Problem Set 2 (Due 3/18/2025 before class)

Late homework will NOT be accepted, unless you have notified course instructor 3 days BEFORE deadline.

Part I (60%)

3.61 •• A projectile is being launched from ground level with no air resistance. You want to avoid having it enter a temperature inversion layer in the atmosphere a height h above the ground. (a) What is the maximum launch speed you could give this projectile if you shot it straight up? Express your answer in terms of h and g. (b) Suppose the launcher available shoots projectiles at twice the maximum launch speed you found in part (a). At what maximum angle above the horizontal should you launch the projectile? (c) How far (in terms of h) from the launcher does the projectile in part (b) land?

IDENTIFY: The equations for h and R from Example 3.8 can be used.

SET UP: $h = \dfrac{v_0^2 \sin^2\alpha_0}{2g}$ and $R = \dfrac{v_0^2 \sin 2\alpha_0}{g}$. If the projectile is launched straight up, $\alpha_0 = 90°$.

EXECUTE: **(a)** $h = \dfrac{v_0^2}{2g}$ and $v_0 = \sqrt{2gh}$.

(b) Calculate α_0 that gives a maximum height of h when $v_0 = 2\sqrt{2gh}$. $h = \dfrac{8gh\sin^2\alpha_0}{2g} = 4h\sin^2\alpha_0$.

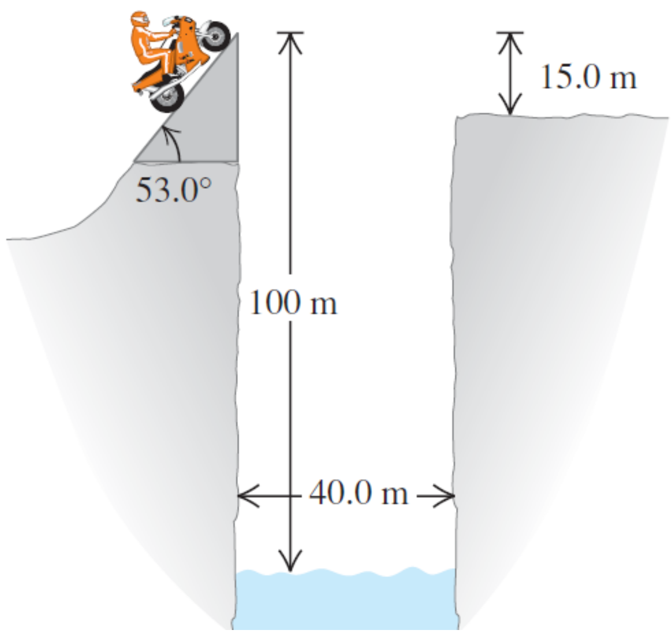
$\sin\alpha_0 = \frac{1}{2}$ and $\alpha_0 = 30.0°$.

(c) $R = \dfrac{(2\sqrt{2gh})^2 \sin 60.0°}{g} = 6.93h$.

EVALUATE: $\dfrac{v_0^2}{g} = \dfrac{2h}{\sin^2\alpha_0}$ so $R = \dfrac{2h\sin(2\alpha_0)}{\sin^2\alpha_0}$. For a given α_0, R increases when h increases. For $\alpha_0 = 90°$, $R = 0$ and for $\alpha_0 = 0°$, $h = 0$ and $R = 0$. For $\alpha_0 = 45°$, $R = 4h$.

3.67 •• **Leaping the River II.** A physics professor did daredevil stunts in his spare time. His last stunt was an attempt to jump across a river on a motorcycle (Fig. P3.67). The takeoff ramp was inclined at 53.0°, the river was 40.0 m wide, and the far bank was 15.0 m lower than the top of the ramp. The river itself was 100 m below the ramp. You can ignore air resistance. (a) What should his speed have been at the top of the ramp to have just made it to the edge of the far bank? (b) If his speed was only half the value found in part (a), where did he land?

Figure **P3.67**



(a) IDENTIFY: Projectile motion.

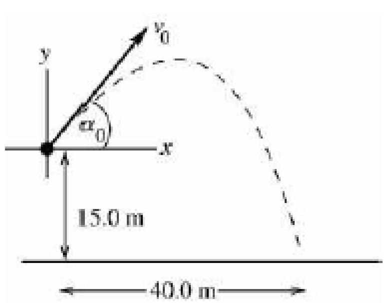


Figure 3.67

Take the origin of coordinates at the top of the ramp and take $+y$ to be upward.

