Fundamentals of Electric Circuits

CHAPTER 5 Operational Amplifiers

Lingling Cao, PhD, Associate Professor

Email: caolingling@hit.edu.cn

CHAPTER 5 Operational Amplifiers

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5.2 Operational Amplifier

- Typically called 'Op Amp' for short
- Active circuit element
- It is an electronic device consisting of a complex arrangement of resistors, transistors, capacitors, and diodes.
- The op amp is an electronic unit that behaves like a voltage-controlled voltage source
- It can also be used in making a voltage- or current-controlled current source
- The op amp can perform many mathematical operations, such as addition, subtraction, multiplication, division, differentiation, and integration

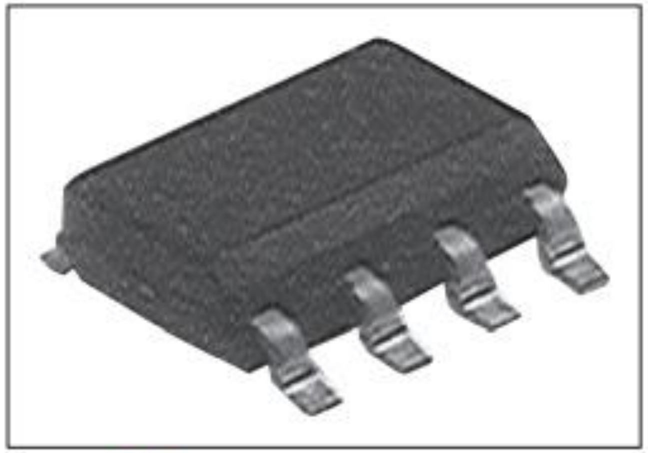
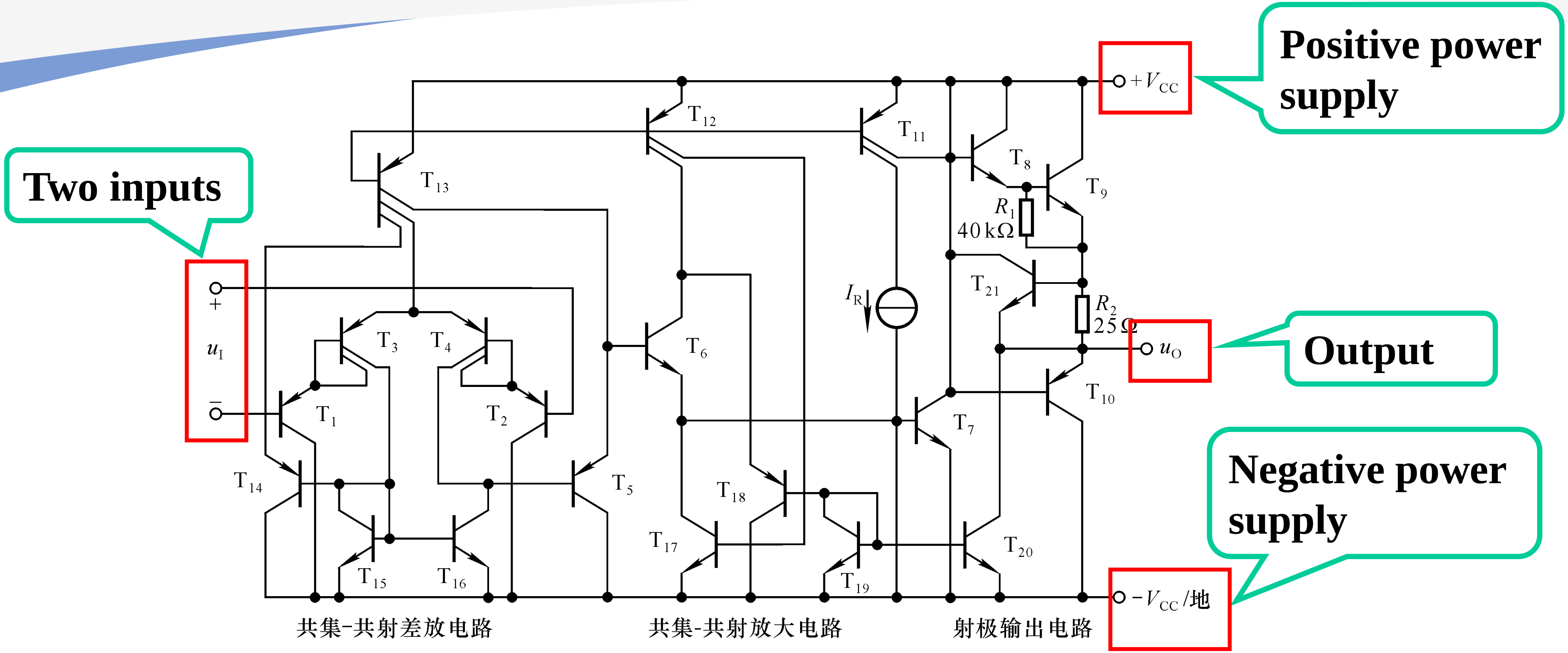


Figure 5.1
A typical operational amplifier.
Courtesy of Tech America.

The pin diagram in Fig. 5.2(a) corresponds to the 741 general-purpose op amp made by Fairchild Semiconductor.

5.2 Typical internal structure of an op amp



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Operational Amplifier

- There are five important terminals on all op-amps
  - The inverting input, appear inverted at the output
  - The noninverting input, appear with the same polarity at the output
  - The output
  - The positive and negative power supplies

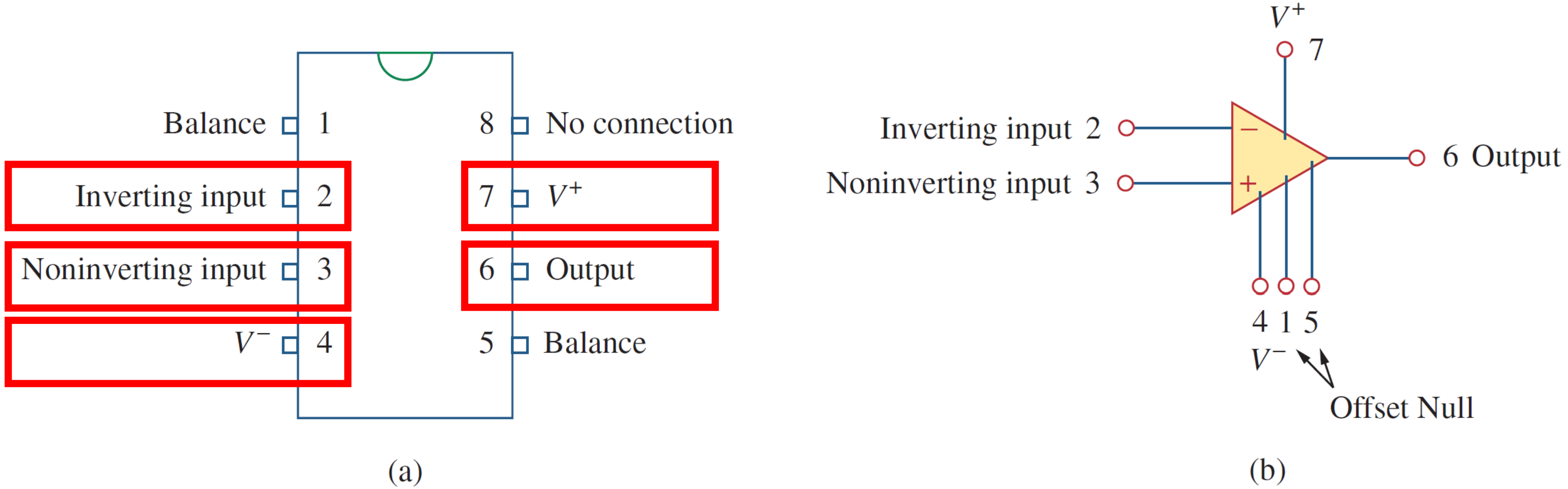
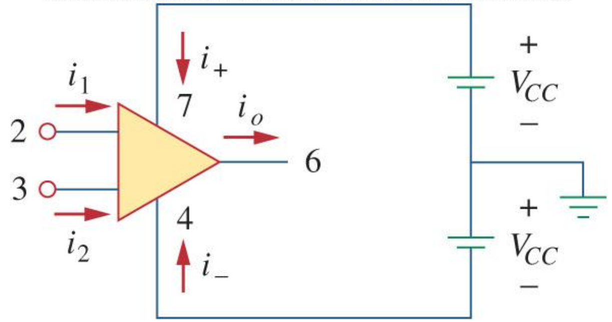


Figure 5.2
A typical op amp: (a) pin configuration, (b) circuit symbol.

Powering an Op-amp

- As an active element, the op-amp requires a power source
- The power supply terminals are often ignored in op amp circuit diagrams for simplification
- Most op-amps use two voltage sources, with a ground reference between them, which gives a positive and negative supply voltage

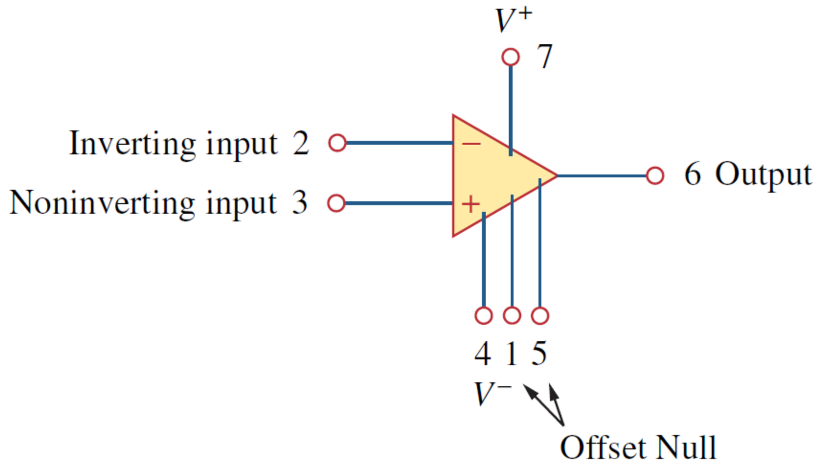


Output Voltage

- The output v_o of an op-amp is proportional to the difference between the noninverting and inverting inputs

$$v_0 = Av_d = A(v_2 - v_1)$$

- v_d is the differential input voltage;
- v_1 is the voltage between the inverting terminal and ground;
- v_2 is the voltage between the noninverting terminal and ground.
- *A* is the open-loop voltage gain, ideally it is infinite



Output Voltage

- The output v_o of an op-amp is proportional to the difference between the noninverting and inverting inputs

$$v_0 = Av_d = A(v_2 - v_1)$$

- The equivalent circuit model: a voltage-controlled source Av_d in series with the output resistance R_o

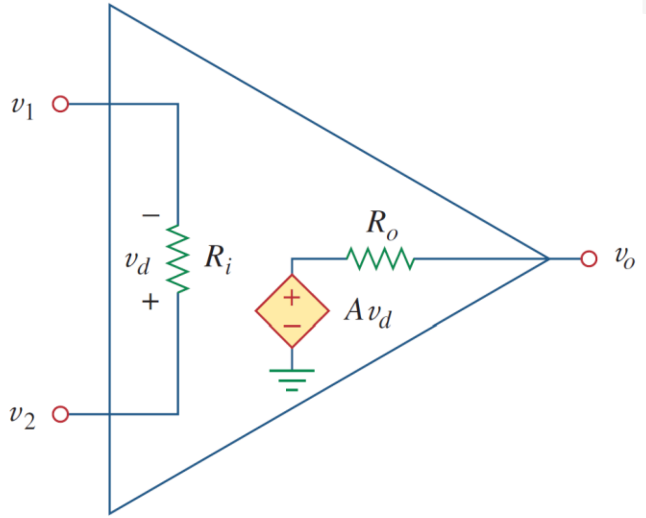


Figure 5.4
The equivalent circuit of the nonideal op amp.

Parameter	**Typical range**	**Ideal values**
Open-loop gain, A	10^5 to 10^8	∞
Input resistance, R_i	10^5 to $10^{13}\Omega$	$\infty\Omega$
Output resistance, R_0	10 to 100 Ω	0Ω
Supply voltage, V_{cc}	5 to 24 V	

Voltage Saturation

- The magnitude of the output voltage cannot exceed |Vcc|
- The output voltage is dependent on and is limited by the power supply voltage
- When an output exceeds the possible voltage range, the output remains at either the maximum or minimum supply voltage, which is called saturation
- Outputs between these limiting voltages are referred to as the linear region

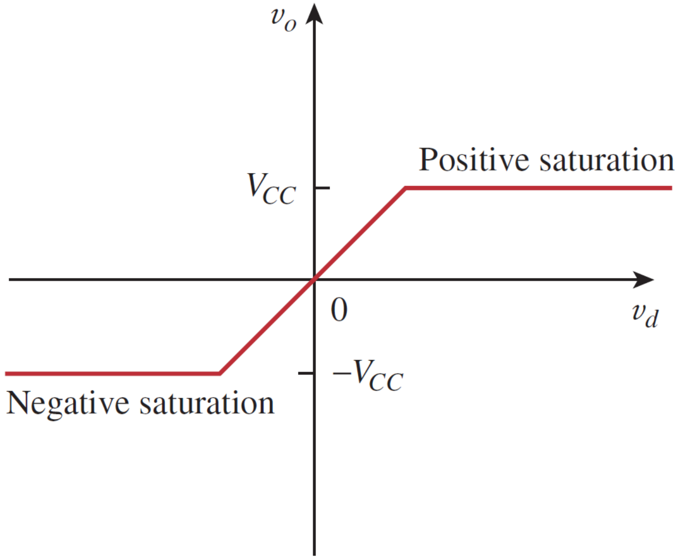


Figure 5.5
Op amp output voltage v_o as a function of the differential input voltage v_d.

$$v_0 = Av_d = A(v_2 - v_1)$$

Feedback

- When there is a feedback path from output to input, the ratio of the output voltage to the input voltage is called the closed-loop gain
- A negative feedback is achieved when the output is fed back to the inverting terminal of the op amp
- Positive feedback would lead to oscillations

Example

Example 5.1

A 741 op amp has an open-loop voltage gain of 2×10^5, input resistance of 2 MΩ, and output resistance of 50 Ω. The op amp is used in the circuit of Fig. 5.6(a). Find the closed-loop gain v_o/v_s. Determine current i when $v_s = 2$ V.

