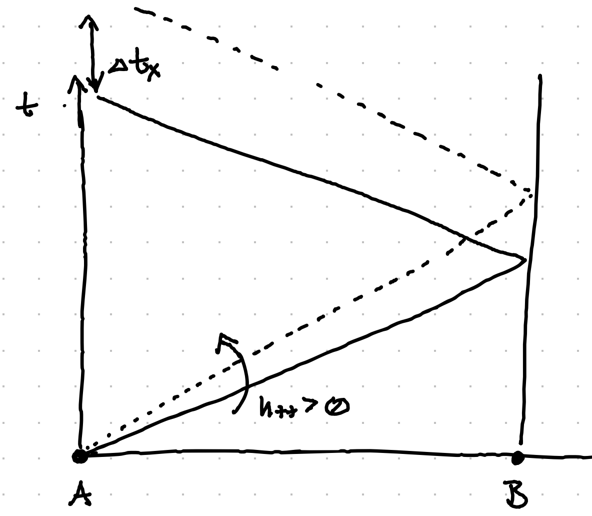
Lecture 4: Gravitational Wave Detector Physics

topics: comoving coordinates
Michelson interferometers
(+ Fabry-Perot cavities)
~~frequency-dependent response~~
detector tensor, peanut patterns
~~noise sources~~
~~technical vs. fundamental noise~~
connections btwn "key parameters" and "observables"
amplitude vs. phase
triangulation
key degeneracies (& how to break them)

Gravitational Waves and Co-moving Observers

working in the transverse-traceless gauge, initially stationary observers will always remain stationary (i.e., at fixed coordinates).

→ however, the distance between them can change!



$$ds^2 = -c^2 dt^2 + (1+h_{xx})dx^2 = 0$$

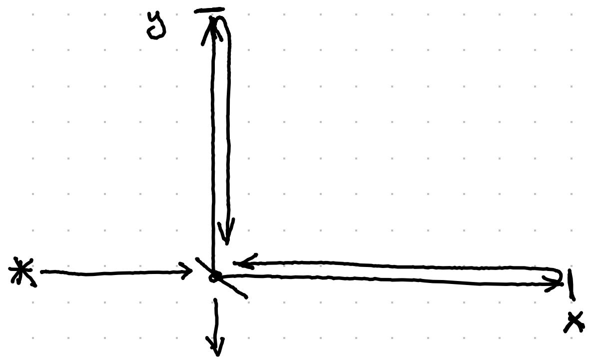
$$\Rightarrow \frac{dx}{dt} = \frac{c}{\sqrt{1+h_{xx}}}$$

strain increases the proper distance btwn A & B

→ light has further to travel

→ the return is <u>delayed</u> relative to an unperturbed system

Now, consider an L-shaped Michelson Interferometer



the round-trip time seen along the x-arm: $t_x = \frac{2L}{c}\sqrt{1+h_{xx}}$

y-arm: $t_y = \frac{2L}{c}\sqrt{1+h_{yy}}$

for the "+ polarization"

∴ the difference in arrival times btwn the arms

$$t_x - t_y = \frac{2L}{c}\left(\sqrt{1+h_{xx}} - \sqrt{1+h_{yy}}\right)$$

$$\approx \frac{2L}{c}\left(\frac{1}{2}h_{xx} - \frac{1}{2}h_{yy}\right)$$

$$\Delta t \approx \frac{2L}{c}(h_+) \quad \text{b/c} \quad h = \begin{bmatrix} h_+ & & \\ & -h_+ & \\ & & 0 \end{bmatrix}$$

we therefore have a way to sense changes in proper separation by relative <u>timing</u> differences for light circulating w/in the cavities.

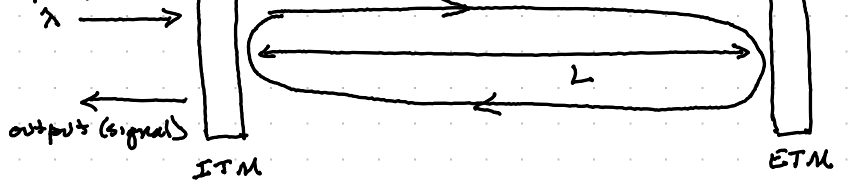
In general, we can write the <u>Detector tensor</u>

$$D^{ij} = \frac{1}{2}(x^i x^j - y^i y^j)$$

and compute the induced strain w/in a detector via

$$h = D^{ij} h_{ij}$$

Current detectors are actually Michelson Interferometers w/ Fabry-Perot cavities w/in each arm.



we match the $\vec{E}$ & $\vec{B}$ fields of travelling waves @ the mirrors to determine the output signal ⇒ resonance when $(L = n\lambda)$

b/c the mirrors are very reflective, we build up a lot of power w/in the cavity where the time-of-flight is most affected by the GW

interference is then btwn the <u>phase</u> of the output signal from different arms.

Key parameters and observables

We record 1-D time series from each interferometric observatory (IFO) and typically decompose this into the <u>amplitude</u> and <u>phase</u>

$$h(t) = A(t)\cos[\phi(t)]$$

or, in the Fourier domain

$$\tilde{h}(f) = \int dt\, e^{-2\pi i f t}\, h(t) = A(f)\, e^{i\psi(f)}$$

As a rule of thumb, intrinsic parameters (masses, spins, etc) affect $\psi(f)$ whereas extrinsic parameters (location, orientation, etc) affect $A(f)$.

