Precision Machine Design

Topic 11

Error Budgets

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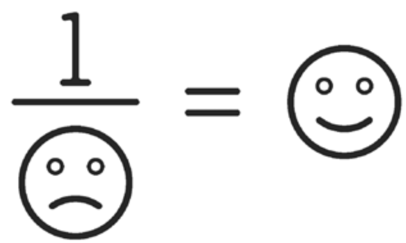
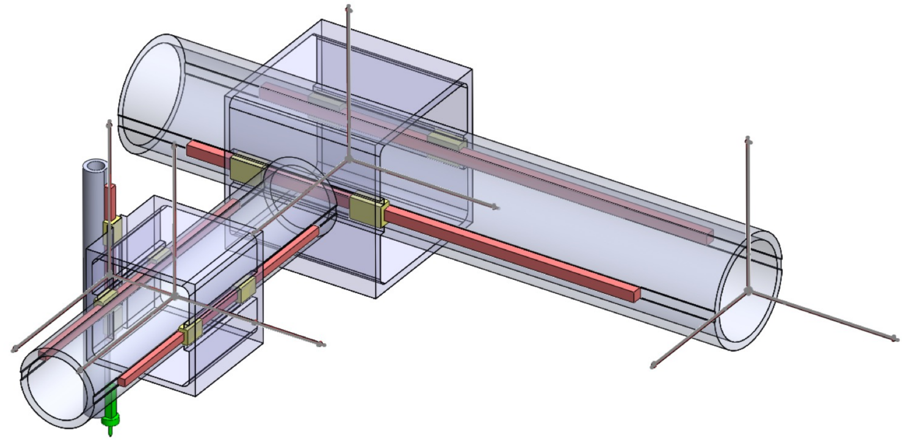
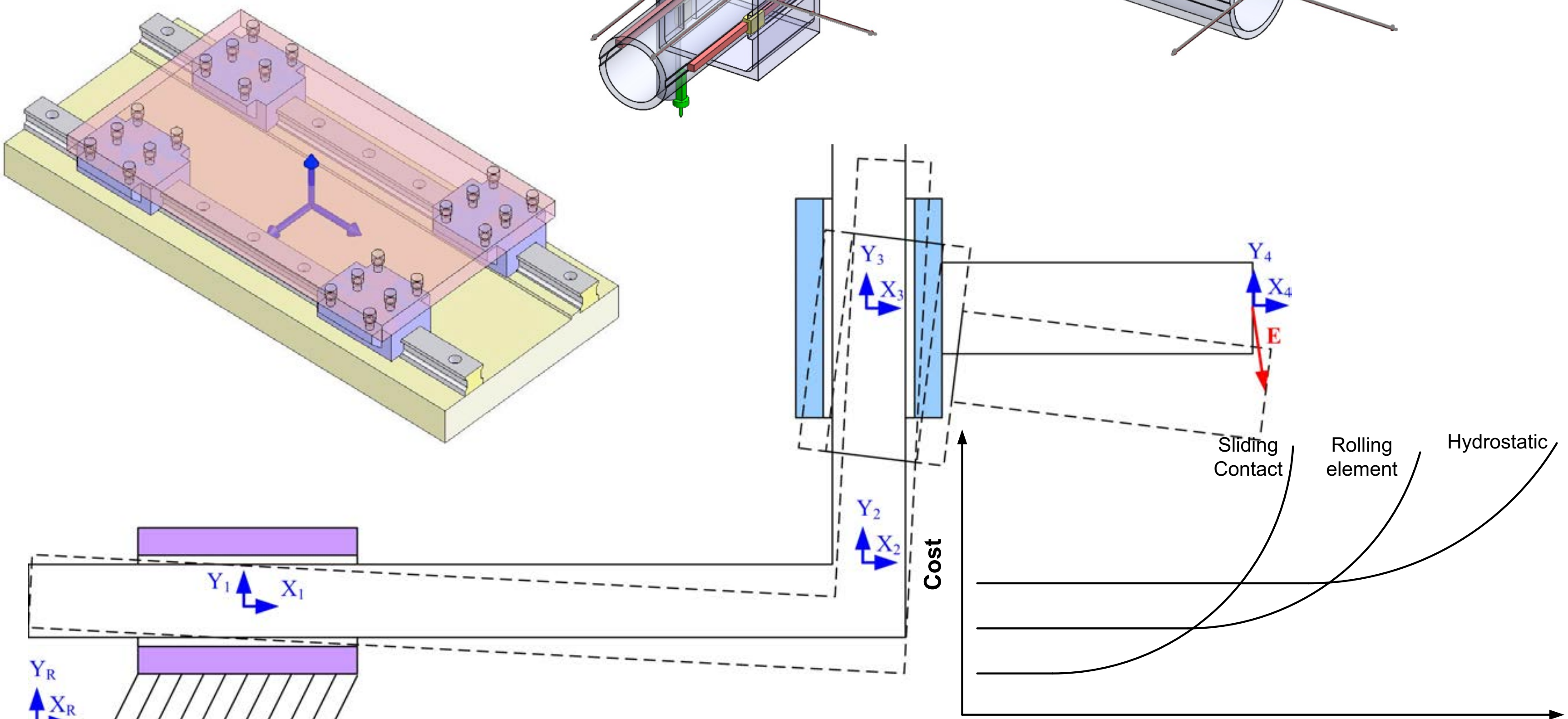
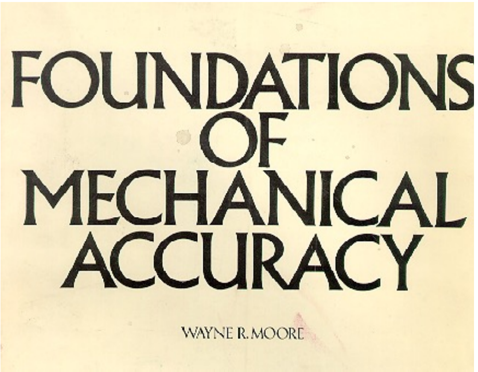


Topic 8 Error Budgets





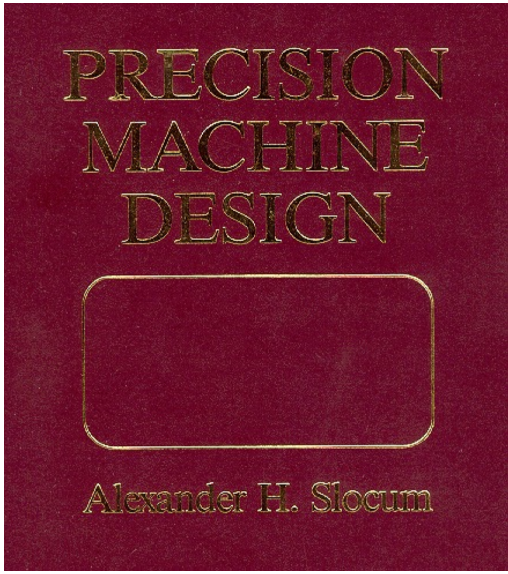
Background:
Find the time to immerse yourself in some Books

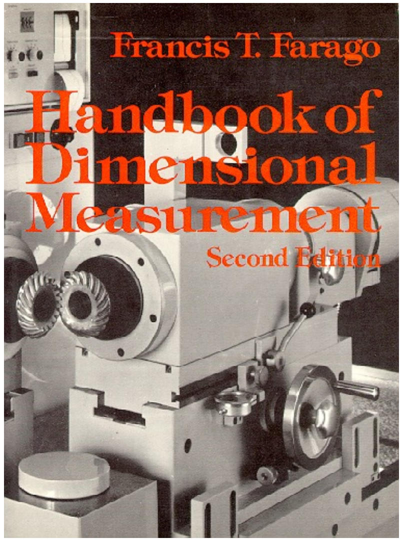


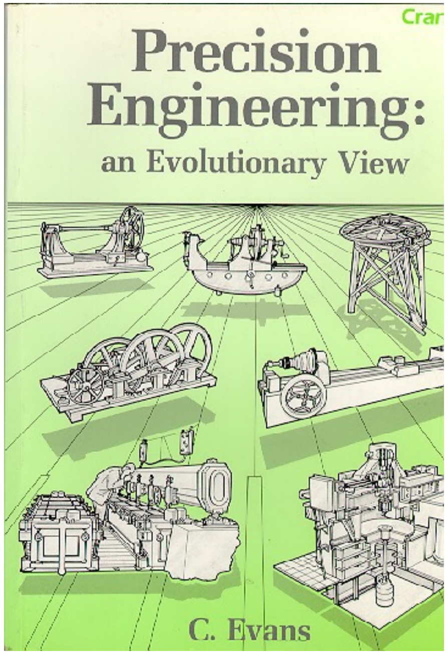
FUNdaMENTALS of Design

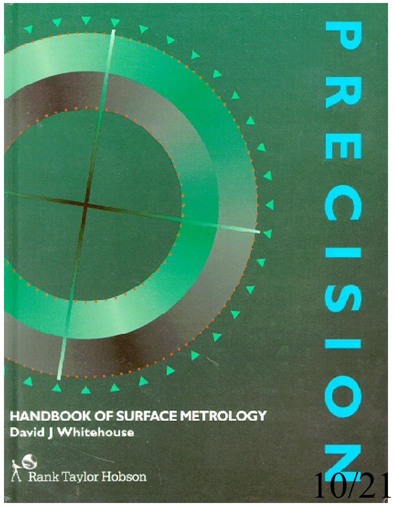
http://web.mit.edu/2.75/resources/FUNdaMENTALS.html

Free on-line text!









