Nodal Analysis

- It is possible to use KCL to analyze a circuit in frequency domain.
- The first step is to convert a time domain circuit to frequency domain by calculating the impedances of the circuit elements at the operating frequency.
- Impedances will be expressed as complex numbers.
- Sources will have amplitude and phase.
- At this point, KCL analysis can proceed as normal.
- It is important to bear in mind that complex values will be calculated, but all other treatments are the same.

Nodal Analysis

Find i_x in the circuit of Fig. 10.1 using nodal analysis.

Example 10.1

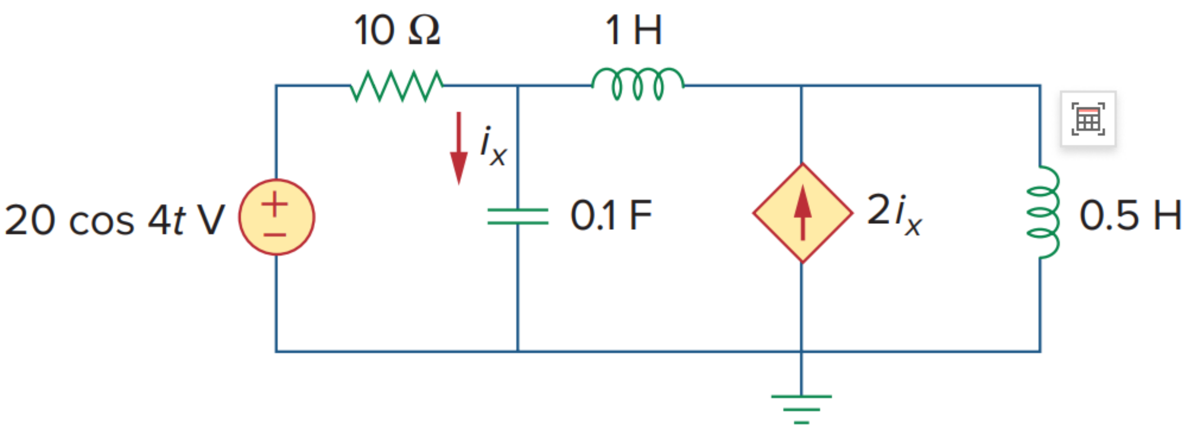


Figure 10.1
For Example 10.1.

Example 10.1

Find i_x in the circuit of Fig. 10.1 using nodal analysis.

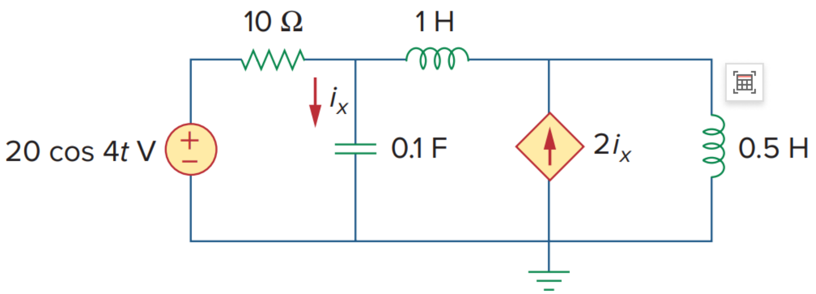


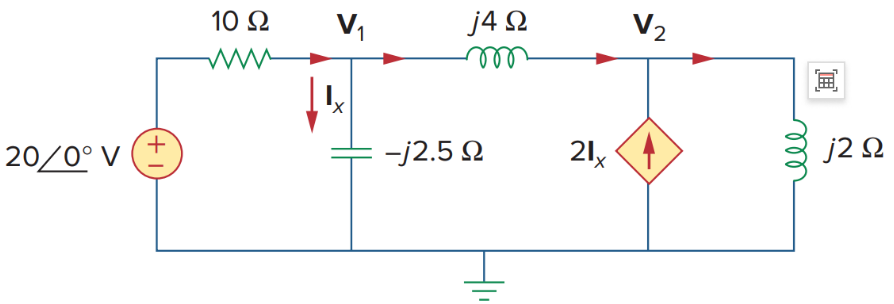
Figure 10.1
For Example 10.1.

