PDEs

$$\phi(x,t), \quad \phi(x,y,z,t)$$

$$\phi(x,y)$$

General:

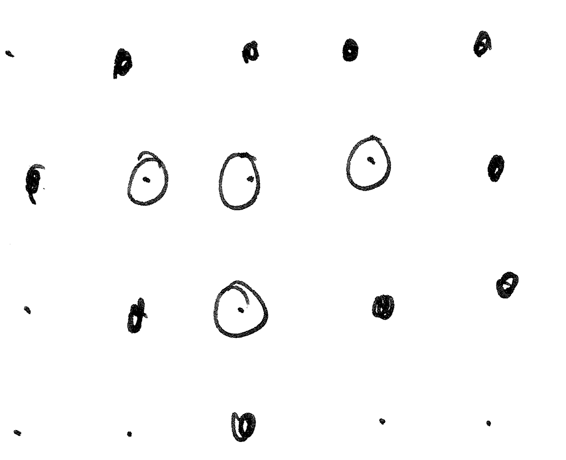
$$A\frac{\partial^2\phi}{\partial x^2} + 2B\frac{\partial^2\phi}{\partial x\partial y} + C\frac{\partial^2\phi}{\partial y^2} + D\frac{\partial\phi}{\partial x} + E\frac{\partial\phi}{\partial y} = F$$

$A = A(x,y)\ldots$

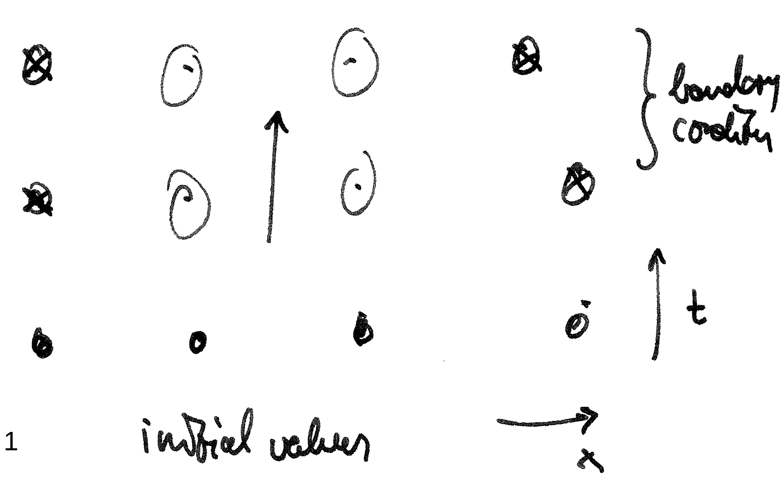
Mathematician:
discriminant $d = AC - B^2$

	d	name	example	
static boundary value	>0	elliptic	$\underline{\nabla}^2\phi(\underline{r}) = -4\pi\rho(\underline{r})$ $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi = -4\pi\rho(x,y)$	Poisson
time evolution initial value	$=0$	parabolic	$D\underline{\nabla}^2\phi(\underline{r}) = \frac{\partial\phi}{\partial t}$	diffusion, heat $\quad \underline{\nabla} D\underline{\nabla}\phi = \frac{\partial\phi}{\partial t}$
	<1	hyperbolic	$\underline{\nabla}^2\phi(\underline{r}) = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$	wave

boundary value problem



initial value problem



Boundary conditions:

values of ϕ on boundary: Dirichlet

values of $\frac{\partial \phi}{\partial x}$ normal to bound: Neumann

+ others

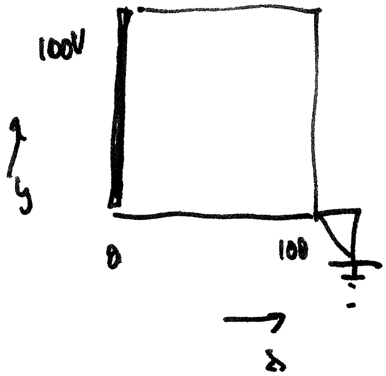
Different types of equation require different boundary conditions for unique solutions.

Solving PDEs:

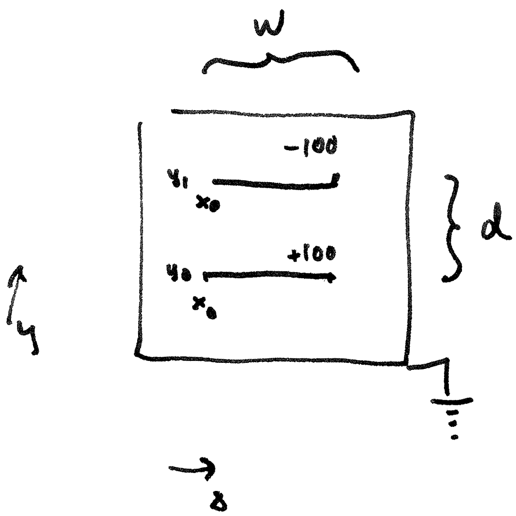
- harder than ODEs : all ODEs:

$$\frac{d\underline{y}(t)}{dt} = \underline{f}(\underline{y}, t) \quad \leftarrow RK4\ldots$$

- specific to problem and boundary conditions



wire on a box



capacitor in a box