Lecture 8

Impulse and Momentum

Date: 4/3/2024

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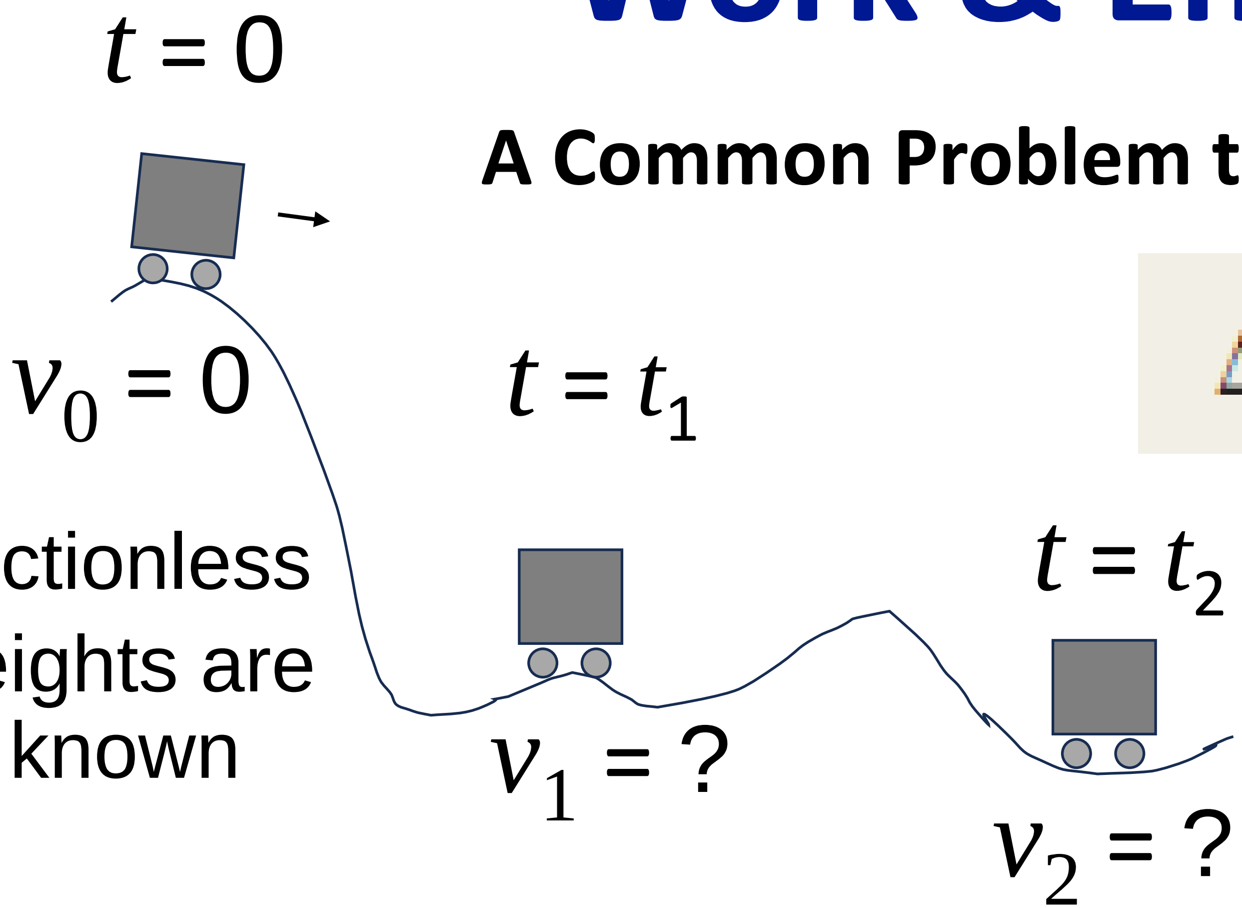
Previously we have learned Work & Energy

A Common Problem to consider:



$$\Delta K + \Delta U + \Delta U_{\text{int}} = 0$$

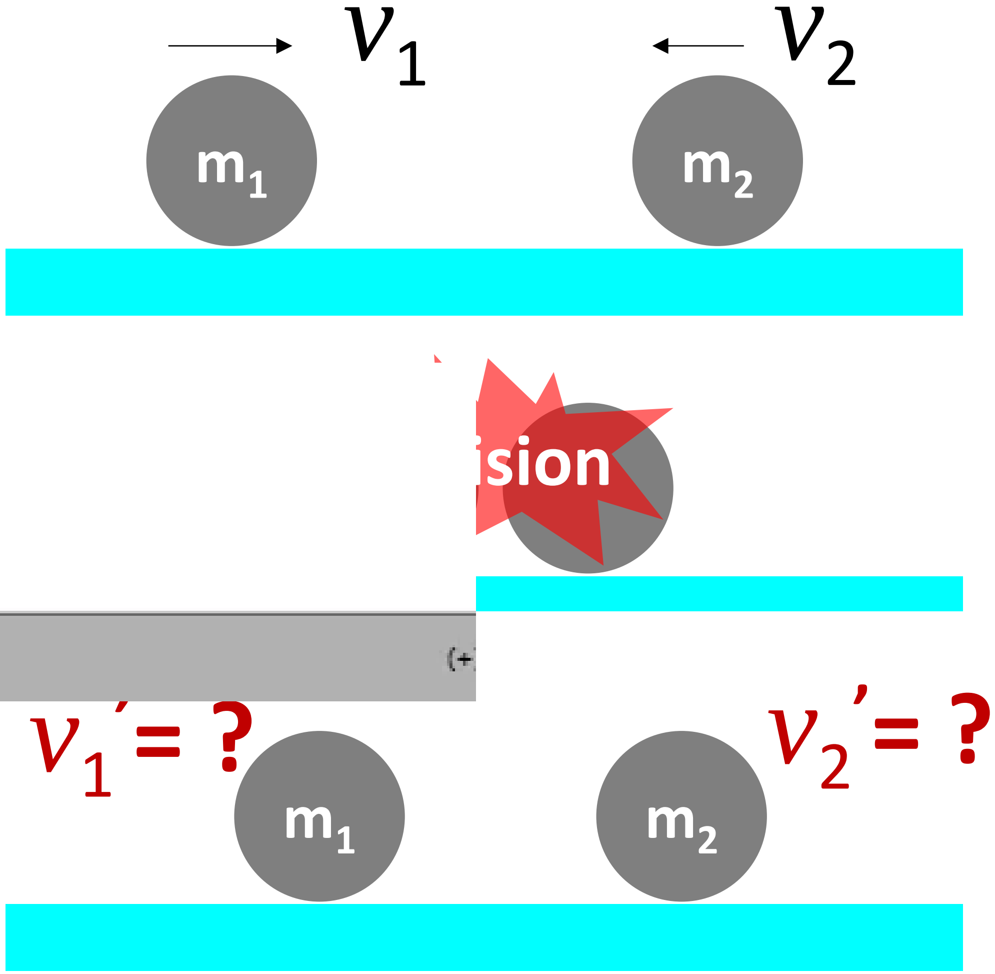
- Frictionless
- Heights are all known

What problems could be *still* hard to solve?

This Lecture

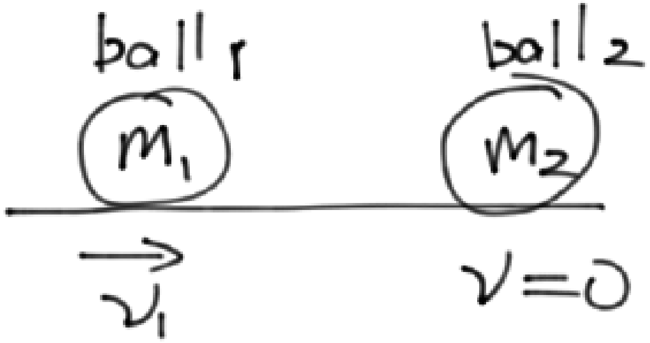
What physics are involved?

Start with 1D:



What problems could be still hard to solve?

Think about a sphere collision



We know immediately that

$$K_1 = \frac{1}{2} m_1 v_1^2$$

but what are v_1' v_2' after collision?

Energy conservation

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

More than 1 solution

But if you do the experiment there is only one outcome

New Tools: Momentum (& Impulse)

Recall Newton's Second Law

$$\sum \vec{F} = m\frac{d\vec{v}}{dt} \; \boxed{\text{Because } \vec{a} = d\vec{v}/dt} \rightarrow \; \sum \vec{F} = m\vec{a}$$

What if we group ***m*** and ***v*** together?

$$\sum \vec{F} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v})$$

We'll call this combination the **momentum,** or **linear momentum**

$$\vec{p} = m\vec{v} \qquad \text{(definition of momentum)}$$

Momentum as Vectors

We'll call this combination the **momentum,** or **linear momentum**

$$\boxed{\vec{p} = m\vec{v}} \qquad \text{(definition of momentum)}$$

$$p_x = mv_x \qquad p_y = mv_y \qquad p_z = mv_z$$

The net force (vector sum of all forces) acting on a particle equals the time rate of change of momentum of the particle. This, not $\sum \vec{F} = m\vec{a}$, is the form in which Newton originally stated his second law

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \qquad \text{(Newton's second law in terms of momentum)}$$

Momentum and Impulse: Defintion

Given *constant* net force $\sum \vec{F}$ applied to a particle Δt from t_1 to t_2

The **impulse** of the net force, denoted by $\vec{J}$

$$\vec{J} = \boxed{\sum \vec{F}(t_2 - t_1)} = \sum \vec{F}\,\Delta t \qquad \text{(assuming constant net force)}$$

SI unit newton-second $(\mathrm{N \cdot s})$

Recall what we just derived, and assume constant force

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \sum \vec{F} = \frac{\vec{p}_2 - \vec{p}_1}{t_2 - t_1} \Rightarrow \boxed{\sum \vec{F}(t_2 - t_1)} = \vec{p}_2 - \vec{p}_1$$

Momentum and Impulse: Defintion

Given *constant* net force $\sum\vec{F}$ applied to a particle Δt from t_1 to t_2

The **impulse** of the net force, denoted by $\vec{J}$

$$\vec{J} = \vec{p}_2 - \vec{p}_1 \qquad \text{(impulse–momentum theorem)}$$

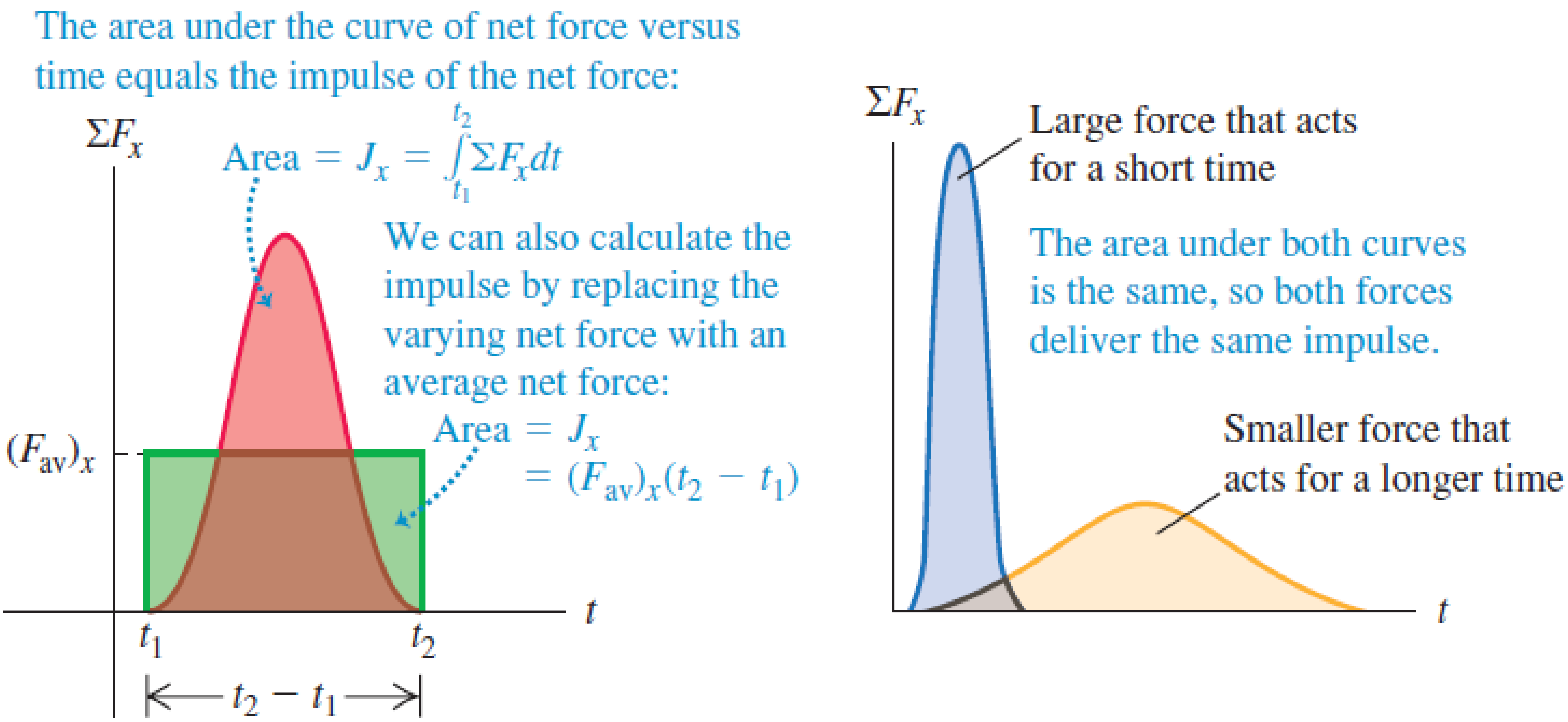
Applicable to ***varying forces***?

