

One Method for Convex Optimization on Square

I. Kuruzov¹ F. Stonyakin^{1,2}

¹Moscow Institute of Physics and Technology

²V.I.Vernadsky Crimean Federal University

COG, 2019

Method Description

Task:

$$\min_{(x,y)} \{f(x,y) | (x,y) \in Q\},$$

where f is a convex function, $Q = [a, b] \times [c, d] \in \mathbb{R}^2$.

One iteration

- Find minimum with accuracy δ on central horizontal segment in square and calculate gradient at this point

Task:

$$\min_{(x,y)} \{f(x,y) | (x,y) \in Q\},$$

where f is a convex function, $Q = [a, b] \times [c, d] \in \mathbb{R}^2$.

One iteration

- Find minimum with accuracy δ on central horizontal segment in square and calculate gradient at this point
- Choose rectangle which anti-gradient looks in

Task:

$$\min_{(x,y)} \{f(x,y) | (x,y) \in Q\},$$

where f is a convex function, $Q = [a, b] \times [c, d] \in \mathbb{R}^2$.

One iteration

- Find minimum with accuracy δ on central horizontal segment in square and calculate gradient at this point
- Choose rectangle which anti-gradient looks in
- Analogically for vertical segment in the rectangle

Plan

- 1 Strategy for segment accuracy
- 2 Convergence
- 3 Convergence
- 4 Tests

Let f be convex and has L -Lipschitz continuous gradient.

$$\text{sign } f'_y(x_0) = \text{sign } f'_y(x_{\text{current}})$$

For this it is sufficient:

$$|f'_y(x_0) - f'_y(x_{\text{current}})| \leq |f'_y(x_0)|$$

Let f be convex and has L -Lipschitz continuous gradient.

$$\text{sign } f'_y(x_0) = \text{sign } f'_y(x_{\text{current}})$$

For this it is sufficient:

$$|f'_y(x_0) - f'_y(x_{\text{current}})| \leq |f'_y(x_0)|$$

$$\boxed{\delta < \frac{|f'_y(x_0)|}{L}}$$

$$\text{sign } f'_y(x_0) = \text{sign } f'_y(x_{\text{current}})$$

For this it is sufficient:

$$|f'_y(x_0) - f'_y(x_{\text{current}})| \leq |f'_y(x_{\text{current}})|$$

$$\text{sign } f'_y(x_0) = \text{sign } f'_y(x_{\text{current}})$$

For this it is sufficient:

$$|f'_y(x_0) - f'_y(x_{\text{current}})| \leq |f'_y(x_{\text{current}})|$$

$$\boxed{\delta < \frac{|f'_y(x_{\text{current}})|}{L}}$$

Convergence

Function f is convex. Size of square Q is equal to a . One takes a center of a current square as approximate solution.

Estimate through Lipschitz function constant

Let function f be L_f -Lipschitz continuous. Then for accuracy ϵ on function it is sufficient:

$$N = \left\lceil \log_2 \frac{L_f a}{\sqrt{2}\epsilon} \right\rceil. \quad (1)$$

Estimate through Lipschitz gradient constant

Let function f have L_g -Lipschitz continuous gradient. Moreover, solution of initial task is **internal point**. Then for accuracy ϵ on function it is sufficient:

$$N = \left\lceil \frac{1}{2} \log_2 \frac{L_g a^2}{4\epsilon} \right\rceil. \quad (2)$$

If following inequality is met

$$\frac{2L_f^2}{L_g} \geq \epsilon$$

then the estimate through L_g is better than through L_f .

- Quadratic Function

$$f(x, y) = (Ax + By)^2 + Cy^2 + Dx + Ey$$

$$(x^*, y^*) \in Q$$

$$L_f = \max_{(x,y) \in Q} \|\nabla f(x, y)\|$$

$$L_g = \max |\lambda(H(f))|$$

- Quadratic Function

$$f(x, y) = (Ax + By)^2 + Cy^2 + Dx + Ey$$

$$(x^*, y^*) \in Q$$

$$L_f = \max_{(x,y) \in Q} \|\nabla f(x, y)\|$$

$$L_g = \max |\lambda(H(f))|$$

- LSM for exponent

$$f(x, y) = \sum_{i=1}^n (x \exp(ya_i) - b_i)$$

$$a_i \in [-1, 1], b_i = \tilde{x} \exp(\tilde{y}a_i) \text{ for } \tilde{x} \in [2, 4], \tilde{y} \in [-1, 1]$$

$$Q = [2, 4] \times [-1, 1]$$

Experiments

Comparison Of Strategies

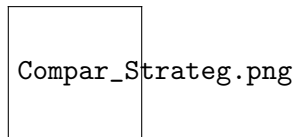




Figure: Comparison of strategies

Summary

- The **first main message** of your talk in one or two lines.
- The **second main message** of your talk in one or two lines.
- Perhaps a **third message**, but not more than that.
- Outlook
 - Something you haven't solved.
 - Something else you haven't solved.

For Further Reading I

-  A. Author.
Handbook of Everything.
Some Press, 1990.
-  S. Someone.
On this and that.
Journal of This and That, 2(1):50–100, 2000.