College Algebra and Trigonometry

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① Relation

Often there are situations where one variable is somehow linked to the value of another variable. For example:

- An individual's level of education is linked to annual income.
- The test score that a student earns is related to the number of hours of study.
- Engine size is linked to gas mileage.

Determine whether a Relation is a Function

Definition of a Relation

A set of ordered pairs (x, y) is called a relation in x and y.

- **The set of x values in the ordered pairs is called the domain of the relation.**
- **The set of y values in the ordered pairs is called the range of the relation.**

Example 1:

For the table shown:

a) Write the set of ordered pairs that defines the relation.

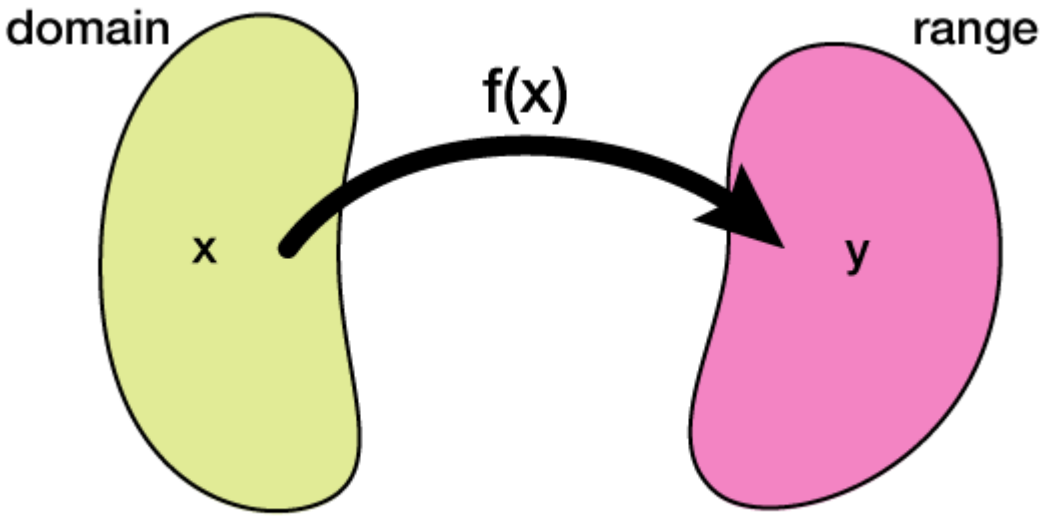
b) Write the domain and range.

x	3	-2	5	1
y	-4	0	3	0

Definition of a Function

Given a relation in *x* and *y*, we say that *y* is a function of *x* if for each value of *x* in the domain, there is exactly one value of *y* in the range.

- **A function is a mapping (rule) that maps every element in the domain to exactly one corresponding element in the range as shown in the figure in the following.**



A function "maps the domain onto the range," one y for every x.

Example 2:

Determine if the relation defines *y* as a function of *x*.

a) {(3, 1), (2, 5), (-4, 2), (-1, 0), (3, -4)}

b) {(-1, 4), (2, 3), (3, 4), (-4, 5)}

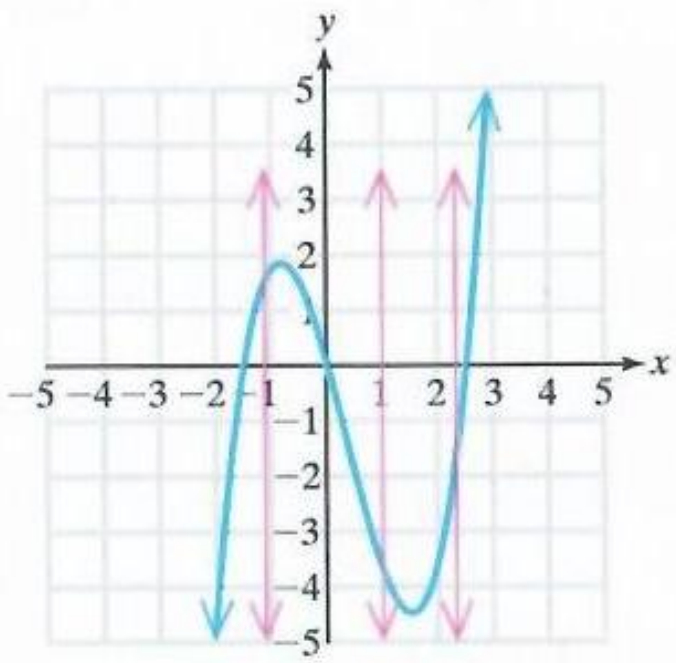
Using the Vertical Line Test

Consider a relation defined by a set of points (*x*, *y*) graphed on a rectangular coordinate system. The graph defines *y* as a function of *x* if no vertical line intersects the graph in more than one point.