The change in momentum of a particle during a time interval equals the impulse of the net force that acts on the particle during that interval.

$$\boxed{\int_{t_1}^{t_2} \sum\vec{F}\,dt} = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt}\,dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} = \vec{p}_2 - \vec{p}_1$$

$$\vec{J} = \int_{t_1}^{t_2} \sum\vec{F}\,dt \qquad \text{(general definition of impulse)}$$

Impulse: Graphics View



Key Concepts to Know

$$\vec{p} = m\vec{v} \qquad \text{(definition of momentum)}$$

$$\vec{J} = \int_{t_1}^{t_2} \Sigma\vec{F}\,dt \qquad \text{(general definition of impulse)}$$

$$\vec{J} = \Sigma\vec{F}(t_2 - t_1) = \Sigma\vec{F}\,\Delta t \qquad \text{(assuming constant net force)}$$

$$\vec{J} = \vec{p}_2 - \vec{p}_1 \qquad \text{(impulse–momentum theorem)}$$

The change in momentum of a particle during a time interval equals the impulse of the net force that acts on the particle during that interval.

Momentum $\vec{p}$ vs Kinetic Energy K

$\vec{J} = \vec{p}_2 - \vec{p}_1$ <u>*Momentum change*</u>

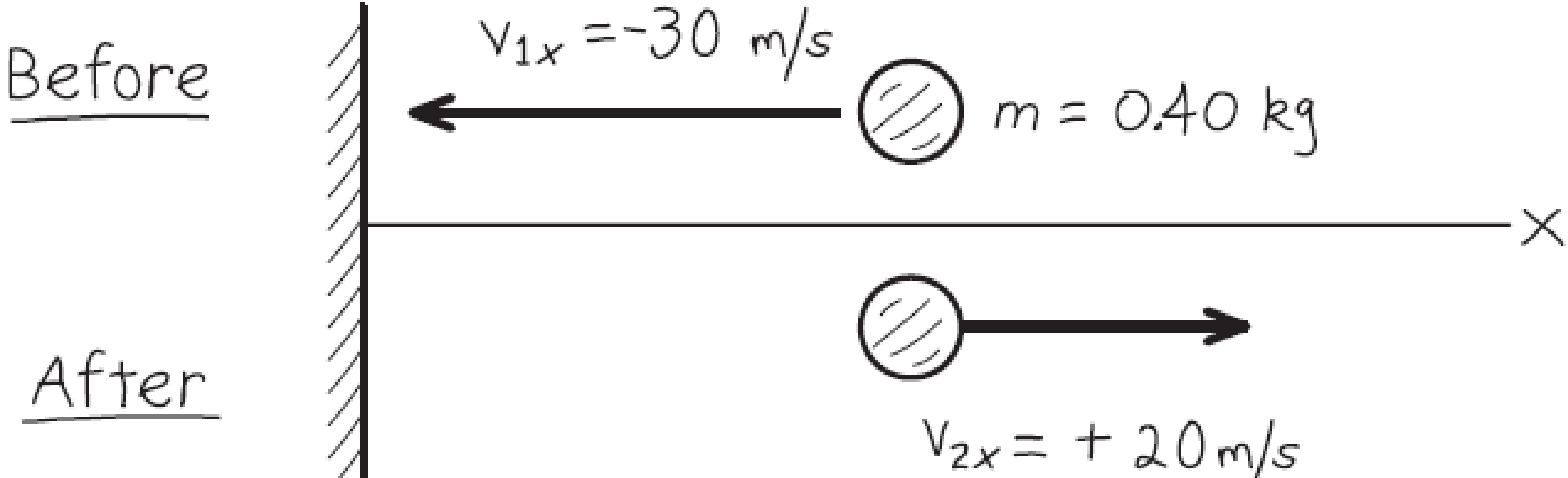
depends on the *time* the net force acts

$W_{tot} = K_2 - K_1$ <u>*Kinetic energy change*</u>

depends on the *distance* over which the net force acts

Example 8.2 A Ball Hits a Wall

You throw a ball with a mass of 0.40 kg against a brick wall. It hits the wall moving horizontally to the left at 30 m/s and rebounds horizontally to the right at 20 m/s. (a) Find the impulse of the net force on the ball during its collision with the wall. (b) If the ball is in contact with the wall for 0.010 s, find the average horizontal force that the wall exerts on the ball during the impact.



Example 8.2 A Ball Hits a Wall

EXECUTE: (a) With our choice of x-axis, the initial and final x-components of momentum of the ball are <u>*Which equation to use?*</u>

$$\vec{p} = m\vec{v}$$

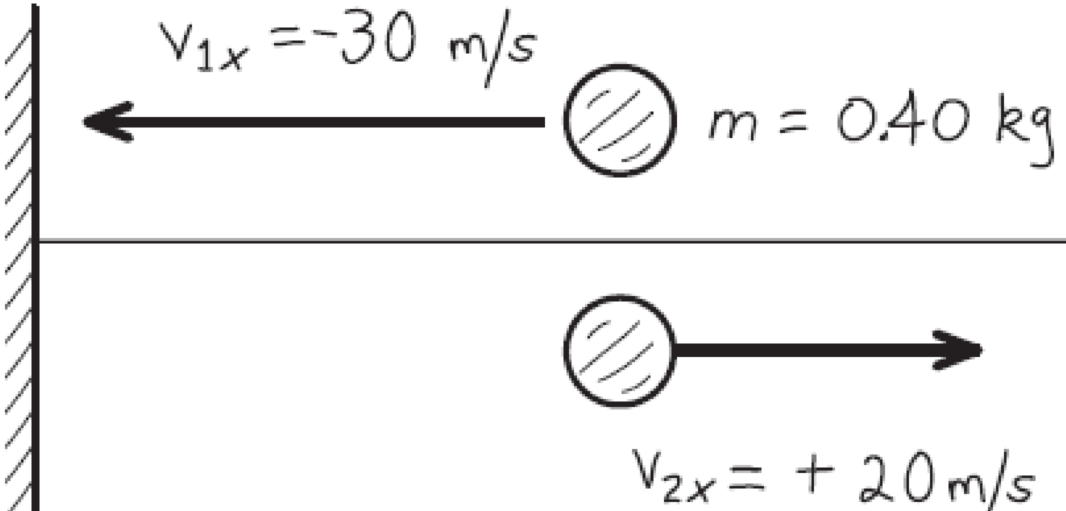
$$= (0.40\ \text{kg})(-30\ \text{m/s}) = -12\ \text{kg}\cdot\text{m/s}$$

$$p_{2x} = mv_{2x} = (0.40\ \text{kg})(+20\ \text{m/s}) = +8.0\ \text{kg}\cdot\text{m/s}$$

$$\vec{J} = \vec{p}_2 - \vec{p}_1$$

change in the x-momentum:

$$J_x = p_{2x} - p_{1x}$$
$$= 8.0\ \text{kg}\cdot\text{m/s} - (-12\ \text{kg}\cdot\text{m/s})$$
$$= 20\ \text{kg}\cdot\text{m/s} = 20\ \text{N}\cdot\text{s}$$



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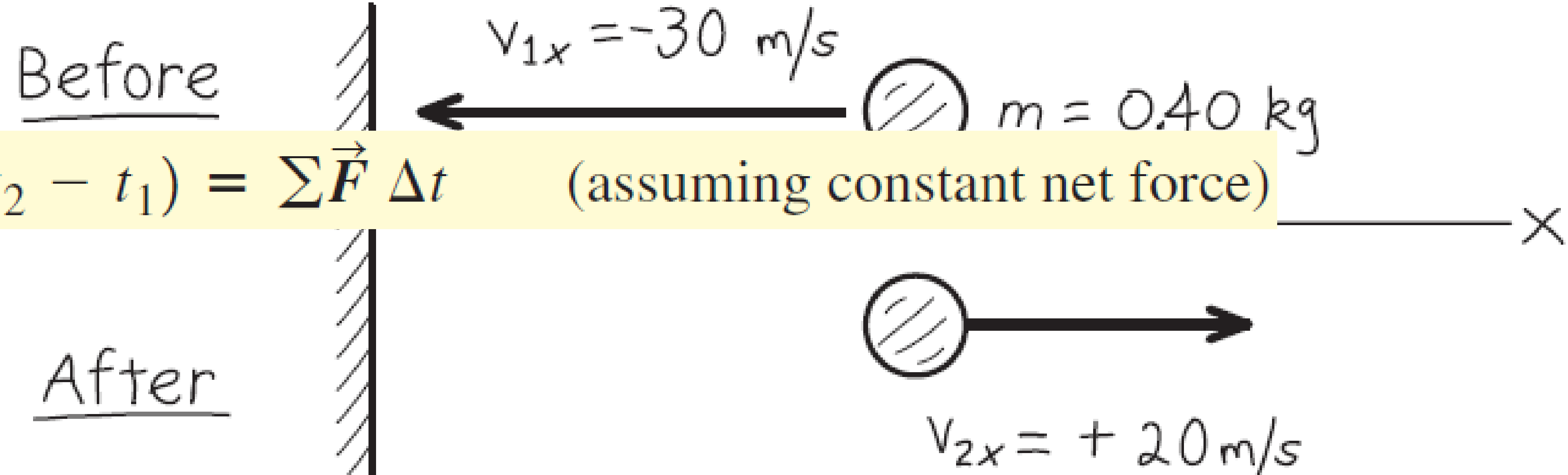
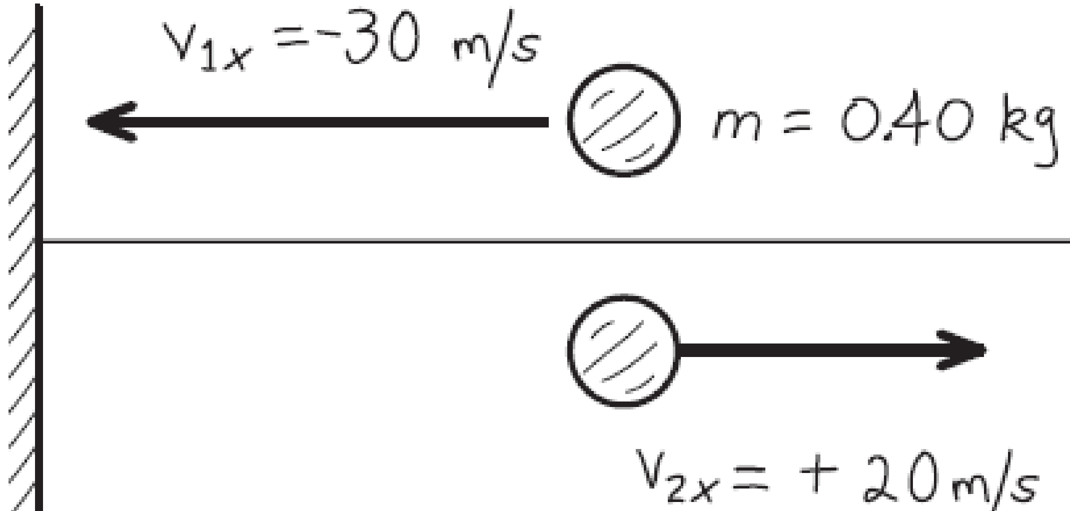


$$\vec{J} = \Sigma\vec{F}(t_2 - t_1) = \Sigma\vec{F}\,\Delta t \qquad \text{(assuming constant net force)}$$

Example 8.2 A Ball Hits a Wall

(b) The collision time is $t_2 - t_1 = \Delta t = 0.010\text{ s}$. From the x-equation in Eqs. (8.9), $J_x = (F_{av})_x(t_2 - t_1) = (F_{av})_x \Delta t$, so

$$(F_{av})_x = \frac{J_x}{\Delta t} = \frac{20\text{ N}\cdot\text{s}}{0.010\text{ s}} = 2000\text{ N}$$



8.1 • (a) What is the magnitude of the momentum of a 10,000-kg truck whose speed is 12.0 m/s? (b) What speed would a 2000-kg SUV have to attain in order to have (i) the same momentum? (ii) the same kinetic energy?

(a) $P_1 = m_1 v_1 = 1.2 \times 10^5 \ \mathrm{kg \cdot m/s}$

$m_2 = \frac{1}{5} m_1$

(b) for $P_2 = P_1 = m_1 v_1 = m_2 v_2 \Rightarrow v_2 = \frac{m_1}{m_2} v_1$

$= 5 v_1$

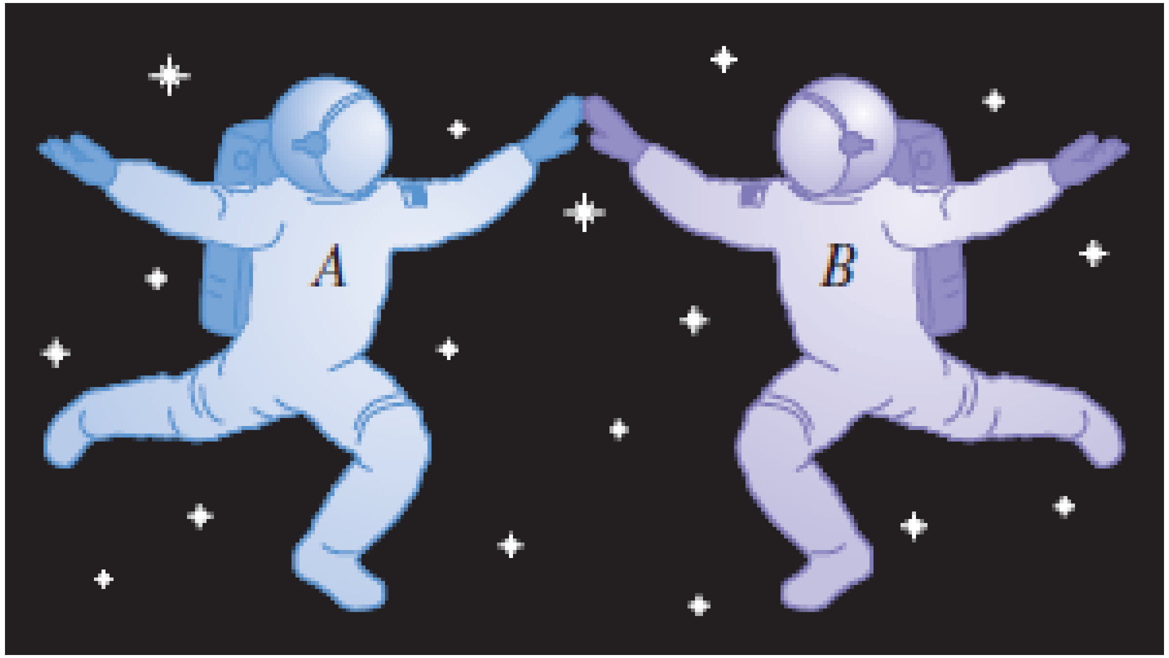
$= 60.0 \ \mathrm{m/s}$

for $K_2 = K_1$

$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2 \qquad v_2 = \sqrt{\frac{m_1}{m_2}} \, v_1 = 12\sqrt{5} \ \mathrm{m/s}$

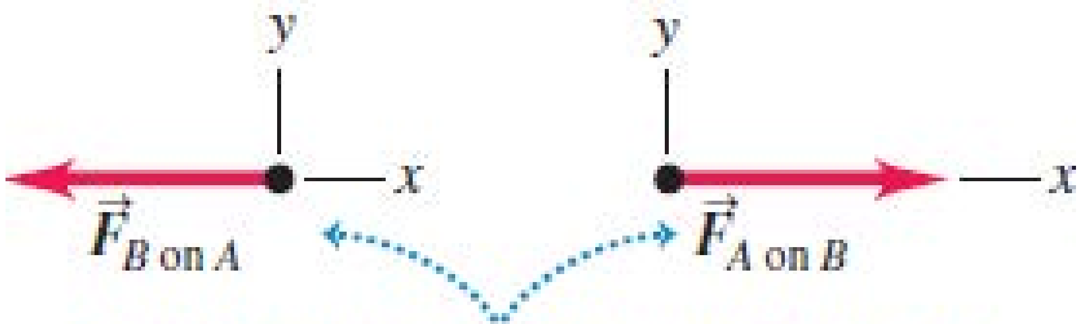
$= 26.83 \ \mathrm{m/s}$

Conservation of Momentum



No external forces act on the two-astronaut system, so its total momentum is conserved.

