College Algebra and Trigonometry

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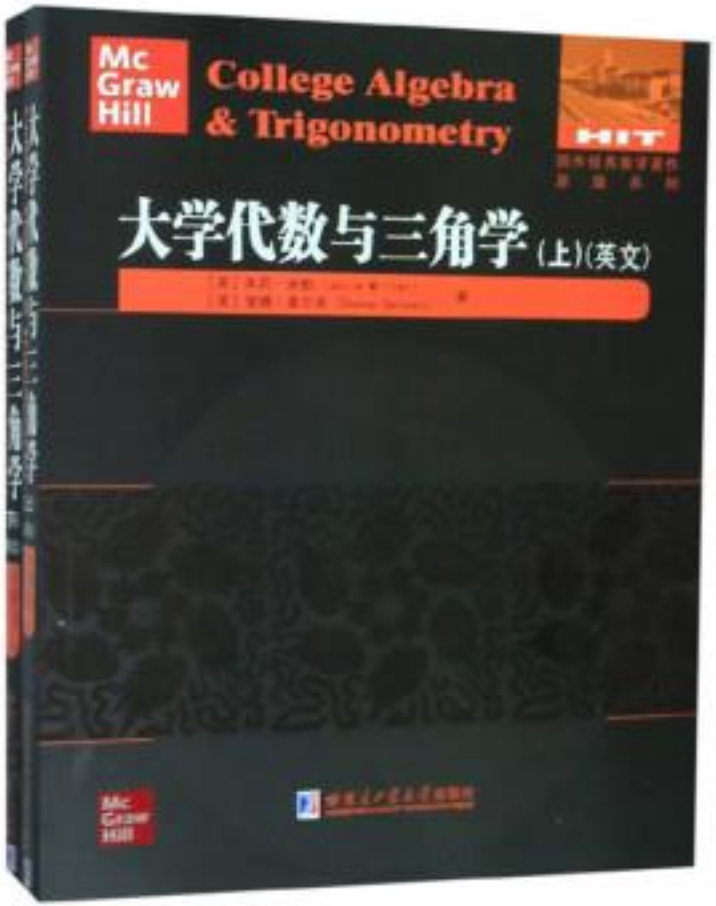
About this course

- *College Algebra and Trigonometry* is a course designed to expose you to polynomials, factoring, rational expressions, complex numbers, rational exponents, radicals, solutions of equations, linear and quadratic inequalities, functions and graphs, exponential and logarithmic functions, synthetic division, basic algebraic and transcendental functions.
- This course prepares you for Calculus and higher courses in Mathematics. It will help you develop your critical thinking, problem solving, and quantitative reasoning skills. This course provides a quantitative foundation for life-long learning.

About this course (II)

Textbook



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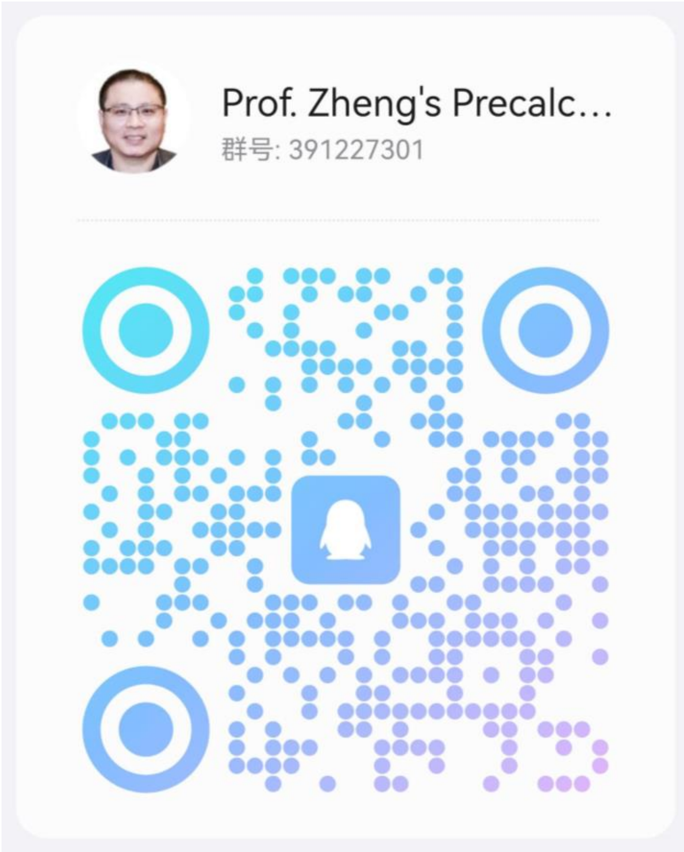

