Lecture 16

Direct-Current Circuit

Date:

Course Instructor:
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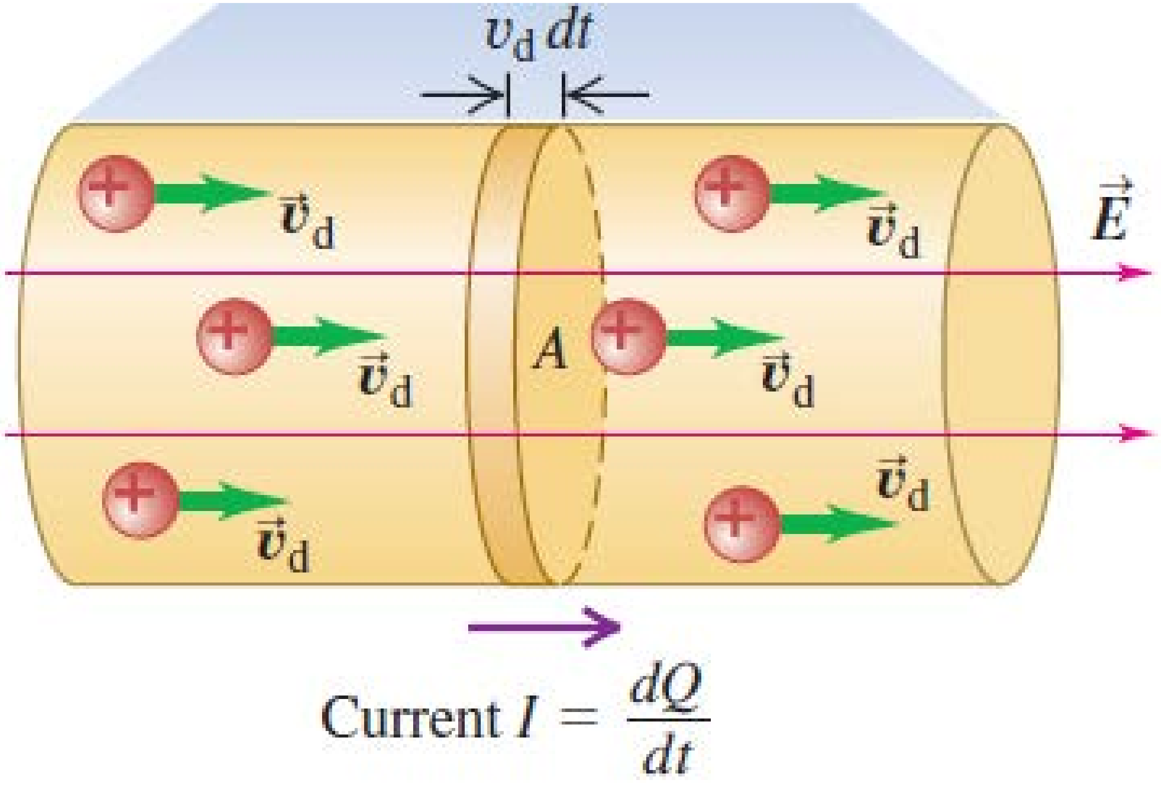
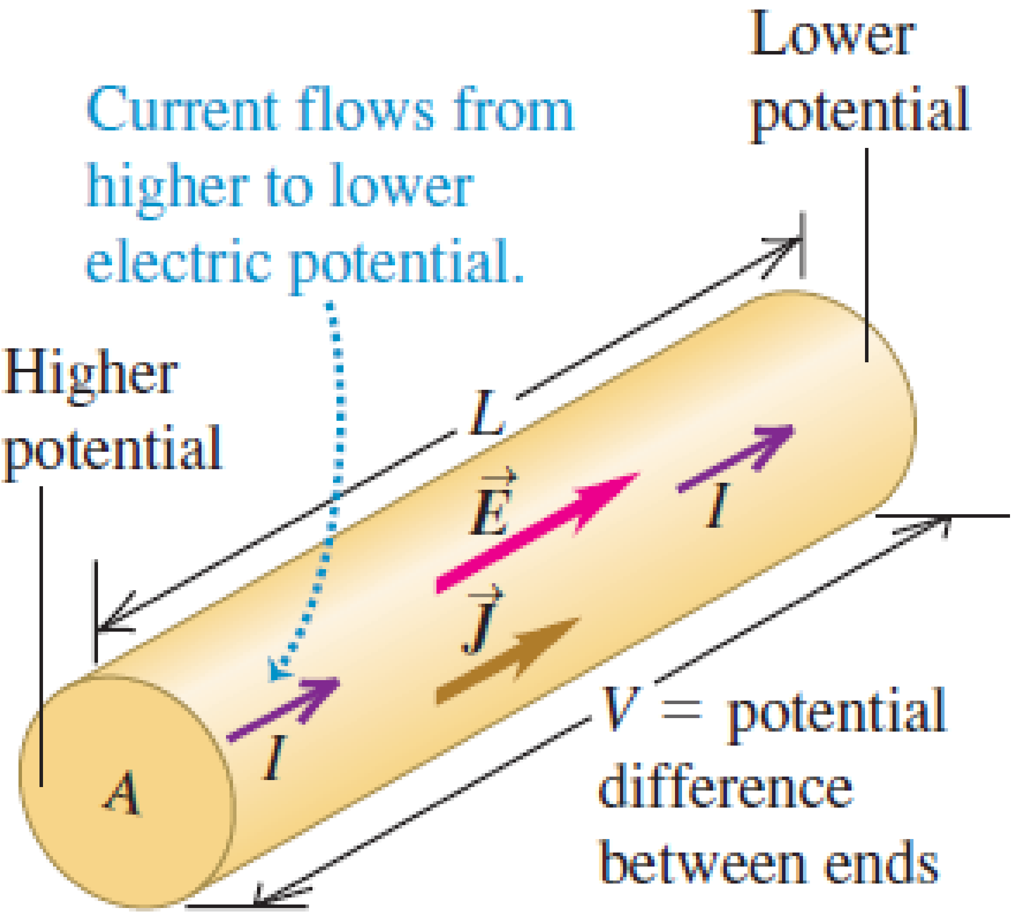
Previous Lecture: Current, Resistance, & Electromotive Force

$$R = \frac{V}{I}$$

$$V_{ab} = \mathcal{E} - Ir$$

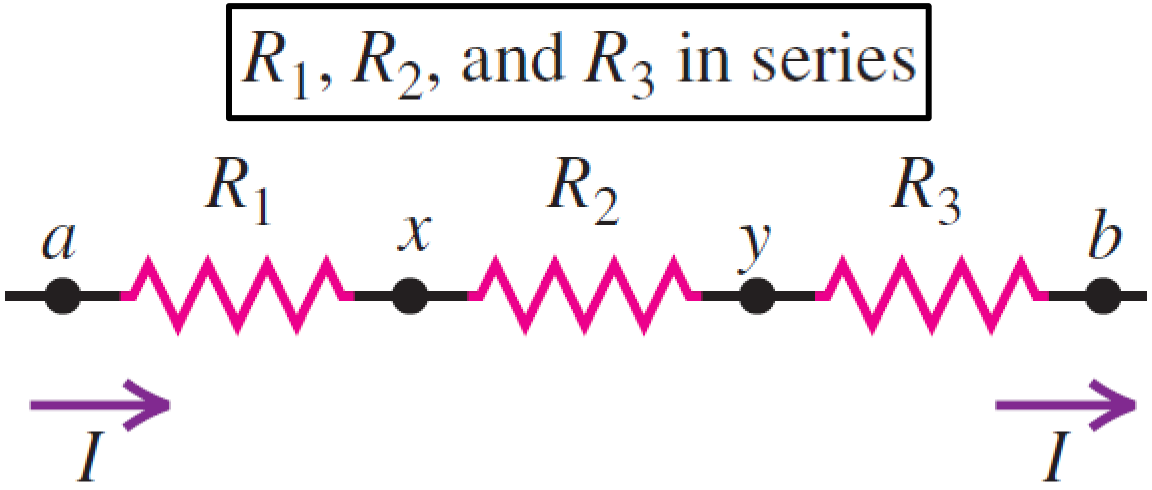
$$P = V_{ab} I = I^2 R = \frac{V_{ab}{}^2}{R}$$

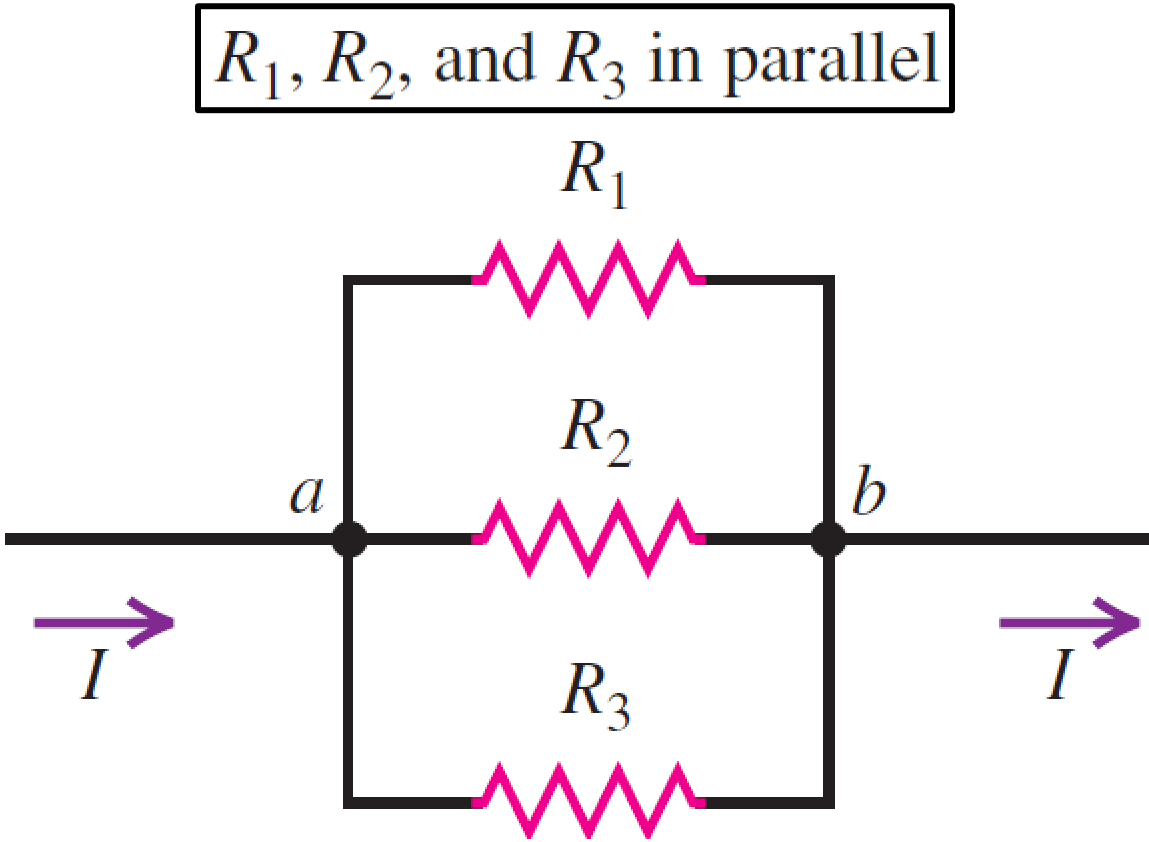




Direct-Current Circuit

- **Alternating current** (ac): current oscillates back and forth
- **Direct-current** (dc) circuits: direction of current does not change with time





The resistance of the resistors in total R_{eq} is called the **equivalent resistance** of the combination

Direct-Current Circuit

- **Alternating current** (ac): current oscillates back and forth
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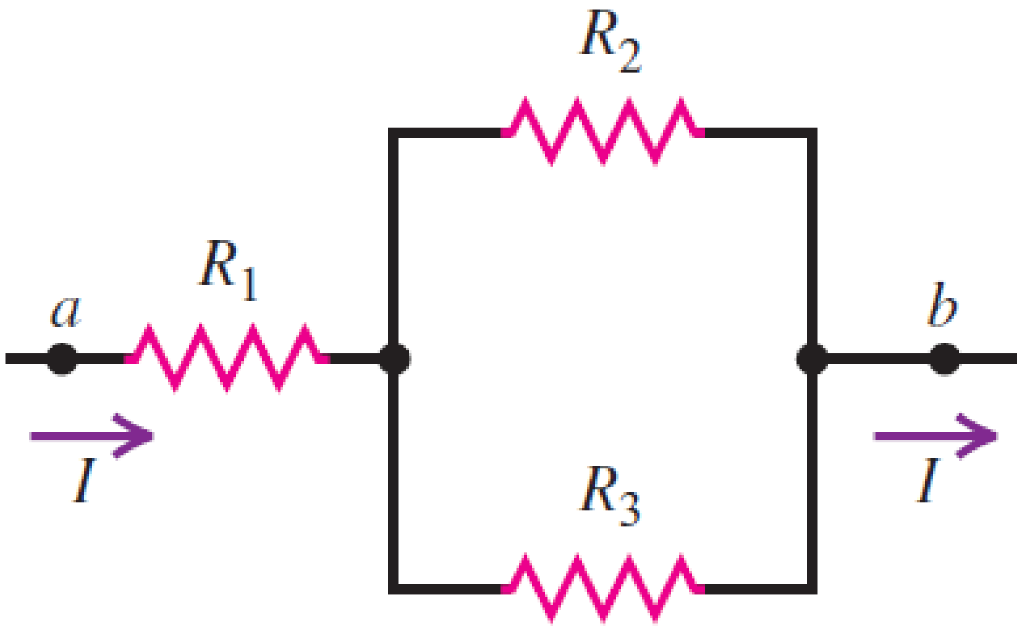
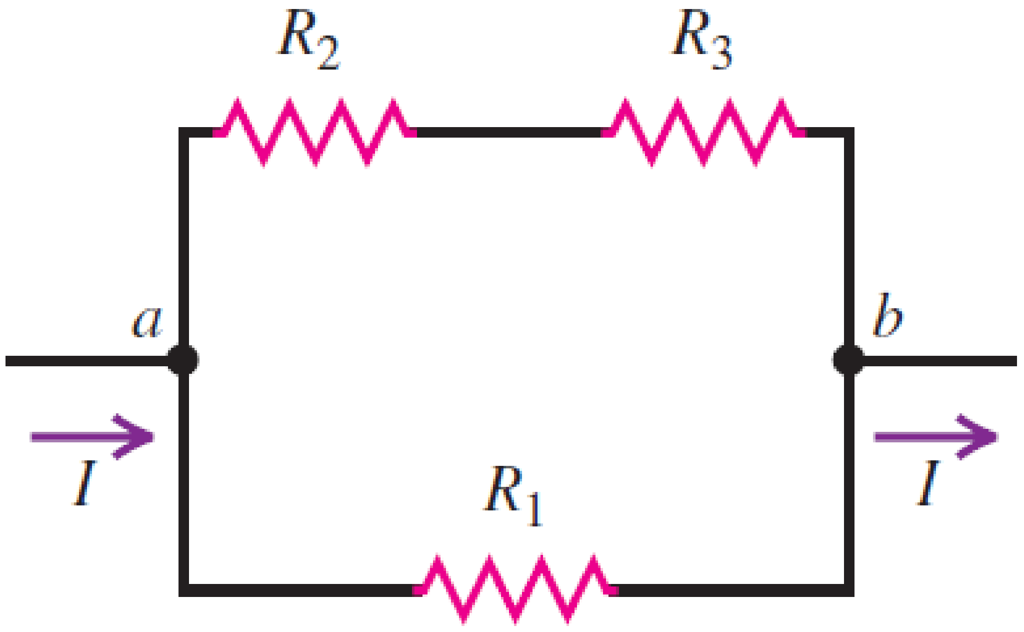
R_1 in series with parallel combination of R_2 and R_3

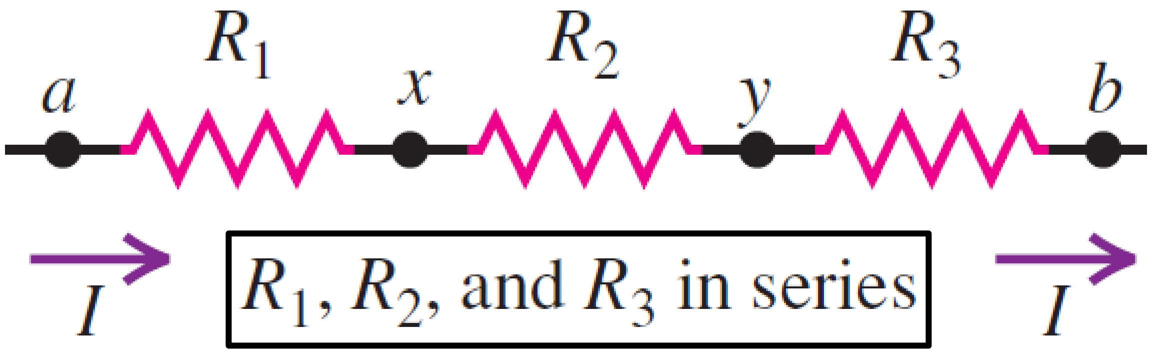


R_1 in parallel with series combination of R_2 and R_3



Resistors in Series

$$V_{ab} = IR_{eq} \quad \text{or} \quad R_{eq} = \frac{V_{ab}}{I}$$



The potential difference V_{ab} across the entire combination is the sum of these individual potential differences:

$$V_{ab} = V_{ax} + V_{xy} + V_{yb} = I(R_1 + R_2 + R_3)$$

So $$\frac{V_{ab}}{I} = R_1 + R_2 + R_3$$

So the potential drop across each resistor:

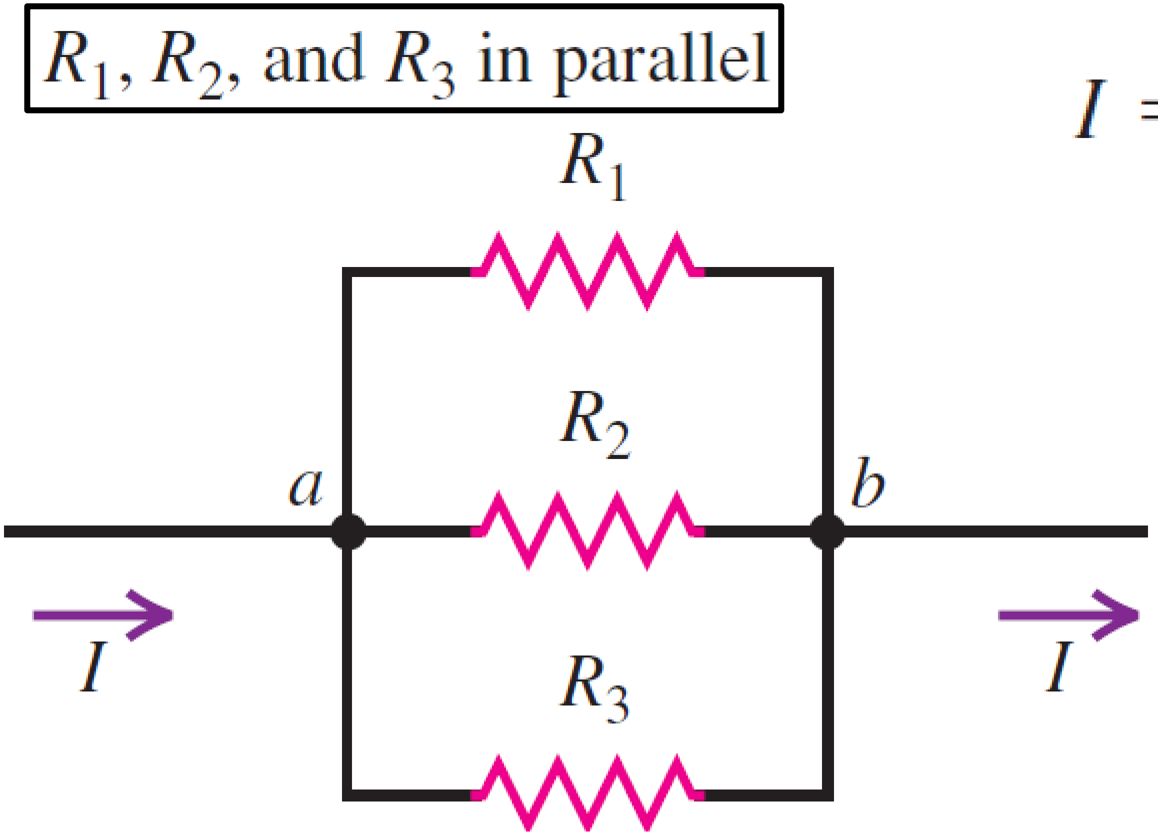
$$V_{ax} = IR_1 \qquad V_{xy} = IR_2 \qquad V_{yb} = IR_3$$

The ratio is, by definition, R_{eq}

$$R_{eq} = R_1 + R_2 + R_3 + \cdots \qquad \text{(resistors in series)}$$

The equivalent resistance of *any number* of resistors in series equals the sum of their individual resistances.

