

Modern Experimental Design



THE WILEY BICENTENNIAL—KNOWLEDGE FOR GENERATIONS

Each generation has its unique needs and aspirations. When Charles Wiley first opened his small printing shop in lower Manhattan in 1807, it was a generation of boundless potential searching for an identity. And we were there, helping to define a new American literary tradition. Over half a century later, in the midst of the Second Industrial Revolution, it was a generation focused on building the future. Once again, we were there, supplying the critical scientific, technical, and engineering knowledge that helped frame the world. Throughout the 20th Century, and into the new millennium, nations began to reach out beyond their own borders and a new international community was born. Wiley was there, expanding its operations around the world to enable a global exchange of ideas, opinions, and know-how.

For 200 years, Wiley has been an integral part of each generation's journey, enabling the flow of information and understanding necessary to meet their needs and fulfill their aspirations. Today, bold new technologies are changing the way we live and learn. Wiley will be there, providing you the must-have knowledge you need to imagine new worlds, new possibilities, and new opportunities.

Generations come and go, but you can always count on Wiley to provide you the knowledge you need, when and where you need it!

WILLIAM J. PESCE
PRESIDENT AND CHIEF EXECUTIVE OFFICER

PETER BOOTH WILEY
CHAIRMAN OF THE BOARD

Modern Experimental Design

THOMAS P. RYAN

Acworth, GA



WILEY-INTERSCIENCE
A JOHN WILEY & SONS, INC., PUBLICATION

Copyright © 2007 by John Wiley & Sons, Inc. All rights reserved.

Published by John Wiley & Sons, Inc., Hoboken, New Jersey

Published simultaneously in Canada

No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400, fax 978-750-4470, or on the web at www.copyright.com. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, 201-748-6011, fax 201-748-6008, or online at <http://www.wiley.com/go/permission>.

Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives or written sales materials. The advice and strategies contained herein may not be suitable for your situation. You should consult with a professional where appropriate. Neither the publisher nor author shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

For general information on our other products and services or for technical support, please contact our Customer Care Department within the United States at 877-762-2974, outside the United States at 317-572-3993 or fax 317-572-4002.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic formats. For more information about Wiley products, visit our web site at www.wiley.com.

Library of Congress Cataloging-in-Publication Data:

Ryan, Thomas P.

Modern experimental design / by Thomas P. Ryan

p. cm.

Includes bibliographical references and index.

ISBN 978-0-471-21077-1

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Contents

Preface	xv
1 Introduction	1
1.1 Experiments All Around Us	2
1.2 Objectives for Experimental Designs	3
1.3 Planned Experimentation versus Use of Observational Data	5
1.4 Basic Design Concepts	6
1.4.1 Randomization	6
1.4.2 Replication versus Repeated Measurements	7
1.4.3 Example	8
1.4.4 Size of an Effect That Can be Detected	11
1.5 Terminology	12
1.6 Steps for the Design of Experiments	13
1.6.1 Recognition and Statement of the Problem	14
1.6.2 Selection of Factors and Levels	14
1.6.2.1 Choice of Factors	14
1.6.2.2 Choice of Levels	15
1.7 Processes Should Ideally be in a State of Statistical Control	18
1.8 Types of Experimental Designs	20
1.9 Analysis of Means	20
1.10 Missing Data	22
1.11 Experimental Designs and Six Sigma	22
1.12 Quasi-Experimental Design	23
1.13 Summary	23
References	23
Exercises	26

