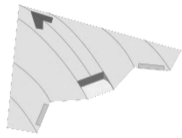
Guidelines on Developing a Lifting Line Excel Program



Overview:

This document will guide you through the process of creating a lifting line code in Excel to calculate the aerodynamic properties of unswept trapezoidal wings with aspect ratios greater than four. All the information you need to complete this task will be contained within this sheet and the lecture notes. For further information, please consider reading:

- Anderson, Fundamentals of Aerodynamics, Chapter 5.
- Katz and Poltkin, Low-Speed Aerodynamics: From Wing Theory to Panel Methods, Chapter 8.

Method

The code developed here is based on Glauert's method and will calculate the first ten Fourier coefficients to define the load distribution. This process will involve the calculation of a 10x10 matrix of Aerodynamic Influence Coefficients (AIC). Excel will handle all the mathematics of the matrix inversion using array functions. The following instructions will guide you through the use of these array functions.

Step 1 – Inputs

Create four inputs for the calculator, these will be:

- Span (b)
- Aspect Ratio (AR)
- Taper Ratio (λ)
- Geometric Angle of Attack (α) [Note: make sure to use the correct units!]
- 2D Aerofoil

Whilst you are creating the cells for these inputs, think if there are any bounds on the upper and lower values of these inputs. Please note that in this code, we will only consider wings with no geometric or aerodynamic twist. No geometric twist implies that your geometric angle of attack is the same along all wing stations. No aerodynamic twist implies that your zero-lift angle of attack of the wing's 2D section as well as the 2d lift curve slope are the same along all wing stations. As a future challenge (not required for this assessment, consider attempting to develop this code to include both twist types!).

Step 2 – Calculation of required geometrical characteristics

Use the input fields you have just created, in the previous step, to calculate the following geometrical properties using the formulae given below:

Wing Area (S):	$\frac{b^2}{AR}$
Root Chord (C_r):	$\frac{b(1+\lambda)}{AR}$
Tip Chord (C_t):	λC_r
Zero lift Angle of Attack ($\alpha_{L=0}$):	Based on data of the 2D aerofoil that the wing is made of.

Step 3 – Calculation of AIC matrix

From the lecture notes, you should know that the angle of attack at any spanwise station can be given by:

$$\alpha(\theta) - \alpha_{L=0} = \sum_{n=1}^{N} A_n \left(\frac{2b}{\pi c(\theta)} \sin(n\theta) + \frac{n\sin(n\theta)}{\sin(\theta)} \right) \quad \text{M}$$

We will write the above equation in matrix form to allow for a simple solution to find the Fourier coefficients (A_n).

The matrix can be created using the following steps:

1. Create a column with the numbers from 1-10; this will help us to construct the variables which are needed for the calculation of each element. These will be the index of each spanwise station.
2. In the next column, use the station indices to calculate the value of θ at that station. θ is from 0 at the port wing tip, to π at the starboard wing tip. As we don't want to include the values of 0 and π in our stations, we can calculate $\theta_i = \frac{i\pi}{11}$.
3. In the next column, using the relation $y = -\frac{b}{2}\cos(\theta)$, calculate the y-coordinate (y_i) at each spanwise station.
4. In the next column, using y_i, C_r, and C_t calculate the local chord at each spanwise station using linear interpolation.
5. In the next columns, in the row above your calculated variables create a row of the numbers 1-10; these will be the values of n.

Your sheet should now look something like this:

				n									
	Theta	y	chord	1	2	3	4	5	6	7	8	9	10
1	θ	y	c										
2	θ	y	c										
3	θ	y	c										
4	θ	y	c										
5	θ	y	c										
6	θ	y	c										
7	θ	y	c										
8	θ	y	c										
9	θ	y	c										
10	θ	y	c										

Now, calculate the value of each element in the AIC matrix based on the equation:

$$\left(\frac{2b}{\pi c(\theta)} \sin(n\theta) + \frac{n\sin(n\theta)}{\sin(\theta)} \right)$$

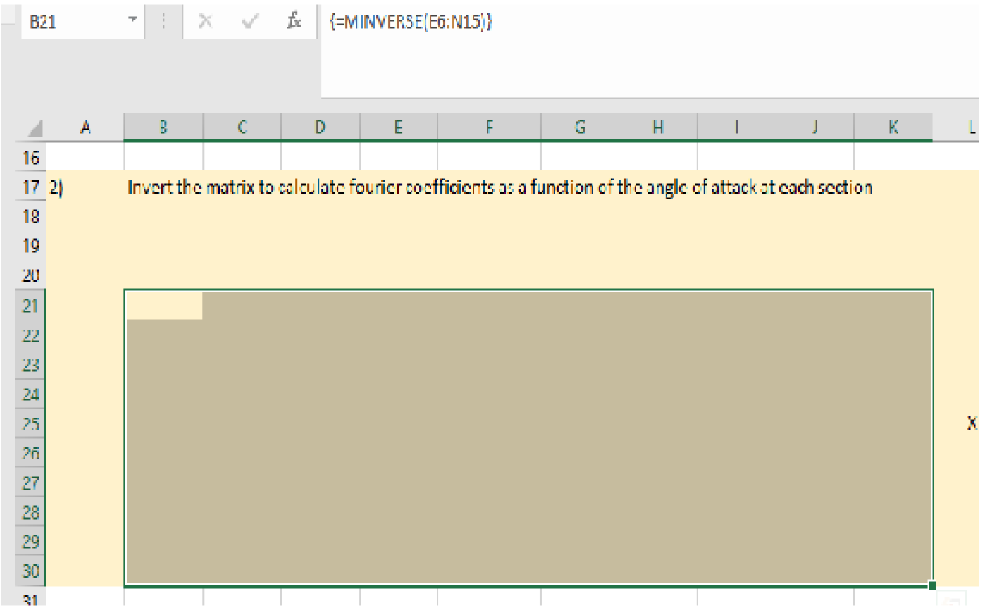
You can save time by anchoring the appropriate variables and dragging the formula across using the small square in the bottom left corner (for more help on this see https://keyskillset.com/anchoring-formula-in-excel/).

