Lecture 14

Capacitance & Dielectrics

Date: 5/27/2025

Course Instructor:
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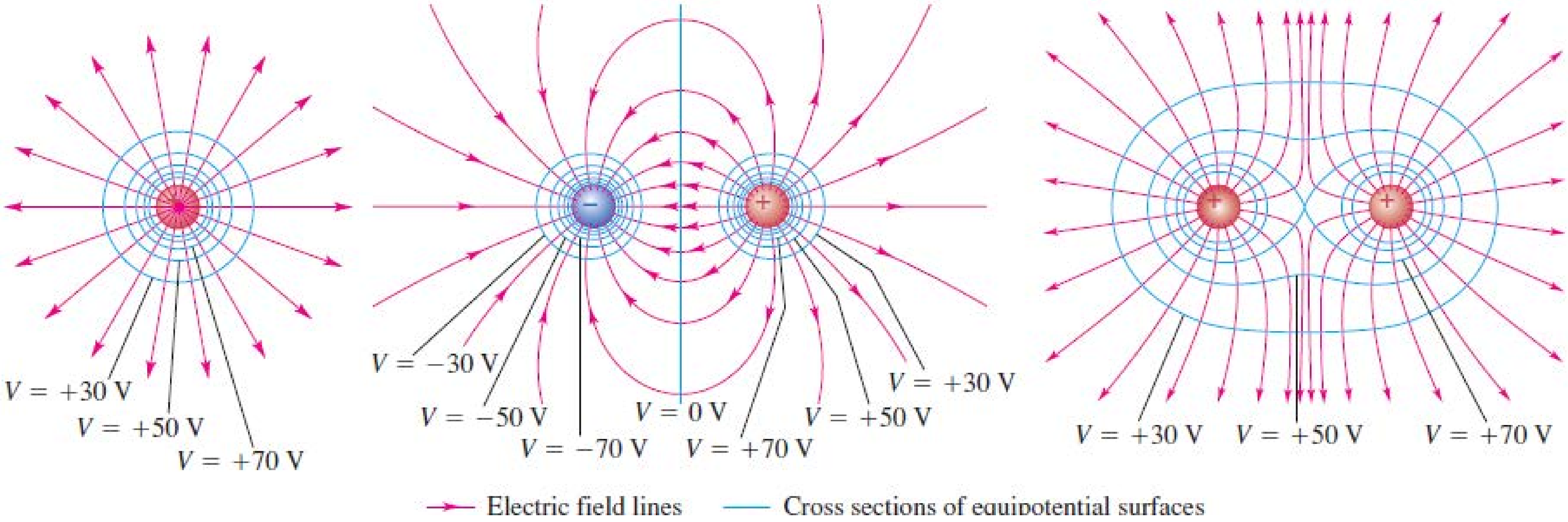
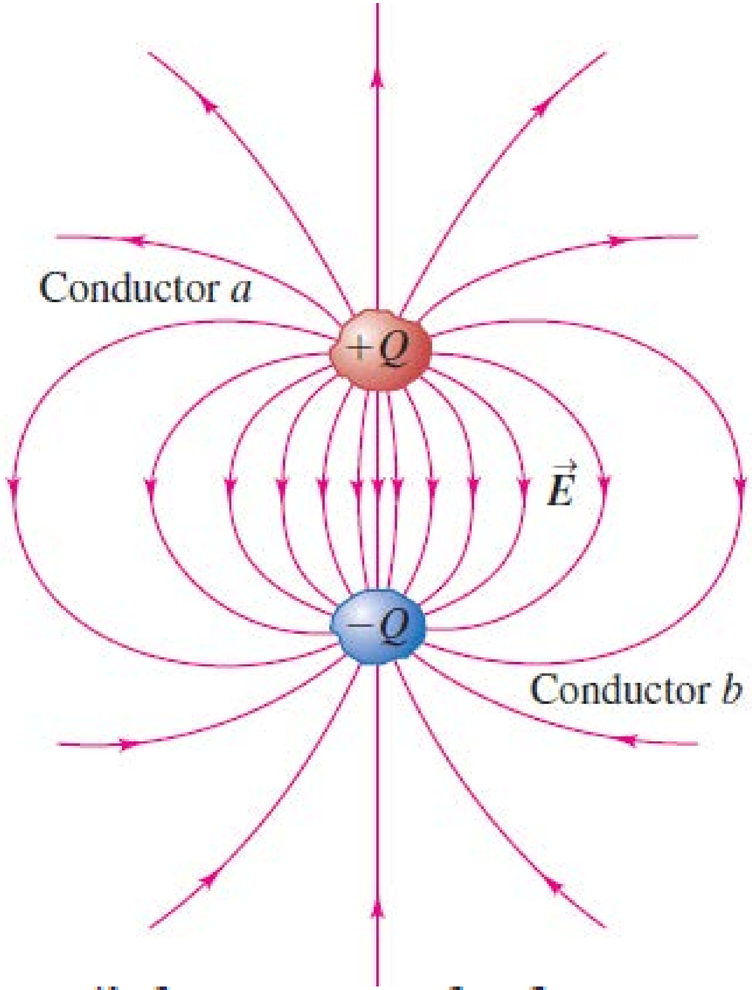
Previous Lecture: Electric Potential

Calculate potential energy from charge distribution ?



Capacitance: Definition and Unit

Any two conductors separated by an insulator (or a vacuum) form a **capacitor** which can store charges with a potential across it.

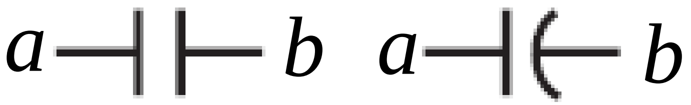


With a fixed *potential difference* V_{ab}, the stored charge Q *is related by*

$$C = \frac{Q}{V_{ab}} \qquad \text{(definition of capacitance)}$$

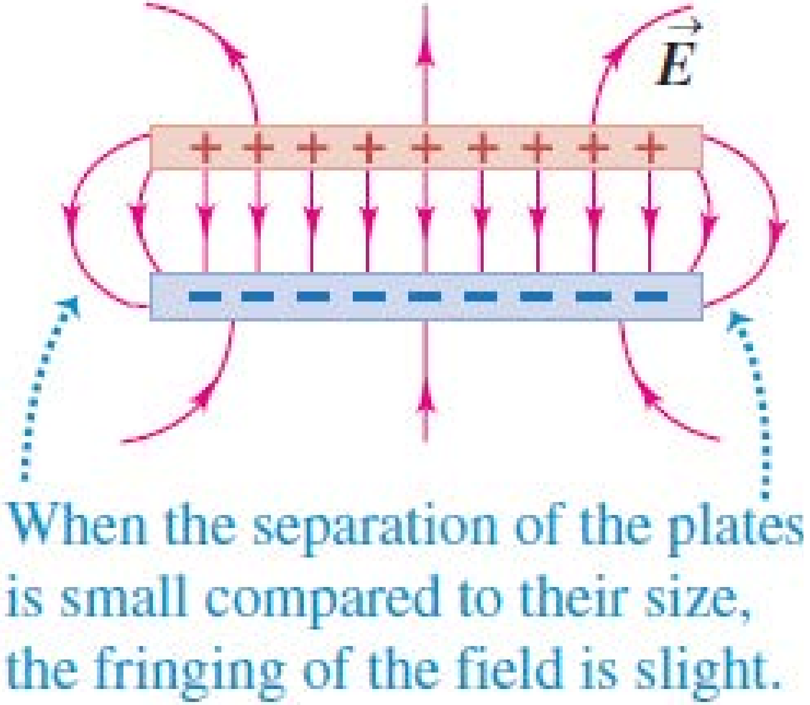
Unit: $1\ \mathrm{F} = 1\ \text{farad} = 1\ \mathrm{C/V} = 1\ \text{coulomb/volt}$

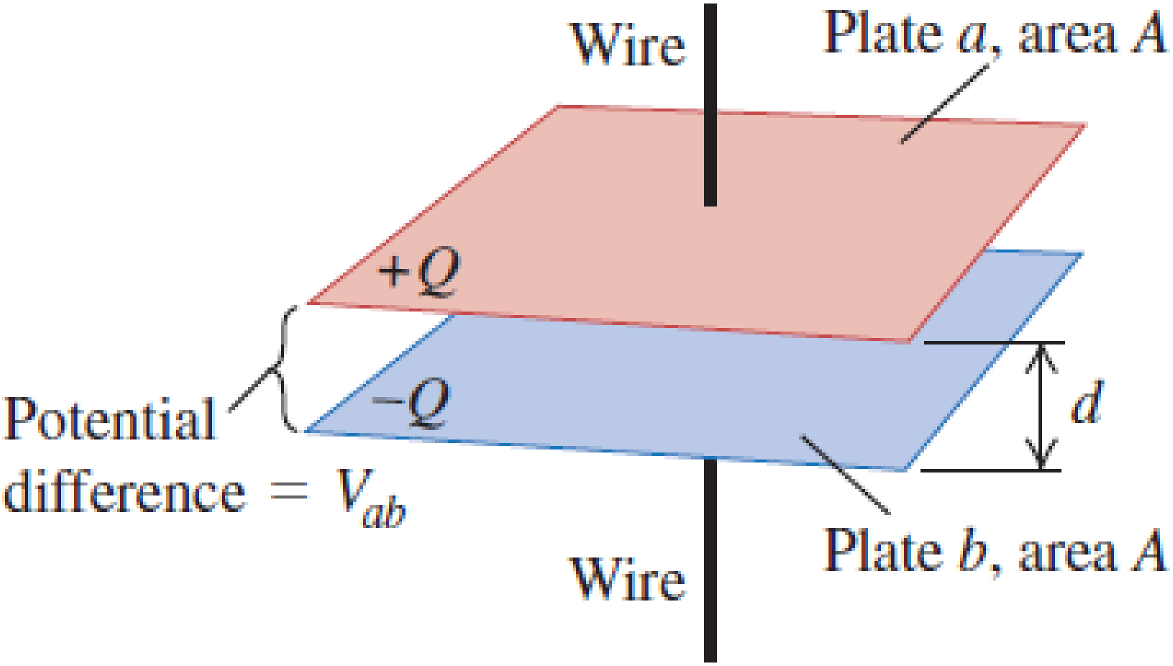
In circuit diagrams a capacitor is represented by either of these symbols:

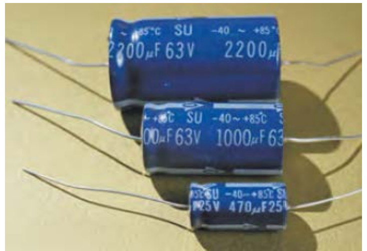


Capacitance: parallel-plate capacitor

Simplest form of capacitor consists of two parallel conducting plates, each with area A separated by a distance d that is **small** in comparison with their dimensions. ***This is the only capacitor model for this class!***







$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}$$ (capacitance of a parallel-plate capacitor in vacuum)

Example 24.1 Size of a 1-F capacitor

The parallel plates of a 1.0-F capacitor are 1.0 mm apart. What is their area?

IDENTIFY and SET UP: This problem uses the relationship among the capacitance C, plate separation d, and plate area A (our target variable) for a parallel-plate capacitor. We solve Eq. (24.2) for A.

Recall what we just learned $C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}$

where ϵ_0 is a constant $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$$A = \frac{Cd}{\epsilon_0} = \frac{(1.0\ \mathrm{F})(1.0 \times 10^{-3}\ \mathrm{m})}{8.85 \times 10^{-12}\ \mathrm{F/m}} = 1.1 \times 10^{8}\ \mathrm{m}^2$$

Example 24.2 Properties of a parallel-plate capacitor

The plates of a parallel-plate capacitor in vacuum are 5.0 mm apart and 2.0 m^2 in area. A10.0-kV potential difference is applied across the capacitor. Compute (a) the capacitance; (b) the charge on each plate; and (c) the magnitude of the electric field between the plates.

Approach: an easy start is to write up the equation and identify the knowns and unknowns

$$C = \frac{\boxed{Q}}{\boxed{V_{ab}}} = \epsilon_0 \frac{\boxed{A}}{\boxed{d}} \qquad \text{(capacitance of a parallel-plate capacitor in vacuum)}$$

$$C = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}\ \mathrm{F/m}) \frac{(2.00\ \mathrm{m}^2)}{5.00 \times 10^{-3}\ \mathrm{m}}$$

$$= 3.54 \times 10^{-9}\ \mathrm{F} = 0.00354\ \mu\mathrm{F}$$

Example 24.2 Properties of a parallel-plate capacitor

The plates of a parallel-plate capacitor in vacuum are 5.0 mm apart and 2.0 m^2 in area. A10.0-kV potential difference is applied across the capacitor. Compute (a) the capacitance; (b) the charge on each plate; and (c) the magnitude of the electric field between the plates.

Approach: an easy start is to write up the equation and identify the knowns and unknowns

$$C = \frac{\boxed{Q}}{\boxed{V_{ab}}} = \epsilon_0 \frac{\boxed{A}}{\boxed{d}} \qquad \text{(capacitance of a parallel-plate capacitor in vacuum)}$$

$$Q = CV_{ab} = (3.54 \times 10^{-9}\ \mathrm{C/V})(1.00 \times 10^{4}\ \mathrm{V})$$
$$= 3.54 \times 10^{-5}\ \mathrm{C} = 35.4\ \mu\mathrm{C}$$

Example 24.2 Properties of a parallel-plate capacitor

The plates of a parallel-plate capacitor in vacuum are 5.0 mm apart and 2.0 m^2 in area. A10.0-kV potential difference is applied across the capacitor. Compute (c) the magnitude of the electric field between the plates.

(c) The electric-field magnitude is

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} = \frac{3.54 \times 10^{-5}\ \mathrm{C}}{(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(2.00\ \mathrm{m^2})}$$

$$= 2.00 \times 10^{6}\ \mathrm{N/C}$$

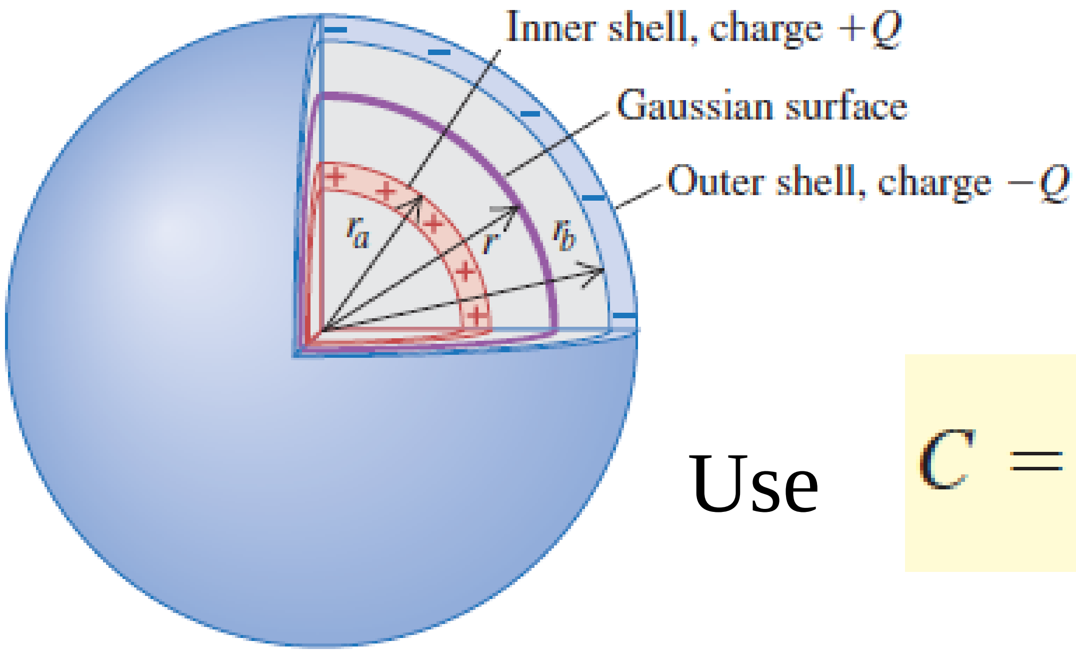
$$E = \frac{V_{ab}}{d} = \frac{1.00 \times 10^{4}\ \mathrm{V}}{5.00 \times 10^{-3}\ \mathrm{m}} = 2.00 \times 10^{6}\ \mathrm{V/m}$$

Example 24.3 A spherical capacitor

Two concentric spherical conducting shells are separated by vacuum. The inner shell has total charge +Q and outer radius r_a and the outer shell has charge -Q and inner radius r_b. Find the capacitance of this spherical capacitor.

Solution: Seems complicated by recall last chapter.