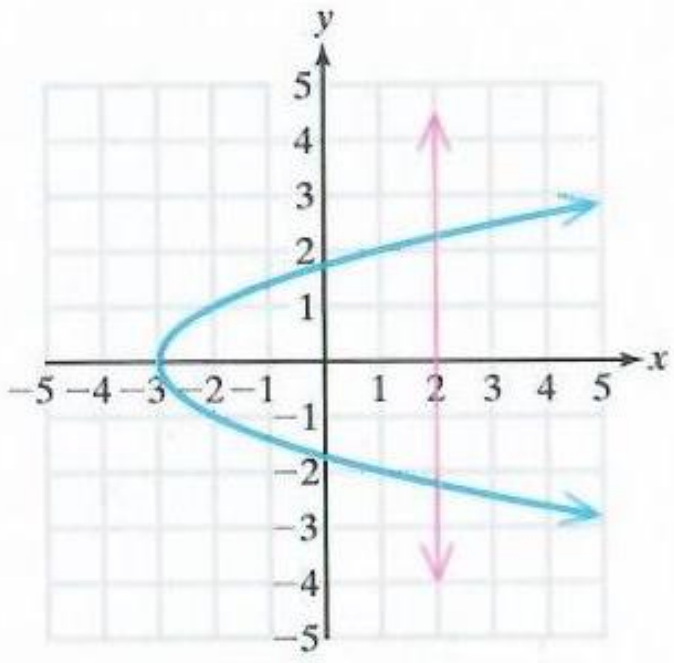
Example 3:

The graph of three relations are shown in blue. In each case, determine if the relation defines *y* as a function of *x*.

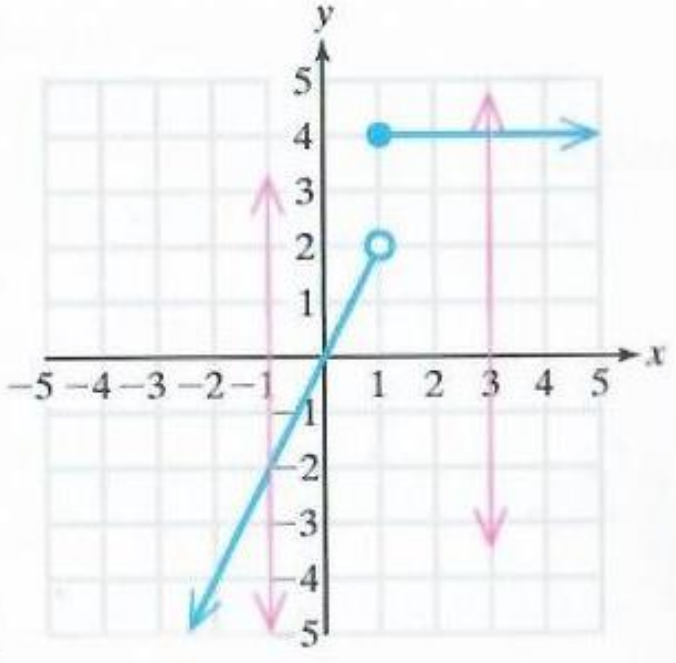
a.



b.

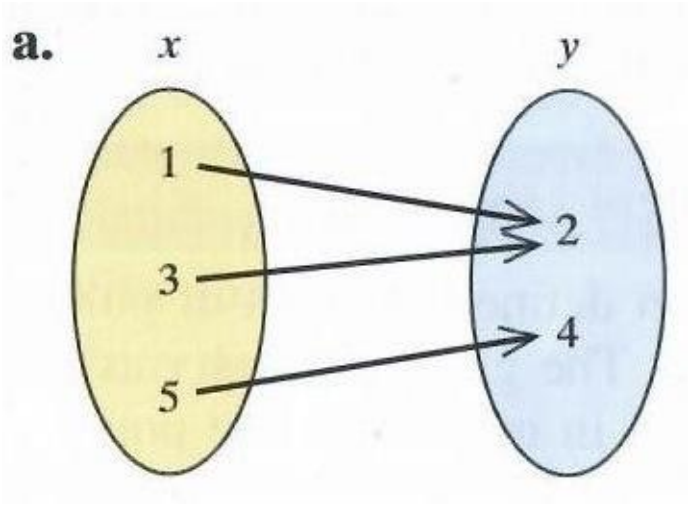


c.

