Nuclear properties and the astrophysical *r* process

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Nuclear properties and the astrophysical *r* process

What is the astrophysical origin of the heaviest elements??

Class #	Subject
1	Introduction, chemical abundances
2	Origins of the elements up to the iron peak
3	Neutron capture nucleosynthesis, *r*-process dynamics
4	The r process: nuclear masses and lifetimes
5	The r process: neutron capture rates and fission

Nuclear properties and the astrophysical *r* process

What is the astrophysical origin of the heaviest elements??

Class #	Subject
1	Introduction, chemical abundances
2	Origins of the elements up to helium
3	Origins of the elements up to iron, *r*-process dynamics
4	Astrophysical sites of the *r* process
5	Nuclear data for the *r* process

Equilibrium nucleosynthesis: the Saha equation

If the creation and destruction reactions

$X + b \leftrightarrow Y + \gamma$

are in equilibrium, the relative abundances of species X, b, and Y are given by a nuclear Saha equation:

$$\frac{N_Y}{N_X N_b} = \frac{g_Y}{g_X g_b} (\hbar c)^3 \left(\frac{2\pi}{\mu c^2 kT} \right)^{3/2} e^{E/kT}$$

where N_X, N_b, N_Y are the number densities of species X, b, and Y,

g_x are the spin factors $(2j_x + 1)$,

μ is the reduced mass of reactants X and b,

and $E = \left(M_X + M_b - M_Y \right) c^2$ is the mass difference.

Nucleosynthesis network equations

$$\frac{dY_j}{dt} = Y_k Y_l \rho N_A \langle \sigma v \rangle_{kl,j} - Y_j Y_l \rho N_A \langle \sigma v \rangle_{jl,n} + Y_i \lambda_{i,j} - Y_j \lambda_{j,m} + \ldots$$

Y_j : abundance of species j (where $N_j = \rho N_A Y_j$)

ρ: baryon density

$\langle \sigma v \rangle_{kl,j}$: astrophysical reaction rate for reaction $k + l \rightarrow j + \gamma$

$\lambda_{i,j}$: decay rate for $i \rightarrow j$

Radioactive decay rates

$$N(t) = N_0 e^{-\lambda t}$$

λ decay constant/decay rate

τ mean lifetime, $\tau = 1/\lambda$

$T_{1/2}$ half-life, $T_{1/2} = \ln 2/\lambda$

Calculated from…

Fermi's 'Golden Rule'

$$\lambda = \frac{2\pi}{\hbar}\left|\left\langle f \middle| H_{\text{int}} \middle| i \right\rangle\right|^2 \rho(E)$$

f, i final and initial state wavefunctions

H_{int} weak interaction Hamiltonian

$\rho(E)$ density of states for the final particles

Nuclear reaction rates

Start with the reaction cross section:

$$\sigma_{ij}(v) = \frac{\text{number of reactions per nucleus } i \text{ per second}}{\text{flux of incoming projectiles } j}$$

$$\sigma_{ij}(v) = \frac{r_{ij} / N_i}{N_j v_{ij}}$$

r_{ij} number of interactions $i(j,k)l$ per second

v_{ij} relative velocity of particles i, j

Nuclear reaction rates

The cross section for the reaction $i + j \rightarrow k + l$ is:

$$\sigma_{ij}(v) = \frac{\text{number of reactions per nucleus } i \text{ per second}}{\text{flux of incoming projectiles } j}$$

$$\sigma_{ij}(v) = \frac{r_{ij} / N_i}{N_j v_{ij}}$$

r_{ij} number of interactions $i(j,k)l$ per second

v_{ij} relative velocity of particles i, j

The reaction rate per volume is:

$$r_{ij} = N_i N_j v_{ij} \sigma_{ij}(v)$$

Nuclear reaction rates

In astrophysical environments the relative velocity v_{ij} is not constant, but instead there exists a distribution of relative velocities, which can be described by the probability function *P(v)*, where:

$$\int_0^\infty P(v)dv = 1$$

So the reaction rate can be generalized to:

$$r_{ij} = n_i n_j \int_0^\infty vP(v)\sigma_{ij}(v)dv$$

$$r_{ij} = n_i n_j \langle \sigma v \rangle_{ij}$$

Nuclear reaction rates

If the nuclei are nonrelativistic and nondegenerate, their velocities can be described by a Maxwell-Boltzmann distribution

$$P(v)dv = \left(\frac{m_{ij}}{2\pi kT}\right)^{3/2} e^{-m_{ij}v^2/2kT}\, 4\pi v^2 dv$$

where:

m_{ij} reduced mass, $m_{ij} = m_i m_j/(m_i + m_j)$

T temperature

k Boltzmann constant, $k = 8.6173 \times 10^{-5}$ eV/K

The velocity distribution can be written as an energy distribution, since $E = m_{ij}v^2/2$

$$P(v)dv = P(E)dE = \frac{2}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} \sqrt{E}\, e^{-E/kT} dE$$

Nuclear reaction rates

So the reaction rate per particle pair becomes:

$$\langle \sigma v \rangle_{ij} = \int_0^\infty v P(v) \sigma(v) dv = \int_0^\infty v P(E) \sigma(E) dE$$

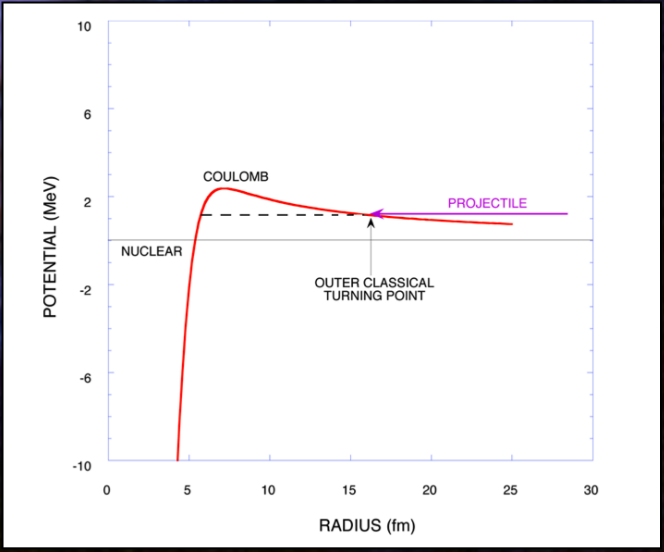
$$= \left(\frac{8}{\pi m_{ij}} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E \sigma(E) e^{-E/kT} dE$$

The cross section $\sigma(E)$ is often parameterized in terms of the astrophysical S-factor:

$$\sigma(E) \equiv \frac{1}{E} e^{-2\pi\eta} S(E)$$

s-wave Coulomb barrier transmission probability

$$\eta = \frac{Z_i Z_j \alpha}{\sqrt{2E/m_{ij}}}$$

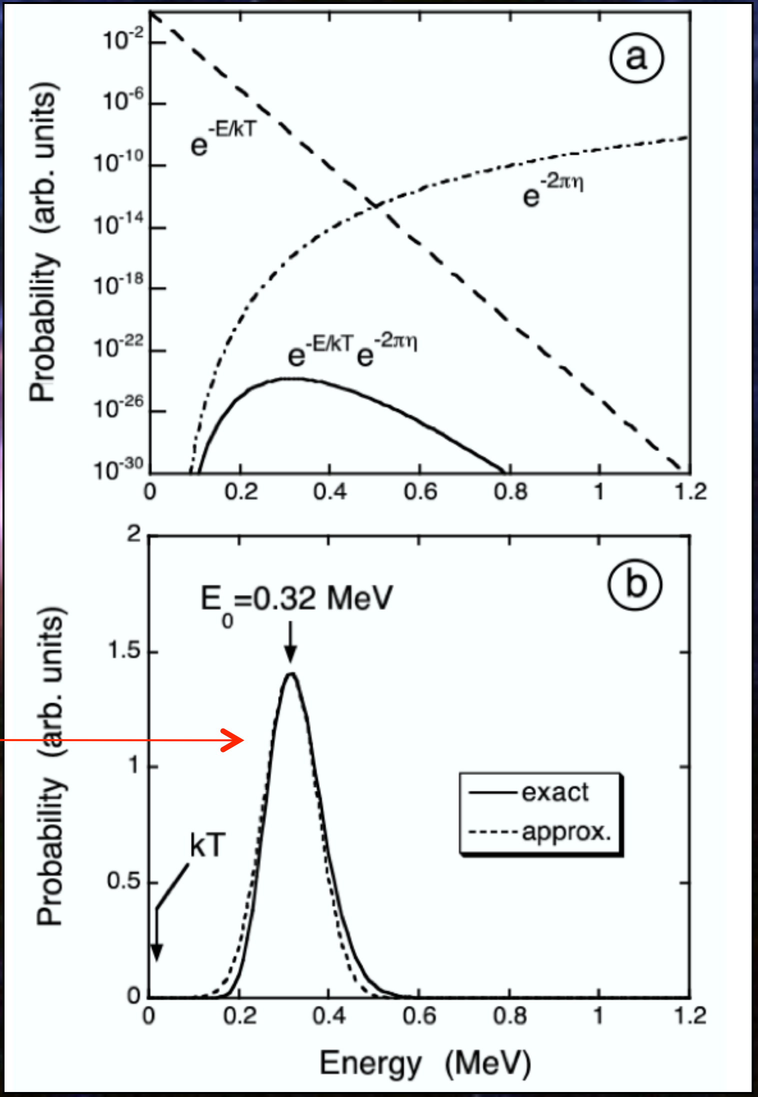


Nuclear reaction rates

Then we have:

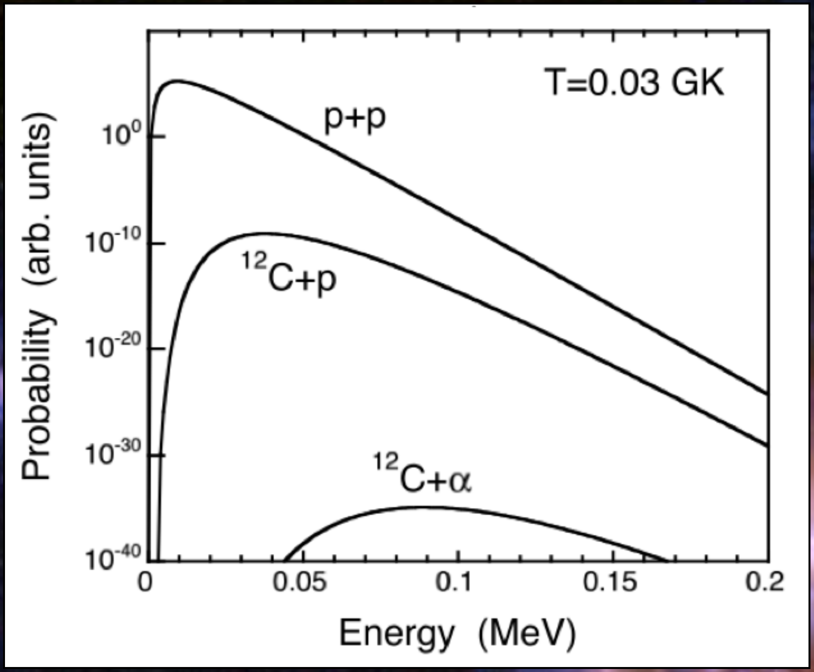
$$\langle \sigma v \rangle_{ij} = \left(\frac{8}{\pi m_{ij}} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty e^{-2\pi\eta} S(E) e^{-E/kT} dE$$

Gamow peak



Nuclear Physics of Stars, Illiadis (2007)

The Gamow peak

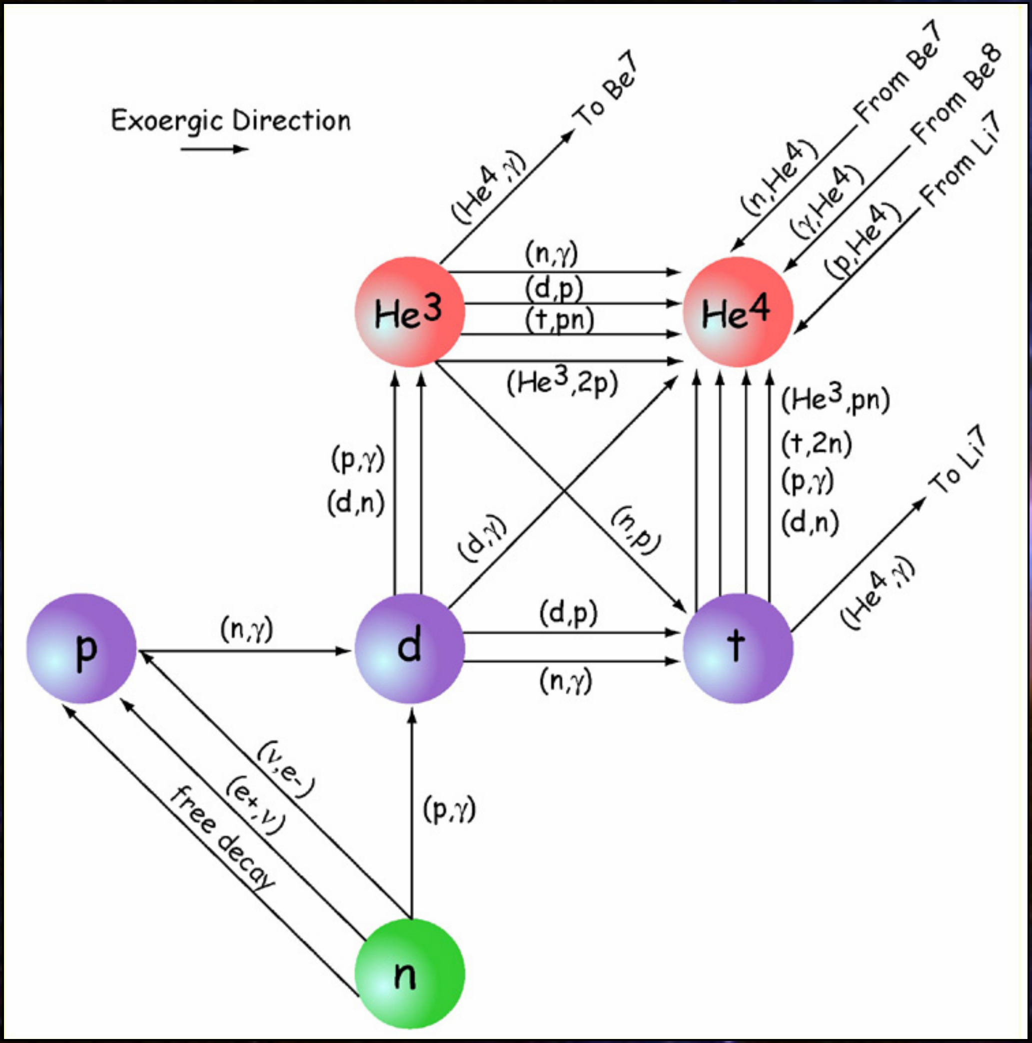


Nuclear Physics of Stars, Illiadis (2007)

Note: The Gamow peak shifts up in energy with increasing Z_i, Z_j, and the area under peak decreases

Therefore in stellar plasmas, the fusion of light nuclei is more probable and releases more energy than fusion of heavier nuclei

a full BBN network

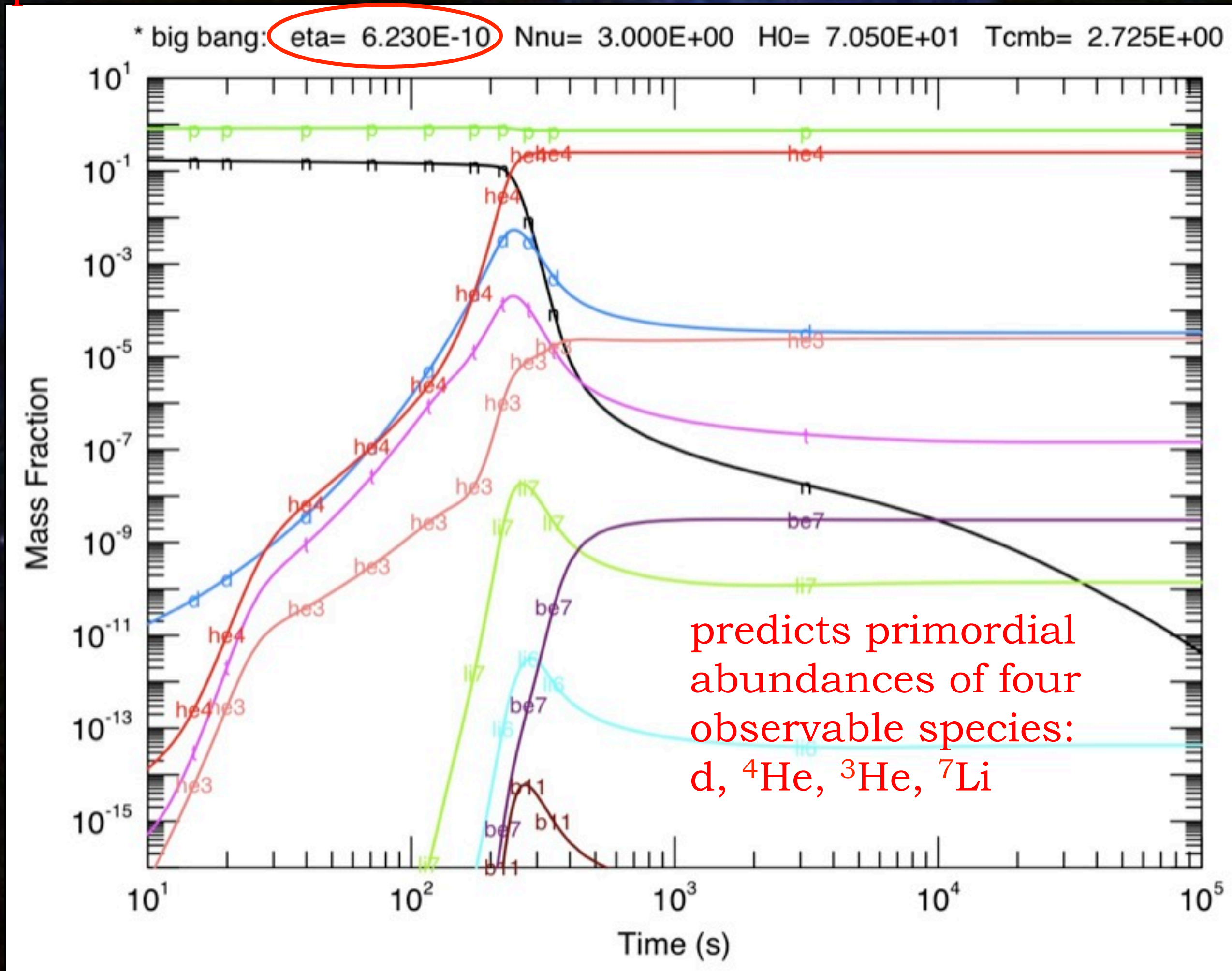


http://cococubed.asu.edu/code_pages/net_bigbang.shtml

Includes:
All relevant nuclear species, reactions, and decays
Dynamically evolving T, ρ
Adjustable physics inputs

time evolution of BBN abundances

one free parameter – η, the baryon to photon ratio



predicts primordial abundances of four observable species: d, ^{4}He, ^{3}He, ^{7}Li

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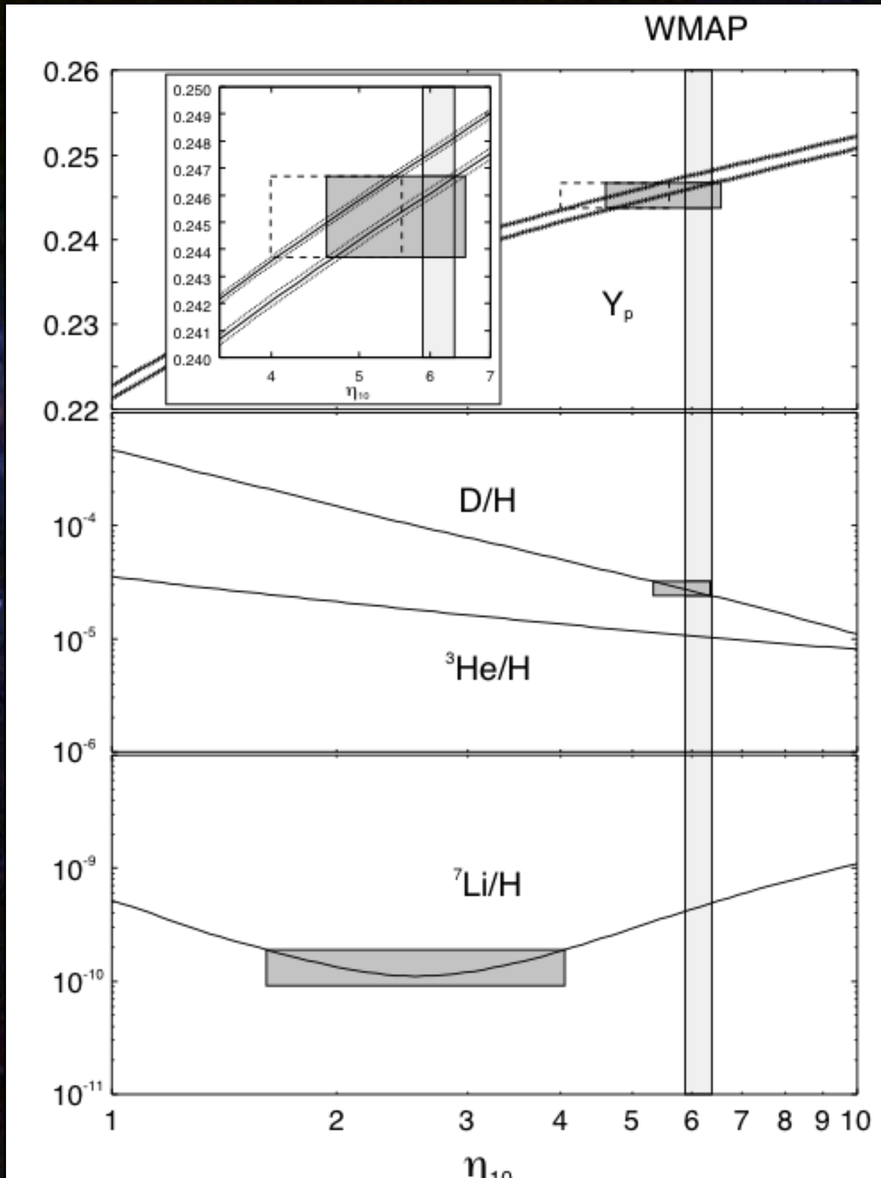
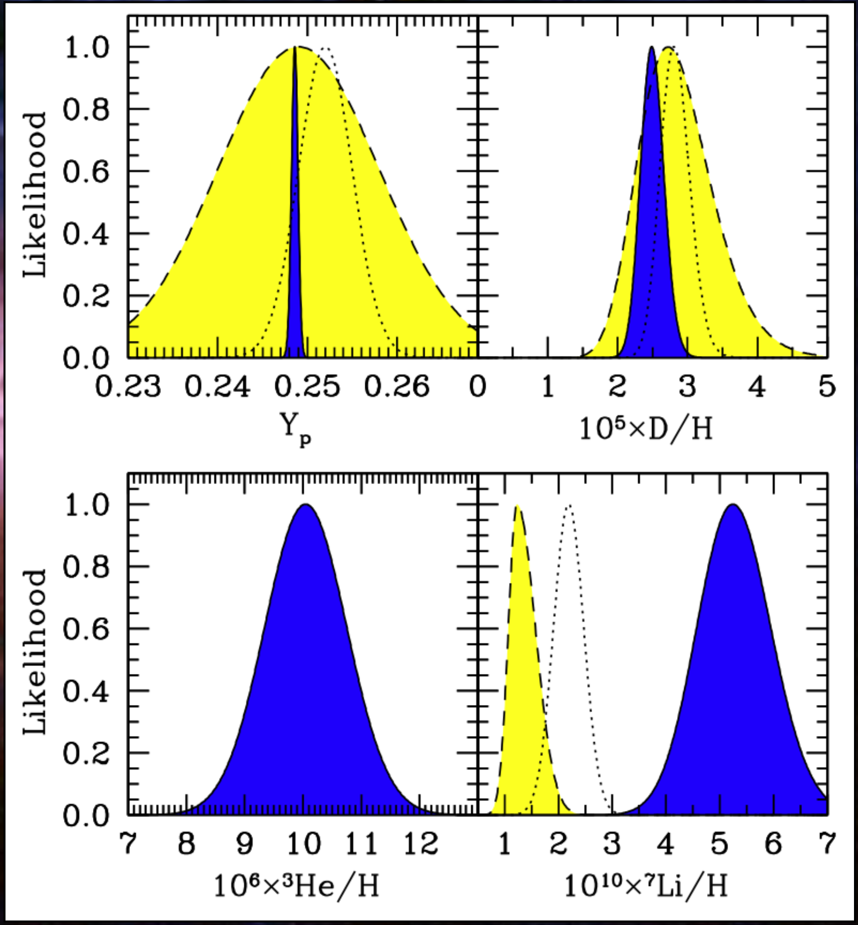
BBN with η fixed from WMAP



Mathews, Kajino, Shima (2005)



Cyburt, Fields, Olive (2008)

BBN nuclear physics uncertainties

12 key reactions:

Table 1. Abundance sensitivity: $\partial \log Y / \partial \log < \sigma v >$ at WMAP baryonic density.

