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Statistical Methods in Natural Language Processing (NLP)

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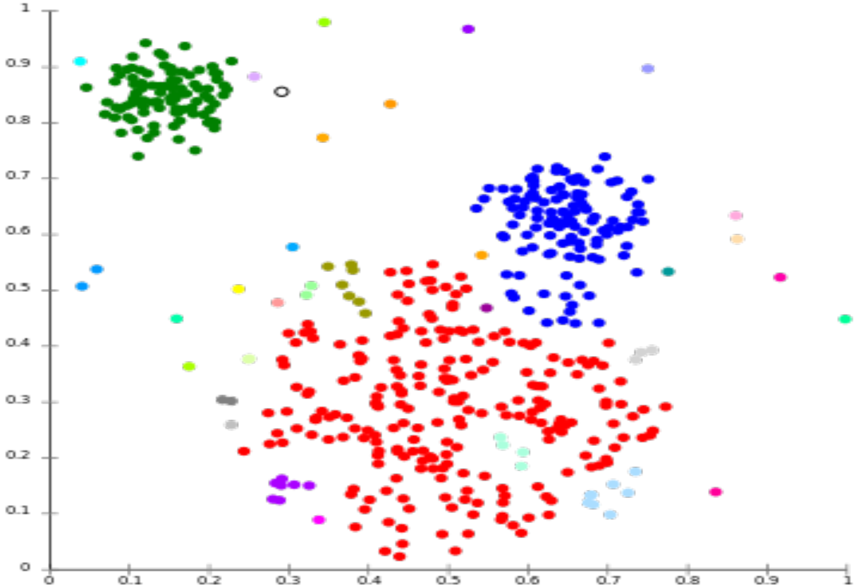
Clustering Algorithms

K-means
Gaussian Mixture Model

Clustering

- Kind of Unsupervised Learning problem
- Given : N unlabelled examples : $\{x_1, \ldots, x_N\}$, each with D dimensions (features) : $\{f_1, \ldots, f_D\}$, Number of clusters K
- **Target**: how to group all the examples into the K portions.
- Example:
  - K-means
  - Gaussian Mixture Model



K-mean Algorithm

- Nonprobabilistic technique
- Hard assignment : each example must belongs to one cluster
- **Cluster** a group of data points whose inter-point distances are small compared with the distances to points outside of the cluster.

K-mean Algorithm

- Given
  - X data set $\{x_1, \ldots, x_N\}$ consisting of N observations of a random D-dimensional Euclidean variable x.
  - Goal is to partition the data set into some number K of clusters
- Define
  - set of D-dimensional vectors μ_k, where $k = 1, \ldots, K$, in which μ_k is a prototype associated with the k^{th} cluster.
- **Goal** :
  - find an assignment of data points to clusters
  - such that the sum of the squares of the distances of each data point to its closest vector μ_k, is a minimum.

K-mean (*Objective Function*)

- For each data point x_n, we introduce a corresponding set of binary indicator variables $r_n^k \in \{0, 1\}$, where $k = 1, \ldots, K$ describing which of the K clusters the data point x_n is assigned to

- objective function, sometimes called a ***distortion measure***,

$$J = \sum_{n=1}^{N} \sum_{k=1}^{K} r_{nk} \|\mathbf{x}_n - \boldsymbol{\mu}_k\|^2$$

- Goal: find values for the $\{ r_n^k \}$ and the $\{ \mu_k, \}$ to minimize J

K mean algorithm

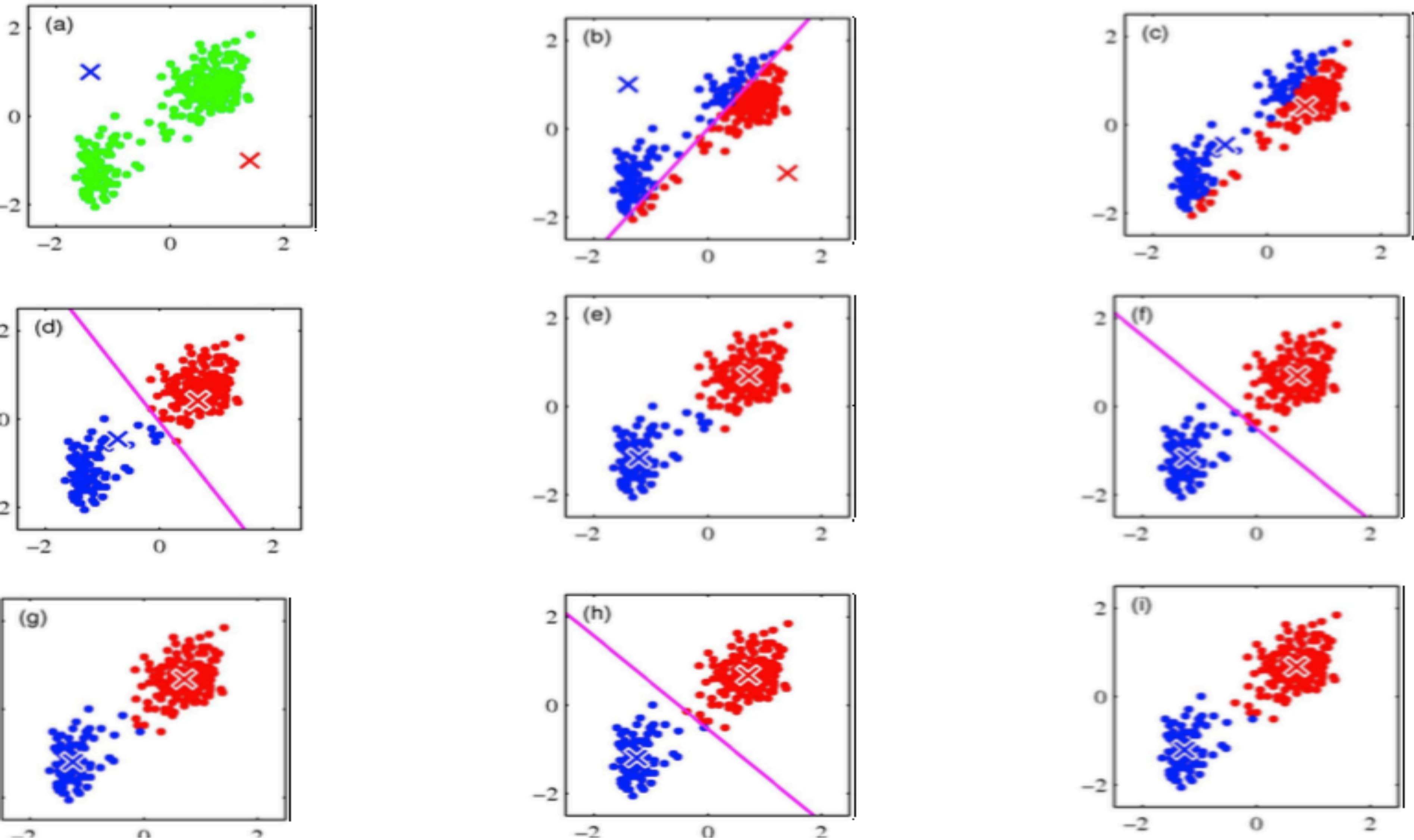
- Randomly initialize *k* centers
  - $\mu^{(0)} = \mu_1^{(0)}, \ldots, \mu_k^{(0)}$
- **Assignment step**: Assign each point $i \in \{1, \ldots, m\}$ to nearest center:

