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ARMA Models for Earthquake Ground Motions

Seismic Safety Margins Research Program

Mark K. Chang, Jan W. Kwiatkowski, Robert F. Nau, Robert M. Oliver, Karl S. Pister
University of California, Berkeley, California

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Prepared by
Mark K. Chang, Jan W. Kwiatkowski, Robert F. Nau, Robert M. Oliver, Karl S. Pister
University of California, Berkeley, California

Lawrence Livermore Laboratory
7000 East Avenue
Livermore, CA 94550

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ABSTRACT

This report contains an analysis of four major California earthquake records using a class of discrete linear time-domain processes commonly referred to as ARMA (Autoregressive/Moving-Average) models. It has been possible to analyze these different earthquakes, identify the order of the appropriate ARMA model(s), estimate parameters and test the residuals generated by these models. It has also been possible to show the connections, similarities and differences between the traditional continuous models (with parameter estimates based on spectral analyses) and the discrete models with parameters estimated by various maximum likelihood techniques applied to digitized acceleration data in the time domain. The methodology proposed in this report is suitable for simulating earthquake ground motions in the time domain and appears to be easily adapted to serve as inputs for nonlinear discrete time models of structural motions.

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CHAPTER 1

INTRODUCTION AND SUMMARY OF FINDINGS

1.1 Introduction

This report contains an analysis of four major California earthquake records using a class of discrete linear time-domain processes commonly referred to as ARMA* models. These models enjoy an extensive literature and have been used in a large number of applications in diverse fields of engineering and statistics. Since there is only a small number of reported studies of these models in connection with earthquake ground motion, it seemed appropriate to analyze several different earthquakes, identify the order of the appropriate ARMA model(s), estimate parameters and test the residuals generated by these models.

The stated purpose of the research project was threefold:

1. To study and report on mathematical models currently being used to analyze earthquake ground motions. In particular, we were asked to make an effort to classify the major similarities and differences of the more popular and useful models.
2. To attempt formal identifications of appropriate ARIMA models which could be used to analyze and simulate earthquake ground motions. The major questions to be addressed were identification of the model and estimation of the parameters of the model with particular emphasis on analysis of the residuals generated by the models.

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*Autoregressive/Moving-Average. These are a special case of the more general ARIMA (Autoregressive/Integrated-Moving-Average) class of discrete models in which differencing of the data is used to achieve stationarity.

3. To show the connections, similarities and differences between the traditional continuous models (with parameter estimates based on spectral analyses) and the discrete models with parameters estimated by various maximum likelihood techniques applied to digitized acceleration data in the time domain.

One of the most critical decisions that has to be made in the design of structures to sustain seismic loading or in the analysis of existing structures to ascertain their probability of survival, is that of specifying the ground motion to which the structure may be exposed. Uncertainties in earthquake initiation and transmission, coupled with uncertainties that are site-specific, demand that the problem be examined nondeterministically. The sparseness of measured records applicable to a particular site normally precludes direct determination of acceptable design models from field data. An alternative approach is to develop a representative class of ground motions for "design" earthquakes by simulation and by classification of various key parameters in analytical models of ground motion. The basic idea is to select a suitable class of mathematical models of ground motion whose characteristics can be related to the physics of earthquakes, the transmission of ground waves and which can be improved and modified as more data and knowledge about earthquakes is acquired.

By examining field data, model parameters can be estimated and tested against actual data. The models can then be used to simulate ground motion and these, in turn, used as inputs for determining the resulting random response of structures exposed to such earthquakes. The methodology, if it is to be used in the design process, must have the properties that it:

1. Is capable of refinement as new data and experience is obtained.
2. Is capable of characterizing ground motion with a small number of parameters.
3. Is compatible with algorithms for simulating structural response of complex linear and nonlinear stochastic systems.

At the present time the majority of procedures proposed to simulate ground motion utilize spectral methods. When coupled with *linear* structural response models, such procedures are satisfactory for prediction of structural response. However, for nonlinear responses, as might occur with strong motion earthquakes, spectral methods can not be easily used, at least at present. The methodology and models proposed in this report are suitable for simulating earthquake ground motions in the time domain and not only appear to satisfy the criteria stated above but also appear to be easily adapted to serve as inputs for nonlinear discrete time structural models. It should be reemphasized that no amount of refinement and sophistication in the structural design process can reduce the need for models and simulations to reproduce various levels of ground motion.

1.2 Highlights and Summary of Findings

Four earthquake records were studied during the course of the research. These included the El Centro 1940 and 1956 earthquakes recorded at the same station, and the 1971 San Fernando earthquake, recorded at two different stations.

1. It was found that digitized records for all four earthquakes could be well-fitted by ARMA models of relatively low order during the entire history of each earthquake. That is to say, model identification remained constant during the buildup and decay periods of each earthquake even though parameter estimates changed somewhat from beginning to end.
2. One earthquake record (El Centro 1940) was satisfactorily fitted to a second-order autoregressive/first-order-moving-average model similar to many second-order damped linear oscillator models in the literature in which the forcing function is white noise. However all of the other earthquake records required a fourth-order-autoregressive/first-order-moving-average model (ARMA (4,1)) to pass conventional goodness-of-fit tests. This result suggests that a fourth-order differential equation is required to describe the underlying continuous process. In some portions of each record we find that the fourth-order model can be interpreted as a pair of linear oscillators in series where the correlated noise output of the first filter is used as input for the second. We find that the natural frequency of the first filter is in the range 2 to 7 hertz and the natural frequency of the second

filter is in the range 10 to 17 hertz with a strongly (low-pass) filtered white noise input. We have utilized the fact that a 4th-order ARMA process with at least one complex conjugate pair of characteristic roots can be uniquely factored into two second order processes, and a second-order process with complex or positive real roots can in turn be expressed in terms of the damping coefficient and natural frequency of a 2nd order linear oscillator. Thus it is possible to estimate the parameters of the ARMA models using maximum likelihood techniques in combination with earthquake records and then from these directly calculate estimates of the natural frequency and damping coefficient of a possible underlying physical process. In such cases there is no need to estimate the spectral power density function or the power spectrum.

3. The principal source of nonstationarity for these earthquakes appears to be in the time-dependent variance (envelope) of the white noise process. There is some evidence that the coefficients of the ARMA process also change with time but from the results obtained in (2) above we find that the resonant frequencies and damping terms appear to remain relatively constant over time.

4. The ARMA models and parameter estimates obtained directly from the data can be translated into efficient simulation models. Moreover, a simple augmentation of these models involving the use of differenced white noise input in conjunction with an additional first-order filter term, can be used to ensure that the simulated accelerograms when integrated will yield mean-square

velocities tending to zero, resulting in a simulation model which is representative of real earthquakes over an extremely wide frequency range but which includes a desirable base-line correction.

1.3 Contents of the Report

In addition to this introductory section there are three chapters and two appendices. Chapter 2 is a brief summary of the ARMA (2,1) and ARMA (4,1) models which have been identified in Chapter 3 of the report. We point out that these models have only been used to identify successfully a limited number of earthquakes; one should not therefore assume that they will be equally successful in other applications. However, as we point out in Chapter 2 there are some direct relationships between the natural frequency and damping terms of the second-order differential equation model and the autoregressive parameters of the ARMA (2,1) and ARMA (4,1) models. In the second chapter we also show how the statistical properties of the correlated outputs of these models are determined by their parameters.

In Chapter 3 there is a detailed summary of our findings for the El Centro (1940 and 1956) and San Fernando (1971) earthquakes. We also show the results of estimating the time-varying coefficients of the time series data for accelerograms.

In Appendix A we have a comparison and survey of the more popular models used to describe earthquake ground motions. As we mentioned earlier most of these models are developed in continuous time with emphasis on spectral characteristics. We find that the majority of models can be classified in three or four major categories with slight variations in assumptions about the nature of the random forcing functions.

Appendix B contains a detailed bibliography in which the topics are classified according to whether they are works of theoretical interest (T), response of structures (R), data (D) or analyses of ground motion (G).

CHAPTER 2

ARMA MODELS FOR EARTHQUAKE GROUND MOTIONS

2.1 Introduction

Most existing models for the analysis and simulation of earthquake ground motion records (see Appendix A) are formulated in continuous time, using linear differential equations with inhomogeneous forcing functions given by white noise which, in certain cases, has been assumed to be filtered. Typically, the order of the linear differential equations and the degree of correlation in the noisy forcing function is specified for theoretical or practical reasons. The coefficients of these differential equations are expressed in terms of the natural frequencies and damping constants of second-order harmonic oscillators where appropriate values for these parameters are usually obtained by matching certain predominant spectral characteristics of real earthquake records with those obtained from the differential equation models. Simulated accelerograms are then generated digitally by numerical integration of the differential equation or impulse response function (with white noise input), or else by using the theoretical Fourier amplitude spectrum (based on the transfer function) to weight a superposition of a large number of sinusoids at equispaced frequencies with randomly generated phase angles. The white noise input or filtered noise output is generally multiplied by an appropriate envelope function to incorporate non-stationary characteristics (i.e., buildup and decay).

In view of the current availability of large quantities of uniformly digitized earthquake acceleration data for analysis, and the widespread interest in generating artificial digitized accelerograms for structural response studies, it appeared worthwhile to consider the use of models formulated explicitly in discrete time. An important class of discrete models are the autoregressive/moving-average (ARMA) models, which can be represented as stochastic linear difference equations of finite order. The ARMA models are of equal generality with linear continuous-time models, (differential equations), but they have a number of significant advantages for purposes of digital analysis and simulation. A large body of literature, exemplified by the work of Box and Jenkins (T[2]), gives systematic procedures for identifying the order of the ARMA model which best describes a particular time series (such as a digitized accelerogram) based on time-domain analysis of the actual data (i.e., without *a priori* assumptions). Moreover, maximum-likelihood techniques are available for estimating optimal parameter values directly from the data, with specifiable confidence intervals for the estimates. The sequence of *residuals*--i.e., deviations from the fitted model, or "one-step-ahead forecast errors"--provides a basis for quantitative statistical tests of goodness of fit, and represents a direct estimate of the underlying noise sequence driving the observed process. These time-domain analytic techniques are somewhat less sensitive than frequency-domain techniques (e.g., spectral analysis) to certain violations of stationarity assumptions and to the effects of digitizing a continuous record. ARMA models can be used directly for discrete simulation by simple iteration of the difference equations, with appropriate discrete noise input, thus simplifying the procedure of obtaining artificial accelerograms with characteristics similar to specified real accelerograms.

In this study the ARMA model-identification and parameter-estimation techniques of Box and Jenkins were applied to a number of California earthquake records, and the results (presented in the next chapter) suggest that two particular ARMA models are worth discussing in some detail. These are the second-order-autoregressive/first-order-moving-average (ARMA (2,1)) model and the fourth-order-autoregressive/first-order-moving-average (ARMA (4,1)) model, which may be considered to correspond to continuous-time models described, respectively, by second- and fourth-order differential equations.

2.2 The ARMA (2,1) Model

The ARMA (2,1) model for a stationary correlated process a_t is defined by the 2nd-order-autoregressive/1st-order moving-average difference equation:

$$a_t - \phi_1 a_{t-1} - \phi_2 a_{t-2} = e_t - \theta_1 e_{t-1} , \tag{2.1}$$

in which it is assumed that $e_t \sim N\left(0,\sigma_e^2\right)$ is independently and identically distributed. That is to say, the input is stationary discrete white noise. In terms of the backward shift operator B $\left(\text{defined by } B^k x_t = x_{t-k}\right)$ (2.1) can be rewritten:

$$\left(1 - \phi_1 B - \phi_2 B^2\right) a_t = (1 - \theta_1 B) e_t , \tag{2.2}$$

or equivalently in the factored form:

$$(1 - r_1 B)(1 - r_2 B) a_t = (1 - \theta_1 B) e_t , \tag{2.3}$$

where r_1 and r_2 are the solutions of the characteristic equation,

$$r^2 - \phi_1 r - \phi_2 = 0 . \tag{2.4}$$

A requirement for stationarity (stability) of the process a_t is that the autoregressive roots r_1 and r_2 lie within the unit circle, or equivalently, that $|\phi_2| < 1$, $\phi_1 + \phi_2 < 1$ and $\phi_2 - \phi_1 < 1$.

The autocorrelation function of the process a_t is symmetric in lag k so that

$$\rho_k = \text{corr } [a_t, a_{t+k}]$$

(2.5a)

$$= \sigma_a^{-2} \text{ cov } [a_t, a_{t+k}] \qquad k = 1,2, \ldots$$

where the variance of the output process a_t , is given by

(2.5b)
$$\sigma_a^2 = \text{var } [a_t] = \frac{1 - \phi_2}{1 + \phi_2} \cdot \frac{\left(1 + \theta_1^2\right)\sigma_e^2}{(1 - \phi_2)^2 - \phi_1^2} ,$$

is proportional to the variance σ_e^2 of the random forcing function e_t . It is well known that for $k \geq 2$ the autocorrelation function ρ_k must satisfy the homogeneous difference equation of (2.1) or,

(2.6)
$$\rho_k - \phi_1\rho_{k-1} - \phi_2\rho_{k-1} = 0 \qquad k \geq 2$$

with initial values given by

$$\rho_0 = 1$$

(2.7)
$$\rho_1 = \frac{\phi_1\left(1 + \theta_1^2 - \theta_1\phi_1\right) - \theta_1\left(1 - \phi_2^2\right)}{(1 - \phi_2)\left(1 + \theta_1^2 - \theta_1\phi_1\right) - \theta_1\phi_1(1 + \phi_2)} .$$

We note that ρ_1 depends on the moving average parameter θ_1 but that for $k \geq 2$ the difference equation in (2.6) does not explicitly include θ_1 . In the ARMA (2,1) model of (2.6) we are dealing with a system of 2nd-order linear difference equations whose solutions can be written as

(2.8a)
$$\rho_k = c_1 r_1^k + c_2 r_2^k \qquad r_1 , r_2 \text{ distinct}$$

where c_1 , c_2 are derived from the initial values in (2.7).

When $\phi_1^2 < -4\phi_2$ the characteristic roots of (2.4), r_1 and r_2, are complex conjugates. The autocorrelation function can then be written in the form

$$\rho_k = (-\phi_2)^{k/2} \frac{\cos(k\lambda_d - \mu_d)}{\cos(-\mu_d)} \qquad k \geq 0 \tag{2.8b}$$

where

$$\lambda_d = \cos^{-1}\left(\frac{\phi_1}{2\sqrt{-\phi_2}}\right) \tag{2.8c}$$

has the interpretation of a frequency and

$$\mu_d = \tan^{-1}\left(\frac{2\rho_1 - \phi_1}{\sqrt{-\phi_1^2 - 4\phi_2}}\right) \tag{2.8d}$$

has the interpretation of a phase angle. Since the autocorrelation of lag 1 depends on both autoregressive *and* moving average parameters in (2.7), it follows that only the phase μ_d depends on the moving average parameter. One of the useful results of this report derives from the expressions in (2.8b), (2.8c), (2.8d) and the close analogies that the discrete model frequency and phase have with their continuous differential equation counterparts. Up to this point the discrete model does not include an explicit time dimension for the lags $k = \pm 1, \pm 2, \ldots$, etc. If we let $\tau = k\Delta t$, i.e., each lag is separated by a time interval of length Δt, then (2.8b) can be rewritten in the form

$$\rho(\tau) = e^{\log(-\phi_2)\frac{\tau}{2\Delta t}} \frac{\cos\left(\frac{\lambda_d}{\Delta t}\tau - \mu_d\right)}{\cos\,(-\mu_d)}$$

(2.9)

$$= e^{-\xi\omega_o\tau} \frac{\cos\left(\left(\omega_0\sqrt{1-\xi^2}\right)\tau - \mu_d\right)}{\cos\,(-\mu_d)} \quad \text{for} \quad \tau = \Delta t, 2\Delta t, \ldots$$

where ω_0 has the interpretation of a natural frequency, and ξ that of a damping ratio. $\frac{\lambda_d}{\Delta t} = \omega_0\sqrt{1-\xi^2}$ may be thought of as the resonant frequency with ω_0 given by

$$\omega_0 = \frac{1}{\Delta t}\sqrt{\frac{1}{4}\,(\log\,(-\phi_2))^2 + \lambda_d^2} \tag{2.10a}$$

and ξ by

$$\xi = \frac{-\log\,(-\phi_2)}{2\sqrt{\frac{1}{4}\,(\log\,(-\phi_2))^2 + \lambda_d^2}}\,. \tag{2.10b}$$

We again note that only μ_d, the phase, depends on the moving average parameter, θ_1; the natural frequency ω_0 and the damping constant ξ depend only on the autoregressive coefficients. It is clear from Equations (2.8c), (2.8d), (2.10a), (2.10b) that one can obtain the frequency ω_0, damping constant ξ, and phase μ_d directly from estimates of the autoregressive and moving average parameters ϕ_1, ϕ_2, θ_1 of the discrete models in (2.1) and (2.6) without any need to estimate the spectrum of the underlying process. These ideas will be further clarified in the next section where we derive the exact mathematical relationships between the autocorrelation function of a discrete sampled process a_t and the continuous autocorrelation function of an underlying continuous process.

To give the reader some idea of the magnitudes that are involved we refer to the El Centro (1940) earthquake which is identified as an ARMA (2,1) model in Chapter 3. Table 3.1 in that chapter shows that in the first five seconds of the (IIA001 S90W) accelerogram records the estimates $\hat{\phi}_1$, $\hat{\phi}_2$ and $\hat{\theta}_1$ are 1.55, -.66, 0.33 respectively. Equations (2.8c), (2.8d) and (2.10a), (2.10b) yield $\omega_0 = 17.88$ (radians/sec.), $\xi = 0.59$ and a damped frequency $\omega_0\sqrt{1 - \xi^2} = 14.44$ (radians/sec.). In the interval between 20 and 25 seconds the parameter estimates are $\hat{\phi}_1 = 1.61$, $\hat{\phi}_2 = -.78$ and $\hat{\theta}_1 = -.01$ with $\omega_0 = 22.42$, $\xi = .27$ and $\omega_0\sqrt{1 - \xi^2} = 20.79$.

