

Experimental Design

Lecture 3

Dr. Jacob Ong'ala

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Designs of Experiment

- There are three designs of experiments that we will discuss in this lecture.
 - » Completely randomized design
 - » Randomized complete block design
 - » Latin Square
- Within each design, there are numerous arrangements that can be used
 - » Hierarchical arrangements
 - » Factorial arrangements

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Completely randomized design

- In a completely randomized design (CRD), v treatments are assigned completely at random so that each experimental unit has the same chance of receiving any one treatment.
- Any difference among experimental units receiving the same treatment is considered as experimental error
- Hence, CRD is appropriate only for experiments with homogeneous experimental units, such as laboratory experiments

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Completely randomized design

- All experimental units are considered the same and no division or grouping among them exist.
- Design is entirely flexible in the sense that any number of treatments or replications may be used.
- Number of replications for different treatments need not be equal and may vary from treatment to treatment depending on the knowledge (if any) on the variability

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Completely randomized design

A	B	A	C	B
C	B	B	A	C
B	C	A	C	A

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Completely randomized design

Layout

- Determine the total number of experimental plots (n) as the product of the number of treatments (t) and the number of replications (r); that is, $n = rt$.
- Assign a plot number to each experimental plot say from 1 to n .
- Assign the treatments to the experimental plots randomly
- No local control measure is provided as such except that the error variance can be reduced by choosing a homogeneous set of experimental units

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Completely randomized design

Procedure

- Let v be number of treatments and n_i be number replication for treatment i , then $n = \sum_{i=1}^v n_i$
- Select n_1 units out of n units randomly and apply treatment 1 to these n_1 units.
- Select n_2 units out of $(n - n_1)$ units randomly and apply treatment 2.
- And so on

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Analysis of CRD

a) Explanation

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Randomized Block Design

- If large number of treatments are to be compared, then large number of experimental units are required
- when the experimental material is not homogeneous and there are v treatments to be compared, then it may be possible to:
 - group the experimental material into blocks of sizes v units.
 - Blocks are constructed such that the experimental units within a block are relatively homogeneous and resemble to each other more closely than the units in the different blocks.
 - If there are b such blocks, we say that the blocks are at b levels. Similarly if there are v treatments, we say that the treatments are at v levels. The responses from the b levels of blocks and v levels of treatments can be arranged in a two-way layout.

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Randomized Block Design

	Blocks						Block Totals	
	1	2	...	j	...	b		
Treatments	1	y_{11}	y_{12}	...	y_{1j}	...	y_{1b}	$B_1 = y_{11}$
	2	y_{21}	y_{22}	...	y_{2j}	...	y_{2b}	$B_2 = y_{21}$

	j	y_{j1}	y_{j2}	...	y_{jj}	...	y_{jb}	$B_j = y_{j1}$

v	y_{1v}	y_{2v}	...	y_{bv}	...	y_{bv}	$B_b = y_{1b}$	
Treatment Totals	$T_1 = y_{10}$	$T_2 = y_{20}$...	$T_j = y_{j0}$...	y_{v0}	Grand Total $G = y_{00}$	

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Randomized Block Design

Layout:

- A two-way layout is called a randomized block design (RBD) or
- a randomized complete block design (RCBD) if within each block, the v treatments are randomly assigned to v experimental units such that each of the $v!$ ways of assigning the treatments to the units has the same probability of being adopted in the experiment and the assignment in different blocks are statistically independent.

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Randomized Block Design

- The RBD utilizes the principles of design - randomization, replication and local control - in the following way: 1. Randomization:
 - Number the v treatments $1, 2, \dots, v$.
 - Number the units in each block as $1, 2, \dots, v$.
 - Randomly allocate the v treatments to v experimental units in each block.
 - Replication** Since each treatment is appearing in the each block, so every treatment will appear in all the blocks. So each treatment can be considered as if replicated the number of times as the number of blocks.
 - Local control** Local control is adopted in RBD in following way:
 - First form the homogeneous blocks of the experimental units.
 - Then allocate each treatment randomly in each block.
- NB:** The error variance now will be smaller because of homogeneous blocks and some variance will be parted away from the error variance due to the difference among the blocks..

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Randomized Block Design

• Analysis

- Explanation

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Latin Square Design

- The treatments in the RBD are randomly assigned to b blocks such that each treatment must occur in each block rather than assigning them at random over the entire set of experimental units as in the CRD.
- There are only two factors - block and treatment effects - which are taken into account and the total number of experimental units needed for complete replication are bv where b and v are the numbers of blocks and treatments respectively.
- If there are three factors and suppose there are b , v and k levels of each factor, then the total number of experimental units needed for a complete replication are bvk .
- This increases the cost of experimentation and the required number of experimental units over RBD.

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Latin Square Design

- In LSD, the experimental material is divided into rows and columns, each having the same number of experimental units which is equal to the number of treatments.
- The treatments are allocated to the rows and the columns such that each treatment occurs once and only once in the each row and in the each column.
- A Latin square of order p is an arrangement of p symbols in p^2 cells arranged in p rows and p columns such that each symbol occurs once and only once in each row and in each column.
- For example, to write a Latin square of order 4, choose four symbols - A, B, C and D. These letters are Latin letters which are used as symbols. Write them in a way such that each of the letters out of A, B, C and D occurs once and only once in each row and each column.

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Latin Square Design

A	B	C	D
B	C	D	A
C	D	A	B
D	A	B	C

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Latin Square Design

- **Example:** Suppose different brands of petrol are to be compared with respect to the mileage per liter achieved in motor cars. Important factors responsible for the variation in the mileage are
 - difference between individual cars.
 - difference in the driving habits of drivers.
 - difference in petrol brand
- We have three factors - cars, drivers and petrol brands. Suppose we have
 - 4 types of cars denoted as 1, 2, 3, 4.
 - 4 drivers that are represented by as a, b, c, d.
 - 4 brands of petrol are indicated by as A, B, C, D.
- A complete replication will require $4 \times 4 \times 4 = 64$ no. of experiments but choose only 16 experiments. To choose such 16 experiments, we take the help of Latin square.

Dr. Ong'ala, J.O - ongalajacob@gmail.com

Latin Square Design

		CARS			
		1	2	3	4
DRIVERS	a	A	B	C	D
	b	B	C	D	A
	c	C	D	A	B
	d	D	A	B	C

Driver "a" will use petrol A in car 1.

Driver "b" will use petrol C in car 2.

Driver "d" will use petrol C in car 4.

Dr. Ong'ala, J.O - ongalajacob@gmail.com