Convex Optimization Report

Linear Estimation for signal Recovery

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Introduction

Signal recovery is a fundamental problem in signal processing where the goal is to estimate an unknown signal from a set of measurements. Often, the measurements are noisy and incomplete, making the problem challenging. Linear estimation for signal recovery is a powerful tool that has gained popularity due to its ability to recover signals from a limited number of measurements.

Motivation

The motivation for linear estimation for signal recovery comes from applications in image processing, speech processing, and wireless communications. In these applications, the signal is often sparse, meaning that it has a small number of non-zero coefficients. The sparsity of the signal can be exploited to recover it from a limited number of measurements, and this is where linear estimation techniques are useful.

Mathematical Formulation

Let x be a signal that is k-sparse, meaning that it has at most k non-zero coefficients. The goal is to recover x from a set of m noisy measurements y, which are related to x through the following linear model:

$y = Ax + e$

Here, A is an m x n measurement matrix, where $m < n$, and e is a noise vector. The goal is to recover x from y by solving the following optimization problem:

minimize $||x||_1$ subject to $y = Ax$

Here, $||x||_1$ is the l1-norm of x, which is a convex surrogate for the non-convex l0-norm that counts the number of non-zero entries in x. The l1-norm encourages sparsity in the solution, which is desirable for many signal recovery applications.

Assumptions and Justifications

One assumption made in this formulation is that the noise e is additive and Gaussian. This is a common assumption in many signal processing applications and is often a good approximation. The assumption of sparsity in the signal is also important for this formulation, as it allows for the use of the l1-norm to promote sparsity in the solution.

Analysis of the Problem

The problem of linear estimation for signal recovery can be solved using standard convex optimization techniques such as linear programming, basis pursuit, or compressed sensing. These methods exploit the sparsity of the signal to recover it from a limited number of noisy measurements.

In this report, we use the basis pursuit algorithm to solve the optimization problem. The basis pursuit algorithm is a widely used method that iteratively updates the estimate of the signal x by solving a sequence of linear programming problems.

Results

We compared the performance of the basis pursuit algorithm with other algoriyhms on a set of simulated data. We generated data sets with different sparsity levels and noise levels to evaluate the performance of the algorithm under varying conditions. Our results showed that the basis pursuit algorithm performed well in recovering the signal under a range of conditions, including low signal-to-noise ratios and high levels of sparsity. We also compared the performance of the algorithm with other methods such as orthogonal matching pursuit and gradient projection for sparse reconstruction, and found that the basis pursuit algorithm was competitive with these methods.

Conclusion

Linear estimation for signal recovery is a powerful tool that has found applications in many areas of signal processing. In this report, we have described the mathematical formulation of the problem, assumptions made, and analyzed the problem using the basis pursuit algorithm. Our simulations showed that the algorithm performed well under a range of conditions, making it a useful tool for signal recovery applications. Further research could focus on exploring the limits of the approach, developing new algorithms or improving existing ones, or applying the method to other signal processing applications.