The problem specifies that the object is displaced 40.0 m to the right when it is 15.0 m below the origin.

We don't know t, the time in the air, and we don't know v_0. Write down the equations for the horizontal and vertical displacements. Combine these two equations to eliminate one unknown.

Set Up: y-component:

$y - y_0 = -15.0\text{ m},\ \ a_y = -9.80\text{ m/s}^2,\ \ v_{0y} = v_0 \sin 53.0°$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

Execute: $-15.0\text{ m} = (v_0 \sin 53.0°)\,t - (4.90\text{ m/s}^2)\,t^2$

Set Up: x-component:

$x - x_0 = 40.0\text{ m},\ \ a_x = 0,\ \ v_{0x} = v_0 \cos 53.0°$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

Execute: $40.0\text{ m} = (v_0 t)\cos 53.0°$

The second equation says $v_0 t = \dfrac{40.0\text{ m}}{\cos 53.0°} = 66.47\text{ m}.$

Use this to replace $v_0 t$ in the first equation:

$-15.0\text{ m} = (66.47\text{ m})\sin 53° - (4.90\text{ m/s}^2)\,t^2$

$$t = \sqrt{\frac{(66.46\text{ m})\sin 53° + 15.0\text{ m}}{4.90\text{ m/s}^2}} = \sqrt{\frac{68.08\text{ m}}{4.90\text{ m/s}^2}} = 3.727\text{ s}.$$

Now that we have t we can use the x-component equation to solve for v_0:

$$v_0 = \frac{40.0\text{ m}}{t\cos 53.0°} = \frac{40.0\text{ m}}{(3.727\text{ s})\cos 53.0°} = 17.8\text{ m/s}.$$

Evaluate: Using these values of v_0 and t in the $y = y_0 = v_{0y} + \frac{1}{2}a_y t^2$ equation verifies that $y - y_0 = -15.0\text{ m}.$

(b) Identify: $v_0 = (17.8\text{ m/s})/2 = 8.9\text{ m/s}$

This is less than the speed required to make it to the other side, so he lands in the river.
Use the vertical motion to find the time it takes him to reach the water:

Set Up: $y - y_0 = -100\text{ m};\ \ v_{0y} = +v_0 \sin 53.0° = 7.11\text{ m/s};\ \ a_y = -9.80\text{ m/s}^2$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-100 = 7.11t - 4.90t^2$

Execute: $4.90t^2 - 7.11t - 100 = 0$ and $t = \frac{1}{9.80}\left(7.11 \pm \sqrt{(7.11)^2 - 4\,(4.90)(-100)}\right)$

$t = 0.726\text{ s} \pm 4.57\text{ s}$ so $t = 5.30\text{ s}.$

The horizontal distance he travels in this time is

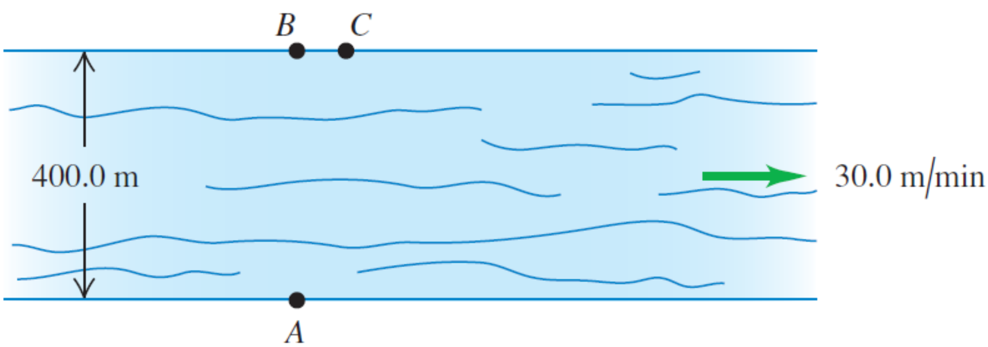
$$x - x_0 = v_{0x}t = (v_0 \cos 53.0°)\,t = (5.36\text{ m/s})(5.30\text{ s}) = 28.4\text{ m}.$$

He lands in the river a horizontal distance of 28.4 m from his launch point.

