MAE-598

Design Optimization

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- Homework - 3

(1)

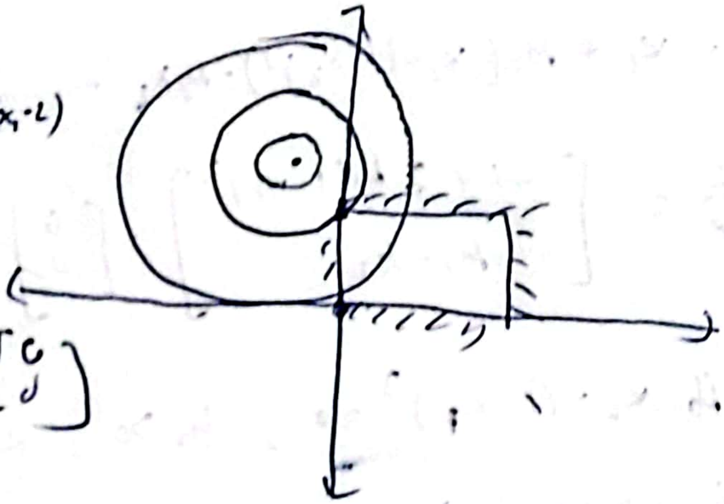
$$\min_{x_1,x_2} f(x) = (x_1+1)^2 + (x_2-2)^2$$

st $g_1: x_2 - 2 \le 0$ $\quad g_3 = -x_1 \le 0$

$g_2: x_2 - 1 \le 0$ $\quad g_4: -x_2 \le 0$

no.

$$L = (x_1+1)^2 + (x_2-2)^2 - \mu_1 x_1 - \mu_2 x_2 + \mu_3(x_1-2)$$
$$+ \mu_4 (x_2-1)^2$$



$$\frac{\partial L}{\partial x} = \begin{bmatrix} 2(x_1+1) - \mu_1 + \mu_3 \\ 2(x_2-2) + \mu_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

μ

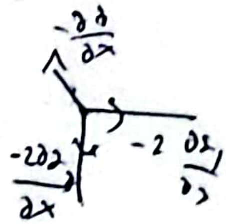
$-x_1 = 0, \mu_1 > 0 \quad , -x_1 < 0, \mu_1 = 0$

$-x_2 = 0, \mu_2 > 0 \quad -x_2 < 0, \mu_2 = 0$

$x_1 - 2 = 0, \mu_3 > 0, \quad x_1 - 2 < 0, \mu_3 = 0$

$x_2 - 1 = 0, \mu_4 > 0, \quad x_2 - 1 < 0, \mu_4 = 0$

$x = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow x_1 = 0 \quad x_2 = 1$



$\mu_1 > 0 \quad \mu_2 = 0$

$\mu_3 = 0 \quad \mu_4 \ge 0$

$$\frac{\partial L}{\partial x}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 - \mu_1 \\ -2 + \mu_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \mu_1 = 2 > 0$$

$$\mu_4 = 2 > 0$$

balance of forces. $\frac{\partial f}{\partial x} + \mu_1 \frac{\partial g_1}{\partial x} + \cancel{\mu_2 \frac{\partial g_3}{\partial x}} + \cancel{\mu_3 \frac{\partial g}{\partial x}} + \mu_4 \frac{\partial g_4}{\partial x}$

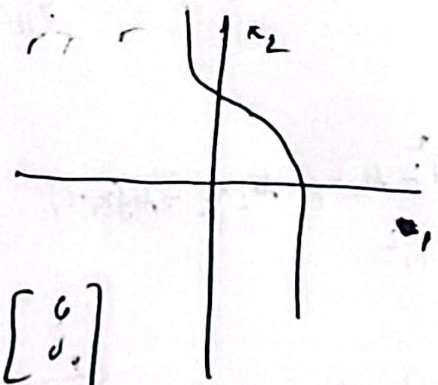
$$\begin{bmatrix} 2 \\ -2 \end{bmatrix} + 2\begin{bmatrix} -1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

② given,

$$\min_{x_1, x_2} f = -x_1$$

s.t. to $g_1 = x_2 - (1-x_1)^3 \leq 0$

$x_2 \geq 0$



so,

$L = -x_1 + \mu_1(x_2 - (1-x_2)^3) - \mu_2 x_2$

$$\frac{\partial L}{\partial x} = \begin{bmatrix} -1 + 3\mu_1(1-x_1)^2 \\ \mu_1 - \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