2	Completely Randomized Design	31
2.1	Completely Randomized Design	31
2.1.1	Model	32
2.1.2	Example: One Factor, Two Levels	33
2.1.2.1	Assumptions	33
2.1.3	Examples: One Factor, More Than Two Levels	35
2.1.3.1	Multiple Comparisons	36
2.1.3.2	Unbalanced and Missing Data	39
2.1.3.3	Computations	40
2.1.4	Example Showing the Effect of Unequal Variances	41
2.2	Analysis of Means	42
2.2.1	ANOM for a Completely Randomized Design	43
2.2.1.1	Example	44
2.2.2	ANOM with Unequal Variances	45
2.2.2.1	Applications	47
2.2.3	Nonparametric ANOM	47
2.2.4	ANOM for Attributes Data	47
2.3	Software for Experimental Design	48
2.4	Missing Values	48
2.5	Summary	48
	Appendix	49
	References	49
	Exercises	51
3	Designs that Incorporate Extraneous (Blocking) Factors	56
3.1	Randomized Block Design	56
3.1.1	Assumption	57
3.1.2	Blocking an Out-of-Control Process	60
3.1.3	Efficiency of a Randomized Block Design	61
3.1.4	Example	61
3.1.4.1	Critique	63
3.1.5	ANOM	64
3.2	Incomplete Block Designs	65
3.2.1	Balanced Incomplete Block Designs	65
3.2.1.1	Analysis	66
3.2.1.2	Recovery of Interblock Information	68
3.2.1.3	ANOM	68
3.2.2	Partially Balanced Incomplete Block Designs	69
3.2.2.1	Lattice Design	70
3.2.3	Nonparametric Analysis for Incomplete Block Designs	70
3.2.4	Other Incomplete Block Designs	70
3.3	Latin Square Design	71
3.3.1	Assumptions	72
3.3.2	Model	74

3.3.3	Example	74
3.3.4	Efficiency of a Latin Square Design	77
3.3.5	Using Multiple Latin Squares	77
3.3.6	ANOM	79
3.4	Graeco–Latin Square Design	80
3.4.1	Model	80
3.4.2	Degrees of Freedom Limitations on the Design Construction	81
3.4.3	Sets of Graeco–Latin Square Designs	82
3.4.4	Application	82
3.4.5	ANOM	83
3.5	Youden Squares	84
3.5.1	Model	85
3.5.2	Lists of Youden Designs	86
3.5.3	Using Replicated Youden Designs	86
3.5.4	Analysis	86
3.6	Missing Values	86
3.7	Software	89
3.8	Summary	90
	References	91
	Exercises	93
4	Full Factorial Designs with Two Levels	101
4.1	The Nature of Factorial Designs	101
4.2	The Deleterious Effects of Interactions	106
4.2.1	Conditional Effects	107
4.2.1.1	Sample Sizes for Conditional Effects Estimation	113
4.2.2	Can We “Transform Away” Interactions?	114
4.3	Effect Estimates	114
4.4	Why Not One-Factor-at-a-Time Designs?	115
4.5	ANOVA Table for Unreplicated Two-Factor Design?	116
4.6	The 2^3 Design	119
4.7	Built-in Replication	122
4.8	Multiple Readings versus Replicates	123
4.9	Reality versus Textbook Examples	124
4.9.1	Factorial Design but not “Factorial Model”	124
4.10	Bad Data in Factorial Designs	127
4.10.1	ANOM Display	134
4.11	Normal Probability Plot Methods	136
4.12	Missing Data in Factorial Designs	138
4.12.1	Resulting from Bad Data	139
4.12.2	Proposed Solutions	140
4.13	Inaccurate Levels in Factorial Designs	140
4.14	Checking for Statistical Control	141
4.15	Blocking 2^k Designs	142

4.16	The Role of Expected Mean Squares in Experimental Design	144
4.17	Hypothesis Tests with Only Random Factors in 2^k Designs? Avoid Them!	146
4.18	Hierarchical versus Nonhierarchical Models	147
4.19	Hard-to-Change Factors	148
4.19.1	Software for Designs with Hard-to-Change Factors	150
4.20	Factors Not Reset	150
4.21	Detecting Dispersion Effects	150
4.22	Software	151
4.23	Summary	151
	Appendix A Derivation of Conditional Main Effects	152
	Appendix B Relationship Between Effect Estimates and Regression Coefficients:	153
	Appendix C Precision of the Effect Estimates	153
	Appendix D Expected Mean Squares for the Replicated 2^2 Design	153
	Appendix E Expected Mean Squares, in General	155
	References	157
	Exercises	162
5	Fractional Factorial Designs with Two Levels	169
5.1	2^{k-1} Designs	170
5.1.1	Which Fraction?	176
5.1.2	Effect Estimates and Regression Coefficients	177
5.1.3	Alias Structure	177
5.1.4	What if I Had Used the Other Fraction?	179
5.2	2^{k-2} Designs	181
5.2.1	Basic Concepts	185
5.3	Designs with $k - p = 16$	187
5.3.1	Normal Probability Plot Methods when $k - p = 16$	187
5.3.2	Other Graphical Methods	188
5.4	Utility of Small Fractional Factorials vis-à-vis Normal Probability Plots	188
5.5	Design Efficiency	190
5.6	Retrieving a Lost Defining Relation	190
5.7	Minimum Aberration Designs and Minimum Confounded Effects Designs	192
5.8	Blocking Factorial Designs	194
5.8.1	Blocking Fractional Factorial Designs	195
5.8.1.1	Blocks of Size 2	200
5.9	Foldover Designs	201
5.9.1	Semifolding	203
5.9.1.1	Conditional Effects	208
5.9.1.2	Semifolding a 2^{k-1} Design	210

