

Let μ denote the true mean decrease in time through the maze. Then the researchers are testing $H_0: \mu = 0$ versus $H_A: \mu > 0$. If the null hypothesis holds, then the sampling distribution of \bar{X} is normal with mean 0 and standard error $3/\sqrt{20} = 0.6708$. Using a one-sided test at $\alpha = 0.05$, the researchers will reject the null hypothesis if the z-score of the test statistic satisfies $Z \geq 1.645$. This corresponds to $Z = (\bar{X} - 0)/0.6708 \geq 1.645$ or $\bar{X} \geq 1.1035$. So, if the true mean decrease in time is 2 s, what is the probability of correctly rejecting the null hypothesis of $\mu = 0$?

$$\begin{aligned} 1 - \beta &= P(\text{Reject } H_0 \mid H_A \text{ true}) \\ &= P(\bar{X} \geq 1.1035 \mid \mu = 2) \\ &= P\left(\frac{\bar{X} - 2}{0.6708} \geq \frac{1.1035 - 2}{0.6708}\right) \\ &= P(Z \geq -1.3365) \\ &= 0.9093. \end{aligned}$$

Thus, the researchers have a 91% chance of correctly concluding the drug is effective if the true average decrease in time is 2 s. □