Reminder: Precision Machine Design Roadmap

Determine Functional Requirements

- Deflections
- Loads
- Power

Error Apportionment

Axis_error_apportionment_estimator

Develop Design

Stick Figures
Structural Loops

Error Analysis

Error_Budget_Spreadsheet

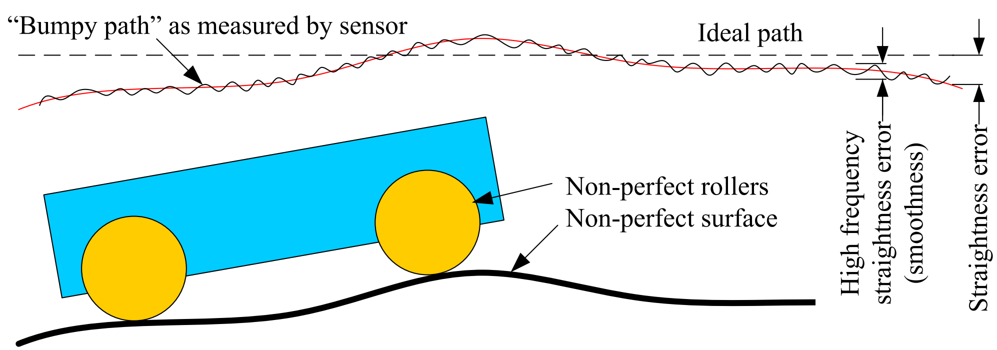
Detailed design

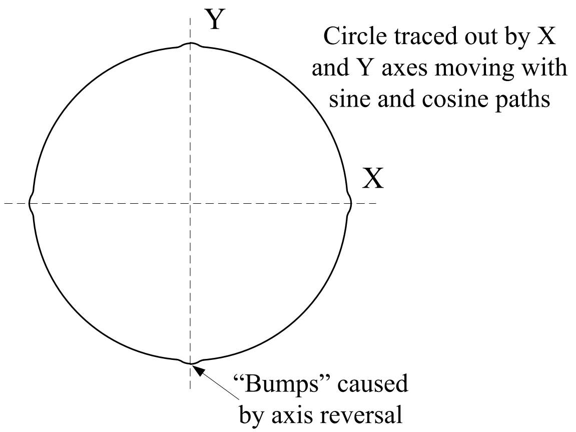
Error Budgets

- A fast low cost method to help evaluate concepts before detailed solid models, FEA (which will not catch geometric errors…), because:
  - Nothing is perfect
  - Need to estimate accuracy and repeatability of concepts
  - Need to better predict loads/life of bearings!
- Start with basics
  - Stick figures
  - Structural loop
  - Error Budget Spreadsheets
    - Homogeneous Transformation Matrices
  - Model Error Motions

"If in other sciences we should arrive at certainty without doubt and truth without error, it behooves us to place the foundations of knowledge in mathematics"

Roger Bacon

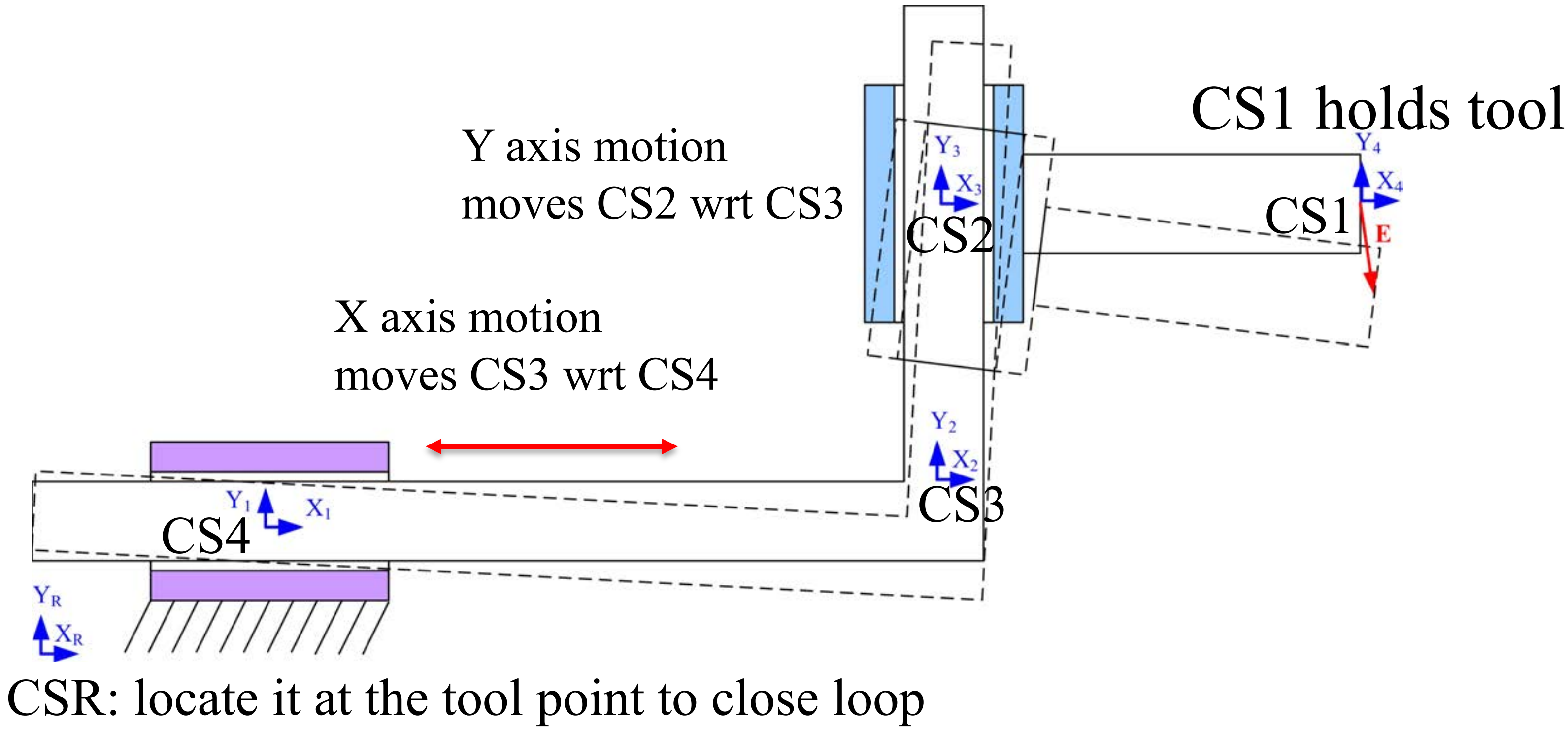




Error Budgets

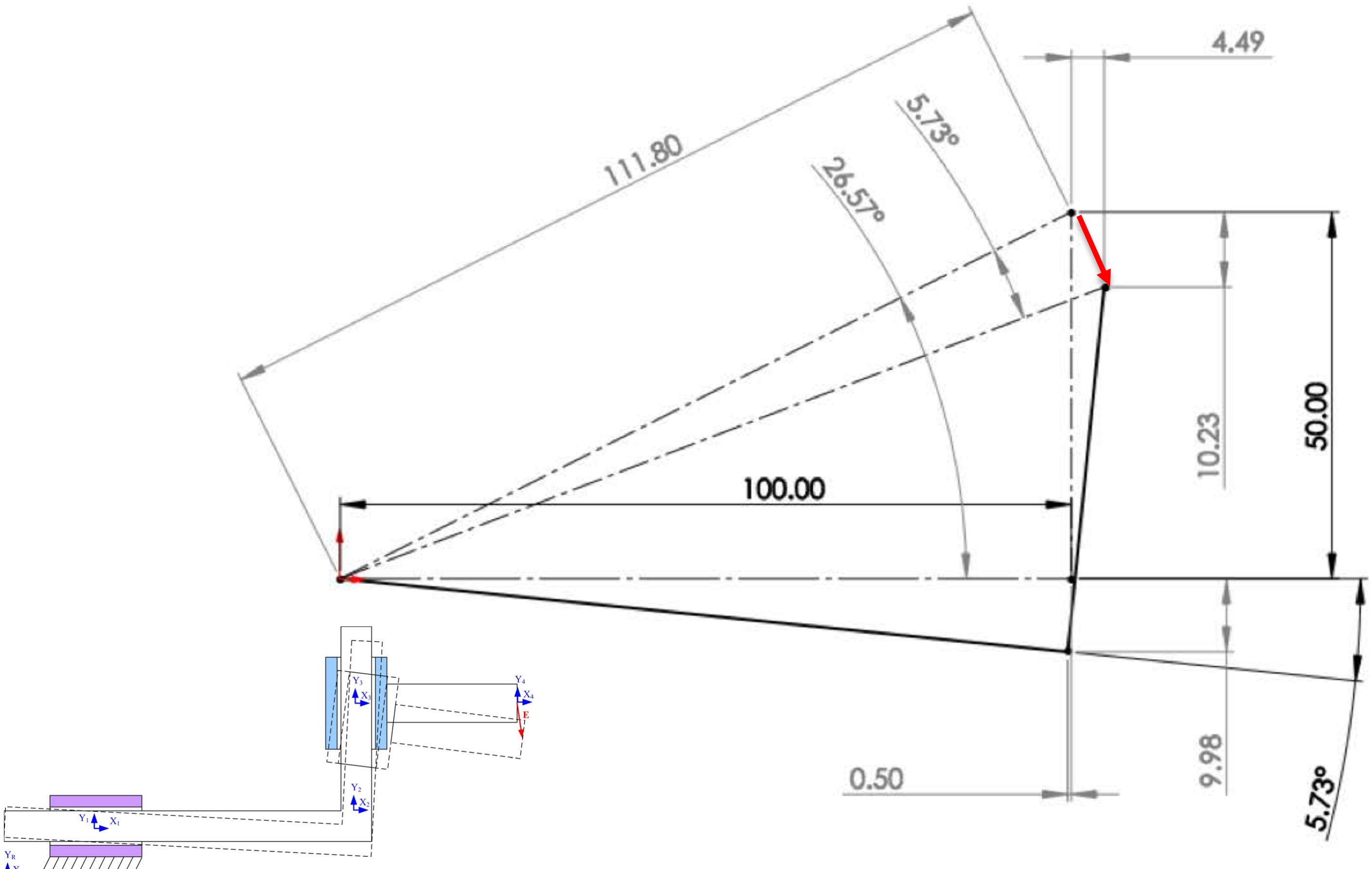
- Error budgets are useful for estimating the accuracy and/or repeatability of a machine
    - They can also be useful for helping to predict misalignment loads on bearings
- Four primary types of error include
  - Geometric
  - Load-induced
  - Thermal
  - Process
- For high precision machines, the magnitude of each will be about equal if there is a balanced allocation of resources
  - During the concept phase, develop the geometric-based error budget based on error allocation (e.g., geometric)
    - Spreadsheet uses Homogeneous Transformation Matrices to sum up errors from modules (coordinate systems)
      - This lets you investigate the overall geometry (and spacing) of elements as well as initial stiffness and load estimates
  - Next, use solid models and FEA to check on load-induced and thermal deflections