5.9.1.3	General Strategy?	215
5.9.1.4	Semifolding with Software	215
5.10	John's $3/4$ Designs	216
5.11	Projective Properties of 2^{k-p} Designs	219
5.12	Small Fractions and Irregular Designs	220
5.13	An Example of Sequential Experimentation	222
5.13.1	Critique of Example	224
5.14	Inadvertent Nonorthogonality—Case Study	225
5.15	Fractional Factorial Designs for Natural Subsets of Factors	226
5.16	Relationship Between Fractional Factorials and Latin Squares	228
5.17	Alternatives to Fractional Factorials	229
5.17.1	Designs Attributed to Genichi Taguchi	229
5.18	Missing and Bad Data	230
5.19	Plackett–Burman Designs	230
5.20	Software	230
5.21	Summary	233
	References	234
	Exercises	238
6	Designs With More Than Two Levels	248
6.1	3^k Designs	248
6.1.1	Decomposing the $A*B$ Interaction	251
6.1.2	Inference with Unreplicated 3^k Designs	252
6.2	Conditional Effects	255
6.3	3^{k-p} Designs	257
6.3.1	Understanding 3^{k-p} Designs	259
6.3.2	Constructing 3^{k-p} Designs	260
6.3.3	Alias Structure	262
6.3.4	Constructing a 3^{3-1} Design	262
6.3.5	Need for Mixed Number of Levels	263
6.3.6	Replication of 3^{k-p} Designs?	264
6.4	Mixed Factorials	264
6.4.1	Constructing Mixed Factorials	265
6.4.2	Additional Examples	266
6.5	Mixed Fractional Factorials	274
6.6	Orthogonal Arrays with Mixed Levels	275
6.7	Minimum Aberration Designs and Minimum Confounded Effects Designs	277
6.8	Four or More Levels	278
6.9	Software	280
6.10	Catalog of Designs	284
6.11	Summary	284
	References	284
	Exercises	286

7	Nested Designs	291
7.1	Various Examples	294
7.2	Software Shortcomings	295
7.2.1	A Workaround	295
7.3	Staggered Nested Designs	298
7.4	Nested and Staggered Nested Designs with Factorial Structure	300
7.5	Estimating Variance Components	300
7.6	ANOM for Nested Designs?	302
7.7	Summary	302
	References	302
	Exercises	304
8	Robust Designs	311
8.1	“Taguchi Designs?”	312
8.2	Identification of Dispersion Effects	314
8.3	Designs with Noise Factors	316
8.4	Product Array, Combined Array, or Compound Array?	318
8.5	Software	320
8.6	Further Reading	322
8.7	Summary	322
	References	323
	Exercises	326
9	Split-Unit, Split-Lot, and Related Designs	330
9.1	Split-Unit Design	331
9.1.1	Split-Plot Mirror Image Pairs Designs	336
9.1.2	Split-Unit Designs in Industry	336
9.1.3	Split-Unit Designs with Fractional Factorials	340
9.1.4	Blocking Split-Plot Designs	342
9.1.5	Split-Unit Plackett-Burman Designs	343
9.1.6	Examples of Split-Plot Designs for Hard-to-Change Factors	343
9.1.7	Split-Split-Plot Designs	345
9.2	Split-Lot Design	345
9.2.1	Strip-Plot Design	346
9.2.1.1	Applications of Strip-Block (Strip-Plot) Designs	347
9.3	Commonalities and Differences Between these Designs	349
9.4	Software	350
9.5	Summary	351
	References	351
	Exercises	354

