CALCULUS

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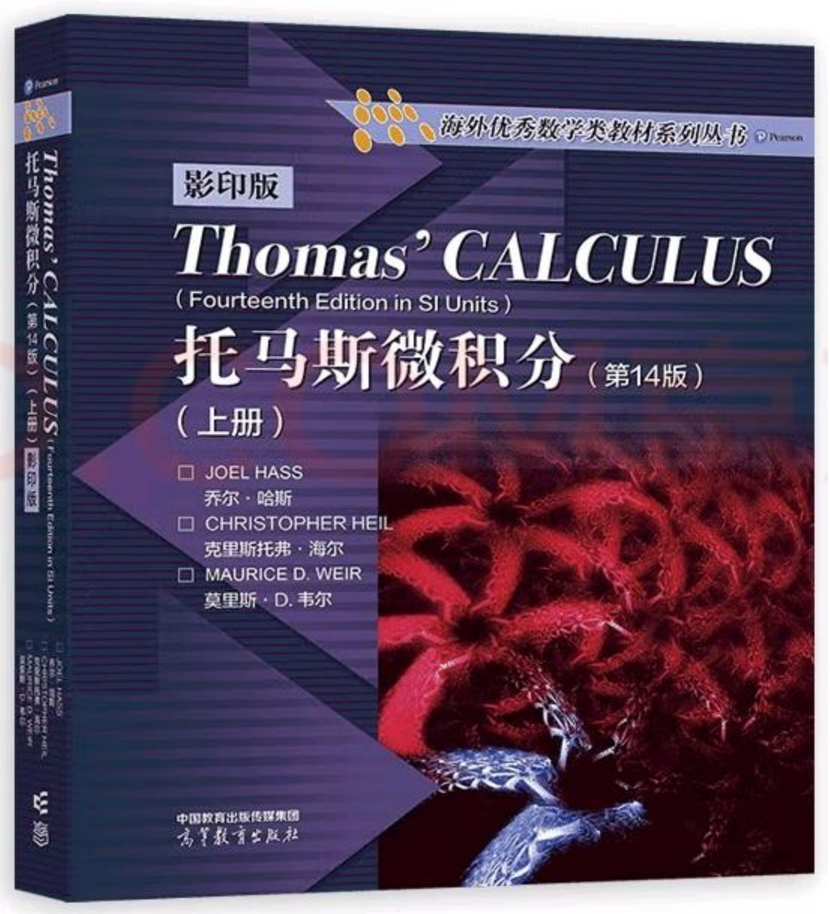
About this course (II)

哈爾濱工業大學(深圳)
HARBIN INSTITUTE OF TECHNOLOGY, SHENZHEN

Textbook



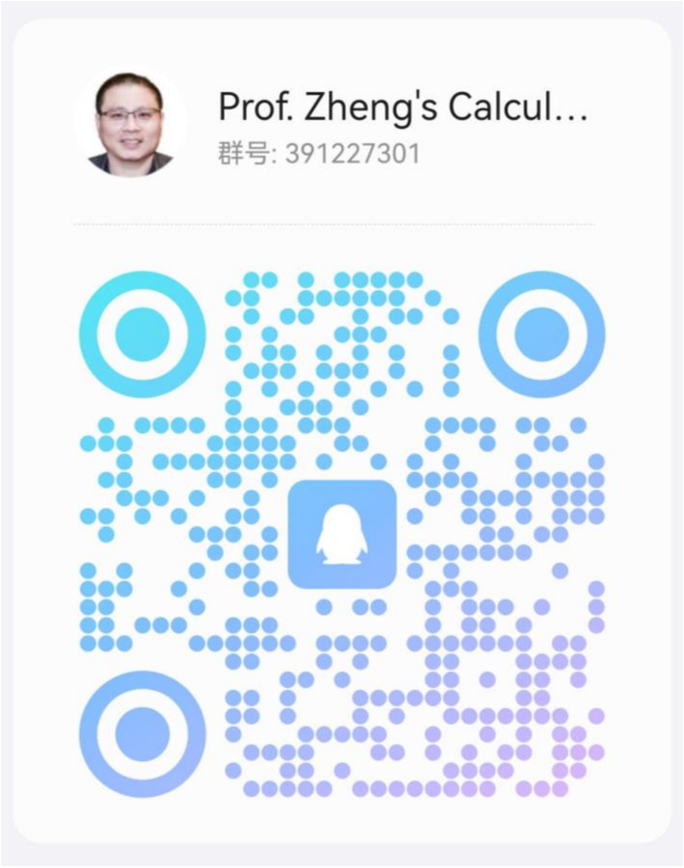
Grading

Homework: 20%

Midterm: 30%

Final: 50%

QQ group code



Course Subject

哈爾濱工業大學(深圳)
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- What is calculus?