Some basic notations

a) $\forall$: This symbol means "any".

b) $\in$: This symbol means "is an element of".

c) $\mathbb{N}=\{1,\ 2,\ 3,\ \cdots\}$: All natural numbers.

b) $\mathbb{Z}=\{\cdots,-3,-2,-1,\ 0,\ 1,\ 2,\ 3,\ \cdots\}$: All integers.

c) $\mathbb{Q}=\left\{\left.\dfrac{p}{q}\right|\ p,q\in\mathbb{Z}\ \text{and}\ q\neq 0\right\}$: All rational numbers.

d) $\mathbb{R}$: Real numbers.

e) $\mathbb{H}$: irrational numbers: CANNOT be expressed as a ratio of integers.

Whole numbers:
W = {0, 1, 2, 3, …}

irrational numbers:
π = 3.141592653589…
e = 2.718281828459…
More…???

Recall

- $2^3 = 2 \cdot 2 \cdot 2 = 8$. In general, let $b \in \mathbb{R}$, and $n \in \mathbb{N}$. Then

$$b^n = \underbrace{b \cdot b \cdot b \cdot \ldots \cdot b}_{n}.$$

b^n is read as "b to the n-th power". b is the base and n is the exponent.

Properties of Exponents

Let a and $b \in \mathbb{R}$, and m and $n \in \mathbb{Z}$.

a) $b^m \cdot b^n = b^{m+n}$.

b) $\dfrac{b^m}{b^n} = b^{m-n}$.

c) $\left(b^m\right)^n = b^{mn}$.

d) $(ab)^m = a^m b^m$.

e) $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m},\ b \neq 0.$

- The equation $x^2 = a$ $(\geqslant 0)$ has two roots, called the square roots of a. A radical sign $\sqrt{\ }$ is used to denote the principal square root $(\geqslant 0)$ of a number a.

For example,

$$x^2 = 25 \Rightarrow x = -5, x = 5.$$

So, we have $\sqrt{25} = 5$.

① Evaluate *n*th-Roots

First, we extend the principal square roots to the principal *n*th-roots. From the recall, we know that for $a \geq 0$, $\sqrt{a} = b$ if $b^2 = a$ and $b \geq 0$.

DEFINITION The Principal *n*th-root

For a positive integers $n > 1$, the principal *n*th-root of real number a, denoted by $\sqrt[n]{a}$, is a real number b such that

$$\sqrt[n]{a} = b \text{ means that } b^n = a.$$

If n is even, then we require that $a \geq 0$ and $b \geq 0$.

For the expression $\sqrt[n]{a}$, the symbol $\sqrt[n]{}$ is called a radical sign.

a is called the **radicand**, and n is called the **index**.

