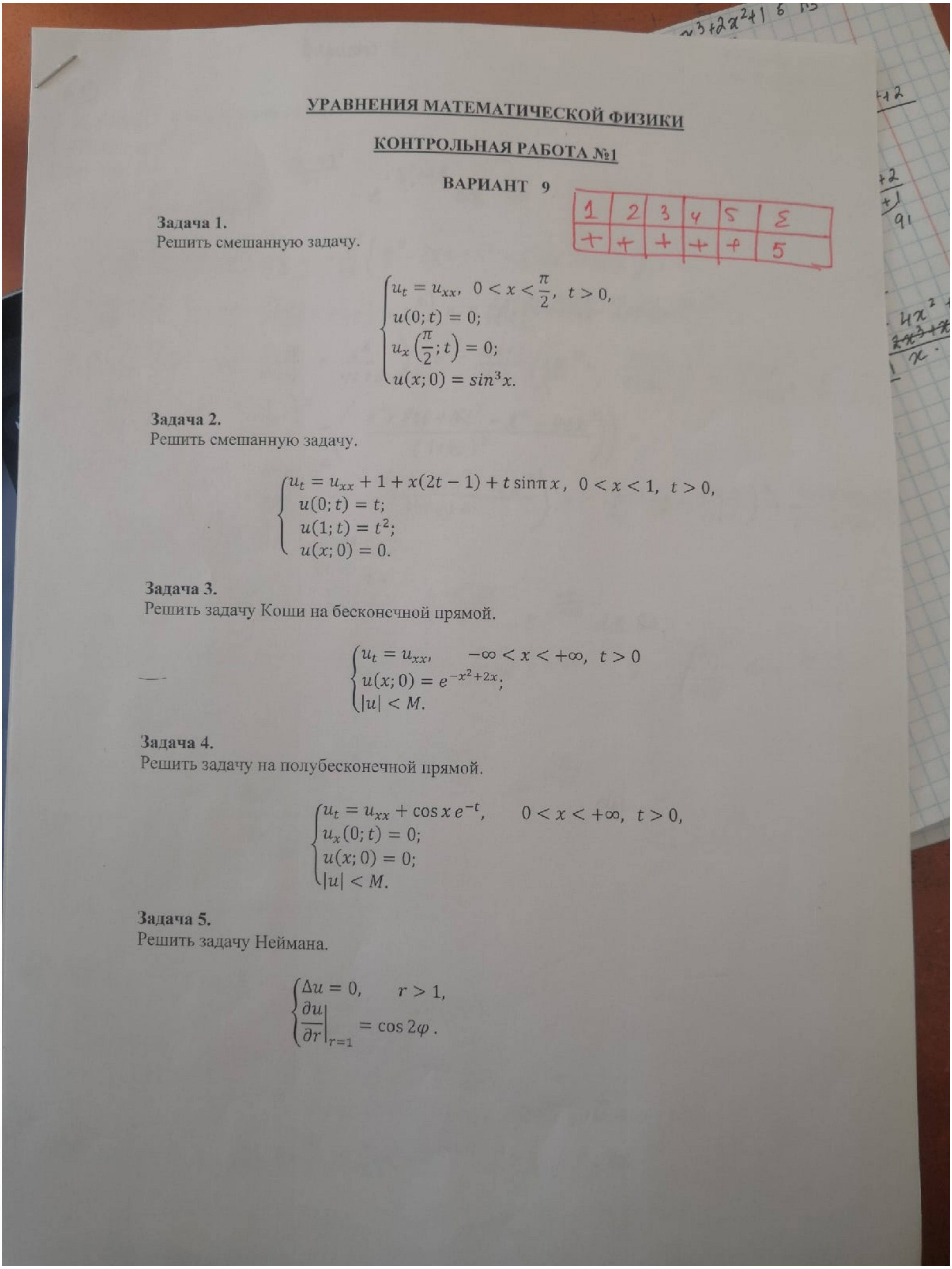
Фотография 1



УРАВНЕНИЯ МАТЕМАТИЧЕСКОЙ ФИЗИКИ

КОНТРОЛЬНАЯ РАБОТА №1

ВАРИАНТ 9

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Задача 1.
Решить смешанную задачу.

