

# Recommender Systems in Large Scale ML

Шугаев Ильянур

VK.com  
Performance Advertising Team  
ilnur.shug@gmail.com

Higher School of Economics, 2020

### RS [16]

*Recommender Systems (RSs) are software tools and techniques that provide suggestions for items that are most likely of interest to a particular user*

### RS [16]

*Recommender Systems (RSs) are software tools and techniques that provide suggestions for items that are most likely of interest to a particular user*

### Notations

- ▶  $U$  — set of subjects (users)
- ▶  $I$  — set of objects (items)
- ▶  $D = \{(u_t, i_t, y_t)\}_{t=1}^m \subset U \times I \times Y$  — transactions, where  $Y$  — set of transactions descriptions

- ▶ E-commerce

- ▶  $U$  - clients of online shop
- ▶  $I$  - products (books, movies, music, etc)
- ▶  $r_{ui} = \mathbb{1}\{u \text{ bought } i\}$

- ▶ E-commerce
  - ▶  $U$  - clients of online shop
  - ▶  $I$  - products (books, movies, music, etc)
  - ▶  $r_{ui} = \mathbb{1}\{u \text{ bought } i\}$
- ▶ Social network
  - ▶  $U$  - users of the site
  - ▶  $I$  - posts, communities
  - ▶  $r_{ui} = \mathbb{1}\{u \text{ visited } i\}$

- ▶ E-commerce
  - ▶  $U$  - clients of online shop
  - ▶  $I$  - products (books, movies, music, etc)
  - ▶  $r_{ui} = \mathbb{1}\{u \text{ bought } i\}$
- ▶ Social network
  - ▶  $U$  - users of the site
  - ▶  $I$  - posts, communities
  - ▶  $r_{ui} = \mathbb{1}\{u \text{ visited } i\}$
- ▶ Movies recommendation
  - ▶  $U$  - clients of the platform
  - ▶  $I$  - movies
  - ▶  $r_{ui} = \text{rating } u \text{ gave to } i$

Users transactions (feedback)  $D$  can be divided into two types

	<b>Explicit</b>	<b>Implicit</b>
$r_{ui}$	explicit rating for item $i$ by user $u$	fact (number of times) that $u$ interacted with $i$

Users transactions (feedback)  $D$  can be divided into two types

	<b>Explicit</b>	<b>Implicit</b>
$r_{ui}$	explicit rating for item $i$ by user $u$	fact (number of times) that $u$ interacted with $i$

Gathering  
complexity



Users transactions (feedback)  $D$  can be divided into two types

	<b>Explicit</b>	<b>Implicit</b>
$r_{ui}$	explicit rating for item $i$ by user $u$	fact (number of times) that $u$ interacted with $i$
Gathering complexity	Hard to collect	Easy to collect

Users transactions (feedback)  $D$  can be divided into two types

	<b>Explicit</b>	<b>Implicit</b>
$r_{ui}$	explicit rating for item $i$ by user $u$	fact (number of times) that $u$ interacted with $i$
Gathering complexity	Hard to collect	Easy to collect
Amount		

Users transactions (feedback)  $D$  can be divided into two types

	<b>Explicit</b>	<b>Implicit</b>
$r_{ui}$	explicit rating for item $i$ by user $u$	fact (number of times) that $u$ interacted with $i$
Gathering complexity	Hard to collect	Easy to collect
Amount	Small	Large

Users transactions (feedback)  $D$  can be divided into two types

	<b>Explicit</b>	<b>Implicit</b>
$r_{ui}$	explicit rating for item $i$ by user $u$	fact (number of times) that $u$ interacted with $i$
Gathering complexity	Hard to collect	Easy to collect
Amount	Small	Large

## Remark

*It is straightforward to convert explicit to implicit, for example:*

- ▶ *if  $r_{ui}$  is a movie rating then  $p_{ui} = \mathbb{1}\{r_{ui} \geq 3\}$  - implicit feedback*

*If not stated otherwise we assume explicit feedback*

# Table of Contents

1. Evaluation
2. Matrix Completion
3. Link Prediction
4. Session-based Recommendations
5. Learning to Rank
6. Resume
7. RecSys at VK

- ▶ System should be able to compute  $\rho(u, i), \rho(u, u'), \rho(i, i')$ , where  $\rho$  — similarity (relevance) function
- ▶ Given user  $u$  system should be able to rank  $I$  according to  $\rho(u, \cdot)$

- ▶ We have different formulations of RS problem thus different solutions
- ▶ Main goal is to compare different solutions

### Protocol

1. Order all transactions  $D$  by time
2. Split  $D$  into  $D_{train}$  /  $D_{valid}$  /  $D_{test}$  sets by timestamp
3. Fit models on  $D_{train}$
4. HPO on  $D_{valid}$
5. Report resulting metrics on  $D_{test}$



### Protocol

1. Order all transactions  $D$  by time
2. Split  $D$  into  $D_{train}$  /  $D_{valid}$  /  $D_{test}$  sets by timestamp
3. Fit models on  $D_{train}$
4. HPO on  $D_{valid}$
5. Report resulting metrics on  $D_{test}$

### Problems

- ▶ Cold-start users — users which appear only in  $D_{test}$  set
- ▶ Cold-start items — items which appear only in  $D_{test}$  set