2.3 Relations Between Discrete and Continuous 2nd-Order Models

If a time series a_t representing a segment of a discretized acceleration record is identified as an ARMA (2,1) process with complex or positive real autoregressive roots, then the underlying continuous acceleration process $a(t)$ may be considered to be described by a system of 2nd-order differential equations of the form:

$$\text{(2.11a)} \qquad a(t) = \ddot{z}(t)$$

$$\text{(2.11b)} \qquad \ddot{z}(t) + 2\xi\omega_0\dot{z}(t) + \omega_0^2 z(t) = c_0\omega_0^2 x(t) + 2c_1\xi\omega_0\dot{x}(t)$$

$$\text{(2.11c)} \qquad \ddot{x}(t) = I(t)$$

where $I(t)$ is a function with constant spectral density--i.e., continuous white noise. Equation (2.11b) describes the following physical system:

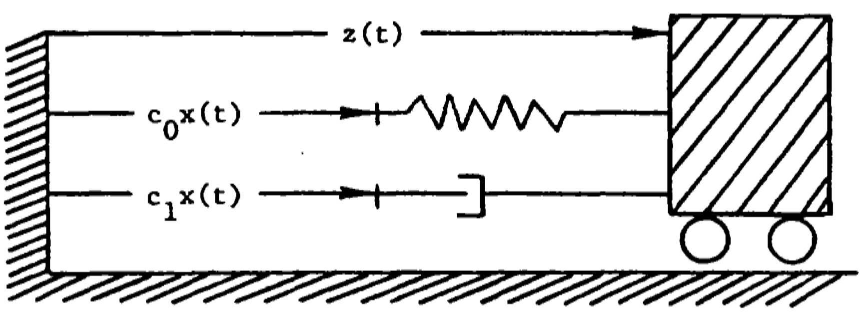

FIGURE 2.1

A GENERAL GROUND MOTION MODEL

in which $z(t)$ is the displacement of an object from its frame of reference. This model describes a one-degree-of-freedom linear oscillator with natural frequency ω_0 and damping factor ξ, with input displacement $x(t)$ applied separately to the spring and the dashpot in proportions c_0 and c_1 respectively.

Many earthquake acceleration models in the literature have been based on 2nd-order linear filters corresponding to the system of Equations (2.11a), (2.11b), (2.11c). The following three special cases have received particular attention:

(i) $c_0 = 1$, $c_1 = 0$ (spring forced, dashpot fixed)

(ii) $c_0 = 0$, $c_1 = 1$ (spring fixed, dashpot forced),

(iii) $c_0 = c_1 = 1$ (spring and dashpot forced equally).

For example, cases (i) and (ii) correspond to filters used by Shinozuka and Sato (G[16]) and Levi, Kozin and Moorman (G[7]). Case (iii) corresponds to the filter suggested by Tajima (R[16]) and used by Housner and Jennings (G[17]) and Ruiz and Penzien (R[12]), among others. In some studies a basic model representing one of the above cases has been refined by additional filtering, to improve the correspondence between the spectra of simulated and real accelerograms. For example, Levi, Kozin and Moorman use an additional 1st-order filter to attenuate higher frequencies in the input to the case (ii) model. Housner, Jennings and Tsai (G[15]) and Murakami and Penzien (R[5]) use an additional 2nd-order filter (of a different form) to attenuate very low frequencies in the output of the case (iii) model.

There is a well-known correspondence between the statistical characteristics of the discrete (sampled) process a_t and the underlying continuous process $a(t)$. In particular, the discrete autocorrelation function (acf) of a_t , defined as $\rho_k = \operatorname{corr}[a_t, a_{t+k}]$, and the continuous acf of $a(t)$, defined as $\rho(\tau) = \operatorname{corr}[a(t), a(t+\tau)]$, are equal at all points where both are defined, i.e., at integral multiplies of the sampling interval. That is, (2.13) $$\rho_k = \rho(k\Delta t) \ , \ k = 0,1,2, \ldots$$

where Δt is the sampling interval, and $\rho_0 \equiv \rho(0) \equiv 1$. If $a(t)$ is low-pass-filtered prior to sampling to eliminate power at frequencies greater than half the sampling frequency $\left(\text{i.e., at all } \omega > \frac{\pi}{\Delta t}\right)$--or if $a(t)$ initially contains neglible power at these frequencies--then the power spectral density functions (psdf's) of a_t and $a(t)$, which are the Fourier transforms of the corresponding acf's, will approximately coincide for frequencies in the range $0 \leq |\omega| \leq \frac{\pi}{\Delta t}$.

Table 2.1 gives the expressions for the acf's, psdf's, and transfer functions for the 2nd-order discrete and continuous random processes a_t and $a(t)$. Note that the acf's are damped cosine waves, i.e., each is completely defined by three parameters: a resonant frequency, a phase, and a damping coefficient.* These three parameters are uniquely determined by the three independent parameters of the corresponding equations of motion: ϕ_1 , ϕ_2 , and θ_1 in the discrete case, and ω_0 , ξ , and c_0/c_1 in the continuous case. As indicated in (2.10a), (2.10b), the frequency and damping coefficient of the discrete model are completely determined by the two autoregressive parameters, ϕ_1 and ϕ_2 ; the frequency and damping coefficient of the continuous autocorrelation function are the same as the resonant (damped) frequency and damping coefficient of the continuous process. The phases of the acf's are affected by the moving-average parameter, θ_1 , in the discrete case, and by the spring-dashpot input ratio, c_0/c_1 , in the continuous case.

*In the overdamped case, i.e., $\xi > 1$, the resonant frequency is imaginary and the acf degenerates to a mixture of exponentials.

In general, a continuous random process described by an n^{th} order differential equation, when sampled at regular intervals Δt, gives rise to a discrete time series which is exactly described as an ARMA $(n,n-1)$ process. Various formulas can be used to obtain approximate conversion relationships between the parameters of the differential equation and the parameters of the corresponding ARMA model--e.g., the differential operator, d/dt, can be approximated by a rational function of the back-shift operator B, such as the backward difference, $(1 - B)/\Delta t$, or the trapezoidal formula, $2(1 - B)/(1 + B)\Delta t$. However, in the second-order case the exact conversion relationships can be readily obtained by enforcing Equation (2.13), using the expressions for λ_d and μ_d given in Table 2.1. In particular, if the sampling frequency is at least twice the resonant frequency $\left(\text{i.e., } \frac{\pi}{\Delta t} \geq \text{Real}\left\{\omega_0\sqrt{1-\xi^2}\right\}\right)$, then the frequencies and damping factors of the discrete and continuous acf's can be equated separately in (2.13) to yield the following one-to-one conversion relationships between the autoregressive parameters (ϕ_1,ϕ_2) of the discrete process and the frequency and damping parameters (ω_0,ξ) of the continuous process:

(2.14) $$\phi_2 = -\exp(-2\omega_0\xi\Delta t)$$

(2.15a) $$\phi_1 = 2\exp(-\omega_0\xi\Delta t)\cos\left(w_0\sqrt{1-\xi^2}\Delta t\right) \quad \text{if} \quad \xi \leq 1$$

(2.15b) $$\phi_1 = 2\exp(-\omega_0\xi\Delta t)\cosh\left(w_0\sqrt{\xi^2-1}\Delta t\right) \quad \text{if} \quad \xi \geq 1$$

and conversely:

(2.16) $$\omega_0\xi = -\log(-\phi_2)/2\Delta t$$

$$\text{(2.17a)} \qquad \omega_0\sqrt{1-\xi^2} = \cos^{-1}(\phi_1/2\sqrt{-\phi_2})/\Delta t \quad \text{if} \quad \phi_1^2 \leq -4\phi_2$$

$$\text{(2.17b)} \quad \omega_0\sqrt{\xi^2-1} = \log\left(\left(\phi_1 + \sqrt{\phi_1^2+4\phi_2}\right)/2\sqrt{-\phi_2}\right)/\Delta t \quad \text{if} \quad \phi_1^2 \geq -4\phi_2 \; .$$

In conjunction with the above relations, it is also possible to uniquely relate θ_1 in the discrete process to c_0/c_1 in the continuous process by equating the phase angles of the respective acf's, subject to the conditions $-1 \leq \theta_1 \leq 1$ and $c_0/c_1 \geq 0$. In the continuous-to-discrete conversion, after determining ϕ_1 and ϕ_2 from ω_0 and ξ (see above), θ_1 can be determined from ϕ_1 , ϕ_2 and ρ_1 (where $\rho_1 = \rho(\Delta t)$) by solving:

$$\text{(2.18)} \qquad \theta_1^2 + \left[\frac{2\rho_1\phi_1 - \phi_1^2 + \phi_2^2 - 1}{\phi_1 - \rho_1(1-\phi_2)}\right]\theta_1 + 1 = 0 \; , \quad |\theta_1| \leq 1 \; .$$

Equation (2.18) is equivalent to Equation (2.7) expressing the autocorrelation of lag 1 in terms of model parameters.

In the discrete-to-continuous conversion, after determining ω_0 and ξ from ϕ_1 and ϕ_2 , by solving (2.16) and (2.17), the ratio c_0/c_1 can be determined from ξ and $\tan \mu_d$ (where the latter depends on ϕ_1 , ϕ_2 , and θ_1) according to:

$$\text{(2.19)} \qquad \frac{c_0}{c_1} = 2\xi\sqrt{\frac{\xi + \sqrt{1-\xi^2}\tan\mu_d}{\xi - \sqrt{1-\xi^2}\tan\mu_d}} \; .$$

Thus, if a time series representing a sampled continuous process is fitted by ARMA (2,1) models and values of the parameters ϕ_1 , ϕ_2 , and θ_1 have been estimated, then estimates of the parameters of the underlying differential equation can be derived using (2.16), (2.17) and (2.19). Conversely, a model which has been formulated in terms of a 2nd-order differential equation with given parameters can be transformed directly into a discrete model (e.g., for simulation purposes) using (2.14), (2.15) and (2.18). Table 2.2 gives examples of the ARMA (2,1) parameters corresponding to the continuous process in which $\omega_0 = 6\pi$ radians/sec., $\xi = .5$, and $\Delta t = .02$ sec., for several different values of c_0/c_1 . Note that the continuous acf phase is minimized when $c_0/c_1 = 0$ $(c_0 = 0 \, , \, c_1 = 1)$, maximized when $c_0/c_1 = \infty$ $(c_0 = 1 \, , \, c_1 = 0)$, and is zero when $c_0/c_1 = 2\xi$. The discrete acf phase is minimized when $\theta_1 = 1$, maximized when $\theta_1 = -1$, and is zero when θ_1 is such that $\rho_1 = \frac{1}{2}\phi_1$. The range of discrete acf phase angles for $-1 \leq \theta_1 \leq 1$ is slightly larger than the range of continuous acf phase angles for $0 \leq c_0/c_1 \leq \infty$, however the acf phase angles are relatively insensitive to the values of these parameters near the endpoints of their respective ranges.

TABLE 2.1

COMPARISON OF DISCRETE AND CONTINUOUS 2ND-ORDER RANDOM PROCESSES

Process	$a_t - \phi_1 a_{t-1} - \phi_2 a_{t-2} = e_t - \theta_1 e_{t-1}$	$a(t) = \ddot{z}(t)$ $\ddot{z}(t) + 2\xi\omega_0\dot{z}(t) + \omega_0^2 z(t) =$ $c_0\omega_0^2 x(t) + 2c_1\xi\omega_0\dot{x}(t)$ $\ddot{x}(t) = I(t)$
Transfer Function of Filter	$H_d(B) = \dfrac{1 - \theta B}{1 - \phi_1 B - \phi_2 B^2}$	$H_c(s) = \dfrac{c_0\omega_0^2 + 2c_1\xi\omega_0 s}{s^2 + 2\xi\omega_0 s + \omega_0^2}$
Power Spectral Density Function	$f(\omega) = A\lvert H_d(e^{-i\omega\Delta t})\rvert^2 =$ $\dfrac{A'(1+\theta^2 - 2\theta\cos(\omega\Delta t))}{\left(1+\phi_1^2+\phi_2^2\right) - 2\phi_1(1-\phi_2)\cos(\omega\Delta t) - 2\phi_2\cos(2\omega\Delta t)}$	$g(\omega) = D\lvert H_c(i\omega)\rvert^2 =$ $\dfrac{D'\left(c_0^2 + 4c_1^2\xi^2\omega^2/\omega_0^2\right)}{\left(1 - \omega^2/\omega_0^2\right)^2 + 4\xi^2\omega^2/\omega_0^2}$
Autocorrelation Function	$\rho_k = \rho(k\Delta t) = (-\phi_2)^{\frac{k}{2}} \dfrac{\cos(\lambda_d k - \mu_d)}{\cos(-\mu_d)}$ $\lambda_d = \cos^{-1}\left(\dfrac{\phi_1}{2\sqrt{-\phi_2}}\right)$ $\mu_d = \tan^{-1}\left[\dfrac{2\rho_1 - \phi_1}{\sqrt{-\phi_1^2 - 4\phi_2}}\right]$ with $\rho_1 = \dfrac{\phi_1\left(1+\theta_1^2 - \theta_1\phi_1\right) - \theta_1\left(1-\phi_2^2\right)}{(1-\phi_2)\left(1+\theta_1^2 - \theta_1\phi_1\right) - \theta_1\phi_1(1+\phi_2)}$	$\rho(\tau) = e^{-\xi\omega_0\tau}\left(\dfrac{\cos(\lambda_c\tau - \mu_c)}{\cos(-\mu_c)}\right)$ $\lambda_c = \omega_0\sqrt{1-\xi^2}$ $\mu_c = \tan^{-1}\left[\dfrac{c_0^2 - 4\xi^2 c_1^2}{c_0^2 + 4\xi^2 c_1^2}\dfrac{\xi}{\sqrt{1-\xi^2}}\right]$
Stability Condition	$\lvert r_{1,2}\rvert < 1$ where $r_{1,2}$ satisfy: $r^2 - \phi_1 r - \phi_2 = 0$	$R(r_{1,2}) < 0$ where $r_{1,2}$ satisfy: $r^2 + 2\xi\omega_0 r + \omega_0^2 = 0 \rightarrow \xi > 0$

Notes: (1) A, A', D and D' are normalizing constants to make $\int_{-\frac{\pi}{\Delta t}}^{\frac{\pi}{\Delta t}} f(\omega)d\omega$ and $\int_{-\infty}^{\infty} g(\omega)d\omega = 1$.

(2) a_t and $a(t)$ are the discrete and continuous output accelerations; e_t and $I(t)$ are the discrete and continuous white noise input.

TABLE 2.2

EXAMPLE OF CONVERSION BETWEEN CONTINUOUS
2ND-ORDER MODEL AND DISCRETE ARMA (2,1) MODEL
(Δt = .02 SEC.)

CONTINUOUS MODEL		COMMON ACF		DISCRETE MODEL	
Frequency & Damping	c_0/c_1	Phase μ (degrees)	$\rho_1 = \rho(\Delta t)$	θ_1	AR Parameters
	- -	-31	.628	+1	
$\omega_0 = 6\pi$ rads/sec.	0	-30	.631	.96	$\phi_1 = 1.57$
	1	0	.784	.68	
$\xi = .5$	6.5	29	.931	0	$\phi_2 = -.69$
	∞	30	.938	-.27	
	- -	31	.942	-1	

2.4 The ARMA (4,1) Process and Its Interpretation

The ARMA (4,1) process is defined by the 4th-order-autoregressive/1st-order-moving-average linear difference equation:

$$a_t - \phi_1 a_{t-1} - \phi_2 a_{t-2} - \phi_3 a_{t-3} - \phi_4 a_{t-4} = e_t - \theta_1 e_{t-1} \tag{2.20}$$

in which $e_t \sim N\left(0,\sigma_e^2\right)$ as before. In terms of the backward shift operator this can be rewritten:

$$\left(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 - \phi_4 B^4\right) a_t = (1 - \theta_1 B) e_t \ . \tag{2.21}$$

The completely factored form is:

$$(1 - r_1 B)(1 - r_2 B)(1 - r_3 B)(1 - r_4 B) a_t = (1 - \theta_1 B) e_t \tag{2.22}$$

where r_1 , r_2 , r_3 , and r_4 are the roots of the characteristic polynomial

$$r^4 - \phi_1 r^3 - \phi_2 r^2 - \phi_3 r - \phi_4 = 0 \ . \tag{2.23}$$

If at least one pair of roots is complex, say $r_1 = \bar{r}_2$, then the 4th-order autoregressive polynomial in (2.21) can be uniquely expressed as the product of two 2nd-order factors with real coefficients, yielding a further equivalent representation:

$$\left(1 - \phi_{11} B - \phi_{12} B^2\right)\left(1 - \phi_{21} B - \phi_{22} B^2\right) a_t = (1 - \theta_1 B) e_t \tag{2.24}$$

where $\phi_{11} = (r_1 + r_2)$, $\phi_{12} = (-r_1 r_2)$, $\phi_{21} = (r_3 + r_4)$, and $\phi_{22} = (-r_3 r_4)$.

Physically, the ARMA (4,1) process as represented in (2.24) can be considered to arise from the action of an ARMA (2,1) filter and an AR (2) filter in series, as follows: white noise e_t first passes through an ARMA (2,1) filter to produce the intermediary process b_t :

$$\left(1 - \phi_{11}B - \phi_{12}B^2\right)b_t = (1 - \theta_1 B)e_t \; ; \tag{2.25}$$

then b_t serves as the input to an AR (2) filter whose output is a_t :

$$\left(1 - \phi_{21}B - \phi_{22}B^2\right)a_t = b_t \; . \tag{2.26}$$

It should be noted that, from a formal standpoint, the order in which the ARMA (2,1) filter and the AR (2) filter operate is irrelevant, i.e., the AR (2) filter could just as well come first. Moreover, it does not matter which 2nd-order autoregressive factor is associated with the 1st-order moving-average factor in forming the ARMA (2,1) filter. These considerations may be important, however, if the exact nature of the intermediary process b_t is of interest. It should also be noted that the ARMA (4,1) model could be considered as an approximate representation of the action of two ARMA (2,1) filters in series, whose moving average parameters are both small (i.e., the product of which is negligible). In such cases it would not be valid to associate the moving-average factor with only one of the 2nd-order autoregressive factors.

If a time series is identified as an ARMA (4,1) process and the estimates of the autoregressive parameters are such that the characteristic polynomial has at least one complex pair of roots, then a unique factorization of the 4th-order autoregressive factor into two 2nd-order factors can be performed as described above. This establishes a basis for interpreting the underlying continuous process in terms of two 2nd-order filters (linear oscillators) acting in series. Natural frequencies and damping factors for the oscillators corresponding to the two second-order autoregressive factors can be computed as described earlier. However, it may not be possible to estimate the parameters of the forcing functions of the oscillators (e.g., c_0 and c_1, as used above) without external information or physical reasoning, owing to the ambiguous status of the moving-average parameters. In fact, the discrete process resulting from sampling a 4th-order continuous random process (that is, a process formed by passing continuous white noise through a filter described by a 4th-order linear differential equation) is in general exactly described by an ARMA (4,3) model.

CHAPTER 3

ANALYSIS OF EL CENTRO AND SAN FERNANDO EARTHQUAKE RECORDS

3.1 Data

The data consisted of California Institute of Technology corrected accelerograms (D[1]), digitized at .02 second. Four earthquake records were studied, comprising two different earthquakes recorded at the same station (El Centro), and two recordings of the same earthquake (San Fernando, 1971) made at different stations. One horizontal component from each of three of the records was studied, and all three components of the fourth, for a total of six components. These were:

Record No.	Earthquake	Station	Date	Component(s)
IIA001	Imperial Valley	El Centro	5-18-40	S90W
IIA011	El Alamo, B. C.	El Centro	2-09-56	S90W
IIC041	San Fernando	Pacoima Dam	2-09-71	S16E
IID056	San Fernando	Castaic Rt.	2-09-71	N21E, N69W, Down

The first 40 seconds of each component was analyzed. Plots of these components appear in Figure 3.1.