Example: Two Axis System



Key Issue: Angular Errors

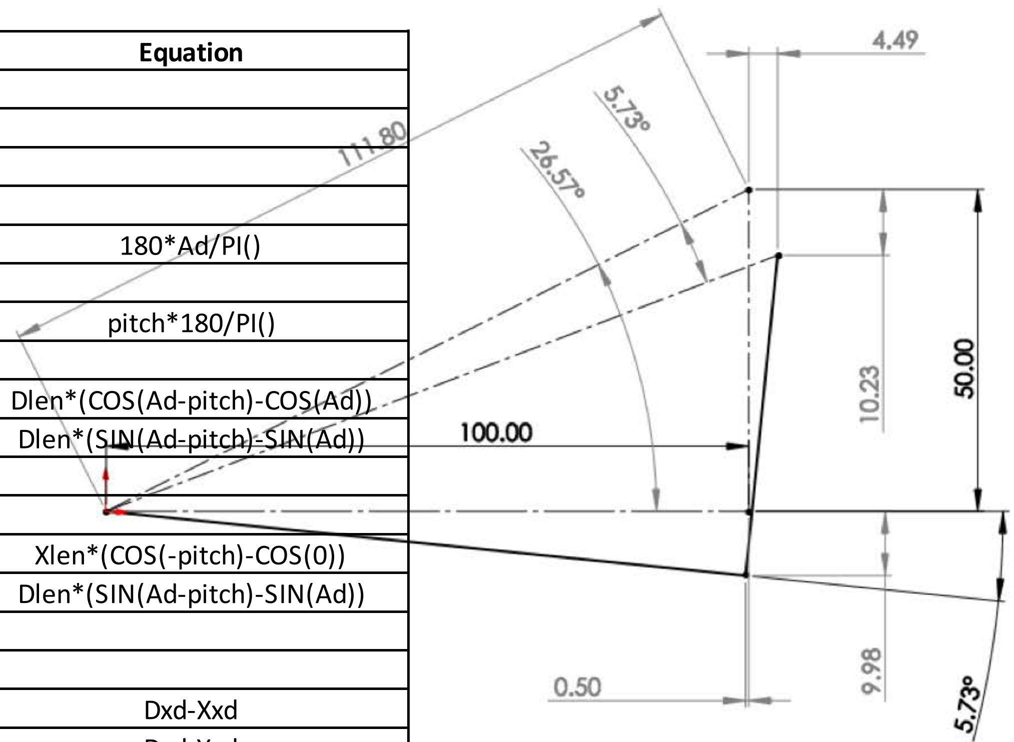
- As axes are stacked on top of each other, keeping track of the affect of small angular errors on toolpoint motion becomes daunting
  - ”Sine errors” are not so simple, and even “cosine” errors start to add up



Key Issue: Angular Errors

- As axes are stacked on top of each other, keeping track of the affect of small angular errors on toolpoint motion becomes daunting
  - ”Sine errors” are not so simple, and even “cosine” errors start to add up

	Value	Units	Variable	Equation
X length	**100**	mm	Xlen	
Y length	**50**	mm	Ylen	
Diagonal	**111.8**	mm	Dlen	
Angle	**0.464**	radians	Ad	
	26.57	degrees		180*Ad/PI()
Imposed pitch	**0.1**	radians	pitch	
	5.73	degrees		pitch*180/PI()
Tip displacement using diagonal				
X direction	**4.49**	mm	Dxd	Dlen*(COS(Ad-pitch)-COS(Ad))
Y direction	**-10.23**	mm	Dyd	Dlen*(SIN(Ad-pitch)-SIN(Ad))
Tip displacement using X direction length only				
X direction	**-0.50**	mm	Xxd	Xlen*(COS(-pitch)-COS(0))
Y direction	**-9.98**	mm	Xyd	Dlen*(SIN(Ad-pitch)-SIN(Ad))
Error in using simple X distance				
X direction	**4.99**	mm	dDX	Dxd-Xxd
Y direction	**-0.25**	mm	dDY	Dyd-Yyd

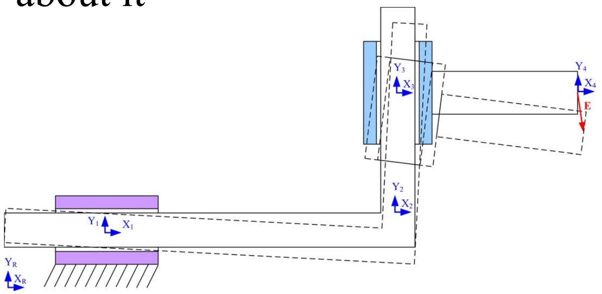


Keeping track of all the errors: *Homogeneous Transformation Matrices*

- Abbe errors (i.e., angular errors amplified by distances) are root cause of biggest errors
  - All those rotations about all those axes can become very confusing.
- HTMs help to model how motions in one link in a serial chain reflect through the chain

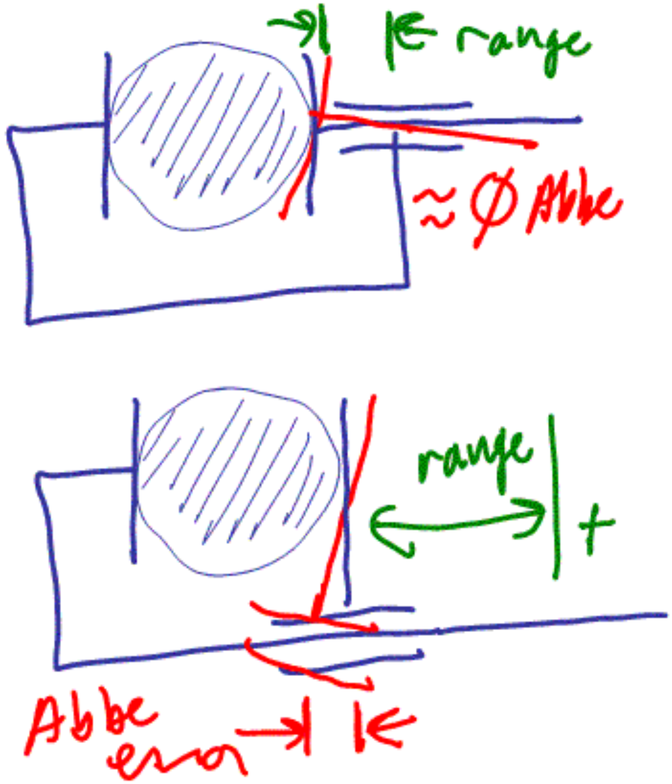
$$\begin{bmatrix} X_N \\ Y_N \\ Z_N \\ 1 \end{bmatrix} = {}^{N}T_{N+1} \begin{bmatrix} X_{N+1} \\ Y_{N+1} \\ Z_{N+1} \\ 1 \end{bmatrix} \qquad {}^{N}T_{N+1} = \begin{bmatrix} O_{ix} & O_{iy} & O_{iz} & P_x \\ O_{jx} & O_{jy} & O_{jz} & P_y \\ O_{kx} & O_{ky} & O_{kz} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

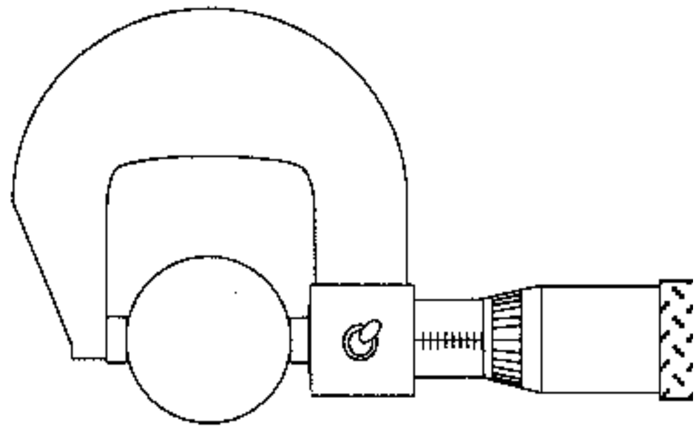
- Each column represents the *direction cosines* of the rotated axes
  - To avoid confusion about order of rotation, one HTM per axis of rotation
  - For small angular error motions, order not important
  - The spreadsheet does the math, so you do not have to worry about it
    - Just use good modeling technique



Homogeneous Transformation Matrices

- HTMs in particular enable modeling of "Abbe errors" as they cascade through the machine





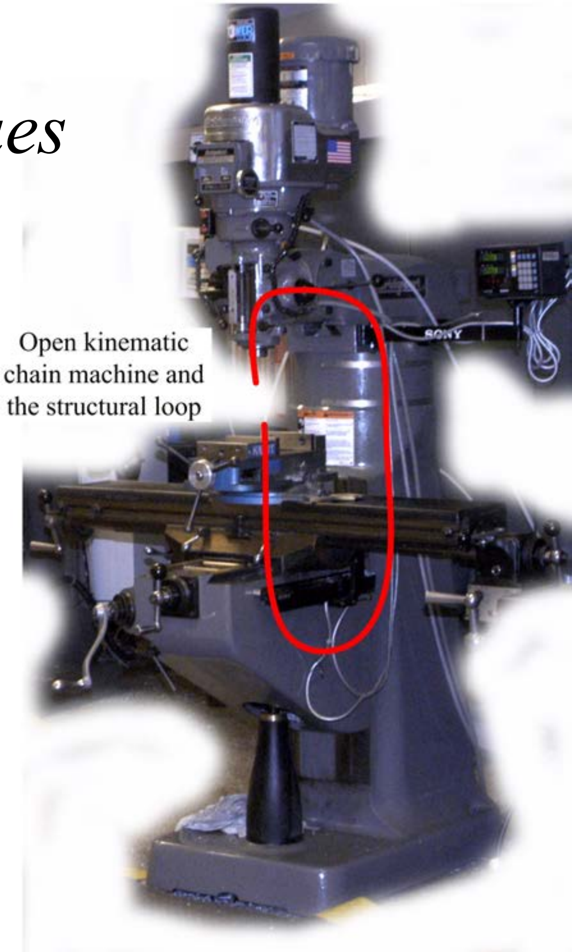
Example: A Single Axis' HTMs

- Consider a single axis moving in the X direction, with a tool tip extending from the plane of the center of stiffness, and $\varepsilon_x = \varepsilon_y = \varepsilon_z = 0.001$ radians
- The individual HTMs to provide the location of the tool tip in the center of stiffness coordinate system are:

45	Tool tip coordinate vector				
46	xt	1			
47	yt	1			
48	zt	1			
49		1			
50	HTM for epsx_1				
51		1	0	0	0
52		0	0.9999995	-0.001	0
53		0	0.001	0.9999995	0
54		0	0	0	1
55					
56	HTM for epsy_1				
57		0.9999995	0	0.001	0
58		0	1	0	0
59		-0.001	0	0.9999995	0
60		0	0	0	1
61					
62	HTM for epsz_1				
63		0.9999995	-0.001	0	0
64		0.001	0.9999995	0	0
65		0	0	1	0
66		0	0	0	1

Error Budgets:
Spreadsheets & Modeling Techniques

- Open kinematic chain machines are straightforward to model
- Closed kinematic chains require local calculation of error motion or an equivalent open-chain model
  - E.g., kinematic coupling error motions
  - Bridge-type machines
    - Method 1:
      - Widely spaced bearings that simply support a bridge component are condensed to a single "very accurate" bearing that supports a cantilever
      - For example, the accuracy of this bearing is based on error motions of a carriage supported by bearings which are spaced the bridge-width apart
    - Method 2:
      - Place the center of stiffness (and the CS) essentially in the middle of the bridge…