8.8 Two astronauts push each other as they float freely in the zero-gravity environment of space.



The forces the astronauts exert on each other form an action–reaction pair.

Consider the two astronauts as a *single system*

This system is called an ***isolated system***.

Forces that these astronauts exert on each other are called **internal forces**

Conservation of Momentum

In this ***isolated system***:

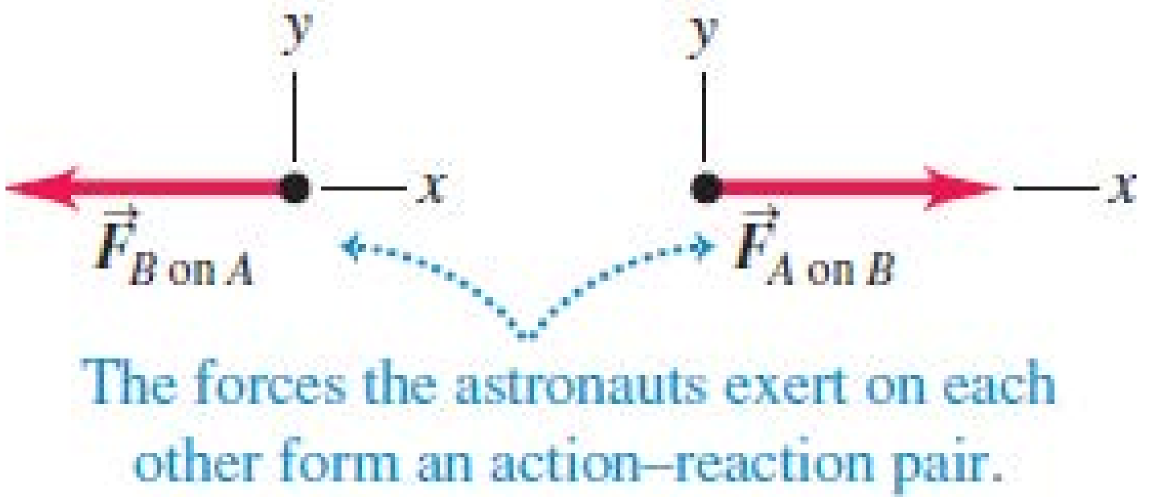
$$\vec{F}_{B\text{ on }A} = \frac{d\vec{p}_A}{dt} \qquad \vec{F}_{A\text{ on }B} = \frac{d\vec{p}_B}{dt}$$

Newton's third law $\vec{F}_{B\text{ on }A} = -\vec{F}_{A\text{ on }B}$

$$\vec{F}_{B\text{ on }A} + \vec{F}_{A\text{ on }B} = \frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt}$$

$$= \frac{d(\vec{p}_A + \vec{p}_B)}{dt} = 0$$

8.8 Two astronauts push each other as they float freely in the zero-gravity environment of space.



total momentum

$$\vec{P} = \vec{p}_A + \vec{p}_B$$

Conservation of Momentum

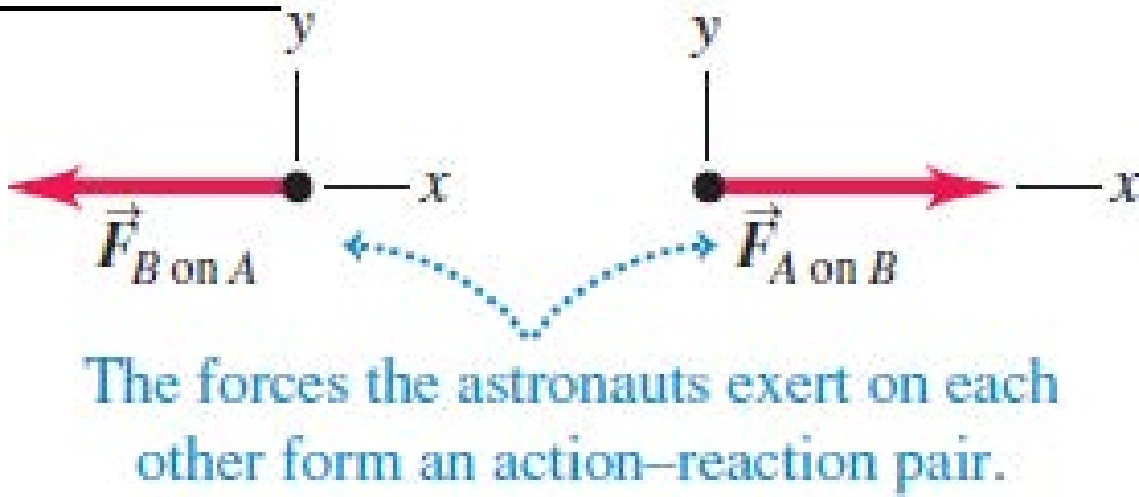
$$\vec{F}_{B \text{ on } A} + \vec{F}_{A \text{ on } B} = \frac{d\vec{P}}{dt} = 0$$

time rate of change of the *total* momentum $\vec{P}$ is zero

If the vector sum of the external forces on a system is zero, the total momentum of the system is constant.

<u>Principle of conservation of momentum</u>

8.8 Two astronauts push each other as they float freely in the zero-gravity environment of space.

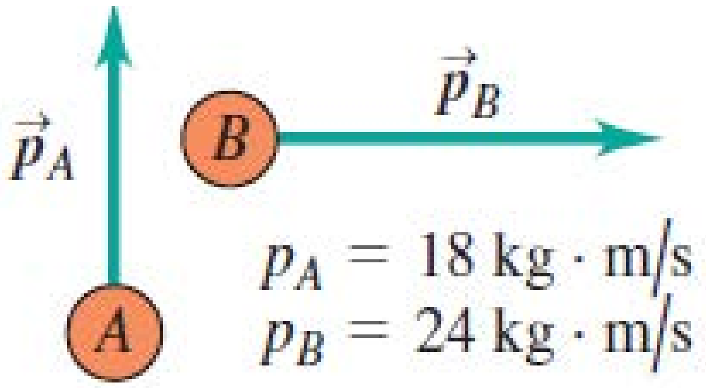


Momentum Conservation in Vector Form

$$\vec{P} = \vec{p}_A + \vec{p}_B + \cdots = m_A\vec{v}_A + m_B\vec{v}_B + \cdots$$

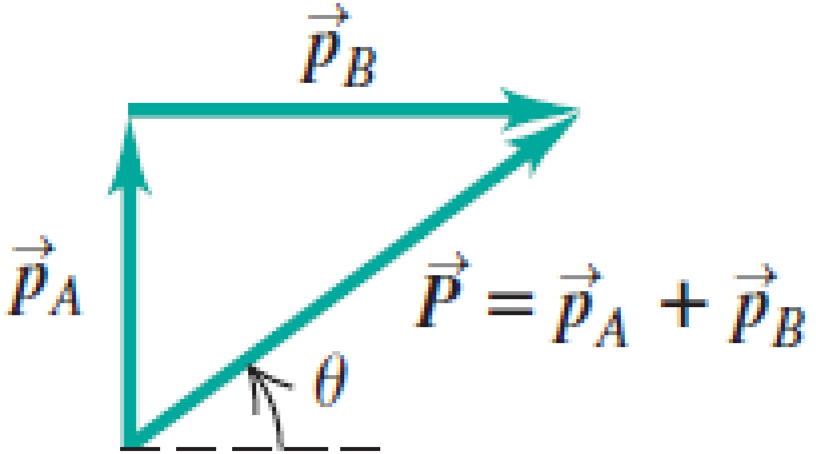
Conservation of momentum means conservation of its components

$$P_x = p_{Ax} + p_{Bx} + \cdots$$
$$P_y = p_{Ay} + p_{By} + \cdots$$
$$P_z = p_{Az} + p_{Bz} + \cdots$$



A system of two particles with momenta in different directions

$$P = |\vec{p}_A + \vec{p}_B| = 30\ \text{kg}\cdot\text{m/s at } \theta = 37^\circ$$



◀ RIGHT!

Conservation of Momentum

IDENTIFY *the relevant concepts:* Confirm that the vector sum of the external forces acting on the system of particles is zero. If it isn't zero, you can't use conservation of momentum.

1. Treat each body as a particle. Draw "before" and "after" sketches, including velocity vectors. Assign algebraic symbols to each magnitude, angle, and component. Use letters to label each particle and subscripts 1 and 2 for "before" and "after" quantities. Include any given values such as magnitudes, angles, or components.
2. Define a coordinate system and show it in your sketches; define the positive direction for each axis.
3. Identify the target variables.

Conservation of Momentum

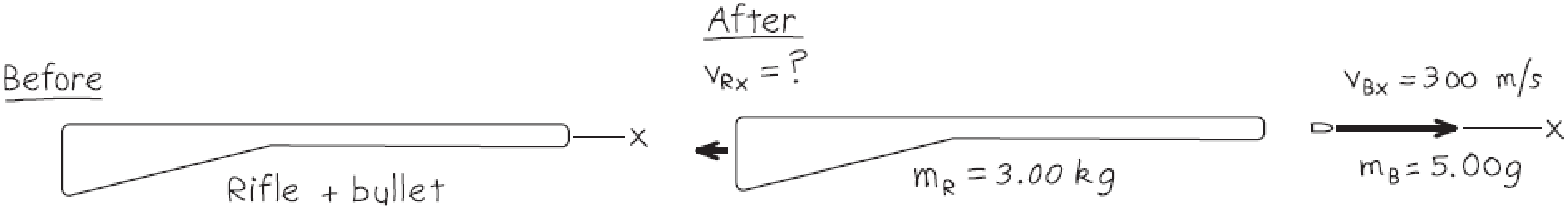
EXECUTE *the solution:*

1. Write an equation in symbols equating the total initial and final x-components of momentum, using $p_x = mv_x$ for each particle. Write a corresponding equation for the y-components. Velocity components can be positive or negative, so be careful with signs!
2. In some problems, energy considerations (discussed in Section 8.4) give additional equations relating the velocities.
3. Solve your equations to find the target variables.

Example 8.4 Recoil of a Rifle

A marksman holds a rifle of mass $m_R = 3.00$ kg loosely, so it can recoil freely. He fires a bullet of mass $m_B = 5.00$ g horizontally with a velocity relative to the ground of $v_{Bx} = 300$ m/s. What is the recoil velocity v_{Rx} of the rifle? What are the final momentum and kinetic energy of the bullet and rifle?

IDENTIFY and SET UP: If the marksman exerts negligible horizontal forces on the rifle, then there is no net horizontal force on the system (the bullet and rifle) during the firing, and the total horizontal momentum of the system is conserved.



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EXECUTE: Conservation of the x-component of total momentum

$$P_x = 0 = m_B v_{Bx} + m_R v_{Rx}$$

$$v_{Rx} = -\frac{m_B}{m_R} v_{Bx} = -\left(\frac{0.00500 \text{ kg}}{3.00 \text{ kg}}\right)(300 \text{ m/s}) = -0.500 \text{ m/s}$$

The negative sign means that the recoil is in the direction opposite to that of the bullet.