The corrected accelerograms were based on uncorrected accelerograms obtained by hand-digitization of mechanical-optical records at unequal intervals. The correction procedures included equal spacing by linear interpolation, band-pass filtering between 0.07 and 25 hz, and other instrument and baseline corrections. Details of these procedures are given in D[1], D[4], D[5] and D[6].

3.2 Identification and Estimation Techniques

Identification of ARMA models and estimation of parameters was performed according to the systematic procedures of Box and Jenkins (T[2]) using the TIMES program documented in Reference D[3]. In the Box-Jenkins procedures a tentative model-identification is initially made on the basis of the qualitative characteristics of the sample autocorrelation function (acf), partial autocorrelation function (pacf), and spectrum of the data. Parameters are then estimated using the Marquardt algorithm (nonlinear regression) for minimizing the sum of squared residuals. The resulting estimates together with the sample acf, pacf, and spectrum of the residuals are then studied to identify which, if any, revisions to the model are needed. This estimation/identification cycle is repeated until a satisfactory fit is achieved using a parsimonious model--i.e., a minimum number of parameters. Goodness of fit is evaluated on the basis of how closely the statistics of the residuals (acf, pacf, spectrum, distribution) resemble those of discrete white noise. A rough quantitative measure of the goodness of fit is provided by the "Q" statistic:

$$Q = N \sum_{k=1}^{n} r_k^2 \,, \tag{3.1}$$

where r_k is the sample autocorrelation at lag k of the residuals, N is the number of data points, and $n \cong \frac{N}{5}$. Under the hypothesis that the residuals are completely uncorrelated (white noise), Q should be distributed approximately as a chi-square statistic with the number of degrees of freedom equal to n minus the number of parameters estimated. As a general rule, a value of Q not much larger than the number of d.f. is considered to indicate a good fit.

The Box-Jenkins procedures are strictly applicable only to stationary time series. For applications to nonstationary series such as earthquake accelerograms, in which the underlying noise variance and possibly also the nature of the filtering may be time-dependent, it is necessary to adopt a moving-window approach--i.e., to separately analyze segments of the record which are short enough to be considered approximately stationary but still long enough to contain sufficient data points for stable estimation. In this study the identification and estimation procedures were initially tested on segments of various lengths from different parts of the records. It was found that five-second segments (250 data points) provided a satisfactory window, and therefore in the later analysis the 40-second components were each divided into eight segments approximately five seconds long.*

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*Adaptive estimation techniques are also available for continuously "tracking" a nonstationary process in real time. An application of the Kalman filter to the adaptive estimation of time-varying autoregressive parameters is discussed by Nau and Oliver (T[1]). Extensions of this technique to include time-varying moving-average parameters and noise variance, and its application to earthquake accelerograms, will be the subject of a later report.

3.3 General Results

In the course of the identification and estimation procedures, various ARMA models of up to fifth-order-autoregressive and fourth-order-moving-average were tested, and it was found:

(i) that each component was well fitted over its entire length (i.e., in all eight 5-second segments) by ARMA models of the same order; and

(ii) that five of the six components (including all those of the San Fernando earthquake) were best fitted overall by ARMA (4,1) models; the remaining component, IIA001 S90W (El Centro, 1940), was best fitted overall by ARMA (2,1) models.

The parameter estimates of these models for all eight segments of each component are given in Tables 3.1-3.6. These tables also show, for each segment:

(a) the standard deviations of the data and of the residuals (the latter is an estimator of the envelope of the underlying noise process);

(b) the "Q" statistic and its associated number of degrees of freedom for the chi-square test (an indicator of goodness of fit, described above);

(c) the representation of each fourth-order-autoregressive (AR (4)) factor with at least one complex pair of characteristic roots as the product of two second-order (AR (2)) factors, as discussed in Chapter 2; i.e.,

$$\underbrace{\left(1-\hat{\phi}_1 B-\hat{\phi}_2 B^2-\hat{\phi}_3 B^3-\hat{\phi}_4 B^4\right)}_{\substack{\text{AR (4) factor}\\ \text{(estimated)}}} = \underbrace{\left(1-\hat{\phi}_{11} B-\hat{\phi}_{12} B^2\right)}_{\substack{\text{AR (2) factor 1}\\ \text{(derived)}}}\underbrace{\left(1-\hat{\phi}_{21} B-\hat{\phi}_{22} B^2\right)}_{\substack{\text{AR (2) factor 2}\\ \text{(derived)}}} \;;$$

(d) the natural frequency (in radians/sec) and damping coefficient corresponding to each AR (2) factor (whether estimated, as in an ARMA (2,1) model, or derived, as in an ARMA (4,1) model) which has complex or positive real characteristic roots, computed according to the formulas given in Chapter 2.

3.4 Discussion of ARMA (2,1) Models

All eight segments of the component IIA001 S90W are fitted by ARMA (2,1) models. The estimated moving-average parameter is positive for the 0-5 sec. segment and is nearly zero for the remaining segments, indicating that a simple AR (2) model is probably sufficient for the latter segments. The two estimated autoregressive parameters are relatively constant from segment to segment.

The sample acf and spectrum of the 5-10 sec. segment of IIA001 S90W are shown in Figures 3.2a and 3.2b, respectively. The sample spectrum can be compared to the theoretical spectrum of the ARMA (2,1) model fitted to this segment, which is plotted in Figure 3.2c. Although the sample and theoretical spectra appear very similar, it should be recalled that the direct comparison of the sample spectrum with the theoretical spectra of hypothetical models is not a part of the Box-Jenkins method, which instead relies principally on the sample acf and pacf in making an initial identification, and which emphasizes analysis of the residuals in evaluating goodness of fit. The sample spectrum of the residuals for this segment and model is shown in Figure 3.2d, and is seen to be nearly constant, resembling the spectrum of white noise.

The natural frequencies corresponding to the estimated AR (2) factors of the ARMA (2,1) models fitted to all eight segments of IIA001 S90W are seen, in Table 3.1, to be in the range 17-26 radians/sec. (2.4-4.2 hz), and the corresponding damping coefficients are in the range .27-.60. These values are very similar to those used in many linear-oscillator earthquake acceleration models in the literature. (See Reference R[5]).

The "input ratio" parameter, c_0/c_1 (as defined in Chapter 2), was also computed for each of these ARMA (2,1) models. The resulting values were in the range 3.0–8.2, suggesting that the appropriate form of the linear-oscillator model for this earthquake is intermediate between cases (i) and (iii) mentioned in Chapter 2.

3.5 Discussion of the ARMA (4,1) Models

All segments of the five components other than IIA001 S90W are fitted by ARMA (4,1) models.* In general, the estimated autoregressive parameters decrease in absolute value and alternate in sign from $\hat{\phi}_1$ to $\hat{\phi}_4$, and the estimated moving-average parameter is significantly negative. The parameter estimates for each segment are highly correlated; a typical correlation matrix for one of these models is shown in Table 3.7. The estimated parameters of the models for the two horizontal components of IID056 are very similar (Tables 3.4 and 3.5); each of the remaining three components fitted by ARMA (4,1) models has a distinctive range of parameter values among its segments, although the models are all structurally similar. The variation in parameter values from segment to segment within a component does not appear highly significant, and is discussed in more detail in the next section.

The sample acf and spectrum of a typical segment fitted by an ARMA (4,1) model, IIC041 S16E 5-10 sec., are shown in Figures 3.3a and 3.3b, respectively. The sample spectrum is again seen to be very similar to the theoretical spectrum of the model, which is plotted in Figure 3.3c. The spectrum of the residuals for the model applied to this segment is shown in Figure 3.3d, and is again seen to be nearly constant, indicating a good fit.

The fact that a fourth-order discrete linear filter may have a unique representation as a pair of second-order filters in series does not necessarily mean the latter representation has physical significance;

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*The 0-5 sec. segment of IIA011 S90W is somewhat exceptional in that an ARMA (3,1) model is fitted to the first difference of the data in order to obtain more stable parameter estimates, since the data appear very nearly nonstationary. This model is equivalent to an ARMA (4,1) model with one characteristic autoregressive root fixed at unity.

however, this representation has been emphasized in this study to facilitate comparison with other continuous and discrete models. The characteristic autoregressive roots of the ARMA (4,1) models, which form the basis of the representations of the AR (4) factors as products of pairs of AR (2) factors in Tables 3.2-3.6, were obtained using a polynomial factoring program. It must be emphasized that this procedure is extremely sensitive to errors or variations in the original autoregressive parameter estimates. In particular, small variations in the estimates of ϕ_3 and/or ϕ_4, well within reasonable confidence limits, can in many cases give rise to significant changes in the nature of the characteristic roots, e.g., determining whether two pairs of complex roots are found or only one. Computation of natural frequencies and damping coefficients corresponding to the AR (2) factors derived by taking roots further magnifies the estimation errors, therefore confidence intervals for these natural frequencies and damping coefficients are unknown and probably large. Nonetheless, the overall pattern of results obtained by representing the AR (4) factors as products of AR (2) factors, and computing the frequencies and damping coefficients corresponding to the latter, is of interest in making qualitative comparisons between the ARMA (4,1) models and other empirical and theoretical models in the literature.

Of the 40 segments fitted by ARMA (4,1) models, all but one segment (IIC041 S16E 0-5 sec.) yield a model with at least one complex pair of characteristic autoregressive roots, enabling a unique decomposition of the AR (4) factor into a product of two AR (2) factors to be performed. Of these 39 models with at least one complex pair of characteristic autoregressive roots, 26 also have a second pair of complex roots, suggesting that the underlying continuous process might be interpreted as a pair of (underdamped) linear oscillators acting in series. These models with two complex pairs of characteristic autoregressive roots include the models for the 5-10 sec. and 10-15 sec. segments of all five components, which are probably the most important segments since they generally exhibit the strongest shaking and are the most nearly stationary in variance. The second pair of roots in the remaining 13 models is real. A real pair of roots within the unit interval can be considered to correspond to an overdamped oscillator (i.e., with a damping coefficient greater than unity); however, in several of the cases one of the real roots is negative, which does not correspond to any stationary continuous process and is probably therefore attributable to the propagation of estimation errors, as noted above.

In all of the models with two complex pairs of characteristic autoregressive roots, one derived AR (2) factor corresponds to an oscillator with a natural frequency in the range 10-36 radians/sec. (1.6-5.7 hz), and the other derived AR (2) factor corresponds to an oscillator with a natural frequency in the range 63-110 radians/sec. (10.0-17.5 hz). The range of corresponding damping coefficients for both factors covers almost the entire unit interval; however, most values are in the range .2-.7. The lower-frequency derived AR (2) factor in these models is thus seen to be very similar to the single, directly estimated AR (2) factor of the ARMA (2,1) models fitted to the segments of IIA001 S90W. Recall that the estimated moving-average parameter is significantly negative ($\hat{\theta}_1 \cong -.5$) in the ARMA (4,1) models, whereas it is nearly zero (i.e., effectively absent) in the ARMA (2,1) models. It thus appears that the essential difference between these two classes of models for the components studied here lies in the presence, in the former, of an additional AR (2) factor corresponding to a higher-frequency oscillator, together with an MA (1) factor in which $\hat{\theta}_1$ is significantly negative. (This MA (1) factor, considered separately, would represent a first-order low-pass filter.) In other words, it may be useful to consider the ARMA (4,1) model to represent the series action of an AR (2) filter, corresponding to a low-frequency oscillator, and an ARMA (2,1) filter, corresponding to a higher frequency oscillator together with a first-order low-pass filter. (It was shown in Chapter 2 that, in an ARMA (2,1) model corresponding to a linear oscillator, the moving-average factor may reflect the nature of the coupling between the oscillator and its input, rather than an independent first-order filter.) The effect of the "additional" filtering in the ARMA (4,1) model is to slightly amplify intermediate frequencies (say, 10-15 hz) and to more sharply attenuate the higher frequencies.

It should be noted that the seismographs on which the data were recorded contain mechanical transducers consisting of linear oscillators with natural frequencies in the range 10-20 hz and damping factors of about 0.6. In particular, the transducer natural frequencies were about 20 hz for the four San Fernando earthquake components, 16 hz for the El Centro 1956 component and 10 hz for the El Centro 1940 component. The instrument correction applied to the data supposedly compensated for the filtering effects of the transducers, however it may be significant that the higher-frequency AR (2) factors of the ARMA (4,1) models represent oscillators with parameters similar to the corresponding transducers, and that the one component fitted by ARMA (2,1) models was the one with the lowest transducer frequency.

3.6 Time-Variation of Parameter Estimates Within Components

As noted in Appendix A, various earthquake models in the recent literature have attempted to account for observed time-varying spectral characteristics of real accelerograms. It is therefore of interest in this study to determine whether the filtering properties of the fitted models--i.e., the ARMA parameter estimates--are significantly time-varying. *A priori* it might be supposed that apparent spectral changes could be due either to variable filtering or else simply due to the transient response of a time-invariant filter to a noise input with a highly nonstationary variance. The ARMA model-identification and parameter-estimation techniques are potentially able to distinguish between these two kinds of effects, since they do not require the accelerogram to be reshaped to a constant variance prior to analysis.

The nonoverlapping moving window approach employed in this study (i.e., the subdivision of each component into eight segments) largely isolates the effect of nonstationary noise variance. In fact, the variance (or, equivalently, standard deviation) of the residuals is found to be by far the most significantly time-varying of the parameters within each component. In Tables 3.1-3.6 it is seen that for each component the standard deviation of the residuals generally decreases by about a factor of ten from the first segment to the last.

The fact that relatively good fits by ARMA models are obtained for all the segments indicates that the amount of nonstationarity of variance remaining within segments is not problematic. However, it might be supposed *a priori* that, if nonstationarity of the variance within segments were to bias or distort the ARMA parameter estimates in any way, then this effect would be most pronounced in the initial (0-5 sec.) segment of each component, in which the variance always builds up from essentially zero to some finite (perhaps even maximal) value. In this regard, it has already been noted that the models for the initial segments of three of the components are in some way exceptional. (IIA001 S90W 0-5 sec. is the only segment whose ARMA (2,1) model features a significantly positive estimated moving-average parameter; IIA011 S90W 0-5 sec. is the only segment whose ARMA (4,1) model requires one characteristic autoregressive root fixed at unity to obtain stable parameter estimates; and IIC041 S16E 0-5 sec. is the segment whose ARMA (4,1) parameter estimates yield four real characteristic autoregressive roots.) Excepting these 0-5 sec. segments, however, there does not appear to be significant variation of the ARMA parameter estimates from segment to segment within any component.* In fact, for each component it appears that a set of "average" ARMA parameter values could be determined which would fall well within the 95% confidence intervals for the parameter estimates of nearly every segment.**

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*One further exception appears to be the 30-35 sec. segment of IIC041 S16E, in which a significant shift in estimated ARMA parameters occurs, reflecting a decrease in the equivalent damping coefficients to nearly zero. Inspection of the data reveals very-nearly-sinusoidal oscillations occurring during a portion of this segment.

**The covariance matrix of the ARMA parameter estimates for each segment, from which the correlation matrices and individual confidence intervals are derived, is computed based on the estimated variance of the residuals and the regression matrix used at the final iteration of the Marquardt algorithm. (See Box and Jenkins, T[2], p. 502.) Confidence intervals are based on the assumption of an approximately multivariate-normal distribution.

More precisely, it should be recalled that the ARMA parameter estimates, especially in the ARMA (4,1) models, are highly correlated. That is, these models contain a certain amount of redundancy--a small variation in one parameter can be largely compensated by simultaneous changes in the remaining parameters, as determined by the correlation matrix, with little effect on the overall filtering properties of the model. In particular, it has been noted that the estimated AR parameters generally alternate in sign ($\hat{\phi}_1$ and $\hat{\phi}_3$ positive, $\hat{\phi}_2$ and $\hat{\phi}_4$ negative) and the estimated MA parameter ($\hat{\theta}_1$) is always negative. Table 3.7 shows that typically $\hat{\theta}_1$ is strongly positively correlated with $\hat{\phi}_1$ and $\hat{\phi}_3$, and strongly negatively correlated with $\hat{\phi}_2$ and $\hat{\phi}_4$. This indicates that a small increase in the absolute value of $\hat{\theta}_1$ could be largely compensated by simultaneous decreases in the absolute values of all four AR parameter estimates, and vice versa. In fact, the observed variations in parameter estimates from segment to segment within each component are generally consistent with this pattern (e.g., if $\hat{\theta}_1$ increases in absolute value, then the AR parameter estimates all decrease in absolute value), and therefore they do not suggest fundamental variations in the filtering properties of the underlying process.

3.7 Conclusions

The application of the time-domain analytic techniques of Box and Jenkins to segments of digitized earthquake accelerograms appears to be a potentially useful method of characterizing recorded earthquakes by linear models with a small number of parameters. It should be emphasized that the detailed discussions of the ARMA (2,1) and ARMA (4,1) models in this report were entirely motivated by the experimental results--no *a priori* assumptions were made concerning the order of appropriate ARMA models for earthquake analysis. The fact that the Box-Jenkins method includes systematic model-identification techniques which do not require such assumptions is one of its principal advantages relative to other model-fitting procedures commonly applied to earthquake data. Therefore the application of Box-Jenkins techniques to other earthquake records should not be expected to yield only ARMA (2,1) and ARMA (4,1) models. However, the fact that five of the six components studied here were best fitted in all their segments by ARMA (4,1) models suggests that this model may be of great generality for California earthquakes. Moreover, these models have appealing connections with simple hypothetical physical models discussed elsewhere in the literature. The ARMA (2,1) model may be considered to include the various basic forms of the linear-oscillator model; and the ARMA (4,1) model, in its representation as two linear oscillators with different natural frequencies acting in series, is somewhat more complex. It appears that the principal source of nonstationarity of the earthquake acceleration data lies in the time-dependence of variance of the driving noise process, rather than the filtering parameters.