Calculus, a mathematical concept, is the study of differentiation and integration of functions in advanced mathematics.

- What is the three important part of calculus?

The three important part of calculus is limits, differentiation, integration.

- What is the research object and key tool of calculus?

The research object of calculus is functions and the key tool for studying calculus is the limit.

- Calculus has extensive applications in almost all fields such as physics, astronomy, optics mechanics, and thermodynamics.

- **Functions** are fundamental to the study of calculus. A review of functions and their graphs is necessary and beneficial.

DEFINITION: A function f from a set D to a set Y is a rule that assigns a unique (single) element $y \in Y$ to each element $x \in D$. In this case, we say that "y is a function of x" and write this symbolically as $y = f(x)$.

The set D of all possible input values is called the domain of the function. The set

$$R = \{y \mid y = f(x), \forall x \in D\}$$

is called the range of the function.

- When we define a function $y = f(x)$ with a formula and the domain is not stated explicitly or restricted by context, the domain is assumed to be the largest set of real x-values for which the formula gives real y-values, which is called the natural domain.

$$\text{natural domain} = \{x \in \mathbb{R} \mid f(x) \text{ is a real number.}\}$$

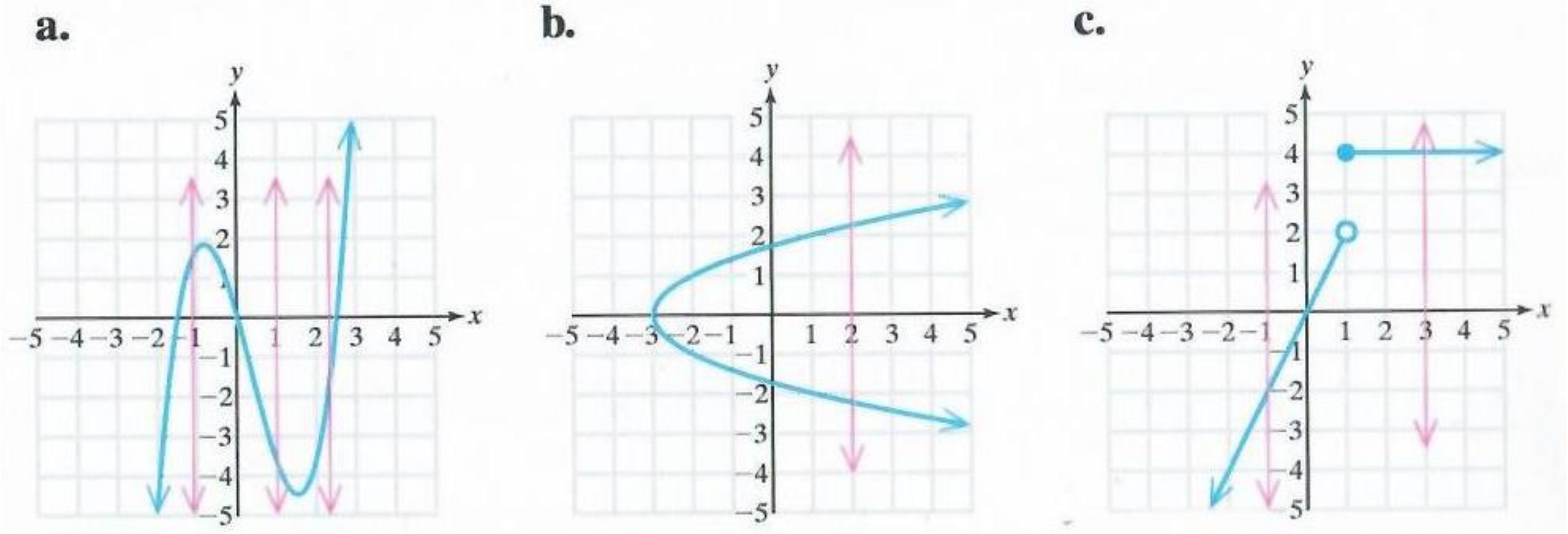
Example 1:

Let's verify the natural domains and associated ranges of some simple functions.

