

# Chapter 4. Classification methods

## Part 5

April 4, 2007

### 1 Boosting Methods

Boosting is one of the most powerful learning idea introduced in the last ten years; see Freund and Schapire (1997)<sup>1</sup>. It was originally designed for classification problems. But it can be extended to regression as well. The motivation for boosting was a procedure that combines the outputs of many “weak” classification methods to produce a powerful “committee”. Some other idea has been proposed based on the idea.

For any methods we discussed above, it can be simply denoted by  $G(x)$ , we class a new sample according to the value of  $G(X)$ . We call  $G(x)$  a classifier.

#### Justification of Boosting

- In practice, it is easy to find quite correct rules of thumb, however hard to find single highly accurate prediction rule.

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<sup>1</sup>Y. Freund and R.E. Schapire (1997) "A decision-theoretic generalization of on-line learning and an application to boosting," Journal of Computer and System Sciences, 55(1):119-139

- If the training examples are few and the hypothesis space is large then there are several equally accurate classifiers.
- Hypothesis space does not contain the true function, but it has several good approximations.
- Exhaustive global search in the hypothesis space is expensive so we can combine the predictions of several locally accurate classifiers.

### **What is different with the previous methods**

- Boosting: Successive classifiers depends upon its predecessors. Previous methods : Individual classifiers were independent.
- Training Examples may have unequal weights.
- Look at errors from previous classifier step to decide how to focus on next iteration over data
- Set weights to focus more on “hard” examples. (the ones on which we committed mistakes in the previous iterations)

**Boosting Algorithm** Consider two classes  $Y \in \{-1, 1\}$  Given a vector of predictor variables  $X$ , a classifier  $G(X)$  produce a prediction taking values  $\{-1, 1\}$ . The error rate is defined as

$$err = n^{-1} \sum_{i=1}^n I(y_i \neq G(X_i))$$

Suppose we are going to combine  $M$  classifiers

$$G(X) = \sum_{m=1}^M G_m(X)$$

The following is the most popular “AdaBoost” algorithm

1. Initially assign uniform weights  $w_i = 1/n$
2. For  $m = 1$  to  $M$ 
  - (a) Fit a classifier  $G_m(x)$  to the training data using weights  $w_i$
  - (b) Compute  $err_m$  the error rate as
$$err_m = [\sum_{i=1}^n w_i * I(y_i \neq G_m(x_i))]/ \sum_{i=1}^n w_i]$$
  - (c) weight  $\alpha_m = \log((1 - err_m)/err_m)$
  - (d) Set  $w_i \leftarrow w_i * \exp(\alpha_m * I(y_i \neq G_m(x_i)))$ ,  $i = 1, \dots, n$

3. output

$$G_{final}(x) = \text{sign}[\sum_{m=1}^M \alpha_m G_m(x)]$$

**Example 1.1** *Classification in genetics For the leukemia gene expression data ((**training points**)). There are 38 cells with 250 genes (selected from about 7000 genes). they are from two types of cells.*

*We arbitrarily choose cell 21-33 for test and the others for learning*

*Boosting method: All are correctly clasified. ((**code**))*

The procedure can be summarized as follows:

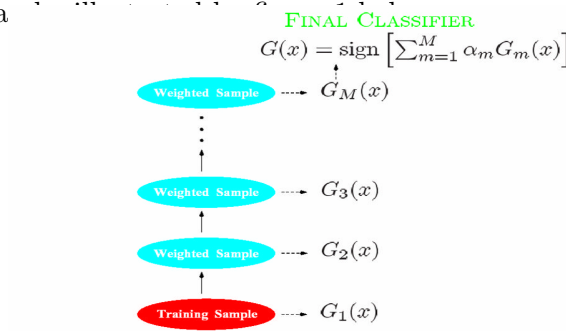


Figure 1: An illustration of the algorithm

As an example, we consider a classification problem as shown in figures 2-5.

