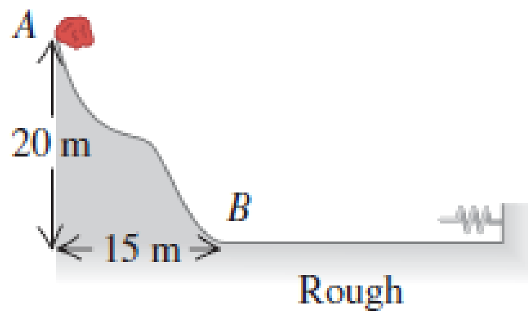
Problem Set 6 (Due 4/8/2025 before class)

Late homework will NOT be accepted, unless you have notified the course instructor 3 days BEFORE deadline.

Part I (60%)

7.49 •• A 15.0-kg stone slides down a snow-covered hill (Fig. P7.49), leaving point *A* with a speed of 10.0 m/s. There is no friction on the hill between points *A* and *B*, but there is friction on the level ground at the bottom of the hill, between *B* and the wall. After entering the rough horizontal region, the stone travels 100 m and then runs into a very long, light spring with force constant 2.00 N/m. The coefficients of kinetic and static friction between the stone and the horizontal ground are 0.20 and 0.80, respectively. (a) What is the speed of the stone when it reaches point *B*? (b) How far will the stone compress the spring?

Figure **P7.49**



Only (a) is to be graded

IDENTIFY: Apply Eq. (7.7) to the motion of the stone.

SET UP: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

Let point 1 be point *A* and point 2 be point *B*. Take $y = 0$ at point *B*.

EXECUTE: $mgy_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2$, with $h = 20.0$ m and $v_1 = 10.0$ m/s

$v_2 = \sqrt{v_1^2 + 2gh} = 22.2$ m/s

EVALUATE: The loss of gravitational potential energy equals the gain of kinetic energy.

(b) IDENTIFY: Apply Eq. (7.8) to the motion of the stone from point *B* to where it comes to rest against the spring.

7.14 •• An ideal spring of negligible mass is 12.00 cm long when nothing is attached to it. When you hang a 3.15-kg weight from it, you measure its length to be 13.40 cm. If you wanted to store 10.0 J of potential energy in this spring, what would be its *total* length? Assume that it continues to obey Hooke's law.

IDENTIFY: Use the information given in the problem with $F = kx$ to find k. Then $U_{el} = \frac{1}{2}kx^2$.

SET UP: x is the amount the spring is stretched. When the weight is hung from the spring, $F = mg$.

EXECUTE: $k = \dfrac{F}{x} = \dfrac{mg}{x} = \dfrac{(3.15\text{ kg})(9.80\text{ m/s}^2)}{0.1340\text{ m} - 0.1200\text{ m}} = 2205\text{ N/m}.$

$x = \pm\sqrt{\dfrac{2U_{el}}{k}} = \pm\sqrt{\dfrac{2(10.0\text{ J})}{2205\text{ N/m}}} = \pm 0.0952\text{ m} = \pm 9.52\text{ cm}.$ The spring could be either stretched 9.52 cm or compressed 9.52 cm. If it were stretched, the total length of the spring would be $12.00\text{ cm} + 9.52\text{ cm} = 21.52\text{ cm}.$ If it were compressed, the total length of the spring would be $12.00\text{ cm} - 9.52\text{ cm} = 2.48\text{ cm}.$

EVALUATE: To stretch or compress the spring 9.52 cm requires a force $F = kx = 210\text{ N}$.

7.35 •• **CALC** A force parallel to the x-axis acts on a particle moving along the x-axis. This force produces potential energy $U(x)$ given by $U(x) = \alpha x^4$, where $\alpha = 1.20\text{ J/m}^4$. What is the force (magnitude and direction) when the particle is at $x = -0.800\text{ m}$?

IDENTIFY: Apply Eq. (7.16).

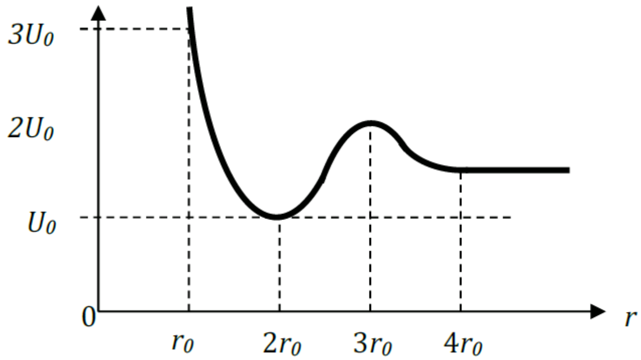
SET UP: The sign of F_x indicates its direction.

