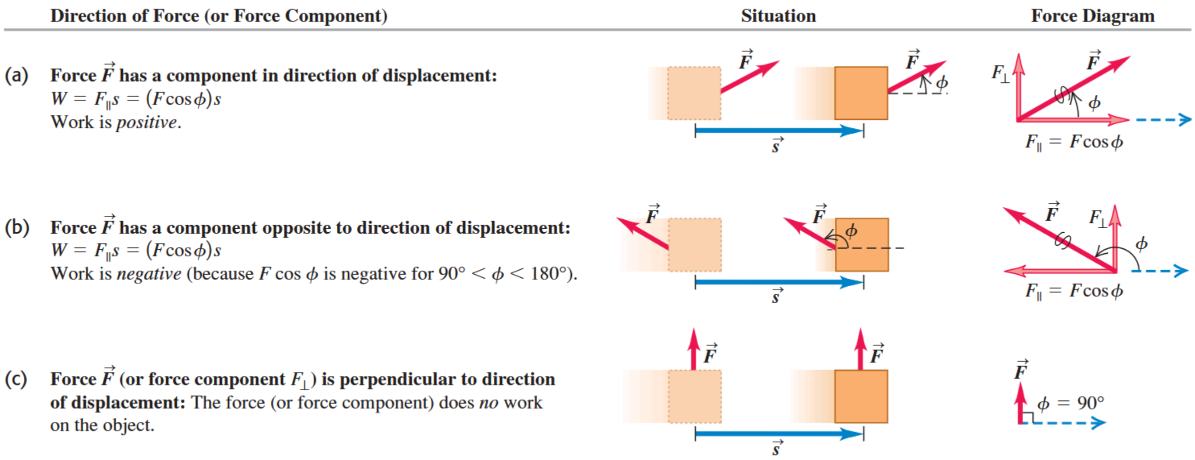
6. Work and Kinetic Energy

Work (with constant force)

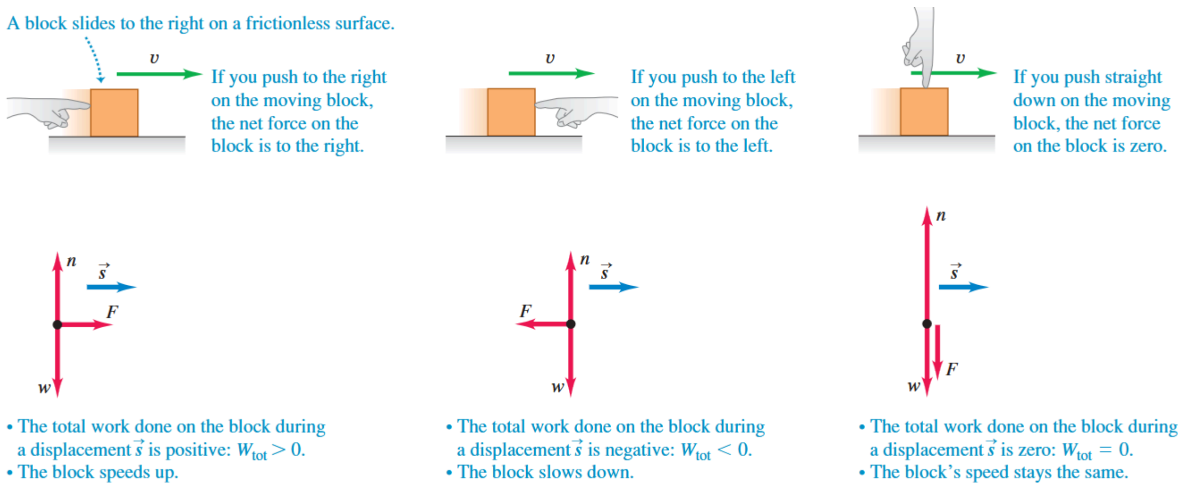
If a body moves through displacement $\overrightarrow{s}$ while a constant force $\overrightarrow{F}$ acts on it, a work is done by the force on the body:

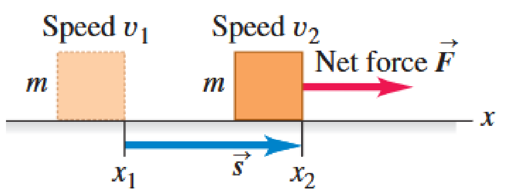
$$W = \overrightarrow{F} \cdot \overrightarrow{s} \cdot \cos\theta$$