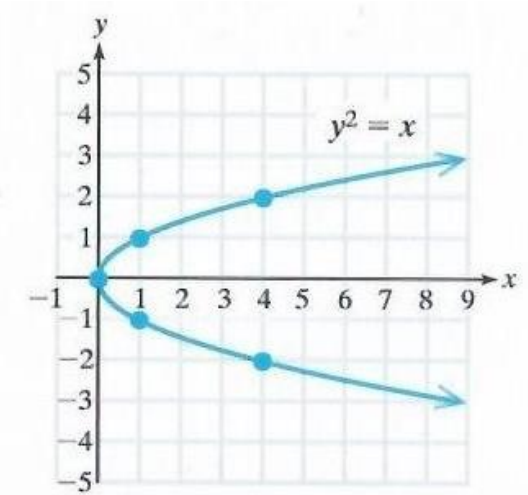


Example 4:

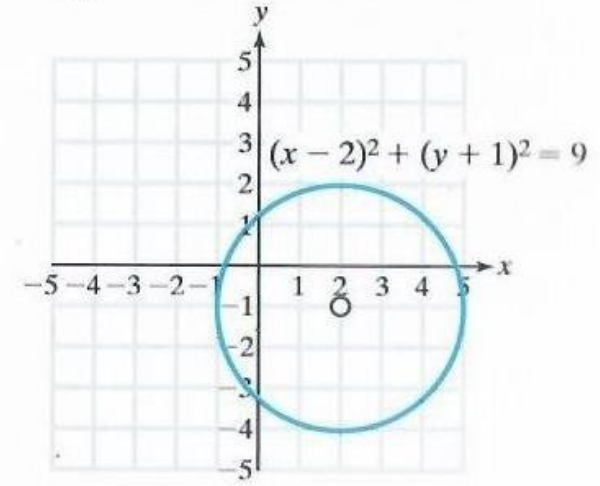
Determine if the relation defines y as a function of x.



b. $y^2 = x$



c. $(x-2)^2 + (y+1)^2 = 9$



② Apply Function Notation

Function Notation

$$f(x) = x-2$$

- ***f* is the name of the function,**
- ***x* is an input variable from the domain,**
- ***f*(*x*) is the function value corresponding to *x*.**

Functions of x: $y = x^2$ $y = |x|$ $y = x^3$

Not Functions of x: $x = y^2$ $x^2 + y^2 = 1$ $x = |y|$

Example 5:

Evaluate the function defined by $f(x) = 3x^2 + 2x$ for the given values of x.

a) $f(a)$ **b) $f(a+h)$**

Skill Practice:

Find the minimum value of the function defined by

$$f(x) = x^2 + 6x + 8$$

③ Determine *x*- and *y*- Intercepts of a Function defined by $y = f(x)$

Finding Intercepts using Function Notation

Given a function defined by $y = f(x)$,

- **The *x*-intercepts are the real solutions to the equation $f(x) = 0$.**
- **The *y*-intercept is given by $f(0)$.**

Example 6:

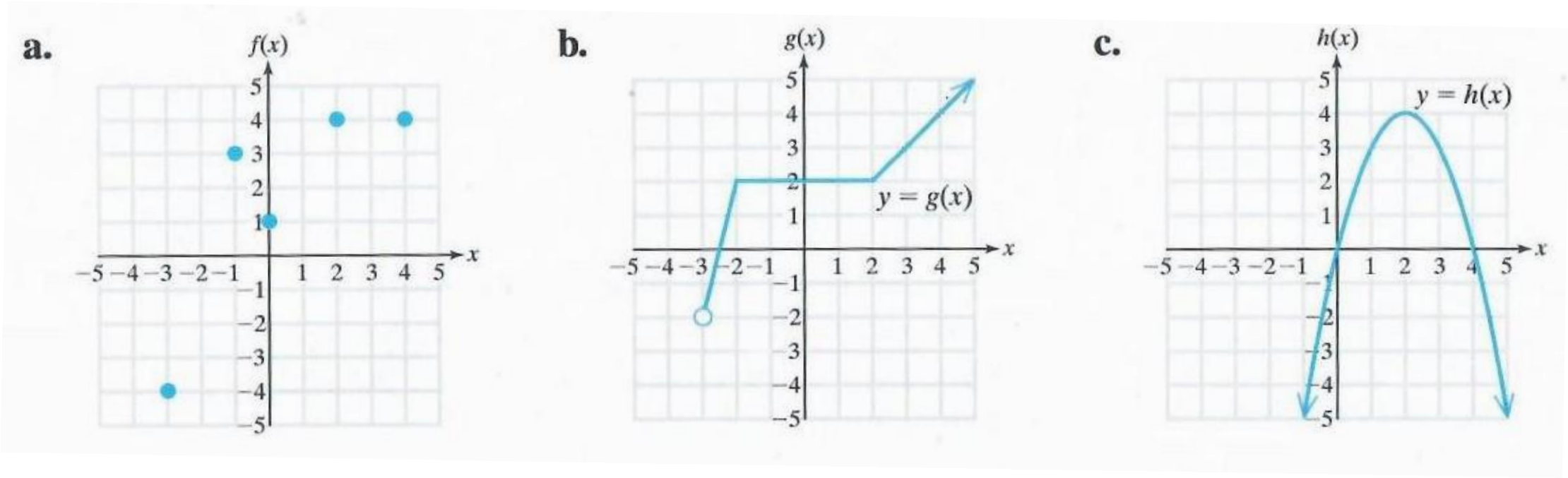
Find the *x*- and *y*-intercepts of the function defined by

$$f(x) = x^2 + 2x$$

④ Determine the domain and range of a Function

Example 7:

Determine the domain and range of the functions shown.



