

# Task 1: Introduction to Probability Theory

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## 1 PART 1. COMBINATORICS

1. In the consonantal inventory of a language the following sounds can form a word: /p/, /a/, /t/, /i/, /d/, /e/ so that
  - (a) /p/ and /a/ are adjacent?
  - (b) /p/ precedes /a/?
  - (c) /p/ precedes /a/ and /a/ precedes /t/?
  - (d) /p/ is before /a/ and /t/ is before /i/?
  - (e) /p/ and /a/ are adjacent and /t/ and /i/ are also adjacent?
  - (f) /d/ can never be in word-initial position?
2. The president of a country has to choose an *Education minister*, an *Energy minister*, and an *Environment minister* from the 20 members of the cabinet. How many different selections of ministers are possible if:
  - (a) the president can appoint any person without restrictions?
  - (b) E must be an minister?
  - (c) F will serve only if he is the minister of Education?
  - (d) A and B can not serve together?
  - (e) C and D can serve together or not at all?
3. In the “Old Tongue” language of Westeros, 3 verbs, 4 nouns, and 3 adjectives can be found in a row, how many arrangements are possible when words of the same part of speech must be found next to each other?
4. In how many ways a person can donate 9 books among 5 people—two children and two adults—if the two children are to receive 3 books and the adults 2 each?
5. How many different 7-letter words are possible when 3 of the sounds are vowels and 4 are consonants? Assume that repetition of vowels and consonants is possible and that there are no phonotactic constraints on where the vowels or consonants can be placed.

6. In an exam students have to answer 6 out of 8 questions.
  - (a) How many options do they have?
  - (b) How many options do they have, if they must answer at least 3 of the first 5 questions?

## 2 PART 2. Probabilities

1. A travel website offers flight selections for a trip using a natural language component. The trip consists of three stops in Europe:
  - (a) **First stop:** Rome - Paris
  - (b) **Second stop:** Milan - Barcelona - Athens
  - (c) **Third stop:** Amsterdam - Palermo - London - Berlin

A user is to make one selection for each stop:

- (a) List all the selections in the event *First Stop + Second Stop*.
- (b) If *Event A* is the event that Amsterdam is selected; how many outcomes are in *Event A* ?
- (c) If *Event B* is the event that Rome is selected; how many outcomes are in *Event B*?
- (d) If the *Third Stop* is the event that London is selected; how many selections are in *Third Stop*?
- (e) List all the outcomes in the event *First Stop - Second Stop - Third Stop*.
- (f) How many overall outcomes are in the sample space?

## 3 PART 3. Conditional Probability

1. A child is going to select two sweets for a dessert. Suppose that the probability that she will like the ice-cream is .7, the probability that she will like the chocolate is .4, and the probability that she will like both is .4. Calculate the conditional probability that she will like the chocolate given that she did not like the ice-cream.
2. A urn contains 30 green and 20 black balls, we select randomly balls from the urn. What is the probability that we remove the black balls before the green ones?
3. Another urn contains 30 green, 20 black balls, and 9 blue balls.
  - (a) What is the probability that we remove the black balls before the green ones?

- (b) What is the probability that we remove the blue balls first, then the black balls, and then the blue ones (in this order)?
- (c) What is the probability that we remove the green balls first?