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The final momenta and kinetic energies are

$$p_{Bx} = m_B v_{Bx} = (0.00500 \text{ kg})(300 \text{ m/s}) = 1.50 \text{ kg} \cdot \text{m/s}$$

$$K_B = \tfrac{1}{2} m_B v_{Bx}{}^2 = \tfrac{1}{2}(0.00500 \text{ kg})(300 \text{ m/s})^2 = 225 \text{ J}$$

$$p_{Rx} = m_R v_{Rx} = (3.00 \text{ kg})(-0.500 \text{ m/s}) = -1.50 \text{ kg} \cdot \text{m/s}$$

$$K_R = \tfrac{1}{2} m_R v_{Rx}{}^2 = \tfrac{1}{2}(3.00 \text{ kg})(-0.500 \text{ m/s})^2 = 0.375 \text{ J}$$

Elastic and Inelastic Collisions

Elastic collision

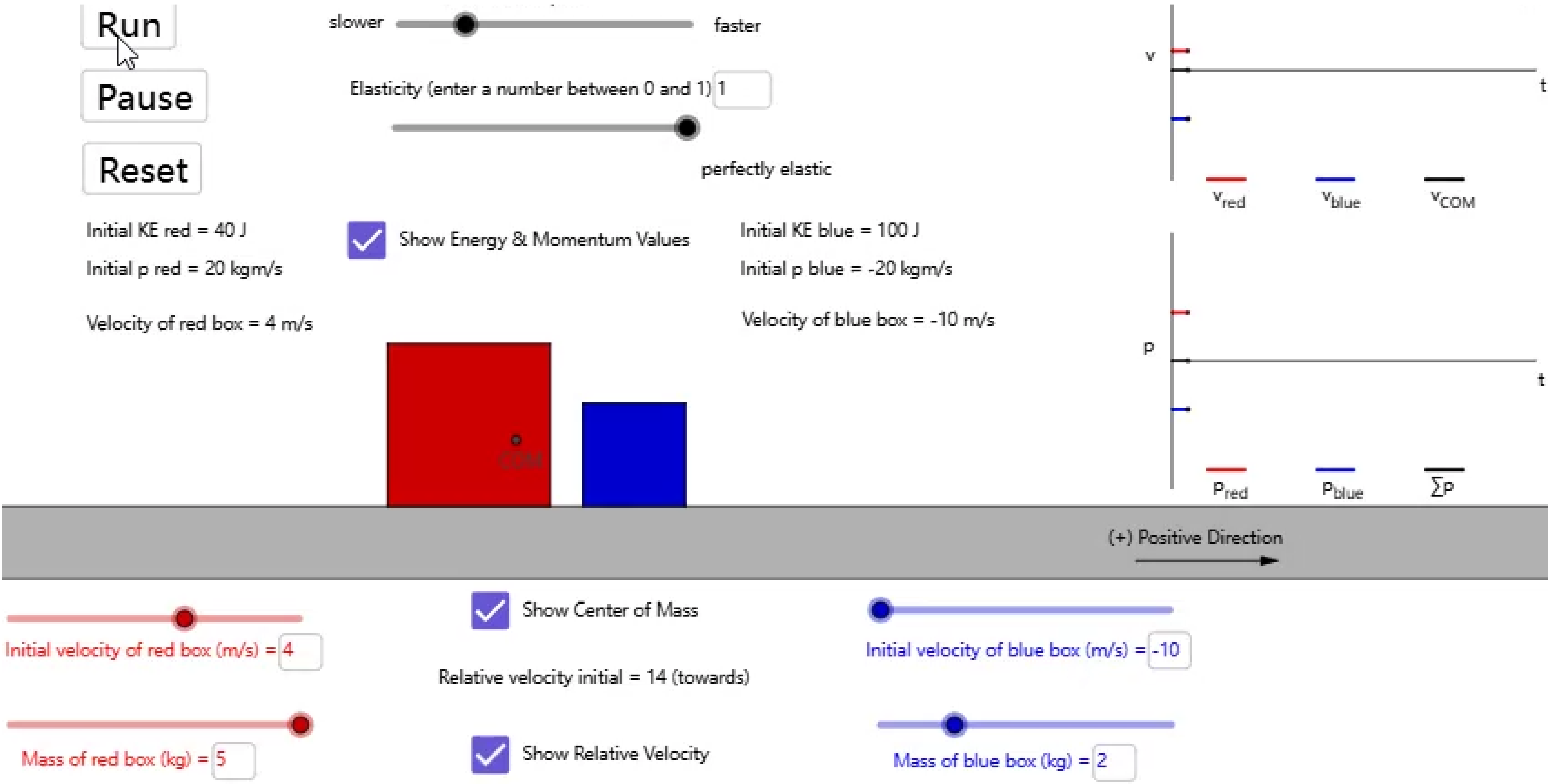
- Forces between the bodies are also *conservative*,
- No mechanical energy is lost or gained in the collision,
- Total *kinetic* energy is the same after the collision as before

Inelastic collision

- Total kinetic energy after the collision is less than before

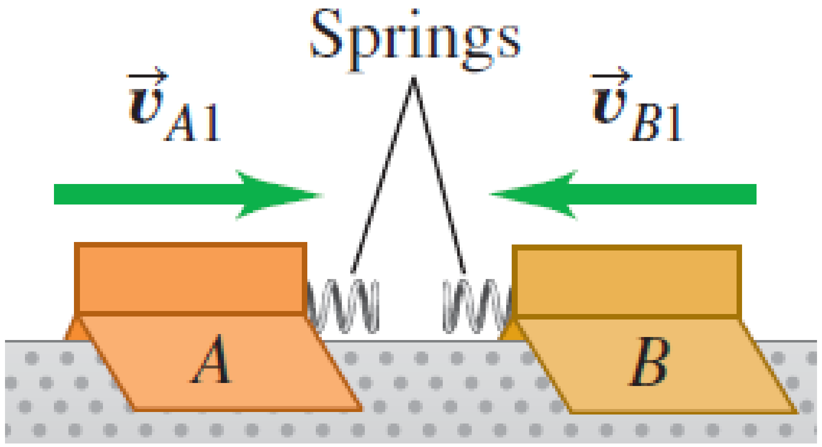
An inelastic collision in which the colliding bodies stick together and move as one body after the collision is often called a **completely inelastic collision.**

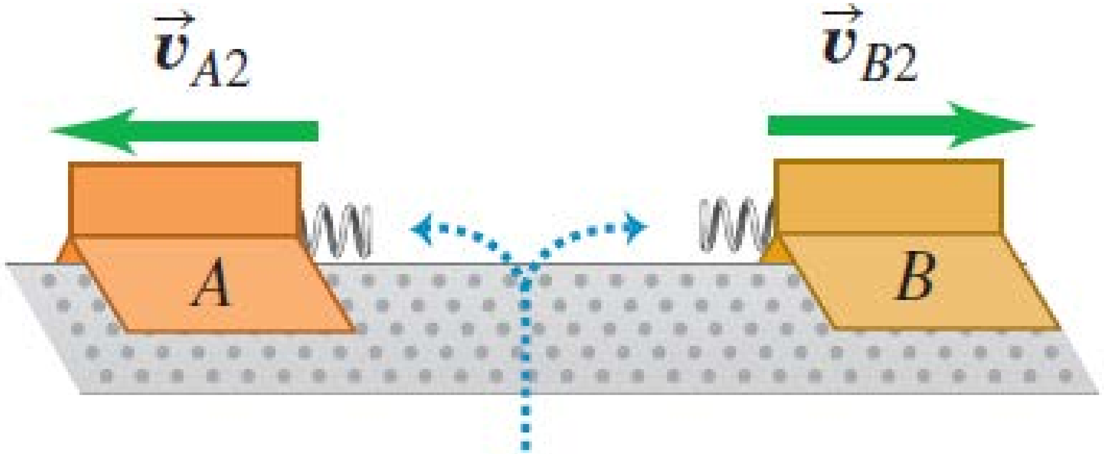
An **inelastic collision** doesn't have to be **completely inelastic**



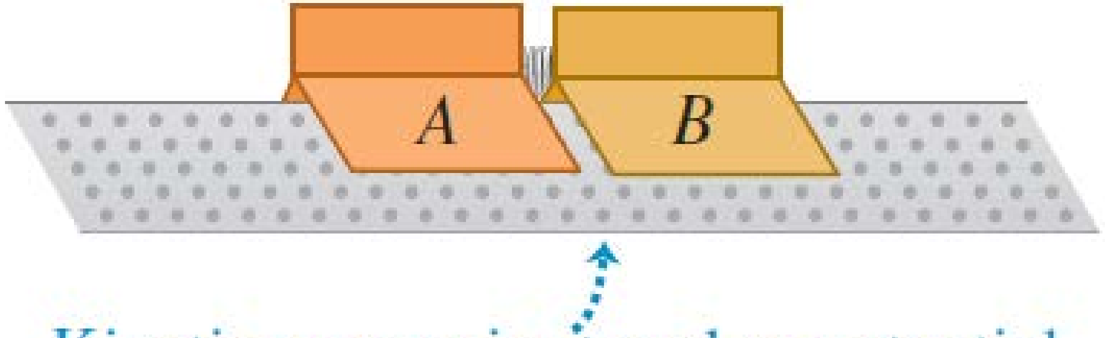
https://ophysics.com/e2.html

Elastic & Inelastic Collisions: Examples





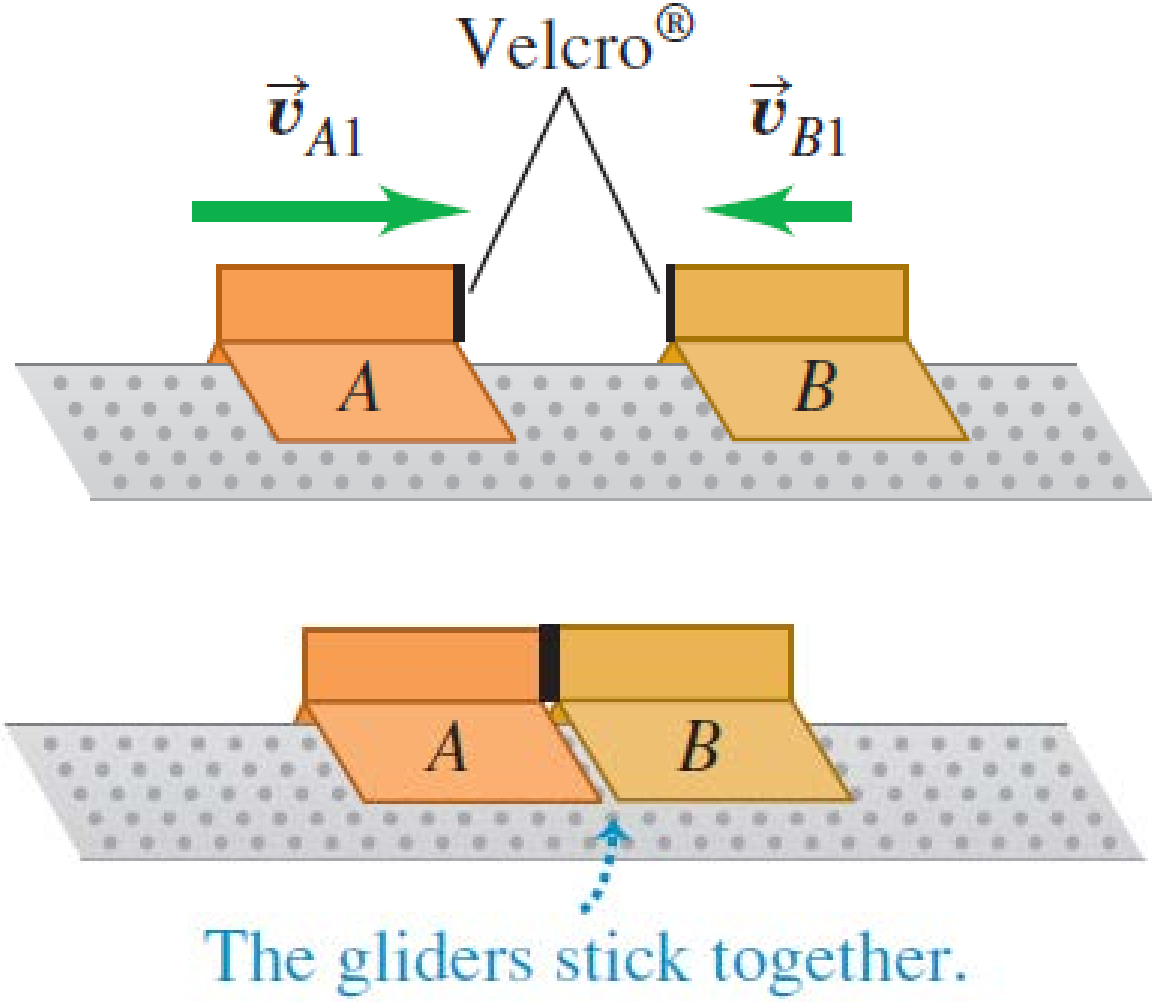
The system of the two gliders has the same kinetic energy after the collision as before it.

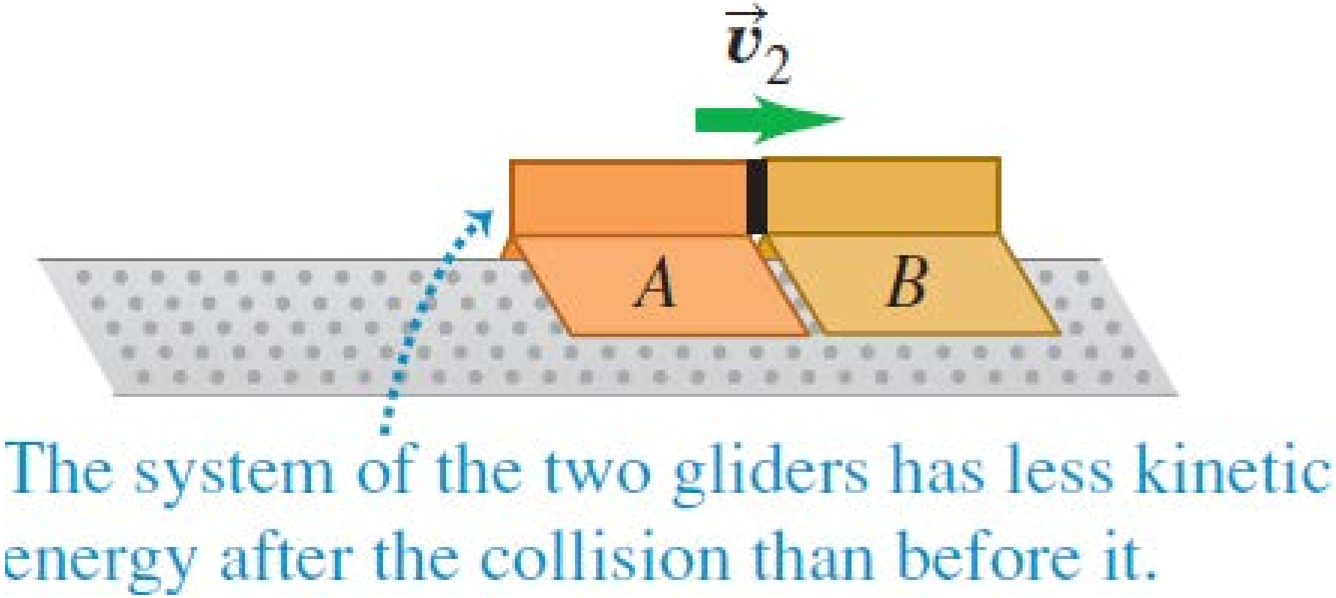


Kinetic energy is stored as potential energy in compressed springs.

