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ML & DL : Experiment - 6

Aim: Apply K-Means and Hierarchical Clustering on sample datasets.

Dataset Description:

The **Wholesale Customers Dataset** is a comprehensive business-to-business (B2B) analytics dataset containing the annual spending of diverse clients of a wholesale distributor. The primary objective is to enable the identification of natural customer segments based on purchasing behavior across multiple product categories, such as fresh food, groceries, and dairy.

This dataset captures essential attributes related to commercial consumption patterns, providing insights into the scale and variety of inventory required by different business types (e.g., Hotels, Restaurants, or Retailers). Unlike supervised learning, this dataset lacks a predefined target variable; instead, it is designed for unsupervised techniques like **K-Means** and **Hierarchical Clustering** to discover groupings based on volume-based similarity measures.

Due to its continuous numerical features and high variance, the dataset is ideal for demonstrating the necessity of **Log Transformation** and **Standardization** in clustering workflows. By applying these preprocessing steps, the dataset allows for the meaningful identification of high-value vs. low-value business segments and the interpretation of strategic purchasing trends.

- **File Type:** CSV (Comma Separated Values)
+1
- **Dataset Size:** 440 rows × 8 columns
- **Target Variable:** Not applicable (unsupervised learning)

Dataset Source:

<https://archive.ics.uci.edu/ml/machine-learning-databases/00292/Wholesale%20customers%20data.csv>

K - Means Clustering

Theory:

Hierarchical Clustering is an unsupervised learning technique used to group data points into clusters based on their similarity, without requiring a predefined number of clusters. Unlike partition-based methods such as K-Means, hierarchical clustering builds a multi-level hierarchy of clusters, which is typically visualized using a dendrogram. This tree-like structure represents how individual data points are progressively merged (or split) into larger clusters based on a

chosen distance metric and linkage criterion.

$$X = \{x_1, x_2, x_3, \dots, x_n\}, \quad x_i \in \mathbb{R}^d$$

The goal of K-Means is to partition the dataset into K disjoint clusters:

$$C = \{C_1, C_2, \dots, C_K\}$$

The most commonly used form in practical applications is agglomerative hierarchical clustering (bottom-up approach). In this method, each data point initially forms its own cluster, and pairs of clusters are iteratively merged based on their similarity until all points belong to a single cluster. The similarity between clusters is computed using different linkage methods such as single linkage, complete linkage, average linkage, or Ward's method. Each linkage strategy defines how the distance between clusters is measured and influences the final cluster structure.

Given a dataset with n data points and d features, hierarchical clustering aims to organize the data into a nested sequence of clusters that captures the underlying structure of the data. By analyzing the dendrogram, an appropriate number of clusters can be selected by cutting the tree at a suitable height. This makes hierarchical clustering especially useful when the optimal number of clusters is not known in advance.

Each cluster C_k is represented by a centroid μ_k , defined as the mean of all points assigned to that cluster:

$$\mu_k = \frac{1}{|C_k|} \sum_{x_i \in C_k} x_i$$

Limitations:

1. High Computational Complexity:

Hierarchical clustering has a computational complexity of $O(n^2)$, which makes it resource-intensive for large datasets. As the number of customers increases, the time and memory required to compute pairwise distances and construct the dendrogram grow rapidly. This limits its scalability for very large retail databases.

2. Sensitivity to Noise and Outliers:

Outliers can significantly influence the cluster formation process in hierarchical clustering. Since early merges or splits cannot be undone, the presence of anomalous customer records may distort the dendrogram structure and lead to less meaningful clusters.

3. Irreversibility of Cluster Merging:

In agglomerative hierarchical clustering, once two clusters are merged, the decision cannot be reversed. If an incorrect merge occurs at an early stage, it can propagate through subsequent levels of the hierarchy and negatively affect the final cluster structure.

4. Dependence on Distance Metric and Linkage Method:

The quality of clustering results is highly dependent on the chosen **distance metric** (e.g., Euclidean distance) and **linkage criterion** (single, complete, average, or Ward's method). Different choices can lead to different dendrogram structures and cluster interpretations, requiring careful experimentation and validation.

Workflow

1. Data Collection: The Wholesale Customers dataset is loaded into a Pandas DataFrame from the UCI Machine Learning Repository. The dataset contains annual spending records across categories like Fresh, Milk, Grocery, Frozen, Detergents_and_Paper, and Delicassen.

2. Data Cleaning and Preparation: Categorical identifiers such as 'Channel' and 'Region' are removed to focus strictly on behavioral spending data. The dataset is inspected for missing values and inconsistencies to ensure data quality.

3. Feature Selection: Relevant numerical features representing purchasing volume across all six product categories are selected. These attributes best represent the consumption behavior and profiles of different business clients.

4. Log Transformation (Dataset Specific): To handle the heavy skewness and high variance common in wholesale spending data, a log transformation is applied. This compresses the range of values, making the distribution more Gaussian and improving the performance of distance-based algorithms.

5. Feature Scaling: All selected numerical features are standardized using `StandardScaler` to ensure each variable contributes equally to distance calculations. This prevents categories with higher absolute spending (e.g., Grocery) from dominating the model.

6. Distance Computation: Pairwise distances between wholesale clients are computed using the Euclidean distance metric to measure similarity between data points.

7. Model Selection (Hierarchical Clustering): Agglomerative Hierarchical Clustering is applied using **Ward's linkage**. This method merges clusters by minimizing the increase in within-cluster variance, which is ideal for creating compact and well-separated segments.

8. Dendrogram Analysis: A dendrogram is constructed to visualize the hierarchical relationships among clients. The optimal number of clusters is determined by identifying the largest vertical gap (the cut level) in the dendrogram where significant increases in linkage distance occur.

9. Final Cluster Formation: Based on the identified dendrogram cut, each wholesale client is assigned a final cluster label. This structural validation ensures that the groupings correspond to natural partitions in the data.

