Assignment for Section 4.1: Spaces of Vectors

(1) The matrix $A = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$ is a "vector" in the space $\mathbb{M}$ of all 2 by 2 matrices.

    (a) Write down the "zero vector" in this space, the "vector" $\frac{1}{2}A$ and the "vector" $-A$.

    (b) What matrices are in the smallest subspace containing A?

(2) For which right sides are these systems solvable?
$$(a) \quad \begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix},$$
$$(b) \quad \begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

Find a condition on b_1, b_2, b_3 for each system.