Population Dynamics
by: Cameron Howard, Uma Vrudhula, and Michael Nuccio

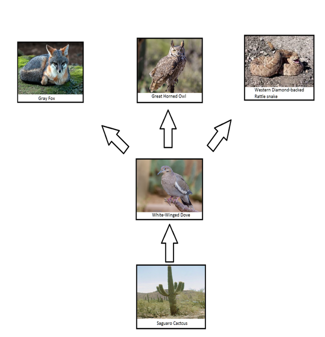
Background

While mankind has an impact on animal populations through long-term effects like climate change, we can also cause more immediate impacts through things like over-hunting, deforestation, and the introduction of foreign animals and disease into animal populations.

Inspired by this, our goal for this project was to model a realistic food web, with creatures of the local Sonoran desert forming our example, and model what impact the sudden severe reduction of one population might have on the rest within the system.

$$\frac{dx}{dt} = \alpha x - \beta xy$$

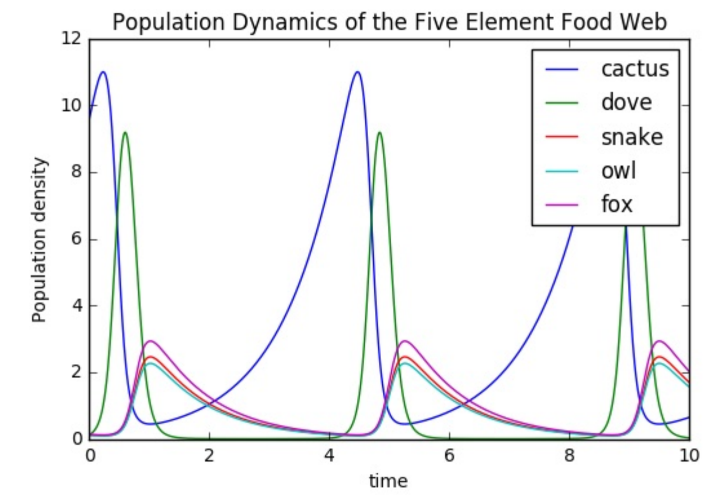
$$\frac{dy}{dt} = \delta x - \gamma xy$$

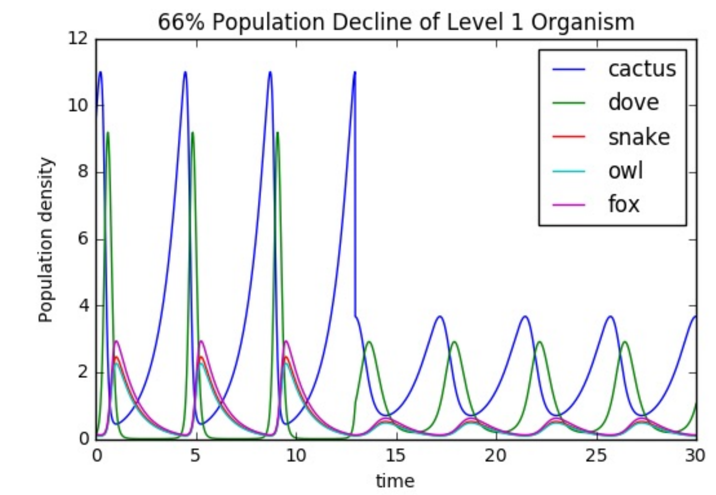


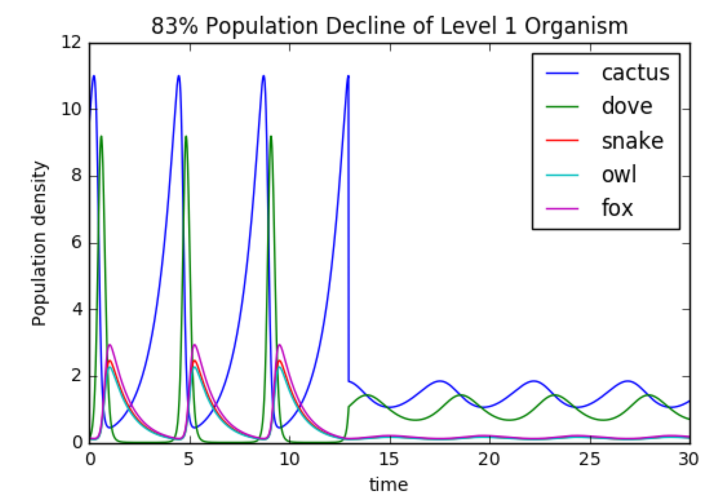
Methods

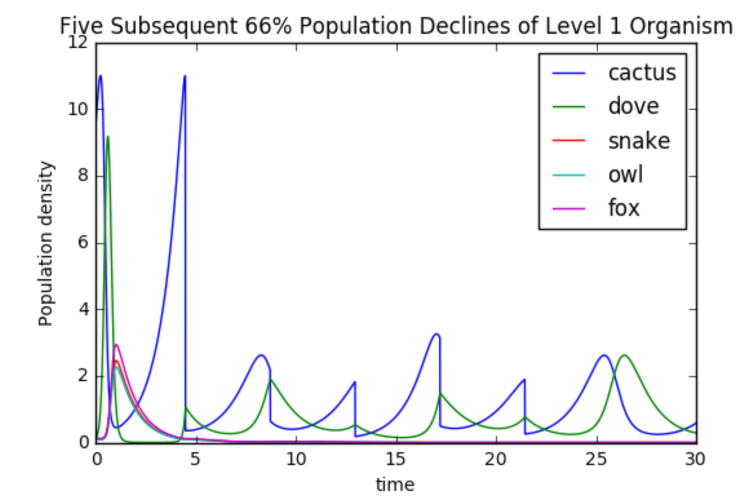
In order to properly integrate the aforementioned Lotka-Volterra equations the Runge-Kutta 4th order method was used. Initially the Euler method was used when difficulties arose implementing the RK4 method. This was quickly ditched when the accumulated error became to great to accurately resemble the system.

