# PH3: TRANSITION PROBABILITIES AS A PROBE FOR NUCLEAR STRUCTURE LECTURE 1

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Jyväskylä Summer School 2017
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# Contents

- 14.-18.8.2017 at 14:15-16:00 (2 × 45 min, a short break in between) at the lecture theatre FYS 2
- 3 lectures (Mon, Tue & Wed)
- Exercise session on Thursday or Friday ( 2 groups, same time & place)
- Compulsory attendance for 1 ECTS credit

Please register JYU Moodle: https://moodle.jyu.fi/course/view.php?id=1343 using enrolment key: JSS2017_PH3

Note: *course material and exercises will be available only on Moodle*.

# Run-through of quantum mechanics

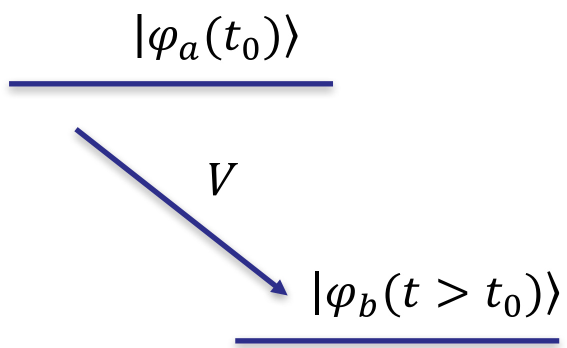
**Time-dependent perturbation theory:** what is the probability to find a system which is at the state $|\varphi_a(t)\rangle$ when $t = t_0$, at the state $|\varphi_b(t)\rangle$ when $t > t_0$ after interaction (perturbation) $V$?

We use the *Fermi's Golden Rule*:

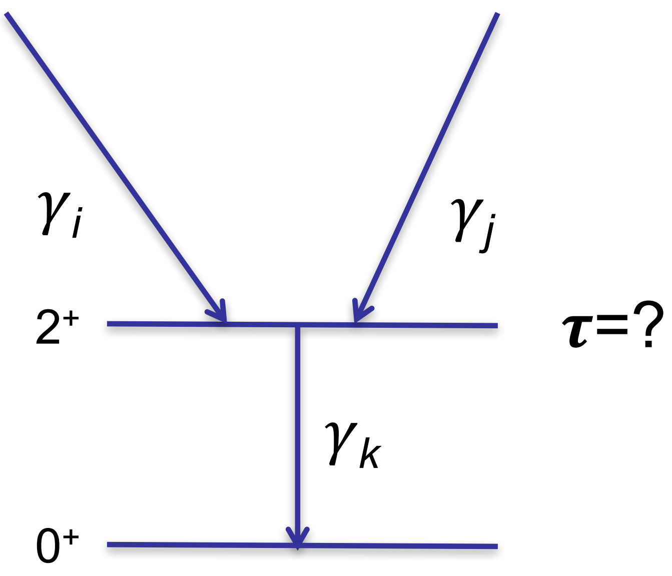
$$\lambda_{ab} = \frac{2\pi}{\hbar} \left|\langle\varphi_b|\hat{V}|\varphi_b\rangle\right|^2 \rho(E_b = E_a)$$

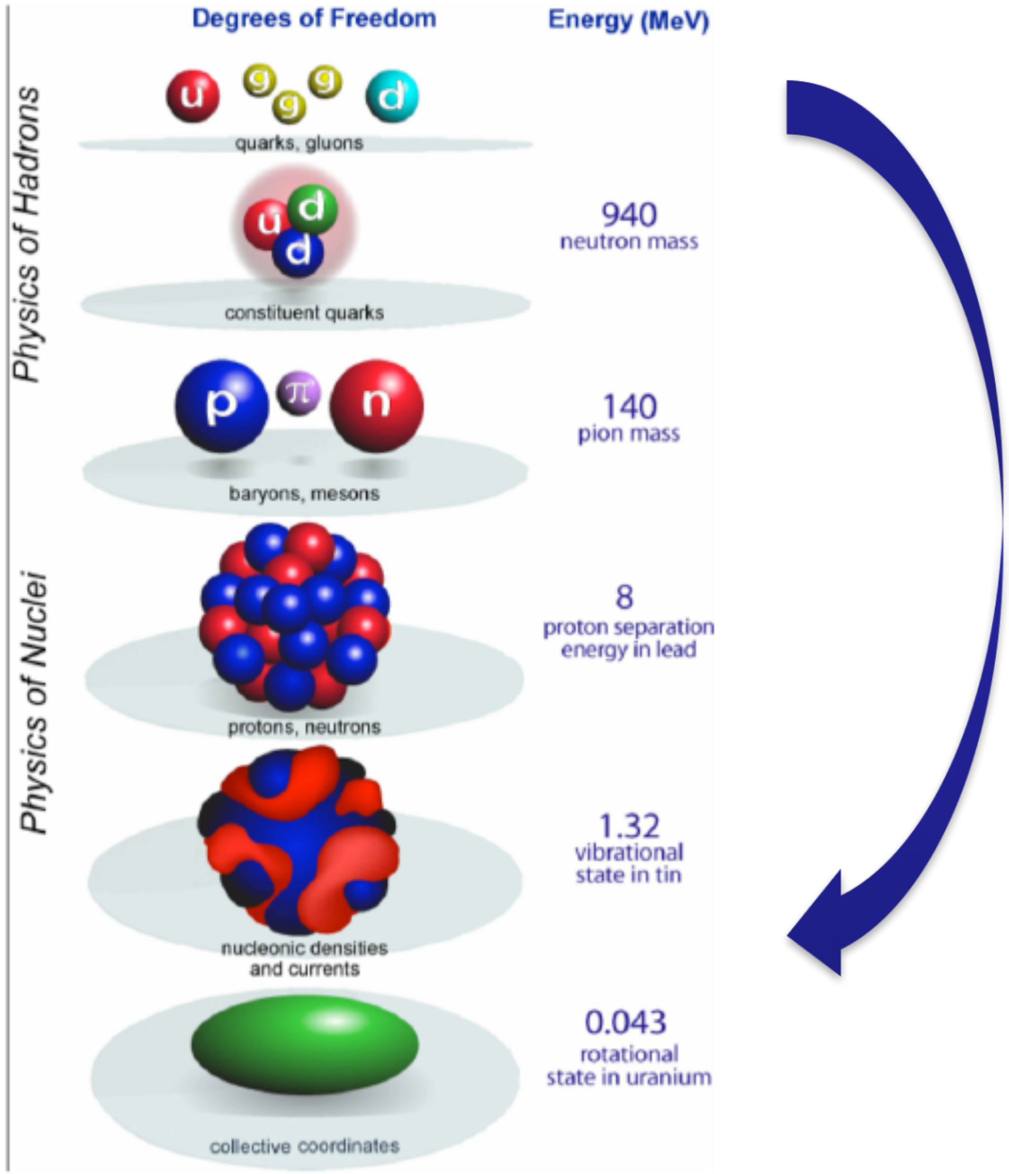
that gives the transition probability per unit time between the two states. Here $\rho(E_b = E_a)$ is the *density of states* and $\langle\varphi_b|\hat{V}|\varphi_b\rangle$ is the *matrix element*.

Note that $V$ can be for instance electromagnetic (EM) or weak interaction.



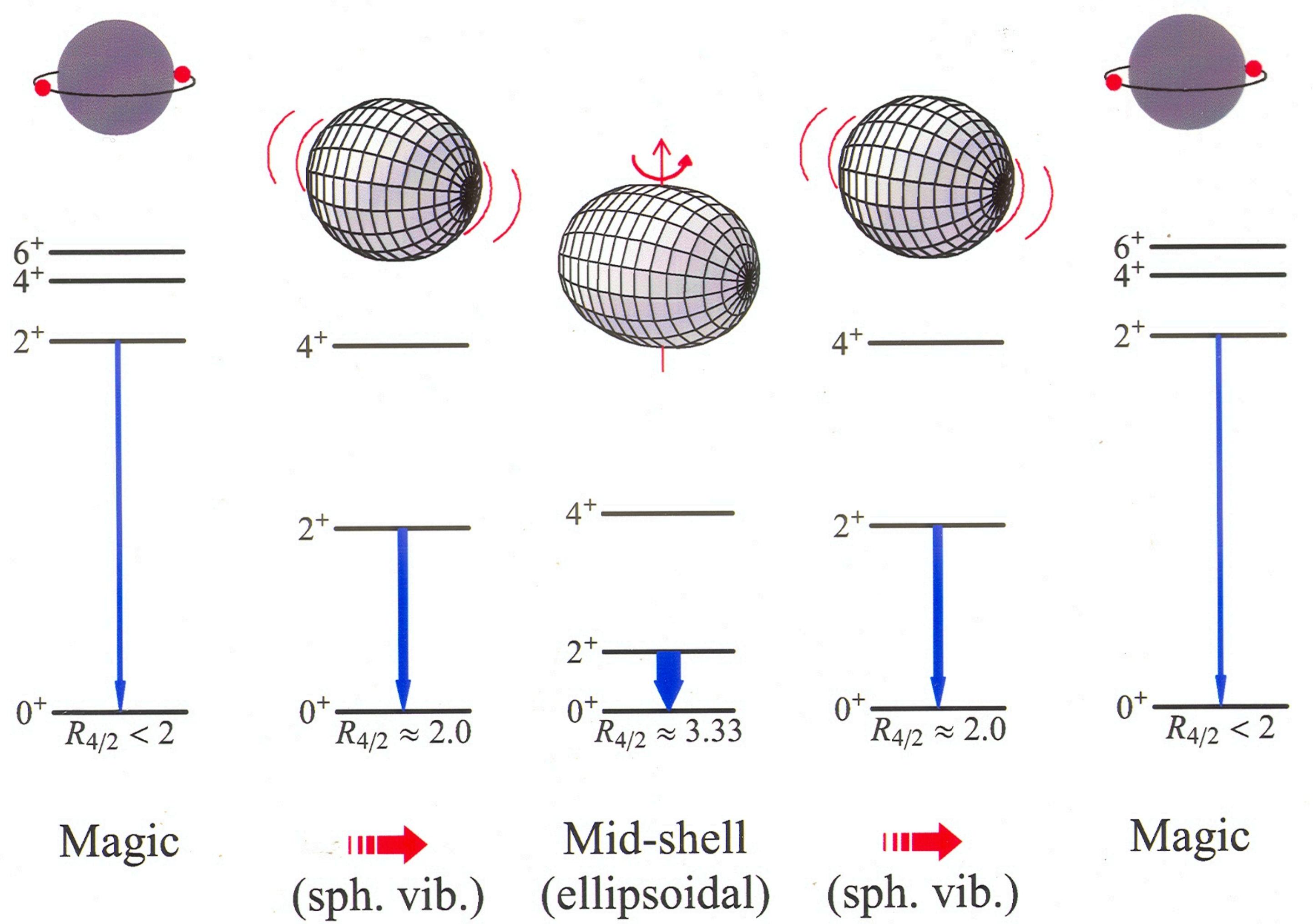
# Why to measure lifetimes of excited states in nuclei?



