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Project on performance of airplanes

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Section 1 BN 4

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لله الفضل والمنة

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Very Important Notes

1. This is a report of determining some aircrafts performance parameters, I chose five airplanes to determine theirs, and I have made some assumptions in this report:
2. The oswald efficiency factor of each aircraft is approached to the plane I see most alike in this report (Estimating oswald efficiency factor from basic aircraft geometrical parameters, M. Nita and D. Scholz, Hamburg university)[1]
3. All aerodynamics properties are obtained at reynold's number = $3.0 * 10^6$
4. The values of absolute ceiling, service ceiling, range and endurance are not good for most aircrafts, even though their equations are taken from the reference and should have given accurate answers, but anyway I believe the fault is not mine.
5. I had so much more to do but didn't have time, maybe I can show the new version later...

1 <https://www.fzt.haw-hamburg.de/pers/Scholz/OPerA/OPerA_PUB_DLRK_12-09-10.pdf>

1. F-16A

1.1. Table of information

#	Characteristic	Description or Value S.I. Units
1	A/C Name	F-16A
2	A/C Type	Fighter
3	A/C Length	15.1
4	A/C Height	5.08
5	Airfoil Section and Wing Data	Unknown
6	b	9.9568
7	S	37.17669984
8	$AR= b^2/S$	2.666666667
9	W (Max Takeoff)	166,600
10	W_E (Empty)	72,422
11	W/S	4481.30148
12	W/b	16732.28346
13	C_{D0}	0.01
14	K	0.147365688
15	e=1/(∏KAR)	0.81
16	L/D max	13.02482258
17	High Lift Device(s)	Don't Know
18	$(C_L)_{max}$ [Cruise]	1.5
19	Engine Name	PW F100-PW-200
20	Engine Type & Number	One Jet engine
21	$(T_A)_{max}$ @Sea-Level	106000
22	$(P_A/W)_{max}$ Or($T_A/W)_{max}$	0.636254502
23	Variation of Power /Thrust w Altitude	Don't Know
24	Varition of Power /Thrust w Speed	Probably no variation
25	Fuel Weight (Capcity) W_F	94,178
26	TSFC	0.001988438
27	Analysis h	1406.846646

28 vcruise 257.942

1.2. Table of performance parameters

Parameter	Real	Calculated
Top speed km/h	2164.568	2451.727178
Stall speed km/h	79	251.4235220
Min speed km/h	-------------------------------------	148.4667211
Max R/C **m/s**	-------------------------------------	164.3575833
Angle at max R/C degrees	-------------------------------------	24.5196
Max angle of climb m/s	-------------------------------------	34.01970732
L/D max	-------------------------------------	13.02482258
Min angle of gliding degrees	-------------------------------------	4.390355151
Range km	2264.347	824.9339327
Endurance h	-------------------------------------	1.086201810
Service ceiling km	7.62	1.406846646
Time to climb to serv ceil. s	-------------------------------------	49.62243029