$$C^{(t)}(j) \leftarrow \arg\min_i ||\mu_i - x_j||^2$$

- **Update step**: μ_i becomes centroid of its point:

$$\mu_i^{(t+1)} \leftarrow \arg\min_{\mu} \sum_{j:C(j)=i} ||\mu - x_j||^2$$

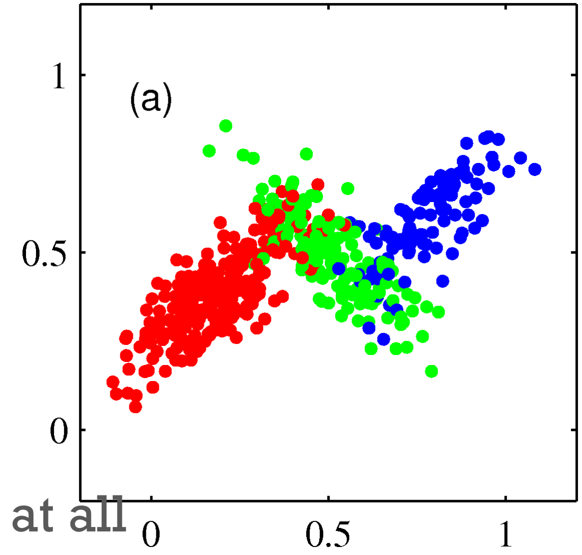
K-mean Example



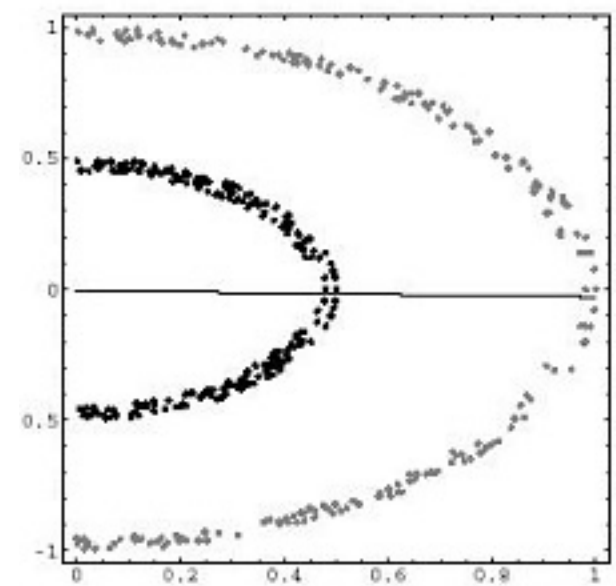
K mean Advantages

- Fast, robust and easier to understand.
- Relatively efficient: O(tknd), where n is # objects, k is # clusters, d is # dimension of each object, and t is # iterations. Normally, k, t, d << n.
- Gives best result when data set are distinct or well separated from each other.

Limitation



- Makes hard assignments of points to clusters
  A point either totally belongs to a cluster or not at all
- No notion of a soft/fractional assignment (i.e., probability of being assigned to each cluster: say K = 3 and for some point x_n, p1 = 0.7,p2 = 0.2,p3 = 0.1)
- K-means often doesn't work when clusters are not round shaped, and/or may overlap, and/or are unequal
- Unable to handle nonlinear or noisy data and outliers.