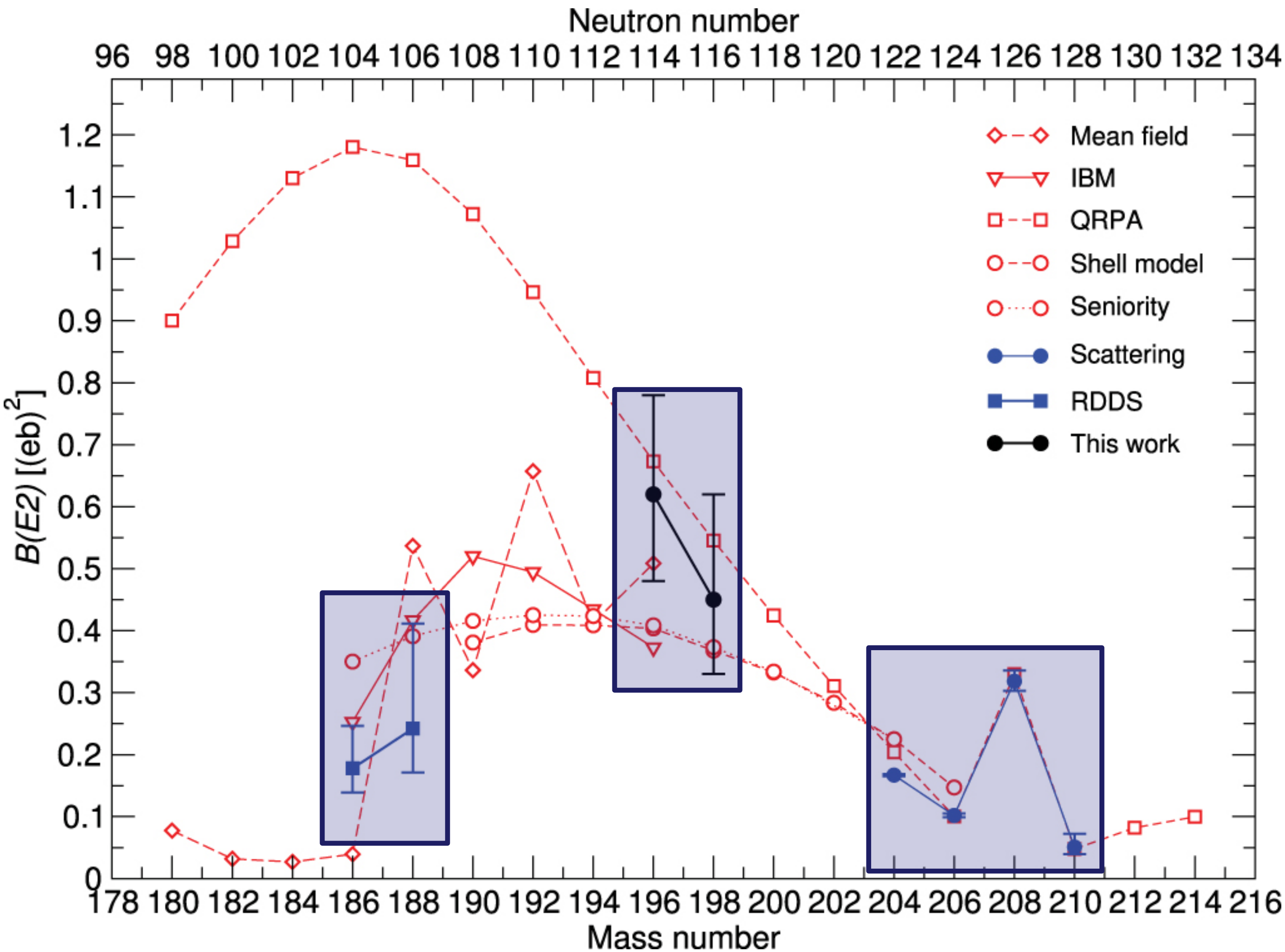



- Exact solution often impossible!
- Data are needed to model nuclei properly
- Are level energies sensitive enough…

# Evolution of nuclear structure (as a function of nucleon number)

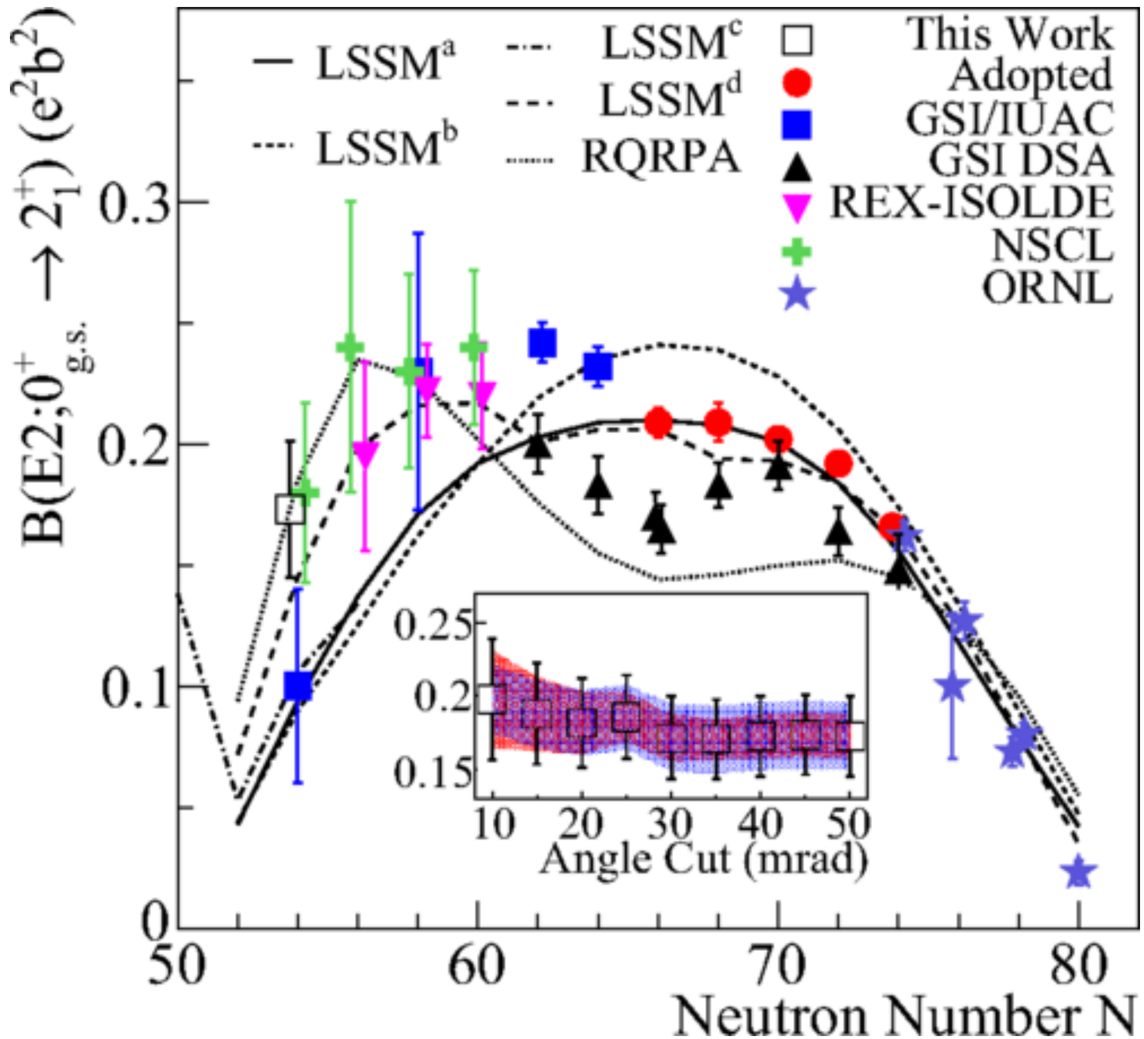


# Neutron-deficient *Z*=82 nuclei

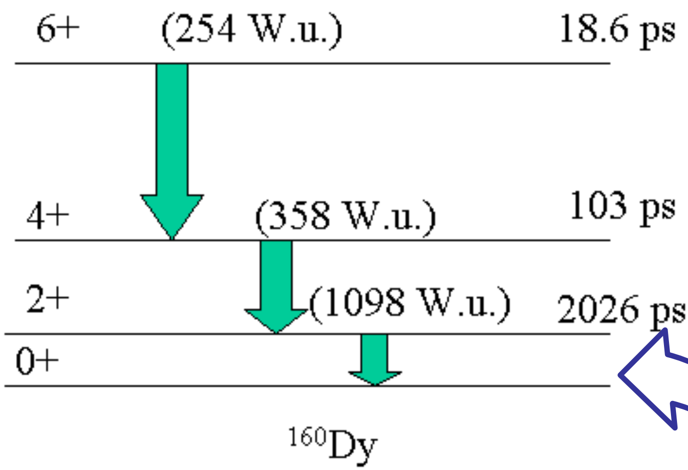


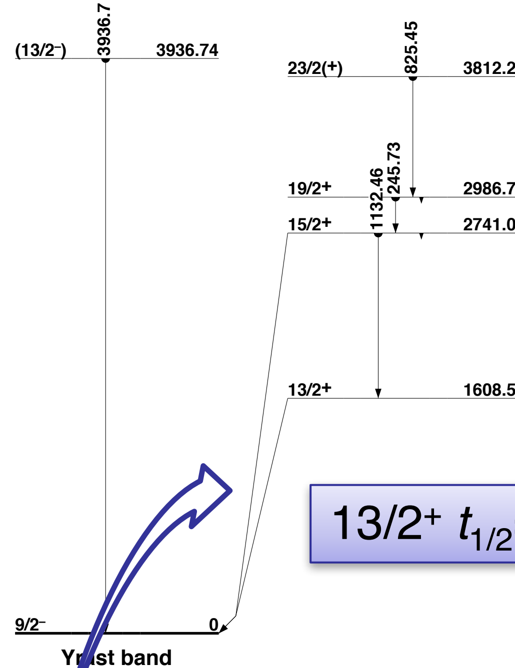

From: J. Pakarinen, T. Grahn et al., J. Phys. G: Nucl. Part. Phys. 44, 064009 (2017)

# Collectivity in *Z*=50 nuclei




From: P. Doornenbal et al., Phys. Rev. C 90, 061302(R) (2014)




$13/2^+ \ t_{1/2}$=0.27 ns

$E_\gamma(2^+ \rightarrow 0^+)$ = 86.8 keV

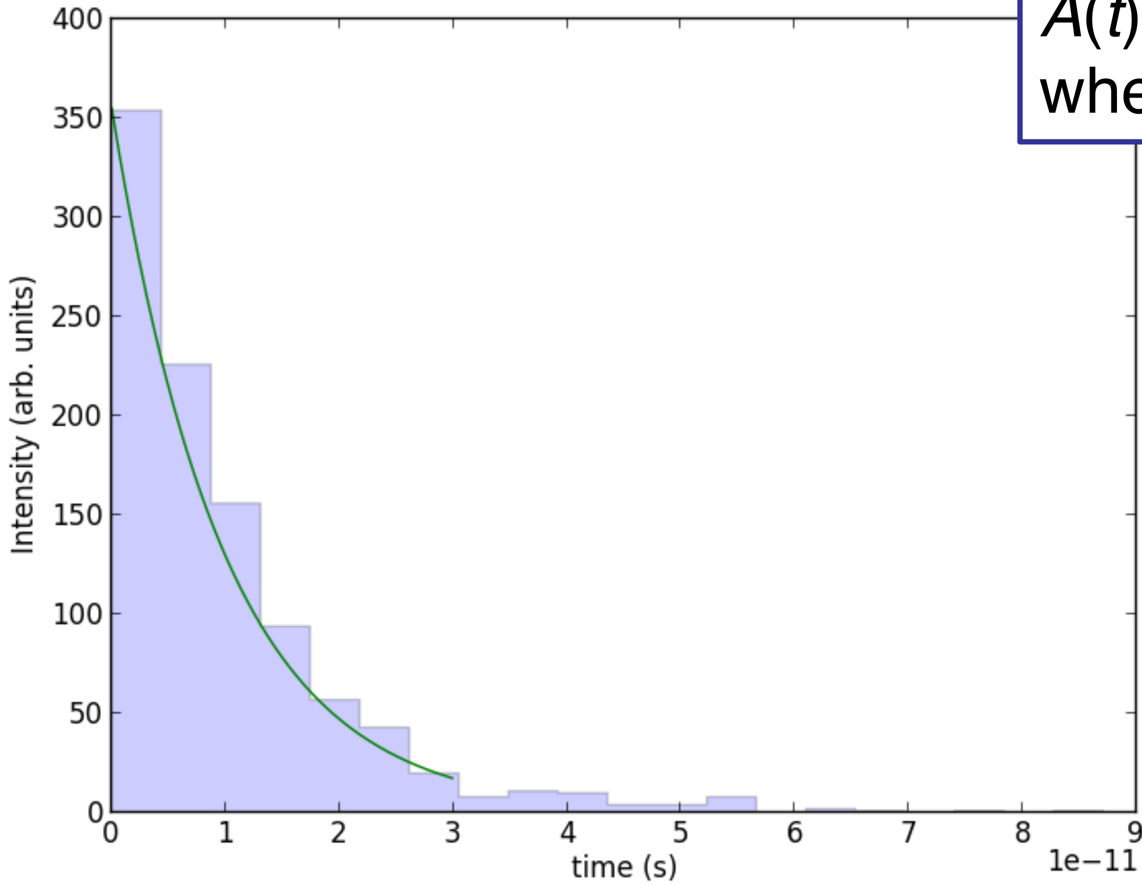
$^{209}_{83}$Bi

Very collective transition

M2/E3 mixture, single-particle transition

## Radioactive decay

$N(t)=N_0 e^{-\lambda t}$ or
$A(t)=A_0 e^{-\lambda t}$,
where $\lambda=1/\tau=\ln 2/t_{1/2}$