10. Cluster Profiling and Visualization: Cluster-wise summary statistics (mean spending per category) are computed to interpret behavioral differences. Principal Component Analysis (PCA) is used to project the high-dimensional spending data into a 2D scatter plot to illustrate the clear separation between segments.

Performance Analysis:

The performance of the **Hierarchical Clustering** model on the **Mall Customers dataset** is evaluated using **internal validation techniques and visual analysis**, as no ground-truth labels are available in unsupervised learning. The evaluation focuses on cluster separation, structural clarity, stability, and interpretability using **dendrogram analysis, distance-based inspection, and cluster profiling**.

1. **Determination of Optimal Number of Clusters (Dendrogram Analysis):** The dendrogram provides a hierarchical view of how individual customers are merged into larger clusters. A clear increase in linkage distance at higher levels of the dendrogram indicates natural separation points in the data. By selecting a suitable cut level in the dendrogram, an optimal number of customer clusters is identified. This cut reveals distinct groupings that balance cluster compactness and interpretability, without over-fragmenting the dataset.

2. Cluster Separation and Visualization:

Two-dimensional visualizations based on selected features such as **Annual Income vs. Spending Score** show clear separation between customer clusters. Most customers are grouped into compact and well-defined regions, with limited overlap between clusters. This indicates that hierarchical clustering successfully captures meaningful structure in customer purchasing behavior.

3. Cluster Size Distribution:

The distribution of customers across clusters shows that each cluster contains a reasonable number of data points, with no extremely small or disproportionately large clusters. This balanced distribution suggests that the clustering solution is stable and not dominated by noise or outliers.

4. Cluster Profiling and Interpretation:

Cluster-wise summary statistics (such as average income, mean spending score, and age distribution) highlight clear differences in purchasing behavior across clusters. For example, one cluster may represent **high-income, high-spending customers**, while another may correspond to **moderate-income, moderate-spending customers**. These distinctions validate the practical usefulness of the clustering results for customer segmentation and targeted marketing strategies.

Hyperparameter Tuning:

Hierarchical clustering does not require explicit specification of the number of clusters during model fitting; however, the quality of clustering depends heavily on the choice of **distance metric** and **linkage method**. Therefore, careful tuning and evaluation of these parameters are essential to obtain meaningful and stable customer segments.

The following aspects are considered during model tuning:

1. Linkage Method Selection:

Different linkage strategies such as **single linkage**, **complete linkage**, **average linkage**, and **Ward's method** are evaluated. Ward's method generally produces more compact and well-separated clusters by minimizing within-cluster variance, making it suitable for customer segmentation tasks.

2. Distance Metric:

The **Euclidean distance** metric is commonly used for numerical features such as income and spending score. Alternative distance measures may be explored to analyze their impact on cluster formation and separation.

3. Dendrogram Cut-Level (Number of Clusters):

The final number of clusters is determined by selecting an appropriate cut level on the dendrogram. This cut-level acts as an implicit hyperparameter, controlling the granularity of segmentation. The chosen cut balances interpretability with cluster compactness.

4. Feature Selection and Scaling:

The choice of input features and proper **feature scaling** significantly influence clustering results. Standardization ensures that all features contribute equally to distance computation, leading to more meaningful and stable clusters.

Code & Output:

```
import pandas as pd

import numpy as np

import matplotlib.pyplot as plt

import seaborn as sns

from sklearn.preprocessing import StandardScaler

from sklearn.cluster import KMeans

from sklearn.metrics import silhouette_score

from sklearn.decomposition import PCA

from scipy.cluster.hierarchy import dendrogram, linkage

# 1. Load Dataset directly for Google Colab

# Using the Wholesale Customers dataset from UCI Repository

url =
```

```
"https://archive.ics.uci.edu/ml/machine-learning-databases/00292/Wholesale%20customers%20data.csv"

df = pd.read_csv(url)

# 2. Preprocessing

# We focus on the 6 product categories for clustering

# Dropping 'Channel' and 'Region' as they are categorical identifiers

cols_to_drop = ['Channel', 'Region']

df_numeric = df.drop(columns=cols_to_drop)

# NOTE: Wholesale data is often heavily skewed.

# Applying Log Transformation helps distance-based algorithms like K-Means/Hierarchical.

df_log = np.log1p(df_numeric)

# Handle any potential missing values (though this dataset is usually clean)

df_log = df_log.fillna(df_log.mean())

print("Columns used for clustering:", df_log.columns.tolist())
print(df_log.head())

# 3. Feature Scaling

# Standardization is critical for distance-based clustering [cite: 504, 505]

scaler = StandardScaler()

X_scaled = scaler.fit_transform(df_log)
```

```
# 4. Finding Optimal K (Elbow Method & Silhouette Score)

wcss = []

silhouette_scores = []

K_range = range(2, 11)

for k in K_range:

    kmeans = KMeans(n_clusters=k, random_state=42, n_init=10)

    labels = kmeans.fit_predict(X_scaled)

    wcss.append(kmeans.inertia_)

    silhouette_scores.append(silhouette_score(X_scaled, labels))

# Plotting Elbow and Silhouette side-by-side [cite: 598]

plt.figure(figsize=(14, 5))

plt.subplot(1, 2, 1)

plt.plot(range(2, 11), wcss, marker='o', color='blue')

plt.title('Elbow Method (Wholesale Data)')

plt.xlabel('Number of Clusters (K)')

plt.ylabel('WCSS')

plt.subplot(1, 2, 2)

plt.plot(K_range, silhouette_scores, marker='o', color='purple')

plt.title('Silhouette Score vs K')

plt.xlabel('Number of Clusters (K)')

plt.ylabel('Silhouette Score')

plt.show()
```

```
# Optimal K selection based on the highest Silhouette Score [cite: 613]

optimal_k = K_range[np.argmax(silhouette_scores)]

print(f"Optimal number of clusters based on Silhouette Score: {optimal_k}")

# 5. Final K-Means Clustering

kmeans = KMeans(n_clusters=optimal_k, random_state=42, n_init=10)

cluster_labels = kmeans.fit_predict(X_scaled)

# 6. PCA for 2D Visualization [cite: 619, 620]

# Reducing 6 features to 2 components for visual inspection

pca = PCA(n_components=2)

X_pca = pca.fit_transform(X_scaled)

plt.figure(figsize=(8, 6))

scatter = plt.scatter(X_pca[:, 0], X_pca[:, 1], c=cluster_labels,
                      cmap='viridis', s=60, edgecolor='k')

plt.xlabel("PCA Component 1")

plt.ylabel("PCA Component 2")

plt.title(f"Wholesale Segments (K-Means K={optimal_k})")

plt.colorbar(scatter, label="Cluster ID")

plt.show()

# 7. Hierarchical Clustering (Dendrogram) [cite: 630, 751]

# Using Ward Linkage to minimize within-cluster variance [cite:
```

