AI1103: Assignment 2

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and latex-tikz codes from

https://github.com/ASHWITHA−11008/Assignment−2/blob/main/Assignment−2.tex

1 Problem

$X(t)$ is a random process with a constant mean value of 2 and the autocorrelation function $R_x(\tau) = 4\left(e^{-0.2|\tau|} + 1\right)$. Let Y and Z be the random variables obtained by sampling $X(t)$ at $t = 2$ and $t = 4$ respectively. Let $W = Y - Z$. The variance of W is

(a) 13.36 (b) 9.36 (c) 2.64 (d) 8.00

2 Solution

Given $W = Y - Z$

Variance of W,

$$\sigma_W^2 = E[Y-Z]^2 - (E[Y-Z])^2 \tag{2.0.1}$$
$$= E[Y^2] + E[Z^2] - 2E[YZ] \tag{2.0.2}$$

$E[Y - Z] = \mu_Y - \mu_Z = 2 - 2 = 0$

We know,

$$R_X(\tau) = E[X(t)X(t+\tau)] \tag{2.0.3}$$
$$R_X(0) = E[X^2(t)] \tag{2.0.4}$$

$$Let \;\; t_1 = 2 \;\; and \;\; t_2 = 4 \tag{2.0.5}$$
$$Y = X(t_1) \tag{2.0.6}$$
$$Z = X(t_2) \tag{2.0.7}$$

So,

$$\sigma_W^2 = E[X^2(t_1)] + E[X^2(t_2)] - 2E[X(t_1)X(t_2)] \tag{2.0.8}$$
$$= 2E[X^2(t_1)] - 2E[X(t_1)X(t_1+2)] \tag{2.0.9}$$
$$= 2R_X(0) - 2R_X(2) \tag{2.0.10}$$
$$= 8\left(e^{-0.2|0|} + 1\right) - 2 \times 4\left(e^{-0.2|2|} + 1\right) \tag{2.0.11}$$
$$= 8(2) - 2[4(0.67 + 1)] \tag{2.0.12}$$
$$= 2.64 \tag{2.0.13}$$

So,option c is correct.