Solution:

We first convert the circuit to the frequency domain:

$$\begin{aligned} 20\cos 4t \quad &\Rightarrow \quad 20\underline{/0^\circ}, \qquad \Omega = 4 \text{ rad/s} \\ 1\text{ H} \quad &\Rightarrow \quad j\Omega L = j4 \\ 0.5\text{ H} \quad &\Rightarrow \quad j\Omega L = j2 \\ 0.1\text{ F} \quad &\Rightarrow \quad \frac{1}{j\Omega C} = -j2.5 \end{aligned}$$

Thus, the frequency domain equivalent circuit is as shown in Fig. 10.2.



Applying KCL at node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

or

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20 \tag{10.1.1}$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

By simplifying, we get

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0 \tag{10.1.2}$$

Equations (10.1.1) and (10.1.2) can be put in matrix form as

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \qquad \Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\underline{/18.43^\circ} \text{ V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\underline{/198.3^\circ} \text{ V}$$

The current $\mathbf{I}_x$ is given by

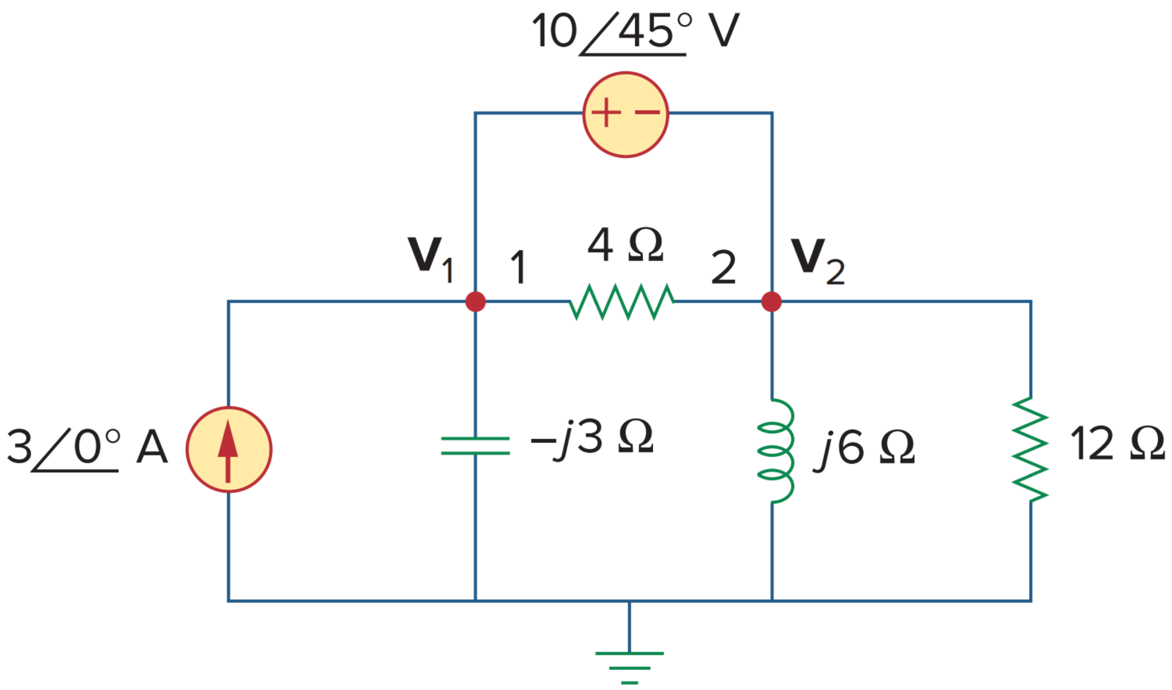
$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\underline{/18.43^\circ}}{2.5\underline{/-90^\circ}} = 7.59\underline{/108.4^\circ} \text{ A}$$

