

$$\begin{aligned}
 2^2 &= 4 = 5 \times 0 + 4 \rightarrow 2^2 \equiv 4 \\
 3^2 &= 9 = 5 \times 1 + 4 \rightarrow 3^2 \equiv 4 \\
 4^2 &= 16 = 5 \times 3 + 1 \rightarrow 4^2 \equiv 1 \\
 5^2 &= 25 = 5 \times 5 + 0 \rightarrow 5^2 \equiv 0 \\
 6^2 &= 36 = 5 \times 7 + 1 \rightarrow 6^2 \equiv 1 \\
 7^2 &= 49 = 5 \times 9 + 4 \rightarrow 7^2 \equiv 4 \\
 8^2 &= 64 = 5 \times 12 + 4 \rightarrow 8^2 \equiv 4 \\
 9^2 &= 81 = 5 \times 16 + 1 \rightarrow 9^2 \equiv 1 \\
 10^2 &= 100 = 5 \times 20 + 0 \rightarrow 10^2 \equiv 0 \\
 11^2 &= 121 = 5 \times 24 + 1 \rightarrow 11^2 \equiv 1 \\
 12^2 &= 144 = 5 \times 28 + 4 \rightarrow 12^2 \equiv 4 \\
 13^2 &= 169 = 5 \times 33 + 4 \rightarrow 13^2 \equiv 4 \\
 14^2 &= 196 = 5 \times 39 + 1 \rightarrow 14^2 \equiv 1 \\
 15^2 &= 225 = 5 \times 45 + 0 \rightarrow 15^2 \equiv 0 \\
 16^2 &= 256 = 5 \times 51 + 1 \rightarrow 16^2 \equiv 1 \\
 17^2 &= 289 = 5 \times 57 + 4 \rightarrow 17^2 \equiv 4 \\
 18^2 &= 324 = 5 \times 64 + 4 \rightarrow 18^2 \equiv 4 \\
 19^2 &= 361 = 5 \times 72 + 1 \rightarrow 19^2 \equiv 1 \\
 20^2 &= 400 = 5 \times 80 + 0 \rightarrow 20^2 \equiv 0
 \end{aligned}$$

Number 5 relation $n^2 \mid 0^2 = 0 = 5 \times 0 + 0 \rightarrow +0$

$$\begin{aligned}
 1^2 &= 1 = 5 \times 0 + 1 \rightarrow x^0 + 1 \\
 2^2 &= 4 = 5 \times 1 - 1 \rightarrow x^1 - 1 \\
 3^2 &= 9 = 5 \times 2 - 1 \rightarrow x^2 - 1 \\
 4^2 &= 16 = 5 \times 3 + 1 \rightarrow x^3 + 1 \\
 5^2 &= 25 = 5 \times 5 + 0 \rightarrow x^5 + 0 \\
 6^2 &= 36 = 5 \times 7 + 1 \rightarrow x^7 + 1 \\
 7^2 &= 49 = 5 \times 10 - 1 \rightarrow x^{10} - 1 \\
 8^2 &= 64 = 5 \times 13 - 1 \rightarrow x^{13} - 1 \\
 9^2 &= 81 = 5 \times 16 + 1 \rightarrow x^{16} + 1 \\
 10^2 &= 100 = 5 \times 20 + 0 \rightarrow x^{20} + 0 \\
 11^2 &= 121 = 5 \times 24 + 1 \rightarrow x^{24} + 1 \\
 12^2 &= 144 = 5 \times 29 - 1 \rightarrow x^{29} - 1 \\
 13^2 &= 169 = 5 \times 34 - 1 \rightarrow x^{34} - 1 \\
 14^2 &= 196 = 5 \times 39 + 1 \rightarrow x^{39} + 1 \\
 15^2 &= 225 = 5 \times 45 + 0 \rightarrow x^{45} + 0 \\
 16^2 &= 256 = 5 \times 51 + 1 \rightarrow x^{51} + 1 \\
 17^2 &= 289 = 5 \times 58 - 1 \rightarrow x^{58} - 1 \\
 18^2 &= 324 = 5 \times 65 - 1 \rightarrow x^{65} - 1 \\
 19^2 &= 361 = 5 \times 72 + 1 \rightarrow x^{72} + 1 \\
 20^2 &= 400 = 5 \times 80 + 0 \rightarrow x^{80} + 0
 \end{aligned}$$