24.5 A spherical capacitor.



$$V_{ab} = V_a - V_b = \frac{Q}{4\pi\epsilon_0 r_a} - \frac{Q}{4\pi\epsilon_0 r_b}$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = \frac{Q}{4\pi\epsilon_0}\frac{r_b - r_a}{r_a r_b}$$

Use $C = \dfrac{Q}{V_{ab}}$ So $C = \dfrac{Q}{V_{ab}} = 4\pi\epsilon_0 \dfrac{r_a r_b}{r_b - r_a}$

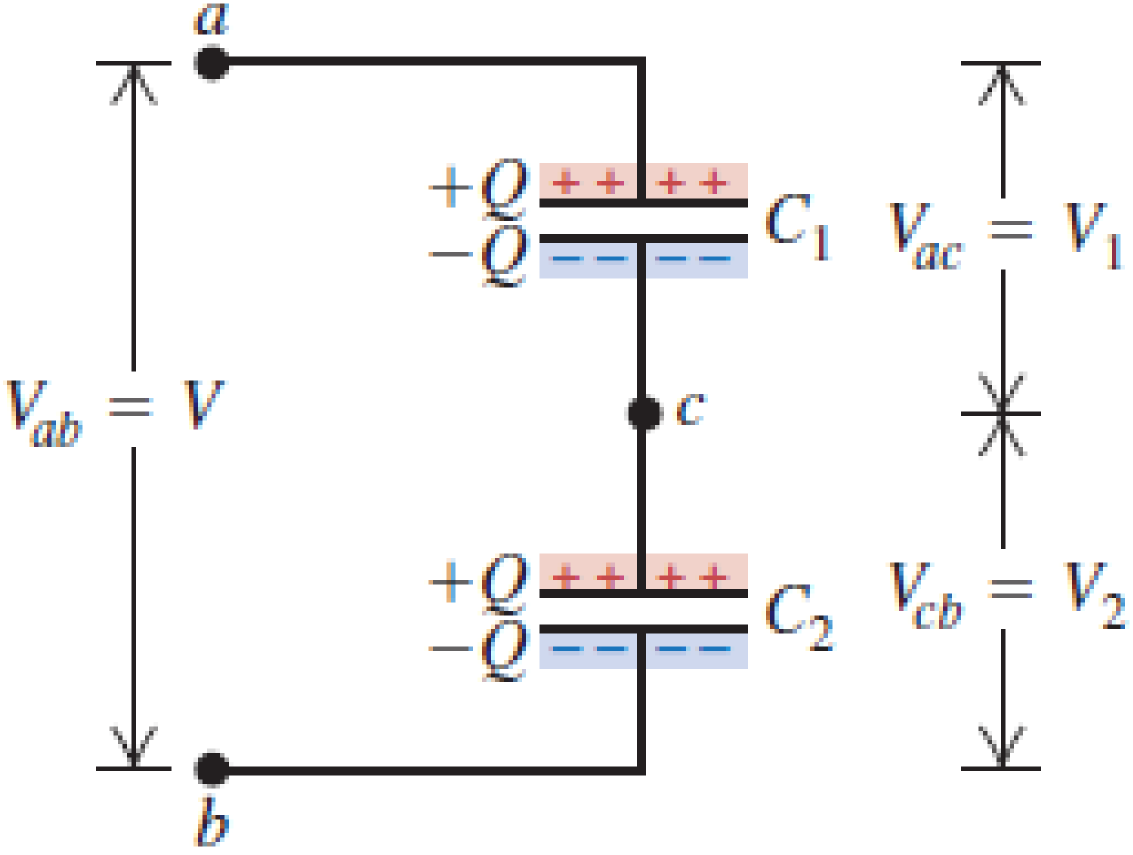
Capacitors in Series

Capacitors in series:

- The capacitors have the same charge
- Their potential differences add:

$$V_{ac} + V_{cb} = V_{ab}.$$

Why? Charge conservation



$$V_{ac} = V_1 = \frac{Q}{C_1} \qquad V_{cb} = V_2 = \frac{Q}{C_2}$$

$$V_{ab} = V = V_1 + V_2 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

So: $$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitors in Series

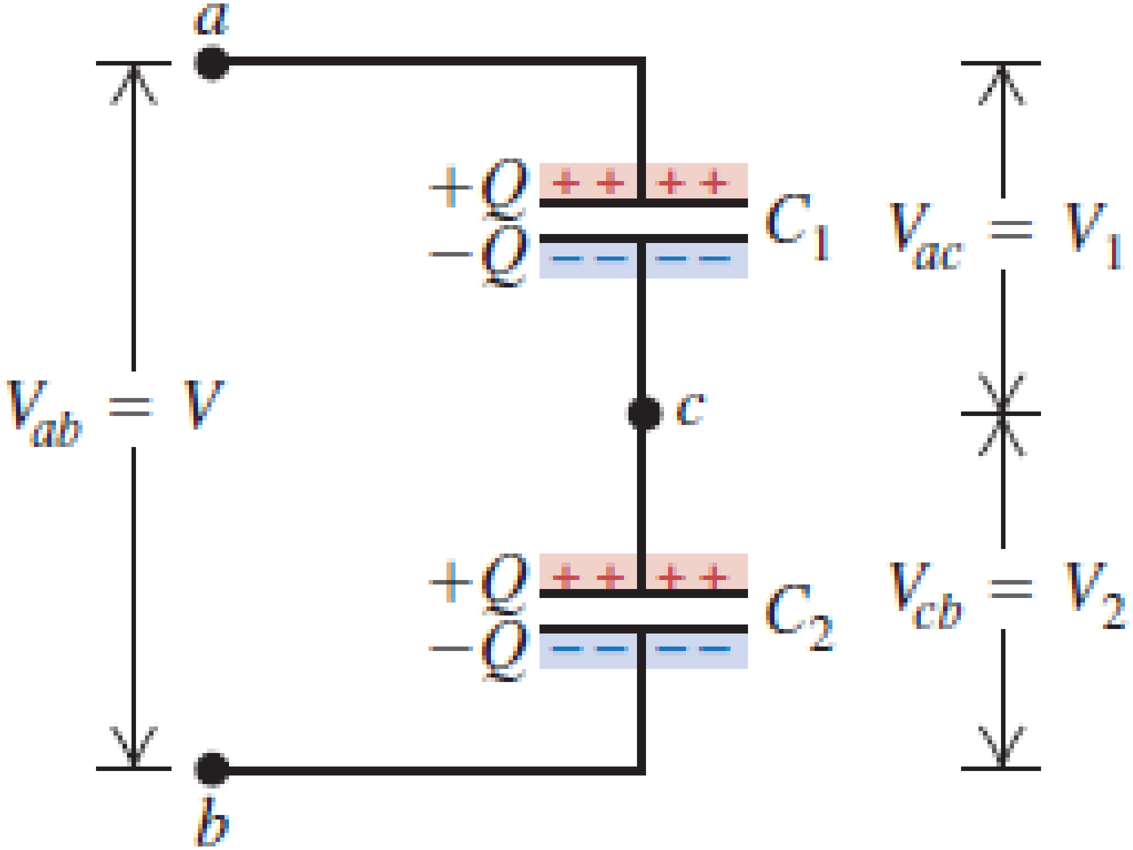
Capacitors in series:

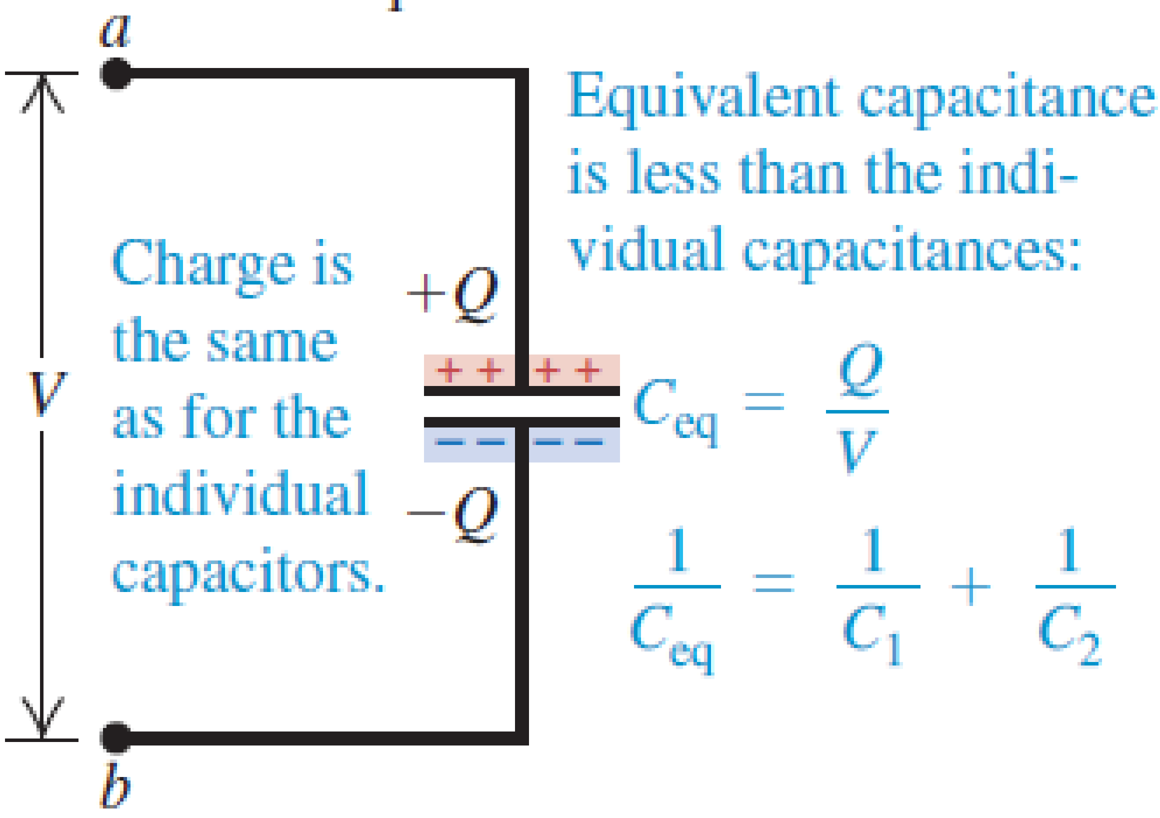
- The capacitors have the same charge
- Their potential differences add:

$V_{ac} + V_{cb} = V_{ab}.$

$$C_{eq} = \frac{Q}{V} \quad \text{or} \quad \frac{1}{C_{eq}} = \frac{V}{Q}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$





Capacitors in Series

The reciprocal of the equivalent capacitance of a series combination equals the sum of the reciprocals of the individual capacitances. In a series connection the equivalent capacitance is always *less than* any individual capacitance.

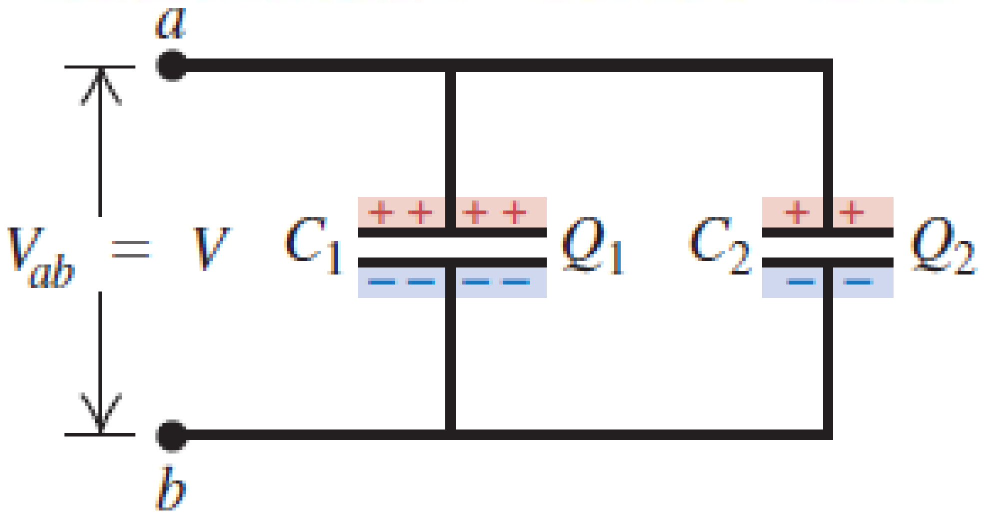
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \qquad \text{(capacitors in series)}$$

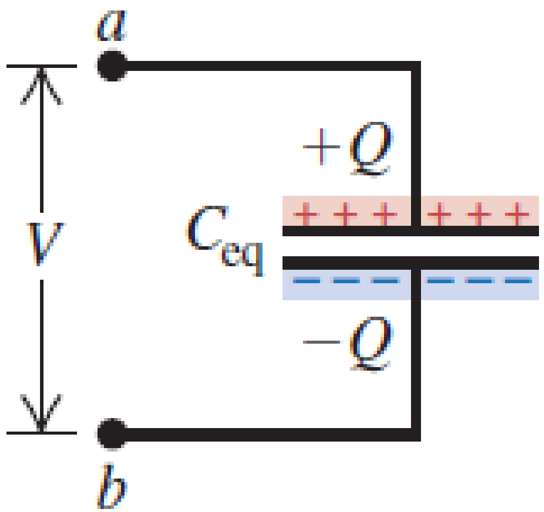
$$V_{total} = V_1 + V_2 + V_3 + \cdots$$

Capacitors in Parallel

Capacitors in parallel:

- The capacitors have the same potential V.
- The charge on each capacitor depends on its capacitance: $Q_1 = C_1V$, $Q_2 = C_2V$.





Charge is the sum of the individual charges:
$Q = Q_1 + Q_2$

Equivalent capacitance:
$C_{eq} = C_1 + C_2$

Hence *in a parallel connection the potential difference for all individual capacitors is the same:* $V_{ab} = V$

$$Q_1 = C_1V \quad \text{and} \quad Q_2 = C_2V$$

$$Q = Q_1 + Q_2 = (C_1 + C_2)V$$

$$\frac{Q}{V} = C_1 + C_2$$

$$C_{eq} = C_1 + C_2$$

Capacitors in Parallel

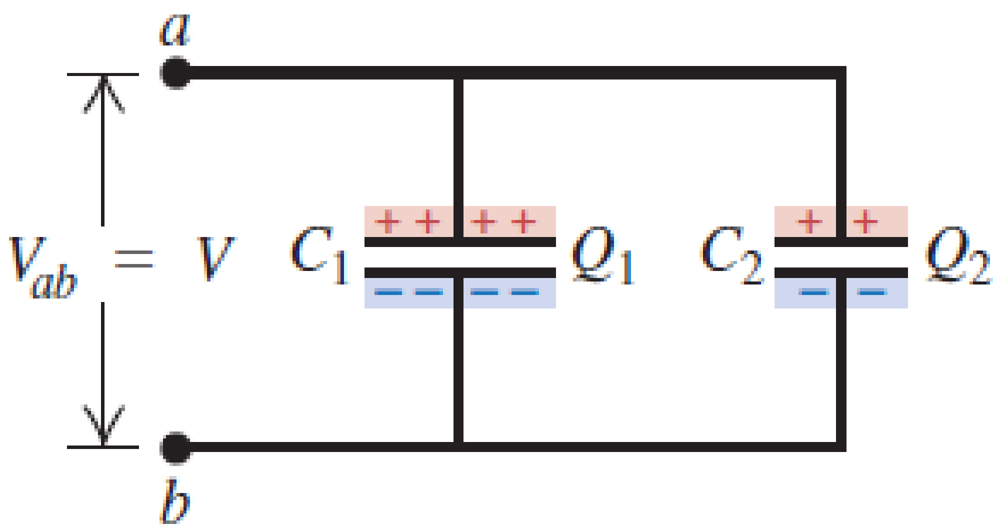
$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(capacitors in parallel)} \qquad (24.7)$$

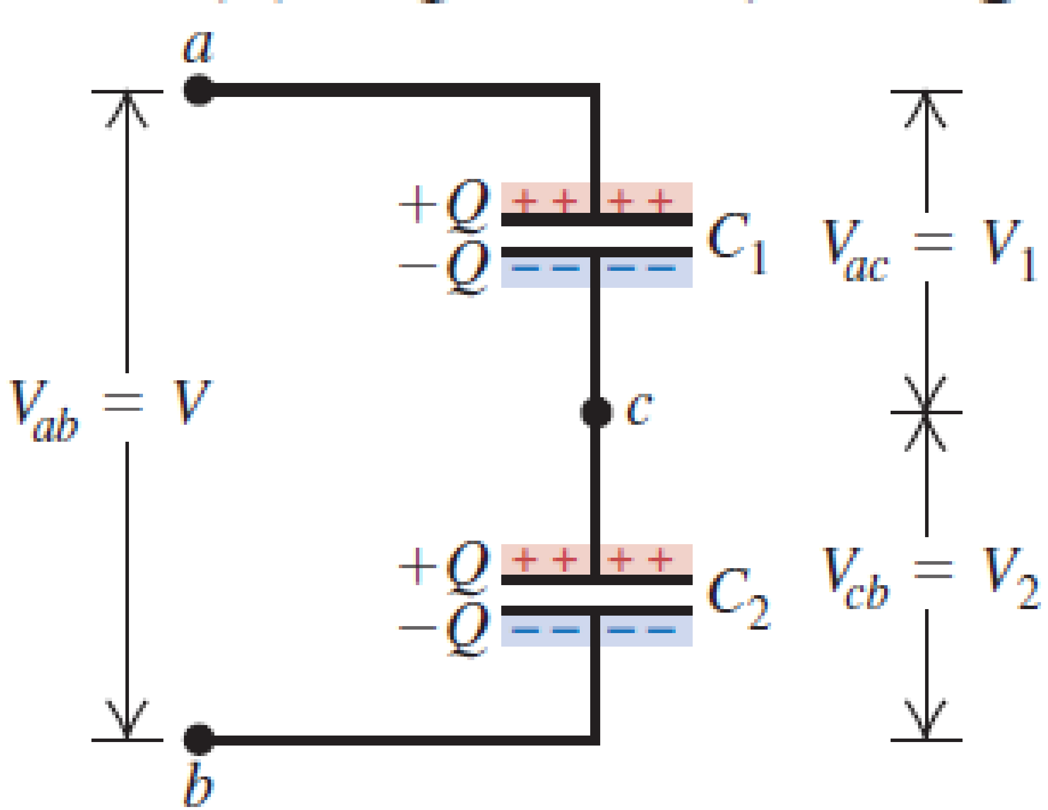
The equivalent capacitance of a parallel combination equals the *sum* of the individual capacitances. In a parallel connection the equivalent capacitance is always *greater than* any individual capacitance.

