AI1103: Assignment 3

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and latex-tikz codes from

https://github.com/ASHWITHA−11008/Assignment−3/blob/main/Assignment−3.tex

1 Problem

10 balls are placed in 10 boxes independently at random. Assuming that all 10 boxes were initially empty,what is the expected number of boxes that remain empty ?

(a)$\left(\frac{9}{10}\right)^9$

(b)$\frac{9^9}{10^{10}}$

(c)$\frac{9^{10}}{10^9}$

(d)$\left(\frac{9}{10}\right)^{10}$

2 Solution:

Let X be the random variable for the number of boxes to be empty. $i = 1, 2, 3, ..., 10$,define X_i by $X_i = 1$ if box i ends up with zero balls, and $X_i = 0$ otherwise

$X = \sum_{i=1}^{i=10} X_i$

$E[X] = E[\sum_{i=1}^{i=10} X_i]$

For any i, $X_i = 1$. So,
$E[X_i] = P(X_i = 1)$

Probability of box i is empty,
$P(X_i = 1) = \left(\frac{9}{10}\right)^{10}$

Thus, $E[X_i] = \left(\frac{9}{10}\right)^{10}$

Expected number of boxes that remain empty are,

$$\begin{aligned} E[X] &= \sum_{i=1}^{i=10}\left(\frac{9}{10}\right)^{10} \\ &= 10\left(\frac{9^{10}}{10^{10}}\right) \\ &= \frac{9^{10}}{10^9} \end{aligned}$$

So, option c is correct.