10	Response Surface Designs	360
10.1	Response Surface Experimentation: One Design or More Than One?	362
10.2	Which Designs?	364
10.3	Classical Response Surface Designs versus Alternatives	364
10.3.1	Effect Estimates?	369
10.4	Method of Steepest Ascent (Descent)	370
10.5	Central Composite Designs	373
10.5.1	CCD Variations	377
10.5.2	Small Composite Designs	377
10.5.2.1	Draper–Lin Designs	378
10.5.3	Additional Applications	383
10.6	Properties of Space-Filling Designs	384
10.7	Applications of Uniform Designs	386
10.8	Box–Behnken Designs	386
10.8.1	Application	388
10.9	Conditional Effects?	389
10.10	Other Response Surface Designs	390
10.10.1	Hybrid Designs	390
10.10.2	Uniform Shell Designs	393
10.10.3	Koshal Designs	393
10.10.4	Hoke Designs	394
10.11	Blocking Response Surface Designs	394
10.11.1	Blocking Central Composite Designs	394
10.11.2	Blocking Box–Behnken Designs	396
10.11.3	Blocking Other Response Surface Designs	396
10.12	Comparison of Designs	397
10.13	Analyzing the Fitted Surface	398
10.13.1	Characterization of Stationary Points	401
10.13.2	Confidence Regions on Stationary Points	402
10.13.3	Ridge Analysis	403
10.13.3.1	Ridge Analysis with Noise Factors	404
10.13.4	Optimum Conditions and Regions of Operability	404
10.14	Response Surface Designs for Computer Simulations	404
10.15	ANOM with Response Surface Designs?	405
10.16	Further Reading	405
10.17	The Present and Future Direction of Response Surface Designs	406
10.18	Software	406
10.19	Catalogs of Designs	408
10.20	Summary	408
	References	409
	Exercises	414

11	Repeated Measures Designs	425
11.1	One Factor	426
11.1.1	The Example in Section 2.1.2	428
11.2	More Than One Factor	428
11.3	Crossover Designs	429
11.4	Designs for Carryover Effects	432
11.5	How Many Repeated Measures?	437
11.6	Further Reading	438
11.7	Software	438
11.8	Summary	439
	References	439
	Exercises	444
12	Multiple Responses	447
12.1	Overlaying Contour Plots	448
12.2	Seeking Multiple Response Optimization with Desirability Functions	449
12.2.1	Weight and Importance	451
12.3	Dual Response Optimization	452
12.4	Designs Used with Multiple Responses	452
12.5	Applications	453
12.6	Multiple Response Optimization Variations	463
12.7	The Importance of Analysis	469
12.8	Software	469
12.9	Summary	471
	References	472
	Exercises	474
13	Miscellaneous Design Topics	483
13.1	One-Factor-at-a-Time Designs	483
13.2	Cotter Designs	487
13.3	Rotation Designs	488
13.4	Screening Designs	489
13.4.1	Plackett–Burman Designs	489
13.4.1.1	Projection Properties of Plackett–Burman Designs	493
13.4.1.2	Applications	494
13.4.2	Supersaturated Designs	498
13.4.2.1	Applications	499
13.4.3	Lesser-Known Screening Designs	500
13.5	Design of Experiments for Analytic Studies	500
13.6	Equileverage Designs	501
13.6.1	One Factor, Two Levels	502
13.6.2	Are Commonly Used Designs Equileverage?	502

