

$$\textcircled{4} \begin{cases} u_t = \frac{1}{4} u_{xx} + 2t + e^{-t} \\ -\infty < x < \infty, t > 0 \\ u(x, 0) = 1 + e^{-x} \end{cases}$$

$$\begin{cases} v_t = \frac{1}{4} v_{xx} + 2t + e^{-t} \\ v(x, 0) = 1 \end{cases}$$

$$v(x, t) = v(t)$$

$$\begin{cases} v' = 2t + e^{-t} \\ v(0) = 1 \end{cases}$$

$$v = t^2 - e^{-t} + C$$

$$v(0) = -1 + C = 1$$

$$v(x, t) = t^2 - e^{-t} + 2$$

$$\begin{cases} w_t = \frac{1}{4} w_{xx} \\ w(x, 0) = e^{-x} \end{cases}$$

$$w = T(t) e^{-x}$$

$$\begin{cases} T' = \frac{T}{4} \\ T(0) = 1 \end{cases}$$

$$T = C e^{\frac{t}{4}}$$

$$T(0) = C = 1$$

$$w(x, t) = e^{\frac{t}{4} - x}$$

$$u(x, t) = t^2 - e^{-t} + e^{\frac{t}{4} - x} + 2$$