College Algebra and Trigonometry

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① Identify One-to-One Functions

Definition of a One-to-One Function:

A function f is a one-to-one function, if for a and b in the domain of f,

if $a \neq b$, then $f(a) \neq f(b)$, or equivalently, if $f(a) = f(b)$, then $a = b$.

Example 1: **Determine whether the function is one-to-one.**

a) $f = \{(1, 4), (2, 3), (-2, 4)\}$

b) $g = \{(-3, 4), (1, -1), (2, 0)\}$

c) $f(x) = 2x - 3$

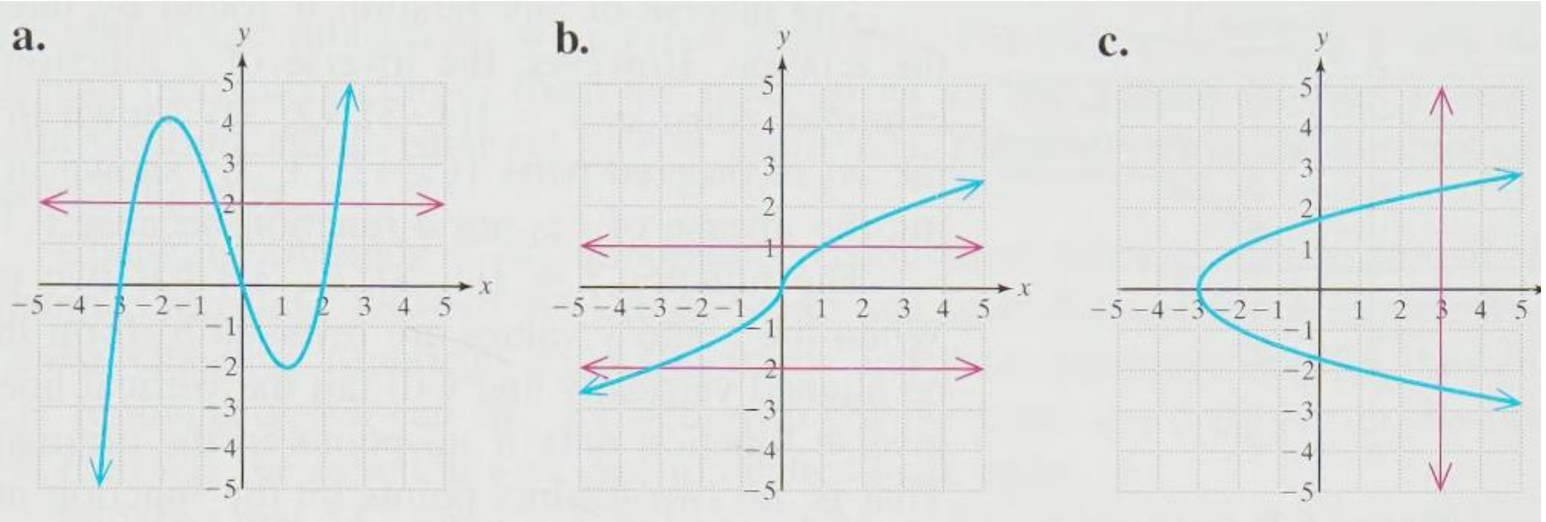
d) $f(x) = x^2 + 1$

Horizontal Line Test for a One-to-One Function:

A function defined by $y = f(x)$ is a one-to-one function if no horizontal line intersects the graph more than one point.

Example 2 :

Use the horizontal line test to determine if the graph defines y as a one-to-one function of x.



② Determine whether Two Functions are Inverses

Definition of an Inverse Function:

Let f be a one-to-one function. Then g is the inverse of f if both of the following conditions are true.

1) $(f \circ g)(x) = x$ for all x in the domain of g.

2) $(g \circ f)(x) = x$ for all x in the domain of f.

Example 4:

Determine whether the functions are inverses.

a) $f(x) = 100 + 12x$ **and** $g(x) = \frac{x-100}{12}$

b) $h(x) = \sqrt[3]{x-1}$ **and** $k(x) = x^3 - 1$

③ Find the Inverse of a Function

Procedures to find an equation of an inverse of a function:

For a one-to-one function defined by $y = f(x)$, the equation of the inverse can be found as follows:

Step 1: Replace $f(x)$ by y.

Step 2: Interchange x and y.

Step 3: Solve for y.

Step 4: Replace y by $f^{-1}(x)$.

Example 5 and 6:

Write an equation for the inverse function for the following functions:

5) $f(x) = 3x - 1$

6) $f(x) = \frac{3-x}{x+3}$

Example 7 and 8:

Find the equation of the inverse of the following functions:

7) Given $m(x) = x^2 + 4$ for $x \geq 0$.

8) Given $f(x) = \sqrt{x - 1}$.