$$0 = \begin{cases} \frac{n}{5} \pm 1 \text{ will apply } +1 \\ \frac{n}{5} \text{ will apply } +0 \\ \frac{n}{5} \pm (2, 3) -1 \end{cases}$$

$$\begin{aligned}
 \text{Proofs} = & \\
 \text{if } n \bmod 5 = 4 & \text{ then } +1 \\
 \text{if } n \bmod 5 = 1 & \text{ then } +1 \\
 \text{if } n \bmod 5 = 0 & \text{ then } +0 \\
 \text{if } n \bmod 5 = (2, 3) & \text{ then } -1
 \end{aligned}$$

3 → -1	2
2 → -1	1
1 → 0	0
0 → 0	0
1 → 0	0
2 → 1	1
3 → 1	2
4 → 1	3
5 → 2	5
6 → 2	7
7 → 3	10
8 → 3	13
9 → 3	16
10 → 4	20
11 → 4	24
12 → 5	29
13 → 5	34
14 → 5	39
15 → 6	45
16 → 6	51
17 → 7	58
18 → 7	65
19 → 7	72
20 → 8	80

~~$T(n) = 5a(n)$~~
 $T(n) + 5a(n) = n^2$

$a(n) = \left\lfloor \frac{n^2 + 1}{5} \right\rfloor$

$$T(n) = \begin{cases} +1 & \text{if } n \equiv 1, 4 \pmod{5} \\ 0 & \text{if } n \equiv 0 \pmod{5} \\ -1 & \text{if } n \equiv 2, 3 \pmod{5} \end{cases}$$

$$b(n) = 2 \cdot \left\lfloor \frac{n}{5} \right\rfloor + \begin{cases} 0 & \text{if } n \pmod{5} < 2 \\ 1 & \text{if } n \pmod{5} \geq 2 \end{cases}$$

$a(n) = \left\lfloor \frac{n^2 + 1}{5} \right\rfloor$

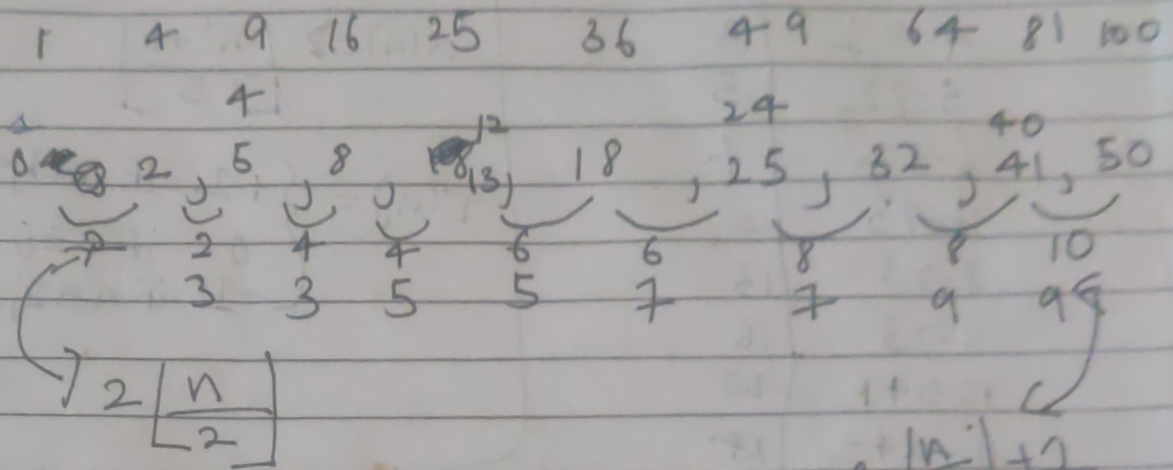
$\left\lfloor \frac{n^2 + 1}{5} \right\rfloor$

