5. Applying Newton’s Laws

Potential Exam Questions/Scenarios

5.2 Dynamics of Particles

- Apply Newton’s Second Law to bodies on which the net force is *not* zero. These bodies are *not* in equilibrium and hence are accelerating.

$$\sum \vec{F} = m\vec{a}$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y$$

5.3 Friction Forces

$$f_k = \mu_k n$$

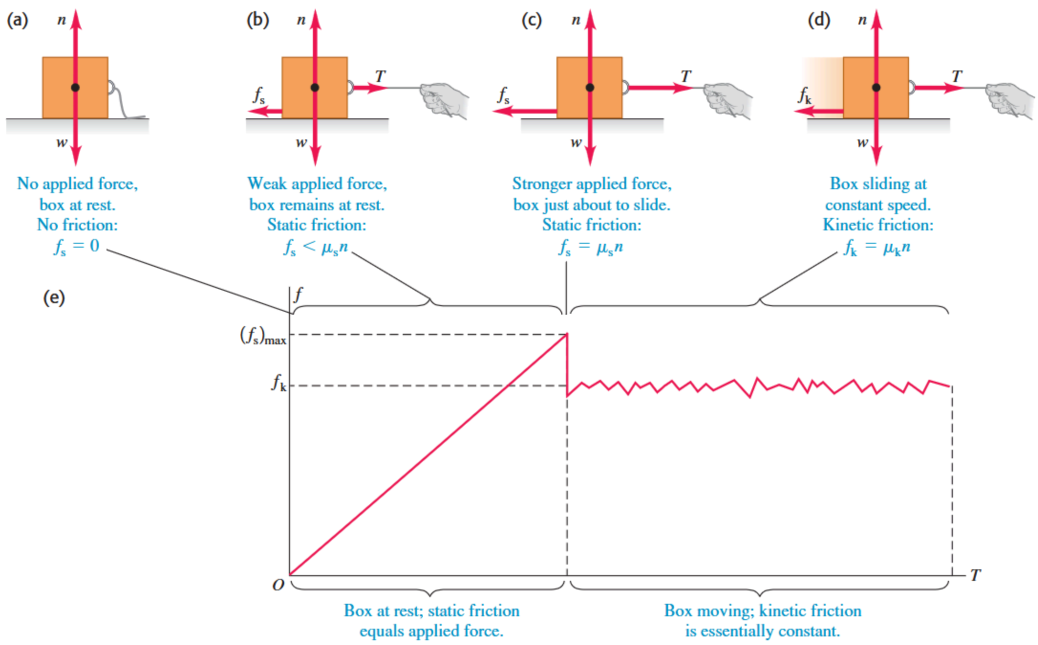
$$f_s \leq \mu_s n$$

- As the pulling angle changes, the normal force decreases, which in turn decreases the friction force ($f_s = \mu_k n$). Reduced normal force makes the dragging easier.

The **kinetic friction does not change** regardless of the force being applied.

The
 static force is exactly equivalent to the force applied, until it reaches its maximum value.

- The static and the kinetic frictions could have the same magnitude, but the distinction lies in whether the object is moving or not.



- Friction force $\vec{f}$: a force exerted by the surface (parallel with the surface) that is always opposite the direction of motion.

- Set the x and y directions **perpendicular (/parallel)** to the slope for easier calculations.

Dynamics of Circular motion

$$a = \frac{v^2}{R} \qquad v = \frac{2\pi R}{T}$$

then

$$\vec{F} = m\vec{a} = m\frac{v^2}{R}$$

Converting Revolutions to T

The time period T is equal to the time it takes for an particle to make one complete revolution.

