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VIBRATION-BASED DAMAGE ACCUMULATION MODELING

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ABSTRACT

A simple methodology is presented which takes input data from wireless accelerometers used in continuous monitoring of large numbers of machines such as pumps, motors, gearboxes, and fans, and outputs a damage accumulation metric which can be used for early warning indication for a broad range of faults. Based on the output, maintenance visits can be made to inspect the machines. Emphasis is on ease of use and broad applicability. It is assumed that the accelerometer data reflects damage occurrence. The approach here is based on time series vibration data analysis that estimates the rate at which damage is accumulated at a given location. This indicator accounts for time-varying symptoms in machines which are often overlooked by traditional vibration diagnostic frequency analysis. As fatigue analysis is the foundation for the damage metric, contribution of repeated load reversal cycles to component damage and the nonlinearity in the relationship between damage and vibration amplitude, are incorporated. A MATLAB code has been developed and validated from simple examples in the literature. The methodology is then applied to a finite element model of a defective shaft-bearing assembly, and in a high pressure pumping field application.

1. INTRODUCTION

A simple methodology is presented which takes input data from wireless accelerometers used in continuous monitoring of large numbers of machines such as pumps, motors, gearboxes, and fans, and outputs a damage accumulation metric which can be used for early warning indication for a broad range of faults. Based on the output, maintenance visits can be made to inspect the machines. Emphasis is on ease of use and broad applicability. The reader is referred to [1-2] among others for in-depth discussion on this general topic. The approach here is based on time series vibration data analysis that estimates the rate at which damage is accumulated at a given location. This indicator accounts for time-varying symptoms in machines which are often overlooked by traditional vibration diagnostic frequency [3-4]. Further, as fatigue analysis is the foundation for the damage metric, contribution of repeated load reversal cycles to component damage and the nonlinearity in the relationship between damage and vibration amplitude, are incorporated.

Typically, sensors acquire a burst of acceleration data every few minutes. Here, an index is presented which is a single number that represents an estimate of the severity of the damage for each burst of data, and this index is used to estimate the rate at which damage is occurring in time. Rainflow counting on the acceleration signal is implemented to obtain number, mean and amplitude of amplitude cycles. Following this, fatigue based damage accumulation is used to compute a damage index. Damage from a combination of variable amplitude loading is accumulated based on Miner's rule. Although components of the methodology presented here are well known, their integration into a MATLAB (and C) code along with interface to accelerometers developed at KCF [https://www.kcftech.com/] have led to a robust approach, at least based on current experience, viz. simple examples in the literature, application to a finite element model of a defective shaft-bearing assembly, and in a high pressure pumping field application.

2. APPROACH TAKEN AND ASSUMPTIONS INVOLVED

In this section, basic concepts are presented and an outline of the damage accumulation method. Details are provided in subsequent sections. Consider a single degree of freedom (SDOF) model in Fig. 1. The structure consists of a viscoelastic rod based on the Voight model along with an end mass. If we refer to the SDOF model, then accelerometer reading, $A(t) = A_1(t)$ relates to strain-cycling and hence damage in the structure. This paper and related computer code takes acceleration data $A(t)$, that is provided by the engineer, and outputs a damage vs. time plot along with estimate of rate of damage accumulation. Referring to Fig. 1, assume the motion $x(t)$ vs t, which is essentially strain $\varepsilon(t)$ vs. t since $\varepsilon(t)=x(t)/L$, is as shown in Fig. 2a. The corresponding stress-strain plot is shown in Fig. 2b, where stress $\sigma(t) = (k^*x(t) + c\, dx/dt)/A$, where A = area of cross-section of rod. We see from Fig. 2b that the total hysteresis count is a full cycle with stress range 2-3 and a half-cycle with stress range 1-4. As another example to illustrate cycle counting, consider the SDOF system in Fig. 1 subjected to a force $F(t)$ as shown in Fig. 3a. The corresponding acceleration $A(t)$ and stress-strain plot, generated using a computer code, are shown in Figs. 3b, 3c respectively. The methodology here is to treat $A(t)$ as the <u>*measured*</u> input to the code rather than $F(t)$. Thus, the steps for the SDOF system will be as follows.