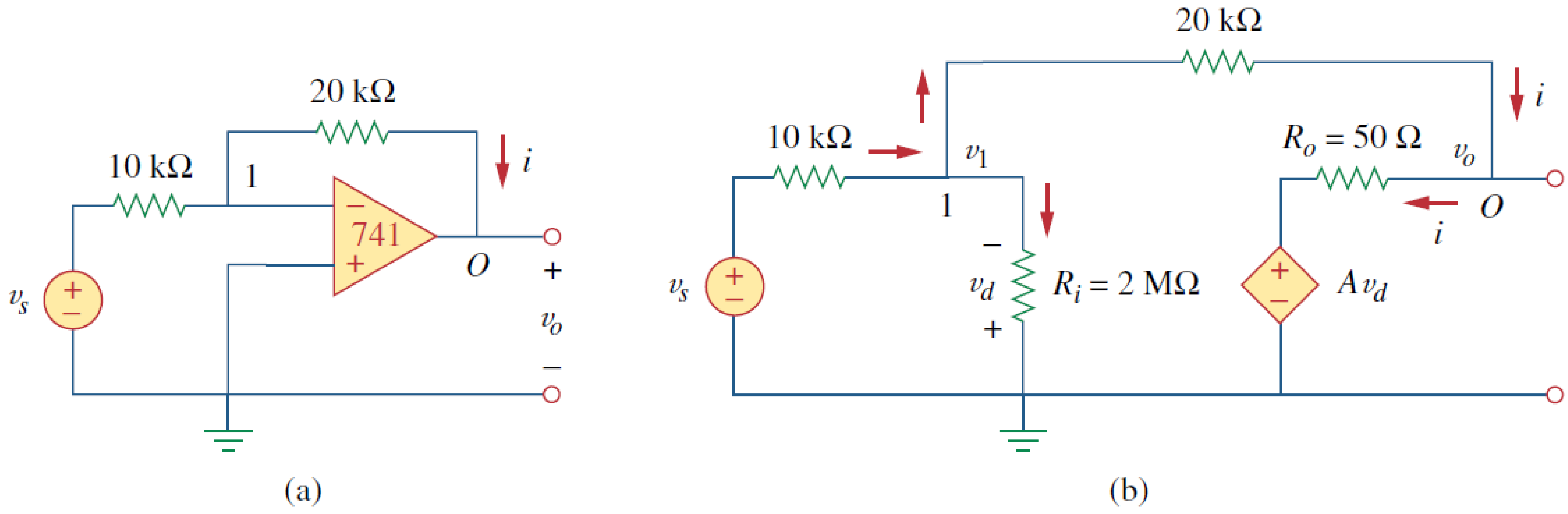
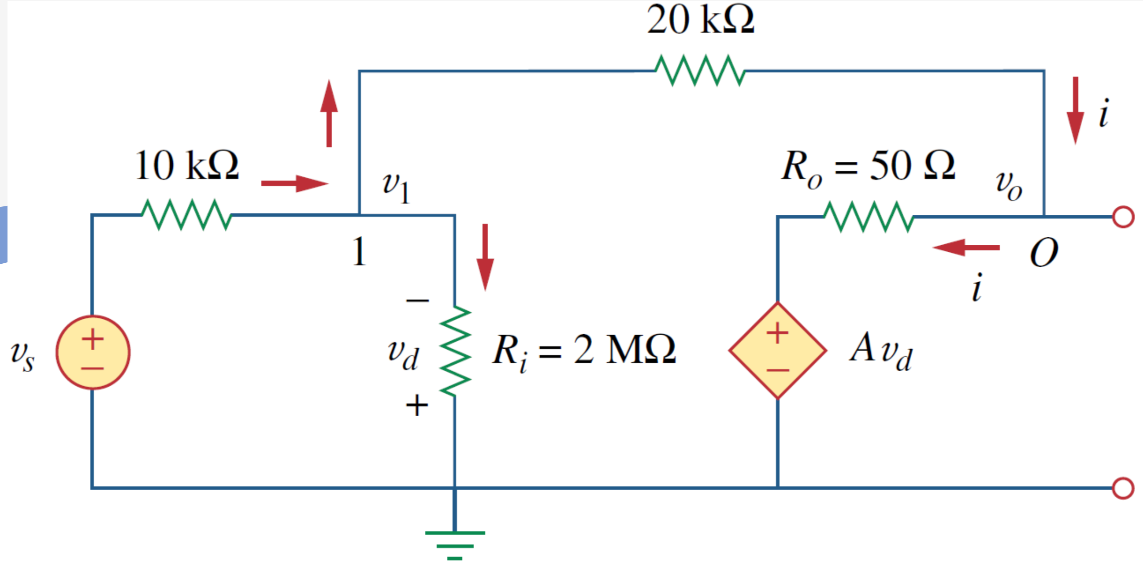


Figure 5.6
For Example 5.1: (a) original circuit, (b) the equivalent circuit.

Example



Solution:

Using the op amp model in Fig. 5.4, we obtain the equivalent circuit of Fig. 5.6(a) as shown in Fig. 5.6(b). We now solve the circuit in Fig. 5.6(b) by using nodal analysis. At node 1, KCL gives

$$\frac{v_s - v_1}{10 \times 10^3} = \frac{v_1}{2000 \times 10^3} + \frac{v_1 - v_o}{20 \times 10^3}$$

Multiplying through by 2000×10^3, we obtain

$$200v_s = 301v_1 - 100v_o$$

or

$$2v_s \simeq 3v_1 - v_o \quad \Rightarrow \quad v_1 = \frac{2v_s + v_o}{3} \qquad \textbf{(5.1.1)}$$

At node O,

$$\frac{v_1 - v_o}{20 \times 10^3} = \frac{v_o - Av_d}{50}$$

But $v_d = -v_1$ and $A = 200{,}000$. Then

$$v_1 - v_o = 400(v_o + 200{,}000v_1) \qquad \textbf{(5.1.2)}$$

Substituting v_1 from Eq. (5.1.1) into Eq. (5.1.2) gives

$$0 \simeq 26{,}667{,}067v_o + 53{,}333{,}333v_s \quad \Rightarrow \quad \frac{v_o}{v_s} = -1.9999699$$

This is closed-loop gain, because the 20-kΩ feedback resistor closes the loop between the output and input terminals. When $v_s = 2$ V, $v_o = -3.9999398$ V. From Eq. (5.1.1), we obtain $v_1 = 20.066667\ \mu$V. Thus,

$$i = \frac{v_1 - v_o}{20 \times 10^3} = 0.19999 \text{ mA}$$

It is evident that working with a nonideal op amp is tedious, as we are dealing with very large numbers.

5.3 Ideal Op Amp

- We assume an ideal op-amp if it has the following characteristics:
- Infinite open-loop gain A
- Infinite input resistance R_{i}, which means it will not affect any node it is attached to
- Zero output resistance R_{o}, which means it is load independent according to Thevenin's theorem

Parameter	**Typical range**	**Ideal values**
Open-loop gain, A	10^5 to 10^8	∞
Input resistance, R_i	10^5 to $10^{13}\,\Omega$	$\infty\Omega$
Output resistance, R_0	10 to 100 Ω	0Ω
Supply voltage, V_{cc}	5 to 24 V	

Ideal Op-amp

- Many modern op-amps come close to the ideal values:

  Most have very large gains, greater than *one million*

  Input impedances are often in the *giga-Ohm* to *terra-Ohm* range

- We assume every op amp is ideal

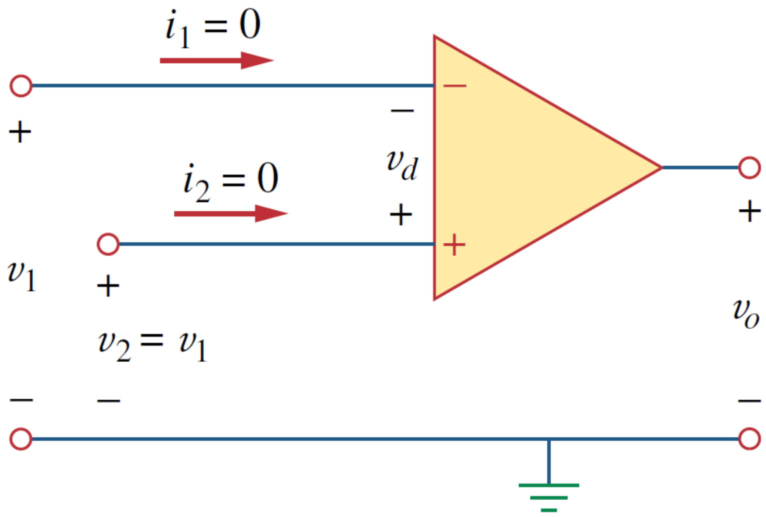


Figure 5.8
Ideal op amp model.

Two important properties of the ideal Op-amp

1. **The currents into both input terminals are zero**

$$-i_1 = i_2 = \frac{v_d}{R_i} = \frac{v_d}{\infty} = 0$$

$$\boxed{i_1 = i_2 = 0}$$

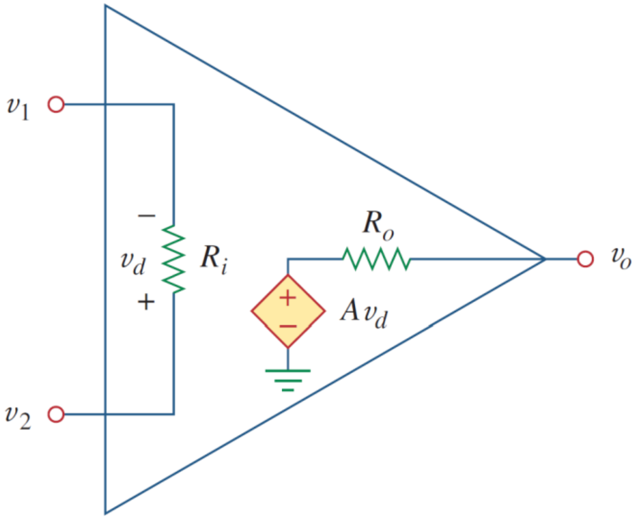


Figure 5.4
The equivalent circuit of the nonideal op amp.

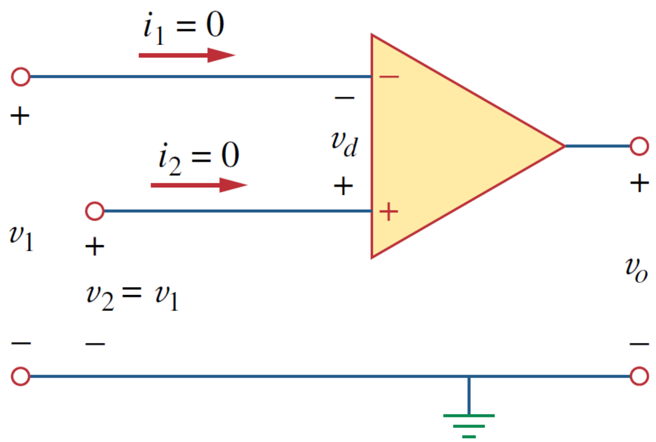


Figure 5.8
Ideal op amp model.

Two important properties of the ideal Op-amp

2. The voltage across the input terminals is equal to zero

$$v_o = Av_d = A(v_2 - v_1)$$

$$v_2 - v_1 = \frac{v_o}{A} = \frac{v_o}{\infty} = 0$$

$$\boxed{v_1 = v_2}$$

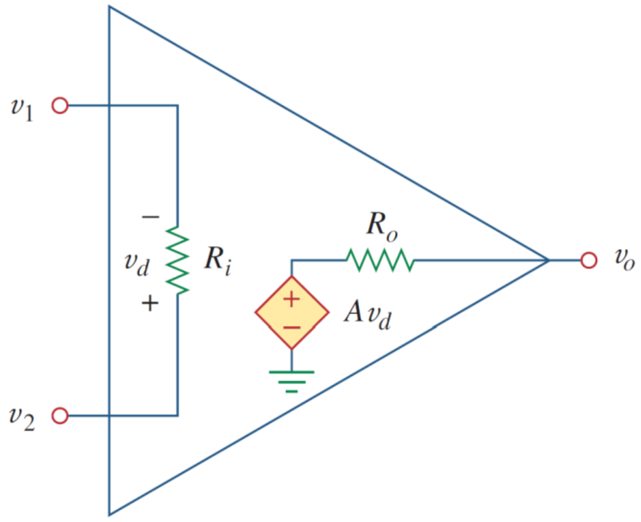


Figure 5.4
The equivalent circuit of the nonideal op amp.

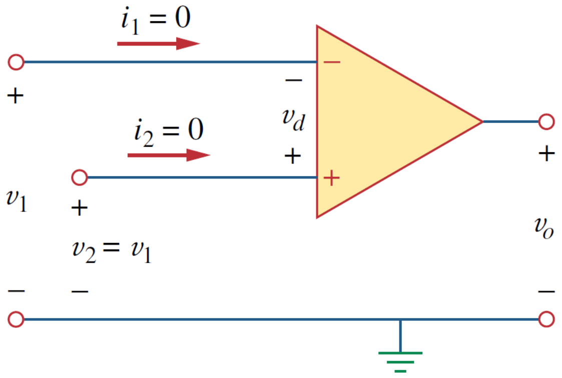


Figure 5.8
Ideal op amp model.