819]

```
linked = linkage(X_scaled, method='ward')

plt.figure(figsize=(12, 7))

dendrogram(linked, truncate_mode='level', p=5)

plt.title("Hierarchical Clustering Dendrogram (Wholesale Data)")

plt.xlabel("Customer Clusters/Indices")

plt.ylabel("Euclidean Distance")

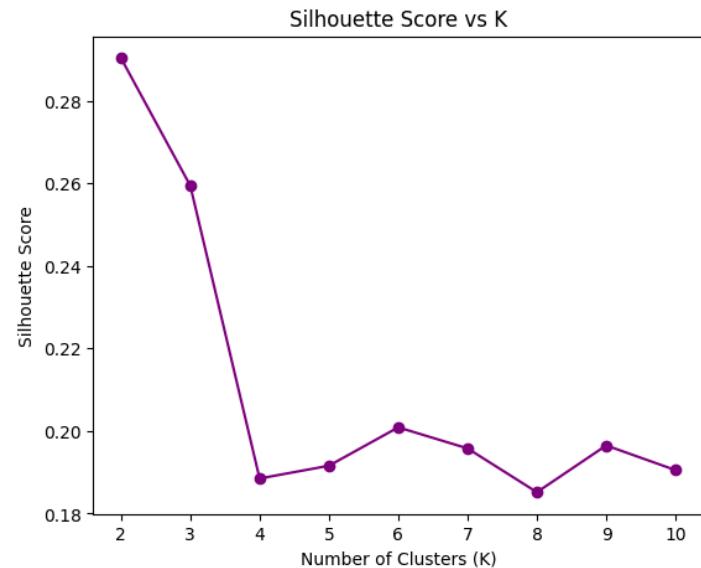
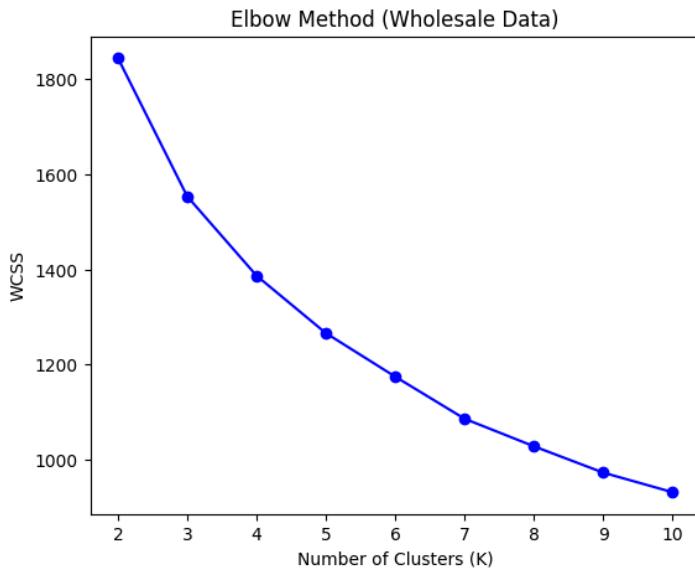
# Visual threshold for optimal cut

# Adjusting threshold y-value based on the log-scaled distribution

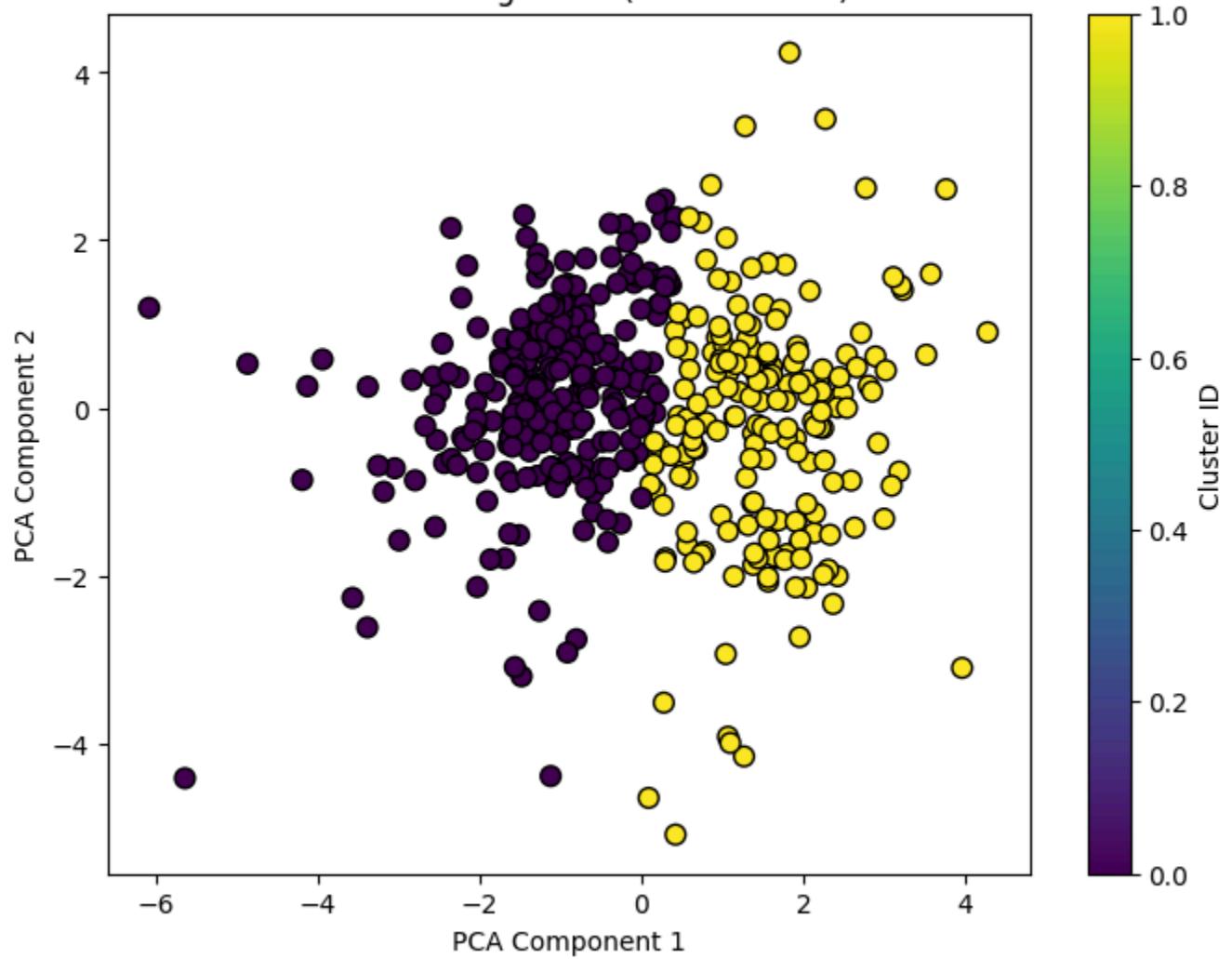
plt.axhline(y=20, color='r', linestyle='--')

plt.show()
```

