TUTORIAL 1 24/10/2012

THE LIFE TABLE

#1 Let $l_{x+t} = a + bt + ct^2$ for $-1 \leq t \leq 1$

Show that

$$q_x = \frac{m_x}{1 - q_{x-1}} \cdot \frac{12 - 13 q_{x-1}}{12 + 5 m_x}$$

#2. The following life-table for Londoners was constructed in 1662 by John Graunt

Age (Yrs)	Survivors	Age (Yrs)	Survivors
0	100	46	10
6	64	56	6
16	40	66	3
26	25	76	1
36	16	86	0

Estimate

a) (i) person-years lived between age 0 and 6;
(ii) the probability of dying between ages 6 and 11

b) For a person on his 26th birthday
(i) the expectation of life,
(ii) the probability of survival to age 36.

#3. Using the English Life-Table, 1950-52 shown on the next page, find answers to the following:
a) What are the probabilities that male and female live births will not survive to their 20th birthday?
b) What is the probability that a woman aged twenty will survive to the age of fifty?

c) How does the chance of a man of seventy becoming a centenarian compare with that of a man of sixty?

d) In what decade of life do most (i) men and (ii) women die?

e) At what age, of those shown in the table, is the difference between the probabilities of death in the next 10 years of men and women greatest (i) absolutely and (ii) relatively?

English Life-Table 1950-52

Age	Males		Females	
x	l_x	e_x	l_x	e_x
0	100000	64.4	100000	71.5
5	96186	64.0	97019	68.7
10	95866	59.2	96794	63.9
15	95601	54.4	96608	59.0
20	95151	49.6	96300	54.2
30	93820	40.3	95311	44.7
40	91968	31.0	93778	35.3
50	87591	22.2	90656	26.3
60	75873	14.8	83646	18.1
70	52350	9.0	67835	11.0
80	21130	4.9	36118	5.8
90	2184	2.6	6079	3.1
100	23	1.7	161	2.1

#4 Below are shown age-specific death rates, ${}_nm_x$ of males in England-Wales 1950-52, and the corresponding probabilities of dying in the age-group, ${}_nq_x$. From the specific death rates for Aberdeen City, also given, calculate the survivors at different ages from a thousand births (the l_x column of an abridged life-table) on the assumption that for each age-group ratios of ${}_nq_x$ to ${}_nm_x$ in Aberdeen City and England-Wales are the same. The infant mortality rate in Aberdeen in 1950-52 was 30.2 per 100 births.

Death Rates of Males, 1950-52

Age-Group in Years	England and Wales ${}_nm_x$	${}_nq_x$	Aberdeen City ${}_nm_x$
1-4	0.00136	0.00567	0.00131
5-14	0.00061	0.00608	0.00059
15-24	0.00116	0.01142	0.00106
25-34	0.00159	0.01570	0.00143
35-44	0.00288	0.02891	0.00365
45-54	0.00824	0.08116	0.01019
55-64	0.02310	0.20981	0.02690
65-74	0.05519	0.43791	0.06211
75-84	0.12775	0.76472	0.14078

#5 Of 21500 persons who reach age 72,
11 die within a month;
16 die within the second month;
22 die within the third month;
30 die within the fourth month;
41 die within the fifth month;
and so on.
Determine

a) the number of years lived by the persons who die between age 72 and 73;
b) the death rate at age 72
c) the central death rate at age 72
d) the force of mortality at age $72\frac{1}{2}$.

#6. Using the table below calculate
a) the probability that a woman aged exactly 20 years will survive until her 40th birthday;
b) the infant mortality rate, q_0;
c) the life expectancy at birth and at exact age 1 ~~year~~ year
d) the probability that a girl who survives until her first birthday will die between her 10th and 20th birthdays;
e) the expected age at death of those who die between their 20th and 30th birthday.
Use two alternative ways. Comment.
f) the expected age at death of those who die when they are aged under 1 year.
Use two alternative ways. Comment

Exact age x	Survivors to age x, l_x	Woman-Years Lived above age x, T_x
0	100000	7700187
1	99016	7601014
10	98746	6711410
20	98497	5725004
30	98105	4741877
40	97346	3764073.