Chapter 2

Multiple Regression Model

Overview

- Multiple Linear Regression
  - $y_i = \beta_0 + \beta_1 x_{1i} + \cdots + \beta_p x_{p,i} + \epsilon_i, \; i = 1, \cdots, n$
  - $\epsilon_i \sim \mathrm{N}(0, \sigma^2)$ independently
  - Matrix form $\underline{y} = X\underline{\beta} + \underline{\epsilon}$
- Least Squares Estimator $\hat{\underline{\beta}} = (X'X)^{-1}X'\underline{y}$
  - Properties
- Analysis of Variance Table
- Inference for $\underline{\beta}$, $E\left(y \middle| \underline{x}_0\right)$ and $Y|\underline{x}_0$
- Statistical software: Use of SAS and R

2.1 Multiple Regression Model

- A regression model that involves more than one independent variables is called a multiple regression model.

- The general form of a regression model with p independent variables $x_1, \ldots, x_p$ is given by

$$y_i = \beta_0 + \beta_1 x_{1i} + \ldots + \beta_p x_{pi} + \varepsilon_i, \quad i = 1, 2, \ldots, n, \quad (2.1)$$

  where $\beta_0, \beta_1, \ldots, \beta_p$ are the regression coefficients that need to be estimated.

2.1 Multiple Regression Model

Remarks

- The independent variables x_1, ..., x_p may all be different basic variables, or some may be functions of a few basic variables.

- For example

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon,$$

where $x_2 = x_1{}^2$.

Example 1

- Suppose we want to investigate how weight varies with height and age for children with a particular kind of nutritional deficiency.

- The dependent variable is weight, y.

- The basic independent variables are height, x_1 and age, x_2.

2.2 More Examples of Multiple Regression Model

Some possible models are as follows:

(i) $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$;

(ii) $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$,

where $x_3 = x_1^2$. (i.e. x_3 is a function of the basic variable x_1);

(iii) $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \varepsilon$,

where $x_3 = x_1^2$, $x_4 = x_1 x_2$ and $x_5 = x_2^2$.

More Examples (Continued)

- Models (ii) and (iii) are called **polynomial** models. (polynomial in basic independent variables)

Remark:

- Model (2.1) is also called a multiple linear regression model since (2.1) is a linear function of the unknown parameters $\beta_0, \beta_1, \ldots, \beta_p$.

Note:

$y = e^{\beta_1 x_1} + \beta_2 x_2 + \epsilon$ is not a multiple linear regression model.

2.3 Interpretation of regression coefficients

- For a fixed i, $i = 1, \ldots, p$, β_i is the slope of $E(y)$ with x_i by holding other variables constant.

  It represents the change in **expectation of y** with a unit change in x_i, while other variables are kept at fixed values.

- β_0 represents the true average of y when $x_1, x_2, \ldots, x_p$ are all zero.

  It may not have any meaning if the region of the values of the independent variables does not include the point $(x_1, x_2, \ldots, x_p) = (0, 0, \ldots, 0)$.

2.4 Matrix representation of the model

Model given by Equation (2.1) can be expressed in matrix form as follows:

$$\underline{y} = X\underline{\beta} + \underline{\varepsilon};$$

where

$$\underline{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, \ X = \begin{pmatrix} 1 & x_{11} & \cdots & x_{p1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{1n} & \cdots & x_{pn} \end{pmatrix}, \ \underline{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{pmatrix}, \ \underline{\epsilon} = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{pmatrix}$$

Matrix representation of the model (Continued)

- Note: x_{ij} denotes the j-th observation from the i-th independent variable.

- Also $\underline{\varepsilon}$ satisfies $\mathrm{E}(\underline{\varepsilon}) = 0$ and $\mathrm{Var}(\underline{\varepsilon}) = \sigma^2 I_n$.

- We further assume that $\boldsymbol{\varepsilon_i}$**'s are independent** normally distributed.

  (i.e. $\varepsilon_i \sim N(0, \sigma^2)$ independently for all i).

