Introduction to Deep Neural Networks

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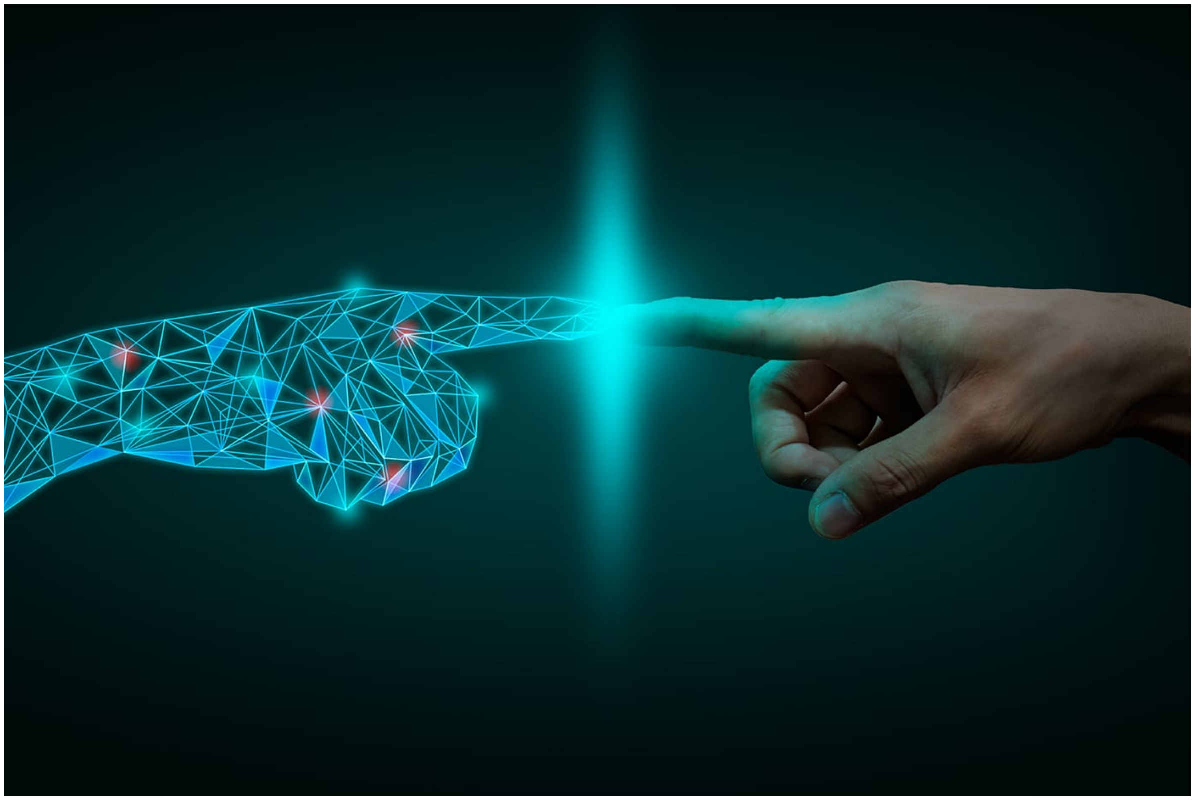
Overview of Deep Learning

Historical Background (1)

- Today, in April 2023, our world is convulsed by the explosion of Artificial Intelligence.
- It has probably been in the last months (weeks), since ChatGPT has arrived, that everybody has an opinion, or a fear on the topic.



Historical Background (2)

- We don't discuss ethical, technological or sociological aspects, but is "*most of this is about prediction*".
- Most AI engines rely on powerful prediction systems that use statistical learning methods.

Deep learning

- Deep learning is a successful AI model which has powered many application such as *self-driving cars, voice assistants, and medical diagnosis systems.*
- Essentially, deep learning extends the basic principles of artificial neural networks by
  - Adding complex architectures and algorithms and
  - At the same time becoming more automatical
- We won't delve into the history of ANN, but a quick look at it may help fully grasp its current capabilities.

The early history of AI (1)

- The origins of AI, and as such of DL can be traced almost one century backwards;
- A Quick History of AI, ML and DL

Milestones in the history of DL

We can see several hints worth to account for:

- The **Perceptron** and the first **Artificial Neural Network** where the basic building block was introduced.
- The **Multilayered perceptron** and **backpropagation** where complex architechtures were suggested to improve the capabilities.
- **Deep Neural Networks**, with many hidden layers, and auto-tunability capabilities.

