CHAPTER 19

Cascading Behavior in Networks

19.1 Diffusion in Networks

A basic issue in the preceding several chapters has been the way in which an individual's choices depend on what other people do. This has informed our use of information cascades, network effects, and rich-get-richer dynamics to model the processes by which new ideas and innovations are adopted by a population. When we perform this type of analysis, the underlying social network can be considered at two conceptually very different levels of resolution: one in which we view the network as a relatively amorphous population of individuals and look at effects in aggregate, and another in which we move closer to the fine structure of the network as a graph and look at how individuals are influenced by their particular network neighbors. Our focus in these past few chapters has been mainly on the first of these levels of resolution – capturing choices in which each individual is at least implicitly aware of the previous choices made by everyone else, and everyone takes these into account. In the next few chapters, we bring the analysis closer to the detailed network level.

What do we gain by considering this second level of resolution, oriented around network structure? To begin with, we can address a number of phenomena that can't be modeled well at the level of homogeneous populations. Many of our interactions with the rest of the world happen at a local, rather than a global, level: we often don't care as much about the full population's decisions as about the decisions made by friends and colleagues. For example, in a work setting we may choose technology to be compatible with the people we directly collaborate with, rather than the universally most popular technology. Similarly, we may adopt political views that are aligned with those of our friends, even if they are nationally in the minority.

In this way, considering individual choices with explicit network structure merges the models of the past several chapters with a distinct line of thinking begun in Chapter 4, when we examined how people link to others who are like them and in turn can become more similar to their neighbors over time. The framework in Chapter 4 dealt explicitly with network connections, but it did not explore the individual decision making that leads people to become similar to their neighbors: a tendency toward favoring similarity was invoked there as a basic assumption rather than being derived from more fundamental principles. In contrast, the past several chapters have developed principles that show how, at an aggregate population level, becoming similar to one's neighbors can arise from the behavior of individuals who are seeking to maximize their utility in given situations. We saw in fact that there are two distinct kinds of reasons why imitating the behavior of others can be beneficial: *informational effects*, based on the fact that the choices made by others can provide indirect information about what they know, and *direct-benefit effects*, in which there are direct payoffs from copying the decisions of others (for example, payoffs that arise from using compatible technologies instead of incompatible ones).

We now connect these two approaches by exploring some of the principles that can be used to model individual decision making in a social network, leading people to align their behaviors with those of their network neighbors.

The Diffusion of Innovations. We will consider specifically how new behaviors, practices, opinions, conventions, and technologies spread from person to person through a social network, as people influence their friends to adopt new ideas. Our understanding of how this process works is built on a rich area of empirical work in sociology known as the *diffusion of innovations* [115, 351, 382]. A number of now-classic studies done in the middle of the twentieth century established a basic research strategy for studying the spread of a new technology or idea through a group of people, and analyzing the factors that facilitated or impeded its progress.

Some of these early studies focused on cases in which the person-to-person influence was due primarily to informational effects: as people observed the decisions of their network neighbors, they obtained indirect information that led them to try the innovation as well. Two of the most influential early pieces of research to capture such informational effects were Ryan and Gross's study of the adoption of hybrid seed corn among farmers in Iowa [358] and Coleman, Katz, and Menzel's study of the adoption of tetracycline by physicians in the United States [115]. In Ryan and Gross's study, they interviewed farmers to determine how and when they decided to begin using hybrid seed corn; they found that while most of the farmers in their study first learned about hybrid seed corn from salesmen, most were first convinced to try using it based on the experience of neighbors in their community. Coleman, Katz, and Menzel went further when they studied the adoption of a new drug by doctors, in that they mapped out the social connections among the doctors making decisions about adoption. While these two studies clearly concerned very different communities and very different innovations, they – like other important studies of that period – shared a number of basic ingredients. In both cases, the novelty and initial lack of understanding of the innovation made it risky to adopt, but it was ultimately highly beneficial; in both cases, the early adopters had certain general characteristics, including higher socioeconomic status and a tendency to travel more widely; and in both cases, decisions about adoption were made in the context of a social structure where people could observe what their neighbors, friends, and colleagues were doing.

Other important studies in the diffusion of innovations focused on settings in which decisions about adoption were driven primarily by direct-benefit effects rather than informational ones. A long line of diffusion research on communication technologies has explored such direct-benefit effects; the spread of technologies such as the telephone, the fax machine, and e-mail has depended on the incentives people have to communicate with friends who have already adopted the technology [162, 285].

As studies of this type began proliferating, researchers started to identify some of the common principles that applied across many different domains. In his influential book on the diffusion of innovations, Everett Rogers gathered together and articulated a number of these principles [351], including a set of recurring reasons why an innovation can fail to spread through a population, even when it is has significant *relative advantage* compared to existing practices. In particular, the success of an innovation also depends on its *complexity* for people to understand and implement; its *observability*, so that people can become aware that others are using it; its *trialability*, so that people can mitigate its risks by adopting it gradually and incrementally; and, perhaps most crucially, its overall *compatibility* with the social system that it is entering. Related to these issues, the principle of homophily that we encountered in earlier chapters can sometimes act as a barrier to diffusion: since people tend to interact with others who are like themselves, while new innovations tend to arrive from "outside" the system, it can be difficult for these innovations to make their way into a tightly-knit social community.

With these considerations in mind, we now begin the process of formulating a model for the spread of an innovation through a social network.

19.2 Modeling Diffusion through a Network

We build our model for the diffusion of a new behavior in terms of a more basic, underlying model of individual decision making: as individuals make decisions based on the choices of their neighbors, a particular pattern of behavior can begin to spread across the links of the network. To formulate such an individual-level model, it is possible to start from either informational effects [2, 38, 186] or direct-benefit effects [62, 147, 308, 420]. In this chapter, we will focus on the latter, beginning with a natural model of direct-benefit effects in networks due to Stephen Morris [308].

Network models based on direct-benefit effects involve the following underlying consideration: you have certain social network neighbors – friends, acquaintances, or colleagues – and the benefits to you of adopting a new behavior increase as more and more of these neighbors adopt it. In such a case, simple self-interest will dictate that you should adopt the new behavior once a sufficient proportion of your neighbors have done so. For example, you may find it easier to collaborate with co-workers if you are using compatible technologies; similarly, you may find it easier to engage in social interaction – all else being equal – with people whose beliefs and opinions are similar to yours.

A Networked Coordination Game. These ideas can be captured very naturally by using a coordination game – a concept we first encountered in Section 6.5. In an underlying social network, we will study a situation in which each node has a choice between two possible behaviors, labeled A and B. If nodes v and w are linked by an edge, then there is an incentive for them to have their behaviors match. We represent

		w	
		A	B
v	A	a, a	$0, 0$
	B	$0, 0$	b, b

Figure 19.1. A-B coordination game.

this using a game in which v and w are the players and A and B are the possible strategies. The payoffs are defined as follows:

- if v and w both adopt behavior A, they each get a payoff of $a > 0$;
- if they both adopt B, they each get a payoff of $b > 0$; and
- if they adopt opposite behaviors, they each get a payoff of 0.

We can write this in terms of a payoff matrix, as in Figure 19.1. Of course, it is easy to imagine many more general models for coordination, but for now we are trying to keep things as simple as possible.

This describes what happens on a single edge of the network, but the point is that each node v is playing a copy of this game with each of its neighbors, and its payoff is the sum of its payoffs in the games played on each edge. Hence, v's choice of strategy will be based on the choices made by all of its neighbors, taken together.

The basic question faced by v is the following: suppose that some of its neighbors adopt A, and some adopt B; what should v do in order to maximize its payoff? This clearly depends on the relative number of neighbors doing each, and on the relation between the payoff values a and b. With a little bit of algebra, we can make up a decision rule for v quite easily, as follows. Suppose that a p fraction of v's neighbors have behavior A, and a $(1 - p)$ fraction have behavior B; that is, if v has d neighbors, then pd adopt A and $(1 - p)d$ adopt B, as shown in Figure 19.2. So if v chooses A, it

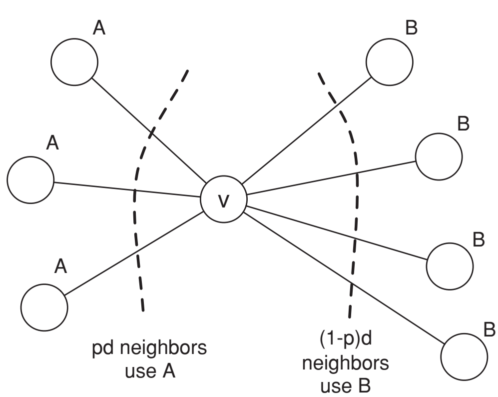


Figure 19.2. Node v must choose between behavior A and behavior B, based on what its neighbors are doing.

gets a payoff of pda, and if it chooses B, it gets a payoff of $(1-p)db$. Thus, A is the better choice if

$$pda \geq (1-p)db,$$

or, rearranging terms, if

$$p \geq \frac{b}{a+b}.$$

We'll use q to denote this expression on the right-hand side. This inequality describes a very simple threshold rule: it says that if a fraction of at least $q = b/(a+b)$ of your neighbors follow behavior A, then you should, too. And it makes sense intuitively: when q is small, then A is the much more enticing behavior, and it only takes a small fraction of your neighbors engaging in A for you to do so as well. However, if q is large, then the opposite holds: B is the attractive behavior, and you need a lot of your friends to engage in A before you switch to A. There is a tie-breaking question when exactly a q fraction of a node's neighbors follow A; in this case, we will adopt the convention that the node chooses A rather than B.

Notice that this is in fact a very simple – and in particular, myopic – model of individual decision making. Each node is optimally updating its decision based on the immediate consideration of what its neighbors are currently doing, but it is an interesting research question to think about richer models in which nodes try to incorporate more long-range considerations into their decisions about switching from B to A.

Cascading Behavior. In any network, there are two obvious equilibria to this network-wide coordination game: one in which everyone adopts A, and another in which everyone adopts B. Guided by diffusion questions, we want to understand how easy it is, in a given situation, to "tip" the network from one of these equilibria to the other. We also want to understand what other "intermediate" equilibria look like – states of coexistence where A is adopted in some parts of the network and B is adopted in others.

Specifically, we consider the following type of situation. Suppose that everyone in the network is initially using B as a default behavior. Then a small set of "initial adopters" all decide to use A. We will assume that the initial adopters have switched to A for some reason outside the definition of the coordination game – they have somehow switched due to a belief in A's superiority, rather than by following payoffs – but we'll assume that all other nodes continue to evaluate their payoffs using the coordination game. Given the fact that the initial adopters are now using A, some of their neighbors may decide to switch to A as well, and then some of their neighbors may switch, and so forth, in a potentially cascading fashion. When does this result in every node in the entire network eventually switching over to A? And when this isn't the result, what causes the spread of A to stop? Clearly the answer depends on the network structure, the choice of initial adopters, and the value of the threshold q that nodes use for deciding whether to switch to A.

The preceding discussion describes the full model. An initial set of nodes adopts A while everyone else adopts B. Time then runs forward in unit steps; in each step, each

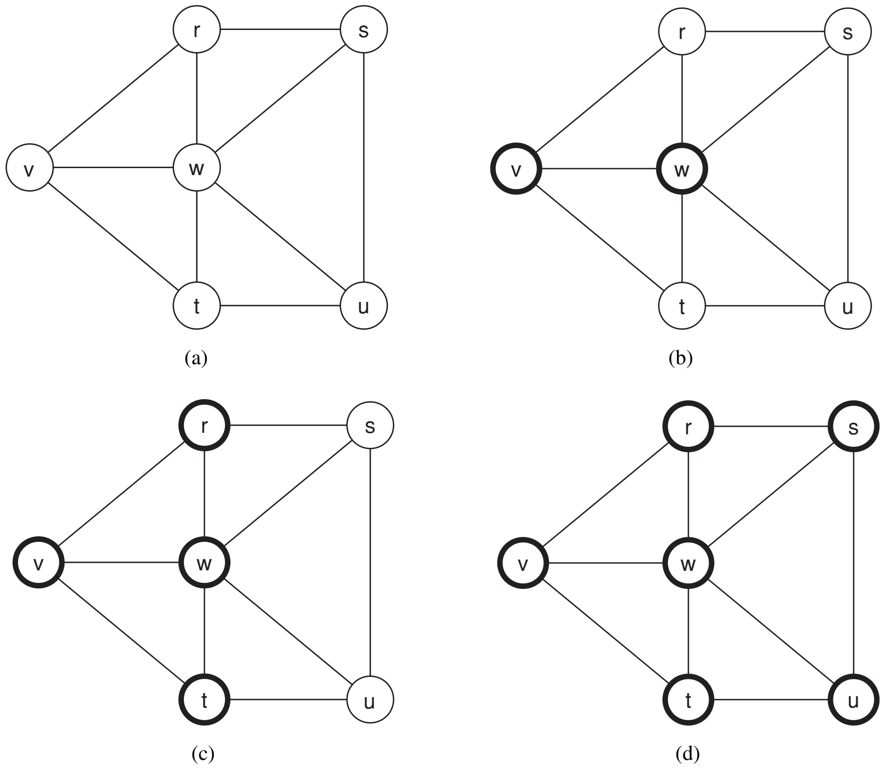


Figure 19.3. Starting with v and w as the initial adopters, and payoffs $a = 3$ and $b = 2$, the new behavior A spreads to all nodes in two steps. Nodes adopting A in a given step are drawn with dark borders; nodes adopting B are drawn with light borders. (a) The underlying network; (b) two nodes are the initial adopters; (c) after one step, two more nodes have adopted; and (d) after a second step, everyone has adopted.

node uses the threshold rule to decide whether to switch from B to A.[1] The process stops either when every node has switched to A or when we reach a step where no node wants to switch, at which point things have stabilized on coexistence between A and B.

