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Introduction

Simulation is a powerful technique for solving a wide variety of problems. To simulate is to copy the behaviour of a system or phenomenon under study. Strictly speaking, we will be dealing with only *numerical sequential simulation;* numerical because there are other forms of simulation—for example, electrical analogue or physical simulation; and sequential because the calculations proceed in a time sequence. Some of the basic ideas in simulation can be best understood by performing actual simulations. Let us, therefore, consider the following two very simple examples and see how simulation is actually done.

1-1. Simulation of a pure pursuit problem—an example

A fighter aircraft sights an enemy bomber and flies directly toward it, in order to catch up with the bomber and destroy it. The bomber (the target) continues flying (along a specified curve) so the fighter (the pursuer) has to change its direction to keep pointed toward the target. We are interested in determining the attack course of the fighter and in knowing how long it would take for it to catch up with the bomber.

If the target flies along a straight line, the problem can be solved directly with analytic techniques. (The proof of such a closed-form expression which gives the course of the pursuer, when the target flies in a straight line, is left as an exercise for you. Problem 1-2.)

However, if the path of the target is curved, the problem is much more difficult and normally cannot be solved directly. We will use simulation to solve this problem, under the following simplifying conditions:

1. The target and the pursuer are flying in the same horizontal plane when the fighter first sights the bomber, and both stay in that plane. This makes the pursuit model two-dimensional.
2. The fighter's speed *VF* is constant (20 kms/minute).
3. The target's path (i.e., its position as a function of time) is specified.
4. After a fixed time span Δt (every minute, in this case) the fighter changes its direction in order to point itself toward the bomber.

Let us introduce a rectangular coordinate system coincident with the horizontal plane in which the two aircraft are flying. We choose the point due south of the fighter and due west of the target (at the beginning of the pursuit) as the origin of this coordinate system. Let the distances be given in kilometers and the time in minutes. We start measuring the time when the fighter first sights the bomber. (See Fig. 1-1.)

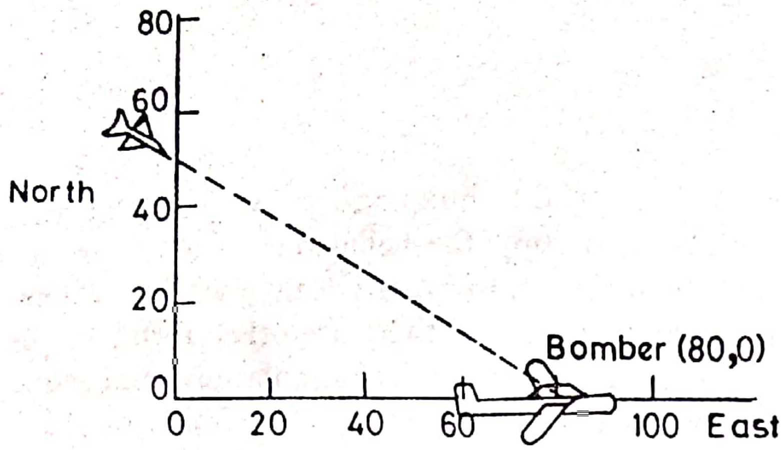

Fig. 1-1: Positions of Pursuer and Target at Time Zero.

We will represent the path of the bomber (which is known to us in advance) by two arrays, the east coordinates and the north coordinates at specified moments (each minute). We call these coordinates $XB(t)$ and $YB(t)$, respectively. They are presented in the form of a table (in kilometers) below.

Time, t	0	1	2	3	4	5	6	7	8	9	10	11	12
$XB(t)$	80	90	99	108	116	125	133	141	151	160	169	179	180
$YB(t)$	0	−2	−5	−9	−15	−18	−23	−29	−28	−25	−21	−20	−17

Table 1-1.

