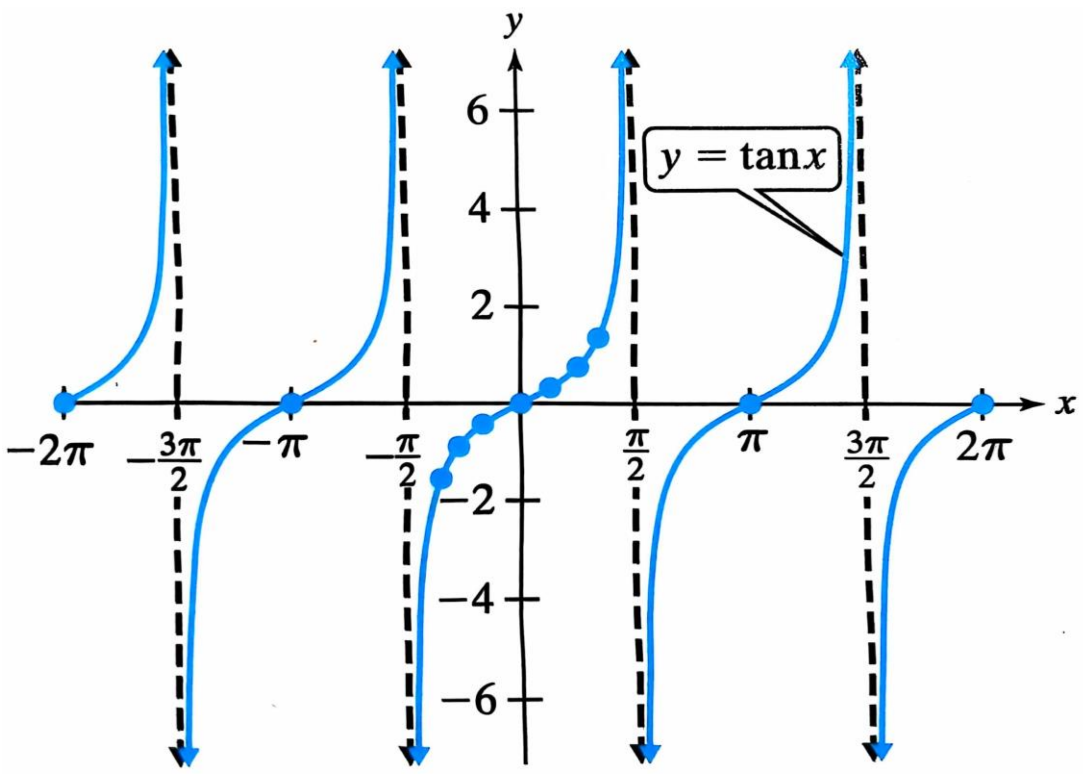
Example 2:

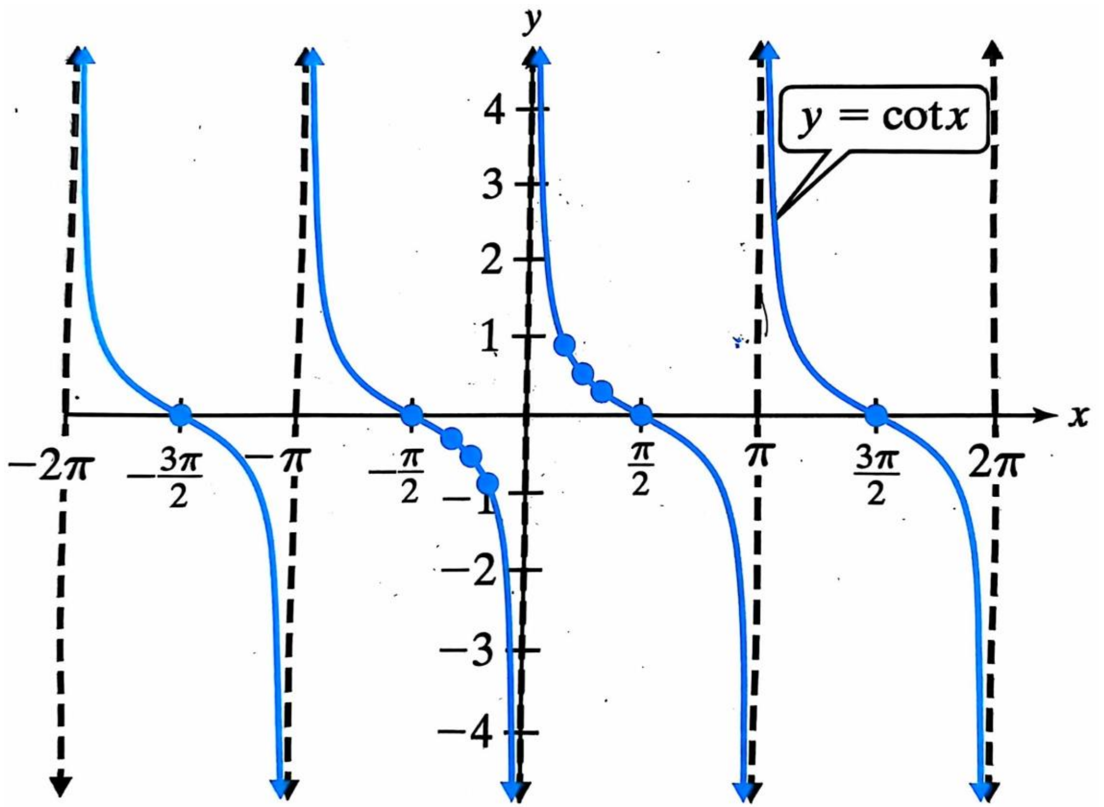
Graph $\quad y = \csc\left(x - \frac{\pi}{4}\right) + 3$