Let's consider an example of this process using the social network in Figure 19.3(a).

- Suppose that the coordination game is set up so that $a = 3$ and $b = 2$; that is, the payoff to nodes interacting using behavior A is $\frac{3}{2}$ times what it is with behavior B. Using the threshold formula, we see that nodes will switch from B to A if at least a fraction $q = \frac{2}{(3+2)} = \frac{2}{5}$ of their neighbors are using A.

[1] Although we won't go through the details here, it is not hard to show that no node that switches to A at some point during this process will ever switch back to B at a later point – hence, what we're studying is indeed a strictly progressive sequence of switches from A to B. Informally, this fact is based on the observation that, for any node that switches to A at some point in time, the number of neighbors of this node that follow A only continues to increase as time moves forward beyond this point – so if the threshold rule said to switch to A at some point in time, it will only say this more strongly at future times. This is the informal version of the argument, but it is not hard to turn this into a proof.

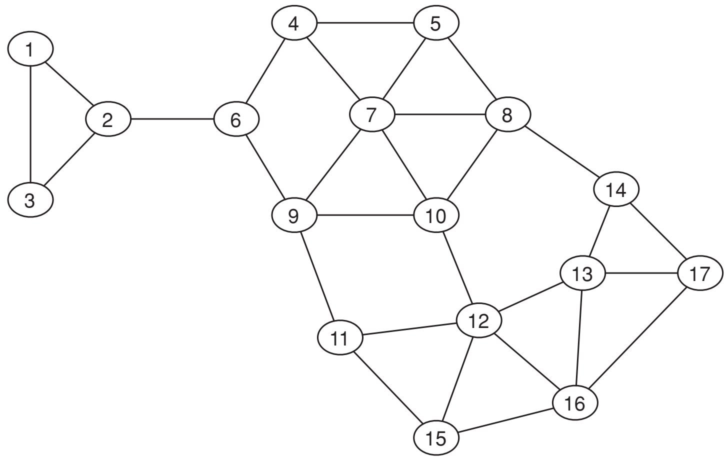


Figure 19.4. A larger example of a graph on which a new behavior may spread.

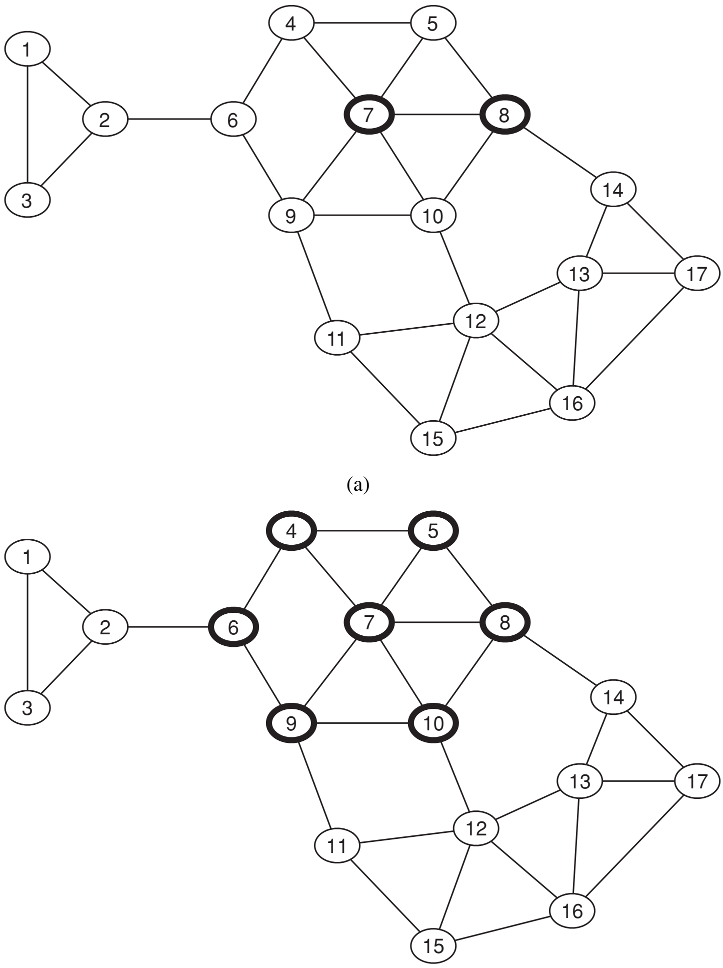
- Now, suppose that nodes v and w form the set of initial adopters of behavior A, while everyone else uses B. [See Figure 19.3(b), where dark circles denote nodes adopting A and lighter circles denote nodes adopting B.] Then after one step, in which each of the other nodes evaluates its behavior using the threshold rule, nodes r and t will switch to A: for each of them, $\frac{2}{3} > \frac{2}{5}$ of their neighbors are now using A. Nodes s and u do not switch, on the other hand, because for each of them, only $\frac{1}{3} < \frac{2}{5}$ of their neighbors are using A.
- In the next step, however, nodes s and u each have $\frac{2}{3} > \frac{2}{5}$ of their neighbors using A, and so they switch. The process now comes to an end, with everyone in the network using A.

Notice how the process really is a chain reaction: nodes v and w aren't able to get s and u to switch by themselves, but once they've converted r and t, this provides enough leverage.

It's also instructive to consider an example in which the adoption of A continues for a while but then stops. Consider the social network in Figure 19.4, and again let's suppose that in the A-B coordination game we have $a = 3$ and $b = 2$, leading to a threshold of $q = \frac{2}{5}$. If we start from nodes 7 and 8 as initial adopters [Figure 19.5(a)], then in the next three steps we will first see (respectively) nodes 5 and 10 switch to A, then nodes 4 and 9, and then node 6. At this point, no further nodes will be willing to switch, leading to the outcome in Figure 19.5(b).

We call this chain reaction of switches to A a *cascade* of adoptions of A, and we'd like to distinguish between two fundamental possibilities: (i) that the cascade runs for a while but stops while there are still nodes using B, or (ii) that there is a *complete cascade*, in which every node in the network switches to A. We introduce the following terminology for referring to the second possibility.

> Consider a set of initial adopters who start with a new behavior A, while every other node starts with behavior B. Nodes then repeatedly evaluate the decision to switch from B to A using a threshold of q. If the resulting cascade



(a)

(b)

Figure 19.5. Starting with nodes 7 and 8 as the initial adopters, the new behavior A spreads to some but not all of the remaining nodes. (a) The two nodes labeled 7 and 8 are the initial adopters and (b) the process ends after three steps.

of adoptions of A eventually causes every node to switch from B to A, then we say that the set of initial adopters *causes a complete cascade at threshold* q.

Cascading Behavior and "Viral Marketing." There are a few general observations to note about the larger example in Figure 19.5. First, it nicely illustrates a point from the opening section that tightly-knit communities in the network can work to hinder the spread of an innovation. Summarizing the process informally, A was able to spread to a set of nodes where there was sufficiently dense internal connectivity, but it was never able to leap across the "shores" in the network that separate nodes 8–10 from nodes 11–14 or that separate node 6 from node 2. As a result, we get coexistence between A and B, with boundaries in the network where the two meet. One can see reflections of this in many instances of diffusion – for example, in different dominant political

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views between adjacent communities. Or, in a more technological setting, consider the ways in which different social networking sites are dominated by different age groups and lifestyles – people have an incentive to be on the sites their friends are using, even when large parts of the rest of the world are using something else. Similarly, certain industries heavily use Apple Macintosh computers despite the general prevalence of Windows: if most of the people you directly interact with use Apple software, it's in your interest to do so as well, despite the increased difficulty of interoperating with the rest of the world.

This discussion also suggests some of the strategies that might be useful if A and B in Figure 19.5 were competing technologies and the firm producing A wanted to push its adoption past the point at which it has become stuck in Figure 19.5(b). Perhaps the most direct way, when possible, would be for the maker of A to raise the quality of its product slightly. For example, if we change the payoff a in the underlying coordination game from $a = 3$ to $a = 4$, then resulting threshold for adopting A drops from $q = \frac{2}{5}$ down to $q = \frac{1}{3}$. With this threshold, we could check that all nodes would eventually switch to A starting from the situation in Figure 19.5(b). In other words, at this lower threshold, A would be able to break into the other parts of the network that are currently resisting it. This captures an interesting sense in which making an existing innovation slightly more attractive can greatly increase its reach. It also shows that our discussion about the coexistence between A and B along a natural boundary in the network depended not just on the network structure but also on the relative payoffs of coordinating on A versus B.

When it's not possible to raise the quality of A – in other words, when the marketer of A can't change the threshold – a different strategy for increasing the spread of A would be to convince a small number of key people in the part of the network using B to switch to A, choosing these people carefully so as to get the cascade going again. For example, in Figure 19.5(b), we can check that if the marketer of A were to focus its efforts on convincing node 12 or 13 to switch to A, then the cascading adoption of A would start up again, eventually causing all of nodes 11–17 to switch. On the other hand, if the marketer of A spent effort getting node 11 or 14 to switch to A, then it would have no further consequences on the rest of the network; all other nodes using B would still be below their threshold of $q = \frac{2}{5}$ for switching to A. This indicates that the question of how to choose the key nodes to switch to a new product can be subtle, and based intrinsically on their position in the underlying network. Such issues are important in discussions of "viral marketing" [230] and have been analyzed in models of the type we are considering here [71, 132, 240, 309, 348].

Finally, it is useful to reflect on some of the contrasts between population-level network effects in technology adoption, as we formulated them in Chapter 17, and network-level cascading adoption as illustrated here. In a population-level model, when everyone is evaluating their adoption decisions based on the fraction of the entire population that is using a particular technology, it can be very hard for a new technology to get started, even when it is an improvement over the status quo. In a network, however, where you only care about what your immediate neighbors are doing, it's possible for a small set of initial adopters to essentially start a long fuse running that eventually spreads the innovation globally. This idea that a new idea is initially propagated at a local level along social network links is something one sees in many settings where an innovation gains eventual widespread acceptance.

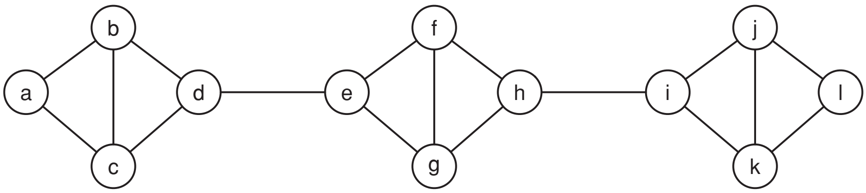


Figure 19.6. A collection of four-node clusters, each of density $\frac{2}{3}$.

19.3 Cascades and Clusters

We continue exploring some of the consequences of our simple model of cascading behavior from the previous section: now that we've seen how cascades form, we look more deeply at what makes them stop. Our specific goal will be to formalize something that is intuitively apparent in Figure 19.5 – that the spread of a new behavior can stall when it tries to break into a tightly-knit community within the network. This in fact will provide a way of formalizing a qualitative principle discussed earlier – that homophily can often serve as a barrier to diffusion by making it hard for innovations to arrive from outside densely connected communities.

As a first step, let's think about how to make the idea of a "densely connected community" precise, so that we can talk about it in the context of our model. A key property of such communities is that when you belong to one, many of your friends also tend to belong. We can take this as the basis of a concrete definition, as follows.

> We say that a cluster of density p is a set of nodes such that each node in the set has at least a fraction p of its network neighbors in the set.

For example, the set of nodes a, b, c, d forms a cluster of density $\frac{2}{3}$ in the network in Figure 19.6. The sets e, f, g, h and i, j, k, l each form clusters of density $\frac{2}{3}$ as well.

As with any formal definition, it's important to notice the ways in which it captures our motivation as well as some of the ways in which it may not. Each node in a cluster does have a prescribed fraction of its friends residing in the cluster as well, implying some level of internal "cohesion." On the other hand, our definition does not imply that any two particular nodes in the same cluster necessarily have much in common. For example, in any network, the set of *all* nodes is always a cluster of density 1 – after all, by definition, all your network neighbors reside in the network. Also, if you have two clusters of density p, then the union of these two clusters (i.e., the set of nodes that lie in at least one of them) is also a cluster of density p. These observations are consistent with the notion that clusters in networks can exist simultaneously at many different scales.

The Relationship Between Clusters and Cascades. The example in Figure 19.7 hints at how the cluster structure of a network may tell us something about the success or failure of a cascade. In this example, we see two communities, each of density $\frac{2}{3}$, in the network from Figure 19.4. These correspond precisely to the parts of the network that the cascading behavior A was unable to break into, starting from nodes 7 and 8 as initial adopters. Could this be a general principle?

