

Linear Programming Problem – Optimization Problem

Example: Consider a chocolate manufacturing company which produces only two types of chocolate – A and B. Both the chocolates require Milk and Choco only. To manufacture each unit of A and B, following quantities are required:

- Each unit of A requires 1 unit of Milk and 3 units of Choco
- Each unit of B requires 1 unit of Milk and 2 units of Choco

The company kitchen has a total of 5 units of Milk and 12 units of Choco. On each sale, the company makes a profit of

- Rs 6 per unit A sold
- Rs 5 per unit B sold.

Now, the company wishes to maximize its profit. How many units of A and B should it produce respectively?

Solution: The first thing I'm gonna do is represent the problem in a tabular form for better understanding.

	Milk	Choco	Profit per unit
A	1	3	Rs 6
B	1	2	Rs 5
Total	5	12	

Let the total number of units produced of A be = X

Let the total number of units produced of B be = Y

Now, the total profit is represented by Z

The total profit the company makes is given by the total number of units of A and B produced multiplied by its per unit profit Rs 6 and Rs 5 respectively.

$$\text{Profit: Max } Z = 6X + 5Y$$

which means we have to maximize Z .

The company will try to produce as many units of A and B to maximize the profit. But the resources Milk and Choco are available in limited amount.

As per the above table, each unit of A and B requires 1 unit of Milk. The total amount of Milk available is 5 units. To represent this mathematically, $X + Y \leq 5$

Also, each unit of A and B requires 3 units & 2 units of Choco respectively. The total amount of Choco available is 12 units. To represent this mathematically, $3X + 2Y \leq 12$

Also, the values for units of A can only be integers.

So we have two more constraints, $X \geq 0$ & $Y \geq 0$

For the company to make maximum profit, the above inequalities have to be satisfied. This is called formulating a real-world problem into a mathematical model.

LPP problem is

X - total number of units produced of A

Y - total number of units produced of B

Z - Profit

$$\text{Max } Z = 6X + 5Y$$

subject to

$$\text{s.t. } X + Y \leq 5$$

$$3X + 2Y \leq 12$$

$$X \geq 0 \text{ \& } Y \geq 0$$

$$\text{min. } -6X - 5Y$$

$$A \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$$

$$b = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

$$u = \begin{bmatrix} 0 & 0 \end{bmatrix} \quad u_0 = \begin{bmatrix} 1 \end{bmatrix}$$

$$A_0 = \begin{bmatrix} 1 \end{bmatrix}$$

$$b_0 = \begin{bmatrix} 1 \end{bmatrix}$$

$$Z = -27 = \boxed{-27}$$

$$x_1 = 2$$

$$y = 3$$

$$\text{eg. } 3X + 2Y = 0$$