$$\begin{cases} u_t = u_{xx}, \ 0 < x < \frac{\pi}{2}, \ t > 0, \\ u(0;t) = 0; \\ u_x\left(\frac{\pi}{2};t\right) = 0; \\ u(x;0) = sin^3 x. \end{cases}$$

Задача 2.
Решить смешанную задачу.

$$\begin{cases} u_t = u_{xx} + 1 + x(2t-1) + t\sin\pi x\,, \ 0 < x < 1, \ t > 0, \\ u(0;t) = t; \\ u(1;t) = t^2; \\ u(x;0) = 0. \end{cases}$$

Задача 3.
Решить задачу Коши на бесконечной прямой.

$$\begin{cases} u_t = u_{xx}, \qquad -\infty < x < +\infty, \ t > 0 \\ u(x;0) = e^{-x^2+2x}; \\ |u| < M. \end{cases}$$

Задача 4.
Решить задачу на полубесконечной прямой.

$$\begin{cases} u_t = u_{xx} + \cos x\, e^{-t}, \qquad 0 < x < +\infty, \ t > 0, \\ u_x(0;t) = 0; \\ u(x;0) = 0; \\ |u| < M. \end{cases}$$

Задача 5.
Решить задачу Неймана.

$$\begin{cases} \Delta u = 0, \qquad r > 1, \\ \left.\frac{\partial u}{\partial r}\right|_{r=1} = \cos 2\varphi\,. \end{cases}$$

Фотография 2

Вариант 9.

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328 группа

(3)

$$\begin{cases} u_t = u_{xx} & -\infty < x < +\infty,\ t>0 \\ u(x,0) = e^{-x^2+2x} \\ |u| < M \end{cases}$$

$$u(x,t) = \int_{-\infty}^{+\infty} \frac{1}{2\sqrt{\pi t}} e^{-\frac{(\xi-x)^2}{4t}} \cdot e^{-\xi^2+2\xi} d\xi =$$

$$-\frac{1}{4t}(\xi-x)^2 - \xi^2 + 2\xi = -\frac{1}{4t}\left(\xi^2 - 2\xi x + x^2 + \xi^2 4t - 8t\xi\right) =$$

$$= -\frac{1}{4t}\left(\xi^2(1+4t) - 2\xi(x+4t) + x^2\right) = $$

$$= -\frac{4t+1}{4t}\left(\xi^2 - 2\xi\frac{x+4t}{1+4t} + \frac{x^2}{4t+1}\right) = -\frac{4t+1}{4t}\left(\left(\xi - \frac{x+4t}{1+4t}\right)^2 - \frac{(x+4t)^2}{(1+4t)^2} + \frac{x^2}{4t+1}\right) =$$

$$= -\frac{4t+1}{4t}\left(\left(\xi - \frac{x+4t}{1+4t}\right)^2 - \left(\frac{x^2+8xt+16t^2-x^2-4tx^2}{(1+4t)^2}\right)\right) =$$

$$= -\frac{4t+1}{4t}\left(\left(\xi - \frac{x+4t}{1+4t}\right)^2 - \frac{8xt+16t^2-4tx^2}{(1+4t)^2}\right) = -\frac{4t+1}{4t}\left(\xi - \frac{x+4t}{1+4t}\right)^2 + \frac{4t(2x+4t-x^2)}{(1+4t)4t} =$$

$$= -\frac{4t+1}{4t}\left(\xi - \frac{x+4t}{1+4t}\right)^2 + \frac{2x+4t-x^2}{1+4t}$$