Resistors in Parallel



$$I = I_1 + I_2 + I_3 = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Divide both sides by V_{ab}

$$\frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

But: R_{eq}, $I/V_{ab} = 1/R_{eq}$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

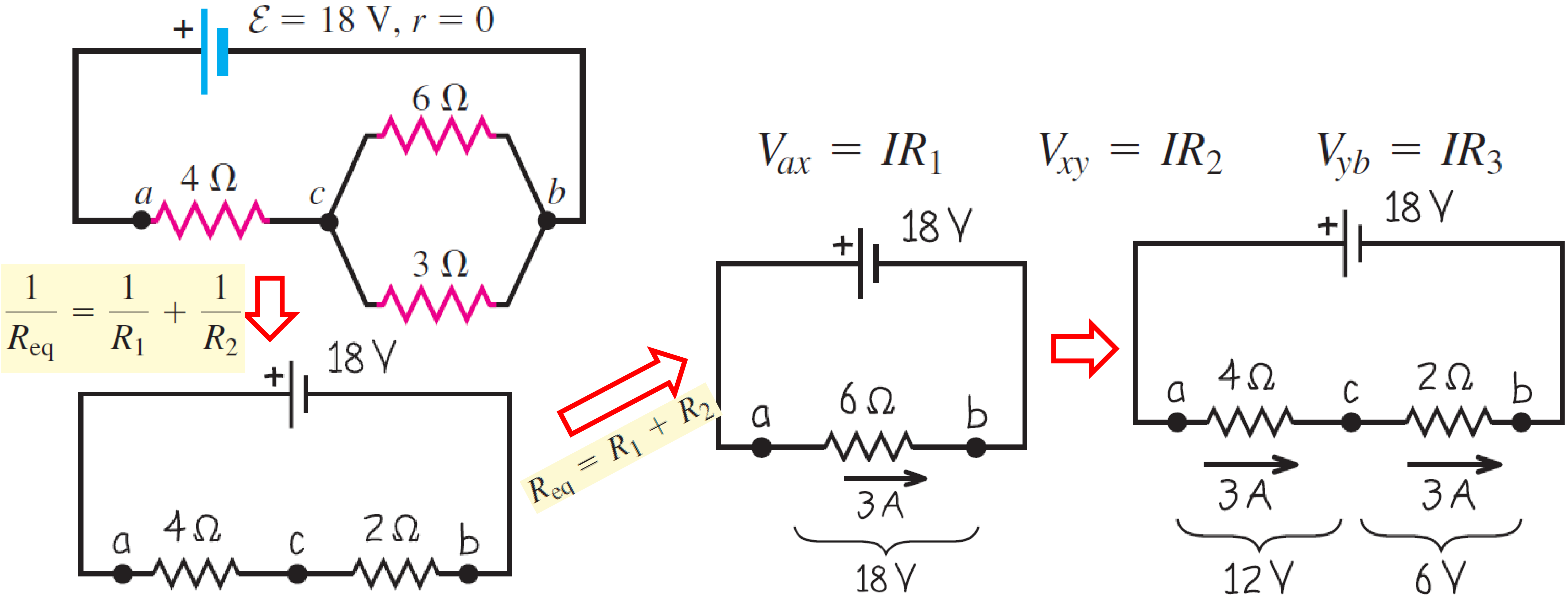
(resistors in parallel)

The voltage V_{ab} must be the same, but the current

$$I_1 = \frac{V_{ab}}{R_1} \qquad I_2 = \frac{V_{ab}}{R_2} \qquad I_3 = \frac{V_{ab}}{R_3}$$

Example 26.1 Equivalent resistance

Find the equivalent resistance of the network in Fig. 26.3a and the current in each resistor. The source of emf has negligible internal resistance.



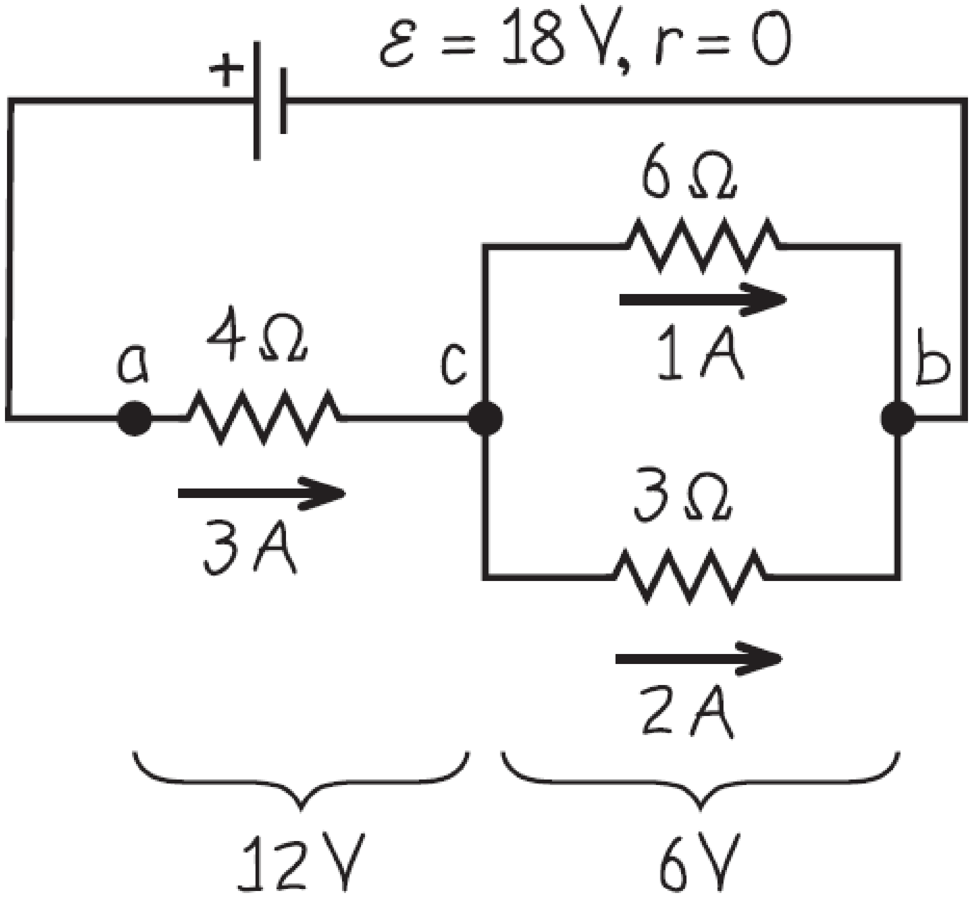
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = R_1 + R_2$$

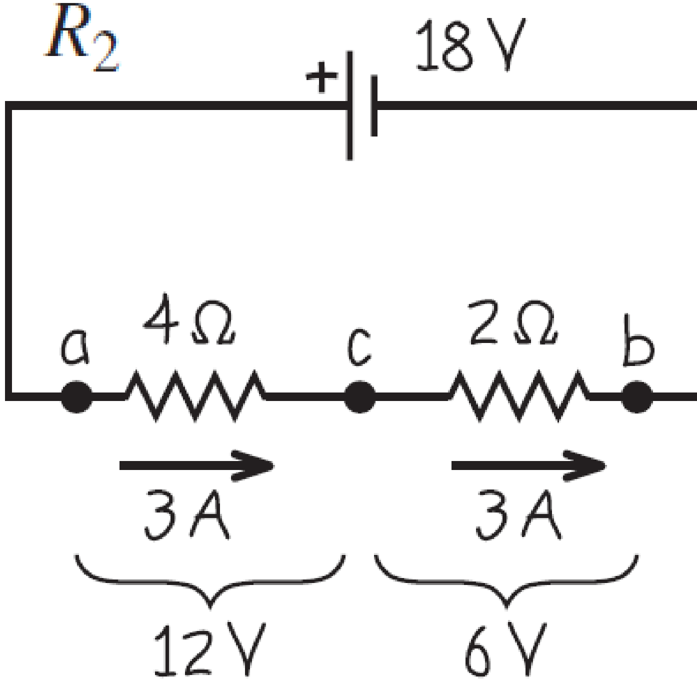
$$V_{ax} = IR_1 \qquad V_{xy} = IR_2 \qquad V_{yb} = IR_3$$

Example 26.1 Equivalent resistance

Find the equivalent resistance of the network in Fig. 26.3a and the current in each resistor. The source of emf has negligible internal resistance.



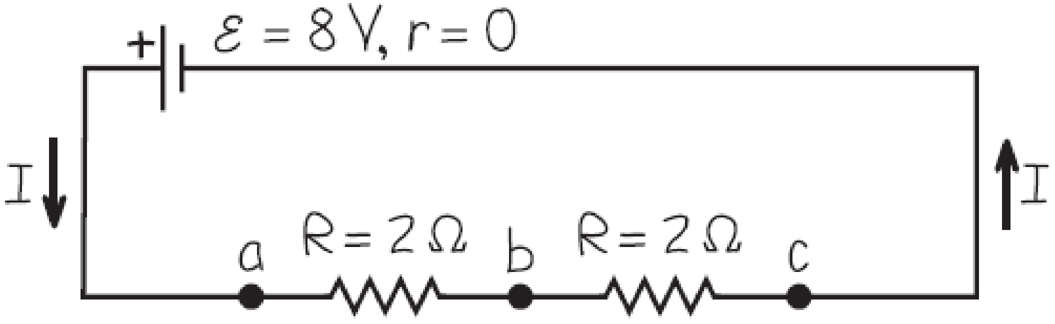
$$I_1 = \frac{V_{ab}}{R_1} \qquad I_2 = \frac{V_{ab}}{R_2}$$



Example 26.2 Series vs parallel combinations

Two identical light bulbs, each with resistance $R = 2\ \Omega$ are connected to a source with $\mathcal{E} = 8$ V and negligible internal resistance. Find the current through each bulb, the potential difference across each bulb, and the power delivered to each bulb and to the entire network if the bulbs are connected (a) in series and (b) in parallel. (c) Suppose one of the bulbs burns out; that is, its filament breaks and current can no longer flow through it. What happens to the other bulb in the series case? In the parallel case?

(a) Light bulbs in series



$$I = \frac{V_{ac}}{R_{eq}} = \frac{8\ \text{V}}{4\ \Omega} = 2\ \text{A}$$

$$P = I^2R = (2\ \text{A})^2(2\ \Omega) = 8\ \text{W}$$

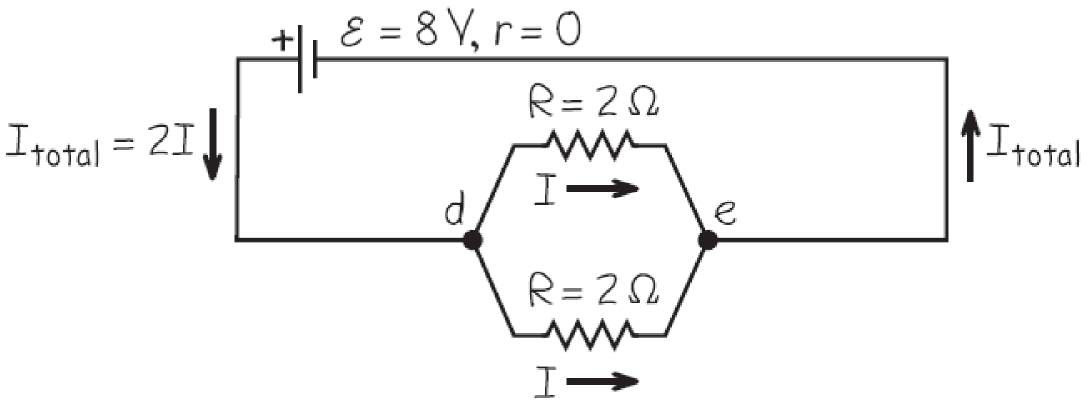
$$P = \frac{{V_{ab}}^2}{R} = \frac{{V_{bc}}^2}{R} = \frac{(4\ \text{V})^2}{2\ \Omega} = 8\ \text{W}$$

In the series case the same current flows through both bulbs. If one bulb burns out, there will be no current in the circuit, and neither bulb will glow.

Example 26.2 Series vs parallel combinations

Two identical light bulbs, each with resistance $R = 2\ \Omega$ are connected to a source with $\mathcal{E} = 8$ V and negligible internal resistance. Find the current through each bulb, the potential difference across each bulb, and the power delivered to each bulb and to the entire network if the bulbs are connected (a) in series and (b) in parallel. (c) Suppose one of the bulbs burns out; that is, its filament breaks and current can no longer flow through it. What happens to the other bulb in the series case? In the parallel case?

(b) Light bulbs in parallel



$$I = \frac{V_{de}}{R} = \frac{8\ \text{V}}{2\ \Omega} = 4\ \text{A}$$

$$P = I^2R = (4\ \text{A})^2(2\ \Omega) = 32\ \text{W}$$

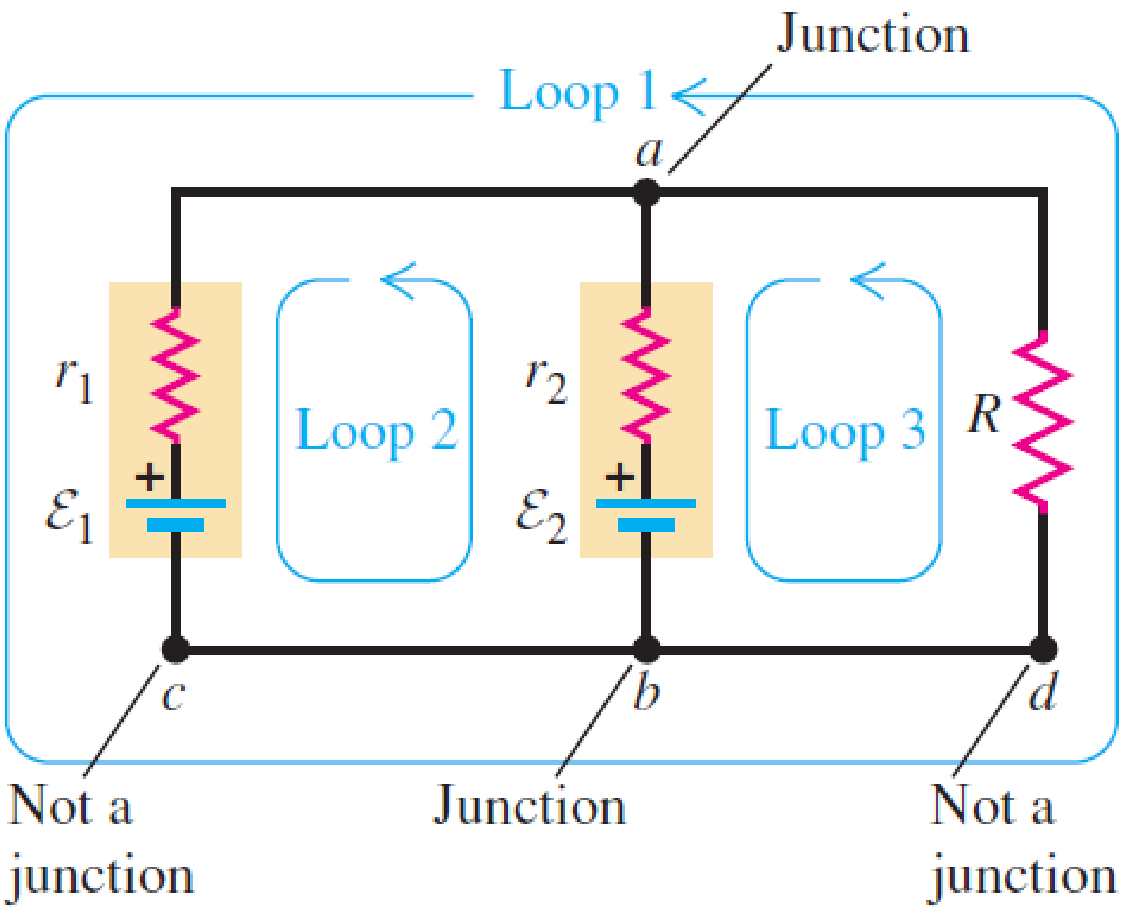
$$P = \frac{{V_{de}}^2}{R} = \frac{(8\ \text{V})^2}{2\ \Omega} = 32\ \text{W}$$

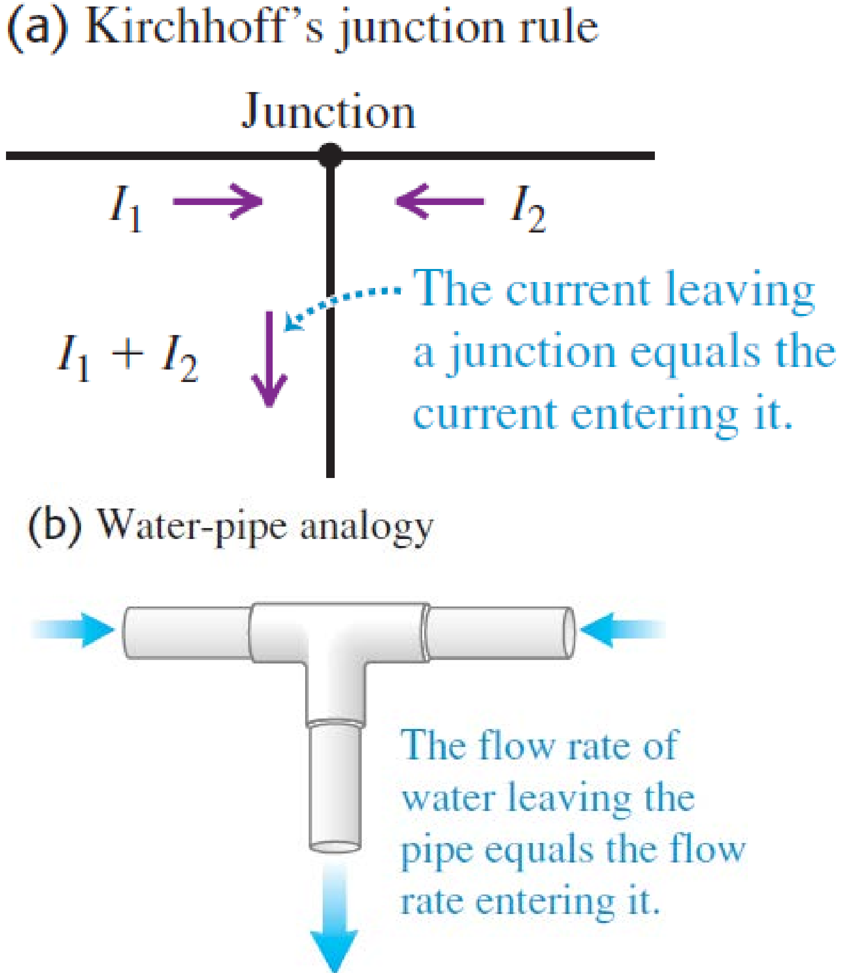
In the parallel case the potential difference across either bulb is unchanged if a bulb burns out. The current through the functional bulb and the power delivered to it are unchanged.