Reaction	^{4}He	D	^{3}He	^{7}Li	$E_0(\ \Delta E_0/2)$ (MeV)
n↔p	-0.73	0.42	0.15	0.40	
^{1}H(n,γ)^{2}H	0	-0.20	0.08	1.33	
^{2}H(p,γ)^{3}He	0	-0.32	0.37	0.57	0.11(0.11)
^{2}H(d,n)^{3}He	0	-0.54	0.21	0.69	0.12(0.12)
^{2}H(d,p)^{3}H	0	-0.46	-0.26	0.05	0.12(0.12)
^{3}H(d,n)^{4}He	0	0	-0.01	-0.02	0.13(0.12)
^{3}H(α,γ)^{7}Li	0	0	0	0.03	0.23(0.17)
^{3}He(n,p)^{3}H	0	0.02	-0.17	-0.27	
^{3}He(d,p)^{4}He	0	0.01	-0.75	-0.75	0.21(0.15)
^{3}He(α,γ)^{7}Be	0	0	0	0.97	0.37(0.21)
^{7}Li(p,α)^{4}He	0	0	0	-0.05	0.24(0.17)
^{7}Be(n,p)^{7}Li	0	0	0	-0.71	

Coc & Vangioni (2010)

Particle Data Group (2008)

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theory

experiment

Reaction	^{4}He	D	^{3}He	^{7}Li	$E_0(\ \Delta E_0/2)$ (MeV)
n$\leftrightarrow$p	-0.73	0.42	0.15	0.40	
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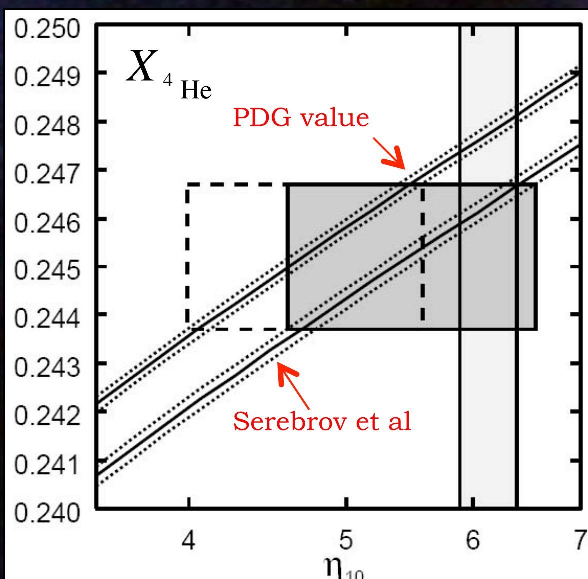
Coc & Vangioni (2010)

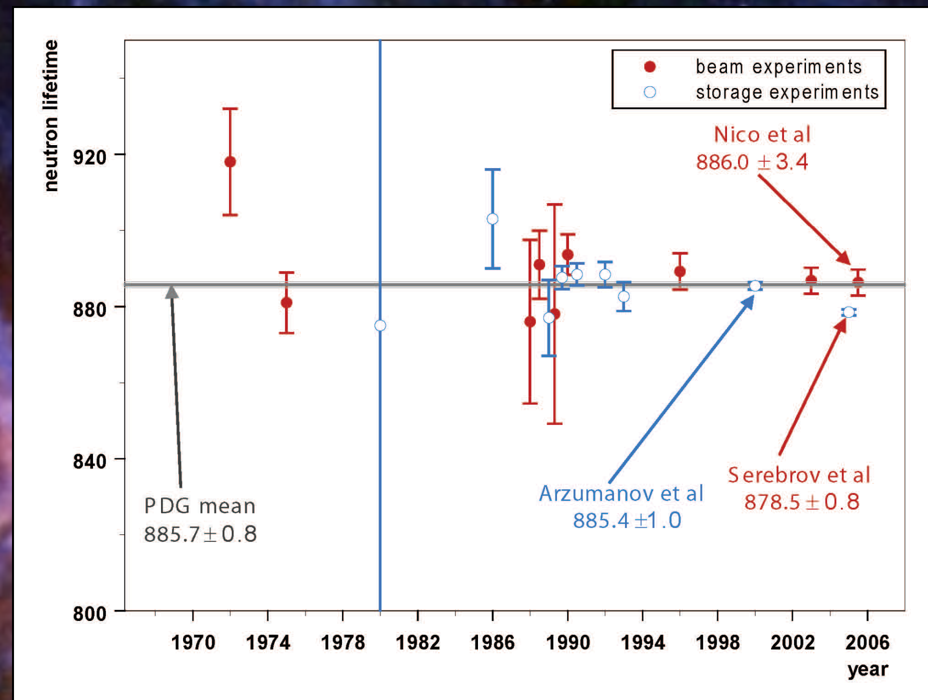
Particle Data Group (2008)

BBN nuclear physics uncertainties

neutron lifetime

Mathews, Kajino, Shima (2005)





Paul (2009)

Particle Data Group (2008)

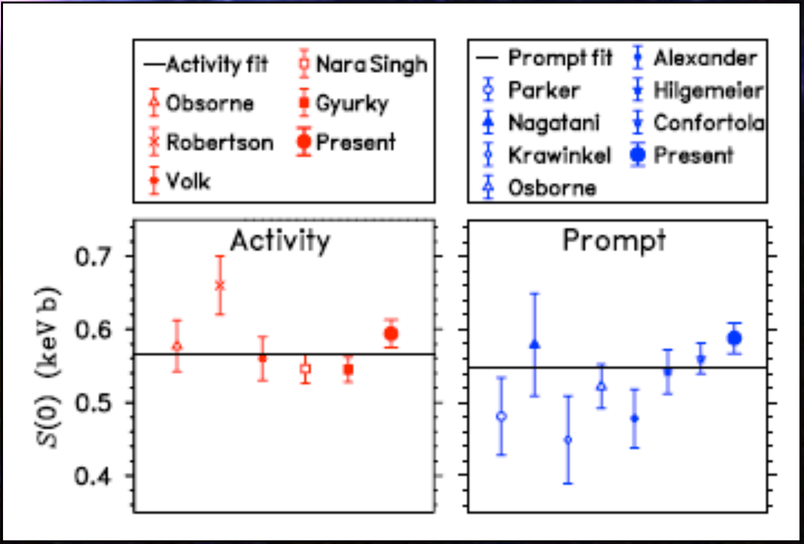
BBN nuclear physics uncertainties

$^{3}He(\alpha,\gamma)^{7}Be$

can count the resulting ^{7}Be activity or the prompt γ rays

		Ref.	$S(0)$ (keV b)
Osborne *et al.*	PRL **48** (1982)	[18]	0.535 ± 0.040
Robertson *et al.*	PRC **27** (1983)	[19]	0.63 ± 0.04
Volk *et al.*	ZPA **310** (1983)	[20]	0.56 ± 0.03
Nara Singh *et al.*	PRL **93** (2004)	[21]	0.53 ± 0.02
present work			0.547 ± 0.017
Weighted average, all activation studies			0.553 ± 0.012
Weighted average, all prompt-γ studies		[3]	0.507 ± 0.016

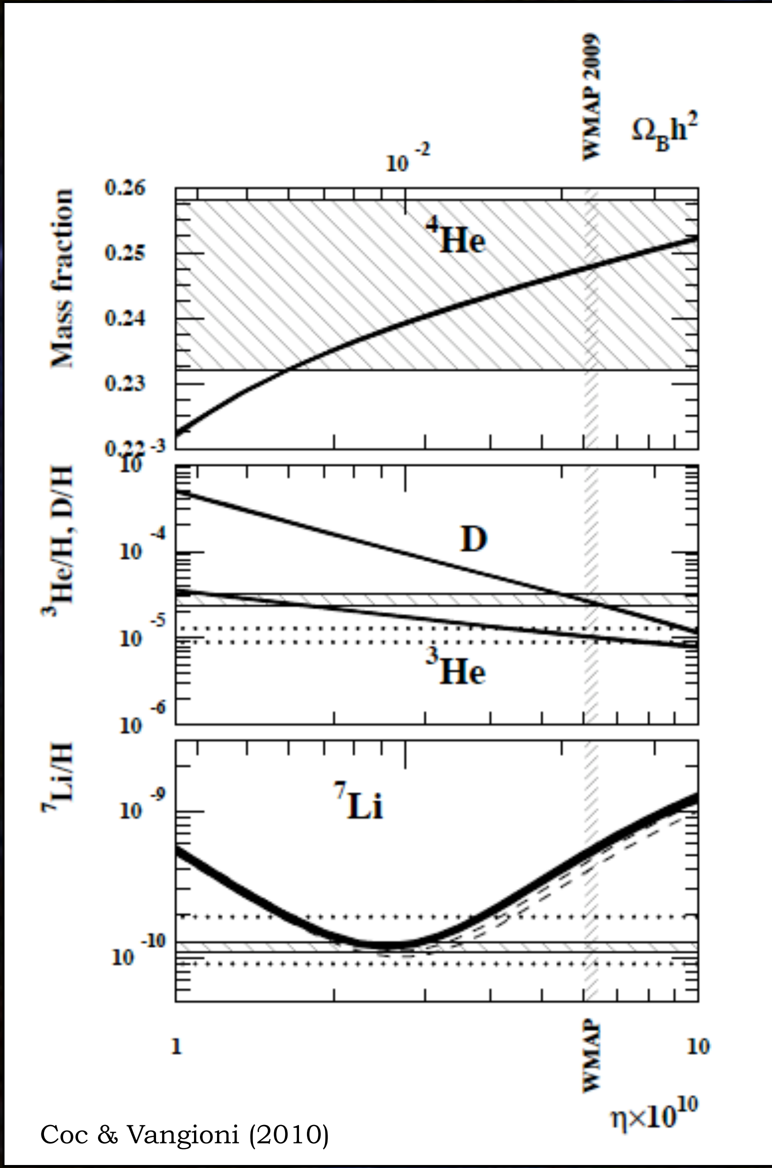
Gyuyrky (2007)



Brown et al (2008)

Particle Data Group (2008)

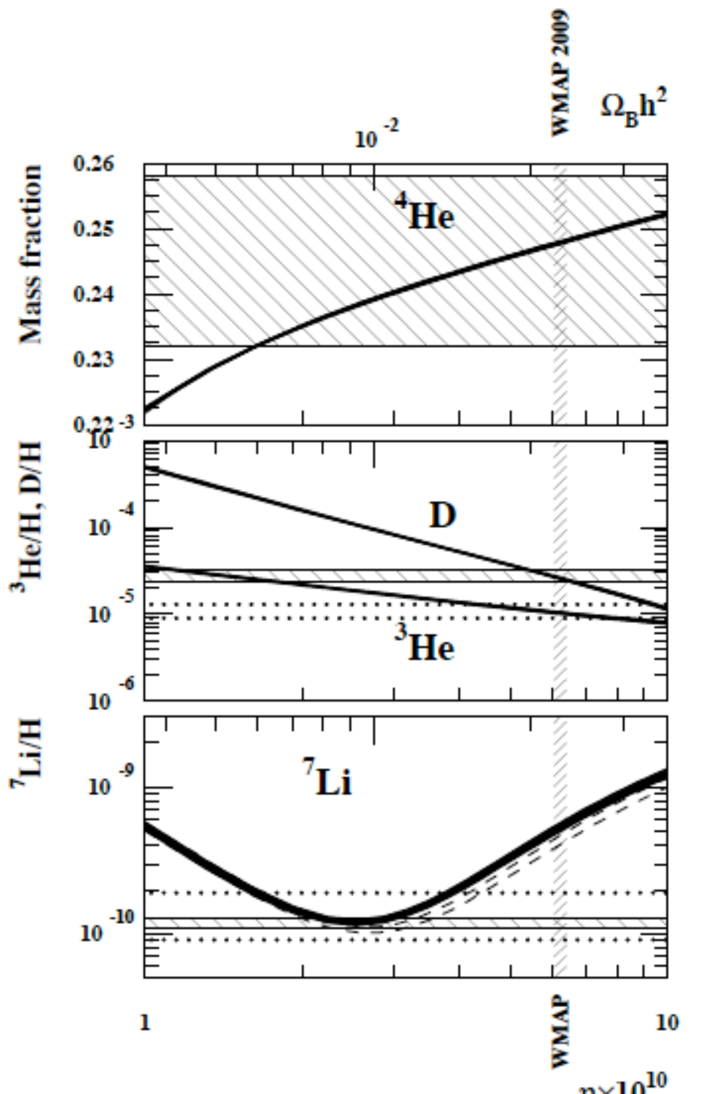
BBN nuclear physics uncertainties



——— with modern rates for ^{1}H(n,γ) and ^{3}He(α,γ)

- - - - - - - - with older values

BBN nuclear physics uncertainties



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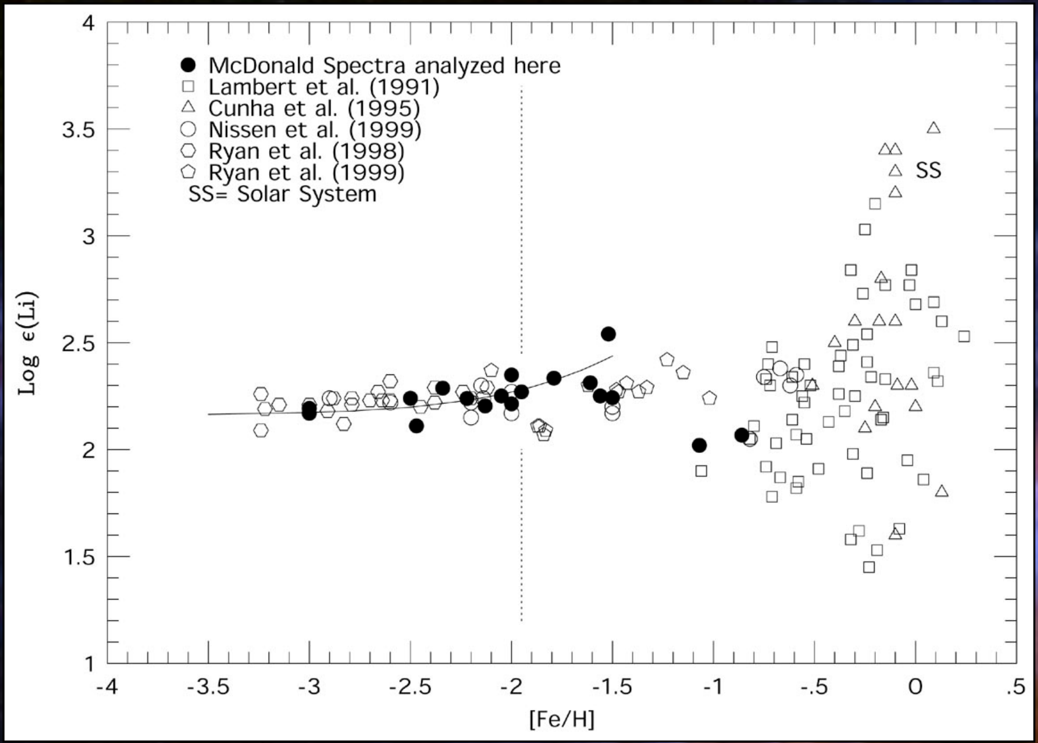
with modern rates for $^1H(n,\gamma)$ and $^3He(\alpha,\gamma)$

with older values

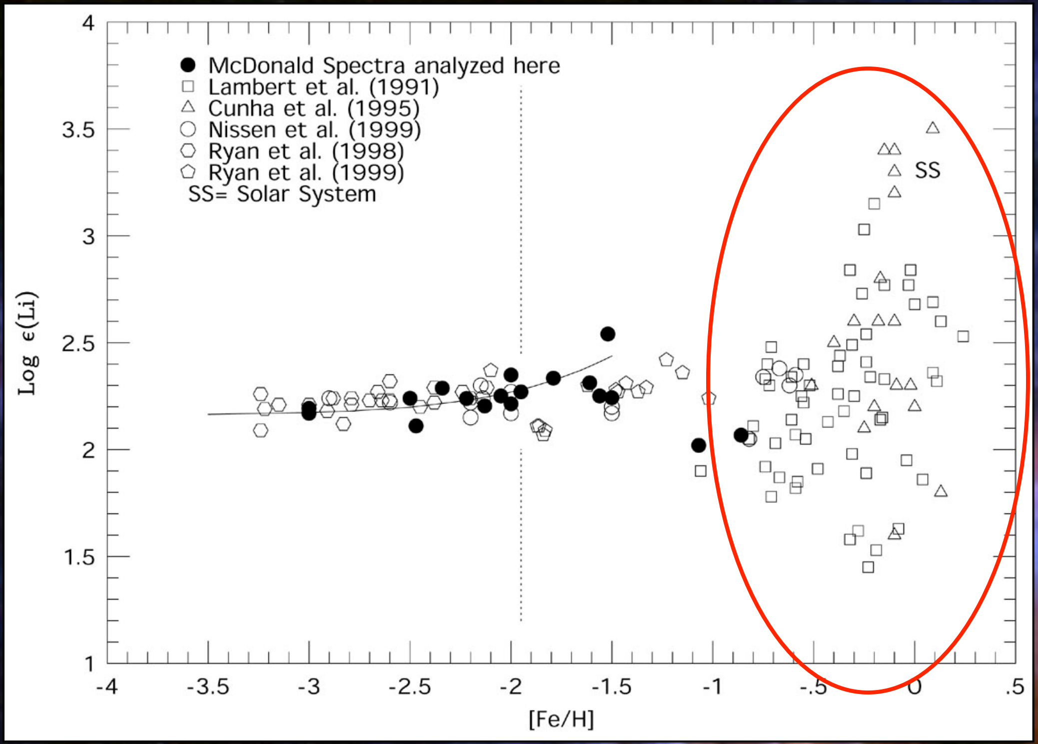
A nuclear physics solution to the ^{7}Li problem is unlikely

An astrophysical solution??

lithium observations



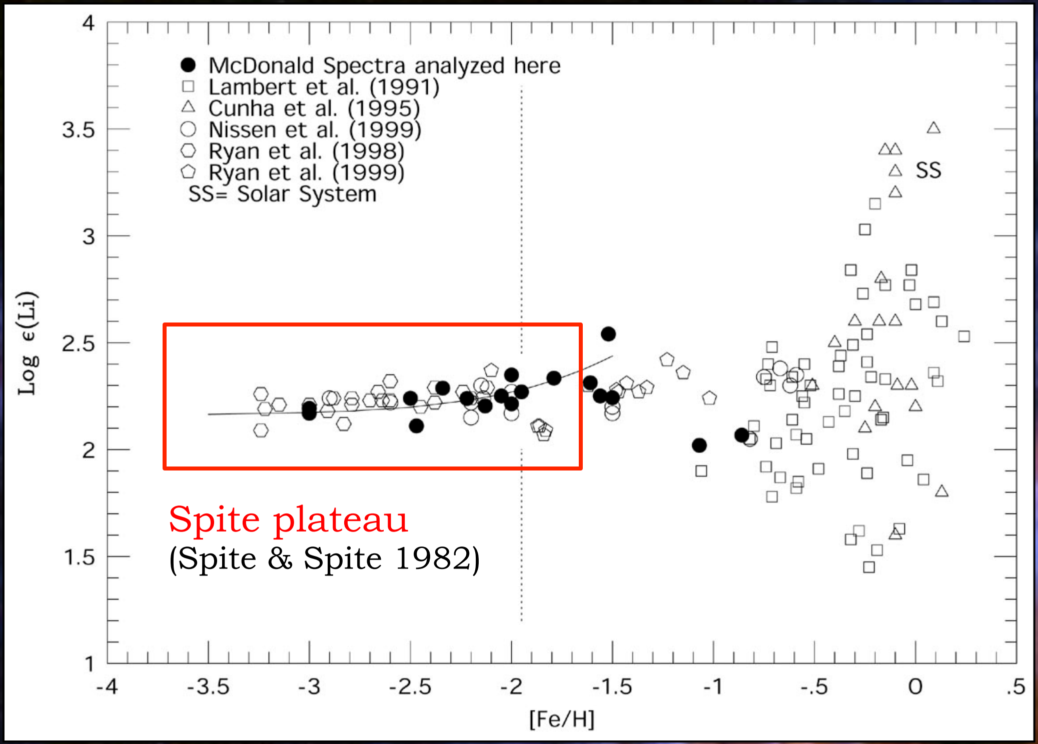
lithium observations



Lots of scatter at high metallicity

Li easily destroyed via nuclear burning in stars, can be synthesized via spallation reactions on CNO nuclei

lithium observations

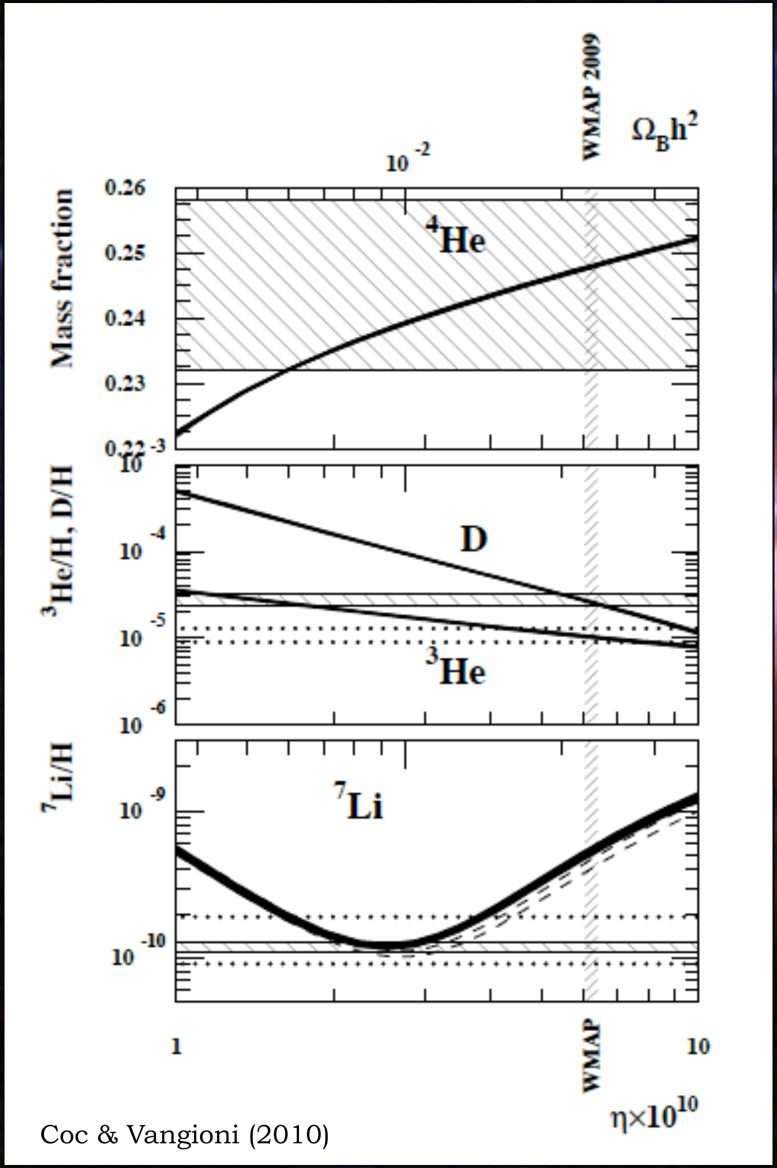


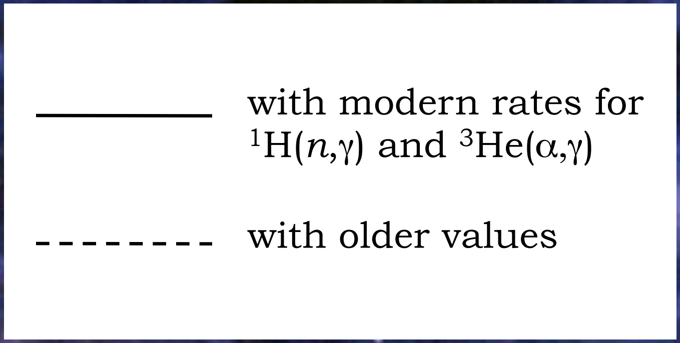
Plateau at low metallicity

→likely a measure of primordial Li

But observations of Li/H are a factor of ~3 lower than predicted

BBN nuclear physics uncertainties





A nuclear physics solution to the ^{7}Li problem is unlikely

An astrophysical solution is also unlikely

Points to physics beyond the Standard Model?