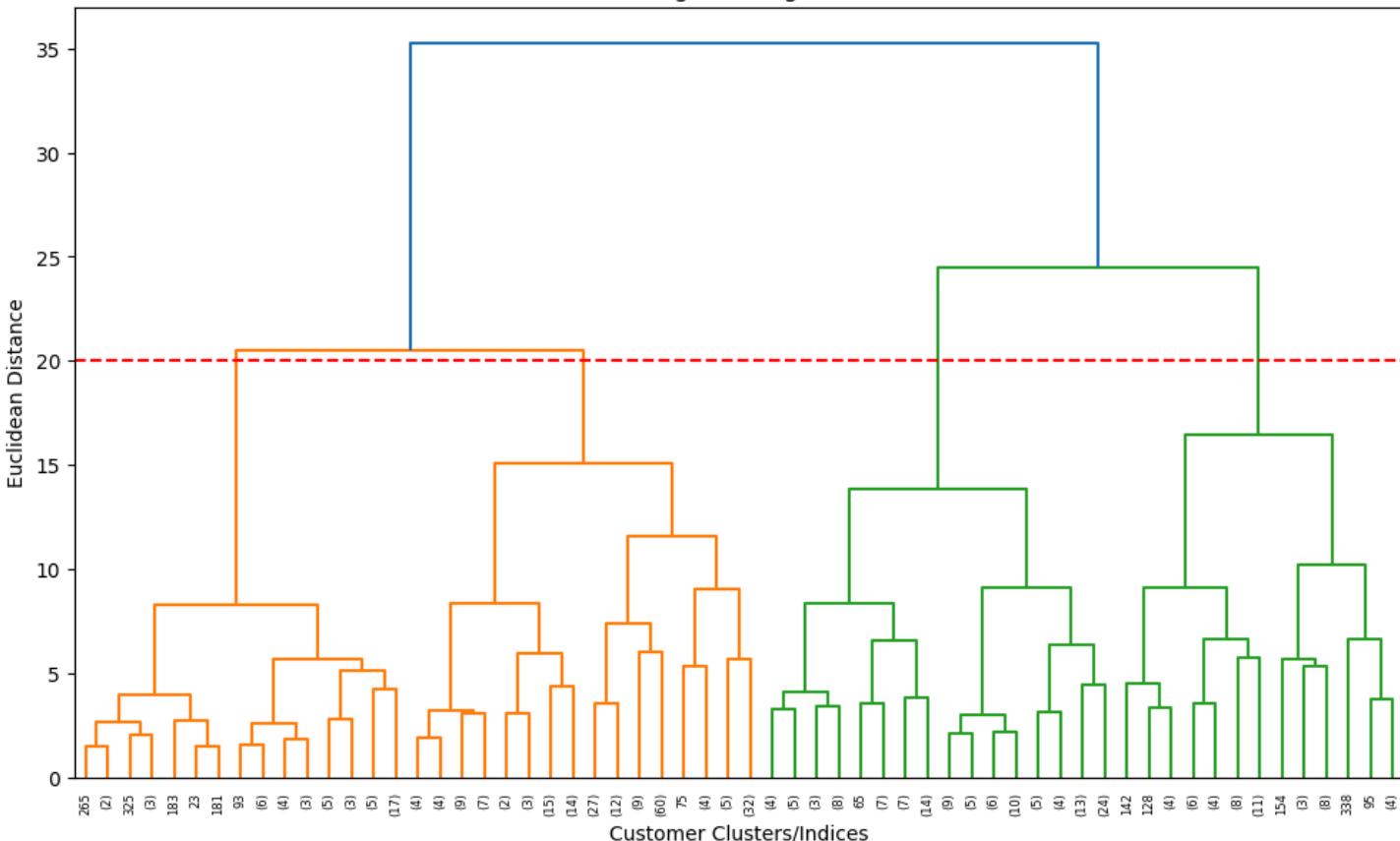
```
Columns used for clustering: ['Fresh', 'Milk', 'Grocery', 'Frozen', 'Detergents_Paper', 'Delicassen']
   Fresh      Milk    Grocery     Frozen  Detergents_Paper  Delicassen
0  9.446992  9.175438  8.930891  5.370638          7.891705  7.199678
1  8.861917  9.191259  9.166284  7.474772          8.099858  7.482682
2  8.756840  9.083529  8.947026  7.785721          8.165364  8.967632
3  9.492960  7.087574  8.348064  8.764834          6.230481  7.489412
4 10.026413  8.596189  8.881697  8.272826          7.483244  8.553718
```