The work is positive if the force is in the direction of the displacement, negative if opposite, and **zero if perpendicular** (as $\cos 90^\circ = 0$):



Work and Kinetic Energy





constant net force does work on a moving body

from point x_1 to x_2. Using a constant-acceleration equation, Eq. (2.13), and replacing v_{0x} by v_1, v_x by v_2, and $(x - x_0)$ by s, we have

$$v_2^2 = v_1^2 + 2a_x s$$

$$a_x = \frac{v_2^2 - v_1^2}{2s}$$

When we multiply this equation by m and equate ma_x to the net force F, we find

$$F = ma_x = m\frac{v_2^2 - v_1^2}{2s} \quad \text{and}$$

$$Fs = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \qquad (6.4)$$

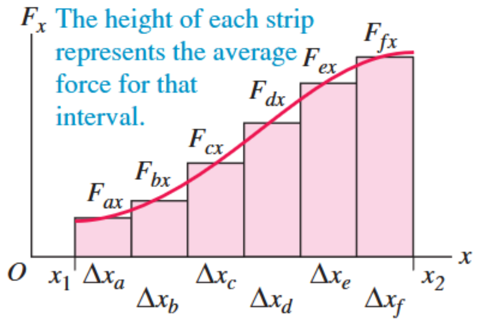
$$K = \frac{1}{2}mv^2$$

The work done by the net force on a particle equals the change in the particle's kinetic energy:

$$W_{\text{tot}} = K_2 - K_1 = \Delta K$$

When the total work done is *positive*, the kinetic energy *increases* (the final kinetic energy is greater than the initial) and the particle is going faster at the end of the displacement. When the total work done is *negative*, the kinetic energy *decreases* (the final kinetic energy is smaller) and the particle is going slower at the end.

Work and Energy with Varying Forces



Using CALCULUS!!!, the total work done by a varying force can be calculated. Using the force F_x, the infinitesimal displacement dx, and the beginning and the ending points x_1 and x_2.

$$W = \int_{x_1}^{x_2} F_x \cos\theta \, dx$$

Force required to stretch a spring

$$F_x = kx$$

where k is the spring constant.

Work Done ON a spring

$$W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

where x_1 is the initial position and the x_2 is the final position.

CAUTION **Work done *on* a spring vs. work done *by* a spring** Note that Eq. (6.10) gives the work that *you* must do *on* a spring to change its length. For example, if you stretch a spring that's originally relaxed, then $x_1 = 0$, $x_2 > 0$, and $W > 0$: The force you apply to one end of the spring is in the same direction as the displacement, and the work you do is positive. By contrast, the work that the *spring* does on whatever it's attached to is given by the *negative* of Eq. (6.10). Thus, as you pull on the spring, the spring does negative work on you. Paying careful attention to the sign of work will eliminate confusion later on!

Work Done BY a spring

$$W = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

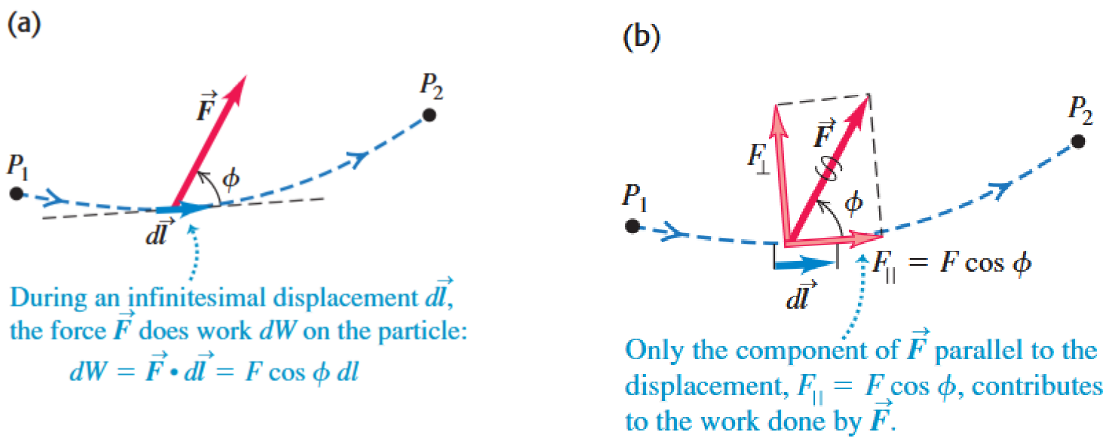
The work done BY and ON a spring are two different things; the signs of the work done change whenever you interchange between the two. So pay attention to the question being asked and determine which one you would use to solve the problems.

