## 2 — PHY 494: Makeup assignment 2 (20 points total)

Due Tuesday, May 3, 2016, 5pm.

This is a **optional Makeup Assignment**. If you choose to hand it in by the deadline, it will be graded like a normal homework assignment. If its grade is better than your worst homework grade then it will replace that grade.

Submission is to your private GitHub repository.

Enter the repository and run the script scripts/update.sh (replace *YourGitHubUser-name* with your GitHub username):

cd assignments-2016-YourGitHubUsername
bash ./scripts/update.sh

It should create three subdirectories makeup\_02/Submission, makeup\_02/Grade, and makeup\_02/Work and also pull in the PDF of the makeup and an additional file.

To submit your makeup assignment, commit and push Python code inside the makeup\_02/Submission directory. Commit any other additional files exactly as required in the problems.

Failure to adhere to the following requirements may lead to homework being returned ungraded with 0 points for the problem.

- Only submit code.
- All code should be in a file makeup02.py.
- Code will be tested against the unit tests in test\_makeup02.py. The grade will be approximately proportional to the number of tests that pass successfully so your code *must* be able run under the tests. (Failing tests for the Bonus problem can be ignored.)

## 2.1 Temperature distribution in a wall of a house in Phoenix [20 points]

We want to determine the temperature distribution T(x,t) (1d) in the wall of a house that is kept at a constant temperature of  $T_{\rm in} = 293$  K (20°C, 68°F) on the inside. On the outside we want to model the diurnal temperature variation of a typical Phoenix day in summer (July) with average lows of 83°F (28.3°C, 301.4 K) and average highs of 106°F (41.1°C, 314.1 K)  $^2$ .

We need to solve the heat equation for the brick wall of thickness L,

$$\frac{\partial T(x,t)}{\partial t} = \frac{K}{C\rho} \frac{\partial^2 T(x,t)}{\partial x^2} \tag{1}$$

<sup>&</sup>lt;sup>1</sup>If the script fails, file an issue in the Issue Tracker for PHY494-assignments-skeleton and just create the directories manually.

<sup>&</sup>lt;sup>2</sup>Data from http://www.usclimatedata.com/climate/phoenix/arizona/united-states/usaz0166

material	$K (W \cdot m^{-1} \cdot K^{-1})$	$C (J \cdot kg^{-1} \cdot K^{-1})$	$\rho \; (\mathrm{kg} \cdot \mathrm{m}^{-3})$
brick	1	900	2000
aluminium	205	870	2700
glass window	0.96	840	2600
wood	0.147	250	500

Table 1: Thermal parameters for selected building materials. Source: http://www. engineeringtoolbox.com

under boundary conditions

$$T(0,t) = \bar{T} + (T_{\text{max}} - T_{\text{min}}) \cos[\omega(t - t_{\text{max}})]$$
(2)

with 
$$\bar{T} := \frac{1}{2}(T_{\text{max}} - T_{\text{min}})$$
 (3)

with 
$$\bar{T} := \frac{1}{2}(T_{\text{max}} - T_{\text{min}})$$
 (3)  
and  $\omega = \frac{2\pi}{t_{\text{day}}}$ 

$$T(L,t) = T_{\rm in}. (5)$$

Eq. 2 roughly models the change in temperature during a day  $t_{\rm day} = 24 \times 60 \times 60$  s, fluctutating between the minimum and the maximum temperature and peaking around noon, i.e.,  $t_{\text{max}} = 12 \times 60 \times 60 \text{ s.}$ 

As initial condition at t = 0 (midnight) we can choose any sensible value or distribution but a simple choice is the outside temperature,

$$T(x,0) = T(0,0) \equiv T_{\min}.$$
 (6)

To solve this problem, write a Python module makeup02.py that contains the functions

- T\_diurnal(t, Tmin, Tmax, t\_max=12\*3600) This function takes the current time t (in seconds), the min and max temperatures  $T_{\min}$  and  $T_{\max}$  and the time at which the temperature peaks  $(t_{\text{max}})$  as parameters. It returns the current temperature, as described by Eq. 2.
- CrankNicholson\_T(L, Dx, Dt, step, t\_max, Kappa, CHeat, rho, Tin, Tmin, Tmax) This function solves Eq. 1 with the given boundary conditions using the Crank-Nicholson algorithm. It returns a tuple (T\_plot, parameters) with the saved temperature profile every step iterations and parameters = (Dx, Dt, step).

Use the provided skeleton code to get started and to implement the required function calls. Unit tests are provided in test\_makeup02.py (which you can run in the usual way). Grading of this assignment will primarily rely on all tests passing.

- (a) Assume that the brickwall has a thickness L=0.3 m and has the thermal parameters given in Table. 1. Compute the temperature distribution T(x,t) over three days. Plot T(x,t). Submit your code and a plot (use the provided function plot\_surface(T\_plot, \*parameters))(Do you see how the brick wall stores heat during the night and then provides heat to its surroundings?) [20 points]
- (b) Bonus: Investigate other materials with parameters in Table 1. Assume realistic lengths.  $^3$  [bonus  $+2^*$ ]
- (c) Bonus: Investigate heat flow through the wall.  $^{4}$  [bonus  $+0^{*}$ ]

<sup>&</sup>lt;sup>3</sup>No unit tests, free form submission. Submission of PDF figures with titles that identify what you did will be sufficient.

<sup>&</sup>lt;sup>4</sup>No unit tests, no points, just do it if you're curious