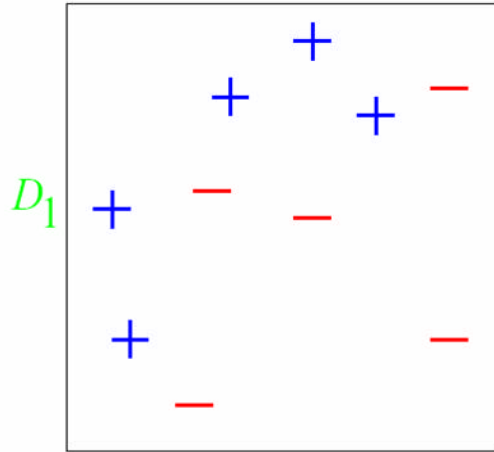


Figure 2: An example for classification

Round 1

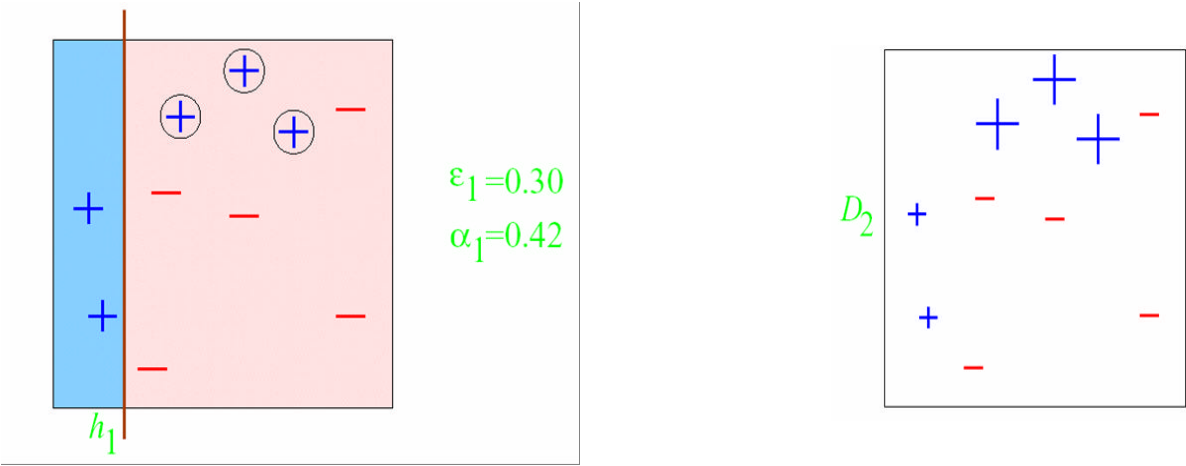


Figure 3: Example: The misclassified cases (circled) are given bigger weight

Round 2

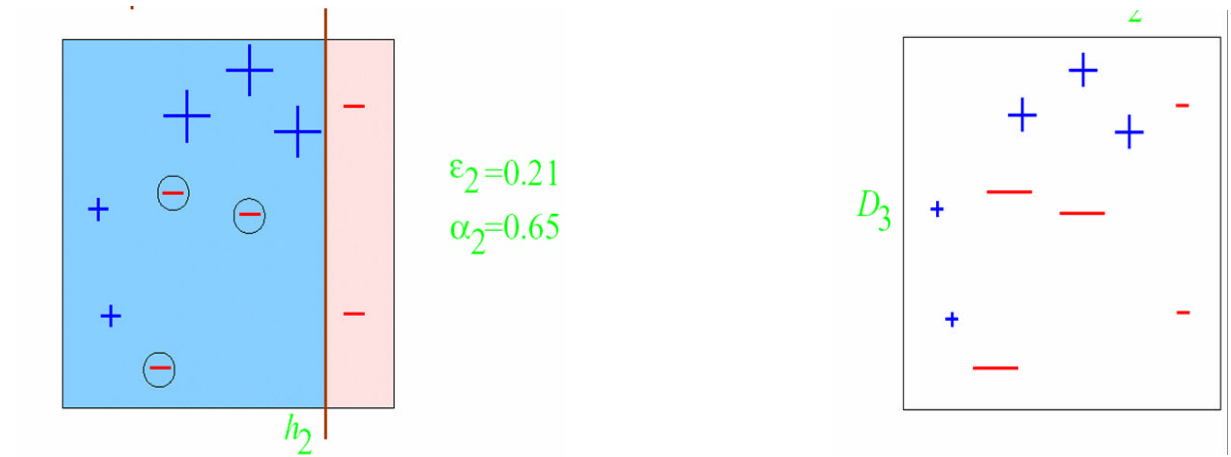
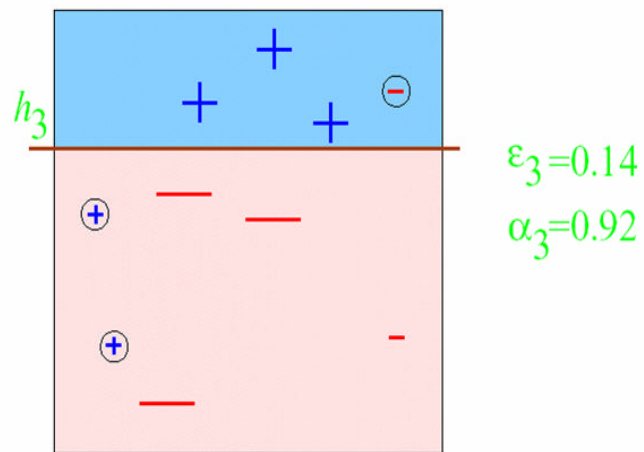


Figure 4: Example: The misclassified cases (circled) are given bigger weight

Round 3



The final classifier is

$$H_{\text{final}} = \text{sign} \left( 0.42 \left[ \text{Diagram 1} \right] + 0.65 \left[ \text{Diagram 2} \right] + 0.92 \left[ \text{Diagram 3} \right] \right)$$

The diagram shows three square regions representing the output of individual weak classifiers. Each square is divided by a vertical line. The first square has a blue left half and a red right half, with the weight 0.42. The second square has a blue left half and a red right half, with the weight 0.65. The third square has a blue top half and a red bottom half, with the weight 0.92.

Figure 5: Example: (cont')