Escaping the Killer by Always Getting Closer

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Problem Setup

Imagine two points in space, A and B. You are standing at point B. A path starts at point A and moves step by step toward B. There is only one rule imposed on the path: after every step, the current position must be closer to B than it was in the previous step. There are no restrictions on how the path bends, which direction it turns, or how small the progress toward B is, as long as the distance to B always decreases.

At first glance, this rule feels very strong. Our intuition tells us that if something keeps getting closer to its target at every step, then it should reach the target in a reasonable amount of time and distance. This intuition comes from straight lines and shortest paths, where moving closer also means making real progress.

The Killer Analogy

Now think of the same setup in a more concrete way. You are standing at point B. A killer starts at point A and will kill you the moment he reaches B. The killer is dumb but perfectly obedient. You are allowed to program him with a rule. The only rule you give is this: after every step, he must be closer to you than he was before. You are also allowed to decide how much closer he gets at each step and in which direction he moves.

The killer always follows the rule. At every step, the distance between him and you decreases. There is no cheating. He never moves farther away from you.

The Question

The natural belief is that such a rule guarantees your death. After all, if the killer keeps getting closer, how can he not eventually reach you? But this belief hides an assumption: that getting closer also limits how much useless movement is possible.

The real question is this: if you are allowed to choose how slowly the distance decreases and how the direction changes, can you force the killer to walk an extremely large total distance before reaching you, possibly an infinite one, even though he gets closer to you at every step?

Why This Is Interesting

This question is not just a thought experiment. It captures a deep idea about systems that follow local rules. A rule that looks reasonable and goal-oriented at each step can still fail badly when viewed over the entire process. For example, gradient descent algorithms in machine learning can get stuck taking tiny steps that never reach the optimal solution, despite always 'improving' locally.

Example with $R = 6$, $m = 0.05$

$$\frac{m}{2\pi R} = \frac{0.05}{2\pi \cdot 6} \approx 0.00133$$

$$\sqrt{1+0.00133^2} \approx 1.0000009$$

First term:

$$\frac{\pi \cdot 36}{0.05} \cdot 1.0000009 \approx 2262$$

Second term is negligible (≈ 0.15).

$$\boxed{L \approx 2262 \text{ units}}$$

Using the simplified formula, the following table summarizes the computed values. The columns correspond to:

R	m	$\frac{m}{2\pi R}$	$\sqrt{1+\left(\frac{m}{2\pi R}\right)^2}$	L
6	1	0.02653	1.000352	113.4811274610
6	0.8	0.02122	1.000225	141.6929067015
6	0.5	0.01326	1.000088	226.4144385030
6	0.2	0.00531	1.000014	565.5890496889
6	0.1	0.00265	1.000004	1131.0300571827
6	0.05	0.00133	1.000001	2261.9778194722
6	0.001	0.00003	1.000000	113097.3364627192