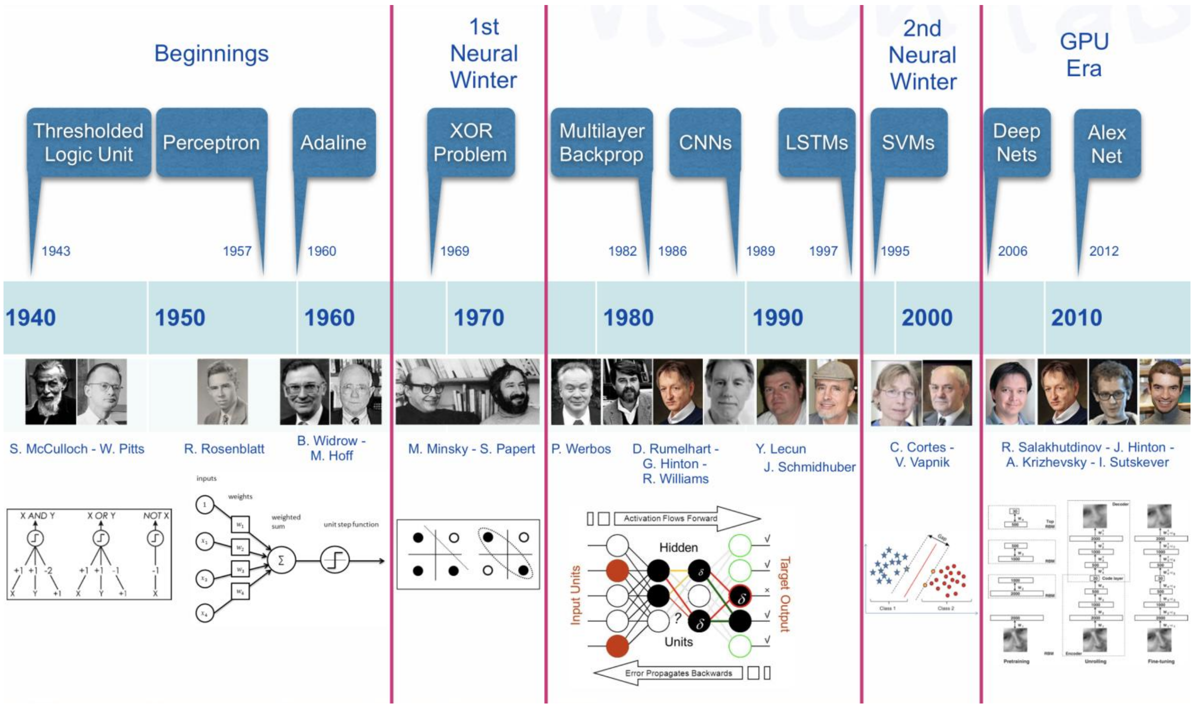
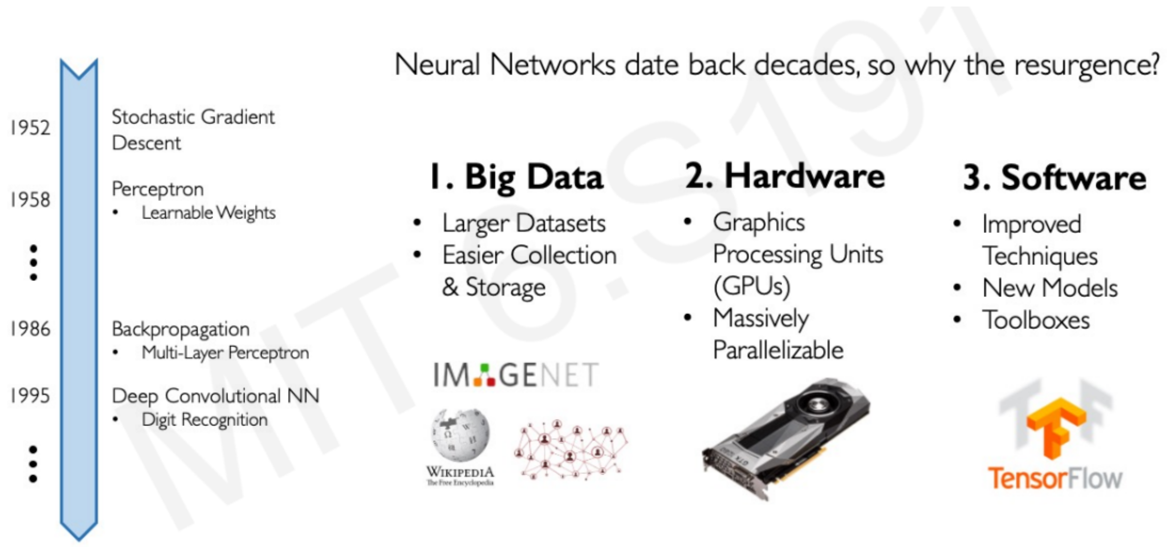


Figure 1: A Brief History of AI from 1940s till Today

From Artificial Neural Networks to Deep learning



1952 Stochastic Gradient Descent

1958 Perceptron
- Learnable Weights

1986 Backpropagation
- Multi-Layer Perceptron

1995 Deep Convolutional NN
- Digit Recognition

Neural Networks date back decades, so why the resurgence?

1. Big Data
- Larger Datasets
- Easier Collection & Storage

2. Hardware
- Graphics Processing Units (GPUs)
- Massively Parallelizable

3. Software
- Improved Techniques
- New Models
- Toolboxes

Source: Alex Amini's 'MIT Introduction to Deep Learning' course (introtodeeplearning.com)

