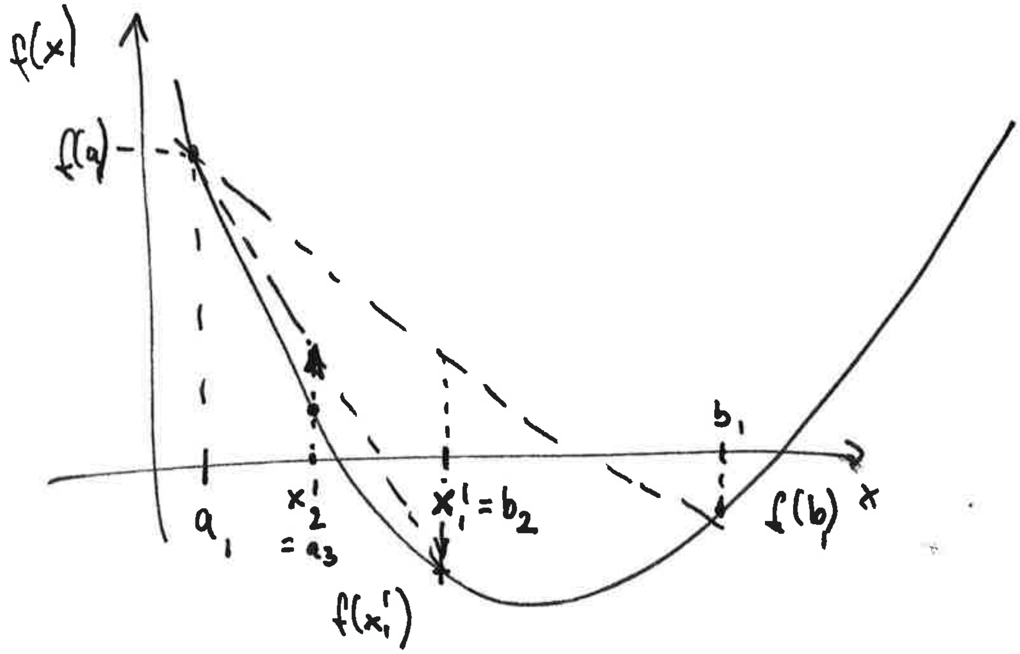
2019-03-21

Bisection



Trial & error to find $f(x_0) = 0$

1. guess x_1 (trial)
2. $f(x_1) \stackrel{?}{=} 0$ (error)
3. improve $x_1 \to x_2$

until error acceptable or iterations exceeded.

Bisection

$a < x_0 < b$ $\quad \left\{ \begin{array}{l} f(a) > 0 \text{ and } f(b) < 0 \\ f(a) < 0 \text{ and } f(b) > 0 \end{array} \right.$