Kirchhoff's Rules

Kirchhoff's junction rule: *The algebraic sum of the currents into any junction is zero*

$$\sum I = 0 \qquad \text{(junction rule, valid at any junction)}$$

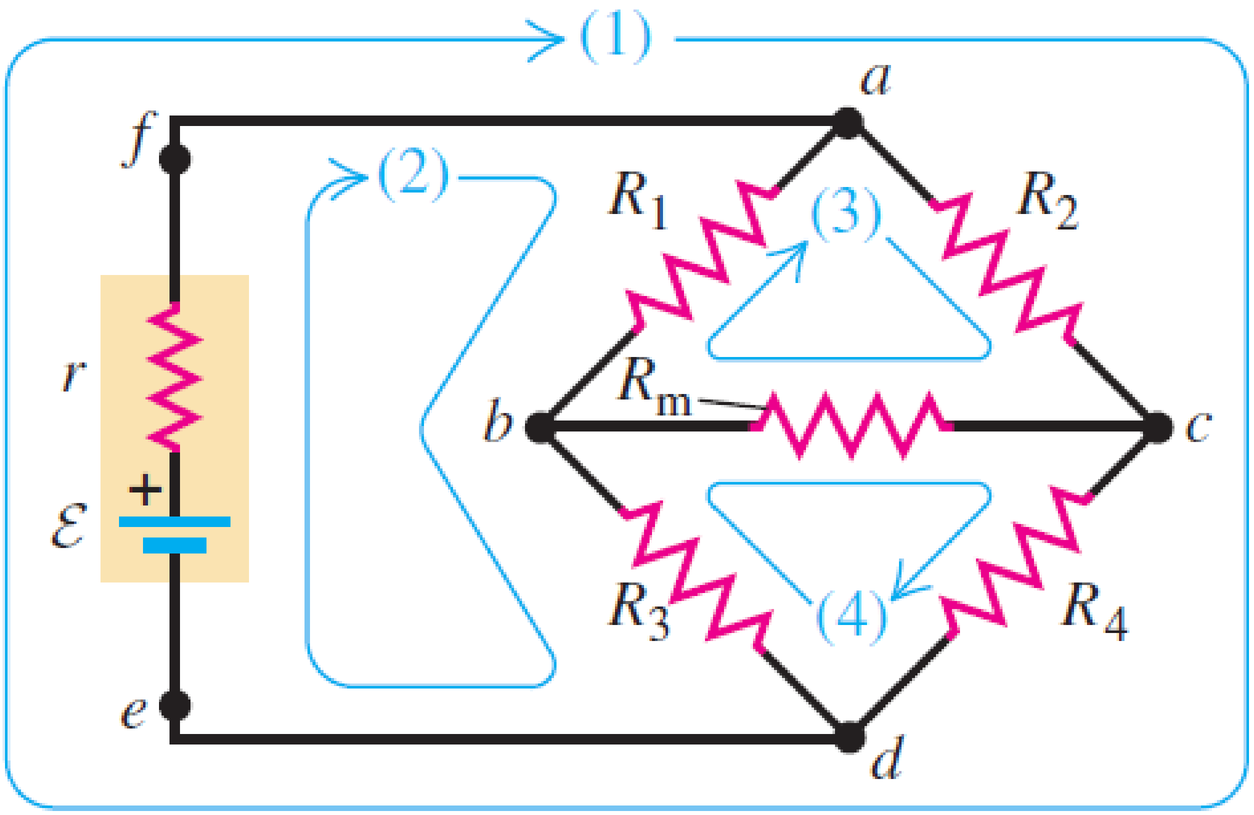




Kirchhoff's Rules

Kirchhoff's loop rule: *The algebraic sum of the potential differences in any loop,* including those associated with emfs and those of resistive elements, *must equal zero.*

$$\sum V = 0 \qquad \text{(loop rule, valid for any closed loop)}$$



The loop rule is a statement that the electrostatic force is *conservative.* Suppose we go around a loop, measuring potential differences across successive circuit elements as we go. When we return to the starting point, we must find that the *algebraic sum* of these differences is zero

Sign Conventions for the Loop Rule

Kirchhoff's loop rule: *The algebraic sum of the potential differences in any loop,* including those associated with emfs and those of resistive elements, *must equal zero.*

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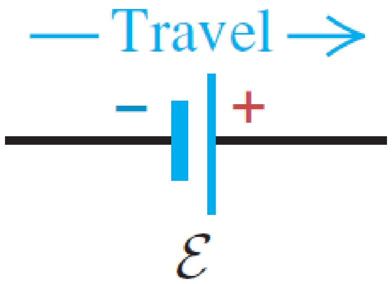
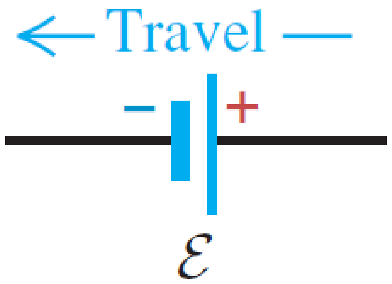
$+\mathcal{E}$: Travel direction from – to +:

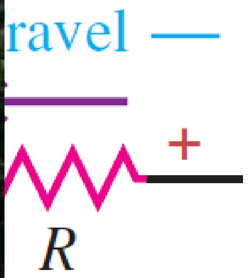


$-\mathcal{E}$: Travel direction from + to –:



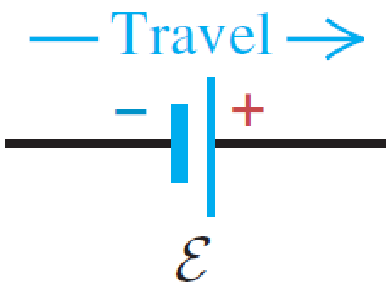
$+IR$: Travel *opposite to* current direction:

$-IR$: Travel *in* current direction:

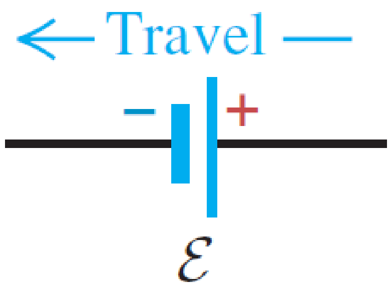


Sign Conventions for the Loop Rule

$+\mathcal{E}$: Travel direction from – to +:



$-\mathcal{E}$: Travel direction from + to –:



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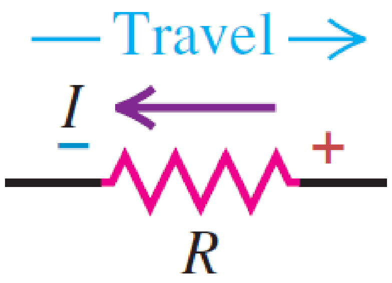
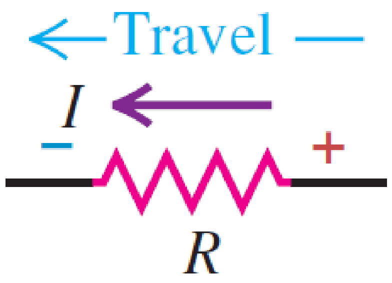
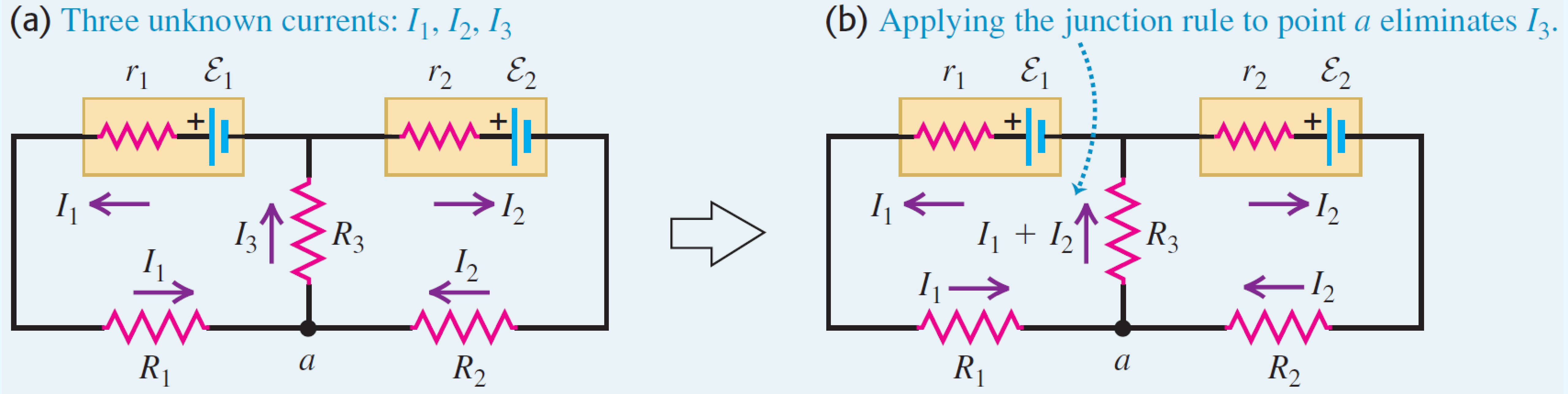


$-IR$: Travel *in* current direction:



Pumps that pulls water to higher potential

Sign Conventions for the Loop Rule



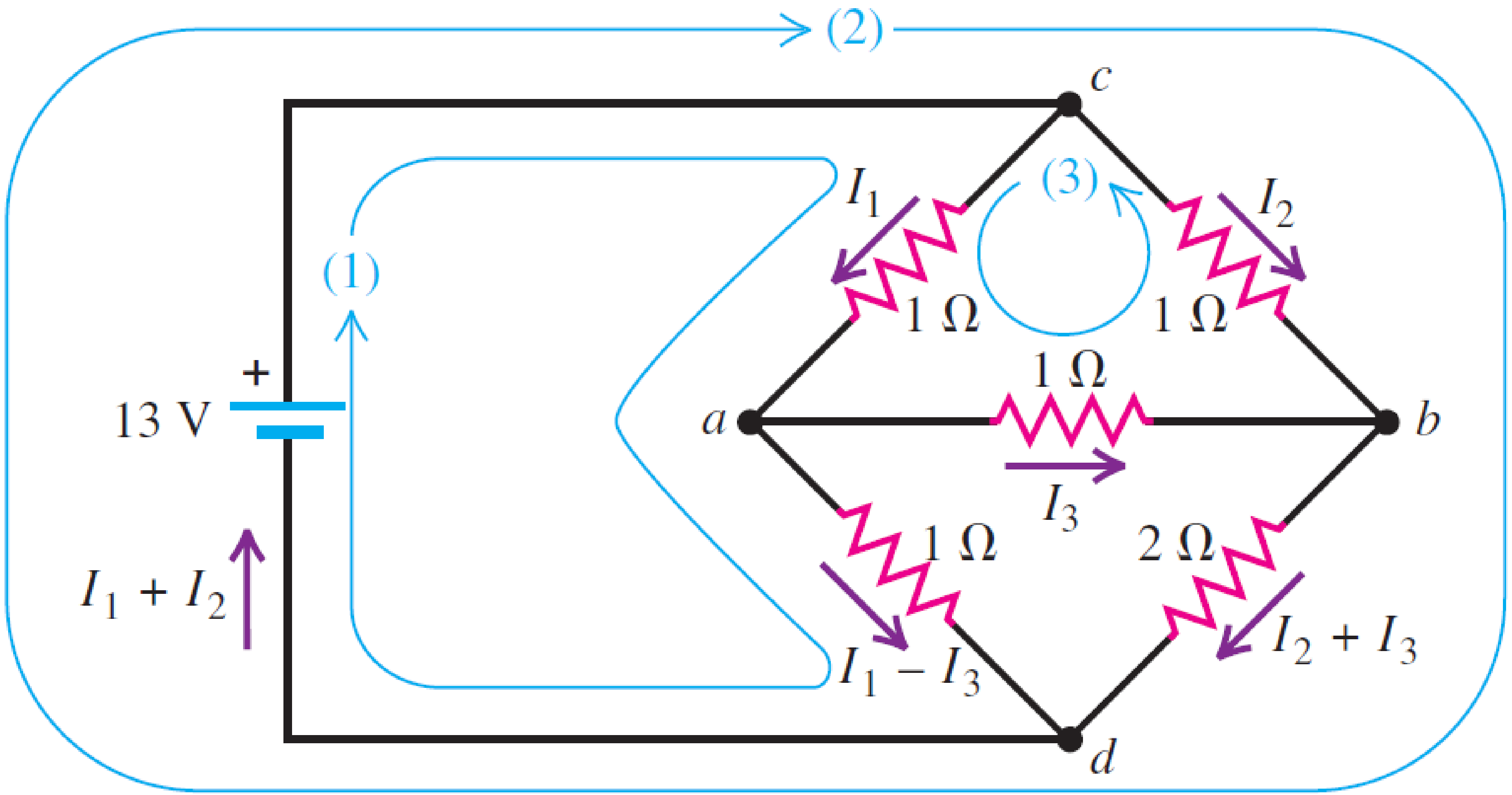
Example 26.6 A complex network

Figure 26.12 shows a “bridge” circuit of the type described at the beginning of this section (see Fig. 26.6b). Find the current in each resistor and the equivalent resistance of the network of five resistors.

