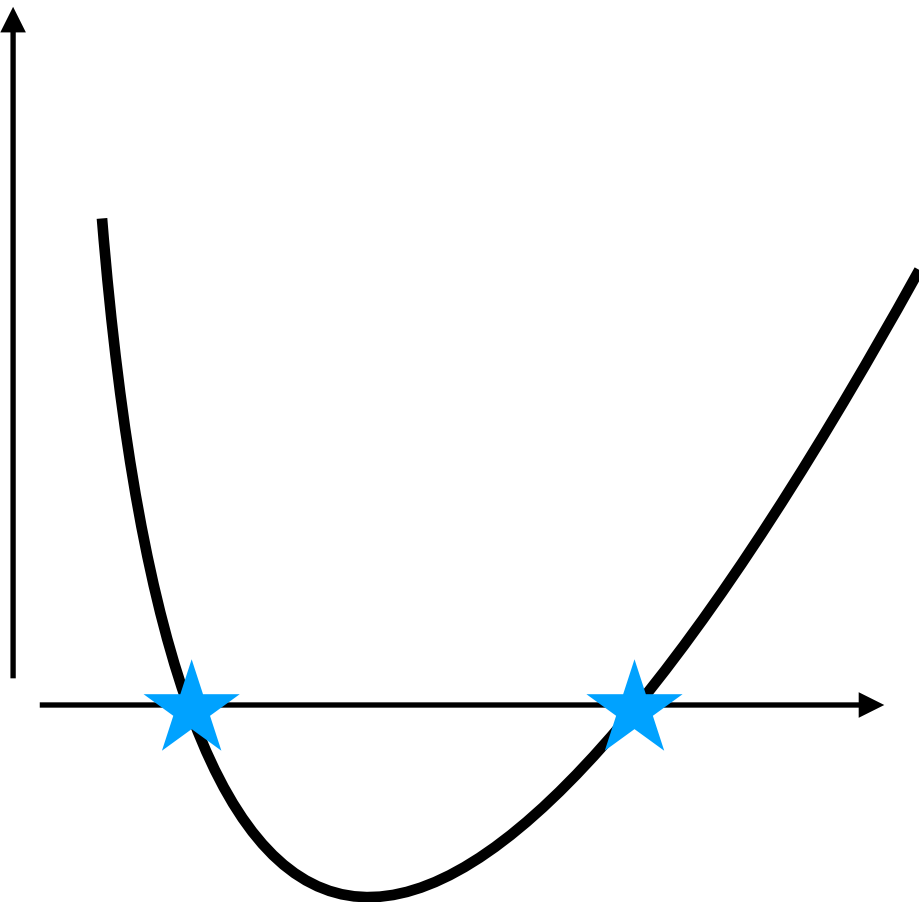# Root finding

Trial and error

1. guess x_1 (trial)
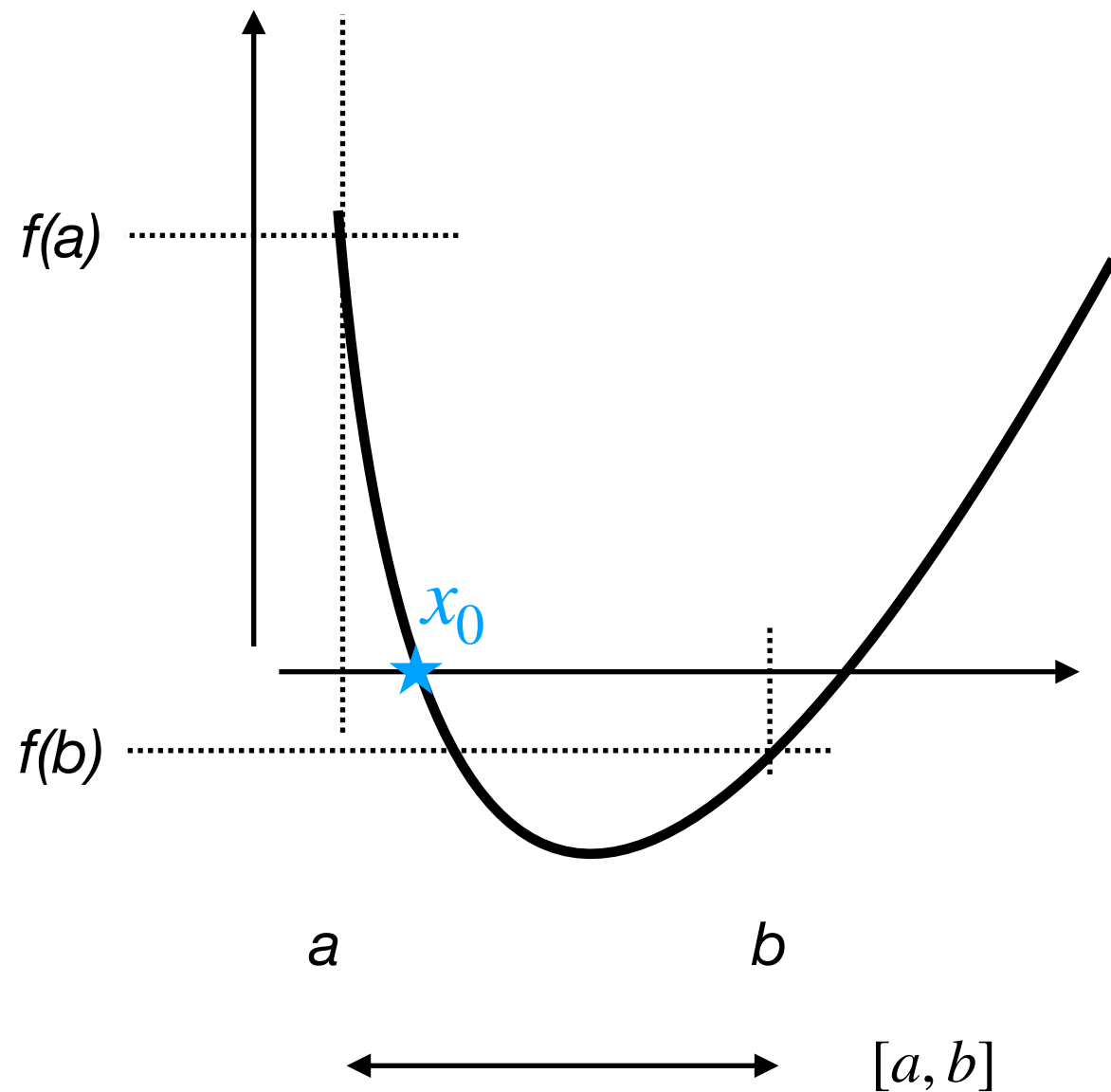
2. Is $f(x_1) = 0$? (error)

