Chapter 0

Basic Prerequisite Knowledge

1. Distributions

- **Normal Distribution, N(μ, σ^2)**

- **Chi-square Distribution, $\chi^2(n)$**
  - Let $U = Z^2$. If $Z \sim \mathrm{N}(0, 1)$, then

$$U \sim \chi^2(1)$$

  - Let $W = X_1 + \cdots + X_n$. If $X_i \sim \chi^2(1)$, $i = 1, \ldots, n$, **independently**, then

$$W \sim \chi^2(n)$$

1. Distributions

- ***t*-Distribution, *t*(*m*)**
  - If $Z \sim \mathrm{N}(0, 1)$ and $V \sim \chi^2(m)$ independently, then

$$T = \frac{Z}{V/\sqrt{m}} \sim t(m)$$

- ***F*-Distribution, *F*(*m*, *n*)**
  - If $V \sim \chi^2(m)$ and $W \sim \chi^2(n)$ independently, then

$$F = \frac{V/m}{W/n} \sim F(m, n)$$

2. Confidence Interval

- If $\hat{\theta}$ is a point estimate of θ, which follows a normal or an approximate normal distribution,
- then a 100(1 – α)% confidence interval for θ is given by

$$\hat{\theta} \pm t_{v,1-\alpha/2}\,\text{s.e.}(\hat{\theta})$$

where s.e.($\hat{\theta}$) is the standard error (i.e. the estimated standard deviation) of $\hat{\theta}$

3. Review of Matrices

3.1 Notation

- $A = (a_{ij})_{i\,=\,1,\,\ldots,\,p;\ j\,=\,1,\,\ldots,\,q}$ denotes a p x q matrix.
- $\underline{a} = \begin{pmatrix} a_1 \\ \vdots \\ a_p \end{pmatrix}$ denotes a p-dimensional vector.
- $A' = (a_{ji})$ = transpose of A.
- $I_p = p$ x p identity matrix.
- $\underline{1}_p = p$-dimensional vector of 1's.

3. Review of Matrices

3.1 Notation

If A is a p x p matrix, then

- $|A|$ = determinant of A (or det(A)),
- A^{-1} = inverse of A. (i.e. $AA^{-1} = A^{-1}A = I_p$.)

3.2 Expectation

- Let $\underline{X} = \begin{pmatrix} X_1 \\ \vdots \\ X_p \end{pmatrix}$ and $\underline{Y} = \begin{pmatrix} Y_1 \\ \vdots \\ Y_p \end{pmatrix}$ be random vectors

- Then $E(\underline{X}) = \begin{pmatrix} E(X_1) \\ \vdots \\ E(X_p) \end{pmatrix}$ and $E(\underline{Y}) = \begin{pmatrix} E(Y_1) \\ \vdots \\ E(Y_p) \end{pmatrix}$

- If A is a q x p matrix of constants and $\underline{b}$ is a q x 1 vector of constants, then

$$E(A\,\underline{X} + \underline{b}) = A\,E(\underline{X}) + \underline{b}$$

3.3 Covariance Matrix

- Covariance matrix is defined as

$$Cov(\underline{X},\underline{Y}) = E\left[\left(\underline{X} - E(\underline{X})\right)\left(\underline{Y} - E(\underline{Y})\right)'\right]$$

$$= E\begin{pmatrix} \vdots & \cdots & \vdots \\ \cdots & (X_i - E(X_i))(Y_j - E(Y_j)) & \cdots \\ \vdots & \cdots & \vdots \end{pmatrix}$$

$$= \begin{pmatrix} \vdots & \cdots & \vdots \\ \cdots & E[(X_i - E(X_i))(Y_j - E(Y_j))] & \cdots \\ \vdots & \cdots & \vdots \end{pmatrix}$$

$$= \left(Cov(X_i, Y_j)\right)_{i=1,\cdots,p;j=1,\cdots,p}$$

3.3 Covariance Matrix (Continued)

- When $\underline{Y} = \underline{X}$, $Cov(\underline{X}, \underline{X})$ is called the variance-covariance matrix of $\underline{X}$ (or dispersion matrix) and is denoted by $V(\underline{X})$.

$$V(\underline{X}) = Cov(\underline{X}, \underline{X})$$

$$= \begin{pmatrix} Var(X_1) & Cov(X_1, X_2) & \cdots & Cov(X_1, X_p) \\ Cov(X_2, X_1) & Var(X_2) & \cdots & Cov(X_2, X_p) \\ \vdots & \vdots & \ddots & \vdots \\ \cdots & \cdots & \cdots & Var(X_p) \end{pmatrix}$$

3.3 Covariance Matrix (Continued)

- Note: $V(\underline{X})$ is a symmetric matrix.

- If A is a p x p matrix of constants and $\underline{b}$ is a p x 1 vector of constants, then

$$V(A\,\underline{X} + \underline{b}) = A\,V(\underline{X})\,A'.$$

- In particular, $V(A\,\underline{X}) = A\,V(\underline{X})\,A'$.

- Note: Adding a constant vector $\underline{b}$ to the random vector $A\underline{X}$ does not change the variance of $A\underline{X}$.

3.4 Some properties of matrices

1. $AB \neq BA$

2. $(A')' = A$ and $(AB)' = B'A'$

3. $(AB)^{-1} = B^{-1}A^{-1}$ providing A^{-1} and B^{-1} exist

4. $(A')^{-1} = (A^{-1})'$