13.7	Optimal Designs	503
13.7.1	Alphabetic Optimality	504
13.7.2	Applications of Optimal Designs	507
13.8	Designs for Restricted Regions of Operability	508
13.9	Space-Filling Designs	514
13.9.1	Uniform Designs	515
13.9.1.1	From Raw Form to Coded Form	518
13.9.2	Sphere-Packing Designs	518
13.9.3	Latin Hypercube Design	519
13.10	Trend-Free Designs	521
13.11	Cost-Minimizing Designs	522
13.12	Mixture Designs	522
13.12.1	Optimal Mixture Designs or Not?	523
13.12.2	ANOM	523
13.13	Design of Measurement Capability Studies	523
13.14	Design of Computer Experiments	523
13.15	Design of Experiments for Categorical Response Variables	524
13.16	Weighing Designs and Calibration Designs	524
13.16.1	Calibration Designs	525
13.16.2	Weighing Designs	526
13.17	Designs for Assessing the Capability of a System	528
13.18	Designs for Nonlinear Models	528
13.19	Model-Robust Designs	528
13.20	Designs and Analyses for Non-normal Responses	529
13.21	Design of Microarray Experiments	529
13.22	Multi-Vari Plot	530
13.23	Evolutionary Operation	531
13.24	Software	531
13.25	Summary	532
	References	533
	Exercises	542
14	Tying It All Together	544
14.1	Training for Experimental Design Use	544
	References	545
	Exercises	546
	Answers to Selected Exercises	551
	Appendix: Statistical Tables	565
	Author Index	575
	Subject Index	587

Preface

Although there is a moderate amount of data analysis, especially in certain chapters, the emphasis in this book is on the statistical design of experiments. Such emphasis is justified by the widely held view that data from a well-designed experiment are easy to analyze. Certain types of designs are not simple, however, such as those covered in Chapters 7, 8, and 11, and the problem is compounded by the fact that some popular statistical software packages have quite limited capability for those designs.

The book would be suitable for an undergraduate one-semester course in design of experiments. For a course taught to nonstatistics majors, an instructor may wish to cover Chapters 1–4, part of Chapter 5, and then pick and choose from the other chapters in accordance with the needs of the students. The selection might include either or both of Chapters 10 and 12 and then cover sections of interest in Chapter 13.

For statistics majors, the book would be suitable for use in an advanced undergraduate course, perhaps covering Chapters 1–5, 7, 8, and much of Chapter 13. There is also enough advanced material for the book to be useful as a reference book in a graduate course taught to statistics majors, and might also be used in a graduate course for nonstatistics majors, depending on the needs and backgrounds of the students.

There is also enough material for a two-semester course, with the first course perhaps covering Chapters 1–6 and the second course covering Chapters 7–12 and 14, and parts of Chapter 13.

There is a considerable amount of material that is not covered to any extent, if at all, in other books on the subject, and some or all of this material might be used in special topics courses. These topics include conditional effects, uniform designs, and designs for restricted operating regions. (I have covered this material in an Internet course.)

A two-semester course in statistical methods should provide more than enough background for the book since the emphasis is on designs rather than statistical concepts. Matrix algebra is used in various places in the book, although it is not used extensively. Nevertheless, proficiency in the basics of matrix algebra is necessary for following some of the material.

One of the special features of the book is the emphasis on conditional effects in Chapters 4, 5, 6, and 10. This is an important topic that is not covered to any extent in most books and is addressed in very few journal articles. Another somewhat unique feature is moderate use of URLs, especially links to published articles that are available to the general public as well as article preprints and technical reports. There are other links for articles that are available to certain groups, such as members of the American Society for Quality. Some of those URLs might of course become outdated but I decided to list them since many of them, such as links to journal articles, will probably not become outdated in the near future. They make available to the reader a considerable amount of important resource material.

It is worth noting that this book does not contain catalogs of designs, as are given in some other books on the subject. Rather, the emphasis is on understanding design concepts and properties, the software that is available for generating specific designs and when to use those designs, and as stated, a moderate amount of analysis of data from experiments in which the designs are used, with extensive analysis provided in some case studies. Although there is some hand computation, the emphasis is on using appropriate software to generate output and interpret the output.

It is also worth noting that whereas there are case studies and a moderate amount of data analyses, there is not a “full” analysis of any dataset as that would include checking for outliers and influential observations, testing assumptions, and so on, which are covered in books on statistical methods. This is important but comes under the heading of data analysis rather than design and analysis of experiments. Although this book has more analysis than most books on design of experiments, it is not intended to be a handbook on data analysis.

I wish to gratefully acknowledge my editor, Steve Quigley, who motivated me to write this book, in addition to the contributions of associate editor Susanne Steitz, production editor Rosalyn Farkas, and colleagues who have made helpful comments, including Dennis Lin and Ivelisse Aviles, plus the helpful comments of three anonymous reviewers.

THOMAS P. RYAN