Success stories

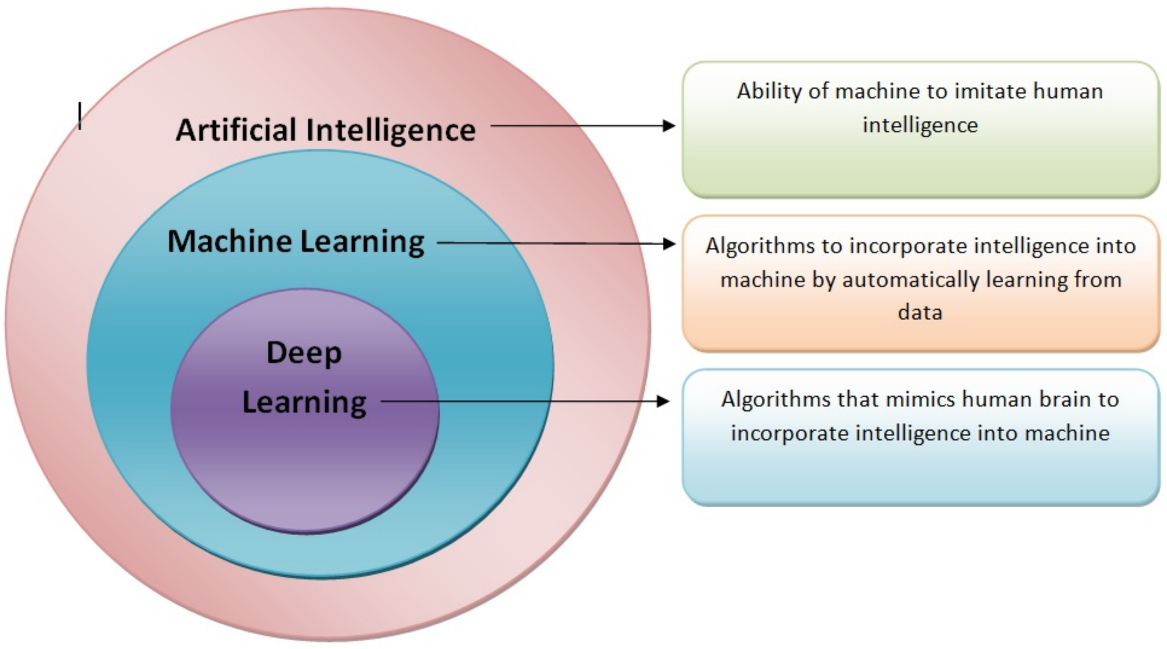
Success stories such as

- the development of self-driving cars,
- the use of AI in medical diagnosis, and
- the creation of personalized recommendations in online shopping

have also contributed to the widespread adoption of AI.

Comparison with Traditional Machine Learning

- A reasonable question is "*how are ArtificiaI Intelligence, Machine Learning and Deep learning related*"?
- A standard answer can be found in the image below that has a myriad variations:

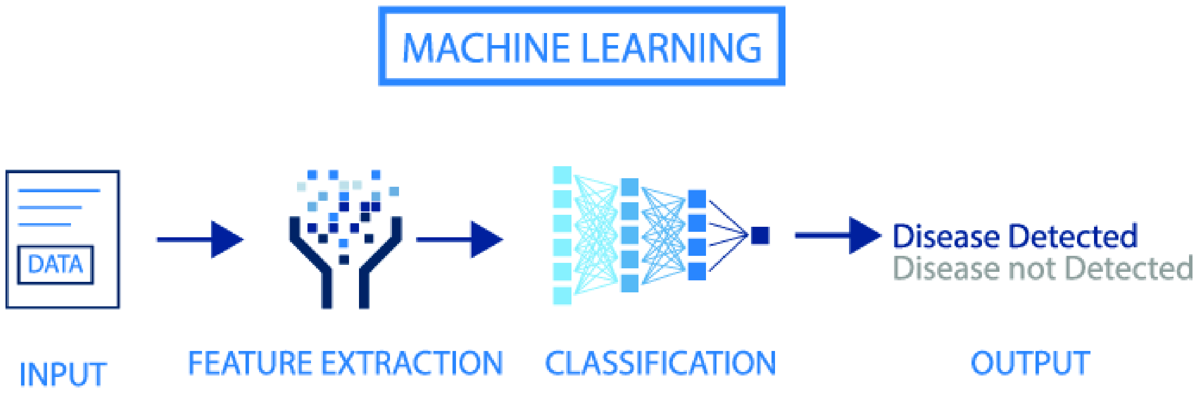

Artificial Intelligence
Machine Learning
Deep Learning
Ability of machine to imitate human intelligence
Algorithms to incorporate intelligence into machine by automatically learning from data
Algorithms that mimics human brain to incorporate intelligence into machine

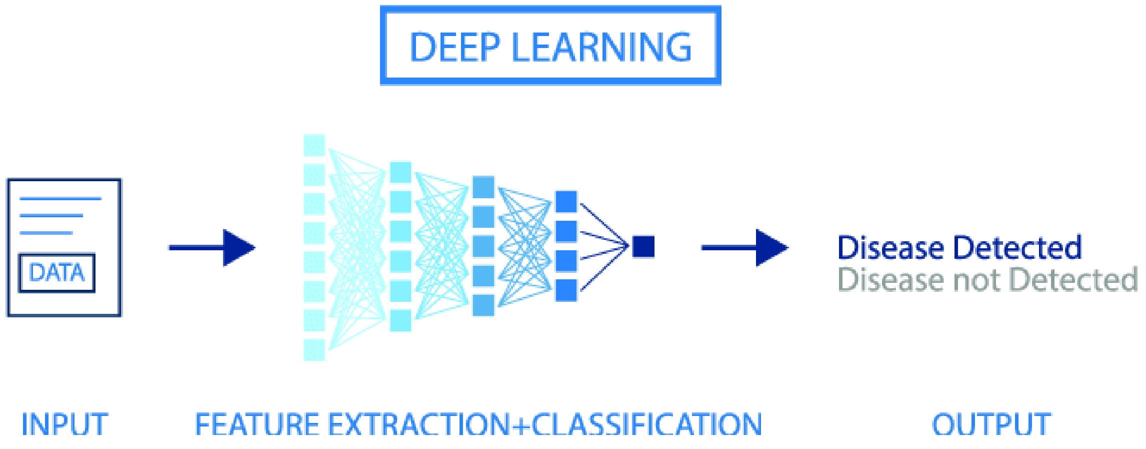
AI, ML, DL

We can keep three definitions:

- Artificial intelligence is the ability of a computer to perform tasks commonly associated with intelligent beings.
- Machine learning is the study of algorithms that learn from examples and experience instead of relying on hard-coded rules and make predictions on new data
- Deep learning is a subfield of machine learning focusing on learning data representations as successive successive layers of increasingly meaningful representations.

ML vs DL





- DNN combine feature extraction and classification in a way that does not require (or dramatically decreases) human intervention.
- The power of DNN requires in its ability to improve with data availability, without seemingly reaching plateaus as ML.

Size is important!

