# Lecture 4

# Newton’s Laws of Motion

## Date: 3/11/2025


**Course Instructor:**

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# What are Newton's laws?

## FIRST LAW

An object at **rest** remains at rest, and an object in **motion** remains in motion at constant speed and in a straight line unless acted on by an unbalanced force.

## SECOND LAW

The **acceleration** of an object depends on the mass of the object and the amount of force applied.

## THIRD LAW

Whenever one object **exerts a force** on another object, the second object exerts an equal and opposite on the first.

# Kinematics so far

Relations between:

- Displacement $\vec{r}$
- Velocity $\vec{v}$
- Acceleration $\vec{a}$

But what produces $\vec{r}$, $\vec{v}$, and $\vec{a}$?

Aristotle believed that forces cause motions.

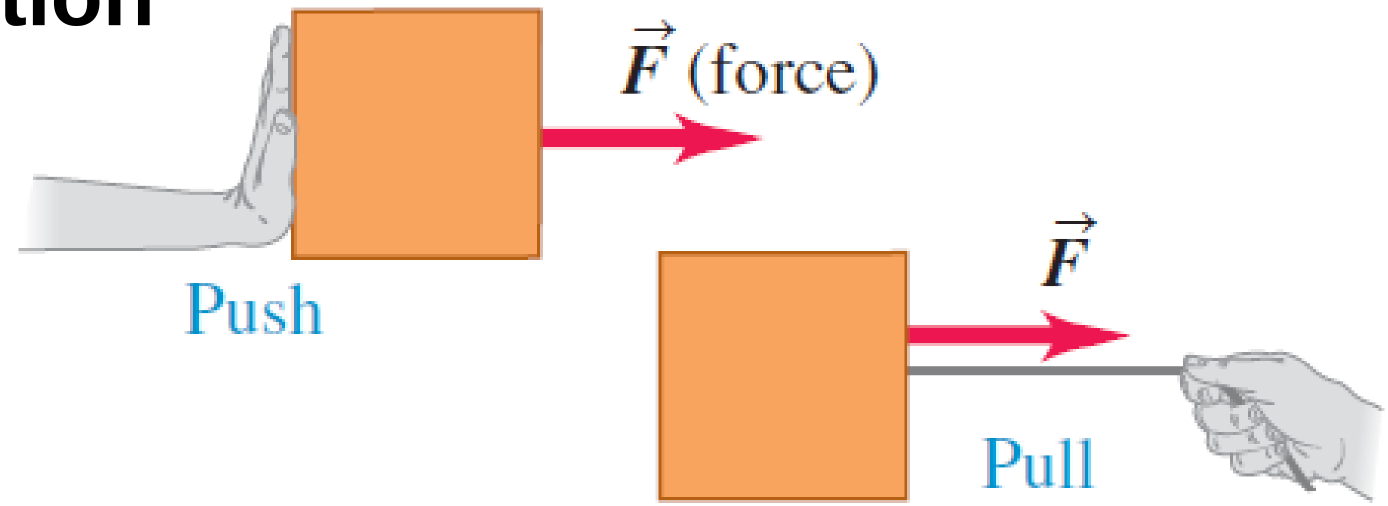
Is that accurate? He was wrong on free falls.

Next, we will address this question with Newton's laws

# Force: An Introduction

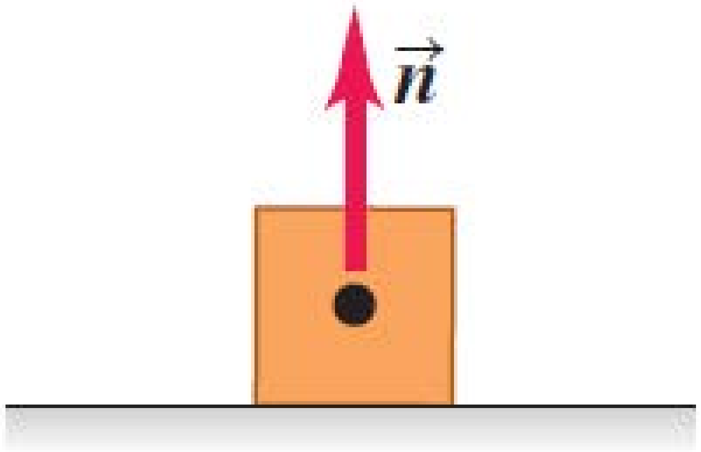
**Before learning physics, what do we mean by "force"?**

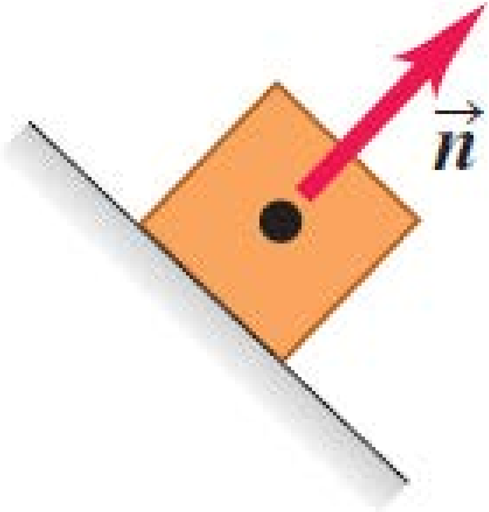
- **A push or a pull**
- **An interaction between two objects or between an object and its environment (think about gravity)**
- **With magnitude & direction – a vector!**



# Types of Forces: 1. Normal force $\vec{n}$

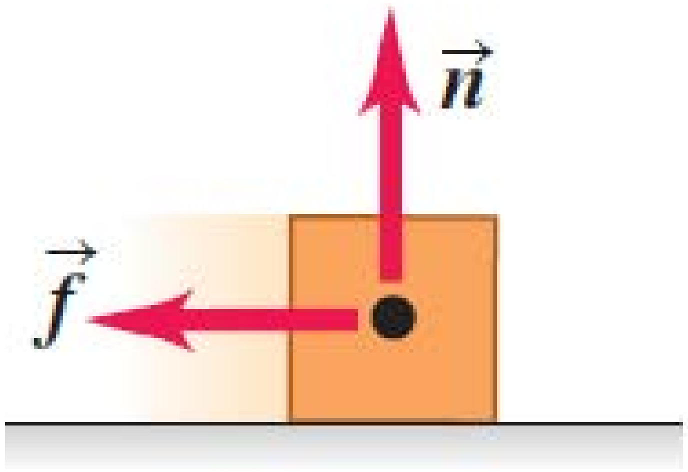
(a) **Normal force $\vec{n}$:** When an object rests or pushes on a surface, the surface exerts a push on it that is directed perpendicular to the surface.



# Types of Forces: 2. Friction Force $\vec{f}$

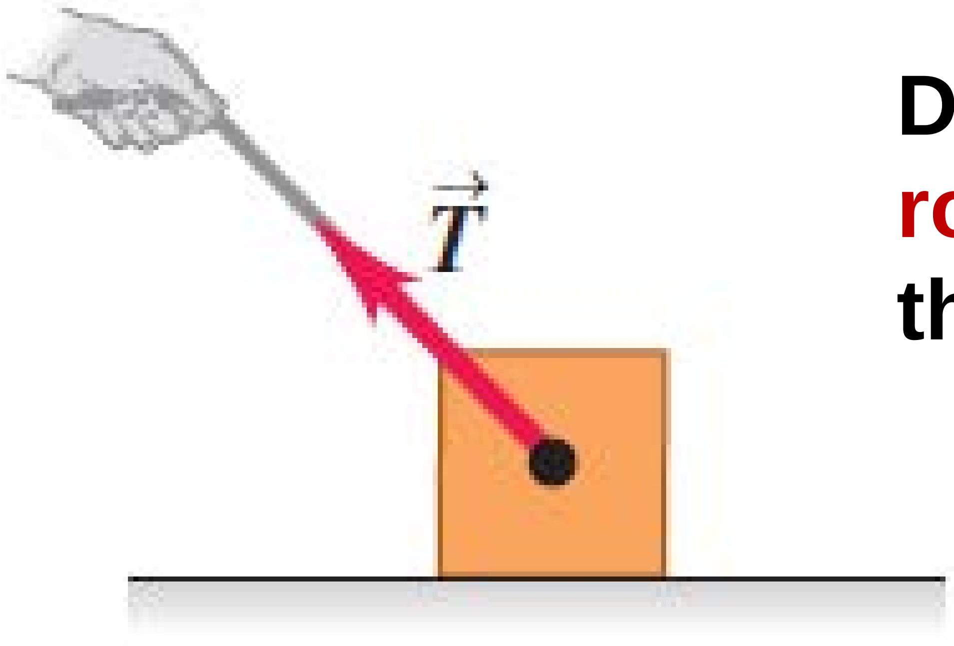
(b) **Friction force $\vec{f}$:** In addition to the normal force, a surface may exert a frictional force on an object, directed parallel to the surface.




**Direction counters the direction of the (tendency of) motion relative to the surface**

# Types of Forces: 3. Tension $\vec{T}$

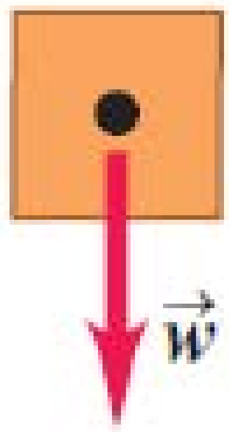
(c) **Tension force $\vec{T}$:** A pulling force exerted on an object by a rope, cord, etc.




**Direction goes along the rope, pointing out from the object of interest**

# Types of Forces : 4. Weight $\vec{w}$

(d) **Weight $\vec{w}$:** The pull of gravity on an object is a long-range force (a force that acts over a distance).



