No: 1.7

$$\begin{cases} \Delta u = 0 & 0<r<3 \\ u(r,0)=0 \\ u(r,2)=0 \\ u(3,\varphi)=\sin 3\pi\varphi \end{cases}$$

$$R_n(r) = A_n r^{\frac{\pi n}{\alpha}}$$

$$u(r,\varphi) = R(r)\Phi(\varphi) = \sum_{n=1}^{\infty} R_n(r)\Phi_n(\varphi); \quad \Phi_n(\varphi) = \sin\frac{\pi n\varphi}{2} \quad \text{т.к. } \alpha=2$$

$$u(3,\varphi) = \sin(3\pi\varphi) = \sum_{n=1}^{\infty} R_n(r)\sin\frac{\pi n \varphi}{2} \Rightarrow n\neq 6:\ R_n(3)=0$$

$$n=6:\ R_n(3)=1$$

$$u(3,\varphi) = A_6 3^{\frac{\pi 6}{2}} \sin 3\pi\varphi = A_6 3^{3\pi}\sin(3\pi\varphi)$$

$$A_6 = \frac{1}{3^{3\pi}}$$

$$u(r,\varphi) = \sin(3\pi\varphi)\frac{1}{3^{3\pi}} r^{3\pi}$$

No: 2.7

$$\begin{cases} \Delta u = 0 \\ u_x(0,y) = u_x(\pi,y) = u(x,0) = 0 \\ u(x,2\pi) = 2\cos 3x \end{cases}$$

$$\begin{cases} X'' + \lambda X = 0 \\ X'(0)=0 \\ X'(\pi)=0 \end{cases} \qquad \begin{cases} Y'' - \lambda Y = 0 \\ Y(0)=0 \end{cases}$$

$$Y = A_n \operatorname{sh} ny + B_n \operatorname{sh} n(y-2\pi)$$

$$Y(0) = 0 = -B_n \operatorname{sh} n2\pi \Rightarrow B_n = 0$$

$$X_n(x) = C_1\sin(\sqrt{\lambda}x) + C_2\cos(\sqrt{\lambda}x)$$

$$X'(0) = \sqrt{\lambda}C_1 = 0 \Rightarrow C_1 = 0 \qquad X'(\pi) = \sqrt{\lambda}C_2\sin(\sqrt{\lambda}\pi) = 0 \qquad \sqrt{\lambda} = n \qquad C_2 = 1$$

$$u(x,2\pi) = \sum_{n=0}^{\infty}\cos nx\, A_n \operatorname{sh} n2\pi \qquad n=3:\ A_3\operatorname{sh}6\pi\cos 3x = 2\cos 3x \Rightarrow A_3 = \frac{2}{\operatorname{sh}6\pi}$$

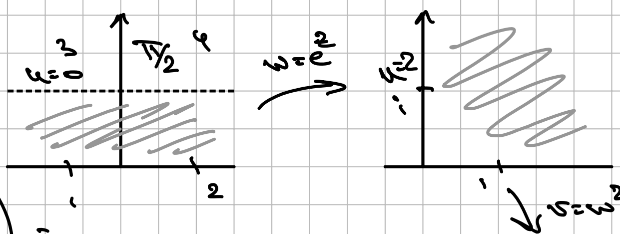
$$u(x,y) = \cos 3x \frac{2}{\operatorname{sh}(6\pi)}\operatorname{sh}3y$$

No: 3.7

$$\begin{cases} \Delta u = 0, & -\infty<x<+\infty \\ & 0<y<\frac{\pi}{2} \\ u(x,0)=0 \\ u(x,\frac{\pi}{2}) = 0, & x<0 \\ u(x,\frac{\pi}{2}) = 2, & x\geq 0 \end{cases}$$

$$z = x+iy \qquad w = e^{x+iy}$$

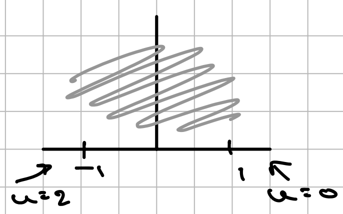
$$v = w^2 = e^{2x+2iy} = e^{2x}(\cos y + i\sin y)^2 = e^{2x}(\cos 2y + i\sin 2y) \Rightarrow$$



