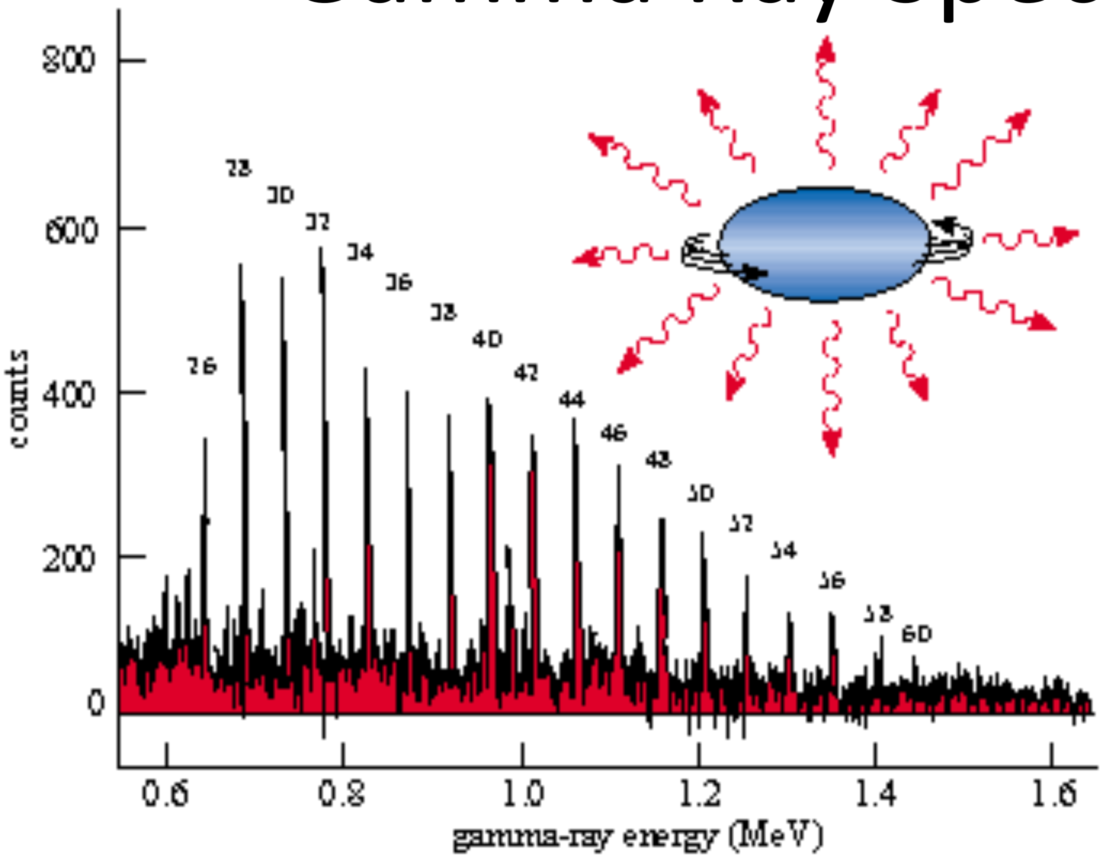
# Gamma Ray Spectroscopy



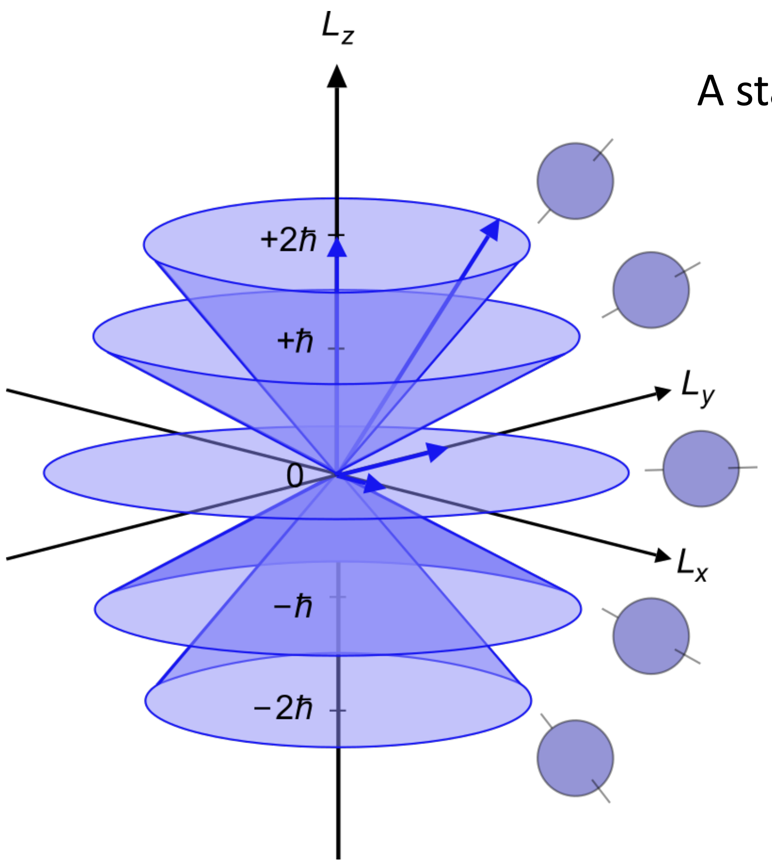

Original SD γ-ray spectrum from 1986 (Daresbury)

A rotating charge distribution acts as an antenna and creates time-varying EM fields: This radiates energy in the form of EM waves. As the amplitude of a wave of multipolecharacter L goes like $1/r^{2L+1}$ it is clear that outside the atom only the lowest multipole order plays a role. For an axially symmetric quadrupole deformed nucleus that is E2 radiation.

Compare to atoms: Electrons and nuclei can form an electric dipole: E1 is possible.

Q: Why is E1 unlikely in nuclei?

# Reminder Angular Momentum:

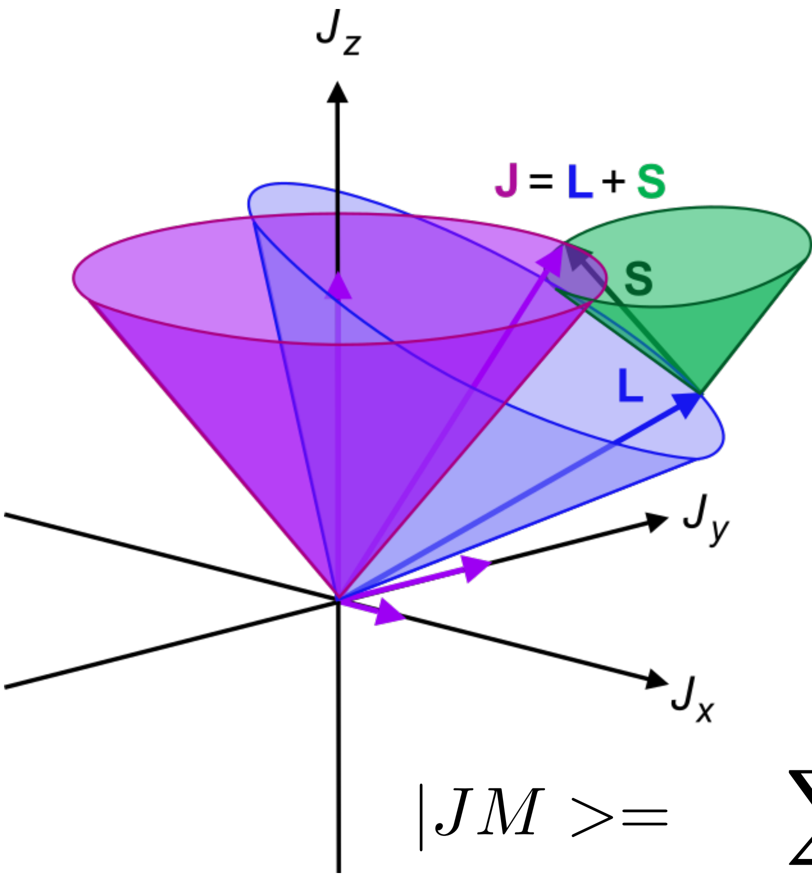



A state with angular momentum L and projection M:

$$\hat{L}^2|LM>=\hbar^2 L(L+1)|LM>$$

$$\hat{L}_z|LM>=\hbar M|LM>$$

# angular momentum addition




E.g. spin-orbit coupling

$$\begin{matrix} |j_1 m_1> \\ |j_2 m_2> \end{matrix} \xrightarrow{?} |JM>$$

I need Clebsch Gordon coefficients:

$$|JM> = \sum_{m_1+m_2=M} \mathcal{C}^{J,M}_{j_1,m_1,j_2,m_2} |j_1 m_1> |j_2 m_2>$$

$$\mathcal{C}^{J,M}_{j_1,m_1,j_2,m_2} \equiv < j_1, m_1, j_2, m_2 | J, M >$$

# Example

Two states with spin 3 and 2 are coupled to spin 4

General:

$$|4M>=\sum_{m_1+m_2=M}\mathcal{C}^{4,M}_{3,m_1,2,m_2}|3m_1>|2m_2>$$

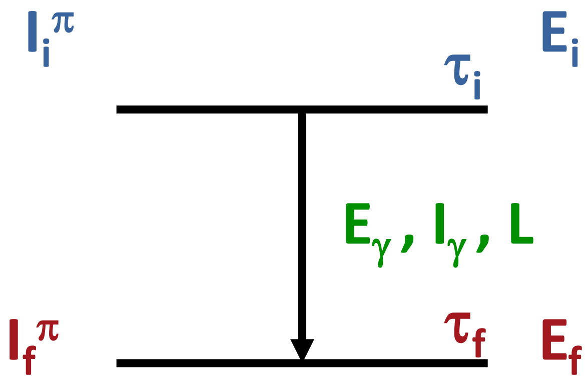
Specific M=-4:

$$|4,-4>=\sum_{m_1+m_2=-4}\mathcal{C}^{4,-4}_{3,m_1,2,m_2}|3,m_1>|2,m_2>$$

$$=\mathcal{C}^{4,-4}_{3,-3,2,-1}|3,-3>|2,-1>+\mathcal{C}^{4,-4}_{3,-2,2,-2}|3,-2>|2,-2>$$

$$=-\sqrt{\frac{3}{5}}|3,-3>|2,-1>+\sqrt{\frac{2}{5}}|3,-2>|2,-2>$$

# Gamma-ray transitions



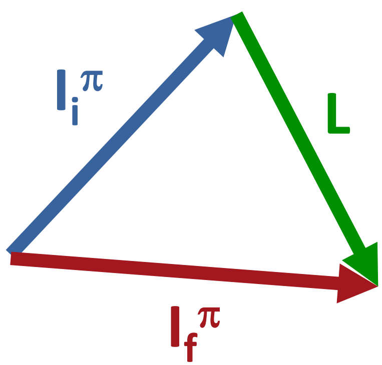

$$E_\gamma = E_i - E_f$$

$$|I_i - I_f| \leq L \leq I_i + I_f$$

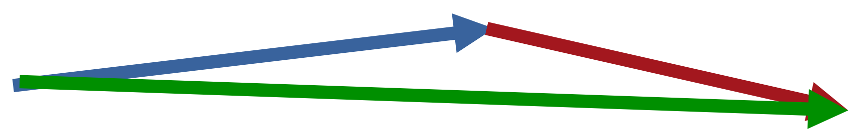
Low L is favored over high L

Three angular momentum vectors: $I_i$, $I_f$ and L

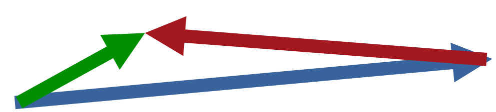
They must obey a triangle rule to conserve angular momentum:



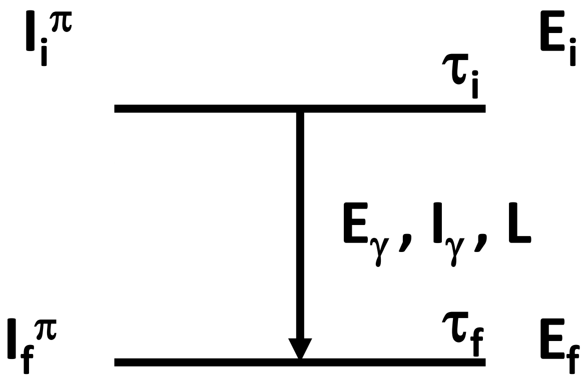

Aligned (“Stretched”):



Anti-aligned:



# Gamma-ray transitions




$$E_\gamma = E_i - E_f$$

$$|I_i - I_f| \leq L \leq I_i + I_f$$

A photon carries spin 1 and has positive parity.
Orbital angular momentum L has parity $(-1)^L$
This leads to selection rules:

$$\Delta\pi(EL) = (-1)^L$$

$$\Delta\pi(ML) = (-1)^{L+1}$$

| | | | | | |
|---|---|---|---|---|---|
| $\Delta\pi$ | YES | E1 | M2 | E3 | M4 |
| | NO | M1 | E2 | M3 | E4 |

Low L is favored over high L

# Some useful relations:

$$e^2 = \frac{e^2}{4\pi\epsilon_0} = 1.44\ \text{MeV fm}$$

$$\hbar c = 197.3\ \text{MeV fm} \approx 200\ \text{MeV fm}$$

$$c \approx 1\,\frac{\text{ft}}{\text{ns}} \approx 30\ \text{cm/ns}$$

# Electromagnetic transitions

$$T_{f\to i}(\sigma\lambda) = \frac{2(L+1)}{\varepsilon_0 L[(2L+1)!!]^2}\left(\frac{E_\gamma}{c}\right)^{2L+1} \left|\langle \psi_f | M(\sigma\lambda) | \psi_i \rangle\right|^2$$

Reduced transition probability B(σλ)

$$B(\pi\lambda; J_i^{\pi} \to J_i^{\pi'}) = C(\pi\lambda)\frac{2J_i+1}{2J_f+1}\frac{\hbar}{\tau}\frac{1}{E_\gamma^{2\lambda+1}}$$

| πλ | M1 | E1 | E2 |
|---|---|---|---|
| C(πλ) | 0.0864 $\mu_N^2$ | $0.955\times10^{-3}$ $e^2fm^2$ | 1245 $e^2fm^4$ |