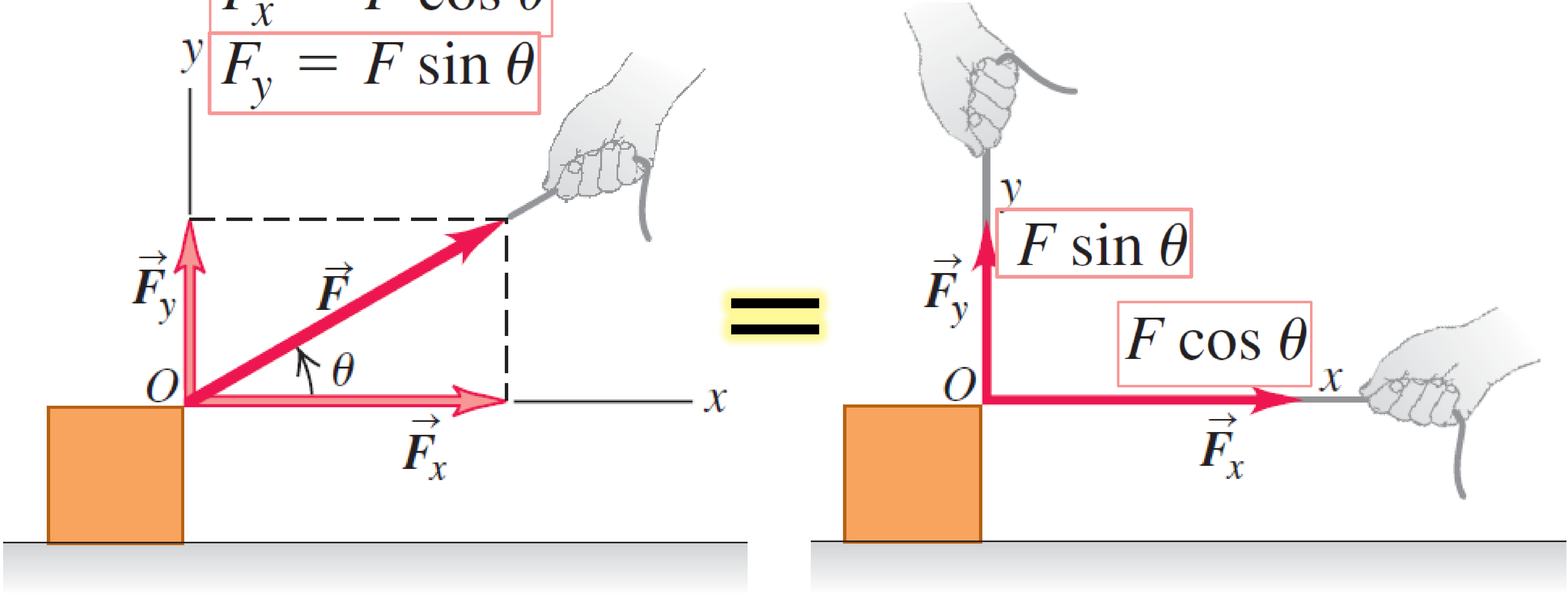

**Direction points to ground from the object of interest**

# Superposition of Forces

Component vectors: $\vec{F}_x$ and $\vec{F}_y$

$$\boxed{F_x = F\cos\theta}$$

$$\boxed{F_y = F\sin\theta}$$



# Superposition of Forces

**R**esultant force: sum of multiple forces

Symbol for sum

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

**Net Force**

**R**esultant force: also consists of *x* and *y* components

$$R_x = \sum F_x$$

$$R_y = \sum F_y$$

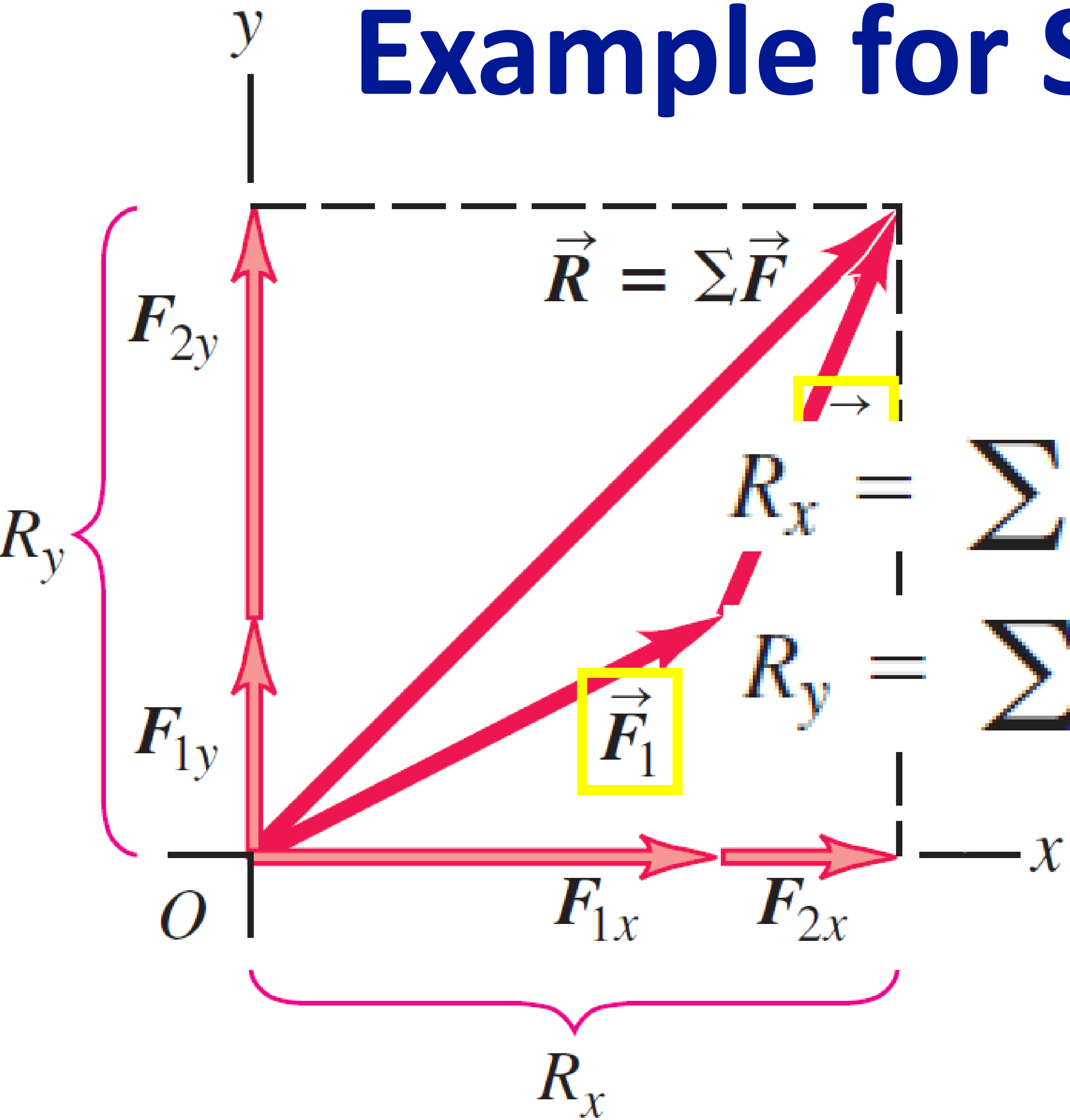
2D space: magnitude calculation

$$R = \sqrt{R_x^{\,2} + R_y^{\,2}}$$

3D space: $R_z = \sum F_z$

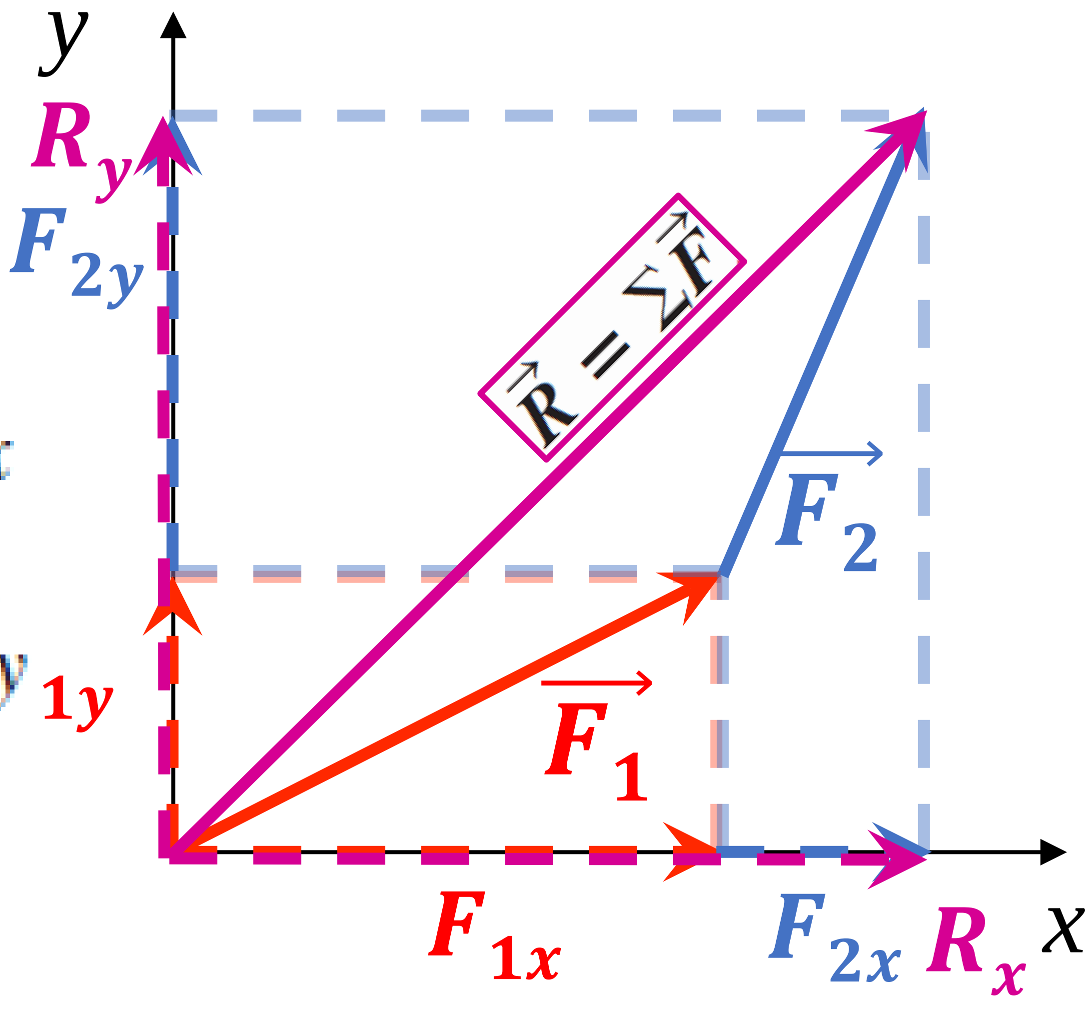
$$R = \sqrt{R_x^{\,2} + R_y^{\,2} + R_z^{\,2}}$$

# Example for Superposition




$$R_x = \sum F_x$$

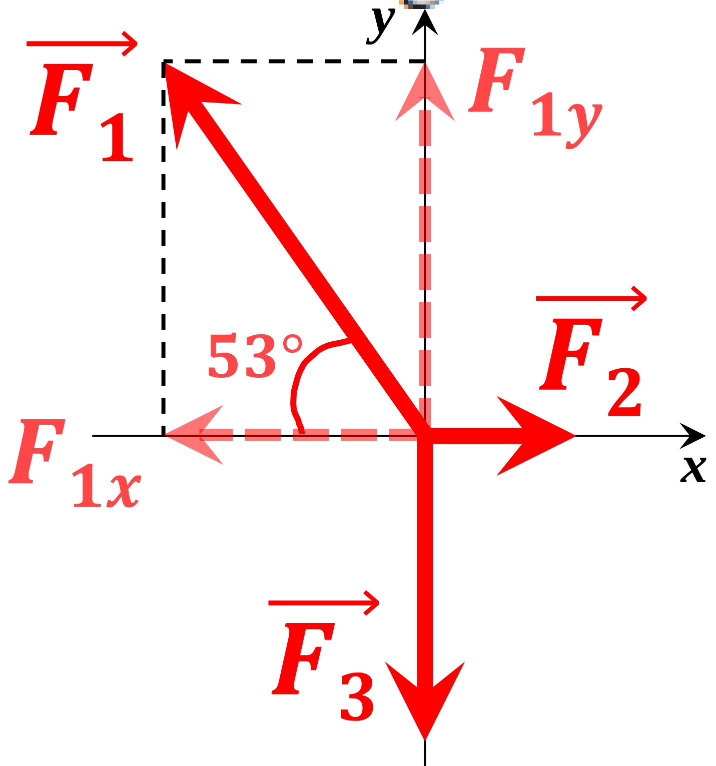
$$R_y = \sum F_y$$




$$\vec{R} = (R_x, R_y)$$

# Example

Three professional wrestlers are fighting over a champion's belt. Figure 4.8a shows the horizontal force each wrestler applies to the belt, as viewed from above. The forces have magnitudes $F_1 = 250$ N, $F_2 = 50$ N, and $F_3 = 120$ N. Find the *x*- and *y*-components of the net force on the belt, and find its magnitude and direction.

