

AI1103: Assignment 3

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and latex-tikz codes from

[https://github.com/ASHWITHA-11008/
Assignment-3/blob/main/Assignment-3.tex](https://github.com/ASHWITHA-11008/Assignment-3/blob/main/Assignment-3.tex)

Expected number of boxes that remain empty are,

$$\begin{aligned} E[X] &= \sum_{i=1}^{i=10} \left(\frac{9}{10}\right)^{10} \\ &= 10 \left(\frac{9^{10}}{10^{10}}\right) \\ &= \frac{9^{10}}{10^9} \end{aligned}$$

1 PROBLEM

10 balls are placed in 10 boxes independently at random. Assuming that all 10 boxes were initially empty, what is the expected number of boxes that remain empty ?

- 1) $\left(\frac{9}{10}\right)^9$
- 2) $\frac{9^9}{10^{10}}$
- 3) $\frac{9^{10}}{10^9}$
- 4) $\left(\frac{9}{10}\right)^{10}$

So, option c is correct.

2 SOLUTION:

Let X be the random variable for the number of boxes to be empty. $i = 1, 2, 3, \dots, 10$, define X_i by $X_i = 1$ if box i ends up with zero balls, and $X_i = 0$ otherwise.

$$\Rightarrow X = \sum_{i=1}^{i=10} X_i$$

$$E[X] = E\left[\sum_{i=1}^{i=10} X_i\right]$$

For any i , $X_i = 1$. So,
 $E[X_i] = P(X_i = 1)$

Probability of box i is empty,

$$P(X_i = 1) = {}^{10}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{10-0}$$

$$\text{Thus, } E[X_i] = \left(\frac{9}{10}\right)^{10}$$