Project 2: Soccer Physics for the WIN!

Project *Comp. Methods Physics* ASU PHY494 (2018)*

March 26, 2018 – April 9, 2018

Abstract In *soccer* (or *football* for the world outside the US) a goal is scored when the entire ball fully crosses the goal line between the goal posts and under the goal cross bar. There are a number of set-piece situations that can directly lead to goals. Here we look at two such opportunities that start from a ball at rest: the **direct free kick** and the **corner kick**. You will write Python code to simulate a realistic flight trajectory of the ball (including spin and air resistance) and solve the problem to choose initial kick parameters that could lead to directly scoring a goal. You will write a *short report* to communicate, discuss and summarize your reasoning and your results. The work is carried out in teams of two or three students.

Due Monday, April 9, 2018, 11:59pm.

- Students work in teams of two or three students.
- **Admissible Collaboration:** Students are allowed to talk to other students in the class about the project and exchange ideas and tips. However, sharing/copying reports or full code solutions is not allowed. **Help from other students must be acknowledged in an Acknowledgments section**. Direct help from outside the class is not allowed (except instructor/TA), e.g., you cannot ask for solutions (online or in person) but you can use books and resources on the internet to solve problems. **Cite all sources**. Code from the class can be used without explicit citation or acknowledgement.
- Each team should commit their report (see Section 3) in **PDF** format to the team's **GitHub repository**; alternatively, combining report and code in a Jupyter notebook is also possible as long as the notebook can be read like a

*Current version of this document: March 26, 2018. See Appendix 6 for a list of changes since v1 from March 26, 2018.

report (i.e., not just bullet points or short comments). If possible, also generate a PDF from your notebook and commit it together with everything else.

- The report *must* contain a section **Contributions** at the end where the contributions of all team members are summarized.

- Each team should commit and push **all code** (see Section 4) that is required to reproduce the results in the report to their **GitHub repository**. Include a text file `README.txt` that describes the commands to run calculations. The code must run in the standard anaconda-based environment used for the class. If it is a Jupyter notebook then it should be possible to *Kernel* → *Restart & Run All* and to produce all the required figures and output.

Grading will take the following into consideration:

- The code runs and produces correct output.

- The report clearly and succinctly describes the question, approach, and results and contains sufficient evidence that the requirements (see below) have been met.

- All team members contributed to the work: assessed by (1) Contributions section in the report, (2) commit history of the repository and comments in code, (3) short oral examination of team members (at instructor's discretion if deemed necessary).

- Code from outside sources (see Admissible Collaboration) and help is thoroughly attributed (Acknowledgements and References).

- BONUS: Additional work that you want to include in an appendix to the report or additional simulations for the main report will be treated as bonus material and can be awarded bonus points.

- BONUS: Elegant and fast code can be awarded bonus points.

Contents

1 Submission instructions

Submission is to your private **team GitHub repository**. Follow the link provided to you by the instructor in order for the repository to be set up: It will have the name *ASU-CompMethodsPhysics-PHY494/project-2-2018*-YourTeamName and will only be visible your team and the instructor/TA. Follow the instructions below to submit this project.

Read the following instructions carefully. Ask if anything is unclear.

1. `git clone` your project repository (change *YourTeamName* to your team's name)

   ```
   repo="project-2-2018-YourTeamName.git"
   git clone https://github.com/ASU-CompMethodsPhysics-PHY494/${repo}
   ```

   or, if you already have done so, `git pull` from within your assignments directory.

2. Create three sub-directories `Submission`, `Grade`, and `Work`.

3. You can try out code in the `Work` directory but you don't have to use it if you don't want to. Your grade with comments will appear in `Grade`.

4. Create your solution in `Submission`. Use Git to `git add` files and `git commit` changes.

   You can create a PDF file or Jupyter notebook inside the `Submission` directory as well as Python code (if required). **Name your files** `project02.pdf` **or or** `project02.ipynb`, depending on how you format your work. Files with code (if requested) should be named exactly as required in the assignment.

5. When you are ready to submit your solution, do a final `git status` to check that you haven't forgotten anything, commit any uncommitted changes, and `git push` to your GitHub repository. Check on *your* GitHub repository web page[1] that your files were properly submitted.

   You can push more updates up until the deadline. Changes after the deadline will not be taken into account for grading.

Work must be legible and intelligible or may otherwise be returned ungraded with 0 points.

2 Problem description

This section describes the model that we will use to study the *free kick with wall* and the *corner kick* in the ball game of soccer.