See Tuomas Grahn's lectures

# E0?

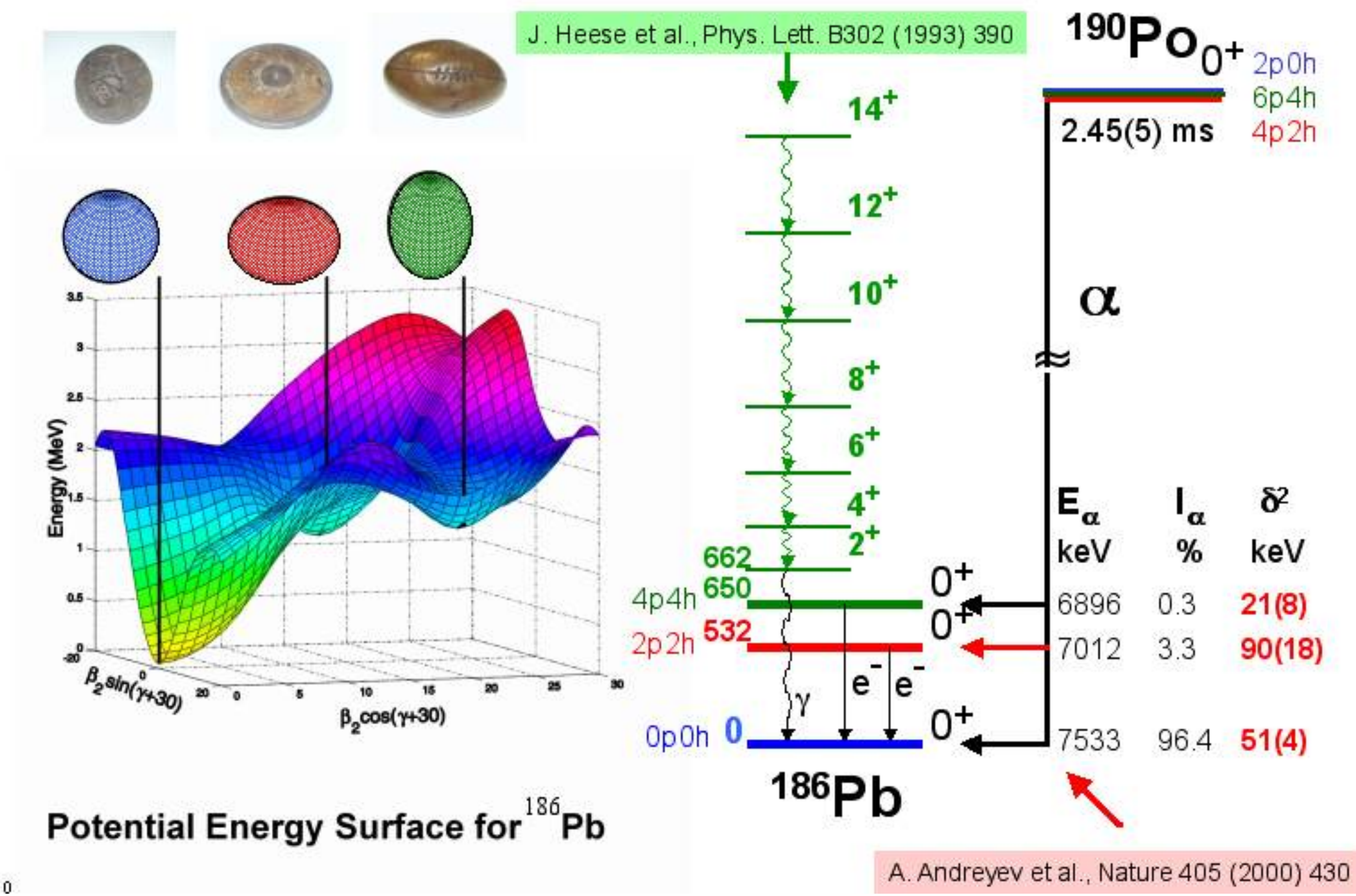
- No $0^+$ to $0^+$ transitions can happen via gamma transitions

- Why?

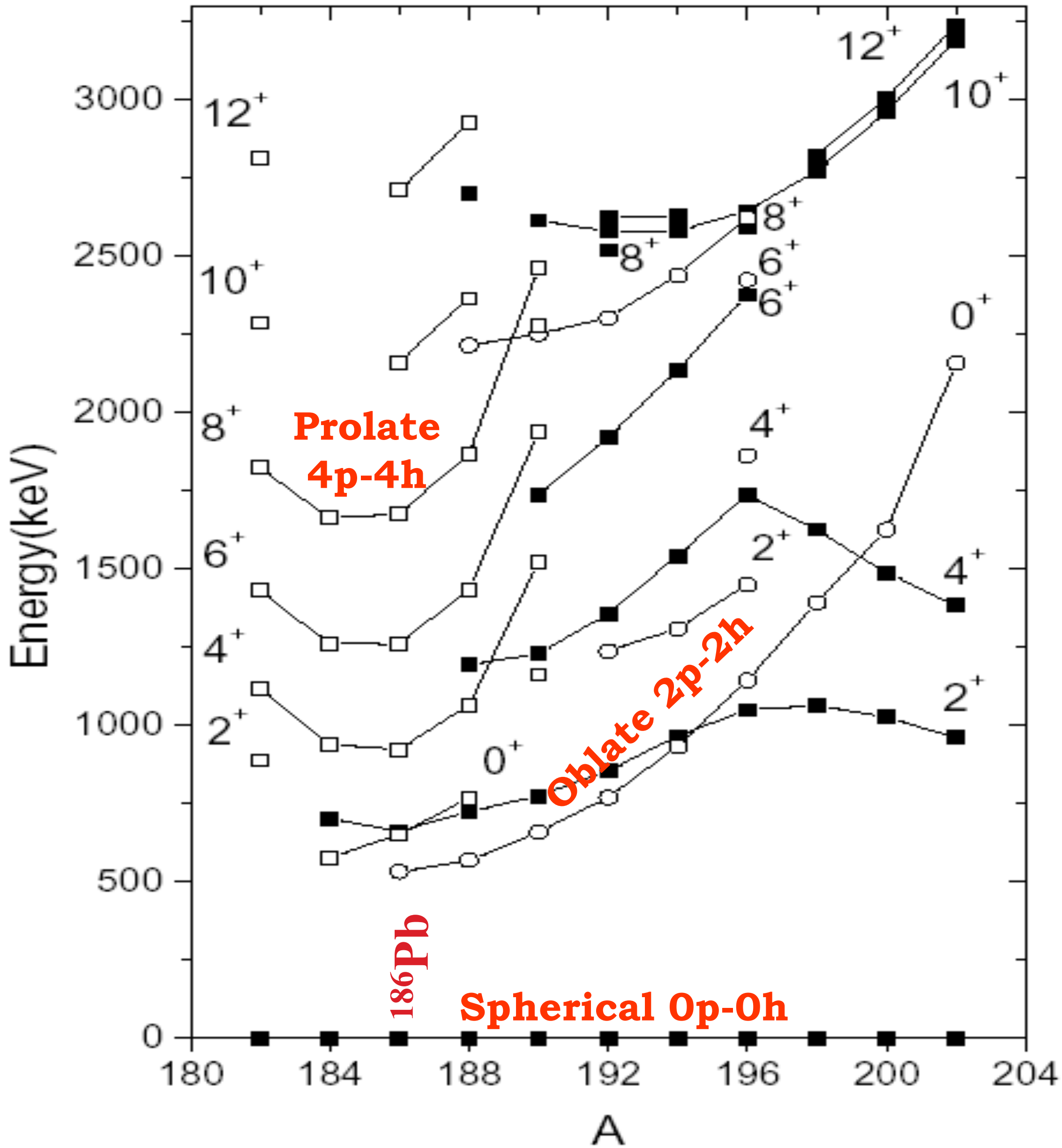
The photon carries spin 1

# Shape coexistence

## Different shapes, co-existing at low excitation energy



# Pb




Even-mass Pb excitation level systematics

# M0?

PHYSICAL REVIEW C VOLUME 47, NUMBER 5 MAY 1993

## Search for an $M0$ transition in $^{170}$Yb

A. Kuhnert, E. A. Henry, T. F. Wang, M. J. Brinkman, M. A. Stoyer, J. A. Becker, and D. R. Manatt
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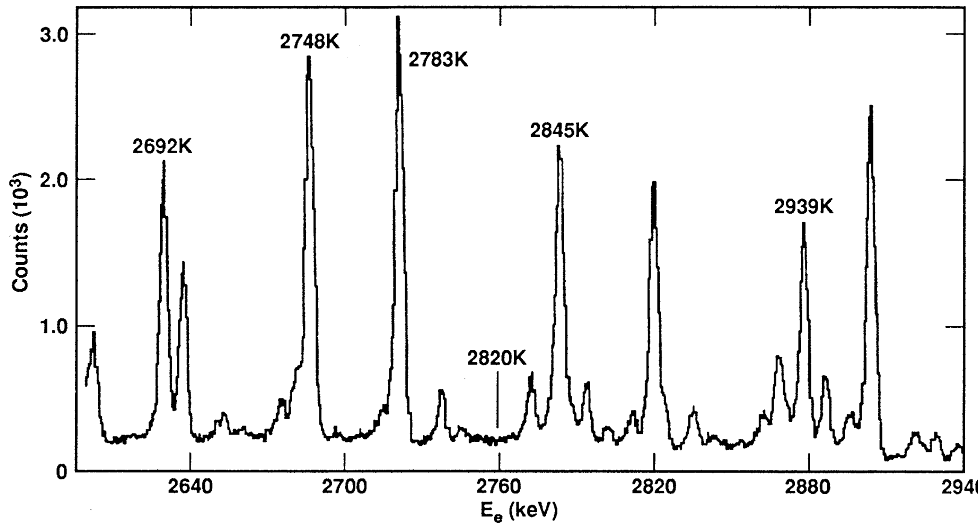
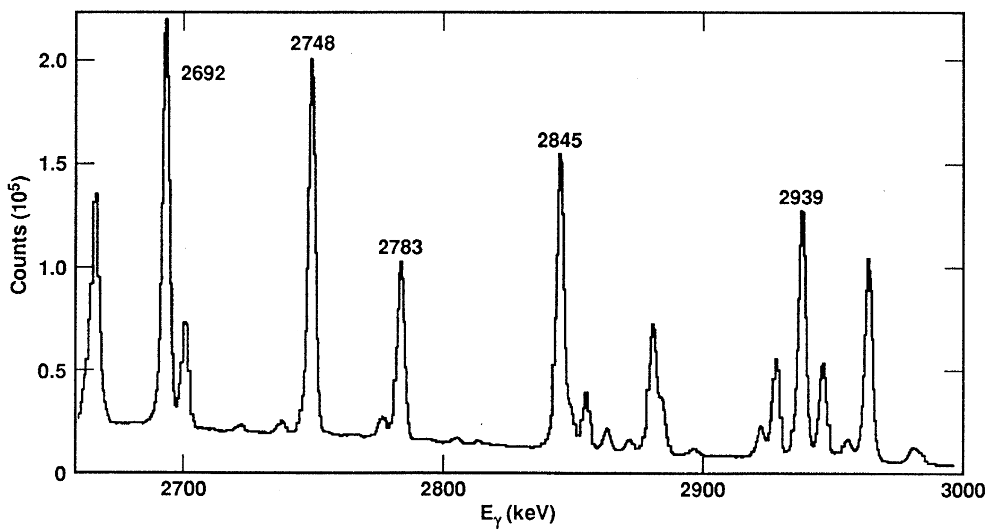
(Received 10 September 1992)

We have examined the suggestion that an $M0$ transition occurs between the 2819.6-keV $0^-$ level of $^{170}$Yb and the $0^+$ ground state by internal-conversion electron emission. Gamma rays, internal-conversion electrons, and internal $e^+e^-$ pairs were detected from the $\beta^+$ and electron capture decay of two-day $^{170}$Lu ($0^+$). At an upper limit of $2\times10^{-8}$ electron per $^{170}$Lu decay, no evidence was obtained for $M0$ decay of the 2819.6-keV level of $^{170}$Yb by single-electron emission.

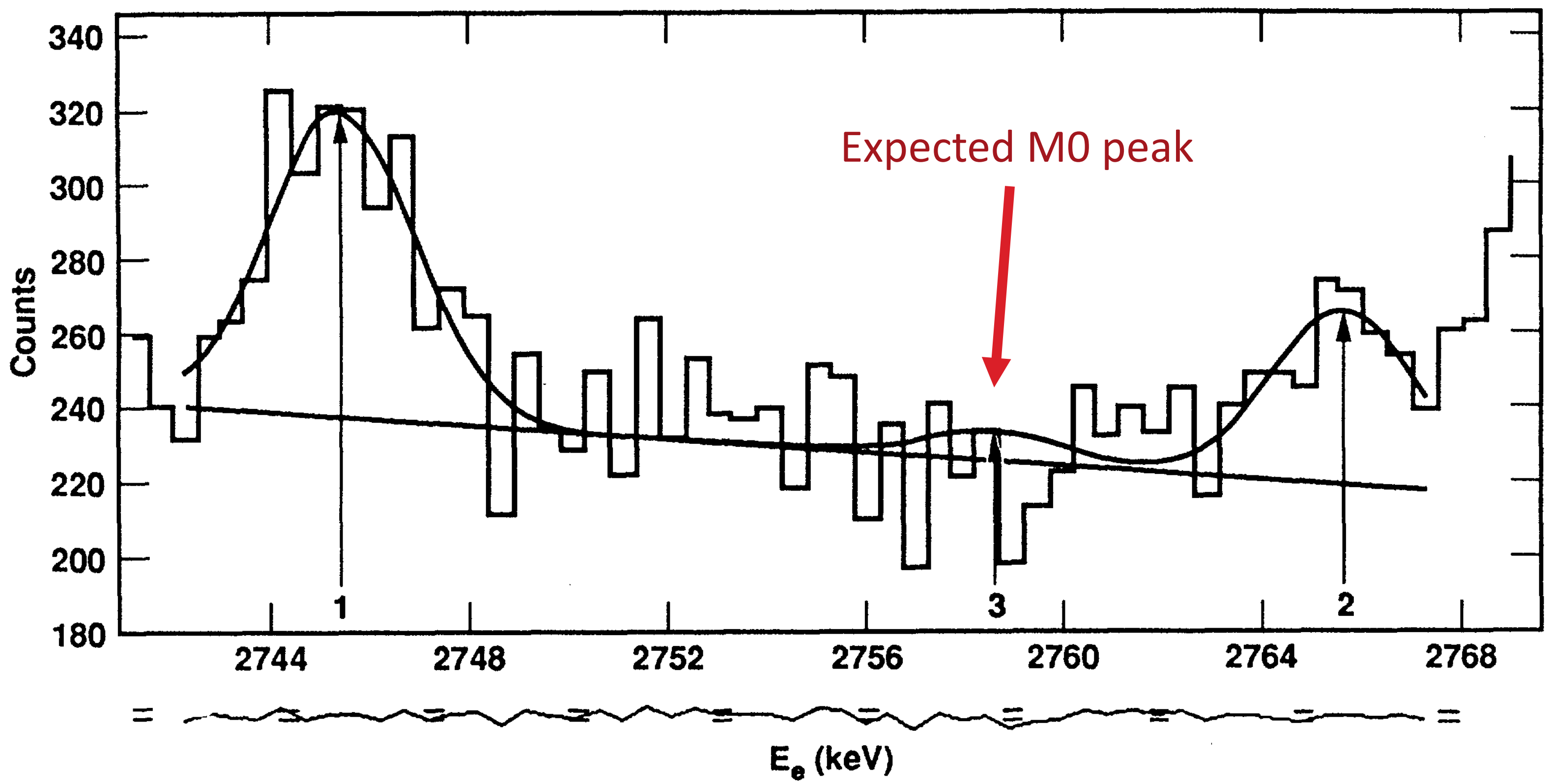
PACS number(s): 23.20.−g, 23.20.Nx, 27.70.+q