Example 5.2

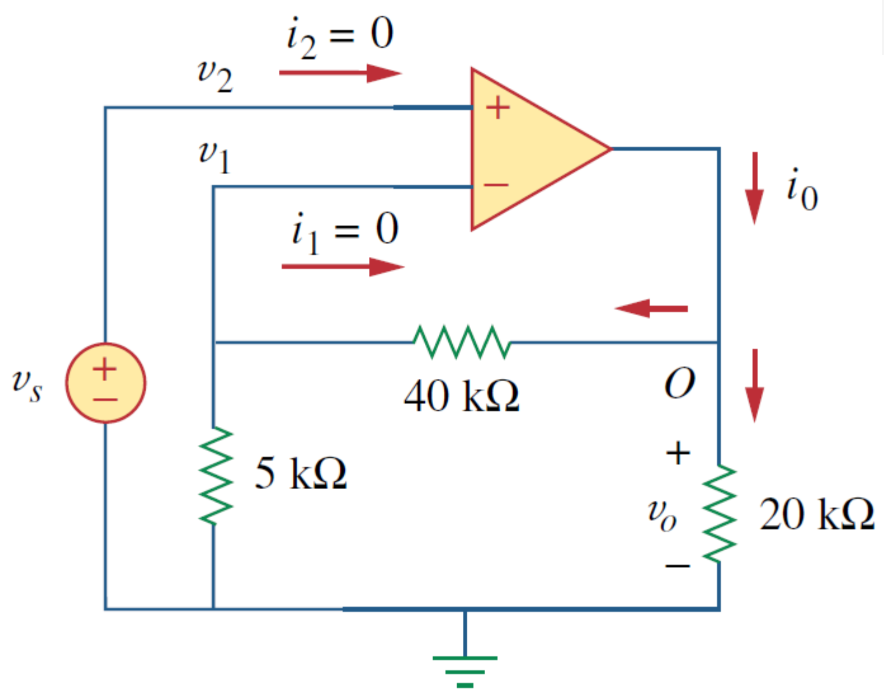


Figure 5.9
For Example 5.2.

Calculate the closed-loop gain v_o/v_s, find i_o when v_s=1V.

$$v_2 = v_s \tag{5.2.1}$$

Since $i_1 = 0$, the 40-kΩ and 5-kΩ resistors are in series; the same current flows through them. v_1 is the voltage across the 5-kΩ resistor. Hence, using the voltage division principle,

$$v_1 = \frac{5}{5+40}v_o = \frac{v_o}{9} \tag{5.2.2}$$

According to Eq. (5.7),

$$v_2 = v_1 \tag{5.2.3}$$

Substituting Eqs. (5.2.1) and (5.2.2) into Eq. (5.2.3) yields the closed-loop gain,

$$v_s = \frac{v_o}{9} \quad \Rightarrow \quad \frac{v_o}{v_s} = 9 \tag{5.2.4}$$

At node O,

$$i_o = \frac{v_o}{40+5} + \frac{v_o}{20}\text{mA} \tag{5.2.5}$$

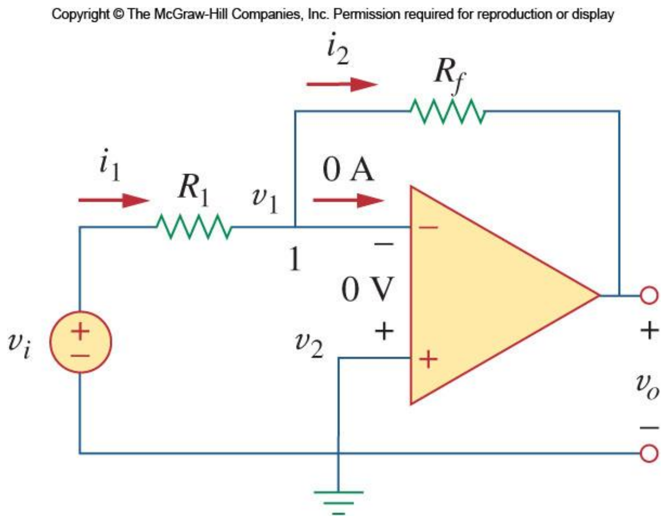
From Eq. (5.2.4), when $v_s = 1$ V, $v_o = 9$ V. Substituting for $v_o = 9$ V in Eq. (5.2.5) produces

$$i_o = 0.2 + 0.45 = 0.65 \text{ mA}$$

5.4 Inverting Amplifier

- In the circuit, the noninverting input is grounded
- The input $\boldsymbol{v_i}$ is connected to the inverting input through resistor $\boldsymbol{R_1}$
- The inverting input is connected to the output voltage $\boldsymbol{v_o}$ via a feedback resistor $\boldsymbol{R_f}$

  (negative feedback)



The closed-loop gain (The relationship between v_i and v_o)

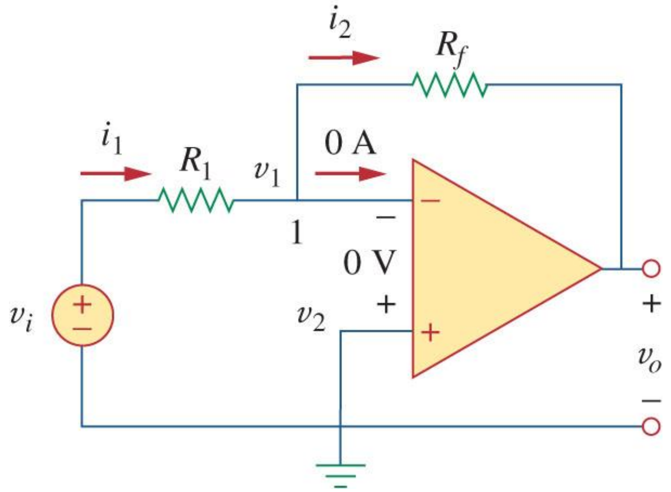
- By applying KCL to node 1, one can see that:

$$i_2 = i_1 \Rightarrow \frac{v_i - v_1}{R_1} = \frac{v_1 - v_0}{R_f}$$

- The noninverting terminal is grounded, v_2=0
- For an ideal op amp, we have v_2=v_1, so v_1= v_2=0
- This yields:

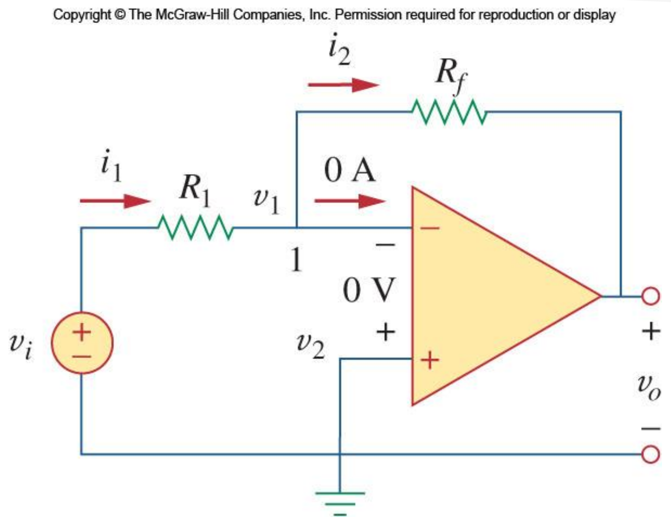
$$\frac{v_i}{R_1} = -\frac{v_0}{R_f}$$

$$\boxed{v_o = -\frac{R_f}{R_1} v_i}$$



The closed-loop gain (The relationship between v_i and v_o)

$$\boxed{v_o = -\frac{R_f}{R_1} v_i}$$

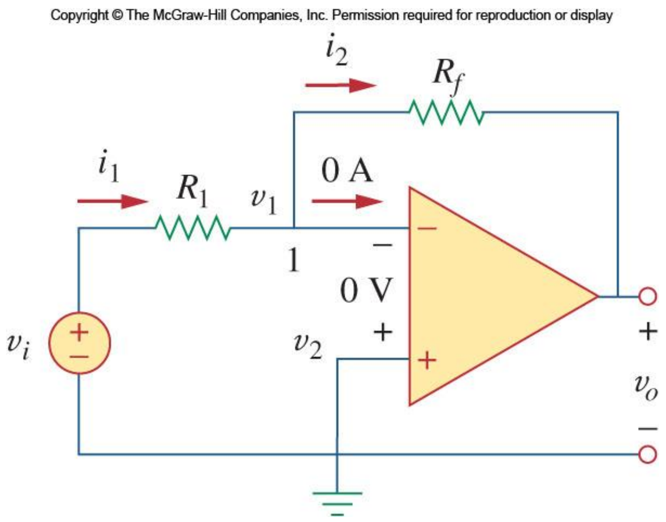


From this we can see:

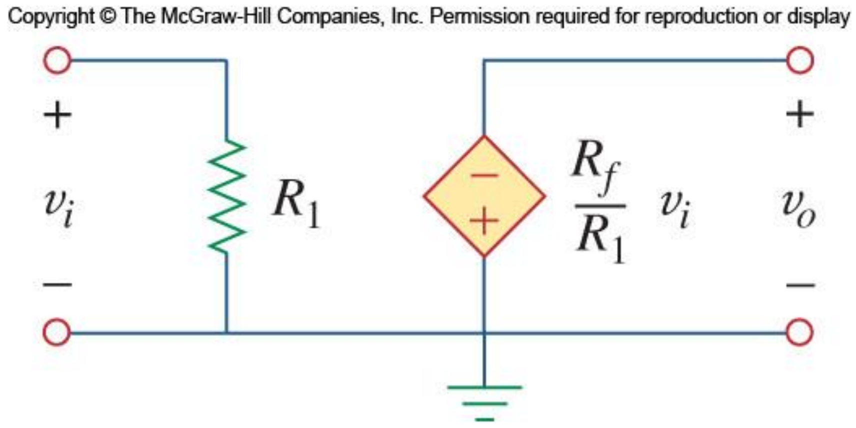
- The closed-loop gain is the ratio of the feedback resistor $\boldsymbol{R_f}$ to the input resistor $\mathbf{R_1}$, which only depends on the external elements connected and is insensitive to the open-loop gain *A* of the op amp.
- The polarity of the output is the reverse of the input, thus the name “inverting” amplifier

Equivalent Circuit for the inverting amplifier

- The input resistance is $\boldsymbol{R_1}$



Inverting Amplifier



Equivalent Circuit

Example

Example 5.3

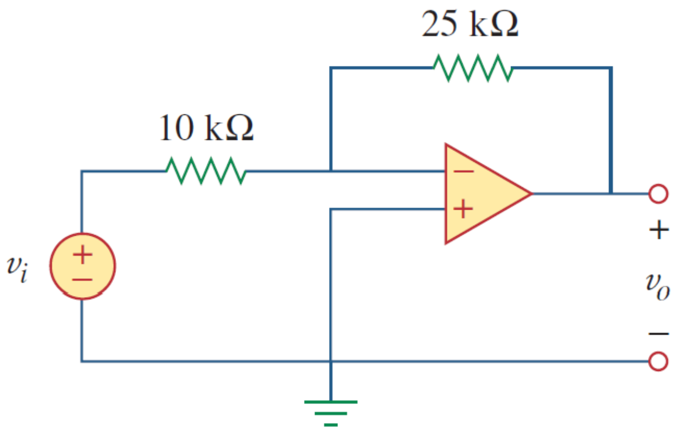


Figure 5.12
For Example 5.3.

Refer to the op amp in Fig. 5.12. If $v_i = 0.5$ V, calculate: (a) the output voltage v_o, and (b) the current in the 10-kΩ resistor.

Solution:

(a) Using Eq. (5.9),

$$\frac{v_o}{v_i} = -\frac{R_f}{R_1} = -\frac{25}{10} = -2.5$$

$$v_o = -2.5v_i = -2.5(0.5) = -1.25 \text{ V}$$

(b) The current through the 10-kΩ resistor is

$$i = \frac{v_i - 0}{R_1} = \frac{0.5 - 0}{10 \times 10^3} = 50\ \mu\text{A}$$

Example

Practice Problem 5.3

Find the output of the op amp circuit shown in Fig. 5.13. Calculate the current through the feedback resistor.

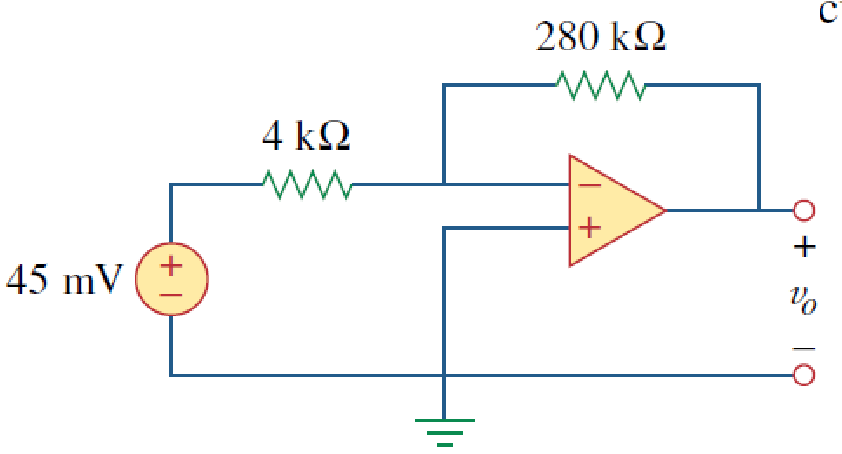


Figure 5.13
For Practice Prob. 5.3.

Answer: -3.15V, 11.25µA

Example

Determine v_o in the op amp circuit shown in Fig. 5.14.

Example 5.4

Solution:

Applying KCL at node a,

$$\frac{v_a - v_o}{40\text{ k}\Omega} = \frac{6 - v_a}{20\text{ k}\Omega}$$

$$v_a - v_o = 12 - 2v_a \quad \Rightarrow \quad v_o = 3v_a - 12$$

But $v_a = v_b = 2$ V for an ideal op amp, because of the zero voltage drop across the input terminals of the op amp. Hence,

$$v_o = 6 - 12 = -6 \text{ V}$$

Notice that if $v_b = 0 = v_a$, then $v_o = -12$, as expected from Eq. (5.9).

