AI1103: Assignment 3

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and latex-tikz codes from

1 PROBLEM

10 balls are placed in 10 boxes independently at random. Assuming that all 10 boxes were initially empty,what is the expected number of boxes that remain empty ?

1) $\left(\frac{9}{10}\right)^9$

2) $\frac{9^9}{10^{10}}$

3) $\frac{9^{10}}{10^9}$

4) $\left(\frac{9}{10}\right)^{10}$

2 SOLUTION:

Let X be the random variable for the number of boxes to be empty. $i = 1, 2, 3, ..., 10$,define X_i by $X_i = 1$ if box i ends up with zero balls, and $X_i = 0$ otherwise.

$$\implies X = \sum_{i=1}^{i=10} X_i \quad (2.0.1)$$

$$E[X] = E[\sum_{i=1}^{i=10} X_i] \quad (2.0.2)$$

$For\ any\ i\ ,\ X_i = 1.\ So,\ E[X_i] = P(X_i = 1)$
$$(2.0.3)$$

Probability of box i is empty,

$$P(X_i = 1) = {}^{10}C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^{10-0}$$

Thus, $E[X_i] = \left(\frac{9}{10}\right)^{10}$

Expected number of boxes that remain empty are,

$$E[X] = \sum_{i=1}^{i=10}\left(\frac{9}{10}\right)^{10} \quad (2.0.4)$$

$$= 10\left(\frac{9^{10}}{10^{10}}\right) \quad (2.0.5)$$

$$= \frac{9^{10}}{10^9} \quad (2.0.6)$$

So, option 3 is correct.