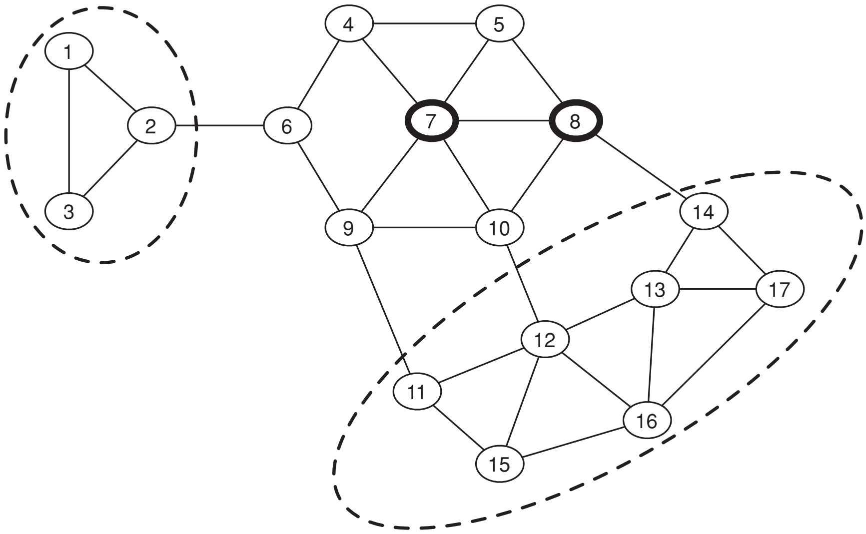


Figure 19.7. Two clusters of density $\frac{2}{3}$ in the network from Figure 19.4.

In fact it is, at least within the context of the model we've developed. We now formulate a result saying, essentially, that a cascade comes to a stop when it runs into a dense cluster, and, furthermore, that this is the only thing that causes cascades to stop [308]. In other words, clusters are the natural obstacles to cascades. Here is the precise statement, phrased in terms of the set of initial adopters and the *remaining network* – the portion of the network consisting of all nodes other than these initial adopters.

> *Claim:* Consider a set of initial adopters of behavior A, with a threshold of q for nodes in the remaining network to adopt behavior A.
>
> (i) If the remaining network contains a cluster of density greater than $1 - q$, then the set of initial adopters will not cause a complete cascade.
> (ii) Moreover, whenever a set of initial adopters does not cause a complete cascade with threshold q, the remaining network must contain a cluster of density greater than $1 - q$.

It is appealing how this result gives a precise characterization for the success or failure of a cascade, in our simple model, using a natural feature of the network structure. Furthermore, it does so by concretely formalizing a sense in which tightly-knit communities block the spread of cascades.

We now prove this result by separately establishing parts (i) and (ii). In going through the proofs of the two parts, it's useful to think about them both in general, and also in light of the example in Figure 19.7, where clusters of density greater than $1 - \frac{2}{5} = \frac{3}{5}$ block the spread of A at threshold $\frac{2}{5}$.

We begin with part (i).

Part (i): Clusters Are Obstacles to Cascades. Consider an arbitrary network in which behavior A is spreading with threshold q, starting from a set of initial adopters. Suppose

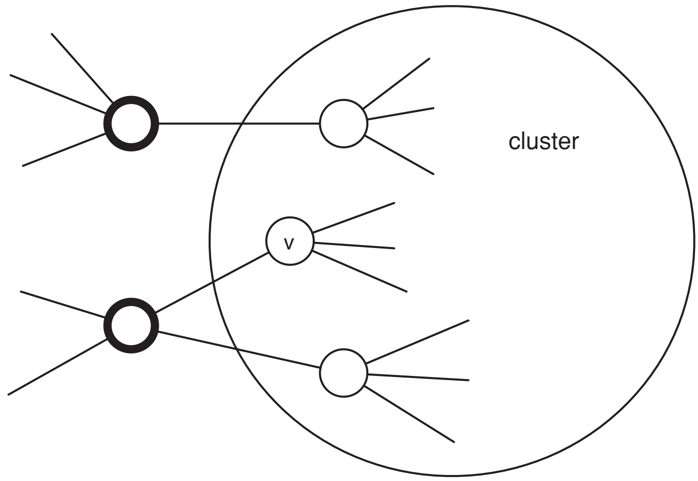


Figure 19.8. The spread of a new behavior, when nodes have threshold q, stops when it reaches a cluster of density greater than $(1 - q)$.

that the remaining network contains a cluster of density greater than $1 - q$. We now argue that no node inside the cluster will ever adopt A.

Indeed, assume the opposite – that some node inside the cluster does eventually adopt A – and consider the earliest time step t at which some node inside the cluster does so. Let v be the name of a node in the cluster that adopts A at time t. The situation is depicted schematically in Figure 19.8 – essentially, we want to argue that, at the time v adopted, it could not possibly have had enough neighbors using A to trigger its threshold rule. This contradiction will show that v in fact could not have adopted.

Here is how we do this. At the time that v adopted A, its decision was based on the set of nodes who had adopted A by the end of the previous time step, $t - 1$. Since no node in the cluster adopted before v did (that's how we chose v), the only neighbors of v that were using A at the time it decided to switch were *outside* the cluster. But since the cluster has density greater than $1 - q$, more than a $1 - q$ fraction of v's neighbors are inside the cluster, and hence less than a q fraction of v's neighbors are outside the cluster. Since these are the only neighbors who could have been using A, and since the threshold rule requires at least a q fraction of neighbors using v, this is a contradiction. Hence, our original assumption, that some node in the cluster adopted A at some point in time, must be false.

Having established that no node in the cluster ever adopts A, we are done, since this shows that the set of initial adopters does not cause a complete cascade.

Part (ii): Clusters Are the Only Obstacles to Cascades. We now establish part (ii) of our claim, which says in effect that not only are clusters a natural kind of obstacle to cascades – they are in fact the *only* kind of obstacle. From a methodological point of view (although all the details are different), this is reminiscent of a question we asked with matching markets: having found that constricted sets are natural obstacles to perfect matchings, we went on to find that they are in fact the only obstacle.

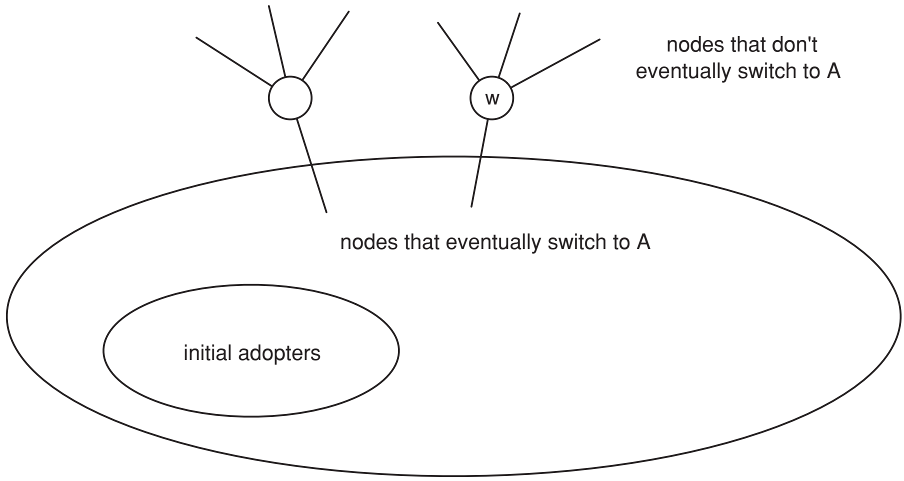


Figure 19.9. If the spread of A stops before filling out the whole network, the set of nodes that remain with B form a cluster of density greater than $1 - q$.

To prove part (ii) we show that whenever a set of initial adopters fails to cause a complete cascade with threshold q, there is a cluster in the remaining network of density greater than $(1 - q)$. In fact, this is not difficult: consider running the process by which A spreads, starting from the initial adopters, until it stops. It stops because there are still nodes using B, but none of the nodes in this set want to switch, as illustrated in Figure 19.9.

Let S denote the set of nodes using B at the end of the process. We want to claim that S is a cluster of density greater than $1 - q$, which will finish the proof of part (ii). To see why this is true, consider any node w in this set S. Since node w doesn't want to switch to A, it must be that the fraction of its neighbors using A is less than q; hence, the fraction of its neighbors using B is greater than $1 - q$. But the only nodes using B in the whole network belong to the set S, so the fraction of w's neighbors belonging to S is greater than $1 - q$. Since this holds for all nodes in S, it follows that S is a cluster of density greater than $1 - q$.

This wraps up our analysis of cascades and clusters; the punch line is that in this model, a set of initial adopters can cause a complete cascade at threshold q if and only if the remaining network contains no cluster of density greater than $1 - q$. So in this sense, cascades and clusters truly are natural opposites: clusters block the spread of cascades, and whenever a cascade comes to a stop, there's a cluster that can be used to explain why.

19.4 Diffusion, Thresholds, and the Role of Weak Ties

One of the fundamental things we learn from studying diffusion is that there is a crucial difference between learning about a new idea and actually deciding to adopt it. This contrast was already important in the early days of diffusion research. For example, Figure 19.10 comes from the original Ryan–Gross study of hybrid seed corn [358]; it

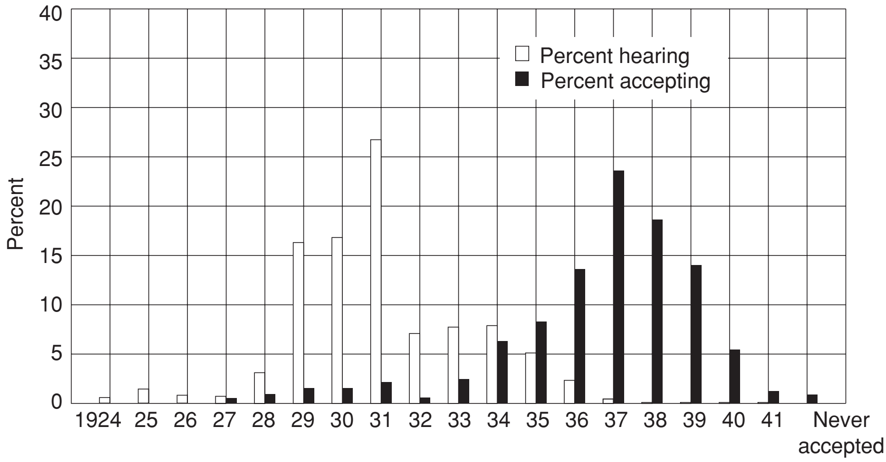


Figure 19.10. The years of first awareness and first adoption for hybrid seed corn in the Ryan–Gross study. (Image from [358]. Scarecrow Press, Inc.)

shows a clear wave of awareness of this innovation that significantly precedes the wave of adoptions.

Our models also illustrate this contrast. If we imagine that people first hear about an innovation when any of their neighbors first adopts, then we see, for example, in Figure 19.5 that nodes 4 and 9 are aware of A as a new behavior right away, but it takes further time for them to actually adopt it. In an even stronger direction, nodes 2 and 11–14 eventually become aware of A but never adopt it.

Centola and Macy [101] and Siegel [369] make the interesting observation that threshold models for diffusion thus highlight an interesting subtlety in the strength-of-weak-ties theory that we discussed in Chapter 3. Recall that the strength of weak ties is rooted in the idea that *weak* social connections, to people we see infrequently, often form local bridges in a social network. They therefore provide access to sources of information – things like new job opportunities – that reside in parts of the network we otherwise wouldn't have access to. To take a canonical picture from Chapter 3, shown here in Figure 19.11, the u-w and v-w edges span tightly-knit communities that wouldn't otherwise be able to communicate, and thus we expect v, for example, to receive information from his edge to w that he wouldn't get from his other edges.

But things look very different if we consider the spread of a new behavior that requires not just awareness but an actual threshold for adoption. Suppose, for example, w and x in Figure 19.11 are the initial adopters of a new behavior that is spreading with a threshold of $\frac{1}{2}$. Then we can check that everyone else in their tightly-knit six-node community will adopt this behavior, but u and v will not. (Nor, therefore, will anyone else lying beyond them in the network.)

This illustrates a natural double-edged aspect to bridges and local bridges in a social network: they are powerful ways to convey awareness of new things, but they are weak at transmitting behaviors that are in some way risky or costly to adopt – behaviors where you need to see a higher threshold of neighbors doing it before you do it as well. In this sense, nodes u and v in Figure 19.11 have strong informational advantages over

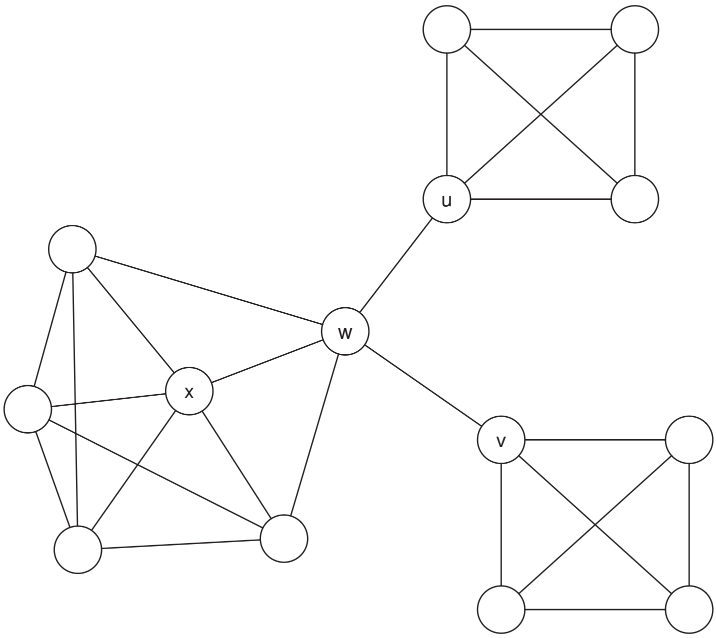


Figure 19.11. The *u-w* and *v-w* edges are more likely to act as conduits for information than for high-threshold innovations.

other members of their respective tightly-knit communities – they can learn from node w about a new behavior currently spreading in w's community – but for behaviors with higher thresholds they will still want to align themselves with others in their own community. If we think about it, this is actually remarkably consistent with the picture from Chapter 3, in which local bridges and positions near structural holes can provide access to information that you're not otherwise learning about from your own cluster in the network. For behaviors that spread with high thresholds, a local bridge may well connect you to someone whose network neighborhood has caused them to settle on a different behavior than you have.

