College Algebra and Trigonometry

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Fall 2024

Definition of a Quadratic Equation :

- **A quadratic equation in the variable is an equation of the form:**

$$ax^2 + bx + c = 0$$

① Solve by using the Zero product Property

Zero Product Property :

If $mn = 0$, then $m = 0$ or $n = 0$.

Example 1: Apply the Zero Product Property to solve the equations:

a) $x^2 - 8x = 0$ **b) $2x(2x - 7) = -12.$**

② Solve by using the Square Root Property

Square Root Property :

If $x^2 = k$, then $x = \sqrt{k}$ or $-\sqrt{k}$.

Example 2: Apply the Zero Product Property to solve:

a) $x^2 = 36$ **b) $9y^2 + 25 = 0$** **c) $(w - 3)^2 = 50$**

③ Complete the Square

Perfect Square Trinomial		Factored Form
$x^2+10x+25$	→	$(x+5)^2$
$t^2-8t+16$	→	$(t-4)^2$
$z^2+12z+36$	→	$(z+6)^2$

Example 3: **Complete the Square**

a) x^2+12x **b)** $x^2-\frac{3}{5}x$ **c)** $2x^2-9x$

Solve a Quadratic Equation $ax^2 + bx + c = 0$ by completing the Square and Apply the Square Root Property

Example 4:

a) $2x^2 + 3x - 6 = 0$

b) $x^2 - 3 = -10x$

④ Solve Quadratic Equations by using The Quadratic Formula

$$ax^2+bx+c=0 \quad \Rightarrow \quad x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$\Rightarrow \quad (x+\frac{b}{2a})^2=(\frac{b}{2a})^2-\frac{c}{a}=\frac{b^2-4ac}{4a^2}$$

$$\Rightarrow \quad \boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

Example 5: **Solve by using the Quadratic Formula**

a) $\boldsymbol{x(x-6)=3}$

b) $\boldsymbol{\frac{3}{10}x^2-\frac{2}{5}x+\frac{7}{10}=0}$

⑤ Use the Discriminant

The solutions to a quadratic equation are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The radicand, $b^2 - 4ac$, is called the *discriminant*.

The value of the discriminant can tell the number and type of solutions to the equation.

Discriminant $\Delta = b^2 - 4ac$	Number and Type of Solutions	Examples
$\Delta < 0$	2 nonreal solutions	$2x^2 - 3x + 5 = 0$
$\Delta = 0$	2 repeated solutions	$x^2 - 8x + 16 = 0$
$\Delta > 0$	2 distinct real solutions	$3x^2 + 5x - 2 = 0$

⑥ Solve an Equation for a specific variable

Example 6:

Solve for r $\quad V = \frac{1}{3}\pi r^2 h \quad (r > 0)$

Example 7:

Solve for t $\quad mt^2 + nt = z$

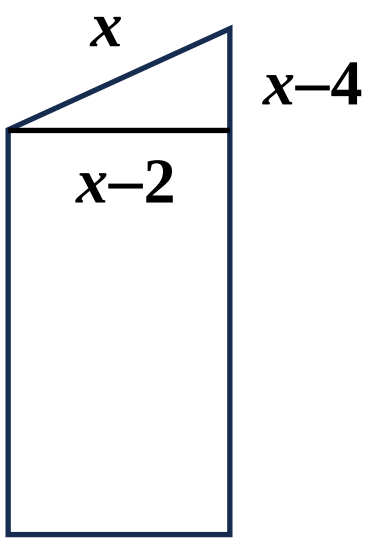
Example 8:

Solve for y $\quad A = \pi h y^2 + 2\pi h y$

① Solve Applications involving Quadratic Equations and Geometry

Example 1:

A window is in the shape of a rectangle with an adjacent right triangle above (see figure). The lengths of two legs of the right triangle are 2 ft and 4 ft less than the length of the hypotenuse, respectively. Find the lengths of the sides.



② Solve Applications involving Quadratic Models

In the study of physical science, a common model used to represent the vertical position of an object moving vertically under the influence of gravity is given by:

$$s = -\frac{1}{2}gt^2 + v_0 t + s_0$$

where:

g: the gravitational acceleration; **t: the time of travel.**

v_0: the initial velocity; **s_0: the initial vertical position.**

Example 2:

A toy rocket is shot straight upward from a launch pad of 3 m above the ground level with an initial velocity of 25 m/s.

a) **Write a model to express the height of the rocket s (in meters) above the ground level.**

b) **Find the time (in seconds) at which the rocket is at a height of 23 m.**

Example 3:

The sum of the first *n* natural numbers, $S = 1 + 2 + 3 + \ldots + n$, is given by:

$$S = \frac{1}{2}n(n+1)$$

If the sum *S* is 120, find the value of *n*.