CAUTION **Capacitors in parallel** The potential differences are the same for all the capacitors in a parallel combination; however, the charges on individual capacitors are *not* the same unless their individual capacitances are the same. The charges on the individual capacitors add to give the total charge on the parallel combination: $Q_{total} = Q_1 + Q_2 + Q_3 + \cdots$. [Compare these statements to those in the "Caution" paragraph following Eq. (24.5).]

Example 24.5 Capacitors in series & in parallel

In Figs. 24.8 and 24.9, let $C_1 = 6.0\ \mu\text{F}$, $C_2 = 3.0\ \mu\text{F}$, and $V_{ab} = 18\ \text{V}$. Find the equivalent capacitance and the charge and potential difference for each capacitor when the capacitors are connected (a) in series (see Fig. 24.8) and (b) in parallel (see Fig. 24.9).





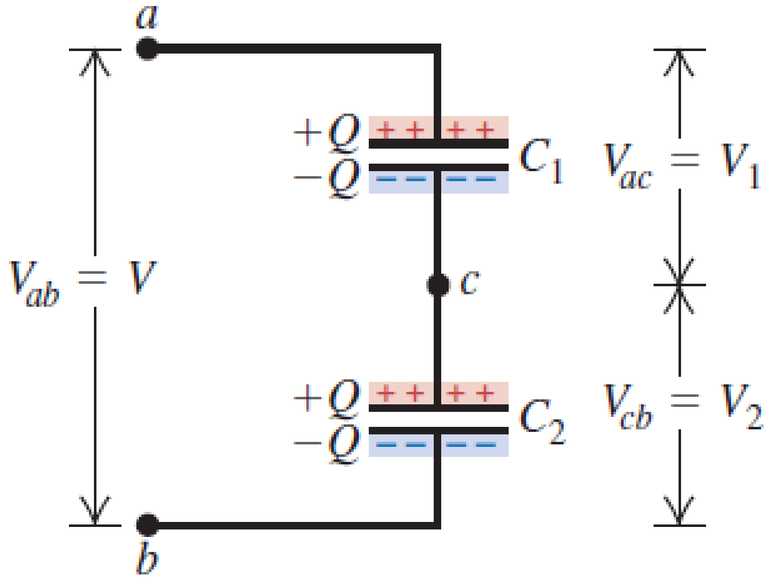
Example 24.5 Capacitors in series & in parallel

In Figs. 24.8 and 24.9, let $C_1 = 6.0\ \mu\text{F}$, $C_2 = 3.0\ \mu\text{F}$, and $V_{ab} = 18\ \text{V}$. Find the equivalent capacitance and the charge and potential difference for each capacitor when the capacitors are connected (a) in series (see Fig. 24.8) and (b) in parallel (see Fig. 24.9).

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{6.0\ \mu\text{F}} + \frac{1}{3.0\ \mu\text{F}} \qquad C_{eq} = 2.0\ \mu\text{F}$$

The charge Q on each capacitor in series is the same as that on the equivalent capacitor.

$$Q = C_{eq}V = (2.0\ \mu\text{F})(18\ \text{V}) = 36\ \mu\text{C}$$



Example 24.5 Capacitors in series & in parallel

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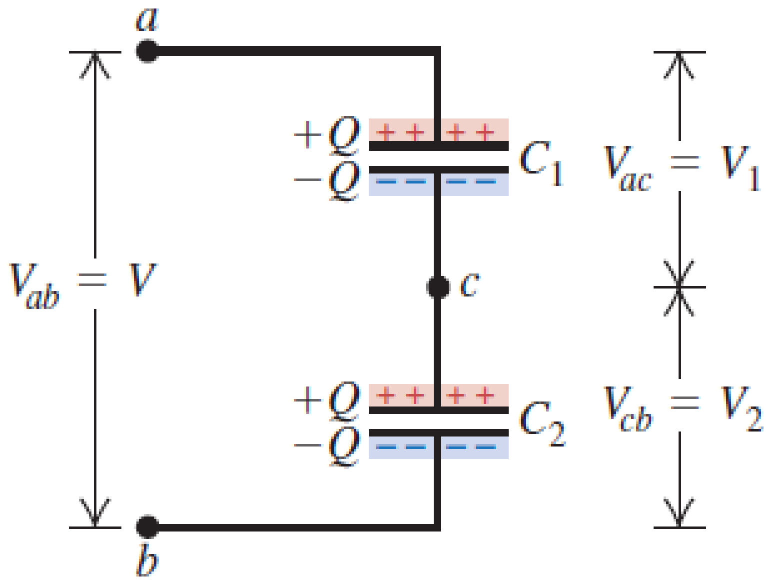
$$V_{ac} = V_1 = \frac{Q}{C_1} = \frac{36\ \mu\text{C}}{6.0\ \mu\text{F}} = 6.0\ \text{V}$$

$$V_{cb} = V_2 = \frac{Q}{C_2} = \frac{36\ \mu\text{C}}{3.0\ \mu\text{F}} = 12.0\ \text{V}$$



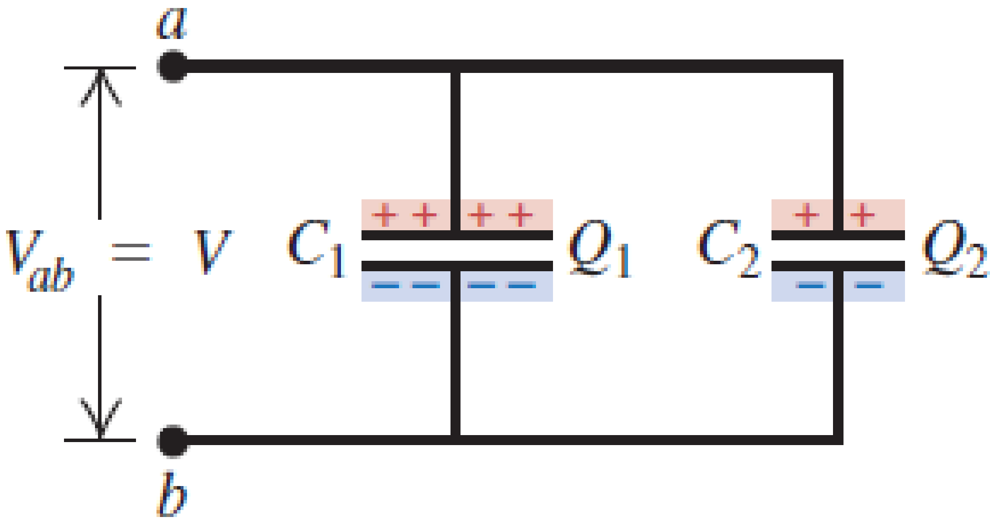
Example 24.5 Capacitors in series & in parallel

In Figs. 24.8 and 24.9, let $C_1 = 6.0\ \mu\text{F}$, $C_2 = 3.0\ \mu\text{F}$, and $V_{ab} = 18$ V. Find the equivalent capacitance and the charge and potential difference for each capacitor when the capacitors are connected (a) in series (see Fig. 24.8) and (b) in parallel (see Fig. 24.9).

$$C_{eq} = C_1 + C_2 = 6.0\ \mu\text{F} + 3.0\ \mu\text{F} = 9.0\ \mu\text{F}$$

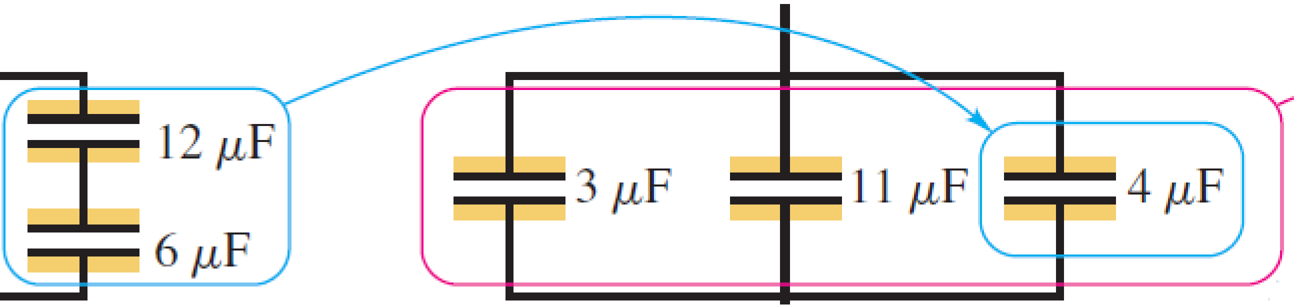
$$Q_1 = C_1 V = (6.0\ \mu\text{F})(18\ \text{V}) = 108\ \mu\text{C}$$

$$Q_2 = C_2 V = (3.0\ \mu\text{F})(18\ \text{V}) = 54\ \mu\text{C}$$

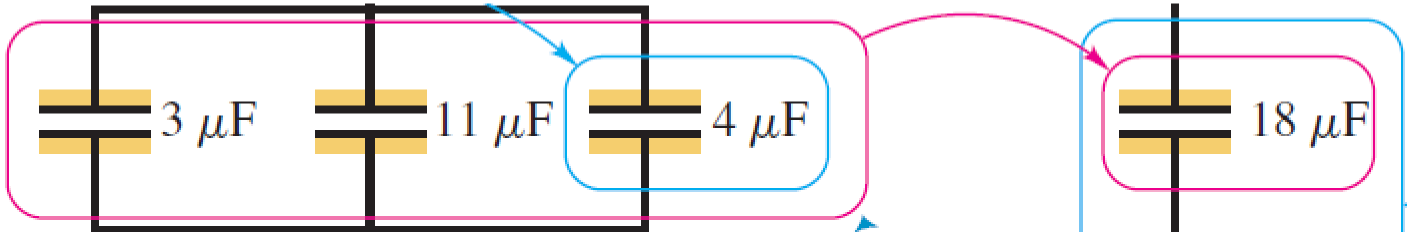


Example 24.6 A capacitor network

Find the equivalent capacitance of the five-capacitor network



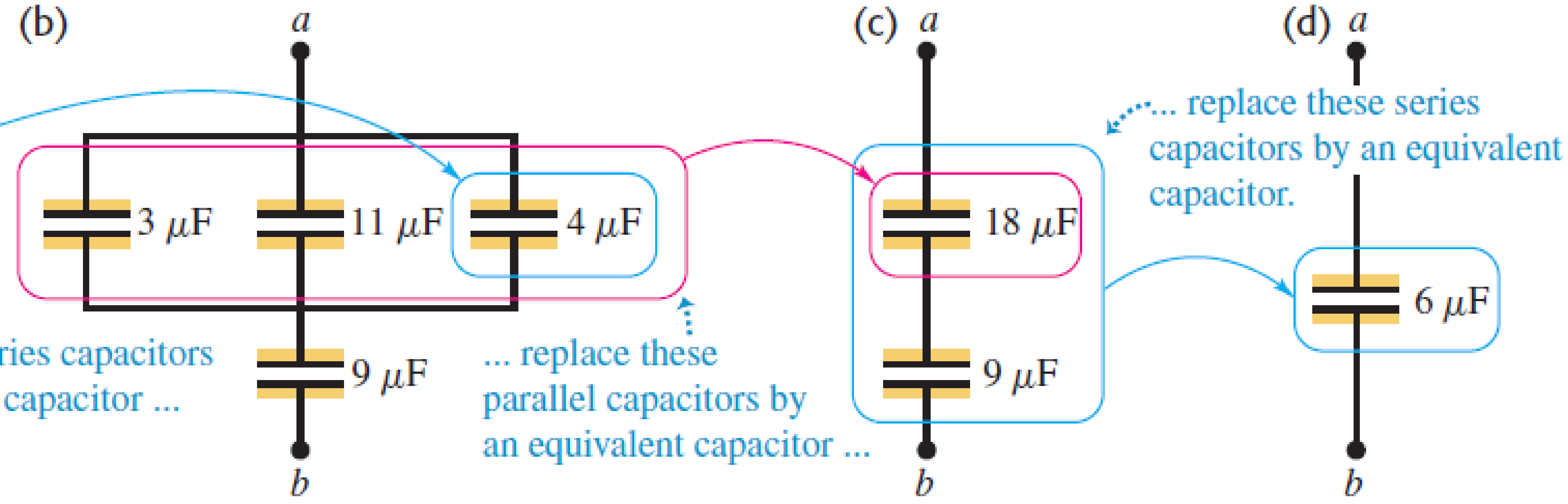
$$\frac{1}{C'} = \frac{1}{12\ \mu\text{F}} + \frac{1}{6\ \mu\text{F}} \qquad C' = 4\ \mu\text{F}$$



$$C'' = 3\ \mu\text{F} + 11\ \mu\text{F} + 4\ \mu\text{F} = 18\ \mu\text{F}$$

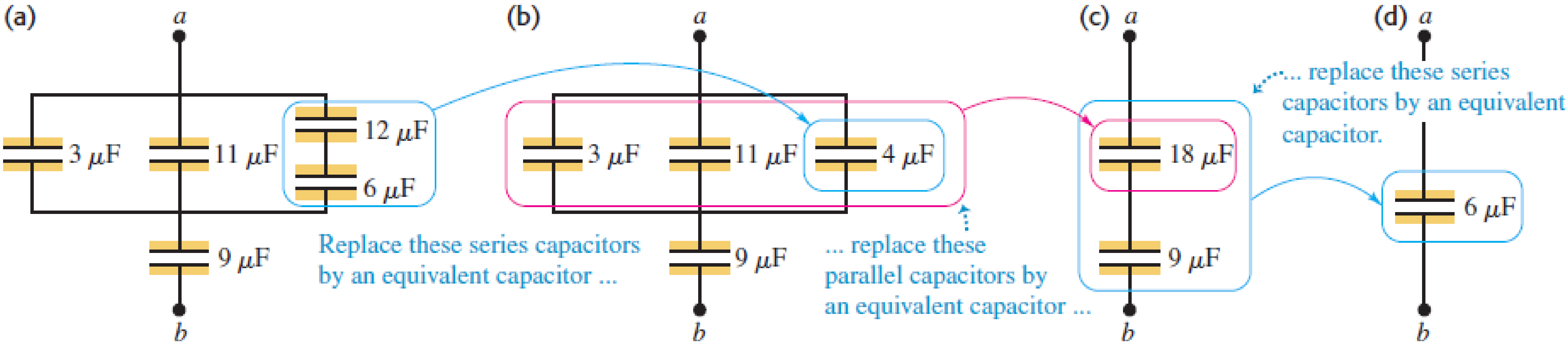
$$\frac{1}{C_{eq}} = \frac{1}{18\ \mu\text{F}} + \frac{1}{9\ \mu\text{F}} \qquad C_{eq} = 6\ \mu\text{F}$$

Example 24.6 A capacitor network



$$\frac{1}{C_{eq}} = \frac{1}{18\ \mu F} + \frac{1}{9\ \mu F} \qquad C_{eq} = 6\ \mu F$$

Example 24.6 A capacitor network



Energy Storage in Capacitors & Electric-Field Energy

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV$$ (potential energy stored in a capacitor)

$$u = \tfrac{1}{2}\epsilon_0 E^2$$ (electric energy density in a vacuum)

Two only equation for this section

Energy Storage in Capacitors & Electric-Field Energy

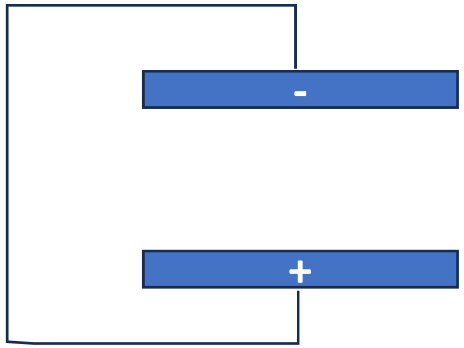
Suppose that when we are done charging the capacitor, the final charge is *Q* and the final potential difference is *V* and

$$V = \frac{Q}{C}$$

Let *q* and *v* be the charge and potential difference, respectively, at an intermediate stage during the charging process; then $v = q/C$

$$dW = v\,dq = \frac{q\,dq}{C}$$

$$W = \int_0^W dW = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$



The intuition is: to charge the capacitor, the more charge it stores, the harder it is to put more energy/charge inside

Energy Storage in Capacitors & Electric-Field Energy

To develop this relationship, let's find the energy *per unit volume* in the space between the plates of a parallel-plate capacitor with plate area and separation *d*. We call this the **energy density**, denoted by *u*

$$u = \text{Energy density} = \frac{\frac{1}{2}CV^2}{Ad}$$

But capacitance C is given by $C = \epsilon_0 A/d$

and $V = Ed$

So we get $\boxed{u = \frac{1}{2}\epsilon_0 E^2}$

Energy Storage in Capacitors & Electric-Field Energy

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV \qquad \text{(potential energy stored in a capacitor)}$$

$$u = \tfrac{1}{2}\epsilon_0 E^2 \qquad \text{(electric energy density in a vacuum)}$$

CAUTION Electric-field energy is electric potential energy It's a common misconception that electric-field energy is a new kind of energy, different from the electric potential energy described before. This is *not* the case; it is simply a different way of interpreting electric potential energy. We can regard the energy of a given system of charges as being a shared property of all the charges, or we can think of the energy as being a property of the electric field that the charges create. Either interpretation leads to the same value of the potential energy. ❙

Example 24.7 Transferring charge & energy between capacitors

We connect a capacitor $C_1 = 8.0\ \mu\text{F}$ to a power supply, charge it to a potential difference $V_0 = 120\ \text{V}$, and disconnect the power supply (Fig. 24.12). Switch S is open. (a) What is the charge Q_0 on C_1? (b) What is the energy stored in C_1? (c) Capacitor $C_2 = 4.0\ \mu\text{F}$ is initially uncharged. We close switch S. After charge no longer flows, what is the potential difference across each capacitor, and what is the charge on each capacitor? (d) What is the final energy of the system?

