College Algebra and Trigonometry

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- An equation is a statement that says two expressions are equal.
  inequality are NOT

Examples of equations :

$$3x-1=5 \qquad x^2+2x-8=0 \qquad e^x \sin x-2\cos x=1$$

Examples of inequalities :

$$\frac{x}{2}-\frac{x}{6}\le 5 \qquad \frac{5}{6}x^3-\frac{x}{2}\ge 9 \qquad \ln(x+1)-3\ln x<2$$

Linear Equations and Rational Equations

Examples of linear equations :

$$2x-3=9 \qquad\qquad \frac{x}{3}+\frac{1}{2}=\frac{5}{6}x-\frac{3}{8}$$

Examples of rational equations :

$$\frac{3}{x}-1=\frac{5}{6x}+\frac{2}{9} \qquad\qquad \frac{6}{y^2+8y+15}-\frac{2}{y+3}=\frac{-4}{y+5}$$

- To solve an equation means to find all the values of x that make the equation true. These values are called solutions, or roots, of the equation.
- If no values of x make the equation true, this equation is called a contradiction.

$$2(3x-1)=3(2x-2)$$

- An equation that is true for any value of the variable x is called an identity.

$$2(3x-2)+1=3(2x-1)$$

- Two equations that have the same solutions are called equivalent equations.

$$3x-1=5 \qquad 2x+2=6 \qquad \frac{x}{2}=\frac{x}{3}+\frac{1}{3}$$

① Solve Linear Equations in One Variable

DEFINITION Linear Equations in One Variable

A linear equation in one variable x is an equation that can be written in the form

$$ax + b = 0$$

where a and b are real numbers and $a \neq 0$.

- What makes this equation linear is that x is raised to the first power. We can also classify a linear equation as a first-degree equation.

Linear Equation in one Variable	Not a Linear Equation in one Variable
$5x+35=0$	$5x^2+35=0$
$\frac{x}{2}-5=0$	$\frac{2}{x}-5=0$
$3x+4=7$	$3x+4y=7$
$0.7x-0.8=0.1$	$0.7x-0.8-0.1$

To solve equations, we need to find a simpler equivalent equation whose solution is obvious. The properties used to produce equivalent equations include the addition and multiplication properties of equality.

Properties of Equality

Let a , b and c are real-valued expressions.

Addition property of equality $$a = b \Leftrightarrow a + c = b + c$$

Multiplication property of equality $$a = b \Leftrightarrow ac = bc \ \ (c \neq 0)$$

Distributive property of equality $$c(a+b) = (a+b)c = ac + bc$$

To solve a linear equation in one variable, isolate the variable by following the following steps.

1) Simplify the algebraic expressions on both sides of the equation.

2) Gather all variable terms on one side of the equation and all constant terms on the other side.

3) Isolate the variable.

Example 1 Solve the equation $3x+4=16$.

Example 2 Solve the equation $-3(x-4)+5=10-(x+1)$.

Example 3 Solve the equation $\dfrac{x-2}{5}-\dfrac{x-4}{2}=\dfrac{x+5}{15}+2$.

② Solving Rational Equations

- A rational equation is an equation that contains one or more rational expressions (the ratio of two polynomials).

Linear Equation

$$\frac{x}{2}-\frac{1}{3}=\frac{2x}{3}-\frac{1}{2}$$

Rational Equation

$$\frac{2}{x}-\frac{1}{6}=\frac{3}{x+1}-\frac{2x}{x-1}$$

Example 4 Solve the equation and check the solution. $\dfrac{12}{x}=\dfrac{6}{2x}+3$

Example 5 Solve the equation and check the solution. $\dfrac{x}{x-4}=\dfrac{4}{x-4}-\dfrac{4}{5}$

Example 6 Solve the equation and check the solution.

$$\frac{6}{y^2+8y+15}-\frac{2}{y+3}=\frac{-4}{y+5}$$

③ Solving an Equation for special variable

Example 7 $3x+2y=6 \text{ for } y$

Example 8 $ax+by=cx+d \quad \text{for} \quad x$

Other Examples

Example 9

$$\frac{x+1}{3x-3}=\frac{2}{5}$$

Example 10

$$\frac{x^2-9}{x^2-4x-21}=\frac{9}{5}$$

Example 11:

Start from rest, an automobile's velocity *v* (in m/s) is given by:

$$v = \frac{180t}{t+40}$$

where *t* is the time in seconds after the car starts to move forward.

Determine the time required for the car to reach a speed of 20 m/s.