Wholesale Segments (K-Means K=2)



Hierarchical Clustering Dendrogram (Wholesale Data)



Conclusion: K-Means clustering was effectively used to segment customers based on income, spending, and purchasing behavior. Hyperparameter tuning and validation identified two well-separated clusters, representing low-value and high-value customers. The results confirm the suitability of K-Means for practical customer segmentation and marketing analysis.

Hierarchical (Agglomerative) Clustering

Theory:

Hierarchical clustering is an **unsupervised machine learning technique** that organizes data points into a hierarchy of clusters by progressively combining similar groups or, less commonly, by recursively splitting larger groups. Unlike partition-based approaches such as K-Means, hierarchical clustering does **not require the number of clusters to be fixed in advance**. Instead, it constructs a hierarchical structure known as a **dendrogram**, which visually represents how individual data points and clusters are merged at different levels of similarity.

This method is particularly useful for **exploratory data analysis**, as it reveals relationships at multiple levels of granularity and allows analysts to choose an appropriate number of clusters by selecting a cut level on the dendrogram. The clustering process relies on a **distance metric** (commonly Euclidean distance for numerical data) to measure similarity between customers and a **linkage criterion** (such as single, complete, average, or Ward's linkage) to determine how distances between clusters are computed.

Given a dataset with n customer records and d features, hierarchical clustering produces a

nested sequence of cluster partitions that captures the underlying structure of customer similarities. In the context of the **Mall Customers dataset**, this approach helps uncover meaningful **customer segments** based on attributes such as annual income, spending score, and age, providing insights into purchasing behavior and market segmentation.

Limitations:

1. High Computational and Memory Requirements:

Hierarchical clustering requires the computation and storage of a pairwise distance matrix, which leads to high time and space complexity. As the number of customers increases, memory usage and computational cost grow rapidly, making this method less suitable for very large datasets.

2. Irreversibility of Merging Decisions:

In agglomerative clustering, once two clusters are merged, the decision cannot be undone. Incorrect merges occurring at early stages due to noise or data irregularities may propagate through the hierarchy and affect the final clustering structure.

3. Sensitivity to Noise and Outliers:

Outliers can significantly influence distance calculations and linkage decisions. A few extreme customer records may distort the dendrogram and result in less meaningful clusters or the formation of singleton clusters.

4. Dependence on Distance Metric and Linkage Strategy:

The final cluster structure depends strongly on the selected distance measure and linkage method. Different combinations can yield different dendograms and cluster interpretations, requiring careful experimentation and validation to obtain stable and meaningful customer segments.

Performance Analysis

The performance of the **Hierarchical (Agglomerative) Clustering** model on the **Customer Personality Analysis dataset** is evaluated using **internal validation techniques and structural visualization**, since unsupervised learning does not involve ground-truth labels. The evaluation focuses on **cluster separation, stability, and interpretability** using silhouette-based tuning, dendrogram analysis, PCA visualization, and cluster profiling.

1. Optimal Number of Clusters Selection

Silhouette score analysis is used to determine the optimal number of clusters. Among the tested values of K (ranging from 2 to 10), **K = 2** yields the highest silhouette score, indicating the best balance between intra-cluster cohesion and inter-cluster separation.

The **dendrogram generated using Ward linkage** supports this choice by showing a large vertical distance between the final merge steps. This significant gap indicates that merging the two major clusters would cause a sharp increase in within-cluster variance. Hence, cutting the dendrogram at this level produces a **natural and stable two-cluster structure**.

2. Cluster Separation and Visualization (PCA Analysis)

To visualize cluster separation, **Principal Component Analysis (PCA)** is applied to project the high-dimensional feature space into two dimensions. The PCA scatter plot shows **clear visual separation between the two hierarchical clusters**, primarily along the first principal component.

The minimal overlap between clusters suggests strong **inter-cluster dissimilarity**, confirming that the hierarchical algorithm has discovered meaningful structure in the data rather than arbitrary groupings.

3. Dendrogram Structural Validation

The dendrogram provides a hierarchical view of customer relationships and confirms that the final clusters are **internally cohesive**. The large linkage distance observed at the final merge stage reflects strong separation between the two clusters, validating that the selected clustering solution is **structurally meaningful and not artificially forced**.

Hyperparameter Tuning

Hierarchical clustering is sensitive to **hyperparameter selection**, particularly the number of clusters and the linkage strategy, which directly influence how data points are grouped and merged. Inappropriate parameter choices can result in unstable or poorly separated clusters. Since hierarchical clustering is an **unsupervised learning method**, internal validation metrics are used instead of classification-based performance measures. In this implementation, multiple configurations are evaluated to ensure meaningful segmentation and strong cluster structure.

