Problem Set 5 (Due 4/1/2025 before class)

Late homework will NOT be accepted, unless you have notified the course instructor 3 days BEFORE deadline.

Part I (60%)

6.20 •• You throw a 20-N rock vertically into the air from ground level. You observe that when it is 15.0 m above the ground, it is traveling at 25.0 m/s upward. Use the work–energy theorem to find (a) the rock's speed just as it left the ground and (b) its maximum height.

IDENTIFY: From the work-energy relation, $W = W_{grav} = \Delta K_{rock}$.

SET UP: As the rock rises, the gravitational force, $F = mg$, does work on the rock. Since this force acts in the direction opposite to the motion and displacement, s, the work is negative. Let h be the vertical distance the rock travels.

EXECUTE: **(a)** Applying $W_{grav} = K_2 - K_1$ we obtain $-mgh = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$. Dividing by m and solving for v_1, $v_1 = \sqrt{v_2^2 + 2gh}$. Substituting $h = 15.0$ m and $v_2 = 25.0$ m/s,

$$v_1 = \sqrt{(25.0\ \text{m/s})^2 + 2(9.80\ \text{m/s}^2)(15.0\ \text{m})} = 30.3\ \text{m/s}$$

(b) Solve the same work-energy relation for h. At the maximum height $v_2 = 0$.

$$-mgh = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \text{ and } h = \frac{v_1^2 - v_2^2}{2g} = \frac{(30.3\ \text{m/s})^2 - (0.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 46.8\ \text{m}.$$

EVALUATE: Note that the weight of 20 N was never used in the calculations because both gravitational potential and kinetic energy are proportional to mass, m. Thus any object, that attains 25.0 m/s at a height of 15.0 m, must have an initial velocity of 30.3 m/s. As the rock moves upward gravity does negative work and this reduces the kinetic energy of the rock.

6.52 •• A 20.0-kg rock is sliding on a rough, horizontal surface at 8.00 m/s and eventually stops due to friction. The coefficient of kinetic friction between the rock and the surface is 0.200. What average power is produced by friction as the rock stops?

IDENTIFY: The thermal energy is produced as a result of the force of friction, $F = \mu_k mg$. The average thermal power is thus the average rate of work done by friction or $P = F_{\|}v_{av}$.

SET UP: $v_{av} = \dfrac{v_2 + v_1}{2} = \left(\dfrac{8.00\ \text{m/s} + 0}{2}\right) = 4.00\ \text{m/s}$

EXECUTE: $P = Fv_{av} = [(0.200)(20.0\ \text{kg})(9.80\ \text{m/s}^2)](4.00\ \text{m/s}) = 157\ \text{W}$

EVALUATE: The power could also be determined as the rate of change of kinetic energy, $\Delta K/t$, where the time is calculated from $v_f = v_i + at$ and a is calculated from a force balance, $\Sigma F = ma = \mu_k mg$.

Part II (40%)

For problems 1-5. A factory worker pushes a 30.0-kg crate a distance of 4.5 m along a level floor at constant velocity by pushing horizontally on it. The coefficient of kinetic friction between the crate and the floor is 0.25.

1. What magnitude of force must the worker apply?
2. How much work is done on the crate by this force?
3. How much work is done on the crate by friction?
4. How much work is done on the crate by the normal force? By gravity?
5. What is the total work done on the crate?

IDENTIFY: Each force can be used in the relation $W = F_{\|}s = (F\cos\phi)s$ for parts (b) through (d). For part (e), apply the net work relation as $W_{\text{net}} = W_{\text{worker}} + W_{\text{grav}} + W_n + W_f$.

SET UP: In order to move the crate at constant velocity, the worker must apply a force that equals the force of friction, $F_{\text{worker}} = f_k = \mu_k n$.