<u>Damage estimation for SDOF</u>

step 1. Measure $A(t)$

step 2. Integrate $A(t)$ to obtain velocity dx/dt and position $x(t)$, using numerical integration techniques such as Trapezoidal method

step 3. Define strain $\varepsilon(t)=x(t)/L$, stress $\sigma(t) = (k^*x(t) + c\, dx/dt)/A$

step 4. From the results in step 3, use Rainflow counting to obtain number, mean and amplitude of stress cycles (Section 3).

step 5. Use fatigue based damage accumulation as given in detail in Section 4 to obtain damage and damage rate. <u>*The damage law involves two constants which, as per fatigue theory, are material properties obtained from a laboratory 'rotating beam test'*</u> [9].

For complex systems, we assume that the accelerometer reading is proportional to strain cycling. Thus, the following steps are used in lieu of the above.

<u>Damage estimation for Complex Systems</u>

step 1. Measure $A(t)$. It is assumed that the accelerometer has been properly placed, or else the measurement is a suitable difference between accelerometer readings as noted with reference to Fig. 1b, so as to reflect damage occurrence.

step 2. Use Rainflow counting on the signal $A(t)$ to obtain number, mean and amplitude of acceleration amplitude cycles (Section 3).

step 3. Use fatigue based damage accumulation as given in detail in Section 4 to obtain damage and damage rate. The damage law involves two constants which, for complex systems, <u>*have to be determined from calibration with field data.*</u>

Whether SDOF or complex multi DOF systems, damage from a combination of variable amplitude loading is accumulated based on Miner's rule. Miner's rule predicts the same resulting damage regardless of whether the low amplitude loading is applied before, after, or simultaneously with the high amplitude loading. While this is a significant assumption, it is universally adopted owing to the lack of any proven substitute. A MATLAB code has been developed to implement the above.

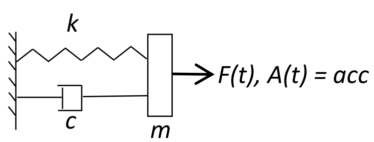
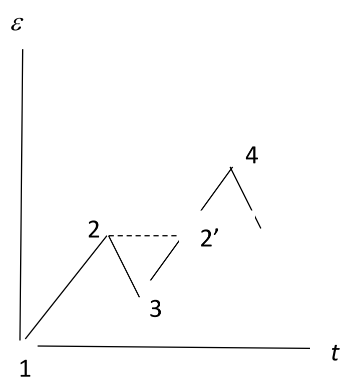
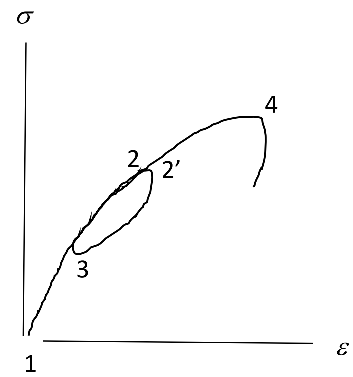


Figure 1. A SDOF Mechanical System

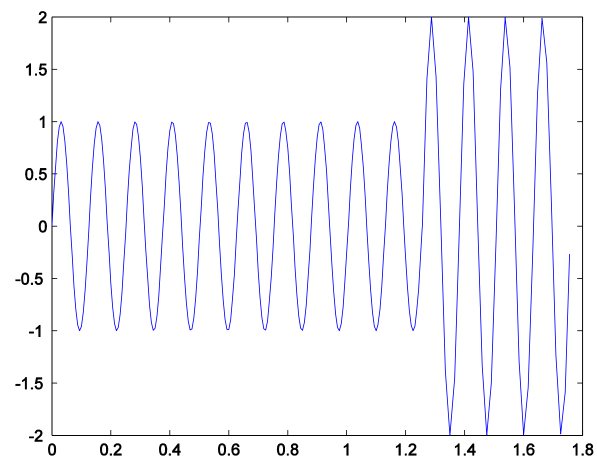


(a)