To explore more…

…check out some of the available online BBN tools

Big Bang Online

http://bigbangonline.org

Frank Timmes' site

http://cococubed.asu.edu/code_pages/net_bigbang.shtml

Coc & Vangioni (2010)


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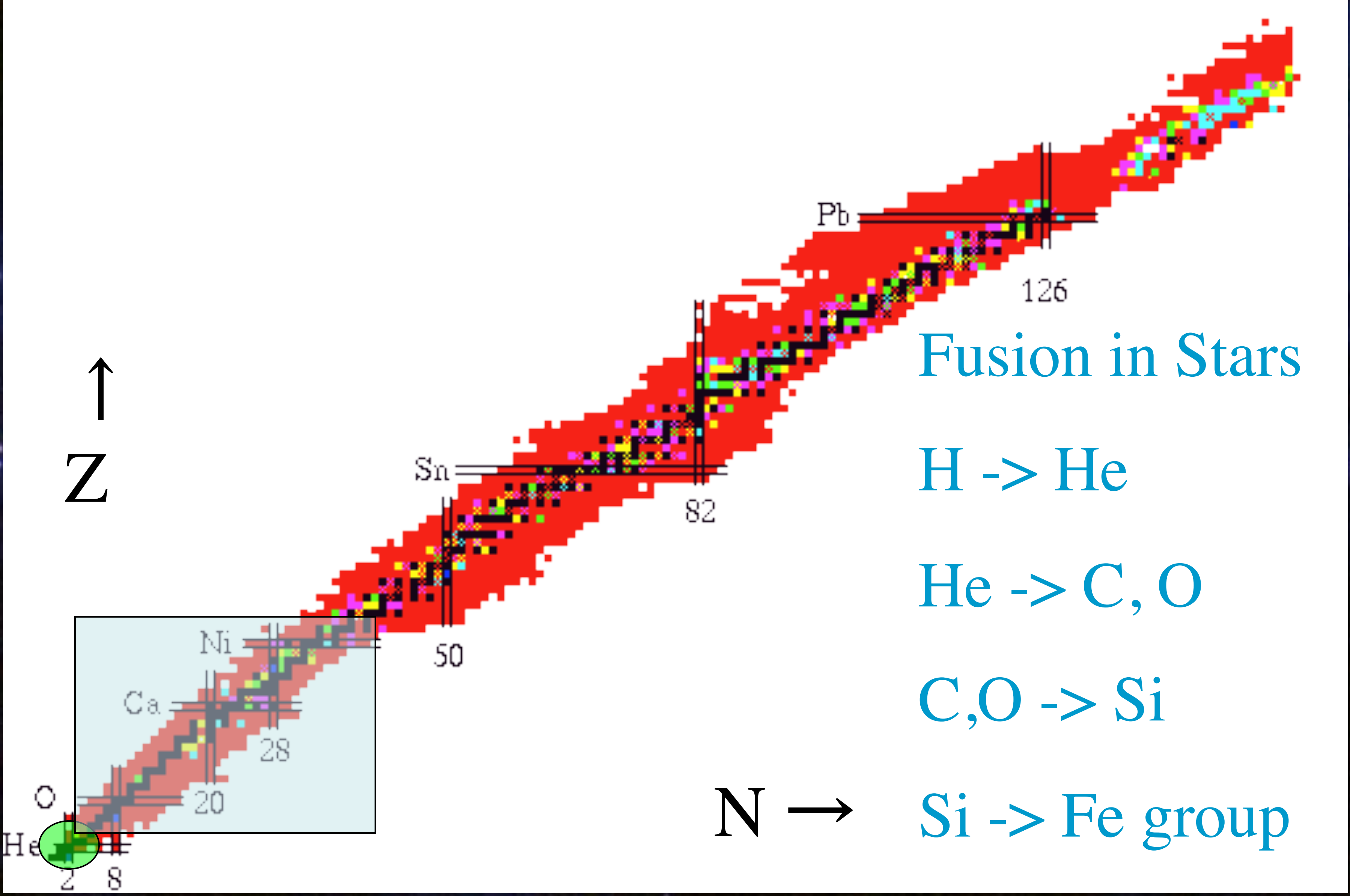
chart of the nuclides



Fusion in Stars

H -> He

He -> C, O

C,O -> Si

Si -> Fe group

consider the Sun…

The Sun emits 3.839×10^{26} J of energy every second and was formed about 4.6 billion years ago (as suggested from the ages of the oldest rocks/meteorites in the solar system)

Use these two observations to show that the Sun cannot be powered by, e.g., gravitational contraction. Show instead that fusion of hydrogen into helium can supply the required energy, and estimate the lifetime of the Sun assuming the inner 10% undergoes H->He fusion.

Hint: you will need to look up a few standard constants

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consider the Sun…

Gravitational contraction?

Sun's lifetime ~ (total energy available)/(luminosity)

$$\sim \frac{U_g}{L_{\mathrm{solar}}} \sim \frac{3}{5}\frac{GM^2}{R}\frac{1}{L_{\mathrm{solar}}}$$

$$\sim \frac{3}{5}\frac{(6.7\times10^{-11}\ \mathrm{Nm^2/kg^2})(2\times10^{30}\ \mathrm{kg})^2}{7\times10^{8}\ \mathrm{m}}\frac{1}{4\times10^{26}\,\mathrm{J/s}}$$

$$\sim 5\times10^{14}\ \mathrm{s} \sim \text{some millions of years}$$

Coc & Vangioni (2010)

consider the Sun…

Fuse H to He?

$$\Delta m = 4 \times m_{^1\text{H}} - m_{^4\text{He}} = 4(1.0078250\text{ u}) - 4.002602\text{ u}$$

$$\Delta m = 0.028698\text{ u} = 0.7\%\text{ of the mass of the 4 H atoms}$$

Sun's lifetime ~ (total energy available)/(luminosity)

$$\sim \frac{E_{\text{nuclear}}}{L_{\text{solar}}} \sim \frac{0.1 \times 0.007 \times M_{\text{solar}} c^2}{L_{\text{solar}}} \sim \frac{1.3 \times 10^{44}\text{ J}}{3.8 \times 10^{26}\text{ J/s}}$$

$$\sim 3 \times 10^{17}\text{ s} \sim 10\text{ billion yrs}$$

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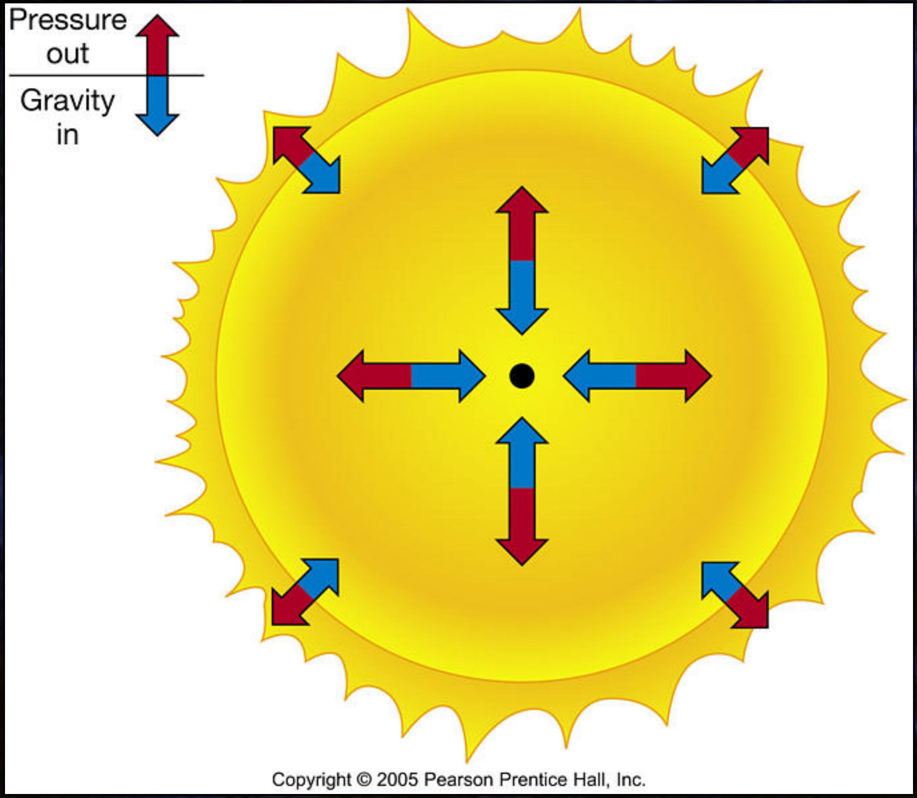
consider the Sun…

The solar radius and luminosity are (roughly) constant

➔ Sun is in hydrostatic equilibrium



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fundamental equations of stellar structure

Condition of hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2}$$

Mass conservation

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

Luminosity gradient

$$\frac{dL_r}{dr} = 4\pi r^2 \rho\varepsilon$$

$\varepsilon =$ total energy released per kg per s

Equation of state

$$P = P_{\text{gas}} + P_{\text{radiation}} = \frac{\rho kT}{\overline{m}} + \frac{1}{3}aT^4$$

$\overline{m} =$ average mass of a gas particle

Coc & Vangioni (2010)

fundamental equations of stellar structure

Condition of hydrostatic equilibrium

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Check out:
http://www.astro.wisc.edu/~townsend/static.php?ref=ez-web
for an online stellar structure/evolution computational tool

Mass conservation

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the basic proton-proton chain

Conversion of ^{1}H to ^{4}He via:

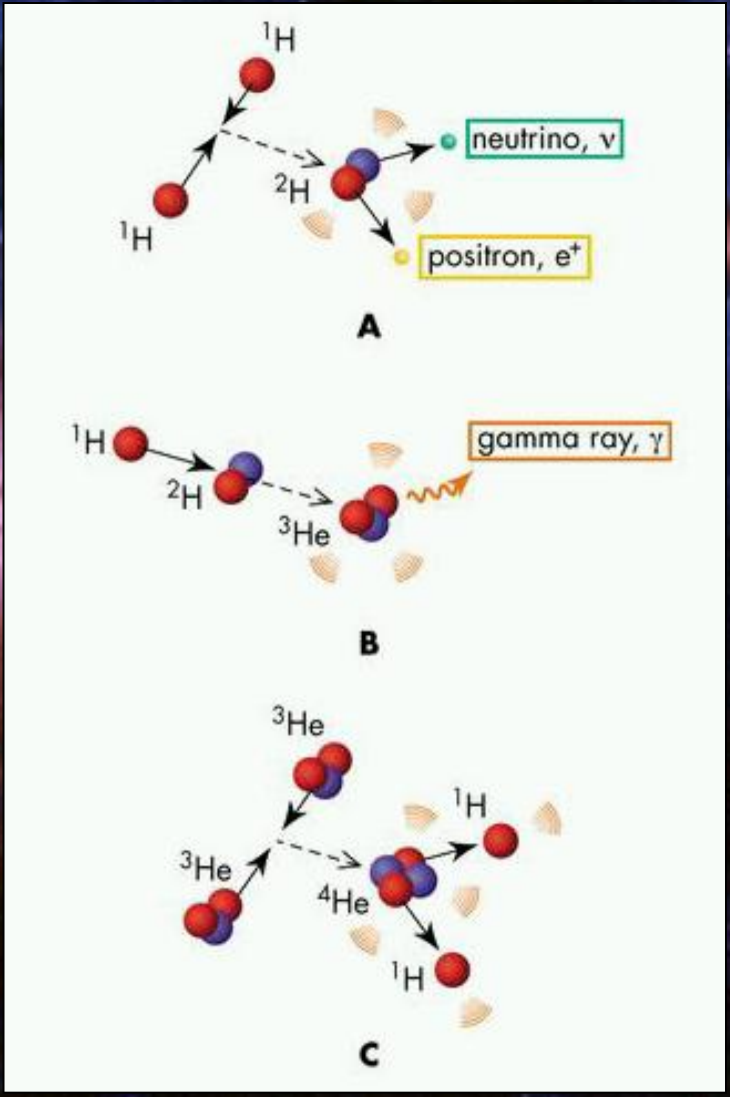
$$^{1}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H} + e^{+} + \nu_e$$

$$^{2}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + \gamma$$

$$^{3}\mathrm{He} + {}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + {}^{1}\mathrm{H} + {}^{1}\mathrm{H}$$

with a net energy release of

$$\begin{aligned}\Delta mc^2 &= \left(4 \times m_{^{1}\mathrm{H}} - m_{^{4}\mathrm{He}}\right)c^2 \\ &= (0.028698\ \mathrm{u}) \times 931.5\ \mathrm{MeV/u} \\ &= 26.7\ \mathrm{MeV}\end{aligned}$$



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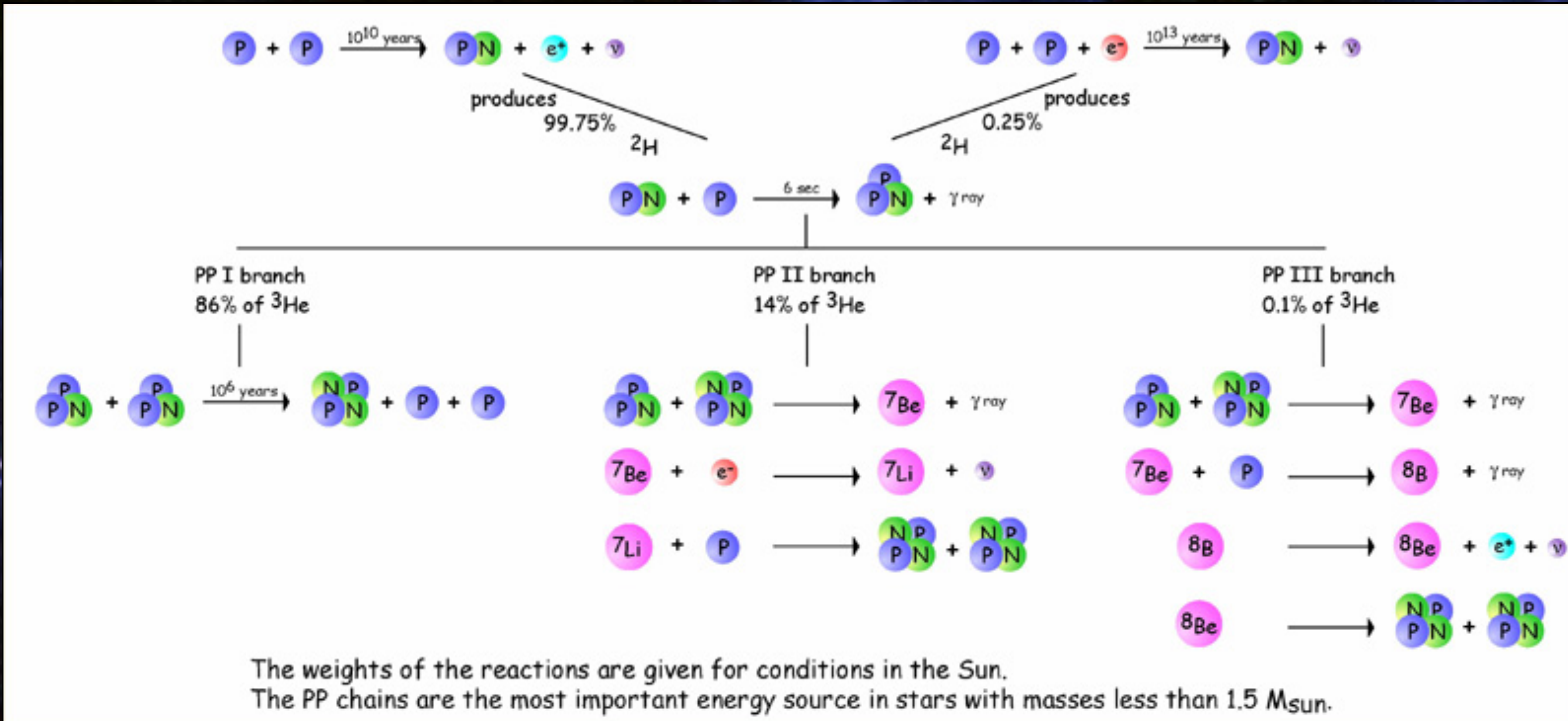
PPI/PPII/PPIII networks



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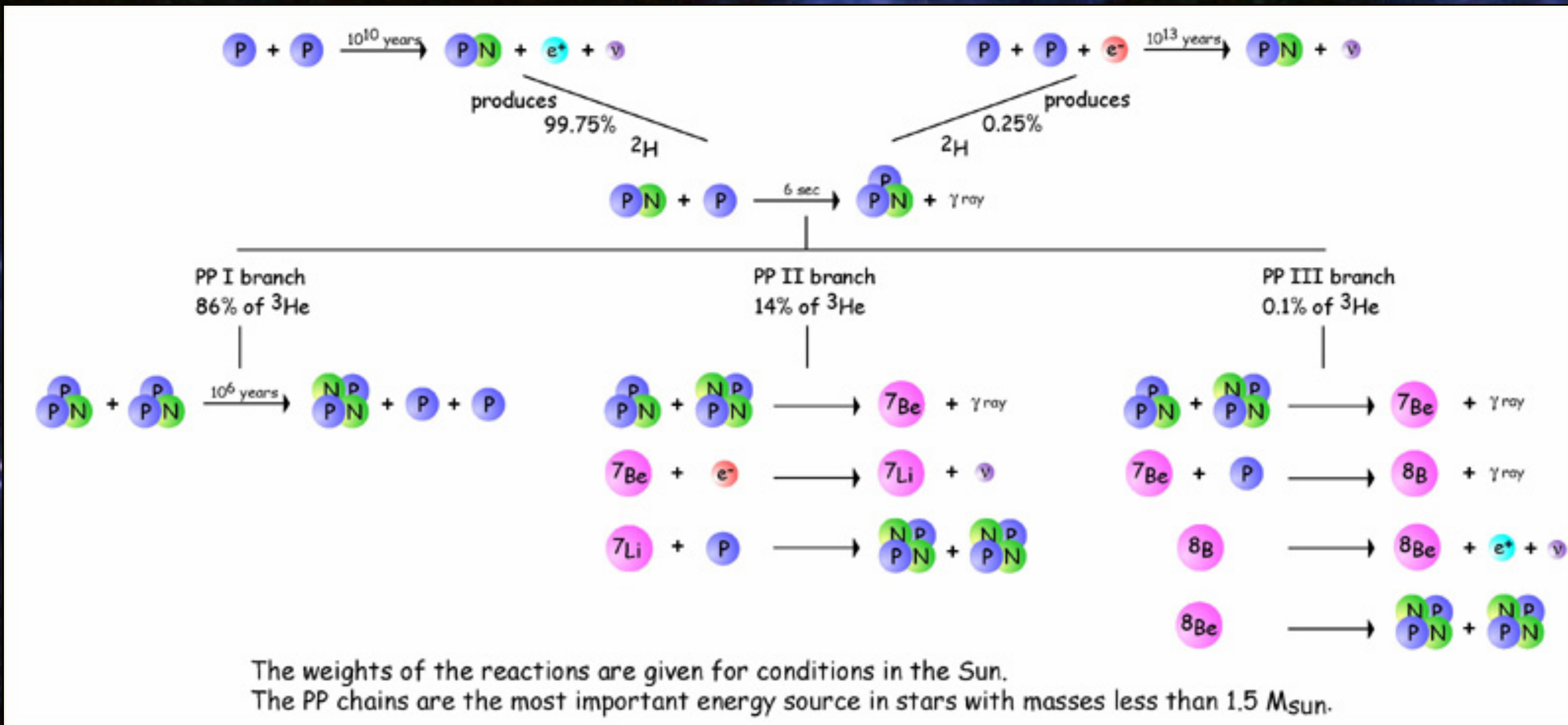

Do we have any direct evidence that the pp chains are operating in the Sun?

Coc & Vangioni (2010)

What is the direct evidence that pp chains are operating in the core of the Sun?

A – We see the composition of the Sun change as a function of time.

B – We observe gamma rays coming from the Sun with appropriate energies for fusion reactions.

C – We detect the positrons emitted in H + H fusion.

D – We detect some of the neutrinos emitted by the weak reactions in the pp chains.

