1-2. A system and its model

In any simulation experiment we have a system whose behaviour we are studying. Loosely speaking, a *system* is a collection of distinct objects which interact with each other. In the pure-pursuit example, the system consists of the two aircraft. In order to study a system we generally gather some relevant information about it. For most studies it is not necessary to consider all the details of the system. For example, in the example in Sec. 1-1 we did not get the information about the sizes or weights of the aircraft. Such a collection of pertinent information about a system is called a *model* of the system.

The construction of an appropriate model of a system under study is a delicate and an all-important affair in simulation (as it is in analytic study). For example, we have assumed that the target takes no evasive action, nor does the pursuer use any predictive technique. We assumed that the pursuer corrects its course only once a minute. Thus we know the 'law' that governs the system. The same system may be described by different models. Several variations of the pure-pursuit model are considered in problems at the end of this chapter.

Constructing appropriate mathematical models of complex, real-life systems is a vast and intricate subject in itself. It requires a thorough knowledge of the system as well as of modelling techniques. In this book, we will in most cases assume that a model has been arrived at and our concern is how to program the computer so that it behaves like the model as the time progresses.

For a given system there are a number of variables that describe what is known as the *state of the system* at any given time. For example, the state of our system (of pure pursuit) is described by the positions of the two aircraft and their velocities.

For further understanding these and other concepts, let us simulate another very simple system, which is different in a fundamental sense from the pure-pursuit system.

1-3. Simulation of an inventory problem

Suppose you work in a retail store and it is your responsibility to keep replenishing a certain item (say, automobile tyres) in the store by ordering it from the wholesaler. You want to adopt a simple policy for ordering new supplies:

> 'When my stock goes down to P items (called *reorder point*), I will order Q more items (called *reorder quantity*) from the wholesaler.'

If the demand on any day exceeds the amount of inventory on hand, the excess represents lost sale and loss of goodwill. On the other hand, overstocking implies increased carrying cost (i.e., cost of storage, insurance, interest, deterioration, etc.). Ordering too frequently will result in excessive reorder cost. Assume the following conditions:

1. There is a 3-day lag between the order and arrival. The merchandise is ordered at the end of the day and is received at the beginning of the fourth day. That is, merchandise ordered on the evening of the ith day is received on the morning of the $(i+3)$rd day.
2. For each unit of inventory the carrying cost for each night is Re. 0.75.
3. Each unit out of stock when ordered results into a loss of goodwill worth Rs. 2.00 per unit plus loss of Rs. 16.00 net income, that would have resulted in its sale, or a total loss of Rs. 18.00 per unit. Lost sales are lost forever; they cannot be backordered.
4. Placement of each order costs Rs. 75.00 regardless of the number of units ordered.
5. The demand in a day can be for any number of units between 0 and 99, each equiprobable.
6. There is never more than one replenishment order outstanding.
7. Initially we have 115 units on hand and no reorder outstanding.