3. improve x_1

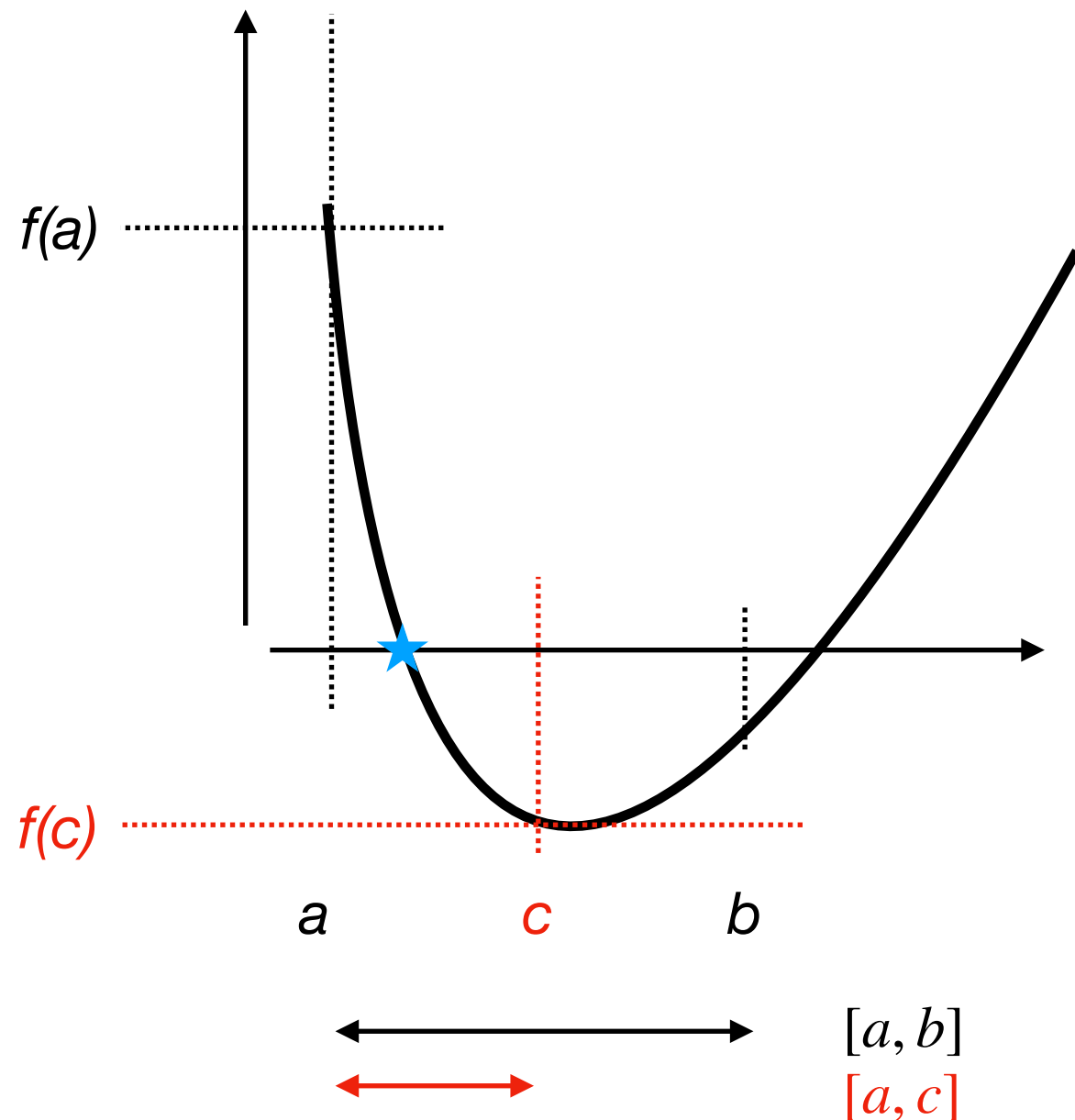until error < eps
(or iterations > max)