1. Number of Clusters (K)

The number of clusters is tuned by training Agglomerative Hierarchical Clustering models for values of **K ranging from 2 to 10**. For each configuration, cluster assignments are evaluated using the **Silhouette Score**, which measures how well each data point fits within its own cluster compared to neighboring clusters.

The value of **K that yields the highest silhouette score is selected as optimal**, indicating strong intra-cluster similarity and good inter-cluster separation.

2. Linkage Method

The model uses **Ward linkage**, which merges clusters by minimizing the increase in **within-cluster variance** at each step. Ward's method is well-suited for **numerical customer data** and typically produces compact and well-separated clusters. This linkage strategy enhances **cluster stability and interpretability**, making the resulting customer segments more meaningful for analysis.

3. Distance Metric

Ward linkage implicitly relies on **Euclidean distance** to measure similarity between data points. Since clustering decisions are based on geometric proximity, all features are **standardized prior to clustering** to ensure that variables with larger numerical ranges do not dominate the distance calculations.

4. Model Validation

In addition to silhouette-based optimization, the **dendrogram structure** is used as a qualitative validation tool. A large vertical gap at the final merge stage indicates that the selected number of clusters corresponds to a **natural partition in the data** rather than an arbitrary cut, thereby supporting the robustness of the clustering solution.

Code & Output:

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.preprocessing import StandardScaler
from sklearn.cluster import AgglomerativeClustering
from sklearn.metrics import silhouette_score
from sklearn.decomposition import PCA
from scipy.cluster.hierarchy import dendrogram, linkage

# 1. Load Dataset directly for Google Colab
# Using the Wholesale Customers dataset from UCI Repository
url =
"https://archive.ics.uci.edu/ml/machine-learning-databases/00292/Wholesale
%20customers%20data.csv"
df = pd.read_csv(url)

# 2. Preprocessing
# Drop categorical identifiers 'Channel' and 'Region' to focus on spending
behavior
df_numeric = df.drop(columns=['Channel', 'Region'], errors='ignore')

# Handle skewness with Log Transformation - critical for Wholesale data
df_log = np.log1p(df_numeric)

# Handle potential missing values
df_log.fillna(df_log.mean(), inplace=True)

# Standardize the features for distance-based clustering
scaler = StandardScaler()
X_scaled = scaler.fit_transform(df_log)

# 3. Finding Best K for Hierarchical Clustering
cluster_range = range(2, 11)
silhouette_scores = []

for k in cluster_range:
    # Applying Agglomerative Clustering with Ward Linkage [cite: 750, 819]
```

```

model = AgglomerativeClustering(n_clusters=k, linkage='ward')
labels = model.fit_predict(X_scaled)
# Measuring intra-cluster cohesion and inter-cluster separation [cite: 799, 816]
score = silhouette_score(X_scaled, labels)
silhouette_scores.append(score)

# Plot Silhouette vs K
plt.figure(figsize=(8,5))
plt.plot(cluster_range, silhouette_scores, marker='o', color='green')
plt.xlabel("Number of Clusters (K)")
plt.ylabel("Silhouette Score")
plt.title("Silhouette Score vs Number of Clusters (Wholesale Hierarchical)")
plt.grid(True)
plt.show()

best_k = cluster_range[np.argmax(silhouette_scores)]
print(f"Best Number of Clusters based on Silhouette: {best_k}")
print(f"Best Silhouette Score: {round(max(silhouette_scores), 4)}")

# 4. Final Hierarchical Clustering
hc = AgglomerativeClustering(n_clusters=best_k, linkage='ward')
final_labels = hc.fit_predict(X_scaled)

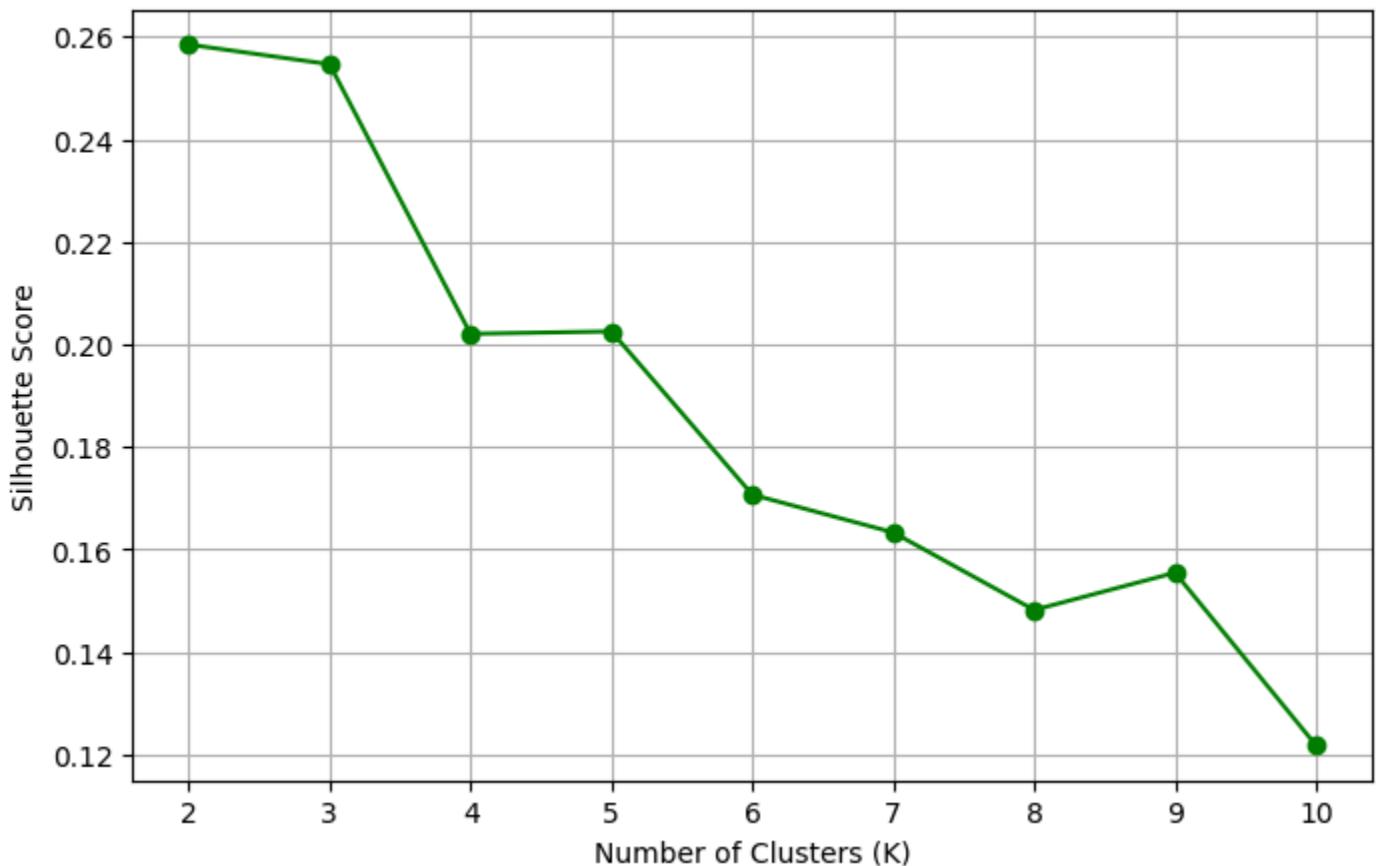
# 5. PCA for 2D Visualization
# Reducing the 6 spending categories to 2 components for visual separation [cite: 804, 805]
pca = PCA(n_components=2)
X_pca = pca.fit_transform(X_scaled)