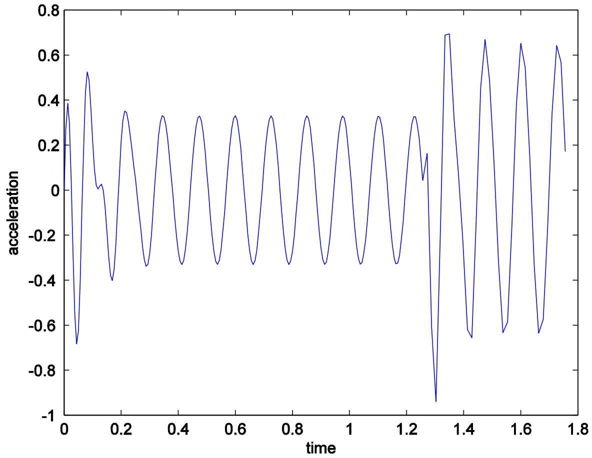


(b)

Figure 2. Example of Rainflow Cycle Counting: (a) Strain vs time, (b) Corresponding Hysteresis Loop



(a)



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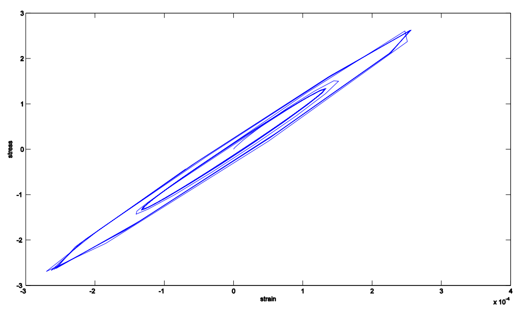

(c)

Figure 3. (a) SDOF loading, (b) Corresponding Acceleration (note: direct measurement of A(t) is starting point for damage estimation of complex systems in method herein), and (c) Corresponding Stress-Strain Cycling

3. Rainflow Counting Algorithm

ASTM E1049 – 85 [5] has been implemented in MATLAB code. A sample input/output from the code is given below (among other validations that were carried out):

```
input        [PV]=[-2 1 -3 5 -1 3 -4 4 -2];
% peaks/valleys of signal
output
** full cycles **
        i1   i2   range     mean
        5    6     4.00     1.00

              ** half cycles **
        i1   i2   range     mean
        1    2     3.00     0.50
        2    3     4.00     1.00
        3    4     8.00     1.00
        4    7     9.00     0.50
        7    8     8.00     0.00
        8    9     6.00     1.00
```

we see from above that there are 1.5 cycles with range = 4, 0.5 cycles with range = 3, 0.5+0.5=1 cycle with range = 8, 0.5 cycles with range = 9, and 0.5 cycles with range = 6, which agrees with ASTM E1049 [5] output:

Range	Cycle Counts
9	.5
8	1
6	.5
4	1.5
3	.5

4. DAMAGE ACCUMULATION THEORY AND IMPLEMENTATION ORIGINATING FROM FATIGUE THEORY

We denote s_a to represent the amplitude, i.e. purely alternating or reversed component, of the acceleration signal, s_m to represent mean. Thus, $s_{max} = s_m + s_a$, $s_{min} = s_m - s_a$, range = $(s_{max}-s_{min}) = 2s_a$. Given any signal, the peaks and valleys are determined, and then rainflow counting is used to provide the number of cycles n_i having amplitude s_a^i. Based on fatigue concepts, mean and alternating stresses for each amplitude are combined into a single equivalent alternating stress, which will also be denoted here by s_a. An S-N diagram schematically shown in Fig. 4, typically generated using a fatigue experiment, gives number of cycles to failure $n_f^{\,i}$.