① Definition of an Exponential Function

Let b be a constant real number such that $b > 0$ and $b \neq 1$. Then for any real number x, a function of the form $f(x) = b^x$ is called an exponential function of base b.

Exponential Functions:

$f(x) = 3^x$

$g(x) = (1/3)^x$

$h(x) = (\sqrt{2})^x$

Not Exponential Functions:

$m(x) = x^3$ **base is not constant**

$n(x) = (-1/3)^x$ **base is negative**

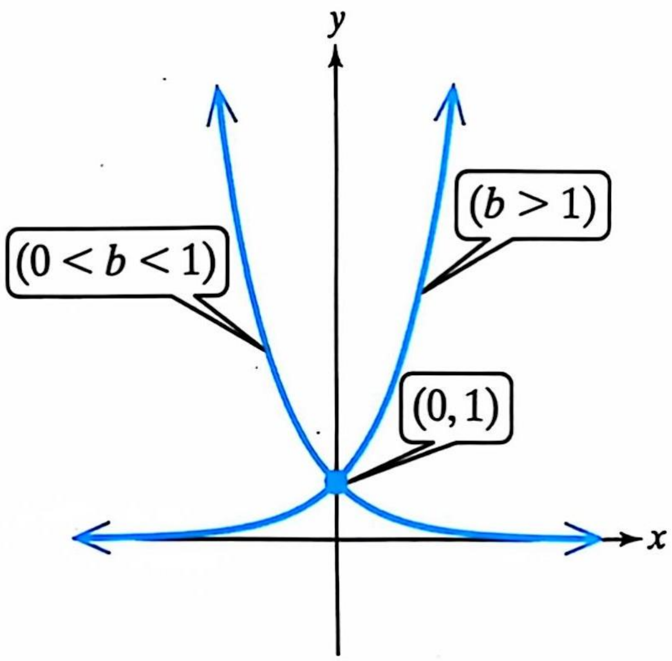
$f(x) = 1^x$ **base is 1**

Graphs of $f(x) = b^x$

The graph of an exponential function defined by $f(x) = b^x$ ($b > 0$ and $b \neq 1$) has the following properties.

1. If $b > 1$, f is an *increasing* exponential function, sometimes called an **exponential growth function.**

   If $0 < b < 1$, f is a *decreasing* exponential function, sometimes called an **exponential decay function.**



2. The domain is the set of all real numbers, $(-\infty, \infty)$.
3. The range is $(0, \infty)$.
4. The line $y = 0$ (x-axis) is a horizontal asymptote.
5. The function passes through the point $(0, 1)$ because $f(0) = b^0 = 1$.

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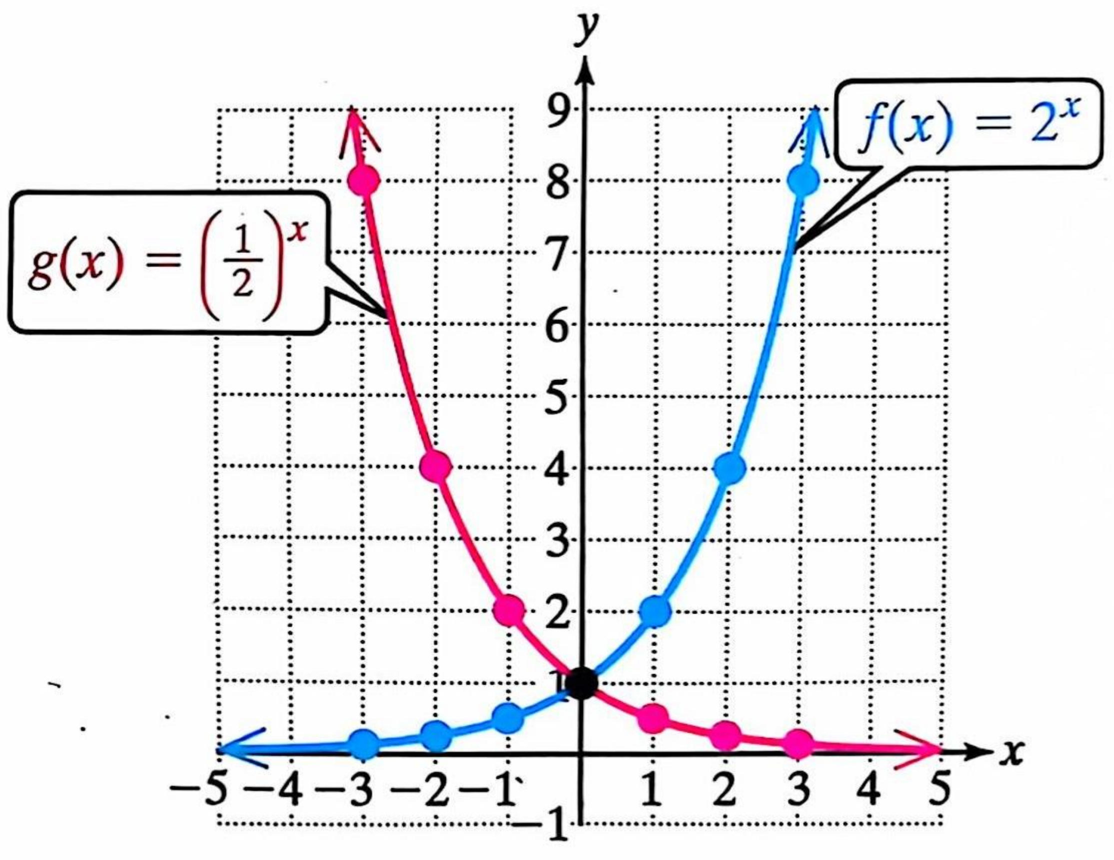
Example 1:

Graph the following functions.

a) $f(x) = 2^x$

b) $g(x) = \left(\frac{1}{2}\right)^x$



② Graph an Exponential Function by Transformation

If $h > 0$, shift to the right.
If $h < 0$, shift to the left.

$$f(x) = ab^{x-h} + k$$

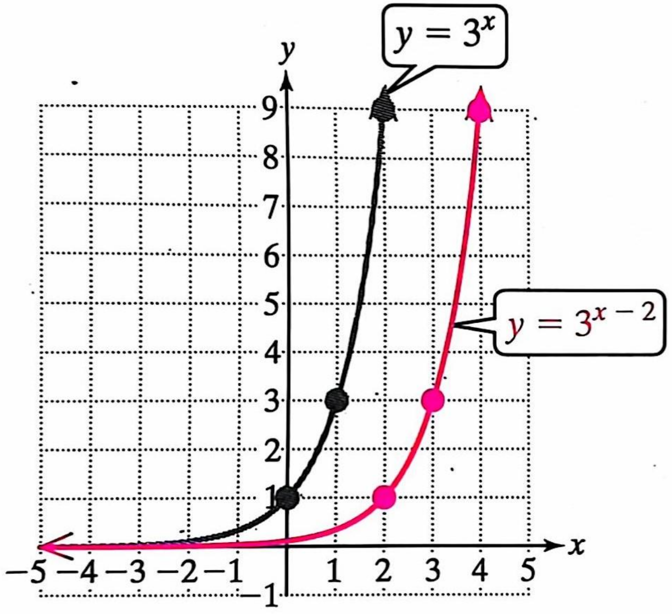
If $a < 0$, reflect across the x-axis.
Shrink vertically if $0 < |a| < 1$.
Stretch vertically if $|a| > 1$.

If $k > 0$, shift upward.
If $k < 0$, shift downward.

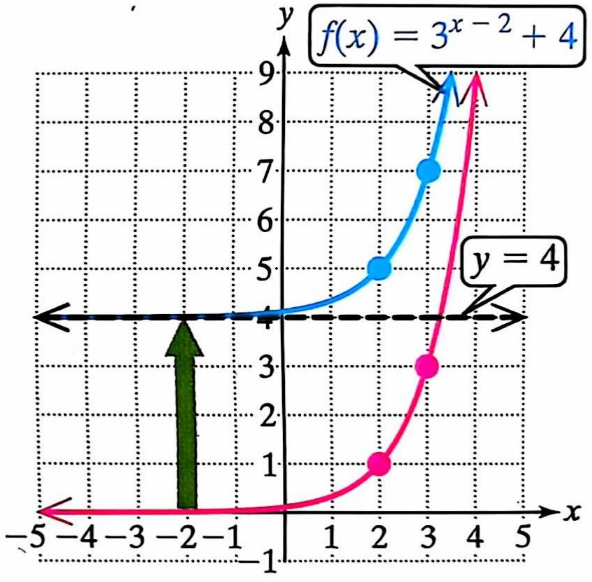
Example 2:

Graph the function by transformation: $f(x) = 3^{x-2} + 4$

x	$y = 3^x$
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9



Shift 2 units to the right. For example, the point (0, 1) on $y = 3^x$ corresponds to (2, 1) on $y = 3^{x-2}$.



Shift the graph of $y = 3^{x-2}$ up 4 units. Notice that with the vertical shift, the new horizontal asymptote is $y = 4$.