Example 24.7 Transferring charge & energy between capacitors

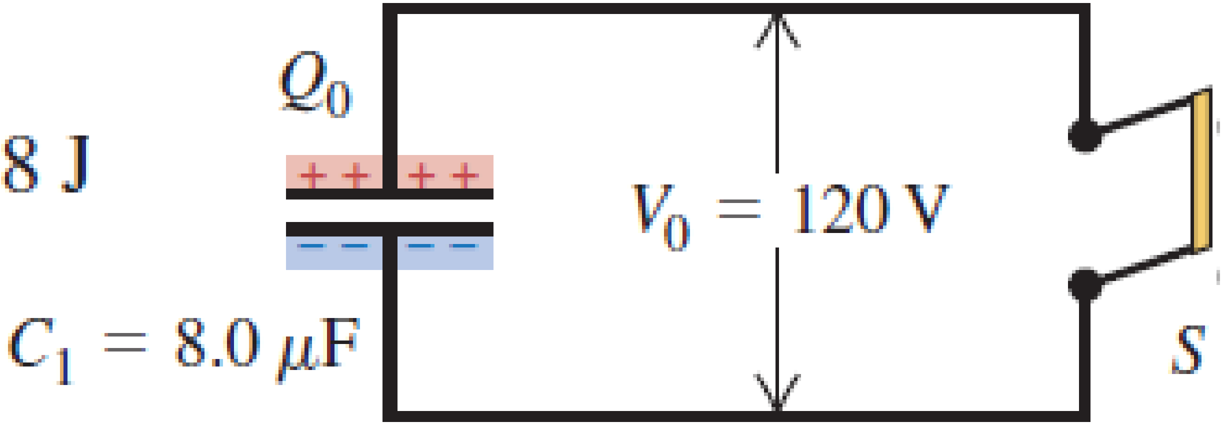
We connect a capacitor $C_1 = 8.0\ \mu\text{F}$ to a power supply, charge it to a potential difference $V_0 = 120\text{ V}$, and disconnect the power supply (Fig. 24.12). Switch S is open. (a) What is the charge Q_0 on C_1? (b) What is the energy stored in C_1? (c) Capacitor

(a) The initial charge Q_0 on C_1 is

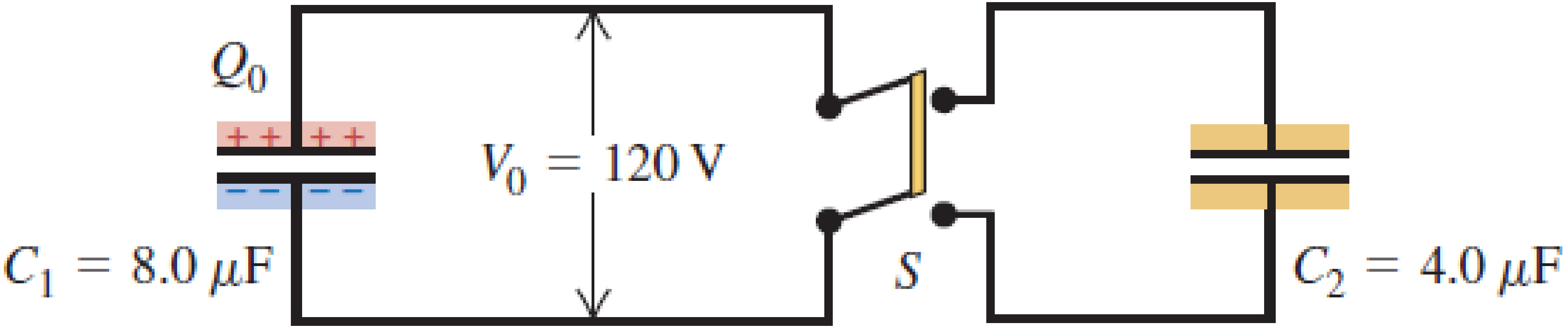
$$Q_0 = C_1 V_0 = (8.0\ \mu\text{F})(120\text{ V}) = 960\ \mu\text{C}$$

The energy initially stored in C_1

$$U_{\text{initial}} = \tfrac{1}{2} Q_0 V_0$$
$$= \tfrac{1}{2}(960 \times 10^{-6}\text{ C})(120\text{ V}) = 0.058\text{ J}$$



Example 24.7 Transferring charge & energy between capacitors

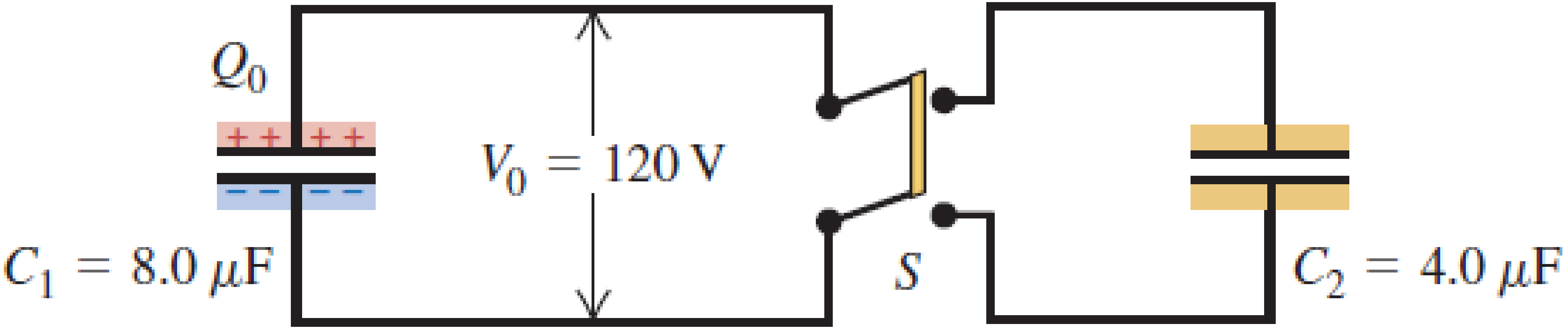


When the switch is closed, the charged capacitor is connected to an uncharged capacitor. The center part of the switch is an insulating handle; charge can flow only between the two upper terminals and between the two lower terminals.

- **Until when? When the potential across the two sides of the switch becomes the same** and this potential is not V_0
- Total charge conserves and the sum is always Q_0

$$Q_1 + Q_2 = Q_0 \qquad Q_1 = C_1V \text{ and } Q_2 = C_2V$$

Example 24.7 Transferring charge & energy between capacitors

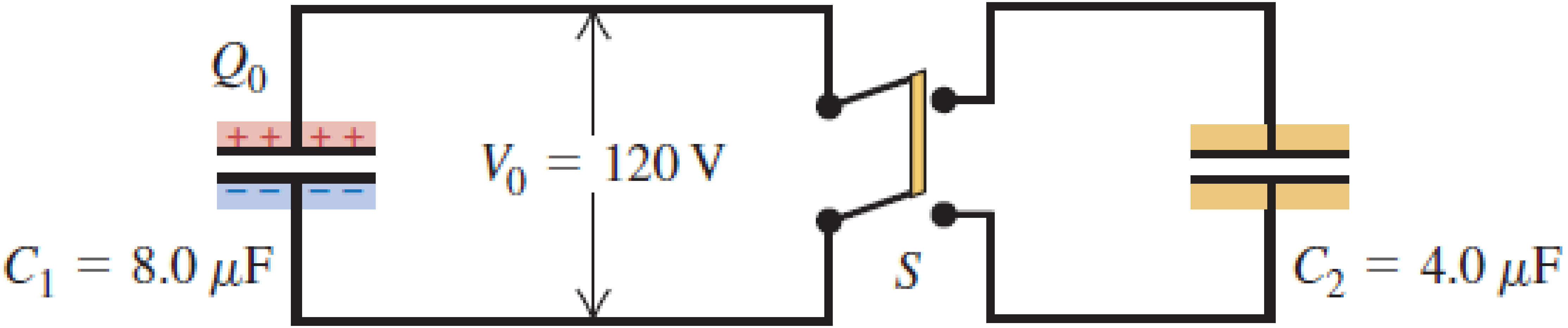


$$Q_1 + Q_2 = Q_0 \qquad Q_1 = C_1 V \text{ and } Q_2 = C_2 V$$

$$V = \frac{Q_0}{C_1 + C_2} = \frac{960\ \mu\text{C}}{8.0\ \mu\text{F} + 4.0\ \mu\text{F}} = 80\ \text{V}$$

$$Q_1 = 640\ \mu\text{C} \qquad Q_2 = 320\ \mu\text{C}$$

Example 24.7 Transferring charge & energy between capacitors



The final energy of the system is

$$U_{\text{final}} = \tfrac{1}{2}Q_1V + \tfrac{1}{2}Q_2V = \tfrac{1}{2}Q_0V$$

$$= \tfrac{1}{2}(960 \times 10^{-6}\ \text{C})(80\ \text{V}) = 0.038\ \text{J}$$

Example 24.8 Electric-field energy

(a) What is the magnitude of the electric field required to store 1.00 J of electric potential energy in a volume of 1.0 m^3 in vacuum? (b) If the field magnitude is 10 times larger than that, how much energy is stored per cubic meter?

Solution (a): we know $u = 1.00\ \mathrm{J/m^3}$

combine these 2 $u = \frac{1}{2}\epsilon_0 E^2$

$$E = \sqrt{\frac{2u}{\epsilon_0}} = \sqrt{\frac{2(1.00\ \mathrm{J/m^3})}{8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}}}$$

$$= 4.75 \times 10^5\ \mathrm{N/C} = 4.75 \times 10^5\ \mathrm{V/m}$$

Example 24.8 Electric-field energy

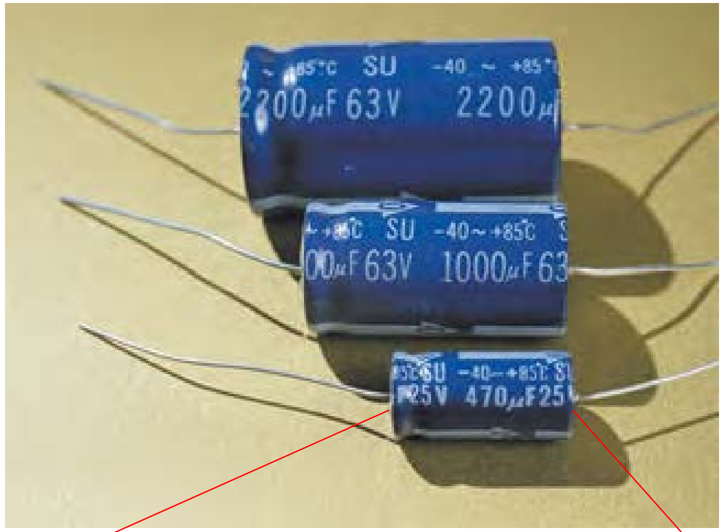
(a) What is the magnitude of the electric field required to store 1.00 J of electric potential energy in a volume of 1.0 m^3 in vacuum? (b) If the field magnitude is 10 times larger than that, how much energy is stored per cubic meter?

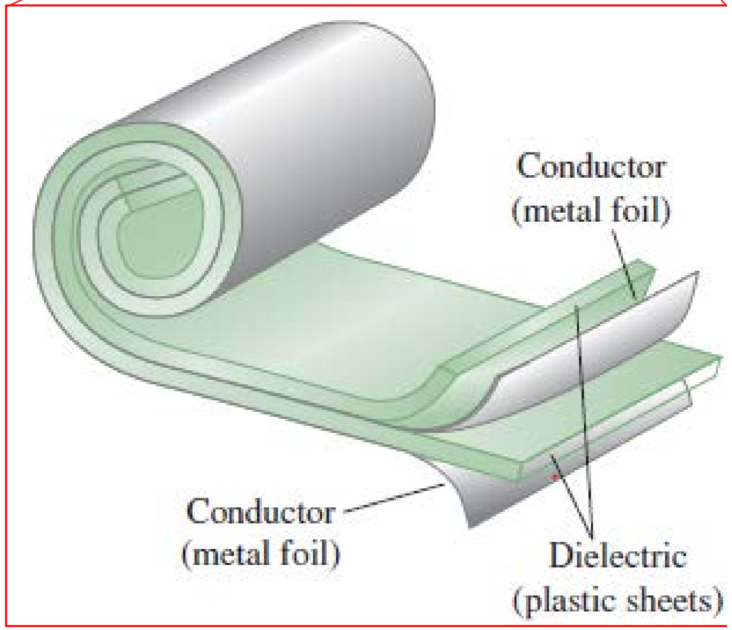
Solution (b): $u = \frac{1}{2}\epsilon_0 E^2$

u is proportional to E^2

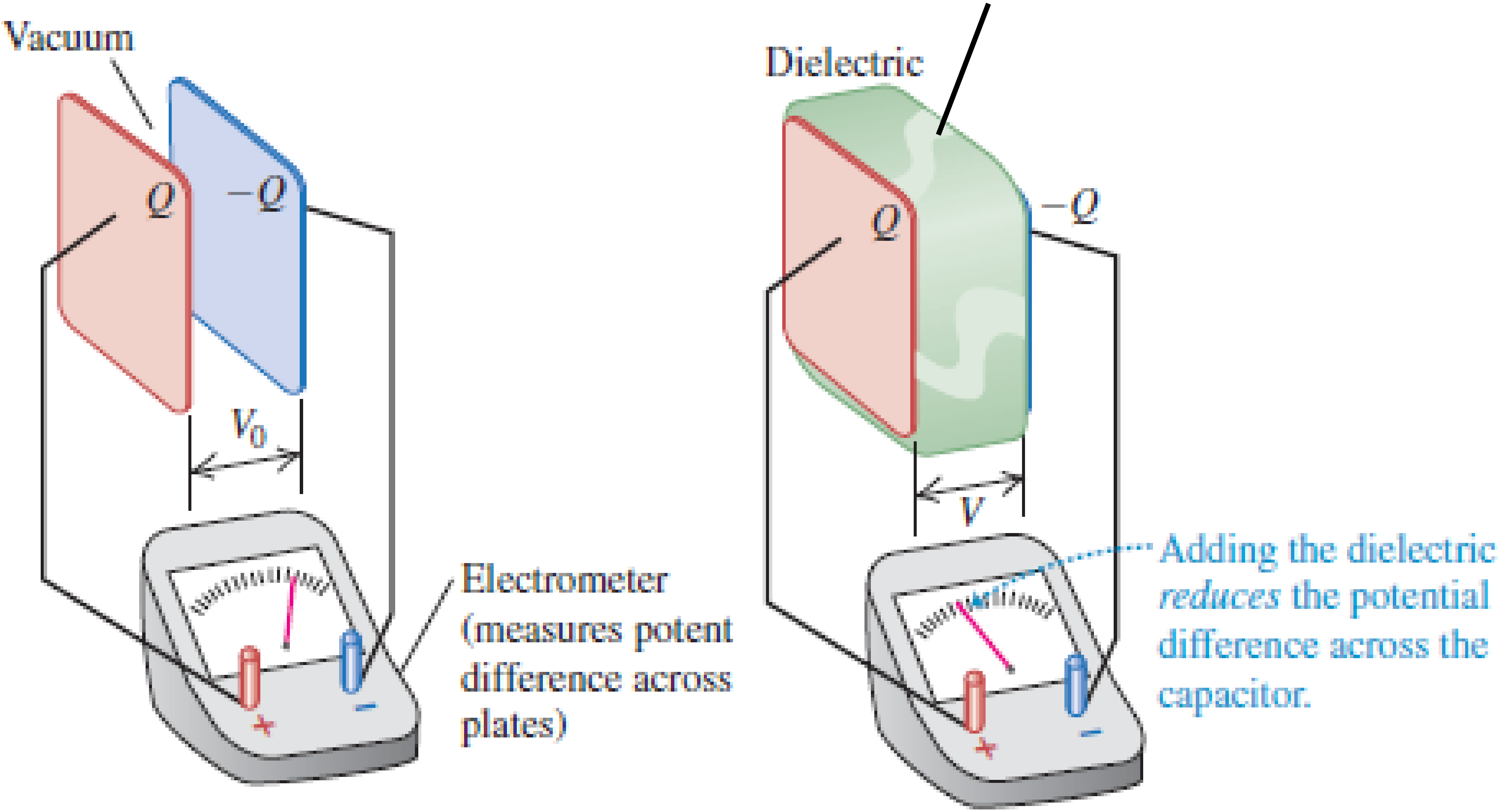
So the energy density becomes $u = 100$ J/m^3

