Problem Set 1 (Due 3/11/2025 before class)

Late homework will NOT be accepted, unless you have notified course instructor 3 days BEFORE deadline.

Part I (60%)

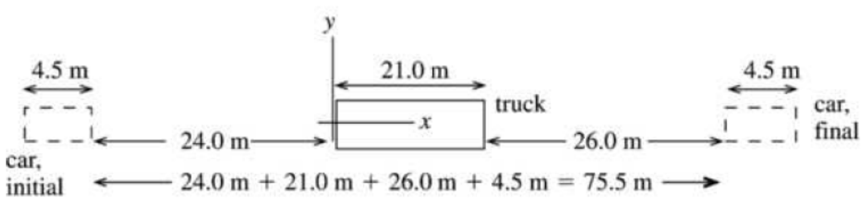
2.77 •• Passing. The driver of a car wishes to pass a truck that is traveling at a constant speed of 20.0 m/s (about 45 mi/h). Initially, the car is also traveling at 20.0 m/s and its front bumper is 24.0 m behind the truck's rear bumper. The car accelerates at a constant 0.600 m/s^2, then pulls back into the truck's lane when the rear of the car is 26.0 m ahead of the front of the truck. The car is 4.5 m long and the truck is 21.0 m long. (a) How much time is required for the car to pass the truck? (b) What distance does the car travel during this time? (c) What is the final speed of the car?

IDENTIFY: Apply constant acceleration equations to each vehicle.

SET UP: **(a)** It is very convenient to work in coordinates attached to the truck.

Note that these coordinates move at constant velocity relative to the earth. In these coordinates the truck is at rest, and the initial velocity of the car is $v_{0x} = 0$. Also, the car's acceleration in these coordinates is the same as in coordinates fixed to the earth.

EXECUTE: First, let's calculate how far the car must travel relative to the truck: The situation is sketched in Figure 2.77.



The car goes from $x_0 = -24.0$ m to $x = 51.5$ m. So $x - x_0 = 75.5$ m for the car.

Calculate the time it takes the car to travel this distance:

$a_x = 0.600\ \text{m/s}^2,\ v_{0x} = 0,\ x - x_0 = 75.5\ \text{m},\ t = ?$

$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$t = \sqrt{\frac{2(x - x_0)}{a_x}} = \sqrt{\frac{2(75.5\ \text{m})}{0.600\ \text{m/s}^2}} = 15.86\ \text{s}$$

It takes the car 15.9 s to pass the truck.

(b) Need how far the car travels relative to the earth, so go now to coordinates fixed to the earth. In these coordinates $v_{0x} = 20.0$ m/s for the car. Take the origin to be at the initial position of the car.

$v_{0x} = 20.0 \text{ m/s},\ a_x = 0.600 \text{ m/s}^2,\ t = 15.86 \text{ s},\ x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (20.0 \text{ m/s})(15.86 \text{ s}) + \frac{1}{2}(0.600 \text{ m/s}^2)(15.86 \text{ s})^2$

$x - x_0 = 317.2 \text{ m} + 75.5 \text{ m} = 393 \text{ m}.$

(c) In coordinates fixed to the earth:

$v_x = v_{0x} + a_x t = 20.0 \text{ m/s} + (0.600 \text{ m/s}^2)(15.86 \text{ s}) = 29.5 \text{ m/s}$

EVALUATE: In 15.86 s the truck travels $x - x_0 = (20.0 \text{ m/s})(15.86 \text{ s}) = 317.2 \text{ m}.$ The car travels $392.7 \text{ m} - 317.2 \text{ m} = 75 \text{ m}$ farther than the truck, which checks with part (a). In coordinates attached to the truck, for the car $v_{0x} = 0,\ v_x = 9.5 \text{ m/s}$ and in 15.86 s the car travels $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = 75 \text{ m},$

2.95 • CALC Two cars, A and B, travel in a straight line. The distance of A from the starting point is given as a function of time by $x_A(t) = \alpha t + \beta t^2$, with $\alpha = 2.60 \text{ m/s}$ and $\beta = 1.20 \text{ m/s}^2$. The distance of B from the starting point is $x_B(t) = \gamma t^2 - \delta t^3$, with $\gamma = 2.80 \text{ m/s}^2$ and $\delta = 0.20 \text{ m/s}^3$. (a) Which car is ahead just after they leave the starting point? (b) At what time(s) are the cars at the same point? (c) At what time(s) is the distance from A to B neither increasing nor decreasing? (d) At what time(s) do A and B have the same acceleration?

