College Algebra and Trigonometry

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Ch 2 Functions and Relations

① Plot Points on a Rectangular Coordinate System

Cartesian coordinate system: named after French Mathematician Rene Descartes (pronounced "day cart") (1597-1650).

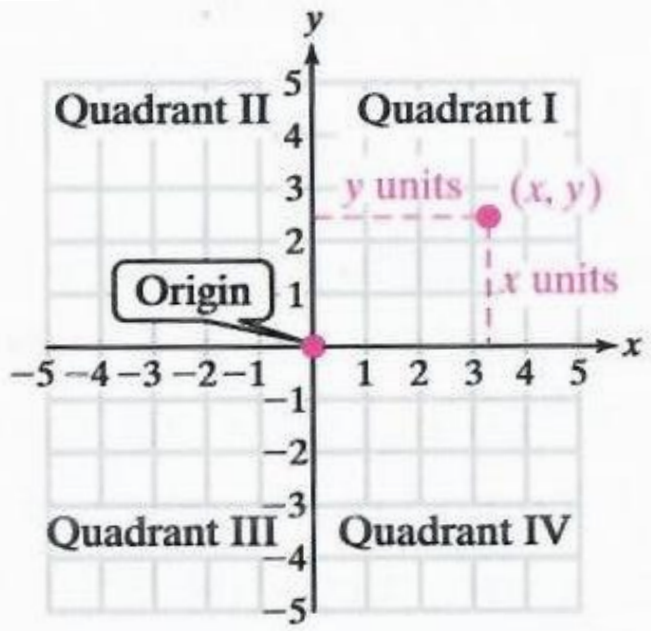


Figure 2-1

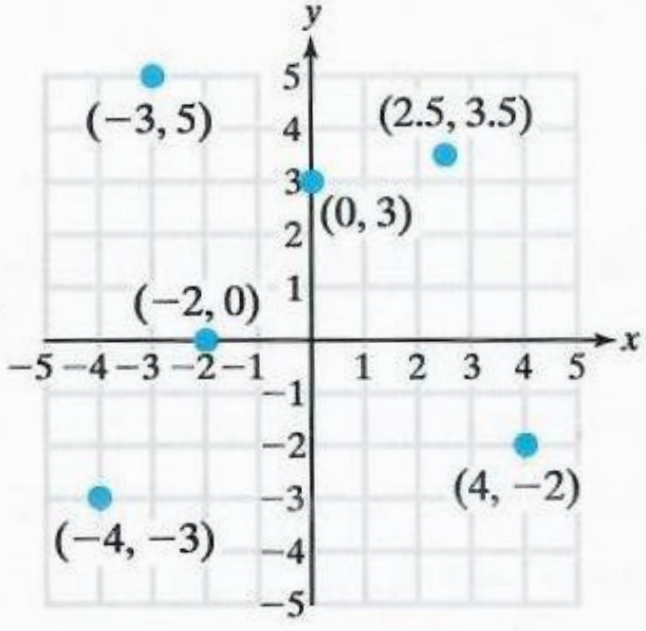


Figure 2-2

② Use the Distance and Midpoint Formulas

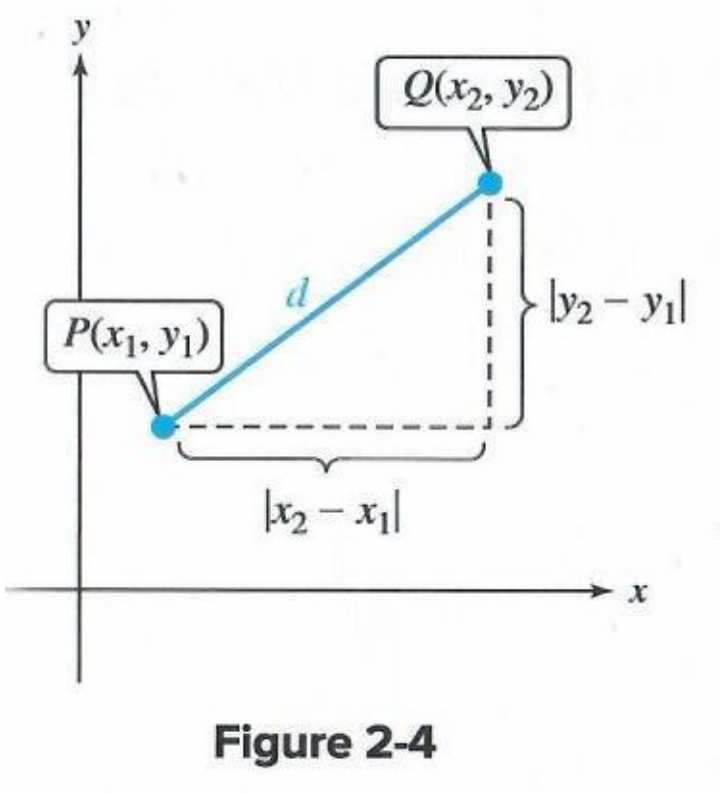