1.3. Images of the airplane

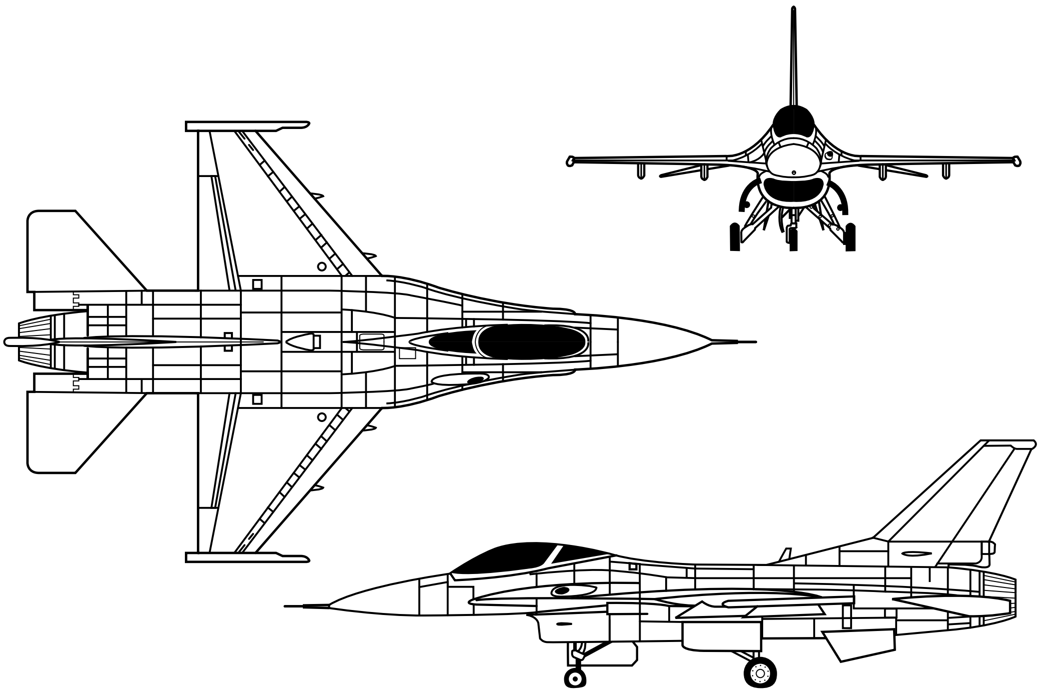


Figure 1: Public Domain, https://commons.wikimedia.org/w/index.php?curid=1167332

2. Embraer 120

2.1. Table of information

#	Characteristic	Description
1	A/C Name	EMBRAER 120
2	A/C Type	twin-turboprop commuter airliner
4	A/C length	20
5	A/C height	6.35
6	Airfoil Section and Wing Data	NACA 23012
7	b	19.78
8	S	39.4
9	AR= b2/S	9.93016243654822
11	WE (Empty)	69286
10	W (Max Takeoff)	112700
12	W/S	292.848020434
13	W/b	748.612471433
14	CD0	0.01
15	K	0.043297747605475
16	e (span efficiency)	0.81
17	(L/D)max	24.52849924
18	High Lift Device(s)	Flaps
19	(CL)max	1.25
20	Engine Name	Pratt & Whitney Canada PW118 / Pratt & Whitney Canada PW118A / Pratt & Whitney Canada PW118B turboprop engines
21	Engine Type & Number	Two , turboprops
22	(PA)max@sea-Level	2680000
23	(PA/W)max	11.8899733806566
24	Variation of thrust with altitude	Don’t know
25	Variation of thrust with velocity	Probably no variation
26	Fuel name and Specific gravity	
27	Fuel Weight (Capacity) WF	43414
28	PSFC	4.25*10^-5
29	vcruise	153.333333333333

2.2. Table of performance parameters

Parameter	Real	Calculated
Top speed km/h	608	780.6686795
Stall speed km/h	162	220.0433737
Min speed km/h	-------------------------------------	27.97958431
Max R/C **m/s**	-------------------------------------	20.37403225
Angle at max R/C degrees	-------------------------------------	16.3561

Max angle of climb m/s	-------------------------------------	49.85402370
L/D max	-------------------------------------	24.52849924
Min angle of gliding degrees	-------------------------------------	2.334593191
Range km	1,750	150.8077741
Endurance h	-------------------------------------	0.3092960682
Service ceiling km	9.085	0.8435092067
Time to climb to serv ceil. s	-------------------------------------	156.7421070

