Problem Set 7 (Due 4/15/2025 before class)

Late homework will NOT be accepted, unless you have notified the course instructor 3 days BEFORE deadline.

Part I (60%)

8.42 •• A 5.00-g bullet is fired horizontally into a 1.20-kg wooden block resting on a horizontal surface. The coefficient of kinetic friction between block and surface is 0.20. The bullet remains embedded in the block, which is observed to slide 0.230 m along the surface before stopping. What was the initial speed of the bullet?

IDENTIFY: Apply conservation of momentum to the collision. Apply conservation of energy to the motion of the block after the collision.

SET UP: Conservation of momentum applied to the collision between the bullet and the block: Let object A be the bullet and object B be the block. Let v_A be the speed of the bullet before the collision and let V be the speed of the block with the bullet inside just after the collision.

Figure 8.42a

P_x is constant gives $m_A v_A = (m_A + m_B)V$.

Conservation of energy applied to the motion of the block after the collision:

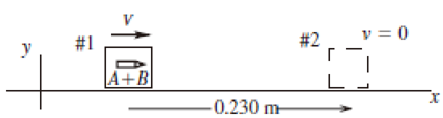


Figure 8.42b

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

EXECUTE: Work is done by friction so $W_{\text{other}} = W_f = (f_k \cos\phi)s = -f_k s = -\mu_k mgs$

$U_1 = U_2 = 0$ (no work done by gravity)

$K_1 = \frac{1}{2}mV^2$; $K_2 = 0$ (block has come to rest)

Thus $\frac{1}{2}mV^2 - \mu_k mgs = 0$

$V = \sqrt{2\mu_k gs} = \sqrt{2(0.20)(9.80\text{ m/s}^2)(0.230\text{ m})} = 0.9495\text{ m/s}$

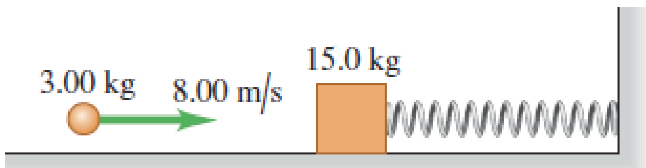
Use this in the conservation of momentum equation

$$v_A = \left(\frac{m_A + m_B}{m_A}\right)V = \left(\frac{5.00\times10^{-3}\text{ kg} + 1.20\text{ kg}}{5.00\times10^{-3}\text{ kg}}\right)(0.9495\text{ m/s}) = 229\text{ m/s}$$

EVALUATE: When we apply conservation of momentum to the collision we are ignoring the impulse of the friction force exerted by the surface during the collision. This is reasonable since this force is much smaller than the forces the bullet and block exert on each other during the collision. This force does work as the block moves after the collision, and takes away all the kinetic energy.

8.44 •• **Combining Conservation Laws.** A 15.0-kg block is attached to a very light horizontal spring of force constant 500.0 N/m and is resting on a frictionless horizontal table. (Fig. E8.44). Suddenly it is struck by a 3.00-kg stone traveling horizontally at 8.00 m/s to the right, whereupon the stone rebounds at 2.00 m/s horizontally to the left. Find the maximum distance that the block will compress the spring after the collision.

Figure **E8.44**



IDENTIFY: During the collision, momentum is conserved. After the collision, mechanical energy is conserved.

SET UP: The collision occurs over a short time interval and the block moves very little during the collision, so the spring force during the collision can be neglected. Use coordinates where $+x$ is to the right. During the collision, momentum conservation gives $P_{1x} = P_{2x}$. After the collision, $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$.

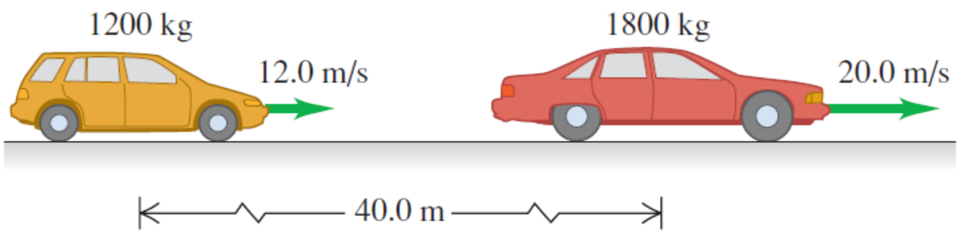
EXECUTE: *Collision*: There is no external horizontal force during the collision and $P_{1x} = P_{2x}$, so $(3.00\text{ kg})(8.00\text{ m/s}) = (15.0\text{ kg})v_{\text{block, 2}} - (3.00\text{ kg})(2.00\text{ m/s})$ and $v_{\text{block, 2}} = 2.00\text{ m/s}$.