$$T = \frac{1}{f}$$

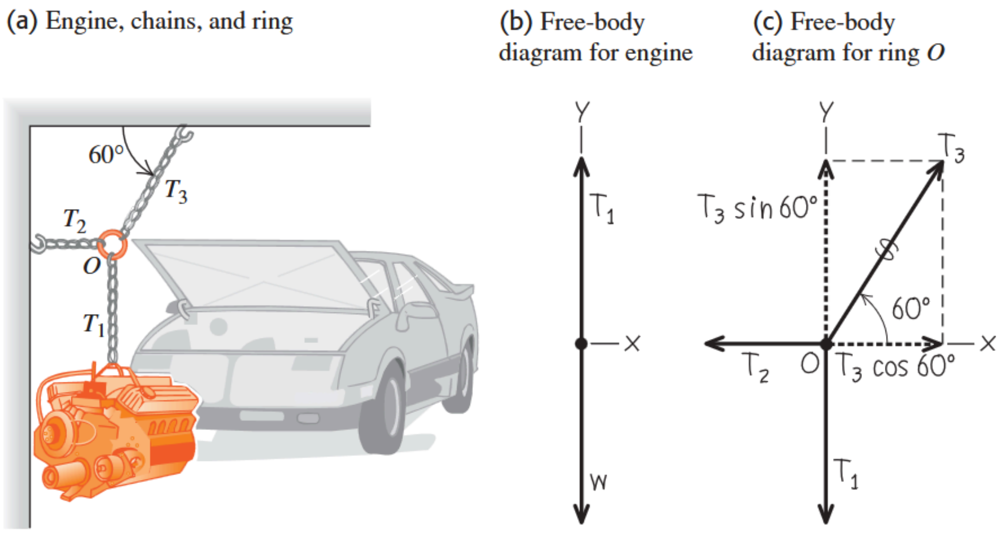
where $f = \text{rev s}^{-1}$

e.g.

$$5\ \text{rev min}^{-1} = \frac{1}{5} \cdot (60) = 12\ \text{s}$$

Some Scenarios

Tension(s) on a rope



One of the simplest systems, draw the free-body diagram for the ring (or just the point where the different ropes are tied together) and the object, isolate the $x - y$ components. Given that the systems are in equilibrium (question asks for tensions in terms of w):

System 1:

$$\sum F_x = 0$$

$$\sum F_y \quad : \quad T_1 = w$$

System 2:

$$\sum F_x \quad : \quad T_2 = T_3 \cos 60^\circ$$

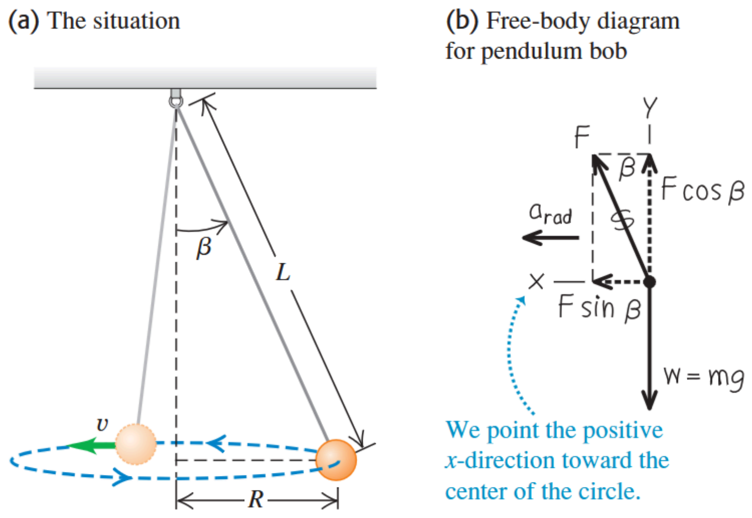
$$\sum F_y \quad : \quad T_1 = T_3 \sin 60^\circ$$

Rearranging the equations:

$$T_1 = w \ \Rightarrow \ w = T_3 \sin 60^\circ \ \Rightarrow T_3 = \frac{w}{\sin 60^\circ} = 1.2w$$

$$T_2 = \left(\frac{w}{\sin 60^\circ}\right) \cos 60^\circ = w \cot 60^\circ = 0.58w$$

Conical Pendulum



Once you figure out the free diagram, the only thing you need to be concerned is the $x - y$ components of the system. Here:

$$\sum F_x \quad : \quad F \sin \beta = m a_{\text{rad}}$$

$$\sum F_y \quad : \quad F \cos \beta = mg$$

We are asked to find the tension force F and the time period T when the mass is moving with a constant speed v. Just rearranging the equations gives:

$$F \cos \beta = mg \Rightarrow F = \frac{mg}{\cos \beta}$$

$$F \sin \beta = m a_{\text{rad}} \Rightarrow \left(\frac{mg}{\cos \beta}\right) \sin \beta = m a_{\text{rad}}$$

$$\Rightarrow mg \tan \beta = m \frac{4\pi^2 R}{T^2} \Rightarrow T^2 = \frac{4\pi^2 R}{g \tan \beta}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{R}{g \tan \beta}}$$