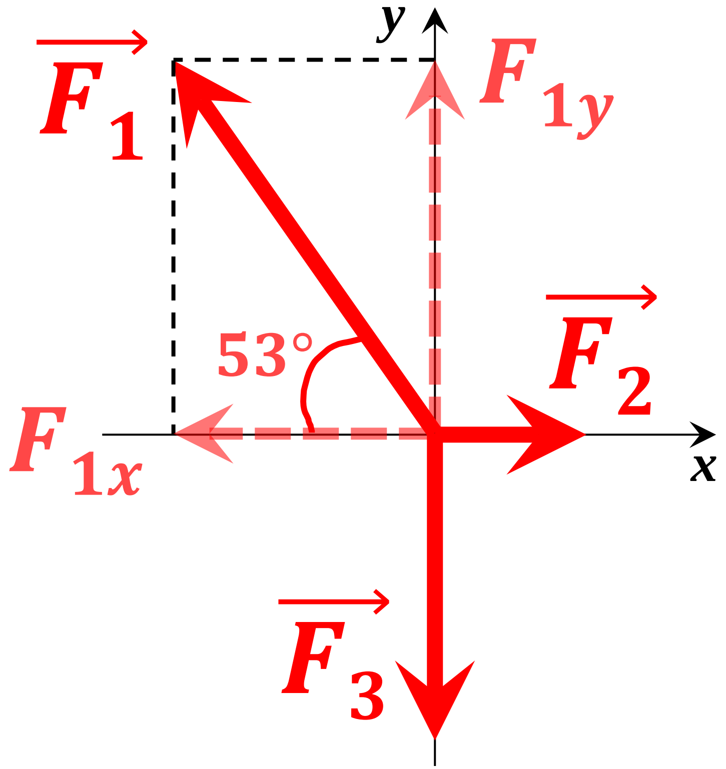



**How to get started with the problem?**

**Think about superposition!**

$$R_x = \sum F_x \qquad R_y = \sum F_y$$

# Example




**How to get started with the problem?**

**Think about superposition!**

$$R_x = \sum F_x \qquad R_y = \sum F_y$$

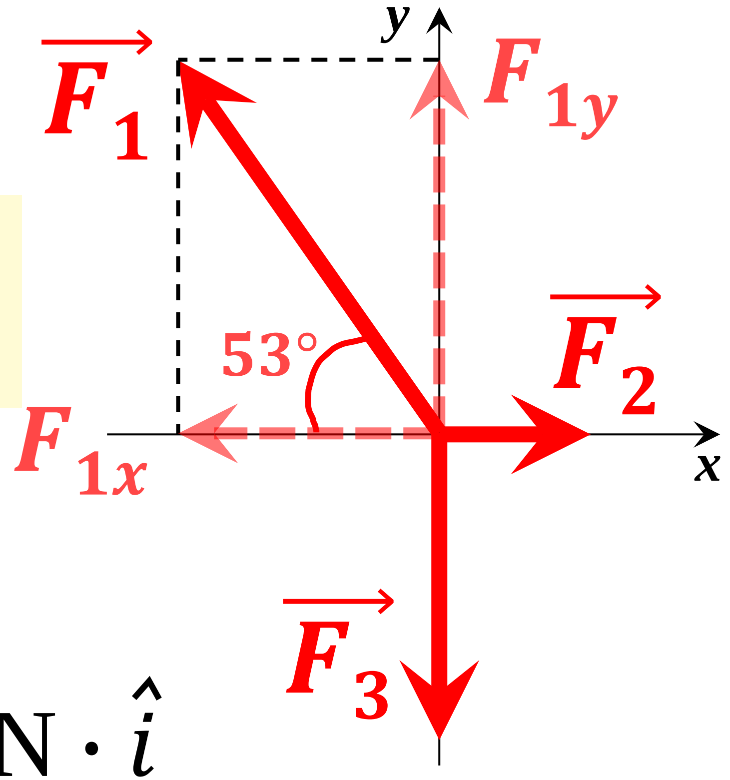
**Graphically we can identify:**

In $x$ direction, the total force: $\boldsymbol{R_x = |F_2| - |F_{1x}|}$

In $y$ direction: the total force: $\boldsymbol{R_y = |F_{1y}| - |F_3|}$

**Making sense?**

# Derivation




$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum \vec{F}$$

$$\overrightarrow{F_1} = F_{1x} \cdot \hat{i} + F_{1y} \cdot \hat{j}$$

$$\overrightarrow{F_2} = F_{2x} \cdot \hat{i} + F_{2y} \cdot \hat{j} \quad \Rightarrow \quad \overrightarrow{F_2} = F_2 \cdot \hat{i} = 50\ \mathrm{N} \cdot \hat{i}$$

$$\overrightarrow{F_3} = F_{3x} \cdot \hat{i} + F_{3y} \cdot \hat{j} \quad \Rightarrow \quad \overrightarrow{F_3} = F_3 \cdot \hat{j} = -120\ \mathrm{N} \cdot \hat{j}$$

**What about $F_1$?**

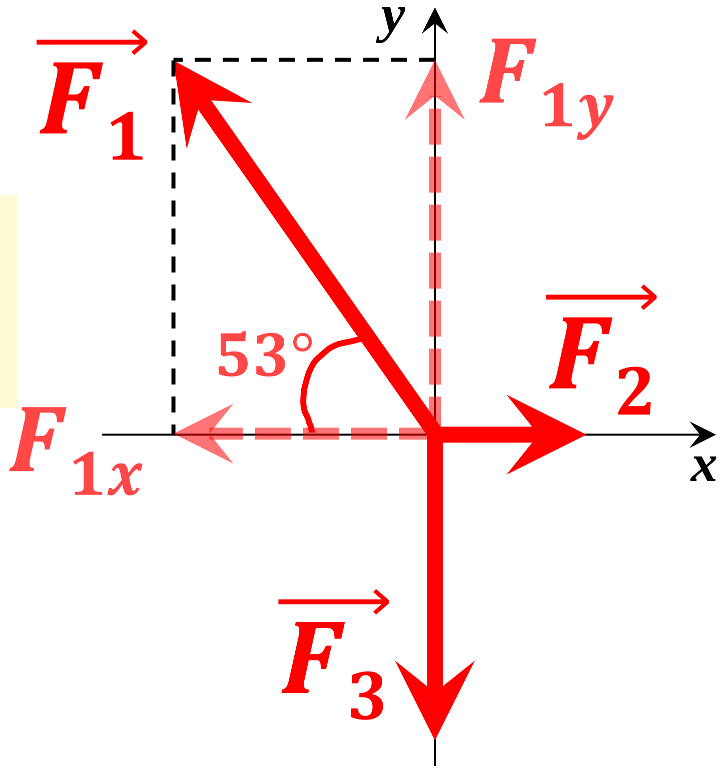
$$F_{1x} = F_1 \cos(180° - 53°) = -F_1 \cos(53°) = -150\ N$$

$$F_{1y} = F_1 \sin(180° - 53°) = F_1 \sin(53°) = 200\ N$$

# Derivation

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum \vec{F}$$

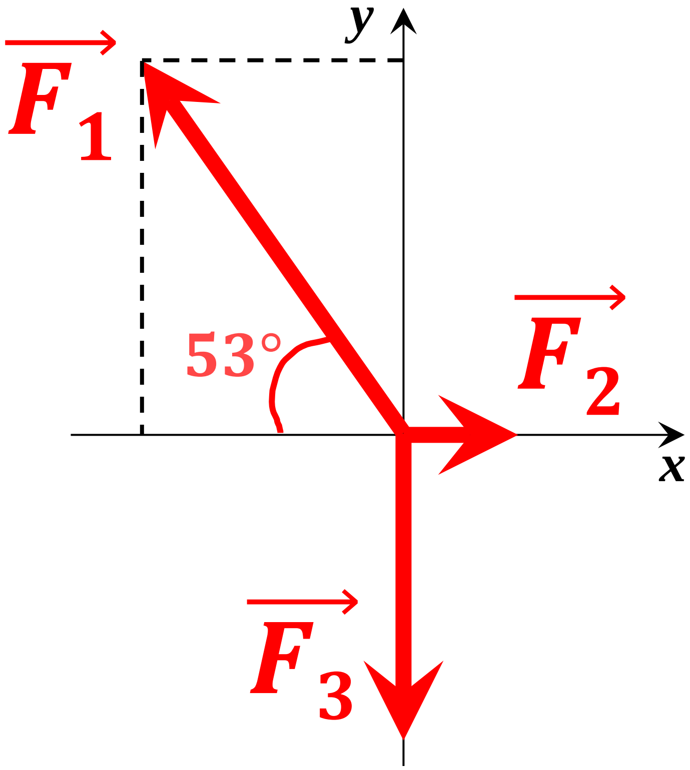
$$R_x = \sum F_x \qquad R_y = \sum F_y$$




$$R_x = F_{1x} + F_{2x} + F_{3x} = -150N + 50N + 0 = -100\ N$$

$$R_y = F_{1y} + F_{2y} + F_{3y} = 200N + 0 - 120N = 80N$$

# Remarks on the result

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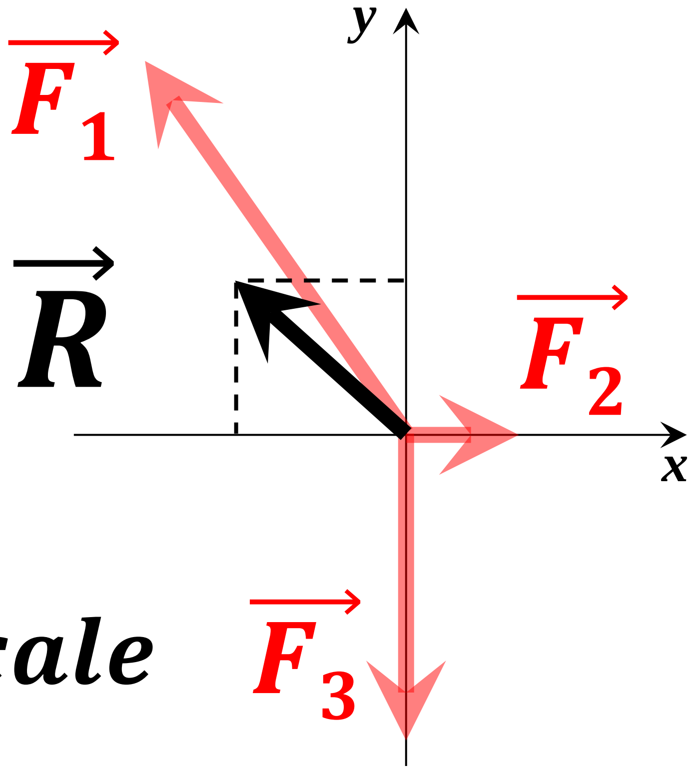



***Not to scale***

$$R_x = F_{1x} + F_{2x} + F_{3x} = -150N + 50N + 0 = -100\ N$$

$$R_y = F_{1y} + F_{2y} + F_{3y} = 200N + 0 - 120N = 80N$$

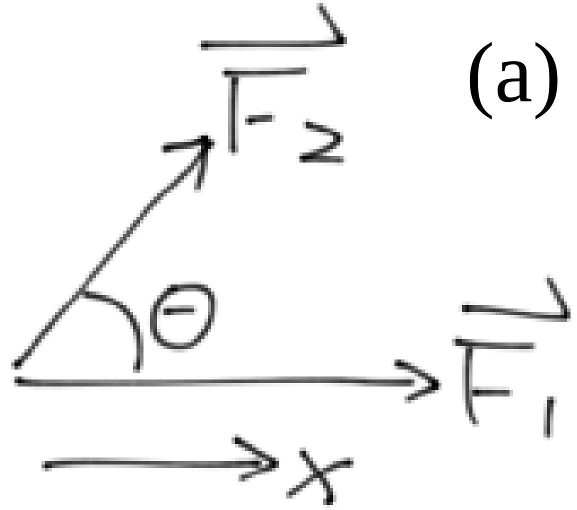
**Exercise**: Two forces have the same magnitude $F$. What is the angle between the two vectors if their sum has a magnitude of (a) $2F$? (b) $\sqrt{2}F$? (c) 0? Sketch the three vectors in each case。

**Solution:** So here is a simplest special case for vector addition,

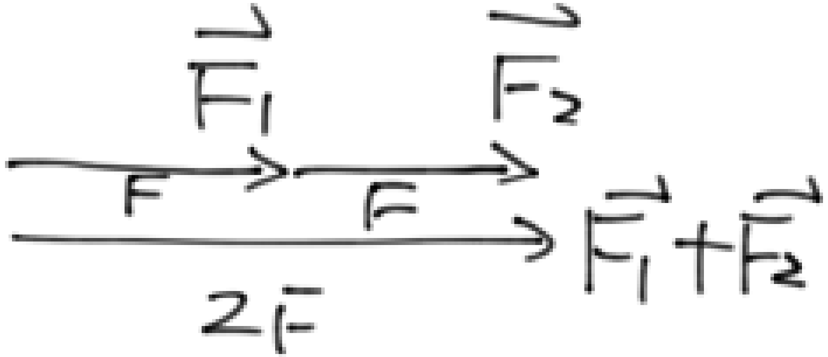
$$\left|\overrightarrow{F_1}\right| = \left|\overrightarrow{F_2}\right| = F$$

The first thing to do for vector addition is to **define a coordinate system**

$\overrightarrow{F_1} = F \cdot \hat{\imath}$ in $x$ direction, so $\overrightarrow{F_2}$ is: $\vec{F_2} = F \cdot \cos\theta \cdot \hat{i} + F \cdot \sin\theta \cdot \hat{j}$




(a) The answer is simple and can follow our intuition




$\theta = 0$

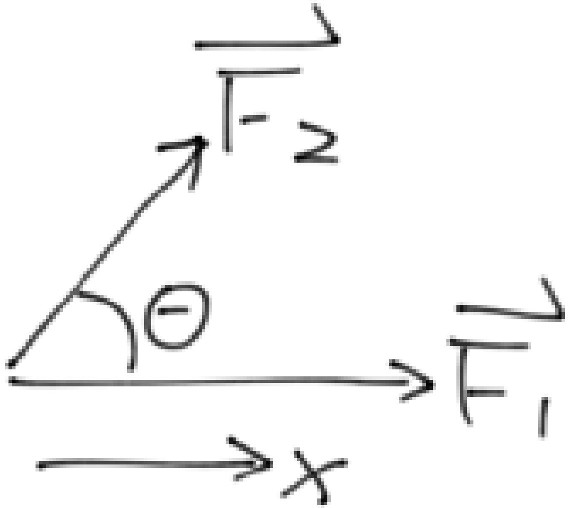
Exercise: Two forces have the same magnitude $F$. What is the angle between the two vectors if their sum has a magnitude of (a) $2F$? (b) $\sqrt{2}F$? (c) 0? Sketch the three vectors in each case。