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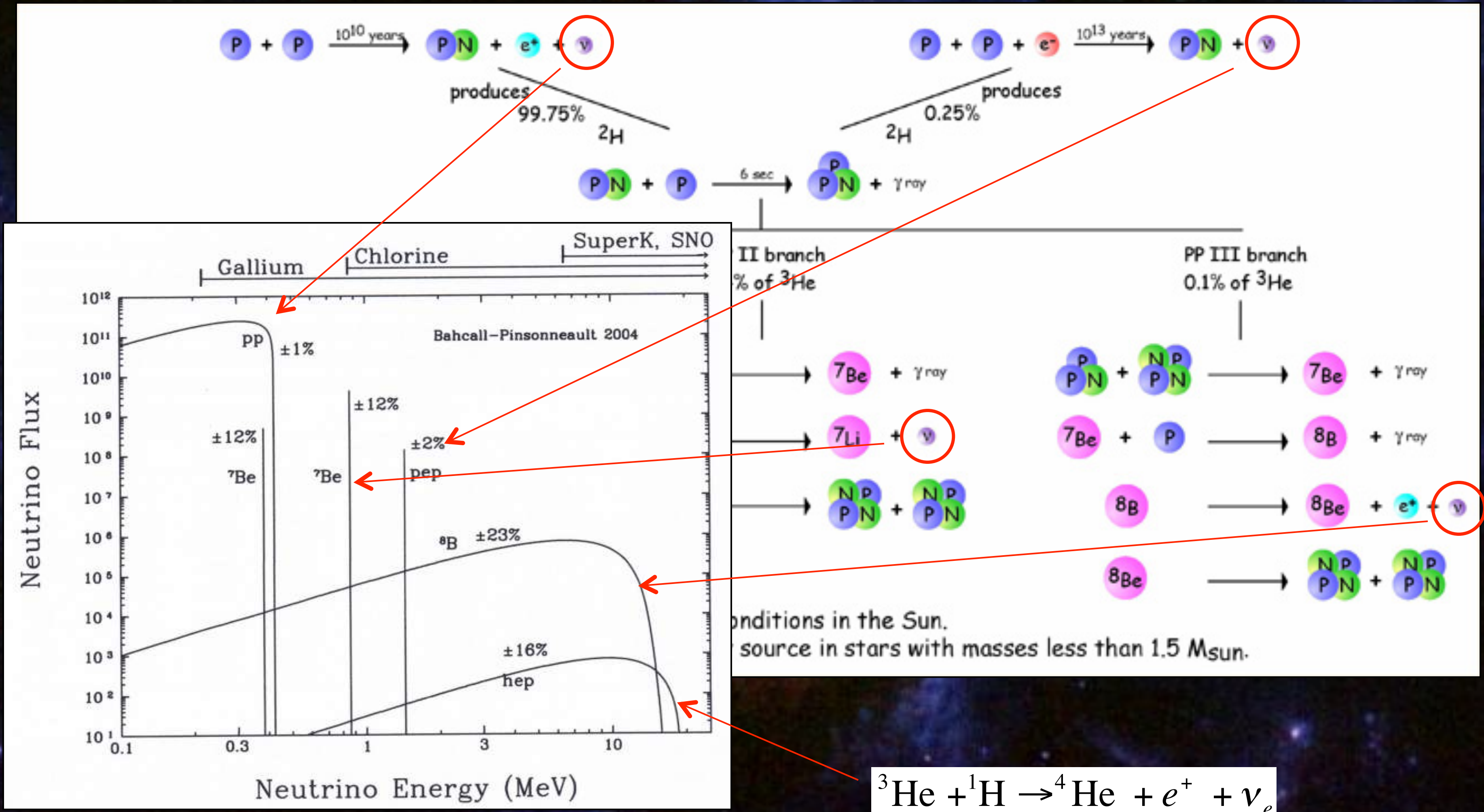
PPI/PPII/PPIII networks: solar neutrino emission

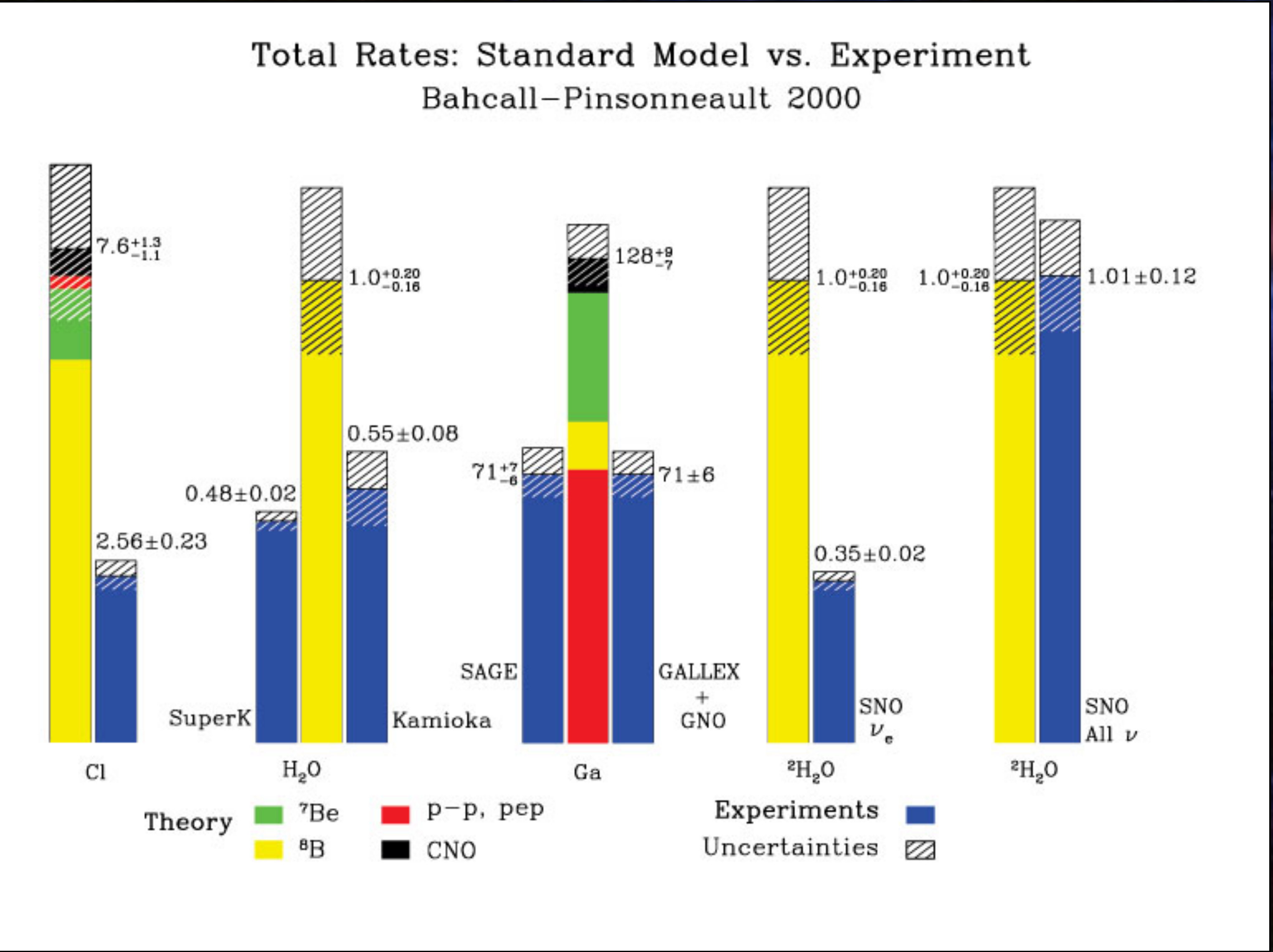


$$^{3}\mathrm{He} + {}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + e^{+} + \nu_e$$

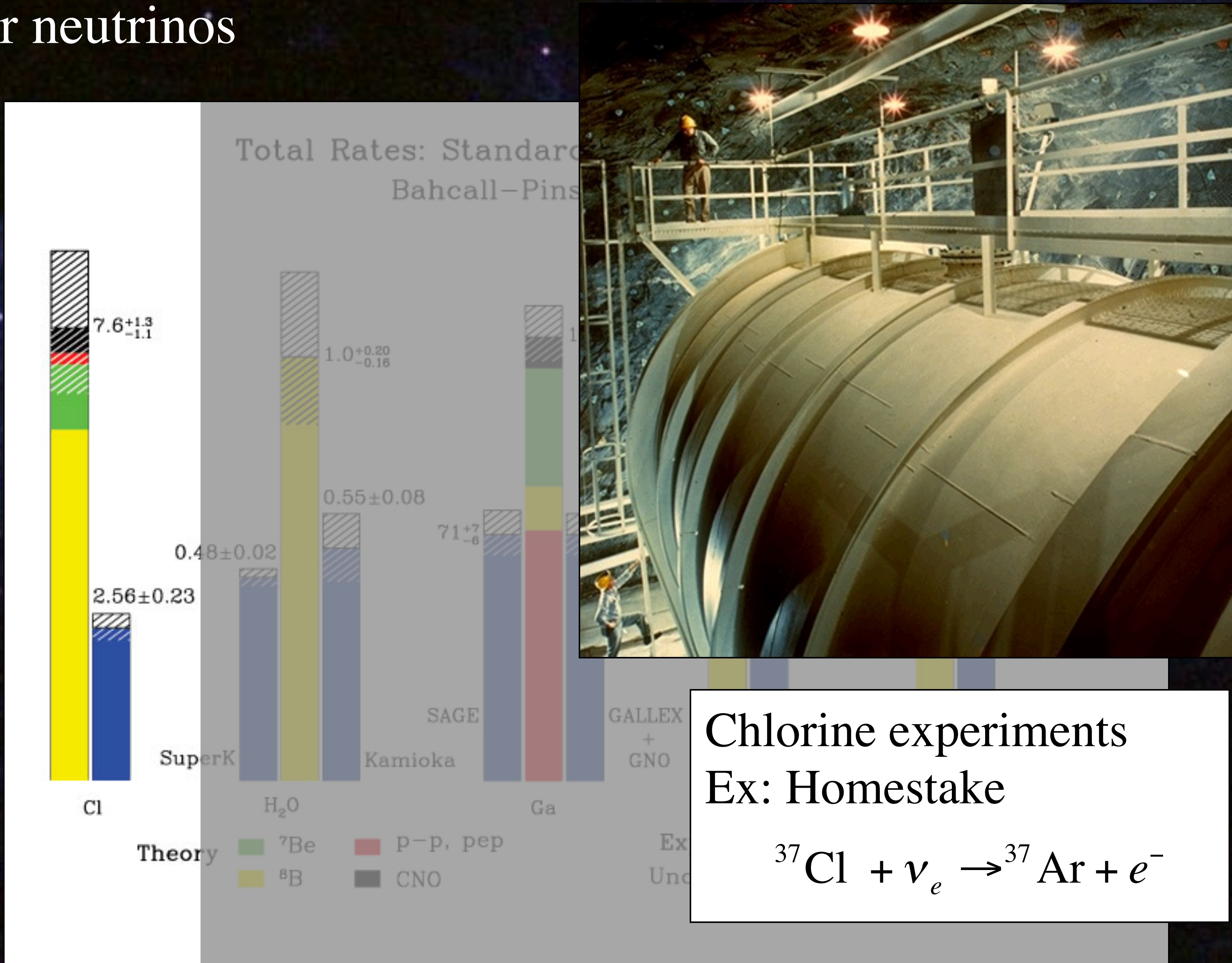
$$\left(\text{only } 0.00005\% \text{ of } {}^{3}\mathrm{He}\right)$$

Coc & Vangioni (2010)

solar neutrinos



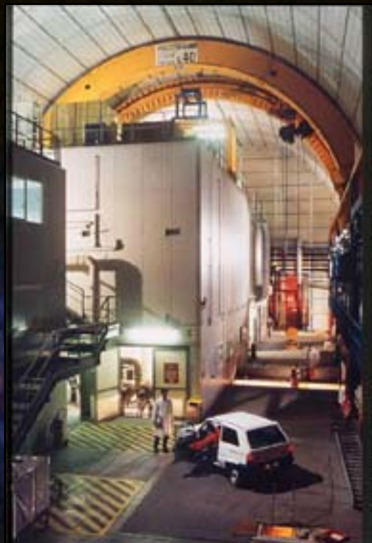
solar neutrinos

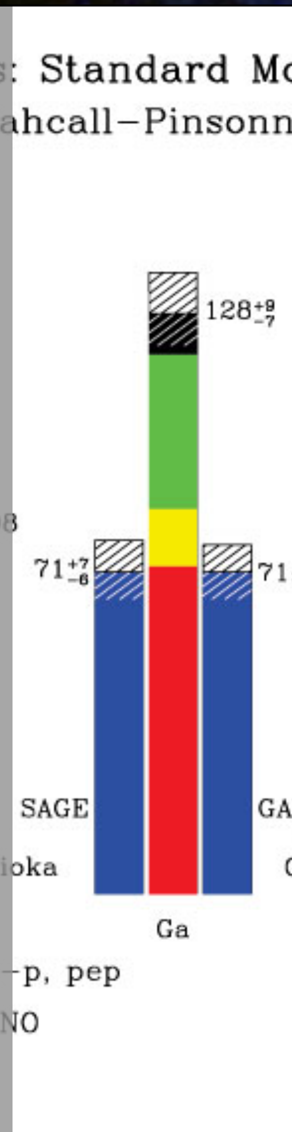


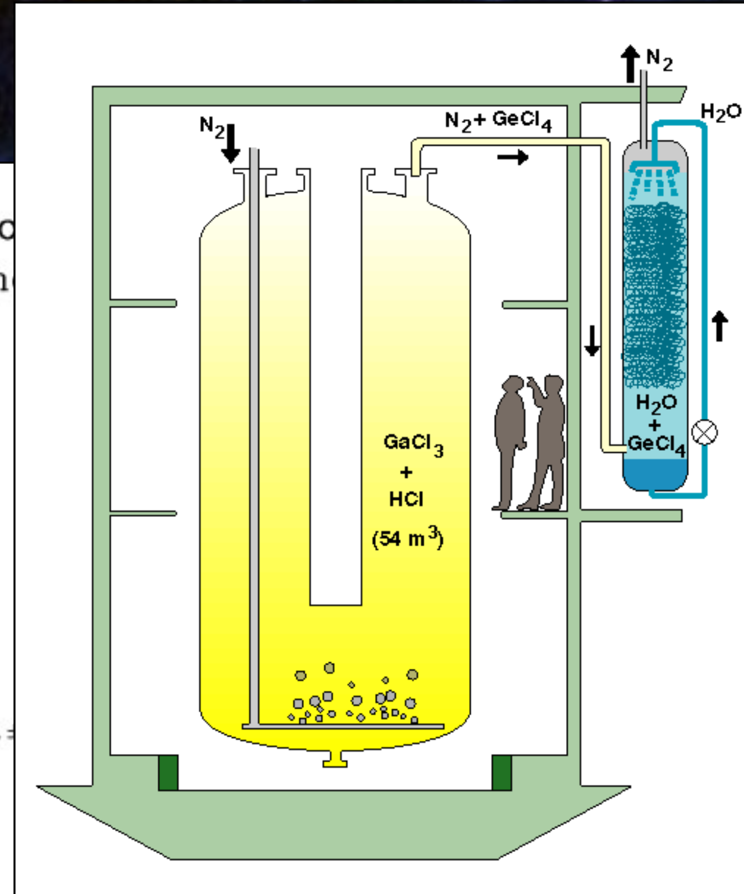
Chlorine experiments
Ex: Homestake

$$^{37}\mathrm{Cl} + \nu_e \rightarrow {}^{37}\mathrm{Ar} + e^-$$

solar neutrinos



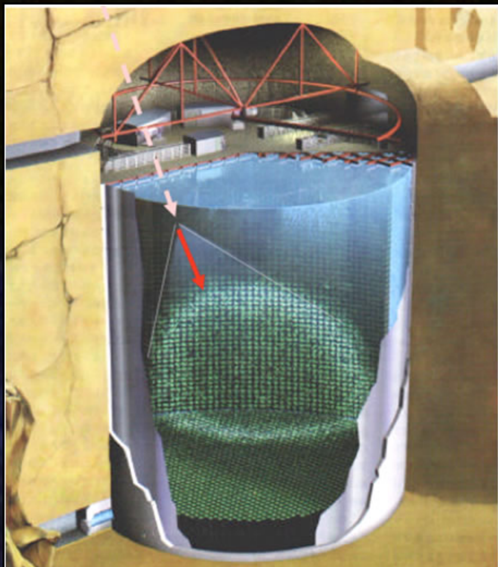


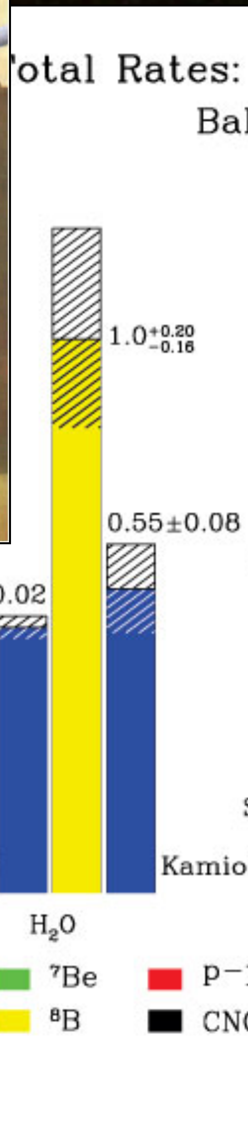


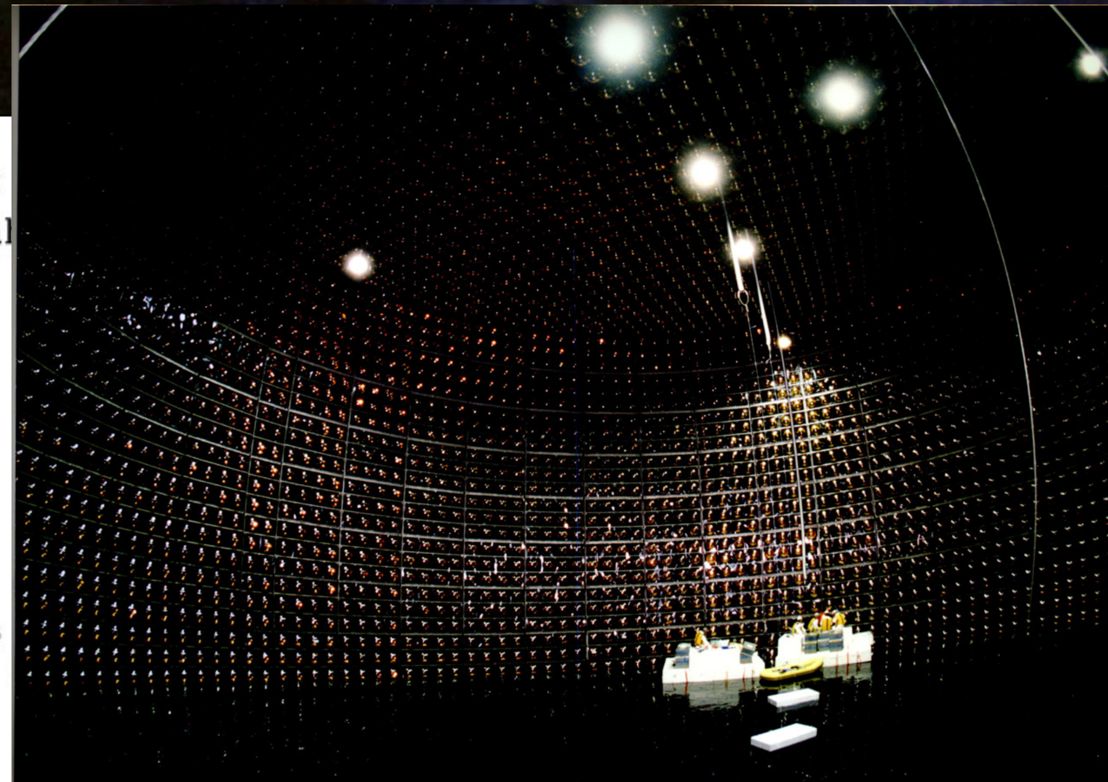
Gallium experiments
Ex: SAGE/GALLEX/GNO

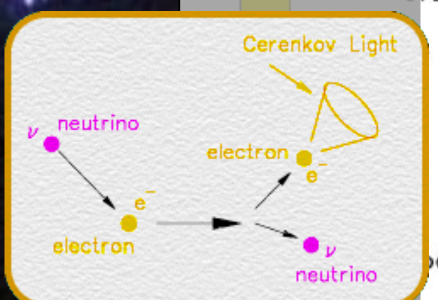
$$^{71}\mathrm{Ga} + \nu_e \rightarrow {}^{71}\mathrm{Ge} + e^-$$

solar neutrinos





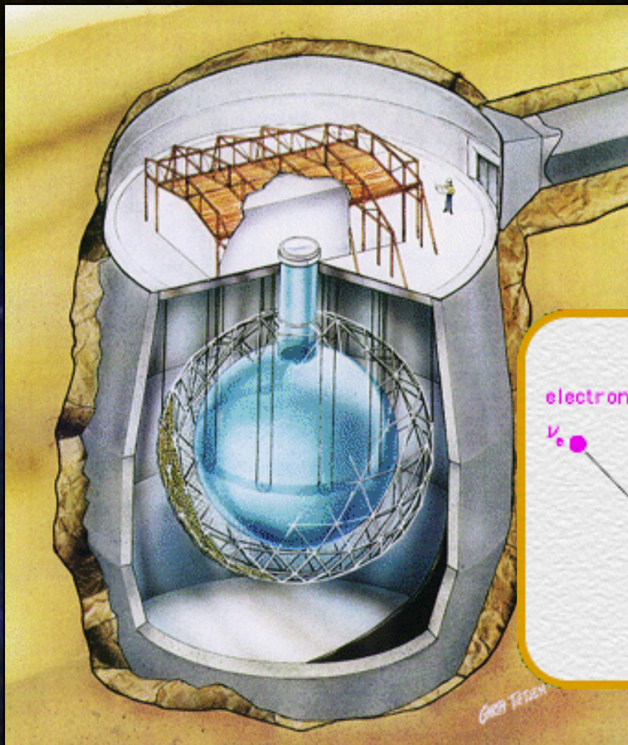


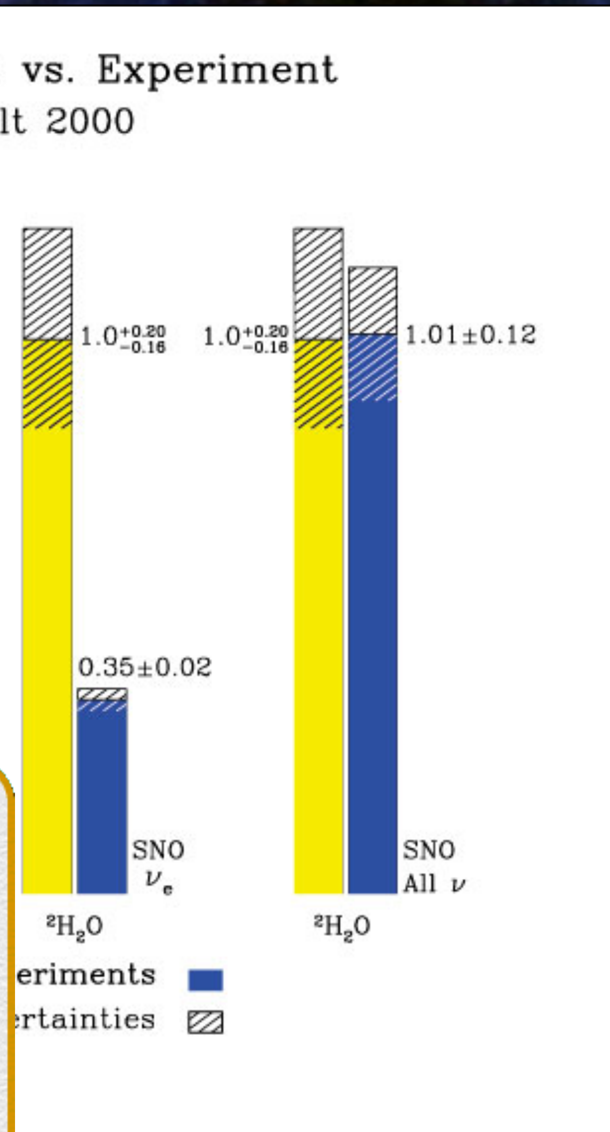


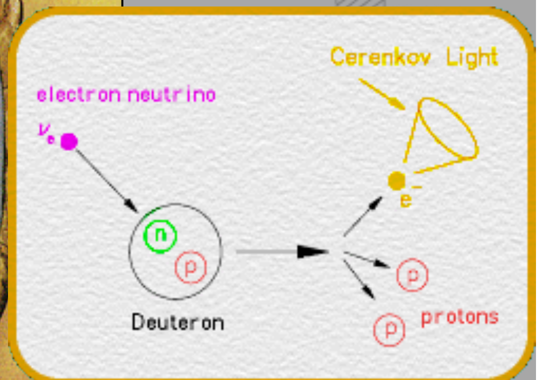
water Cherenkov detector
ex: Super-Kamiokande

