Calculus Model Question for Final Exam

Follow all directions and show your work throughout this assessment. Clearly identify all answers.

1. Find the area of the region bounded by the $y = -x^2 + 3x$ and the curve $y = 2x^3 - x^2 - 5x$.

2. Given $f(x) = 6x - 1$, find the inverse f^{-1} of $f(x)$.

3. Let $f(x) = x^3 + x^2 + 6x + 1, x \geq 0$. Find the value of $\dfrac{df^{-1}}{dx}$ at the point $a = 1 = f(0)$.

4. Solve the following equations for x.
   (a) $e^{3x-6} = 9$.

   (b) $\ln(x - 8) = 5$.

5. show that $1 \leq \displaystyle\int_0^1 \sqrt{x^2 + 1}dx \leq \sqrt{2}$

6. Evaluate each integral in the following by using a substitution, completing the square, reducing an improper function, integration by parts or partial fractions.
   (a) $\displaystyle\int \frac{4xdx}{\sqrt{2x^2 + 1}}$

(b) $\int \frac{dx}{\sqrt{-x^2+4x-3}}$

(c) $\int x \sin x dx$

(d) $\int x^5 \ln x dx$

(e) $\int e^x \cos x dx$

(f) $\int x e^x dx$

(g) $\int \frac{2x-6}{x^2-6x+8} dx$

(h) $\int \frac{2x-7}{x^2-7x+12} dx$

(i) $\int \frac{x+1}{(x-1)(x-2)(x-3)} dx$

(j) $\int \frac{x^3+3x^2+2x-7}{x+2} dx$

(k) $\int_{-1}^{1} (x^{2024} \sin x + \frac{x^7}{1+x^2}) dx$

(l) $\int_{-1}^{1} (\sin x^9 + \cos 2x + 7x^7 \cos^8 x + x^{12}) dx$

(m)* $\displaystyle\int \frac{x^2+1}{x(x-1)^2}\ln x dx$

(n)* $\displaystyle\int \frac{xe^x}{(1+x)^2}dx$

(o)* $\displaystyle\int \frac{1+2x^2}{x^2(1+x^2)}dx$

7. Evaluate each trigonometric integral in the following.

(a) $\displaystyle\int \sin^5 x dx$

(b) $\displaystyle\int \sin^6 x\cos^3 x dx$

(c) $\displaystyle\int \cos^2 x\sin^2 x dx$

8. Solve the following separable differential equation.

$\displaystyle\frac{dy}{dx} = \sin x e^{-y}$

9. Find the derivative of y with respect to x.

(a) $\displaystyle y = \frac{x^5}{4}\ln x - \frac{x^4}{4}$

(b) $\displaystyle y = \ln\frac{1}{x^2\sqrt{x+1}}$

(c) $y = \ln\left(\dfrac{(x^2+1)^6}{\sqrt{1-x}}\right)$

(d) $y = (9x^2 - 6x + 2)e^{3x}$

(e) $y = \ln(3x^3 e^{x^2})$

(f) $y = \ln(x^3 + 1)e^x$

(g) $y = \arctan(\sin x)$

(h) $y = \dfrac{(x^4+3x)^4\sqrt[5]{x+3}}{e^{x^3+3x}\sin x}$

10. Use L'Hôpital's Rule to evaluate the following limits.

(a) $\lim\limits_{x\to 1} \dfrac{x^3-1}{4x^3-x-3}$

(b) $\lim\limits_{t\to 0} \dfrac{\sin t^2}{t^2}$

(c) $\lim\limits_{x\to 1} \dfrac{x-1}{\ln x - \sin \pi x + x - 1}$

(d) $\lim\limits_{x\to 0} \dfrac{\tan^2 x(1-\cos x)}{x^4}$

11. Evaluate the following integral by eliminating square root.

(a) $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{1+\cos 2x}\,dx$

(b) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{1-\cos 2x}$

12. Find the values.

(a) $\sin(\arccos(\frac{\sqrt{2}}{2}))$

(b) $\tan(\arcsin(-\frac{1}{2})$