Answer Key: Exercise 2

Math Bootcamp

Instructions

- *Please show your work! You may lose points by simply writing in the answer. If the problem requires you to execute commands in `R`, please include the code you used to get your answers. Please also include the `.R` file that contains your code. If you are not sure if work needs to be shown for a particular problem, please ask.*

Question 1

Which of the following functions are continuous? If not, where are the discontinuities?

1. $f(x) = \frac{9x^3 - x}{(x-1)(x+1)}$

   Discontinuous at $x = 1$, $x = -1$

2. $f(x) = e^{-x^2}$

   Continuous

3. $h(x, y) = \frac{xy}{x+y}$

   Discontinuous at (0,0)

4. $f(x) = e^{-x^2}$

   Continuous

5. $g(y, z) = \frac{6y^4z^3 + 3y^2z - 56}{12y^5 - 3zy + 18z}$

   All ratios of polynomials are continuous unless a denominator is zero (y = 0, z = 0 here).

6. $f(y) = y3 - y2 + 1$

   All polynomials are continuous

7. $f(a,b) = \begin{cases} x^3 + 1x & x > 0 \\ \frac{1}{2} & x = 0 \\ -x^2x & x < 0 \end{cases}$

Discontinuous at x=0

Question 2

A very famous sequence of numbers is called the Fibonacci sequence, which starts with 0 and 1 and continues according to:

$$0, 1, 1, 2, 3, 5, 8, 13, 21, ...$$

Figure out the logic behind the sequence and write it as a function using subscripted values like x_j for the jth value in the sequence.

It is easy to see that numbers are produced from adding consecutive values. This is described by:

$$x_n = x_{n-1} + x_{n-2},$$

for all $n \geq 2$ with the following conditions:

$$x_0 = 0, x_1 = 1$$

Question 3

- *Using the change of base formula for logarithms, change $log_6(36)$ to $log_3(36)$.*

$$log_a(x) = \frac{log_b(x)}{log_b(a)}$$

$$log_a(x) = \frac{log_6(36)}{log_6(3)} = log_3(36)$$

- *Sociologists Holland and Leinhardt (1970) developed measures for models of structure in interpersonal relations using ranked clusters. This approach requires extensive use of factorials to express personal choices. The authors defined the notation $x^{(k)} = x(x-1)(x-2) * s(x-k+1)$. Show that $x^{(k)}$ is just $\frac{x!}{(x-k)!}$.*

  We might expand the top factorial just enough to make the identity clear:

$$
\begin{aligned}
x! &= x(x-1)(x-2)\cdots(x-k+1)(x-k)\cdots 3\cdot 2\cdot 1 && (1)\\
\frac{x!}{(x-k)!(x-k)!} &= x(x-1)(x-2)\cdots(x-k+1)(x-k)\cdots 3\cdot 2\cdot 1 && (2)\\
&= \frac{x(x-1)(x-2)\cdots(x-k+1)[(x-k)\cdots 3\cdot 2\cdot 1]}{(x-k)!} && (3)\\
&= \frac{x(x-1)(x-2)\cdots(x-k+1)[(x-k)!]}{(x-k)!} && (4)\\
&= \frac{x(x-1)(x-2)\cdots(x-k+1)\cancel{[(x-k)!]}}{\cancel{(x-k)!}} && (5)\\
&= x(x-1)(x-2)\cdots(x-k+1) && (6)\\
&= x^{(k)} && (7)
\end{aligned}
$$

We might also fully expand the denominator instead of recombining the numerator:

$$
\begin{aligned}
x! &= x(x-1)(x-2)\cdots(x-k+1)(x-k)\cdots 3\cdot 2\cdot 1 && (8)\\
\frac{x!}{(x-k)!(x-k)!} &= x(x-1)(x-2)\cdots(x-k+1)(x-k)\cdots 3\cdot 2\cdot 1 && (9)\\
&= \frac{x(x-1)(x-2)\cdots(x-k+1)[(x-k)\cdots 3\cdot 2\cdot 1]}{[(x-k)\cdots 3\cdot 2\cdot 1]} && (10)\\
&= \frac{x(x-1)(x-2)\cdots(x-k+1)\cancel{[(x-k)\cdots 3\cdot 2\cdot 1]}}{\cancel{[(x-k)\cdots 3\cdot 2\cdot 1]}} && (11)\\
&= x(x-1)(x-2)\cdots(x-k+1) && (12)\\
&= x^{(k)} && (13)
\end{aligned}
$$