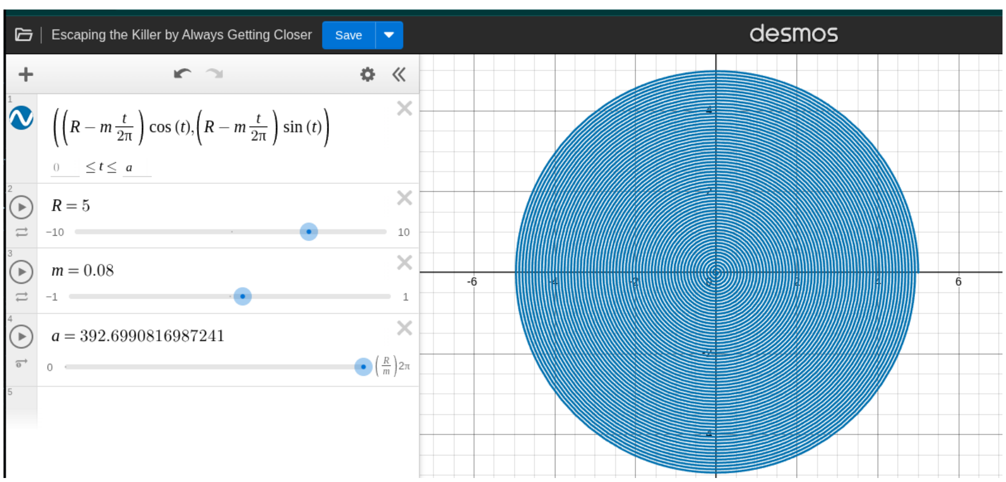

Figure 1: Visualization of the behavior of the function. Interactive experimentation available at Desmos Calculator.

Local Progress vs. Global Convergence

This problem is a fascinating exploration of the difference between **local progress** (getting closer at each step) and **global convergence** (actually reaching the target). Even with a "greedy" rule that mandates getting closer, you can mathematically manipulate the path to be arbitrarily long or even infinite.

1. The Geometry of "Getting Closer"

- **The Constraint**: The rule "distance to B must always decrease" simply means that after each step, the killer must move into the interior of a circle centered at B with a radius equal to his current distance.

- **The Exploit**: The constraint only restricts radial distance, not tangential movement. The killer can circle almost indefinitely while decreasing radius microscopically.
- **Total Distance vs. Displacement**: Displacement is the straight-line distance from A to B. Total distance is the sum of every step taken. In a spiral, the total distance can be massive compared to the displacement because the path "wastes" movement by circling the target.

2. Connection to the Archimedean Spiral

An Archimedean spiral provides the perfect visual representation is the perfect visual representation of this problem.

- **The Spiral Path**: Your killer follows the spiral line. At any point on that line, as the parameter t increases, the radius $\left(R - m \cdot \frac{t}{2\pi}\right)$ decreases.
- **Meeting the Rule**: Because the radius is always shrinking, the killer is technically always "getting closer" to the center $(0, 0)$.
- **Forcing a Long Walk**: By making the decrease rate (m) very small, you force the killer to take more rotations $\left(N = \frac{R}{m}\right)$ to reach the center. Each rotation adds a huge amount of total walking distance while only achieving a small m amount of progress toward the center.

3. Can the Path Be Infinite?

Yes. If you choose the steps such that the distance decreases but the **sum of the steps** diverges, the killer will walk forever without ever reaching you.

- **The Mathematical Trap**: Imagine the killer's distance to you at step n is D_n. If the distance decreases by $d_n = \frac{1}{n}$ at step n, then the total distance

  $$\sum_{n=1}^{\infty} \frac{1}{n}$$

  diverges (harmonic series), even though the position converges.
- **Infinite Length**: You can even design a spiral that gets closer to the center but has an infinite arc length. Unlike the logarithmic spiral where infinite rotations yield finite distance, the Archimedean spiral can achieve infinite distance with decreasing radius.

The General Arc Length Integral

Having established that such paths are theoretically possible, we now prove this rigorously using calculus. We'll derive the exact arc length formula for an Archimedean spiral and show that it diverges as the decrease rate approaches zero.

For an Archimedean spiral

$$r(\theta) = R - \frac{m\theta}{2\pi},$$

the arc length from $\theta = 0$ to $\theta = 2\pi N$ (where $N = R/m$ rotations) is:

$$L = \int_0^{\frac{2\pi R}{m}} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Substituting

$$r = R - \frac{m\theta}{2\pi}, \qquad \frac{dr}{d\theta} = -\frac{m}{2\pi},$$

we obtain:

$$L = \int_0^{\frac{2\pi R}{m}} \sqrt{\left(R - \frac{m\theta}{2\pi}\right)^2 + \left(\frac{m}{2\pi}\right)^2}\, d\theta$$

Step-by-Step Solution

Let

$$u = R - \frac{m\theta}{2\pi}.$$

Then:

$$du = -\frac{m}{2\pi}\, d\theta, \qquad d\theta = -\frac{2\pi}{m}\, du.$$

Limits:

$$\theta = 0 \Rightarrow u = R, \qquad \theta = \frac{2\pi R}{m} \Rightarrow u = 0.$$