Matrix representation of the model (Continued)

Remark:

It follows from the assumptions on $\underline{\varepsilon}$ that

$$E\left(\underline{y}\right) = X\underline{\beta}, \qquad Var\left(\underline{y}\right) = \sigma^2 I_n,$$

and

$$y_i | \underline{x}_i \sim N\left(\mu_{y_i|\underline{x}_i}, \sigma^2\right), \qquad i = 1, 2, \cdots, n$$

where

$$\underline{x}_i{}' = \begin{pmatrix} 1 & x_{1i} & \cdots & x_{pi} \end{pmatrix}$$

and

$$\mu_{y_i|\underline{x}_i} = \underline{x}_i{}'\underline{\beta} = \beta_0 + \beta_1 x_{1i} + \cdots + \beta_p x_{pi}.$$

2.5 Estimation of the regression coefficients

- Assume that the matrix $X'X$ is non-singular. (i.e. $(X'X)^{-1}$ exists.) The least squares estimator of $\underline{\beta}$ is obtained by minimizing $\underline{\varepsilon}'\underline{\varepsilon}$ with respect to $\underline{\beta}$.

$$\underline{\epsilon}'\underline{\epsilon} = \left[\underline{\beta} - (X'X)^{-1}X'\underline{y}\right]'(X'X)\left[\underline{\beta} - (X'X)^{-1}X'\underline{y}\right] - \underline{y}'X(X'X)^{-1}X'\underline{y} + \underline{y}'\underline{y}$$

- Let $\hat{\underline{\beta}} = (X'X)^{-1}X'\underline{y}$. Then $\underline{\epsilon}'\underline{\epsilon}$ is at minimum if $\underline{\beta} = \hat{\underline{\beta}}$

  Therefore $\hat{\underline{\beta}}$ is the least squares estimator of $\underline{\beta}$.

  [$(X'X)\hat{\underline{\beta}} = X'\underline{y}$ is called the **normal equation**.]

  Note: $\hat{\underline{\beta}} = \begin{pmatrix}\hat{\beta}_0 & \hat{\beta}_1 & \cdots & \hat{\beta}_p\end{pmatrix}'$

2.6 Properties of $\underline{\hat{\beta}}$

1. $E\left(\underline{\hat{\beta}}\right) = \underline{\beta}$ (Unbiasedness)
2. $Var\left(\underline{\hat{\beta}}\right) = \sigma^2 (X'X)^{-1}$
3. Let $\underline{a}_i{}' = (a_{i1} \quad \cdots \quad a_{in})$ denote the i-th row of the matrix $(X'X)^{-1}X'$ (p x n matrix). i.e

$$\begin{pmatrix} \underline{a}_0{}' \\ \underline{a}_1{}' \\ \vdots \\ \underline{a}_p{}' \end{pmatrix} = (X'X)^{-1}X'$$

Then $\hat{\beta}_i = \underline{a}_i{}'\underline{y}$.

Properties of $\hat{\underline{\beta}}$ (Continued)

- Since y_i's are normally distributed and $\hat{\beta}_i$ is a linear combination of y_i's, therefore
$$\hat{\beta}_i \sim N\left(\beta_i, \sigma_i^2\right)$$
where σ_i^2 is the $(i + 1, i + 1)^{th}$ entry of $Var\left(\hat{\underline{\beta}}\right)$

- The equation of regression line is
$$\hat{y} = \underline{x}'\hat{\underline{\beta}}, \qquad \text{where } \underline{x}' = (1 \quad x_1 \quad \cdots \quad x_p)$$
- The vector of fitted values is
$$\hat{\underline{y}} = X\hat{\underline{\beta}} = X(X'X)^{-1}X'\underline{y}.$$

2.7 Example 1 (Continued)

- Refer to Example 1 on p2-4.
- Suppose that a random sample consists of 12 children who attend a clinic is chosen.
- The weight, height and age data obtained for each child are given as follows:

Child	1	2	3	4	5	6	7	8	9	10	11	12
Weight, y	64	71	53	67	55	58	77	57	56	51	76	68
Height, x_1	57	59	49	62	51	50	55	48	42	42	61	57
Age, x_2	8	10	6	11	8	7	10	9	10	6	12	9

- Estimate the regression equation.