$$= \int_{-\infty}^{+\infty} \frac{1}{2\sqrt{\pi t}} e^{-\frac{4t+1}{4t}\left(\xi - \frac{x+4t}{1+4t}\right)^2} \cdot e^{\frac{-x^2+2x}{1+4t}} \cdot e^{\frac{4t}{1+4t}} d\xi =$$

$$\frac{4t+1}{4t}\left(\xi - \frac{x+4t}{1+4t}\right)^2 = a^2 \qquad \frac{\sqrt{4t+1}}{\sqrt{4t}}\left(\xi - \frac{x+4t}{1+4t}\right) = a \qquad \xi = \frac{\sqrt{4t}}{\sqrt{4t+1}}a + \frac{x+4t}{1+4t}$$

$$d\xi = \frac{\sqrt{4t}}{\sqrt{4t+1}} da$$

$$= \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi}\sqrt{4t+1}} e^{-a^2} \cdot e^{-\frac{x^2+2x}{1+4t}} \cdot e^{\frac{4t}{1+4t}} da =$$

$$\boxed{= \frac{1}{\sqrt{4t+1}} e^{-\frac{x^2+2x}{1+4t}} \cdot e^{\frac{4t}{1+4t}}}$$ +

№1

$$\begin{cases} u_t = u_{xx} & 0 < x < \frac{\pi}{2} \\ u(0,t) = 0 \\ u_x(\frac{\pi}{2}, t) = 0 \\ u(x,0) = \sin^3 x \end{cases}$$

$$u(x,t) = X(x)\,T(t)$$

$$X'' + \lambda X = 0$$

$$X(x) = C_1 \cos\sqrt{\lambda}\, x + C_2 \sin\sqrt{\lambda}\, x$$

$$X'(x) = -C_1\sqrt{\lambda}\sin\sqrt{\lambda}\, x + C_2\sqrt{\lambda}\cos\sqrt{\lambda}\, x$$

$$X(0) = C_1 = 0 \qquad X'(\tfrac{\pi}{2}) = C_2\sqrt{\lambda}\cos\tfrac{\pi}{2}\sqrt{\lambda} = 0 \qquad \frac{\pi}{2}\sqrt{\lambda} = \frac{\pi}{2} + \pi n$$

$$\lambda_n = (1+2n)^2$$

$$X(x) = \sin(1+2n)x$$

стр. 1

всего 3 стр

Фотография 3

$$\frac{T'(t)}{T(t)} = -\lambda \quad T_n(t) = C_n e^{-(1+2n)^2 t}$$

$$U_n(x,t) = \sum_{n=0}^{\infty} C_n e^{-(1+2n)^2 t} \sin(1+2n)x$$

+

$$U(x,0) = \sin^3 x = \frac{3}{4}\sin x - \frac{1}{4}\sin 3x$$

$$\boxed{U(x,t) = \frac{3}{4} e^{-t} \sin x - \frac{1}{4} e^{-9t} \sin 3x}$$

№2
$$\begin{cases} U_t = U_{xx} + 1 + x(2t-1) + t\sin\pi x & 0<x<1,\ t>0 \\ U(0,t) = t \\ U(1,t) = t^2 \\ U(x,0) = 0 \end{cases}$$

$$U(x,t) = V(x,t) + a(t)x + b(t)$$

$$a(t)\cdot 0 + b(t) = t \to b(t) = t \qquad \to U(x,t) = V(x,t) + x(t^2-t) + t$$

$$a(t)\cdot 1 + t = t^2 \to a(t) \ t^2 - t$$

$$V_t + \cancel{x(2t-1)} + \cancel{1} = V_{xx} + \cancel{1} + \cancel{x(2t-1)} + t\sin\pi x$$

$$\begin{cases} V_t = V_{xx} + t\sin\pi x \\ V(0,t) = 0 \\ V(1,t) = 0 \\ V(x,0) = 0 \end{cases} \qquad V(x,t) = \sum_{n=1}^{\infty} T_n(t)\sin\pi n x \qquad T_n' = -$$