### Protocol

1. Order all transactions  $D$  by time
2. Split  $D$  into  $D_{train}$  /  $D_{valid}$  /  $D_{test}$  sets by timestamp
3. Fit models on  $D_{train}$
4. HPO on  $D_{valid}$
5. Report resulting metrics on  $D_{test}$

### Problems

- ▶ Cold-start users — users which appear only in  $D_{test}$  set
- ▶ Cold-start items — items which appear only in  $D_{test}$  set

### Common solutions

- ▶ Leave only users and items which appear in all sets

### Protocol

1. Split  $D$  into set of sessions  $\mathcal{S} = \{S_u\}_{u \in U}$
2. For each user  $u$  split  $S_u$  into  $S_{u,train}, S_{u,valid}, S_{u,test}$  (using timestamp)
3. Therefore  $D_s = \cup_{u \in U} S_{u,s}$  for  $s \in \{train, valid, test\}$
4. Fit models on  $D_{train}$
5. HPO on  $D_{valid}$
6. Report resulting metrics on  $D_{test}$

### Protocol

1. Split  $D$  into set of sessions  $\mathcal{S} = \{S_u\}_{u \in U}$
2. For each user  $u$  split  $S_u$  into  $S_{u,train}, S_{u,valid}, S_{u,test}$  (using timestamp)
3. Therefore  $D_s = \cup_{u \in U} S_{u,s}$  for  $s \in \{train, valid, test\}$
4. Fit models on  $D_{train}$
5. HPO on  $D_{valid}$
6. Report resulting metrics on  $D_{test}$

### Problems

- Peeking into the future

### Protocol

1. Split  $D$  into set of sessions  $\mathcal{S} = \{S_u\}_{u \in U}$
2. For each user  $u$  split  $S_u$  into  $S_{u,train}, S_{u,valid}, S_{u,test}$  (using timestamp)
3. Therefore  $D_s = \cup_{u \in U} S_{u,s}$  for  $s \in \{train, valid, test\}$
4. Fit models on  $D_{train}$
5. HPO on  $D_{valid}$
6. Report resulting metrics on  $D_{test}$

### Problems

- ▶ Peeking into the future

### Common solutions

- ▶ Use previous scenario instead

- ▶ For each user  $u$  we have a set of  $N_u$  ground truth relevant items  $D_u = \{i_1, i_2, \dots, i_{N_u}\}$  and
- ▶ List of  $Q_u$  recommended items (according to  $\rho(u, \cdot)$ )  $R_u = \{r_1, r_2, \dots, r_{Q_u}\}$ , in order of decreasing relevance

---

<sup>1</sup>[https://en.wikipedia.org/wiki/Evaluation\\_measures\\_\(information\\_retrieval\)](https://en.wikipedia.org/wiki/Evaluation_measures_(information_retrieval))

- ▶ For each user  $u$  we have a set of  $N_u$  ground truth relevant items  $D_u = \{i_1, i_2, \dots, i_{N_u}\}$  and
- ▶ List of  $Q_u$  recommended items (according to  $\rho(u, \cdot)$ )  $R_u = \{r_1, r_2, \dots, r_{Q_u}\}$ , in order of decreasing relevance

## Popular Ranking Metrics<sup>1</sup>

Metric	Definition	Notes
Precision@k	$\frac{1}{ U } \sum_{u \in U} \frac{1}{k} \sum_{i=1}^{\min(k, Q_u)} \mathbb{1}\{r_i \in D_u\}$	(# of recommended items @k that are relevant) / (# of recommended items @k)

<sup>1</sup>[https://en.wikipedia.org/wiki/Evaluation\\_measures\\_\(information\\_retrieval\)](https://en.wikipedia.org/wiki/Evaluation_measures_(information_retrieval))

- ▶ For each user  $u$  we have a set of  $N_u$  ground truth relevant items  $D_u = \{i_1, i_2, \dots, i_{N_u}\}$  and
- ▶ List of  $Q_u$  recommended items (according to  $\rho(u, \cdot)$ )  $R_u = \{r_1, r_2, \dots, r_{Q_u}\}$ , in order of decreasing relevance

## Popular Ranking Metrics<sup>1</sup>

Metric	Definition	Notes
Precision@k	$\frac{1}{ U } \sum_{u \in U} \frac{1}{k} \sum_{i=1}^{\min(k, Q_u)} \mathbb{1}\{r_i \in D_u\}$	(# of recommended items @k that are relevant) / (# of recommended items @k)
Recall@k	$\frac{1}{ U } \sum_{u \in U} \frac{1}{ D_u } \sum_{i=1}^{\min(k, Q_u)} \mathbb{1}\{r_i \in D_u\}$	(# of recommended items @k that are relevant) / (total # of relevant items)

<sup>1</sup>[https://en.wikipedia.org/wiki/Evaluation\\_measures\\_\(information\\_retrieval\)](https://en.wikipedia.org/wiki/Evaluation_measures_(information_retrieval))



- ▶ For each user  $u$  we have a set of  $N_u$  ground truth relevant items  $D_u = \{i_1, i_2, \dots, i_{N_u}\}$  and
- ▶ List of  $Q_u$  recommended items (according to  $\rho(u, \cdot)$ )  $R_u = \{r_1, r_2, \dots, r_{Q_u}\}$ , in order of decreasing relevance