Bisection

$$a < x_0 < b$$

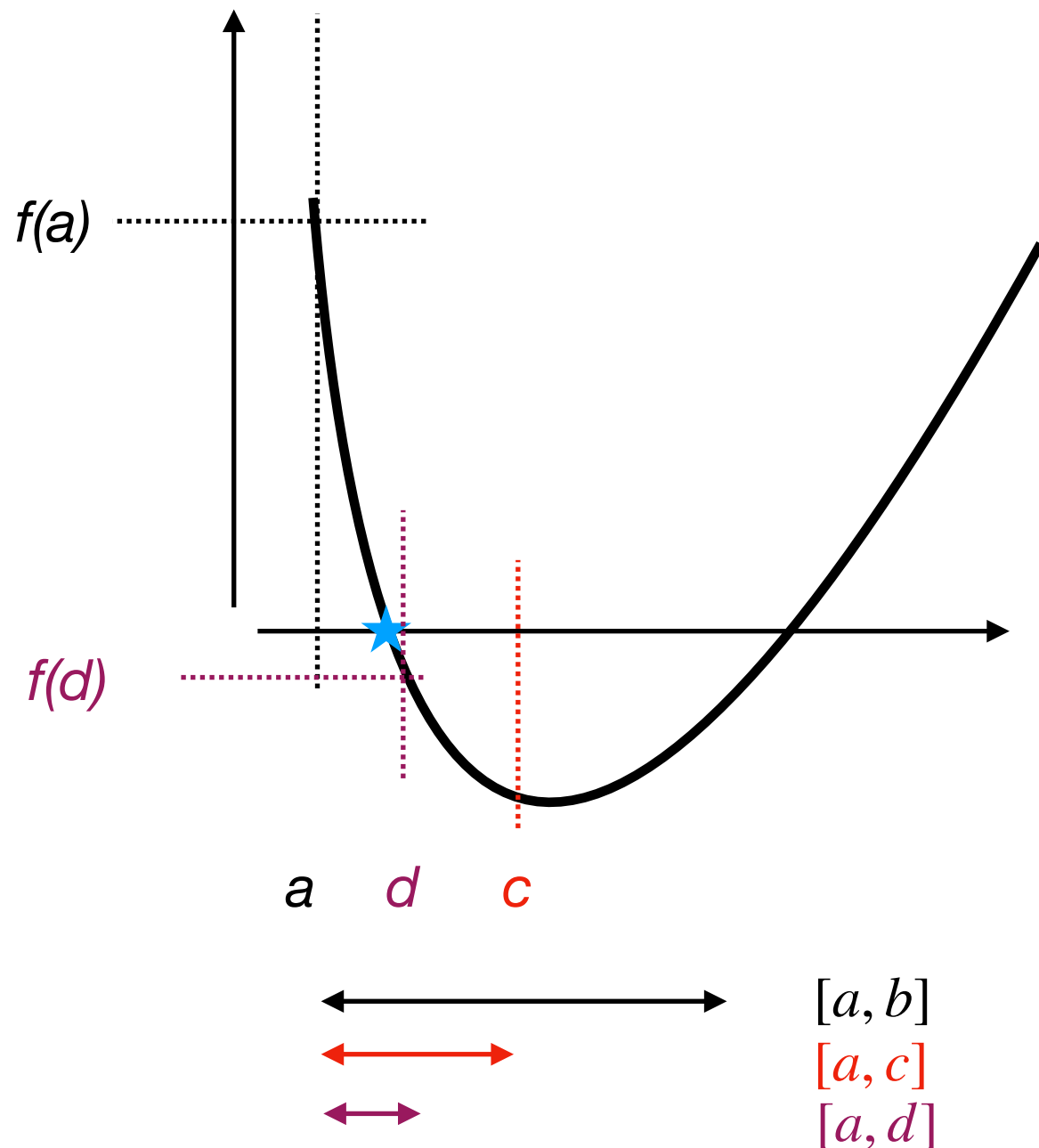$$f(a) > 0 \quad \textbf{and} \quad f(b) < 0$$

Bisection

$f(a) > 0$ **and** $f(c) < 0$

$a < x_0 < c$

Note:

$f(c) < 0$ **and** $f(b) < 0$

so root x_0 is *not* in [c, b]

Bisection

$f(a) > 0$ **and** $f(d) < 0$

$a < x_0 < d$

Note:

$f(c) < 0$ **and** $f(d) < 0$

so root x_0 is *not* in [d, c]