Example 1 (Continued)

- Consider the model $\underline{y} = X\underline{\beta} + \underline{\varepsilon}$,

  where $\underline{y}$ is a 12 x 1 vector, X is a 12 x 3 matrix, $\underline{\beta} = (\beta_0, \beta_1, \beta_2)$ and $\underline{\varepsilon}$ is a 12 x 1 random vector.

  i.e. $y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \varepsilon_i \quad i = 1, \ldots, 12.$

- The least squares estimator of $\underline{\beta}$ is given by

$$\underline{\hat{\beta}} = (X'X)^{-1}X'\underline{y}$$

Example 1 (Continued)

- From the data, we have

$$X'X = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 57 & 59 & \cdots & 57 \\ 8 & 10 & \cdots & 9 \end{pmatrix} \begin{pmatrix} 1 & 57 & 8 \\ 1 & 59 & 10 \\ \vdots & \vdots & \vdots \\ 1 & 57 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & 633 & 106 \\ 633 & 33903 & 5679 \\ 106 & 5679 & 976 \end{pmatrix}$$

$$|X'X| = 151956$$

Example 1 (Continued)

Therefore

$$(X'X)^{-1} = \frac{1}{151956}\begin{pmatrix} 838287 & -15834 & 1089 \\ -15834 & 476 & -1050 \\ 1089 & -1050 & 6147 \end{pmatrix}$$

Note: The (i, j)th entry of the above matrix is given by multiplying $(-1)^{i+j}$ to the determinant of $(X'X)$ with the j-th row and i-th column deleted.

p2-28

Example 1 (Continued)

$$X'\underline{y} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 57 & 59 & \cdots & 57 \\ 8 & 10 & \cdots & 9 \end{pmatrix} \begin{pmatrix} 64 \\ 71 \\ \vdots \\ 68 \end{pmatrix} = \begin{pmatrix} 753 \\ 40270 \\ 6796 \end{pmatrix}$$

- Therefore

$$\underline{\hat{\beta}} = (X'X)^{-1}X'\underline{y}$$

$$= \frac{1}{151956} \begin{pmatrix} 838287 & -15834 & 1089 \\ -15834 & 476 & -1050 \\ 1089 & -1050 & 6147 \end{pmatrix} \begin{pmatrix} 753 \\ 40270 \\ 6796 \end{pmatrix}$$

$$= \begin{pmatrix} 6.553 \\ 0.722 \\ 2.050 \end{pmatrix}$$

Example 1 (Continued)

- The regression equation is

$$\hat{y} = 6.553 + 0.722x_1 + 2.050x_2$$

- If age, x_2, is held constant, one unit increase in height would result in an estimated increase of 0.722 unit in the expectation of weight.

- Similarly, if height, x_1, is held constant, one unit increase in age results in an estimated increase of 2.05 units in expectation of weight.

Example 1 (Continued)

Note:

$$X'X = \begin{pmatrix} n & \sum x_{1i} & \sum x_{2i} \\ \sum x_{1i} & \sum x_{1i}^2 & \sum x_{1i}x_{2i} \\ \sum x_{2i} & \sum x_{1i}x_{2i} & \sum x_{2i}^2 \end{pmatrix}$$

$$X'\underline{y} = \begin{pmatrix} \sum y_i \\ \sum x_{1i}y_i \\ \sum x_{2i}y_i \end{pmatrix}$$

2.8 Analysis of Variance

- As in the simple regression model, the total corrected sum of squares can be expressed into the **sum** of two sums of squares,
  - the sum of squares due to error, *SSE*, and
  - the sum of squares due to regression, *SSR*.

$$SST = SSR + SSE$$

$SST = \sum_{i=1}^{n}(y_i - \bar{y})^2 = \underline{y}'\underline{y} - n\bar{y}^2$ with $n - 1$ d.f.

$SSR = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 = \underline{\hat{\beta}}'X'\underline{y} - n\bar{y}^2$ with $n - p - 1$ d.f.

$SSE = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \underline{y}'\underline{y} - \underline{\hat{\beta}}'X'\underline{y}$ with p d.f.

Analysis of Variance (Continued)

- It can be shown that $E[MSE] = \sigma^2$, thus MSE serves as an unbiased estimator of σ^2.

- Also it can be shown that

$$\frac{SSE}{\sigma^2} \sim \chi^2(n-p-1)$$

Analysis of Variance (Continued)

- To test whether there is **any** relationship between the dependent variable and the independent variables is equivalent to testing

  H_0: $\beta_1 = \beta_2 = \ldots = \beta_p = 0$ against

  H_1: $\beta_j \neq 0$ for some $j \in \{1, 2, \ldots, p\}$.

- It can be shown that under H_0,

$$E[MSR] = \sigma^2 \quad \text{and} \quad SSR/\sigma^2 \sim \chi^2(p).$$

- Also it can be shown that SSR and SSE are independent.

