College Algebra and Trigonometry

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① Convert between Logarithmic and Exponential Forms

Equation	Solution
$5^x = 5$	$x = 1$
$5^x = 15$	$x = ?$
$5^x = 25$	$x = 2$
$5^x = 75$	$x = ?$
$5^x = 125$	$x = 3$

Definition of a Logarithmic Function:

If x and b are positive real numbers such that $b \neq 1$, then $y = \log_b x$ is called the logarithmic function base b, where:

$$y = \log_b x \quad \text{is equivalent to} \quad b^y = x$$

Notes:

- **The value y is called the logarithm, b is called the base, and x is called the argument.**
- **The expression $y = \log_b x$ is called the logarithmic form, and $b^y = x$ is called the exponential form.**

Example 1:

Write each equation in exponential form.

a) $\log_2 16 = 4$ b) $\log_{10}\left(\frac{1}{100}\right) = -2$ c) $\log_7 1 = 0$

Example 2:

Write each equation in logarithmic form.

a) $3^4 = 81$ b) $10^6 = 1000000$ c) $\left(\frac{1}{5}\right)^{-1} = 5$

② Evaluate Logarithmic Expressions

Equivalence Property of Exponential Expressions:

If b, x, and y are real numbers with $b > 0$ and $b \neq 1$, then:

$$b^x = b^y \quad \text{implies that} \quad x = y$$

Example 3:

Evaluate the following logarithmic expressions.

a) $\log_4 16$ b) $\log_9 3$ c) $\log_{1/2} 8$

Definition of Common and Natural Logarithmic Functions

- **The logarithmic function base 10 is called the common logarithmic function, denoted as:**

$$y = \log_{10} x = \log x$$

- **The logarithmic function base e is called the natural logarithmic function, denoted as:**

$$y = \log_e x = \ln x$$

Example 4:

Evaluate the following logarithmic expressions.

a) $\log 100000$ **b)** $\log 0.001$ **c)** $\ln e^4$ **d)** $\ln \frac{1}{e}$

③ Apply Basic Properties of Logarithms

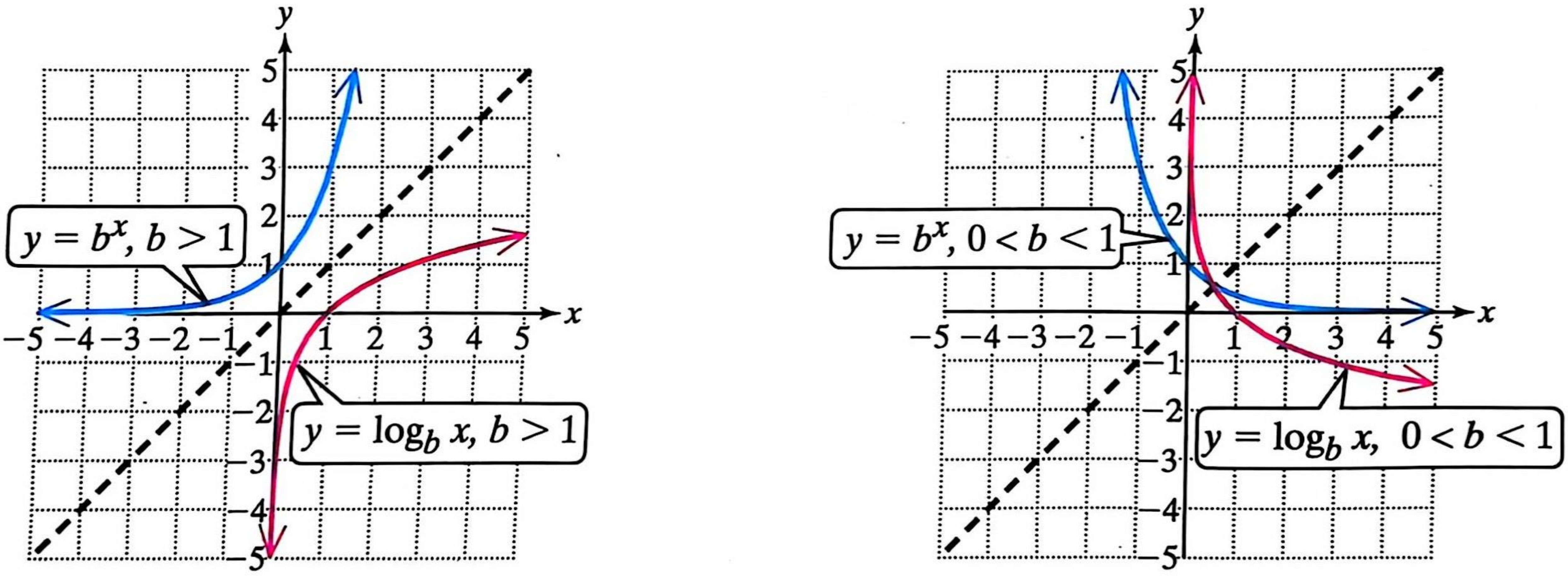
Property	Reason
1. $\log_b 1 = 0$	$b^0 = 1$
2. $\log_b b = 1$	$b^1 = b$
3. $\log_b b^x = x$	$b^x = b^x$
4. $b^{\log_b x} = x$	$\log_b x = \log_b x$