$$e^- + \nu_e \rightarrow e^- + \nu_e$$

solar neutrinos



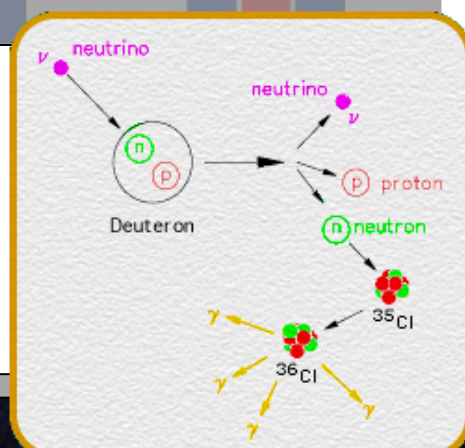




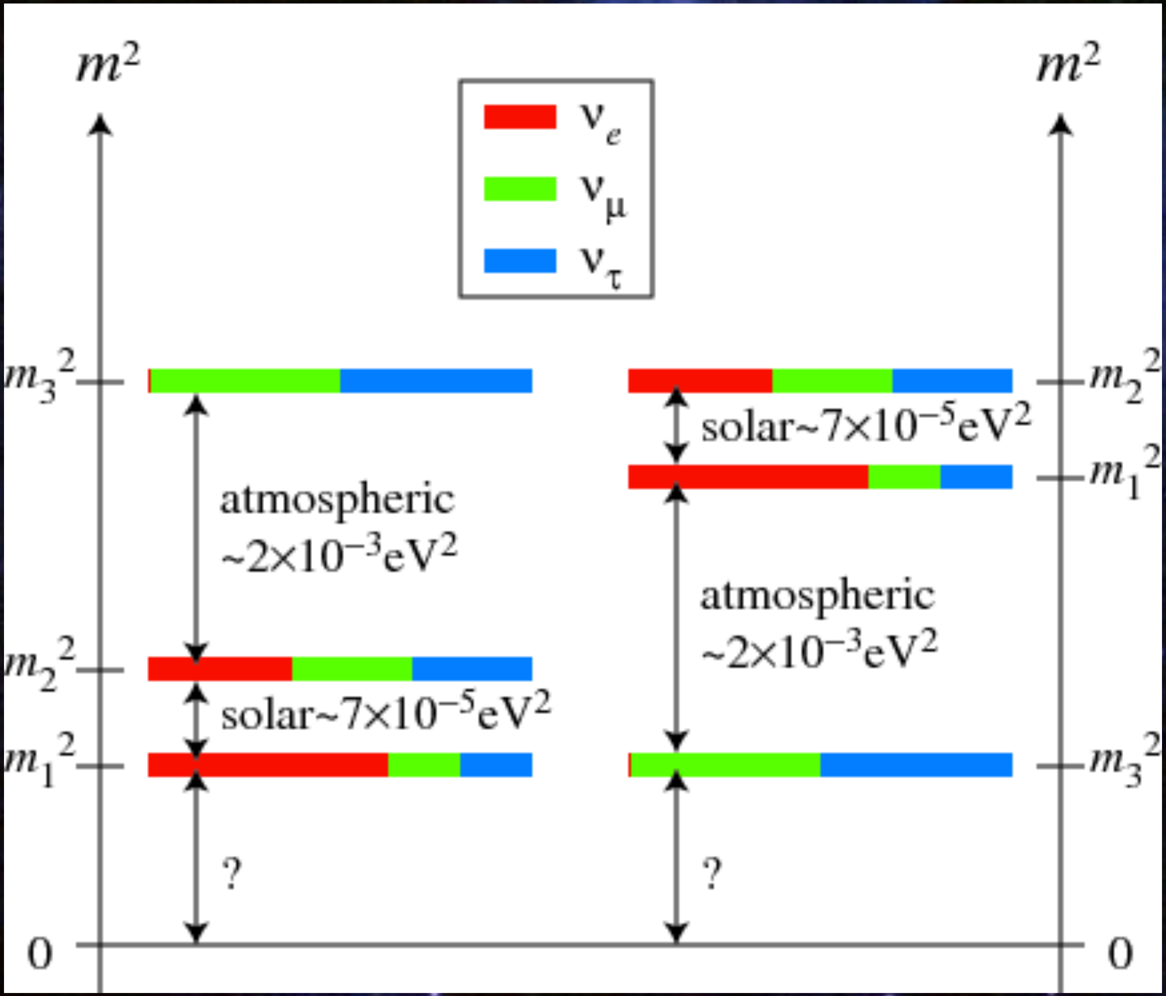
heavy water detector

$$d + \nu_e \rightarrow p + p + e^-$$

$$d + \nu_x \rightarrow p + n + \nu_x$$



neutrino oscillations



neutrino oscillations

mass eigenstates (1,2,3) flavor eigenstates (ε,μ,τ)

$$|\nu_i\rangle = \sum_\alpha U_{\alpha i} |\nu_\alpha\rangle$$

where

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{bmatrix} \begin{bmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13} e^{-i\delta} & 0 & \cos\theta_{13} \end{bmatrix} \begin{bmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

neutrino oscillations

Parameter	best-fit ($\pm 1\sigma$)
Δm^2_{21} [10^{-5} eV ^{2}]	$7.54^{+0.26}_{-0.22}$
$\lvert\Delta m^2\rvert$ [10^{-3} eV ^{2}]	$2.43^{+0.06}_{-0.10}$ ($2.42^{+0.07}_{-0.11}$)
$\sin^2\theta_{12}$	$0.307^{+0.018}_{-0.016}$
$\sin^2\theta_{23}$	$0.386^{+0.024}_{-0.021}$ ($0.392^{+0.039}_{-0.022}$)
$\sin^2\theta_{13}$ [173]	0.0241 ± 0.0025 ($0.0244^{+0.0023}_{-0.0025}$)

$$|\nu_i\rangle = \sum_\alpha U_{\alpha i}|\nu_\alpha\rangle$$

where

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$

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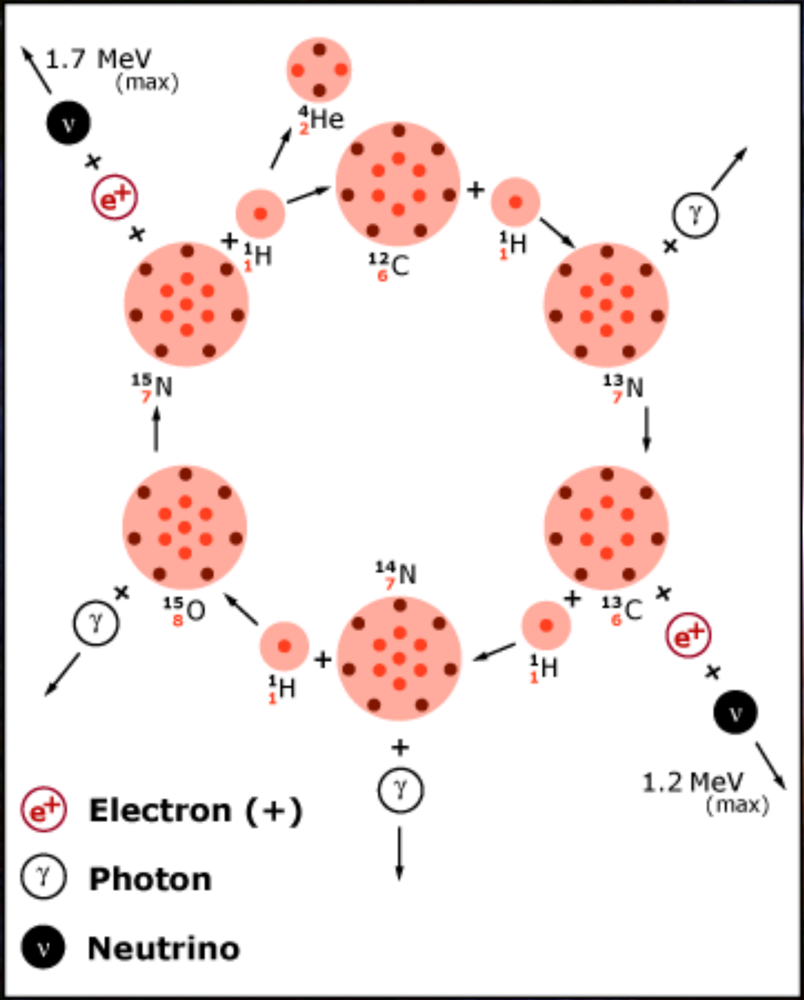
CN cycle



$$^{12}C + {}^1H \rightarrow {}^{13}N + \gamma$$

$$^{13}N \rightarrow {}^{13}C + e^+ + \nu_e$$

$$^{13}C + {}^1H \rightarrow {}^{14}N + \gamma$$

$$^{14}N + {}^1H \rightarrow {}^{15}O + \gamma$$

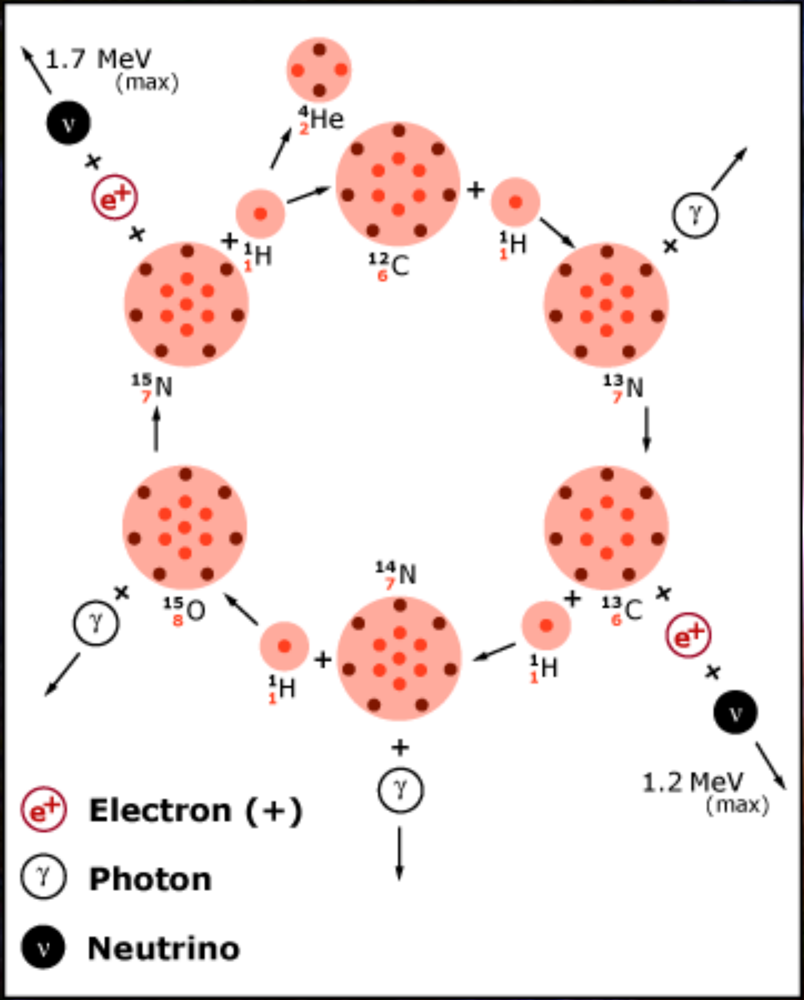
$$^{15}O \rightarrow {}^{15}N + e^+ + \nu_e$$

$$^{15}N + {}^1H \rightarrow {}^4He + {}^{12}C$$

Net :

$$^{12}C + 4{}^1H \rightarrow {}^{12}C + {}^4He + 2e^+ + 2\nu_e$$

CN cycle



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$$^{13}C + ^{1}H \rightarrow ^{14}N + \gamma$$

$$\boxed{^{14}N + ^{1}H \rightarrow ^{15}O + \gamma}$$

$$^{15}O \rightarrow ^{15}N + e^{+} + \nu_{e}$$

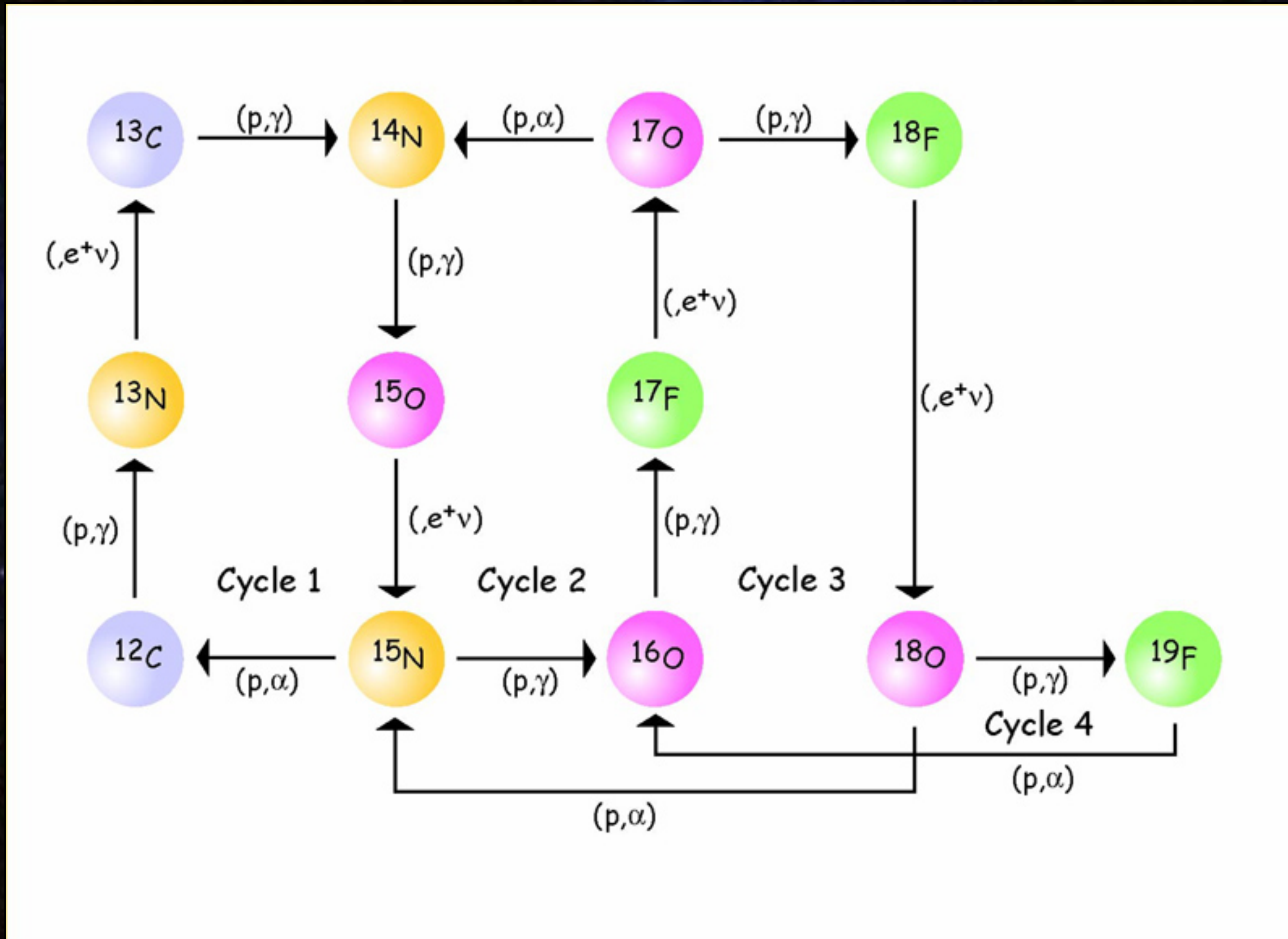
$$^{15}N + ^{1}H \rightarrow ^{4}He + ^{12}C$$

slowest reaction, sets timescale

Net :

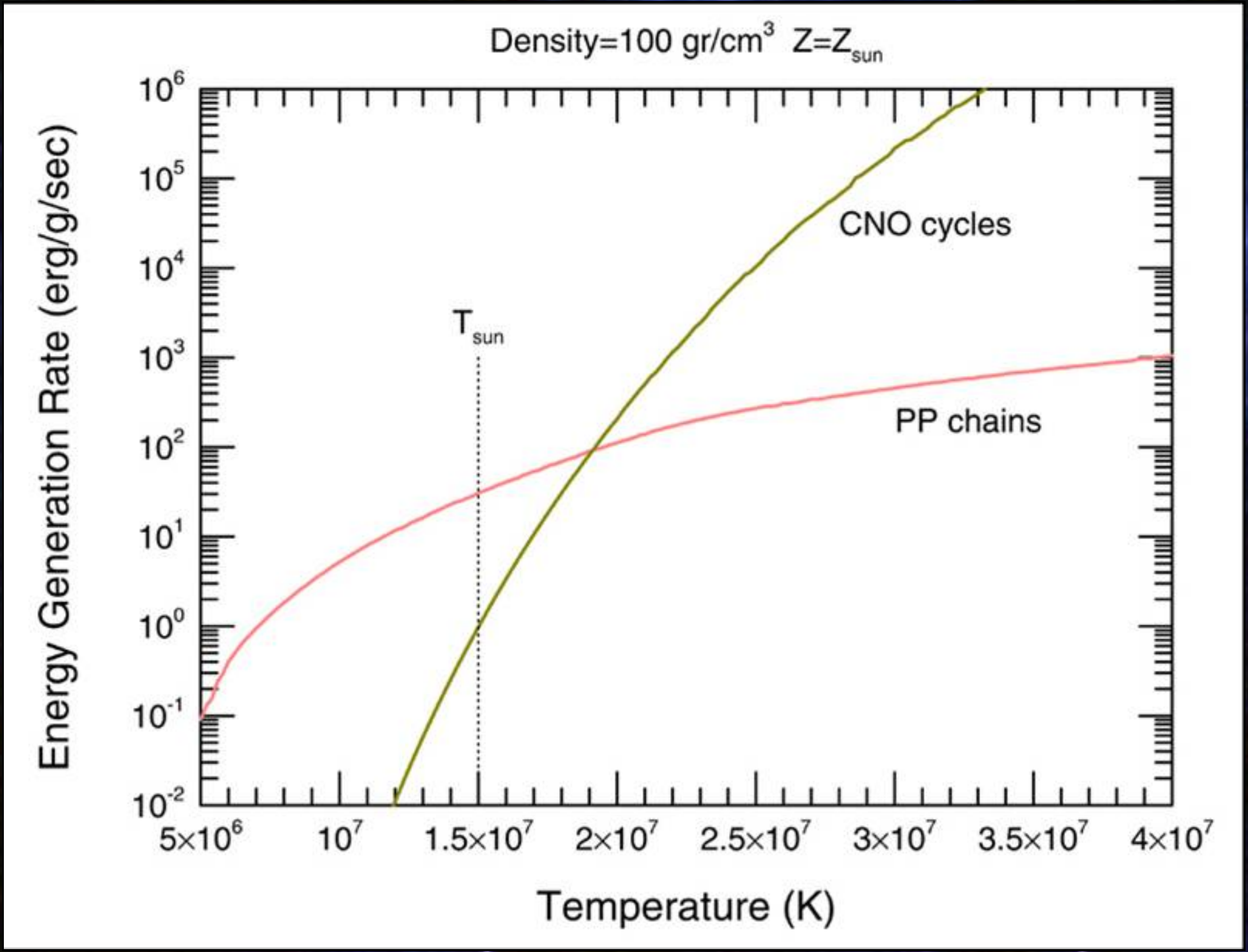
$$^{12}C + 4\,^{1}H \rightarrow ^{12}C + ^{4}He + 2e^{+} + 2\nu_{e}$$

CNO cycles



13C
(p,γ)
14N
(p,α)
17O
(p,γ)
18F
(,e+ν)
(p,γ)
(,e+ν)
(,e+ν)
13N
15O
17F
(p,γ)
(,e+ν)
(p,γ)
Cycle 1
Cycle 2
Cycle 3
12C
15N
16O
18O
19F
(p,α)
(p,γ)
(p,γ)
Cycle 4
(p,α)
(p,α)

CNO cycles vs PP chains



end of H fusion?

Sun has been fusing H into He for ~4.6 billion years

There is probably enough H in core for another ~4 billion years

How can we learn what happens next??

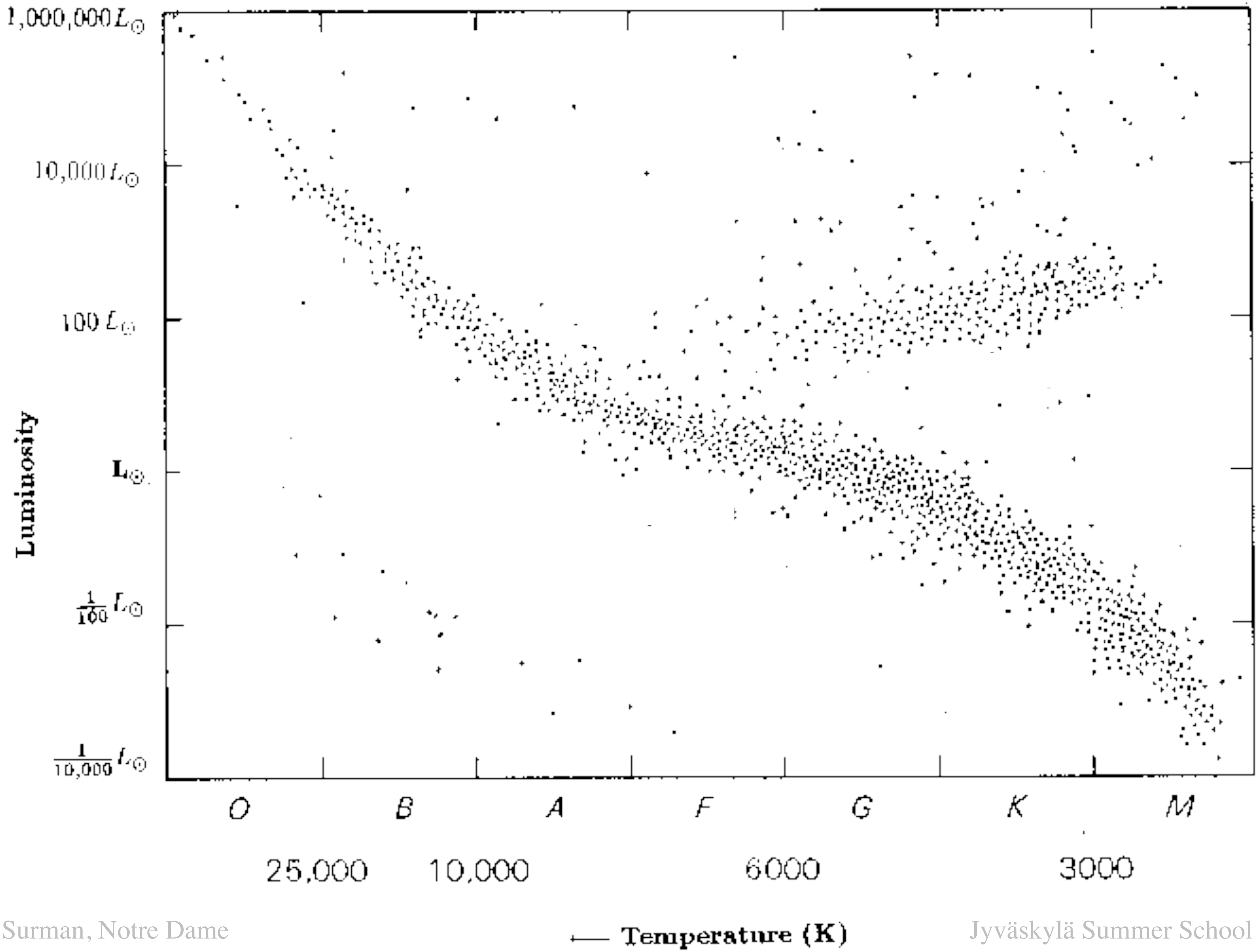
end of H fusion?