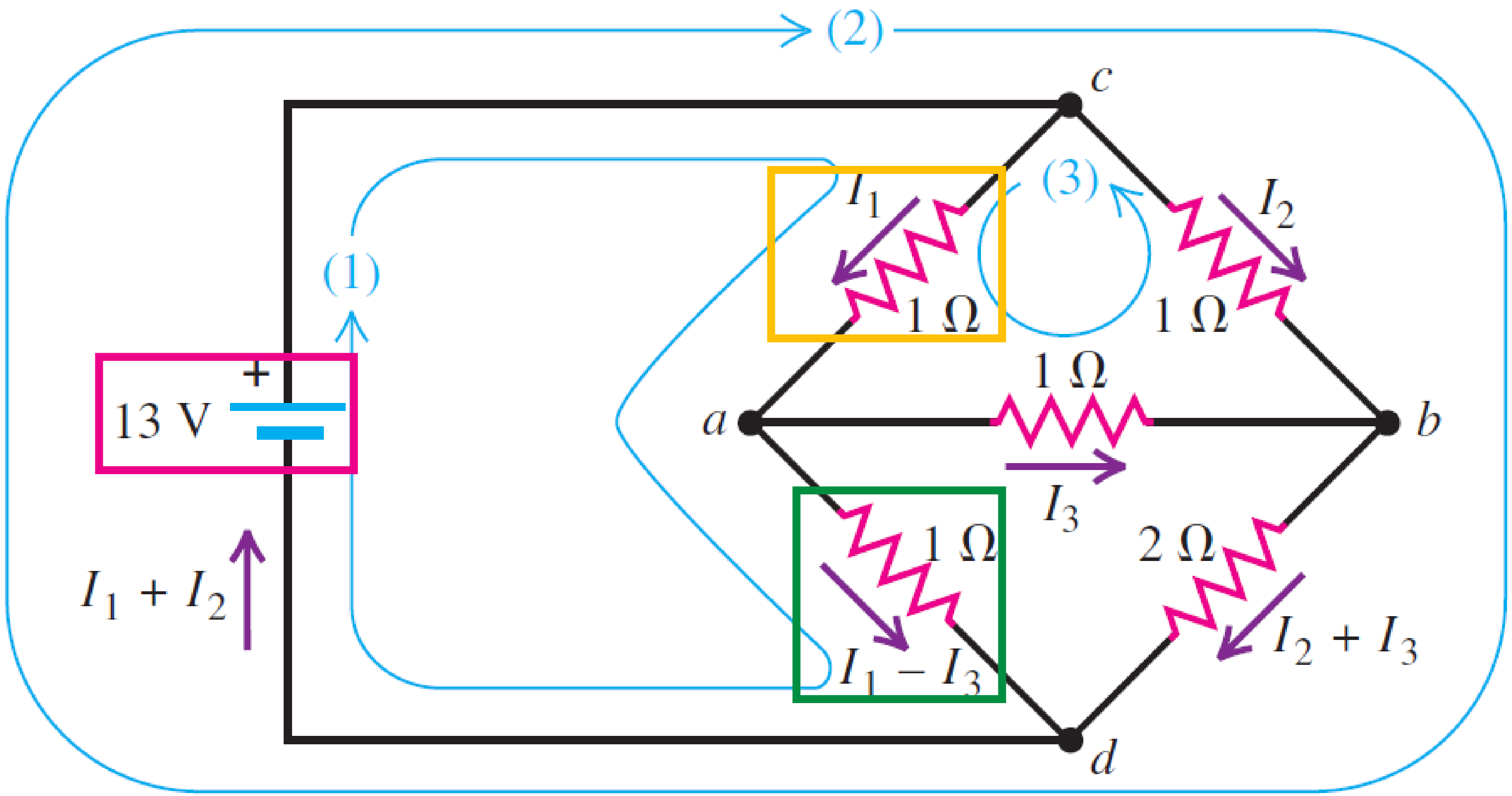
1. Assign the loop numers

2. Use the junction rule

$$\sum I = 0$$



Example 26.6 A complex network

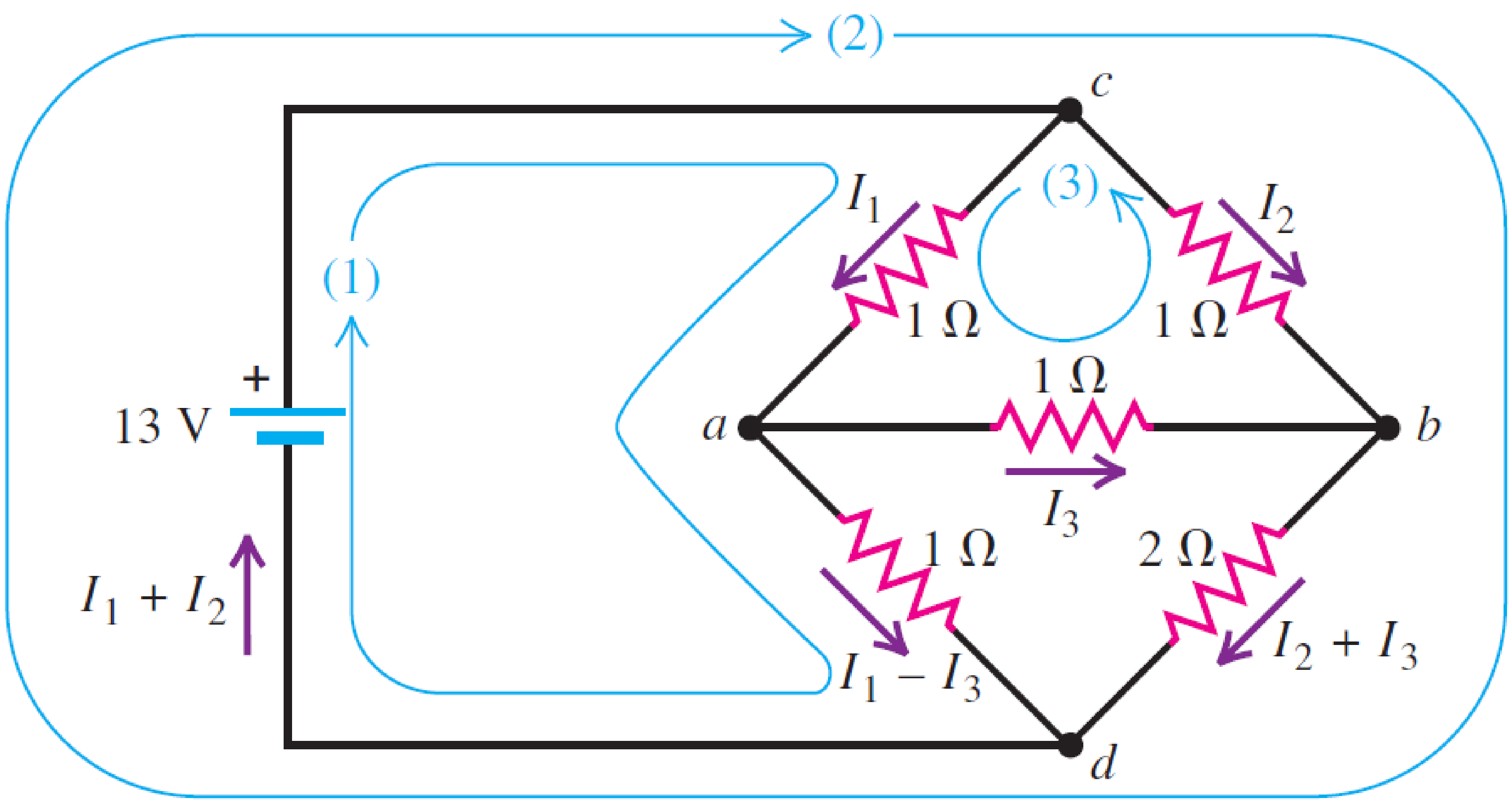


$$\boxed{13\ \text{V}}\ \boxed{-\ I_1(1\ \Omega)}\ \boxed{-\ (I_1 - I_3)(1\ \Omega)} = 0 \qquad (1)$$

3. Use loop rule, starting from loop 1

Two variables, 1 equation, NEED 1 more!

Example 26.6 A complex network



$$13\text{ V} - I_1(1\ \Omega) - (I_1 - I_3)(1\ \Omega) = 0 \qquad (1)$$

Loop 2 $$-I_2(1\ \Omega) - (I_2 + I_3)(2\ \Omega) + 13\text{ V} = 0 \qquad (2)$$

Loop 3 $$-I_1(1\ \Omega) - I_3(1\ \Omega) + I_2(1\ \Omega) = 0 \qquad (3)$$

3 variables
3 equation

Example 26.6 A complex network

$$13\text{ V} - I_1(1\ \Omega) - (I_1 - I_3)(1\ \Omega) = 0 \qquad (1)$$

$$-I_2(1\ \Omega) - (I_2 + I_3)(2\ \Omega) + 13\text{ V} = 0 \qquad (2)$$

$$-I_1(1\ \Omega) - I_3(1\ \Omega) + I_2(1\ \Omega) = 0 \qquad (3) \Rightarrow I_2 = I_1 + I_3;$$

$$\Rightarrow 13\text{ V} = I_1(3\ \Omega) + I_3(5\ \Omega) \qquad (2')$$

$$13\text{ V} = I_1(2\ \Omega) - I_3(1\ \Omega) \qquad (1')$$

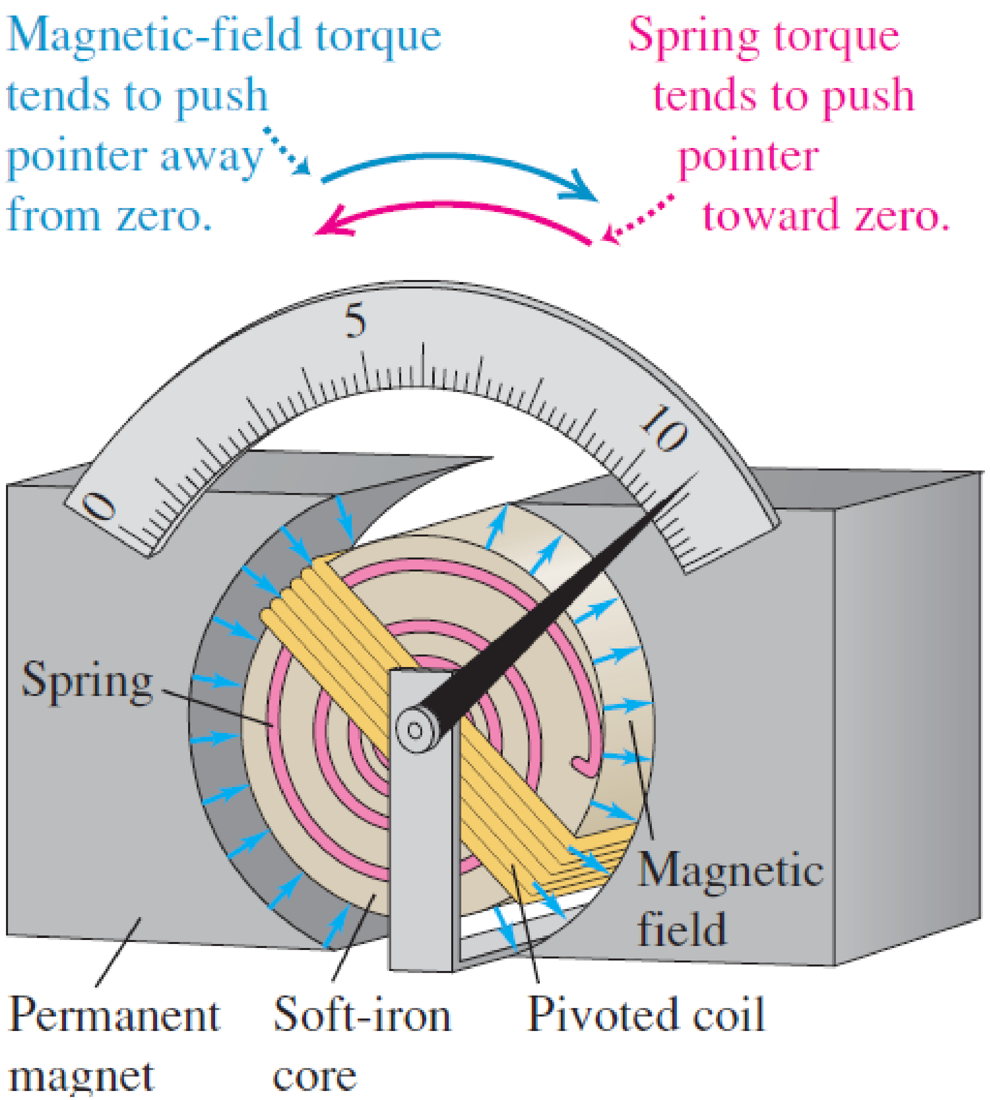
Now we can eliminate I_3 by multiplying Eq. (1') by 5 and adding the two equations. We obtain

$$78\text{ V} = I_1(13\ \Omega) \qquad I_1 = 6\text{ A}$$

Plug in 1' and 3 to get $I_3 = -1\text{ A} \quad I_2 = 5\text{ A}$

Electrical Measuring Instruments

d'Arsonval galvanometer, often called just a meter.



Angular deflection of the coil and pointer is directly proportional to the coil current, and the device can be calibrated to measure current

full scale when the current in its coil is I_{fs}

coil has a resistance R_c

The corresponding potential difference for full-scale deflection is

$$V = I_{fs}R_c$$

Ammeters

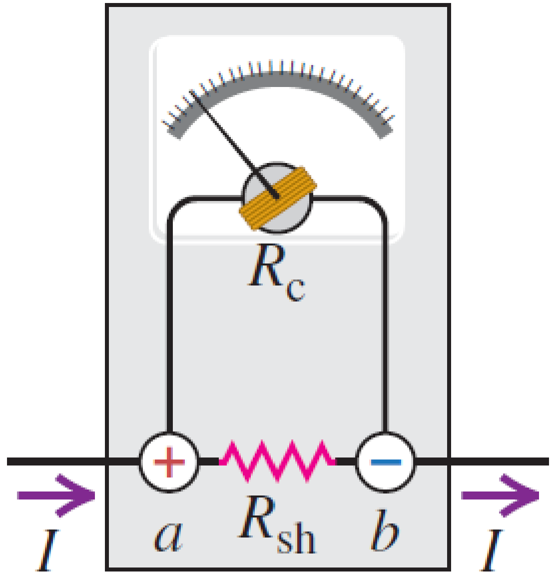
An ammeter always measures the current passing through it, ***ideally having 0 resistance.*** Real ammeters always have some finite resistance, but it is always desirable to have ***as little resistance as possible.***

Shunt resistor or simply a *shunt*, denoted as R_{sh}

- Used to adjust the scale of the ammeters/voltmeters

$$V_{ab} = I_{fs}R_c = (I_a - I_{fs})R_{sh} \quad \text{(for an ammeter)}$$

(a) Moving-coil ammeter



Example 26.8 Designing an ammeter

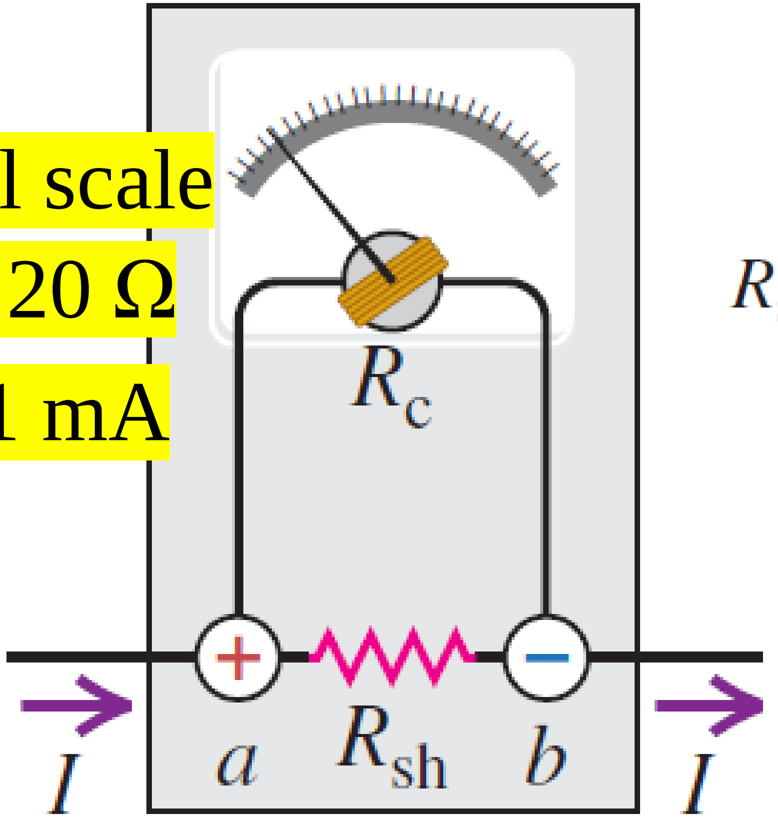
What shunt resistance is required to make the 1.00-mA, 20-Ω meter described above into an ammeter with a range of 0 to 50.0 mA?

Hint: learn to interpret this condition into V and I in the circuit!

at full scale

$R_c = 20\ \Omega$

$I_c = 1$ mA



$$I_{fs}R_c = (I_a - I_{fs})R_{sh} \qquad \text{(for an ammeter)}$$

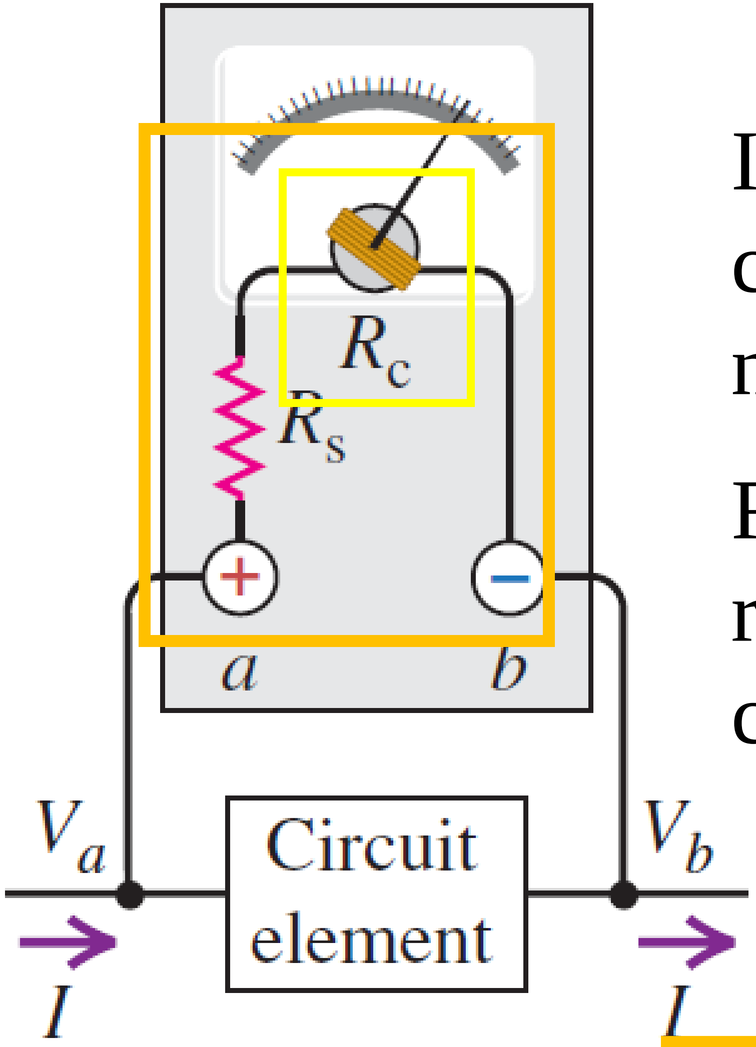
$$R_{sh} = \frac{I_{fs}R_c}{I_a - I_{fs}} = \frac{(1.00 \times 10^{-3}\ \text{A})(20.0\ \Omega)}{50.0 \times 10^{-3}\ \text{A} - 1.00 \times 10^{-3}\ \text{A}}$$

$$= 0.408\ \Omega$$

What we want: I = 50 mA in total

so: $I_{sh} = I - I_c = 50$ mA in total

Voltmeters



Ideal voltmeter would have *infinite* resistance, so connecting it between two points in a circuit would not alter any of the currents.

Real voltmeters always have finite but large enough resistance that connecting it in a circuit does not change the other currents appreciably

full-scale deflection is $I_{fs}R_c$

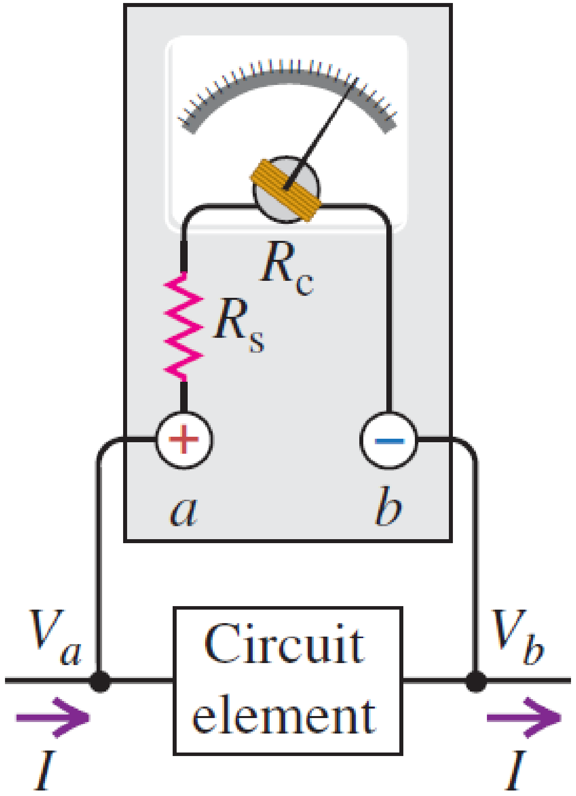
connecting a resistor R_s in *series* with the coil

$$V_V = I_{fs}(R_c + R_s) \qquad \text{(for a voltmeter)}$$

Example 26.9 Designing a voltmeter

What series resistance is required to make the 1.00-mA, 20-Ω meter described above into a voltmeter with a range of 0 to 10.0 V?