Dielectrics





Dielectric ≈ non-metal materials:
water/oil/plastic/glass/diamond



Definition of ***Dielectric Constant***

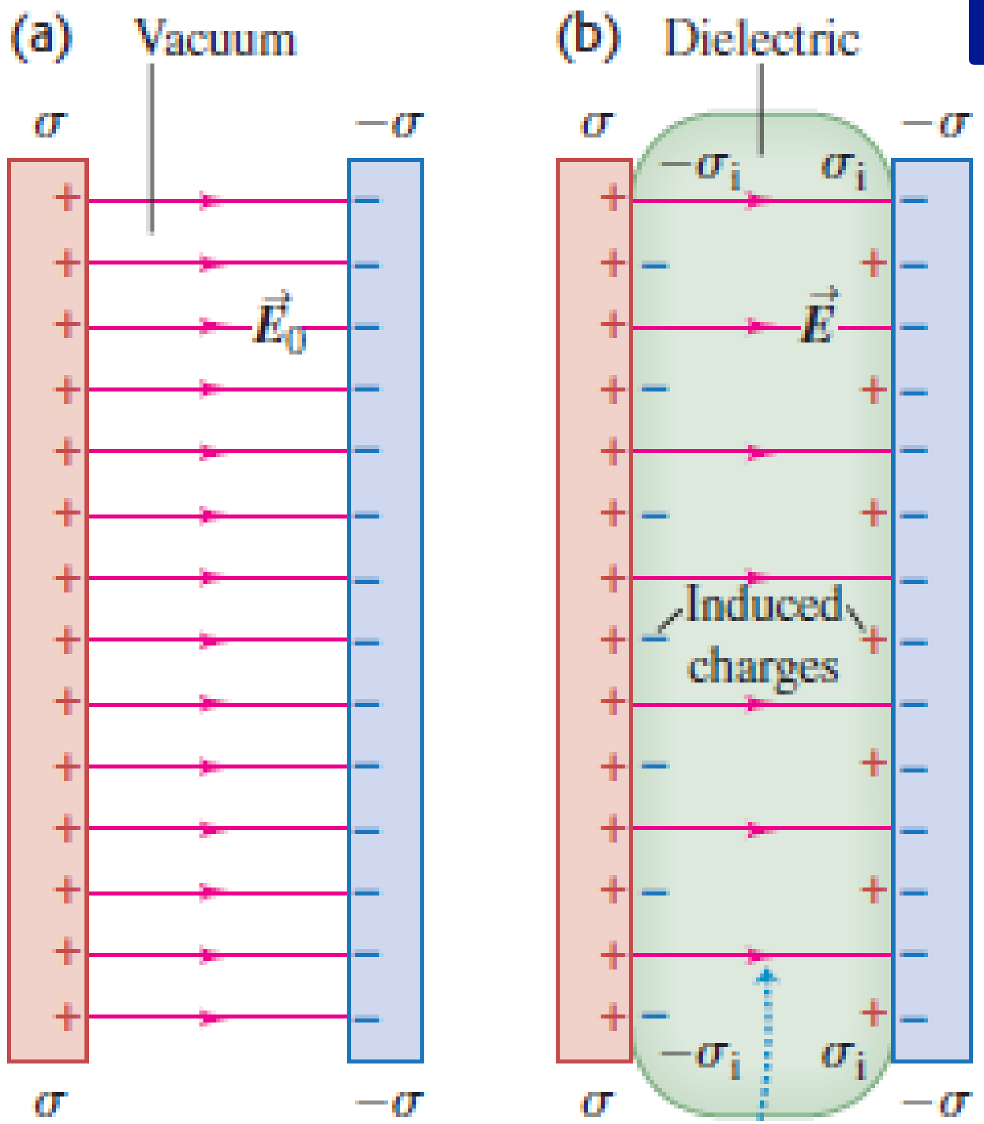
When the space between plates is completely filled by the dielectric, the ratio of C to C_0 (equal to the ratio of V_0 to V) is called the ***dielectric constant*** of the material K:

$$K = C/C_0 \qquad (24.12)$$

Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas®	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

Dielectrics



For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

The surface charge on the conducting plates does not change, but an *induced* charge of the opposite sign appears on each surface of the dielectric (Fig. 24.15). The dielectric was originally electrically neutral and is still neutral; the induced surface charges arise as a result of *redistribution* of positive and negative charge within the dielectric material, a phenomenon called **polarization.**

$$E = \frac{E_0}{K}$$

Dielectrics

The product $K\epsilon_0$ is the **permittivity** of the dielectric, denoted by ϵ.

$$\epsilon = K\epsilon_0 \qquad \text{(definition of permittivity)}$$

In terms of ϵ we can express the electric field within the dielectric as

$$E = \frac{\sigma}{\epsilon}$$

The capacitance when the dielectric is present is given by:

$$C = KC_0 = K\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d} \qquad \text{(parallel-plate capacitor, dielectric between plates)}$$

For the energy density u:

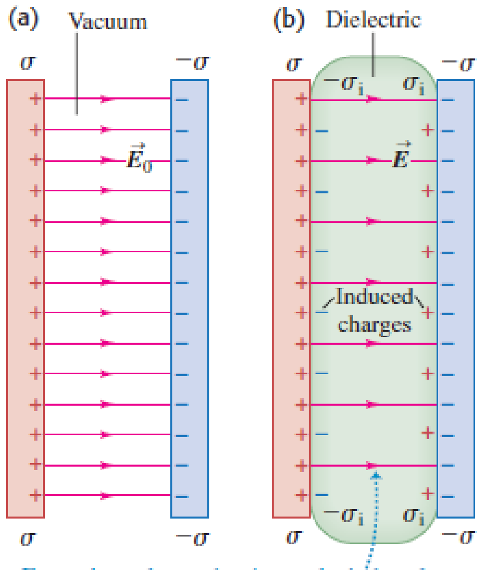
$$u = \tfrac{1}{2}K\epsilon_0 E^2 = \tfrac{1}{2}\epsilon E^2 \qquad \text{(electric energy density in a dielectric)}$$

Example 24.10 A capacitor with and without a dielectric

Suppose the parallel plates in Fig. 24.15 each have an area of 2000 cm^2 and are 1.0 cm apart. We connect the capacitor to a power supply, charge it to a potential difference $V_0 = 3.0$ kV and disconnect the power supply. We then insert a sheet of insulating plastic material between the plates, completely filling the space between them. We find that the potential difference decreases to 1.00 kV while the charge on each capacitor plate remains constant. Find (a) the original capacitance C_0 (b) the magnitude of charge Q on each plate; (c) the capacitance C after the dielectric is inserted; (d) the dielectric constant K of the dielectric; (e) the permittivity of the dielectric; (f) the magnitude of the induced charge Q_i on each face of the dielectric; (g) the original electric field E_0 between the plates; and (h) the electric field E after the dielectric is inserted.

Example 24.10 A capacitor with and without a dielectric

Suppose the parallel plates in Figure below each have an area of 2000 cm^2 and are 1.0 cm apart. We connect the capacitor to a power supply, charge it to a potential difference V_0 = 3.0 kV and disconnect the power supply. We then insert a sheet of insulating plastic material between the plates, completely filling the space between them. We find that the potential difference decreases to 1.00 kV while the charge on each capacitor plate remains constant. Find (a) the original capacitance C_0.



Solution: (a)

$$C_0 = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}\ \text{F/m}) \frac{2.00 \times 10^{-1}\ \text{m}^2}{1.00 \times 10^{-2}\ \text{m}}$$

$$= 1.77 \times 10^{-10}\ \text{F} = 177\ \text{pF}$$

Example 24.10 A capacitor with and without a dielectric

Suppose the parallel plates in Figure below each have an area of 2000 cm^2 and are 1.0 cm apart. We connect the capacitor to a power supply, charge it to a potential difference V_0 = 3.0 kV and disconnect the power supply. We then insert a sheet of insulating plastic material between the plates, completely filling the space between them. We find that the potential difference decreases to 1.00 kV while the charge on each capacitor plate remains constant. Find (b) the magnitude of charge *Q* on each plate

Solution: (b) From the definition of capacitance, Eq. (24.1),

$$Q = C_0 V_0 = (1.77 \times 10^{-10}\ \text{F})(3.00 \times 10^{3}\ \text{V})$$
$$= 5.31 \times 10^{-7}\ \text{C} = 0.531\ \mu\text{C}$$

Example 24.10 A capacitor with and without a dielectric

Suppose the parallel plates in Figure below each have an area of 2000 cm^2 and are 1.0 cm apart. We connect the capacitor to a power supply, charge it to a potential difference V_0 = 3.0 kV and disconnect the power supply. We then insert a sheet of insulating plastic material between the plates, completely filling the space between them. We find that the potential difference decreases to 1.00 kV while the charge on each capacitor plate remains constant. Find (c) the capacitance *C* after the dielectric is inserted.

Solution: (c) When the dielectric is inserted, *Q* is unchanged but the potential difference decreases to $V = 1.00$ kV

$$C = \frac{Q}{V} = \frac{5.31 \times 10^{-7}\ \mathrm{C}}{1.00 \times 10^{3}\ \mathrm{V}} = 5.31 \times 10^{-10}\ \mathrm{F} = 531\ \mathrm{pF}$$

Example 24.10 A capacitor with and without a dielectric

Suppose the parallel plates in Figure below each have an area of 2000 cm^2 and are 1.0 cm apart. We connect the capacitor to a power supply, charge it to a potential difference $V_0 = 3.0$ kV and disconnect the power supply. We then insert a sheet of insulating plastic material between the plates, completely filling the space between them. We find that the potential difference decreases to 1.00 kV while the charge on each capacitor plate remains constant. Find (d) the dielectric constant K of the dielectric.

Solution: (d) the dielectric constant is

$$K = \frac{C}{C_0} = \frac{5.31 \times 10^{-10}\ \mathrm{F}}{1.77 \times 10^{-10}\ \mathrm{F}} = \frac{531\ \mathrm{pF}}{177\ \mathrm{pF}} = 3.00$$

Alternatively, from Eq. (24.13),

$$K = \frac{V_0}{V} = \frac{3000\ \mathrm{V}}{1000\ \mathrm{V}} = 3.00$$

Example 24.10 A capacitor with and without a dielectric

Suppose the parallel plates in Figure below each have an area of 2000 cm^2 and are 1.0 cm apart. We connect the capacitor to a power supply, charge it to a potential difference V_0 = 3.0 kV and disconnect the power supply. We then insert a sheet of insulating plastic material between the plates, completely filling the space between them. We find that the potential difference decreases to 1.00 kV while the charge on each capacitor plate remains constant. Find (e) the permittivity of the dielectric.

Solution: (e) Using from part (d) in Eq. (24.17), the permittivity is

$$\begin{aligned}\epsilon &= K\epsilon_0 = (3.00)(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}) \\ &= 2.66 \times 10^{-11}\ \mathrm{C^2/N \cdot m^2}\end{aligned}$$

Example 24.10 A capacitor with and without a dielectric

Suppose the parallel plates in Figure below each have an area of 2000 cm^2 and are 1.0 cm apart. We connect the capacitor to a power supply, charge it to a potential difference V_0 = 3.0 kV and disconnect the power supply. We then insert a sheet of insulating plastic material between the plates, completely filling the space between them. We find that the potential difference decreases to 1.00 kV while the charge on each capacitor plate remains constant. Find (f) the magnitude of the induced charge Q_i on each face of the dielectric

Solution: (f) Multiplying both sides of Eq. (24.16) by the plate area A gives the induced charge $Q_i = \sigma_i A$

$$Q_i = Q\left(1 - \frac{1}{K}\right) = (5.31 \times 10^{-7}\ \text{C})\left(1 - \frac{1}{3.00}\right) = 3.54 \times 10^{-7}\ \text{C}$$

Example 24.10 A capacitor with and without a dielectric

Suppose the parallel plates in Figure below each have an area of 2000 cm^2 and are 1.0 cm apart. We connect the capacitor to a power supply, charge it to a potential difference V_0 = 3.0 kV and disconnect the power supply. We then insert a sheet of insulating plastic material between the plates, completely filling the space between them. We find that the potential difference decreases to 1.00 kV while the charge on each capacitor plate remains constant. Find (g) the original electric field E0 between the plates

Solution: (g) Since the electric field between the plates is uniform, its magnitude is the potential difference divided by the plate separation:

$$E_0 = \frac{V_0}{d} = \frac{3000\ \text{V}}{1.00 \times 10^{-2}\ \text{m}} = 3.00 \times 10^5\ \text{V/m}$$

Example 24.10 A capacitor with and without a dielectric

Find (h) the electric field E after the dielectric is inserted

Solution: (h) After the dielectric is inserted

$$E = \frac{V}{d} = \frac{1000\ \text{V}}{1.00 \times 10^{-2}\ \text{m}} = 1.00 \times 10^{5}\ \text{V/m}$$

or, from Eq. (24.18),

$$E = \frac{\sigma}{\epsilon} = \frac{Q}{\epsilon A} = \frac{5.31 \times 10^{-7}\ \text{C}}{(2.66 \times 10^{-11}\ \text{C}^2/\text{N} \cdot \text{m}^2)(2.00 \times 10^{-1}\ \text{m}^2)}$$

$$= 1.00 \times 10^{5}\ \text{V/m}$$

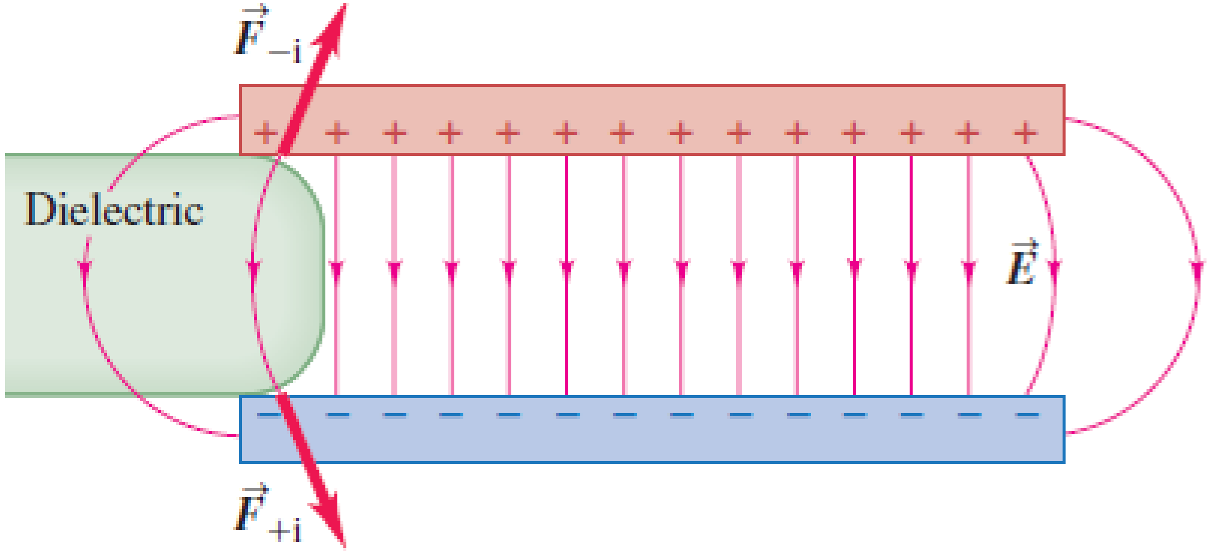
or, from Eq. (24.14),

$$E = \frac{E_0}{K} = \frac{3.00 \times 10^{5}\ \text{V/m}}{3.00} = 1.00 \times 10^{5}\ \text{V/m}$$

Example 24.11 Energy storage with and without a dielectric

Find the energy stored in the electric field of the capacitor in Example 24.10 and energy density, before and after the dielectric sheet is inserted.

IDENTIFY and SET UP: Consider the ideas of energy stored in a capacitor and of electric-field energy density. Use Eq. (24.9) to find the stored energy and Eq. (24.20) to find the energy density.



Stored energies U_0 and U without and with the dielectric in place are

$$U_0 = \tfrac{1}{2}C_0V_0^2 = \tfrac{1}{2}(1.77 \times 10^{-10}\ \mathrm{F})(3000\ \mathrm{V})^2 = 7.97 \times 10^{-4}\ \mathrm{J}$$

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(5.31 \times 10^{-10}\ \mathrm{F})(1000\ \mathrm{V})^2 = 2.66 \times 10^{-4}\ \mathrm{J}$$

Example 24.11 Energy storage with and without a dielectric

Find the energy stored in the electric field of the capacitor in Example 24.10 and energy density, before and after the dielectric sheet is inserted.

IDENTIFY and SET UP: Consider the ideas of energy stored in a capacitor and of electric-field energy density. Use Eq. (24.9) to find the stored energy and Eq. (24.20) to find the energy density.