Thus:

$$L = \int_R^0 \sqrt{u^2 + \left(\frac{m}{2\pi}\right)^2} \left(-\frac{2\pi}{m}\right) du$$

$$L = \frac{2\pi}{m} \int_0^R \sqrt{u^2 + \left(\frac{m}{2\pi}\right)^2}\, du$$

Standard Integral Formula

$$\int \sqrt{u^2 + a^2}\, du = \frac{u}{2}\sqrt{u^2 + a^2} + \frac{a^2}{2} \ln\left|u + \sqrt{u^2 + a^2}\right| + C$$

where

$$a = \frac{m}{2\pi}.$$

Applying the Formula

$$L = \frac{2\pi}{m} \left[\frac{u}{2}\sqrt{u^2 + a^2} + \frac{a^2}{2} \ln\left|u + \sqrt{u^2 + a^2}\right|\right]_0^R$$

Evaluating the limits:
At $u = R$:

$$\frac{R}{2}\sqrt{R^2 + \left(\frac{m}{2\pi}\right)^2} + \frac{1}{2}\left(\frac{m}{2\pi}\right)^2 \ln\left|R + \sqrt{R^2 + \left(\frac{m}{2\pi}\right)^2}\right|$$

At $u = 0$:

$$\frac{1}{2}\left(\frac{m}{2\pi}\right)^2 \ln\left|\frac{m}{2\pi}\right|$$

Final General Formula

$$L = \frac{2\pi}{m} \left[\frac{R}{2}\sqrt{R^2 + \left(\frac{m}{2\pi}\right)^2} + \frac{1}{2}\left(\frac{m}{2\pi}\right)^2 \ln\left|\frac{R + \sqrt{R^2 + \left(\frac{m}{2\pi}\right)^2}}{m/(2\pi)}\right|\right]$$

Simplifying:

$$L = \frac{\pi R}{m}\sqrt{R^2 + \left(\frac{m}{2\pi}\right)^2} + \frac{m}{4\pi} \ln\left|\frac{R + \sqrt{R^2 + \left(\frac{m}{2\pi}\right)^2}}{m/(2\pi)}\right|$$

Cleaner Form

Let

$$\alpha = \frac{m}{2\pi}.$$

Then:

$$L = \frac{\pi R}{m}\sqrt{R^2+\alpha^2} + \alpha \ln\left|\frac{R+\sqrt{R^2+\alpha^2}}{\alpha}\right|$$

Simplified Formula! Plug and Play Formula

$$L = \frac{\pi R^2}{m}\sqrt{1+\left(\frac{m}{2\pi R}\right)^2} + \frac{m}{4\pi}\ln\left|\frac{2\pi R}{m}\left(1+\sqrt{1+\left(\frac{m}{2\pi R}\right)^2}\right)\right|$$

Just substitute any R and m values and calculate!

Taking the Limit. As m approaches 0, the dominant term $\frac{\pi R^2}{m}$ grows without bound. Since R is fixed and $m \to 0^+$, we have

$$\lim_{m\to 0^+} \frac{\pi R^2}{m} = \infty.$$

Therefore, the killer can be forced to walk an infinite distance while always getting closer.

Key Insights.

- **Local Rule $\neq$ Global Result**: A rule that seems to guarantee an end-goal (getting closer) does not necessarily guarantee the goal will be reached in a finite time or distance.
- **Poorly Specified Objectives**: This illustrates why optimization algorithms need well-defined objectives. An algorithm that 'always moves closer to the goal' sounds reasonable, but without bounding the path complexity, it can behave arbitrarily inefficiently. This appears in gradient descent with poorly tuned learning rates, where the algorithm technically improves at each step but takes an impractically long path to convergence.
- **The Power of Direction**: This spiral demonstrates that rotation allows that **rotation** allows for nearly infinite movement within a very small, finite space.