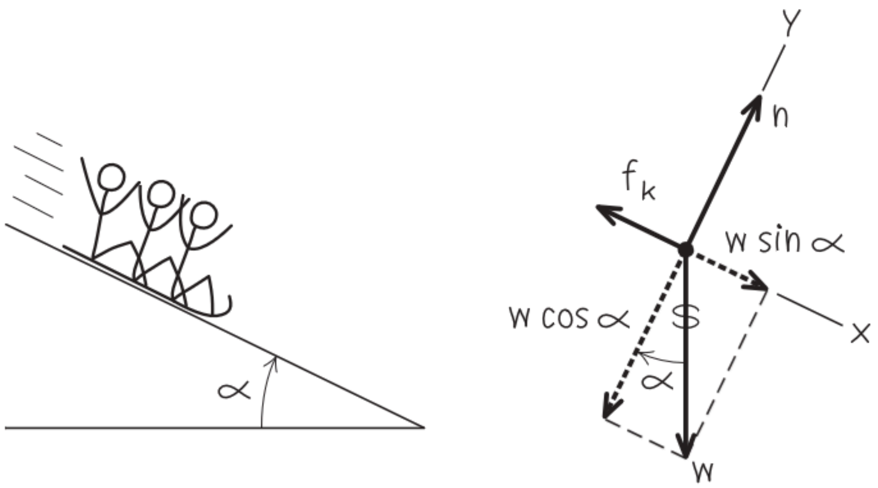
Recall that $a_{\text{rad}} = \frac{4\pi^2 R}{T^2}$.

Mass on a slope (inclination)

Depending on where the motion (mainly acceleration) is headed, the coordinates of the free-body diagram could be drawn differently.

Here in these two cases, the motion of the body occurs along the slope, so defining the $x - y$ coordinates parallel with the slope is preferred:

Case 1: with **constant velocity** and **kinetic friction**



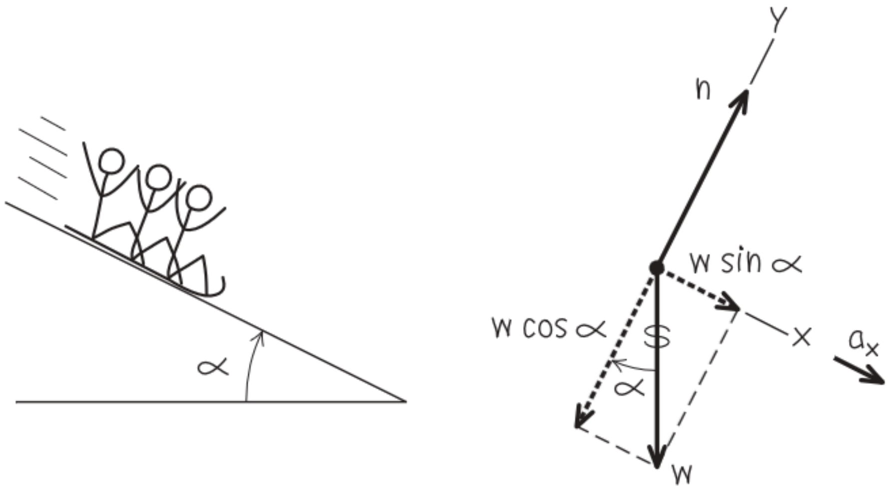
$$\sum F_x \quad : \quad f_k = w \sin \alpha$$

$$\sum F_y \quad : \quad n = w \cos \alpha$$

in the x component, the kinetic friction is equal to the horizontal component of the weight.

in the y component, the normal force and the vertical vector of the weight cancel each other out, hence the system is in equilibrium and the velocity is proven to be **constant (no acceleration)**.

Case 2: speeding up with no friction



$$\sum F_x \quad : \quad w \sin \alpha = ma_x$$

$$\sum F_y \quad : \quad n = w \cos \alpha$$

according to the Newton's second law $(F = ma)$, in the x component, as there is no friction to cancel out the horizontal component of the weight, the **body is in acceleration along the slope**.

in the y component, the normal force and the vertical vector of the weight still cancel each other.

