Simulation Course (ETS061) 2016-2017
Homework 2: Linear and Integer Programming

* The deadline for this homework is May 26, 2017.

* The answers must be delivered in a written report.

* All source files of your implementations must be delivered. Please, name each source file using the problem and task number to which the file belongs (for example 1.a, 2.c, etc.).

Problem 1: UniCAM is a company producing a certain type of DSLR camera. UniCAM has devised 5 different packages to sell its cameras. The packages are labeled as PI, PII, PIII, PIV, and PV. They differ in the type and number of professional lenses offered with the camera. UniCAM has a limited number of each lens in stock. Each package has a certain price, and projected to be sold to a maximum number of customers (i.e. demands). See the following two tables for more complete information:

Table 1: Capacity and price information

	PI	**PII**	**PIII**	**PIV**	**PV**	**#In Stock (in 1000s)**
Macro lens	2	0	0	0	0	36
Prime lens	0	2	2	2	1	216
Wide lens*	0.2	1	0	0.5	0	18
Price (in 10000 SEK)	4	3	2	2	1	
* The interpretation of the third row is that Wide lens is provided in 20% of PI packages, in all of PII packages, in half of PIV packages, and in none of PIII and PV packages.						

Table 2: Demand information

	PI	**PIII**	**PI + PII + PIII**	**PIV + PV**
Maximum demands (in 1000s)	16	2	34	28

UniCAM is interested in determining the number of each package to be sold, such that its total profit is maximized.

a) Formulate a linear program (LP) for the problem described above.
b) Implement your LP in MATLAB, solve it and obtain the optimal number of each package, and the total profit.
c) Implement sensitivity analysis and find the shadow prices for each constraint. This must be done for both capacity constraints (Table 1) and demand constraints (Table 2).

d) Using the shadow price corresponding to the maximum demand for package PIII, perform an extended experiment by incrementing the maximum demand for PIII with a step size of 0.6, run the LP with the new demand, and record the achieved profit. Repeat this until the profit becomes constant (i.e. flattened). Then, using your recorded data, draw a diagram showing the relationship between the change of maximum demand for PIII and the total profit. Explain your observations from this diagram about the number of times the shadow price changes before the profit becomes constant.

e) In this part, our goal is to find a range in which the variation of price for package PI does not affect the optimal solution found in part (b) (**note:** by optimal solution, we mean the optimal number of each package, not the amount of optimal profit). More specifically, using the original price of PI (that is 4, according to Table 1), you should find two values α and β such that $\alpha \leq 4 \leq \beta$ and the optimal solution does not change within this range.

Hint: implement an iterative simulation procedure, which is initiated with the original price 4. Then, to find β, the procedure increases the price (by a very small value in each iteration), finds the new solution and checks if this solution remains unchanged. Similar procedure is applied to find α, but using iterative decrease of the price.

Problem 2: an integer program is given as follows:

Maximize $z = x_1 + 5x_2$
subject to: $2x_1 - x_2 \leq 4$
$\qquad\qquad -x_1 + x_2 \leq 1$
$\qquad\qquad x_1 + 4x_2 \leq 12$
$\qquad\qquad x_1 \in Z^+, x_2 \in Z^+$ (positive integers)

a) Implement this integer program in MATLAB, solve it, and report the optimal solution (i.e. x_1, x_2, and z).

b) Relax the problem to a linear program (i.e. x_1 and x_2 be real numbers). Solve this problem using MATLAB, and obtain the solution. From this optimal solution, identify all possible roundings of x_1 and x_2 to integer values. Which pair of feasible rounding for x_1 and x_2 gives a maximum value for the objective function? How is this solution compared to the one obtained in part (a)?

c) Again, relax the problem to real-valued variables. Our goal is to obtain the optimal integer solution in a semi-automatic way. You are allowed to use MATLAB solver to solve linear programs only (not any integer program). Therefore, you need to apply branch-and-bound method to determine a new sub-problem in each step, formulate this sub-problem as a linear program in MATLAB and solve it. If the solution is fractional, then make new branches, and so on so forth (see Lecture-O2 to find an example). In your answer to this task, you should draw a branch-and-bound tree, which depicts all steps of the branch-and-bound method you applied, and the solution obtained in each step. In case, you terminate (i.e. cut) a branch in the tree, you should explain why it is terminated.