Exercise 5

Math Bootcamp

Instructions

- Please show your work! You may lose points by simply writing in the answer. If the problem requires you to execute commands in `R`, please include the code you used to get your answers. Please also include the `.R` file that contains your code. If you are not sure if work needs to be shown for a particular problem, please ask.
- You should submit your work electronically on GitHub in `.pdf` form.

Question 1

Determine the rank of the following matrix:

$$\begin{pmatrix} 1 & 2 & 0 & 1 & 0 \\ 2 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 1 & 2 & 1 & -1 & 1 \end{pmatrix}$$

Question 2

Solve the following systems of equations for x, y, and z:

1.

$$\begin{aligned} x + y + 2z &= 2 \\ 3x - 2y + z &= 1 \\ y - z &= 3 \end{aligned}$$

2.

$$\begin{aligned} 2x + 3y - z &= -8 \\ x + 2y - z &= 2 \\ -x - 4y + z &= -6 \end{aligned}$$

3.

$$
\begin{aligned}
x - y + 2z &= 2 \\
4x + y - 2z &= 10 \\
x + 3y + z &= 0
\end{aligned}
$$

Question 3

Another method for solving linear systems of equations of the form $\mathbf{A}^{-1}\mathbf{y} = \mathbf{x}$ is Cramer's rule. Define $\mathbf{A}_j$ as the matrix where y is plugged in for the$j$th column of $\mathbf{A}$. Perform this for every column $1, \ldots, q$ to produce q of these matrices, and the solution will be the vector $\left[\frac{|\mathbf{A}_1|}{\mathbf{A}}, \frac{|\mathbf{A}_2|}{\mathbf{A}}, \ldots, \frac{|\mathbf{A}_q|}{\mathbf{A}}\right]$. Show that performing these steps on the matrix in the example on page 159 gives the same answer.