Bisection
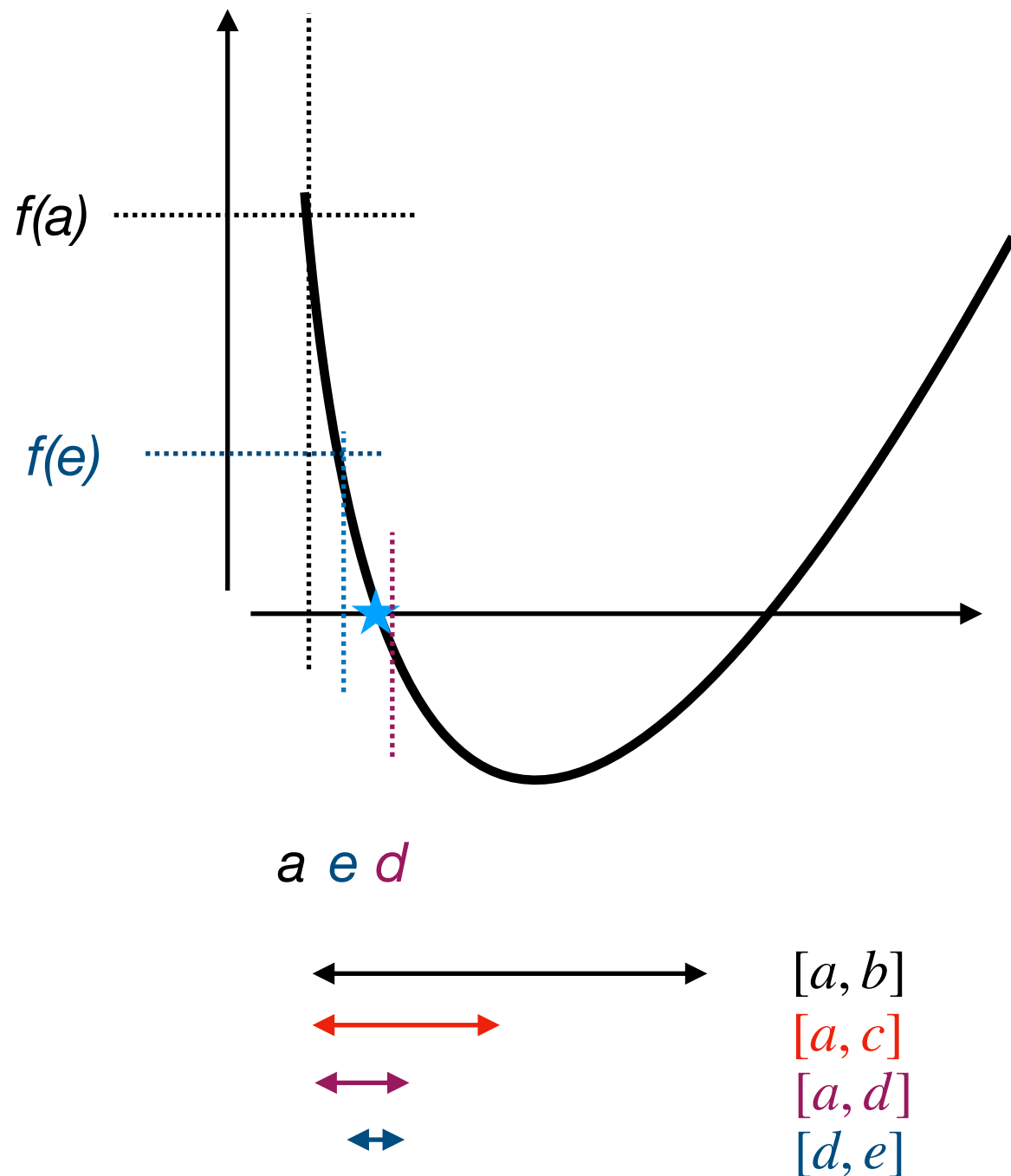
$f(e) > 0$ **and** $f(d) < 0$

$e < x_0 < d$

Note:

$f(a) > 0$ **and** $f(e) > 0$

so root x_0 is *not* in [a, e]

Bisection algorithm

1. bisect

2. pick half with sign change

3. $|f(x)| < $ eps ?