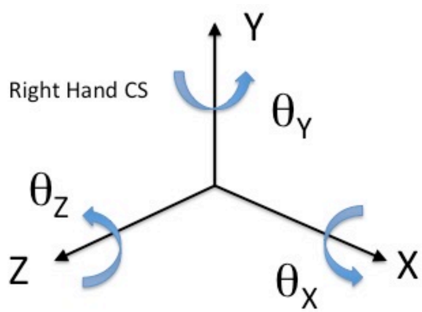


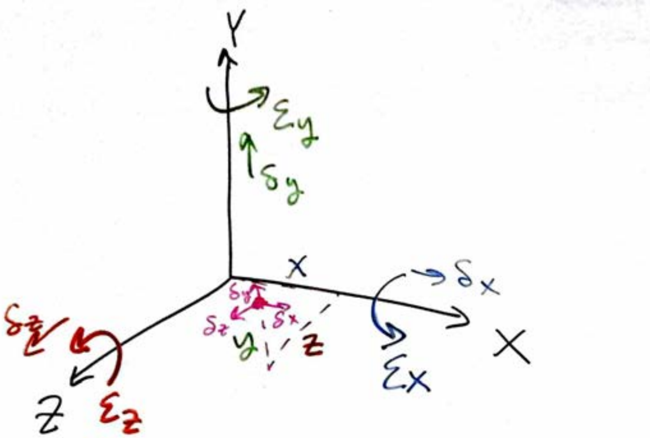
Open kinematic chain machine and the structural loop

Error Gains

- “Error Gains” enable accounting for the sign of errors
  - A random error has no sign, yet there is an implied sign that occurs with random rotational error motions due to the way rotations are done mathematically
  - From *Error_Budget_Spreadsheet.xls: Axes' CS and Loads*
- Errors at the tool tip are calculated by multiplying each error’s gain by each error as it occurs within the machine
  - All the errors and their contributions to the total can thus be kept separate
  - In the case of random errors, their absolute values can then be combined without one error cancelling another (e.g., for average of errors)
-

Error gain matrix: rotational errors' effects (in this coordinate sy			
	epsx's gains	epsy's gains	epsz's gains
deltaX	0.000	-213.000	45.000
deltaY	213.000	0.000	10.000
deltaZ	-45.000	-10.000	0.000

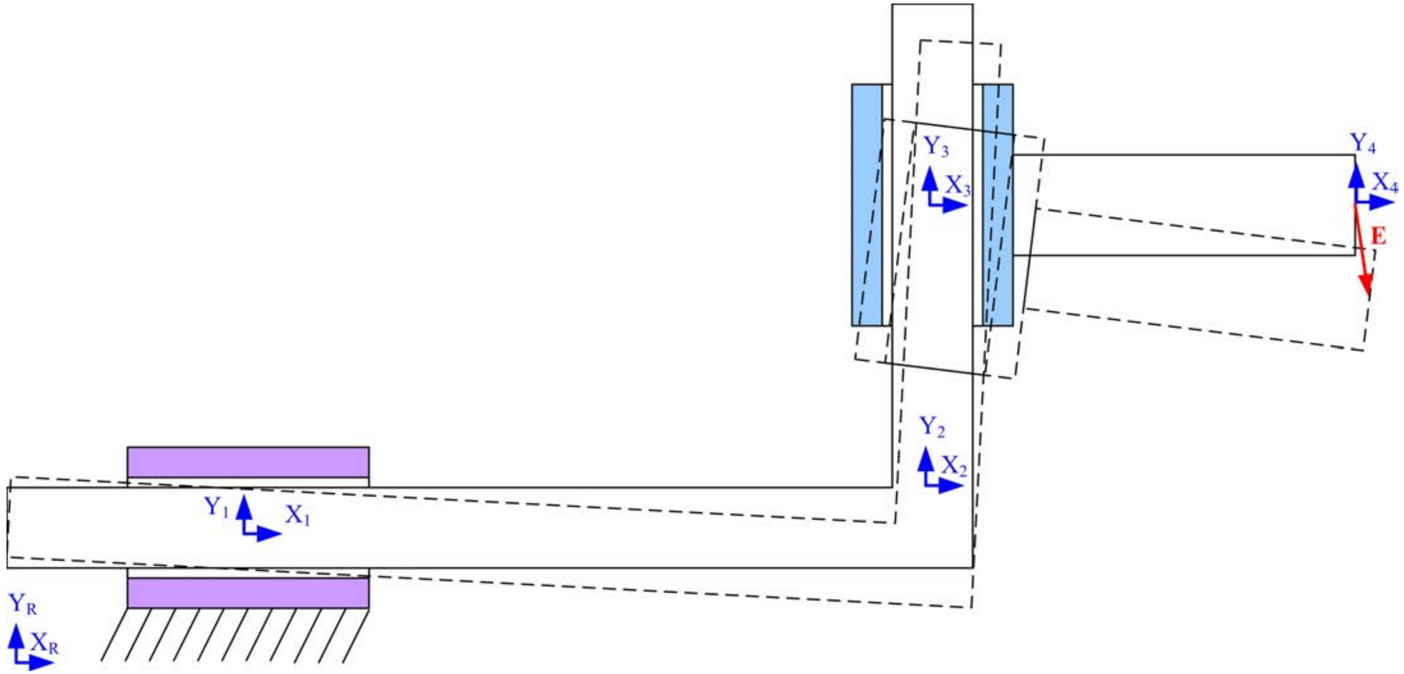




$$\delta_x \approx \varepsilon_y z - \varepsilon_z y + \delta_x$$
$$\delta_y \approx -\varepsilon_x z + \varepsilon_z x + \delta_y$$
$$\delta_z \approx \varepsilon_x y - \varepsilon_y x + \delta_z$$

Error Gain

- The gain for a rotation error about the Z1 axis will depend on the location of the tool tip (Coordinate System 4) with respect to the reference coordinate system (CS1 = CSR in simplest case)

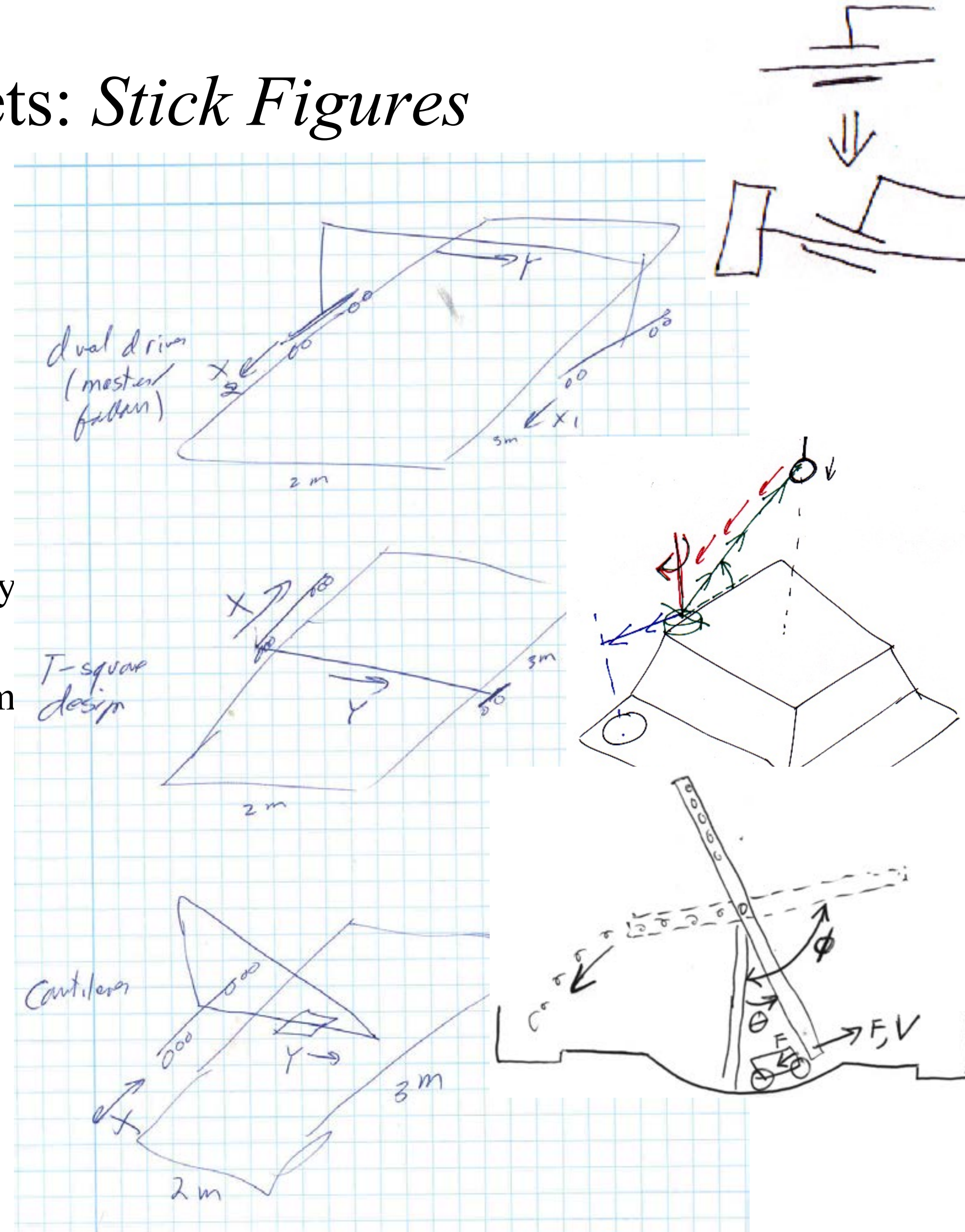


Using Error Gains for Total Error Estimation

- For a particular machine geometry, a spreadsheet can compute the Error Gains
  - Each error in each axis has a unique *gain* that depends on machine axes' positions
  - Gains have a sign: indicate magnitude and direction of the resulting toolpoint error
  - Random errors in a coordinate system's location are multiplied by their *gain* to create an assessment of all random errors including their "signs" (direction)
- Different combination methods can be applied
  - Sum of all errors may lead to random errors cancelling (too liberal)
  - Sum of absolute value of all errors (too conservative)
  - Root Square Sum of all errors (liberal)
  - Average of Sum of absolute value of all errors & RSS (fair estimate)
  - Ideally probabilistic methods would be used in a Matlab or Python based program
    - E.g., Monte Carlo simulation
    - See "An error budgeting framework for engineering design" MIT SM thesis, Andrew Duenner
- Which method?
  - Develop experience and intuition by design, predict, measure, compare results with predictions!

