

College Algebra and Trigonometry

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6.3 Double-Angle, Power-Reducing, and Half-Angle Formulas Angle Angle Sangle S



1 Apply the Double-Angle Formulas

$$sin2\theta = sin(\theta + \theta) = sin\theta cos\theta + cos\theta sin\theta = 2sin\theta cos\theta$$
 $cos2\theta = cos(\theta + \theta) = cos\theta cos\theta - sin\theta sin\theta$
 $= cos^2\theta - sin^2\theta = 2cos^2\theta - 1 = 1 - 2sin^2\theta$

$$tan2\theta = tan(\theta + \theta) = \frac{tan\theta + tan\theta}{1 - tan\theta tan\theta} = \frac{2tan\theta}{1 - tan^2\theta}$$

Example 1:

Given $\sin \theta = 3/5$ for θ in Quadrant II, find the exact values of :

a) $\sin 2\theta$

- b) $\cos 2\theta$ c) $\tan 2\theta$

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The double-angle formula can be used with angles other than 2θ :

$$sin4\theta = 2sin2\theta cos2\theta$$
 $cos6x = cos^23x - sin^23x$

$$sin(\alpha-1) = 2sin\frac{\alpha-1}{2}cos\frac{\alpha-1}{2}$$

$$tanx = \frac{2tan\frac{x}{2}}{1 - tan^2\frac{x}{2}}$$

Example 2:

Verify the identity:

$$\sin 3x = 3\sin x - 4\sin^3 x$$

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Apply the Power-Reducing Formulas

$$sin^2\theta = \frac{1 - cos2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$tan^{2}\theta = \frac{sin^{2}\theta}{cos^{2}\theta} = \frac{1 - cos2\theta}{1 + cos2\theta}$$

Proof.

Example 3:

Write $\sin^4 x + \cos^2 x$ in terms of first powers of cosine. Range = ?

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(3) Apply the Half-Angle Formulas

$$sin\frac{\alpha}{2} = \pm \sqrt{\frac{1 - cos\alpha}{2}}$$
 $cos\frac{\alpha}{2} = \pm \sqrt{\frac{1 + cos\alpha}{2}}$ $tan\frac{\alpha}{2} = \pm \sqrt{\frac{1 - cos\alpha}{1 + cos\alpha}} = \frac{1 - cos\alpha}{sin\alpha} = \frac{sin\alpha}{1 + cos\alpha}$

Proof: ("tan" will be proved in Example 5)

Example 4:

Use the half-angle formula to find the exact values of

a) sin67.5°

b) cos112.5°

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Example 5:

Show that:

$$tan\frac{\alpha}{2} = \pm \sqrt{\frac{1 - cos\alpha}{1 + cos\alpha}} = \frac{1 - cos\alpha}{sin\alpha} = \frac{sin\alpha}{1 + cos\alpha}$$

Example 6:

If $sin\alpha = -\frac{4}{5}$ and $\pi < \alpha < \frac{3\pi}{2}$, find the exact values:

- a) $sin\frac{\alpha}{2}$ b) $cos\frac{\alpha}{2}$ c) $tan\frac{\alpha}{2}$



1 Apply the Product-to-Sum Formulas

$$sinu \ sinv = \frac{1}{2} [cos(u - v) - cos(u + v)]$$

$$cosu \ cosv = \frac{1}{2} [cos(u - v) + cos(u + v)]$$

$$sinu \ cosv = \frac{1}{2} [sin(u + v) + sin(u - v)]$$

$$cosu \ sinv = \frac{1}{2} [sin(u + v) - sin(u - v)]$$

Example 1:

Write the product as a sum or difference.

a) $\sin 7x \cos 3x$

b) $\sin(-x)\sin 4x$



Example 2:

Use the product-to-sum formula to find the exact value of:

Note:

$$\sin 15^{\circ} \cos 75^{\circ} = \sin 15^{\circ} \sin 15^{\circ} = \sin^2 15^{\circ} = \frac{1 - \cos 30^{\circ}}{2}$$

Skill Practice:

Use the product-to-sum formula to find the exact value of:

cos15°sin75°



2 Apply the Sum-to-Product Formulas

$$cosx - cosy = -2sin\frac{x+y}{2}sin\frac{x-y}{2}$$

$$cosx + cosy = 2cos\frac{x+y}{2}cos\frac{x-y}{2}$$

$$sinx - siny = 2sin\frac{x-y}{2}cos\frac{x+y}{2}$$

$$sinx + siny = 2sin\frac{x+y}{2}cos\frac{x-y}{2}$$

Example 3:

Write each expression as a product.

a)
$$\sin 7\theta + \sin 3\theta$$

b)
$$\cos\alpha - \cos 3\alpha$$



Example 4:

Use a sum-to-product formula to find the exact value of

$$\cos 255^{\circ} + \cos 195^{\circ}$$

Example 5:

Verify the identity:

$$\frac{\sin 5x + \sin x}{\cos 5x + \cos x} = \tan 3x$$



1 Solve Trigonometric Equations in Linear Form

Example 1:

Solve
$$2tanx = \sqrt{3} - tanx$$

a) Over $[0, 2\pi)$.

b) Over the set of real numbers.

Skill Practice:

Solve
$$3\cos x = 2\sqrt{2} - \cos x$$

- a) Over $[0, 2\pi)$.
- b) Over the set of real numbers.



2 Solve Trigonometric Equations Involving Multiple or Compound Angles

Example 2:

$$2\sin 2x - \sqrt{3} = 0$$

- a) Write the solution set for the general solution.
- b) Write the solution set on the interval $[0, 2\pi)$.

Example 3:

$$\sin\frac{x}{2}-1=0$$

- a) Write the solution set for the general solution.
- b) Write the solution set on the interval $[0, 2\pi)$.



Example 4:

$$sin\left(x-\frac{\pi}{3}\right)+\frac{\sqrt{2}}{2}=0$$

- a) Write the solution set for the general solution.
- b) Write the solution set on the interval $[0, 2\pi)$.

Skill Practice:

$$cot\left(x-\frac{\pi}{4}\right)=-1$$

- a) Write the solution set for the general solution.
- b) Write the solution set on the interval $[0, 2\pi)$.



3 Solve Higher-degree Trigonometric Equations

Example 5:

Solve the equation on the interval $[0, 2\pi)$:

$$2sin^2x + 7sinx - 4 = 0$$

Example 6:

Solve the equation on the interval $[0, 2\pi)$:

$$\cot^2 x - 3 = 0$$

Example 7:

Solve the equation on the interval $[0, 2\pi)$:

$$tanx sin^2x = tanx$$



4 Use Identities to Solve Trigonometric Equations

Example 8:

Solve the equation on the interval $[0, 2\pi)$:

$$sec^2x - tanx = 1$$

Example 9:

Solve the equation on the interval $[0, 2\pi)$:

$$cos3x + cosx = 0$$

Example 10:

Solve the equation on the interval $[0, 2\pi)$:

$$sinx + 1 = cosx$$