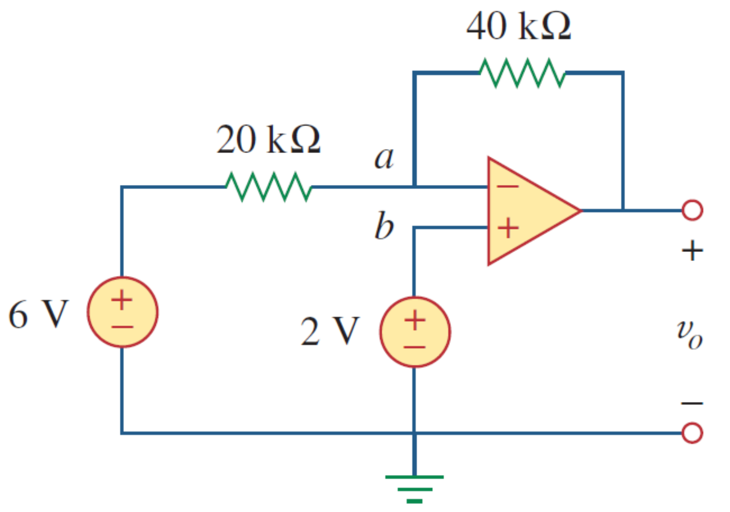


Figure 5.14
For Example 5.4.

Example

Practice Problem 5.4

Two kinds of current-to-voltage converters (also known as *transresistance amplifiers*) are shown in Fig. 5.15.

(a) Show that for the converter in Fig. 5.15(a),

$$\frac{v_o}{i_s} = -R$$

(b) Show that for the converter in Fig. 5.15(b),

$$\frac{v_o}{i_s} = -R_1\left(1 + \frac{R_3}{R_1} + \frac{R_3}{R_2}\right)$$

Answer: Proof.

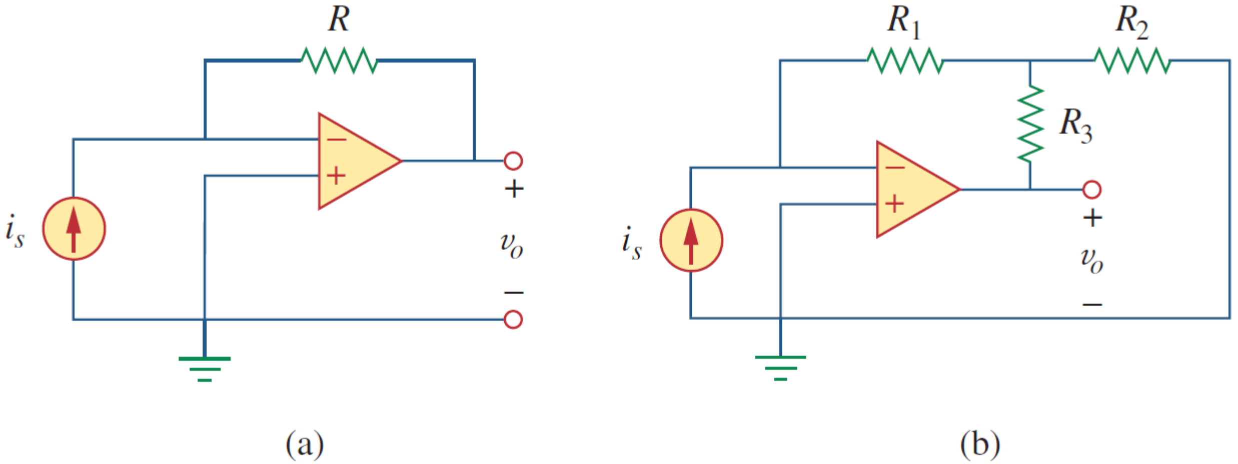
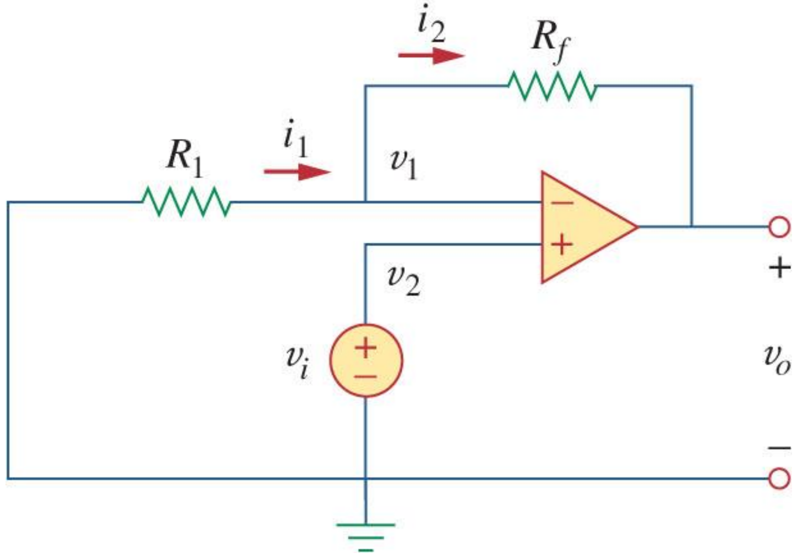


Figure 5.15
For Practice Prob. 5.4.

5.5 Non-Inverting Amplifier

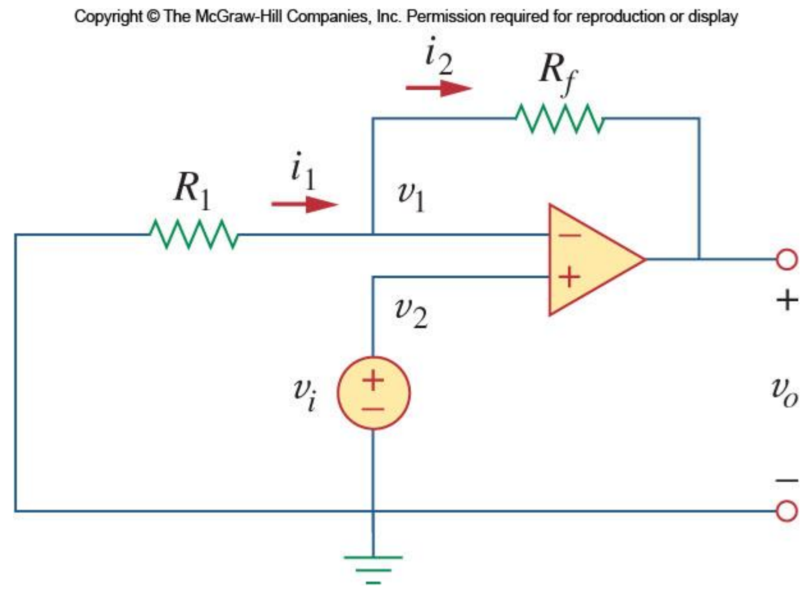
- The input $\boldsymbol{v_i}$ is connected to the noninverting input
- Resistor R_1 is connected between the inverting terminal and the ground
- The inverting input is connected to the output voltage $\boldsymbol{v_o}$ via a feedback resistor $\boldsymbol{R_f}$ **(negative feedback)**



Non-Inverting Amplifier

Applying KCL to the inverting terminal gives:

$$i_1 = i_2 \Rightarrow \frac{0 - v_1}{R_1} = \frac{v_1 - v_0}{R_f}$$



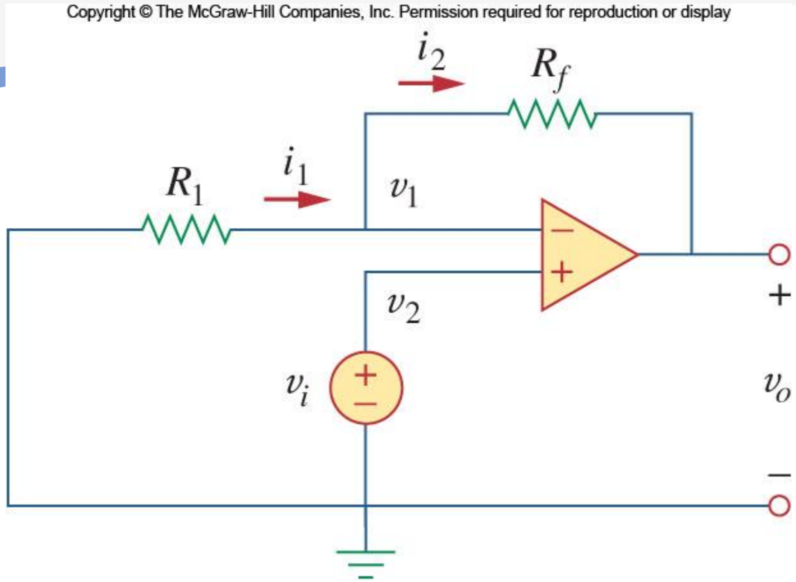
We have $v_1 = v_2 = v_i$, so this gives the following relationship:

$$\frac{-v_i}{R_1} = \frac{v_i - v_0}{R_f}$$

The output voltage is thus:

$$v_0 = \left(1 + \frac{R_f}{R_1}\right) v_i$$

Non-Inverting Amplifier



$$v_0 = \left(1 + \frac{R_f}{R_1}\right) v_i$$

- Note that the gain here is positive, thus the output and the input have the same polarity, and the amplifier is noninverting.
- The gain depends only on the external resistors.
- Also note that this amplifier retains the infinite input resistance of the op-amp.
- The amplifier's gain can never go below 1.

Voltage follower

$$v_0 = \left(1 + \frac{R_f}{R_1}\right) v_i$$

- If the feedback resistor $R_f = 0$ (short circuit) or $R_1 = \infty$ (open circuit) or both, the gain becomes 1. This configuration is called a voltage follower or a unity gain amplifier.

- It is good to separate two circuits while allowing the signal to pass through.
- Such a circuit has a very high input impedance and is therefore useful as an intermediate-stage (or buffer) to isolate one circuit from another. The voltage follower minimizes interaction between the two stages and eliminates interstage loading.

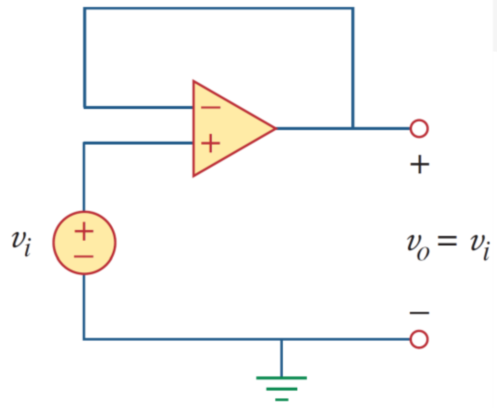


Figure 5.17
The voltage follower.

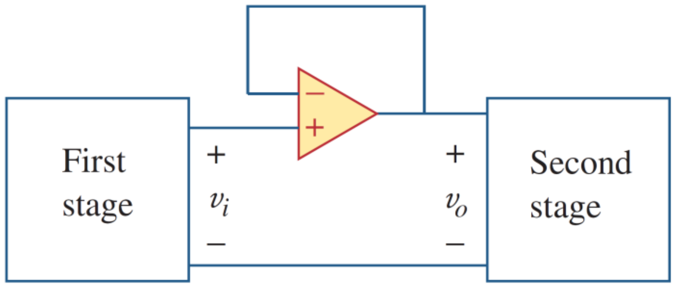
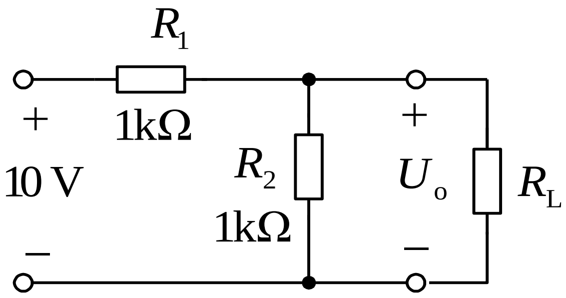


Figure 5.18
A voltage follower used to isolate two cascaded stages of a circuit.

Example Voltage divider (1) without 1kΩ load; (2) with 1kΩ load. Find the output voltage.

【**Solution**】（1）without 1kΩ load

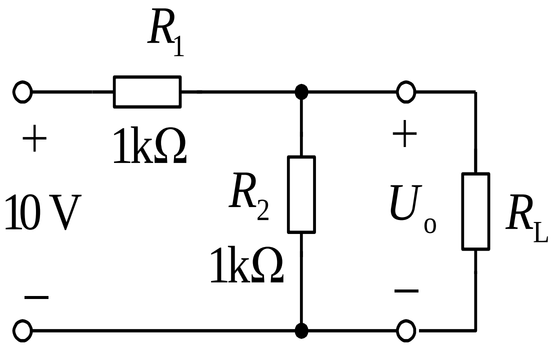


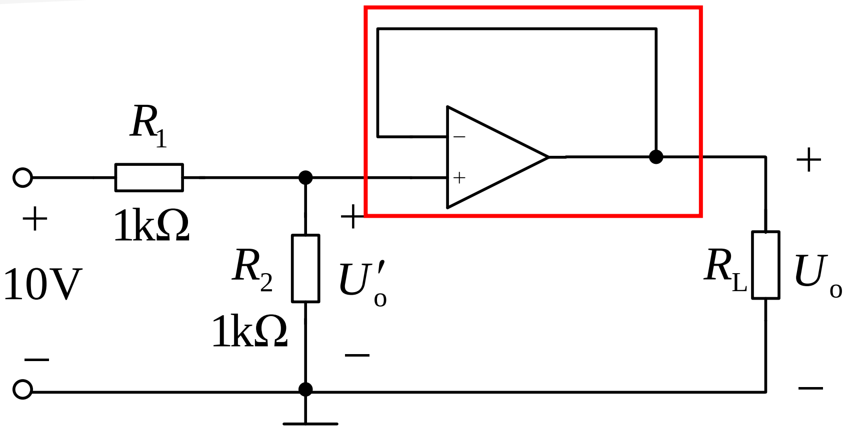
$$U_o = \frac{R_2}{R_1 + R_2} \times 10 = \frac{1}{1+1} \times 10 = 5\text{V}$$

（2）with 1kΩ load

$$U_o = \frac{R_2 // R_L}{R_1 + R_2 // R_L} \times 10 = \frac{0.5}{1+0.5} \times 10 = 3.3\text{V}$$

Example





$$U_o' = \frac{R_2}{R_1 + R_2} \times 10 = \frac{1}{1+1} \times 10 = 5\text{V}$$

$$U_o = U_o' = 5\text{V}$$

With the voltage follower, no matter how the load changes, the load voltage remains unchanged

Example

Example 5.5

For the op amp circuit in Fig. 5.19, calculate the output voltage v_o.