The trade-offs inherent in this picture have been used to motivate some of the reasons why many social movements tend to build support locally and relatively slowly. Although a world-spanning system of weak ties in the global friendship network is able to spread awareness of a joke or an online video with remarkable speed, political mobilization moves more sluggishly, needing to gain momentum within neighborhoods and small communities. Thresholds provide a possible reason: social movements tend to be inherently risky undertakings, and hence individuals tend to have higher thresholds for participating; under such conditions, local bridges that connect very different parts of the network are less useful. Such considerations provide a perspective on other well-known observations about social movements in the diffusion literature, such as Hedstrom's findings that such movements often spread geographically [215] and McAdam's conclusion that strong ties, rather than weak ties, played the more significant role in recruitment to student activism during Freedom Summer in the 1960s [290, 291].

		w A	w B
v	A	a_v, a_w	$0, 0$
v	B	$0, 0$	b_v, b_w

Figure 19.12. A-B coordination game.

19.5 Extensions of the Basic Cascade Model

Our discussion thus far has shown how a very simple model of cascades in networks can capture a number of qualitative observations about how new behaviors and innovations diffuse. We now consider how the model can be extended and enriched, keeping its basic points the same while hinting at additional subtleties.

Heterogeneous Thresholds. Thus far we have kept the underlying model of individual behavior as simple as possible – everyone has the same payoffs and the same intensity of interaction with their network neighbors. But we can easily make these assumptions more general while still preserving the structure of the model and the close connection between cascades and clusters.

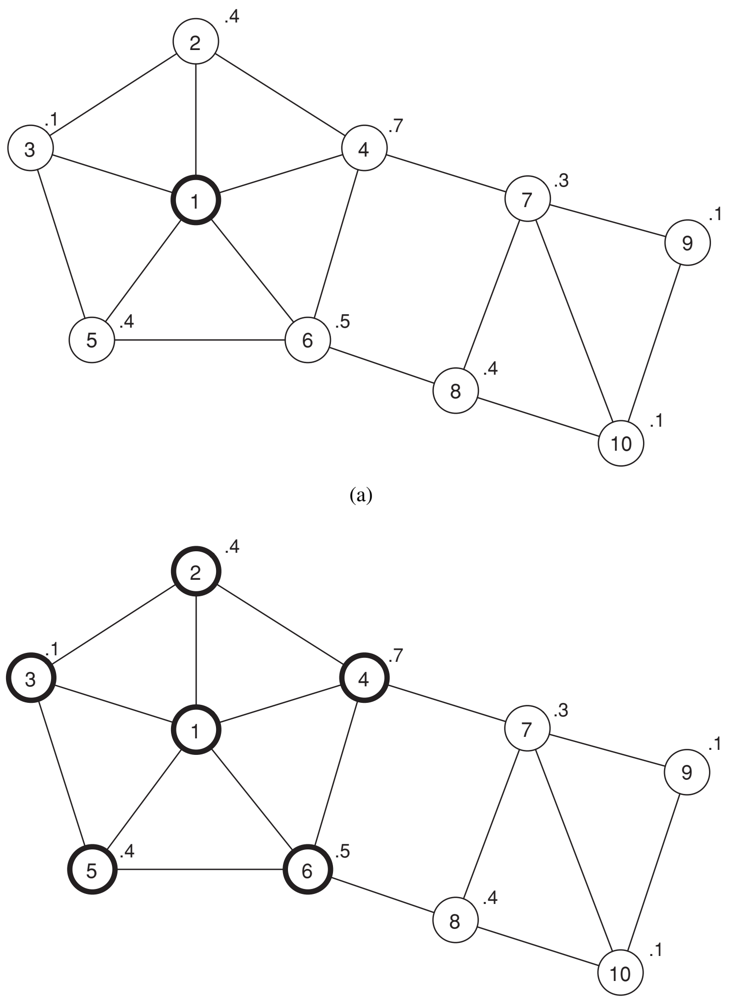
As the main generalization we consider, suppose that each person in the social network values behaviors A and B differently. Thus, for each node v, we define a payoff a_v – labeled so that it is specific to v – that it receives when it coordinates with someone on behavior A, and we define a payoff b_v that it receives when it coordinates with someone on behavior B. When two nodes v and w interact across an edge in the network, they are thus playing the coordination game in Figure 19.12.

Almost all of the previous analysis carries over with only small modifications; we now briefly survey how these changes go. When we first defined the basic coordination game, with all nodes agreeing on how to value A and B, we next asked how a given node v should choose its behavior based on what its neighbors are doing. A similar question applies here, leading to a similar calculation. If v has d neighbors, of whom a p fraction have behavior A and a $(1 - p)$ fraction have behavior B, then the payoff from choosing A is pda_v while the payoff from choosing B is $(1 - p)db_v$. Thus, A is the better choice if

$$p \geq \frac{b_v}{a_v + b_v}.$$

Using q_v to denote the right-hand side of this inequality, we again have a very simple decision rule – now, each node v has its own *personal* threshold q_v, and it chooses A if at least a fraction q_v of its neighbors have done so. Moreover, the variation in this set of heterogeneous node thresholds has an intuitive meaning in terms of the variation in payoffs: if a node values A more highly relative to B, its threshold q_v is correspondingly lower.

The process now runs as before, starting from a set of initial adopters, with each node evaluating its decision according to its own threshold rule in each time step and



(a)

(b)

Figure 19.13. Starting with node 1 as the unique initial adopter, the new behavior A spreads to some but not all of the remaining nodes. (a) One node is the initial adopter, and (b) the process ends after four steps.

switching to A if its threshold is reached. Figure 19.13 shows an example of this process (where each node's threshold is drawn to the upper right of the node itself).

A number of interesting general observations are suggested by what happens in Figure 19.13. First, the diversity in node thresholds clearly plays an important role that interacts in complex ways with the structure of the network. For example, despite node 1's "central" position, it would not have succeeded in converting anyone at all to A were it not for the extremely low threshold on node 3. This relates closely to a point made in work by Watts and Dodds [409], who argue that for understanding the spread of behaviors in social networks, we need to take into account not just the power of influential nodes but also the extent to which these influential nodes have access to easily *influenceable* people.

It is also instructive to look at how the spread of A comes to a stop in Figure 19.13 and to ask whether the notion of clusters as obstacles to cascades can be extended to hold even in the case when thresholds are heterogeneous. In fact, this is possible, by formulating the notion of a cluster in this setting as follows. Given a set of node thresholds, let's say that a *blocking cluster* in the network is a set of nodes for which each node v has more than a $1 - q_v$ fraction of its friends also in the set. (Notice how the notion of cluster density – like the notion of thresholds – becomes heterogeneous as well: each node has a different requirement for the fraction of friends it needs to have in the cluster.) By a fairly direct adaptation of the analysis from Section 19.3, one can show that a set of initial adopters will cause a complete cascade – with a given set of node thresholds – if and only if the remaining network does not contain a blocking cluster.

19.6 Knowledge, Thresholds, and Collective Action

We now switch our discussion to a related topic that integrates network effects at both the population level and the local network level. We consider situations for which coordination across a large segment of the population is important, and the underlying social network serves to transmit information about people's willingness to participate.

Collective Action and Pluralistic Ignorance. A useful motivating example is the problem of organizing a protest, uprising, or revolt under a repressive regime [109, 110, 192]. Imagine that you are living in such a society, and you are aware of a public demonstration against the government that is planned for tomorrow. If an enormous number of people show up, then the government will be seriously weakened, and everyone in society – including the demonstrators – will benefit. But if only a few hundred show up, then the demonstrators will simply all be arrested (or worse), and it would have been better had everyone stayed home. In such circumstances, what should you do?

This is an example of a *collective action* problem, where an activity produces benefits only if enough people participate. In this way, it is reminiscent of our analysis in Chapter 17 of population-level network effects: as with joining a large-scale demonstration, you only want to buy a fax machine if enough other people do. The starker setting of the present example highlights a few points, however. In the case of a fax machine, you can watch the experience of early adopters; you can read reviews and advertisements; you can canvass a wide array of friends and colleagues to see what they plan to do. Due to the much stronger negative payoffs associated with opposing a repressive government, many of these options are closed to you – you can talk about the idea with a small number of close friends whom you trust, but beyond this your decision about whether to show up for the demonstration is made difficult by a lack of knowledge of other people's willingness to participate, or of their criteria for deciding whether to participate.

These considerations illustrate some of the reasons why repressive governments work so hard to limit communication among their citizens. It is possible, for example, that a large fraction of the population is strong enough in its opposition to be willing to take extreme measures, but that most of these people believe they're in a small minority,

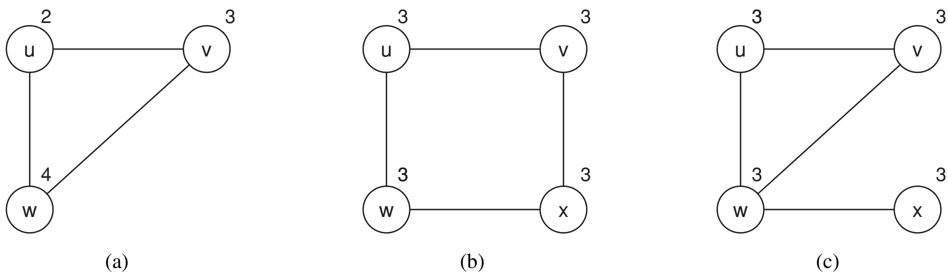


Figure 19.14. Each node in the network has a threshold for participation but only knows the threshold of itself and its neighbors. For three different networks, an uprising will not occur in (a) or (b), but can occur in (c).

and hence view opposition as too risky. In this way, a government could survive long after there is enough strong opposition in principle to get rid of it.

This phenomenon is known as *pluralistic ignorance* [330], in which people have wildly erroneous estimates about the prevalence of certain opinions in the population at large. It is a principle that applies widely, not just in settings where a central authority is actively working to restrict information. For example, a survey conducted in the United States in 1970 (and replicated several times in the surrounding years with similar results) showed that while only a minority of white Americans at that point personally favored racial segregation, significantly more than 50% believed that it was favored by a majority of white Americans in their region of the country [331].

A Model for the Effect of Knowledge on Collective Action. Let's consider how the structure of the underlying social network can affect the way people make decisions about collective action, following a model and a set of illustrative examples proposed by Michael Chwe [109, 110]. Suppose that each person in a social network knows about a potential upcoming protest against the government, and she has a personal *threshold* which encodes her willingness to participate. A threshold of k means, "I will show up for the protest if I am sure that at least k people in total (including myself) will show up."

The links in the social network encode strong ties, where the two endpoints of each link trust each other. Thus, we assume that each person in the network knows the thresholds of all her neighbors in the network, but – due to the risky nature of communication about dissent in this society – does not know the thresholds of anyone else. Now, given a network with a set of thresholds, how should we reason about what is likely to happen?

Let's consider the examples in Figure 19.14, which show some of the subtleties that arise here. Scaling down our notion of "uprising" to a size commensurate with these 3- to 4-person examples, suppose that each node represents one of the senior vice presidents at a company, each of whom must decide whether to actively confront the unpopular CEO at the next day's board meeting. It would be disastrous to do so without reasonable support from the others, so each is willing to confront the CEO provided that at least a certain number of them do so in total. We'll also assume that each node knows what the social network looks like.

First, Figure 19.14(a) indicates some of the reasoning that nodes must do about the decisions being made by other nodes. Here, node w would only join the protest if at least four people do; because there are only three people in total, this means he will never join. Node v knows that w's threshold is 4, so v knows that w won't participate. Because v requires three people in order to be willing to join, v won't participate either. Finally, u only requires two people in order to participate, but she knows the thresholds of both other nodes and hence can determine that neither will participate. So she doesn't either. Hence, the protest doesn't happen.

Figure 19.14(b) introduces even more subtle considerations, in which nodes must reason about what other nodes *know* in order to reason about what they will do. In particular, consider the situation from u's perspective (since it's symmetric for all nodes). She knows that v and w each have a threshold of 3, and so each of u, v, and w would feel safe taking part in a protest that contained all three of them. But she also knows that v and w don't know each other's thresholds, and so they can't engage in the same reasoning that she can.

Is it safe for u to join the protest? The answer is no, for the following reason. Since u doesn't know x's threshold, there's the possibility that it's something very high, like 5. In this case, node v, seeing neighbors with thresholds of 3 and 5, would not join the protest. Neither would w. So in this case, if u joined the protest, she'd be the only one – a disaster for her. Hence, u can't take this chance, and so she doesn't join the protest.

Since the situation is symmetric for all four nodes in Figure 19.14(b), we can conclude that no node will join the protest, and so no protest happens. There is something striking about this: each node in the network knows the fact that there are three nodes with thresholds of 3 – enough for a protest to form – but each holds back because they cannot be sure that any other nodes know this fact.