Guidelines to find Domain of a function:

To determine the implied domain of a function defined by $y = f(x)$.

- **Exclude values of x that make the denominator of a fraction zero.**
- **Exclude values of x that make the radicand negative within an even-indexed root.**

Example 8:

Write the domain of each function in interval notation.

a) $f(x) = \frac{x+3}{2x-5}$

b) $g(x) = \frac{x}{x^2-4}$

c) $h(t) = \sqrt{2-t}$

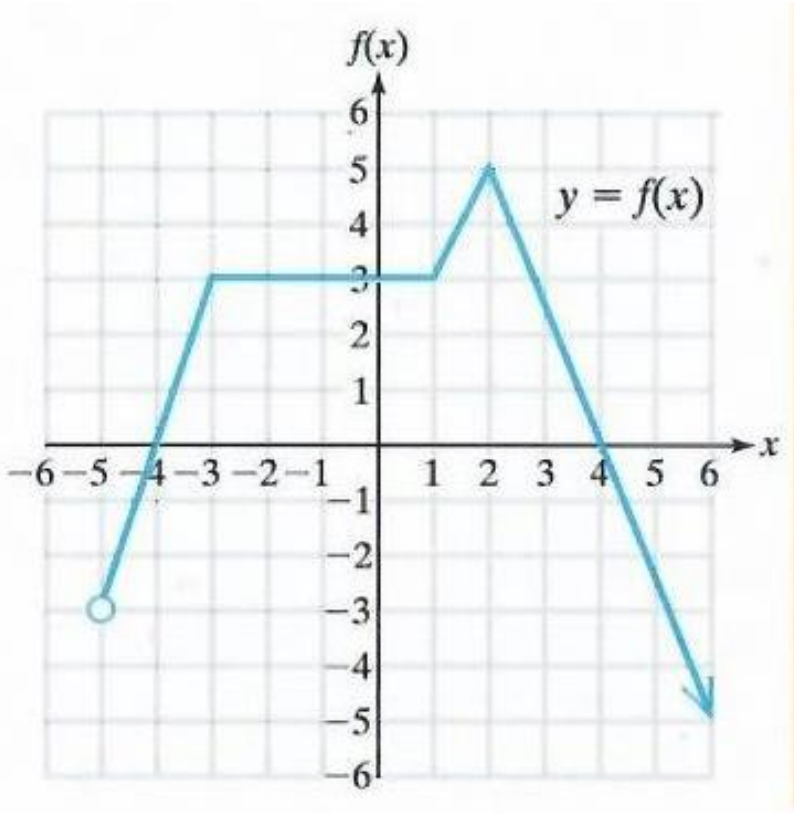
d) $m(r) = |r-1|$

⑤ Intercept a Function Graphically

Example 9:

Use the function f pictured to answer the questions.

a. Determine $f(2)$.
b. Determine $f(-5)$.
c. Find all x for which $f(x) = 0$.
d. Find all x for which $f(x) = 3$.
e. Determine the x-intercept(s).
f. Determine the y-intercept.
g. Determine the domain of f.
h. Determine the range of f.



① Graph Linear Equations in Two Variables

Linear Equation in Two Variables

A linear equation in two variables x and y is the equation that is written in the standard form as:

$$Ax + By = C.$$

where A, B, C are real numbers in which both A and B are nonzero.

Example 1:

Graph the following linear equations.

a) $2x + 3y = 6$

b) $\frac{x}{2} - \frac{y}{3} = 1$

② Determine a Slope of a Line

Slope of a Line:

The slope of a line passing through the distinct points (x_1, y_1) and (x_2, y_2) is:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$ **provided that x_1 and x_2 are non equal.**

Example 2:

Find the slope of the line passing through the given points.

a) (-3, -2) and (2, 5)

b) (1, -7) and (-5/2, 0)

Example 3:

Find the slope of horizontal and vertical lines.

a) $x = -2$ **b)** $y = 3$

Linear Equations and Slopes of Lines:

$Ax + By = C$

$(A \neq 0$ and $B \neq 0)$

Slanted line

Positive Slope **Negative Slope**

$x = k$

(k **is a constant)**

Vertical line

Undefined Slope

$y = C$

(C **is a constant)**

Horizontal line

Zero Slope