1 Tests functions

Functions $-\sin\frac{\pi x}{a}$ and $-\sin\frac{\pi x}{b}$ are convex on square $Q = [0,1]^2$ when $a, b \geq 1$. Therefore, a function $-A\sin\frac{\pi x}{a} - B\sin\frac{\pi y}{b}$ is convex for all $A, B \geq 0$ as cone combination of convex function.

Functions x^n are convex and monotonously non-decreasing on $[0,1]$ for all $n \in \mathbb{N}$ that's why functions $\left(-A\sin\frac{\pi x}{a} - B\sin\frac{\pi y}{b} + A + B + D\right)^n$ are convex for all $D \geq 0$.

Therefore, following function is convex:

$$f(x,y) = -A_1\sin\frac{\pi x}{a_1} - B_1\sin\frac{\pi x}{b_1} + \sum_{n=2}^{N}\left(-A_n\sin\frac{\pi x}{a_n} - B_n\sin\frac{\pi y}{b_n} + A_n + B_n + D_n\right)^n,$$

where $A_i, B_i.D_i \geq 0$ and $a_i, b_i \geq 1$ for all $i = \overline{1,n}$.

The function f is differentiable infinite times and we can use it to test the method.

Let's take $a_1 = \cdots = a_n = a$ and $b_1 = \cdots = b_n = b$:

$$f(x,y) = -A_1\sin\frac{\pi x}{a} - B_1\sin\frac{\pi x}{b} + \sum_{n=2}^{N}\left(-A_n\sin\frac{\pi x}{a} - B_n\sin\frac{\pi y}{b} + A_n + B_n + D_n\right)^n,$$

where $A_i, B_i.D_i \geq 0$ for all $i = \overline{1,n}$ and $a, b \geq 1$.

Then functions derivative is:

$$f'_x(x,y) = \left(-A_1 - \sum_{n=2}^{N} nA_n\left(-A_n\sin\frac{\pi x}{a} - B_n\sin\frac{\pi y}{b} + A_n + B_n + D_n\right)^{n-1}\right)\cdot$$

$$\cdot\frac{\pi}{a}\cos\frac{\pi x}{a}$$

$$f'_y(x,y) = \frac{\pi}{b}\left(-B_1 - \sum_{n=2}^{N} nB_n\left(-A_n\sin\frac{\pi x}{a} - B_n\sin\frac{\pi y}{b} + A_n + B_n + D_n\right)^{n-1}\right)\cdot$$

$$\cdot\frac{\pi}{b}\cos\frac{\pi y}{b}$$

$$f''_{xy}(x,y) = \left(\sum_{n=2}^{N} n(n-1)A_n B_n \left(-A_n \sin \frac{\pi x}{a} - B_n \sin \frac{\pi y}{b} + A_n + B_n + D_n \right)^{n-2} \right) \cdot$$

$$\cdot \frac{\pi^2}{ab} \cos \frac{\pi x}{a} \cos \frac{\pi y}{b}$$

Using written above expressions we can give estimates for derivatives:

$$|f'_x|\Big|_{x=x_0} \geq \left(A_1 + \sum_{n=2}^{N} n A_n D_n^{n-1} \right) \frac{\pi}{a} \left| \cos \frac{\pi x_0}{a} \right|$$

$$|f'_x|\Big|_{y=y_0} \geq \left(B_1 + \sum_{n=2}^{N} n B_n D_n^{n-1} \right) \frac{\pi}{b} \left| \cos \frac{\pi y_0}{b} \right|$$

$$|f''_{xy}| \leq \frac{\pi^2}{ab} \left(\sum_{n=2}^{N} n(n-1)A_n B_n \left(A_n + B_n + D_n \right)^{n-2} \right)$$

Also we know solution of this task:

$$x^* = \begin{cases} 1, \text{if } a \geq 2, \\ \frac{a}{2}, \text{else} \end{cases} \tag{1}$$

$$y^* = \begin{cases} 1, \text{if } b \geq 2, \\ \frac{b}{2}, \text{else} \end{cases} \tag{2}$$

and task's value:

$$f^* = f(x^*, y^*).$$