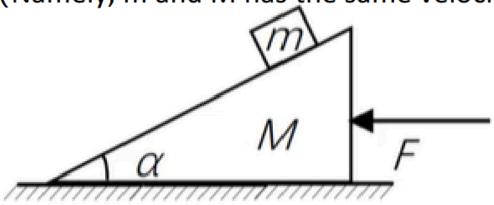
However, a different case is when the motion occurs in align with the floor (or the earth):

一木块放在光滑的斜面体上，木块质量为m，斜面体质量为M，斜面的倾角为α，如图所示，欲使木块相对斜面静止、所用水平推力应是：（地面阻力不计）

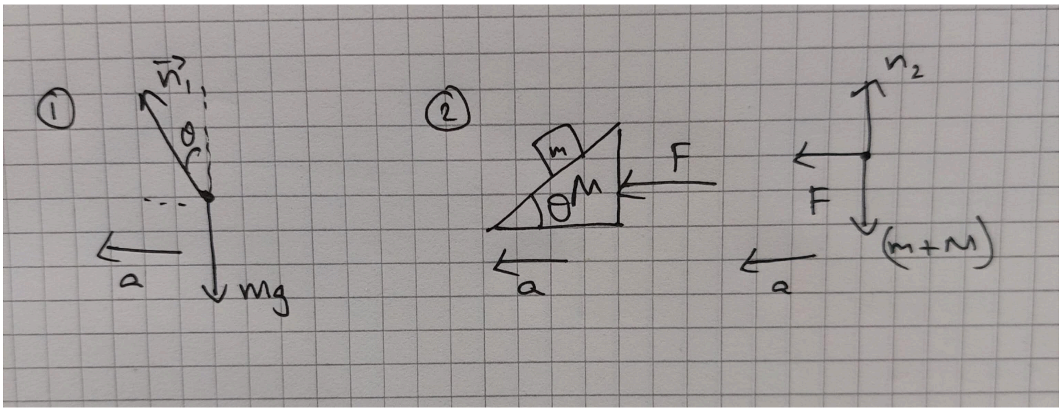
A．$M g \tan\alpha$　　B．$(M+m) g \tan\alpha$

C．$M g \sin\alpha$　　D．$(M+m) g \sin\alpha$

Translation: If all surfaces are **frictionless**, what is ***F*** so that the block *m* does NOT move relative to *M*? （Namely, m and M has the same velocity）



Since there is two objects, two systems should be drawn out:



For system 1:

$$\sum F_x \quad : \quad n_1 \sin\theta = ma$$

$$\sum F_y \quad : \quad n_1 \cos\theta = mg$$

For system 2:

$$\sum F_x \quad : \quad F = (M+m)a$$

$$\sum F_y \quad : \quad n_2 = (M+m)g$$

Rearranging the equations, since the acceleration a in the both systems are the same:

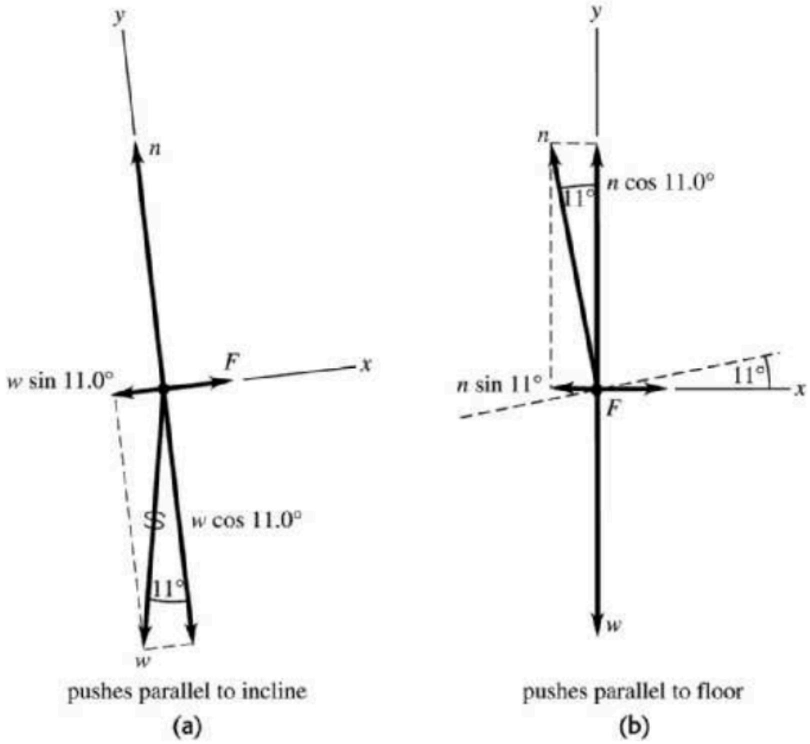
$$n_1 \cos\theta = mg \Rightarrow n_1 = \frac{mg}{\cos\theta} \Rightarrow \left(\frac{mg}{\cos\theta}\right)\sin\theta = ma$$