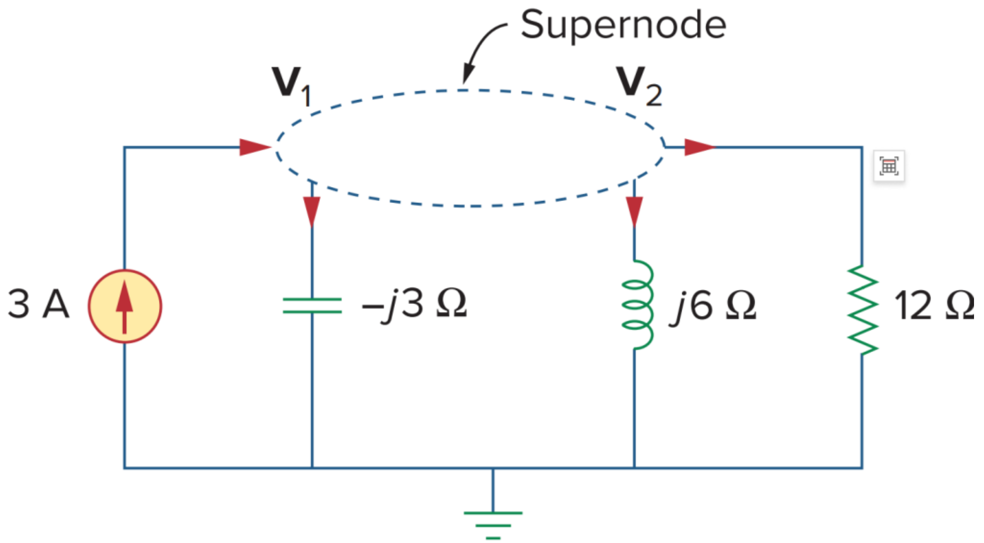
Transforming this to the time domain,

$$i_x = 7.59\cos(4t + 108.4^\circ) \text{ A}$$

Nodal Analysis

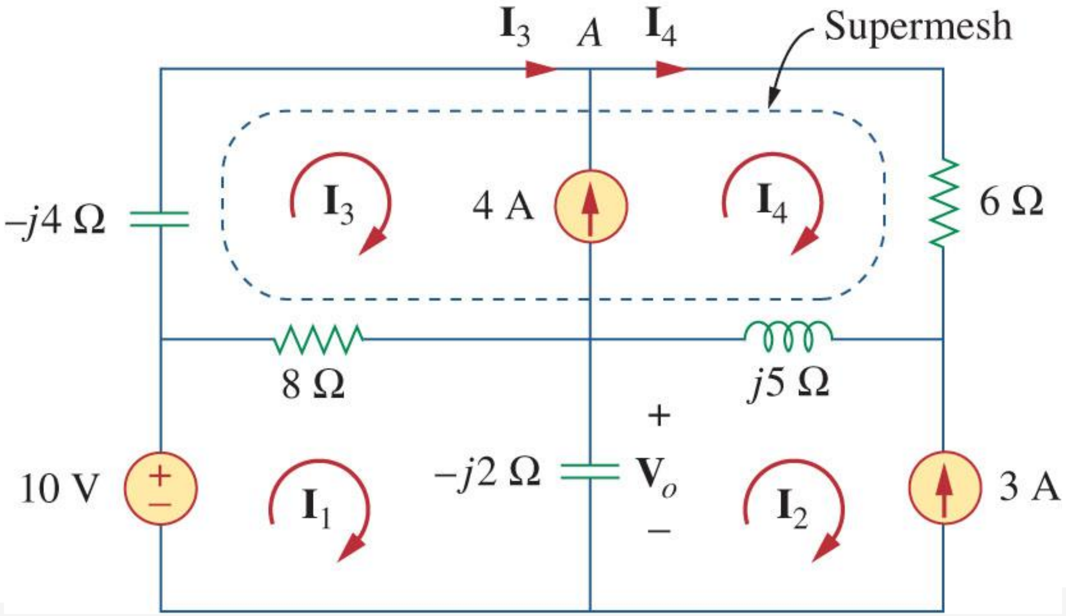
- The equivalency of the frequency domain treatment compared to the DC circuit analysis includes the use of supernodes.





Mesh Analysis

- Just as in KCL, the KVL analysis also applies to phasor and frequency domain circuits.
- The same rules apply: Convert to frequency domain first, then apply KVL as usual.
- In KVL, supermesh analysis is also valid.



Mesh Analysis

Determine current $\mathbf{I}_o$ in the circuit of Fig. 10.7 using mesh analysis.