## Popular Ranking Metrics<sup>1</sup>

Metric	Definition	Notes
Precision@k	$\frac{1}{ U } \sum_{u \in U} \frac{1}{k} \sum_{i=1}^{\min(k, Q_u)} \mathbb{1}\{r_i \in D_u\}$	(# of recommended items @k that are relevant) / (# of recommended items @k)
Recall@k	$\frac{1}{ U } \sum_{u \in U} \frac{1}{ D_u } \sum_{i=1}^{\min(k, Q_u)} \mathbb{1}\{r_i \in D_u\}$	(# of recommended items @k that are relevant) / (total # of relevant items)
HR@k	$\frac{1}{ U } \sum_{u \in U} \mathbb{1}\{R_{u,1:k} \cap D_u \neq \emptyset\}$	(# of times top k contains relevant) / (# of users)

<sup>1</sup>[https://en.wikipedia.org/wiki/Evaluation\\_measures\\_\(information\\_retrieval\)](https://en.wikipedia.org/wiki/Evaluation_measures_(information_retrieval))

- ▶ For each user  $u$  we have a set of  $N_u$  ground truth relevant items  $D_u = \{i_1, i_2, \dots, i_{N_u}\}$  and
- ▶ List of  $Q_u$  recommended items (according to  $\rho(u, \cdot)$ )  $R_u = \{r_1, r_2, \dots, r_{Q_u}\}$ , in order of decreasing relevance

## Popular Ranking Metrics<sup>1</sup>

Metric	Definition	Notes
Precision@k	$\frac{1}{ U } \sum_{u \in U} \frac{1}{k} \sum_{i=1}^{\min(k, Q_u)} \mathbb{1}\{r_i \in D_u\}$	(# of recommended items @k that are relevant) / (# of recommended items @k)
Recall@k	$\frac{1}{ U } \sum_{u \in U} \frac{1}{ D_u } \sum_{i=1}^{\min(k, Q_u)} \mathbb{1}\{r_i \in D_u\}$	(# of recommended items @k that are relevant) / (total # of relevant items)
HR@k	$\frac{1}{ U } \sum_{u \in U} \mathbb{1}\{R_{u,1:k} \cap D_u \neq \emptyset\}$	(# of times top k contains relevant) / (# of users)
MAP	$\frac{1}{ U } \sum_{u \in U} \frac{1}{N_u} \sum_{i=1}^{Q_u} \frac{1}{i} \mathbb{1}\{r_i \in D_u\}$	how many of the recommended documents are in the set of true relevant documents

<sup>1</sup>[https://en.wikipedia.org/wiki/Evaluation\\_measures\\_\(information\\_retrieval\)](https://en.wikipedia.org/wiki/Evaluation_measures_(information_retrieval))

- ▶ For each user  $u$  we have a set of  $N_u$  ground truth relevant items  $D_u = \{i_1, i_2, \dots, i_{N_u}\}$  and
- ▶ List of  $Q_u$  recommended items (according to  $\rho(u, \cdot)$ )  $R_u = \{r_1, r_2, \dots, r_{Q_u}\}$ , in order of decreasing relevance

## Popular Ranking Metrics<sup>1</sup>

Metric	Definition	Notes
Precision@k	$\frac{1}{ U } \sum_{u \in U} \frac{1}{k} \sum_{i=1}^{\min(k, Q_u)} \mathbb{1}\{r_i \in D_u\}$	(# of recommended items @k that are relevant) / (# of recommended items @k)
Recall@k	$\frac{1}{ U } \sum_{u \in U} \frac{1}{ D_u } \sum_{i=1}^{\min(k, Q_u)} \mathbb{1}\{r_i \in D_u\}$	(# of recommended items @k that are relevant) / (total # of relevant items)
HR@k	$\frac{1}{ U } \sum_{u \in U} \mathbb{1}\{R_{u,1:k} \cap D_u \neq \emptyset\}$	(# of times top k contains relevant) / (# of users)
MAP	$\frac{1}{ U } \sum_{u \in U} \frac{1}{N_u} \sum_{i=1}^{Q_u} \frac{1}{i} \mathbb{1}\{r_i \in D_u\}$	how many of the recommended documents are in the set of true relevant documents
NDCG		

<sup>1</sup>[https://en.wikipedia.org/wiki/Evaluation\\_measures\\_\(information\\_retrieval\)](https://en.wikipedia.org/wiki/Evaluation_measures_(information_retrieval))

- ▶ Precision@k, Recall@k, HR@k — the order of the recommendations is not taken into account
- ▶ MAP, NDCG — metric takes into account the order of the recommendations

- ▶ Precision@k, Recall@k, HR@k — the order of the recommendations is not taken into account
- ▶ MAP, NDCG — metric takes into account the order of the recommendations

**Q:** Why not to use ROC AUC?

# Table of Contents

## 1. Evaluation

## 2. Matrix Completion

Formulation

Singular Value Decomposition

Alternating Least Squares

Large Scale Matrix Factorization

## 3. Link Prediction

## 4. Session-based Recommendations

## 5. Learning to Rank

## 6. Resume

## 7. RecSys at VK

Aggregated data:

- ▶  $R = (r_{ui})_{u \in U, i \in I}$  — cross-tabulation matrix, where
- ▶  $r_{ui} = \text{agg}\{(u_t, i_t, y_t) \in D \mid u_t = u \wedge i_t = i\}$

Aggregated data:

- ▶  $R = (r_{ui})_{u \in U, i \in I}$  — cross-tabulation matrix, where
- ▶  $r_{ui} = \text{agg}\{(u_t, i_t, y_t) \in D \mid u_t = u \wedge i_t = i\}$

Task:

- ▶ Fill missing values  $r_{ui}$



Aggregated data:

- ▶  $R = (r_{ui})_{u \in U, i \in I}$  — cross-tabulation matrix, where
- ▶  $r_{ui} = \text{agg}\{(u_t, i_t, y_t) \in D \mid u_t = u \wedge i_t = i\}$

Task:

- ▶ Fill missing values  $r_{ui}$

Examples of  $r_{ui}$ :

- ▶ rating from user  $u$  to movie  $i$
- ▶ number of times user  $u$  visited page  $i$

- ▶ Content-based methods
- ▶ Collaborative filtering
  - ▶ User/Item based
  - ▶ Matrix Factorizations (SVD, PMF [12], ALS [8, 9])
- ▶ Neural Architectures (NCF [7], CB2CF [1])

### Definition (Latent Factor Model via Matrix Factorization)

*Given data  $D$ , our goal is to find matrices  $P = (p_{ut})_{|U| \times |T|}$  and  $Q = (q_{it})_{|I| \times |T|}$ , where  $T$  - set of latent factors ( $|T| \ll |U|, |T| \ll |I|$ ) such that*

$$R = P\Delta Q^T,$$

$$\Delta = \text{diag}(\pi_1, \dots, \pi_{|T|})$$

## Low-rank approximation

$$R_k \equiv P_k \Sigma_k Q_k^T \quad : \quad \|R - R_k\|_F \rightarrow \min,$$

where  $k$  - rank

## Low-rank approximation

$$R_k \equiv P_k \Sigma_k Q_k^T \quad : \quad \|R - R_k\|_F \rightarrow \min,$$

where  $k$  - rank

### Remark

*$R_k$  is the best rank- $k$  approximation of the matrix  $R$  in terms of  $F$  norm (RMSE)<sup>a</sup>*

---

<sup>a</sup>[https://en.wikipedia.org/wiki/Low-rank\\_approximation](https://en.wikipedia.org/wiki/Low-rank_approximation)

# Singular Value Decomposition (SVD)

## Model

### Low-rank approximation

$$R_k \equiv P_k \Sigma_k Q_k^T \quad : \quad \|R - R_k\|_F \rightarrow \min,$$

where  $k$  - rank

### Remark

*$R_k$  is the best rank- $k$  approximation of the matrix  $R$  in terms of  $F$  norm (RMSE)<sup>a</sup>*

<sup>a</sup>[https://en.wikipedia.org/wiki/Low-rank\\_approximation](https://en.wikipedia.org/wiki/Low-rank_approximation)

**Q:** Before we apply SVD we need to fill unobserved values in  $R$ .  
How to do that?

# Singular Value Decomposition (SVD)

## Model

### Low-rank approximation

$$R_k \equiv P_k \Sigma_k Q_k^T \quad : \quad \|R - R_k\|_F \rightarrow \min,$$

where  $k$  - rank

### Remark

*$R_k$  is the best rank- $k$  approximation of the matrix  $R$  in terms of  $F$  norm (RMSE)<sup>a</sup>*

<sup>a</sup>[https://en.wikipedia.org/wiki/Low-rank\\_approximation](https://en.wikipedia.org/wiki/Low-rank_approximation)

**Q:** Before we apply SVD we need to fill unobserved values in  $R$ .  
How to do that?

**A:** Popular choices

- ▶  $r_{ui} = 0$  if value is unobserved

- ▶ Missing values in  $R$  are mixture of true negative values and absence of interactions. Therefore  $r_{ui} = 0$  is not best possible choice.



- ▶ Missing values in  $R$  are mixture of true negative values and absence of interactions. Therefore  $r_{ui} = 0$  is not best possible choice.
- ▶ **Q:** Suppose that  $R_k = R$ . Is this model useful for recommendations?

- ▶ Missing values in  $R$  are mixture of true negative values and absence of interactions. Therefore  $r_{ui} = 0$  is not best possible choice.
- ▶ **Q:** Suppose that  $R_k = R$ . Is this model useful for recommendations?  
**A:** No, therefore we need regularization

## Likelihood of data

$$\mathbb{P} \{R \mid P, Q, \sigma^2\} = \prod_{u \in U} \prod_{i \in I} [\mathcal{N}(r_{ui} \mid \mathbf{p}_u \mathbf{q}_i, \sigma^2)]^{I_{ui}},$$

where  $I_{ui} = \mathbb{1}\{r_{ui} \neq 0\}$

## Likelihood of data

$$\mathbb{P} \{R \mid P, Q, \sigma^2\} = \prod_{u \in U} \prod_{i \in I} [\mathcal{N}(r_{ui} \mid \mathbf{p}_u \mathbf{q}_i, \sigma^2)]^{I_{ui}},$$

where  $I_{ui} = \mathbb{1}\{r_{ui} \neq 0\}$

## Priors

$$\mathbb{P} \{P \mid \sigma_P^2\} = \prod_{u \in U} \mathcal{N}(\mathbf{p}_u \mid 0, \sigma_P^2 \mathbf{I}) \quad \mathbb{P} \{Q \mid \sigma_Q^2\} = \prod_{i \in I} \mathcal{N}(\mathbf{q}_i \mid 0, \sigma_Q^2 \mathbf{I})$$