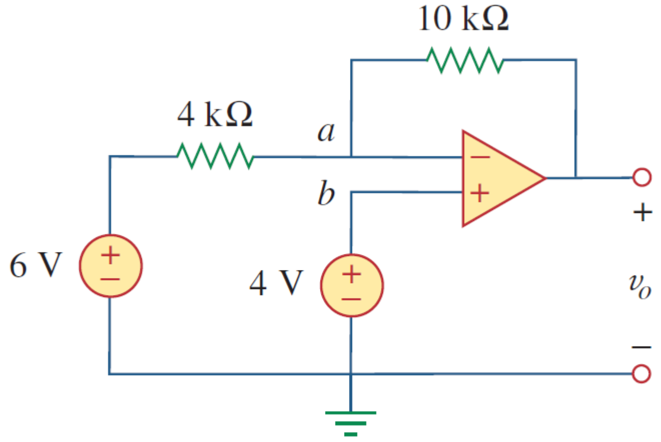


Figure 5.19
For Example 5.5.

METHOD 2 Applying KCL at node a,

$$\frac{6 - v_a}{4} = \frac{v_a - v_o}{10}$$

But $v_a = v_b = 4$, and so

$$\frac{6 - 4}{4} = \frac{4 - v_o}{10} \quad \Rightarrow \quad 5 = 4 - v_o$$

or $v_o = -1$ V, as before.

Example

Example 5.5

For the op amp circuit in Fig. 5.19, calculate the output voltage v_o.

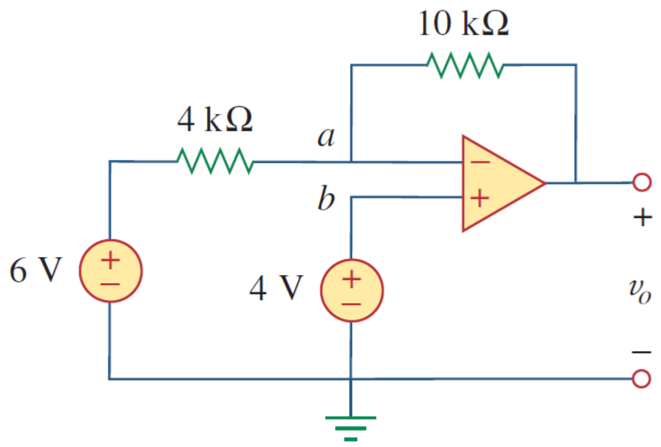


Figure 5.19
For Example 5.5.

Solution:
We may solve this in two ways: using superposition and using nodal analysis.

■ **METHOD 1** Using superposition, we let

$$v_o = v_{o1} + v_{o2}$$

where v_{o1} is due to the 6-V voltage source, and v_{o2} is due to the 4-V input. To get v_{o1}, we set the 4-V source equal to zero. Under this condition, the circuit becomes an inverter. Hence Eq. (5.9) gives

$$v_{o1} = -\frac{10}{4}(6) = -15 \text{ V}$$

To get v_{o2}, we set the 6-V source equal to zero. The circuit becomes a noninverting amplifier so that Eq. (5.11) applies.

$$v_{o2} = \left(1 + \frac{10}{4}\right)4 = 14 \text{ V}$$

Thus,

$$v_o = v_{o1} + v_{o2} = -15 + 14 = -1 \text{ V}$$

Example

Calculate v_o in the circuit of Fig. 5.20.

Practice Problem 5.5

Answer: 7 V.

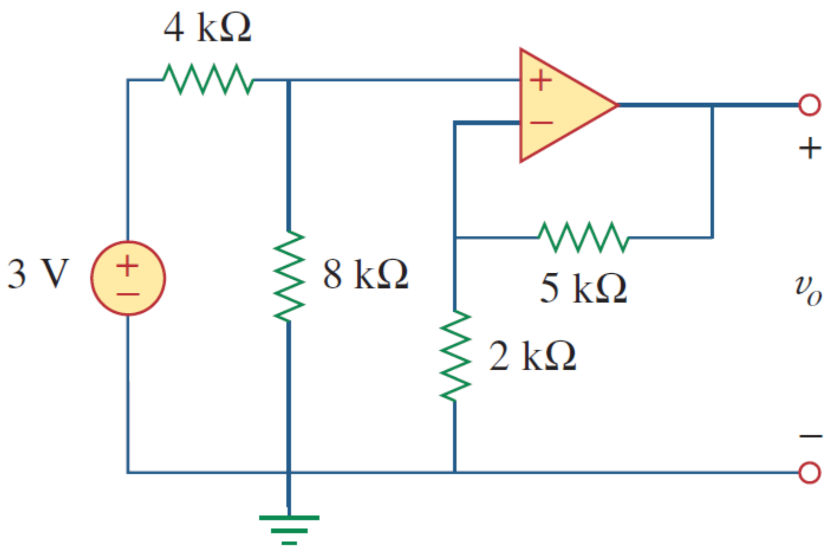


Figure 5.20
For Practice Prob. 5.5.

Example Find the output voltage $\boldsymbol{u_o}$ in the circuit； if $u_o = 0.5u_i$, how to design the amplifier?

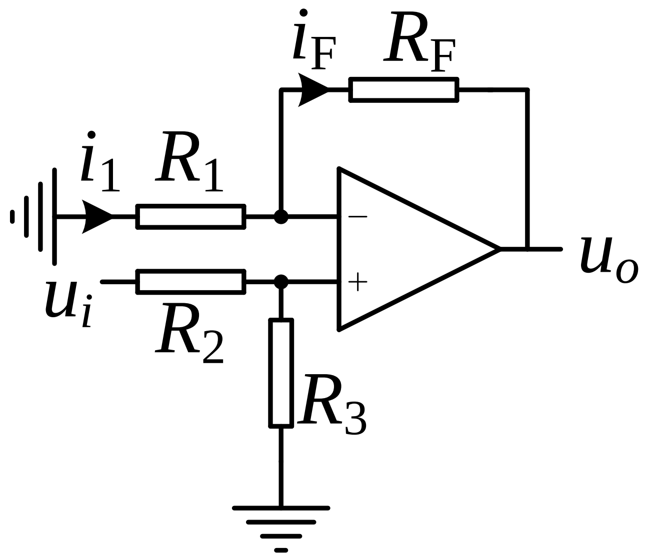
【**Solution**】

$$i_1 = i_F$$

$$\frac{0-u_-}{R_1} = \frac{u_- - u_o}{R_F}$$

$$u_- = u_+$$

$$u_+ = \frac{R_3}{R_2+R_3}u_i$$



$$\boxed{u_o = (1+\frac{R_F}{R_1})u_+ = (1+\frac{R_F}{R_1})\frac{R_3}{R_2+R_3}u_i}$$

5.6 Summing Amplifier

- The summing amplifier is a variation of the inverting amplifier.
- It combines several inputs and produces an output that is the weighted sum of the inputs.
- The current entering each input of the op amp is zero. Applying KCL at node a gives

$$i = i_1 + i_2 + i_3$$

- The current from each input is proportional to the applied voltage and the input resistance

$$i_1 = \frac{(v_1 - v_a)}{R_1} \quad i_2 = \frac{(v_2 - v_a)}{R_2} \quad i_3 = \frac{(v_3 - v_a)}{R_3} \quad i = \frac{(v_a - v_0)}{R_f}$$

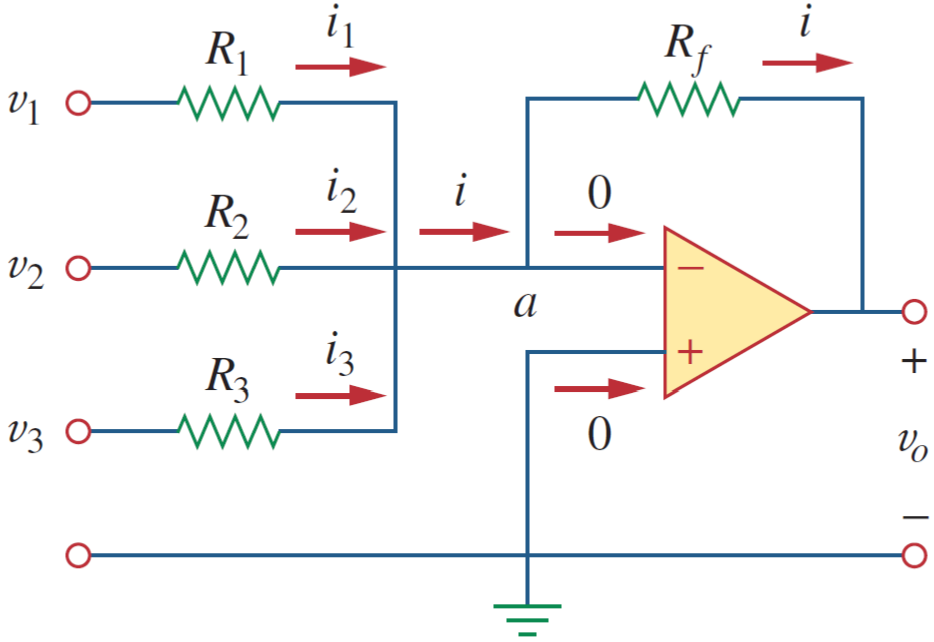


Figure 5.21
The summing amplifier.

Summing Amplifier

- We get the following relationship:

$$v_0 = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$

- Note that the output is a weighted sum of the inputs, it is also called a summer.
- The number of inputs is not limited.
- When R_1=R_2=R_3,

$$v_0 = -\frac{R_f}{R_1}(v_1 + v_2 + v_3)$$

- When R_1=R_2=R_3=R_f,

$$v_0 = -(v_1 + v_2 + v_3)$$

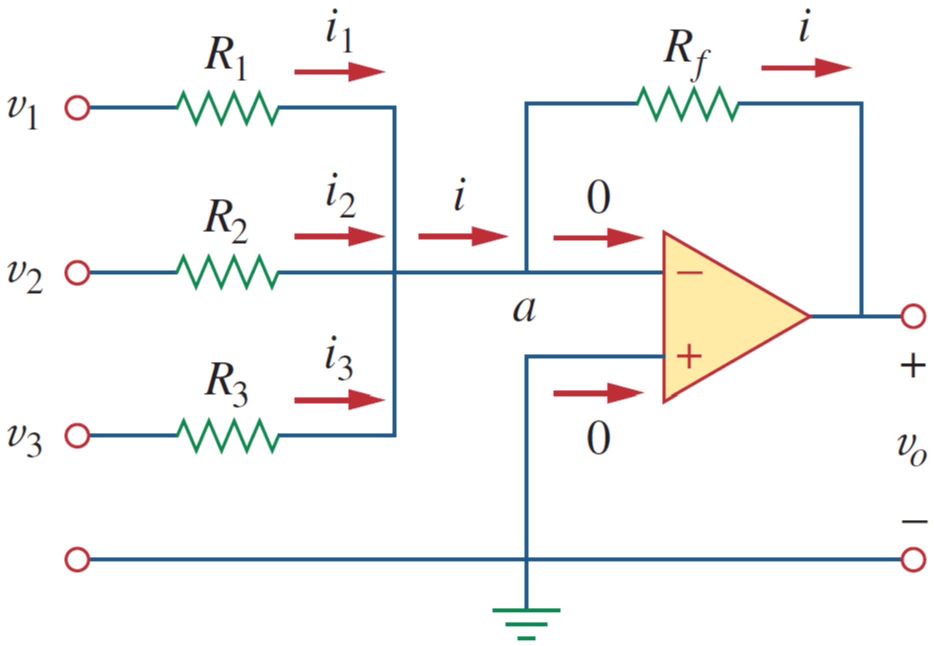
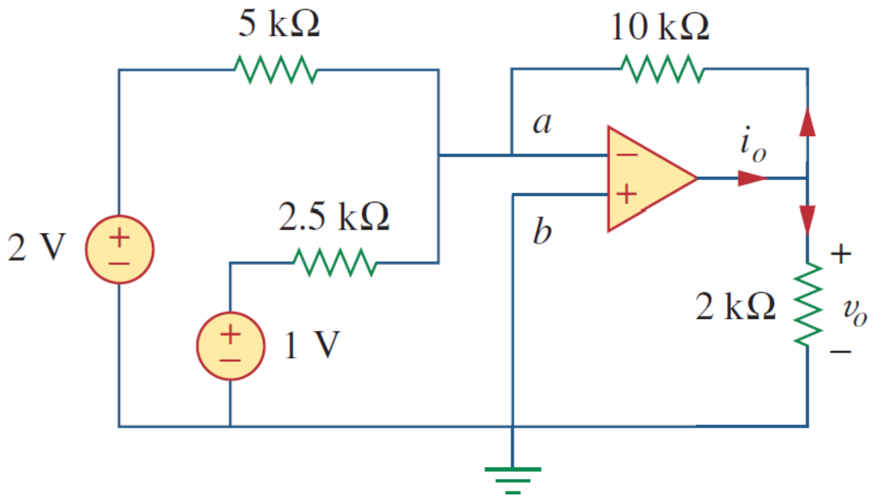


Figure 5.21
The summing amplifier.

ExampleExample 5.6

Calculate v_o and i_o in the op amp circuit in Fig. 5.22.



igure 5.22
or Example 5.6.

Solution:

This is a summer with two inputs. Using Eq. (5.15) gives

$$v_o = -\left[\frac{10}{5}(2) + \frac{10}{2.5}(1)\right] = -(4 + 4) = -8 \text{ V}$$

The current i_o is the sum of the currents through the 10-kΩ and 2-kΩ resistors. Both of these resistors have voltage $v_o = -8$ V across them, since $v_a = v_b = 0$. Hence,

$$i_o = \frac{v_o - 0}{10} + \frac{v_o - 0}{2} \text{ mA} = -0.8 - 4 = -4.8 \text{ mA}$$

Example

Find v_o and i_o in the op amp circuit shown in Fig. 5.23.

