

# *College Algebra and Trigonometry*

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### ① Graph Linear Equations in Two Variables

#### Linear Equation in Two Variables

A linear equation in two variables  $x$  and  $y$  is the equation that is written in the **standard form** as:

$$Ax + By = C.$$

where  $A, B, C$  are real numbers in which both  $A$  and  $B$  are nonzero.

#### Example 1:

Graph the following linear equations.

a)  $2x + 3y = 6$

b)  $\frac{x}{2} - \frac{y}{3} = 1$

### ② Determine a Slope of a Line

#### Slope of a Line:

The slope of a line passing through the distinct points  $(x_1, y_1)$  and  $(x_2, y_2)$  is:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{provided that } x_1 \text{ and } x_2 \text{ are non equal.}$$

#### Example 2:

Find the slope of the line passing through the given points.

a)  $(-3, -2)$  and  $(2, 5)$

b)  $(1, -7)$  and  $(-5/2, 0)$

### Example 3:

Find the slope of horizontal and vertical lines.

a)  $x = -2$

b)  $y = 3$

### Linear Equations and Slopes of Lines:

$$Ax + By = C$$

( $A \neq 0$  and  $B \neq 0$ )

Slanted line



Positive Slope



Negative Slope

$$x = k$$

( $k$  is a constant)

Vertical line



Undefined Slope

$$y = C$$

( $C$  is a constant)

Horizontal line



Zero Slope

### ③ Apply the Slope-Intercept Form of a Line

#### **Slope-Intercept Form of a Line:**

Given a line with slope  $m$  and  $y$ -intercept  $(0, b)$ , the **slope-intercept form** of the line is given by:

$$y = mx + b$$

#### **Example 4:**

Use the slope and  $y$ -intercept to graph a line given  $3x + 4y = 4$ .

- a)** Write the equation in the slope-intercept form.
- b)** Determine the slope and  $y$ -intercept.
- c)** Graph the line by using the slope and  $y$ -intercept.

### Definition of Linear and Constant Functions

Let  $m$  and  $b$  represent real numbers where  $m \neq 0$ . Then:

- A function defined by  $f(x) = mx + b$  is a linear function whose graph is a slanted line.
- A function defined by  $f(x) = b$  is a constant function whose graph is a horizontal line.

### Example 5:

Write an equation of the line with a slope of 3 and passing through the point (2, 1). Then write the linear equation using the function notation  $y = f(x)$ .

### ④ Compute Average Rate of Change

#### Average Rate of Change:

If a function  $f(x)$  is defined on an interval  $[x_1, x_2]$ , then the **average rate of change** of  $f(x)$  on this interval is the slope of the secant line containing  $(x_1, y_1)$  and  $(x_2, y_2)$ , written as:

$$\text{Average rate of change} \quad m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

#### Example 6:

Given the function  $f(x) = x^2 - 1$ , Determine the average rate of change from  $x_1 = -2$  to  $x_2 = 0$ .

### ① Apply the Point-Slope Formula

$$\frac{y - y_1}{x - x_1} = m$$

**Slope Formula**



$$y - y_1 = m(x - x_1)$$

**Point-Slope Formula**

### Point-Slope Formula

The **point-slope formula** for a line is given by  $y - y_1 = m(x - x_1)$ , where  $m$  is the slope of the line and  $(x_1, y_1)$  is a point on the line.



### **Example 1:**

**Use the point-slope formula to find an equation for the line passing through the point  $(2, -3)$  and having the slope of  $-4$ . Write the answer in the slope-intercept form.**

### **Example 2:**

**Use the point-slope formula to find an equation for the line passing through the points  $(4, -6)$  and  $(-1, 2)$ . Write the answer in the slope-intercept form.**

### ② Determine the Slopes of Parallel and Perpendicular Lines

#### Slopes of Parallel and Perpendicular Lines

- If  $m_1$  and  $m_2$  represent the slopes of two **nonvertical** parallel lines, then  $m_1 = m_2$ .
- If  $m_1$  and  $m_2$  represent the slopes of two **nonvertical** perpendicular lines, then  $m_1 = -\frac{1}{m_2}$ , or equivalently  $m_1 m_2 = -1$ .

### **Example 3:**

**Write an equation of the line passing through the point  $(-4, 1)$  and parallel to the line defined by  $x + 4y = 3$ . Write the answer in the slope-intercept form and the standard form.**

### **Example 4:**

**Write an equation of the line passing through the point  $(2, -3)$  and perpendicular to the line defined by  $y = x/2 - 4$ . Write the answer in the slope-intercept form and the standard form.**