Evaluate: He has half the minimum speed and makes it only about halfway across.

3.78 • A 400.0-m-wide river flows from west to east at 30.0 m/min. Your boat moves at 100.0 m/min relative to the water no matter which direction you point it. To cross this river, you start from a dock at point *A* on the south bank. There is a boat landing directly opposite at point *B* on the north bank, and also one at point *C*, 75.0 m downstream from *B* (Fig. P3.78). (a) Where on the north shore will you land if you point your boat perpendicular to the water current, and what distance will you have traveled? (b) If you initially aim your boat directly toward point *C* and do not change that bearing relative to the shore, where on the north shore will you

Figure **P3.78**



IDENTIFY: All velocities are constant, so the distance traveled is $d = v_{B/E}t$, where $v_{B/E}$ is the magnitude of the velocity of the boat relative to the earth. The relative velocities $\vec{v}_{B/E}$, $\vec{v}_{B/W}$ (boat relative to the water) and $\vec{v}_{W/E}$ (water relative to the earth) are related by $\vec{v}_{B/E} = \vec{v}_{B/W} + \vec{v}_{W/E}$.

SET UP: Let $+x$ be east and let $+y$ be north. $v_{W/E-x} = +30.0$ m/min and $v_{W/E-y} = 0$. $v_{B/W} = 100.0$ m/min. The direction of $\vec{v}_{B/W}$ is the direction in which the boat is pointed or aimed.

EXECUTE: **(a)** $v_{B/W-y} = +100.0$ m/min and $v_{B/W-x} = 0$. $v_{B/E-x} = v_{B/W-x} + v_{W/E-x} = 30.0$ m/min and $v_{B/E-y} = v_{B/W-y} + v_{W/E-y} = 100.0$ m/min. The time to cross the river is

$t = \dfrac{y - y_0}{v_{B/E-y}} = \dfrac{400.0 \text{ m}}{100.0 \text{ m/min}} = 4.00$ min. $x - x_0 = (30.0 \text{ m/min})(4.00 \text{ min}) = 120.0$ m. You will land 120.0 m east of point *B*, which is 45.0 m east of point *C*. The distance you will have traveled is $\sqrt{(400.0 \text{ m})^2 + (120.0 \text{ m})^2} = 418$ m.

(b) $\vec{v}_{B/W}$ is directed at angle ϕ east of north, where $\tan\phi = \dfrac{75.0\text{ m}}{400.0\text{ m}}$ and $\phi = 10.6°$.

$v_{B/W-x} = (100.0\text{ m/min})\sin 10.6° = 18.4\text{ m/min}$ and $v_{B/W-y} = (100.0\text{ m/min})\cos 10.6° = 98.3\text{ m/min}$.

$v_{B/E-x} = v_{B/W-x} + v_{W/E-x} = 18.4\text{ m/min} + 30.0\text{ m/min} = 48.4\text{ m/min}$.

$v_{B/E-y} = v_{B/W-y} + v_{W/E-y} = 98.3\text{ m/min}$. $t = \dfrac{y - y_0}{v_{B/E-y}} = \dfrac{400.0\text{ m}}{98.3\text{ m/min}} = 4.07\text{ min}$.

$x - x_0 = (48.4\text{ m/min})(4.07\text{ min}) = 197\text{ m}$. You will land 197 m downstream from *B*, so 122 m downstream from *C*.

(c) (i) If you reach point *C*, then $\vec{v}_{B/E}$ is directed at $10.6°$ east of north, which is $79.4°$ north of east. We don't know the magnitude of $\vec{v}_{B/E}$ and the direction of $\vec{v}_{B/W}$. In part (a) we found that if we aim the boat due north we will land east of *C*, so to land at *C* we must aim the boat west of north. Let $\vec{v}_{B/W}$ be at an angle ϕ of north of west. The relative velocity addition diagram is sketched in Figure 3.78. The law of sines says $\dfrac{\sin\theta}{v_{W/E}} = \dfrac{\sin 79.4°}{v_{B/W}}$. $\sin\theta = \left(\dfrac{30.0\text{ m/min}}{100.0\text{ m/min}}\right)\sin 79.4°$ and $\theta = 17.15°$. Then