To transport above fatigue theory to vibration based structural health monitoring, a damage measure is introduced as

$$D_i = \frac{n_i}{n_f^i} \qquad (1)$$

Evidently, $D_i \geq 1$ implies failure. If say $D_i = 0.1$, then this means the number of cycles n_i can be increased by a factor of 10 before failure occurs. To combine the different cycles obtained from rainflow counting, Miner's rule can be used as

$$D = \sum_i D_i = \sum_i \frac{n_i}{n_f^i} \qquad (2)$$

Thus, for example, if the stress loading history or 'signal' consists of 10 cycles at s_a^1 and 20 cycles at s_a^2, and D = 0.2, then this means the component can be subjected to 1/D = 5 repetitions of the loading or 50 cycles at s_a^1 and 100 cycles at s_a^2 until failure. Or, for instance, if D = 0.2 over an interval T=1s, then if the machine is operational over a 60s interval, D=(.2x60)=12.

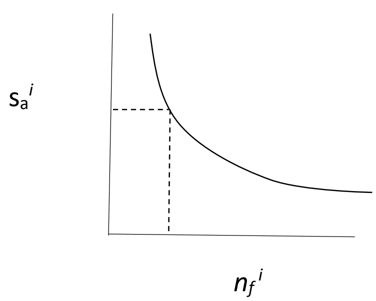

Figure 4. S-N curve (often referred to as the Wohler curve)

The relationship between s_a^i and n_f^i schematically shown in Fig. 4 can be approximated by the Basquin equation as

$$n_f^i = \left(\frac{s_a^i}{S_f}\right)^{\frac{1}{b}} \qquad (3)$$

where S_f and b are material properties [6]. Writing Eq. (3) above as

$$n_f^i = A\, s_a^{-m} \equiv \left(\frac{s_a}{s_{ref}}\right)^{-m}$$

where $m > 0$, we obtain

$$D_i = \frac{n_i}{n_f^i} = n_i \left(\frac{s_a}{s_{ref}}\right)^m \qquad (4)$$

and Miner's rule is $D = \sum_i D_i$. In rainflow counting, n_i = 1 for a full cycle and = 0.5 for a half-cycle.

Damage Index, *m*, s_{ref}

Choice of m and s_{ref} can be based on calibrating Eq. (4) with failure data. However, the objective here is to trend the data and estimate severity of the damage, and estimate the rate at which damage is occurring in time. These values can be further refined by calibration, however. Here, the damage measure in Eq. (4) is re-defined as simply a damage index DI. DI > 1 does not having any relation to whether failure occurs or not. s_{ref} = 1.0 is used. Choice of m is user-defined in the code. Values found in the literature are given below, although best choice involves calibration with specific problems and field data.

Table 1. Exponent m related to damage index used in Literature

value of *m*	Description (in literature)
3	standard value for crack-growth-dominated process, applicable to sharply notched or welded structures [7], also used in wind turbine tower made of welded steel [3, pg. 86]
5	valid for many engineering components
6	glass fiber [8], although *m*=10 used in [3] for turbine blade made of fiberglass

5. VALIDATION EXAMPLES AND A FIELD APPLICATION

Examples 1 and 2 are simple examples where results from the code can be compared with results in literature. Example 3 relates to data from simulating failures using a finite element model. Field Application refers to using the existing methodology to a practical problem.

Example 1.
[9, pg. 291]
s_a = 40,000 psi, s_m = 35,000 psi
Endurance limit S_e = 50,000 psi
Ultimate strength S_u = 100,000 psi.
Number of cycles to failure n_f = 87,800

In MATLAB code, a stress signal y = 35000 + 40000 sin(2π *f t*), with *f* = 20 cps gives 1/D = 4469 = no of repetitions. Thus, n_f = 4469*20 = 89,378. The result in the textbook can be obtained by noting that this sin curve has two ¼-cycles on either side of the peaks/valleys.

Example 2.
Ref. [10]
y1 = sin(2π 100 *t*)
y2 = sin(2π 190 *t*)
y3 = sin(2π 360 *t*)
D is determined as per Eq. (4), with m = 4, s_{ref} = 1

Signal	DI from ref. 8, for 180s	DI based on MATLAB Code result for 1 s x 180s
y1	18,000	17,876
y1+y2	143,000	143,190
y1+y2+y3	613,000	605,448

Example 3. Data from Simulating Failure using Finite Element Analysis

To simulate the machine response for a machine fault, a rotor-bearing test assembly (Fig. 5) has been used as a test example. The shaft-rotor-bearing assembly, a very common and vital part of most machines, is used to transmit mechanical power. Any kind of machine faults present in this assembly can easily be monitored from the acceleration signal at bearing. Here, imbalance is incorporated and simulated for accelerator response at a bearing at one of the ends.

