Simulating Reaction-Diffusion Textures Using Turing's Partial Differential Equations from Chemical Basis of Morphogenesis

Background:

The great mathematician Alan Turing, known for being the father of computer science as well as cracking the Nazis' Enigma code during WWII amongst other things, attempted to theorize about <u>animal development from chemical substrates</u>.

Question:	Is it possible to suggest that certain well-known physical laws are sufficient to account for reaction-diffusion textures in nature?

Central Idea/Model:

Suppose there are two homogeneously distributed substances, P and S, within a certain space (Fig 2 in ppt). Also suppose P stimulates the production of itself while S acts as a long-range inhibitor of which inhibits the diffusion of P. This causes P to diffuse slower than S. The system of these two different interacting molecules, called morphogens, establish chemical gradients through a reaction-diffusion system.

Propagation depends on four variables per morphogen (Fig 2 in ppt): rate of production, rate of degradation, rate of diffusion, and strength of the activating/inhibiting interactions. After a certain amount of time, a stable pattern of standing waves form into the patterns we know and love in nature.

Approach/Methods/Goals:

Given Turing's original reaction-diffusion PDEs:

$$\frac{\partial u}{\partial t} = F(u,v) - d_u v + D_u \Delta u \quad \rightarrow \quad \frac{\partial a}{\partial t} = C_s(\alpha - ab) + C_a \nabla^2 a$$

$$\frac{\partial v}{\partial t} = G(u,v) - d_v v + D_v \Delta v \quad \rightarrow \quad \frac{\partial b}{\partial t} = C_s(ab - b - \beta) + C_b \nabla^2 b$$

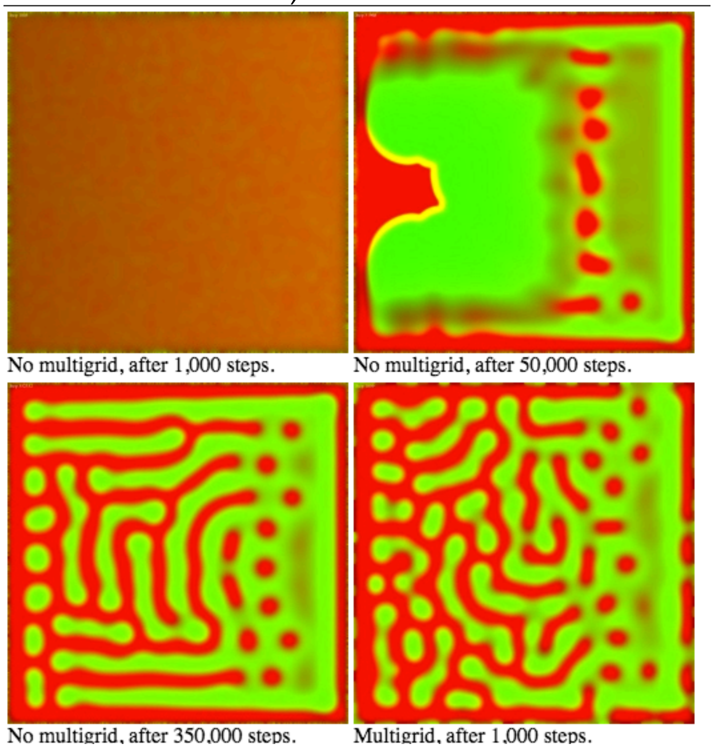
where C_s is the reaction rate parameter, C_a and C_b are diffusion rate parameters, α is a growth parameter, and β is a decay parameter. The coefficients need to be scaled in order to produce spatial features according to desired units.

After that, some type of initial conditions need to be established. Need to figure out the parameter/scale coefficients. Assuming that the system starts out with two homogeneously distributed substances, a grid type of structure may be appropriate.

Next, the continuous equations need to be discretized via a finite difference method. An appropriate numerical analysis technique that can model both the 1st order time derivatives and 2nd order spatial derivatives is the Crank-Nicolson method.

$$\frac{\partial U}{\partial t}\left(x, t + \frac{\Delta t}{2}\right) \approx \frac{U(x, t + \Delta t) - U(x,t)}{\Delta t} + O(\Delta t^2)$$

Referring back to the structure, need to create a finite-sized grid with periodic boundary conditions. Research has suggested the usefulness of a multi-grid in helping accelerate the convergence of standing wave patterns, and dramatically cutting the number of steps (which are also to be determined).



No multigrid, after 1,000 steps. No multigrid, after 50,000 steps.
No multigrid, after 350,000 steps. Multigrid, after 1,000 steps.

Plots; the fun part:

A decent goal would be to figure out what determines the types of textures that a system will produce. Target textures would be as follows: stripes, spots (possibly two kinds), the ***donut/rings, and the ***giraffe-esque mosaic patterns. There is a possibility that different shapes may emerge as seen in Figure 3 of the ppt.