1) $y = x^2$. 2) $y = \sqrt{x}$. 3) $y = \dfrac{1}{x}$. 4) $y = \sqrt{4 - x}$. 5) $y = \sqrt{1 - x^2}$.

The Vertical Line Test for a Function

Consider a relation defined by a set of points (*x*, *y*) graphed on a rectangular coordinate system. The graph defines *y* as a function of *x* if no vertical line intersects the graph in more than one point.



Common Functions:

① Linear Function:

$$f(x) = mx + b$$

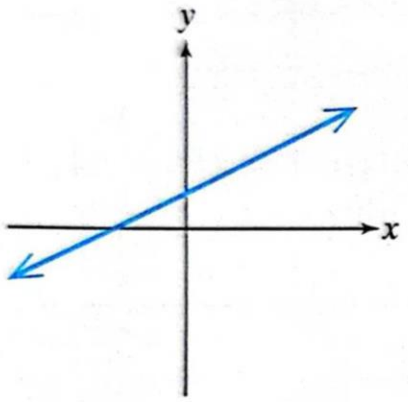
where m and b are constants.

Special cases:

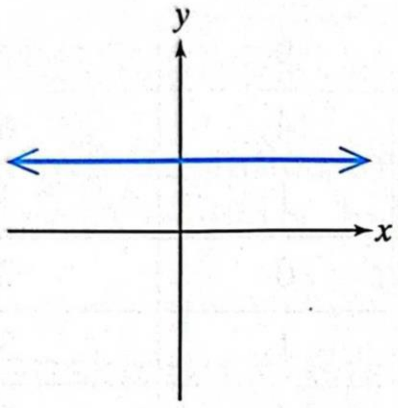
1) $m = 0$: $f(x) = b$ constant function.

2) $m = 1$, $b = 0$: $f(x) = b$ identity function.