$\Rightarrow$ counter example: $A(f) \sim \dfrac{\mathcal{M}^{5/6}}{D_L} f^{-7/6}\, \Omega(\text{orientation, polarization})$

$$\left.\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/2}}\right\}\text{ chirp mass}$$

By measuring $\psi(f)$, we can obtain estimates of $(m_1, m_2, \vec{S}_1, \vec{S}_2, \text{etc})$ which then lets us constrain extrinsic parameters thru A

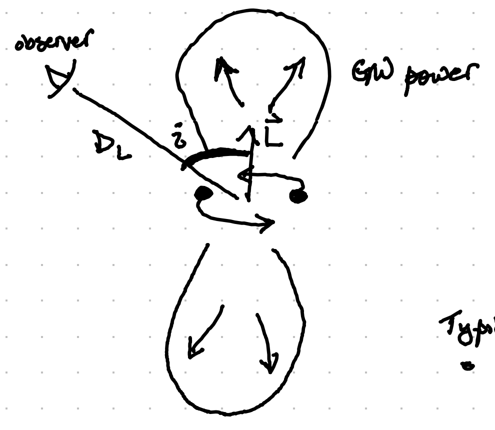
$\Rightarrow$ in general, it is much easier to measure ψ than A b/c we <u>interfere</u> the observed signal w/ theoretical models & or data from other IFOs

$\Rightarrow$ $\psi(f)$ is a rapidly varying function of f

$\Rightarrow$ $A(f)$ is not...

Some key degeneracies:

<u>D_L & orientation (inclination)</u>



GW radiation is <u>anisotropic</u>

so the observed amplitude could correspond to many combinations of D_L and inclination.

(det. response is also anisotropic...)

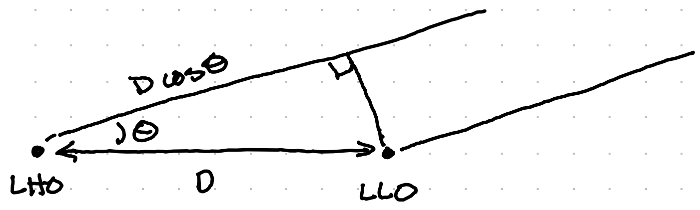
Typical ways to break this:

- inclination dependence is different for each polarization (h_+ vs. $h_\times$)
- inclination dependence is different for higher order modes

<u>Localization</u> (triangulation)

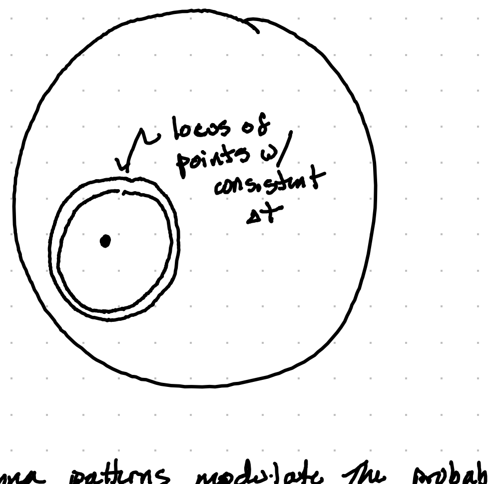
det. response is anisotropic, but typically varies slowly over the sky $\Rightarrow$ IFOs have "broad fields of view" or "high acceptance"

however, GWs travel @ c and therefore arrive at spatially separated IFOs at different times.

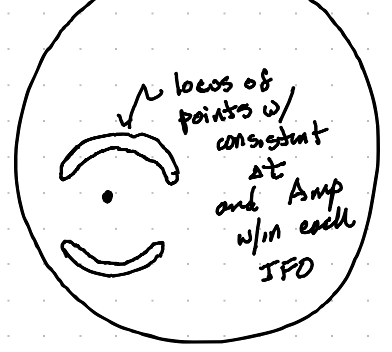


the time delay btwn IFOs is $\underline{\Delta t = \frac{D}{c}\cos\theta}$

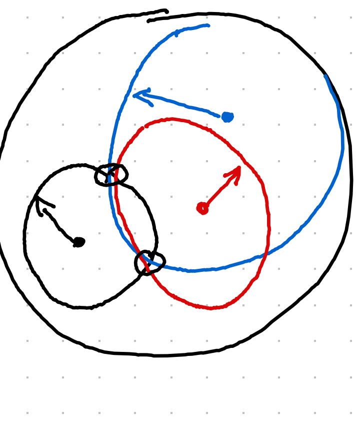
however, this only constrains 1 angle $\rightarrow$ triangulation <u>rings</u>



antenna patterns modulate the probability around the ring (bananas/arcs instead of full rings)



Multiple baselines provide multiple triangulation rings $\rightarrow$ these only intersect at a few points



3 IFOs $\rightarrow \binom{3}{2} = 3$ baselines

$\exists$ reflection symmetry across the plane defined by IFOs

$\Rightarrow$ localize to 2 dots

antenna patterns can then modulate the weights of the dots