Energy densities without and with the dielectric:

$$u_0 = \tfrac{1}{2}\epsilon_0 E_0^2 = \tfrac{1}{2}(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(3.00 \times 10^5\ \mathrm{N/C})^2$$

$$= 0.398\ \mathrm{J/m^3}$$

$$u = \tfrac{1}{2}\epsilon E^2 = \tfrac{1}{2}(2.66 \times 10^{-11}\ \mathrm{C^2/N \cdot m^2})(1.00 \times 10^5\ \mathrm{N/C})^2$$

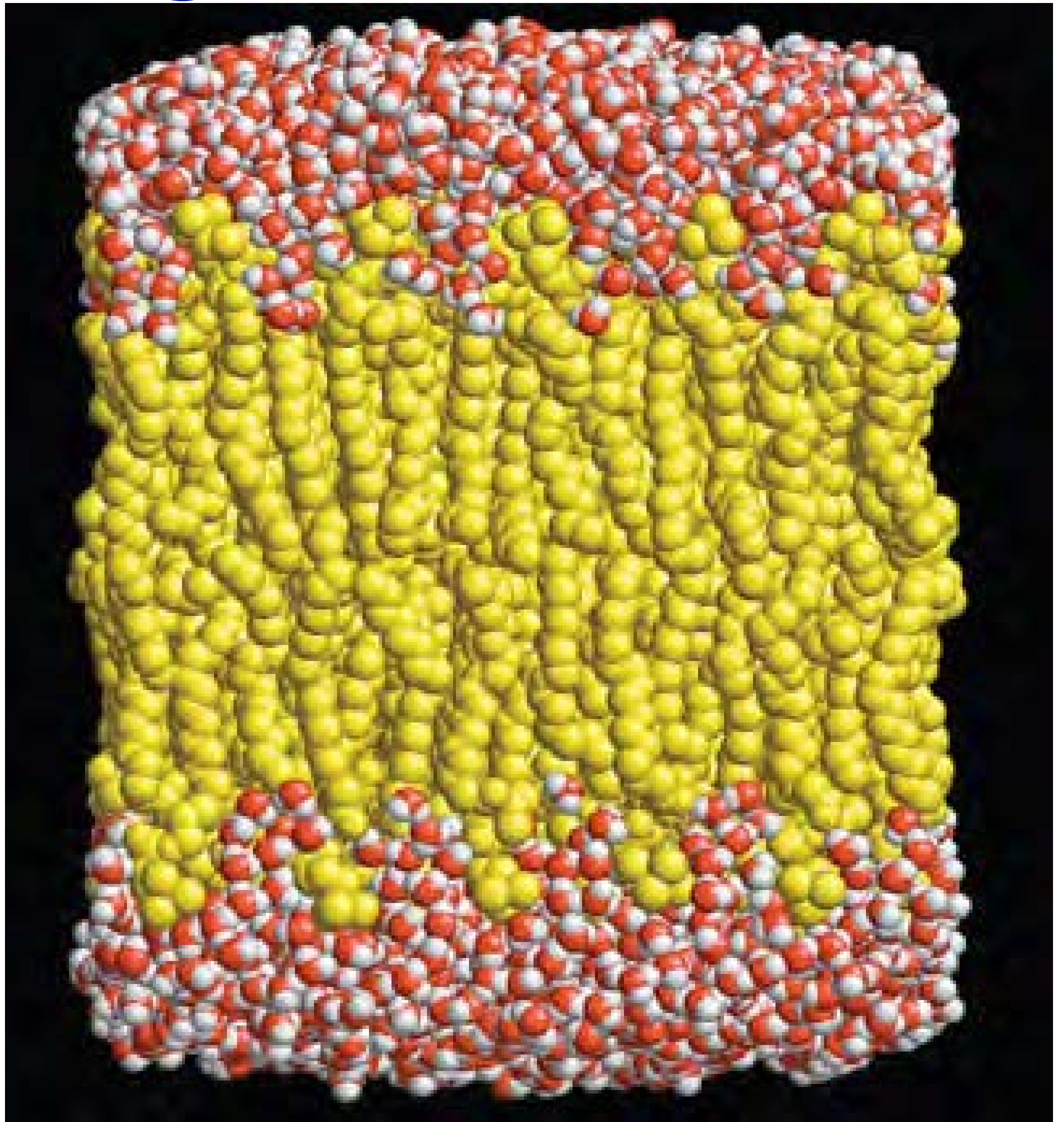
$$= 0.133\ \mathrm{J/m^3}$$

Dielectric Constant and Dielectric Strength of Some Insulating Materials

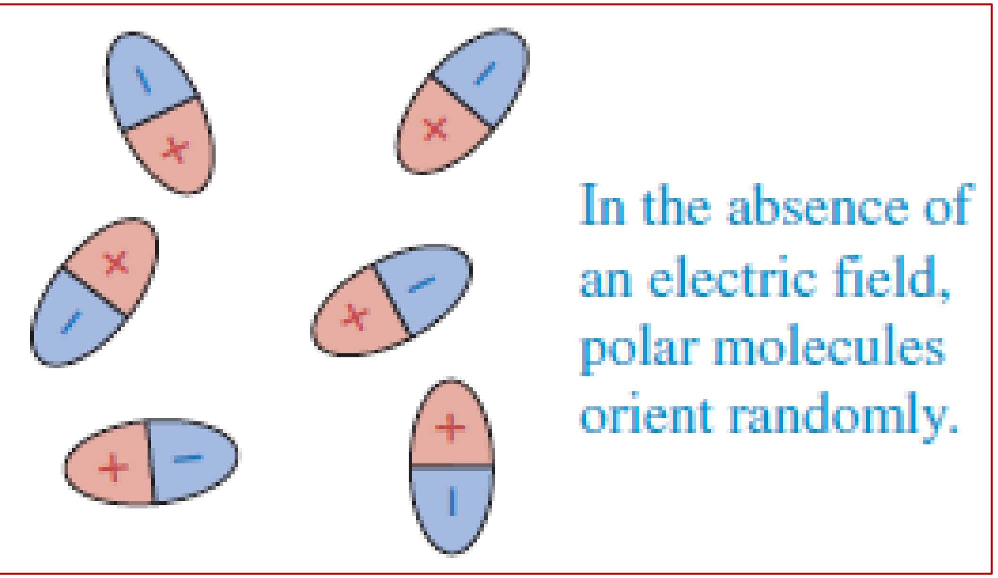
Material	Dielectric Constant, K	Dielectric Strength, E_m (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7

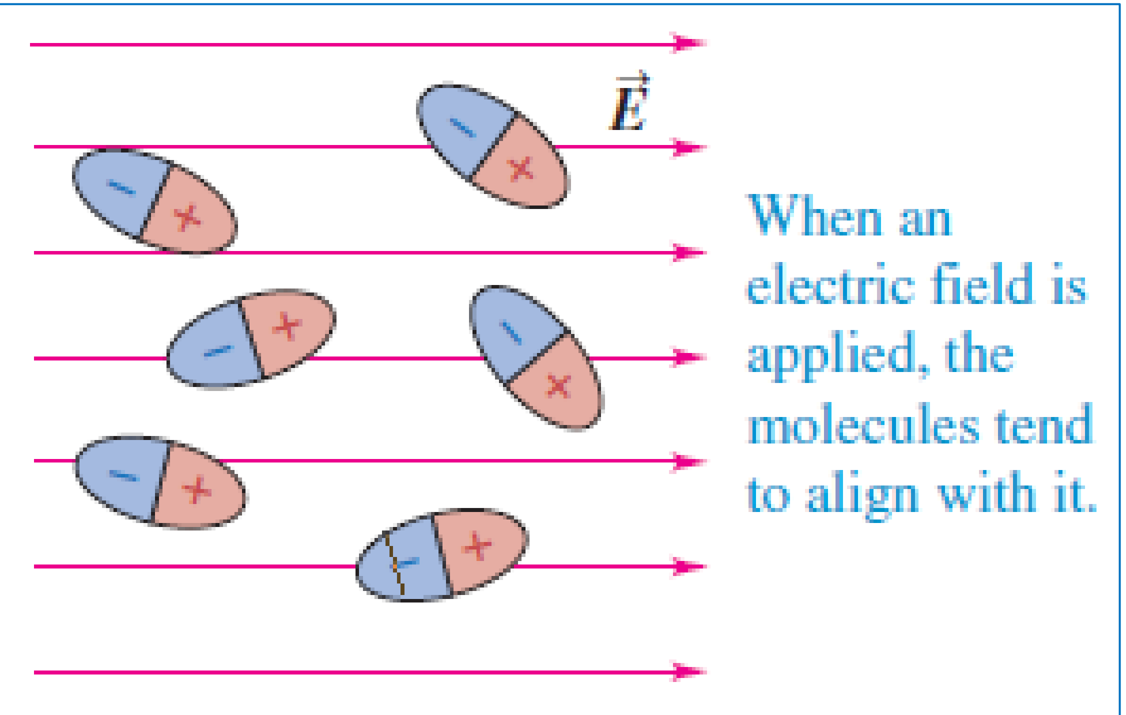
Dielectric Constant and Dielectric Strength of Some Insulating Materials

The membrane of a living cell behaves like a dielectric between the plates of a capacitor. The membrane is made of two sheets of lipid molecules, with their water-insoluble ends in the middle and their water-soluble ends (shown in red) on the surfaces of the membrane. The conductive fluids on either side of the membrane (water with negative ions inside the cell, water with positive ions outside) act as charged capacitor plates, and the nonconducting membrane acts as a dielectric with K of about 10. The potential difference V across the membrane is about 0.07 V and the membrane thickness d is about 7×10^{-9} m, so the electric field $E = V/d$ in the membrane is about 10^7 V/m—close to the dielectric strength of the membrane. If the membrane were made of air, V and E would be larger by a factor of $K \approx 10$ and dielectric breakdown would occur.



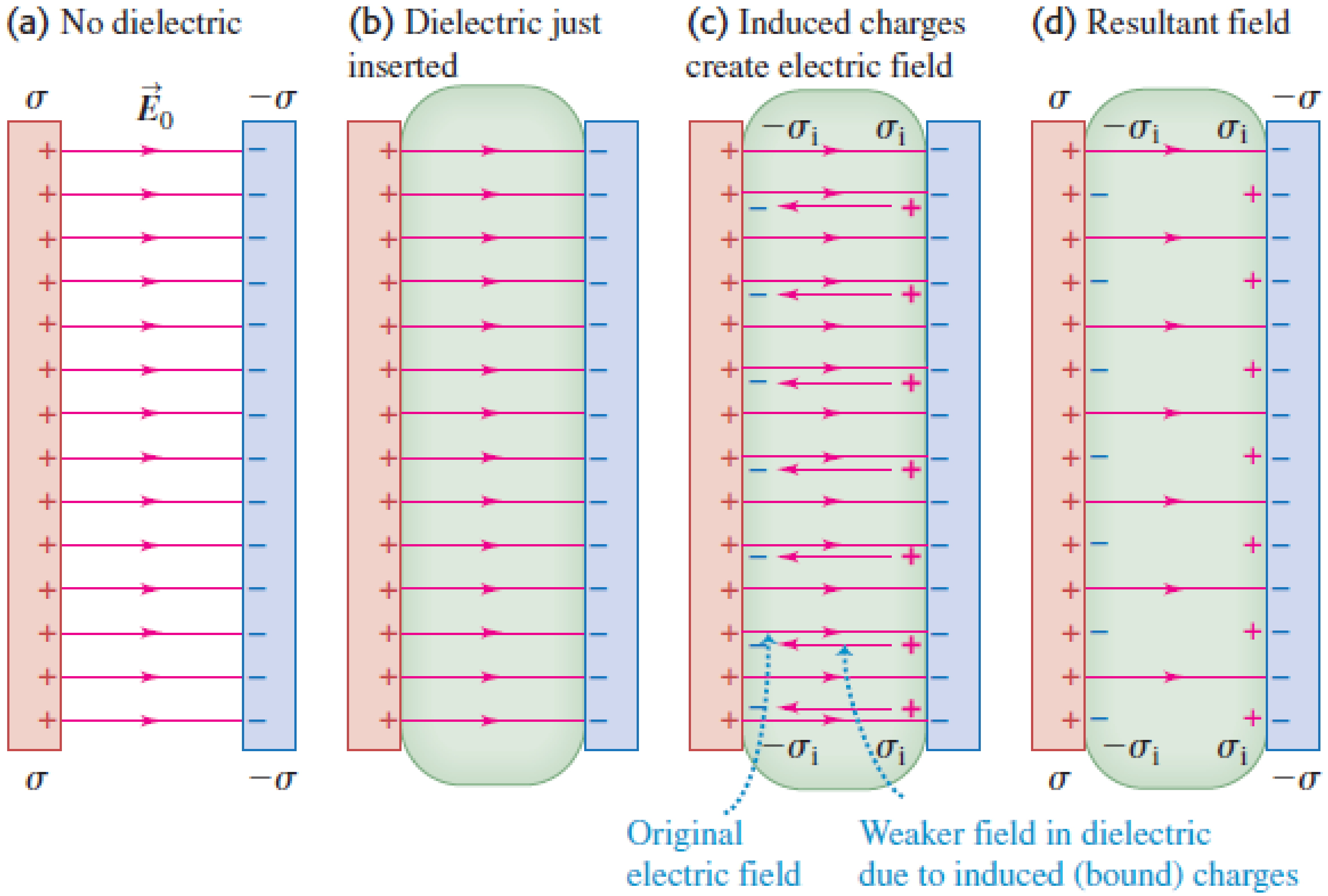
Molecular Model of Induced Charge





At the *molecular* level. Some molecules, have equal amounts of + and - charges but a lopsided distribution, with excess positive charge concentrated on one side of the molecule and negative charge on the other forming an *electric dipole*, and the molecule is called a *polar molecule*. When no electric field is present in a gas or liquid with polar molecules, the molecules are oriented randomly. When they are placed in an electric field, however, they tend to orient themselves as a result of the electric-field torques. Because of thermal agitation, alignment of the molecules with is not perfect.

Molecular Model of Induced Charge

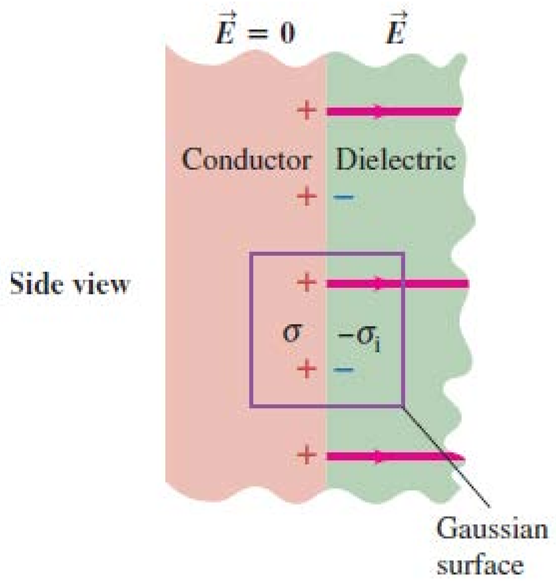


This field is *opposite* to the original field, but it is not great enough to cancel the original field completely.

Gauss's Law in Dielectrics (Not to Test)

$$\oint K\vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl-free}}}{\epsilon_0} \qquad \text{(Gauss's law in a dielectric)}$$

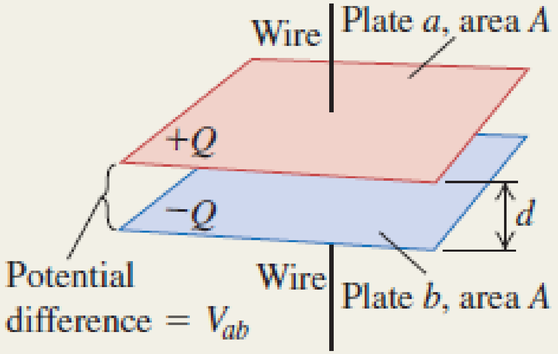
flux of $K\vec{E}$, not $\vec{E}$



Summary

$$C = \frac{Q}{V_{ab}} \qquad (24.1)$$

$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d} \qquad (24.2)$$

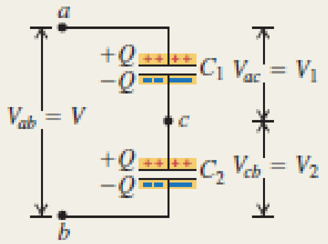


$$U = \frac{Q^2}{2C} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV$$

$$u = \tfrac{1}{2}\epsilon_0 E^2$$

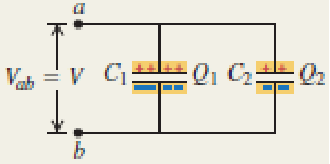
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \qquad (24.5)$$

(capacitors in series)



$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad (24.7)$$

(capacitors in parallel)

