

Thus,

$$\begin{aligned} 1 - \beta &= P(\text{Reject } H_0 \mid H_A \text{ true}) \\ &= P\left(\bar{X} > \mu_0 + q \frac{\sigma}{\sqrt{n}} \mid \mu = \mu_1\right) \\ &= P\left(\frac{\bar{X} - \mu_1}{\sigma/\sqrt{n}} > \frac{\mu_0 - \mu_1 + q(\sigma/\sqrt{n})}{\sigma/\sqrt{n}}\right) \\ &= P\left(Z > \frac{\mu_0 - \mu_1}{\sigma/\sqrt{n}} + q\right) \\ &= P\left(Z > q - \frac{(\mu_1 - \mu_0)}{\sigma/\sqrt{n}}\right). \end{aligned} \tag{8.2}$$

Thus, we see that the power of a test,  $1 - \beta$ , is determined by the size of the lower bound of  $Z$  in Equation 8.2,

$$Z > q - \frac{(\mu_1 - \mu_0)}{\sigma/\sqrt{n}}.$$

The smaller that lower bound, the larger the power. The population standard deviation  $\sigma$  is not controllable by the analyst. The other factors that determine the power are:

- *Effect size*: The difference between the hypothesized mean  $\mu_0$  and the actual mean  $\mu_1$ . The larger the difference, the more likely we would detect the difference.
- *Denominator*  $\sigma/\sqrt{n}$ : The larger the sample size, the smaller the denominator and hence the larger the amount being subtracted. This too increases power.
- *Significance Level*  $\alpha$ : Increasing  $\alpha$  decreases the quantile  $q$  and pushes that lower bound to the left, increasing power.