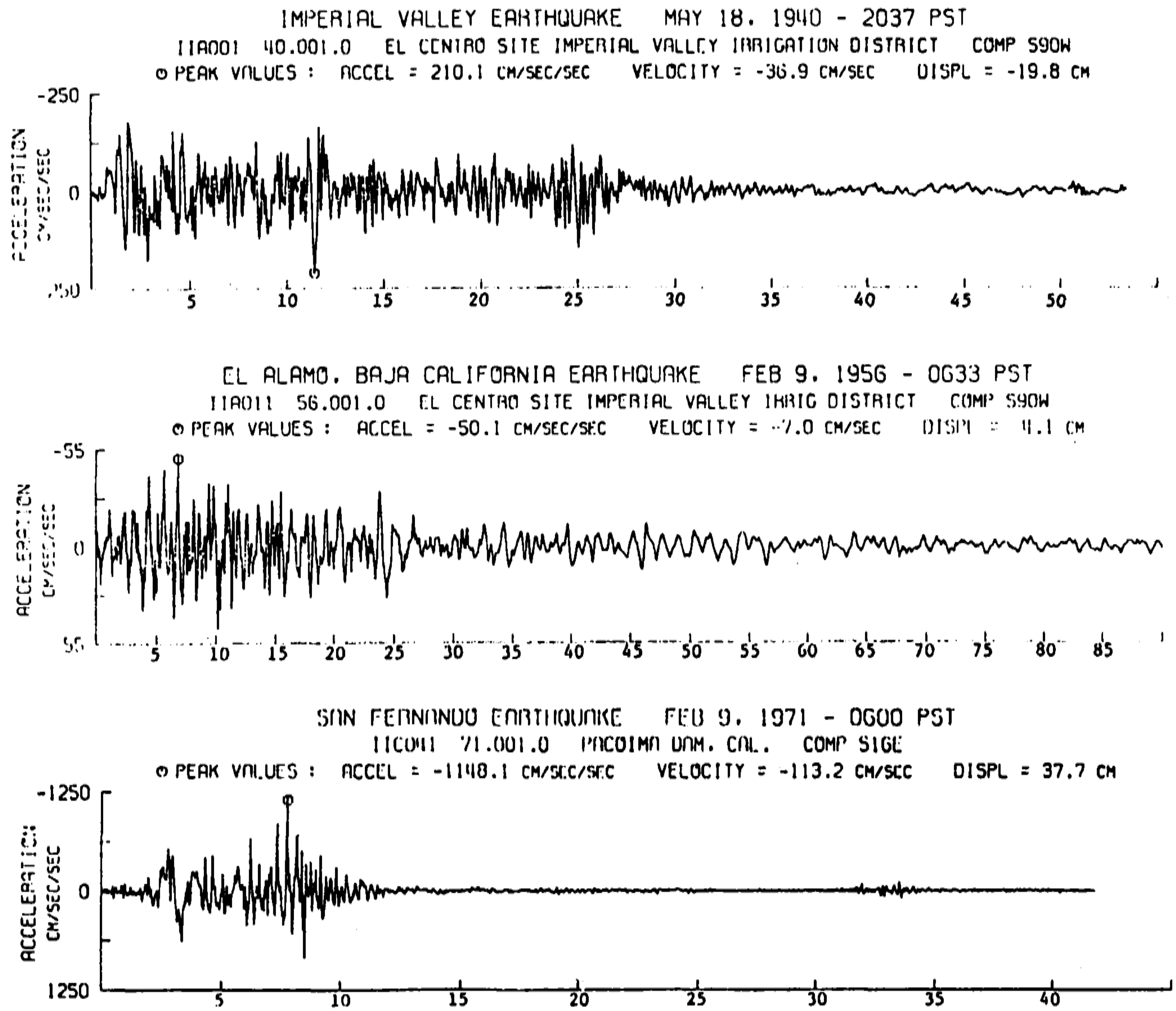

FIGURE 3.1a

ACCELEROGRAMS OF EARTHQUAKES STUDIED IN THIS REPORT

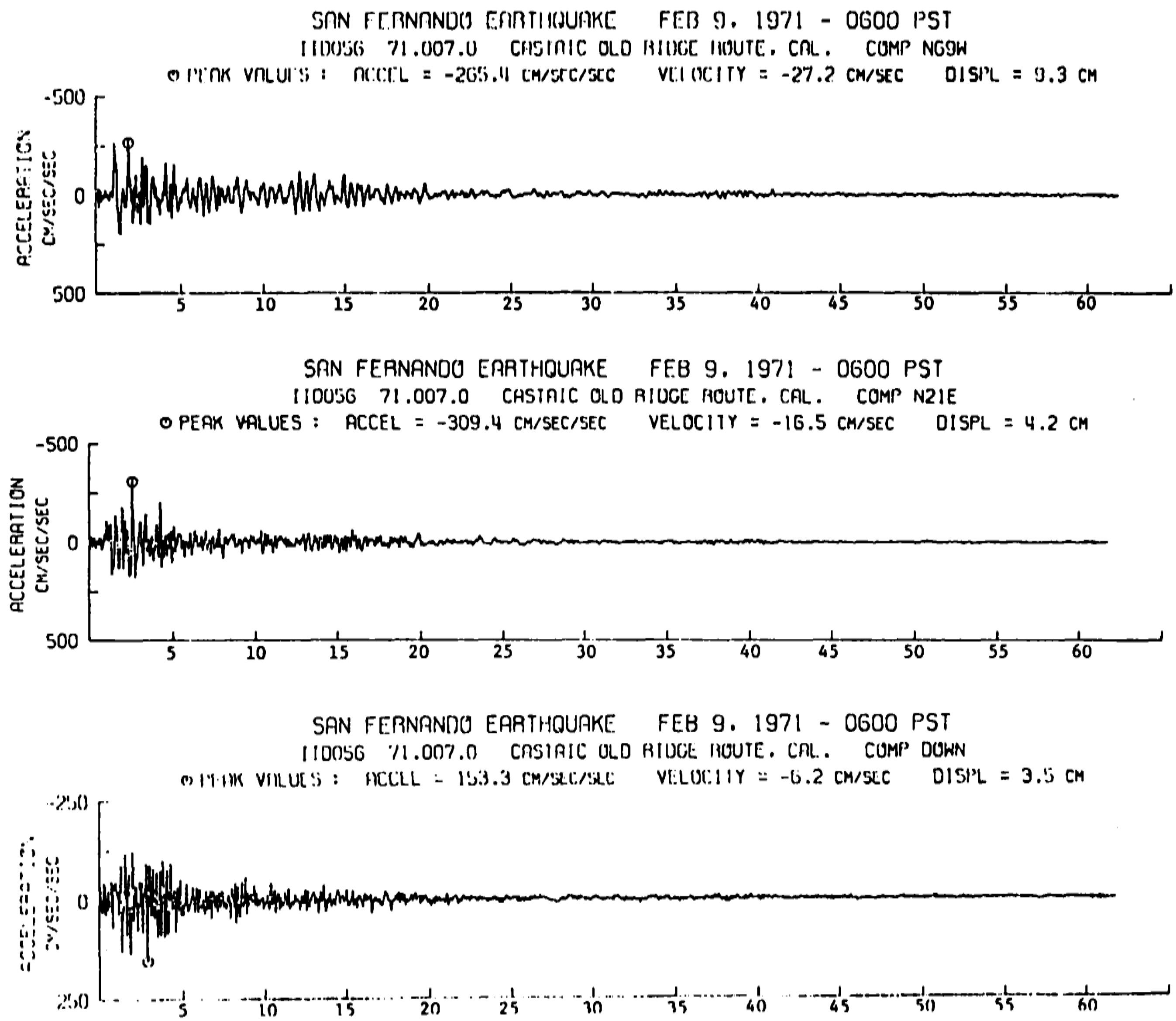

FIGURE 3.1b

ACCELEROGRAMS OF EARTHQUAKES STUDIED IN THIS REPORT

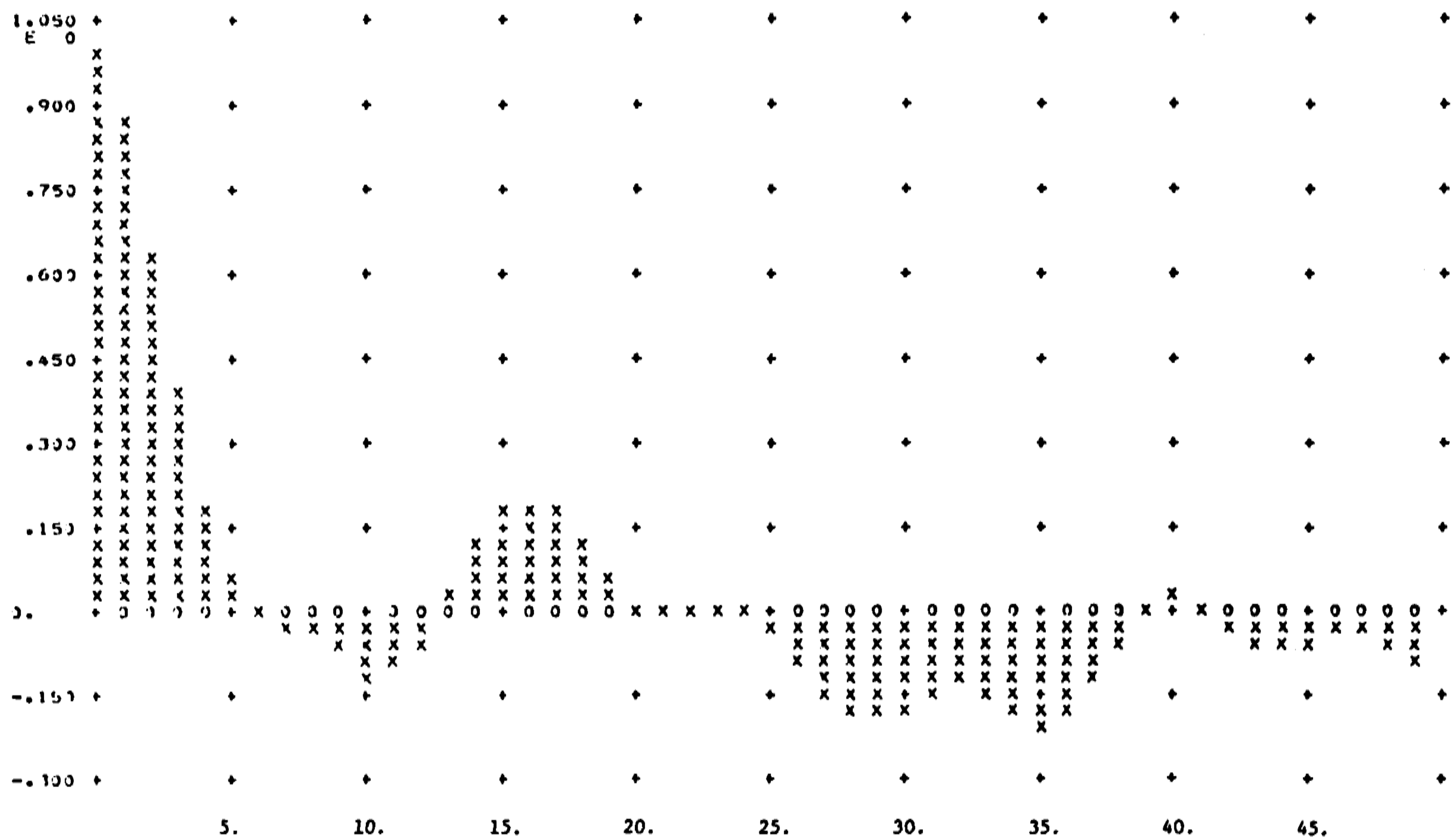

FIGURE 3.2a

IIA001 S90W ACCELERATION DATA 5-10 SECONDS

AUTOCORRELATION FUNCTION

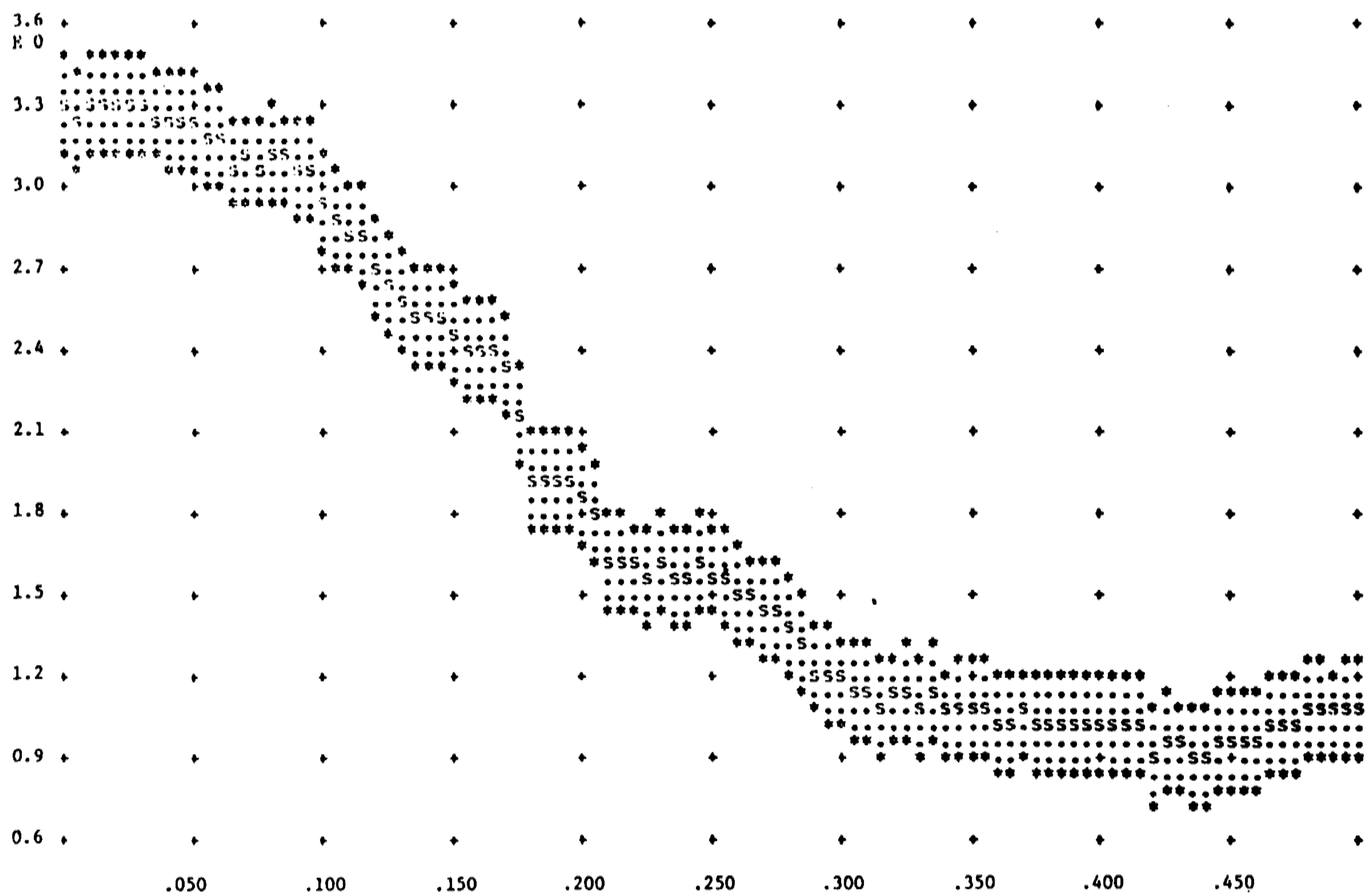

FIGURE 3.2b

IIA001 S90W ACCELERATION DATA 5-10 SECONDS

LOG 10 SPECTRUM (95 P.C. CONFIDENCE LIMITS, SMOOTHING BANDWIDTH = 0.1)