# Nuclear structure vs. lifetime

Transition probability for EM transitions, following from the Fermi's Golden Rule:

$$\lambda(L) = \frac{1}{\tau} = \frac{8\pi(L+1)}{\hbar L[(2L+1)!!]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} B(\mathcal{M}L; I_i \rightarrow I_f),$$

$$B(\mathcal{M}L; I_i \rightarrow I_f) = \frac{1}{2I_i+1}|\langle I_f||\hat{m}L||I_i\rangle|^2$$

Reduced transition probability $B(\mathcal{M}L)$ can be extracted from a mean lifetime $\tau$.The $B(\mathcal{M}L)$ is a model independent observable. As electromagnetic operator is well understood, wave functions can be–in principle–extracted.

$$t_{1/2} \propto \frac{1}{E_\gamma^{(2L+1)}}$$

Note the strong energy dependence!

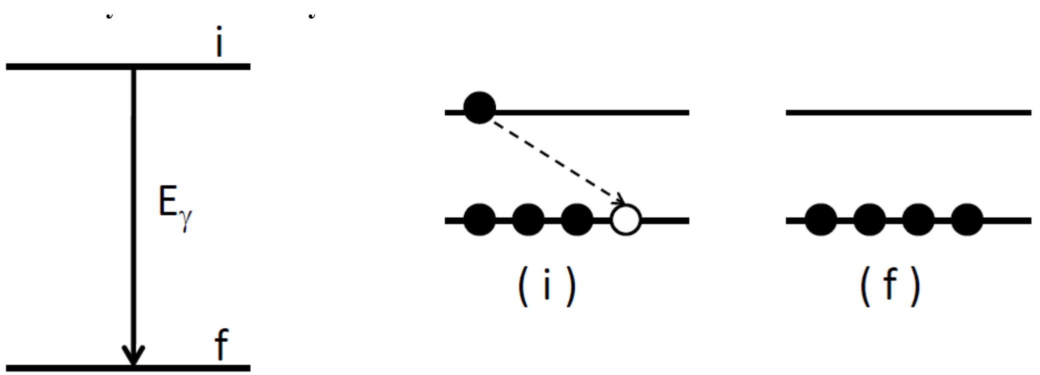
Note: throughout the lectures *lifetime* is the <u>mean lifetime</u> $\boldsymbol{\tau}=t_{1/2}/\ln 2$

# Calculation of Transition Rates

$$\lambda(L) = \frac{1}{\tau} = \frac{8\pi(L+1)}{\hbar L[(2L+1)!!]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} B(\mathcal{M}L; I_i \to I_f),$$

$$B(\mathcal{M}L; I_i \to I_f) = \frac{1}{2I_i+1}|\langle I_f||\hat{m}L||I_i\rangle|^2$$

- Calculation of matrix element requires wavefunctions of initial and final state – complicated
- Make a simplification – assume transition due to single proton moving between two shell-model states:



# Calculation of Transition Rates

- Approximation made by Victor Weisskopf:

$$B_{sp}(EL) = \frac{e^2}{4\pi}\left(\frac{3R^L}{L+3}\right)^2$$

$$B_{sp}(ML) = 10\left(\frac{\hbar}{m_p cR}\right)^2 B_{sp}(EL)$$

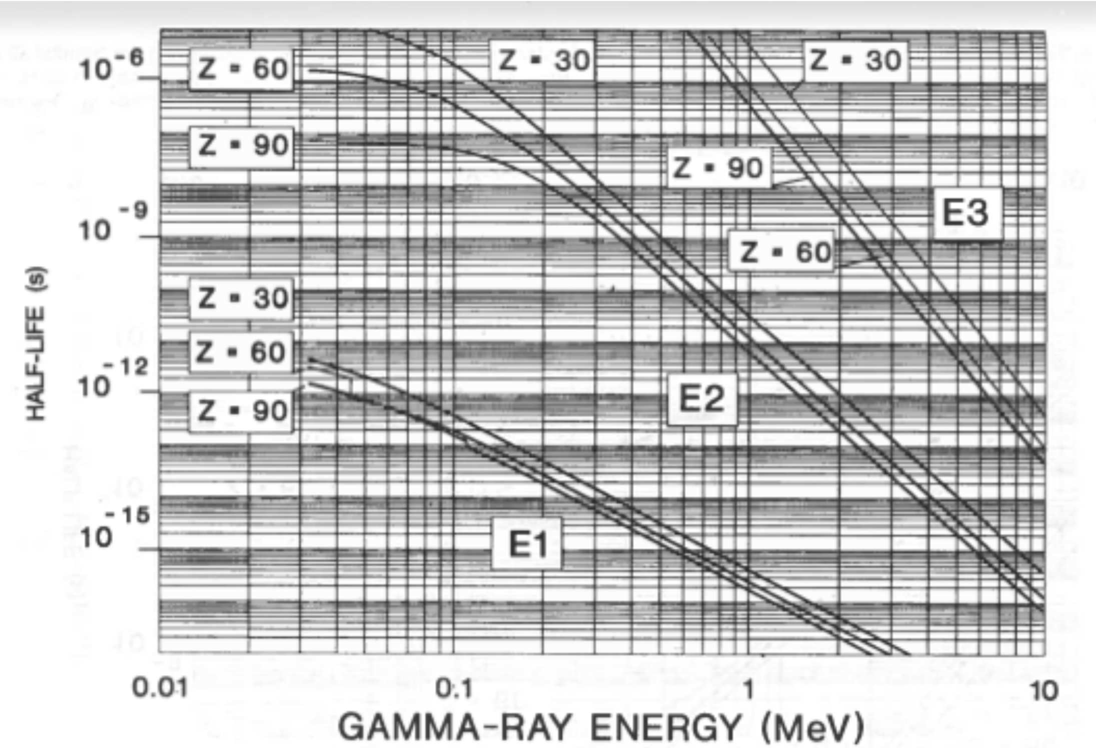
- $R$ - nuclear radius ($R=R_0A^{1/3}$), $m_p$ - mass of proton
- Get following expressions:

$$B_{sp}(EL) = \frac{e^2}{4\pi}\left(\frac{3}{L+3}\right)^2 (R_0)^{2L} A^{2L/3}$$

$$B_{sp}(ML) = \frac{10}{\pi}\left(\frac{e\hbar}{2m_p c}\right)^2 \left(\frac{3}{L+3}\right)^2 (R_0)^{2L-2} A^{(2L-2)/3}$$

# Weisskopf estimates for the EM transition rates

| Multipole | $E$ | $M$ |
|---|---|---|
| $l$ | $\lambda\,(s^{-1})$ | $\lambda\,(s^{-1})$ |
| 1 | $1.03 \times 10^{14}\ A^{2/3}E_\gamma^3$ | $3.15 \times 10^{13}\ E_\gamma^3$ |
| 2 | $7.28 \times 10^{7}\ A^{4/3}E_\gamma^5$ | $2.24 \times 10^{7}\ A^{4/3}E_\gamma^5$ |
| 3 | $3.39 \times 10^{1}\ A^{2}E_\gamma^7$ | $1.04 \times 10^{1}\ A^{4/3}E_\gamma^7$ |
| 4 | $1.07 \times 10^{-5}\ A^{8/3}E_\gamma^9$ | $3.27 \times 10^{-6}\ A^{2}E_\gamma^9$ |
| 5 | $2.40 \times 10^{-12}\ A^{10/3}E_\gamma^{11}$ | $7.36 \times 10^{-13}\ A^{8/3}E_\gamma^{11}$ |



If a nucleus can be assumed to be a rotating and axially symmetric in shape, then

$$B(E2; KI_i \rightarrow KI_f) = \frac{5}{16\pi} e^2 Q_t^2 \langle I_f K 2 0 | I_i K \rangle^2,$$

where $Q_t$ is the *transitional* quadrupole moment.
(see eg. Bohr & Mottelsson, Nuclear Structure, Vol. 2, p. 45)

Electric dipole *E*2 is the most common EM mode, so therefore we discuss more on that.
The $B(E2)$ value is usually expressed in $e^2$b$^2$ or in Weisskopf units (W.u.).
Typical $B(E2)$ values:
~1 W.u. single-particle transition
~10-100 W.u. collective vibrational transition
~100-few hundred collective rotation

Level mean lifetime $\tau$ is directly proportional to the nuclear structure by means of EM operator.