~~364~~
 $\frac{37}{5} = 7$

$\frac{400 + 1}{5} = 80$
 $364 = 72$

$\left\lfloor \frac{5 \cdot \frac{n^2 + 1}{5}}{5} \right\rfloor$

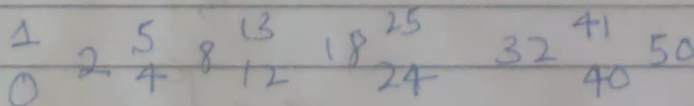
$$2 \times 0 + 1$$



upper $\rightarrow \lfloor \frac{n^2}{2} \rfloor + 1$ lower $\rightarrow \lfloor \frac{n^2}{2} \rfloor$

if $n \bmod 2 = 1$ then $\begin{cases} +1 \text{ even too small} \\ -1 \text{ odd too big} \end{cases}$
 if $n \bmod 2 = 0$ then $+0$

$n^2 = \begin{cases} \text{if } n \bmod 2 = 1 \text{ then } \begin{cases} \text{Even } 2 \lfloor \frac{n^2}{2} \rfloor + 1 \\ \text{odd } 2 \lfloor \frac{n^2}{2} \rfloor - 1 \end{cases} \\ \text{if } n \bmod 2 = 0 \text{ then } 2 \lfloor \frac{n^2}{2} \rfloor \text{ or } 2 \lfloor \frac{n^2}{2} \rfloor + 2 \end{cases}$



a,

$$4 \rightarrow \text{up} \quad \left[\frac{n^2}{4} \right]^{-3}$$

$$\rightarrow \text{down} \quad \left[\frac{n^2}{4} \right]^{+1}$$

$$5 \rightarrow \text{up} \quad \left[\frac{n^2}{5} \right]^{-4, -1}$$

$$\rightarrow \text{down} \quad \left[\frac{n^2}{5} \right]^{+1, +4}$$

$$3 \rightarrow \text{up} \quad \left[\frac{n^2}{3} \right]^{-2}$$

$$\rightarrow \text{down} \quad \left[\frac{n^2}{3} \right]^{+1}$$

$$2 \rightarrow \text{up} \quad \left[\frac{n^2}{2} \right]^{-1}$$

$$\rightarrow \text{down} \quad \left[\frac{n^2}{2} \right]^{+1}$$

$$6 \rightarrow \text{up} \quad \left[\frac{n^2}{6} \right]^{-3, -2, -1}$$

$$\rightarrow \text{down} \quad \left[\frac{n^2}{6} \right]^{+1, +4, +9}$$

4 relab'm

10 20 30 40 50 60 70 80 90 100

up

up

1	4	9	16	25	36	49	64	81	100
0	1	3	4	7	9	13	16	21	25
		2		6		12		20	

up

1	2	1	3	2	4	3	5	4	6	5	7	6	8	7	9	8
1	1	2	2	3	3	4	4	5	5	6	6					

up $\rightarrow \left\lfloor \frac{n^2}{4} \right\rfloor + n$

down $\rightarrow \left\lfloor \frac{n^2}{4} \right\rfloor + n$

up \rightarrow then \downarrow then $n+2$ then \downarrow if even \downarrow if odd

random $\rightarrow \left\lfloor \frac{n}{2} \right\rfloor$

up up $\left\lfloor \frac{n}{2} \right\rfloor + (n-1)/2 \left\lfloor \frac{(n+1)}{2} \right\rfloor + n \bmod 2$

5's

1 4 9 16 25 36 49 64 81 100

Δ 0 1 2 4 5 8 10 13 ~~17~~ 20 25 29
 0 0 1 3 5 7 9 12 16 24 27

$$\text{up } a_1 \left\lceil \frac{n^2}{5} \right\rceil$$

$$\text{down } \left\lfloor \frac{n^2}{5} \right\rfloor$$

6th

1 4 9 16 25 36 49 64 81 100 121 144

Δ 1 1 2 3 5 6 9 11 14 17 21 24
 0 0 1 2 4 6 8 10 13 16 20

$$\text{if } \rightarrow \left\lceil \frac{n^2}{6} \right\rceil^{-5, -2, -3,}$$

$$\text{down } \Rightarrow \left\lfloor \frac{n^2}{6} \right\rfloor^{+1, +4, +3,}$$