→ If $x_2 - (1-x_1)^3 = 0$, $\mu_1 > 0$. $x_2 - (1-x_1) \leq 0$, $\mu_1 = 0$

→ If $-x_2 = 0$, $\mu_2 > 0$. $-x_2 < 0$, $\mu_2 = 0$

→ If $x_2 - (1-x_1)^3 = 0$, $\mu_1 > 0$, $-x_2 = 0$, $\mu_2 > 0$

$$x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\frac{\partial L}{\partial x} = \begin{bmatrix} -1 \\ \mu_1 - \mu_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \times$$

⇒ $x_2 - (1-x_1)^3 = 0$, $\mu_1 > 0$, $-\cancel{\mu_2} < 0$, $\mu_2 = 0$

$$\frac{\partial L}{\partial x} = \begin{bmatrix} \mu_1 - 0 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \times$$

→ $x_2 - (1-x_1)^3 < 0$; $\mu_1 = 0$, $\mu_2 = 0$ ×

KKT conditions are not sufficient

∴ No possible scenario to this solution.

③ giv

$$\max_{x_1,x_2,x_3} f = x_1x_2 + x_2x_3 + x_1x_3$$

$$\text{st} \quad h = x_1+x_2+x_3-3=0$$

max, min

$$L = -x_1x_2 - x_2x_3 - x_1x_3 + \lambda(x_1+x_2+x_3-3).$$

$$\frac{\partial L}{\partial x} = \begin{bmatrix} -x_2-x_3+\lambda \\ -x_1-x_3+\lambda \\ -x_2-x_1+\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \quad \begin{matrix} x_1=1 \\ x_2=1 \\ x_3=1 \\ \lambda=2 \end{matrix}$$

$$\frac{\partial L}{\partial \lambda} = x_1+x_2+x_3-3=0$$

$$L_{xx} = \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix} < 0$$

$$dx^t L_{xx}\, dx = \begin{bmatrix} dx_1 & dx_2 & dx_3 \end{bmatrix} \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{bmatrix}$$

$$= -2\,dx_1\,dx_2 - 2\,dx_1\,dx_3 - 2\,dx_2\,dx_3 \quad \text{— (*)}$$

$$\frac{dh}{dx}dx = 0 \Rightarrow \begin{bmatrix} \frac{dh}{dx_1} & \frac{dh}{dx_2} & \frac{dh}{dx_3} \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{bmatrix} = 0$$

$$\Rightarrow dx_1 + dx_2 + dx_3 = 0$$

$\Rightarrow d x_1 = -d x_2 - d x_3$

do sub $d x_1$ in ($*$)

$$d x^T L_{xv} d x = -2\left((1 - d x_2 - d x_3) d x_2\right) + \left((-d x_2 - d x_3) d x_3 + d x_2 d x_3\right) + \left(\frac{3}{4} d x_3^{3}\right)$$

$$= 2\left(d x_2^2 + d x_2 d x_3 + d x_3^2\right)$$

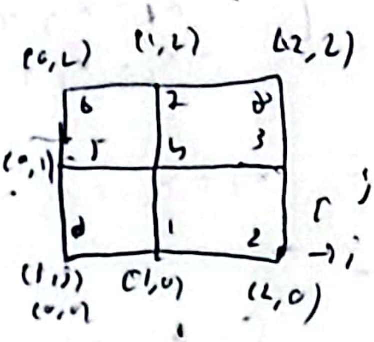
$$= 2\left(\left(d x_2 + \frac{1}{2} d x_3\right)^2 + \frac{3}{4} d x_3^2\right) > 0$$

(5) a:

$N_{sites} \rightarrow N_i \rightarrow x$-dire

$N_j \rightarrow y$-dire

c_{ij} = cost of moving from nod i to j



$$\min \sum_{i=0}^{N_i} c_{ij} \quad \forall j = 0,1,\ldots, N_j$$

let, x_{ijn} be location of truck at n^{th} site

obj: $\min \sum_{i=0}^{N_i} c_{ij} \; \forall j = 0,1,\ldots, N_j$

→ ensure ~~all~~ that all sites & connection nodes are used, so the c_{ij} will be minimized

and: $\left. x_{ijn} \right|_{n=0} = 0$ } ensures truck starts at $(n=0)$ and

$\left. x_{ijn} \right|_{n=N} = 0$ } finishes at $n=N$ at site 0