Model Final Exam Paper

1. Given $f(x) = 3x^2 + 12x + 5$,
   a) Write the function in vertex form.
   b) Identify the vertex (h, k).
   c) Find any x-intercepts.
   d) Find the y-intercepts.
   e) Sketch the graph.
   f) Determine the axis of symmetry.
   g) Determine the maximum or minimum value of f.
   h) Write the domain and range in interval notation.

2. Give $f(x) = x^3 - 9x$,
   a) Determine the end behavior of the graph of the function.
   b) Find the x- and y-intercepts of the graph of the function.
   c) Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.
   d) Use the information in a) through c) to draw a complete graph of the function.

3. Use long division to divide. $\left(-5 + x + 4x^2 + 2x^3 + 3x^4\right) \div \left(x^2 + 2\right)$

4. Use synthetic division to divide. $\left(-2x + 4x^3 + 18 + x^4\right) \div \left(x + 2\right)$

5. Use the remainder theorem to determine if the given number c is a zero of polynomial,

   $(\text{a})\ f(x) = 2x^3 - 4x^2 - 13x - 9;\ c = 4$

   $(\text{b})\ f(x) = x^3 + x^2 - 3x - 3;\ c = \sqrt{3}$

6. Use the factor theorem to determine if the given polynomials are factors of $f(x) = x^4 - x^3 - 11x^2 + 11x + 12$.

   (a) $x-3$ (b) $x+2$

7. Factor $f(x) = 3x^3 + 25x^2 + 42x - 40$, given that –5 is a zero of $f(x)$.

8. Write a polynomial $f(x)$ of degree 3 that has the zeros, $\frac{1}{2}, \sqrt{6}$ and $-\sqrt{6}$.

9. Find the zeros and their multiplicities. $f(x) = 2x^4 + 5x^3 - 2x^2 - 11x - 6$

10. Given $f(x) = x^4 - 6x^3 + 28x^2 - 18x + 75$, and that $3-4i$ is a zero of $f(x)$,

    a) Find the remaining zeros.
    b) Factor $f(x)$ as a product of linear factors.
    c) Solve the equation. $x^4 - 6x^3 + 28x^2 - 18x + 75 = 0$.

11. Given $f(x) = \dfrac{4x}{x^2 - 4}$,

    a) Find the domain of the rational function.
    b) Find the y-intercept by evaluating $f(0)$.
    c) Find the x-intercepts by solving the zeros of the numerator of R that are in the domain of R. Determine the behavior of the graph of R near each x-intercept.
    d) Determine the vertical asymptotes. Graph each vertical asymptote using a dashed line.
    e) Determine the horizontal or slant asymptote, if one exists. Graph the asymptote using a dashed line.
    f) Plot at least one point on the intervals defined by the x-intercepts, vertical asymptotes, and points at which the graph of R intersects the asymptote.
    g) Sketch the graph.

12. Find the inverse of the function for the one-to-one function defined by

$$f(x)=3x-1.$$

13. Graph the function.

a) $f(x)=2^x$ b) $g(x)=\left(\frac{1}{2}\right)^x$

14. Write each equation in exponential form.

a) $\log_2 16=4$ b) $\log_{10}\left(\frac{1}{100}\right)=-2$ c) $\log_7 1=0$

15. Write each equation in logarithmic form.

a) $3^4=81$ b) $10^6=1000000$ c) $\left(\frac{1}{5}\right)^{-1}=5$

16. Graph the functions.

a) $y=\log_2 x$ b) $\log_{\frac{1}{4}} x$

17. Write the expression as the sum or difference of logarithms.

a) $\log_2\left(\frac{z^3}{xy^5}\right)$ b) $\log\sqrt[3]{\frac{(x+y)^2}{10}}$

18. Write the expression as a single logarithm and simplify the result if possible.

$$\log_2 560-\log_2 7-\log_2 5$$

19. Solve.

a) $3^{2x-6}=81$ b) $25^{4-t}=\left(\frac{1}{5}\right)^{3t+1}$ c) $8^{x+2}=16^{x+1}$