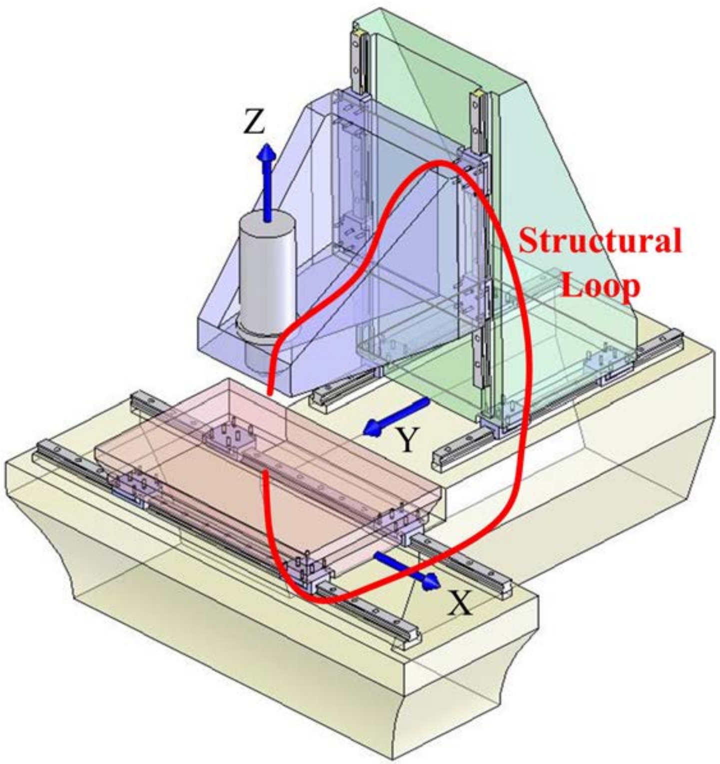
Error Budgets: *Stick Figures*

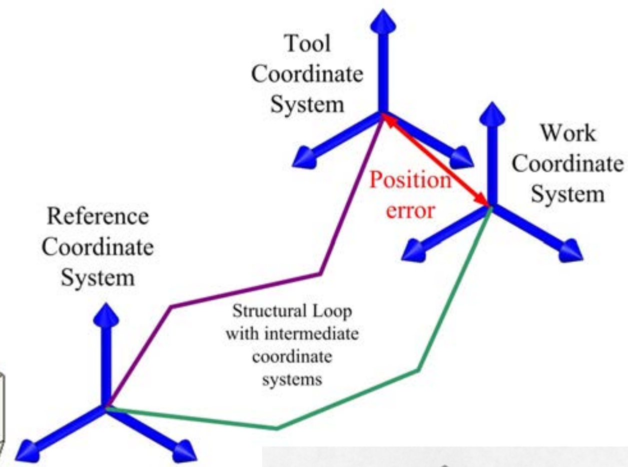
- Stick figure:
  - The sticks join at centers of stiffness, mass, friction, and help to:
    - Define the sensitive directions in a machine
    - Locate coordinate systems
    - Set the stage for error budgeting
  - The designer is no longer encumbered by cross section size or bearing size
    - It helps to prevent the designer from locking in too early on a concept
- Error budget and preliminary load analysis can indicate required stiffness required for each "stick" and "joint"
  - Appropriate cross sections and bearings can then be deterministically selected

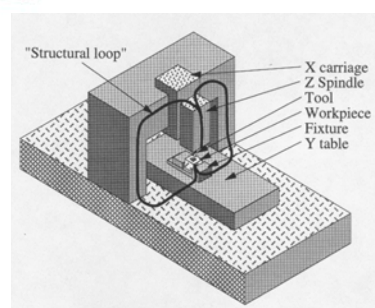


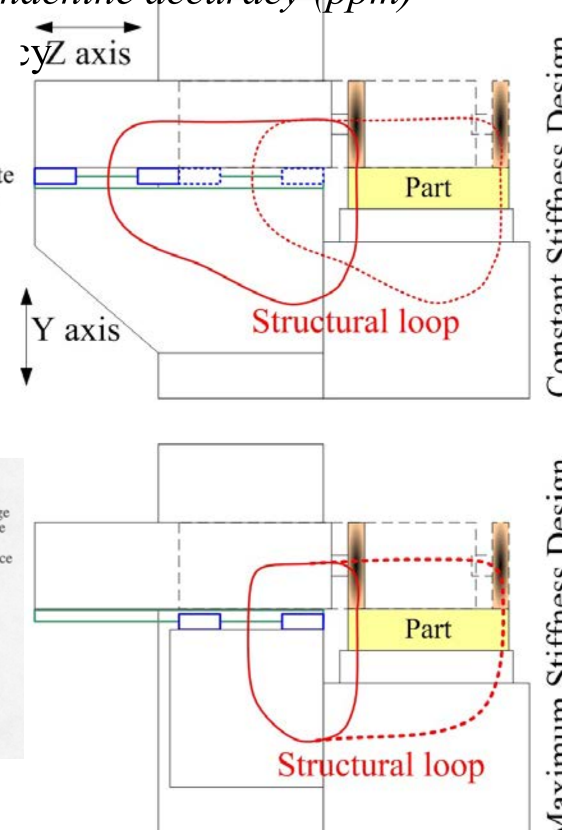
Error Budgets: Structural Loops

- The *Structural Loop* is the path that a load takes from the tool to the work as it passes through axes' Centers of Stiffness (CoS)
  - It contains joints and structural elements that locate the tool with respect to the workpiece
  - It can be represented as a stick-figure to enable a design engineer to create a ***concept***
  - Subtle differences can have a HUGE effect on the performance of a machine
  - The *structural loop* gives an indication of machine stiffness and accuracy
    - *The product of the length of the structural loop and the characteristic manufacturing and component accuracy (e.g., parts per million) is indicative of machine accuracy (ppm)*



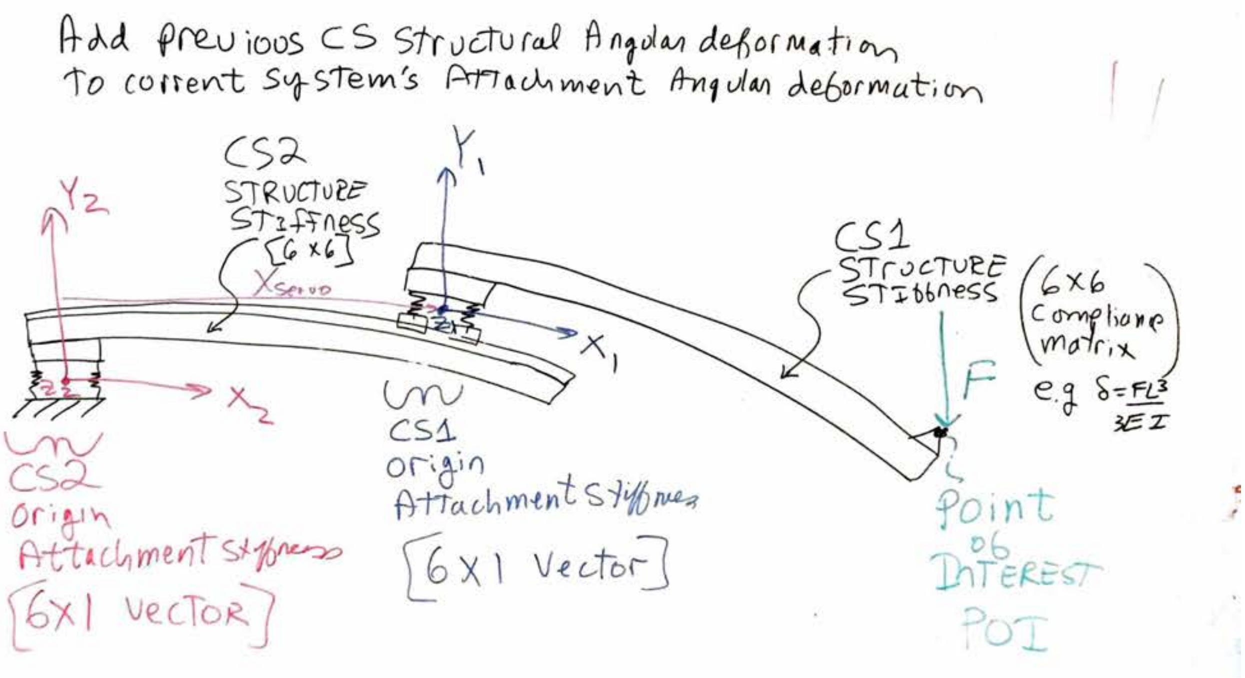






Coordinate Systems and System Stiffness

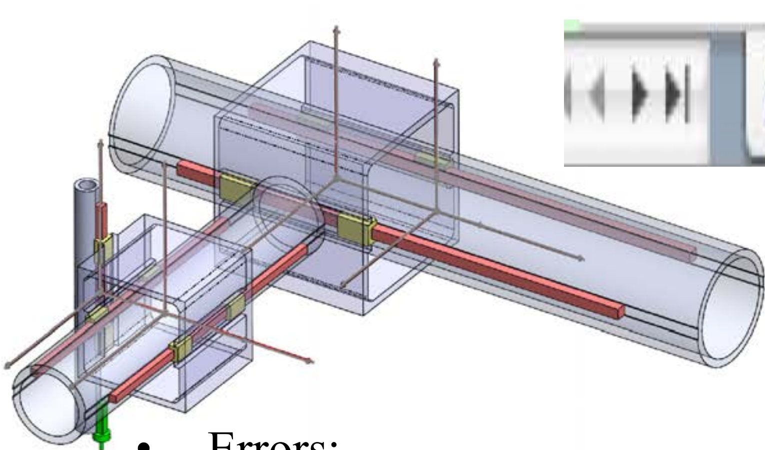
- There are TWO stiffness regimes to consider
  - Stiffness of the connection (e.g., bearings) at the Coordinate System origin (Center of Stiffness) caused by forces and moments applied to the CoS
    - Linear stiffness
    - Rotary stiffness
  - Stiffness of the structure (e.g., cantilever beam)
    - 6x6 stiffness matrix



Error Budgets: *Spreadsheets*

- Modules from Tool to work position are each defined with respect to adjacent modules all the way back to a reference coordinate system (CS)
    - Start with CS1 as the CS that defines the position of the tool

