

Calculus Model Question for Mid-term Exam

Follow all directions and show your work for full credits. Clearly identify all answers.

1. Find the following limits.

(a) $\lim_{x \rightarrow 5} \frac{x + 5}{x^2 - 20}$

(b) $\lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{x - 2}$

(c) $\lim_{x \rightarrow 4} \frac{x - 4}{\sqrt{x} - 2}$

(d) $\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 12} - 4}{x - 2}$

(e) $\lim_{x \rightarrow \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$

2. Given function

$$f(x) = \begin{cases} 1 - x^2, & x \neq 1; \\ 2, & x = 1. \end{cases}$$

(a) find $\lim_{x \rightarrow 1^+} f(x)$ and $\lim_{x \rightarrow 1^-} f(x)$.

(b) does $\lim_{x \rightarrow 1} f(x)$ exist? If so, what is it? If not, why not?

3. Find a in a way that extends

$$f(x) = \begin{cases} x^2 - 2x + 3, & x < 3; \\ 2ax, & x \geq 3. \end{cases}$$

to be continuous at $x = 3$.

4. Using the definition to find the derivative of $y = -2x^2$ at $x = 1$.

5. Find the horizontal and vertical asymptotes for the function

$$y = \frac{(x+3)(x+1)}{(x+2)(x-1)}.$$

6. Find the slope of the function $f(x) = x^2 + 1$ at $(1, 2)$ and then find an equation for the tangent line and the normal line to $f(x)$ at $(1, 2)$.

7. Find the first derivative.

(a) $y = (x^2 + 1)(x + 1 + \frac{1}{x})$

(b) $y = \frac{x+5}{3x+2}$

(c) $y = \frac{(x^2+x)(x^2-x+1)}{x^4}$

(d) $y = (\sin x + \cos x)(x^2 + 2x)$

(e) $y = x^2 \sin x + 2x \cos x - 2 \sin x$

8. Find the first derivative for the following functions.

(a) $y = \left(\frac{x^2}{2} + x - \frac{1}{x}\right)^4$

(b) $y = \sin^4 x$

(c) $y = x^2 \sin^4 x + x \cos^{-2} x$

9. Use implicit differentiation to find $\frac{dy}{dx}$ for the following equations.

(a) $x^2y + xy^2 = 8$

(b) $2xy + y^2 + x^2 = x + y$

(c) $y \sin\left(\frac{1}{x}\right) = 1 - xy$

(d) $y^3 = \frac{x-1}{x+1}$

(e) $x^2(x+y)^2 = x^2 + y^2$

10. Find the absolute maximum and minimum values of

$$f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + 1$$

on $[0, 3]$.

11. Find the critical points of

$$f(x) = x^3 - \frac{3}{2}x^2 - 6x + 3.$$

Identify the open intervals on which $f(x)$ is increasing and decreasing. Find the function's local and absolute extreme values.

12. For the function of

$$f(x) = -x^3 + 6x^2 - 9x + 3,$$

- (a) Find the critical points of $f(x)$, if any, and identify the intervals on which $f(x)$ is increasing and decreasing.
 - (b) Find the points of inflection, if any occur, and identify the intervals on which $f(x)$ is concave up and concave down.
 - (c) Identify any asymptotes that may exist.
 - (d) Sketch the graph.
 - (e) Find any local and absolute values of $f(x)$.
13. A rectangle plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 400 m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions?
14. Use Newton's method to find the positive root of the equation

$$x^4 - 2 = 0.$$

Start $x_0 = 1$ and find x_1 .