K mean DEMO

http://syskall.com/kmeans.js/

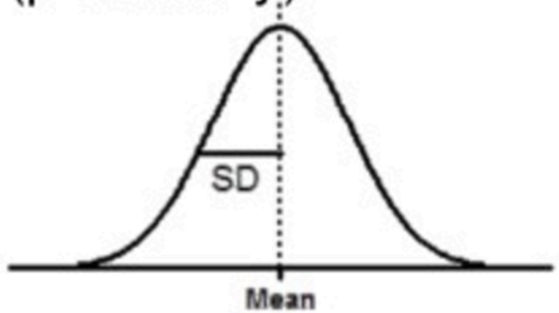
Soft Assignment

- **K-means algorithm** :
  - every data point is assigned uniquely to one, and only one, of the clusters.
  - Some data points that lie roughly midway between cluster centers.
- **Solution**: adopting a probabilistic approach, we obtain '**soft**' assignments of data points to clusters in a way that reflects the level of uncertainty over the most appropriate assignment.

Gaussian Mixture Model GMM

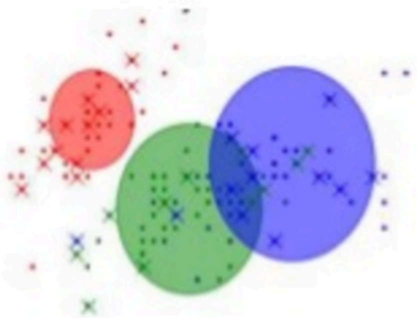
- Gaussian

"Gaussian is a characteristic symmetric "bell curve" shape that quickly falls off towards 0 (practically)"



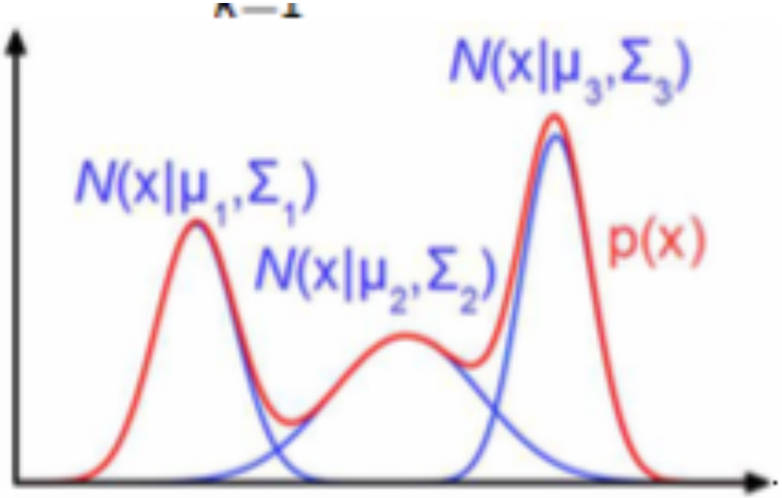
- Mixture Model

"mixture model is a probabilistic model which assumes the underlying data to belong to a mixture distribution"



