Number 1

$$a = l_x,\quad b = \frac{1}{2}l_{x+1} - \frac{1}{2}l_{x-1},\quad c = \frac{1}{2}l_{x+1} + \frac{1}{2}l_{x-1} - l_x$$

$$m_x = \frac{d_x}{L_x} = \frac{d_x}{l_x}\cdot\frac{l_x}{L_x} = q_x \cdot \frac{l_x}{L_x}$$

$$L_x = \int_0^1 l_{x+t}\,dt = \int_0^1 (a+bt+ct^2)\,dt = a + \frac{b}{2} + \frac{c}{3}$$

$$q_x = m_x \cdot \frac{L_x}{l_x} = \left(a + \frac{b}{2} + \frac{c}{3}\right)\cdot\frac{m_x}{l_x}$$

$$= \left(l_x + \frac{1}{4}l_{x+1} - \frac{1}{4}l_{x-1} + \frac{1}{6}l_{x+1} + \frac{1}{6}l_{x-1} - \frac{1}{3}l_x\right)\cdot\frac{m_x}{l_x}$$

$$= \left(\frac{2}{3}l_x + \frac{5}{12}l_{x+1} - \frac{l_{x-1}}{12}\right)\cdot\frac{m_x}{l_x}$$

$$= \frac{1}{12}\left(8l_x + 5l_{x+1} - l_{x-1}\right)\cdot\frac{m_x}{l_x}$$

$$= \frac{1}{12}\left(8 + 5(1-q_x) - \frac{1}{1-q_{x-1}}\right)\cdot m_x$$

$$= \frac{1}{12}\left(13 - 5q_x - \frac{1}{1-q_{x-1}}\right)\cdot m_x$$

$$= \frac{1}{12}\left(\frac{13(1-q_{x-1})-1}{1-q_{x-1}} - 5q_x\right)\cdot m_x$$

$$q_x = \frac{1}{12}\left(\frac{12-13q_{x-1}}{1-q_{x-1}} - 5q_x\right)\cdot m_x$$

$$q_x = \frac{m_x}{12}\left(\frac{12-13q_{x-1}}{1-q_{x-1}}\right) - \frac{5}{12}q_x \cdot m_x$$

$$q_x + m_x q_x \frac{5}{12} = \frac{m_x}{12}\left(\frac{12-13q_{x-1}}{1-q_{x-1}}\right)$$

$$q_x\left(1+\frac{5}{12}m_x\right) = \frac{m_x}{12}\left(\frac{12-13q_{x-1}}{1-q_{x-1}}\right).$$

$$q_x\left(\frac{12+5m_x}{12}\right) = \frac{m_x}{12(1-q_{x-1})}\left(12-13q_{x-1}\right)$$

$$q_x = \frac{m_x}{1-q_{x-1}}\cdot\left(\frac{12-13q_{x-1}}{12+5m_x}\right).$$

Scanned by CamScanner

Number 2

Estimate

a) i) Person-years lived between age 0 and 6

$$L_x = \frac{n}{2}(l_x + l_{x+n})$$

$$= \frac{6}{2}(100 + 64)$$

$$= 492$$

ii) Probability of dying between ages 6 and 11

$$q_x = \frac{d_x}{l_x} = \frac{l_x - l_{x+n}}{l_x} = \frac{64 - 40}{64} = 0.375$$

b)

i The expectation of Life

$$e^\circ_x = \frac{T_x}{l_x}$$

Age	l_x	${}_nL_x$	T_x	$e^\circ_x = T_x / l_x$
0	100	492	1822	18.22
6	64	520	1330	20.78
16	40	325	810	20.25
26	25	205	485	19.4
36	16	130	280	17.5
46	10	50	150	15.0
56	6	45	70	11.67
66	3	20	25	8.3
76	1	5	5	5.0
86	0			

$$e^\circ_{26} = \frac{485}{25} = 19.4 \text{ years}$$

b)

ii) Probability of Survival to age 36

$$_{n}P_{x} = \frac{l_{x+1}}{l_x} = \frac{16}{25} = 0.64$$

Number 3

a) Male;

$$_{n}q_{x} = \frac{_{n}d_x}{l_x} = \frac{l_x - l_{x+n}}{l_x} = \frac{100000-95151}{100,000} = 0.04849.$$

female

$$_{n}q_{x} = \frac{100000-96300}{100,000} = 0.037$$

b) Probability of Survival

$$_{n}P_{x} = \frac{l_{x+n}}{l_x} = \frac{90656}{96300} = 0.9414$$

c) Centenarian

At seventy: $_{n}P_{x} = \frac{l_{x+n}}{l_x} = \frac{23}{52350} = 0.0004394$

At Sixty: $_{n}P_{x} = \frac{23}{75823} = 0.000303$

At 70 years has got higher chance of living to 100 years Compared to one at 60 years

This is due to the probability of 70 years is greater than 60 years

$$P_{70} > P_{60} \quad (0.0004394 > 0.000303)$$

d) In what decade

Age(x)	$d_x M$	$d_x F$	$q_{x}M$	$q_{x}F$	$q_{x}M - q_{x}F$	Ration = q_xM / q_xF
0–10	4134	3206	0.0413	0.0321	0.0092	1.29
10–20	715	494	0.0075	0.0051	0.0024	1.47
20–30	1331	989	0.0140	0.0103	0.0037	1.36
30–40	1852	1533	0.0197	0.0161	0.0036	1.22
40–50	4377	3122	0.0476	0.0333	0.0143	1.43
50–60	11768	7010	0.1344	0.0773	0.0571	1.74
60–70	23493	15811	0.3096	0.1890	0.1206	1.64
70–80	31220	31717	0.5964	0.4676	0.1288	1.28
80–90	18946	30039	0.8966	0.8317	0.0649	1.08
90–100	2161	5918	0.9895	0.9735	0.0160	1.02