3⁵

up +1,

down -2,

~~1 4 9 16 25 36 49 64 81~~

~~1 4 9 16 25 36 49 64 100~~

1 4 9 16 25 36 49 64 81 100

1	1	3	5	7	9	12	16	21	27	33
0	2	3	6	9	12	17	22	27	34	
	2	2	2	3	4	4	5	6	6	
1	2	2	2	3	4	4	5	6	6	
2	1	3	3	3	3	5	5	5	7	7

up up $\rightarrow \left\lfloor \frac{2n}{3} \right\rfloor$

down down

~~$\left\lceil \frac{n+2}{3} \right\rceil$~~

~~$\left\lceil \frac{n+2}{3} \right\rceil + 1$~~

up $\rightarrow \left\lceil \frac{n^2}{3} \right\rceil \left\lceil \frac{n^2}{3} \right\rceil$

$2 \left\lfloor \frac{n}{3} \right\rfloor + 1$

down $\rightarrow \left\lfloor \frac{n^2}{3} \right\rfloor \left\lfloor \frac{n^2}{3} \right\rfloor$

1	2	3	6	9	12	17	22	27	34
0	1		5	8		16	21		33

<u>2's</u>											
0	1	2	5	8	13	18	25	32	41	50	61
0	0		4		12	18	24	32	40	60	84

up up $\lceil \frac{n^2}{2} \rceil$

up up 1 3 3 5 5 7 7 9 9 11 11 13 13

up up $\lceil \frac{n}{2} \rceil * 2 + 1$

low/down $\lfloor \frac{n^2}{2} \rfloor$

Wallis formula

down/down $\rightarrow 0, 2, 2, 4, 4, 6, 6, 8, 8, 10, 10, 12, 12$

down down $\lfloor \frac{n+1}{2} \rfloor * 2$

~~$\lceil \frac{n}{2} \rceil * 2 + 1$~~

~~$\lfloor \frac{n^2}{2} \rfloor * 2$~~
 ~~$\lfloor \frac{n}{2} \rfloor * 2 + 1$~~

$$\pi = 2 * \prod_{n=1}^{\infty} \frac{\lfloor \frac{n+1}{2} \rfloor * 2}{\lfloor \frac{n}{2} \rfloor * 2 + 1}$$

$$\Rightarrow \pi = 2 * \prod_{n=1}^{\infty} \frac{\lfloor \frac{n+1}{2} \rfloor * 2}{\lfloor \frac{n}{2} \rfloor * 2 + 1}$$

Wallis = mine

$$\pi = 2 \prod_{n=1}^{\infty} \frac{(2n)^2}{(2n-1)(2n+1)} = 2 \prod_{n=1}^{\infty} \frac{\lfloor \frac{n+1}{2} \rfloor * 2}{\lfloor \frac{n}{2} \rfloor * 2 + 1}$$

3)

$$\frac{1}{0 \frac{1}{2}}$$

1	2	3	6	9	12	17	22	27	34	41
0	1		5	8		16	21		33	40

$$\text{up} \rightarrow \left\lfloor \frac{n^2}{3} \right\rfloor$$

up up $\rightarrow 1, 1, 3, 3, 3, 5, 5, 5, 7, 7$

$$2 \text{ up up} \rightarrow 2 * \left\lfloor \frac{n}{3} \right\rfloor + 1$$

$$\text{down} \rightarrow \left\lfloor \frac{n^2}{3} \right\rfloor$$

down down $\rightarrow 1, 1, 2, 3, 4, 4, 5, 6, 6, 7, 8, 8, 9, 10, 10, 11$

$$\text{down down} \left\lfloor \frac{n+2}{3} \right\rfloor \quad \left\lfloor \frac{2 * (n+1)}{3} \right\rfloor$$

$$2 * \prod_{n=1}^{\infty} \left\lfloor \frac{2 * (n+1)}{3} \right\rfloor = \pi$$

↑'s

1 1 3 4 7 9 13 16 21 25
0 2 2 6 9 12 16 20

$$\text{up} \rightarrow \left\lceil \frac{n^2}{4} \right\rceil$$

up up \rightarrow 0, 2, 1, 3, 2, 4, 3, 5, 4,
6, 5, 7, 6, 8, 7, 9

$$\text{up up} \rightarrow \left\lceil \frac{n}{4} \right\rceil + 1$$

$$2 * \left\lceil \frac{n-1}{2} \right\rceil - \left\lfloor \frac{n-1}{2} \right\rfloor$$