2.1 Soccer physics

2.1.1 Equations of motion

The ball obeys Newton's equations of motion with the following forces to be included:

1. gravity $\mathbf{F}_G = -g\,\hat{\mathbf{e}}_y$
2. quadratic air resistance $\mathbf{F}_\mathbf{d}$ (see Eq. 1)
3. lift due to spin (the Magnus force $\mathbf{F}_M$, see Eq. 3)

(The coordinate system was assumed to have the x-direction from the ball towards the goal and the y-direction perpendicular to the ground and pointing to the sky.)

Air resistance An approximately quadratic dependence of the drag force on the velocity occurs at high Reynolds numbers, i.e., turbulent flow. An approximate expression is

$$\mathbf{F}_d = -\frac{1}{2} C_D(v) \rho A v^2 \frac{\mathbf{v}}{v} \tag{1}$$

The quadratic drag coefficient C_D depends on the translational (center of mass) velocity of the object and on the spin parameter S (see Eq. 5 below). If it spins slowly, it exhibits a "drag crisis" whereby its aerodynamic drag sharply *decreases* at a critical velocity v_c when the air flow switches from the laminar to the turbulent regime, as shown in Figure 1 in Goff and Carré (2010). At higher spin with velocities above v_c, the drag coefficient

[1] `https://github.com/ASU-CompMethodsPhysics-PHY494/project-2-2018-`*`YourTeamName`*

mainly depends on S. The experimental data can be parametrized by the dimensionless drag coefficient $C_D(v, S)$ as (Goff and Carré, 2010)

$$C_D(v,S) = \begin{cases} a + \frac{b}{1+\exp[(v-v_c)/v_s]}, & \text{if} \quad v < v_c \quad \text{or} \quad S < 0.05 \\ cS^d, & \text{if} \quad v \geqslant v_c \quad \text{and} \quad S \geqslant 0.05 \end{cases} \tag{2}$$

$$v_c = 12.19\,\mathrm{m\,s^{-1}}, \qquad v_s = 1.309\,\mathrm{m\,s^{-1}}, \qquad a = 0.155, \qquad b = 0.346,$$
$$c = 0.4127, \qquad d = 0.3056.$$

Spin and lift The airflow is changed around a spinning object. The Magnus force is

$$\mathbf{F}_M = \alpha \boldsymbol{\omega} \times \mathbf{v} \tag{3}$$

where $\boldsymbol{\omega}$ is the ball's angular velocity in $\mathrm{rad\,s^{-1}}$ (e.g., up to about $75.4\,\mathrm{rad\,s^{-1}}$ or about $12\,\mathrm{rev\,s^{-1}}$ (revolutions per second) for a typical soccer ball).[2] For a sphere the proportionality constant α can be written

$$\mathbf{F}_M = \frac{1}{2} C_L \rho A \frac{v}{\omega} \boldsymbol{\omega} \times \mathbf{v} \tag{4}$$

where C_L is the lift coefficient, ρ the air density, A the ball's cross section.

C_L is mainly a function of the *spin parameter*

$$S = \frac{r\omega}{v} \tag{5}$$

with the radius r of the ball. $S = v_{\mathrm{spin}}/v$ is the ratio of the speed of a point on the ball's surface to the translational velocity of the ball. In general we write

$$\mathbf{F}_M = \frac{1}{2} C_L(S) \frac{\rho A r}{S} \boldsymbol{\omega} \times \mathbf{v}. \tag{6}$$

The lift coefficient is modelled as (Wang, 2015) (based on data in Goff and Carré (2009))[3]

$$C_L(S) = \frac{1}{2} S^{0.4} \tag{7}$$

where S is the spin parameter.

2.1.2 Parameters and constants

The following constants should be used in setting up the simulations.

[2] To convert from ω in $\mathrm{rad\,s^{-1}}$ to $\mathrm{rev\,s^{-1}}$ divide by 2π:

$$\frac{\omega}{\mathrm{rad\,s^{-1}}} \times \frac{1}{2\pi} = \frac{\omega}{\mathrm{rev\,s^{-1}}}.$$

[3] A more accurate expression for the lift coefficient C_L based on the newer data in Goff and Carré (2010) would need to be fitted and was not available.