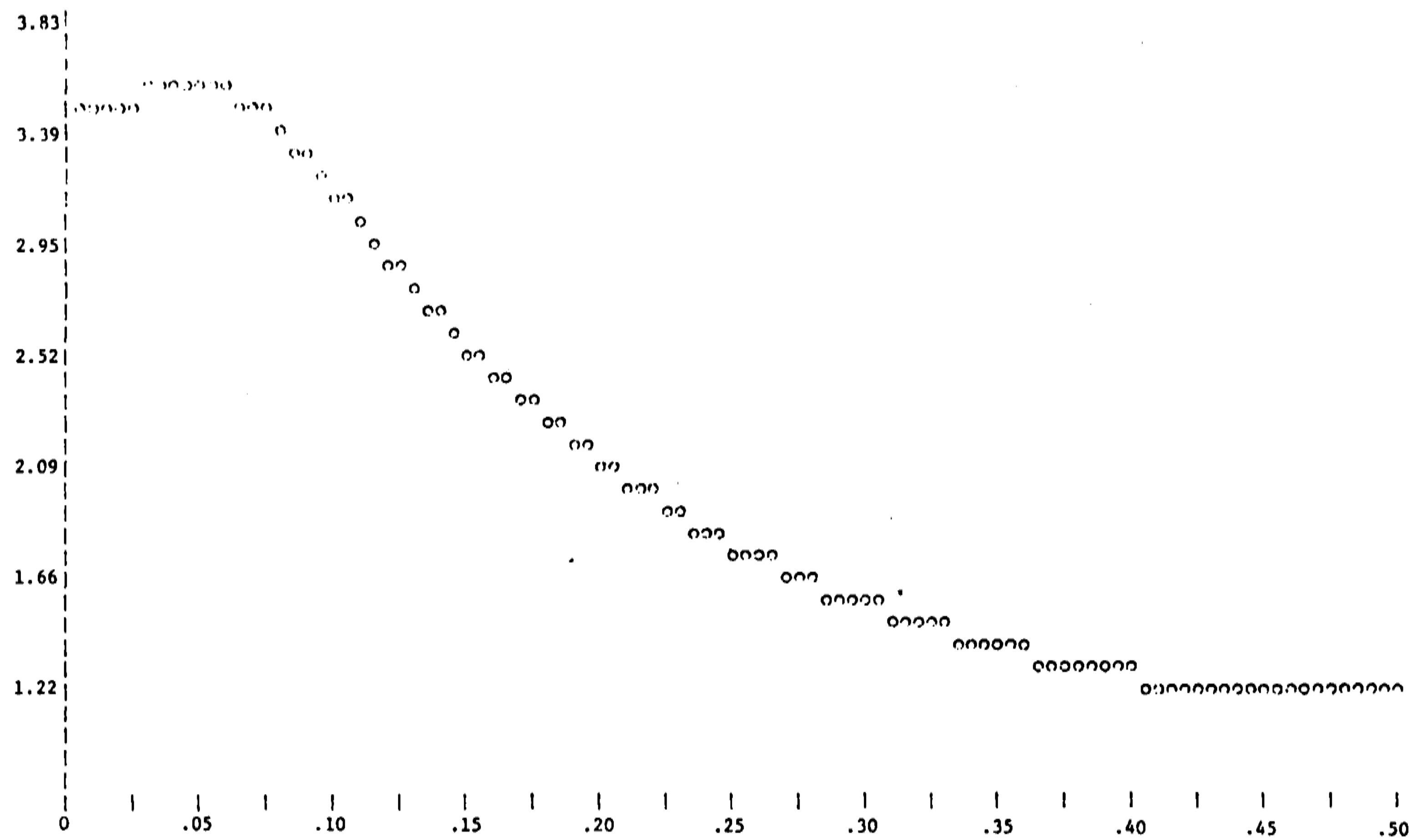

FIGURE 3.2c

THEORETICAL SPECTRUM OF ARMA (2,1) MODEL FOR

IIA001 S90W 5-10 SECONDS

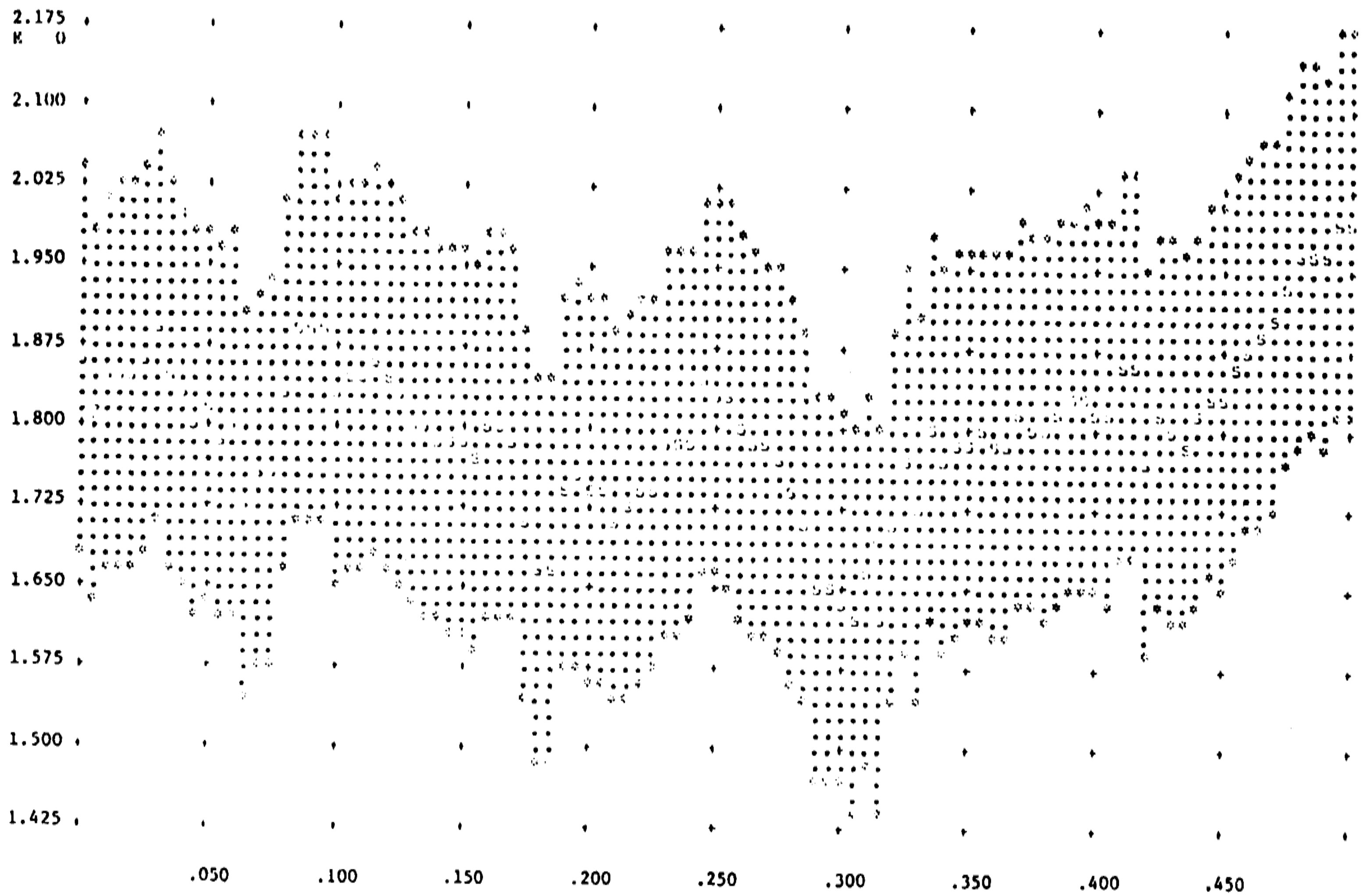

FIGURE 3.2d

I1A001 S90W ARMA (2,1) MODEL RESIDUALS 5-10 SECONDS

LOG 10 SPECTRUM (95 P.C. CONFIDENCE LIMITS, SMOOTHING BANDWIDTH = 0.1)

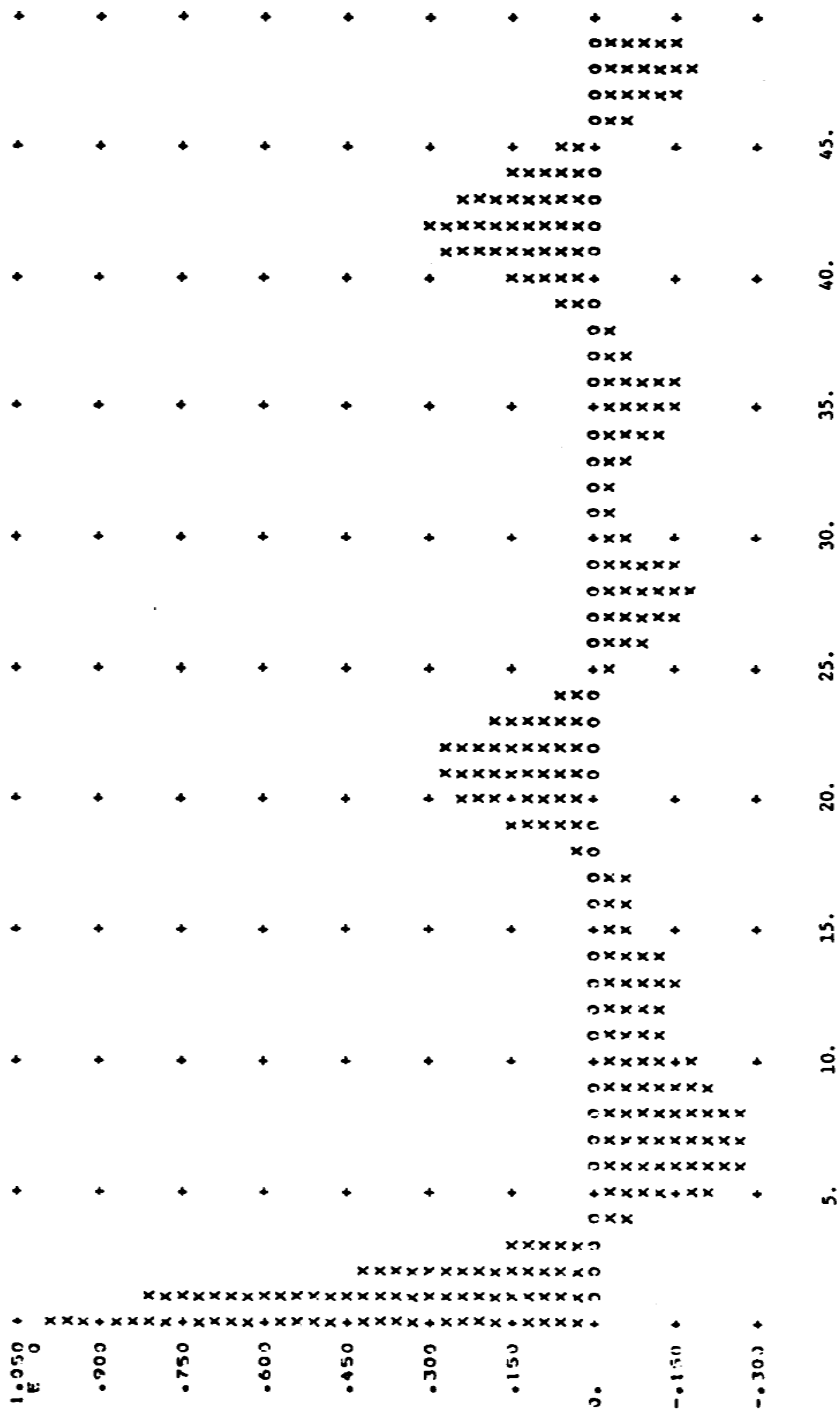

FIGURE 3.3a

IIC041 S16E ACCELERATION DATA 5-10 SECONDS

AUTOCORRELATION FUNCTION

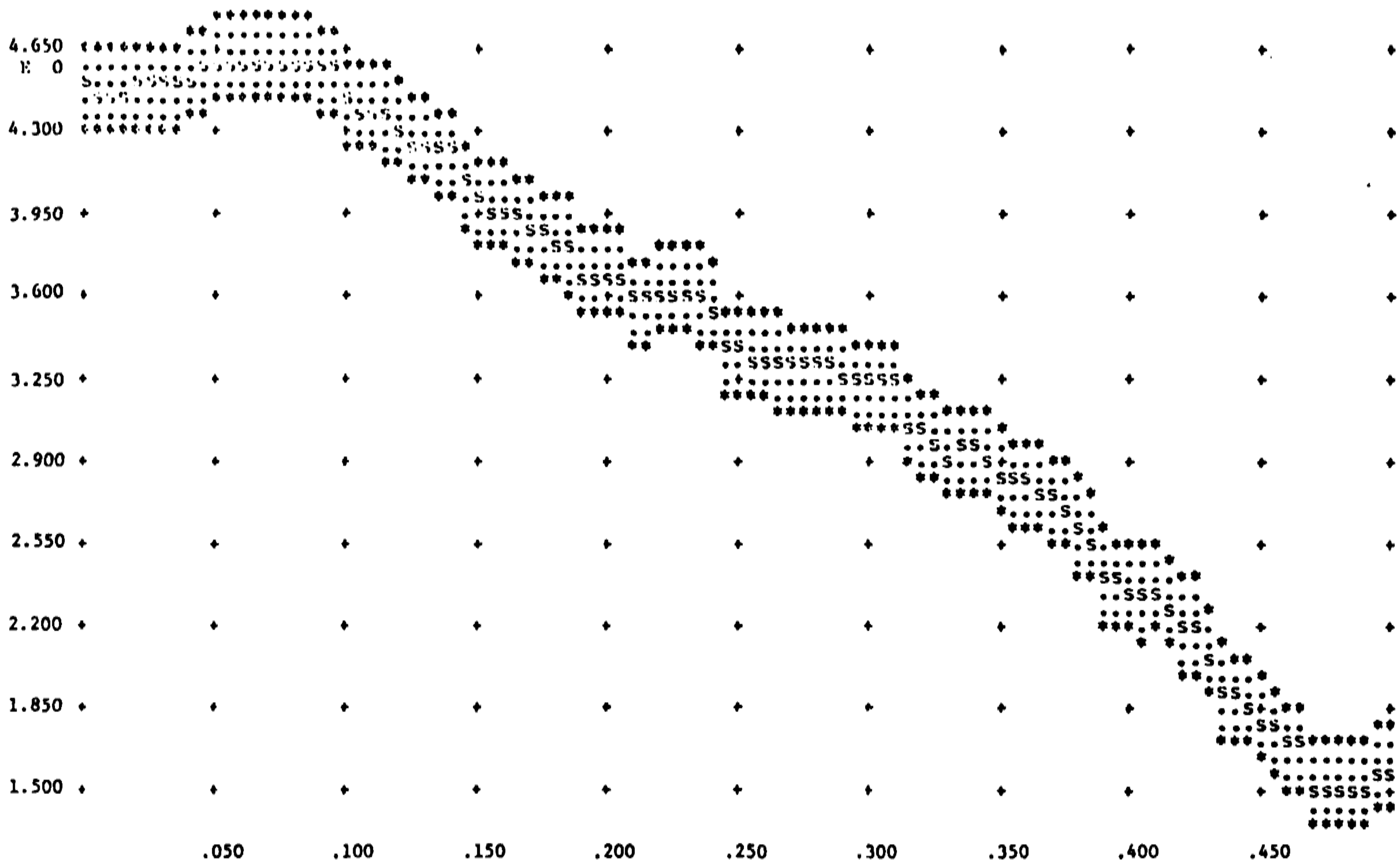

FIGURE 3.3b

IIC041 S16E ACCELERATION DATA 5-10 SECONDS

LOG 10 SPECTRUM (95 P.C. CONFIDENCE LIMITS, SMOOTHING BANDWIDTH = 0.1)

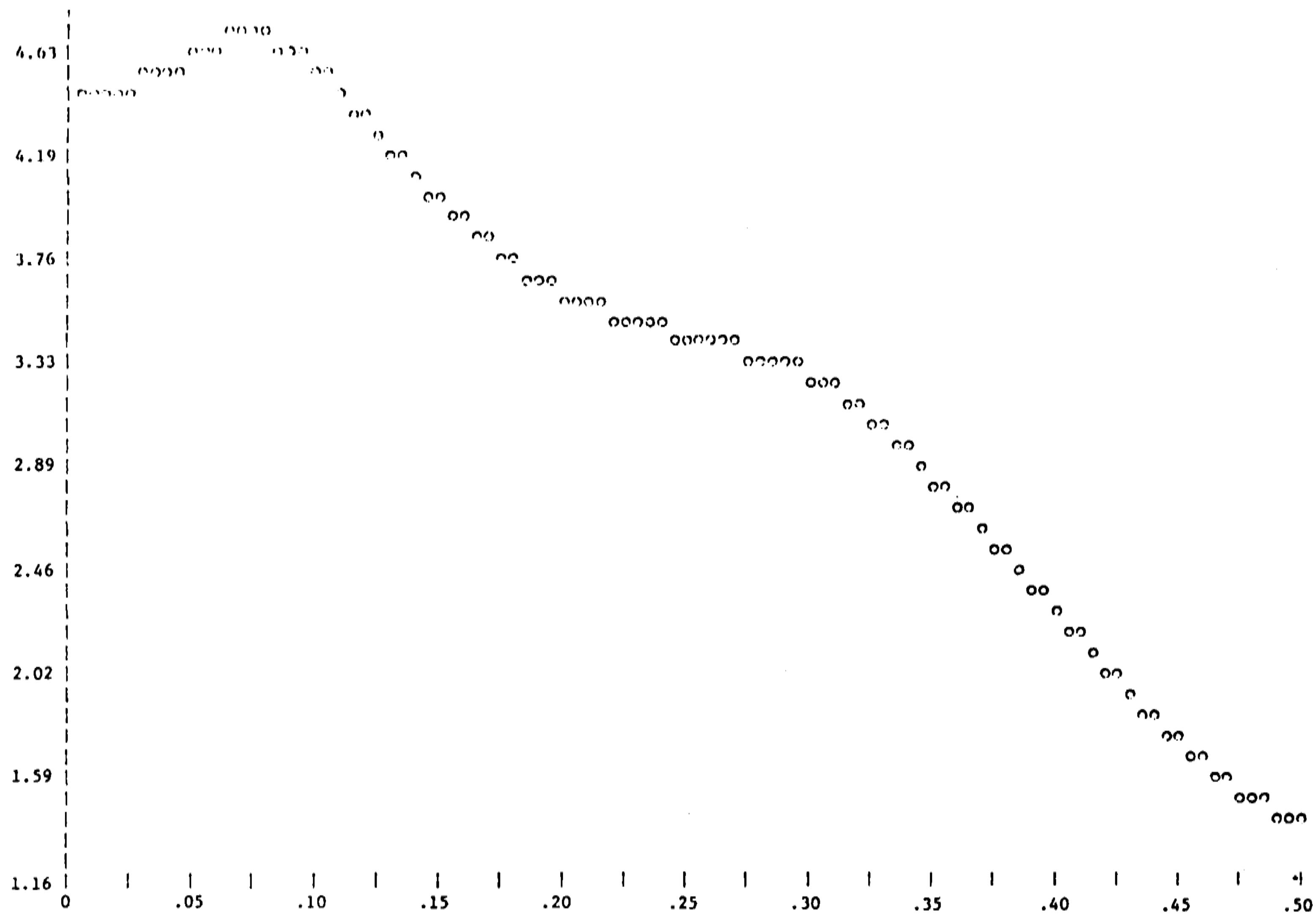

FIGURE 3.3c

THEORETICAL SPECTRUM OF ARMA (4,1) MODEL FOR

IIC041 S16E 5-10 SECONDS

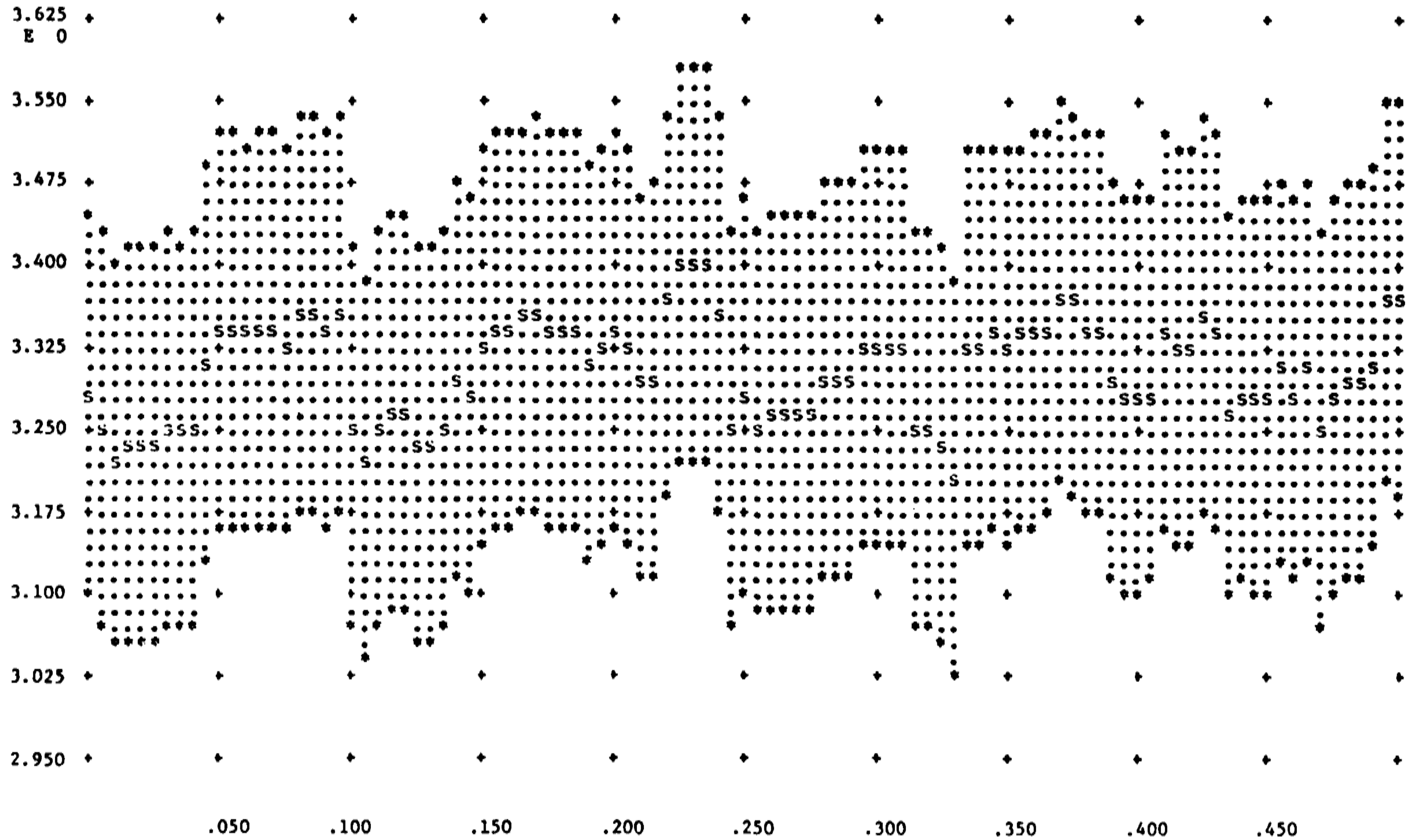

FIGURE 3.3d

IIC041 S16E ARMA (4,1) MODEL RESIDUALS 5-10 SECONDS

LOG 10 SPECTRUM (95 P.C. CONFIDENCE LIMITS, SMOOTHING BANDWIDTH = 0.1)