plt.figure(figsize=(8,6))
sns.scatterplot(
    x=X_pca[:,0],
    y=X_pca[:,1],
    hue=final_labels,
    palette='viridis',
    s=60,
    edgecolor='black',
    legend='full'
)
plt.xlabel("PCA Component 1")
plt.ylabel("PCA Component 2")

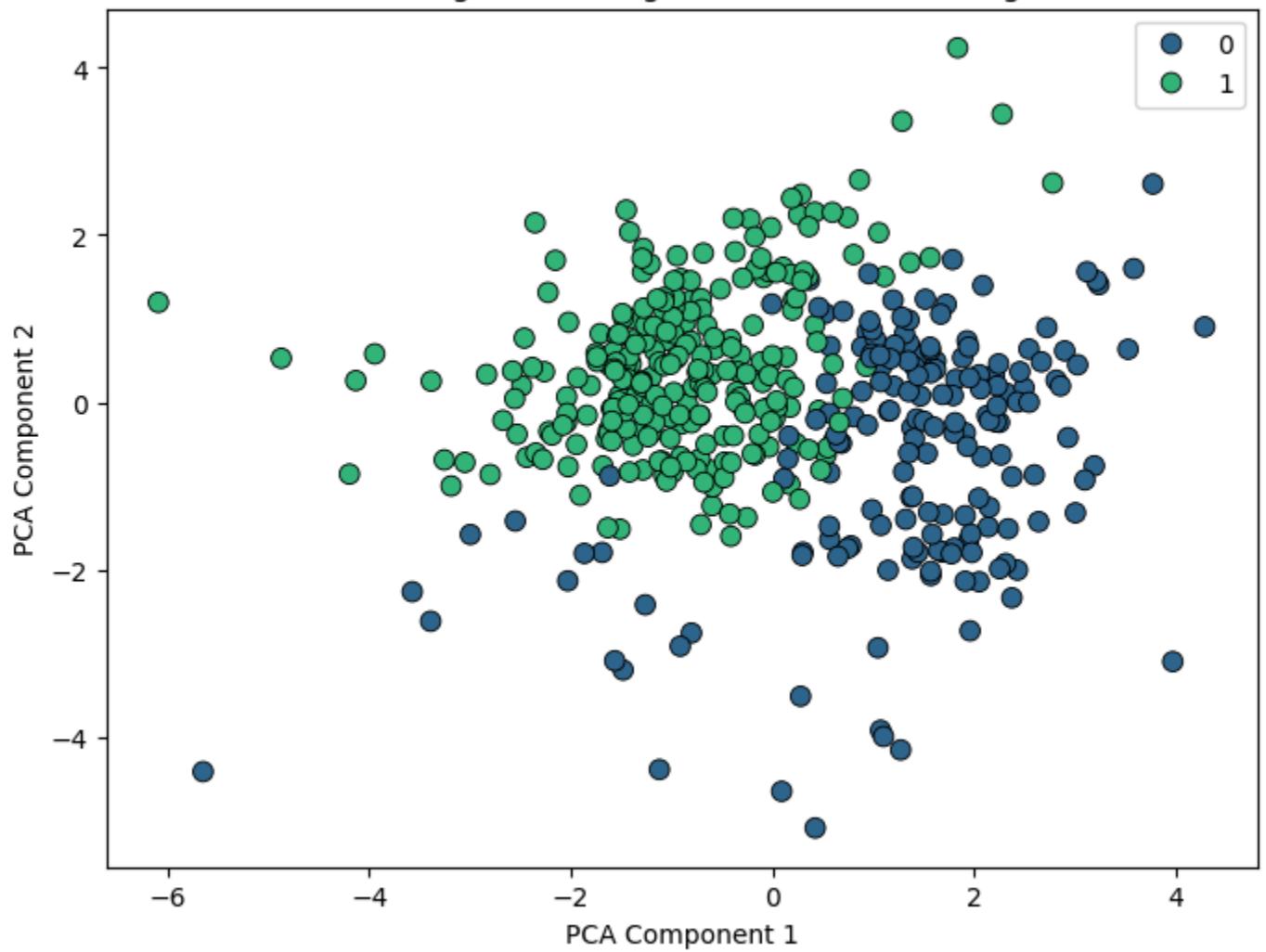
```

```
plt.title(f"Wholesale Segments using Hierarchical Clustering  
(K={best_k})")  
plt.show()  
  