Work-Energy Theorem for Motion Along a Curve

Again using CALCULUS!!!, if a particle moves from P_1 to P_2 on a curved path, we can calculate the work done using force $\overrightarrow{F}$ and the infinitesimal displacement $d\overrightarrow{l}$, and θ is assumed to be the angle between $\overrightarrow{F}$ and $d\overrightarrow{l}$:

$$dW = F\cos\theta\, dl = F_{||}\, dl = \overrightarrow{F} \cdot d\overrightarrow{l}$$

$$W = \int_{P_1}^{P_2} F\cos\theta \, dl = \int_{P_1}^{P_2} F_{||}\, dl = \int_{P_1}^{P_2} \overrightarrow{F} \cdot d\overrightarrow{l}$$



Power

When ΔW quantity of work is done during a time interval Δt, the average work done per unit time or **average power** P_{av} is:

$$P_{\mathrm{av}} = \frac{\Delta W}{\Delta t}$$

instantaneous power:

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

$$P = \overrightarrow{F} \cdot \overrightarrow{v}$$

Scenarios

To calculate the work done you can either:

Two approaches:

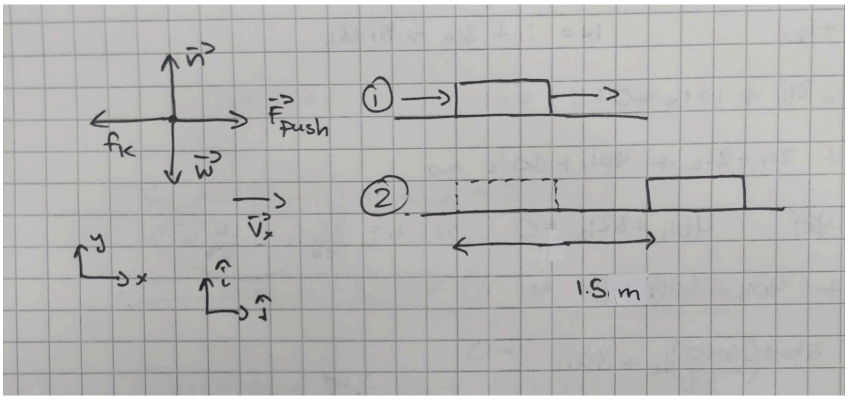
1. Compute work done by **individual forces** first and **algebraic sum** of them is the total work
2. Compute the **net force** first, and then the total work

Simple Push & Pull

6.1 • You push your physics book 1.50 m along a horizontal tabletop with a horizontal push of 2.40 N while the opposing force of friction is 0.600 N. How much work does each of the following forces do on the book: (a) your 2.40-N push, (b) the friction force, (c) the normal force from the tabletop, and (d) gravity? (e) What is the net work done on the book?

6.2 • A tow truck pulls a car 5.00 km along a horizontal roadway using a cable having a tension of 850 N. (a) How much work does the cable do on the car if it pulls horizontally? If it pulls at 35.0° above the horizontal? (b) How much work does the cable do on the tow truck in both cases of part (a)? (c) How much work does gravity do on the car in part (a)?

Exercise 6.1:



remember $W = \overrightarrow{F} \cdot \overrightarrow{s} \cos\theta$

$$W_{total} = W_F + W_f + W_n + W_w$$

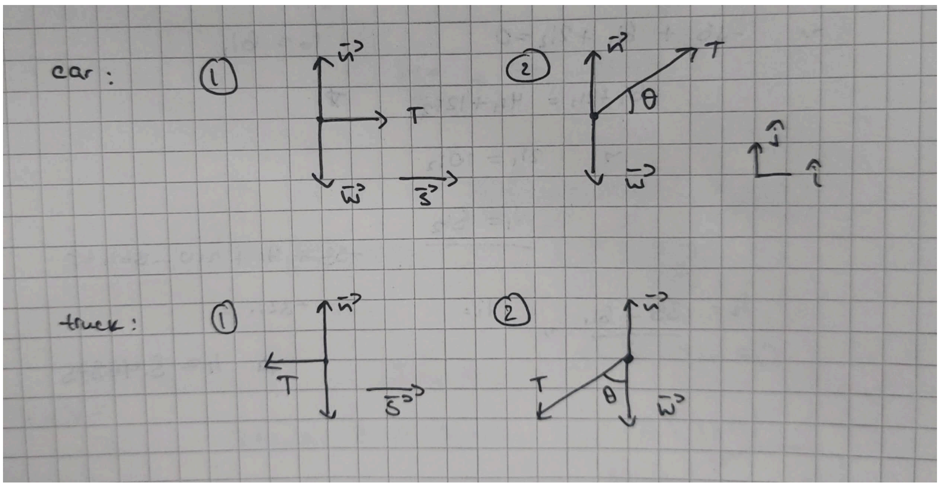
$$W_F = 2.4 \text{ N } (\hat{i}) \cdot 1.5 \text{ m}(\hat{i}) \cdot \cos 0^\circ = 3.6 \text{ J}$$