③ Evaluate the Exponential Function Base *e*

- **There is an important exponential function whose base is an irrational number called *e*.**
- **Consider the expression $f(x) = \left(1 + \frac{1}{x}\right)^x$. The value of *f*(*x*) for increasingly large values of *x* approaches a constant called *e*.**
- **As $x \to \infty$,**

$$f(x) = \left(1 + \frac{1}{x}\right)^x \to e \approx 2.718281828$$

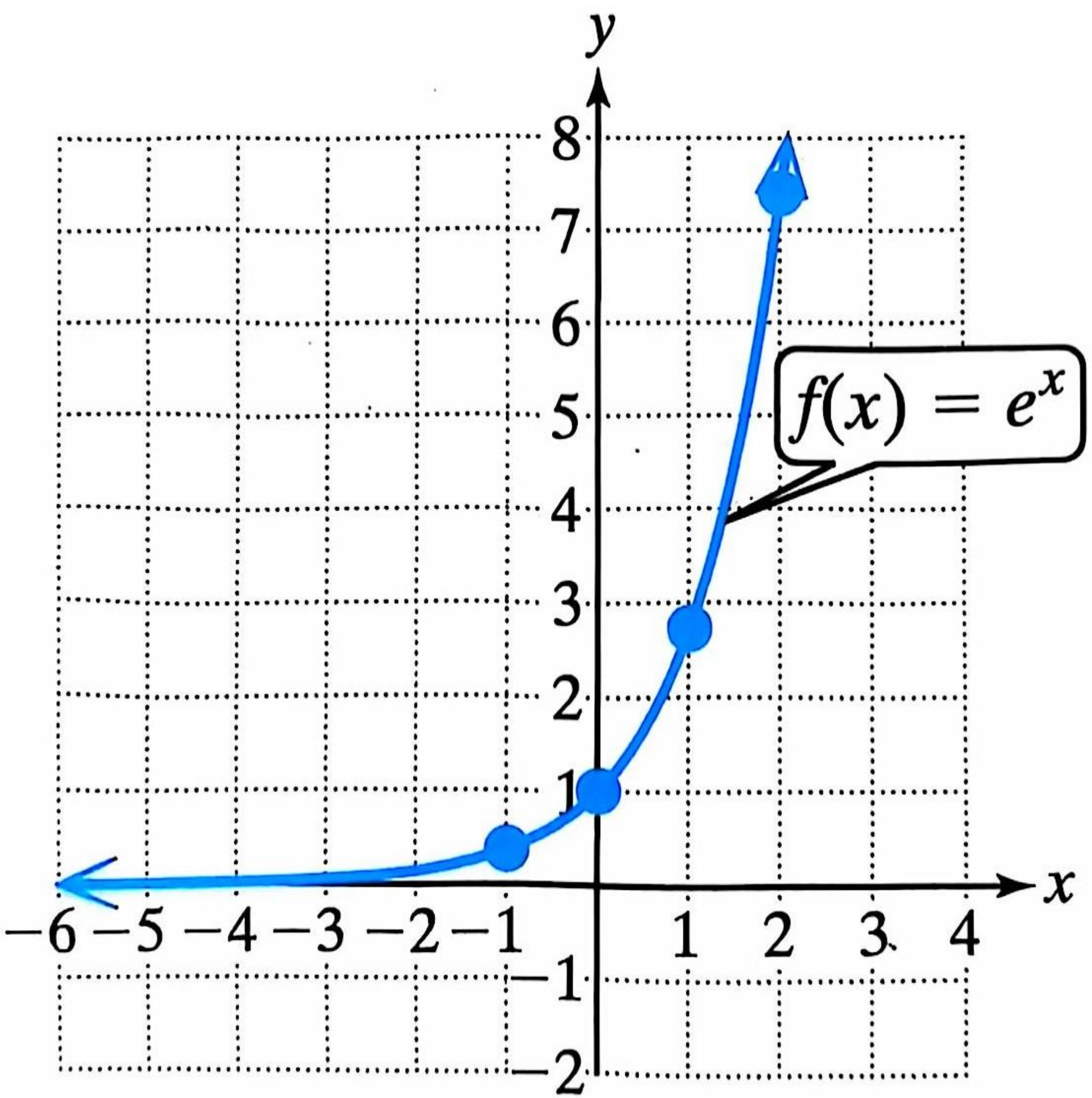
Table 4-6

x	$\left(1 + \frac{1}{x}\right)^x$
100	2.70481382942
1000	2.71692393224
10,000	2.71814592683
100,000	2.71826823717
1,000,000	2.71828046932
1,000,000,000	2.71828182710

Example 3:

Graph: $f(x) = e^x$

x	$f(x) = e^x$
-3	0.050
-2	0.135
-1	0.368
0	1.000
1	2.718
2	7.389
3	20.086



Example 4:

Use the properties of exponents to simplify:

a) $(e^x)^2$ b) $e^x \cdot e^{-x}$ c) $(e^x - e^{-x})^2$

Example 5:

a) **Factor** $e^{x+3h} - e^{x+h}$.

b) **Solve** $3x^2e^x - 2xe^x - e^x = 0$.