Environment Physical constants due to the environment:

air density $\rho = 1.2\,\mathrm{kg\,m^{-3}}$

acceleration due to gravity $g = 9.81\,\mathrm{kg\,m\,s^{-2}}$

Ball Choose values for the 2006 World Cup ball (Adidas Teamgeist) (Goff and Carré, 2010) and parameters that are typical for professional soccer players (Goff and Carré, 2009):

mass $m = 0.436\,\mathrm{kg}$

diameter $D = 0.218\,\mathrm{m}$ (i.e., radius $r = 0.109\,\mathrm{m}$)

speed $4.5\,\mathrm{m\,s^{-1}} \leqslant v_0 \leqslant 31\,\mathrm{m\,s^{-1}}$ (these are values that can be observed in a wide range of situations; for the specific problem here, a more narrow range might occur)

spin (angular velocity of the ball) $0 \leqslant \omega_0 \leqslant 12\,\mathrm{rev\,s^{-1}} = 75.4\,\mathrm{rad\,s^{-1}}$ (these are values that can be observed in a wide range of situations; for the specific problem here, a more narrow range might occur; for David Beckham's famous free kick goals, $\omega_0 \approx 10\,\mathrm{rev\,s^{-1}}$ were estimated.)

Geometry The geometry of the situation for the **free kick** with a wall of defending players is shown in Fig. 1. This is just one of many possible positions to take the shot. We assume that the kicker scores if he or she can shoot the ball over or around the wall into the target area, as specified in Fig. 1b.

The geometry for the **corner kick** is shown in Fig. 2. We assume the kicker scores when she or he can shoot the ball into the target area inside the goal near the far post. In order to accomplish this, the ball must have some spin so that it may curve back towards the goal.

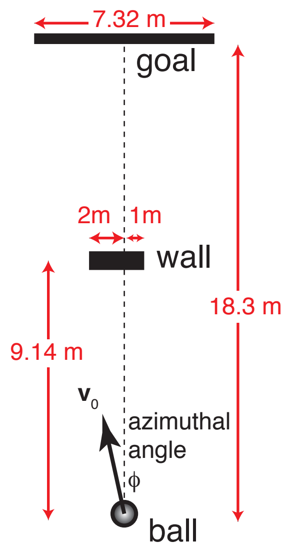
2.2 Ball trajectories

Your code should be able to generate realistic trajectories for balls kicked in two set piece situations: the free kick (Section 2.2.1) and the corner kick (Section 2.2.2). You should plot a range of trajectories for different initial conditions. The plots should show the elements on the field (location of goal, target zone, and e.g., the wall of defending players).

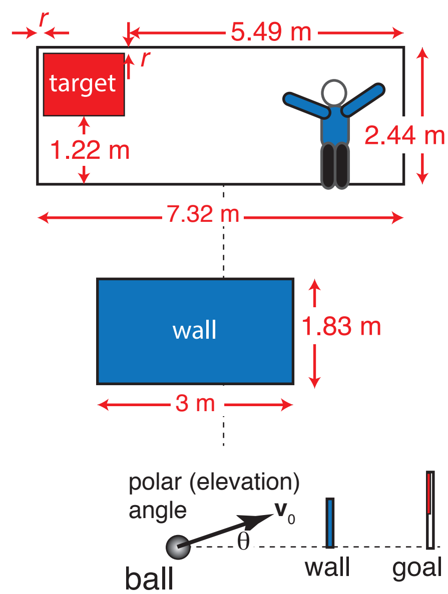
2.2.1 Free kick

The geometry of the situation is described in Fig. 1. A trajectory should start at the position of the ball (18.3 m from the center of the goal). A trajectory should end when the ball

1. leaves the field across the goal line



(a) Top view. The initial velocity of the ball $\mathbf{v}_0$ has an azimuthal (direction) angle ϕ that is positive when measured from the center line (dashed) in counterclockwise direction.



(b) Side view. The initial velocity of the ball $\mathbf{v}_0$ has a polar (launch) angle θ that is positive when measured above ground (dashed line).

Figure 1. Sketch of the model for the free kick. The free kick is taken at the center at a distance of 18.3 m from the goal. The wall made from defending players is positioned off center (dashed line) to guard the left half of the goal while the goal keeper defends the right half. The *target* area for a scoring free kick (out of reach of the goalie and inside the goal) is in the upper left corner of the goal (as seen by the kicker); note that the target area is offset by the radius of the ball r from the top and left edges. The goal keeper is only shown for comparison.

