$$u_t = u_{xx} + 2\sin x \cos 2x = u_{xx} + \sin 3x - \sin x \quad \Big| \; \sin 3x - \sin x = 2\sin x \cos 2x$$

$$u(0,t) = 0 = u(\pi,t)$$

$$u(x,0) = \sin x$$

$$u = v + w$$

$$\begin{cases} v_t = v_{xx} - \sin x \\ 0 \ldots \\ v(x,0) = \sin x \end{cases}$$

$$v = T(t)\sin x$$

$$T'(t)\cancel{\sin x} = -T(t)\cancel{\sin x} - \cancel{\sin x}$$

$$T'(t) = -T(t) - 1$$

$$T_2 = A = -1$$

$$0 = -A - 1 \Rightarrow A = -1$$

$$T = Ce^{-t} - 1$$

$$T(0) = 1 = C - 1 = 1 \Rightarrow C = 2.$$

$$T = 2e^{-t} - 1$$

$$v = (2e^{-t} - 1)\sin x$$

$$\begin{cases} w_t = w_{xx} + \sin 3x \\ 0 \ldots \\ 0 \end{cases}$$

$$w = T(t)\sin 3x$$

$$T'(t) = -9T(t) + 1 \qquad 0 = -9A + 1 \qquad A = \frac{1}{9}$$

$$T(t) = Ce^{-9t} + \frac{1}{9}$$

$$T(0) = 0 = C + \frac{1}{9} \Rightarrow T(t) = \frac{1}{9}\left(1 - e^{-9t}\right)$$

$$w = \frac{\sin 3x}{9}\left(1 - e^{-9t}\right)$$

Ответ: $u(x,t) = (2e^{-t} - 1)\sin x + \dfrac{1 - e^{-9t}}{9}\sin 3x$

$$u_{tt} = a^2 u_{xx} \qquad 0 < x, t < +\infty$$

$$u_x(0,t) = 0$$

$$u(x,0) = \frac{1}{1+x^2}$$