Example 10.3

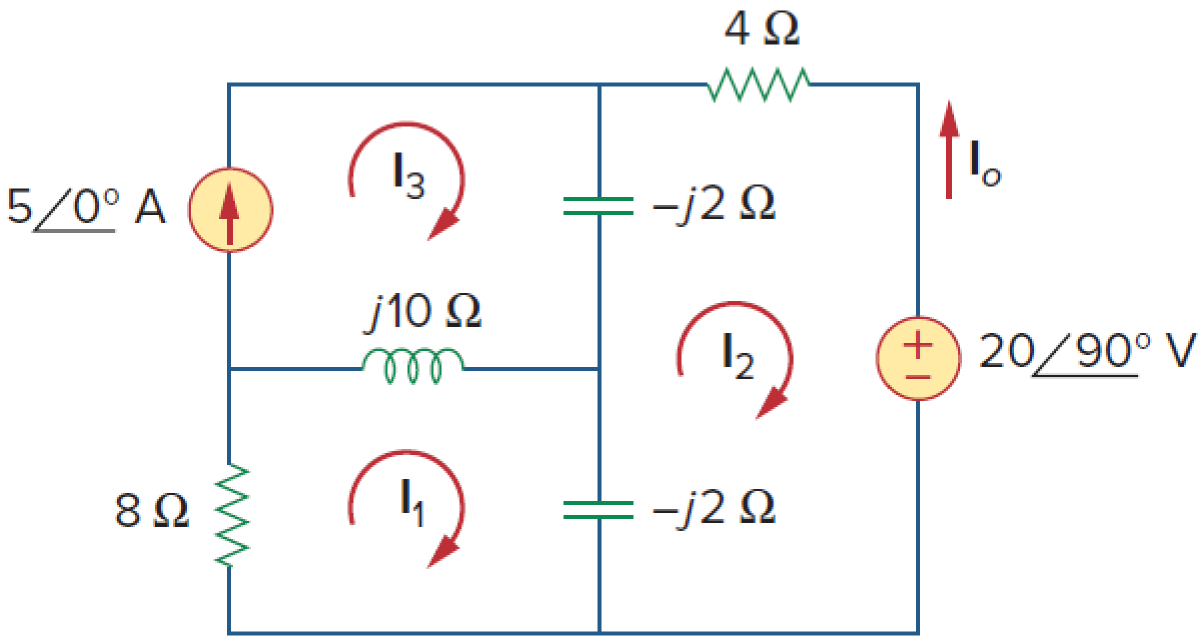


Figure 10.7
For Example 10.3.

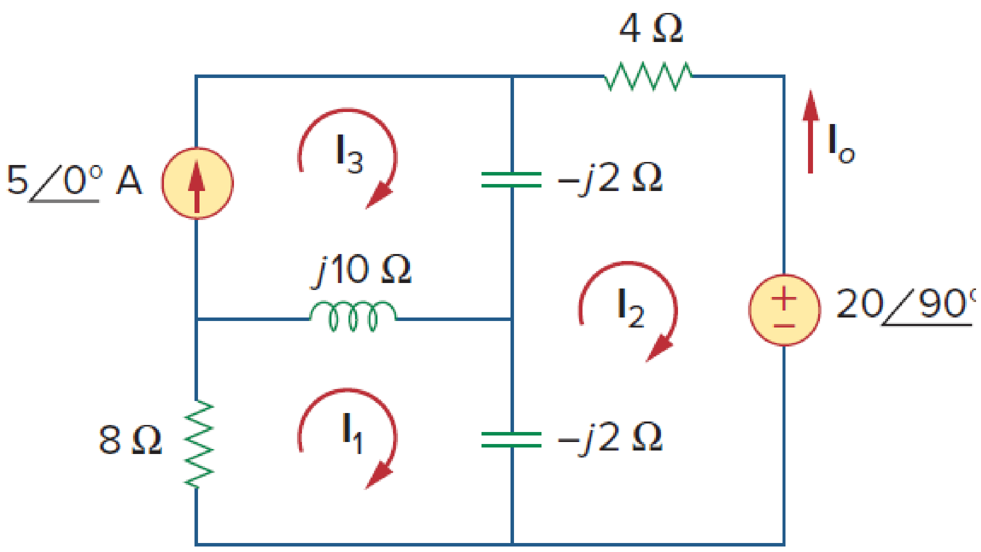


Figure 10.7
For Example 10.3.