i) The decade where there is more death for men and women is between 70 and 80 years

e) Greatest

i) Absolutely

The greatest absolute difference is between age 70-80 with a difference of 0.1288

ii) Relatively

The greatest ratio lies between age 50-60 with a ratio of 1.74

Number 4

Age-Group	${}_nM_x$	${}_nq_x$	${}_nM_x$	${}_nq_x$	l_x
1-4	0.00136	0.00567	0.00131	0.00546	698
5-14	0.00061	0.00608	0.00059	0.00588	694
15-24	0.00116	0.01142	0.00106	0.01044	689
25-34	0.00159	0.01570	0.00143	0.0141	681
35-44	0.00288	0.02891	0.00365	0.0366	671
45-54	0.00824	0.08116	0.01019	0.1004	646
55-64	0.02310	0.20981	0.02690	0.2443	581
65-74	0.05519	0.43791	0.06211	0.4925	439
75-84	0.12775	0.76472	0.14076	0.8426	222

Infant Mortality Rate $= \dfrac{{}_nd_x}{{}_nl_x} = \dfrac{30.2}{100} = 0.302 = \dfrac{{}_nd_x}{{}_nl_x}$

$q_x = \dfrac{d_x}{l_x}$ $\quad 0.302 \times 1000 = 302$

$l_x\ (1-4) = 1000 - 302 = 698$

$(5-14) = 698\,(1 - 0.00546) = 694$

$(15-24) = 694\,(1 - 0.00588) = 689$

$(25-34) = 689\,(1 - 0.01044) = 681$

$(35-44) = 681\,(1 - 0.0141) = 671$

$(45-54) = 671\,(1 - 0.0366) = 646$

$(55-64) = 646\,(1 - 0.1004) = 581$

$(65-74) = 581\,(1 - 0.2443) = 439$

$(75-84) = 439\,(1 - 0.4925) = 222$

Number 5

No of deaths
11 die within a month
16 die within the second and month
22 die within 3rd
30 die within 4th
41 die within 5th
56 die within 6th
76 die within 7th
102 die within 8th
135 die within 9th
176 die within 10th
226 die within 11th
286 die within 12th

Age	${}_nd_x$	Seq	l_x	l_{x+1}
0-1	11	5	21500	21489
1-2	16	6	21489	21473
2-3	22	8	21473	21451
3-4	30	11	21451	21421
4-5	41	15	21421	21380
5-6	56	20	21380	21324
6-7	76	26	21324	21248
7-8	102	33	21248	21146
8-9	135	41	21146	21011
9-10	176	50	21011	20835
10-11	226	60	20835	20609
11-12	286	71	20609	20323
	1177			

a) Determine the number of years lived by the person who die between age 72 and 73

$${}_n a_x = \frac{{}_n L_x - n\, l_{x+n}}{{}_n d_x}$$

$${}_n L_x = \frac{n}{2}(l_x + l_{x+n}) = \frac{12}{2}(21500 + 20323) = 250938$$

$${}_n a_x = \frac{250938 - (12 \times 20323)}{1177} = 6_{yrs} \text{ Ans.}$$

b) The death rate at age 72

$${}_n q_x = \frac{l_x - l_{x+n}}{l_x} = \frac{21500 - 20323}{21500} = 0.0547$$

c) The central death rate at age 72

$${}_n M_x = \frac{{}_n d_x}{{}_n L_x} = \frac{1177}{250938} = 0.00469$$

Number 6

Calculate:

a) $P_x = \frac{l_{40}}{l_{20}} = \frac{97346}{98497} = 0.988$

b) $q_0 = \frac{100{,}000 - 99016}{100{,}000} = 0.00984$

c) Life expectancy at birth:

At e_0 $\quad e_0 = \frac{T_0}{l_0} = \frac{7700187}{100{,}000} = 77.00187$

$e_1 = \frac{T_1}{l_1} = \frac{7601014}{99016} = 76.77$

d) $q_x = \frac{98746 - 98497}{98746} = 0.00252$

e) i)

$${}_na_x = \frac{{}_nL_x - n\,l_{x+n}}{{}_nd_x} =$$

$n = 10 \quad l_{x+n} = 98105 \quad \therefore n\,l_{x+n} = 981050$

$${}_nd_x = 98497 - 98105 = 392$$

$${}_nL_x = \frac{n}{2}(98497 + 98105) = 983{,}010$$

$$\therefore {}_na_x = \frac{983010 - 981050}{392} = 5$$

$\therefore$ Expected age at death $= {}_n a_x + 20 = 25$

e (ii) Using median;

$$= \left(\frac{20+30}{2}\right) = 25$$

Comment: Both methods i and ii give out the same output.

f). i.)

$${}_nL_x = n * l_{x+n} + {}_n a_x \, {}_n d_x$$

$${}_nL_x = \frac{n}{2}\left(l_x + l_{x+n}\right).$$

$$= \frac{1}{2}\left(100{,}000 + 99016\right)$$

$${}_nL_x = 99508$$

$n = 1$

$l_{x+n} = 99016$

$${}_nd_x = l_x - l_{x+n}$$

$${}_nd_x = 100000 - 99016$$

$${}_nd_x = 984$$

$$99508 = 1 * 99016 + {}_n a_x * 984$$

$$99508 - 99016 = 984 \, {}_n a_x$$

$$\frac{492}{984} = \frac{984}{984} \, {}_n a_x$$

$${}_n a_x = 0.5$$

$\Rightarrow 0 + 0.5 = 0.5$ yrs Ans.

f) ii. Using Median $\left(\frac{0+1}{2}\right) = 0.5$ yrs Ans.

Both methods yield the same result.