② Graph $y = \tan x$ and $y = \cot x$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\tan x$	0	1	undefined	-1	0	1	undefined	-1	0
$\cot x$	undefined	1	0	-1	undefined	1	0	-1	undefined



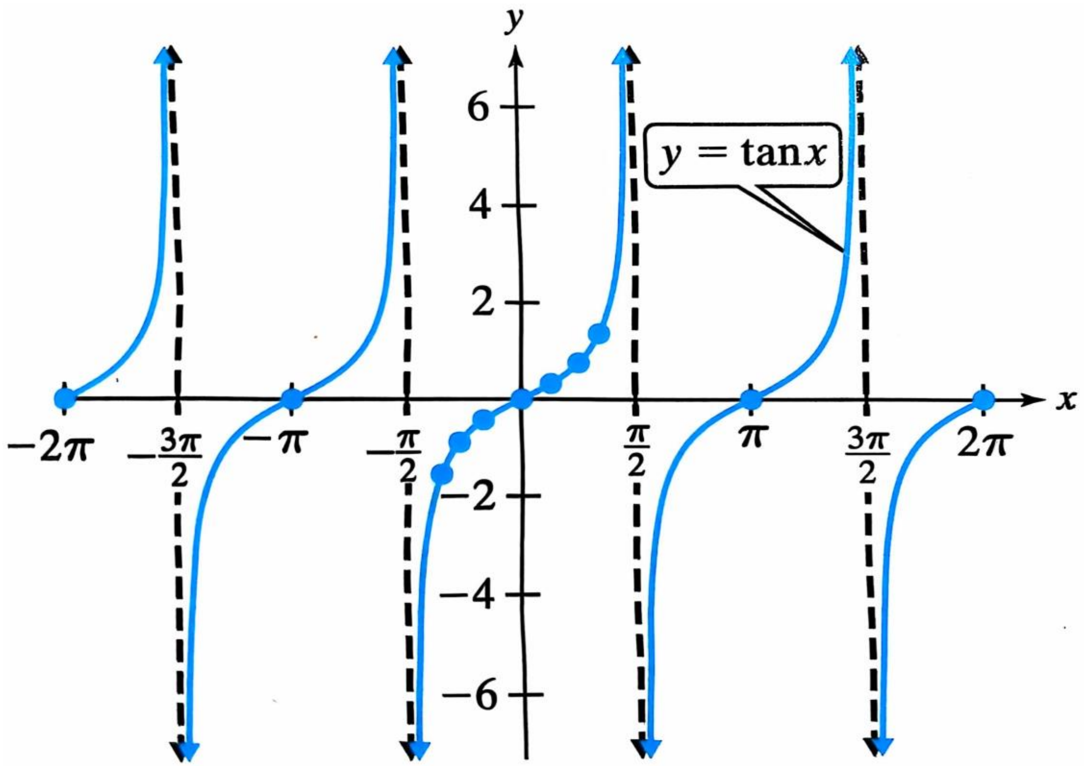


Graphs of the Tangent and Cotangent Functions

	$y = \tan x$	$y = \cot x$
Domain	$\{x \mid x \neq \frac{(2n+1)\pi}{2}$ for all integers $n\}$	$\{x \mid x \neq n\pi$ for all integers $n\}$
Range	All real numbers	All real numbers
Amplitude	None ($y = \tan x$ is unbounded.)	None ($y = \cot x$ is unbounded.)
Period	π	π
Vertical Asymptotes	$x = \frac{(2n+1)\pi}{2}$ (odd multiples of $\frac{\pi}{2}$)	$x = n\pi$ (multiples of π)
Symmetry	Origin (The tangent function is an odd function.)	Origin (The cotanget function is an odd function.)

Example 4:

Graph $y = 3tan2x$



Example 5:

Graph $y = \cot\left(x + \frac{\pi}{4}\right) + 2$