

AI1103: Assignment 3

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and latex-tikz codes from

[https://github.com/ASHWITHA-11008/
Assignment-3/blob/main/Assignment-3.tex](https://github.com/ASHWITHA-11008/Assignment-3/blob/main/Assignment-3.tex)

Expected number of boxes that remain empty are,

$$E[X] = \sum_{i=1}^{i=10} \left(\frac{9}{10}\right)^{10} \quad (2.0.6)$$

$$= 10 \left(\frac{9^{10}}{10^{10}}\right) \quad (2.0.7)$$

$$= \frac{9^{10}}{10^9} \quad (2.0.8)$$

So, option 3 is correct.

1 PROBLEM

10 balls are placed in 10 boxes independently at random. Assuming that all 10 boxes were initially empty, what is the expected number of boxes that remain empty ?

- 1) $\left(\frac{9}{10}\right)^9$
- 2) $\frac{9^9}{10^{10}}$
- 3) $\frac{9^{10}}{10^9}$
- 4) $\left(\frac{9}{10}\right)^{10}$

2 SOLUTION:

Let X be the random variable for the number of boxes to be empty. $i = 1, 2, 3, \dots, 10$, define X_i by $X_i = 1$ if box i ends up with zero balls, and $X_i = 0$ otherwise.

$$\Rightarrow X = \sum_{i=1}^{i=10} X_i \quad (2.0.1)$$

$$X_i = 1 \quad \forall i = 1, 2, 3, \dots, 10. \text{ So, } E[X_i] = P(X_i = 1) \quad (2.0.2)$$

Probability of box i is empty,

$$P(X) = {}^nC_k \times \left(\frac{1}{10}\right)^k \times \left(\frac{9}{10}\right)^{n-k} \quad (2.0.3)$$

$$P(X_i = 1) = {}^{10}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{10-0} \quad (2.0.4)$$

Thus,

$$E[X_i] = \left(\frac{9}{10}\right)^{10} \quad (2.0.5)$$