# $^{170}$Yb conversion electrons

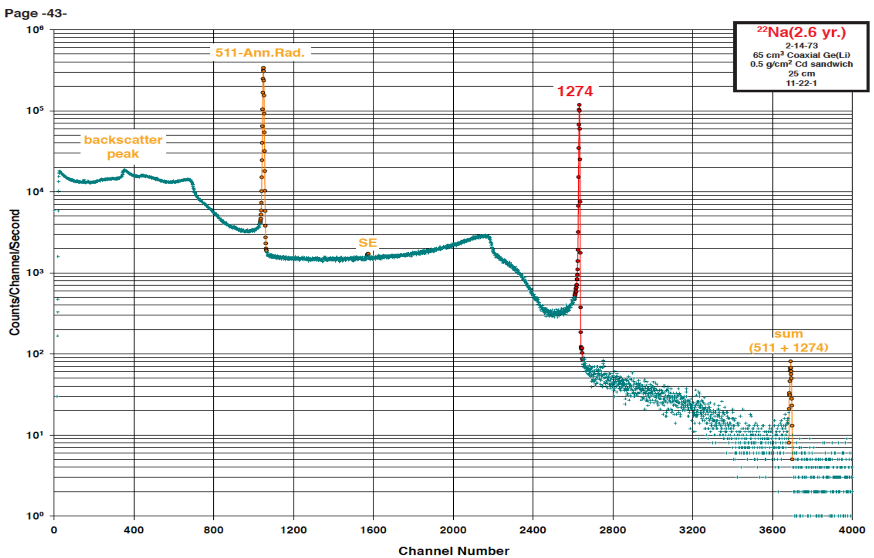


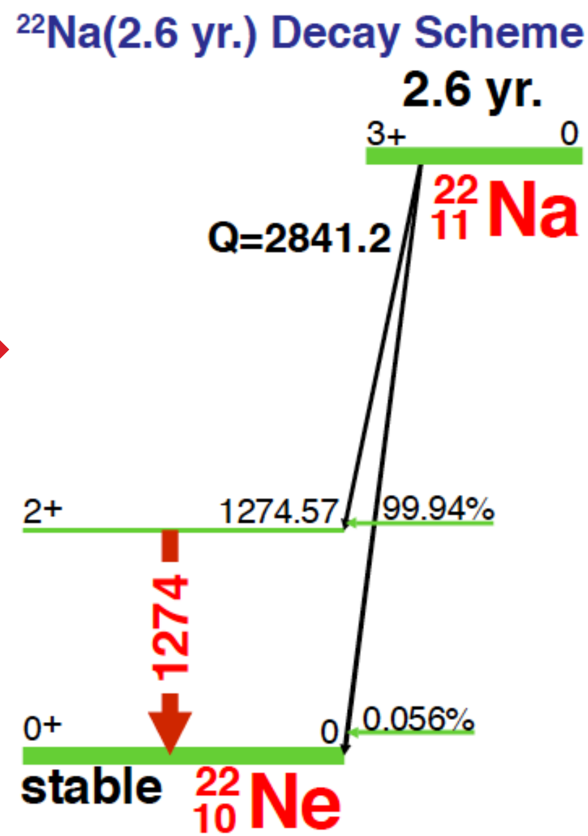



# Upper limit


Expected M0 peak
Counts
340
320
300
280
260
240
220
200
180
1
3
2
2744
2748
2752
2756
2760
2764
2768
$E_e$ (keV)

# From Spectrum to Level Scheme

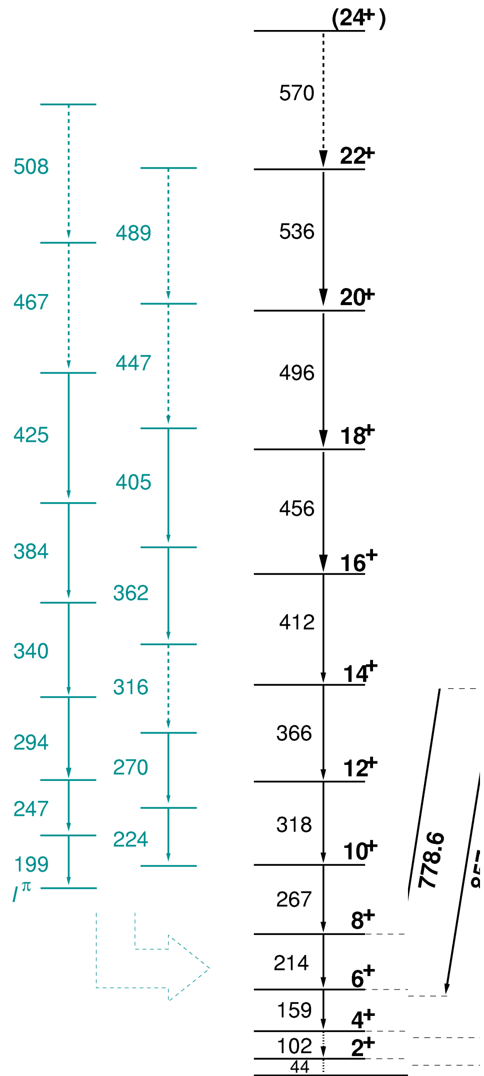


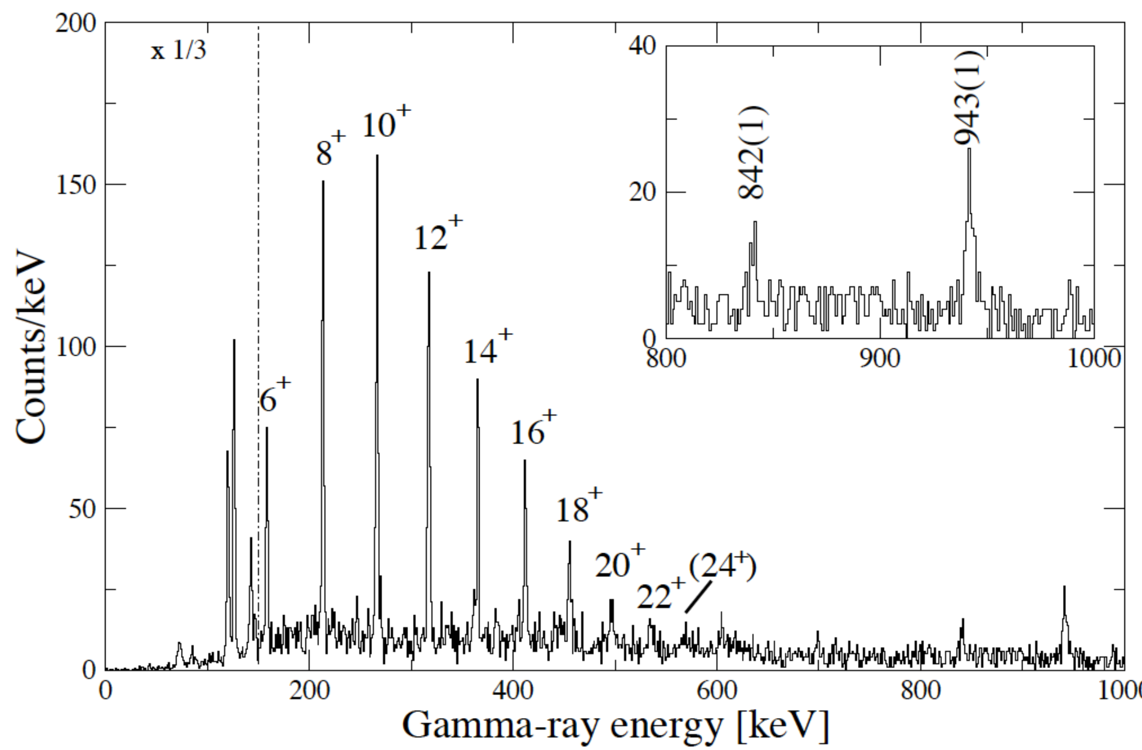



# Frequently occurring cases

Rotational bands

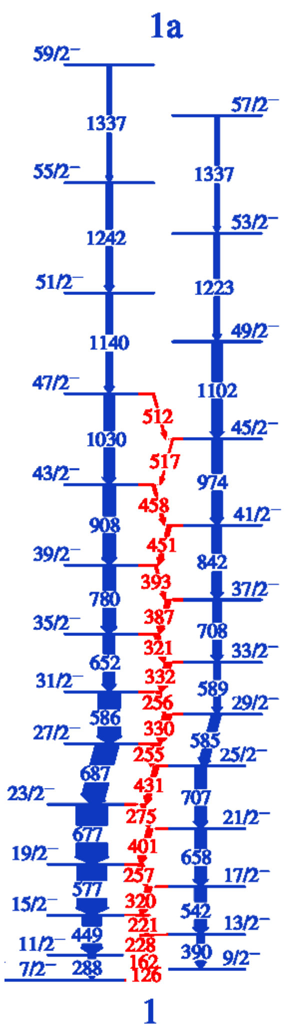
Consists entirely of stretched E2 transitions

