

Model Test For Mid-Term

1 Calculate the following matrices

(1) Calculate the multiplication

$$A = \begin{pmatrix} -1 & 4 & 3 & 0 \\ 2 & 3 & -7 & 8 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 \\ 0 & -4 & -3 \\ 2 & 0 & 5 \\ 3 & -1 & 2 \end{pmatrix} \begin{pmatrix} 9 & -3 \\ -6 & 2 \\ 3 & -1 \end{pmatrix}.$$

(2) Calculate the matrix $f(A)$, where

$$A = \begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \quad f(x) = x^3 + x^2 + x + 1.$$

2 Do elementary row transformations on the following matrices:

(1) Transfer the following matrix into an echelon form and calculate its rank

$$\begin{pmatrix} 2 & -1 & 6 & 5 \\ 3 & 8 & -4 & -2 \\ 1 & 10 & 16 & 12 \end{pmatrix}$$

(2) Calculate the inverse matrix of the following matrix

$$\begin{pmatrix} 13 & -2 & 3 \\ 5 & -1 & 1 \\ 4 & -1 & 1 \end{pmatrix}.$$

3 Solve the following systems of linear equations:

(1) Solve the following homogeneous system

$$\begin{cases} 3x_1 - 3x_2 + 4x_3 - 4x_4 = 0, \\ 5x_1 + x_2 + 2x_3 + 2x_4 = 0, \\ 2x_1 - x_2 - x_3 - 7x_4 = 0. \end{cases}$$

(2) Solve the following system of linear equations

$$\begin{cases} 2x_1 + 6x_2 - 2x_3 + 9x_4 = 20, \\ -3x_1 + 7x_2 - 13x_3 - 5x_4 = -13, \\ 4x_1 - 2x_2 + 10x_3 + 3x_4 = 10, \\ x_1 + 5x_2 - 3x_3 - 3x_4 = -5. \end{cases}$$

4 Find out a maximal linear independent subset of the following vectors:

$$\alpha_1 = \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} -1 \\ 1 \\ 7 \end{pmatrix}, \quad \alpha_3 = \begin{pmatrix} 5 \\ -8 \\ -18 \end{pmatrix}, \quad \alpha_4 = \begin{pmatrix} 6 \\ 4 \\ 9 \end{pmatrix}$$