1. Linear functions $f(x) = mx + b$

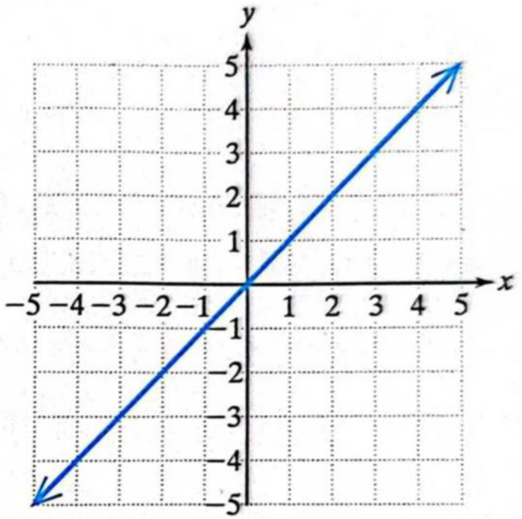


Constant functions $f(x) = b$



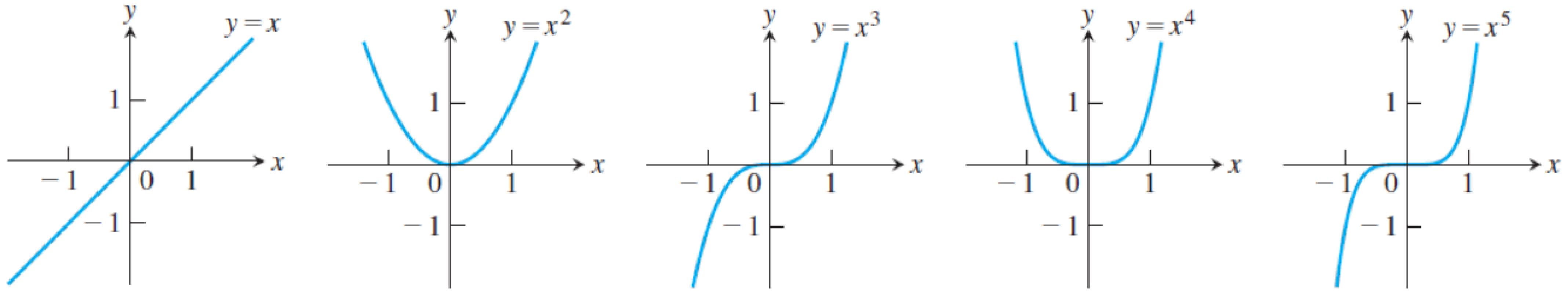
2. Identity function: $f(x) = x$

x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

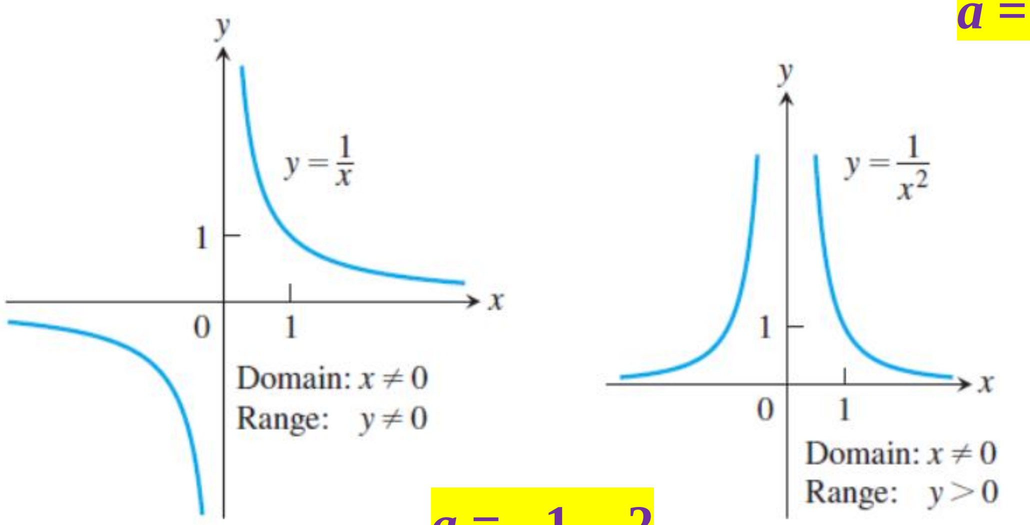


Common Functions:

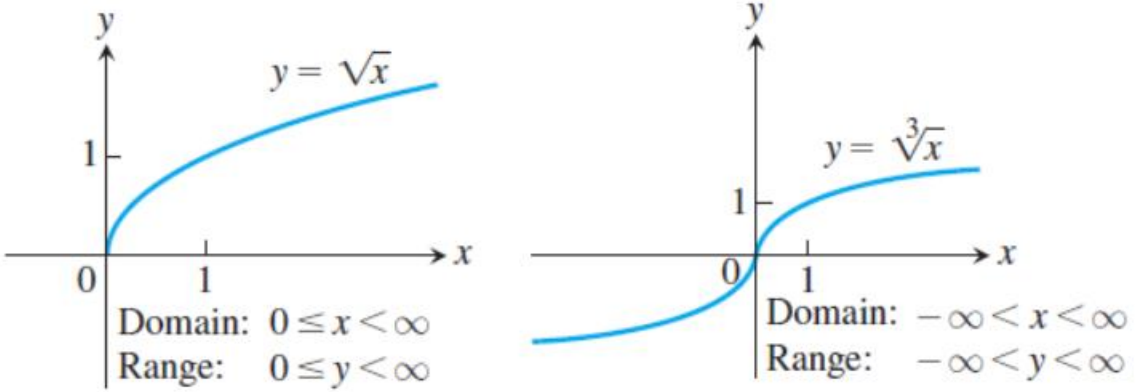
② **Power Function:** $f(x) = x^a$ where a is a constant.



a = 1, 2, 3, 4, 5



a = –1, –2



a = 1/2, 1/3

Common Functions:

③ Polynomial Function:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where n be a whole number and a_n , …… , a_0 are real numbers ($a_n \neq 0$).

