2 — PHY 494: Makeup assignment 2 (20 points total)

Due Wednesday, May 4, 2016, 5pm.[1]

This is a **optional Makeup Assignment**. If you choose to hand it in by the deadline, it will be graded like a normal homework assignment. If its grade is better than your worst homework grade then it will replace that grade.

Submission is to your **private GitHub repository**.

Enter the repository and run the script `scripts/update.sh` (replace *YourGitHubUsername* with your GitHub username):

```
cd assignments-2016-YourGitHubUsername
bash ./scripts/update.sh
```

It should create three subdirectories[2] `makeup_02/Submission`, `makeup_02/Grade`, and `makeup_02/Work` and also pull in the PDF of the makeup and an additional file.

To submit your makeup assignment, commit and push Python code inside the `makeup_02/Submission` directory. *Commit any other additional files exactly as required in the problems.*

Failure to adhere to the following requirements may lead to homework being returned ungraded with 0 points for the problem.

- Only submit code.
- All code should be in a file `makeup02.py`.
- Code will be tested against the unit tests in `test_makeup02.py`. The grade will be approximately proportional to the number of tests that pass successfully so your code *must* be able run under the tests. (Failing tests for the Bonus problem can be ignored.)

2.1 Temperature distribution in a wall of a house in Phoenix [20 points]

We want to determine the temperature distribution $T(x, t)$ (1d) in the wall of a house that is kept at a constant temperature of $T_{\text{in}} = 293$ K (20°C, 68°F) on the inside. On the outside we want to model the diurnal temperature variation of a typical Phoenix day in summer (July) with average lows of 83°F (28.3°C, 301.4 K) and average highs of 106°F (41.1°C, 314.1 K) [3].

We need to solve the heat equation for the brick wall of thickness L,

$$\frac{\partial T(x,t)}{\partial t} = \frac{K}{C\rho}\frac{\partial^2 T(x,t)}{\partial x^2} \tag{1}$$

[1] Corrections: 2016-04-28: Fixed Eq. 2 (thanks to @andrewdurkiewicz). 2016-05-02: Fixed test `test_makeup02.TestCrankNicholson_T.test_solution` and extended submission deadline by 1 day. 2016-05-05 (after submission): changed initial conditions Eq. 6 to what was implemented in the solution...

[2] If the script fails, file an issue in the Issue Tracker for PHY494-assignments-skeleton and just create the directories manually.

[3] Data from http://www.usclimatedata.com/climate/phoenix/arizona/united-states/usaz0166

material	K ($\mathrm{W \cdot m^{-1} \cdot K^{-1}}$)	C ($\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$)	ρ ($\mathrm{kg \cdot m^{-3}}$)
brick	1	900	2000
aluminium	205	870	2700
glass window	0.96	840	2600
wood	0.147	250	500

Table 1: Thermal parameters for selected building materials. Source: http://www.engineeringtoolbox.com

under boundary conditions

$$T(0,t) = \bar{T} + \frac{1}{2}(T_{\text{max}} - T_{\text{min}})\cos[\omega(t - t_{\text{max}})] \tag{2}$$

$$\text{with} \quad \bar{T} := \frac{1}{2}(T_{\text{max}} + T_{\text{min}}) \tag{3}$$

$$\text{and} \quad \omega = \frac{2\pi}{t_{\text{day}}} \tag{4}$$

$$T(L,t) = T_{\text{in}}. \tag{5}$$

Eq. 2 roughly models the change in temperature during a day $t_{\text{day}} = 24 \times 60 \times 60$ s, fluctutating between the minimum and the maximum temperature and peaking around noon, i.e., $t_{\text{max}} = 12 \times 60 \times 60$ s.

As initial condition at $t = 0$ (midnight) we can choose any sensible value or distribution but a simple choice is the outside temperature on the left boundary and the inside temperature everywhere else[4]

$$T(0,0) = T_{\text{min}} \tag{6}$$

$$T(x,0) = T_{\text{in}}, \quad x > 0. \tag{7}$$

To solve this problem, write a Python module `makeup02.py` that contains the functions

T_diurnal(t, Tmin, Tmax, t_max=12*3600) This function takes the current time t (in seconds), the min and max temperatures T_{min} and T_{max} and the time at which the temperature peaks (t_{max}) as parameters. It returns the current temperature, as described by Eq. 2.

[4] An alternative condition is to have everywhere the outside temperature

$$T(x,0) = T(0,t=0) \equiv T_{\text{min}}.$$

In principle, the solution at larger times does not depend sensitively on the initial choice. However, the *tests* were written based on a solution that implements Equations 6 and 7 and thus these initial conditions are prescribed in this problem.

CrankNicholson_T(L, Dx, Dt, step, t_max, Kappa, CHeat, rho, Tin, Tmin, Tmax) This function solves Eq. 1 with the given boundary conditions using the Crank-Nicholson algorithm. It returns a tuple `(T_plot, parameters)` with the saved temperature profile every `step` iterations and `parameters = (Dx, Dt, step)`.

Use the provided skeleton code to get started and to implement the required function calls. Unit tests are provided in `test_makeup02.py` (which you can run in the usual way). Grading of this assignment will primarily rely on all tests passing.

(a) Assume that the brickwall has a thickness $L = 0.3$ m and has the thermal parameters given in Table. 1. Compute the temperature distribution $T(x,t)$ over three days. Plot $T(x,t)$. Submit your code and a plot (use the provided function `plot_surface(T_plot, *parameters)`)(Do you see how the brick wall stores heat during the night and then provides heat to its surroundings?) **[20 points]**

(b) Bonus: Investigate other materials with parameters in Table 1. Assume realistic lengths.[5] **[bonus +2*]**

(c) Bonus: Investigate heat flow through the wall.[6] **[bonus +0*]**

[5] No unit tests, free form submission. Submission of PDF figures with titles that identify what you did will be sufficient.

[6] No unit tests, no points, just do it if you're curious