Two-Dimensional Billiard Simulation

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Problem:

This simulation would show how a group of particles would behave in a two-dimensional system if they only interacted through elastic collisions. If the simulated system matches any properties of gasses in the real world then this model could be used to potentially save a great deal of computational time compared to other methods. The computation should be simpler if all particles can be assumed to travel in a straight path until a collision, at which point all that needs to be calculated is the new directions and velocities. Those can be found with conservation of momentum and solving for the deflection angle. Holes can also be places along the boundary that would remove a particle if it enters

Approach:

At least one particle has to be given an initial velocity relative to the walls at the start of the simulation. Each particle would need a few parameters: mass, radius, x/y position, and x/y velocity. Holes would just need radius and position. Velocities should remain constant until a collision occurs. Each time step would move each particle based on its component velocities and then check if a collision happens. Collisions between particles can simply check if the distance between the two centers is shorter than the sum of the radii. Collisions with the walls are a bit more tricky. If the boundary shape is a box or a circle then they can be taken care of with special cases, otherwise it would require a more complex algorithm. In all scenarios, at each time step the simulation would check if the radius added on to the position falls outside of the boundaries of the walls. For a box, that would just be checking each of the four directions along the axis. For a circle, the direction would be along the line between the center of the circle to the particle. Other shapes might require checking angles all around the particle.

Collisions between particles and against the walls would be treated differently because the walls are static boundaries in this case. Between walls, the speed will be the same but reflected along the tangent line of where it collides with the wall. Collisions between particles can just use momentum conservation to find the new velocities. Holes can be dealt with similarly to detecting collisions by seeing if the sum of the separation distance and the radius of the particle is smaller than the radius of the hole. Holes might also need some special interaction with walls so that particles don’t collide with walls before ever being able to reach a hole.

Objectives:

Success of creating this simulation can be quantified by the following goals. Implementing an elastic collision algorithm between particles and with the walls. Removing particles through the holes and ensuring holes and walls don’t conflict with each other. Determining how long the simulation needs to run before all particles disappear through the holes. Visualizing the simulation. Adding friction to slow the velocities.