Solution:
Applying KVL to mesh 1, we obtain

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0$$

For mesh 2,

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\,\underline{/90^\circ} = 0 \qquad \textbf{(10.3.2)}$$

For mesh 3, $\mathbf{I}_3 = 5$. Substituting this in Eqs. (10.3.1) and (10.3.2), we get

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50 \qquad \textbf{(10.3.3)}$$

$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10 \qquad \textbf{(10.3.4)}$$

Equations (10.3.3) and (10.3.4) can be put in matrix form as

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

from which we obtain the determinants

$$\Delta = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68$$

$$\Delta_2 = \begin{vmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\,\underline{/-35.22^\circ}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\,\underline{/-35.22^\circ}}{68} = 6.12\underline{/-35.22^\circ}\text{ A}$$

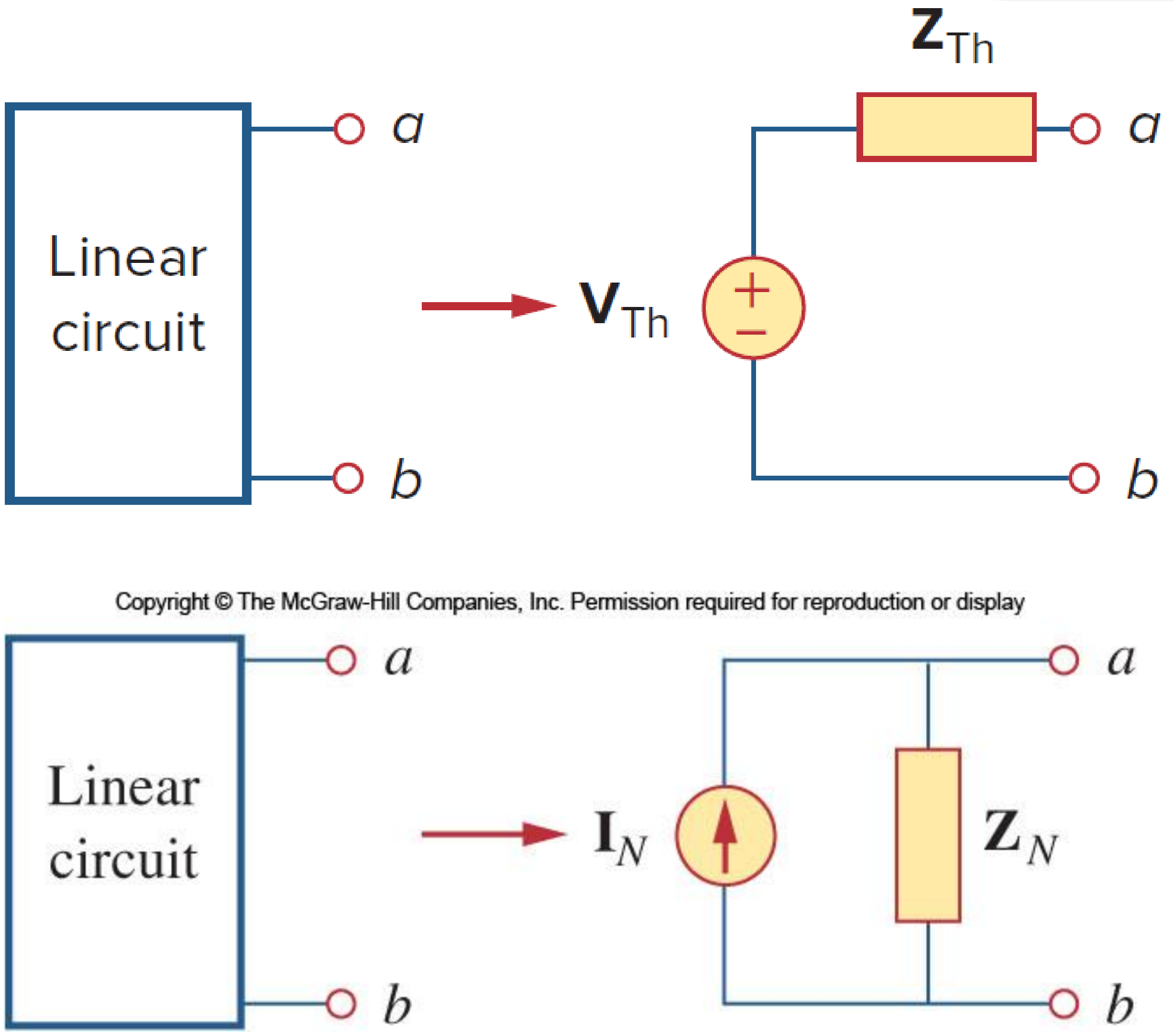
The desired current is

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\underline{/144.78^\circ}\text{ A}$$