$$x = \frac{1}{2}(a + b)$$

if $f(a)f(x) < 0$

$x_0 \in [a, x]$

$b \leftarrow x$

else

$x_0 \in [x, b]$

$a \leftarrow x$

Root finding with trial and error
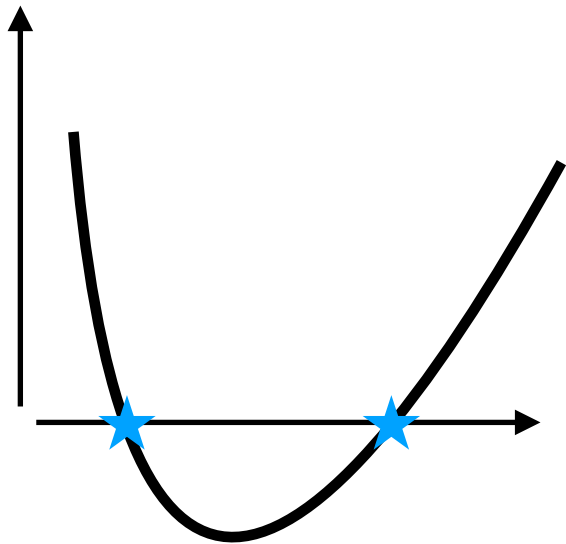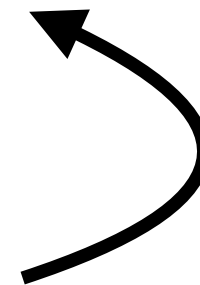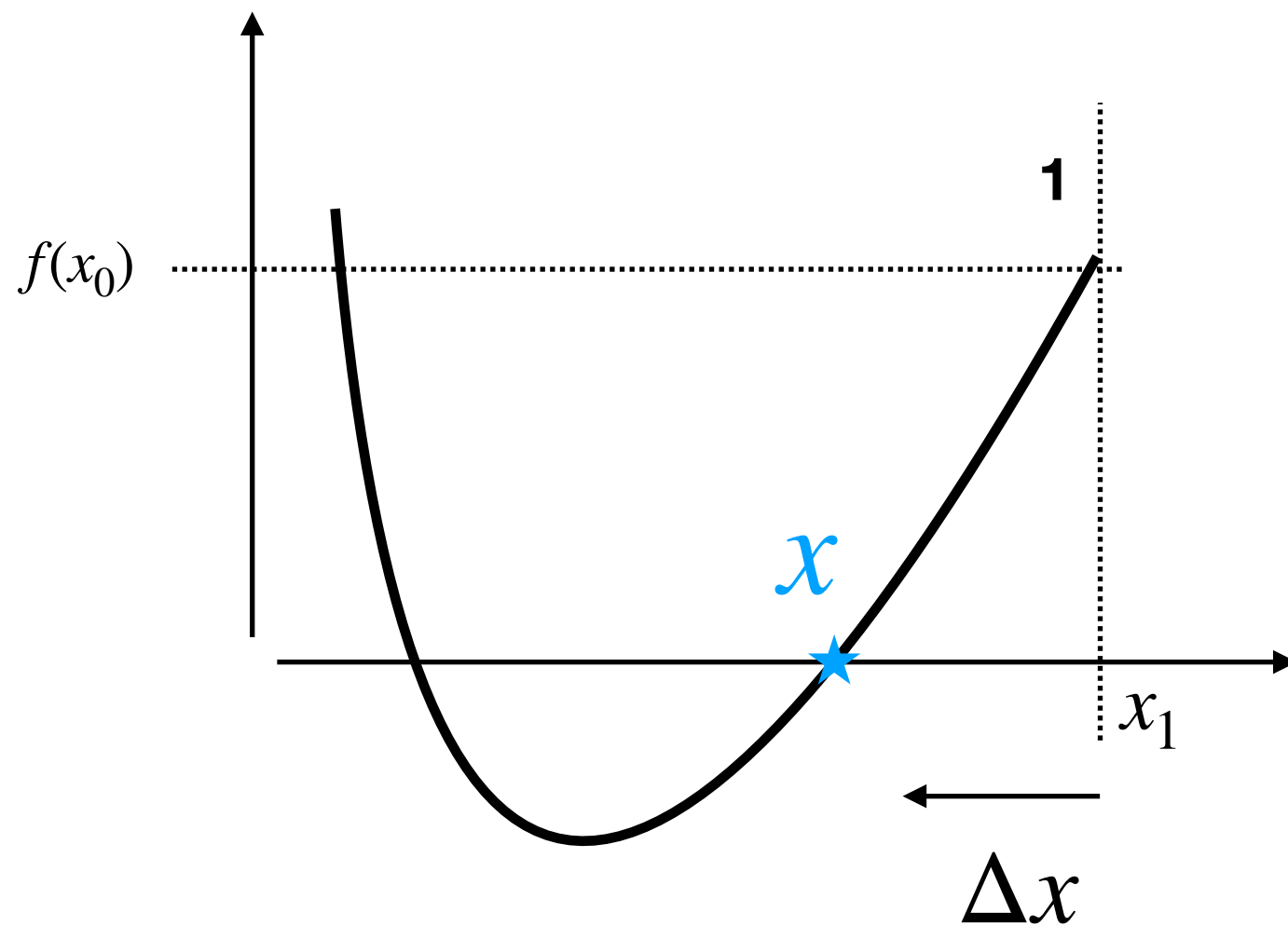
1. guess x_1 (trial)
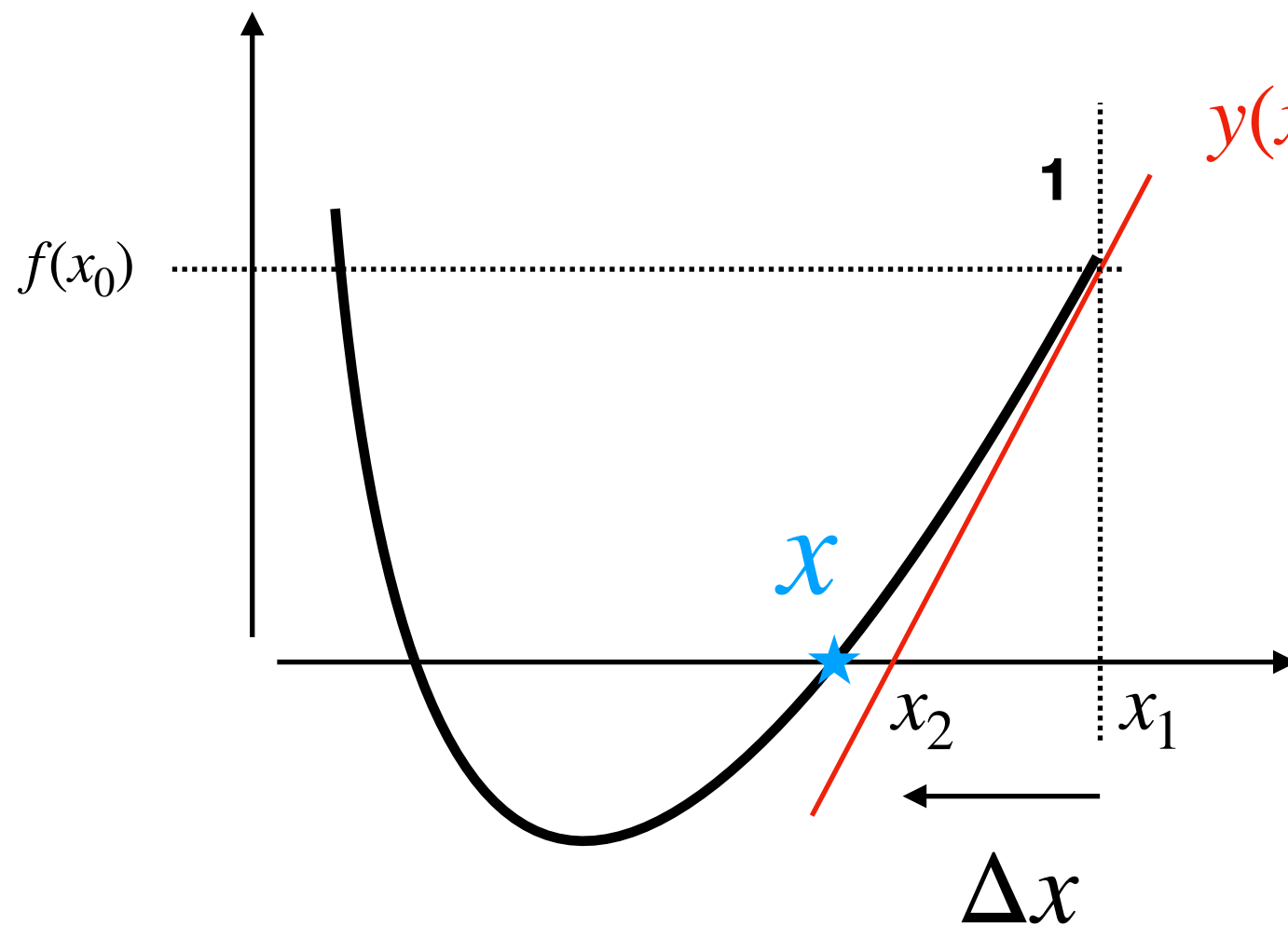
2. Is $f(x_1) = 0$? (error)

3. improve x_1

until error < eps
(or iterations > max)