(b) To get a more general expression:

$$\vec{R} = \vec{F_1} + \vec{F_2} = F(1+\cos\theta)\hat{i} + F\cdot\sin\theta\cdot\hat{j}$$

$$|\vec{R}|^2 = F^2(1+\cos\theta)^2 + F^2\cdot\sin^2\theta$$

$$= (2+2\cos\theta)\cdot F^2$$




for b: $R = \sqrt{2}F$ $\quad R^2 = 2F^2$

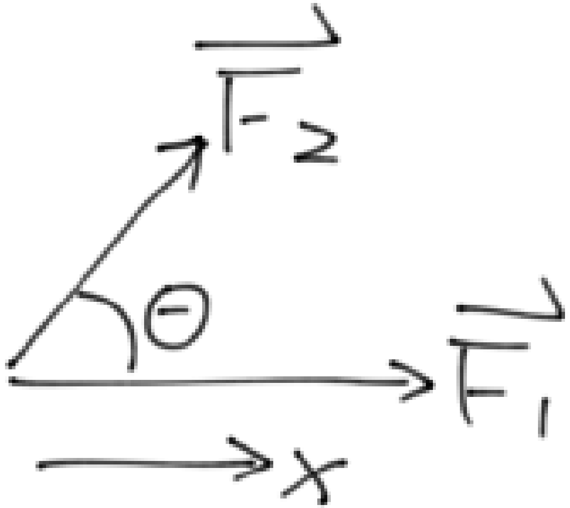
$$\cos\theta = 0 \Rightarrow \theta = \pm 90^\circ$$

Exercise: Two forces have the same magnitude $F$. What is the angle between the two vectors if their sum has a magnitude of (a) $2F$? (b) $\sqrt{2}F$? (c) 0? Sketch the three vectors in each case。

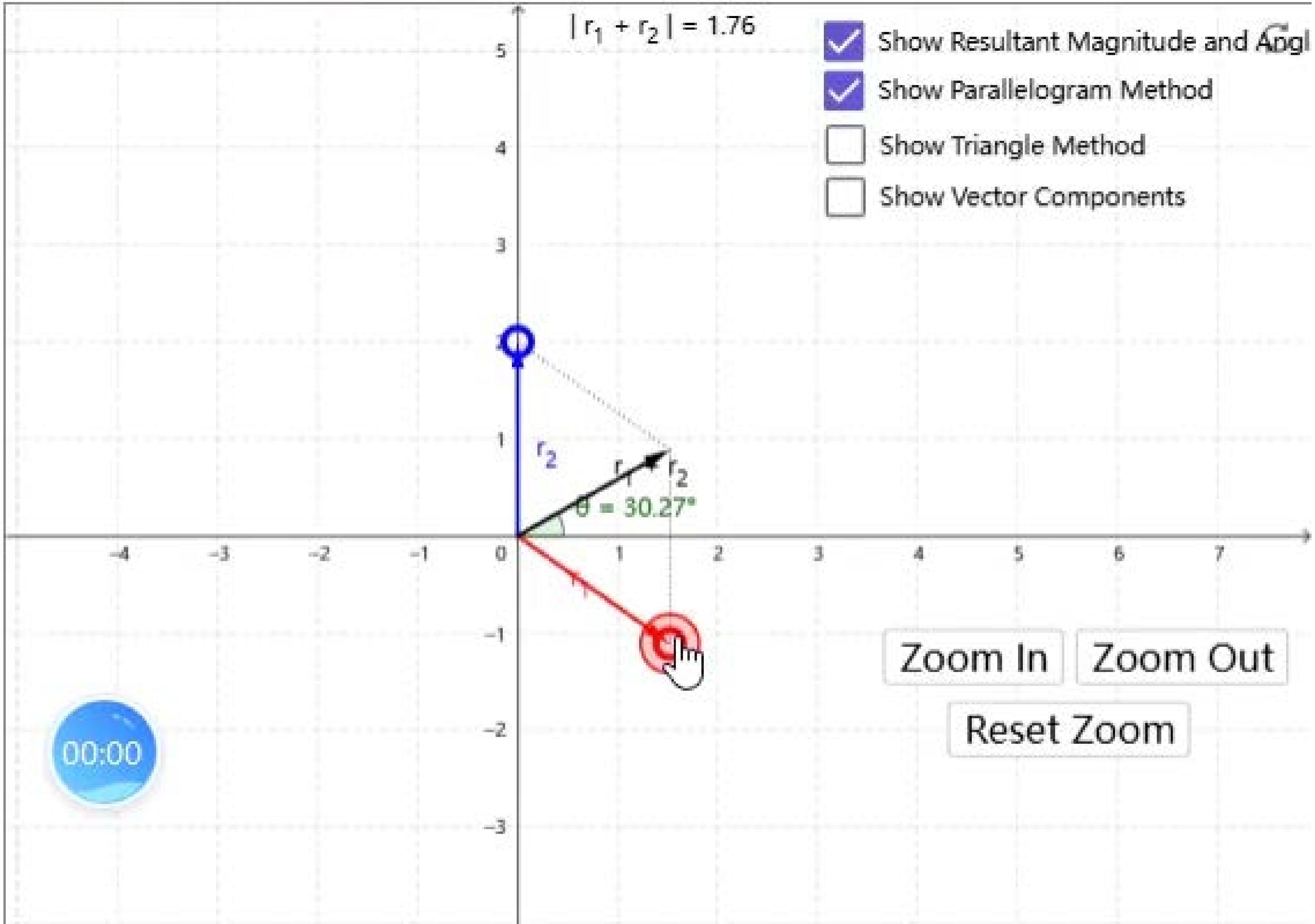
(b) To get a more general expression:

$$\vec{R} = \vec{F_1} + \vec{F_2} = F(1+\cos\theta)\hat{i} + F\cdot\sin\theta\cdot\hat{j}$$

for c: $\left.\begin{array}{l}\sin\theta = 0\\ \cos\theta + 1 = 0\end{array}\right] \rightarrow \theta = 180^\circ$

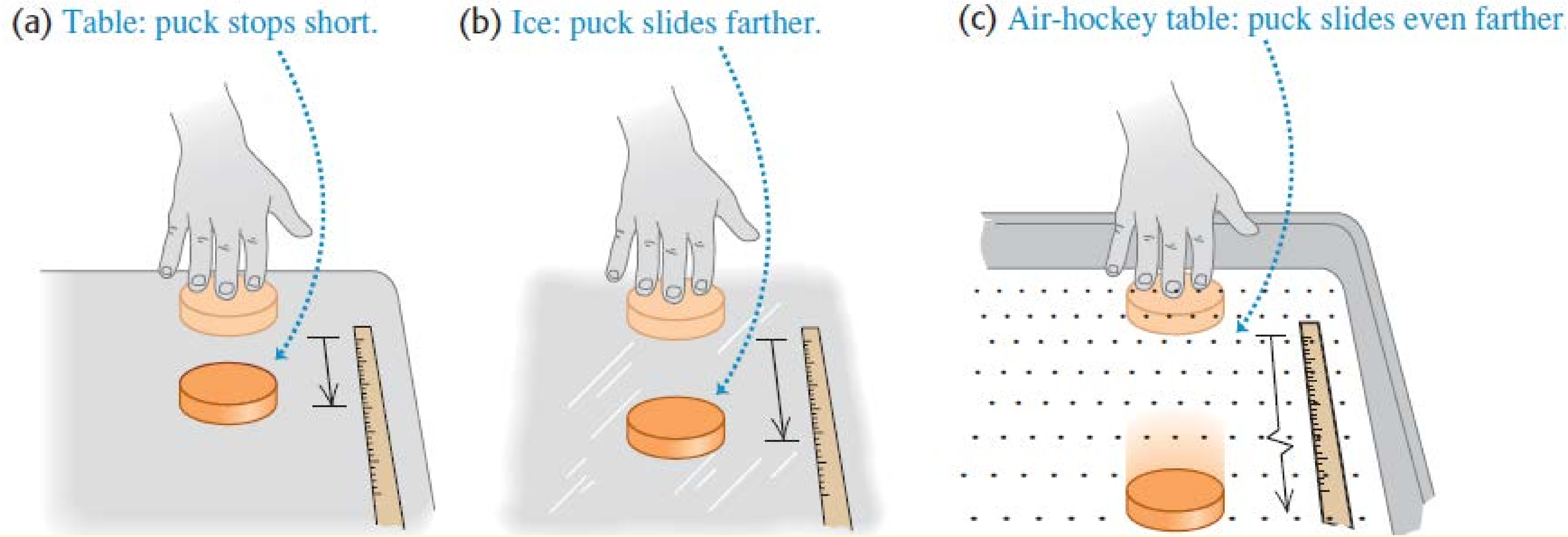


# Vector Addition



# Newton's First Law

## How do the forces that act on a body affect its motion?



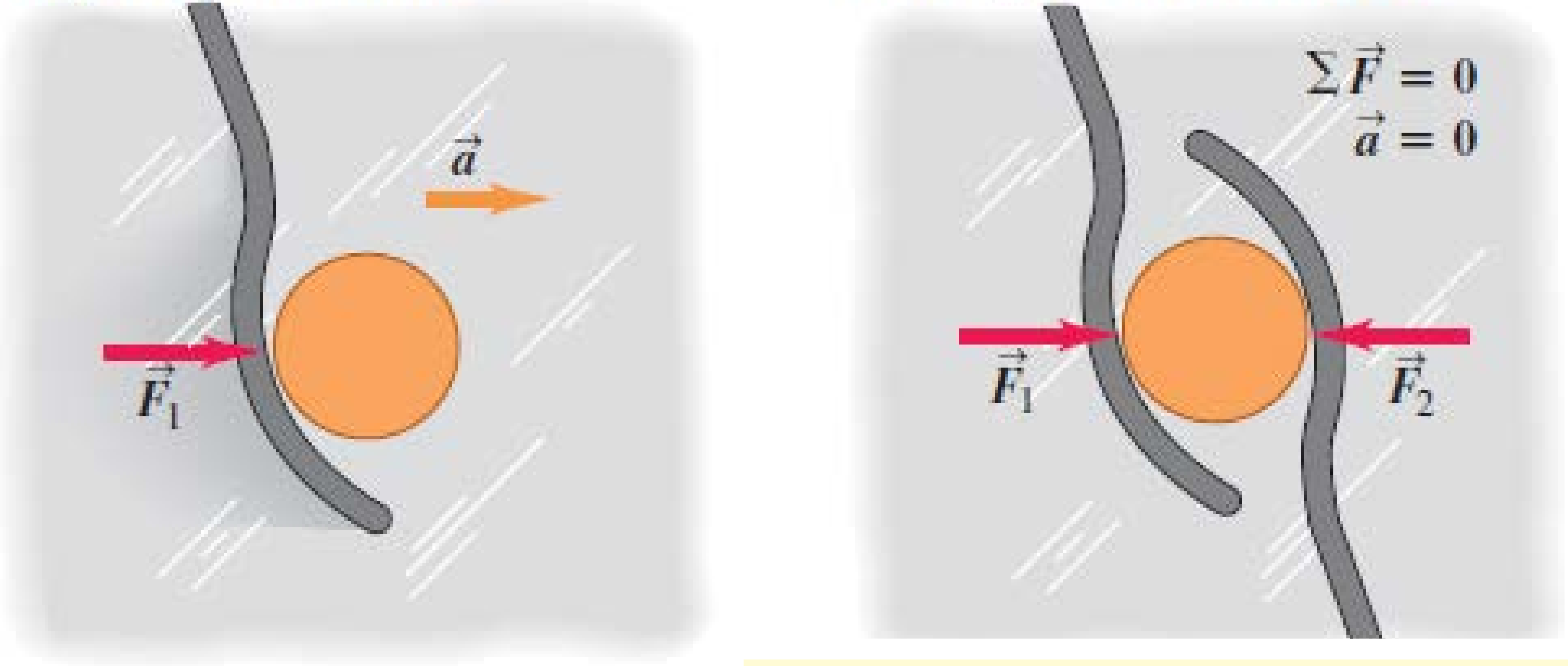

**Newton's first law of motion:** A body acted on by no net force moves with constant velocity (which may be zero) and zero acceleration.

# Newton's First Law

## How do the forces that act on a body affect its motion?

(a) A puck on a frictionless surface accelerates when acted on by a single horizontal force.