Things would turn out very differently if the link from v to x were shifted to instead connect v and w, resulting in the network of Figure 19.14(c). Now, each of u, v, and w not only knows the fact that there are three nodes with thresholds of 3, but this fact is *common knowledge* [29, 154, 276]: among the set of nodes consisting of u, v, and w, each node knows this fact, each node knows that each node knows it, each node knows that each node knows that each node knows it, and so on indefinitely. We touched on common knowledge briefly in the context of game theory in Chapter 6; as we see here, it also plays an important role in interactions designed to achieve coordination.

So the differences between the examples in Figures 19.14(b) and 19.14(c) are subtle and come down to the different networks' consequences for the knowledge that nodes have about what others know. This contrast also highlights another way of thinking about the power of strong ties and tightly-knit communities for encouraging participation in high-risk activities, a topic that we discussed in Section 19.4. Weak ties have informational advantages since your strong ties are to people who know things that heavily overlap with what you know. But for collective action, such overlaps in knowledge can be precisely what is needed.

This model for common knowledge and coordination has been developed in further research [110]; understanding the precise interaction of knowledge with collective action remains an interesting direction of study.

Common Knowledge and Social Institutions. Building on these models, Chwe and others have argued that a broad range of social institutions in fact serve the role of helping people achieve common knowledge [111]. A widely publicized speech, or an article in a high-circulation newspaper, has the effect not just of transmitting a message, but of making the listeners or readers realize that many others have gotten the message as well.

This is a useful context for thinking about freedom of the press and freedom of assembly, and their relationship to open societies. But institutions relatively far from the political sphere can also have strong roles as generators of common knowledge. For example, Chwe argues that Super Bowl commercials are often used to advertise products where there are strong network effects – things like cell-phone plans and other goods where it's in your interest to be one of a large population of adopters [111]. For example, the Apple Macintosh was introduced during the 1984 Super Bowl in a commercial directed by Ridley Scott. (Years later, it was declared the "Greatest Television Commercial of All Time" by both *TV Guide* and *Advertising Age* magazine.) As Chwe writes of the event, "The Macintosh was completely incompatible with existing personal computers: Macintosh users could easily exchange data only with other Macintosh users, and if few people bought the Macintosh, there would be little available software. Thus, a potential buyer would be more likely to buy if others bought them also; the group of potential Macintosh buyers faced a coordination problem. By airing the commercial during the Super Bowl, Apple did not simply inform each viewer about the Macintosh; Apple also told each viewer that many other viewers were informed about the Macintosh" [111].

Recently, David Patel used principles of common knowledge to argue that differences between the organization of Sunni and Shiite religious institutions can help explain much about the power dynamics that followed the 2003 U.S. invasion of Iraq [339]. In particular, strong organizational structures enabled Friday sermons at Shiite mosques to be centrally coordinated, while the Sunni religious establishment lacked comparable structures: "Shiite Ayatollahs, controlling hierarchical networks of clerical deputies, can reliably and consistently disseminate similar messages in different mosques, generating common knowledge and coordination across dispersed Shiite congregations on national-level issues like federalism and voting strategies. Through mosque networks, Shiites reliably know what Shiites in far distant areas know" [339]. Patel thus argues that these mechanisms for facilitating shared knowledge enabled Shiites to achieve coordination on goals at a national scale, in a way that other groups in post-invasion Iraq lacked the institutional power to do.

Through all of this, we're seeing that social networks don't simply allow for interaction and the flow of information, but that these processes in turn allow individuals to base decisions on what others know, and on how they expect others to behave as a result. The potential of this framework for studying social processes and social institutions is still being actively explored.

19.7 Advanced Material: The Cascade Capacity

If we go back to the basic model of this chapter, in which nodes choose between behaviors A and B based on thresholds derived from a networked coordination game, an interesting perspective is to understand how different network structures are more or less hospitable to cascades. A first version of this perspective is the analysis in

Figure 19.15. An infinite path with a set of early adopters of behavior A (shaded).

Section 19.3, where we showed that clusters in the network structure form the natural obstacles to cascades. Here we take a different approach; given a network, we ask: what is the largest threshold at which any "small" set of initial adopters can cause a complete cascade? This maximum threshold is thus an inherent property of the network, indicating the outer limit on its ability to support cascades; we will refer to it as the *cascade capacity* of the network.

To make this idea work at a technical level, we clearly need to be careful about what we mean by a "small" set. For example, clearly if we take the set of initial adopters to be the full set of nodes, or (in most cases) something that is almost the full set of nodes, then we can get cascades even at thresholds approaching or equal to 1.

It turns out that the cleanest way to formalize the question is in fact to consider infinite networks in which each node has a finite number of neighbors. We can then define the cascade capacity as the largest threshold at which a *finite* set of nodes can cause a complete cascade. In this way, "small" will mean finite, in the context of a network where the full node set is infinite.

A. Cascades on Infinite Networks

With this goal in mind, we now describe the model in general. The social network will be modeled as a connected graph on an infinite set of nodes; although the node set is infinite, each individual node is only connected to a finite number of other nodes.

The model of node behavior is the same one that we defined earlier in the chapter. The fact that the node set is infinite doesn't pose any problems, since each node only has a finite set of neighbors, and it only makes decisions based on the behavior of these neighbors. To be concrete, initially, a finite set S of nodes has behavior A (this is the small set of early adopters), and all other nodes adopt B. Time then runs forward in steps $t = 1, 2, 3, \ldots$. In each step t, each node other than those in S uses the decision rule with threshold q to decide whether to adopt behavior A or B. (As before, we assume that the nodes in S are committed to A and they never reevaluate this decision.) Finally, we say that the set S *causes a complete cascade* if, starting from S as the early adopters of A, every node in the network eventually switches permanently to A. (Given the fact that the node set is infinite, we must be careful to be clear on what this means: for every node v, there is some time t after which v is always using behavior A.)

The Cascade Capacity. The key definition is now the following. We say that the *cascade capacity* of the network is the largest value of the threshold q for which some finite set of early adopters can cause a complete cascade. To illustrate this definition, let's consider two simple examples. First, in Figure 19.15, we have a network consisting of a path that extends infinitely in both directions. Suppose that the two shaded nodes are early adopters of A, and that all other nodes start out adopting B. What will happen?

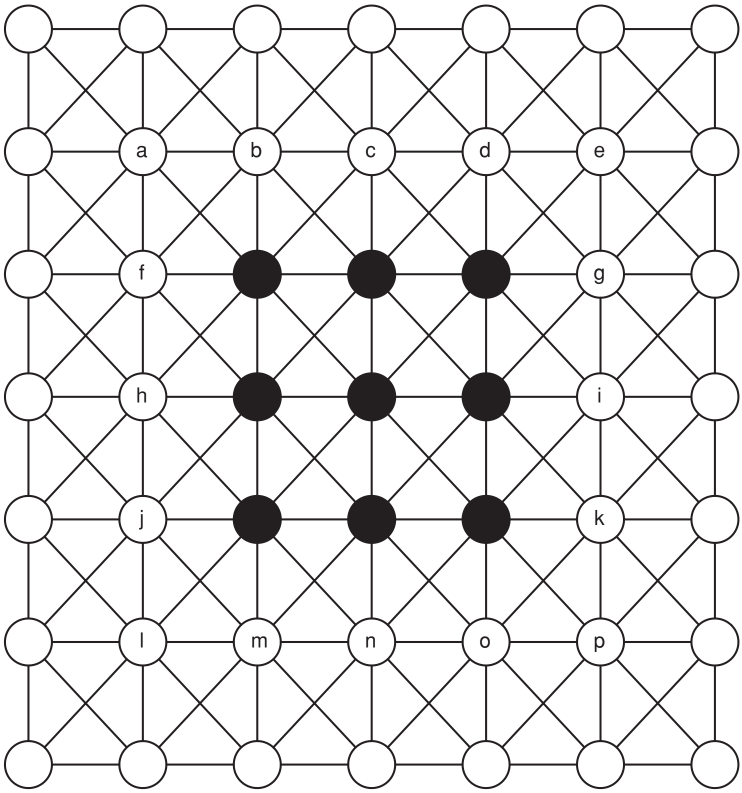


Figure 19.16. An infinite grid with a set of early adopters of behavior A (shaded).

It's not hard to check that if $q \leq \frac{1}{2}$, then nodes u and v will switch to A, after which nodes w and x will switch, and the switches will simply propagate all the way down the path: for each node, there will come some time at which it chooses to switch permanently to A. So the cascade capacity of the infinite path is at least $\frac{1}{2}$, since we have just seen a finite set of initial adopters that causes a complete cascade at threshold $\frac{1}{2}$. In fact, $\frac{1}{2}$ is the exact value of the cascade capacity of the infinite path: with $q > \frac{1}{2}$, *no* finite set of initial adopters can get any node to their right to switch to A, and so A clearly cannot spread to all nodes.

Figure 19.16 shows a second simple example: a network consisting of an infinite grid in which each node is connected to its eight nearest neighbors. Suppose that the nine shaded nodes are early adopters of A and that all other nodes start out adopting B. You can check that if the threshold q is at most $\frac{3}{8}$, then behavior A gradually pushes its way out to the neighbors of the shaded nodes: first to the nodes labeled c, h, i, and n; then to the nodes b, d, f, g, j, k, m, and o; and then to other nodes from there, until every node in the grid is eventually converted to A. (With a smaller threshold – when $q \leq \frac{2}{8}$, for example – behavior A spreads even faster.) We can check that in fact $\frac{3}{8}$ is the cascade capacity of the infinite grid: given any finite set of initial adopters, they are contained in some rectangle of the grid, and if $q > \frac{3}{8}$, no node outside this rectangle will ever choose to adopt A.

Note that the cascade capacity is an intrinsic property of the network itself. A network with a large cascade capacity is one in which cascades happen more "easily"; in other words, they happen even for behaviors A that don't offer much payoff advantage over the default behavior B. As we discussed in Section 19.2, the fact that a small set of initial adopters can eventually cause the whole population to switch illustrates how a better technology (A, when $q < \frac{1}{2}$) can displace an existing, inferior one (B). Viewed in this sense, the example of the grid in Figure 19.16 can be viewed as a kind of failure of social optimality. The fact that the cascade capacity of the grid is $\frac{3}{8}$ means that, when q is strictly between $\frac{3}{8}$ and $\frac{1}{2}$, A is the better technology, but the structure of the network makes B so heavily entrenched that no finite set of initial adopters of A can cause A to win.

We now consider the following fundamental question: how large can a network's cascade capacity be? The infinite path shows that there are networks in which the cascade capacity can be as large as $\frac{1}{2}$: a new behavior A can displace an existing behavior B even when the two confer essentially equivalent benefits (with A having only the "tie-breaking" advantage such that when a node has an equal number of neighbors using A and B, it chooses A). Does there exist any network with a higher cascade capacity? This would be a bit surprising, since such a network would have the property that an inferior technology can displace a superior one, even when the inferior technology starts at only a small set of initial adopters.

In fact, we will show that no network has a cascade capacity larger than $\frac{1}{2}$. In other words, regardless of the structure of the underlying network, if a new behavior requires 51% of someone's friends to adopt it before they do, then it can't spread very far through the population. Despite the fact that this is perhaps an intuitively natural fact, proving it is a bit subtle; it requires a way to bound the extent of a behavior that is spreading at a threshold beyond $\frac{1}{2}$.

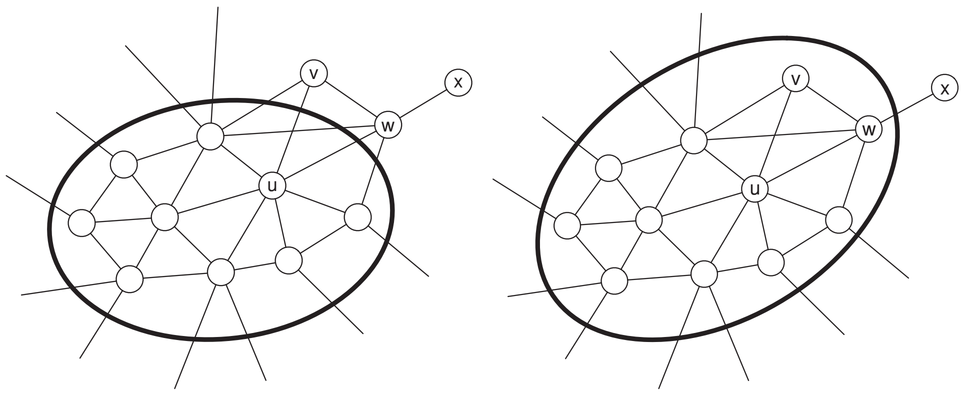
B. How Large Can the Cascade Capacity Be?

We now formulate and prove this basic fact about the cascade capacity.

> *Claim:* There is no network in which the cascade capacity exceeds $\frac{1}{2}$.

Although we motivated this claim as a natural one just above, it is less clear why it is true. After all, it's certainly imaginable a priori that there could be some cleverly constructed network, set up in just the right way, so that even though each node needs 51% of its neighbors to adopt before it does, the cascade rolls on steadily, eventually causing everyone to switch. What we really need to show is the following: if $q > \frac{1}{2}$, then regardless of what the underlying network looks like, a new behavior starting at a finite set of nodes will not spread to every other node.

Analyzing the Interface. We're going to approach this question by tracking the "interface" where adopters of A are linked to adopters of B. At a very high level, we're going to show that, as the process runs, this interface becomes narrower and narrower, eventually shrinking to the point where the process must stop, having failed to reach all nodes.



(a) (b)

Figure 19.17. Let the nodes inside the dark oval be the adopters of A. Part (a) shows the situation before one step of the process, during which nodes v and w adopt A. After they adopt, as shown in (b), the size of the interface has strictly decreased. In general, the size of the interface strictly decreases with each step of the process when $q > \frac{1}{2}$.