Elastic collision

Elastic & Inelastic Collisions: Examples





<u>(Completely) Inelastic collision</u>

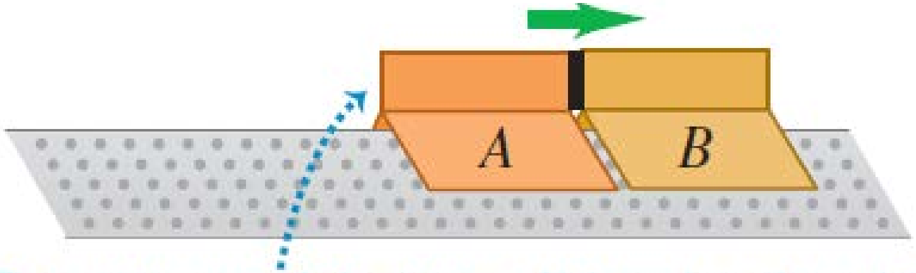
Completely Inelastic Collisions: Momentum Conservation

Momentum Conservation states

$$m_A\vec{v}_{A1} + m_B\vec{v}_{B1} = (m_A + m_B)\vec{v}_2$$

Suppose (just a special condition)
B was at rest, or $v_{B1} = 0$

$$\vec{v}_{A2} = \vec{v}_{B2} = \vec{v}_2$$



The system of the two gliders has less kinetic energy after the collision than before it.

Final velocity

$$\boxed{v_{2x} = \frac{m_A}{m_A + m_B}v_{A1x}}$$

Before collision $$K_1 = \frac{1}{2}m_A v_{A1x}^2$$

After collision

$$K_2 = \frac{1}{2}(m_A + m_B)v_{2x}^2$$

$$= \frac{1}{2}(m_A + m_B)\left(\frac{m_A}{m_A + m_B}\right)^2 v_{A1x}^2$$

What is the lost in kinetic energy?

Completely Inelastic Collisions: Energy Change

Momentum Conservation states

$$m_A\vec{v}_{A1} + m_B\vec{v}_{B1} = (m_A + m_B)\vec{v}_2$$

Suppose (just a special condition)

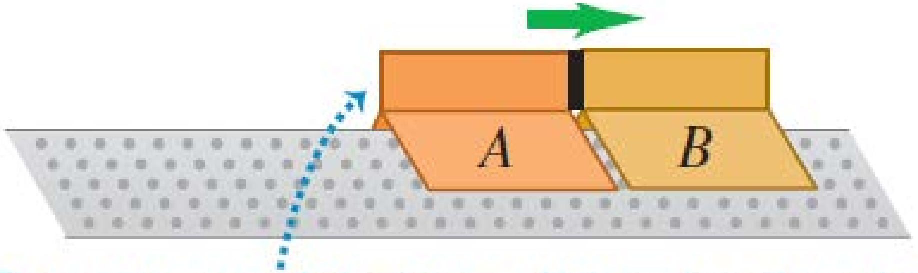
B was at rest, or $v_{B1} = 0$

Final velocity

$$\boxed{v_{2x} = \frac{m_A}{m_A + m_B} v_{A1x}}$$

What is the lost in kinetic energy?

$$\vec{v}_{A2} = \vec{v}_{B2} = \vec{v}_2$$

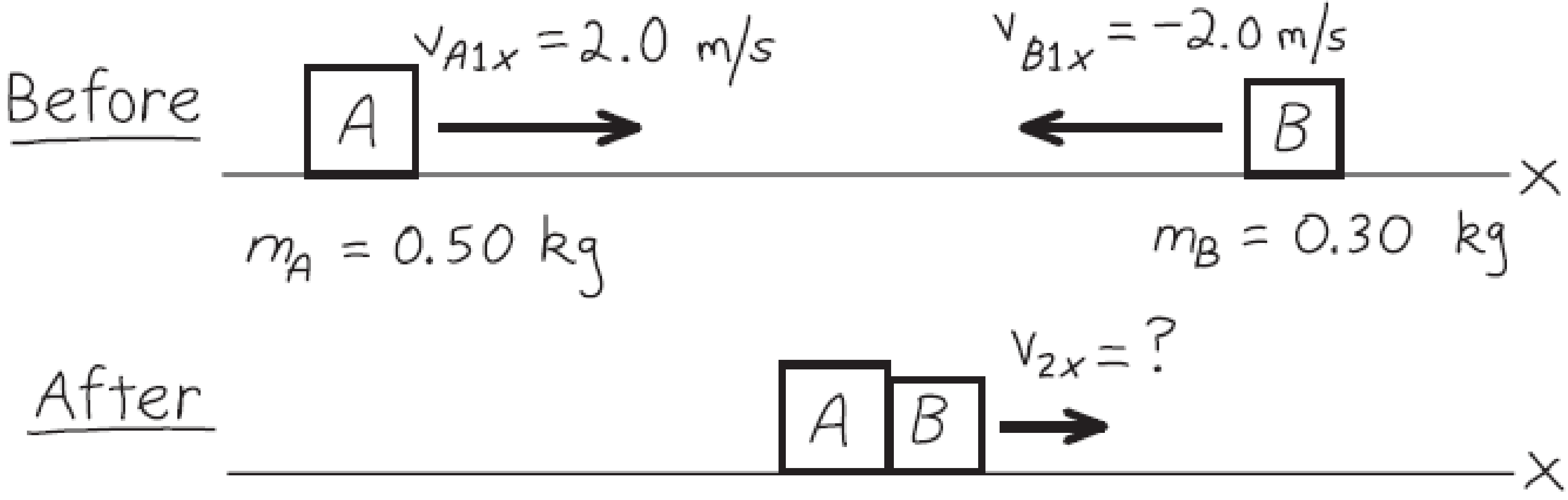


The system of the two gliders has less kinetic energy after the collision than before it.

$$\frac{K_2}{K_1} = \frac{m_A}{m_A + m_B}$$

Example 8.7 Completely inelastic collision

We repeat the collision described in Example 8.5 (Section 8.2), but this time equip the gliders so that they stick together when they collide. Find the common final *x*-velocity, and compare the initial and final kinetic energies of the system.



Example 8.7 Completely inelastic collision

We repeat the collision described in Example 8.5 (Section 8.2), but this time equip the gliders so that they stick together when they collide. Find the common final *x*-velocity, and compare the initial and final kinetic energies of the system.

$$m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B) v_{2x}$$

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(0.50\ \text{kg})(2.0\ \text{m/s}) + (0.30\ \text{kg})(-2.0\ \text{m/s})}{0.50\ \text{kg} + 0.30\ \text{kg}}$$

$$= 0.50\ \text{m/s}$$

Example 8.7 Completely inelastic collision

Because v_{2x} is positive, the gliders move together to the right after the collision. Before the collision, the kinetic energies are:

$$K_A = \tfrac{1}{2} m_A v_{A1x}{}^2 = \tfrac{1}{2}(0.50\ \text{kg})(2.0\ \text{m/s})^2 = 1.0\ \text{J}$$

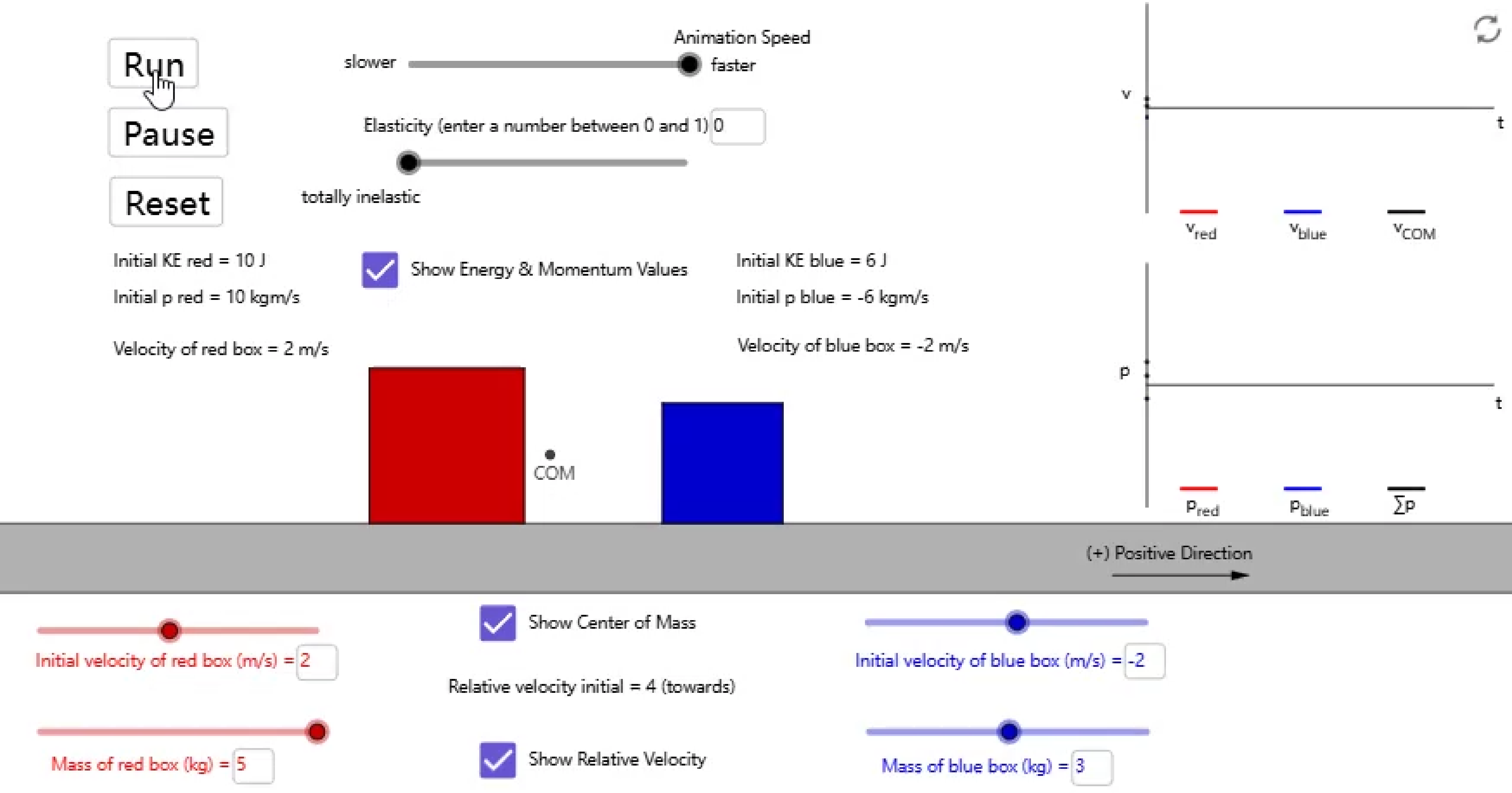
$$K_B = \tfrac{1}{2} m_B v_{B1x}{}^2 = \tfrac{1}{2}(0.30\ \text{kg})(-2.0\ \text{m/s})^2 = 0.60\ \text{J}$$

The total kinetic energy before the collision is 1.6 J.
The kinetic energy after the collision is:

$$K_2 = \tfrac{1}{2}(m_A + m_B) v_{2x}{}^2 = \tfrac{1}{2}(0.50\ \text{kg} + 0.30\ \text{kg})(0.50\ \text{m/s})^2$$

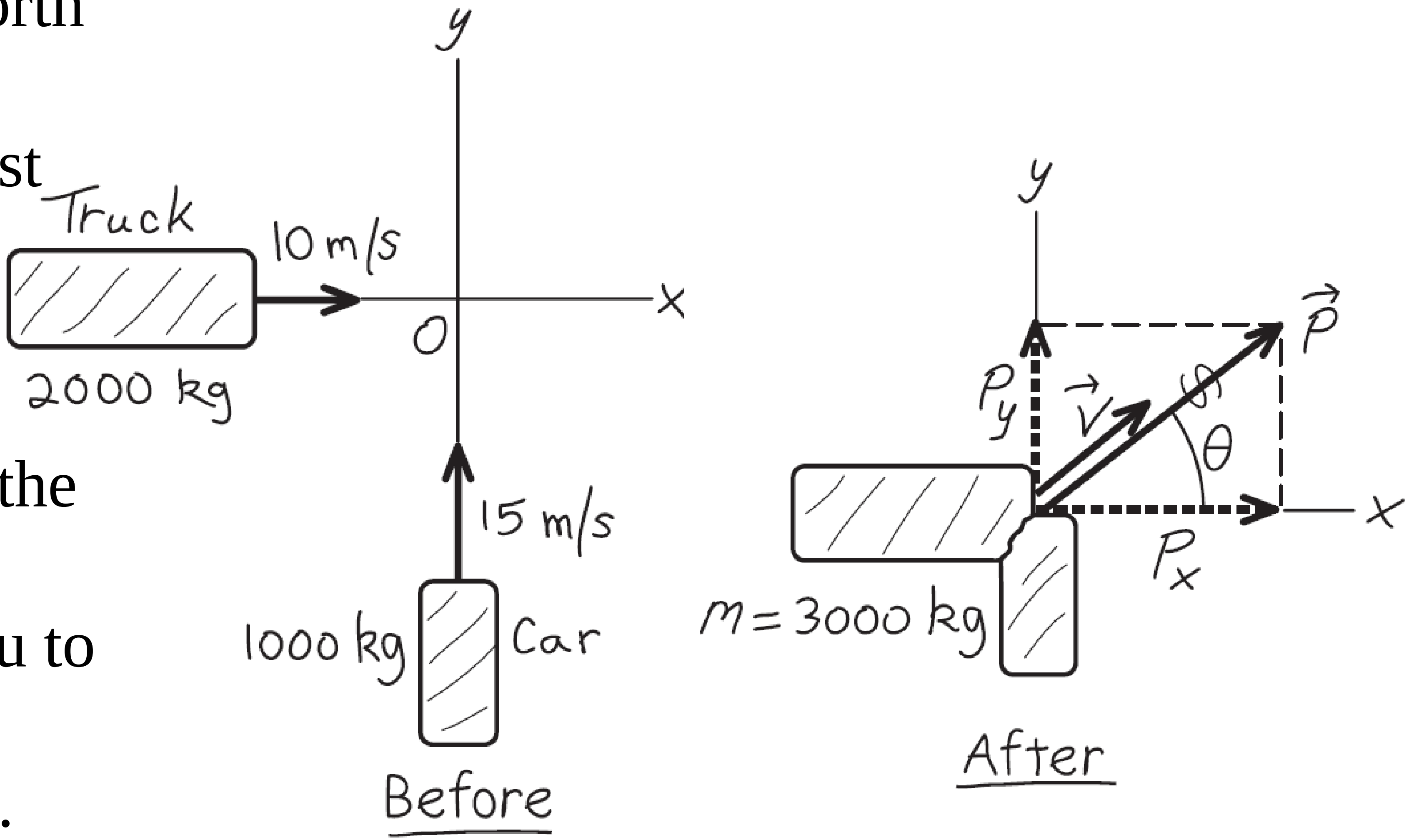
$$= 0.10\ \text{J}$$

Lost 1.5 J!