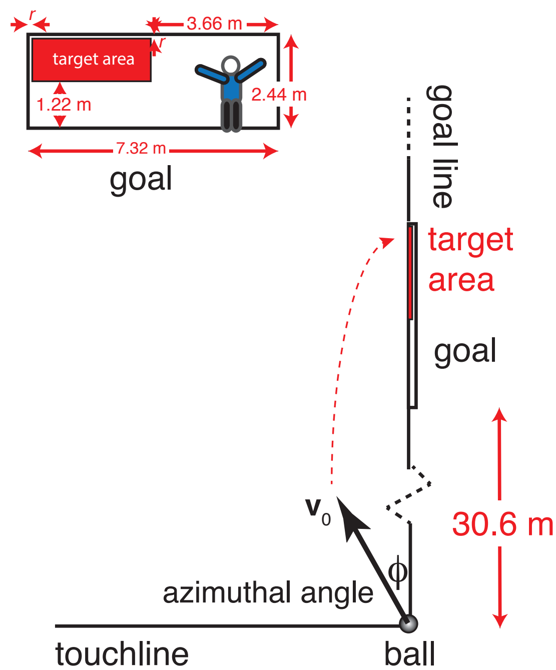


Figure 2. Sketch of the model for scoring from a corner kick. The ball has to be shot into the *target area* (shown in the inset) near the far goal post (away from the corner). It is assumed that shots on goal but outside the target area can be easily caught by the goal keeper. For the shot to count as a goal, the ball may not cross the touchline or the goal line before entering the target area from the playing field side. The initial direction of the ball with velocity $\mathbf{v}_0$ is characterized by the azimuthal angle ϕ (in the plane of the field, here counted as positive into the field to the left) and the polar angle θ (launch angle measured from the ground up, not shown). Note that the target area is offset by the radius of the ball r from the top and left edges. The goal keeper is only shown for comparison.

2. flies over the goal

3. crosses the goal line inside the goal (includes hitting the target area)

4. hits the wall

Initial conditions Sensible ranges of parameters are (Cook and Goff, 2006)[4]

- initial velocity $24\,\mathrm{m\,s^{-1}} \leqslant v_0 \leqslant 30\,\mathrm{m\,s^{-1}}$
- initial spin $0\,\mathrm{rev\,s^{-1}} \leqslant \omega_0 \leqslant 12\,\mathrm{rev\,s^{-1}}$ and for the orientation of the spin you may focus on full sideways spin, i.e., the spin axis $\boldsymbol{\omega}$ is perpendicular to the ground and parallel to the direction of gravity.
- polar (launch) angle $10^\circ \leqslant \theta \leqslant 20^\circ$
- azimuthal (direction) angle $-2^\circ \leqslant \phi \leqslant 5^\circ$

2.2.2 Corner kick

The geometry of the situation is described in Fig. 2. A trajectory should start at the position of the ball on the corner of the field; assume that the ball sits on the corner. A trajectory should end when the ball

1. leaves the field across the goal line or touchline

2. flies over the goal

3. crosses the goal line inside the goal (includes hitting the target area)

Initial conditions Sensible starting ranges of parameters are shown below (based on Cook and Goff (2006)) although you should feel to explore a wider range of parameters if necessary.

- initial velocity $24\,\mathrm{m\,s^{-1}} \leqslant v_0 \leqslant 30\,\mathrm{m\,s^{-1}}$
- initial spin $0\,\mathrm{rev\,s^{-1}} \leqslant \omega_0 \leqslant 12\,\mathrm{rev\,s^{-1}}$ and for the orientation of the spin you may focus on full sideways spin, i.e., the spin axis $\boldsymbol{\omega}$ is perpendicular to the ground and parallel to the direction of gravity.
- polar (launch) angle $20^\circ \leqslant \theta \leqslant 30^\circ$
- azimuthal (direction) angle $-4^\circ \leqslant \phi \leqslant 4^\circ$

[4] For a related treatment of the problem see Bray and Kerwin (2003).

2.3 Goal-scoring trajectories

The aim of the project is to find parameters for the kicks that lead to goals. In our model we consider a trajectory to be *goal-scoring* when it hits a *target area*. This target area is presumed to be a very difficult catch for the goal keeper. The target areas are different for the two situations and are defined in Figures 1 and 2.

You can use any number of approaches to find goal-scoring trajectories. Possible approaches are

exhaustive parameter scan Run the simulation for combinations of parameters and store for each parameter combination if the trajectory was goal scoring; this is the approach taken by Cook and Goff (2006).

shooting algorithm Combine the simulation algorithm with a root finding algorithm such as bisection to automatically find solutions to the problem; see Chapter 3.7 in Wang (2015).

manual experimentation Manually rerun your simulation with different parameters, using intuition and previous trials to guide you. Record parameter sets that score a goal.