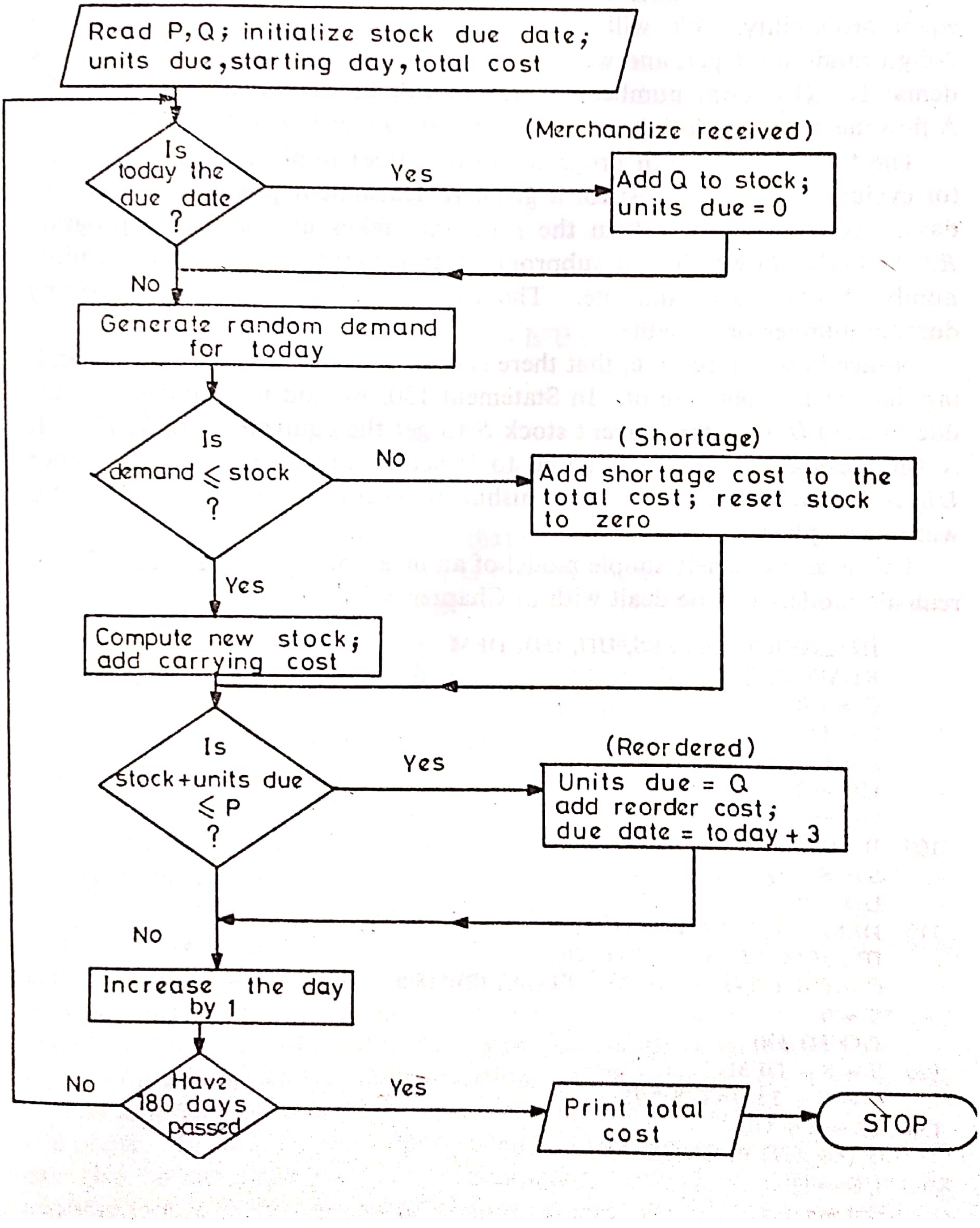
With these conditions in force you have been asked to compare the following five replenishment policies and select the one that has the minimum total cost (i.e., reorder cost + carrying cost + lost sales cost).

	P (reorder point)	Q (reorder quantity)
Policy I	125	150
Policy II	125	250
Policy III	150	250
Policy IV	175	250
Policy V	175	300

(Since we are interested in simulation here and not in inventory theory, we will not investigate the reasonableness of the replenishment policy too critically. Ours is undoubtedly a very simplified model.)

The problem does not easily lend itself to an analytic solution; it is best therefore to solve it by simulation. Let us simulate the running of the store for about six months (180 days) under each of the five policies and then compare their costs.

A simulation model of this inventory system can be easily constructed by stepping time forward in the fixed increment of a day, starting with Day 1, and continuing up to Day 180. On a typical day, Day i, first we check to see if merchandise is due to arrive today. If yes, then the existing stock S is increased by Q (the quantity that was ordered). If DEM is the demand for today, and $DEM \leqslant S$, our new stock at the end of today will be $(S - DEM)$ units. If $DEM > S$, then our new stock will be zero. In either case, we calculate the total cost resulting from today's transactions, and add it to the total cost C incurred till yesterday. Then we determine if the inventory on hand plus units on order is greater than P, the reorder point. If not, place



an order (to be delivered three days hence), by stating the amount ordered and the day it is due to be received. We repeat this procedure for 180 days.

Initially we set day number $i = 1$, stock $S = 115$, number of units due $UD = 0$ (because there is no outstanding order), and the day they are due $DD = 0$.

The demand, DEM, for each day is not a fixed quantity but a random variable. It could assume any integral value from 00 to 99 and each with equal probability. We will use a special subroutine, which generates a 2-digit random integer, and will use the numbers thus produced as the daily demands. (Random number generators will be discussed in Chapter 3.) A flowchart for simulating this problem is shown in Fig. 1-3 (page 7).

The following Fortran program (format free) implements the flowchart for evaluating the total cost for a given replenishment policy (P, Q) for 180 days. Statement No. 110 in the program makes use of the subprogram $RNDY1\,(DUM)$ which is a subprogram to generate a real pseudorandom number between zero and one. The argument of this function can be any dummy number or variable.

Notice how condition 6, that there is no more than one reorder outstanding, has been taken care of. In Statement 130, we add the number of units due (if any) UD to the current stock S to get the equivalent stock, ES. It is this number which is compared to P before an order is placed. Since $UD > P$ if we already have a replenishment order outstanding another order will not be placed.