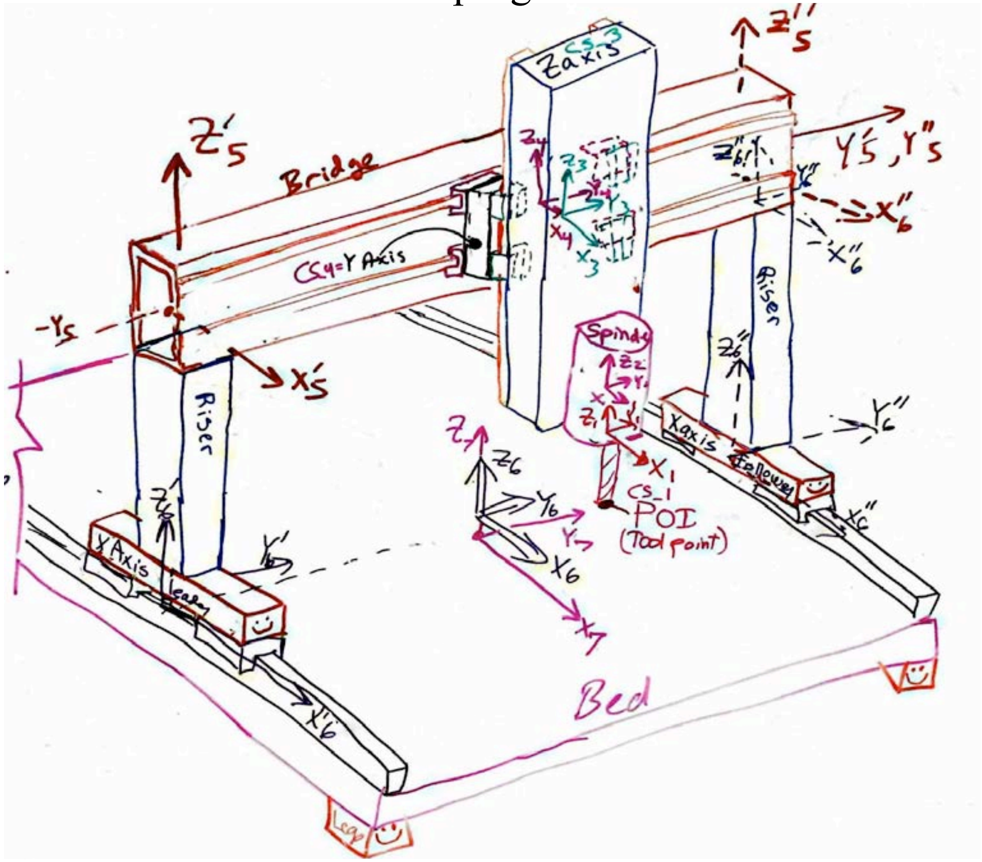
…



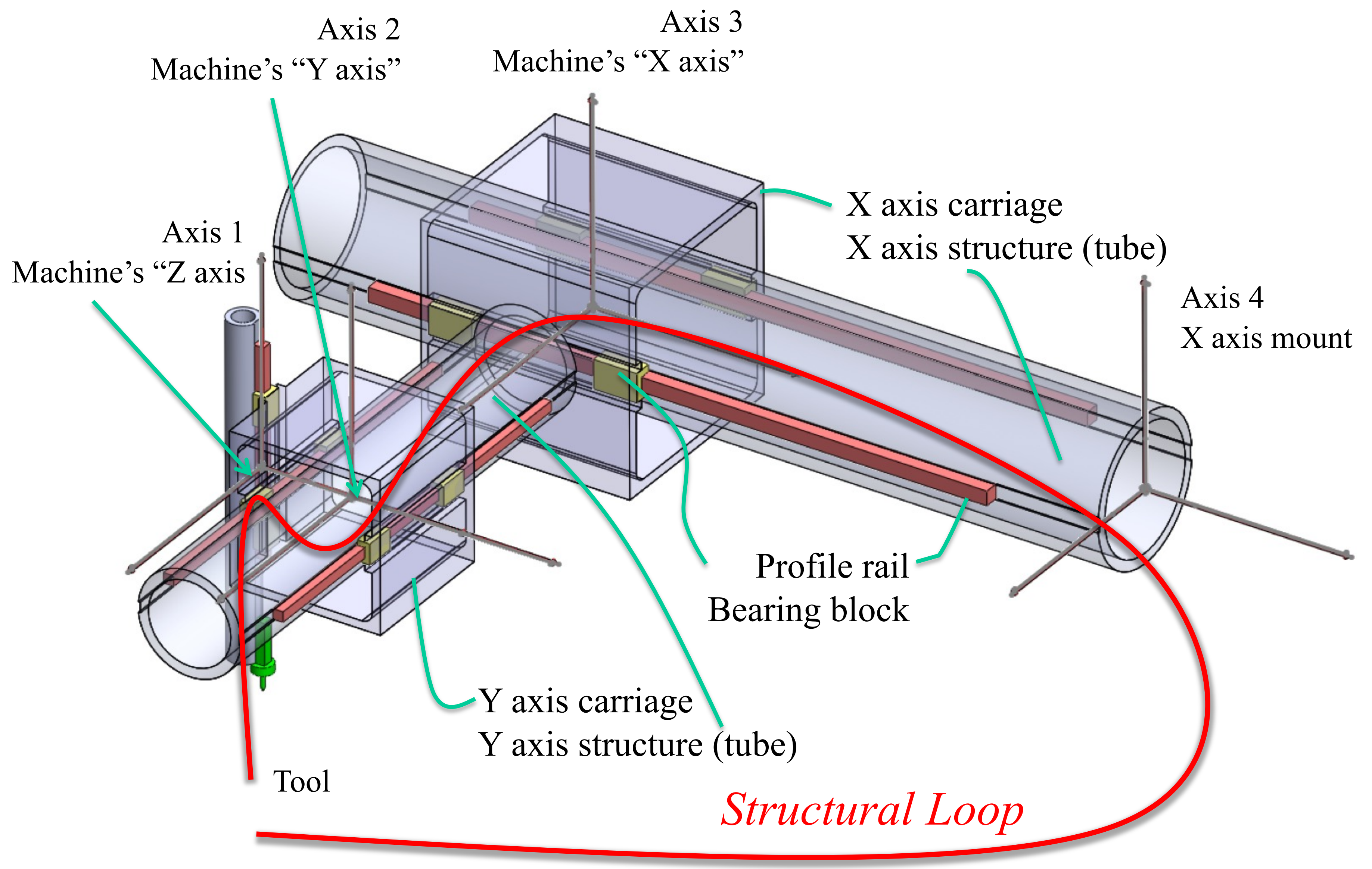
- Errors:
    - Random errors:
        - Bearing motions, mfg errors, thermal
    - Systematic errors:
        - Load induced (force, thermal), mfg errors (to be mapped)
            - Deflections modeled using linear & rotational springs at center of stiffness
            - Bearings and structural elements can be included
            - Simple first order approximations can be obtained to roughly size elements
    - Solid model and FEA used later to get better estimates

How to Use Error Budget Spreadsheet

- See the text "How to use Error budget spreadsheet.doc"
- The dilemma is does one use a generic program, or carefully write one's own for the specific design at hand?
  - It may take longer to learn and tweak the generic program than to just learn from it and start from scratch with a dedicated program



Example: Coordinate System Location



Example: Coordinate System Location

- Axis 1
  - Origin placed at the Center of Stiffness (CoS) of the two Z axis bearing carriages
  - The tool is located wrt the CoS
- Axis 2
  - Origin placed at the CoS of the four Y axis bearing carriages
  - Axis 1 origin is located wrt the CoS
- Axis 3
  - Origin placed at the CoS of the four X axis bearing carriages
  - Axis 2 origin is located wrt the CoS
- Axis 4
  - Origin placed at the CoS of the X axis structure
    - This forms the reference CS
      - Deflections or mounting errors of the X axis tube cause motions of axis 3 origin and reflect on up to the toolpoint
- Next step—walk through the details on the spreadsheet

Intuition check: Stiffness Along the Loop

- Type of beam?

round_square_tube_compare.xls				
To estimate size of a motor given desired torque				
By Alex Slocum, Last modified 2020.10.11 by Alex Slocum				
Enters numbers in **BOLD,** Results in **RED**				
	units	name	value	
Structural beam modulus of elasticity	MPa	Es	**6.67E+04**	
Shear Modulus	MPa	Gs	**2.58E+04**	
Round			**X axis**	**Y axis**
OD	mm	R_OD	**250**	**200**
wall	mm	R_wall	**25**	**20**
Area	mm^2	R_Area	**17,671**	**11,310**
I	mm^4	R_I	**1.E+08**	**5.E+07**
Ipolar	mm^4	R_Ip	**2.E+08**	**9.E+07**
Square				
OD	mm	S_OD	**200**	
wall	mm	S_wall	**20**	
Area	mm^2	S_Area	**14,400**	
Iyy	mm^4	S_I	**7.87E+07**	
Ipolar	mm^4	S_IP	**1.17E+08**	
Comparison				
Bending stiffness round/square	**1.44**			
torsional stiffness round/square	**1.94**			
weight round/square	**1.23**			

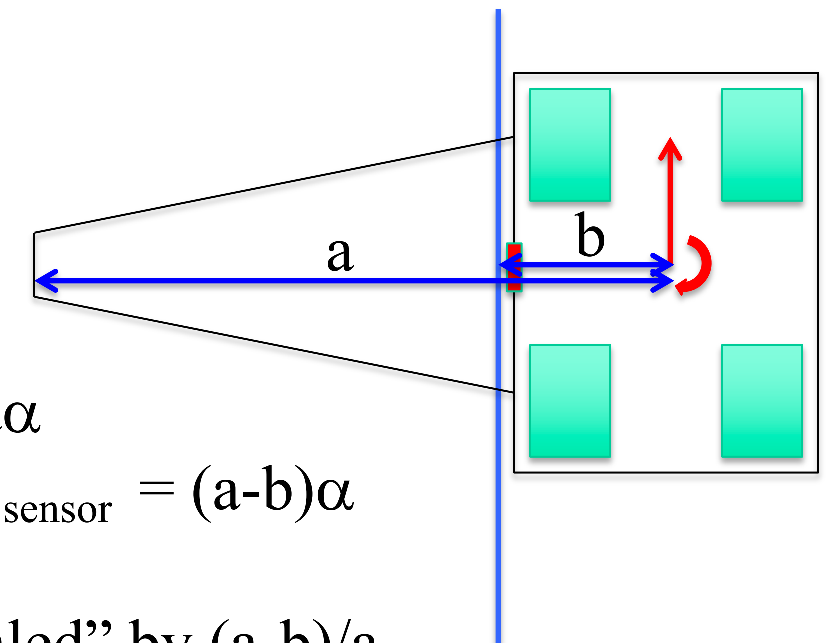
Stiffness Along the Loop

- Effect of linear bearings?
- Bearings are often sized for load/life
  - Linear_Motion_Guide_Life.xls
- End up being much stiffer than the structure when mounted on structure sized for stiffness