## Likelihood of data

$$\mathbb{P} \{R \mid P, Q, \sigma^2\} = \prod_{u \in U} \prod_{i \in I} [\mathcal{N}(r_{ui} \mid \mathbf{p}_u \mathbf{q}_i, \sigma^2)]^{I_{ui}},$$

where  $I_{ui} = \mathbb{1}\{r_{ui} \neq 0\}$

## Priors

$$\mathbb{P} \{P \mid \sigma_P^2\} = \prod_{u \in U} \mathcal{N}(\mathbf{p}_u \mid 0, \sigma_P^2 \mathbf{I}) \quad \mathbb{P} \{Q \mid \sigma_Q^2\} = \prod_{i \in I} \mathcal{N}(\mathbf{q}_i \mid 0, \sigma_Q^2 \mathbf{I})$$

## Posterior

$$\ln \mathbb{P} \{P, Q \mid R, \sigma^2, \sigma_P^2, \sigma_Q^2\} \rightarrow \max \quad \Longleftrightarrow$$
$$\mathcal{L} = \underbrace{\frac{1}{2} \sum_{u \in U, i \in I} I_{ui} (r_{ui} - \mathbf{p}_u \mathbf{q}_i)^2}_{\text{sum over observer } r_{ui}} + \underbrace{\frac{\lambda_P}{2} \sum_{u \in U} \|\mathbf{p}_u\|_F^2}_{L_2 \text{ reg}} + \underbrace{\frac{\lambda_Q}{2} \sum_{i \in I} \|\mathbf{q}_i\|_F^2}_{L_2 \text{ reg}} \rightarrow \min$$

## Recall FM model

$$\phi_{FM}(\mathbf{w}, \mathbf{x}) = \phi_{LM}(\mathbf{w}, \mathbf{x}) + \sum_{j_1=1}^n \sum_{j_2=j_1+1}^n (\mathbf{w}_{j_1} \cdot \mathbf{w}_{j_2}) x_{j_1} x_{j_2},$$

where  $\mathbf{x} \in \mathbb{R}^n$  and loss function  $\mathcal{L} = \frac{1}{2} (y - \phi_{FM}(\mathbf{w}, \mathbf{x}))^2$

## Recall FM model

$$\phi_{FM}(\mathbf{w}, \mathbf{x}) = \phi_{LM}(\mathbf{w}, \mathbf{x}) + \sum_{j_1=1}^n \sum_{j_2=j_1+1}^n (\mathbf{w}_{j_1} \cdot \mathbf{w}_{j_2}) x_{j_1} x_{j_2},$$

where  $\mathbf{x} \in \mathbb{R}^n$  and loss function  $\mathcal{L} = \frac{1}{2} (y - \phi_{FM}(\mathbf{w}, \mathbf{x}))^2$

Assume that  $\mathbf{x} = [\mathbf{u}, \mathbf{i}]$ , where

- ▶  $\mathbf{u}$  — one-hot vector for user
- ▶  $\mathbf{i}$  — one-hot vector for item

## Recall FM model

$$\phi_{FM}(\mathbf{w}, \mathbf{x}) = \phi_{LM}(\mathbf{w}, \mathbf{x}) + \sum_{j_1=1}^n \sum_{j_2=j_1+1}^n (\mathbf{w}_{j_1} \cdot \mathbf{w}_{j_2}) x_{j_1} x_{j_2},$$

where  $\mathbf{x} \in \mathbb{R}^n$  and loss function  $\mathcal{L} = \frac{1}{2}(y - \phi_{FM}(\mathbf{w}, \mathbf{x}))^2$

Assume that  $\mathbf{x} = [\mathbf{u}, \mathbf{i}]$ , where

- ▶  $\mathbf{u}$  — one-hot vector for user
- ▶  $\mathbf{i}$  — one-hot vector for item

Therefore, if we *skip LM term*(**Q**: what if not?) we will get

$$\phi_{FM}(P, Q, \mathbf{x}) = \sum_{u=1}^{|U|} \sum_{i=1}^{|I|} \mathbb{1}\{\mathbf{u}_u = 1 \wedge \mathbf{i}_i = 1\} \mathbf{p}_u \cdot \mathbf{q}_i$$



## Recall FM model

$$\phi_{FM}(\mathbf{w}, \mathbf{x}) = \phi_{LM}(\mathbf{w}, \mathbf{x}) + \sum_{j_1=1}^n \sum_{j_2=j_1+1}^n (\mathbf{w}_{j_1} \cdot \mathbf{w}_{j_2}) x_{j_1} x_{j_2},$$

where  $\mathbf{x} \in \mathbb{R}^n$  and loss function  $\mathcal{L} = \frac{1}{2}(\mathbf{y} - \phi_{FM}(\mathbf{w}, \mathbf{x}))^2$

