№:1.3

Найти ф-цию гармон. в кольце $1<r<3$ такую, что

$$u|_{r=1}=2 \qquad ; \ u|_{r=3}=\sin 3\varphi$$

$$u(r,\varphi)=C+C_0\ln r+\sum_{n=1}^{\infty}\left(r^n(A_n\cos n\varphi+B_n\sin n\varphi)+r^{-n}(C_n\cos n\varphi+D_n\sin n\varphi)\right)$$

$$u(1,\varphi)=2=C+\underbrace{C_0\ln 1}_{=0}+\sum_{n=0}^{\infty}(A_n+C_n)\cos n\varphi+(B_n+D_n)\sin n\varphi$$

$C=2 \qquad B_3+D_3=0 \qquad A_n+C_n=0\ \forall n$

$B_n+C_n=0 \quad \forall n\neq 3$

$$u(3,\varphi)=\sin 3\varphi=2+C_0\ln 3+\sum_{n=1}^{\infty}3^n(A_n+C_n)\cos n\varphi+\frac{1}{3^n}(B_n+D_n)\sin n\varphi$$

$2+C_0\ln 3=0 \Rightarrow C_0=-\frac{2}{\ln 3} \qquad \sin 3\varphi=(27B_3+\frac{1}{27}D_3)\sin 3\varphi$

$$\begin{cases}27B_3+\frac{1}{27}D_3=1\\ B_3+D_3=0 \quad |\cdot 27 \ \ominus\end{cases} \Rightarrow \quad B_3=\frac{27}{728}, \quad D_3=-\frac{27}{728}$$

$$\underline{\underline{u(r,\varphi)}}=2-\frac{2}{\ln 3}\ln r+\left(r^3\frac{27}{728}-r^{-3}\frac{27}{728}\right)\sin 3\varphi$$

№:2.3

Найти гарм. ф. в прямоуг. $0<x<a,\ 0<y<b$ если $u|_{x=0}=3\sin\frac{\pi y}{b}$; $u|_{x=a}=u|_{y=0}=0$, $u|_{y=b}=0$

$$\begin{cases}\Delta u=0\\ u(0,y)=3\sin\frac{\pi y}{b}\\ u(a,y)=0\\ u(x,0)=0\\ u(x,b)=0\end{cases}$$

Углы не выкалываем

$u(0,0)=3\sin\frac{\pi 0}{b}=0$ (смотрим, чтобы были 0)

$u=v_1+v_2+v_3+v_4 \qquad v_2,v_3,v_4=0$ т.к. гр.усл. $=0$

$$\begin{cases}\Delta v_1=0\\ v_1(0,y)=3\sin\frac{\pi y}{b}\\ v_1(a,y)=0\\ v_1(x,0)=0\\ v_1(x,b)=0\end{cases}$$

$v_1=X(x)Y(y) \qquad \frac{X''}{X}=-\frac{Y''}{Y}=\lambda$

$X''Y+XY''=0$

$$\begin{cases}X''-\lambda X=0\\ X(a)=0\end{cases} \qquad \begin{cases}Y''+\lambda Y=0\\ Y(0)=0\\ Y(b)=0\end{cases} \qquad p^2+\lambda=0 \quad p=\pm\sqrt{\lambda}\,i$$

$Y_n=C_1\sin(\sqrt{\lambda}y)+C_2\cos(\sqrt{\lambda}y)$

$$X_n(x)=A_ne^{\sqrt{\lambda}x}+B_ne^{-\sqrt{\lambda}x}=C_n\,sh\frac{\pi n}{b}x+D_n\,sh\frac{\pi n}{b}(x-a)$$

$Y(0)=0=C_2$

$Y(b)=0=C_1\sin(\sqrt{\lambda}y) \qquad \sqrt{\lambda}=\frac{\pi n}{b}$

$Y=\sin(\frac{\pi n}{b}y)$

$$\sum_{n=1}^{\infty}\left(C_n\,sh\frac{\pi n}{b}x+D_n\,sh\frac{\pi n}{b}(x-a)\right)\sin\frac{\pi n}{b}y$$

$$v_1(0,y)=3\sin\frac{\pi y}{b}=\sum_{n=1}^{\infty}C_n\underbrace{sh 0}_{=0}+D_n\,sh\frac{\pi n}{b}a\,\sin\frac{\pi n}{b}y$$