Step 4 – Invert the AIC matrix

We will now use excel to invert our AIC matrix using the steps below:

1. Click and drag to highlight an area which is 10 rows down and 10 columns across.
2. Type in the formula =MINVERSE(, then highlight your original AIC matrix and type the close bracket ,), but **do not press enter**

3. Press ctrl + shift + enter together. You should see the area which you originally highlighted populate with the inverse of your AIC matrix. In the formula bar you should see your formula is enclosed by {curly brackets}.

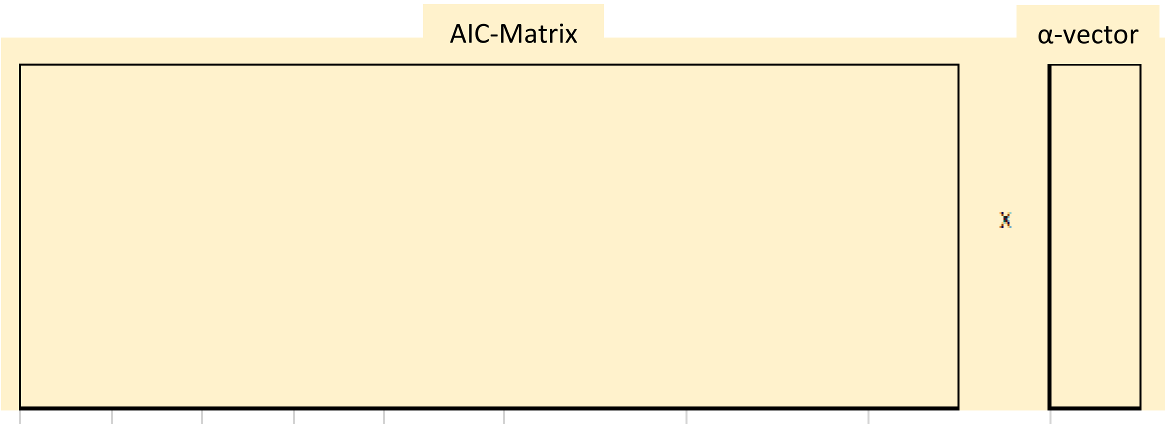


Step 5 – Create a vector of the local geometric angle of attack

The vector will be based on:

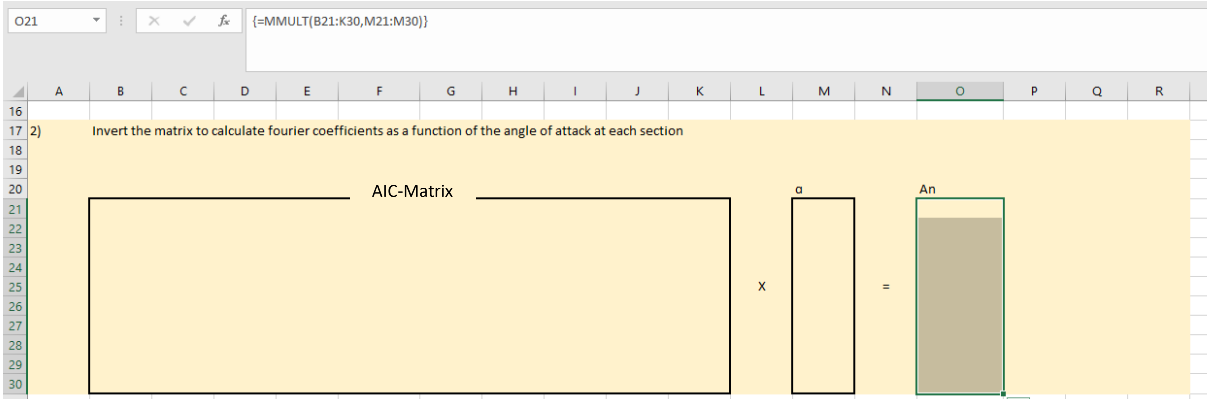
$$\alpha - vector = \alpha(\theta) - \alpha_{L=0}(\theta)$$

This vector of alphas will be multiplied by the inverse of the AIC matrix to calculate the Fourier coefficients. Because your wing does not have twist, this vector will be constant.



Step 6 – Calculate the Fourier coefficients

Use the MMULT function to multiply the inverse of the AIC matrix and the angle of attack vector. This is an array function so should be implemented in the same way as step 4.



Step 7 – Calculate the aerodynamic coefficients

Based on the expressions provided in Topic 12, use the Fourier coefficients to calculate the lift coefficient, lift curve slope, induced drag coefficient, wing span efficiency factor, as well as the distributions for circulation and induced angle of attack along the wing span. Once finished, make sure to test/verify your programme on some solved lifting line examples. You can find these in our recommended textbooks.