2.3. Images of the airplane

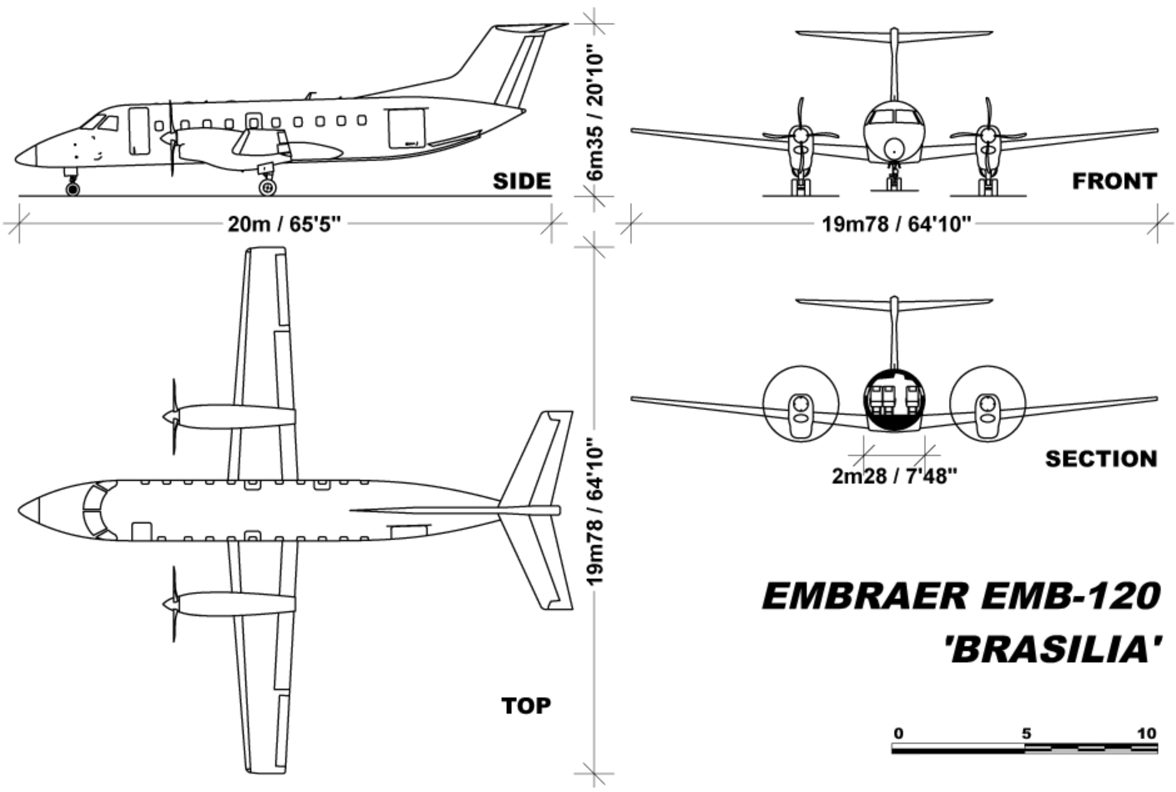


Figure 2: By Julien.scavini - Own work, CC BY 3.0, https://commons.wikimedia.org/w/index.php?curid=20555164

4. Data sheet

1. Jet Engines

1. Analytical

1.1. Maximum velocity

$$v_{max} = \sqrt{\frac{(\frac{T_A}{W})_{max}(\frac{W}{S}) + (\frac{W}{S})\sqrt{(\frac{T_A}{W})^2 - \frac{4C_{Do}}{\pi eAR}}}{\rho_\infty C_{Do}}} \qquad \text{(Equation 1)}$$

1.2. Minimum velocity

$$v_{min} = \sqrt{\frac{(\frac{T_A}{W})_{max}(\frac{W}{S}) - (\frac{W}{S})\sqrt{(\frac{T_A}{W})^2 - \frac{4C_{Do}}{\pi eAR}}}{\rho_\infty C_{Do}}} \qquad \text{(Equation 2)}$$

1.3. Stall velocity

$$v_{stall} = \sqrt{\frac{W}{\frac{1}{2}\rho_\infty C_{L,max}}} \qquad \text{(Equation 3)}$$

1.4. Rate of Climb

$$R/C = \frac{T_A V - T_R V}{W} \qquad \text{(Equation 4)}$$

$$v_{@R/C_{max}} = Solve[\frac{d}{dv}R/C = 0] \qquad \text{(Equation 5)}$$

$$R/C_{max} = \frac{(T_A v - T_R v)_{v@R/C_{max}}}{W} \qquad \text{(Equation 6)}$$

$$\gamma \text{ (max angle of climb) } = sin^{-1}(\frac{T_A - T_R}{W}) \qquad \text{(Equation 7)}$$

1.5. Range

$$Range = 2\sqrt{\frac{2}{\rho_\infty s}}\frac{1}{c_t}\frac{C_L^{\frac{1}{2}}}{C_D}(w_0^{\frac{1}{2}} - w_1^{\frac{1}{2}}) \qquad \text{(Equation 8)}$$

1.6. Endurance

$$Endurance = \frac{1}{c_t}\frac{C_L}{C_D}ln\frac{w_0}{w_1} \qquad \text{(Equation 9)}$$

1.7. Required thrust

$$T_R = \frac{1}{2}\rho_\infty v_\infty^2 SC_{Do} + \frac{1}{2}\rho_\infty v_\infty^2 S\frac{C_L^2}{\pi eAR} \qquad \text{(Equation 10)}$$

1.8. Minimum Angle of Gliding

$$\theta = \tan^{-1}(\frac{1}{L/D}) \quad \text{(Equation 11)}$$

2. Piston Propeller engine

1. Analytical

First, consider these parameters to make the equation look better: a, b, c, d, e and t.

AR = $\frac{b^2}{s}$; $k = \frac{1}{\pi eAR}$; $WS = \frac{W}{S}$; $PAMW = \frac{PA}{W}$; $v_D = \sqrt{\frac{WS}{\rho_{infty}}}$;

a=$\frac{2PAMWv_D^2}{C_{Do}}$; $b = \frac{4kvD^4}{C_{Do}}$; $c = 81a^4 - 768b^3$;

$d = 108 * a^2 + 12\sqrt{c}$;

$e = d^{\frac{1}{3}} + 48bd^{\frac{-1}{3}}$; $t = \sqrt{\frac{1}{ed^{\frac{1}{3}}}(12\sqrt{\frac{6}{e}}ad^{\frac{1}{3}} - d^{\frac{2}{3}} - 48b)}$;

1.1. Maximum velocity

$$v_{max} = \frac{\sqrt{6e}}{12}(1+t) \quad \text{(Equation 1)}$$

1.2. Minimum velocity

$$v_{min} = \frac{\sqrt{6e}}{12}(1-t) \quad \text{(Equation 2)}$$

1.3. Stall velocity

$$v_{stall} = \sqrt{\frac{W}{\frac{1}{2}\rho_\infty C_{L,max}}} \quad \text{(Equation 3)}$$