Example 1 Simplify.

a) $\sqrt[5]{32}$ b) $\sqrt{\dfrac{49}{64}}$ c) $\sqrt[3]{-0.008}$ d) $\sqrt[4]{-1}$ e) $-\sqrt[4]{1}$

② Simplify Expressions of the Forms $a^{\frac{1}{n}}$ and $a^{\frac{m}{n}}$

Next, we want to define an expression of the form a^n, where n is a rational number. Furthermore, we want a definition for which the properties of integer exponents can be extended to rational exponents. For example, we want

$$\left(25^{\frac{1}{2}}\right)^2 = 25^{\frac{1}{2}\cdot 2} = 25.$$

So, $25^{\frac{1}{2}}$ must be a square root of 25, because when squared, it equals 25. Hence,

$$25^{\frac{1}{2}} = \sqrt{25}.$$

DEFINITION The Forms $a^{\frac{1}{n}}$ and $a^{\frac{m}{n}}$

Let m and n be integers such that $\frac{m}{n}$ is a rational number in lowest terms and $n > 1$.

Then

$$a^{\frac{1}{n}} = \sqrt[n]{a} \quad \text{and} \quad a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m.$$

If n is even, we require that $a \geq 0$.

Example 2 Write the expressions using radical notation and simplify if possible.

a) $25^{\frac{1}{2}}$ b) $\left(\dfrac{64}{27}\right)^{\frac{1}{3}}$ c) $(-81)^{\frac{1}{4}}$ d) $32^{\frac{3}{5}}$ e) $(-27)^{\frac{2}{3}}$

③ Simplify Expressions with Rational Exponents

The properties of integer exponents can be extended to expressions with rational exponents.

Properties of Rational Exponents

Let a and $b \in \mathbb{R}$, and m and $n \in \mathbb{Q}$.

a) $b^m \cdot b^n = b^{m+n}$.

b) $\dfrac{b^m}{b^n} = b^{m-n}$.

c) $\left(b^m\right)^n = b^{mn}$.

d) $(ab)^m = a^m b^m$.

e) $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$, $b \neq 0$.

Example 3 Simplify. Assume that all variables represent positive real numbers.

a) $\dfrac{x^{\frac{4}{7}}x^{\frac{2}{7}}}{x^{\frac{1}{7}}}$

b) $\left(\dfrac{5c^{\frac{3}{4}}}{d^{\frac{1}{2}}}\right)^{2}\left(\dfrac{d^{\frac{5}{3}}}{2c^{\frac{1}{2}}}\right)$

c) $81^{-3/4}$

④ Simplify Radicals

■ Properties of Radicals

Let a and $b \in \mathbb{R}$, and $n > 1$.

a) If n is even, $\sqrt[n]{a^n} = |a|$.

b) If n is odd, $\sqrt[n]{a^n} = a$.

c) $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$.

d) $\left(\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}\right) = \sqrt[n]{\dfrac{a}{b}}$.

e) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$.

Simplified Form of a Radical

1. All exponents in the radicand must be less than the index.
2. No fractions may appear in the radicand.
3. No denominator of a fraction may contain a radical.
4. The exponents in the radicand may not all share a common factor with the index.

Example 4-5 Simplify. Assume that all variables represent positive real numbers.

a) $\sqrt{\dfrac{x^3}{9}}$

b) $\dfrac{\sqrt[3]{3x^7y}}{\sqrt[3]{81xy^4}}\left(x, y \neq 0\right)$

c) $\sqrt[3]{c^5}$

d) $\sqrt[4]{32x^9y^6}$

⑤ Multiply Single-term Radical Expressions

Example 6:

Multiply. Assume that x represents a positive real number.

a) $\sqrt{6} \cdot \sqrt{10}$

b) $(2\sqrt[4]{x^3})(5\sqrt[4]{x^7})$

⑥ Add and Subtract Radicals

Example 7:

All variables represent positive real numbers.

a) $5\sqrt{7t}-2\sqrt{7t}+\sqrt{7t}$

b) $x\sqrt{98x^3y}+5\sqrt{18x^5y}$

c) $3\sqrt{5x}+2x\sqrt{5x}$

Like Radicals

$3\sqrt{2x}$ and $-5\sqrt{2x}$

$3\sqrt{2y}$ and $-2\sqrt{8y}$

Not Like Radicals

$3\sqrt{2x}$ and $-5\sqrt{2y}$

$3\sqrt{2y}$ and $-2\sqrt[3]{2y}$