$n\neq 1:\ D_n=0$

$n=1:\ D_1\,sh\frac{\pi a}{b}=3 \qquad D_1=\frac{3}{sh\frac{\pi a}{b}}$

$$v_1(a,y)=0=\sum_{n=0}^{\infty}C_n\,sh\frac{\pi n}{b}a\,\sin\frac{\pi n y}{b}\Rightarrow C_n=0$$

$v_1(x,0)=0$

$$\underline{\underline{u(x,y)}}=\frac{3}{sh\frac{\pi a}{b}}sh\frac{\pi(x-a)}{b}\sin\frac{\pi}{b}y$$

No: 5.3

$$\begin{cases} u_{tt} = 4u_{xx} + u - 2tx + 2\cos^2 x \\ u_x(0,t) = 2t \\ u_x(\frac{\pi}{2}, t) = 2t \\ u(x,0) = \cos 2x \\ u_t(x,0) = 2x+1 \end{cases}$$

$u = v + ax^2 + bx$ $\quad u_x(0,t) = v_x(0,t) + b = 2t \Rightarrow b = 2t$

$u_x(\frac{\pi}{2}, t) = v_x(\frac{\pi}{2}, t) + 2\frac{\pi}{2} a + \cancel{2t} = \cancel{2t} \Rightarrow a = 0$

$u = v + 2tx$

$u(x,0) = \cos 2x = v(x,0)$

$u_t(x,0) = v_t(x,0) + \cancel{2x} = \cancel{2x} + 1$

$$\begin{cases} v_{tt} = 4v_{xx} + v + \cancel{2tx} - \cancel{2tx} + 1 + \cos 2x \\ v_x(0,t) = v_x(\frac{\pi}{2}, t) = 0 \\ v(x,0) = \cos 2x \\ v_t(x,0) = 1 \end{cases}$$

$v(x,t) = X(x)T(t)$ $\quad X_n(x) = 1; X_n(x) = \cos\frac{\pi n x}{l} =$

$= \cos 2nx$

$n=0$: $X_0(x) = 1$ $\quad \begin{cases} T'' = T + 1 \\ T(0) = 0 \\ T'(0) = 1 \end{cases}$ $\quad T'' - T = 1$ $\quad \lambda^2 - 1 = 0$ $\quad \lambda = 1$ $\quad \lambda = -1$

$T_{o.p} = C_1 e^{t} + C_2 e^{-t}$

$T_{r.p} = -1$ $\quad T(t) = C_1 e^{t} + C_2 e^{-t} - 1$

$T(0) = 0 = C_1 + C_2 - 1$ $\quad C_1 + C_2 = 1$ $\quad C_1 = \frac{3}{2}$

$T'(0) = 1 = C_1 - C_2 - 1$ $\quad C_1 - C_2 = 2$ $\quad C_2 = -\frac{1}{2}$

$T(t) = \frac{3}{2}e^{t} - \frac{1}{2}e^{-t} - 1$

$n=1$: $X_1(x) = \cos 2x$ $\quad \begin{cases} T'' = -16T + T + 1 \\ T(0) = 1 \\ T'(0) = 0 \end{cases}$ $\quad T'' + 15T = 1$

$\lambda^2 + 15 = 0 \Rightarrow \lambda = \pm i\sqrt{15}$;

$T_o = C_1 \cos(\sqrt{15}t) + C_2 \sin(\sqrt{15}t)$

$r.p = \frac{1}{15}$ $\quad T(t) = C_1 \cos(\sqrt{15}t) + C_2 \sin(\sqrt{15}t) + \frac{1}{15}$

$T(0) = 1 = C_1 + \frac{1}{15} \Rightarrow C_1 = \frac{14}{15}$ $\quad T(t) = \frac{14}{15}\cos(\sqrt{15}t) - \frac{1}{15\sqrt{15}}\sin(\sqrt{15}t) + \frac{1}{15}$

$T'(0) = 0 = \sqrt{15}C_2 + \frac{1}{15} \Rightarrow C_2 = -\frac{1}{15\sqrt{15}}$

No: 3.3