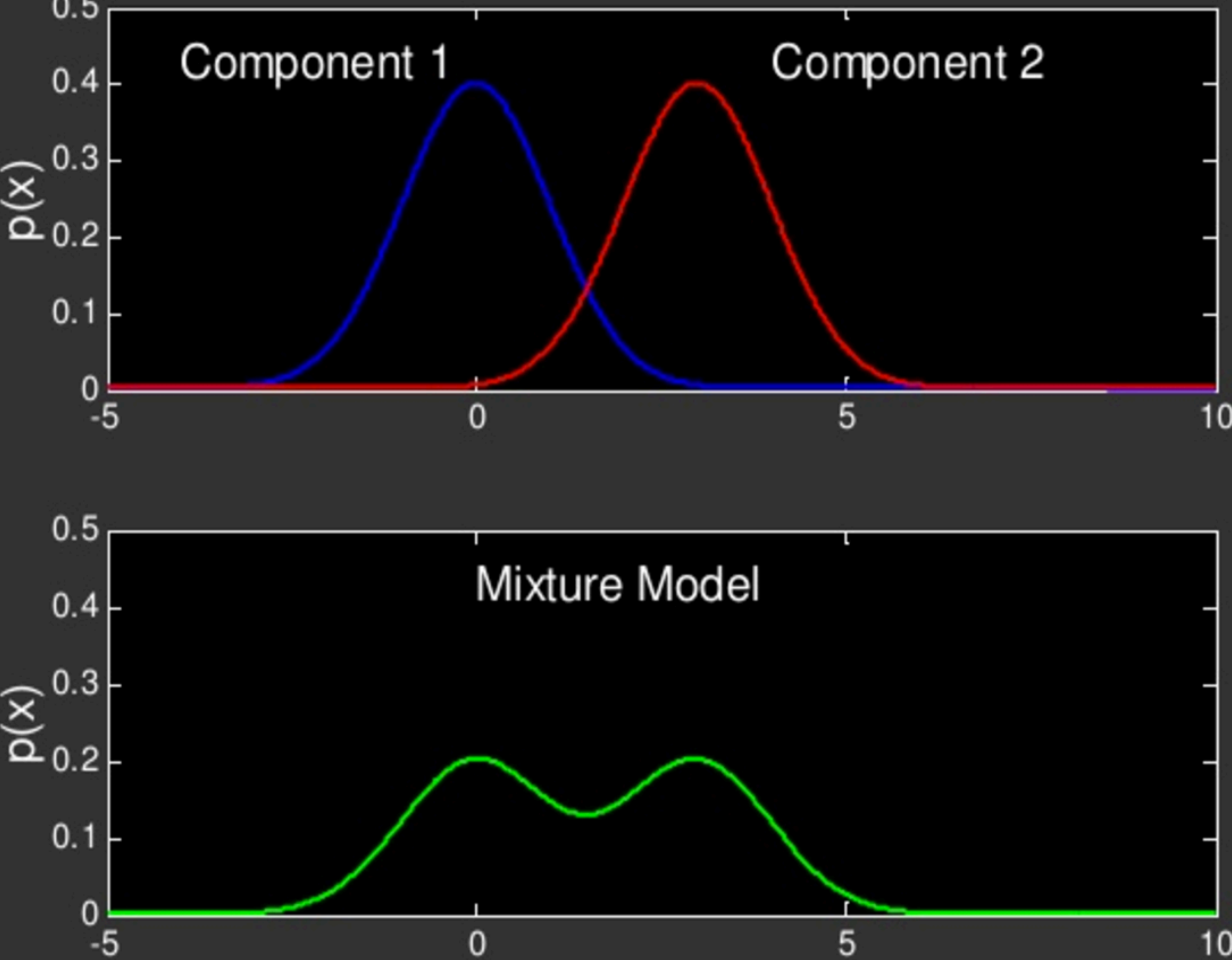
Gaussian Mixture Model GMM

- **Gaussian (normal) distribution**: model for the distribution of continuous variables
- **Mixture Distribution** : probability distribution of a random variable that is derived from a collection of other random variable.
- A **probability density p**(x) represents a mixture distribution or mixture model



Gaussian Mixture Model GMM

- A Gaussian mixture model is a **probabilistic model** that assumes all the data points are generated from a **mixture** of a finite number of Gaussian distributions with unknown parameters.



Component 1
Component 2
p(x)
0.5
0.4
0.3
0.2
0.1
0
-5
0
5
10
Mixture Model
p(x)
0.5
0.4
0.3
0.2
0.1
0
-5
0
5
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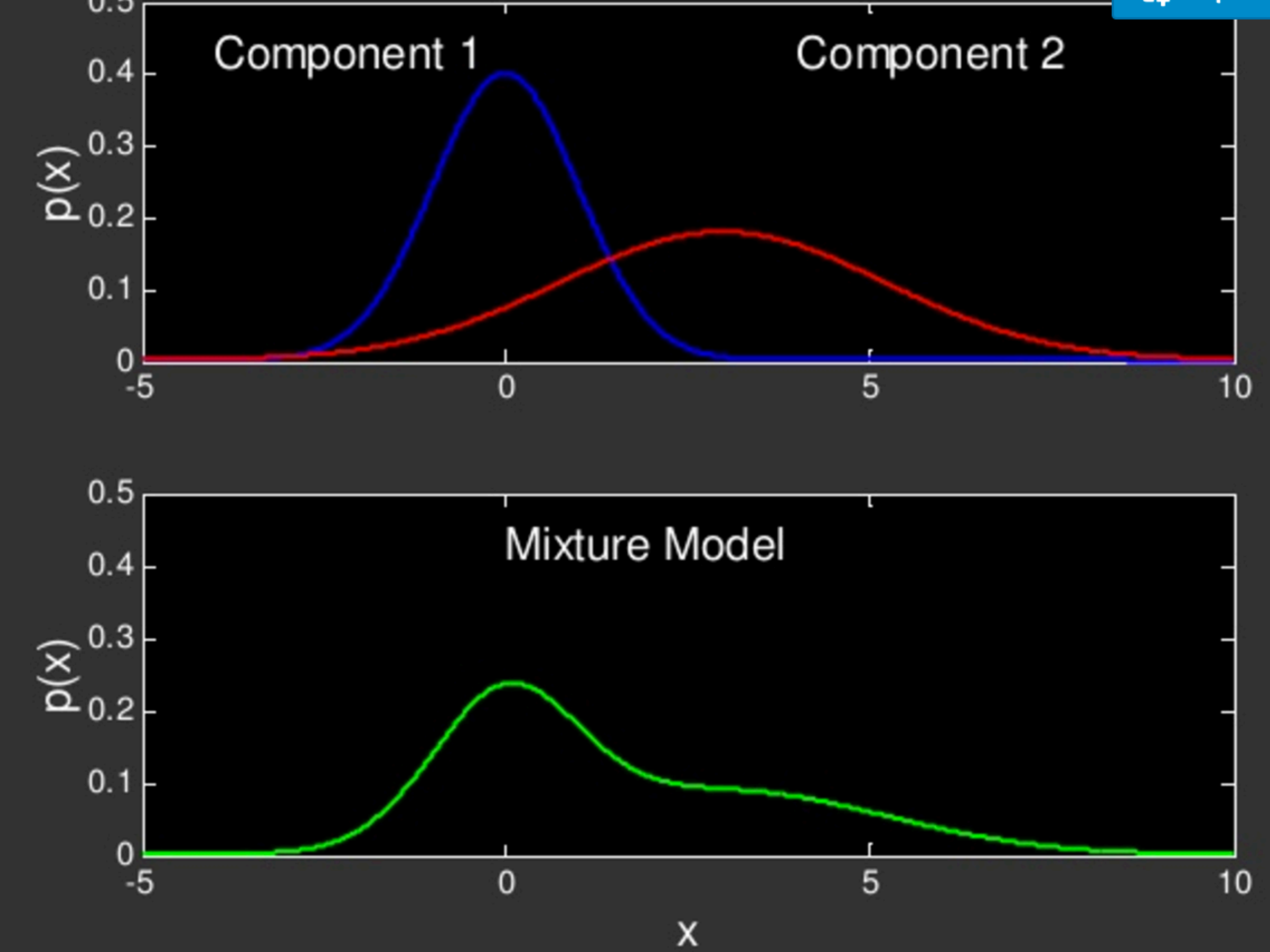