Analysis of Variance (Continued)

- Let

$$F = \frac{MSR}{MSE} = \frac{(SSR/\sigma^2)/p}{(SSE/\sigma^2)/(n-p-1)}$$

- Then under H_0,

$$F \sim F(p, n-p-1)$$

- Therefore we reject H_0 if

$$F_{\text{obs}} > F_{\alpha}(p, n-p-1)$$

Analysis of Variance (Continued)

- ANOVA Table

Source	SS	Df	MS	F-ratio	p-value
Regression	$SSR = \underline{\hat{\beta}}'X'\underline{y} - n\bar{y}^2$	p	$\frac{SSR}{p}$	$F_{obs} = \frac{MSR}{MSE}$	$\Pr\left(F(p, n-p-1) > F_{obs}\right)$
Error	$SSE = \underline{y}'\underline{y} - \underline{\hat{\beta}}'X'\underline{y}$	$n-p-1$	$\frac{SSE}{n-p-1}$		
Total	$SST = \underline{y}'\underline{y} - n\bar{y}^2$	$n-1$			

Example 1 (Continued)

$$SST = \underline{y}'\underline{y} - n\bar{y}^2 = 48319 - \frac{753^2}{12} = 888.25$$

$$\begin{aligned} SSR &= \underline{\hat{\beta}}'X'\underline{y} - n\bar{y}^2 \\ &= \begin{pmatrix} 6.553 & 0.722 & 2.050 \end{pmatrix} \begin{pmatrix} 753 \\ 40270 \\ 6746 \end{pmatrix} - \frac{753^2}{12} \\ &= 692.823 \end{aligned}$$

$$SSE = SST - SSR = 888.25 - 692.823 = 195.427$$

Analysis of Variance (Continued)

ANOVA Table

Source	SS	df	MS	F	p-value
Regression	692.823	2	346.411	15.95	0.0011
Error	195.427	9	21.714		
Total	888.25	11			

- Since the observed $F > F_{0.05}(2, 9) = 4.26$ (or p-value = 0.0011 < 0.05), we reject the null hypothesis that there is no significant relationship between y and x_1 and x_2 at the 5% significance level.

2.9 Inference concerning β_i's

A $100(1 - \alpha)\%$ confidence interval of β_i is given by

$$\hat{\beta}_i \pm t_{\alpha/2}(n-p-1)s.e.(\hat{\beta}_i)$$

Example 1 (cont'd)

A 95% confidence interval of β_0 is given by

$$\hat{\beta}_0 \pm t_{0.025}(9)\sqrt{\hat{\sigma}^2\left((X'X)^{-1}\right)_{11}}$$

$$= 6.553 \pm 2.262\sqrt{21.7142\frac{838287}{151956}}$$

$$= 6.553 \pm 2.262\,(10.9448)$$

$$= (-18.2042, 31.3102)$$

Refer to p.2-17 for the value of $(X'X)^{-1}$. p2-17

Inference concerning β_i's (Continued)

- Similarly, a 95% confidence interval of β_1 is given by

$$0.722 \pm 2.262 \sqrt{21.7142 \frac{476}{151956}}$$
$$= 0.772 \pm 2.262\,(0.2608) = (0.1321, 1.3119)$$

and a 95% confidence interval of β_2 is given by

$$2.0501 \pm 2.262 \sqrt{21.7142 \frac{6147}{151956}}$$
$$= 2.0501 \pm 2.262\,(0.9372)$$
$$= (-0.0699, 4.1703)$$

2.10 Inference concerning $\mu_{Y|x_0}$

A 100(1 – α)% **confidence interval** for $\mu_{Y|\underline{x}_0}$ is given by

$$\underline{x}_0{}'\underline{\hat{\beta}} \pm t_{\alpha/2}(n-p-1)\sqrt{\hat{\sigma}^2 \underline{x}_0{}'(X'X)^{-1}\underline{x}_0}$$

Example 1 (Continued)

Given that $\underline{x}_0' = (1 \quad 59 \quad 10)$, find a 95% confidence interval for $\mu_{Y|\underline{x}_0}$

