

## Population Dynamics

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### Problem:

The important problem being examined is that of population dynamics. More specifically, it will be to see how a sudden change in the population at the lowest levels of a food chain affect the populations at the top of the food chain. The equations that are currently used to model such scenarios are known as the Lotka-Volterra equations. These are a pair of first-order, non-linear differential equations where one represents a prey species and the other the predator species. These equations can be seen below in lines one and two.

$$\frac{dx}{dt} = \alpha x - \beta xy \quad (1)$$

$$\frac{dy}{dt} = \delta xy - \gamma y \quad (2)$$

In these equations  $x$  and  $y$  represent the prey and predator populations respectively. Their time derivatives naturally represent the rate of change of the population. The other values are constants used to describe the interaction between predator and prey.

### Approach:

To solve these equations for multiple tiers of a given food chain these equations will first be applied to the lowest organism and its predator using the Runge-Kutta 4 integration method. At this point the equations will be used again to describe the previous predator as a prey animal and introduce a new predator. This can be done because the first set of equations tell us the population of the animal at any given time. By knowing the population of the chains first predator, once it is treated like a prey animal, the prey equations can be applied to it. At this point a new predator can be introduced which preys upon the first predator. This will be repeated for all steps in the chain.

### Objectives:

- 1.) Successfully perform the integration, and plot the population vs time for a single pair of predators and prey.
- 2.) Extend our methods to include many animals in a direct line food chain
- 3.) Introduce competition (where more than one animal feeds on the same prey, this can also be done to see what effect invasive species have on a habitat) in the food chain, effectively making it a “food tree.”
- 4.) Introduce disease or some other factor that quickly reduces the population at different tiers of the food chain. Observe and quantify the effects of these sudden reductions in population on other animals in the chain
- 5.) Stretch goal: Compute the error in the model by comparing to an analytical solution for the “food tree”