Run
Pause
Reset
Animation Speed
slower
faster
Elasticity (enter a number between 0 and 1) 0
totally inelastic
Initial KE red = 10 J
Initial p red = 10 kgm/s
Show Energy & Momentum Values
Initial KE blue = 6 J
Initial p blue = -6 kgm/s
Velocity of red box = 2 m/s
Velocity of blue box = -2 m/s
COM
v
t
v_{red}
v_{blue}
v_{COM}
p
t
p_{red}
p_{blue}
$\sum p$
(+) Positive Direction
Show Center of Mass
Initial velocity of red box (m/s) = 2
Relative velocity initial = 4 (towards)
Initial velocity of blue box (m/s) = -2
Mass of red box (kg) = 5
Show Relative Velocity
Mass of blue box (kg) = 3

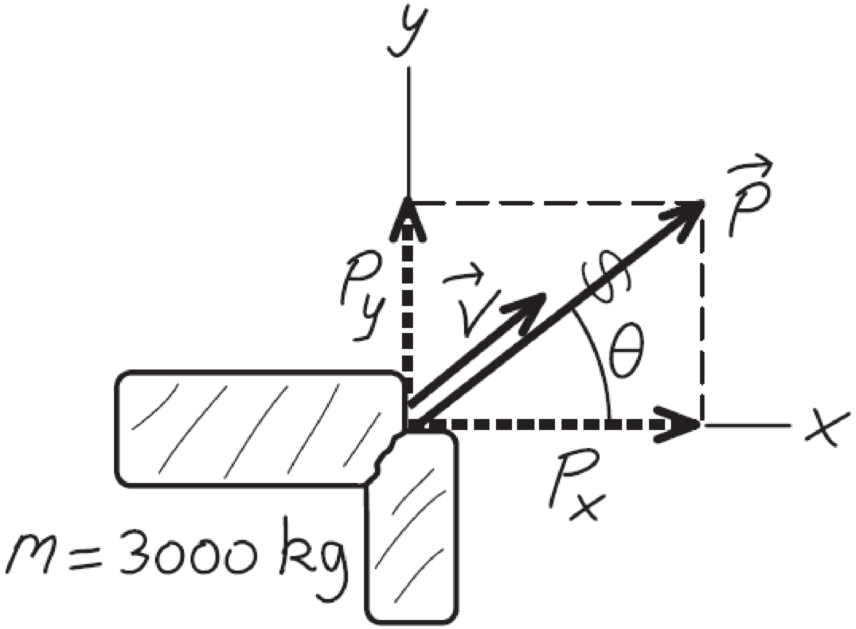
Example 8.9 An automobile collision

A 1000-kg car traveling north at 15 m/s collides with a 2000-kg truck traveling east at 10 m/s The occupants, wearing seat belts, are uninjured, but the two vehicles move away from the impact point as one. The insurance adjustor asks you to find the velocity of the wreckage just after impact. What is your answer?



Example 8.9 An automobile collision

IDENTIFY and SET UP: We'll treat the cars as an isolated system, so that the momentum of the system is conserved before and after the collision – Kinetic Energy is not!



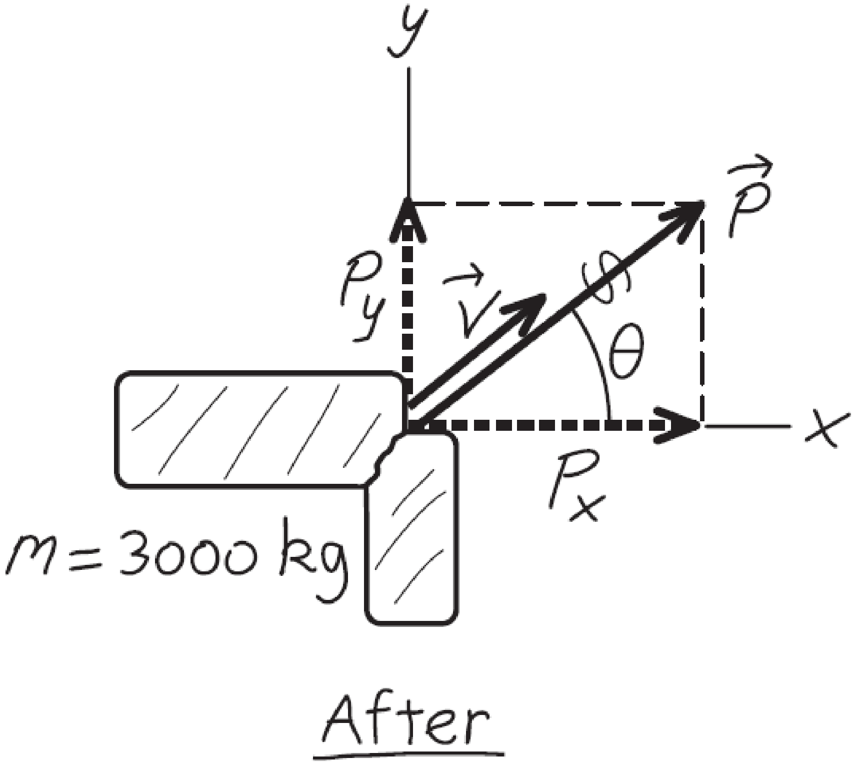
The momentum has the same value just after the collision; hence we can find the velocity just after the collision (our target variable) using:

$\vec{P} = M\vec{V}$, where $M = m_C + m_T = 3000$ kg is the mass of the wreckage.

Example 8.9 An automobile collision

$$P_x = p_{Cx} + p_{Tx} = m_C v_{Cx} + m_T v_{Tx}$$
$$= (1000\ \text{kg})(0) + (2000\ \text{kg})(10\ \text{m/s})$$
$$= 2.0 \times 10^4\ \text{kg} \cdot \text{m/s}$$

$$P_y = p_{Cy} + p_{Ty} = m_C v_{Cy} + m_T v_{Ty}$$
$$= (1000\ \text{kg})(15\ \text{m/s}) + (2000\ \text{kg})(0)$$
$$= 1.5 \times 10^4\ \text{kg} \cdot \text{m/s}$$



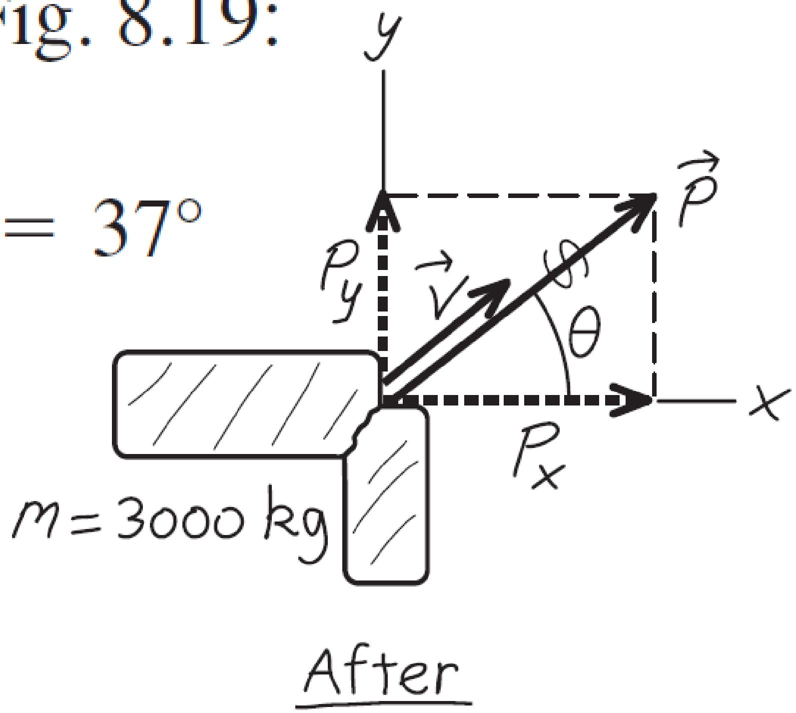
Example 8.9 An automobile collision

The magnitude of $\vec{\boldsymbol{P}}$ is

$$P = \sqrt{(2.0 \times 10^4\ \mathrm{kg \cdot m/s})^2 + (1.5 \times 10^4\ \mathrm{kg \cdot m/s})^2}$$
$$= 2.5 \times 10^4\ \mathrm{kg \cdot m/s}$$

and its direction is given by the angle θ shown in Fig. 8.19:

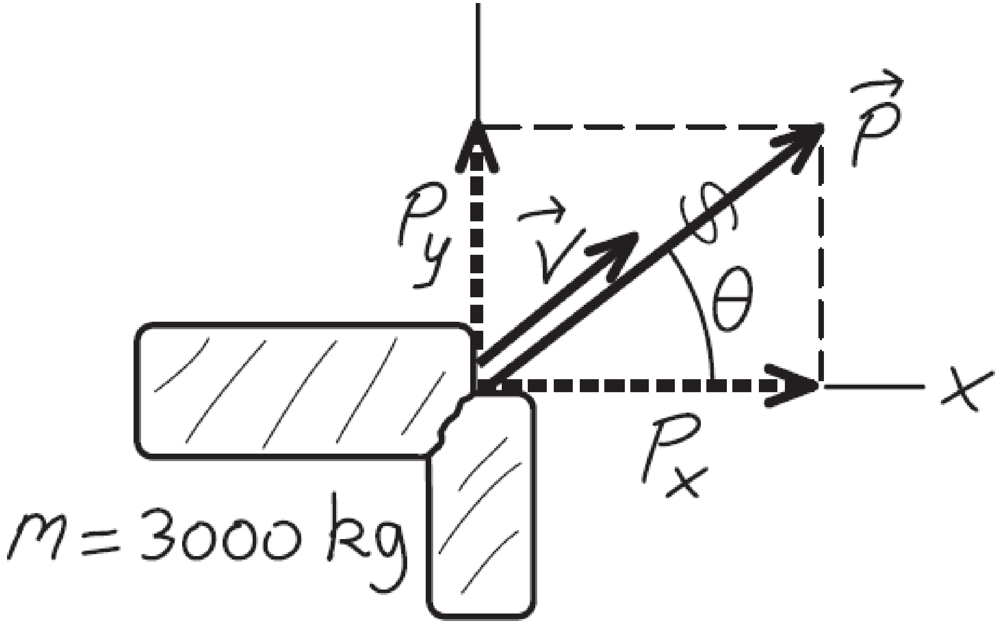
$$\tan\theta = \frac{P_y}{P_x} = \frac{1.5 \times 10^4\ \mathrm{kg \cdot m/s}}{2.0 \times 10^4\ \mathrm{kg \cdot m/s}} = 0.75 \qquad \theta = 37^\circ$$



Example 8.9 An automobile collision

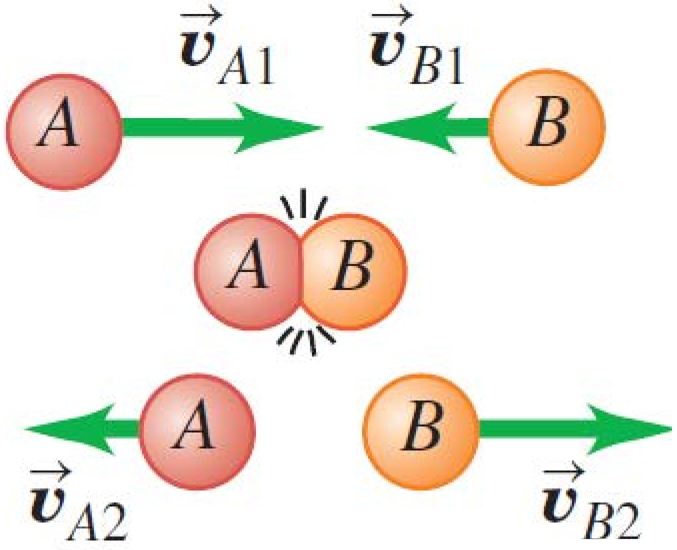
From $\vec{\boldsymbol{P}} = M\vec{\boldsymbol{V}}$, the direction of the velocity $\vec{\boldsymbol{V}}$ just after the collision is also $\theta = 37°$. The velocity magnitude is

$$V = \frac{P}{M} = \frac{2.5 \times 10^4 \text{ kg} \cdot \text{m/s}}{3000 \text{ kg}} = 8.3 \text{ m/s}$$



Classifying Collisions

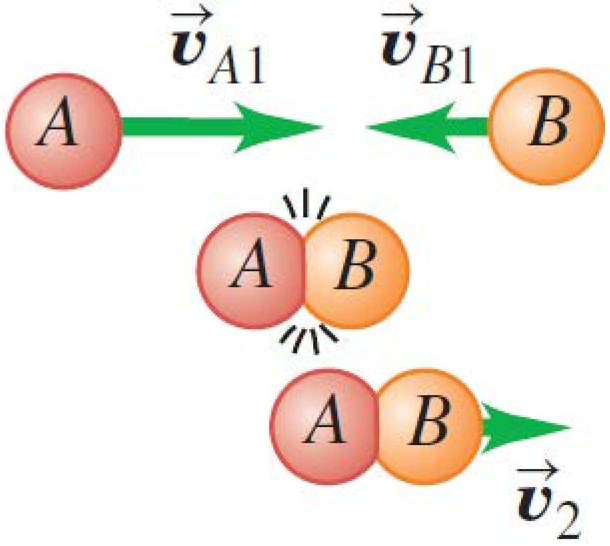
Elastic:
Kinetic energy conserved.