1.4. Rate of Climb

$$R/C = \frac{P_A - P_R}{W} \quad \text{(Equation 4)}$$

$$\mathrm{v}_{@R/C_{max}} = Solve[\frac{d}{dv}R/C = 0] \quad \text{(Equation 5)}$$

$$R/C_{max} = \frac{(P_A - P_R)_{v@R/C_{max}}}{W} \quad \text{(Equation 6)}$$

$$\gamma \text{ (max angle of climb) } = sin^{-1}(\frac{P_A/v - P_R/v}{W}) \quad \text{(Equation 7)}$$

1.5. Range

$$Range = \frac{\eta}{c}\frac{C_L^{\frac{3}{2}}}{C_D} ln\frac{W_0}{W_1} \quad \text{(Equation 8)}$$

1.6. Endurance

$$E = \frac{\eta}{c}\frac{C_L^{\frac{3}{2}}}{C_D}(2\rho_\infty s)^{\frac{1}{2}}(W_1^{\frac{-1}{2}} - W_0^{\frac{-1}{2}}) \quad \text{(Equation 9)}$$

1.7. Required power

$$PR = \frac{1}{2}\rho_\infty v_\infty^3 SC_{Do} + \frac{1}{2}\rho_\infty v_\infty S\frac{C_L^2}{\pi eAR} \quad \text{(Equation 10)}$$

1.8. Minimum Angle of Gliding

$$\theta = \tan^{-1}(\frac{1}{L/D}) \quad \text{(Equation 11)}$$

4.2. Graphical

Velocity

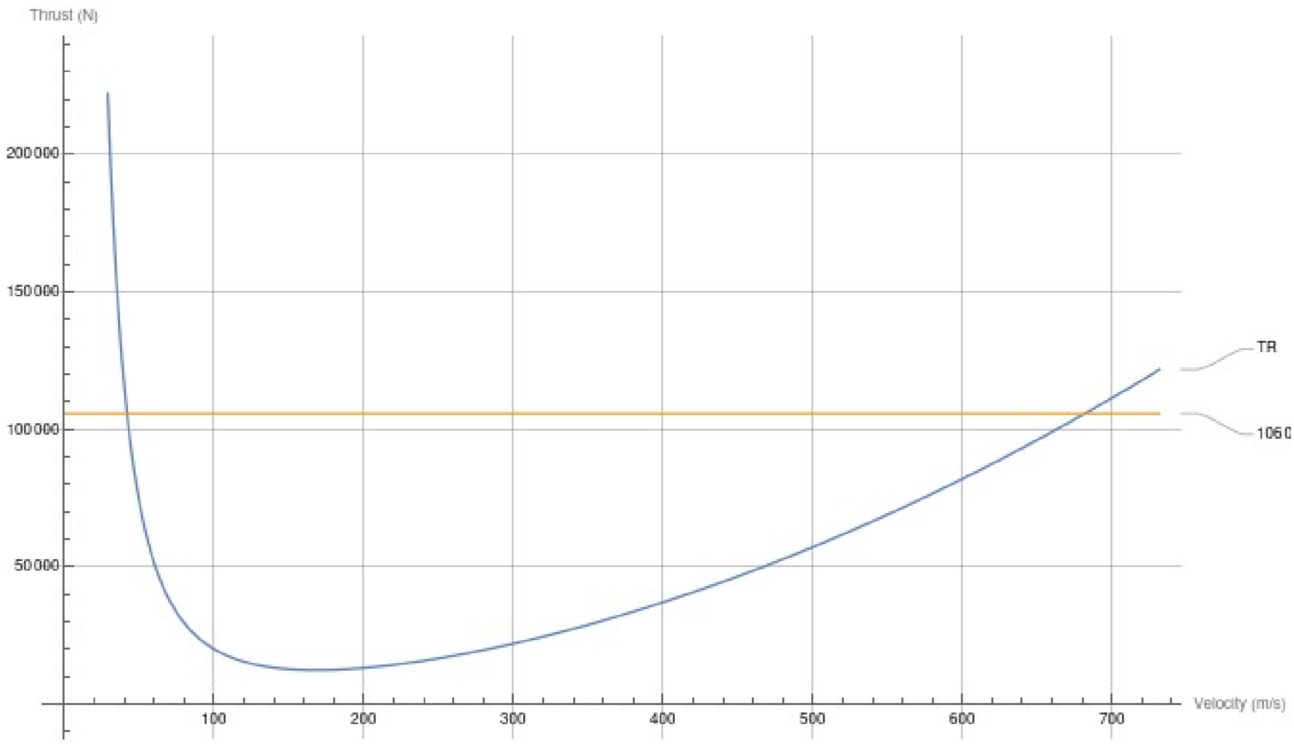
We notice the intersection of the two curves: the required thrust curve and the available thrust of the engine, so the max velocity is determined directly, while true minimum speed is:

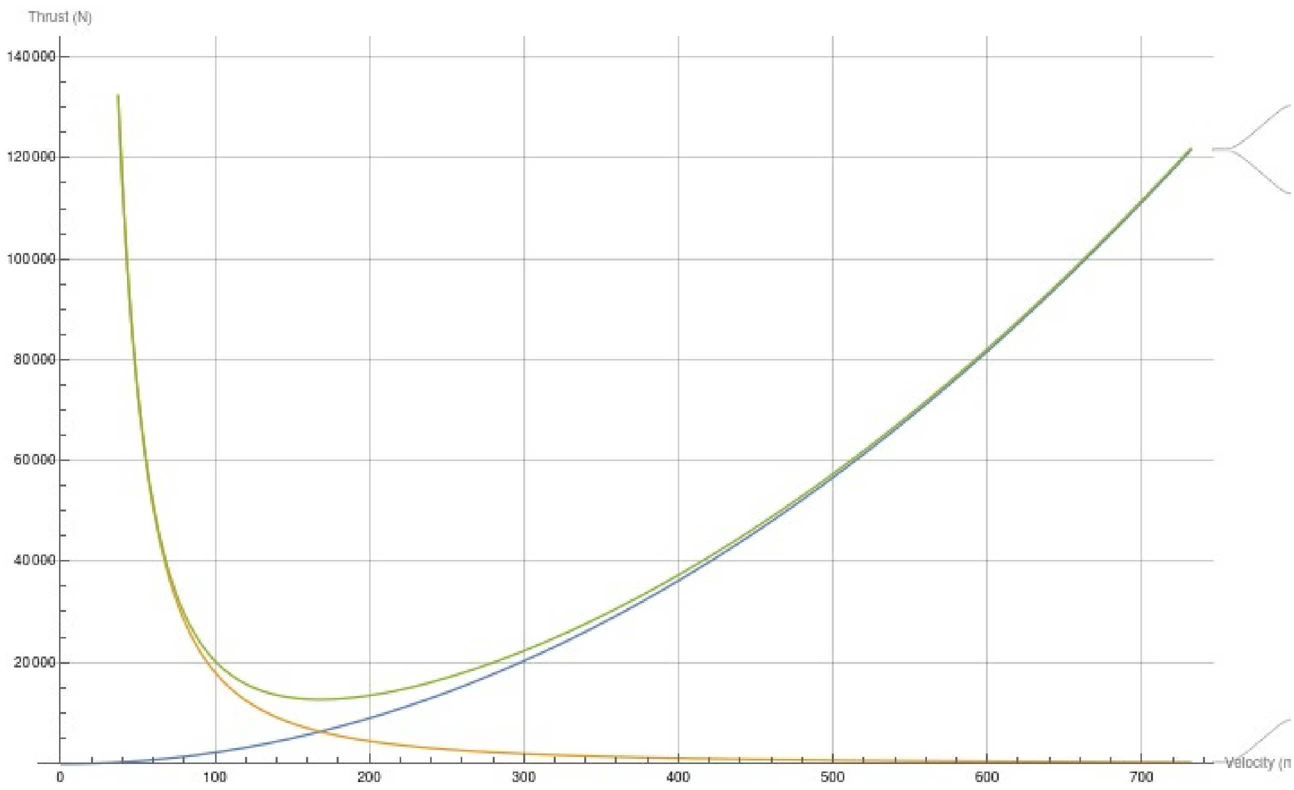
$v_{min} = min(v_{min}, v_{stall})$

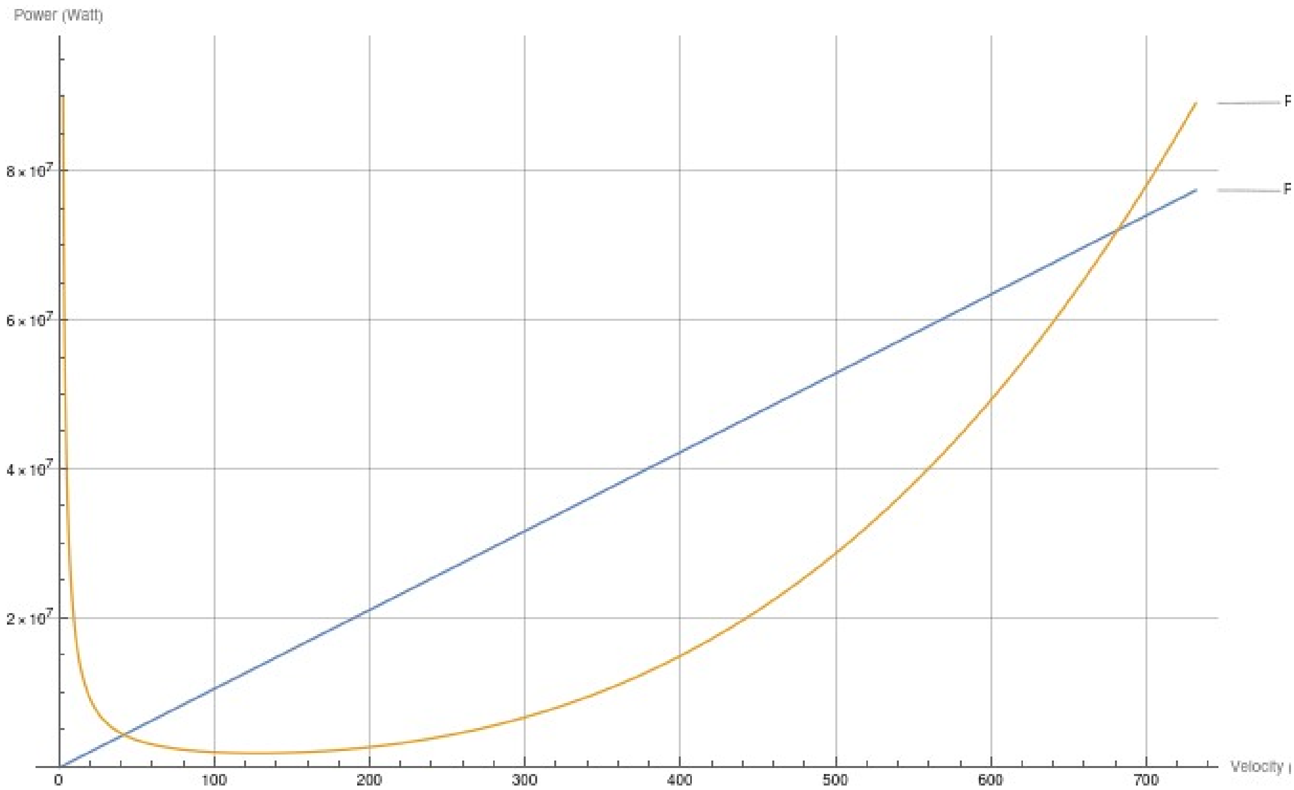
The required thrust curve is deduced from the addition of these two curves in the graph, let those two curves be A and B, where:

To deduce the required thrust curve, consider:

$A = \frac{1}{2}\rho_\infty v_\infty^2 SC_{Do}$, while B $= \frac{C_L^2}{\pi eAR}$

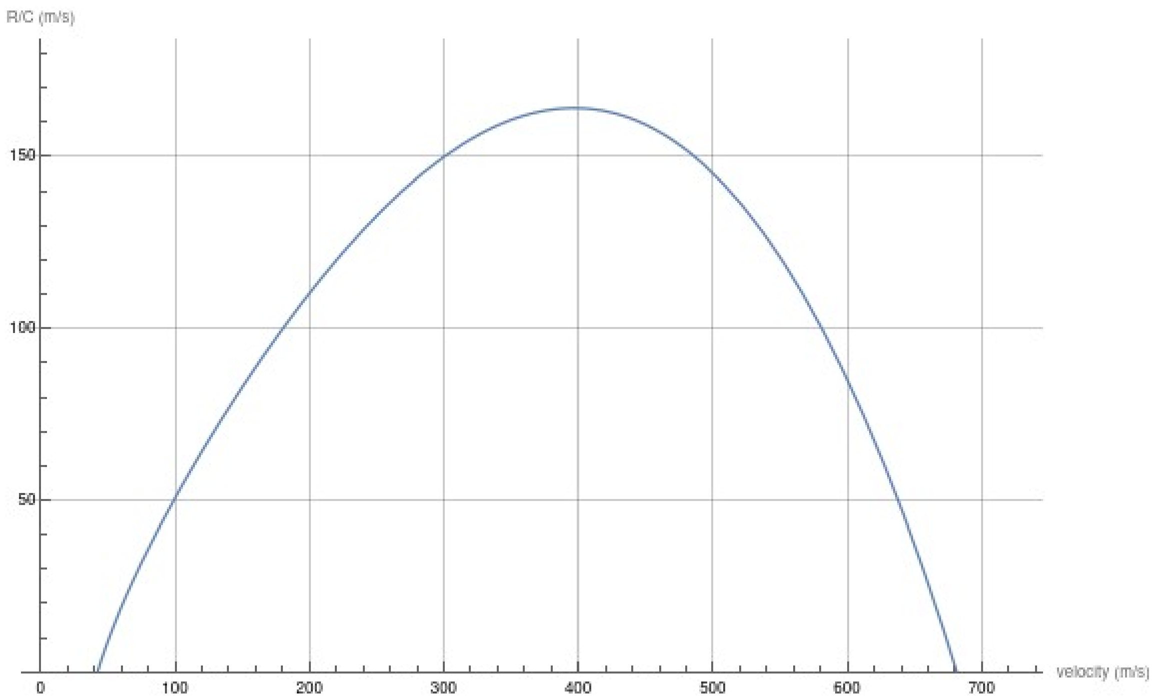

Thrust (N)
200000
150000
100000
50000
100
200
300
400
500
600
700
Velocity (m/s)
TR
1060