Addition of size 25 bearing rails		
width	mm	**23**
height	mm	**23**
Area	mm^2	**529**
Moment of Inertia	mm^4	**23,320**
Modulus of elasticity	MPa	**2.00E+05**
distance of centroid to centroid of circle	mm	**131.5**
Machined groove for bearing rails		
depth	mm	**5**
area	mm^2	**115**
Moment of Inertia	mm^4	**1198**
Plane round section stiffness (EI)	N-mm^2	**7.5.E+12**
Increase in round section stiffness (EI) by steel linear guide rails on top and bottom	N-mm^2	**9.2.E+12**
Increase in stiffness by mounting steel rails at top and bottom location on round rail		**1.22**
Assume simply supported X axis with Y axis loading it		
X axis length	mm	**1600**
Force from Y axis	N	**1000**
Deflection (currently)	mm	**0.0113**
Deflection if rails mounted top and bottom	mm	**0.0093**
difference	mm	**0.0021**
Size 25 linear guide carriages		
Stiffness	N/micron	**600**
Number of carriages per rail		**2**
Total translational stiffness	N/micron	**2400**
spacing between carriages on rails on opposite sides of tube	mm	**263**
Angular stiffness	N-m/rad	**4.15E+07**
Torsional stiffness of X axis tube structure	N-m/rad	**1.46.E+07**
Ratio linear bearing torsional stiffness/X axis		**2.84**
Assume Y axis Cantilevered		
Y axis length	mm	**600**
Distance from load applied to base of beam	mm	**500**
Deflection due to Y axis beam bending	mm	**0.013**
Distance from load applied to center of X axis	mm	**625**
Deflection due to X axis structure torsion	mm	**0.000027**
Deflection due to X axis bearings torsion	mm	**0.000009**
Total torsional deflection	mm	**0.000036**
Ratio bending deflection / torsional deflection		**373**

Sensor Placement Effects

- If a linear encoder is used to measure an axis' position, it can reduce the Abbe error
- To model the effect of sensor placement, scale the angular error that affects error motions in the direction of sensor action
  - Add sensor location effect as an input in the Error Budget spreadsheet



$$\delta_{Abbe} = a\alpha$$

$$\delta_{\text{Abbe with sensor}} = (a-b)\alpha$$

α is "scaled" by (a-b)/a

Error_Budget_Spreadsheet.xls: Summary Worksheet

- Outputs for system and each axis, and input servo axis motions

Error_Budget_Spreadsheet.xls

Suumary of Results

Written by Aex Slocum 2015.08.30, last edit 2016.05.24 by AHS

3 DoF Bridge Machine

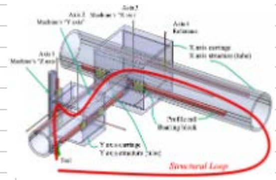
Last use: 2020.10.11	By:	Alex Slocum	*Please enter edits in "Edit History" Worksheet*	
Inputs in BLACK in Yellow backgrour	Input	BE CONSISTANT WITH UNITS!		
Outputs in RED in Green backgrounc	Output			

Axes' coordinates and applied loads are defined in worksheet *"Axes' CS & Loads"*

Axes' coordinates and applied loads are defined in worksheet *"Axes' CS & Loads"*

All Axes

	For the Entire Machine, length dimenson				mm
Number of axes	All axes' Geometric Errors				
4	Random			Systematic	F=kX displacement
	Sum	RSS	Avg(SUM, RSS)	Sum	Sum
deltaX	0.131	0.091	0.111	-0.816	0.009
deltaY	0.070	0.046	0.058	0.583	-0.012
deltaZ	0.123	0.093	0.108	0.984	-0.064
Vector displacement	0.193	0.138	0.165	1.405	0.066



Servo cells are **BOLD PURPLE** other numbers for quick ref and all are used workbook CSs and Loads

Setting up the structural loop and the CS's locations wrt each other, using servo motions to drive changes

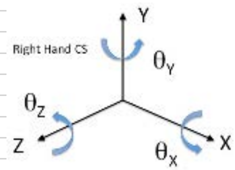
	X	Y	Z	Description
POI location in CS_1 coordinates	0	0	-200	Tool point wrt Z carriage
CS_1 location in CS_2 coordinates	-100	0	0	Z carriage CoS wrt Y carriage CoS
CS_2 location in CS_3 coordinates	0	500	0	Y carriage CoS wrt X carriage CoS
CS_3 location in CS_4	-150	200	0	X carriage CoS wrt X axis anchor to "rigid frame"
CS_4 coordinates wrt the toolpoint	250	-700	200	Anchor point wrt the cutting point
Sum (must be zero)	0	0	0	
X servo (0-600 mm)		200	X servo	Only moves to middle of axis (simply supported tube)
Y servo motion (0 to 700 mm)		200	Y servo	
Z servo motion (0-200)		0	Z servo	

This is a section the user creates to model motion of the machine. It is used to vary relative locations of the Coordinate systems on the workbook tab *CSs and Loads*

Error_Budget_Spreadsheet.xls: *Axes' CS and Loads Worksheet*

- Inputs for each axis' geometry and loads; outputs for each axis' load & error calcs

Error Budget Spreadsheet
Axes' Coordinate Systems and Loads Applied
Written by Aex Slocum 2015.08.30



Inputs in BLACK in Yellow background cells:	Input from Summary)	
Outputs in RED in Green background cells:	Output	
Axes' coordinates and applied loads are defined in worksheet *"Axes' CS & Loads"*		
Axes' coordinates and applied loads are defined in worksheet *"Axes' CS & Loads"*		
Units of length	mm	
Number of axes used (NOT including reference frame), Naxes	4	*Note that axes beyond this number are automatically "turned off"*

NOTE axis 1 is the axis (structure) that HOLDS the tool, it doe NOT have to be located at tool point)

CS 1	Coordinates of point of interest (POI) in this CS (0,0,0 is at CS center of stiffness)		
	X coordinate of point of interest in this CS	0	
	Y coordinate of point of interest in CS	0	
	Z coordinate of point of interest in CS	-200	
1	Coordinates of this CS in the next CS		
1	Rotation about this CS's X axis to align Y axes (degrees)	0	
Enter name below	Rotation about resulting Y axis to align axes (degrees)	0	
	rotation about resulting Z axis to align axes (degrees)	0	
	X translation of this CS's origin along *next* CS's X axis	-100	*Location of CS_1's origin in CS_2*
	Y translation of aligned CS along *next* CS's Y axis	0	
	Z translation of aligned CS along *next* CS's Z axis	0	
Z motion carriage where tool tip is POI	Resulting ideal coordintes of point of interest in the next CS		
	X coordinate of point of interest in *next* CS	-100.000	
	Y coordinate of point of interest in *next* CS	0.000	
	Z coordinate of point of interest in *next* CS	-200.000	
	Applied loads at point of interest in this axis' CS		
	Fx=	0	
	Fy=	0	
	Fz=	-20	
	unity	1	
	Mx=	0	
	My=	0	
	Mz=	0	
	unity	1	

Applied loads at Center of Stiffness in this axis' CS		Resulting loads at this CS Center of Stiffness	
Fx=	0	Fx=	0
Fy=	0	Fy=	0
Fz=	-100	Fz=	-120
unity	1	unity	1
Mx=	0	Mx=	0
My=	0	My=	0
Mz=	0	Mz=	0
unity	1	unity	1

Error motions of CS Origin along & about its axes. Values from Axis_1 worksheet

Random Geometric Errors		Systematic Geometric Errors	
deltaX	0.005000	deltaX	0.050000
deltaY	0.005000	deltaY	0.050000
deltaZ	0.005000	deltaZ	0.050000
EpsilonX	0.000050	EpsilonX	0.000500
EpsilonY	0.000050	EpsilonY	0.000500
EpsilonZ	0.000050	EpsilonZ	0.000500

This CS origin F=kx Displacements + structural angular displ from next CS		Stiffness of this CS origin wrt its axes		Next CS CS Origin structural angular displ		Next CS CS Origin structural angular displ in this CS	
deltaX	0.000000	KX	2312800	EpsilonX	0.000000	EpsilonX	0
deltaY	0.000000	KY	6335.545185	EpsilonY	0.000000	EpsilonY	0
deltaZ	-0.000052	KZ	2312800	EpsilonZ	0.000000	EpsilonZ	0
EpsilonX	0.000000	KrotX	23262791667	Unity	1	Unity	1
EpsilonY	0.000000	KrotY	23532375000				
EpsilonZ	0.000000	KrotZ	23262791667				

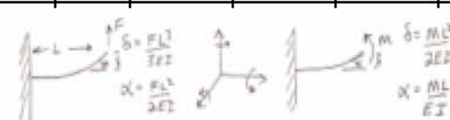
This CS POI F=kx structural displacements		POI displacements = Structure compliance matrix * Force & Moment at POI					
deltaX	0.000000	2.09E-05	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
deltaY	0.000000	0.00E+00	6.64E-06	0.00E+00	0.00E+00	0.00E+00	4.98E-08
deltaZ	-0.000417	0.00E+00	0.00E+00	2.09E-05	0.00E+00	-1.56E-07	0.00E+00
EpsilonX	0.000000	0.00E+00	0.00E+00	0.00E+00	6.72E-10	0.00E+00	0.00E+00
EpsilonY	0.000003	1.56E-07	0.00E+00	-1.56E-07	0.00E+00	1.56E-09	0.00E+00
EpsilonZ	0.000000	0.00E+00	4.98E-08	0.00E+00	0.00E+00	0.00E+00	6.72E-10

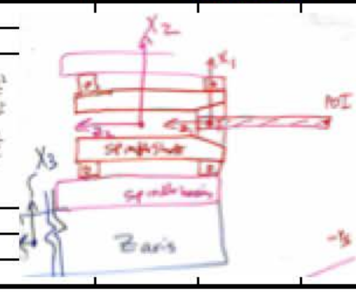
	NOTE axis 1 is the axis (structure) that HOLDS the tool, it doe NOT have to be located at tool point)				
CS 1	**Coordinates of point of interest (POI) in this CS (0,0,0 is at CS center of stiffness)**			**Error motions of CS Origin along & about its ax**	
	X coordinate of point of interest in this CS	0		**Random Geometric Errors**	
	Y coordinate of point of interest in CS	0		deltaX	0.005000
	Z coordinate of point of interest in CS	-200		deltaY	0.005000
1	**Coordinates of this CS in the next CS**			deltaZ	0.005000
1	Rotation about this CS's X axis to align Y axes (degrees)	0		EpsilonX	0.000050
Enter name below	Rotation about resulting Y axis to align axes (degrees)	0		EpsilonY	0.000050
	rotation about resulting Z axis to align axes (degrees)	0		EpsilonZ	0.000050
	X translation of this CS's origin along *next* CS's X axis	-100	*Location of CS_1's origin in CS_2*	**This CS origin F=kx Displacements + structural angular displ from next CS**	
	Y translation of aligned CS along *next* CS's Y axis	0		deltaX	0.000000
	Z translation of aligned CS along *next* CS's Z axis	0		deltaY	0.000000
Z motion carriage where tool tip is POI	**Resulting ideal coordintes of point of interest in the next CS**			deltaZ	-0.000052
	X coordinate of point of interest in *next* CS	-100.000		EpsilonX	0.000000
	Y coordinate of point of interest in *next* CS	0.000		EpsilonY	0.000000
	Z coordinate of point of interest in *next* CS	-200.000		EpsilonZ	0.000000
	Applied loads at point of interest in this axis' CS			**This CS POI F=kx structural displacements**	
	Fx=	0		deltaX	0.000000
	Fy=	0		deltaY	0.000000
	Fz=	-20		deltaZ	-0.000417
	unity	1		EpsilonX	0.000000
	Mx=	0		EpsilonY	0.000003
	My=	0		EpsilonZ	0.000000
	Mz=	0			
	unity	1			
	Applied loads at Center of Stiffness in this axis' CS		**Resulting loads at this CS Center of Stiffness**		
	Fx=	0	Fx=	0	
	Fy=	0	Fy=	0	
	Fz=	-100	Fz=	-120	
	unity	1	unity	1	
	Mx=	0	Mx=	0	
	My=	0	My=	0	
	Mz=	0	Mz=	0	
	unity	1	unity	1	

