College Algebra and Trigonometry

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① Solve Exponential Equations $a^x = b$

Equivalence Property of Exponential Expressions:

If b, x, and y are real numbers with $b > 0$ and $b \neq 1$, then:

$$b^x = b^y \quad \text{implies that} \quad x = y$$

Example 1:

Solve exponential equations using the equivalence property.

a) $3^{2x-4} = 81$

b) $25^{4-t} = \left(\frac{1}{5}\right)^{3t+1}$

Steps to Solve Exponential Equation by using Logarithms:

1. **Isolate the exponential expression on one side of the equation.**
2. **Take a logarithm of the same base on both sides of the equation.**
3. **Use the power property of logarithms to "bring down" the exponent.**
4. **Solve the resulting equation.**

Example 2:

Solve an exponential equation using logarithms.

$$3^x = 36$$

Note: any base can be used as long as it is positive and not equal to 1.

Example 3:

Solve exponential equations using logarithms.

a) $10^{5-2x} + 810 = 1810$

b) $12 = 36e^{-0.5x}$

Example 4:

Solve exponential equations.

a) $4^{2x-7} = 5^{3x+1}$

b) $3^{1+x} + 3^{2+x} = 108$

Example 5:

Solve exponential equations in quadratic forms.

a) $e^{2x} + 5e^{x} - 36 = 0$

b) $2^{1+x} + 2^{2+2x} = 20$

② Solve Logarithmic Equations $\log_a x = b$

Equivalence Property of Logarithmic Expressions:

If b, x, and y are real numbers with $b > 0$ and $b \neq 1$, then:

$$\log_b x = \log_b y \quad \textbf{implies that} \quad x = y$$

Example 6 and 7:

Solve exponential equations using the equivalence property.

6) $\log_2(3x - 4) = \log_2(x + 2)$

7) $\ln(x - 4) = \ln(x + 6) - \ln x$

Steps to Solve Logarithmic Equations by using Exponential Form:

1. **Isolate the logarithms on one side of the equation.**
2. **Use the properties of logarithms to write the equation in the form** $\log_b x = k$, **where** k **is a constant.**
3. **Write the equation in exponential form.**
4. **Solve the equation.**
5. **Check the potential solution(s) in the original equation.**

Example 8:

Solve the equations:

a) $4\log_3(2t-7)=8$

b) $\log m+\log(m+3)=1$

Example 9:

Solve logarithmic equations.

a) $\log_2 x=3-\log_2(x-2)$

b) $ln(x+1)+ln(x-1)=2$