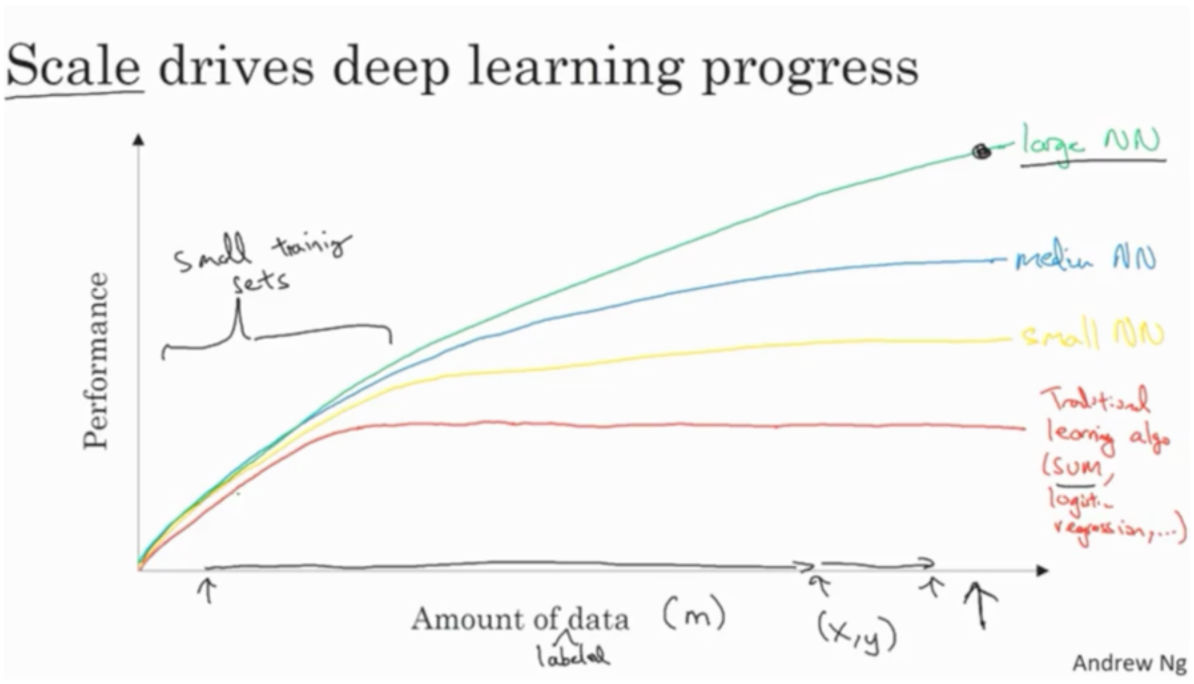


Figure 2: An illustration of the performance comparison between deep learning (DL) and other machine learning (ML) algorithms, where DL modeling from large amounts of data can increase the performance

The impact of Deep learning

- Near-human-level image classification
- Near-human-level speech transcription
- Near-human-level handwriting transcription
- Dramatically improved machine translation
- Dramatically improved text-to-speech conversion
- Digital assistants such as Google Assistant and Amazon Alexa

- Near-human-level autonomous driving
- Improved ad targeting, as used by Google, Baidu, or Bing
- Improved search results on the web
- Ability to answer natural language questions
- Superhuman Go playing

Not all that glitters is gold ...

- According to F. Chollet, the developer of Keras,
  - "*we shouldn't believe the short-term hype, but should believe in the long-term vision.*
  - *It may take a while for AI to be deployed to its true potential—a potential the full extent of which no one has yet dared to dream*
  - *but AI is coming, and it will transform our world in a fantastic way*".

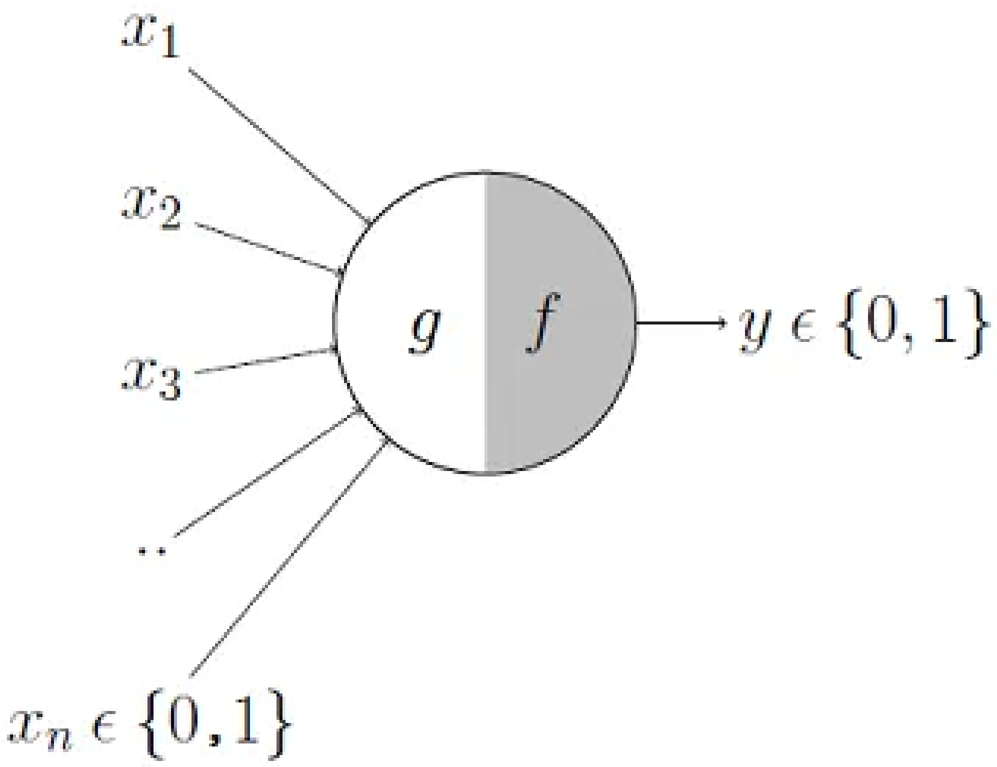
Artificial Neural Networks

The perceptron, the building block

- The perceptron, was introduced in the 50's (one version of the perceptron at least), as a mathematical model that might emulate a neuron.
- The idea is: *how can we produce a model that given some inputs, and an appropriate set of examples, learn to produce the desired output*?