Motion after the collision: When the spring has been compressed the maximum amount, all the initial kinetic energy of the block has been converted into potential energy $\frac{1}{2}kx^2$ that is stored in the compressed spring. Conservation of energy gives $\frac{1}{2}(15.0\text{ kg})(2.00\text{ m/s})^2 = \frac{1}{2}(500.0\text{ kg})x^2$, so $x = 0.346\text{ m}$.

EVALUATE: We cannot say that the momentum was converted to potential energy, because momentum and energy are different types of quantities.

8.54 • A 1200-kg station wagon is moving along a straight highway at 12.0 m/s. Another car, with mass 1800 kg and speed 20.0 m/s, has its center of mass 40.0 m ahead of the center of mass of the station wagon (Fig. E8.54). (a) Find the position of the center of mass of the system consisting of the two automobiles. (b) Find the magnitude of the total momentum of the system from the given data. (c) Find the speed of the center of mass of the system. (d) Find the total momentum of the system, using the speed of the center of mass. Compare your result with that of part (b).

Figure **E8.54**



IDENTIFY: Apply Eqs. 8.28, 8.30 and 8.32. There is only one component of position and velocity.

SET UP: $m_A = 1200$ kg, $m_B = 1800$ kg. $M = m_A + m_B = 3000$ kg. Let $+x$ be to the right and let the origin be at the center of mass of the station wagon.

EXECUTE: **(a)** $x_{\text{cm}} = \dfrac{m_A x_A + m_B x_B}{m_A + m_B} = \dfrac{0 + (1800\text{ kg})(40.0\text{ m})}{1200\text{ kg} + 1800\text{ kg}} = 24.0\text{ m}.$

The center of mass is between the two cars, 24.0 m to the right of the station wagon and 16.0 m behind the lead car.

(b) $P_x = m_A v_{A,x} + m_B v_{B,x} = (1200\text{ kg})(12.0\text{ m/s}) + (1800\text{ kg})(20.0\text{ m/s}) = 5.04\times10^4\text{ kg}\cdot\text{m/s}.$

(c) $v_{\text{cm},x} = \dfrac{m_A v_{A,x} + m_B v_{B,x}}{m_A + m_B} = \dfrac{(1200\text{ kg})(12.0\text{ m/s}) + (1800\text{ kg})(20.0\text{ m/s})}{1200\text{ kg} + 1800\text{ kg}} = 16.8\text{ m/s}.$

(d) $P_x = Mv_{\text{cm}-x} = (3000\text{ kg})(16.8\text{ m/s}) = 5.04\times10^4\text{ kg}\cdot\text{m/s}$, the same as in part (b).

EVALUATE: The total momentum can be calculated either as the vector sum of the momenta of the individual objects in the system, or as the total mass of the system times the velocity of the center of mass.

Part II (40%)

1. A 68.5-kg astronaut is doing a repair in space on the orbiting space station. She throws a 2.25-kg tool away from her at 3.2 m/s relative to the space station. With what speed and in what direction will she begin to move?

   IDENTIFY: Apply conservation of momentum to the system of the astronaut and tool.
   SET UP: Let A be the astronaut and B be the tool. Let $+x$ be the direction in which she throws the tool, so $v_{B2x} = +3.20$ m/s. Assume she is initially at rest, so $v_{A1x} = v_{B1x} = 0$. Solve for v_{A2x}.
   EXECUTE: $P_{1x} = P_{2x}$. $P_{1x} = m_A v_{A1x} + m_B v_{B1x} = 0$. $P_{2x} = m_A v_{A2x} + m_B v_{B2x} = 0$ and $v_{A2x} = -\frac{m_B v_{A2x}}{m_A} = -\frac{(2.25\text{ kg})(3.20\text{ m/s})}{68.5\text{ kg}} = -0.105$ m/s. Her speed is 0.105 m/s and she moves opposite to the direction in which she throws the tool.
   EVALUATE: Her mass is much larger than that of the tool, so to have the same magnitude of momentum as the tool her speed is much less.

