Answer Key: Exercise 1

Math Bootcamp

Instructions

- *Please show your work! You may lose points by simply writing in the answer. If the problem requires you to execute commands in* `R`*, please include the code you used to get your answers. Please also include the* `.R` *file that contains your code. If you are not sure if work needs to be shown for a particular problem, please ask.*

Question 1

Indicate the level of measurement and which measure(s) of central tendency can be used for the following:

1. *college education: none, BA/BS, MA, Ph.D.*: Nominal
2. *letter grades*: Ordinal
3. *income given as 0–10K, 10–20K, 30–50K, 50–80K, 100K+*: Ordinal
4. *distance of commute from home to work*: Ratio
5. *marital status: single, married, widowed, divorced* Nominal
6. *working status: employed, unemployed, retired, student*: Nominal
7. *governmental level: local, state, federal, international*: Ordinal
8. *party: Conservative, Labour, Liberal Democrats, Brexit Party*: Nominal

For Nominal variables, the mode is the appropriate measure of central tendency. For Ordinal variables, the median is the appropriate measure of central tendency (though the mode can also be computed). For Interval and Ratio data, both median and mean are appropriate measures of central tendency; if there is a view that the data is skewed or possesses outliers, a median may be desireable.

Question 2

Simplify and solve the following expressions:

1. $(-x^4y^2)^2 = x^8y^4$
2. $\frac{x4}{x3} = x$
3. $y^7y^6y^5y^4 = y^22$
4. $(2a^2)(4a^4) = 8a^6$
5. $\left(\frac{1}{27b^3}\right)^{1/3} = \frac{1}{3b}$
6. $(z^2)^4 = z^8$
7. $9(3^0) = 9$
8. $(-2)^{7-4} = -8$
9. $\frac{2a/7b}{11b/5a} = \frac{10a^2}{77b^2}$
10. $(a+b)^2+(a-b)^2+2(a+b)(a-b)-3a^2 = a^2$
11. $\sqrt[3]{2^3} = 2$
12. $\sqrt[3]{27} = 3$
13. $\sqrt[4]{625} = 5$

Question 3

1. *Prove de Morgan's Laws for two sets A and B. Formally, the rules are written as:*

   $\neg(P \cup Q) \iff (\neg P) \cap (\neg Q), \neg(P \cup Q) \iff (\neg P) \cap (\neg Q),$

   and

   $\neg(P \cap Q) \iff (\neg P) \cup (\neg Q), \neg(P \cap Q) \iff (\neg P) \cup (\neg Q)$

   where P and Q are propositions:

   $\neg$ *is the negation logic operator (NOT),*

   $\cap$ *is the conjunction logic operator (AND),*

   $\cup$ *is the disjunction logic operator (OR),*

   $\iff$ *is a metalogical symbol meaning "can be replaced in a logical proof with"*

   Show $\neg(A \cup B) = (\neg A) \cap (\neg B)$:

   DeMorgan's law could be demonstrated using a Venn Diagram. If we would like to instead use logic, we can do so as follows:

   (a) Suppose: $X \in \neg(A \cup B)$
   (b) Then: $X \notin A$ and $X \notin B$
   (c) So: $X \in \neg(A) \cap \neg(B)$

   Show $\neg(A \cap B) = \neg(A) \cup \neg(B)$:

   Observe that the logical prepositions here are equivalent to the "visual partition" of the Venn Diagram:

   (a) Suppose: $X \in \neg(A \cap B)$
   (b) Then: X is in one of $(A\ B), (B\ A), (\neg(A) \cap \neg(B))$.
   (c) A partition of the sample space consists of these three sets along with $A \cap B$
   (d) So: $X \in \neg(A \cap B)$

2. *For some set A, explain $A \cup A$ and $A \cap A$.*

   The question asks about the union of a set with itself, and the intersection of a set with itself. The union of two sets A and B is the set of elements which are in A, in B, or in both A and B, $A \cup B = \{x : x \in A \text{ or } x \in B$. The intersection of two sets contains all elements of A that also belong to B or equivalently, all elements of B that also belong to A. So, if we have only one set, the union and intersection of a set with itself is just the set itself.