CALCULUS

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- What are the dimensions of a rectangle with fixed perimeter having *maximum area*? What are the dimensions for the *least expensive* cylindrical can of a given volume? How many items should be produced for the *most profitable* production run?
- Each of these questions asks for the best, or optimal, value of a given function. In this section we use derivatives to solve a variety of optimization problems in mathematics, physics, economics, and business.

◆ Solving Applied Optimization Problems

1. *Read the problem*. Read the problem until you understand it. What is given? What is the unknown quantity to be optimized?

2. *Draw a picture*. Label any part that may be important to the problem.

3. *Introduce variables*. List every relation in the picture and in the problem as an equation or algebraic expression, and identify the unknown variable.

4. *Write an equation for the unknown quantity*. If you can, express the unknown as a function of a single variable or in two equations in two unknowns. This may require considerable manipulation.

5. *Test the critical points and endpoints in the domain of the unknown*. Use what you know about the shape of the function's graph. Use the first and second derivatives to identify and classify the function's critical points.

① Maximum Area

Example 1

A scientist wants to enclose a rectangular study plot. He has 200 m of fencing. Using this fencing, what are the dimensions of the study plot that will have the largest area?

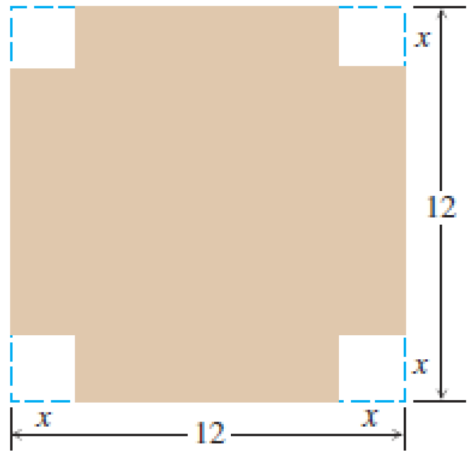
Deep Thinking:

What if this rectangular study plot is built against the wall?

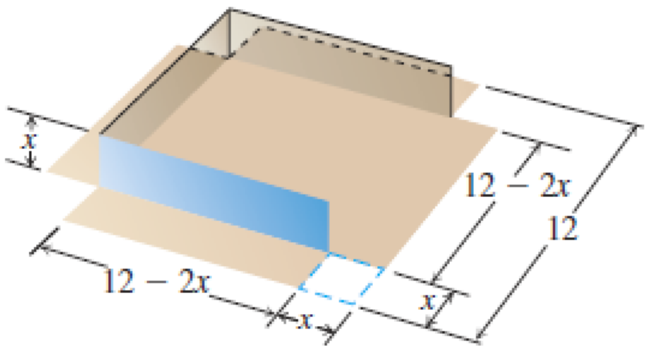
② Maximum Volume

Example 2

An open-top box is to be made by cutting small congruent squares from the corners of a 12 m × 12 m sheet of tin and bending up the sides. How large should the squares cut from the corners be to make the box hold as much as possible?



(a)

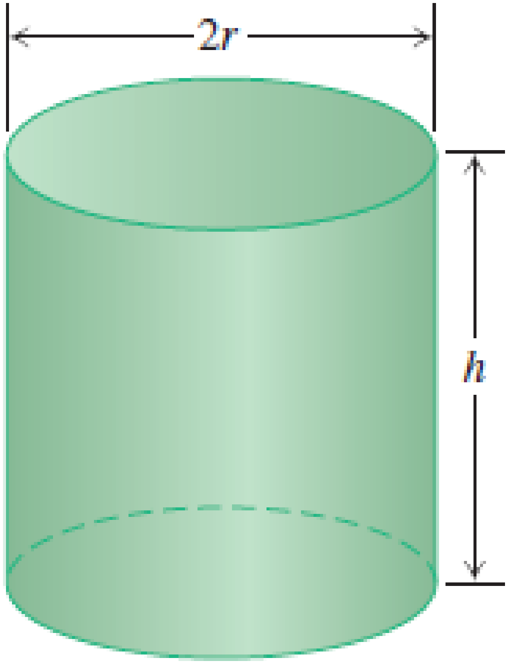


(b)

③ Minimizing Material

Example 3

You have been asked to design a 250π-cm^3 can shaped like a right circular cylinder. What dimensions (in centimeters) will use the least material?



Hint:

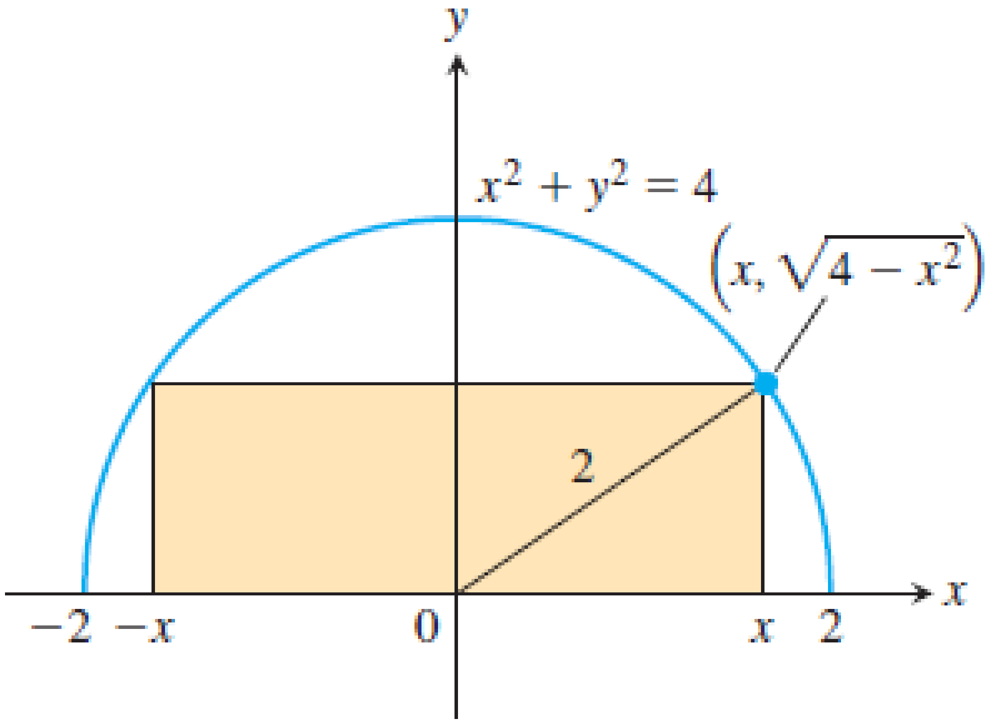
"The least material" means the least surface area.