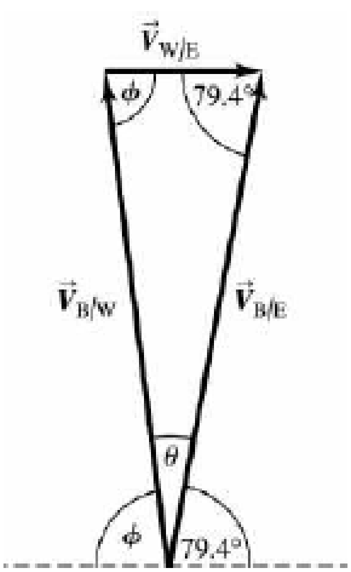
$\phi = 180° - 79.4° - 17.15° = 83.5°$. The boat will head $83.5°$ north of west, so $6.5°$ west of north.

$v_{B/E-x} = v_{B/W-x} + v_{W/E-x} = -(100.0\text{ m/min})\cos 83.5° + 30.0\text{ m/min} = 18.7\text{ m/min}$.

$v_{B/E-y} = v_{B/W-y} + v_{W/E-y} = (100.0\text{ m/min})\sin 83.5° = 99.4\text{ m/min}$. Note that these two components do give the direction of $\vec{v}_{B/E}$ to be $79.4°$ north of east, as required. (ii) The time to cross the river is

$t = \dfrac{y - y_0}{v_{B/E-y}} = \dfrac{400.0\text{ m}}{99.4\text{ m/min}} = 4.02\text{ min}$. (iii) You travel from *A* to *C*, a distance of

$\sqrt{(400.0\text{ m})^2 + (75.0\text{ m})^2} = 407\text{ m}$. (iv) $v_{B/E} = \sqrt{(v_{B/E-x})^2 + (v_{B/E-y})^2} = 101\text{ m/min}$. Note that $v_{B/E}t = 406\text{ m}$, the distance traveled (apart from a small difference due to rounding).



Part II (40%) Basic Problems

1. An object accelerates uniformly from rest at a rate of 1.9 m/s^2 west for 5.0 s. Find:

   (a) the displacement

   (b) the final velocity

   (c) the distance traveled

   (d) the final speed

Solution: The given data,

$$\text{Initial velocity, } v_0 = 0$$
$$\text{Acceleration, } a = 1.9\,\mathrm{m/s^2}$$
$$\text{Time interval, } t = 5\,\mathrm{s}$$

(a) Use the following equation,

$$x - x_0 = \frac{1}{2}at^2 + v_0 t$$

$$x - 0 = \frac{1}{2}(1.9)(5)^2 + 0(5)$$

$$x = \boxed{23.75\,\mathrm{m/s}}$$

In the second line, for convenience, we simply adopt the initial position (x_0) at time $t = 0$ as 0.

(b) the equation below gives the velocity at the end of time interval

$$\begin{aligned} v &= v_0 + at \\ v &= 0 + (1.9)(5) \\ &= +9.5\,\mathrm{m/s} \qquad \text{West} \end{aligned}$$

(c) In a straight-line motion, if the velocity and acceleration have the same signs, the speed of the moving object increases. Here, by establishing a coordinate system and choosing west as a positive direction, we can see the acceleration and velocity are in the same direction.

Therefore, the object moves west without any changing direction. In this type of motion, in which the object does not change its direction, the displacement (vector) and distance traveled are the same.

Thus, as calculated in part (a), the total distance is approximately $\boxed{24\,\mathrm{m}}$.

(d) As reasoning of (c), since the direction of the motion does not change so the magnitude of vector quantities shows the value of scalar ones. Here, the magnitude of the final velocity ($v = 9.5\,\mathrm{m/s}$) is equal to the final speed.

2. A 3.0 kg ball is thrown vertically into the air with an initial velocity of 15 m/s. What is the maximum height of the ball?