Error_Budget_Spreadsheet.xls: Axes' Errors and Compliances Worksheet

- Input connection's coordinates, parameters for computing stiffness, and errors of elements

A	B	C	D	E	F	G	H	I	J	K	L	M
CS 1	Tool Holder											
Geometric errors at CS origin and location and stiffness of points (or bearings) of attachment between coordinate systems												
Error motions of CS Origin along & about its axes. Values read used by "Coordinate Systems" worksheet				***CAREFUL as this compliance matrix is referenced in the worksheet "CSs and Loads"***								
Element	**Tool**	**Bearings (if values needed for this CS)**		**Tool in toolholder (cantilever beam): X lateral, Y transverse, Z axial**								
Structure type	**cantilever**	Size (e.g., rail width)	25	**dx**		1.74E-04	0	0	0	-2.61E-06	0	**Fx**
Material	**steel tool**	Fmax (N)	17346	**dy**		0	1.74E-04	0	2.61E-06	0	0	**Fy**
Young's Modulus (N/mm^2)	2.00E+05	k @ medium preload (N/mm)	647000	**dz**		0	0	2.04E-06	0	0	0	**Fz**
Poison ratio	0.29	ball diameter (mm)	3	**thetaX**	**-=-**	0	2.61E-06	0	5.22E-08	0	0	**Mx**
Shear modulus (N/mm^2)	77519	% deflection max load	0.01	**thetaY**		-2.61E-06	0	0	0	5.22E-08	0	**My**
Beam properties (rectangular tube, or round, else customize)		stiffness est (N/mm)	578200	**thetaZ**		0	0	0	0	0	6.73E-08	**Mz**
Position along beam	100	rail width (mm)	25									
Length (mm)	100	roll rotary stiffness (Nmm/radian)	101093750									
Height (mm), or OD	25	pitch rotary stiffness (Nmm/radian)	33697917									
Width (mm), ID or fraction of solid cylinder	50%	yaw rotary stiffness (Nmm/radian)	33697917									
wall (mm)		**servo system (if values needed for this CS)**										
Area (mm^2)	245	Ballscrew										
I width dimension of beam (mm^4)		Root diameter (mm)	18									
I height dimension of beam (mm^4)	9587	Length (mm)	2500									
Ipolar (mm^4)	19175	Modulus	2.00E+05									
I/A stiffness to weight ratio	39	shaft stiffness	2.04E+04									
ref, simply supported stiffness about Izz (N/mm)	5752	factor to derate stiffness of shaft for nu	3									
Notes: Y axis is + into the page so a + X direction force causes a - rotation about Y axis. Cells not used are colored blue												
Net stiffness of CS origin (POA@Center of Stiffness) due to attachment points		**Servo stiffness (if applicable, else make sure to enter "0"**										
Linear stiffness												
KX	1.00E+06	0.00E+00										
KY	1.00E+06	0.00E+00				**Application note: IF using a purchased linear table with mfg specs for stu**						
KZ	1.00E+06	0.00E+00				**CRITICAL: Enter Total number of bearing or attachment points (up to 12) AND**						
Rotation stiffness				**Number of attach points =**			1					
KrotX	1.00E+08	0.00E+00					**Attachment points between current CS origin and next CS: usually**					
KrotY	1.00E+08	0.00E+00				**location in**	1	2	3	4	5	6
KrotZ	1.00E+08	0.00E+00				X	0	0	0	0	0	0
Application note: IF using a purchased axis with mfg specs for stiffness and accuracy, just enter "1" for number of attach points and then the stiffness and error motion data from the catalog for that "point" (the entire table)						Y	0	0	0	0	0	0
						Z	0	0	0	0	0	0
						Linear stiffness at the point						
Additional angular error motions (in event of using a modular axis)						KX	1.00E+06	0	0	0	0	0
Random						KY	1.00E+06	0	0	0	0	0
EpsilonX (radians)	0					KZ	1.00E+06	0	0	0	0	0
EpsilonY (radians)	0					**Rotation stiffness at the point**						



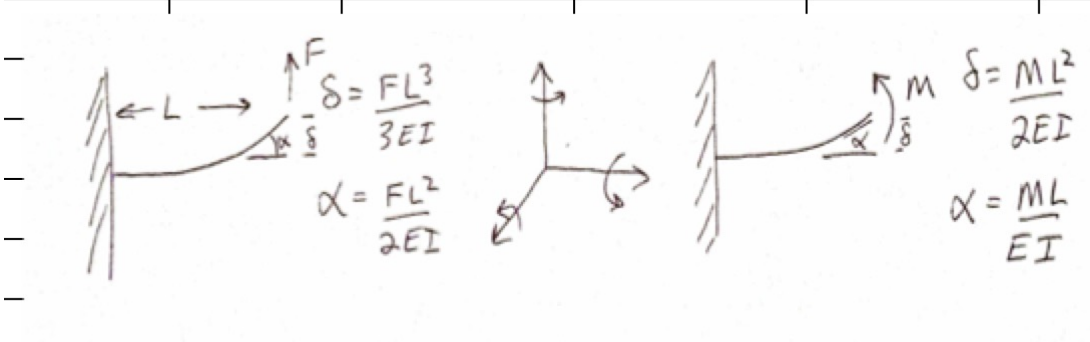


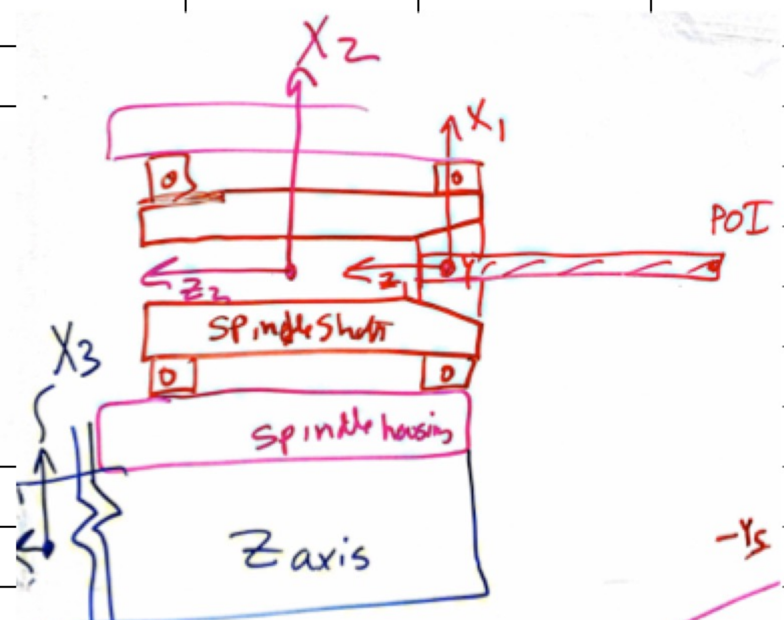
	A	B	C	D
1	CS 1	Tool Holder		
2	Geometric errors at CS origin and location and stiffness of points (or bearings) of attachment between coordinate systems			
3	Error motions of CS Origin along & about its axes. Values read used by "Coordinate Systems" worksheet			
4	Element	Tool	Bearings (if values needed for this CS)	
5	Structure type	cantilever	Size (e.g., rail width)	25
6	Material	steel tool	Fmax (N)	17346
7	Young's Modulus (N/mm^2)	2.00E+05	k @ medium preload (N/mm)	647000
8	Poison ratio	0.29	ball diameter (mm)	3
9	Shear modulus (N/mm^2)	77519	% deflection max load	0.01
10	Beam properties (rectangular tube, or round, else customize)		stiffness est (N/mm)	578200
11	Position along beam	100	rail width (mm)	25
12	Length (mm)	100	roll rotary stiffness (Nmm/radian)	101093750
13	Height (mm), or OD	25	pitch rotary stiffness (Nmm/radian)	33697917
14	Width (mm), ID or fraction of solid cylinder	50%	yaw rotary stiffness (Nmm/radian)	33697917
15	wall (mm)		servo system (if values needed for this CS)	
16	Area (mm^2)	245	Ballscrew	
17	I width dimension of beam (mm^4)		Root diameter (mm)	18
18	I height dimension of beam (mm^4)	9587	Length (mm)	2500
19	Ipolar (mm^4)	19175	Modulus	2.00E+05
20	I/A stiffness to weight ratio	39	shaft stiffness	2.04E+04
21	ref, simply supported stiffness about Izz (N/mm)	5752	factor to derate stiffness of shaft for nu	3
22	Notes: Y axis is + into the page so a + X direction force causes a - rotation about Y axis. Cells not used are colored blue			

CAREFUL as this compliance matrix is referenced in the worksheet "CSs and Loads"

Tool in toolholder (cantilever beam): X lateral, Y transverse, Z axial

dx		1.74E-04	0	0	0	-2.61E-06	0	**Fx**
dy		0	1.74E-04	0	2.61E-06	0	0	**Fy**
dz		0	0	2.04E-06	0	0	0	**Fz**
thetaX	_=_	0	2.61E-06	0	5.22E-08	0	0	**Mx**
thetaY		-2.61E-06	0	0	0	5.22E-08	0	**My**
thetaZ		0	0	0	0	0	6.73E-08	**Mz**





Example

- Straight tubular beam
- Create a new workbook to enter dimensions and calculate stiffnesses

Axis 1, Cantilevered Round tube	
OD (mm)	100
Wall (mm)	15
Iyy (mm^4)	3730150
Ipolar (mm^4)	7460301
Length (mm)	1000
Modulus (N/mm^2)	7.00E+04
Rotational spring stiffness model	
Krot_x	5.22E+08
Krot_y	5.22E+08
Krot_z	5.22E+08

- Assume size 20 linear motion guide (profile rail bearing)
- The axis' beam stiffness is accounted for with a rotational model, divided equally at the four bearing blocks
  - The bearing blocks individual rotational stiffness's are ignored
  - They are very small wrt the rotational stiffness from spacing the blocks apart