TABLE 3.1

ARMA (2,1) MODELS FOR IIA001 S90W COMPONENT
(IMPERIAL VALLEY 1940 EARTHQUAKE, EL CENTRO STATION)

Segment (Sec.)	# Points	Standard Deviation (cm/sec^2)		Goodness of Fit	ARMA Parameter Estimates With Approx. 95% Confidence Limits			Parameters of Corresponding Linear-Oscillator Model (Natural Frequency, Damping Coefficient and Input Ratio)		
		Data	Residuals	Q/# d.f.	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\theta}_1$	ω_0	ξ	c_0/c_1
0-5	248	73.1	27.9	65/45	1.55 $\pm$.23	-.66 $\pm$.18	.33 $\pm$.25	18	.59	3.4
5-10	248	52.1	21.6	48/45	1.35 $\pm$.20	-.54 $\pm$.18	.01 $\pm$.24	26	.59	5.3
10-15	256	61.9	23.0	69/47	1.43 $\pm$.19	-.58 $\pm$.17	.06 $\pm$.23	22	.60	5.5
15-20	248	34.7	10.9	51/45	1.56 $\pm$.14	-.71 $\pm$.13	.09 $\pm$.19	21	.41	3.7
20-25	248	43.3	11.7	38/45	1.61 $\pm$.10	-.78 $\pm$.10	-.01 $\pm$.16	22	.27	3.0
25-30	256	38.2	9.24	57/47	1.63 $\pm$.10	-.77 $\pm$.10	-.01 $\pm$.16	21	.31	3.7
30-35	248	13.1	2.95	60/45	1.65 $\pm$.10	-.76 $\pm$.10	.00 $\pm$.16	19	.35	4.4
35-40	248	7.30	1.45	61/45	1.66 $\pm$.10	-.76 $\pm$.10	-.10 $\pm$.16	17	.40	8.2

TABLE 3.2

ARMA (4,1) MODELS FOR IIA011 S90W COMPONENT
(EL ALAMO B.C. 1956 EARTHQUAKE, EL CENTRO STATION)

Segment (Sec.)	# Points	Standard Deviations (cm/sec^2) Data	Residuals	Goodness of Fit Q/# d.f.	ARMA Parameter Estimates With Approx. 95% Confidence Limits $\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\phi}_3$	$\hat{\phi}_4$	$\hat{\theta}_1$	AR (2) Factor 1 ϕ_{11} / ω_0	ϕ_{12} / ξ	AR (2) Factor 2 ϕ_{21} / ω_0	ϕ_{22} / ξ
0-5	248	13.3	1.42	49/45	.94 ±.16	-.40 ±.23	-.02 ±.16	**	-.81 ±.10	.99 41	-.44 .49	.96 *	.04 *
5-10	248	17.6	1.96	42/43	2.03 ±.15	-1.67 ±.33	.80 ±.32	-.23 ±.14	-.67 ±.12	1.65 15	-.73 .51	.38 68	-.32 .43
10-15	256	16.0	1.66	70/45	2.06 ±.16	-1.70 ±.36	.79 ±.36	-.23 ±.15	-.73 ±.12	1.67 15	-.75 .47	.38 68	-.30 .44
15-20	248	12.8	.947	58/43	2.15 ±.18	-1.64 ±.44	.51 ±.42	-.05 ±.16	-.66 ±.14	1.57 15	-.64 .74	.58 62	-.08 1.00
20-25	248	12.3	.637	61/43	2.06 ±.17	-1.33 ±.43	.20 ±.42	.06 ±.16	-.81 ±.12	1.31 24	-.47 .80	.75 *	.12 *
25-30	256	5.66	.403	42/45	1.85 ±.28	-.90 ±.68	-.17 ±.63	.19 ±.23	-.71 ±.24	1.31 28	-.54 .54	.79 *	.12 *
30-35	248	6.13	.335	35/43	1.95 ±.29	-1.06 ±.71	.01 ±.64	.07 ±.22	-.64 ±.26	1.73 10	-.76 .67	.23 *	.10 *
35-40	248	4.69	.236	43/43	2.36 ±.38	-2.08 ±.91	.95 ±.74	-.26 ±.21	-.13 ±.39	1.91 10	-.94 .15	.45 65	-.28 .49

*One real root is negative; no corresponding frequency and damping.

**0-5 sec. segment estimated as ARMA (3,1) on first difference of data; i.e., $\left(1-\phi_1B-\phi_2B^2-\phi_3B^3\right)(1-B)a_t = (1-\theta_1B)e_t$. This is equivalent to ARMA (4,1) model with one AR root fixed at unity.

TABLE 3.3

ARMA (4,1) MODELS FOR IIC041 S16E COMPONENT
(SAN FERNANDO 1971 EARTHQUAKE, PACOIMA DAM STATION)

Segment (Sec.)	# Points	Standard Deviations (cm/sec^2): Data	Residuals	Goodness of Fit: Q/# d.f.	ARMA Parameter Estimates With Approx. 95% Confidence Limits: $\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\phi}_3$	$\hat{\phi}_4$	$\hat{\theta}_1$	AR (2) Factor 1: ϕ_{11} / ω_0	ϕ_{12} / ξ	AR (2) Factor 2: ϕ_{21} / ω_0	ϕ_{22} / ξ
0-5	248	185	61.5	46/43	.89 ±.19	-.04 ±.29	-.06 ±.26	.00 ±.16	-.82 ±.13	* / *	* / *	* / *	* / *
5-10	248	261	115	56/43	.89 ±.19	-.37 ±.27	.24 ±.24	-.22 ±.15	-.69 ±.16	1.34 / 29	-.59 / .45	-.45 / 100	-.38 / .24
10-15	256	54.7	19.0	58/45	.93 ±.17	-.26 ±.26	.13 ±.24	-.23 ±.14	-.74 ±.14	1.45 / 27	-.68 / .36	-.52 / 106	-.33 / .26
15-20	248	15.5	6.14	43/43	.88 ±.17	-.25 ±.25	.016 ±.23	-.18 ±.14	-.78 ±.12	1.38 / 31	-.69 / .30	-.51 / 100	-.26 / .30
20-25	248	13.4	4.92	73/43	.95 ±.21	-.26 ±.33	.06 ±.28	-.25 ±.14	-.62 ±.18	1.50 / 28	-.78 / .23	-.56 / 108	-.32 / .26
25-30	256	5.67	2.82	69/45	.66 ±.15	-.25 ±.20	.10 ±.19	-.25 ±.14	-.79 ±.11	1.27 / 34	.64 / .32	-.61 / 107	-.39 / .22
30-35	248	26.9	13.6	73/43	.82 ±.16	-.69 ±.18	.53 ±.16	-.60 ±.10	-.33 ±.19	1.43 / 34	-.84 / .13	-.61 / 97	-.72 / .08
35-40	248	5.87	2.68	49/43	.82 ±.22	-.28 ±.32	.12 ±.26	-.25 ±.14	-.64 ±.22	1.38 / 31	-.68 / .31	-.56 / 106	-.36 / .24

*All 4 characteristic roots are real; factorization into 2nd-order terms cannot be performed uniquely.

TABLE 3.4

ARMA (4,1) MODELS FOR IID056 N69W COMPONENT
(SAN FERNANDO 1971 EARTHQUAKE, CASTAIC STATION)

Segment (Sec.)	# Points	Standard Deviations (cm/sec^2) Data	Residuals	Goodness of Fit Q/# d.f.	ARMA Parameter Estimates With Approx. 95% Confidence Limits $\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\phi}_3$	$\hat{\phi}_4$	$\hat{\theta}_1$	AR (2) Factor 1 ϕ_{11} / ω_0	ϕ_{12} / ξ	AR (2) Factor 2 ϕ_{21} / ω_0	ϕ_{22} / ξ
0-5	248	82.7	21.3	38/43	1.50 ±.17	-1.05 ±.32	.47 ±.32	-.11 ±.16	-.76 ±.12	1.18 28	-.37 .89	.32 71	-.30 .43
5-10	248	45.7	8.06	45/43	1.90 ±.19	-1.63 ±.41	.96 ±.37	-.36 ±.15	-.45 ±.19	1.66 18	-.77 .37	.24 72	-.47 .26
10-15	256	46.9	5.83	50/45	2.11 ±.19	-1.93 ±.41	1.13 ±.39	-.40 ±.15	-.40 ±.19	1.76 15	-.84 .29	.34 69	-.48 .27
15-20	248	29.3	3.58	98/43	2.04 ±.20	-1.71 ±.46	.81 ±.43	-.24 ±.17	-.54 ±.18	1.66 17	-.76 .39	.38 68	-.31 .43
20-25	248	12.1	1.79	67/43	2.01 ±.24	-1.69 ±.53	.80 ±.49	-.21 ±.19	-.46 ±.22	1.53 18	-.63 .64	.48 64	-.33 .44
25-30	256	9.45	1.07	41/45	2.13 ±.29	-1.84 ±.64	.93 ±.55	-.27 ±.19	-.28 ±.27	1.70 12	-.75 .58	.43 65	-.36 .39
30-35	248	7.18	1.06	50/43	1.96 ±.25	-1.55 ±.55	.68 ±.50	-.17 ±.19	-.49 ±.23	1.56 18	-.66 .60	.41 67	-.26 .50
35-40	248	8.72	1.28	71/43	1.87 ±.21	-1.35 ±.48	.50 ±.45	-.15 ±.18	-.60 ±.19	1.63 20	-.77 .33	.23 77	-.20 .53

TABLE 3.5

ARMA (4,1) MODELS FOR IID056 N21E COMPONENT
(SAN FERNANDO 1971 EARTHQUAKE, CASTAIC STATION)

		Standard Deviations (cm/sec^2)		Goodness of Fit	ARMA Parameter Estimates With Approx. 95% Confidence Limits					AR (2) Factor 1		AR (2) Factor 2	
Segment (Sec.)	# Points	Data	Residuals	Q/# d.f.	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\phi}_3$	$\hat{\phi}_4$	$\hat{\theta}_1$	ϕ_{11} ω_0	ϕ_{12} ξ	ϕ_{21} ω_0	ϕ_{22} ξ
0-5	248	80.1	20.3	56/43	1.61 ±.19	-1.22 ±.38	.53 ±.36	-.16 ±.17	-.65 ±.16	1.34 28	-.59 .47	.27 73	-.27 .44
5-10	248	30.1	6.64	64/43	1.82 ±.17	-1.65 ±.35	.86 ±.34	-.27 ±.15	-.61 ±.15	1.39 28	-.64 .40	.43 65	-.42 .33
10-15	256	24.3	4.94	69/45	1.80 ±.18	-1.48 ±.38	.66 ±.36	-.21 ±.16	-.62 ±.16	1.48 27	-.72 .31	.32 71	-.29 .43
15-20	248	22.0	3.62	68/43	1.87 ±.23	-1.35 ±.52	.34 ±.49	.00 ±.20	-.61 ±.19	1.33 31	-.63 .37	.54 127	-.004 1.08
20-25	248	9.54	1.58	32/43	1.75 ±.26	-1.10 ±.58	.19 ±.53	.09 ±.21	-.63 ±.22	1.08 40	-.55 .37	.67 *	.16 *
25-30	256	7.30	.99	41/43	1.73 ±.22	-.96 ±.48	.15 ±.35	.02 ±.18	-.67 ±.18	1.09 31	-.32 .93	.64 *	.05 *
30-35	248	4.44	.99	49/43	1.75 ±.23	-1.38 ±.46	.64 ±.43	-.16 ±.18	-.56 ±.21	1.33 25	-.51 .66	.42 66	-.32 .43
35-40	248	5.93	1.27	44/43	1.91 ±.19	-1.78 ±.40	-.96 ±.38	-.29 ±.17	-.53 ±.17	1.43 26	-.64 .43	.48 63	-.45 .31

*One real root is negative; no corresponding frequency and damping coefficient.

TABLE 3.6

ARMA (4,1) MODELS FOR IID056 DOWN COMPONENT
(SAN FERNANDO 1971 EARTHQUAKE, CASTAIC STATION)

Segment (Sec.)	# Points	Standard Deviations (cm/sec^2) Data	Residuals	Goodness of Fit Q/# d.f.	ARMA Parameter Estimates With Approx. 95% Confidence Limits $\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\phi}_3$	$\hat{\phi}_4$	$\hat{\theta}_1$	AR (2) Factor 1 ϕ_{11} / ω_0	ϕ_{12} / ξ	AR (2) Factor 2 ϕ_{21} / ω_0	ϕ_{22} / ξ
0-5	248	50.4	17.7	61/43	1.17 ±.19	-.82 ±.32	.41 ±.28	-.36 ±.14	-.56 ±.19	1.46 / 32	-.81 / .17	-.29 / 92	-.44 / .22
5-10	248	20.1	6.83	65/43	1.24 ±.16	-.93 ±.28	.34 ±.27	-.20 ±.15	-.72 ±.13	1.28 / 36	-.69 / .26	-.04 / 86	-.29 / .36
10-15	256	14.5	4.32	50/45	1.36 ±.16	-.90 ±.29	.34 ±.27	-.18 ±.14	-.68 ±.13	1.40 / 30	-.69 / .30	-.03 / 87	-.26 / .39
15-20	248	9.96	2.45	46/43	1.57 ±.18	-1.10 ±.35	.34 ±.34	-.03 ±.16	-.75 ±.13	.94 / 44	-.46 / .44	.63 / 68	-.06 / 1.04
20-25	248	5.23	1.33	52/43	1.58 ±.20	-1.10 ±.39	.33 ±.38	-.04 ±.17	-.64 ±.16	1.01 / 41	-.44 / .51	.57 / 63	-.08 / 1.00
25-30	256	3.62	.858	85/45	1.66 ±.35	-1.21 ±.70	.39 ±.64	.06 ±.28	-.51 ±.32	.89 / 50	-.62 / .24	.77 / *	.10 / *
30-35	248	3.16	1.06	47/45	1.11 ±.19	-.53 ±.32	.18 ±.30	.02 ±.17	-.76 ±.14	.44 / 65	-.31 / .44	.67 / *	.08 / *
35-40	248	3.18	1.02	46/45	1.39 ±.26	-.92 ±.47	.27 ±.43	.00 ±.20	-.64 ±.22	.79 / 51	-.45 / .39	.59 / *	.001 / *

*One real root is negative; no corresponding frequency and damping coefficient.

TABLE 3.7

CORRELATION MATRIX OF THE ARMA (4,1)

PARAMETER ESTIMATES FOR IID056-N21E (0-5 SECONDS)

	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\phi}_3$	$\hat{\phi}_4$	$\hat{\theta}_1$
$\hat{\phi}_1$	1				
$\hat{\phi}_2$	-.94	1			
$\hat{\phi}_3$	.81	-.94	1		
$\hat{\phi}_4$	-.62	.77	-.91	1	
$\hat{\theta}_1$	.75	-.79	.76	-.65	1

CHAPTER 4

DIRECTIONS FOR FUTURE RESEARCH

Figure 4.1 gives a flow chart of the several stages in which data analyses and models of earthquake ground motion can be compared and described. As we discuss in Appendix A most of the models in the published literature concern themselves with stages 1, 2, 5 and 6. There has been much less emphasis on stages 3, 4 and 7 which include parameter estimation, statistical tests for goodness of fit, time-varying and non-stationary effects and correlations of site-specific characteristics with earthquake data. We also find that because the ARMA models are identified and their parameters are estimated directly from data in the time domain, they provide an extremely direct method of proceeding from the analysis of actual accelerograms to the generation of artificial accelerograms with the same characteristics. The number of steps from analysis to simulation is reduced since it is not necessary to convert back and forth between discrete and continuous models, or between time-domain and frequency-domain characterizations.

In the models that we discussed in Chapters 2 and 3 the emphasis has been on identifying both the order of the linear model and estimates of model parameters which satisfactorily describe actual earthquake records. Possibly the most important use of these models is in simulating typical or atypical earthquakes possibly in conjunction with structural response models. The idea behind such simulations would be to first correlate white noise through the use of one or more ARMA (2,1) or ARMA (4,1) filters of the type discussed in earlier chapters and

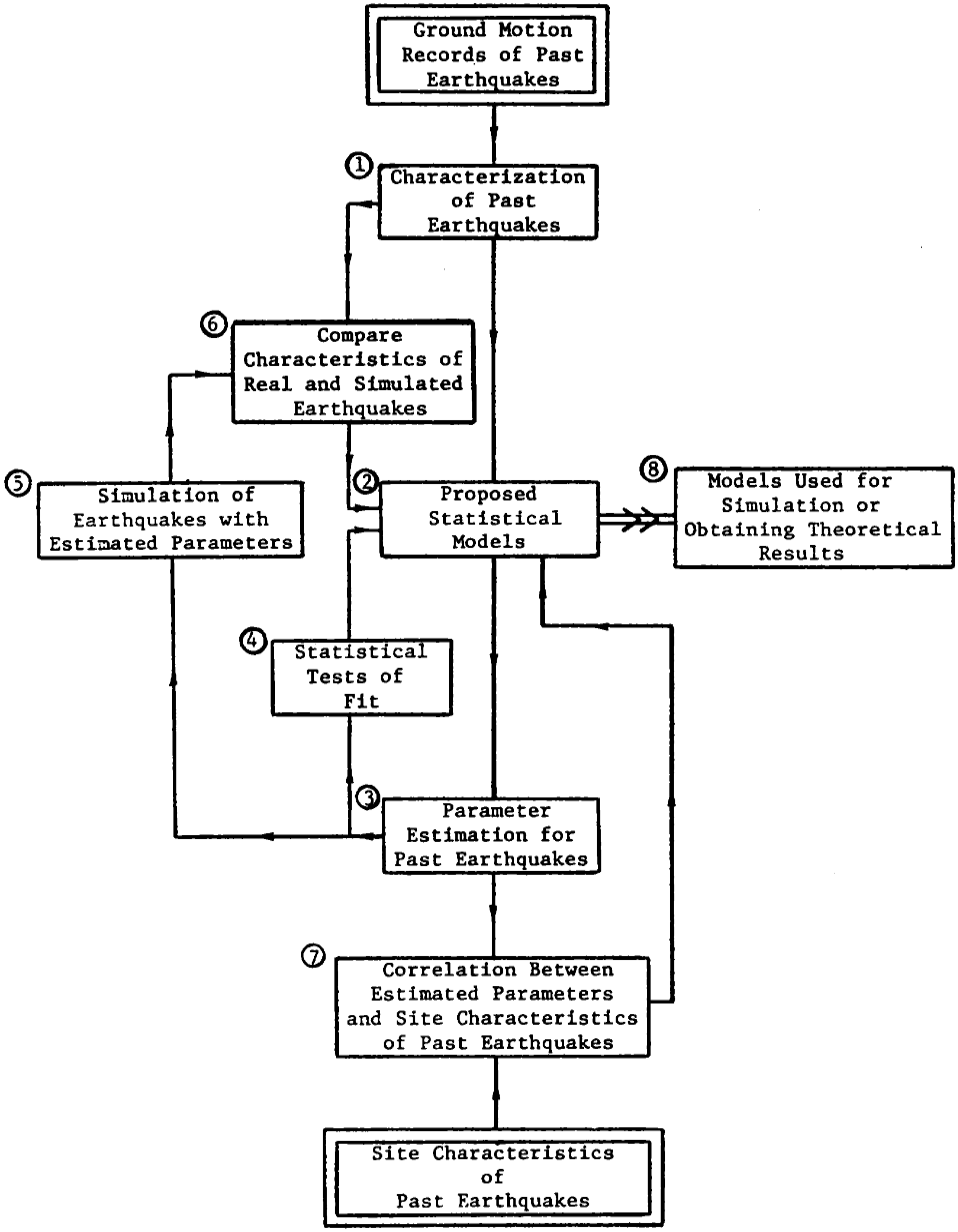

FIGURE 4.1

STAGES IN EARTHQUAKE MODELLING

eventually use the output of these filters as inputs for the forcing functions of the structural response equations.