You are free to use whichever approach solves the problem, including coming up with your own approach. However, you should not just ask other students for the answer... (Make sure you read the notes on the *Acknowledgments* section in Section 3 below if you heavily rely on the help of other people.)

At least one parameter set that leads to goal scoring trajectory should be reported together with plots of the trajectory. The plots must show clearly how the trajectory hits the target area.

3 Report

Write a report in which you address the objectives in Section 5 below in the context of the problem (Section 2). The report should contain all results (figures, tables, equations). It must contain a *title*, *author's name*, sections *Background* (problem description, definitions, any equations that you use), *Results and Discussion* (description and interpretation of results), and *Summary* (short summary of the main results).

The report *must* contain a section *Contributions* where you summarize the contributions of each team member. For example, for a team consisting of three members with initials A.B., C.D.E., and F.G., the beginning of this section could be written along the lines of

> A.B. wrote the code to initialize the planet positions and velocities with help from C.D.E. C.D.E. wrote the integration routine, A.B., C.D.E., and F.G. together wrote the simulation function `simulate_soccerball()`. F.G. with help from A.B. wrote the orbit plotting code and produced figures 1 and 2. F.G. also wrote the Background section ...

(Note that all team members should have commits in the team reporsitory.)

If you had any form of outside help you must describe it in an *Acknowledgments* section.

If you use code or material from elsewhere you *must cite the source* (add a *References* section).

Any bonus material can be shown in an optional *Appendix*.

The report must be written in full sentences and read as a coherent piece of work. Figures must have legends, labels, and captions. Type set in an 11pt font with single line spacing (captions, labels, legend may have smaller font sizes but must still be legible) and leave at least 1 in margins.

Overall, a length of about four pages is expected for a written document produced with a word processor; the report should not be less than three pages. The written portion (excluding figures) should generally not exceed six pages.

4 Code

For all numerical calculations use Python 3.x. You may use any of the Python packages that are part of the Anaconda 3 distribution such as `numpy` and `matplotlib`.

Include all the code that is needed to generate the results shown in your report. This can consist of Python programs, modules, a Jupyter notebook, or a mixture thereof. **Include a separate file `README.txt` that explains how to run your code** in order to generate the results in your report. **Your code must run without errors** in order for you to be awarded full marks.

5 Objectives

Address the following objectives in your report while taking all requirements in Section 2 into account:

(a) Briefly describe the behavior of the drag and lift coefficients of a soccer ball. Plot $C_D(v, S)$ (Eq. 2) in the range $0 < v < 30\,\mathrm{m\,s^{-1}}$ (1) for fixed $S = 0$ and (2) for fixed $S = 0.2$. Plot $C_L(S)$ (Eq. 7) for $0 < S < 0.7$.

(b) Simulate soccer ball trajectories of the

   (i) *free kick with wall* (2.2.1) and the

   (ii) *corner kick* (2.2.2)

   with your own Python program.

(c) For each case, show at least three typical trajectories using 2D plots (top and side views); as a bonus you can also show 3D plots. Describe the trajectories and list the parameters that were used to create them. "Typical" means that they should show a broad range of outcomes such as "hits the goal", "hits the wall", "flies besides or over the goal", ...

(d) For each of the two cases, find at least one set of parameters that results in a *goal-scoring trajectory* (see Section 2.3). For each goal-scoring trajectory

   (i) list the parameters

   (ii) show plots of the trajectory

   (iii) include a plot of a trajectory with the same parameters as the scoring trajectory *except* that spin is set to zero ($\omega_0 = 0$) to demonstrate the influence of spin

6 History

Changes and updates to this document.

2018-03-26 initial version

References

Goff J E and Carré M J, 2010 Soccer ball lift coefficients via trajectory analysis. *European Journal of Physics* **31** 775–784.

Wang J, 2015 *Computational Modelling and Visualization of Physical Systems with Python* (John Wiley & Sons, Hoboken, NJ).

Goff J E and Carré M J, 2009 Trajectory analysis of a soccer ball. *American Journal of Physics* **77** 1020–1027.

Cook B G and Goff J E, 2006 Parameter space for successful soccer kicks. *European Journal of Physics* **27** 865, http://stacks.iop.org/0143-0807/27/i=4/a=017.

Bray K and Kerwin D, 2003 Modelling the flight of a soccer ball in a direct free kick. *Journal of Sports Sciences* **21** 75–85, pMID: 12630787.