Practice Problem 5.6

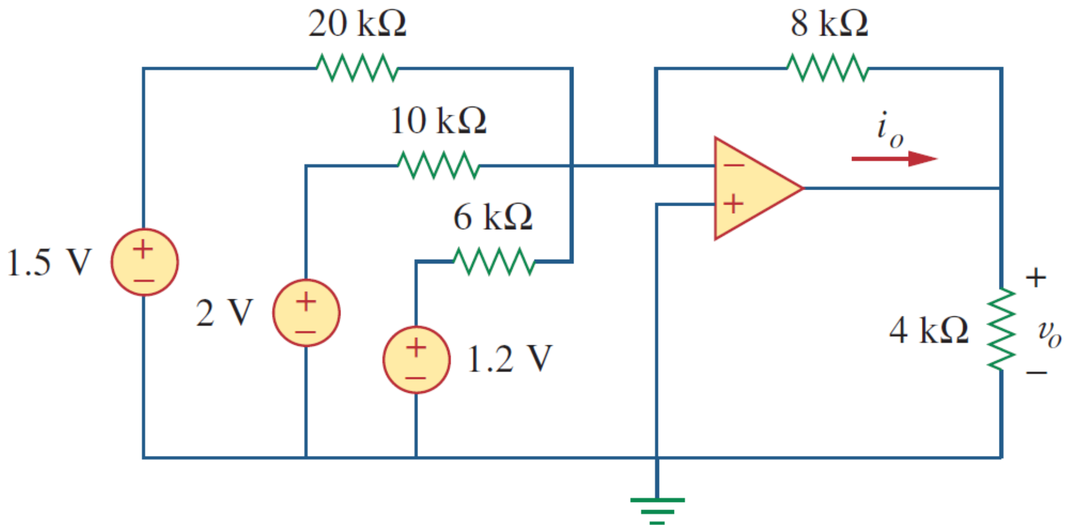


Figure 5.23
For Practice Prob. 5.6.

Answer: −3.8 V, −1.425 mA.

Example Find the output voltage u_0

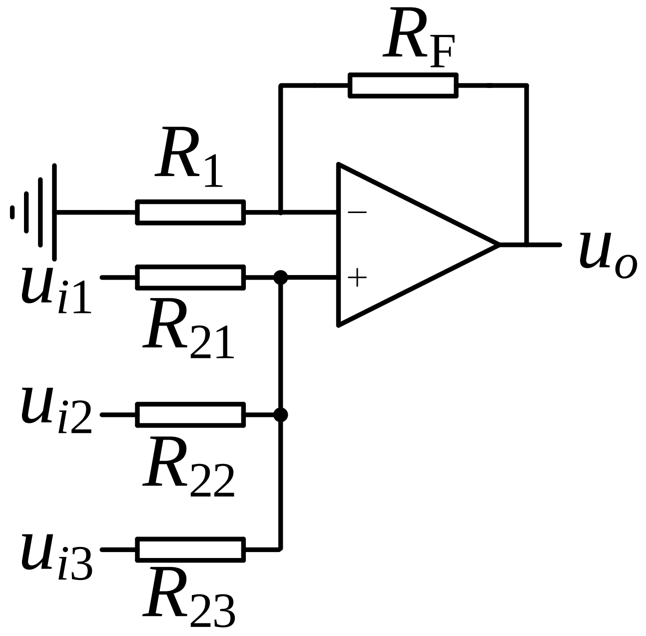
【Solution】 Superposition

$$u_o' = (1+\frac{R_F}{R_1})\frac{R_{22} // R_{23}}{R_{21}+R_{22} // R_{23}}u_{i1}$$

$$u_o'' = (1+\frac{R_F}{R_1})\frac{R_{21} // R_{23}}{R_{22}+R_{21} // R_{23}}u_{i2}$$

$$u_o''' = (1+\frac{R_F}{R_1})\frac{R_{21} // R_{22}}{R_{23}+R_{21} // R_{22}}u_{i3}$$

$$u_o = u_o' + u_o'' + u_o'''$$

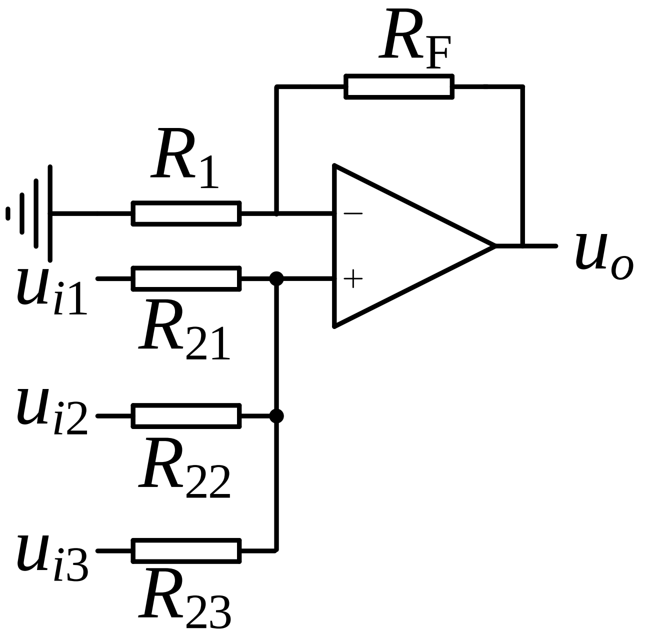


Example Find the output voltage u_0

【Solution】 Nodal analysis

$$u_o = (1+\frac{R_F}{R_1})u_+$$

$$u_+ = \frac{\frac{u_{i1}}{R_{21}}+\frac{u_{i2}}{R_{22}}+\frac{u_{i3}}{R_{23}}}{\frac{1}{R_{21}}+\frac{1}{R_{22}}+\frac{1}{R_{23}}}$$



$$\boxed{u_o = (1+\frac{R_F}{R_1})\,u_+ = (1+\frac{R_F}{R_1})\frac{\frac{u_{i1}}{R_{21}}+\frac{u_{i2}}{R_{22}}+\frac{u_{i3}}{R_{23}}}{\frac{1}{R_{21}}+\frac{1}{R_{22}}+\frac{1}{R_{23}}}}$$

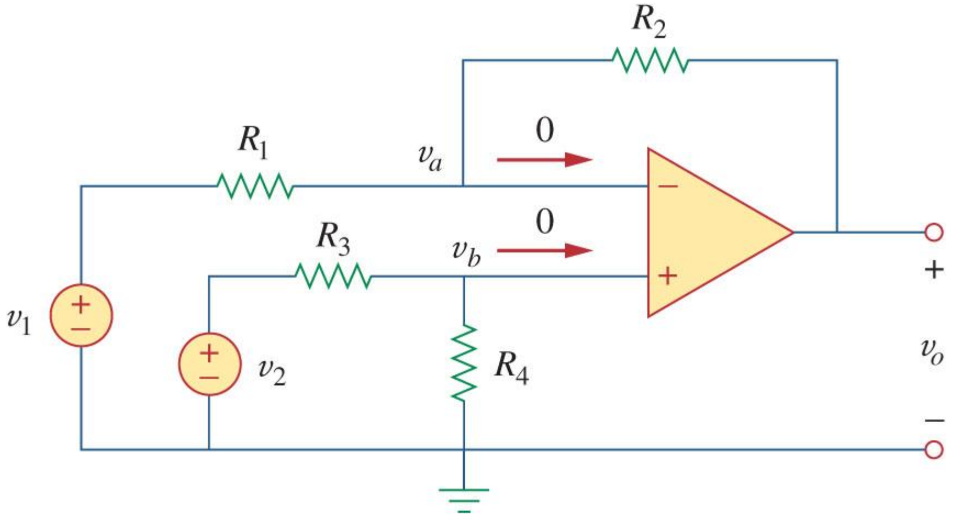
5.7 Difference Amplifier

- Difference (or differential) amplifiers are used in variations where there is a need to amplify the difference between two input signals.

- Applying KCL to node *a* in the circuit shown gives:

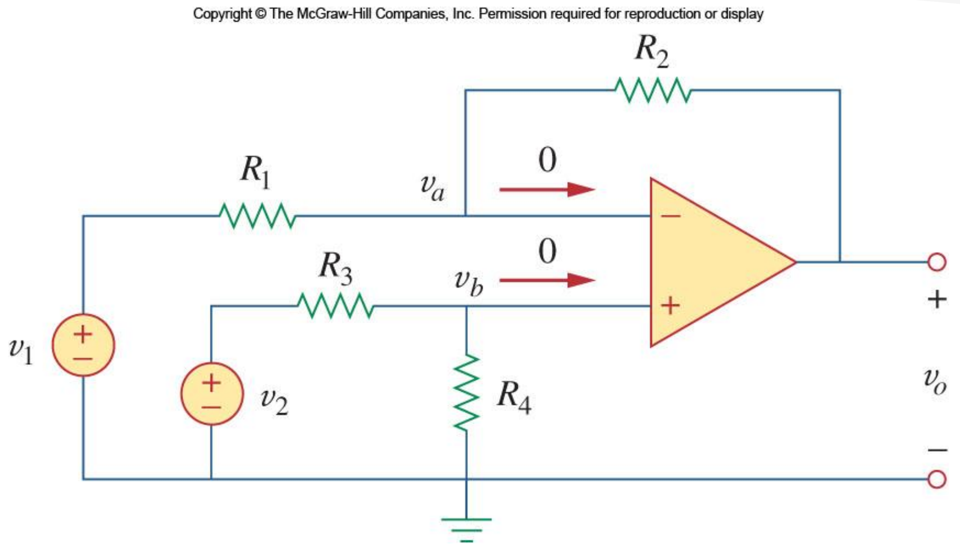
$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2}$$

$$v_0 = \left(\frac{R_2}{R_1} + 1\right) v_a - \frac{R_2}{R_1} v_1$$



Difference Amplifier

$$v_0 = \left(\frac{R_2}{R_1} + 1\right) v_a - \frac{R_2}{R_1} v_1$$



- Applying KCL to node b gives:

$$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4}$$

$$v_b = \frac{R_4}{R_3 + R_4} v_2$$

- $v_a = v_b$ resulting in the following relationship:

$$v_0 = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1$$

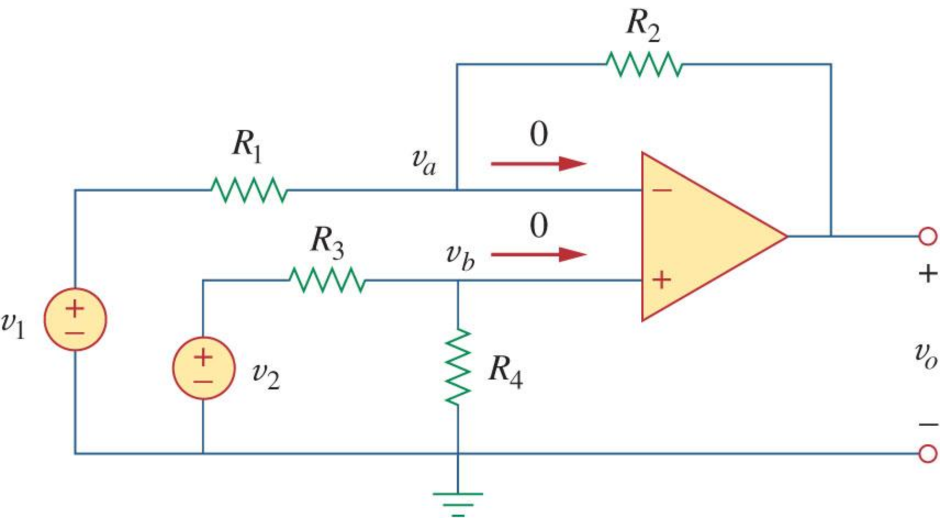
- *If* $R_1/R_2 = R_3/R_4$

$$v_0 = \frac{R_2}{R_1}(v_2 - v_1)$$

- *If* $R_1/R_2 = R_3/R_4 = 1$ $\quad v_0 = v_2 - v_1$

Example 5.7

Design an op amp circuit with inputs v_1 and v_2 such that $v_o = -5v_1 + 3v_2$.



$$v_0 = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1$$

Solution:

The circuit requires that

$$v_o = 3v_2 - 5v_1 \tag{5.7.1}$$

This circuit can be realized in two ways.

Design 1 If we desire to use only one op amp, we can use the op amp circuit of Fig. 5.24. Comparing Eq. (5.7.1) with Eq. (5.18), we see

$$\frac{R_2}{R_1} = 5 \quad \Rightarrow \quad R_2 = 5R_1 \tag{5.7.2}$$

Also,

$$5\frac{(1 + R_1/R_2)}{(1 + R_3/R_4)} = 3 \quad \Rightarrow \quad \frac{\frac{6}{5}}{1 + R_3/R_4} = \frac{3}{5}$$

or

$$2 = 1 + \frac{R_3}{R_4} \quad \Rightarrow \quad R_3 = R_4 \tag{5.7.3}$$

If we choose $R_1 = 10\ \text{k}\Omega$ and $R_3 = 20\ \text{k}\Omega$, then $R_2 = 50\ \text{k}\Omega$ and $R_4 = 20\ \text{k}\Omega$.

Example 5.7

Design an op amp circuit with inputs v_1 and v_2 such that $v_o = -5v_1 + 3v_2$.

Design 2 If we desire to use more than one op amp, we may cascade an inverting amplifier and a two-input inverting summer, as shown in Fig. 5.25. For the summer,

$$v_o = -v_a - 5v_1 \tag{5.7.4}$$

and for the inverter,

$$v_a = -3v_2 \tag{5.7.5}$$

Combining Eqs. (5.7.4) and (5.7.5) gives

$$v_o = 3v_2 - 5v_1$$

which is the desired result. In Fig. 5.25, we may select $R_1 = 10\ \text{k}\Omega$ and $R_3 = 20\ \text{k}\Omega$ or $R_1 = R_3 = 10\ \text{k}\Omega$.