IDENTIFY and SET UP: Use $v_x = dx/dt$ and $a_x = dv_x/dt$ to calculate $v_x(t)$ and $a_x(t)$ for each car. Use these equations to answer the questions about the motion.

EXECUTE: $x_A = \alpha t + \beta t^2,\ v_{Ax} = \dfrac{dx_A}{dt} = \alpha + 2\beta t,\ a_{Ax} = \dfrac{dv_{Ax}}{dt} = 2\beta$

$x_B = \gamma t^2 - \delta t^3,\ v_{Bx} = \dfrac{dx_B}{dt} = 2\gamma t - 3\delta t^2,\ a_{Bx} = \dfrac{dv_{Bx}}{dt} - 2\gamma - 6\delta t$

(a) IDENTIFY and SET UP: The car that initially moves ahead is the one that has the larger v_{0x}.

EXECUTE: At $t = 0,\ v_{Ax} = \alpha$ and $v_{Bx} = 0.$ So initially car A moves ahead.

(b) IDENTIFY and SET UP: Cars at the same point implies $x_A = x_B$.

$\alpha t + \beta t^2 = \gamma t^2 - \delta t^3$

EXECUTE: One solution is $t = 0,$ which says that they start from the same point. To find the other solutions, divide by t: $\alpha + \beta t = \gamma t - \delta t^2$

$\delta t^2 + (\beta - \gamma)t + \alpha = 0$

$t = \dfrac{1}{2\delta}\left(-(\beta - \gamma) \pm \sqrt{(\beta - \gamma)^2 - 4\delta\alpha}\right) = \dfrac{1}{0.40}\left(+1.60 \pm \sqrt{(1.60)^2 - 4(0.20)(2.60)}\right) = 4.00 \text{ s} \pm 1.73 \text{ s}$

So $x_A = x_B$ for $t = 0,\ t = 2.27 \text{ s}$ and $t = 5.73 \text{ s}.$

EVALUATE: Car A has constant, positive a_x. Its v_x is positive and increasing. Car B has $v_{0x} = 0$ and a_x that is initially positive but then becomes negative. Car B initially moves in the $+x$-direction but then slows down and finally reverses direction. At $t = 2.27 \text{ s}$ car B has overtaken car A and then passes it. At $t = 5.73 \text{ s},$ car B is moving in the $-x$-direction as it passes car A again.

(c) Identify: The distance from A to B is $x_B - x_A$. The rate of change of this distance is $\frac{d(x_B - x_A)}{dt}$. If this distance is not changing, $\frac{d(x_B - x_A)}{dt} = 0$. But this says $v_{Bx} - v_{Ax} = 0$. (The distance between A and B is neither decreasing nor increasing at the instant when they have the same velocity.)

Set Up: $v_{Ax} = v_{Bx}$ requires $\alpha + 2\beta t = 2\gamma t - 3\delta t^2$

Execute: $3\delta t^2 + 2(\beta - \gamma)t + \alpha = 0$

$$t = \frac{1}{6\delta}\left(-2(\beta-\gamma) \pm \sqrt{4(\beta-\gamma)^2 - 12\delta\alpha}\right) = \frac{1}{1.20}\left(3.20 \pm \sqrt{4(-1.60)^2 - 12(0.20)(2.60)}\right)$$

$t = 2.667\text{ s} \pm 1.667\ s$, so $v_{Ax} = v_{Bx}$ for $t = 1.00\text{ s}$ and $t = 4.33\text{ s}$.

Evaluate: At $t = 1.00\text{ s}$, $v_{Ax} = v_{Bx} = 5.00\text{ m/s}$. At $t = 4.33\text{ s}$, $v_{Ax} = v_{Bx} = 13.0\text{ m/s}$. Now car B is slowing down while A continues to speed up, so their velocities aren't ever equal again.

