

## 2 — PHY 494: Makeup assignment 2 (20 points total)

Due Tuesday, May 3, 2016, 5pm.<sup>1</sup>

This is a **optional Makeup Assignment**. If you choose to hand it in by the deadline, it will be graded like a normal homework assignment. If its grade is better than your worst homework grade then it will replace that grade.

Submission is to your **private GitHub repository**.

Enter the repository and run the script `scripts/update.sh` (replace *YourGitHubUsername* with your GitHub username):

```
cd assignments-2016-YourGitHubUsername
bash ./scripts/update.sh
```

It should create three subdirectories<sup>2</sup> `makeup_02/Submission`, `makeup_02/Grade`, and `makeup_02/Work` and also pull in the PDF of the makeup and an additional file.

To submit your makeup assignment, commit and push Python code inside the `makeup_02/Submission` directory. *Commit any other additional files exactly as required in the problems.*

Failure to adhere to the following requirements may lead to homework being returned ungraded with 0 points for the problem.

- Only submit code.
- All code should be in a file `makeup02.py`.
- Code will be tested against the unit tests in `test_makeup02.py`. The grade will be approximately proportional to the number of tests that pass successfully so your code *must* be able run under the tests. (Failing tests for the Bonus problem can be ignored.)

### 2.1 Temperature distribution in a wall of a house in Phoenix [20 points]

We want to determine the temperature distribution  $T(x, t)$  (1d) in the wall of a house that is kept at a constant temperature of  $T_{\text{in}} = 293 \text{ K}$  ( $20^\circ\text{C}$ ,  $68^\circ\text{F}$ ) on the inside. On the outside we want to model the diurnal temperature variation of a typical Phoenix day in summer (July) with average lows of  $83^\circ\text{F}$  ( $28.3^\circ\text{C}$ ,  $301.4 \text{ K}$ ) and average highs of  $106^\circ\text{F}$  ( $41.1^\circ\text{C}$ ,  $314.1 \text{ K}$ )<sup>3</sup>.

We need to solve the heat equation for the brick wall of thickness  $L$ ,

$$\frac{\partial T(x, t)}{\partial t} = \frac{K}{C\rho} \frac{\partial^2 T(x, t)}{\partial x^2} \quad (1)$$

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<sup>1</sup>Corrections: 2016-04-28: Fixed Eq. 2 (thanks to @andrewdurkiewicz)

<sup>2</sup>If the script fails, file an issue in the [Issue Tracker for PHY494-assignments-skeleton](#) and just create the directories manually.

<sup>3</sup>Data from <http://www.usclimatedata.com/climate/phoenix/arizona/united-states/usaz0166>

material	$K$ ( $\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$ )	$C$ ( $\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$ )	$\rho$ ( $\text{kg} \cdot \text{m}^{-3}$ )
brick	1	900	2000
aluminium	205	870	2700
glass window	0.96	840	2600
wood	0.147	250	500

Table 1: Thermal parameters for selected building materials. Source: <http://www.engineeringtoolbox.com>

under boundary conditions

$$T(0, t) = \bar{T} + \frac{1}{2}(T_{\max} - T_{\min}) \cos[\omega(t - t_{\max})] \quad (2)$$

$$\text{with } \bar{T} := \frac{1}{2}(T_{\max} + T_{\min}) \quad (3)$$

$$\text{and } \omega = \frac{2\pi}{t_{\text{day}}} \quad (4)$$

$$T(L, t) = T_{\text{in}}. \quad (5)$$

Eq. 2 roughly models the change in temperature during a day  $t_{\text{day}} = 24 \times 60 \times 60$  s, fluctuating between the minimum and the maximum temperature and peaking around noon, i.e.,  $t_{\max} = 12 \times 60 \times 60$  s.

As initial condition at  $t = 0$  (midnight) we can choose any sensible value or distribution but a simple choice is the outside temperature,

$$T(x, 0) = T(0, 0) \equiv T_{\min}. \quad (6)$$

To solve this problem, write a Python module `makeup02.py` that contains the functions

**T\_diurnal(t, Tmin, Tmax, t\_max=12\*3600)** This function takes the current time  $t$  (in seconds), the min and max temperatures  $T_{\min}$  and  $T_{\max}$  and the time at which the temperature peaks ( $t_{\max}$ ) as parameters. It returns the current temperature, as described by Eq. 2.

**CrankNicholson\_T(L, Dx, Dt, step, t\_max, Kappa, CHeat, rho, Tin, Tmin, Tmax)** This function solves Eq. 1 with the given boundary conditions using the Crank-Nicholson algorithm. It returns a tuple (`T_plot`, `parameters`) with the saved temperature profile every `step` iterations and `parameters = (Dx, Dt, step)`.

Use the provided skeleton code to get started and to implement the required function calls. Unit tests are provided in `test_makeup02.py` (which you can run in the usual way). Grading of this assignment will primarily rely on all tests passing.

- (a) Assume that the brickwall has a thickness  $L = 0.3$  m and has the thermal parameters given in Table. 1. Compute the temperature distribution  $T(x, t)$  over three days. Plot  $T(x, t)$ . Submit your code and a plot (use the provided function `plot_surface(T_plot, *parameters)`)(Do you see how the brick wall stores heat during the night and then provides heat to its surroundings?) **[20 points]**
- (b) BONUS: Investigate other materials with parameters in Table 1. Assume realistic lengths.<sup>4</sup> **[bonus +2\*]**
- (c) BONUS: Investigate heat flow through the wall.<sup>5</sup> **[bonus +0\*]**

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<sup>4</sup>No unit tests, free form submission. Submission of PDF figures with titles that identify what you did will be sufficient.

<sup>5</sup>No unit tests, no points, just do it if you're curious