$$\text{down} \rightarrow \left\lfloor \frac{n^2}{4} \right\rfloor$$

down down \rightarrow 1, 1, 2, 2, 3, 3, 4, 4, 5

$$\text{down down} \rightarrow \left\lceil \frac{n}{2} \right\rceil$$

$$\prod_{n=1}^{\infty} \frac{\left\lceil \frac{n+1}{2} \right\rceil}{2 * \left\lceil \frac{n}{2} \right\rceil - \left\lfloor \frac{n}{2} \right\rfloor} = 1$$

if limit is even of odd
or for large value, it will tend to 1

5's	1	1	2	4	5	8	10	13	17	20	25	29
	0	0	1	3		7	9	12	16		24	28

up $\lceil \frac{n}{5} \rceil$

up up $\rightarrow 0 \ 1 \ 2 \ 1 \ 3 \ 2 \ 3 \ 4 \ 3 \ 5 \ 4 \ 5 \ 6 \ 5 \ 7 \ 6$

1 1 -1 2 -1 1 1 -1 2 -1

up
for all

$$\lceil \frac{n}{5} \rceil + \lceil \frac{n-1}{5} \rceil - \lceil \frac{n-2}{5} \rceil + 2 \lceil \frac{n-3}{5} \rceil - \lceil \frac{n-4}{5} \rceil$$

$$\lceil \frac{1}{5} \rceil + \lceil \frac{0}{5} \rceil - \lceil \frac{-1}{5} \rceil + 2 \lceil \frac{-2}{5} \rceil - \lceil \frac{-3}{5} \rceil$$

$$1 + 0 - 0 + 0 - 0$$

$$\lceil \frac{2}{5} \rceil + \lceil \frac{1}{5} \rceil - \lceil \frac{0}{5} \rceil + 2 \lceil \frac{-1}{5} \rceil - \lceil \frac{-2}{5} \rceil$$

$$1 + 1 - 0 + 0 - 0 = 2$$

$$\lceil \frac{7}{5} \rceil + \lceil \frac{6}{5} \rceil - \lceil \frac{5}{5} \rceil + 2 \lceil \frac{4}{5} \rceil - \lceil \frac{3}{5} \rceil$$

$$2 + 2 - 1 + 2 - 1 = 4$$

actual
for all

$$\lceil \frac{n-1}{5} \rceil + \lceil \frac{n-2}{5} \rceil - \lceil \frac{n-3}{5} \rceil + 2 \lceil \frac{n-4}{5} \rceil - \lceil \frac{n-5}{5} \rceil$$

Pdown 0 0 1 3 5 7 9 12 16 20 24

down down 0 1 2 2 2 2 3 4 4 4 5 6 6
 6 6 7 8 8 8 8

down down $\rightarrow \left\lceil \frac{n-1}{5} \right\rceil + \left\lceil \frac{n}{5} \right\rceil$

$$\prod_{n=1}^{\infty} \frac{\left\lceil \frac{n-1}{5} \right\rceil + \left\lceil \frac{n}{5} \right\rceil}{\left\lceil \frac{n}{5} \right\rceil + \left\lceil \frac{n-1}{5} \right\rceil - \left\lceil \frac{n-2}{5} \right\rceil + 2 \left\lceil \frac{n-3}{5} \right\rceil - \left\lceil \frac{n-4}{5} \right\rceil}$$

mu (up 4, low 4)

- $mu=5 \rightarrow (3,3) \text{ or } (5,5)$
- $mu=6 \rightarrow (3,3) \text{ or } (6,6)$
- $mu=7 \rightarrow (3,3) \text{ or } (7,7)$
- $mu=8 \rightarrow (5,5) \text{ or } (9,9)$
- $mu=10 \rightarrow (4,4) \text{ or } (10,10)$
- $mu=11 \rightarrow (4,4) \text{ or } (11,11)$
- $mu=13 \rightarrow (6,6) \text{ or } (13,13)$
- $mu=14 \rightarrow (6,6) \text{ or } (14,14)$
- $mu=15 \rightarrow (6,6) \text{ or } (15,15)$
- $mu=17 \rightarrow (8,8) \text{ or } (17,17)$

- $mu=8 (5,5)$
- $mu=12 (7,7)$
- $mu=16 (8,9,9)$

$$\frac{mu+2}{2}$$

7

for ~~mu~~
 mu when its multiple
of 4 or 4 its factor