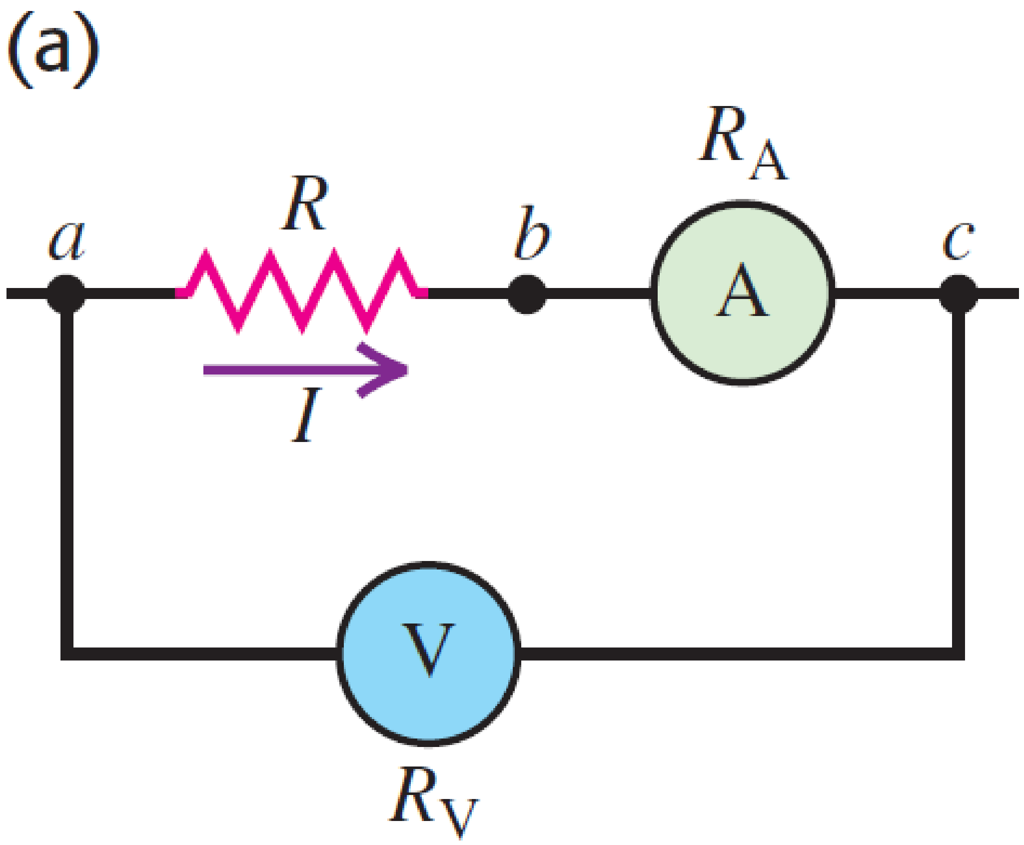
$$\boxed{V_V} = \boxed{I_{fs}}(\boxed{R_c} + R_s) \qquad \text{(for a voltmeter)}$$

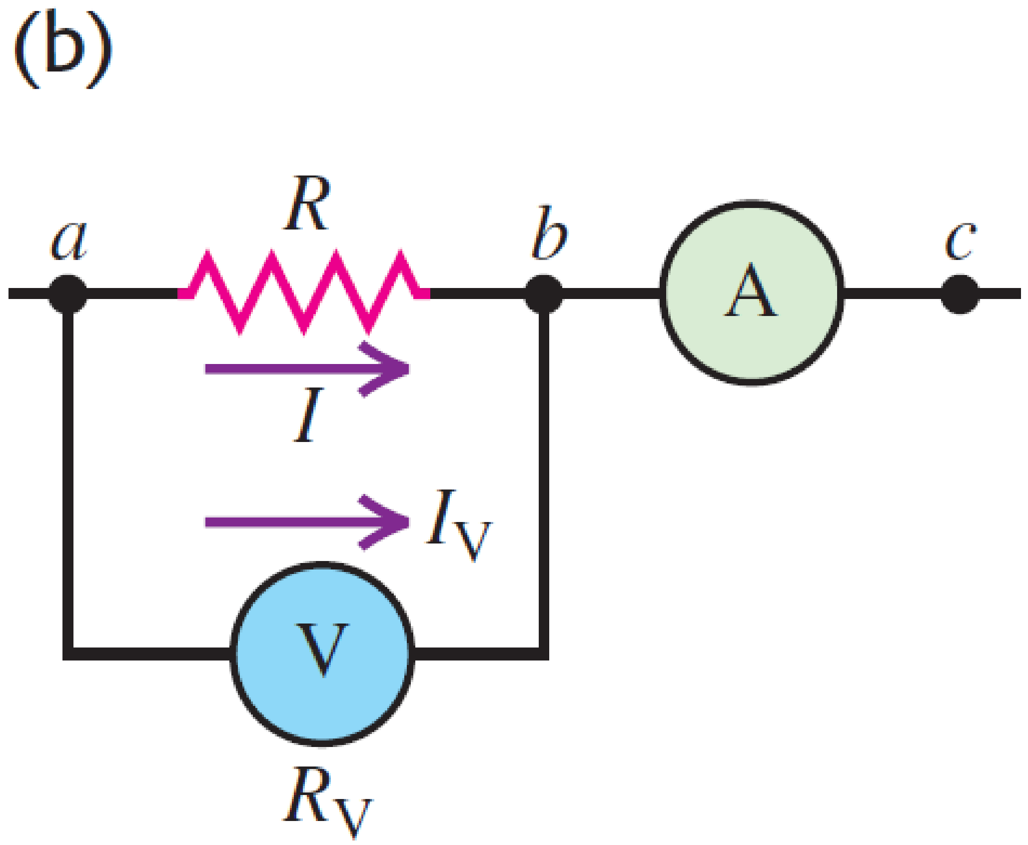


$$R_s = \frac{V_V}{I_{fs}} - R_c = \frac{10.0\ \text{V}}{0.00100\ \text{A}} - 20.0\ \Omega = 9980\ \Omega$$

Ammeters and Voltmeters in Combination

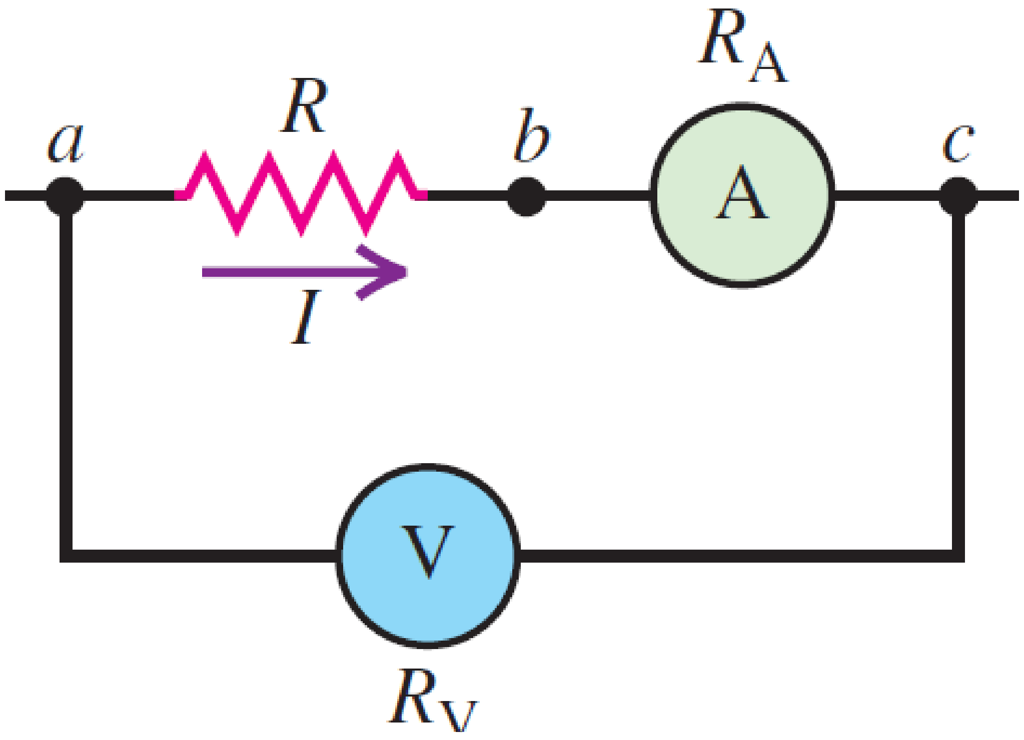
Ammeter–voltmeter method for measuring resistance.





Example 26.10 Measuring resistance I

The voltmeter in the circuit reads 12.0 V and the ammeter reads 0.100 A. The meter resistances are $R_v = 10000\ \Omega$ (for the voltmeter) and $R_A = 2.0\ \Omega$ (for the ammeter). What are the resistance R and the power dissipated in the resistor?



From Ohm's law, $V_{bc} = 2.00$ V :

$$V_{bc} = IR_A = (0.100\ \text{A})(2.00\ \Omega)$$

Since $V_{ab} = IR$, where R is unknown.

But $V = V_{ac} = 12$ V, which is the reading of the voltmeter。

$$V_{ab} = V - V_{bc} = 11.8\ \text{V}$$

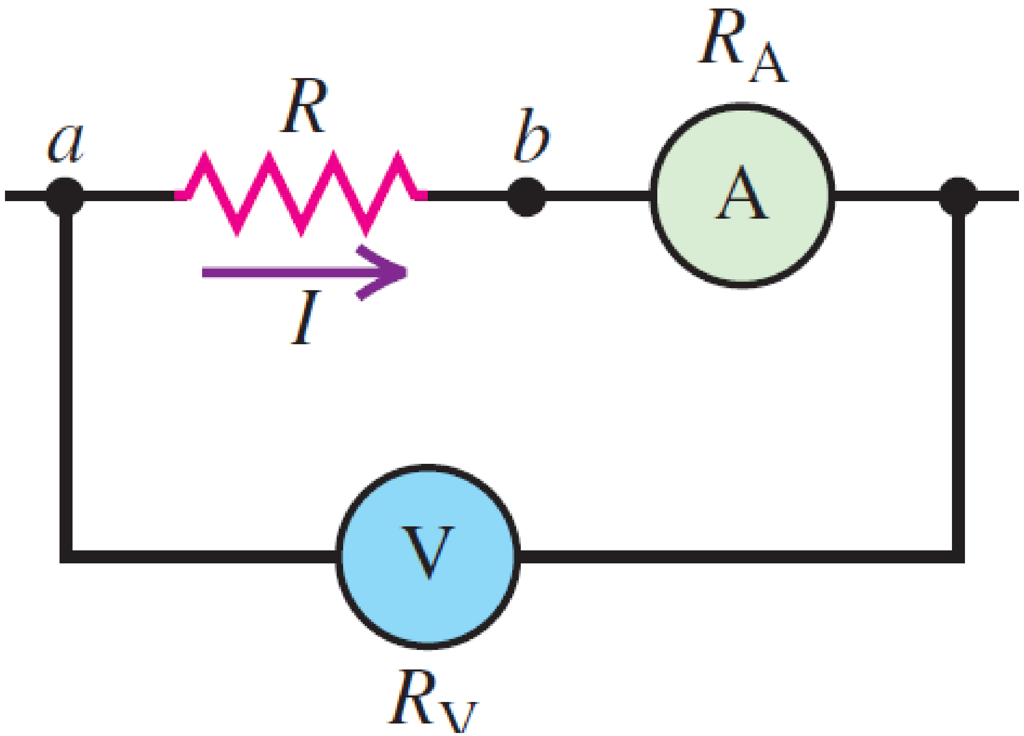
So $$R = \frac{V_{ab}}{I} = \frac{11.8\ \text{V}}{0.100\ \text{A}} = 118\ \Omega$$

Example 26.10 Measuring resistance I

The voltmeter in the circuit reads 12.0 V and the ammeter reads 0.100 A. The meter resistances are $R_v = 10000\ \Omega$ (for the voltmeter) and $R_A = 2.0\ \Omega$ (for the ammeter). What are the resistance R and the power dissipated in the resistor?

The power dissipated in this resistor is

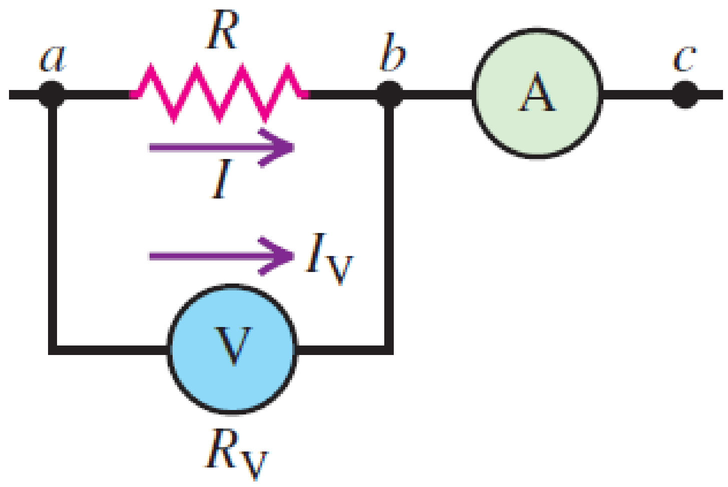
$$P = V_{ab}I = (11.8\ \text{V})(0.100\ \text{A}) = 1.18\ \text{W}$$



The key to solve this problem is to interpr the readings. What is the voltmeter measuring? V_{ab}, V_{ac}, or V_{bc}?

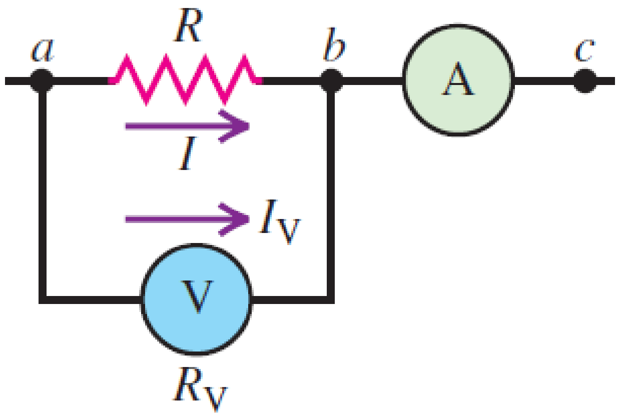
Example 26.11 Measuring resistance II

Suppose the meters of Example 26.10 are connected to a different resistor as shown in Fig. 26.16b, and the readings obtained on the meters are the same as in Example 26.10. What is the value of this new resistance R, and what is the power dissipated in the resistor?



Example 26.11 Measuring resistance II

Suppose the meters of Example 26.10 are connected to a different resistor as shown in Fig. 26.16b, and the readings obtained on the meters are the same as in Example 26.10. What is the value of this new resistance R, and what is the power dissipated in the resistor?



EXECUTE: We have $I_V = V/R_V = (12.0\ \text{V})/(10{,}000\ \Omega) = 1.20\ \text{mA}$. The actual current I in the resistor is $I = I_A - I_V = 0.100\ \text{A} - 0.0012\ \text{A} = 0.0988\ \text{A}$, and the resistance is

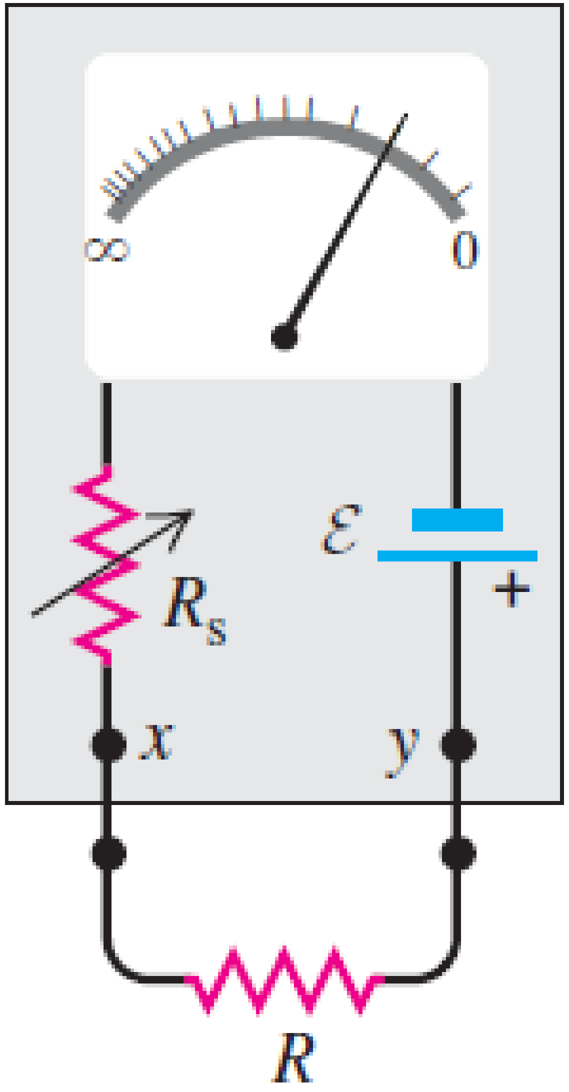
$$R = \frac{V_{ab}}{I} = \frac{12.0\ \text{V}}{0.0988\ \text{A}} = 121\ \Omega$$

The power dissipated in the resistor is

$$P = V_{ab}I = (12.0\ \text{V})(0.0988\ \text{A}) = 1.19\ \text{W}$$

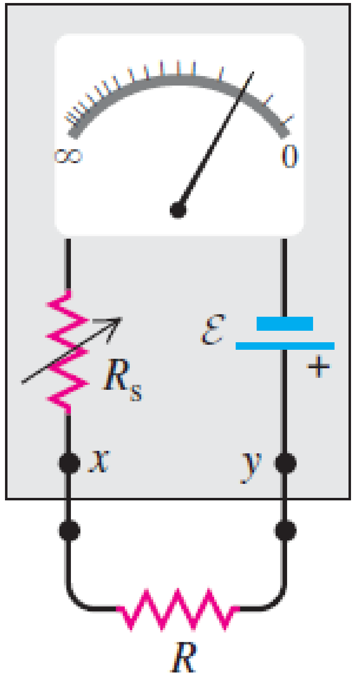
Ohmmeters

An alternative method for measuring resistance is to use a d'Arsonval meter in an arrangement called an **ohmmeter.** It consists of a meter, a resistor, and a source (often a flashlight battery) connected in series. The resistance *R* to be measured is connected between terminals *x* and *y*.



