AI1103: Assignment 3

BATHINI ASHWITHA
CS20BTECH11008

and latex-tikz codes from

https://github.com/ASHWITHA−11008/Assignment−3/blob/main/Assignment−3.tex

1 Problem

10 balls are placed in 10 boxes independently at random. Assuming that all 10 boxes were initially empty,what is the expected number of boxes that remain empty ?

1) $\left(\frac{9}{10}\right)^9$
2) $\frac{9^9}{10^{10}}$
3) $\frac{9^{10}}{10^9}$
4) $\left(\frac{9}{10}\right)^{10}$

2 Solution:

Let X be the random variable for the number of boxes to be empty. $i = 1, 2, 3, ..., 10$,define X_i by $X_i = 1$ if box i ends up with zero balls, and $X_i = 0$ otherwise.

$$X_i = \begin{cases} 1, & \text{box } i \text{ is empty} \\ 0, & \text{otherwise} \end{cases} \tag{2.0.1}$$

$$\implies X = \sum_{i=1}^{i=10} X_i \tag{2.0.2}$$

$$X_i = 1 \quad \forall i = 1, 2, 3, ...10. \quad So, \quad E[X_i] = P(X_i = 1) \tag{2.0.3}$$

Using binomial distribution,

$$P_x(k) = {}^nC_k \times p^k \times q^{n-k} \tag{2.0.4}$$

Here,

$$n = 10 \tag{2.0.5}$$

$$p = \frac{1}{10} \tag{2.0.6}$$

$$q = \frac{9}{10} \tag{2.0.7}$$

Probability of box i is empty,

$$P_x(0) = P(X_i = 1) = {}^{10}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{10-0} \tag{2.0.8}$$

Thus,

$$E[X_i] = \left(\frac{9}{10}\right)^{10} \tag{2.0.9}$$

Expected number of boxes that remain empty are,

$$E[X] = \sum_{i=1}^{i=10} \left(\frac{9}{10}\right)^{10} \tag{2.0.10}$$

$$= 10\left(\frac{9^{10}}{10^{10}}\right) \tag{2.0.11}$$

$$= \frac{9^{10}}{10^9} \tag{2.0.12}$$

So, option 3 is correct.