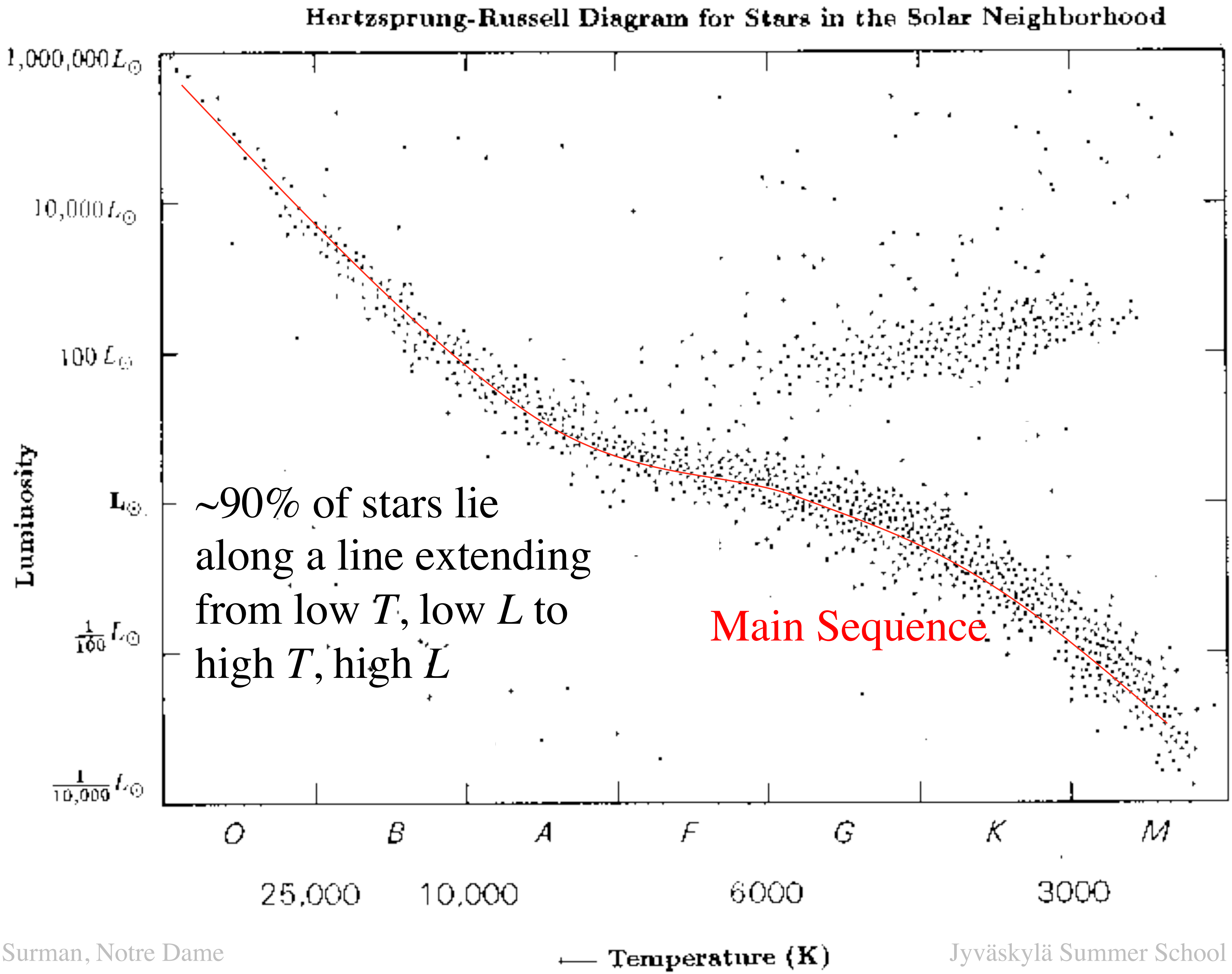
Sun has been fusing H into He for ~4.6 billion years
There is probably enough H in core for another ~4 billion years

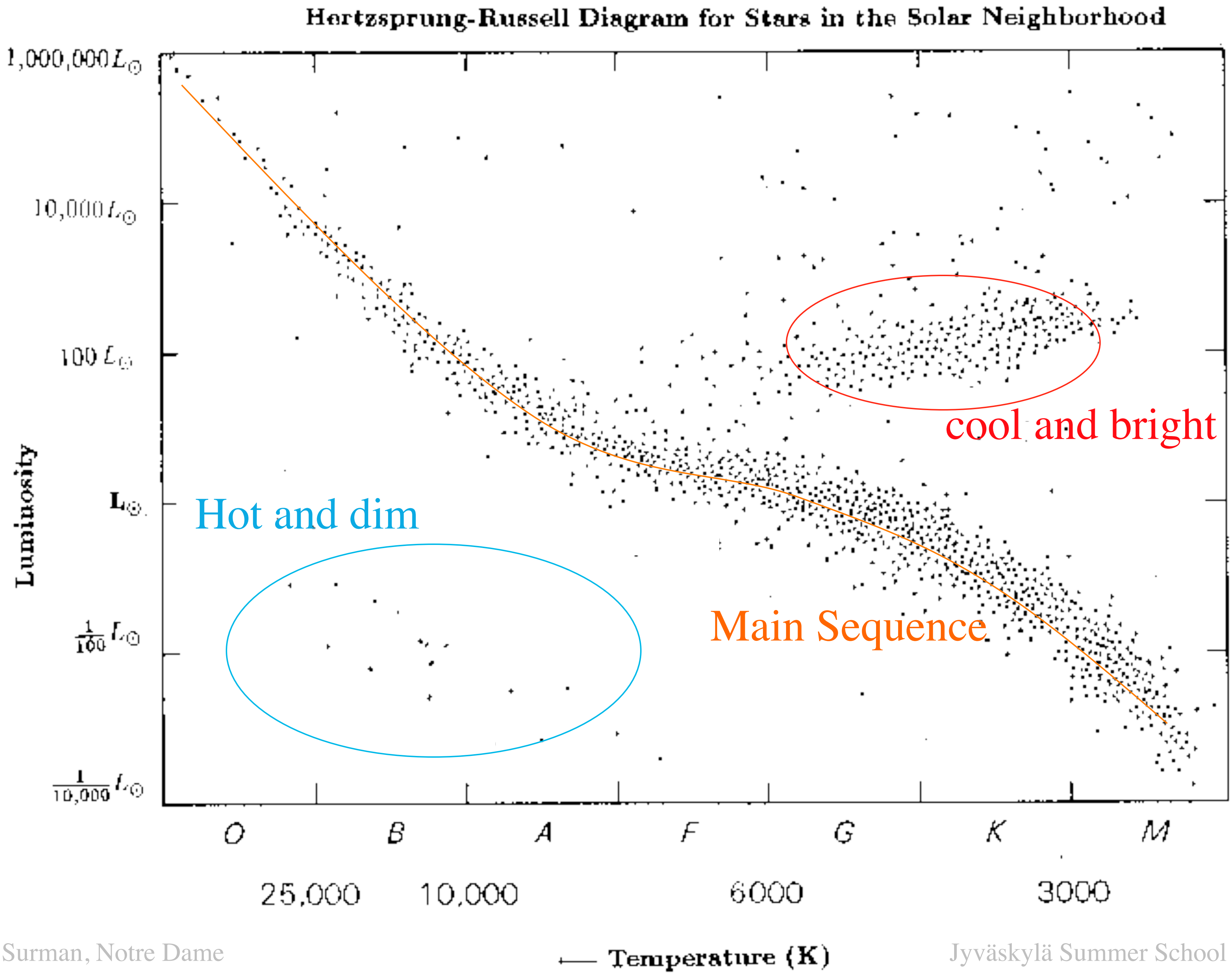
How can we learn what happens next??
=> study other stars, try to observe ‘life cycle’ by analyzing stellar properties

What we can measure: luminosity, mass, temperature, size, elemental composition

Hertzsprung-Russell Diagram for Stars in the Solar Neighborhood




Hertzsprung-Russell Diagram for Stars in the Solar Neighborhood
1,000,000 $L_\odot$
10,000 $L_\odot$
100 $L_\odot$
$L_\odot$
$\frac{1}{100} L_\odot$
$\frac{1}{10,000} L_\odot$
Luminosity
~90% of stars lie along a line extending from low T, low L to high T, high L
Main Sequence
O
B
A
F
G
K
M
25,000
10,000
6000
3000
← Temperature (K)



Hertzsprung-Russell Diagram for Stars in the Solar Neighborhood
1,000,000 $L_\odot$
10,000 $L_\odot$
100 $L_\odot$
$L_\odot$
$\frac{1}{100} L_\odot$
$\frac{1}{10,000} L_\odot$
Luminosity
cool and bright
Hot and dim
Main Sequence
O
B
A
F
G
K
M
25,000
10,000
6000
3000
← Temperature (K)

correlate with other stellar properties…

- Radius

hot, dim stars => very small — white dwarfs

cool, bright stars => very large — red giants

top of MS => big, bright, hot stars

bottom of MS => cool, dim, small stars

correlate with other stellar properties…

- Radius

  hot, dim stars => very small

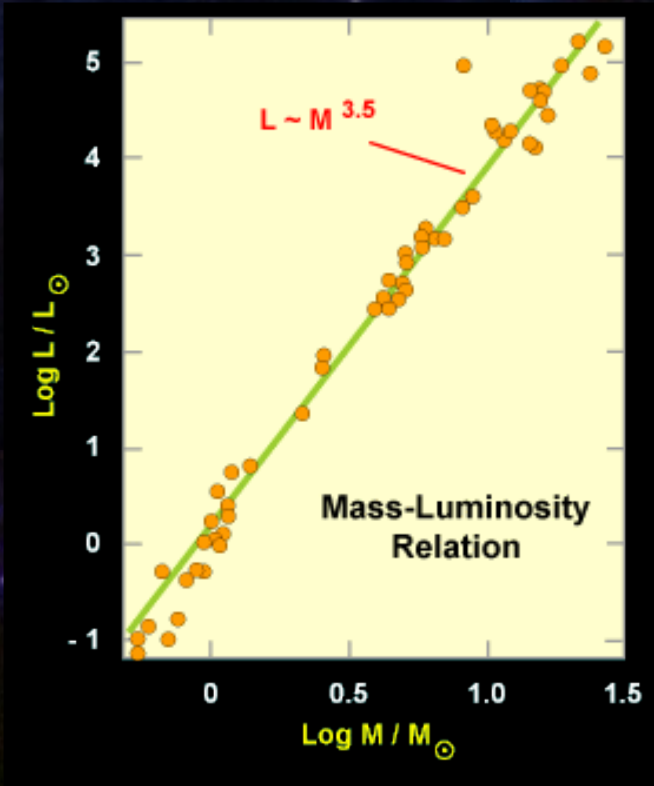
  cool, bright stars => very large

  top of MS => big, bright, hot stars

  bottom of MS => cool, dim, small stars

- Mass

  for main sequence stars, mass and luminosity are directly proportional



Which of the following would lengthen the amount of time a star is able to fuse hydrogen at its center?

A – The temperature in the core of the star is increased.

B – The gas in the core of the star is enriched with hydrogen.

C – The mass of the star is increased.

D – none of the above

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larger mass -> greater pressure required to support star -> higher temp, density -> faster fusion

$$\frac{t}{t_{\text{solar}}} = \left(\frac{M}{M_{\text{solar}}}\right)^{-2.5}$$

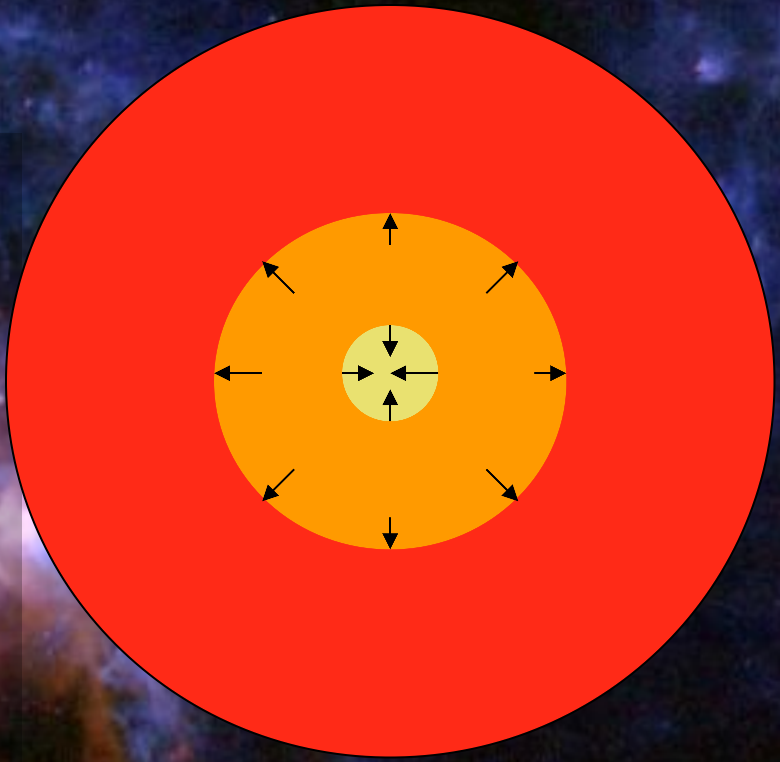
0.4 M_solar	~100 billion years
1.0 M_solar	~10 billion years
40 M_solar	~ few million years

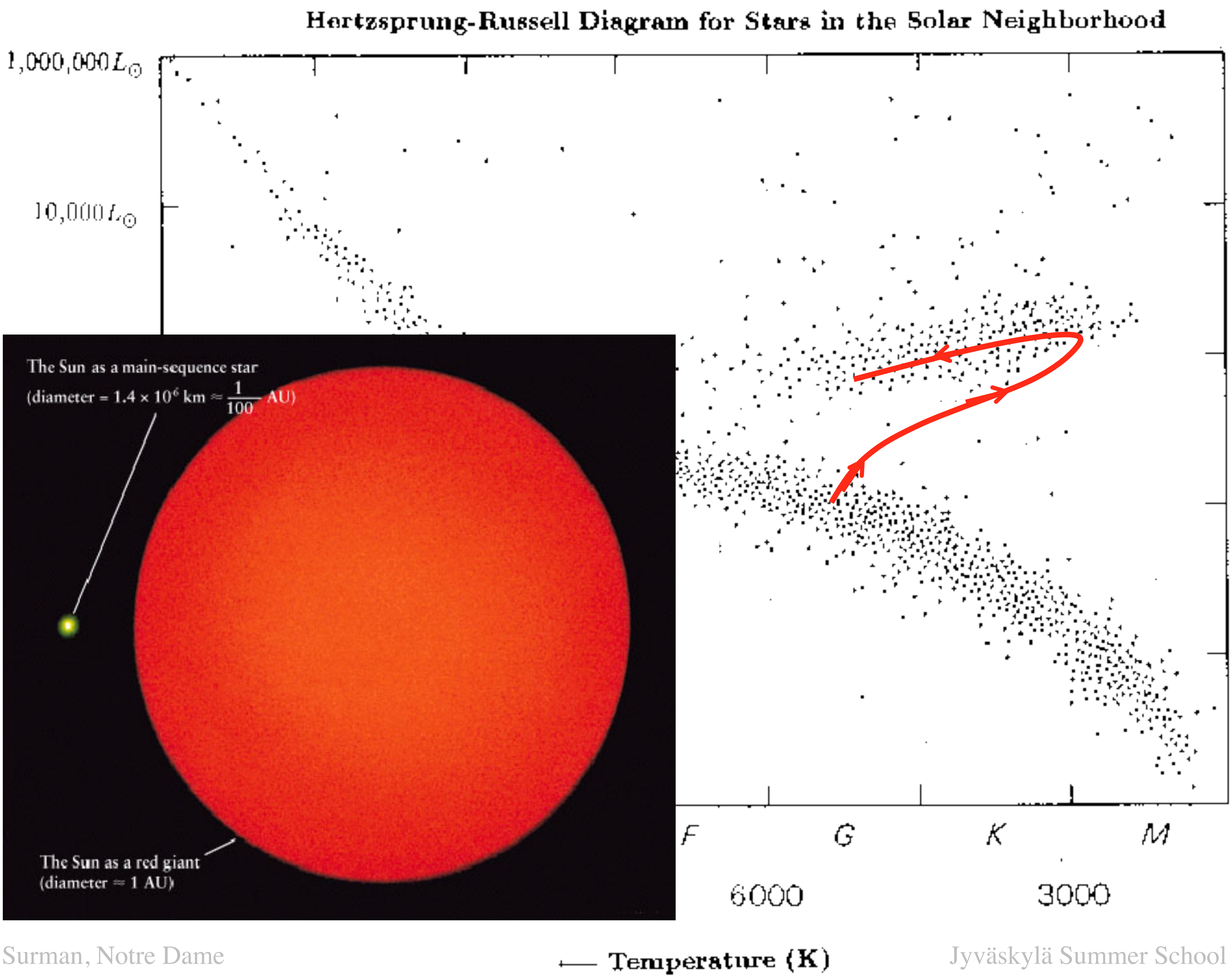
What happens to a main sequence star when its core H runs out?

H fusion in core slows/stops
- Core pressure drops
- Core compresses

H fusion ignites in shell around core
- Shell pressure rises
- Shell expands

=> Red giant

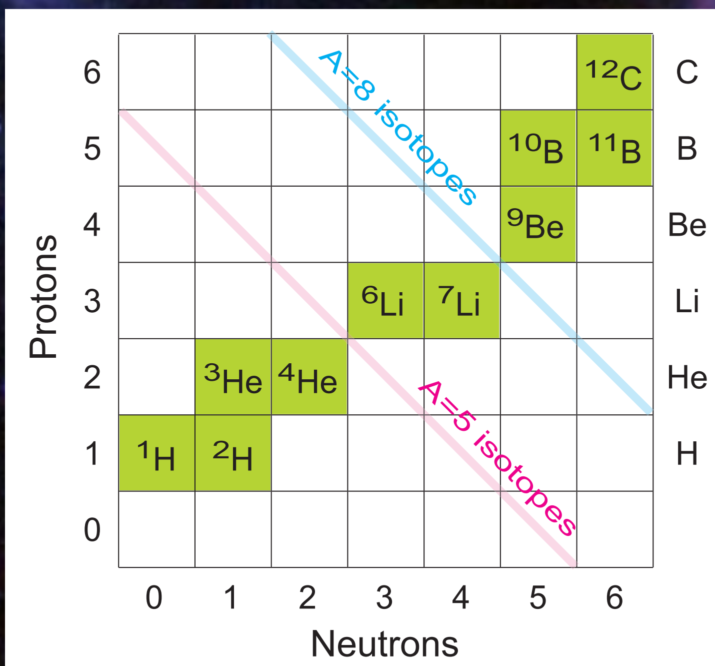


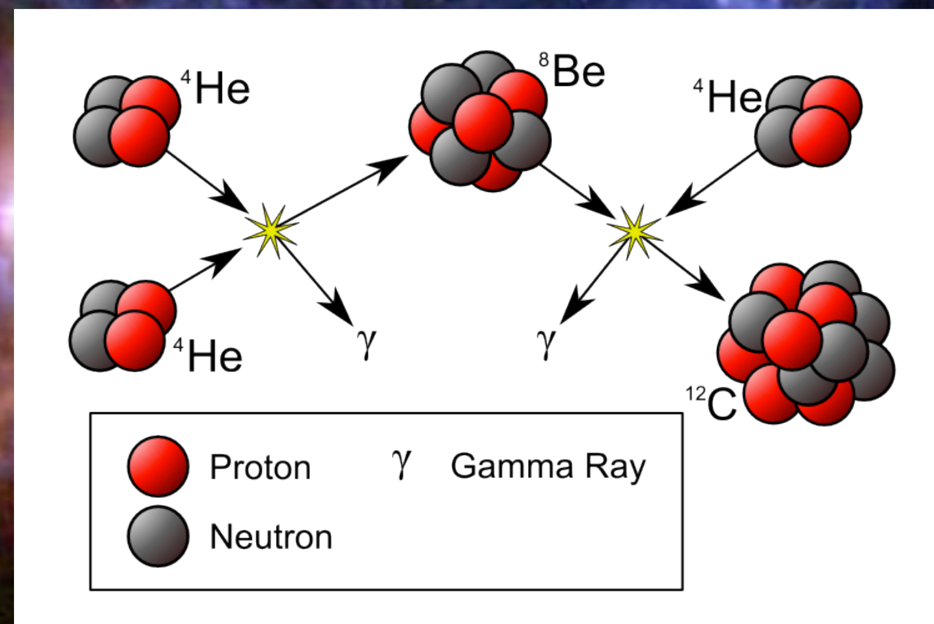

Hertzsprung-Russell Diagram for Stars in the Solar Neighborhood
1,000,000 $L_\odot$
10,000 $L_\odot$
F
G
K
M
6000
3000
← Temperature (K)
The Sun as a main-sequence star
(diameter = 1.4 × 10⁶ km ≈ 1/100 AU)
The Sun as a red giant
(diameter = 1 AU)

He core fusion: triple alpha process

$$^{4}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{8}\mathrm{Be}^{*} \qquad (-93.7\ \mathrm{keV})$$

$$^{8}\mathrm{Be} + {}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma \qquad (+7.367\ \mathrm{MeV})$$





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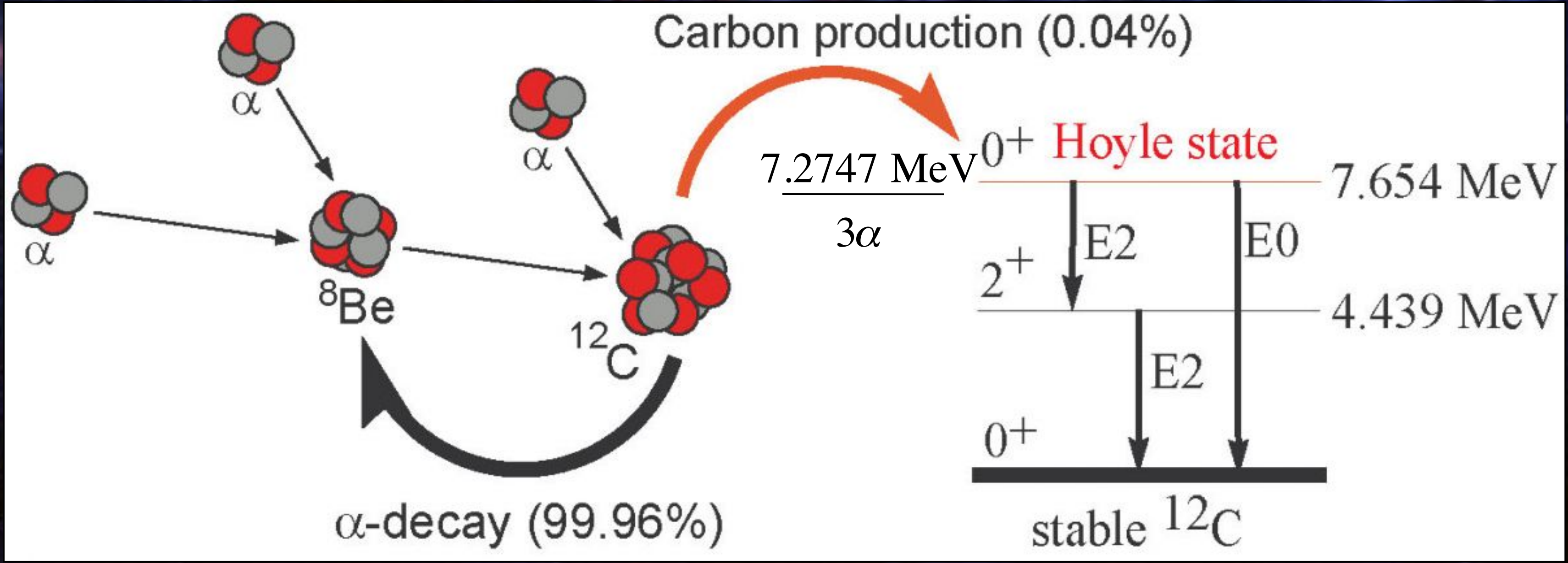