(b) An object acted on by forces whose vector sum is zero behaves as though no forces act on it.




$$\sum \vec{F} = 0 \qquad \text{(body in equilibrium)}$$

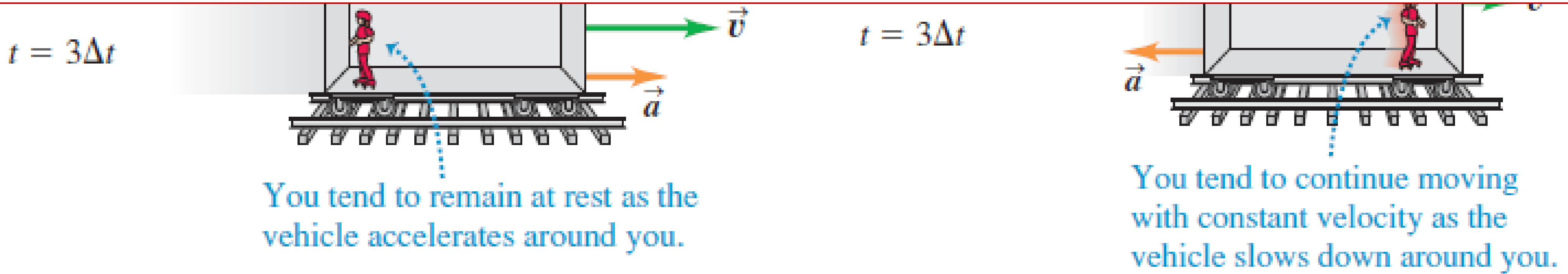
# Wait! What about relative acceleration?

We previously introduced the concept of
***frame of reference in kinematics***

Suppose you are in a bus that is traveling on a straight road and speeding up. If you could stand in the aisle on roller skates, you would **start moving *backward*** relative to the bus as the bus gains speed. Here is no net force acting on you, yet your velocity changes relative to the bus! Is newton's law valid?

# Inertial Frames of Reference


What is wrong? The bus is undergoing acceleration with non-zero net force!

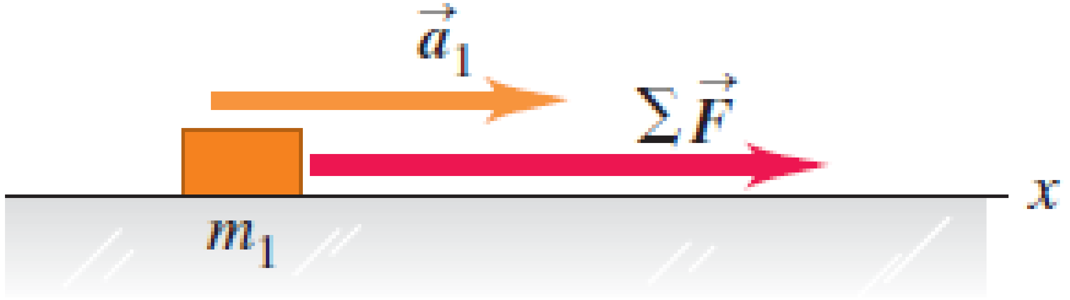


# What is the acceleration when $\sum \vec{F} \neq 0$

# Newton's Second Law!

$$a = \frac{|\sum \vec{F}|}{m} \longleftrightarrow |\sum \vec{F}| = ma$$

(a) A known force $\sum \vec{F}$ causes an object with mass $m_1$ to have an acceleration $\vec{a}_1$.

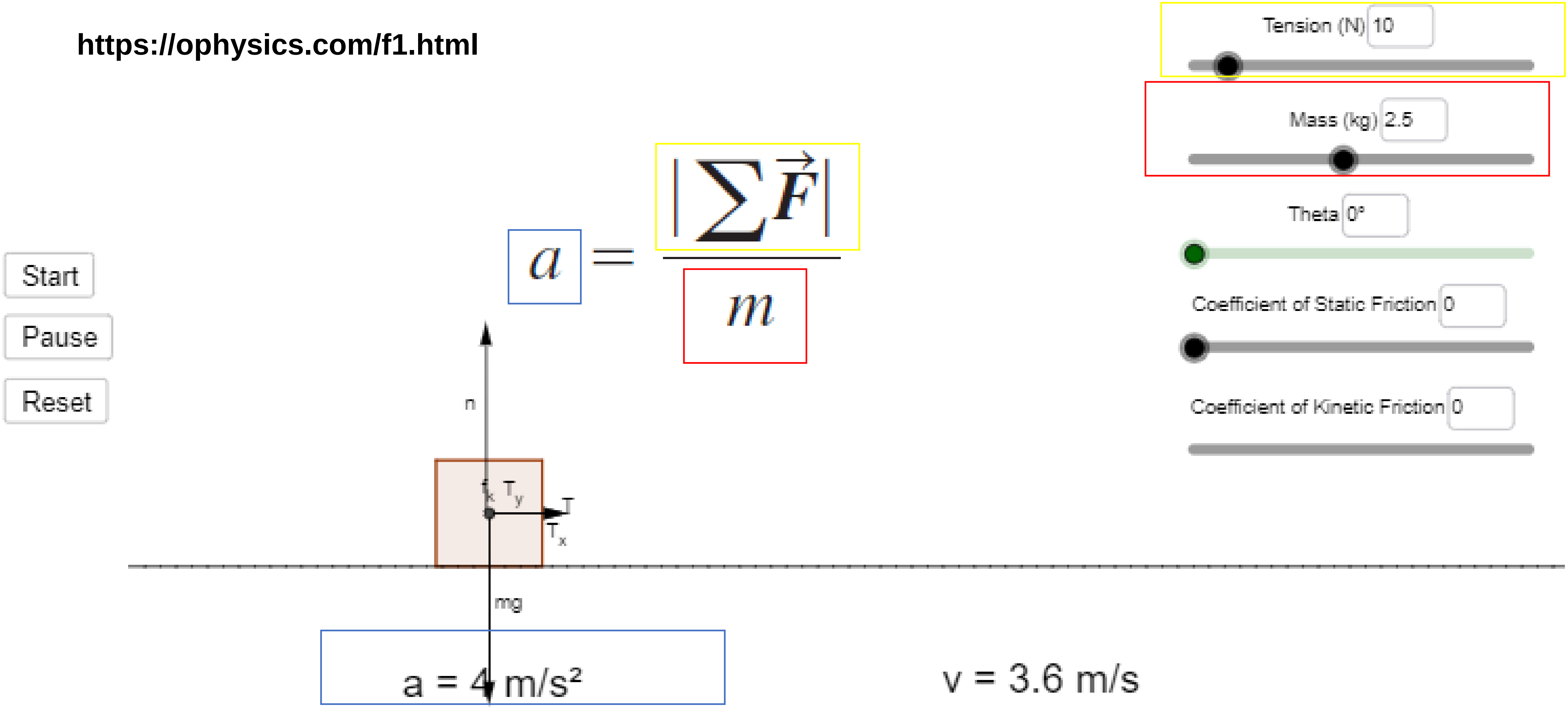


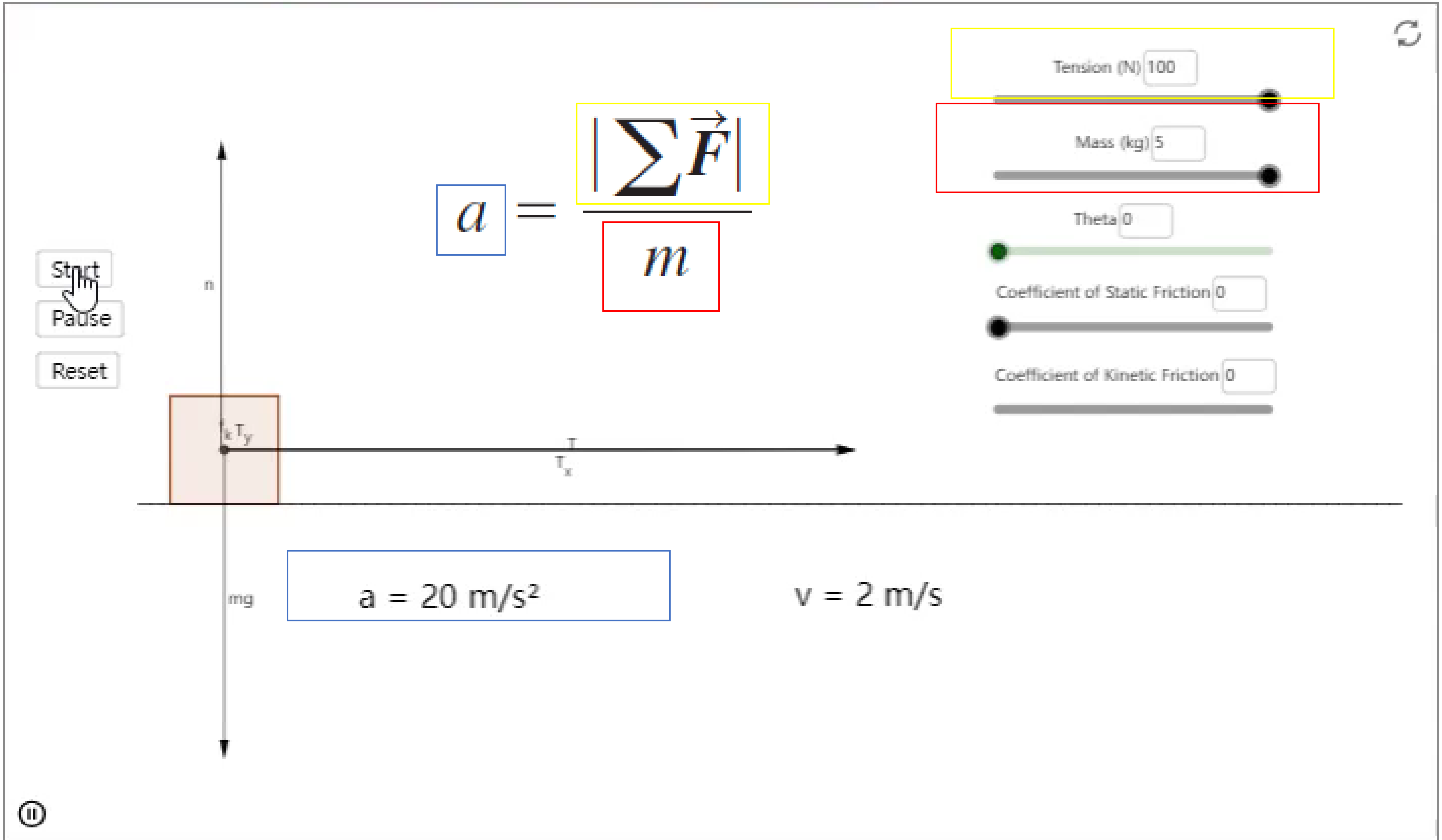

(b) Applying the same force $\sum \vec{F}$ to a second object and noting the acceleration allow us to measure the mass.

# Illustrations with O-Physics Again

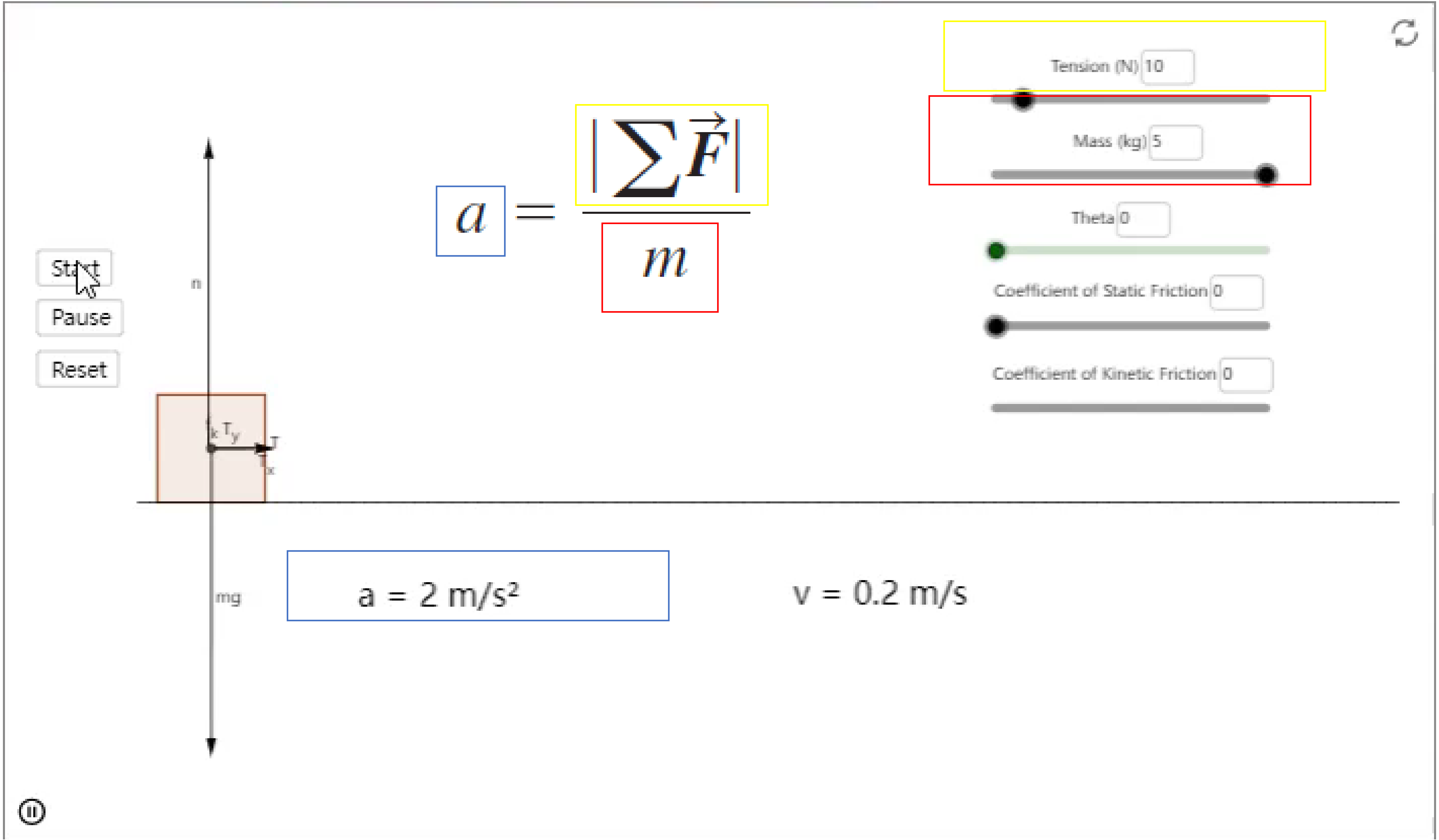
**https://ophysics.com/f1.html**

$$a = \frac{\left|\sum \vec{F}\right|}{m}$$





$a = \dfrac{\left|\sum \vec{F}\right|}{m}$
Tension (N) 100
Mass (kg) 5
Theta 0
Coefficient of Static Friction 0
Coefficient of Kinetic Friction 0
Start
Pause
Reset
n
$T_y$
T
$T_x$
mg
$a = 20\ m/s^2$
$v = 2\ m/s$