# 6. Dendrogram (Full Dataset)  
# Ward linkage minimizes within-cluster variance [cite: 819, 820]  
linked = linkage(X_scaled, method='ward')  
  
plt.figure(figsize=(15, 8))  
dendrogram(  
    linked,  
    truncate_mode='lastp', # Shows only the last p merged clusters [cite:  
898]  
    p=20,  
    leaf_rotation=90,  
    leaf_font_size=12,  
    show_contracted=True  
)  
# A large vertical gap at the final merge indicates a natural partition  
[cite: 801, 827]  
plt.axhline(y=20, color='red', linestyle='--', label='Cut Threshold')  
plt.xlabel("Cluster Groups")  
plt.ylabel("Euclidean Distance")  
plt.title("Hierarchical Clustering Dendrogram (Ward Linkage - Wholesale  
Data)")  
plt.legend()  
plt.show()
```

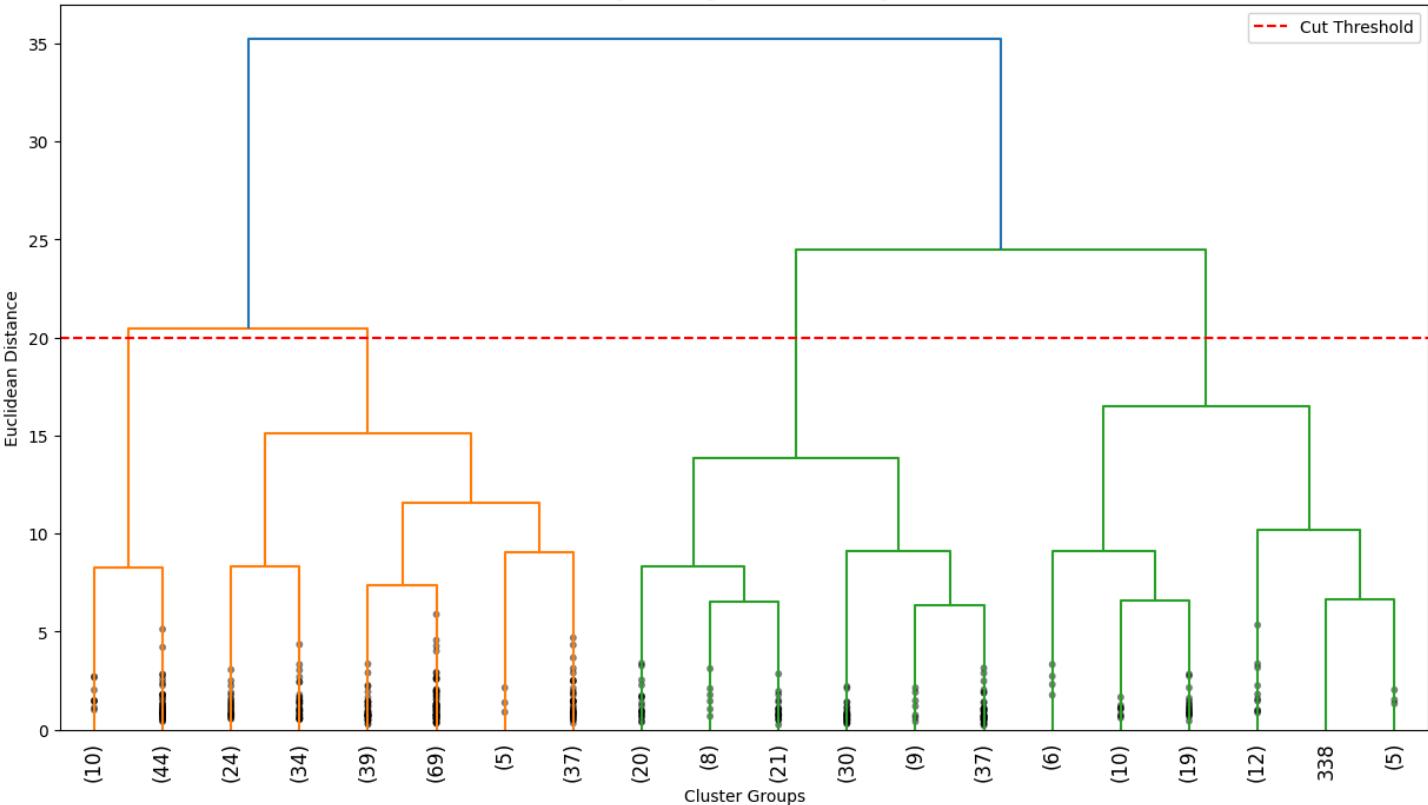
Silhouette Score vs Number of Clusters (Wholesale Hierarchical)



Wholesale Segments using Hierarchical Clustering (K=2)



Hierarchical Clustering Dendrogram (Ward Linkage - Wholesale Data)



Conclusion

Hierarchical (Agglomerative) Clustering successfully identified **two distinct and well-separated customer segments** based on key attributes such as income, spending behavior, purchasing activity, and engagement patterns. The combination of **silhouette score-based tuning and dendrogram analysis** confirmed that the chosen number of clusters provides a **stable and meaningful partition** of the customer dataset.

The clear separation observed in PCA visualizations and the strong structural gaps in the dendrogram indicate that the model captured **natural groupings present in the data** rather than producing arbitrary clusters. These findings demonstrate that hierarchical clustering is an effective and interpretable technique for **practical customer segmentation**, making it valuable for applications such as targeted marketing, customer profiling, and strategic decision-making in real-world business scenarios.