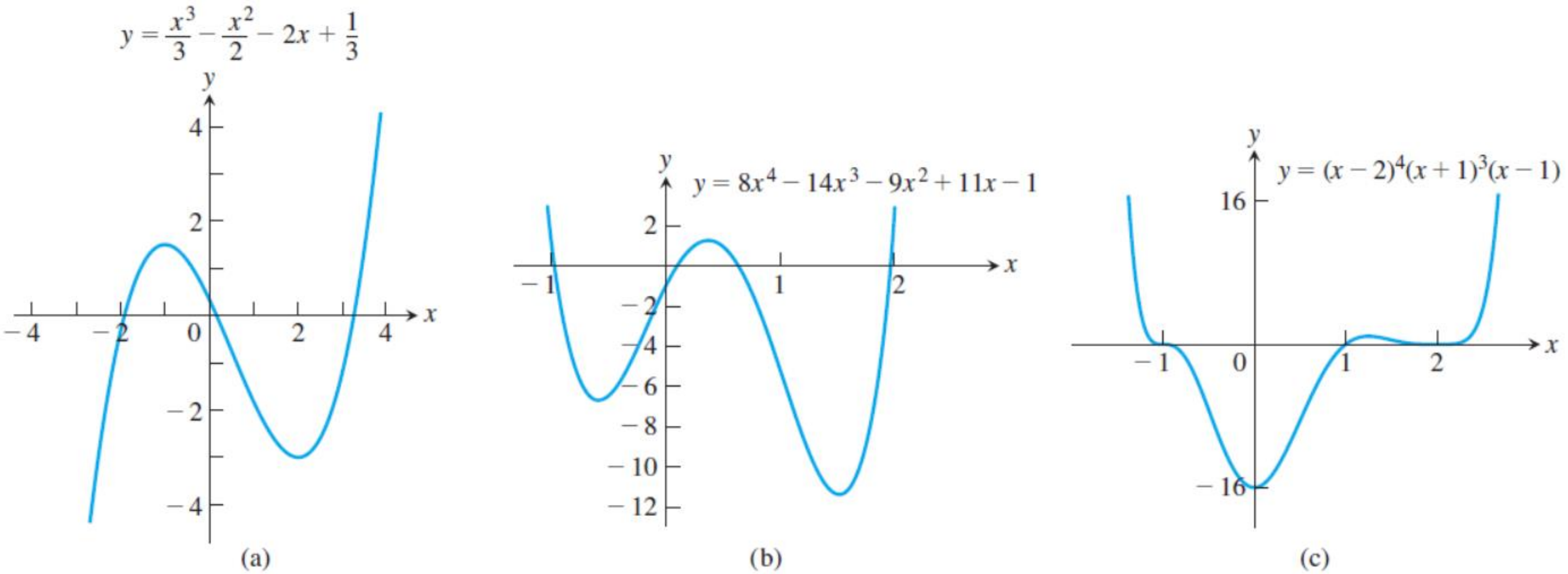


FIGURE 1.18 Graphs of three polynomial functions.

Common Functions:

④ Rational Function:

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomials.