Discrete models with constant parameters above can be used to model short segments of earthquake ground acceleration records over which the statistical characteristics are approximately constant. However, to model longer records it is necessary to incorporate nonstationary features, especially the build-up and decay in amplitude characteristic of real earthquakes. In filtered-noise models in the literature this is usually accomplished by applying an envelope (or "time-multiplier") function either to the white noise input or to the filtered noise output. The former approach has the advantage that it allows the transient response from nonstationary inputs to be reflected in the output of the filter. In an ARMA model this can be achieved by letting the noise input variance be an explicit function of time, i.e., $e_t \sim N\left(0,\sigma_e^2(t)\right)$. Some models in the literature (e.g., Amin and Ang, G[12]) have also attempted to explicitly account for nonstationary spectral characteristics by using filters with time-varying parameters. In an ARMA model this effect can also be achieved by letting the autoregressive and/or moving-average parameters be time-varying. Techniques such as the Kalman filter (described in Reference T[1]) can be used to estimate the time-histories of ARMA parameters in real accelerograms if they are thought to be time-varying.

In our opinion there appears to be several potentially productive areas for continuing research:

1. The identification, analysis and parameter estimation of a number of low and high intensity earthquakes at distinct sites including three directional acceleration data.
2. The design and development of discrete-time simulation models for earthquake ground motions yielding acceleration data outputs suitable for use as input data to linear and nonlinear models of structures.
3. The design and analysis of baseline correction filters to be used in conjunction with the simulation models described above. There are several promising first and second order ARMA filters which can be used to achieve baseline corrections.
4. The design and analysis of discrete time models that include nonstationary features in the variance of the white noise forcing functions and in the structure of the autoregressive and moving average parameters of these models.
5. Statistical comparison and validation of measures of goodness-of-fit between simulated and actual ground motions. These include comparison of the distributions of acceleration data, extreme value analysis, chi-square tests of forecast residuals, autocorrelations and partial autocorrelations as well as the more familiar spectral characteristics.
6. The design and analysis of scaling laws and indices which can be useful to the engineer in his or her analysis of "worst possible" ground motions based on extrapolation of existing historical observations and predicted earthquake magnitudes.

APPENDIX A

EXISTING MODELS FOR EARTHQUAKE GROUND MOTIONS

A.1 Introduction

This appendix describes some of the published methods that have been used to describe the accelerations of earthquake ground motions. In Figure 4.1 we gave a diagrammatic representation of the complete process. Most papers on models of earthquake ground motions are only concerned with particular aspects of the overall problem; however, in Iyengar and Iyengar (G[11]) most of the steps are included. Section 2 summarizes some common methods of characterizing earthquakes; Section 3 describes several models that have been proposed; and Section 4 lists different models for simulating structural responses to earthquakes. Simulated earthquakes may be used *either* for testing given models, *or*, when models are adequate, as inputs to structural response simulations. Although this report is not primarily concerned with structural response, we have included a short section on the subject because many papers on structural response refer to the use of simulated ground motions.

A.2 Analysis and Characterization of Earthquakes

Since, for the purposes of structural design, the dynamic response of many buildings may be approximated as that of a linear oscillator, it is common to characterize earthquakes in terms of their frequency contents. Two such characterizations of earthquakes with finite duration, T, are the *Response Spectrum* and the *Fourier Amplitude Spectrum*.

The *Response Spectrum*, for a given damping coefficient ξ , is defined as the maximum absolute "response" of a linear oscillator to the given earthquake, as a function of the oscillator natural frequency ω_o . The "response" may be defined as the output displacement $x(t)$, the pseudo velocity $x(t) \cdot \omega$, the pseudo acceleration $x(t) \cdot \omega^2$, or the quantity $r(t)$ given by $r^2(t) = (\omega \cdot x(t))^2 + (\dot{x}(t) + \xi \cdot \omega \cdot x(t))^2$. The *Fourier Amplitude Spectrum* is the quantity $F(\omega) = |\int_0^T e^{i\omega t} a(t)dt|$ as a function of frequency ω , where $a(t)$ is the ground acceleration.

Both these quantities are *properties* of the *particular* earthquake being analyzed. Response and Fourier Amplitude Spectra for several recorded earthquakes, together with the digitized accelerations, velocities, and displacements of ground motions have been compiled at the California Institute of Technology (Reference D[1]).

None of the above functions incorporates the time-varying nature of an earthquake. Among the functions which do take this into account are the *Response Envelope Spectrum* and the *Moving Window Fourier Amplitude Spectrum.*

The *Response Envelope Spectrum* (R.E.S.) (G[9]) is obtained, like the Response Spectrum, by using the earthquake record as input to a succession of linear oscillators (sometimes known as the "multifilter" technique), but plotting the "envelope" of the output as a function of both time *and* oscillator frequency. For the *Moving Window Fourier Amplitude Spectrum* (M.W.F.A.S.) (G[1]), one calculates the Fourier Amplitude Spectrum of the earthquake over relatively small time intervals, and plots this as a function of both time and frequency.

Besides frequency domain characteristics, there are time domain properties of an earthquake which are useful. One such property is the *Sample Autocorrelation Function* $\rho(s) = \frac{1}{T\sigma_a^2} \int_s^T a(\tau)a(\tau - s)d\tau$ where $\sigma_a^2 = \frac{1}{T} \int_0^T a^2(\tau)d\tau$. Another characterization, for nonstationary processes, is given in T[6] and T[7], using the concept of a *time-dependent autocorrelation function.*

The quantities mentioned above are not only useful in characterizing certain features of *given* earthquake records, they may also be used in *estimation* of the statistical properties of earthquakes in general. The distinction between the actual samples (empirical values) and the underlying statistical properties (e.g., expected values) is not always made clear in the literature.

For *stationary* and *ergodic* stochastic processes, the Response Spectrum may be used as an estimate of the *Expected Response Spectrum*, and the quantity $\frac{F(\omega)^2}{T}$ may be used as an estimate of the P.S.D.F. Similarly the Sample Autocorrelation Function may be used as an estimate of the true *Autocorrelation Function* of the random process of which the earthquake is a sample realization.

As mentioned in Chapters 1 and 2, descriptions of time series in terms of their spectral properties have equivalent formulations in terms of filtered white noise. For example, if $a(t)$ consists of white noise with variance σ_w^2 , filtered through an impulse response $h(t)$ with transfer function $H(\omega)$, the *Power Spectral Density Function* (P.S.D.F.) of $a(t)$ is given by $G(\omega) = \frac{\sigma_w^2}{2\pi} |H(\omega)|^2$. These frequency domain properties also have equivalent formulations in the time domain. For example, the P.S.D.F. is the Fourier Transform of the autocorrelation function.

For *nonstationary* processes, the R.E.S. may be used to estimate the *Expected Response Envelope Spectrum*, and the M.W.F.A.S. for estimating the *Evolutionary Power Spectral Density Function* (E.P.S.D.F.) as defined in T[11] and T[12]. Other estimates of the E.P.S.D.F. are given in G[3] (using the multifilter technique), and, for special cases, in G[10].

A.3 Existing Statistical Models for Earthquake Ground Motions

The models listed here include some which were invented without reference to earthquakes, but which were subsequently used in this connection. The types of models considered are those which are suitable for motions usually associated with strong earthquakes on firm ground, for which there are no dominating frequency components (T[5], Chapter 7.4).

For modelling ground motions, given that an earthquake has occurred, it is necessary to regard some quantities as *parameters* of the earthquake, and to model the ground motions as a random process *conditional on* the parameters. Duration of the earthquake and intensity are examples of what may be used as parameters.

Most probabilistic models for earthquake ground motions are based either on "shot noise" (G[5], G[22], R[1], R[3], T[3]), which is a stochastic process consisting of randomly arriving impulses with random amplitudes, or on "white noise" (differentiated Brownian motion), a limiting form of shot noise (T[5], Chapter 9.3). Ground motions are described by these models as being generated by a shot- or white-noise sequence.

For example, earthquakes are sometimes regarded as "bursts" of white noise, of finite duration. This model is used in R[4], R[7], and R[8]. A refinement to this model gives ground accelerations as finite segments of white noise passed through a linear time-invariant filter (G[20], G[21]).

However, since earthquakes are clearly nonstationary, in order to model ground motions more accurately, some time-variation must be introduced. The simplest method is to model the acceleration of an earthquake of given duration as a segment of a stationary random process (e.g., filtered noise) multiplied by a predetermined "time-multiplier" or "envelope function" $\psi(t)$:

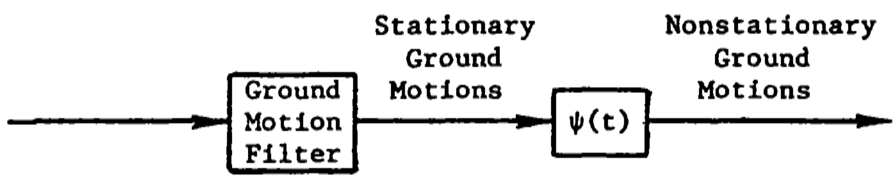

FIGURE A.1

USE OF A TIME MULTIPLIER

One commonly used shape for $\psi(t)$ is (G[14], G[15]):

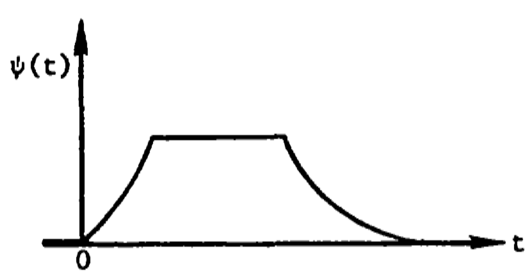

FIGURE A.2

EXAMPLE OF A TIME MULTIPLIER

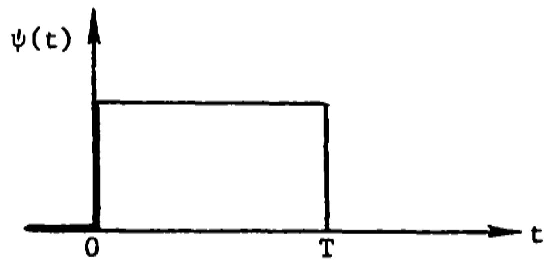

FIGURE A.3

TIME MULTIPLIER FOR BURST-OF-WHITE-NOISE MODEL

For a summary of other suggested shapes, see G[1] and G[5]. This type of model has been used for calculating theoretical response in R[6], R[11] and R[14].

Another approach is to suppose that the noise process which generates the motion is multiplied by a time-multiplier *before* passing through the ground motion filter (G[11], G[12]). A summary of the different time-multipliers that have been used is given in G[5].

Most of the ground motion filters used are simple 1-degree of freedom oscillators, of which the following is an example:

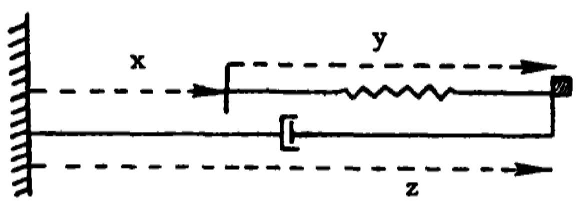

FIGURE A.4

A SIMPLE OSCILLATOR

z is the ground displacement, x is an "input" displacement, and

(A.1) $$\ddot{z} + 2\xi_c\omega_c\dot{z} + \omega_c^2 z = \omega_c^2 x \ .$$

Letting $a = \ddot{z}$, the ground acceleration, Equation (A.1) may be written as

$$\ddot{a} + 2\xi_c\omega_c\dot{a} + \omega_c^2 a = \omega_c^2 b \tag{A.2}$$

where b is the input acceleration, or as

$$\ddot{a} + 2\xi_c\omega_c\dot{a} + \omega_c^2 a = \omega_c^2\dot{c} \tag{A.3}$$

where c is the input velocity.

One kind of model (G[12]) assumes b is white noise, so that, in terms of white noise input, the frequency response function is

$$H(i\omega) = \frac{1}{\left(1 - \frac{\omega^2}{\omega_c^2}\right) + 2\xi_c \frac{i\omega}{\omega_c}} \tag{A.4}$$

with impulse response function

$$h(t) = Ce^{-\xi_c\omega_c t} \sin\left(\sqrt{1 - \xi_c^2}\,\omega_c t\right) . \tag{A.5}$$

Another model (G[17]) assumes that c is white noise. The frequency response in terms of white noise input is now

$$H(i\omega) = \frac{\omega}{\left(1 - \frac{\omega^2}{\omega_c^2}\right) + 2\xi_c \frac{i\omega}{\omega_c}} \tag{A.6}$$

with impulse response function

$$h(t) = De^{-\xi_c \omega_c t} \cos\left(\sqrt{1 - \xi_c^2}\,\omega_c t - \beta\right) \tag{A.7}$$

where

$$\beta = \tan^{-1}\left[\frac{1 - 2\xi_c^2}{2\xi_c \sqrt{1 - \xi_c^2}}\right] \tag{A.8}$$

As noted in Chapter 2, this model is equivalent to applying white noise input acceleration to the end of the dashpot, while holding the spring fixed.

The autocorrelation functions and power spectra for these processes may be found in Chapter 2. The latter model has the advantage that, for time-multipliers that tend to 0 as $t \to \infty$, the mean square velocity also tends to 0.

If, instead of Figure A.4, the oscillator is one found in many structural response studies:

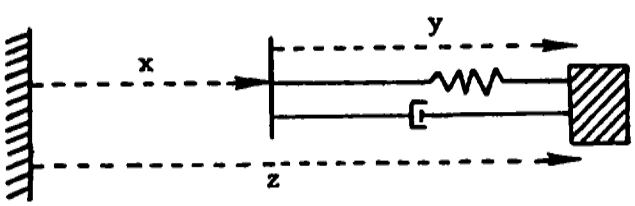

FIGURE A.5

ANOTHER SIMPLE OSCILLATOR

then Equation (A.2) and (A.4) become

$$\ddot{a} + 2\xi_c \omega_c \dot{a} + \omega_c^2 a = 2\xi_c \omega_c \dot{b} + \omega_c^2 b \ . \tag{A.9}$$

(A.10)
$$H(i\omega) = \frac{1 + 2\xi_c \frac{i\omega}{\omega_c}}{\left(1 - \frac{\omega^2}{\omega_c^2}\right) + 2\xi_c \frac{i\omega}{\omega_c}} .$$

In G[7] a model is given in which a white noise process is filtered before *and* after passing through a time-multiplier:

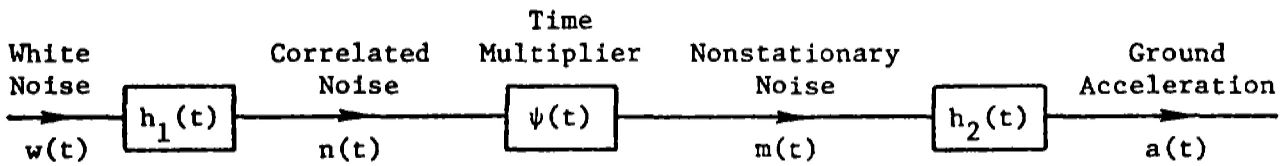

FIGURE A.6

GROUND MOTIONS AS FILTERED WHITE NOISE

Another method is to pass white noise through a *time-varying filter*. An example is given in G[20], based on the concept of the evolutionary spectrum (T[11] and T[12]), in which

(A.11)
$$G(\omega,t) \propto \frac{\left\{1 + 4\xi_c^2 \frac{\omega^2}{\omega_c^2(t)}\right\} \frac{1}{\omega_c(t)}}{\left\{\left(1 - \frac{\omega^2}{\omega_c^2(t)}\right)^2 + 4\xi_c^2 \frac{\omega^2}{\omega_c^2(t)}\right\}}$$

where $\omega_c(t)$ is a deterministic function of time:

(A.12)
$$\omega_c(t) = \frac{\gamma}{\beta t^c + 1} .$$

In G[1] the resulting process is multiplied by a function of the type $\psi(t) = (a_1 + a_2 t)e^{-\alpha t}$:

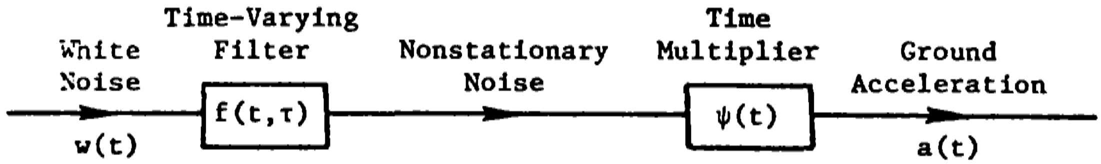

FIGURE A.7

GROUND MOTIONS AS FILTERED WHITE NOISE

A similar model, given in R[15], is

$$a(t) = \sum_{j=1}^{n} ta_j e^{-\alpha_j t} \cos (\omega_j t + \phi_j) \tag{A.13}$$

where a_j , α_j , and ω_j are given quantities and ϕ_j are random variables.