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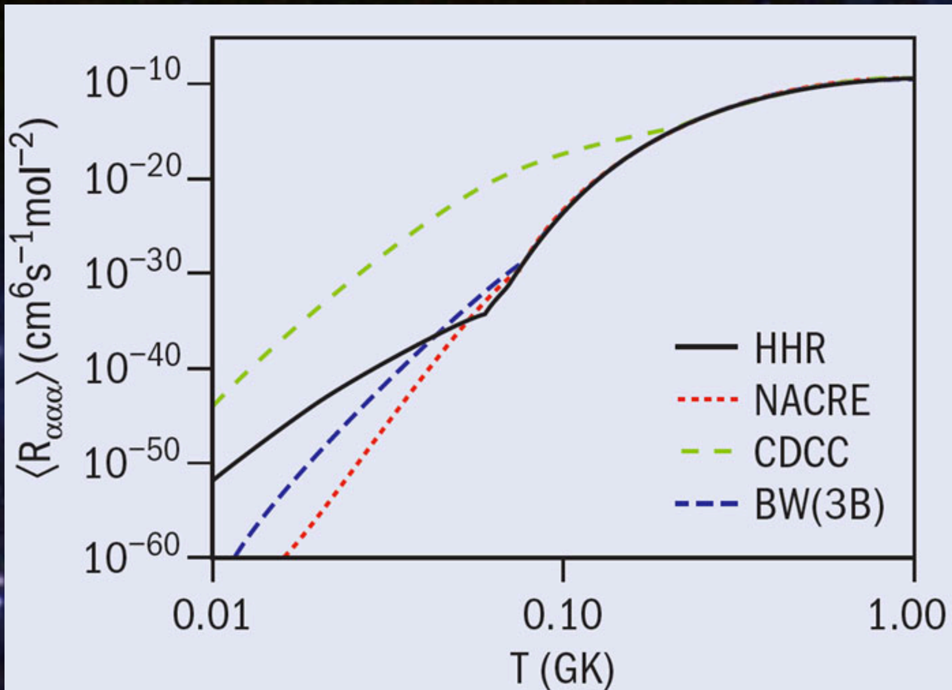
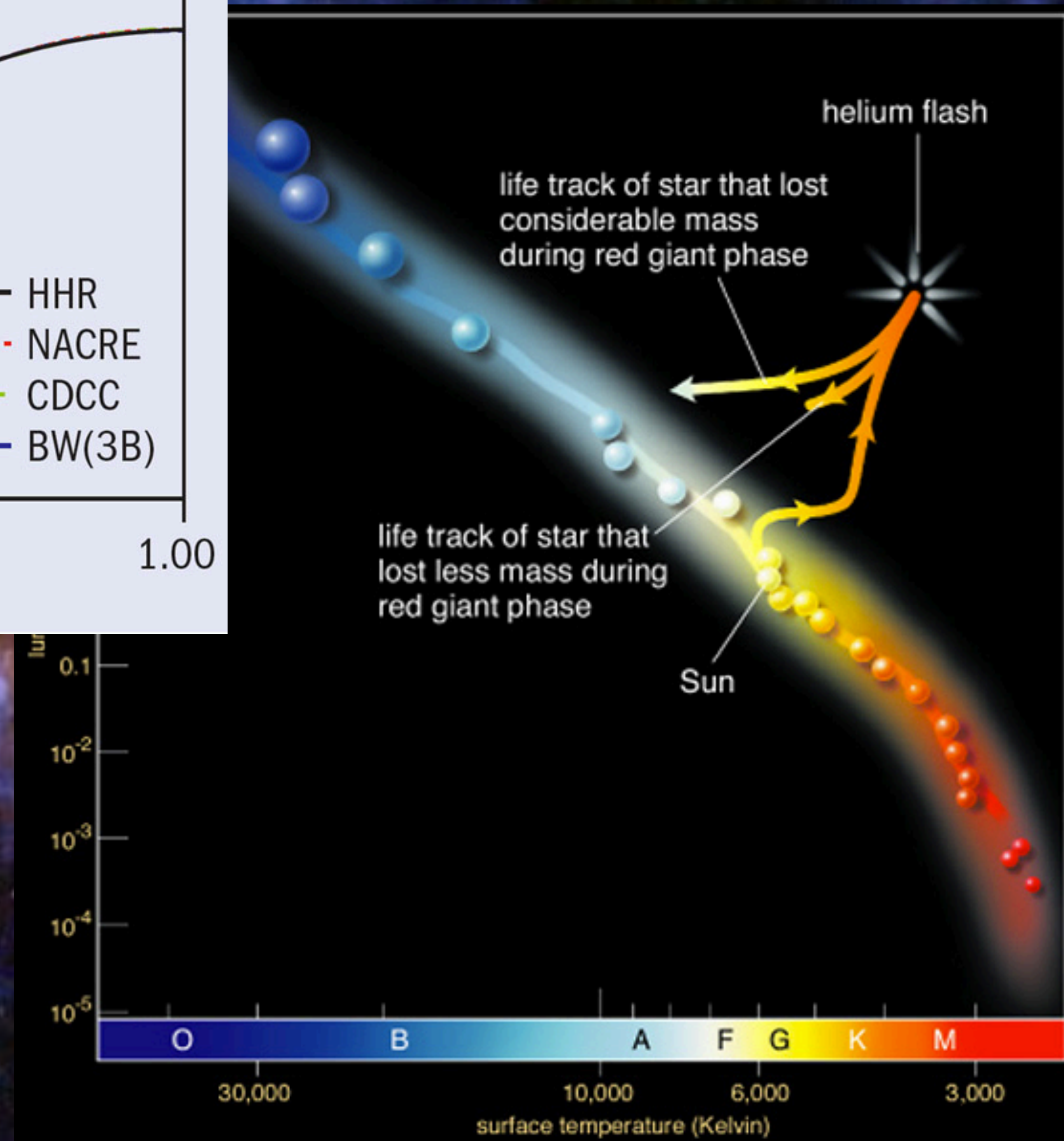
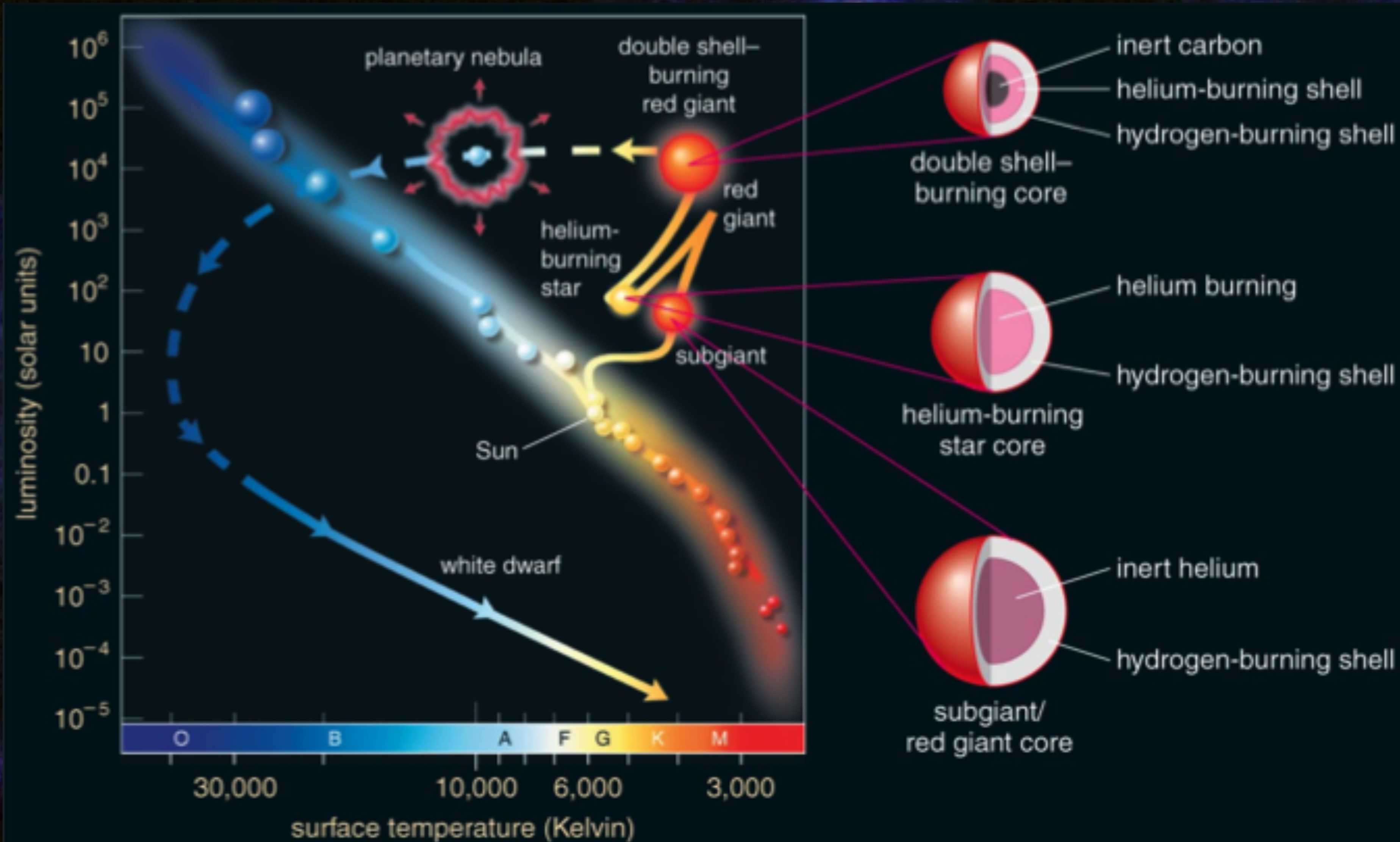




evolution of low mass stars

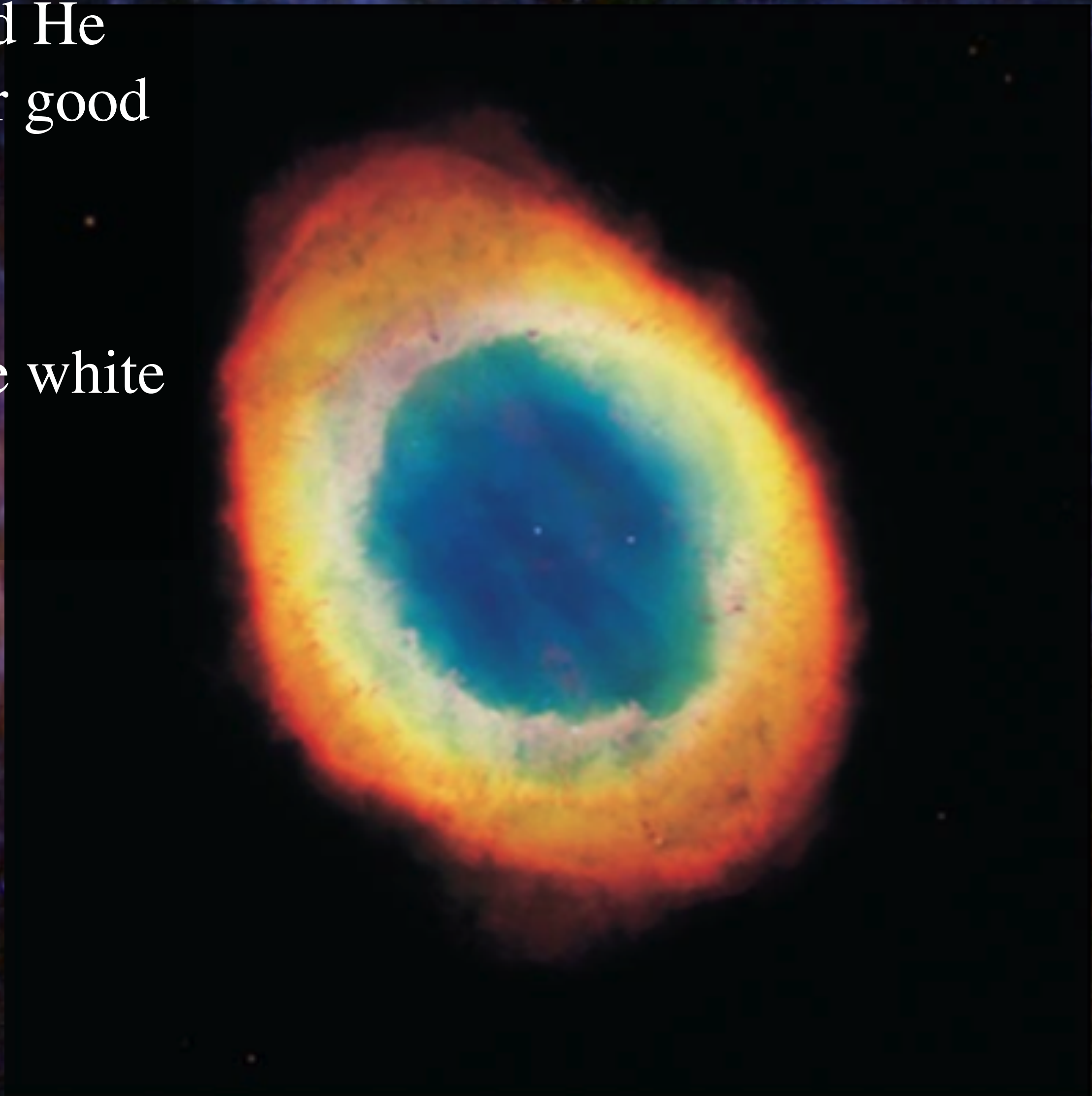


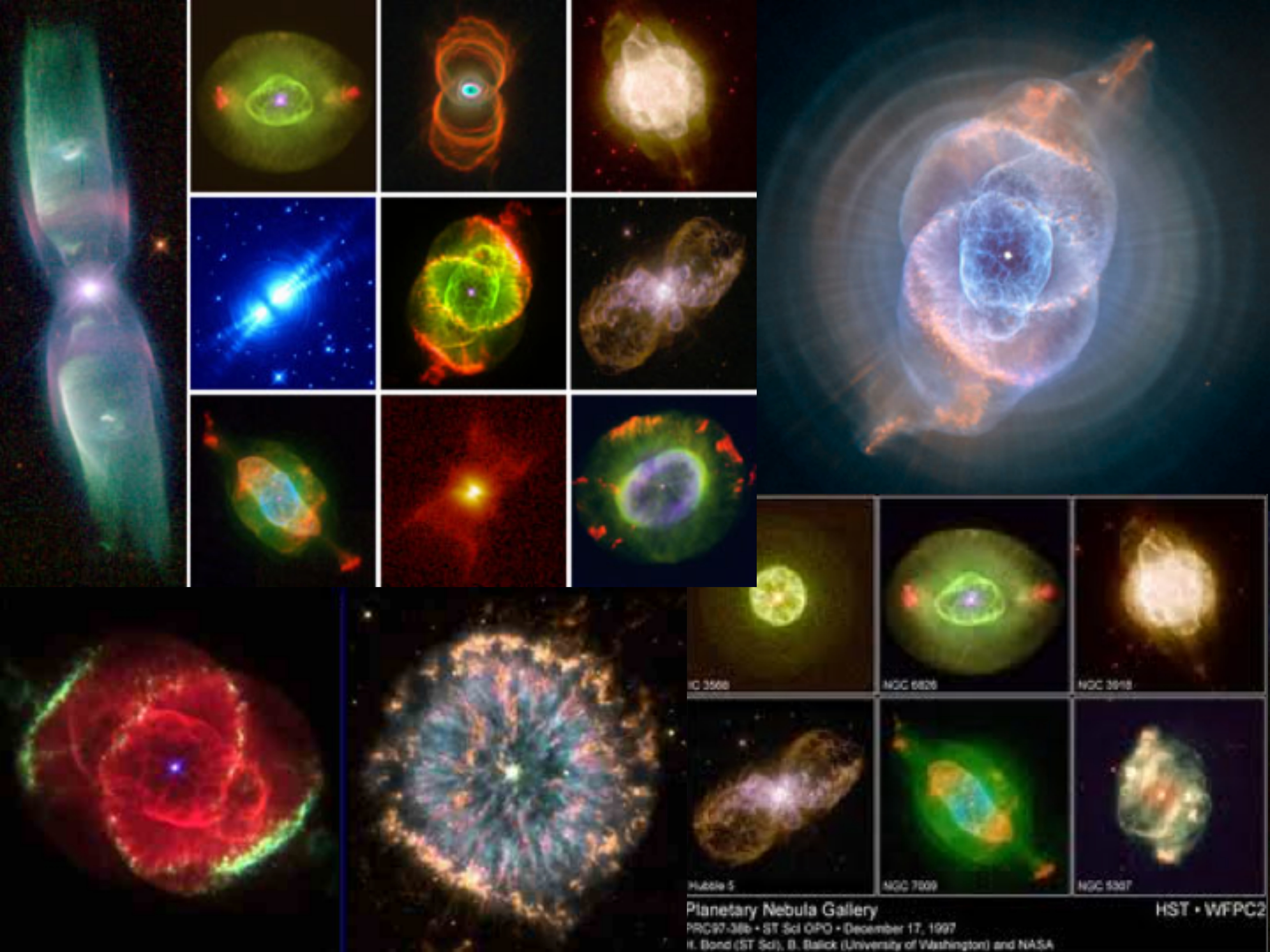
planetary nebula

double shell burning ends with a pulse that ejects the H and He outer layers into space for good

⇒ a planetary nebula forms

⇒ core left behind is a dense white dwarf




C 3568
NGC 6826
NGC 3918
Hubble 5
NGC 7009
NGC 5307
Planetary Nebula Gallery
HST • WFPC2
PRC97-38b • ST ScI OPO • December 17, 1997
H. Bond (ST ScI), B. Balick (University of Washington) and NASA

The helium burning phase for a star of a given mass is much shorter than the hydrogen burning phase primarily because…

A – The helium mass fraction in the core is less than the hydrogen mass fraction when the star was young.

B – The triple alpha process releases less energy per reaction than the pp chains/CNO cycles.

C – The star becomes a white dwarf before it can use most of its helium.

D – The core temperature never rises high enough for complete helium burning.

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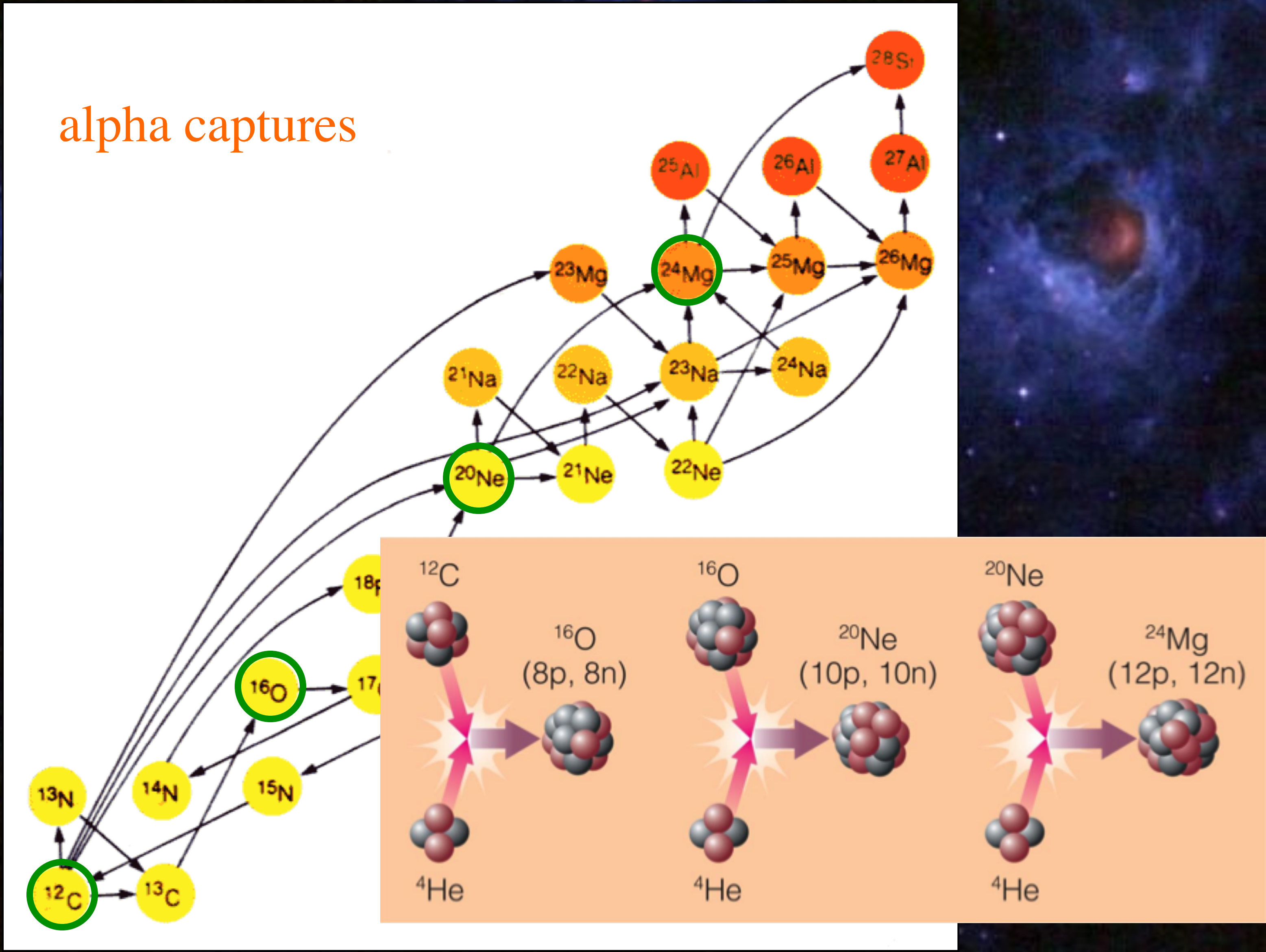
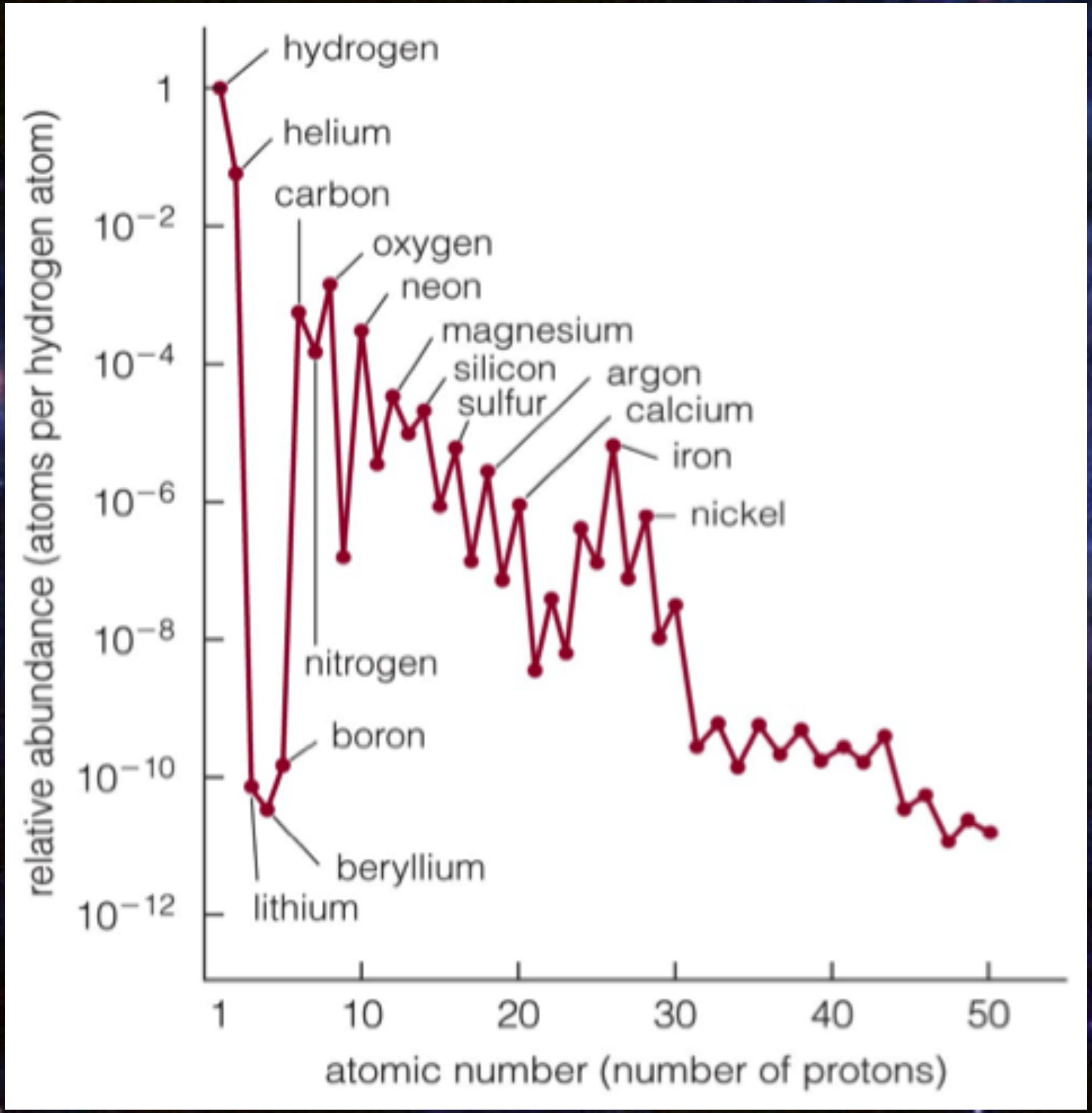
advanced burning stages: carbon, oxygen, neon