Solution: Place the origin of the coordinate system at the ball's thrown point so $y_0 = 0$. Apply the following kinematic equation to find the maximum height where the vertical velocity is zero, $v_y = 0$,

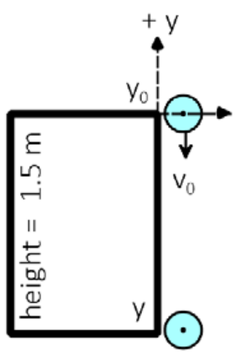
$$v_y^2 - v_{0y}^2 = 2(-g)(y - y_0)$$

$$0 - (15)^2 = 2(-9.8)(h_{\max} - 0)$$

$$\Rightarrow \boxed{h_{\max} = 11.47\,\mathrm{m}}$$

3. A window is 1.50 m high. A stone falling from above passes the top of the window with a speed of 3.00 m/s. When will it pass the bottom of the window? (Take the acceleration due to gravity to be 10 m/s^2.)

Solution: The stone is fallen from the upper edge of the window so place the origin of the coordinate system at this point ($y_0 = 0$). Since the vector of initial velocity is downward and the window's bottom edge is located $1.5\,\mathrm{m}$ below the origin, so we set $v_{0y} = -3\,\mathrm{m/s}, y = -1.5\,\mathrm{m}$ in kinematic equations.



$$\begin{aligned} y - y_0 &= \frac{1}{2}(-g)t^2 + v_{0y}t \\ (-1.5) - 0 &= \frac{1}{2}(-10)t^2 + (-3)t \\ -1.5 &= -5t^2 - 3t \end{aligned}$$

After rearranging the above equation, we arrive at $t^2 + 0.6t - 0.3 = 0$ whose solution is obtained as

$$t^2 + 0.6t - 0.3 = 0$$

$$t = \frac{-0.6 \pm \sqrt{(0.6)^2 - 4(1)(-0.3)}}{2(1)}$$

$$t_1 = 0.342\,\mathrm{s}$$

$$t_2 = -0.924\,\mathrm{s}$$

The above quadratic equation has two roots but the physical solution is the one with positive sign. The negative one indicates a time before we dropped the stone! Thus, we choose the positive solution i.e. $\boxed{t = 0.342\,\mathrm{s}}$.

4. A ball is thrown horizontally from the roof of a building 50−m-tall and lands 45 m from the base. What was the ball's initial speed?

Solution: This is a projectile motion problem with launch angle $\alpha = 0°$, so the projectile equations which are the x and y components of velocity and displacement vectors are written as below

$$x = v_{0x}t = v_0 \cos \alpha t$$

$$y = -\frac{1}{2}gt^2 + \underbrace{v_0 \sin \alpha}_{v_{0y}} t + y_0$$

$$v_x = v_0 \cos \alpha$$

$$v_y = \underbrace{v_0 \sin \alpha}_{v_{0y}} - gt$$

If we choose the releasing point as the reference then the coordinate of the point of impact is $(x = 45, y = -50 \text{ m})$. First, find the total flight time, then substitute it into the x component of the projectile.

$$y = -\frac{1}{2}gt_{tot}^2 + \underbrace{v_0 \sin \alpha}_{v_{0y}} t_{tot} + y_0$$

$$\Rightarrow \ -50 = -\frac{1}{2}(9.8)t_{tot}^2 + v_0 \sin 0° t_{tot} + 0$$

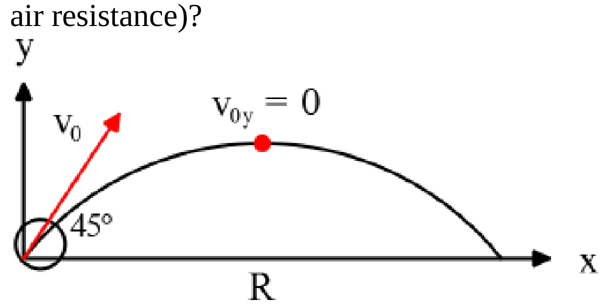
$$\Rightarrow t_{tot} = \sqrt{\frac{2 \times 50}{9.8}} = 3.19 \quad \text{s}$$

Therefore,

$$x = v_0 \cos \alpha t_{tot}$$

$$\Rightarrow v_0 = \frac{x}{\cos 0° \, t_{tot}} = \frac{45}{3.19} = 14.1 \quad \text{m/s}$$

5. A 1 kg projectile is fired from a cannon with an initial speed of $10\sqrt{2}$m/s. The cannon has an elevation angle of 45°. How far does the projectile go before striking the ground (neglect the air resistance)?