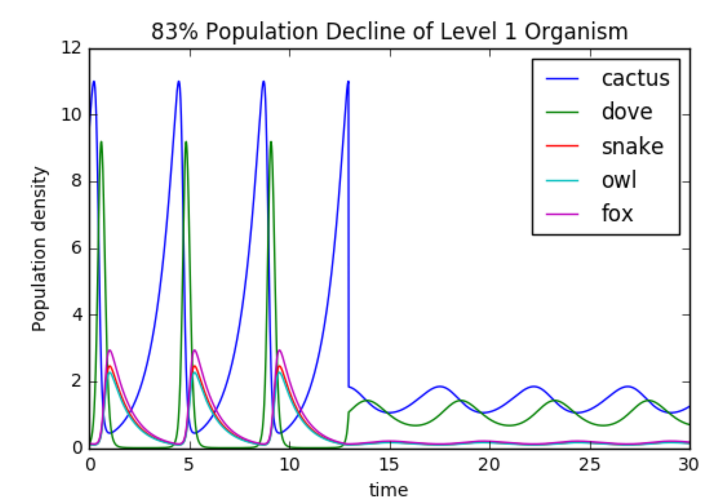
The equations used require initial population densities for each species as well as constants representing reproductive success, predatory success, and rate of natural death. Initial population densities were calculated from census data for the five animals.

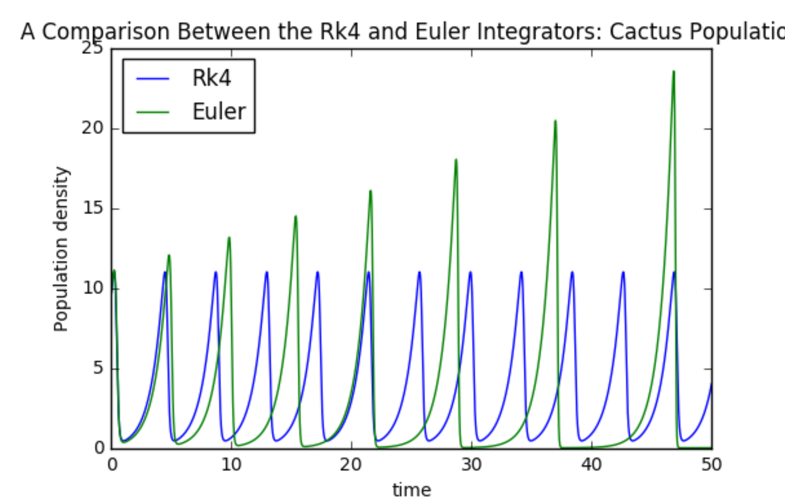












Code available at:
https://github.com/ASU-CompMethodsPhysics-PHY494/final-2017-team-az/

Methods cont.

The initial hope was that these coefficients would be available, or at least derivable from known data. This was not the case. Instead, as this information was unavailable, the coefficients were all set to one. This at least provided a stable system to work with.

At this point, for various species and various points in time, the population of a given animal was reduced by dividing its population by a number between 2 and 10 at a critical time step in the RK4 iterations. The effect this population change had was then examined by graphing the populations over time.

Methods Cont.

```
y = np.array([c0, d0, …])
t = 0
populations = []

def f(t, y):

    c = alpha_c*y[0] - beta_c*y[0]*y[1]
    d = alpha_d*y[0]*y[1] – beta_d*y[1]
    …
    return np.array([c, d, …])

for i in range(Nmax):
    populations.append([t, y[0], y[1],…])
    y = rk4(y, f, t, h)
    t += h

return np.array(populations)
```

Results

The Euler integrator produced error that accumulated as the number of iterations increased. The rk4 algorithm produced stable, constant population density maximums that were sustained through 50,000 iterations.

The cyclic curves exhibits one common period of 4.24 units of time, but their phases vary based on trophic level. It was found that initial conditions within certain intervals will always lead to a specific equilibrium state.

The effect a population decline had on the system depended on when it was implemented, with maximum change occurring at a population peak. The subsequent amplitudes decrease as the severity of the decline increases. This effect is magnified with multiple population reductions and lower trophic levels.

Summary

Our findings show that our system is resilient to extinction of species due individual impacts to populations, but rarely ever does the system return to the same population levels. Multiple reductions in population over time, however, quite clearly do lead to the extinction of species within the system. For future work, greater numbers of species could be added, and accuracy of results could be improved by sourcing further real-world data for calculating coefficients for our system of equations.

Contributions

-RK4 code was provided by professor Oliver Beckstein.