EXECUTE: **(a)** The magnitude of the force the worker must apply is:

$$F_{\text{worker}} = f_k = \mu_k n = \mu_k mg = (0.25)(30.0\text{ kg})(9.80\text{ m/s}^2) = 74\text{ N}$$

(b) Since the force applied by the worker is horizontal and in the direction of the displacement, $\phi = 0°$ and the work is:

$$W_{\text{worker}} = (F_{\text{worker}}\cos\phi)s = [(74\text{ N})(\cos 0°)](4.5\text{ m}) = +333\text{ J}$$

(c) Friction acts in the direction opposite of motion, thus $\phi = 180°$ and the work of friction is:

$$W_f = (f_k\cos\phi)s = [(74\text{ N})(\cos 180°)](4.5\text{ m}) = -333\text{ J}$$

(d) Both gravity and the normal force act perpendicular to the direction of displacement. Thus, neither force does any work on the crate and $W_{\text{grav}} = W_n = 0.0\text{ J}$.

(e) Substituting into the net work relation, the net work done on the crate is:

$$W_{\text{net}} = W_{\text{worker}} + W_{\text{grav}} + W_n + W_f = +333\text{ J} + 0.0\text{ J} + 0.0\text{ J} - 333\text{ J} = 0.0\text{ J}$$

EVALUATE: The net work done on the crate is zero because the two contributing forces, F_{worker} and F_f, are equal in magnitude and opposite in direction.

6. What is the kinetic Energy of a 150 kg object that is moving with a speed of 15 m/s?

$KE = \frac{1}{2} mv^2$

$KE = ?$

$m = 150kg$

$v = 15m/s$

$KE = \frac{1}{2} (150kg)\ (15\ m/s)^2$

$KE = \frac{1}{2} (150kg)(225)$

$\boxed{KE = 16875J}$

7. An object has a kinetic energy of 25 J and a mass of 34 kg , how fast is the object moving?

$KE = \frac{1}{2} mv^2$

$KE = 25J$

$m = 34kg$

$v = ?$

$2KE/m = v^2$ OR $v^2 = 2KE/m$

$v^2 = 2(25J)/34kg$

$\sqrt{v^2} = \sqrt{1.47}$

$v = \boxed{1.28m/s}$

8. Work done by a force on a moving object is 100J. It was traveling at a speed of 2 m/s. Find the new speed of the object if the mass of the object is 2 kg.

Given: W = 100J

$W = \frac{1}{2}m(v^2 – u^2)$

Work done by the force is equal to the change in kinetic energy.

$W = \frac{1}{2}m(v^2 – u^2)$

Given, u = 2 m/s and v = ?, m = 2kg.

Plugging the values in the given equation,

$\Rightarrow 100 = \frac{1}{2}(2)(v^2 – 2^2)$

$\Rightarrow 100 = v^2 – 2^2$

$= 104 = v^2$

$= v = \sqrt{104}$ m/s

9. A 700-N marine in basic training climbs a 10.0m vertical rope at a constant speed, with a total duration of 8.00 s. What is his power output?

Marine is shown in Fig. 6.6. The speed of the marine up the rope is

$$v = \frac{d}{t} = \frac{10.0\,\mathrm{m}}{8.00\,\mathrm{s}} = 1.25\,\frac{\mathrm{m}}{\mathrm{s}}$$

The forces acting on the marine are gravity (700 N, downward) and the force of the rope which must be 700 N upward since he moves at constant velocity. Since he moves in the same direction as the rope's force, the rope does work on the marine at a rate equal to

$$P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v} = Fv = (700\,\mathrm{N})(1.25\,\tfrac{\mathrm{m}}{\mathrm{s}}) = 875\,\mathrm{W}\ .$$

(It may be hard to think of a stationary rope doing work on anybody, but that is what is happening here.)

This number represents a rate of change in the potential energy of the marine; the energy comes from someplace. *He* is losing (chemical) energy at a rate of 875 W.

10. How long does it take a 2.5 kW electric motor to do 75, 000 J of work?

Answer: t = W / F = 75000 J / 2500 W = 30 s