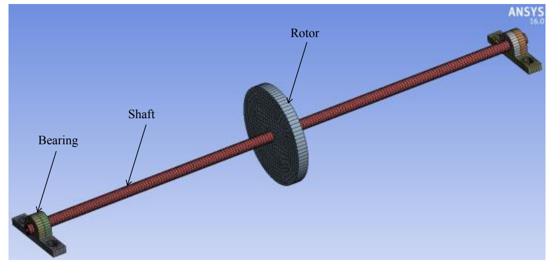

Figure 5: Finite Element Model of a Shaft-Rotor-Bearing Assembly, in ANSYS 16.0 (shell element thicknesses are shown in figure)

The finite element model of the test example is shown in Figure 5. The shaft (Outer diameter = 19.05mm and Inner diameter=9.525mm) is mounted between two identical bearings separated by 952.5mm. The rotor (Diameter = 177.8mm and thickness = 19.05mm) is mounted at halfway distance between the bearings. Considering manufacturing and installation error, an inherent mass imbalance is incorporated in the model by keeping the rotor center 0.05mm away from the shaft center. Bonded contact is defined between shaft and rotor, and frictionless contact is defined between shaft and bearing. Rotational speed of 1500 rpm is applied to the shaft, which is less than natural frequency of shaft-rotor-bearing assembly. Transient structural analysis is carried out where shaft along with rotor spins in real time. The rotational velocity is applied to the shaft using APDL command language in ANSYS 16.0. The accelerator signal with time is recorded at the bearing housing. To reduce the computation time, shell elements are used for all the parts. All the parts are made of stainless steel, density = 8.03g/cc, Young's modulus=200GPa and Poisson's ratio =0.24. Both bearing feet are fixed and a point mass (Fig. 6) is added at outer perimeter of the rotor.

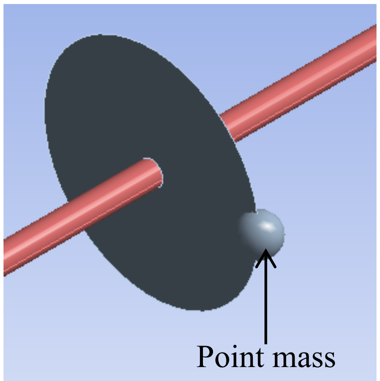

Figure 6: Showing point mass at outer periphery of rotor

The time signals (magnitudes) are shown in Fig. 7. The damage index is shown in Fig. 8, as also a cumulative plot, and values of damage rate = slope of cumulative plot. Increased DI is evidently validated corresponding to increased imbalance.

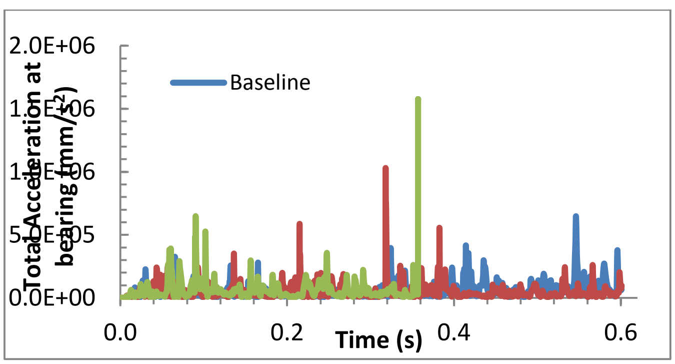

Figure 7: Absolute Acceleration Signal at Bearing for Baseline, 5g imbalance and 8g imbalance