(d) Identify and **Set Up:** $a_{Ax} = a_{Bx}$ requires $2\beta = 2\gamma - 6\delta t$

Execute: $t = \frac{\gamma - \beta}{3\delta} = \frac{2.80\text{ m/s}^2 - 1.20\text{ m/s}^2}{3(0.20\text{ m/s}^3)} = 2.67\text{ s}.$

Evaluate: At $t = 0$, $a_{Bx} > a_{Ax}$, but a_{Bx} is decreasing while a_{Ax} is constant. They are equal at $t = 2.67\text{ s}$ but for all times after that $a_{Bx} < a_{Ax}$.

Part II (40%) Basic Problems

1. How fast will an object (in motion along the x-axis) be moving at t = 10 s if it had a speed of 2 m/s at t = 0 and a constant acceleration of 2 m/s?

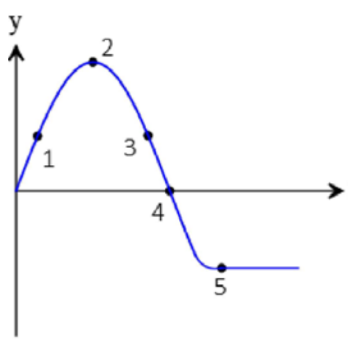
   $v = v_0 + at$

   $v = 2 + 2(10)$

   $v = 22$ m/s

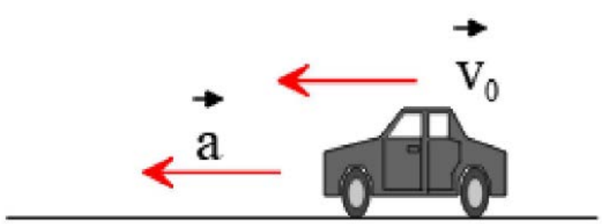
2. A car accelerates from rest at 4 m/s2. What is the velocity of the car after 4 seconds? (Answer: 16 m/s)

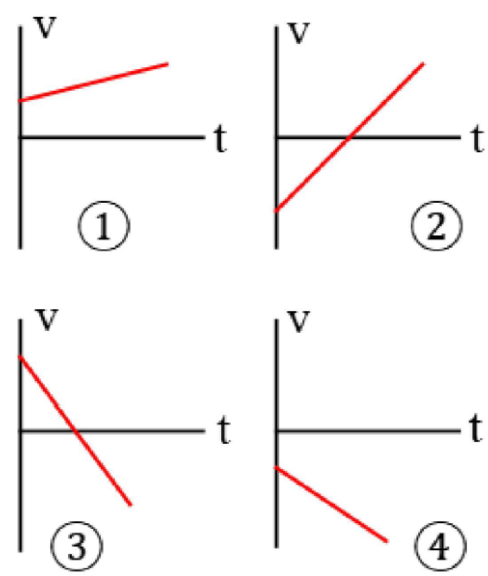
3. Consider the position vs. time graph for a moving object, as shown in the figure below. At which numbered points does the object have the greatest speed?



Solution: The slope of a tangent line at each point on an $x - t$ graph represents the magnitude (speed) and direction of the velocity vector at that point. If we start drawing slopes at the origin or point 4, we notice that as we get closer to the top of the graph, the slopes become smaller and smaller. This continuous decrease in the slope of the curves indicates that the velocities are also decreasing until they reach the top of the curve, where the velocity becomes zero. Based on this reasoning, we conclude that the slope at the numbered point 4 must be the greatest.

4. See figure below (lower left) a car is moving at a constant rate along the 2 axis. Which of the following velocity vs. time graphs (lower right) describes the motion of the car? Take **right** as **positive** direction





Solution: Take the positive to the right. The car is moving to the left, so the direction of its velocity is negative. Therefore, the first choice is incorrect since all the car's velocities are positive, indicating movement to the right. The car begins to move non-stop to the left (in the negative direction), so its velocity is negative at all times. Thus, choices (2) and (3) are also incorrect.

Note that the phrase "at a constant rate" implies constant acceleration.

On the other hand, the acceleration is also directed to the left. We know that the slope of the velocity-time graph provides the direction and magnitude of the acceleration. If the angle that the slope makes with the horizontal is obtuse ($\alpha > 90^\circ$), then the direction of the acceleration is negative.

Hence, the correct answer is 4

5. A car is rolling toward a cliff with an initial speed of 15 m/s. The maximum negative acceleration that the brakes can provide is -0.3 m/s. If the cliff is 350 m from the initial position of the car, will the car go over the cliff?

   In order to stop, the final speed must be zero before the car moves 350 m.

