Project Proposal

In the Life of a Protostar

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Problem:

When a molecular cloud is massive, cold, and dense enough, gravity overcomes the thermal pressure of the cloud and it starts to collapse. The critical size of the cloud is called the Jean's radius (or length) and is calculated by[1]

$$R_j = \frac{G\mu M}{5kT}, \tag{1}$$

where G is the universal gravitational constant, μ is the mean particle mass, M is the cloud mass enclosed by R_j, k is the Stefan-Boltzmann constant, and T is the cloud temperature. As the cloud collapses, half of the gravitational potential energy is radiated away and half of it is used to heat the interior of the cloud. The additional thermal pressure at the core fights back against gravity and if a balance between gravity and pressure is achieved before the core is hot enough for stable nuclear fusion, then the cloud will live out its life as a brown dwarf; if the core does reach temperatures hot enough for stable nuclear fusion, then a main-sequence star will have been born.

In this project we want to build a model determine if a given cloud will collapse into brown dwarf or a main sequence star and how long the process will take.

Approach:

In the model, we will construct several clouds; each cloud will have the same μ and initial T, but different M. For each cloud we will calculate the initial size and, considering the cloud to be initially uniform, we will break up the cloud into N spherical shells. For each time slice Δt, the outward pressure and gravitational attraction felt by each shell will be calculated and updated accordingly.

Since gravity "communicates" at the speed of light, each shell will be calculated so that it "feels" the effects of gravity from all the interior shells that are $c\Delta t$ away. On the other hand, thermal pressure communicates at the speed of sound in the gas, which means that each shell will "feel" the pressure from shells $v_t\Delta t$ away, where v_t is the isothermal speed of sound in the shell, which is found using

$$v_t = \sqrt{\frac{\gamma kT}{\mu}}. \tag{2}$$

The model for each cloud will be considered complete if either the temperature from the innermost shell reaches some critical temperature, $T_c = 10^7\ K$,[2] where nuclear fusion can take place, or if the difference between pressure and gravity is below some tolerance ϵ for all shells.

Objectives:

The goal is to produce a figure showing the core temperature vs time for each model cloud and to compare the results against theoretical values.[3] A bonus objective is to produce animated movies for each cloud that show the temperature evolution of a 2D slice of the cloud over time.

[1] adapted from Equation 16 on pg 457 of Carroll's "An Introduction to Modern Astrophysics" by exchanging ρ_0 for $M = 4/3\pi R_j^3$ and solving for R_j.

[2] taken as $T_{quantum}$ from pg 392 of Carroll's "An Introduction to Modern Astrophysics"

[3] compare with Table 1 on pg 471 of Carroll's "An Introduction to Modern Astrophysics"

Useful Equations and relations:

hydrogen mass $m_H = 1.6737236 \times 10^{-27}\ kg$

helium mass $m_{He} = 6.6464764 \times 10^{-27}\ kg$

speed of light $c = 2.99792458 \times 10^{8}\ m/s$

universal gravitational constant $G = 6.67408 \times 10{-}11\ Nm^2/kg^2$

Planck's constant $h = 6.62606876 \times 10^{-34}\ Js$

Boltzmann's constant $k = 1.3806503 \times 10^{-23}\ J/K$

Stefan Boltzmann's constant $\sigma = \frac{2\pi^5 k^4}{15c^2h^3}$

radiation constant $a = \frac{4\sigma}{c}$

heat capacity ratio for monatomic ideal gas $\gamma = 5/3$

average molecular mass, χ is fraction of particles

$$\begin{aligned} \mu &= \sum_{n=1}^{N} \chi_n m_n \\ &= \frac{3}{4} m_H + \frac{1}{4} m_{He} \end{aligned} \tag{3}$$

Jean's Radius

$$R_j = \frac{G\mu M_{cloud}}{5kT}, \tag{4}$$

Volume of shell

$$V_{shell} = \frac{4}{3}\pi[(r_0 + \Delta r)^3 - r_0^3] \tag{5}$$

speed of sound in ideal gas

$$v_{gas} = \sqrt{\frac{\gamma kT}{\mu}} \tag{6}$$

radiation pressure of shell

$$P_{rad} = \frac{1}{3} a T_{shell}^4 \tag{7}$$

gas pressure

$$P_{gas} = \frac{m_{shell}}{\mu} \frac{kT_{shell}}{V_{shell}} \tag{8}$$

gravity pressure

$$P_{grav} = -\frac{GM(r) m_{shell}}{r_{shell} A_{shell}} \tag{9}$$