Tension (N) 10
Mass (kg) 5
Theta 0
Coefficient of Static Friction 0
Coefficient of Kinetic Friction 0
$a = \frac{|\sum \vec{F}|}{m}$
Start
Pause
Reset
n
$T_y$
T
$T_x$
mg
a = 2 m/s²
v = 0.2 m/s

$$a = \frac{\left|\sum \vec{F}\right|}{m}$$

Start

Pause

Reset

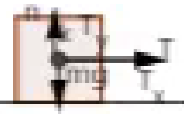
Tension (N) 10

Mass (kg) 0.5

Theta 0

Coefficient of Static Friction 0

Coefficient of Kinetic Friction 0



a = 20 m/s²

v = 0.6 m/s

# Newton's Second Law in 2D/3D

$$\vec{a} = \frac{\sum \vec{F}}{m}$$

$$\sum \vec{F} = m\vec{a} \qquad \text{(Newton's second law of motion)}$$

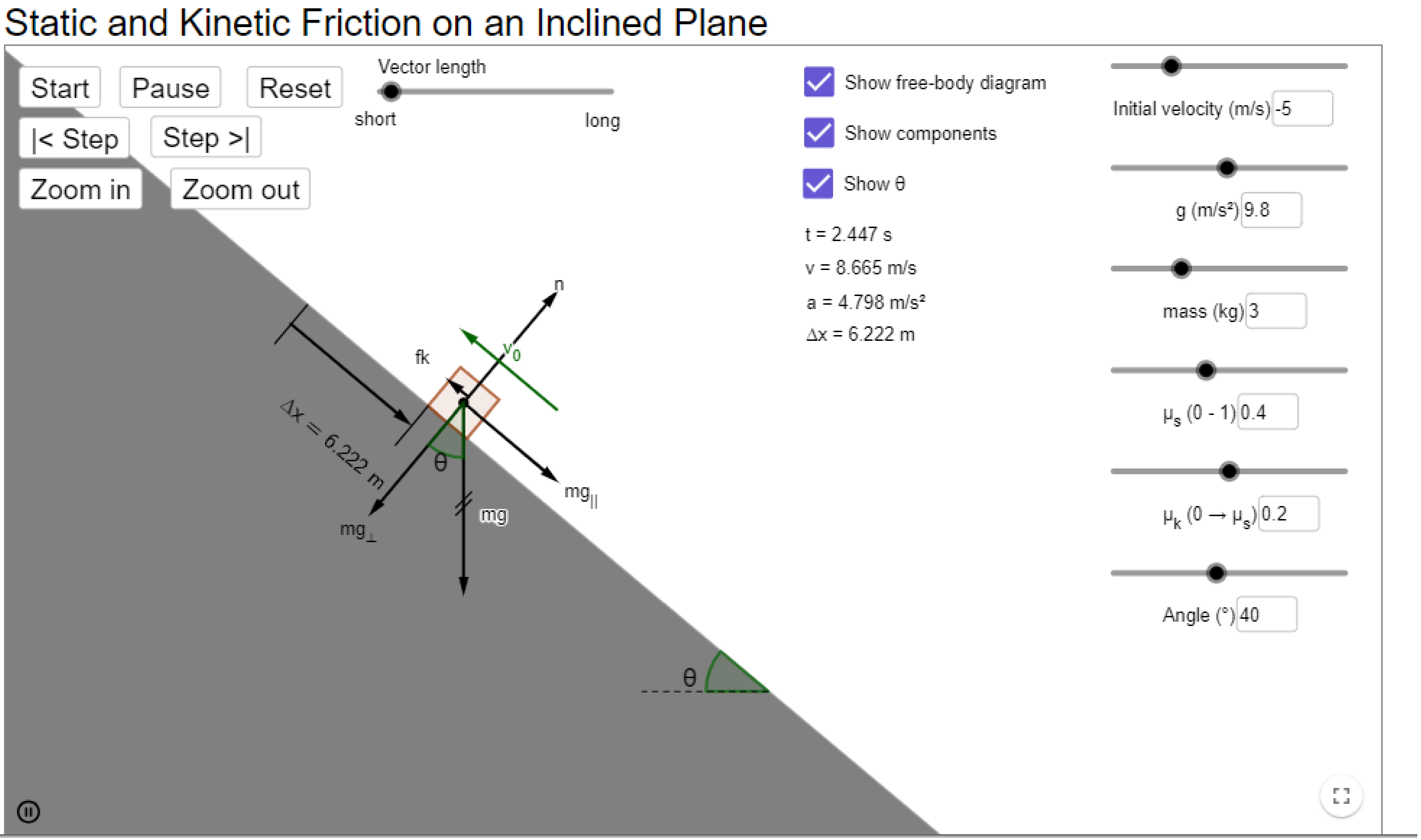
x, y, and z components can be calculated independently!

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

# Illustrations with O-Physics Again

**https://ophysics.com/f2.html**

**Won't go in details before we study more friction**

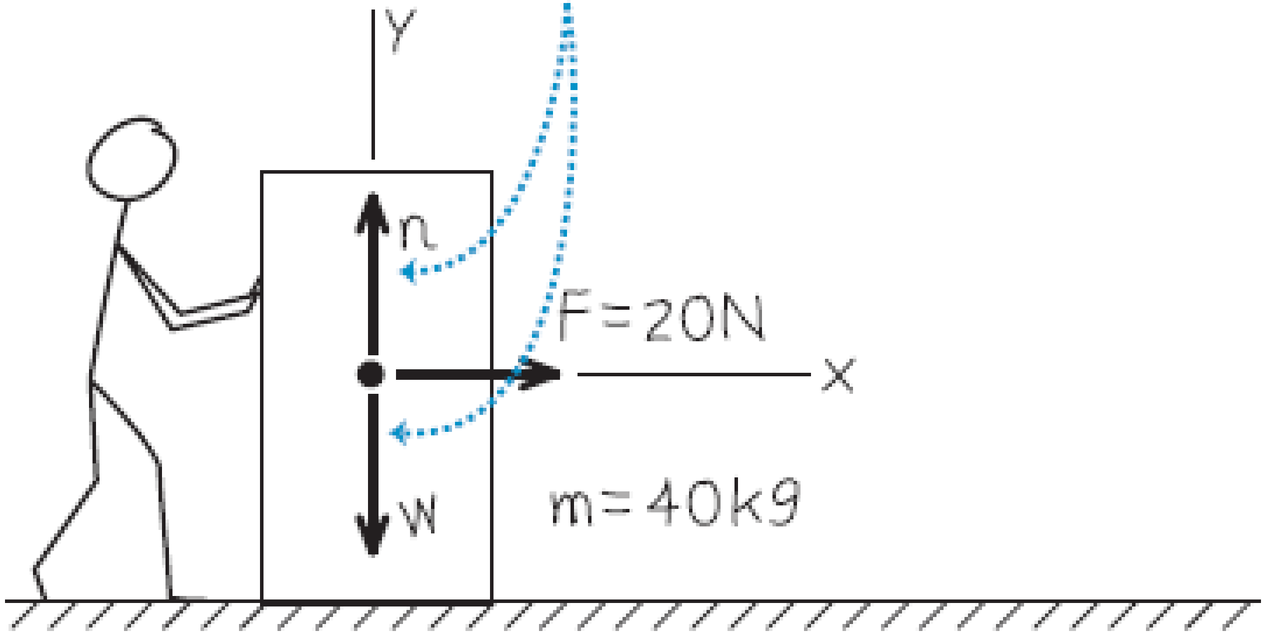


# Example 4.4

A worker applies a constant horizontal force with magnitude 20 N to a box with mass 40 kg resting on a level floor with negligible friction. What is the acceleration of the box?

Hint: set up a **coordinate system** and sketch out the problem. It is usually convenient to take one axis **either along or opposite** the direction of the body's acceleration.

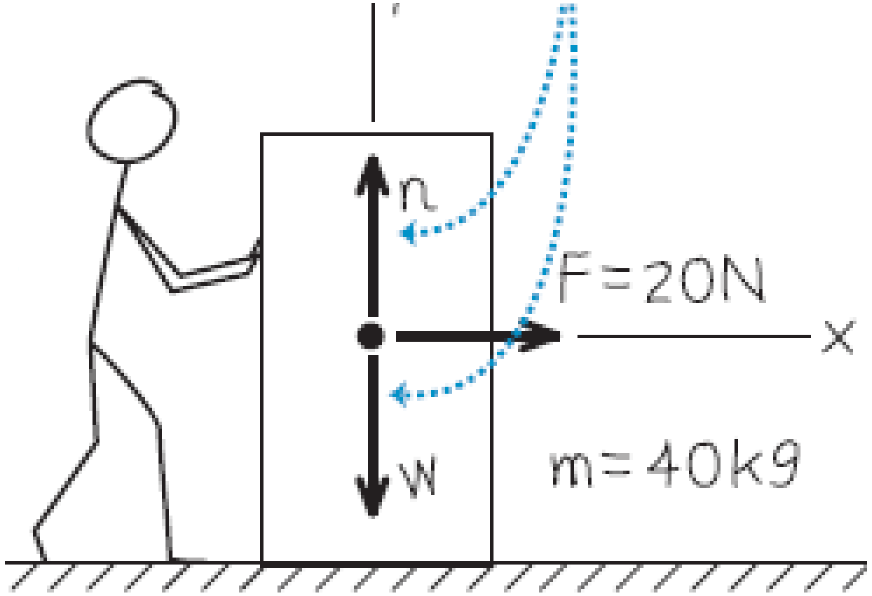
The box has no vertical acceleration, so the vertical components of the net force sum to zero. Nevertheless, for completeness, we show the vertical forces acting on the box.



