

Title: Stern-Gerlach Experiment simulation  
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In 1922 Otto Stern and Walther Gerlach performed a revolutionary experiment that would confirm the quantization of the electron spin. The experiment was a big accomplishment for the quantum theory of the atom, as it confirmed there was two orientations for electron spin ( $+1/2$ ,  $-1/2$ ). The goal of this simulation would be to recreate results seen by the revolutionary Stern-Gerlach experiment. The experiment consisted of passing silver atoms, used as silver has an unpaired 5s electron, through an inhomogeneous magnetic field. The electron has an electron spin magnetic moment, which wants to align itself with this magnetic field producing a torque. But apart from the torque, there is also a force on a magnetic dipole defined as by equation 1. This means that if we introduce an electron to an inhomogeneous magnetic field, there will be a split depending on the sign of the spin. Saving all the mathematical details in between for later, we arrive with the final z-axis force in equation 3. This result was the core to Stern-Gerlach's experiment, and the results replicated them. This results demonstrated that electrons with a spin  $+1/2$  experience a force in the upward direction, while spin  $-1/2$  in the downward direction. This split was the key result that confirmed many theories in quantum mechanics and opened it to becoming a now widely accepted theory.

Equation 1:  $\mathbf{F} = \nabla(\boldsymbol{\mu} \cdot \mathbf{B})$

Equation 2:  $\mathbf{B} = \langle \alpha x, 0, (B_0 + \alpha z) \rangle$

Equation 3:  $F_z = \gamma \mu_B S_z$ ;  $S_z = (\pm 1/2)$  Thus, F is either up, or down along z axis.

Resource:

**"Introduction to Quantum Mechanics", Prentice Hall, 1995. By: David Griffiths**

Approach

Electrons *Either* have spin  $1/2$  or  $-1/2$ , there is no other solution. In our array of spin values, we need to simulate this as such. This can be done by assorting random values for each index in the spin array, and using the modulus function to dictate if that index would be spin  $1/2$  or  $-1/2$ . Using equation 3, along with an updating position array, we should be able to find the final position of each electron. The biggest challenge will be to find an updating integrator that can handle such small time-steps. This also needs to replicate no force with a constant magnetic field.

Objectives

1. Produce code that allocates each array with a random spin ( $+1/2$ ,  $-1/2$ ).
2. Since this is a simulation, we need some way of integrating over very small amounts of time. These atoms travel very fast, so varlet integration may not be the best in regards to updating position.
3. Updating positions will allow us to see how these atoms are reacting to the magnetic field, purely based on the sign of their spin. Do these match what we would expect?
4. Create a 3d simulation that shows the atom going into the magnetic field, and either go up or down. These should leave a mark for each trial. So we get the classic Stern-Gerlach electron split.