   $v^2 = v_o^2 + 2a(x - x_o)$

   $0 = 15^2 + 2(-0.3)(x - 0)$

   $x = 375$ m

   Thus, the car cannot stop in time and does go over the cliff.

6. Cart A moves with a uniform speed past point 1 on a straight track at 0.3 m/s. At the same time, Cart B moves past point 1 at 0.1 m/s but is uniformly accelerating at 0.1 m/s. Point 2 is 1.0 m past point 1. Which cart gets to point 2 first?
   Scheme:

Find the time for each cart to reach point 2:

Cart A:

$x - x_o = v_o t + \frac{1}{2}at^2$

$1.0 - 0 = 0.3t + 0$

$t = 3.3$ s

Cart B:

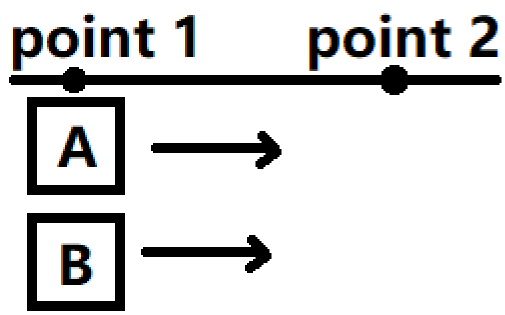
$x - x_o = v_o t + \frac{1}{2}at^2$

$1.0 - 0 = 0.1t + \frac{1}{2}(0.1)t^2$

Use the quadratic equation to deterime t

$t = 3.6$ s

Cart A arrives at point 2 first.



7. A small ball is released from a window at t = 0. Assuming free-fall conditions, how far does it travel in 2.8 seconds? If the ball had more mass would it fall a greater distance?

Assume $y_o = 0$

$y - y_o = v_o t + ½gt^2$

$y - 0 = 0 + -4.9(2.8)^2$

$y = -38.4$ m

This means that the ball has fallen 38.4 m below its initial position.

Free-fall acceleration is independent of mass, so the distance fallen by a more massive object would be the same.

8. A car moving at 20 m/s passes a street corner. The car maintains this speed even though the speed limit is 10 m/s. The police car that was sitting at the corner begins to chase the car by accelerating at 2 m/s. How long will it take for the police car to catch the speeder? How far from the corner is the catch-up point? How fast will the police car be traveling at that time?

For the car: $x = 20t$

For the police car: $x = ½(2)t^2$

At catch up point: $20t = ½(2)t^2$

Therefore, t = 20sec

position at catch up: $x_{catch\ up} = 20t = 20(20) = 400$ m

Speed at catch-up: $v = v_o + at = 0 + (2)(20) = 40$ m/s

9. Determine the distance between two steel spheres (after 1.4 s) dropped from a tower if the second sphere was dropped 0.5 seconds after the first. Assume free-fall and that the spheres are dropped from rest.

Sphere 1 after 1.4 s:

$y_1 = y_o + v_o t + ½gt^2$

$y_1 = 0 + 0 - 4.9(1.4)^2$

$y_1 = -9.6$ m

Sphere 2 after 0.9 s:

$y_2 = y_o + v_o t + ½gt^2$

$y_2 = 0 + 0 - 4.9(0.9)^2$

$y_2 = -4.0$ m

The spheres will be 5.6 m apart at that time.

10. A car accelerates uniformly from rest to a velocity of 101 km/h east in 8.0 s. What is the magnitude of its acceleration?

Solution: The given data are

$$\text{initial velocity}, v_0 = 0$$

$$\begin{aligned}\text{final velocity}, v &= 101\,\mathrm{Km/h} \\ &= 101\left(\frac{1000\,\mathrm{m}}{3600\,\mathrm{s}}\right) \\ &= 28.06\,\mathrm{m/s}\end{aligned}$$

$$\begin{aligned}\text{time interval , t} &= 8\,\mathrm{s} \\ \text{acceleration , } &=?\end{aligned}$$

The relevant kinematic equation which relates those together is $v = v_0 + a\,t$. So

$$v = v_0 + a\,t$$

$$28.06 = 0 + a(8)$$

$$\Rightarrow a = \frac{28.06 - 0}{8}$$

$$= \boxed{3.51\,\mathrm{m/s^2}}$$