2. A 1300 kg race car is traveling at 80 m/s while a 15,000 kg truck is traveling at 20 m/s. Which has the greater momentum

   Solution: Car: 104 000 kg*m/s; truck: 300 000 kg*m/s

3. When the momentum of an object doubles and its mass remains constant, how much does its kinetic energy change?

   Justification: Momentum of an object is defined as the product of its mass and its velocity: p = mv

   If the momentum is doubled while the mass remains constant then the velocity must be doubled: $2\boldsymbol{p} = m(2\boldsymbol{v})$

   The kinetic energy of an object is defined by the formula: $E_K = \frac{1}{2}m\boldsymbol{v}^2$

   Substituting the new velocity, 2v, into this equation:

   $$E_K = \frac{1}{2}m(2\boldsymbol{v})^2$$
   $$E_K = \frac{1}{2}m(4\boldsymbol{v}^2)$$
   $$E_K = 4(\frac{1}{2}m\boldsymbol{v}^2)$$

   Therefore, by doubling the momentum of an object we quadruple its kinetic energy

4. How much force is required to stop a 60 kg person traveling at 30 m/s during a time of 5 second?

Givens:

$m = 60kg$

$v_{initial} = 30\frac{m}{s}$

$v_{final} = 0\frac{m}{s}$

$t = 5.0\ s$

$F = ?$

Equation: $F = \frac{\Delta p}{\Delta t}$ *and* $\Delta p = p_{final} - p_{initial}$ *and* $p = mv$

Calculations:

$$F = \frac{\Delta p}{\Delta t}$$

$$F = \frac{p_{final} - p_{initial}}{\Delta t}$$

$$F = \frac{mv_{final} - mv_{initial}}{\Delta t}$$

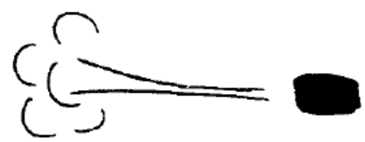
So:

$$F = \frac{(60kg)\left(0\frac{m}{s}\right) - (60kg)\left(30\frac{m}{s}\right)}{5.0s}$$

$$F = -360\frac{kg * m}{s^2}$$

$$\boxed{F = -360\ N}$$

5. Wayne hits a stationary 0.12-kg hockey puck with a force that lasts for 0.01 s and makes the puck shoot across the ice with a speed of 20.0 m/s, scoring a goal for the team. With what force did Wayne hit the puck?



Given: $m = 0.12$ kg
$\Delta v = 20.0$ m/s
$\Delta t = 1.0 \times 10^{-2}$ s

Unknown: $F = ?$
Original equation: $F\Delta t = m\Delta v$

$$\textit{Solve: } F = \frac{m\Delta v}{\Delta t} = \frac{(0.12\text{ kg})(20.0\text{ m/s})}{1.0 \times 10^{-2}\text{ s}} = 240\text{ kg}\cdot\text{m/s}^2 = \mathbf{240\ N}$$

For problems 6 and 7: A tennis ball traveling at 10.0 m/s is returned by Venus Williams. It leaves her racket with a speed of 36.0 m/s in the opposite direction from which it came.

6. What is the change in momentum of the tennis ball?

Solution: In this exercise, the tennis ball is coming toward Venus with a speed of 10.0 m/s in one direction, but she hits it back with a speed of 36.0 m/s in the opposite direction. Therefore, you must think about velocity

a. *Given:* $v_o = -10.0\ \text{m/s}$
$v_f = 36.0\ \text{m/s}$
$m = 0.060\ \text{kg}$

Unknown: $\Delta p = ?$
Original equation: $\Delta p = m\Delta v = m(v_f - v_o)$

Solve: $\Delta p = m(v_f - v_o) = (0.060\ \text{kg})[36.0\ \text{m/s} - (-10.0\ \text{m/s})] = \mathbf{2.8\ kg{\cdot}m/s}$

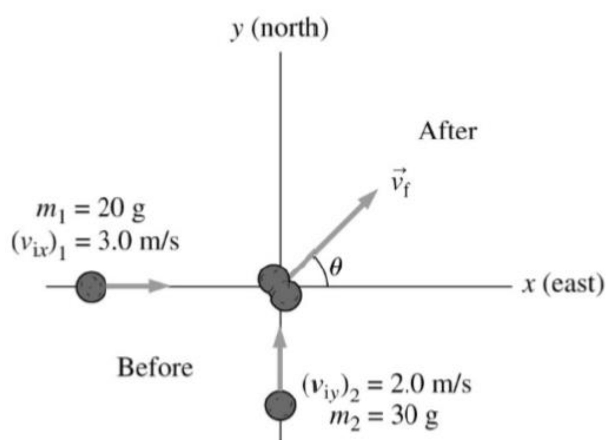
7. If the 0.060-kg ball is in contact with the racket for 0.020 s, with what average force has Venus hit the ball?

b. *Given:* $m = 0.060\ \text{kg}$
$\Delta v = 46.0\ \text{m/s}$
$\Delta t = 0.020\ \text{s}$

Unknown: $F = ?$
Original equation: $F\Delta t = m\Delta v$

Solve: $F = \dfrac{m\Delta v}{\Delta t} = \dfrac{(0.060\ \text{kg})(46.0\ \text{m/s})}{(0.020\ \text{s})} = \mathbf{140\ N}$

8. A 20 g ball of clay traveling east at 3.0 m/s collides with a 30 g ball of clay traveling north at 2.0 m/s. What are the speed and the direction of the resulting 50 g ball of clay?