Thevenin and Norton Theorem

- Both Thevenin and Norton's theorems are applied to AC circuits the same way as DC.
- The only difference is the fact that the calculated values will be complex.

$V_{Th} = Z_N I_N \quad Z_{Th} = Z_N$



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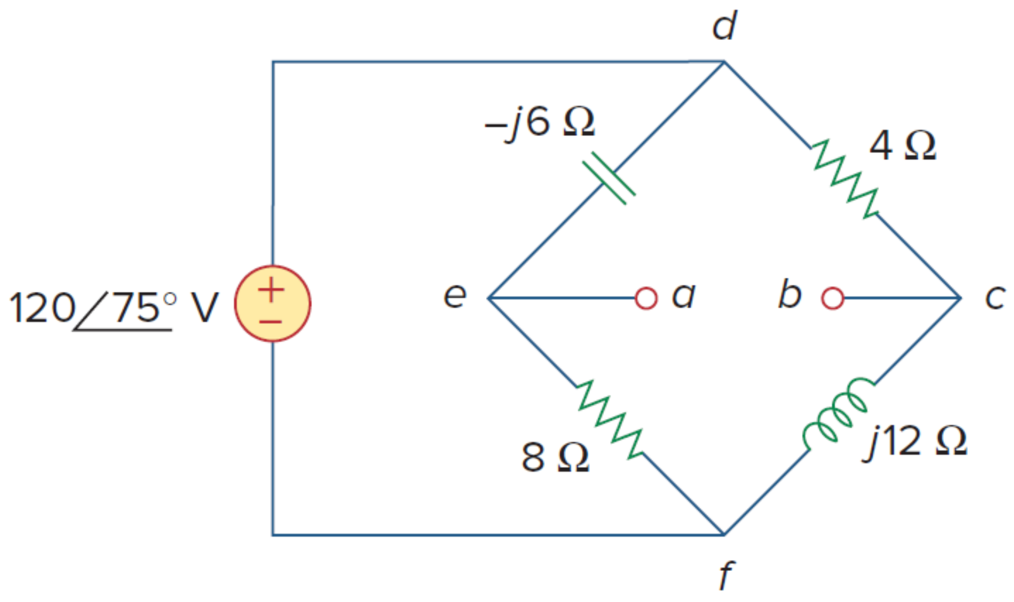
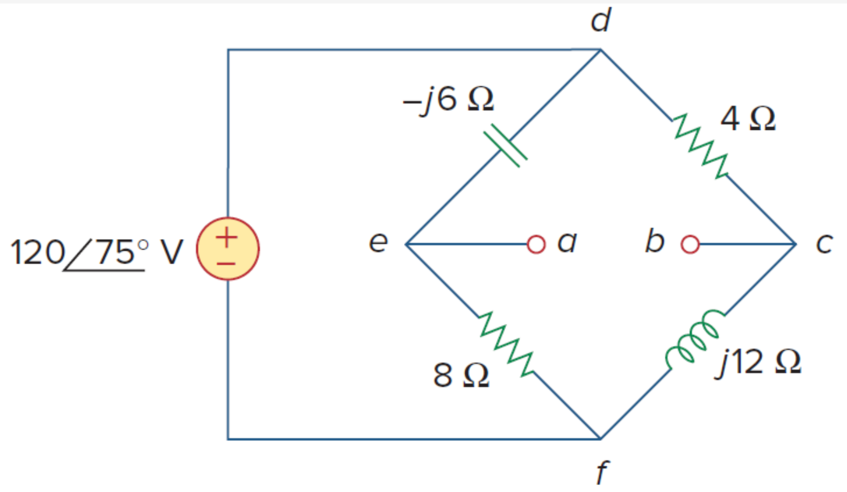
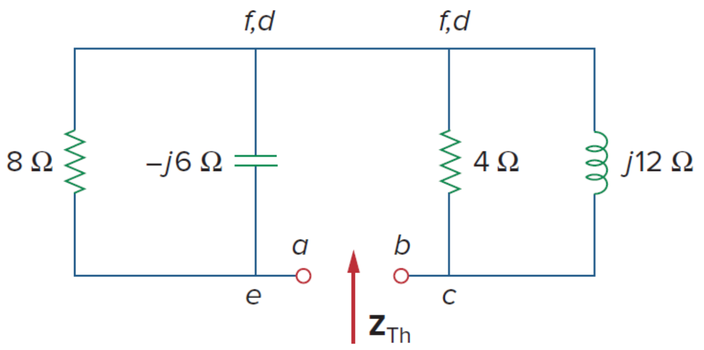
Thevenin and Norton Theorem