# `Shape´ of the nucleus–collective transitions

For a well-behaving rotor *intrinsic* quadrupole moment $Q_0 \approx Q_t$

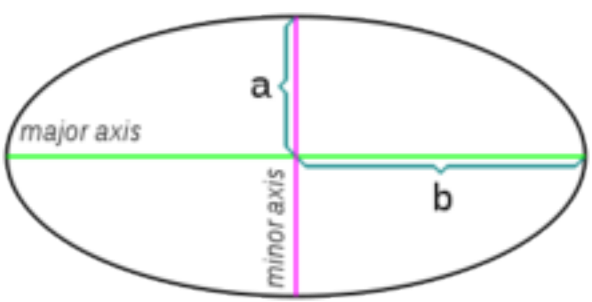
$$Q_0 = \sqrt{\frac{16\pi}{5}} \langle IK|E2|IK \rangle.$$

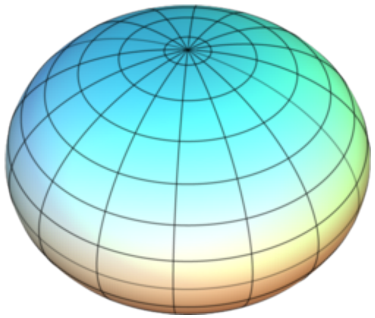
`Shape´ of an axially deformed nucleus can be described with a parameter $\beta_2$

$$\beta_2 = \frac{4}{3}\sqrt{\frac{\pi}{5}}\frac{\Delta R}{R_0 A^{1/3}},$$

where $\Delta R$ is the difference between semi-major and semi-minor axes of an ellips
Uniform charge distribution:

$$Q_0 = \frac{3}{\sqrt{5\pi}} Z R^2 \beta_2 (1 + 0.16\beta_2) e$$



# Semi-empirical formula, `Grodzins rule´

For the $2^+ \rightarrow 0^+$ $E2$ transition from the first $2^+$ state to the ground state:

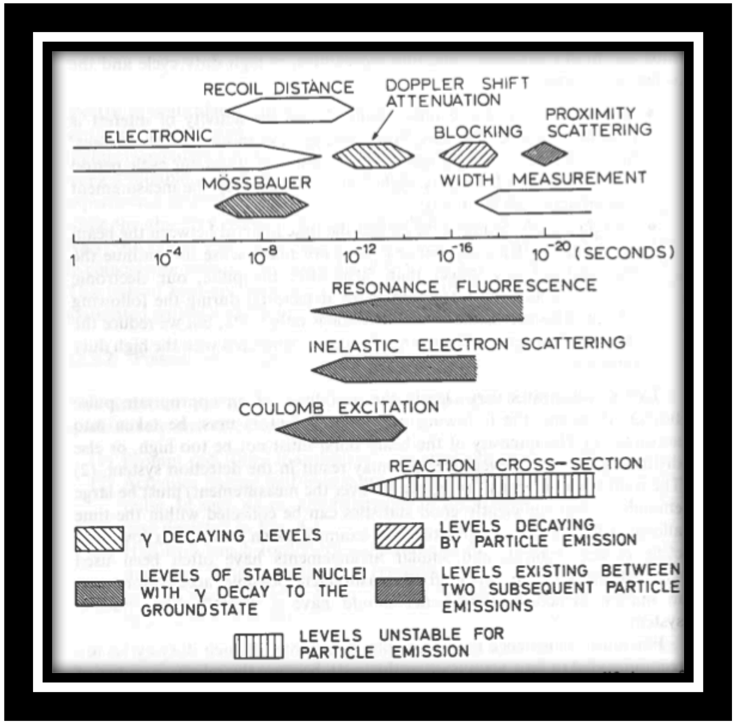
$$\lambda_\gamma(2^+ \rightarrow 0^+) = (3 \pm 1)10^{10}[E(2^+)]^4 Z^2 A^{-1}$$

(L. Grodzins, Physics Letters, Vol. 2, p. 88, 1962)
The 2+ energies vary from few tens of keV to few MeV.
Consequently, lifetimes of these states vary even more (several orders of magnitude)!

$\Rightarrow$Several methods required

More to read:
Kantele: Handbook of Nuclear Spectrometry,
PJ Nolan and JF Sharpey-Schafer, Rep. Prog. Phys. 42, 1 (1979)

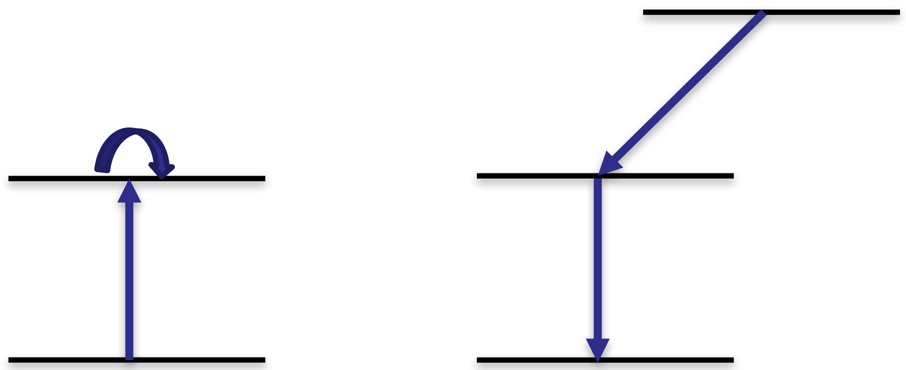
**Isomeric state**:
a `metastable´ with a lifetime of ~1 ns or longer



During this course, we will study the basics of three methods to measure transition rates:

1) **Direct (electronic) measurement, i.e. the fast-timing method**
2) **Recoil Distance Doppler-shift method**
3) **Coulomb excitation**

After the course one will understand the foundations of these methods and their use with stable and radioactive beams.



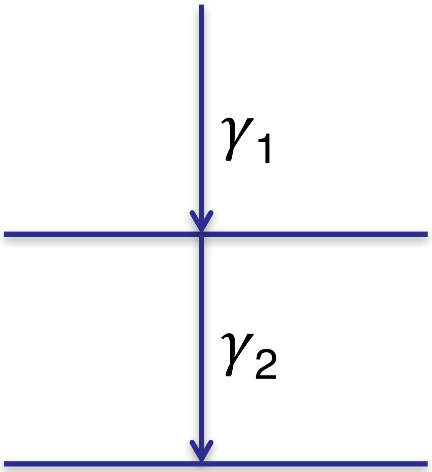
# 1. Direct (electronic) method (fast timing)

A simplified system:

- Two $\gamma$ rays populate and depopulate a state with a mean lifetime $\tau$
- Measure the time between the $\gamma$ rays

STEPS:

- Detector signal: time pick-off in fast timing: constant fraction discriminator CFD
- TAC: amplitude is proportional to a difference between \`start´ and \`stop´




$\tau = t_{1/2}/\ln 2$

# CFD (constant fraction discriminator):

- Timing signal is produced (triggered) independently of the pulse amplitude
- Optimum time resolution over wide range of pulse height

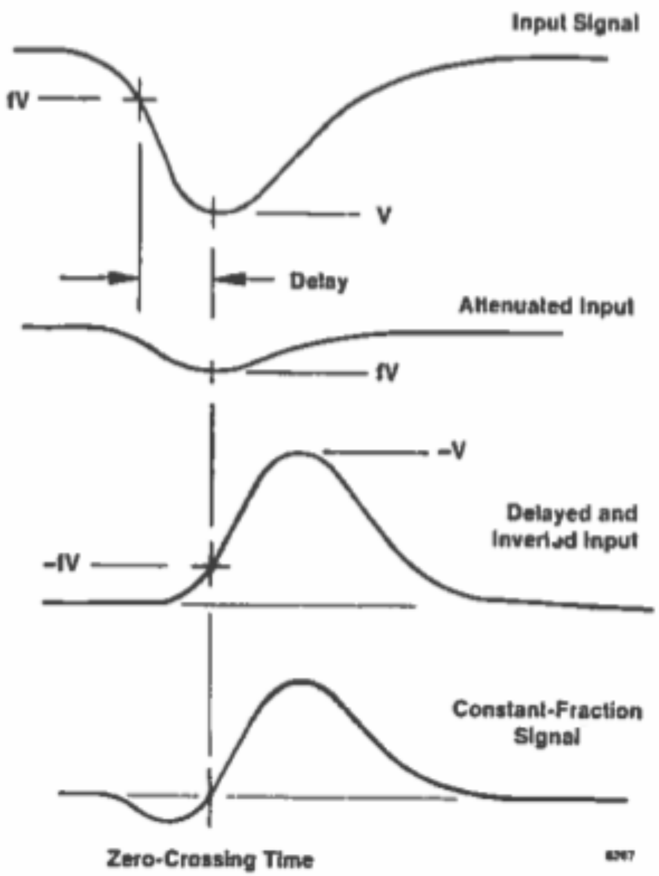
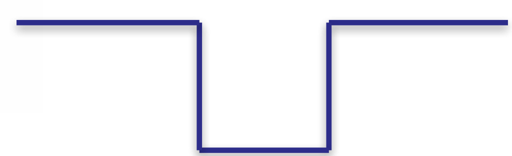


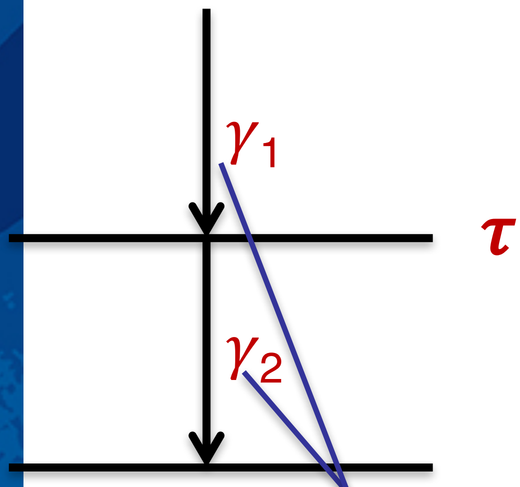

Fig. 3. Formation of the Constant-Fraction Signal.

Input: amplifier pulse


Output
(triggered at zero-crossing time):
logic pulse

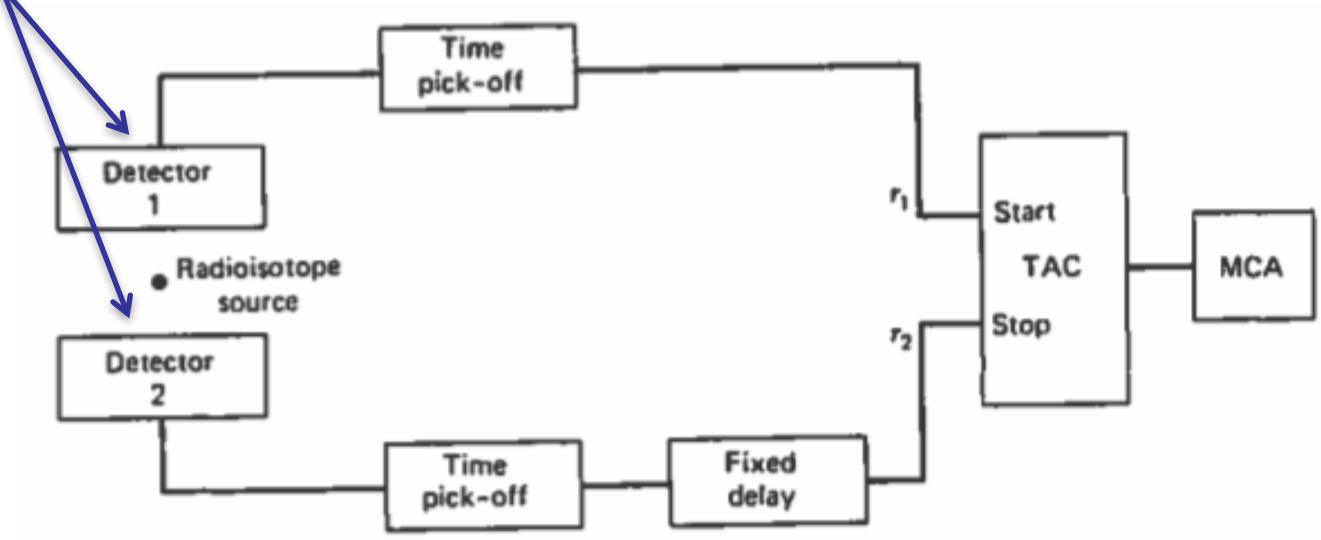


# Fast timing with two detectors




Measure the time difference between emissions of the two consecutive $\gamma$-rays.