Component 1
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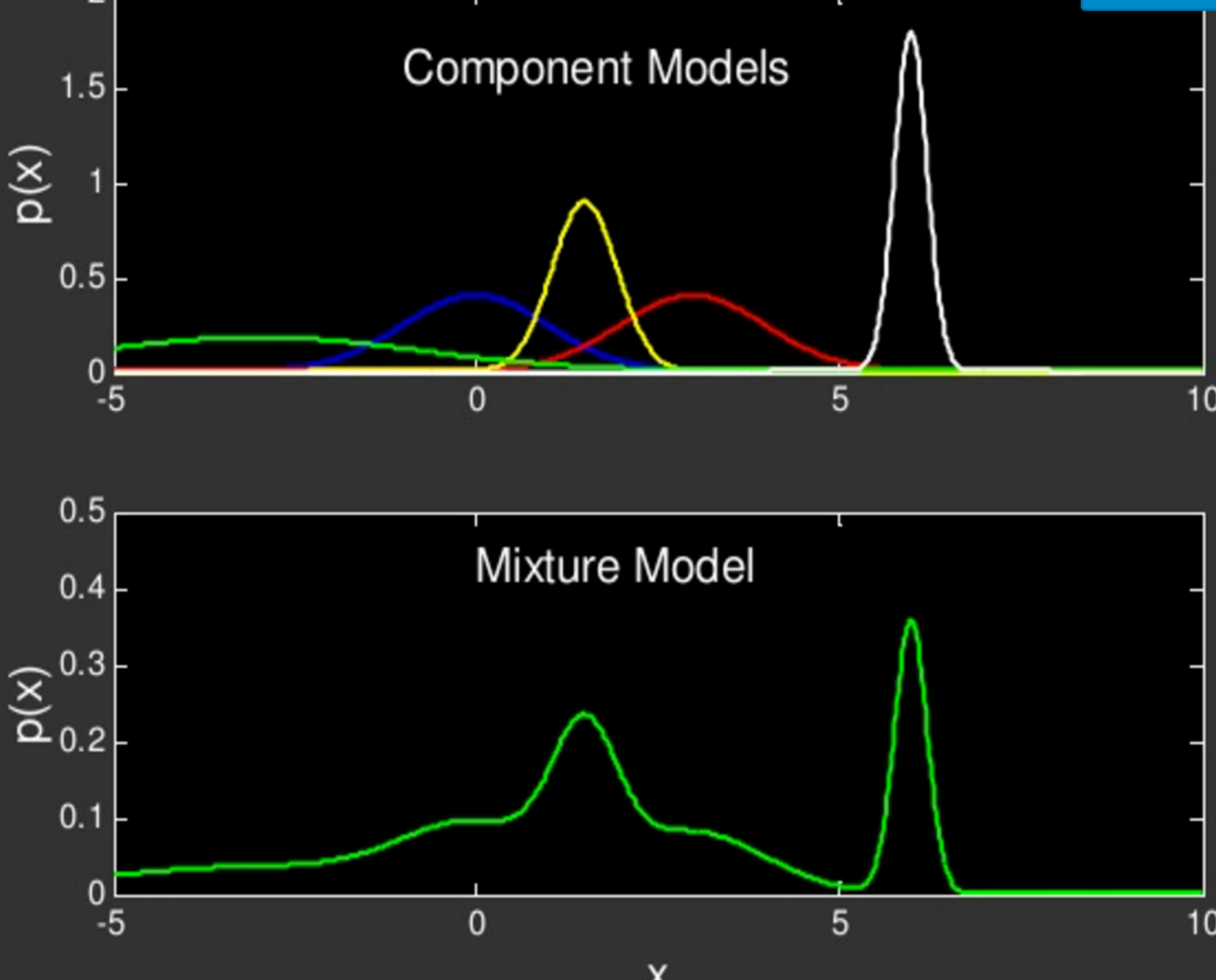
x


Component Models
p(x)
1.5
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-5
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10
Mixture Model
0.5
0.4
0.3
0.2
0.1
0
-5
0
5
10
x

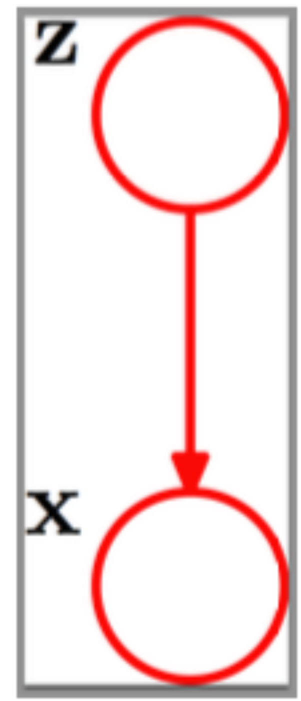
Gaussian Mixture Model: Joint Distribution

- Factorize joint according to Bayes net:

$$p(x,z) = p(z)p(x \mid z) = \pi_z N(x \mid \mu_z, \Sigma_z)$$

- π_z is probability of choosing cluster z.

- $X \mid Z = z$ has distribution $N(\mu_z, \Sigma_z)$.