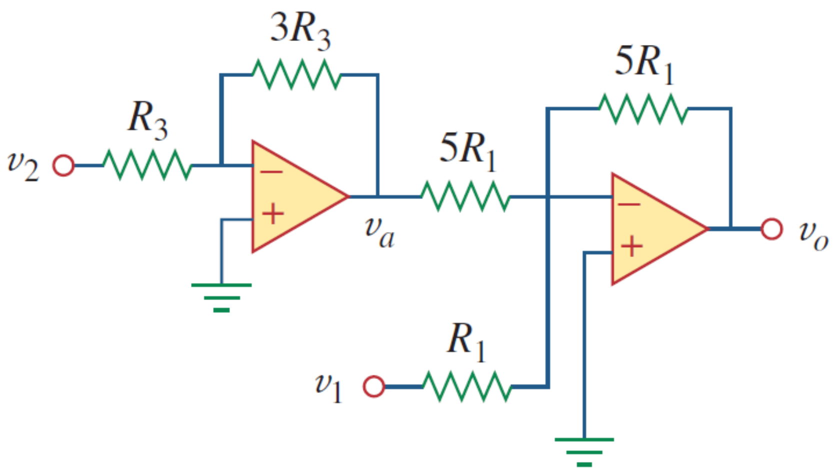


Figure 5.25
For Example 5.7.

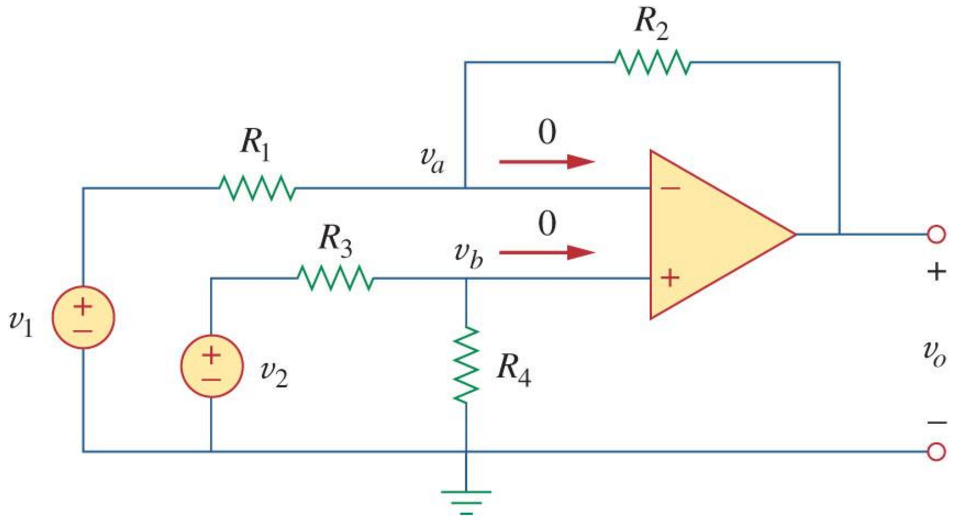
Practice Problem 5.7

Design a difference amplifier with gain 7.5.

Answer: Typical: $R_1 = R_3 = 20\text{k}\Omega$, $R_2 = R_4 = 150\ \text{k}\Omega$.

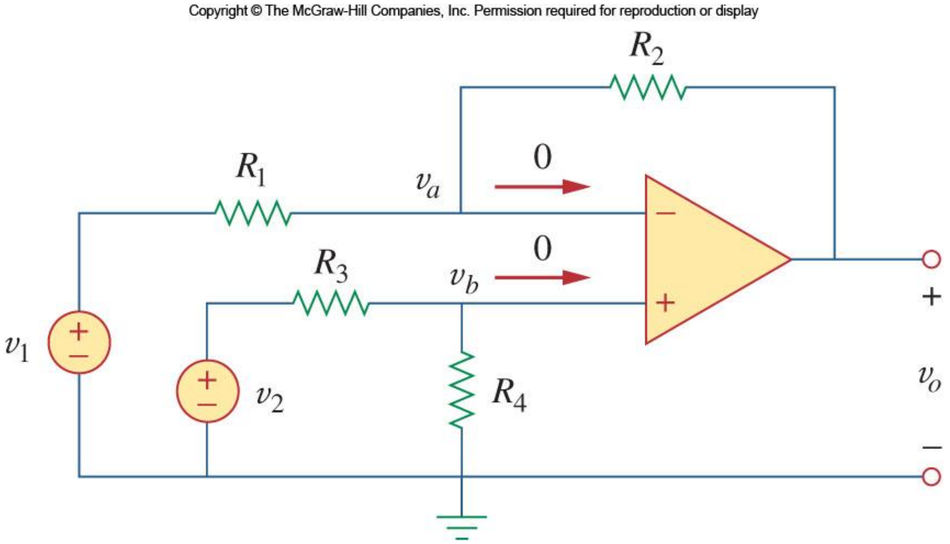
Example: Instrumentation Amplifier

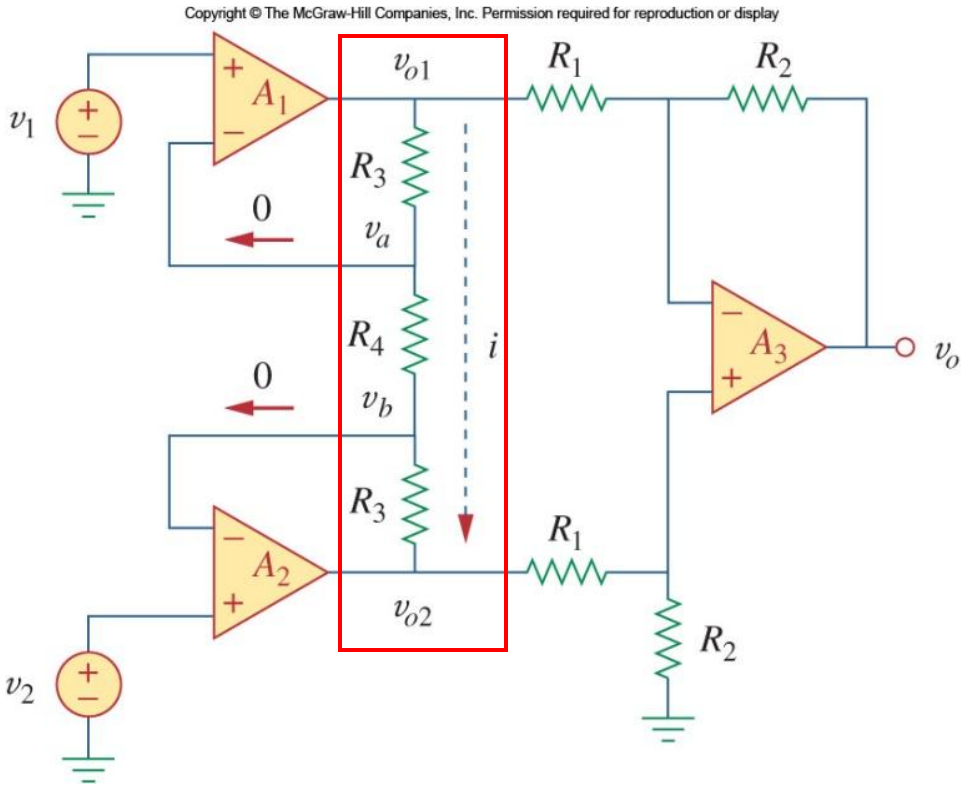
The difference amplifier has one significant drawback: the input resistance is low



Example: Instrumentation Amplifier

- By placing a noninverting amplifier stage before the difference amplifier, the problem can be resolved





Example: Instrumentation Amplifier

Example 5.8

An *instrumentation amplifier* shown in Fig. 5.26 is an amplifier of low-level signals used in process control or measurement applications and commercially available in single-package units. Show that

$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

Solution:

We recognize that the amplifier A_3 in Fig. 5.26 is a difference amplifier. Thus, from Eq. (5.20),

$$v_o = \frac{R_2}{R_1}(v_{o2} - v_{o1}) \qquad \textbf{(5.8.1)}$$

Since the op amps A_1 and A_2 draw no current, current i flows through the three resistors as though they were in series. Hence,

$$v_{o1} - v_{o2} = i(R_3 + R_4 + R_3) = i(2R_3 + R_4) \qquad \textbf{(5.8.2)}$$

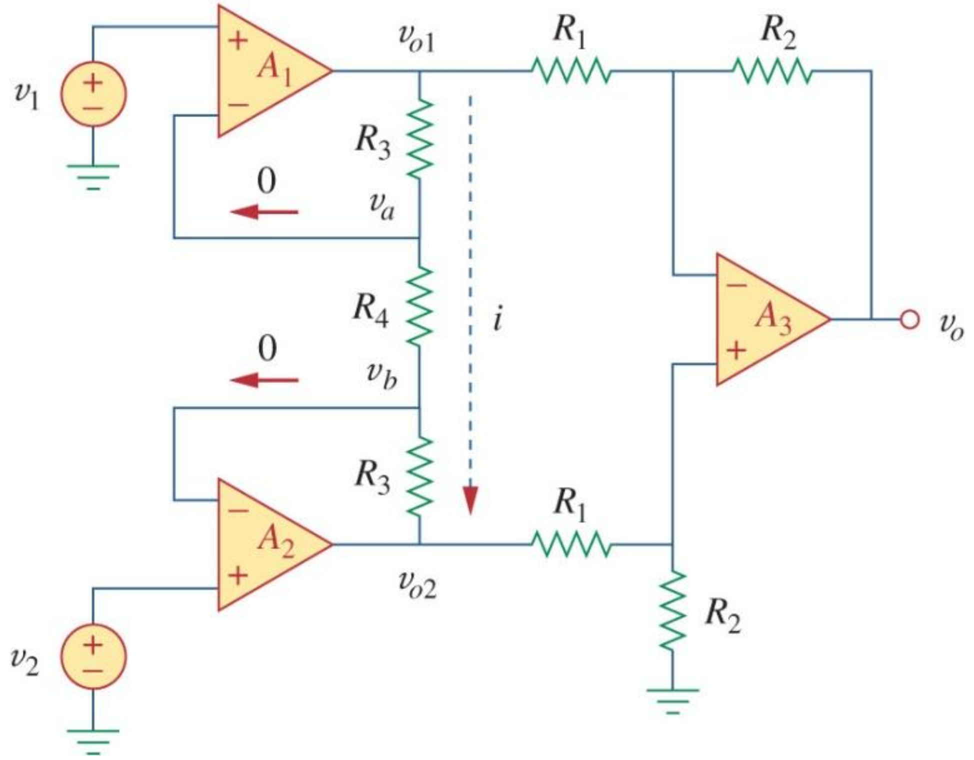
$$i = \frac{v_a - v_b}{R_4}$$

and $v_a = v_1$, $v_b = v_2$. Therefore,

$$i = \frac{v_1 - v_2}{R_4} \qquad \textbf{(5.8.3)}$$

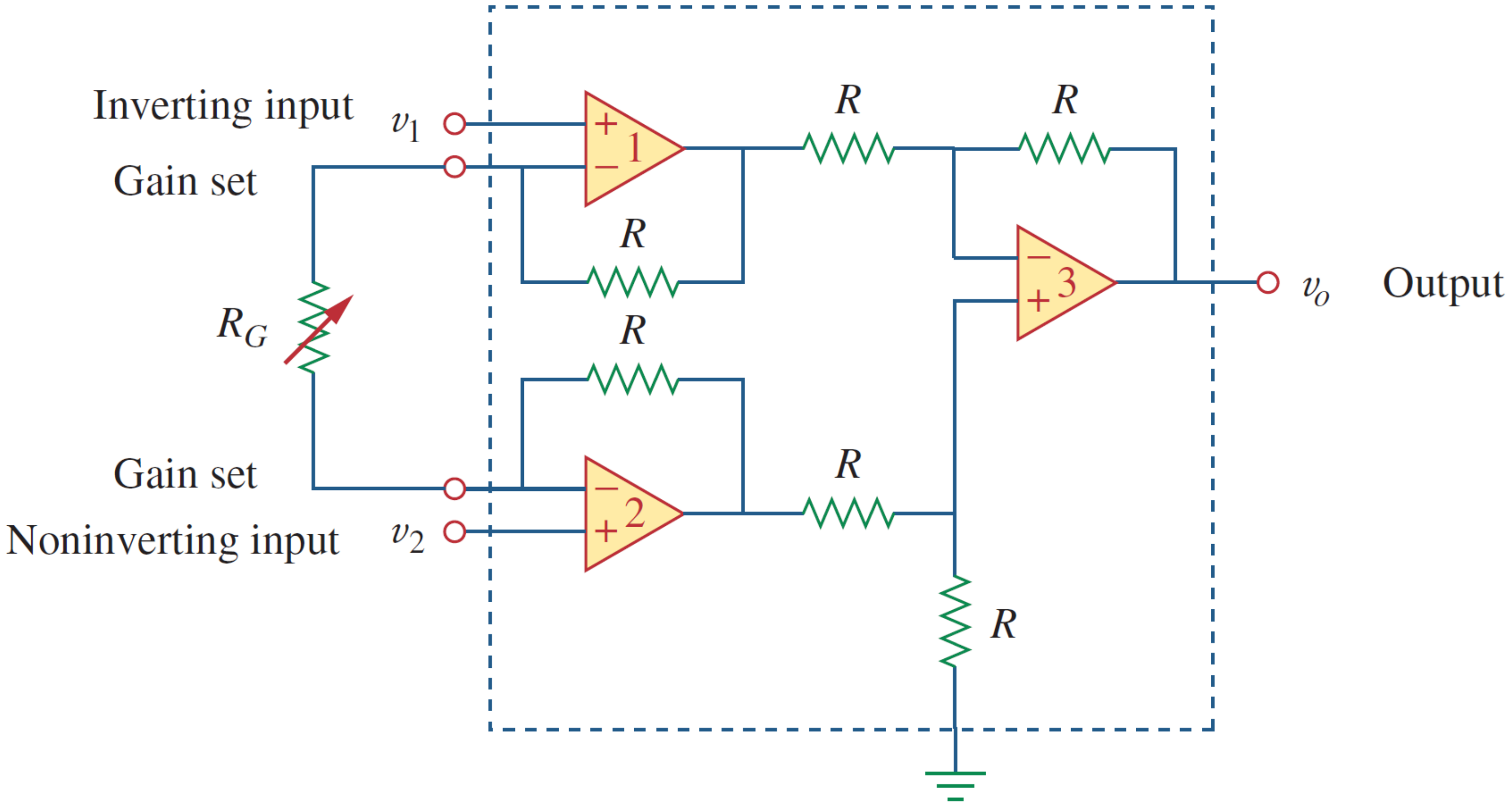
Inserting Eqs. (5.8.2) and (5.8.3) into Eq. (5.8.1) gives

$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$



Example: Instrumentation Amplifier

They are often packaged as a single chip with the only external component being the gain resistor, which are widely used in measurement systems.