EXECUTE: $F_x = -\dfrac{dU}{dx} = -4\alpha x^3 = -(4.8\text{ J/m}^4)x^3.$ $F_x(-0.800\text{ m}) = -(4.8\text{ J/m}^4)(-0.80\text{ m})^3 = 2.46\text{ N}.$ The force is in the $+x$-direction.

EVALUATE: $F_x > 0$ when $x < 0$ and $F_x < 0$ when $x > 0$, so the force is always directed towards the origin.

Part II (40%)

1. The graph above represents the potential energy U as a function of position r for a particle of mass m. If the particle is released from rest at position r_o, what will its speed be at position $3r_0$?



Solution:

This is a conservation of energy problem, in which we examine the relationship between the particle's potential and kinetic energies.

$$U_i + K_i = U_f + K_f$$

$$3U_0 + 0 = 2U_0 + \tfrac{1}{2}mv^2$$

$$U_0 = \tfrac{1}{2}mv^2$$

$$v = \sqrt{\frac{2U_0}{m}}$$

2. The behavior of a non-linear spring is described by the relationship F = $-2kx^3$, where x is the displacement from the equilibrium position and *F* is the force exerted by the spring. How much potential energy is stored in the spring when it is displaced a distance *x* from equilibrium?

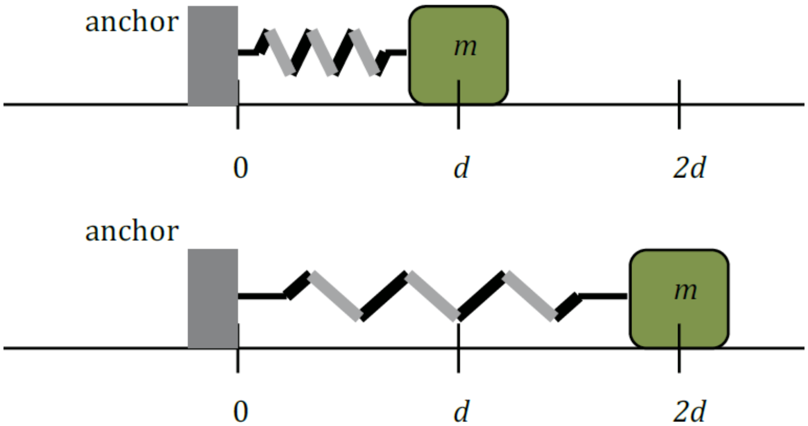
Answer: The potential energy stored in the spring is calculated using the Work integral:

$$U = -\int_{x_i}^{x_f} F \bullet dx$$

$$U = -\int_{0}^{x} -2kx^3 \bullet dx$$

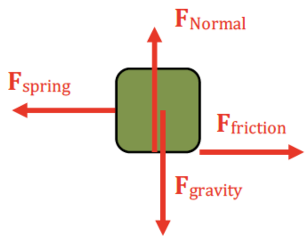
$$U = 2k \left. \frac{x^4}{4} \right|_0^x = \frac{1}{2}kx^4$$

For problems 3-9: A block of mass *m* rests on a rough surface, and has a light spring of spring constant *k* and unstretched length *d* attached to one side as shown, with the other end of the spring attached to an anchor. There is a static coefficient of friction μ_s between the surface and the block, and when the block is placed to the right at position 2*d*, it remains stationary on the surface. Express answers in terms of *m*, *k*, *d*, and fundamental constants.



3. Draw a free-body diagram of the block at the time when it is located at position 2d.

The block isn't moving yet, so the force exerted by the spring to the left equals the force exerted by friction to the right. Vectors should be labeled and drawn with lengths proportional to their magnitude. Some instructors wish for vectors to be drawn with their point of origin beginning at the location where that force is applied, and some instructors prefer that force vectors be drawn originating from the center of the (point) mass.

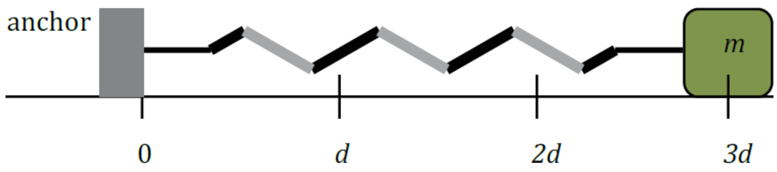


4. Determine the friction force acting on the block when it is located at position 2d.

The friction force acting on the block at this point is equal in magnitude to the force applied by the spring:

$$F_{net} = ma$$
$$-F_{spring} + F_{friction} = 0$$
$$F_{friction} = kx = kd$$

To continue 5-9, the block is now moved to position 3d and released, where it remains at rest. When the block is moved slightly past this position, the block begins to slide along the surface with a kinetic coefficient of friction μ_k