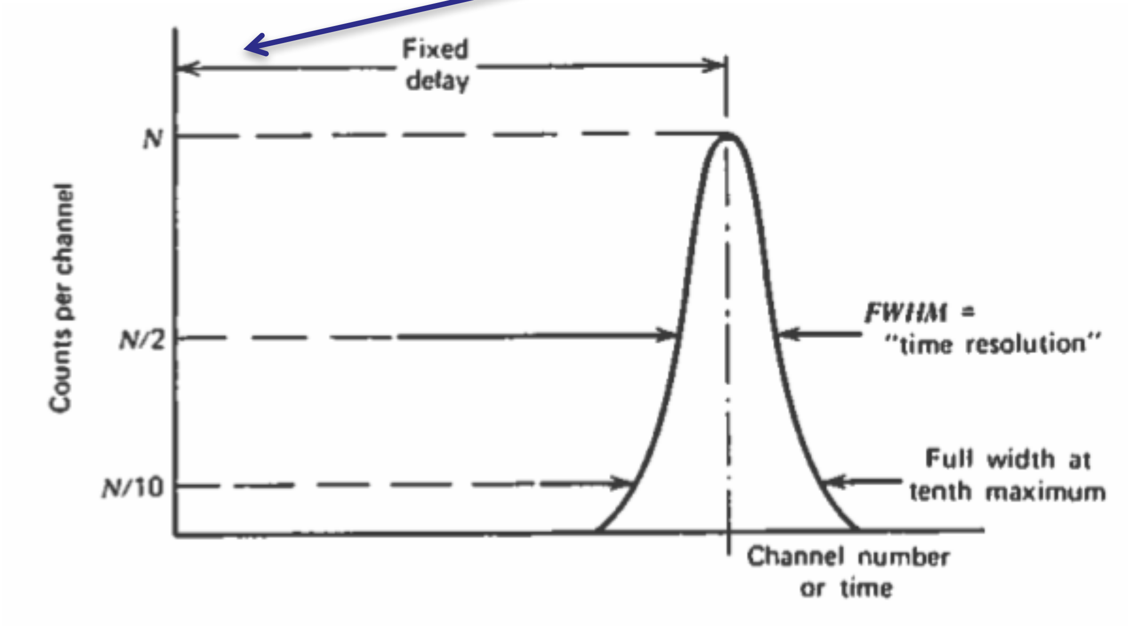
- Fast detectors–scintillators e.g. $LaBr_3$:Ce
- Start-stop \`clock´: TAC (time-to-amplitude converter), or TDC (time-to-digital converter)



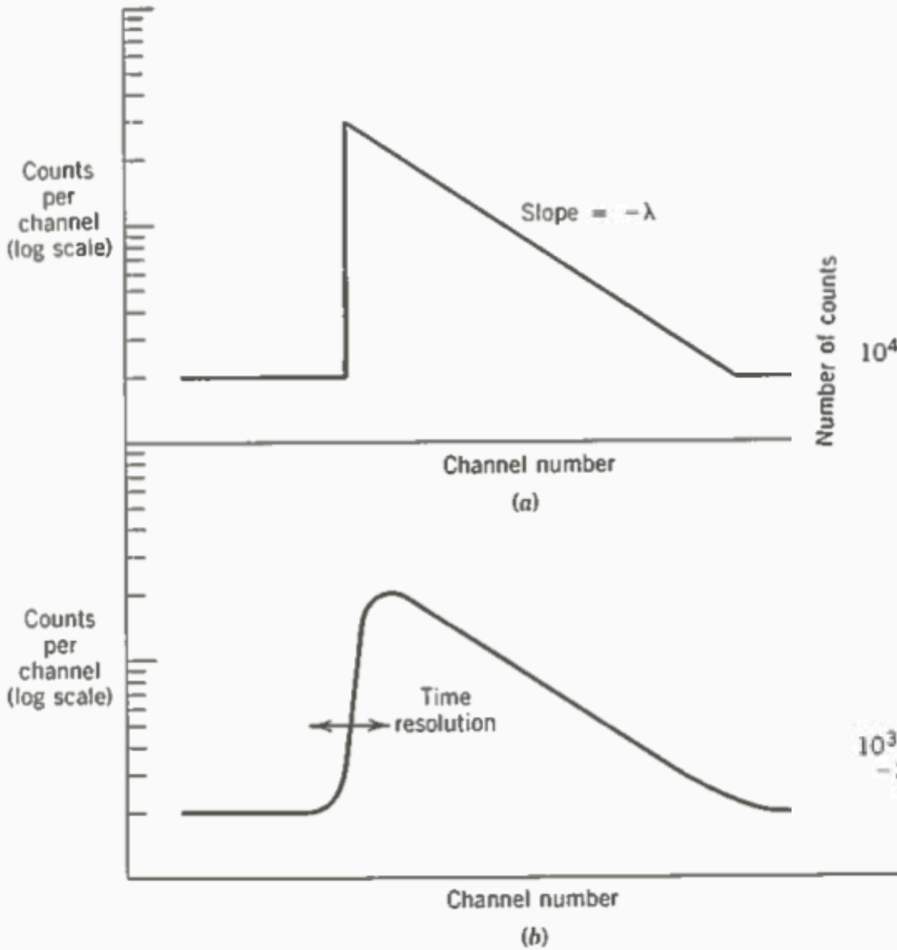
# TAC (time-to-amplitude converter)

**Input**: `star ´ and `stop´ logic signals from the CFDs
**Output**: a signal whose amplitude is proportional to the time difference of the input and output signals

Spectrum consists of differential number of intervals (d*N*/d*t*, y-axis) whose length lies within d*t*, which is the width of a channel in a data acquisition system (DAQ).



# TAC spectrum



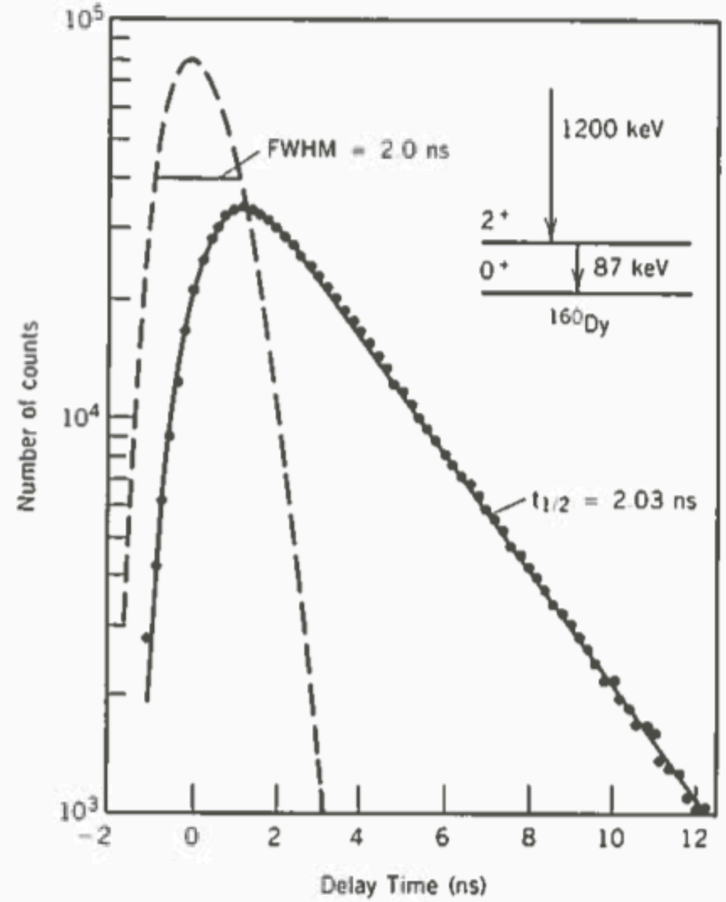
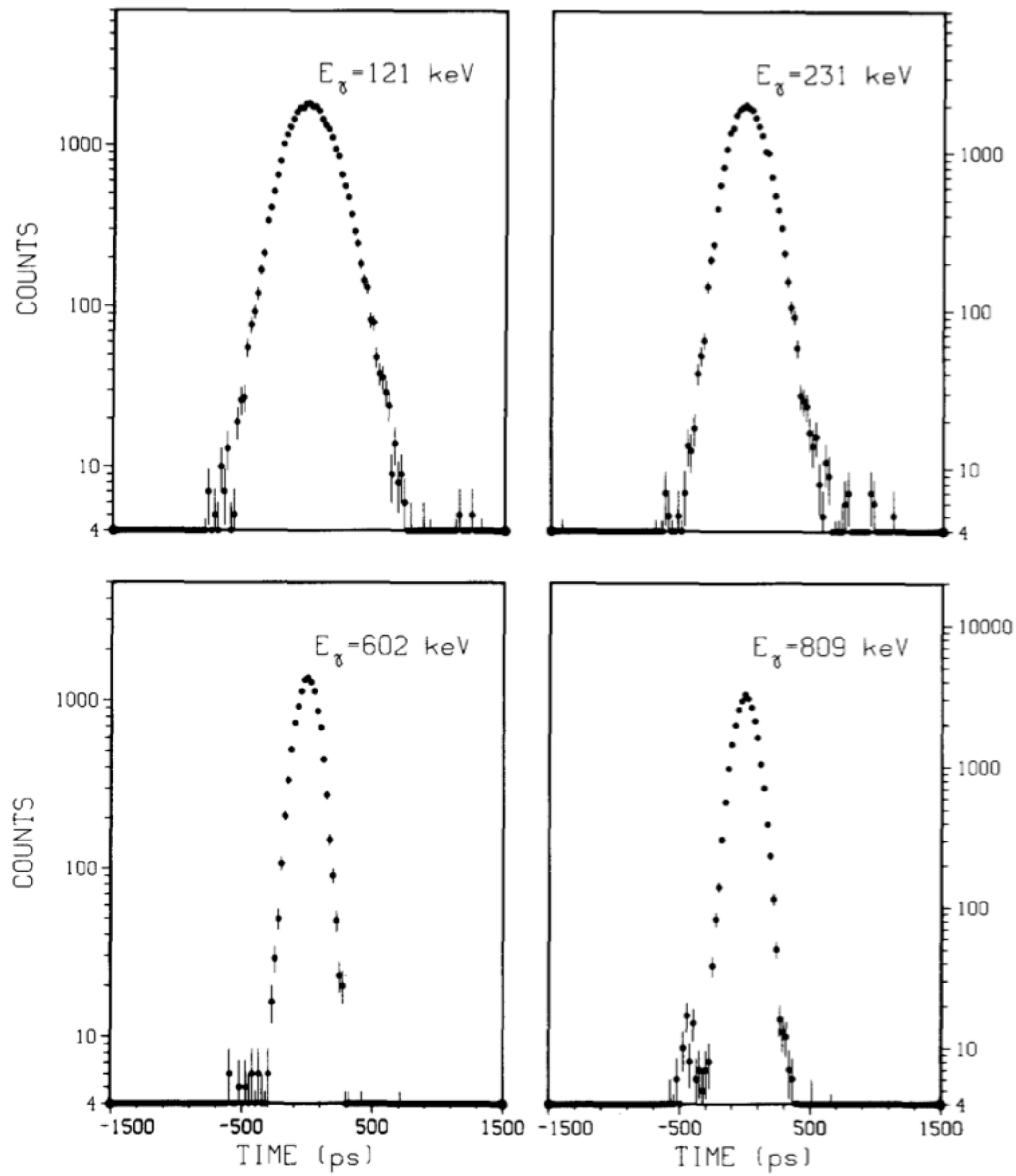




**Figure 7.32** If the nuclear state between the first and second radiations has a half-life that is *not* negligibly short compared with the time resolution, we can observe its exponential decay. These TAC spectra should be compared with those of Figure 7.30 to see the effect of the decay of the state. (*a*) Ideal detectors, (*b*) real detectors.

