

**NOTES: Finding Remainders of Big Powers (Power Mod)**

**1. What is MOD?** MOD simply means the **remainder** after division. Example:  $7 \bmod 5 = 2$  because  $7 = 5 \cdot 1 + 2$  (remainder 2)

**2. Key Idea:** To find remainder of very big powers like  $2^{31}$ ,  $7^{19}$ ,  $9^{100}$ :  $\rightarrow$  We look for a repeating pattern in remainders.

**3. Example:  $2^{31} \bmod 5$**  First find small powers:  $2^1 = 2 \rightarrow$  remainder 2  $2^2 = 4 \rightarrow$  remainder 4  $2^3 = 8 \rightarrow$  remainder 3  $2^4 = 16 \rightarrow$  remainder 1 Cycle = [2, 4, 3, 1] (length 4)

**4. Reduce the exponent:**  $31 \bmod 4 = 3$  So  $2^{31}$  has same remainder as  $2^3$ .  $2^3 = 8 \rightarrow 8 \bmod 5 = 3$  **Final Answer: 3**

**5. Why you cannot reduce 2 to 1?** Because  $2 \bmod 5 = 2$  (NOT 1). Number smaller than divisor keeps itself as remainder.

**6. Summary Shortcut:** 1. Find remainders of first few powers 2. Identify the cycle 3. Divide exponent by cycle length 4. Use the position in cycle to get remainder