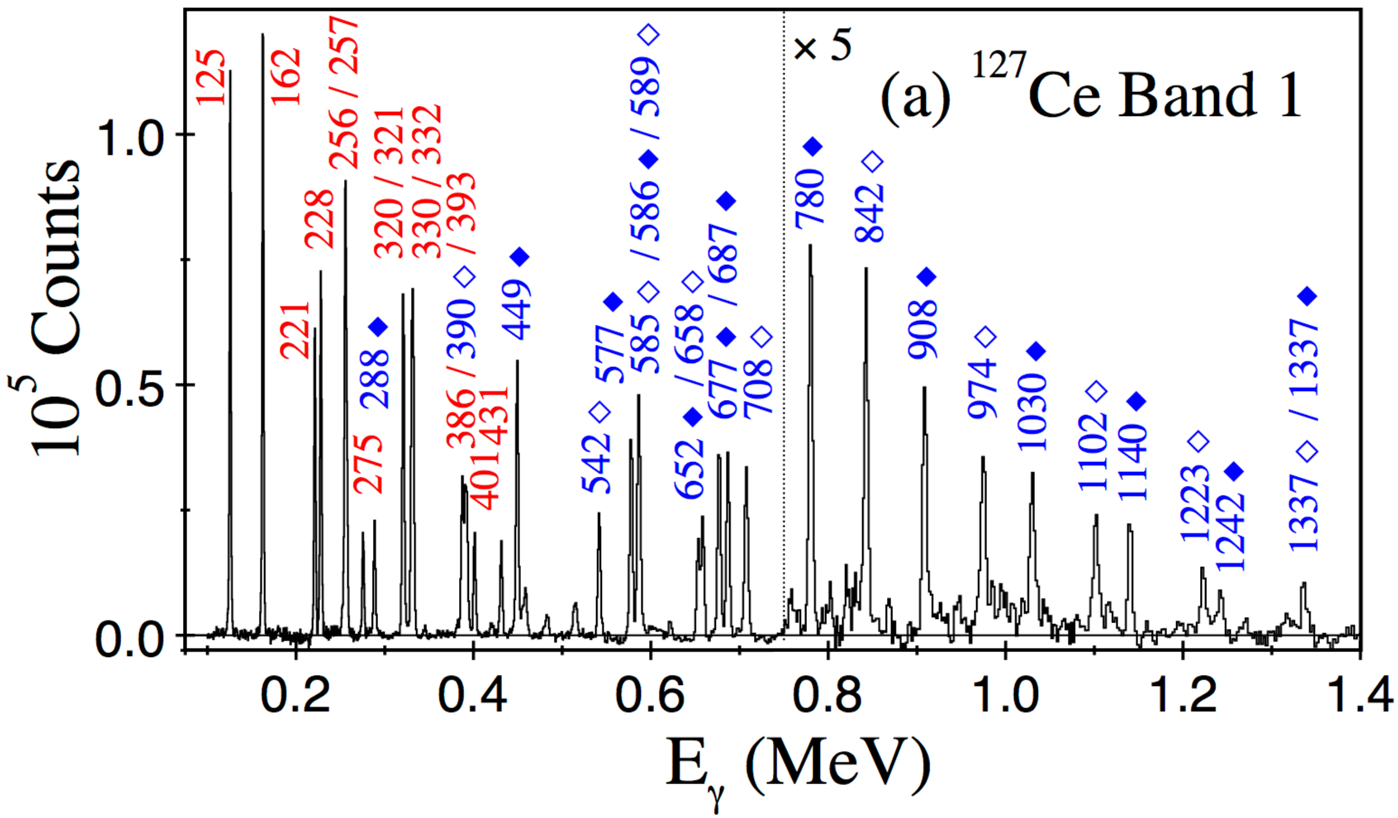


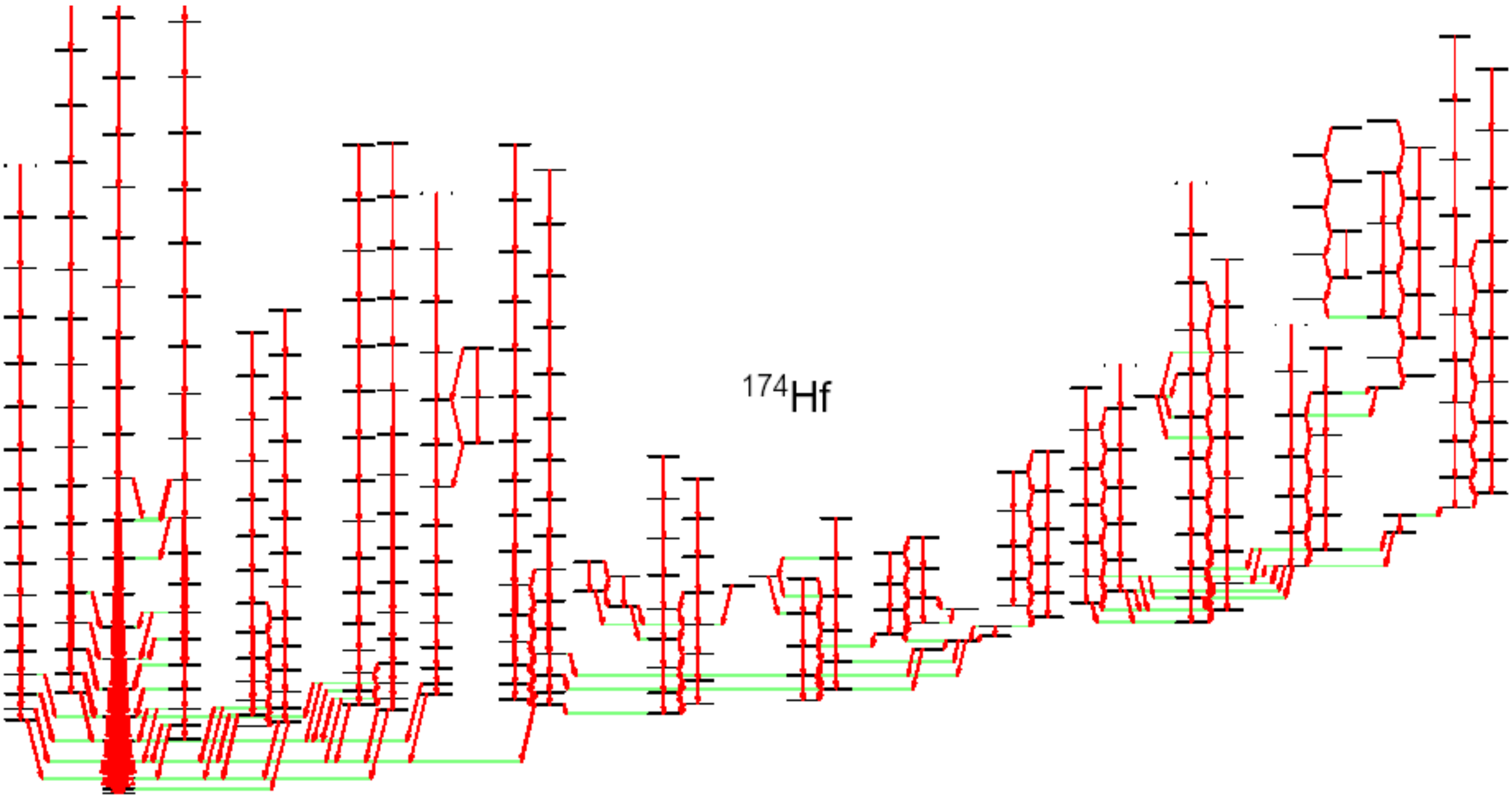
$^{127}$Ce

# Frequently occurring cases



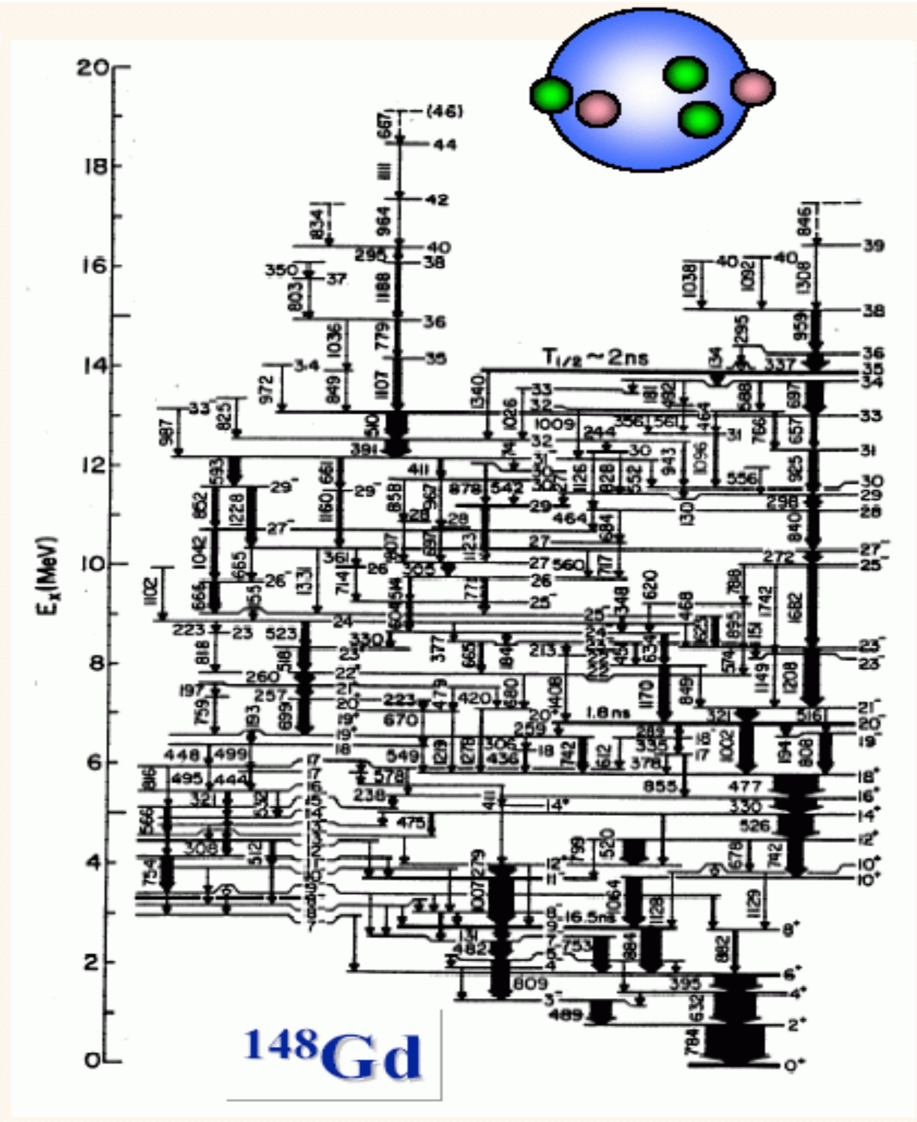



# Collective level scheme




This nucleus has 347 known levels and 516 gamma rays !

# Noncollective level scheme




- $^{148}$Gd is an example of a nucleus showing single-particle behaviour
- Complicated set of energy levels
- No regular features e.g. band structures
- Some states are isomeric

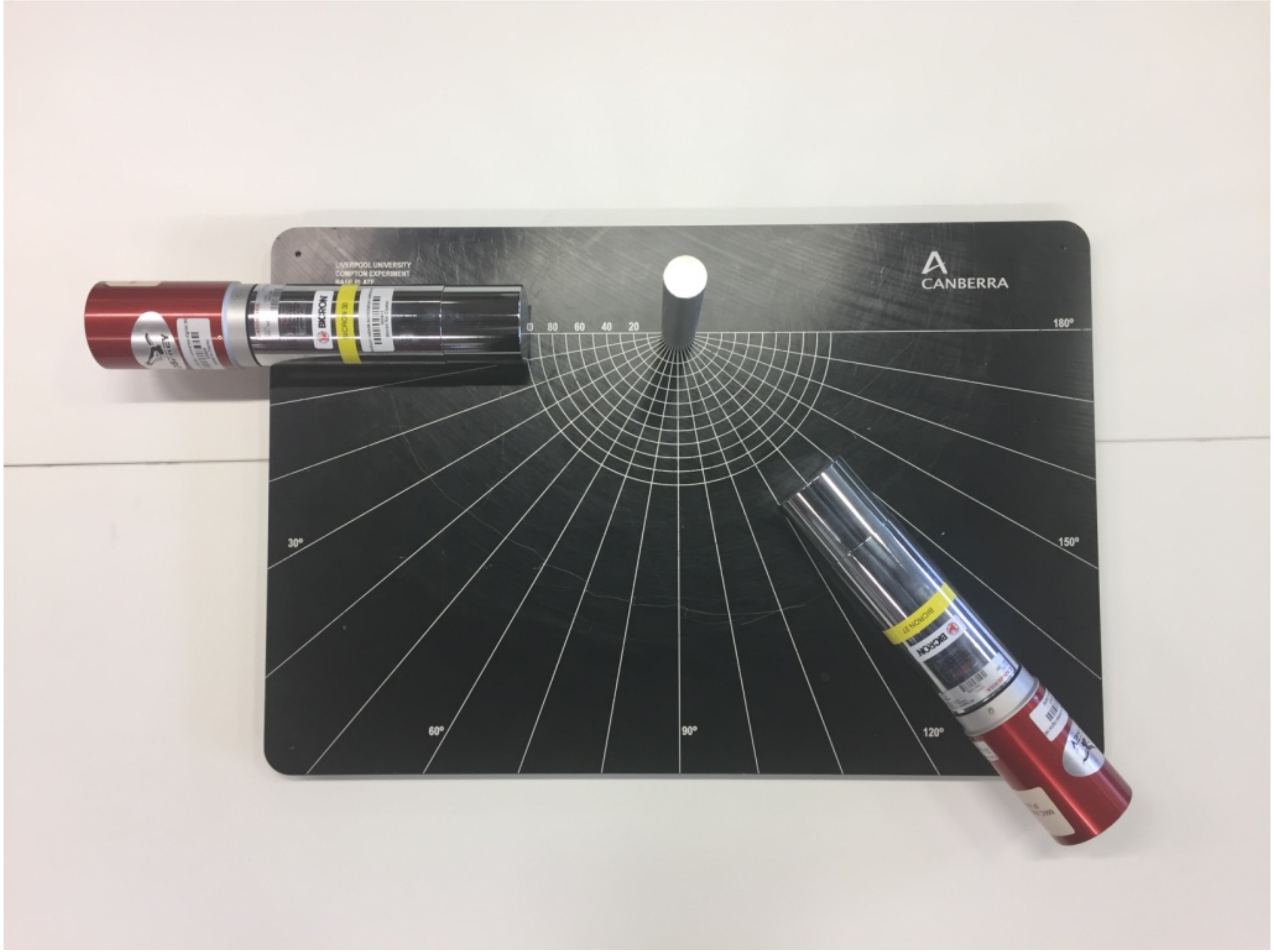
# Angular Corellations

Once a gamma ray has been emitted, we have a quantisation axis!

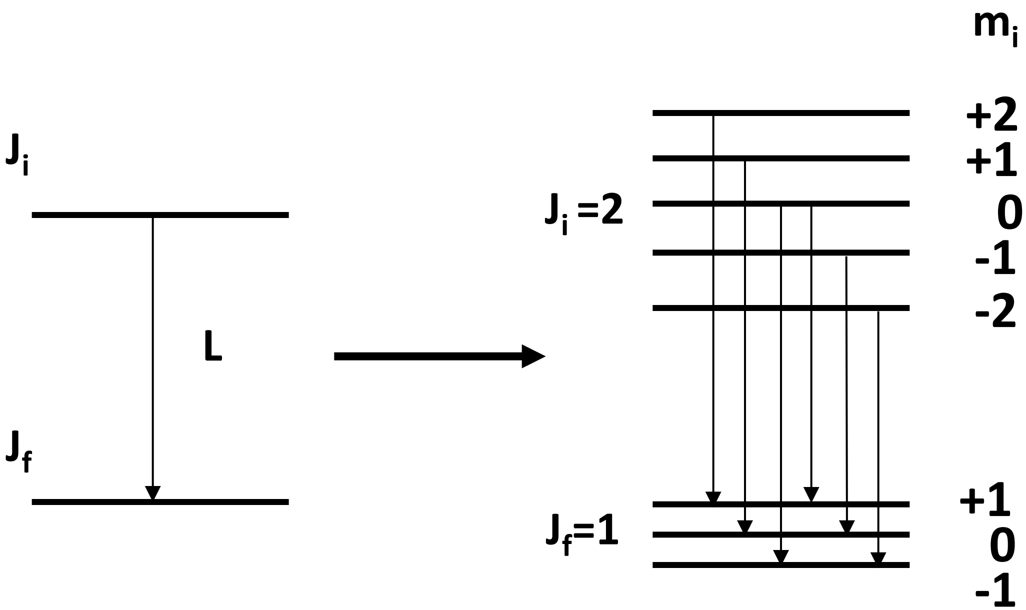
Where should we place a second detector to detect the next gamma in the cascade?

What does the angular corellation tell us about the multipole character?

# Setup



## Degeneracy of m substates




Electromagnetic transitions can be treated exactly.

$$\left| J_f m_f \right\rangle = R_f^*(r) Y_{J_f m_f}^*(\Omega)$$

$$\left\langle J_i m_i \right| = R_i(r) Y_{J_i m_i}(\Omega)$$

**Dipole transition. E1**

Dipole moment = $$ez = er\cos\theta = er\sqrt{\frac{4\pi}{3}}\,Y_{10}(\Omega)$$

Transition $\left\langle J_f m_f \right| \hat{T}(E1) \left| J_i m_i \right\rangle$

$$= e\sqrt{\frac{4\pi}{3}} \iiint \boxed{R_f^*(r)\, r\, R_i(r)\, r^2}\, \boxed{Y_{J_f m_f}^*(\Omega)\, Y_{10}(\Omega)\, Y_{J_i m_i}(\Omega) \sin\theta\, d\phi\, d\theta\, dr}$$

**Nuclear Structure Part** **Angular Part (solve explicitly)**

Evaluate $$J_0 = \int_\Omega Y^*_{J_f m_f}(\Omega) Y_{10}(\Omega) Y_{J_i m_i}(\Omega)\, d\Omega$$

$$Y_{10} Y_{J_i m_i} = \sum_{JM} C^{JM}_{10\,J_i m_i} Y_{JM}$$ recoupling

$$J_0 = \sum_{JM} \int_\Omega C^{JM}_{10\,J_i m_i} Y^*_{J_f m_f} Y_{JM}\, d\Omega$$

$$= \sum_{JM} C^{JM}_{10\,J_i m_i} \delta_{J_f J} \delta_{m_f M}$$

$$= C^{J_f m_f}_{10\,J_i m_i}$$

Total intensity $$G = e^2 \left(\frac{4\pi}{3}\right) \left\langle R_f \left| r^3 \right| R_i \right\rangle^2 \left(C^{J_f m_f}_{10 J_i m_i}\right)^2$$

The transitions between all different m states can be calculated through the simple use of a Clebsch-Gordan Coefficient.

$$G\left(J_i m_i \to J_f m_f ; LM\right) \propto \left(C^{J_f m_f}_{LMJ_i m_i}\right)^2$$

This gives the intensity distribution over all m state contributions.

