8. Impulse and Momentum

Momentum

$$\vec{p} = m\vec{v}$$

$$\Sigma\vec{F} = m\frac{dv}{dt} = \frac{d\vec{p}}{dt}$$

$$p_x = mv_x \quad p_y = mv_y \quad p_z = mv_z$$

Impulse

The **change in momentum** caused by a force over time is defined as the impulse.

$$\vec{J} = \Delta\vec{p}_2 - \vec{p}_1$$

The impulse is a vector quantity, same as the net force $\sum \vec{F}$, its magnitude is the product of the magnitude of the net force and the length of time.

$$\overrightarrow{J} = \Sigma \vec{F} \Delta t$$

$$\int_{t_1}^{t_2} \vec{F} \, dt$$

A rapid change in momentum requires a large net force, while a gradual change in momentum requires less net force.

Elastic Collisions

elastic collision in an isolated system is one which **kinetic energy and momentum are conserved**

(in the simplest words, most of the time, elastic collisions refers to a collision where the bodies colliding separate and go in the opposite directions)

$$K_{A1} + K_{B1} = K_{A2} + K_{B2}$$

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

and further

$$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$$

the velocities of two bodies should flip in an elastic collision, the body initially moving faster will become slower, and vice versa.

Inelastic Collision

Inelastic collision is where only the momentum is conserved in the system, and there is a loss in the kinetic energy.

the bodies in the system become entangled (stuck together) and go with the same velocity in the same direction

$$v_{B2} = v_{A2}$$

Center of Mass

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$y_{\text{cm}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i y_i}{\sum_i m_i} \qquad \text{(center of mass)}$$

$$\overrightarrow{r}_{cm} = \frac{m_1 \overrightarrow{r}_1 + m_2 \overrightarrow{r}_2 + m_3 \overrightarrow{r}_3 + \ldots}{m_1 + m_2 + m_3} = \frac{\sum_i m_i \overrightarrow{r}}{\sum_i m_i}$$

it looks very complicated but it just quite simple: the position vector $\overrightarrow{r}_{cm}$ of center of mass is simply the sum of all the positions vectors $\overrightarrow{r}_1, \overrightarrow{r}_2 \ldots$ divided by the total.

for most scenarios: calculate the $x - y$ components of the position vectors and divide them by the total mass separately, and then combine them in a single position vector $\vec{r}_{cm}$