d) $4^{2x-7}=5^{3x+1}$ e) $7^x=60$

20. Solve.

a) $\log_2(3x-4)=\log_2(x+2)$ b) $\ln(x-4)=\ln(x+6)-\ln x$ c) $4\log_2(2t-7)=8$

d) $\log(w+47)=2.6$ e) $\log_2 x=3-\log_2(x-2)$

21. Let $(-2, -5)$ be a point on the terminal side of angle θ drawn in standard position. Find the exact value of each of the six trigonometric functions of θ.

22. Find the reference angle θ'.

a) $\theta=315^\circ$ b) $\theta=-\dfrac{13\pi}{12}$ c) $\theta=3.5$ d) $\theta=\dfrac{25\pi}{4}$

23. Evaluate the functions.

a) $\sin\dfrac{4\pi}{3}$ b) $\tan(-225^\circ)$ c) $\sec\dfrac{11\pi}{6}$

d) $\sec\dfrac{9\pi}{2}$ e) $\sin(-510^\circ)$

24. Given that $\sin\theta=-\dfrac{4}{7}$ and $\cos\theta>0$, find $\cos\theta$ and $\tan\theta$.

25. Given that $\cos\theta=-\dfrac{4}{7}$ for θ in Quadrant II, find $\sin\theta$ and $\tan\theta$.

26. Given $f(x) = 4\sin 3x$,

a) Identify the amplitude and period.

b) Graph the function and identify the key points on one full period.

30. Given $y=\cos\left(2x+\dfrac{\pi}{2}\right)$,

a) Identify the amplitude and period.

b) Graph the function and identify the key points on one full period.

27. Find the exact values or state that the expression is undefined.

a) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ b) $\arcsin\frac{1}{2}$ c) $\sin^{-1}2$

d) $\cos^{-1}\left(-\frac{1}{2}\right)$ e) $\tan^{-1}\sqrt{3}$ f) $\arctan(-1)$

28. Verify that the equation is an identity.

$$\frac{\cos(-x)\tan(-x)}{\sin x} = -1$$

29. Verify that the equation is an identity.

$$\frac{1}{1-\cos x} - \frac{1}{1+\cos x} = 2\cot x \csc x$$

30. Find the exact values.

a) $\cos 15^\circ$ b) $\sin\frac{11\pi}{12}$

31. Find the exact values of $\cos(\alpha-\beta)$ given that $\sin\alpha = -\frac{4}{5}$ and $\cos\beta = -\frac{5}{8}$ for α in Quadrant III and β in Quadrant II.

32. Find the exact values of $\tan 255^\circ$.

33. Write $5\sin x - 12\cos x$ in the form $k\sin(x+\alpha)$.

34. Given that $\sin\theta = \frac{2}{3}$ for θ in Quadrant II, find the exact function values.

a) $\sin 2\theta$ b) $\cos 2\theta$ c) $\tan 2\theta$

35. Write $\sin^4 x + \cos^2 x$ in terms of first power of cosine.

36. If $\sin\alpha = -\frac{4}{5}$ and $\pi < \alpha < \frac{3\pi}{2}$, find the exact values of each expression.

a) $\sin\frac{\alpha}{2}$ b) $\cos\frac{\alpha}{2}$ c) $\tan\frac{\alpha}{2}$

37. Solve $2\tan x = \sqrt{3} - \tan x$ over $[0, 2\pi)$.

38. Given $2\sin 2x - \sqrt{3} = 0$,

a) Write the solution set for the general solution.

b) Write the solution set on the interval $[0, 2\pi)$.

39. Given $-1 + \sin\frac{x}{2} = 0$,

a) Write the solution set for the general solution.

b) Write the solution set on the interval $[0, 2\pi)$.