$$W_f = 0.6 \text{ N } (-\hat{i}) \cdot 1.5 \text{ m } (\hat{i}) \cdot \cos 0^\circ = -0.9 \text{ J}$$

$$W_n = W_w = 0 \qquad \text{as perpendicular to displacement } : \cos 90^\circ = 0$$

$$W_{\text{total}} \text{ or } W_{\text{net}} = (3.6 - 0.9) \text{ J} = 2.7 \text{ J}$$

Example 6.2:



$$1.\ W_{T,\text{car}} = 850\ \text{N}\ (\hat{\imath}) \cdot 5000\ \text{m}\ (\hat{\imath}) \cdot \cos 0^\circ\ = 4250\ \text{kJ}$$

$$1.\ W_{T,\text{truck}} = 850\ \text{N}\ (-\hat{\imath}) \cdot 5000\ \text{m}\ (\hat{\imath}) \cdot \cos 0^\circ\ = -4250\ \text{kJ}$$

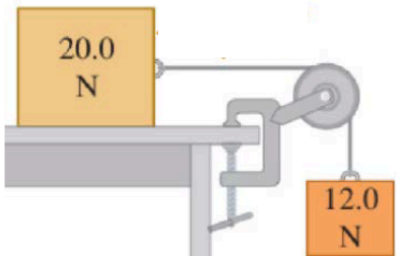
$$2.\ W_{T,\text{car}} = 850\ \text{N}\ (\hat{\imath}) \cdot 5000\ \text{m}\ (\hat{\imath}) \cdot \cos 35^\circ\ = 3480\ \text{kJ}$$

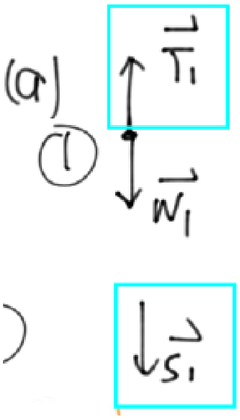
$$2.\ W_{T,\text{truck}} = 850\ \text{N}\ (-\hat{\imath}) \cdot 5000\ \text{m}\ (\hat{\imath}) \cdot \cos 35^\circ\ = -3480\ \text{kJ}$$

$$W_{grav} = 0$$

Pulley

6.7 • Two blocks are connected by a very light string passing over a massless and frictionless pulley (Fig. E6.7). Traveling at constant speed, the 20.0-N block moves 75.0 cm to the right and the 12.0-N block moves 75.0 cm downward. During this process, how much work is done (a) on the 12.0-N block by (i) gravity and (ii) the tension in the string? (b) On the 20.0-N block by (i) gravity, (ii) the tension in the string, (iii) friction, and (iv) the normal force? (c) Find the total work done on each block.



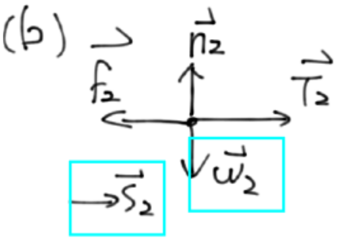


$$W_{tot} = W_{T_1} + W_{w_1}$$

$$W_{w_1} = 12\ \mathrm{N}(\hat{j}) \cdot 0.75\ \mathrm{m}(\hat{j}) \cdot \cos 0^\circ = 9\ \mathrm{J}$$

$$\text{since } T_1 = w \ : \ \text{ as constant speed} = \text{no acceleration}$$

$$W_{T_1} = -W_w = -9\ \mathrm{J} \ : \ \text{ as displacement is opposite } T_1$$



$$T_2 = T_1 = f_k$$
$$\text{since constant speed} = \text{no acceleration}$$

$$W_{T_2} = 12\ \mathrm{N}(\hat{\imath}) \cdot 0.75\ \mathrm{m}(\hat{\imath}) \cdot \cos 0^\circ = 9\ \mathrm{J}$$

