# One Method for Convex Optimization on Square

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# Method Description

Task:

$$\min_{(x,y)} \{f(x,y) | (x,y) \in Q\},$$

where $f$ is a convex function, $Q = [a,b] \times [c,d] \in \mathbb{R}^2$.

## One iteration

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- Find minimum with accuracy $\delta$ on central horizontal segment in square and calculate gradient at this point
- Choose rectangle which anti-gradient looks in
- Analogically for vertical segment in the rectangle

# Plan

1. Strategy for segment accuracy
2. Convergence
3. Convergence
4. Tests

Let $f$ be convex and has $L$-Lipschitz continuous gradient.

$$\operatorname{sign} f'_y(x_0) = \operatorname{sign} f'_y(x_{current})$$

For this it is sufficient:

$$|f'_y(x_0) - f'_y(x_{current})| \leq |f'_y(x_0)|$$

# Strategies
## True Gradient

Let $f$ be convex and has $L$-Lipschitz continuous gradient.

$$\operatorname{sign} f'_y(x_0) = \operatorname{sign} f'_y(x_{current})$$

For this it is sufficient:

$$|f'_y(x_0) - f'_y(x_{current})| \leq |f'_y(x_0)|$$

$$\boxed{\delta < \frac{|f'_y(x_0)|}{L}}$$

# Strategies
## Current Gradient

$$\text{sign } f_y'(x_0) = \text{sign } f_y'(x_{current})$$

For this it is sufficient:

$$|f_y'(x_0) - f_y'(x_{current})| \le |f_y'(x_{current})|$$

# Strategies

## Current Gradient

$$\operatorname{sign} f'_y(x_0) = \operatorname{sign} f'_y(x_{current})$$

For this it is sufficient:

$$|f'_y(x_0) - f'_y(x_{current})| \leq |f'_y(x_{current})|$$

$$\boxed{\delta < \frac{|f'_y(x_{current})|}{L}}$$

# Convergence

Function $f$ is convex. Size of square $Q$ is equal to $a$. One takes a center of a current square as approximate solution.

## Estimate through Lipschitz function constant

Let function $f$ be $L_f$-Lipschitz continuous. Then for accuracy $\epsilon$ on function it is sufficient:

$$N = \left\lceil \log_2 \frac{L_f a}{\sqrt{2}\epsilon} \right\rceil. \tag{1}$$

## Estimate through Lipschitz gradient constant

Let function $f$ have $L_g$-Lipschitz continuous gradient. Moreover, solution of initial task is **internal point**. Then for accuracy $\epsilon$ on function it is sufficient:

$$N = \left\lceil \frac{1}{2} \log_2 \frac{L_g a^2}{4\epsilon} \right\rceil. \tag{2}$$

# Convergence

If following inequallity is met

$$\frac{2L_f^2}{L_g} \geq \epsilon$$

then the estimate through $L_g$ is better than through $L_f$.

- Quadratic Function

$$f(x,y) = (Ax+By)^2 + Cy^2 + Dx + Ey$$

$$(x^*, y^*) \in Q$$

$$L_f = \max_{(x,y)\in Q} \|\nabla f(x,y)\|$$

$$L_g = \max \left|\lambda\left(H(f)\right)\right|$$

- Quadratic Function

$$f(x,y) = (Ax+By)^2 + Cy^2 + Dx + Ey$$
$$(x^*, y^*) \in Q$$
$$L_f = \max_{(x,y)\in Q} \|\nabla f(x,y)\|$$
$$L_g = \max |\lambda(H(f))|$$

- LSM for exponent

$$f(x,y) = \sum_{i=1}^{n} (x\exp(ya_i) - b_i)$$
$$a_i \in [-1,1], b_i = \tilde{x}\exp(\tilde{y}a_i) \text{ for } \tilde{x} \in [2,4], \tilde{y} \in [-1,1]$$
$$Q = [2,4] \times [-1,1]$$

# Experiments

## Comparison Of Strategies

Compar_Strateg.png

Figure: Comparison of strategies

# Summary

- The first main message of your talk in one or two lines.
- The second main message of your talk in one or two lines.
- Perhaps a third message, but not more than that.

- Outlook
  - Something you haven't solved.
  - Something else you haven't solved.

# For Further Reading I


A. Author.
*Handbook of Everything*.
Some Press, 1990.

S. Someone.
On this and that.
*Journal of This and That*, 2(1):50–100, 2000.