The total angular distribution from a state $J_i \rightarrow J_f$ via $\pi L$ transition is then:

$M = m_i - m_f$

$$F_{Tot,L}(\theta) = \sum_{m_i m_f} P(m_i)\left[C^{J_i m_i}_{J_f m_f LM}\right]^2 F_L^M(\theta)$$

Relative population of initial m-states

Relative intensity

Angular Distribution Function

The $F_L^M(\theta)$ can be calculated from the Poynting Vector and values for different LM are tabulated.

Usually nuclei are randomly oriented in space and gamma rays have an isotropic angular distribution.

The angular distributions are very different for unequal populations of m states.

They can be calculated from the Poynting Vector and values for different LM are tabulated.

$$F_L^M(\theta)$$

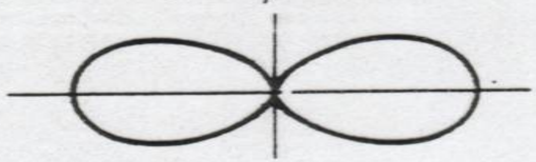
$$F_1^0 = \frac{3}{8\pi}\sin^2\vartheta\,; \qquad F_1^{\pm 1} = \frac{3}{16\pi}(1 + \cos^2\vartheta)\,,$$

$$F_2^0 = \frac{15}{8\pi}\sin^2\vartheta\cos^2\vartheta\,; \qquad F_2^{\pm 2} = \frac{5}{16\pi}(1 - \cos^4\vartheta)\,;$$

$$F_2^{\pm 1} = \frac{5}{16\pi}(1 - 3\cos^2\vartheta + 4\cos^4\vartheta)\,.$$

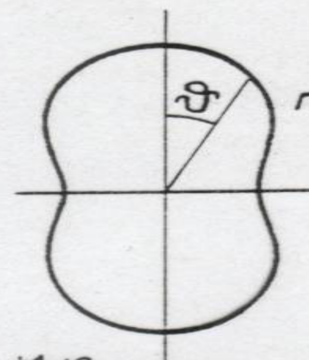
<u>Dipol</u>

$l = 1,\ m = 0$



$|x_1^0|^2 \sim \sin^2\vartheta$

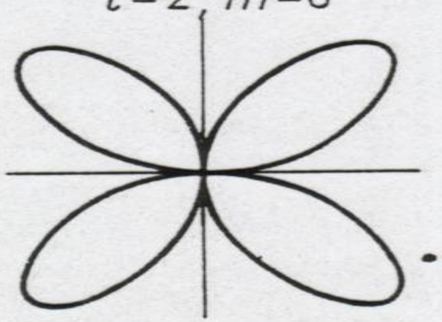
$l = 1$
$m = \pm 1$




$|x_1^{\pm 1}|^2 \sim (1 + \cos^2\vartheta)$

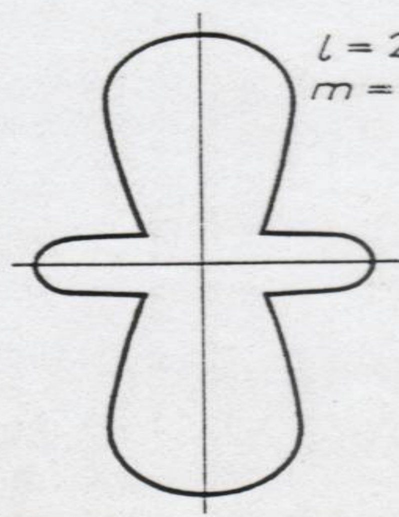
<u>Quadrupol</u>

$l = 2,\ m = 0$



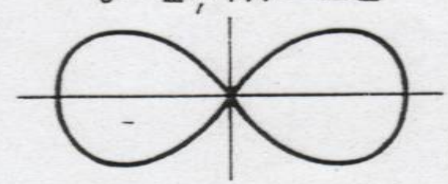
$|x_2^0|^2 \sim \sin^2\vartheta\cos^2\vartheta$

$l = 2$
$m = \pm 1$



$|x_2^{\pm 1}|^2 \sim (1 - 3\cos^2\vartheta + 4\cos^4\vartheta)$

$l = 2\,;\ m = \pm 2$



$|x_2^{\pm 2}|^2 \sim (1 - \cos^4\vartheta)$

# TABLE OF LEGENDRE POLYNOMIALS AND SPHERICAL HARMONICS

## TABLE A1

### Ordinary Legendre Polynomials

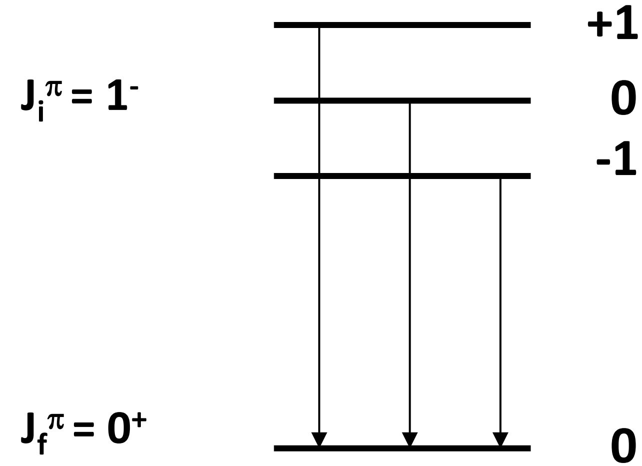
| | |
|---|---|
| $P_0(\cos\theta) = 1$ | $P_3(\cos\theta) = \frac{1}{2}(5\cos^3\theta - 3\cos\theta)$ |
| $P_1(\cos\theta) = \cos\theta$ | $P_4(\cos\theta) = \frac{1}{8}(35\cos^4\theta - 30\cos^2\theta + 3)$ |
| $P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$ | |

## TABLE A2

### Normalized Spherical Harmonics

| | |
|---|---|
| $Y_0{}^0(\theta, \varphi) = \sqrt{1/4\pi}$ | $Y_3{}^{\pm 2}(\theta, \varphi) = \frac{1}{4}\sqrt{105/2\pi}\cos\theta(1 - \cos^2\theta)\,e^{\pm 2i\varphi}$ |
| $Y_1{}^0(\theta, \varphi) = \frac{1}{2}\sqrt{3/\pi}\cos\theta$ | $Y_3{}^{\pm 3}(\theta, \varphi) = \mp\frac{1}{8}\sqrt{35/\pi}\sin^3\theta\,e^{\pm 3i\varphi}$ |
| $Y_1{}^{\pm 1}(\theta, \varphi) = \mp\frac{1}{2}\sqrt{3/2\pi}\sin\theta\,e^{\pm i\varphi}$ | $Y_4{}^0(\theta, \varphi) = \frac{3}{16}\sqrt{1/\pi}(35\cos^4\theta - 30\cos^2\theta + 3)$ |
| $Y_2{}^0(\theta, \varphi) = \frac{1}{4}\sqrt{5/\pi}(3\cos^2\theta - 1)$ | $Y_4{}^{\pm 1}(\theta, \varphi) = \mp\frac{3}{8}\sqrt{5/\pi}\sin\theta(7\cos^3\theta - 3\cos\theta)\,e^{\pm i\varphi}$ |
| $Y_2{}^{\pm 1}(\theta, \varphi) = \mp\frac{1}{2}\sqrt{15/2\pi}\cos\theta\sin\theta\,e^{\pm i\varphi}$ | $Y_4{}^{\pm 2}(\theta, \varphi) = -\frac{3}{8}\sqrt{5/2\pi}(\cos^4\theta - 8\cos^2\theta + 1)\,e^{\pm 2i\varphi}$ |
| $Y_2{}^{\pm 2}(\theta, \varphi) = \frac{1}{4}\sqrt{15/2\pi}(1 - \cos^2\theta)\,e^{\pm 2i\varphi}$ | $Y_4{}^{\pm 3}(\theta, \varphi) = \mp\frac{3}{8}\sqrt{35/\pi}\sin\theta(\cos\theta - \cos^3\theta)\,e^{\pm 3i\varphi}$ |
| $Y_3{}^0(\theta, \varphi) = \frac{1}{4}\sqrt{7/\pi}(5\cos^3\theta - 3\cos\theta)$ | $Y_4{}^{\pm 4}(\theta, \varphi) = \frac{3}{16}\sqrt{35/2\pi}\sin^4\theta\,e^{\pm 4i\varphi}$ |
| $Y_3{}^{\pm 1}(\theta, \varphi) = \mp\frac{1}{8}\sqrt{21/\pi}(5\cos^2\theta - 1)\sin\theta\,e^{\pm i\varphi}$ | |

**Example**




$$F_{Tot,L}(\theta) = \sum_{m_i m_f} P(m_i)\left[C^{J_i m_i}_{J_f m_f L M}\right]^2 F_L^M(\theta)$$

$$F_{Tot,L}(\theta) = P(1)\left[C^{11}_{0011}\right]^2 F_1^1(\theta) + P(0)\left[C^{10}_{0010}\right]^2 F_1^0(\theta) + P(-1)\left[C^{1-1}_{001-1}\right]^2 F_1^{-1}(\theta)$$

$$= p_0\left[\frac{6}{16\pi}\left(1+\cos^2\theta\right) + \frac{3}{8\pi}\left(\sin^2\theta\right)\right]$$

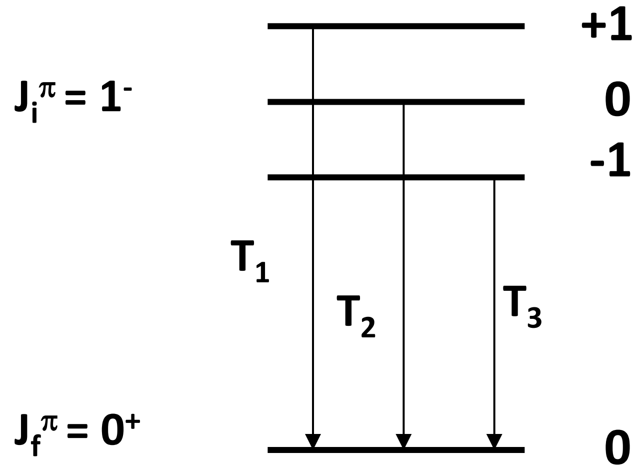
$$= p_0\,\frac{3}{8\pi}\left(1+\cos^2\theta+\sin^2\theta\right) = \boxed{p_0\,\frac{3}{4\pi}}$$

**Independent of $\theta$**

If $m_i$ are all equally populated, the radiation is isotropic!

i.e There is no useful information for determining the multipolarity!

**Example**




$$T_1 \propto \left[C^{11}_{00\,11}\right]^2 = 1$$

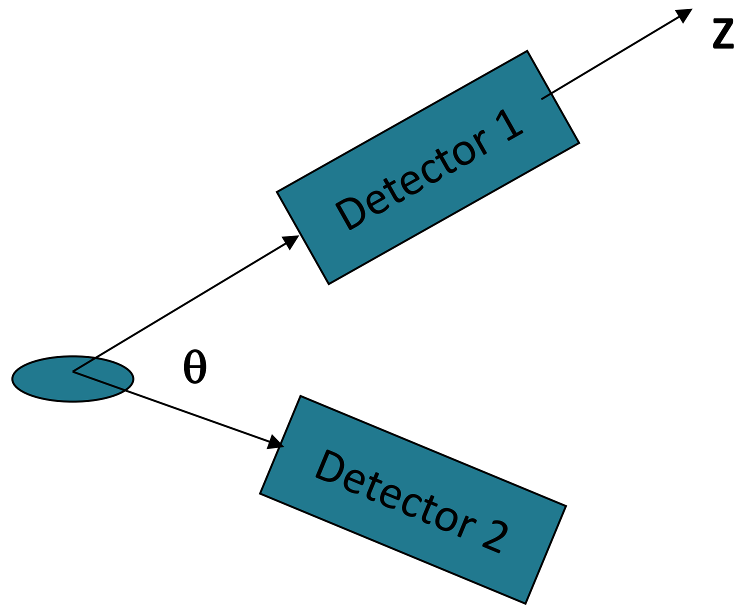
$$T_2 \propto \left[C^{10}_{00\,10}\right]^2 = 1$$

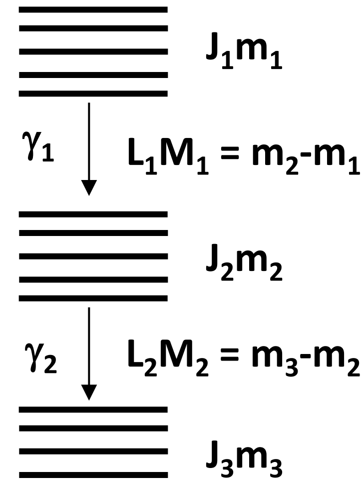
$$T_3 \propto \left[C^{1-1}_{00\,1-1}\right]^2 = 1$$

**Equal intensities.**

To obtain unequal distributions, the nucleus must be oriented.

The easiest way to orient a nucleus is by gamma-ray emission.
i.e. **angular correlations**.