advanced burning stages: carbon, oxygen, neon



evidence for alpha captures =>

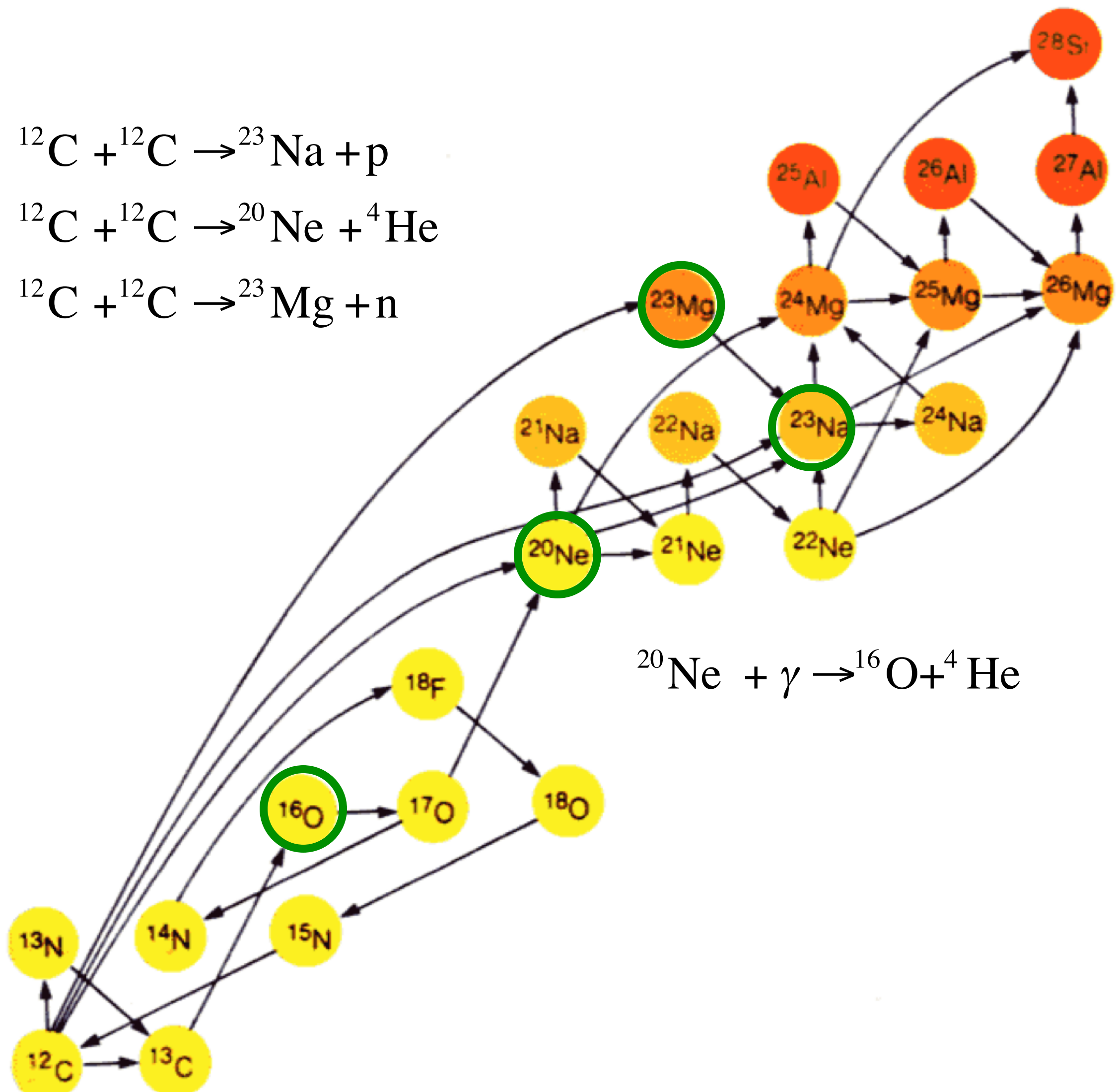
higher abundances of elements with even proton number

advanced burning stages: carbon, oxygen, neon

$$^{12}C + ^{12}C \rightarrow ^{23}Na + p$$

$$^{12}C + ^{12}C \rightarrow ^{20}Ne + ^{4}He$$

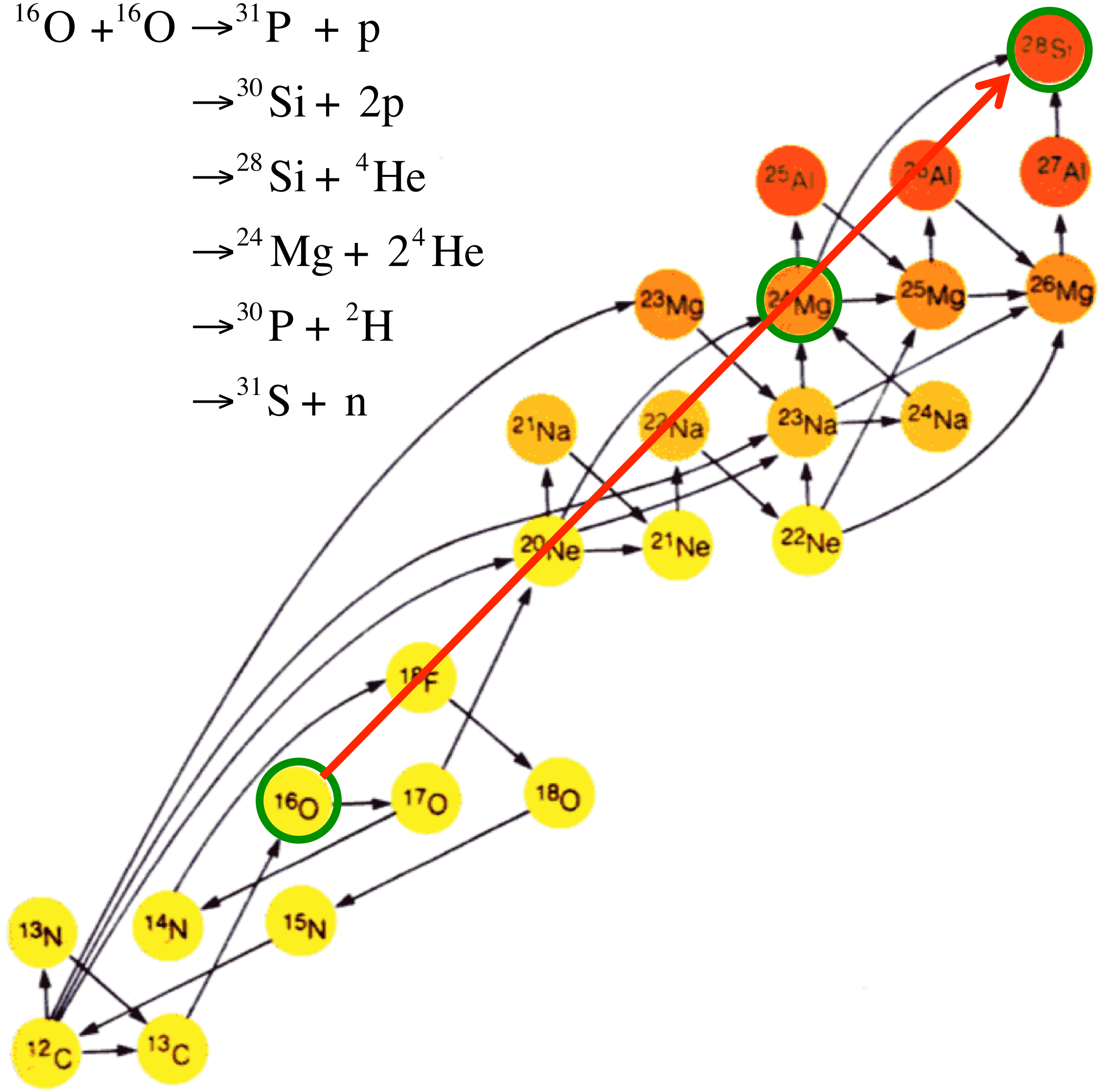
$$^{12}C + ^{12}C \rightarrow ^{23}Mg + n$$



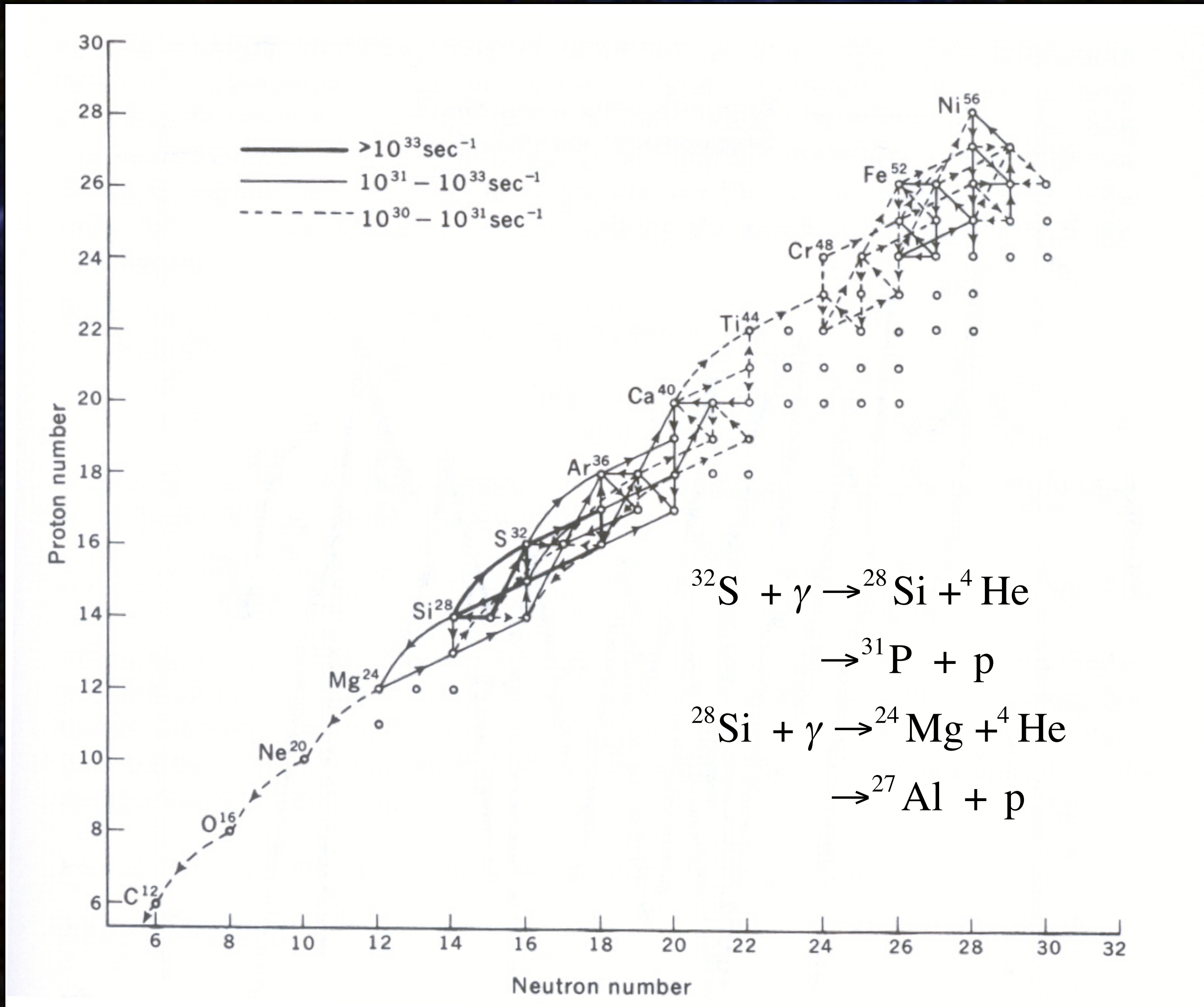
$$^{20}Ne + \gamma \rightarrow ^{16}O + ^{4}He$$

advanced burning stages: carbon, neon, oxygen

$$^{16}O + ^{16}O \rightarrow ^{31}P + p$$
$$\rightarrow ^{30}Si + 2p$$
$$\rightarrow ^{28}Si + ^{4}He$$
$$\rightarrow ^{24}Mg + 2\,^{4}He$$
$$\rightarrow ^{30}P + ^{2}H$$
$$\rightarrow ^{31}S + n$$



advanced burning stages: silicon



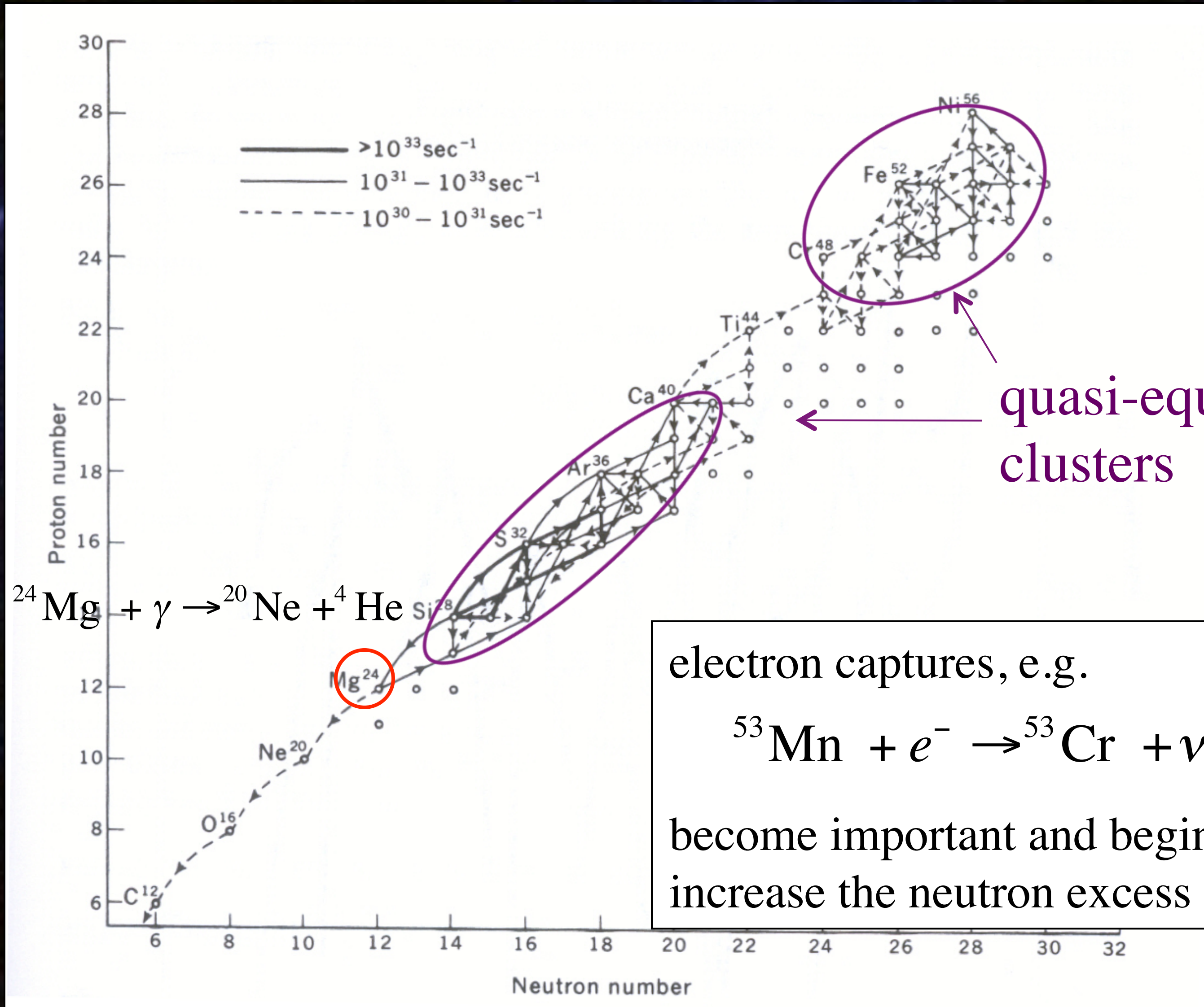
$$^{32}\mathrm{S} + \gamma \rightarrow {}^{28}\mathrm{Si} + {}^{4}\mathrm{He}$$

$$\rightarrow {}^{31}\mathrm{P} + p$$

$$^{28}\mathrm{Si} + \gamma \rightarrow {}^{24}\mathrm{Mg} + {}^{4}\mathrm{He}$$

$$\rightarrow {}^{27}\mathrm{Al} + p$$

advanced burning stages: silicon



$$^{24}\text{Mg} + \gamma \rightarrow {}^{20}\text{Ne} + {}^{4}\text{He}$$

quasi-equilibrium clusters

electron captures, e.g.

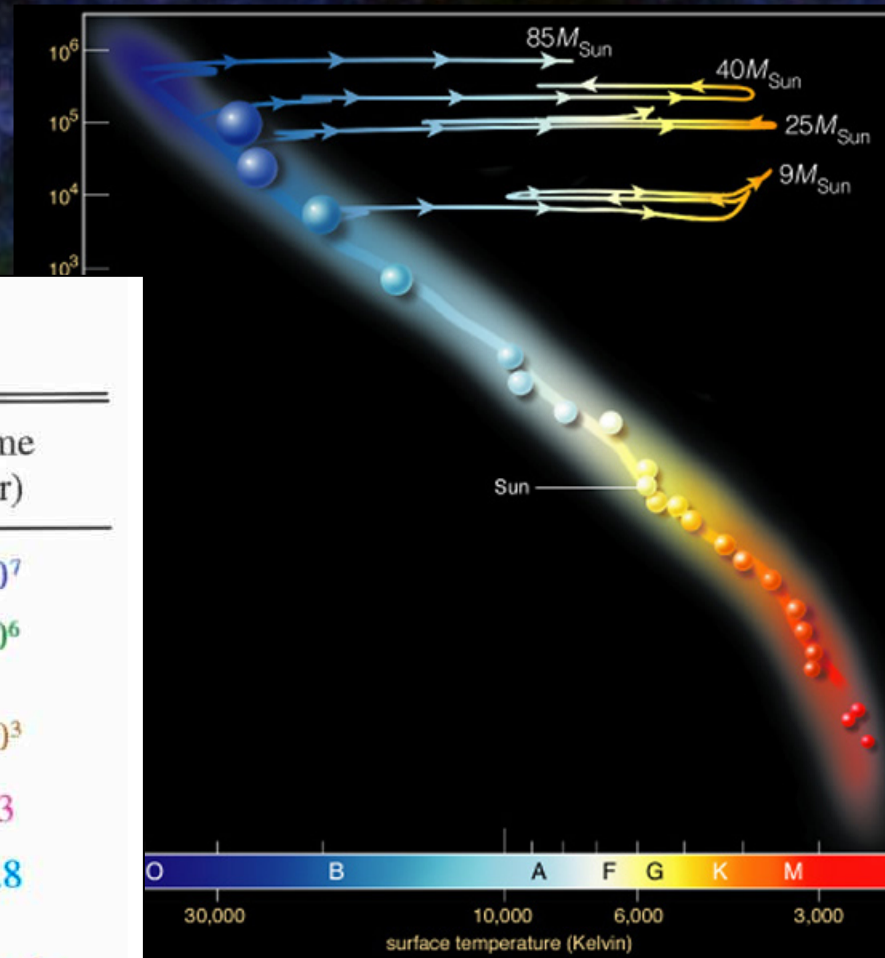
$$^{53}\text{Mn} + e^{-} \rightarrow {}^{53}\text{Cr} + \nu_e$$

become important and begin to increase the neutron excess

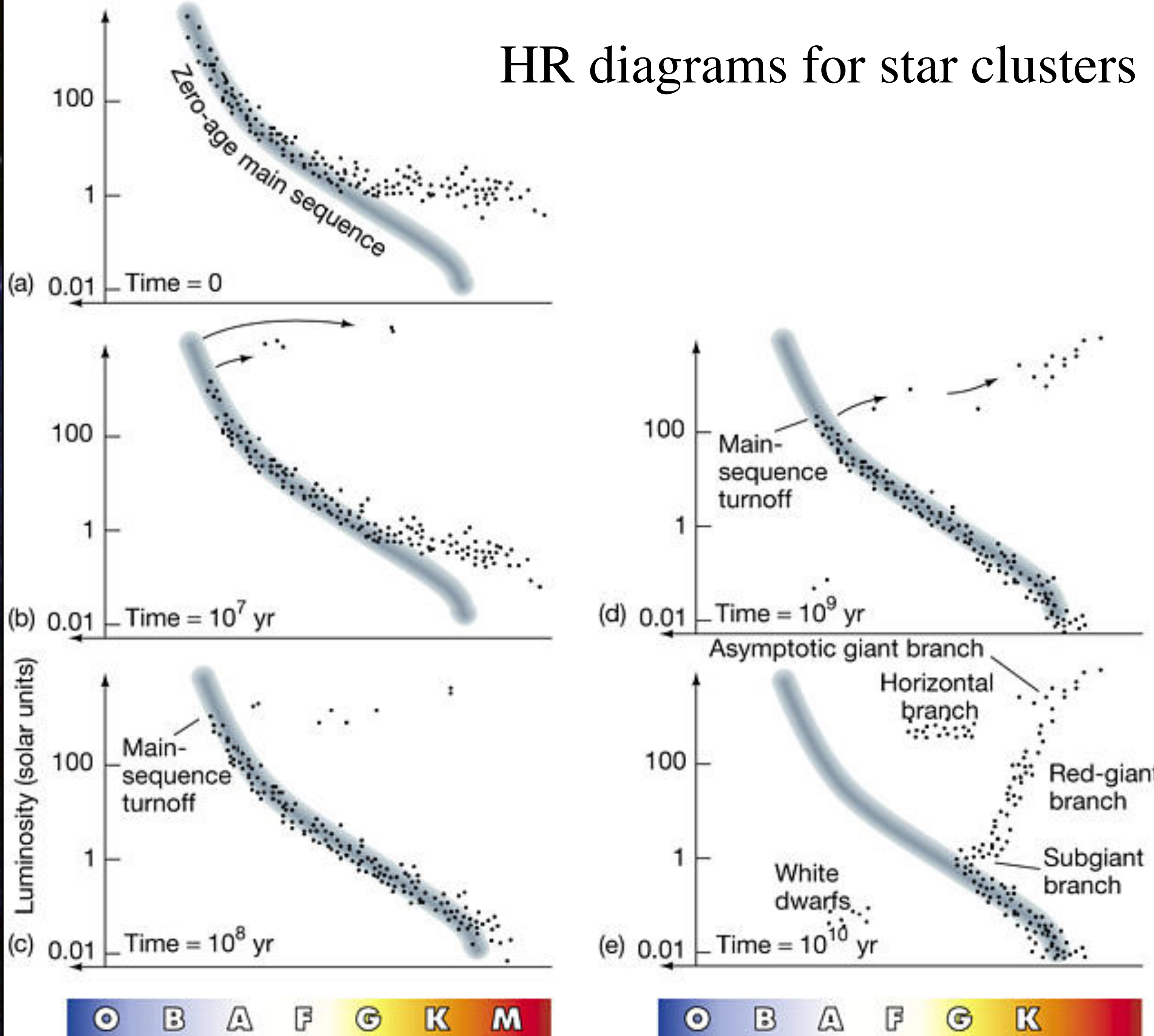
advanced burning stages: overview

Advanced Nuclear Burning Stages
(e.g., 20 solar masses)

Fuel	Main Product	Secondary Products	Temp (10^9 K)	Time (yr)
H	He	^{14}N	0.02	10^7
He	C,O	^{18}O, ^{22}Ne s- process	0.2	10^6
C	Ne, Mg	Na	0.8	10^3
Ne	O, Mg	Al, P	1.5	3
O	Si, S	Cl, Ar K, Ca	2.0	0.8
Si	Fe	Ti, V, Cr Mn, Co, Ni	3.5	1 week



HR diagrams for star clusters



Partners for the computational exercises

1	Beckmann Kristine	Jokiniemi Lotta
2	Debarsy Paul	Kostensalo Joel
3	Färber Michelle	Haverinen Tiia
4	Lorenz Christian	Vilen Markus
5	Oleksii Lukianchuk	Scafes Adela
6	Morlacchi Silvia	Calverley Tom
7	Piersa Monika	Salvioni Gianluca
8	Roth Anton	Susayev Yaroslav
9	Canete Laetitia	Franks Jordan
10	Hukkanen Marjut	Tali Maris
11	Geldof Sarina	Hilton Joshua