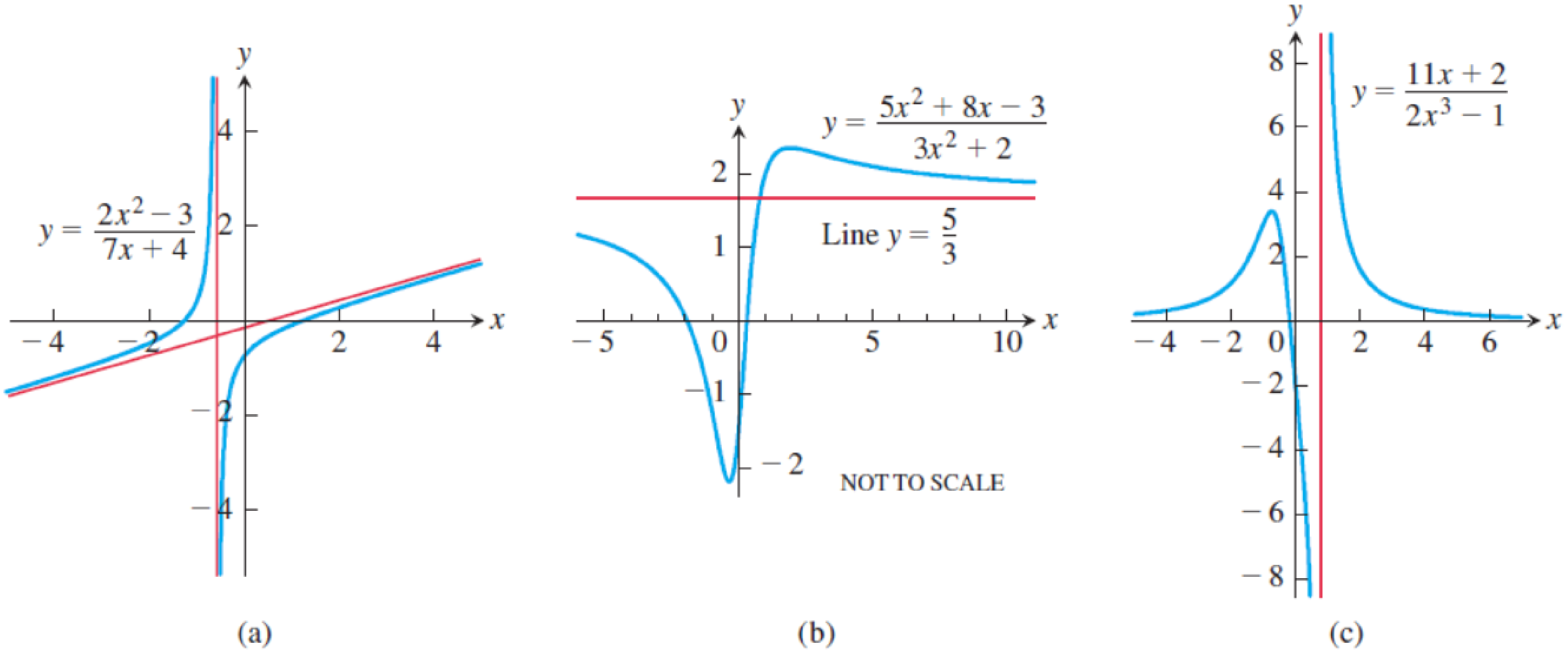


FIGURE 1.19 Graphs of three rational functions. The straight red lines approached by the graphs are called

Common Functions:

⑤ Trigonometric Functions:

$$\sin x = \frac{y}{r}$$

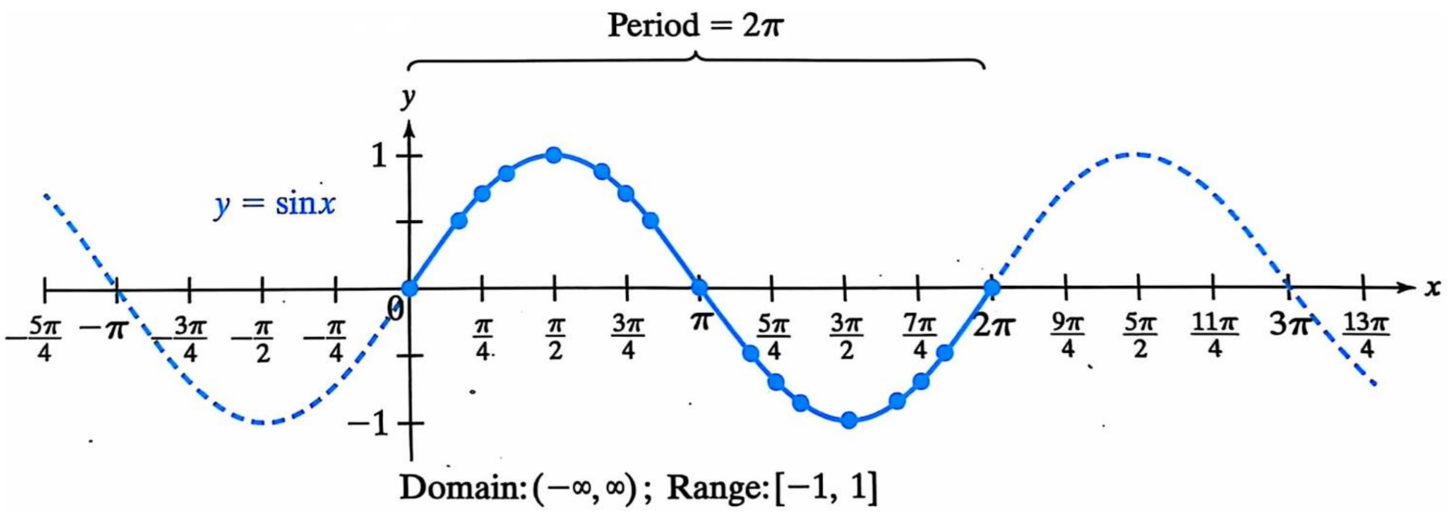
$$\cos x = \frac{x}{r}$$

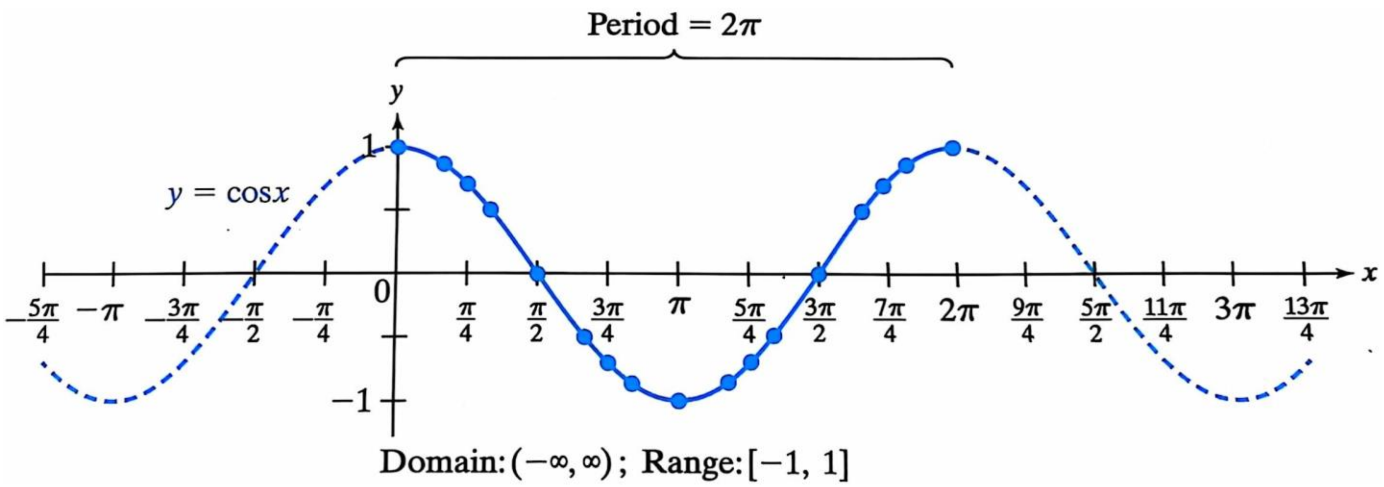
$$\tan x = \frac{\sin x}{\cos x} = \frac{y}{x}$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{x}{y}$$

$$\sec x = \frac{1}{\cos x} = \frac{r}{x}$$

$$\csc x = \frac{1}{\sin x} = \frac{r}{y}$$

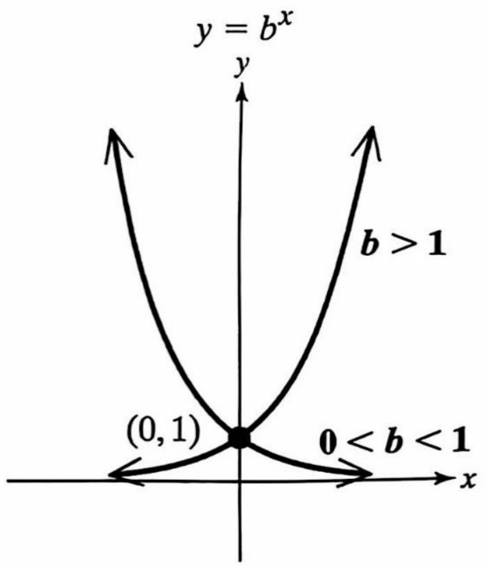




Pythagorean Identity: $\sin^2 x + \cos^2 x = 1$

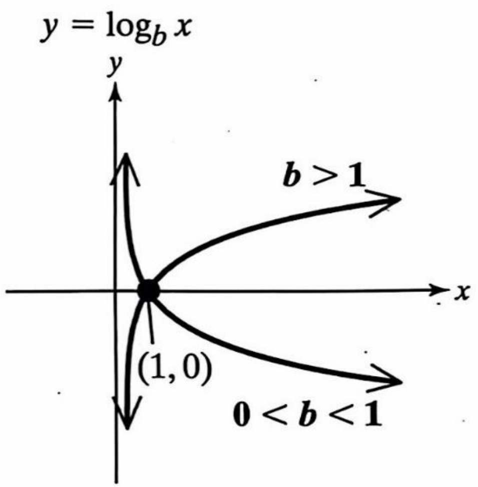
Common Functions: ⑥ Exponential Functions and ⑦ Logarithmic Functions

Exponential Functions



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Horizontal asymptote: $y = 0$
Passes through $(0, 1)$
If $b > 1$, the function is increasing.
If $0 < b < 1$, the function is decreasing.