$$mg\tan\theta = ma \;\Rightarrow\; g\tan\theta = a$$

$$F = (M+m)a \Rightarrow a = \frac{F}{(M+m)}$$

$$g\tan\theta = \frac{F}{(M+m)} \Rightarrow F = (M+m)g\tan\theta$$

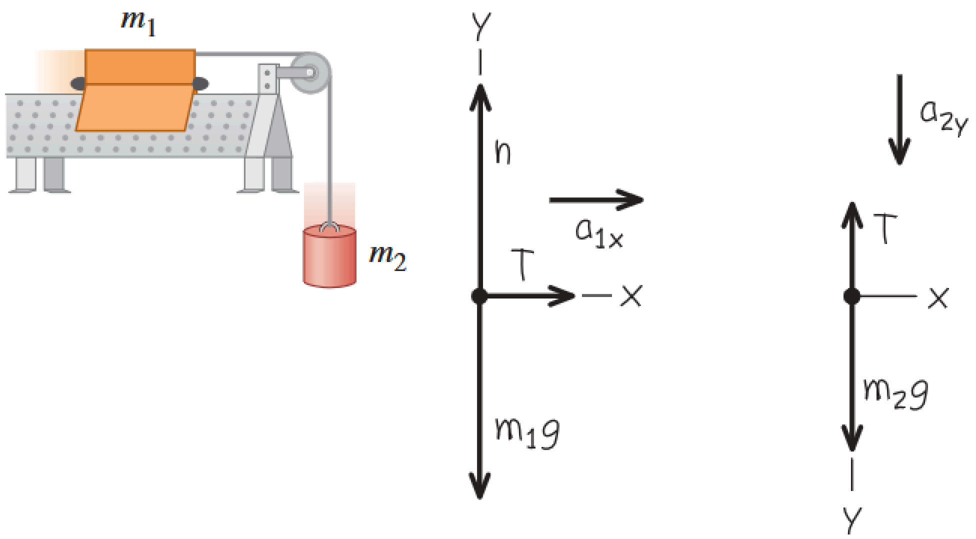
Free Body Diagram for Mass on Slope scenarios:



a. where the coordinate is drawn **relative to the inclination**
b. where the coordinate is drawn **relative to the floor (or earth**)

Pulley problems

Most pulley problems state that the rope is of negligent mass and of the same length throughout the motion. So you can assume that the tension in the systems (of the same rope) is the same:



Here, there are two objects so two free-body diagrams are drawn. As stated above, the tension in each system is equal to each other. So:

System 1:

$$\sum F_x \quad : \quad T = m_1 a_{1x}$$

$$\sum F_y \quad : \quad n = m_1 g$$

the vertical components cancel each other out, and there is acceleration (to the right) in the x component

System 2:

$$\sum F_x \quad : \text{no horizontal component}$$

$$\sum F_y \quad : \quad m_2 g - T = m_2 a_{2y}$$

as the acceleration is downwards (hence the motion is downwards), the tension T is negative and the weight $m_2 g$ is positive and their net difference is, according Newtons 2nd law, $m_2 a$

Since the **acceleration is the same** for both the system, rearranging the equations (find the tension T and the acceleration a) :

$$m_2 g - T = m_2 a \Rightarrow T = m_2 g - m_2 a \text{ and } T = m_1 a$$

$$\Rightarrow m_1 a = m_2 g - m_2 a \Rightarrow m_1 a + m_2 a = m_2 g \Rightarrow a(m_1 + m_2) = m_2 g$$

$$\Rightarrow a = \frac{m_2 g}{m_1 + m_2}$$

$$T = m_1 \left(\frac{m_2 g}{m_1 + m_2} \right) = \frac{m_1 m_2}{m_1 + m_2} g$$