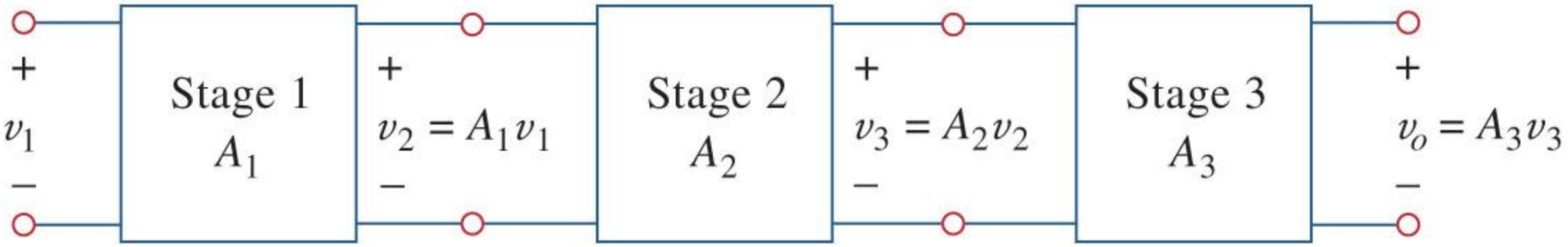


$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

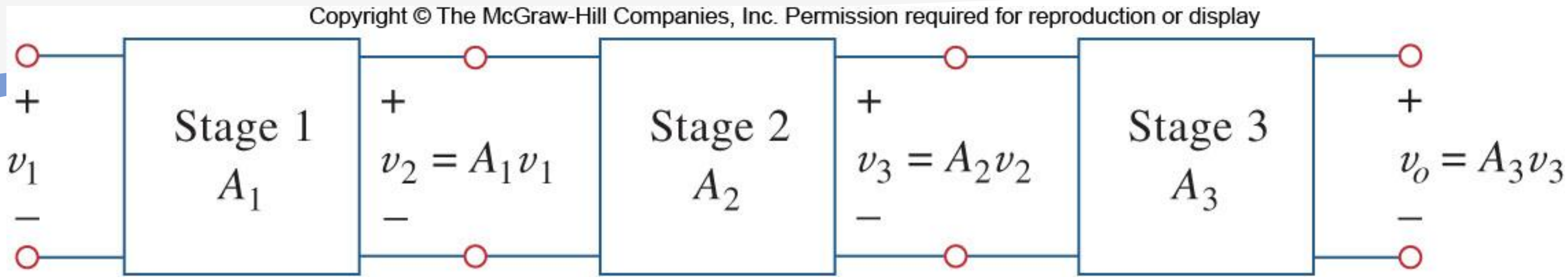
$$A_v = 1 + \frac{2R}{R_G}$$

5.8 Cascaded Op Amps

- It is common in practical applications to connect op-amp circuits in cascade to achieve a large overall gain.
- Each amplifier is then called a "stage".
- A cascade connection is a *head-to-tail* arrangement of two or more op-amp circuits such that the output of one stage is the input to the next stage.



Cascaded Op Amps



- As the ideal op-amp circuit has infinite input resistance and zero output resistance, stages can be cascaded without changing their input-output relationships.

- The overall gain of the cascaded connection is the product of the individual gains:

$$A = A_1 A_2 A_3$$

Example

Find v_o and i_o in the circuit in Fig. 5.29.

Example 5.9

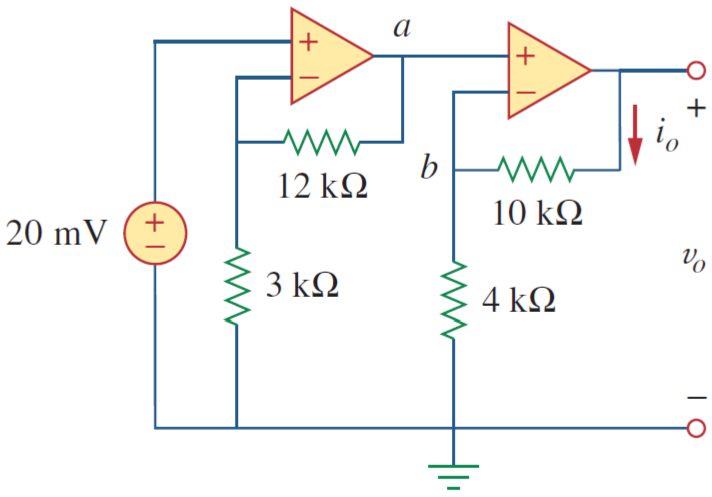


Figure 5.29
For Example 5.9.

Solution:
This circuit consists of two noninverting amplifiers cascaded. At the output of the first op amp,

$$v_a = \left(1 + \frac{12}{3}\right)(20) = 100 \text{ mV}$$

At the output of the second op amp,

$$v_o = \left(1 + \frac{10}{4}\right)v_a = (1 + 2.5)100 = 350 \text{ mV}$$

The required current i_o is the current through the 10-kΩ resistor.

$$i_o = \frac{v_o - v_b}{10} \text{ mA}$$

But $v_b = v_a = 100$ mV. Hence,

$$i_o = \frac{(350 - 100) \times 10^{-3}}{10 \times 10^3} = 25\ \mu\text{A}$$

Example

Practice Problem 5.9

Determine v_o and i_o in the op amp circuit in Fig. 5.30.

Answer: 6 V, 24 μA.

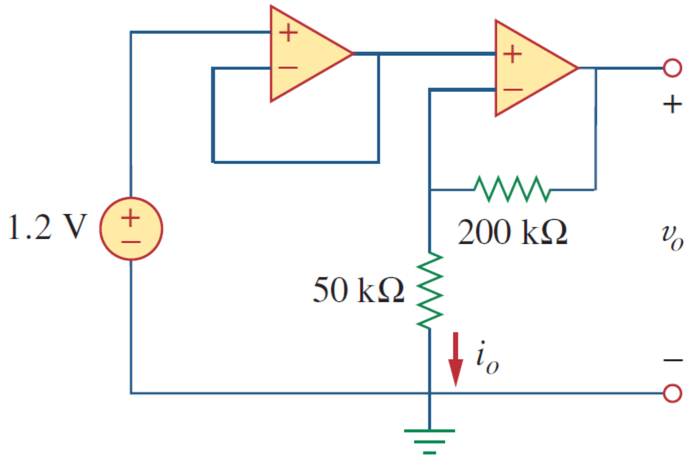


Figure 5.30
For Practice Prob. 5.9.

Example

Example 5.10

If $v_1 = 1$ V and $v_2 = 2$ V, find v_o in the op amp circuit of Fig. 5.31.

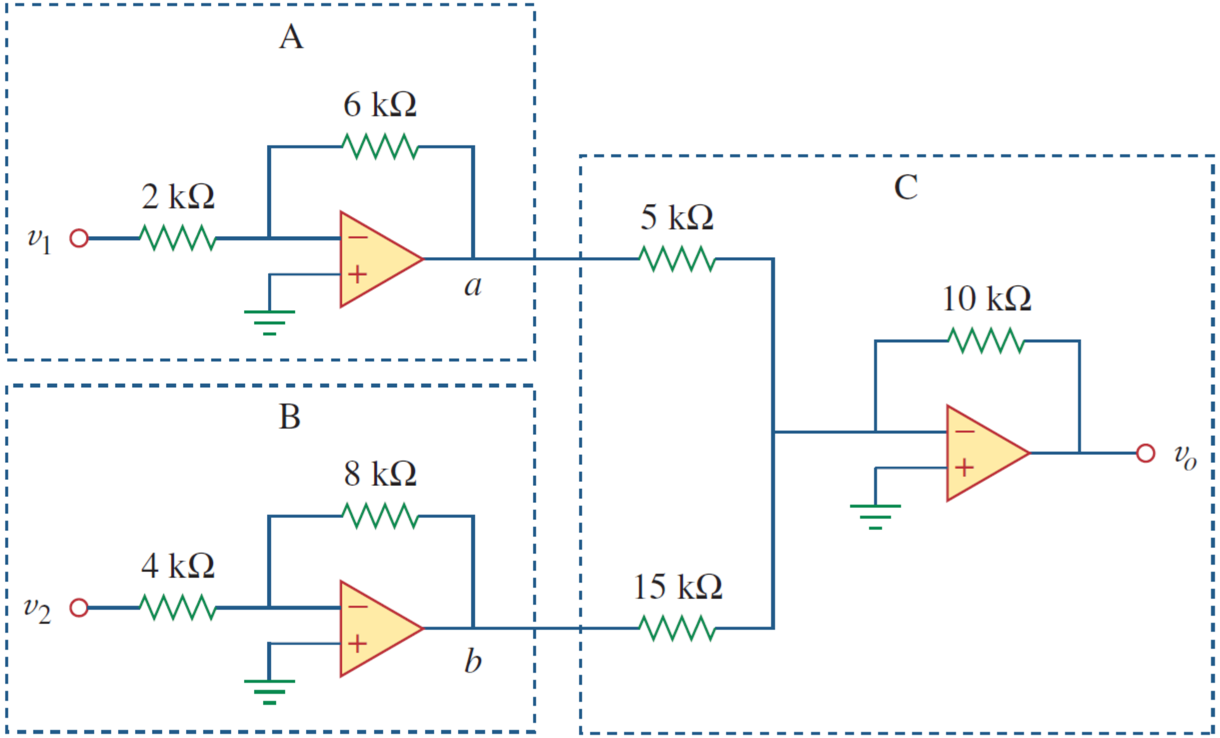


Figure 5.31
For Example 5.10.

【Solution】

Attempt. Let the output of the first op amp circuit be designated as v_{11} and the output of the second op amp circuit be designated as v_{22}. Then we get

$$v_{11} = -3v_1 = -3 \times 1 = -3 \text{ V},$$
$$v_{22} = -2v_2 = -2 \times 2 = -4 \text{ V}$$

In the third circuit we have

$$\begin{aligned} v_o &= -(10\text{ k}\Omega/5\text{ k}\Omega)v_{11} + [-(10\text{ k}\Omega/15\text{ k}\Omega)v_{22}] \\ &= -2(-3) - (2/3)(-4) \\ &= 6 + 2.667 = \mathbf{8.667\ V} \end{aligned}$$

Example

Practice Problem 5.10

If $v_1 = 7$ V and $v_2 = 3.1$ V, find v_o in the op amp circuit of Fig. 5.33.

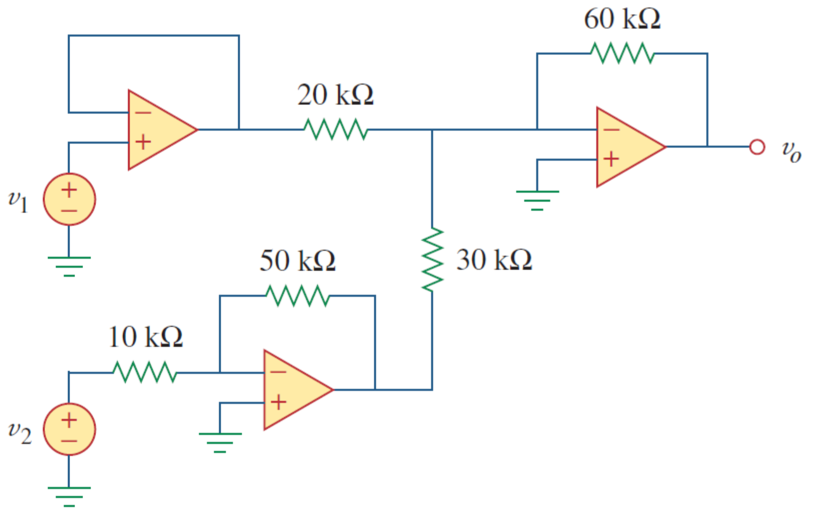


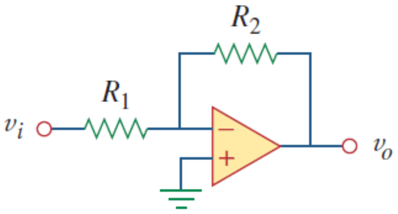
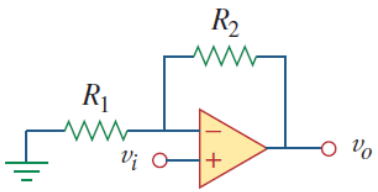
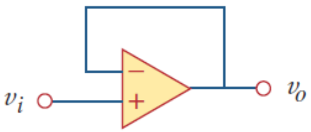
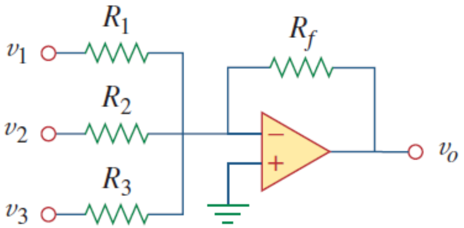
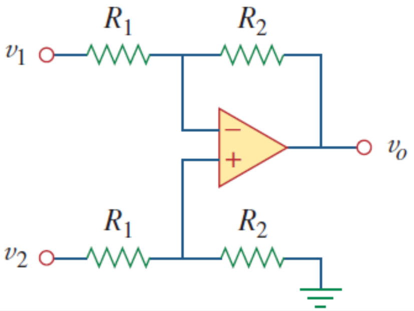
Figure 5.33
For Practice Prob. 5.10.

Answer: 10 V.

Summary

TABLE 5.3

Summary of basic op amp circuits.

Op amp circuit	Name/output-input relationship
	Inverting amplifier $v_o = -\frac{R_2}{R_1} v_i$
	Noninverting amplifier $v_o = \left(1 + \frac{R_2}{R_1}\right) v_i$
	Voltage follower $v_o = v_i$
	Summer $v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3\right)$
	Difference amplifier $v_o = \frac{R_2}{R_1}(v_2 - v_1)$