Mc Cullough's neurone

- The first computational model of a neuron was proposed by Warren MuCulloch (neuroscientist) and Walter Pitts (logician) in 1943.


x_1
x_2
x_3
..
$x_n \epsilon \{0,1\}$
g
f
$y \epsilon \{0,1\}$

Mc Cullough's neurone

- It may be divided into 2 parts.
  - The first part, g,takes an input (ahem dendrite ahem),
  - It performs an aggregation and
  - based on the aggregated value the second part, f, makes a decision.

See the source of this picture for an illustration on how this can be used to emulate logical operations such as AND, OR or NOT, but not XOR.

Limitations

This first attempt to emulate neurons succeeded but with limitations:

- What about non-boolean (say, real) inputs?
- What if all inputs are not equal?
- What if we want to assign more importance to some inputs?
- What about functions which are not linearly separable? Say XOR function

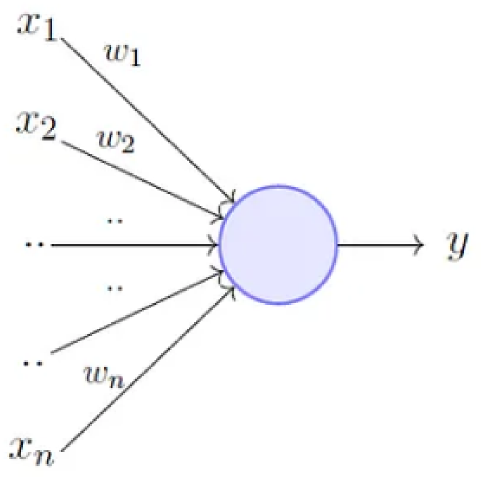
Overcoming the limitations

- To overcome these limitations Frank Rosenblatt, proposed the classical perception model, the *artificial neuron*, in 1958.
- It is more generalized computational model than the McCulloch-Pitts neuron where weights and thresholds can be learnt over time.
- Rosenblatt's perceptron is very similar to an M-P neuron but
  - It takes a weighted sum of the inputs and
  - It sets the output as one only when the sum is more than an arbitrary threshold (***theta***).

Rosenblatt's perceptron

Rosenblatt's perceptron (1)

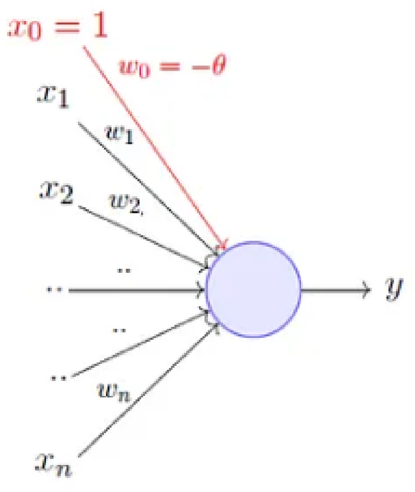
- Instead of hand coding the thresholding parameter θ,
- It is added as one of the inputs, with the weight $w_0 = -\theta$.



$$y = 1 \quad if \sum_{i=1}^{n} w_i * x_i \geq \theta$$
$$= 0 \quad if \sum_{i=1}^{n} w_i * x_i < \theta$$

Rewriting the above,

$$y = 1 \quad if \sum_{i=1}^{n} w_i * x_i - \theta \geq 0$$
$$= 0 \quad if \sum_{i=1}^{n} w_i * x_i - \theta < 0$$



A more accepted convention,

$$y = 1 \quad if \sum_{i=0}^{n} w_i * x_i \geq 0$$
$$= 0 \quad if \sum_{i=0}^{n} w_i * x_i < 0$$

$where, \quad x_0 = 1 \quad and \quad w_0 = -\theta$

Comparison between the two

McCulloch Pitts Neuron
(assuming no inhibitory inputs)

$$y = 1 \quad if \sum_{i=0}^{n} x_i \geq 0$$
$$= 0 \quad if \sum_{i=0}^{n} x_i < 0$$

Perceptron

$$y = 1 \quad if \sum_{i=0}^{n} w_i * x_i \geq 0$$
$$= 0 \quad if \sum_{i=0}^{n} w_i * x_i < 0$$

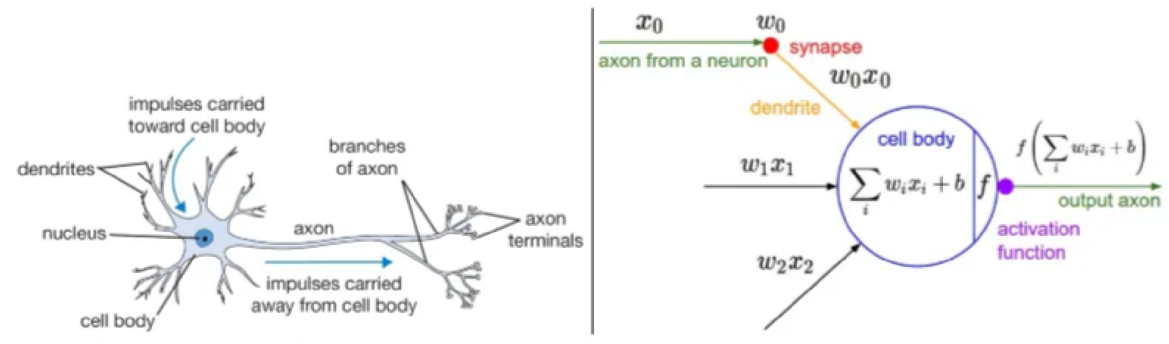
Comparison between the two

- This is an improvement because
  - both, weights and threshold, can be learned and
  - the inputs can be real values
- But there is still a drawback in that a single perceptron can only be used to implement linearly separable functions.
- Artificial Neural Networks improve on this by introducing *Activation Functions*