i.e. root x_0 in $[a, b]$

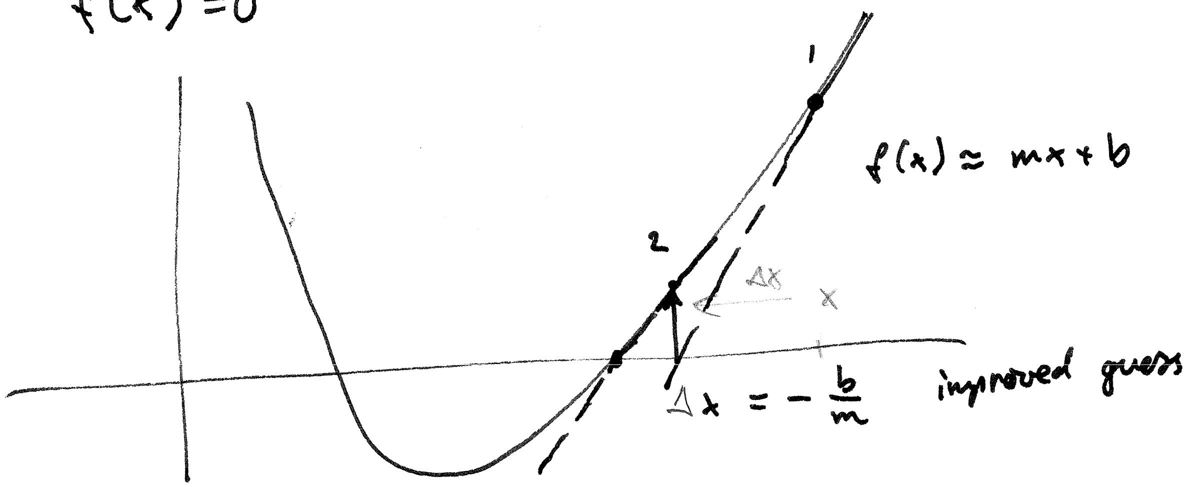
~~Eg. f(a)~~

1. bisect $\qquad x' = \frac{1}{2}(a+b)$
2. pick half with sign change

   if $f(a)f(x) > 0$:
   $a = x \qquad x_0$ in $[x, b]$
   else:
   $b = x \qquad x_0$ in $[a, x]$
3. $|f(x)| < \varepsilon$?

Newton – Raphson

$f(x) = 0$



Algorithm: Derivation

x_0 = old guess Δx : unknown correction

$x = x_0 + \Delta x$ = (unknown) new guess

Expand $f(x)$:

$$f(x = x_0 + \Delta x) \approx f(x_0) + \Delta x \left.\frac{df}{dx}\right|_{x_0}$$

Determine correction: intercept of lin approx w/ x-axis

$f(x_0 + \Delta x) = 0$

$$f(x_0) + f'(x_0)\,\Delta x = 0$$

$$\Delta x = -\frac{f(x_0)}{f'(x_0)}$$

Repeat!

1) can use analytical derivative
2) or numerical forward difference

$$\frac{df}{dx} \approx \frac{f(x + h) - f(x)}{h}$$

or central $$\frac{df}{dx} = \frac{f(x + \frac{h}{2}) - f(x - \frac{h}{2})}{h}$$

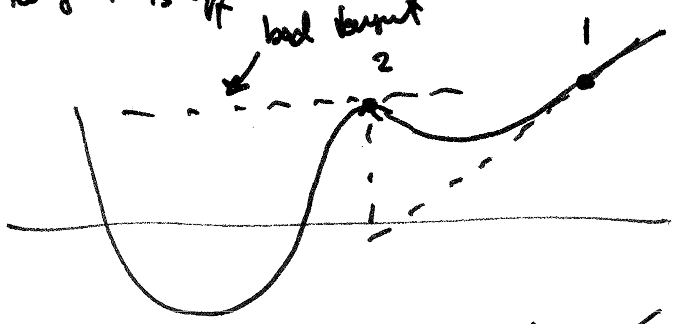
while $|f(x) > \epsilon|$:

$\Delta x = -\frac{f(x)}{f'(x)}$

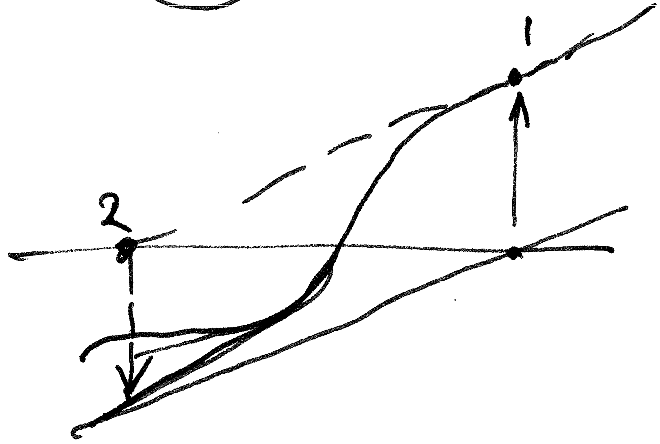
$x \leftarrow x + \Delta x$

Possible issues

- Initial guess must be close
- tangent is off



local min/max



inf. loop

$\Rightarrow$ Solutions:

1) start with bisection
2) implement backtracking

If new guess increases magnitude (i.e. error increases)

$$|f(x+\Delta x)|^2 > |f(x)|^2$$

then go back to x and try smaller guess

$$x \to x + \frac{\Delta x}{2}$$

Reduce Δx more if necessary.

Advantages

- quadratical convergence
- fast