Volume of the can = 250π cm^3

④ Examples from Mathematics and Physics

Example 4

A rectangle is to be inscribed in a semicircle of radius 2. What is the largest area the rectangle can have, and what are its dimensions?

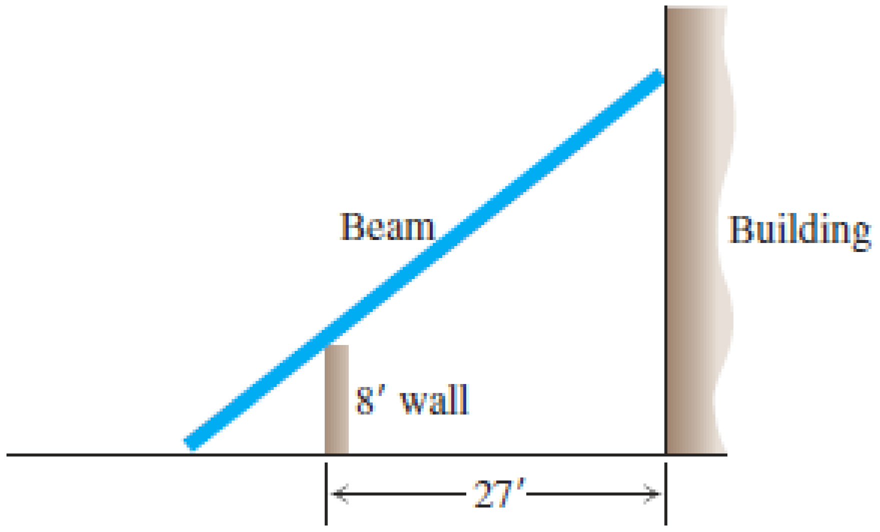


Example 5

Find a positive number for which the sum of its reciprocal and four times its square is the smallest possible.

Example 6

The 8-ft wall shown here stands 27 ft from the building. Find the length of the shortest straight beam that will reach to the side of the building from the ground outside the wall.



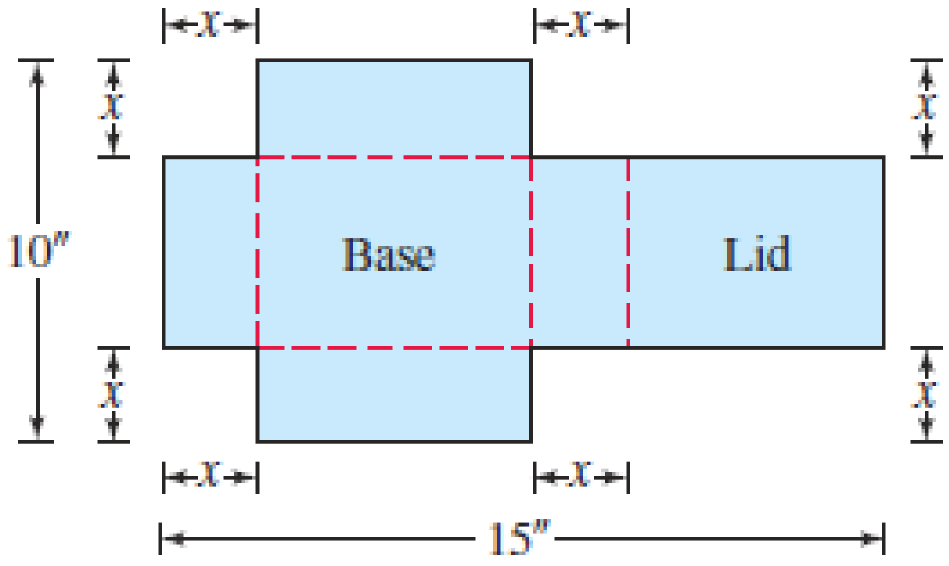
Skill Practice 1 Designing a box with a lid

A piece of cardboard measures 10 in. by 15 in. Two equal squares are removed from the corners of a 10-in. side as shown in the figure. Two equal rectangles are removed from the other corners so that the tabs can be folded to form a rectangular box with lid.

a) Write a formula $V(x)$ for the volume of the box.

b) Find the domain of $V(x)$ for the problem and graph V over this domain.

c) Find the maximum volume and the value of x that gives it.



Skill Practice 2 Motion on a line

The positions of two particles on the s-axis are $s_1 = \sin t$ and $s_2 = \sin(t + \pi/3)$, with s_1 and s_2 in meters and t in seconds.

a. At what time(s) in the interval $0 \leq t \leq 2\pi$ do the particles meet?

b. What is the farthest apart that the particles ever get?

Skill Practice 3 Shortest beam

Find the point on the line $\frac{x}{a} + \frac{y}{b} = 1$ (a, b > 0) that is closest to the origin.