This is an extremely simple model of an inventory-control system. More realistic models will be dealt with in Chapter 6

```
      INTEGER P, Q, S, ES, UD, DD, DEM
      READ, P, Q
      C = 0.0
      S = 115
      I = 1
      UD = 0
      DD = 0
100   IF (DD .NE. I) GO TO 110
      S = S + Q
      UD = 0
110   DEM = RNDY1 (DUM)* 100.0
      IF (DEM .LE. S) GO TO 120
      C = C + (FLOAT (DEM) - FLOAT (S))*18.0
      S = 0
      GO TO 130
120   S = S - DEM
      C = C + FLOAT (S)*.75
130   ES = S + UD
      IF (ES .GT. P) GO TO 140
      UD = Q
      DD = I + 3
      C = C + 75.0
```

```
140  I= I+1
     IF (I .LE. 180) GO TO 100
     PRINT, P, Q, C
     STOP
     END
```

The program yielded the following cost figures for the five inventory policies:

P	Q	Cost in Rs.
125	150	38679.75
125	250	31268.25
150	250	29699.25
175	250	26094.00
175	300	27773.25

Thus, Policy IV ($P = 175$, $Q = 250$) is the best amongst the five considered.

1-4. The basic nature of simulation

There does not seem to be much that is common between the two problems and the methods of solving them. The first problem (the pure pursuit) is basically continuous, in the sense that its state changes continuously with time; whereas the second problem is discrete, because the arrival and sale of merchandise occurs only in discrete steps. The first problem is deterministic, while the second one is stochastic in nature. Yet there are some common features and these are the features essential to simulation.

In both cases we started from a mathematical model of the system under study. Some initial conditions (state at time zero) were assumed in both cases. The change of state occurred in accordance with some equations (rules or laws). Using these rules numerically we continued changing the state of the system as time moved forward. This was done for as long a period as needed. At the end of this period we collected the desired information about the system (which is the solution to the problem). These calculations were programmed for a digital computer. Thus the computer was made to simulate or mimic the real-life system as its variables changed. Through this process we managed to get around the necessity of obtaining an analytic solution and therein lies the great advantage of simulation.

It should be noted that the simulation in both examples (pure pursuit as well as inventory control) could have been performed manually with pencil and paper. In theory any system that can be simulated on a digital computer can also be simulated by hand. In practice, however, simulation as an analytic tool is useful only when done on a computer. This is because the practical problems that require simulation are complex and need a very large number of simple, repetitive calculations. In fact, simulation is one of the most important uses of the digital computer.

To simulate is to experiment: Simulation is basically an experimental technique. It is a fast and relatively inexpensive method of doing an 'experiment' on the computer. Consider, for example, the inventory-control problem. Instead of trying out in the actual store the five replenishment policies, each for a period of six months, and then selecting the best one, we conducted the experiment on the computer and obtained the results within a few minutes, at a very small cost (provided, of course, our model reflects reality). This is why computer simulation is often referred to as performing a *simulation experiment*.

No unified theory: There is no unifying theory of computer simulation. Learning simulation does not consist of learning a few fundamental theorems and then using them and their various corollaries to solve problems. There are no underlying principles guiding the formulation of simulation models. Each application of simulation is *ad hoc* to a great extent. In this sense simulation is an art, and one often hears the expression 'the art of simulation.'

1-5. When to simulate

All of us in our daily lives encounter problems, which although mathematical in nature, are too complex to lend themselves to exact mathematical analysis. The performance of such a system (say, weather or traffic jam) may be difficult to predict, either because the system itself is complex or the theory is not yet sufficiently developed. The difficulties in handling such problems (by means of classical mathematical tools) may also arise due to the effect of uncertainties, or due to dynamic interactions between decisions and subsequent events, or due to complex interdependencies among variables in the system, or due to some combination of these. Formerly (i.e., before the days of computer simulation), in such situations either an intuitive decision was made, or, if the stakes were too high to rely on intuition, elaborate laboratory experiments had to be conducted, which were usually expensive and time consuming. Simulation provides a third alternative which is cheap and fast, and thus fills the gap between exact analysis and physical intuition. Occasionally, simulation is also used even when an exact analytic solution is possible, but it is too expensive in terms of computation time.

Simulation in science and engineering research: Simulation has changed, in a very fundamental sense, the way in which research is conducted today. Earlier most experiments were carried out physically in the laboratories. Thousands and even millions were spent on physical models (e.g., wind tunnels, river basin models, network analyzers, aircraft flight simulators) and expensive experiments. Today a majority of these experiments are simulated on a computer. 'Computer experiments' besides being much faster, cheaper, and easier, frequently provide better insight into the system than laboratory experiments do. Not all laboratory experiments, of course,