Assume that  $\mathbf{x} = [\mathbf{u}, \mathbf{i}]$ , where

- ▶  $\mathbf{u}$  — one-hot vector for user
- ▶  $\mathbf{i}$  — one-hot vector for item

Therefore, if we *skip LM term*(**Q**: what if not?) we will get

$$\phi_{FM}(P, Q, \mathbf{x}) = \sum_{u=1}^{|U|} \sum_{i=1}^{|I|} \mathbb{1}\{\mathbf{u}_u = 1 \wedge \mathbf{i}_i = 1\} \mathbf{p}_u \cdot \mathbf{q}_i$$

If we plug it in square-loss and add  $L_2$  reg. we will obtain exactly loss for PMF

- ▶ Slow to train when number of observations is very large, therefore
- ▶ Slow convergence rate

Let  $\mathcal{K}$  be a set of  $(u, i)$  pairs for which  $r_{ui}$  is known, therefore

$$\mathcal{L} = \frac{1}{2} \sum_{(u,i) \in \mathcal{K}} (r_{ui} - \mathbf{p}_u \mathbf{q}_i)^2 + \lambda (\|\mathbf{p}_u\|_F^2 + \|\mathbf{q}_i\|_F^2)$$

Optimization of  $\mathcal{L}$  can be done more efficiently due to the following

## Observation

*If during optimization we fix  $P$  in  $\mathcal{L}$  then optimization problem becomes convex wrt to  $Q$  and vice versa. Therefore can be solved in closed-form*

Faster than SGD convergence

- ▶  $p_{ui} = \mathbb{1}\{r_{ui} > 0\}$  — implicit feedback (observed interaction)

- ▶  $p_{ui} = \mathbb{1}\{r_{ui} > 0\}$  — implicit feedback (observed interaction)
- ▶ If  $p_{ui} = 0$  our beliefs are associated with varying confidence levels

- ▶  $p_{ui} = \mathbb{1}\{r_{ui} > 0\}$  — implicit feedback (observed interaction)
- ▶ If  $p_{ui} = 0$  our beliefs are associated with varying confidence levels
- ▶ Zero values of  $p_{ui}$  are associated with low confidence

- ▶  $p_{ui} = \mathbb{1}\{r_{ui} > 0\}$  — implicit feedback (observed interaction)
- ▶ If  $p_{ui} = 0$  our beliefs are associated with varying confidence levels
- ▶ Zero values of  $p_{ui}$  are associated with low confidence
- ▶ As  $r_{ui}$  grows we have a stronger indication that  $u$  indeed likes  $i$

- ▶  $p_{ui} = \mathbb{1}\{r_{ui} > 0\}$  — implicit feedback (observed interaction)
- ▶ If  $p_{ui} = 0$  our beliefs are associated with varying confidence levels
- ▶ Zero values of  $p_{ui}$  are associated with low confidence
- ▶ As  $r_{ui}$  grows we have a stronger indication that  $u$  indeed likes  $i$

Popular choices for confidence  $c_{ui}$

- ▶  $1 + \alpha r_{ui}$
- ▶  $1 + \alpha \log(1 + r_{ui}/\epsilon)$



- ▶  $p_{ui} = \mathbb{1}\{r_{ui} > 0\}$  — implicit feedback (observed interaction)
- ▶ If  $p_{ui} = 0$  our beliefs are associated with varying confidence levels
- ▶ Zero values of  $p_{ui}$  are associated with low confidence
- ▶ As  $r_{ui}$  grows we have a stronger indication that  $u$  indeed likes  $i$

Popular choices for confidence  $c_{ui}$

- ▶  $1 + \alpha r_{ui}$
- ▶  $1 + \alpha \log(1 + r_{ui}/\epsilon)$

We obtain following loss

$$\mathcal{L} = \frac{1}{2} \sum_{(u,i) \in U \times I} c_{ui} (p_{ui} - \mathbf{p}_u \mathbf{q}_i)^2 + \frac{\lambda_P}{2} \sum_{u \in U} \|\mathbf{p}_u\|_F^2 + \frac{\lambda_Q}{2} \sum_{i \in I} \|\mathbf{q}_i\|_F^2$$

Can be solved via ALS (see [9] for details)

**Q:** How do we usually scale SGD?

**Q:** How do we usually scale SGD?

**A:** Parameter Server, Data/Model Parallelism

**Q:** How do we usually scale SGD?

**A:** Parameter Server, Data/Model Parallelism

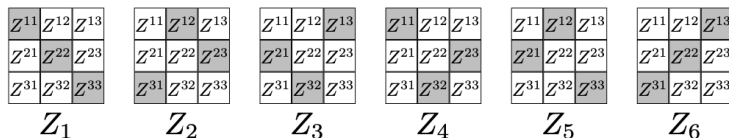
**Q:** How can we apply them to SGD MF?