Thrust (N)
140000
120000
100000
80000
60000
40000
20000
0
100
200
300
400
500
600
700
Velocity (m



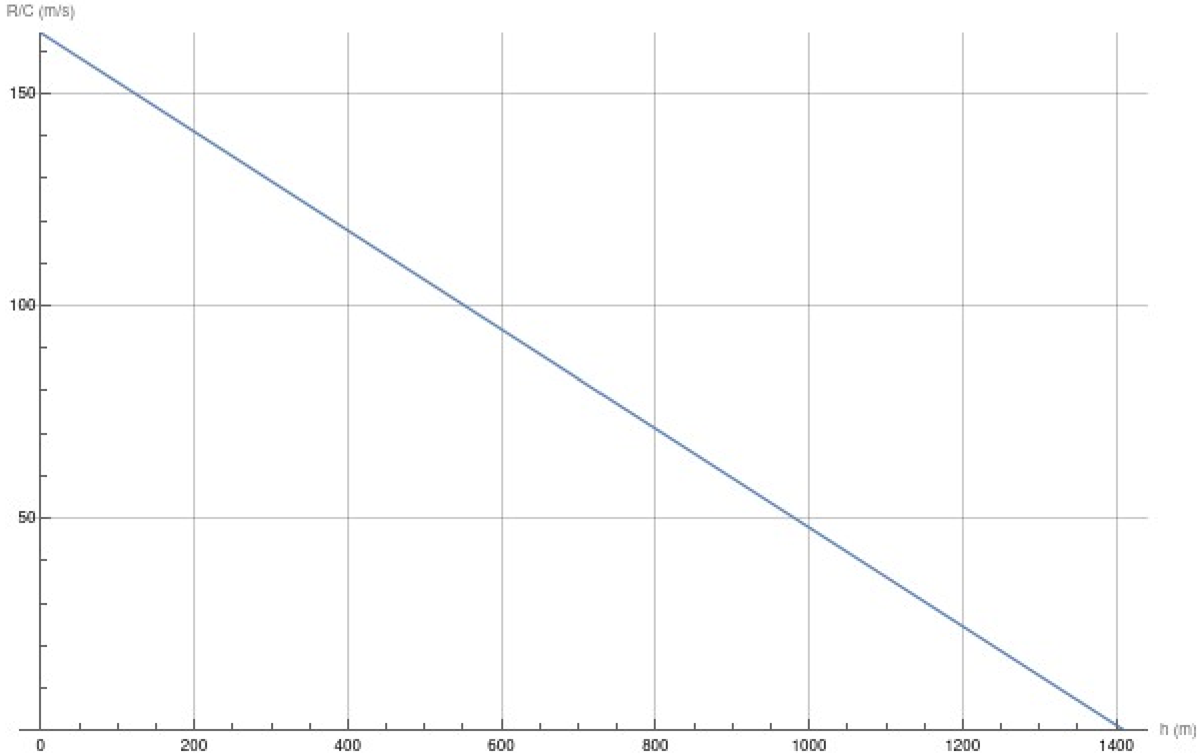
Power (Watt)
8 × 10^7
6 × 10^7
4 × 10^7
2 × 10^7
0
100
200
300
400
500
600
700
Velocity (
P
P