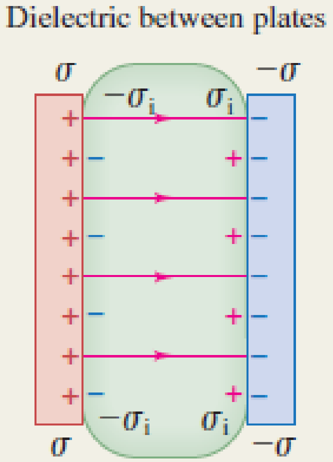


$$C = KC_0 = K\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$$

(parallel-plate capacitor filled with dielectric) (24.19)

$$u = \tfrac{1}{2}K\epsilon_0 E^2 = \tfrac{1}{2}\epsilon E^2 \qquad (24.20)$$

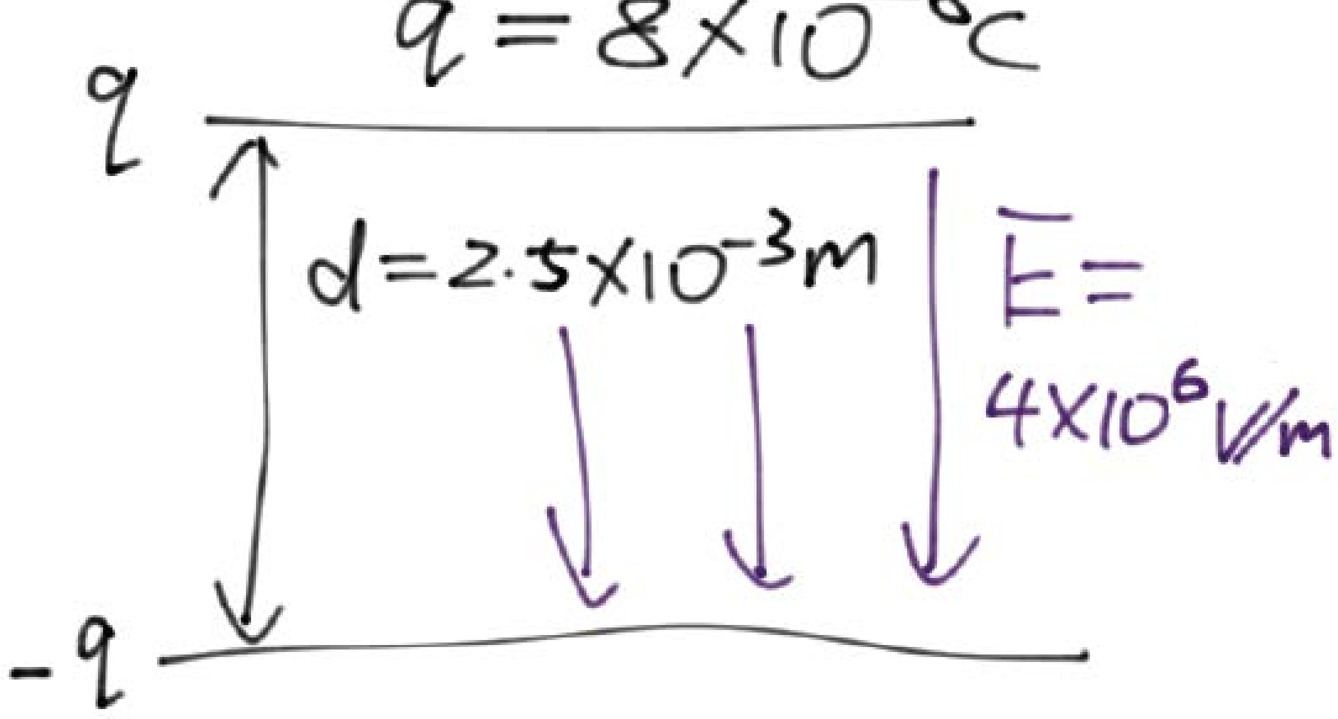
$$\oint K\vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl-free}}}{\epsilon_0} \qquad (24.23)$$



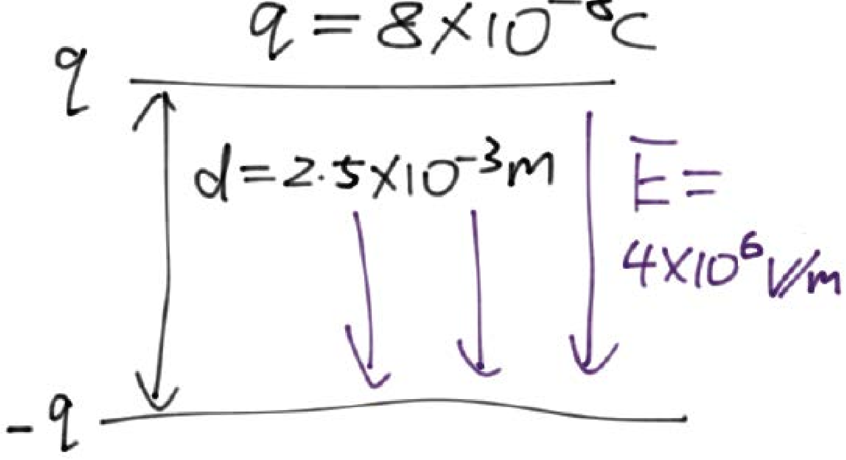
24.1 • The plates of a parallel-plate capacitor are 2.50 mm apart, and each carries a charge of magnitude 80.0 nC. The plates are in vacuum. The electric field between the plates has a magnitude of 4.00×10^6 V/m. (a) What is the potential difference between the plates? (b) What is the area of each plate? (c) What is the capacitance?

(a) $|\Delta V| = |E \cdot d|$

$= 10^4 \ V$



24.1 • The plates of a parallel-plate capacitor are 2.50 mm apart, and each carries a charge of magnitude 80.0 nC. The plates are in vacuum. The electric field between the plates has a magnitude of 4.00×10^6 V/m. (a) What is the potential difference between the plates? (b) What is the area of each plate? (c) What is the capacitance?



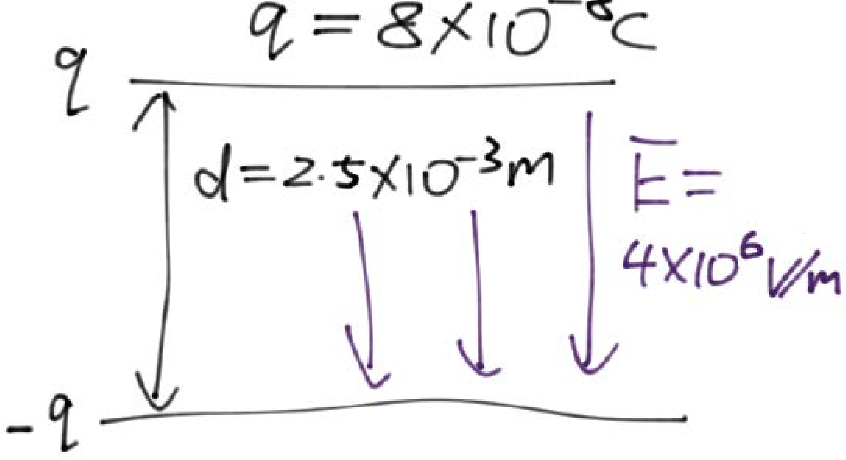
(b)

$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}$$

$$A = \frac{Q}{V_{ab}} \cdot \frac{d}{\epsilon_0} = \frac{8 \times 10^{-8}\,C}{1 \times 10^{4}\,V} \cdot \frac{2.5 \times 10^{-3}\,m}{8.85 \times 10^{-12}\,C^2/N \cdot m^2}$$

$$= 2.26 \times 10^{-3}\,m^2$$

24.1 • The plates of a parallel-plate capacitor are 2.50 mm apart, and each carries a charge of magnitude 80.0 nC. The plates are in vacuum. The electric field between the plates has a magnitude of 4.00×10^6 V/m. (a) What is the potential difference between the plates? (b) What is the area of each plate? (c) What is the capacitance?



(c) $C = \frac{Q}{V_{ab}} = 8\times10^{-12}$ F

24.5 • A 10.0-μF parallel-plate capacitor with circular plates is connected to a 12.0-V battery. (a) What is the charge on each plate? (b) How much charge would be on the plates if their separation were doubled while the capacitor remained connected to the battery? (c) How much charge would be on the plates if the capacitor were connected to the 12.0-V battery after the radius of each plate was doubled without changing their separation?

(a)

$$C = \frac{Q}{V_{ab}} \Rightarrow Q = C \cdot V_{ab}$$

$C = 10^{-5}\,F$

$+ + + + +$

$- - - - - -$

$$= 10 \times 10^{-6}\,F \cdot 12\,V$$

$$= 1.2 \times 10^{-4}\,C$$

24.5 • A 10.0-μF parallel-plate capacitor with circular plates is connected to a 12.0-V battery. (a) What is the charge on each plate? (b) How much charge would be on the plates if their separation were doubled while the capacitor remained connected to the battery? (c) How much charge would be on the plates if the capac-

(b)

$$C = \frac{Q'}{\underline{V_{ab}}} = \epsilon_0 \frac{A}{d'} \leftarrow \text{doubles}$$

same

same

$$d' = 2d$$

$$Q' = \frac{Q}{2}$$

$$= 6 \times 10^{-5}\,C$$

24.5 • A 10.0-μF parallel-plate capacitor with circular plates is connected to a 12.0-V battery. (a) What is the charge on each battery? (c) How much charge would be on the plates if the capacitor were connected to the 12.0-V battery after the radius of each plate was doubled without changing their separation?

(c) $A = \pi r^2$, $r'' = 2r$

$\Rightarrow A'' = \pi r''^2 = \pi (2r)^2 = 4A$

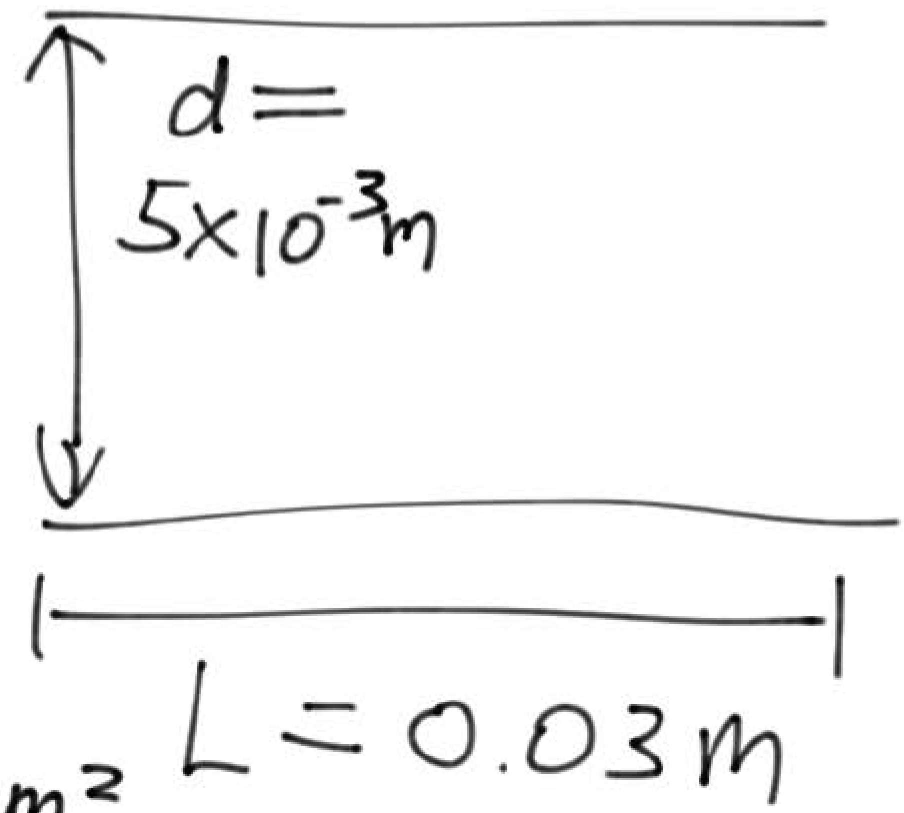
Since $C = \epsilon_0 \frac{A}{d}$ $\quad C'' = \epsilon_0 \frac{A''}{d} = 4C$

so $Q'' = C'' \cdot V_{ab} = 4Q = 4.8 \times 10^{-4} C$

24.4 •• Capacitance of an Oscilloscope. Oscilloscopes have parallel metal plates inside them to deflect the electron beam. These plates are called the *deflecting plates*. Typically, they are squares 3.0 cm on a side and separated by 5.0 mm, with vacuum in between. What is the capacitance of these deflecting plates and hence of the oscilloscope? (*Note:* This capacitance can sometimes have an effect on the circuit you are trying to study and must be taken into consideration in your calculations.)

This equation again:

$$C = \epsilon_0 \frac{A}{d}. \quad \text{What is } A?$$

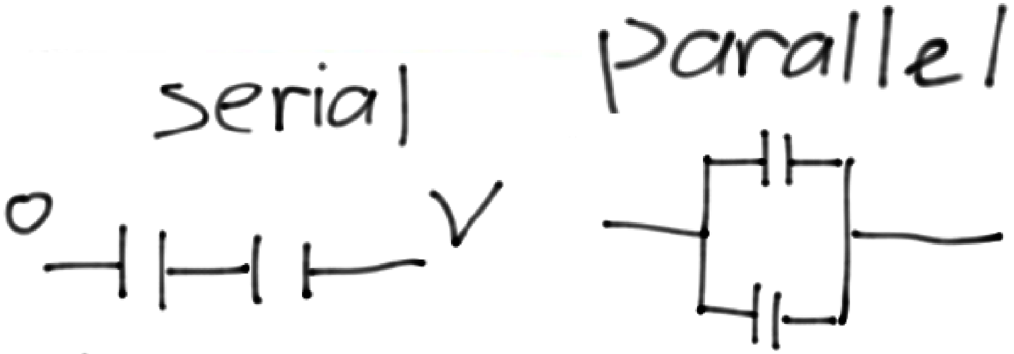


$$A = L^2 = 9\times10^{-4}\,m^2$$

$$C = 8.85\times10^{-12}\,C^2/N\cdot m^2 \cdot \frac{9\times10^{-4}\,m^2}{5\times10^{-3}\,m}$$

$$= 1.6\times10^{-12}\,F$$

24.15 • BIO Electric Eels. Electric eels and electric fish generate large potential differences that are used to stun enemies and prey. These potentials are produced by cells that each can generate 0.10 V. We can plausibly model such cells as charged capacitors. (a) How should these cells be connected (in series or in parallel) to produce a total potential of more than 0.10 V? (b) Using the connection in part (a), how many cells must be connected together to produce the 500-V surge of the electric eel?



$V = V_1 + V_2$ $\quad$ $V = V_1 = V_2$

(a) Asking: series vs parallel

total potential adds up ✓

total potential stays the same ✗

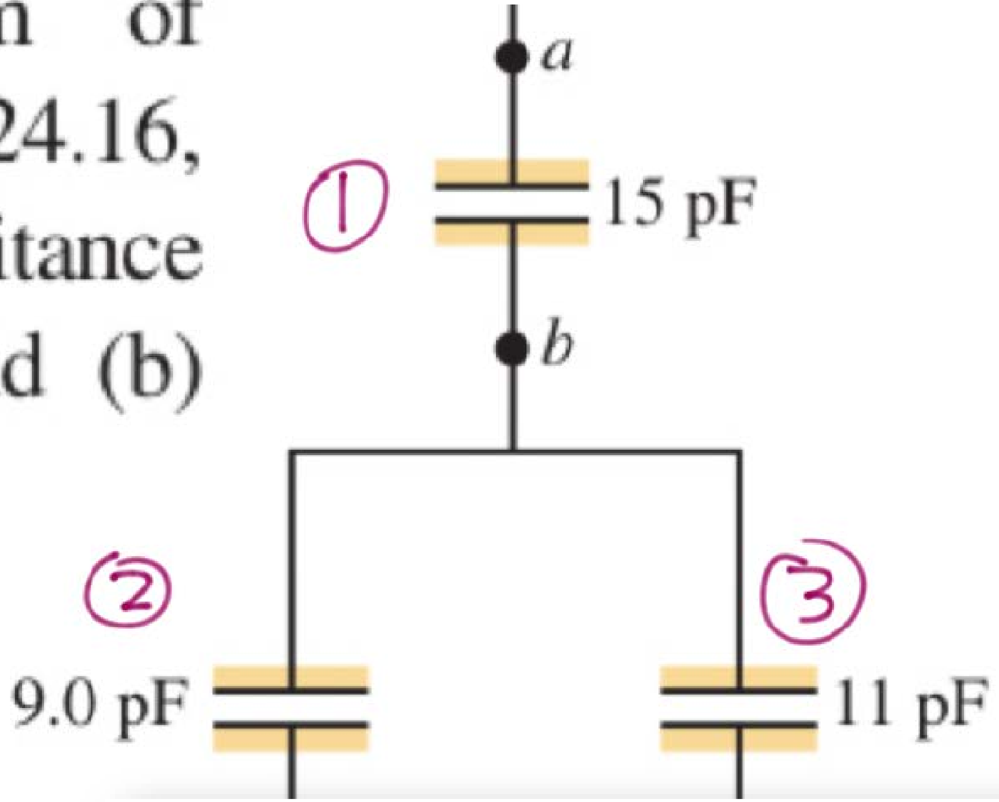
(b) $V_{total} = \sum_{i=1,2,\cdots n} V_i$, where $V_i = 0.1\ V$

(so) $= n \cdot V_i$

Rearrange

$n = \frac{V_{total}}{V_i} = 5000$

24.16 • For the system of capacitors shown in Fig. E24.16, find the equivalent capacitance (a) between b and c, and (b) between a and c.



(a) $C_{bc} = C_2 + C_3$

$= 20\ \text{pF}$

$= 2 \times 10^{-11}\ \text{F}$

When you forget about this for parallel connection remember: Area adds up!

$C_{bc} = \frac{A}{d} = \frac{Q}{V_{bc}}$

24.16 • For the system of capacitors shown in Fig. E24.16, find the equivalent capacitance (a) between b and c, and (b) between a and c.