Collisions are classified according to energy considerations.

Energy lost is maximized (without external force)

Completely inelastic:
Bodies have same final velocity.



Elastic Collisions

Let's look at an elastic collision between two bodies *A* and *B*. We start with a one-dimensional collision, **in *x* only**. The key is the ***energy conservation***:

$$\boxed{\frac{1}{2}m_A v_{A1x}^2 + \frac{1}{2}m_B v_{B1x}^2 = \frac{1}{2}m_A v_{A2x}^2 + \frac{1}{2}m_B v_{B2x}^2}$$

In addition to: $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$

Conclusion first: in an **elastic collision** of two bodies, the relative velocities before and after the collision have the **same magnitude** but **opposite sign**.

$$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$$

Use instead of energy conservation

Observation in a Special Case

Elastic Collisions, One Body Initially at Rest

$$\frac{1}{2}m_A v_{A1x}{}^2 = \frac{1}{2}m_A v_{A2x}{}^2 + \frac{1}{2}m_B v_{B2x}{}^2$$

$$m_A v_{A1x} = m_A v_{A2x} + m_B v_{B2x}$$

First we rearrange Eqs. (8.19) and (8.20) as follows:

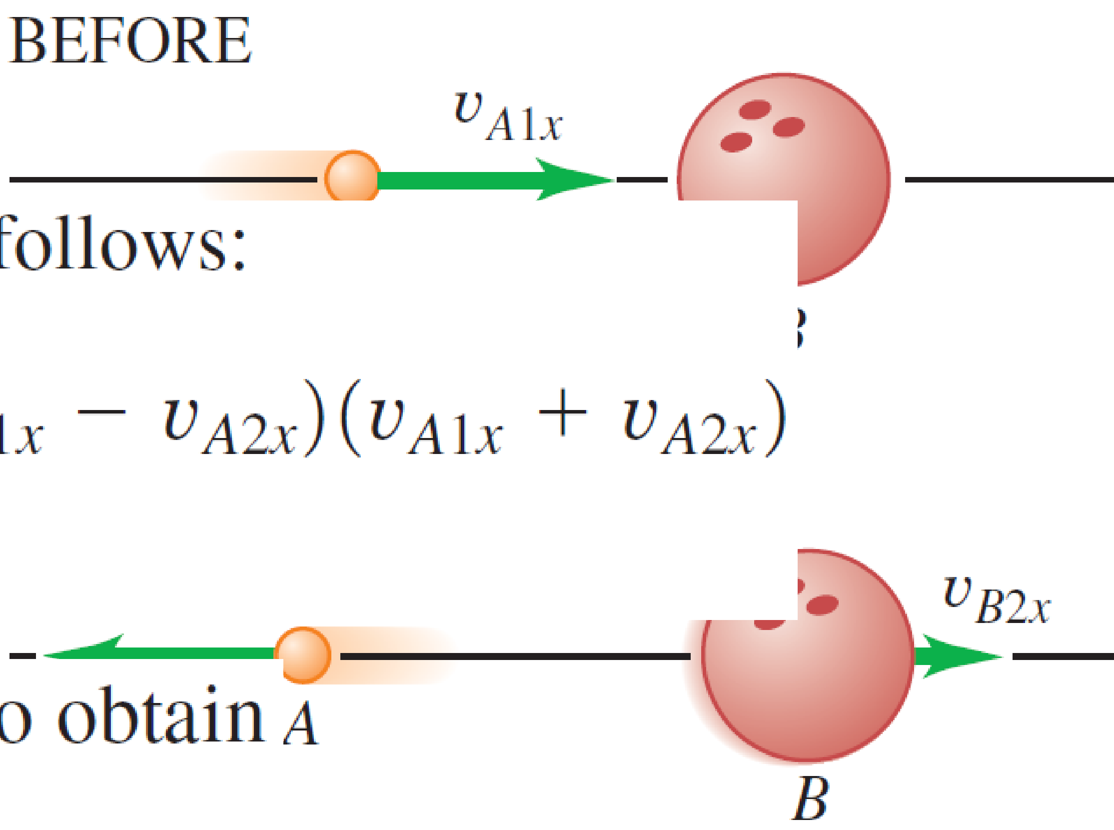
$$m_B v_{B2x}{}^2 = m_A(v_{A1x}{}^2 - v_{A2x}{}^2) = m_A(v_{A1x} - v_{A2x})(v_{A1x} + v_{A2x})$$

$$m_B v_{B2x} = m_A(v_{A1x} - v_{A2x})$$

Now we divide Eq. (8.21) by Eq. (8.22) to obtain

$$v_{B2x} = v_{A1x} + v_{A2x}$$

(a) Ping-Pong ball strikes bowling ball.



Observation in a Special Case

Elastic Collisions, One Body Initially at Rest

We substitute this expression back into Eq. (8.22) to eliminate v_{B2x} and then solve for v_{A2x}:

$$m_B(v_{A1x} + v_{A2x}) = m_A(v_{A1x} - v_{A2x})$$

$$v_{A2x} = \frac{m_A - m_B}{m_A + m_B} v_{A1x} \tag{8.24}$$

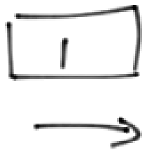
Finally, we substitute this result back into Eq. (8.23) to obtain

$$v_{B2x} = \frac{2m_A}{m_A + m_B} v_{A1x} \tag{8.25}$$

Okay, don't take notes on this example

8.46 •• A 0.150-kg glider is moving to the right on a frictionless, horizontal air track with a speed of 0.80 m/s. It has a head-on collision with a 0.300-kg glider that is moving to the left with a speed of 2.20 m/s. Find the final velocity (magnitude and direction) of each glider if the collision is elastic. Kinetic energy conserves

$m_1 = 0.15\ \text{kg}$



$\vec{v}_1 = 0.8\ \text{m/s}\ \hat{i}$

$m_2 = 0.3\ \text{kg}$

$\vec{v}_2 = 2.2\ \text{m/s} \cdot (-\hat{i})$

Momentum conservation:

$$\vec{P}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1{}' + m_2\vec{v}_2{}' \quad (1)$$

$\vec{v}_1{}'$ $\vec{v}_2{}'$

$$K_0 = \tfrac{1}{2}m_1 v_1^2 + \tfrac{1}{2}m_2 v_2^2 = \tfrac{1}{2}m_1 v_1'^2 + \tfrac{1}{2}m_2 v_2'^2 \quad (2)$$

Two equations. only two unknowns v_1' v_2'

from ① $v_1' = \frac{P_0}{m_1} - \frac{m_2}{m_1} v_2' = \frac{1}{m_1}(P_0 - m_2 v_2')$

Plug in ② $K_0 = \frac{1}{2} m_1 \cdot \frac{1}{m_1^2}(P_0^2 - 2P_0 m_2 v_2' + m_2^2 v_2'^2) + \frac{1}{2} m_2 v_2'^2$

$$K_0 = \frac{P_0^2}{2m_1} - \frac{m_2}{m_1} P_0 v_2' + \frac{m_2^2}{2m_1} v_2'^2 + \frac{1}{2} m_2 v_2'^2$$

$$K_0 = \frac{P_0^2}{2m_1} - \frac{m_2}{m_1} P_0 v_2' + \frac{m_2^2}{2m_1} v_2'^2 + \frac{1}{2} m_2 v_2'^2$$

$2m_1$

$$2m_1 K_0 = P_0^2 - 2m_2 P_0 v_2' + (m_2^2 + m_1 m_2) v_2'^2$$

Rearrange into $\underline{a}x^2 + \underline{b}x + \underline{c} = 0$ $\quad x \equiv v_2'$

$$\underbrace{(m_2^2+m_1m_2)}_{a}\, v_2'^2 - \underbrace{2m_2}_{b} p_0 v_2' + \underbrace{p_0^2 - 2m_1K_0}_{c} = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow$$

$$v_2' = \frac{\cancel{2}m_2p_0 \pm \sqrt{\cancel{4}m_2^2p_0^2 - \cancel{4}(p_0^2-2m_1K_0)(m_2^2+m_1m_2)}}{\cancel{2}(m_2^2+m_1m_2)}$$

This part $= \cancel{m_2^2p_0^2} - (\cancel{m_2^2p_0^2} - 2m_1m_2^2K_0 + m_1m_2p_0^2 - 2m_1^2m_2K_0)$

$= 2m_1m_2^2K_0 - m_1m_2p_0^2 + 2m_1^2m_2K_0$

$= m_1m_2\left[2(m_1+m_2)K_0 - p_0^2\right]$ $\quad K_0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$p_0 = m_1v_1 + m_2v_2$

$= \cancel{m_1^2v_1^2} + m_1m_2v_1^2 + m_1m_2v_2^2 + \cancel{m_2^2v_2^2} - (\cancel{m_1^2v_1^2} + 2m_1m_2v_1v_2 + \cancel{m_2^2v_2^2})$

$= m_1m_2(v_1-v_2)^2$

Bravo!

$$v_2' = \frac{\cancel{m_2}(m_1v_1+m_2v_2) \pm m_1\cancel{m_2}(v_1-v_2)}{m_2^{\cancel{2}}+m_1\cancel{m_2}}$$

$$= \frac{(m_1 \pm m_1)v_1 + (m_2 \mp m_1)v_2}{m_1+m_2}$$

When "$\pm$" takes "$+$", "$\mp$" takes "$-$"

$$v_2' = \frac{2m_1v_1+(m_2-m_1)v_2}{m_1+m_2}$$

But $m_1v_1'+m_2v_2' = m_1v_1+m_2v_2$

$$mv_1' = m_1v_1+m_2v_2 - \frac{2m_1m_2v_1+(m_2^2-m_1m_2)v_2}{m_1+m_2}$$

$$= \frac{m_1^2v_1+m_1m_2(v_1+v_2)+\cancel{m_2^2v_2}-2m_1m_2v_1-\cancel{m_2^2v_2}+m_1m_2v_2}{m_1+m_2}$$

But $m_1v_1' + m_2v_2' = m_1v_1 + m_2v_2$

$$m v_1' = m_1v_1 + m_2v_2 - \frac{2m_1m_2v_1 + (m_2^2 - m_1m_2)v_2}{m_1+m_2}$$

$$= \frac{m_1^2v_1 + m_1m_2(v_1+v_2) + \cancel{m_2^2v_2} - 2m_1m_2v_1 - \cancel{m_2^2v_2} + m_1m_2v_2}{m_1+m_2}$$

$$= \frac{m_1^2v_1 - m_1m_2v_1 + 2m_1m_2v_2}{m_1+m_2}$$ divide by m_1

$$v_1' = \frac{m_1v_1 - m_2v_1 + 2m_2v_2}{m_1+m_2} = \boxed{\frac{2m_2v_2 + (m_1-m_2)v_1}{m_1+m_2}}$$

$$v_2' = \frac{2m_1v_1 + (m_2-m_1)v_2}{m_1+m_2}$$

What is $v_1' - v_2'$ here? $-(v_1 - v_2)$

Elastic Collisions

Let's look at an elastic collision between two bodies *A* and *B*. We start with a one-dimensional collision, **in *x* only**. The key is the ***energy conservation***:

$$\boxed{\frac{1}{2}m_A v_{A1x}^{\,2} + \frac{1}{2}m_B v_{B1x}^{\,2} = \frac{1}{2}m_A v_{A2x}^{\,2} + \frac{1}{2}m_B v_{B2x}^{\,2}}$$

In addition to: $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$

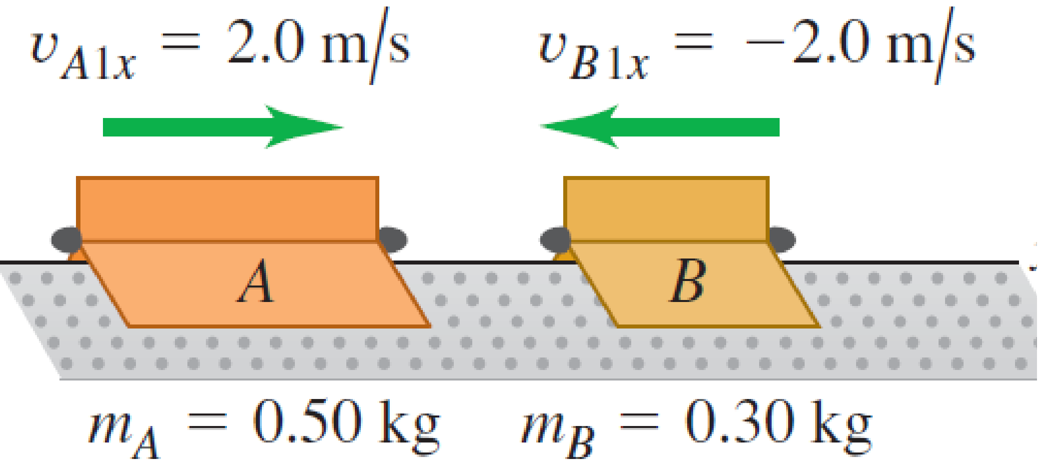
Conclusion first: in an **elastic collision** of two bodies, the relative velocities before and after the collision have the **same magnitude** but **opposite sign**.