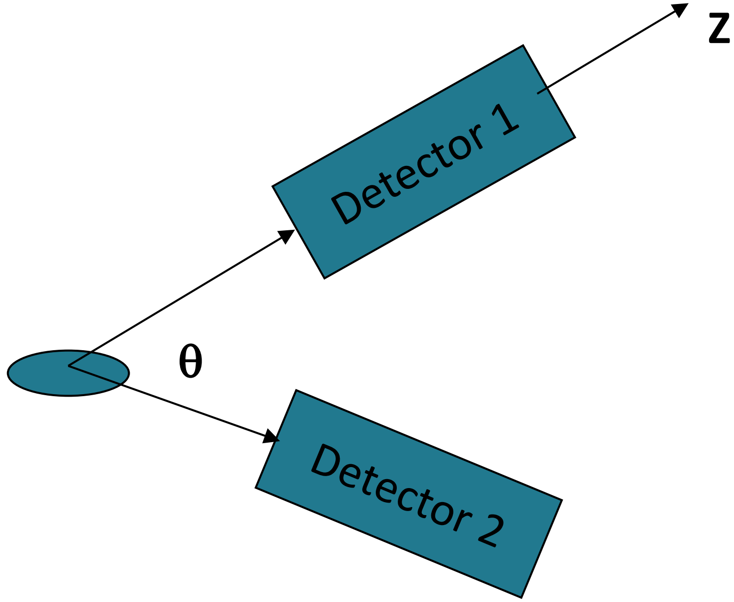


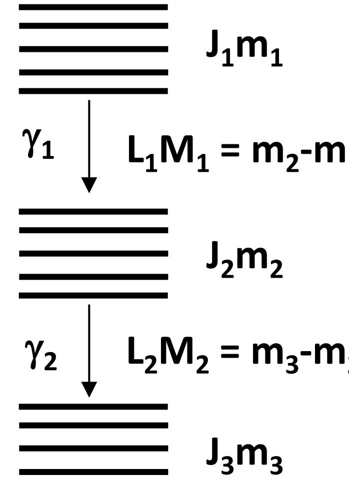




The first gamma ray leaves the nucleus in a defined m state. We choose Z axis to point in the same direction.

$$W(\theta, L_1, L_2, J_1, J_3)$$

$$F_{Tot,L_2} = \sum_{m_2} P(m_2)\left[C^{J_2 m_2}_{J_3 m_3 L_2 M_2}\right]^2 F^{M_2}_{L_2}(\theta)$$




We want $$W(\theta, L_1, L_2, J_1, J_3)$$

We know $$F_{Tot, L_2} = \sum_{m_2} P(m_2) \left[ C^{J_2 m_2}_{J_3 m_3 L_2 M_2} \right]^2 F^{M_2}_{L_2}(\theta)$$

Assume all $m_1$ states are equally populated. Therefore

$$P(m_2) = \sum_{m_1} \left[ C^{J_1 m_1}_{J_2 m_2 L_1 M_1} \right]^2 F^{M_1}_{L_1}(\theta = 0)$$

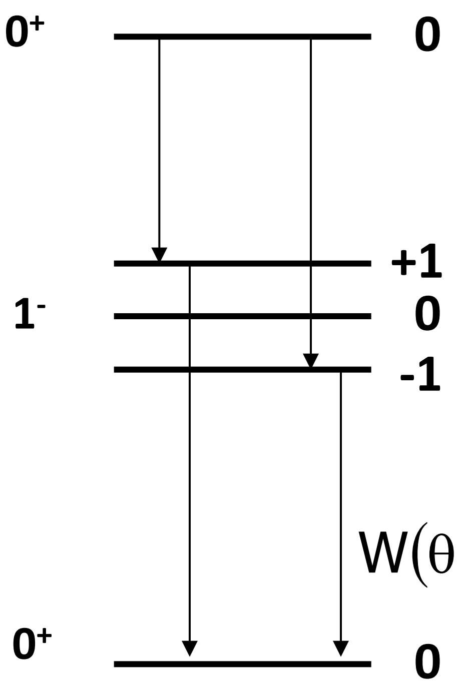
Choice of z-axis means photon can only make $M_1 = \pm 1$ transitions

$$P(m_2) = \sum_{m_1} \left[ C^{J_1 m_1}_{J_2 m_2 L_1 \pm 1} \right]^2 F^{\pm 1}_{L_1}(\theta = 0)$$

## Total angular correlation

$$W(\theta, L_1, L_2, J_1, J_3) = \sum_{m_1 m_2 m_3} \left[C^{J_1 m_1}_{J_2 m_2 L_1 \pm 1}\right]^2 F^{\pm 1}_{L_1}(\theta = 0) \left[C^{J_2 m_2}_{J_3 m_3 L_2 M_2}\right]^2 F^{M_2}_{L_2}(\theta)$$

## Example



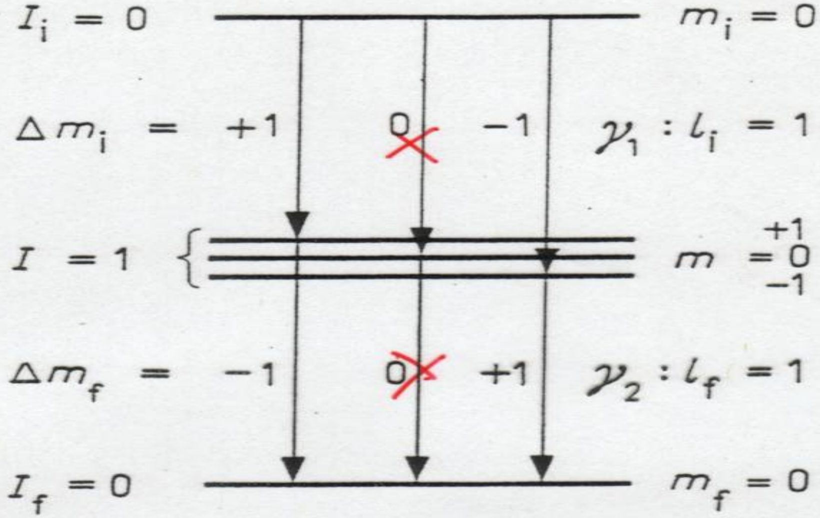

$$F^{\pm 1}_{1}(0) = \frac{3}{8\pi}$$

$$F^{\pm 1}_{1}(\theta) = \frac{3}{16\pi}\left(1 + \cos^2\theta\right)$$

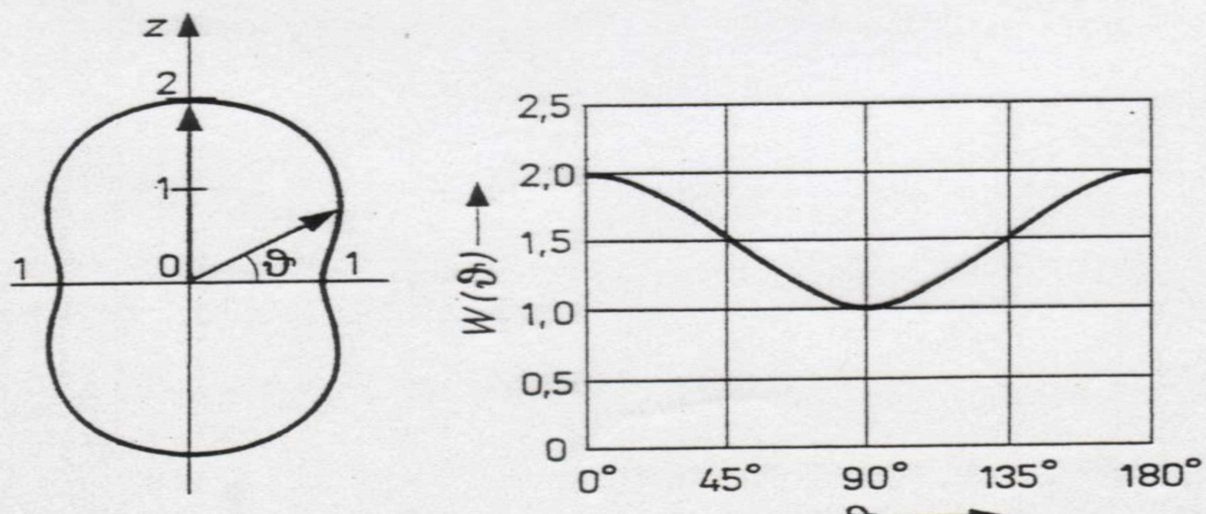
$$C^{00}_{1\pm 1 1 \pm 1} = \frac{1}{\sqrt{3}}$$

$$W(\theta) = \frac{3}{8\pi}\left[\left(\frac{1}{\sqrt{3}}\right)^2 1^2 . \frac{3}{16\pi}\left(1 + \cos^2\theta\right) + \left(\frac{1}{\sqrt{3}}\right)^2 1^2 . \frac{3}{16\pi}\left(1 + \cos^2\theta\right)\right]$$

$$W(\theta) = \frac{1}{3}\left(\frac{3}{8\pi}\right)^2 \left(1 + \cos^2\theta\right)$$



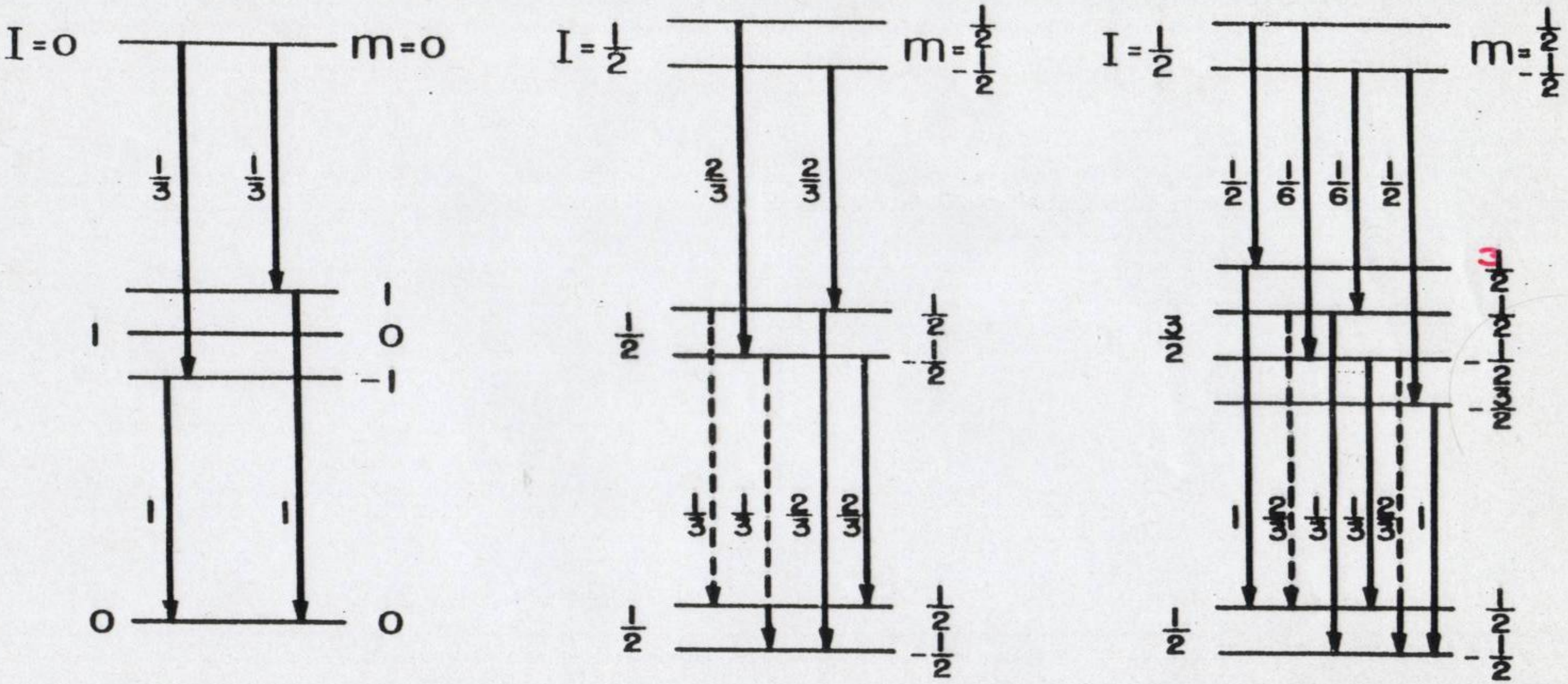
$I_i = 0$
$m_i = 0$
$\Delta m_i = +1 \quad 0 \quad -1$
$\gamma_1 : l_i = 1$
$I = 1$
$m = +1, 0, -1$
$\Delta m_f = -1 \quad 0 \quad +1$
$\gamma_2 : l_f = 1$
$I_f = 0$
$m_f = 0$

z
2
1
0
1
ϑ
1
2,5
2,0
1,5
1,0
0,5
0
$W(\vartheta)$
0°
45°
90°
135°
180°
ϑ


a) b)

# ANGULAR CORRELATIONS

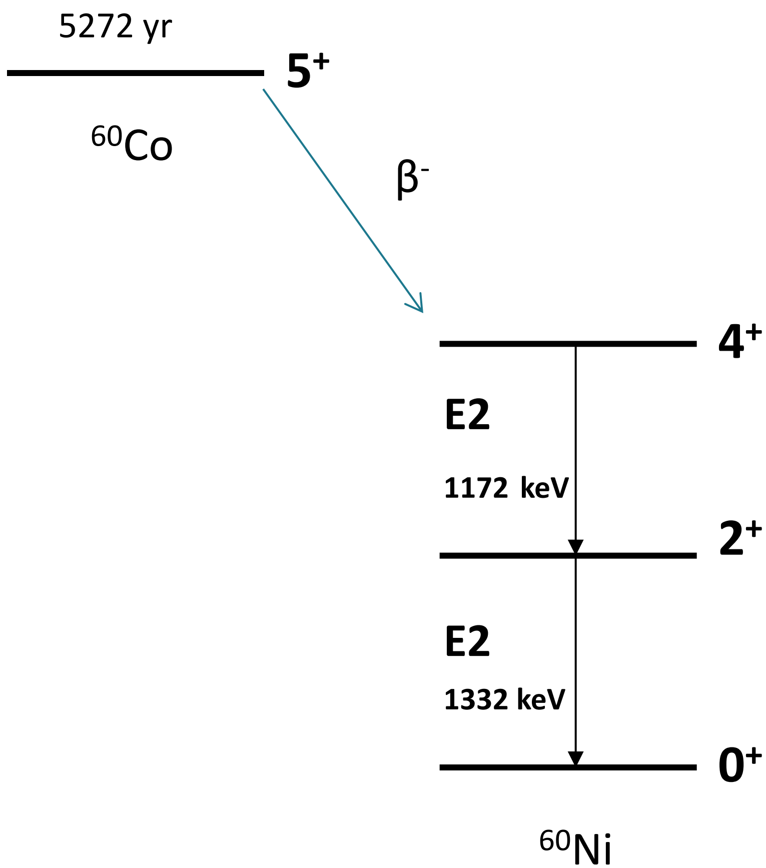



$W(\theta) = 8 + 4P_2(\cos\theta)$
$\propto 1 + \frac{1}{2}P_2(\cos\theta)$

$W(\theta) = 24/3$
$\propto$ CONST.

$W(\theta) = 8 + 2P_2(\cos\theta)$
$\propto 1 + \frac{1}{4}P_2(\cos\theta)$

# Homework: $^{60}$Co




Find the angular correlation function for the 1332 keV Gamma Ray relative to the 1172 keV Gamma Ray

This is easy and straightforward but becomes unwieldy.

