# MAE-598

## Design Optimization

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## Homework - 4

(2) thrust, $L = \alpha$

$$f = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ -g + \frac{\alpha}{x_3} \\ -k\alpha \end{bmatrix}$$

$$H = -L + \lambda^T f = -\alpha + \lambda_1 \dot{x}_1 + \lambda_2 \dot{x}_2 + \lambda_3 \dot{x}_3$$

$$= -\alpha + \lambda_1 x_2 - \lambda_2 g + \lambda_2 \frac{\alpha}{x_3} - \lambda_3 k\alpha$$

$$= \alpha\left(-1 + \frac{\lambda_2}{x_3} - \lambda_3 k\right) + \lambda_1 x_2 - \lambda_2 g$$

It is in form of $b(t)\alpha + a(t)$

$$\alpha^* = 0 \text{ if } b \le 0$$

$$\alpha^* = 1 \text{ if } b > 0$$

now,

$$\dot{\lambda} = -\frac{\partial H}{\partial x} = -\begin{bmatrix} \frac{\partial H}{\partial x_1} \\ \frac{\partial H}{\partial x_2} \\ \frac{\partial H}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 0 \\ -\lambda_1 \\ \frac{\lambda_2 \alpha}{x_3^2} \end{bmatrix}$$

$$b = -1 + \frac{\lambda_2}{x_3} - \lambda_3 k$$

$$\frac{db}{dt} = \frac{\dot{\lambda}_2 x_3 - \dot{x}_3 \lambda_2}{x_3^2} - \dot{\lambda}_3 k = \frac{-\lambda_1}{x_2}$$

$$x_3 > 0$$

$$\dot{x}_1 = 0$$

Optimal control policy is

$\alpha^* = 0 \quad \text{if } b \leq 0 \quad t \in [0, t_s)$

$\alpha^* = 1 \quad \text{if } b > 0 \quad t \in [t_s, t^*]$

$t_s$ = switching time

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ -g \\ 0 \end{bmatrix} \quad \alpha = 0$$

now

$$x_1 = -\frac{1}{2} g t^2 + v_0 t + h_0$$

$$x_2 = -gt + v_0$$

$$\dot{x} = \begin{bmatrix} x_2 \\ -g + \frac{1}{x_3} \\ -k \end{bmatrix} \quad \alpha = 1$$

$$x_3 = -k(t - t_s) + m_0$$

$$x_2 = -g(t - t^*) + \frac{1}{k} \ln \left[ \frac{k(t^* - t_s) - m_0}{k(t - t_s) - m_0} \right]$$