# Fluid Dynamics in 2 Dimensions

Nathan Simon

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## 1 Problem

In general, the problem of fluid simulations is described by the following equations after some simplifcation:

$$\begin{aligned} \nu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{1}{\rho}\frac{\partial P}{\partial x}&=f \\ \nu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{1}{\rho}\frac{\partial P}{\partial y}&=g \\ \frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}&=0 \end{aligned} \tag{1}$$

These describes the conservation of momentum for a system of compressible fluids. The fluid velocities in the x and y directions are represented by $u$ and $v$ respectively while pressure is represented by $P$. The numerical solution to these equations describes the flow velocities for every component of our discretized system. The key differentiating component will be the proposed fluid properties and boundary conditions. With respect to the first, some fluid conditions will allow for quite a bit of simplification. In particular, taking a fluid to be incompressable will remove time dependence from the the third equation (the continuity equation). If we further take $\nu$ (fluid viscosity) to be relatively large, then the linear terms of the first two equations can also be approximated as 0. Moving on to boundary conditions, these will largely define the long-term behavior of the system. Specifically sources of flow and obstacles in the environment will define currents throughout.

## 2 Approach

This problem first requires the creation of specific boundary conditions to describe the environment. That taken care of the two key methods are being proposed to solve the above equations are the finite element and finite difference methods. Each has its own advantages for this problem with the additional work varying between code setup and equation manipulation. Finite difference tends to be preferred as the construction of basis vectors for finite element method can become difficult beyond 1 dimension but finite element may be useful as it relies far less on manually descretizing differential components (which there are a lot of) in the original equation.

## 3 Objectives

- Determine a steady-state solution for arbitrary boundary/initial conditions
- Visualize dynamic solution evolution to steady-state
- Simulate compressible fluids
- Quantitatively estimate the accuracy of the simulation