$$\Rightarrow v = \xi + i\eta$$

$$\xi = e^{2x}\cos 2y$$

$$\eta = e^{2x}\sin 2y$$



$$u(\xi,\eta) = \frac{2}{\pi}\left(\int_{-\infty}^{-1}\frac{2\,dt}{(t-\xi)^2+\eta^2}\right) =$$

$$= \frac{2}{\pi}\left(\frac{2}{\eta}\arctan\frac{(t-\xi)}{\eta}\Big|_{-\infty}^{-1}\right) =$$

$$= \frac{2}{\pi}\left(\arctan\frac{-1-\xi}{\eta} + \frac{\pi}{2}\right) =$$

$$= 1 - \frac{2}{\pi}\arctan\frac{e^{2x}\cos 2y + 1}{e^{2x}\sin 2y}$$

No: 5.7

$$\begin{cases} u_{tt} = u_{xx} + 3u_t + 2u_x + e^{3t-x}\sin x \\ u(0,t) = 0 & 0<x<\pi \\ u(\pi,t) = 0 \\ u(x,0) = e^{-x}\sin x \\ u_t(x,0) = 0 \end{cases}$$

$$u(x,t) = X(x)T(t) \qquad X_n = \sin\frac{\pi n x}{\pi} = \sin nx$$

$$u = e^{\alpha x+\beta t} v$$

$$u_t = e^{\alpha x+\beta t}(v_t + \beta v)$$

$$u_{tt} = e^{\alpha x+\beta t}(v_{tt} + \beta v_t + \beta v_t + \beta^2 v)$$

$u_x = e^{\alpha x+\beta t}(v_x+\alpha v)$

$u_{xx} = e^{\alpha x+\beta t}(v_{xx}+2\alpha v_x+\alpha^2 v)$

$\cancel{e^{\alpha x+\beta t}} v_{tt} + \overset{=6}{2\beta} v_t \cancel{e^{\alpha x+\beta t}} + \overset{=9}{\beta^2} v \cancel{e^{\alpha x+\beta t}} = \cancel{e^{\alpha x+\beta t}} v_{xx} + \overset{-2}{2\alpha} v_x \cancel{e^{\alpha x+\beta t}} + \overset{1}{\alpha^2} v \cancel{e^{\alpha x+\beta t}} + \overset{3}{3} \cancel{e^{\alpha x+\beta t}}(v_t+\beta v) +$

$+2\cancel{e^{\alpha x+\beta t}}(v_x+\alpha v) + \cancel{e^{3t-x}} \sin x \;\Big|: e^{3t-x}$ $\qquad \alpha=-1 \quad \beta=3$

$v_{tt} + \overset{3}{\cancel{6}} v_t + \cancel{9v} = v_{xx} - \cancel{2v_x} + v + \cancel{3v_t} + \cancel{9v} + \cancel{2v_x} - 2v + \sin x$

$$\begin{cases} v_{tt}+3v_t = v_{xx} - v + \sin x \\ v(0,t)=0=v(\pi,t) \\ v(x,0)=\sin x \\ v_t(x,0)=0 \end{cases}$$

$v(x,t)=X(x)T(t) \qquad X_n(x) = \sin\frac{\pi n x}{\pi} = \sin nx$

$n=1: \begin{cases} T''+3T' = -T-T+1 \\ T(0)=1 \\ T'_t(0)=0 \end{cases} \qquad \lambda^2+3\lambda+2=0 \qquad \lambda_1=-1, \; \lambda_2=-2$

$T_{r.p.} = \frac{1}{2} \qquad T_{op.} = C_1e^{-t}+C_2e^{-2t}+\frac{1}{2}$

$$\left.\begin{array}{l} T(0)=1=C_1+C_2+\frac{1}{2} \\ T'(0)=0=-C_1-2C_2 \end{array}\right| \Rightarrow \begin{array}{ll} -C_2=\frac{1}{2} & C_2=-\frac{1}{2} \\ C_1=-2C_2 & C_1=1 \end{array} \qquad T(t)= e^{-t}-\frac{1}{2}e^{-2t}+\frac{1}{2}$$

$u(x,t) = e^{3t-x}\sin x\left(e^{-t}-\frac{1}{2}e^{-2t}+\frac{1}{2}\right)$