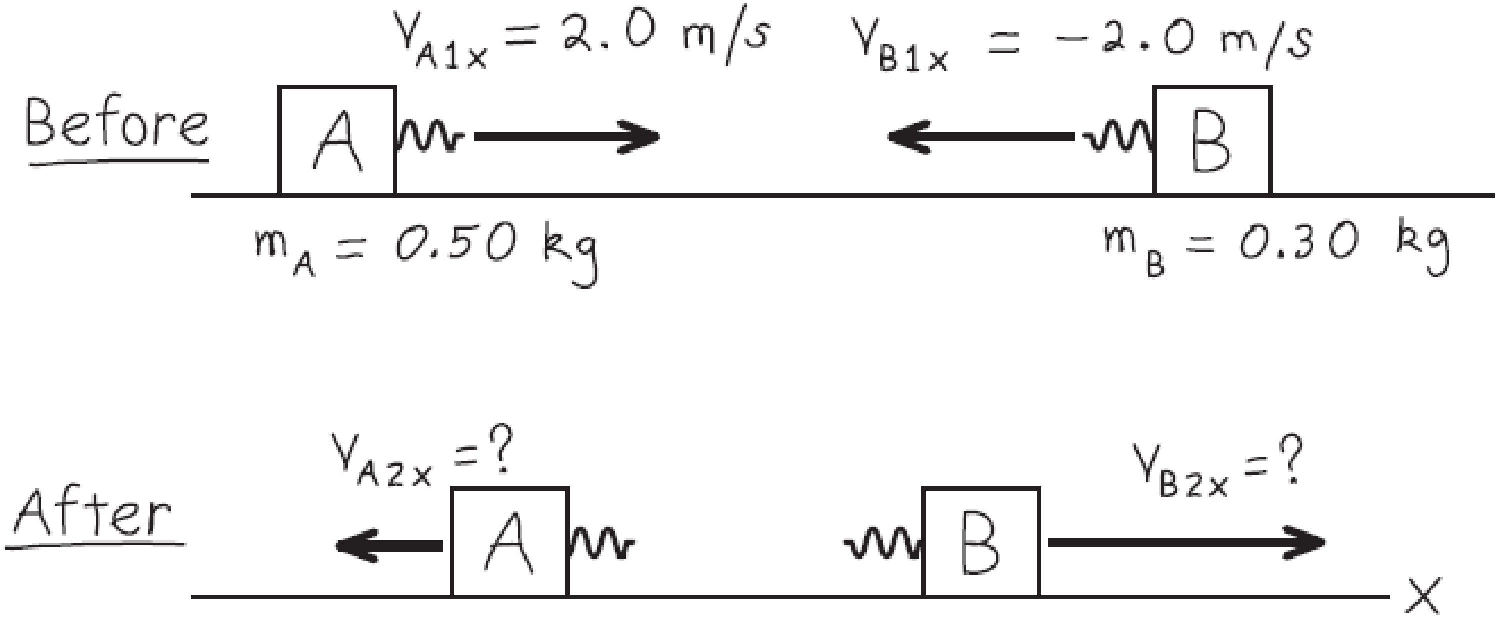
$$\boxed{v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})}$$

Use instead of energy conservation

Example 8.10: Elastic straight-line collision

We repeat the air-track collision of Example 8.5 (Section 8.2), but now we add ideal spring bumpers to the gliders so that the collision is elastic. What are the final velocities of the gliders?





Example 8.10: Elastic straight-line collision

We repeat the air-track collision of Example 8.5 (Section 8.2), but now we add ideal spring bumpers to the gliders so that the collision is elastic. What are the final velocities of the gliders?

From conservation of momentum,

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

Plug in numbers:

$$(0.50\ \text{kg})(2.0\ \text{m/s}) + (0.30\ \text{kg})(-2.0\ \text{m/s})$$
$$= (0.50\ \text{kg})v_{A2x} + (0.30\ \text{kg})v_{B2x}$$

Simplify to:

$$0.50 v_{A2x} + 0.30 v_{B2x} = 0.40\ \text{m/s}$$

Example 8.10: Elastic straight-line collision

We repeat the air-track collision of Example 8.5 (Section 8.2), but now we add ideal spring bumpers to the gliders so that the collision is elastic. What are the final velocities of the gliders?

$$0.50 v_{A2x} + 0.30 v_{B2x} = 0.40 \text{ m/s}$$

$$\begin{aligned} v_{B2x} - v_{A2x} &= -(v_{B1x} - v_{A1x}) \\ &= -(-2.0 \text{ m/s} - 2.0 \text{ m/s}) = 4.0 \text{ m/s} \end{aligned}$$

Solving these equations simultaneously, we find

$$v_{A2x} = -1.0 \text{ m/s} \qquad v_{B2x} = 3.0 \text{ m/s}$$

Example 8.10: Elastic straight-line collision

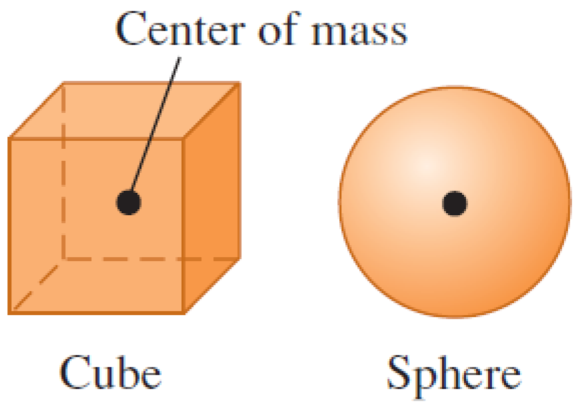
The total kinetic energy before the collision (which we calculated in Example 8.7) is 1.6 J. The total kinetic energy after the collision is

$$\tfrac{1}{2}(0.50\ \text{kg})(-1.0\ \text{m/s})^2 + \tfrac{1}{2}(0.30\ \text{kg})(3.0\ \text{m/s})^2 = 1.6\ \text{J}$$

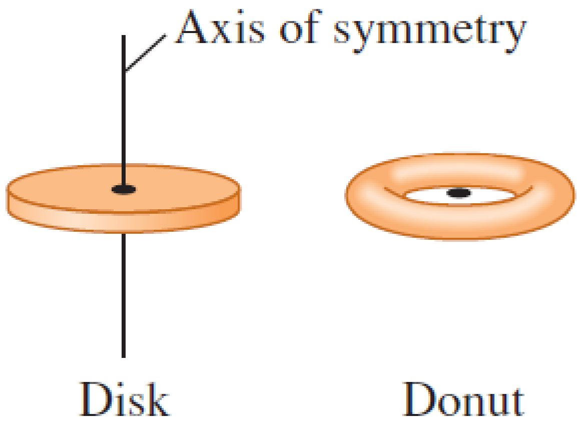
Center of Mass

We can restate the principle of conservation of momentum in a useful way by using the concept of **center of mass**

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$



$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

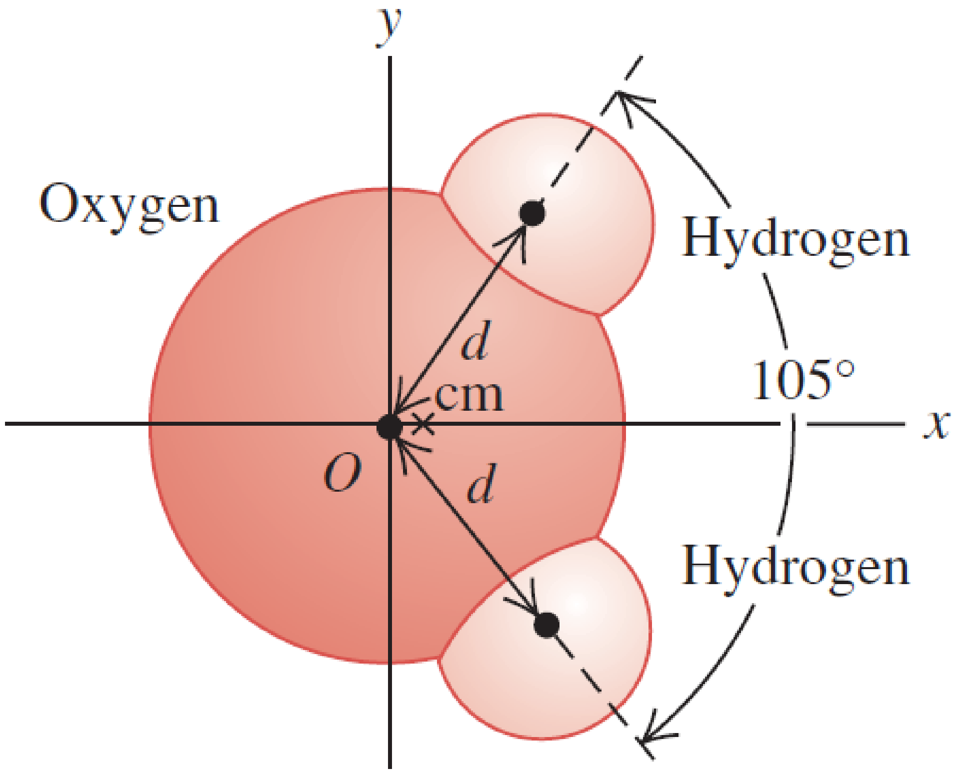


Example 8.13 Center of mass of a water molecule

Figure 8.27 shows a simple model of a water molecule. The oxygen-hydrogen separation is $d = 9.57 \times 10^{-11}$ m. Each hydrogen atom has mass 1.0 u, and the oxygen atom has mass 16.0 u. Find the position of the center of mass.

EXECUTE: The oxygen atom is at $x = 0$, $y = 0$.

- The x-coordinate of hydrogen atom is $d \cos (105°/2)$;
- the y-coordinates are $d \sin(105°/2)$



Example 8.13 Center of mass of a water molecule

$$x_{cm} = \frac{\begin{bmatrix} (1.0\text{ u})(d\cos 52.5°) + (1.0\text{ u}) \\ \times (d\cos 52.5°) + (16.0\text{ u})(0) \end{bmatrix}}{1.0\text{ u} + 1.0\text{ u} + 16.0\text{ u}} = 0.068d$$

$$y_{cm} = \frac{\begin{bmatrix} (1.0\text{ u})(d\sin 52.5°) + (1.0\text{ u}) \\ \times (-d\sin 52.5°) + (16.0\text{ u})(0) \end{bmatrix}}{1.0\text{ u} + 1.0\text{ u} + 16.0\text{ u}} = 0$$

Substituting $d = 9.57 \times 10^{-11}$ m, we find

$$x_{cm} = (0.068)(9.57 \times 10^{-11}\text{ m}) = 6.5 \times 10^{-12}\text{ m}$$

Motion of the Center of Mass

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots = \vec{P}$$

Total momentum is equal to the total mass times the velocity of the center of mass

External Forces & Center-of-Mass Motion

Accelerations are related in the same way

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots$$

$$\Sigma\vec{F} = \Sigma\vec{F}_{ext} + \Sigma\vec{F}_{int} = M\vec{a}_{cm}$$

Because of Newton's third law, the internal forces all cancel in pairs,

$$\Sigma\vec{F}_{int} = 0.$$

We get:

$$\Sigma\vec{F}_{ext} = M\vec{a}_{cm} \qquad \text{(body or collection of particles)}$$

External Forces & Center-of-Mass Motion

When a body or a collection of particles is acted on by external forces, the center of mass moves just as though all the mass were concentrated at that point and it were acted on by a net force equal to the sum of the external forces on the system.

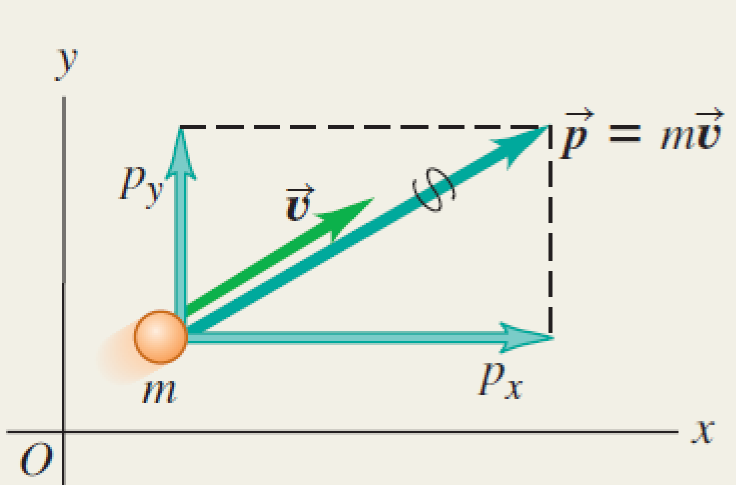
We get:

$$\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{cm}} \qquad \text{(body or collection of particles)}$$

Summary

$$\vec{p} = m\vec{v}$$

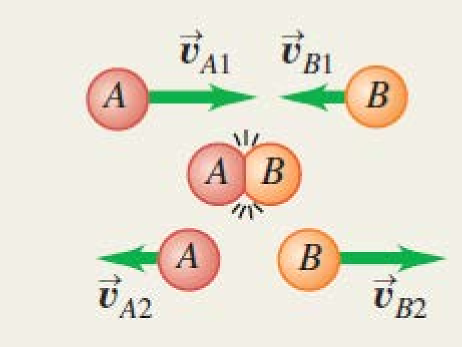
$$\sum\vec{F} = \frac{d\vec{p}}{dt}$$



$$\vec{J} = \sum\vec{F}(t_2 - t_1) = \sum\vec{F}\,\Delta t$$

$$\vec{J} = \int_{t_1}^{t_2} \sum\vec{F}\,dt$$

$$\vec{J} = \vec{p}_2 - \vec{p}_1$$



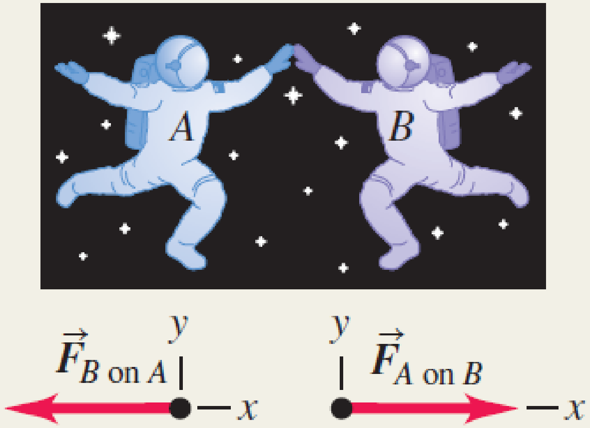
$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i} \quad (8.29)$$

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots = M\vec{v}_{cm} \quad (8.32)$$

$$\sum\vec{F}_{ext} = M\vec{a}_{cm} \quad (8.34)$$

$$\vec{P} = \vec{p}_A + \vec{p}_B + \cdots = m_A\vec{v}_A + m_B\vec{v}_B + \cdots \quad (8.14)$$

If $\sum\vec{F} = \mathbf{0}$, then $\vec{P}$ = constant.



$\vec{P} = \vec{p}_A + \vec{p}_B$ = constant