$$\sum_{n=1}^{\infty} f_n \sin\pi n x = t\sin\pi n x$$

$$f_n = \begin{cases} t, & n=1 \\ 0, & \text{иначе} \end{cases}$$

$$n \neq 1: \begin{cases} T_n' + \pi^2 n^2 T_n = 0 \\ T_n(0) = 0 \end{cases} \Rightarrow 0$$

$$n=1: \begin{cases} T_1' + \pi^2 T_1 = t \\ T_1(0) = 0 \end{cases} \qquad T_{1,0} = Ce^{-\pi^2 t} \qquad \Rightarrow \quad A + \pi^2(At+B) = t$$

$$T_{1,ч} = At + B \qquad A = \frac{1}{\pi^2}$$

$$A + \pi^2 B = 0 \qquad B = -\frac{1}{\pi^4}$$

$$\Rightarrow T = \frac{1}{\pi^2}t - \frac{1}{\pi^4}$$

$$T_1 = Ce^{-\pi^2 t} + \frac{1}{\pi^2}t - \frac{1}{\pi^4} \qquad T_1(0) = 0 = C - \frac{1}{\pi^4} = 0 \Rightarrow C = \frac{1}{\pi^4}$$

$$T_1 = \frac{1}{\pi^4} e^{-\pi^2 t} + \frac{1}{\pi^2}t - \frac{1}{\pi^4}$$

$$V(x,t) = \left(\frac{1}{\pi^4} e^{-\pi^2 t} + \frac{1}{\pi^2}t - \frac{1}{\pi^4}\right)\sin\pi x$$

$$\boxed{U(x,t) = \left(\frac{1}{\pi^4} e^{-\pi^2 t} + \frac{1}{\pi^2}t - \frac{1}{\pi^4}\right)\sin\pi x + x(t^2-t) + t}$$ +

СТР 2

№5
$$\begin{cases} \Delta U = 0 & r>1 \\ \left.\frac{\partial U}{\partial r}\right|_{r=1} = \cos 2\varphi \end{cases}$$

$$U(r,\varphi) = C + \sum_{n=1}^{\infty} \frac{1}{r^n}(A_n\cos n\varphi + B_n\sin n\varphi)$$

$$\frac{\partial U}{\partial r}(r,\varphi) = \sum_{n=1}^{\infty} (-1)\frac{n}{r^{n+1}}(A_n\cos n\varphi + B_n\sin n\varphi)$$

$$\left.\frac{\partial U}{\partial r}\right|_{r=1} = \sum_{n=1}^{\infty} -n\cdot(A_n\cos n\varphi + B_n\sin n\varphi) = \cos 2\varphi \Rightarrow A_1 = B_1 = 0 \quad A_2 = -\frac{1}{2} \quad B_2 = A_3 = B_3 = \dots = 0$$

$$\boxed{U(r,\varphi) = -\frac{1}{2r^2}\cos 2\varphi}$$ +

Фотография 4

$$\begin{cases} u_t = u_{xx} + \cos x \; e^{-t} & -\infty < x < +\infty ,\ t > 0 \\ u_x(0,t) = 0 \\ u(x,0) = 0 \end{cases}$$

$u(x,t) = \cos x \cdot T(t)$

$\cos x \cdot T' = -\cos x \, T(t) + \cos x \, e^{-t}$

$$\begin{cases} T' = -T + e^{-t} \\ T(0) = 0 \end{cases}$$

$T' = -T \quad T = c e^{-t}$

$c' e^{-t} = e^{-t} \Rightarrow c = t + C_1$

$\cos x \cdot C_1 = 0 \Rightarrow C_1 = 0$

$$\boxed{u(x,t) = \cos x \cdot t \cdot e^{-t}}$$

+