Application of Linear Programming for Optimal Use of Raw Materials in Bakery

ME 4126D

Optimization Methods in Engineering

Project Report

तमसो मा ज्योतिर्गमय

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INTRODUCTION

Linear programming is a family of mathematical programming that is concerned with or useful for allocation of scarce or limited resources to several competing activities on the basis of given criterion of optimality. In statistics, linear programming (LP) is a special techniques employed for the purpose of optimization of linear function subject to linear equality and inequality constraint. Linear programming determines the way to achieve best outcome, such as maximum profit or minimum cost in a given mathematical model and given some list of requirement as a linear equation. The technique of linear programming is used in a wide range as applications, including agriculture, industry, transportation, economics, health system

NEED FOR OPTIMIZATION

Sometimes many production companies are faced with problems of how to utilize the available resources in order to maximize profit; this is because the use of linear programming which brings a suitable quantitative approach of decision-making has not been fully applied.The decision of most production managers are based on the total input used in the production and output proceed. This method of decision making is always biased, that is it brings about reduction in the accuracy of forecasting for the future, such as price fluctuation and shortage of raw materials or available resources.The problem of decision making based on the use of limited resource is a major factor that brought the application of linear programming model which is now one of the most powerful tools which all decision makers must apply before achieving effective decision.

OUTLINE OF THE REPORT

An optimum solution for maximizing the profit of a bakery in producing three types of bread loaves from the collected raw materials data using linear programming solved through LINGO Software.

LITERATURE SURVEY

The reference papers which were referred during the course of project work have been inserted below

Sl - no	Title of the paper	Authors , year of publication	Major finding
1	An introduction to linear programming problem	Steven J. Miller,2007	Reason for the great versatility of linear programming is due to the ease at which constraints can be incorporated into the linear programming model.
2	Optimizing profit with the linear programming model: A focus on Golden plastic industry limited, Enugu, Nigeria	Benedict I. Ezema, Ozochukwu Amakon ,2012	The problem of industries all over the world is a result of shortage of production inputs which result in low capacity utilization and consequently low outputs.
3	Application of linear programming to semi-commercial arable and fishery enterprises in Abia State, Nigeria	Igwe, K.C, C.E Onyenweaku, J.C. Nwaru ,2011	linear programming is a relevant technique in achieving efficiency in production planning, particularly in achieving increased agricultural productivity

LINEAR PROGRAMMING MODEL

The general linear programming model with n decision variables and m constraints can be stated in the following.

$$\text{Optimize } (\max \text{ or } \min)\ Z = c_1x_1 + c_2x_2 + \dots\dots\dots\dots + c_nx_n$$

$$s.t$$

$$a_{11}x_1 + a_{12}x_2 + \dots\dots\dots\dots + a_{1n}x_n\ (\le,\ =,\ \ge)\ b_1$$

$$a_{12}x_1 + a_{22}x_2 + \dots\dots\dots\dots + a_{2n}x_n\ (\le,\ =,\ \ge)\ b_2$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$a_{m1}x_1 + a_{m2} + \dots\dots\dots\dots\dots + a_{mn}x_n\ (\le,\ =,\ \ge)\ b_m$$

Where c1, c2, --------cn represent the per unit profit (or cost) of decision variables x1, x2, ----,xn to the value of the objective function. And a11, a12 ------, a2n, -------, am, am2, ---- amn represent the amount of resource consumed per unit of the decision variables. The bi represents the total availability of the ith resource ‘Z’

represent the measure – of – performance which can be either profit, or cost or reverence etc.

Standard form of a Linear Programming Model:

The use of the simplex method to solve a linear programming problem requires that the problem be converted into its standard form. The standard form of a linear programming problem has the following properties.

I. the constraints should be expressed as equations by adding slack or plus variables.
II. The right-hand side of each constraint should be made of non-negative (if not). This is done by multiplying
III. The objective function should be of a maximization type.

For n decision variables and m constraints, the standard form of the linear programming model can be formulated as follows.

$$\text{Optimize } (\max)\ Z = c_1x_1 + c_2x_2 + \ldots\ldots\ldots + c_nx_n + 0s_1 + 0s_2 + \ldots\ldots\ldots + 0s_m$$

Subject to the linear constraints

$$\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots\ldots + a_{1n}x_n = b_1 \\
&a_{12}x_1 + a_{22}x_2 + \ldots\ldots\ldots\ldots + a_{2n}x_n = b_2 \\
&\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
&\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
&a_{m1}x_1 + a_{m2} + \ldots\ldots\ldots\ldots + a_{mn}x_n + s_m = b_m
\end{aligned}$$

$$x_1, x_2, \ldots\ldots., x_n, s_1, s_2, \ldots\ldots\ldots, s_m \geq 0$$

This can be stated in a more compact form as:

$$\text{Optimize } (\max)\ z = \sum_{j=1}^{n} c_j x_i + \sum_{i=1}^{m} 0 s_i$$

Subject to the linear constraints

$$\sum_{j=1}^{n} a_{ij} x_j s_i = b_i, i = 1, 2, \ldots\ldots m \text{ and}$$

$$x_j, s_i \geq 0 \text{ (for all i and j)}$$

Assumptions

It is assumed that the raw materials required for production of bread are limited (scarce).
It is assumed that an effective allocation of raw materials to the variables (big loaf, giant loaf and small loaf) will aid optimal production and at the same time maximizing the profit of then bakery.
It is assumed that the qualities of raw materials used in bread production are standard (not inferior).

