AI1103: Assignment 2

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and latex-tikz codes from

https://github.com/ASHWITHA−11008/Assignment−2/blob/main/Assignment−2.tex

1 Problem

$X(t)$ is a random process with a constant mean value of 2 and the autocorrelation function $R_x(\tau) = 4\left(e^{-0.2|\tau|} + 1\right)$. Let Y and Z be the random variables obtained by sampling $X(t)$ at $t = 2$ and $t = 4$ respectively. Let $W = Y - Z$. The variance of W is

(a) 13.36 (b) 9.36 (c) 2.64 (d) 8.00

2 Solution

Given $W = Y - Z$

Variance of W

$$\begin{aligned}\sigma_W^2 &= E[Y-Z]^2 - (E[Y-Z])^2 \\ &= E[Y^2] + E[Z^2] - 2E[YZ]\end{aligned}$$

Since given mean value is constant
$E[Y - Z] = \mu_Y - \mu_Z = 0$

We know,

$$\begin{aligned}R_X(\tau) &= E[(X)(X+\tau)] \\ E[X^2(t)] &= R_X(0) \\ E[X^2(2)] &= E[X^2(4)] = R_X(0)\end{aligned}$$

So,

$$\begin{aligned}\sigma_W^2 &= R_X(0) + R_X(0) - 2R_X(2) \\ &= 4\left(e^{-0.2|0|} + 1\right) + 4\left(e^{-0.2|0|} + 1\right) - 2 \times 4\left(e^{-0.2|2|} + 1\right) \\ &= 8 + 8 - 2[4(0.67 + 1)] \\ &= 2.64\end{aligned}$$

So,option c is correct.