Known:

$m_1 = 20g = 0.020kg$

$(v_{ix})_1 = 3\frac{m}{s}$

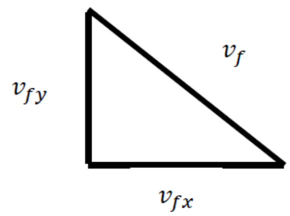
$m_2 = 30g = 0.030kg$

$(v_{iy})_2 = 2\frac{m}{s}$

Find:

v_f=?

θ =?



Use the Law of Conservation of Momentum ($\boldsymbol{P_f = P_i}$) to solve for the clay's velocity

$$P_f = P_i$$

X-direction: $m_1(v_{ix})_1 + m_2(v_{ix})_2 = (m_1 + m_2)v_{fx}$

Rewrite v_{fx} in terms of v_f using trigonometry.

$$\cos(\theta) = \frac{v_{fx}}{v_f} \rightarrow v_{fx} = v_f\cos(\theta)$$

$$\rightarrow m_1(v_{ix})_1 + m_2(v_{ix})_2 = (m_1 + m_2)v_f\cos(\theta)$$

$$\rightarrow v_f\cos(\theta) = \frac{m_1(v_{ix})_1 + m_2(v_{ix})_2}{(m_1 + m_2)}$$

$$v_f\cos(\theta) = 1.2\frac{m}{s}$$

Y-direction: $m_1(v_{iy})_1 + m_2(v_{iy})_2 = (m_1 + m_2)v_{fy}$

Rewrite v_{fy} in terms of v_f using trigonometry.

$$\sin(\theta) = \frac{v_{fy}}{v_f} \rightarrow v_{fy} = v_f\sin(\theta)$$

$$\rightarrow m_1(v_{iy})_1 + m_2(v_{iy})_2 = (m_1 + m_2)v_f\sin(\theta)$$

$$\rightarrow v_f\sin(\theta) = \frac{m_1(v_{iy})_1 + m_2(v_{iy})_2}{(m_1 + m_2)}$$

$$v_f\sin(\theta) = 1.2\frac{m}{s}$$

Use the Pythagorean theorem ($\boldsymbol{c^2 = a^2 + b^2}$) to find v_f, thus

$$v_f^2 = v_{fx}^2 + v_{fy}^2$$

Substitute v_{fx} and v_{fy} (which was found above):

$$\rightarrow v_f = \sqrt{(v_f\cos(\theta))^2 + (v_f\sin(\theta))^2}$$

$$v_f = \sqrt{\left(1.2\frac{m}{s}\right)^2 + \left(1.2\frac{m}{s}\right)^2} \approx 1.69\frac{m}{s}$$

$$\tan(\theta) = \frac{v_{fy}}{v_{fx}}$$

$$\rightarrow \theta = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right)$$

$$\theta = \tan^{-1}\left(\frac{1.2\frac{m}{s}}{1.2\frac{m}{s}}\right) = 45°$$

The clay has a velocity of 1.69 m/s heading northeast at a 45-degree angle.

9. A 2 kg mass is at (x, y)=(1 m, 2 m), and a 4 kg mass is at (x, y)=(5 m, 6 m). Calculate the center of mass.

Start with the formulas:

$$x_{COM} = \frac{\sum(m_i x_i)}{\sum m_i} \qquad y_{COM} = \frac{\sum(m_i y_i)}{\sum m_i}$$

Now solve for x_{COM}:

$$x_{COM} = \frac{(2)(1) + (4)(5)}{2 + 4} = \frac{22}{6} \approx 3.67\ m$$

Then solve for y_{COM}:

$$y_{COM} = \frac{(2)(2) + (4)(5)}{2 + 4} = \frac{28}{6} \approx 4.67\ m$$

Answer: The center of mass is approximately (3.67 m, 4.67 m). At the 3.67 meter point in the X axis and 4.67 meter point in the Y axis.

10. A uniform rod of length 2 meters and mass 8 kg has one end at x=0 m and the other at x=2 m. Find the center of mass.

Recognize that for uniform objects, the center of mass lies at the geometric center. Since the rod spans from x = 0 m, the center of mass is:

$$x_{COM} = \frac{0 + 2}{2} = 1\ m$$

Answer: The center of mass is at x=1 m.