Housing Structural Heat Analysis

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Problem

Over the course of a year in Arizona, internal household temperatures can be staggering without any electrical aid, such as air conditioning. Therefore, this problem will physically model the effects of various structural materials for houses in 2D and (potentially) 3D by using the *heat equation*, a partial differential equation (PDE) that describes the distribution of heat in a given region over time. The equation is defined as

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u,$$

where $u(\vec{x}, t)$ is the temperature at location $\vec{x}$ at time t and α is the thermal diffusivity of a material/fluid. The problem will investigate the equilibrium reached when the internal temperature is initially a given constant and the boundary conditions vary from simulation to simulation based on the material and household structure being replicated. These boundary conditions may range from thermal functions on a particular boundary with respect to time based on weather or functions representing the heat flux through the boundary due to flawed insulation material.

This problem will prove to be extensive due to the amount of data collection necessary to simulate realistic households as well as the number of potential combinations of boundary conditions (including unrealistic scenarios for eccentric households). For simplicity, the problem will investigate 2D houses initially. Nevertheless, the problem will prove interesting since it will provide a way to determine the efficiency of various materials at reducing heat incurred over time, which would be ideal knowledge when updating an actual house's structure.

Approach

In order to solve the 2D heat equation, the Crank-Nicolson algorithm is used in which a finite difference method is used to numerically solve the PDE for a spatial grid of positions over time. The algorithm essentially divides the domain into a grid-like structure where each cell will have its future temperature evaluated based on the surrounding cells' current temperature, creating an interdependent system of data. If 3D simulations are implemented, the algorithm can be extended without having to start from scratch.

The various material properties investigated will have their data collected from various places, such as the following two articles on insulation materials:

- http://www.greenspec.co.uk/building-design/insulation-materials-thermal-properties/
- http://www.open.edu/openlearn/nature-environment/the-environment/energy-buildings/content-section-2.2.3
- https://www.energycodes.gov/sites/default/files/becu/2009_iecc_residential.pdf

Additionally, more information may be found at construction facilities, such as Home Depot, where these kind of physical properties are displayed.

Information regarding the average sizes of households can be found through the census (below), and then based on the size (and subsequently area), various house shapes will be fabricated consistent with the data.

- https://www.census.gov/const/C25Ann/sftotalmedavgsqft.pdf.
- https://www.census.gov/data/datasets/2014/econ/mhs/puf.html

Lastly, the simulations will be run for two 48-hour scenarios: a summer and winter day in the Phoenix area. These scenarios will be implemented by defining the boundary heat function as a varying sinusoidal function ranging from minimum to maximum temperatures as documented below as an average per month:

- https://www.currentresults.com/Weather/Arizona/Places/phoenix-temperatures-by-month-average.php

With the simulator and the boundary conditions, no other information is needed to solve the heat equation unless there are other factors to consider, such as windows, though those will simply add to the potential boundary conditions used.

Objectives

1. Collect data for boundary conditions (material properties, household structural effects, etc.)
2. (Prototype) Create and test/debug a 1D simulator to numerically solve the heat equation.
3. Extend simulator into 2D.
4. Determine the average temperature of the household during a 48-hours period in summer or winter when varying one property from the following set: $\{$materials/insulation, house shape/size, wall thickness$\}$.
   - There will be a set of results for the summer scenario and another for the winter scenario.
   - The results will be visually rendered: the 1D simulator will show the effects of materials/insulation and wall thicknesses by plotting $T(x,t)$ while the 2D simulator will depict the effects of the house shape/size by creating heat maps of $T(\vec{x},t)$ for t at the end of the 48-hour period.
5. (Reach) Create an animated set of results that shows the variation over time.
6. (Reach) Consider windows, doors, and other significant structural factors (and collect relevant data) on boundaries when applying conditions.
7. (Reach) Extend simulator into 3D.
   - Collect additional data for floor and roof boundary conditions.