Project Proposal

In the Life of a Protostar

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Problem:

When a molecular cloud is massive, cold, and dense enough, gravity overcomes the thermal pressure of the cloud and it starts to collapse. The critical size of the cloud is called the Jean's radius (or length) and is calculated by[1]

$$R_j = \frac{G\mu M}{5kT}, \tag{1}$$

where G is the universal gravitational constant, μ is the mean particle mass, M is the cloud mass enclosed by R_j, k is the Stefan-Boltzmann constant, and T is the cloud temperature. As the cloud collapses, half of the gravitational potential energy is radiated away and half of it is used to heat the interior of the cloud. The additional thermal pressure at the core fights back against gravity and if a balance between gravity and pressure is achieved before the core is hot enough for stable nuclear fusion, then the cloud will live out its life as a brown dwarf; if the core does reach temperatures hot enough for stable nuclear fusion, then a main-sequence star will have been born.

In this project we want to build a model determine if a given cloud will collapse into brown dwarf or a main sequence star and how long the process will take.

Approach:

In the model, we will construct several clouds; each cloud will have the same μ and initial T, but different M. For each cloud we will calculate the initial size and, considering the cloud to be initially uniform, we will break up the cloud into N spherical shells. For each time slice Δt, the outward pressure and gravitational attraction felt by each shell will be calculated and updated accordingly.

Since gravity "communicates" at the speed of light, each shell will be calculated so that it "feels" the effects of gravity from all the interior shells that are $c\Delta t$ away. On the other hand, thermal pressure communicates at the speed of sound in the gas, which means that each shell will "feel" the pressure from shells $v_t\Delta t$ away, where v_t is the isothermal speed of sound in the shell, which is found using

$$v_t = \sqrt{\frac{\gamma kT}{\mu}}. \tag{2}$$

The model for each cloud will be considered complete if either the temperature from the innermost shell reaches some critical temperature, $T_c = 10^7\ K$,[2] where nuclear fusion can take place, or if the difference between pressure and gravity is below some tolerance ϵ for all shells.

Objectives:

1 Create model clouds of initial masses ($M_\odot$): .7, .8, 1, 1.5, 2, 3, 5, 9, 15, 25, and 60.

2 Brake up each model cloud into 1000 shells and set up a time array so there will be 1000 time steps for the simulation. The maximum time, t_{max} will be 100 times the benchmark "Contraction Time (Myr)" listed for the corresponding initial cloud masses in Table 1.[3]

[1] adapted from Equation 16 on pg 457 of Carroll's "An Introduction to Modern Astrophysics" by exchanging ρ_0 for $M = 4/3\pi R_j^3$ and solving for R_j.

[2] taken as $T_{quantum}$ from pg 392 of Carroll's "An Introduction to Modern Astrophysics"

[3] compare with Table 1 on pg 471 of Carroll's "An Introduction to Modern Astrophysics"

3 Calculate the gas conditions in each shell (radius of shell, temperature, gas pressure, radiation pressure, gravity pressure, velocity, and mean free path) for each time step.

4 The simulation will complete if:

  A The inner shells reach $\approx 10^7\ K$, enough for stable nuclear fusion to take place. This signifies that the protostar has formed a main-sequence star.

  B If the maximum change in shell radii between time steps is less than some tolerance, ϵ. This signifies that the protostar has reached equilibrium without turning into a main sequence star and is a brown dwarf.

  C If t_{max} is reached. This means that the cloud has not reached equilibrium in the allotted time, which signifies that either tolerances or the model needs to be tuned.

5 Compare contraction times with Figure 1. Success is within a few orders of magnitude. Due to some simplifications in the model, it is not expected to be exact; however, the model should exhibit the general trend shown in Figure 1.

6 Create a single figure that shows core temperature vs time for all model protostars.

7 Bonus, create small movies that shows the luminosity of a 2D slice of the protostars over time.

Initial Mass ($M_\odot$)	Contraction Time (Myr)
60	0.0282
25	0.0708
15	0.117
9	0.288
5	1.15
3	7.24
2	23.4
1.5	35.4
1	38.9
0.8	68.4

Figure 1: Benchmark Initial Mass and Contraction Time.