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assignment 2

$\boxed{1}$ $p(X) = 1/6$ $p(\neg x) = 5/6$
$p(Y) = 1/2$ $p(\neg y) = 1/2$

$E(x) = \cancel{- p_x \log_2(p_x)}$ $E(Y) = \cancel{- P(y) \log_2(P(y))}$
$= \cancel{0.431}$ $= \cancel{0.5}$

$E(x) = -\frac{1}{6}\log_2(1/6) - \frac{5}{6}\log(\frac{5}{6})$

$= 0.65$

$E(y) = -\frac{1}{2}\log_2(1/2) - \frac{1}{2}\log_2(\frac{1}{2})$
$= 1$

A $\therefore E(y) > E(x)$

$\boxed{2}$ $p(y=1) = 5/9$
$p(y=0) = 4/9$

$p(y=1 | x_1 = 1 \cap x_2 = a \cap x_3 = q)$ vs $p(y=0 | x_1 = 1 \cap x_2 = a \cap x_3 = q)$

$p(y=1 | x_1 = 1 \cap x_2 = a \cap x_3 = q) = p(y=1 | x_1 = 1) \cdot p(y=1 | x_2 = a)$
$p(y_1 = 1 | x_3 = q) \cdot p(y=1)$

$= \frac{1}{5} \times \frac{1}{5} \times \frac{2}{5} \times \frac{5}{9}$

$= 1/225$

$$P(y=0 \mid x_1=1 \cap x_2=a \cap x_3=q) = P(y=0 \mid x_1=1) \cdot P(y=0 \mid x_2=a) \cdot P(y=0 \mid x_3=q) \cdot P(y=0)$$

$$= \frac{1}{4} \times \frac{2}{4} \times \frac{1}{4} \times \frac{4}{9} = \frac{1}{72}$$

(Ans).

(3) (i) 'A' and 'B' are independent

$$\therefore \; P(B|A) = 0.5$$

$$\therefore P(B) = 0.5$$

$$P(B) = P(A \cap B) + P(\neg A \cap B)$$

$$\Rightarrow 0.5 = 0.1 + x + 0.2 + 0.1$$

$$\therefore x = 0.1$$

(ii) $0.1 + 0.2 + 0.2 + Y + x + 0.1 + 0.1 + 0.1 = 1$

$\therefore Y = 0.2$

(iii) $$p(A \mid B \cap C) = \frac{p(A \cap B \cap C)}{p(B \cap C)}$$

$$= \frac{0.1}{0.1 + 0.2}$$

$$= 0.33$$

④ Test A,

$p(\text{True} \mid \text{virus}) = 0.95$

$p(\neg\text{True} \mid \text{virus}) = 0.05$

$p(\text{True} \mid \neg\text{virus}) = 0.10$

$p(\neg\text{True} \mid \neg\text{virus}) = 0.90$

Test B,

$p(\text{True} \mid \text{virus}) = 0.90$

$p(\neg\text{True} \mid \text{virus}) = 0.10$

$p(\text{True} \mid \neg\text{virus}) = 0.05$

$p(\neg\text{True} \mid \neg\text{virus}) = 0.95$

Test A, $p(\text{virus} \mid \text{Test}) = \dfrac{p(\text{Test} \mid \text{virus})\, p(\text{virus})}{p(\text{Test})}$

$$= \frac{p(\text{Test} \mid \text{virus})\, p(\text{virus})}{p(\text{Test} \cap \text{virus}) + (\text{Test} \cap \neg\text{virus})}$$

$$= \frac{p(\text{Test} \mid \text{virus})\, p(\text{virus})}{p(\text{Test} \mid \text{virus})\, p(\text{virus}) + p(\text{Test} \mid \neg v) * p(\neg\text{virus})}$$

$$= \frac{0.95 \times 0.01}{0.95 \times 0.01 + 0.05 * 0.99}$$

$$= 0.08755$$

Test B, $p(\text{virus} \mid \text{Test}) = \dfrac{p(\text{Test} \mid \text{virus})\, p(\text{virus})}{p(\text{Test} \mid \text{virus})\, p(\text{virus}) + p(\text{Test} \mid \neg\text{virus})\, p(\neg\text{virus})}$

$$= \frac{0.90 \times 0.01}{0.9 \times 0.01 + 0.05 \times 0.99}$$

$$= 0.1538$$

B test is more indicative.