Neurons and Activation Functions

- An *activation function* is a function that is added into an artificial neurone in order to help it learn complex patterns in the data.
- When comparing with a neuron-based model that is in our brains (when neurons are connected), the activation function is at the end deciding *what is to be fired to the next neuron.*
- That is exactly what an activation function does in an ANN as well.
  - It takes in the output signal from the previous cell
  - and converts it into some form that
  - can be taken as input to the next cell.

Activation function



A cartoon drawing of a biological neuron (left) and its mathematical model (right).

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Artificial Neuron

With all these ideas in mind we can now define an Artificial Neuron as a *computational unit* that :

- takes as input $x = (x_0, x_1, x_2, x_3)$ ($x_0 = +1$, called bias),
- outputs $h_\theta(x) = f(\theta^\top x) = f(\sum_i \theta_i x_i)$,
- where $f : \mathbb{R} \mapsto \mathbb{R}$ is called the **activation function**.

Activation functions

- Goal of activation function is to provide the neuron with *the capability of producing the required outputs*.
  - For instance, if the output has to be a probability, the activation function will only produce values between 0 and 1.
- With this idea in mind activation functions are chosen from a set of pre-defined functions:
  - the sigmoid function:
  - the hyperbolic tangent, or `tanh`, function
  - The ReLU

The sigmoid function:

$$f(z) = \frac{1}{1 + e^{-z}}$$

- A useful property: *If* $f(z) = 1/(1 + e^z)$ is the sigmoid function, then its derivative is given by $f'(z) = f(z)(1 - f(z))$.

the hyperbolic tangent,

Also called `tanh`, function:

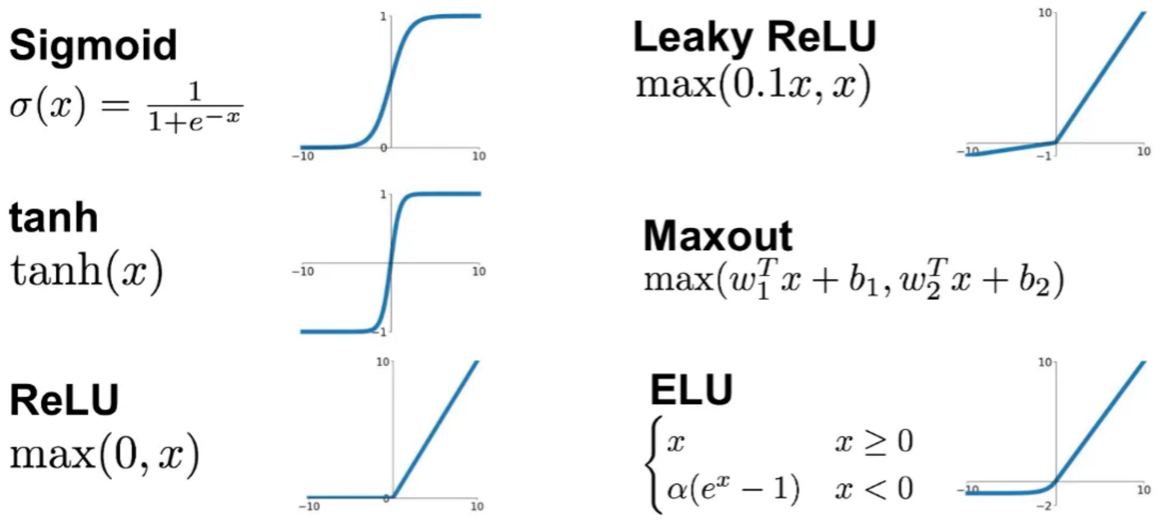
$$f(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

- The `tanh(z)` function is a rescaled version of the sigmoid, and its output range is $[-1, 1]$ instead of $[0, 1]$.
- – A useful property: *if* f is the `tanh` function, then its derivative is given by $f'(z) = 1 - (f(z))^2$.

The ReLU

- In modern neural networks, the default recommendation is to use the *rectified linear unit* or ReLU defined by the activation function $f(z) = \max\{0, z\}$.
- This function remains very close to a linear one, in the sense that is a piecewise linear function with two linear pieces.
- Because rectified linear units are nearly linear, they preserve many of the properties that make linear models easy to optimize with gradient based methods. They also preserve many of the properties that make linear models generalize well.