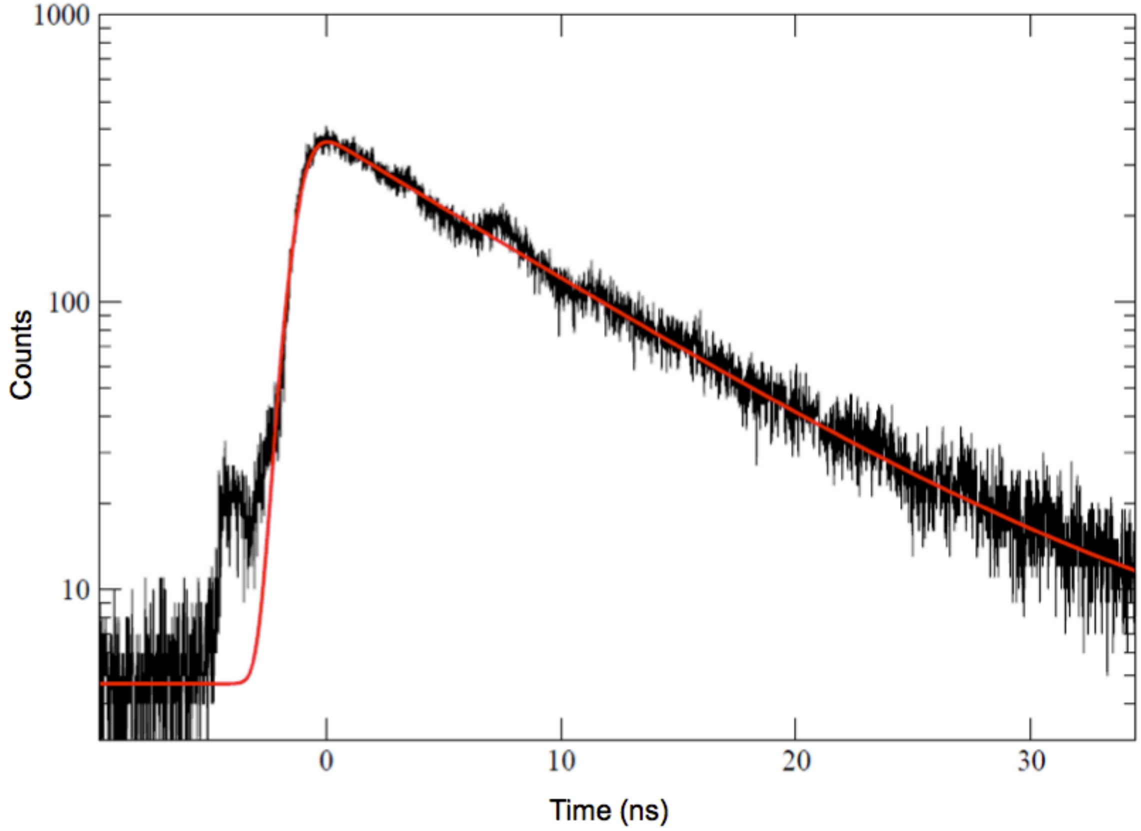
# TAC spectrum

Prompt response function (PRF):
Energy dependent, determined
usually with `prompt´ calibration
source and represent `time zero´ in a sense that it is the detection limit.



# TAC spectrum

## Half-life of $^{5}/_{2}{}^{+}$ state in $^{133}$Cs

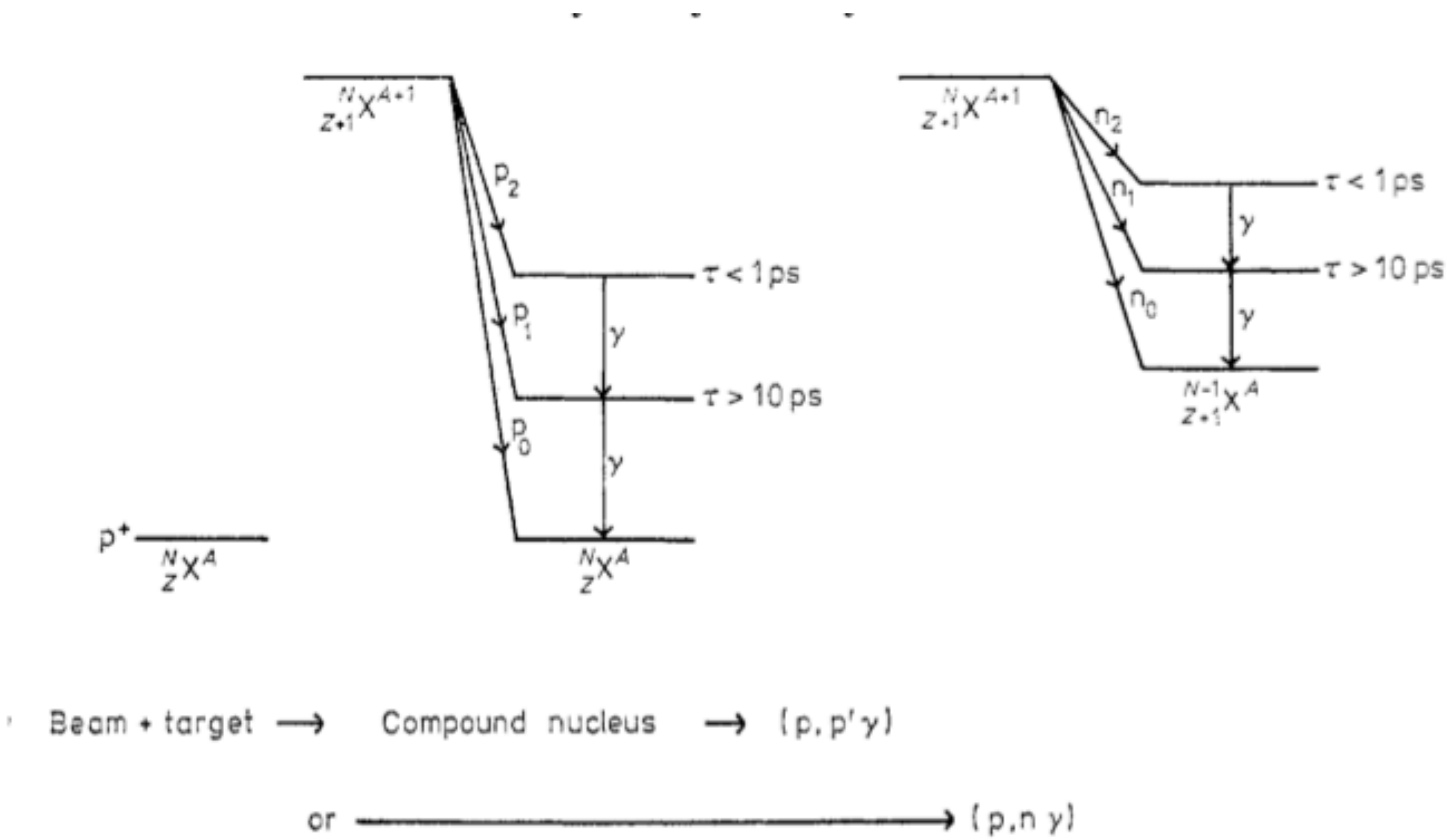



Measured Half-life
5.994(93) *ns*

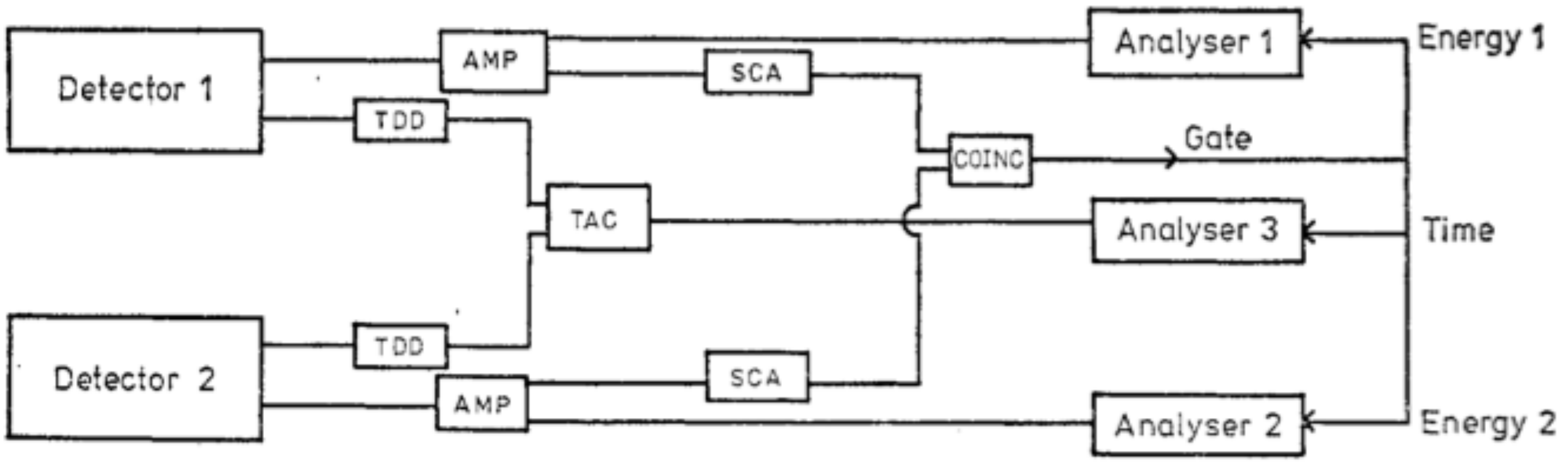
Half-life (NNDC)
6.283(14) *ns*

# Fast timing

Often (in reality) the situation is more complex:
Several $\gamma$ rays in the spectrum
$\Rightarrow$energy gates and coincidences required to clean the spectrum

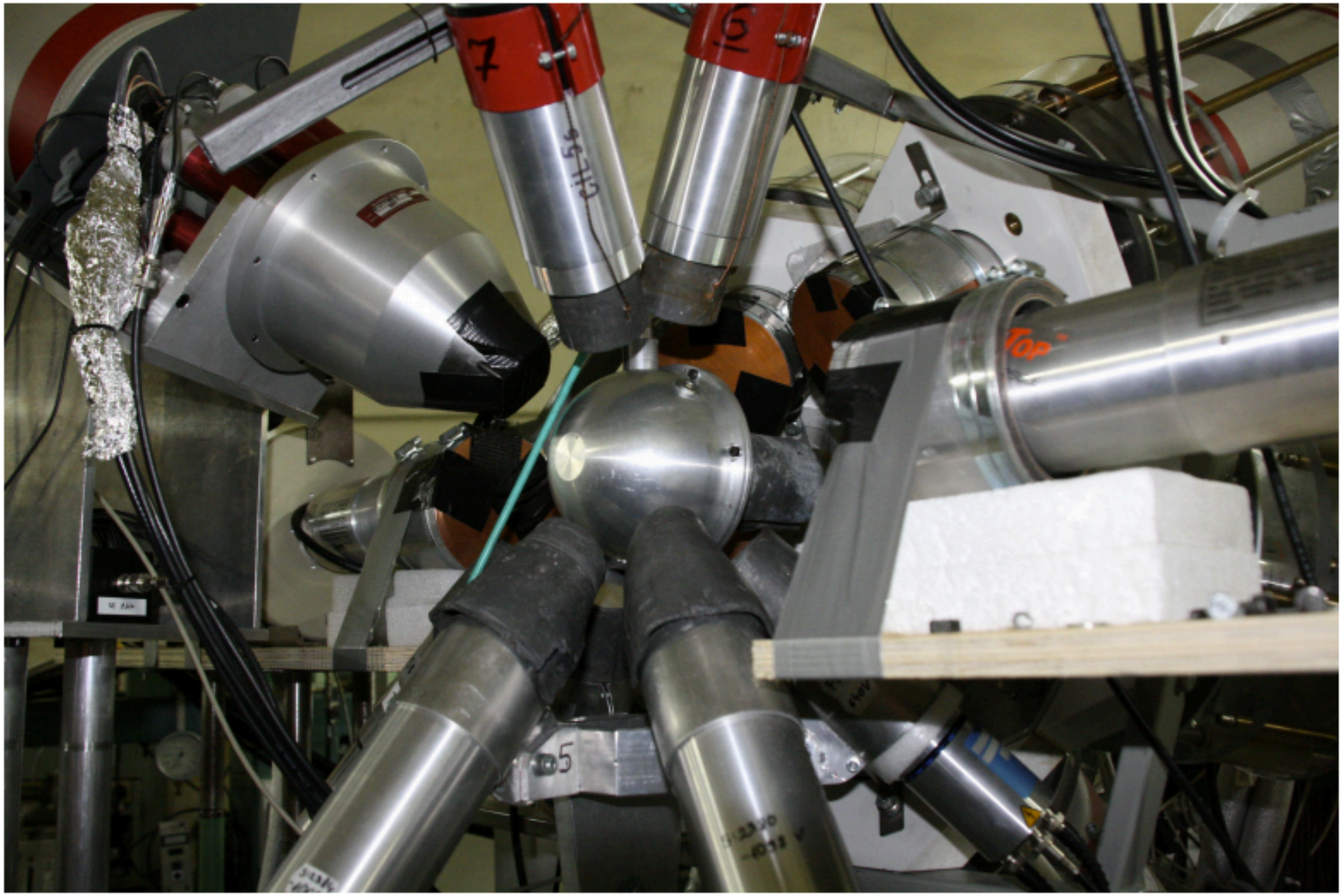


Record both energy and time information from the detector.
Energy gates (here SCA, single channel analyser) and their coincidence condition (coinc, coincidence unit)
can be used to trigger the DAQ.




