2. **Решить задачу:**

$\Delta u(x,y)=0,\ 0<x<\pi,\ 0<y<\pi,$

$u(x=0,y)=0,\ u(x=\pi,y)=\sin 4y,\ u(x,y=\pi)=0,\ u(x,y=0)=\sin 2x.$

$$\begin{cases}\Delta v = 0\\ v\big|_{x=\pi} = \sin 4y\\ v\big|_{x=0,\ y=0,\ y=\pi} = 0\end{cases} \qquad \begin{cases}\Delta w = 0\\ w\big|_{y=0} = \sin 2x\\ w\big|_{x=0,\ x=\pi,\ y=\pi} = 0\end{cases}$$

- $v = \sin 4y \cdot X(x)$

$\Delta v = -16 \sin 4y\, X(x) + \sin 4y\, X''(x) = 0$

$$\begin{cases}X'' - 16X = 0\\ X(0)=0\\ X(\pi)=1\end{cases} \Rightarrow \quad X(x) = C_1 \operatorname{ch} 4x + C_2 \operatorname{sh} 4x$$

$X(0) = C_1 = 0$

$X(\pi) = C_2 \operatorname{sh} 4\pi = 1 \Rightarrow C_2 = \dfrac{1}{\operatorname{sh} 4\pi}$

$$v(x,y) = \frac{\operatorname{sh} 4x}{\operatorname{sh} 4\pi} \sin 4y$$

- $w(x,y) = \sin 2x \cdot Y(y) =$

$$\begin{cases}Y'' - 4Y = 0\\ Y(0)=1\\ Y(\pi)=0\end{cases} \Longrightarrow \quad Y(y) = C_1 \operatorname{ch} 2x + C_2 \operatorname{sh} 2x$$

$Y(0) = C_1 = 1$ $\qquad C_2 = -\dfrac{\operatorname{ch} 2\pi}{\operatorname{sh} 2\pi}$

$Y(\pi) = \operatorname{ch} 2\pi + C_2 \operatorname{sh} 2\pi = 0$

$$Y(y) = \frac{\operatorname{ch} 2x \cdot \operatorname{sh} 2\pi - \operatorname{sh} 2x \cdot \operatorname{ch} 2\pi}{\operatorname{sh} 2\pi} = \frac{\operatorname{sh} 2(x-\pi)}{\operatorname{sh} 2\pi}$$

$$u(x,y) = \frac{\operatorname{sh} 4x}{\operatorname{sh} 4\pi} \sin 4y + \frac{\operatorname{sh} 2(x-\pi)}{\operatorname{sh} 2\pi}$$

4. **Решить задачу:**

$$u_{tt} = u_{xx} + \sin\frac{5x}{2}, \quad 0 < x < \pi, \quad t > 0,$$
$$u(0,t) = 0, \quad u_x(\pi,t) = t, \quad t > 0,$$
$$u(x,0) = 2, \quad u_t(x,0) = x, \quad 0 < x < \pi.$$

$$v(x,t) = u(x,t) - xt$$

$$\begin{cases} v_{tt} = v_{xx} + \sin\frac{5x}{2} \\ v(0,t) = v_x(\pi,t) = 0 \\ v(x,0) = 2 \\ v_t(x,0) = 0 \end{cases}$$

$$v = w + h$$

$$\begin{cases} w_{tt} = w_{xx} \\ w(0,t) = w_x(\pi,t) = 0 \\ w(x,0) = 2 \\ w_t(x,0) = 0 \end{cases}$$

$$w = X(x)T(t)$$
$$\frac{X''}{X} = \frac{T''}{T} = -\lambda$$
$$\lambda_n = \left(\frac{2n-1}{2}\right)^2$$
$$X_n(x) = \sin\frac{2n-1}{2}x$$

$$T'' + \lambda T = 0$$
$$T_n = A_n \cos\frac{2n-1}{2}t + B_n \sin\frac{2n-1}{2}t$$
$$w = \sum_{n=1}^{\infty}\left(A_n \cos\frac{2n-1}{2}t + B_n \sin\frac{2n-1}{2}t\right)\sin\frac{2n-1}{2}x$$
$$w(x,0) = \sum_{n=1}^{\infty} A_n \sin\frac{2n-1}{2}x = 2$$
$$A_n = \frac{4}{\pi}\int_0^{\pi}\sin\frac{2n-1}{2}x\,dx = \frac{8}{\pi(2n-1)}$$
$$w_t(x,0) = \sum_{n=1}^{\infty} nB_n \sin\frac{2n-1}{2}x = 0 \implies B_n = 0$$

$$\begin{cases} h_{tt} = h_{xx} + \sin\frac{5x}{2} \\ h(0,t) = h_x(\pi,t) = 0 \\ h(x,0) = 0 \\ h_t(x,0) = 0 \end{cases}$$

c.φ. $\Rightarrow h(x,t) = T(t)\sin\frac{5x}{2}$

тут +

$$\begin{cases} T'' - \frac{25}{4}T = 1 \\ T(0) = 0 \\ T'(0) = 0 \end{cases}$$

$$T = \overbrace{C_1 e^{\frac{5}{2}t} + C_2 e^{-\frac{5}{2}t}}^{T_0} \underbrace{- \frac{4}{25}}_{T_2}$$

$$T(0) = C_1 + C_2 - \frac{4}{25} = 0$$

$$T'(0) = \frac{5}{2}C_1 - \frac{5}{2}C_2 = 0$$

$$C_1 = C_2 = C \Rightarrow 2C = \frac{4}{25} \Rightarrow T = \frac{2}{25}\left(e^{\frac{5}{2}t} + e^{-\frac{5}{2}t}\right) - \frac{4}{25}$$

$$u(x,t) = \frac{8}{\pi}\sum_{n=1}^{\infty} \frac{\sin\frac{2n-1}{2}x \, \sin\frac{2n-1}{2}t}{2n-1} + \frac{2}{25}\left(e^{\frac{5}{2}t} + e^{-\frac{5}{2}t} - \frac{1}{2}\right)\sin\frac{5x}{2}$$