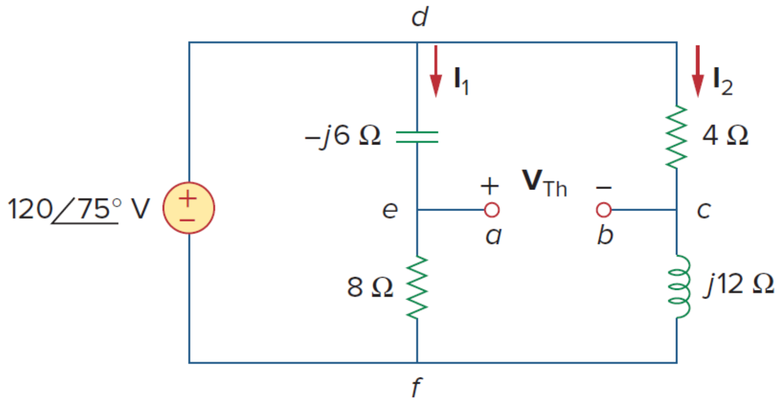
Example 10.8 Obtain the Thevenin equivalent at terminals *a-b* of the circuit in Fig. 10.22.



Thevenin theorem







Solution:

We find $\mathbf{Z}_{\text{Th}}$ by setting the voltage source to zero. As shown in Fig. 10.23(a), the 8-Ω resistance is now in parallel with the $-j6$ reactance, so that their combination gives

$$\mathbf{Z}_1 = -j6 \parallel 8 = \frac{-j6 \times 8}{8 - j6} = 2.88 - j3.84\ \Omega$$

Similarly, the 4-Ω resistance is in parallel with the $j12$ reactance, and their combination gives

$$\mathbf{Z}_2 = 4 \parallel j12 = \frac{j12 \times 4}{4 + j12} = 3.6 + j1.2\ \Omega$$

The Thevenin impedance is the series combination of $\mathbf{Z}_1$ and $\mathbf{Z}_2$; that is,

$$\mathbf{Z}_{\text{Th}} = \mathbf{Z}_1 + \mathbf{Z}_2 = 6.48 - j2.64\ \Omega$$

To find $\mathbf{V}_{\text{Th}}$, consider the circuit in Fig. 10.23(b). Currents $\mathbf{I}_1$ and $\mathbf{I}_2$ are obtained as

$$\mathbf{I}_1 = \frac{120\underline{/75^\circ}}{8 - j6}\ \text{A}, \qquad \mathbf{I}_2 = \frac{120\underline{/75^\circ}}{4 + j12}\ \text{A}$$

Applying KVL around loop *bcdeab* in Fig. 10.23(b) gives

$$\mathbf{V}_{\text{Th}} - 4\mathbf{I}_2 + (-j6)\mathbf{I}_1 = 0$$

or

$$\mathbf{V}_{\text{Th}} = 4\mathbf{I}_2 + j6\mathbf{I}_1 = \frac{480\underline{/75^\circ}}{4 + j12} + \frac{720\underline{/75^\circ + 90^\circ}}{8 - j6}$$

$$= 37.95\underline{/3.43^\circ} + 72\underline{/201.87^\circ}$$

$$= -28.936 - j24.55 = 37.95\underline{/220.31^\circ}\ \text{V}$$