# Example 4.4

A worker applies a constant horizontal force with magnitude 20 N to a box with mass 40 kg resting on a level floor with negligible friction. What is the acceleration of the box?

$$a_x = \frac{\sum F_x}{m}$$

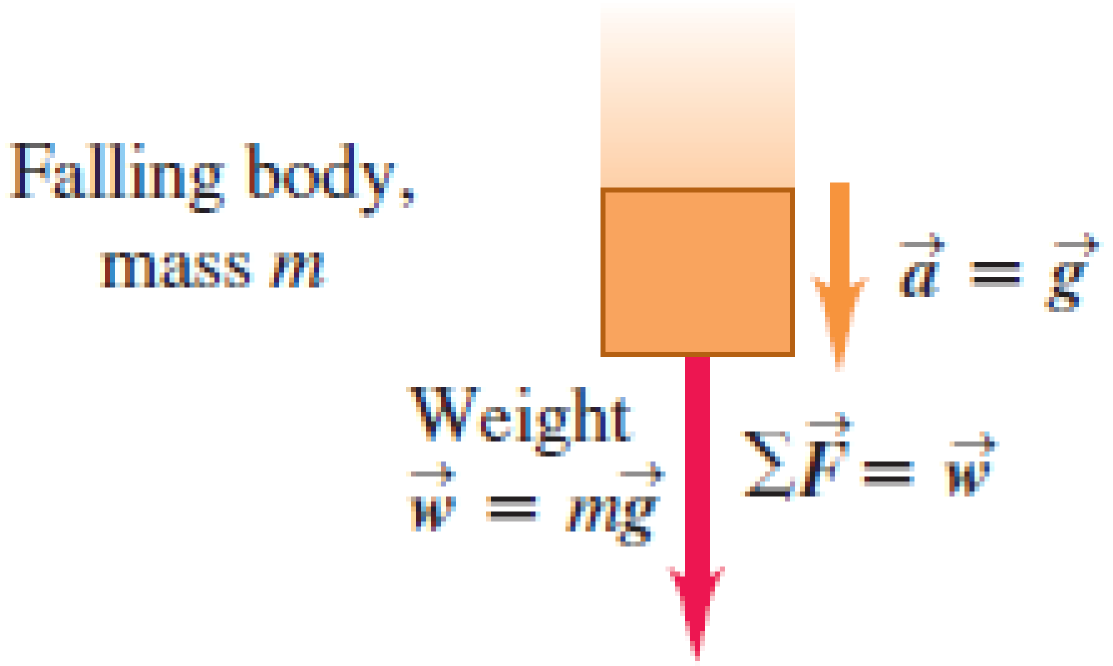


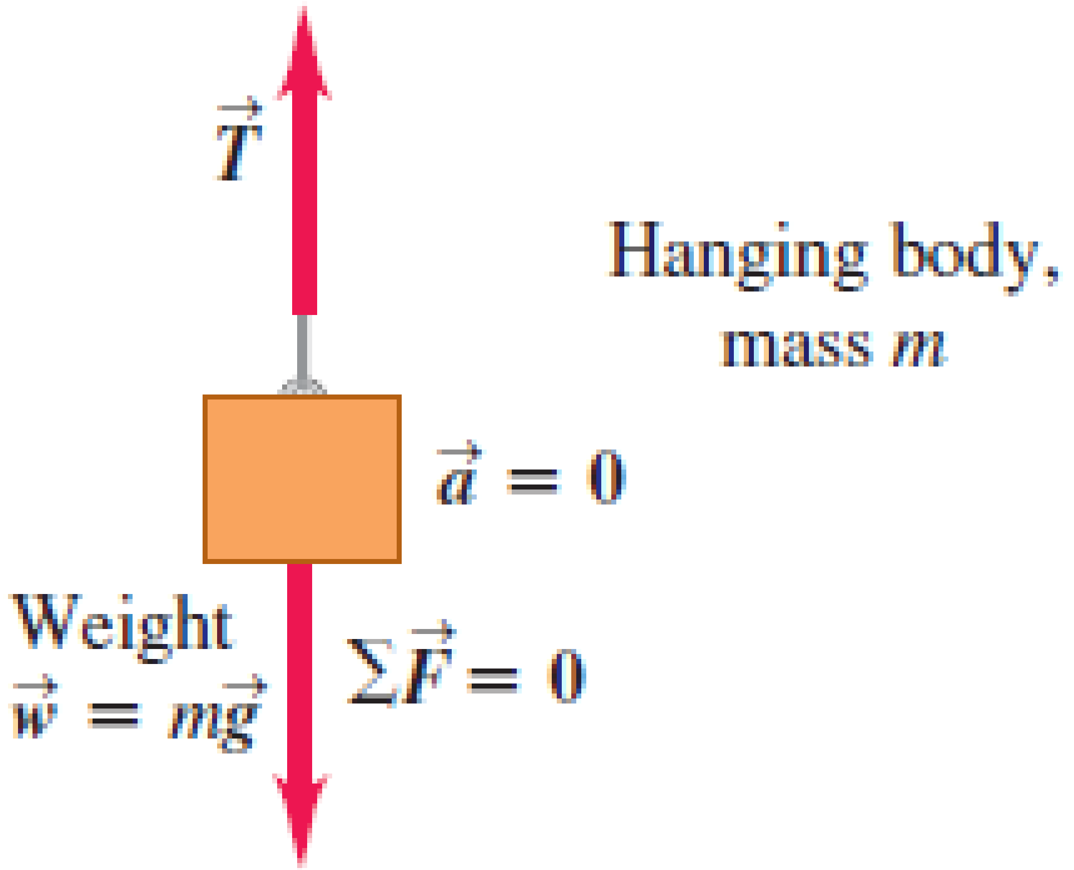

To determine $a_x$, we didn't need the $y$-component of Newton's second law from Eqs. (4.8), $\sum F_y = ma_y$.

# A note on mass versus weight

In 1D: $w = mg$

In 2D/3D: $\vec{w} = m\vec{g}$



# A note on mass versus weight

**CAUTION** **Don't confuse mass and weight** The SI units for mass and weight are often misused in everyday life. Incorrect expressions such as "This box weighs 6 kg" are nearly universal. What is meant is that the *mass* of the box, probably determined indirectly by *weighing*, is 6 kg. Be careful to avoid this sloppy usage in your own work! In SI units, weight (a force) is measured in newtons, while mass is measured in kilograms.

**Why is it important in physics?**
**Imagine if you are on the moon…**
**Mass does not change, but weight varies.**

# Example 4.7

A $2.49 \times 10^4$ N Rolls-Royce Phantom traveling in the $+x$-direction makes an emergency stop; the $x$-component of the net force acting on it is $-1.83 \times 10^4$ N. What is its acceleration?

**Hints:** **Solution:**

$$a_x = \frac{\sum F_x}{m}$$

$$w = mg$$

**EXECUTE:** The mass of the car is

$$m = \frac{w}{g} = \frac{2.49 \times 10^4\ \text{N}}{9.80\ \text{m/s}^2} = \frac{2.49 \times 10^4\ \text{kg} \cdot \text{m/s}^2}{9.80\ \text{m/s}^2}$$
$$= 2540\ \text{kg}$$

# Example 4.7

A $2.49 \times 10^4$ N Rolls-Royce Phantom traveling in the $+x$-direction makes an emergency stop; the $x$-component of the net force acting on it is $-1.83 \times 10^4$ N. What is its acceleration?

**Solution:**

Then $\sum F_x = ma_x$ gives

$$a_x = \frac{\sum F_x}{m} = \frac{-1.83 \times 10^4 \text{ N}}{2540 \text{ kg}} = \frac{-1.83 \times 10^4 \text{ kg} \cdot \text{m/s}^2}{2540 \text{ kg}}$$

$$= -7.20 \text{ m/s}^2$$

**4.7** •• A 68.5-kg skater moving initially at 2.40 m/s on rough horizontal ice comes to rest uniformly in 3.52 s due to friction from the ice. What force does friction exert on the skater?

Assume $\vec{f}$ is constant

$\vec{f}$ $\quad m = 68.5\,\text{kg}$

$\longrightarrow \vec{v}_0$

$t_0 = 0$, $v_0 = 2.4\,\text{m/s}$

$\longrightarrow t_1 = 3.52\,\text{s} \quad v_1 = 0$

Equations $\quad \vec{F} = m\vec{a} \overset{1D}{\Rightarrow} f = ma$

Kinematics $\quad \Delta\vec{v} = \vec{a}\cdot\Delta t \overset{1D}{\Rightarrow} a = \dfrac{\Delta v}{\Delta t} = \dfrac{v_1 - v_0}{t_1 - t_0}$

Combine: $\quad f = m \cdot \dfrac{v_1 - v_0}{\Delta t} = 68.5\,\text{kg} \times \dfrac{0 - 2.4\,\text{m/s}}{3.52\,\text{s}}$

$$= -46.7\,\text{N}$$

If we define $+x$ as the direction of $v_0$

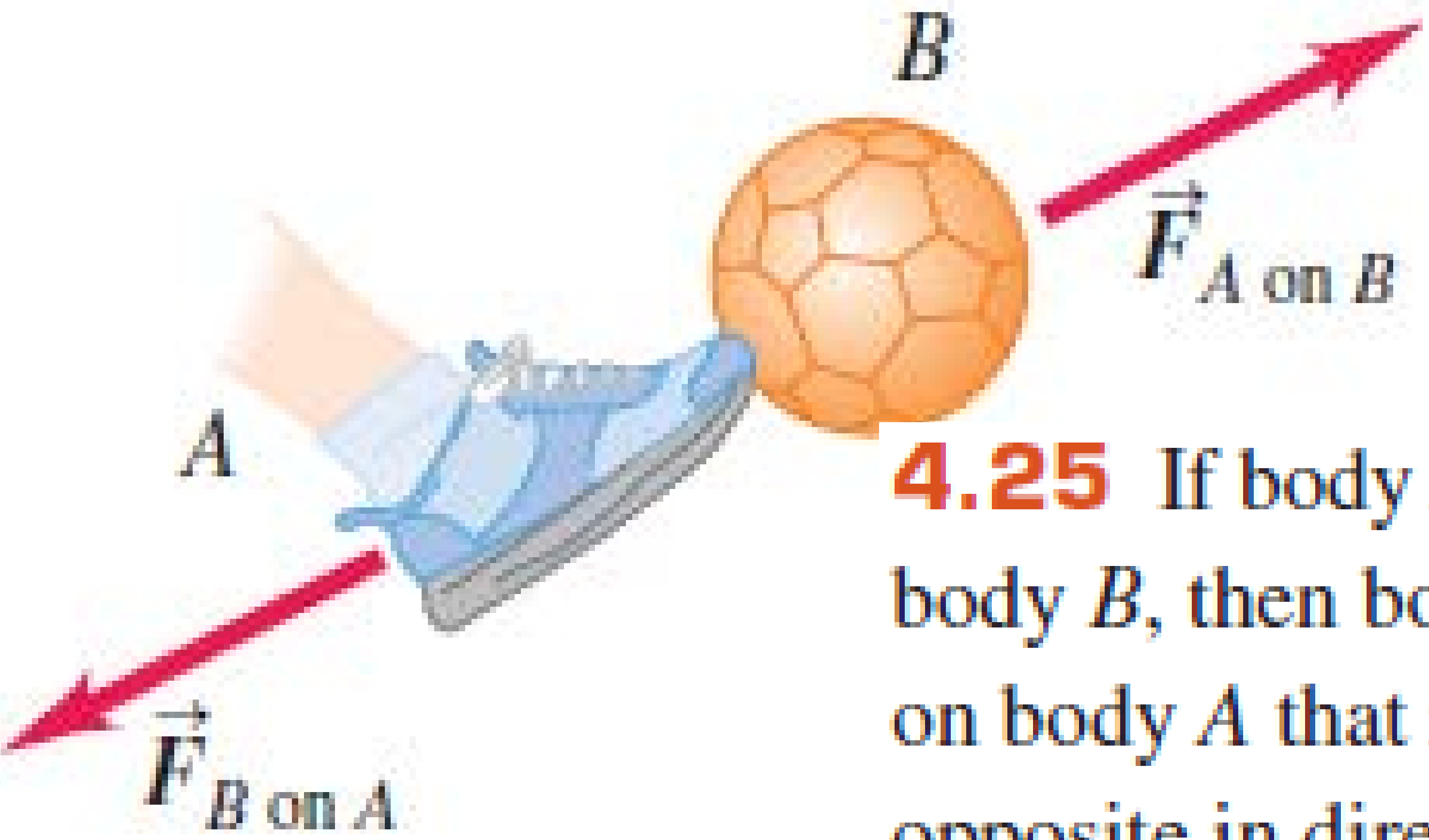
# Newton's Third Law

**Newton's third law of motion:** If body *A* exerts a force on body *B* (an "action"), then body *B* exerts a force on body *A* (a "reaction"). These two forces have the same magnitude but are opposite in direction. These two forces act on *different* bodies.

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A} \qquad \text{(Newton's third law of motion)}$$

# Newton’s Third Law

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A} \qquad \text{(Newton's third law of motion)}$$




**4.25** If body $A$ exerts a force $\vec{F}_{A \text{ on } B}$ on body $B$, then body $B$ exerts a force $\vec{F}_{B \text{ on } A}$ on body $A$ that is equal in magnitude and opposite in direction: $\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$.

# One note (that will be tested in exams)

For example, in Fig. 4.25 $\vec{F}_{A \text{ on } B}$ is the force applied *by* body $A$ (first subscript) *on* body $B$ (second subscript), and $\vec{F}_{B \text{ on } A}$ is the force applied *by* body $B$ (first subscript) *on* body $A$ (second subscript). The mathematical statement of Newton's third law is

**Action and reaction must be acted on two different objects interacting mechanically**

# Free-Body Diagrams

1. *Newton's first and second laws apply to a specific body.* Whenever you use Newton's first law, $\sum\vec{F} = 0$, for an equilibrium situation or Newton's second law, $\sum\vec{F} = m\vec{a}$, for a nonequilibrium situation, you must decide at the beginning to which body you are referring. This decision may sound trivial, but it isn't.