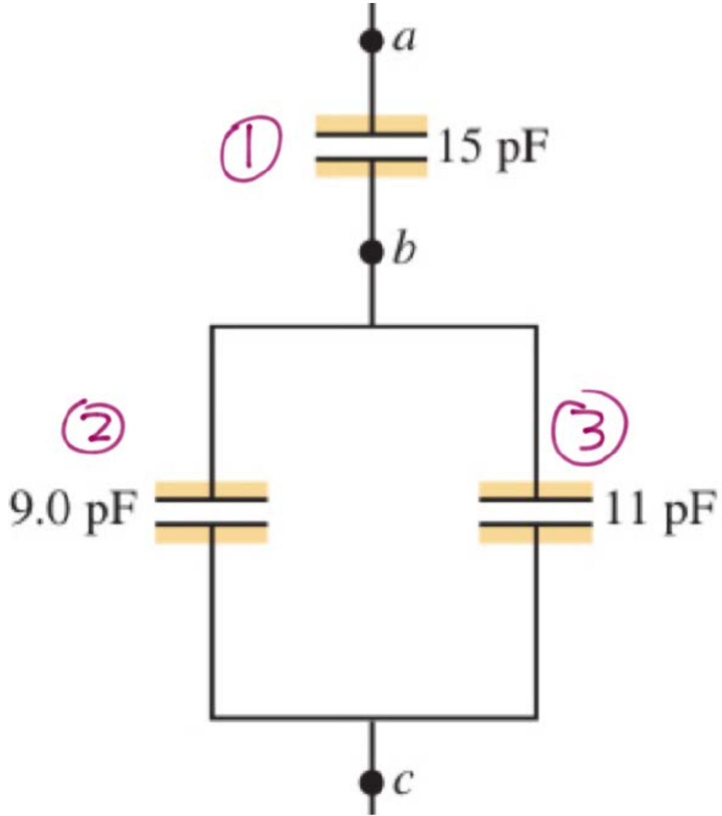
(b) Treat bc as a <u>single</u> capacitor C_{bc}

connected in series with C_{ab}

$$\frac{1}{C_{ac}} = \frac{1}{C_{ab}} + \frac{1}{C_{bc}} \quad \text{so}$$

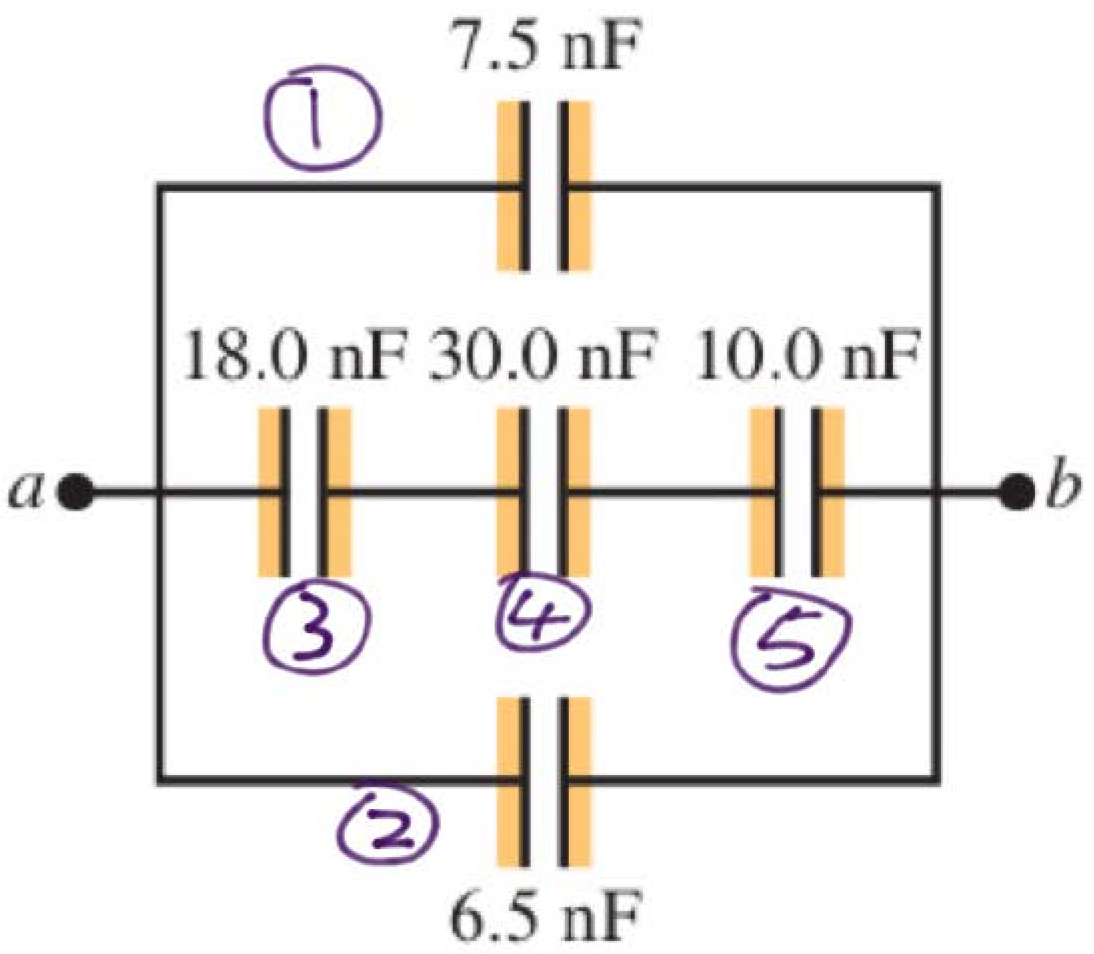
$$C_{ac} = \left(\frac{1}{15} + \frac{1}{20}\right)^{-1} \text{pF}$$

$$= 8.57 \times 10^{-12}\ \text{F}$$



24.21 •• For the system of capacitors shown in Fig. E24.21, a potential difference of 25 V is maintained across *ab*. (a) What is the equivalent capacitance of this system between *a* and *b*? (b) How much charge is stored by this system? (c) How much charge does the 6.5-nF capacitor store? (d) What is the potential difference across the 7.5-nF capacitor?

Figure **E24.21**



(a) $C_{345} = (C_3^{-1} + C_4^{-1} + C_5^{-1})^{-1}$ in series

$$= \left(\frac{1}{18} + \frac{1}{30} + \frac{1}{10}\right)^{-1} nF = 5.3\,nF$$

$C_{ab} = C_1 + C_2 + C_{345}$ in parallel

$$= 19.3\,nF$$

(b) $Q = C \cdot V_{ab} = 19.3 \times 10^{-9} F \cdot 25V$

$$= 4.8 \times 10^{-7} C$$

(c) Isolate out capacitor 2.

What is V_2? Still V_{ab}! 2 is directly connected to a and b with wires.

$$Q_2 = C_2 Q_{ab} = 6.5 \times 10^{-9} F \cdot 25V = 1.6 \times 10^{-7} C$$

(d) still $V_{ab} = 25V$

A more difficult question is: what is the potential across the 18 nF, 30 nF and the 10 nF capacitor.

The answer should add up to V_{ab}

$$V_3 + V_4 + V_5 = V_{ab}$$

Also, charges should conserve:

$$Q_{345} = Q_3 + Q_4 + Q_5$$

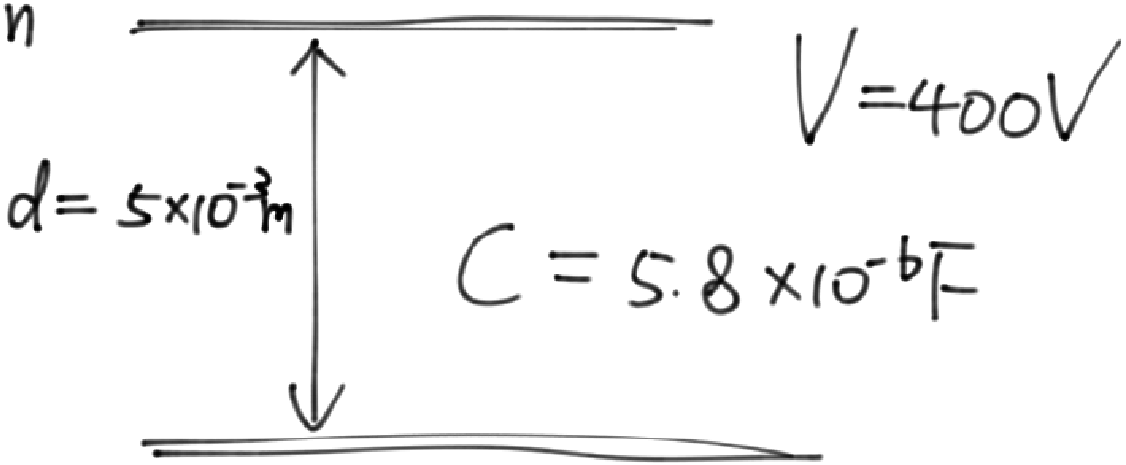
or $C_{345} \cdot V_{ab} = C_3 V_3 + C_4 \cdot V_4 + C_5 \cdot V_5$

$$V_3 = \frac{\frac{1}{C_3}}{C_3^{-1} + C_4^{-1} + C_5^{-1}} V_{ab}$$

etc. My guess

24.25 • A 5.80-μF, parallel-plate, air capacitor has a plate separation of 5.00 mm and is charged to a potential difference of 400 V. Calculate the energy density in the region between the plates, in units of J/m^3.

Now use the equation we just learned



$$u = \frac{1}{2}\epsilon_0 E^2$$

But what is E?

$$E = \frac{V}{d}$$

Recall potential of previous chapter?

24.25 • A 5.80-μF, parallel-plate, air capacitor has a plate separation of 5.00 mm and is charged to a potential difference of 400 V. Calculate the energy density in the region between the plates, in units of J/m^3.

$$\text{so } u = \frac{1}{2}\epsilon_0 \frac{V^2}{d^2}$$

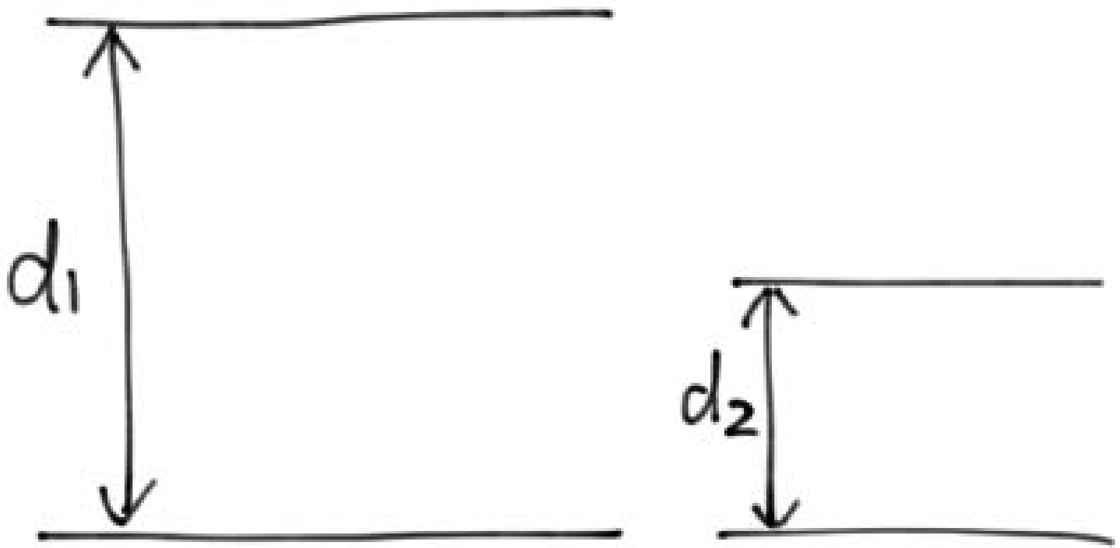
$$= 0.5 \cdot 8.85\times10^{-12}\ C^2/N\cdot m^2 \cdot \frac{(400V)^2}{(5\times10^{-3}m)^2}$$

$$= 0.028\ J/m^3$$

24.28 •• A parallel-plate vacuum capacitor has 8.38 J of energy stored in it. The separation between the plates is 2.30 mm. If the separation is decreased to 1.15 mm, what is the energy stored (a) if the capacitor is disconnected from the potential source so the charge on the plates remains constant, and (b) if the capacitor remains connected to the potential source so the potential difference between the plates remains constant?

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

(a)



$\underline{\underline{Q}} = CV$ constant

$$\left.\begin{aligned} C_1 &= \epsilon_0 \frac{A}{d_1} \\ C_2 &= \epsilon_0 \frac{A}{d_2} \\ d_2 &= \frac{1}{2} d_1 \end{aligned}\right\} \Rightarrow C_2 = 2C_1$$

but $C \cdot V$ needs to be constant

so $V_2 = \frac{1}{2} V_1$

$U_1 = \frac{1}{2} C_1 V_1^2 \qquad U_2 = \frac{1}{2} C_2 V_2^2$

$$U_1 = \frac{1}{2} C_1 V_1^2 \qquad U_2 = \frac{1}{2} C_2 V_2^2$$

$$= \frac{1}{2}(2C_1)\left(\frac{1}{2}V_1\right)^2 = \frac{1}{2}U_1$$

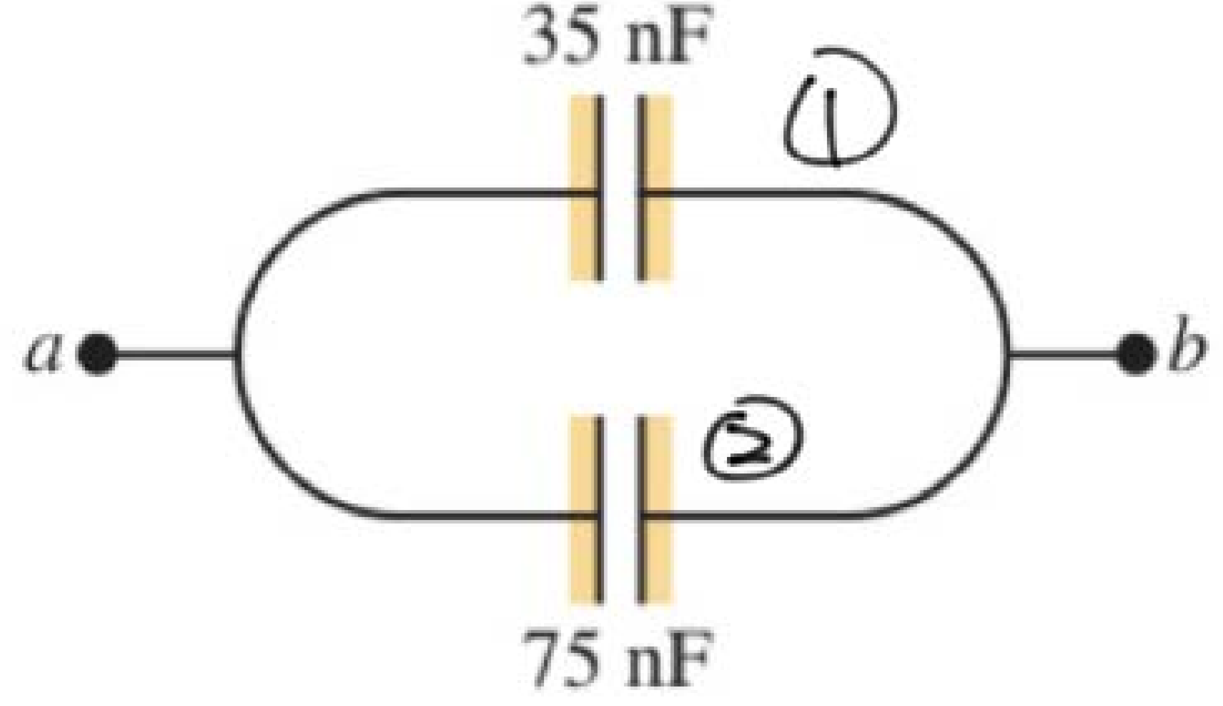
$$= 4.19\,J$$

(b) $C_2 = 2C_1 \qquad V_1 = \sqrt{\frac{2U_1}{C_1}} = V_2$

$$U_2 = \frac{1}{2} C_2 V_1^2 = \frac{1}{2}(2C_1)V_1^2 = 2U_1 = 16.76\,J$$

24.31 • For the capacitor network shown in Fig. E24.31, the potential difference across ab is 220 V. Find (a) the total charge stored in this network; (b) the charge on each capacitor; (c) the total energy stored in the network; (d) the energy stored in each capacitor; (e) the potential difference across each capacitor.

Figure **E24.31**



(a) $V_{ab} = 220\,V$ $\quad C_{ab} = C_1 + C_2 = 110\,nF$

$$Q = C_{ab} V_{ab} = 220V \cdot 1.1 \times 10^{-7} F$$
$$= 2.42 \times 10^{-5} C$$

(b) Again, when connected in parallel, potential is still V_{ab}

$$Q_1 = C_1 \cdot V_{ab} = 7.7 \times 10^{-6} C$$

$$Q_2 = C_2 \cdot V_{ab} = 1.65 \times 10^{-5} C$$

Notice that the $\dfrac{Q_1}{Q_2} = \dfrac{C_1}{C_2}$ as expected.

(c) $U_{ab} = \frac{1}{2} C_{ab} V_{ab}^2$ Our new friend right?

$= 0.5 \cdot (1.1 \times 10^{-7}\ F) \cdot (220V)^2$

$= 2.66 \times 10^{-3} J$

(d) No electronic elements other than wires exist between a/b and capacitors $V = V_{ab} = 220V$

$U_1 = \frac{1}{2} C_1 V^2 = 0.5 \cdot 35 \times 10^{-9} F \cdot (220V)^2$

$= 8.47 \times 10^{-4} J$

$U_2 = \frac{1}{2} C_2 V^2 = 0.5 \cdot 75 \times 10^{-9} F \cdot (220V)^2$

$= 1.82 \times 10^{-3} J$

(e) 220 V