More precisely, suppose the behavior A spreads from a finite initial set S with threshold $q > \frac{1}{2}$. As time moves forward in steps $t = 1, 2, 3, \ldots$, potentially larger and larger sets become adopters of A. At any given point in time, each edge in the network can be described as an A-A edge (connecting two adopters of A), a B-B edge (connecting two adopters of B), or an A-B edge (connecting an adopter of A to an adopter of B). We define the *interface* to be the set of A-B edges. Figure 19.17 shows a useful way to picture the interface: if the set of adopters of A consists of the nodes inside the dark oval, then the edges in the interface are the ones that cross the oval.

What we're going to show is that, in each step, the size of the interface i.e., the number of edges it contains – must strictly decrease. This will be enough to show what we need, for the following reason. The size of the interface clearly starts at some number I_0: since there is some finite set of initial adopters S, and since each of these has a finite set of neighbors, the set of A-B edges is finite and has some size I_0. The size of the interface is always a nonnegative whole number, so if it strictly decreases in each step, the spread of A can run for at most I_0 steps before terminating. Since each step only results in a finite number of nodes converting to A, the process terminates with only a finite set of nodes having adopted A. (So in fact we'll get something stronger than we needed: not only does A not spread everywhere, it only reaches a finite set starting from S.)

The Size of the Interface Decreases in Each Step. So the crux of this is to consider one step of the process and show that the size of the interface strictly decreases. What happens in one step of the process? Figure 19.17 illustrates a way to think about this question. Certain nodes that are currently adopters of B discover, for the first time, that at least a q fraction of their neighbors are now adopters of A, and so they too switch to A.

This causes the interface to change in the following way. When a node w switches from B to A, its edges to nodes that remain with B change from being B-B edges to being A-B edges – so this causes them to join the interface. (An example is the edge linking w and x in Figure 19.17.) On the other hand, the edges from w to nodes that were already with A change from being A-B edges to being A-A edges; in other words, they leave the interface. (See, for example, the edge linking u and w.) Each edge that joins or leaves the interface in this step can be accounted for in this way by exactly one node that switches from B to A.

So to analyze the change in the size of the interface, we can separately consider the contribution from those edges accounted for by each individual node that switches. Thus, consider a node w that switches; suppose that, before the switch, it had a edges to nodes that were already adopters of A, and b edges to nodes that will remain adopters of B at the end of the step. So node w accounts for b edges joining the interface and a edges leaving it. But since $q > \frac{1}{2}$, and node w decided to switch to A in this step, it must be that w had more edges to adopters of A than to adopters of B; therefore $a > b$, and hence w accounts for more edges leaving the interface than edges joining the interface. But this is true for each node that switches in this step, and so the overall size of the interface goes down.

This is what we needed to show. Chaining back through the earlier arguments, since the interface starts at some fixed size I_0, the process can only go for at most I_0 steps before running out of steam and stopping, not having reached all nodes.

Some Final Thoughts. We've shown that when $q > \frac{1}{2}$, no finite set of nodes can cause a complete cascade, in any network. In terms of an underlying story about users choosing between technologies A and B, the situation in which $q > \frac{1}{2}$ corresponds intuitively to the case in which the new technology A is in fact worse: the payoff from an A-A interaction is lower than that of a B-B interaction, and so you only switch to A in cases where more than half your friends already have. So at least in the simple model we've been studying here, a worse technology will not displace a better technology that's already in widespread use. (However, recall the connection with our earlier discussion of network effects: in networks where the cascade capacity is strictly less than $\frac{1}{2}$, it is possible for a better technology to be unable to displace a worse one that is already in widespread use.)

It is also interesting to reflect a bit on the way in which we argued that A can't spread to all nodes when $q > \frac{1}{2}$; there's a methodological parallel here to our discussion of matching markets (though again the details are completely different). There too we had a process – the bipartite auction procedure that updated prices – and we wanted to show that it must come to a halt. Lacking any obvious measure of progress on the process, we invented a nonobvious one – a kind of "potential energy" that steadily drained out of the process as it ran, eventually forcing it to terminate. In retrospect, we used a very similar strategy here, with the size of the interface serving as the potential energy function that steadily decreases until the process has to stop.

C. Compatibility and Its Role in Cascades

We've gotten a lot of mileage in this chapter from taking a game that is fundamentally very simple – a coordination game with two possible strategies – and analyzing how it is played across the edges of a potentially complex network. There are many directions in which the game can be extended and generalized, and most of these lead quickly to current research and open questions. To illustrate how even small extensions to the underlying game can introduce new sources of subtlety, we discuss here an extension that takes into account the notion that a single individual can sometimes choose a combination of two available behaviors [225].

To illustrate what we mean, let's go back to the extended example we considered in Figure 19.5, and the discussion at the end of Section 19.2 of how behaviors A and B ended up coexisting in the network. Coexistence is a common outcome, and it is interesting to ask what things look like along the boundaries between A and B. For example, A and B could be different languages coexisting along a national border, or A and B could be social networking sites that appeal respectively to students in college and to students in high school. Our current model says that anyone positioned along the interface between A and B in the network – for example, nodes 8–14 in Figure 19.5 – will receive positive payoffs from neighbors who adopt the same behavior, but payoffs of zero from their interactions with neighbors who adopt different behaviors.

Experience suggests that when people are actually faced with such situations, they often choose an option that corresponds to neither A nor B – rather, they become *bilingual*, adopting both A and B. In some cases, bilinguality is meant literally: for example, someone who lives near speakers of both French and German is reasonably likely to speak (some amount of) both. But technological versions of bilinguality abound as well: people with friends on two incompatible Instant Messenger systems, or two different social networking sites, will likely have accounts on both; people whose work requires dealing with two different computer operating systems will likely have a way to run both. The common feature of all these examples is that an individual chooses to use some form of both available behaviors, trading off the greater ease of interaction with people of multiple types against the cost of having to acquire and maintain both forms of behavior (i.e., the costs of having to learn an additional language, maintain two different versions of a technology, and so forth). What effect does this bilingual option have on the spread of a behavior through a network?

Modeling the Bilingual Option. In fact, it is not hard to set up a model that captures the possibility that a node will choose to be bilingual. On each edge, connecting two nodes v and w, we still imagine a game being played, but now there are three available strategies: A, B, and AB. The strategies A and B are the same as before, whereas the strategy AB represents a decision to adopt both behaviors. The payoffs follow naturally from the intuition discussed earlier: the nodes can interact with each other using any behavior that is available to both of them. If they interact using A, they each get a payoff of a, while if they interact using B, they each get a payoff of b. In other words, two bilingual nodes can interact using the better of the two behaviors; a bilingual node and a monolingual node can only interact using the monolingual node's behavior; and two monolingual nodes can only interact at all if they have the same behavior. Written as a payoff matrix, the game is shown in Figure 19.18, where we use the notation $(a, b)^+$ to denote the larger of a and b.

It's easy to see that AB is a dominant strategy in this game: why not be bilingual when it gives you the best of both worlds? However, to model the trade-off discussed earlier, we need to also incorporate the notion that bilinguality comes with a cost.

		w		
		A	B	AB
	A	a, a	$0, 0$	a, a
v	B	$0, 0$	b, b	b, b
	AB	a, a	b, b	$(a, b)^+, (a, b)^+$

Figure 19.18. A coordination game with a bilingual option. Here the notation $(a, b)^+$ denotes the larger of a and b.

The meaning of the cost varies with the context, but the cost in general corresponds to the additional effort and resource expenditure needed to maintain two different behaviors. Thus, we assume that each node v will play a copy of this three-strategy bilingual coordination game with each of its neighbors; as in our models earlier in the chapter, v must use the same strategy in each copy of the game it plays. Its payoff will be equal to the sum of its payoffs in its game with each neighbor, minus a single cost of c if v chooses to play the strategy AB. It is this cost that creates incentives not to play AB, balancing the incentives that exist in the payoff matrix to play it.

The remainder of the model works as before. We assume that every node in an infinite network starts with the default behavior B, and then (for nonstrategic reasons) a finite set S of initial adopters begins using A. We now run time forward in steps $t = 1, 2, 3, \ldots$; in each of these steps, each node outside S chooses the strategy that will provide it the highest payoff, given what its neighbors were doing in the previous step. We are interested in how nodes choose strategies as time progresses, and particularly which nodes eventually decide to switch permanently from B to A or AB.

An Example. To get some practice with the model, let's try it on the infinite path shown in Figure 19.19. Let's suppose that nodes r and s are the initial adopters of A, and that the payoffs are defined by the quantities $a = 2$, $b = 3$, and $c = 1$.

Here is how nodes behave as time progresses. In the first time step, the only interesting decisions are the ones faced by nodes u and v, since all other nodes are either initial adopters (who are hard-wired to play A) or nodes that have all neighbors using B. The decisions faced by u and v are symmetric; for each of them, we can check that the strategy AB provides the highest payoff. (It yields a payoff of $2 + 3 - 1 = 4$ from being able to interact with both neighbors, but having to pay a cost of 1 to be bilingual.) In the second time step, nodes w and x have a fresh decision to make, since they now have neighbors using AB, but we can check that B still yields the highest payoff for each of them. From here on, no node will change its behavior in any future time steps. So with these payoffs, the new behavior A does not spread very far: the decision by the initial adopters to use A caused their neighbors to become bilingual, but after that further progress stopped.

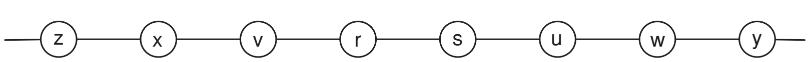
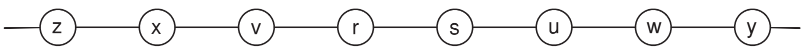


Figure 19.19. An infinite path, with nodes r and s as initial adopters of A.



Start	B	B	B	A	A	B	B	B
Step 1	B	B	AB	A	A	AB	B	B
Step 2	B	AB	AB	A	A	AB	AB	B
Step 3	AB	AB	A	A	A	A	AB	AB
Step 4	AB	A	A	A	A	A	A	AB

Figure 19.20. With payoffs $a = 5$ and $b = 3$ for interaction using A and B, respectively, and a cost $c = 1$ for being bilingual, the strategy A spreads outward from the initial adopters r and s through a two-phase structure. First, strategy AB spreads, and then behind it, nodes switch permanently from AB to A.

We can further experiment with this example by keeping the network the same but changing the payoffs so that A becomes much more desirable: specifically, let's set $a = 5$, and keep $b = 3$ and $c = 1$. What happens in this case is more complex, as depicted in Figure 19.20. (For the discussion that follows, we will only talk about what happens to the right of the initial adopters, since what's going on to the left is symmetric.)

- In the first step, node u will switch to AB, since it receives a payoff of $5 + 3 - 1 = 7$ from doing so. As a result, in the second step, node w also switches to AB.
- From the third step onward, strategy AB continues to move to the right, one node at a time. However, something additional happens starting in the third step. Because node w switched to AB in the second step, node u faces a new decision: it has one neighbor using A and the other using AB, and so now u's best choice is to switch from AB to A. Essentially, there's no point in being bilingual anymore if all your neighbors now have the higher-payoff behavior available to them (A in this case).
- In the fourth step, node w also switches from AB to A and, more generally, strategy A moves to the right, two steps behind strategy AB. No other changes in strategy happen, so each node switches first to AB (as the wave of bilinguality passes through it), and then permanently switches to A (the higher-payoff monolingual option) two steps later.

Here is one way to view what is happening in this version of the example: as AB spreads through the nodes, B becomes *vestigial* – there is no longer any point for a node to use it. Thus, nodes abandon B completely over time, and so in the long run only A persists.

A Two-Dimensional Version of the Cascade Capacity. In the basic model earlier in this chapter, with the underlying coordination game based just on strategies A and B, we formulated the following question. We are given an infinite graph; for which payoff values a and b is it possible for a finite set of nodes to cause a complete cascade of adoptions of A? Phrased this way, the question appears to depend on two numbers (a and b), but we saw earlier that in fact it depends only on the single number $q = b/(a + b)$.

We can ask the analogous question for our model that includes the strategy AB: given an infinite graph, for which payoff values a, b, and c is it possible for a finite set of nodes to cause a complete cascade of adoptions of A? As with our earlier question, we can eliminate one of the numbers from this question quite easily. The easiest way to do this is to note that the answer to our question remains the same if we were to multiply each of a, b, and c by the same fixed factor. (For example, it does not matter if we multiply each of a, b, and c by 100 and measure the payoffs in cents instead of dollars.) Therefore, we can assume that $b = 1$, fixing this as our basic "unit of currency," and ask how the possibility of a cascade depends on A and C. Choosing b as the number that we fix equal to 1 makes some intuitive sense, since it is the payoff from using the default behavior B; in this way, we're essentially asking: how much better does the new behavior A have to be (the payoff a) and how compatible should it be with B (the payoff c) in order for a cascade to have a possibility of forming?

This question has recently been studied for graphs in general [225], and an interesting qualitative conclusion arises from the model: strategy A does better when it has a higher payoff (this is natural), but in general it has a particularly hard time cascading when the level of compatibility is "intermediate" – when the value of c is neither too high nor too low. Rather than describing the general analysis of this phenomenon, we show how it happens on the infinite path, where the analysis is much simpler and where the main effects are already apparent. We then discuss some possible interpretations of this effect.

