College Algebra and Trigonometry

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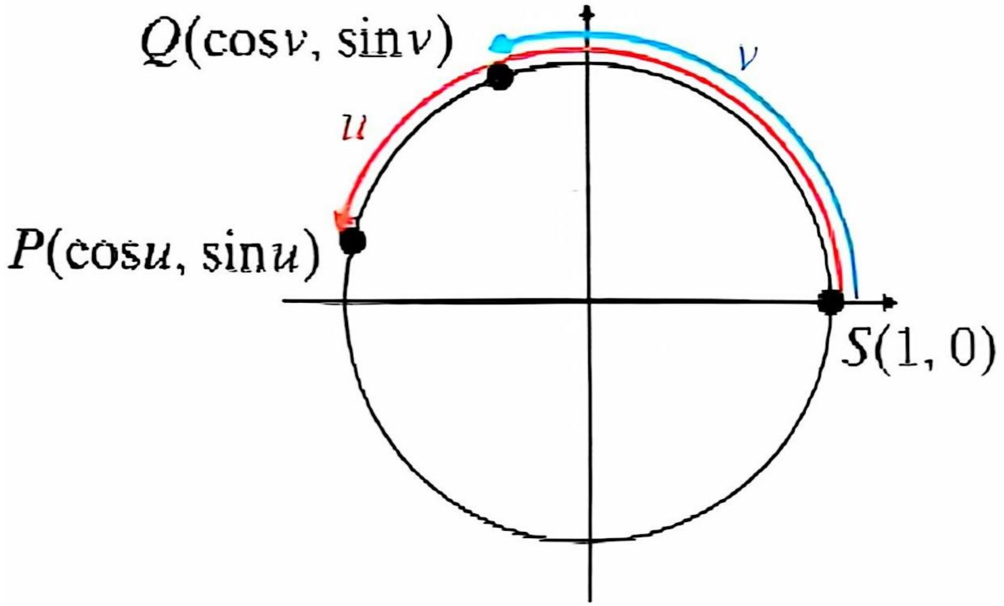
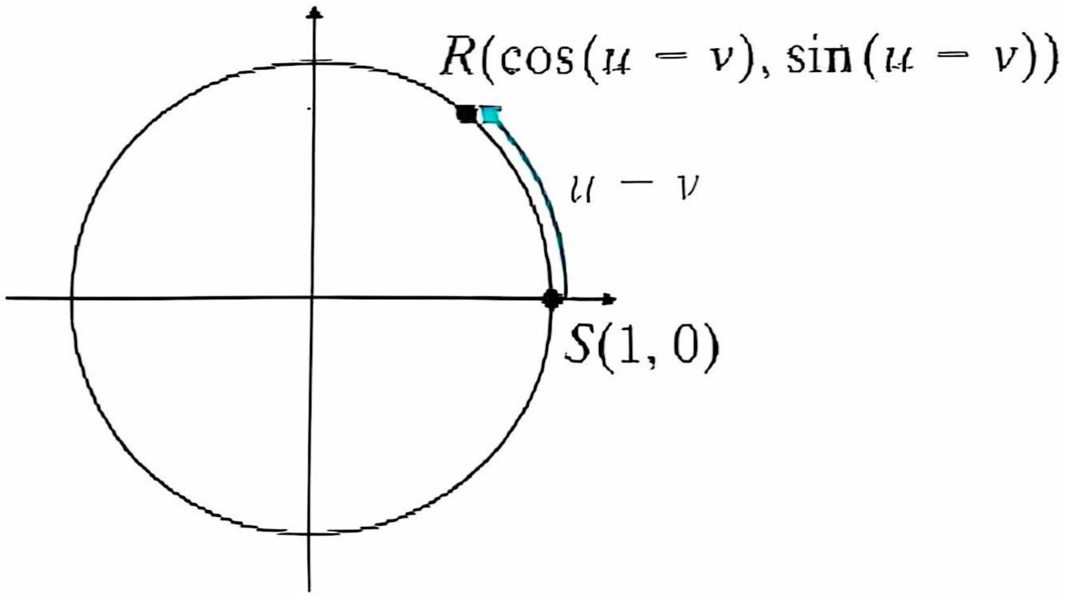
① Apply Sum and Difference Formulas for Sine and Cosine

A unit circle $x^2 + y^2 = 1$ in which $x = \cos\theta$ and $y = \sin\theta$. P, Q, R, and S are the points on this unit circle.





$$|PQ| = |RS| \Rightarrow \cos(u-v) = \cos u\cos v + \sin u\sin v$$

$$\Rightarrow \sin(u+v) = \cos\left[\left(\frac{\pi}{2}-u\right)-v\right] = \cos\left(\frac{\pi}{2}-u\right)\cos v + \sin\left(\frac{\pi}{2}-u\right)\sin v$$

Sine Formulas:

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

Cosine Formulas:

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

Tangent Formulas:

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

$$\sin(\pi+u) = -\sin u \qquad \sin(\pi-u) = \sin u$$

$$\cos(\pi+v) = -\cos v \qquad \cos(\pi-v) = -\cos v$$

$$\tan(\pi+v) = \tan v \qquad \tan(\pi-v) = -\tan v$$

Example 1: Find the exact values.

a) $\cos 15°$ b) $\sin \frac{11\pi}{12}$

Example 2:

Find the exact value of the expression.

$$\sin 25° \cos 35° + \cos 25° \sin 35°$$

Example 3:

Find the exact value of $\cos(\alpha - \beta)$ given that $\sin\alpha = -\frac{3}{5}$ and $\cos\beta = -\frac{12}{13}$ for α and β are both in Quadrant III.

② Apply Sum and Difference Formulas for Tangent

We can use the identities for $\sin(u \pm v)$ and $\cos(u \pm v)$ to derive similar identities for $\tan(u \pm v)$.

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - (\tan u)(\tan v)}$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + (\tan u)(\tan v)}$$

Example 4:

Find the exact values of $\tan 255°$ and $\tan\frac{5\pi}{12}$.

③ Use Sum and Difference Formulas to Verify Identities

Cofunction Identities*

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right) \qquad \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right) \qquad \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

$$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right) \qquad \csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$$

Periodic Identities*

$$\sin(\theta + 2\pi) = \sin\theta$$
$$\cos(\theta + 2\pi) = \cos\theta$$

$$\tan(\theta + \pi) = \tan\theta$$
$$\cot(\theta + \pi) = \cot\theta$$

$$\csc(\theta + 2\pi) = \csc\theta$$
$$\sec(\theta + 2\pi) = \sec\theta$$

*All statements can be made using 90° for $\frac{\pi}{2}$, 180° for π, and 360° for 2π.

Example 5:

Verify the cofunction identity.

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$

Example 6:

Verify the cofunction identity.

$$\cos(x+y)\cos(x-y)=\cos^2 x-\sin^2 y$$

④ Use Sum and Difference Formulas to Verify Identities

Sum of $A\sin x$ and $B\cos x$

For the real numbers A, B, and x,

$$A\sin x + B\cos x = k\sin(x + \alpha)$$

where $k = \sqrt{A^2 + B^2}$, and α satisfies $\cos\alpha = \dfrac{A}{k}$ and $\sin\alpha = \dfrac{B}{k}$.

Example 7:

Write $9\sin x + 12\cos x$ in the form $k\sin(x+\alpha)$.