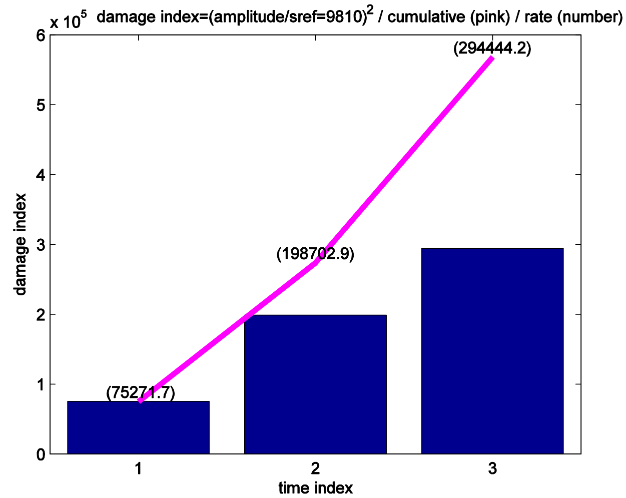

Figure 8. DI Plot for increasing imbalance in shaft-bearing assembly ('time index' refers to time during which data was recorded for baseline, unbalanced 5g and unbalanced 8g)

Field Application

For a high pressure pumping field application, DI is computed for each of 23 columns of 1800-point accelerometer data, five minutes apart. Results: a graph of DI vs time is plotted, as also a cumulative plot, and values of damage rate = slope of cumulative plot (Fig.9). Steep DI rates in the DI plot correspond well to observed intermittent failures in the field observations. While details of the failures have not been obtainable other than whether failures occurred or not, the correlation with recorded intermittent failures does underline a contribution made here which is that a simple methodology is presented that can be used for early warning indication for a broad range of faults, with ease of use and broad applicability. The contribution here is on presenting a damage index from accelerometer data. As noted in Section 2, proper placement and/or suitable processing of data from multiple accelerometers may enhance the ability of the data to reflect damage occurrence, in which case the DI presented here will be even more accurate a prediction.

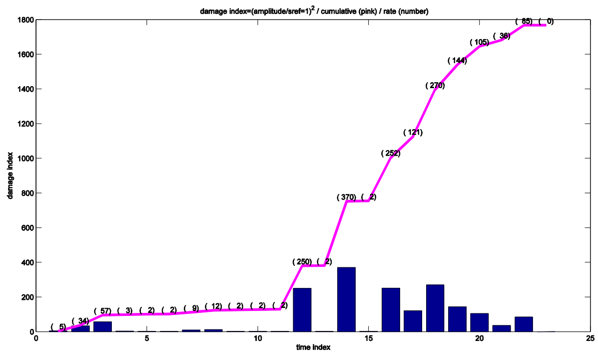

Figure 9. Damage Index for Field Application ('time index' refers to 5 minute intervals during which burst of data was recorded)

6. CONCLUSIONS

A simple methodology is presented which takes input data from wireless accelerometers used in continuous monitoring of large numbers of machines such as pumps, motors, gearboxes, and fans, and outputs a damage accumulation metric which can be used for early warning indication for a broad range of faults. Based on the output, maintenance visits can be made to inspect the machines. Emphasis is on ease of use and broad applicability.

It is assumed that the accelerometer data reflects damage occurrence. This paper and related computer code takes acceleration data that is provided by the engineer, and outputs a damage vs. time plot along with estimate of rate of damage accumulation. Typically, sensors acquire a burst of acceleration data every few minutes. Here, an index is presented which is a single number that represents an estimate of the severity of the damage for each burst of data, and this index is used to estimate the rate at which damage is occurring in time. Rainflow counting on the acceleration signal is implemented to obtain number, mean and amplitude of amplitude cycles. Following this, fatigue based damage accumulation is used to compute a damage index. Damage from a combination of variable amplitude loading is accumulated based on Miner's rule. Although components of the methodology presented here are well known, their integration into a MATLAB code along with interface to accelerometers developed at KCF [https://www.kcftech.com/] have led to a robust approach. Results have been validated on rainflow counting as well as on damage examples in the literature. This indicator accounts for time-varying symptoms in machines which are often overlooked by traditional vibration diagnostic frequency analysis. The methodology is applied to a finite element model of a defective shaft-bearing assembly, and in a high pressure pumping field application. With regard to the field application, results from the code match well to the intermittent failures observed. Further implementation and use of the MATLAB code in the field will lead to further calibration and enhancements.

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