When Do Cascades Happen on an Infinite Path? The infinite path is an extremely simple graph, and we saw earlier in this section that in the model with only the strategies A and B, the condition for A to cascade is correspondingly very simple: a cascade of A's can occur precisely when the threshold q is at most $\frac{1}{2}$ – or, equivalently, when $a \geq b$. In other words, a better technology will always spread on the path.

Once we add strategy AB as an option, however, the situation becomes more subtle. Because we are only concerned with whether some finite set of initial adopters can cause a complete cascade of A's, we can assume that this set of initial adopters forms a contiguous interval of nodes on the path. (If not, we can take the left- and rightmost initial adopters and study the situation in which every node in between is also an initial adopter – this set is still finite, and it has just as good a chance of causing a complete cascade.) So changes in nodes' strategies will spread outward symmetrically to the left and right of the initial adopters, and we simply need to account for the possible decisions that nodes make in evaluating their strategies as this happens. Because of the symmetry, we will only think about how strategy changes occur to the right of the initial adopters, since what is going on to the left is the same.

There are two kinds of node-level decisions that are particularly useful for our analysis.

- First, we have to think about nodes like w in Figure 19.21, with a left neighbor using A and a right neighbor using B. (For example, this happens in the first step of

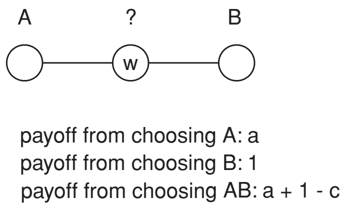
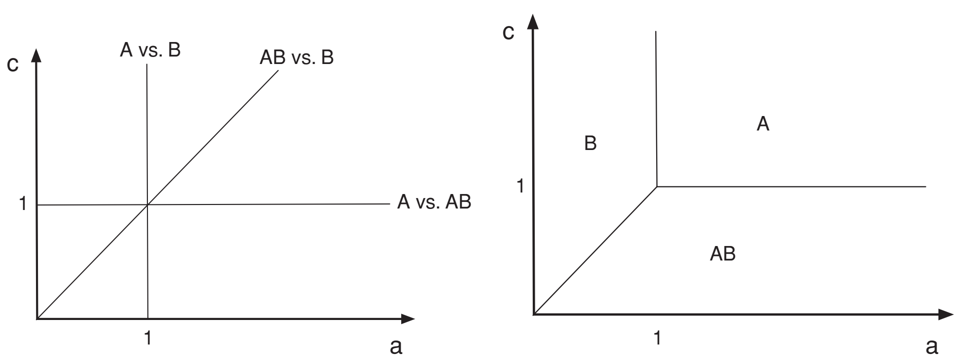


Figure 19.21. The payoffs to a node on the infinite path with two neighbors using A and B.

the cascade with the node immediately to the right of the initial adopters.) In this situation, node w receives a payoff of a from choosing A (because it can interact with its left neighbor), a payoff of 1 from choosing B (because it can interact with its right neighbor), and a payoff of $a + 1 - c$ from choosing AB (because it can interact with both neighbors, but pays a cost of c to be bilingual).

Node w will choose the strategy that provides the highest payoff, and that's determined by the relationship between a and c. In other words, we should be asking the following: for which values of a and c will node w choose A, for which will it choose B, and for which will it choose AB? This question can be answered easily if we plot the comparisons among the payoffs in the (a, c)-plane as shown in Figure 19.22(a), with the value of a on the x-axis and the value of c on the y-axis. The break-even point between strategies AB and B, for example, is given by the line defined by setting the two payoffs equal: $a + 1 - c = 1$, or equivalently $a - c = 0$. This is the diagonal line in the figure. Similarly, we draw lines for the break-even point between strategies A and B ($a = 1$) and between strategies A and AB ($a = a + 1 - c$, or equivalently $c = 1$).

These three lines all meet at the point (1, 1), and so we see that they divide the (a, c)-plane into six regions. As shown in Figure 19.22(b), A is the best strategy



(a) (b)

Figure 19.22. Given a node with neighbors using A and B, the values of a and c determine which of the strategies A, B, or AB it will choose. (Here, by rescaling, we can assume $b = 1$.) We can represent the choice of strategy as a function of a and c by dividing up the (a, c)-plane into regions corresponding to different choices. (a) Lines showing break-even points between strategies and (b) regions defining the best choice of strategy.

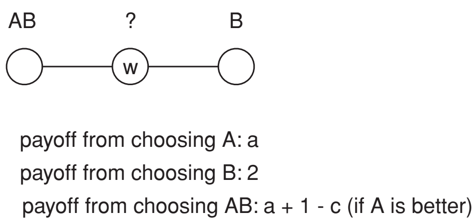


Figure 19.23. The payoffs to a node on the infinite path with two neighbors using AB and B.

in two of these regions, B is the best strategy in two of them, and AB is the best strategy in two of them.

- If AB begins to spread, then we'll also have to think about the situation pictured in Figure 19.23: a node whose left-hand neighbor is using AB and whose right-hand neighbor is using B.

  Now, if $a < 1$, then B provides node w with the highest payoff regardless of the value of the cost c (as long as it is positive). So let's consider the more interesting alternative, when $a \geq 1$. This is very similar to the previous case, when w's left-hand neighbor was using A; the one change is that the payoff to w for using B has now gone up to 2, because now w can use B to interact with both neighbors rather than just one.

  As a result, the lines in the (a, c)-plane defining the break-even points between B and the other strategies shift to the right (they are now $a = 2$ and $a + 1 - c = 2$). This in turn shifts the three regions of the (a, c)-plane that define which strategy will be chosen by w, as shown in Figure 19.24.

We are now in a position to determine the values of a and c for which a cascade of A's can occur. We start with a contiguous interval of initial adopters of A, and we consider the node u immediately to the right of the initial adopters. (Again, everything here also applies to the left of the initial adopters by symmetry.)

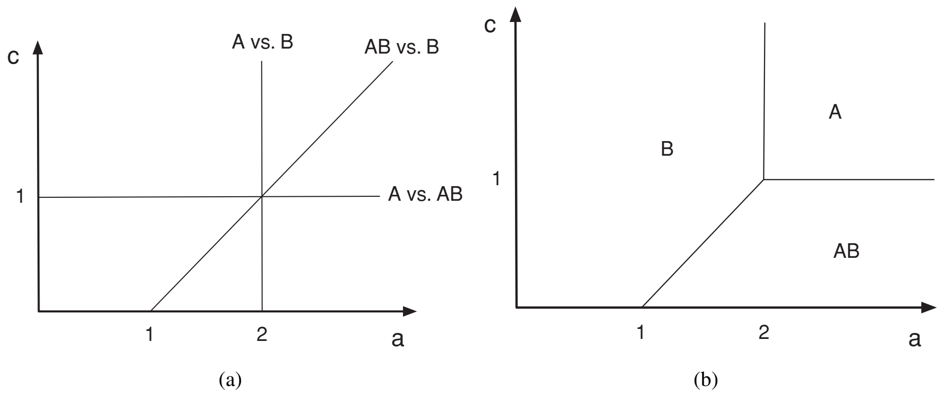


Figure 19.24. Given a node with neighbors using AB and B, the values of a and c determine which of the strategies A, B, or AB it will choose, as shown by this division of the (a, c)-plane into regions. (a) Lines showing break-even points between strategies and (b) regions defining the best choice of strategy.

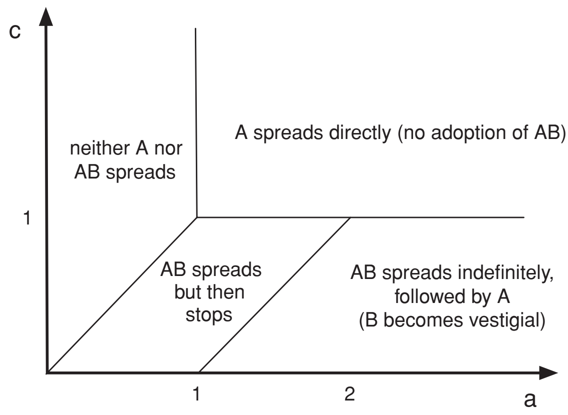


Figure 19.25. There are four possible outcomes for how A spreads or fails to spread on the infinite path, indicated by this division of the (a, c)-plane into four regions.

- If we are in the B region of Figure 19.22(b), then node u will favor B as its strategy, so it will stick with this and the new strategy A will not spread at all.
- If we are in the A region of Figure 19.22(b), then node u will favor A as its strategy, and it will switch to A. So in the next time step we have exactly the same situation shifted one node to the right, and as a result the new strategy A spreads all the way down the path: a cascade occurs.
- Most interestingly, suppose we are in the AB region of Figure 19.22(b). Then, in the next time step, the situation will look different: the crucial decision will now be faced by the next node w to the right of u, who will have its left-hand neighbor (u) now using AB, and its right-hand neighbor still using B.

  To understand what w will do, based on values of a and c, we consult the regions in Figure 19.24(b). But crucially, since we know that AB was the best choice in the first step, we know that the values of a and c lie in the AB region from Figure 19.22(b) – so when we consider Figure 19.24(b), we are concerned not with how its regions carve up the full (a, c)-plane, but only how they carve up the AB region from Figure 19.22(b).

  In fact, they divide the AB region from Figure 19.22(b) by a diagonal line segment from the point $(1, 0)$ to the point $(2, 1)$, as shown in Figure 19.25. To the left of this line segment, B wins and the cascade stops. To the left of this line segment, AB wins – so AB continues spreading to the right, and behind this wave of ABs, nodes will steadily drop B and use only A. This is the scenario that we saw in our example, where B fails to persist because it becomes vestigial in a bilingual world.

Figure 19.25 in fact summarizes the four possible cascade outcomes, based on the values of a and c [i.e., where they lie in the (a, c)-plane]: (i) B is favored by all nodes outside the initial adopter set, (ii) A spreads directly without help from AB, (iii) AB spreads for one step beyond the initial adopter set, but then B is favored by all nodes after that, or (iv) AB spreads indefinitely to the right, with nodes subsequently switching to A.

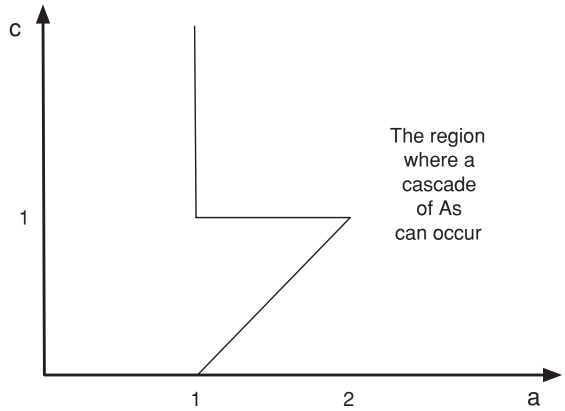


Figure 19.26. The set of values for which a cascade of As can occur defines a region in the (a, c)-plane consisting of a vertical line with a triangular cutout.

So a cascade of As can occur if the pair of values (a, c) lies in one of the two regions described by outcomes (ii) and (iv). This means that the portion of the (a, c)-plane where a cascade can occur looks as depicted in Figure 19.26: it lies to the right of a vertical line with a strange triangular "cutout." The vertical line makes a lot of sense: it corresponds to $a \geq 1$, or, in other words, the requirement that interaction using A produces a higher payoff than interaction using B. But what does the triangular cutout mean? Formally, it says that when the cost of bilinguality is neither too high nor too low, the new strategy A has to be "extra good" – it must produce a payoff a significantly higher than 1 – in order to spread. Moreover, although we won't consider more complex graphs here, the region of the (a, c)-plane where a cascade of As can occur in any graph turns out to have some kind of indentation analogous to the triangular cutout, though the particular boundary of the indentation depends on the structure of the graph [225].

This triangular cutout region has a natural qualitative interpretation that provides potential insight into how compatibility and bilinguality affect the process of diffusion in a network. We discuss this interpretation now.

Interpretations of the Cascade Region. One way to appreciate what's going on in the triangular cutout region is to consider the following question, phrased in terms of technology adoption. Suppose that you're the firm manufacturing the default technology B, and the payoff from interacting via B is equal to 1. Now a new technology A with payoff $a = 1.5$ begins to appear. For which values of the bilinguality cost c should you expect B to survive?

Even without performing any concrete calculations, you could reason as follows. If it's extremely easy to maintain both technologies simultaneously, then adoption of AB will become widespread, and once it is sufficiently widespread, people will begin dropping B altogether, since A is better and it's possible to interact with everyone using A. Essentially, A will have won through "infiltration," working its way into the population via coexistence with B. On the other hand, if it's extremely hard to maintain both technologies simultaneously, then people on the boundary between the two user populations – those who have friends using both technologies – will have to simply choose one or the other. And in this case, you could expect that they may well choose A, because it's in fact better. In this case, A will win through a kind of "direct conquest," simply eliminating B as it goes.

But in between – when it's neither extremely easy nor extremely hard to maintain both technologies – something more favorable to B can happen. Specifically, a bilingual "buffer zone" may form between people who adopt only A and those who adopt only B. On the B side of this buffer zone, no one will have an incentive to change what they're doing, because by using B they can interact with all their neighbors – the bilingual ones and the ones using only B – rather than interacting with only a fraction of their neighbors by switching to the marginally better technology, A. In other words, the inferior technology B has survived because it was neither too compatible nor too incompatible with A – rather, by partially accommodating A, it prevented A from spreading too far.[2]

One can tell this story about nontechnological settings as well. For example, discourse in a succession of geographically adjacent towns may switch from a traditional language B to a more global language A that confers benefits beyond the immediate community – or it may end up with bilingual inhabitants who use both. In a related vein, one could even consider how a more traditional set of cultural practices (B) may persist in the face of more modern ones (A), depending on how easy it is for a person to observe both.