2.10 Inference concerning $\mu_{Y|\underline{x}_0}$

A 95% confidence interval for $\mu_{Y|\underline{x}_0}$ is given by

$$(1 \quad 59 \quad 10)\begin{pmatrix}6.553\\0.722\\2.050\end{pmatrix}$$

$$\pm\, 2.262\sqrt{21.714(1 \quad 59 \quad 10)(X'X)^{-1}\begin{pmatrix}1\\59\\10\end{pmatrix}}$$

$$= 69.655 \pm 2.262(1.8639) = 69.655 \pm 4.216$$

$$= (65.439, 73.871)$$

2.11 Inference concerning $Y|\underline{x}_0$

A $100(1 - \alpha)\%$ **prediction interval** for $Y|\underline{x}_0$ is given by

$$\underline{x}_0'\underline{\hat{\beta}} \pm t_{\alpha/2}(n-p-1)\sqrt{\hat{\sigma}^2\left(1+\underline{x}_0'(X'X)^{-1}\underline{x}_0\right)}$$

Example 1 (Continued)

Given that $\underline{x}_0' = (1 \quad 59 \quad 10)$, find a 95% confidence interval for $Y|\underline{x}_0$

2.11 Inference concerning $Y|\underline{x}_0$

A 95% prediction interval for $Y|\underline{x}_0$ is given by

$$(1 \quad 59 \quad 10)\begin{pmatrix} 6.553 \\ 0.722 \\ 2.050 \end{pmatrix}$$

$$\pm\ 2.262\sqrt{21.714\left[1+(1 \quad 59 \quad 10)(X'X)^{-1}\begin{pmatrix} 1 \\ 59 \\ 10 \end{pmatrix}\right]}$$

$$= 69.655 \pm 2.262(5.0188) = 69.655 \pm 11.3525$$

$$= (58.3021, 81.0071)$$

2.12 Confidence Region for $\underline{\beta}$

A 100(1 – α)% confidence region for $\underline{\beta}$ is given by

$$\left\{ \underline{\beta} : \left(\underline{\beta} - \underline{\hat{\beta}}\right)' X'X \left(\underline{\beta} - \underline{\hat{\beta}}\right) \le (p+1)\hat{\sigma}^2 F_\alpha(p+1, n-p-1) \right\}$$

Example 1 (Continued)

A 95% confidence region for $\underline{\beta}$ is given by

$$\begin{pmatrix} \beta_0 - 6.553 & \beta_1 - 0.722 & \beta_2 - 2.050 \end{pmatrix} \begin{pmatrix} 12 & 633 & 106 \\ 633 & 33903 & 5679 \\ 106 & 5679 & 976 \end{pmatrix} \begin{pmatrix} \beta_0 - 6.553 \\ \beta_1 - 0.722 \\ \beta_2 - 2.050 \end{pmatrix}$$

$$\le 3\,(21.7142)(3.86)$$

$$\Rightarrow \quad 12(\beta_0 - 6.553)^2 + 33903(\beta_1 - 0.722)^2 + 976(\beta_2 - 2.050)^2$$
$$+ 2(633)(\beta_0 - 6.553)(\beta_1 - 0.722) + 2(106)(\beta_0 - 6.553)(\beta_2 - 2.050)$$
$$+ 2(5679)(\beta_1 - 0.722)(\beta_2 - 2.050) \le 251.45$$

2.12 SAS program

(Using the data in Example 1)

```
data ch2ex1;
   input weight height age;
datalines;
64 57 8
71 59 10
......
68 57 9
. 59 10
proc glm data=ch2ex1;
 model weight = height age /i p clm;
 output out=ch2ex1out p=yhat r=res;
run;
```

"." represents a missing observation
The additional data line is for computing confidence interval and prediction at height = 59 and age = 10.

(Printout given on p.2-35 to 2-37)

SAS program (Continued)

Alternative procedure:

```
proc reg data=ch2ex1;
   model weight = height age;
run;
```

Some other useful procedures:

```
proc plot data=ch2ex1out;
  plot res*height;
  plot res*age;
proc print data=ch2ex1out;
run;
```

Partial SAS Output

X'X Inverse Matrix

	Intercept	height	age	weight
Intercept	5.5166429756	-0.104201216	0.0071665482	6.5530482508
height	-0.104201216	0.0031324857	-0.006909895	0.7220379584
age	0.0071665482	-0.006909895	0.0404524994	2.0501263524
weight	6.5530482508	0.7220379584	2.0501263524	195.42739346