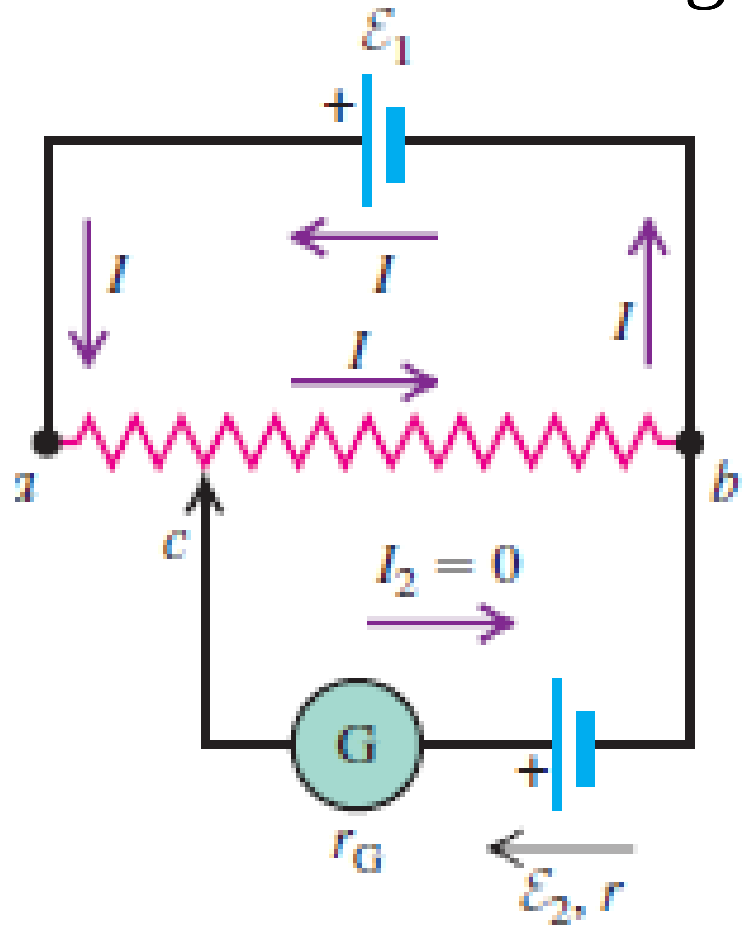
Ohmmeters

The series resistance R_s is variable; it is adjusted so that when terminals x and y are short-circuited (that is, when $R = 0$), the meter deflects full scale. When nothing is connected to terminals x and y, so that the circuit between x and y is *open* (that is, when $R \to \infty$), there is no current and hence no deflection. For any intermediate value of R the meter deflection depends on the value of R, and the meter scale can be calibrated to read the resistance R directly. Larger currents correspond to smaller resistances, so this scale reads backward compared to the scale showing the current.



The Potentiometer

The *potentiometer* is an instrument that can be used to measure the emf of a source without drawing any current from the source; it also has a number of other useful applications. Essentially, it balances an unknown potential difference against an adjustable, measurable potential difference.



ε_1 is known, and to measure ε_2, contact *c* is moved until a position is found at which the galvanometer G shows no deflection: no current passes through ε_2

$$\mathcal{E}_2 = IR_{cb}$$

R-C Circuits

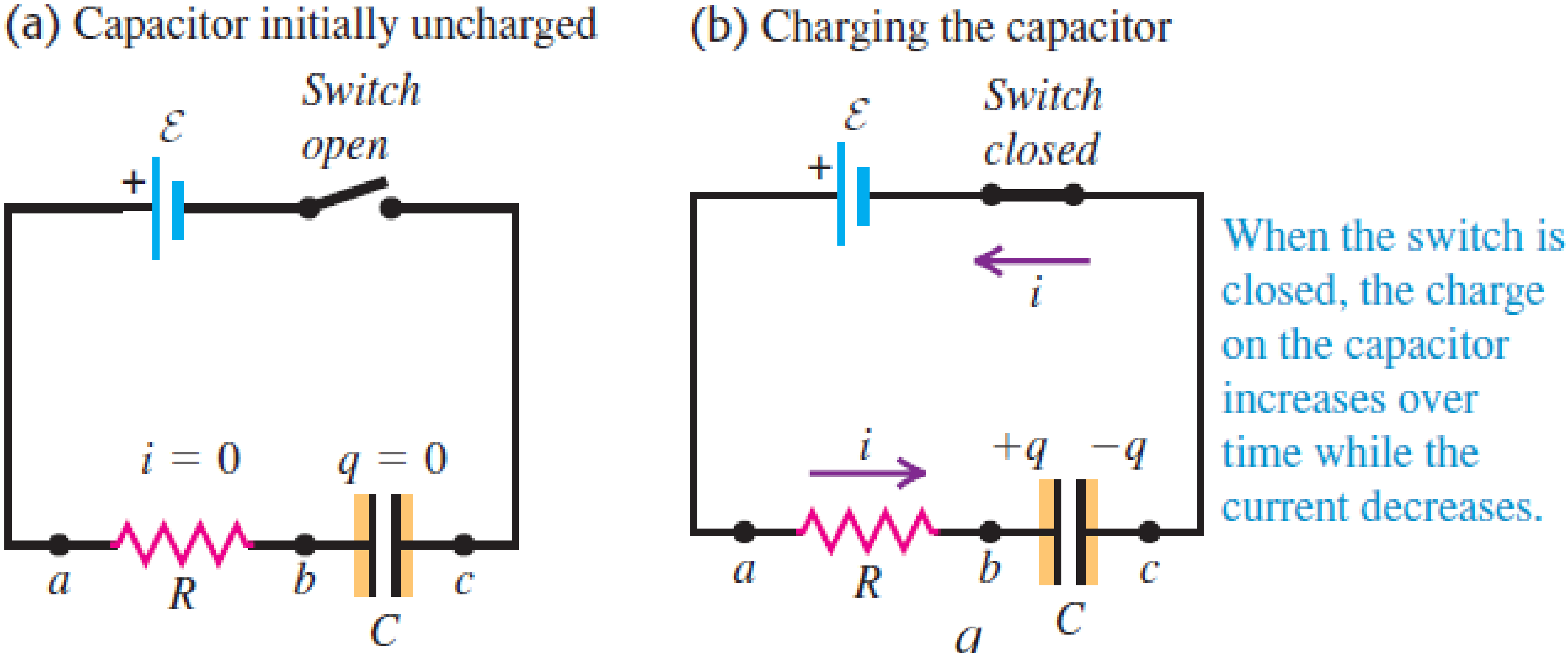
Previously, we have assumed that all the emfs and resistances are *constant* (time independent) so that all the potentials, currents, and powers are also independent of time.

A circuit such as this that has a resistor and a capacitor in series is called an ***R-C* circuit.**

CAUTION **Lowercase means time-varying** Up to this point we have been working with constant potential differences (voltages), currents, and charges, and we have used *capital* letters V, I, and Q, respectively, to denote these quantities. To distinguish between quantities that vary with time and those that are constant, we will use *lowercase* letters v, i, and q for time-varying voltages, currents, and charges, respectively. We suggest that you follow this same convention in your own work.

R-C Circuits

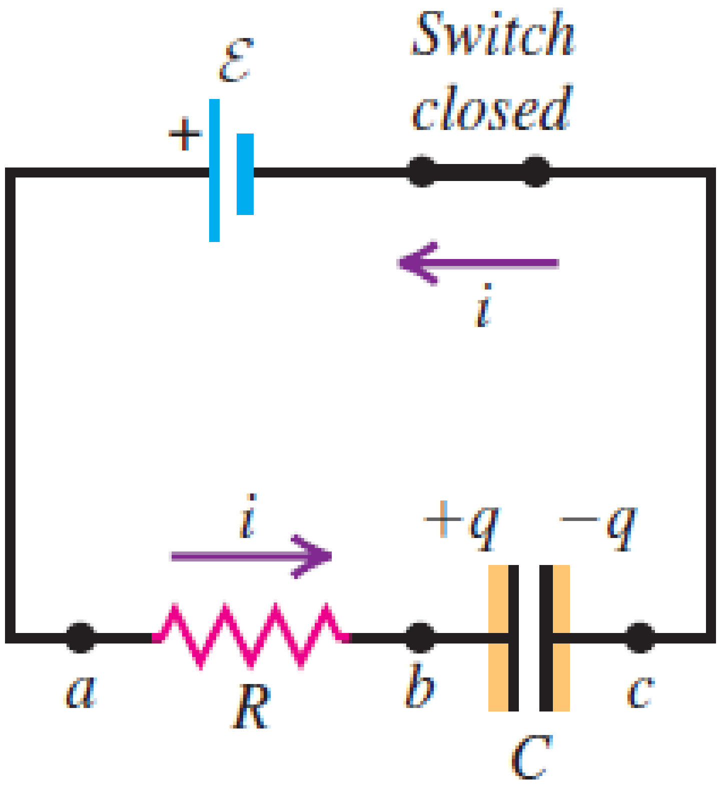
A circuit such as this that has a resistor and a capacitor in series is called an ***R-C* circuit.**



$$v_{ab} = iR \qquad v_{bc} = \frac{q}{C}$$

R-C Circuits

(b) Charging the capacitor



When the switch is closed, the charge on the capacitor increases over time while the current decreases.

$$v_{ab} = iR \qquad v_{bc} = \frac{q}{C}$$

Using these in Kirchhoff's loop rule,

$$\mathcal{E} - iR - \frac{q}{C} = 0$$

The potential drops by an amount *iR* as we travel from *a* to *b* and by *q*/C as we travel from *b* to *c*. Solving the equation for *i*, we find

$$i = \frac{\mathcal{E}}{R} - \frac{q}{RC}$$

But: $i = dq/dt$

So: $$\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} = -\frac{1}{RC}(q - C\mathcal{E})$$

R-C Circuits

We can rearrange this to

$$\frac{dq}{q - C\mathcal{E}} = -\frac{dt}{RC}$$

and then integrate both sides. We change the integration variables to q' and t' so that we can use q and t for the upper limits. The lower limits are $q' = 0$ and $t' = 0$:

$$\int_0^q \frac{dq'}{q' - C\mathcal{E}} = -\int_0^t \frac{dt'}{RC}$$

When we carry out the integration, we get

$$\ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

R-C Circuits

Exponentiating both sides (that is, taking the inverse logarithm) and solving for q, we find

$$\frac{q - C\mathcal{E}}{-C\mathcal{E}} = e^{-t/RC}$$

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right) \quad \text{(}R\text{-}C\text{ circuit, charging capacitor)} \tag{26.12}$$

The instantaneous current i is just the time derivative of Eq. (26.12):

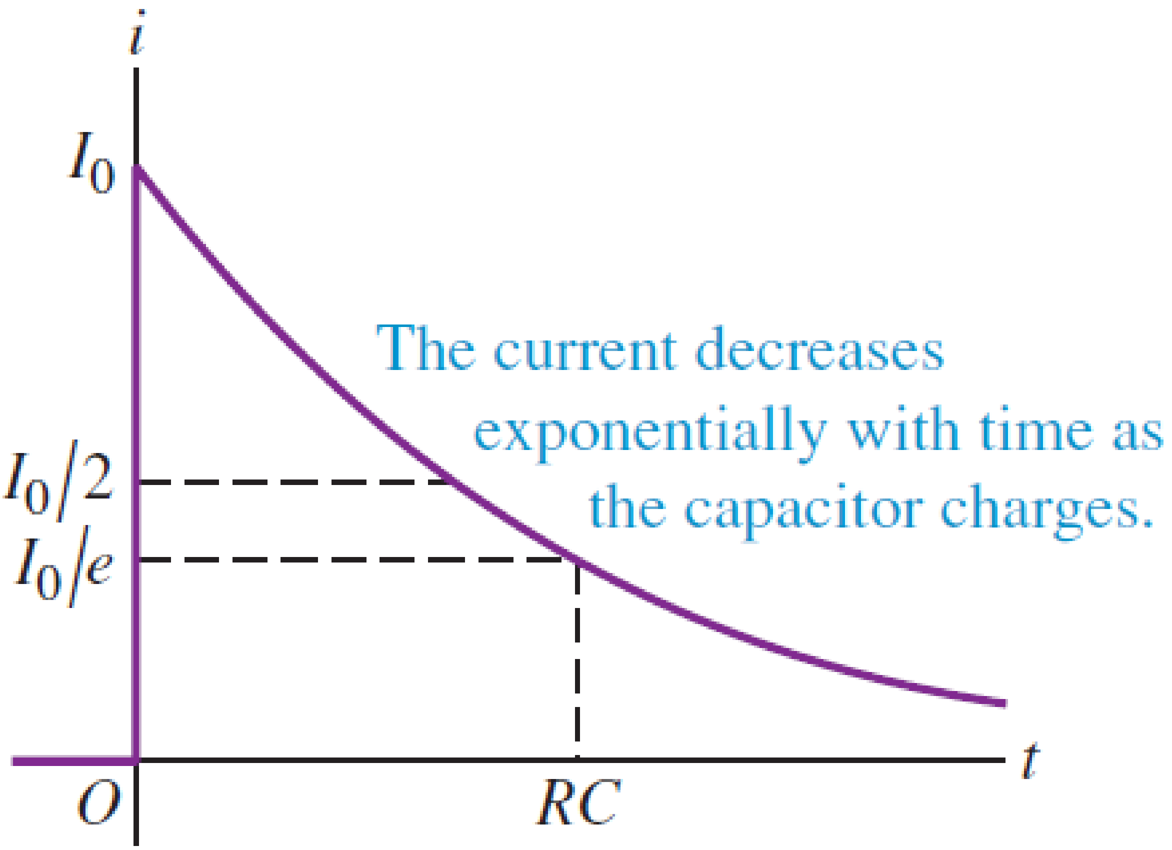
$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC} \quad \text{(}R\text{-}C\text{ circuit, charging capacitor)} \tag{26.13}$$

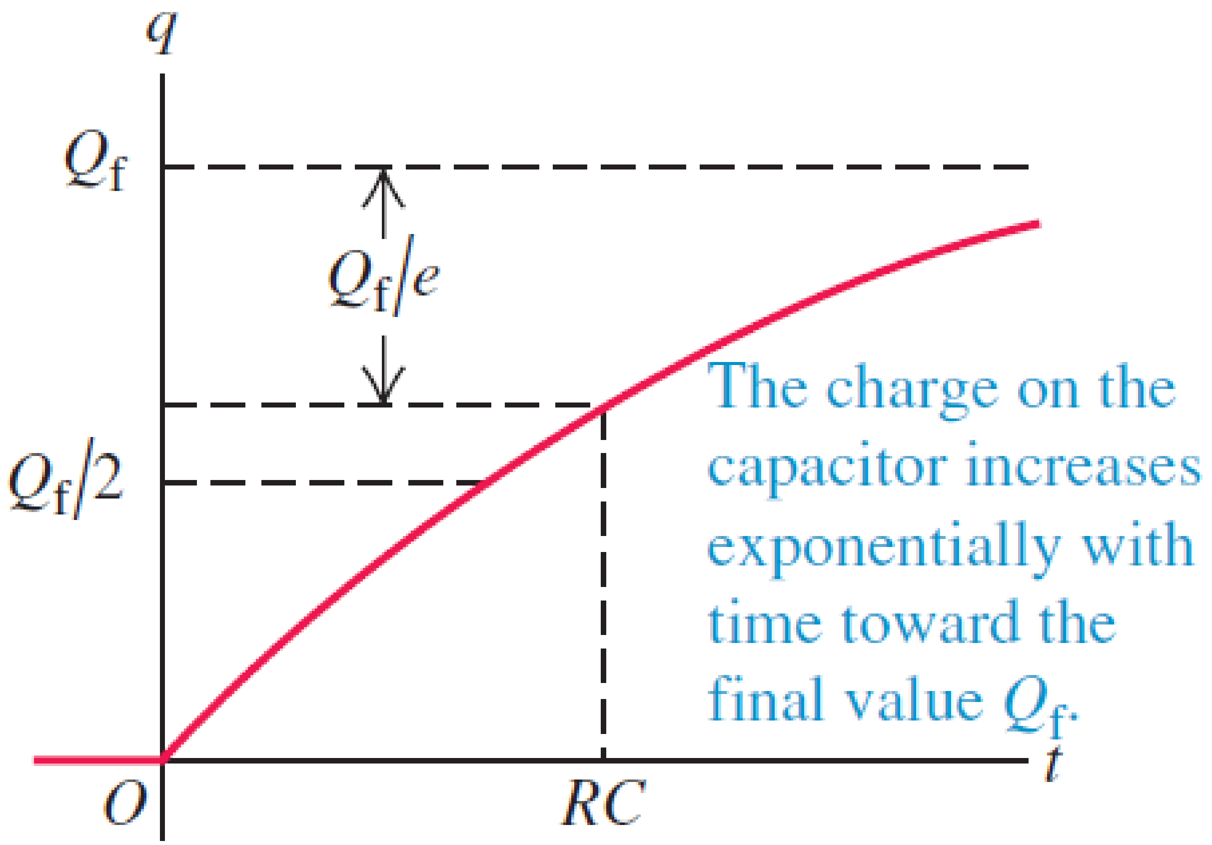
The charge and current are both *exponential* functions of time.

R-C Circuits

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC} = I_0 e^{-t/RC}$$

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right) = Q_f\left(1 - e^{-t/RC}\right)$$



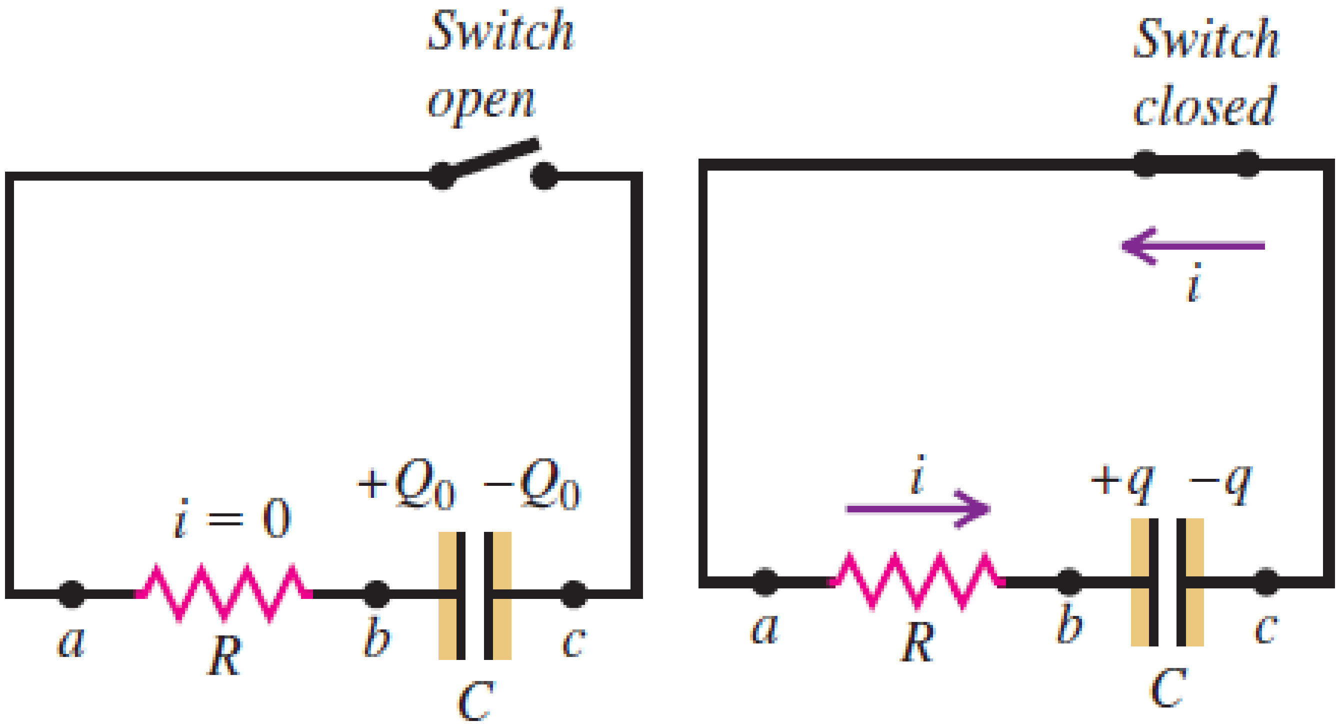


Time Constant in R-C Circuits

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC} = I_0 e^{-t/RC}$$

We call *RC* the **time constant,** or the **relaxation time,** of the circuit, denoted by $\tau = RC$ (time constant for *R*-*C* circuit)