Solution: In the projectile language, the distance from the launching to the striking point is called the range of the projectile that is found by substituting the total flight time into the x component of the projectile motion that is $X = v_0 \cos\theta \; t$.

From the definition of kinetic energy, one can find the initial velocity of the projectile below

$$K = \frac{1}{2} m v_0^2 \to 10^4 = \frac{1}{2}(1)\, v_0^2$$

$$\Rightarrow \boxed{v_0 = \sqrt{2} \times 10^2 \text{ m/s}}$$

Consider the starting and landing points to be on the same level. In this case, using the kinematic equation $v_y = v_0 \sin\theta - gt$ and knowing the fact that at the highest point, the vertical component of the projectile's velocity is zero, i.e., $v_y = 0$, find half of the total flight time that is $t_{tot} = 2t$ (since there is no air resistance).

$$v_y = v_0 \sin\theta - gt = 0$$

$$\to t = \frac{v_0 \sin\theta}{g} = \frac{\left(\sqrt{2} \times 10^2\right) \sin 45^\circ}{9.8} = 10.2 \text{ s}$$

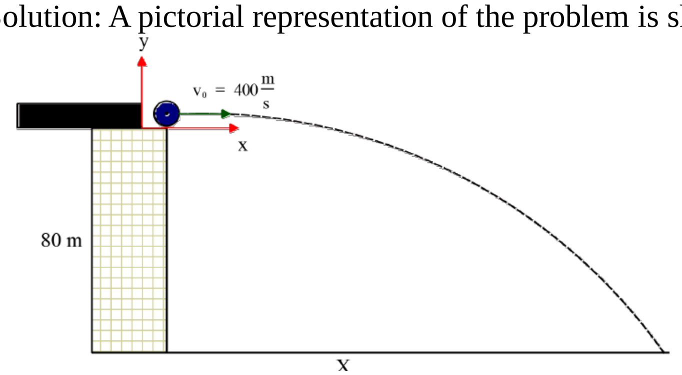
This is the elapsed time to the highest point.

$$\Rightarrow t_{tot} = 2t = 20.4 \text{ s}$$

Therefore,

$$\begin{aligned} Range = X &= v_0 t \cos\theta \\ &= \left(\sqrt{2} \times 10^2\right) \cos 45^\circ \;\; \times 20.4 \\ &= 2040 \text{ m} \end{aligned}$$

6. A bullet is fired horizontally from the top of a cliff which is 80m above a big lake. If the bullet muzzle (initial) speed is 400 m/s, how far from the bottom of the cliff does the bullet strike the surface of the lake? Neglect air resistance.
   Solution: A pictorial representation of the problem is shown in the figure below.



Put a coordinate at the starting point. Since the hitting point of the bullet is 80 m below the coordinate, so the coordinate of the landing point is ($x = ?, y = -80\,\mathrm{m}$). First, using the equation $y = -\frac{1}{2}gt^2 + v_{0y}t + y_0$ find the time required to reach the bullet to the ground.

$$y = -\frac{1}{2}gt^2$$

$$\rightarrow \quad t = \sqrt{\frac{2y}{-g}} = \sqrt{2 \times \frac{(-80)}{(-10)}} = 4\,\mathrm{s}$$

Now using the relation between uniform velocity and displacement, i.e., $x = vt$ we obtain

$$x = vt = (400)(4) = 1600\,\mathrm{m} \rightarrow \mathrm{x} = 1\mathrm{mile}$$

7. A rock is tossed at a 42° angle at an initial height of 1.2 m from the ground. 1.6 seconds after release, the rock reaches its maximum height. Find the initial velocity, the maximum height and the overall speed at maximum height.

