

AI1103: Assignment 2

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and latex-tikz codes from

[https://github.com/ASHWITHA-11008/
Assignment-2/blob/main/Assignment-2.tex](https://github.com/ASHWITHA-11008/Assignment-2/blob/main/Assignment-2.tex)

1 PROBLEM

$X(t)$ is a random process with a constant mean value of 2 and the autocorrelation function $R_x(\tau) = 4(e^{-0.2|\tau|} + 1)$. Let Y and Z be the random variables obtained by sampling $X(t)$ at $t = 2$ and $t = 4$ respectively. Let $W = Y - Z$. The variance of W is

- (a) 13.36 (b) 9.36 (c) 2.64 (d) 8.00

2 SOLUTION

Given $W = Y - Z$

Variance of W ,

$$\begin{aligned}\sigma_W^2 &= E[Y - Z]^2 - (E[Y - Z])^2 \\ &= E[Y^2] + E[Z^2] - 2E[YZ]\end{aligned}$$

$$E[Y - Z] = \mu_Y - \mu_Z = 2 - 2 = 0$$

We know,

$$R_X(\tau) = E[X(t)X(t + \tau)]$$

$$R_X(0) = E[X^2(t)]$$

Let $t_1 = 2$ and $t_2 = 4$

$$Y = X(t_1)$$

$$Z = X(t_2)$$

So,

$$\begin{aligned}\sigma_W^2 &= E[X^2(t_1)] + E[X^2(t_2)] - 2E[X(t_1)X(t_2)] \\ &= 2E[X^2(t_1)] - 2E[X(t_1)X(t_1 + 2)] \\ &= 2R_X(0) - 2R_X(2) \\ &= 8(e^{-0.2|0|} + 1) - 2 \times 4(e^{-0.2|2|} + 1) \\ &= 8(2) - 2[4(0.67 + 1)] \\ &= 2.64\end{aligned}$$

So, option c is correct.