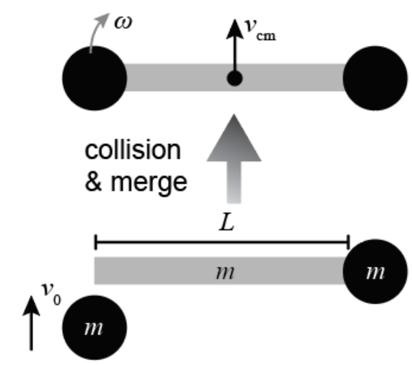
Name: ______________________ Student ID: ________________________

Quiz 9 Rotations 2

As shown in the scheme on the right, a sphere (assume negligible size) of mass known $\boldsymbol{m}$ and initial velocity $\boldsymbol{v_0}$ collides with a stick with length known length $\boldsymbol{L}$ and mass $\boldsymbol{m}$, which has another sphere connected on the right, both with mass $\boldsymbol{m}$. Assume this collision is completely inelastic and the sphere sticks to the rod afterwards. Ignoring other forces such as gravity and friction, what are the angular velocity $\boldsymbol{\omega}$ and the straight-line velocity $\boldsymbol{v}_{\text{cm}}$ after the collision?



To continue, given that the moment of inertia of a rod of mass $\boldsymbol{m}$ and length $\boldsymbol{L}$ is $I_{rod} = \frac{1}{12}mL^2$, and assume the two spheres have negligible sizes, what fraction of the kinetic energy before collision has been lost during the collision? What fractions are converted to the straight-line and rotational kinetic energy of the combined system?

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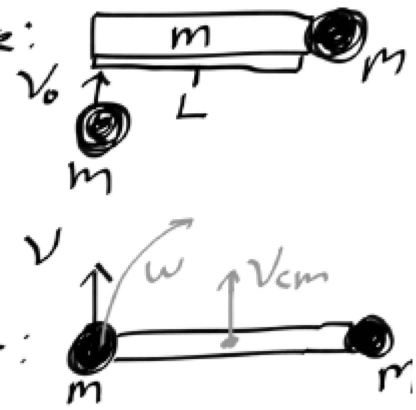
Momentum conservation still applies to center of mass

$m v_0 = 3m v_{cm}$ → center of mass for the whole

so $v_{cm} = \frac{1}{3} v_0$

Before:



After:

Now let's consider the rotational motion

Before collision: only the left ball is moving. Relative to point A: $I_0 = m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{4}$

$\omega_0 = \frac{v_0}{L/2} = \frac{2v_0}{L}$

so initial angular momentum is $I_0 \omega_0 = \frac{m v_0 L}{2}$

Afterwards $I = 2\underbrace{I_0}_{\text{two spheres}} + \underbrace{\frac{mL^2}{12}}_{\text{rod}} = \frac{7}{12} mL^2$

So: $I_0 \omega_0 = I\omega$ (angular momentum conserve

$\omega = \frac{I_0}{I}\omega_0 = \frac{m v_0 L}{2} \Big/ \frac{7}{12} mL^2 = \frac{6}{7}\frac{v_0}{L}$

Initial $K_0 = \frac{1}{2} m v_0^2$

Afterwards $K_{cm} = \frac{1}{2}(3m)\, v_{cm}^2 = \frac{1}{6} m v_0^2 = \frac{K_0}{3}$

$K_{rot} = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{7}{12}mL^2\right)\cdot\left(\frac{6}{7}\frac{v_0}{L}\right)^2 = \frac{3}{14} m v_0^2 = \frac{3}{7} K_0$

so $K = K_{cm} + K_{rot} = \frac{16}{21} K_0$ lost $\frac{5}{21} K_0$

Without rotation will lost $K_0 - K_{cm} = \frac{2}{3} K_0$