Of course, the model we are discussing is extremely simple, and the full story in any of these scenarios would include many additional factors. For example, in studying competition between technology firms, there has been a long line of work on the role that compatibility and incompatibility can play [143, 235, 415], including case studies of technologies including instant messaging [158] and electronic imaging [283]. But, as with many of our earlier analyses, the streamlined nature of the model helps provide insight into principles that have reflections in more complex settings as well. In this particular case, the model also shows how detailed network structure can play a role in a setting that has otherwise been analyzed primarily at the population level, treating individuals as interacting in aggregate.

Finally, the discussion shows how the basic diffusion model – based on a simple coordination game – is amenable to extensions that capture additional features of real situations where diffusion can take place. Even small extensions such as the one considered here can introduce significant new sources of complexity, and the development of even richer extensions is an open area of research.

[2] On the infinite path, the bilingual buffer zones that form are very simple – just one node thick. But in general graphs, the buffer zones can have a more complex structure. In fact, it is possible to prove an analog of the result from Section 19.3, where we showed that clusters are the only obstacle to cascades in the two-strategy model. The more general result is that, with an additional bilingual option AB, a structure consisting of a cluster and a bilingual buffer zone accompanying it is the only obstacle to a cascade of As [225].

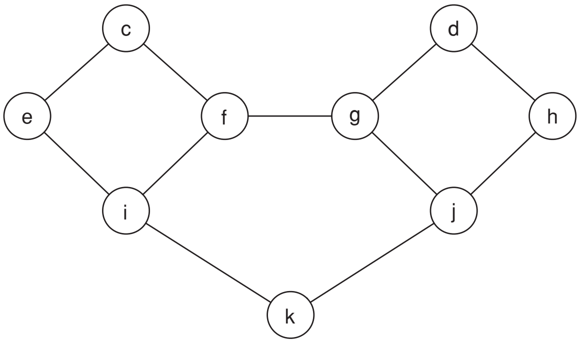


Figure 19.27. Starting from nodes e and f, the new behavior A fails to spread to the entire graph.

19.8 Exercises

1. Consider the network depicted in Figure 19.27; suppose that each node starts with the behavior B, and each node has a threshold of $q = \frac{1}{2}$ for switching to behavior A.

(a) Now, let e and f form a two-node set S of initial adopters of behavior A. If other nodes follow the threshold rule for choosing behaviors, which nodes will eventually switch to A?

(b) Find a cluster of density greater than $1 - q = \frac{1}{2}$ in the part of the graph outside S that blocks behavior A from spreading to all nodes, starting from S, at threshold q.

2. Consider the model from Chapter 19 for the spread of a new behavior through a social network. Suppose we have the social network depicted in Figure 19.28; suppose that each node starts with the behavior B, and each node has a threshold of $q = \frac{2}{5}$ for switching to behavior A.

(a) Now, let c and d form a two-node set S of initial adopters of behavior A. If other nodes follow the threshold rule for choosing behaviors, which nodes will eventually switch to A? Give a brief (one- to two-sentence) explanation for your answer.

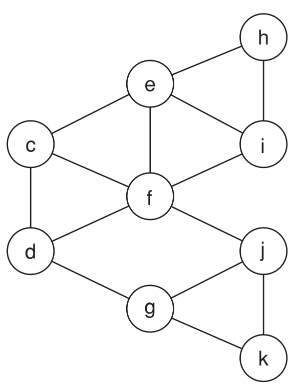


Figure 19.28. Starting from nodes c and d, the new behavior A fails to spread to the entire graph.

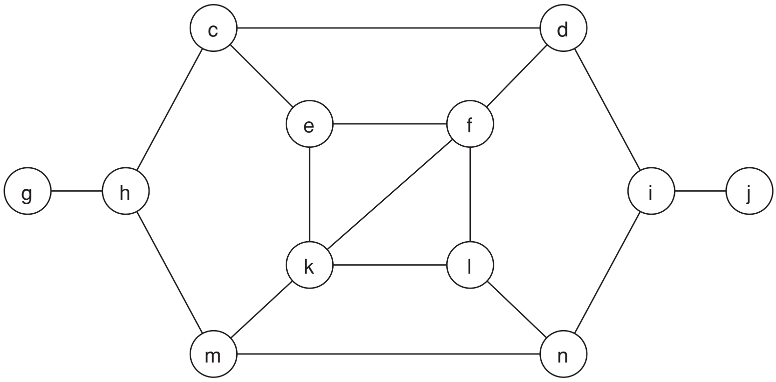


Figure 19.29. A social network in which a new behavior is spreading.

(b) Find a cluster of density greater than $1 - q = \frac{3}{5}$ in the part of the graph outside S that blocks behavior A from spreading to all nodes, starting from S, at threshold q. Give a brief (one- to two-sentence) explanation for your answer.

(c) Suppose you were allowed to add a single edge to the given network, connecting one of nodes c or d to any one node that it is not currently connected to. Could you do this in such a way that now behavior A, starting from S and spreading with a threshold of $\frac{2}{5}$, would reach all nodes? Give a brief explanation for your answer.

3. Consider the model from Chapter 19 for the diffusion of a new behavior through a social network. Recall that for this we have a network, a behavior B that everyone starts with, and a threshold q for switching to a new behavior A; that is, any node will switch to A if at least a fraction q of its neighbors have adopted A.

Consider the network depicted in Figure 19.29; suppose that each node starts with the behavior B, and each node has a threshold of $q = \frac{2}{5}$ for switching to behavior A. Now, let e and f form a two-node set S of initial adopters of behavior A.

(a) If other nodes follow the threshold rule for choosing behaviors, which nodes will eventually switch to A?

(b) Find a cluster of density greater than $1 - q = \frac{3}{5}$ in the part of the graph outside S that blocks behavior A from spreading to all nodes, starting from S, at threshold q.

(c) Suppose you're allowed to add one node to the set S of initial adopters, which currently consists of e and f. Can you do this in such a way that the new three-node set causes a cascade at threshold $q = \frac{2}{5}$?

Provide an explanation for your answer, either by giving the name of a third node that can be added, together with an explanation for what will happen, or by explaining why there is no choice for a third node that will work to cause a cascade.

4. Consider the model from Chapter 19 for the diffusion of a new behavior through a social network.

Suppose that initially everyone is using behavior B in the social network in Figure 19.30, and then a new behavior A is introduced. This behavior has a threshold of $q = \frac{1}{2}$: any node will switch to A if at least $\frac{1}{2}$ of its neighbors are using it.

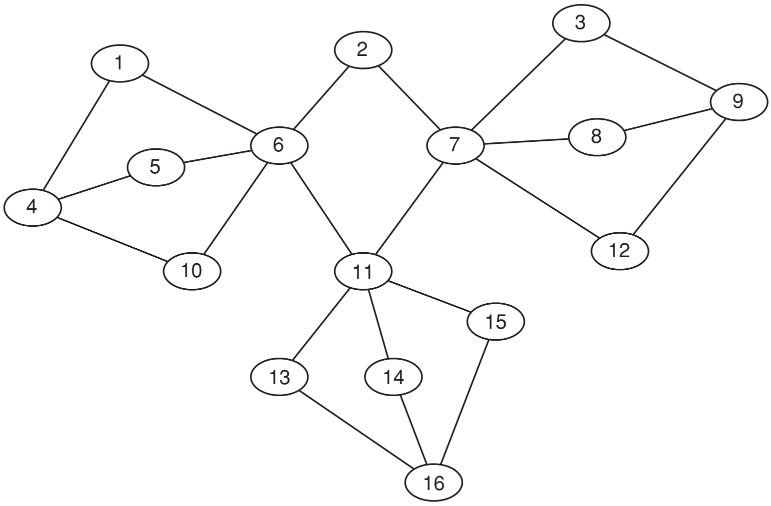


Figure 19.30. A social network through which a new behavior diffuses.

(a) Find a set of three nodes in the network with the property that if they act as the three initial adopters of A then it will spread to all nodes. (In other words, find three nodes that are capable of causing a cascade of adoptions of A.)
(b) Is the set of three nodes you found in part (a) the only set of three initial adopters capable of causing a cascade of A, or can you find a different set of three initial adopters that could also cause a cascade of A?
(c) Find three clusters in the network, each of density greater than $\frac{1}{2}$, with the property that no node belongs to more than one of these clusters.
(d) How does your answer to part (c) help explain why there is no set consisting of only two nodes in the network that would be capable of causing a cascade of adoptions of A (that is, only two nodes that could cause the entire network to adopt A)?

5. Continuing with the diffusion model from Chapter 19, recall that the threshold q was derived from a coordination game that each node plays with each of its neighbors. Specifically, if nodes v and w are each trying to decide whether to choose behaviors A and B, then

- if v and w both adopt behavior A, they each get a payoff of $a > 0$;
- if they both adopt B, they each get a payoff of $b > 0$; and
- if they adopt opposite behaviors, they each get a payoff of 0.

The total payoff for any one node is determined by adding up the payoffs it gets from the coordination game with each neighbor.

Let's now consider a slightly more general version of the model, in which the payoff for choosing opposite behaviors is not 0, but some small positive number x. Specifically, suppose we replace the third point above with the following:

- if they adopt opposite behaviors, they each get a payoff of x, where x is a positive number that is less than both a and B.

Here's the question: in this variant of the model with these more general payoffs, is each node's decision still based on a threshold rule? Specifically, is it possible to write a formula for a threshold q, in terms of the three quantities a, B, and x, so that each node v will adopt behavior A if at least a fraction q of its neighbors are adopting A, and it will adopt B otherwise?

In your answer, either provide a formula for a threshold q in terms of a, B, and x, or explain why, in this more general model, a node's decision can't be expressed as a threshold in this way.

6. A group of twenty students living on the third and fourth floors of a college dorm like to play online games. When a new game appears on campus, each of these students needs to decide whether to join, by registering, creating a player account, and taking a few other steps necessary in order to start playing.

When a student evaluates whether to join a new online game, she bases her decision on how many of her friends in this group are involved in the game as well. (Not all pairs of people in this twenty-person group are friends, and it is more important if your friends are playing than if many people in the group overall are playing.)

To make the story concrete, let's suppose that each game goes through the following "life cycle" within this group of students:

(a) The game has some initial players in the group, who have discovered it and are already involved in it.
(b) Each other student outside this set of initial players is willing to join the game if at least half of her friends in the group are playing it.
(c) Rule (b) is applied repeatedly over time, as in our model from Chapter 19 for the diffusion of a new behavior through a social network.

Suppose that in this group of twenty students, ten live on the third floor of the dorm and ten live on the fourth floor. Suppose that each student in this group has two friends on their own floor, and one friend on the other floor. Now, a new game appears, and five students all living on the fourth floor each begin playing it.

If the other students use the preceding rule to evaluate whether to join the game, will this new game eventually be adopted by all twenty students in the group? There are three possible answers to this question: yes, no, or there is not information in the setup of the question to be able to tell. Say which answer you think is correct, and explain.

7. Some friends of yours have gone to work at a large online game company, and they're hoping to draw on your understanding of networks to help them better understand the user population in one of their games.

Each character in the game chooses a series of *quests* to go on, generally as part of a group of characters who work together on them; there are many options for quests to choose from, but once a character goes on a quest with a group, it can generally last for a couple of weeks.

Your friends working at the game company have also mapped the social network of the game, and they've invented what they find is a useful way of classifying each player's friends: a *reinforced* friend is one with whom the player has at least one other friend in common, and an *unreinforced* friend is one with whom the player has no other friends in common. For example, Figure 19.31 shows the friends of a player A: players B, C, and D would count as reinforced friends, while player E would be an unreinforced friend.

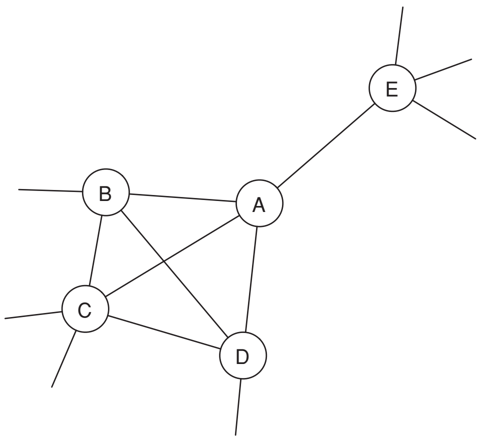


Figure 19.31. A small portion of the social network in an online game.

Now, your friends are particularly interested in what causes players to choose particular quests instead of others, and they are also interested in how players learn about particular methods of cheating along the way – general tricks outside the rules of the game that make it easier to accumulate points, usually regardless of which particular quest they're on. To do some market research on this, they've anonymously surveyed players of the game, asking them two questions:

(a) How did you first learn about the current quest that you're taking part in?
(b) How have you learned about ways of cheating in the game?

To their surprise, the answers to these questions were quite different. For question (a), 80% of respondents said that they first found out about the current quest they're on from a reinforced friend, whereas for question (b), 60% of respondents said that they found out about ways of cheating from an unreinforced friend.

Your friends thought you might be able to shed some light on these findings. Why did the answers to these two questions turn out differently? Is the difference specific to this particular game, or could it be predicted from general principles of social networks? In one to two paragraphs, describe how particular ideas from the book can shed light on why the answers to these questions turned out the way they did.