**Q:** How do we usually scale SGD?

**A:** Parameter Server, Data/Model Parallelism

**Q:** How can we apply them to SGD MF?

**A:**



**Рис.:** Blocks in grey can be processed independently.  $Z_i$  —  $i$ -th sub-epoch.  $Z_1, \dots, Z_6$  — single epoch

# Table of Contents

1. Evaluation
2. Matrix Completion
3. Link Prediction
  - Formulation
  - Large Scale Link Prediction
4. Session-based Recommendations
5. Learning to Rank
6. Resume
7. RecSys at VK

## Aggregated data:

- ▶  $G = \langle U \cup I, E \rangle, w: E \rightarrow \mathbb{R}$  — weighted bipartite graph, where
- ▶  $E = \{(u, i) \mid \exists t: (u_t, i_t, y_t) \in D \wedge u_t = u \wedge i_t = i\} \subset U \times I$
- ▶  $w(u, i) = r_{ui} = \text{agg}\{(u_t, i_t, y_t) \in D \mid u_t = u \wedge i_t = i\}$

Aggregated data:

- ▶  $G = \langle U \cup I, E \rangle, w: E \rightarrow \mathbb{R}$  — weighted bipartite graph, where
- ▶  $E = \{(u, i) \mid \exists t: (u_t, i_t, y_t) \in D \wedge u_t = u \wedge i_t = i\} \subset U \times I$
- ▶  $w(u, i) = r_{ui} = \text{agg}\{(u_t, i_t, y_t) \in D \mid u_t = u \wedge i_t = i\}$

Task:

- ▶ Predict weight  $w(u, i)$  of non-existing edge  $(u, i)$



In more general case  $G$  is a Heterogeneous Information Network (HIN)

### Definition (Heterogeneous Information Network)

$G = \langle V, E, \Phi, \Psi, w \rangle$  is a Heterogeneous Information Network (HIN), where  $\Phi: V \rightarrow A$  — mapping from vertex to its type,  $\Psi: E \rightarrow X$  — mapping from edge to its type, such that  $|A| > 1$  or  $|X| > 1$ .

In more general case  $G$  is a Heterogeneous Information Network (HIN)

## Definition (Heterogeneous Information Network)

$G = \langle V, E, \Phi, \Psi, w \rangle$  is a Heterogeneous Information Network (HIN), where  $\Phi: V \rightarrow A$  — mapping from vertex to its type,  $\Psi: E \rightarrow X$  — mapping from edge to its type, such that  $|A| > 1$  or  $|X| > 1$ .

HIN example (movies recommendations):

- ▶  $V = \text{Actors} \cup \text{Movies} \cup \text{Users} \cup \text{Genres}$ , vertices types  
 $A = \{\text{actor}, \text{movie}, \text{user}, \text{genre}\}$
- ▶ Edges types  $X = \{\text{starred\_in}, \text{rated}, \text{belongs\_to}\}$

- ▶ DeepWalk [13], Node2Vec [6]
- ▶ Graph Representation Learning (GCMC [2])
- ▶ HINs (metapath2vec [3], HIN2vec [4])

- ▶ Pytorch BigGraph [10]
- ▶ GraphVite [21]

# Table of Contents

1. Evaluation
2. Matrix Completion
3. Link Prediction
- 4. Session-based Recommendations**
5. Learning to Rank
6. Resume
7. RecSys at VK

Aggregated data:

- ▶  $S_u = \langle (i_1, y_1), (i_2, y_2), \dots, (i_{n_u}, y_{n_u}) \rangle$  —  $u$ -th user session chronologically ordered

Aggregated data:

- ▶  $S_u = \langle (i_1, y_1), (i_2, y_2), \dots, (i_{n_u}, y_{n_u}) \rangle$  —  $u$ -th user session chronologically ordered

Task:

- ▶  $P(i_{n_u+1} = i \mid S_u)$  — probability of the next event given user session  $S_u$

- ▶ RNN based [18], BERT2Rec [17]



- ▶ RNN based [18], BERT2Rec [17]

### Remark

*But be careful! Most neural architectures can not outperform simple solutions. See [11].*

# Table of Contents

1. Evaluation
2. Matrix Completion
3. Link Prediction
4. Session-based Recommendations
5. Learning to Rank
6. Resume
7. RecSys at VK



# Learning to Rank

Popular Solutions

# Table of Contents

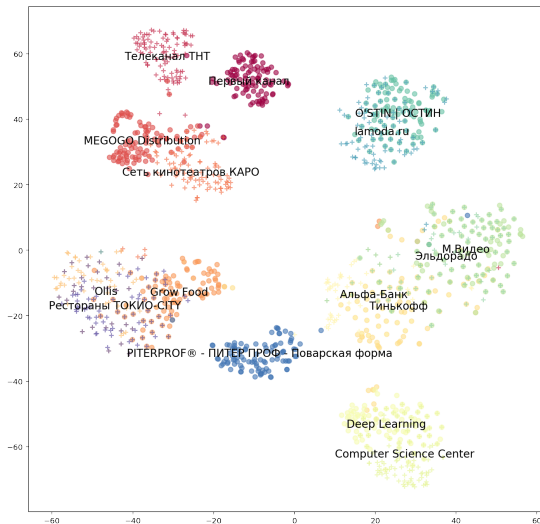
1. Evaluation
2. Matrix Completion
3. Link Prediction
4. Session-based Recommendations
5. Learning to Rank
- 6. Resume**
7. RecSys at VK

