# CALCULUS

Prof. Liang ZHENG

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- The Fundamental Theorem tells us how to evaluate a definite integral once we have an antiderivative for the integrand function. However, finding antiderivatives (or indefinite integrals) is not as straightforward as finding derivatives.
- In this chapter we study a number of important techniques that apply to finding integrals for specialized classes of functions such as trigonometric functions, products of certain functions, and rational functions. We also study the integrals whose domain or range are infinite, called *improper integrals*.

# 8.1 Using Basic Integration Formulas



## ① Basic integration formulas

1. $\int k\,dx = kx + C$ (any number $k$)
2. $\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
3. $\int \frac{dx}{x} = \ln|x| + C$
4. $\int e^x dx = e^x + C$
5. $\int a^x dx = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$
6. $\int \sin x\,dx = -\cos x + C$
7. $\int \cos x\,dx = \sin x + C$
8. $\int \sec^2 x\,dx = \tan x + C$
8. $\int \csc^2 x\,dx = -\cot x + C$
10. $\int \sec x \tan x\,dx = \sec x + C$
11. $\int \csc x \cot x\,dx = -\csc x + C$
12. $\int \tan x\,dx = \ln|\sec x| + C$
13. $\int \cot x\,dx = \ln|\sin x| + C$
14. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
15. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$
16. $\int \sinh x\,dx = \cosh x + C$
17. $\int \cosh x\,dx = \sinh x + C$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$
21. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C \quad (a > 0)$
22. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C \quad (x > a > 0)$

## ② Rewriting an integral to match it to a standard formula

**Example 1** Find $\int_1^5 \frac{2x-3}{\sqrt{x^2-3x+6}}dx$

**Example 2** Find $\int_0^{\pi/3} 3(cosxsin2x + sinxcos2x)dx$

**Example 3** Find $\int_0^{\pi/4} \frac{dx}{1-sinx}$

## ③ Making a substitution to evaluate

**Example 4** Evaluate $\int \frac{3}{\sqrt{16-9x^2}}dx$

**Example 5** Evaluate $\int \frac{2}{9+4x^2}dx$

**Example 6** Evaluate $\int \frac{1}{e^x+e^{-x}}dx$

## ④ Completing the square to evaluate

**Example 7** Evaluate $\int_2^6 \frac{dx}{\sqrt{8x - x^2}}$

**Example 8** Evaluate $\int_{-2}^0 \frac{dx}{x^2 + 2x + 2}$

**Example 9** Evaluate $\int_4^5 \frac{2dx}{(x-1)\sqrt{x^2 - 2x - 3}}$

## ⑤ Reducing an improper function

**Note:** The integrand is an improper fraction if the degree of the numerator is equal to or greater than the degree of the denominator.

**Example 10** Evaluate $\int_1^3 \frac{2x}{x+1}dx$

**Example 11** Evaluate $\int \frac{3x^2-x}{3x+2}dx$

**Example 12** Evaluate $\int_0^2 \frac{2x^3-x^2+8x-2}{x^2+4}dx$

## ④ Miscellaneous

**Example 13** Evaluate $\int \frac{3x+2}{\sqrt{1-x^2}}dx$

**Example 14** Find $\int \frac{dx}{(1+\sqrt{x})^3}$

**Example 15** Evaluate $\int_{-\pi/2}^{\pi/2} x^3 cosx dx$

**Skill Practice 1** Evaluate $\int_{-1}^{0} \frac{2dx}{2x^2+2x+1}$

**Skill Practice 2** Find $\int \frac{\sqrt{x}}{1+x^3}dx$

**Skill Practice 3** Evaluate $\int_{-\pi/3}^{\pi/3} \frac{1}{1-cos\theta}d\theta$