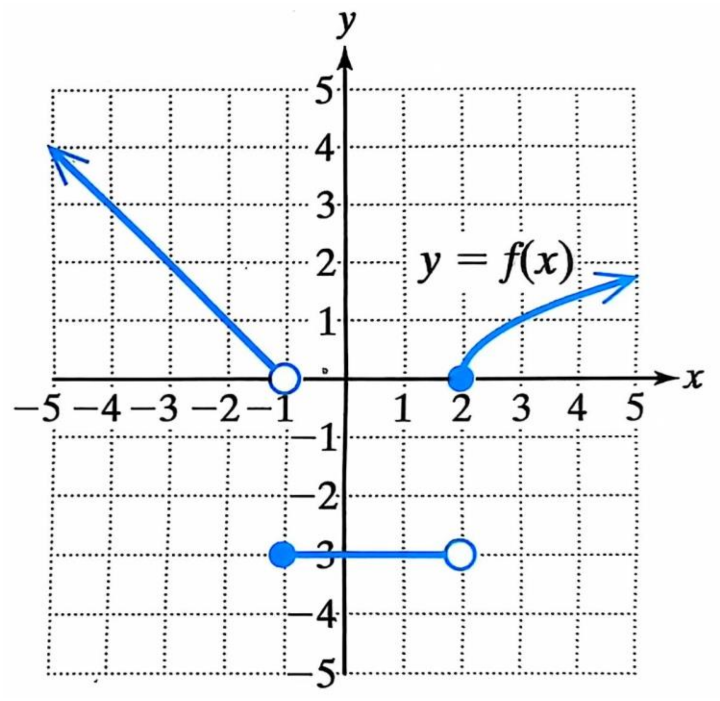
Logarithmic Functions



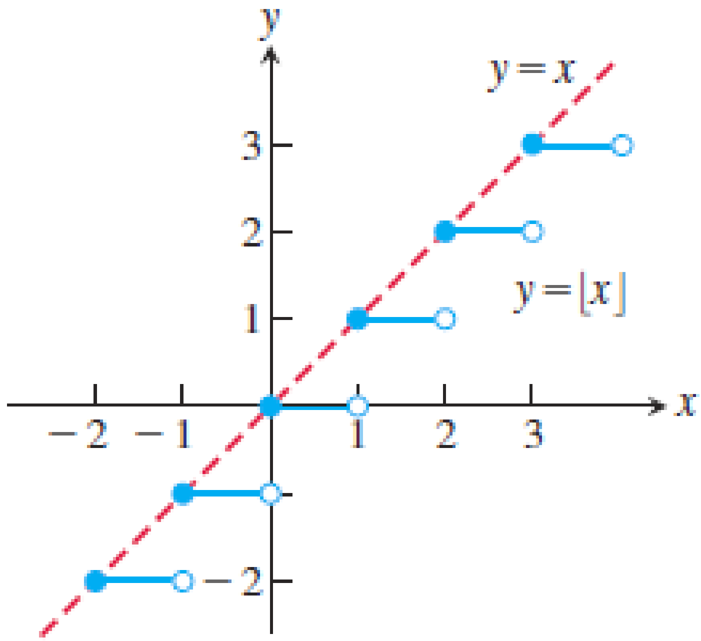
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Vertical asymptote: $x = 0$
Passes through $(1, 0)$
If $b > 1$, the function is increasing.
If $0 < b < 1$, the function is decreasing.

Common Functions:

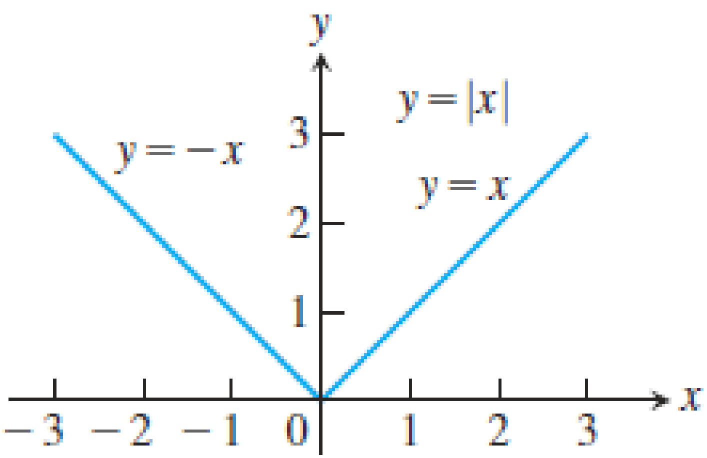
⑧ Piecewise-Defined Functions



General Piecewise-Defined Function



Greatest Integer Function



Absolute Value Function