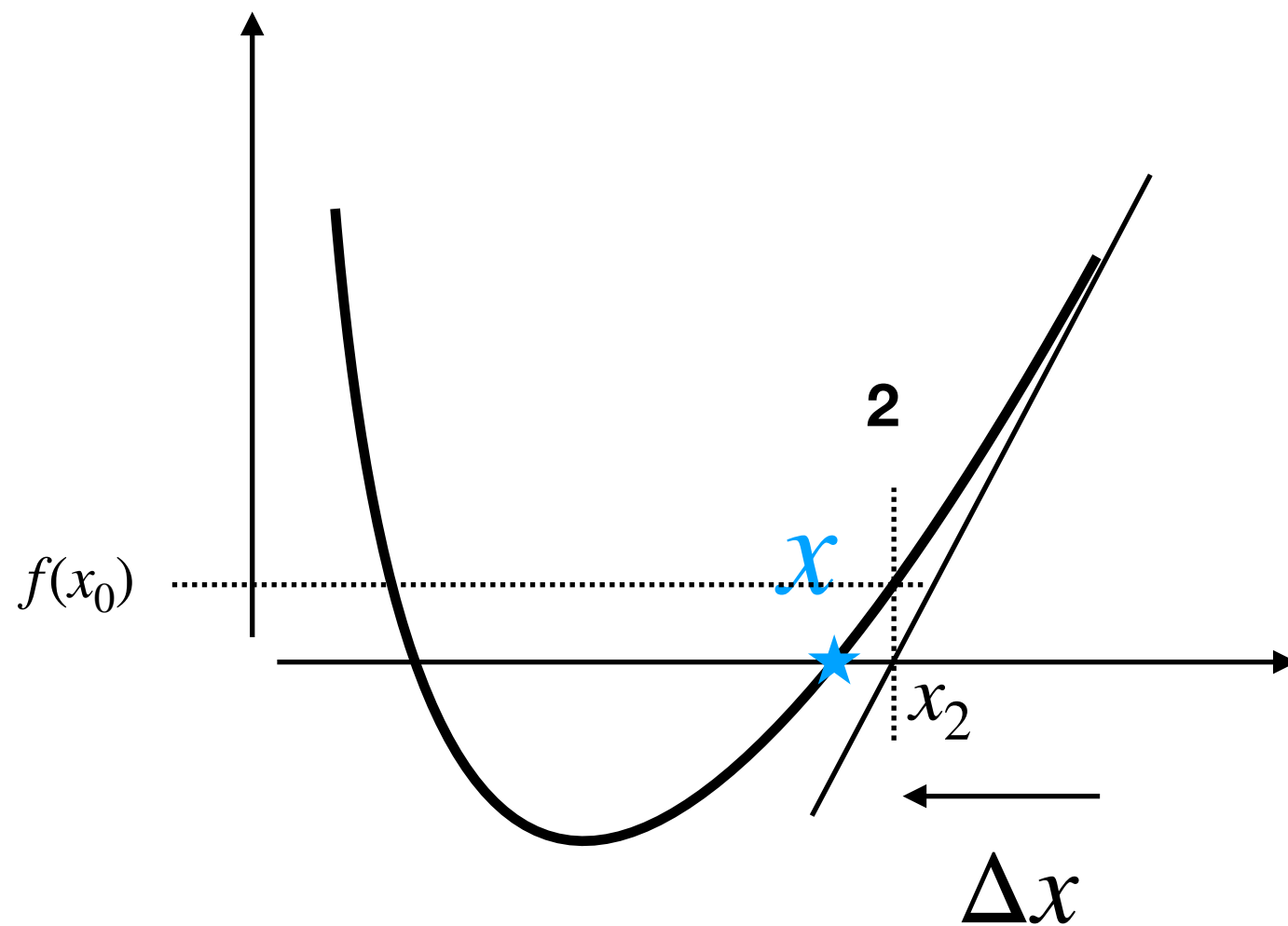# Newton-Raphson

Newton-Raphson
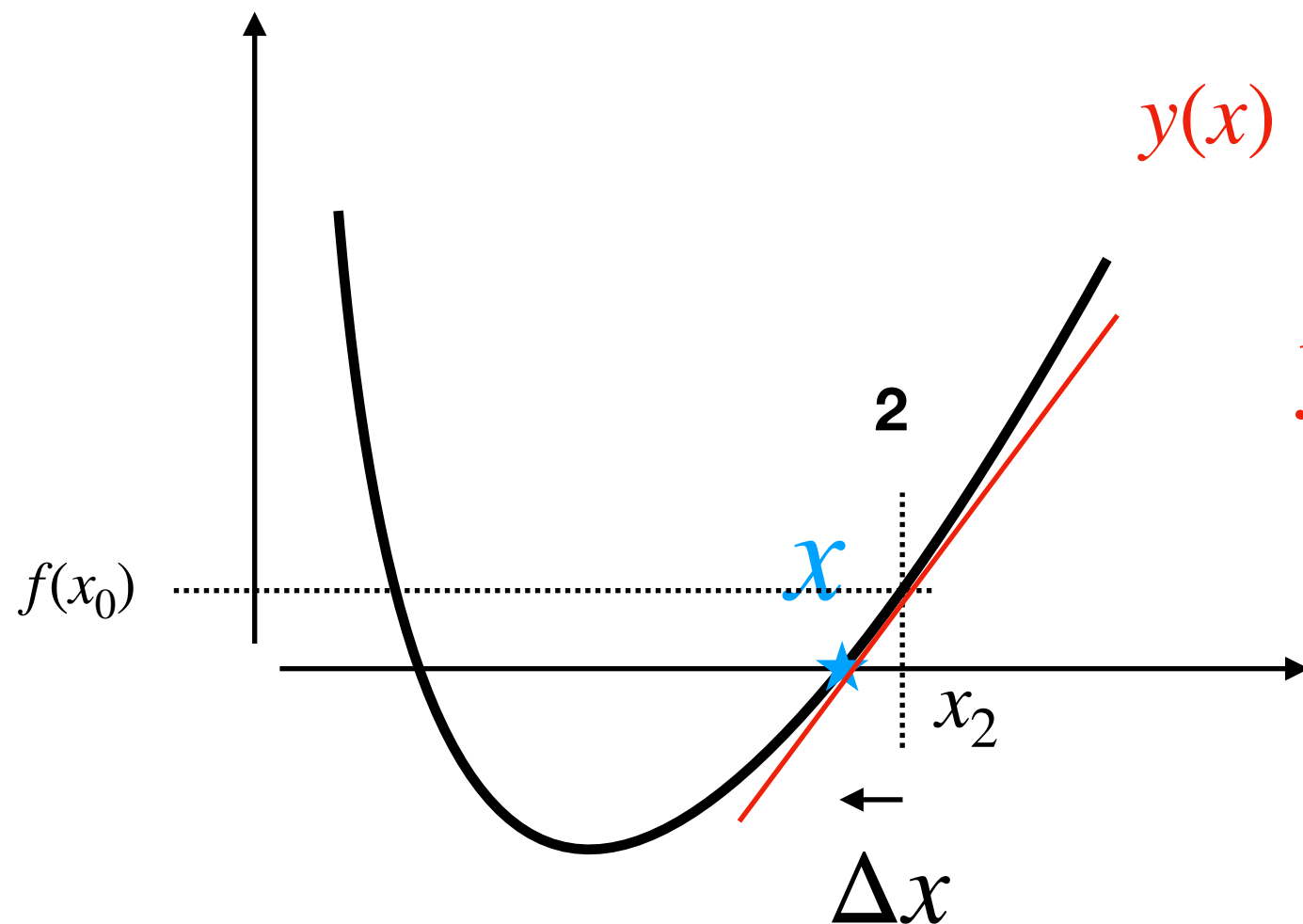

$$y(x) = mx + b$$

$$y(x) = f'(x_1)x + x_2$$

$$x_2 = x_1 + \Delta x$$

Newton-Raphson

Newton-Raphson

$$y(x) = mx + b$$

$$y(x) = f'(x_2)x + x_3$$
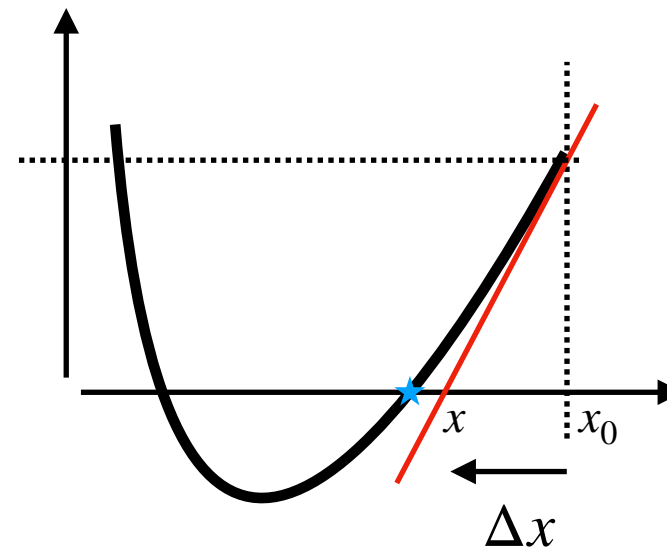
$$x_3 = x_2 + \Delta x$$

NR algorithm

x_0 **initial guess for root**

x **updated guess**