A.4 Structural Response

Theoretical work on structural response to earthquakes may broadly be divided into two categories: analytical results (R[1], R[3], R[4], R[6], R[7], R[11]) and results from simulation (R[2], R[8], R[12]). Many of the works referred to are not written with direct reference to structural response to earthquakes, but are phrased in more abstract terms.

Some works are concerned with maximum responses for linear and nonlinear oscillators (R[2], R[16]), others with first passage probabilities (R[3], R[9]). A variety of input models of the type discussed in Section 3 are used for representing ground motion. In G[2] and R[4] it is shown, based on work by Caughey and Stumpf [T[14]) and Hammond (T[9]), how the spectral properties of the response vary with time for certain types of transient input. An interesting approach, given in R[13] and R[10], finds the input, out of all possible inputs in a certain class, which gives the worst response to certain structures.

APPENDIX B

BIBLIOGRAPHY

The works listed here have been classified according to their main interests. The categories are Ground Motion (G), Data (D), Response of Structures (R), and Theoretical Background (T). Within categories, works are listed in reverse chronological order. These categories are very broad, and overlap in many cases.

Ground Motion (G)

G[1] Kubo, T. and J. Penzien (1976), "Time and Frequency Domain Analyses of Three-Dimensional Ground Motions, San Fernando Earthquake," EERC 76-6, University of California, Berkeley.

G[2] Gasparini, D. and E. H. Vanmarcke (1976), "Simulated Ground Motions Compatible with Prescribed Response Spectra," R76-4, Department of Civil Engineering, Massachusetts Institute of Technology.

G[3] Kameda, H. (1975), "Evolutionary Spectra of Seismogram by Multifilter," A.S.C.E., Vol. 101, No. EM6.

G[4] Udwadia, F. E. and M. D. Trifunac (1974), "Characterization of Response Spectra Through the Statistics of Oscillator Response," B.S.S.A., Vol. 64, No. 1.

G[5] Shinozuka, M. (1973), "Digital Simulation of Ground Accelerations," Paper 360, Fifth World Conference on Earthquake Engineering, Rome, Italy.

G[6] Shinozuka, M. and C.-M. Jan (1972), "Digital Simulation of Random Processes and Its Applications," Journal of Sound and Vibration, Vol. 25, No. 1.

G[7] Levy, R., F. Kozin and R. B. B. Moorman (1971), "Random Processes for Earthquake Simulation," A.S.C.E., Vol. 97, No. EM2.

G[8] Trifunac, M. D. (1971), "Introduction to Volume II in 'Strong Motion Earthquake Accelerograms,'" EERL 73-03, California Institute of Technology.

G[9] Trifunac, M. D. (1970), "Response Envelope Spectrum of Strong Earthquake Ground Motions," EERL 70-06, California Institute of Technology.

G[10] Liu, S. C. (1970), "Evolutionary Power Spectral Density of Strong-Motion Earthquakes," B.S.S.A., Vol. 60, No. 3.

G[11] Iyengar, R. N. and K.T.S.R. Iyengar (1969), "A Non-Stationary Random Process Model for Earthquake Accelerograms," B.S.S.A., Vol. 59, No. 3.

G[12] Amin, M. and H. S. Ang (1968), "A Nonstationary Model for Strong Motion Earthquakes," Structural Research Series No. 306, Civil Engineering Studies, University of Illinois.

G[13] Toki, K. (1968), "Simulation of Earthquake Motion and Its Application," Bulletin of the Disaster Prevention Research Institute, Vol. II-A, March 1968, Kyoto University, (in Japanese).

G[14] Liu, S. C. (1968), "Statistical Analysis and Stochastic Simulation of Ground Motion Data," Bell Systems Technical Journal, December 1968.

G[15] Jennings, P. C., G. W. Housner and N. C. Tsai (1968), "Simulated Earthquake Motions," Earthquake Engineering Research Laboratory, California Institute of Technology.

G[16] Shinozuka, M. and Y. Sato (1967), "Simulation of Non-stationary Random Processes," A.S.C.E., Vol. 93, No. EM1.

G[17] Housner, G. W. and P. C. Jennings (1964), "Generation of Artificial Earthquakes," A.S.C.E., Vol. 90, No. EM1.

G[18] Hudson, D. E. (1962), "Some Problems in the Application of Spectrum Techniques to Strong-Motion Earthquake Analysis," B.S.S.A., Vol. 52.

G[19] Berg, G. V. and G. W. Housner (1961), "Integrated Velocity and Displacement of Strong Earthquake Ground Motion," Bulletin of the Seismological Society of America, Vol. 51, No. 2.

G[20] Kanai, K. (1957), "Some Empirical Formulas for the Seismic Characteristics of the Ground," Bulletin of the Earthquake Research Institute, Vol. 35.

G[21] Thomson, W. T. (1959), "Spectral Aspects of Earthquakes," B.S.S.A., Vol. 49, No. 1.

G[22] Housner, G. W. (1947), "Characteristics of Strong Motion Earthquakes," B.S.S.A., Vol. 37, No. 1.

Data (D)

D[1] Hudson, D. E., A. G. Brady and M. D. Trifunac (1971, 1972, 1973), "Strong Motion Accelerograms, Digitized and Plotted Data, Volume II: Corrected Accelerograms and Integrated Velocity and Displacement Curves." Part A, EERL 71-50; Part C, EERL 72-51; Part D, EERL 72-52. Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena.

D[2] California Institute of Technology, Earthquake Engineering Research Laboratory, "Index to Strong Motion Earthquake Accelerograms," EERL 76-02.

D[3] Willie, R. R. (1977), "Everyman's Guide to TIMES," ORC 77-2, Operations Research Center, University of California, Berkeley.

D[4] Trifunac, M. D., F. E. Udwadia and A. G. Brady (1971), "High Frequency Errors and Instrument Corrections of Strong Motion Accelerograms," EERL 71-05, Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena.

D[5] Trifunac, M. D. (1970), "Low Frequency Digitization Errors and a New Method for Zero Baseline Correction of Strong Motion Accelerograms," EERL 70-07, Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena.

D[6] Trifunac, M. D. and D. E. Hudson (1970), "Laboratory Evaluations and Instrument Corrections of Strong Motion Accelerographs," EERL 70-04, Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena.

Response of Structures (R)

R[1] Wen, J.-K. (1976), "Methods for Random Vibration of Hysteretic Systems," A.S.C.E., Vol. 102, No. EM2.

R[2] Chokshi, N. C. and L. D. Lutes (1976), "Maximum Response Statistics for Yielding Oscillator," A.S.C.E., Vol. 102, No. EM6.

R[3] Yan, J.-N. (1975), "Approximations to First Passage Probability," A.S.C.E., Vol. 101, No. EM4.

R[4] Corotis, R. B. and A. M. Vanmarcke (1975), "Time-Dependent Content of System Response," A.S.C.E., Vol. 101, No. 5.

R[5] Murakami, M. and J. Penzien (1975), "Nonlinear Response Spectra for Probabilistic Seismic Design and Damage Assessment of Reinforced Concrete Structures," EERC 75-38, University of California, Berkeley.

R[6] Holman, R. E. and G. G. Hart (1974), "Nonstationary Response of Structural Systems," A.S.C.E., Vol. 100, No. EM2.

R[7] Lutes, L. D. and H. Takemiya (1974), "Random Vibration of Yielding Oscillators," A.S.C.E., Vol. 100, No. EM2.

R[8] Lutes, L. D. and V. S. Shah (1973), "Transient Random Response of Bilinear Oscillators," A.S.C.E., Vol. 99, No. EM4.

R[9] Corotis, R. B., A. M. Vanmarcke and C. A. Cornell (1972), "First Passage of Nonstationary Random Processes," A.S.C.E., Vol. 98, No. EM2.

R[10] Iyengar, R. N. (1972), "Worst Inputs and a Bound on the Highest Peak Statistics of a Class of Non-Linear Systems," Journal of Sound and Vibration, Vol. 25, No. 1.

R[11] Hasselman, T. K. (1972), "Linear Response to Nonstationary Random Excitation," A.S.C.E., Vol. 98, No. EM3.

R[12] Ruiz, P. and J. Penzien (1971), "Stochastic Seismic Response of Structures," A.S.C.E., Vol. 97, No. EM2.

R[13] Shinozuka, M. (1970), "Maximum Structural Response to Seismic Excitations," A.S.C.E., Vol. 96, No. EM5.

R[14] Hart, G. G. (1970), "Stochastic Frame Response Using Modal Truncation," A.S.C.E., Vol. 96, No. EM5.

R[15] Bogdanoff, J. L., J. E. Goldberg and M. C. Bernard (1961), "Response of a Simple Structure to a Random Earthquake-Type Disturbance," B.S.S.A., Vol. 51.

R[16] Tajima, H. (1960), "A Statistical Model of Determining the Maximum Response of a Building Structure During an Earthquake," Proceedings of the Second World Conference on Earthquake Engineering, Tokyo and Kyoto.

Theoretical Background (T)

T[1] Nau, R. F. and R. M. Oliver (1978), "Adaptive Filtering Revisited," ORC 78-11, Operations Research Center, University of California, Berkeley.

T[2] Box, G. E. P. and G. M. Jenkins (1976), TIMES SERIES ANALYSIS: FORECASTING AND CONTROL, Revised Edition, Holden-Day, San Francisco.

T[3] Shinozuka, M., P. Wai and R. Vaicaitis (1976), "Simulation of a Filtered Poisson Process," Technical Report No. 6, Department of Civil Engineering and Engineering Mechanics, Columbia University.

T[4] Brillinger, D. R. (1975), TIME SERIES DATA ANALYSIS AND THEORY, Holt, Rhinehart and Winston.

T[5] Newmark, N. M. and E. Rosenblueth (1971), FUNDAMENTALS OF EARTHQUAKE ENGINEERING, Prentice-Hall, New Jersey.

T[6] Bendat, J. S. and A. G. Piersol (1971), RANDOM DATA: ANALYSIS AND MEASUREMENT ANALYSIS, Wiley, New York.

T[7] Mark, W. D. (1970), "Spectral Analysis of the Convolution and Filtering of Non-Stationary Stochastic Processes," Journal of Sound and Vibration, Vol. 2.

T[8] Jenkins, G. M. and D. G. Watts (1968), SPECTRAL ANALYSIS AND ITS APPLICATIONS, Holden-Day, San Francisco.

T[9] Hammond, J. K. (1968), "On the Response of Single and Multidegree of Freedom Structures to Nonstationary Random Vibrations," Journal of Sound and Vibration, Vol. 7.

T[10] Cramer, H. and M. R. Leadbetter (1967), STATIONARY AND RELATED STOCHASTIC PROCESSES, Wiley, New York.

T[11] Priestly, M. B. (1967), "Power Spectral Analysis of Non-stationary Processes," Journal of Sound and Vibration, Vol. 6.

T[12] Priestly, M. B. (1965), "Evolutionary Spectra and Non-stationary Processes," Journal of the Royal Statistical Society, Series B, Vol. 27.

T[13] Yaglom, A. M. (1962), AN INTRODUCTION TO THE THEORY OF STATIONARY RANDOM FUNCTIONS, Prentice-Hall, New Jersey.

T[14] Caughey, T. K. and H. J. Stumpf (1961), "Transient Response of a Dynamic System Under Random Excitations," Journal of Applied Mechanics, Vol. 28.

T[15] Bartlett, M. S. (1946), "On the Theoretical Specification and Sampling Properties of Autocorrelated Time Series," Journal of the Royal Statistical Society, Series B, Vol. 27.

T[16] Rice, S. O. (1944), "Mathematical Analysis of Random Noise," Bell Systems Technical Journal.

EARTHQUAKE ENGINEERING RESEARCH CENTER REPORTS

NOTE: Numbers in parenthesis are Accession Numbers assigned by the National Technical Information Service; these are followed by a price code. Copies of the reports may be ordered from the National Technical Information Service, 5285 Port Royal Road, Springfield, Virginia, 22161. Accession Numbers should be quoted on orders for reports (PB --- ---) and remittance must accompany each order. Reports without this information were not available at time of printing. Upon request, EERC will mail inquirers this information when it becomes available.

EERC 67-1 "Feasibility Study Large-Scale Earthquake Simulator Facility," by J. Penzien, J.G. Bouwkamp, R.W. Clough and D. Rea - 1967 (PB 187 905)A07

EERC 68-1 Unassigned

EERC 68-2 "Inelastic Behavior of Beam-to-Column Subassemblages Under Repeated Loading," by V.V. Bertero - 1968 (PB 184 888)A05

EERC 68-3 "A Graphical Method for Solving the Wave Reflection-Refraction Problem," by H.D. McNiven and Y. Mengi - 1968 (PB 187 943)A03

EERC 68-4 "Dynamic Properties of McKinley School Buildings," by D. Rea, J.G. Bouwkamp and R.W. Clough - 1968 (PB 187 902)A07

EERC 68-5 "Characteristics of Rock Motions During Earthquakes," by H.B. Seed, I.M. Idriss and F.W. Kiefer - 1968 (PB 188 338)A03

EERC 69-1 "Earthquake Engineering Research at Berkeley," - 1969 (PB 187 906)A11

EERC 69-2 "Nonlinear Seismic Response of Earth Structures," by M. Dibaj and J. Penzien - 1969 (PB 187 904)A08

EERC 69-3 "Probabilistic Study of the Behavior of Structures During Earthquakes," by R. Ruiz and J. Penzien - 1969 (PB 187 886)A06

EERC 69-4 "Numerical Solution of Boundary Value Problems in Structural Mechanics by Reduction to an Initial Value Formulation," by N. Distefano and J. Schujman - 1969 (PB 187 942)A02

EERC 69-5 "Dynamic Programming and the Solution of the Biharmonic Equation," by N. Distefano - 1969 (PB 187 941)A03

EERC 69-6 "Stochastic Analysis of Offshore Tower Structures," by A.K. Malhotra and J. Penzien - 1969 (PB 187 903)A09

EERC 69-7 "Rock Motion Accelerograms for High Magnitude Earthquakes," by H.B. Seed and I.M. Idriss - 1969 (PB 187 940)A02

EERC 69-8 "Structural Dynamics Testing Facilities at the University of California, Berkeley," by R.M. Stephen, J.G. Bouwkamp, R.W. Clough and J. Penzien - 1969 (PB 189 111)A04

EERC 69-9 "Seismic Response of Soil Deposits Underlain by Sloping Rock Boundaries," by H. Dezfulian and H.B. Seed 1969 (PB 189 114)A03

EERC 69-10 "Dynamic Stress Analysis of Axisymmetric Structures Under Arbitrary Loading," by S. Ghosh and E.L. Wilson 1969 (PB 189 026)A10

EERC 69-11 "Seismic Behavior of Multistory Frames Designed by Different Philosophies," by J.C. Anderson and V. V. Bertero - 1969 (PB 190 662)A10

EERC 69-12 "Stiffness Degradation of Reinforcing Concrete Members Subjected to Cyclic Flexural Moments," by V.V. Bertero, B. Bresler and H. Ming Liao - 1969 (PB 202 942)A07

EERC 69-13 "Response of Non-Uniform Soil Deposits to Travelling Seismic Waves," by H. Dezfulian and H.B. Seed - 1969 (PB 191 023)A03

EERC 69-14 "Damping Capacity of a Model Steel Structure," by D. Rea, R.W. Clough and J.G. Bouwkamp - 1969 (PB 190 663)A06

EERC 69-15 "Influence of Local Soil Conditions on Building Damage Potential during Earthquakes," by H.B. Seed and I.M. Idriss - 1969 (PB 191 036)A03

EERC 69-16 "The Behavior of Sands Under Seismic Loading Conditions," by M.L. Silver and H.B. Seed - 1969 (AD 714 982)A07

EERC 70-1 "Earthquake Response of Gravity Dams," by A.K. Chopra - 1970 (AD 709 640)A03

EERC 70-2 "Relationships between Soil Conditions and Building Damage in the Caracas Earthquake of July 29, 1967," by H.B. Seed, I.M. Idriss and H. Dezfulian - 1970 (PB 195 762)A05

EERC 70-3 "Cyclic Loading of Full Size Steel Connections," by E.P. Popov and R.M. Stephen - 1970 (PB 213 545)A04

EERC 70-4 "Seismic Analysis of the Charaima Building, Caraballeda, Venezuela," by Subcommittee of the SEAONC Research Committee: V.V. Bertero, P.F. Fratessa, S.A. Mahin, J.H. Sexton, A.C. Scordelis, E.L. Wilson, L.A. Wyllie, H.B. Seed and J. Penzien, Chairman - 1970 (PB 201 455)A06

EERC 70-5 "A Computer Program for Earthquake Analysis of Dams," by A.K. Chopra and P. Chakrabarti - 1970 (AD 723 994)A05

EERC 70-6 "The Propagation of Love Waves Across Non-Horizontally Layered Structures," by J. Lysmer and L.A. Drake 1970 (PB 197 896)A03

EERC 70-7 "Influence of Base Rock Characteristics on Ground Response," by J. Lysmer, H.B. Seed and P.B. Schnabel 1970 (PB 197 897)A03

EERC 70-8 "Applicability of Laboratory Test Procedures for Measuring Soil Liquefaction Characteristics under Cyclic Loading," by H.B. Seed and W.H. Peacock - 1970 (PB 198 016)A03

EERC 70-9 "A Simplified Procedure for Evaluating Soil Liquefaction Potential," by H.B. Seed and I.M. Idriss - 1970 (PB 198 009)A03

EERC 70-10 "Soil Moduli and Damping Factors for Dynamic Response Analysis," by H.B. Seed and I.M. Idriss - 1970 (PB 197 869)A03

EERC 71-1 "Koyna Earthquake of December 11, 1967 and the Performance of Koyna Dam," by A.K. Chopra and P. Chakrabarti 1971 (AD 731 496)A06

EERC 71-2 "Preliminary In-Situ Measurements of Anelastic Absorption in Soils Using a Prototype Earthquake Simulator," by R.D. Borcherdt and P.W. Rodgers - 1971 (PB 201 454)A03

EERC 71-3 "Static and Dynamic Analysis of Inelastic Frame Structures," by F.L. Porter and G.H. Powell - 1971 (PB 210 135)A06

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This report contains an analysis of four major California earthquake records using a class of discrete linear time-domain processes commonly referred to as ARMA (Autoregressive/Moving-Average) models. It has been possible to analyze these different earthquakes, identify the order of the appropriate ARMA model(s), estimate parameters and test the residuals generated by these models. It has also been possible to show the connections, similarities and differences between the traditional continuous models (with parameters estimated by various maximum likelihood techniques applied to digitized acceleration data in the time domain. The methodology proposed in this report is suitable for simulating earthquake ground motions in the time domain and appears to be easily adapted to serve as inputs for nonlinear discrete time models of structural motions.

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