Ans
(5)

$p(\cancel{\text{me}}\ \text{smoker} \mid \text{male}) = 0.70$

$p(\neg \text{smoker} \mid \text{male}) = 0.30$

$$\text{Entropy}(\text{Smoking} \mid \text{male}) = -0.7 \log_2 0.7 - 0.3 \log_2 0.3$$
$$= 0.8813$$

(6) $E(\text{decision}) = -\frac{3}{4}\log_2\left(\frac{3}{4}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right)$

$$= 0.8113$$

$$E(\text{Rain}=\text{yes} \mid \text{Temp}=\text{cold}) = -\tfrac{1}{2}\log_2 \tfrac{1}{2} - \tfrac{1}{2}\log_2 \tfrac{1}{2}$$
$$= 1$$

$$E(\text{Rain}=\text{No} \mid \text{Temp}=\text{cold}) = -\tfrac{2}{2}\log_2 \tfrac{2}{2} - \tfrac{0}{2}\log_2 \tfrac{0}{2}$$
$$= 0$$

$$IG(\text{Rain}) = 0.813 - \tfrac{1}{2}\times 1 - \tfrac{1}{2}\times 0$$
$$= 0.313$$

$E(RT = \text{a few} \mid \text{cold}) = -1/2 \log_2 1/2 - 1/2 \log_2 1/2$
$= 1$

$E(RT = \text{a lot} \mid \text{cold}) = -2/2 \log_2 2/2 - 0/2 \log_2 0/2$
$= 0$

$IG(RT) = 0.8113 - 1/2 \times 1 - 1/2 - 0$
$= 0.313$

as, $IG(RT) = IG(Rain)$
any of them can be selected.

7) $y = mX + c$

$m = 9$
$c = 2$
$\alpha = 0.001$

$$SSR = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$
$$= [11-(9\times2+2)]^2 + [23-(9\times4+2)]^2 + [28-(9\times6+2)]^2$$
$$= 1090$$

$$\frac{d}{dm}(SSR) = 2[y_1 - (mx_1 + c)]\cdot(x_1) + 2[y_2 - (mx_2 + c)](x_2) + 2[y_3 - (mx_3 + c)](x_3)$$
$$= 492$$

$\text{Step size} = \frac{d}{dm}(SSR) \times \alpha = 492 \times 0.001 = .492$

$m_{new} = m_{old} - \text{stepsize}$

$= 8.508$

$$\frac{d}{dc}(ssr) = 2\left[y_1 - (mx_1 + c)\right] + 2\left[y_2 - (mx_2 + c)\right] + 2\left[y_3 - (mx_3 + c)\right]$$

$= 104$

$c_{new} = 2 - 104 \times 0.001 = 1.896$

⑧ $h_1 = 4x_1 - x_2$ | $x_1 = 5$

$h_2 = 3x_1 + 2x_2$ | $x_2 = 1$

$h_1 = \text{Relu}(4 \times 5 - 1)$

$= 19$

$h_2 = \text{Relu}(3 \times 5 + 2) = 17$

Output, $Z = 3h_1 + 2h_2$

$= 3 \times 19 + 17 \times 2 = 91$

$$y = \sigma(z) = \frac{1}{1 + e^{-91}} \approx 1$$

Activation function adds non-linearity, as such multiplayer nets can model complex functions, control gradient flow and helps interpret output.

9. (i) Regression predicts continuous values
it's loss functions are SSR, MSE, MAE
and evaluation metrices are R^2, MAE, RMSE

classification predicts discrete values.
it's loss function's are Binary Cross Entropy.
and evaluation metrices are precision, roc-auc.

(ii) Decrease the learning rate or optimize with adaptive step size

(iii) squared error is smooth and differentiable everywhere, give gradient proportional to error. Absolute error has an defined derivative at '0' and ~~constand~~ constant magnitude of gradients

(iv) Cross entropy yields stronger gradient when predictions are wrong, leads to faster learning and stronger probability, MSE assumes Gaussian noise and can saturate with small gradients.

(10)

x	y	$z = mx+1$	a
0.2	0	1.6	0.83
0.4	0	2.2	0.9
0.8	1	3.4	0.97

$m = 3$
$b = 1$

$$a = \sigma(z) = \frac{1}{1+e^{-z}}$$

$$L = -\frac{1}{n}\sum_{i=1}^{n}\left[y\log a + (1-y)\log(1-a)\right]$$

$$L = \frac{[0+(1-0)\log(1-.83)] + [0+\log(1-0.9)] + [\log 0.97]}{3}$$

$= 1.363$

$$\frac{dL}{dm} = \frac{1}{3}\sum (a-y)x$$

$$= \frac{0.83 \times 0.2 + 0.9 \times 0.4 + (0.97-1) \times 0.8}{3}$$

$$= 0.1673$$

$$\frac{dL}{db} = \frac{1}{3}\sum (a-y)$$

$$= \frac{0.83 + 0.9 + (0.97-1)}{3}$$

$$= 0.567$$

$$m_{new} = 3 - (0.1673 \times 0.1) = 2.98$$

$$b_{new} = 1 - (0.567 \times 0.1) = 0.9433$$