$x = x_0 + \textcolor{red}{\Delta x}$ **correction?**

$y(\xi) = f'(x_0)\xi + x$

$x_2 = x_1 + \Delta x$

$f(x = x_0 + \Delta x) \approx f(x_0) + \Delta x \dfrac{df}{dx}\bigg|_{x_0}$

$f(x_0) + f'(x_0)\Delta x = 0$

$\textcolor{red}{\Delta x = -\dfrac{f(x_0)}{f'(x_0)}}$

while $|f(x) > \epsilon|$

$\Delta x = -\dfrac{f(x)}{f'(x)}$

$x \leftarrow x + \Delta x$
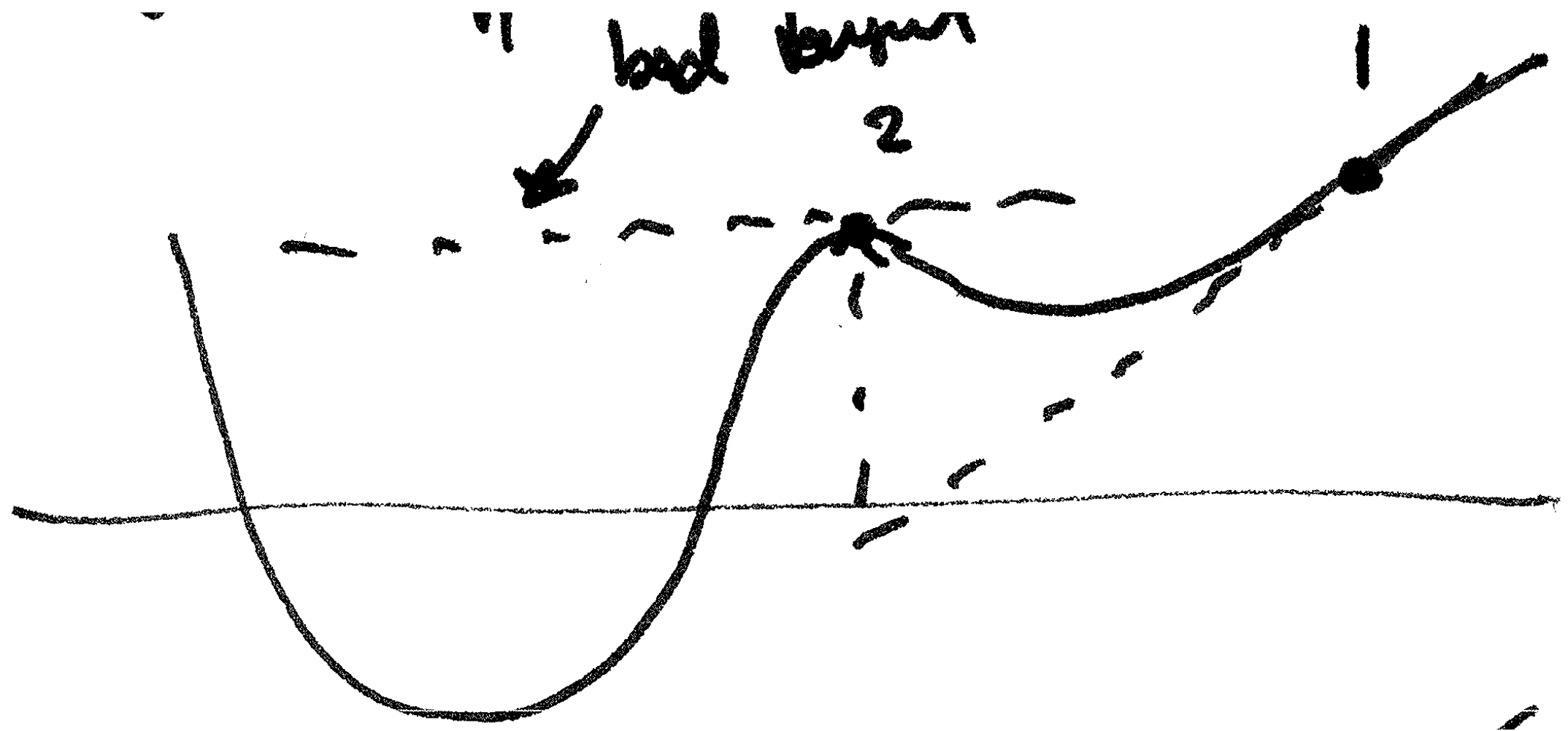
Newton-Raphson