Learning the Gaussian Mixture Model - likelihood function

- we only observe X, we need the **marginal distribution**:

$$
\boxed{
\begin{aligned}
p(x) &= \sum_{z=1}^{k} p(x, z) \\
&= \sum_{z=1}^{k} \pi_z \mathcal{N}(x \mid \mu_z, \Sigma_z)
\end{aligned}
}
$$

Learning the Gaussian Mixture Model - likelihood function

- The model **likelihood** for $D = \{x_1, \ldots, x_n\}$ is

$$
\begin{aligned}
L(\pi, \mu, \Sigma) &= \prod_{i=1}^{n} p(x_i) \\
&= \prod_{i=1}^{n} \sum_{z=1}^{k} \pi_z \mathcal{N}(x_i \mid \mu_z, \Sigma_z).
\end{aligned}
$$

- As usual, we'll take **objective function** to be the log of this:

$$
J(\pi, \mu, \Sigma) = \sum_{i=1}^{n} \log \left\{ \sum_{z=1}^{k} \pi_z \mathcal{N}(x_i \mid \mu_z, \Sigma_z) \right\}
$$

Gaussian Mixture Model: Conditional Distribution

- We observe X = x, the **conditional distribution** of the cluster Z given X = x is

$$\mathbf{p(z \mid X = x) = p(x,z)/p(x)}$$

- The conditional distribution is a **soft assignm**ent to clusters.
- $z \in \{1,\ldots,k\}$, Cluster assignment Z is called a **hidden variable**

Learning the Gaussian Mixture Model

- Build GMM

  1. Cluster probabilities $\pi = (\pi_1,\ldots,\pi_k)$
  2. Cluster means $\mu = (\mu_1,\ldots,\mu_k)$
  3. Cluster covariance matrices: $\Sigma = (\Sigma_1,\ldots\Sigma_k)$

- We have a probability model: let's find the MLE.

- Suppose we have data $D = \{x_1,\ldots,x_n\}$.
  We need the **model likelihood** for D.

Learning the Gaussian Mixture Model

- Denote the probability that observed value x_i comes from cluster j by $\gamma_{ji} = P(Z = j \mid X = x_i)$.
- The responsibility that cluster j takes for observation xi . Computationally,
  - $\gamma_{ji} = P(Z = j \mid X = x_i)$
  - $\gamma_{ji} = p(Z=j, X = x_i)/p(x) = \pi_j N(xi \mid \mu j, \Sigma j) /p(x)$
- The vector $\gamma_1,\ldots,\gamma_k$ is exactly the soft assignment for x_i.

Exception Maximization Learning

1. Initialize parameters μ, π, Σ.

2. "E step". Evaluate the responsibilities using current parameters

$$\gamma_i^j = \frac{\pi_j \mathcal{N}(x_i \mid \mu_j, \Sigma_j)}{\sum_{c=1}^{k} \pi_c \mathcal{N}(x_i \mid \mu_c, \Sigma_c)},$$

   - for i = 1,…,n and j = 1,…,k.

3. "M step". Re-estimate the parameters using responsibilities

$$\mu_c^{\text{new}} = \frac{1}{n_c} \sum_{i=1}^{n} \gamma_i^c x_i$$

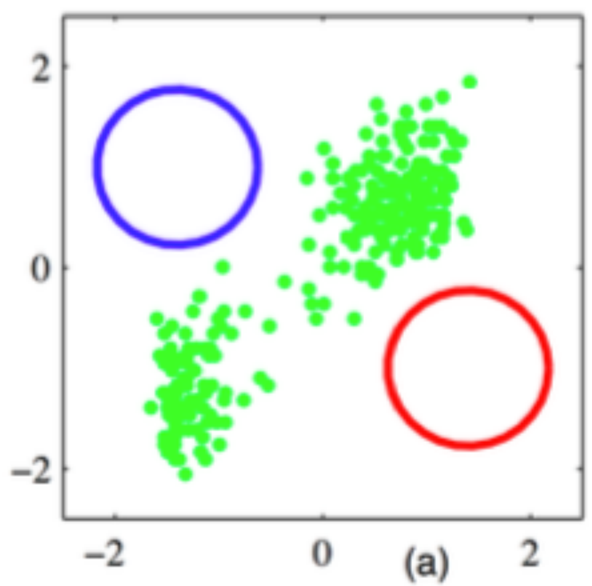
$$\Sigma_c^{\text{new}} = \frac{1}{n_c} \sum_{i=1}^{n} \gamma_i^c (x_i - \mu_{\text{MLE}})(x_i - \mu_{\text{MLE}})^T$$

$$\pi_c^{\text{new}} = \frac{n_c}{n},$$

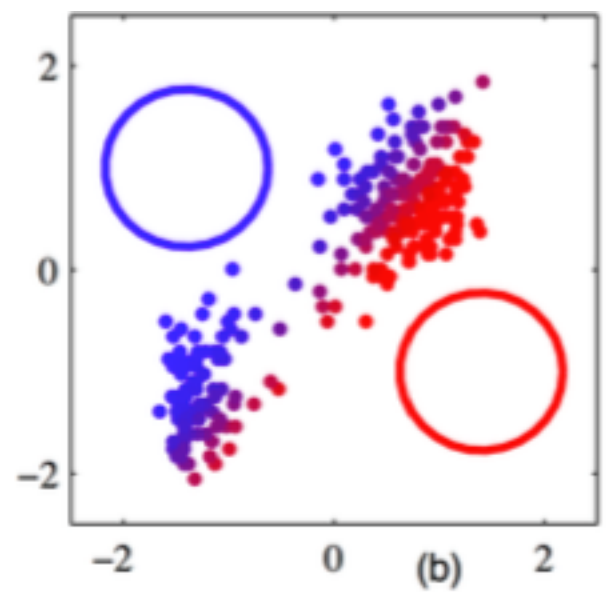
4. Repeat from Step 2, until log-likelihood converges.

