Fourier Transforms with Wave Equations: Final Proposal

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1 Problem

Which method of using Python to find the momentum space Fourier transforms of position space wave functions is most efficient?

2 Approach

We will be testing 4 methods of finding the momentum space wave function. These methods are:

1. Calculate the momentum space transform by hand and use Python to graph the resulting function – this will serve as a control.

2. Use the DFT function in Numpy to numerically evaluate the transform and then graph the result.

3. Use the FFT function in Numpy to the same effect.

4. Design our own DFT function to evaluate the transform.

3 Objectives

Objective 1: We will analytically transform the position space functions of the following potentials:

1. The 1st, 2nd, and 3rd stationary states of the infinite square well.

2. The 1st, 2nd, and 3rd stationary states of the simple harmonic oscillator.

3. The stationary state of the dirac delta potential.

4. A free particle with initial wave function $\Psi(x, 0) = \frac{A}{x^2+a^2}$

Objective 2: Numerically evaluate the transforms of the same wave functions using the DFT function in Numpy.

Objective 3: Numerically solve the transforms again using the FFT function of Numpy instead.

Objective 4: Design and test a basic DFT algorithm of our own to find the Fourier transforms of the same position-space functions. We will test the efficiency of our function by increasing or decreasing the step size used in calculating the position-space wave function values until the time of calculation for our own DFT function roughly matches that of the Numpy DFT function, and then compare the results graphically and, if possible, analytically in some way.

Stretch goals:

1. Design a FFT function to increase efficiency.

2. Numerically solve the anharmonic oscillator potential ($V = \frac{1}{2}\omega^2 x^2 + \alpha x^4$) and find its transform through all four methods.