Activation function plots



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Putting it all together

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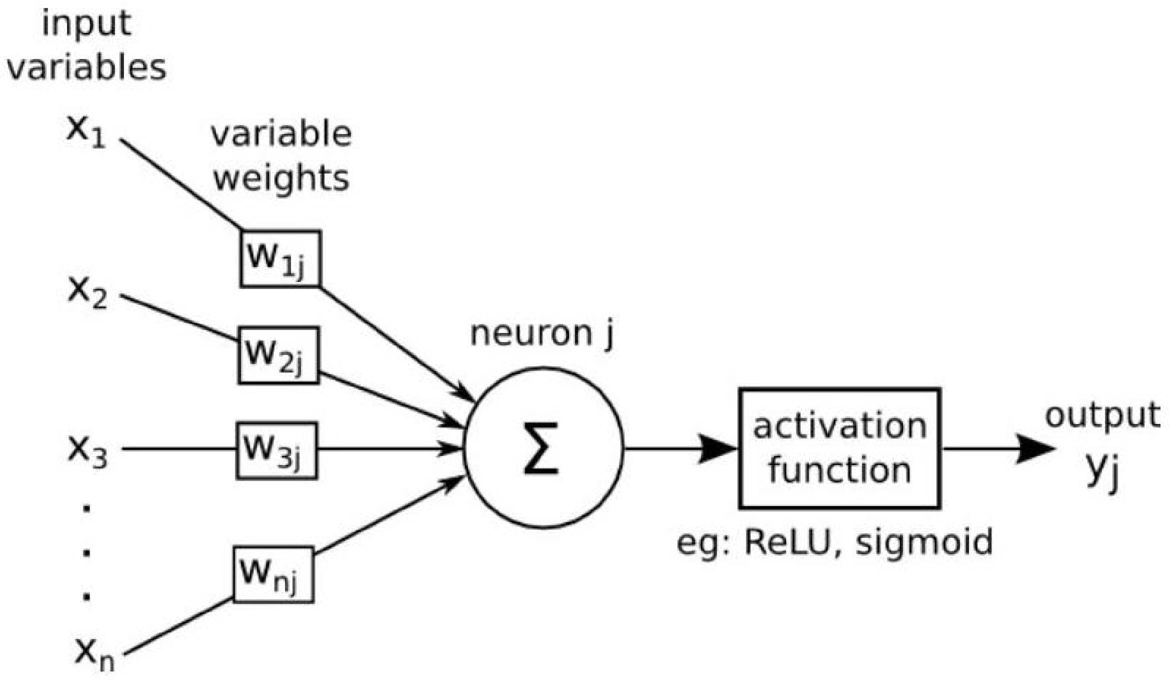

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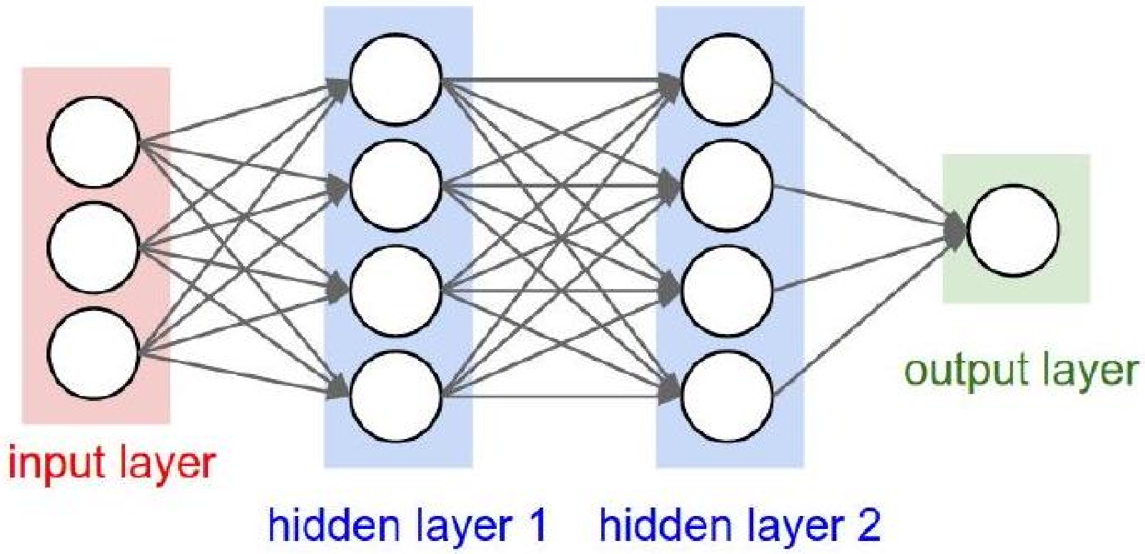
$\Sigma = \langle w_j, x \rangle + b_j$

Artificial Neural Networks

- A multilayer perceptron (or Artificial neural network) is a structure composed by *several hidden layers of neurons* where the output of a neuron of a layer becomes the input of a neuron of the next layer.
- Moreover, the output of a neuron can also be the input of a neuron of the same layer or of neuron of previous layers (this is the case for recurrent neural networks).
- One the last layer, called output layer, we may apply a different activation function as for the hidden layers depending on the type of problems we have at hand : regression or classification.

An ANN

[]

The architechture of ANN

- Multilayers perceptrons have a basic architecture since each unit (or neuron) of a layer is linked to all the units of the next layer but has no link with the neurons of the same layer.
- The parameters of the architecture are:
  - the number of hidden layers and
  - the number of neurons in each layer.
  - The activation functions

The architecture of ANN

- For the output layer, as mentioned previously, the activation function is generally different from the one used on the hidden layers. For example:.
  - For regression, we apply no activation function on the output layer.
  - For binary classification, the output gives a prediction of $\mathbb{P}(Y = 1/X)$ since this value is in $[0, 1]$ and the sigmoid activation function is generally considered.
  - For multi-class classification, the output layer contains one neuron per class (i), giving a prediction of $\mathbb{P}(Y = i/X)$. The sum of all these values has to be equal to 1 . The sum of all these values has to be equal to 1
- The multidimensional function *softmax* is generally used

$$\operatorname{softmax}(z)_i = \frac{\exp(z_i)}{\sum_j \exp(z_j)}$$

Mathematical formulation

We can summarize the mathematical formulation of a multilayer perceptron with (L) hidden layers as follows:

- Set $h^{(0)}(x) = x$ For $k = 1, \ldots, L$ (hidden layers),

$$\begin{aligned} a^{(k)}(x) &= b^{(k)} + W^{(k)} h^{(k-1)}(x) \\ h^{(k)}(x) &= \phi\left(a^{(k)}(x)\right) \end{aligned}$$

For $k = L + 1$ (output layer)

$$\begin{aligned} a^{(L+1)}(x) &= b^{(L+1)} + W^{(L+1)} h^{(L)}(x) \\ h^{(L+1)}(x) &= \psi\left(a^{(L+1)}(x)\right) := f(x, \theta). \end{aligned}$$

where ϕ is the activation function and ψ is the output layer activation function (for example softmax for multiclass classification).