1. There are many ways to formulate RecSys problem as ML problem
  - ▶ Matrix Completion
  - ▶ Link Prediction
  - ▶ Session-based
  - ▶ Learning to Rank
2. Additionally each formulation can be further adjusted to deal with different kinds of user feedback
  - ▶ Explicit
  - ▶ Implicit
3. Evaluation (Protocols, Metrics)

# Table of Contents

1. Evaluation
2. Matrix Completion
3. Link Prediction
4. Session-based Recommendations
5. Learning to Rank
6. Resume
7. RecSys at VK
  - Similar Communities and Domains Search
  - Тематические Ленты

# Similar Communities and Domains Search





---

<sup>2</sup>[https://vk.com/video-187376020\\_456239017](https://vk.com/video-187376020_456239017)

- [1] O. Barkan, N. Koenigstein, E. Yogev, and O. Katz. Cb2cf: a neural multiview content-to-collaborative filtering model for completely cold item recommendations. In *Proceedings of the 13th ACM Conference on Recommender Systems*, pages 228-236, 2019.
- [2] R. v. d. Berg, T. N. Kipf, and M. Welling. Graph convolutional matrix completion. *arXiv preprint arXiv:1706.02263*, 2017.
- [3] Y. Dong, N. V. Chawla, and A. Swami. metapath2vec: Scalable representation learning for heterogeneous networks. In *Proceedings of the 23rd ACM SIGKDD international conference on knowledge discovery and data mining*, pages 135-144, 2017.
- [4] T.-y. Fu, W.-C. Lee, and Z. Lei. Hin2vec: Explore meta-paths in heterogeneous information networks for representation learning. In *Proceedings of the 2017 ACM on Conference on Information and Knowledge Management*, pages 1797-1806, 2017.
- [5] R. Gemulla, E. Nijkamp, P. J. Haas, and Y. Sismanis. Large-scale matrix factorization with distributed stochastic gradient descent. In *Proceedings of the 17th ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 69-77, 2011.

- [6] A. Grover and J. Leskovec. node2vec: Scalable feature learning for networks. In *Proceedings of the 22nd ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 855–864, 2016.
- [7] X. He, L. Liao, H. Zhang, L. Nie, X. Hu, and T.-S. Chua. Neural collaborative filtering. In *Proceedings of the 26th international conference on world wide web*, pages 173–182, 2017.
- [8] X. He, H. Zhang, M.-Y. Kan, and T.-S. Chua. Fast matrix factorization for online recommendation with implicit feedback. In *Proceedings of the 39th International ACM SIGIR conference on Research and Development in Information Retrieval*, pages 549–558, 2016.
- [9] Y. Hu, Y. Koren, and C. Volinsky. Collaborative filtering for implicit feedback datasets. In *2008 Eighth IEEE International Conference on Data Mining*, pages 263–272. IEEE, 2008.
- [10] A. Lerer, L. Wu, J. Shen, T. Lacroix, L. Wehrstedt, A. Bose, and A. Peysakhovich. Pytorch-biggraph: A large-scale graph embedding system. *arXiv preprint arXiv:1903.12287*, 2019.

- [11] M. Ludewig, N. Mauro, S. Latifi, and D. Jannach. Performance comparison of neural and non-neural approaches to session-based recommendation. In *Proceedings of the 13th ACM Conference on Recommender Systems*, pages 462–466, 2019.
- [12] A. Mnih and R. R. Salakhutdinov. Probabilistic matrix factorization. In *Advances in neural information processing systems*, pages 1257–1264, 2008.
- [13] B. Perozzi, R. Al-Rfou, and S. Skiena. Deepwalk: Online learning of social representations. In *Proceedings of the 20th ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 701–710, 2014.
- [14] S. Rendle. Factorization machines. In *2010 IEEE International Conference on Data Mining*, pages 995–1000. IEEE, 2010.
- [15] S. Rendle, C. Freudenthaler, Z. Gantner, and L. Schmidt-Thieme. Bpr: Bayesian personalized ranking from implicit feedback. *arXiv preprint arXiv:1205.2618*, 2012.
- [16] F. Ricci, L. Rokach, and B. Shapira. Introduction to recommender systems handbook. In *Recommender systems handbook*, pages 1–35. Springer, 2011.

- [17] F. Sun, J. Liu, J. Wu, C. Pei, X. Lin, W. Ou, and P. Jiang. Bert4rec: Sequential recommendation with bidirectional encoder representations from transformer. In *Proceedings of the 28th ACM International Conference on Information and Knowledge Management*, pages 1441–1450, 2019.
- [18] Y. K. Tan, X. Xu, and Y. Liu. Improved recurrent neural networks for session-based recommendations. In *Proceedings of the 1st Workshop on Deep Learning for Recommender Systems*, pages 17–22, 2016.
- [19] A. Van den Oord, S. Dieleman, and B. Schrauwen. Deep content-based music recommendation. In *Advances in neural information processing systems*, pages 2643–2651, 2013.
- [20] J. Weston, S. Bengio, and N. Usunier. Wsabie: Scaling up to large vocabulary image annotation. In *Twenty-Second International Joint Conference on Artificial Intelligence*, 2011.
- [21] Z. Zhu, S. Xu, J. Tang, and M. Qu. Graphvite: A high-performance cpu-gpu hybrid system for node embedding. In *The World Wide Web Conference*, pages 2494–2504, 2019.