Now suppose that after charging the has acquired a charge Q_0:



When the switch is closed, the charge on the capacitor and the current both decrease over time.

$$i = \frac{dq}{dt} = -\frac{q}{RC}$$

Time Constant in R-C Circuits

To find q as a function of time, we rearrange Eq. (26.15), again change the names of the variables to q' and t', and integrate. This time the limits for q' are Q_0 to q. We get

$$\int_{Q_0}^{q} \frac{dq'}{q'} = -\frac{1}{RC} \int_0^t dt'$$

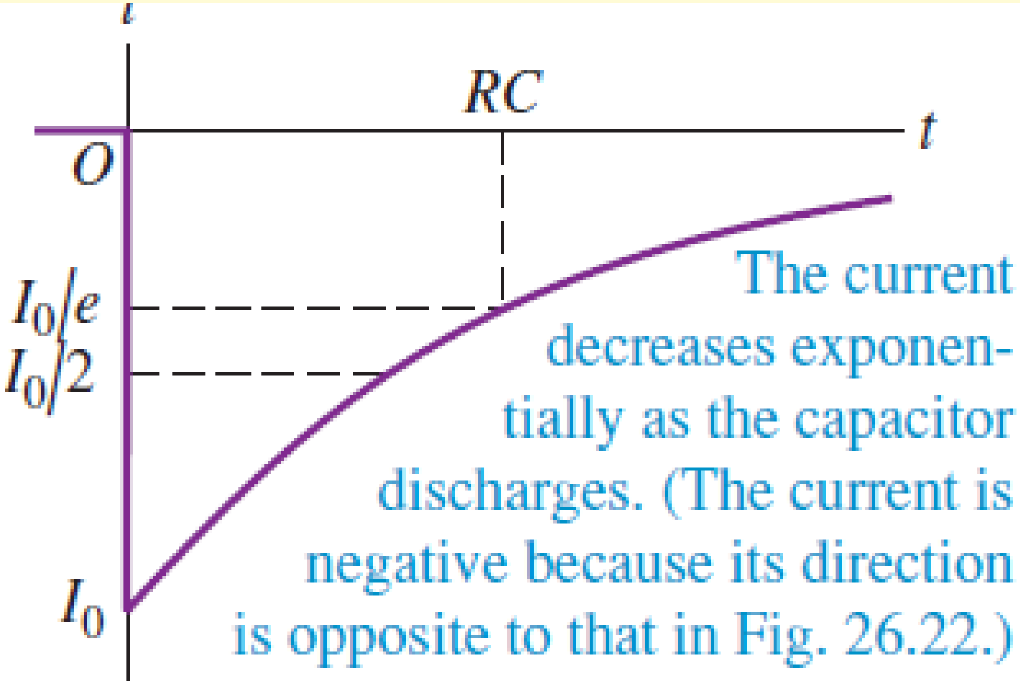
$$\ln \frac{q}{Q_0} = -\frac{t}{RC}$$

$$q = Q_0 e^{-t/RC} \qquad (R\text{-}C \text{ circuit, discharging capacitor}) \qquad (26.16)$$

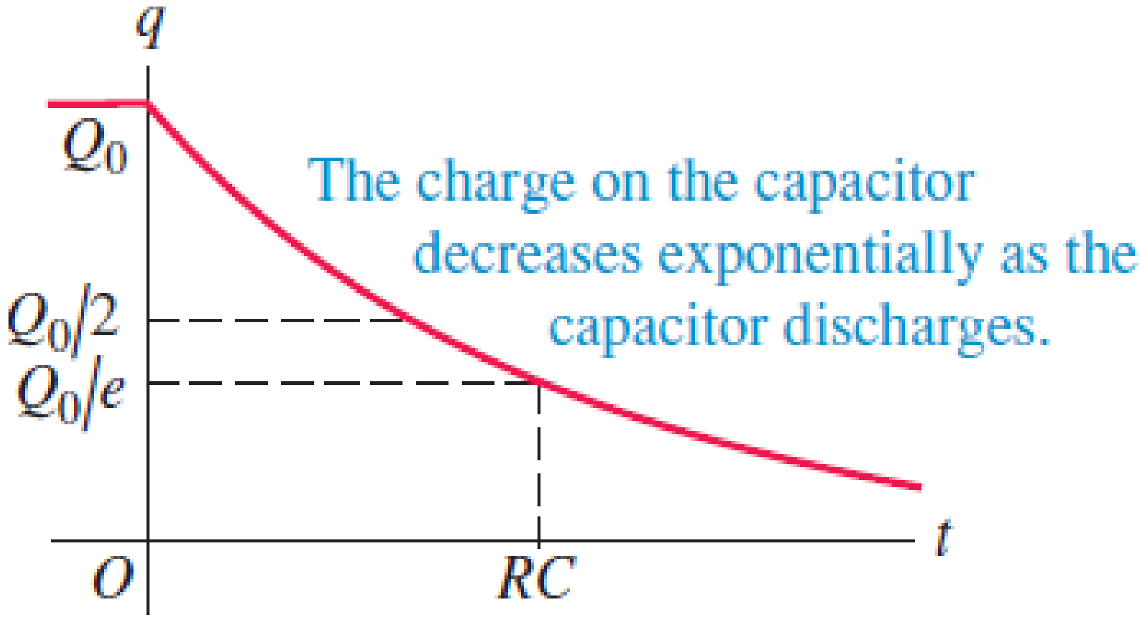
$$i = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}$$

Time Constant in R-C Circuits

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-t/RC} = I_0 e^{-t/RC}$$



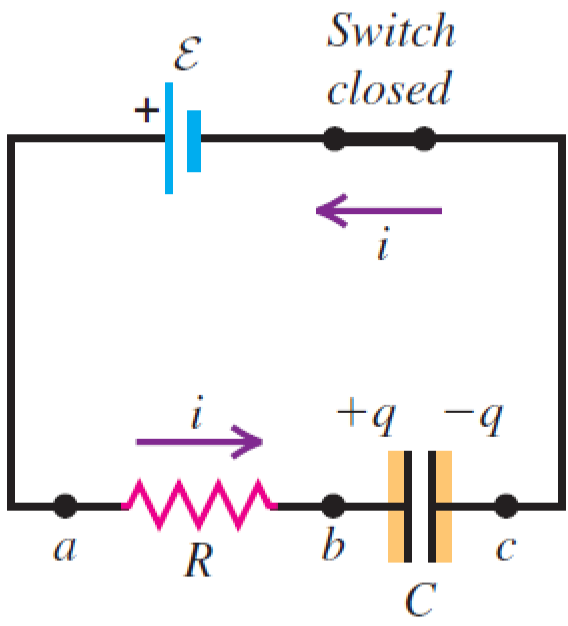
$$q = Q_0 e^{-t/RC}$$



Example 26.12 Charging a capacitor

A 10-MΩ resistor is connected in series with a 1.0-μF capacitor and a battery with emf 12.0 V. Before the switch is closed at time $t = 0$, the capacitor is uncharged. (a) What is the time constant? (b) What fraction of the final charge Q_f is on the capacitor at $t = 46$ s? (c) What fraction of the initial current I_0 is still flowing at $t = 46$ s?

IDENTIFY and SET UP: This is the same situation as shown in Fig. 26.20, with $R = 10$ MΩ, $C = 1.0$ μF, and $\mathcal{E} = 12.0$ V. The charge q and current i vary with time as shown in Fig. 26.21. Our target variables are (a) the time constant τ, (b) the ratio q/Q_f at $t = 46$ s, and (c) the ratio i/I_0 at $t = 46$ s. Equation (26.14) gives τ.



Example 26.12 Charging a capacitor

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(a) From Eq. (26.14),

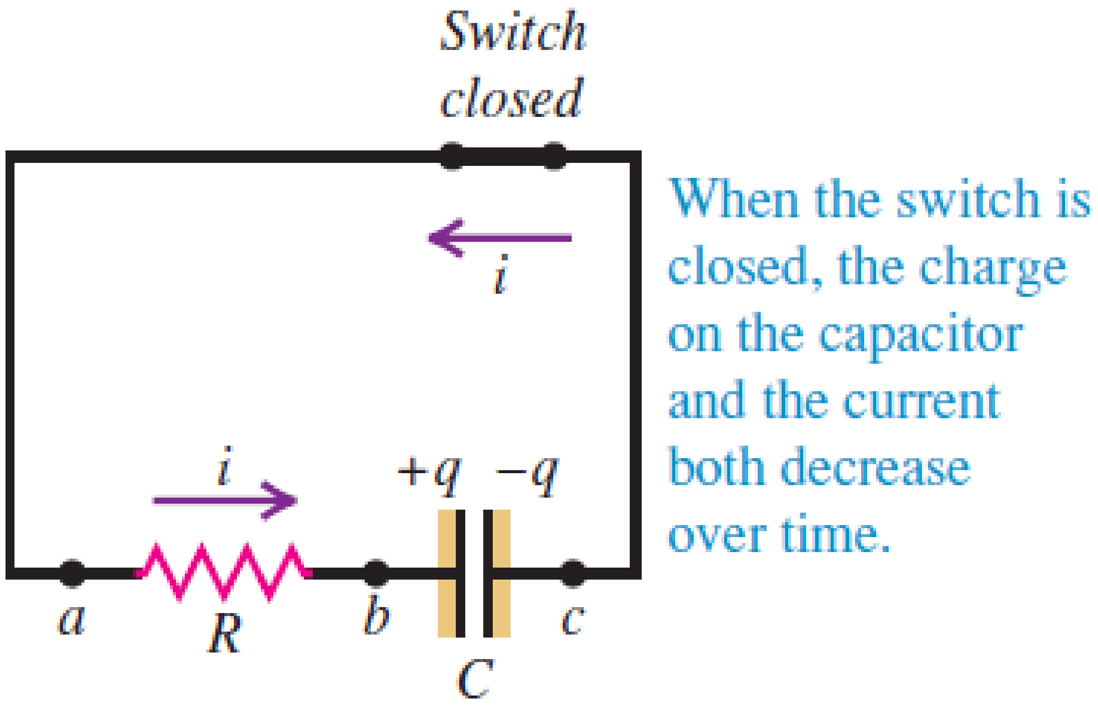
$$\tau = RC = (10 \times 10^6\ \Omega)(1.0 \times 10^{-6}\ \text{F}) = 10\ \text{s}$$

$$\frac{q}{Q_f} = 1 - e^{-t/RC} = 1 - e^{-(46\ \text{s})/(10\ \text{s})} = 0.99$$

$$\frac{i}{I_0} = e^{-t/RC} = e^{-(46\ \text{s})/(10\ \text{s})} = 0.010$$

Example 26.13 Discharging a capacitor

The resistor and capacitor of Example 26.12 are reconnected as shown in Fig. 26.22. The capacitor has an initial charge of 5.0 μC and is discharged by closing the switch at $t = 0$. (a) At what time will the charge be equal to 0.50 μC? (b) What is the current at this time?



Example 26.13 Discharging a capacitor

EXECUTE: (a) Solving Eq. (26.16) for the time t gives

$$t = -RC \ln \frac{q}{Q_0} = -(10\ \mathrm{s}) \ln \frac{0.50\ \mu\mathrm{C}}{5.0\ \mu\mathrm{C}} = 23\ \mathrm{s} = 2.3\tau$$

(b) From Eq. (26.17), with $Q_0 = 5.0\ \mu\mathrm{C} = 5.0 \times 10^{-6}\ \mathrm{C}$,

$$i = -\frac{Q_0}{RC} e^{-t/RC} = -\frac{5.0 \times 10^{-6}\ \mathrm{C}}{10\ \mathrm{s}} e^{-2.3} = -5.0 \times 10^{-8}\ \mathrm{A}$$

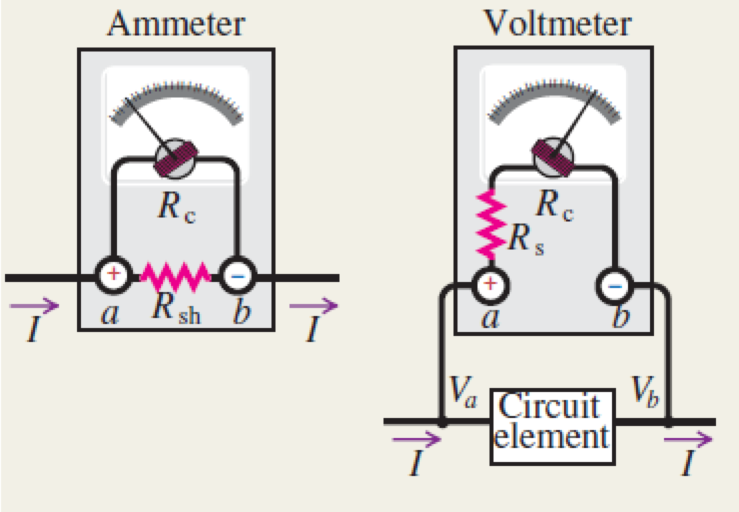
Summary

$$R_{eq} = R_1 + R_2 + R_3 + \cdots \qquad (26.1)$$

(resistors in series)

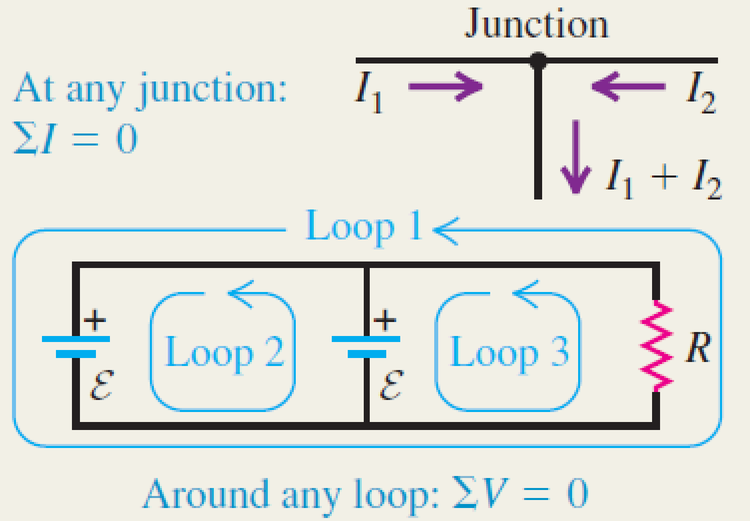
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \qquad (26.2)$$

(resistors in parallel)



$$\sum I = 0 \qquad \text{(junction rule)} \qquad (26.5)$$

$$\sum V = 0 \qquad \text{(loop rule)} \qquad (26.6)$$



Capacitor charging:

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

$$= Q_f\left(1 - e^{-t/RC}\right)$$

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC}$$

$$= I_0 e^{-t/RC}$$

Capacitor discharging:

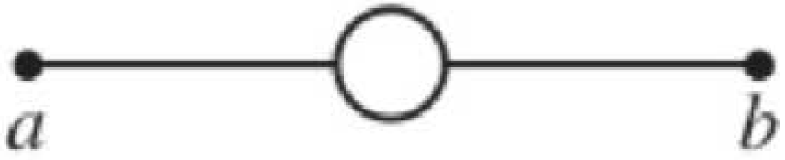
$$q = Q_0 e^{-t/RC}$$

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-t/RC}$$

$$= I_0 e^{-t/RC}$$

26.1 •• A uniform wire of resistance R is cut into three equal lengths. One of these is formed into a circle and connected between the other two (Fig. E26.1). What is the resistance between the opposite ends a and b?

Figure **E26.1**

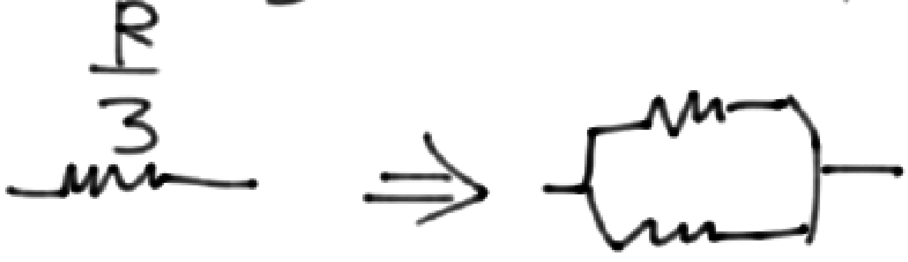


How do we get started? Think about connection in series vs parallel.