5. In terms of the variables given, what is the value of μ_s?

The coefficient of static friction is based on the maximum force of static friction that the block-surface can support. In this case:

$$\mu_{static} = \frac{F_{friction}}{F_{Normal}} = \frac{kx}{mg} = \frac{k(2d)}{mg}$$

6. How much potential energy is stored in the mass-spring system just before the block begins to move?

The potential energy stored in the spring is simply based on $U_{spring} = \frac{1}{2}kx^2$:

$$U_{spring} = \frac{1}{2}kx^2$$

$$U_{spring} = \frac{1}{2}k(2d)^2 = 2kd^2$$

7. The block slides a total distance of d before coming to a halt again. Determine the coefficient of kinetic friction μ_k

The coefficient of kinetic friction can be determined by using a Conservation of Energy analysis, taking into account the potential energy of the mass-spring system at the beginning and end, as well as the energy lost to heat in the block's slide:

$$U_{spring-initial} - \Delta E_{internal} = U_{spring-final}$$

$$\frac{1}{2}kx_i^{\,2} - F_{friction}d = \frac{1}{2}kx_f^{\,2}$$

$$\frac{1}{2}kx_i^{\,2} - \mu_k mgd = \frac{1}{2}kx_f^{\,2}$$

$$\frac{1}{2}k(2d)^2 - \mu_k mgd = \frac{1}{2}kd^2$$

$$\mu_k = \frac{3kd}{2mg}$$

Note that this result reveals that $\mu_k = \frac{3}{4}\left(\frac{2kd}{mg}\right) = \frac{3}{4}\mu_x$, which is consistent with the principal that coefficients of kinetic friction are less than coefficients of static friction.

8. At what position does the block have its maximum velocity as it slides?

The block has its maximum velocity where the slope of the velocity-time curve is 0, ie. where acceleration is 0. At that position, the force from the spring and the force of friction are equal to each other:

$$F_{net} = ma$$

$$F_{spring} - F_{friction} = 0$$

$$kx - \mu_k F_{Normal} = kx - \mu_k mg = 0$$

where x is the displacement of the spring from its unstretched length at d. Substituting in μ_k from the problem before:

$$kx - \mu_k mg = 0$$

$$kx - \left(\frac{3kd}{2mg}\right)mg = 0$$

$$kx - \frac{3}{2}kd = 0$$

$x = \frac{3d}{2}$, relative to unstretched length at d

Relative to the origin:

$$\frac{3d}{2} + d = \frac{5}{2}d$$

9. What is the maximum velocity of the block as it slides?

Now that we know the position of the maximum velocity, we can use Conservation of Energy again to calculate that velocity:

$$U_{s-initial} - \Delta E_{\text{int}} = K + U_{s-final}$$

$$\frac{1}{2}kx_i^2 - F_{friction}x = \frac{1}{2}mv^2 + \frac{1}{2}kx_f^2$$

$$\frac{1}{2}k(2d)^2 - \mu_k mg\left(\frac{d}{2}\right) = \frac{1}{2}mv^2 + \frac{1}{2}k\left(\frac{3}{2}d\right)^2$$

$$\frac{1}{2}k(2d)^2 - \left(\frac{3kd}{2mg}\right)mg\left(\frac{d}{2}\right) = \frac{1}{2}mv^2 + \frac{1}{2}k\left(\frac{3}{2}d\right)^2$$

Solve to get $v = \frac{d}{2}\sqrt{\frac{k}{m}}$

10. A mass of 10 kg is taken from the ground for 10 m uphill on the wedge. The wedge makes an angle of 30°with the ground. Find the potential energy of the block.

Solution:

The potential energy of a mass ‘*m*’ at the height ‘*h*’ is given by:

P = *mgh*

This wedge is in the form of a right-angled triangle.

Let’s say, *h* is the vertical height at which the box reaches, let the slanted length be *L*

L = 10 m

$$sin(30°) = \frac{h}{L}$$
$$= \frac{1}{2} = \frac{h}{L}$$
$$= \frac{10}{2} = h$$
$$= \frac{5}{2} = h$$
$$= h = 5m$$

Given: m = 5 kg and g = 10 m/s^2 and h = 5 m.

Aim: Find the potential energy.

Plugging in the values in the formula.

P = mgh

⇒ P = (5 kg)(10 m/s^2)(5 m)

⇒P = 250 J

Thus, the potential energy of the object is 250 J.