R/C

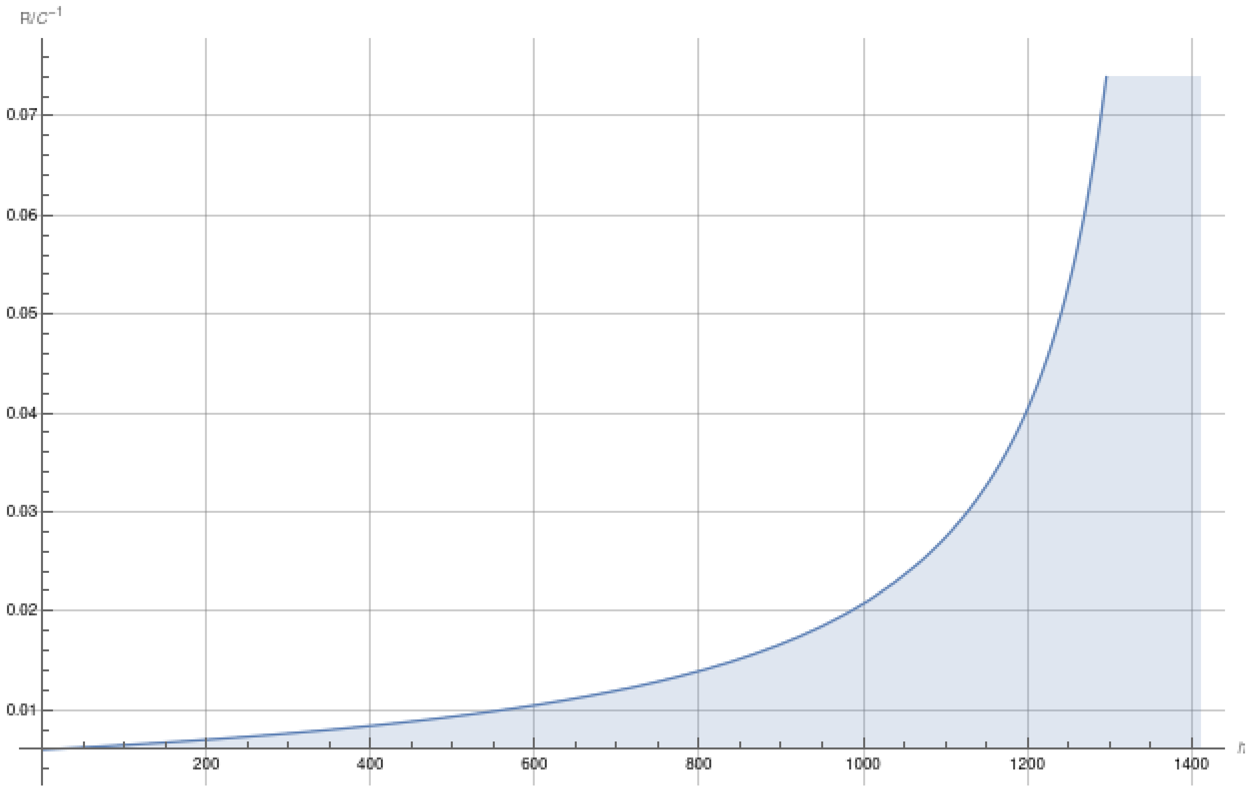


R/C (m/s)
150
100
50
0
100
200
300
400
500
600
700
velocity (m/s)

Service ceiling



Time to climb to service ceiling



References

- Summary of airfoil data, war-time report, naca March 1945
- Commercial Airplane Design Principles, 2014
<https://www.sciencedirect.com/topics/engineering/lift-to-drag-ratio>
- FlightMechanics4Pilots (c) Agostino De Marco, Università degli Studi di Napoli Federico II
<https://agodemar.github.io/FlightMechanics4Pilots/mypages/thrust-power-required/>
- General Aviation Aircraft Design, 2014
<https://www.sciencedirect.com/topics/engineering/specific-fuel-consumption>
- <https://www.saloodo.com/logistics-dictionary/gross-weight/>
- <https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=&cad=rja&uact=8&ved=2ahUKEwjO5quF37L3AhU3hf0HHYo8B2MQFnoECAcQAw&url=https%3A%2F%2Fcode7700.com%2F1986_maximum_range.htm&usg=AOvVaw23brH3R3liN3yS5IXy8BWS>
- <https://www.aircraftflightmechanics.com/AircraftPerformance/RangeandEndurance.html>
- <https://ocw.mit.edu/ans7870/16/16.unified/propulsionS04/UnifiedPropulsion4/UnifiedPropulsion4.htm#:~:text=For%20a%20vehicle%20in%20steady,climb%20(increasing%20potential%20energy).>
- International Journal of Naval Architecture and Ocean Engineering, Aerodynamic characteristics of NACA 4412 airfoil section with flap in extreme ground effect
via science direct: <https://www.sciencedirect.com/science/article/pii/S2092678216303776>
- <https://www.stratosjets.com/blog/cruise-speed/>
- <https://www.statista.com/statistics/614178/cruising-speed-of-most-common-airliners/>
- (article: specific fuel consumption, Layton, J. Preston, 2014)
- (artice: FUEL CONSUMPTION DUE TO SHAFT POWER OFF-TAKES FROM THE ENGINE, Dieter Scholz, Ravinka Seresinhe, Ingo Staack, Craig Lawson2)
- (article: Breguet's Formulas for Aircraft Range & Endurance: An Application of Integral Calculus, Colonel Kip P. Nygren, Major Robert R. Schulz, United States Military Academy)
- <https://museumofaviation.org/portfolio/f-16a-fighting-falcon/#:~:text=PERFORMANCE%3A,Cruising%20speed%3A%20577%20mph.>
- <https://en.wikipedia.org/wiki/General_Dynamics_F-16_Fighting_Falcon_variants>
- <https://en.wikipedia.org/wiki/Cessna_152>
- <https://en.wikipedia.org/wiki/Embraer_EMB_120_Brasilia>
- (Introduction to flight, anderson)