EM Example

1. Initialization

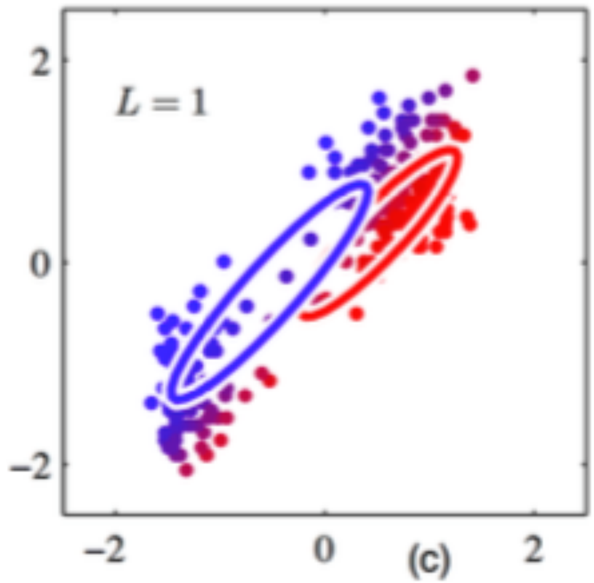


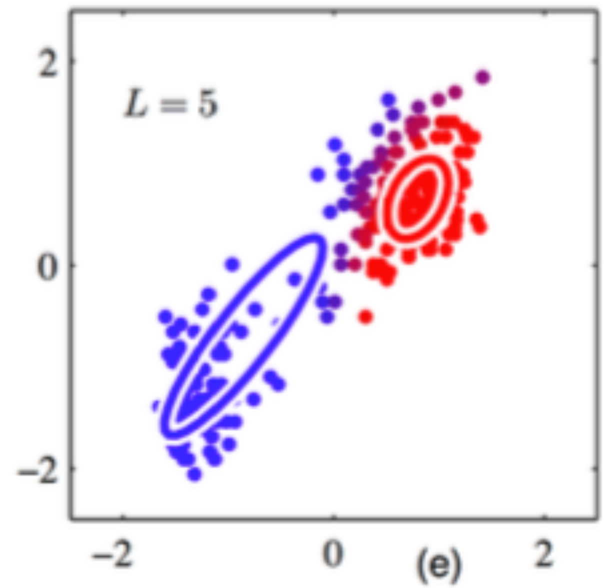
2. First soft assignment



EM Example

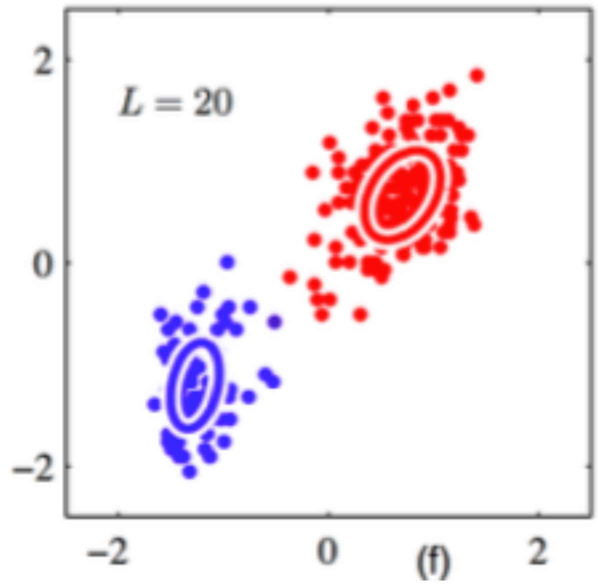
3. Updating Parameters:
4. Repeat Assignment





EM Example

5. After 20 rounds:



Advantages/Dis of GMM

Strength

- ✓ GMM model accommodates mixed membership
- ✓ Flexible in terms of cluster covariance

Weakness:

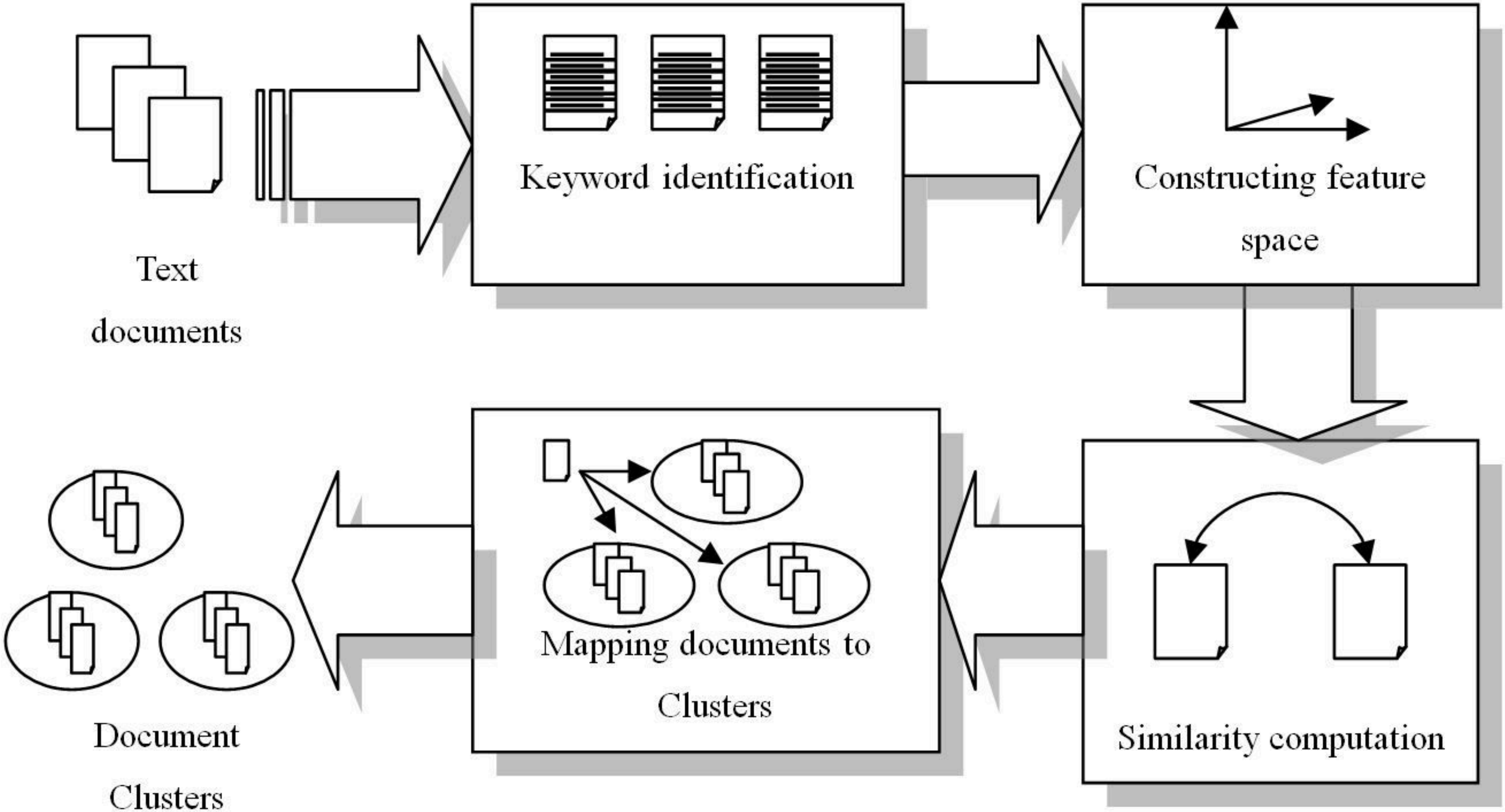
- x Computationally expensive if the number of distribution is large, or the data set contains very few observed data points
- x Hard to estimate number of clusters

GMM Demo

https://lovasoa.github.io/expectation-maximization/dist/

+ Clustering– NLP example

- Document clustering
  - The similarity measure is the key point to the clustering problem.
  - represent each document as an array of numbers Using VSM
    - Count up the number of times each word appears in the document.
    - Choose a set of "feature" words that will be included in your vector. This should exclude extremely common words ("stopwords") like "the", "a", etc.
    - Make a vector for each document based on the counts (. TF X IDF) of the feature words.
  - Apply k-means , EM to cluster documents by maximizing the likelihood of the unlabeled documents


Text documents
Keyword identification
Constructing feature space
Similarity computation
Mapping documents to Clusters
Document Clusters

+

Clusetring related documents

[PDF] A survey on various approaches in **document clustering**
…, V Preamsudha, MP **Scholar** - International …, 2011 - pdfs.semanticscholar.org
Abstract Document clustering is the process of segmenting a particular collection of texts into subgroups including content based similar ones. The purpose of document clustering is to meet human interests in information searching and understanding. Nowadays all paper
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A new unsupervised method for **document clustering** by using WordNet lexical and conceptual relations
DR Recupero - Information Retrieval, 2007 - Springer
… improve the preprocessing since it is the most critical step for the generation of an appropriate **document** representation. The other one is how the use of WordNet benefits the cluster labeling task: having **documents** represented by concepts … Frequent term-based text **clustering**. …
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Ant-based and swarm-based **clustering**
J Handl, B Meyer - Swarm Intelligence, 2007 - Springer
… Homogeneous ants for web **document** similarity modeling and categorization. … A stochastic heuristic for visualising graph **clusters** in a bi-dimensional space prior to partitioning. Journal of Heuristics, 5(3), 327–351. … Antclust: ant **clustering** and web usage mining. …
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+ NLP example

- D1: there is a dog who chased a cat
- D2: someone ate pizza for lunch
- D3: the dog and a cat walk down the street toward another dog
- **feature words** are [dog, cat, street, pizza, lunch]
- **[1, 1, 0, 0, 0]** // dog 1 time, cat 1 time
- [0, 0, 0, 1, 1] // pizza 1 time, lunch 1 time
- **[2, 1, 1, 0, 0]** // dog 2 times, cat 1 time, street 1 time
- **Apply Clustering Algorithm Cosine similarity between vectors**

Document clustering using K-means

- Assuming we have data with no labels for **news** and **technical** data
- We want to be able to categorize a new document into **one** of the 2 classes (K=2)
- We can extract represent document as feature vectors
  - Features can be word id or other NLP features such as POS tags, word context etc (D=total dimension of Feature vectors)
  - N documents are available
- Randomly initialize 2 class means
- Compute square distance of each point (x_n)(D) to class means (μ_k)
- Assign the point to K for which μ_k is lowest
- Re-compute μ_k and re-iterate until converge

References

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