Expand $W(\theta)$ in terms of the Legendre Polynomials, $P_L(\cos\theta)$.

$$W(\theta) = 1 + A_{22}P_2(\cos\theta) + A_{44}P_4(\cos\theta) + \ldots A_{k\max\, k\max}P_{k\max}(\cos\theta)$$

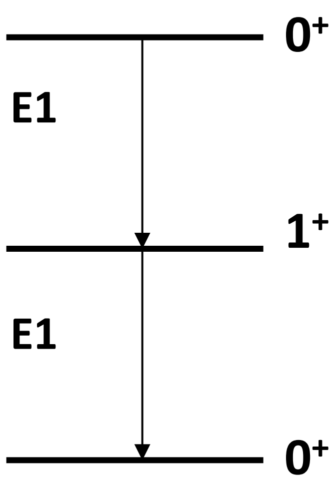
Only even terms because unpolarised radiation is always symmetrical to 90º therefore only $P_0$, $P_2$, $P_4$ ....

$$k_{\max} = Min(2J_2, 2L_1, 2L_2)$$

$$A_{kk} = F_k(L_1, L_2, J_1, J_2)F_k(L_2, L_2, J_3, J_2)$$

These are tabulated

**Example**

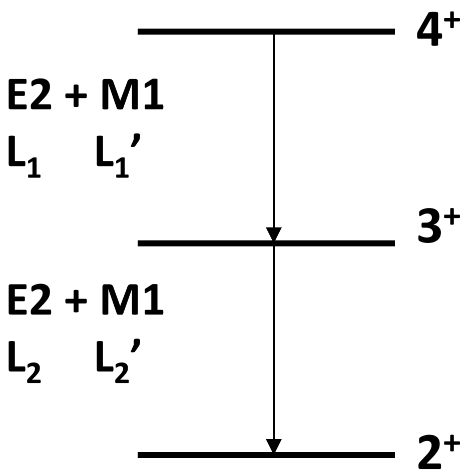



$$W(\theta) = 1 + F_2(1101)F_2(1110)P_2(\cos\theta)$$

$$W(\theta) = 1 + 0.707 \cdot 0.707 P_2(\cos\theta)$$
$$= 1 + 0.5P_2(\cos\theta)$$

$= 1 + \frac{3}{4}\cos^2(\theta) - \frac{1}{4} = \text{const}^*(1+\cos^2(\theta))$ as before.

**Example**




$$W(\theta) = 1 + A_{22}P_2(\cos\theta) + A_{44}P_4(\cos\theta)$$

$$A_k^{Max} = A_{kk} = A_k(L_1, L_1', J_1, J_2) + A_k(L_2, L_2', J_3, J_2)$$

Statistical tensor

$$A_k(L_1, L_1', J_1, J_2) = \frac{B_k(j_1)}{1+\delta^2}\left[F_k(L_1, L_1', J_1, J_2) + 2\delta F_k(L_1, L_1', J_1, J_2) + \delta^2 F_k(L_1, L_1', J_1, J_2)\right]$$

Multipole mixing ratio

$$A_k(L_1, L_1', J_1, J_2) = \ldots$$

**All that is needed to describe any situation are the $F_k$ coefficients, which are tabulated**

$$F_k(L, L', J_1, J_2) = (-1)^{J_1+J_2+L-L'}\sqrt{(2L+1)(2L'+1)(2J_2+1)}\; C^{k0}_{L_1 L' -1} \begin{Bmatrix} L & L' & K \\ J_2 & J_2 & J_1 \end{Bmatrix}$$

# TABLES OF $F$-COEFFICIENTS

Numerical values of $F_k(LL'I'I)$ are tabulated for $k=1,2,3,4$, for intermediate spins $I=1, \frac{3}{2}\, 2, \frac{5}{2}, 3, \frac{7}{2}, 4$, and for $L \leqslant L' \leqslant 3$. The $F$-coefficients are symmetric in the multipole orders $L$ and $L'$: $F_k(LL'I'I) = F_k(L'LI'\,I)$. For $k=0$, $F_0(LL'I'I) = \delta_{L'L}$.

TABLE B1 ($I=\frac{1}{2}$)

| $L$ | $L'$ | $I'$ | $I$ | $k=1$ |
|---|---|---|---|---|
| 0 | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | 1.732 |
| 1 | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | -1.000 |
| 1 | 1 | $\frac{3}{2}$ | $\frac{1}{2}$ | 0.500 |
| 1 | 2 | $\frac{3}{2}$ | $\frac{1}{2}$ | 0.866 |
| 2 | 2 | $\frac{3}{2}$ | $\frac{1}{2}$ | -0.500 |
| 2 | 2 | $\frac{5}{2}$ | $\frac{1}{2}$ | 0.333 |
| 2 | 3 | $\frac{5}{2}$ | $\frac{1}{2}$ | 0.943 |
| 3 | 3 | $\frac{5}{2}$ | $\frac{1}{2}$ | -0.333 |
| 3 | 3 | $\frac{7}{2}$ | $\frac{1}{2}$ | 0.250 |

TABLE B2 ($I=1$)

| $L$ | $L'$ | $I'$ | $I$ | $k=1$ | $k=2$ |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 1 | -1.225 | 0.707 |
| 0 | 1 | 1 | 1 | 1.732 | 0 |
| 0 | 2 | 1 | 1 | 0 | -2.236 |
| 1 | 1 | 1 | 1 | -0.612 | -0.354 |
| 1 | 2 | 1 | 1 | 0.612 | -1.061 |
| 2 | 2 | 1 | 1 | -0.612 | -0.354 |
| 1 | 1 | 2 | 1 | 0.612 | 0.071 |
| 1 | 2 | 2 | 1 | 0.822 | 0.474 |
| 2 | 2 | 2 | 1 | -0.204 | 0.354 |
| 2 | 3 | 2 | 1 | 0.730 | -0.632 |
| 3 | 3 | 2 | 1 | -0.408 | -0.424 |
| 2 | 2 | 3 | 1 | 0.408 | -0.101 |
| 2 | 3 | 3 | 1 | 0.873 | 0.378 |
| 3 | 3 | 3 | 1 | -0.102 | 0.530 |
| 3 | 3 | 4 | 1 | 0.306 | -0.177 |

TABLE B3 ($I=\frac{3}{2}$)

| $L$ | $L'$ | $I'$ | $I$ | $k=1$ | $k=2$ | $k=3$ |
|---|---|---|---|---|---|---|
| 1 | 1 | $\frac{1}{2}$ | $\frac{3}{2}$ | -1.118 | 0.500 | 0 |
| 1 | 2 | $\frac{1}{2}$ | $\frac{3}{2}$ | 0.387 | -0.866 | 0.775 |
| 2 | 2 | $\frac{1}{2}$ | $\frac{3}{2}$ | -0.671 | -0.500 | 0.895 |
| 0 | 1 | $\frac{3}{2}$ | $\frac{3}{2}$ | 1.732 | 0 | 0 |
| 0 | 2 | $\frac{3}{2}$ | $\frac{3}{2}$ | 0 | -2.236 | 0 |
| 1 | 1 | $\frac{3}{2}$ | $\frac{3}{2}$ | -0.447 | -0.400 | 0 |
| 1 | 2 | $\frac{3}{2}$ | $\frac{3}{2}$ | 0.693 | -0.775 | -0.346 |
| 2 | 2 | $\frac{3}{2}$ | $\frac{3}{2}$ | -0.477 | 0 | -0.894 |
| 2 | 3 | $\frac{3}{2}$ | $\frac{3}{2}$ | 0.566 | -0.632 | -0.283 |
| 3 | 3 | $\frac{3}{2}$ | $\frac{3}{2}$ | -0.447 | -0.600 | 0.447 |
| 1 | 1 | $\frac{5}{2}$ | $\frac{3}{2}$ | 0.671 | 0.100 | 0 |
| 1 | 2 | $\frac{5}{2}$ | $\frac{3}{2}$ | 0.794 | 0.592 | 0.076 |
| 2 | 2 | $\frac{5}{2}$ | $\frac{3}{2}$ | -0.075 | 0.357 | 0.383 |
| 2 | 3 | $\frac{5}{2}$ | $\frac{3}{2}$ | 0.806 | -0.338 | 0.227 |
| 3 | 3 | $\frac{5}{2}$ | $\frac{3}{2}$ | -0.261 | 0.150 | -0.671 |
| 2 | 2 | $\frac{7}{2}$ | $\frac{3}{2}$ | 0.447 | -0.143 | -0.064 |
| 2 | 3 | $\frac{7}{2}$ | $\frac{3}{2}$ | 0.828 | +0.463 | -0.069 |
| 3 | 3 | $\frac{7}{2}$ | $\frac{3}{2}$ | 0 | +0.500 | 0.226 |
| 3 | 3 | $\frac{9}{2}$ | $\frac{3}{2}$ | 0.335 | -0.250 | -0.075 |

## TABLE B4 ($I=2$)

| $L$ | $L'$ | $I'$ | $I$ | $k=1$ | $k=2$ | $k=3$ | $k=4$ |
|---|---|---|---|---|---|---|---|
| 2 | 2 | 0 | 2 | -0.707 | -0.598 | 1.414 | -1.069 |
| 1 | 1 | 1 | 2 | -1.061 | 0.418 | 0 | 0 |
| 1 | 2 | 1 | 2 | 0.473 | -0.935 | 0.632 | 0 |
| 2 | 2 | 1 | 2 | -0.589 | -0.299 | 0 | 0.713 |
| 2 | 3 | 1 | 2 | 0.421 | -0.535 | -0.316 | 0.996 |
| 3 | 3 | 1 | 2 | -0.472 | -0.717 | 0.707 | 0.089 |
| 0 | 1 | 2 | 2 | 1.732 | 0 | 0 | 0 |
| 0 | 2 | 2 | 2 | 0 | -2.236 | 0 | 0 |
| 1 | 1 | 2 | 2 | -0.353 | -0.418 | 0 | 0 |
| 1 | 2 | 2 | 2 | 0.725 | -0.612 | -0.414 | 0 |
| 2 | 2 | 2 | 2 | -0.354 | 0.128 | -0.808 | -0.305 |
| 2 | 3 | 2 | 2 | 0.676 | -0.571 | -0.084 | -0.798 |
| 3 | 3 | 2 | 2 | -0.353 | -0.179 | 0.354 | -0.134 |
| 1 | 1 | 3 | 2 | 0.708 | 0.120 | 0 | 0 |
| 1 | 2 | 3 | 2 | 0.775 | 0.655 | 0.111 | 0 |
| 2 | 2 | 3 | 2 | 0 | 0.341 | 0.505 | 0.076 |
| 2 | 3 | 3 | 2 | 0.828 | -0.175 | 0.242 | 0.326 |
| 3 | 3 | 3 | 2 | -0.177 | 0.329 | -0.471 | 0.089 |
| 2 | 2 | 4 | 2 | 0.471 | -0.171 | -0.101 | -0.008 |
| 2 | 3 | 4 | 2 | 0.797 | 0.505 | -0.100 | -0.063 |
| 3 | 3 | 4 | 2 | 0.059 | 0.448 | 0.471 | -0.030 |
| 3 | 3 | 5 | 2 | 0.353 | -0.299 | -0.118 | 0.004 |

## TABLE B5 ($I=\frac{5}{2}$)

| $L$ | $L'$ | $I'$ | $I$ | $k=1$ | $k=2$ | $k=3$ | $k=4$ |
|---|---|---|---|---|---|---|---|
| 2 | 2 | $\frac{1}{2}$ | $\frac{5}{2}$ | -0.683 | -0.535 | 1.095 | -0.617 |
| 2 | 3 | $\frac{1}{2}$ | $\frac{5}{2}$ | 0.138 | -0.378 | -0.258 | 1.091 |
| 3 | 3 | $\frac{1}{2}$ | $\frac{5}{2}$ | -0.488 | -0.802 | 0.913 | 0.154 |
| 1 | 1 | $\frac{3}{2}$ | $\frac{5}{2}$ | -1.025 | 0.374 | 0 | 0 |
| 1 | 2 | $\frac{3}{2}$ | $\frac{5}{2}$ | 0.520 | -0.949 | 0.555 | 0 |
| 2 | 2 | $\frac{3}{2}$ | $\frac{5}{2}$ | -0.537 | -0.191 | -0.313 | 0.705 |
| 2 | 3 | $\frac{3}{2}$ | $\frac{5}{2}$ | 0.528 | -0.587 | -0.262 | 0.326 |
| 3 | 3 | $\frac{3}{2}$ | $\frac{5}{2}$ | -0.415 | -0.441 | 0.091 | -0.077 |
| 0 | 1 | $\frac{5}{2}$ | $\frac{5}{2}$ | 1.732 | 0 | 0 | 0 |
| 0 | 2 | $\frac{5}{2}$ | $\frac{5}{2}$ | 0 | -2.236 | 0 | 0 |
| 1 | 1 | $\frac{5}{2}$ | $\frac{5}{2}$ | -0.293 | -0.428 | 0 | 0 |
| 1 | 2 | $\frac{5}{2}$ | $\frac{5}{2}$ | 0.741 | -0.507 | -0.445 | 0 |
| 2 | 2 | $\frac{5}{2}$ | $\frac{5}{2}$ | -0.293 | 0.191 | -0.704 | -0.397 |
| 2 | 3 | $\frac{5}{2}$ | $\frac{5}{2}$ | 0.727 | -0.498 | +0.013 | -0.798 |
| 3 | 3 | $\frac{5}{2}$ | $\frac{5}{2}$ | -0.293 | 0.027 | -0.517 | -0.077 |
| 1 | 1 | $\frac{7}{2}$ | $\frac{5}{2}$ | 0.732 | 0.134 | 0 | 0 |
| 1 | 2 | $\frac{7}{2}$ | $\frac{5}{2}$ | 0.761 | 0.694 | 0.136 | 0 |
| 2 | 2 | $\frac{7}{2}$ | $\frac{5}{2}$ | 0.049 | 0.325 | 0.574 | 0.118 |
| 2 | 3 | $\frac{7}{2}$ | $\frac{5}{2}$ | 0.835 | -0.071 | 0.236 | 0.447 |
| 3 | 3 | $\frac{7}{2}$ | $\frac{5}{2}$ | -0.098 | 0.401 | -0.304 | 0.103 |
| 2 | 2 | $\frac{9}{2}$ | $\frac{5}{2}$ | 0.488 | -0.191 | -0.130 | -0.015 |
| 2 | 3 | $\frac{9}{2}$ | $\frac{5}{2}$ | 0.774 | 0.530 | -0.121 | -0.102 |
| 3 | 3 | $\frac{9}{2}$ | $\frac{5}{2}$ | 0.098 | 0.401 | 0.517 | -0.044 |
| 3 | 3 | $\frac{11}{2}$ | $\frac{5}{2}$ | 0.073 | -0.334 | -0.152 | 0.007 |

