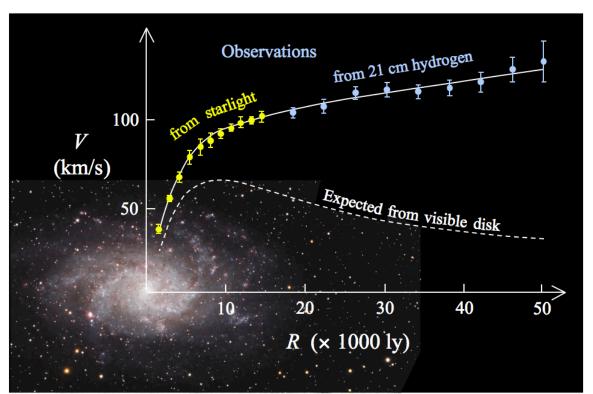


Alexander Warren

Galactic Rotation Curve Problem



- Rotational velocity of galaxies ≠ predictions.
- Matter rotating too fast for their orbits to be stable.
- Explained by "dark matter" or modified gravity laws.

Dilemma! – We must either:

- 1) Invent a new type of matter that has never been seen before but nevertheless completely dominates over normal matter in the universe, or
- 2) Modify Newton's Universal Law of Gravity (and GR).

Modified Newtonian Dynamics (MOND)

- $\overrightarrow{F}_{Gravity} = -G \frac{m_1 m_2}{r^2} \hat{r} = m\mu(a_0, a) \overrightarrow{a}$
- $\mu(a_o, a)$ modifies acceleration, variants include

$$\left(1+\frac{a_o}{a}\right)^{-1}$$
 and $\left(1+\left(\frac{a_o}{a}\right)^2\right)^{-1/2}$

- Physical constant $a_o = 1.2 \times 10^{-10} ms^{-2}$ introduced. Oddly $a_o \approx \frac{cH_o}{2\pi} \approx c^2 \sqrt{\Lambda/3}$ and $\frac{c^2}{a_o} \approx 10^{26} m$, the current Hubble diameter.
- 2nd Law & Equivalence Principle Violated (scandalous!)





Crazy Idea? Maybe Not!

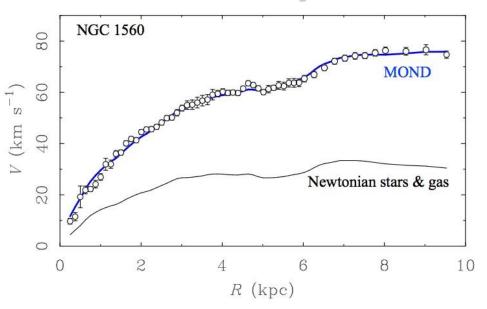


Fig. 1: NGC 1560
Comparison of the observed rotational velocities of material in galaxy NGC 1560 (white data points) with the Newtonian and MOND predictions.

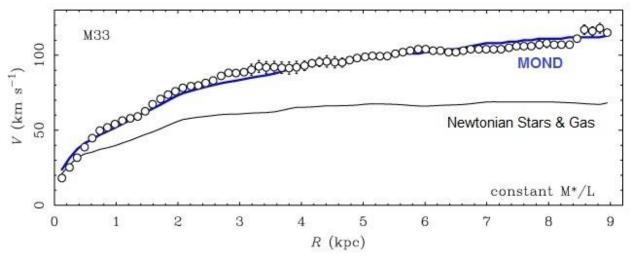


Fig. 2: M33 Same comparison for galaxy M33.

Approach

Simulate M33, populate initial lattice with particles according to mass distribution model:

$$M(r)M_{\odot} = \begin{cases} 460e^{(-\frac{r}{1.56kpc})}, & r \le 6kpc \\ 9.33e^{(+\frac{r}{125kpc})}, & r > 6kpc \end{cases}$$

- Calculate initial velocities for each particle to maintain circular orbits in Newtonian gravity and MOND with both variants of $\mu(a_o, a)$.
- Time evolve each system with velocity Verlet.
- Calculating acceleration will require root finding each time step i.e. $\frac{F}{m} = a \left(1 + \frac{a_o}{a}\right)^{-1}$, solve for a.

Goals

- Implement a 2D lattice of particles that replicates the distribution of baryonic matter throughout galaxy M33.
- 2. Calculate the initial velocities needed for the particles to maintain circular orbits in this configuration under:
 - i. Classical Newtonian gravitation
 - ii. MOND with $\mu(a_o, a) = \left(1 + \frac{a_o}{a}\right)^{-1}$
 - iii. MOND with $\mu(a_o, a) = \left(1 + \left(\frac{a_o}{a}\right)^2\right)^{-1/2}$.
- 3. Implement velocity Verlet loop to simulate the evolution of each system over time.
- 4. Plot the velocity for a particle at a given radius from the core for each system at different time steps, compare against observational. Is the behavior stable?
- 5. Measure average kinetic and potential energy for each system over time.