One Method for Convex Optimization on Square

I. Kuruzov[1] F. Stonyakin[1,2]

[1]Moscow Institute of Physics and Technology

[2]V.I.Vernadsky Crimean Federal University

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Method Description

Task:

$$\min_{(x,y)} \{f(x,y) | (x,y) \in Q\},$$

where f is a convex function, $Q = [a,b] \times [c,d] \in \mathbb{R}^2$.

One iteration

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- Choose rectangle which anti-gradient looks in
- Analogically for vertical segment in the rectangle

Plan

Let f be convex and has L-Lipschitz continuous gradient.

$$\text{sign}\, f'_y(x_0) = \text{sign}\, f'_y(x_{current})$$

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Convergence

Function f is convex. Size of square Q is equal to a. One takes a center of a current square as approximate solution.

Estimate through Lipschitz function constant

Let function f be L_f-Lipschitz continuous. Then for accuracy ϵ on function it is sufficient:

$$N = \left\lceil \log_2 \frac{L_f a}{\sqrt{2}\epsilon} \right\rceil. \tag{1}$$

Estimate through Lipschitz gradient constant

Let function f have L_g-Lipschitz continuous gradient. Moreover, solution of initial task is **internal point**. Then for accuracy ϵ on function it is sufficient:

$$N = \left\lceil \frac{1}{2} \log_2 \frac{L_g a^2}{4\epsilon} \right\rceil. \tag{2}$$

Convergence

If following inequallity is met

$$\frac{2L_f^2}{L_g} \geq \epsilon$$

then the estimate through L_g is better than through L_f.

- Quadratic Function

$$f(x,y) = (Ax+By)^2 + Cy^2 + Dx + Ey$$

$$(x^*, y^*) \in Q$$

$$L_f = \max_{(x,y)\in Q} \|\nabla f(x,y)\|$$

$$L_g = \max |\lambda (H(f))|$$

Experiments

Iterations Number

Theoretical Iteration Number through function constant 40

Theoretical Iteration Number through gradient constant 20

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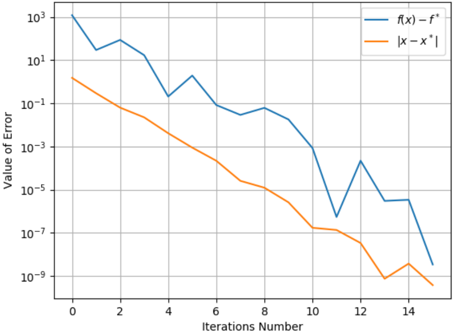

Experiments

Iterations Number

$$f(x,y) = (x+y)^2 + x^2,\ Q = [1,2]^2,\ (x^*, y^*) = 1, 1$$

Theoretical Iteration Number through function constant 30
Theoretical Iteration Number through gradient constant 14

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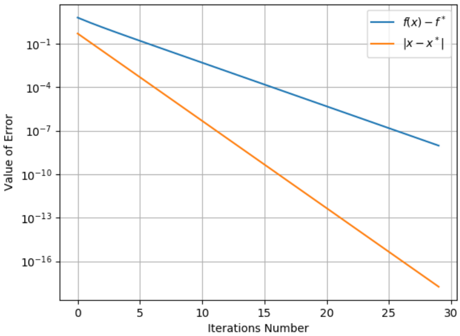
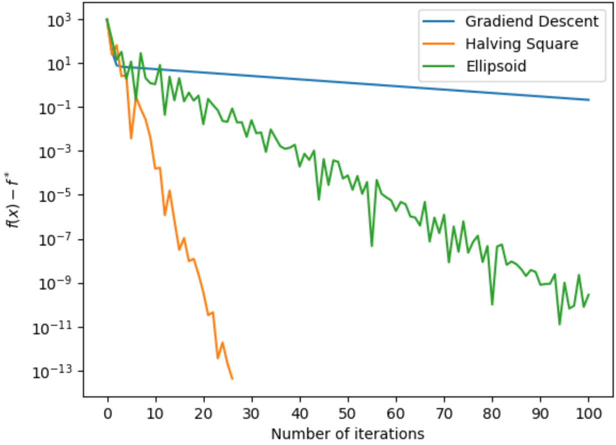


Experiments

Comparison Of Methods



Work time

Gradiend Descent 0.0045

Halving Square 0.0209

Ellipsoid 0.0283

Summary

- Strategy for solution on segment
- Convergence Results
- Experiments for this method
  - Iterations number
  - Comparison with different methods