R equivalent

$\frac{R}{3} + \frac{R}{3} + \frac{R}{3}$ in series

Turning one of the $\frac{R}{3}$ into a ring means



two $\frac{R}{6}$ resistors in parallel

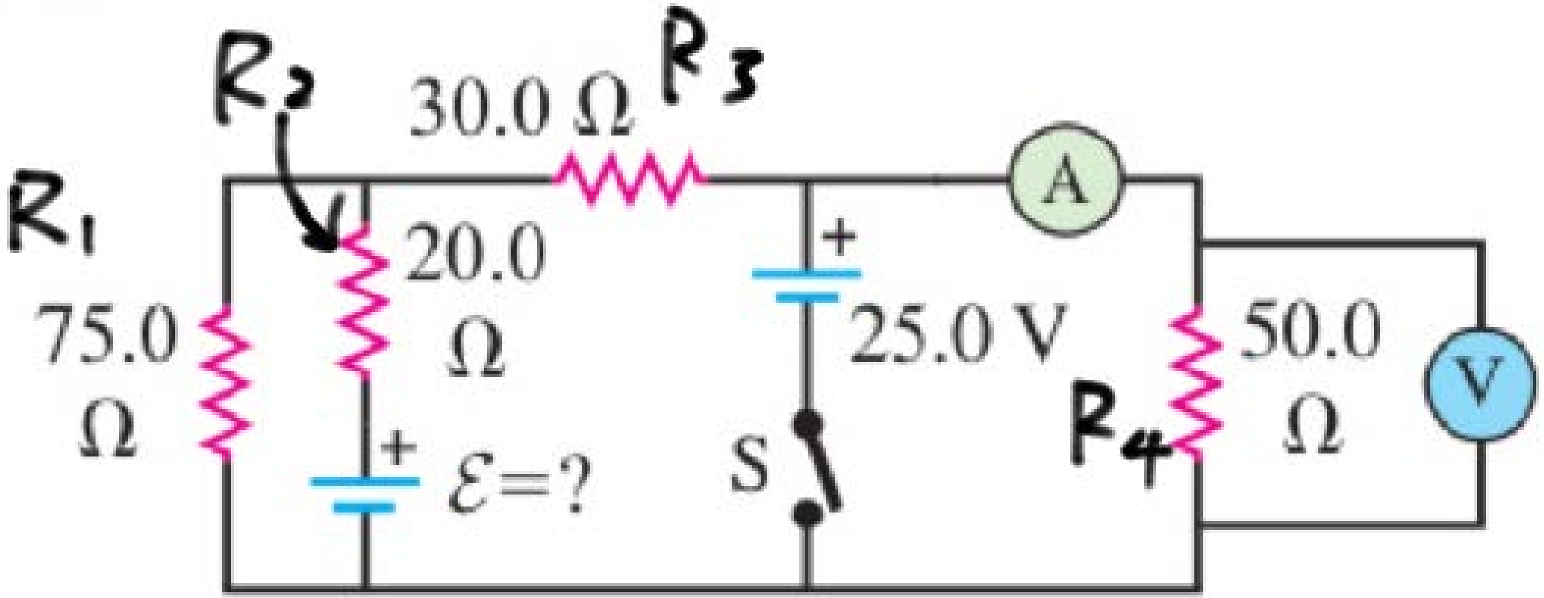
half length $\frac{R}{6}$

$$R_{ring} = \left(\left(\frac{R}{6}\right)^{-1}+\left(\frac{R}{6}\right)^{-1}\right)^{-1} = \frac{R}{12}$$

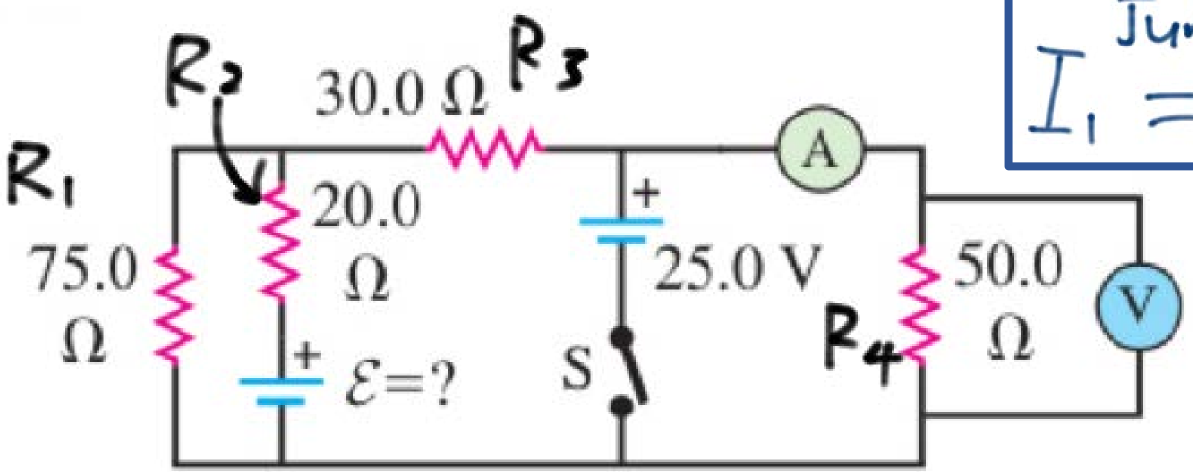
So the total resistance is

$$\frac{R}{3}+\frac{R}{3}+\frac{R}{12} = \frac{9}{12}R = \frac{3}{4}R$$

26.31 •• In the circuit shown in Fig. E26.31 the batteries have negligible internal resistance and the meters are both idealized. With the switch S open, the voltmeter reads 15.0 V. (a) Find the emf $\mathcal{E}$ of the battery. (b) What will the ammeter read when the switch is closed?



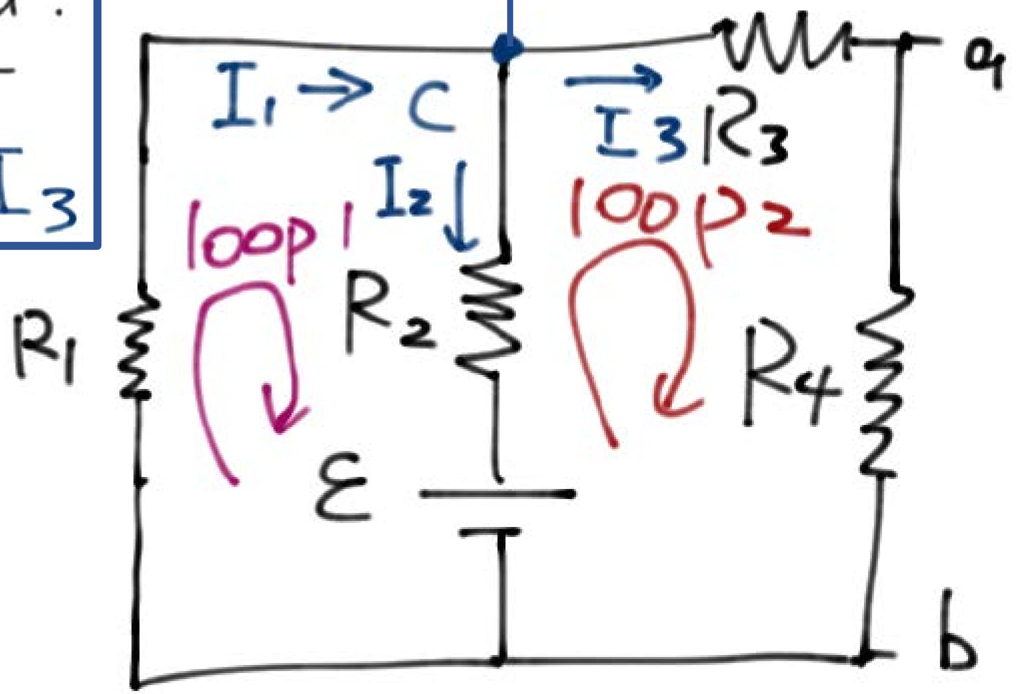
26.31 •• In the circuit shown in Fig. E26.31 the batteries have negligible internal resistance and the meters are both idealized. With the switch S open, the voltmeter reads 15.0 V. (a) Find the emf $\mathcal{E}$ of the battery. (b) What will the ammeter read when the switch is closed?

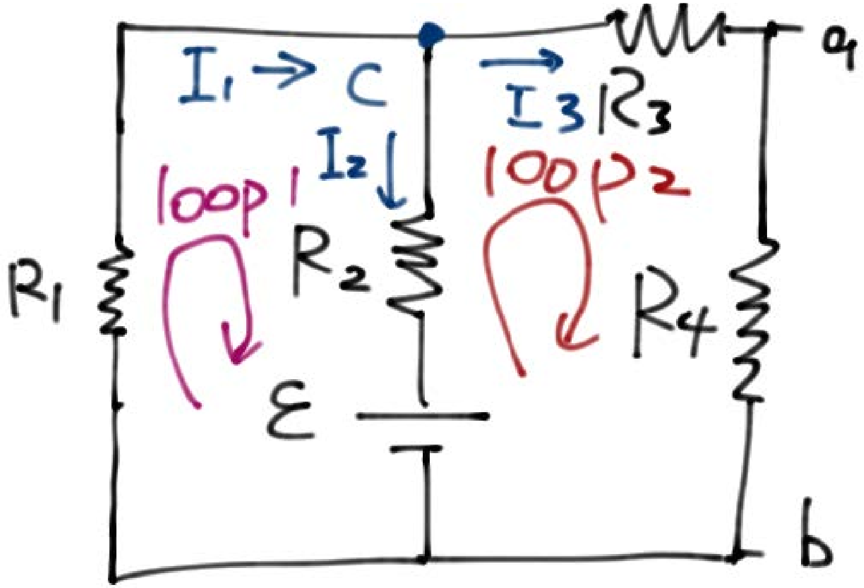


Simplified.

Junction c

$I_1 = I_2 + I_3$





Loop 1 : $-I_1R_1 - I_2R_2 - \mathcal{E} = 0$ ②

Loop 2 : $\mathcal{E} + I_2R_2 - I_3R_3 - I_3R_4 = 0$ ③

Unknowns : I_1 , I_2 , I_3 , $\mathcal{E}$

But we have only 3 equations

Simplified.

Junction c

$$I_1 = I_2 + I_3$$

4th equation : $V_4 = I_3 R_4 \Rightarrow I_3 = \frac{V_4}{R_4} = 0.3A$

$I_1 = I_2 + 0.3A$ ①

② + ③ gives (eliminates $\mathcal{E}$ and I_2)

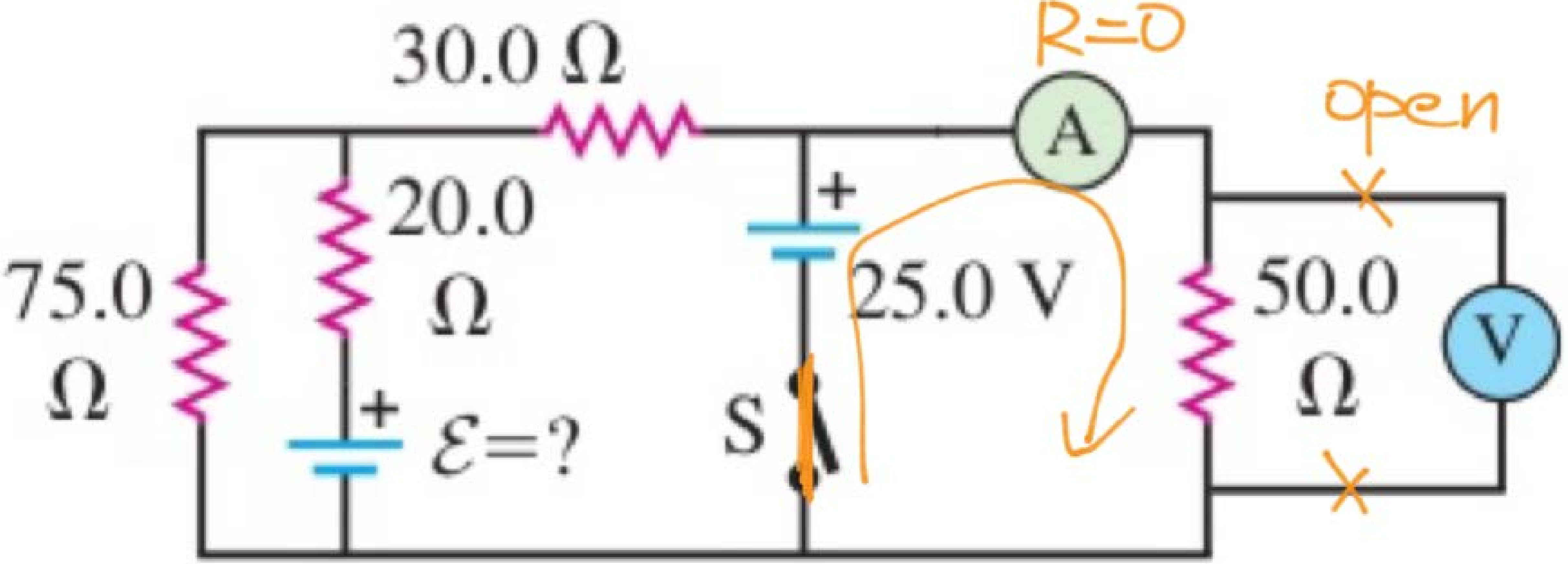
From loop

$$V_4 = V_{ab} = 15V$$

$$\mathcal{E} = -I_1R_1 - I_2R_2 = 0.32A \cdot 75\Omega + 0.62A \cdot 20$$

$$= 36.4 V$$

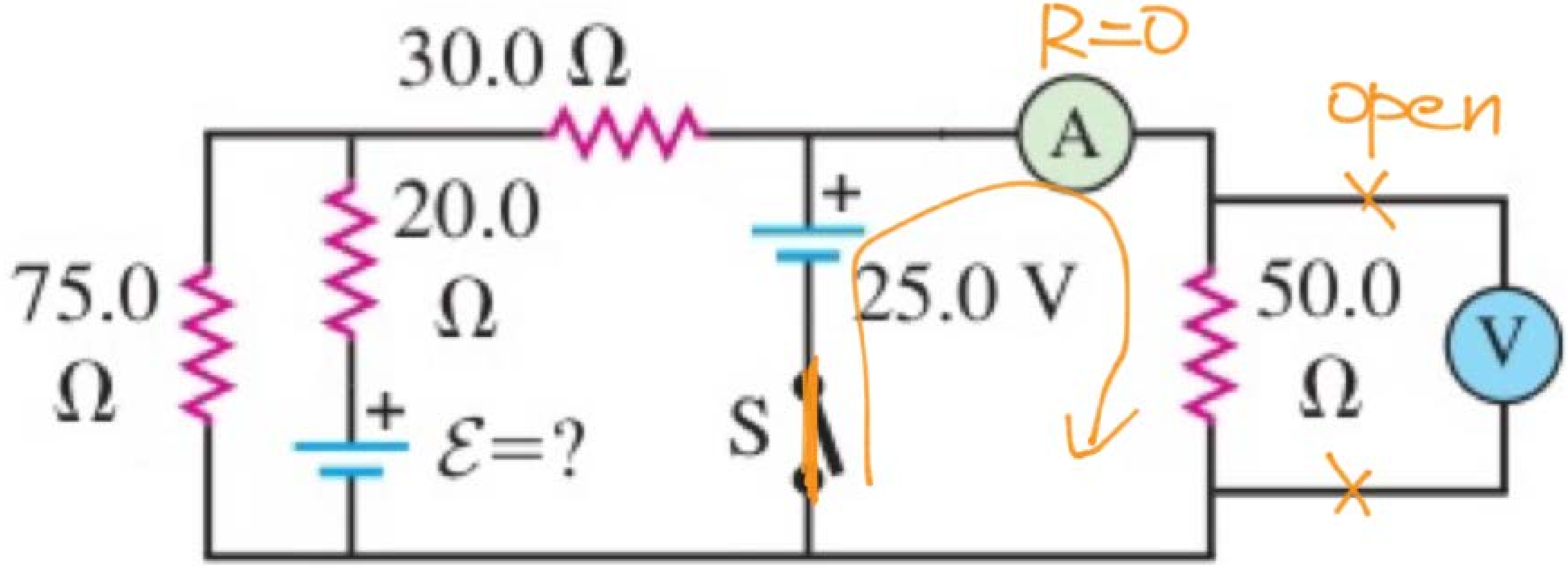
(b) Here is why the loop is convenient



SO: $\mathcal{E}_2 - I_3' \cdot R_4 = 0$

emf $\mathcal{E}$ of the battery. (b) What will the ammeter read when the switch is closed?

(b) Here is why the loop is convenient



so: $\mathcal{E}_2 - I_3' \cdot R_4 = 0$

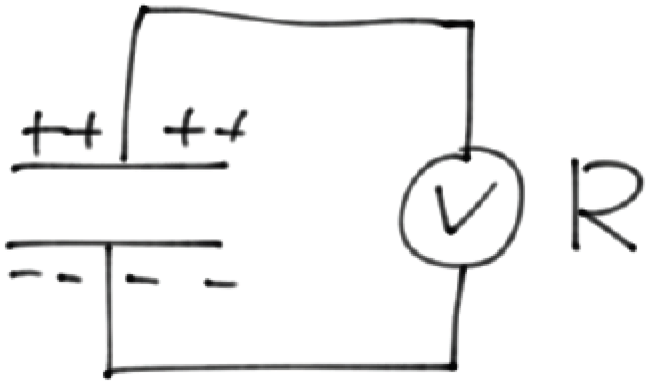
so: $\varepsilon_2 - I_3' \cdot R_4 = 0$

$$I_3' = \frac{\varepsilon_2}{R_4} = \frac{25\,V}{50\,\Omega} = 0.5\,A$$

No need to look at the other parts of the circuit. Isn't it convenient?

26.41 • A capacitor is charged to a potential of 12.0 V and is then connected to a voltmeter having an internal resistance of 3.40 MΩ. After a time of 4.00 s the voltmeter reads 3.0 V. What are (a) the capacitance and (b) the time constant of the circuit?

$3.4\ M\Omega = 3.4\times10^{6}\,\Omega.$



Recall $i = I_0 e^{-\frac{t}{RC}}$

at $t_1 = 4.0\,s$ $V_1 = 3.0\,V$

$$I_1 = I_0 e^{-\frac{t_1}{RC}} = \frac{V_1}{R} \quad (1)$$

I_0 is known $\quad I_0 = \frac{V_0}{R} = \frac{12.0\,V}{3.4\times10^6\,\Omega}$

$= 3.5\times10^{-6}\,A$

Rearrange ① $\quad e^{-\frac{t_1}{RC}} = \frac{V_1}{I_0 R}$

Take ln $\quad -\frac{t_1}{RC} = \ln\frac{V_1}{I_0 R}$

so $C = -\dfrac{t_1}{R\ln\frac{V_1}{I_0 R}}$

$$\text{so } C = -\frac{t_1}{R\ln\frac{V_1}{I_0R}} \qquad \ln\frac{V_1}{I_0R} = \ln\frac{V_1}{V_0}$$

$$= \frac{t_1}{R\ln\frac{V_0}{V_1}} = \frac{4.0\,s}{3.4\times10^6\,\Omega\cdot\ln 4} = 8.5\times10^{-7}F$$

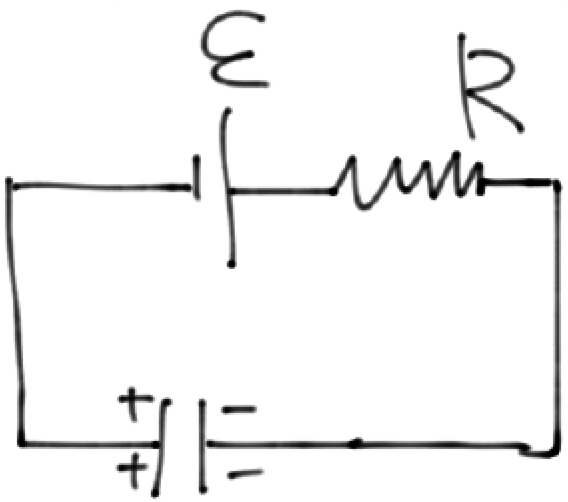
(b)

$$\tau = RC = 8.5\times10^{-7}F\cdot 3.4\times10^6\,\Omega$$

$$= 2.9\ s$$

26.45 • An emf source with $\mathcal{E} = 120$ V, a resistor with $R = 80.0\ \Omega$, and a capacitor with $C = 4.00\ \mu$F are connected in series. As the capacitor charges, when the current in the resistor is 0.900 A, what is the magnitude of the charge on each plate of the capacitor?

$$q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) \quad ①$$



Only t is unknown here

$$i = I_0 \cdot e^{-\frac{t}{RC}} \longrightarrow \text{rearrange} \quad e^{-\frac{t}{RC}} = \frac{i}{I_0}$$

i: 0.9A

I_0: $\frac{\varepsilon}{R} = 1.5\text{A}$

$$= \frac{Ri}{\varepsilon}$$

26.45 • An emf source with $\mathcal{E} = 120$ V, a resistor with $R = 80.0\ \Omega$, and a capacitor with $C = 4.00\ \mu\text{F}$ are connected in series. As the capacitor charges, when the current in the resistor is 0.900 A, what is the magnitude of the charge on each plate of the capacitor?

Plug $e^{-\frac{t}{RC}}$ into ①

$$q = C\mathcal{E}\left(1 - \frac{Ri}{\mathcal{E}}\right)$$

$$= 4.0\times10^{-6}\text{F} \cdot 120\text{V}\left(1 - \frac{80\Omega \cdot 0.9\text{A}}{120\text{V}}\right)$$

$$= 1.92\times10^{-4}\ \text{C}$$