## TABLE B6 ($I=3$)

| $L$ | $L'$ | $I'$ | $I$ | $k=1$ | $k=2$ | $k=3$ | $k=4$ |
|---|---|---|---|---|---|---|---|
| 3 | 3 | 0 | 3 | −0.500 | −0.866 | 1.080 | 0.213 |
| 2 | 2 | 1 | 3 | −0.667 | −0.495 | 0.926 | −0.447 |
| 2 | 3 | 1 | 3 | 0.356 | −0.463 | −0.289 | 1.045 |
| 3 | 3 | 1 | 3 | −0.458 | −0.650 | 0.540 | 0.036 |
| 1 | 1 | 2 | 3 | −1.000 | 0.346 | 0 | 0 |
| 1 | 2 | 2 | 3 | 0.548 | −0.949 | 0.507 | 0 |
| 2 | 2 | 2 | 3 | −0.500 | −0.124 | −0.463 | 0.670 |
| 2 | 3 | 2 | 3 | 0.585 | −0.592 | −0.211 | 0 |
| 3 | 3 | 2 | 3 | −0.375 | −0.274 | −0.180 | −0.107 |
| 0 | 1 | 3 | 3 | 1.732 | 0 | 0 | 0 |
| 0 | 2 | 3 | 3 | 0 | −2.236 | 0 | 0 |
| 1 | 1 | 3 | 3 | −0.250 | −0.433 | 0 | 0 |
| 1 | 2 | 3 | 3 | 0.750 | −0.433 | −0.463 | 0 |
| 2 | 2 | 3 | 3 | −0.250 | 0.227 | −0.617 | −0.447 |
| 2 | 3 | 3 | 3 | 0.756 | −0.436 | −0.068 | −0.739 |
| 3 | 3 | 3 | 3 | −0.250 | 0.144 | −0.540 | −0.036 |
| 1 | 1 | 4 | 3 | 0.750 | 0.144 | 0 | 0 |
| 1 | 2 | 4 | 3 | 0.750 | 0.722 | 0.154 | 0 |
| 2 | 2 | 4 | 3 | 0.083 | 0.309 | 0.617 | 0.149 |
| 2 | 3 | 4 | 3 | 0.836 | 0 | 0.226 | 0.520 |
| 3 | 3 | 4 | 3 | −0.083 | 0.433 | −0.180 | 0.104 |
| 2 | 2 | 5 | 3 | 0.500 | −0.206 | −0.154 | −0.020 |
| 2 | 3 | 5 | 3 | 0.756 | 0.546 | −0.136 | −0.134 |
| 3 | 3 | 5 | 3 | 0.125 | 0.361 | 0.540 | −0.055 |
| 3 | 3 | 6 | 3 | 0.375 | −0.361 | −0.180 | 0.010 |

## TABLE B7 ($I=\frac{7}{2}$)

| $L$ | $L'$ | $I'$ | $I$ | $k=1$ | $k=2$ | $k=3$ | $k=4$ |
|---|---|---|---|---|---|---|---|
| 3 | 3 | $\frac{1}{2}$ | $\frac{7}{2}$ | −0.490 | −0.818 | 0.957 | 0.171 |
| 2 | 2 | $\frac{3}{2}$ | $\frac{7}{2}$ | −0.654 | −0.468 | 0.821 | −0.358 |
| 2 | 3 | $\frac{3}{2}$ | $\frac{7}{2}$ | 0.404 | −0.505 | −0.295 | 0.967 |
| 3 | 3 | $\frac{3}{2}$ | $\frac{7}{2}$ | −0.436 | −0.546 | 0.319 | −0.019 |
| 1 | 1 | $\frac{5}{2}$ | $\frac{7}{2}$ | −0.982 | 0.327 | 0 | 0 |
| 1 | 2 | $\frac{5}{2}$ | $\frac{7}{2}$ | 0.567 | −0.945 | 0.474 | 0 |
| 2 | 2 | $\frac{5}{2}$ | $\frac{7}{2}$ | −0.473 | −0.078 | −0.547 | 0.637 |
| 2 | 3 | $\frac{5}{2}$ | $\frac{7}{2}$ | 0.622 | −0.583 | −0.171 | −0.186 |
| 3 | 3 | $\frac{5}{2}$ | $\frac{7}{2}$ | −0.346 | −0.164 | −0.319 | −0.108 |
| 0 | 1 | $\frac{7}{2}$ | $\frac{7}{2}$ | 1.732 | 0 | 0 | 0 |
| 0 | 2 | $\frac{7}{2}$ | $\frac{7}{2}$ | 0 | −2.236 | 0 | 0 |
| 1 | 1 | $\frac{7}{2}$ | $\frac{7}{2}$ | −0.218 | −0.436 | 0 | 0 |
| 1 | 2 | $\frac{7}{2}$ | $\frac{7}{2}$ | 0.873 | −0.378 | −0.474 | 0 |
| 2 | 2 | $\frac{7}{2}$ | $\frac{7}{2}$ | −0.218 | 0.249 | −0.547 | −0.478 |
| 2 | 3 | $\frac{7}{2}$ | $\frac{7}{2}$ | 0.774 | −0.387 | 0.103 | −0.673 |
| 3 | 3 | $\frac{7}{2}$ | $\frac{7}{2}$ | −0.218 | 0.218 | −0.522 | −0.007 |
| 1 | 1 | $\frac{9}{2}$ | $\frac{7}{2}$ | 0.609 | 0.153 | 0 | 0 |
| 1 | 2 | $\frac{9}{2}$ | $\frac{7}{2}$ | 0.742 | 0.742 | 0.169 | 0 |
| 2 | 2 | $\frac{9}{2}$ | $\frac{7}{2}$ | 0.087 | 0.296 | 0.647 | 0.174 |
| 2 | 3 | $\frac{9}{2}$ | $\frac{7}{2}$ | 0.835 | 0.052 | 0.215 | 0.567 |
| 3 | 3 | $\frac{9}{2}$ | $\frac{7}{2}$ | −0.044 | 0.447 | −0.087 | 0.102 |
| 2 | 2 | $\frac{11}{2}$ | $\frac{7}{2}$ | 0.508 | −0.218 | −0.174 | −0.025 |
| 2 | 3 | $\frac{11}{2}$ | $\frac{7}{2}$ | 0.742 | 0.556 | −0.148 | −0.161 |
| 3 | 3 | $\frac{11}{2}$ | $\frac{7}{2}$ | 0.146 | 0.327 | 0.551 | −0.063 |
| 3 | 3 | $\frac{13}{2}$ | $\frac{7}{2}$ | 0.382 | −0.382 | −0.203 | 0.012 |

## TABLE B8 ($I=4$)

| $L$ | $L'$ | $I'$ | $I$ | $k=1$ | $k=2$ | $k=3$ | $k=4$ |
|---|---|---|---|---|---|---|---|
| 3 | 3 | 1 | 4 | -0.484 | -0.783 | 0.874 | 0.145 |
| 2 | 2 | 2 | 4 | -0.645 | -0.448 | 0.749 | -0.304 |
| 2 | 3 | 2 | 4 | 0.436 | -0.530 | -0.295 | 0.900 |
| 3 | 3 | 2 | 4 | -0.420 | -0.470 | 0.175 | -0.048 |
| 1 | 1 | 3 | 4 | -0.968 | 0.313 | 0 | 0 |
| 1 | 2 | 3 | 4 | 0.581 | -0.940 | 0.449 | 0 |
| 2 | 2 | 3 | 4 | -0.452 | -0.045 | -0.599 | 0.609 |
| 2 | 3 | 3 | 4 | 0.647 | -0.571 | -0.139 | -0.304 |
| 3 | 3 | 3 | 4 | -0.323 | -0.085 | -0.397 | -0.101 |
| 0 | 1 | 4 | 4 | 1.732 | 0 | 0 | 0 |
| 0 | 2 | 4 | 4 | 0 | -2.236 | 0 | 0 |
| 1 | 1 | 4 | 4 | -0.194 | -0.439 | 0 | 0 |
| 1 | 2 | 4 | 4 | 0.760 | -0.335 | -0.481 | 0 |
| 2 | 2 | 4 | 4 | -0.194 | 0.265 | -0.490 | -0.498 |
| 2 | 3 | 4 | 4 | 0.786 | -0.347 | 0.126 | -0.614 |
| 3 | 3 | 4 | 4 | -0.194 | 0.269 | -0.493 | 0.013 |
| 1 | 1 | 5 | 4 | 0.774 | 0.160 | 0 | 0 |
| 1 | 2 | 5 | 4 | 0.735 | 0.757 | 0.181 | 0 |
| 2 | 2 | 5 | 4 | 0.129 | 0.285 | 0.667 | 0.194 |
| 2 | 3 | 5 | 4 | 0.832 | 0.092 | 0.205 | 0.601 |
| 3 | 3 | 5 | 4 | -0.032 | 0.453 | -0.016 | 0.098 |
| 2 | 2 | 6 | 4 | 0.632 | -0.228 | -0.191 | -0.030 |
| 2 | 3 | 6 | 4 | 0.730 | 0.564 | -0.157 | -0.184 |
| 3 | 3 | 6 | 4 | 0.161 | 0.299 | 0.556 | -0.069 |
| 3 | 3 | 7 | 4 | 0.387 | -0.399 | -0.222 | 0.014 |

$$F_k(LL'I'I) = F_k(L'LI'I).$$

For $k=0$, $F_0(LL'I'I) = \delta_{L'L}$.

# Geometric and attenuation factors

To account for the finite opening angle of the detector $\Delta\theta$, $A_{kk}$ must be modified by a geometric factor $Q_{k}$.

Also, to account for incomplete alignment of the magnetic substates attenuation factors $\alpha_k$ are introduced.

$$a_k \to Q_k \alpha_k A_{kk}$$

# Multipole Mixing Ratio

- Because of the relative multipole transition probabilities, we only need to consider M1/E2 mixing
- For a $\Delta I = 1$ transition, M1 radiation accounts for $1 / [1+\delta^2]$ (typically 95%) of the intensity, while E2 radiation accounts for $\delta^2 / [1+\delta^2]$ (typically 5%) of the intensity
- The mixing ratio can be positive or negative and perturbs the angular distribution

# Angular Distributions

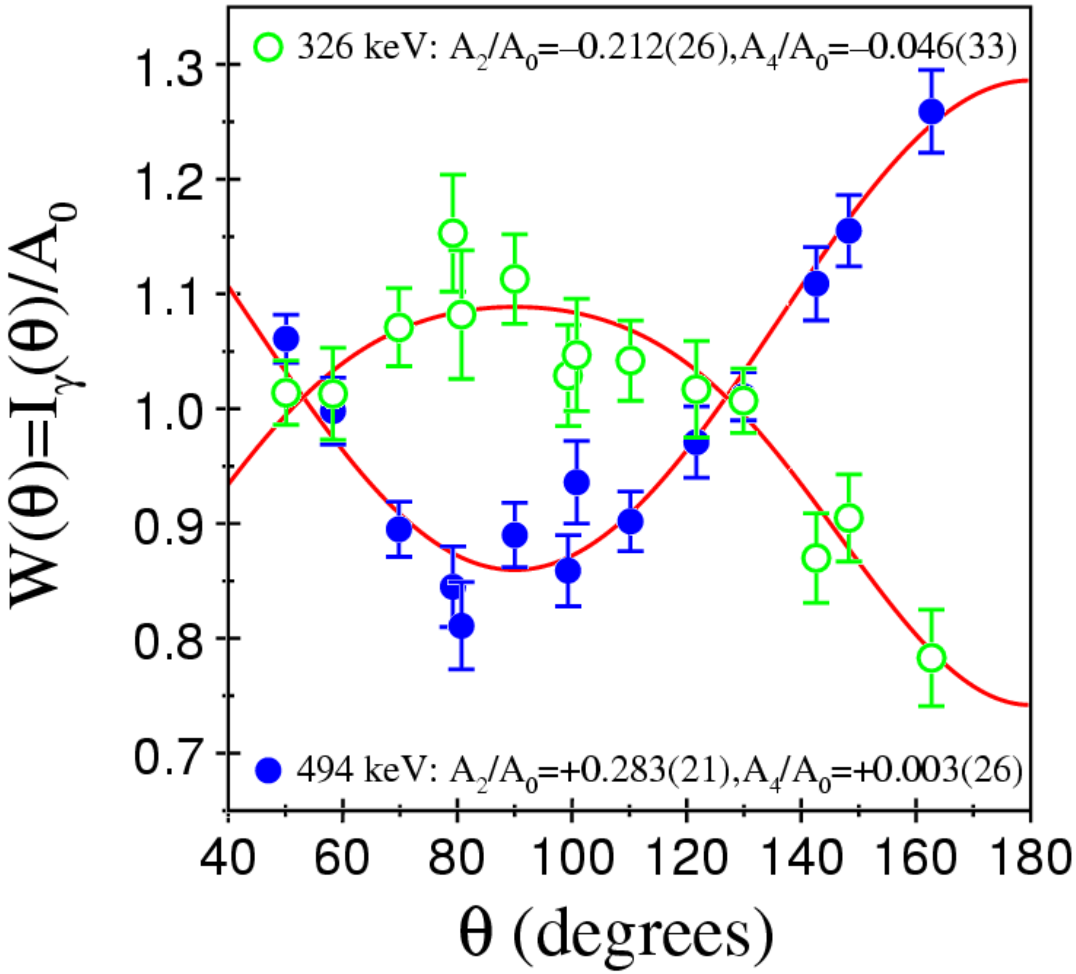
- The general form for the angular distribution function of radiation emitted following a heavy-ion fusion-evaporation reaction is:

$$W(\theta) = A_0 \,[\, 1 + Q_2\{A_2/A_0\}P_2(\cos\theta) + Q_4\{A_4/A_0\}P_4(\cos\theta) \,]$$

where $Q_k$ are geometrical attenuation coefficients which account for the finite size of the detectors and $P_k(\cos\theta)$ are Legendre polynomials

- The measured $A_k/A_0$ coefficients are compared to theory for different types of radiation

# Angular Distributions in $^{109}Te$




- Typically $A_4/A_0$ is close to zero
- $A_2/A_0$ ~ +0.3 for a pure quadrupole ($\Delta I = 2$) transition
- $A_2/A_0$ ~ -0.3
- for a pure dipole
- ($\Delta I = 1$) transition

A J Boston et al., PRC 61 (1999) 064305.