Example 6:

Simplify: a) $\log_3 3^{10}$ b) $\ln e^3$ c) $\log_{11} 121$ d) $\log 1000$

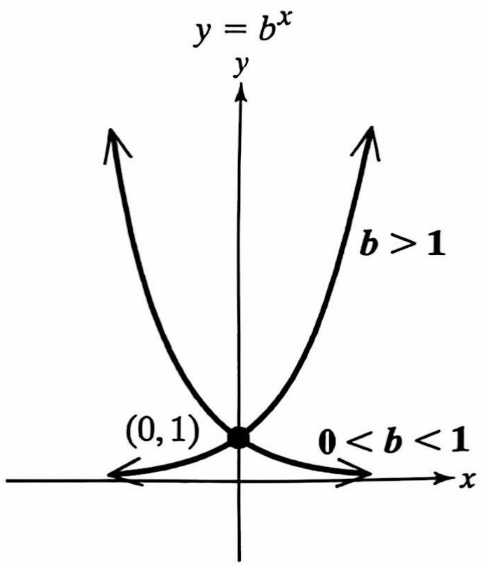
e) $\log_{\sqrt{7}} 1$ f) $\ln 1$ g) $5^{\log_5(a^2+1)}$ h) $10^{\log(b^2)}$

④ Graphs of Logarithmic Functions



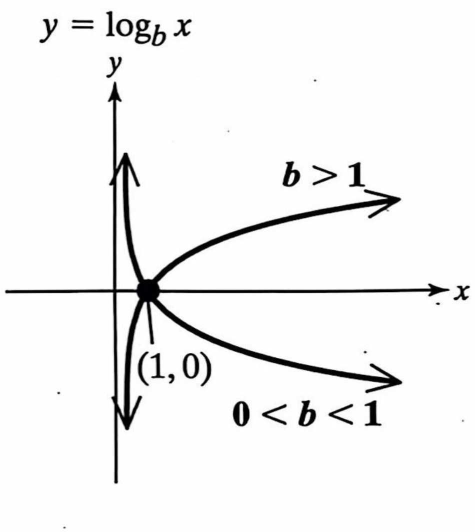
Graphs of Exponential and Logarithmic Functions

Exponential Functions



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Horizontal asymptote: $y = 0$
Passes through $(0, 1)$
If $b > 1$, the function is increasing.
If $0 < b < 1$, the function is decreasing.

Logarithmic Functions



Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Vertical asymptote: $x = 0$
Passes through $(1, 0)$
If $b > 1$, the function is increasing.
If $0 < b < 1$, the function is decreasing.