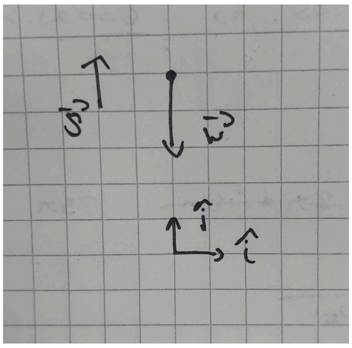
$$W_{T_2} = -W_{f_k} = -9\ \mathrm{J} \ : \ \text{ and } W_{grav} = W_n = 0 \text{ since } \cos 90^\circ = 0$$

Kinetic Energy change

6.20 •• You throw a 20-N rock vertically into the air from ground level. You observe that when it is 15.0 m above the ground, it is traveling at 25.0 m/s upward. Use the work–energy theorem to find (a) the rock's speed just as it left the ground and (b) its maximum height.

$$\text{remember :} \qquad W_{tot} = K_2 - K_1$$

A. When the rock is 15m above the ground:



$$W_{tot} = \overrightarrow{w} \cdot \overrightarrow{s}$$

$$\Rightarrow W_{tot} = mg(-\hat{j}) \cdot 15 \text{ m}(\hat{j}) = -15mg \text{ J}$$

$$-15mg = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \Rightarrow \frac{2(15mg) + mv_2^2}{m} = v_1^2$$

$$\Rightarrow v_1 = \sqrt{30g + v_2^2}$$

$$v_1 = \sqrt{30 \cdot 9.8 + 25^2} = 30.3 \text{ ms}^{-1}$$

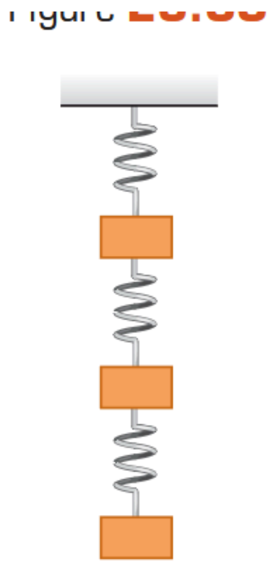
B. Max height means the final kinetic energy $K_2 = 0$ as the velocity at the max height is 0.

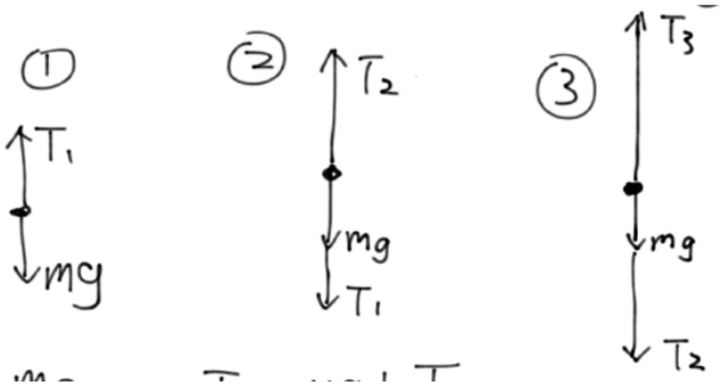
$$-mgH = K_2 - K_1 \Rightarrow -mgH = -\frac{1}{2}mv_1^2$$

$$\Rightarrow H = \frac{v_1^2}{2g} = \frac{919}{2 \cdot 9.8} = 46.9 \text{ m}$$

Springs

6.33 • Three identical 6.40-kg masses are hung by three identical springs, as shown in Fig. E6.33. Each spring has a force constant of 7.80 kN/m and was 12.0 cm long before any masses were attached to it. (a) Draw a free-body diagram of each mass. (b) How long is each spring when hanging as shown? (*Hint:* First isolate only the bottom mass. Then treat the bottom two masses as a system. Finally, treat all three masses as a system.)





remember $F = k \cdot \Delta x$

$$T_1 = mg = k\Delta x_1$$
$$T_2 = mg + T_1 = 2mg = k\Delta x_2$$
$$T_3 = mg + T_2 = 3mg = k\Delta x_3$$

$$\Delta x_1 = \frac{mg}{k} \Rightarrow \frac{6.4 \cdot 9.8}{7.4 \times 10^{-3}} = 0.0008$$