Likewise, we will represent the path of the fighter plane by two arrays $XF(t)$ and $YF(t)$. In this example, initially we are given

$$YF(0) = 50 \text{ kms}, \quad XF(0) = 0 \text{ kms}.$$

Our purpose is to compute the positions of the pursuer, namely, $XF(t)$, $YF(t)$ for $t = 1, 2, \ldots, 12$, or until the fighter catches up with the bomber. We will assume that once the fighter is within 10 kms of the bomber, the fighter shoots down its target by firing a missile, and the pursuit is over. In case the target is not caught up within 12 minutes, the pursuit is abandoned, and the target is considered escaped. From the time $t = 0$ till the target is shot down, the attack course is determined as follows:

The fighter uses the following simple strategy: It looks at the target at instant t, aligns its velocity vector with the line of sight (i.e., points itself toward the target). It continues to fly in that direction for one minute, till instant $(t+1)$. At time $(t+1)$ it looks at the target again and realigns itself.

The distance DIST (t) at a given time t between the target and the pursuer is given by

$$\text{DIST}\ (t) = \sqrt{(YB(t) - YF(t))^2 + (XB(t) - XF(t))^2} \qquad \ldots(1\text{-}1)$$

The angle θ of the line from the fighter to the target at a given time t is given by

$$\text{Sin}\ \theta = \frac{YB(t) - YF(t)}{\text{DIST}\ (t)} \qquad \ldots(1\text{-}2)$$

$$\text{Cos}\ \theta = \frac{XB(t) - XF(t)}{\text{DIST}\ (t)} \qquad \ldots(1\text{-}3)$$

Using this value of the position of the fighter at time $(t+1)$ is determined by

$$XF(t+1) = XF(t) + VF\ \text{Cos}\ \theta \qquad \ldots(1\text{-}4)$$

$$YF(t+1) = YF(t) + VF\ \text{Sin}\ \theta \qquad \ldots(1\text{-}5)$$

With these new coordinates of the pursuer, its distance from the target is again computed using Eq. (1-1). If this distance is 10 kms. or less the pursuit is over, otherwise θ is recomputed, and the process continues.

A flowchart of the logic of this problem is given in Fig. 1-2 (page 4).

The following FORTRAN program (a format-free version) will implement the flowchart.

```
      DIMENSION XB (25), YB (25), XF (25), YF (25)
      INTEGER T, J
      READ, (XB (T), YB (T), T = 1,13)
      READ, XF (1), YF (1), VF
      T = 1
100   DIST = SQRT ((YB (T) – YF (T))** 2 + (XB (T) – XF (T))** 2)
      IF (DIST. LE. 10.0) GO TO 110
      IF (T.GT.12) GO TO 120
      XF (T + 1) = XF(T) + VF* (XB(T) – XF (T))/DIST
      YF (T + 1) = YF (T) + VF* (YB (T) – YF (T))/DIST
      T = T + 1
      GO TO 100
110   PRINT 990, T, DIST
990   FORMAT (10X, 10H CAUGHT AT, 13, 8H MTS AND, F10.3, 4H KMS)
      STOP
120   PRINT 1000
1000  FORMAT (10X, 16H TARGET ESCAPED)
      STOP
      END
```

(Note that since Fortran does not permit 0 as an index we had to set $T = 1$ to correspond to $t = 0$ in the flowchart.)

The output of this program for the specified data is as follows:

CAUGHT AT 10 MTS AND 2.969 KMS

This is an example of simulation. We had to resort to simulation be-

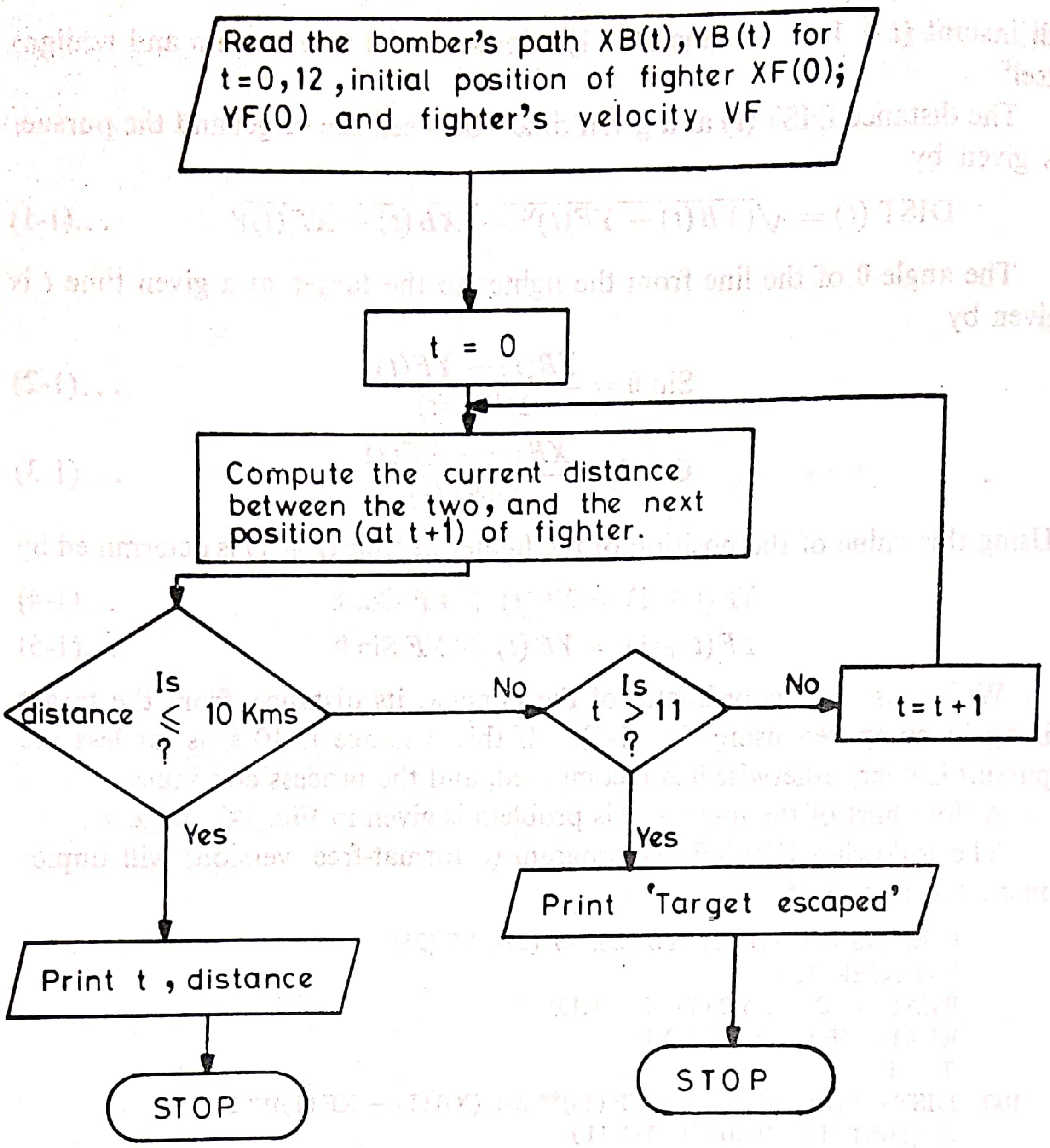

Fig. 1-2: Flowchart of pursuit problem.

cause analytically we could not make a long-term prediction about the path that the fighter plane would take (given the initial position and path of the target). But by simulation we were able to make the computer go through the instant-to-instant predictions for as many instants as we wanted. This was possible only because we knew the basic process involved, namely, how the fighter plane behaves at any particular instant. Such knowledge of the basic process of the system under study is essential for all simulation experiments.

The simple strategy, of pursuer redirecting himself toward the target at fixed intervals of time, while the target goes on its predetermined path without making any effort to evade the pursuer, is called *pure pursuit*. Although in many situations, the strategy used by the pursuer is more sophisticated, this basic approach can be used for any pursuit problem as long as we know