Example 7:

Graph:

a) $\boldsymbol{y} = \log_{\mathbf{2}} \boldsymbol{x}$ **b)** $\boldsymbol{y} = \log_{\mathbf{1/2}} \boldsymbol{x}$

Example 8:

Identify the domain in interval notation:

a) $\boldsymbol{f}(\boldsymbol{x}) = \log_{\mathbf{2}}(\mathbf{2}\boldsymbol{x} + \mathbf{4})$ **b)** $\boldsymbol{g}(\boldsymbol{x}) = \ln(\mathbf{5} - \boldsymbol{x})$ **c)** $\boldsymbol{h}(\boldsymbol{x}) = \log(\boldsymbol{x}^{\mathbf{2}} - \mathbf{9})$

① Apply the Product, Quotient, and Power Properties of Logarithms

Product Property of Logarithms

- **Let b, x, and y be positive real numbers where $b \neq 1$. Then**

$$\log_{\boldsymbol{b}}(\boldsymbol{xy}) = \log_{\boldsymbol{b}} \boldsymbol{x} + \log_{\boldsymbol{b}} \boldsymbol{y}$$

Example 1:

Write the logarithm as a sum and simplify if possible. Assume x and y are positive real numbers.

a) $\log_2(\boldsymbol{8x})$ **b)** $\ln(\boldsymbol{ye^3})$ **c)** $\log(\boldsymbol{10xy})$

Quotient Property of Logarithms

- **Let b, x, and y be positive real numbers where $b \neq 1$. Then**

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

Example 2:

Write the logarithm as a difference and simplify if possible.

Assume x and y are positive real numbers.

a) $\log_3\left(\frac{x}{9}\right)$ **b)** $\log\frac{y}{1000}$ **c)** $\ln\frac{xe}{y}$

Power Property of Logarithms

- **Let b and x be positive real numbers where $b \neq 1$. Let p be any real number. Then**

$$\log_{\boldsymbol{b}} \boldsymbol{x}^{\boldsymbol{p}} = \boldsymbol{p}\log_{\boldsymbol{b}} \boldsymbol{x}$$

Example 3:

Simplify the logarithms. Assume x and y are positive real numbers.

a) $\ln \sqrt[5]{x^2}$ **b)** $\log(100y^2)$ **c)** $\log_{\boldsymbol{3}}(\boldsymbol{27x^3y^2})$

② Write a Logarithmic Expression in Expanded Form

$$\log(\boldsymbol{abc}) = \log \boldsymbol{a} + \log \boldsymbol{b} + \log \boldsymbol{c}$$

$$\ln\left(\frac{\boldsymbol{ab}}{\boldsymbol{c}}\right) = \ln \boldsymbol{a} + \ln \boldsymbol{b} - \ln \boldsymbol{c}$$

Example 4:

Expand the logarithms. All variables are positive real numbers.

a) $\log_2\left(\frac{z^3}{xy^5}\right)$

b) $\log \sqrt[3]{\frac{(x+y)^2}{10}}$

③ Write a Logarithmic Expression in Condensed Form

$$\log \boldsymbol{a} + \log \boldsymbol{b} + \log \boldsymbol{c} = \log(\boldsymbol{abc})$$

$$\ln \boldsymbol{a} + \ln \boldsymbol{b} - \ln \boldsymbol{c} = \ln\left(\frac{\boldsymbol{ab}}{\boldsymbol{c}}\right)$$

Example 5:

Expand the logarithms. All variables are positive real numbers.

a) $\log_2 \mathbf{560} - \log_2 \mathbf{7} - \log_2 \mathbf{5}$

b) $\mathbf{3}\log \boldsymbol{a} + \mathbf{2}\log \boldsymbol{b} - \frac{\mathbf{1}}{\mathbf{2}}\log \boldsymbol{c}$

④ Apply the Change-of-Base Formula

Change-of-Base Formula

- **Let a and b be positive real numbers where $a \neq 1$ and $b \neq 1$. Then for any positive real number x :**

$$\log_b x = \frac{\log_a x}{\log_a b}$$

$$\Rightarrow \quad \log_b x = \frac{\log x}{\log b} \quad \text{and} \quad \log_b x = \frac{\ln x}{\ln b}$$

Example 6:

Use the change-of-base formula to simplify:

$$(\log_2 12)(\log_{12} 8)$$