Mathematical formulation

- At each step, $W^{(k)}$ is a matrix with
  - number of rows equal to the number of neurons in the layer k and
  - number of columns the number of neurons in the layer $k-1$.
- By
  - organizing our parameters in matrices and
  - using matrix-vector operations, we can take advantage of fast linear algebra routines to quickly perform calculations in our network.

An example

A predictive ANN

We use the `neuralnet` package to build a simple neural network to predict if a type of stock pays dividens or not.

```
if (!require(neuralnet))
  install.packages("neuralnet", dep=TRUE)
```

```
Loading required package: neuralnet
```

```
Warning: package 'neuralnet' was built under R version 4.2.3
```

And use the `dividendinfo.csv` dataset from https://github.com/MGCodesandStats/datasets

```
mydata <- read.csv("https://raw.githubusercontent.com/MGCodesandStats/datasets/master/divi
str(mydata)
```

```
'data.frame':	200 obs. of  6 variables:
 $ dividend       : int  0 1 1 0 1 1 1 0 1 1 ...
 $ fcfps          : num  2.75 4.96 2.78 0.43 2.94 3.9 1.09 2.32 2.5 4.46 ...
 $ earnings_growth: num  -19.25 0.83 1.09 12.97 2.44 ...
 $ de             : num  1.11 1.09 0.19 1.7 1.83 0.46 2.32 3.34 3.15 3.33 ...
 $ mcap           : int  545 630 562 388 684 621 656 351 658 330 ...
 $ current_ratio  : num  0.924 1.469 1.976 1.942 2.487 ...
```

Data preprocessing

```
scaleddata<-scale(mydata)
normalize <- function(x) {
  return ((x - min(x)) / (max(x) - min(x)))
}
maxmindf <- as.data.frame(lapply(mydata, normalize))
```

Finally we break our data in a test and a training set:

```
trainset <- maxmindf[1:160, ]
testset <- maxmindf[161:200, ]
```

Training a neural network

Setting the parameters of a neural network reqyuires experience and understanding of their meaning, and even so, cahnges in the parameters can lead to similar results.

```
#Neural Network
library(neuralnet)
nn <- neuralnet(dividend ~ fcfps + earnings_growth + de + mcap + current_ratio,
                data=trainset,
                hidden=c(2,1),
                linear.output=FALSE,
                threshold=0.01)
```

The output of the procedure is a neural network with estimated weights

```
plot(nn)
```

Plotting the network

The object `nn`contains information on the process:

```
nn$result.matrix
```

```
                                    [,1]
error                       1.006659e+00
reached.threshold           9.642892e-03
```

```
steps                           6.630000e+03
Intercept.to.1layhid1          -2.126190e+00
fcfps.to.1layhid1              -5.740918e-01
earnings_growth.to.1layhid1 -8.428624e+00
de.to.1layhid1                 -3.671520e+00
mcap.to.1layhid1                8.859204e+00
current_ratio.to.1layhid1       6.616783e+00
Intercept.to.1layhid2           3.663689e+00
fcfps.to.1layhid2              -2.308812e+00
earnings_growth.to.1layhid2 -3.948246e+00
de.to.1layhid2                  7.804959e-01
mcap.to.1layhid2               -2.182738e+00
current_ratio.to.1layhid2      -2.046124e+00
Intercept.to.2layhid1          -1.829718e+00
1layhid1.to.2layhid1           -8.121411e+00
1layhid2.to.2layhid1            1.546760e+01
Intercept.to.dividend           4.371156e+01
2layhid1.to.dividend           -9.268492e+01
```

Testing the accuracy of the model

```
#Test the resulting output
temp_test <- subset(testset, select =
                      c("fcfps","earnings_growth",
                        "de", "mcap", "current_ratio"))
head(temp_test)
```

```
        fcfps earnings_growth        de       mcap current_ratio
161 0.2008114      0.09654591 0.3562341 0.26580460     0.5148975
162 0.2636917      0.51642797 0.3486005 0.91235632     0.5573954
163 0.1318458      0.32737995 0.7048346 0.26867816     0.5037547
164 0.4908722      0.44953665 0.4605598 0.05172414     0.2187935
165 0.1967546      0.24448189 0.6768448 0.04885057     0.5360724
166 0.6125761      0.47750632 0.8829517 0.97844828     0.3964783
```

```
nn.results <- compute(nn, temp_test)
results <- data.frame(actual =
                  testset$dividend,
                  prediction = nn.results$net.result)
head(results)
```

```
    actual   prediction
161      0 2.069528e-21
162      1 1.000000e+00
163      0 5.502585e-22
164      0 5.559009e-22
165      0 5.398124e-22
166      1 1.000000e+00
```

Evaluating the model

A confusion matrix can bu built to evaluate the predictive ability of the network:

```r
roundedresults<-sapply(results,round,digits=0)
roundedresultsdf=data.frame(roundedresults)
attach(roundedresultsdf)
table(actual,prediction)
```

```
      prediction
actual  0  1
     0 17  0
     1  3 20
```