   At max height $v_y = 0$

   $v_y = v_{oy} + gt$

   $0 = v_{oy} - 9.8(1.6)$

   $v_{oy} = 15.7$ m/s

   We also know that $v_{oy} = v_o \sin\theta_o$

   $15.7 = v_o \sin(42)$

   $v_o = 23.5$ m/s

   To find max height: $v_y = 0$

   $v_y{}^2 = v_{oy}{}^2 + 2g(\Delta y)$

   $0 = (15.7)^2 - 19.6(y_{max} - 1.2)$

   $y_{max} = 13.8$ m

   To find overall speed at max height: $v = v_x = v_{ox}$

   $v = v_o \cos\theta_o = 23.5\cos(42)$

   $v = 17.5$ m/s

8. A particle is moving at a constant speed in a circular trajectory centered on the origin of an x-y coordinate system. At one point (x = 4 m, y = 0 m) the particle has a velocity of -5.0 $\hat{\boldsymbol{j}}$ m/s. Determine the velocity and acceleration when the particle is at: (a) x = 0, y = -4 m. (b) x = -4 m, y = 0. (c) x = -2.83, y = 2.83 m.

a. x = 0, y = -4 m

The velocity is always tangent to the trajectory.

v = -5**i** m/s (because the circular motion is clockwise on the axis system)

$a = v^2/r = 5^2/4 = 6.25\ m/s^2$

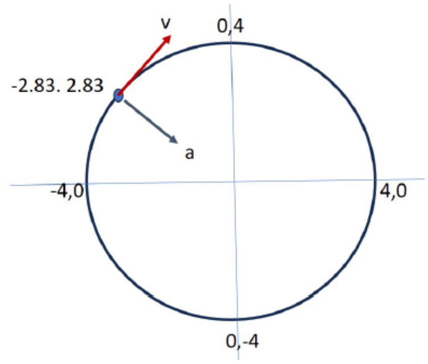
a = 6.25**j** m/s^2(always directed toward the center)

b. x = -4 m, y = 0

v = 5**j** m/s

The acceleration magnitude is constant, so we still have a = 6.25 m/s^2

a = 6.25**i** m/s^2 (always directed toward the center)



d. x = -2.83, y = 2.83 m

The velocity will still have a magnitude of 5 m/s, and will still be tangent to the trajectory. At the point referred to, the angle of the tangent is 45°.

v = 5m/s(cos45**i** + sin45**j**)

v = 3.54**i** +3.54**j** m/s

The acceleration will still have the same magnitude, and again, must point toward the center. In this case, at a 315° angle.

a = 6.25 m/s^2(cos315**i** + sin315**j**)

a = 4.42**i** - 4.42**j** m/s^2

9. A stunt pilot executes a uniform speed circular path in an airplane. The initial velocity (in m/s) of the plane is given by v = 2500$\hat{\imath}$+ 3000$\hat{\jmath}$ m/s. One minute later the velocity of the plane is v = -2500$\hat{\imath}$ - 3000 j m/s. Find the magnitude of the acceleration.

   First find the speed:

   $v = (2500^2 + 3000^2)^{1/2} = 3905\ m/s$

   For uniform circular motion:

   $a = v^2/r$ and $T = 2\pi r/v$

   Combine to show that $a = 2\pi v/T$

   Since T is the period of the motion, and the given data report that it takes one minute to reverse the velocity (the components have reversed), the period is 2 minutes (120 s).

   $a = 2\pi(3905)/120$

   $a = 204\ m/s^2$

10. An 5−kg object moves around a circular track of a radius of 18 cm with a constant speed of 6 m/s. Find The magnitude and direction of the acceleration of the object.

Solution: When an object moves around a circular path at a constant speed, the only acceleration that experiences is the centripetal acceleration or radial acceleration.

This kind of acceleration is always toward the center of the circle and its magnitude is found by the following formula

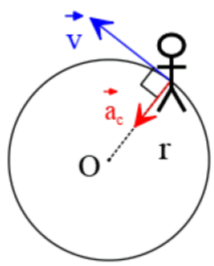
$$a_c = \frac{v^2}{r}$$

where v is the constant speed with which the object revolves the circle, and r is the radius of the circle.

(a) The track is circular and the speed of the object is constant, so a centripetal acceleration directed toward the center is applied to the object whose magnitude is as follows

$$a_c = \frac{6^2}{0.18\,\mathrm{m}} = 50\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

In the figure below, a top view of the motion is sketched.



(b) By applying Newton's second law along the direction of the centripetal acceleration, we can find the magnitude of the net force causing the acceleration as follows

$$F_{net} = m\frac{v^2}{r}$$

Therefore,

$$F_{net} = 5 \times 50 = 250 \quad \mathrm{N}$$

Note: At each point along the circular path, the instantaneous velocity of the revolving object is tangent to the path. The direction of this velocity changes, but its magnitude remains constant.