Modern systems aim to use digitizers, that sample the detector signal into the digital form. However, the required clock frequency is still on the horizon.

# Latest advances in fast-timing measurements




N. Mărginean et al. European Physical Journal A 46,329 (2010)

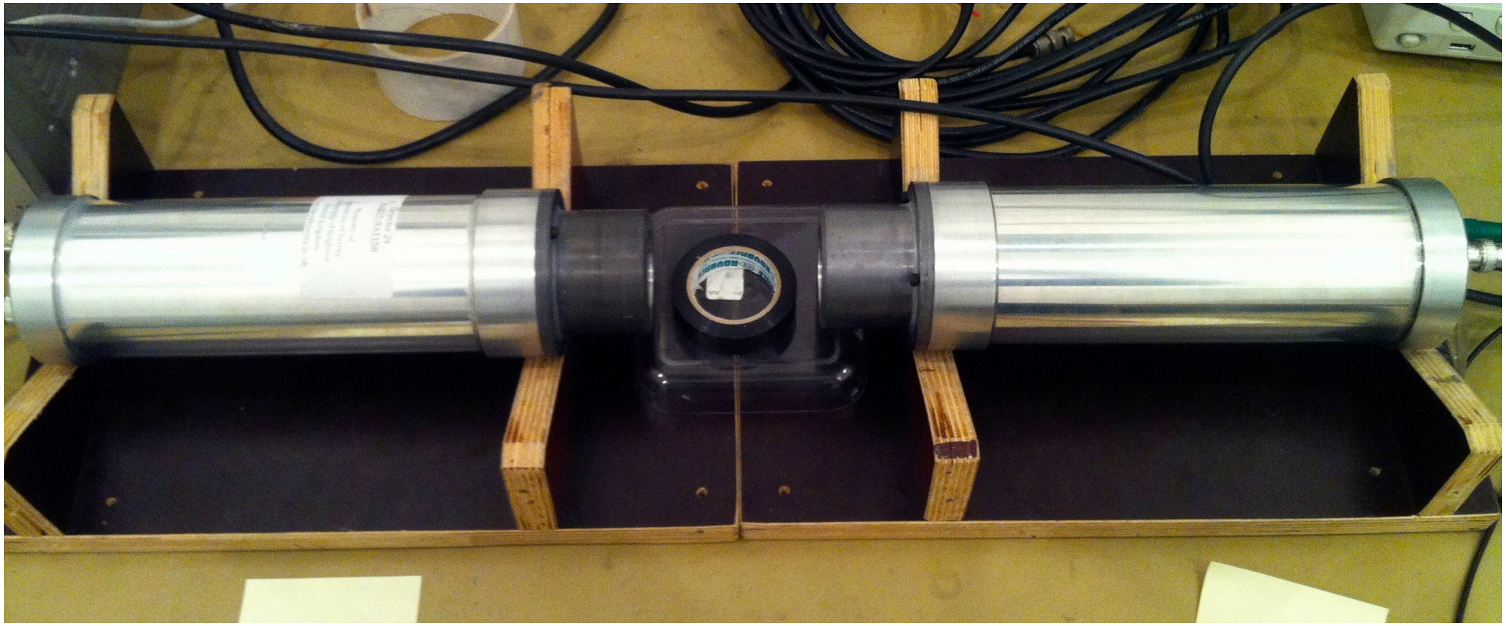
# In-beam fast-timing lifetime measurements

Difficulties:

- Fast scintillator detectors have earlier had poor energy resolution $\Rightarrow$gating with energy has been very challenging
- Large number of crystals required for reasonable efficiency
- Emitting nucleus is in flight: point of emission of $\gamma$ rays vary (speed of light = 30 cm/ns)

Advances:

- $LaBr_3$:Ce Relatively new scintillator material in $\gamma$-ray spectroscopy
- Large, efficient crystals for fast-timing



# Bucharest accelerator laboratory: 7 Ge detectors and 5 $LaBr_3$:Ce detectors



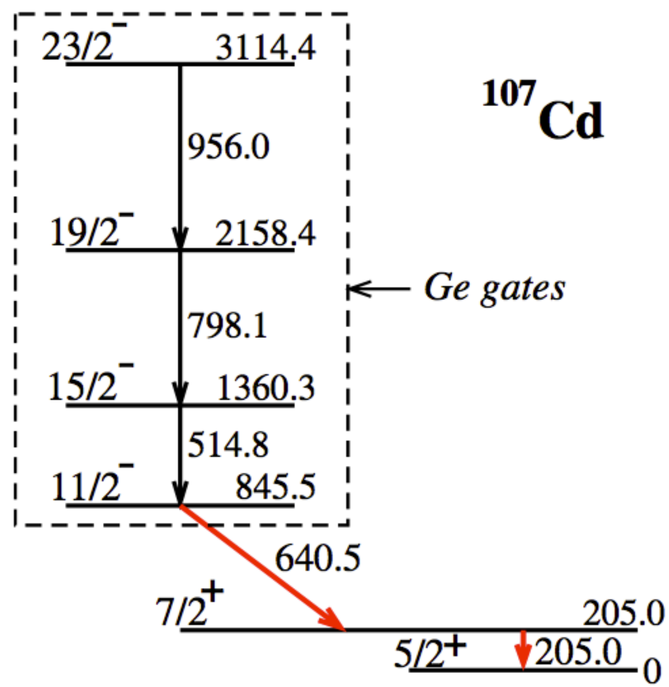



**Fig. 3.** Relevant part of the level scheme of $^{107}$Cd [9]. The dashed rectangle comprises the three transitions used for gating in the HPGe detector spectra.

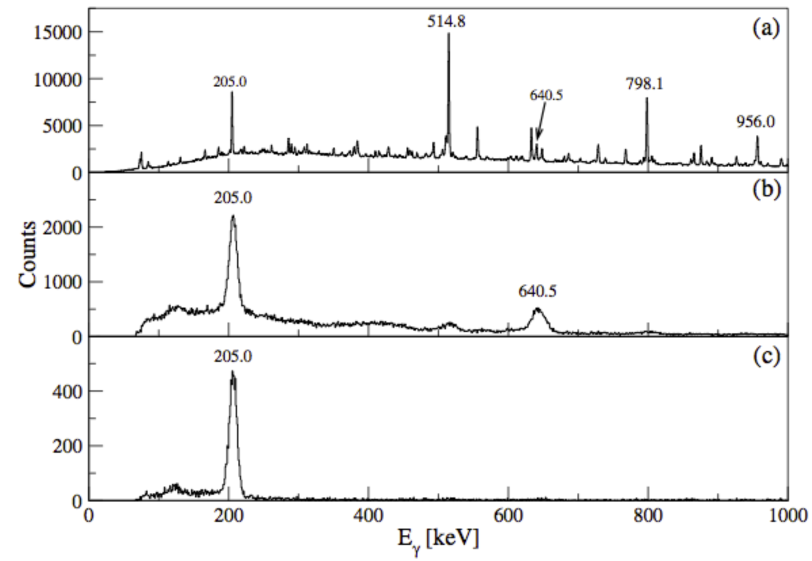



**Fig. 4.** (a) Projection of the HPGe axis of a $\gamma\gamma\gamma$ (HPGe-$LaBr_3$-$LaBr_3$) coincidence cube; (b) the energy projection of a $E_\gamma E_\gamma \Delta t$ cube constructed for the $LaBr_3$:Ce detectors, gated by the transitions 515, 798, and 956 keV detected in the HPGe detectors (highlighted in fig. 3); (c) gate on the 640.5 keV transition in spectrum (b), obtained from the $\gamma\gamma$ coincidence ($LaBr_3$-$LaBr_3$) matrix projected from the cube described at (b).

N. Mărginean et al. [European Physical Journal A 46,329 (2010)]

Two ways to extract the lifetime:

1. *Slope method*. Exponential decay $e^{-\lambda t}$ fit of time spectrum `tail´
2. *Centroid shift* (especially for short lifetimes). Difference in peak centroids (start/stop swapped) is twice the mean lifetime $\tau$.

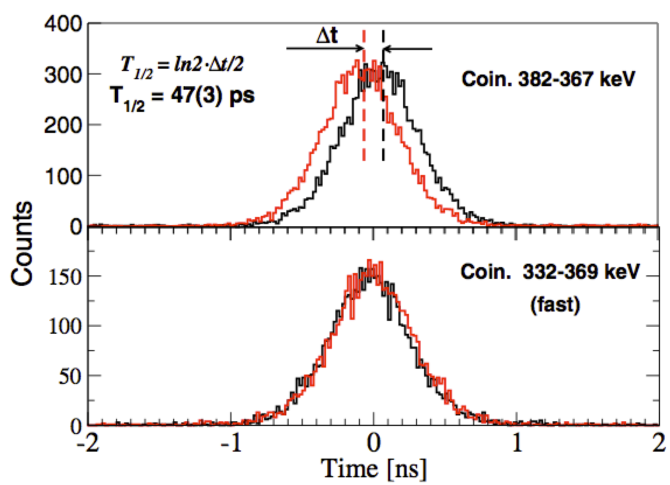



**Fig. 8.** Time spectra for the pair of $\gamma$-ray transitions 382-367 keV (top graph), and 332-369 keV (bottom graph). The two time distributions in each case were obtained by gating on the two transitions as start and stop in both possible ways. The bottom spectrum corresponds to a "prompt" coincidence, the 1118.3 keV level (fig. 6) having a short lifetime, indiscernible with the present method, while the top spectrum shows a clear shift of the centroid (first-order momentum of the distribution), corresponding to the lifetime of the 367 keV level, as indicated.