Figure 2-4

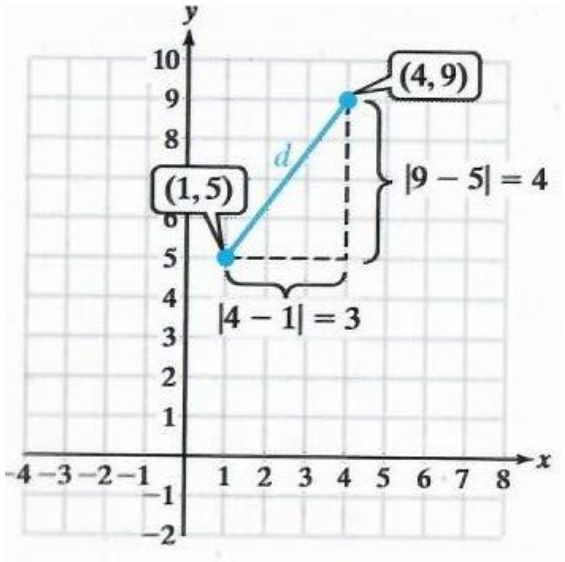
Distance Formula

The distance between points (x_1, y_1) and (x_2, y_2) is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

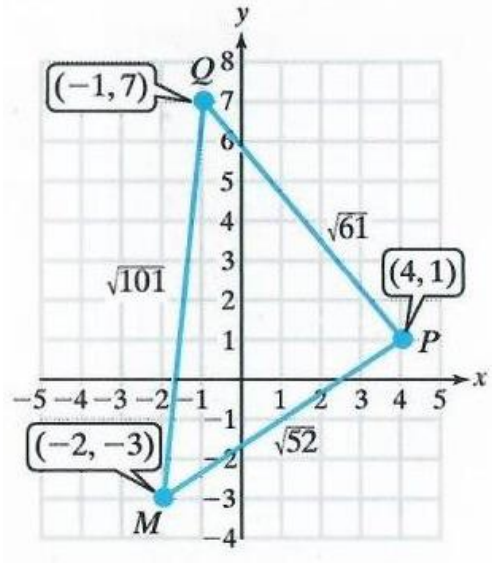
Example 1:

Find the Distance between Two Points (1, 5) and (4, 9).



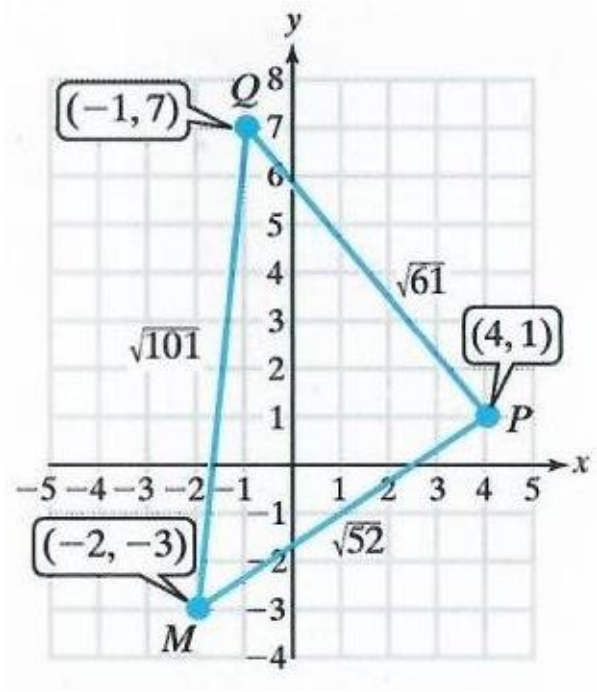
Example 2:

Determine if three points *M*(-2, -3), *P*(4, 1), and *Q*(-1, 7) form the vertices of a right triangle.