**CAUTION** **Forces in free-body diagrams** When you have a complete free-body diagram, you *must* be able to answer this question for each force: What other body is applying this force? If you can't answer that question, you may be dealing with a nonexistent force. Be especially on your guard to avoid nonexistent forces such as "the force of acceleration" or "the $m\vec{a}$ force," discussed in Section 4.3.

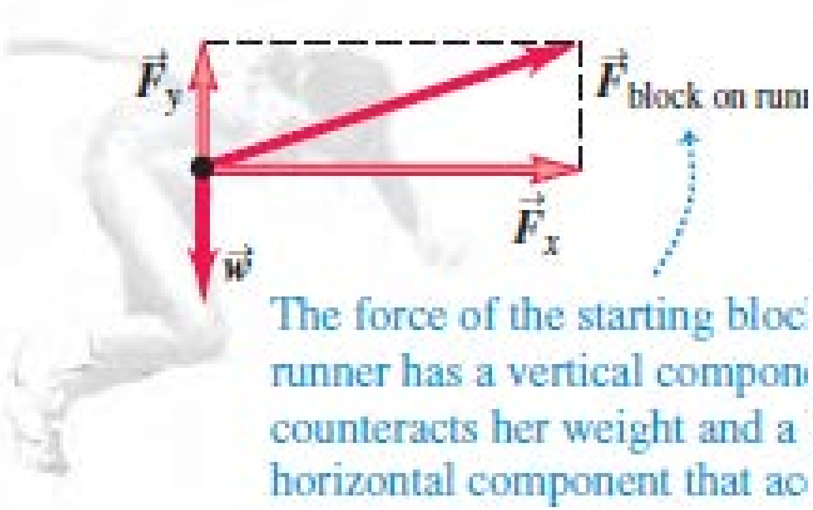
# Free-Body Diagrams

*Only forces acting on the body matter.* The sum $\sum\vec{F}$ includes all the forces that act *on* the body in question. Hence, once you've chosen the body to analyze, you have to identify all the forces acting on it. Don't get confused between the forces acting on a body and the forces exerted by that body on some other body. For example, to analyze a person walking, you would include in $\sum\vec{F}$ the force that the ground exerts on the person as he walks, but *not* the force that the person exerts on the ground (Fig. 4.29). These forces form an action–reaction pair and are related by Newton's third law, but only the member of the pair that acts on the body you're working with goes into $\sum\vec{F}$.

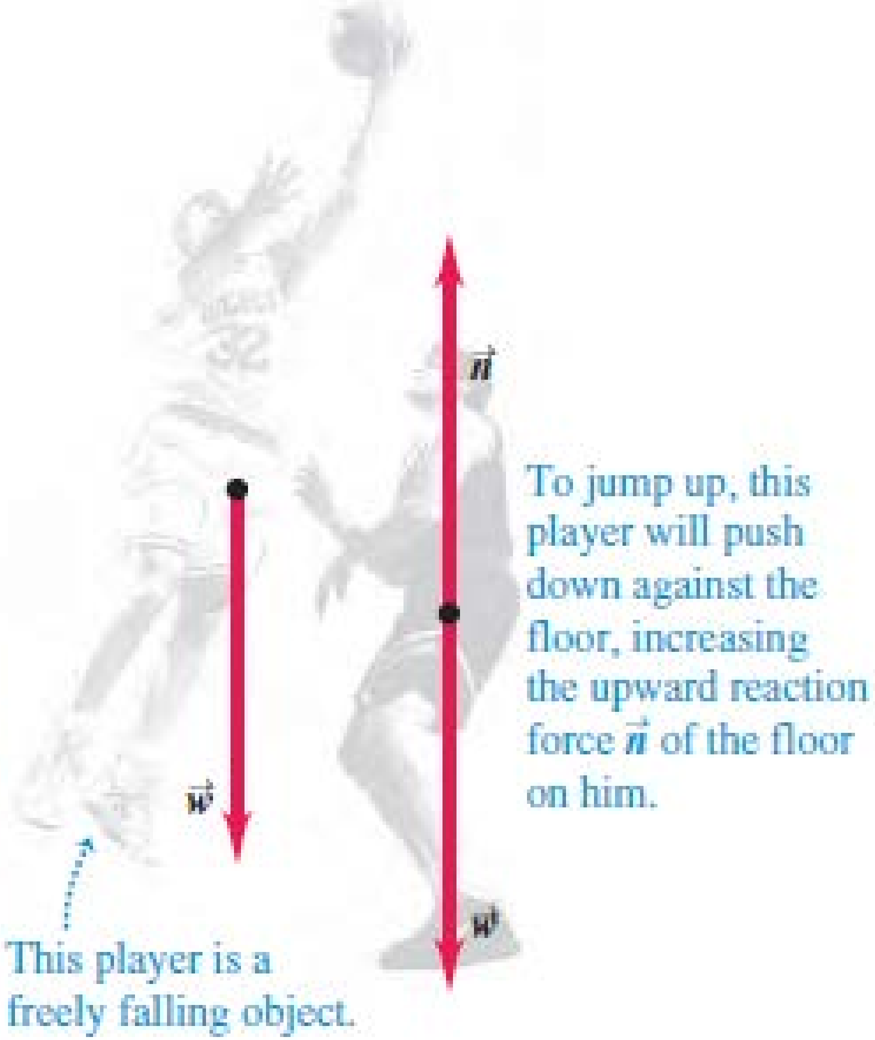
# Free-Body Diagrams

*Free-body diagrams are essential to help identify the relevant forces.* A **free-body diagram** is a diagram showing the chosen body by itself, "free" of its surroundings, with vectors drawn to show the magnitudes and directions of all the forces applied to the body by the various other bodies that interact with it. We have already shown some free-body diagrams in Figs. 4.18, 4.19, 4.21, and 4.26a. Be careful to include all the forces acting *on* the body, but be equally careful *not* to include any forces that the body exerts on any other body. In particular, the two forces in an action–reaction pair must *never* appear in the same free-body diagram because they never act on the same body. Furthermore, forces that a body exerts on itself are never included, since these can't affect the body's motion.

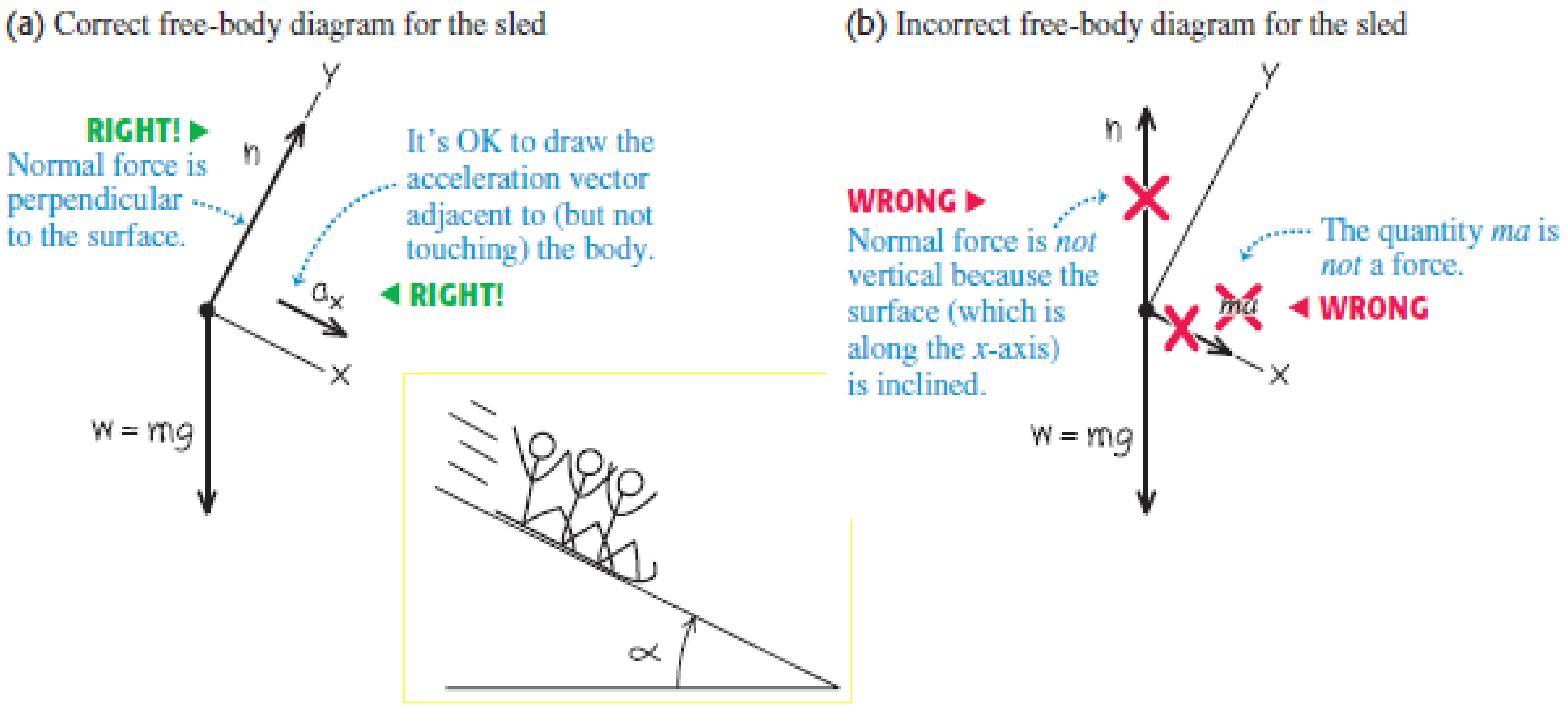
# Free-Body Diagrams





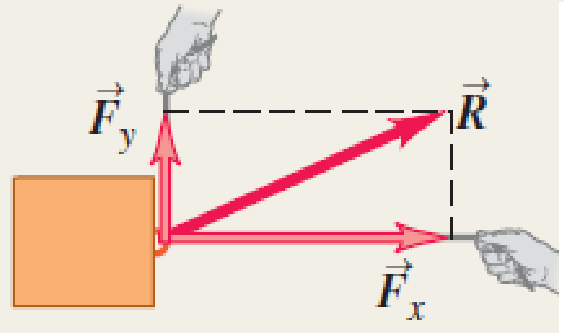


# Free-Body Diagrams

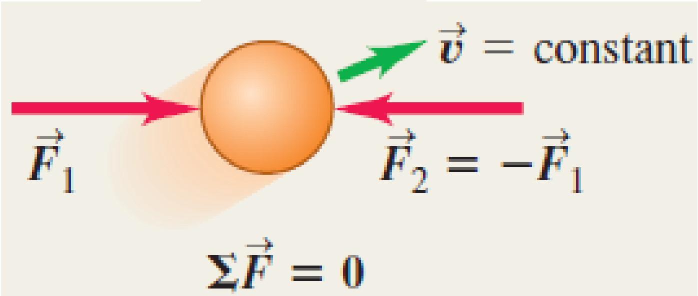


# Summary

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum \vec{F} \quad (4.1)$$




$$\sum \vec{F} = 0$$

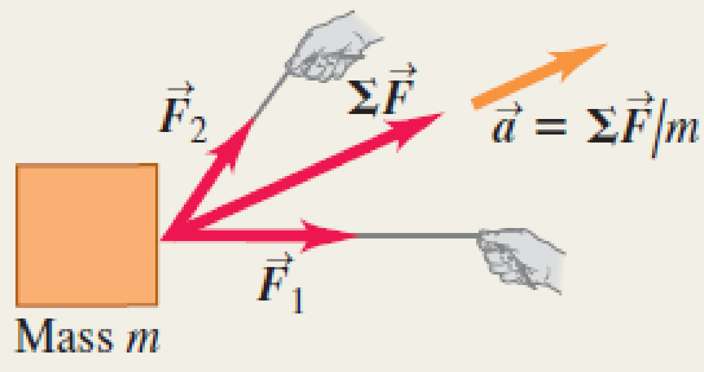



$$\sum \vec{F} = m\vec{a}$$

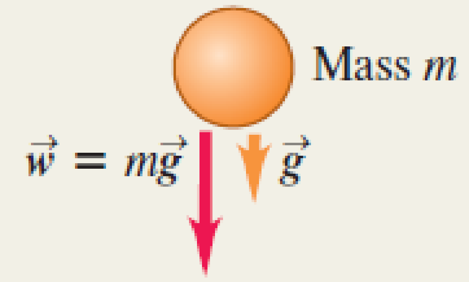
$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

$$\sum F_z = ma_z$$




$$w = mg$$




$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$