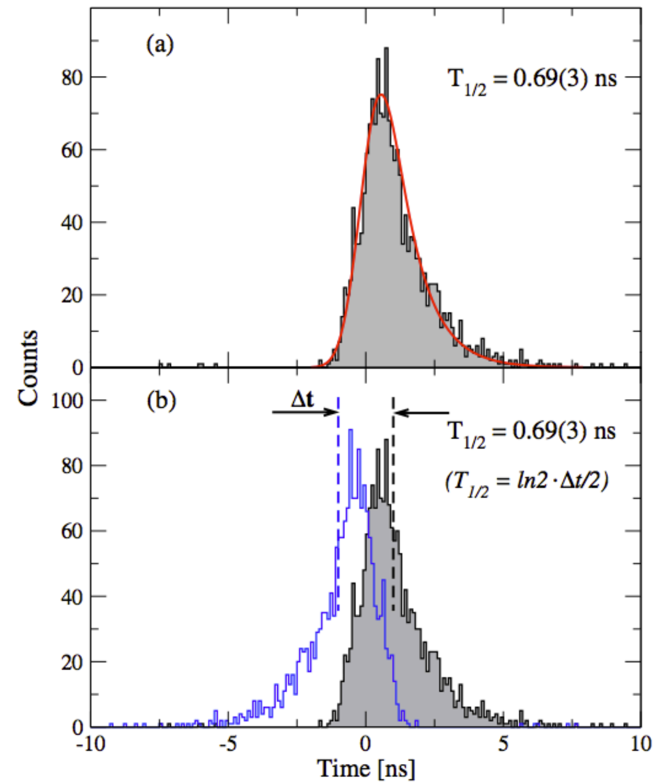
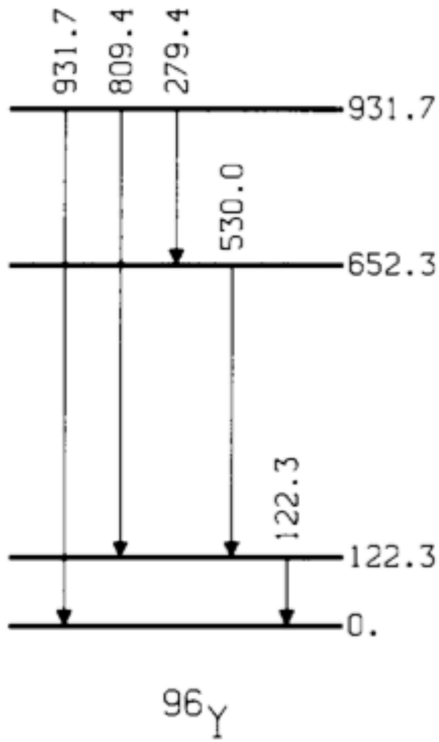
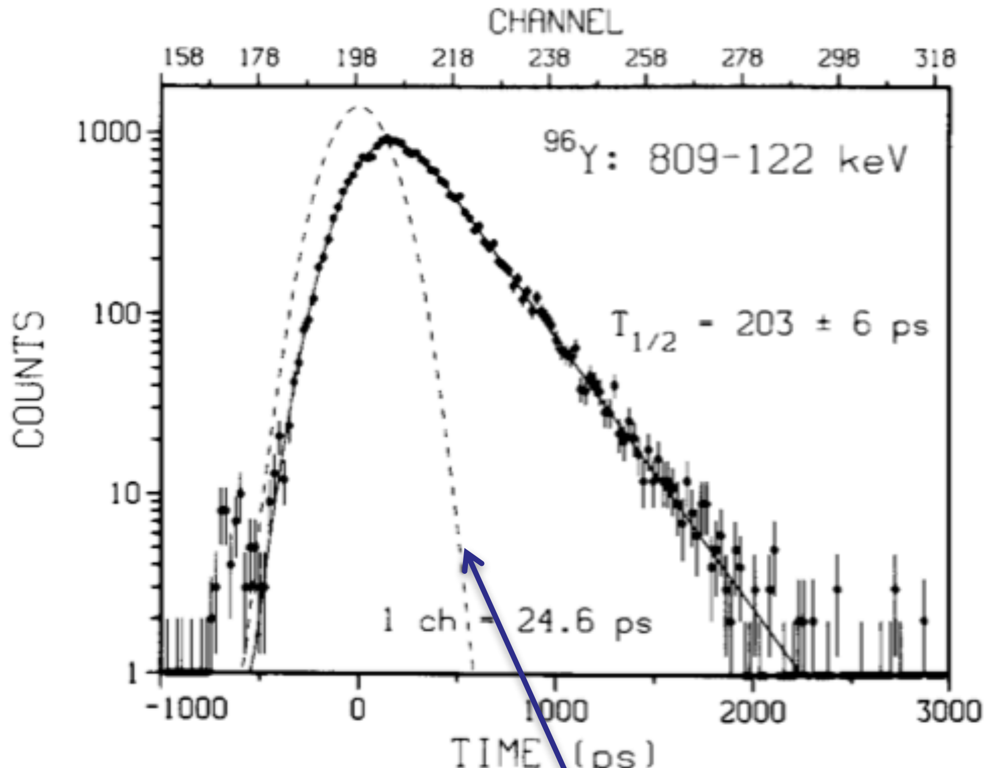



**Fig. 5.** (a) Delayed coincidence time spectrum of the 205.0 keV level. The continuous line is a fit to the data with an exponential decay convoluted with a Gaussian representing the prompt coincidence time spectrum, that provides a half-life of $0.69 \pm 0.03$ ns. (b) Delayed coincidence time spectra obtained for the 205.0 keV level with two alternatives: i) start with the 640.5 keV transition and stop with the 205.0 keV one (this is identical with the spectrum shown in (a)); ii) start with the 205.0 keV transition and stop with the 640.5 keV one (see fig. 3). The dashed lines indicate the centroids of the two distributions, their time difference being twice the lifetime of the level.

# More examples




809 keV-122 keV time distribution ⇒ lifetime of the 122 keV state

Slope method works best when $\tau \gg$ FWHM of PRF

Prompt response function (PRF)

From: H. Mach et al., Nucl. Intrum. Methods A280, 49 (1989)

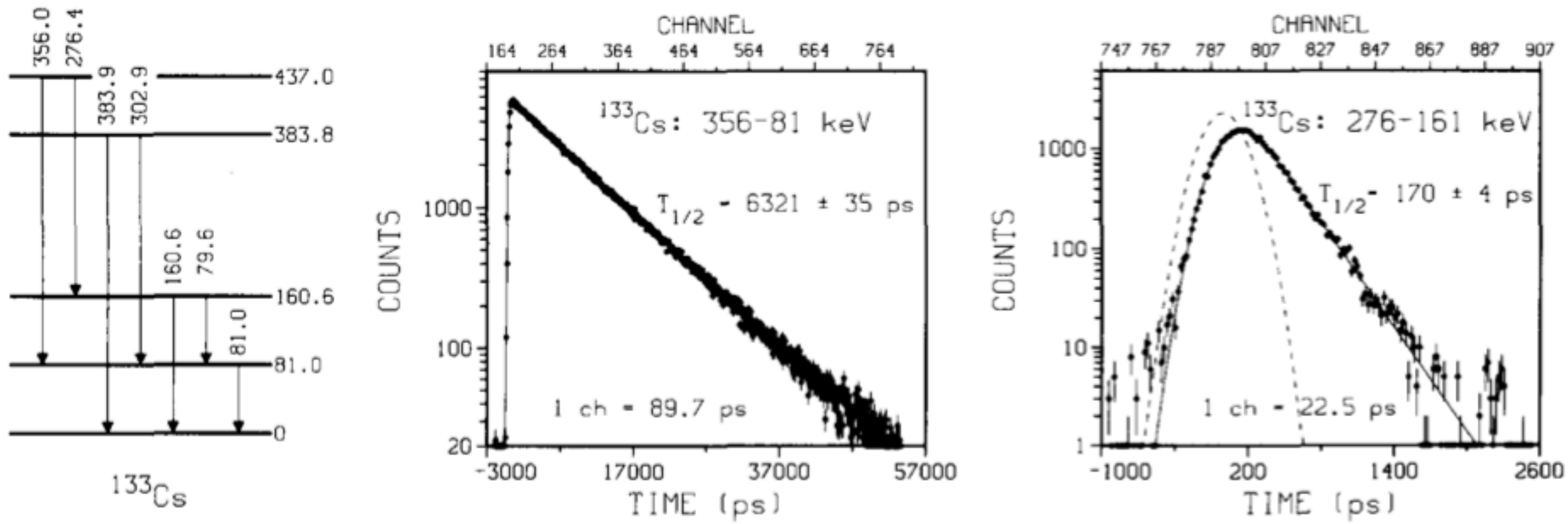
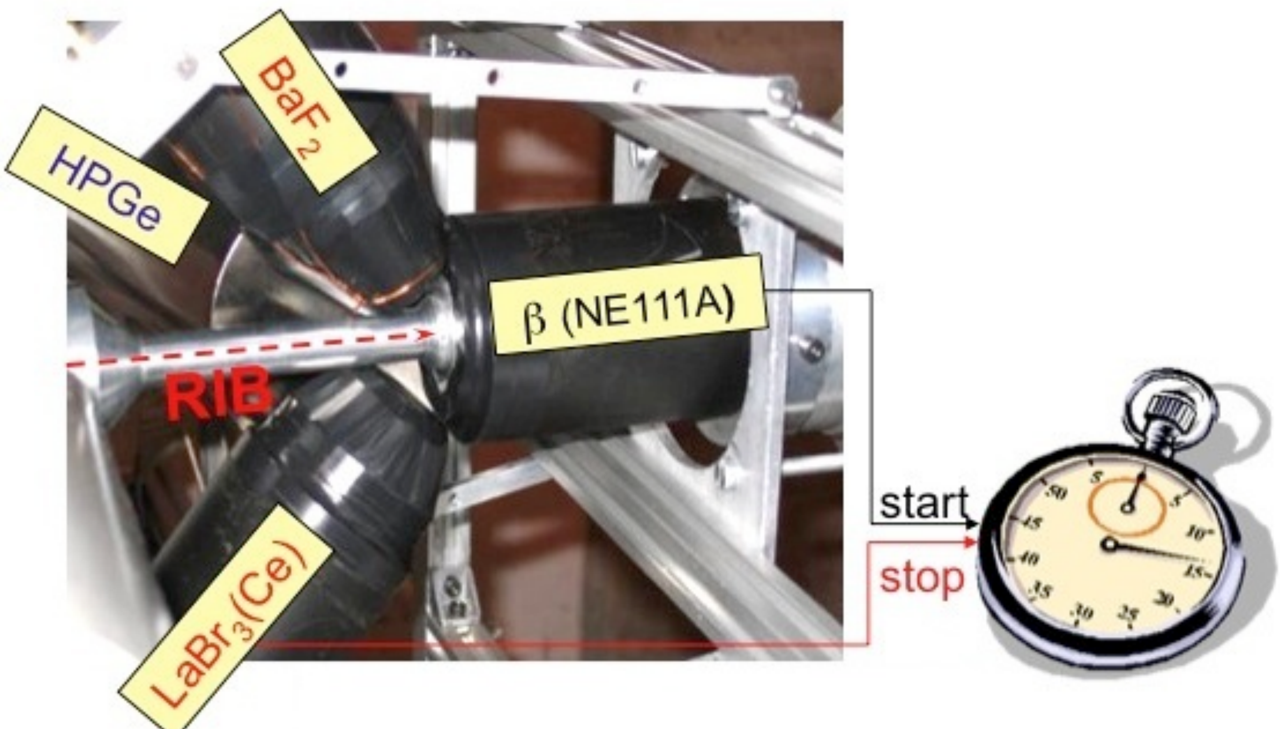



Fig. 16. A partial level scheme for the $^{133}Ba \rightarrow {}^{133}Cs$ decay is shown on the left. The center and right panels show the deconvolution of the $356 \rightarrow 81$ and $276 \rightarrow 161$ time distributions, respectively.

Note that $^{133}$Ba is a standard calibration source

# Future applications of the fast-timing

- Radioactive ion-beam (RIB) laboratories (SPIRAL2, HIE-ISOLDE, FAIR…): modular arrays of the $LaBr_3$:Ce detectors for low-intensity measurements
- In combination with other spectrometers such as β-decay and/or neutron detectors
- Focal planes of magnetic separators (RITU and MARA at JYFL)
- Combined with a large Ge-detector array such as Gammasphere at Argonne National Lab.



# Contact information

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