Skill Practice:

If three points $M(-2, -3)$, $P(4, 1)$, and $Q'(x, 7)$ form the vertices of a right triangle. Determine the value of x.



Midpoint Formula

The midpoint of the line segment with endpoints (x_1, y_1) and (x_2, y_2) is:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

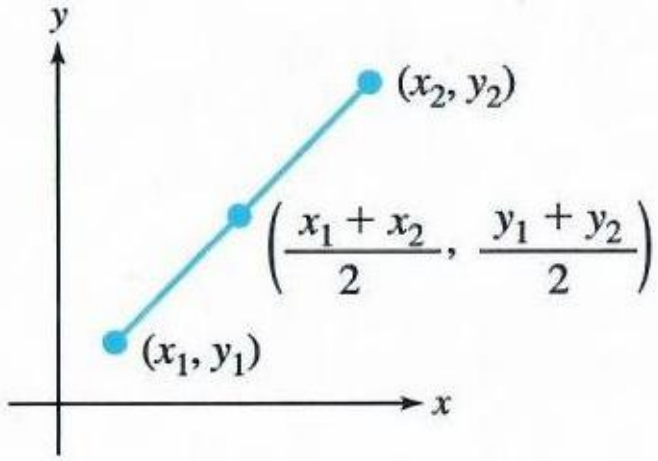


Figure 2-5

Example 3:

Find the midpoint of the line segment with endpoints (2, 6) and (8, 12).

③ Graph Equations by Plotting Points

Example 4:

Graph the equation by plotting points:

$$y - |x| = -1$$

Skill practice:

Graph the equation by plotting points:

$$y - x^2 = 1$$

④ Identify *x*- and *y*-Intercepts

Determine *x*- and *y*-Intercepts from an Equation.

Given an equation in *x* and *y*,

- **Find the *x*-intercept(s) by substituting 0 for *y* in the equation and solving for *x*.**
- **Find the *y*-intercept(s) by substituting 0 for *x* in the equation and solving for *y*.**

Example 6:

Given the equation $y = |x| - 1$

a) Find the *x*-intercept(s).

a) Find the *y*-intercept(s).

Skill Practice:

Given the equation $y = x^2 - 2x - 3$

a) Find the *x*-intercept(s).

a) Find the *y*-intercept(s).

Definition of a Circle

A circle is the set of all points in a plate that are equidistant from a fixed point called the center. The fixed distance from any point on the circle to the center is called the radius.

Standard Form of an equation of a Circle

Given a circle centered at (h, k) with radius r, the standard form of an equation of the circle (also called the center-radius form) is given by:

$$(x-h)^2+(y-k)^2=r^2 \quad \text{where} \quad r>0$$

① Write an Equation of a Circle in Standard Form

Example 1:

Write the standard form of an equation of a circle with center (3, 4) and radius 5.

Example 2:

Write the standard form of an equation of a circle with endpoints of a diameter (-1, 0) and (3, 4).

② Write the General Form of an Equation of a Circle

General Form of an equation of a Circle

An equation of a circle written in the form $x^2 + y^2 + Ax + By + C = 0$

is called the general form of an equation of a circle.

Example 3:

A circle is written in the standard form as:

$$x^2 + y^2 + 10x - 6y + 25 = 0$$

Identify the center and radius.

Example 4:

Determine if the following equation represents the graph of a circle:

$$x^2 + y^2 - 14y + 49 = 0$$

Then identify the solution set.

Example 5:

Find the shortest distance from the origin to a point on the circle:

$$x^2 + y^2 + 4x - 12y + 31 = 0$$

Question: the longest distance from the origin to a point on a circle?

Example 6:

1) Determine if the following equation represents the graph of a circle:

$$(x+1)(x-7)+(y+2)(y-4)=0$$

2) Identify the center and radius.

3) Find the shortest distance from the origin to a point on a circle?