Data Presentation and Analysis

The data for this research project was collected from Gorretta bakery limited, Nigeria. The data consist of total amount of raw materials (sugar, flour, yeast, salt, wheat gluten and soybean oil) available for daily production of three different sizes of bread (big loaf, giant loaf and small loaf) and profit contribution per each unit size of bread produced. The content of each raw material per each unit product of bread produced is shown below.

Flour

Total amount of flour available = 200kg

Each unit of big loaf requires 0.2kg of flour

Each unit of giant loaf requires 0.24kg of flour

Each unit of small loaf requires 0.14kg of flour

Sugar

Total amount of sugar available = 160g

Each unit of big loaf requires 0.14g of sugar

Each unit of giant loaf requires 0.20g of sugar

Each unit of small loaf requires 0.16g of sugar

Yeast

Total amount of yeast available = 20kg

Each unit of big loaf requires 0.02kg of yeast

Each unit of giant loaf requires 0.02kg of yeast

Each unit of small loaf requires 0.02kg of yeast

Salt

Total amount of salt available = 8.5g

Each unit of big loaf requires 0.0011g of salt

Each unit of giant loaf requires 0.00105g of salt

Each unit of small loaf requires 0.00017g of salt

Wheat gluten

Total amount of wheat gluten = 15.0g

Each unit of big loaf requires 0.000167g of wheat gluten

Each unit of giant loaf requires 0.002g of wheat gluten

Each unit of small loaf requires 0.00012g of wheat gluten

Soybean Oil

Total amount (volume) of soybean available = 10.0L

Each unit of big loaf requires 0.0157L of soybean oil

Each unit of giant loaf requires 0.021L of soybean oil

Each unit of small loaf requires 0.0098L of soybean oil

Profit contribution per unit product (size) of bread produced

Each unit of big loaf = N30

Each unit of giant loaf = N40

Each unit of small loaf = N20

The above data can be summarized in a tabular form.

Raw material	**Product**			**Total available raw material**
	Big loaf	**Giant loaf**	**Small loaf**	
Flour (kg)	0.20	0.24	0.14	200.0
Sugar (g)	0.14	0.20	0.16	160.0
Yeast (kg)	0.02	0.02	0.02	20.0
Salt (g)	0.0011	0.00105	0.00017	8.5
Wheat gluten (g)	0.000167	0.002	0.00012	15.0
Soybean oil (L)	0.015	0.021	0.0098	10.0
Profit (N)	30	40	20	

Model formulation

Let the quantity of big loaf to be produce = x1

Let the quantity of giant loaf to be produce = x2

Let the quantity of small loaf to be produce = x3

Let Z denote the profit to be maximize

The linear programming model for the above production data is given by

$$\text{Max } Z = 30x_1 + 40x_2 + 20x_3$$

$$S.t$$

$$0.20x_1 + 0.24x_2 + 0.14x_3 \leq 200$$

$$0.14x_1 + 0.20x_2 + 0.16x_3 \leq 160$$

$$0.02x_1 + 0.02x_2 + 0.02x_3 \leq 20$$

$$0.0011x_1 + 0.00105x_2 + 0.00017x_3 \leq 8.5$$

$$0.000167x_1 + 0.002x_2 + 0.00012x_3 \leq 15$$

$$0.015x_1 + 0.021x_2 + 0.0098x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

Converting the model into its corresponding standard form;

$$Max\ Z = 30x_1 + 40x_2 + 20x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 + 0s_6$$

$$S.t$$

$$0.20x_1 + 0.24x_2 + 0.14x_3 + s_1 = 200$$

$$0.14x_1 + 0.20x_2 + 0.16x_3 + s_2 = 160$$

$$0.02x_1 + 0.02x_2 + 0.02x_3 + s_3 = 20$$

$$0.0011x_1 + 0.00105x_2 + 0.00017x_3 + s_4 = 8.5$$

$$0.000167x_1 + 0.002x_2 + 0.00012x_3 + s_5 = 15$$

$$0.015x_1 + 0.021x_2 + 0.0098x_3 + s_6 = 10$$

$$x_1, x_2, x_3, s_1, s_2, s_3, s_4, s_5, s_6 \geq 0$$

The above linear programming model was solved using LINGO software, which gives an optimal solution of:

X1 = 38.0, X2 = 0.0, X3 = 962.0

Z = 20385.0

Interpretation of Result

Based on the data collected the optimum result derived from the model indicates that two sizes of bread should be produced, small loaf and big loaf. Their production quantities should be 962.0 and 38.0 units respectively.

This will produce a maximum profit of N20,385.0

SUMMARY

The objective of this research work was to apply linear programming for optimal use of raw material in bread production. Goretta bakery limited was used as our case study. The decision variables in this research work are the three different sizes of bread (big loaf, giant loaf and small loaf) produced by Goretta bakery limited. The researcher focused mainly on six raw materials (flour, sugar, yeast, salt, wheat gluten and soybean oil)used in the production and the amount of raw material required of each variable (bread size). The result shows that 962 unit of small loaf, 38 unit of big loaf and 0 unit of giant loaf should be produce respectively which will give a maximum profit of 20385.0

CONCLUSION

Based on the analysis carried out in this research work and the result shown, Goretta bakery limited should produce the three sizes of bread (big loaf, giant loaf and small loaf) in order to satisfy her customers. Also, more of small loaf and big loaf should be produce in order to attain maximum profit, because they contribute mostlyto the profit earned by the company

REFERENCES

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