The GLM Procedure

Dependent Variable: weight

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	692.8226065	346.4113033	15.95	0.0011
Error	9	195.4273935	21.7141548		
Corrected Total	11	888.2500000			

R-Square	Coeff Var	Root MSE	weight Mean
0.779986	7.426048	4.659845	62.75000

Partial SAS Output (Continued)

Source	DF	Type I SS	Mean Square	F Value	Pr > F
height	1	588.9225232	588.9225232	27.12	0.0006
age	1	103.9000834	103.9000834	4.78	0.0565

Source	DF	Type III SS	Mean Square	F Value	Pr > F
height	1	166.4297494	166.4297494	7.66	0.0218
age	1	103.9000834	103.9000834	4.78	0.0565

Parameter	Estimate	Standard Error	t Value	Pr > |t|
Intercept	6.553048251	10.94482708	0.60	0.5641
height	0.722037958	0.26080506	2.77	0.0218
age	2.050126352	0.93722561	2.19	0.0565

Partial SAS Output (Continued)

Observation	Observed	Predicted	Residual
1	64.00000000	64.11022270	-0.11022270
2	71.00000000	69.65455132	1.34544868
3	53.00000000	54.23366632	-1.23366632
4	67.00000000	73.87079154	-6.87079154
5	55.00000000	59.77799495	-4.77799495
6	58.00000000	57.00583064	0.99416936
7	77.00000000	66.76639948	10.23360052
8	57.00000000	59.66200742	-2.66200742
9	56.00000000	57.37990603	-1.37990603
10	51.00000000	49.17940062	1.82059938
11	76.00000000	75.19887994	0.80112006
12	68.00000000	66.16034905	1.83965095
13 *	.	69.65455132	.

Partial SAS Output (Continued)

Observation	95% Confidence Limits for Mean Predicted Value	
1	59.20029579	69.02014960
2	65.43820064	73.87090200
3	48.40925309	60.05807956
4	68.45162916	79.28995393
5	56.43016113	63.12582876
6	52.61739008	61.39427119
7	63.14572800	70.38707097
8	55.36619240	63.95782244
9	48.72759206	66.03221999
10	42.95068544	55.40811579
11	69.04195838	81.35580149
12	62.34142127	69.97927683
13 *	65.43820064	73.87090200

* Observation was not used in this analysis

Sum of Residuals	-0.0000000
Sum of Squared Residuals	195.4273935
Sum of Squared Residuals - Error SS	-0.0000000
PRESS Statistic	299.1176996
First Order Autocorrelation	0.1114070
Durbin-Watson D	1.7598064

2.13 R program

```
> ch2ex1=read.table("d:/ST3131/Lecture/ch2ex1.txt",
   header=T)
> attach(ch2ex1)
> model1=lm(weight ~ height+age)
> anova(model1)
Analysis of Variance Table

Response: weight
          Df Sum Sq Mean Sq F value    Pr(>F)
height     1 588.92  588.92 27.1216 0.0005582 ***
age        1 103.90  103.90  4.7849 0.0564853 .
Residuals  9 195.43   21.71
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’
   0.1 ‘ ’ 1
```

R program (Continued)

```
> summary(model1)

Call:
lm(formula = weight ~ height + age)

Residuals:
    Min      1Q  Median      3Q     Max
-6.8708 -1.7004  0.3454  1.4642 10.2336

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   6.5530    10.9448   0.599   0.5641
height        0.7220     0.2608   2.768   0.0218 *
age           2.0501     0.9372   2.187   0.0565 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.66 on 9 degrees of freedom
Multiple R-squared:  0.78,      Adjusted R-squared: 0.7311
F-statistic: 15.95 on 2 and 9 DF,  p-value: 0.001099
```

R program (Continued)

```
> confint(model1,"(Intercept)",level=0.95)
                2.5 %   97.5 %
(Intercept) -18.20587 31.31197
> confint(model1,"height",level=0.95)
           2.5 %  97.5 %
height 0.1320559 1.31202
> confint(model1,"age",level=0.95)
          2.5 %   97.5 %
age -0.07002526 4.170278
> newx=data.frame(height=c(59),age=c(10))
> predict(model1,newx,interval="conf",level=0.95)
       fit     lwr     upr
1 69.65455 65.4382 73.8709
> predict(model1,newx,interval="predict",level=0.95)
       fit      lwr      upr
1 69.65455 58.30129 81.00782
```