Advantages

- converges very quickly (quadratical convergence!!)

- fast

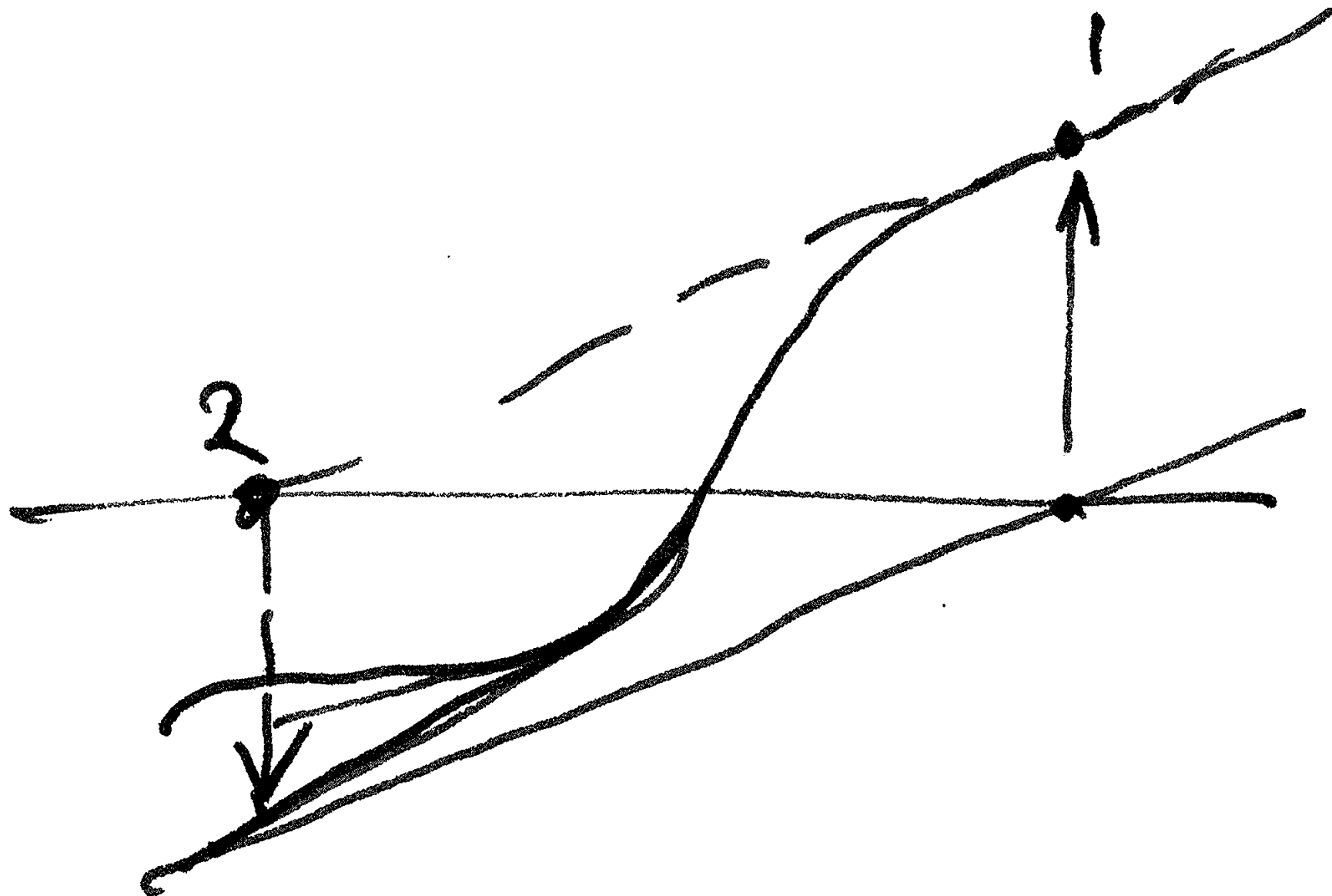- works best with analytical derivative (but can use numerical ones)

Disadvantages

- guess must be close to root

- can fail/loop in certain situations:

NR – FAILS!

NR – FAILS!

Improvements

- start with bisection to get close to root, then home in with Newton-Raphson

- implement *backtracking* : if new guess increases error then go back and try smaller guess

$$x \leftarrow x + \Delta x / 2$$

Newton-Raphson

$f(x) = 0$

$f(x) \approx mx + b$

$\Delta x = -\dfrac{b}{m}$ improved guess