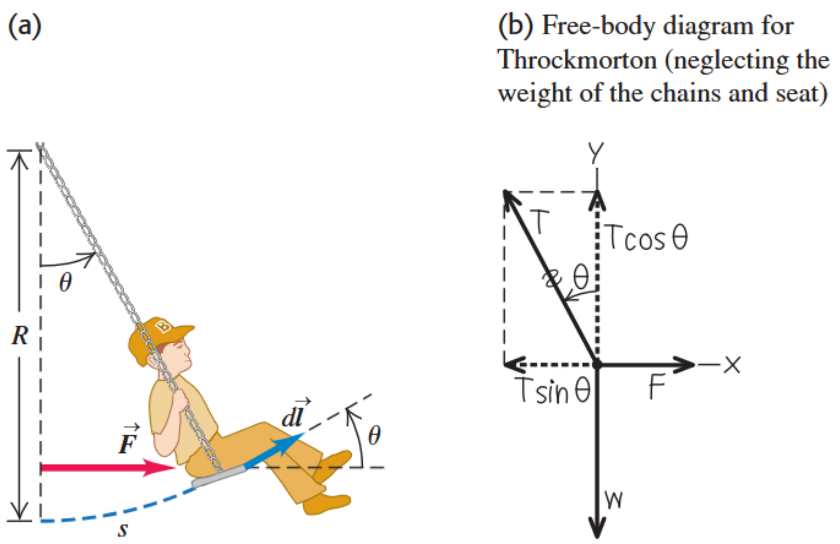
$2 \cdot \Delta x_1 = \Delta x_2$ $\quad$ $3 \cdot \Delta x_1 = \Delta x_3$ $\quad$ $\Delta x_2 = 0.016$ $\quad$ $\Delta x_3 = 0.025$

$$l_1 = 0.12 + 0.0084 = 0.128 \text{ m}$$

$$l_2 = 0.12 + 0.016 = 0.136 \text{ m}$$

$$l_3 = 0.12 + 0.025 = 0.145 \text{ m}$$

Varying force with Curved Path



To compute the work done by $\vec{F}$, we need to know how this force varies with the angle θ. The net force on Throcky is zero, so $\sum F_x = 0$ and $\sum F_y = 0$. From Fig. 6.24b,

$$\sum F_x = F + (-T\sin\theta) = 0$$
$$\sum F_y = T\cos\theta + (-w) = 0$$

From the free-body diagram, the $x - y$ and component forces.

angle $d\theta$ has a magnitude $dl = ds = R\ d\theta$. The work done by $\vec{F}$ is then

$$W = \int \vec{F} \cdot d\vec{l} = \int F \cos\theta\ ds$$

Now we express F and ds in terms of the angle θ, whose value increases from 0 to θ_0:

$$\begin{aligned} W &= \int_0^{\theta_0} (w \tan\theta) \cos\theta\ (R\ d\theta) = wR \int_0^{\theta_0} \sin\theta\, d\theta \\ &= wR(1 - \cos\theta_0) \end{aligned}$$

Using $W = \int \overrightarrow{F} \cdot d\overrightarrow{l}$, and substituting $\overrightarrow{F}$ and $d\overrightarrow{l}$, to find the work done by F along the curved path.

$\hat{\imath}\ ds\cos\theta + \hat{\jmath}\ ds\sin\theta$. Similarly, we can write the three forces as

$$\begin{aligned} \vec{T} &= \hat{\imath}(-T\sin\theta) + \hat{\jmath}T\cos\theta \\ \vec{w} &= \hat{\jmath}(-w) \\ \vec{F} &= \hat{\imath}F \end{aligned}$$

We use Eq. (1.21) to calculate the scalar product of each of these forces with $d\vec{l}$:

$$\begin{aligned} \vec{T} \cdot d\vec{l} &= (-T\sin\theta)(ds\cos\theta) + (T\cos\theta)(ds\sin\theta) = 0 \\ \vec{w} \cdot d\vec{l} &= (-w)(ds\sin\theta) = -w\ \sin\theta\, ds \\ \vec{F} \cdot d\vec{l} &= F(ds\cos\theta) = F\cos\theta\, ds \end{aligned}$$

Since $\vec{T} \cdot d\vec{l} = 0$, the integral of this quantity is zero and the work done by the chain tension is zero, just as we found above. Using $ds = R\ d\theta$, we find the work done by the force of gravity is

$$\begin{aligned} \int \vec{w} \cdot d\vec{l} &= \int (-w\sin\theta) R\, d\theta = -wR \int_0^{\theta_0} \sin\theta\, d\theta \\ &= -wR(1 - \cos\theta_0) \end{aligned}$$